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# بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

In the name of Allah, the Gracious, the Merciful

مَا أَصَابَ مِنْ مُصِيبَةٍ فِي الْأَرْضِ وَلَا فِي أَنْفُسِكُمْ إِلَّا فِي كِتَابٍ مِنْ قَبْلِ أَنْ نَبْرَأَهَا إِنَّ ذَلِكَ عَلَى اللَّهِ يَسِيرٌ. لِكَيْلَا تَأْسَوْا عَلَى مَا فَاتَكُمْ وَلَا تَفْرَحُوا بِمَا آتَاكُمْ وَاللَّهُ لَا يُحِبُّ كُلَّ مُخْتَالٍ فَخُورٍ.

No disaster can befall on the earth or in your souls but it is in a book before We bring it into being; that is easy for Allah. In order that you may not grieve for what has escaped you, nor be exultant at what He has given you; and Allah does not love any prideful boaster. (The Holy Qur'an, 057: 022-023)

اللَّهُمَّ صَلِّ عَلَى سَيِّدِنَا مُحَمَّدٍ الْوَظْفِ وَالْوَحْيِ وَالرِّسَالَةِ وَالْحِكْمَةِ وَعَلَى آلِهِ وَصَحْبِهِ وَسَلِّمْ تَسْلِيمًا

May Allah send blessings on our Master whose quality, revelation, message, and wisdom are praised [Muhammad], and on his lineage and companions, and salute him with a perfect salutation

# **First Visibility of the Lunar Crescent and Other Problems in Historical Astronomy**

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by

Louay J. Fatoohi

July 1998

A thesis submitted to the University of Durham for the degree of Doctor of Philosophy



الى

أستاذي الصوفي ومُرشدِي الروحي

حضرة الشيخ محمد الكَسَنَزاني الحُسَيْنِي قَدَّسَ اللهُ سره العزيز

أستاذ الطريقة العليّة القادريّة الكسنزانيّة

For

My Sufi Master and spiritual guide

Shaikh Muhammad al-Casnazani al-Husseini

The Master of Tariqa ‘Aliyyah Qadiriyyah Casnazaniyyah



## **Abstract**

The first part of this dissertation investigates methods of predicting the first visibility of the lunar crescent: an astronomical problem that has attracted the interest of man since ancient times. Many early nations used lunar calendars, the months of which began on the evening of the first sighting of the lunar crescent after conjunction. In modern times, the resolution of this astronomical problem is of special importance - both for historians who need to determine ancient dates exactly and for Muslims around the world, whose religious calendar is lunar. The interest in this matter over the centuries has resulted in the appearance of a number of solutions by a variety of authors for predicting the first visibility of the lunar crescent. The purpose of the first part of this dissertation is to assess the accuracy of these prediction models using ancient, mediaeval and modern observational data and to explore possible improvement. The study concludes that the concept of a “zone of uncertainty” must be incorporated into any lunar visibility criterion; it further applies this conclusion to the widely used modern criterion of true lunar altitude versus azimuthal difference between the sun and moon. The observational data show that developing a “zone of uncertainty” in this particular criterion yields the best results of all.

The second part of the dissertation is an investigation of six problems in historical astronomy. These are: (i) assessing the accuracy of solar eclipse observations made by Jesuit astronomers in China; (ii) assessing the accuracy of lunar eclipse observations made by Jesuit astronomers in China; (iii) dating the solar eclipse of Thales; (iv) determining the modern equivalent of the Babylonian angular units of measurement; (v) dating the eclipses of Thucydides; and (vi) dating the solar eclipse of Plutarch. All papers have been published or are currently in press.

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## **Part 2. Problems in Historical Astronomy**

**1 Accuracy of Solar Eclipse Observations made by Jesuit Astronomers in China. (published paper)**

**2 Accuracy of Lunar Eclipse Observations made by Jesuit Astronomers in China. (published paper)**

**3 Thales's Prediction of a Solar Eclipse. (published paper)**

**4 Angular Measurements in Babylonian Astronomy. (paper in press)**

**5 The Eclipses Recorded by Thucydides. (paper in press)**

**6 The Total Solar Eclipse Recorded by Plutarch. (paper in press)**

### **Additional Published Studies Referenced in this Dissertation**

**1 Lunar Eclipse Times Recorded in Babylonian History.**

**2 Accuracy of Early Estimates of Lunar Eclipse Magnitude.**

**3 The Babylonian Unit of Time.**

## **Part One**

### **The First Visibility of the Lunar Crescent**



## Introduction

The problem of predicting the first visibility of the lunar crescent attracted attention throughout history from many nations who used lunar calendars to regulate their activities. The oldest available records which reveal organised interest in this matter date back almost three thousand years to the time of the Babylonians. Predicting the first visibility of the lunar crescent received great interest from medieval Muslim astronomers, largely because timings of religious practices in Islam - such as the beginning and end of the fasting month of Ramadhan - are determined by a lunar calendar.

In modern times, scientific interest in understanding the visibility of the lunar crescent has been motivated mainly by two factors: (i) the need to accurately convert dates of historical records of nations that used the lunar calendar; (ii) the need of Muslims to ascertain when the lunar crescent may be visible for the first time after conjunction with the sun - and hence to look for it, and also to know when it cannot be seen. The following quotation expresses how one of the contemporary investigators of the question of first visibility of the lunar crescent described its present cultural and religious significance:

With roughly  $1 \times 10^9$  people of the Islamic faith following the Islamic calendar, this problem is likely to be the one (non-trivial) problem in astronomy that has the greatest impact on our modern world (Schaefer, 1996: 759).

### 1.1 Overview

Predicting the earliest visibility of the lunar crescent after conjunction is a matter of considerable complexity. It is a problem where astronomical, atmospheric and human factors (especially visual acuity) are all at work. The fact that even modern astronomers cannot agree on the best criterion for determining the first visibility of the lunar crescent only attests to the complex nature of this matter.



Throughout history, each attempt to put forward a criterion has followed either an empirical or theoretical approach. The empirical approach, which is more frequently employed, is based on analysing a collection of observational data and then formulating a criterion that best fits the observations. On the other hand, the theoretical approach is embodied in attempts to resolve the problem through considering the various factors affecting crescent visibility and designing a descriptive mathematical model. While the Babylonian criterion was empirical, the Arab astronomers took mostly a theoretical approach. Recent studies on the subject have presented prediction models from both aspects: empirical and theoretical.

In this study, I have compiled a large number of reliable observations - both ancient and recent - from the available astronomical literature. I have carefully checked these data and then used them to test the accuracy of the most important ancient and modern prediction models. Whether a model is theoretical or empirical, the test by observational data must have the final word on its accuracy. The study shows that the various tested criteria all have poor accuracy and concludes that the concept of a “zone of uncertainty” must be incorporated into any lunar visibility criterion. Also developed here is a new criterion of lunar visibility that incorporates the concept of a “zone of uncertainty” and is therefore much more accurate than the previously suggested models.

## **1.2 Literature Review**

The oldest literature on the problem of the first visibility of the lunar crescent comes from ancient Babylon. During the 1870s and 1880s a large number of clay tablets that contain astronomical information (written in cuneiform) were excavated from the site of ancient Babylon in Iraq. It seems that no other astronomical texts have been unearthed from the ruins of Babylon since the end of the last century. The British Museum acquired virtually all of the known tablets; there was little interest from other museums. The most important of the excavated texts (including nearly all the datable ones) have been translated and transliterated recently and they have therefore become much more accessible to researchers.

The second major source of literature on lunar crescent visibility is the Arab world of the Middle Ages. Few actual observations are available, but there is a large number of astronomical tables and texts which are partially or totally dedicated to discussing this

problem. Although many of these sources are still in manuscript form, many others have been published or partially introduced through recent publications.

The third major - and most important - source of literature on the question of the visibility of the lunar crescent is modern publications. In the present century scientists have shown renewed interest in this matter and several tens of research papers have so far been published, addressing various aspects of the visibility of the lunar crescent.

The present investigation is based on the study of accessible works from the three main sources listed above.

### 1.3 Abbreviations

I have used a number of abbreviations and symbols throughout the text. These are defined below:

Solar longitude:  $\lambda_s$

Lunar longitude:  $\lambda_m$

Difference in longitude between the sun and moon:  $\Delta\lambda$

Lunar latitude:  $\beta$

Difference in azimuth between the sun and moon:  $\Delta Z$

Geocentric lunar altitude:  $h$

Solar depression:  $s$

Arc of vision (*Arcus Visionis*) or Arc of Descent which is the difference in altitude between the moon and the setting sun (neglecting refraction and parallax):  $H$ . In general,  $H=h+s$ , whereas at sunset  $H=h$ .

Arc of light or the geocentric elongation which is the angular separation between the sun and moon):  $L$

Topocentric elongation is the angular distance between the sun and moon, with allowance for lunar parallax:  $L'$

Moonset lagtime (the time interval between sunset and moonset); this quantity is expressed either in minutes of time or in equatorial degrees where  $1^\circ$  equals 4 minutes:  $S$

Geocentric width of the crescent:  $W$

Topocentric width of the crescent:  $W'$ .



## **1.4 Acknowledgement**

This study would not have been possible, and my understanding of the subject would have been greatly impaired, without the generous help and patience of my supervisor Professor F.R. Stephenson, to whom I am deeply indebted. I also gratefully acknowledge the support of the Physics Department at the University of Durham and the Airey Neave Trust (London).

## The Observational Data

The availability of reliable observations is vital for the investigation of the problem of the first visibility of the lunar crescent. I have, therefore, compiled a large number of observations of the lunar crescent from ancient, medieval and modern astronomical literature. I have extracted the ancient data from the “astronomical diaries” of the Babylonians. The Babylonians showed much interest in predicting the date of the first visibility of the young moon because they used a lunar calendar whose month began at sunset on the day in which the lunar crescent was first visible. I have collected the medieval data that I used in this study from a book by the Andalusian traveller ibn Jubayr and the modern data from recent astronomical publications. This chapter explains the procedure of compiling and verifying the data, starting with the Babylonian observations.

### 2.1 The Babylonian “Astronomical Diaries”

The ancient Babylonians developed great interest in astronomical observations. This interest was mainly motivated by their concern with astrology, though calendrical needs contributed as well. In fact, there was never any distinction between the *astronomers* who made observations and the *astrologers* who interpreted the observations; both tasks were performed by the same people (Britton & Walker, 1996).

From the eighth century BC onward, the Babylonians systematically and continuously recorded their astronomical observations on clay tablets. The Babylonian heritage of astronomical cuneiform texts is usually classified, after Sachs (1948), into four categories: (i) “Almanacs”, which are yearly lists of various predicted lunar and planetary phenomena, solstices and equinoxes, etc; (ii) “Goal-Year Texts”, which were designed for the prediction of lunar and planetary phenomena based on certain fundamental periods and were prepared from the “Astronomical Diaries” (see below); (iii) “Normal-Star Almanacs”, which are texts on the positions of thirty one stars close to the ecliptic which the Babylonians used for reference and which were denoted *Normalsterne* (“Normal stars”) by Epping (1889): a list of these stars,

with longitude and latitude at the epoch 164 B.C., is given by Stephenson and Walker (1985); (iv) the “Astronomical Diaries”. For the purpose of this study, only the last category of Babylonian astronomical texts will be of interest.

The “Astronomical Diaries”, or more briefly “diaries”, is the modern term used to refer to the tablets known in Akkadian as *nasaru ša ginê* which means “regular watching”. These diaries represent records of daily astronomical observations made in the Neo-Babylonian period by *professionals* who, according to excavated late documents, were employed and paid specifically to make these observations. Their job also included recording their observations in the diaries and preparing astronomical tables and yearly almanacs. A diary usually covered about 6 months of observation. The entries for each month typically include information on the following: the length of the previous month; lunar and solar eclipses; lunar and planetary conjunctions with each other or with Normal stars; solstices and equinoxes; heliacal rising and setting of planets and Sirius; meteors; and comets.

In the diaries, the Babylonians also systematically recorded the six time intervals termed by A. Sachs “Lunar Sixes”. These may be described as follows: On the first day of the month the Babylonians recorded the time between sunset and moonset (*na*). Around the middle of the month they recorded four intervals related to the full moon which are as follows: the time interval between moonset and sunrise when the moon set for the last time before sunrise (*ŠU*); the interval between sunrise and moonset when the moon set for the first time after sunrise (*na*); the interval between moonrise and sunset when the moon rose for the last time before sunset (*ME*); and the interval between sunset and moonrise when the moon rose for the first time after sunset (*GE<sub>6</sub>*). Finally, near the end of the month the Babylonians recorded the time between moonrise and sunrise when the waning crescent moon was visible for the last time (*KUR*). When clear skies prevailed, these intervals were measured, but when clouds or mist intervened, they were predicted.

In addition to the astronomical data, the diaries also contain some non-astronomical information: on the weather, the prices of six basic commodities, the height of the river Euphrates, and certain historical events.

It should be emphasised that although the major bulk of celestial phenomena referred to in the diaries are actual observations, some of the recorded events are not observations but rather predictions based on some mathematical calculations. This is sometimes stated clearly whereas in others it is implicit, as in the case when the sky is mentioned to have been overcast.



Most of the available tablets containing the diaries are damaged to varying degrees - often extensively. In some cases the date of the tablet is broken away. Such tablets can often be dated by using a unique combination of astronomical data which they record - for example, eclipses and lunar and planetary positions. This is how Sachs and Hunger determined many of the dates of the diaries which they recently published in transliteration and translation in three volumes (Sachs & Hunger, 1988, 1989, 1996). These volumes, which form the exclusive source of the Babylonian data of the present study, cover diaries from 652 B.C. to 61 B.C.

The following is an example of the diary reports for the first seven days of the lunar month whose first day corresponds to B.C. 163 August 11 (round brackets denote editorial comment, square brackets indicate damaged text that has been restored by the editors, whereas the number at the beginning of each paragraph indicates the line number in the text):

- 1 Year 149 (Seleucid), king Antiochus. Month V, (the 1st of which was identical with) the 30th (of the preceding month), sunset to moonset: 10°, it was very low; measured (despite) mist.
- 2 Night of the 2nd, the moon was 1 cubit behind  $\gamma$  Virginis. Night of the 3rd, the moon was 1 cubit above  $\alpha$  Virginis, the moon having passed 0.5 cubit
- 3 to the east. The 3rd, the north wind blew. Night of the 4th, the moon was 4 cubits in front of  $\alpha$  Librae. The 4th, the north wind blew. Night of the 5th,
- 4 beginning of the night, the moon was 2.5 cubits below  $\beta$  Librae. The 5th, the east wind blew. Night of the 6th, beginning of the night, the moon was 20 fingers above  $\beta$
- 5 Scorpii. The 6th, ZI IR (unidentified), the east wind blew. Night of the 7th, beginning of the night, the moon was 3 cubits in front of  $\theta$  Ophiuchi,
- 6 the moon being 2.5 cubits high to the north, it stood 1 cubit 8 fingers in front of Mars to the west, the moon being 2 cubits high to [the north;]
- 7 last part of the night, Venus was 4 cubits below  $\epsilon$  Leonis. The 7th, clouds were in the sky, ZI IR, the east wind blew (trans. Sachs & Hunger, 1996: 25).

As seen in the above example, a typical diary starts with a mention of the Babylonian year and month. This is followed by a phrase stating that the first day of that month was either “identical with” or “followed”<sup>1</sup> the 30th of the preceding month, indicating that the previous month contained either 29 or 30 days, respectively. (The Babylonian months invariably contained only 29 or 30 days). After that there is a mention of the measured or predicted *na*

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<sup>1</sup>The phrases “identical with” and “followed”, which describe the length of the month, have been

interval, which is the time between sunset and moonset of the first day of the month - usually known as “moonset lagtime” in modern terminology. This is one of the six quantities termed “Lunar Sixes” already mentioned.

During each month, the Babylonian observers recorded when the moon and planets passed near to each other or near to normal stars. In a diary, the relative position of one celestial body to another may be described by one of the terms “above” (*e*), “below” (*šap*), “in front of” (*ina IGI*), or “behind” (*ar*). The terms “behind” and “in front of” are roughly synonymous with “to the east of” and “to the west of”, respectively, following the apparent rotation of the celestial sphere.

For the measurement of angles, such as the position of celestial bodies and magnitudes of eclipses (Stephenson & Fatoohi, 1994) the Babylonians used the units “finger” (*SI*) and “cubit” (*KUŠ*) which contained twenty four fingers in the Neo-Babylonian period (Sachs & Hunger, 1988: 22). It was previously suggested that the cubit was approximately equivalent to 2° (Kugler, 1909/10: 547-550; Neugebauer, 1955: 39; Sachs & Hunger, 1988: 22). However, a recent investigation of Babylonian measurements of close planetary conjunctions has shown that the cubit closely equalled 2.2° (Fatoohi & Stephenson, 1998). This last study has also shown that the Babylonians did not use horizon coordinates (altitude and azimuth), but there was little evidence to determine whether ecliptical or equatorial coordinates were used. However, because of the Babylonians’ introduction of the concept of the zodiac around B.C. 400 it appears more reasonable that the Babylonian astronomers used an ecliptical system.

For the measurement of time intervals shorter than a day, such as the durations of the phases of an eclipse (Stephenson & Fatoohi, 1993) the Babylonians used the unit *uš*. According to Neugebauer, “The “degree” (*uš*) is the fundamental unit for the measurement not only of arcs, especially for the longitude, but also for the measurement of time, corresponding to our modern use of right ascension. Therefore, 1 degree = 4 minutes of time” (Neugebauer, 1955: 39). Accordingly, Sachs and Hunger, who translate *uš* as “time degree”, have converted all measurements in *uš* in the diaries, especially those of the Lunar Sixes, into time-degrees. Professor Stephenson and I have confirmed, through the investigation of Babylonian records of lunar eclipse durations, that the modern equivalence of the *uš* is accurately 4 minutes and have shown that the definition of this unit showed no variations over the centuries covered by the Late Babylonian astronomical texts (Stephenson & Fatoohi, 1994).

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introduced by Hunger in substitution of the very brief original cuneiform text.



## 2.2 Conversion of Dates from the Babylonian to the Julian Calendar

The Babylonians used a luni-solar calendar whose month began on the evening of the first visibility of the lunar crescent. Because the Babylonians considered the month to consist of either 29 or 30 days (“hollow” or “full” month respectively), the new month was begun after the thirtieth day of the previous month if the new crescent was not seen. By the fifth century B.C. the Babylonians were able to predict the beginning of the month by computation of the expected first visibility of the lunar crescent (Britton & Walker, 1996: 45). Such calculation was used whenever the young moon was obscured by clouds or mist.

As the lunar year (of 12 months) is some eleven days shorter than the solar year, the Babylonians resorted to intercalation in order to keep the lunar months in line with the seasons so that the first month *Nisanu* would not fall much away from the spring of the year, i.e. between late March and early April. This was done by adding a thirteenth month to some years. In the third millennium B.C., the choice of the intercalary month was arbitrary, but from the early second millennium B.C. the intercalary month was added either after the sixth month of *Ululu* in the form of a second *Ululu* or after the twelfth month *Addaru* as a second *Addaru*.

Before 500 B.C. intercalation was irregular, but after this date the Babylonians recognised that 235 lunar months (about 6939.688 days) have almost the same number of days as 19 solar years (about 6939.605 days), almost a century before the Greek astronomer Meton of Athens who is often credited with the discovery of this cycle in 432 B.C (Britton & Walker, 1996: 52). In the first quarter of the fourth century B.C., the Babylonians embodied their recognition of the “Metonic cycle” in their calendrical calculations, thus systematically adding seven lunar months over each nineteen-years period.

Sachs and Hunger have given the Julian year and Babylonian lunar month in each of the published astronomical diaries. In order to use the data of the diaries in the present study, the dates of the observations had first to be converted into Julian dates. This could have been achieved using the specially prepared tables of Parker and Dubberstein (1956) which cover the period 626 B.C. to A.D. 75. However, the use of these manual tables would not be very practical when a large number of data is involved. Therefore, I used only the intercalary scheme from these tables, i.e. the recorded positions of the thirteenth months. I then integrated this scheme in a specially designed Fortran program that reads in the Babylonian date and converts it to its Julian equivalent, totally independent of the tables and more accurate (see

comments in §3.4). Dr Raymond Mercier has informed me that he has written a commercial program that converts Babylonian into Julian dates. However, to the best of my knowledge, program BABYLON0.FOR which I have designed is the first of its kind to be published and put in the public domain (Fatoohi, 1998).

I used the lunar visibility criterion suggested by Schoch (1928) to determine the expected dates of first visibility of the crescents. The use of a specific lunar visibility criterion for this purpose is of no critical importance because the converted dates, whether found manually by tables or by the program, could be considered only a first approximation anyway. The reason is that the date of actual observation of the crescent in any given month, which is the date that really matters for the purpose of this study, is not necessarily the same as that predicted by any theoretical calculation. For instance, a crescent that in theory should have been easily spotted could have set unseen due to unfavourable weather and its actual first visibility could have occurred the next evening. Therefore, in each instance the calculated date of first visibility must be checked against real observational data - usually in the form of time or positional measurement from the month under consideration (see §2.4). In this way, one can be sure whether the theoretically calculated date is exact or in need of amendment. In practice, such amendments never exceeded a single day, but even such a seemingly small discrepancy is crucial to account for for the purpose of crescent visibility studies.

## 2.3 The Babylonian Data

I have thoroughly scanned the Sachs and Hunger's three volumes (Sachs & Hunger, 1988, 1989, 1996) and compiled a list of dates of Julian years and Babylonian months in which the moon was first *sighted*. This is not simply a list of each year and month cited in the extant diaries because, as already mentioned, the Babylonians did not depend solely on observation when determining the first day of the month, though this seems to have been the practice in ideal weather. The Babylonian astronomers did use mathematical methods for determining the first day of the month, at least when visibility of the lunar crescent was prevented by unfavourable weather conditions. Since my purpose was to collect dates of actual observations rather than predictions of first visibility of lunar crescents I have selected only the entries that contain explicit statements confirming that the moon was indeed sighted. Terms and phrases used by the Babylonians to indicate actual sighting of the moon include "visible", "seen", "first



appearance”, and “earthshine”. Descriptions of the position of the moon or its brightness, such as “low”, “could be seen”, “was low to the sun”, “faint” and “bright”, are also indications of actual observations. Below are examples from different years of reported first sightings of the lunar crescent:

Month V, (the 1st of which was identical with) the 30th (of the preceding month), first appearance of the moon; sunset to moonset: 12°; the moon was 2 cubits in front of Mercury (Sachs & Hunger, 1988: 115). [Julian date is B.C. 373 July 23]

[Month V,] the 1st (of which followed the 30th of the preceding month), sunset to moonset: 15.5°; the moon was 1.66 cubits in front of  $\alpha$  Virginis (Sachs & Hunger, 1988: 167). [Julian date is B.C. 334 August 12]

Month IX, the 1st (of which followed the 30th of the preceding month), sunset to moonset: 15°, measured; the moon stood 1.5 cubits in front of Mercury to the west (Sachs & Hunger, 1988: 341). [Julian date is B.C. 274 December 4]

Month IX, (the 1st of which was identical with) the 30th (of the preceding month), sunset to moonset: 17.5°; it was bright, earthshine, measured; it was low to the sun (Sachs & Hunger, 1989: 205). [Julian date is B.C. 204 December 10]

[Month V, (the 1st of which was identical with) the 30th (of the preceding month), sunset to moonset: [nn°]; it was faint, it was low to the sun; (the moon) [stood] 3 cubits in front of Mars, 5 cubits in front of Saturn to the west (Sachs & Hunger, 1989: 451). [Julian date is B.C. 171 August 9]

In order to confine myself to actual sighting of the lunar crescent, I have excluded all entries where the text contained explicit statements and terms implying invisibility of the moon, such as “I did not watch”, “I did not see the moon”, “overcast”, “mist”, and “clouds”. I have also ruled out all entries in which the moonset lagtime or interval between sunset and moonset (*na*) is said to have been predicted as this might well be due to the fact that the moon was not seen. As an essential measure of extra caution, I have discounted any entry that does not contain an explicit statement that the moon was seen, even if it does not contain any explicit or implicit indication to the contrary. Accordingly, the final list of acceptable entries, though numbering as many as 209 in total, was unavoidably only a small part of the original material. The following



are examples of the kinds of entries that have been discarded for one or more of the reasons mentioned above:

[Month XI, (the 1st of which was identical with) the 30th (of the preceding month),] sunset to moonset:  $14^{\circ}$ ; there were dense clouds, so that I did not see the moon (Sachs & Hunger, 1988: 59). [Julian date is B.C. 453 February 12]

Month VIII, the 1st (of which followed the 30th of the preceding month), sunset to moonset:  $18.5^{\circ}$ . Night of the 1st, clouds crossed the sky (Sachs & Hunger, 1988: 351). [Julian date is B.C. 271 November 2]

Month II, (the 1st of which was identical with) the 30th (of the preceding month, sunset to moonset):  $13^{\circ}$ ; dense clouds, I did not watch. Night of the 1st, [clouds] crossed the sky (Sachs & Hunger, 1989: 19). [Julian date is B.C. 256 April 23]

[Diaries from month VII to the end] of month XII, year 113, which is the year 177, king Arsaces. Month VII, the 1st (of which followed the 30th of the preceding month), sunset to moonset:  $11.5^{\circ}$ ; mist [...] (Sachs & Hunger, 1996: 193). [Julian date is B.C. 135 September 30]

## **2.4 Determination of the Julian Date of First Visibility of the Lunar Crescent for Babylonian Observations**

Having collected all reliable dates of first sightings of the moon after conjunction, I made a preliminary conversion of all dates to their Julian equivalent using program BABYLON0.FOR. The calculated dates, however, would not necessarily coincide with the real dates of observation, but could be a day in error - as explained above. In order to determine the precise date of each observation, I made use of a recorded astronomical event in the same month, the date of which could be determined exactly. In 136 of the 209 entries that I compiled, the measured moonset lagtime is given; since the lagtime changes from one day to another by an average of 54 minutes (some  $13.5^{\circ}$ ) then this quantity could be used to determine the exact date of first sighting of the lunar crescent. The following are two different explanatory examples:

Month III, (the 1st of which was identical with) the 30th (of the preceding month), the moon became visible behind Cancer; it (i.e. the crescent) was thick; sunset to moonset: 20° (Sachs & Hunger, 1988: 49).

This observation is from year -567. According to BABYLON0.FOR, the Julian date of this event is -567 June 20. From my further computations (see comments on program SUNMOON0.FOR in §3.2), the moonset lagtime on that day was 89 minute, i.e. 22.25 time-degrees, which is close to that given in the Babylonian text; hence B.C. 568 June 20 is confirmed to be the exact Julian date of observation.

Month III, (the 1st of which was identical with) the 30th (of the preceding month), sunset to moonset: 12° 40'; measured (despite) mist (Sachs & Hunger, 1988: 251).

This entry belongs to year -301. The Julian date of this event according to BABYLON0.FOR is B.C. 302 June 18. However, the computed moonset lagtime on that date is -18 minutes, i.e. -4.5° time-degrees, with the negative sign indicating that the sun-moon conjunction had not even taken place. Therefore, the exact date of observation was in fact the next day, i.e. B.C. 302 June 19. On this latter date the lagtime was 51 minutes, i.e. 12.75° time-degrees - almost exactly the same quantity as measured by the Babylonians. The results of checked computation of the Julian dates of the Babylonian dates where the lagtime is given are shown in table 2.1, which gives in order the following information:

Columns 1-2: the Julian year and the Babylonian month of the observation as given in the translated diaries.

Columns 3-5: the Julian year, month, and day of the first day of the above month as computed by BABYLON0.FOR, including any necessary correction based on the comparison between the computed and the textual moonset lagtime; i.e. this is the finalised date.

Column 6: the correction to the preliminary date of the event according to BABYLON0.FOR; +1 indicates that the real date of observation was found to be one day later than the computed date, and -1 indicates that the date of observation is one day before the computed date; a blank cell indicates that the date of observation is the same as the computed date. Whether the date computed by BABYLON0.FOR had to be altered or not was determined by comparison of the observed and computed lagtime in the following columns.

Column 7: the observed moonset lagtime (*na*) in time-degrees, i.e. as given in the astronomical diaries.



**Table 2.1** The results of dating Babylonian observations for which the measured time from sunset to moonset is recorded.

1	2	3	4	5	6	7	8	9
-567	3	-567	6	20		20	22.25	-2.25
-567	11	-566	2	12		14.5	17.25	-2.75
-567	12	-566	3	14		25	25.75	-0.75
-418	8	-418	10	20		20	18.75	1.25
-381	4	-381	7	4		27	23.25	3.75
-378	8	-378	10	27		14.5	14.75	-0.25
-375	11	-374	1	20		15	14.75	0.25
-375	13	-374	3	20		14	19.75	-5.75
-373	12	-372	2	27		15.66	14	1.66
-372	5	-372	7	23		12	10.5	1.5
-370	5	-370	8	1		14.5	11.75	2.75
-370	8	-370	10	28		13.5	13	0.5
-368	4	-368	7	10		14.5	12.25	2.25
-366	3	-366	6	18		15	13.75	1.25
-366	5	-366	8	17		19	17	2
-346	9	-346	12	2		20	20.75	-0.75
-346	12	-345	3	1		22.5	23.5	-1
-342	10	-342	12	17		18	18	0
-333	3	-333	6	14		13.5	14	-0.5
-333	5	-333	8	13		15.5	16.5	-1
-328	7	-328	10	13	-1	16	11.75	4.25
-328	9	-328	12	12		15	18	-3
-324	1	-324	4	6		20	21.25	-1.25
-324	5	-324	8	2		15	18.25	-3.25
-324	7	-324	9	30		14	16	-2
-322	9	-322	12	7		16	21.75	-5.75
-322	10	-321	1	5		16	19.5	-3.5
-321	1	-321	4	3		26	19.75	6.25
-321	5	-321	7	30		15	16.25	-1.25
-321	6	-321	8	29		12	13.5	-1.5
-307	4	-307	6	26		16.5	18.5	-2
-307	6	-307	8	24		18	19.25	-1.25
-302	6	-302	8	28		11	11	0
-302	8	-302	10	27		11.33	10.75	0.58
-302	9	-302	11	26		17.66	17	0.66
-302	10	-302	12	26		27	28	-1
-302	11	-301	1	24		21	21.75	-0.75
-301	3	-301	6	19	-1	12.66	12.75	-0.09
-294	11	-293	1	25		16	17.25	-1.25
-291	2	-291	5	1		18	17.25	0.75
-291	6	-291	8	26	-1	12.5	10.25	2.25
-289	3	-289	6	8		22	20.25	1.75
-283	8	-283	10	27		17	17.25	-0.25
-281	8	-281	11	4		19.5	18.5	1

1	2	3	4	5	6	7	8	9
-277	2	-277	4	26		19	19	0
-277	3	-277	5	26		25	24	1
-273	9	-273	12	4		15	15.75	-0.75
-266	8	-266	10	19		13	13.25	-0.25
-266	9	-266	11	18		17	17.5	-0.5
-264	7	-264	9	26	-1	9.5	7.25	2.25
-255	1	-255	3	25		22	22	0
-255	62	-255	9	17		12	13	-1
-251	7	-251	10	3		13	13.5	-0.5
-251	12	-250	2	28		25.5	26.25	-0.75
-247	11	-246	1	15		12.5	15	-2.5
-246	1	-246	4	14		18.5	17.5	1
-246	2	-246	5	14		28	28.75	-0.75
-246	7	-246	10	8		13.66	13.5	0.16
-245	4	-245	7	1		20	21	-1
-237	5	-237	8	1		15	15	0
-234	7	-234	9	26		20	18.75	1.25
-234	9	-234	11	24		19	23.25	-4.25
-234	12	-233	2	20		20	21	-1
-232	7	-232	10	3		15.5	17.5	-2
-232	12	-231	2	28		23	25.25	-2.25
-226	11	-225	1	23		20	23.5	-3.5
-218	8	-218	10	28		18	19.5	-1.5
-218	12	-217	2	23		21	21.5	-0.5
-203	9	-203	12	10		17.5	18.5	-1
-201	10	-201	12	18		17	20.25	-3.25
-201	13	-200	3	16		19	18.5	0.5
-198	4	-198	6	21		18	18.25	-0.25
-198	12	-197	3	14		25	26.25	-1.25
-197	8	-197	11	5		16	15	1
-197	11	-196	2	2		19	21.25	-2.25
-195	8	-195	11	12		12.44	14.75	-2.31
-195	10	-194	1	11		17.5	22.5	-5
-194	3	-194	6	7		19	22	-3
-194	7	-194	10	3		17	18.75	-1.75
-193	2	-193	4	28		15	16.25	-1.25
-193	3	-193	5	28		24	27.75	-3.75
-192	6	-192	9	11		16	16.25	-0.25
-190	1	-190	3	26		24	25.5	-1.5
-188	1	-188	4	2		20	22.5	-2.5
-187	7	-187	10	16		15	15	0
-187	8	-187	11	14		10	12.25	-2.25
-186	12	-185	3	2		22	23.5	-1.5
-183	2	-183	5	7		19	19.75	-0.75
-183	5	-183	8	4		17	16	1
-183	8	-183	10	31		22	22.5	-0.5
-182	12	-181	2	15		21	21.5	-0.5

1	2	3	4	5	6	7	8	9
-178	5	-178	8	8		21.5	20.25	1.25
-178	6	-178	9	6		15.5	14.5	1
-176	7	-176	10	13		18	16.5	1.5
-175	2	-175	5	9		24	27.5	-3.5
-175	9	-175	12	1		18	22.5	-4.5
-173	9	-173	12	9		12	14.25	-2.25
-170	7	-170	10	8		9	9	0
-169	2	-169	5	2		16	18.75	-2.75
-168	5	-168	8	17		12	14	-2
-168	9	-168	12	13		14	17.75	-3.75
-164	3	-164	6	5		14	16.5	-2.5
-163	2	-163	4	25		13	14	-1
-163	3	-163	5	26	+1	26	29.25	-3.25
-162	5	-162	8	11		10	11.25	-1.25
-162	6	-162	9	10		13.5	14.75	-1.25
-161	7	-161	9	29		20	19.5	0.5
-158	4	-158	6	29		23	25	-2
-158	6	-158	8	26		12	14.5	-2.5
-155	11	-154	1	18		18	23.25	-5.25
-152	13	-151	3	15		19	21.5	-2.5
-149	8	-149	11	14		15	17	-2
-144	7	-144	9	21	+1	18	20.75	-2.75
-144	8	-144	10	20		15	18	-3
-144	9	-144	11	18		13	14.5	-1.5
-142	9	-142	11	26		17	16	1
-140	4	-140	7	9		17	19	-2
-134	8	-134	10	30		12	14	-2
-134	12	-133	2	25		21	21.75	-0.75
-133	6	-133	8	20		11	10.75	0.25
-133	7	-133	9	19		11	10.25	0.75
-133	8	-133	10	19		12	13.25	-1.25
-133	13	-132	3	15		22	22.5	-0.5
-132	7	-132	10	7		11.5	11.25	0.25
-131	8	-131	10	26		16	15.25	0.75
-123	3	-123	6	2		17	18.75	-1.75
-119	3	-119	6	17		14	17	-3
-112	13	-111	3	22		16.5	18.75	-2.25
-107	1	-107	4	7		24	21.75	2.25
-105	1	-105	4	16		22	25.5	-3.5
-105	6	-105	9	9		11	12	-1
-96	2	-96	5	5		23	24.75	-1.75
-87	5	-87	7	23		14	14.75	-0.75
-87	13	-86	3	17		22	24.75	-2.75
-77	3	-77	6	4		17	21.75	-4.75
-77	6	-77	9	1		17	15.25	1.75



Column 8: the computed lagtime which I calculated using program SUNMOON0.FOR and converted into time-degrees.

Column 9: the difference between the computed and observed lagtime in time-degrees. When this difference was large I changed the computed date either to the next or previous day, whichever gave smaller, acceptable difference; this is the date given in columns 3-5.

Table 2.1 shows that there are 9 entries where the difference between the measured and the computed lagtime was more than  $4^\circ$ , i.e. more than 16 minutes of time. The difference could well be due to inaccurate measurement of the lagtime - a difficult quantity to measure owing to the marginal visibility of the moon - or scribal error in the original text and does not necessarily indicate an error in the date. Measurement of the *na* interval would be a difficult task since often the young crescent moon can only be seen for a short time about midway between sunset and moonset. However, as a measure of caution, I re-checked these entries using additional data from the text. For this purpose, I used observations of lunar east-west separation (i.e. when the moon is “behind”, “east”, “in front of”, or “west”) from a star or planet recorded during the same lunar month. Because the moon traverses about  $13^\circ$  every day, the date of any reported lunar conjunction in the month can be exactly determined, and this date can be used as a reference for verifying the date of the first day of the month, i.e. the date of the observation. However, if the text did not mention the horizontal separation I used the north-south separation (i.e. when the moon is “above” or “below”) because the latter would be given only when the moon was horizontally close to the planet or star.

The results are given in table 2.2 which contains the following information:

Columns 1-3: the Julian year and the Babylonian month and day of the observation as given in the translated diaries.

Column 4: the time of the event in the night (when specified in the text).

Column 5: the reported separation between the moon and the planet or star in degrees after converting the original measurements expressed in cubits or fingers by taking the cubit to be equal to  $2.2^\circ$  and the finger  $0.092^\circ$  (Fatoohi & Stephenson, 1998).

Column 6: the position of the moon with respect to the planet or star.

Column 7: the name of the planet or star whose separation from the moon is given.

Columns 8-10: the Julian year, month, and day of the event as computed by BABYLON0.FOR, including any necessary correction based on the comparison between the computed and the textual lunar-planetary/stellar separation.

Column 11: the local time of the lunar conjunction (in hours and decimals). This was taken to be the time of sunset when the observation is quoted to have occurred in the “beginning” or “first part” of the night, and the time of sunrise when the observation had occurred in the “last part” of the night. Since the purpose is only to determine the approximate and not exact distance between the moon and planet or star, then the time of sunset or sunrise can be taken without modification as the time of the observation. In four cases the time of observation was missing, yet it is obviously vital to know whether it was the “beginning or first part of the night” or “last part of the night”. However, because of the rapid motion of the moon only one of these times would be compatible with both the recorded direction and amount of separation between the moon and the star/planet. I have found that in the four cases the observation must have been made in the beginning of the night, and in each case there is good agreement between the recorded description and calculation.

**Table 2.2** Checking of the dates of the observations whose measured lagtime differs from computation by more than 4 degrees.

1	2	3	4	5	6	7	8	9	10	11	12	13	14
-375	13	2	-	0.9	front	Eta Tauri	-374	3	21	18.0		-1.0	1.0
-328	7	7	beginning	3.7	above	Delta Capricorni	-328	10	20	17.7		0.1	4.4
-322	9	22	last	3.3	behind	Alpha Librae	-322	12	29	7.0		3.4	-0.5
-321	1	5	beginning	1.5	front	Beta Geminorum	-321	4	7	18.3		-1.4	-1.6
-234	9	1	-	-	above	Jupiter	-234	11	24	17.2		-1.5	3.0
-195	10	7	beginning	3.3	front	Eta Tauri	-194	1	17	17.1		-4.2	-8.1
-175	9	1	-	2.2	front	Beta Capricorni	-175	12	1	17.1		-2.0	-8.0
-155	11	8	-	4.4	front	Beta Tauri	-154	1	25	17.2		-2.4	-3.1
-77	3	4	beginning	4.4	behind	Alpha Leonis	-77	6	7	19.1		5.4	1.5

Column 12: the correction to the date of the event according to BABYLON0.FOR. None of the computed dates needed correction.

Column 13: the difference in longitude (in degrees) between the moon and the planet or star.

Column 14: the difference in latitude (in degrees) between the moon and the planet or star. The longitude and latitude of the moon have been computed using program



SUNMOON0.FOR, whereas the coordinates of planets and stars were computed using VSOP870.FOR (see comments in §3.3) and STELLAR0.FOR, respectively. The latter is based on *Sky Catalogue 2000.0* (Hirshfeld & Sinnott, 1982).

Comparison between the lunar-planet/stellar separation as reported by the Babylonian astronomers (column 5) and the computed separation (either column 13 or 14, depending on the direction given in column 6) for each observation shows that there is only a small difference in each case. This confirms that the computed dates are correct.

In the other 73 of the compiled 209 entries, the observed lagtime was missing, mainly because the text is broken away. In this case, I used other astronomical data from the same month to verify the date, exactly as in table 2.2. The results are given in table 2.3 whose columns give the same information of table 2.2. Only for the eighth month of year -164 no lunar conjunction is reported, but the text states that on “the 12th, moonset to sunrise 12°” (Sachs & Hunger, 1989: 497), which could be used instead. I found that the computed date of -164 December 11 requires no correction as the computed value of that “Lunar Six” was 12.3°, and accordingly the correct date of the first day of the month is -164 October 31, as calculated by BABYLON0.FOR.

Thus, tables 2.1, 2.2 and 2.3 contain the exact Julian dates of the 209 Babylonian observations of the lunar crescent mentioned in the astronomical diaries.

## 2.5 The Medieval Data

Many medieval Arabic books do mention the Julian dates of the beginnings of months of different years or give clues, such as naming the corresponding weekdays, that would enable the calculation of these dates. However, such dates cannot be assumed to be based on reliable observations of the lunar crescent. For example, a month could have begun without seeing the crescent because of unfavourable weather conditions. Additionally, as will be later explained, untrue claims of observations of the new moon have been made and are still being made by people for a number of reasons.

I have found, however, a reliable source of observations of the lunar crescent. This is the Arabic classical book “The travels of ibn Jubayr” which contains the diaries of the Andalusian traveller Muhammad ibn Jubayr during his journey of pilgrimage that took him from his



**Table 2.3** Checking of the dates of the observations whose measured lagtime is unavailable in the text.

1	2	3	4	5	6	7	8	9	10	11	12	13	14
-567	1	8	beginning	2.2	front	Beta Virginis	-567	4	29	18.6		-2.6	3.3
-567	2	1	-	8.8	below	Beta Geminorum	-567	5	22	18.9		0.8	-7.1
-381	2	3	-	-	behind	Saturn	-381	5	8	18.8		6.4	0.1
-372	1	12	first	4.4	behind	Alpha Virginis	-372	4	8	19.3		3.5	-3.0
-332	7	6	beginning	4.4	behind	Beta Capricorni	-332	10	4	18.0		5.9	-0.2
-324	4	7	beginning	4.4	front	Alpha Librae	-324	7	9	19.2	-1	-4.8	3.8
-322	11	16	first	2.2	behind	Gamma Virginis	-321	2	18	18.5		0.6	-0.2
-302	5	22	last	0.7	below	Beta Tauri	-302	8	20	5.3	-1	0.8	-0.9
-294	2	1	-	-	front	η Geminorum	-294	5	4	18.7		-1.4	-1.6
-291	4	3	beginning	1.1	behind	Beta Virginis	-291	7	1	19.2		0.5	-0.7
-286	3	11	beginning	-	-	Beta Scorpii	-286	6	14	19.2		-0.7	0.6
-284	8	10	beginning	4.4	front	Eta Piscium	-284	11	15	17.3	-1	-5.5	-6.7
-277	1	9	beginning	4.4	behind	Rho Leonis	-277	4	5	18.3		2.0	-4.8
-249	5	16	last	4.4	front	Eta Piscium	-249	8	29	5.4		-0.8	-9.4
-245	2	1	-	8.8	front	Eta Geminorum	-245	5	3	18.7		-7.1	6.1
-237	4	3	-	7.7	front	Gamma Virginis	-237	7	5	19.2		-6.4	-5.4
-234	13	6	beginning	1.1	west	Alpha Geminorum	-233	3	26	18.1		-2.7	-12.8
-210	4	3	-	2.2	behind	Beta Virginis	-210	7	6	19.2		3.4	3.3
-209	3	10	beginning	5.5	front	Alpha Librae	-209	6	2	19.1		-7.2	-0.9
-207	2	9	beginning	2.2	front	Beta Virginis	-207	5	9	18.8	-1	-2.9	-0.5
-207	1	6	beginning	3.3	behind	Beta Geminorum	-207	4	7	18.3		4.6	-2.0
-197	7	6	beginning	1.5	behind	Beta Capricorni	-197	10	12	17.8		1.7	-2.4
-193	8	6	beginning	2.2	behind	Delta Capricorni	-193	10	27	17.6		2.8	-2.8
-193	11	16	last	3.3	behind	Gamma Virginis	-192	2	4	6.7		3.3	1.2
-193	12	17	last	3.3	behind	Beta Librae	-192	3	5	6.2		2.7	-7.4
-193	13	4	beginning	4.4	front	Beta Tauri	-192	3	21	18.0		-3.3	-4.0
-190	3	4	beginning	2.2	behind	Epsilon Leonis	-190	5	27	19.0		2.7	-5.0
-189	7	5	beginning	3.3	behind	Saturn	-189	10	13	17.8		5.3	-4.4
-189	8	12	beginning	-	front	Alpha Tauri	-189	11	18	17.3		-4.3	10.3
-179	1	13	beginning	2.2	behind	Mars	-179	4	5	18.3		1.7	-0.1
-179	5	7	-	4.4	front	Beta Scorpii	-179	7	26	19.0		-6.4	3.6
-178	7	3	-	4.4	above	Theta Ophiuchi	-178	10	8	17.9		1.6	6.0
-176	6	24	last	-	front	Rho Leonis	-176	10	7	6.1		-2.0	3.7
-173	8	1	-	4.4	behind	Mercury	-173	11	10	17.4		3.8	-2.2
-173	11	1	beginning	-	behind	Jupiter	-172	2	6	17.4		6.4	-2.0
-170	5	19	last	3.3	behind	Eta Tauri	-170	8	28	5.4		2.7	1.1
-170	8	3	beginning	4.4	front	Beta Capricorni	-170	11	9	17.4		-3.3	-8.6
-170	11	16	last	2.2	behind	Alpha Virginis	-169	2	19	6.5		3.5	-1.7
-163	9	17	last	3.3	front	Alpha Leonis	-163	12	6	6.9		-1.1	-4.9
-163	13	7	beginning	1.5	behind	Beta Geminorum	-162	3	22	18.0		3.8	-11.8
-156	9	3	-	3.3	front	Saturn	-156	12	3	17.1		-6.0	-3.5
-146	10	21	last	4.4	front	Delta Scorpii	-145	1	30	6.8		-4.3	1.5
-146	11	1	-	2.2	behind	Venus	-145	2	7	17.4		3.0	5.7
-143	5	5	beginning	3.3	front	Beta Scorpii	-143	8	15	18.8		-4.3	2.6

1	2	3	4	5	6	7	8	9	10	11	12	13	14
-143	6	4	beginning	2.2	front	Alpha Scorpii	-143	9	12	18.3		-3.0	8.7
-143	7	23	last	5.5	front	Gamma Virginis	-143	11	1	6.5		-3.6	-3.2
-141	3	1	-	13.2	below	Alpha Geminorum	-141	5	23	19.0		1.1	-13.5
-141	7	7	beginning	2.2	behind	Delta Capricorni	-141	10	23	17.6		3.2	2.5
-140	1	7	beginning	0.6	behind	Alpha Leonis	-140	4	18	18.5		0.3	0.6
-140	9	17	last	3.3	behind	Epsilon Leonis	-140	12	20	7.0		4.3	-7.9
-140	11	6	beginning	3.3	behind	Eta Tauri	-139	2	6	17.4		0.7	-8.5
-137	10	23	last	3.3	front	Theta Ophiuchi	-136	1	23	6.8		-4.7	0.7
-136	7	2	-	3.3	front	Delta Scorpii	-136	9	23	18.1		-2.9	1.1
-129	4	8	beginning	4.4	behind	Alpha Scorpii	-129	7	16	19.1		3.2	1.6
-124	9	16	last	2.2	behind	Alpha Leonis	-124	12	23	7.0		4.6	-2.2
-124	11	8	beginning	4.4	behind	Zeta Tauri	-123	2	11	17.4		4.4	-2.8
-119	1	1	-	2.2	front	Saturn	-119	4	19	18.5		-1.6	0.0
-118	2	1	-	2.2	front	Mu Geminorum	-118	5	8	18.8		-0.1	3.0
-117	8	4	beginning	5.5	behind	Mars	-117	10	25	17.6		5.7	-3.5
-111	3	6	beginning	-	behind	Gamma Virginis	-111	6	24	19		2.6	-8.1
-111	5	1	-	4.4	front	Alpha Virginis	-111	8	18	19		-4.1	-3.2
-105	2	5	beginning	2.2	front	Rho Leonis	-105	5	19	19		-0.1	-2.2
-105	3	3	-	4.4	front	Venus	-105	6	15	19		-0.7	-3.9
-105	7	13	beginning	2.2	front	Beta Arietis	-105	10	21	18		-1.2	-11.6
-104	6	4	beginning	2.2	behind	Beta Scorpii	-104	9	1	18		3.4	4.0
-95	3	1	-	3.3	front	Alpha Geminorum	-95	5	24	19		-1.4	-5.7
-87	7	26	last	4.4	front	Alpha Virginis	-87	10	16	6.3		-4.9	3.1
-86	8	10	beginning	3.3	front	Eta Piscium	-86	11	16	17		-1.3	-8.6
-83	4	5	beginning	4.4	front	Alpha Virginis	-83	7	13	19		-4.4	7.1
-77	5	3	-	1.1	front	Alpha Virginis	-77	8	4	19		0.3	-0.7
-77	8	12	beginning	5.5	front	Eta Tauri	-77	11	10	17		-5.6	0.7
-73	4	3	-	-	below	Gamma Virginis	-73	7	21	19		0.4	-8.1



home town of Granada to Macca and then back again to Granada. The journey lasted for just over two years from A.D. 1183 February to 1185 April.

Ibn Jubayr is very careful, elaborate and accurate in the details he gives in his book. This is particularly notable when he reports on observing the lunar crescent. There are a number of alleged sightings of the crescent which he goes to a great length in refuting (ibn Jubayr, 1907: 167-168). When reporting a sighting of the lunar crescent of a certain month, ibn Jubayr uses the cliché “Its crescent appeared on” followed by the name of the weekday and, except in one case, the corresponding Julian date. Therefore, there can be no ambiguity as for the Julian date of each observation. However, given that the Muslim calendar differs from the Julian calendar and yet resembles the Babylonian calendar in its practice of reckoning the day from sunset to sunset, I have found that ibn Jubayr is inconsistent in naming the weekdays. According to the Islamic tradition, the night precedes the morning and, therefore, for instance, while “Wednesday morning” would refer to the same period of the day in both calendars, “Wednesday night” in the Julian calendar corresponds to “Thursday night” in the Islamic calendar, and so on. In all of his reports of lunar observations, including the two eclipses mentioned below, ibn Jubayr follows the Julian convention when naming the weekdays (i.e. counting at each midnight). In some other instances, however, he follows the Islamic tradition of naming the weekday (e.g., pp. 35, 65).

The following two examples of the new moons of A.D. 1183 July 21 and 1184 February 13, respectively, show the accuracy with which ibn Jubayr reported his observations and explain the way with which I have dealt with his material:

[Month Rabi' ath-Thani of year 579 H.] Its crescent appeared on Saturday night when we were in that island. It did not appear to the eye on that night because of the clouds but it appeared in the second night large and high so we became sure that it had appeared on the night of the mentioned Saturday which is the twenty second of July (ibn Jubayr, 1907: 74).

In this case, I considered only the positive observation of Sunday night.

[Month Thu al-Qi'da of year 579 H.] Its crescent appeared on Wednesday night the fourteenth of February, according to a testimony on its sighting that the judge accepted. As for the majority of the people of al-Masjid al-Haram, they did not see anything and their waiting was extended until the time of the sunset prayer. There were among them those who would

imagine it and then point at it. But when they would try to verify it it would vanish from their sight and the news (about seeing it) turns out to be untrue (ibn Jubayr, 1907: 162).

I took this instance to indicate a negative observation on that particular date at al-Masjid al-Haram in Macca.

Additional checks of the general accuracy of ibn Jubayr’s accounts of lunar events can be made using his reports of two lunar eclipses. This is a translation of his account of the first eclipse:

[Month Safar of year 579 H.] Its crescent appeared on Wednesday night the fifteenth of May [A.D. 1183] when we were in Qus..... And on Wednesday which corresponded to the fifteenth of [the month], when we were in the mentioned place of al-Hajir, the moon totally eclipsed in the beginning of the night and continued until people had gone to sleep (ibn Jubayr, 1907: 65).

The date given by ibn Jubayr corresponds to 1183 July 6 A.D., for which  $\Delta T$  (see §3.1) would have been about 790 second. The quantities needed to assess the accuracy of ibn Jubayr’s report, computed for Qus (32.7 E, 26.1 N) where ibn Jubayr witnessed the eclipse, are as follows:

Local time of first contact and lunar altitude at that time:	20.33	16.9°
Local time of beginning of phase of totality and lunar altitude:	21.55	28.2°
Local time of end of phase of totality and lunar altitude:	22.49	34.7°
Local time of last contact and lunar altitude:	23.72	39.0°
Magnitude of maximum phase:	1.142	
Local time of sunset:	18.34	

Obviously, ibn Jubayr’s account is in satisfactory agreement with calculations.

Ibn Jubayr’s report of the second lunar eclipse may be translated as follows:

[Month Sha‘ban of year 579 H.] Its crescent appeared on Saturday night the nineteenth of November [A.D. 1183].... And on the dawn of Thursday the thirteenth [of the month], which corresponded to the first day of December, the moon was eclipsed after dawn. The eclipse began while people were performing the dawn prayer in the honourable Masjid [in Macca] and it set eclipsed. Two thirds of the moon was eclipsed (ibn Jubayr, 1907: 139).



This eclipse of A.D. 1183 December 1 was seen from Macca (39.8 E, 21 N) and hence we have the following calculations:

Local time of first contact and lunar altitude:	5.36     14.7°
Local time of last contact and lunar altitude:	8.24     -19.7°
Magnitude of maximum phase:	0.707
Local time of sunrise:	6.51
Local time of moonset:	6.64

The prayer of dawn could have started when the solar depression was about 18°, and this would have occurred about 5.25 a.m. It is clear, therefore, that the moon was “eclipsed after dawn” and that this happened “while people were performing the dawn prayer”. It is also true that the moon set eclipsed and that when it set it was some two thirds eclipsed. For this and the other reasons that I have already mentioned, I concluded that ibn Jubayr’s accounts of sightings of the lunar crescent are reliable.

I extracted from ibn Jubayr’s diaries twenty six observations two of which are negative observations of the lunar crescent. These observations are included in table 3.1.

**2.6 The Modern Data**

In addition to the Babylonian and medieval data, I have compiled observations of the lunar crescent from the modern astronomical literature. As in the case of the Babylonian data, these modern observations were made by experienced observers and have carefully been checked and published by Schaefer (1988, 1996) and Doggett and Schaefer (1994) who compiled them from a large number of publications as well as from Moonwatches that they organised. The original total number of the observations compiled by Schaefer and Doggett is 294 with 23 of them being observations of last visibility of old moon rather than first visibility of new moon. Since it is the case of new moon that concerns calendrical calculation, and given that Fotheringham and Maunder (BAA, 1911: 345, 347) have both indicated that observing new moon might be easier than observing the old moon, I have neglected the 23 morning observations. They are very few in comparison with the evening observations that I have compiled, and it would be safer to have a consistent set of observations.

One very important aspect of the remaining 271 modern observations is that they are not all positive sightings but 82 of them represent negative observations, i.e. unsuccessful attempts to

spot the new moon. Such negative observations, which are of exceptional importance in determining the limits of visibility, are unfortunately missing from the Babylonian collection. The Babylonian diaries do not state explicitly when the crescent was looked for but not seen despite good weather conditions, and this cannot be inferred indirectly. The total 506 Babylonian, medieval and modern observations are all included in their historical order in table 3.1. This is the largest set of observations that has been used in any modern study of the problem of predicting the first visibility of the lunar crescent.

## The Computations

I have designed four major programs, in Fortran, for computing various quantities needed for verifying the observational data and parameters required in the study of the earliest visibility of the moon. This chapter explains each of these programs and presents in a tabulated form, for each of the 506 observations of the lunar crescent, the calculated parameters that are most cited in the next chapters.

In addition to the above four main programs, I designed a number of other secondary programs that I used to prepare the binary data files employed by the main programs. The four main programs are very lengthy, making some 35 pages. Therefore, after consulting my supervisor Professor Stephenson, I have not included them in the thesis.

For computing the positions of the 31 “Normal stars” that the Babylonians used to observe, and whose conjunctions with the moon were needed for dates verification, I used program STELLAR0.FOR which is based on *Sky Catalogue 2000.0* (Hirshfeld & Sinnott, 1982).

### 3.1 Computation of the Dates of New Moons

Program HILAL0.FOR computes the nearest date of conjunction of the sun and moon for any given date. It computes the solar ecliptic coordinates using the VSOP82 (*Variations Séculaires des Orbites Planétaires*) planetary theory (Bretagnon, 1982; Bretagnon & Simon, 1986). Although this solution contains a relatively small number of periodic terms for the calculation of the solar longitude and radius vector, it yields an accuracy of 2.2" between years 0 and +2800 and of 3.2" between -4000 and +8000.

I computed the lunar ecliptic coordinates using the semi-analytical lunar ephemeris ELP2000-85 (Chapront-Touzé & Chapront, 1988) (ELP stands for *Ephémérides lunaires Parisiennes*, 2000 indicates that the epoch of reference is the year J2000 whereas 85 refers to the year of preparation of the ephemeris). This ephemeris has been built from the older version ELP2000-82 which is more precise and complete than the Improved Lunar Ephemeris (ILE) of Eckert, Jones and Clark (1954) and which introduces modern values of the lunar parameters



and other physical quantities (Chapront-Touzé and Chapront, 1991: iii). ELP2000-82 is also considered to be the most fully developed lunar theory and closest to numerical integrations (Cook, 1988: 162). The internal precision of ELP2000-85 is estimated to vary from 0.5" to about 10" over the time span (1500 B.C. - 2000 A.D.).

Rather than using the abridged version of ELP2000-85 given in the previous reference, I contacted one of the authors who kindly provided me with the complete solution on disk. I have converted the periodic terms of the theory, which are 218 for the longitude, 188 for the latitude, and 155 for the distance, into one binary file that is read in by the program when calculating the coordinates of the moon and its distance from the earth.

Although Chapront-Touzé and Chapront (1988) suggest that ELP2000-85 is valid over a time span of a few thousand years, using this theory for ancient times requires a significant modification. The ELP2000-85 solution assumes a value of  $-23.8946 \text{ arcsec/cy}^2$  for the tidal secular acceleration of the moon. In 1991 Chapront-Touzé and Chapront published lunar tables based on ELP2000-85 and, having referred to the fact that the lunar coordinates strongly depend on the adopted value of the tidal secular acceleration, they justified their adoption of the above value on the basis that it "does not significantly differ from the value [then] involved in most of the published ephemerides" (Chapront-Touzé and Chapront, 1991: 3). However, since 1992, lunar laser ranging (LLR) has yielded results for the lunar acceleration close to  $-26 \text{ arcsec/cy}^2$ , with the most recent result obtained by Dickey and collaborators being  $-25.88 \pm 0.5 \text{ arcsec/cy}^2$  (Dickey *et al*, 1994). More recently, Stephenson (1997) reported a personal communication in which J. L. Williams of the LLR team claims that consistent results for lunar acceleration are being found in the range  $-25.8$  to  $-26.0 \text{ arcsec/cy}^2$ .

Although the difference between these recent results and that assumed by ELP2000-85 may seem small, it does nevertheless accumulate significant errors over a long period as in the case of the Babylonian data. For instance, calculations based on a value of  $-23.8946 \text{ arcsec/cy}^2$  for the tidal secular acceleration would have the moon in the year 500 B.C. about  $0.2^\circ$  of longitude ahead of its position using the presently accepted value of  $-26 \text{ arcsec/cy}^2$ . The difference in latitude is less by about an order of magnitude. I have remedied this situation by using a special formula given in the *Astronomical Almanac* which accounts for the deficiency in the tidal secular acceleration of the moon by modifying the Julian date of the event so that the computed lunar coordinates are for a lunar acceleration of  $-26 \text{ arcsec/cy}^2$  (*Astronomical Almanac*, 1998: K8). Given  $N$  is the original tidal secular acceleration, the formula of the time in days that should be added to the Julian date is:  $(-0.000091 (N + 26) (\text{year}-1955)^2) / 86400$ . For example,



the correction for the year -500 is -1155 seconds, i.e. -0.0134 days.

In order for the calculations to be valid for an ancient epoch such as the Babylonian, it is even more necessary to make allowance for the cumulative effect of changes in the length of the day ( $\Delta T$ ) which results from variations in the earth's rate of rotation due to tides and other causes (Morrison & Stephenson, 1997). For example,  $\Delta T$  is estimated to have been as much as about 16800 seconds (4.66 hours) in the year -500 which corresponds to about  $2.5^\circ$  and  $0.2^\circ$  change in the lunar longitude and latitude, respectively. I have incorporated into HILAL0.FOR a subroutine that computes  $\Delta T$  using the values recently derived by Stephenson and Morrison (1995) from their analysis of historical records of astronomical events - mainly eclipses, including those from Babylon.

I have incorporated in the program a subroutine for converting the Julian day number into Julian/Gregorian date based on the algorithm given by Hatcher (1984). For the conversion of Julian/Gregorian date into Julian day number, I have used the formulae given by Muller (1975). The program also includes a subroutine for converting the terrestrial time into local time as it is more practical to use the latter in certain cases.

### 3.2 Computation of Solar, Lunar and Other Astronomical Parameters

I have designed the comprehensive program SUNMOON0.FOR for the purpose of calculating solar, lunar and other astronomical parameters needed for the study of first visibility of the lunar crescent. This program gives first the option of doing the calculations for a certain date either at sunset, at a given terrestrial time or at the "best time" of observation of the lunar crescent (this concept is explained in §7.8). The program can also very easily be adopted to do the calculations for any value of solar depression. After choosing one of the options, the program requires input data of the geographical longitude and latitude in degrees of the place of observation and its height in meters relative to sea level. Finally, the program asks for the Julian/Gregorian year, month and day of the event. If the second option is chosen, the required terrestrial time should also be fed in. I have used in SUNMOON0.FOR all necessary elements and subroutines that have been used in program HILAL0.FOR.

The program SUNMOON0.FOR calculates the following parameters for both of the moon and sun:

- (i) The *true geocentric* longitude, latitude, right ascension and declination (i.e. neglecting

nutration, solar aberration and the equatorial horizontal parallax ( $\pi$ )).

(ii) The *apparent geocentric* longitude, latitude, right ascension, declination, azimuth and altitude (i.e. allowing for nutration, as well as aberration in the case of the sun, but neglecting the equatorial horizontal parallax and refraction in altitude). I used the formulae of Meeus (1991) for computing the nutration in longitude and in obliquity and the expression given by Laskar (1986) for the calculation of the mean obliquity of the ecliptic.

For calculating refraction as a function of true altitude, I used the simple formula of Saemundsson (1986) which assumes that the observation is made at sea level, when the atmospheric pressure is 1010 millibars and when the temperature is 10° Celsius. There are two reasons for using this simplified formula. Firstly, it is not possible to know for most observations, certainly not for the ancient ones, the atmospheric pressure and temperature at the time of observation. Secondly, the inclusion of such calculations would be spurious given the limitations on the accuracy of determining the time of sunset (see below for details), for which calculations predicting the first visibility of the lunar crescent are often made.

(iii) The *topocentric* longitude, latitude, right ascension, declination and altitude (i.e. allowing for nutration, aberration of the sun, the equatorial horizontal parallax and refraction). I have neglected the parallax in azimuth because it is always very small. At the horizon, the parallax in azimuth is always less than  $\pi/300$ , where  $\pi$  the equatorial horizontal parallax of the body; i.e. it is less than about 12" even in the case of the moon.

(iv) The geocentric and topocentric semidiameter.

(v) The distance from earth (in Astronomical units for the sun and in kilometres for the moon).

(vi) The local times of rising and setting. Since coordinates' calculations are made for the centre of the celestial body, and given that the calculated rising and setting times refer to the upper limb of the disk, allowance must be for the effect of refraction and semidiameter. I used the generally adopted mean value of 34' for the effect of refraction at the horizon, and I used 16' for the semidiameter of the sun. Hence, sunset calculations are made for *true* solar depression of 0.83° plus the depression due to the elevation of the horizon above sea level which equals the square root of the height in meters multiplied by 0.0353°. In the case of the moon, the situation is more complicated because the moon's semidiameter changes with parallax (Yallop & Hohenkerk, 1992: 487).

Since most criteria of lunar visibility are designed for calculations of lunar and solar parameters made at sunset, it is important to stress the limitations inherent in these calculations



because of the uncertainty in determining the time of sunset itself. Sinnott (1989) deduced that a change of temperature from winter to summer of some  $25^{\circ}$  changes the time of sunset by about 20 seconds and that change in the air's index of refraction from 1.000284 to 1.000300 due to change in atmospheric pressure also affects the time of sunset by a dozen seconds. However, even these results were later found to be an underestimation. In an empirical study that was conducted from a number of different locations and throughout the year and which involved the measurement of refraction on the horizon at 144 different times of sunset, Schaefer and Liller (1990) have shown that the variation of refraction on the horizon can be substantially larger than has previously been realised. These authors have found from their measurements that the average refraction on the horizon is  $0.551^{\circ}$ , which is very close to generally adopted mean value of refraction of  $34'$  ( $0.567^{\circ}$ ) that I have used in my program. However, Schaefer and Liller also found that their measurements fluctuated from  $0.234^{\circ}$  to  $1.678^{\circ}$ . In fact, at the 95% confidence level, the total refraction varied over a range of  $0.64^{\circ}$ , meaning that times of sunrise and sunset can be predicted only with an accuracy of several minutes.

(vii) The width of the illuminated part of the disk of the moon.

(viii) Ultimately, the program computes the following key parameters: (a) the moonset lagtime, i.e. the difference between moonset and sunset; (b) the "arc of light" (elongation), which is the angular separation between the sun and moon; (c) the "arc of separation" (*Arcus Appartionis*), which is the separation in right ascension; (d) the "arc of descent" or "arc of vision" (*Arcus Visionis*), which designates the difference in altitude between the moon and the set sun (without parallax); and (e) the azimuthal difference. These and other angles for the time of sunset are shown in figure 3.1.

(ix) The program also computes the Julian day number,  $\Delta T$ , the equation of time and the terrestrial or local time of the event, whichever is needed.

### 3.3 Computation of Planetary Coordinates

In order to check the lunar conjunctions with planets in the case of some of the Babylonian data, I have designed program VSOP870.FOR. This program is based on the analytical theory VSOP87 (Bretagnon & Francou, 1988). This is in fact the same VSOP82 solution that was presented in elliptic variables (Bretagnon, 1982; Bretagnon & Simon, 1986), but the new

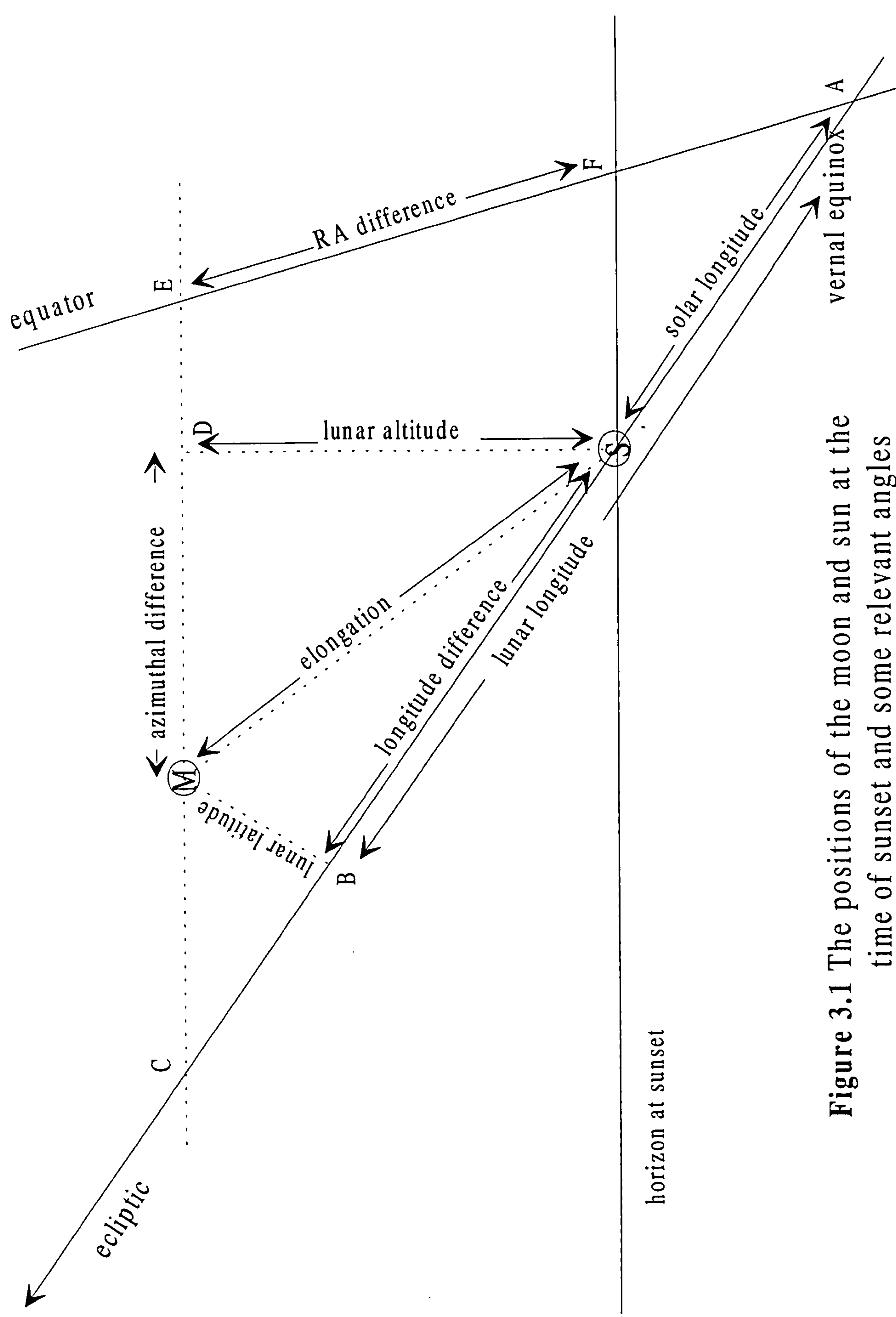


Figure 3.1 The positions of the moon and sun at the time of sunset and some relevant angles

version is in spherical variables (longitude, latitudes and radius vector) which are more convenient for calculating planetary positions. But the main aspect of superiority of VSOP87 to VSOP82 concerns the control of precision of calculations. One drawback in VSOP82 is that it is not possible to determine where to truncate its several series to achieve a selected degree of precision, while in the case of the VSOP87 solution, whose terms count in tens of thousands, the degree of precision of calculations can be computed by the amazingly simple formula  $Z(n)^{1/2}A$ , where  $n$ ,  $A$  and  $Z$  are the number of retained terms, the amplitude of the smallest retained number and  $Z$  a number smaller than 2, respectively. The substitution of  $Z$  with 2 yields the greatest possible error in the heliocentric longitude.

I have downloaded from the official anonymous FTP site on the internet of the Bureau des Longitudes, France, the periodic terms of version D of VSOP87, i.e. VSOP87D, which is expressed in heliocentric variables longitude, latitude and distance and reckoned to mean ecliptic and equinox of date. I have converted the periodic terms into a binary file that is read by the program. In addition to asking for the date and local time of calculations, the program also requires input of the precision with which the calculations are carried out in order to decide the cut-off limit of the terms to be included in the calculations. I used in my calculations a precision of 1" which is far beyond the accuracy required, as these calculations were merely to verify dates of lunar conjunctions. I have also used in VSOP870.FOR the relevant subroutines from HILAL0.FOR and SUNMOON0.FOR.

### 3.4 Conversion of the Babylonian Dates into Julian Dates

I designed program BABYLON0.FOR to convert the Babylonian dates into their Julian equivalents. It uses an external binary file which contains a list of the Julian day numbers of the *provisional* dates of first visibility of every crescent during the period B.C. 626 - A.D. 75, which is the interval covered by the tables of Parker and Dubberstein (1956), and performs the conversion using pure mathematical procedure. The *expected* Julian day numbers of first visibility of the lunar crescents in the binary file were determined by first finding the dates of solar-lunar conjunctions of that period using program HILAL0.FOR and then applying the lunar visibility criterion of Schoch (1928). I have applied an appropriate formula that reduced the Julian day numbers to two-digits numbers in order to reduce the size of the binary file.

The input of the program is the date as given by Sachs and Hunger in the translated diaries,



i.e. Julian year and the Babylonian month and day, together with a reference number for identifying whether the time of the event is between sunset and midnight or between midnight and sunset of the following day. The reason for reading the latter into the program is that the Babylonian day falls in two successive Julian days - roughly 6 hours in one Julian day (between sunset and midnight) and 18 hours in the next.

Using the 209 Babylonian dates, I have found that BABYLON0.FOR has the same  $\pm 1$  days precision of the tables of Parker and Dubberstein. This program, however, has even a better accuracy than the tables. Comparison with the tables of Parker and Dubberstein reveals that the program converted accurately 199 dates (i.e. 95.2%), and only 8 (i.e. 3.8%) and 2 (i.e. 1%) converted dates were wrong by -1 and +1 day, respectively, while the corresponding figures for the tables were 195 (i.e. 93.3%), 8 (i.e. 3.8%) and 6 (i.e. 2.9%). The reason that BABYLON0.FOR is of a slightly better accuracy than the manual tables despite the fact that both methods use the visibility criterion of Schoch must be inaccuracies in the calculations of solar and lunar parameters by Parker and Dubberstein. In fact, the latter do concede that the accuracy of their tables depends upon the accuracy of Schoch's (1928) astronomical tables which they used in their calculations (Parker & Dubberstein, 1956).

It should be stressed that the  $\pm 1$  days accuracy barrier cannot be overcome by any computer program or manual tables that computes the Julian dates of a *truly* lunar calendar, i.e. whose month is begun after *actual* sighting of the lunar crescent. The reason is that it is not possible to know by mere theoretical calculations whether the crescent of a certain month would be visible or not on a particular day after conjunction. Firstly, there is no ideal lunar visibility criterion that can determine with utter certainty the state of visibility of *every* crescent. And secondly, as has already been stated, even if it was possible to know theoretically with certainty if the crescent would be visible on a certain date, in real life situations the moon can fail to be sighted due to a number of reasons, such as unfavourable weather. Accordingly, the date of actual observation of the crescent for the first time in any given month is not necessarily the same as that predicted by any particular theoretical criterion. A crescent that in theory should have been easily spotted could have set unseen due to unfavourable weather and its actual first visibility could have occurred the next evening.

### 3.5 Results of Computations

Using HILAL0.FOR and SUNMOON0.FOR I have computed various key parameters at sunset for the Babylonian, medieval and modern data of first sighting of the lunar crescent. In the case of the Babylonian data, I assumed that the observations were made from a little above the level of the walls of Babylon which were about 15 meters in height (Ravn, 1942: 22, 28).

Some of the frequently used results of computations are shown in table 3.1 along with other information; columns in this table give the following information for each observation:

- Column 1: a reference number.
- Columns 2-4: the Julian year, month, and local day.
- Columns 5-7: the longitude (in degrees west of Greenwich), latitude (in degrees) and the altitude (in meters) above sea level of the observing site.
- Column 8: the result of observation by the unaided eye, where “V” indicates visibility and “I” invisibility.
- Column 9: the result of observation by binoculars or telescope, where “B” indicates visibility using binoculars, “T” means that the crescent was sighted by a telescope, “V” indicates visibility by binoculars or telescope (not specified in the original source), “I” means that the crescent was not seen despite the use of binoculars or telescope, and blank indicates non-use of optical instrument.
- Column 10: the Julian date of local sunset for which the calculations are made.
- Column 11: the moonset lagtime (the difference between moonset and sunset) (in minutes).
- Column 12: the apparent geocentric longitude of the moon.
- Column 13: the difference between the longitudes of the sun and moon.
- Column 14: the apparent arc of light or elongation (the angular separation between the sun and moon).
- Column 15: the true lunar altitude.
- Column 16: the true azimuthal difference between the sun and moon.
- Column 17: the geocentric width of the illuminated part of the disk of the moon (in arc minutes).
- Column 18: the Julian date of the appropriate (new moon) conjunction.
- Column 19: the age of the moon (in hours) at sunset of the day of observation, i.e. the difference between columns 10 and 18.

It is interesting to note that none of the 422 positive sightings of the lunar crescent shows

any contradiction with any basic condition of visibility, such as the presence of the moon above the horizon at the time of sunset. All of the 506 observations occurred after conjunction. In the case of the Babylonian data, this further confirms that these were real observations of the lunar crescent and not mere predictions, though this does not rule out the possibility that the visibility of at least some of the crescents - if not all - was predicted first and then followed up by observation.



Table 3.1 Computations of various parameters needed for the investigation of lunar visibility for the ancient and modern observations

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	-567	4	22	-44.4	32.6	15	V		1514073.353	62	40.0	14.7	14.3	12.6	6.8	0.54	1514072.324	24.7
2	-567	5	22	-44.4	32.6	15	V		1514103.367	114	78.8	24.8	24.0	21.9	9.8	1.44	1514101.606	42.3
3	-567	6	20	-44.4	32.6	15	V		1514132.378	89	101.9	20.3	19.5	16.8	10.0	0.97	1514130.901	35.4
4	-566	2	12	-44.4	32.6	15	V		1514369.321	69	333.7	15.8	15.8	12.8	9.2	0.64	1514367.999	31.7
5	-566	3	14	-44.4	32.6	15	V		1514399.336	103	10.2	22.8	22.3	20.7	8.5	1.26	1514397.548	42.9
6	-418	10	20	-44.4	32.6	15	V		1568676.286	75	225.3	23.1	22.7	14.3	17.8	1.27	1568674.179	50.6
7	-381	5	6	-44.4	32.6	15	V		1582023.327	73	56.6	17.0	16.3	14.6	7.4	0.70	1582022.138	28.5
8	-381	7	4	-44.4	32.6	15	V		1582082.348	93	118.0	22.1	21.5	17.8	12.2	1.16	1582080.725	39.0
9	-378	10	27	-44.4	32.6	15	V		1583293.274	59	232.9	23.4	23.0	10.4	20.7	1.29	1583291.606	40.0
10	-374	1	20	-44.4	32.6	15	V		1584474.274	59	309.3	13.0	12.1	10.5	6.2	0.40	1584473.351	22.2
11	-374	3	20	-44.4	32.6	15	V		1584533.306	79	11.3	16.5	16.1	16.0	1.6	0.69	1584532.126	28.3
12	-372	2	27	-44.4	32.6	15	V		1585242.295	56	345.2	11.5	11.4	11.2	2.3	0.36	1585241.414	21.2
13	-372	3	28	-44.4	32.6	15	V		1585272.309	100	23.5	20.5	20.1	20.0	1.8	1.04	1585270.826	35.6
14	-372	7	23	-44.4	32.6	15	V		1585389.345	42	132.3	17.3	17.1	8.5	14.9	0.73	1585388.054	31.0
15	-370	8	1	-44.4	32.6	15	V		1586128.341	47	145.7	22.4	22.5	9.7	20.4	1.24	1586126.764	37.9
16	-370	10	28	-44.4	32.6	15	V		1586216.272	52	228.4	17.8	17.3	9.5	14.5	0.75	1586214.930	32.2
17	-368	7	10	-44.4	32.6	15	V		1586837.346	49	121.6	19.1	19.2	9.9	16.5	0.91	1586835.721	39.0
18	-366	6	18	-44.4	32.6	15	V		1587545.343	55	97.7	16.7	16.7	10.7	12.8	0.70	1587543.856	35.7
19	-366	8	17	-44.4	32.6	15	V		1587605.332	68	162.6	23.8	23.3	13.9	18.8	1.33	1587603.140	52.6
20	-346	12	2	-44.4	32.6	15	V		1595017.253	83	265.4	18.9	18.5	14.5	11.5	0.87	1595015.508	41.9
21	-345	3	1	-44.4	32.6	15	V		1595106.293	94	356.8	20.7	20.3	18.4	8.6	1.05	1595104.648	39.5
22	-342	12	17	-44.4	32.6	15	V		1596493.254	72	281.5	19.7	19.6	11.4	16.0	0.96	1596491.814	34.5
23	-333	6	14	-44.4	32.6	15	V		1599594.337	56	92.7	15.5	15.1	10.9	10.5	0.59	1599592.985	32.4
24	-333	8	13	-44.4	32.6	15	V		1599654.329	66	165.5	30.6	30.5	13.5	27.6	2.22	1599651.990	56.1
25	-332	9	29	-44.4	32.6	15	V		1600067.290	60	206.5	24.4	23.9	11.8	20.9	1.39	1600065.410	45.1
26	-328	10	13	-44.4	32.6	15	V		1601542.277	47	207.6	11.4	11.6	9.4	6.7	0.36	1601541.271	24.2



1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
27	-328	12	12	-44.4	32.6	15	V		1601602.251	72	272.5	15.3	14.8	12.7	7.6	0.58	1601600.841	33.8
28	-324	4	6	-44.4	32.6	15	V		1602813.306	86	31.2	19.1	18.6	17.3	6.4	0.89	1602811.547	42.2
29	-324	7	3	-44.4	32.6	15	V		1602901.339	52	107.1	11.0	10.7	9.7	4.3	0.32	1602900.400	22.5
30	-324	8	2	-44.4	32.6	15	V		1602931.333	73	142.9	17.9	17.7	14.4	10.4	0.81	1602929.911	34.1
31	-324	9	30	-44.4	32.6	15	V		1602990.288	64	203.7	20.5	20.1	12.8	15.6	1.01	1602988.811	35.4
32	-322	12	7	-44.4	32.6	15	V		1603788.249	86	276.2	24.5	24.5	13.2	20.8	1.46	1603786.425	43.8
33	-321	1	5	-44.4	32.6	15	V		1603817.257	78	300.1	18.8	18.8	12.6	14.0	0.89	1603815.915	32.2
34	-321	2	3	-44.4	32.6	15	V		1603846.274	58	323.9	13.4	13.1	10.5	8.0	0.46	1603845.335	22.5
35	-321	4	3	-44.4	32.6	15	V		1603905.304	79	25.4	16.9	16.0	15.9	1.5	0.68	1603904.067	29.7
36	-321	7	30	-44.4	32.6	15	V		1604023.334	65	138.0	16.6	16.3	12.8	10.1	0.68	1604021.844	35.8
37	-321	8	29	-44.4	32.6	15	V		1604053.316	54	170.2	19.5	19.0	11.2	15.4	0.90	1604051.512	43.3
38	-307	6	26	-44.4	32.6	15	V		1609103.336	74	107.4	18.0	17.3	14.1	10.0	0.77	1609101.877	35.0
39	-307	8	24	-44.4	32.6	15	V		1609162.317	77	168.4	22.0	21.8	15.9	15.2	1.19	1609160.726	38.2
40	-302	7	29	-44.4	32.6	15	V		1610962.332	48	132.4	11.6	11.3	9.4	6.3	0.35	1610961.404	22.3
41	-302	8	28	-44.4	32.6	15	V		1610992.314	44	166.3	16.2	15.7	9.1	12.8	0.62	1610990.922	33.4
42	-302	10	27	-44.4	32.6	15	V		1611052.262	43	230.8	20.7	20.8	7.5	19.4	1.05	1611050.350	45.9
43	-302	11	26	-44.4	32.6	15	V		1611082.248	68	263.4	22.7	22.8	10.6	20.3	1.27	1611080.169	49.9
44	-302	12	26	-44.4	32.6	15	V		1611112.250	112	297.2	26.0	25.7	17.6	18.8	1.61	1611109.961	54.9
45	-301	1	24	-44.4	32.6	15	V		1611141.265	87	319.5	19.0	18.3	15.6	9.6	0.86	1611139.670	38.3
46	-301	6	19	-44.4	32.6	15	V		1611287.334	51	91.7	9.5	9.5	9.5	1.2	0.26	1611286.673	15.9
47	-294	5	4	-44.4	32.6	15	V		1613798.313	96	60.3	21.6	20.9	19.0	8.7	1.11	1613796.705	38.6
48	-293	1	25	-44.4	32.6	15	V		1614064.265	69	315.2	13.5	13.2	13.2	1.0	0.48	1614063.219	25.1
49	-291	5	1	-44.4	32.6	15	V		1614891.312	69	52.9	16.8	16.7	13.9	9.3	0.72	1614890.029	30.8
50	-291	6	29	-44.4	32.6	15	V		1614950.334	80	115.7	23.3	22.8	15.7	16.7	1.29	1614948.683	39.6
51	-291	8	26	-44.4	32.6	15	V		1615008.313	41	162.2	13.7	13.2	8.4	10.2	0.45	1615007.332	23.6
52	-289	6	8	-44.4	32.6	15	V		1615659.328	80	91.4	19.6	18.9	15.5	10.9	0.91	1615657.645	40.4
53	-286	6	4	-44.4	32.6	15	V		1616751.326	102	89.0	20.7	20.1	18.9	6.7	1.03	1616749.560	42.4
54	-284	11	6	-44.4	32.6	15	V		1617637.253	36	237.9	17.0	17.2	6.2	16.0	0.72	1617635.681	37.7



1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
55	-283	10	27	-44.4	32.6	15	V		1617992.259	69	239.3	28.8	28.8	11.5	26.5	1.98	1617989.767	59.8
56	-281	11	4	-44.4	32.6	15	V		1618730.254	74	246.1	28.0	27.9	12.1	25.3	1.87	1618728.204	49.2
57	-277	3	28	-44.4	32.6	15	V		1619970.295	105	24.8	21.9	21.1	21.1	0.5	1.14	1619968.685	38.6
58	-277	4	26	-44.4	32.6	15	V		1619999.307	76	47.1	16.2	15.3	15.0	2.7	0.62	1619998.051	30.1
59	-277	5	26	-44.4	32.6	15	V		1620029.321	96	81.9	22.4	21.7	18.7	11.1	1.19	1620027.467	44.5
60	-273	12	4	-44.4	32.6	15	V		1621682.242	64	261.6	12.8	12.8	11.5	5.6	0.44	1621681.180	25.5
61	-266	10	19	-44.4	32.6	15	V		1624193.263	53	224.8	22.6	22.3	9.8	20.1	1.21	1624191.233	48.7
62	-266	11	18	-44.4	32.6	15	V		1624223.245	70	256.9	24.2	24.2	11.1	21.5	1.41	1624221.015	53.5
63	-264	9	26	-44.4	32.6	15	V		1624901.282	29	196.7	17.1	17.0	5.8	16.0	0.71	1624900.047	29.6
64	-255	3	25	-44.4	32.6	15	V		1628003.291	88	20.1	19.4	18.7	17.8	5.4	0.90	1628001.595	40.7
65	-255	9	17	-44.4	32.6	15	V		1628179.289	52	188.3	17.9	17.4	10.6	13.9	0.76	1628178.014	30.6
66	-251	10	3	-44.4	32.6	15	V		1629656.274	54	201.2	14.8	14.8	11.1	9.8	0.57	1629654.990	30.8
67	-250	2	28	-44.4	32.6	15	V		1629804.279	105	359.1	23.1	22.7	20.5	9.8	1.30	1629802.663	38.8
68	-249	8	13	-44.4	32.6	15	V		1630335.317	81	157.7	22.1	22.0	16.4	14.7	1.20	1630333.461	44.5
69	-246	1	15	-44.4	32.6	15	V		1631221.252	60	306.6	14.5	14.4	10.2	10.2	0.54	1631219.923	31.9
70	-246	4	14	-44.4	32.6	15	V		1631310.298	71	34.6	14.7	14.1	14.0	0.2	0.53	1631309.039	30.2
71	-246	5	14	-44.4	32.6	15	V		1631340.311	115	70.6	22.1	21.7	21.5	2.4	1.19	1631338.549	42.3
72	-246	10	8	-44.4	32.6	15	V		1631487.269	54	217.0	25.8	25.6	10.0	23.7	1.58	1631485.400	44.9
73	-245	5	3	-44.4	32.6	15	V		1631694.306	89	55.1	17.3	17.1	17.0	0.3	0.76	1631692.769	36.9
74	-245	7	1	-44.4	32.6	15	V		1631753.328	84	113.0	18.8	18.3	15.9	9.1	0.86	1631751.816	36.3
75	-237	7	3	-44.4	32.6	15	V		1634677.327	86	123.8	27.7	27.4	17.3	21.5	1.83	1634675.287	48.9
76	-237	8	1	-44.4	32.6	15	V		1634706.321	60	147.7	23.8	23.4	12.3	20.0	1.34	1634704.627	40.7
77	-234	9	26	-44.4	32.6	15	V		1635858.278	75	201.1	21.8	21.7	15.2	15.6	1.17	1635856.485	43.0
78	-234	11	24	-44.4	32.6	15	V		1635917.238	93	261.3	22.2	21.7	16.1	14.7	1.18	1635915.605	39.2
79	-233	2	20	-44.4	32.6	15	V		1636005.272	84	346.6	18.5	18.1	16.1	8.4	0.85	1636003.951	31.7
80	-233	3	21	-44.4	32.6	15	V		1636034.286	55	9.6	13.0	13.1	11.1	7.0	0.46	1636033.327	23.0
81	-232	10	3	-44.4	32.6	15	V		1636596.271	70	208.0	21.2	20.9	14.0	15.6	1.09	1636594.319	46.8
82	-231	2	28	-44.4	32.6	15	V		1636744.276	101	358.7	22.3	21.9	19.8	9.4	1.22	1636742.659	38.8



