## *An enquiry into the predictive value of grammar school entrance examinations*

Rutter, Dryden

# An Enquiry Into the Predictive Value of Grammar School Entrance Examinations.

by

Dryden Rutter, M.Sc.

Thesis Submitted for Degree of M.Ed.

The University,

Durham.

March 1949.

## Preface.

My sixteen years teaching experience has been varied. I have taught pupils in the primary school, before thay have taken the grammar school entrance examination, in the grammar school after they have been successful, and in the senior elementary school after they have been rejected. Like many of my colleagues, I have often had misgivings on both the methods and the results of selection. I have found that the grammar school staffs are usually less pleased with the results of selection than those of the primary school. There seems to be a tendency in the grammar school to judge pupils, on entry, from the standard of the "A" pupil and, if I may venture a criticism, grammar school teaching is too often biased in favour of these "A" pupils. This, no doubt, is due to the fact that all roads in the grammar school lead to the school certificate examination. The new examination system may give the grammar schools freedom for development in spheres other than academic.

The object of this treatise is to investigate, in some detail, the methods of selection used in this county over a number of years. Unfortunately, I have been able to deal with only the selected pupils, as information on the unselected pupils, for the years in question, is unavailable. Tracing the scholarship results as far back as 1934, and the school certificate results back to 1939 has been a tedious operation. However, I am greatly indebted

to the Director of Education and the Headmaster of the school for permission to use the results. I am also very grateful to the Northern Universities Joint Matriculation Board who so readily gave me permission to use their school certificate results. My debt to Professor Peel cannot be overestimated, for without his ready guidance, the work would have probably foundered on some statistical reef.

D. Rutter,
The University,
Durham.

March 1949.

# An Enquiry Into the Prediction Value of Grammar School Entrance Examinations.

## Contents

## Chapter One.

### Introduction: The Problem of Selection.

One of the most difficult and controversial problems confronting educationalists today is that of deciding what type of secondary education is best suited to the individual child. Since the demand for grammar school education far exceeds the supply, the problem has resolved itself into one of deciding which children are best fitted for grammar school education. As a result, the entrance examination for these schools has become highly competitive. Various methods of selection have been, and are being tried, all, having the same aim, that of choosing those pupils who would benefit most from an acedemic education. To estimate the success of any method of selection, we must follow the careers of the pupils selected through the grammar school. However, no matter what criteria we use as a measure of success in the grammar school, we must not expect too high a comparison with the entrance examination. The latter, in far too many cases, occupies only an hour or two, and in such a short time the effect of extraneous factors can be considerable. Many authorities rectify this, to a certain extent, by a system of transference between the types of secondary education. Schemes of transference are worthy of much more consideration than many authorities are giving them.

Several authorities have inaugurated systems of selection using qualifying examinations incorporated with a record of the pupils' progress in the primary school and the head teachers' estimate of fitness for entry into the grammar school. An interesting method is that used by the Borough of Walsall since 1944. (Ref. 1).

All candidates are given two intelligence tests with a three weeks interval, the average result being taken. The grammar school places are then shared out, each primary school being awarded the number of places corresponding to the number of its pupils gaining the minimum I.Q. necessary to fill the grammar schools. The places are then awarded to the individual pupils on the combined result of the intelligence tests, the school report and the primary school examinations. The intelligence tests are weighted in the ratio 3 : 2 against the combination of the other two. This method of selection avoids the extremely difficult task of comparing assessments of head teachers.

A similar scheme involving primary school ratings has been recently introduced in Middlesex. The primary school ratings are on a fifteen point scale and are based on the average mark of the pupils of each particular school in a qualifying examination. Awards are made on the total of marks in the qualifying examination and the "grading marks" of the primary school.

Experiment in the method of selection is a very welcome feature. However, the value of any method of selection is best measured by a comparison of the achievement of the pupils in the grammar school with their achievement in the qualifying tests. It is the aim of this work to make such a comparison. There have been several investigations of a similar nature, many of which have made valuable suggestions for the improvement of the technique of selecting pupils for the grammar school.

A.D. Amos (Ref. 2) came to the conclusion that the best prediction of secondary school success was from a combination

of intelligence tests and ordinary school examinations. J.W. Collier (Ref. 3) compared entrance results consisting of attainment tests in English and Arithmetic and an Intelligence Test (M.H.T.3) with marks gained in the secondary school. His results showed that the Intelligence Test was the best predictor, the Arithmetic Test was the worst, while the best prediction was obtained by a combination of all three. He also suggested a weighting in favour of the Intelligence Test and the English Test. A.G. Hughes (Ref.4) supported the use of a "carefully constructed intelligence test" in addition to the English and the Arithmetic examinations.

C.W. Valentine (Ref. 5) investigated the progress of several pupils in the secondary school. He compared their order of merit in the entrance examination with their order of merit at various stages of the secondary school. He found practically no relation between the order in the entrance examination and the school certificate examination. A better prediction of school certificate order was given by the order of merit after one year in the secondary school, a result which was to be expected. Valentine was an early advocate of the transference of pupils unsuited to the grammar school, to make way for those better fitted for this type of education.

T.E. Stubbins (Ref. 6) conducted a follow-up of two groups of 180 boys entering grammar schools in 1930 and 1931, and taking the school certificate examination five years later in 1935 and 1936 respectively. The entrance examination consisted of attainment tests in English and Arithmetic, an Intelligence Test and an estimate mark supplied by the primary school headteacher. For the pupils concerned, he found that the English mark in the

the entrance examination was the most reliable predictor of school certificate marks: the headteachers' estimate was of practically no predictive value; the Intelligence Test was the best predictor of Mathematics and Physics, while the Arithmetic marks were of little value, except when used with the Intelligence Test to predict school certificate Mathematics.

Probably the first large scale investigation into the problem of selection was carried out in 1937 in the West Riding of Yorkshire (Ref. 7). The object was to find the relative value of two types of examination, the first consisting of a Moray House Intelligence Test with ordinary English and Arithmetic Tests, and the second consisting of three Moray House tests in Intelligence, English and Arithmetic. The results in these tests was compared with the order of merit as estimated (before the tests) by each elementary school headteacher. The order of merit was reversed and treated as ordinary marks. For the unselected pupils the multiple correlations of each examination with the headteachers' ranking were almost equal. For the unselected pupils the multiple correlation of the standardised tests was slightly better than that of the ordinary tests. In each case the Arithmetic had the highest predictive value, with the English the least. Amongst other recommendations, the report suggested that the entrance examination results should be considered along with the elementary school headteachers' ranking (made before the examination) plus a grading of the pupils on a five point scale.

The West Riding provided material for a similar enquiry by W.G. Emmett (Ref. 8). He compared the entrance results of about 1,000 pupils in 1933 and 1934, with their order of merit

estimated by the grammar school headteachers two years later. A further comparison of about 750 of these was made with their estimated order of merit after three years in the grammar school. The entrance examination consisted of attainment tests in English and Arithmetic with a Moray House Intelligence Test. These were compared with the grammar school headteachers' ranks, as before, the ranks were reversed and treated as ordinary marks. His results showed that the best single measure of later performance was the Intelligence Test. Considering the three entrance examinations as a team for prediction purposes, the Intelligence Test was almost equal to the English and Arithmetic papers in their best weighted combination.

