An assessment of aerial surveyed string digital ground models

Heatherington, S.

AN ASSESSMENT OF AERIAL SURVEYED STRING DIGITAL GROUND MODELS

BY

S. HEATHERINGTON B.Sc.

Thesis submitted to the University of Durham April, 1974 for the award of M.Sc.

An Assessment of Aerial Surveyed String Digital Ground Models

ABSTRACT

The Thesis reports research carried out in the Surveyor's Department of Durham County Council, into the use of ground models in highway design. The research isolates the various sources of error that contribute to the ultimate error in earthworks quantities, obtained from an aerial surveyed string digital ground model, and assesses their relative importance.

The research covers the following topics :

1. Isolation of the various sources of error contributing to error in calculated quantities e.g.

   (a) error in preparation of model.

   (b) error inherent in the type of model specified.

   (c) error in technique used to interpolate from the model.

2. Detailed investigation of each source of error to compare magnitude and type (i.e. random or systematic).

3. Specification of a procedure for testing a model to assess its accuracy both for design quantities and ultimately for contractors acceptance.

The use of the aerial surveyed string ground model is justified as being an invaluable aid to the practising highway engineer and a practical method of proving the accuracy of a model is provided. To support the research two test areas have been precisely levelled and the results are discussed, together with modifications to data collection and data retrieval techniques.

ACKNOWLEDGEMENTS

I am completely aware that without the guidance and advice of a number of people the project would not have left the ground. Particular thanks are extended to the following :

1. J. R. Tully, B.Sc., F.I.Mun.E., M.I.C.E. County Engineer and Surveyor, Durham County Council for allowing the time, facilities and expertise of his staff, to undertake the work involved.
2. Durham University for accepting the topic as a subject of study for a part time course.
3. G. S. Craine, B.Sc., M.I.C.E., M.I.Mun.E., for many hours of fruitful discussion and practical advice, and for suggesting the topic of research. Without Gordon's experience and enthusiasm literally nothing would have been achieved.
4. Dr. A. G. Hawkes, Senior Reader at Durham University who has been my course supervisor and helped translate the practical realities into a theoretical basis from which results could be formulated.
5. B. K. S. (Air Surveys) Ltd., for the provision of an aerial surveyed ground model knowing it was to undergo extensive criticism, albeit constructive. I was very much the benefactor of a visit to their premises in Coleraine, where I gained an important insight into the production of aerial surveyed maps and models so that the improvements I suggest could be of practical credence.
6. R. E. Beels, A.R.I.C.S. for ensuring the field information was collected in a precise manner.

7. Mrs. Janis Wood for the conscientious and accurate typing of a difficult mathematical text on a non-mathematical typewriter.

There are many others who have been of help but cannot be singled out and to all I am most grateful.

Stuart Heatherington.

STUART HEATHERINGTON.

CONTENTS

CHAPTER 1 INTRODUCTION

The increase in mobility which has gone hand in hand with the developments of science and technology has necessitated improved transportation systems. Road networks are important for the effective exploitation of natural resources both within a region and between regions. The basic unit in the network viz. a road, is built to serve as a line of communication between two or more places.

The design of a road poses many and varied problems. Factors to be considered include traffic studies, route demand, geographical, geological, physical obstructions (e.g. factories, historical monuments etc.) design standards and construction costs. The civil engineer is therefore faced with a substantial amount of information on which to base the final design and for that design to be the optimum, the information needs to be both accurate and useable.

Determining earthworks quantities is a major problem in road construction. Estimates of earthworks volumes are required at various stages of a scheme design, often for several alternatives and they must be produced with reasonable speed and accuracy. When an alignment is finalised accuracy is often more important than speed but the engineer is confronted with the problem of collecting ground data to satisfy both these requirements.

Ground models have been developed in recent years which introduce the flexibility to enable assessment of alternative schemes but the accuracy of the resulting volumes has often been questioned when compared with the previous methods of setting out and levelling cross-sections.

It is impossible to collect and store sufficient data to be able to determine any ground level with 100% certainty and for this reason a model needs to be sought to enable earthworks estimates to be made to the desired accuracy. The forces of nature are so complicated that one cannot hope to simulate the land surface by even a complex mathematical function. The most that can be expected is that a multitude of three-dimensional points can be collected in a random or specified manner and stored so that intermediate points may be interpolated, either by a linear relationship between two adjacent points, or, where curvature is noticeable, by a low order equation.

Using such a model the continuous land surface is represented by a discrete set of values and inaccuracies immediately become apparent. The inaccuracies may be kept within specified bounds by the density of the points, and the method which links the points together. The project has been concerned with these inaccuracies and has had the following terms of reference :

1. Justify the use of a ground model in calculating earthworks quantities.
2. Investigate the methods of storage and retrieval of information and suggest additions or amendments to the present techniques wherever necessary.
3. Investigate testing procedures for acceptance of contracted ground models by aerial survey and suggest an appropriate test on which to base the acceptance.

The work has been concerned with String Digital Ground Models produced by aerial survey. This bias is towards string model techniques because they are basically considered to provide greater engineering potential but the model testing and assessment procedures can be applied to other forms of model.

The results of the project show that the aerial surveyed S.D.G.M. (string digital ground model) is a very sound concept and it's use in calculating earthworks quantities is justified. The models can have an important role in both the preliminary and final stages of highway design. The most significant error is shown to be in the initial preparation of the model and is systematic. The investigation of the storage and retrieval of information has revealed some inadequacies in the present practice but the modifications which will be outlined for the S.D.G.M. should provide better use of the collected information. Arising from the analysis of the errors a testing procedure is described which is simple to apply and uses the relevant statistics of mean and variance. The improved specification and testing procedures should provide better ground models from which realistic information will be supplied to the practising engineer.

CHAPTER 2 BACKGROUND

2.1 Introduction

Traditional practise in calculating proposed earthworks quantities was to measure in the field, the levels across cross-sections perpendicular to the alignment of the designed road, and calculate the areas of the cross-sections by trapeziums. The cross-sections were regularly spaced along the length of a scheme and were connected to give a volume.

With the development of land survey, aerial survey and computer techniques together with the increased stringency of positioning the roadway, it was found necessary to have sufficient data to enable consideration of alternative designs. A ground model was therefore required independent of cross-sections from which the levels of the cross-sections could be removed.

The concept of the Digital Ground Model (D.G.M.) relies on measuring the levels of co-ordinated points thereby defining the ground topography. Working from this system of three-dimensional points other co-ordinated points are able to be levelled by interpolation either linearly or according to a mathematical function of higher degree.

Various ground models have been designed. The measured points may be regularly spaced or irregularly spaced and they may also be connected to one another or unconnected. The main types of D.G.M. are, the triangular D.G.M., square grid D.G.M. the string D.G.M., and the semis de points D.G.M. These are shown in FIG. 1.

FIG. 1 TYPES OF D.G.M.

Measured Points

Regularly Spaced. — Irregularly Spaced

Unconnected. (Square Grid)

Connected String/Triangular)

Unconnected (Semis De-Points)

2.2 Semis de Points D.G.M.

This system, developed in France and in practical use since 1968 is based upon fitting a surface to the measured points using a second degree function. The points are measured completely at random and any required level is interpolated from those measured points closest to it. The theory of the system is described by Baussart(2) and Deligny (3).

The system pays no regard for the terrain features and the density of the measured points which is predetermined is irregular and independent of the general nature of the ground.

2.3 Square Grid D.G.M.

The Square Grid Model has had wide use in the United Kingdom through its promotion by the B.I.P.S.* suite of computer programs. The method has had wide documentation (1,4). The ground is represented by a series of levels taken at the nodes of a square grid superimposed on the terrain. The width of the square is a variable between different blocks of the model. Although the superimposition of the grid pays no attention to localised features it is a regular sample of the ground. When the model is prepared by aerial survey each point is individually located and the accuracy depends more on the flying height at which the photograph was taken than on the operator of the sterographic

* B.I.P.S. - British Integrated Program Suite.

equipment** which produces the model. Even though maximum accuracy is obtained for each point the individual measurement of the point is time consuming of the photogrammetist and the accuracy may not be reflected in the use to which the data is put.

Data retrieval is by means of double linear interpolation for each point level required. The four node points surrounding the required level are assumed to be points on a hyperbolic paraboloid. Each cross-section required consists of regularly spaced interpolated points. The use of the hyperbolic paraboloid (which is effectively double interpolation) disregards the features of the ground and tends to smooth all predominant local "bumps". This effect can be reduced by having a small grid width but the cost of this could become prohibitive. Manual editing of points and cross-sections is always necessary to guarantee reliable information.

2.4 Triangular D.G.M.

The Triangular D.G.M. is an improvement on the data collection task for the Square Grid Model. Randomly positioned triangles have levels taken at their nodes, the levels usually being measured by ground survey as opposed to aerial survey. The triangles are considered to represent planes and the land surface is built up of many such planes. Ideally cross-sections should be extracted from the triangles but when developed the processing overhead was considered excessive and the transformation of the model into a square grid was adopted*. The square grid model thus created has a much smaller grid width, than would normally be given. The method is useful for producing a perspective view

** A description of the photogrammetric method appears in Reference 5.

* A computer program is now in existence which does use the data direct.

of the ground but considerable editing of the resulting square grid model is necessary to show missing details.

2.5 String D.G.M.

Essentially the string digital ground model consists of a series of strings of co-ordinates. The ground is represented by strings which define the ground features. There are two types of string; breaklines which are three-dimensional and depict all angular features, and contours which are two-dimensional and together with a master level define general ground curvature. Both forms of string are complementary to one another. The contours are specified at a regular level difference but where they become sparse additional break-lines are used to provide better coverage.

The accuracy of the two types of string are different. The 3-D strings are as accurate as the individual level points but the contour strings are produced in a continuous fashion and are slightly less accurate. The frequency of points defining a contour is usually determined by a timing device set at a fixed interval to guarantee sufficient points. This means that when the photogrammetist is leading the floating mark* over undulating ground he is moving slower and more points are given than over even land. The frequency of points on the 3-D strings must be sufficient to adequately define the feature within a required tolerance.

Data retrieval is influenced by the purpose for which the data is required and in present practise, this is the cross-section for subsequent earthwork evaluation. Levels on the cross-section are only defined where the cross-section cuts the

* A summary of the photogrammetric method is given in Reference 5.

features (i.e. strings) stored in the model** and there is no double interpolation as there is in the square grid model.

A basic assumption is that a feature is adequately represented by the series of straight lines joining all the points on a string and the method of storage also assumes a plane between adjacent string lines. The reliability of the strings depend on the number of points used to define them and this is a key to both the usefulness and accuracy of the model.

The string model concept may be used to advantage in defining a ground model by either land survey methods or aerial survey. When the model is produced by land survey it will only consist of 3-D strings and this can be very useful in urban situations. Another of the advantages of the aerial survey technique is that it provides a regular sample of the ground by its use of contours and 3-D strings. In areas of changing slope the contours become more dense and where curvature is noticeable (crossing a railway embankment etc.) the 3-D strings describe it. The direct plotting of the contours and 3-D strings in the projective plane provides an immediate check on both the acceptability of the model and shows up any "blunders" made with the provision of the data.

2.6 Conclusions

There are three factors to be considered in deciding the potential of a ground model :-

1. Ground definition technique.
2. Accuracy of ground measurement.
3. Method of data retrieval.

** This is the method as used at the start of the project. However modifications to the method indicated by the project are now included. A fuller description of the improvements are given in Chapter 5.

A further factor, which is now assuming less importance but which greatly influenced early models is the availability of large and fast computers. Early models were a compromise between ideal model requirements as defined and satisfied by the above three factors and an efficient means of data storage and mathematical processing.

Perhaps the most difficult problem has been that of taking into account the influence of terrain break-lines such as channels, existing roads, and other angular features in the landscape. Those systems using linear interpolation based on terrain elements as break-lines or contours have overcome this problem. Methods using spot heights in the form of a regular grid or random points immediately introduce interpolation difficulties over undulating or angular terrain. This requires the manual editing in of features to overcome model definition inadequacies, and this in turn means a disproportionate increase in the time and cost of processing.

The overwhelming result of the consideration of the factors involved show that String Ground Models have the most potential of the methods at present available.

CHAPTER 3. THEORY

3.1 Introduction

Earthworks quantities are computed from a knowledge of the levels across cross-sections. These are combined to produce a cross-sectional area. The cross-sections are regularly spaced along the length of the design road and the areas are multiplied by the length between them to give a volume. From this it can be seen that the earthworks errors are constituted from the area of the cross-section and the spacing between the cross-sections. The following sections investigate these sources of error and develop the basis for a useful test on the acceptability of provided data.

The volumetric programs work from a file of design line co-ordinates consisting of chainage, easting, northing, whole circle bearing of tangent, and sometimes, radius of curvature. Channel lines, verges and other relevant lines are all designed with respect to this line, being offset perpendicularly from it. The design line is also used for extracting cross-sections from the ground model. When the areas of the individual cross-sections have been found they are multiplied by the chainage interval of the design line to produce the volume.

3.2 Longitudinal Spacing

Although the design line is often the centre line of the road the chainage is not necessarily the true factor to use in calculating the volume. The inaccuracy involved cannot be ignored on bends and when calculating volumes for irregular features such as interchanges.

For example FIG. 3.2(i) shows a typical section of a road. The shaded area represents the natural ground shape. The accepted method of calculating the amount of earthworks required is to calculate the differences in areas of the end cross-sections and multiply their mean by the distance between them.