1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
83	-225	1	23	-44.4	32.6	15	V		1638899.254	94	319.6	19.5	18.6	17.1	7.4	0.90	1638897.840	33.9
84	-218	10	28	-44.4	32.6	15	V		1641734.249	78	232.0	20.3	20.1	14.6	13.9	1.02	1641732.825	34.2
85	-217	2	23	-44.4	32.6	15	V		1641852.271	86	349.8	18.7	17.8	17.1	4.8	0.82	1641850.726	37.1
86	-210	7	4	-44.4	32.6	15	V		1644540.323	105	120.7	23.2	22.9	20.0	11.4	1.32	1644538.551	42.5
87	-209	5	24	-44.4	32.6	15	V		1644864.311	60	69.3	11.2	11.2	11.1	0.1	0.35	1644863.305	24.1
88	-207	4	2	-44.4	32.6	15	V		1645543.288	75	24.4	15.6	15.2	15.1	1.8	0.62	1645542.050	29.7
89	-207	5	1	-44.4	32.6	15	V		1645572.300	49	46.2	9.5	9.8	9.4	2.5	0.27	1645571.508	19.0
90	-203	12	10	-44.4	32.6	15	V		1647256.233	74	271.8	15.9	15.4	12.9	8.4	0.62	1647254.798	34.4
91	-201	12	18	-44.4	32.6	15	V		1647994.234	81	279.3	15.7	15.6	14.4	5.9	0.64	1647993.038	28.7
92	-200	3	16	-44.4	32.6	15	V		1648083.280	74	8.9	16.3	15.4	15.1	2.4	0.63	1648081.809	35.3
93	-198	6	21	-44.4	32.6	15	V		1648910.320	73	106.1	20.9	20.6	14.2	14.9	1.05	1648908.436	45.2
94	-197	3	14	-44.4	32.6	15	V		1649176.278	105	13.2	23.3	22.8	21.2	8.6	1.31	1649174.624	39.7
95	-197	10	7	-44.4	32.6	15	V		1649383.263	79	212.8	22.4	22.4	15.6	16.0	1.24	1649381.220	49.0
96	-197	11	5	-44.4	32.6	15	V		1649412.241	60	234.0	14.3	14.2	11.4	8.5	0.53	1649410.983	30.2
97	-196	2	2	-44.4	32.6	15	V		1649501.256	85	328.6	18.5	17.8	15.7	8.6	0.82	1649499.884	32.9
98	-195	11	12	-44.4	32.6	15	V		1650150.237	59	243.7	16.4	15.9	10.6	11.8	0.64	1650148.743	35.9
99	-194	1	11	-44.4	32.6	15	V		1650210.243	90	308.9	20.4	20.1	15.2	13.1	1.01	1650208.387	44.6
100	-194	6	7	-44.4	32.6	15	V		1650357.315	88	90.8	19.0	18.2	16.5	7.7	0.86	1650355.961	32.5
101	-194	10	3	-44.4	32.6	15	V		1650475.266	75	211.1	24.4	24.0	14.8	19.1	1.42	1650473.357	45.8
102	-193	4	28	-44.4	32.6	15	V		1650682.297	65	47.9	14.5	13.6	13.0	4.0	0.50	1650681.098	28.8
103	-193	5	28	-44.4	32.6	15	V		1650712.311	111	84.7	22.6	21.9	20.8	6.9	1.22	1650710.559	42.1
104	-193	10	22	-44.4	32.6	15	V		1650859.250	70	230.5	24.9	24.5	12.8	21.0	1.46	1650857.390	44.6
105	-192	1	19	-44.4	32.6	15	V		1650948.247	106	319.6	23.5	23.3	18.2	14.6	1.35	1650946.175	49.7
106	-192	2	17	-44.4	32.6	15	V		1650977.265	67	340.4	15.3	15.1	12.8	7.9	0.59	1650975.874	33.4
107	-192	3	18	-44.4	32.6	15	V		1651007.280	83	13.0	18.4	17.7	16.9	5.0	0.81	1651005.582	40.7
108	-192	9	11	-44.4	32.6	15	V		1651184.285	65	188.4	23.2	22.7	13.2	18.6	1.27	1651182.634	39.6
109	-190	3	26	-44.4	32.6	15	V		1651745.282	102	23.5	21.5	20.8	20.6	1.1	1.10	1651743.465	43.6
110	-190	5	24	-44.4	32.6	15	V		1651804.309	100	77.3	18.8	18.6	18.3	2.6	0.89	1651802.600	41.0



1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
111	-189	10	9	-44.4	32.6	15	V		1652307.260	52	219.0	26.6	26.7	9.5	25.0	1.70	1652304.975	54.9
112	-189	11	7	-44.4	32.6	15	V		1652336.239	42	241.4	19.6	19.9	7.0	18.6	0.96	1652334.645	38.3
113	-188	4	2	-44.4	32.6	15	V		1652483.285	90	27.8	18.5	18.2	18.1	1.8	0.86	1652481.958	31.9
114	-187	10	16	-44.4	32.6	15	V		1653045.254	60	227.4	27.4	27.4	10.6	25.4	1.80	1653042.738	60.4
115	-187	11	14	-44.4	32.6	15	V		1653074.235	49	247.9	18.4	18.3	8.3	16.3	0.82	1653072.533	40.8
116	-185	3	2	-44.4	32.6	15	V		1653547.271	94	357.8	19.5	19.3	19.1	1.5	0.96	1653545.615	39.7
117	-183	5	7	-44.4	32.6	15	V		1654344.300	79	60.4	17.7	17.0	15.5	6.9	0.75	1654342.715	38.0
118	-183	8	4	-44.4	32.6	15	V		1654433.312	64	155.8	28.1	27.9	13.2	24.8	1.88	1654431.208	50.5
119	-183	10	31	-44.4	32.6	15	V		1654521.242	90	240.6	25.4	24.9	16.1	19.1	1.52	1654519.444	43.1
120	-181	2	15	-44.4	32.6	15	V		1654993.262	86	341.4	18.0	17.2	17.0	2.4	0.77	1654991.888	33.0
121	-179	3	24	-44.4	32.6	15	V		1655761.280	83	19.1	18.8	18.3	16.9	7.2	0.87	1655759.954	31.8
122	-179	7	20	-44.4	32.6	15	V		1655879.317	68	134.8	21.5	20.9	13.6	15.9	1.09	1655877.460	44.6
123	-178	8	8	-44.4	32.6	15	V		1656263.309	81	155.7	24.3	23.8	16.5	17.3	1.40	1656261.316	47.8
124	-178	9	6	-44.4	32.6	15	V		1656292.288	58	176.4	16.5	16.4	11.9	11.3	0.69	1656290.863	34.2
125	-178	10	6	-44.4	32.6	15	V		1656322.261	69	209.0	19.2	19.2	13.9	13.3	0.93	1656320.528	41.6
126	-176	9	13	-44.4	32.6	15	V		1657030.281	53	181.4	14.1	14.1	10.9	9.0	0.53	1657029.258	24.6
127	-176	10	13	-44.4	32.6	15	V		1657060.254	66	217.4	20.1	19.7	12.9	14.9	0.97	1657058.706	37.2
128	-175	5	9	-44.4	32.6	15	V		1657268.300	110	67.7	23.1	22.3	21.2	6.9	1.26	1657266.384	46.0
129	-175	12	1	-44.4	32.6	15	V		1657474.229	90	271.8	24.9	24.6	14.2	20.2	1.48	1657472.272	47.0
130	-173	11	10	-44.4	32.6	15	V		1658183.235	66	249.2	24.2	24.1	10.8	21.7	1.41	1658181.512	41.3
131	-173	12	9	-44.4	32.6	15	V		1658212.228	57	272.5	18.0	18.1	9.0	15.8	0.82	1658210.970	30.2
132	-172	2	6	-44.4	32.6	15	V		1658271.256	88	333.4	19.2	18.6	16.4	8.8	0.89	1658269.837	34.1
133	-170	8	9	-44.4	32.6	15	V		1659186.308	43	148.6	16.1	15.6	8.7	12.9	0.61	1659184.892	34.0
134	-170	10	8	-44.4	32.6	15	V		1659246.258	36	213.0	21.1	21.4	6.7	20.3	1.10	1659244.309	46.8
135	-170	11	7	-44.4	32.6	15	V		1659276.236	58	245.9	23.7	23.8	9.6	21.8	1.37	1659274.100	51.3
136	-169	2	3	-44.4	32.6	15	V		1659364.254	68	325.9	14.3	13.4	13.0	3.4	0.49	1659363.147	26.6
137	-169	5	2	-44.4	32.6	15	V		1659452.296	75	52.0	14.6	14.4	14.3	1.1	0.56	1659451.280	24.4
138	-168	8	17	-44.4	32.6	15	V		1659925.302	56	168.5	27.7	27.7	11.6	25.3	1.84	1659923.215	50.1



1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
139	-168	12	13	-44.4	32.6	15	V		1660043.228	71	276.6	17.1	16.5	12.0	11.3	0.69	1660041.649	37.9
140	-164	6	5	-44.4	32.6	15	V		1661313.311	66	88.7	18.1	17.9	12.9	12.5	0.81	1661311.654	39.8
141	-164	10	31	-44.4	32.6	15	V		1661461.239	89	240.2	24.6	24.2	16.1	18.2	1.44	1661459.455	42.8
142	-163	4	25	-44.4	32.6	15	V		1661637.292	56	44.5	13.2	12.6	11.3	5.6	0.43	1661636.168	27.0
143	-163	5	26	-44.4	32.6	15	V		1661668.306	117	89.8	28.9	28.7	23.1	17.2	2.00	1661665.711	62.3
144	-163	11	19	-44.4	32.6	15	V		1661845.230	100	257.6	22.9	22.7	17.7	14.3	1.29	1661843.505	41.4
145	-162	3	16	-44.4	32.6	15	V		1661962.275	64	6.7	14.3	13.3	13.0	3.2	0.48	1661961.241	24.8
146	-162	8	11	-44.4	32.6	15	V		1662110.305	45	152.8	18.3	17.9	9.2	15.4	0.80	1662108.618	40.5
147	-162	9	10	-44.4	32.6	15	V		1662140.282	59	185.0	21.1	20.6	12.1	16.8	1.05	1662138.337	46.7
148	-161	9	29	-44.4	32.6	15	V		1662524.265	78	206.3	23.7	23.5	15.7	17.6	1.36	1662522.080	52.5
149	-158	6	29	-44.4	32.6	15	V		1663528.316	100	117.4	24.3	23.6	19.3	13.7	1.39	1663526.574	41.8
150	-158	8	26	-44.4	32.6	15	V		1663586.294	58	164.4	15.2	15.2	11.8	9.6	0.60	1663585.226	25.6
151	-156	12	1	-44.4	32.6	15	V		1664414.226	103	275.2	28.0	27.7	15.7	23.0	1.86	1664412.227	48.0
152	-154	1	18	-44.4	32.6	15	V		1664827.242	93	316.7	20.8	20.6	15.9	13.1	1.07	1664825.768	35.4
153	-151	3	15	-44.4	32.6	15	V		1665979.273	86	9.9	18.1	17.5	17.4	0.8	0.80	1665977.947	31.8
154	-149	11	14	-44.4	32.6	15	V		1666953.230	68	251.0	21.8	21.3	11.4	18.1	1.12	1666951.505	41.4
155	-145	1	9	-44.4	32.6	15	V		1668105.236	110	307.9	21.3	21.0	20.2	5.7	1.13	1668103.739	35.9
156	-145	2	7	-44.4	32.6	15	V		1668134.253	77	331.2	15.4	15.0	15.0	0.2	0.60	1668133.149	26.5
157	-144	9	21	-44.4	32.6	15	V		1668726.270	83	204.0	28.5	28.1	16.4	23.0	1.90	1668723.643	63.0
158	-144	10	20	-44.4	32.6	15	V		1668755.244	72	224.9	20.3	20.0	13.7	14.6	1.00	1668753.422	43.7
159	-144	11	18	-44.4	32.6	15	V		1668784.228	58	246.4	12.3	12.4	10.7	6.2	0.41	1668783.171	25.4
160	-143	8	11	-44.4	32.6	15	V		1669050.303	73	161.1	26.2	25.7	15.0	21.0	1.61	1669048.052	54.0
161	-143	9	9	-44.4	32.6	15	V		1669079.280	52	181.4	18.0	17.6	10.6	14.0	0.77	1669077.665	38.8
162	-143	10	9	-44.4	32.6	15	V		1669109.254	69	213.4	20.1	20.0	13.7	14.5	1.00	1669107.398	44.5
163	-142	11	26	-44.4	32.6	15	V		1669522.225	64	255.5	13.7	13.7	11.6	7.0	0.49	1669520.960	30.4
164	-141	5	23	-44.4	32.6	15	V		1669700.302	100	81.7	23.9	23.4	19.6	12.9	1.37	1669698.588	41.1
165	-141	10	17	-44.4	32.6	15	V		1669847.247	82	223.4	22.6	22.4	15.8	16.0	1.25	1669845.427	43.7
166	-140	4	12	-44.4	32.6	15	V		1670025.284	85	38.7	19.5	19.3	17.2	8.6	0.95	1670023.650	39.2



1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
167	-140	7	9	-44.4	32.6	15	V		1670113.314	76	123.1	19.8	19.1	14.7	12.4	0.93	1670111.883	34.3
168	-140	12	3	-44.4	32.6	15	V		1670260.224	62	264.8	15.4	14.7	10.8	10.0	0.56	1670258.976	29.9
169	-139	2	1	-44.4	32.6	15	V		1670320.249	96	331.4	21.1	20.8	17.6	11.1	1.09	1670318.339	45.9
170	-137	12	31	-44.4	32.6	15	V		1671383.230	91	299.0	21.7	21.6	14.5	16.2	1.17	1671381.708	36.5
171	-136	9	22	-44.4	32.6	15	V		1671649.268	54	197.5	21.0	20.5	10.9	17.3	1.03	1671647.413	44.5
172	-134	10	30	-44.4	32.6	15	V		1672417.236	56	239.0	24.7	24.8	9.5	23.0	1.48	1672414.956	54.7
173	-133	2	25	-44.4	32.6	15	V		1672535.262	87	352.3	18.5	17.7	17.5	2.4	0.82	1672533.816	34.7
174	-133	8	20	-44.4	32.6	15	V		1672711.296	43	161.5	18.3	17.8	8.8	15.5	0.79	1672709.871	34.2
175	-133	9	19	-44.4	32.6	15	V		1672741.271	41	195.7	22.8	22.8	8.2	21.3	1.26	1672739.376	45.5
176	-133	10	19	-44.4	32.6	15	V		1672771.244	52	228.5	25.6	25.7	9.2	24.0	1.58	1672769.001	53.8
177	-132	3	15	-44.4	32.6	15	V		1672919.271	90	11.0	18.9	18.4	18.3	1.2	0.88	1672917.719	37.2
178	-132	10	7	-44.4	32.6	15	V		1673125.254	45	216.1	24.5	24.7	8.2	23.3	1.47	1673123.336	46.0
179	-131	10	26	-44.4	32.6	15	V		1673509.238	61	235.9	25.3	25.2	10.5	23.0	1.54	1673507.355	45.2
180	-129	7	9	-44.4	32.6	15	V		1674130.313	73	128.2	25.5	25.3	14.8	20.7	1.56	1674128.119	52.7
181	-124	12	7	-44.4	32.6	15	V		1676108.222	98	274.5	20.9	20.6	17.1	11.4	1.07	1676106.296	46.2
182	-123	2	4	-44.4	32.6	15	V		1676167.249	70	328.5	15.1	14.3	13.3	4.9	0.54	1676165.892	32.6
183	-123	6	2	-44.4	32.6	15	V		1676285.304	75	87.5	19.6	19.2	14.7	12.6	0.94	1676283.892	33.9
184	-119	4	19	-44.4	32.6	15	V		1677702.284	76	42.9	17.0	16.3	15.3	5.3	0.69	1677700.817	35.2
185	-119	6	17	-44.4	32.6	15	V		1677761.309	68	96.5	14.3	13.8	12.6	5.4	0.50	1677759.993	31.6
186	-118	5	8	-44.4	32.6	15	V		1678086.292	98	63.9	20.1	19.3	18.8	4.2	0.96	1678084.650	39.4
187	-117	10	22	-44.4	32.6	15	V		1678618.240	42	224.8	18.8	18.5	7.7	16.9	0.85	1678616.514	41.4
188	-111	3	22	-44.4	32.6	15	V		1680596.271	75	14.5	15.6	15.4	15.1	2.5	0.63	1680594.909	32.7
189	-111	6	19	-44.4	32.6	15	V		1680685.309	63	101.0	16.8	16.2	12.1	10.7	0.67	1680683.781	36.7
190	-111	8	18	-44.4	32.6	15	V		1680745.294	56	170.4	28.5	28.5	11.5	26.2	1.95	1680742.970	55.8
191	-107	4	7	-44.4	32.6	15	V		1682073.277	87	33.6	19.2	18.3	17.6	5.4	0.87	1682071.919	32.6
192	-105	4	16	-44.4	32.6	15	V		1682812.281	102	45.7	23.1	22.8	20.6	9.8	1.31	1682810.430	44.4
193	-105	5	15	-44.4	32.6	15	V		1682841.294	76	69.4	19.0	18.9	15.2	11.3	0.91	1682839.856	34.5
194	-105	6	13	-44.4	32.6	15	V		1682870.306	50	93.3	15.3	15.2	9.9	11.6	0.60	1682869.196	26.6



1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
195	-105	9	9	-44.4	32.6	15	V		1682958.276	48	178.3	15.2	14.8	9.8	11.1	0.56	1682957.186	26.2
196	-105	10	9	-44.4	32.6	15	V		1682988.249	77	214.8	21.7	21.6	15.1	15.5	1.16	1682986.617	39.2
197	-104	8	29	-44.4	32.6	15	V		1683313.284	65	173.0	19.9	19.6	13.5	14.3	0.97	1683311.877	33.8
198	-96	5	5	-44.4	32.6	15	V		1686119.289	99	60.6	19.0	18.7	18.6	0.4	0.91	1686117.805	35.6
199	-95	5	24	-44.4	32.6	15	V		1686503.297	104	79.8	20.3	19.8	19.4	4.1	1.01	1686501.617	40.3
200	-87	7	23	-44.4	32.6	15	V		1689485.305	59	137.9	21.0	20.5	11.9	16.8	1.04	1689483.819	35.7
201	-87	9	20	-44.4	32.6	15	V		1689544.263	75	198.7	24.1	23.7	15.1	18.4	1.39	1689542.529	41.6
202	-86	3	17	-44.4	32.6	15	V		1689722.266	99	16.0	22.0	21.6	20.1	7.7	1.17	1689720.239	48.7
203	-86	11	7	-44.4	32.6	15	V		1689957.225	70	239.5	16.7	16.6	13.2	10.1	0.71	1689956.037	28.5
204	-83	7	9	-44.4	32.6	15	V		1690932.307	81	122.9	19.4	18.9	15.5	10.8	0.91	1690930.529	42.7
205	-77	6	4	-44.4	32.6	15	V		1693088.299	87	86.4	16.8	16.5	16.0	3.8	0.71	1693086.897	33.7
206	-77	8	2	-44.4	32.6	15	V		1693147.300	58	146.3	20.2	19.6	11.9	15.6	0.96	1693145.787	36.3
207	-77	9	1	-44.4	32.6	15	V		1693177.279	61	184.6	29.1	28.9	12.4	26.3	2.00	1693175.187	50.2
208	-77	10	30	-44.4	32.6	15	V		1693236.229	82	245.9	31.4	31.3	13.2	28.6	2.34	1693234.002	53.4
209	-73	7	19	-44.4	32.6	15	V		1694594.304	63	138.0	25.3	25.2	12.9	21.8	1.54	1694592.097	53.0
210	1183	4	26	-31.3	30.1	116	V		2153264.199	168	77.9	35.7	34.9	33.3	9.7	2.93	2153261.389	67.4
211	1183	5	25	-32.7	26.1	150	V		2153293.202	131	98.9	28.8	28.1	26.3	8.9	1.94	2153290.809	57.4
212	1183	6	24	-36.0	22.7	50	V		2153323.193	139	131.7	33.1	32.6	29.7	13.3	2.57	2153320.296	69.5
213	1183	7	24	-39.2	21.5	6	V		2153353.178	135	163.8	36.6	36.2	30.7	20.0	3.13	2153349.860	79.6
214	1183	8	22	-39.8	21.4	227	V		2153382.163	101	184.3	29.0	28.7	22.7	17.3	2.01	2153379.499	63.9
215	1183	9	21	-39.8	21.0	227	V		2153412.144	109	217.1	32.4	31.9	23.4	21.7	2.45	2153409.198	70.7
216	1183	10	20	-39.8	21.0	227	V		2153441.128	88	238.5	24.9	24.3	17.9	16.1	1.46	2153438.926	52.8
217	1183	10	20	-39.8	21.0	227	I		2153441.128	88	238.5	24.9	24.3	17.9	16.1	1.46	2153438.926	52.8
218	1183	11	19	-39.8	21.0	227	V		2153471.122	119	273.7	29.7	29.1	22.0	19.0	2.07	2153468.641	59.5
219	1183	12	19	-39.8	21.0	227	V		2153501.128	164	310.9	36.4	36.0	30.5	19.1	3.09	2153498.293	68.0
220	1184	1	16	-39.8	21.0	227	V		2153529.141	75	320.4	17.4	17.2	15.1	7.6	0.77	2153527.850	31.0
221	1184	2	14	-39.8	21.0	227	I		2153558.152	49	344.1	11.9	11.9	10.5	5.0	0.39	2153557.308	20.3
222	1184	3	15	-44.2	15.4	1750	V		2153588.149	86	22.9	20.8	20.1	18.7	0.3	1.04	2153586.690	35.0



1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
223	1184	3	15	-44.4	33.3	34	V		2153588.146	93	22.8	20.8	20.1	18.8	7.0	1.04	2153586.690	34.9
224	1184	3	16	-39.8	21.0	227	V		2153589.159	152	38.0	35.0	34.2	33.6	0.7	2.83	2153586.690	59.2
225	1184	4	14	-38.8	23.7	150	V		2153618.169	136	61.2	29.8	29.0	28.5	2.2	2.07	2153616.030	51.3
226	1184	5	14	-44.4	32.6	15	V		2153648.174	181	97.7	37.5	36.9	34.6	13.2	3.25	2153645.358	67.6
227	1184	6	12	-36.2	32.6	350	V		2153677.209	142	119.8	31.9	31.5	26.8	15.9	2.40	2153674.709	60.0
228	1184	7	11	-36.3	33.5	720	V		2153706.209	101	141.1	25.5	25.3	19.0	15.9	1.58	2153704.114	50.3
229	1184	8	10	-36.3	33.5	720	V		2153736.192	96	174.1	29.7	29.5	18.7	22.3	2.09	2153733.603	62.1
230	1184	9	9	-36.3	33.5	720	V		2153766.165	93	206.1	32.5	32.1	17.4	26.8	2.46	2153763.205	71.1
231	1184	10	9	-35.1	32.9	10	V		2153796.139	102	238.1	34.7	34.3	17.7	29.8	2.79	2153792.921	77.2
232	1184	12	7	-15.7	38.1	50	V		2153855.163	123	293.4	30.4	30.2	16.6	25.4	2.19	2153852.509	63.7
233	1185	1	5	-12.5	38.0	50	V		2153884.186	106	316.3	23.7	23.5	16.1	17.1	1.37	2153882.230	46.9
234	1185	2	4	-12.5	38.0	50	V		2153914.208	148	353.7	30.6	30.1	26.0	15.3	2.21	2153911.837	56.9
235	1185	3	6	-12.5	38.0	50	V		2153944.229	195	32.4	39.4	38.6	36.5	12.6	3.54	2153941.331	69.5
236	1859	7	1	-23.7	38.0	122	V		2400227.246	65	115.3	16.0	15.4	11.4	10.2	0.62	2400226.112	27.2
237	1859	10	27	-23.7	38.0	122	V		2400345.149	33	234.5	20.8	21.1	5.1	20.4	1.07	2400343.523	39.0
238	1860	1	23	-23.7	38.0	122	I		2400433.153	30	309.9	7.0	6.2	5.0	3.3	0.12	2400432.512	15.4
239	1860	2	23	-23.7	38.0	122	V		2400464.176	102	354.4	20.1	19.6	19.0	2.5	0.98	2400462.319	44.6
240	1860	6	20	-23.7	38.0	122	V		2400582.245	80	109.3	19.8	19.1	14.1	12.8	0.92	2400580.725	36.5
241	1861	3	12	-23.7	38.0	122	V		2400847.188	63	4.4	12.3	12.3	11.9	1.6	0.41	2400846.068	26.9
242	1861	8	7	-23.7	38.0	122	I		2400995.229	21	150.6	15.5	15.7	3.8	15.1	0.60	2400994.038	28.6
243	1861	8	8	-23.7	38.0	122	V		2400996.228	52	164.8	28.8	28.8	9.8	27.0	1.97	2400994.038	52.6
244	1861	9	7	-23.7	38.0	122	V		2401026.200	68	203.3	38.3	38.3	11.9	36.6	3.41	2401023.425	66.6
245	1861	10	5	-23.7	38.0	122	I		2401054.170	24	212.0	19.6	19.9	4.1	19.4	0.95	2401052.790	33.1
246	1861	11	4	-23.7	38.0	122	V		2401084.143	79	250.2	28.0	27.6	11.8	25.1	1.84	2401082.170	47.3
247	1861	12	3	-23.7	38.0	122	V		2401113.131	84	272.8	21.3	20.7	12.5	16.4	1.07	2401111.596	36.8
248	1862	1	1	-22.9	37.9	122	V		2401142.140	70	295.2	14.2	13.5	11.3	7.0	0.48	2401141.080	25.5
249	1862	3	31	-23.7	38.0	122	V		2401231.201	81	26.3	15.6	15.3	14.9	0.3	0.62	2401229.824	33.0
250	1862	4	29	-23.7	38.0	122	I		2401260.219	42	47.2	8.1	7.8	7.5	0.1	0.18	2401259.477	17.8



1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
251	1862	7	28	-23.7	38.0	122	I		2401350.236	38	147.0	21.7	21.8	7.2	20.6	1.15	2401348.378	44.6
252	1864	1	10	-23.7	38.0	122	V		2401881.143	102	308.5	18.8	18.5	17.2	6.1	0.88	2401879.824	31.7
253	1864	2	8	2.3	53.5	183	V		2401910.214	96	333.0	13.7	13.6	12.6	4.4	0.50	2401909.256	23.0
254	1864	3	9	-23.7	38.0	122	V		2401940.187	106	10.5	21.1	20.4	20.0	2.3	1.07	2401938.666	36.5
255	1864	5	6	-26.2	39.6	122	I		2401998.220	37	55.3	8.9	8.1	6.6	4.4	0.19	2401997.510	17.0
256	1864	6	6	-23.7	38.0	122	V		2402029.241	93	102.6	26.4	26.1	17.0	19.8	1.67	2402026.986	54.1
257	1864	8	4	-23.7	38.0	122	I		2402088.231	39	155.8	23.3	23.4	7.3	22.2	1.32	2402086.107	51.0
258	1864	9	3	-23.7	38.0	122	V		2402118.204	49	188.1	26.7	26.5	9.0	24.9	1.68	2402115.756	58.7
259	1864	11	1	-23.7	38.0	122	V		2402177.145	79	242.8	23.3	22.9	12.9	18.8	1.28	2402175.145	48.0
260	1865	1	28	-23.7	38.0	122	V		2402265.157	93	326.3	17.5	17.1	16.6	3.1	0.77	2402263.897	30.2
261	1865	3	28	-23.7	38.0	122	V		2402324.199	100	28.8	20.8	19.9	18.9	5.4	1.02	2402322.728	35.3
262	1865	4	26	-23.7	38.0	122	V		2402353.217	68	52.0	15.6	14.8	12.6	7.4	0.58	2402352.093	27.0
263	1865	6	24	-23.7	38.0	122	I		2402412.245	43	110.9	17.8	18.0	7.9	16.1	0.80	2402410.831	33.9
264	1865	7	24	-23.7	38.0	122	V		2402442.239	46	144.9	23.2	23.2	8.5	21.5	1.30	2402440.270	47.2
265	1865	10	21	-23.7	38.0	122	V		2402531.154	69	229.6	21.3	21.0	11.7	17.2	1.08	2402529.186	47.2
266	1866	1	17	-23.7	38.0	122	I		2402619.149	56	307.2	9.8	9.9	9.6	0.7	0.28	2402618.359	19.0
267	1866	4	16	-23.7	38.0	122	V		2402708.211	90	46.6	20.1	19.5	16.9	9.3	0.97	2402706.794	34.0
268	1867	2	5	-23.7	38.0	122	I		2403003.163	53	327.0	10.5	9.9	9.4	1.8	0.28	2403002.261	21.7
269	1867	11	27	-23.7	38.0	122	V		2403298.132	77	260.8	15.8	15.8	12.5	9.3	0.65	2403296.716	34.0
270	1868	6	22	-23.7	38.0	122	V		2403506.245	99	121.2	29.7	29.2	18.3	22.9	2.06	2403504.115	51.1
271	1869	5	12	-23.7	38.0	122	I		2403830.228	44	64.5	12.5	12.7	8.0	9.7	0.42	2403829.172	25.3
272	1871	2	20	-23.7	38.0	122	V		2404479.174	57	345.2	13.5	13.6	10.2	8.7	0.48	2404478.076	26.4
273	1871	4	20	-23.7	38.0	122	I		2404538.213	38	40.4	10.2	10.3	7.2	7.1	0.29	2404537.294	22.1
274	1871	5	20	-23.7	38.0	122	I		2404568.232	57	73.1	13.9	13.3	10.2	8.2	0.47	2404566.948	30.8
275	1871	6	18	-23.7	38.0	122	I		2404597.245	24	94.0	7.0	6.3	4.2	4.4	0.12	2404596.604	15.4
276	1871	6	19	-23.7	38.0	122	V		2404598.245	75	105.9	17.9	17.3	13.0	11.1	0.76	2404596.604	39.4
277	1871	8	17	-23.7	38.0	122	V		2404657.221	62	161.4	17.1	17.1	11.7	12.2	0.74	2404655.793	34.3
278	1872	6	7	-23.7	38.0	122	V		2404952.241	82	94.9	17.7	17.0	14.1	9.2	0.75	2404950.641	38.4



1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
279	1872	7	6	-23.7	38.0	122	I		2404981.245	50	115.5	10.6	10.5	8.5	5.7	0.30	2404980.268	23.5
280	1872	9	4	-23.7	38.0	122	V		2405041.203	63	181.3	18.9	18.9	11.9	14.5	0.89	2405039.538	40.0
281	1872	10	3	-23.7	38.0	122	I		2405070.171	41	202.9	12.2	12.0	7.6	9.0	0.38	2405069.146	24.6
282	1872	10	4	-23.7	38.0	122	V		2405071.170	70	215.9	24.2	23.8	12.5	20.2	1.39	2405069.146	48.6
283	1872	12	31	-23.7	38.0	122	V		2405159.138	77	299.4	19.1	18.9	11.3	15.1	0.90	2405157.776	32.7
284	1873	4	27	-23.7	38.0	122	I		2405276.218	43	47.5	10.0	9.2	7.9	4.2	0.24	2405275.446	18.5
285	1873	5	27	-23.7	38.0	122	V		2405306.236	81	82.8	16.4	15.8	14.0	6.7	0.65	2405304.889	32.3
286	1873	12	20	-23.7	38.0	122	I		2405513.133	27	279.6	10.8	11.2	3.9	10.3	0.32	2405512.284	20.4
287	1874	4	17	-23.7	38.0	122	V		2405631.212	77	43.4	15.9	15.0	14.1	4.3	0.60	2405630.078	27.2
288	1875	6	4	2.6	51.5	0	V		2406044.347	82	86.7	13.0	13.0	11.1	7.2	0.45	2406043.432	22.0
289	1875	7	4	-23.7	38.0	122	V		2406074.245	92	123.0	20.7	20.5	16.1	12.4	1.06	2406072.726	36.5
290	1876	2	26	-23.7	38.0	122	V		2406311.178	81	354.5	17.0	16.2	15.0	5.3	0.69	2406309.764	33.9
291	1876	6	22	-23.7	38.0	122	I		2406428.245	62	103.3	11.7	11.7	10.5	4.7	0.38	2406427.428	19.6
292	1876	7	22	-23.7	38.0	122	V		2406458.239	73	141.8	21.6	21.1	13.4	16.2	1.11	2406456.704	36.9
293	1877	3	16	-23.7	38.0	122	V		2406695.191	90	14.2	18.0	17.3	16.9	1.7	0.78	2406693.621	37.7
294	1877	6	12	-23.7	38.0	122	V		2406783.243	79	97.3	15.5	15.2	13.5	6.6	0.61	2406782.106	27.3
295	1877	11	7	-23.7	38.0	122	V		2406931.140	73	254.3	28.9	29.0	10.2	27.1	2.00	2406928.866	54.6
296	1877	12	6	-23.7	38.0	122	V		2406960.130	68	275.3	20.6	20.5	9.6	18.0	1.03	2406958.420	41.1
297	1878	1	5	-23.7	38.0	122	V		2406990.140	109	308.4	23.2	22.6	16.9	14.7	1.27	2406988.086	49.3
298	1878	6	2	-23.7	38.0	122	V		2407138.239	102	92.4	20.4	19.8	17.5	9.0	1.00	2407136.575	39.9
299	1878	7	1	-23.7	38.0	122	V		2407167.246	61	115.5	15.8	15.2	10.7	10.5	0.59	2407166.022	29.4
300	1878	7	31	-23.7	38.0	122	V		2407197.234	58	153.2	24.9	24.5	10.9	21.9	1.46	2407195.403	43.9
301	1878	10	27	-23.7	38.0	122	V		2407285.149	45	237.9	23.8	23.9	6.9	22.9	1.38	2407283.458	40.6
302	1878	11	26	-23.7	38.0	122	V		2407315.132	109	274.3	30.1	29.7	14.9	25.7	2.12	2407312.883	54.0
303	1879	5	23	-23.7	38.0	122	V		2407493.234	135	89.9	27.7	27.1	23.7	12.7	1.80	2407490.743	59.8
304	1879	6	22	-23.7	38.0	122	V		2407523.245	120	124.6	33.8	33.2	22.1	25.0	2.64	2407520.346	69.6
305	1879	7	22	-23.7	38.0	122	V		2407553.240	99	161.1	41.6	41.4	18.9	37.3	3.97	2407549.879	80.7
306	1879	12	15	-23.7	38.0	122	V		2407699.131	139	293.3	30.0	29.4	20.4	21.3	2.09	2407696.962	52.1



1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
307	1881	3	30	2.6	51.5	0	V		2408170.278	69	20.9	10.7	10.6	10.6	1.3	0.32	2408169.439	20.1
308	1885	12	7	-5.7	50.6	213	V		2409883.151	75	268.1	12.4	12.6	8.8	8.6	0.42	2409882.053	26.3
309	1899	3	12	-13.3	52.5	61	V		2414726.213	80	4.0	12.0	12.1	11.8	1.5	0.40	2414725.328	21.2
310	1900	5	29	0.7	38.7	305	V		2415169.307	61	83.0	15.1	14.4	10.4	9.4	0.54	2415168.118	28.6
311	1901	4	19	2.8	50.7	30	V		2415494.301	69	41.8	12.8	12.0	10.4	5.9	0.39	2415493.402	21.6
312	1908	2	3	3.2	56.0	0	V		2417975.200	95	332.5	19.0	18.9	10.2	16.0	0.89	2417973.859	32.2
313	1908	5	1	-3.1	44.1	183	V		2418063.287	57	55.1	14.2	13.9	9.5	9.8	0.50	2418062.148	27.3
314	1909	2	21	0.0	51.1	30	V		2418359.227	86	350.7	18.2	18.2	11.8	13.9	0.84	2418357.953	30.6
315	1911	1	31	0.9	51.0	61	V		2419068.204	65	326.2	15.4	15.7	7.9	13.5	0.62	2419066.906	31.1
316	1911	6	27	-2.3	49.9	15	V		2419215.336	90	113.2	18.2	18.2	12.6	13.3	0.84	2419214.056	30.7
317	1911	8	25	-2.3	49.9	15	V		2419274.287	53	172.5	21.1	20.9	8.5	19.2	1.07	2419272.677	38.6
318	1913	11	28	-18.5	-33.9	91	V		2420100.237	49	254.7	8.8	9.1	8.9	0.4	0.24	2420099.570	16.0
319	1915	3	16	-8.7	49.4	213	V		2420573.232	62	5.5	10.5	10.1	9.5	1.6	0.29	2420572.321	21.8
320	1916	4	3	-8.7	49.4	213	V		2420957.252	83	26.7	13.0	13.0	12.5	1.5	0.46	2420956.181	25.7
321	1918	3	13	-5.1	50.2	91	V		2421666.238	82	5.4	13.1	13.0	12.7	1.5	0.46	2421665.328	21.8
322	1919	4	1	1.6	53.9	183	V		2422050.280	81	23.7	12.7	12.2	11.3	3.7	0.41	2422049.378	21.7
323	1920	4	19	-7.0	43.5	15	V		2422434.264	56	41.1	11.7	10.8	9.9	4.4	0.32	2422433.406	20.6
324	1921	2	8	71.1	42.3	30	V		2422729.423	57	329.5	9.9	9.9	9.8	0.3	0.28	2422728.526	21.5
325	1921	2	8	111.7	35.2	2469	V		2422729.549	58	331.1	11.3	11.2	9.7	0.7	0.35	2422728.526	24.6
326	1921	2	8	118.0	34.2	2012	V	F	2422729.567	58	331.3	11.5	11.4	10.0	0.9	0.36	2422728.526	25.0
327	1921	2	8	6.2	36.5	0	I		2422729.249	42	327.4	8.0	8.2	8.2	1.6	0.20	2422728.526	17.4
328	1921	2	8	9.1	38.8	0	I		2422729.254	44	327.5	8.0	8.3	8.3	1.2	0.20	2422728.526	17.5
329	1921	2	8	110.0	32.2	2441	V		2422729.547	55	331.0	11.3	11.2	9.6	1.4	0.35	2422728.526	24.5
330	1921	8	4	-18.5	-33.9	30	V		2422906.173	60	143.6	11.9	11.6	11.6	0.1	0.38	2422905.346	19.9
331	1921	10	31	-18.5	-33.9	30	I		2422994.218	36	227.1	9.4	8.9	7.2	5.1	0.22	2422993.485	17.6
332	1921	12	30	-18.5	-33.9	30	I		2423054.251	43	295.7	17.2	17.4	8.4	15.2	0.75	2423052.736	36.4
333	1922	1	29	-18.5	-33.9	30	I		2423084.247	40	328.2	19.1	19.0	8.0	17.2	0.89	2423082.492	42.1
334	1922	2	28	-18.5	-33.9	30	V		2423114.226	48	0.4	21.0	20.6	9.8	18.1	1.04	2423112.283	46.6



1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
335	1922	3	29	-18.5	-33.9	30	I		2423143.200	34	21.0	12.7	12.3	6.8	10.1	0.39	2423142.044	27.7
336	1922	4	27	-18.5	-33.9	30	I		2423172.174	23	41.9	5.3	5.2	4.4	2.6	0.09	2423171.710	11.1
337	1922	4	28	-18.5	-33.9	30	V		2423173.174	65	54.6	17.0	16.7	12.0	11.6	0.72	2423171.710	35.1
338	1922	5	27	-18.5	-33.9	30	V		2423202.158	56	76.7	11.1	11.3	10.1	4.8	0.35	2423201.253	21.7
339	1932	4	7	-36.2	50.0	122	V		2426805.180	107	35.2	17.6	17.1	15.8	5.5	0.75	2426803.557	38.9
340	1933	3	27	-7.7	48.6	122	V		2427159.246	109	25.1	18.5	18.0	17.1	4.5	0.84	2427157.640	38.6
341	1933	5	25	-33.9	55.6	213	V		2427218.257	107	78.6	14.6	14.6	11.4	8.7	0.56	2427216.922	32.0
342	1934	3	16	-33.9	55.6	213	V		2427513.161	101	10.3	15.0	14.6	13.5	4.6	0.57	2427512.006	27.7
343	1934	5	14	-36.2	50.0	122	V		2427572.219	92	67.1	13.9	13.8	12.4	5.4	0.50	2427571.021	28.7
344	1934	5	14	-5.1	48.2	244	V		2427572.301	94	68.1	14.8	14.7	13.1	5.6	0.57	2427571.021	30.7
345	1934	5	14	-33.9	55.6	213	V		2427572.243	105	67.4	14.2	14.1	11.7	7.2	0.52	2427571.021	29.3
346	1934	6	13	-33.9	55.6	213	V		2427602.272	90	100.6	18.6	18.2	9.9	15.2	0.83	2427600.592	40.3
347	1935	2	4	-5.1	48.2	244	V		2427838.199	84	329.4	14.4	13.7	12.0	5.9	0.51	2427837.185	24.3
348	1935	4	4	-5.1	48.2	244	V		2427897.261	105	31.2	17.1	16.9	16.2	3.0	0.75	2427896.008	30.1
349	1942	12	8	74.0	40.7	0	V	F	2430702.395	62	267.7	11.5	11.5	9.9	6.1	0.36	2430701.583	19.5
350	1953	4	14	-5.3	51.1	30	V	F	2434482.274	86	37.4	12.9	12.8	12.6	2.3	0.45	2434481.340	22.4
351	1954	3	5	88.0	44.5	244	V		2434807.492	69	356.9	12.1	11.9	11.5	0.2	0.39	2434806.633	20.6
352	1961	1	17	118.3	34.0	305	V		2437317.550	81	314.1	16.4	15.6	14.5	5.1	0.65	2437316.396	27.7
353	1962	4	5	28.2	-25.8	1372	V	F	2437760.329	55	29.7	14.1	14.1	10.9	7.3	0.53	2437759.323	24.1
354	1970	4	6	88.0	44.5	244	V	F	2440683.519	65	27.9	11.1	10.8	10.6	0.8	0.33	2440682.673	20.3
355	1970	4	6	122.0	48.0	152	V		2440683.616	77	29.3	12.4	12.0	11.8	2.0	0.40	2440682.673	22.6
356	1970	6	4	98.2	26.3	0	V		2440742.557	59	85.3	11.3	11.3	11.5	0.2	0.35	2440741.599	23.0
357	1970	6	4	97.0	28.0	30	V		2440742.557	60	85.3	11.3	11.3	11.3	0.3	0.35	2440741.599	23.0
358	1971	3	27	0.0	51.0	30	V		2441038.268	86	20.1	13.6	13.3	13.0	2.5	0.48	2441037.308	23.0
359	1971	4	25	88.2	39.5	183	V	F	2441067.529	70	47.1	11.9	12.0	11.8	0.3	0.40	2441066.669	20.7
360	1972	3	15	117.6	35.5	1128	I	B	2441392.586	42	4.0	8.4	8.6	7.3	2.6	0.21	2441391.983	14.5
361	1972	3	15	117.6	35.5	914	I	I	2441392.586	42	4.0	8.4	8.6	7.4	2.6	0.21	2441391.983	14.5
362	1973	3	5	85.0	40.0	305	V	F	2441747.486	66	357.7	12.5	12.4	11.9	1.0	0.42	2441746.505	23.6



1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
363	1973	7	1	-170.5	-44.0	1189	I	B	2441864.721	50	109.6	10.4	9.7	6.2	6.4	0.26	2441863.986	17.6
364	1976	12	21	91.6	42.0	274	V	F	2443134.446	70	282.0	11.7	11.8	10.2	5.1	0.38	2443133.590	20.6
365	1976	12	21	81.3	29.9	0	V		2443134.438	61	281.8	11.6	11.6	11.5	2.2	0.37	2443133.590	20.4
366	1976	12	21	122.5	37.6	0	V		2443134.539	72	283.3	13.0	12.9	12.2	4.9	0.45	2443133.590	22.8
367	1976	12	21	83.6	42.7	213	V		2443134.422	69	281.6	11.4	11.5	9.9	5.0	0.36	2443133.590	20.0
368	1977	2	18	87.7	43.8	30	V	F	2443193.477	57	340.3	10.1	9.8	9.6	0.8	0.27	2443192.652	19.8
369	1977	12	11	-20.0	47.8	183	V		2443489.120	76	272.2	12.7	12.8	9.7	8.0	0.44	2443488.232	21.3
370	1978	1	9	93.6	41.6	244	V		2443518.463	68	300.6	11.3	11.2	10.4	3.0	0.35	2443517.667	19.1
371	1978	1	9	76.9	38.9	30	V	B	2443518.420	63	300.0	10.7	10.7	10.4	2.1	0.32	2443517.667	18.1
372	1978	1	9	79.8	36.0	305	V	F	2443518.435	61	300.2	10.9	10.9	10.3	1.5	0.33	2443517.667	18.4
373	1978	1	9	79.8	36.0	305	V		2443518.435	61	300.2	10.9	10.9	10.3	1.5	0.33	2443517.667	18.4
374	1978	1	9	79.8	36.0	305	V	F	2443518.435	61	300.2	10.9	10.9	10.3	1.5	0.33	2443517.667	18.4
375	1978	1	9	81.1	34.0	152	V	F	2443518.442	60	300.3	11.0	10.9	10.6	1.1	0.34	2443517.667	18.6
376	1978	1	9	81.1	34.0	152	V	F	2443518.442	60	300.3	11.0	10.9	10.6	1.1	0.34	2443517.667	18.6
377	1978	1	9	84.3	33.9	335	V	F	2443518.452	60	300.5	11.1	11.1	10.5	1.1	0.34	2443517.667	18.8
378	1978	1	9	87.6	41.9	244	V		2443518.446	67	300.4	11.0	11.0	10.2	2.9	0.34	2443517.667	18.7
379	1978	1	9	82.7	27.7	0	V	F	2443518.454	56	300.5	11.2	11.1	11.3	0.3	0.35	2443517.667	18.9
380	1978	1	9	81.3	29.9	0	V		2443518.447	57	300.4	11.1	11.0	11.2	0.2	0.34	2443517.667	18.7
381	1978	1	9	81.3	29.9	0	V		2443518.447	57	300.4	11.1	11.0	11.2	0.2	0.34	2443517.667	18.7
382	1978	1	9	89.8	43.0	274	V	F	2443518.450	69	300.4	11.1	11.1	10.2	3.2	0.34	2443517.667	18.8
383	1978	1	9	90.2	30.0	0	V	F	2443518.472	59	300.8	11.4	11.3	11.5	0.4	0.36	2443517.667	19.3
384	1978	1	9	90.2	30.0	0	V	F	2443518.472	59	300.8	11.4	11.3	11.5	0.4	0.36	2443517.667	19.3
385	1978	1	9	90.2	30.0	0	V		2443518.472	59	300.8	11.4	11.3	11.5	0.4	0.36	2443517.667	19.3
386	1978	1	9	98.1	29.7	152	V		2443518.496	61	301.1	11.8	11.6	11.4	0.4	0.38	2443517.667	19.9
387	1978	3	9	64.2	45.1	9	V		2443577.428	52	359.5	10.5	9.6	9.0	3.4	0.26	2443576.609	19.7
388	1978	3	9	73.8	42.7	183	V	F	2443577.457	53	359.9	10.9	10.0	9.1	3.2	0.28	2443576.609	20.3
389	1978	3	9	72.9	41.3	61	V		2443577.454	52	359.9	10.8	9.9	9.4	2.9	0.28	2443576.609	20.3
390	1978	3	9	73.8	42.7	183	V		2443577.457	53	359.9	10.9	10.0	9.1	3.2	0.28	2443576.609	20.3



1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
391	1978	3	9	89.0	40.5	244	V		2443577.500	55	0.5	11.4	10.5	9.7	3.0	0.31	2443576.609	21.4
392	1978	3	9	93.6	41.6	244	V		2443577.512	57	0.6	11.6	10.7	9.9	3.3	0.32	2443576.609	21.7
393	1978	3	9	93.6	41.6	244	V		2443577.512	57	0.6	11.6	10.7	9.9	3.3	0.32	2443576.609	21.7
394	1978	3	9	89.0	40.5	244	V		2443577.500	55	0.5	11.4	10.5	9.7	3.0	0.31	2443576.609	21.4
395	1978	3	9	119.3	50.3	610	V		2443577.581	67	1.6	12.5	11.6	9.8	5.4	0.37	2443576.609	23.3
396	1978	3	9	72.9	41.3	30	V	F	2443577.454	52	359.8	10.8	9.9	9.4	2.9	0.28	2443576.609	20.3
397	1979	1	28	81.3	29.9	0	V		2443902.458	47	318.3	9.9	9.2	9.4	0.3	0.25	2443901.764	16.7
398	1979	1	28	82.4	29.7	0	V	B	2443902.461	47	318.4	9.9	9.3	9.4	0.3	0.25	2443901.764	16.7
399	1979	1	28	91.7	42.0	244	V	B	2443902.472	56	318.6	10.1	9.5	8.7	2.7	0.26	2443901.764	17.0
400	1979	1	28	90.3	38.7	183	V	B	2443902.473	53	318.6	10.1	9.4	8.9	2.1	0.26	2443901.764	17.0
401	1979	1	28	93.6	42.0	274	V		2443902.478	56	318.6	10.2	9.5	8.7	2.7	0.26	2443901.764	17.1
402	1979	1	28	122.3	47.6	30	V	F	2443902.545	65	319.7	11.1	10.5	9.7	4.5	0.31	2443901.764	18.7
403	1979	1	28	122.4	37.8	61	V	F	2443902.562	58	319.9	11.4	10.7	10.5	2.5	0.32	2443901.764	19.2
404	1980	7	13	70.7	41.4	0	V		2444434.512	58	132.4	20.7	20.2	10.2	17.5	1.01	2444432.782	41.5
405	1982	4	24	84.1	37.2	305	V		2445084.516	68	50.5	16.0	15.7	12.4	9.3	0.64	2445083.354	27.9
406	1983	11	5	84.1	37.2	305	I	V	2445644.444	44	235.8	12.8	12.4	7.2	9.7	0.40	2445643.432	24.3
407	1983	11	5	-35.6	15.6	335	I		2445644.132	34	231.6	8.9	8.3	7.2	3.1	0.20	2445643.432	16.8
408	1983	11	6	-35.6	15.6	335	V		2445645.132	81	245.0	21.4	20.6	16.9	11.1	1.08	2445643.432	40.8
409	1983	12	5	84.1	37.2	305	I	V	2445674.433	60	270.0	16.7	16.2	8.9	13.3	0.67	2445673.019	33.9
410	1983	12	5	-35.6	15.6	335	I		2445674.132	51	266.1	13.2	12.5	10.2	6.2	0.42	2445673.019	26.7
411	1984	1	3	-35.6	15.6	335	I		2445703.141	15	287.2	4.8	4.8	2.7	3.6	0.07	2445702.720	10.1
412	1984	1	3	84.1	37.2	305	I	I	2445703.441	25	290.9	8.1	8.2	3.5	7.1	0.18	2445702.720	17.3
413	1984	1	4	-35.6	15.6	335	V		2445704.141	67	299.4	16.0	15.6	13.5	6.6	0.63	2445702.720	34.1
414	1984	2	2	-35.6	15.6	335	I		2445733.152	28	320.2	7.2	7.8	5.8	4.6	0.17	2445732.491	15.9
415	1984	2	3	-35.6	15.6	335	V		2445734.153	77	332.1	18.1	17.9	16.7	4.9	0.83	2445732.491	39.9
416	1984	3	3	84.1	37.2	305	I	V	2445763.484	56	356.8	13.2	13.3	10.0	8.1	0.47	2445762.271	29.1
417	1984	3	3	-35.6	15.6	335	I		2445763.159	39	352.9	9.6	10.0	8.7	3.7	0.28	2445762.271	21.3
418	1984	3	4	-35.6	15.6	335	V		2445764.159	85	4.8	20.6	20.2	19.4	2.2	1.04	2445762.271	45.3



1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
419	1984	4	2	-35.6	15.6	335	I		2445793.163	52	25.9	12.8	12.4	11.7	1.8	0.42	2445792.007	27.7
420	1984	4	3	-35.6	15.6	335	V		2445794.163	100	38.1	24.1	23.4	22.7	0.4	1.37	2445792.007	51.7
421	1984	5	1	84.1	37.2	305	I	I	2445822.520	43	51.8	10.0	9.3	7.5	4.9	0.24	2445821.656	20.7
422	1984	5	2	-35.6	15.6	335	V		2445823.167	74	60.0	17.6	16.7	16.1	0.3	0.73	2445821.656	36.2
423	1984	5	31	-35.6	15.6	335	I		2445852.173	51	82.3	11.9	11.1	10.5	0.5	0.34	2445851.201	23.3
424	1984	6	1	-35.6	15.6	335	V		2445853.173	110	95.8	24.5	23.7	23.1	1.5	1.42	2445851.201	47.3
425	1984	6	30	-35.6	15.6	335	V		2445882.178	89	119.3	20.2	19.7	18.8	3.6	1.00	2445880.639	36.9
426	1984	7	29	-35.6	15.6	335	V		2445911.176	69	143.0	16.3	16.1	15.0	4.3	0.68	2445909.995	28.3
427	1984	8	27	-35.6	15.6	335	I		2445940.165	49	166.7	12.2	12.1	11.1	3.6	0.41	2445939.310	20.5
428	1984	8	28	-35.6	15.6	335	V		2445941.164	95	181.9	26.4	25.9	21.9	13.1	1.67	2445939.310	44.5
429	1984	9	25	-35.6	15.6	335	I		2445969.149	30	190.1	7.4	7.3	6.7	1.7	0.16	2445968.634	12.4
430	1984	9	26	-35.6	15.6	335	V		2445970.149	75	205.3	21.6	21.0	17.0	11.6	1.11	2445968.634	36.4
431	1984	10	25	-35.6	15.6	335	I		2445999.135	55	228.1	15.7	14.9	11.8	8.6	0.59	2445998.007	27.1
432	1984	10	26	-35.6	15.6	335	V		2446000.135	106	242.7	29.3	28.6	21.8	18.3	2.00	2445998.007	51.1
433	1984	11	23	81.0	34.0	61	I	V	2446028.430	39	254.8	13.0	12.6	6.4	10.7	0.41	2446027.457	23.3
434	1984	11	23	-35.6	15.6	335	I		2446028.130	30	250.5	9.0	8.3	5.9	5.3	0.20	2446027.457	16.2
435	1984	11	24	-35.6	15.6	335	V		2446029.130	86	264.7	22.2	21.5	16.9	12.8	1.17	2446027.457	40.2
436	1984	12	23	-35.6	15.6	335	V		2446058.137	58	286.4	14.4	14.2	11.3	7.8	0.53	2446056.991	27.5
437	1985	1	21	155.0	19.0	4145	V		2446087.679	54	314.9	12.8	12.9	9.2	6.4	0.44	2446086.604	25.8
438	1985	4	20	84.1	37.2	305	I	I	2446176.513	37	39.4	8.5	7.8	6.6	3.4	0.17	2446175.724	19.0
439	1986	10	5	73.2	40.8	30	V		2446709.438	58	220.6	28.3	28.0	9.5	26.4	1.87	2446707.288	51.6
440	1986	12	31	77.0	39.0	30	I	B	2446796.415	32	291.1	11.2	11.8	4.6	10.9	0.36	2446795.633	18.8
441	1987	4	28	77.1	38.9	30	I		2446914.500	57	49.2	11.0	10.5	10.4	1.1	0.31	2446913.566	22.4
442	1987	4	28	77.1	38.9	30	I	B	2446914.500	57	49.2	11.0	10.5	10.4	1.1	0.31	2446913.566	22.4
443	1987	4	28	77.1	38.9	30	I		2446914.500	57	49.2	11.0	10.5	10.4	1.1	0.31	2446913.566	22.4
444	1987	4	28	77.1	38.9	30	V	F	2446914.500	57	49.2	11.0	10.5	10.4	1.1	0.31	2446913.566	22.4
445	1987	4	28	77.1	38.9	30	V	F	2446914.500	57	49.2	11.0	10.5	10.4	1.1	0.31	2446913.566	22.4
446	1987	4	28	81.7	36.2	1402	I		2446914.513	59	49.3	11.1	10.7	9.6	0.7	0.32	2446913.566	22.7



1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
447	1987	4	28	81.7	36.2	1402	V	F	2446914.513	59	49.3	11.1	10.7	9.6	0.7	0.32	2446913.566	22.7
448	1987	4	28	81.7	36.2	1402	V		2446914.513	59	49.3	11.1	10.7	9.6	0.7	0.32	2446913.566	22.7
449	1987	4	28	81.8	26.7	0	V	F	2446914.499	50	49.2	11.0	10.5	10.5	1.5	0.31	2446913.566	22.4
450	1987	4	28	82.5	28.0	0	I		2446914.502	52	49.2	11.0	10.6	10.6	1.2	0.31	2446913.566	22.5
451	1987	4	28	84.4	33.7	335	I		2446914.515	57	49.4	11.2	10.7	10.3	0.1	0.32	2446913.566	22.8
452	1987	4	28	84.5	42.7	244	V	B	2446914.526	66	49.5	11.3	10.9	10.4	2.1	0.33	2446913.566	23.0
453	1987	4	28	84.5	42.7	244	V	F	2446914.526	66	49.5	11.3	10.9	10.4	2.1	0.33	2446913.566	23.0
454	1987	4	28	87.4	33.0	61	I		2446914.521	57	49.4	11.3	10.8	10.7	0.1	0.32	2446913.566	22.9
455	1987	4	28	87.7	40.8	244	I		2446914.532	64	49.6	11.4	10.9	10.5	1.7	0.33	2446913.566	23.2
456	1987	4	28	90.1	30.0	0	V		2446914.525	55	49.5	11.3	10.8	11.0	0.7	0.33	2446913.566	23.0
457	1987	4	28	93.7	41.6	244	I		2446914.550	66	49.8	11.6	11.1	10.7	1.9	0.34	2446913.566	23.6
458	1987	4	28	97.7	30.3	183	V		2446914.548	57	49.8	11.6	11.1	10.8	0.6	0.34	2446913.566	23.6
459	1987	4	28	104.0	30.6	2408	V	F	2446914.570	58	50.0	11.8	11.3	9.8	0.4	0.36	2446913.566	24.1
460	1987	4	28	104.0	30.6	2408	V		2446914.570	58	50.0	11.8	11.3	9.8	0.4	0.36	2446913.566	24.1
461	1987	4	28	111.9	40.7	1311	V		2446914.602	69	50.5	12.2	11.7	10.5	2.0	0.38	2446913.566	24.9
462	1987	4	28	122.0	37.0	1494	V		2446914.626	67	50.8	12.5	12.0	10.8	1.2	0.39	2446913.566	25.4
463	1987	4	28	77.0	38.9	30	V	F	2446914.500	57	49.2	11.0	10.5	10.4	1.1	0.31	2446913.566	22.4
464	1987	5	28	105.5	39.2	1981	V		2446944.603	95	83.7	16.5	16.3	14.1	5.3	0.70	2446943.135	35.2
465	1987	6	26	71.0	-30.1	2774	I	B	2446973.419	17	102.3	7.5	8.6	1.4	8.0	0.19	2446972.735	16.4
466	1987	6	26	84.5	42.7	244	V	B	2446973.559	58	103.9	9.0	9.5	8.2	3.8	0.25	2446972.735	19.8
467	1987	6	26	100.0	30.0	457	I		2446973.576	51	104.2	9.2	9.6	8.8	1.4	0.26	2446972.735	20.2
468	1987	6	26	105.0	39.8	1676	I		2446973.612	58	104.6	9.6	10.0	7.9	3.8	0.28	2446972.735	21.1
469	1987	6	26	111.9	40.7	1311	V	B	2446973.633	59	104.8	9.9	10.2	8.2	4.1	0.29	2446972.735	21.6
470	1987	6	26	112.1	33.5	329	V		2446973.616	54	104.6	9.7	10.0	9.1	2.4	0.28	2446972.735	21.1
471	1987	6	26	122.0	37.0	1494	V		2446973.653	58	105.1	10.1	10.4	8.5	3.5	0.30	2446972.735	22.0
472	1987	6	26	84.1	37.2	305	I	V	2446973.545	53	103.8	8.9	9.3	8.4	2.6	0.25	2446972.735	19.5
473	1987	9	23	84.1	37.2	305	I	I	2447062.484	17	190.3	9.9	9.5	2.8	9.0	0.23	2447061.632	20.5
474	1988	1	19	111.0	32.2	780	I	V	2447180.535	47	310.7	11.5	11.3	7.7	7.6	0.34	2447179.727	19.4



1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
475	1988	4	16	84.1	37.2	305	I	I	2447268.511	36	34.0	6.8	6.6	6.2	1.3	0.14	2447268.000	12.3
476	1988	6	14	84.1	37.2	305	I	I	2447327.543	50	92.0	7.8	8.3	7.8	1.2	0.20	2447326.885	15.8
477	1989	4	6	107.2	34.0	3018	V		2447623.572	67	30.4	13.1	12.8	11.1	1.5	0.45	2447622.648	22.2
478	1989	5	5	85.7	43.0	244	I	V	2447652.536	52	53.1	7.6	8.1	7.9	0.7	0.20	2447651.992	13.1
479	1989	5	5	105.5	39.7	3353	I	V	2447652.592	53	53.9	8.4	8.7	7.0	0.8	0.23	2447651.992	14.4
480	1989	5	5	84.8	42.7	259	I	V	2447652.533	51	53.0	7.6	8.1	7.8	0.8	0.19	2447651.992	13.0
481	1989	5	5	84.8	42.7	259	I	I	2447652.533	51	53.0	7.6	8.1	7.8	0.8	0.19	2447651.992	13.0
482	1989	5	5	97.0	30.3	183	I	V	2447652.550	41	53.3	7.8	8.3	7.6	2.8	0.20	2447651.992	13.4
483	1989	6	3	155.5	19.8	4255	I	I	2447681.715	25	78.7	5.2	6.1	3.2	3.1	0.12	2447681.328	9.3
484	1989	6	4	1.0	50.8	0	V		2447682.341	82	87.6	13.4	13.3	11.1	7.7	0.47	2447681.328	24.3
485	1989	7	4	121.6	37.4	1283	V		2447712.652	89	127.1	24.0	23.4	15.1	17.0	1.35	2447710.708	46.7
486	1989	10	2	108.8	36.1	1981	V		2447802.547	68	223.9	34.1	34.2	9.9	32.2	2.74	2447799.409	75.3
487	1990	2	25	83.5	35.6	1524	V	V	2447948.482	38	345.2	8.1	7.5	6.2	0.5	0.17	2447947.872	14.6
488	1990	2	25	83.5	35.6	1524	I	V	2447948.482	38	345.2	8.1	7.5	6.2	0.5	0.17	2447947.872	14.6
489	1990	2	25	83.5	35.6	1524	I	I	2447948.482	38	345.2	8.1	7.5	6.2	0.5	0.17	2447947.872	14.6
490	1990	4	25	73.7	41.6	30	I	V	2448007.492	66	47.0	11.5	11.6	11.5	0.1	0.37	2448006.687	19.3
491	1990	4	25	121.5	37.7	61	V		2448007.620	75	49.0	13.4	13.3	13.3	0.2	0.48	2448006.687	22.4
492	1990	5	24	118.1	34.2	530	I	V	2448036.624	51	72.7	9.0	9.0	8.5	0.3	0.24	2448035.992	15.2
493	1990	5	24	118.1	34.2	530	V	V	2448036.624	51	72.7	9.0	9.0	8.5	0.3	0.24	2448035.992	15.2
494	1990	5	24	83.5	35.6	1524	I	V	2448036.532	46	71.3	7.7	7.8	6.8	0.1	0.19	2448035.992	13.0
495	1990	5	24	110.5	31.6	1372	I	V	2448036.600	48	72.3	8.7	8.7	7.7	0.3	0.22	2448035.992	14.6
496	1990	11	19	76.8	39.0	79	V		2448215.412	87	264.8	27.5	27.2	12.0	24.4	1.78	2448212.879	60.8
497	1990	12	18	76.8	39.0	79	V		2448244.410	78	285.5	18.7	18.2	11.4	14.0	0.83	2448242.682	41.5
498	1991	3	17	76.8	39.0	79	V		2448333.471	108	17.6	20.7	20.4	20.1	0.6	1.06	2448331.841	39.1
499	1991	5	15	76.8	39.0	79	V		2448392.510	132	80.1	25.4	24.7	22.6	10.1	1.53	2448390.692	43.6
500	1992	3	5	76.8	39.0	79	V		2448687.463	83	1.6	15.9	15.8	15.5	0.4	0.66	2448686.058	33.7
501	1995	1	1	106.0	33.0	1219	I	V	2449719.511	43	288.8	7.7	7.9	7.0	0.1	0.19	2449718.956	13.3
502	1995	6	28	71.0	-30.1	2774	V		2449897.420	51	106.5	9.7	10.0	8.1	0.4	0.28	2449896.536	21.2



1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
503	1996	1	20	111.0	32.4	853	I	V	2450103.536	38	307.4	7.1	7.6	6.7	1.8	0.17	2450103.036	12.0
504	1996	1	20	113.2	32.8	259	I	V	2450103.539	39	307.4	7.2	7.6	7.2	1.7	0.18	2450103.036	12.1
505	1996	1	20	118.3	34.1	530	I	V	2450103.553	41	307.6	7.4	7.8	7.2	1.5	0.18	2450103.036	12.4
506	1996	1	20	118.3	34.1	530	I	I	2450103.553	41	307.6	7.4	7.8	7.2	1.5	0.18	2450103.036	12.4

## The Babylonian Criterion of First Visibility Of the Lunar Crescent<sup>1</sup>

The accurate prediction of the evening of first visibility of the new crescent was of major significance for the Babylonians. This matter was of such an importance that it was the main goal of the Babylonian lunar theory in the Seleucid period (commencing 311 B.C.) (Neugebauer, 1955: 41). The Babylonians succeeded in formulating a truly mathematical lunar theory which they used for predicting various parameters of the lunar motion, as found recorded in the lunar ephemerides they prepared.

Modern investigators of the problem of first visibility of the new crescent, who are not themselves scholars of Babylonian astronomy, have frequently claimed that the Babylonian conditions of visibility were that the new moon is more than 24 hours of age and that the arc of separation should be equal to or greater than  $12^\circ$ , i.e. the moon sets at least 48 minutes after sunset. This supposed Babylonian criterion is also often cited as a single condition:  $S \geq 12^\circ$ . It seems that Bruin was the first modern researcher who attributed this criterion to the Babylonians in his well-known paper on lunar visibility (Bruin, 1977: 333) and that all subsequent researchers who reiterated this claim were simply relying on his account (see for instance Ilyas, 1994; Schaefer 1988). However, it should be noted that Bruin did not cite any reference in support of his claim. Bruin seems to have suggested this because he noted that the simple rule of  $S \geq 12^\circ$  was used by Arab astronomers from the 8th century onward; he believed that it might have transmitted to them from the Hindus who would have learned it from the Babylonians. However, Bruin's claim with regard to the Babylonian condition of lunar visibility is, at best, inaccurate. The  $12^\circ$  equatorial difference is indeed the crescent visibility criterion adopted by the Indian *Suryasiddhanta* (ca. 600) and the *Khandakhadyaka* (650), as pointed out by King (1987). However, even though Babylonian astronomical knowledge had passed to the Indians (by way of the Greeks) this does not necessarily mean that this was *the* Babylonian criterion of crescent visibility.

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<sup>1</sup> A paper based on this chapter and relevant information from chapter two is currently in press (Fatoohi *et al*, 1998a).



Study of the Babylonian lunar ephemerides has revealed that they are based on two somewhat different versions of lunar theory, usually referred to as “System A” and “System B”. According to System A, the sun moves with constant velocities on two different arcs of the ecliptic, whereas System B assumes that the solar velocity changes with time in a linear zigzag function (Neugebauer, 1955). The different representations of solar motion yield differences in the calculation of sun-moon conjunction. The difference between the two theories is usually represented by figure 4.1.

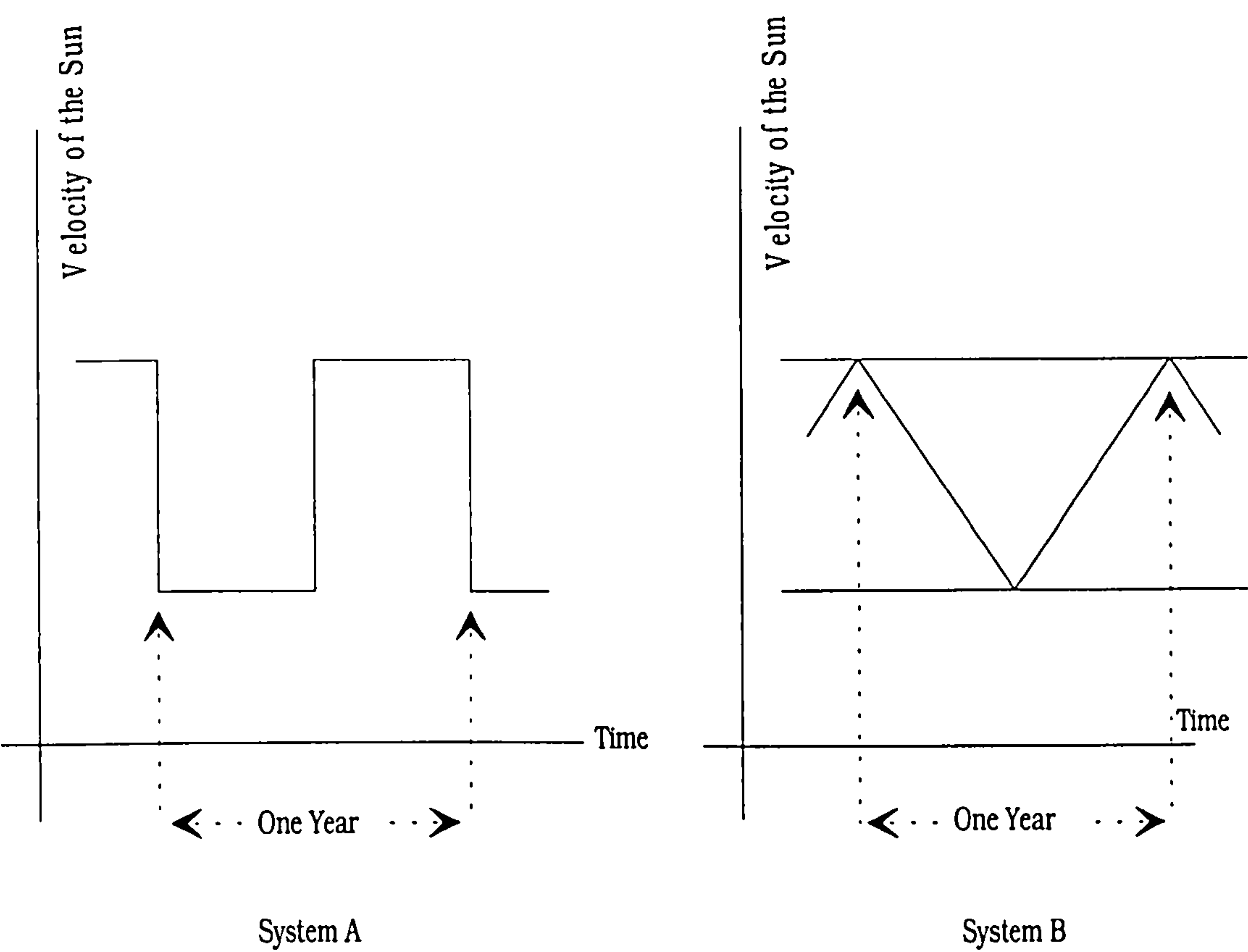


Figure 4.1. The two representations of solar motion in the Babylonian lunar theory

It is interesting to note that although System B must have been an improvement of System A, both Systems were used simultaneously throughout the period 250-50 B.C. in preparing ephemerides. Neugebauer notes that such a practice, which is contrary to our modern scientific concepts where new theories replace old ones, is yet more prominent in the planetary theory (Neugebauer, 1957: 115). The lunar ephemerides were used by the Babylonians to predict the first and last visibility of the moon. A comparative list of the main columns of computations of

a complete ephemeris in the two system is given in table 4.1 (adapted from Neugebauer, 1955: 43).

**Table 4.1** The columns of astronomical calculations included by Babylonian astronomers in each ephemeris of System A and System B. As can be seen, some parameters are calculated in ephemerides of both Systems whereas others are restricted to one System or the other. Although the last four quantities are missing from the tables of System A, preserved procedure texts tell us that they were calculated; they would be necessary for finding the lagtime.

System A	System B
Dates	
Relative velocity of the moon with respect to the sun(?)	
	Velocity of the sun
Longitude of the moon	
Length of daylight	
	Half length of the night
Latitude of the moon	
Magnitude of eclipses	
Velocity of the moon	
Length of the month in first approximation	
	Correction related to the next column
Correction in the length of the month caused by the variability of solar velocity	
Second correction to the length of the month	
Length of the month	
	Date of syzygy, midnight epoch
Date of syzygy, evening epoch	Date of syzygy, evening or morning epoch
	Time difference between syzygy and sunset or sunrise
	Elongation of first or last visibility
	Influence of the obliquity of the ecliptic
	Influence of the latitude
Duration of first or last visibility (lagtime)	

Although the existence of procedure texts which give criteria for determining the first and last visibility of the moon is hard to doubt, so far, unfortunately, no such texts have come to light (Christopher Walker, private communication). Therefore, it is only through the analysis of individual cases in the ephemerides that certain criteria can be concluded.



Contrary to what is commonly assumed about the Babylonian criterion, Neugebauer (1955) found from the study of extant ephemerides that the moonset lagtime *alone* could not have been used as the visibility criterion by the Babylonians in any of the two Systems. He suggests that a criterion of the following form might have been used by the Babylonians for both Systems:

$$\text{elongation (L) + moonset lagtime (in degrees) (S) > constant} \quad (4.1)$$

Neugebauer suggests that the rationale behind the inclusion of the elongation in the criterion of first visibility would be that the elongation determines, in addition to the angular distance between the sun and moon, the width of the visible crescent. Therefore, the above criterion would mean that the probability of sighting the new crescent increases with the width of the crescent and with the time in which the crescent remains above the horizon before setting.

As for the value of the constant in the above criterion, Neugebauer has found from his study of preserved texts that in the case of System A the constant could have been about  $21^\circ$ . In other words, the Babylonian visibility criterion for System A would be:

$$L + S > 21^\circ \quad (4.2)$$

In the case of System B, Neugebauer found two ephemerides which suggest a value of about  $23^\circ$  for the constant whereas another suggests  $\geq 20^\circ$  and a third accepts a value as low as  $\geq 17^\circ$ . This represents a marked range.

Interestingly, Neugebauer notes that the moonset lagtime might have been used alone for predicting the visibility of the new moon in extreme cases. He concludes this from the existence of isolated lists of lagtimes that seem to have been collected for several years in succession. The lowest values found in these texts are  $11.33^\circ$ ,  $11.66^\circ$ , and  $11.83^\circ$ , and these are followed by a phrase of unknown meaning. The highest value of lagtime given is  $25.16^\circ$  without alternative, although an ephemeris preserved for the same year accepts instead  $12^\circ$ . One alternative solution of  $20.5^\circ$  for a full month (30 days) and  $10.5^\circ$  for a hollow month (29 days) is also given (Neugebauer, 1955: 67, 84; 1975: 539-540).

I have found that the smallest value of  $L+S$  in the 209 Babylonian observations which I have compiled is about  $22.9^\circ$  (observation 89), which is almost the same limit suggested by Neugebauer for System B and which is still very close to the limit of  $21^\circ$  that he suggested for System A. The highest value of  $L+S$  that I have found is  $58.6^\circ$  (observation 143). Therefore, while exceeding the  $23^\circ$  limit does not ensure visibility of the lunar crescent, this limit may have been used by the Babylonians as the lowest limit for the visibility of the crescent.

The latitude of Babylon is about  $32.6^\circ$  N. To test the reliability of the above criterion that the Babylonians might have used, I applied it to all entries of latitudes within the range  $\pm$

(30-35) of table 3.1. I assumed that the Babylonian criterion was  $L+S \geq 23^\circ$ , as this is the smallest value in the Babylonian data. I found that the quantity  $L+S$  is less than  $23^\circ$  for only 3 of the 239 positive observations from latitudes close to that of Babylon. But while this criterion thus misjudges only 1.3% of the *positive* observations, it has 6 of the 19 *negative* observation in the visibility zone, i.e.  $L+S$  greater than  $23^\circ$ . The latter result represents a very high percentage of error, 31.6%. The unreliability of this criterion becomes even more manifest when applied to the data from all latitudes. Six of the total 422 positive observations (i.e. 1.4%) are wrongly placed according to the Babylonian criterion, but as many as 38 of the 84 negative observations (i.e. 45.2%) contradict the visibility condition. Certainly, this would be a very bad global criterion, but it might give a useful indication when not to bother looking for the moon.

There have been modern attempts to formulate modern crescent visibility criteria that would predict the dates when the crescent could have been visible in Babylon. These attempts were originally triggered by interest in determining the beginnings of the Babylonian months which would help in determining the equivalent Julian dates of Babylonian records. One such solution was first attempted by Karl Schoch who designed tables for determining the evening of the first sighting of the lunar crescent which are applicable to all places whose latitudes differ little from that of Babylon. Schoch also presented his lunar visibility tables, following Fotheringham (1910), in the form of a curve of true lunar altitude ( $h$ ) versus the azimuthal difference between the sun and moon ( $\Delta Z$ ) at sunset, so that the new moon would be first visible on the first evening after conjunction in which the moon falls above the curve (Schoch, 1928: 95) (see table 4.2). However, the criterion of Schoch suffers from the important flaw of being based on both observations and *predictions* of the lunar crescent (Neugebauer, 1951). Even Schoch's identification of what he considered to have been observations was not totally sound. For instance, Schoch states that, "The most valuable observations for my purpose are the most ancient, belonging to a time when the Babylonians were unable to compute the appearance of the crescent, i.e. the time from Rim-Sin to Ammizaduga and from Nebuchadnezzar to Xerxes" (Schoch, 1928: 98). But the fact that the Babylonians were at some stage of their history unable to predict the first appearance of the crescent does not necessarily mean that they did not follow some simple rules in fixing their calendar, the most probable and simple of such rule is that the month would be of either 29 or 30 days. If such basic a rule was followed, then the length of the Babylonian months determined according to this rule would have no implication whatsoever for the visibility of the moon. (It should be stressed that the skies of Babylon are

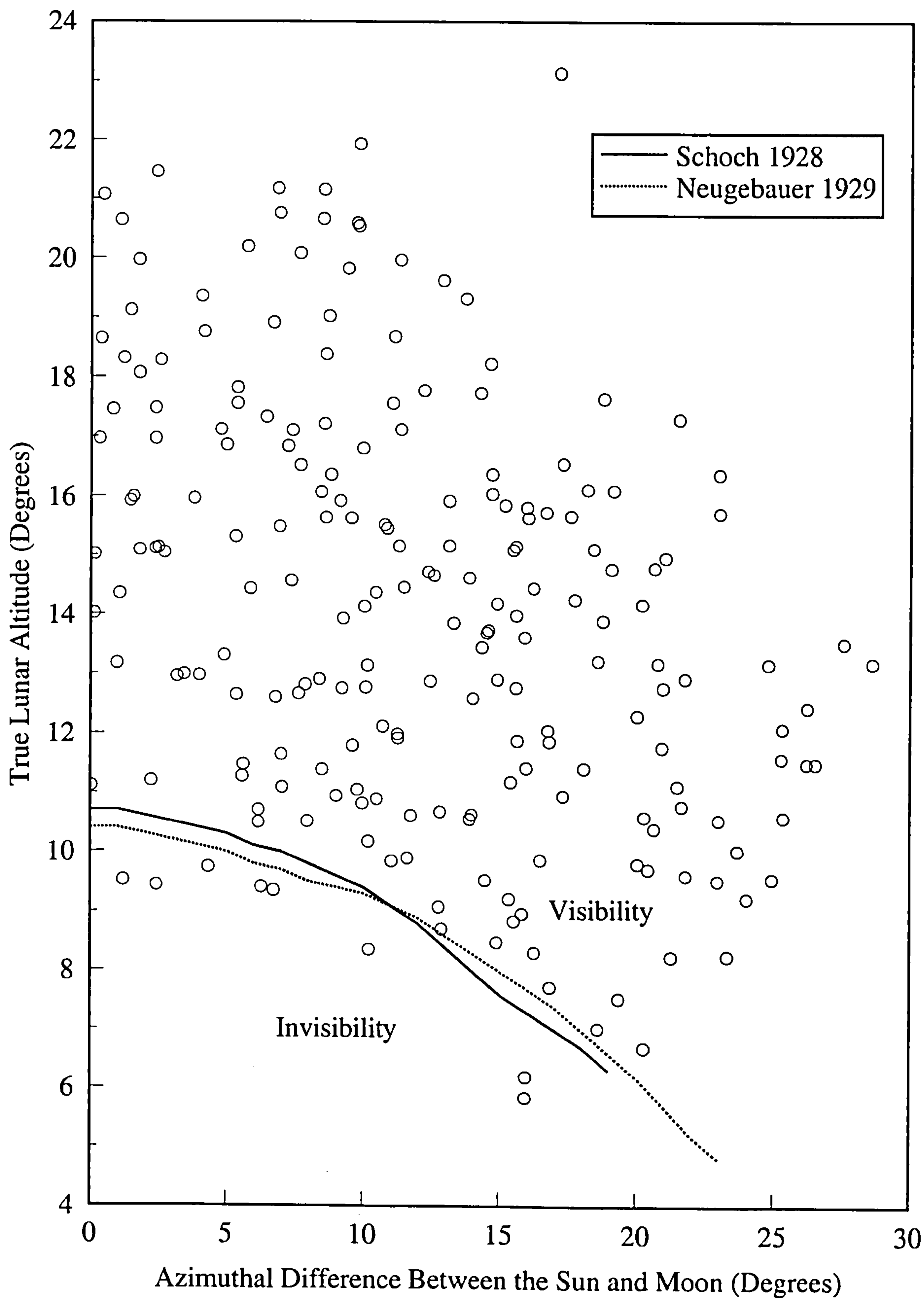


often cloudy in winter, for example). It was exactly to avoid using such pseudo-observational data that for the present project I collected only actual observations of the lunar crescent. Although Fotheringham (1928: 48) expresses his confidence in Schoch's criterion for computing the first visibility of the lunar crescent at Babylon, it seems fair to conclude that Schoch's solution can neither be regarded as observational nor theoretical; hence it is likely to lead to errors in predicting the dates of first sightings of the lunar crescent in Babylon.

**Table 4.2** The criteria of K. Schoch and P. V. Neugebauer. At any specified azimuthal difference from the sun, the crescent is expected to be visible when the moon is not lower than a critical true altitude at sunset.

Azimuthal Difference ( $\Delta Z$ )	Minimum True Lunar Altitude (h)		Azimuthal Difference ( $\Delta Z$ )	Minimum True Lunar Altitude (h)	
	Schoch	Neugebauer		Schoch	Neugebauer
0°	10.7°	10.4°	12°	8.8°	8.9°
1	10.7	10.4	13	8.4	8.6
2	10.6	10.3	14	8.0	8.3
3	10.5	10.2	15	7.6	8.0
4	10.4	10.1	16	7.3	7.7
5	10.3	10.0	17	7.0	7.4
6	10.1	9.8	18	6.7	7.0
7	10.0	9.7	19	6.3	6.6
8	9.8	9.5	20	-	6.2
9	9.6	9.4	21	-	5.7
10	9.4	9.3	22	-	5.2
11	9.1	9.1	23	-	4.8

Another criterion for determining the first visibility of the lunar crescent at Babylon was suggested by P. V. Neugebauer. This solution uses the same two parameters employed by Schoch, i.e.  $\Delta Z$  and h, but the suggested curve lies a little below that of Schoch for smaller  $\Delta Z$  and slightly above it for larger  $\Delta Z$  (Neugebauer, 1929: table E 21). However, the differences



**Figure 4.2** The visibility criteria of Schoch and Neugebauer together with the Babylonian positive observations.



between both curves are too small to be of any significance in practical use. Neugebauer's curve also extends to  $23^\circ$  of  $\Delta Z$  in contrast to that of Schoch which covers only up to  $19^\circ$  of  $\Delta Z$  (see table 4.2 for both criteria). I have not come across any other modern criterion that is based on Babylonian data or is designed to predict the lunar visibility in Babylon in particular. Researchers into the Babylonian calendar have relied on one or the other of the above criteria (see for example Parker and Dubberstein (1956) who use that of Schoch and Huber (1982) who opted for that of Neugebauer).

I have examined both criteria of Schoch and Neugebauer using the 209 observations that I have collected from the Babylonian diaries. Because these are real observations, they can serve as a very reliable indicator of the accuracy of both criteria. I have plotted in figure 4.2 the visibility curves of Schoch and Neugebauer as well as the 209 Babylonian observations. The graph shows that both models are reasonably good in predicting the observations. Of the 209 positive observations, only 8 fell below the visibility curves. In other words, according to the criteria of Schoch and Neugebauer about 3.8% of the sighted crescents would have been invisible. However, if the visibility curve is lowered so that it starts from about  $h=9.5^\circ$  (rather than about  $10.5^\circ$ ) for  $\Delta Z=0^\circ$  then all of the observations would be above the visibility curve, i.e. in the visibility zone.

It should be stressed, however, that the fact that this modified curve would have almost all positive observations in the visibility zone does not tell us anything about the suitability of this criterion for hypothetical negative observations from Babylon. In other words, it is evident that while lowering the dividing line would include all the positive observations in the visibility zone, (i.e. above the curve), the revised curve would have significantly more negative observations in its visibility zone than the original curves would. This drawback in the criteria of Neugebauer and Schoch would have become manifest if the Babylonian data included actual negative observations in addition to the positive. Indeed, when I later discuss in detail modern forms of the  $h$ - $\Delta Z$  visibility criterion I show, using medieval and modern mid-latitudes data from table 3.1, that Neugebauer's model has already 21% of the negative observations in the visibility zone (see §7.2 for details).

## The Islamic Calendar and First Visibility of the Lunar Crescent

The great interest that the Muslims developed in astronomy has its origin in Islam itself. The Holy Qur'an states that astronomical phenomena are for man to make use of. For instance, stars can be used for guidance: **{And by the star they find a way}** [from 016.016].<sup>1</sup>

﴿وَبِالنُّجُومِ هُمْ يَهْتَدُونَ﴾

More importantly, the holy book of Islam imposed on the believer a number of worshipping practices the timing and/or dating of which require astronomical knowledge. For instance, the performance of the five daily prayers requires the determination of the solar altitude or depression beforehand, while it is also necessary to fix the *Qibla*, i.e. the direction of the Ka'ba in Macca which the Muslims everywhere have to face when they pray. At the beginning of the eleventh century, this is how the celebrated Egyptian astronomer ibn Yunus explained the bearing of astronomical knowledge on Islamic religious practices as well as other social matters at the start of his well-known astronomical work called *al-Zij al-Hakimi*<sup>2</sup>:

The observation of heavenly bodies is connected with religious law, since it permits knowledge of the time of prayer, of the time of sunrise which marks the prohibition of drinking and eating for him who fasts, of the moment when daybreak finishes, of the time of sunset whose ending marks the start of the evening meal and cessation of religious obligations, and moreover knowledge of the moment of eclipses so that the corresponding prayers can be made, and also knowledge of the direction of the Ka'ba (towards Macca) for all those who pray, and equally knowledge of the beginning of the months and of days involving doubt, and knowledge of the time of sowing, of the pollination of trees and the harvesting of fruit, and knowledge of

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<sup>1</sup> Qur'anic verses cannot be translated accurately from Arabic, their original language, because no translation can cover the deep meanings of a Qur'anic verse. I have included, therefore, the Arabic text of every cited Qur'anic verse after an appropriate, though inevitably limited, translation of its meanings.

<sup>2</sup> "The term 'zij' is of Persian origin corresponding to the Greek kanôn; in its proper sense it denotes collections of tables of motion for the stars, introduced by explanatory diagrams which enable their compilation; but it is also often used as a generic term for major astronomical treatises which include tables" (Morelon, 1996a: 1). This particular zij of ibn Yunus was dedicated to the Fatimid Caliph al-Hakim (996-1021 A.D.), hence its title. This monumental work is in eighty-one chapters of which only a little more than half is preserved.



the direction of one place from another, and of how to find one's way without going astray (trans. Morelon, 1996a: 15).

Because the Islamic calendar is lunar, the question of the first visibility of the lunar crescent was one of the issues that Muslims astronomers, and also non-Muslim astronomers who worked in Islamic lands, dedicated much time and effort to. But before investigating the contribution of Islamic astronomy to this scientific question, it is in order to look briefly at the history and forms of the Islamic calendar - the regulation of which was the main reason of the Muslims' huge interest in this matter.

## 5.1 Characteristics of the Islamic Calendar

The characteristics of the Islamic calendar are derived from two main sources, the Holy Qur'an and sayings of the Prophet Muhammad (*Salla Allah ta'ala 'alayhi wa sallam*),<sup>3</sup> also known as Prophetic traditions or *hadith*. The Holy Qur'an clearly states that the year of the Islamic calendar consists of twelve months:

**{The number of months in the sight of Allah is twelve - In the book of Allah the day He created the heavens and the earth } [from 009.036].**

﴿إِنَّ عِدَّةَ الشُّهُورِ عِنْدَ اللَّهِ اثْنَا عَشَرَ شَهْرًا فِي كِتَابِ اللَّهِ يَوْمَ خَلَقَ السَّمَاوَاتِ وَالْأَرْضِ﴾.

It also contains several verses indicating that the moon and the sun are both to be used for calendrical calculations:

**{It is He Who appointed the sun a shining brightness and the moon a light, and ordained for it mansions, that you might know the numbering of years, and the reckoning. Allah created not (all) that save in truth. He details the signs for people who have knowledge} [010.005].**

﴿هُوَ الَّذِي جَعَلَ الشَّمْسَ ضِيَاءً وَالْقَمَرَ نُورًا وَقَدَرَهُ مَنَازِلَ لِتَعْلَمُوا عَدَدَ السِّنِينَ وَالْحِسَابَ مَا خَلَقَ اللَّهُ ذَلِكَ إِلَّا بِالْحَقِّ يُفَصِّلُ الْآيَاتِ لِقَوْمٍ يَعْلَمُونَ﴾.

**{And We made the night and the day two signs; then We made the sign of the night dark and We made the sign of the day sight-giving, so that you may seek grace from**

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<sup>3</sup> Muslims mention this phrase after the name of the Prophet Muhammad (*Salla Allah ta'ala 'alayhi wa sallam*) in compliance with a Qur'anic command [033.056]. This phrase cannot be translated because its meanings are not quite understood. A rough translation would be "May Allah (High is He) send blessings and peace on him (the Prophet)".

your Lord, and that you might know the numbering of years and the reckoning; and We have explained everything in detail} [017.012].

﴿وَجَعَلْنَا اللَّيْلَ وَالنَّهَارَ آيَتَيْنِ فَمَحَوْنَا آيَةَ اللَّيْلِ وَجَعَلْنَا آيَةَ النَّهَارِ مُبْصِرَةً لِّتَبْتَغُوا فَضْلًا مِّن رَّبِّكُمْ وَلِتَعْلَمُوا عَدَدَ السِّنِينَ وَالْحِسَابَ وَكُلُّ شَيْءٍ فَصْلَانَا تَفْصِيلًا﴾.

{He causes the dawn to break; and He has made the night for rest, and the sun and the moon for reckoning; this is an arrangement of the Mighty, the Knowing} [006.096].

﴿فَالِقُ الْإِصْبَاحِ وَجَعَلَ اللَّيْلَ سَكَنًا وَالشَّمْسَ وَالْقَمَرَ حُسْبَانًا ذَلِكَ تَقْدِيرُ الْعَزِيزِ الْعَلِيمِ﴾.

{The sun and the moon follow a reckoning} [055.005].

﴿الشَّمْسُ وَالْقَمَرُ بِحُسْبَانٍ﴾.

{And (as for) the moon, We have ordained for it stages till it becomes again as an old dry, bent palm branch. Neither is it allowable to the sun that it should overtake the moon, nor can the night outstrip the day; and each floats in an orbit} [036.039-040].

﴿وَالْقَمَرَ قَدْرَ نَاهُ مَنَازِلَ حَتَّىٰ عَادَ كَالْعُرْجُونِ الْقَدِيمِ. لَا الشَّمْسُ يَنْبَغِي لَهَا أَنْ تُدْرِكَ الْقَمَرَ وَلَا اللَّيْلُ سَابِقُ النَّهَارِ وَكُلٌّ فِي فَلَكٍ يَسْبَحُونَ﴾.

As obvious from the following two verses, it is the moon that should be used for calendrical calculations of the times of religious practices of fasting and pilgrimage. The second verse is also quite clear in stating that first sighting of the lunar crescent should be used for determining the beginning and end of the month:

{They ask you, (O Muhammad), of new moons, say: “They are fixed timings for mankind and for the pilgrimage”} [from 002.189].