On a still larger scale W.W. McClelland (Ref. 9) conducted an enquiry with the object of finding which test or combination of tests was the best predictor of secondary school success. His tests consisted of Intelligence Tests, standardised scholastic tests, ordinary school examinations and teachers' estimates concerning pupils entering the secondary schools in Dundee in 1936. The results of the tests were compared with the secondary school teachers' estimate of success after three years in the secondary school. The investigation indicated that the best battery for predicting secondary school success consisted of the I.Q., the combined English and Arithmetic in the qualifying examination, together with the teachers' estimate in English and Arithmetic scaled on the actual mark gained in these two. It also showed that the total English and Arithmetic mark gained in the qualifying examination had a higher predictive value than the I.Q.

This investigation concerns pupils entering a grammar school from 1934 to 1943, the year 1937 excepted. The actual marks gained by the pupils in the various examinations were compared with the actual marks gained by them in three separate school certificate subjects five years later. Previous investigations have used a single criterion of success in the grammar school, usually an estimated order of merit. In this case the criterion in complex, consisting of three assessments. Using Peel's method of calculating prediction for a complex criterion (Ref. 10) the predictive value of the entrance examination for certain arbitrary weightings of the tests was calculated. In addition, the maximum prediction of the entrance examination and the weightings of tests and criteria associated with it were obtained.

## Chapter Two.

## Method and Scope of the Enquiry.

The school concerned in the enquiry is a large mixed grammar school in an urban area, drawing pupils from a district where the predominant industry is coal-mining. The annual intake is about ninety six being approximately 10% of the age group. The number of pupils in the area going to private grammar schools has always been negligible. There is no technical education and the competition for entry to the grammar school has always been very keen. The age group consists of those pupils who are eleven but not twelve on 1st August of the year in which they sit. The character of the entrance examination has changed considerably over the period of the experiment, and since the object of the investigation is to compare the predictive values, a detailed statement of the methods of selection is warranted.

### 1934, 1935, 1936.

All pupils in the age group were given a qualifying examination in English and Arithmetic. About one-third of these took the competitive examination, which again consisted of unstandardised papers in English and Arithmetic. In each case the English and Arithmetic were weighted in the ratio 3 : 2 and in the Competitive Examination there was an age-allowance of 1% of the actual marks for each complete month of age below 12 years. Final selection was made after an interview which involved a number of pupils equal to 150%

of the places available. The number of pupils whose entry was decided by the interview in any year, was small.

1938 and 1939.

All pupils were given a qualifying examination in English and Arithmetic, weighted in the ratio 3 : 2. Approximately 40% proceeded to the Competitive Examination, which contained in addition to the unstandardised papers in English and Arithmetic, an unstandardised Intelligence Test. The weights of the three tests was almost equal. In 1938, an age-allowance of 2% of the possible marks was given in the Qualifying Examination, but in the Competitive Examination this was reduced to 1% for each month of age below 12. In 1939, the age-allowance was the same for both examinations, 1% of the possible marks for each complete month of age below 12 years of age. There was no interview in either of these years.

1940, 1941, 1942, 1943.

Entry into the grammar schools for these years was decided on a single standardised Moray House Intelligence Test.

The year 1937 is omitted owing to the fact that the entrance results for that year are not available. The method of selection for that year was similar to that for 1938 and 1939. It has always been the practice in this county to provide a supplementary examination for those pupils who are unable to take the usual examinations through illness.

In the grammar school, all pupils enter for the school certificate examination after five years; no pupil is kept out of

the examination, even though his estimated chance of passing is negligible. The school certificate examination is that of the Northern Universities Joint Matriculation Board, who very kindly permitted the use of their examination results. Only those pupils who completed five years in the school and took the school certificate examination in English Language, English Literature, Mathematics and Geography were considered. All were taking the examination for the first time. This reduced the number of pupils in the experiment considerably as premature withdrawals, transfers to other grammar schools and pupils who missed one or more of the examinations were omitted. English Language, English Literature, Mathematics and Geography were used as all pupils in the school take these subjects in the school certificate examination. The following table shows the number of pupils in the investigation :-

| Year | Boys | Girls | Total |
|---|---|---|---|
| 1934 | 18 | 26 | 44 |
| 1935 | 26 | 23 | 49 |
| 1936 | 25 | 21 | 46 |
| 1938 | 24 | 31 | 55 |
| 1939 | 27 | 26 | 53 |
| 1940 | 29 | 24 | 53 |
| 1941 | 36 | 32 | 68 |
| 1942 | 24 | 26 | 50 |
| 1943 | 32 | 22 | 54 |
| Total | 241 | 231 | 472 |

The marks gained in the various parts of the entrance examination, without age allowance, were compared with the marks

gained in the School Certificate Examination in English, Mathematics and Geography. The English mark used was the average of that for the Language and Literature papers. Pearson Product- Moment Coefficients of Correlation were calculated. It should be noted that these correlation coefficients apply to the sample of selected pupils, which constituted only about 10% of the total population. Higher correlation coefficients would have been obtained if it had been possible to correct these to include the unselected pupils. As information regarding the unselected pupils was not available this could not be done, a fact which should be kept in mind when valuing the results.

Entrance 1934 - School Certificate 1939.

n 44 For significance at 5% level .297, at 1% level .384.

| | Entrance | | School Certificate | | |
|---|---|---|---|---|---|
| | Arith. $w_1$ | English $w_2$ | Mathematics $u_1$ | English $u_2$ | Geography $u_3$ |
| $w_1$ | | .2866 | .4762 | .2115 | .3402 |
| $w_2$ | .2866 | | .4145 | .4241 | .3377 |
| $u_1$ | .4762 | .4145 | | .4454 | .6761 |
| $u_2$ | .2115 | .4241 | .4454 | | .6215 |
| $u_3$ | .3402 | .3377 | .6761 | .6215 | |

Entrance 1935 - School Certificate 1940.

n 49. For significance at 5% level .281, and 1% level .364

| | Entrance | | School Certificate | | |
|---|---|---|---|---|---|
| | Arith. $w_1$ | English $w_2$ | Mathematics $u_1$ | English $u_2$ | Geography $u_3$ |
| $w_1$ | | .01948 | .2131 | .4786 | .2830 |
| $w_2$ | .01948 | | .5243 | .2642 | .3367 |
| $u_1$ | .2131 | .5243 | | .6286 | .7178 |
| $u_2$ | .4786 | .2642 | .6286 | | .5348 |
| $u_3$ | .2830 | .3367 | .7178 | .5348 | |

Entrance 1936 - School Certificate 1941.

n 46. For significance at 5% level .291, at 1% level .376.

| | Entrance | | School Certificate | | |
|---|---|---|---|---|---|
| | Arith. $w_1$ | English $w_2$ | Mathematics $u_1$ | English $u_2$ | Geography $u_3$ |
| $w_1$ | | .2342 | .3666 | .3406 | .2801 |
| $w_2$ | .2342 | | .1957 | .4501 | .2987 |
| $u_1$ | .3666 | .1957 | | .5880 | .5905 |
| $u_2$ | .3406 | .4501 | .5880 | | .5840 |
| $u_3$ | .2801 | .2987 | .5905 | .5840 | |

Entrance 1938 - School Certificate 1943.

n 55. For significance at 5% level .265, at 1% level .345.

| | Entrance | | | School Certificate | | |
|---|---|---|---|---|---|---|
| | Arith. $w_1$ | English $w_2$ | Intell. $w_3$ | Maths. $u_1$ | English $u_2$ | Geography $u_3$ |
| $w_1$ | | .00537 | .3196 | .4751 | .1356 | .3318 |
| $w_2$ | .00537 | | .3362 | .2545 | .4008 | .1972 |
| $w_3$ | .3196 | .3362 | | .5303 | .5060 | .5493 |
| $u_1$ | .4751 | .2545 | .5303 | | .5129 | .7132 |
| $u_2$ | .1356 | .4008 | .5060 | .5129 | | .3142 |
| $u_3$ | .3318 | .1972 | .5493 | .7132 | .3142 | |