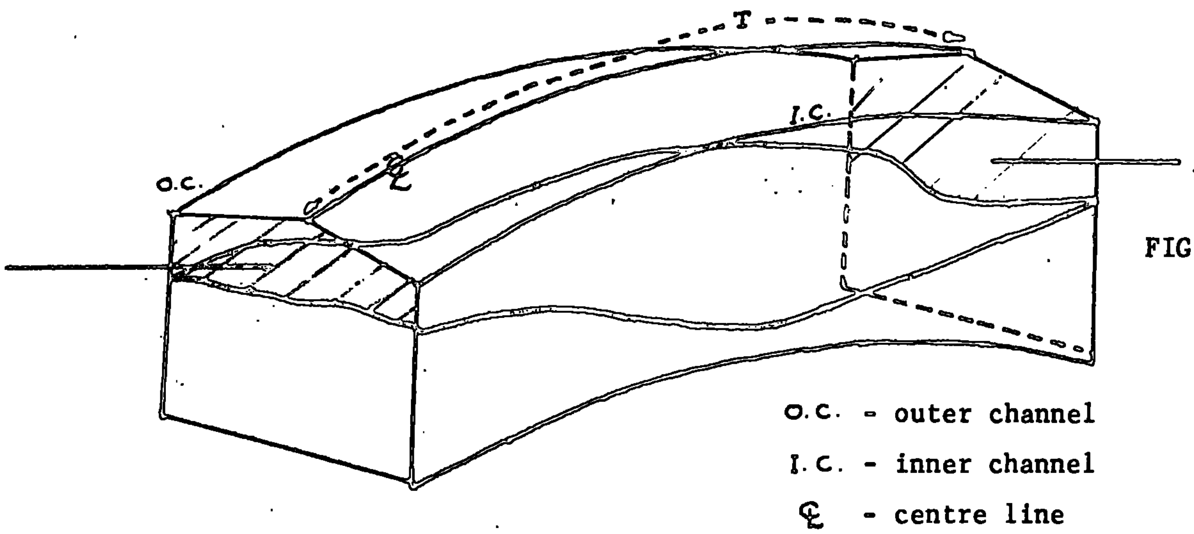


FIG. 3.2(i)

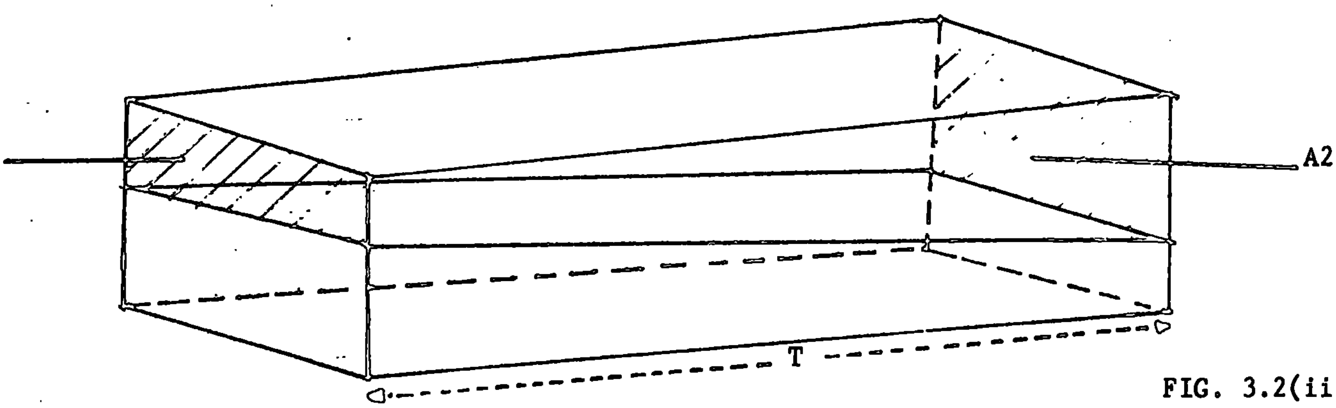


FIG. 3.2(ii)

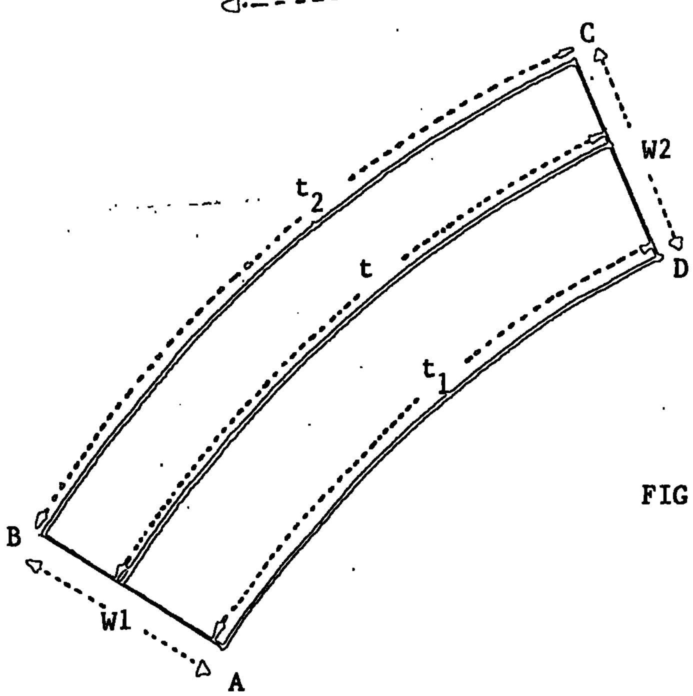


FIG. 3.2(iii)

Effectively FIG 3.2(i) is transformed into FIG 3.2(ii) and the amount of earthworks is $\frac{(A_1 + A_2)}{2}$ T. The loss of accuracy involved in assuming a linear deformation of cross-sections may only be reduced by taking smaller values of T (this will be investigated in 5.2) i.e. taking smaller chainage intervals.

The use of "T" as the multiplying factor is basically the wrong value to use in computing the volume for anything other than a straight road. For example in FIG. 3.2(iii) where the projective plane only is shown, if the design line is the ℄ (centre-line) the value of T taken would be t. Where the design line is the outer channel the value of T taken would be t_2 and for the design line being the inner channel the value taken would be t_1. The effect is further emphasised when the two offset lines are not parallel to the design line.

The traditional approach has been satisfactory in the manual evaluation of earthworks for motorway design but as the emphasis changes to automatic small scheme consideration then a more refined technique is necessary.

If the end area method is to be used the correct value of T may be found by calculating the area enclosed by A, B, C, D in FIG. 3.2(iii) and to divide this area by the means of the two lengths AB and CD (W1 and W2). The following equations show how the true base area ABCD may be computed.

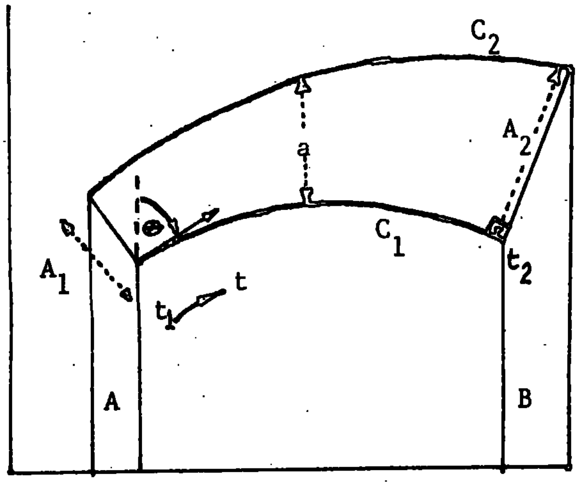


FIG. 3.2(iv)

Considering FIG. 3.2 (iv)

Let $C_1 : x = x(t) ; \; y = y(t)$

Let a vary linearly between the points t_1 and t_2.

$$\therefore \; a = A_1 + (t - t_1) \frac{(A_2 - A_1)}{(t_2 - t_1)}$$

the curve C_2 is offset perpendicularly from C_1.

$$\therefore C_2 : X = x - a\cos\theta \; ; \; Y = y + a\sin\theta$$

but $\cot\theta = y' / x'$

$$\therefore C_2 : X = x - \frac{a}{b} y' \; ; \; Y = y + \frac{a}{b} x' \text{ where } b = (x'^2 + y'^2)^{\frac{1}{2}}$$

thus the area between the two curves C_1 and C_2 including segments

A and B is $\displaystyle\int_{t=t_1}^{t_2} Y \, dx - y \, dX$

$$\text{i.e. area} = \int_{t=t_1}^{t_2} \left(\left(y + \frac{a}{b} x'\right)\left(x' - \frac{a}{b} y'' - \frac{a'y'}{b} + \frac{ay'b'}{b^2}\right) - yx' \right) dt \qquad (1)$$

The area (1) includes the segments A and B (see FIG. 3.2(iv))

Let $x_o = x(t_1)$; $X_o = X(t_1)$; $y_o = y(t_1)$; $Y_o = Y(t_1)$
$x_1 = x(t_2)$; $X_1 = X(t_2)$; $y_1 = y(t_2)$; $Y_1 = Y(t_2)$

$\therefore$ area of $A = \frac{1}{2}(Y_o + y_o)(x_o - X_o)$ area of $B = \frac{1}{2}(Y_1 + y_1)(X_1 - x_1)$

i.e. $$A = \frac{1}{2} \frac{y_o'}{b_1} A_1 (2y_o + \frac{A_1}{b_1} x_o') \qquad (2)$$

$$-B = \frac{1}{2} \frac{A_2}{b_2} y_1' (2y_1 + \frac{A_2}{b_2} x_1') \qquad (3)$$

where at t_1 ; $a = A_1$ $b = b_1$ and at t_2 ; $a = A_2$ and $b = b_2$

The true base area between cross-sections at t_1 and t_2 along the curve C_1 is (1) - (2) - (3)

base area = (1) - (2) - (3) (4)

The solution of equation (4) may be accomplished using numerical integration techniques. However if the areas are reorientated by translation and rotation and the curve C_1 approximated by a circular arc equation (4) may be solved analytically as follows :-

Let $C_1 \triangleq x = R\sin\phi$; $y = R\cos\phi$

where R is the radius and is constant and ϕ is the angle consumed from point (x_o, y_o)

$R = b = (x'^2 + y'^2)^{1/2}$

with these assumptions $a = A_1 + \phi \frac{(A_2 - A_1)}{\phi_1}$

and equation(1)may be written, as

$$\int_{\phi = 0}^{\phi_1} \left(a \left(R + \frac{a}{2}\right)(1 + \cos 2\phi) + \frac{a'}{2} (R + a) \sin 2\phi\right) d\phi \tag{5}$$

and equations (2) and (3) may be written as

$$A = 0 \qquad B = \frac{\sin 2\phi_1}{2} \left(R + \frac{A_2}{2}\right) A_2 \tag{6}$$

equation (5) may be conveniently reduced by integrating by parts the first set of brackets. This enables the second set of brackets to be cancelled and on subtracting (6) the base area reduces to :

$$\text{base area} = \phi \left(\frac{R}{2} (A_1 + A_2) + \frac{(A_2 + A_1)^2}{6} - \frac{A_1 A_2}{6}\right) \tag{7}$$

Where the cross-sections straddle the design line the base area will comprise that area to the left D_L of the design line and that to the right D_R see FIG. 3.2(v).

(It is useful here to introduce the normal sign conventions :

centre of curvature to the left -ve

centre of curvature to the right +ve

offset to the left -ve

offset to the right +ve

area to the left -ve

area to the right +ve)

In FIG. 3.2(v) the following hold

$$D_L = -\emptyset \left(\frac{R}{2} (A_1 + A_2) + \frac{(A_2 + A_1)^2}{6} - \frac{A_1 A_2}{6} \right) \quad (8)$$

$$D_R = \emptyset \left(\frac{R}{2} (A_1* + A_2*) + \frac{(A_2* + A_1*)^2}{6} - \frac{A_1* A_2*}{6} \right) \quad (9)$$

∴ total base area $= D_L + D_R$

the mean width of cross-section in this situation is

$$- \frac{1}{2} (A_1 - A_1* + A_2 - A_2*) \quad (10)$$

and from equations (8), (9) and (10) the length factor to be used in calculating earthworks quantities by the end-area method is :

$$2 \emptyset \left(\frac{R}{2} - \frac{1}{6} \left((A_1 + A_2 + A_1* + A_2*) + \frac{A_1* A_2* - A_1 A_2}{A_1 + A_2 - A_1* - A_2*} \right) \right)$$

NOTE : the absolute value of the length factor ought always to be taken.