﴿يَسْأَلُونَكَ عَنِ الْأَهْلِ قُلْ هِيَ مَوَاقِيتُ لِلنَّاسِ وَالْحَجِّ﴾.

{The month of Ramadhan in which the Qur'an was revealed, a guidance to men and clear proofs of guidance and distinction (between the right and the wrong); therefore whoever of you witnesses the new moon (of this month), let him fast it} [from 002.185].

﴿شَهْرُ رَمَضَانَ الَّذِي أُنْزِلَ فِيهِ الْقُرْآنُ هُدًى لِّلنَّاسِ وَبَيِّنَاتٍ مِّنَ الْهُدَىٰ وَالْفُرْقَانِ فَمَن شَهِدَ مِنْكُمُ الشَّهْرَ فَلْيَصُمْهُ﴾.

Further details stating that the Muslims should begin and end their month with the first sighting of the new moon come from traditions of the Prophet Muhammad (*Salla Allah ta'ala 'alayhi wa sallam*) on when to begin and end the fasting of Ramadhan: (i) “fast when you see it (the crescent) and break your fast when you see it. If it was covered by clouds, make an estimate about it (whether it would be seen)”, and (ii) “fast when you see it and break your fast when you see it. If it was covered by clouds then complete thirty days of Sha‘ban (the month before Ramadhan)”. It is this apparently simple rule of beginning the month with the first sighting of the young crescent that triggered the huge interest of Muslim astronomers down the centuries in establishing criteria for determining the earliest visibility of the lunar crescent after



conjunction.

The possibility of designing criteria for predicting the first sighting of the lunar crescent resulted in Muslim scholars adopting two different interpretations of the stipulation of the Holy Qur'an of "witnessing" and of the Prophet (*Salla Allah ta'ala 'alayhi wa sallam*) of "seeing" the crescent for beginning and ending the month. There are those who insist that there must be a direct sighting by the unaided eye, while others admit the use of calculation when unfavourable weather prevails. Scholars in the first category are themselves divided into two groups. The first group believes that astronomy has nothing to do with the determination of the beginning and end of the Islamic month and that the matter should be resolved only by way of actual sighting of the crescent by the naked eye and by applying the basic rule that the month can be either 29 or 30 days. The second group are more tolerant, giving astronomical knowledge the secondary, but still significant, task of supporting or refuting claims of unaided sighting of the crescent.

The third group represents scholars who opt for the alternative interpretation, accepting the results of calculation where necessary. These men argue that it is legal to begin and end the month on the basis of merely "knowing" that the crescent was visible to the naked eye, even if actual sighting by the eye was not possible, due to unfavourable weather, for example. For instance, Muhammad bin Idris al-Shafi'i (767-820 A.D.), who is the founder of one of the major Islamic legal doctrines,<sup>4</sup> is reported to have said: "When one of those who is able to reach conclusions by (knowledge of) stars and lunar mansions concludes that the crescent is visible, though it was overcast, then it is legal for him to begin the fast or end it" (Kamal ad-Deen, 1996: 33). The adherents of this interpretation of the Qur'anic verse and Prophetic traditions usually cite a number of arguments in support of their position. Firstly, the Prophet (*Salla Allah ta'ala 'alayhi wa sallam*) ordered the Muslims to depend on "seeing" the crescent simply because at the time they had no other means of knowledge to solve this problem and, therefore, the subsequent acquisition of knowledge in this field by Muslims should be taken into consideration. Secondly, although the Prophet (*Salla Allah ta'ala 'alayhi wa sallam*) instructed the people to fast and break their fast upon "seeing" the crescent, the verb "see" itself is also used by the Holy Qur'an and the Prophet (*Salla Allah ta'ala 'alayhi wa sallam*) to

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<sup>4</sup> An Islamic legal doctrine, known in Arabic as *Mathhab*, represents collections of rulings on all sorts of issues related to various aspects of the life of the Muslim as an individual and as a member of society. Each Islamic legal doctrine reflects a particular understanding and interpretation of the Holy Qur'an and Prophetic traditions.

mean “know” as well and not only refers to visual sighting. More importantly, the relevant Qur’anic verse has the verb “witness” which means “know”. Thirdly, Muslims everywhere pray according to time tables prepared by astronomers and do not themselves measure or calculate the altitude or depression of the sun to determine the time of prayers. Thus, determining the visibility of the lunar crescent by certain astronomical criteria is not an unprecedented, heretical innovation as claimed by those who insist on actual seeing of the crescent by the eye.

Obviously, Muslim astronomers who worked on solving the problem of first visibility of the lunar crescent stand on the open-minded side of the argument in their interpretation of the Qur’anic verse and Prophetic traditions, i.e. those who belong to the second and third groups according to the above classification.

Another Prophetic tradition that has implications for the Islamic calendar is the following: “The people of every country have their own visibility (of the lunar crescent)”. This tradition has been interpreted by scholars to mean that the beginnings and the ends of the Islamic months must not be unified for all people, as in the case of prayer when people of different localities must have different times of prayer. This tradition, therefore, seems to indicate the illegality of the attempts to develop a *global* Islamic calendar where the months are begun and ended everywhere, or for a number of countries, after seeing the crescent in a certain place within that area. Scholars have agreed that the only exception to this rule is the beginning of the month of pilgrimage which should be determined according to Macca because the pilgrimage, which is a communal act of worship, takes place in Macca.

However, there are other accounts of events in which the Prophet (*Salla Allah ta'ala 'alayhi wa sallam*) ordered the Muslims to break their fast after the arrival of people from nearby regions who reported seeing the new moon a day before. This seems to be the basis of the response of religious scholars to geo-political developments and their acceptance that the people of any one country can, and in fact must, begin and end their months together. Ilyas (1979) has suggested the possibility of extending this national calendrical system into a regional or even a global system, but there seems to be *no* support for such a concept.

## 5.2 The Controversy of Crescent Visibility in the Islamic World

Determining the first sighting of the lunar crescent would have been an issue of some



importance for the pre-Islamic Arab population of the Arabian peninsula, helping them organise the affairs of their lives. However, the pre-Islamic Arabs did not develop any knowledge of this issue that goes beyond the general knowledge that any inhabitant of a desert would develop out of his personal experience in watching the sky. Astronomy, and indeed science in general, never occupied any space in the life of the Arab of the Arabian desert. What this meant is that when the question of first visibility of the lunar crescent became of vital importance for those Arabs having become Muslims, they had little to rely on other than their basic experiential knowledge of the sky and its phenomena. This is manifested in the following Prophetic tradition: “We are an illiterate nation; we do not know writing nor counting. The month is (therefore) so and so”, and he made with his two hands the number twenty nine and then thirty. This statement shows that the overwhelming majority of early Muslims, including the Prophet (*Salla Allah ta'ala 'alayhi wa sallam*) himself, were not only lacking the knowledge required for calendrical calculations, but there were also illiterate so they could not seek the help of some written sources of knowledge. In any case, written sources would have been non-Arabic, so that the early Muslims had to rely on sighting the crescent and on the basic fact that the month can only be 29 or 30 days. This illiteracy, however, was soon to be changed by Islam.

The interest that the Muslims developed in various aspects of science was motivated by several Qur'anic verses that urge the believer to seek knowledge such as: **{Say are those who know equal with those who know not? Only the men of understanding are mindful}** [from 039.009], as well as a number of Prophetic traditions such as: “Seek knowledge even if it was in China”, “Seek knowledge from the cradle to the grave”, and “Seeking knowledge is an obligatory duty on every Muslim man and Muslim woman”.

﴿قُلْ هَلْ يَسْتَوِي الَّذِينَ يَعْلَمُونَ وَالَّذِينَ لَا يَعْلَمُونَ إِنَّمَا يَتَذَكَّرُ أُولُوا الْأَلْبَابِ﴾.

It should be stressed that the kind of knowledge that Islam encourages is that which has religious or other useful applications, such as medical knowledge. It is in this pro-science environment that Islamic science developed and prospered, with astronomy being one of its major fields.

The advance of Islamic astronomy was made possible by the translation of non-Arabic astronomical books into Arabic (see O'Leary, 1948: 155-175). The first texts that were translated into Arabic in the eighth century were Indian and Persian, whereas the concentration in the following century was on Greek works. From the ninth century, Arab astronomers were

citing all of the four books of Ptolemy (ca. A.D. 150),<sup>5</sup> the commentaries on *Almagest* composed by Pappus (ca. A.D. 320) and by Theon of Alexandria (ca. A.D. 360), in addition to a series of Greek treatises called the “Small astronomy collection” which was considered as an introduction to the reading of *Almagest*.<sup>6</sup>

The first generation of Muslim astronomers cited the following Indian astronomical works: (i) *Aryabhatiya* which was written by Aryabhata in 499 A.D. and known in Arabic under the title *al-Arjabhar*; (ii) *Khandakhadyaka* by Brahmagupta who died after 665 A.D. which is referred to by Muslim astronomers under the title *Zij al-Arkand*; and (iii) *Mahassidhanta* which was written around the end of the seventh or the beginning of the eighth century and which is referred to by Arab authors under the title *Zij al-Sindhind*. As for the Persian sources of Islamic astronomy, Muslim astronomers have referred from the end of the eighth century onward to the *Zij ash-Shah* (Morelon, 1996a).

Muslim astronomers made use of the knowledge that they acquired from Indian, Persian and Greek sources and improved on this knowledge in the course of using it for various purposes. This resulted in numerous astronomical tables and methods for solving various astronomical problems, including the prediction of the first visibility of the lunar crescent. But although it is true that Muslim astronomers showed great interest in this particular astronomical question and invested much effort in solving it, the last word in determining the beginnings of Islamic months was never left to astronomers. For the population of any area, a judge would usually declare the beginning of the new month after witnesses, in whose character and reputation he found no flaws to suspect the reliability of their evidence, came forward to declare under oath that they had sighted the crescent. It was no common practice for judges to call upon science to give its verdict on the reported sighting, even when, for instance, it was possible that the witness might have made an innocent error.

To make matters worse, it is not uncommon for observers to deliberately make false claims that they have sighted the crescent. This can happen, and has indeed happened, for numerous reasons, such as claiming the cash reward that is offered in many places to the first reportee of

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<sup>5</sup> In the order of their composition, these books are: the *Almagest*, the *Planetary Hypotheses*, the *Phaseis* and the *Handy tables*.

<sup>6</sup> The “Small astronomy collection” includes the following books: the *Data*, the *Optics*, the *Catoptrica* and the *Phenomena* of Euclid (before 300 B.C.); the *Spherics*, *On Habitations* and *On Days and Nights* of Theodosius who lived in the second century B.C.; *On the Moving Sphere* and *On Risings and Settings* by Autolycus who lived in the third century B.C.; *On the Sizes and Distances of the Sun and Moon* of Aristarchus of Samos (d. ca. 230 B.C.); *On the Ascensions of Stars* by Hypsicles who



crescent sighting (Ilyas, 1994: 446). An interesting example is given by ibn Jubayr in the book from which I collected the medieval Arab observations. Ibn Jubayr gives a very critical and detailed account of what happened one day when the people of Macca were trying, without success, to spot the new crescent of the month of pilgrimage. One of the frustrated observers suddenly shouted a cry of success in spotting the new crescent, and immediately similar cries were heard from the waiting crowd who started to point in the direction where they claimed seeing it. Yet the problem is that it was very cloudy that there was no way that anyone could have seen the crescent even if it was visible. When a group of people went to the judge to claim spotting the crescent, he ridiculed them and rejected their evidence. In fact, the clouds were so thick that the judge commented: "if someone would claim to have seen the sun from behind those clouds I would not have believed him, let alone seeing a thin crescent"! It is interesting to learn that the specific reason why those people were willing and actively seeking to delude themselves about sighting the crescent. Spotting it on that particular evening would have meant that the ninth of the month of pilgrimage would begin on a Friday, a correspondence of religious significance (ibn Jubayr, 1907: 167-168). In this particular event there were many people who came forward to the judge to give evidence on sighting the crescent, and it was only the thick clouds in the sky that enabled the judge to reject their claims. If the sky was clear, the judge would have no reason to reject the alleged sighting given the number of witnesses. Astronomers would not be consulted.

The difficulty that Islamic religious scholars had with incorporating the continuously developing astronomical knowledge into the religious legal system has continued down to the present time, with the result that the Islamic calendar is often founded on untrue sightings of the lunar crescent. In a critical comment on a bad fifteenth century Arabic table for predicting the first visibility of the lunar crescent, King wrote:

If the predictions for sensitive cases in the table were correct, it was only by chance. It was such astronomers who gave the profession a bad name and who caused the legal scholars, who were responsible for the actual regulation of the calendar, to disregard their pronouncements, a tradition which persisted up to the present day (King, 1991: 237).

King's remark is not quite accurate. Incompetent astronomers could have been partly to blame for the legal scholars' neglect of astronomy, but the role of such astronomers would have

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lived around 150 B.C.; and the *Spherica* of Menelaus (ca. 100 A.D.).

been minor and one that would have totally lost its significance over time. Religious scholars of the present time are well aware of the progress and maturity that sciences, including astronomy, have achieved, yet they are almost as keen as their predecessors in maintaining negative attitudes toward astronomy. In other words, the problem has nothing to do with astronomy or astronomers and has all to do with the religious authorities themselves. The root of the problem is simply the following: the regulation of the calendar has been one of the privileges of the religious scholars who think they would lose it once astronomy is given any part in this function. Unfortunately, the civil authorities in Islamic countries do not seem to be enthusiastic about changing this situation, presumable thinking that it is not worth upsetting the religious authorities. As a result, astronomy remains very much neglected as a means of contributing to the regulation of the Islamic calendar. It is ironical indeed that the Islamic calendar which was significantly responsible for the steady progress of Islamic astronomy over more than seven centuries has been left deprived of the great service that astronomy can offer.

Recently, two Algerian researchers published an interesting study that shows the negative consequences of excluding astronomy from the regulation of the calendar. Guessoum and Meziane (1996) critically studied the accuracy of determining the beginnings of the fasting month of Ramadhan, the following month of an-Nasr, which signals the end of the fasting month, and the month of al-Haj, which is needed for determining the date of pilgrimage and the religious festival or *'Id*. The data compiled by these researchers is for Algeria and covers the period 1963-1994. These authors have found that in almost half of the cases one or more of the fundamental lunar visibility limits, such as the Danjon limit (see §7.2), would have been violated. And when checking the alleged sightings using a number of ancient and new lunar visibility criteria, these researchers have concluded that about 75% of the crescents would have been invisible (Guessoum & Meziane, 1996)!

As I found some inaccuracies in the calculation made by these authors in their paper, I have re-calculated the necessary parameters for the 97 sightings they listed. I have found that in 53 cases (i.e. 54.6%)<sup>7</sup> the alleged observation would have occurred when the crescent was less than 15.0 hours old - which is the currently accepted record of sighting the young crescent (see §7.5). Even more surprising is the fact that the collection included 5 supposed observations of pre-conjunction moons! Obviously, if any visibility criterion would be applied to the remaining observations some of these also would fail the test. It is worth noting also that the possibility of

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<sup>7</sup> The corresponding figure in Guessoum and Meziane's paper is 46.9%.



unfavourable weather has been neglected.

Guessoum and Meziane made the interesting remark that frequently the crescent is claimed to have been spotted in the east of the Arab world, for instance in Saudi Arabia, one day before it is seen in the west of the Arab world, for example in Algeria. This means that the error in determining the beginnings of the Islamic months is even gravest in the eastern Arab countries. Indeed, Ilyas studied a number of alleged sightings in several Asian countries and found that it is not uncommon to find that the crescent is claimed to have been seen before conjunction or when it set before the sun (Ilyas, 1994).

I should also mention that the yearly calendars that are usually produced in the Islamic world by bodies that are mainly scientific rather than religious, are also flawed. Studies of the annular calendars of many countries, including Saudi Arabia, reveal that the first day of the month is considered to be that immediately following the day of conjunction. This seems to indicate a lack of proper understanding of the real nature of the problem of first visibility of the lunar crescent.

Given these circumstances, one can only hope that the authorities in the Islamic world become more aware of the importance of properly integrating astronomy in the regulation of the calendar, at least as a safeguard against erroneous reports of sighting, whether purposeful or unintentional.

### 5.3 Versions of the Islamic Calendar

Although the Holy Qur'an and the Prophet Muhammad (*Salla Allah ta'ala 'alayhi wa sallam*) laid down the foundations of Islamic calendar, during the life of the Prophet (*Salla Allah ta'ala 'alayhi wa sallam*) (570-632 A.D.), and for a few years after him, the earliest Muslims had no calendar that was specially designed to serve their own purposes. The claim of some authors (e.g. O'Neil, 1975: 47) that it was the Prophet Muhammad (*Salla Allah ta'ala 'alayhi wa sallam*) who established the Islamic calendar is historically incorrect. The same is true of the unsubstantiated theory put forward by Alavi (1968) who suggested that when Islam appeared in the Arabian peninsula the people of Macca were already using a luni-solar calendar while a purely lunar calendar was in use in Madina. He proposed that the Muslim emigrants carried their luni-solar calendar with them to Madina and that both calendars were used at the same time until the Prophet (*Salla Allah ta'ala 'alayhi wa sallam*) abrogated the

Maccan calendar. Alavi originally put forward his theory to explain one confusing aspect of the chronology of early Islam which is the historians' attribution of a number of different dates to any single event. However, there is no evidence to support Alavi's theory, and it should be borne in mind that the carelessness of early Islamic historians was not shown only in their dating of events but also in the accounts they gave. There was no "Islamic calendar" during the life time of the Prophet Muhammad (*Salla Allah ta'ala 'alayhi wa sallam*).

After the departure of the Prophet Muhammad (*Salla Allah ta'ala 'alayhi wa sallam*) from this world, Abu Bakr, the first caliph, started sending out military campaigns to various regions outside the Arabian peninsula. These campaigns, which continued during the caliphate of his successor 'Umar bin al-Khattab, resulted in a continuously and rapidly expanding Islamic state, the administration of which was becoming more and more complicated (Baltaji, 1970: 371). The caliph 'Umar, therefore, was alerted to the necessity of adopting an official calendar for the growing state. Some chroniclers have mentioned certain administrative problems which they believe urged 'Umar not to delay the introduction of an official calendar for his state (at-Tabari, 1961: 388-393). Al-Beiruni mentions two specific events that prompted 'Umar to introduce an official calendar for the state. In the first event, 'Umar received a "check"<sup>8</sup> dated the month of Sha'ban, yet he could not know whether the intended Sha'ban was of that same year or of the coming one. In the second event, one of the local rulers of the state complained to 'Umar about the difficulty of recognising the dates of documents that he received from 'Umar (al-Beiruni, 1923: 29-30). The introduction of the new calendar occurred in A.D. 637.

'Umar decided to date with respect to the lunar year of the Hijra [*emigration*] of the Prophet Muhammad (*Salla Allah ta'ala 'alayhi wa sallam*) from Macca to Madina. Accordingly, the new calendar became to be known as the Hijra calendar. 'Umar adopted in the new calendar the same arrangement of the year as in the pre-Islamic Arab calendar. This started with the first day of the month Muharram and ended in the last day of Thu al-Hijja. He also left the old Arabic names of the months without change. Researchers have disagreed on the etymology of the names of the Hijra months, which seem to be relics from a variety of older calendars. However, here is the usually accepted etymology:

1- Muharram: this name is derived from the word "harram" [*forbade*], because this month is one of four months of the year known as "al-Ashhur al-Hurum" during which the Arabs of

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<sup>8</sup> This could have been a document undersigned by a local ruler stating the amount of taxes obtained in that part of the state.



the Arabian peninsula used normally to observe complete peace throughout (see for instance, ibn 'Asakir, 1911: 25).

2- Safar: none of the suggested origins of this name seems to be convincing. Some early historians have suggested that the word "Safar" has come from "Safariyyah" which is the name of a place that the Arabs used to attack (An-Nuwayri, 1923: 158). Friha suggested that the word has originated from "Sifr" [*emptiness*] (Friha, 1988: 62-64).

3- Rabi' al-Awwal: this name means literally "the first spring". However, al-Beiruni states that the word "Rabi'" which means "*spring*" in modern Arabic actually meant "*autumn*" in old Arabic (Al-Beiruni, 1923: 60).

4- Rabi' ath-Thani: this means "the second spring".

5- Jamada al-Aula: Many researchers have suggested that the name of this month is derived from "Jamada" [*froze*], because water freezes in this month ('Atiyyatullah, 1963: 625). The complete name thus means "the first freezing".

6- Jamada al-Akhira: this name means "the last freezing".

7- Rajab: al-Beiruni believes that this name refers to the movement of the Arab tribes, yet not for fighting because this is one of the four "al-Ashhur al-Hurum" (al-Beiruni, 1923: 60). Ibn 'Asakir, on the other hand, suggests that the name has been coined from the Arabs' tradition in this month of supporting long palm trees with lengths of wood or rocks because of the weight of the ripe dates (ibn 'Asakir, 1911: 25).

8- Sha'ban: it is thought that the origin of this name is the word "Sha'aba" [*branched, spread, or divided*] (al-Beiruni, 1923: 325; ibn 'Asakir, 1991: 25), because Arab tribes used to raid on each other in this month. Friha, however, thinks that this name stands for the growth of the branches of trees (Friha, 1988: 73).

9- Ramadhan: researchers agree that the name of this month is derived from "ramdh", a word related to "*heat*" (Friha, 1988: 74).

10- Shawwal: this name is derived from the word "shala" [*raised*] because in this month the camels raise their tails for mating (al-Beiruni, 1923: 235).

11- Thu al-Qi'da: this name is thought to have been derived from the word "qa'ada" [*sat down*] because this is one of the four "al-Ashhur al-Hurum" during which Arab tribes would refrain from raiding one another (al-Beiruni, 1923: 235).

12- Thu al-Hijja: this is derived from "hajja" [*made the pilgrimage*]. Obviously, this is the month of pilgrimage of the Arabs (Friha, 1988: 78). It is the fourth of "al-Ashhur al-Hurum".

'Umar's choice of the pre-Islamic Arab year for the new calendar may not seem unusual. However, the decision to use it without any adaptation remains difficult to understand because it meant that the year of the Hijra calendar would not start with the actual Hijra month. The Prophet (*Salla Allah ta'ala 'alayhi wa sallam*) left Macca at the end of Safar and arrived at Madina around the middle of Rabi' al-Awwal; these are, respectively, the second and third months of the Hijra calendar. This confusing aspect of the Hijra calendar has misled some researchers who are not knowledgeable in Islamic history into thinking that the Hijra happened in Muharram (e.g. Freeman-Grenville, 1977: 1). Other investigators who failed to notice that 'Umar was simply using the same year of the old Arab calendar without adaptation thought, wrongfully, that he chose Muharram as the first month of the year because Muharram of the first Hijra year started with Friday, the sacred day of Islam (Tsybulsky, 1979: 15). Despite the common view that the Hijra calendar is a "religious" calendar, the fact is that the Hijra calendar has no "religious" value; it was introduced ultimately to serve "secular" purposes of the Islamic state.

Although the Hijra calendar is, for historical purposes, the calendar that the majority of Muslims used and still use, there are other calendars in use in the Islamic world, though comparatively little is known about them. One of these is the Persian calendar, known as the Jalali calendar, which resulted from reforming the old Persian calendar by the famous poet and mathematician 'Umar al-Khayyam (ca. A.D. 1038-1123) in A.D. 1074/1075 (Nasr, 1968: 52-53). Like the Hijra calendar, the Jalali calendar kept its non-Islamic month names. Its reference year is the solar year of Hijra. The Jalali year comprises twelve months and its first day corresponds to March 21st in the Gregorian calendar. The first six months of the Jalali year have 31 days each, and each of the last six months has 30 days in a leap year. In a common year, the last month consists of 29 instead of 30 days. The intercalary system of the Jalali calendar keeps its year in phase with the natural solar year. This calendar counts dates from 21/3/622 A.D.

Regardless of the reason(s) behind the caliph 'Umar's choice of the year of Hijra [started on 15/July/622 A.D.] to be the base year of the new calendar, this decision had unfortunate consequences. The new calendar had to overlook one of the most important periods in Islamic history, namely the pre-Hijra era. This neglected period represents most of the lifetime of the Prophet Muhammad (*Salla Allah ta'ala 'alayhi wa sallam*) and includes more than half of the years over which the Holy Qur'an was revealed to the Prophet (*Salla Allah ta'ala 'alayhi wa sallam*). The Jalali calendar suffers from this same flaw. Thus, Islamic history remained



without a calendar to cover it all, and this created the chronological confusion that is discussed below.

When dating Islamic history, chroniclers used to divide it, indirectly, into three parts which are dated with respect to three different reference years. The first part covers the period between the birth of the Prophet (*Salla Allah ta'ala 'alayhi wa sallam*) and the first revelation of the Holy Qur'an. For instance, historians write that the Prophet (*Salla Allah ta'ala 'alayhi wa sallam*) travelled to Syria for trading when he was "twenty five years old" (ibn Hisham, 1937: 202), and that he became the arbiter in a quarrel between Arab tribes that was about to turn into serious fighting when he was "thirty five years old" (ibn Hisham, 1937: 209). Evidently, the base year of this period is the birth year of the Prophet (*Salla Allah ta'ala 'alayhi wa sallam*). The second part of Islamic history includes the period between the first revelation of the Holy Qur'an and the Hijra, and its events are dated with respect to the year of the first revelation. Accordingly, ibn Sa'ad (1905) writes that the first emigration to Abyssinia of a group of Muslims was in the "fifth year after the revelation" (p. 136), and he states that the Prophet (*Salla Allah ta'ala 'alayhi wa sallam*) went out to the city of at-Ta'if in the "tenth year after the revelation" (p. 142). The third part is the period after Hijra, whose events are, of course, dated on the Hijra calendar, i.e. the Hijra year is considered as the base year. Thus, a relatively short period of Islamic history - just over half a century - is dated according to three different reference years, i.e. dated with three different calendars.

In order to resolve these problems and provide a clear and consistent chronology of the Islamic history, two new calendars (one lunar and one solar) that reckon dates with respect to the lunar and solar dates of the birth of the Prophet Muhammad (*Salla Allah ta'ala 'alayhi wa sallam*), and thus covering all of Islamic history, have recently been suggested. By using either of these calendars, all of Islamic history can be dated according to one base year and, in result, the historical order of the different events and the time intervals between them would be obvious. This would also make the calculation of the Julian dates of these events easier.

The lunar calendar, which is a religious calendar, is known as the Miladi Muhammadi [*of the birth of Muhammad*] calendar. It reckons years from the lunar birth month and year of the Prophet Muhammad (*Salla Allah ta'ala 'alayhi wa sallam*) (al-Casnazani *et al*, 1992, 1994a, 1994b). Naturally, the month of the Miladi Muhammadi calendar is reckoned from first visibility of the lunar crescent. Since the birth of the Prophet (*Salla Allah ta'ala 'alayhi wa sallam*) was in the third month of the Hijra calendar, the Miladi Muhammadi year begins two

month after the year of the Hijra calendar. The conversion between the Miladi Muhammadi and Hijra calendars follows this formula:

Miladi Muhammadi date = Hijra date + 53 years + 10 months (5.1)

The names of the months of the Miladi Muhammadi calendar have been derived from the Islamic history so that each month is named after a significant Islamic event that occurred in that month. Table 5.1 shows the names of the Miladi Muhammadi months and their corresponding Hijra months.

Table 5.1 The Miladi Muhammadi months and their Hijra equivalents

Miladi Muhammadi Months		Hijra Months	
No.	Name	No.	Name
1	An-Nur [ <i>the light</i> ]	3	Rabi' al-Awwal
2	Al-Quds [ <i>Jerusalem</i> ]	4	Rabi' ath-Thani
3	Al-Karrar [ <i>the attacker</i> ]	5	Jamada al-Aula
4	Az-Zahra' [ <i>the ever-flowering</i> ]	6	Jamada al-Akhira
5	Al-Isra' [ <i>the night journey</i> ]	7	Rajab
6	Al-Qadisiyya	8	Sha'ban
7	Ramadhan	9	Ramadhan
8	An-Nasr [ <i>the victory</i> ]	10	Shawwal
9	Al-Bay'a [ <i>the pledge</i> ]	11	Thu al-Qi'da
10	Al-Haj [ <i>the pilgrimage</i> ]	12	Thu al-Hijja
11	Al-Hijra [ <i>the immigration</i> ]	1	Muharram
12	Al-Futooh [ <i>the conquests</i> ]	2	Safar

The etymology of the names of the Miladi Muhammadi months is as follows:

1. An-Nur [*the light*]: the Prophet Muhammad (*Salla Allah ta'ala 'alayhi wa sallam*) who is described in the Holy Qur'an as being "light", {**There has come to you from Allah a light and a manifest Book**} [from 005.015], was born in this month [12/1/1 M.M. (Miladi Muhammadi), 2/5/570 A.D.].

﴿قَدْ جَاءَكُمْ مِنَ اللَّهِ نُورٌ وَكِتَابٌ مُبِينٌ﴾



2. Al-Quds [*Jerusalem*]: the Muslims' conquest of Jerusalem occurred in this month [637 M.M., 1183 A.D.]; this city embraces "al-Masjid al-Aqsa" [*al-Aqsa mosque*] which is one of the holiest places of Muslims.

3. Al-Karrar [*the attacker*]: the conquest of Khaybar was in this month, where the Prophet Muhammad (*Salla Allah ta'ala 'alayhi wa sallam*) called al-Imam 'Ali bin abi Talib "al-Karrar" due to his distinguished role in defeating the enemy in this battle [61 M.M., 628 A.D.]. Al-Imam 'Ali bin abi Talib is the spiritual heir of the Prophet Muhammad (*Salla Allah ta'ala 'alayhi wa sallam*).

4. Az-Zahra' [*the ever-flowering*]: this is the title of as-Sayyidah Fatima, the daughter of the Prophet Muhammad (*Salla Allah ta'ala 'alayhi wa sallam*), who was born in this month [38 M.M., 606 A.D.].

5. Al-Isra': this is the Qur'anic name of the "night journey" of the Prophet Muhammad (*Salla Allah ta'ala 'alayhi wa sallam*) from al-Masjid al-Haram in Macca to al-Masjid al-Aqsa in Jerusalem, which was followed by his ascension to the heavens, which occurred in this month (*suras 17 and 53 of the Holy Qur'an*) [52 M.M., 620 A.D.].

6. Al-Qadisiyyah: in this month the Muslims achieved victory over the Persian army in the battle of al-Qadisiyyah [69 M.M., 637 A.D.].

7. Ramadhan: this is the fasting month of Islam, and the name is the same as that used in the Hijra calendar because it is mentioned in the Holy Qur'an.

8. An-Nasr [*the victory*]: in this month Muslims defeated their allied enemies in the decisive battle of "al-Khandaq" [*the ditch*] [59 M.M., 627 A.D.].

9. Al-Bay'a [*the pledge*]: the pledge of ar-Radhwan, when Muslims pledged loyalty to the Prophet (*Salla Allah ta'ala 'alayhi wa sallam*), occurred in this month [60 M.M., 628 A.D.].

10. Al-Haj [*the pilgrimage*]: the month of the yearly pilgrimage to Macca.

11. Al-Hijra [*the immigration*]: this month corresponds to the first month of the Hijra calendar.

12. Al-Futooh [*the conquests*]: several Islamic conquests took place in this month.

McPartlan (1997) has noted that the Miladi Muhammadi calendar "shows distinct advantages over the Hijra calendar for anyone interested in the chronology of the early life of Muhammad and the beginning of Islam", but he made the critical remark that "very little importance seems to be given to fixing the key point of the calendar, namely its beginning" (McPartlan, 1997: 25). It is true that historical sources refer to other possible dates for the birth of the Prophet Muhammad (*Salla Allah ta'ala 'alayhi wa sallam*) beside the date adopted

by the Miladi Muhammadi calendar, which corresponds to 12/Rabi' Al-Awwal/54 before Hijra. However, the above date has attracted more consensus than others. In fact, it is officially considered in almost all Islamic countries as the date of birth of the Prophet (*Salla Allah ta'ala 'alayhi wa sallam*).

The other calendar, which dates with respect to the solar birth month and year of the Prophet (*Salla Allah ta'ala 'alayhi wa sallam*), is the Shamsi Muhammadi [*of the solar date of birth of Muhammad*] calendar. As the birth of the Prophet (*Salla Allah ta'ala 'alayhi wa sallam*) was found to be on 2/5/570 A.D. (al-Casnazani *et al*, 1994c), the first day of the Shamsi Muhammadi calendar, i.e. 1/1/1 S.M., corresponds to 1/5/570 A.D. Therefore, the Shamsi Muhammadi year begins four months after the beginning of the Julian/Gregorian year; May, which is the fifth month of the Julian/Gregorian year, is the first month of the Shamsi Muhammadi year. Thus, the conversion between Shamsi Muhammadi and Julian/Gregorian dates is governed by the following formula:

$$\text{Shamsi Muhammadi date} = \text{Julian/Gregorian date} - 569 \text{ years} - 4 \text{ months} \quad (5.2)$$

The Shamsi Muhammadi calendar is a civil Islamic calendar, hence some of the names of its months have Islamic connotation while others refer to changes in the weather and seasons. The latter group of names are chosen to describe specifically changes in the weather of the Arabian peninsula and the neighbouring countries, as this is the region where the Prophet Muhammad (*Salla Allah ta'ala 'alayhi wa sallam*) was born and from where Islam was promulgated to the world. A list of the names of the Shamsi Muhammadi months and their Julian/Gregorian equivalents is given in table 5.2.

The etymology of the names of the Shamsi Muhammadi months is as follows:

1. Ar-Rahma [*mercy*]: the Prophet Muhammad (*Salla Allah ta'ala 'alayhi wa sallam*) who was born in this month [2/1/1 S.M. (Shamsi Muhammadi), 2/5/570 A.D.] is described in the Holy Qur'an as being: **{Mercy to all peoples}** [from 021.107].

﴿رَحْمَةً لِّلْعَالَمِينَ﴾.

2. Al-Firdaws [*paradise*]: in this month the fields become adorned with fruits, vegetables, and grains.

3. Ash-Shams [*the sun*]: this is the first of the hot months of summer.

4. Ar-Ratab [*palm dates*]: dates ripen in this month. The palm is considered a blessed tree in Islam.

5. Ar-Rihla [*the journey*]: this name refers to the Hijra of the Prophet Muhammad (*Salla Allah ta'ala 'alayhi wa sallam*) from Macca to Madina which took place in this month. The



Prophet (*Salla Allah ta'ala 'alayhi wa sallam*) left Macca on 8/5/53 S.M. (8/9/622 A.D.) and arrived to Madina on 22/5/53 S.M. (22/9/622 A.D.)

- 6. Al-Ghayth [*rain*]: rain starts to fall in this month.
- 7. Al-Bard [*cold*]: this is the first month of winter.
- 8. Ath-Thalj [*snow*]: snow starts to fall in this month.
- 9. Ar-Rih [*wind*]: there are gusty winds in this month.

**Table 5.2** The Shamsi Muhammadi months and their Julian/Gregorian equivalents

Shamsi Muhammadi Months		Gregorian Months	
No.	Name	No.	Name
1	Ar-Rahma [ <i>mercy</i> ]	5	May
2	Al-Firdaws [ <i>paradise</i> ]	6	June
3	Ash-Shams [ <i>the sun</i> ]	7	July
4	Ar-Ratab [ <i>palm dates</i> ]	8	August
5	Ar-Rihla [ <i>the journey</i> ]	9	September
6	Al-Ghayth [ <i>rain</i> ]	10	October
7	Al-Bard [ <i>cold</i> ]	11	November
8	Ath-Thalj [ <i>snow</i> ]	12	December
9	Ar-Rih [ <i>wind</i> ]	1	January
10	Az-Zar' [ <i>planting</i> ]	2	February
11	Al-Buraq	3	March
12	Ar-Rabi' [ <i>spring</i> ]	4	April

- 10. Az-Zar' [*planting*]: the first month of the season of planting summer products.
- 11. Al-Buraq: this is the name of the means which the Prophet Muhammad (*Salla Allah ta'ala 'alayhi wa sallam*) used in his ascension to heavens which occurred in this month (6/11/50 S.M., 6/3/620 A.D.).
- 12. Ar-Rabi' [*spring*]: this is the first month after the vernal equinox.

The introduction of the Miladi Muhammadi and Shamsi Muhammadi calendars can help in writing the history of Islam on a sounder scientific ground.

## The Criterion of First Visibility Of the Lunar Crescent in the Medieval and Later Arab World

In this chapter I review some of the contributions of the Muslim astronomers to the solution of the problem of lunar first visibility. Obviously, this chapter is not intended to test all solutions to the problem as suggested by astronomers from Islamic countries. Practically, this is impossible, and there is little point in taking on such a huge task. What the following sections are intended to do, however, is presenting and testing samples of the different kinds of solutions for the problem of first visibility of the lunar crescent that Muslim and non-Muslim astronomers working in Islamic countries have suggested.

Many Muslim astronomers included their suggested criteria in astronomical handbooks or *zijes* - of which Muslim astronomers compiled some 200, beginning with the Muslims' first encounter with mathematical astronomy in the 8th century. One major survey of Islamic *zijes* has been published by Kennedy (1956).

A large number of Muslim astronomers advocated visibility criteria of the following form:

$$\Delta\lambda + \mu \beta > f(n) \quad (6.1)$$

where  $\Delta\lambda$  is the difference between the lunar and solar longitudes,  $0 < \mu < 1$  is a constant dependent on terrestrial latitude,  $\beta$  is the lunar latitude, and  $f(n)$  is a series of limits for each zodiacal sign ( $n$ ), with the first zodiacal sign covering longitudes 0-30°, the second 30-60° and so on. Although the lunar latitude significantly affects the angular separation between the sun and moon and the moon lagtime, some criteria neglect the lunar latitude, thus reducing inequality 6.1 to the following form:

$$\Delta\lambda > f(n) \quad (6.2)$$

The first three models that are reviewed below are of this latter type while the fourth does take the change in  $\beta$  into consideration.

### 6.1 The $(\lambda_m, \Delta\lambda)$ Criterion of al-Khawarizmi

The earliest known table for predicting crescent visibility was derived by the well-known



astronomer and mathematician of Baghdad Muhammad ibn Musa al-Khawarizmi (born ca. 780 and died ca. 850). This table is preserved in three different sources that have been investigated by King (1987). In the margin of MS Paris Bibliothèque Nationale ar. 6913, which is a copy of an anonymous *zij* called *al-Zij al-Riqani*, the visibility criterion is included in a table entitled *al-ru'ya li-'l-Khawarizmi* which means “visibility (of lunar crescent) according to Al-Khawarizmi”. The entries are shown in table 6.1 whose first column lists the 12 *Buruuj*, i.e. “zodiacal signs”, whereas the second column gives *daraj al-hudud*, i.e. the “degrees of limits”. Although not stated explicitly, King (1987) suggests that this latter quantity represents the critical difference in longitude between the sun and the moon, following the typical form of the lunar visibility criteria. The underlying concept of the table is as follows: when the sun or moon is in a certain zodiacal sign ( $\lambda$ ), then the lunar crescent can be seen when, at sunset, the difference between the lunar and solar longitudes ( $\Delta\lambda$ ) is equal to or greater than the corresponding critical quantity which is a function of  $\lambda$ ; i.e.  $\Delta\lambda > f(\lambda)$ , where  $\lambda$  is either  $\lambda_s$  (solar longitude) or  $\lambda_m$  (lunar longitude).

**Table 6.1** The crescent visibility table of al-Khawarizmi

Zodiacal Sign	Visibility Function ( $f(\lambda)$ )	Zodiacal Sign	Visibility Function ( $f(\lambda)$ )
I	10°;12'	VII	18;36
II	9;58	VIII	16;07
III	10;01	IX	12;58
IV	11;23	X	10;40
V	14;29	XI	9;56
VI	17;44	XII	10;04

King (1987) has analysed the table of al-Khawarizmi and concluded the following:

1. The table is based on Indian visibility theory which states that the crescent can be seen if the time from sunset to moonset (S) (lagtime of moonset) is greater than or equal to 12°. This is an expected finding given that al-Khawarizmi had compiled a set of astronomical tables based largely on *Zij al-Sindhind* which is an Arabic translation of the 7th-8th century Indian

work *Mahassidhanta*. Al-Beiruni (1955: 952) also has stressed that al-Khawarizmi based his criterion on the Indian rule. In his commentary on the zij of al-Khawarizmi, ibn al-Muthanna also referred explicitly to the fact that al-Khawarizmi based his tables on the Indian assumption that  $S=12^\circ$  separates visibility from invisibility. He further proceeded to explain that at this value of  $S$  the illuminated width of the crescent would be  $4/5$  of a digit, on the assumption that this constitutes  $1/15$  of the body of the moon (Goldstein, 1967: 101-102)<sup>1</sup>. In other words, it is assumed that the width of the crescent increases almost linearly with its age, by about  $2'$  daily. However, modern calculations show that this assumption is very inaccurate. Firstly, the change in the width of the crescent with its age is not linear, with the increase and decrease in lunar width in the beginning and end of the month, respectively, being slower. Secondly, the crescent has to be more than 24 hours old (or  $12^\circ$  of lagtime) before it becomes  $2'$  wide. In fact, table 3.1 shows that the smallest lagtime of a lunar crescent that is more than  $2'$  wide is  $16.5^\circ$  (observation 24) and that its lagtime can be  $29.25^\circ$  (observations 143) when the crescent is still  $2'$  in width.

2. The underlying latitude is about  $33^\circ$  (Baghdad,  $33.25^\circ$  N), a parameter used by al-Khawarizmi in various other works.

3. The underlying obliquity is  $23.85^\circ$ , a parameter also securely associated with al-Khawarizmi.

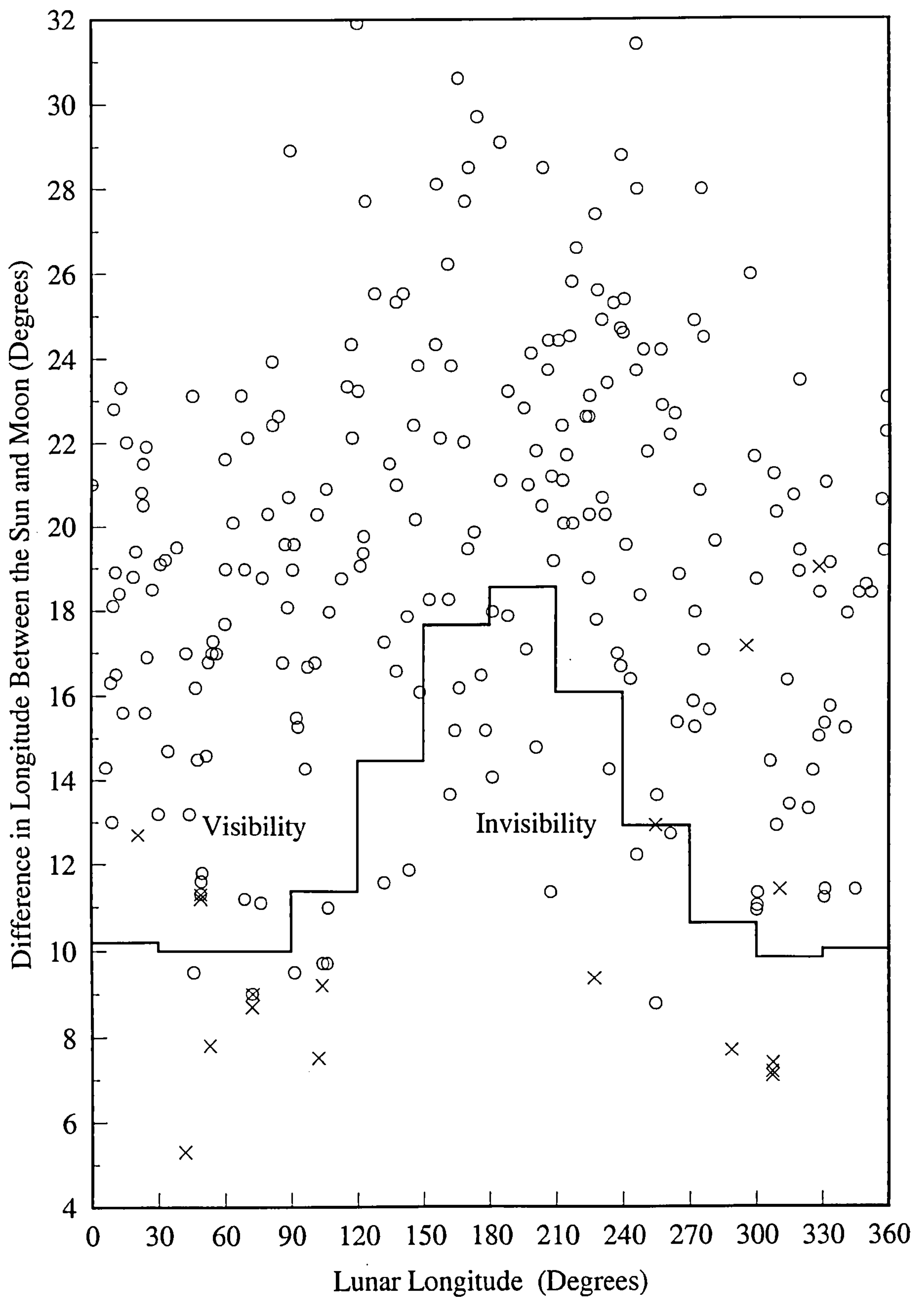
The smallest value of  $\Delta\lambda$  in the table is  $9;56^\circ$  for the zodiacal sign of Aquarius, while the greatest value is  $18;36^\circ$  for the sign of Libra. Since, on average, the longitudinal separation between the sun and moon increases by about  $12^\circ$  per day, the above two figures imply that the crescent cannot be seen when it is less than about 20 hours old and that it may remain invisible for up to about one day and a half after conjunction.

I have investigated the accuracy of al-Khawarizmi's table using observational data from table 3.1. Since the original table was prepared for latitude of about  $33^\circ$ , I have extracted from table 3.1 all entries for latitudes in the range  $\pm(30-35)$  and plotted them with al-Khawarizmi's criterion. In this and all following figures, circles and crosses denote positive and negative observations, respectively. I took the zodiacal sign to be that in which the moon (rather than the sun) was located, as other tables of the same form usually use the lunar longitude. The results are shown in figure 6.1. Six of the 19 negative observations (i.e. 31.6%) and 23 of the 239

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<sup>1</sup> The cited manuscript of ibn al-Muthanna's commentary that Goldstein edited has the figure  $3/5$  instead of  $4/5$ , but this must be a scribal error because the Arabs considered the moon's diameter to be





**Figure 6.1** The criterion of al-Khawarizmi with observations for latitudes  $\pm(30-35)$

positive sightings (i.e. 9.6%) are in contradiction with the criterion. (This graph, and some of the graphs that will follow, may show smaller numbers of points because of coinciding data points). Obviously, the table represents a very unreliable criterion for predicting the first visibility of the lunar crescent. When this criterion is used for other latitudes, its erroneous results further increase for the negative observations and slightly decrease for the positive observations. I applied the solution of al-Khawarizmi to all of the observations of table 3.1 and found that 34 negative observations (i.e. 40.5%) fall in the visibility zone and 33 positive (i.e. 7.8%) fall in the invisibility zone.

### 6.2 The ( $\lambda_m$ , $\Delta\lambda$ ) Criterion of al-Qallas

One of the works of Maslama ibn Ahmad al-Majriti, the famous 10th century astronomer who worked in Cordova, was a recension of the *zij* of al-Khawarizmi. It contains a table for determining crescent visibility (King, 1987: 195). The same table is also found in the *zij* of ibn Ishaq al-Tunisi who worked in Tunis in the early 13th century. Ibn Ishaq attributes the table to an individual named al-Qallas, whose title means “the maker of skull-caps”. Kennedy and Janjanian (1965) named this same lunar table after al-Khawarizmi as it is found in the *zij* attributed to the latter (Kennedy, 1956).

The table contains values of a visibility function  $f(\lambda_m)$  calculated not merely for every zodiacal sign, as in the case of the visibility table of al-Khawarizmi, but rather for each of the three decans within that sign, i.e. for every  $10^\circ$  of  $\lambda_m$ . The values of  $f(\lambda_m)$  given in the table are symmetrical about  $\lambda = 180^\circ$ , and there is no reference to a specific latitude for which the table had been prepared. Kennedy and Janjanian (1965) could not explain this symmetry in terms of the mathematical methods that were in use by early Muslim astronomers. King (1987), on the other hand, suggested that the author of the table computed the visibility function for the first 6 signs and simply assumed the symmetry of the function. This assumption, however, does not seem to have any reasonable justification. This crescent visibility criterion is shown in table 6.2.

The table in the work of al-Majriti explicitly states that the lunar crescent will be seen if  $\Delta\lambda > f(\lambda_m)$ , whereas the text in the *zij* of al-Khawarizmi states that the evening of first

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12 digits.



visibility of the lunar crescent of the month would be the first evening that satisfies the following relation at sunset:  $\Delta\lambda + \beta > f(\lambda_m)$ . However, Kennedy and Janjanian found that the values of  $f(\lambda_m)$  of the table can be reproduced on the assumption that  $\beta = 0$ . Assuming that the table was based on the Indian criterion of  $S \geq 12^\circ$ , Kennedy and Janjanian reproduced the values of  $f(\lambda_m)$  with a maximum error of  $0.389^\circ$ , on the supposition that the calculations were made for a latitude ( $\phi$ ) of  $42.67^\circ$  and obliquity ( $\epsilon$ ) of  $23.50^\circ$  (Kennedy and Janjanian, 1965: 77). They concluded, therefore, that the table cannot have been computed for Baghdad or India; they inferred that it is probably the work of the Andalusian astronomer al-Majriti whom they describe as being the redactor of al-Khawarizmi's *zij*. King (1987) also found that the crescent visibility table of al-Qallas was computed for the latitude of the fifth climate.<sup>2</sup> His reproduction of the values of  $f(\lambda_m)$  are even less accurate than Kennedy and Janjanian's with the error reaching as much as about  $1^\circ$  on the assumption that the precise value of the latitude is related to that of the obliquity according to any pair of the following scheme:

1.  $\epsilon = 23.55^\circ$  and  $\phi = 41.28^\circ$ ;    3.  $\epsilon = 23.85^\circ$  and  $\phi = 40.87^\circ$
2.  $\epsilon = 23.58^\circ$  and  $\phi = 41.23^\circ$ ;    4.  $\epsilon = 24.00^\circ$  and  $\phi = 40.68^\circ$

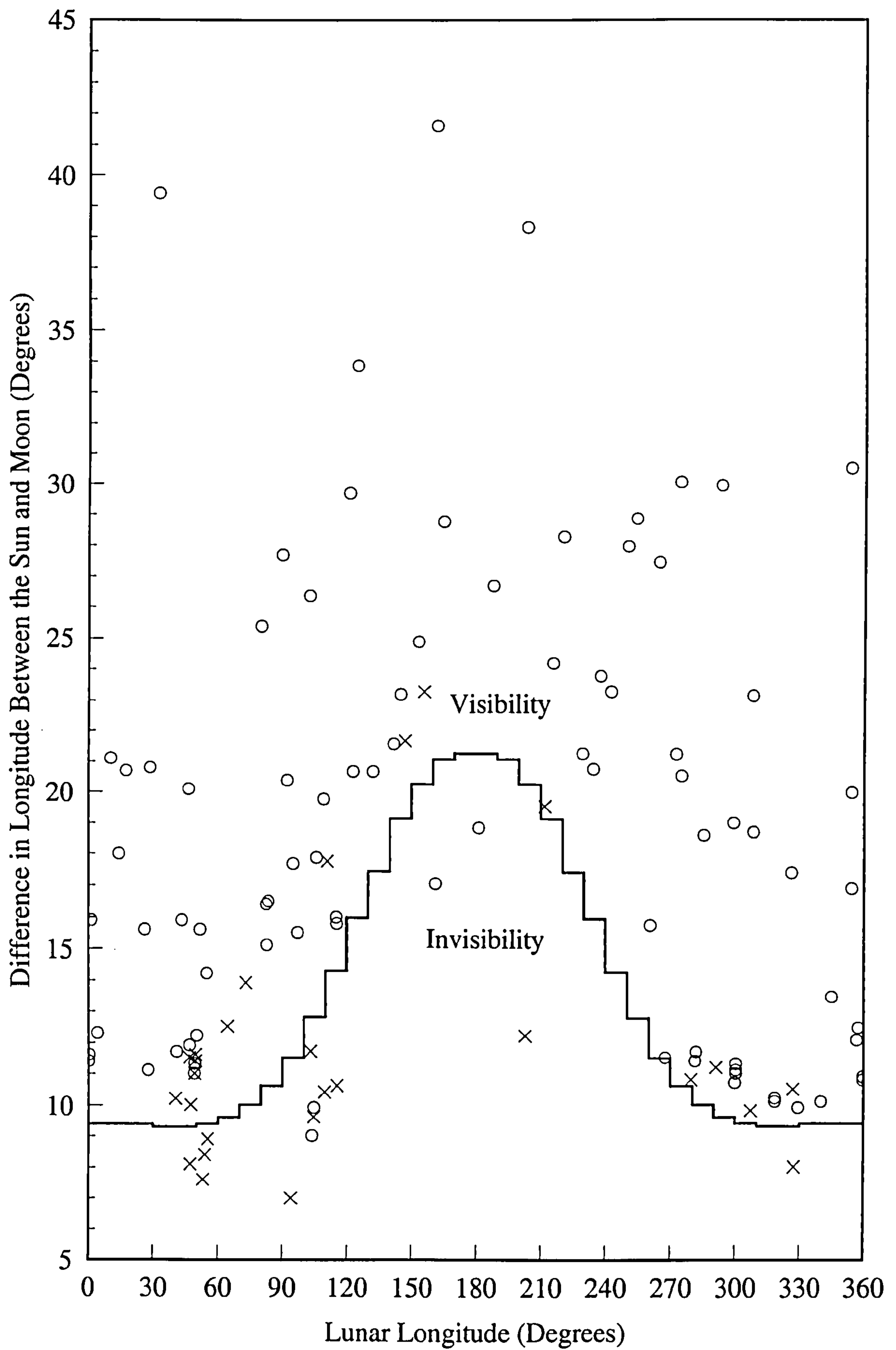
In a more recent investigation of this same table, Hogendijk could recompute the values in the table with a maximum error of 8' on the assumption of  $\epsilon = 23.35^\circ$ , which underlines a set of astrological tables in the Latin version of the *zij*, and  $\phi = 41.35^\circ$ , which is that of the city of Saragossa (Hogendijk, 1988a: 35). This city is known to have been a centre of scientific activity in the eleventh century. Two of its kings, Ahmad al-Muqtadir ibn Hud (d. 1081) and his son Yusuf al-Mu'taman (d. 1085), were themselves active mathematicians and astronomers.

The minimum and maximum values of  $\Delta\lambda$  in the table implies that the crescent can never be seen if it is less than about 18.5 hours old (second decan of signs II and XI) and may still not be seen even when it is some 42.5 hours old (third decan of sign VI and first decan of sign VII).

I have plotted in figure 6.2 al-Qallas' criterion, assuming that the visibility function  $f(\lambda_m)$  applies over all of its respective decan, and have also plotted all entries from table 3.1 for latitudes  $\pm(38-45^\circ)$ . It has 18 discordant negative observations out of a total of 33 (i.e. 54.5%),

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<sup>2</sup> The Arab astronomers borrowed from the Greek the concept of  $\kappa\lambda\iota\mu\alpha\tau\alpha$  (climates). Each  $\kappa\lambda\iota\mu\alpha$  (climate) represents a narrow zone some  $0.6^\circ$  wide centred on a standard parallel of a latitude. Late Greeks, and accordingly the Arabs, identified seven climates. A history of the concept of  $\kappa\lambda\iota\mu\alpha\tau\alpha$  is given by Dicks (1960: 154-164).



**Figure 6.2** The criterion of al-Qallas with observations for latitudes  $\pm(38-45)$



while 4 out of the 96 positive (i.e. 4.2%) are placed in the wrong zone. Applying the criterion to all latitudes, it fails in 36 negative observations (i.e. 42.9%) and 39 positive observation (9.2%). This model is as bad as that of al-Khawarizmi.

**Table 6.2** The crescent visibility table of al-Qallas

Zodiacal Sign	Decan	Visibility Function (f( $\lambda$ ))	Decan	Zodiacal Sign
I	1	9;26°	3	XII
	2	9;25	2	
	3	9;21	1	
II	1	9;19	3	XI
	2	9;18	2	
	3	9;21	1	
III	1	9;33	3	X
	2	9;57	2	
	3	10;37	1	
IV	1	11;29	3	IX
	2	12;48	2	
	3	14;15	1	
V	1	15;58	3	VIII
	2	17;31	2	
	3	19;11	1	
VI	1	20;20	3	VII
	2	21;04	2	
	3	21;17	1	

### 6.3 The ( $\lambda_m$ , $\Delta\lambda$ ) Criterion of al-Lathiqi

In 1698/99, Muhammad al-Lathiqi computed a table of lunar crescent visibility similar to that of al-Khawarizmi (King, 1987: 213). Al-Lathiqi states that he computed the table for latitude 34.5°, which could be that of his home city of Lattakia. The calculation method he used is to compute first the longitudinal difference between the two luminaries at two thirds of an hour after sunset. Then if  $\Delta\lambda$  is greater than or equal to the visibility function of the specific zodiacal sign of the moon, the crescent will be assumed to be visible; if it is less than this it

cannot be seen. Al-Lathiqi’s criterion which is shown in table 6.3 implies that the crescent cannot be seen if it is less than about 18.5 hours old (the sign of Gemini) and may remain invisible for up to 33 hours after conjunction (the sign of Libra).

The criterion of al-Lathiqi is plotted in figure 6.3 with the observations from table 3.1 for latitudes  $\pm(32-37^{\circ})$ . Not unexpectedly, this criterion has percentages of error comparable to al-Khawarizmi's unreliable criterion. Seven of the 22 negative observations (i.e. 31.8%) are in the visibility zone whereas 14 out of 240 positive observations (i.e. 5.8%) are in the invisibility zone. When the model of al-Lathiqi is tested using the observational data for all latitudes, I have found that this criterion wrongly predict 37 negative observations (i.e. 44%) to have been visible and 20 positive observations (i.e. 4.7%) to have been invisible. Al-Lathiqi’s solution is slightly better than the previous two with regard to the positive observations yet its error with the negative observations is greater.

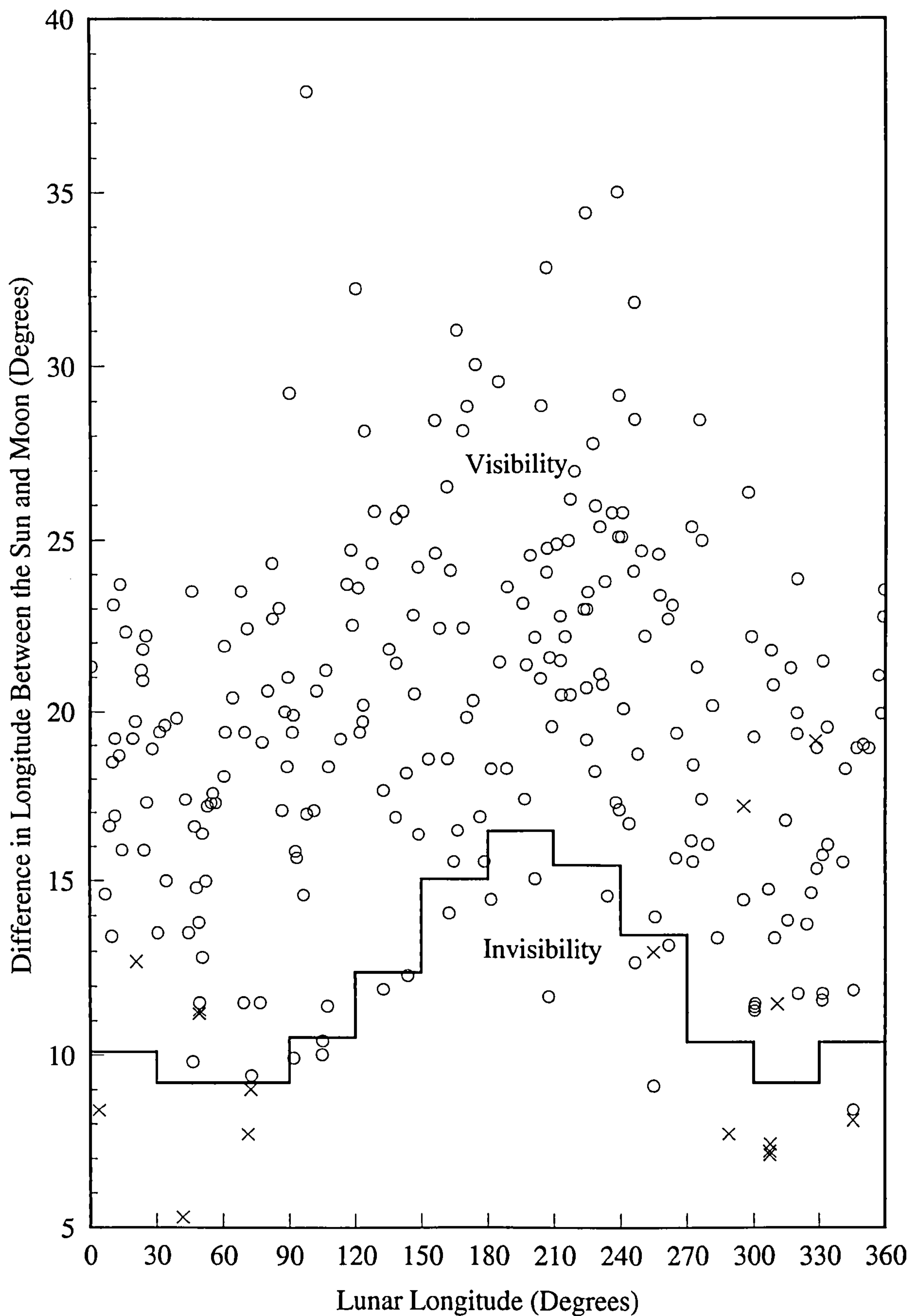
**Table 6.3** The crescent visibility table of al-Lathiqi

Zodiacal Sign	Visibility Function (f( $\lambda$ ))	Zodiacal Sign	Visibility Function (f( $\lambda$ ))
I	10;08°	VII	16;30
II	9;14	VIII	15;27
III	9;09	IX	13;30
IV	10;31	X	10;22
V	12;26	XI	9;11
VI	15;03	XII	10;25

### 6.4 The ( $\lambda_s, \Delta\lambda, \beta$ ) Criterion of al- Sanjufini

In 1366 a certain Abu Muhammad al-Sanjufini completed a *zij* that he dedicated to his patron, Prince Randa, the Mongol viceroy of Tibet and a direct descendant in the seventh generation from Genghis Khan. One of the forty two tables of the *zij* is designed for predicting the first visibility of the lunar crescent. The table is explicitly said to have been computed for





**Figure 6.3** The criterion of al-Lathiqi with observations for Latitudes  $\pm(32-37)$

terrestrial latitude  $38;10^\circ$  which indicates, according to Kennedy and Hogendijk (1988), that the table was computed for the second Mongol capital Yung-ch'ang fu. Al-Sanjufini's table gives the visibility function for each degree of lunar latitude for every  $10^\circ$  of solar longitude. Table 6.4 contains only the entries for  $\beta = -5, 0$  and  $5$ , as the entries for the other lunar latitudes are obtained by linear interpolation.

**Table 6.4** The crescent visibility table of al-Sanjufini

$\lambda_s$	$\Delta\lambda=\lambda_m-\lambda_s$			$\lambda_s$	$\Delta\lambda=\lambda_m-\lambda_s$		
	$\beta=5^\circ$	$\beta=0^\circ$	$\beta=-5^\circ$		$\beta=5^\circ$	$\beta=0^\circ$	$\beta=-5^\circ$
0°	8;37°	9;39°	10;43°	180°	12;50°	19;39°	29;18°
10	8;30	9;37	10;59	190	12;25	19; 0	28;11
20	8;19	9;38	11;16	200	11;55	18; 08	26;45
30	8;07	9;41	11;35	210	11;12	17;03	25;05
40	7;54	9;45	12;09	220	10;37	15;57	23;13
50	7;41	9;44	12;52	230	9;39	14;45	21;20
60	7;28	10;15	13;59	240	9;28	13;33	19;22
70	7;25	10;43	15;18	250	9;18	12;34	17;37
80	7;31	11;25	16;43	260	8;15	11;41	16;03
90	7;56	12;22	18;46	270	8;00	10;57	14;34
100	8;09	13;28	20;29	280	8;01	10;16	13;42
110	9;20	14;48	22;53	290	7;51	10; 0	12;38
120	10; 9	16; 8	25; 4	300	7;58	9;46	12; 0
130	10;53	17;28	27;20	310	8;18	9;31	11;38
140	11;41	18;26	28;37	320	8;15	9;37	11;18
150	12;36	19;21	29;52	330	8;22	9;36	11; 1
160	12;53	19;53	30;31	340	8;30	9;35	10;54
170	12;57	19;53	30;23	350	8;39	9;33	10;50

The entries for  $\lambda_s=70^\circ$  and  $\lambda_s=160^\circ$  suggest, respectively, that the crescent cannot be seen before it is 14.5 hours old and that it may remain invisible even when it is some two days and a



half old.

This table is used in the same way as the other tables. For any given pair of  $\lambda_s$  and  $\beta$ , the crescent is considered to be visible if the difference in longitude between the sun and moon at sunset is greater than the corresponding value in the table. As the table was originally prepared for latitude of about  $38^\circ$ , I checked its accuracy using the data from table 3.1 for latitudes  $\pm (35-40^\circ)$ . Of the 83 positive observations in this range of latitudes, only 4 (i.e. 4.8%) contradict the criterion. However, as many as 14 of the 30 negative observations (i.e. 46.7%) fall in the visibility zone of the criterion. This model does not yield better results when applied to data of all latitudes, with 34 (i.e. 8.1%) and 30 (i.e. 35.7%) of the positive and negative observations, respectively, in contradiction with the criterion. It is interesting to note that the introduction of the lunar latitude as an additional parameter in the criterion has not improved this solution significantly in comparison with the other models which neglect the lunar latitude.

Muslim astronomers have designed many other tables of lunar visibility similar to the four tables that have so far been reviewed (see for instance, Hogendijk, 1988b; King 1987), but there is no reason to expect them to be significantly better than those tested above.

## 6.5 The (S, L, $V_m$ ) Criterion of ibn Yunus

One of the Muslim scientists who studied the problem of predicting the first visibility of the lunar crescent and suggested a multi-parameter criterion is the celebrated Egyptian astronomer ibn Yunus (d. 1008) who lived and worked in Fustat near Cairo. The visibility of the lunar crescent in the theory of ibn Yunus depends on three parameters. The first of these is the angular distance between the sun and moon, or the elongation, which determines the width of the crescent. In his calculations, ibn Yunus uses the quantity  $L/15$  which he refers to as *qaws al-nur* “the arc of light” and which is measured by “digits” where one digit equals  $15^\circ$ . Arab astronomers very often used this quantity. The second quantity is the lagtime of moonset measured on the celestial equator by degrees, where 1 degree equals 4 minutes. The third parameter is the lunar velocity, which varies from slightly less than  $12^\circ$  to just more than  $15^\circ$  per day. This quantity is a function of the distance of the moon from the earth and is related therefore to the brightness of the moon - but only very weakly so.

Ibn Yunus puts his conditions of visibility of the crescent in six statements of the following form: “if the lagtime is  $12^\circ$  we check: if it has one digit of light and it is fast moving then the

crescent can be seen; and if the lagtime is 12° and it has less than one digit of light and it is slow moving then the crescent will not be seen”. Table 6.5 summarises the visibility conditions according to ibn Yunus.

**Table 6.5** The crescent visibility table of ibn Yunus

Condition				Visibility	Condition				Visibility
No.	S	L	V <sub>m</sub>		No.	S	L	V <sub>m</sub>	
1a	12°	15°	fast	Yes	4a	>13°	<15°	slow	Yes
1b	12	<15	slow	No	4b	<13°	<15	slow	No
2a	11	>15	max.	Yes	4c	>13°	10	slow	No
2b	11	>15	fast	No	5a	14	10	fast	Yes
2c	11	<15	max.	No	5b	<14	10	fast	No
3a	13	<15	fast	Yes	6a	15	10	slow	Yes
3b	13	<15	slow	No	6b	<15	10	slow	No
3c	13	10	fast	No	7a	15	any value	any value	Yes

The lunar visibility model of ibn Yunus has recently been published by King in its Arabic origin along with his English translation (King, 1988). This lunar visibility criterion has survived in various manuscripts of Egyptian and Yemeni provenance. King published two versions of this lunar model of visibility. The first version is found in a manuscript in Paris (copied ca. 1300) and two manuscripts from Cairo (copied ca. 1750 and 1700), whereas the second version survives in a unique manuscript in London.

After presenting both versions of the lunar visibility model of ibn Yunus, King expressed doubts about his rendering of certain passages of the Arabic text due to some linguistic ambiguity in the style (King, 1988: 161). It seemed probable to King that the Paris-Cairo version represents the original form of the theory of ibn Yunus, but he does not explain the historical or otherwise basis of his conclusion. However, if originality would be concluded on the basis of consistency and clarity, then the London version would have to be the original.

I have studied both versions and have found that King’s caution about his translation was in



order. The main source of error in King's translation is his failure to identify the parameter that is indirectly referred to by a number of phrases. My translation of the first and the last three statements of the London version is exactly similar to King's, but my translation of the 2nd and 3rd statements are different from his. Table 6.5 represents my translation of the London version of the criterion of ibn Yunus; King (1988) presented his translation in table 2 of his paper.

The limits of visibility and invisibility given in ibn Yunus' table do not provide complete coverage of all possible combinations of the values of lagtime, elongation and lunar velocity. For instance, ibn Yunus does not mention whether the crescent would be visible or not when the lagtime is  $12^\circ$ , the elongation is less than one digit and the lunar velocity is fast. Therefore, it is not possible to conduct a quantitative study of the accuracy of ibn Yunus' criterion.

However, I have checked the upper and lower limits of the lagtime, for the table implies that the crescent cannot be seen if it sets less than  $11^\circ$  after the sun, and it will definitely be visible when its lagtime is greater than  $15^\circ$ . I used observational data from table 3.1 to check the reliability of this criterion. Since ibn Yunus worked in Cairo ( $30.03^\circ$  N,  $31.15^\circ$  E) I applied his visibility conditions only to the entries of latitudes  $\pm(27-33^\circ)$ , though he did not state that his criterion is applicable only to certain latitudes. I have found that while ibn Yunus' criterion assumes that no crescent can be seen if its lagtime is less than  $11^\circ$ , there are in fact 11 positive observations with lagtime less than  $11^\circ$ , with one as low as  $7.5^\circ$  (observation 63). On the other hand, I did not find any invisible crescent with lagtime greater than  $15^\circ$ , in agreement with ibn Yunus' prediction. When data for all latitudes are considered, there are 13 and 6 contradictory positive and negative observations, respectively.

There are, of course, other criteria that employed three parameters. In an eighteenth century Egyptian manuscript, King found a table for the visibility of the crescent which is based on three parameters: the elongation, the difference in setting times of the sun and moon, and the altitude of the crescent (King, 1991). The table, which covers the years 1125-1130 of the Hijra calendar (A.D. 1713-1718), contains calculations for predicting crescent visibility for each of the Hijra months. However, there is no explicit reference to the visibility conditions used.

Another lunar visibility criterion that was based on three variables, and probably among the earliest, is that of the celebrated astronomer Thabit bin Qurra (ca. 824-901) who worked in Baghdad. His criterion involves the calculation of the following three parameters at the time of moonset: the elongation, the solar depression and the azimuthal difference between the sun and moon (Carmody, 1960: 31-36; Kennedy, 1960; Morelon, 1996b). Bin Qurra suggests the

following visibility conditions:

- (i)  $L < 10.8^\circ$ : the crescent cannot be seen.
- (ii)  $L > 25^\circ$ : the crescent will be seen.
- (iii)  $25 \geq L \geq 10.8^\circ$  and solar depression ( $s$ )  $\geq 11.1^\circ$ : the crescent will be seen.
- (iv)  $25 \geq L \geq 10.8^\circ$  and  $s < 11.1^\circ$ : subject to a rather complicated set of calculations.

I have found that already the first and third conditions of Thabit bin Qurra's criterion misjudge more than 20% of the negative observations in table 3.1. Obviously, taking into consideration condition 4 would further increase the percentage error. Therefore, this is also a weak criterion.

## 6.6 The (S, L) Criteria

The above lunar crescent visibility criteria are only a small sample of a large number of solutions that Arab astronomers based on the Indian condition of  $S \geq 12^\circ$ . There have been, however, other criteria which *differed* from the Indian rule. For instance, Ya'qub ibn Tariq, the prominent eighth century astrologer of Baghdad, stated that the crescent will be visible if any of the following conditions applies:

$$S \geq 12^\circ \text{ and } L > 11.25^\circ$$

$$S \geq 10^\circ \text{ and } L > 15^\circ$$

If none of these two conditions applies, the crescent will not be seen. So, a lagtime that is greater than  $12^\circ$  is not sufficient for the crescent to be visible, nor does a lagtime less than  $12^\circ$  necessarily imply invisibility.

Although Kennedy (1968) suggests that ibn Tariq employed the Indian criterion, the latter did in fact introduce another important condition for visibility that must apply for the crescent to be seen, even when the lagtime of moonset is greater than  $12^\circ$ . This additional condition is that the angular distance between the sun and moon should be greater than  $11.25^\circ$ . It was only in the second quarter of this century that the French astronomer Danjon recognised that, regardless of the visibility conditions, the lunar crescent cannot be seen when it is less than  $7^\circ$  from the sun (Danjon, 1932, 1936), a concept which will be considered in detail later. Although ibn Tariq set a rather higher value for the critical distance of the moon from the sun, his introduction of the concept that the lunar crescent cannot be seen if it was less than a critical distance from the sun, even when the other visibility condition is fulfilled, is itself significant.



The use by Thabit bin Qurra of the elongation in his three-parameter criterion came almost a century after Ya'qub ibn Tariq (ibn Yunus' model came another century later).

Since ibn Tariq worked in Baghdad (44.5° E, 33.25° N), I used from table 3.1 the entries of latitudes  $\pm(30-35^\circ)$  to test his criterion. The criterion of ibn Tariq and the 258 observational data are plotted in figure 6.4. There are four negative observations that contradict the criterion. However, two of them are exactly on the visibility curve and may be tolerated. Therefore, there are two clear-cut discordant negative observations out of the total of 19, i.e. 10.5%. On the other hand, there are 11 positive observations in the invisibility zone of the criterion. These make 4.6% of the total of 239. This criterion, therefore, when applied to a certain band of latitudes, is far better than the other Arab criteria that I have tested. However, this criterion shows enormous latitude dependence. When I used all the data of table 3.1 and not only those of latitudes  $\pm(30-35^\circ)$  I found 24 negative observations (i.e. 28.6%) and 29 positive (i.e. 6.9%) contradict the criterion. Nevertheless, it would be a good working solution in the latitudes of Baghdad and Cairo.

Another (S, L) criterion is that proposed by Shams al-Din al-Khalili who worked in Damascus in the late fourteenth century (King, 1988: 163). He suggested that visibility is possible when:

$$0.5 (L + S) \geq 7^\circ$$

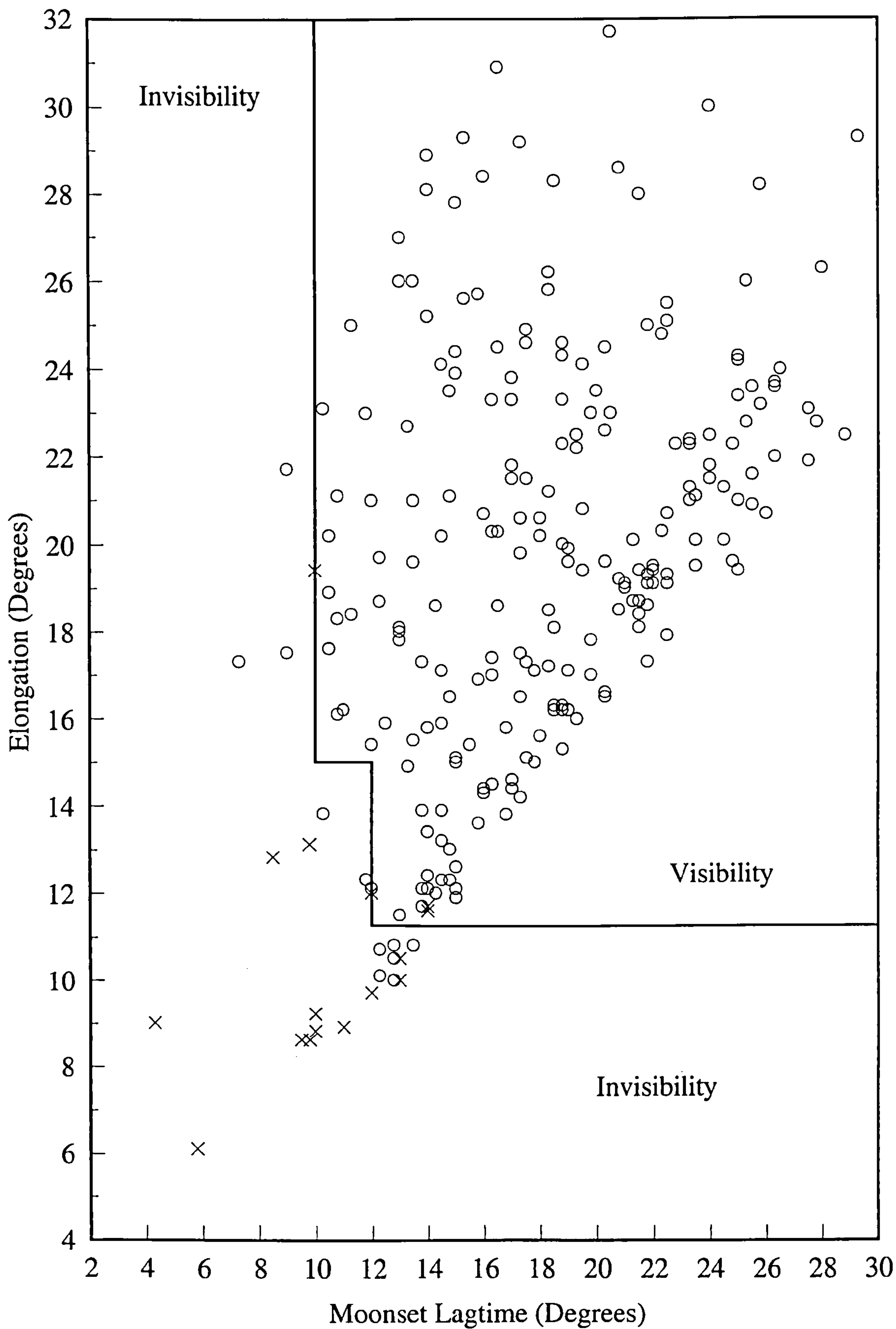
Obviously, the general form of this criteria is very much similar to that of the Babylonians, though its value of  $14^\circ$  for the sum of L and S represents a much lower visibility condition than the Babylonians'. This is manifestly a very bad criterion that predict the overwhelming majority of the negative observations in table 3.1 to be positive.

## 6.7 Maimonides' ( $\lambda_m, \Delta\lambda, S$ ) Criterion

I have found only one criterion of lunar visibility of this kind - that of the renowned Jewish scholar Musa ibn Maimon, also known as Maimonides (born in Cordoba in 1135 and died in Egypt in 1204), who worked in Cairo. The Hebrews used a luni-solar calendar based on the first sighting of the crescent moon, hence their interest in this astronomical question.

Maimonides stated first the following two straightforward visibility rules (Maimonides, 1967):

- 1- When the moon lies between the beginning of the sign of Capricorn and the end of the



**Figure 6.4** The criterion of Ya'qub ibn Tariq with observations for latitudes  $\pm(30-35)$



sign of Gemini: if  $\Delta\lambda$  amounts to exactly  $9^\circ$  or less then the new crescent cannot be seen at all in any part of Palestine; and if  $\Delta\lambda$  is greater than  $15^\circ$  then the crescent would certainly be visible throughout the entire area of Palestine.

2- When the moon lies between the beginning of the sign of Cancer and the end of the sign of Sagittarius: if  $\Delta\lambda$  amounts to  $10^\circ$  or less then the new moon cannot be seen at all in any part of Palestine; and if  $\Delta\lambda$  is greater than  $24^\circ$  then the lunar crescent would surely be seen throughout the entire area of Palestine.

Obviously, the differentiation between both cases comes from the fact that the ecliptic makes variable angles with the horizon such that at the autumn equinox the ecliptic intersects the horizon at a much smaller angle than at the vernal equinox. However, when  $\Delta\lambda$  lies between  $9^\circ$  and  $15^\circ$  in the first case or between  $10^\circ$  and  $24^\circ$  in the second, then a new parameter should be calculated: the lagtime. After giving a lengthy method for computing the lagtime, Maimonides gives the following two visibility rules:

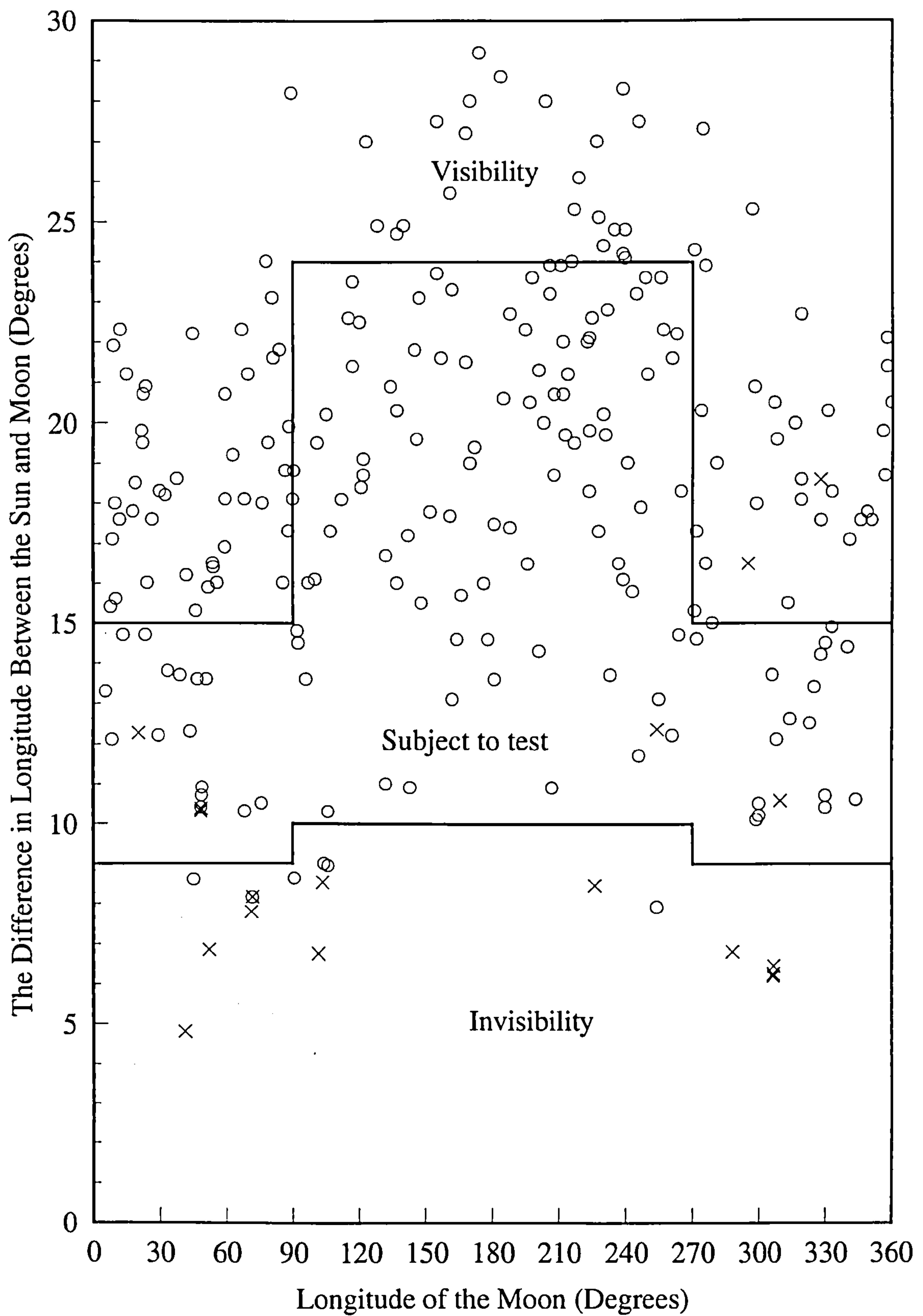
3- If the lagtime is  $9^\circ$  or less then the new moon cannot be possibly seen in any part of Palestine.

4- If the lagtime is greater than  $14^\circ$  then the lunar crescent will of necessity be visible and may well be seen clearly in any part of Palestine.

But if the lagtime is greater than  $9^\circ$  and less than  $14^\circ$  then a new set of visibility conditions, which Maimonides calls “limits of visibility”, apply. These reduce to the following visibility criterion:

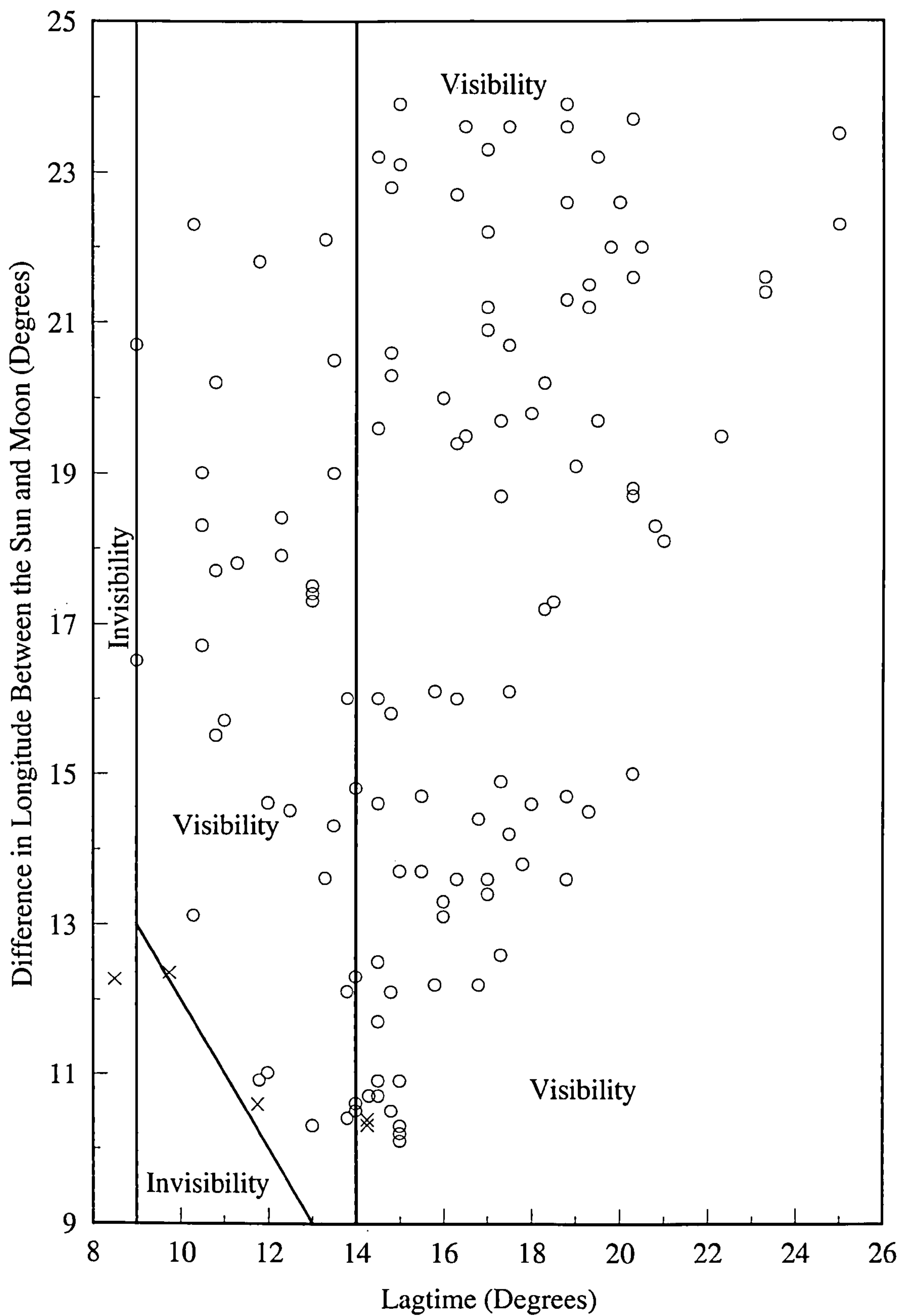
$$5- S + \Delta\lambda \geq 22^\circ$$

I have checked the visibility conditions of Maimonides using the 258 observations from table 3.1 for latitudes  $\pm(30-35^\circ)$ , since Maimonides indicates that his criterion is for Jerusalem ( $31.8^\circ$  N). Figure 6.5 represents the first two conditions of the criterion (points 1 and 2), with the 258 observations. The upper part of the graph represents the visibility zone, whereas the lower part is the invisibility zone. Any observations contained in the extensive region between the visibility and invisibility zones might be negative or positive, and hence they should be subjected to test by additional conditions. Only 2 negative and 6 positive observations contradict the first two visibility conditions of Maimonides. In figure 6.6, I have plotted the 5 negative observations and the 128 positive observations that fell in the area between the visibility and invisibility zones in figure 6.5, in addition to the visibility conditions 3 and 4 (the two vertical lines corresponding to lagtimes  $9^\circ$  and  $14^\circ$ ). The region of  $S \leq 9^\circ$  is an invisibility zone, whereas the region of  $S > 14^\circ$  is a visibility zone. I also plotted the fifth visibility



**Figure 6.5** The first two conditions of the criterion of Maimonides with observations for latitudes  $\pm(30-35)$





**Figure 6.6** The other three conditions of the criterion of Maimonides with observations for latitudes  $\pm(30-35)$

condition, i.e.  $S + \Delta\lambda \geq 22^\circ$ , which is represented by the slanted line. The visibility conditions of figure 6.6 are contradicted by 4 negative and no positive observations. Thus, the total numbers of discordant observations in Maimonides' criterion are 6 for the negative observations (i.e. 31.6%) and 6 for the positive (i.e. 2.5%). Maimonides criterion is, therefore, better than al-Khawarizmi's, al-Qallas' and al-Lathiqi's, but it is of a lower accuracy than the solution of ibn Tariq for the latitudes  $\pm(30-35^\circ)$ .

I have also tested the suitability of Maimonide's criterion for all latitudes. There are only 3 negative observations and 12 positive observations that contradict the first two conditions of the criterion, with 38 negative sightings and 241 positive sighting requiring the second test. The second test revealed further 30 contradictory negative observations but no positive observations. Thus the total number of contradictory negative observations is 33 (i.e. 39.3%) and the number of the positive ones is 12 (i.e. 2.8%). This criterion is, therefore, no better than those already tested.