Entrance 1939 - School Certificate 1944.

n 53. For significance of correlation coefficient at the 5% level .270, at the 1% level .352.

| | Entrance | | | School Certificate | | |
|---|---|---|---|---|---|---|
| | Arith. $w_1$ | English $w_2$ | Intell. $w_3$ | Maths. $u_1$ | English $u_2$ | Geography $u_3$ |
| $w_1$ | | .08696 | .02021 | .5428 | .3624 | .1320 |
| $w_2$ | .08696 | | .4943 | .2242 | .3840 | .2130 |
| $w_3$ | .02021 | .4943 | | .3717 | .5193 | .3818 |
| $u_1$ | .5428 | .2242 | .3717 | | .5411 | .4505 |
| $u_2$ | .3624 | .3840 | .5193 | .5411 | | .3006 |
| $u_3$ | .1320 | .2130 | .3818 | .4505 | .3006 | |

Entrance 1940 - School Certificate 1945.
n 53. For significance at 5% level . 270, at 1% level .352.

| | Entrance Intell. | School Certificate Maths. | English | Geography |
|---|---|---|---|---|
| Intell | | .5552 | .6251 | .4842 |
| S.C. Maths. | .5552 | | .7939 | .7260 |
| S.C. Eng. | .6251 | .7939 | | .5370 |
| S.C. Geog. | .4842 | .7260 | .5370 | |

Entrance 1941 - School Certificate 1946.
n 68. For significance at 5% level .238, at 1% level .310.

| | Entrance Intell. | School Certificate Maths. | English | Geography |
|---|---|---|---|---|
| Intell. | | .4184 | .4141 | .2379 |
| S.C. Maths. | .4184 | | .5357 | .6619 |
| S.C. Eng. | .4141 | .5357 | | .4898 |
| S.C. Geog. | .2379 | .6619 | .4898 | |

Entrance 1942 - School Certificate 1947.
n 50. For significance at 5% level .279, at 1% level .361.

| | Entrance Intell. | School Certificate Maths. | English | Geography. |
|---|---|---|---|---|
| Intell. | | .4289 | .6282 | .4761 |
| S.C. Maths. | .4289 | | .5200 | .6127 |
| S.C. Eng. | .6282 | .5200 | | .7021 |
| S.C. Geog. | .4761 | .6127 | .7021 | |

Entrance 1943 - School Certificate 1948.
n 54. For significance at 5% level, .266, at 1% level .348.

| | Entrance Intell. | School Certificate Maths. | English | Geography. |
|---|---|---|---|---|
| Intell. | | .5171 | .6874 | .4422 |
| S.C. Maths. | .5171 | | .5321 | .7416 |
| S.C. Eng. | .6874 | .5321 | | .5368 |
| S.C. Geog. | .4422 | .7416 | .5368 | |

## Chapter Three.

## Calculation of Regression Coefficients and Prediction Values.

In this chapter will be found the results of calculations based on the previous batteries of correlation coefficients. There are three main groups of calculations, those for 1934, 1935 and 1936 with two selection tests; those for 1938 and 1939 with three selection tests and those for 1940 to 1943 where there was a single test of selection. Regression coefficients were calculated by Aitken's method of pivotal condensation.

Where the criterion was complex, Peel's method of calculating the prediction was used.

Each battery was considered in the form

| | $u'$ | $w'$ |
|---|---|---|
| $u$ | $R_{aa}$ | $R_{ab}$ |
| $w$ | $R_{ba}$ | $R_{bb}$ |

where u denotes the criteria (three school certificate subjects) and w denotes the tests of entry into the grammar school. The value of $R_{ab}R_{bb}^{-1}$ was first calculated by Aithen's method, giving the test weights in terms of the criteria. The maximum prediction was then calculated when the criteria was weighted in a given way. (The ratios 1 : 1 : 1 and 1 : 2 : 1 were used). This is given by

$$r = \sqrt{\frac{u' R_{ab} R_{bb}^{-1} R_{ba} u}{u' R_{aa} U}}$$

Now this is the only maximum prediction obtainable with the given weightings of criteria, and is not the maximum prediction possible in the Hotelling sense. The latter is the highest possible correlation between the batteries of tests and criteria. This value and the weights of tests and criteria associated with it, were

found by solving the equation :-

$$\left| R_{ab} R_{bb}^{-1} R_{ba} - \lambda . R_{aa} \right| = 0$$

This is a symmetrical determinantal equation in $\lambda$ of the same degree as the number of tests in the criterion, its largest positive root being the square of the maximum correlation.

As a check on the working, the position of the tests and the criteria in the battery were interchanged and new regression coefficients were obtained, giving the criteria test weights in terms of the entrance tests. The maximum prediction was calculated from these, together with the weights associated with it. These values agreed with those obtained previously. This was done for each of the years 1934 to 1939.

The reversability of the process can be particularly helpful when the number of tests in the criterion is bigger than the number of predicting tests. Interchanging reduces the degree of the equation to be solved to give maximum correlation between the two batteries. Thus for 1934, 1935 and 1936, reversing involved the solution of a quadratic instead of a cubic, which was necessary by the direct method.

At this stage the results are stated, but detailed working for the years 1936 and 1938 is given in the appendix. The calculations were performed to five or six figures but are recorded to four. Five or six figure working is necessary in calculating the maximum prediction for 1938 and 1939, as these are obtained by solving a determinantal cubic equation.

## Entrance 1934 - School Certificate 1939.

The battery of correlation coefficients was as follows :

| | School Certificate | | | Entrance | |
|---|---|---|---|---|---|
| | Maths. $u_1$ | English $u_2$ | Geography $u_3$ | Arithmetic $w_1$ | English $w_2$ |
| $u_0$ | | .4454 | .6761 | .4762 | .4145 |
| $u_2$ | .4454 | | .6215 | .2115 | .4241 |
| $u_3$ | .6761 | .6215 | | .3402 | .3377 |
| $w_1$ | .4762 | .2115 | .3402 | | .2866 |
| $w_2$ | .4145 | .4241 | .3377 | .2866 | |

From this we have the following test weights :

$$w_1 = .3894\ u_1 + .0980\ u_2 + .2652\ u_3 \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$
$$w_2 = .3029\ u_1 + .3960\ u_2 + .2617\ u_3$$

If the assessments, Mathematics, English and Geography are weighted in the ratio 1 : 1 : 1 we have from (1), $w_1 : w_2 = 3 : 4$, and the maximum prediction with these weights is .5418.

Now if the assessments are weighted in the ratio 1 : 2 : 1, we find $w_1 : w_2 = 2 : 3$, and the maximum prediction is reduced to .4350.

Maximum (Hotelling) prediction was .5803 with $u_1 : u_2 : u_3 = 14 : 6 : -1$, and $w_1 : w_2 = 10 : 11$.

Reversing the criteria and the tests in the calculation the following regression coefficients were obtained :-

$$u_1 = .3274\ w_1 + .4543\ w_2$$
$$u_2 = .3356\ w_1 - .0185\ w_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$
$$u_3 = -.0922\ w_1 + .0445\ w_2$$

These gave the same Hotelling maximum prediction with the same weights as the direct method, using equations (1).