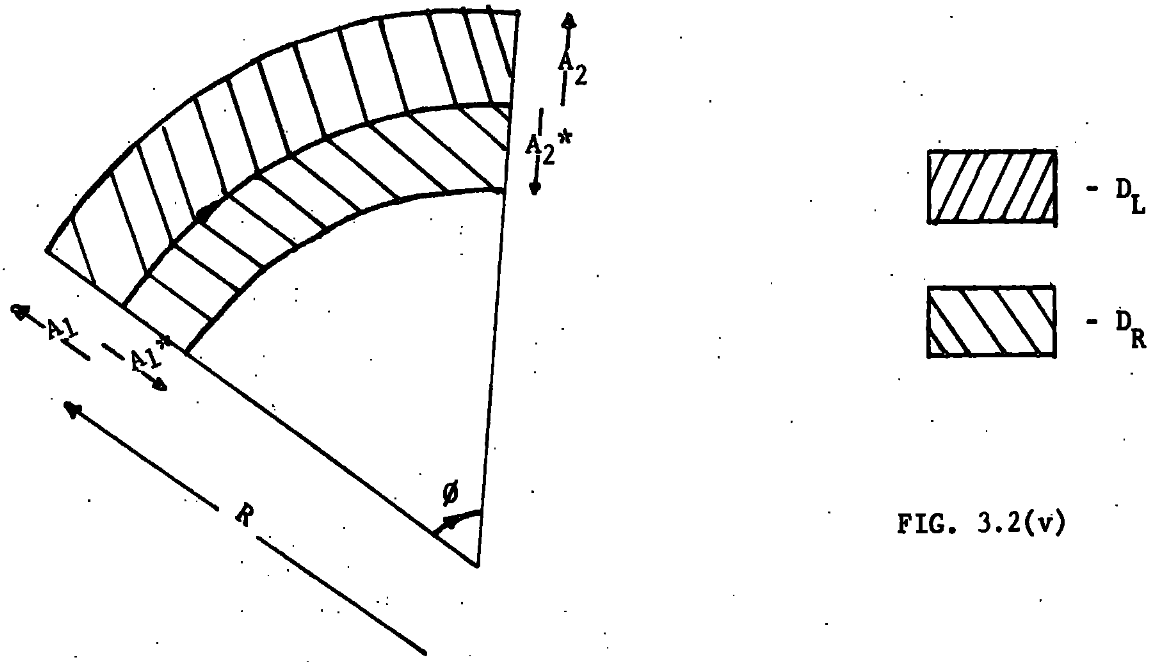


FIG. 3.2(v)

<u>Example</u> - In FIG. 3.2(v) Let :-

$A_1 = -1 \; ; \; A_1{*} = +2 \; ; \; A_2 = -1 \; ; \; A_2{*} = +2 \; ; \; \phi = \frac{1}{2} \; ; \; R = 5$

$$\text{length factor} = \left(\frac{5}{2} - \frac{1}{6}\left((-1 + -1 + 2 + 2) + \frac{4 - 1}{-1 + -1 -2 -2}\right)\right)$$

$$\underline{= 2.25}$$

(the length factor under the traditional method is $R\phi = 2.5$)

In the practical situation the ϕ and R are unknown but they are simply related to the design line parameters which are stored on the computer.

The design line parameters are (c_i, x_i, y_i, θ_i, r_i)

c_i = chainage, x_i = easting, y_i = northing, θ_i = whole circle bearing of tangent line, r_i = instantaneous radius of curvature (not in all computer packages).

the required parameters ϕ and R are related in the following manner.

$$\phi_{i+1} = \theta_{i+1} - \theta_i$$

$$R_{i+1} = \frac{r_{i+1} + r_i}{2} \quad \text{or} \quad R_{i+1} = \frac{L_{i+1}}{2\sin\left\{\frac{\phi_{i+1}}{2}\right\}}$$

where $L_{i+1} = ((x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2)^{1/2}$

Although the equations for the length factor may appear complex it should be noted that at the time of earthworks evaluations all the necessary values for the formula are easily accessible.

3.3 Cross-sectional Area

Consideration of the cross-sectional area reveals that the errors involved are a combination of two factors :-

(1) error in the actual value assigned to the model points

(2) sparsity of the model points (quality of model)

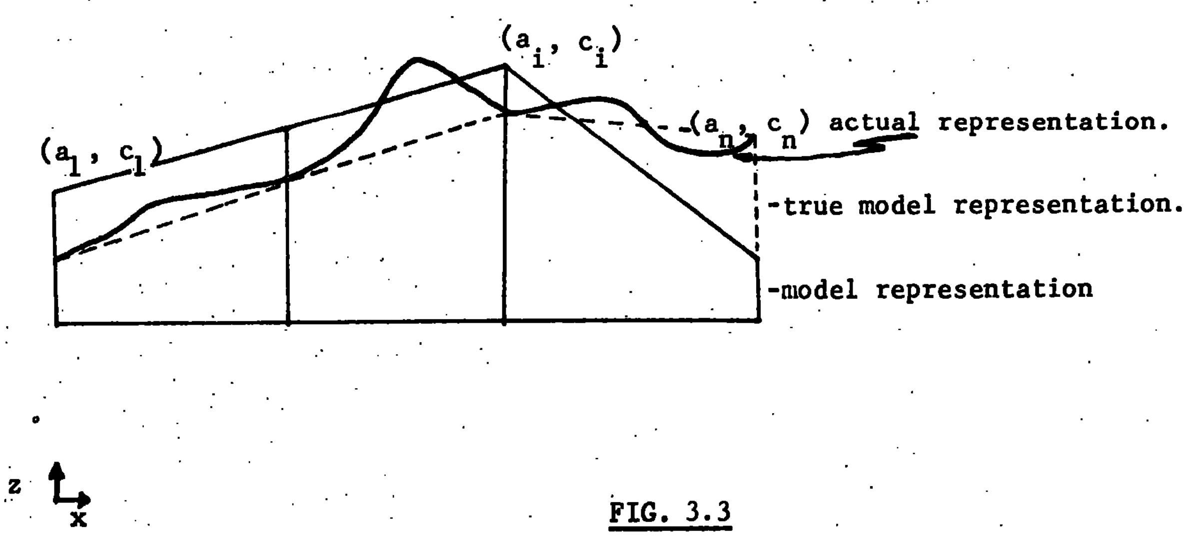


FIG. 3.3

The two factors may be isolated from one another. Having them separate, and knowing the connection between them gives a guide to both improving the model and evolving a testing procedure by which an aerial surveyed ground model may be accepted or rejected.

Across the cross-section the level of the land may be represented as a one-valued function of the distance x along the cross-section; $z = f(x)$.

and the area of the cross-section is

$$A_T = \int_{a_1}^{a_n} f(x)\,dx$$

The model seeks to represent this surface by a function of the model points c_i ; $z = g(x)$

and the area of the cross-section is

$$A_M = \int_{a_1}^{a_n} g(x)\,dx$$

The error in area is therefore the true area minus the model area i.e.

$$E_c = \int_{a_1}^{a_n} f(x)\,dx - \int_{a_1}^{a_n} g(x)\,dx.$$

if g(x) is a finite polynomial of degree m in x then

$g(x) = AX$ where $X = (1, x, x^2, \ldots.. x^m)^t$

and A is a $(m + 1)$ row vector of constant coefficients.

if in the interval (a_i, a_{i+1}) the polynomial is found from the values $c_{i-k}, c_{i-k+1}, \ldots\ldots c_i, c_{i+1}, \ldots\ldots c_{i+k'}$.

$(k + k' = m)$ then the following hold

$$c_{i-k} = AX_{i-k}$$

$$c_{i-k+1} = AX_{i-k+1} \quad \text{where } X_j = (1, a_j, a_j^2, \ldots\ldots a_j^m)^t$$

$$\vdots \qquad\qquad \vdots$$

$$c_{i+k'} = AX_{i+k'}$$

Let $H = (X_{i-k}, X_{i-k+1}, \ldots\ldots X_i, \ldots\ldots X_{i+k'})$

and $C = (C_{i-k}, C_{i-k+1}, \ldots\ldots C_i, \ldots\ldots C_{i+k'})$

and therefore $C = AH$ and hence $A = CH^{-1}$

However $c_j = z_j + e_j$ where z_j is the true value and e_j is the error involved.

$\therefore$ if $E = (e_{i-k}, e_{i-k+1}, \ldots\ldots e_{i+k'})^t$

& $z = (z_{i-k}, z_{i-k+1}, \ldots\ldots z_{i+k'})^t$

then $A = (z + E)H^{-1} = zH^{-1} + EH^{-1}$

Hence $g(x) = AX = (zH^{-1} + EH^{-1})X$

$= zH^{-1}X + EH^{-1}X$

following from this the error in area is

$$E_c = \int_{a_1}^{a_n} f(x)dx - \int_{a_1}^{a_n} zH^{-1}\,Xdx - \int_{a_1}^{a_n} EH^{-1}\,Xdx$$

$\longleftarrow$ A $\longrightarrow$ $\longleftarrow$ B $\longrightarrow$

A is the effect which the model has on the overall error and is independent of the "data preparation error" i.e. the error in preparing the values assigned to the data points.

B is the associated data preparation error.

if $g(x)$ is allowed to be a polynomial of degree 1 i.e. linear; then $X_i = (1, a_i)$

and $H = \begin{vmatrix} 1 & 1 \\ a_i & a_{i+1} \end{vmatrix}$ for g in the interval (a_i, a_{i+1})

thus $H^{-1} = \left(\frac{1}{a_i - a_{i+1}}\right) \times \begin{vmatrix} -a_{i+1} & 1 \\ a_i & -1 \end{vmatrix}$

$z = (z_i, z_{i+1}) = (f(a_i), f(a_{i+1}))$

$\therefore$ in the interval (a_i, a_{i+1})

$$zH^{-1}X = \frac{x(z_i - z_{i+1})}{(a_i - a_{i+1})} + \frac{a_i z_{i+1} - a_{i+1} z_i}{a_i - a_{i+1}}$$

and $\int_{a_i}^{a_{i+1}} zH^{-1} X dx$ reduces to $\frac{(a_{i+1} - a_i)}{2} (z_i + z_{i+1})$

and $$\int_{a_1}^{a_n} zH^{-1} X dx = \frac{1}{2} \sum_{i=1}^{n-1} (a_{i+1} - a_i)(z_i + z_{i+1})$$

for all the cross-section.

similarly $$\int_{a_1}^{a_n} EH^{-1} X dx = \frac{1}{2} \sum_{i=1}^{n-1} (a_{i+1} - a_i)(e_i + e_{i+1})$$

Thus the error in area of cross-section is

$$\int_{a_1}^{a_n} f(x)dx - \frac{1}{2}\sum_{i=1}^{n-1} (a_{i+1} - a_i)(f(a_{i+1}) + f(a_i))$$

$$+ \frac{1}{2}\sum_{i=1}^{m-1} (a_i - a_{i+1})(e_{i+1} + e_i)$$

It is impossible to collect sufficient data to know the exact form of f(x) for a particular cross-section.

If the assumption that a field survey is precise enough to allow for linear interpolation is used then the following results hold.

$$f(x) = \frac{(x - x_j)(Z_{j+1} - Z_j)}{(x_{j+1} - x_j)} + Z_j : x_j \leqslant x \leqslant x_{j+1}$$

where (x_i, Z_i) are the field survey points on the cross-section,

and
$$\int_{a_1}^{a_n} f(x)\, dx = \frac{1}{2}\sum_{j=1}^{m-1} (x_{j+1} - x_j)(Z_{j+1} + Z_j)$$

where $x_1 = a_1$ & $x_m = a_n$

the error term thus reduces to

$$E_C = \frac{1}{2}\sum_{j=1}^{m-1} (x_{j+1} - x_j)(Z_{j+1} + Z_j) - \frac{1}{2}\sum_{i=1}^{n-1} (a_{i+1} - a_i)(f(a_{i+1}) + f(a_i))$$

$$- \frac{1}{2}\sum_{i=1}^{n-1} (a_{i+1} - a_i)(e_{i+1} + e_i)$$

(1)

NOTE : a_i may be found to lie between x_j and x_{j+1} for some value of j, and hence

$$f(a_i) = \frac{(a_i - x_j)(Z_{j+1} - Z_j)}{(x_{j+1} - x_j)} + Z_j \qquad x_j \leqslant a_i \leqslant x_{j+1}$$

Effectively what equation (1) says is that the error in area for each cross-section is equal to the area under the land survey minus the area under the true model minus a linear combination of the model preparation error.

3.4 Construction of Testing Procedure

The model preparation error having been identified may be used to advantage in deciding whether to accept the data which makes up the model. Present day specifications (reference 6) on the accuracy of topological surveys are that 85% of the contoured points should lie within $\pm \frac{1}{2}$ the contour interval. Hence for 1/2500 scale plans the specification is that 85% of the e_i (errors) are less than 1 metre.

The distribution of the errors have been found to be as shown in FIG. 3.4(i). This is of course diagrammatic.

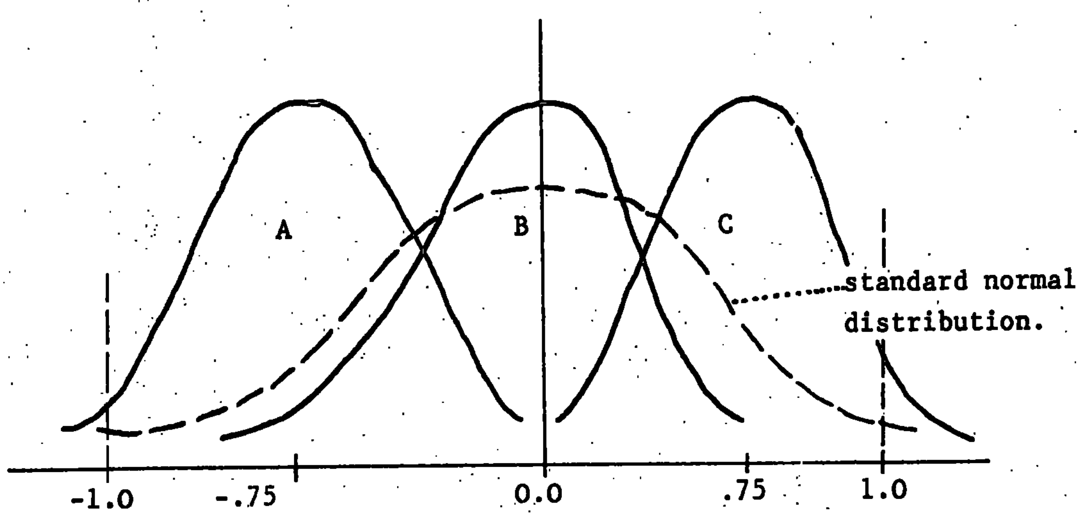


FIG. 3.4(i)

In each case in FIG. 3.4(i) A, B and C are all such that 85% of the values do indeed lie within $\pm \frac{1}{2}$ the contour interval, but the cross-sectional area would be systematically 0.75 square metres per metre high or low respectively for C and A.