In determining the sources of Maimonides' astronomy, Neugebauer pointed out the fact that almost all of Maimonides' numerical tables "either agree exactly with the tables given by al-Battani (d. 929) or can be derived therefrom by means of simple rounding off" (Neugebauer, 1967: 148). Neugebauer also indicated that the possibility of a connection with still later works such as that of ibn Yunus cannot be excluded. However, as far as the form of the last visibility condition of Maimonides is concerned, Neugebauer highlighted the fact that the latter bears obvious similarity to the Babylonian visibility criterion where the sum of elongation and the difference in right ascension is used.

It seems fair to conclude that while some medieval Arab criteria may be considered relatively acceptable to use for certain latitudes, such as ibn Tariq's criterion, they are not reliable for global use. This is so because most of these criteria were originally designed for specific latitudes; changing the latitude by a few degrees materially alters the visibility of the young crescent.



## **Modern Criteria of First Visibility Of the Lunar Crescent**

After the enormous interest of the medieval Muslim astronomers in the visibility of the lunar crescent this astronomical question went into some oblivion for centuries (Ilyas, 1994). The question was revitalised at the beginning of this century by Fotheringham's 1910 paper, which brought this issue to the attention of modern science. This chapter discusses the various efforts made during the present century to resolve the problem of determining the earliest visibility of the lunar crescent. Rather than presenting the various contributions purely in their historical order, I have grouped all related works on any one type of criterion under a section that exclusively addresses that particular criterion.

### **7.1 The Danjon Visibility Limit**

When the distinguished French astronomer André Danjon was the director of Strasbourg Observatory, he became engaged in determining the light curve of the moon. In 1931, he noticed that the crescent moon of August 13, which was only 16.2 hr before new, extended only 75-80° from cusp to cusp. In other words, Danjon found that the outer terminator of the crescent was considerably less than a complete half-circle, which it should have been theoretically. This proved not to be an isolated observation because other observations which he made, and also examination of previous records, showed that this shortening of the crescent was a general and real phenomenon. Danjon also noticed that the shortening diminishes as the angular distance of the moon from the sun increases (Danjon, 1932; Ashbrook, 1972).

Danjon illustrated this phenomenon in a diagram (Danjon, 1932: figure 29), which I have reproduced in figure 7.1, and which he explained as follows:

Let us represent the moon... by its projection on a plane passing through its centre and those of the earth and sun. Light coming from the direction SO illuminates the left half of the globe, limited by the terminator BD. Since the earth is in the direction OE, the hemisphere turned

toward us is bounded by the great circle that projects as AC. On a smooth sphere, the zone AOB would appear sunlit, forming a 180-degree-long crescent with one cusp at O, the other at the diametrically opposite point of the sphere.

But the moon is not smooth, and the mechanism described above displaces the cusp from O to Q. The lunar surface in the little triangle OPQ remains invisible. We call PQ the *deficiency arc*, and evaluate it as follows. If  $a$  is the angular distance of the moon from the sun (taking account of lunar parallax),  $2\omega$  the length of the crescent (which would be  $180^\circ$  on a smooth sphere), the deficiency arc  $\alpha$  is given by the formula

$$\sin \alpha = \sin a \cos \omega \text{ (Danjon, 1932: 60).}$$

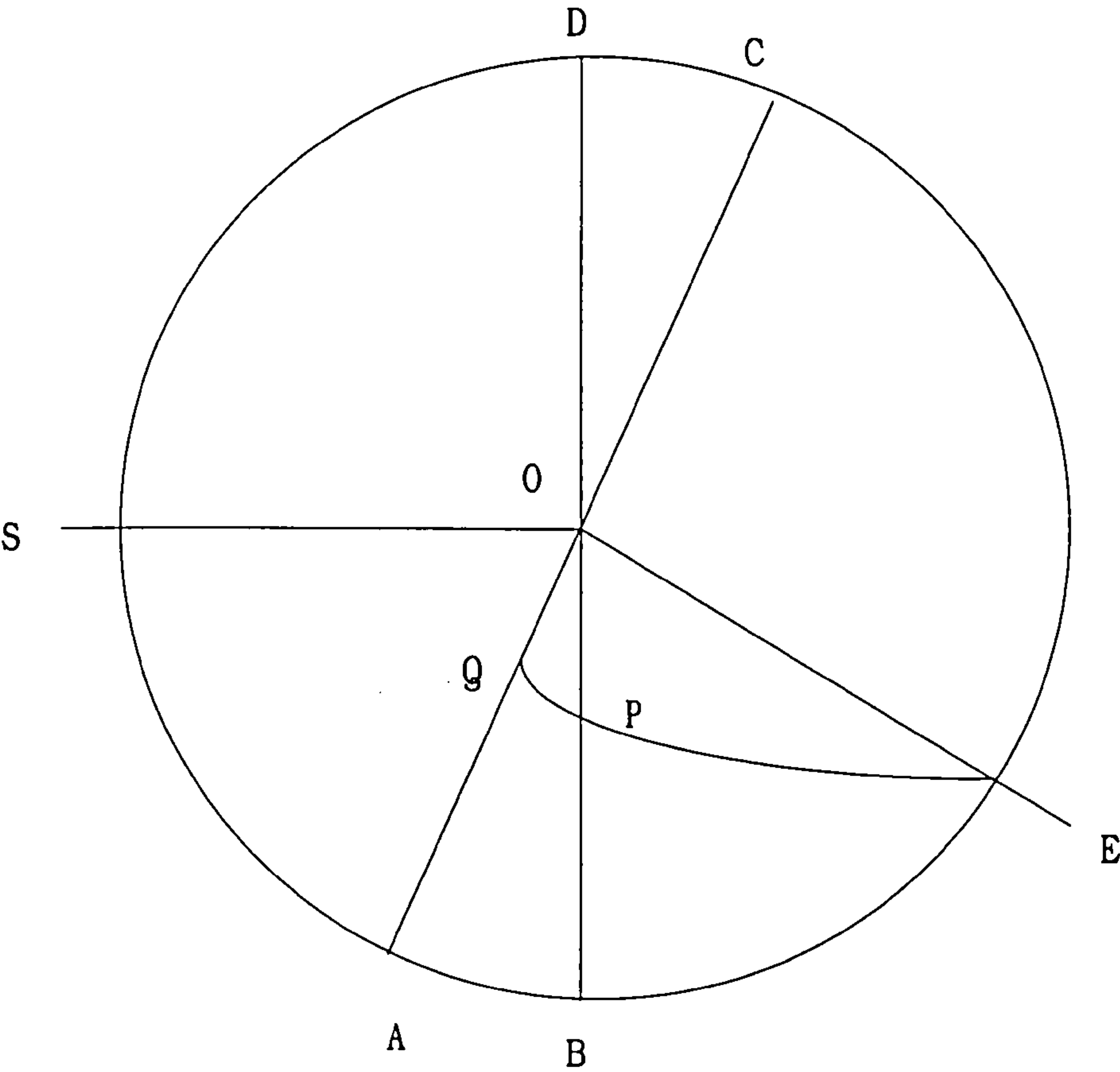


Figure 7.1The diagram of Danjon

Danjon collected 75 measurements and estimates of crescent length and calculated the deficiency arc (the amount of contraction of the sunlit crescent) in each case as a function of the topocentric elongation ( $L'$ ), i.e. taking account of lunar parallax. The result was as shown in figure 7.2, which is a reproduction of figure 30 of (Danjon, 1936) - itself an improvement and different presentation of figure 31 of a previous paper (Danjon, 1932) (I have introduced the comments and dotted lines for illustration). The graph indicates that when the moon is



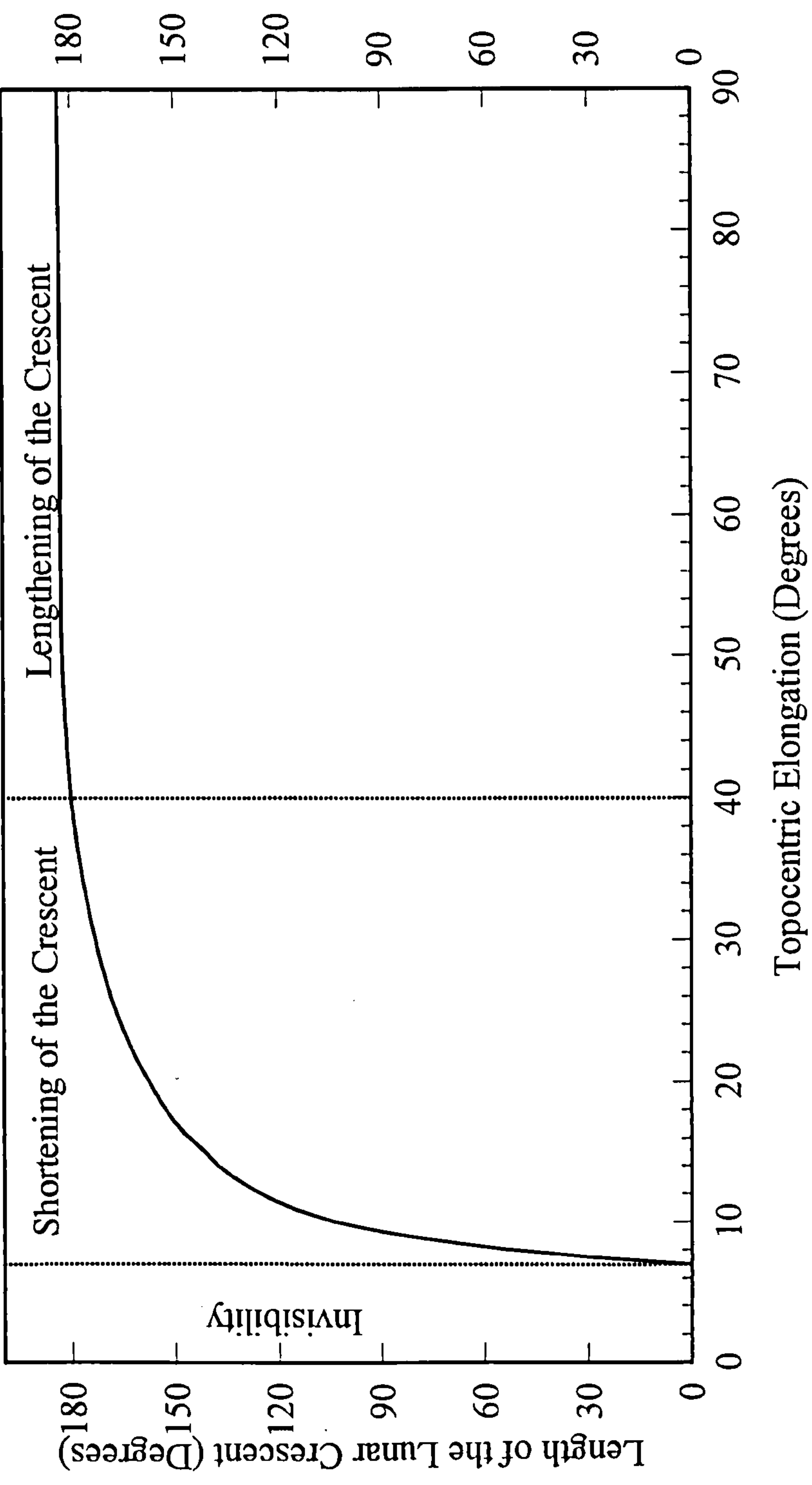


Figure 7.2 Danjon limit

exactly  $7^\circ$  from the sun, then the lunar crescent cannot be seen because no part of it will be sunlit. In other words, according to the above equation, when the topocentric elongation is  $7^\circ$ , the deficiency arc is also  $7^\circ$ . Obviously, when the topocentric elongation is less than  $7^\circ$  there would also be no sunlit crescent visible. However, the arc of deficiency decreases gradually with the increase in topocentric elongation until it starts to be negative after  $40^\circ$  of topocentric elongation, though the change of the negative deficiency arc with topocentric elongation is relatively slow. In summary, when the moon is between  $7^\circ$  and  $40^\circ$  from the sun, the bright cusps of the crescent extend for less than  $180^\circ$ , i.e. the phenomenon is *shortening* of the crescent, whereas when the moon is more than  $40^\circ$  from the sun, its outer terminator extends for slightly more than a semicircle (up to about  $185^\circ$ ), i.e. the effect is *lengthening* of the crescent.

The phenomenon observed by Danjon has important implications for the determination of the first visibility of the lunar crescent. It indicates that, no matter what its age is, the crescent cannot be seen if it is less than  $7^\circ$  from the sun, regardless of any favourable observing circumstances that may exist. The moon of a specific age can be of different elongations from the sun, depending on its latitude and whether near perigee or apogee. Danjon also noted that since the new moon cannot pass more than  $5.5^\circ$  north or south of the sun, which is less than the  $7^\circ$  limit, then the lunar crescent must disappear for a period of time during every lunation.

Danjon suggested that the phenomenon he observed is caused by the shadows of the lunar mountains. However, this explanation has been contested more recently by other researchers. McNally (1983) disagreed with the explanation of Danjon, and also objected to an alternative explanation that attributes the phenomenon to distortions of the figure of the moon. McNally stressed that modern measurements have shown that variations in height of the lunar mountainous terrain and departures from true sphericity are less than 0.6% of the lunar radius; so the arc of shortening cannot be caused by distortions of the moon's shape from sphericity.

McNally has explained the Danjon limit in terms of atmospheric seeing (turbulence). He suggested that the atmospheric seeing causes the crescent to be invisible where the cusp is thinner than the size of the "seeing disk". He thought that if the angular size of the seeing disk is larger than the width of the crescent, then the illumination of the crescent will be spread over a wider area and the illumination per unit area will thereby be reduced. The contrast between crescent and sky having been reduced by the seeing therefore renders the thinner part of the crescent more difficult to detect for both the naked eye and telescope. While conceding that variables such as atmospheric clarity, contrast with sky background and visual response do all



modify how the lunar crescent is actually observed by the human eye or instrument, McNally emphasised that these latter factors are secondary to the effect of the atmospheric seeing. McNally also suggested that, according to his model, the Danjon limit should be  $5^\circ$  rather than  $7^\circ$  as indicated by Danjon (McNally, 1983). Finally, he conceded that while his model can explain the shortening in the length of the lunar crescent it does not explain why the outer terminator of the moon can lengthen at large elongations to rather greater than  $180^\circ$ .

Bradley Schaefer also investigated the phenomenon of the Danjon limit. Though he first accepted the interpretation of Danjon (Doggett, Seidelmann & Schaefer, 1988: 35), he later rejected attributing the phenomenon to shadows of lunar mountains because the required shadow length would have to be a function of the earth's position and because mountain chains would have to average over 12000 meters in height (Schaefer, 1991). Schaefer also gave three reasons for disagreeing with McNally's explanation. Firstly, physiological experiments show that the visibility of unresolved sources does not depend on the smearing imposed on the source. Secondly, the resolution of the human eye ( $42''$  or larger) is always much larger than the size of the seeing disk so that seeing has no appreciable effect on the perceived width - and hence visibility - of the cusp. Thirdly, telescopic and visual observers report essentially the same arc shortening although the perceived cusp widths differ by large amounts (Doggett & Schaefer, 1994).

Schaefer has pointed out that "for naked eye observations, the critical portions of the crescent are always narrower than the resolution of the eye. In such a case, the detection threshold does not depend on the surface brightness of the moon, but on the total brightness integrated across the crescent" (Schaefer, 1991: 271). He, therefore, suggests that the shortening of the length of the lunar crescent is due to sharp falling off of the brightness integrated across the crescent towards the cusps. This he attributes to three reasons, the first of which is that the crescent gets rapidly narrower. The second reason is that the cusps are regions illuminated by the sun when very close to the local horizon and hence on average the polar terrain is illuminated less than equatorial terrain because of greater foreshortening. Thirdly, he suggested that macroscopic roughness of the surface of the moon creates shadows at the lunar poles that cover more of the illuminated surface than at the lunar equator (Schaefer, 1991). On the other hand, Schaefer does confirm that the Danjon limit is  $7^\circ$ .

Recently, Bernard Yallop suggested another possible explanation for the phenomenon observed by Danjon. He believes that "when earthshine on the main disk of the moon matches the brightness of the crescent, which it will do when the crescent is small enough, then there

[will be] no contrast between the disk and the crescent so it cannot be seen” (Yallop, private communication).

The Malaysian scientist Muhammad Ilyas also investigated the magnitude of the Danjon limit. Contrary to McNally who decreased the limit to  $5^\circ$ , Ilyas concluded that the critical elongation should be increased to  $9\text{--}10^\circ$ , below which the crescent cannot be seen because its width would be too small to produce sufficient contrast above the average eye’s threshold (Ilyas, 1982). Later Ilyas increased his limit to  $10.5^\circ$  (Ilyas, 1984, 1988). This is slightly higher than the value suggested by a recent, though not the latest, ruling of the Royal Greenwich Observatory on the visibility of the lunar crescent which states that, “It is unlikely that the new crescent will be visible unless the elongation exceeds  $10^\circ\ldots$ ” (Yallop, 1996).

Ilyas’ derived his  $10.5^\circ$  limit, as discussed later (§7.4), by assuming that the lowest limit of visibility of the lunar width is  $W=0.25'$  and using the somewhat inaccurate formula of Bruin (1977) for converting the elongation to width ( $W=d \sin^2 (L/2)$ ; where  $d$  is the lunar diameter). Bruin’s above simplified formula neglects the fact that the width of the lunar crescent depends to some extent on the distance of the earth from both of the moon and sun. For calculating the width of the lunar crescent I have used the rigorous algorithm of Meeus (1991).

It should be made clear that although Danjon stressed in his publications that his  $7^\circ$  limit is for “topocentric” rather than “geocentric” elongation, neither McNally nor Schaefer have clearly indicated that they have accounted for lunar parallax in their calculations of the elongation. Ilyas, on the other hand, compares his results with Danjon’s yet he is certainly using geocentric elongation. Firstly, he uses Bruin’s above formula which neglects the lunar parallax. Additionally, in one of his papers he gives an equation for conversion between “true” lunar altitude and the elongation, thus implying that he refers to the “geocentric” rather than “topocentric” elongation (Ilyas, 1988).

From the observational and calendrical point of view, the *explanation* of the Danjon limit is not of much interest, but the determination of its magnitude is very important because it is a reliable criterion for rejecting any claim of sighting the lunar crescent when it is less than the Danjon limit from the sun. This does not mean, however, that by the topocentric elongation of the moon merely exceeding the Danjon limit the crescent would *necessarily* be visible. (For instance, Ilyas (1994) refers to such a case of calendrical misuse of the Danjon limit). Other factors, such as the altitude of the moon and atmospheric conditions, may render the lunar crescent invisible despite having a topocentric elongation greater than the Danjon limit.

It might well be the case, of course, that the shortening in the length of a crescent that is  $7^\circ$



from the sun is such that no part of it remains lit, but then this does not mean necessarily that  $7^\circ$  is the lowest visibility limit of the crescent as far as its topocentric angular separation from the sun is concerned. In fact, Danjon's setting of the lowest limit to  $7^\circ$  was not the result of direct measurement of a crescent of just above this topocentric elongation but merely the result of the extrapolation of the curve that he fit to his data. In fact, the smallest topocentric elongation in Danjon's original data was  $8^\circ$  for which the arc of deficiency was  $6.2^\circ$  (Danjon, 1936: 60). In other words, Danjon could have only guessed the arc of deficiency for topocentric elongations less than  $8^\circ$  and the value of topocentric elongation which would have the same value of the arc of deficiency. The line that he fitted to the data could have been shifted in any direction.

I have extracted from table 3.1 all the observations with topocentric elongation less than or equal to  $9.4^\circ$  and have included them in ascending order in table 7.1. Reference to table 3.1 shows that naked eye observations of lunar crescents with topocentric elongations above  $9.4^\circ$  are commonplace, so they are of little relevance to the study of the visibility limit of topocentric elongation. The columns of table 7.1 contain the following information about each of the 45 observations: (1) reference number, (2-4) Julian date, (5-7) coordinates and altitude of the observing site, (8) the result of observing with the unaided eye (I=invisible, V=visible), (9) the result of observing with binoculars or telescope, and finally (10) the topocentric elongation. The table shows that the minimum angular distance between a visible crescent and the sun is  $7.5^\circ$  which is that of the lunar crescent of 1990 February 25 observed from  $83.5^\circ$  W  $35.6^\circ$  N. The table reports three attempts to observe that crescent from the same site, the three of which included the use of binoculars or telescope. Observer 487 succeeded in spotting the young moon by both of the instrument and unaided eye; observer 488 saw the crescent by the optical instrument and could not see it by the naked eye; whereas observer 489 failed to discern the crescent by both means.

The fact that three simultaneous observations of the same crescent produced three different results, as well as the fact that these three observations were made from a place that is as high as 1524 meters above sea level, strongly indicate that a lunar crescent of about of  $7.5^\circ$  topocentric elongation is very difficult to spot, though not necessarily impossible. In other words,  $7.5^\circ$  seems to be the lowest visibility limit of topocentric elongation. This conclusion is further supported by the fact that observer 489 failed to spot the crescent even with the aid of an optical instrument. Table 7.1 shows that all of the 7 lunar crescents with topocentric elongations less than  $7.5^\circ$  were not seen. The  $5^\circ$  limit suggested by McNally (1983) is certainly

**Table 7.1** The entries of table 3.1 with topocentric elongation  $\leq 9.4^\circ$

1	2	3	4	5	6	7	8	9	10
411	1984	1	3	-35.6	15.6	335	I		4.8
336	1922	4	27	-18.5	-33.9	30	I		5.2
483	1989	6	3	155.5	19.8	4255	I	I	6.1
238	1860	1	23	-23.7	38.0	122	I		6.2
275	1871	6	18	-23.7	38.0	122	I		6.3
475	1988	4	16	84.1	37.2	305	I	I	6.6
429	1984	9	25	-35.6	15.6	335	I		7.3
487	1990	2	25	83.5	35.6	1524	V	V	7.5
488	1990	2	25	83.5	35.6	1524	I	V	7.5
489	1990	2	25	83.5	35.6	1524	I	I	7.5
503	1996	1	20	111.0	32.4	853	I	V	7.6
504	1996	1	20	113.2	32.8	259	I	V	7.6
438	1985	4	20	84.1	37.2	305	I	I	7.8
505	1996	1	20	118.3	34.1	530	I	V	7.8
506	1996	1	20	118.3	34.1	530	I	I	7.8
494	1990	5	24	83.5	35.6	1524	I	V	7.8
414	1984	2	2	-35.6	15.6	335	I		7.8
250	1862	4	29	-23.7	38.0	122	I		7.8
501	1995	1	1	106.0	33.0	1219	I	V	7.9
480	1989	5	5	84.8	42.7	259	I	V	8.1
481	1989	5	5	84.8	42.7	259	I	I	8.1
478	1989	5	5	85.7	43.0	244	I	V	8.1
255	1864	5	6	-26.2	39.6	122	I		8.1
412	1984	1	3	84.1	37.2	305	I	I	8.2
327	1921	2	8	6.2	36.5	0	I		8.2
476	1988	6	14	84.1	37.2	305	I	I	8.3
328	1921	2	8	9.1	38.8	0	I		8.3
407	1983	11	5	-35.6	15.6	335	I		8.3
482	1989	5	5	97.0	30.3	183	I	V	8.3
434	1984	11	23	-35.6	15.6	335	I		8.3
361	1972	3	15	117.6	35.5	914	I	I	8.6
360	1972	3	15	117.6	35.5	1128	I	B	8.6
465	1987	6	26	71.0	-30.1	2774	I	B	8.6
495	1990	5	24	110.5	31.6	1372	I	V	8.7
479	1989	5	5	105.5	39.7	3353	I	V	8.7
331	1921	10	31	-18.5	-33.9	30	I		8.9
492	1990	5	24	118.1	34.2	530	I	V	9.0
493	1990	5	24	118.1	34.2	530	V	V	9.0
318	1913	11	28	-18.5	-33.9	91	V		9.1
284	1873	4	27	-23.7	38.0	122	I		9.2
397	1979	1	28	81.3	29.9	0	V		9.2



an underestimation. The same can be said of the  $7^\circ$  limit suggested by Danjon (1932, 1936) and accepted by Schaefer (1991). Di Cicco stressed that, “No sighting has ever penetrated this barrier [ $7^\circ$ ]” (di Cicco, 1989: 322), but in fact by the time of the publication of his paper even the minimum observation of  $7.5^\circ$  had not been made.

On the other hand, the  $10.5^\circ$  limit suggested by Ilyas (1984) is an underestimation of the visibility limit of the human eye. In fact, table 3.1 includes 31 naked eye positive observations of lunar crescents with topocentric elongation less than  $10.5^\circ$ . Even if Ilyas refers to “geocentric” elongation with his figure of  $10.5^\circ$ , i.e. some  $9.5^\circ$  of topocentric elongation, there are still 9 naked eye sightings below that limit. Therefore, according to the available observational data  $7.5^\circ$  is the smallest visible topocentric elongation and the invisibility limit of topocentric elongation could be just below that.

Table 7.1 does also tell us that, with the single exception of observation 487, which was detected at the high altitude of 1524 meters, all the crescents with topocentric elongation in the range  $7.5$ - $8.9^\circ$  were missed by the unaided eye. Of these 28 crescents that escaped sighting by the naked eye, 20 were tried with the help of binoculars or telescope, but still 7 of the 20 evaded detection. The crescent with the second smallest elongation that has been seen by the unaided eye in table 7.1 is  $9.0^\circ$  of observation 493. But this was also spotted with binoculars in addition to the naked eye, and it was sighted from a place which is 530 meters above sea level. The crescent with the smallest elongation that has been seen by the unaided eye and whose detection did not include the use of optical help nor watching from a high place is that of observation 318 which was  $9.1^\circ$  away from the sun at sunset. Table 3.1 shows that the minimum visible topocentric elongation in the Babylonian data is  $9.5^\circ$  of observation 46. However, as already mentioned, the modern data show that naked eye observations of lunar crescents with elongations above  $9.4^\circ$  are commonplace.

From the above discussion, it seems reasonable to conclude that  $7.5^\circ$  is the lowest visibility limit of topocentric elongation (i.e. the limit below which the crescent would be invisible because of the phenomenon of shortening of the outer terminator, regardless of the availability of whatever favourable visibility conditions). However, near sea-level there is little chance that the crescent would be seen when it is less than  $9^\circ$  away from the sun at sunset. This conclusion is in agreement with the recent ruling of RGO that “It is unlikely that the new crescent will be visible unless the elongation exceeds  $10^\circ$ ” (Yallop, 1996), as the RGO figure refers to geocentric elongation. Therefore, it may be concluded that the  $5^\circ$  limit of McNally (1983) and



the 7° limit suggested by Danjon and accepted by Schaefer (1991) are definitely underestimates, whereas the 10.5° limit of Ilyas (1984) is overestimated.

On the basis of such a visibility limit, it can also be concluded that the most favourable time for sighting the crescent is when the moon is near perigee as this would increase its angular distance from the sun for a given age.<sup>1</sup>

## 7.2 The Lunar Altitude-Azimuthal Difference Criterion

As mentioned above, Fotheringham published in 1910 a paper that discussed the problem of the first visibility of the lunar crescent (Fotheringham, 1910), revived the interest of scientists in this matter and took it beyond the friendly competition of observing the youngest crescent. In this paper, Fotheringham suggested a new criterion for predicting the visibility of the lunar crescent which he based on the study of 76 observations of the new moon made by August Mommsen and Julius Schmidt at Athens in the second half of the past century. The collection consisted of 55 positive observations and 21 negative. Fotheringham calculated the true geocentric lunar altitude (i.e. without parallax and refraction) and the azimuthal difference between the sun and moon at sunset for the 76 observations and suggested a visibility criterion in the form of a curve of true lunar altitude (h) versus azimuthal difference ( $\Delta Z$ ). Elongation is a function of these two parameters. The points that define the visibility curve of Fotheringham are shown in table 7.2. He gives the following rough approximation to his conditions of visibility of the moon:

$$\text{Minimum true lunar altitude} = 12.0^\circ - 0.008^\circ \Delta Z^2 \quad (7.1)$$

Fotheringham designed his visibility curve as a dividing line between the positive observations, which lay above the curve, and the negative, which lay below it. In other words, the new moon should be seen when it lies above the curve and should be invisible when it is below the curve. The visibility curves of Schoch (1928) and Neugebauer (1929), who designed their criteria after that of Fotheringham, follow a similar pattern. All but two of the 76 observations that Fotheringham considered were in accordance with his curve. Both of the discordant observations were positive, but fell below the curve. One of them was a morning

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<sup>1</sup> The findings of this section have recently been published in a paper (Fatoohi *et al*, 1998b). Yallop (1998) has accordingly modified the criterion used by the RGO to take into consideration the fact that the lowest visibility limit of topocentric elongation is 7.5°.



observation and he suggested that this may provide evidence that the same rule does not apply to evening and morning observations. This could be the result of different surface brightness of the western and eastern limbs of the moon.

From the fact that only one evening observation violated the visibility curve, Fotheringham concluded that “given a clear sky, the problem is almost purely astronomical”. And having designed a criterion based only on the relative positions of the sun and moon and horizon, he also concluded that his solution of the lunar visibility criterion is independent of geographical latitude and should therefore be “applicable to any place, subject to a slight modification for permanent differences in the clearness of the air” (Fotheringham, 1910: 530).

Fotheringham also suggested that the criterion of Maimonides ( $\lambda_m$ ,  $\Delta\lambda$  and S; see §6.7) has nearly the same effect as his own. In fact, Fotheringham reduced Maimonides’ criterion to a form similar to that of his own (see table 7.2) (Fotheringham, 1928). Fotheringham notes that Maimonides’ visibility curve is slightly lower than his curve, which he attributes to the observation being slightly easier in Jerusalem.

**Table 7.2** The lunar visibility criteria of Fotheringham, Maimonides (as deduced by Fotheringham), Maunder, Schoch and Neugebauer. The calculations are for sunset.

Azimuthal Difference ( $\Delta Z$ )	Minimum True Lunar Altitude (h)				
	Fotheringham	Maimonides	Maunder	Schoch	Neugebauer
0°	12.0°	11.8°	11.0°	10.7°	10.4°
5	11.9	11.3	10.5	10.3	10.0
10	11.4	9.7	9.5	9.4	9.3
15	11.0	9.7	8.0	7.6	8.0
19				6.3	6.6
20	10.0	9.7	6.0		6.2
23	7.7	7.3			4.8

It has been claimed by Ilyas (1994), citing Rizvi (1974), that the h- $\Delta Z$  criterion was originally proposed by the 10th century Muslim astronomer al-Battani (d. 929) and that Fotheringham merely rediscovered it. I am not aware of al-Battani’s work that Ilyas refers to,

which he does not name, and the *zij* of al-Battani (1899) does not contain such a criterion. Fotheringham himself makes no reference to the claimed original solution of al-Battani. It should be stressed, however, that Fotheringham never explained how he came to design his novel criterion which came to be the prototype of a number of similar models.

The altitude-azimuth separation criterion shows that it is not sufficient for the new crescent to be merely above the horizon at sunset to be seen, but it would have to be higher than a certain critical altitude to be just visible. It also suggests that while the minimum lunar altitude from the sun decreases as the azimuthal difference between the two luminaries increases, there is a limiting value of lunar altitude below which the crescent moon would not be seen regardless of the value of the azimuthal separation (see explanation below).

The publication of Fotheringham's criterion was followed the next year by another paper by Walter Maunder who pointed out a serious flaw in the way Fotheringham determined the minimum altitudes of his criterion. Maunder criticised Fotheringham for basing the dividing line of his criterion upon the negative observations whereas it should have been based on the positive. Basing the visibility curve on negative observations is very unreliable because the fact that the moon was not seen on a given occasion does not mean that it was impossible to see it. A crescent could be missed for many reasons, such as unfavourable atmospheric conditions, and not necessarily because it is intrinsically invisible. Further, Fotheringham's criterion could not account for one evening positive observation which, according to his curve, should have been negative.

Maunder also criticised Fotheringham for the fact that the majority of the 76 observations that the latter used in determining his criterion were far above the visibility limit and therefore offered no clue as to the position of the dividing-line between visibility and invisibility. The validity of this criticism of Fotheringham's determination of the dividing line of his visibility criterion was later confirmed by Ashbrook's (1971) note that Schmidt's log of his observations, which Fotheringham analysed, shows that the former seldom saw any extremely young crescent.

In order to improve on Fotheringham's criterion, Maunder collected from the astronomical literature additional 11 positive observations made at various localities. These he added to those of Fotheringham's set which are close to the visibility limit, ruling out the remaining which were irrelevant to the investigation. Maunder plotted these observations and found that the dividing line suggested by Fotheringham cannot be reconciled with an appreciable proportion of the observations. He therefore suggested considerably lowering the dividing line



as seen in table 7.2.

Unlike Fotheringham who positioned his visibility line just above three negative observations, the fact that the data that define Maunder's visibility line (table 7.2) yield a perfect quadratic polynomial fit clearly shows that Maunder used a quadratic to represent his model. The equation that describes Maunder's criterion is as follows:

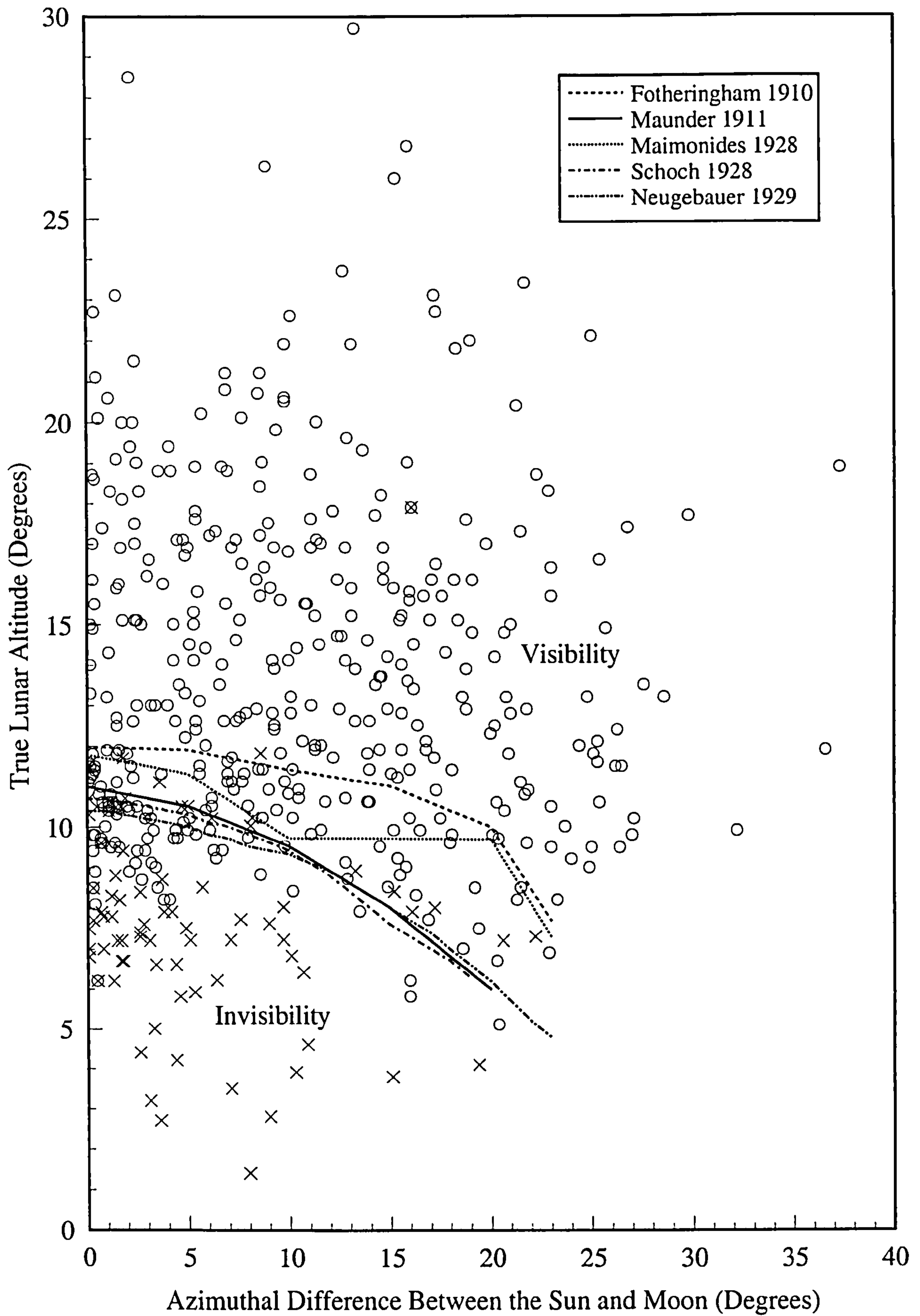
$$\text{Minimum true lunar altitude} = 11 - 0.05 |\Delta Z| - 0.01 \Delta Z^2 \quad (7.2)$$

In addition to the three versions of h-ΔZ criterion of Fotheringham, Maimonides and Maunder (table 7.2), Schoch (1928: 95) and Neugebauer (1929: table E 21) followed Fotheringham and Maunder and suggested another two versions of the same solution whose visibility curves are even lower than that of Maunder. For comparing the various criteria, I have extracted from table 4.2 and included in table 7.2 the relevant entries from the criteria of Schoch and Neugebauer. The criterion used by the Indian Ephemeris and Nautical Almanac (Ashbrook, 1971) is that of Neugebauer, not Schoch as Ilyas (1994) states. The criterion of Neugebauer is represented by equation (7.3):

$$\text{Minimum true lunar altitude} = 10.43 - 0.07 |\Delta Z| - 0.003 \Delta Z^2 - 0.0002 |\Delta Z|^3 \quad (7.3)$$

Fotheringham did not comment on Maunder's criticism of setting the dividing line of his criterion high, but replied to a similar criticism by Schoch who found that a large number of positive observations according to his (Schoch's) formula would be negative according to Fotheringham's. Fotheringham did not concede that his dividing line should have been lower but he rather suggested that Babylon, from which Schoch used data for designing his model, had a greater transparency of air than Athens (Fotheringham, 1928). In fact, as already shown in figure 4.2, Schoch's curve itself is too high to include all positive Babylonian observations and it should be lowered by more than a degree to include all but one of the observations.

I have plotted in figure 7.3 all the observational data of table 3.1 as well as the five forms of the h-ΔZ criterion in table 7.2. The graph shows clearly that the solution of Fotheringham is overly pessimistic. Though it has only two discordant negative observation, Fotheringham's model leaves scores of the positive observations in its invisibility zone. Thus it can be of no practical use for predicting the earliest visibility of the crescent. The relatively large distance between Fotheringham's curve of limiting visibility and the negative observations (except a couple of them) confirms Maunder's criticism. The alternative form of the h-ΔZ criterion which Fotheringham derived from Maimonides' model - although better - is not much different from Fotheringham's. The other three criteria of Maunder, Schoch and Neugebauer are clearly much better than the previous two. These three visibility curves are close to each other.



**Figure 7.3** The various criteria of true lunar altitude versus azimuthal difference with all the observations

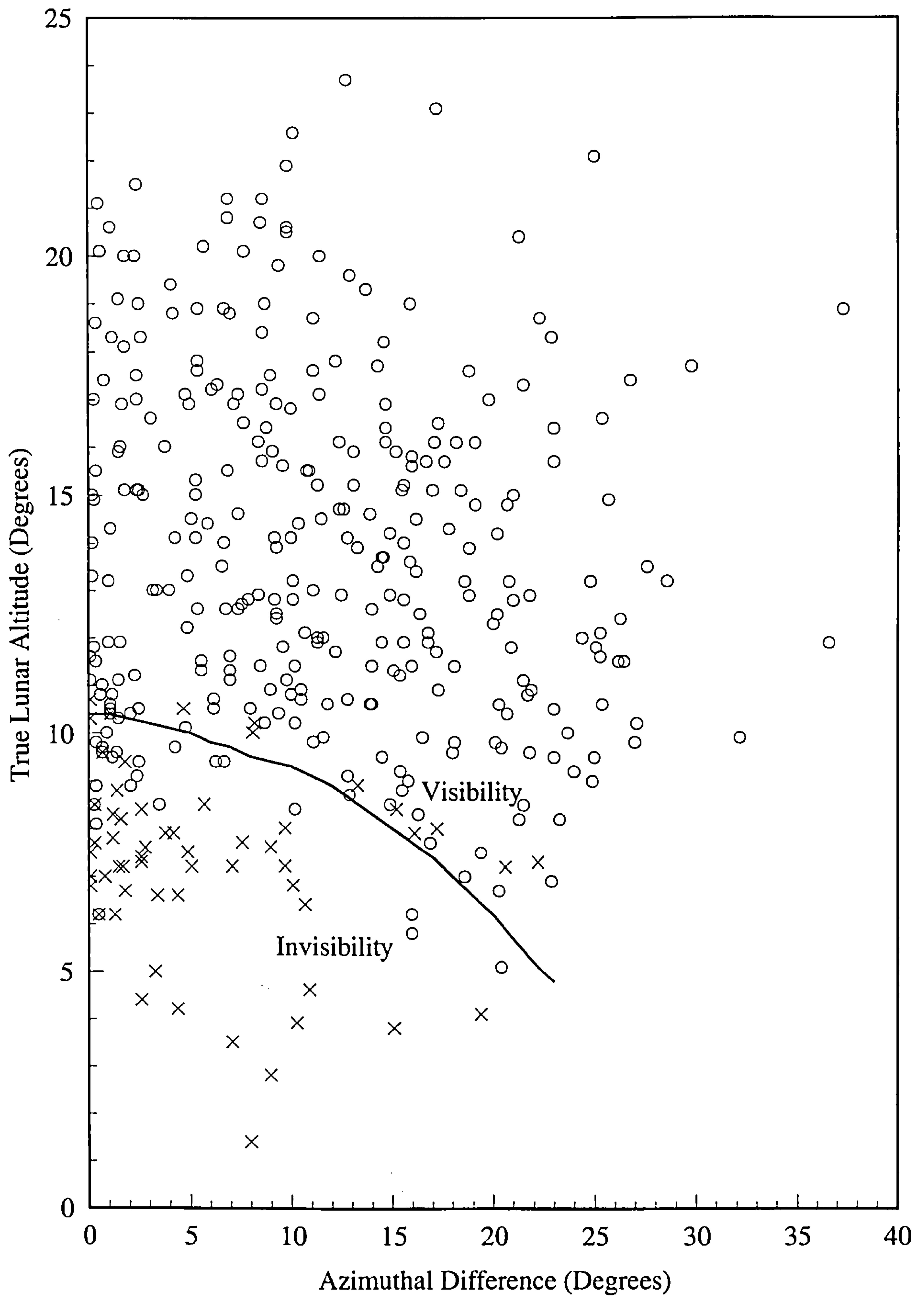


I have checked and compared the accuracy of the highest of the three curves, which is Maunder's, with the lowest curve of Neugebauer. Neugebauer's curve misjudges 24 of the 84 negative observations (i.e. 28.6%) and misses 52 of the 422 positive (i.e. 12.3%). On the other hand, while only 13 negative observations (i.e. 15.5%) fall above Maunder's curve, as many as 75 of the 422 positive observations (i.e. 17.8%) contradict this curve (note that there are several coinciding points in figure 7.3). The significant difference between the error of both models despite the fact that their dividing lines are close to each other is due to the fact that there is a significant number of points in the region that separates both lines. Obviously, neither criterion can be considered satisfactory.

While Schoch suggested that his model is applicable to all latitudes close to that of Babylon, Fotheringham claimed that his criterion is independent of the geographical latitude of the observer. I have used the data in table 3.1 to investigate whether or not the  $h-\Delta Z$  criterion depends on the observer's latitude. Fotheringham, and accordingly Maunder, designed his criterion on observational data from Athens (23.7 W, 38 N), whereas Schoch and Neugebauer formulated their solution to suit observations that are made from latitudes close to that of Babylon (44.4 E, 32.6 N). I have, therefore, separated the observations of table 3.1 into two categories according to the geographical latitude of the observers, including in the first category only the observations made from latitudes  $\pm(30-40^\circ)$ . These consisted of 384 observations, 322 positive and 62 negative. The second group included all other 122 observations, 100 of which are positive and the remaining 22 negative.

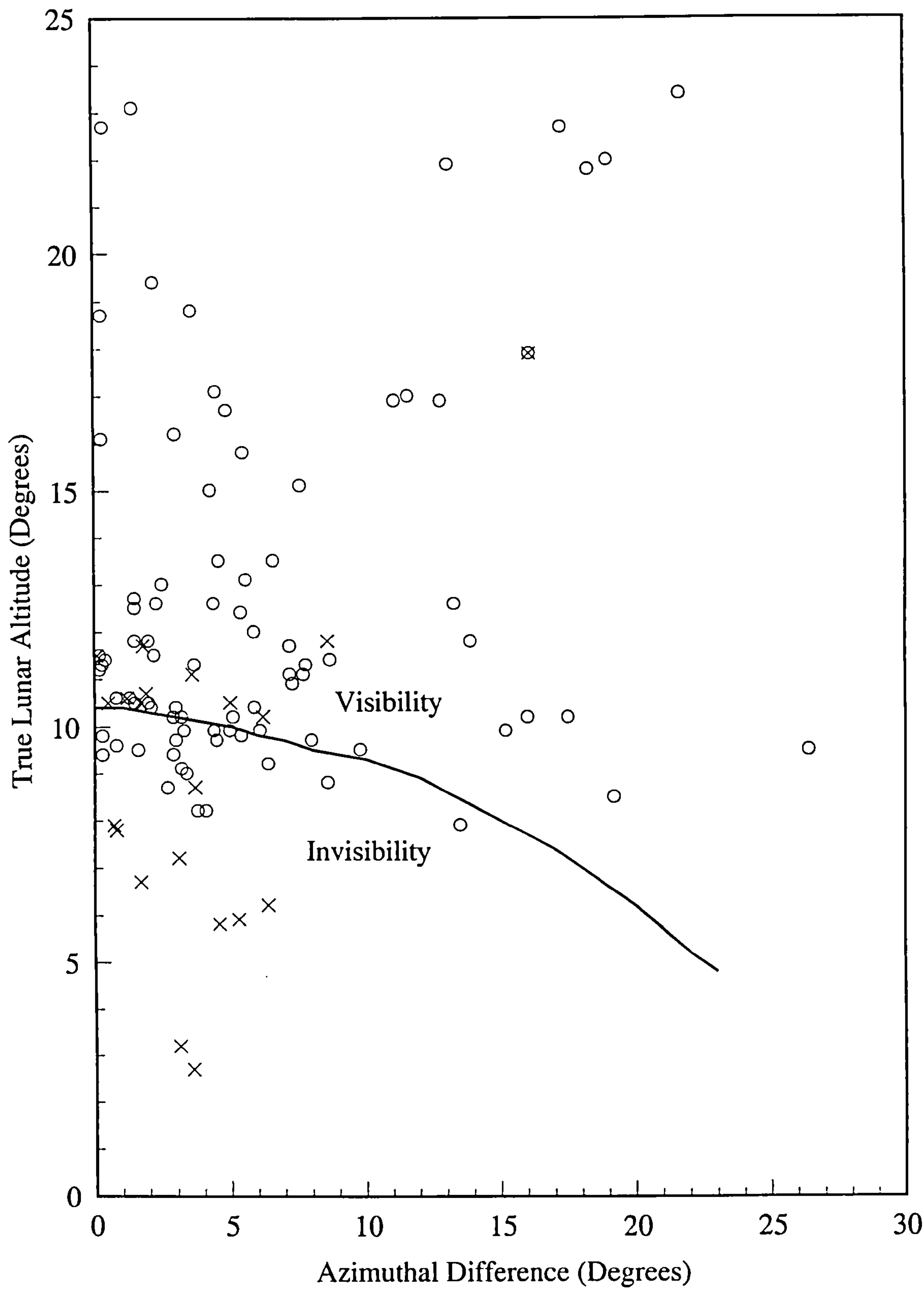
I have plotted in figure 7.4 the mid-latitudes data and Neugebauer's form of the  $h-\Delta Z$  criterion. The curve has 26 of the 322 (i.e. 8.1%) positive observations in the invisibility zone and 13 of the 62 (i.e. 21%) negative observations in the visibility zone. Figure 7.5 is similar to figure 7.4 but it includes the observational data from all the latitudes other than  $\pm(30-40^\circ)$ . Here the curve of Neugebauer has as many as 11 of the 22 (i.e. 50%) negative observations and 26 of the 100 (i.e. 26%) positive in the wrong zone.

It is obvious from figures 7.4 and 7.5 that Neugebauer's criterion gives much larger errors when applied to latitudes away from that of Babylon. This shows that, contrary to Fotheringham's assertion, this type of solution is latitude-dependent. This and the high percentage of error that all forms of this criterion give cannot be overcome by simply lowering or raising the curve or even changing its shape. Any such changes can improve the reliability of the criterion with respect to part of the data but only at the cost of worsening its assessment of the other part. For instance, lowering the curve would decrease the number of positive



**Figure 7.4** The criterion of Neugebauer and the observations for latitudes  $\pm(30-40)$





**Figure 7.5** The criterion of Neugebauer and the observations for latitudes other than  $\pm(30-40)$

observations that are already in the invisibility area, but then this would raise more negative observations to the visibility zone. Similarly, any change to make the criterion more suitable to a certain range of latitudes would make it more unreliable for other latitudes. Table 3.1 shows that at azimuthal difference of  $0.5^\circ$  a crescent that is as low as  $6.2^\circ$  has been seen, which is that of the already discussed observation number 487. This is not an isolated observation. For instance, observer 502 saw a crescent of  $8.1^\circ$  true altitude and  $0.4^\circ$  azimuthal difference with the naked eye only. On the other hand, still for very small  $\Delta Z$ , many crescents that are higher than  $10^\circ$  or even  $11^\circ$  have been missed. Therefore, it seems fair to say that the  $h\text{-}\Delta Z$  criterion, in its present form, is itself inherently of limited utility for predicting the first visibility of the lunar crescent on the global level.

It must also be noted that any visibility curve of the form  $h\text{-}\Delta Z$  has to reach a minimum altitude below which the crescent will not be seen regardless of its azimuthal difference from the sun. The reason for this is the significant diminution in the brightness of the descending moon due to atmospheric extinction. Table 3.1 shows that while the lowest crescent that has been spotted by binoculars or telescope is only  $1.4^\circ$  (observation 465) in true altitude, the  $5.1^\circ$  true altitude crescent of observation 237 is the lowest that has been sighted by the naked eye. Obviously, this is not necessarily the minimum visible altitude and a slightly lower crescent may still be visible. However, Ilyas' suggested altitude limit of about  $4^\circ$  (Ilyas, 1994: 443) seems to be low according to the data that I have compiled.

### 7.3 Bruin's Physical Criterion

All of the above criteria have been criticised for being formulated to satisfy a limited number of observations and implying that all observing sites have exactly the same observing conditions (Doggett & Schaefer, 1994). An attempt to overcome this shortcoming was made by Frans Bruin of the Observatory of the American University of Beirut, Lebanon. Bruin was the first modern researcher to put forward a theoretical, astrophysical criterion for determining the visibility of the lunar crescent, following the theoretical approach of the Muslim astronomers. Bruin built into his pioneer model a number of parameters that take into account the physiology of the human eye, the brightness of the twilight sky, extinction in the atmosphere, and the surface brightness of the moon (Bruin, 1977). However, referring to the fact that the nature of the problem obviates a detailed theoretical approach, Bruin justified his low astronomical

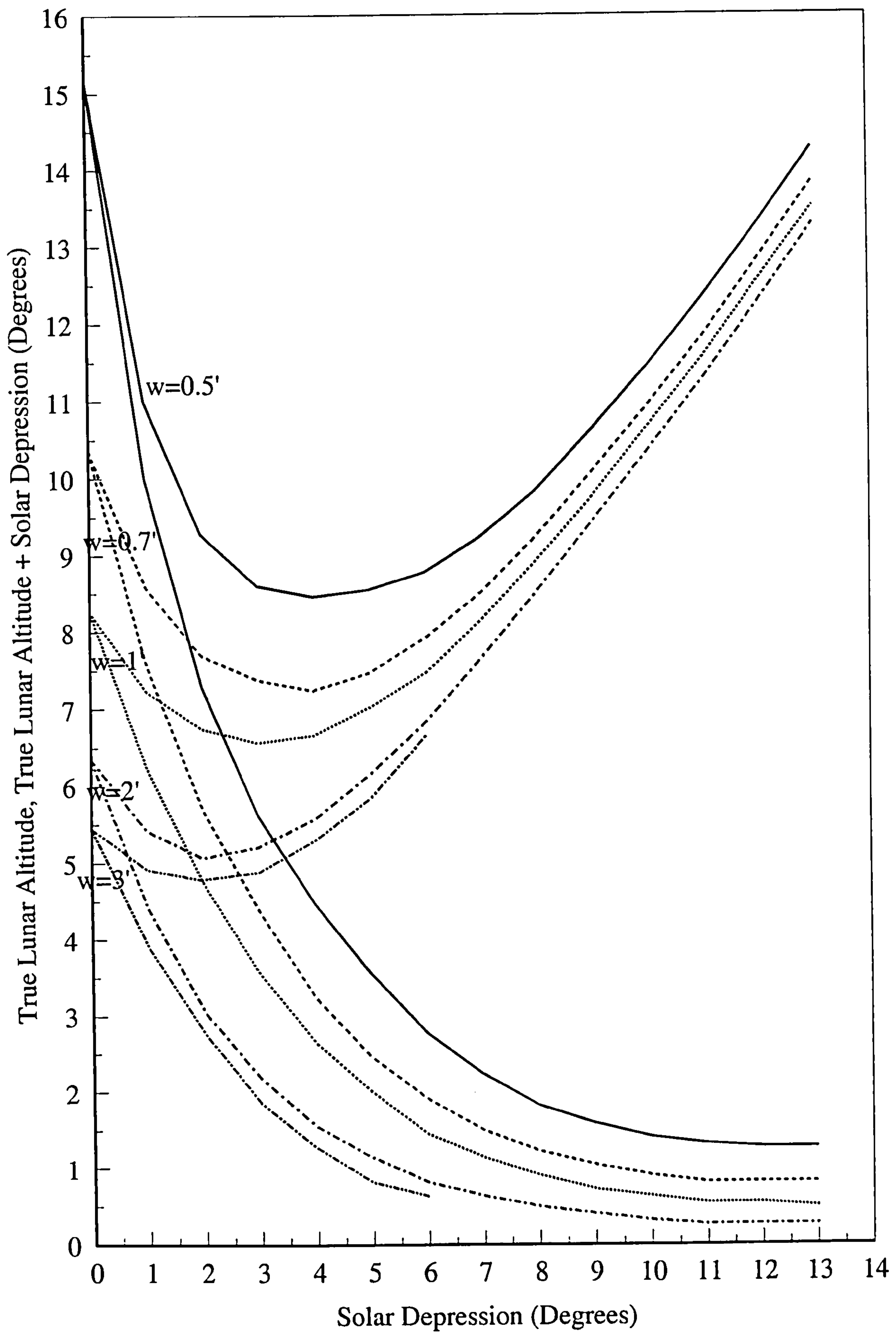


precision when constructing his model.

Bruin set out from the simplified assumption that, at a certain moment, the brightness of the evening sky is independent of azimuth and altitude. In other words, Bruin assumed a western sky of homogeneous brightness which decreases uniformly at every point as the sun descends below the horizon. He deduced his graphical criterion from three diagrams, the first of which is of the mean brightness of the western sky ( $B_s$ ) after sunset as a function of the solar depression ( $s$ ); this diagram is based on the combined results of his own direct measurements and those of another group (Kooman *et al*, 1952) (Bruin, 1977: figure 7). The second diagram is of the brightness of the full moon at night ( $B_m$ ) as a function of the altitude, following the extinction curve of Bemporad (1904) (Bruin, 1977: figure 8). Bruin took the brightness of the moon “to be uniform over its surface, although the slanting light from the sun on the early crescent causes it there to be somewhat brighter” (Bruin, 1977: 339). The third diagram that Bruin made use of is for the minimum contrast observable by the human eye as given by Siedentopf (1940), ignoring the effect of colour. Since Siedentopf’s diagram is for a circular disk whereas the young crescent is a sickle of width less than (half) an arcminute when one day old, Bruin made the assumption that the visibility of a crescent of width  $W$  will be about equivalent to a circular disk of diameter  $W$ , so that he could use Siedentopf’s diagram.

Next, Bruin constructed his diagram of crescent visibility from the above reference diagrams in the following way: firstly, he assumed a certain brightness of the sky,  $B_s$ , and found from the first diagram the corresponding solar depression ( $s$ ); secondly, he found from Siedentopf’s diagram the required minimum contrast of an object of diameter  $W$  (the width of the crescent in this case) in order to see it against a background of sky of the assumed brightness; finally, Bemporad’s extinction curve gave the lunar altitude ( $h$ ) corresponding to that specific lunar brightness ( $B_m$ ). By taking various values of  $B_s$ , Bruin constructed his visibility criterion curve for any particular  $W$ . He made the calculations for five lunar widths: 0.5', 0.7', 1', 2', and 3' (Bruin, 1977: figure 9). Figure 7.6 represents a reproduction of the original visibility curves of Bruin. By plotting his visibility curves starting from  $W=0.5'$ , Bruin implied that this value indicates the thinnest crescent that can be seen by the human eye.

As seen in figure 7.6, Bruin’s approach yielded a simple visibility curve of true lunar altitude as a function of solar depression. Bruin took notice of the fact that, while setting, the arc of descent of the sun and moon,  $H=s+h$ , remains practically constant. So, he attempted to make his results more convenient to use by plotting  $H$  as a function of  $s$  for each of the curves. For each value of  $W$ , both  $h$  and  $H$  are plotted against the solar depression in figure 7.6.



**Figure 7.6** The criterion of Bruin



The use of the visibility diagram of Bruin is quite simple. First, the width of the crescent on the given date must be calculated to select the suitable curve from Bruin's diagram. Now, suppose  $W=0.5'$ , then for an evening after conjunction when  $H=10^\circ$ , the new moon becomes first visible at about  $s=1.6^\circ$ , and remains visible until about  $s=8.2^\circ$ . In other words, the lunar crescent will remain visible for some 30 minutes. On the other hand, when  $H$  is about  $8.6^\circ$ , the new moon will be visible only for a moment, namely when  $s=4^\circ$  and  $h=4.6^\circ$ . Bruin notes that, apart from telling us *when* the crescent will be visible, his curve also tells us *how long* it will be seen.

Bruin stated that in the ten years that preceded the publication of his paper he made careful observations of the new lunar crescent which confirmed that his theory "permits the prediction of visibility, for an evening when the sky is clear, to within five minutes of time". He also added that, "No correction was found to be necessary for our assumption that a sickle of width  $W$  is equivalent to a circular spot of diameter  $W$ ", and therefore concluded that, "It would seem that further refinement of the above theory would serve no practical purpose" (Bruin, 1977: 341). However, the accuracy that Bruin attributes to his criterion and his implied assumption that the human eye cannot see any crescent that is less than  $0.5'$  in width are both refuted by extensive observational data, as shown below.

As for the assumption about the lowest limit of  $W$ , table 3.1 includes 115 sightings of crescents with  $W < 0.5'$ . In 38 of these 115 positive observations the lunar crescent was spotted by both binoculars or telescope and the naked eye, whereas in 77 it was seen by the naked eye only. Therefore, Bruin has grossly underestimated the lowest visible limit of  $W$ .

Table 3.1 also shows that the thinnest lunar crescent that has been spotted using an optical aid is that of observations 487 and 488, and also of observation 503 whose  $W=0.17'$ . The  $0.17'$  wide crescent of observation 487 is also the thinnest lunar crescent that was seen by the naked eye after being first detected by binoculars or telescope. The smallest  $W$  that has been seen by the unaided eye and whose observation did not include the use of binoculars or telescope is  $0.24'$  of observation 318. The thinnest crescent in the Babylonian collection is  $0.26'$  of observation 46. It is also clear that observations of crescents of widths smaller than or equal to  $0.26'$  are not uncommon. There are ten such naked eye sightings, three of which were not preceded by the use of optical aid.

While table 3.1 contains one case of naked eye observation of  $W=0.17'$ , it is obvious that crescents of widths less about  $0.24'$  are unlikely to be seen by unaided eye. This is in accord with Ilyas' suggestion that the lowest limit of  $W=0.5'$  as stated by Bruin is an overestimation

and should be lowered to 0.25' (Ilyas, 1981). However, the observation of the 0.17' crescent indicates that 0.25' is not the lowest lunar width at which the crescent can be seen in ideal circumstances and hence cannot be relied upon to nullify reports of sightings of thinner crescents.

Bruin’s underestimation of the ability of the human eye to sight thin crescents is shown not only in his high lowest limit of W but in his curves of visibility as well (figure. 7.6). I have extracted from figure 7.6 and listed in table 7.3 the minimum vertical separation between the sun and moon that would allow sighting of the moon at each width of Bruin’s five visibility curves. And I have plotted in figure 7.7 the fitted line to Bruin’s criterion as represented in table 7.3 and the observational data of table 3.1. I have extrapolated the fitting line to W=0.17' which is the smallest lunar width that has been seen by the naked eye.

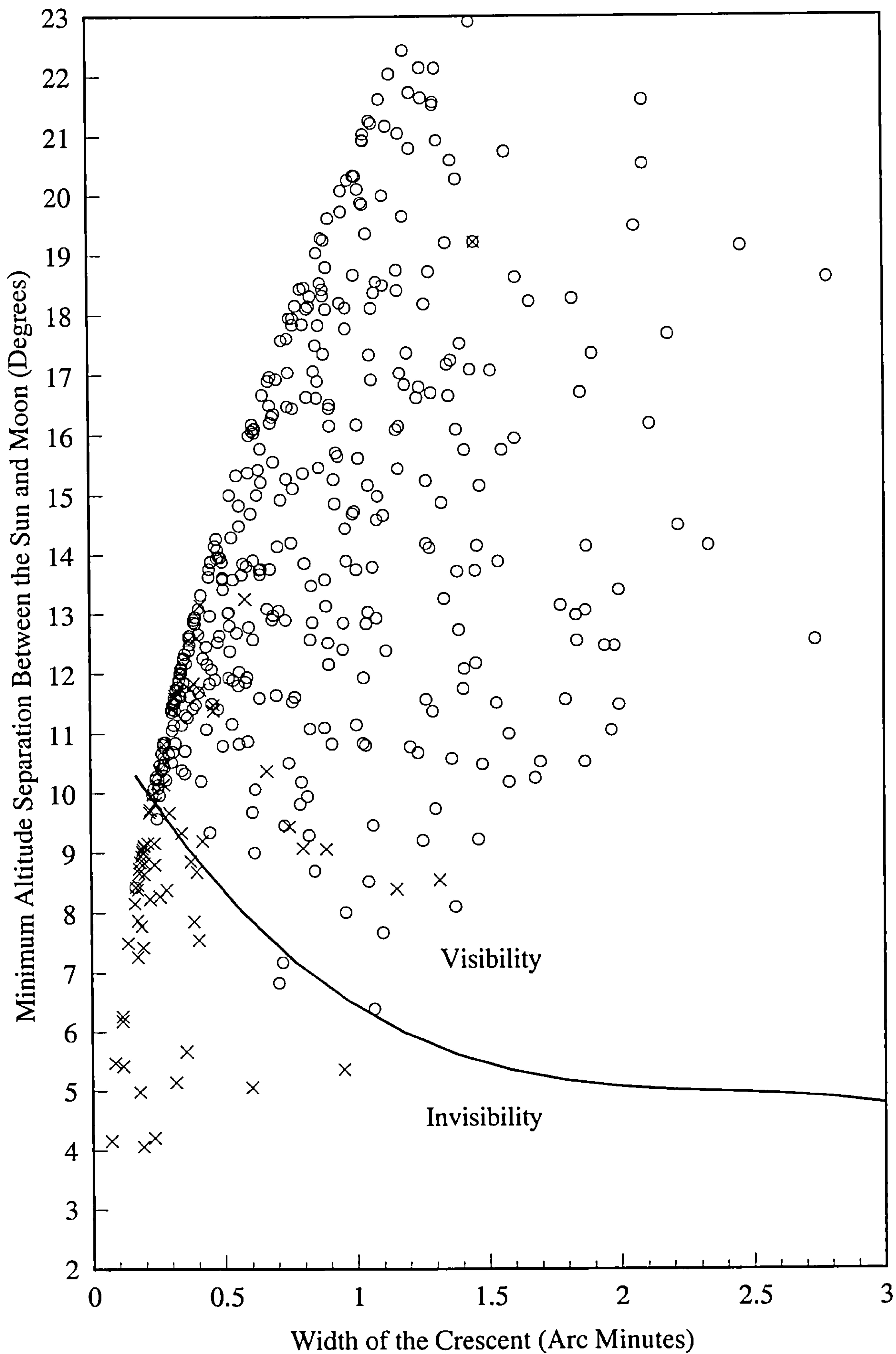
The graph clearly shows the large number of positive observations that Bruin’s cut-off value of W=0.5' would not even consider implying that they would be invisible. There are 115 positive observations with W<0.5' as well other two observations that contradict the model. These 117 data points represent 27.7% of the positive observations. The criterion also wrongly expects 8 negative observations (i.e. 9.5%) to have been visible. The flaws of Bruin’s model are particularly manifest in the zone of small lunar widths and altitudes.

**Table 7.3** Bruin’s minimum arc of descent that would allow the visibility of the moon at each of the five lunar widths.

Lunar Width (W)	Minimum Separation in Altitude Between the Sun and Moon
0.5'	8.45°
0.7	7.23
1	6.55
2	5.05
3	4.77

Taking into account the suggested modification of Bruin’s model of considering 0.17' the minimum width of the lunar disk that a naked human eye may detect, I have found that Bruin’s criterion wrongly expects only 4 positive observations (i.e. 0.9%) to be invisible, yet it has as





**Figure 7.7** Another representation of Bruin's criterion with the observational data

many as 36 of the 84 negative observations (i.e. 42.8%) in the visibility zone.

Interestingly, the percentages of error in both directions in Bruin's original model are almost the same as those of the opposite directions of Neugebauer's criterion. This indicates that the relatively sophisticated theoretical bases of the physical criterion of Bruin did not grant it any accuracy beyond that of the simple, empirical model of Neugebauer. I should also mention here that I am surprised to find that Bruin seems to treat the figures of his visibility graph rather crudely when he reads the coordinates of one of the points as (4,5) rather than (4,4.6). This may reflect Bruin's own assessment of the accuracy of his visibility criterion.

## 7.4 Ilyas' Composite Criterion

Ilyas (1981) conducted a comparison between the empirical  $h-\Delta Z$  criterion of Maunder and the physical criterion of Bruin. He calculated for several new moons in the year 1979 over a terrestrial latitude range of  $\pm 70^\circ$  the longitudes just meeting the minimum visibility requirements of Maunder's set of  $h-\Delta Z$  at local sunset. He also calculated for each of the same latitudes the longitudes just meeting Bruin's minimum visibility limit, i.e. when at sunset the crescent has the minimum visible altitude corresponding to its width (table 7.3). Ilyas found that the first visibility longitudes according to Bruin are shifted to the west of those according to Maunder by about  $70-80^\circ$ . In other words, Bruin's criterion has a significantly higher limit for the first visibility of the lunar crescent than Maunder's. Ilyas attributed this difference to Bruin's lowest cut-off value of  $W=0.5'$ , and found that if this limit is lowered to  $W=0.25'$  (about  $10.5^\circ$  of geocentric elongation according to Bruin's equation) then both criteria would be "consistent and in good agreement at all latitudes" (Ilyas, 1981: 156).

Ilyas came from this comparison with two conclusions. Firstly, since Bruin's criterion is independent of latitude, then the same can be said of Maunder's (Ilyas, 1982: 51), which is in fact that same claim that Fotheringham made when he first suggested his criterion (Fotheringham, 1910). Secondly, since the physical criterion has a "very small uncertainty (a few arcmin), Maunder's form of the first visibility criterion may be used with a similar confidence" (Ilyas, 1981: 159). On the basis of the result of his comparison of the two criteria, Ilyas suggested a visibility criterion in the form of a curve of lunar altitude versus geocentric elongation which extends to about  $25^\circ$  of elongation (Ilyas, 1982). In a subsequent study, Ilyas (1988) extended his curve to cover elongations up to  $60^\circ$  to be usable for high latitudes, and



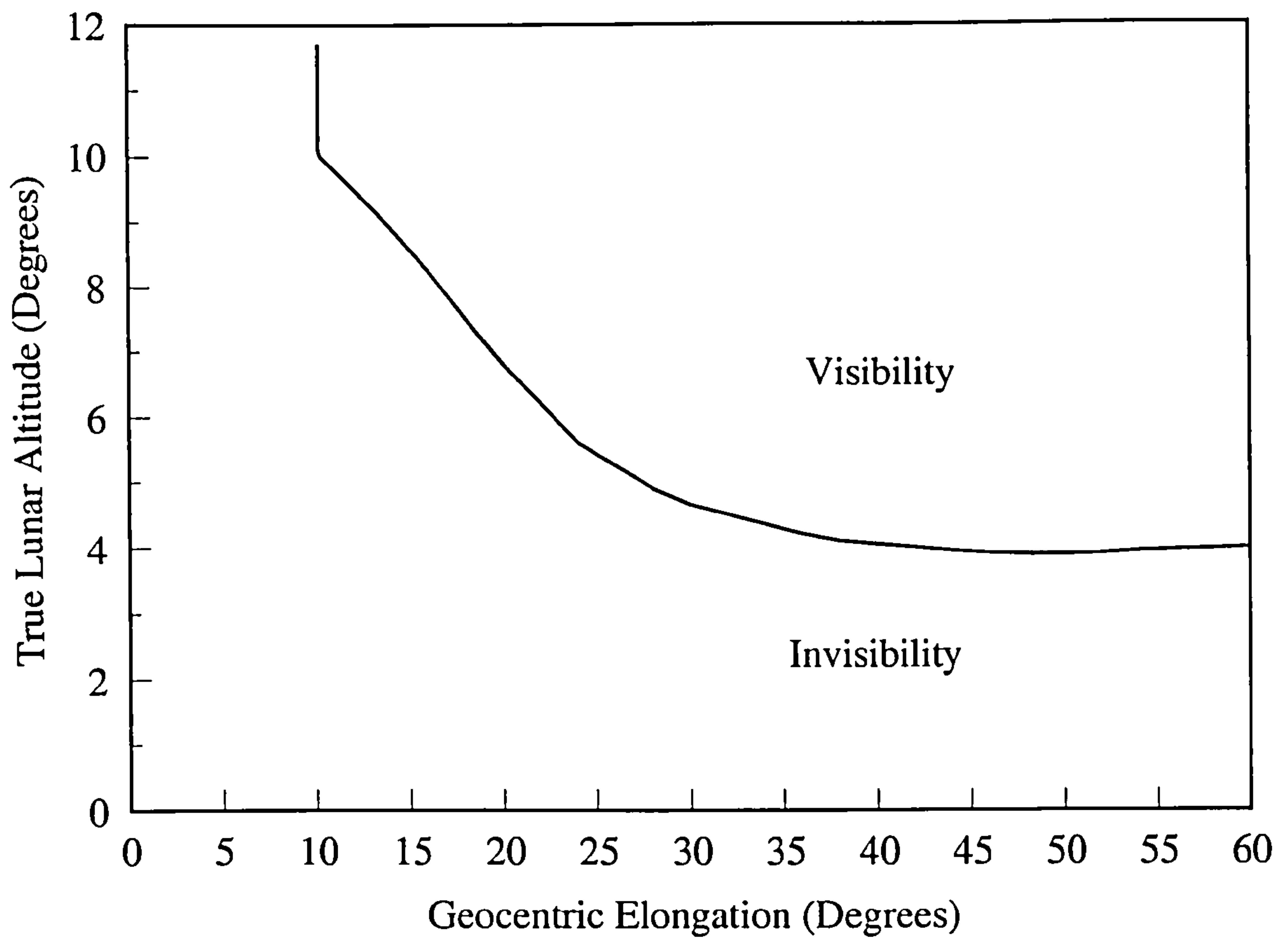
also presented the same criterion in the form of lunar altitude-azimuthal difference. Figures 7.8 and 7.9 are reproductions of both versions of Ilyas’ model.

One aspect of Ilyas’  $h-\Delta Z$  is that although he claims using Maunder’s criterion in developing his own criterion, analysis of Figure 7.9 shows that his curve does not really coincide with that of Maunder but that it is much closer to Neugebauer’s. Ilyas has clearly indicated in the first paper that he published about his composite criterion (Ilyas, 1981) and in some of his subsequent publications (e.g. Ilyas, 1984, 1985, 1994) that he used Maunder’s model. However, he referred in other papers to the “Indian” criterion (e.g. Ilyas, 1982, 1987), a term which he uses to refer to Neugebauer’s model. I cannot explain this apparent contradiction, given the significant difference between both criteria especially for smaller  $\Delta Z$ . I have included in table 7.4 the  $h-\Delta Z$  criteria of Ilyas, Maunder and Neugebauer for comparison.

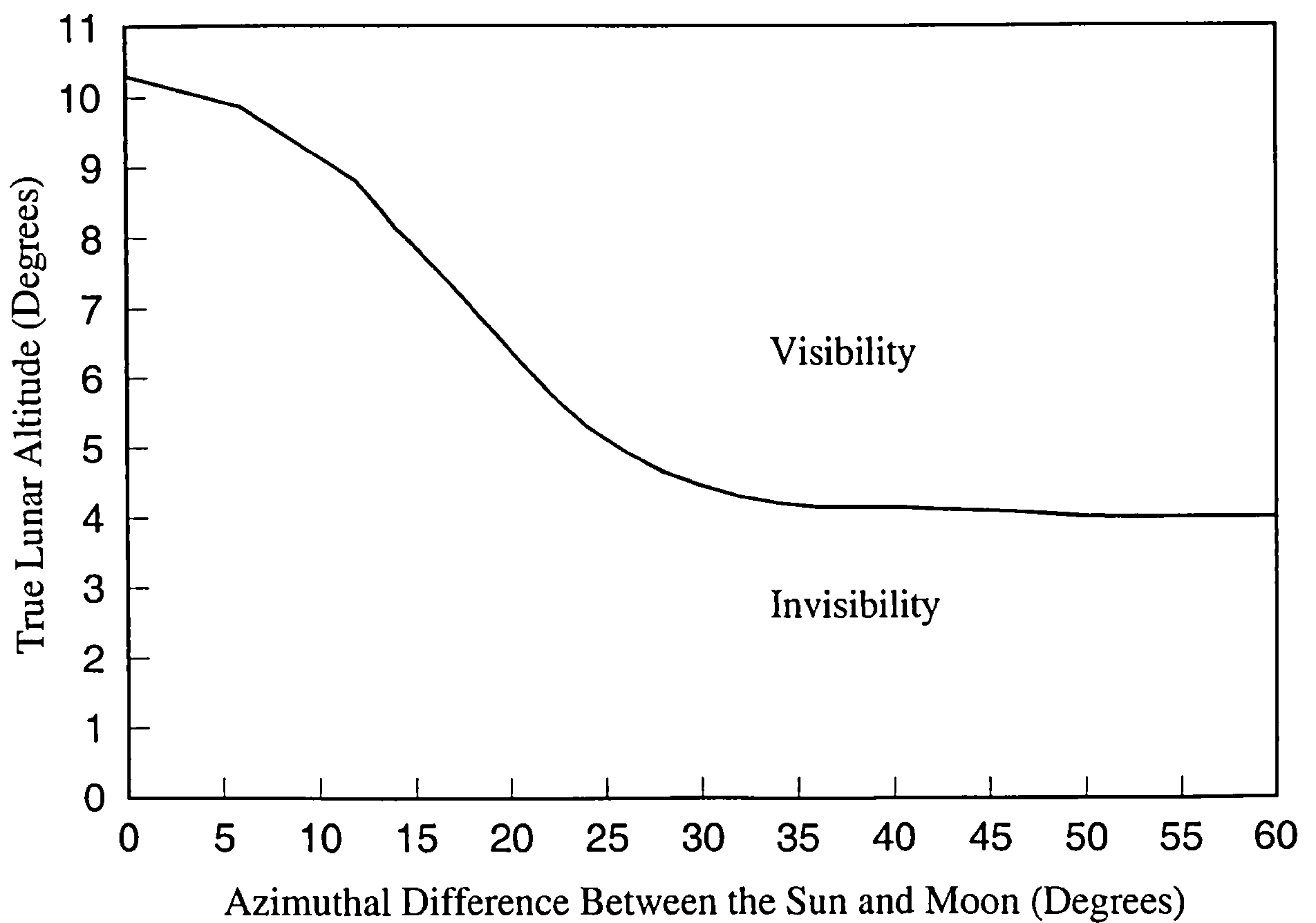
**Table 7.4** the  $h-\Delta Z$  criteria of Ilyas, Maunder and Neugebauer

Azimuthal Difference ( $\Delta Z$ )	Minimum True Lunar Altitude (h)		
	Ilyas	Maunder	Neugebauer
0°	10.3°	11.0°	10.4°
5	9.9	10.5	10.0
10	9.15	9.5	9.3
15	7.9	8.0	8.0
19	-	-	6.6
20	6.4	6.0	6.2
23	5.6	-	4.8

Despite his assertion of using two consistent, independent solutions to develop his own criterion, Ilyas’ approach suffers from a fundamental flaw. Ilyas used Maunder’s (or, in fact, Neugebauer’s) criterion to show that Bruin’s lowest limit of the lunar width should be reduced to  $W=0.25'$  instead of  $0.5'$ , and later he used Bruin’s criterion to extend Maunder’s to larger azimuthal difference. Yet he does not seem to have made any attempt to independently verify the reliability of either of the two criteria which he used to calibrate each other. In other words,



**Figure 7.8** The h-L criterion of Ilyas (1982, 1988)



**Figure 7.9** The h-ΔZ criterion of Ilyas (1988)

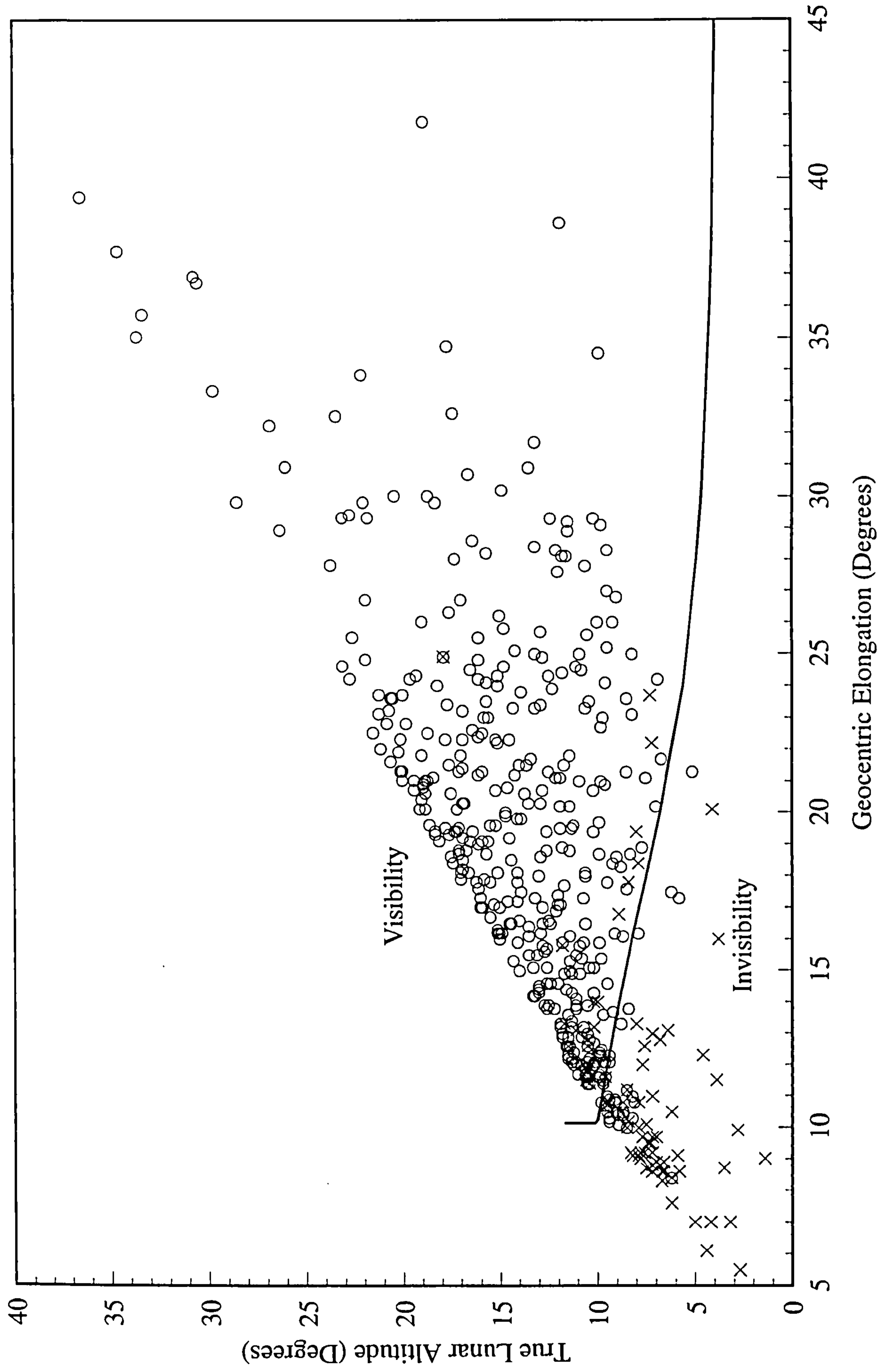


the fact that two independently derived criteria agree with each other does not necessarily imply that they are sound criteria. One should not forget, after all, that Bruin developed his criterion in full knowledge of previous works on the subject - including that of Maunder and the data that the latter used to develop his visibility curve. One obvious false result of Ilyas' approach is his conclusion that since Bruin's model has "very small uncertainty (a few arcmin)" (Ilyas, 1981: 159), then Maunder's criterion has the same accuracy. Ilyas seems to refer here to Bruin's assertion that his criterion "permits the prediction of visibility, for an evening when the sky is clear, to within five minutes of time" (Bruin, 1977: 341). However, the above investigation of Bruin's solution (§7.3) has revealed that it achieves far less than the claimed accuracy.

I have investigated both forms of Ilyas' criterion using the observational data of table 3.1. In figure 7.10, I have plotted the observational data and the h-L criterion of Ilyas (1984, 1987). There are 25 negative observations in the visibility zones (i.e. 29.8%) of the total number. As already mentioned when discussing the Danjon limit, the criterion is flawed in the region  $7.5^\circ \leq L' \leq 9^\circ$  as it has an unrealistic cut-off value of about  $L=10^\circ$  (which Ilyas later even increased to  $10.5^\circ$ ). However, this had no effect on the number of contradictory positive observations because those with small L are already below the dividing line of visibility anyway. This criterion misjudges as many as 33 of the sighted crescents (i.e. 7.8%). In a study that included (positive) observations from the 12th century, McPartlan found that Ilyas' curve should be adjusted down by about  $0.5^\circ$  to include positive observations that fell in the invisibility zone of Ilyas' criterion (McPartlan, 1991, 1996).

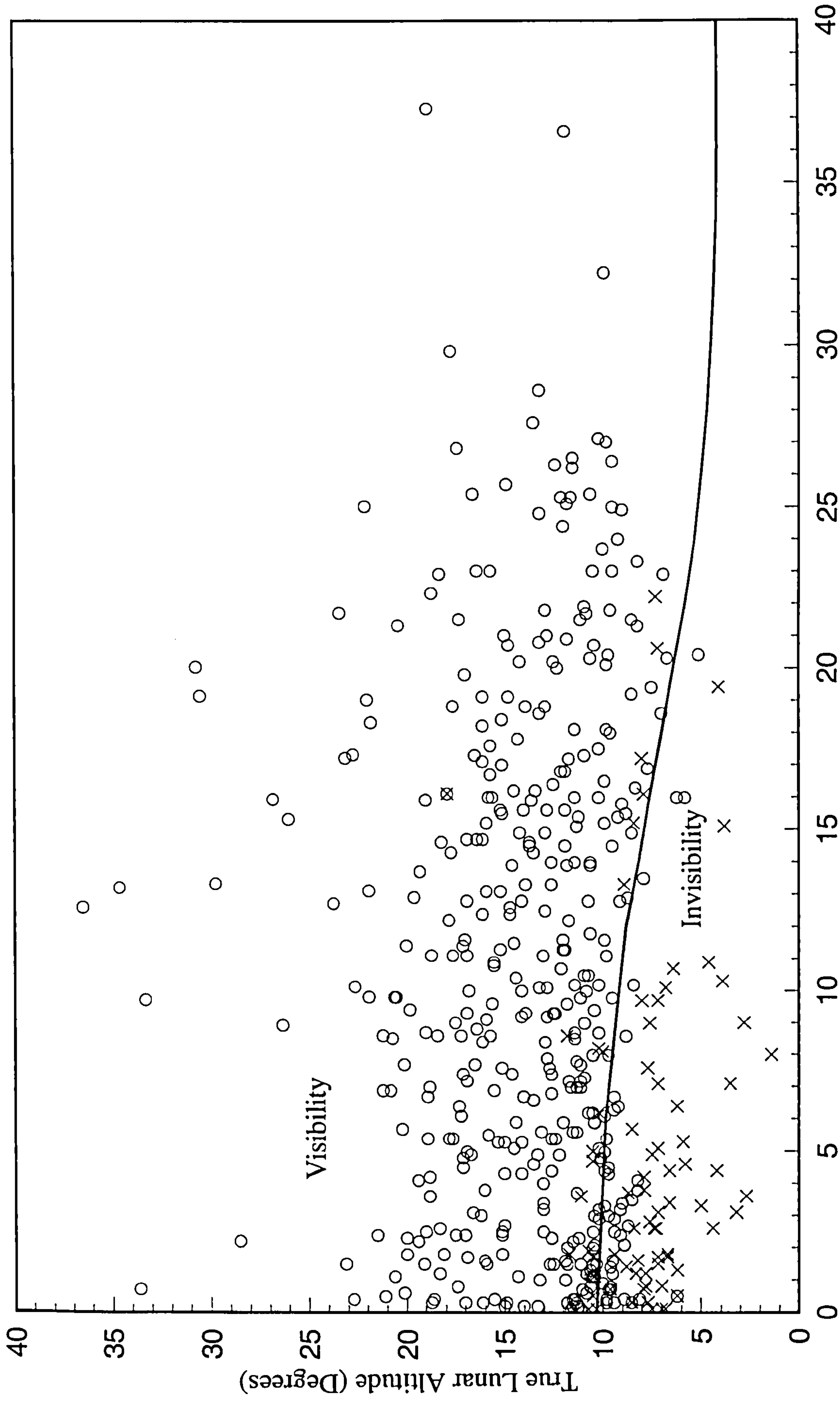
In figure 7.11, I have plotted the observational data and Ilyas' criterion of h- $\Delta Z$ . This model contradicts 24 negative observations (i.e. 28.6%) of the total number and it has 48 discordant positive observations (i.e. 11.3%) below the curve. As expected, the percentages of error differ from those of Maunder's model but are almost exactly the same as those of the model of Neugebauer because Ilyas's criterion is itself based on Neugebauer's not Maunder's.

I also note that neither of Ilyas' models agrees with Bruin's, a result that contradicts Ilyas' claim that he derived his criteria from Bruin. Furthermore, given that Ilyas' model is indeed based on Neugebauer's (not Maunder's), I conclude that Ilyas was wrong in claiming to have found "excellent agreement" between Bruin's and Neugebauer's (Maunder's) models. Given the very inaccurate assumptions that Bruin embedded in his solution and the fact that he applied what he calls a "*gestalt* factor", i.e. some unexplained empirical changes, it would have been indeed surprising if Bruin's model would have come out in "excellent agreement" with



**Figure 7.10** The lunar altitude-elongation criterion of Ilyas with the observations





Azimuthal Difference Between the Sun and Moon (Degrees)

Figure 7.11 The lunar altitude-azimuthal difference criterion of Ilyas with the observations

any other independently developed solution.

## 7.5 The Lunar Age Criterion

It has been suggested that the age of the moon after conjunction was used in ancient times for predicting the visibility of the lunar crescent. Bruin (1977: 333) erroneously claimed that the Babylonians' had two conditions of visibility, namely, that the age of the moon must be greater than 24 hours and that its separation from the sun in right ascension must be more than  $12^\circ$ . Schaefer states that in ancient times it "was canonized as a rule that the moon will be visible if its age is greater than 24 hr" (Schaefer, 1996: 761). Certainly, using the age of the moon only to predict the visibility of the lunar crescent, if reliable, would be one of the simplest such criteria.

Astronomers, professional and amateur, have long found fascination in sighting very young crescents (e.g. Whitmell, 1909; MacKenzie, 1922). But Fotheringham was probably also the first in this century to publish a paper about the effect of the age of the lunar crescent after conjunction on its visibility (Fotheringham, 1921). In his paper, Fotheringham mentions that Carl Schoch had stated in then unpublished material "the rule that from February 1 to April 15 the minimum age of the moon for visibility in terrestrial latitudes between  $45^\circ$  and  $52^\circ$  varies from 20 hours to 23 hours; the former of these values should hold where the moon's mean anomaly is between  $340^\circ$  and  $20^\circ$  and her argument of latitude (the mean distance of the moon from its ascending node) between  $70^\circ$  and  $110^\circ$ , the latter when her anomaly is between  $160^\circ$  and  $200^\circ$ , and her argument of latitude between  $250^\circ$  and  $290^\circ$ " (Fotheringham, 1921: 310). Fotheringham did not try to present a visibility criterion based on the age of the new moon but he was merely interested in showing that as the age of the lunar crescent increases its altitude, which is one of the parameters of his visibility criterion (Fotheringham, 1910), also increases. Fotheringham also indicated that the increase in the age of the lunar crescent is reflected in its increase in brilliance.

Ashbrook has noticed that "observations of the crescent 24 hours before or after new are fairly common, but sightings less than 20 hours from conjunction are very rare" (Ashbrook, 1972: 95). Doggett, Seidelmann & Schaefer (1988: 34) have also stressed that "sightings of the Moon within 20 hours of conjunction are extremely rare". The Royal Greenwich Observatory ruling on the visibility of the lunar crescent states that, "The crescent cannot be seen when the



age is less than 14 hours and is usually visible by the time the age is 30 hours” (Yallop, 1996).

The modern form of lunar age as a criterion for predicting the first visibility of the lunar crescent was presented by Ilyas (1983). Using his h-L criterion, Ilyas computed for each new moon of the years 1979-1984 the geographical longitudes of its first visibility at the geographical latitudes 0, 30 and 60°. He then calculated the age of the lunar crescents at those longitudes of first visibility and plotted for each latitude the age of the crescent as a function of the time of year. Ilyas found that the data for all latitudes show a similar pattern of variation with time of year together with a superimposed variability at a given time. The latter resulted in a band of values which varied in width from almost zero around day numbers 140 and 290 to a maximum around day number 230 (Ilyas, 1983: figure 1). Ilyas considered the width of the band to represent the upper and lower limits of age requirements of visibility.

In addition to its dependence on the time of year, the age criterion showed strong dependence on the geographical latitude, with the amplitude of variation increasing with latitude. Ilyas found that the age criterion of visibility is  $20 \pm 4$  hours at the equator,  $25 \pm 8$  hours at latitude 30°, and about  $40 \pm 20$  hours at latitude 60°. Ilyas concluded that “at the lower latitudes, especially in the equatorial belt, the data should be quite useful in making a first estimation” (Ilyas, 1983: 26). He also inferred that his lowest limit of about 16 hours at the equator is consistent with the record of sighting the youngest crescent (Ilyas, 1983, 1994).