## Entrance 1935 - School Certificate 1940.

The battery of correlation coefficients was as follows :-

| | School Certificate | | | Entrance | |
|---|---|---|---|---|---|
| | Maths. $u_1$ | English $u_2$ | Geography $u_3$ | Arithmetic $w_1$ | English $w_2$ |
| $u_1$ | | .6286 | .7178 | .2131 | .5243 |
| $u_2$ | .6286 | | .5348 | .4786 | .2642 |
| $u_3$ | .7178 | .5348 | | .2830 | .3367 |
| $w_1$ | .2131 | .4786 | .2830 | | .01948 |
| $w_2$ | .5243 | .2642 | .3367 | .01948 | |

From this we have the following weights :-

$$w_1 = .2030\ u_1 + .4736\ u_2 + .2765\ u_3$$
$$w_2 = .5203\ u_1 + .2550\ u_2 + .3313\ u_3 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ (1)$$

If the assessments, Mathematics, English and Geography are weighted in the ratio 1 : 1 : 1 , we have from (1), $w_1 : w_2 = 6 : 7$, and the maximum prediction with these weights is .5669.

Now if the assessments are weighted in the ratio 1 ; 2 ; 1 , we find that $w_1 : w_2 = 1 : 1$ , and the maximum prediction, with these weights is .5727.

Maximum (Hotelling) prediction is .5781 with $u_1 : u_2 : u_3 = 17 : 13 : 3$ and $w_1 : w_2 = 4 : 5$.

Reversing the criteria and the tests the following regression coefficients were obtained :-

$$u_1 = -.2549\ w_1 + .6331\ w_2$$
$$u_2 = .5457\ w_1 - .0991\ w_2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ (2)$$
$$u_3 = .1741\ w_1 - .0647\ w_2$$

These gave the same Hotelling maximum prediction with the same weights as those obtained using equations (1).

## Entrance 1936 - School Certificate 1941.

The following battery of correlation coefficients was obtained :-

| | School Certificate | | | Entrance | |
|---|---|---|---|---|---|
| | Maths. $u_1$ | English $u_2$ | Geography $u_3$ | Arithmetic $w_1$ | English $w_2$ |
| $u_1$ | | .5880 | .5905 | .3666 | .1957 |
| $u_2$ | .5880 | | .5840 | .3406 | .4501 |
| $u_3$ | .5905 | .5840 | | .2801 | .2987 |
| $w_1$ | .3666 | .3406 | .2801 | | .2342 |
| $w_2$ | .1957 | .4501 | .2987 | .2342 | |

From this we have the following test weights :-

$$w_1 = .3394\,u_1 + .2488\,u_2 + .2223\,u_3$$
$$w_2 = .1162\,u_1 + .3918\,u_2 + .2466\,u_3 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Weighting the assessments, Mathematics, English and Geography in the ratio 1 : 1 : 1 , we have from (1), $w_1 : w_2 = 1 : 1$ , and the maximum prediction with these weights is .4816.

Weighting the assessments in the ratio 1 : 2 : 1 , we find that $w_1 : w_2 = 11 : 12$ , and the maximum prediction, with these weights is .4974.

Maximum (Hotelling) prediction is .5177 with $u_1 : u_2 : u_3 = 1 : 30 : 7$ and $w_1 : w_2 = 2 : 3$.

Reversing the positions of the criteria and the tests the following regression coefficients were obtained :-

$$u_1 = .2412\,w_1 - .1467\,w_2$$
$$u_2 = .1794\,w_1 + .4725\,w_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$
$$u_3 = .0325\,w_1 + .1094\,w_2$$

These gave the same Hotelling maximum prediction with the same weights as those obtained using equations (1).

## Entrance 1938 - School Certificate 1943.

The battery of correlation coefficients was as follows :-

| | School Certificate | | | Entrance | | |
|---|---|---|---|---|---|---|
| | Maths. $u_1$ | English $u_2$ | Geography $u_3$ | Arith. $w_1$ | English $w_2$ | Intell. $w_3$ |
| $u_1$ | | .5129 | .7132 | .4751 | .2545 | .5303 |
| $u_2$ | .5129 | | .3142 | .1356 | .4008 | .5060 |
| $u_3$ | .7132 | .3142 | | .3318 | .1972 | .5493 |
| $w_1$ | .4751 ~~.2545~~ | .1356 | .3318 | | .0054 | .3196 |
| $w_2$ | .2545 | .4008 | .1972 | .0054 | | .3362 |
| $w_3$ | .5303 | .5060 | .5493 | .3196 | .3362 | |

From this we have the following test weights :-

$$w_1 = .3548\,u_1 + .0005\,u_2 + .1779\,u_3$$
$$w_2 = .1267\,u_1 + .2601\,u_2 + .0346\,u_3 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$
$$w_3 = .3743\,u_1 + .4184\,u_2 + .4808\,u_3$$

Weighting the assessments, Mathematics, English, Geography in the ratio 1 : 1 : 1 , from (1) we have $w_1 : w_2 : w_3 = 5 : 4 : 12$, and the maximum prediction, with these weights is .6884.

Weighting the assessments in the ratio 1 : 2 : 1 , we have $w_1 : w_2 : w_3 = 4 : 5 : 12\frac{1}{2}$, and the maximum prediction with these weights is .6803.

Maximum (Hotelling) prediction is .6892 with $u_1 : u_2 : u_3 = 3 : 4 : 4$, and $w_1 : w_2 : w_3 = 1 : 1 : 3$.

Reversing the criteria and the tests, the following regression coefficients were obtained :-

$$u_1 = .5728\,w_1 + .0132\,w_2 + .0795\,w_3$$
$$u_2 = -.1488\,w_1 + .3717\,w_2 + .3445\,w_3 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$
$$u_3 = -.0300\,w_1 + .0710\,w_2 + .3844\,w_3$$

The same maximum prediction with the same weights was obtained from these as from equations (1).

## Entrance 1939 - School Certificate 1944.

The following battery of correlation coefficients was obtained :-

| | School Certificate | | | Entrance | | |
|---|---|---|---|---|---|---|
| | Maths. $u_1$ | English $u_2$ | Geography $u_3$ | Arithmetic $w_1$ | English $w_2$ | Intell. $w_3$ |
| $u_1$ | | .5411 | .4505 | .5428 | .2242 | .3717 |
| $u_2$ | .5411 | | .3006 | .3624 | .3840 | .5193 |
| $u_3$ | .4505 | .3006 | | .1320 | .2130 | .3818 |
| $w_1$ | .5428 | .3624 | .1320 | | .0870 | .0202 |
| $w_2$ | .2242 | .3840 | .2130 | .0870 | | .4943 |
| $w_3$ | .3717 | .5193 | .3818 | .0202 | .4943 | |

From this we have the following test weights :-

$$w_1 \equiv .5356\,u_1 + .3418\,u_2 + .1229\,u_3$$
$$w_2 \equiv -.0010\,u_1 + .1336\,u_2 + .0197\,u_3 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$
$$w_3 \equiv .3614\,u_1 + .4463\,u_2 + .3695\,u_3$$

Weighting the assessments in the ratio $1 : 1 : 1$, we have from (1), $w_1 : w_2 : w_3 \equiv 7 : 1 : 8$, and the maximum prediction, with these weights is .6902.

Weighting the assessments in the ratio $1 : 2 : 1$, we have $w_1 : w_2 : w_3 \equiv 5 : 1 : 6$, and the maximum prediction, with these weights is .7073.

Maximum (Hotelling) prediction is .7217, with $u_1 : u_2 : u_3 \equiv 6 : 6 : 1$ and $w_1 : w_2 : w_3 \equiv 7 : 1 : 6$.