For this reason, the specification as at present is neither useful nor meaningful. There is no adequate way of testing if the model fulfills the criteria and even when it is fulfilled accuracy need not necessarily be maintained.

An important property of the model preparation error is found to be that within each sample (i.e. cross-section) individual errors vary only slightly from one another, i.e. the variance for each section is small, although the mean can be markedly different from zero. This indicates the presence of a systematic error and the model preparation error may be represented by :-

$e_{ij} = n_i + \phi_{ij}$ where n_i = systematic error.

ϕ_{ij} = random error.

e_{ij} = model preparation error of the j'th reading on the i'th cross-section.

The random error may be assumed to be normally distributed with mean 0 and variance 6^2 and therefore e_{ij} will also be normally distributed with mean n_i and variance 6^2.

For each sample the three relevant parameters of the mean, sample variance, and number of observations, may be used to determine a practical testing procedure. Two procedures which were considered were the construction of confidence intervals for the mean and the calculation of the mean squared error.

Both these procedures suffer from the disadvantage which is also in the present specification, of not necessarily preventing the acceptance of a bias. The ideal situation to have is that the mean of the errors is zero and the variance has some value 6^2 which is not too large i.e. $6^2 \leqslant w^2$ but certainly not $6^2 > w^2$

In this situation one can have α% confidence that any sample removed from the model will have a mean correct to within $\pm N_\alpha \frac{w}{\sqrt{n}}$ where N_α is the $(1 - \frac{\alpha}{200})$ point of the standard normal distribution.

The test is therefore concerned with the taking of a sample of size n the sample mean $\bar{x}$ and sample variance S^2 (both estimates of the "population" mean and variance) should be within certain defined limits.

i.e. $|\bar{x}| < b$; $S^2 < w^2$

where b and w^2 are the critical bias and variance allowed.

Following the "spirit" of the specification as at present laid down the critical values may be set. The present specification is that :-

80% of all points shall be correct to within $\pm$ half the contour interval.

At 1:500 scale mapping the contour interval is 0.5 metres; thus half that interval is 0.25 and it would seem reasonable to have a critical bias of half this value. i.e. b = 0.125.

It would also seem appropriate to have 80% confidence in the calculation of the mean to lie within $\pm$ half the contour interval.

Therefore $\frac{N_\alpha w}{\sqrt{n}} \leqslant 0.25$ and for $\alpha = 0.8$ $N_\alpha = 1.624$.

If w^2 is taken to be 0.125 then the minimum value of n necessary to ensure the above criterium is 6.

The test would thus appear to be : on a sample of size greater than 6 reject the data if one or both of the following hold :

1. $|\bar{x}| > 0.125.$
2. $s^2 > 0.125.$

The above criteria prevents constant systematic bias of an intolerable degree, varying systematic bias, and large variance of results. If the sample data does not satisfy the constraints then a deeper investigation of the errors is required.

In having a test criteria it is necessary to know how powerful the test is. There are two types of error associated with the test. These are :-

1. accepting the data on the basis of the sample when in fact the data is incorrect.
2. rejecting the data on the basis of the sample when in fact the data is acceptable.

The client for the ground model is concerned that the probability of Type 1 error is minimised whereas the contractor is more concerned that Type 2 error be minimum. The probability of Type 1 error is governed by the choice of "α" and that for Type 2 effectively by the size of sample (n).

The probability of Type 2 error may be expressed as :

$$pr(\text{Type 2}) = pr(|\bar{x}| > b \text{ or } S^2 > w^2 / \mu \leqslant b \text{ and } 6^2 \leqslant w^2)$$

this is equivalent to :

$$pr(\text{Type 2}) = pr\left(\frac{-b-u}{6/\sqrt{n}} > \frac{\bar{x}-u}{6/\sqrt{n}} > \frac{b-u}{6/\sqrt{n}} \text{ or } \frac{(n-1)s^2}{6^2} > w^2\frac{(n-1)}{6^2} \Big/ u, 6^2\right)$$

i.e. $pr\,(Z* > X > Z \text{ or } Y > W/u, \underline{6}^2)$

where the substitutions are obvious

the variables X and Y are independently distributed with respectively standard normal distribution and $X^2(n - 1)$ degrees of freedom.

The probability therefore reduces to

$$pr(\text{Type } 2) = pr\ (Z^* > X > Z/u,6) + pr\ (Y > W/u,\ 6)$$

$$- pr\ (Z^* > X > Z/u,6) \times pr\ (Y > W/u,\ 6)$$

because of the independence property.

This probability may be evaluated for constant b and w^2, and varying n, for different combinations of mean and variance, where the mean u and variance 6^2 are both less than the related critical values. The following tables indicate the results of such computations for the critical values of b = 0.125 w^2 = 0.125.

n = 9

TABLE 3.4(i)

6^2 \ u	0.0	0.05	0.10	0.125	0.150	0.20
0.04	0.062	0.136	0.355	0.501	0.646	0.870
0.09	0.37	0.414	0.532	0.606	0.682	0.819
0.1225	0.593	0.617	0.682	0.725	0.769	0.853
0.125	0.597	0.622	0.685	0.726	0.770	0.853
0.16	0.751	0.764	0.798	0.821	0.845	0.893
0.25	0.921	0.925	0.933	0.938	0.944	0.957

n = 16

TABLE 3.4(ii)

6^2 \ u	0.0	0.05	0.10	0.125	0.150	0.20
0.04	0.012	0.067	0.309	0.500	0.691	0.933
0.09	0.219	0.282	0.457	0.569	0.681	0.863
0.1225	0.516	0.554	0.653	0.716	0.780	0.888
0.125	0.538	0.574	0.668	0.727	0.787	0.891
0.16	0.761	0.778	0.823	0.850	0.880	0.931
0.25	0.960	0.963	0.968	0.972	0.976	0.984

n = 25

TABLE 3.4(iii)

σ^2 \ u	0.0	0.05	0.10	0.125	0.150	0.20
0.04	0.002	0.030	0.266	0.5	0.734	0.970
0.09	0.131	0.195	0.403	0.549	0.695	0.905
0.1225	0.480	0.521	0.641	0.719	0.797	0.920
0.125	0.504	0.544	0.657	0.731	0.805	0.922
0.16	0.793	0.809	0.854	0.883	0.911	0.959
0.25	0.984	0.985	0.988	0.99	0.992	0.995

n = ∞ (infinity)

TABLE 3.4(iv)

	0.0	0.05	0.10	0.125	0.150	0.20
0.04	0.0	0.0	0.0	0.0	1.0	1.0
0.09	0.0	0.0	0.0	0.0	1.0	1.0
0.1225	0.0	0.0	0.0	0.0	1.0	1.0
0.125	0.0	0.0	0.0	0.0	1.0	1.0
0.16	1.0	1.0	1.0	1.0	1.0	1.0
0.25	1.0	1.0	1.0	1.0	1.0	1.0

In Tables 3.4(i), (ii), and (iii) the shaded areas show the extent of the region considered to be acceptable. The double shaded areas show the extremes above which the probability of rejection rises sharply. Table 3.4(iv) shows the ideal situation where the complete model is taken as the sample.

Obviously the larger the sample considered the more confidence one can have that the data will not be rejected when it has a mean and variance acceptable. Again, one is not too concerned that when the true mean and variance is reaching the bounds of acceptability its rejection rate increases. The solution to the problem of how large a sample ought to be taken will probably be as follows :-

When the true mean and true variance are half the allowable constraints (for 1:500 scale mapping hypothesised as 0.125) the probability of rejection of the data should be 0.2.

Investigation of the Tables 3.4(i), (ii) and (iii) shows the answer to be that n lies somewhere between 16 and 25.

Hence for a sample of size n, (lying between 16 and 25) the estimates of the mean and variance will provide a test for which the probability of Type 1 error(probability of accepting the data when it is false) is 0.2 and that of rejecting the data when it is most acceptable (Type 2 error) is also 0.2.

CHAPTER 4. DESIGN OF TESTS

4.1 Introduction

Taking precise measurements in the field and comparing them to those obtained by aerial survey is the obvious method of testing a ground model. It is a time consuming process to test a square grid D.G.M. by setting out accurately the grid and levelling the mesh points manually. When the string D.G.M. is considered the task becomes virtually impossible in setting out the points along a contour, or even along a feature. To set out the contours or 3-D strings only tests the model preparation error, and is not, as is shown in 3.3, the whole solution to the problem. The solution lies in the combined effect of model preparation error, error inherent in the specification of the model and error in the interpolation procedures which together produce the error in cross-section or in any point extracted from the model. To test the model it is consistent to take levels which represent the land surface efficiently and accurately.

4.2 Description of Test Areas

Two test areas were considered; at Bowburn, Co. Durham, and Horsley, Northumberland. Both areas had been flown to provide aerial models, the first for 1:2500 scale mapping, and the second for 1:500 scale mapping. The area at Bowburn was used as a very basic pilot study to provide a general idea of what was required.

The 1:500 scale Horsley area had been flown to give a square grid model. The same overlap photographs, over part of this model, were used by the aerial survey contractors to provide a string model, and they created the model with the knowledge that extensive tests were to be carried out.

4.2.1 Bowburn Test Area (1:2500 scale mapping)

Location - The area selected for the test lies immediately to the north of Quarrington Village and covers land to the west and east of the minor road leading from Quarrington Village to Bowburn Village, in the County of Durham.

Description - The test area was divided into three separate areas of differing character.

(a) To the east of the minor road, the test area comprises steeply sloping permanent pasture, leading up to the ordnance survey "trigonometric" point at Beacon Hill (a fourth order Ordnance Survey Block).

(b) To the west of the road, the terrain is rough and shelves steeply away to the floor of a disused quarry. The vegetation along the radial sections along which measurements were taken, required some clearing in order to obtain uninterrupted lines of sight.

(c) The road channel itself - a "hard" feature constituting a three dimensional string.

4.2.2 Horsley Test Area (1:500 scale mapping)

Location - The area selected for the test lies two kilometres to the west of the village of Horsley, to the south of the A.69 trunk road, and immediately to the east of Whittle Dene in the County of Northumberland. It comprises part of the agricultural land known as Whittle Farm under the ownership of W. A. Dinning.

Description - (i) Size. The boundary of the test area is defined by a square of 300 metres side.

(ii) Features :

(a) Artificial. The area includes a section of the existing A.69 Trunk Road, with associated fences and hedges; the access track to Whittle Farm, the access track to agricultural land from the farm, the farm outbuildings and the cottage known as Whittle Lodge.

(b) Natural. The general nature of the test area is uniform throughout and contains no special features. The area to the west of the access track to Whittle Farm slopes uniformly down from south to north at a gradient of approximately 1 in 28. To the east of the access track the area is similar to that to the west, but is less steep, having an average gradient of 1 in 300.

Both areas, referenced Block A and Block B respectively, are under permanent pasture, but at the time of aerial photography the eastern area, Block B, was under plough (3/4/1969). Both areas are "open" with respect to all parts being air visible, and both exhibit good texture for the photogrammetric plotting process.

4.3 Taking of Measurements

The simplest method of taking measurements in the field and also extracting equivalent levels from the ground model was found to be in setting out cross-sections and levelling along them. The cross-sections were immediately in a form by which the equivalent cross-sections could be removed from the model.

4.3.1 Bowburn Test Area

Initial thoughts were that radial sections set out from one point would provide the best results in the easiest manner. In fact this was not the best method for two reasons. First, all the radials tended to cut the same strings in the model, although obviously in different places and this made the results too localised. Secondly the analysis of the results although not complex, became confusing. However the Bowburn Area measurements did use this technique.

FIG. 4.3.1(i) shows diagrammatically the layout of the cross-sections.