Despite the novelty of the concept behind his development of the age criterion, Ilyas’ results were bound to be unsatisfactory because of the inaccuracy of his original h-L criterion that he used to develop the lunar age criterion. Additionally, his claim, citing Ashbrook (1971), that the youngest sighted crescent is 16 hours is incorrect. In fact, Ashbrook states in the cited article that the sighting record is 14.5 hours reported by Whitmell (1916).

In 1988 Schaefer stated that the record is that of observation 360 (14.9 hours), which he cited as a naked eye as well as binoculars observation (Schaefer, 1988: 520). I checked the original report of that particular observation and found that the reporter explicitly states that “because the crescent was hard to see, [the observer] made no attempt at a naked-eye sighting”, and he continued to view the crescent for three minutes with the binoculars only (McMahon, 1972). This entry is wrongly cited in Schaefer’s compilation as a naked eye positive sighting. I have corrected this information in table 3.1.

In a subsequent paper, Schaefer stated instead that the record for naked eye observation is 15.4 hours, which was set by Julius Schmidt at Athens for a morning crescent (Schaefer, 1992: S34). A later and rather detailed study of the records for young moon sightings (Schaefer,

Ahmad & Doggett, 1993) dismissed the reliability of the record cited by Ashbrook after investigating from historical records the circumstances of the assumed observations. This study re-affirmed Schaefer's claim that the 15.4 hours crescent which was observed by Julius Schmidt is the youngest that has ever been seen, and it additionally found that the record for binoculars observations is 13.47 hours. The supposed Schmidt's observation, however, was later rejected by Loewinger who found that it was not made by the highly skilled and experienced observer Julius Schmidt but rather by his unskilled gardener Friedrich Schmidt in a casual observation (Loewinger, 1996).

In a more recent paper, Schaefer (1996) reported a new sighting record by the experienced John Pierce who sighted the new crescent of 1990 February 25 from Collins Gap in eastern Tennessee (83.5 W, 35.6 N) at about 23:55 UT, i.e. when the crescent was only 15.0 hours old (observation 487; the age shown in table 3.1 is that at sunset whereas the actual observation was after sunset). Observing from a high altitude, Pierce spotted the very young crescent with the unaided eye, and the sighting was confirmed with a 12.5-inch telescope as seen by four people. The moon was also seen with unaided vision by another person, seen by two persons only through binoculars and the telescope, and totally missed by another two. As for observing with a telescope, Schaefer suggested that the record of sighting the youngest crescent is 12.1 hours - for observation 503 (the 12 hours cited in table 3.1 is the age at sunset). It is clear that around the equinox is the most favourable time of the year for viewing a young crescent because at that time the ecliptic crosses the horizon more steeply than other times of the year.

Obviously, Ilyas' criterion which claims that no crescent younger than 16 hours can be seen underestimates the ability of the human eye to detect young crescents. Ilyas' wrong conclusion could have been the result of the 7.8% probability of his h-L criterion reporting a visible crescent as being invisible. This may have resulted from totally neglecting the region  $7.5^{\circ} \leq L' \leq 9.5^{\circ}$  by setting the cut-off limit of geocentric elongation at  $10.5^{\circ}$ . Obviously, if the original criterion of Ilyas were more realistic he would have made a better prediction of the age limit, but then this would mean that the difference between the upper and lower limits of the age criterion would be larger; consequently, this criterion would have been even a weaker prediction tool for the first visibility of the lunar crescent.

Table 3.1 contains three positive observations whose crescents were less than 16 hours old at sunset. These ages are 14.6 (15.0 at the time of observation), 15.2 and 15.9 hours, which relate to observations 487, 493, and 46, respectively. Interestingly, these three entries are from mid-latitudes where, according to Ilyas' criterion, the crescent would have to be at least about



17 hours old to be visible. Additionally, table 3.1 shows that crescents as old as 52.8 and 51.0 hours have been missed (observations 217 and 257, respectively), whereas the youngest and oldest crescents (still at earliest visibility) that have been seen are 15.0 and 80.7 hours (observations 487 and 305, respectively). To avoid the problem of the dependence of a criterion on latitude, one can look at the observations from one location, say that from Babylon. Here again the difference between the youngest and oldest crescents is very large. The youngest crescent in the Babylonian collection is 15.9 hours of observation 46 and the oldest is 63.1 hours of observation 157. The difference is 47.2 hours which represents a difference of two days in the date of earliest visibility of the crescent. It is possible that some of the late sightings of crescents were caused by unfavourable weather conditions, but it is improbable that this factor is totally responsible for the large range of crescent age. It is obvious, therefore, that the age criterion is not a reliable criterion for lunar visibility.

## 7.6 The Moonset Lagtime Criterion

As mentioned earlier, the early Indian astronomers had suggested the simple visibility criterion (often erroneously attributed to the Babylonians) that the crescent would be visible if it sets after the sun by 48 minutes or more. Needless to say, if reliable, this would be the simplest lunar visibility criterion.

This supposed criterion has been criticised in general by almost all modern researchers, except Loewinger who investigated one of Schaefer's collections of observations (Schaefer, 1988) and concluded that "the simple Indian rule for invisibility ( $[S] < 48$ ) [is] a very useful instrument for the elimination of false reports" (Loewinger, 1996: 451). The major drawback of Loewinger's study is that it included a limited number of observations. The much larger number of observations that I have compiled leads to a conclusion that is totally contrary to Loewinger's.

I have extracted from table 3.1 all positive observations (whether by the unaided eye or optical instrument) of lagtime less than 48 minutes and have included them in table 7.5 in their ascending order. The table's columns include the following information about the observations: (1) reference number, (2-4) Julian date, (5-7) coordinates and altitude of the observing site, (8) the result of observing with the unaided eye (I=invisibility, V=visibility), (9) the result of observing with binoculars or telescope, and finally (10) the lagtime. Table 7.5 contains 13

**Table 7.5** The entries of table 3.1 with moonset lagtime < 48 minutes

1	2	3	4	5	6	7	8	9	10
465	1987	6	26	71.0	-30.1	2774	I	B	17
63	-264	9	26	-44.4	32.6	15	V		29
440	1986	12	31	77.0	39.0	30	I	B	32
237	1859	10	27	-23.7	38.0	122	V		33
54	-284	11	6	-44.4	32.6	15	V		36
134	-170	10	8	-44.4	32.6	15	V		36
487	1990	2	25	83.5	35.6	1524	V	V	38
488	1990	2	25	83.5	35.6	1524	I	V	38
503	1996	1	20	111.0	32.4	853	I	V	38
433	1984	11	23	81.0	34.0	61	I	V	39
504	1996	1	20	113.2	32.8	259	I	V	39
51	-291	8	26	-44.4	32.6	15	V		41
175	-133	9	19	-44.4	32.6	15	V		41
482	1989	5	5	97.0	30.3	183	I	V	41
505	1996	1	20	118.3	34.1	530	I	V	41
14	-372	7	23	-44.4	32.6	15	V		42
112	-189	11	7	-44.4	32.6	15	V		42
187	-117	10	22	-44.4	32.6	15	V		42
360	1972	3	15	117.6	35.5	1128	I	B	42
42	-302	10	27	-44.4	32.6	15	V		43
133	-170	8	9	-44.4	32.6	15	V		43
174	-133	8	20	-44.4	32.6	15	V		43
501	1995	1	1	106.0	33.0	1219	I	V	43
41	-302	8	28	-44.4	32.6	15	V		44
406	1983	11	5	84.1	37.2	305	I	V	44
146	-162	8	11	-44.4	32.6	15	V		45
178	-132	10	7	-44.4	32.6	15	V		45
301	1878	10	27	-23.7	38.0	122	V		45
264	1865	7	24	-23.7	38.0	122	V		46
494	1990	5	24	83.5	35.6	1524	I	V	46
15	-370	8	1	-44.4	32.6	15	V		47
26	-328	10	13	-44.4	32.6	15	V		47
397	1979	1	28	81.3	29.9	0	V		47
398	1979	1	28	82.4	29.7	0	V	B	47
474	1988	1	19	111.0	32.2	780	I	V	47



observations of crescents that were seen with the aid of binoculars or telescope but were invisible to the naked eye. The smallest lagtime in this group is 17 minutes (observation 465) and the second smallest lagtime is 32 minutes (observation 440). There are also two crescents which were sighted by both binoculars or telescope and the naked eye. These are observations 487 and 398 whose lagtimes are, respectively, 38 and 47 minutes. More importantly, the table contains 20 positive observations by the naked eye with no help from an instrument. The smallest lagtime in this latter group is 29 minutes (for the Babylonian observation number 63) and the second smallest lagtime is 33 minutes (observation 237). Therefore, there are 22 observations by the naked eye in table 7.5 which violate the Indian 48 minutes limit. These represent as much as 31% of the 71 observations in table 3.1 with lagtimes less than 48 minutes. In other words, a third of the crescents with lagtime less than 48 minutes could be seen. Surely, Loewinger's conclusion about the usefulness of this criterion "for the elimination of false reports" and his rejection of Schaefer's dismissal of its reliability (Schaefer, 1988) are unfounded. Applying the 48 minutes rule to table 3.1, I find that this criterion is contradicted by 35 negative observations, i.e. 40.7%, and 22 positive, i.e. 5.2%.

I should emphasise, however, that my calculations agree with Loewinger's in revealing serious computational errors in Schaefer's (1988) listed values of lagtime; these discrepancies remain inexplicable given that the error in some entries is about 100% of the correct value of the lagtime. The largest lagtime of a crescent that has been missed with the aid of binoculars or telescope is 51 minutes (observation 481), while for the naked eye the corresponding figure is 88 minutes (observation 217). For the unaided eye, the difference between the smallest lagtime for a visible crescent (29 minutes of observation 63) and the largest lagtime for an invisible crescent (observation 217) is 59 minutes. Schaefer notes that the moon will set 54 minutes later each successive day; he then proceeds to conclude that "an uncertainty of over 54 minutes in the critical lagtime implies that no location on earth can have a certain prediction by the moonset lagtime criterion" (Schaefer, 1988: 520). It is true, obviously, that the 48 minutes lagtime criterion cannot be relied upon to reject reports, but then the observational data do not support a conclusion of the form Schaefer presents either. As I have mentioned already in the previous section, a certain large lagtime of first visibility of a lunar crescent could have been the result of unfavourable weather conditions. This may have been the case, for instance, with observation 217 which has 88 minutes lagtime. If this observation is ignored out of caution, the second largest lagtime of a negative observation in table 3.1 is 66 minutes of observations 457 and 490. Therefore, the difference between the smallest lagtime for a visible crescent (29

minutes) and the largest lagtime for an invisible crescent (66 minutes) is only 37 minutes. Schaefer's above comment, therefore, is untenable.

Using the same technique of deriving his age criterion, Ilyas (1985) used his h-L model to establish a lagtime criterion. He concludes that the minimum lagtime visibility criterion is season-dependent, but this dependence is significant only at high latitudes. His modern form of the lagtime criterion is  $41 \pm 2$ ,  $46 \pm 4$ ,  $49 \pm 9$ , and  $55 \pm 15$  minutes for latitudes 0, 30°, 40° and 50°, respectively. Ilyas considers with some details how the "Babylonians" could have reached their "excellent estimation" of 48 minutes as the visibility limit of lagtime (Ilyas, 1985). However, the observations of table 7.5 strongly refutes his claim. Interestingly, the table includes 16 Babylonian positive observations of lagtime less than 48 minutes, 5 of which are less than 42 minutes (with the smallest lagtime being only 29 minutes) in contradiction with Ilyas'  $46 \pm 4$  minutes criterion for latitude 30°.

## 7.7 Schaefer's Criterion

The American scientist Bradley Schaefer (whose work has already been extensively cited) published in 1988 a paper in which he criticised the available lunar visibility criteria and discussed their inadequacy in comparison with a theoretical model for predicting the visibility of the lunar crescent that he designed. However, Schaefer did not give in that paper any details about his own criterion except that he "tried to follow in Bruin's [1977] footsteps while using accurate physical, meteorological and physiological equations" (Schaefer, 1988: 512). None of Schaefer's subsequent relevant publications that I have located (Doggett & Schaefer, 1994; Doggett, Seidelmann & Schaefer, 1988; Schaefer, 1991, 1992, 1993, 1996; Schaefer, Ahmad & Doggett, 1993) - most of which criticise the previous criteria and applaud his own - included any details about his criterion that would enable an independent assessment of the model. The importance of such an assessment is yet further stressed by the fact that large numbers of serious computational errors have been found in publications by Schaefer (Loewinger, 1995; Yallop, 1998) as well as inconsistencies between his individual papers (Yallop, 1998).

I find that Ilyas (1994: 454) and Yallop (private communication) have also complained that while Schaefer presented qualitative, comparative assessments of his and others' criteria, he never revealed actual information about his model. This may be due to the fact that Schaefer has marketed his criterion in the form of a commercial computer program (Schaefer, 1990).



Unfortunately, I cannot, therefore, investigate Schaefer's model and contrast its accuracy with other criteria.

## 7.8 Yallop's Empirical Criterion

The most recent lunar visibility criterion has been suggested by Bernard Yallop<sup>2</sup> (1997, 1998) who prepares HM Nautical Almanac Office (NAO) technical note on crescent visibility. NAO used to rely on a criterion based on Bruin's (1977) model (Yallop, 1996) before deciding to abandon it in 1997, opting for one based on Neugebauer's (1929) model which is referred to by the technical note of NAO as the "Indian". Yallop explains the reason for this change by plotting the criteria of Maunder, Neugebauer and Bruin, expressed in terms of the arc of vision (H) as a function of  $\Delta Z$ . He notes that Bruin's model differs from the other two criteria for  $\Delta Z > 20$ , showing a strong inflexion which translates into far too late predictions of first visibility of the lunar crescent for high latitudes, when the motion of the moon is nearly parallel to the horizon.

In his new criterion, Yallop basically uses the criterion of Neugebauer (1929) expressed as a function of the arc of vision and the width of the lunar crescent so that the crescent can be seen when its height above the sun is greater than  $f(W)$ :

$$H > 11.8371 - 6.3226W' + 0.7319W'^2 - 0.1018W'^3 \quad (7.4)$$

In the above equation,  $W'$  represents the "topocentric" width of the crescent and is calculated using the following set of equations in which  $W$  is the geocentric width of the crescent,  $\pi$  is the equatorial horizontal parallax of the moon,  $SD$  is the semi-diameter of the moon,  $h$  is the geocentric altitude of the moon and  $L$  is the geocentric elongation which is about  $1^\circ$  less than its topocentric counterpart ( $L'$ ):

$$W = 0.7245 \pi \quad (7.5)$$

$$SD' = SD(1 + \sin h \sin \pi) \quad (7.6)$$

$$W' = SD' (1 - \cos L) \quad (7.7)$$

Apart from using  $W'$  instead of  $W$ , a significant change that Yallop introduces to the use of Neugebauer's criterion is that the calculations are not made for the time of sunset but rather for what he calls the "best time" of observation of the lunar crescent. The fact that a certain

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<sup>2</sup> I am very grateful to Dr Bernard Yallop for providing me with a copy of the text of his revised criterion (Yallop, 1998) prior to publication.

empirical criterion is based on calculating parameters for the time of sunset does not imply that the crescent would necessarily be seen at sunset, should the criterion show it to be visible on that date. The observer would usually have to wait for the sun to go down sufficiently below the horizon so that the contrast between the twilight sky and the crescent would be adequate for the moon to be seen. However, the time that favours visibility does not last until moonset because although the continuous setting of the sun below the horizon would render the twilight sky darker, the descending moon would reach an altitude before setting beyond which it would not be seen because of the significant diminishing in its brightness due to atmospheric extinction. So, there is a limited period of time between sunset and moonset that allows sighting of the moon, and it is within this period that the “best time” of observation is determined.

In designing a simple rule for determining the best time of observation, Yallop has relied on Bruin’s model. Yallop has noted that Bruin suggested that the minimum of each of his curves of figure 7.6 represents the optimum situation of visibility. He also noted that a straight line can be drawn to pass through the origin (0,0) and the minima of the series of curves, including the minimum point of the curve of  $W=0.5'$ . Following the text of Bruin in reading the coordinates of the latter point as (4,9), in which case  $h$  would be  $5^\circ$ , rather than as (4,8.6) as seen from the graph itself, in which case  $h$  would be  $4.6^\circ$ , Yallop proceeded to conclude that at the best time of observation  $4h=5s$  and, hence, if  $T_s$  is the time of sunset,  $T_m$  is the time of moonset and  $S$  is the lagtime, the best time  $T_b$  is given by the equation:

$$T_b = (5T_s + 4T_m)/9 = T_s + (4/9) S \quad (7.8)$$

Yallop then designed a visibility test parameter  $q$  based on the representation of inequality 7.4 of Neugebauer's criterion and which is to be used in conjunction with the best time of observation according to equation 7.8 which is derived from the curves of Bruin. He defines  $q$  as follows (the scaling of  $q$  by a factor of 10 is intended to confine it roughly to the range  $\pm 1$ ):

$$q > (H - (11.8371 - 6.3226W' + 0.7319W'^2 - 0.1018W'^3))/10 \quad (7.9)$$

Having calculated parameter  $q$  for the 295 morning and evening observations compiled by Schaefer and Doggett (Doggett & Schaefer, 1994; Schaefer, 1988, 1996), Yallop divided the values of parameter  $q$  into six categories which he empirically calibrated by comparing his values of  $q$  with the visibility code used by Schaefer in his collection of observations, i.e whether the crescent was visible or invisible by the unaided eye and/or binoculars or telescope. Yallop also states that he found it necessary to use theoretical arguments to obtain some of the limiting values for  $q$ , though he does not explain these arguments. Yallop’s model is shown in table 7.6.



The letter “I” in the column of the visibility codes stands for “invisible” while the letters “V” and “F” stand for “visible”. The visibility codes of categories B and C both indicate possible visibility of the crescent with the naked eye and optical instrument but they differ in that in the second case the crescent may need be to found by the optical instrument first before it can be spotted by the naked eye. This is why the letter “F” instead of “V” is used in category C.

**Table 7.6** The criterion of Yallop

Category	Range of q	Explanation	Visibility Code	
			unaided eye	Binoculars or Telescope
A	$q > +0.216$	Easily visible ( $H > 12^\circ$ )	V	
B	$+0.216 \geq q > -0.014$	Visible under perfect conditions	V	V
C	$-0.014 \geq q > -0.160$	May need optical aid to find crescent	V	F
D	$-0.160 \geq q > -0.232$	Will need optical aid to find crescent	I	V
E	$-0.232 \geq q > -0.293$	Not visible with a telescope $L \leq 8.5^\circ$	I	I
F	$-0.293 \geq q$	Not visible, below Danjon limit, $L \leq 8^\circ$		I

The original version of this criterion (Yallop, 1997) included only five categories of q and, naturally, assigned different limiting values for categories D and E. However, Yallop conceded that that version of the criterion was based on a high estimation of the Danjon limit and he modified it after becoming aware of a recent study of the Danjon limit that I have co-authored (Fatoohi *et al*, 1998b) and which concludes that present observational data suggest that the Danjon limit is about 7.5°. Yallop claims that category F of his model is in fact Danjon’s condition of visibility. However, Yallop’s amendment seems to have not been thought out carefully. This can be concluded from the fact that the new version of the model makes a pseudo distinction between two categories, E and F, which are in fact indistinguishable from

each other. It is meaningless to make a distinction between observations that are “not visible with a telescope” and those that are also “not visible” for being “below Danjon limit”, as these are two different wordings that refer to one thing, namely, observations that cannot be seen by the naked eye nor by telescope. In fact, to suggest that category E, where  $L \leq 8.5^\circ$ , represents observations that are “not visible with a telescope” is just another way of saying that the Danjon limit is in fact  $8.5^\circ$  and, accordingly, category F, where  $L < 8^\circ$ , would simply be included in category E.

Yallop’s amalgamation of Neugebauer’s limits of visibility, Bruin’s curves of visibility and the concept of the best time of observation is certainly a new approach to solving the problem of predicting the first visibility of the lunar crescent. However, the reliability of this new criterion is, by definition, entirely dependent on the accuracy of Neugebauer’s model and Bruin’s curves of visibility which Yallop used in defining the best time of observation. In fact, Yallop himself concedes that his simple rule would be reliable if “the derivation of the Bruin curves is sound”. Now, Yallop is well aware of Schaefer and Doggett’s strong yet justifiable criticism of the highly inaccurate assumptions that Bruin used in building his model. Additionally, Yallop himself criticised Bruin and referred to the latter’s allusion to the fact that he manipulated his curves and that they were not drawn strictly on the basis of the astronomical and astrophysical assumptions used, which Yallop views as the reason that he could not reproduce Bruin’s curves. This throws deep doubts on the accuracy of equation 7.8 which defines the best time of observation, itself a fundamental parameter in Yallop’s criterion. All this makes it rather difficult to understand the rationale behind Yallop’s usage of Bruin’s results. Similarly, Yallop does not justify his choice of Neugebauer’s model which I have shown to yield wrong results for 28.6% and 12.3% of the negative and positive observations, respectively.

It is possible to argue that Yallop’s classification of his criterion into six categories of different values of  $q$  could have significantly reduced the contradictory results expected from using Neugebauer’s and Bruin’s models. However, the limits that he chose for parameter  $q$  were such that the criterion has significant errors even when used for the observations that Yallop used to calibrate it. For instance, 12 of the 166 calibration observations (i.e. 7.2%) which belong to category A are in fact negative. These are crescents that, according to Yallop, “should be very easy to see”. Even worse, category B which represents crescents that are “visible under perfect conditions” has 17 negative observations which represent 25% of the 68 observations used for calibrating it. Yallop claims that category F represents observations that



are “below Danjon limit”, yet the elongations of 9 of the 17 observations (52.9%) in this category are in fact above Danjon limit with 3 of the observations (17.6%) visible with optical aid. In other words, the claimed correspondence between the range of  $q$  of category F and the Danjon limit is refuted by the data used itself.

Another problem with Yallop’s criterion is the number of data points used to determine the range of  $q$  of each category. Yallop’s choice of the values of  $q$  meant that of the 295 evening and morning observations that he used for calibration, the significant number of 166 observations belonged to category A, whereas only 68, 26, 14, 4 and 17 points fell in categories B, C, D, E and F, respectively. Having a significant number of observations in each category is essential to validate the categorisation system and the chosen values of  $q$ .

Given the contradiction between the categories and the calibration observations themselves and the limited number of observations used for the categorisation, it is very difficult to justify designing such a criterion, let alone expecting it to be reliably applicable to independent data.

I have tested the criterion of Yallop using the observations of table 3.1. As it contains Babylonian and Arab observations in addition to the data compiled by Schaefer and Doggett which Yallop has already used in calibrating his criterion, this set of data can serve as an independent assessor of the accuracy of Yallop’s criterion. I repeated all calculations for these observations for the best time of observation using program SUNMOON0.FOR. The discrepancies of the various categories of Yallop’s model with the results of computation are shown in table 7.7 whose columns show the following information for each of the categories of Yallop’s model: (1) the category of Yallop’s criterion; (2) the kind of observations represented (i.e. the definition of the category); (3) the original number of observational data from table 3.1 that fall within that range of  $q$ ; (4-5) the number and percentage of contradictory points; (6) the kind of error.

With the exception of category A, each of the other categories has a high percentage error. Obviously, the percentage error must be considered with caution because of the limited number of points, but then this in fact a criticism of the criterion itself which was developed and its categories were determined on the basis of a significantly small number of points. It is of particular interest to note that 7 of the 9 observations of category E have  $L$  greater than  $8.5^\circ$  and 5 of the 10 observations of category F have elongations above the Danjon limit, both in clear contradiction with the definitions themselves of these two categories.

To sum up, the criterion of Yallop is subject to criticism on a number of accounts. Firstly, it includes a pseudo distinction between categories E and F which are, according to their own

definitions, indistinguishable from each other. Secondly, it is based on Neugebauer’s low accuracy criterion of visibility. Thirdly, it determines the “best time” of observation using the equally inaccurate method of Bruin. Fourthly, apart from category A, each of the other categories of this model has a very small number of calibrating points. Fifthly, significant numbers of the original calibration observations themselves contradict the categories that they belong to. And finally, testing this criterion with the data of table 3.1 reveals that it is highly unreliable.

**Table 7.7** The results of testing the criterion of Yallop

1	2	3	4	5	6
A	Easily visible ( $H > 12^\circ$ )	373	13	3.5%	negative observations
B	Visible under perfect conditions	75	18	24%	negative observations
C	May need optical aid to find crescent	24	12	50%	10 missed by the naked eye though seen with optical aid 2 missed by the naked eye and the optical instrument
D	Will need optical aid to find crescent	15	4	26.7%	missed by optical aid
E	Not visible with a telescope $L \leq 8.5^\circ$	9	2	22.2%	seen with optical aid in 7 cases $L > 8.5^\circ$
F	Not visible, below Danjon limit, $L \leq 8^\circ$	10	2	20%	seen with optical aid in 5 cases $L > 8^\circ$



## Discussion and Conclusion

The previous series of investigations shows that each of the various criteria which have been proposed for predicting the first visibility of the lunar crescent has a significant error when tested against both negative and positive observations. I have compared in table 8.1 the percentage errors of fourteen ancient and modern criteria.

It is interesting to note that the better a criterion for the prediction of one kind of observations the worse it is for the other. For instance, the Babylonian  $L+S \geq 23^\circ$  criterion has the second lowest rate of failure in predicting positive observations, only 1.4%, but it has the highest percentage of error in predicting negative observations, 45.2%. On the other hand, Bruin's original solution contradicts only 9.5% of the negative observations, which is the lowest percentage of error for these observations among the fourteen criteria, yet the number of discordant positive observations is the highest at 27.7%. All other twelve models which I have considered fall somewhere between the Babylonian and Bruin's criteria in terms of their inaccuracy for negative and positive observations. It is very important to stress that a significant failure of a criterion in predicting either of the negative or positive observations renders it practically useless. In other words, if any criterion reports a high percentage of error for one kind of observations it is of no real value for predicting the first visibility of the lunar crescent. The test has shown that each of the fourteen criteria of table 8.1 suffers from this fundamental flaw.

A glance at any of the criteria when plotted against all the observations of table 3.1 reveals that, regardless of the astronomical parameters employed by the model, it will always has a zone where both positive and negative observations exist. None of the solutions can be modified to be significantly a better prediction tool for the negative or the positive observations without further worsening its reliability for the reverse situation. This applies to all of the criteria that I have reviewed in this study and which are based on a variety of solar and lunar astronomical parameters. This implies that no specific set of parameters can be used to design a model that can "definitely" predict the visibility or invisibility of *every* lunar crescent. This is a disappointing result but is to be expected given that the visibility of the lunar crescent is a compound problem with astronomical, atmospheric and physiological components. In addition

to the positions of the sun and moon, atmospheric transparency and acuity of the eyesight of the observer directly affect the visibility of the young moon. In fact, even factors such as the age of the observer and his experience in knowing where to look for the crescent can play crucial roles in the success or failure of spotting the thin crescent (Doggett & Schaefer, 1994).

**Table 8.1** A comparison between the percentage errors of a number of different ancient and modern criteria of first visibility of the lunar crescent, using all the observations of table 3.1.

Criterion	Contradictory Observations			
	Negative		Positive	
	Number	Percentage	Number	Percentage
Babylonian $L+S \geq 23^\circ$	38	45.2%	6	1.4%
Indian $S \geq 12^\circ$ (ca. 600)	35	41.7%	22	5.2%
Ibn Tariq's (8th century)	24	28.6%	29	6.9%
Al-Khawarizmi's (9th century)	35	40.5%	33	7.8%
al-Qallas' (10th century)	36	42.9%	39	9.2%
Maimonides' (12th century)	33	39.3%	12	2.8%
Al-Sanjufini's (14th century)	30	35.7%	34	8.1%
al-Lathiqi's (18th century)	37	44%	20	4.7%
Maunder's h-ΔZ (1911)	13	15.5%	75	17.8%
Neugebauer's h-ΔZ (1929)	24	28.6%	52	12.3%
Bruin's original model (Minimum $W=0.5'$ ) (1977)	8	9.5%	117	27.7%
Bruin's modified model (Minimum $W=0.25'$ ) (1977)	36	42.8%	4	0.9%
Ilyas' h-L (1982 & 1988)	25	29.8%	33	7.8%
Ilyas' h-ΔZ (1988)	24	28.6%	48	11.3%

The complicated nature of the problem of the visibility of the lunar crescent and the fact that - at least near the boundary conditions of any criterion - neither visibility nor invisibility



can be “definitely” ensured is embodied in the concept of the “zone of uncertainty”. The existence of this zone was first recognised, though implicitly, by Ilyas after his introduction of the concept of the “International Lunar Date Line” (ILDL), which is similar to the Solar Date Line (Ilyas, 1979).<sup>1</sup> As already mentioned (§7.5), Ilyas used his h-L criterion to locate the geographical longitude at any one latitude where the minimum visibility condition for a certain crescent is just met. He repeated this calculation for other latitudes in a series of steps and then drew a line joining these points - called by him the ILDL (as above) - which could then be used for global calendrical calculation. Ilyas pointed out that visibility of the crescent moon to the west of the ILDL *improves* increasingly with longitudinal separation because of the delay in the sunset by 1 hour for each 15° in longitude; this results in increased angular separation between the sun and moon and higher lunar altitude at sunset. The zone of uncertainty, therefore, is a region centred around the ILDL in which the actual visibility of the crescent is doubtful because of unpredictable changes in local observing conditions and abilities of the observer. In this region, factors that usually have minor contributions to the visibility or invisibility of the lunar crescent have greater effects which can be decisive. Thus, inside this zone the visibility of the crescent cannot be predicted with certainty whereas outside it the visibility or invisibility can be predicted with high confidence.

It is thus clear that any criterion for visibility of the lunar crescent is bound to have a “zone of uncertainty”. This inevitable existence of the zone of uncertainty means that any model that defines a sharp line that separates the visibility and invisibility zones is oversimplified and doomed to have a high percentage of wrong predictions for either negative or positive observations; this is what we indeed found having tested these various criteria. Naturally, the number of discordant observations increases as the limit of visibility is approached. Only Schaefer claims to have incorporated the concept of the zone of uncertainty in the design of his criterion.

Schaefer translated the concept of the “zone of uncertainty” in his model into “probability” of seeing or not seeing the crescent. This probability is one when visibility is ensured, zero when the crescent cannot be seen, and somewhere in between in other cases. Unfortunately, the

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<sup>1</sup> Ilyas has also introduced the following concepts in his attempt to design a global Hijra calendar: (i) the Islamic Lunation Number (ILN) which denotes the number of lunar cycles and is counted from the first month (Muharram) in the first Hijra year; (ii) the Islamic Day Number (IDN) which denotes the cumulative day number of an Islamic year varying from 1 to 354 or 355, with the first of Muharram being IDN 1; (iii) the Hijra Day Number (HDN) which denotes the cumulative number of days, with the first day of Muharram of the first Hijra year being HDN 1.

unavailability of any information about Schaefer's model makes it impossible to evaluate his application of the concept. Schaefer has only revealed (in a joint paper with Doggett) that he takes the ILDL to be "the locus of positions for which the probability of seeing the crescent is 50%", where "Traveling east from the [I]LDL, the probability gradually decreases to zero, while traveling west, the probability increases to nearly unity for clear sky" (Doggett & Schaefer, 1994: 389, 392). He also states that, "The boundaries of the zone of uncertainty depend on the chosen confidence level for sighting the Moon" (Doggett & Schaefer, 1994: 389). Schaefer does not explain the rationale behind his choice of the ILDL to represent the 50% probability points; nor does he give the reasons for choosing any specific confidence level to define the zone of uncertainty of his model as well as the practical implications and justification of choosing that confidence level. I cannot, therefore, comment on Schaefer's application of the concept of zone of uncertainty and will confine myself to discussing Ilyas' treatment of the matter.

Although Ilyas has suggested that the consistency of Maunder's criterion with Bruin's model implies that the former is similar to the latter in having a "very small uncertainty (a few arcmin)" (Ilyas, 1981: 159), he has reconsidered his conclusion in later studies. He now suggests that the criterion has a much greater zone of uncertainty, and refers to the fact that the Indian Ephemeris already lists a certainty region beyond  $+1^\circ$  from the dividing curve of Maunder. Ilyas suggested that this would have implications both for his h-L criterion at lower latitudes, which he calibrated using the Maunder's criterion, and for ILDL. Ilyas explained these implications in the following words:

[I]t would seem safe to assume an error of about  $\pm 1^\circ$  in altitude separation. This means that if the actual altitude separation is greater, by  $1^\circ$ , than the required value of the criterion, positive visibility would be certain and if less, by  $1^\circ$ , than the required value, negative visibility on the specific evening is certain. This would translate into an uncertainty region of about  $\pm 30^\circ$  in longitude in the first visibility longitudes determined using the present criterion....

The effect of this error in the criterion on the predicted data is not very serious. What this means is that around the first visibility longitudes and the associated ILDL, there is a small region of about  $\pm 30^\circ$  longitude around the ILDL where the visibility is uncertain. Over the rest of the global surface, we can determine the date with greater certainty (Ilyas, 1994: 452).

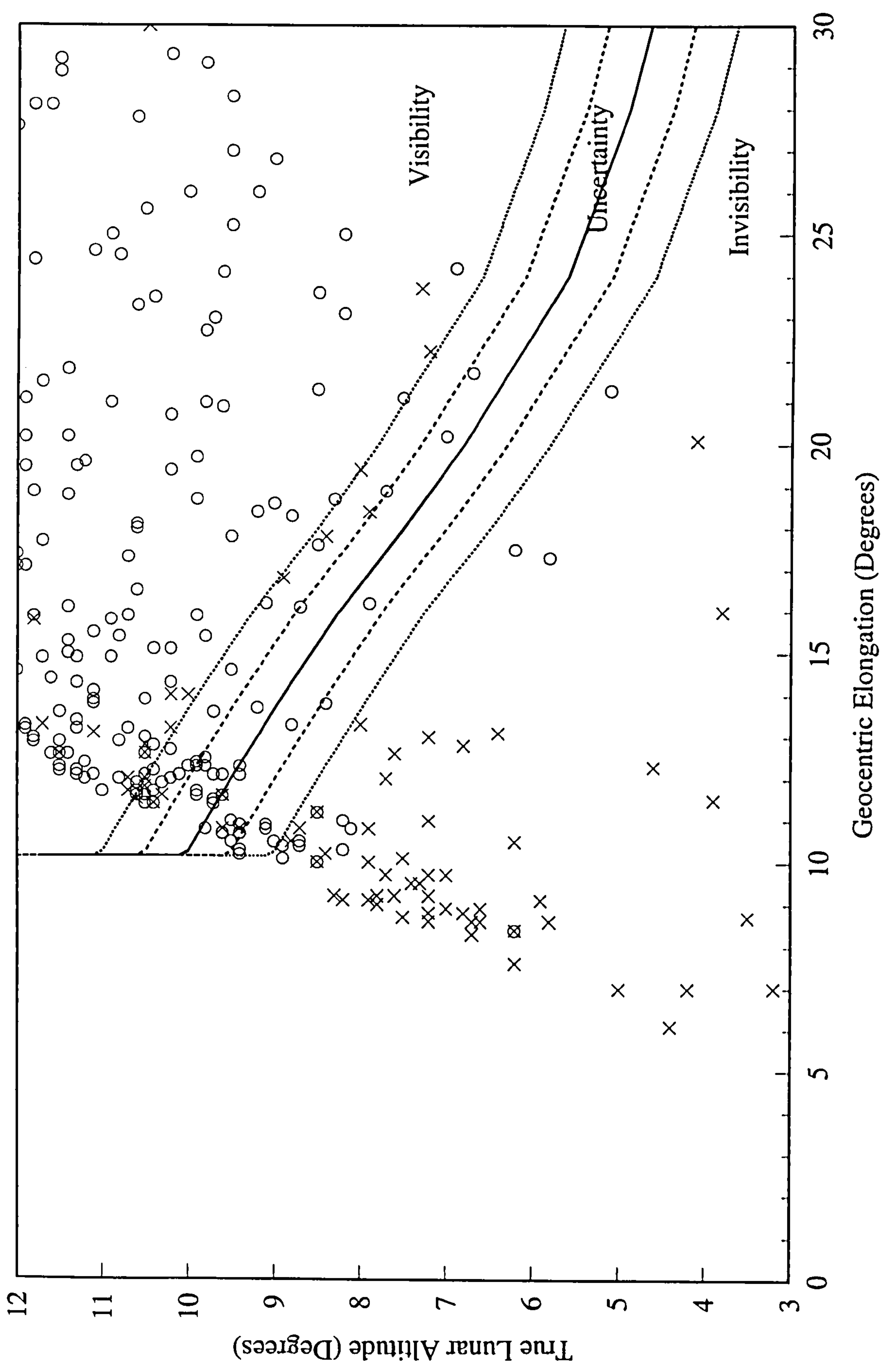
Although Ilyas concedes that the criterion of Maunder has a higher zone of certainty than he first thought and explains that this would mean a zone of uncertainty of  $\pm 1^\circ$  in altitude



separation for his h-L criterion, he does not give a satisfactory explanation of his choice of the specific width of  $\pm 1^\circ$ . In fact, he adds that, "Perhaps an error of about half as much would have about 80-90 per cent confidence level up to middle latitudes" (Ilyas, 1994: 452). Additionally, Ilyas does not tell us why he thinks that the zone of uncertainty would have to be symmetrical around his original h-L curve. The point is that once the inaccuracy of the criterion has been conceded and, accordingly, the existence of a zone of uncertainty has been established, there is no reason to insist that the position of Maunder's curve has anymore a real significance or meaning, such as bisecting the zone of uncertainty. In other words, the supposed original curve of visibility, whether it is Maunder's or any other such line that is wrongly claimed to separate the visibility and invisibility zones, is meaningless; therefore relating concepts to it does not make sense.

In order to study the modification that Ilyas suggests to his h-L criterion, I have plotted in figure 8.1 his original curve and the observational data of table 3.1. (i.e. figure 7.9), along with the surrounding bands of  $\pm 1^\circ$  and  $\pm 0.5^\circ$  of lunar altitude. Since my intention is to study the effect of the suggested zones of uncertainty, I have concentrated in the graph on the regions of interest and neglected other parts of the graph. The graph, therefore, displays only part of the original observational data. I have compared the modified criteria of figure 8.1 with the original criterion of figure 5.9 and have included the results of the comparison in table 8.2.

It is obvious from the table that the  $\pm 0.5^\circ$  wide zone of uncertainty is of no practical use whatsoever because it is not significantly different from the original model (with no zone of uncertainty). Firstly, this criterion has exactly the same high percent of contradictory negative observations of the original model (29.8%). The reduction in the erroneous predictions for the positive observations from 7.8% to 5.0%, or even eliminating them all together, does not grant the criterion any higher reliability than its original form because it retains the main flaw of the original version, which is the high percent of error for predicting the negative observations. The 29.8% error is extremely high and unreasonable for what is supposedly a zone of certainty inside of which prediction should be made with high confidence. Secondly, the fact that there are eight times as many contradictory negative observations as in the zone of uncertainty defies the very definition of the zone of uncertainty and defeats the purpose behind establishing it. For in this case we have a criterion where the percentage of error caused by the presence of negative observations in the visibility zone, which is by definition a zone of certainty, is far higher than the percentage of negative observations in the zone of uncertainty itself! This criterion is unreliable to use even for the data from latitudes  $\pm(30-40)$ . For instance, it has



**Figure 8.1** The h-L criterion of Ilyas with the suggested  $\pm 0.5$  and  $\pm 1$  zones of uncertainty



22.6% of the negative observations in the visibility zone. Ilyas is certainly wrong when he suggests that the  $\pm 0.5^\circ$  wide zone would achieve “about 80-90 per cent confidence level up to middle latitudes”.

**Table 8.2** Comparison between three versions of Ilyas’ h-L criterion and the improved h-ΔZ criterion suggested by this study (the latter is explained below).

Criterion	Contradictory Observations				Observations in Zone of Uncertainty			
	Negative		Positive		Negative		Positive	
	No.	Percent	No.	Percent	No.	Percent	No.	Percent
Ilyas’ original h-L	25	29.8%	33	7.8%	-	-	-	-
Ilyas’ h-L with $\pm 0.5^\circ$ zone of uncertainty	25	29.8%	21	5.0%	3	3.6%	30	7.1%
Ilyas’ h-L with $\pm 1^\circ$ zone of uncertainty	14	16.7%	13	3.0%	14	16.7%	60	14.2%
Improved h-ΔZ (with zone of uncertainty)	5	5.9%	15	3.6%	23	27.4%	69	16.4%

Choice of the  $\pm 1^\circ$  wide zone also does not result in a criterion that is quite free from the above second flaw, for negative observations are equally present (16.7%) in both the visibility zone, which is a zone of certainty, and the uncertainty zone. Moreover, the 16.7% error in predicting the negative observations is certainly still very high. The aim of introducing a zone of uncertainty into a criterion is to significantly reduce the percentage of error in both certainty zones, yet this has not been achieved in this second modified version of Ilyas’ model despite the fact that its zone of uncertainty is  $2^\circ$  wide. The reason that this wide zone of uncertainty did not improve the criterion considerably enough is that the boundaries of this zone have not been determined carefully but rather arbitrarily. As already mentioned, the very concept of establishing the zone of uncertainty so that it is symmetric around a line which was originally designed as a dividing line between the visibility and invisibility zones is nonsensical, because once the existence of the zone of uncertainty has been established the original dividing line

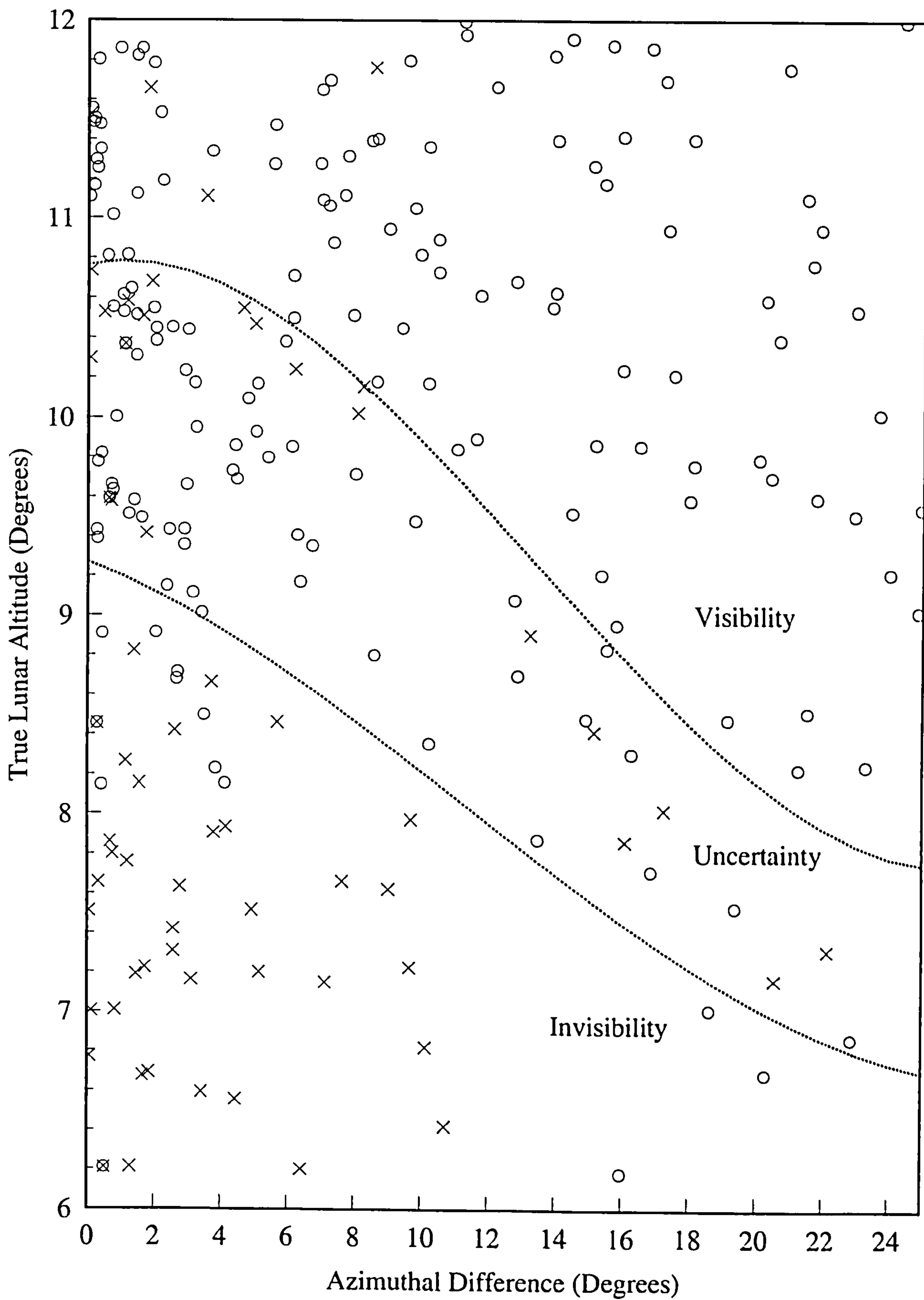
loses its meaning and significance and, therefore, re-defining it as being the line that divides the uncertainty zone in the middle has no justification. Choosing the boundaries of the zone of uncertainty carefully and on a sound basis should lead to a better criterion than that resulting from Ilyas' arbitrary choice. This is shown below.

I have studied the results of applying the concept of the zone of uncertainty to a number of different criteria, including the h-L model, and have found that developing such a zone in the h- $\Delta Z$  criterion yields the best results. In other words, this particular criterion can be better improved by introducing into it a zone of uncertainty than can other solutions that I have experimented with. Rather than choosing the boundaries of this zone randomly, as Ilyas seems to have done, I have used the observational data of table 3.1 to set the boundaries. The revised h- $\Delta Z$  criterion with the suggested zone of uncertainty and the observational data are shown in figure 8.2. This graph also concentrates on the region of interest and displays therefore only part of the data. Extending the zone of uncertainty beyond  $\Delta Z=23^\circ$  should be taken with some caution because of the unavailability of data in this region.

The basic ideas behind defining each of the dividing lines of the zone of uncertainty are essentially the same. In principle, the upper curve that separates the visibility region from the uncertainty region should be drawn so that it has the smallest possible number of negative observations in the visibility zone. Alternatively, the lower curve that separates the invisibility region from the uncertainty region should be drawn so that the smallest possible number of positive observations fall underneath it in the invisibility zone. While aiming toward this objective, it must also be kept in mind that while widening the zone ensures less contradictory observations, it also means more crescents whose (in)visibility would not be predictable; from the calendrical point of view this would lead to a larger region of terrestrial longitudes for which the first date of the month cannot be predicted with high confidence. In other words, the borders of the zone of uncertainty should not be drawn according to extreme points, but a balance should be struck that takes into consideration both of the above facts.

I have drawn the upper line of the zone of uncertainty just above 7 high negative observations. Any slight lowering of the line would move the 7 negative observations to the visibility region, thus undermining the reliability of the criterion. There are five discordant negative observations, four of which are shown in figure 8.2 (the fifth observation is very high, suggesting that the crescent might have well been accidentally missed, not because it was intrinsically invisible). Elevating the line to include the lowest four discordant negative observations in the uncertainty region would widen this region considerably and, in





**Figure 8.2** The improved  $h$ - $\Delta Z$  criterion with data for all latitudes

consequence, would move a very large number of positive observations from the visibility zone to the zone of uncertainty. This would result in a criterion that is practically useless. In fact, even elevating the line by only about  $0.5^\circ$  to move one of the four discordant negative observations into the uncertainty region would move a significant number of positive observations from the visibility region to the zone of uncertainty. The simplest equation that describes the upper line of the zone of uncertainty is as follows:

$$h = 10.7638 + 0.0356 |\Delta Z| - 0.0164 \Delta Z^2 + 0.0004 |\Delta Z|^3 \quad (8.1)$$

The lower line of the zone of uncertainty in figure 8.2 was drawn to have as few as possible of the positive observations in the invisibility region. As is clear from the diagram, moving the line even slightly higher would result in transferring a number of positive observations to the invisibility region. On the other hand, lowering it further to move more positive observations to the zone of uncertainty would affect only a few positive observations, and in the same time move a significant number of negative observations from the zone of invisibility to the zone of uncertainty. In both cases, the criterion would be seriously undermined rather than improved. The equation describing the lower line of the zone of uncertainty is as follows:

$$h = 9.2714 - 0.0644 |\Delta Z| - 0.0058 \Delta Z^2 + 0.0002 |\Delta Z|^3 \quad (8.2)$$

For all these reasons, the two dividing lines in figure 8.2 draw the optimum zone of uncertainty of the  $h$ - $\Delta Z$  according to the available observational data. I have included in table 8.2 a number of statistics of the improved  $h$ - $\Delta Z$  criterion, i.e. after adding the zone of uncertainty, for comparison with Ilyas' criteria. As already mentioned, the Ilyas' criterion with the  $\pm 0.5^\circ$  is fundamentally flawed, therefore the improved  $h$ - $\Delta Z$  criterion should be compared with the criterion of  $\pm 1^\circ$  wide zone of uncertainty.

The percentage error of the improved  $h$ - $\Delta Z$  criterion for the positive observations (3.6%) is almost the same as that of Ilyas' criterion (3.0%). However, the improved  $h$ - $\Delta Z$  has a significantly smaller percentage error for negative observations (5.9%) than Ilyas' model (16.7%). Therefore, the improved  $h$ - $\Delta Z$  is free of the fundamental flaw in Ilyas' solution. As has already been mentioned, there is always the possibility that a crescent was not seen because of circumstances that have nothing to do with it being intrinsically invisible. The improved  $h$ - $\Delta Z$  criterion has almost the same percentage of positive observations in the zone of uncertainty (16.4%) as Ilyas' model (14.2%), but the former has a higher percentage of negative observations in the zone of uncertainty (27.4%) than Ilyas' solution (16.7%). However, this difference is, in fact, the result of the improved  $h$ - $\Delta Z$  having a significantly smaller number of negative observations in the visibility zone (only 5) than Ilyas' model (14).



In fact, the zones of uncertainty of both criteria should be compared in terms of their width as the available data themselves can be biased. The zone of uncertainty of the improved h-ΔZ criterion is significantly narrower than that of Ilyas' model. While the latter has a zone of uncertainty of a constant width of 2°, the maximum width of the former is 1.5° at smaller azimuthal difference and this decreases to about 1° at larger azimuthal difference. Therefore, in comparison with the h-L criterion, the improved h-ΔZ criterion has a much smaller region of geographic longitudes along which the date of the visibility of the lunar crescent cannot be determined with high confidence. This means that the improved h-ΔZ criterion can be used to draw much more accurate ILDLs.

In order to test the implications of the zone of uncertainty of the improved h-ΔZ model for calendrical calculations I have used this criterion to determine the beginnings of the Islamic month for 25 Miladi Muhammadi years 1466-1490 (A.D. 1991-2015). I have applied the test to Macca, which is a city of religious significance, and Baghdad and Casablanca, which are at the eastern and western borders of the Arab world, respectively. I have found that out of the 300 months, the number of months whose beginnings would be uncertain are only 22, 17 and 25 for Macca, Baghdad and Casablanca, respectively. In other words, the zone of uncertainty would result, on average, in only about 7% of the beginnings of the Islamic months being uncertain.

The fact that the width of the zone of uncertainty of the improved h-ΔZ is close to 1° means that in order to appreciate the extent of the superiority of the new criterion it should be contrasted with Ilyas' model of the  $\pm 0.5^\circ$  zone of uncertainty rather than the  $\pm 1^\circ$  wide zone. The huge difference between the improved h-ΔZ criterion and Ilyas' model with  $\pm 0.5^\circ$  zone of uncertainty is the result of determining the zone of uncertainty in the former on a sound basis, fully utilising the available observational data.

The improved h-ΔZ criterion suggested here is the first to depart from the otherwise standard form of being merely a "definite" dividing line or limit between visibility and invisibility. Any such dividing line or limit is meaningless theoretically and of little use in practice as an accurate criterion. The revised criterion presented in this study is based on the sound and empirically verified concept that there should be three and not only two zones in any model: the zones of invisibility, visibility and uncertainty. Additionally, the statistics on the improved h-ΔZ criterion given in table 8.2 suggest that it is a highly reliable tool for prediction of first visibility of the lunar crescent.

However, establishing a systematic programme to watch for the new crescent moon to obtain more observational data would help in further refining the improved  $h-\Delta Z$  criterion. New negative observations would be especially useful as the currently available data consist mainly of positive observations.

It is necessary to emphasise that the improved  $h-\Delta Z$ , or any other lunar visibility criterion, should be used to predict the lunar visibility *in conjunction* with a number of limits that this study has revealed and which determine the minimum visibility requirements for certain parameters for naked eye observations. These limits are the approximately  $7.5^\circ$  of topocentric elongation (observation 487),  $5.1^\circ$  of true lunar altitude (observation 237), 17' width of lunar disk (observation 487), 29 minutes of lagtime (observation 63), and 15 hours of age (observation 487). Although these are the minimum limits of visibility that have been found so far, new observations may show that some of these limits may yet accept slightly lower values. Any such changes, however, are likely to be minimal.

It is hoped that the criterion developed in this study, in conjunction with the extreme limits of visibility, should prove very useful in regulating any lunar calendar that depends on the first visibility of the lunar crescent, such as the Islamic calendar.



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## **Part Two**

### **Problems in Historical Astronomy**

## **Introduction**

This part of the dissertation consists of six studies that I have co-authored with my supervisor Professor F.R. Stephenson during the time of my research for the PhD degree. Three of these papers have already been published and three are in press (one of them is included here in proofs form). Professor Stephenson agrees that I have carried out 50% of the work involved in each of these studies.



## ACCURACY OF SOLAR ECLIPSE OBSERVATIONS MADE BY JESUIT ASTRONOMERS IN CHINA

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### 1. *Introduction*

During the seventeenth and eighteenth centuries, Jesuit astronomers at the Chinese court in Beijing observed many eclipses of the Sun and Moon. For most of these events the times of beginning, middle and end were measured and the magnitudes estimated. Summaries of virtually all of the observations made between A.D. 1644 and 1785 are still preserved. In this paper, the various solar eclipse measurements that the Jesuits made during this period are compared with computation based on modern solar and lunar ephemerides.

### 2. *Sources of Data*

The *Qing-Chao-Wen-Xian-Tung-Kao* (*Qing Dynasty comprehensive study of civilization*),<sup>1</sup> compiled between 1747 and 1785, contains (in Chapters 263 and 264) numerous observations of solar and lunar eclipses made by Jesuit astronomers at Beijing. These cover the date range from 1644 (the beginning of the Qing Dynasty) to 1785.

The *Qing-Chao-Wen-Xian-Tung-Kao* is an update of the original *Wen-Xian-Tung-Kao*, compiled by Ma Duanlin around 1300. Towards the end of last century, the observations made by the Jesuits were summarized by Wylie,<sup>2</sup> who reduced the measured times to hours and minutes. More recently Chen<sup>3</sup> reproduced the records (in their original Chinese), and converted all dates to the Gregorian calendar as well as reducing the measured times. Although neither author compared the times with their computed equivalents, these compilations by Wylie and Chen form useful supplements to the data in the *Qing-Chao-Wen-Xian-Tung-Kao*.

In this paper we have concentrated on the *solar* eclipse observations by the Jesuit astronomers (*Qing-Chao-Wen-Xian-Tung-Kao*, Chap. 263). A discussion of the lunar data will be published on a subsequent occasion.

The observations were all made at the "Ancient Observatory" in Beijing (lat. 39°.92 N, long. 116°.42 E). Dates are given exclusively in terms of the Chinese luni-solar calendar — year of a reign period of a particular emperor, lunar month, day of the month (invariably the first day) and the day of the 60-day cycle (a repetitive cycle independent of any astronomical parameter). They are thus readily reduced to the Gregorian Calendar.

### 3. *Measurements of Magnitude and Time*

For the purpose of expressing magnitude, the solar diameter was divided into 10 *fen* ('divisions'), each subdivided into 60 *miao* ('subdivisions'). In each case estimates of magnitude were made to the nearest *miao*.

In most instances the times of first contact ('beginning of loss'), maximal phase, and last contact ('restoration of fullness') are reported. However, occasionally it is stated that an eclipse was interrupted by clouds or that the Sun rose or set partially obscured. Times are expressed in terms of *shi* ('double hours'), *ke* ('marks') and *fen* ('divisions'), generally following the Chinese style. A few words of explanation are necessary.

The interval from one midnight to the next was divided into twelve equal parts known as *shi*, a system originating in ancient times. There is no evidence that the origin of this division of the day is any way related to the occidental system of 24 hours in a day. The first double hour (*zi*) was centred on local midnight and thus began at 23h local time. Similarly the 7th double hour *wu* was centred on local noon when the Sun was on the meridian.

Each double hour was bisected into a *chu* ('initial') and a *zheng* ('central') half. Thus the initial half of the first double hour extended from 23h to midnight and this was immediately followed by the central half, which lasted from midnight to 1h.

During the entire period covered by the observations, the natural day (midnight to midnight) was divided into 96 equal *ke*. Each of these units was thus exactly equivalent to one quarter of an hour. As related in the *Qing-Chao-Wen-Xian-Tung-Kao* (Chap. 256), this system was introduced in 1628 (the second year of the Tian-chong reign period of Emperor Tai-zong). It replaced an older scheme in which the day had been divided into 100 equal *ke*. Each half of a double hour began with the *chu-ke* ('initial mark'). This was followed by the first, second and third *ke*, the last of these completing the appropriate half-*shi*. A full list of the 12 *shi* and 96 *ke* and their equivalents on the 24-hour clock is given in Table 1.

Individual *ke* were divided into *fen* ('divisions'). During earlier Chinese history, the number of *fen* in a *ke* varied considerably. However, in the period covered by the Jesuit observations under discussion each *ke* contained 15 *fen* so that one division was exactly equal to 1 minute. If an observational report omits the number of *fen*, the time may be inferred as exactly on the quarter hour. One *fen* would mean 1 minute afterwards, and so on up to a maximum of 14 *fen*, following which the next *ke* would commence.

To give an example, the eclipse of 1671 Sep 3 was described as follows:

10th year of the Kang-hsi reign period, 8th month, first day *jimao* (the 16th day of the cycle). The Sun was eclipsed at *Zhang* lunar lodge.... It was eclipsed (to the extent of) 1 *fen* and 59 *miao* (i.e. a magnitude of 0.198). The beginning of loss was at 9 *fen* in the first *ke* of the central half of the hour of *shen*



TABLE 1. The twelve Chinese double hours and their 96 divisions.

Double Hour	INITIAL HALF				CENTRAL HALF			
	Initial	First	Second	Third	Initial	First	Second	Third
	<i>ke</i> h	<i>ke</i> h	<i>ke</i> h	<i>ke</i> h	<i>ke</i> h	<i>ke</i> h	<i>ke</i> h	<i>ke</i> h
zi	23	23;15	23;30	23;45	0	0;15	0;30	0;45
chou	1	1;15	1;30	1;45	2	2;15	2;30	2;45
yin	3	3;15	3;30	3;45	4	4;15	4;30	4;45
mao	5	5;15	5;30	5;45	6	6;15	6;30	6;45
chen	7	7;15	7;30	7;45	8	8;15	8;30	8;45
si	9	9;15	9;30	9;45	10	10;15	10;30	10;45
wu	11	11;15	11;30	11;45	12	12;15	12;30	12;45
wei	13	13;15	13;30	13;45	14	14;15	14;30	14;45
shen	15	15;15	15;30	15;45	16	16;15	16;30	16;45
you	17	17;15	17;30	17;45	18	18;15	18;30	18;45
xu	19	19;15	19;30	19;45	20	20;15	20;30	20;45
hai	21	21;15	21;30	21;45	22	22;15	22;30	22;45

(i.e. 16h 24m). Maximum eclipse was 7 *fen* in the initial *ke* of the initial half of the hour of *you* (i.e. 17h 07m). It was restored to fullness at 14 *fen* in the 2nd *ke* of the hour of *you* (i.e. 17h 44m).

Virtually all other records follow very much this same pattern. (In the above translation we have omitted the position of the Sun in the appropriate lunar lodge — which specifies the right ascension — since it uses the term *fen* in a separate context).

The Jesuit astronomers would have had access to a telescope,<sup>4</sup> but it is not known to what extent this instrument was utilized for eclipse observations — rather than using the unaided eye. Times were measured with the aid of a clepsydra *Hu-Lou*.

#### 4. Computations

We have adopted the reduction of dates on the Chinese calendar to the Gregorian calendar by Chen.<sup>5</sup> In each case the reduced date corresponds to that of a calculated solar eclipse. We have converted the recorded magnitudes to decimals of the solar diameter and the various times of eclipse phases to hours and decimals (local apparent time, i.e. with the Sun on the meridian at 12.00 h). In computing the various times and magnitudes for Beijing, we have utilized specially designed programs. These require only the input of the Gregorian calendar date and the appropriate figure for the Earth's rotational clock error,  $\Delta T$  (arising from changes in the length of the day produced by tides and other causes). Values of  $\Delta T$  were obtained from a recent paper on Earth's past rotation.<sup>6</sup> Over the entire period under discussion, this parameter was very small — ranging from about 50 sec in 1644 to 10 sec in 1785.

In Table 2 we have listed the following details for each eclipse:

TABLE 2. Measured timings of eclipse phases.

Gregorian Date	Magnitude	First Contact (h)	Maximum Phase (h)	Last Contact (h)
1644/9/1	0.28	11.017	12.283	13.483
1648/6/21	0.920	5.883	6.867	8.000
1650/10/25	0.770	10.600	12.017	13.583
1657/6/12	0.662	4.317	5.150	6.067
1658/6/1	0.442	8.900	10.183	11.650
1665/1/16	0.890	15.350	16.617	17.767
1666/7/2	0.978	15.483	16.683	17.733
1669/4/30	0.548	13.133	14.450	15.717
1671/9/3	0.198	16.400	17.117	17.733
1676/6/11			18.250	19.167
1681/9/12	0.382	8.367	9.367	10.583
1685/11/26	0.232	15.133	15.967	16.733
1688/4/30	0.982	8.133	9.317	10.650
1690/9/3	0.273	6.833	7.583	8.433
1691/2/28	0.335	12.033	13.333	14.467
1692/2/17	0.528	11.800	13.233	14.783
1695/12/6	0.855	15.717	16.850	17.950
1697/4/21	1.037	7.883	9.117	10.367
1704/11/27	0.462	12.933	14.250	15.367
1706/5/12	0.638	18.350	19.217	20.050
1708/9/14	0.532	16.867	17.800	18.650
1709/9/4	0.490	6.133	6.983	7.983
1712/7/4	0.568	3.667	4.517	5.417
1715/5/3	0.620	18.183	19.333	19.850
1719/2/19	0.7	15.117	16.033	17.483
1720/8/4	0.703	10.567	12.200	13.750
1721/7/24	0.403	17.117	17.983	18.783
1730/7/15	0.937	11.017	12.767	14.500
1731/12/29	0.918		7.817	9.083
1735/10/16	0.835	7.783	8.983	10.300
1742/6/3	0.707	6.683	7.617	8.633
1745/4/2	0.117	10.950	11.767	12.500
1746/3/22	0.695	9.583	11.083	12.667
1747/8/6	0.235	16.983	17.667	18.300
1751/5/25	0.468	6.817	7.650	8.550
1758/12/30	0.885	15.083	16.333	
1760/6/13	0.970	16.433	17.450	18.383
1762/10/17	0.567	16.833		
1763/10/7	0.712		7.033	8.000
1769/6/4	0.358	17.083	17.783	18.467
1770/5/25	0.388	7.583	8.433	9.367
1773/3/23	0.420	13.300	14.667	15.750
1774/9/6	0.385	7.233	8.200	9.300
1775/8/26	0.455	11.350	12.867	14.283
1776/1/21	0.178	9.600	10.333	11.100
1784/8/16	0.192	5.533	6.233	6.983
1785/8/5	0.428	6.700	7.717	8.883

column 1: the date on the Gregorian calendar (year/month/day)

column 2: the observed magnitude (as a decimal of the solar diameter)

column 3: the measured time of first contact (expressed as local time in hours and decimals)



column 4: the measured local time of greatest phase (hours and decimals)

column 5: the measured local time of last contact (hours and decimals)

On the rare occasions that the Sun was stated to rise or set eclipsed, or if cloud was said to prevent observation, the appropriate entry in Table 2 is omitted.

In Table 3 we have compared the recorded information with that obtained from computation. This table lists the following:

column 1: the year A.D.

column 2: the error in the magnitude (observed – computed)

column 3: the error in the time of first contact (measured – computed)

column 4: the error in the time of greatest phase (measured – computed)

column 5: the error in the time of last contact (measured – computed)

### 5. Discussion of Results

In Figure 1 are plotted the various errors in magnitude (listed in column 2 of Table 3) as a function of the year A.D. For most of the period covered by the diagram (up to about 1750), the scatter is considerable, the standard error of measurement being 0.05. This is a typical performance for the unaided eye. Thus, in a previous paper,<sup>7</sup> we investigated a large number of naked-eye estimates of lunar eclipse magnitude from various parts of the world between about 700 B.C. and A.D. 1600. We obtained a similar scatter and standard error. In Figure 1, the dashed line indicates the computed mean error; this systematic effect is negligible.

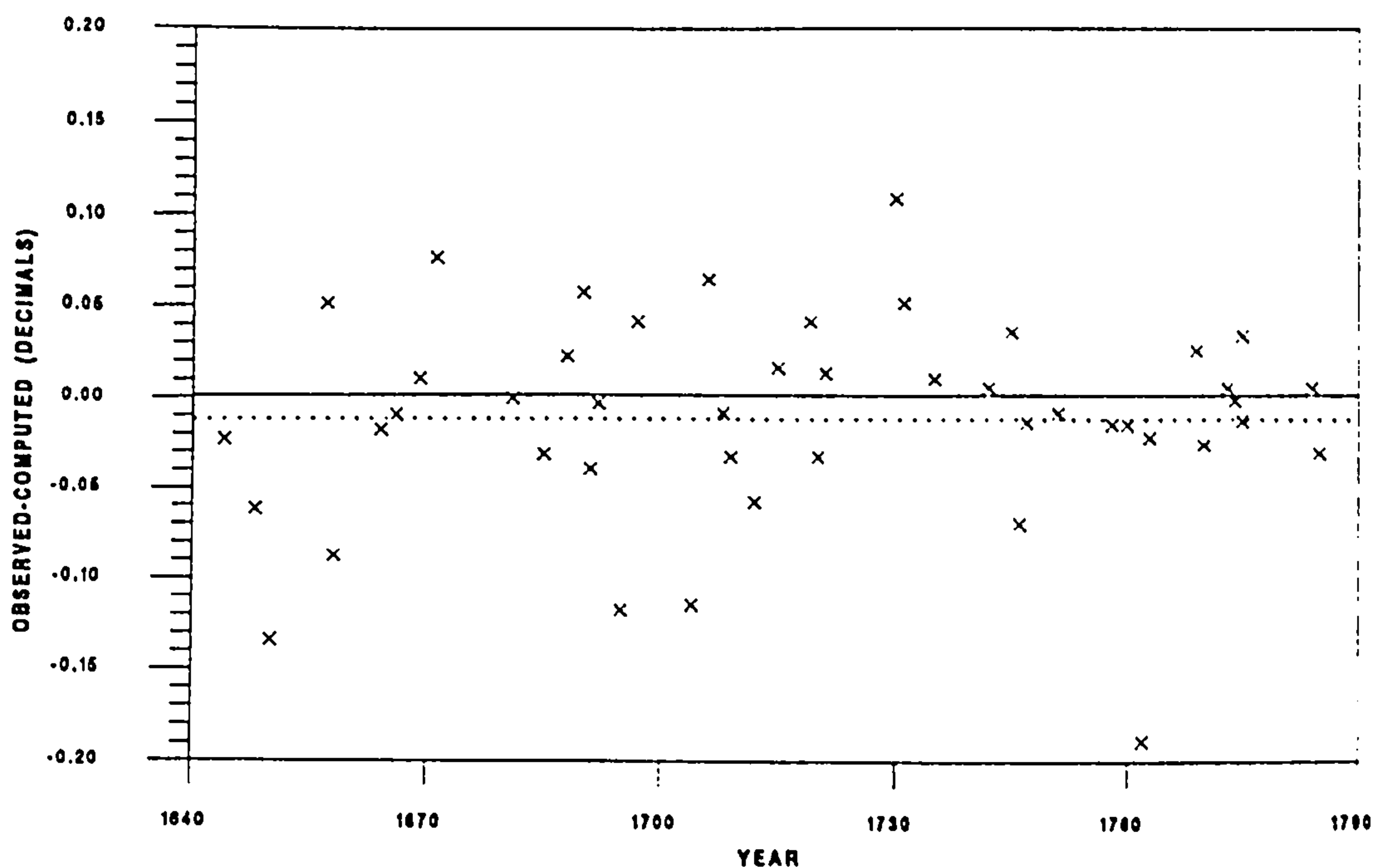


FIG. 1. Errors in magnitude.

TABLE 3. Comparison of measured and computed timings.

Gregorian Year	Magnitude Difference	First Contact Difference (h)	Maximum Phase Difference (h)	Last Contact Difference (h)
1644	-0.023	0.076	0.131	0.129
1648	-0.062	-0.286	-0.346	-0.374
1650	-0.134	0.333	0.414	0.592
1657	0.051	0.016	0.007	0.018
1658	-0.088	-0.184	-0.318	-0.399
1664	-0.018	0.528	0.350	0.249
1666	-0.010	-0.080	-0.081	-0.102
1669	0.010	0.016	0.064	0.139
1671	0.076	-0.026	0.149	0.242
1676			-0.529	-0.040
1681	-0.001	0.166	0.142	0.250
1685	-0.032	0.470	0.489	0.486
1688	0.022	-0.049	0.026	0.155
1690	0.057	0.046	0.107	0.227
1691	-0.040	-0.256	-0.193	-0.214
1692	-0.004	0.524	0.446	0.560
1695	-0.118	0.370	0.332	0.380
1697	0.041	0.088	0.218	0.258
1704	-0.115	0.692	0.626	0.429
1706	0.064	0.089	0.143	0.211
1708	-0.009	0.326	0.335	0.322
1709	-0.033	0.274	0.302	0.423
1712	-0.058	-0.117	-0.167	-0.242
1715	0.016	0.037	0.378	0.135
1719	0.041	0.531	0.064	0.297
1720	-0.033	-0.124	-0.025	0.033
1721	0.013	-0.213	-0.219	-0.229
1730	0.108	-0.080	-0.083	-0.104
1731	0.051		0.408	0.481
1735	0.010	0.087	0.114	0.132
1742	0.005	0.090	0.105	0.122
1745	0.035	0.098	0.271	0.357
1746	-0.070	0.268	0.251	0.237
1747	-0.014	-0.018	-0.033	-0.061
1751	-0.009	0.103	0.107	0.108
1758	-0.015	0.078	0.068	
1760	-0.015	0.121	0.107	0.099
1762	-0.189	0.153		
1763	-0.022		0.100	0.109
1769	0.025	0.099	0.097	0.119
1770	-0.026	0.063	0.039	0.028
1773	0.005	0.285	0.277	0.106
1774	-0.002	-0.026	-0.051	-0.037
1775	-0.013	-0.055	-0.056	-0.051
1775	0.033	0.074	0.136	0.211
1784	0.005	-0.041	-0.037	-0.036
1785	-0.031	0.066	0.026	0.022

From around 1750 the Jesuit observations show a marked improvement (standard error 0.03) — as is clear from Figure 1. This was presumably associated with the acquisition of a new observing instrument by the Imperial Observatory. Known as the *Kan-Shuo-Wang-Ru-Jiao-Yi* ("Instrument for observing eclipses"),



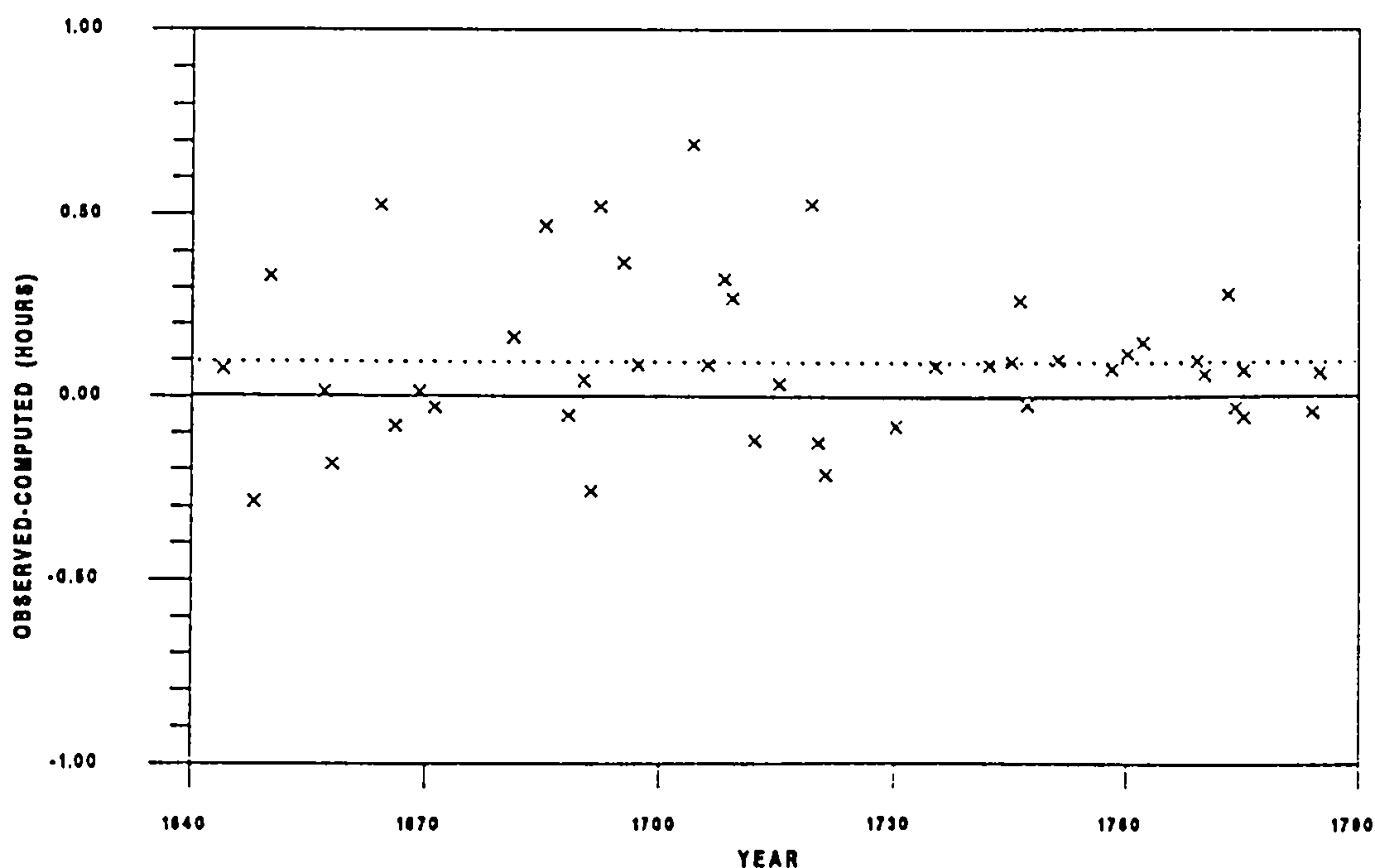


FIG. 2. Errors in the timings of first contact.

this device was set up in 1744 (the 9th year of the Qian-Long reign period), as related in the *Qing-Chao-Wen-Xian-Tung-Kao* (Chap. 258). It consisted of three rings representing the ecliptic, lunar orbit and celestial equator. By use of scales, the magnitudes and also times of the various eclipse phases could be estimated.

In Figures 2, 3 and 4 are plotted the errors in the timings of first contact (Figure 2), greatest phase (Figure 3) and last contact (Figure 4), each as a function of the year A.D. It will be seen from these diagrams that the various measurements of time are of a poor calibre up to about 1750. The standard errors of measurement during this period are close to 0.25 h (15 min) for each set of data. These results are scarcely any better than those achieved by native Chinese astronomers several centuries before.<sup>8</sup> There is also evidence of a small but systematic zero error — shown by a dotted line — amounting to approximately 0.10 h (6 min). This affects timings of each of the three eclipse phases to much the same degree throughout the entire period; in all cases, observations were late relative to computation. Values of  $\Delta T$  at this period are both small and well-established, and it may be inferred that the zero error is associated with faulty standardization of the clepsydra, perhaps as the result of using a slightly misaligned gnomon to indicate noon.

After about 1750 the measurements show a considerable improvement — standard error approximately 0.10 h for each eclipse phase. This gain in accuracy was probably due to the construction of a new three-stage water-clock in 1746 (the 11th year of the Qian-Long reign period), as well as the introduction of the “Instrument for observing eclipses” (mentioned above) two years previously. The new clepsydra is described in Chap. 258 of the *Qing-Chao-Wen-*

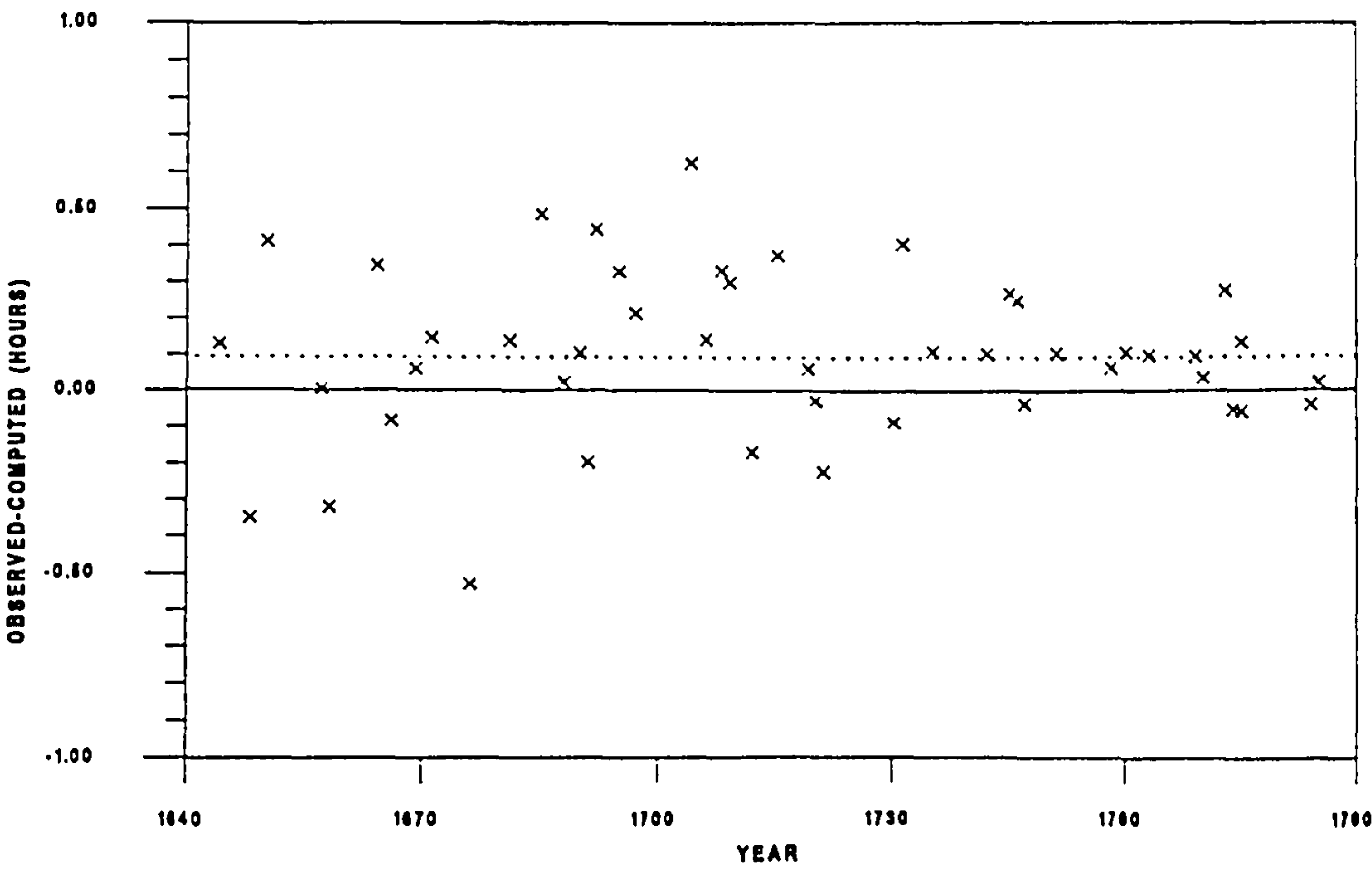


FIG. 3. Errors in the timings of maximum phase.

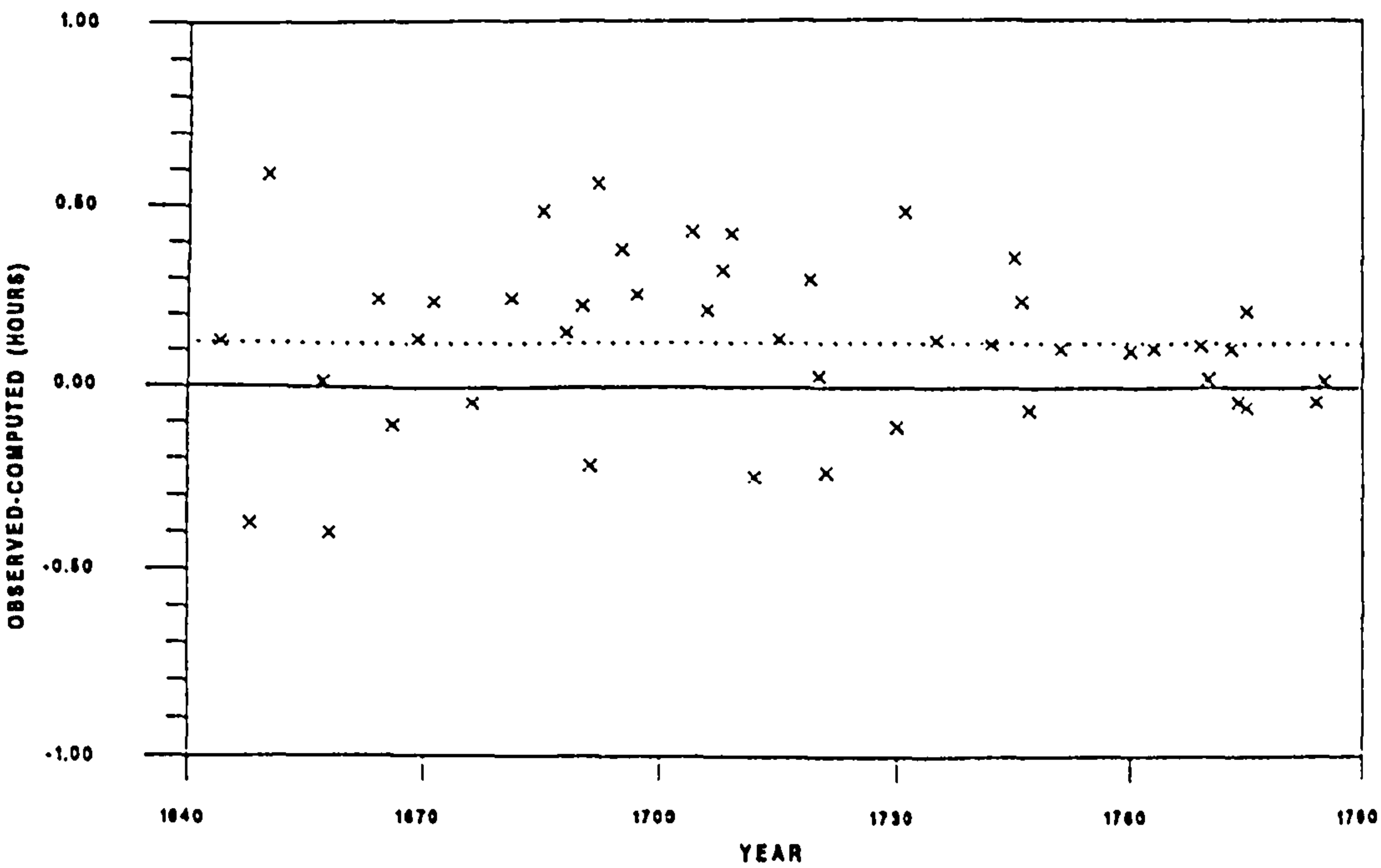


FIG. 4. Errors in the timings of last contact.



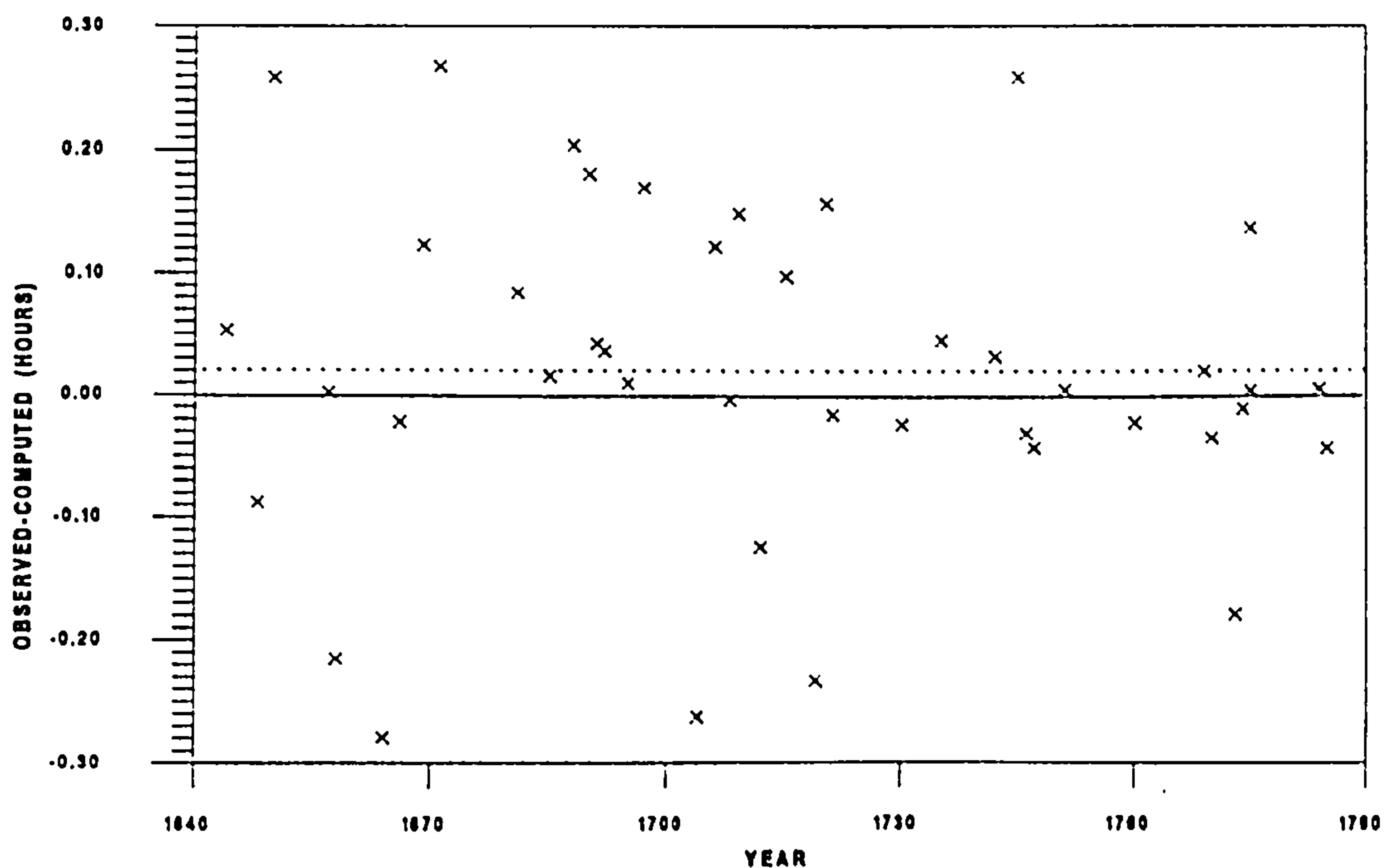


FIG. 5. Errors in the durations of eclipses.

*Xian-Tung-Kao*. Even these later refined measurements are still far inferior to contemporary European results, which show a standard error of no more about 0.02 h.<sup>9</sup>

Finally, in Figure 5 are plotted (on a larger scale than for the previous diagrams) the errors in measuring the durations of each of the eclipses. These show a much smaller standard error (0.10 h) than the timing of individual phases, and (with two notable exceptions) once again reveal improved precision after about 1750. The systematic error (0.02 h) is much smaller than for timings of individual phases.

Errors in measuring time seem likely to be due to defects in the clepsydras themselves — rather than due to poor resolution of the contacts (whether using the unaided eye or a small telescope). If the optical definition was poor, a significant delay in detection of first contact and an advance in the detection of last contact would be expected relative to the computed geometrical circumstances. However, there is no evidence of this in our results; the times of maximal phase show much the same systematic errors as the true contacts (i.e. beginning and end).

It is noteworthy that the eclipse durations (Figure 5) show a much smaller standard error than the individual phases themselves (Figures 2, 3 and 4). The water clocks could probably be effectively standardized — by astronomical means — at only three moments during daylight: sunrise, noon and sunset, each of which would be roughly 6 hours apart. Clock drift over a 6-hour interval would probably be much greater than during the two hours or so that a typical solar eclipse lasts.

### Conclusion

It is clear from our analysis that the eclipse observations made in China by Jesuit astronomers during the seventeenth and eighteenth centuries were of surprisingly poor quality. Apart from the period after about 1750, magnitude estimates were mediocre, and it is evident that the observers were over-ambitious in expressing such results to the nearest  $1/600$  of the solar diameter (1 *jiao*).

Timing errors are serious, typically amounting to  $\frac{1}{4}$  hour up to about 1750. Although there was a significant improvement in accuracy after this date, following the introduction of new measuring devices, the calibre of the measurements fell far below contemporary standards in Europe.

### Acknowledgments

We thank Professor Xu Zhentao of Purple Mountain Observatory, Nanjing (currently visiting the University of Durham) for reading this paper and offering several helpful criticisms.

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## ACCURACY OF LUNAR ECLIPSE OBSERVATIONS MADE BY JESUIT ASTRONOMERS IN CHINA

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### 1. *Introduction*

In a previous paper,<sup>1</sup> we investigated the accuracy of solar eclipse observations made during the seventeenth and eighteenth centuries by Jesuit astronomers at the Chinese court in Beijing. These observations, ranging in date from 1644 to 1785, consisted of both magnitude estimates and timed measurements for the various phases. We showed that in relation to contemporary European measurements, the Jesuit results were of low accuracy, particularly in the case of the timings. Evidently the water clocks that they used were considerably inferior to the mechanical timepieces available to European astronomers. Until about 1745, when a three-stage water-clock was introduced by the Jesuits at Beijing, the standard error of measuring the time of a solar eclipse contact proved to be about 0.25h. A systematic error of approximately 0.10h was also present, observed times being consistently later than expected. For more recent measurements, the standard error had reduced to about 0.10h, although the systematic error was still present.