Reversing the criteria and the tests, the following regression coefficients were obtained :-

$$u_1 \equiv .5509\,w_1 - .0235\,w_2 + .0313\,w_3$$
$$u_2 \equiv .1091\,w_1 + .3622\,w_2 + .4308\,w_3 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$
$$u_3 \equiv -.1490\,w_1 + .1147\,w_2 + .2382\,w_3$$

The same maximum prediction with the same weights was obtained from these as from equations (1)

Entry 1940 to 1943 - School Certificate 1945 to 1948.

The batteries of correlation coefficients were as follows :-

| | School Certificate Maths. | English | Geography | Moray House Intell. Test. |
|---|---|---|---|---|
| 1940 | | | | |
| Maths. | 1 | .7949 | .7260 | .5552 |
| English | .7939 | 1 | .5370 | .6251 |
| Geography | .7260 | .5370 | 1 | .4842 |
| M.H. Intell. | .5552 | .6251 | .4842 | 1 |
| 1941 | | | | |
| Maths. | 1 | .5357 | .6619 | .4184 |
| English | .5357 | 1 | .4898 | .4141 |
| Geography | .6619 | .4898 | 1 | .2379 |
| M.H. Intell. | .4184 | .4141 | .2379 | 1 |
| 1942 | | | | |
| Maths. | 1 | .5200 | .6127 | .4289 |
| English | .5200 | 1 | .7021 | .6282 |
| Geography | .6127 | .7021 | 1 | .4761 |
| M.H. Intell. | .4289 | .6282 | .4761 | 1 |
| 1943 | | | | |
| Maths. | 1 | .5321 | .7416 | .5171 |
| English | .5321 | 1 | .5368 | .6874 |
| Geography | .7416 | .5368 | 1 | .4422 |
| M.H. Intell. | .5171 | .6874 | .4422 | 1 |

The calculations based on these batteries followed similar lines to those given by G.H. Thomson (Ref. 11) except that the position of tests and criteria is reversed. Regression

coefficients and their standard errors were calculated, the latter being obtained from the reciprocal matrix of correlations of tests (in this case School Certificate Mathematics, English and Geography).

The results are summarised in the following table :-

| | School Cert. Maths. | School Cert. English | School Cert. Geog. | Prediction |
|---|---|---|---|---|
| 1940 | | | | |
| Regr. Co. | - .01483 | .52145 | .21494 | .6495 |
| St. Err. | .2201 | .1789 | .1587 | |
| 1941 | | | | |
| Regr. Co. | .3541 | .2923 | -.13998 | .486 |
| St. Err. | .1508 | .1297 | .1461 | |
| 1942 | | | | |
| Regr. Co. | .13985 | .5549 | .0008 | .6395 |
| St. Err. | .1450 | .1610 | .1740 | |
| 1943 | | | | |
| Regr. Co. | .2459 | .5859 | - .05465 | .7111 |
| St. Err. | .1525 | .1213 | .1531 | |

## Chapter Four.

## Observations on the results.

The following table gives a summary of the prediction values and the approximate weights associated with them.

| | Selection | | | Criteria | | | |
|---|---|---|---|---|---|---|---|
| | Arith. | Eng. | Intell. | Maths. | Eng. | Geog. | Pred. |
| 1934 | 3 | 4 | | 1 | 1 | 1 | .5418 |
| | 2 | 3 | | 1 | 2 | 1 | .4350 |
| | 10 | 11 | | 14 | 6 | -1 | .5795 |
| 1935 | 6 | 7 | | 1 | 1 | 1 | .5669 |
| | 1 | 1 | | 1 | 2 | 1 | .5727 |
| | 4 | 5 | | 17 | 13 | 3 | .5781 |
| 1936 | 1 | 1 | | 1 | 1 | 1 | .4816 |
| | 11 | 12 | | 1 | 2 | 1 | .4974 |
| | 2 | 3 | | 1 | 30 | 7 | .5177 |
| 1938 | 5 | 4 | 12 | 1 | 1 | 1 | .6884 |
| | 4 | 5 | 12 | 1 | 2 | 1 | .6803 |
| | 1 | 1 | 3 | 3 | 4 | 4 | .6892 |
| 1939 | 7 | 1 | 8 | 1 | 1 | 1 | .6902 |
| | 5 | 1 | 6 | 1 | 2 | 1 | .7073 |
| | 7 | 1 | 6 | 6 | 6 | 1 | .7217 |
| 1940 | | | / | -1 | 35 | 14 | .6495 |
| 1941 | | | / | 6 | 5 | -2 | .486 |
| 1942 | | | / | 1 | 4 | 0 | .6395 |
| 1943 | | | / | 5 | 12 | -1 | .7111 |

Viewing these results as a whole, we conclude for the results investigated :

(1) The best prediction of grammar school success is given by a battery of unstandardised attainment tests in Arithmetic and English with an unstandardised Intelligence Test.

(2) A single Moray House Intelligence Test gave a prediction almost as good as the above battery.

(3) Both the above were considerably better for prediction purposes than unstandardised attainment tests in Arithmetic and English.

The results for the year 1941 need some explanation. This was the second year that this type of test was used, and it was found that in the county as a whole (approximately 12,000 pupils) the results of the test were better than those for 1940. The reason suggested at the time was the effect of concentrated training which was given in certain schools for this type of test. Such training could have some influence on the results when entry was decided by a single Moray House Test. Another contributory factor to the low prediction for that year is the low correlation of school certificate Geography, for which staffing difficulties are held responsible.

For the years 1934, 1935, 1936, the Arithmetic and English tests are of almost equal value, maximum prediction being obtained by a weighting slightly in favour of the English. However, taking the three years as a whole, very little indication of possible success in the school certificate examination is given by either the Arithmetic or the English.

In 1938 the Entrance Arithmetic gave a fairly good prediction of School Certificate Mathematics, but was of little value in predicting School Certificate English or Geography. In a similar way the Entrance English gave a much better prediction of School Certificate English than School Certificate Mathematics or Geography. The Entrance Intelligence Test, hwoever, gave a better prediction of all three than either the Arithmetic or English.

In 1939, the predictive values of the various entrance tests was roughly the same as those for 1938, except that the Entrance Arithmetic was better than the Entrance Intelligence Test for predicting School Certificate Mathematics.

For the years 1940 to 1943 there was a high degree of correlation between the school certificate subjects and between the entrance tests and the latter. The School Certificate English showed the closest relation to the Entrance Intelligence Test. The multiple correlation of the school certificate subjects, as a team, with the Intelligence test was high, reaching .7111 in 1943. The regression coefficients of Mathematics and Geography were lower than those for English, in some cases being negative. This is due to the high correlations between the tests. Such a tendency is often characteristic of regression coefficients obtained from data in which the ~~tests~~ school subjects correlate highly with each other.

Since 1943 the entrance examination has undergone still further changes. In 1944 the Qualifying Examination was a Moray House Intelligence Test, while the Competitive Examination consisted of two standardised Attainment Tests in Arithmetic and English (Moray

House Tests). These pupils will take the School Certificate Examination in July 1949. It will be interesting to compare the predictive value of these tests with those of preceding years. This will be done in September or October when the school certificate results are available.