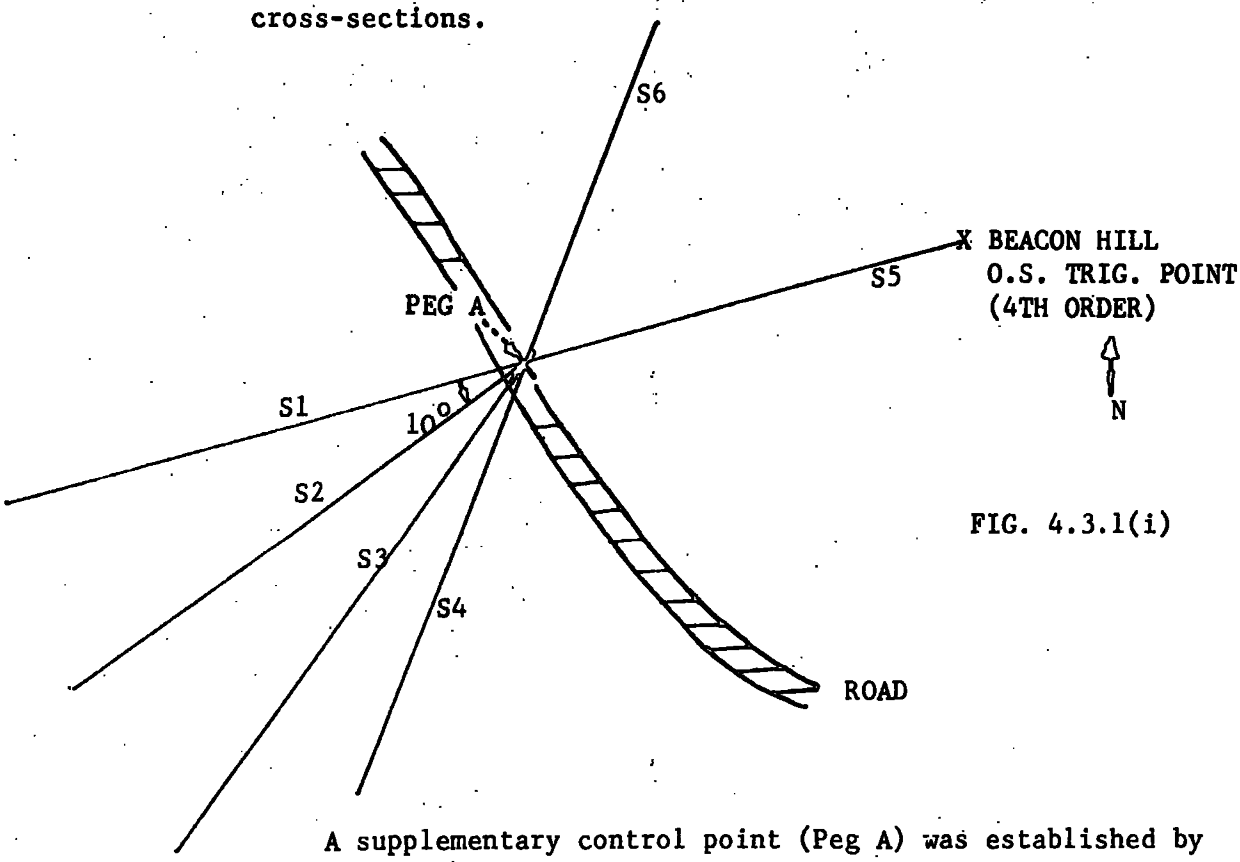


FIG. 4.3.1(i)

A supplementary control point (Peg A) was established by bearing and distance, and checked by triangulation, from an ordnance survey fourth order block known as Beacon Hill. An ordnance survey third order block was used as reference object (R.O.)

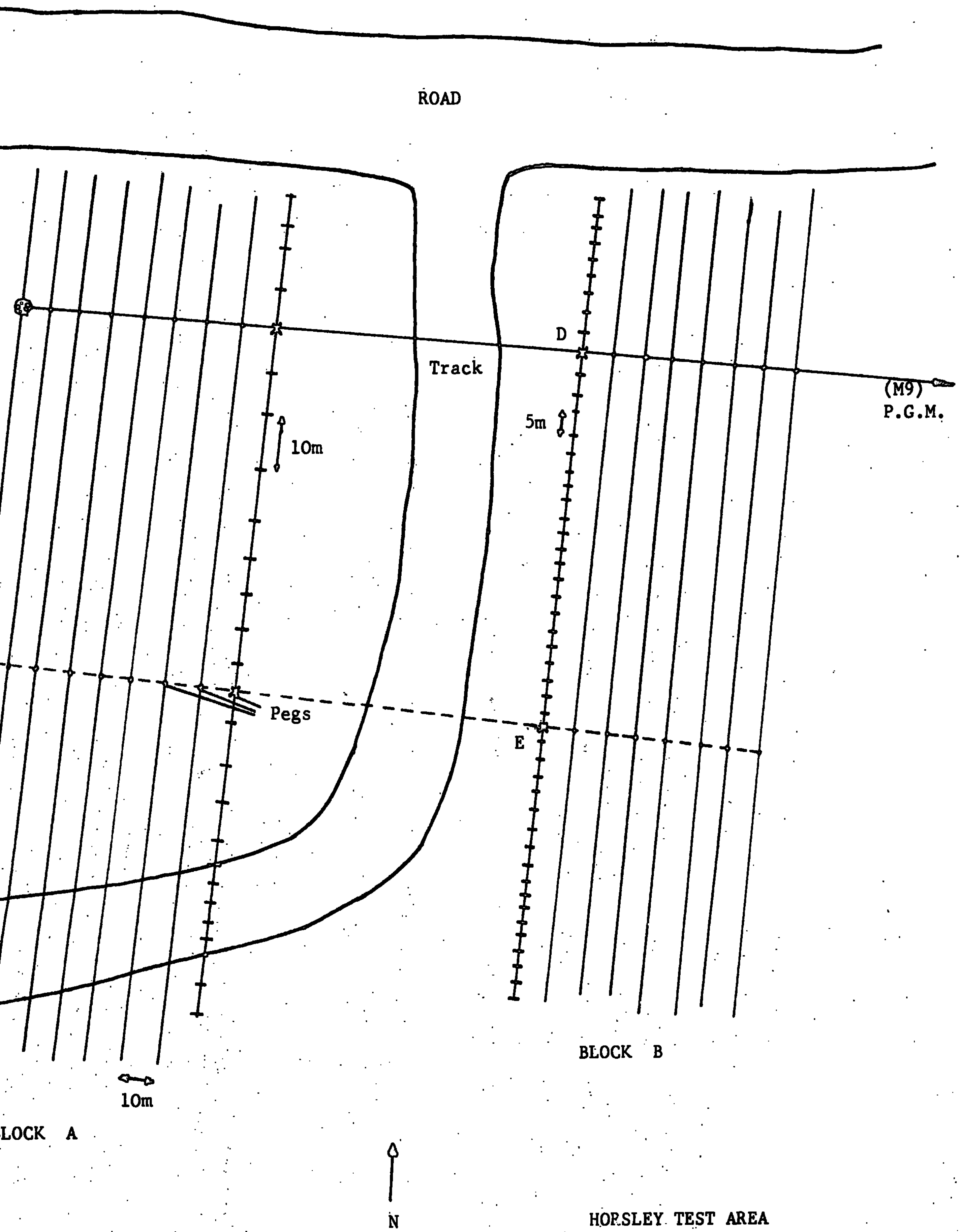
Sections were observed down lines of fixed bearing through Peg A. Sections S1, S2, S3 and S4, led from Peg A into the disused quarry, and were displaced from one another by an angle of 10^{o}. Sections S5 and S6 were continuations of S1 and S.4. For analysis purposes the sections were referenced as follows :-

S1, S5	-	Chain 0
S2	-	Chain 10
S3	-	Chain 20
S4, S6	-	Chain 30

4.3.2 Horsley Test Area

The two blocks A and B were located in eastings, northings and height, relative to existing co-ordinated and levelled control in the form of concrete permanent ground markers (P.G.M.) established in conjunction with the original 1/500 scale aerial survey of the complete route.

The terminal points of the base line, upon which Blocks A and B were established, were defined by P.G.M.'s M5 and M9, at a distance of approximately 385 metres apart and running approximately west to east across the test area. From this base line other lines were set out perpendicularly in the two blocks - Block A and Block B. The lines were set out using steel tape, pegs, and the tacheometer. A right angle was turned at M5 from M5 - M9 and the Peg A placed approximately 180 metres from M5 (See FIG. 4.3.2(i). Two rows of pegs were then positioned along M5 - M9 and along A - E. For both Block A and Block B the lines between the pegs were 10 metres distant from one another. The supplementary control pegs



HORSLEY TEST AREA

FIG. 4.3.2.(i)

A, D and E were accurately coordinated with the aid of a Kern DK - RV tacheometer and Tellurometer MA 100*.

In Block 'A' points were levelled along each perpendicular line on average every 10 metres but for Block B this distance was reduced to 5 metres. Wherever a change in curvature was apparent a point was also levelled. From pegs along M5 - D the maximum distance from the peg to the levelled point was 100 metres. The remaining points were taken off from pegs along AE and the common points distance checked against the tellurometered distance. Thus each line was over 200 metres in length and each could be considered as a road cross-section.

4.4 Precision of Measurements

The comparison between the aerial survey and field survey will only give a meaningful assessment of the accuracy of the aerial survey if the field survey measurements are "precise". Even in the testing procedures it is impossible to discover the exact cross-section profile. The field survey assumes that every significant change in curvature and grade is documented well enough to allow the linear interpolation of level of points intermediate to those measured. Errors made in obtaining data from the field include :

1. non-selection of significant curvature changes.
2. sufficient points to enable linear interpolation are not included.
3. cross-sections are inaccurately coordinated into the coordinate system of the model (national grid).

* A specification of these instruments appear in 4.4.

4. horizontal offset distances along each cross-section are not measured precisely.

Measurements were taken using the following instruments.

1. Tellurometer MA100. This instrument accurately measures the distance between two points using radio waves. The instrument emits radio waves which are reflected back from the reference point to the source to give a reading. The specifications are :

(a) Accuracy - standard deviation of a single measurement 1.5 mm ± 2 parts per million.

(b) Resolution - 0.1 mm.

(c) Radiation Source - Gallium Arsenide Diode.

2. Kern DK-RV - A self reducing tacheometer which is used in conjunction with a special, extended precision vertical staff. Its specifications are :

(a) Accuracy ± 3 to 5 cms/100 metres.

(b) Maximum distance possible 150 metres.

(c) Horizontal circle (angles) 10" with micrometer.

(d) Vertical circle (levels based on a tangent scale) 0.0001 by estimation, 0.001 direct.

Using these instruments the author personally helped an experienced land surveyor (Mr. R. E. Beels (A.R.I.C.S.) level the test areas. Non-selection of significant curvature changes was avoided because of the author's involvement, and this also removed the possibility of insufficient points to enable linear interpolation to be carried out. The use of the tellurometer ensured accurate coordination of the cross-section into the model. The remaining error, - precise measurement of horizontal distances, was overcome by never levelling a point more than 100 metres from the tacheometer.

4.5 Analysis of Data

The land survey provided the "true" cross-sections and the model provided the "assumed" cross-sections. By interpolating the "true" levels to the model points to provide the "true" values for the model a third cross-section was produced. These three representations of each cross-section were the basis for the analysis :

1. true representation.
2. true model representation.
3. actual model representation.

The comparison between (2) the true model representation and (3) the actual model representation gave the model preparation error. That comparison between the true representation (1) and (2) the true model gave the quality of model error. The difference between (1) and (3) produced the overall error or combined error.

(2) - (3) = Data Preparation Error.

(1) - (2) = Quality of Model Error.

(1) - (3) = Overall Error.

The comparisons were investigated by two methods :-

(a) the individual point levels on each cross-section.

(b) the areas contained in each cross-section.

The reduction of the data into the amenable form of means and variances was achieved by the writing of various computer programs.

CHAPTER 5. GENERAL RESULTS AND SUGGESTED AMENDMENTS

5.1 Introduction

In this section a general review and assessment of the results obtained from the analysis is given. Reference is made to the tables in Chapter 8.

It has been shown in Chapter 3 that the errors involved in calculating earthworks quantities may be isolated into those concerned with longitudinal spacing and those with cross-sectional area. Consideration of cross-sectional area allows the errors to be broken down further into the following factors :-

(a) blunders.

(b) quality of model errors.

(c) data preparation error.

(d) overall error (d = a + b + c)

Chapter 3.3 shows how this is theoretically true and Chapter 4 gives an indication of how to achieve relative measurements, although they do not consider the affect of blunders. Blunders are arbitrary and impossible to analyse. However for the string ground model their removal may be accomplished by two methods :

1. consideration of adjacent levels on 3-D strings.
2. plotting of the model.

Assuming the blunders are removed the overall error is calculating earthworks quantities is a combination of the model preparation error and the quality of model error. Using a ground model the continuous land surface is represented by a discrete set of values and inaccuracies immediately become apparent. These errors are termed quality of model errors.

In actually assigning the values to the discrete set of points there is also error involved be it operator error, tilt of the aerial survey photographs, or random error. This is termed data preparation error.

5.2 Longitudinal Spacing

In the end area method of calculation of volumes the areas of the cross-sections removed perpendicularly to the design line are multiplied by the interval on the design line to give a volume. This interval may affect the accuracy of calculations in two ways. First the longer the interval then the less the number of cross-sections and the less the value of the information. Secondly the interval is usually taken from the design line and the widths of the cross-sections may vary and make this value incorrect.

5.2.1 Size of Interval

The general practice has been to take an interval of 20 metres along the design line. String Digital Ground Models are the first continuous ground model and being continuous the accuracy of earthworks calculations may be improved by taking cross-sections at a smaller interval than that which has been considered normal practise.

Theoretically by taking cross-sections at a very small interval very good definition of the ground surface is obtained and may be taken to such an extreme that the finite slices produce a near perfect solution. Greater intervals introduce errors because of the approximation to the surface and the errors may be equally positive or negative and therefore often self-cancelling although within local areas they may have an undesirable effect on cut and fill quantities.

In practise the spacing must be kept to practical limits. It may be true that the string model is continuous but as has been explained in Chapter 2, it is constructed from a discrete set of points. It is meaningless to take out cross-sections at a smaller interval than the spacing of the digitised points. Another factor to be considered is that it is wasteful of resources to commit inessential information to the attention of the design engineer and the smaller the interval the greater the number of cross-sections. For these reasons consideration must be given to a practical limit to the size of interval.

Using the specially digitised model a hypothetical road was designed which had as its design line a straight line 200 metres in length. The vertical profile was flat and completely below the ground surface, there was no super-elevation applied and the side slopes were vertical. The cross-section width did not vary from 100 metres. The whole purpose was merely to extract earthworks quantities at differing chainage intervals and note the effect on the definition of the surface.