The Jesuit astronomers also observed numerous lunar eclipses at Beijing and summaries of their observations — again made between 1644 and 1785 — are preserved in the same sources as for the solar eclipses. In the present paper, the various lunar eclipse measurements that the Jesuits made are compared with the results of present-day computation. Comparison with our conclusions obtained from the study of the solar observations will also be made.

### 2. *Sources of Data*

The observations that we have analysed are all to be found in Chap. 264 of the *Qing-Chao-Wen-Xian-Tung-Kao* (*Qing Dynasty comprehensive study of civilization*),<sup>2</sup> compiled between 1747 and 1785. These observations have been summarized by Wylie,<sup>3</sup> who reduced the measured times to hours and minutes, although he did not convert the dates to the Gregorian calendar. The observations were all made at Beijing (lat. 39°.92 N, long. 116°.42 E).

### 3. *Measurements of Magnitude and Time*

For the purpose of expressing magnitude, the Jesuit astronomers followed the Chinese practice of dividing the lunar (or solar) diameter into 10 *fen* ('divisions').

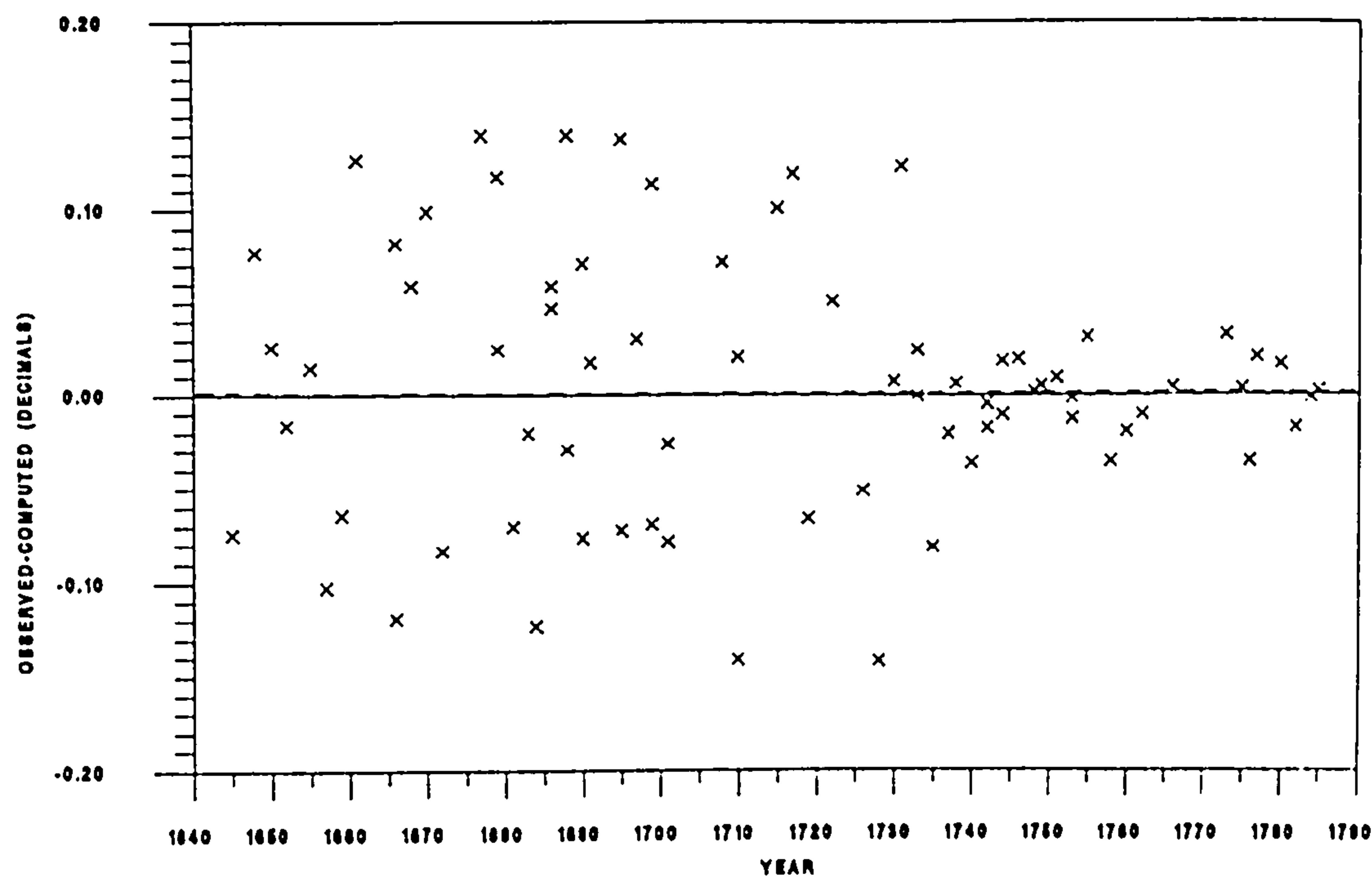


FIG. 1. Errors in magnitude for partial eclipses.

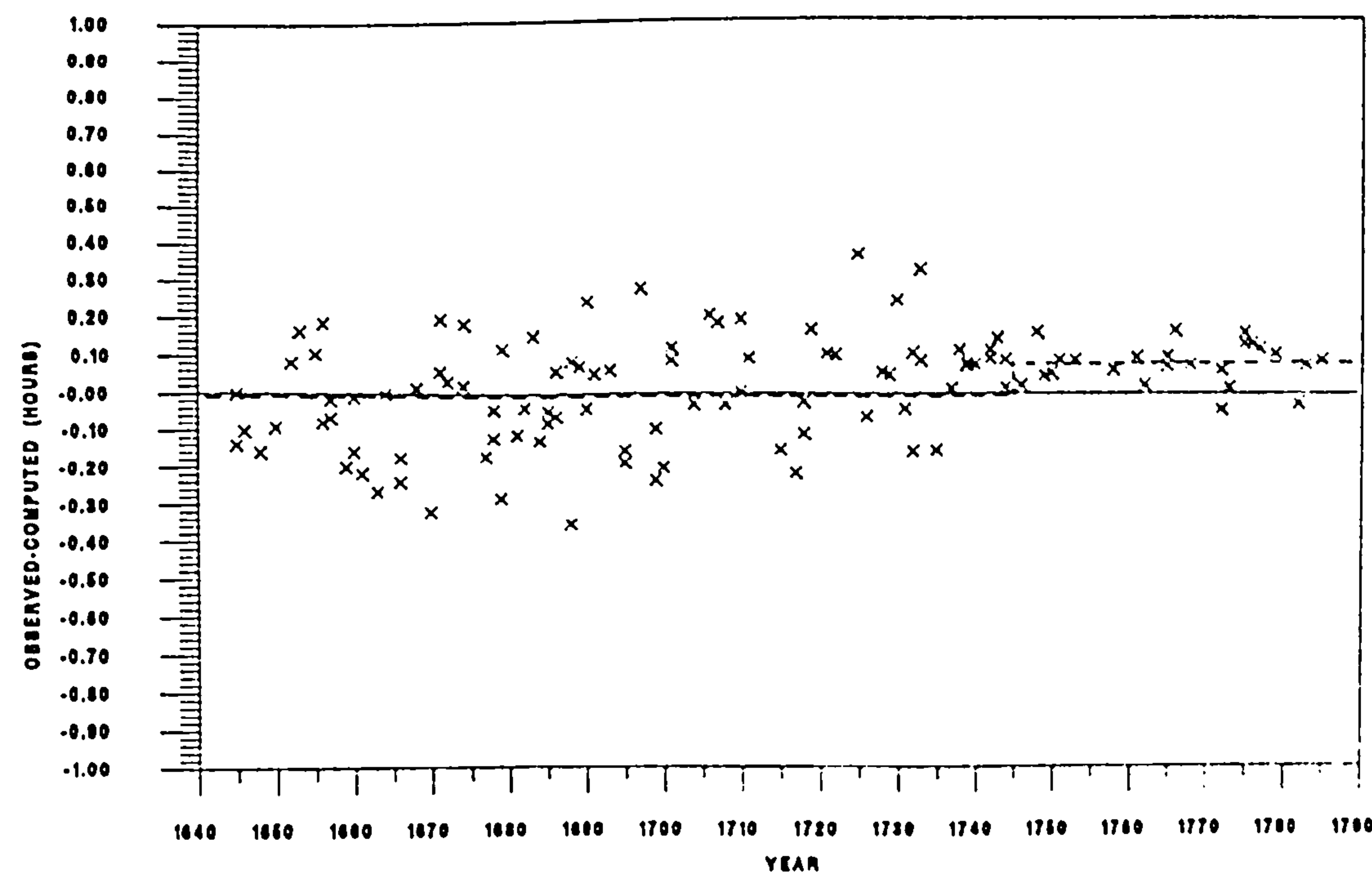


FIG. 2. Errors in the timings of first contact.



Each of these units was subdivided into 60 *miao* ('subdivisions'), magnitudes being consistently estimated to the nearest *miao*, or 1/600 of the disk. In the *Qing-Chao-Wen-Xian-Tung-Kao*, magnitudes are cited for both total and partial eclipses. The magnitude of a total eclipse (which exceeds 10 *fen*) can only be estimated indirectly, for example from the measured duration. However, the method used is not stated. We have thus restricted our attention to the recorded magnitudes of partial eclipses, which presumably were determined by direct observation.

For both total and partial eclipses, the times of the various phases are reported fairly systematically in the *Qing-Chao-Wen-Xian-Tung-Kao*. However, occasionally unfavourable weather or moonrise interrupted observation, and this is stated in the text. The times of first contact and last contact were measured for all eclipses. In the case of total eclipses, the onset and end of totality (respectively, immersion and emersion) were also timed. The time of maximum phase was also regularly recorded for both partial and total eclipses. However, it is apparent that these results were obtained simply by averaging the time of first and last contact. The Jesuits were evidently aware that for a lunar eclipse (unlike a solar obscuration), greatest phase is precisely mid-way between first and last contact. In this paper we have thus concentrated on the measurements made at first contact, immersion, emersion, and last contact.

As in the case of solar obscurations, the measured times are expressed in terms of *shi* ('double hours'), *ke* ('marks') and *fen* ('divisions'), following the contemporary Chinese style. At this period, 1 *ke* was exactly equal to 15 minutes (in present-day units) and 1 *fen* to 1 minute. For details, see our previous paper.<sup>4</sup>

#### 4. Computations

We have converted the dates on the Chinese calendar to the Gregorian calendar using a computer program which we have devised and which is based on the tables of Xue and Ouyang.<sup>5</sup> In each case we have found that the reduced date corresponds exactly to that of a tabular lunar eclipse. We have converted the recorded magnitudes to decimals of the lunar diameter, and the various times of eclipse phases to local apparent time in hours and decimals. For computing the eclipse circumstances we have devised an appropriate computer program that incorporates an accurate lunar and solar ephemeris.

#### 5. Discussion of Results

In Figure 1 are plotted the derived observational errors in magnitude for partial lunar eclipses as a function of the year A.D. The dashed line indicates the computed mean error; this systematic effect is negligible. Up to about 1745, the scatter is considerable, the standard error of measurement being 0.07, a little greater than for solar eclipses.<sup>6</sup> This is a typical performance for the unaided

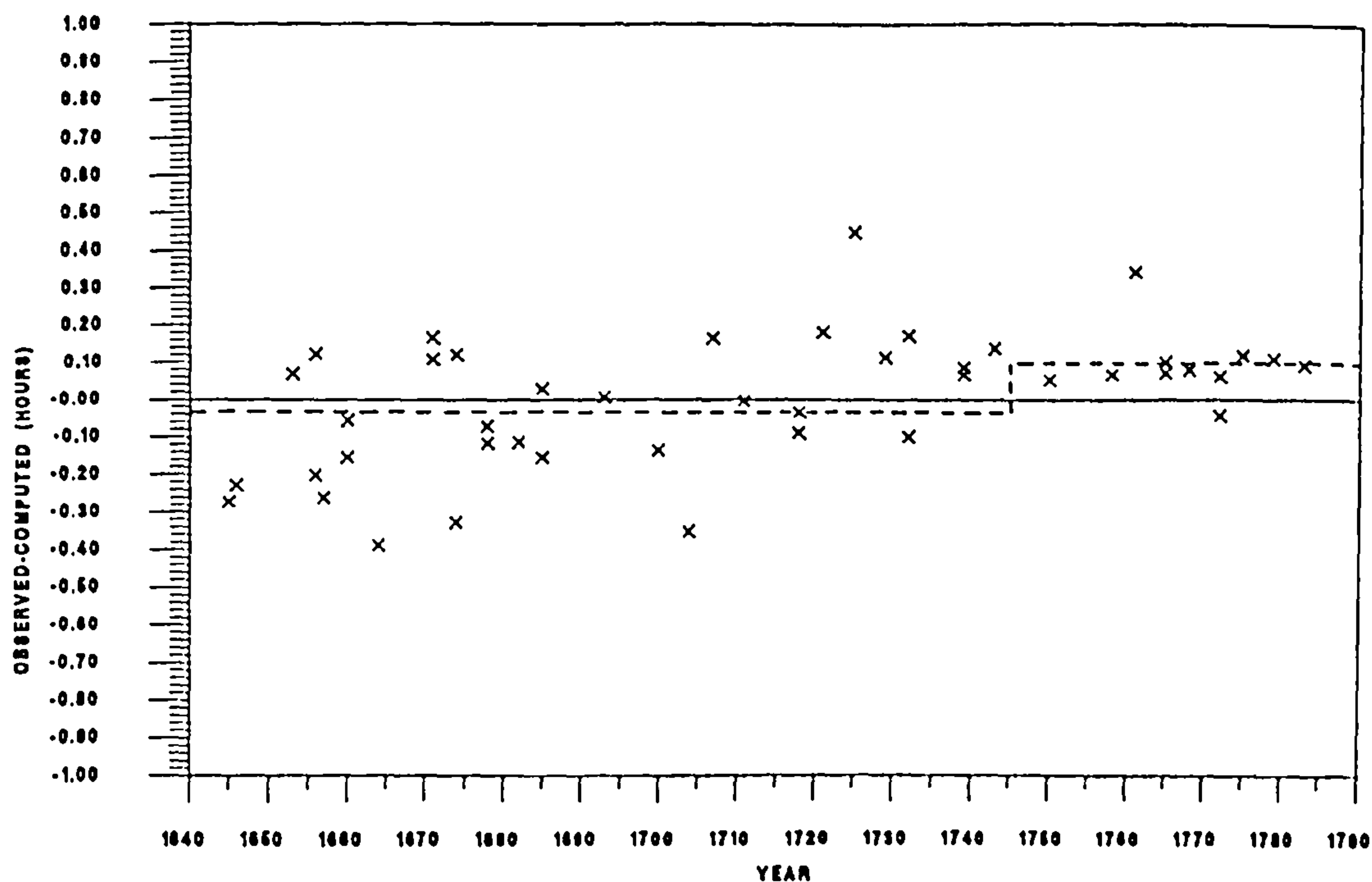


FIG. 3. Errors in the timings of immersion (total eclipses).

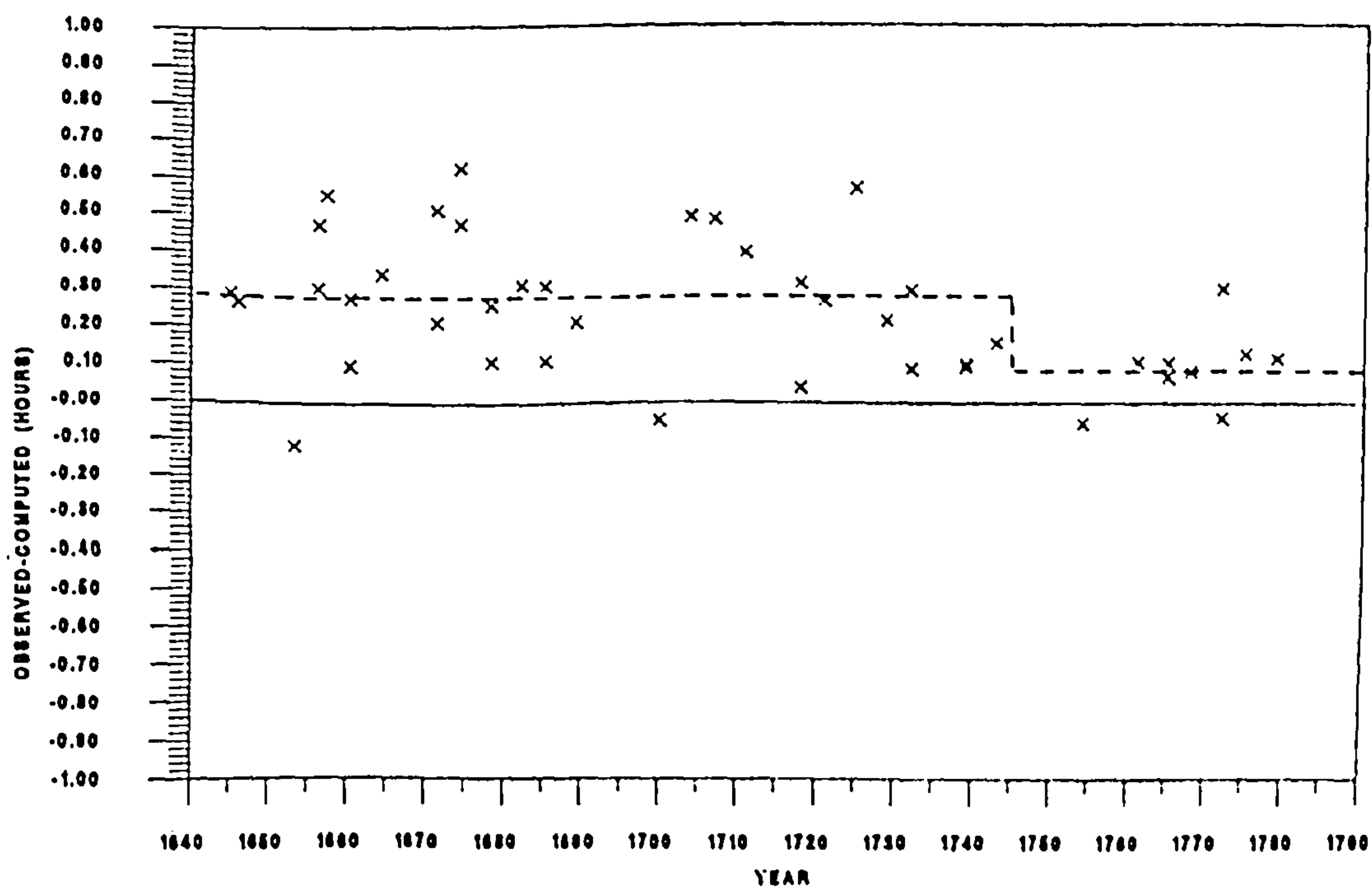


FIG. 4. Errors in the timings of emersion (total eclipses).



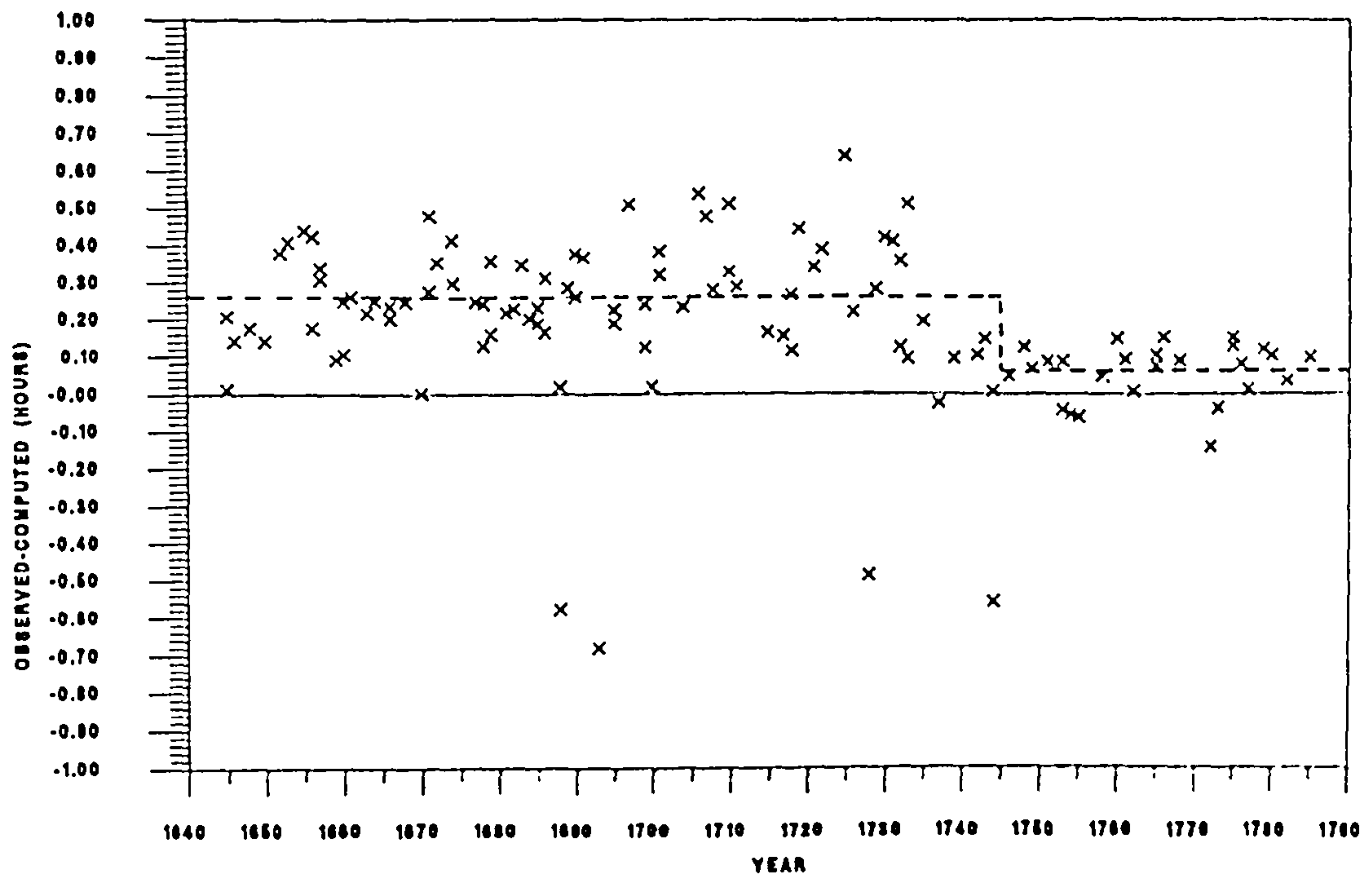


FIG. 5. Errors in the timings of the last contact.

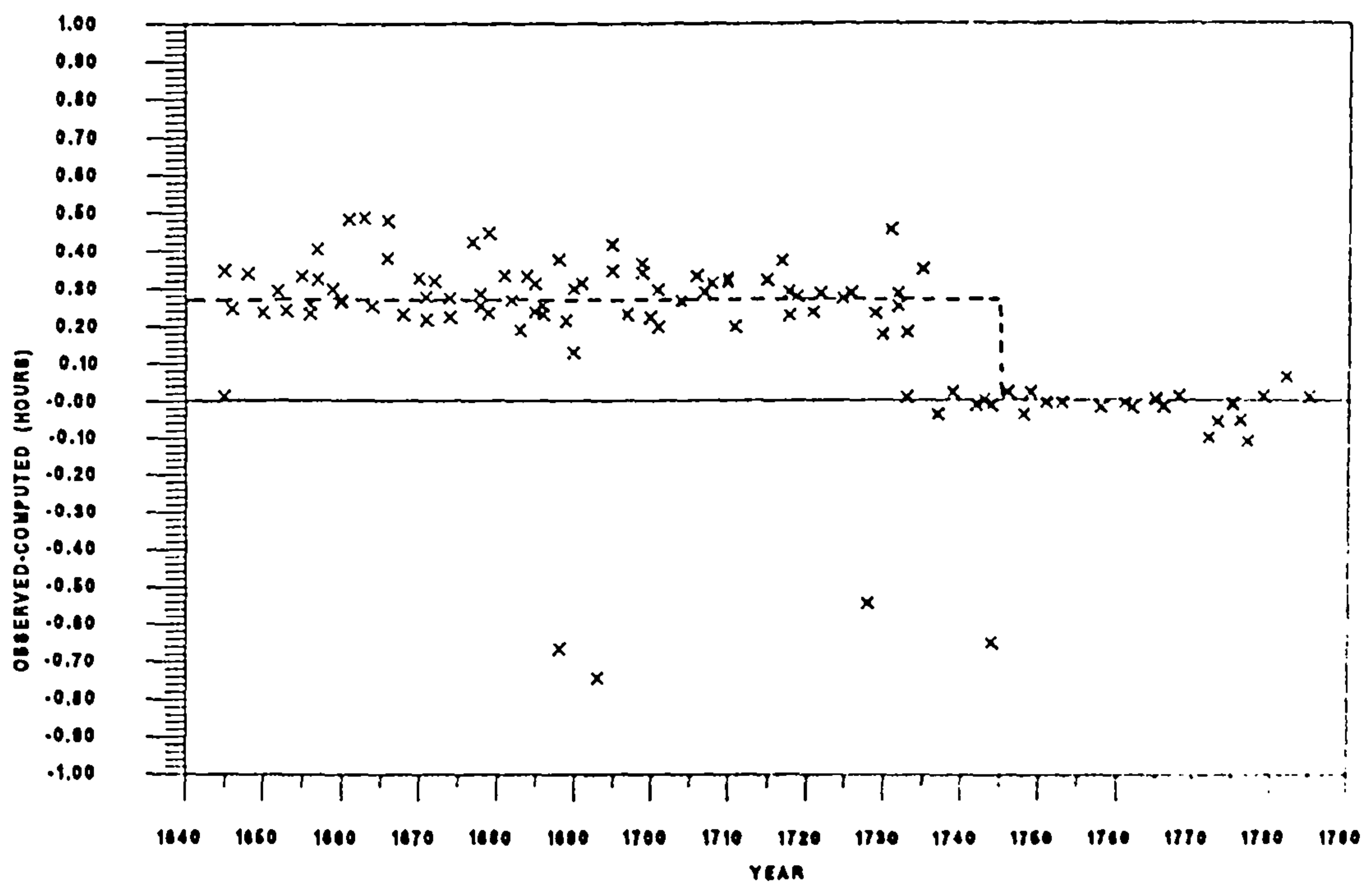


FIG. 6. Errors in timings of first to last contact.

eye.<sup>7</sup> From around 1745, the Jesuit observations show a marked improvement (standard error 0.03 — the same as for solar eclipses). This was presumably associated with the acquisition of a new *Kan-Shuo-Wang-Ru-Jiao-Yi* ("Instrument for observing eclipses"), set up at the Imperial Observatory in 1744. Although little is known about this instrument, the gain in precision may well be commensurate with the use of a small telescope. Nevertheless, to quote estimates to the nearest *jiao* was optimistic.

In Figures 2–5 are plotted the various timing errors: first contact for all eclipses (Figure 2); immersion for total eclipses only (Figure 3); emersion for total eclipses only (Figure 4); and last contact for all eclipses (Figure 5). The dashed lines indicate the computed mean error for each phase, the observations before and after 1745 being separated. It will be seen from the diagrams that the various measurements of time in the period prior to 1745 are of poor calibre. Significant systematic errors for emersion and last contact — amounting to some 0.25h — are present, whereas for first contact and immersion they are negligible. This discrepancy is difficult to account for. No similar feature was apparent in the solar data, which yield systematic errors of +0.12h at first contact and +0.15h at last contact. The individual lunar results show a considerable scatter about the mean; for first and last contact, the standard errors of measurement are close to 0.20h. In the case of immersion and emersion, the uncertainties are somewhat greater, but these phases are poorly defined optically. Systematic errors apart, the results for first and last contact at this period are somewhat better than for the corresponding solar eclipse observations (standard error 0.25h).

After about 1745, the measurements show a marked improvement — standard error approximately 0.05h for each eclipse phase. This gain in accuracy was probably due to the construction of a new three-stage water-clock (*Hu-Lou*) in 1746 as well as the introduction of the "Instrument for observing eclipses" (mentioned above) two years previously. However, there is clear evidence of a systematic error of about 0.10h. Both features were apparent from the solar eclipse data, as shown in our previous paper.<sup>8</sup> Here we inferred that the zero error is associated with faulty standardization of the clepsydra — perhaps through the use of a slightly mis-aligned gnomon (by some 1°.5) to indicate noon. For comparison, contemporary European observations display errors of only about 0.02h.<sup>9</sup>

Finally, in Figure 6 are plotted the errors in measuring the durations from first to last contact for both partial and total eclipses. These observations show a very small scatter in measuring time-intervals for both pre-1745 and post-1745 observations. However, there are marked dissimilarities. The pre-1745 data show a systematic error of as much as 0.30h — in measuring durations of only between about 2 and 4 hours. This can only partially be due to optical effects — for example, confusion between the umbral and penumbral shadows — since first contact observations display negligible bias. The true explanation at present eludes us. After 1745, the systematic error in measuring durations reduces to zero, implying that both first and last contact were clearly resolved.



### Conclusion

Our analysis of both lunar and solar eclipse observations made in China by Jesuit astronomers during the seventeenth and eighteenth centuries shows that they were of surprisingly poor quality. Up to about 1745, when new instruments were introduced at the imperial observatory in Beijing, timing errors were severe — typically amounting to a quarter of an hour; this is no better than the achievements of Chinese astronomers several centuries previously.<sup>10</sup> The later Jesuit observations, although substantially improved, were impaired by systematic clock errors of about 0.10h, which meant that all measured times were consistently late by this amount. However, the problems experienced by the Jesuits in setting up a working observatory so remote from Europe should not be underestimated.

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## THALES'S PREDICTION OF A SOLAR ECLIPSE

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### 1. *Introduction*

In a recent paper published in this journal,<sup>1</sup> Panchenko reconsidered the question of the date of the eclipse said to have been predicted by Thales of Miletus early in the sixth century B.C. The eclipse is recorded as having been seen at some indefinite location in Asia Minor during a battle between the Lydians and the Medes. The identity of the eclipse — which is mentioned by a number of ancient writers including Herodotus, Pliny and Diogenes Laertius — has long been a matter of debate.<sup>2</sup>

Panchenko gave a detailed discussion of Thales's possible method of prediction based on the use of eclipse cycles. In this way, Panchenko deduced the date of the eclipse as either B.C. 582 Sep 21 or 581 Mar 16, the latter alternative being regarded as less likely. However, although his arguments are of considerable interest, Panchenko did not consider in any detail the visibility of these eclipses in Asia Minor. In our view, the question of visibility is a matter of key importance.

### 2. *The Records*

Although Herodotus does not mention an eclipse directly, in his *History* (I, 74) he describes how "the day was turned to night" during a battle between the Lydians and the Medes. He also mentions that the loss of daylight had been predicted by Thales:

After this, seeing that Alyattes would not give up the Scythians to Cyraxes at his demand, there was war between the Lydians and the Medes five years.... They were still warring with equal success, when it chanced, at an encounter which happened in the sixth year, that during the battle the day was turned to night. Thales of Miletus had foretold this loss of daylight to the Ionians, fixing it within the year in which the change did indeed happen. So when the Lydians and the Medes saw the day turned to night, they ceased from fighting, and both were the more zealous to make peace. Those who reconciled them were Syennesis the Cicilian and Labnetus the Babylonian....<sup>3</sup>

Pliny (*Naturalis historia*, II, 53) does not mention the battle, but he makes it clear that the celestial phenomenon foretold by Thales was indeed a solar eclipse. Pliny also specifies the year when the prediction was made and when the eclipse occurred: "The original discovery (of the cause of eclipses) was made in Greece by Thales of Miletus, who in the fourth year of the 48th Olympiad (585/4 B.C.) foretold



the eclipse of the Sun that occurred in the reign of Alyattes, in the 170th year after the foundation of Rome (584/3 B.C.)."<sup>4</sup>

Diogenes Laertius (I, 23), in his *Life of Thales*, makes a passing reference to Thales's prediction, naming Eudemus as his source: "He (Thales) seems by some accounts to have been the first to study astronomy, the first to predict eclipses of the Sun and to fix the solstices."<sup>5</sup>

### 3. General Discussion

As noted above, the year 170 AUC given by Pliny corresponds to 584/3 B.C. Judging from the charts of Ginzel,<sup>6</sup> the only total eclipse which could have been visible in Asia Minor for many years around this time — between 602 and 557 B.C. — occurred on B.C. 585 May 28. This date, which is remarkably close to that indicated by Pliny, was accepted by both Ginzel and Fotheringham.<sup>7</sup>

By contrast, neither of the eclipses of B.C. 582 Sep 21 and B.C. 581 March 16 which Panchenko selected can have been very large in Asia Minor. In this paper we shall investigate in detail the visibility of each of the three eclipses of B.C. 585, 582 and 581 in order to reconsider the question of the date of the eclipse of Thales.

### 4. Solar Eclipse Computations

In order accurately to investigate the visibility of a solar eclipse in the ancient past, due allowance must be made for the effect of long-term changes in the Earth's rate of rotation. These changes are mainly produced by tides raised by the Moon and Sun in the oceans and seas of the Earth, which result in a gradual increase in the length of the day. The length of the day itself has only increased by a small fraction of a second during the whole of the historical period. However, the cumulative effect over many centuries is significant, amounting to a "clock error" (usually known as  $\Delta T$ ) of as much as five hours around 580 B.C. During this interval the Earth rotates through an angle of about  $75^\circ$ .

The variation of  $\Delta T$  in the historical past is best determined from a study of reliable eclipse observations. The most extensive recent study of Earth's past rotation — using historical eclipses — is by Stephenson and Morrison.<sup>8</sup> These authors analysed a wide variety of reliable Babylonian, Chinese, European and Arab eclipse observations — some dating from as early as 700 B.C. Their study enables the value of  $\Delta T$  to be deduced with fairly high precision (within about 2%) at any date in the ancient and medieval past. We have used the results obtained by Stephenson and Morrison — in conjunction with accurate ephemerides of the Moon and Sun — to investigate the circumstances of the three solar eclipses listed above.

### 5. Results

In Figure 1, we show the computed tracks of totality in both 585 and 582 B.C. in the Mediterranean region; the track of the annular eclipse of 581 B.C. lay much to the

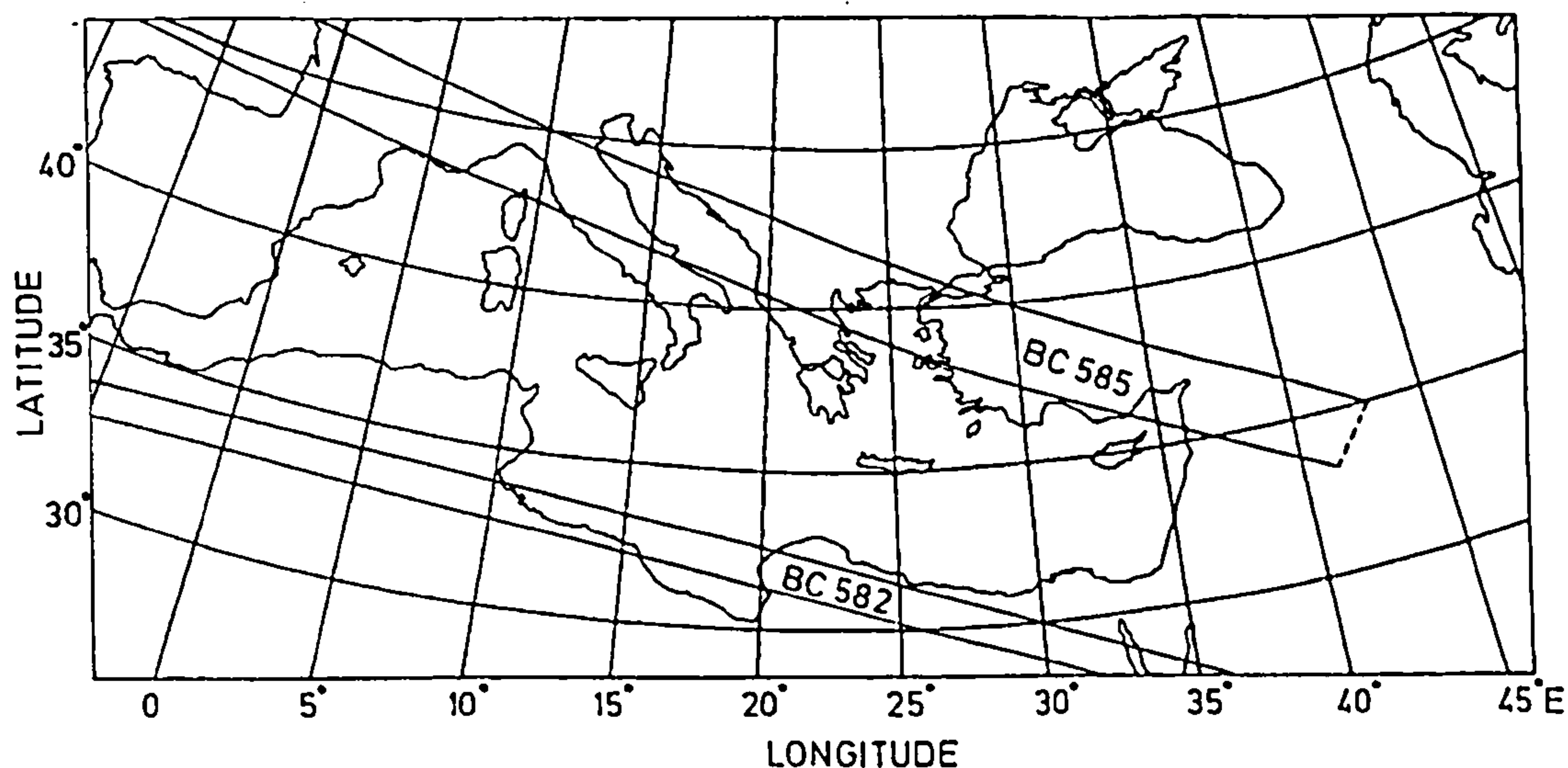


FIG. 1. Chart of the Mediterranean area showing the computed tracks of totality for the solar eclipses of B.C. 585 May 28 and 582 Sep 21.

east of the area covered by map. According to our computations, the eclipse of 585 B.C. was certainly total over much of Asia Minor about an hour before sunset (and incidentally would probably be total at Miletus, where Thales lived). However, the track of totality in 582 B.C. passed far to the south of Asia Minor; the magnitude at any point in the peninsula cannot have exceeded 85%. The eclipse would not cause any appreciable loss of daylight in Asia Minor and may well have passed completely unnoticed. (It is worth pointing out that the zone of totality in 582 B.C. did not in fact reach further north than latitude  $34.0^{\circ}\text{N}$  at any point on the Earth's surface.)

In the central zone of the annular eclipse of 581 B.C., no more than 96% of the Sun could have been covered. However, the computed magnitude reached no more than 63% anywhere in Asia Minor. As a contender for the "Eclipse of Thales", this event thus fares even poorer than that of 582 B.C.

## 6. Conclusion

As contenders for the eclipse of Thales, the events of 582 and 581 B.C. might at first sight seem more attractive on the assumption that Thales made a prediction using a numerical cycle — as suggested by Panchenko. However, these eclipses reached too small a magnitude in Asia Minor to produce the marked loss of daylight described by Herodotus. In our view, the only plausible date for the eclipse of Thales is B.C. 585 May 28. Acceptance of this date requires only a slight dating error by Pliny (or his source) at a period more than 600 years before his own time.



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# Angular measurements in Babylonian astronomy

By L. J. Fatoohi and F. R. Stephenson (Durham)

## 1. Introduction

In estimating the angular separation between two celestial bodies, Babylonian astronomers in the period from at least 600 B. C. to 50 B. C. used two related units, which normally applied to linear measure. These units are the KÙŠ (“cubit”) and SI (“finger”), 1 cubit being composed of 24 fingers in the Neo-Babylonian period. The angular equivalent of the cubit in this period was found by Kugler (1909/10: 547-550) to be approximately 2 deg (see also Neugebauer, 1955: 39; Sachs and Hunger 1988: 22). However, it would appear that a detailed investigation of this ratio using Babylonian astronomical observations has never been published. It is our intention to try to remedy this omission.

In order to determine the adopted angular equivalents of the cubit and finger we have investigated a large sample of neo-Babylonian measurements of separation at conjunction between (i) any two of the planets Mercury, Venus, Mars, Jupiter and Saturn, and (ii) between one of these planets and a bright star. The various observations are recorded on the Late Babylonian astronomical texts which are now mainly in the British Museum. In this paper we have concentrated specifically on *close* approaches (less than 1 cubit) for which the measurements are expressed only in fingers. This alone represents a sizeable set of data – some 200 individual determinations. Such measurements are usually quoted to the nearest one or two of fingers (in the sample we have analysed, individual separations are 1, 2, 3, 4, 5, 6, 8, 10, 14 and 20 of these units.)

## 2. Observational data

Our exclusive source of data has been the transliteration and translation of the Babylonian astronomical diaries published in 3 volumes by Sachs and Hunger (1988, 1989 and 1996).

The principal Babylonian astronomical observations which make use of the finger (and cubit) involve either conjunctions between the Moon and planets or stars, or conjunctions between planets and other planets or stars. In our investigation we have concentrated on observations in the latter category. This is because precise times of observation are never recorded – only rather vague statements: SAG GE<sub>6</sub> (“beginning of the night”); USAN (“first part of the night”); MURUB<sub>4</sub> (“middle part of the night”); and ZALÁG (“last part of the night”). The Moon moves so much more rapidly than any of the planets (roughly 0.5 deg hourly) that fairly careful estimates of the time of conjunction would be needed to make satisfactory use of such data. In the case of the planets, accurate times are largely unnecessary for the purpose of this study. Not only is the motion of these celestial bodies much slower than that of the Moon, but we have not found any case where the term MURUB<sub>4</sub> is used for the time of a conjunction involving the planets; all observations

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evidently took place reasonably soon after sunset or before sunrise.

Although Mercury and Venus move fairly rapidly (about 1 deg daily on average), both planets (especially Mercury) are never seen far from the Sun and are thus only visible for a relatively short time in a dark sky. In investigating observations involving these two planets, we have assumed for Mercury a moment of observation 0.5 h after sunset (when the terms SAG GE<sub>6</sub> or USAN are used) and 0.5 h before sunrise (when the term ZALÁG is applied). For Venus, which is usually considerably further from the Sun, we have doubled (to 1 hour) the appropriate interval after sunset or before sunrise. Sometimes the time of night is not even specified in the record (i. e. broken away). In such cases we have followed the same assumptions, depending on whether the planet was to the east or west of the Sun.

Except when near conjunction with the Sun, the outer planets Mars, Jupiter and Saturn may be visible for a large fraction of the night. In principle, this could lead to a greater uncertainty in the actual time of observation since the terms SAG GE<sub>6</sub>, USAN and ZALÁG are not well-defined. Fortunately, this is unimportant for the slow moving planets Jupiter and Saturn (daily motion 0.05 to 0.1 deg). In these cases we have somewhat arbitrarily interpreted the terms SAG GE<sub>6</sub> and USAN to imply 1 hour after sunset (when the planet was to the east of the Sun) and ZALÁG as 1 hour before sunrise (when the planet was west of the Sun), as for Venus. For Mars (mean daily motion 0.5 deg), a better estimate of the time of night would be desirable but this cannot be achieved in practice unless the planet happened to be fairly close to conjunction with the Sun when the observation was made. We have therefore cautiously adopted the same criteria as for Venus, bearing in mind the absence of any reference to an observation being made in the middle of the night.

In the 3 volumes published by Sachs and Hunger, we have made a careful search for *all* conjunctions between planets with one another or between planets and stars for which the separation is expressed purely in “fingers”. Our aim is to derive a result for the angular equivalent of this unit, and consequently the cubit, which contains 24 fingers (Sachs and Hunger 1988: 22). Examples of the type of record which we have investigated are given below. In some cases the date of the text is fully preserved, as in example (i). However, in many instances the date is broken away and has been derived indirectly by Sachs and Hunger using astronomical computation based on the wide variety of lunar and planetary data in the same text.

(i) “Year 5 of (king) Umakus, month I, night of the 7th, first part of the night, Venus was 8 fingers below  $\beta$  Tauri, Venus having passed 4 fingers to the east”. (Sachs and Hunger, 1988: 63). The date of this observation corresponds to B. C. 420 Apr 1 (see below).

(ii) “[Year 12] of king Alexander... [month IV]....,

night of the 3rd.... first part of the night Venus was 4 fingers in front of  $\beta$  Virginis [...] [Night of the 4th], beginning of the night...., Venus was 2 fingers behind  $\beta$  Virginis, Venus being 2 fingers low to the south". (Sachs and Hunger, 1988: 199). The first date corresponds to B. C. 325 Jul 5.

(iii) "[Year 12] of king Alexander.... [month III]...., Night of the 4th.... First part of the night, Mars [was above  $\beta$ ] Virginis 2 fingers, it came near, Mars being 1 finger back to the west" (Sachs and Hunger, 1988: 197). The date corresponds to B. C. 325 Jun 7.

(iv) "[Year 167 (Seleucid)], month IX...., the 5th, when Saturn became stationary to the [east], it became stationary 1/5 cubit behind  $\alpha$  Leonis, Saturn being 6 fingers high to the north" (Sachs and Hunger 1996: 320). The date of this observation is equivalent to B. C. 145 Nov. 22.

As shown in the above examples, in the Late Babylonian texts, the position of the Moon or a planet relative to a nearby planet or star may be described in one of the following ways: "above" (*e*), "below" (*šap*), "in front of" (*ina* IGI), or "behind" (*ar*). The term "in front of" is synonymous with "to the west of", following the apparent rotation of the celestial sphere. Similarly, "behind" is equivalent to "to the east of". In some texts, *both* linguistic alternatives are occasionally found. As in the above examples, often only a single measurement (i. e. "above" or "below") is given, but frequently two co-ordinates presumably roughly at right angles – are specified.

### 3. Computation

First we had to convert the Babylonian dates of the observations to the Julian calendar. The Babylonians used a lunar calendar of twelve months for common years and thirteen months for the leap ones. At first, leap years were used arbitrarily, but after around 400 B. C. a clear rule based on the Metonic cycle was followed. Like many lunar calendars, the first sighting of the new crescent determined the first night of the month. The day in the Babylonian calendar began at sunset, some 6 hours before the start of the civil date.

Conversion of Babylonian dates into their Julian equivalents can be made using the special tables prepared by Parker and Dubberstein (1956). However, we have used only the intercalary scheme from these tables and have designed a computer program that reads in the Babylonian date and converts it to the Julian calendar, totally independently of Parker and Dubberstein's tables. For the determination of the crescent visibility conditions, we have used the rule given by Neugebauer (1929). This lists altitude limits for a series of azimuth differences, which for the present purpose do not differ significantly from the results of modern studies.

Any theoretical computation of the the first day of a Babylonian month can only be a first approximation.



The Babylonians would not necessarily have seen the lunar crescent on the first night when it should have been visible, for instance due to cloudy sky. Prevention of observation by cloud is clearly stated in some texts – especially in winter. This would mean that the adopted date for the first night of the month could have been one day before or after the computed date.

by To verify the *exact* date of each observation, we compared one ~~lunar observation~~ (e. g. a conjunction between the Moon and a planet or star) on a specific date in the month in question with its computed equivalent.

event/

We also designed a computer program for deducing the co-ordinates of selected stars in the past (applying precession and proper motion). Only 31 stars (known as the “Normal Stars”) seem to have been employed by the Babylonian astronomers for recording lunar and planetary movements. For reference, a list of these stars, with longitude and latitude at the epoch 164 B. C., is given by Stephenson and Walker (1985). For deducing planetary co-ordinates we have designed a program based on the VSOP87 analytical solution by Bretagnon and Francou (1988). This program requires the input of the Julian (or Gregorian) date together with the Terrestrial Time (TT) – formerly known as Ephemeris Time (ET). Because of variations in the Earth’s rate of rotation – due to tides and other causes – it is necessary to make allowance for the cumulative effect of changes in the length of the day (usually termed  $\Delta T$ ). For the calculation of  $\Delta T$  we have used the list of values at 50-year intervals derived recently by Stephenson and Morrison (1995). To give an example, in the year -500 (i. e. 501 B. C.)  $\Delta T$  is estimated to have been as much as 16800 sec (4.67 hours).

— of

We have compared each individual measured distance – expressed in fingers – (whether “above” or “below,” or “behind” or “in front of”) with its computed equivalent on three separate assumptions: (i) that the Babylonians used ecliptical co-ordinates; (ii) that they used equatorial co-ordinates; (iii) that they used horizon co-ordinates (altitude and azimuth). The texts themselves do not give any indication of which – if any – of the above systems were used. We have rejected a very few (some ten in all) observations for which there was an obvious scribal error in the record; in such cases the two celestial bodies involved were found to be many degrees apart on the stated date of conjunction.

We have plotted degrees vs fingers for each of the three assumptions: Fig. 1 (ecliptical); Fig. 2 (equatorial); Fig. 3 (horizon) and in each case have fitted the best straight line to the data. We have used the gradient of this line to derive the equivalent of the finger (and hence of the cubit) in degrees.

Both Fig. 1 and Fig. 2 show much the same form (correlation coefficients respectively 0.85 and 0.76). If ecliptical co-ordinates are assumed (Fig. 1), the angular equivalent of the finger is (to 2 significant figures) 0.092 deg and hence the cubit is 2.2 deg. If equatorial

co-ordinates are assumed (Fig. 2) precisely the same results are obtained (i. e. 0.092 deg and 2.2 deg). In both cases, a number of measured values for the separation between two celestial bodies prove to be of the wrong sign. On the assumption of ecliptical co-ordinates, there are 21 such cases or about 10 per cent of the total; for equatorial co-ordinates the corresponding figures are 32 and 16 per cent. There is clearly little to choose between the two separate alternatives.

On the supposition that horizon co-ordinates were used instead – see Fig. 3 – the scatter is considerably greater (correlation coefficient only 0.21). The resulting value for the finger is as small as 0.040 deg and for the cubit 1.0 deg. On this convention, as many as 79 measurements (39 per cent of the total) would have the wrong sign.

It thus seems highly likely that the Babylonians did not employ horizon co-ordinates and used either an ecliptical or equatorial scheme instead. However, there appears to be insufficient evidence to discriminate between these two latter alternatives. In general, both Neugebauer (1975: 545) and Van der Waerden and Huber (1974: 99), were of the opinion that ecliptical co-ordinates were utilised.

Both graphs of ecliptical (Fig. 1) and equatorial (Fig. 2) co-ordinates show a definite intercept of approximately 0.2 deg. A similar intercept appears even when plotting the data for each planet separately; individual values range from 0.12 (for Jupiter) to 0.36 (for Saturn); in the case of Venus – usually by far the brightest planet – the intercept is 0.15 deg. Our results thus seem to eliminate the possibility of the intercept being due to an optical effect, for example, produced by the “rays” which to most observers appear to surround a bright planet or star due to eye defects. These rays would be expected to cover a wider area than for brighter objects. The intercept may instead indicate a systematic error in the measuring device which the Babylonians used to determine angular separations. (It is also possible that these angles were in fact estimated by the unaided eye rather than measured with the aid of any special tool).

In order to decide whether the angular definition of the finger changed over time, we have plotted in Fig. 4 the equivalent of the finger in degrees from Fig. 1 (ecliptical) versus the year of each measurement. The graph has a very small slope of about 0.0059 (per century), with a correlation coefficient of only 0.05 – implying that the definition of the finger did not change during the period under consideration.

0.0059  $\hat{a}$

### *Conclusion*

Our results for the angular equivalent of the finger and cubit in the Neo-Babylonian period are respectively 0.092 and 2.2 deg, and we find no evidence for any variation of these equivalents during the period



from about 600 B. C. to 50 B. C. In our opinion, the astronomers of the time did not adopt horizon co-ordinates for making their measurements. Whether they used ecliptical or equatorial co-ordinates (or an approximation to either system) remains an open question. However, because of the Babylonian introduction of the concept of the zodiac (around 400 B. C.), it seems more reasonable to us that the astronomers used an ecliptical system.

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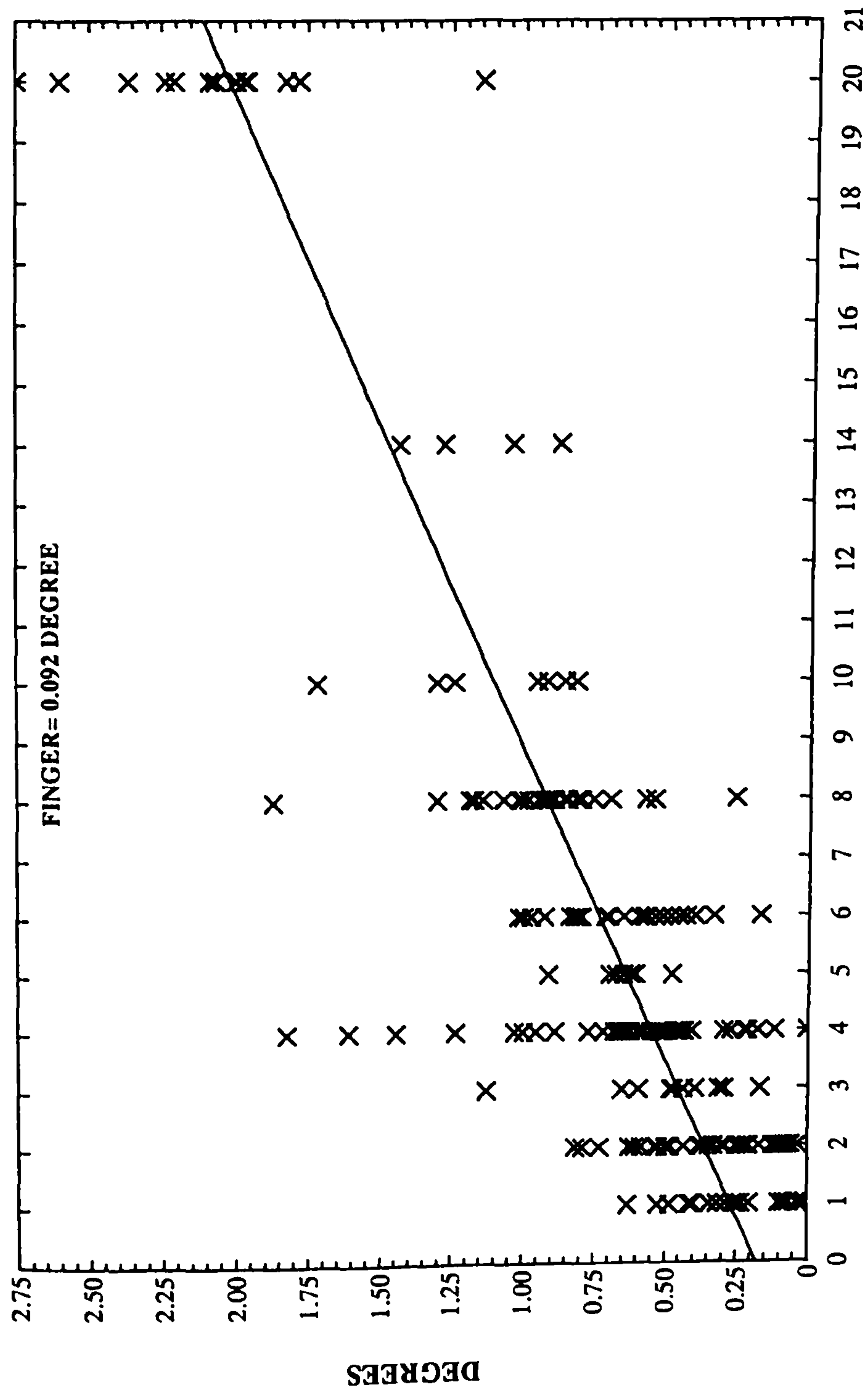
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FIG. 1

ECLIPTICAL CO-ORDINATES



"FINGERS"



# EQUATORIAL CO-ORDINATES

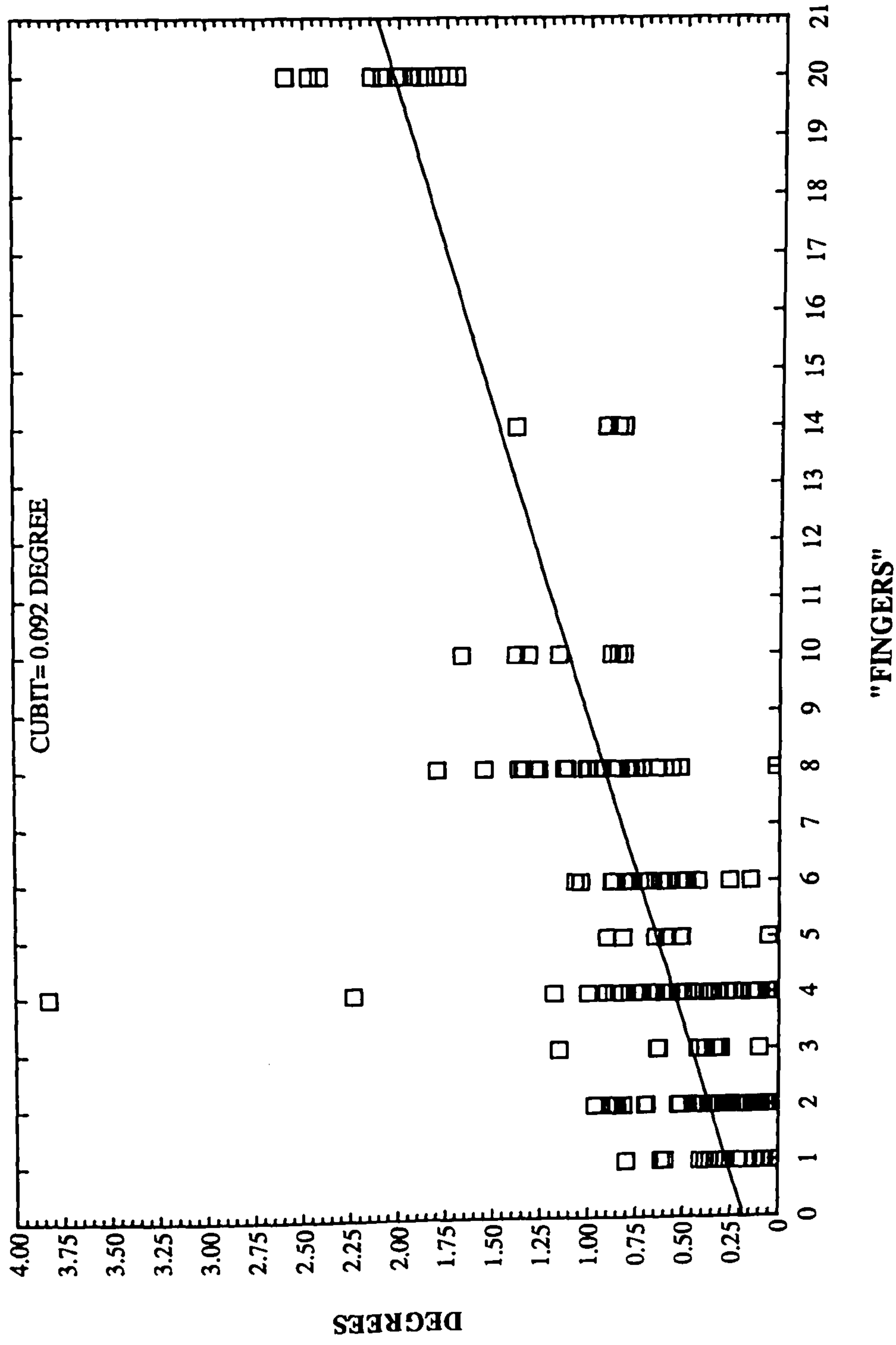
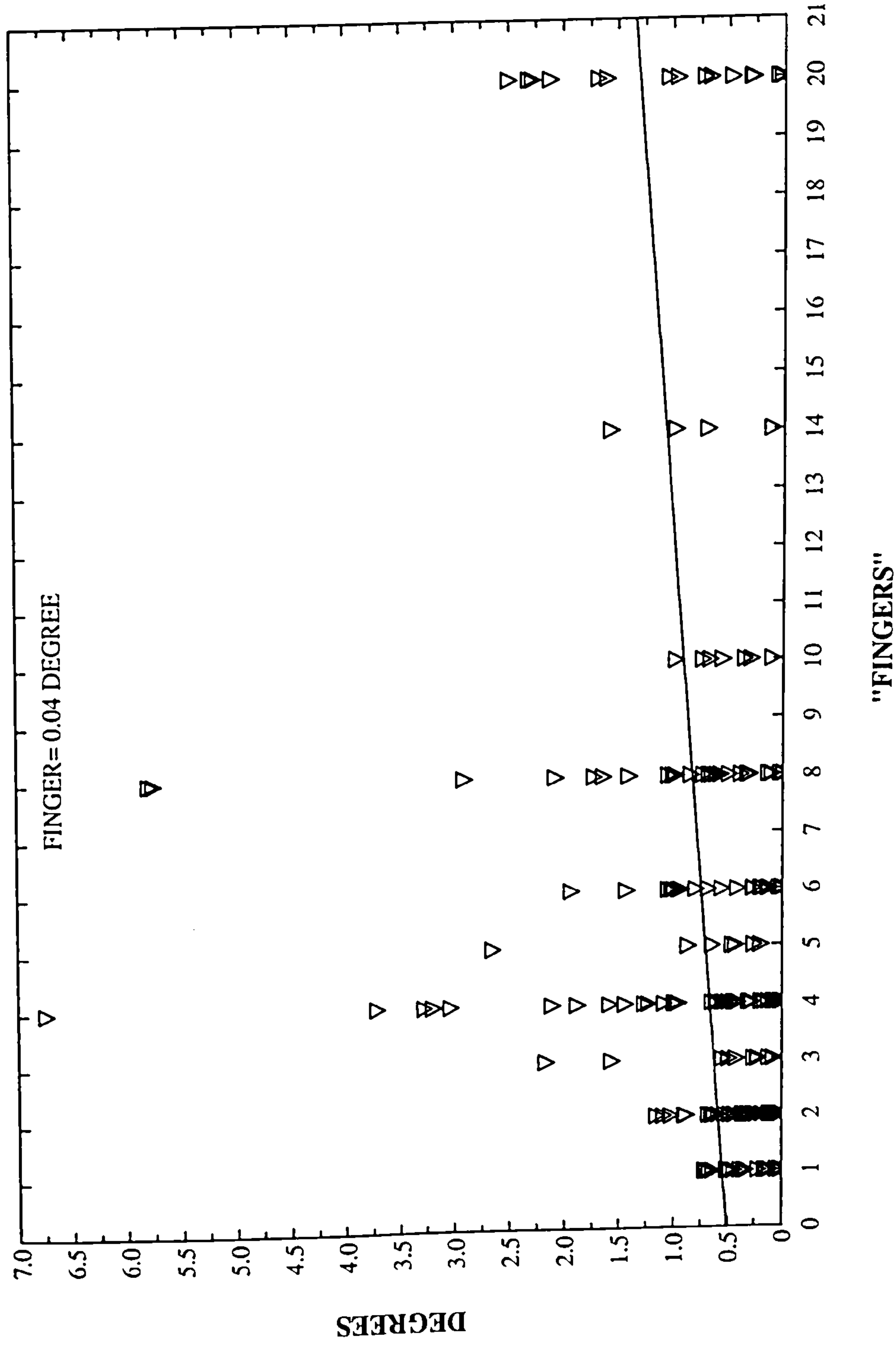


FIG. 3

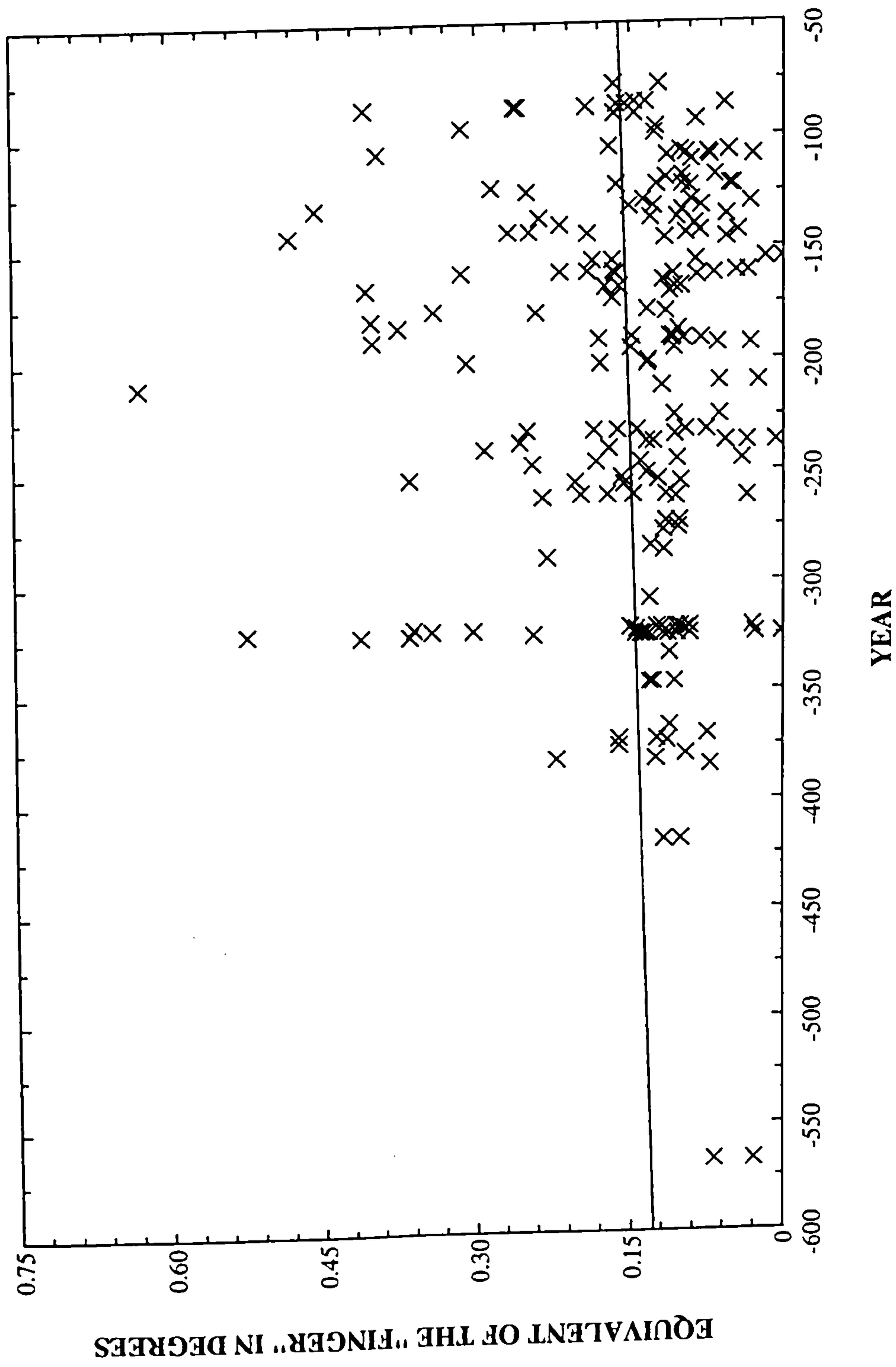
# HORIZON CO-ORDINATES





CONSTANCY OF THE EQUIVALENT OF THE "FINGER"

FIG. 4



# THE ECLIPSES RECORDED BY THUCYDIDES

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## 1. Introduction

In his *History of the Peloponnesian War*, Thucydides records three eclipses: two of the Sun and one of the Moon. These events took place at widely spaced intervals. A century has now elapsed since all three eclipses were last considered in detail [1]. Recent studies of Earth's past rotation [2] enable the precise dates and local circumstances (e.g. magnitudes and times) for all eclipses in a selected period and at a given place to be computed. In the light of this research, it seems appropriate to reconsider the eclipses which Thucydides cites in his celebrated history.

## 2. The eclipse records

Each of the dates of the three eclipses mentioned by Thucydides is quoted exclusively in terms of the year of the Peloponnesian War, which began in 431 B.C. and ended 27 years later. The solar events (see 2.28.1 and 4.52.1) occurred during the first and 8th years, while the lunar obscuration (see 7.50.4) took place in the 19th year. Thucydides followed the practice of dividing each year into two conventionalised seasons, "summer" (including spring and autumn) and "winter". For each of the three eclipses which he cites, the season was specified as summer. The corresponding eclipse dates are thus during the summers of 431, 424 and 413 B.C.

Thucydides seems to have had a special interest in eclipses of the Sun, remarking (1.23.3) that during the Peloponnesian War they "occurred at more frequent intervals than we find recorded of all former times". Surprisingly, he mentions only two of them in his history. Were there others which he failed to record? He specifically notes that the two solar eclipses which he does report occurred at the time of new Moon, while the lunar obscuration happened at full Moon. This may illustrate his knowledge of the true cause of eclipses, which had only lately



been explained by the astronomer Anaxagoras (ca. 500 - 428 B.C.). Interestingly, Thucydides reports both solar eclipses in a matter-of-fact way, but he stresses that the lunar eclipse was regarded by eyewitnesses as a major omen by the Athenians in Sicily (not by Thucydides himself).

(i) The first eclipse is said by Thucydides (2.28.1) to have occurred only a few months after the start of the War. His account has been translated by Smith [3] as follows:

During the same summer at the beginning of a lunar month (the only time, it seems, when such an occurrence is possible), the Sun was eclipsed after midday; it assumed the shape of a crescent (*menooides*), and became full again and during the eclipse some stars (*asteron tinon*) became visible.

This eclipse appears to have been unusually large, and Thucydides' description is especially interesting since it is the earliest account in occidental history to mention the visibility of stars by day during an eclipse. (Among the records of other civilisations, only a single Chinese observation, dating from 444 B.C., makes an earlier allusion to stars seen under these circumstances) [4].

The eclipse described by Thucydides in the above quotation is probably alluded to by both Cicero (*De Republica*, 1.16.25) and Plutarch (*Life of Pericles*, 35.1 - 35.2). Both authors recount a story concerning Pericles (himself a former pupil of Anaxagoras), who died in 429 B.C. It is remarked that a solar eclipse occurred during the Peloponnesian War, bringing on darkness and causing terror. However (as Plutarch relates), Pericles -using his cloak - demonstrated the cause of the eclipse, thus dispelling the alarm. The details of the tale as given by Cicero and Plutarch differ considerably, but both writers lived several centuries after the event. According to Plutarch the story was told in the schools of philosophy. However, only the contemporary account by Thucydides merits further consideration here.

In view of the careful description of the eclipse which Thucydides gives, it seems plausible that he saw it himself, although this cannot be proved. However, there is nothing in his text to indicate where he was when the eclipse occurred. He may well have been in his home city of Athens. He makes no mention of any travels from there until the 8th year of the War, while in the year after the eclipse he tells us (2.48.3) that he caught the plague in Athens. Further discussion of the possible place of observation will be given in section 3 of this paper.

(ii) The second solar obscuration was stated by Thucydides (4.52.1) to have occurred in the

8th year of the War. Following his customary style he ends the previous paragraph (4.51.1) as follows: "And the winter ended, and with it the seventh year of this war of which Thucydides composed the history". His brief description of this second eclipse may be translated as follows:

At the very beginning of the next summer a partial eclipse of the Sun took place at new Moon, and in the early part of the same month an earthquake [5].

This description is appreciably briefer than in the previous instance. The place of observation - as well as the location of the earthquake - is not mentioned. Athens is a possible place of observation. However, not long after the eclipse, Thucydides (4.104.4 ff) led an abortive military exploit from the island of Thasos to relieve the city of Amphipolis in Thrace. This failure led to his banishment for twenty years.

(iii) Finally, Thucydides (7.50.4) implied that the lunar eclipse happened in the summer of the 19th year. This observations was made from Sicily. A translation of Thucydides' account is as follows:

But after all was ready and when they (the Athenians) were about to make their departure (from Syracuse) the Moon, which happened then to be at the full, was eclipsed. And most of the Athenians, taking the incident to heart, urged the generals to wait. Nicias also, who was somewhat too much given to divination and the like, refused even to discuss further the question of their removal until they should have waited thrice nine days, as the soothsayers prescribed. Such, then, was the reason why the Athenians delayed and stayed on [6].

In contrast to the two solar eclipses, in this instance there is a detailed historical context. The indicated place of observation is Syracuse, where the Athenians had landed. The delay urged by the soothsayers was to have disastrous consequences for the Athenians at the hands of the Syracusans (7.51.1 ff).

It is clear from Thucydides' account of the events leading up to the eclipse that it occurred in the high summer. After several allusions to spring (7.19.1 ff), the season is set as summer (7.27.1), the "season of sickness" (7.47.2). Not many days afterwards, the nights were "autumnal and cold" (7.87.1).

This same lunar eclipse is also recorded by both Diodorus Siculus (13.12.6) and Plutarch (*Life of Nicias*, 23.1 - 23.6), but neither writer adds anything significant to the description



given by Thucydides.

3. Solar eclipse computations

In Table 1, we summarise our computations for all of the solar eclipses visible at Athens over an extended period: ranging from 433 to 402 B.C. This period has been selected so as to slightly overlap the date range actually covered by the War (431 to 404 B.C.). For most of the ten eclipses listed in the table, the calculated local circumstances would be similar throughout the Aegean. However, the magnitudes of the unusually large eclipses of 431 and 402 B.C. would be somewhat dependent on location. In the table we have given the following information for each eclipse: Julian date, magnitude (the maximum proportion of the Sun covered by the Moon) as a decimal, local time of greatest phase in hours and minutes, and Sun's altitude in degrees at that time - all for the city of Athens (lat = 37.98 deg N, long = 23.73 deg E). It should be noted that in the case of the eclipse of 405 B.C., the Sun set eclipsed in Greece well before greatest phase was reached, so that only a relatively small eclipse would be actually visible.

Table 1 Solar eclipses visible at Athens around the time of the Peloponnesian War

Date (B.C.)	Magnitude	Time	Altitude
433 Mar 30	0.55	14;15	43°
431 Aug 3	0.88	17;30	18
426 Nov 4	0.32	14;05	30
424 Mar 21	0.71	08;30	27
418 Jun 11	0.12	11;40	74
411 Jan 27	0.35	10;20	28
409 Jun 1	0.47	12;00	73
405 Mar 20	0.38	17;45	0
404 Sep 3	0.73	08;35	36
402 Jan 1	1.04	09;00	17

The first solar eclipse cited by Thucydides (2.28.1) occurred only a few months after the start of the Peloponnesian War. Reference to Table 1 shows that only the eclipse of B.C. 431 Aug 3 fits the established chronology of the War, and this indeed occurred during the summer. It reached a magnitude at Athens of 0.88. The Sun would thus assume a very obvious crescent shape (see Fig 1a), while the diminution of daylight would be marked. The eclipse began well after midday (about 4.30 p.m.), and reached its greatest phase about an hour later (5.30 p.m.). This latter time was roughly 1 1/2 hours before sunset. However, in indicating a time "after midday" Thucydides may have given only a very general indication of the time of day.

The eclipse of 431 B.C. was annular. Within the central zone - which crossed the Black Sea (see Fig 2) - the Sun would be reduced to a narrow ring of light surrounding the dark Moon. However, in the Aegean area the eclipse would only be partial, although the Sun would be very largely obscured. Thucydides makes no mention of the ring phase, instead describing only the crescent shape assumed by the Sun. From the geographical position of the central zone, this is what we would expect.

Even in those places where the eclipse was central, no more than 0.98 of the Sun's diameter would be covered by the Moon. Hence - as compared with a total eclipse, for example - there would be no great darkness. Thucydides' reference to the visibility of "some stars" is thus difficult to explain. Although Venus, some 20 deg to the east of the Sun, would be fairly prominent, no other planet or star should have been detectable. In the late afternoon, both the bright objects Jupiter and Sirius would be below the horizon. Mercury, although fairly close to Venus, would be very faint [7].

Even well outside the zone of annularity - as in the Aegean area itself - Venus should have been fairly readily seen, but no other star. Except in the unlikely event of a bright comet or "new star" gracing the skies, some allowance for exaggeration must be made.

It has been suggested [8] that the observation was made in Thrace, where Thucydides had the right of working gold mines (4.105.1). In the section of his history immediately following the description of the eclipse, Thucydides (2.29.5) mentions an alliance between the Thracians and the Athenians. As shown in Fig 1b, the magnitude at Thrace (0.94) would have been appreciably larger than at Athens; this is also evident from the map in Fig 2. In Thrace, the daylight at mid-eclipse would have been somewhat dimmer than at Athens (although by no means very dark), and Venus would probably have been more prominent. However, Thrace as the place of observation must still remain conjectural.



The second solar eclipse noted by Thucydides (4.52.1) occurred "at the very beginning of summer", seven years after the previous eclipse. From Table 1, it is evident that the eclipse of B.C. 424 Mar 21 must be referred to here. This event took place at the appropriate time of year and would be quite significant: magnitude 0.71 at Athens and much the same throughout the Aegean (for instance 0.74 at Thrace) at about 8.30 a.m. The eclipse would be much less impressive than that of 431 B.C. and the decrease in daylight would be too small to render any stars visible.

As mentioned above, Thucydides (1.23.3) remarked on the frequency of solar eclipses during the Peloponnesian War. In Fig 3 we have shown diagrammatically the maximum degree of obscuration for all of the solar eclipses visible during the 27 years of the War: from 431 to 404 B.C. (see Table 1 for fuller details). There were as many as eight such events, but apart from the recorded eclipses of 431 and 424 B.C. and the further eclipse of 404 B.C. (at the very end of the War), all were small: in each case less than half of the Sun was obscured. Such minor eclipses have negligible effect on the daylight and might well have passed unnoticed.

In order to investigate Thucydides' assertion on the unusual frequency of solar eclipses during the War, we have made a comparison between the number of such events which according to computation should have been visible in the Aegean (weather permitting) during the Peloponnesian War with the corresponding number in the previous 50-year period. From Table 2, it can be seen that in the previous half-century, 16 solar eclipses were visible at Athens, compared with 8 during the 27 years of the War. Hence the average frequency was much the same. Hence based on the results of astronomical computation, Thucydides' statement is not accurate.

#### 4. Lunar eclipse computations

The single lunar eclipse recorded by Thucydides (7.50.4) took place in the summer of the 19th year of the War; the date indicated is thus the summer of 413 B.C. From any given region of the Earth, lunar eclipses are seen much more frequently than their solar counterparts. During the whole of the War, more than 20 lunar eclipses would be visible throughout the Eastern Mediterranean region.

We compute that during a period of more than three years encompassing 413 B.C. (between March, 414 B.C. and July, 411 B.C.), only two lunar eclipses were visible at Syracuse (lat =

37.07 deg N, long = 15.30 deg E). These both occurred during the year 413 B.C. itself: a large partial eclipse in the early spring (Mar 4) and a total obscuration of the Moon in the summer (Aug 27). As noted above (section 2), the eclipse of the Moon recorded by Thucydides (VII.50.4) took place during the high summer. Hence there seems no alternative to adopting the date B.C. 413 Aug 27 for the event witnessed by the Athenians at Syracuse. This would start at 8.15 p.m. (about 1 1/2 hours after sunset) and end towards midnight (11.40 p.m.). Totality would last for about 45 minutes (between 9.35 and 10.20 p.m.) and during that time the sky would be considerably darkened. Following the characteristic pattern of total lunar eclipses, the Moon would probably turn blood red in colour, or may possibly have even disappeared from sight for a while.

**Table 2 Solar eclipses visible at Athens in the 50 years preceding the Peloponnesian War**

Date (B.C.)	Magnitude	Time	Altitude
480 Oct 2	0.58	13;55	43
478 Feb 17	0.95	11;30	38
477 Aug 1	0.07	14;05	58
470 Mar 20	0.31	15;25	26
466 Jul 2	0.09	6;10	16
466 Dec 26	0.7	7;15	0
463 Apr 30	0.96	14;50	45
458 Aug 2	0.4	10;25	62
455 May 31	0.19	8;25	41
453 Oct 3	0.78	17;45	2
450 Mar 9	0.24	6;40	4
448 Jul 12	0.2	15;00	48
447 Dec 26	0.13	7;50	5
437 Jun 10	0.46	17;40	18
434 Oct 4	0.74	6;25	4
433 Mar 30	0.55	14;15	3



These changes may well have led to the alarm experienced by the Athenians, although there can surely be little doubt that many of the soldiers would have witnessed similar phenomena before. For instance, we compute that between the start of the Peloponnesian War and 413 B.C. as many as ten total lunar eclipses would have been visible in the Mediterranean region.

## Conclusion

The dates which we have derived for the three eclipses recorded by Thucydides are respectively 431 Aug 3, 424 Mar 21 and 413 Aug 27. These are, in fact, the same as those given long ago by Ginzel [9]. However, Ginzel was unable to make satisfactory allowance for changes in the Earth's rate of rotation and we have thus been able to deduce more reliable information on local circumstances. Our calculations indicate that the descriptive details of all three events which Thucydides gives - brief though they are - are reliable. However, his remarks on the unusual frequency of solar eclipses during the War seem exaggerated.

## References

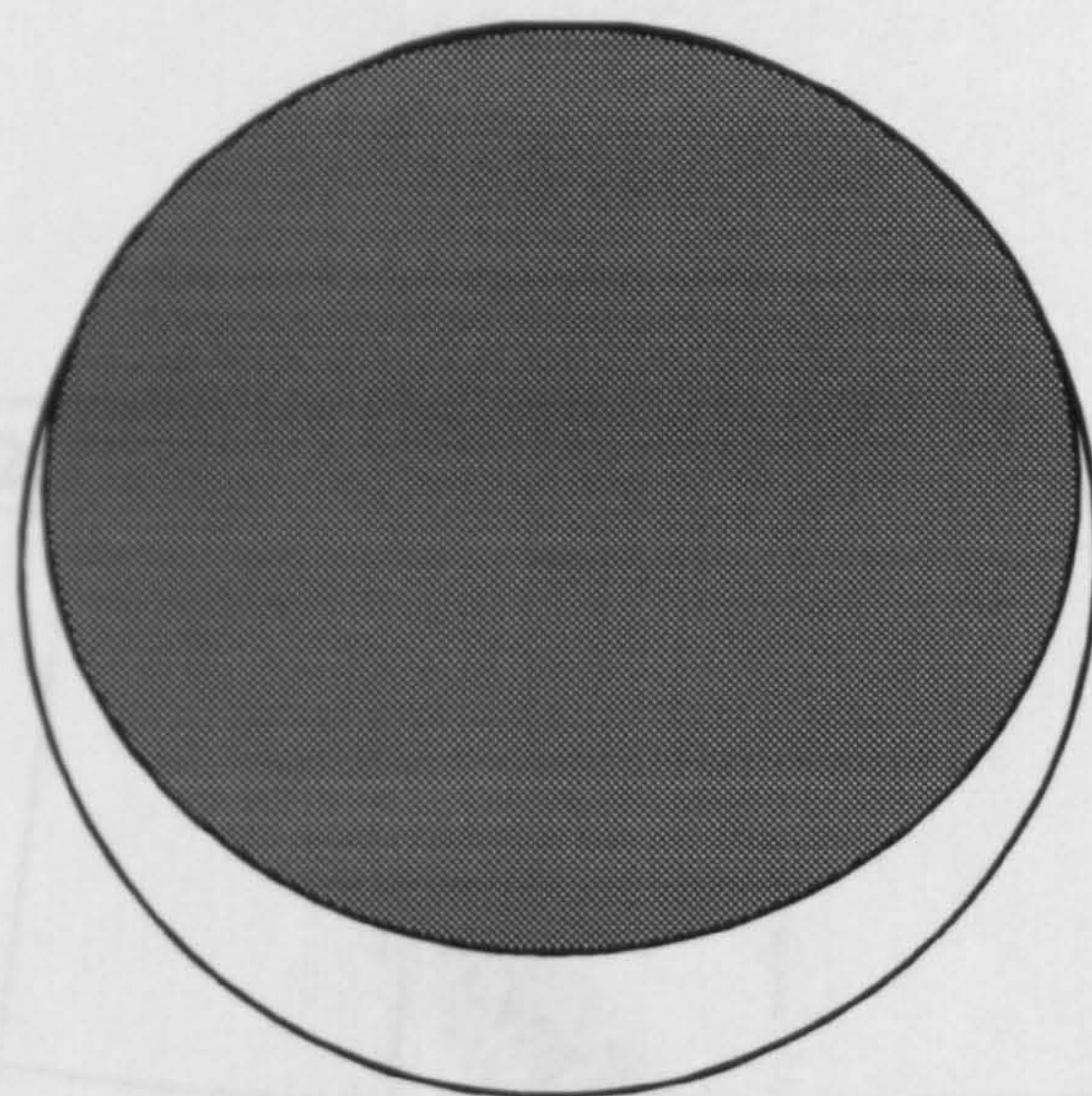
1. F.K. Ginzel, *Spezieller Kanon der Sonnen-und Mondfinsternisse* (Berlin 1899) 176-179.
2. F.R. Stephenson and L.V. Morrison, "Long-term fluctuations in the Earth's rotation: 700 B.C. to A.D. 1990", *Philosophical Transactions of the Royal Society of London: A* 351 (1995) 165-202; F.R. Stephenson, *Historical eclipses and Earth's rotation* (Cambridge 1997).
3. C.F. Smith, *Thucydides: History of the Peloponnesian War I* (London 1919) 309-311.
4. Stephenson, *Historical eclipses* (as in n. 2) 227 and 346.
5. C.F. Smith, *Thucydides: History of the Peloponnesian War II* (London 1920) 299.
6. C.F. Smith, *Thucydides: History of the Peloponnesian War IV* (London 1923) 101.
7. J.K. Fotheringham, "A solution of ancient eclipses of the Sun", *Monthly Notices of the Royal Astronomical Society* 81 (1920) 104-126.
8. Fotheringham, *ibid.*
9. Ginzel, *Spezieller Kanon* (as in n. 1) 176-179.

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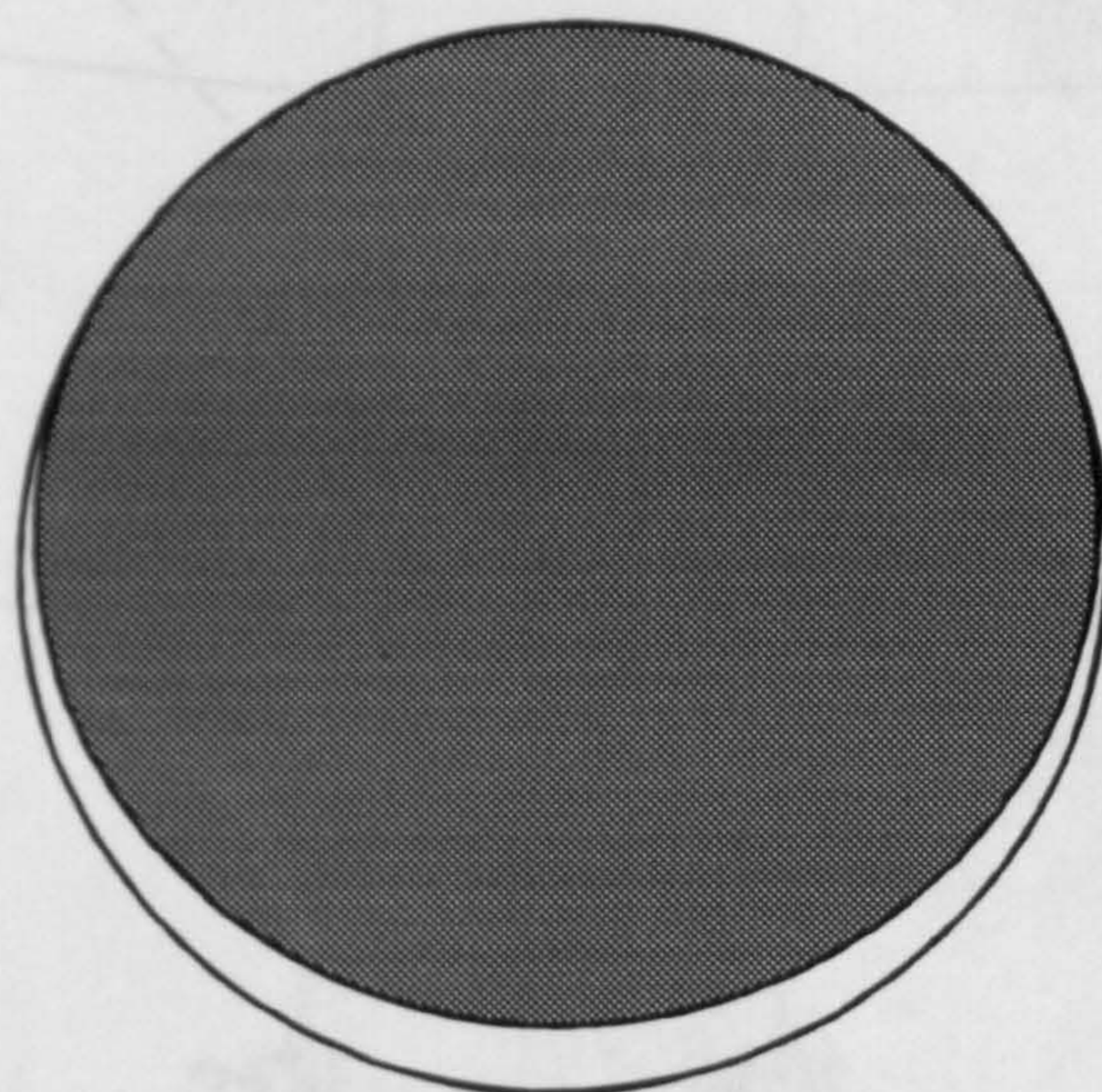


(a)



ATHENS (0.88)

(b)



THRACE (0.94)

Fig 1. The solar eclipse of B.C. 431 Aug 3 as seen from Athens (a) and Thrace (b).