The following table shows how the method of selection for grammar schools has varied in this county over the last fifteen years.

| Year | Qualifying Exam. | Competitive Exam. |
|---|---|---|
| 1934<br>1935<br>1936 | Unstandardised Tests in Arithmetic and English | Unstandardised Tests in Arithmetic and English with an interview |
| 1937<br>1938<br>1939 | Unstandardised Tests in Arithmetic and English | Unstandardised Tests in Arith. and English with an unstand. Intell. Test. |
| 1940<br>1941<br>1942<br>1943 | None | Standardised Moray House Intelligence Test |
| 1944 | Standardised Moray House Intell. Test | Standardised Moray House Tests in Arith. and Eng. |
| 1945<br>1946 | Unstandardised tests in Arith. and Eng. with a Stand. Moray House Intelligence Test | Standardised Moray House Tests in Arithmetic and English. |
| 1947<br>1948<br>1949 | Unstandardised tests in Arithmetic and English | Standardised Moray House Tests in Arithmetic, English and Intelligence. |

The Qualifying Examination has in the fifteen years completed the circle, returning to unstandardised attainment tests in Arithmetic and English. During this time, the Competitive

examination has developed from two unstandardised tests to three standardised tests. Such changes provide abundant material for investigation into the merits of the respective methods of selection. Mine has been an individual experiment and I feel that much valuable information on the problem of selection for secondary grammar education could be obtained from a team research. This should, if possible, consider the unslected pupils as W.G. Emmett did in his enquiry.

## Appendix A.

### Detailed Working for 1936 - 1941.

Consider the battery of correlation coefficients in the following forms :-

| | School Certificate | | | Entrance | |
|---|---|---|---|---|---|
| | Maths. $u_1$ | English $u_2$ | Geography $u_3$ | Arith. $w_1$ | English $w_2$ |
| $u_1$ | 1 | .5880 | .5905 | .3666 | .1957 |
| $u_2$ | .5880 | 1 | .5840 | .3406 | .4501 |
| $u_3$ | .5905 | .5840 | 1 | .2801 | .2987 |
| $w_1$ | .3666 | .3406 | .2801 | 1 | .2342 |
| $w_2$ | .1957 | .4501 | .2987 | .2342 | 1 |

| | $u'$ | $w'$ |
|---|---|---|
| $u$ | $R_{aa}$ | $R_{ab}$ |
| $w$ | $R_{ba}$ | $R_{bb}$ |

The regression coefficients are obtained by evaluating $R_{ab} R_{bb}^{-1}$, using Aitken's method of pivotal condensation. From these we calculate the maximum prediction with the school certificate subjects equally weighted, and the maximum prediction when the School Certificate English has double the weight of either Mathematics or Geography. Lastly, we calculate maximum prediction in the Hotelling sense. The method is that given by Peel for the calculation of the prediction of a complex criterion.

To calculate $R_{ab} R_{bb}^{-1}$

| | | | | |
|---|---|---|---|---|
| 1 | .2342 | -1 | | .2342 |
| .2342 | 1 | | -1 | .2342 |
| .3666 | .1957 | | | .5623 |
| .3406 | .4501 | | | .7907 |
| .2801 | .2987 | | | .5788 |
| | .94515 | .2342 | -1 | .17935 |
| | 1 | .247791 | -1.058033 | .189758 |
| | .109842 | .3666 | | .476442 |
| | .370331 | .3406 | | .710931 |
| | .233101 | .2801 | | .513201 |
| | | .339382 | .116216 | .455598 |
| | | .248835 | .391822 | .640657 |
| | | .222340 | .246629 | .468969 |

We then have the test weights in terms of the criteria weights.

$$w_1 = .339382\ u_1 + .248835\ u_2 + .222340\ u_3$$
$$w_2 = .116216\ u_1 + .391822\ u_2 + .246629\ u_3 \qquad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

The maximum prediction with arbitrary weightings of the three criteria is given by

$$r = \sqrt{\frac{u'\, R_{ab} R_{bb}^{-1} R_{ba} u}{u'\, R_{aa} u}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

First let us weight the three school certificate subjects equally, then $u_1 : u_2 : u_3 = 1 : 1 : 1$, and from (1) we have

$$w_1 = .810557 \quad \text{and} \quad w_2 = .754667 \qquad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Then
$$u' R_{ab} R_{bb}^{-1} R_{ba} u = \begin{bmatrix} .81056 & .75467 \end{bmatrix} \begin{bmatrix} .3666 & .3406 & .2801 \\ .1957 & .4501 & .2987 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$
$$= \begin{bmatrix} .81056 & .75467 \end{bmatrix} \begin{bmatrix} .9873 \\ .9445 \end{bmatrix} = 1.513052 \quad \ldots\ldots (4)$$

Also
$$u' R_{aa} u = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & .5880 & .5905 \\ .5880 & 1 & .5840 \\ .5905 & .5840 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2.1785 \\ 2.1720 \\ 2.1745 \end{bmatrix}$$
$$= 6.5250 \quad \ldots\ldots\ldots (5)$$

Hence the maximum prediction is $\sqrt{\dfrac{1.5131}{6.5250}} = .4816$

---

Now let us weight the three criteria in the ratio 1 : 2 : 1 .

From (1) we have
$$w_1 = 1.05939 \quad \text{and} \quad w_2 = 1.14649$$

Then
$$u' R_{ab} R_{bb}^{-1} R_{ba} u = \begin{bmatrix} 1.05939 & 1.14649 \end{bmatrix} \begin{bmatrix} .3666 & .3406 & .2801 \\ .1957 & .4501 & .2987 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$
$$= 3.00566 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Also
$$u' R_{aa} u = \begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & .5880 & .5905 \\ .5880 & 1 & .5840 \\ .5905 & .5840 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = 11.869 \quad \ldots\ldots\ldots (7)$$

The maximum prediction with these weights is $\sqrt{\dfrac{3.0060}{11.869}} = .4974$

---

Interchanging the tests and the criteria is equivalent to calculating $R_{ba} R_{aa}^{-1}$.

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | .5880 | .5905 | -1 | | | 1.1785 |
| .5880 | 1 | .5840 | | -1 | | 1.1720 |
| .5905 | .5840 | 1 | | | -1 | 1.1745 |
| .3666 | .3406 | .2801 | | | | .9873 |
| .1957 | .4501 | .2987 | | | | .9445 |
| | .654256 | .236786 | .5880 | -1 | | .479042 |
| | 1 | .361916 | .898731 | -1.528454 | | .732193 |
| | .236786 | .651310 | .5905 | | -1 | .478596 |
| | .125039 | .063623 | .3666 | | | .555262 |
| | .335028 | .183139 | .1957 | | | .713868 |
| | | .565613 | .377693 | .361917 | -1 | .305223 |
| | | 1 | .667759 | .639866 | -1.767993 | .539632 |
| | | .018369 | .254224 | .191116 | | .463709 |
| | | .061887 | -.1054 | .512075 | | .468562 |
| | | | .241958 | .179362 | .032476 | .453796 |
| | | | -.146726 | .472476 | .109416 | .435166 |

This gives us the criteria weights in terms of the test weights.

$$u_1 = .241958\, w_1 - .146726\, w_2$$
$$u_2 = .179362\, w_1 + .472476\, w_2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots (8)$$
$$u_3 = .032476\, w_1 + .109416\, w_2$$

Maximum prediction, in the Hotelling sense, is obtained by solving

$$\left| R_{ba} R_{aa}^{-1} R_{ab} - \lambda R_{bb} \right| = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

The maximum prediction is the square root of the largest root of this equation (9).

$$R_{ba} R_{aa}^{-1} R_{ab} = \begin{bmatrix} .241958 & .179362 & .032476 \\ -.146726 & .472476 & .109416 \end{bmatrix} \begin{bmatrix} .3666 & .1957 \\ .3406 & .4501 \\ .2801 & .2987 \end{bmatrix}$$

$$= \begin{bmatrix} .158889 & .137782 \\ .137782 & .216630 \end{bmatrix}$$

Thus we have to solve the quadratic :-

$$\begin{vmatrix} .15889 - \lambda & .13778 - .2342\lambda \\ .13778 - .2342\lambda & .21663 - \lambda \end{vmatrix} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (10)$$

The larger root of equation (10) is .2681, and the maximum prediction is the square root of this, .5178.