Table 8.17 and 8.18 shows the results obtained by the test which removed cross-sections at 1, 5, 10, and 20 metre spacings and compared the amounts of cut. It is fairly obvious from the results that a substantial increase in accuracy is accomplished by reducing the cross-section spacing from 20 metres to 10 metres.

The increased accuracy is due to the cross-sections defining the curvature in the longitudinal direction of both the road and ground and even if the ground were flat the road curvature would produce errors. In the case of the test example the road was on level grade and only the ground model was tested and this means that in the practical situation the error will only be a proportion of the actual error.

It is difficult to express the differences as a percentage error because earthworks vary in shape and depth, and it is more realistic to consider the compounded effect over a length of road. A quick calculation on the basis of Table 8.18 reveals that :-

An average road of total width 40 metres and of length 100 metres would result in a volumetric error of 40 x 0.036 x 100 cubic metres by taking 20 metre cross-sections as opposed to 40 x 0.006 x 100 cubic metres by taking 10 metres cross-sections i.e. 144 cubic metres against 24 cubic metres.

The "choice of interval error" is directly compatible with the "quality of model" error* and it is general practise to take 20 metre intervals. As a result of the tests any criticisms of ground models must be viewed in this context.

* The quality of model error will be discussed more fully in later sections.

5.2.2 Length Factor of Interval

Cross-sections are removed perpendicularly to the design line (usually the centre-line) of a proposed road and the product of the areas of two adjacent cross-sections and the chainage interval gives the volume. The chainage interval is calculated from the design line and this has been called the "length factor".
e.g. volume = length x breadth x height.
the breadth and height are combined in the calculation by using the cross-sectional area: the length is the chainage interval.

This is the traditional practise and for general motorway design is acceptable, complicated junction design being done manually. However designs are becoming more complex and computerised designs of considerably more detail are required.

The inaccuracy involved in the above method probably cannot be ignored on bends and when calculating volumes for irregular features such as interchanges. It is difficult to make a general analysis of the effect of differing offsets especially when the design line is the centre line. The need to use the true length factor (as opposed)to the "traditional" length factor) may only be proved or disproved in the design office in practical circumstances.

Research carried out on a small number of urban designs would reveal the necessity or otherwise of using the true length factor in the design.

5.3 Blunders

Blunders occur at the data preparation stage and are both arbitrary and impossible to analyse. The more serious blunders may be shown up by considering adjacent levels on three dimensional strings and by plotting the model.

Three-dimensional strings are included in the model when changes in gradient (railway embankments etc.) are noticeable and when the slope of the ground is so small that the contours provide insufficient definition. A railway embankment or similar feature will be described by strings which run along the feature rather than across it. Thus a difference in level of more than say, 2.5 metres on a 1 : 500 scale model should cause concern and will probably indicate that a blunder has occurred.

One of the major advantages of the aerial surveyed string D.G.M. is that it provides a regular sample of the ground by its use of contours and 3-D strings. In areas of changing slope the contours become more dense and where curvature is noticeable (crossing a railway embankment etc.) the 3-D strings describe it. The direct plotting of the contours and 3-D strings in the projective plane provides an immediate check on both the acceptability of the model and shows up any "blunder" made with the provision of the data. Examples of checks which may be made by reference to the plot are :-

1. absence of contours or partial absence.
2. absence of 3-Dstrings or partial absence.
3. inaccurate overlapping of closed contours.
4. double digitising of sections of contours.
5. crossing of contours.
6. unintelligible 3-D strings.

These examples are not unusual and are faily easy to recognise when a plot is available. However without the plot (produced by computer) the task is impossible. If these blunders remain in the model then inaccurate results are bound to occur.

5.3.1 Absence of Contours or Partial Absence

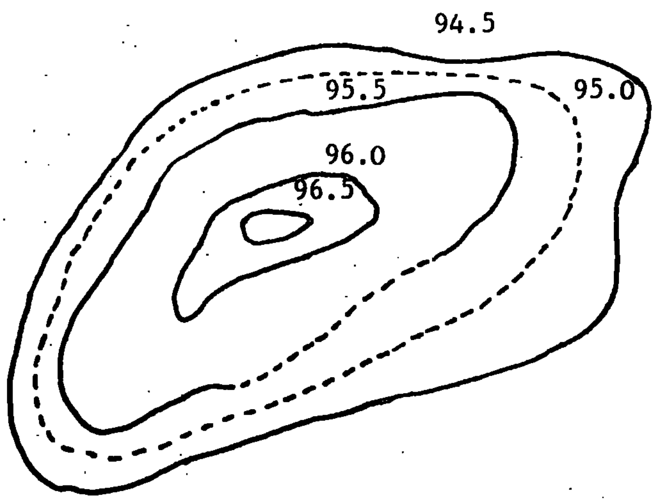


FIG 5.3(i)

Fig. 5.3(i) shows that contour level 95.0 is missing and contour level 95.5 is incomplete.

5.3.2 Absence of 3-D Strings or Partial Absence

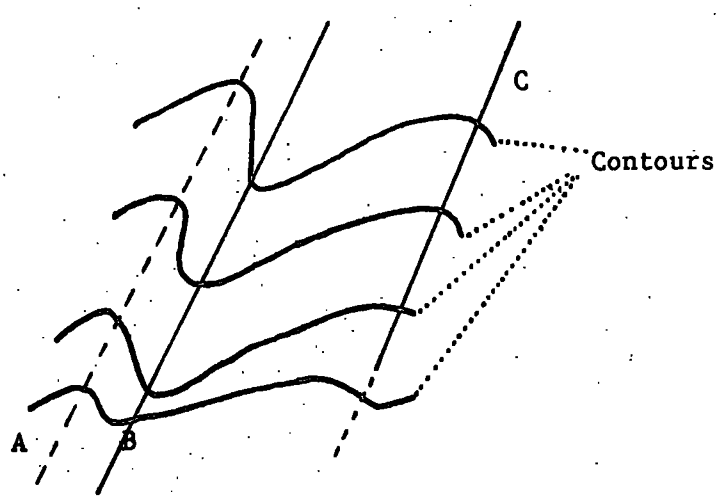


FIG. 5.3(iia)

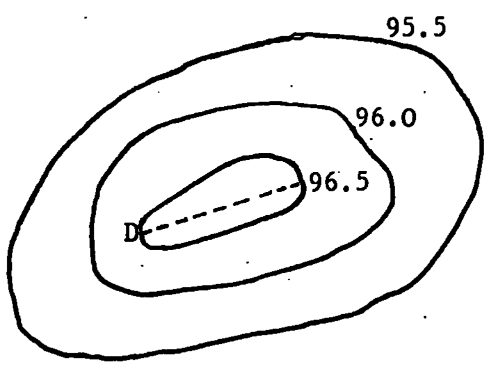


FIG. 5.3 (iib)

In FIG. 5.3(iia) B is a complete 3-D string; A is a string which is missing altogether; and C is a 3-D string which is only partially complete. The fact that A is a 3-D string known to be missing is explained more fully in Chapter 5.4.5. It is necessary for detailed design for all 3-D strings to be included especially where vee-shapes occur in the contours.

A further need is for a 3-D string to be included across the innermost closed contour of a set of nested contours. Again this is explained in Chapter 5.4.5. FIG. 5.3(iib) demonstrates that 3-D string D is missing. By infilling this string adequate detail for the crest of a hill or the bottom of a hollow is ensured.

5.3.3 Inaccurate Overlapping of Closed Contours

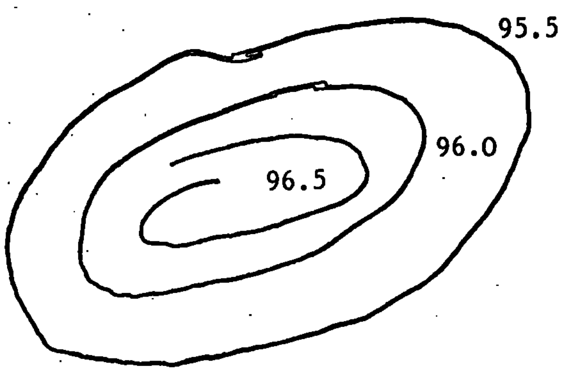


FIG. 5.3 (iii)

Slight overlapping of the ends of a closed contours such as contour level 95.5 in FIG. 5.3(iii) and also contour level 96.0, is unavoidable but accepting contour level 96.5 as defining the ground accurately would severely prejudice the accuracy of any cross-section cutting these contours.

5.3.4 Double Digitising of Sections of Contours

Reference to FIG. 5.3(iv) shows that points 6 and 7 are obviously points which have been mistakenly picked possibly by an accidental jerking of the stereoscopic equipment. If these points are removed the sequence of points 1, 2, 3, 4, 5, 8 will provide the correct shape of the contour.

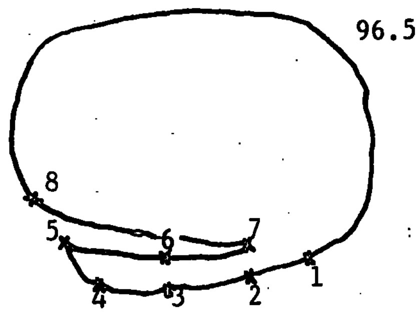


FIG. 5.3(iv)

5.3.5. Crossing of Contours

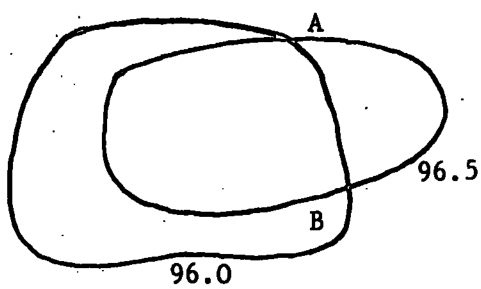


FIG. 5.3(v)

This again is a fairly obvious blunder since points A and B would appear to simultaneously have the levels 96.0 and 96.5. This is topographically impossible unless the model is describing a cliff overhang in which case three dimensional strings ought to have been used.

5.3.6 Unintelligible 3-D Strings

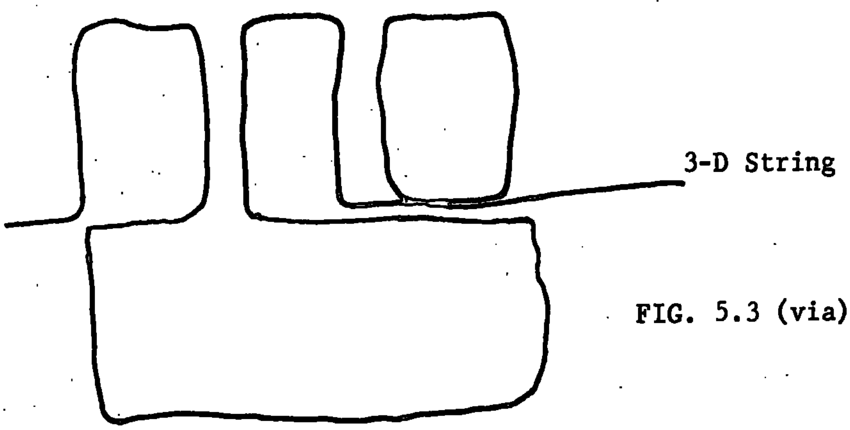


FIG. 5.3 (via)

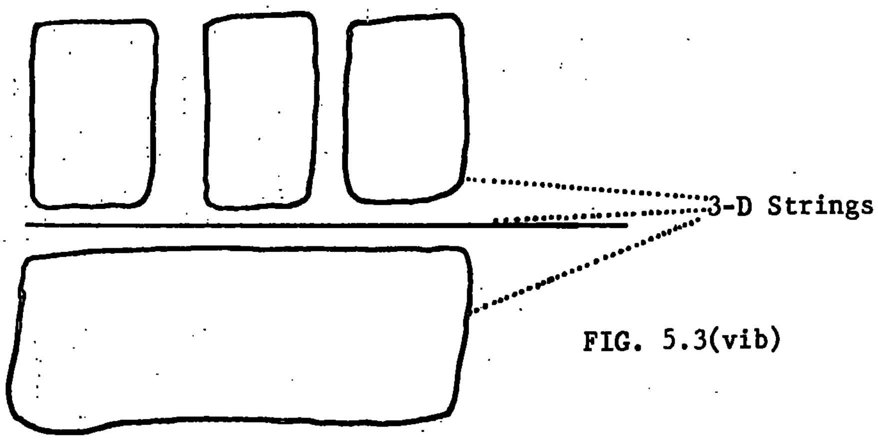


FIG. 5.3(vib)

It is much clearer if the three-dimensional string which describes the levels around three houses and a factory (say) in FIG. 5.3(via) is defined by five separate 3-D strings as in FIG. 5.3(vib). This improves the stored model visually and leaves less room for blunders such as shown in FIG. 5.3(vic) since the digitising is done systematically.

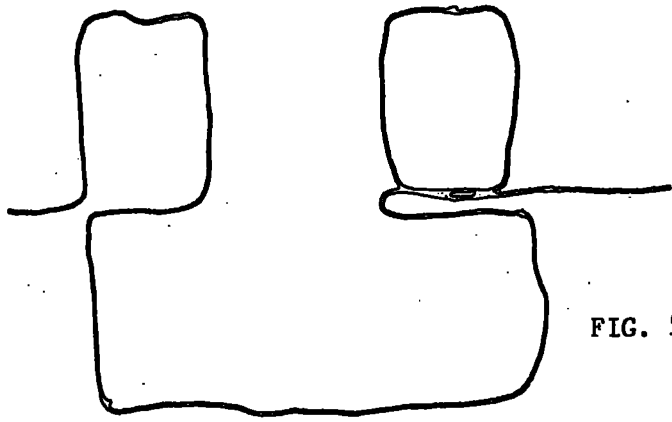


FIG. 5.3(vic)

Although the types of blunders as detailed in 5.3.1 to 5.3.6 are illustrated diagrammatically they were all found in the general investigation of plots of actual ground models in the practical situation.