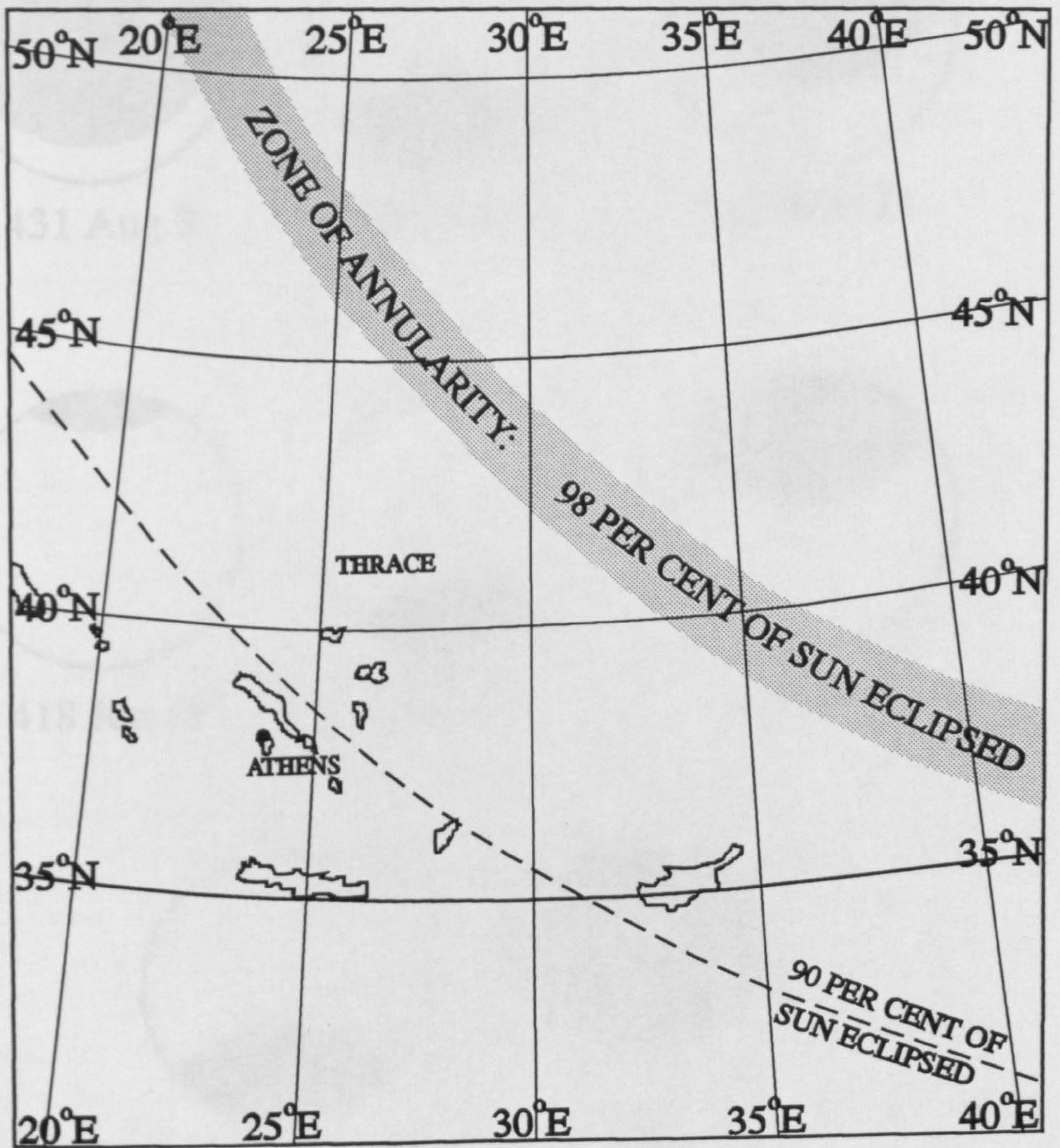
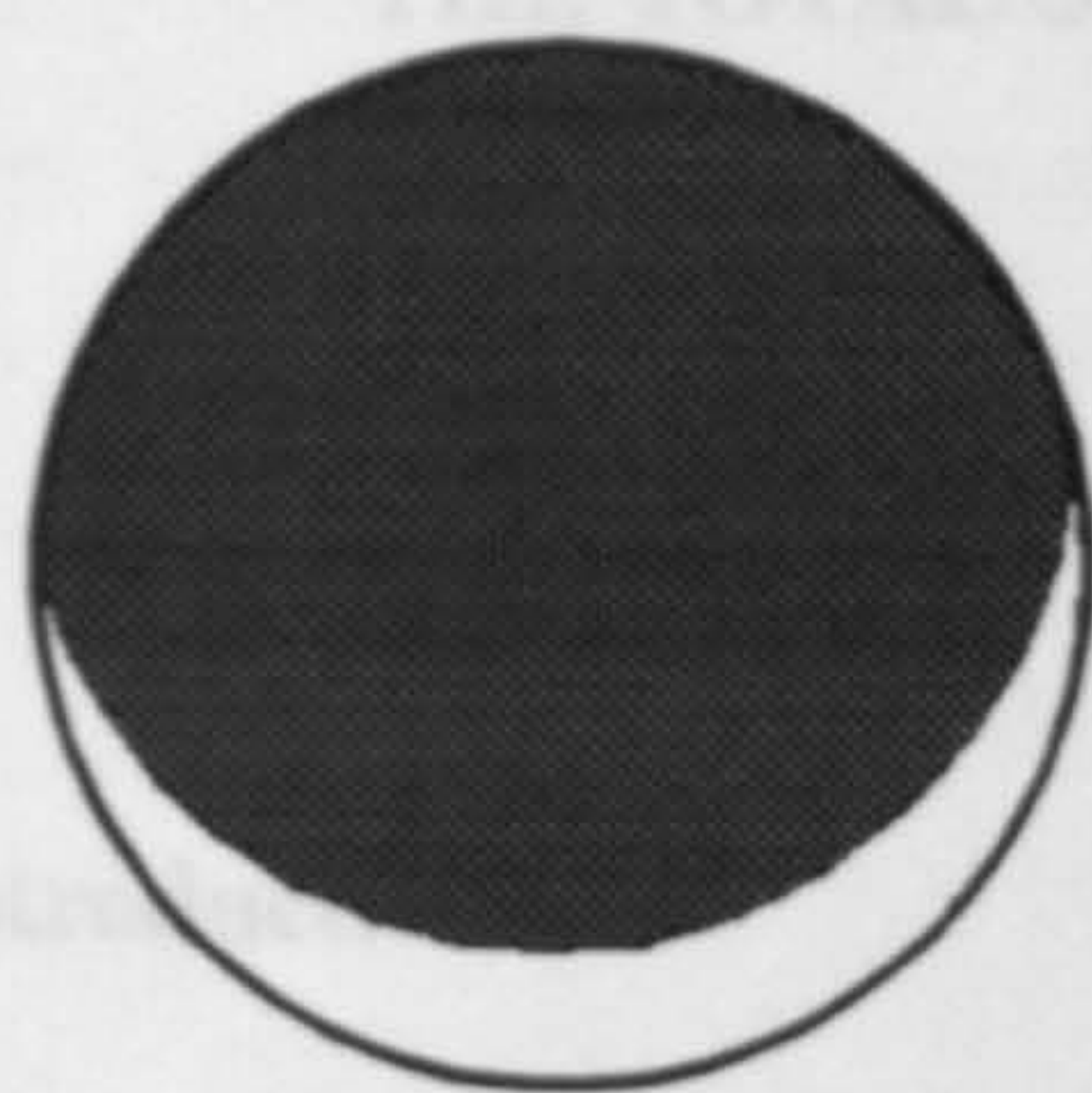
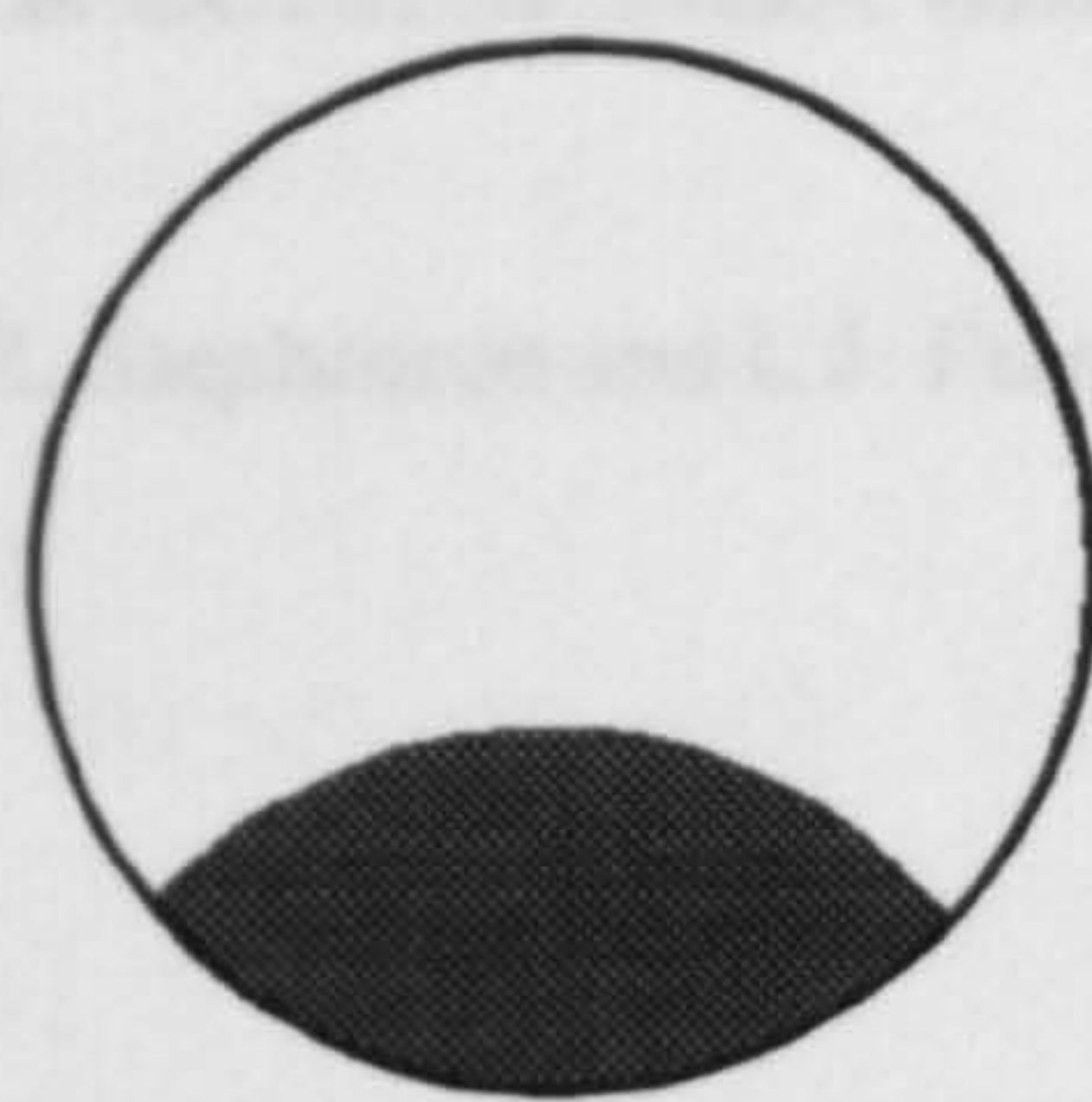


Fig 2. Map showing the track of the annular eclipse of B.C. 431 Aug 3.

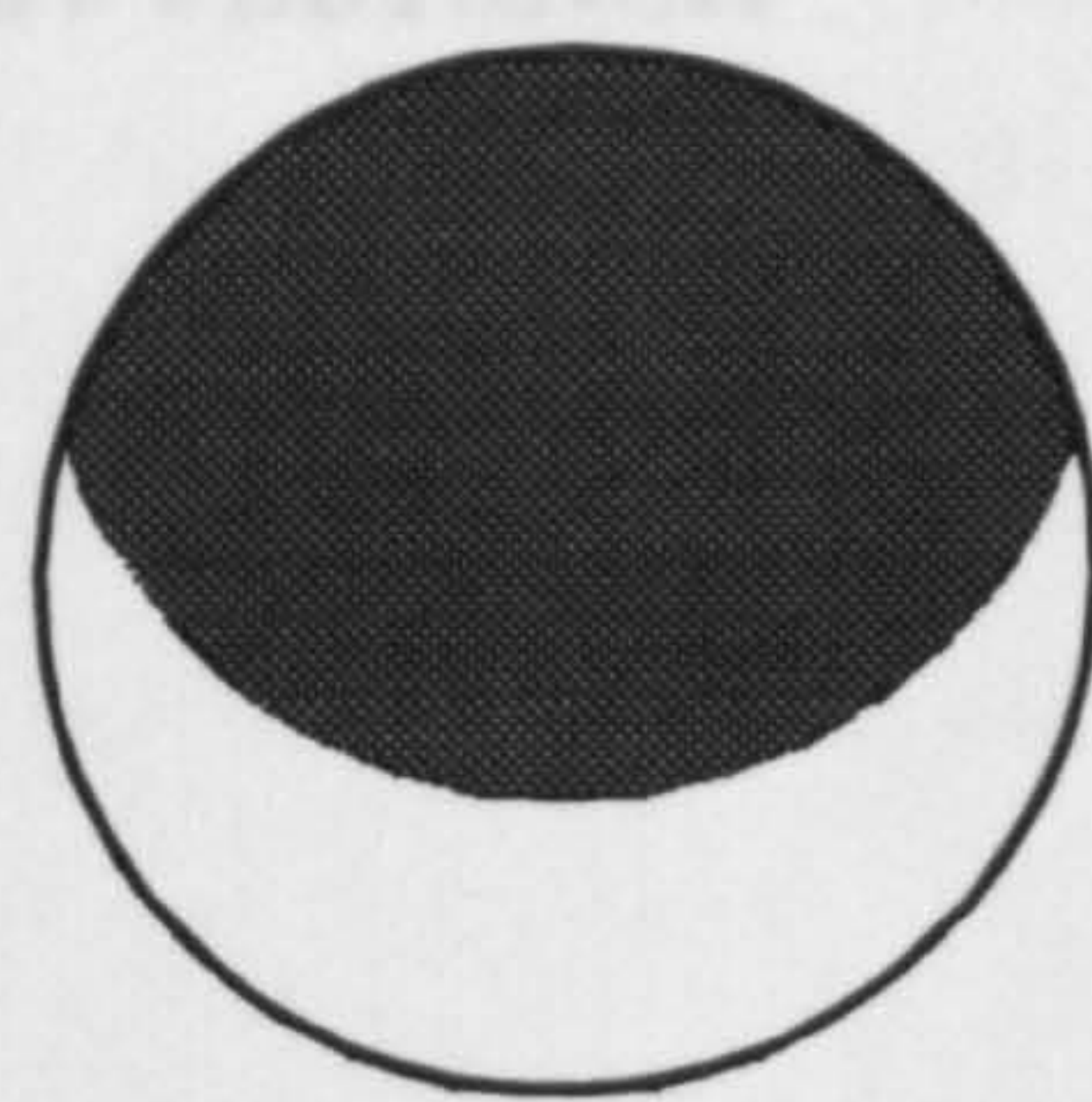




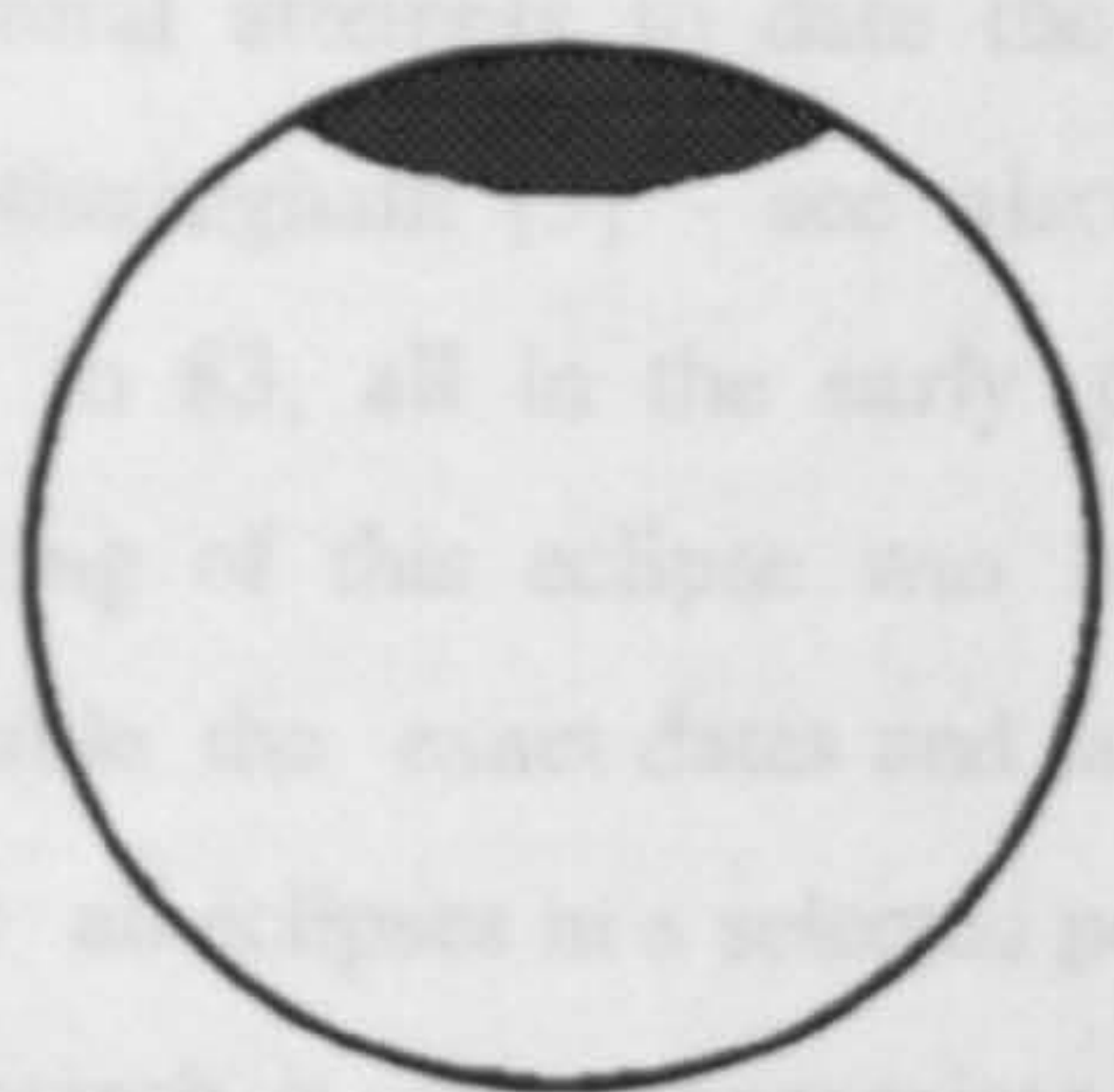
431 Aug 3



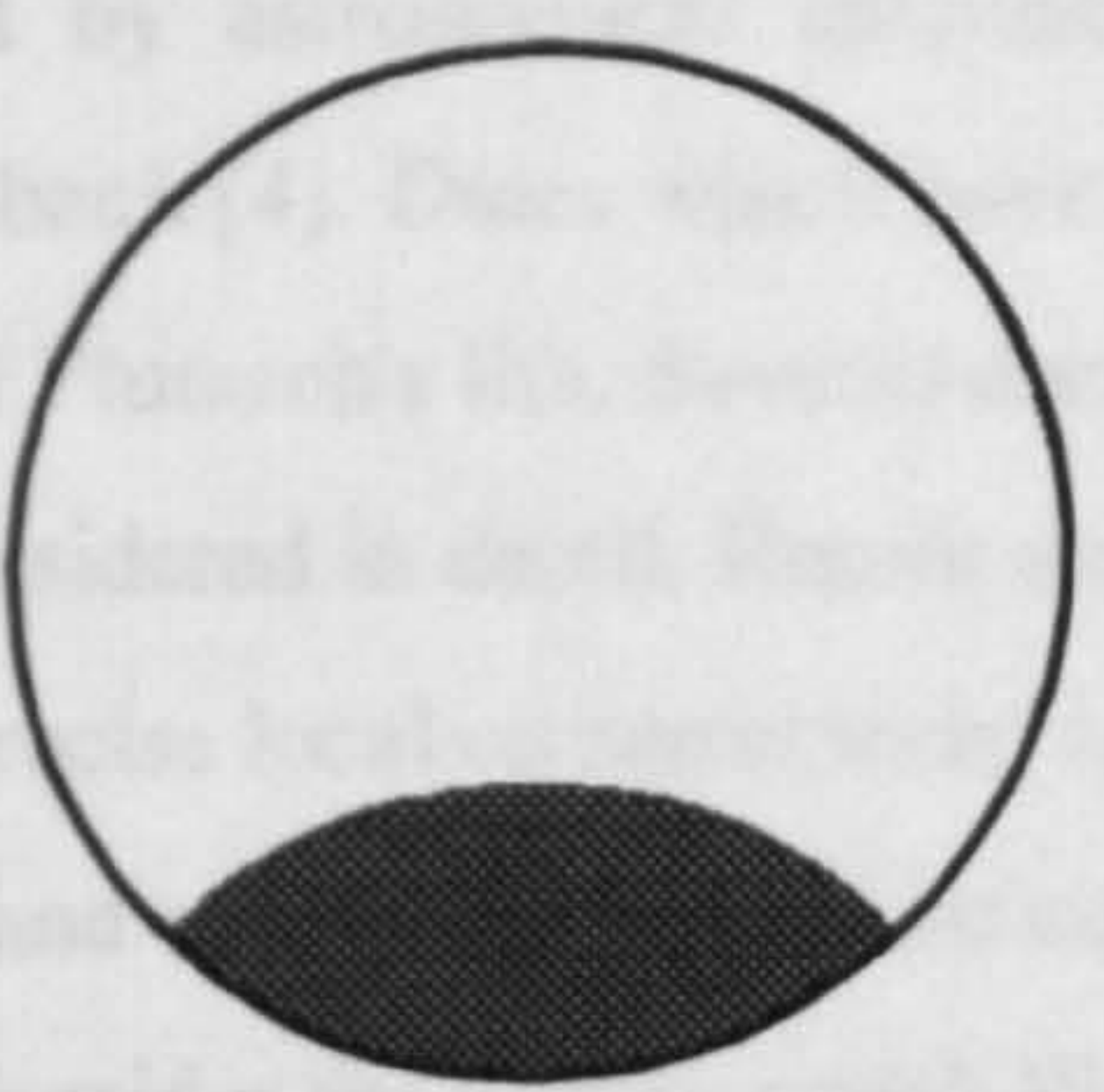
426 Nov 4



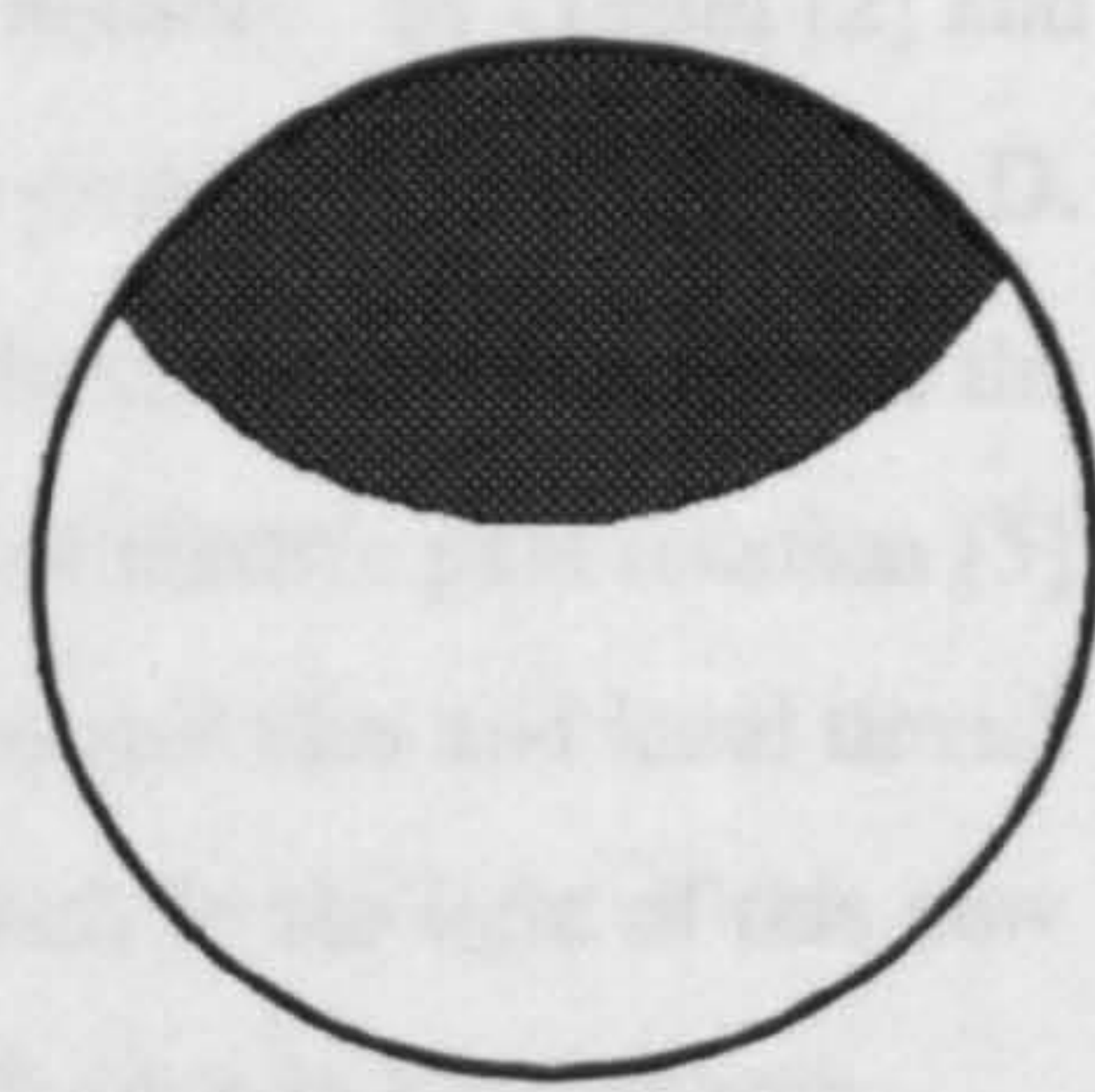
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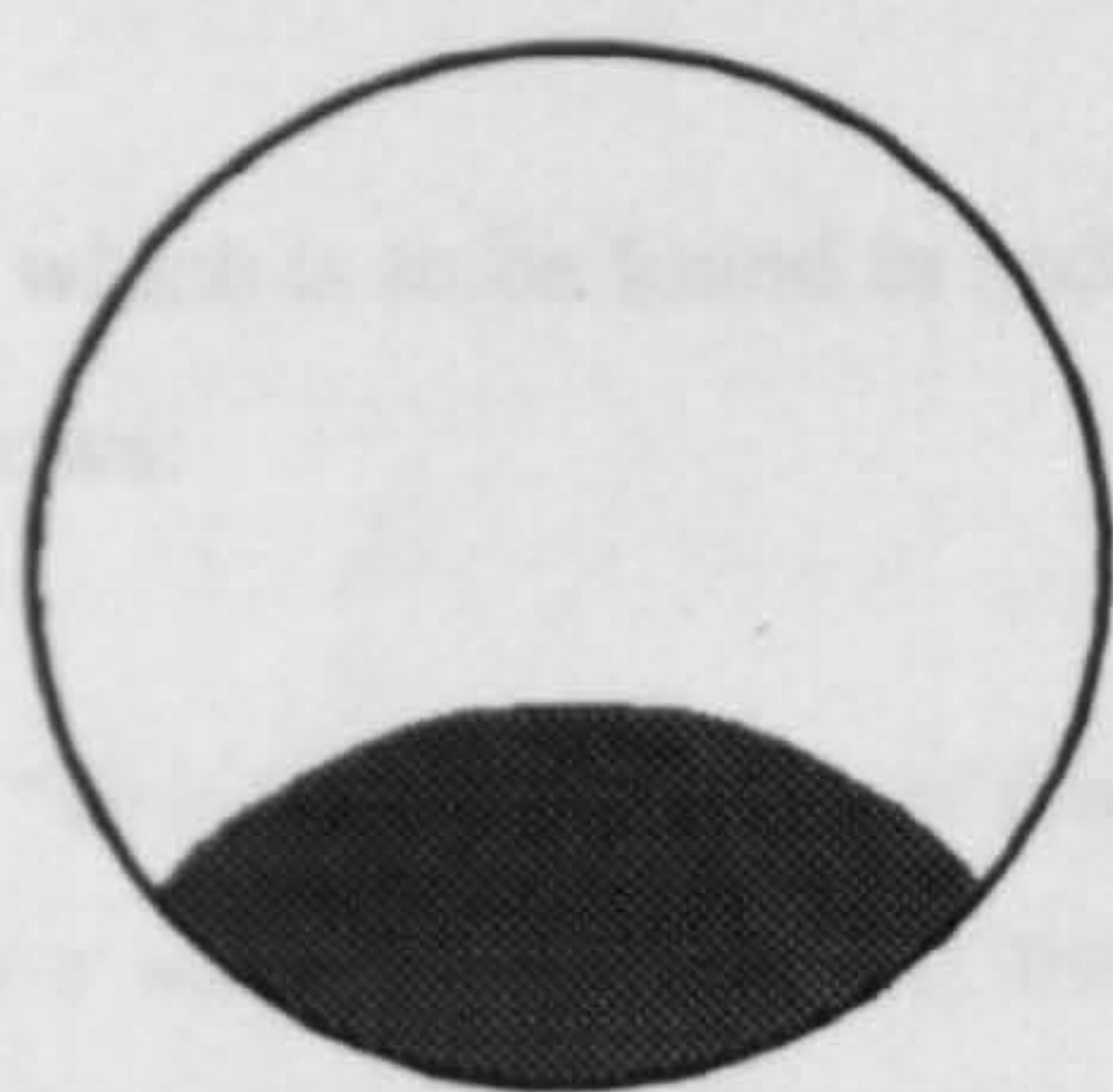
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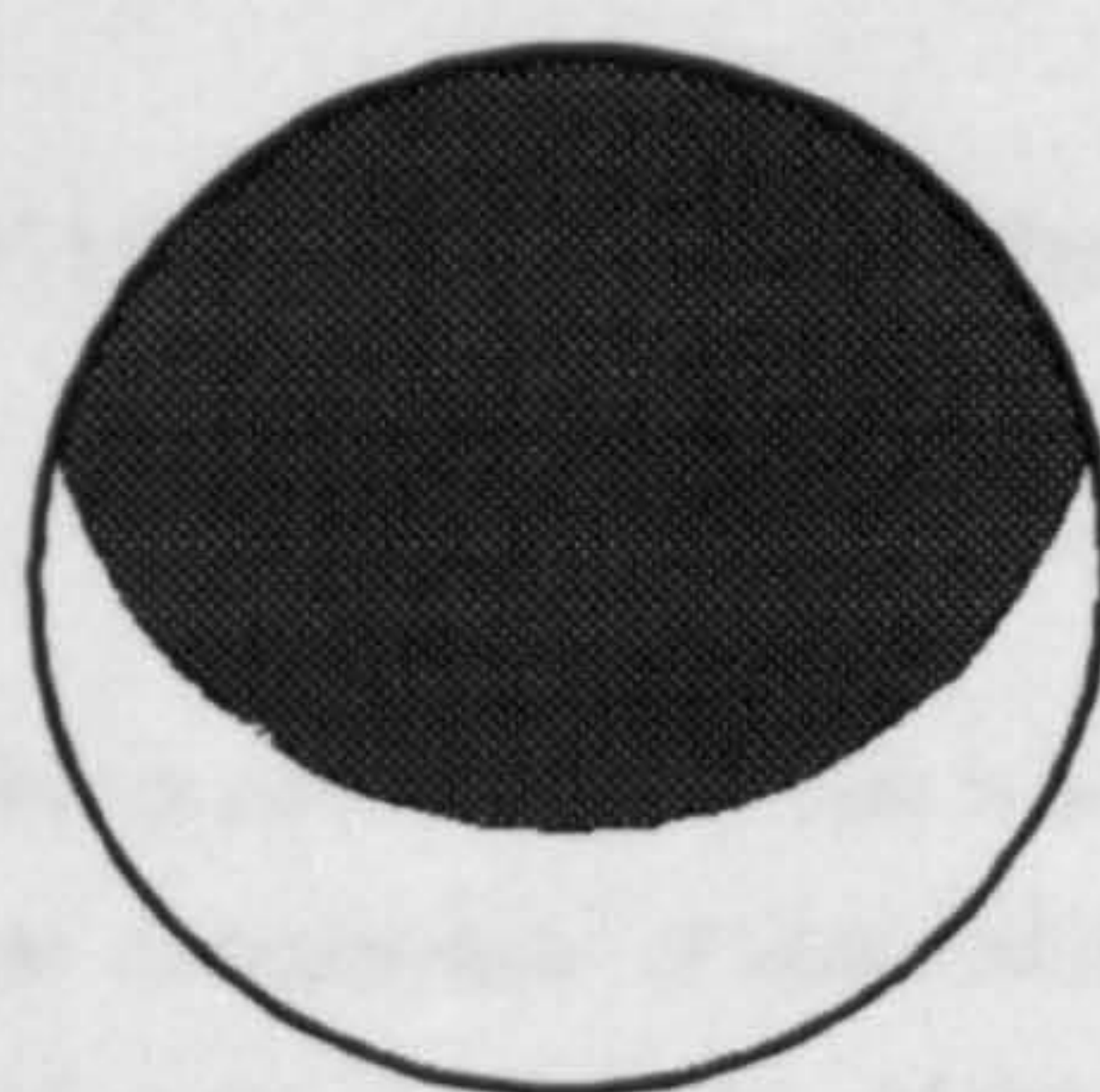
411 Jan 27



409 Jun 1



405 Mar 20  
(Sunset)



404 Sep 3

**Fig 3. Solar eclipses visible at Athens during the Peloponnesian War (431 - 404 B.C.).**



# THE TOTAL SOLAR ECLIPSE DESCRIBED BY PLUTARCH

F.R. Stephenson and L.J. Fatoohi

## Introduction

In his dialogue *De facie in orbe lunae* [1], Plutarch (ca. A.D. 46 - after 119) gives a vivid account of a major eclipse of the Sun. On the feasible assumption that Plutarch's description refers to a real observation, probably of an eclipse which was fully total, there have been several attempts to date the event by astronomical calculation: notably by Ginzel [2] and Fotheringham [3] - see also Sandbach [4]. Dates which have been proposed range from A.D. 71 to 83, all in the early part of Plutarch's life. Several decades have now elapsed since the dating of this eclipse was last considered in detail. Recent studies of Earth's past rotation [5] enable the exact dates and fairly precise local circumstances (e.g. magnitudes and local times) for all eclipses in a selected period and at a given place to be computed. In the light of this new research, it seems appropriate to reconsider the eclipse which Plutarch cites in his dialogue.

## Plutarch's description of the eclipse

This account, which is to be found in sections 931D-E of *De Facie*, has been translated by Prickard [6] as follows:

Lucius said... "Grant me that no-one of the phenomena relating to the Sun is so like another as an eclipse to a sunset, remembering that recent concurrence of Sun and Moon, which, beginning just after noon, showed us plainly many stars in all parts of the heavens, and produced a chill in the temperature like that of twilight. If you have forgotten it, Theon here will bring up Mimnermus and Cydias, and Archilochus, and Stesichorus and Pindar besides, all bewailing at eclipse time 'the brightest star stolen from the sky' and 'night with us at midday', speaking of the ray of the Sun as 'a track of darkness'....

This will be referred to as text (i) below. Although the complete disappearance of the Sun is not alluded to at the "recent concurrence of Sun and Moon", no other phase would render *many*



stars visible by day. Incidentally, the eclipse descriptions by Archilochus [7] and Pindar [8] are well known; the brief quotations which Plutarch gives here are clearly from these two authors. Regrettably, the relevant works by the poets Mimnermus, Cydias and Stesichorus - all of whom lived ca. 600 B.C. - no longer survive.

Later in his dialogue (932B), Plutarch also makes a brief reference to what is clearly the solar corona [9]:

... whereas if the Moon sometimes hides the Sun entirely, yet the eclipse does not last long and has no breadth; but a certain brightness is apparent round the rim, which does not allow the shadow to be deep and absolute.

It is plausible that this account, which will be cited as text (ii), may be associated with the previous description of the total eclipse itself. Plutarch is unique among Classical authors in mentioning the corona, the extended outer atmosphere of the Sun. This is only visible at an eclipse which is either total or virtually so. Cherniss and Helmbold [10] argued that if Plutarch was indeed referring to the corona in text (ii), his description is "remarkably tame". They suggested instead that the account is more likely to refer to an annular (i.e. ring) eclipse. However, surprising as it often seems to modern astronomers who have witnessed total obscurations of the Sun, the corona does not appear to have left much of an impression on observers in ancient and medieval times. Before A.D. 1600, only one other account of a total eclipse (A.D. 968) definitely mentions the corona, even though many detailed descriptions of great eclipses are preserved in medieval European and Arabic chronicles [11]. The record from A.D. 968, which originates from Constantinople, was written by the contemporary historian Leo Deaconus [12]. His account likens the corona to "a certain dull and feeble glow, like a narrow headband, shining around the extreme portion of the edge of the disk". Plutarch's description is, in fact, not too dissimilar from this much later record. By comparison, during a central annular eclipse the unobscured portion of the Sun is dazzling in brightness. Such an event could scarcely be described as preventing the shadow from becoming "deep and absolute" since often there is hardly any noticeable reduction in daylight, even during the ring phase.

## Nature of the eclipse

In Plutarch's dialogue, the eclipse was the basis of an intellectual - if somewhat entertaining - discussion. Most of the characters identified in the text are known to have been associates of Plutarch himself. Newton [13] was of the opinion that statement (i) was merely some product of Plutarch's imagination, but he seems to have been very much a lone voice. By contrast, Ginzel [14], Fotheringham [15] and Sandbach [16] - and more recently Cherniss and Helmbold [17] - all regarded the account as a reference to a real event. Muller [18], who had personally witnessed several total eclipses from various sites, thought that account had "the definite flavour of personal experience and eye-witness description". He added that "the probability that this is a real record is very high".

There are sound reasons for believing that the eclipse was indeed authentic. In addition to alluding to the corona, Plutarch is the only ancient writer to note a fall in temperature at an eclipse. During a major eclipse the temperature may drop by several degrees celsius; however, not until recent centuries do we find similar effects recorded. It seems that the main concern of most early observers was to describe the awe-inspiring darkness which accompanied the disappearance of the Sun. It is also worth pointing out that only two other Classical writers apart from Plutarch note the visibility of stars in the daytime during a large eclipse. These are Thucydides [19], who casually notes the appearance of "some stars" at a solar eclipse in 431 B.C. [20], and Phlegon of Tralles [21], who asserts that "stars actually appeared in the sky" at an eclipse which probably occurred in A.D. 29. In comparison with these reports, Plutarch's unrivalled description that the eclipse "showed us plainly many stars in all parts of the heavens" is graphic in its detail.

Although we have no way of knowing what other sources might have been at Plutarch's disposal, based on the ancient literature which is extant today he could not have obtained access to details such as the above from any other literary source. Plutarch's whole account in (i) above is so original that it seems quite likely that he himself was an eye-witness to the events which he so vividly describes. Accordingly, in the remainder of this paper we shall assume that the eclipse to which he refers was indeed a real event and occurred during Plutarch's own lifetime. However, both the place of observation and the date require careful discussion.



## Place of observation

The beginning of Plutarch's dialogue *De Facie* is lost and, with it, any indication of date or place. In principle, the eclipse could have occurred at any time in his adult lifetime. Plutarch was born at Chaeronea in Boeotia around A.D. 46 and died after A.D. 119. Although he was normally resident in Chaeronea throughout his life, he is known to have travelled throughout much of Greece. He also paid at least two official visits to Rome, where he lectured on philosophy. Plutarch had close links with the Athenian Academy, while from about A.D. 95 he held a priesthood for life at Delphi - not far from Chaeronea. Plutarch's many dialogues are usually set in various places in Greece, but sometimes in Rome - the places with which he himself was familiar. In view of the fact that the "recent concurrence of Sun and Moon" was so clearly remembered, it seems highly likely that totality was witnessed at one or other of these locations. There is a small possibility that the eclipse was seen instead at Alexandria, which Plutarch visited at some point in his career. The place of observation of the eclipse will thus be assumed to be either Greece, or Rome - or with less likelihood - Alexandria.

## Computational results

It may be computed that during the lifetime of Plutarch, only four eclipses could have been total in the central or eastern Mediterranean: A.D. 59 Apr 30, 71 Mar 20, 75 Jan 5 and 83 Dec 27. The first of these occurred when Plutarch was aged only about 13, and could have scarcely been described as "recent" when he wrote *De Facie*. Although we shall consider this further, it appears to be an unlikely choice.

Two annular eclipses were also visible in this same period (A.D. 67 May 31 and 80 Mar 10) but neither of these was very large. For the first of these events, no more than 90 per cent of the Sun's disk would be covered, even where the ring phase was visible, so that the loss of daylight would be scarcely noticeable. In the eclipse of A.D. 80, up to 96 per cent of the solar disk would be obscured by the Moon but the fall in daylight would not be very significant. On this occasion, among the bright planets and stars only Venus would be above the horizon; certainly "many stars" would not "shine out from many parts of the sky". (It should be emphasised that during a total eclipse, the sky brightness falls to little more than one-millionth of its normal level - a spectacular event indeed).

In all probability, a choice must be made between one of the four total solar obscurations listed above (A.D. 59, 71, 75 or 83). On the basis of the detailed investigation of Earth's past rotation by Stephenson and Morrison [22], we have listed in Table 1 the following details for each eclipse: computed magnitude (as a percentage of the Sun's diameter), local time (in hours and minutes) and solar altitude (in degrees) for each eclipse. We have made calculations for three selected locations: Athens (taken as representative of Greece), Rome and Alexandria. The corresponding tracks of totality in the central and eastern Mediterranean are shown in Fig 1. In interpreting these maps, it should be borne in mind that due to irregularities in the Earth's rate of rotation, it is not possible to compute the geographical positions of ancient eclipse tracks with high precision. We estimate that errors of up to about 2 degrees in longitude for a given latitude remain a possibility. As a result, the eclipse tracks shown in Fig 1 could plausibly be displaced in an easterly or westerly direction by this amount. We now proceed to discuss in chronological order the circumstances of each of the four eclipses of A.D. 59, 71, 75 and 83.

Referring to Table 1, the eclipse of A.D. 59, which occurred when Plutarch was a boy, can only have been partial in all three cities of Athens, Rome and Alexandria. As depicted in Fig 1, the track of totality lay far to the south of Rome and the Italian peninsula, and much to the north of Alexandria. Since this track ran almost parallel to the equator, allowances for uncertainties caused by variations in the Earth's rotation rate would have negligible effect on visibility. In particular, the eclipse can never have been total north of latitude 36.0 deg. The track thus passed significantly to the south of the Peloponnesus, and at Sparta (the scene of several dialogues by Plutarch) - which on this occasion was much better placed than Athens - the magnitude cannot have exceeded 96 per cent. This is far from sufficient to produce the effects described by Plutarch. It is therefore clear that the eclipse of A.D. 59 cannot be that described by Plutarch.

The eclipse of A.D. 71 was small in both Alexandria (80 per cent) and Rome (78 per cent). No stars would be visible at either location. In particular, the track of totality passed far to the south of the Italian peninsula. However, at Athens the computed magnitude was as large as 99.5 per cent; only a little to the south of this city the eclipse would be total. The only difficulty with the eclipse of A.D. 71 as seen from Athens or neighbouring cities in Greece, is that greatest phase would occur around 10;50 h, rather than "just after noonday". However, at 10;50 h, the Sun would then be almost at its maximum height - altitude 48 deg, or only 3 deg less than the meridian altitude. There is no suggestion in the record that time was carefully measured; to the casual bystander this eclipse would be regarded as occurring close to midday.



Although the duration of totality would be very short - not exceeding 15 seconds - this would be sufficient to render several stars visible, with Venus and Sirius prominent to the east of the Sun.

**Table 1** The four solar eclipses during the lifetime of Plutarch that were visible in the central or eastern Mediterranean

	Date AD	Magnitude (%)	Local Time (hr;min)	Solar Altitude (degree)
Athens	59 Apr 30	94	15;10	42
Alexandria		88	15;50	35
Rome		81	14;05	51
Athens	71 Mar 20	99.5	10;50	48
Alexandria		80	11;15	56
Rome		78	10;00	39
Athens	75 Jan 5	87	16;05	7
Alexandria		61	16;35	5
Rome		91	15;10	12
Athens	83 Dec 27	80	14;05	22
Alexandria		98	14;45	22
Rome		70	12;55	23

The circumstances in A.D. 75 are unfavourable at all three selected locations. At Alexandria, only 61 per cent of the solar disc would be covered. The magnitude was larger at both Athens (87 per cent) and Rome (91 per cent), but far from total at these locations. The zone of totality fell far (about 350 km) to the south-east of Rome. Further, in those areas of the Mediterranean where the eclipse was indeed total, greatest phase would occur only about an hour before sunset. The event could thus scarcely be described as “beginning just after

noonday”, while it would seem inappropriate to compare a sunset eclipse with a sunset. We therefore feel that this date can also be eliminated.

Finally, the eclipse of A.D. 83 would not have been total in Italy or in Greece. At Rome, the computed magnitude was only 70 per cent, while at Athens the magnitude was only slightly larger (80 per cent). On the other hand, at Alexandria computations show that the eclipse was very large indeed - nearly 98 per cent at about 14;45 h - and just possibly might have been total there. Although this identification cannot be ruled out on astronomical grounds, Plutarch only once visited Alexandria and in general seldom appeals to his Alexandrian experiences in his numerous dialogues.

## Conclusion

In summary, there would appear to be only two contenders for the “eclipse of Plutarch”: A.D. 71 and 83. If evidence can be found for a visit to Alexandria by Plutarch in A.D. 83, then the eclipse of Dec 27 in that year deserves consideration. However, we regard the eclipse of A.D. 71 Mar 20, which was total in Greece - the centre of Plutarch's cultural life - as by far the more likely candidate.

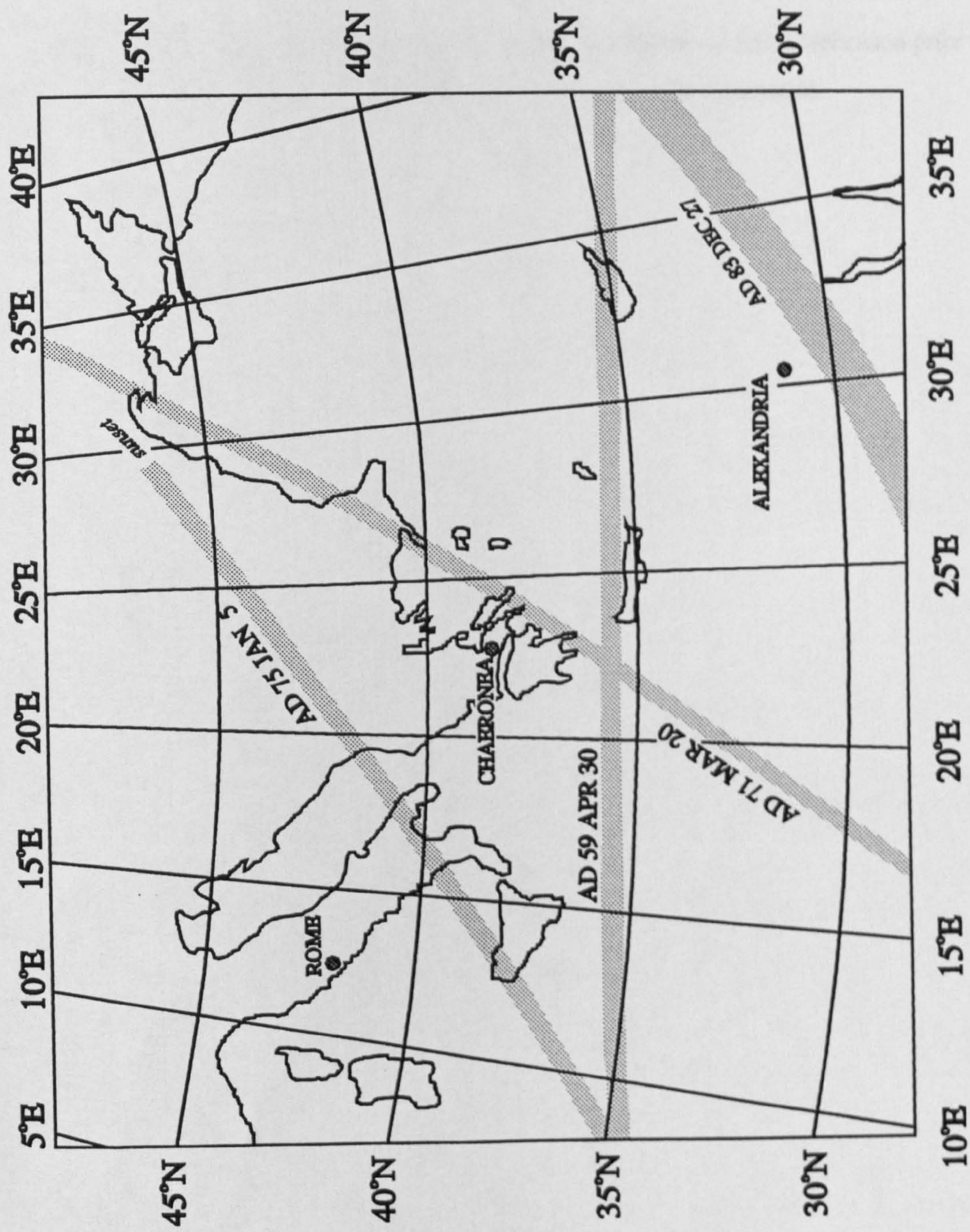
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**FIG 1. Total Solar Eclipses in the Central and Eastern Mediterranean Region During the Lifetime of Plutarch.**



## Introduction

These are three papers that I have published jointly with Professor F.R. Stephenson prior to my registration for the PhD degree, and which are referenced in the dissertation.

## LUNAR ECLIPSE TIMES RECORDED IN BABYLONIAN HISTORY

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### 1. *Introduction*

Numerous observations of lunar eclipses are recorded on the Late Babylonian Texts that were recovered from the site of Babylon rather more than a century ago and are now very largely in the British Museum. Nearly all these date from between 700 and 50 B.C. Although most tablets are very fragmentary, many texts record (among other details) either or both of the following measurements for lunar eclipses: (i) the durations of the various phases of both total and partial eclipses; and (ii) the times of onset relative to sunrise or sunset. Relatively few solar eclipse observations from Babylon are preserved and these will not be considered.

Comparison of both sets of measurements (i) and (ii) with the results of modern computation provides useful information on the accuracy with which the astronomers of Babylon were able to measure intervals of time. Investigation of the durations of the various eclipse phases has the advantage over the intervals relative to sunrise and sunset that the calculated results are independent of changes in the Earth's rate of rotation. However, the sunrise and sunset intervals are typically much longer and it is possible to make use of these measurements by making due allowance for long-term variations in the length of the day.

In this paper we make a detailed investigation of the available observations in both categories, restricting our attention to those records for which a reliable date can be established.

### 2. *Eclipse Observations on the Late Babylonian Texts*

The astronomical texts from which our observations are derived came to light at the site of Babylon during the 1870s and 1880s. For the most part they were discovered accidentally and hence there is little archaeological context.<sup>1</sup> Only a few tens of these cuneiform tablets were ever excavated officially (by Hormuzd Rassam on behalf of the Trustees of the British Museum<sup>2</sup>). The rest, numbering about 2,000 texts, seem to have been dug up by inhabitants of the nearby communities such as Hillah. These were eventually sold to antique dealers in Baghdad. Soon afterwards, the British Museum acquired virtually all of the known tablets since at the time no other academic institution was concerned to purchase such material. Even the British Museum collection is very incomplete, but it seems that no other astronomical texts have been unearthed from the ruins of Babylon since the end of last century.



In 1948, the late Abraham Sachs attempted the first detailed classification of the Late Babylonian astronomical texts, following pioneering translation work on selected texts by Franz Kugler, s.J. and others.<sup>3</sup> Subsequently, Sachs and Schaumberger published drawings of many of the tablets in the British Museum; these had been executed at the end of last century by T. G. Pinches and J. N. Strassmaier, s.J.<sup>4</sup> Sachs had also listed the recorded or computed dates for about half of these tablets. Sachs and Schaumberger now classified the texts into nine groups: (i) those devoted to mathematical astronomy; (ii) astronomical diaries; (iii) normal-star almanacs; (iv) almanacs; (v) "goal-year texts"; (vi) tables of planetary and lunar observations; (vii) miscellaneous astronomical texts; (viii) astrological texts; and (ix) tablets devoted to aspects of mathematics.

Eclipse observations are recorded on texts in three of the above categories: diaries, goal-year texts and eclipse tables. Although both solar and lunar eclipses are noted on the various tablets, the latter are much more numerous and cover a considerably longer time-span. This is possibly the result of chance; it is known that only a small proportion of the original archive has come to light.

Among the three relevant categories of astronomical texts, the diaries contain the most original material. These tablets record day-to-day observations of celestial phenomena and typically covered six or seven months. During the Hellenistic period (late fourth century B.C. onwards), the Babylonian astronomers abstracted material from earlier diaries to use in the preparation of yearly almanacs. Before producing the almanac for a selected year, the observations that would assist in making the necessary predictions were assembled in the so-called goal-year texts, the planned year being the goal-year. Such texts might cite eclipses from 18 years previously, Venus data from 8 years beforehand, and so on.

The main motive behind these activities was astrological. Eclipse tables (also produced from the diaries during the Hellenistic period) contain lists of eclipses, usually at 18-year intervals. Sometimes these extend back several centuries. It seems clear from the content of these subsidiary texts that in general the Late Babylonian astronomers had access to series of diaries covering vast periods of time.

Although less than about 10% of the content of the original diaries has survived (largely in the period from about 380 to 70 B.C.), many additional observations, particularly of lunar eclipses, are to be found in surviving goal-year texts and eclipse tables. Reports of lunar and solar eclipses appear with comparable frequency in the extant diaries and goal-year texts. However, whereas many lengthy lunar eclipse tables are still preserved, their solar counterparts are virtually non-existent.

Hermann Hunger, following the extensive work of Sachs — which remained largely unpublished at his death in 1983 — has recently published<sup>6</sup> photographs, transliterations and translations of all the datable diaries down to 164 B.C., and he is currently continuing this task beyond that date. However, the only comparable work for the goal-year texts and eclipse tables



is the unpublished manuscript of Peter Huber,<sup>7</sup> which he has freely circulated. This compilation also abstracts from copies of diaries that were available to Huber. Huber assembled all the solar and lunar eclipse material he could find, largely using the drawings of tablets published by Sachs and Schaumberger.<sup>8</sup> However, his compilation — although extensive — is by no means complete.

In the present investigation we have analysed all the datable lunar eclipse observations in the works of Sachs-Hunger and Huber. However, Hermann Hunger has kindly supplied us with some additional unpublished material from 163 B.C. onwards which was unknown to Huber. These observations will be published by Hunger in future volumes of the Sachs-Hunger series.

### *3. Remarks on the Babylonian Calendar*

Until the Seleucid Era (311 B.C.), Babylonian dates were counted from the accession of each monarch but continuous counting of years was adopted thereafter. For the period covered by the texts (from about 700 B.C. onwards), the dates of accession for each ruler are accurately known. Although intercalation was rather irregular before about 400 B.C., the dates of many intercalary months (always following the 6th or 12th month) are still preserved in Babylonian records.<sup>9</sup> Hence even at this early period, conversion of dates to the Julian Calendar presents few difficulties. After about 400 B.C. a systematic scheme based on the Metonic cycle was used for intercalation; the details of this are well known.

The Babylonian year commenced around the time of the vernal equinox. When the sky was clear, each month began with the first sighting of the crescent moon in the evening sky — as for the beginning of the months of Ramadan and an-Naṣr in the Islamic world today. If cloud prevailed, the Babylonian astronomers used instead a fairly accurate rule to predict whether the crescent should have been visible or not. Months consisted of either 29 or 30 days, each day beginning at sunset. Tables for the rapid and accurate conversion of dates from the Babylonian lunar calendar to the Julian calendar in the entire period from 626 B.C. to A.D. 75 — taking into account visibility of the lunar crescent — have been constructed by Parker and Dubberstein.<sup>10</sup>

Dating problems often occur with damaged texts. In the case of a diary — which usually covers six or seven months — if the date is broken off it is frequently possible to restore it by comparing the recorded planetary and lunar observations with calculation.<sup>11</sup> However, numerous small fragments of diaries currently remain undated. In the case of goal-year texts, dates of individual phenomena are often scattered over the tablet so that usually dating presents few problems. Lunar eclipse tables normally list eclipses at 18-year intervals, the various eclipses in each particular year being cited. This periodicity has enabled Sachs and Schaumberger and also Huber to restore accurate sequences of dates, especially when certain historical events — for example the deaths of rulers — are noted in sequence. We have used astronomical



computation to check that the Julian dates of lunar eclipses listed by Sachs-Hunger and Huber are valid and we have noted very few errors indeed.

#### 4. *Character of the Lunar Eclipse Observations*

Originally each lunar eclipse observation was recorded with the date expressed relative to the lunar calendar, followed by a summary (sometimes quite detailed) of the observation itself. Examples are as follows, LBA numbers being those cited in the publication of Sachs and Schaumberger<sup>12</sup> and BM numbers their British Museum references:

##### (i) B.C. 424 Sep 28–29 (partial)

“[Year 41 (Artaxerxes I)], month VI, day 14. 50 deg after sunset, beginning on the north-east side. After 22 deg, 2 fingers lacked to totality. 5 deg duration of maximal phase. In 23 deg toward [west it became bright] 50 deg total duration...” [LBA 1422 (= BM 34787); transl. Huber, pp. 32–33].

##### (ii) B.C. 316 Dec 13–14 (total)

“[Philip, year 7], month IX, 15, beginning in the south-east side. After 19(?) deg total. 5 deg duration of maximal phase. In 16 deg on the north-east side it became bright. 40 deg total duration.... It was eclipsed  $1\frac{1}{2}$  cubits (roughly 3 deg) in front of  $\beta$  Gem. (Began) at 44 deg after sunset. Month IX, year(?) 7 Philip(?), (the following year is) year 2 Antigonos (I), son of ...” [LBA 1414 (= BM 32238); transl. Huber, pp. 51–52; note that although the ‘19’ in the text is indistinct, it can be restored from the remaining time-intervals, the sum of which is 40 deg].

##### (iii) B.C. 215 Dec 25 (total)

“Year 97 (SE), month IX, night of the 13th(?).... When  $\alpha$  Per culminated, lunar eclipse, beginning on the east side. In 21 deg of night, all of it became covered; 16 deg of night (duration of) totality; when it began to become clear, it cleared in 19 deg of night from north-east to west(?). 56 deg onset, totality [and clearing]. (Began) at one-half *beru* (i.e. 15 deg) after sunset...” [LBA 294 (= BM 36402) + BM 36865; transl. Hunger, ii, 156–7].

The first two texts are lunar eclipse tables listing these events at 18-year intervals. Their dates were derived from the 18-year sequences.<sup>13</sup> The third tablet is an astronomical diary. Fortunately its date (in terms of the Seleucid Era) is preserved.

In Late Babylonian astronomical practice, the standard units of time were *beru* and *uš*, where 1 *beru* was equal to 30 *uš*. Unlike the time-units for civil purposes, such as the three equal night-watches, the *beru* and *uš* showed no seasonal fluctuation; they were respectively equivalent to two hours and to four minutes in modern measure. Since 1 *uš* was the interval for the celestial sphere to turn through 1 deg, it is customary to translate *uš* as “degree”. Presumably these various measurements were made with some sort of clepsydra, although little information survives.<sup>14</sup> There is no evidence that alti-



tude measurements were ever used; no altitude determinations are preserved on any of the extant Late Babylonian astronomical texts.

As in the examples given above, the durations of each of the three phases of a *total* eclipse — from first contact to second contact (immersion), totality itself, and third contact (emersion) to last contact — were usually reported. Often the complete duration from start to finish was also stated; this provides useful confirmation of the veracity of the individual sub-durations. If one of the phases was interrupted by moonrise or moonset, this is usually recorded.

In the case of a *partial* eclipse, three separate times were usually noted: (i) the interval from first contact to the moment when the eclipse appeared to reach its height; (ii) the interval around maximal phase when no change in the degree of obscuration of the moon could be detected; and (iii) the interval from the end of maximal phase to last contact. Again, the interval from beginning to end was frequently stated. Greatest phase typically was estimated to last from 5 deg to 10 deg. In the former interval, the degree of obscuration of the lunar diameter increases and decreases by no more than 5% (roughly half a digit). As far as we are aware, this notion of a discrete maximal phase — rather than a momentary maximum — is without parallel in other early civilizations. It suggests a particularly careful watch by the Babylonian astronomers.

In practice, many texts are broken so that often no more than one or two of the possible intervals are preserved. Nevertheless, several complete observations also survive.

Prior to about 600 B.C., timing of eclipses was in general fairly crude in Babylon, most intervals being estimated only to the nearest 10 deg. However, by early in the sixth century B.C., time intervals were consistently expressed to the nearest degree. As a result we have only considered observations made from this slightly later period down to the very latest observations in about 40 B.C.

Durations of the various phases of both total and partial eclipses will be investigated in Section 5. As in the above examples, the majority of lunar eclipse observations specify the time of onset relative to sunrise or sunset (whichever was nearer) and also the durations of the various phases. Hence the time of each individual phase relative to sunrise or sunset may be obtained. These latter observations will be considered in Section 6. A few late texts (from the middle of the third century B.C. onwards) also give the time of onset relative to the meridian transit of a selected *ziqupu* or culminating star. However, we have preferred to concentrate here on the much more numerous sunrise and sunset data.

### 5. Analysis of Eclipse Durations

Unlike solar eclipses, the duration of each phase of a lunar obscuration is independent of the observer's location. Hence computation need take no account of the errors in local time arising from irregularities in the Earth's



TABLE 1. Investigation of durations for total eclipses.

Julian Date			Observed Intervals				Computed Intervals			
			1	2	3	T	1	2	3	T
−561	3	2	..	25	18	..	15.8	26.1	15.8	57.7
−554	10	6	17	28	20	65	16.5	24.6	16.5	57.6
−500	11	7	15	25	25	65	16.7	23.6	16.7	57.0
−406	10	21	21	12?	15	48	16.6	21.8	16.6	55.0
−405	4	15	25?	19	..	..	16.4	18.3	16.4	51.1
−377	4	6	15	21	19	55	15.8	19.2	15.8	50.8
−370	11	11	22	20	21	63	17.0	21.2	17.0	55.2
−352	11	22	23	18	..	..	17.0	21.2	17.0	55.2
−316	12	13	19?	5	16	40	17.1	21.0	17.0	55.0
−272	2	16	..	19	22	..	15.6	19.4	15.6	50.6
−225	8	1	17	10	15	42	16.8	16.1	16.8	9.7
−214	12	25	21	16	19	56	15.8	20.7	15.8	52.3
−188	2	17	16	..	..	..	18.3	19.8	18.3	56.4
−149	7	3	20	12	..	..	20.3	13.1	20.3	53.7
−123	8	13	19?	24	19	62	16.9	24.3	16.9	58.1
−119	6	1	24	6	24	54	21.3	5.0	21.3	47.6

The columns of this table list the following quantities:  
1–3. Year, month and day (Julian).  
4. Measured interval of first partial phase (between first and second contact).  
5. Measured duration of totality.  
6. Measured interval of last partial phase (between third and fourth contact).  
7. The measured total duration of the eclipse.  
8–11. Computed intervals for comparison with the data in columns 4–7.

rate of rotation. It is thus possible to compare these measured intervals directly with their computed equivalents.

In computing the various eclipse parameters we have used Newcomb’s solar theory<sup>15</sup> and the lunar theory of the Improved Lunar Ephemeris<sup>16</sup> — with further amendments to correspond to a lunar orbital acceleration of −26 arcsec/cy/cy;<sup>17</sup> this latter result is close to that obtained from current lunar laser-ranging measurements. In computing the intervals between the various lunar eclipse contacts, we have applied the customary increment of 2% to the Earth’s shadow radius to allow for the terrestrial atmosphere.

In Table 1, we have compared the available measured durations of all phases (together with the time from start to finish) of recorded total eclipses, with the values obtained by computation. We have rejected all examples where a reading is questionable on account of textual damage. Interestingly, the recorded time-intervals between the initial phase (first contact to immersion) and last phase (emersion to fourth contact) are rarely found to be the same. This may be due partly to faulty timing apparatus, but the difficulty of distinguishing the true contacts — especially immersion (second contact) and emersion (third contact) — is probably also responsible. (Whatever the explanation, the astronomers seem to have recorded their results with the minimum of bias.)

In Table 2 we have compared the measured times of the various phases of recorded partial eclipses with computation. The intervals considered are as

TABLE 2. Investigation of durations for partial eclipses.

Julian Date			Observed Intervals			Computed Intervals		
			1	2	T	1	2	T
-423	9	26	24.5	25.5	50	25.6	25.6	51.2
-409	12	21	...	...	60	23.9	23.9	47.7
-407	10	31	...	...	27	11.6	11.6	23.1
-396	4	5	...	...	27	8.2	8.2	16.4
-345	1	18	...	...	23	12.0	12.0	24.0
-238	4	28	20	20	40	17.7	17.7	35.4
-184	11	24	...	...	44	5.3	5.3	10.7
-162	3	30	...	...	20	10.2	10.2	20.4
-153	3	21	23	21	44	24.8	24.8	49.5
-142	2	17	22.5	...	...	23.9	23.9	47.9
-128	11	5	21	19	40	21.5	21.5	42.9
- 79	4	11	23.5	...	...	20.8	20.8	41.6
- 66	1	19	...	19	...	24.2	24.2	48.3

The columns of this table list the following quantities:

1-3. Year, month and day (Julian).

4. Measured interval between first contact and mid-eclipse.

5. Measured interval between mid-eclipse and last contact.

6. The measured total duration of the eclipse.

7-9. Computed intervals for comparison with the data in columns 4-6.

follows: (i) first contact to middle of maximal phase; (ii) middle of maximal phase to last contact; and (iii) duration from start to finish. Where the recorded duration of maximal phase was given, we have halved this interval and added it to the recorded duration of first or last phase.

In Figure 1 we have plotted for all of the durations of the various lunar eclipse phases the discrepancy (with the sign ignored) between measurement and computation. Identifying the three successive phases of a total eclipse as  $a$ ,  $b$  and  $c$  and the two successive phases of a partial eclipse as  $d$  and  $e$ , we have represented in the diagram errors in both these individual quantities and the various sums  $(a+b)$ ,  $(b+c)$ ,  $(a+b+c)$  and  $(d+e)$  as a function of the computed interval.

Clearly the scatter in Figure 1 is very large. The mean discrepancy between measured and computed time-intervals is as much as 7 deg or about half-an-hour. Comparison may be made with a recent investigation by Stephenson and Said<sup>18</sup> of a series of eclipse timings by medieval Arab astronomers. Most of these latter measurements were made indirectly using altitude determinations. Stephenson and Said found that the typical error in timing an eclipse contact was no more than about 5 minutes, roughly equivalent to the basic Babylonian time-unit (the  $u\bar{s}$ ). This suggests that most of the Babylonian errors arose from measurement of time-intervals rather than merely poor contact definition with the unaided eye.

In Figure 1, on the assumption that the longer the time-interval measured with a primitive clock the greater the likely error, we have fitted the best straight line to the data which passes through the origin. Although this line is a poor fit, the gradient is appreciable — about 13%. This may give an



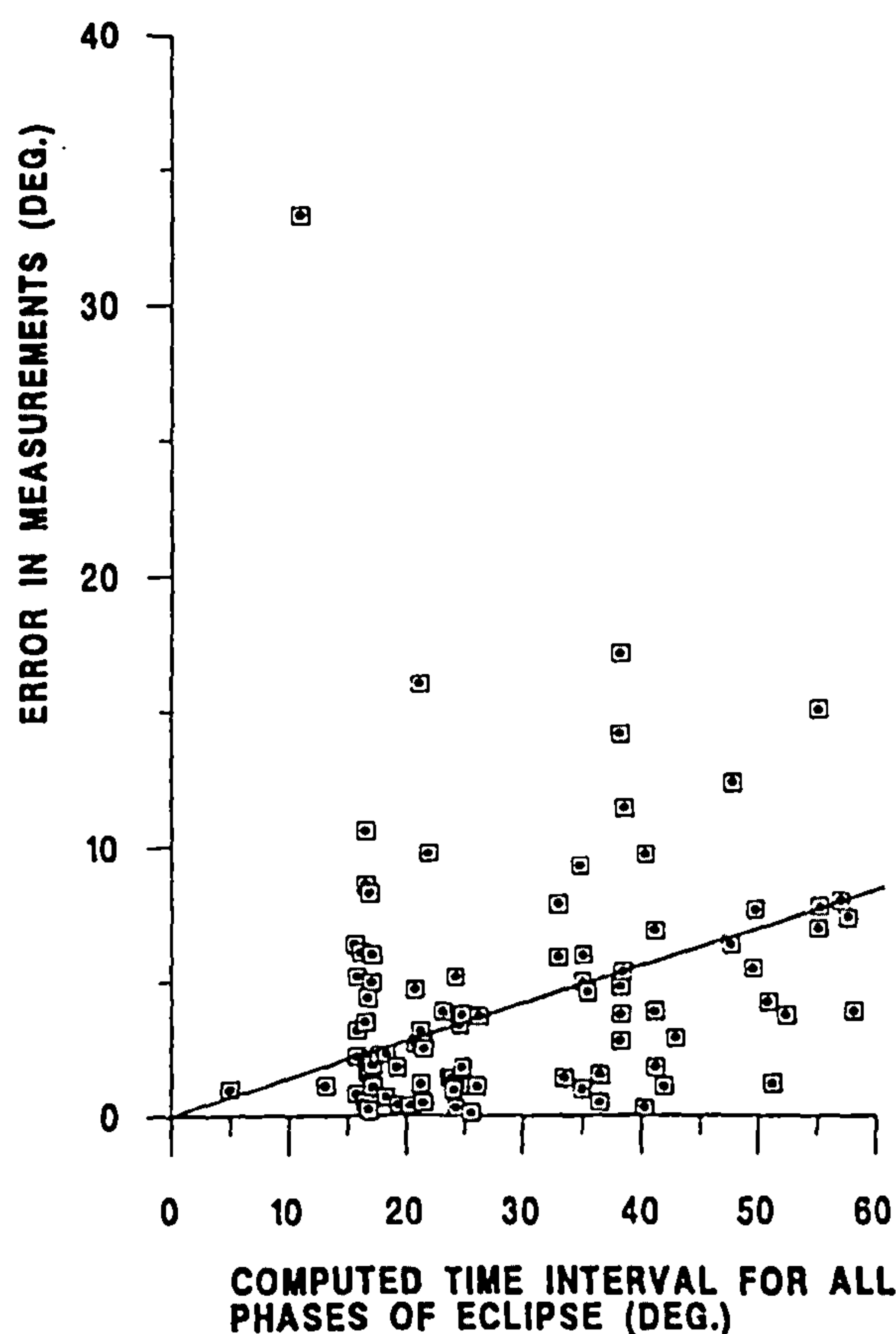


FIG. 1. Discrepancies between measured and computed durations for Late Babylonian observations of total and partial lunar eclipses (all contacts).

indication of the likely drift rate over several hours for a typical timing device as used by the Babylonian astronomers. In Section 6, this result will be compared with that obtained from the intervals measured relative to sunrise or sunset.

#### 6. *Analysis of Time-intervals Relative to Sunrise or Sunset*

As noted earlier, in comparing the measured local time of an eclipse contact with the computed time, allowance must be made for variations in the Earth's rate of rotation. Modern computations yield the Terrestrial Time (TT) — defined by the motion of the Moon — of an eclipse contact; this was formerly known as Ephemeris Time (ET). However, observations yield the local time of this contact which can be readily reduced to Universal Time (UT), as measured relative to the rotating Earth. The difference between TT and UT (known as  $\Delta T$ ) is a measure of the accumulated clock error caused by changes in the length of the day — due to tides and other mechanisms. Although during the Late Babylonian period, the length of the mean solar day was only about 0.05 sec shorter than at present, roughly one million days have elapsed since then. Hence the accumulated clock error  $\Delta T$  is large, amounting to several hours. If tides were the only significant mechanism,  $\Delta T$

TABLE 3. Comparison of measured and computed time interval for first contact relative to sunrise or sunset.

Julian Date			Time	Measured	Computed	Difference
-694	5	1	B.R.	30	42.15	-12.15
-685	4	22	A.S.	100	112.2	-12.20
-684	10	3	A.S.	20	17.85	+02.15
-665	4	10	A.S.	3	10.5	-07.50
-602	10	27	A.S.	55	57.6	-02.60
-600	4	11	A.S.	95	93	+02.00
-598	2	19	A.S.	105	88.2	+17.20
-586	1	8	B.R.	35	32.7	+02.30
-579	8	14	A.S.	45	48.6	-03.60
-572	4	2	A.S.	90	89.55	+00.45
-561	3	2	A.S.	90	81.9	+08.01
-554	10	6	A.S.	55	54.9	+00.10
-536	10	17	B.R.	14	9.45	+04.55
-522	7	16	A.S.	50	45.3	+04.70
-521	1	10	B.R.	75	64.05	+10.95
-500	11	7	A.S.	77	67.8	+09.20
-482	11	19	B.R.	10	8.85	+01.15
-420	2	2	A.S.	19	16.5	+02.50
-407	10	31	A.S.	15	12	+03.00
-406	10	21	B.R.	48	52.2	-04.20
-405	10	10	B.R.	14	11.7	+02.30
-396	4	5	A.S.	48	46.65	+01.35
-377	4	6	A.S.	37	39.15	-02.15
-370	5	17	A.S.	66	54.9	+11.10
-370	11	11	A.S.	30	32.7	-02.70
-366	8	30	B.R.	56	39	+17.00
-352	11	22	B.R.	47	44.25	+02.75
-316	6	18	A.S.	10	13.8	-03.80
-316	12	13	A.S.	44	51.9	-07.90
-307	7	9	B.R.	10	12.3	-02.30
-239	11	3	B.R.	3	1.2	+01.80
-238	4	28	A.S.	80	59.7	+20.30
-225	8	1	A.S.	52	69	-17.00
-214	12	25	A.S.	15	33.3	-18.30
-211	4	30	B.R.	20	26.4	-06.40
-193	11	5	B.R.	12	10.05	+01.95
-188	2	17	B.R.	34	43.35	-09.35
-162	3	30	B.R.	85	98.4	-13.40
-159	1	26	A.S.	48	54	-06.00
-153	3	21	A.S.	4	4.8	-00.80
-142	2	17	A.S.	7	7.65	-00.65
-133	3	10	B.R.	9	14.55	-05.55
-128	11	5	B.R.	55	57	-02.00
-119	6	1	A.S.	66	67.65	-01.65
-108	4	30	A.S.	8	8.55	-00.55
-095	8	3	A.S.	57	63.00	-06.00
-079	4	11	B.R.	40	40.5	-00.50
-079	10	5	A.S.	30	31.95	-01.95

A.S.: after sunset.

B.R.: before sunrise.



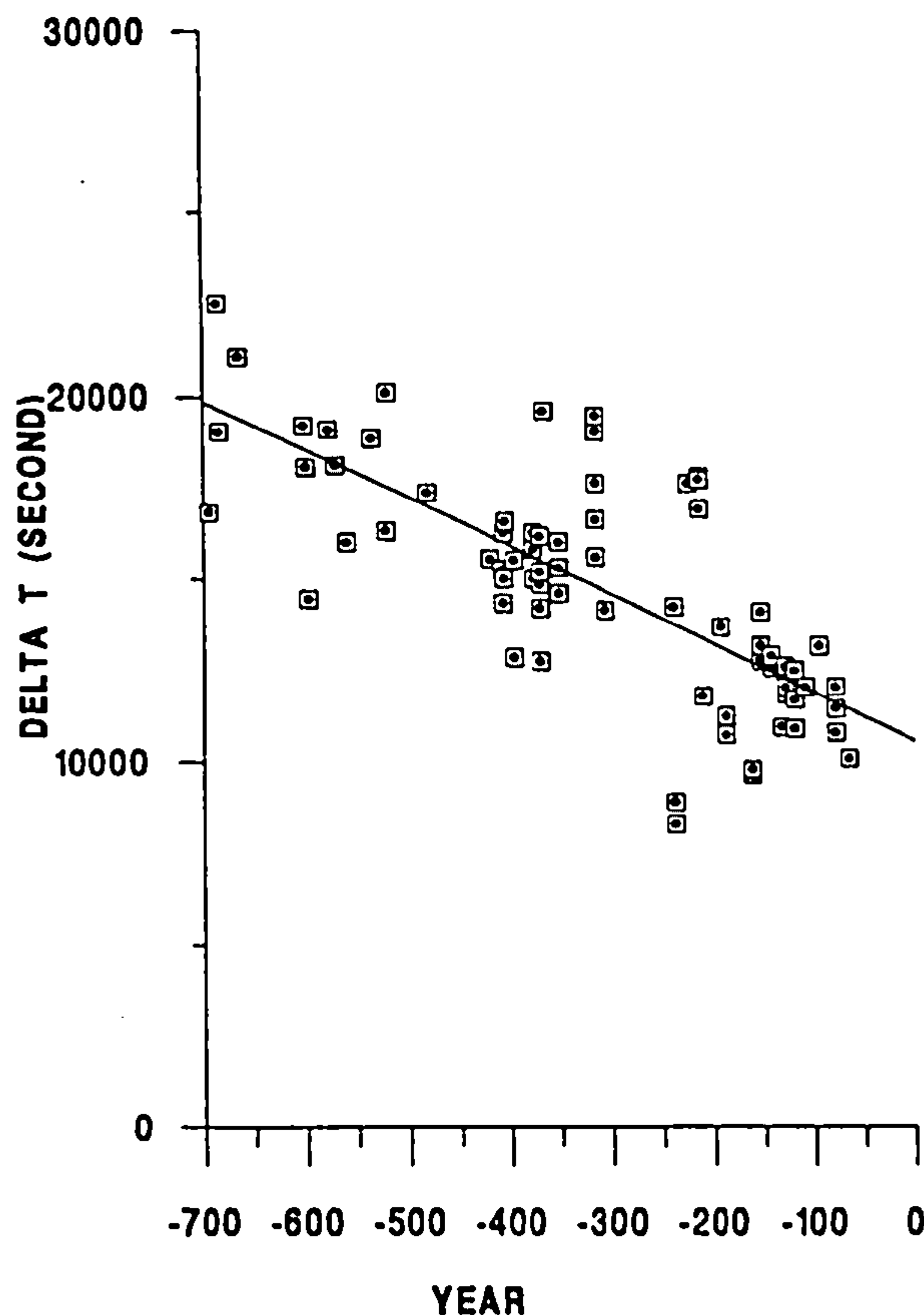


FIG. 2. Variation of the Earth's rotational clock error ( $\Delta T$ ) in the Late Babylonian period and least squares straight line fit to the data.

would be represented by a closely parabolic equation. However, there are significant non-tidal mechanisms.<sup>19</sup>

As the period covered by the Late Babylonian observations is relatively short compared with the number of centuries elapsed since then, we have felt it sufficient for the present purpose to derive a mean linear equation for  $\Delta T$  during the required interval. This was achieved as follows. For each preserved contact timing we first computed the local apparent time of sunrise or sunset at Babylon. The measurement was then converted to UT by adjusting for the equation of time and the geographic longitude. The appropriate value for  $\Delta T$  indicated by the measurement is then given by the difference  $TT - UT$ .

Using this approach, we tabulated the  $\Delta T$  results derived from the Babylonian timings as a function of date. We then fitted the best straight line to these results (see Figure 2, which is a graph of  $\Delta T$  in seconds of time *versus* years B.C.). This line has the equation

$$\Delta T = 10620 - 13.2t \quad (1)$$

where  $t$  is in years from 1 B.C. (the year 0 on the astronomical dating system).

TABLE 4. Comparison of measured and computed intervals relative to sunrise or sunset for contacts of eclipse other than the first contact.

Julian Date			Time	Measurements			Computations			Difference		
				2nd	3rd	4th	2nd	3rd	4th	2nd	3rd	4th
−554	10	6	B.R.	72	100	120	71.4	96	112.5	+00.90	+00.4	+00.75
−500	11	7	A.S.	92	117	142	84.45	108.15	124.65	+07.55	+08.85	+17.35
−407	10	31	A.S.	...	...	42	.....	.....	35.1	.....	.....	+06.90
−406	10	21	B.R.	27	...	...	35.7	.....	.....	−08.7	.....	.....
−396	4	5	A.S.	...	...	75	.....	.....	63	.....	.....	+12.00
−377	4	6	A.S.	52	73	92	54.9	74.1	89.85	−02.90	−01.10	+02.15
−370	11	11	A.S.	52	72	93	49.65	70.95	87.75	+02.35	+01.05	+05.25
−352	11	22	B.R.	24	6	...	27.15	6.15	.....	−03.15	−00.05	.....
−316	12	13	A.S.	63	68	84	69	90	107.1	−06.00	−22.00	−23.10
−238	4	28	A.S.	...	...	120	.....	.....	95.1	.....	.....	+24.90
−225	8	1	A.S.	69	79	94	85.8	101.85	118.5	−16.80	−22.85	−24.50
−214	12	25	A.S.	36	52	71	49.05	69.6	85.5	−13.05	−17.60	−14.50
−188	2	17	B.R.	18	...	...	25.2	.....	.....	−09.35	.....	.....
−162	3	30	B.R.	...	...	65	.....	.....	77.75	.....	.....	−12.55
−153	3	21	A.S.	...	...	48	.....	.....	54.3	.....	.....	−06.30
−128	11	5	B.R.	...	...	15	.....	.....	13.95	.....	.....	+01.05
−119	6	1	A.S.	...	...	120	.....	.....	115.05	.....	.....	+04.95
−066	1	19	A.S.	...	...	23	.....	.....	16.65	.....	.....	+06.35

A.S.: after sunset.  
B.R.: before sunrise.

TABLE 5. Comparison of measured and computed intervals relative to sunrise or sunset of eclipse maxima.

Julian Date			Time	Measurement	Computed	Difference
−238	4	28	A.S.	100	77.40	+22.60
−153	3	21	A.S.	27	29.55	−02.55
−142	2	17	A.S.	29.5	31.65	−02.15
−128	11	5	B.R.	34	35.48	−01.48
−119	6	1	A.S.	93	91.28	+01.72
−079	4	11	B.R.	16.5	19.65	−03.15

A.S.: after sunset.  
B.R.: before sunrise.

Most of the recorded timings relate to first contact, but in a significant number of instances details are preserved for other contacts too. Initially, we grouped our data into three categories: first contacts, other contacts, and maxima for partial eclipses. For observations in the second category we summed the individual durations. In the case of eclipse maxima, we considered true greatest phase to be reached halfway through the supposed duration of maximum.

Many of the recorded time intervals are extremely long — some exceeding 100 deg (6<sup>h</sup>40<sup>m</sup>). Hence this data set affords a particularly useful opportunity for testing the reliability of Babylonian clocks.

We have used Equation (1) to recompute what each of the local times of



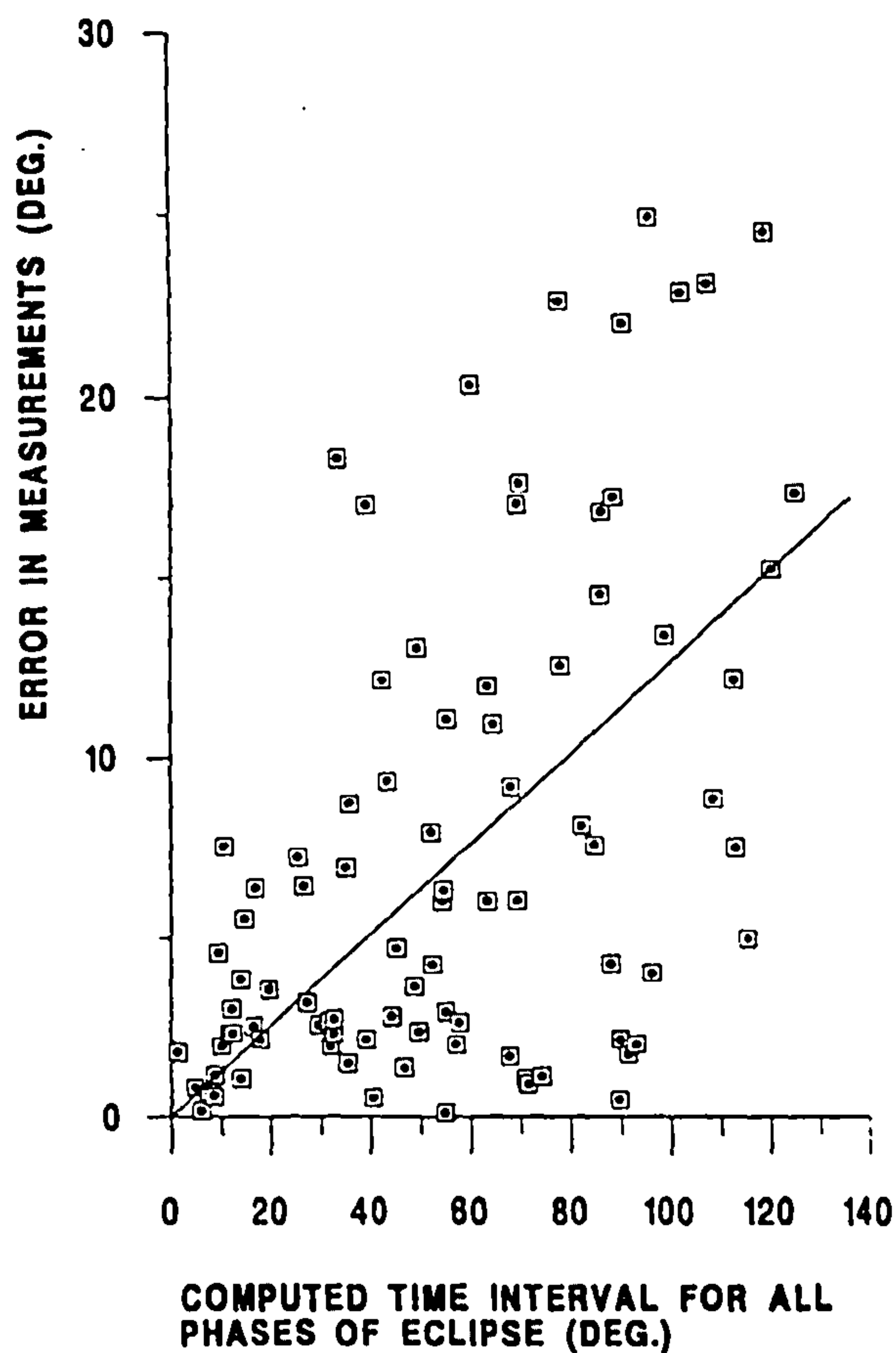


FIG. 3. Discrepancies between measured and computed intervals relative to sunset or sunrise for Late Babylonian observations of total and partial lunar eclipses (all contacts).

the various contacts should have been on the basis of theory alone. We have designed a program which computes the local time of any selected eclipse contact by first deriving the TT, then converting to UT using Equation (1) and subsequently reducing to local time by allowing for the equation of time and geographic longitude. Finally, time intervals relative to sunrise or sunset in deg are deduced.

The various results of this part of our investigation are shown in Tables 3, 4 and 5. Of these, Table 3 is restricted to first contact determinations, Table 4 to other contacts, and Table 5 to eclipse maxima. The data in these tables are plotted in Figure 3, which shows the error in measurement as a function of the computed time-interval (relative to sunrise or sunset). Both axes are again marked in deg.

The scatter in Figure 3 is also very large; the mean discrepancy between measured and computed time-intervals is some 12 deg or almost 50 minutes. There is a noticeable general increase in the deviation with increasing computed time-intervals. As in Figure 1, we have fitted the best straight line to the data through the origin. The gradient of this line is close to 13%. This happens to be identical to the result obtained from Figure 1 but the significance of this agreement must remain doubtful.

## 7. Conclusion

Late Babylonian timings of lunar eclipse contacts provide interesting evidence on the accuracy of time measurement by the astronomers of the period. In this paper, we have analysed two separate types of measurement: durations of the various phases of eclipses, and contact times expressed relative to sunrise or sunset. Although the Babylonian astronomers rounded the time-intervals that they determined for these phenomena to the nearest four minutes, the real accuracy they achieved was far less than this. Typical errors of at least half-an-hour in measuring intervals of no more than six hours represents a poor performance by any reasonable standards — even allowing for the occasional scribal error. Not until medieval Arab astronomers introduced altitude measurements as an alternative technique to direct timing did the measurement of time in any part of the world significantly improve.

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## Accuracy of Early Estimates of Lunar Eclipse Magnitudes

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### SUMMARY

Estimates of lunar eclipse magnitudes made with the unaided eye by astronomers in antiquity are analysed, with a view to determining the accuracy with which the eye can estimate such a quantity. These observations are recorded in Babylonian, Chinese, Arabic and European history. It is shown that the discrepancies between observation and computation follow a remarkably skew distribution. In general, the magnitudes of small eclipses (less than half of the disk covered) are over-estimated, while with large eclipses the reverse is true.

### 1 INTRODUCTION

Many fairly careful observations of both lunar and solar eclipses made with the unaided eye by astronomers are recorded in history. In ancient times, the Babylonians systematically reported times of occurrence and other details for these phenomena while during the medieval period Chinese and Arab astronomers were particularly active in this field. Later, with the onset of the Renaissance, European astronomers followed much the same practice and naked eye observations of eclipses continued on a regular basis until the telescope became widely disseminated. Early records of eclipses often include an estimate of the magnitude (the maximum degree of obscuration of the disk) and it is the purpose of the present investigation to compare such determinations with the results of computation.

Most preserved estimates of magnitude are for lunar eclipses; similar details are comparatively rare for their solar counterparts. This circumstance is partly the result of the less frequent occurrence of solar eclipses at a given place but difficulties in viewing the brilliant solar disk are a further contributing factor. In addition, by historical accident some of the extant historical sources – notably the material from Babylon – show a bias towards the preservation of lunar rather than solar eclipse records. For these reasons we have confined our attention in this paper to the recorded estimates of lunar eclipse magnitudes.

The investigation of early determinations of the magnitudes of *lunar* eclipse has other advantages. Thus, the computed magnitude of a solar eclipse depends very much on the observer's location on the terrestrial surface and is also a function of the Earth's rotational clock error  $\Delta T$  (the difference between Terrestrial Time and Universal Time). By contrast, the maximum degree of obscuration for a lunar eclipse is independent of these factors, so that it is possible to make direct comparison between observation and present-day calculation with the minimum of assumptions.



## 2 LUNAR ECLIPSE MAGNITUDES

Lunar eclipses which are observable by the unaided eye fall into two main categories: partial and total; penumbral obscurations are seldom noticeable. Only partial eclipses will concern us in this paper. Their magnitudes can be readily estimated without optical aid. Although the degree of obscuration of the Moon at a total eclipse can vary from unity up to about 1.89 (because the Earth's shadow is so much larger than the Moon), the magnitude cannot of course be judged directly.

For a partial eclipse, the magnitude is currently defined as the fraction of the lunar diameter covered at greatest phase. However, occasional early estimates give surface magnitude (the proportion of the area of the disk covered) instead. It is not clear from historical records just how widespread this practice was in antiquity.

Ancient Babylonian astronomers estimated the magnitudes of lunar eclipses to the nearest twelfth of the disk. This same practice later spread to the Greeks and thence to the medieval Arabs; it was still in vogue in Europe until relatively recent times. However, in China the fraction of the disk covered was usually estimated in fifteenths, evidence of an independent tradition. It appears that in making these various determinations no instruments were utilized; the observer simply made an eye-judgment.

We have made a compilation of more than 70 estimates of the magnitudes of partial lunar eclipses from a wide variety of historical sources. In each case there is nothing in the text to suggest that the observer was prevented from viewing the Moon around maximum phase – either by cloud or the Moon being below the horizon.