Substituting the value $\lambda = .2681$ in equation (10) we have

$$\begin{vmatrix} -.10921 & .07499 \\ .07499 & -.05147 \end{vmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (11)$$

From which we get $w_1 : w_2 = .05147 : .07499 = 2 : 3$ (Approx.)

Using these ratios for $w_1$ and $w_2$ in equations (8) we find that

$u_1 : u_2 : u_3 = .014484 : .44667 : .098782 = 1 : 30 : 7$ (approx.)

Now if we used these ratios $u_1 : u_2 : u_3 = 1 : 30 : 7$ in equations (1) we find

$$w_1 = 9.3607 \quad \text{and} \quad w_2 = 13.5971$$

These are in the approximate ratio 2 : 3 . Calculating the maximum prediction in the usual way we obtain .5177. Thus both the maximum prediction and the weights associated with it can be obtained from either equations (1) or equations (8).

If we had obtained the maximum prediction by using

equations (1), we would have had to solve

$$\left| R_{ab} \; R_{bb}^{-1} \; R_{ba} - \lambda R_{aa} \right| = 0$$

This would be a symmetrical determinantal cubic equation and its largest root would be the same as the larger root of the quadratic (10).

## Appendix B.

## Detailed working for 1938 - 1943.

Consider the battery of correlation coefficients in the following form :-

| | School Certificate | | | Entrance | | |
|---|---|---|---|---|---|---|
| | Maths. $u_1$ | English $u_2$ | Geography $u_3$ | Arith. $w_1$ | English $w_2$ | Intell. $w_3$ |
| $u_1$ | 1 | .5129 | .7132 | .4751 | .2545 | .5303 |
| $u_2$ | .5129 | 1 | .3142 | .1356 | .4008 | .5060 |
| $u_3$ | .7132 | .3142 | 1 | .3318 | .1972 | .5493 |
| $w_1$ | .4751 | .1356 | .3318 | 1 | .0054 | .3196 |
| $w_2$ | .2545 | .4008 | .1972 | .0054 | 1 | .3362 |
| $w_3$ | .5303 | .5060 | .5493 | .3196 | .3362 | 1 |

| | $u'$ | $w'$ |
|---|---|---|
| $u$ | $R_{aa}$ | $R_{ab}$ |
| $w$ | $R_{ba}$ | $R_{bb}$ |

The calculation of regression coefficients and the prediction values, with the weights associated with them, follows in the same way as those for Appendix A.

Calculation of $R_{ab} R_{bb}^{-1}$.

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | .0054 | .3196 | -1 | | | .3250 |
| .0054 | 1 | .3362 | | -1 | | .3416 |
| .3196 | .3362 | 1 | | | -1 | .6558 |
| .4751 | .2545 | .5303 | | | | 1.2599 |
| .1356 | .4008 | .5060 | | | | 1.0424 |
| .3318 | .1972 | .5493 | | | | 1.0783 |
| | .99997 | .33447 | .0054 | -1 | | .33984 |
| | 1 | .33448 | .0054 | -1.00003 | | .33985 |
| | .33447 | .89786 | .3196 | | -1 | .55193 |
| | .25193 | .37846 | .4751 | | | 1.10549 |
| | .40007 | .46266 | .1356 | | | .99833 |
| | .19541 | .44326 | .3318 | | | .97047 |
| | | .78598 | .31779 | .33448 | -1 | .43826 |
| | | 1 | .40435 | .42555 | -1.27229 | .55760 |
| | | .29419 | .47374 | .25194 | | 1.01987 |
| | | .32884 | .13344 | .40008 | | .86237 |
| | | .37790 | .33074 | .19542 | | .90406 |
| | | | .35478 | .12675 | .37430 | .85583 |
| | | | .00047 | .26014 | .41839 | .67900 |
| | | | .17794 | .03460 | .48080 | .69334 |

Thus the test weights are given by :-

$$w_1 = .35478\,u_1 + .00047\,u_2 + .17794\,u_3$$
$$w_2 = .12675\,u_1 + .26014\,u_2 + .03460\,u_3 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$
$$w_3 = .37430\,u_1 + .41839\,u_2 + .48080\,u_3$$

Weighting School Certificate Mathematics, English and Geography equally, we have $u_1 : u_2 : u_3 = 1 : 1 : 1$, and from (1)

$$w_1 = .53319 \ , \ w_2 = .42149 \ , \ w_3 = 1.27348 \ .$$

Then
$$u' R_{ab} R_{bb}^{-1} R_{ba} u = \begin{bmatrix} .53319 & .42149 & 1.27348 \end{bmatrix} \begin{bmatrix} .4751 & .1356 & .3318 \\ .2545 & .4008 & .1972 \\ .5303 & .5060 & .5493 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= 2.88108$$

Also
$$u' R_{aa} \cdot u = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & .5129 & .7132 \\ .5129 & 1 & .3142 \\ .7132 & .3142 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 6.0806$$

Hence the maximum prediction with the school subjects equally weighted is
$$\sqrt{\frac{2.88108}{6.0806}} = .6884$$

Weighting the school subjects in the ratio $1 : 2 : 1$, we have from equation (1), $w_1 = .53366$ , $w_2 = .68163$ , $w_3 = 1.69187$ .

Then
$$u' R_{aa} R_{bb}^{-1} R_{ba} u = \begin{bmatrix} .53366 & .68163 & 1.69187 \end{bmatrix} \begin{bmatrix} .4751 & .1356 & .3318 \\ .2545 & .4008 & .1972 \\ .5303 & .5060 & .5493 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$$= 4.96831$$

Also
$$u' R_{aa} u = \begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & .5129 & .7132 \\ .5129 & 1 & .3142 \\ .7132 & .3142 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = 10.7348$$

Hence the maximum prediction with the weight of the English doubled is
$$\sqrt{\frac{4.9683}{10.7348}} = .6803$$

To find the Hotelling maximum prediction we evaluate :

$$R_{ab} R_{bb}^{-1} R_{ba} \simeq \begin{bmatrix} .35478 & .12675 & .374295 \\ .00047 & .26014 & .41839 \\ .17994 & .03460 & .48080 \end{bmatrix} \begin{bmatrix} .4751 & .1356 & .3318 \\ .2545 & .4008 & .1972 \\ .5303 & .5060 & .5493 \end{bmatrix}$$

$$\simeq \begin{bmatrix} .39930 & .28830 & .34831 \\ .28830 & .31603 & .28128 \\ .34831 & .28128 & .33000 \end{bmatrix}$$

The maximum prediction is then the square root of the largest root of the following cubic equation :-

$$\begin{vmatrix} .39930 - \lambda & .28830 - .5129\lambda & .34831 - .7132\lambda \\ .28830 - .5129\lambda & .31603 - \lambda & .28128 - .3142\lambda \\ .34831 - .7132\lambda & .28128 - .3142 & .33000 - \lambda \end{vmatrix} \simeq 0 \ldots\ldots (2)$$

The largest positive root of this equation is .4750, and hence the maximum prediction is .6892.