5.4 Cross-Sectional Area

Consideration of the overall error composed of the data preparation error and the quality of model error revealed some inadequacies both in the storage of the model and the retrieval of the data for a particular cross-section. Although the two components could be isolated from one another modifications arising from the preliminary results had the effect of changing both errors simultaneously. For that reason the two types of error are considered together and their joint affect on the overall error discussed.

5.4.1 There is no need to compare differences in area to comprehend the errors involved- point errors are sufficient.

The advantage of using point errors is that from them a mean and a variance (standard error) may be calculated. The variance of the errors of the interpolated points gives a quick guide to any blunders since where blunders occur a sharp increase in variance is virtually inevitable. When the area errors alone are considered for each cross-section there is only one area and no variance can be calculated. Apart from all this the area of the cross-section is calculated using the point levels so that the point levels are the primary source of earthworks volumes.

The mean interpolation error values, as shown in tables 8, 9, 10 (Chapter 8) compare very favourably with the normalised error in area. The area errors are normalised so that the dimension is square metres per metre length of cross-section. A cursory glance at the tables reveals the differences to be about one tenth of the normalised area error except in a small number of occasions.

The differences as tabulated in tables 8.8, 8.9, 8.10 may themselves be analysed. Assuming a normal distribution for the differences confidence intervals for the mean differences may be constructed from the "students 't' - distribution" (both the mean and the variance need to be calculated). Table 8.15 shows the 90% confidence intervals on the mean differences and from these tables it is quite reasonable to take the interpolation error as being directly compatible with the area error.

5.4.2 **Secondary Interpolation is a need which is not difficult to accomplish.**

This is a very important modification to the existing computer programs which the research has revealed. The component of the area error which was termed quality of model error is itself composed of two types of error :

1. the actual choice of discrete points making up the model.
2. error due to interpolation procedures.

The number of model points stored may be very large and the overall definition of the land surface good, thereby making the "quality" of model good. However, the computer programs which extract individual cross-sections would find it neither easy nor economic to utilise every piece of data. For this reason errors in deciding which data points to consider (i.e. interpolation procedures) must be taken into account.

The simplest way of using the data in the model is by using linear interpolation across the strings. Along the string linear interpolation is already assumed by the specification of how the data is to be collected*. To date cross-section programs have only detected where actual strings cut the cross-section e.g. FIG. 5.4.2.(i)

* The specification lays down that sufficient points are collected along strings to ensure linear interpolation between adjacent points.

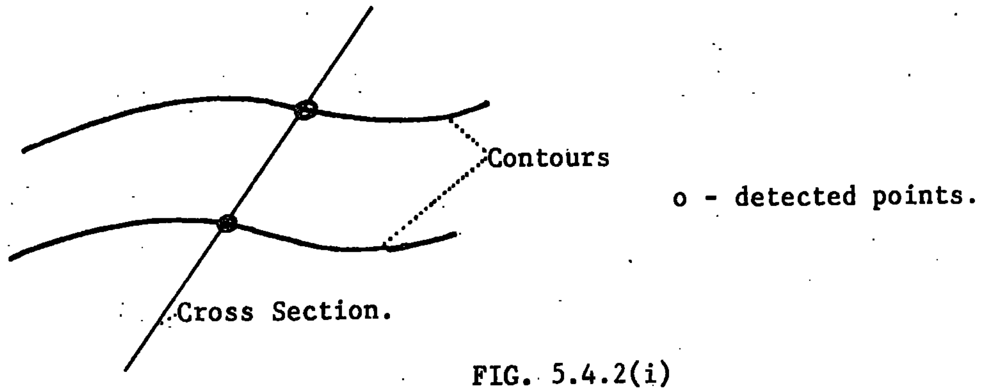


FIG. 5.4.2(i)

This technique does not make maximum use of the stored data to detect local transverse curvature and in certain circumstances manual editing was indicated by the program e.g. FIG. 5.4.2.(ii).

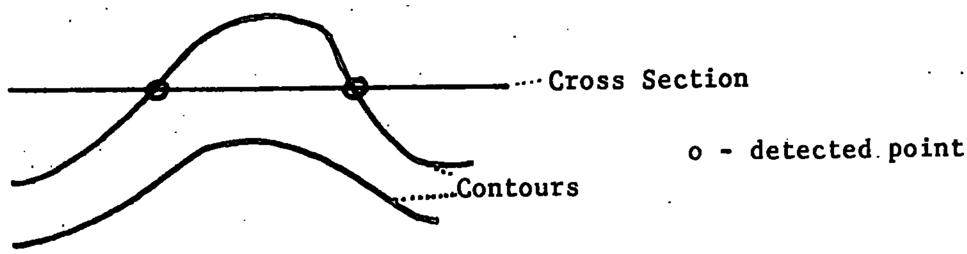
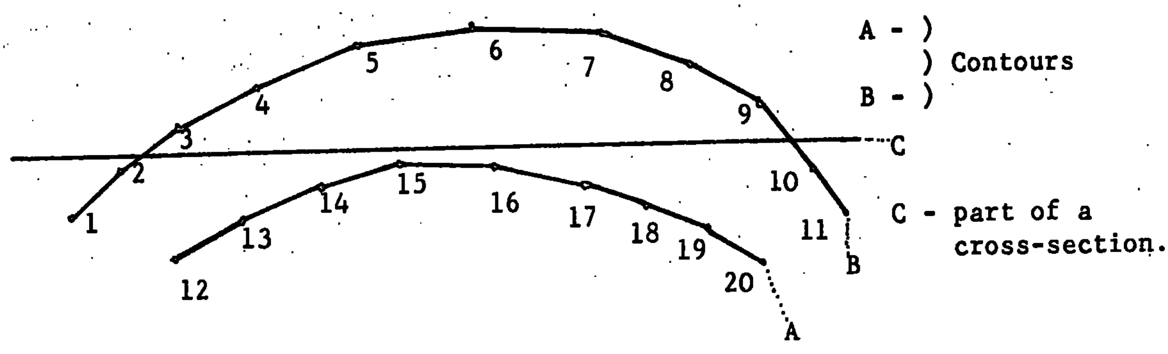


FIG. 5.4.2.(ii)

These disadvantages are not short-comings of the model but short comings of the technique used to extract data. A further example may be useful in illuminating this problem more clearly.



points 1 - 11 are the data points stored in the model defining string B : points 12 - 20 are those data points stored in the model for string A.

FIG. 5.4.2.(iii)

In FIG. 5.4.2.(iii) specifying that only at those points where the cross-section cuts the model are levels taken, infers in effect that points 4, 5, 6, 7 and 8 are all closer to the cross-section than points 14, 15, 16, 17 and 18. Obviously this is not true.

Another situation where the present techniques prove inadequate is where the cross-section lies parallel or near-parallel to the contours with no 3-D strings cutting the area.

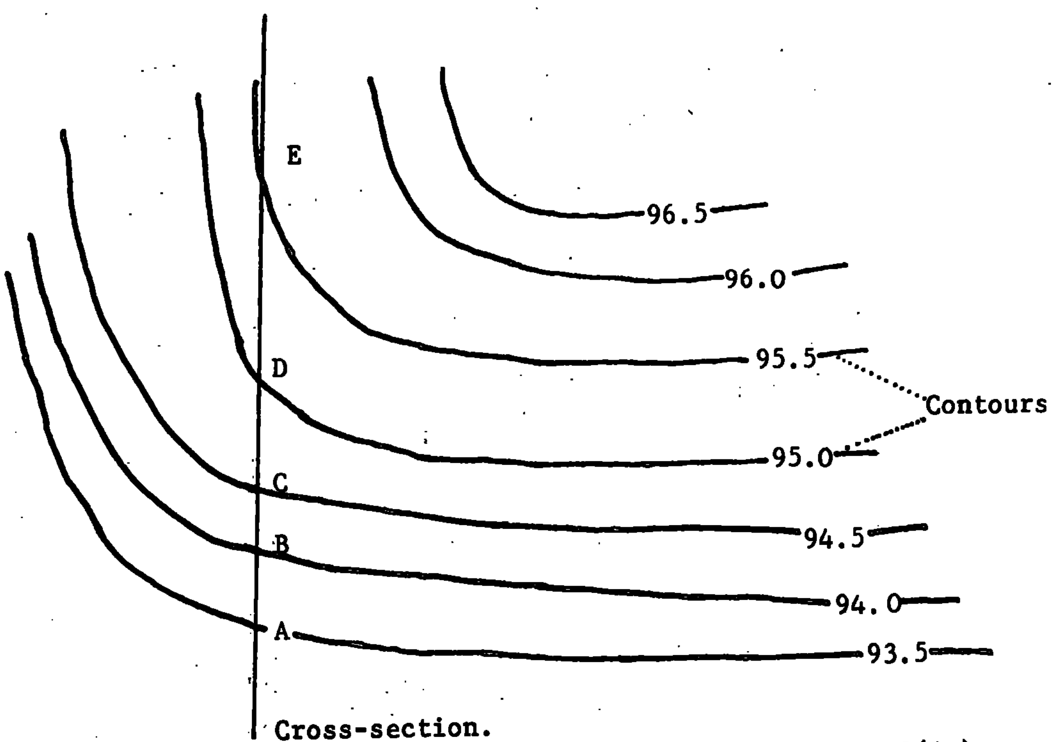


FIG. 5.4.2(iv)

For example in FIG. 5.4.2.(iv) although there are no cuts which are such that two adjacent levels are the dame and although cuts A, B, C and probably D are sufficiently close together to give adequate definition to the cross-section, point E is too far distant from D to allow satisfactory interpolation of an intermediate point.

The "primary" process of searching through the model and finding all detail strings actually cutting the cross-sections may be supplemented by a "secondary" process, where deemed necessary from an inspection of primary output, to find the level of intermediate points. The secondary process defines points as required to be included where :-

1. two adjacent cuts have the same level - find the level of a point which is halfway between the two points of equal level.
2. two adjacent cuts are sufficiently far distant from one another to exceed a specified tolerance - find the level of a point halfway between the two cuts. If the distance between the adjacent cuts and the included point still exceeds the tolerance repeat the process until the tolerance is not exceeded.

If both (1) and (2) occur simultaneously the process as defined in (2) is carried out.

The levels of the secondary points are found by creating dummy cross-sections through the point and perpendicular to the original cross-section. The model is then searched again with these transverse sections and from the cuts which they make with the strings either side of the point the level is determined by interpolation See FIG. 5.4.2.(v)

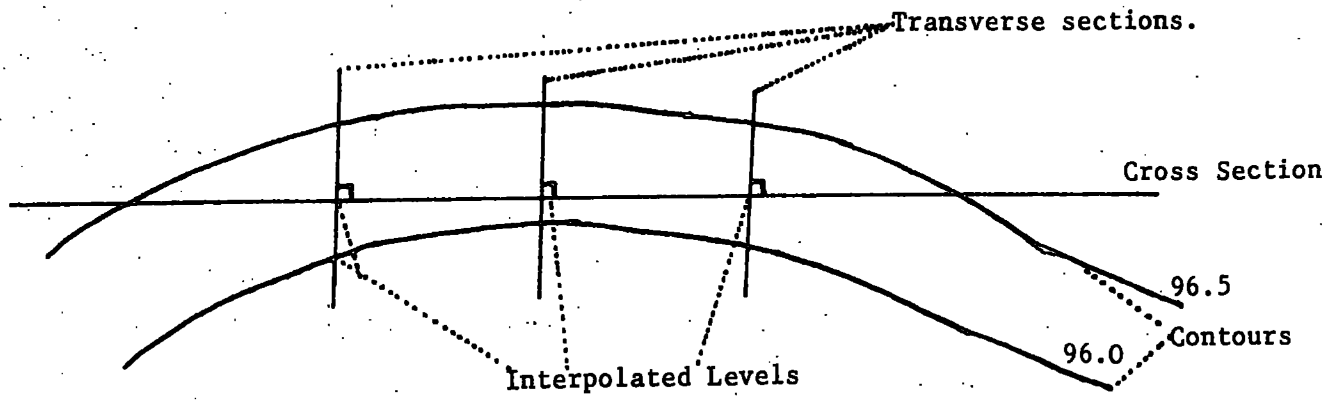


FIG. 5.4.2(v)

This procedure detects transverse curvature (transverse to the direction of the cross-section) and ensures maximum use is made of the stored model data.

The frequency at which these secondary points are inserted depends on the curvature contained in the model surface and taken to its extreme limit the more points then the greater the reliability of the section. However for road design considerations at 1/500 scale 20 metres is a realistic figure. Thus where inspection of the primary output data indicates adjacent points at the same level then an intermediate point is inserted by secondary means. Furthermore if the distance between any points on the cross-section exceeds 20 metres and also if no points are detected sufficient points must be introduced to satisfy the 20 metre criteria.

The figure of 20 metres is empirical and is the same as that quoted in the specification for maximum distance between adjacent strings for models stored at 1/500 scale. Obviously the value could be relaxed for larger scales.