## 3 HISTORICAL SOURCES

Our main sources may be summarized as follows: (1) The Late Babylonian astronomical texts (Huber, personal communication; Sachs & Hunger 1988–1989); (2) Ptolemy's *Almagest* (trans. Toomer 1984), which contains both Babylonian and Greek data; (3) the dynastic histories of China – data from which have recently been compiled by a team of Chinese scholars (Beijing Obs 1988); (4) the *Zij* (astronomical handbook) by the medieval Egyptian astronomer ibn Yunus (d. AD 1009) – for details, see Stephenson & Said (1991); (5) the compilation of 17th century European observations assembled by Pingre in the late 18th century but published in edited form in 1901; many of the earlier observations cited in this work were made with the unaided eye.

Brief comments about each source and the material which we have selected from it are as follows.

### 3.1 *The Late Babylonian astronomical texts*

The cuneiform astronomical tablets which were discovered at the site of Babylon more than a century ago are now largely in the British Museum. Huber (1973) – in an unpublished memoir – made a special study of the eclipse records from this source; he also gave full translations. Further details are also to be found in the compilations of Sachs & Hunger (1988–1989). The observational texts recovered from Babylon – many of which are in the form



of day to day astronomical diaries – range in date from about 700 to 50 BC. Most of these tablets are badly damaged and statistical estimates indicate that only about 10 per cent of the original material has ever been recovered.

Babylonian astronomers were in the habit of estimating lunar eclipse magnitudes in *si* ('fingers'), each equivalent to one-twelfth of the Moon's disk. The use of 'fingers' suggests linear units, but there appears to be nothing in extant history to confirm this. An example from the compilation by Huber is as follows:

Year 175 (SE), intercalary 12th month, night of the 15th.... When  $\alpha$  Coronae culminated, lunar eclipse, beginning on the south-east side. In 18 deg of night it made 7 fingers. (Began) at 1 *beru* (2 hours) before sunrise [British Museum Tablet WAA 34034, trans. Huber (1973), p. 69].

The date in terms of the Seleucid calendar corresponds to 136 BC April 1 (Parker & Dubberstein 1956). This date agrees exactly with that of an eclipse which according to modern calculation would be visible in Babylon. Dates are now missing from many tablets but these can often be restored using astronomical calculations based on the lunar and planetary data which they contain (Sachs & Hunger, 1988–1989). In all we have been able to assemble about 20 separate Babylonian estimates of lunar eclipse magnitude from the astronomical texts; these range in date from 713 to 66 BC.

### 3.2 Ptolemy's *Almagest*

Only five ancient Greek estimates of lunar eclipse magnitude are preserved. These are all in the *Almagest* (books IV and VI) and date from between 174 BC and AD 136. The observations were first investigated by Fotheringham (1909). Four of the five determinations are expressed either in digits (equivalent to the Babylonian fingers) or in sixths of the disk. Ptolemy also cites six Babylonian estimates of lunar eclipse magnitudes between 720 and 491 BC (*Almagest*, IV and V). None of these are still found on the extant tablets discovered at the site of Babylon. We have included these observations with the data obtained from the Babylonian texts themselves.

It might be mentioned here that Ptolemy notes that use of *surface* magnitude was frequent in his time:

But most of those who observe the [weather] indications derived from eclipses measure the size of the obscuration, not by the diameters of the disks [of Sun and Moon], but, on the whole, by [the amount of] the total surface of the disks since, when one approaches the problem naively, the eye compares the whole part of the surface which is visible with the whole of that which is invisible. [*Almagest*, VI, 7 – trans. Toomer, 1984, p. 302.]

However, Toomer in a footnote to the above quotation remarks:

Although there is no reason to doubt Ptolemy's statement, I know of no surviving ancient eclipse magnitude which is unambiguously given in area digits.

The following account of the eclipse of 27 Jan 141 BC given by Ptolemy, illustrates the style of the few Greek records:

Again, in the 37th year of the Third Kallippic Cycle, which is the 507th from Nabonassar [on the day] Tybi [V] 2/3 in the Egyptian Calendar [27/28 Jan 141 B.C.], at the beginning of the fifth hour [of night] in Rhodes, the Moon began to be eclipsed; the maximum obscuration was 3 digits from the south. [*Almagest*, VI – trans. Toomer (1984), p. 284.]



Tybi was the fifth month of the Egyptian Calendar and the eclipse occurred on the night between the 2nd and 3rd of that month.

### 3.3 Chinese observations

Although systematic observation of solar eclipses commenced in China as early as 700 BC, few lunar eclipses are reported before AD 400. However, from this latter date, fairly consistent records are available. These are mainly to be found in the astronomical and calendrical treatises of the dynastic histories, works which have been printed and re-printed many times. The compilation of astronomical records from these and other sources produced at Beijing Observatory (1988) contains a substantial section devoted to lunar eclipse observations; this has proved of considerable utility.

Chinese estimates of eclipse magnitude are usually expressed in *fen* (divisions) – normally fifteenths of the disk of the luminary. However, other combinations, expressed directly as fractions, do occur.

Twelve estimates of lunar eclipse magnitude are recorded in Chinese history between AD 440 and 595 and we have included these in our investigation. Observations made after that date are extremely rare until the last (Qing) dynasty – beginning in AD 1644. During this latter period the Jesuits made many such observations. As they were possibly made with the aid of a telescope, we have not included them in our study.

As an example of a typical Chinese record of a lunar eclipse magnitude we may quote the following:

Yuan-chia reign period, 17th year, 9th month, 16th day, full Moon. The Moon was eclipsed.... The eclipse began at the first division of the 2nd watch. At the third division (of the 2nd watch) the eclipse reached 12/15. (The Moon) was situated at 1.5 deg in Mao (lunar lodge). [*Sung-shu*, chap. 12 (Beijing Obs 1988, p. 265.)]

The date reduces to AD 440 Oct 26 (Chueh & Ou-yang 1956). Calculation shows that the eclipse of the Moon which occurred on this same day would be visible in China.

### 3.4 Arab estimates

The *Zij* of ibn Yunus contains many observations of solar and lunar eclipses made in Baghdad and Cairo (Stephenson & Said 1991). (For a full translation of the treatise by ibn Yunus into French, see Caussin 1804.) In all, ibn Yunus records 10 estimates of lunar eclipse magnitude ranging in date from AD 854 to 990; the most recent examples of these he observed himself. To this material we have added further estimates by al-Battani in AD 883 and al-Biruni in 1003 (Stephenson & Said 1991).

The following example from ibn Yunus, summarizing his observations of the lunar eclipse of AD 979 November 7, illustrates the type of material available:

369 A.H. (month) Rabi II (al-Quds), (day) 13, Friday .... Several scholars met in order to observe this eclipse (at Cairo). They estimated the portion of the surface eclipsed to be 10 digits; altitude when they noticed its eclipse, 64 1/2 deg in the east; altitude when its clearance was reached, 65 deg in the west [trans. Stephenson & Said 1991].

When reduced to the Julian calendar (Freeman-Grenville 1977), the



TABLE I

*The difference between the computed and measured magnitude of eclipses observed by the Babylonians*

Julian Date	Magnitude		
	Observed	Computed	Difference
-719 3 8	$3/12 = 0.25$	0.11	-0.14
-719 9 1	$7/12 = 0.58$	0.50	-0.08
-712 4 19	$1/2 \text{ diam} = 0.50$	0.62	0.12
-685 4 22	$2/3 \text{ diam} = 0.67$	0.55	-0.12
-620 4 21	$1/4 = 0.25$	0.16	-0.09
-602 10 27	$1/2 \text{ diam} = 0.50$	0.58	0.08
-600 10 5	$9/12 = 0.75$	0.52	-0.23
-598 2 19	$2/3 \text{ diam} = 0.75$	0.75	0.00
-522 7 16	$1/2 \text{ diam} = 0.50$	0.54	0.04
-501 11 19	$1/4 \text{ diam} = 0.25$	0.20	-0.05
-490 4 26	$2/12 = 0.17$	0.10	-0.07
-439 2 2	$7/12 = 0.58$	0.44	-0.14
-423 9 28	$10/12 = 0.83$	0.93	0.10
-409 12 21	$11/12 = 0.92$	0.95	0.03
-407 10 31	$1/4 \text{ diam} = 0.25$	0.18	-0.07
-396 4 5	$1/4 \text{ diam} = 0.25$	0.10	-0.15
-211 10 24	$10/12 = 0.83$	0.94	0.11
-193 11 5	$2/3 \text{ diam} = 0.67$	0.92	0.25
-162 3 30	$3/12 = 0.25$	0.12	-0.13
-153 3 21	$10/12 = 0.83$	0.85	0.02
-142 2 17	$9/12 = 0.75$	0.88	0.13
-135 4 1	$7/12 = 0.58$	0.73	0.15
-108 5 1	$6/12 = 0.50$	0.52	0.02
-079 4 11	$6/12 = 0.50$	0.60	0.10
-065 12 28	$5/12 = 0.42$	0.34	-0.08

recorded date proves to be precisely correct. This is one of the very few instances where a surface magnitude seems definitely intended. A literal translation of the first clause of the second sentence above is: 'They estimated the eclipse of the surface of the circle of the Moon to be 10 digits'. Only on one another occasion (AD 856) do we find a similar assertion. Here it is stated that 'it was found that there remained of (the Moon's) globe which was not included in the eclipse more than one-quarter and less than one-third'. In most other Arab texts consulted the lunar diameter is specifically implied.

### 3.5 Early 17th century European observations

Pingre (1901) made a careful search of European astronomical works for references to eclipses and other celestial phenomena in the 17th century. The list which he compiled is very exhaustive. Although many late 16th century observations of lunar eclipses are no doubt scattered in the literature of the period, we have restricted our attention to the much more accessible material assembled by Pingre. We have concentrated only on those observations from the first two decades of the 17th century. All estimates made in the first half of our selected period are necessarily made with the unaided eye. Dissemination of the telescope in the following decades was slow, while early

TABLE II  
*The difference between the computed and measured magnitude of eclipses observed by the ancient Greeks*

Julian Date	Magnitude		
	Observed	Computed	Difference
−173 4 30	7 dig = 0.58	0.62	0.04
−140 1 27	3 dig = 0.25	0.26	0.01
+125 4 5	1/6 = 0.17	0.15	−0.02
+134 10 20	5/6 = 0.83	0.83	0.00
+136 3 5	1/2 = 0.50	0.45	−0.05

TABLE III  
*The difference between the computed and measured magnitude of eclipses observed by the Chinese*

Julian Date	Magnitude		
	Observed	Computed	Difference
440 10 26	12/15 = 0.80	0.82	0.02
485 12 7	1/3 = 0.33	0.43	0.10
489 4 1	7/15 = 0.47	0.53	0.06
493 1 18	2/7 = 0.29	0.15	−0.14
500 3 1	3/15 = 0.20	0.20	0.00
505 10 28	10/15 = 0.67	0.70	0.03
509 2 20	12/15 = 0.80	0.88	0.08
516 9 26	8/15 = 0.53	0.45	−0.08
520 1 20	10/15 = 0.67	0.68	0.01
590 10 18	4/5 = 0.80	0.78	−0.02
592 8 28	2/3 = 0.67	0.61	−0.06
595 12 22	2/3 = 0.67	0.64	−0.03

telescopes had such a narrow field of view that they would not be suitable for viewing eclipses. Unless an entry specifically states that a telescope was used, we have assumed unaided eye observations. It should be noted that often several observers in different parts of Europe made independent estimates of the magnitude of the same eclipse.

Michael Maestlin was one of several European astronomers who systematically observed eclipses around the year AD 1600. Pingre summarized his report of the eclipse of AD 1607 in the following words:

A.D. 1607 Sep 26. Maestlin at Tübingen took... the altitude of Sirius as 13 deg in the east when the eclipse began. Therefore it commenced at 15 h 34 m (local time). By direct vision he concluded that the eclipse had exceeded three quarters of the Moon's diameter. The Moon set before the end of the eclipse. [Pingre (1901), p. 24.]

4 COMPUTATIONS

In computing eclipse magnitudes we have used Newcomb's solar theory (1895) and the lunar theory of the Improved Lunar Ephemeris (Eckert, Jones & Clark 1954) – the latter with further amendments to correspond to a lunar orbital acceleration of  $-26$  arcsec/cy/cy (Morrison & Ward 1975, Morrison 1979). This acceleration is very close to that obtained from current lunar



TABLE IV

*The difference between the computed and measured magnitude of eclipses observed by the Arabs*

Julian Date	Magnitude		
	Observed	Computed	Difference
854 2 16	8.5/10 = 0.85	0.92	0.07
856 6 22	8.5/12 surf = 0.70	0.59	-0.11
883 7 23	10.5/12 = 0.88	0.95	0.07
923 6 1	9.5/12 = 0.79	0.66	-0.13
927 9 14	3.5/12 = 0.29	0.22	-0.07
979 5 14	8.5/12 = 0.71	0.70	-0.01
979 11 6	10.0/12 surf = 0.80	0.84	0.04
981 4 22	1/4 = 0.25	0.18	-0.07
981 10 16	5.0/12 = 0.42	0.36	-0.06
986 12 19	10.0/12 = 0.83	0.91	0.08
990 4 12	7.5/12 = 0.63	0.74	0.11
1003 2 19	1/4 = 0.25	0.14	-0.11
1003 8 15	3.5/12 = 0.29	0.14	-0.15

TABLE V

*The difference between the computed and measured magnitude of eclipses observed by the Europeans without the aid of a telescope*

Gregorian Date	Magnitude		
	Observed	Computed	Difference
1601 12 9	10.0/12 = 0.83	0.92	0.09
1603 5 24	7.5/12 = 0.63	0.62	-0.01
1603 5 24	8.0/12 = 0.67	0.62	-0.05
1603 11 18	2.5/12 = 0.21	0.22	0.01
1603 11 18	2.5/12 = 0.21	0.22	0.01
1603 11 18	3.0/12 = 0.25	0.22	-0.03
1603 11 18	3.0/12 = 0.25	0.22	-0.03
1605 9 26	9.5/12 = 0.79	0.67	-0.12
1605 9 26	8.5/12 = 0.71	0.67	-0.04
1605 9 26	8.5/12 = 0.71	0.67	-0.04
1609 1 19	11.0/12 = 0.92	0.81	-0.11
1609 1 19	9.5/12 = 0.79	0.81	0.02
1612 5 14	6.5/12 = 0.54	0.56	0.02
1612 5 14	8.5/71 = 0.56	0.56	-0.15
1612 5 14	6.5/12 = 0.54	0.56	0.02
1612 5 14	6.1/12 = 0.51	0.56	0.05
1619 6 25	1.0/12 = 0.08	0.11	0.03
1619 12 19	3/4 = 0.75	0.91	0.16
1619 12 19	4/5 = 0.80	0.91	0.11

laser ranging measurements (Williams, Newhall & Dickey 1992). We have applied the customary increment of 2 per cent to the Earth's shadow radius to make approximate allowance for the effect of the terrestrial atmosphere. In practice, the magnitude of each individual eclipse is influenced to a minor extent by conditions prevailing in the Earth's upper atmosphere – through which the sunlight reaching the unobscured portion of the Moon is refracted. Nevertheless, the precision of our computations should be considerably

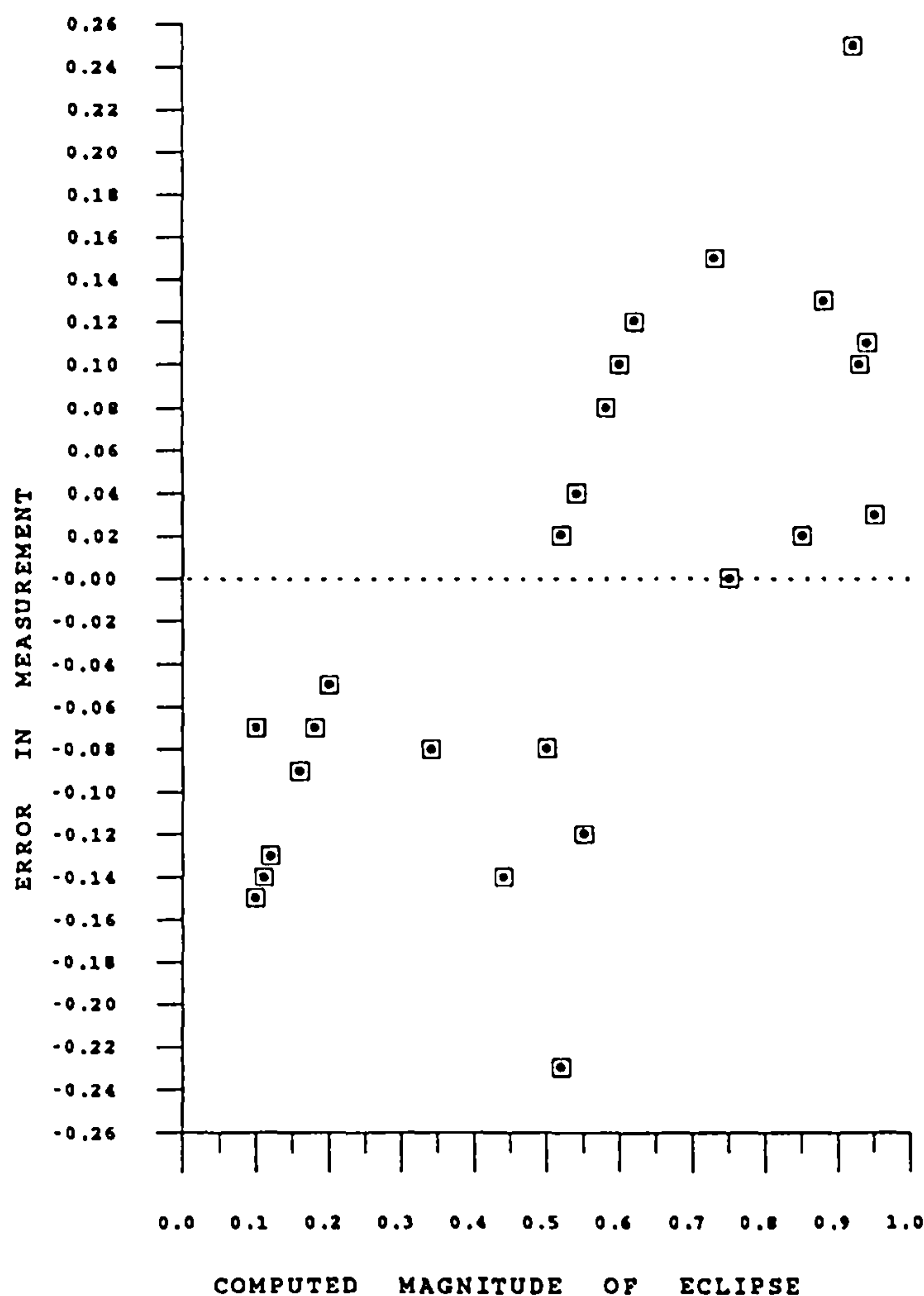


FIG. 1. Comparison of computations and measurements of magnitudes of lunar eclipses observed by the Babylonians.

greater than that with which the various magnitude estimates were themselves made.

We have designed a computer program which not only deduces the magnitude for a selected eclipse but also derives the altitude of the Moon at that moment for the appropriate place of observation. For this latter purpose we have utilized the  $\Delta T$  equations published by Stephenson & Morrison (1984). For each eclipse we have used this program to verify that the Moon was above the horizon at greatest phase.

## 5 RESULTS

The various observations which we have studied, along with our computed magnitudes, are summarized in Tables I–V. These respectively are devoted to Babylonian, Greek, Chinese, Arabic and European data. In each table we have given the following details: (1) the date according to the Julian (up to AD 1582) or Gregorian Calendar; (2) the observed magnitude expressed either in its original form or in a rationalized version of the original (see below); (3) the observed magnitude expressed as a decimal; (4) the computed



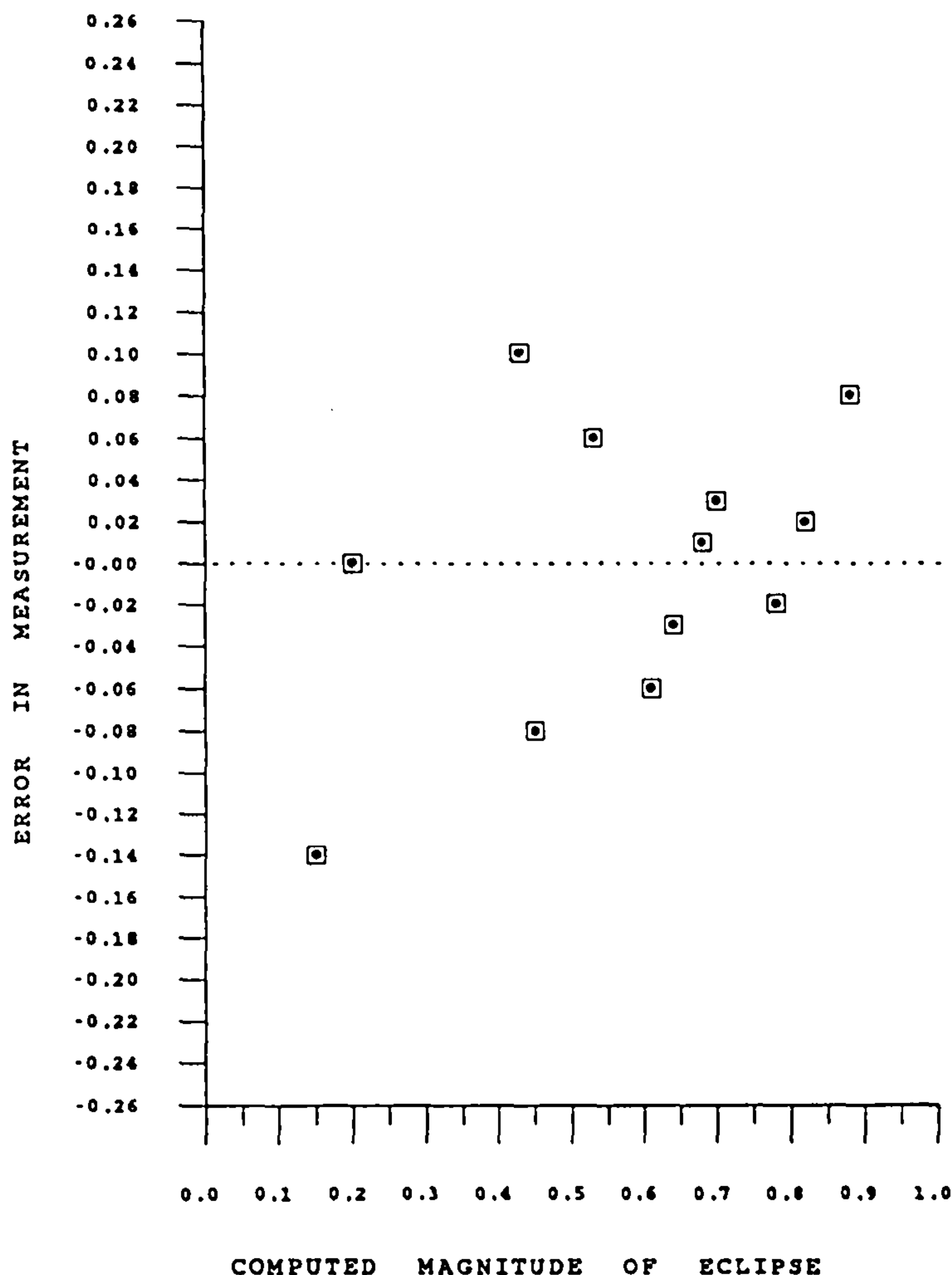


FIG. 2. Comparison of computations and measurements of magnitudes of lunar eclipses observed by the Chinese.

magnitude also given as a decimal; and (5) the discrepancy between the observed and computed magnitude in the sense (computed—observed).

Several Arab observations state that the magnitude was either less than or greater than a certain specified fraction – for instance ‘greater than 9 digits’ in AD 923 or ‘greater than  $\frac{1}{4}$ ’ in AD 1003. In order to interpret these estimates, we have assumed that the basic unit of magnitude was the digit (i.e.  $\frac{1}{12}$  of the diameter). Thus in the above examples, we have inferred that the implied magnitude was  $9\frac{5}{12}$  in 923 and  $3\frac{5}{12}$  in 1003 – see column (2) of Table 4. Similar remarks apply to the other Arab observations expressed in this way. In Table 4 we have converted two recorded *surface* measurements in AD 856 and 979 to diameter. In a small proportion of the Chinese records only, the date was incorrectly given and we have been unable to suggest a viable alternative, leading to rejection of the observation.

In Figs 1 to 4 we have plotted the results of our analysis for each of the Babylonian, Chinese, Arabic and 17th century European sets of data; the ancient Greek data are too few to warrant a graphical display. Each diagram shows the observational error (in the sense computed—observed magnitude) as a function of the computed magnitude.

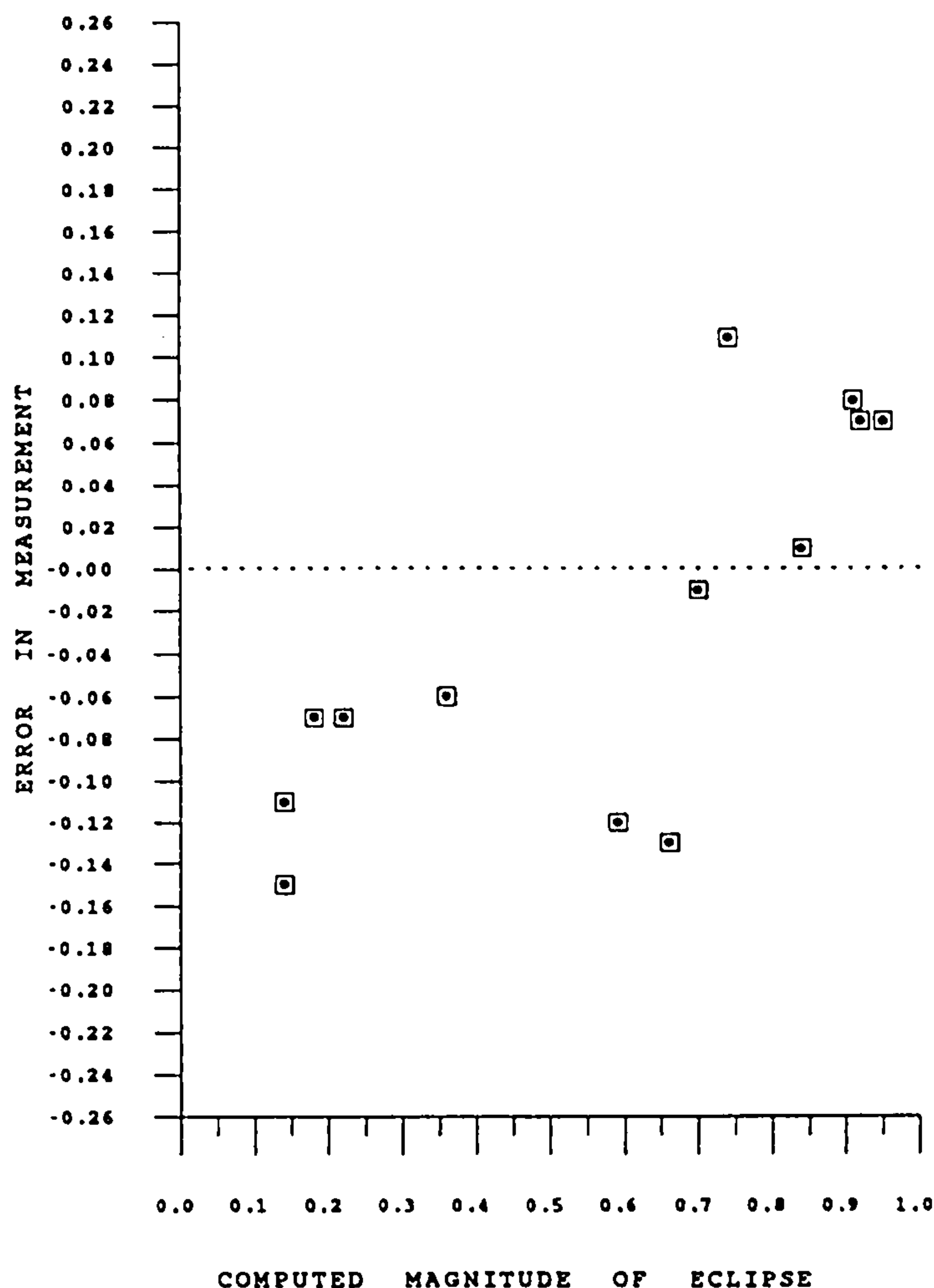


FIG. 3. Comparison of computations and measurements of magnitudes of lunar eclipses observed by the Arabs.

Figure 5 is a similar plot for all the available observations, including the Greek data. Observations from each different source are denoted by separate symbols. The solid line represents the best fitting straight line to the data. The distribution of points in this diagram is remarkable. For computed magnitudes less than about 0.5 the observers clearly tended to over-estimate the degree of obscuration of the Moon while for large magnitudes (above about 0.7) the reverse is true. Among the individual data sets, Fig. 1 (Babylonian) and Fig. 3 (Arabic) particularly reveal this trend. If this feature is disregarded, it may be simply concluded that when a determination of lunar eclipse is made with the unaided eye, the standard deviation is about 0.08. However, the trend is so obvious (mean gradient 0.19) that further consideration is needed.

We have investigated the possibility that part of the skew distribution in Fig. 5 is due to most observations being expressed in terms of surface rather than linear magnitude. However, as we have already noted, the Arab data – nearly all of which is specifically quoted in relation to diameter – clearly reveals much the same feature. On the other hand, just how the Babylonian and Chinese astronomers defined magnitude (i.e. relative to



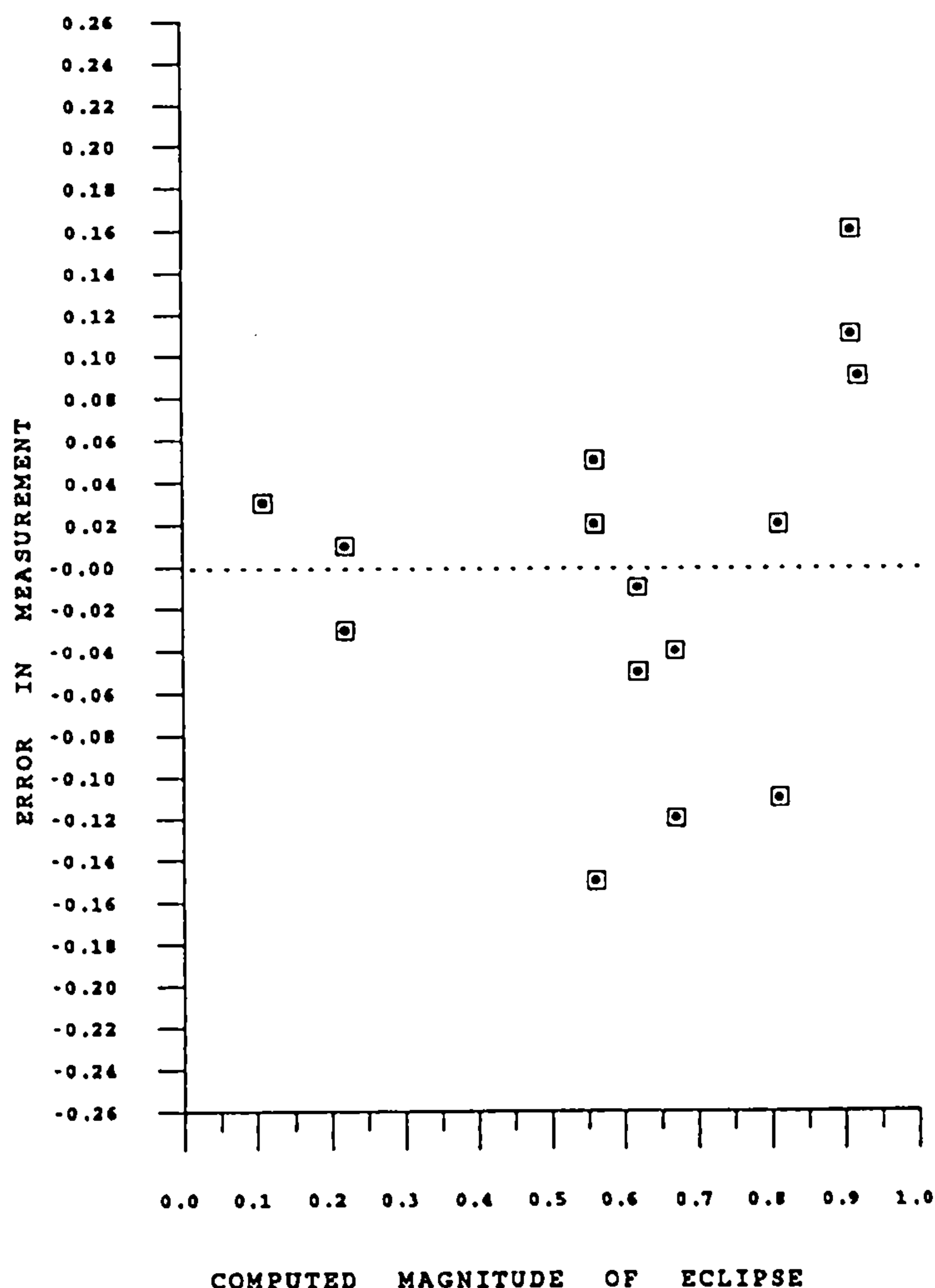


FIG. 4. Comparison of computations and measurements of magnitudes of lunar eclipses observed by the Europeans.

diameter or area) is not clear while even the 17th century European observations do not specifically state which method was used.

Figure 6 is similar to Fig. 5, but here we have converted all *computed* linear magnitudes to their surface equivalent on the assumption that the various observed magnitudes were definitely in terms of area rather than diameter. Once again, the best fitting straight line is shown. It can be seen that individual points show an even more skew distribution, the gradient of the mean straight line being as great as 0.29. It thus seems more likely that the original measurements were in general made in terms of diameter.

Part of the distribution in Fig. 5 is probably physiological. However, the effect of the deep penumbral shadow may provide a plausible explanation for the magnitudes of small eclipses (less than about 0.5) being consistently over-estimated. Photographs of small partial eclipses which we have inspected clearly show the deep penumbral shadow at a significant distance beyond the umbra. Unaided eye observers have noted confusion between the umbral and penumbral shadows. For example, after observing the eclipse of 1605 April 3, Wendelin criticized his contemporary Lansberg for not having properly distinguished between the umbra and the penumbra; as a result, the time

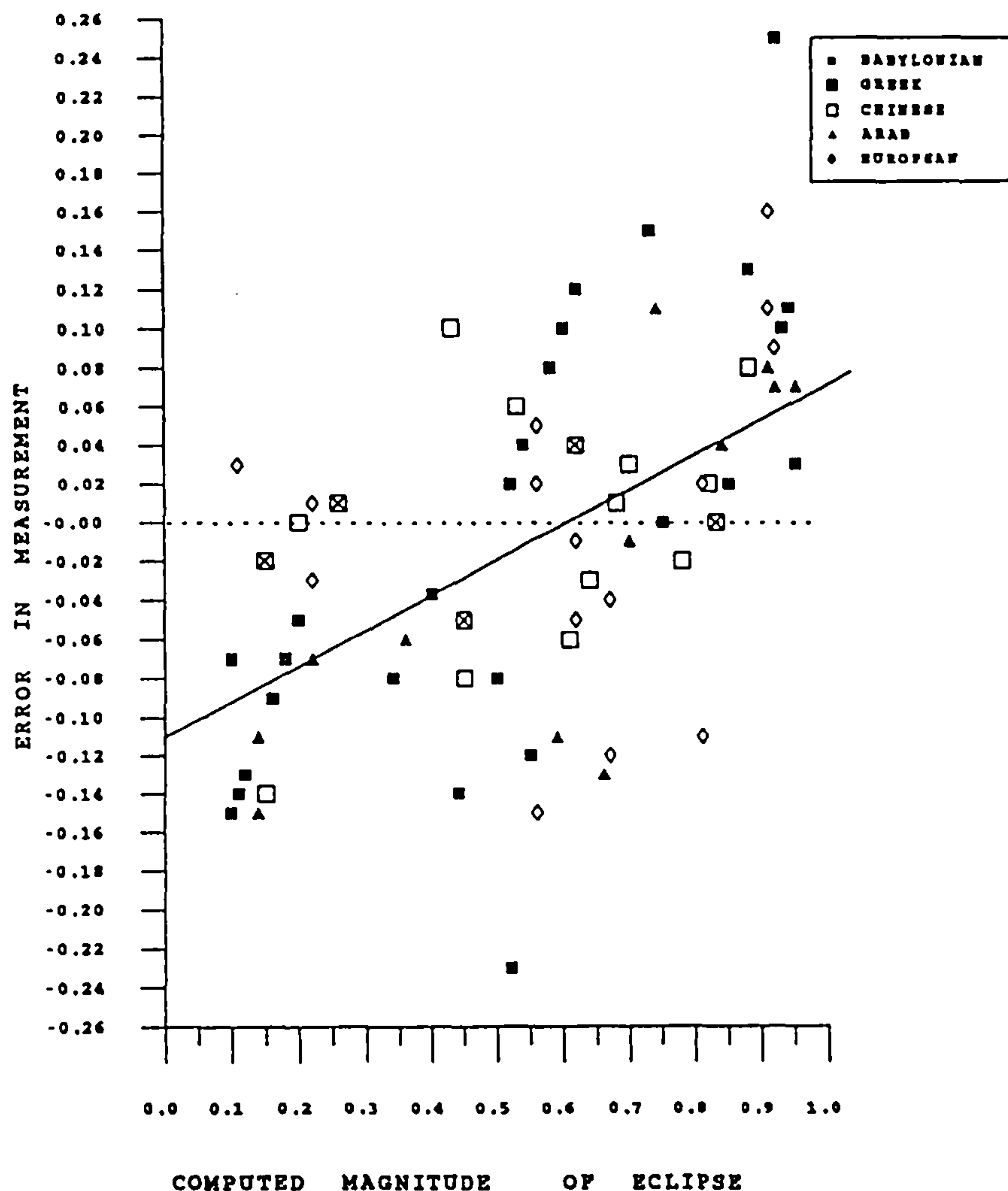


FIG. 5. Comparison of computations and measurements of magnitudes of lunar eclipses observed in different civilizations in the pre-telescopic era.

which Lansberg recorded for the moment of last contact was too late (Pingre 1901, p. 20). Morrison & Stephenson (1982) showed from an analysis of the durations of early 17th century observations of lunar eclipses that confusion by the unaided eye between the umbra and penumbra appeared to be common; recorded durations were often significantly longer than expected.

For large eclipses, in which the magnitude was systematically underestimated, there is no such simple explanation. The effect of contrast may confuse the observer as may the fairly small angle between the cusps of the remaining crescent. Whatever the true explanation, the trend shown in Fig. 5 is impressive.

The scatter about the best fitting straight line in Fig. 5 is considerably smaller than that about the abscissa, the standard deviation being 0.05. If allowance is made for systematic effects common to observers in general, individual estimates of eclipse magnitude would appear to be tolerably accurate.



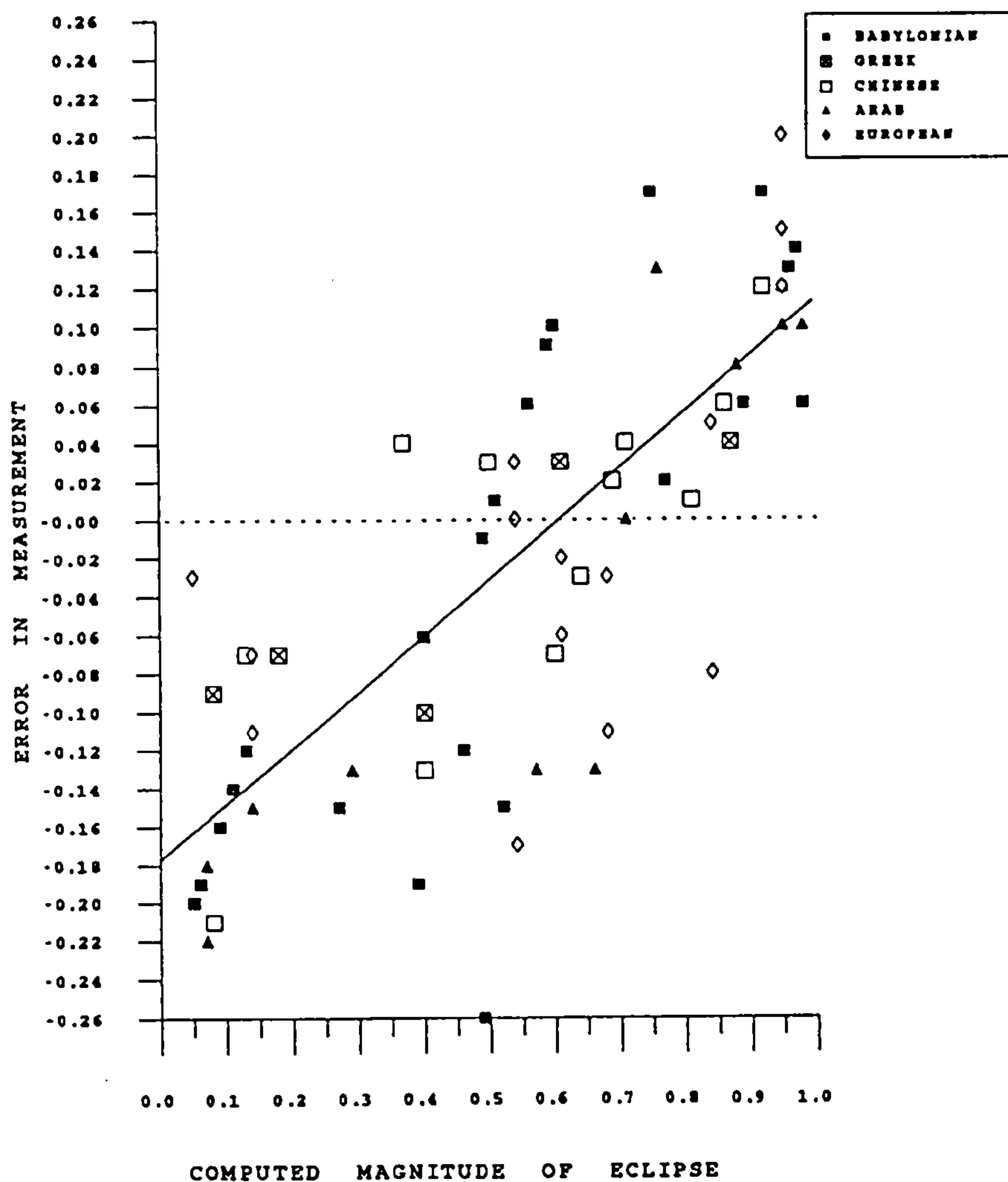


FIG. 6. Comparison of computations and measurements of magnitudes of lunar eclipses observed in different civilizations in the pre-telescopic era (surface area).

## 6 CONCLUSION

When unaided eye estimates of lunar eclipse magnitudes recorded in history are compared with their computed equivalents, they show a strongly biased distribution. The magnitudes of small eclipses are consistently over-estimated, while for large eclipses the reverse is generally true. The assumption that observers tended to use surface rather than linear magnitude is not a viable explanation since then errors would be even more serious. In our view, the deep penumbral shadow is to some degree responsible for the apparent over-estimate of small magnitudes but probably physiological effects cannot be ignored.

Some years ago one of us (Stephenson 1972) asked a group of navigation students to estimate the magnitudes of a series of artificial *solar* eclipses in which at least 70 per cent of the diameter was obscured. This related to an attempt to date the so-called eclipse of Hipparchus which was said to be  $\frac{1}{5}$  covered at Alexandria (as well as total at the Hellespont). It was found that observed magnitudes were systematically 0.04 too low. With the above results based on early determinations in mind, it may be of interest to try

similar experiments for a variety of phases – both with artificial eclipses and at forthcoming partial lunar eclipses such as those of 1994 May 25 and 1995 April 15.

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## THE BABYLONIAN UNIT OF TIME

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### 1. *Introduction*

Throughout the period covered by the Late Babylonian astronomical texts (approximately 750 B.C. to A.D. 75), the fundamental unit of time was the *uš*, usually translated as 'degree'. Many of the extant inscriptions from this major archive, now very largely in the British Museum, contain measurements of the times of various lunar and planetary phenomena in terms of this unit (or its multiple, the *bēru*, equal to 30 *uš*). Hence accurate knowledge of the modern equivalent is of considerable importance.

According to Neugebauer,<sup>1</sup> "The 'degree' (*uš*) is the fundamental unit for the measurement not only of arcs, especially for the longitude, but also for the measurement of time, corresponding to our use of right ascension. Therefore 1 degree = 4 minutes of time". Translations of the Late Babylonian astronomical texts (e.g. by Sachs and Hunger<sup>2</sup>) customarily render the term *uš* directly as degree.<sup>3</sup> The implications are, equally, that there were 360 of these units in a combined day and night.

Despite such apparent consistencies, it seems desirable to consider in detail the relation between the *uš* and modern units of time, and also any possible change in definition down the centuries or, alternatively, any seasonal variation. For example, the Late Babylonian astronomical texts (hereafter: LBAT) often quote durations of solar eclipses as so many units of day (*uš-me*) and of lunar eclipses in terms of units of night (*uš-ge<sub>6</sub>*). The question whether, for instance, the hours of both daylight and darkness may each have been divided into 180 *uš* needs to be addressed. Thus, seasonal hours (ᾠραι καίρικαι), twelve to the day and twelve to the night, were regularly used by the ancient Greeks.<sup>4</sup> Alternatively, on account of the obliquity of the ecliptic, the daily motion of the Sun in right ascension varies cyclically during the course of the year (reaching extremes at the solstices and equinoxes). There is thus a possibility that the *uš* might have shown a variation of this form.

Interpretation of the *uš* is of more than historical interest. Babylonian measurements of the times of *onset* of the various phases of both lunar and solar eclipses relative to sunrise or sunset are systematically expressed in terms of this unit. These observations provide perhaps the most reliable ancient data for investigating long-term changes in the length of the day — due to tides and other causes.<sup>5</sup> This topic is currently of considerable interest in geophysics.

The calculated *durations* of the various phases of lunar eclipses (unlike their



times of onset) are independent of changes in the Earth's rate of rotation, and as a result, accurate computation of durations is possible even for remote Antiquity.<sup>6</sup> Hence the numerous measurements of this kind that are preserved on the LBAT provide perhaps the best data for investigating the equivalence and constancy of the *uš* — by comparing recorded intervals with computation.

## 2. Late Babylonian Observations of Lunar Eclipses

Observations of lunar eclipses are found in four types of LBAT: diaries, goal-year texts, eclipse tables, and texts specifically devoted to a single eclipse. Transliterations and translations (along with Julian dates) of many Babylonian eclipse records from about 700 B.C. to 50 B.C. are given by Huber<sup>7</sup> in an unpublished manuscript. In addition, the Babylonian astronomical diaries transliterated and translated by Sachs and Hunger<sup>8</sup> contain several observations omitted by Huber. Although Sachs and Hunger's publications so far extend only down to 165 B.C., Professor Hunger has kindly supplied us, in advance of publication, with further material originating from after this date.

Unless the Moon rose or set eclipsed, Babylonian astronomers systematically measured the following time-intervals for total eclipses: (A) the duration of entrance of the Moon into the umbral shadow (i.e. from first contact to immersion); (B) the duration of totality (from immersion to emersion); and (C) the duration of exit from the umbral shadow (from emersion to last contact). Usually the entire interval from first to last contact was also noted; this is, of course, merely the sum of the intervals (A), (B) and (C). If the Moon rose or set whilst eclipsed, the Babylonians estimated the durations of the *visible* phases. Measurements were presumably made with the aid of a water clock, although little information is available on the instruments they used.

For a partial eclipse, three intervals were also regularly measured: (D) duration from first contact until no further increase in phase was detectable by the observers; (E) the interval during which no change in phase was noticeable; and (F) the interval from the moment when a decline in phase was first noticed to last contact. No other ancient or medieval civilization seems to have distinguished phase (E), which although arising from the limited acuity of the unaided eye, requires considerable care in observation. Recorded estimates of the duration of this phase are typically around 5 to 7 *uš*. Computation of the change in the degree of obscuration of the Moon in 3 degrees (roughly half of the above interval) yields a result close to 2% of the lunar diameter for all but the smallest eclipses. In angular measure, this corresponds to only about 0.5 arcmin.

Although the duration of phase (E) cannot be computed since it is purely an optical effect, when the full duration of an eclipse is not preserved — e.g. on account of textual damage or the Moon's rising or setting eclipsed — a reasonable estimate of the interval from either start to mid-eclipse (G) or mid-eclipse to end (H) may be obtained by adding half of (E) to either (D) or (F). All of the intervals (A), (B), (C), (G) and (H) can be readily compared with computation.



In principle, interval (A) should be equal to (C) and (G) to (H), by symmetry. However, as their measurements reveal, the Babylonians did not necessarily assume this to be the case: clock drift and the limited acuity of the unaided eye, as well as the diffuseness of the edge of the terrestrial shadow, are likely to have been responsible for the discrepancies. As the contacts of small partial eclipses are difficult to define with the unaided eye since the Moon then enters the Earth's umbra very obliquely, we have throughout this paper rejected all eclipses whose magnitude (proportion of the lunar diameter obscured) was less than 0.2.

For each of the reported durations, we have compared the measured interval with our computed equivalent. A detailed discussion of the method of computation used by the authors was given in a recent paper<sup>9</sup> and need not be repeated here. In this same paper, the authors compiled a list of Babylonian lunar eclipse durations and these data form the basis of the present investigation. However, before analysing this material, it is important to consider in detail its suitability for our purpose. On account of the effect of the terrestrial atmosphere, the edge of the Earth's umbral shadow is far from sharp, so that lunar eclipse contacts are often poorly defined. For comparison, we have investigated two sets of independent observations made with the unaided eye in which the interpretation of the unit of time is not problematical. These data are from China and Europe.

### 3. Analysis of Lunar Eclipse Durations Recorded in the History of China and Europe

We have compiled Chinese measurements of eclipse durations (all between A.D. 400 and 1300) from two main sources: (i) the *Shou-shiliyi* ("Treatise on the season-granting calendar"), in the official history of the Yuan Dynasty;<sup>10</sup> and (ii) a recent compilation by Beijing Observatory of eclipse records in other early Chinese sources.<sup>11</sup> Medieval Chinese astronomers systematically measured durations of the various phases of lunar eclipses in either of two units: *ke* ('divisions') or *geng* ('night watches'). It is well established that the *ke* was of fixed length, precisely 100 of these units making up a complete day and night at almost all periods of Chinese history before the Jesuit era.<sup>12</sup> The interval between dusk and dawn was divided into five equal *geng* (night-watches), and hence, unlike the *ke*, these units showed seasonal fluctuations.<sup>13</sup> Eclipse durations were commonly quoted to the nearest fifth of a *geng* (from about 0.4 to 0.6 hours).

We have reduced each of the recorded measurements to hours and have compared these results with the computed values. The results of our investigation of this material are summarized in Table 1. This gives in order the following information:

- column 1: Julian date;
- column 2: the computed eclipse magnitude;
- column 3: the measured interval expressed in hours (after reduction from the original units);
- column 4: our computed equivalent interval in hours; and
- column 5: the ratio of the results in columns 3 and 4 (for reference).

TABLE 1. Analysis of Chinese measurements for lunar eclipses.

Date	Mag.	M	C	M/C
434/09/04	1.465	0.81	1.10	0.74
437/12/28	1.335	1.02	1.17	0.87
440/10/26	-0.824	0.94	1.48	0.64
585/01/20	-0.765	1.50	1.42	1.06
585/01/20	-0.765	1.00	1.42	0.70
595/12/22	0.637	1.81	1.42	1.28
595/12/22	0.637	1.56	1.42	1.22
596/12/10	-1.765	3.87	3.90	0.99
1071/12/09	0.426	0.96	1.13	0.85
1073/04/24	0.637	1.20	1.33	0.90
1073/04/24	0.637	1.52	1.33	1.14
1074/10/08	1.785	1.37	1.07	1.28
1099/11/30	-1.637	2.00	1.88	1.06
1099/11/30	-1.637	2.00	1.88	1.06
1106/01/21	-0.802	1.28	1.48	0.86
1168/03/25	1.776	1.00	1.03	0.97
1270/04/08	0.831	1.28	1.50	0.85
1270/04/08	0.831	1.44	1.50	0.96
1272/08/10	0.570	1.44	1.28	1.13
1272/08/10	0.570	1.28	1.28	1.00
1277/05/18	-1.275	1.28	1.10	1.16
1277/05/18	-1.275	0.96	1.22	0.79
1277/05/18	-1.275	1.28	1.10	1.16

M: measured interval (in hours)

C: computed interval (in hours)

In this table, duplication of dates refers to different phases of the same eclipse.

The mean ratio of the various results in Table 1 is  $0.99 \pm 0.04$ . This figure will be compared with that obtained from the European data.

European measurements that we have investigated were all taken from a single source, the compilation of Pingré.<sup>14</sup> We have restricted our attention to observations ranging in date from 1601 to 1620. In the earlier half of this period, observations were necessarily made with the unaided eye. Virtually all of the later measurements were probably also made in the same way since dissemination of the telescope was slow and early instruments had such a narrow field of view that they were unsuitable for eclipse observations. Durations were, of course, measured in hours.

The results of our investigation of the European data are summarized in Table 2. This gives in order the following information:

column 1: Julian date;

column 2: the computed eclipse magnitude;

column 3: the measured interval expressed in hours (the standard mode);

column 4: our computed equivalent interval in hours; and

column 5: the ratio of the results in columns 3 and 4 (for reference).

In this table, duplication of dates refers to measurements of the same eclipse by different observers.



TABLE 2. Analysis of European measurements for lunar eclipses.

Date	Mag.	M	C	M/C
1601/12/09	0916	3.19	3.02	1.06
1601/12/09	0916	3.60	3.02	1.19
1601/12/09	0916	3.08	3.02	1.02
1603/05/24	0.621	2.98	2.80	0.99
1603/05/24	0.621	2.80	2.80	1.00
1605/04/03	-0.994	3.30	3.13	1.05
1605/04/03	-0.994	3.33	3.13	1.06
1605/04/03	-0.994	3.27	3.13	1.04
1605/04/03	-0.994	3.33	3.13	1.06
1605/09/26	0.674	2.97	2.98	1.00
1609/01/19	-0.809	3.17	3.12	1.02
1609/01/19	-0.809	3.10	3.12	0.99
1610/12/29	0.387	2.17	2.13	1.02
1612/05/14	0.561	2.47	2.65	0.93
1616/08/26	-1.038	3.47	3.15	1.10
1616/08/26	-1.038	3.17	3.15	1.01
1616/08/26	-1.038	3.57	3.15	1.13
1616/08/26	-1.038	3.13	3.15	0.99
1616/08/26	-1.038	3.20	3.15	1.02
1616/08/26	-1.038	3.22	3.15	1.02
1617/08/16	1.405	1.40	1.41	0.99
1617/08/16	1.405	1.23	1.41	0.87
1617/08/16	1.405	1.28	1.41	0.91
1619/12/19	0.906	2.93	3.01	0.97
1620/06/14	-1.518	3.85	3.85	1.00
1620/06/14	-1.518	3.83	3.85	0.99
1620/06/14	-1.518	1.53	1.63	0.94
1620/06/14	-1.518	1.53	1.63	0.94
1620/06/14	-1.518	1.72	1.63	1.06
1620/06/14	-1.518	1.48	1.60	0.96
1620/12/09	-1.601	1.53	1.60	0.96
1620/12/09	-1.601	1.63	1.60	1.02
1620/12/09	-1.601	1.58	1.60	0.99
1620/12/09	-1.601	3.60	3.58	1.01

The mean ratio of the various results in Table 2 is  $1.01 \pm 0.01$ .

The above results for both Chinese and European lunar eclipses give mean values extremely close to unity for the ratio between the measured time-intervals and our computed results. Hence the validity of our method of investigation is confirmed. It thus seems fully justifiable to extend the same technique to the analysis of Babylonian data, in order to consider their definition of the unit of time.

#### 4. Analysis of Lunar Eclipse Durations Recorded on the Late Babylonian Astronomical Texts

In Table 3 are listed the results of our investigation of the Babylonian data. Each individual measurement was expressed to the nearest *uš*. We have considered

TABLE 3. Analysis of Babylonian measurements for lunar eclipses.

Date	Night	Interval	Mag.	M	C	M/C
-561/03/02	12.68	B	-1.772	25.0	1.740	14.368
-561/03/02	12.68	C	-1.772	18.0	1.053	17.089
-554/10/06	12.12	A	1.530	17.0	1.100	15.455
-554/10/06	12.12	B	1.530	28.0	1.640	17.073
-554/10/06	12.12	C	1.530	20.0	1.100	18.182
-500/11/07	13.20	A	1.474	15.0	1.113	13.473
-500/11/07	13.20	B	1.474	25.0	1.573	15.890
-500/11/07	13.20	C	1.474	25.0	1.113	22.455
-423/09/26	11.82	G	-0.929	24.5	1.707	14.355
-423/09/26	11.82	H	-0.929	24.5	1.707	14.355
-409/12/21	14.01	G+H	-0.952	60.0	3.180	18.868
-406/10/21	12.67	A	1.386	21.0	1.107	18.976
-406/10/21	12.67	B	1.386	12.0	1.453	08.257
-406/10/21	12.67	C	1.386	15.0	1.107	13.554
-405/04/15	11.20	A	1.273	25.0	1.093	22.866
-405/04/15	11.20	B	1.273	19.0	1.220	15.574
-377/04/06	11.49	A	1.317	15.0	1.053	14.241
-377/04/06	11.49	B	1.317	21.0	1.280	16.406
-377/04/06	11.49	C	1.317	19.0	1.053	18.038
-370/11/11	13.32	A	1.357	22.0	1.133	19.412
-370/11/11	13.32	B	1.357	20.0	1.413	14.151
-370/11/11	13.32	C	1.357	21.0	1.133	18.529
-352/11/22	13.62	A	1.349	23.0	1.133	20.294
-352/11/22	13.62	B	1.349	18.0	1.413	12.736
-345/01/18	13.79	G+H	-0.199	23.0	1.600	14.375
-316/12/13	13.79	A	1.337	19.0	1.140	16.667
-316/12/13	13.79	C	1.337	16.0	1.133	14.118
-272/02/16	13.05	B	-1.335	19.0	1.293	14.691
-272/02/16	13.05	C	-1.335	22.0	1.040	21.154
-238/04/28	10.75	G	0.407	20.0	1.180	16.949
-238/04/28	10.75	H	0.407	20.0	1.180	16.949
-225/08/01	10.13	A	-1.208	17.0	1.120	15.179
-225/08/01	10.13	B	-1.208	10.0	1.073	09.317
-255/08/01	10.13	C	-1.208	15.0	1.120	13.393
-214/12/25	14.02	A	1.374	21.0	1.053	19.937
-214/12/25	14.02	B	1.374	16.0	1.380	11.594
-214/12/25	14.02	C	1.374	19.0	1.053	18.038
-188/02/17	13.00	A	-1.278	16.0	1.220	13.115
-153/03/21	11.96	G	0.850	23.0	1.653	13.911
-153/03/21	11.96	H	0.850	21.0	1.653	12.702
-149/07/03	09.71	A	-1.110	20.0	1.353	14.778
-149/07/03	09.71	B	-1.110	12.0	0.873	13.740
-142/02/17	12.98	G	-0.883	22.5	1.593	14.121
-128/11/05	13.21	G	0.629	21.0	1.433	14.651
-128/11/05	13.21	H	0.629	19.0	1.433	13.256
-123/08/13	10.48	A	-1.489	19.0	1.127	16.864
-123/08/13	10.48	B	-1.489	24.0	1.620	14.815
-123/08/13	10.48	C	-1.489	19.0	1.127	16.864
-119/06/01	09.89	A	-1.019	24.0	1.420	16.901
-119/06/01	09.89	B	-1.019	06.0	0.333	18.000
-119/06/01	09.89	C	-1.019	24.0	1.420	16.901
-079/04/11	11.23	G	-0.600	23.5	1.387	16.947
-066/01/19	13.72	H	-0.806	19.0	1.613	11.777



only observations made after about 560 B.C. since before then it was the practice of the astronomers to round all measurements to the nearest 5 or 10  $\mu\check{s}$ .

The various columns of Table 3 give in order the following information:

- column 1: Julian date;
- column 2: the computed length of the night on that date in hours;
- column 3: a letter identifying the interval as defined in Section 2 above;
- column 4: the computed eclipse magnitude;
- column 5: the measured interval expressed in  $\mu\check{s}$ ;
- column 6: our computed equivalent interval in hours; and
- column 7: the ratio of the results in columns 5 and 6 (for reference).

The mean number of  $\mu\check{s}$  per hour over the entire period covered by this table, is  $15.8 \pm 0.4$ . This is marginally greater than the 'accepted' figure of 15.0, but we shall discuss this further below.

In Figure 1 are plotted all the data in column 7 of Table 3 as a function of the year. The diagram shows a fair degree of scatter, but some of this could well be due to scribal errors in copying or re-copying. It will be seen that the straight line of best fit has a slight gradient (of  $-0.4$  per century) but in our opinion this is spurious. A very gradual change in the definition of a fundamental unit seems extremely unlikely. In particular, the graph gives no evidence for any *sudden* changes in the definition of the  $\mu\check{s}$ .

In Figures 2(a) and 2(b) we have separated the Babylonian data into two groups: before and after the approximate mean epoch of 300 B.C. Each graph shows the straight line of best fit. In the earlier group, the mean result for the above ratio is as large as  $16.3 \pm 0.6$ . However, for the later group, the result is very close to

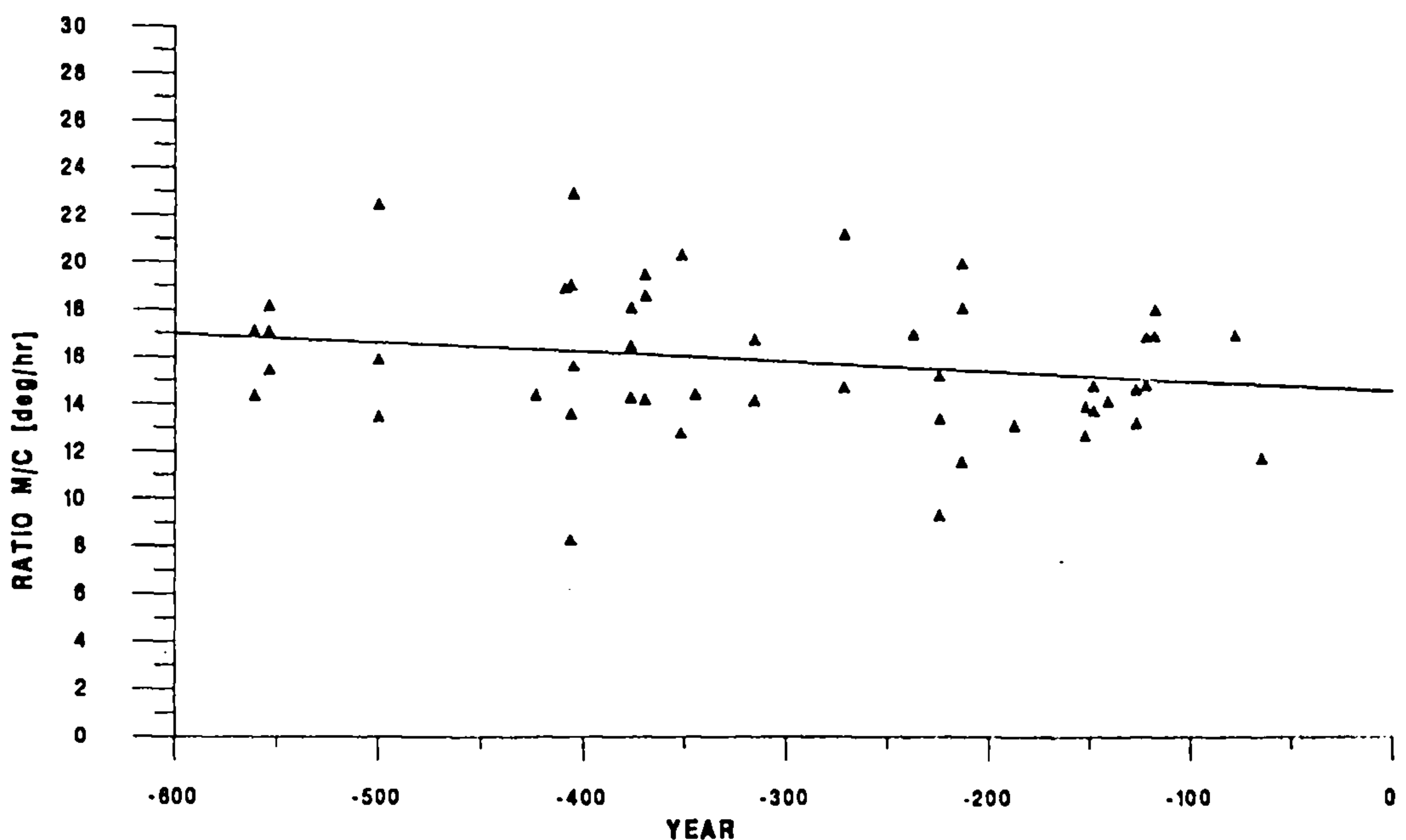


FIG. 1. Graph showing individual results for the number of  $\mu\check{s}$  in an hour as obtained from analysis of Late Babylonian lunar eclipse durations.

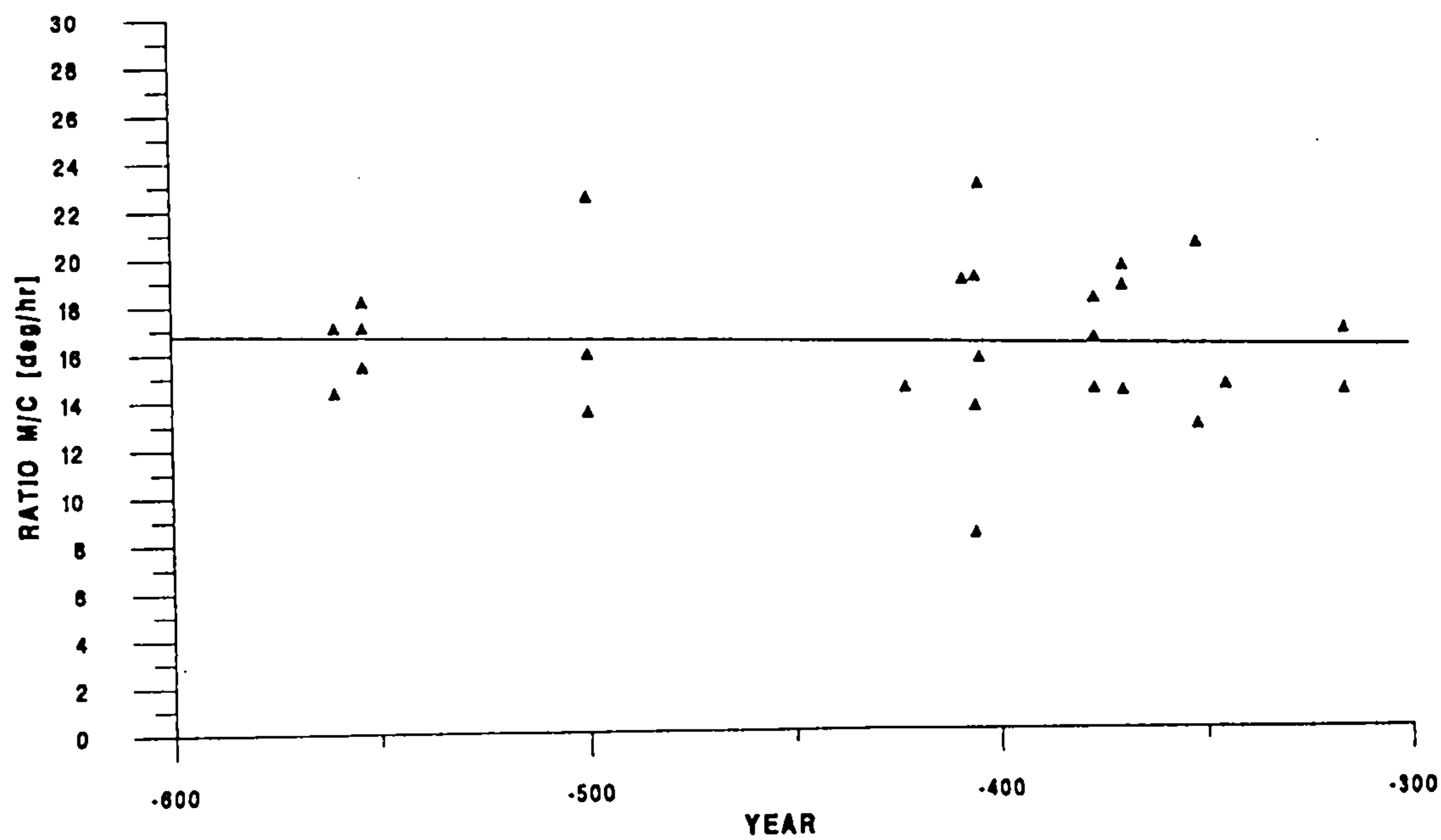


FIG. 2(a). As for Fig. 1 but for all data before before 300 B.C.

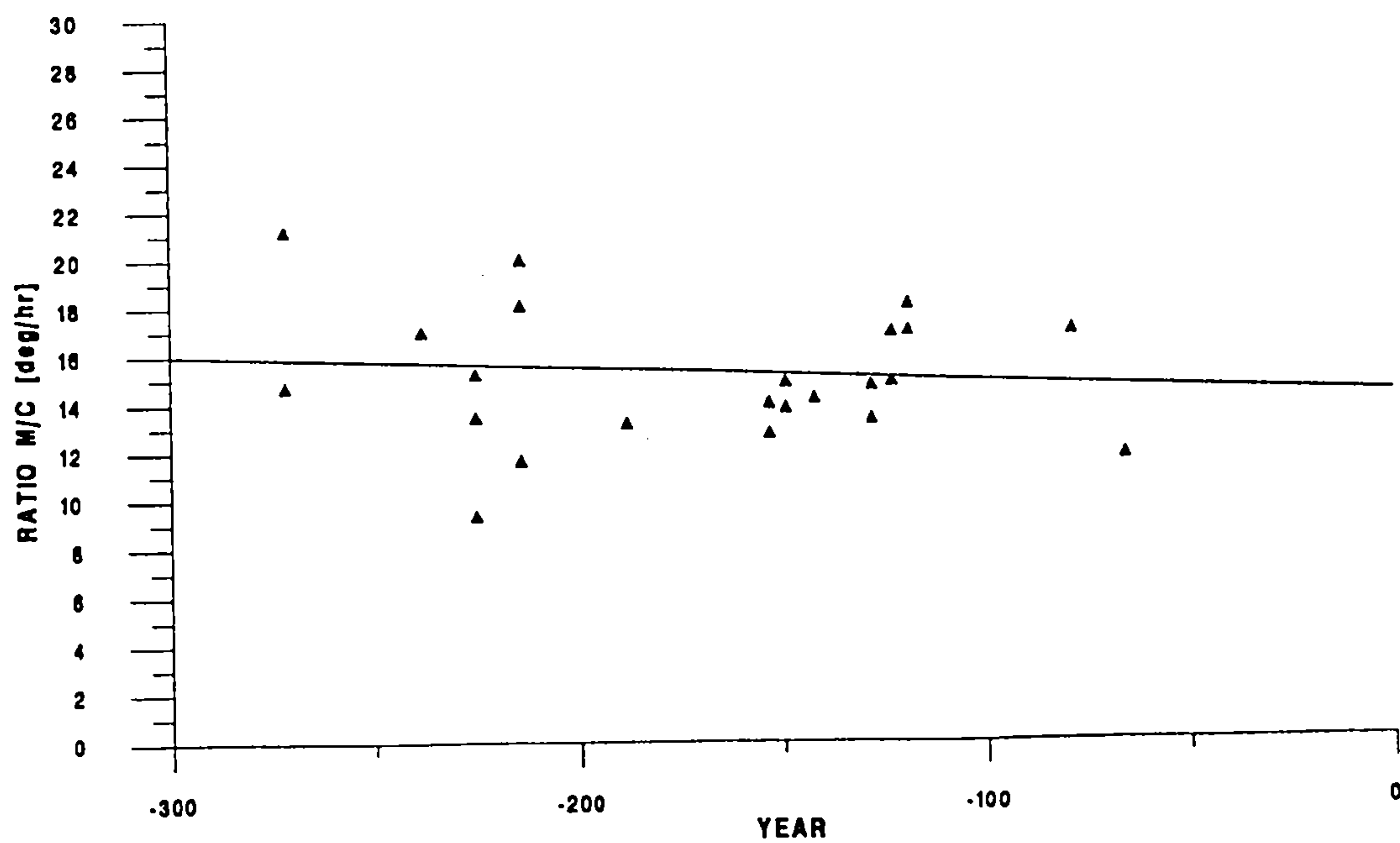


FIG. 2(b). As for Fig. 1 but for all data after 300 B.C.



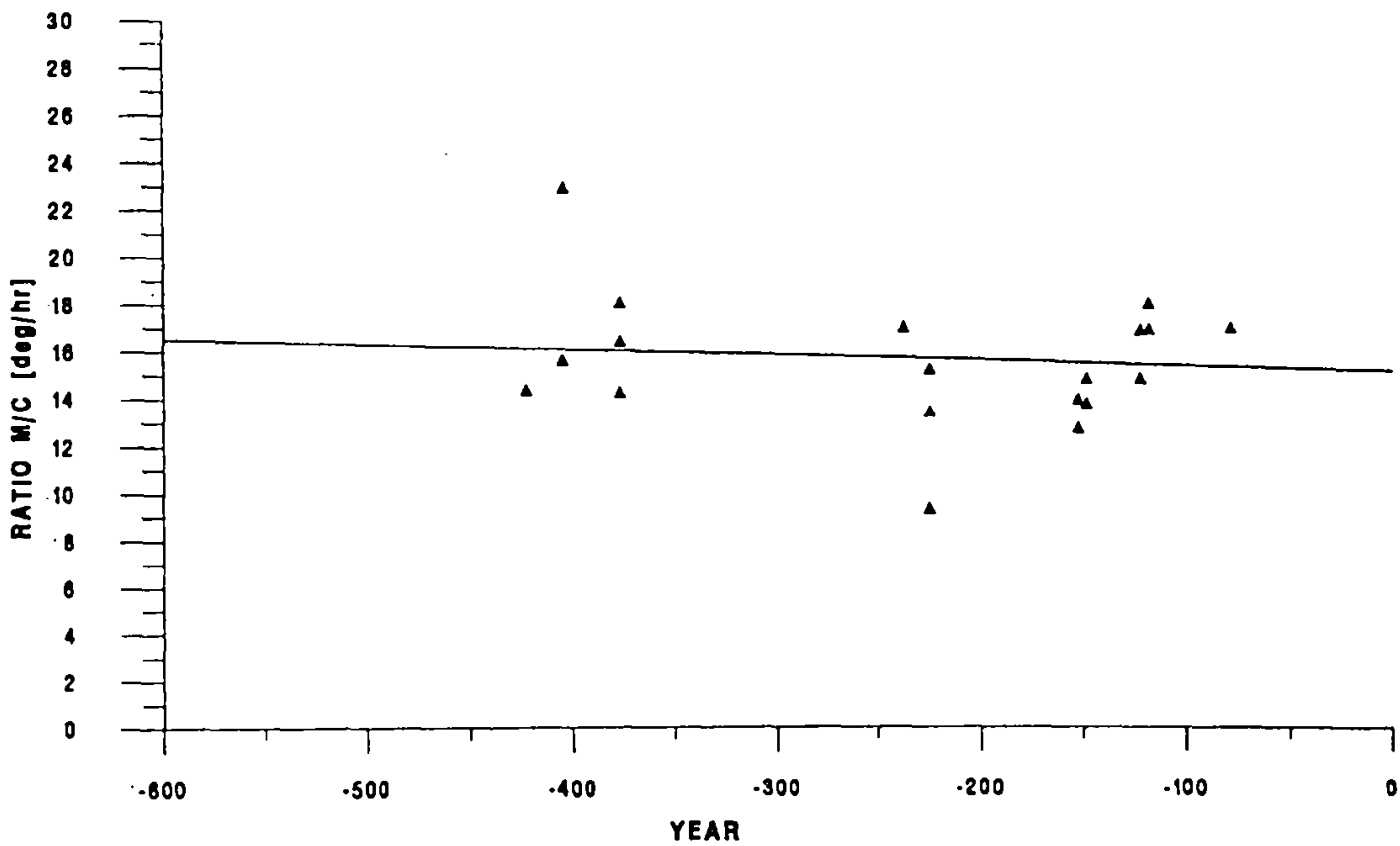


FIG. 3(a). Graph showing individual results for the number of  $\mu\tilde{s}$  in an hour as obtained from all 'summer' data.

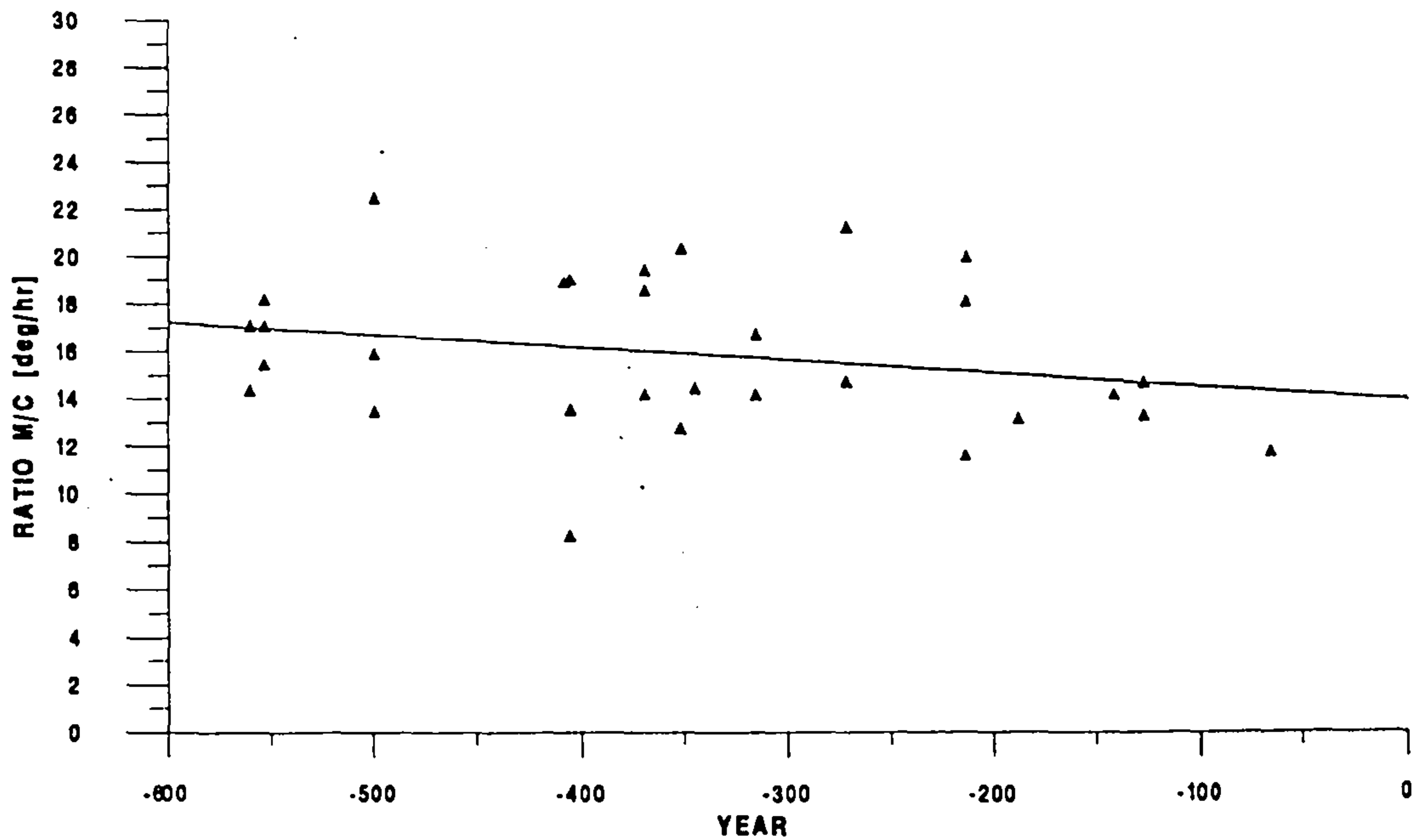


FIG. 3(b). Graph showing individual results for the number of  $\mu\tilde{s}$  in an hour as obtained from all 'winter' data.

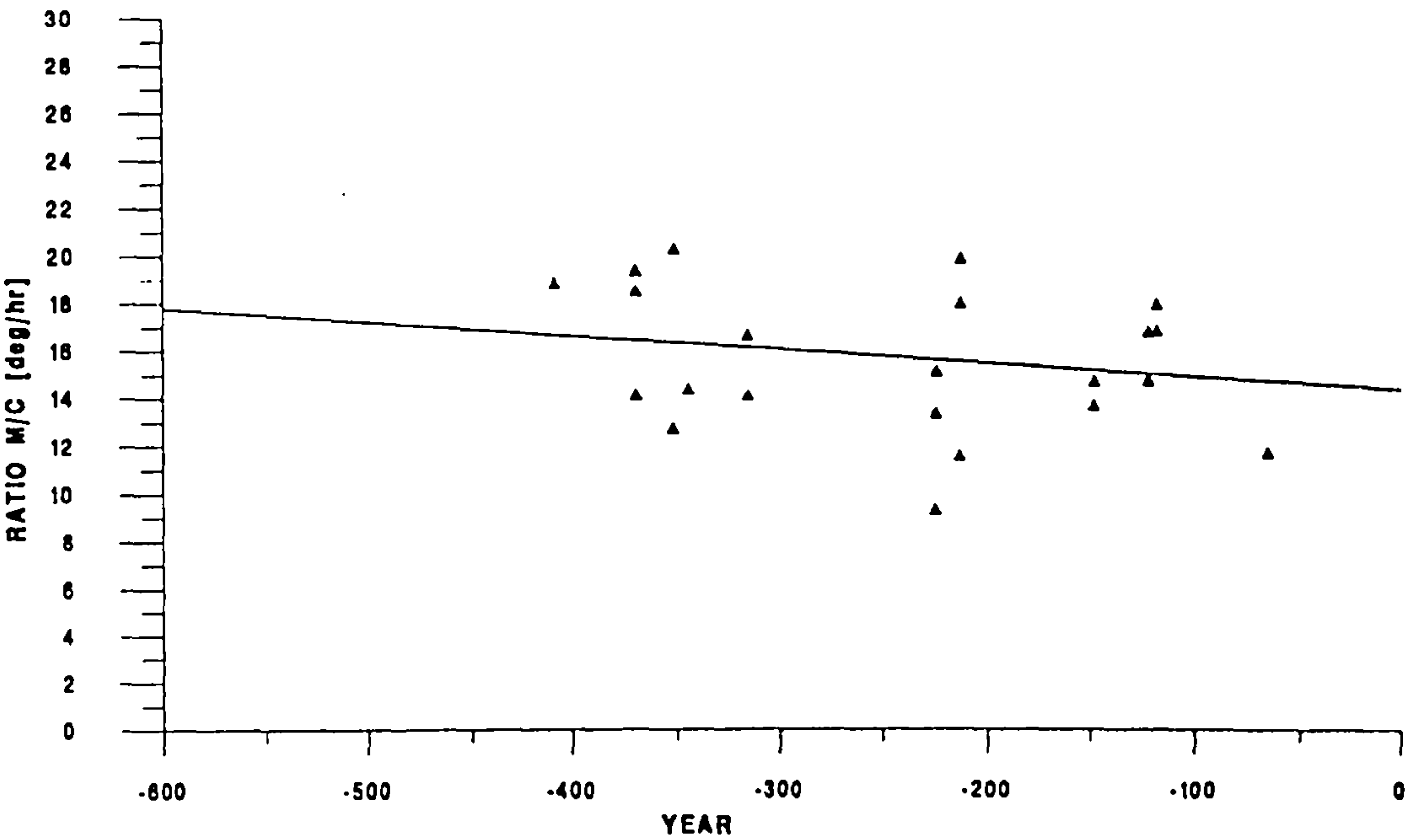


FIG. 4(a). Graph showing individual results for the number of  $\mu s$  in an hour as obtained from all 'solstice' data.

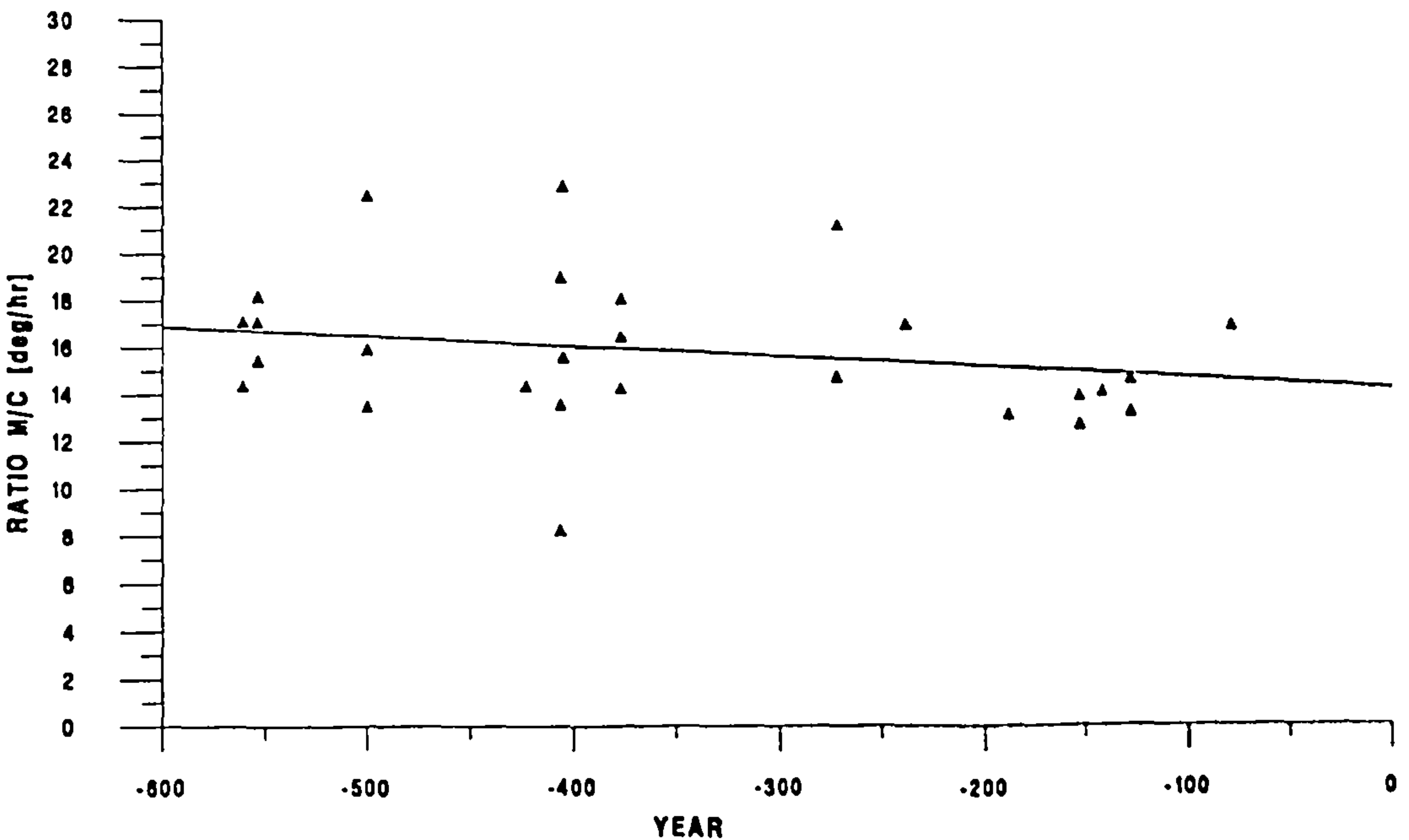


FIG. 4(b). Graph showing individual results for the number of  $\mu s$  in an hour as obtained from all 'equinox' data.



15.0 — actually  $15.3 \pm 0.5$ . We feel that there is sufficient evidence here to conclude that over the period covered by the two diagrams there were precisely 15 *uš* in an hour.

The above conclusion ignores possible seasonal effects, which we shall now investigate.

In Figures 3(a) and 3(b) we have made plots similar to Figure 1 but we have now separated the data into 'summer' and 'winter' components. For the purpose of this analysis, we define 'summer' as the period when nights are shorter than 12.0 hours and "winter" when nights are longer than 12.0 hours. Summer nights at Babylon can actually be as short as 10.0 hours and winter nights as long as 14.0 hours, the mean ratio for the length of nights in Figures 3(a) and 3(b) being 0.71: 1. Our result for the mean number of *uš* in an hour from summer observations alone is  $15.7 \pm 0.5$  whereas for winter measurements it is  $15.9 \pm 0.6$  — virtually identical results. Hence it may be concluded that the evidence is strongly against any seasonal variation in the *uš* between summer and winter.

In Figures 4(a) and 4(b) we have separated 'solstice' and 'equinox' data. We define 'solstice' observations as those made when nights were either shorter than 10.75 hours or longer than 13.25 hours. Data for which the nights were of intermediate duration were included in the 'equinox' category. These limits were chosen so that there were roughly equal numbers of data in the two sets. The mean number of *uš* in an hour for solstice data proves to be  $15.7 \pm 0.6$  whereas for equinox data it is  $15.8 \pm 0.6$ . Here again the evidence is contrary to any seasonal variation in the *uš* between solstices and equinoxes.

## 5. Conclusion

Our investigation of lunar eclipse durations shows clearly that the Babylonian time unit known as the *uš* shows no seasonal variations, and there is furthermore no evidence of any change in its definition over the centuries covered by the Late Babylonian astronomical texts. Our result for the mean number of *uš* in an hour ( $15.8 \pm 0.4$ ) is sufficiently close to the accepted figure of 15.0 to provide satisfactory confirmation, especially since the later observations indicate an even closer figure ( $15.3 \pm 0.5$ ). It does seem rather curious that all our individual results are marginally above 15.0 but this presumably arises from the limited number of data used. Babylonian arithmetic was sexagesimal so that in a combined day and night any number of *uš* close to (but not equal to) 360 would seem most inappropriate — especially since the *bēru* was defined as 30 *uš*.

The few published discussions of the Babylonian water-clock<sup>15</sup> concern an instrument used to mark the duration of a 'watch' of the night (i.e. one-third of the night). This was filled with varying amounts of water according to the seasons of the year, the outflow being variable and the end of a watch corresponding with the complete emptying of the water-clock. The length of the Babylonian 'double-hour' traditionally had a corresponding seasonal variation. Accurate measurement of unpredictable time-intervals (for example eclipse phases),

however, requires the use of a water-clock with a constant inflow and outflow of water. Our investigation seems to confirm what has long been assumed, that Babylonian astronomical observations were measured by means of this second type of water-clock, for which there has until now been no direct published evidence.

We feel that the exact equivalence  $1 \text{ } \mu\text{š} = 4.0$  minutes of time can be confidently accepted in any investigation of Late Babylonian astronomical time measurements.

### Acknowledgement

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