Substituting the value $\lambda \simeq .4750$ in equation (2), and working to four figures, we obtain the determinant :

$$\begin{vmatrix} -.0757 & .0447 & .0095 \\ .0447 & -.1590 & .1321 \\ .0095 & .1321 & -.1450 \end{vmatrix}$$

The ratio of the minors of this determinant will give the ratio of the criteria weights for maximum prediction. Hence, we have

$$u_1 : u_2 : u_3 \simeq 5574 : 7745 : 7430 \simeq 3 : 4 : 4 \quad \text{(approx)}. \ldots\ldots\ldots (3)$$

Using the ratios in (3), we obtain from equations (1)

$$w_1 : w_2 : w_3 \simeq 3303 : 2978 : 8899 \simeq 1 : 1 : 3 \quad \text{(approx)}. \_\_\_\_\_ (4)$$

We now reverse the positions of the criteria and the entrance tests in the calculation, and proceed in a similar way.

Calculation of $R_{ba} R_{aa}^{-1}$.

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | .5129 | .7132 | -1 | | | 1.2261 |
| .5129 | 1 | .3142 | | -1 | | .8271 |
| .7132 | .3142 | 1 | | | -1 | 1.0274 |
| .4751 | .1356 | .3318 | | | | .9425 |
| .2545 | .4008 | .1972 | | | | .8525 |
| .5303 | .5060 | .5493 | | | | 1.5856 |
| | .73693 | -.0516 | .5129 | -1 | | .19823 |
| | 1 | -.07002 | .69600 | -1.35700 | | .26900 |
| | -.0516 | .49135 | .7132 | | -1 | .15295 |
| | -.10808 | -.00704 | .4751 | | | .35998 |
| | .27027 | .01569 | .2545 | | | .54046 |
| | .23401 | .17109 | .5303 | | | .93540 |
| | | .48773 | .74911 | -.07002 | -1 | .16683 |
| | | 1 | 1.53591 | -.14356 | -2.05030 | .34204 |
| | | -.01461 | .55032 | -.14666 | | .38905 |
| | | .03461 | .06640 | .36675 | | .46776 |
| | | .18748 | .36743 | .31754 | | .87245 |
| | | | .57276 | -.14876 | -.02995 | .39405 |
| | | | .01323 | .37171 | .07097 | .45592 |
| | | | .07949 | .34446 | .38438 | .80833 |

Hence the school certificate test weights are given by :-

$$u_1 = .57276\, w_1 + .01323\, w_2 + .07949\, w_3$$
$$u_2 = -.14876\, w_1 + .37171\, w_2 + .34446\, w_3 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$
$$u_3 = -.02995\, w_1 + .07097\, w_2 + .38438\, w_3$$

Maximum prediction in the Hotelling sense is found by first evaluating

$$\begin{bmatrix} .57276 - .14876 - .02995 \\ .01323 \quad .37171 \quad .07097 \\ .07949 \quad .34446 \quad .38438 \end{bmatrix} \begin{bmatrix} .4751 & .2545 & .5303 \\ .1356 & .4008 & .5060 \\ .3318 & .1972 & .5493 \end{bmatrix}$$

$$= \begin{bmatrix} .24201 & .08024 & .21201 \\ .08024 & .16635 & .23409 \\ .21201 & .23409 & .42759 \end{bmatrix}$$

The maximum prediction is the square root of the largest root of the equation :-

$$\begin{vmatrix} .24201 - \lambda & .08024 - .0054\lambda & .21201 - .3196\lambda \\ .08024 - .0054\lambda & .16635 - \lambda & .23409 - .3362\lambda \\ .21201 - .3196\lambda & .23409 - .3362\lambda & .42759 - \lambda \end{vmatrix} = 0$$

.475 was found to be a root of this equation. Substituting this value for $\lambda$ we obtain the following determinant :-

$$\begin{vmatrix} -.23299 & .07757 & .06020 \\ .07757 & -.30856 & .074393 \\ .06020 & .07439 & -.047412 \end{vmatrix}$$

The ratios of the minors of this determinant will give the ratio of the entrance tests for maximum prediction. Thus :-

$$W_1 : W_2 : W_3 \simeq 909959 : 815639 : 2435193 \simeq 1 : 1 : 3 \text{ (approx.)}$$

Using these ratios in equations (5) we find that $u_1, u_2, u_3$ are in the approximate ratios 3, 4, 4, which is in agreement with equations (3) and (4).

We thus arrive at the same Hotelling maximum prediction with the same weights by either the direct or reverse process.

## Appendix C.

### Solving the Cubic.

Equation (2) in Appendix B was :-

$$\begin{vmatrix} .3993 - \lambda & .28830 - .5129\lambda & .34831 - .7132\lambda \\ .28830 - .5129\lambda & .31603 - \lambda & .28128 - .3142\lambda \\ .34831 - .7132\lambda & .28128 - .3142\lambda & .33000 - \lambda \end{vmatrix} = 0 \quad \ldots(1)$$

This reduces to,

$$-.3594\lambda^3 + .2363\lambda^2 - .03279\lambda + .00077 = 0$$

i.e. $$\lambda^3 - .6575\lambda^2 + .09124\lambda - .00214 = 0$$

Substituting $$x + .2192 = \lambda$$

we have $$x^3 - .05286x - .00320 = 0 \quad \ldots(2)$$

Since $$4 \cdot (.05286)^3 > 27 \cdot (0032)^2$$

all the roots are real, and the largest root is given by :-

$$x = \frac{2}{\sqrt{3}} \sqrt{.05286} \quad \text{Cos} \frac{\theta}{3}$$

where $$\text{Cos}\,\theta = \frac{3^{\frac{3}{2}}}{2} \cdot \frac{.0032}{(.05286)^{\frac{3}{2}}}$$

From which we get the root of equation (2) is .2558, and hence the largest root of equation (1) is .4750.

## References and Bibliography.


1. Moore, V.J. (pp. 5) 'Symposium on the Selection of Pupils for Different Types of Secondary Schools. III. A method used in a County Borough. Brit. Jour. of Ed. Psych., Feb., 1948.

2. Amos, A.D. (pp. 6) Examination and Intelligence Test Forecasts of School Achievement. Brit. Jour. of Ed. Psych., Vol. 1, 1931.

3. Collier, J.W. (pp. 7) The Predictive Value of Intelligence Tests for Secondary Education. Brit. Jour. of Ed. Psych., Vol. 3, 1933.

4. Hughes, A.G. (pp. 7) Discrepancies between results of Intelligence Tests and Entrance Examinations to Secondary Schools. Brit. Jour. of Ed. Psych., Vol. 4, 1934.

5. Valentine, C.W. (pp. 7) The Reliability of Examinations.

6. Stubbins, T.E. (pp. 7) The Prognostic Value of Subjects of a Secondary School Entrance Examination Br. Jour. of Ed. Psych., Feb. 1940.

7. West Riding Report (pp. 8) Special Place Examinations.

8. Emmett, W.G. (pp. 8) An Enquiry into the Prediction of Secondary School Success.

9. McClelland, W. (pp. 9) Selection for Secondary Education.

10. Peel, E.A. (pp. 10) Prediction of a Complex Criterion and Battery Reliability. Brit. Jour. Psych. Statistical Section, July 1948.

11. Thomson, G.H. (pp. 25) The Factorial Analysis of Human Ability.

| | | |
|---|---|---|
| 12. | Whittaker | Calculus of Observations. |
| 13. | Lindquist | Statistical Analysis in Educational Research. |
| 14. | Aitken | Determinants and Matrices. |
| 15. | Davies and Jones | The Selection of Children for Secondary Education. |
| 16. | Brereton | The Case for Examinations. |