5.4.3 Further Implications of Secondary Interpolation

(a) Secondary interpolation should be restricted to maximum spacing between contours.

Logically if one is going to include a secondary point when the spacing between primary cuts exceeds 20 metres (say) then it is wrong to interpolate that value from secondary cuts if the distance between them itself exceeds 20 metres.

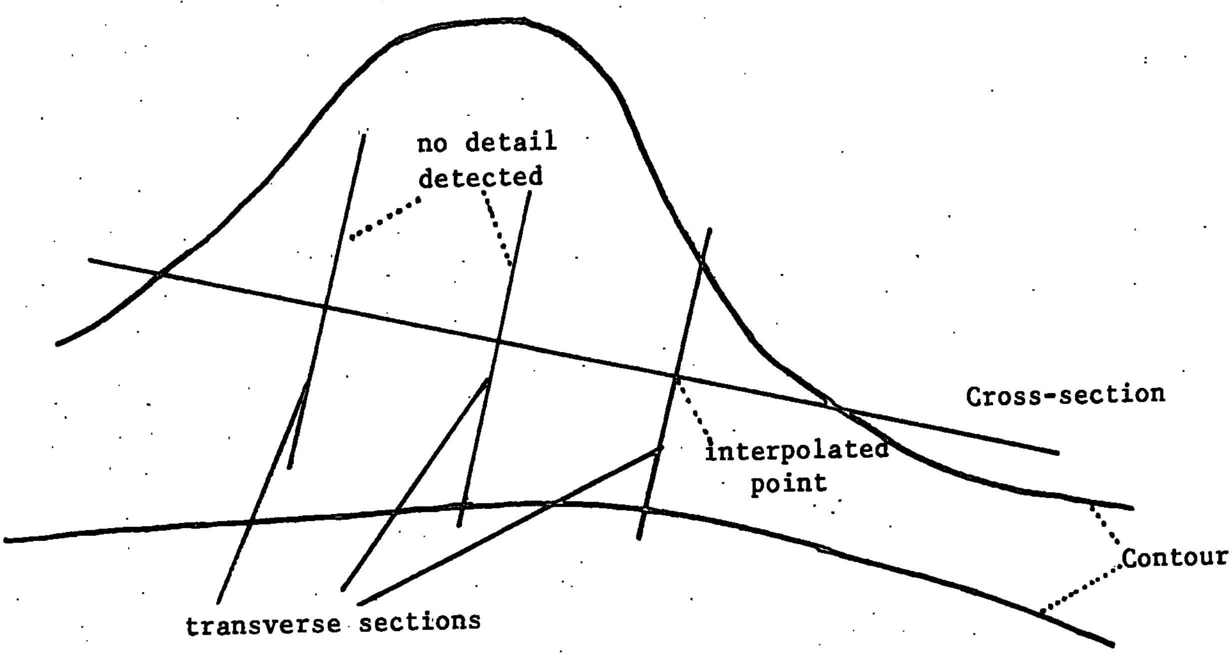


FIG. 5.4.3.(i)

This restricts secondary interpolation to being a local procedure and if no left and/or right offsets are found within the tolerance specified for the inclusion of secondary points then no point is introduced, which means the transverse curvature is insignificant and primary interpolation takes precedence e.g. FIG. 5.4.3.(i).

(b) Data preparation error is similar for secondary interpolation as it is for primary interpolation.

The same basic assumptions are made for secondary interpolation as for primary, so this appeared to be true. Reference to tables 8.1, 8.5, 8.12 show this reasoning to be correct.

The histograms in Table 8.1 may lead one to believe there are more errors under secondary interpolation. However it must be borne in mind that secondary interpolation includes more points and there were more "out-liers" (errors outside the limits shown in the graphs) under primary interpolation than when amendments (i.e. secondary interpolation) were included. This was especially true for tables 8.1a, b and c and was partially due to the absence of the improvements which will be detailed in 5.4.5.

It was thought impractical to devise a parametric test to verify the fact that secondary interpolation did not radically affect the data preparation error. Instead a population of means and variances with the binomial distribution was used. Table 8.16(a) shows the result of this non-parametric test.

One argument which may be used against the form of secondary interpolation employed is that interpolation from points perpendicular to the original cross-section does not provide the best results. FIG. 5.4.3. (ii) shows the basis for this argument.

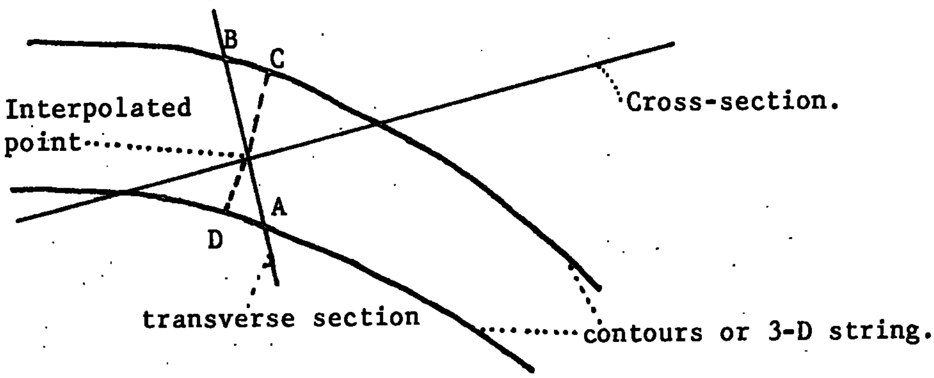


FIG. 5.4.3.(ii)

It could be suggested that interpolating between points C and D would be better, because they are closer together, than interpolating between points A and B. However the above results show that although this may indeed be better it is unnecessary. To search the model in such a way as to find the points C and D, (which may not be collinear with the required point) would involve considerably more computer processing than the method proposed and would not enhance the accuracy past the basic accuracy of the model. One must also remember the ultimate objective i.e. calculation of earthworks quantities and the accuracies involved consist of those across the cross-section and also those between cross-sections. Reference to the results of Chapter 5.2 show that the data preparation error compares favourably with the errors involved between cross-sections, i.e. longitudinal spacing.

5.4.4. The Use of 3-D Strings

The specification (*) for the digitising of strings is as follows :-

"There are two dimensional and three dimensional strings. The 2-D strings will represent contours and 3-D strings will represent any other features e.g. the tops and bottoms to railway embankments, river embankments and ditches; The centre line and channels of existing roads.

Where contours are sparse (greater than 30 metres apart at 1/500 scale) a 3-D string can be placed to cover the area and so represent the surface.

Where contours are very close together, the appropriate string may be ended and restarted as a new string elsewhere.

Contours will be digitised up to highway boundaries or similar features but the highway will be represented by 3-D strings only".

This specification is sufficiently vague for the operator of the stereoscopic equipment to use his subjective judgement, and although this is important some refinements are necessary to make the collection of data more systematic. The refinements suggested which are easy to incorporate improve the quality of the model and therefore the overall error of the model and may be highlighted by the use of the model plot.

* See Ref. 7

The stricter specifications were touched on in 5.3.2 and were that three dimensional strings for all features other than contours should be included and in particular :

1. where regular vee-shapes occur in the contours.
2. across the innermost closed contour of any set of nested contours.
3. where spacing between contours become excessive (i.e. over flat ground) in a more systematic manner.

5.3.6 also explained the importance of having intelligible 3-D strings ensuring a more systematic approach and therefore (one would hope) a more accurate approach.

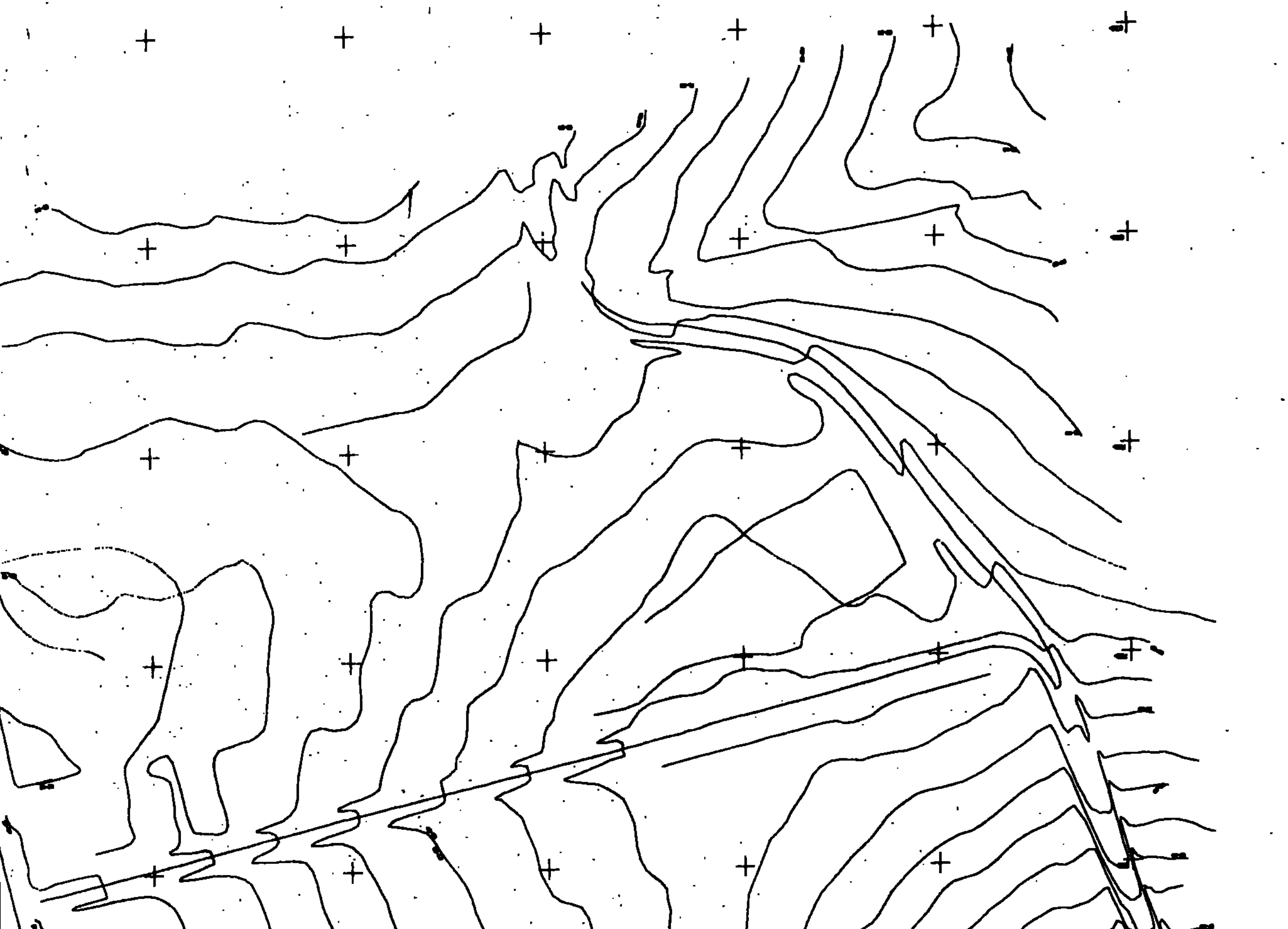
5.4.4.(a) 3-D strings need always be included where regular vee shapes occur in the contours.

Detailed investigation of the individual errors making up each mean showed apparent blunders at points where the cross-section cut a contour as that contour was crossing a 3-D feature. The effect was adequately removed by infilling the vee-shapes. It is imperative that whenever there is an appreciable sharp change in gradient a 3-D string is included to define the feature. A striking advantage of the contoured string model is that a plot generally reveals (where the direction of contour changes abruptly) where 3-D strings are required.

The results for Block A of the Heddon Test Area were closely examined and it was found that appreciable differences were occurring at the ditches running east-west across the area at both the north end and the south end of the site. In these places (see plan 5.4.4(i)) regular vee-shapes were apparent in the contours. Using a table digitiser more 3-D strings were added into the model by joining the vertices of the vee-shapes to one-another and attaching the level of the contours to the vertices. The "revised" model was then used to extract the same cross-sections as before.

One anomaly was immediately apparent in that the inclusion of more 3-D strings should not of its own improve the data preparation error but this indeed happened. The explanation of this is that the end-points of a cross-section need necessarily to be interpolated from the adjacent cuts either side, and where more detail is included this interpolation is improved.

Reference to tables 8.5(d), 8.6(d), 8.7(d) and comparison of these with tables 8.5(b), 8.6(b) and 8.7(b) show the improvements to the model by the inclusion of 3-D strings along regular vee-shapes. This improvement together with that of secondary interpolation radically improved the overall error both for mean and variance (compare table 8.7(b) before secondary interpolation with table 8.7(d) after secondary interpolation.



THROCKLEY TEST AREA

FIG. 5.4.4.(i)

It will be noted that the quality of model error without secondary interpolation for the "basic" model is identical to that for the "revised" model and there is no obvious improvements to the model results. The overall errors contradict this. Again the revised model does not seem to improve the variances of the data preparation errors with secondary interpolation. These discrepancies are allowable because of the particular positioning of the added 3-D strings with respect to the cross-sections. It does seem, however that a definite indication is given of the advantages to be gained by making this specification.

The effect on the overall error is rather surprising after the constituent errors have been considered. One should remember that although theoretically the data preparation error may be isolated, in practise it depends somewhat on the model itself. e.g. the cut of a contour is described by the intersection of the cross-section with the line drawn between two consecutive model points.

Thus the data preparation error can in fact be correlated with the quality of model error in producing the overall error. A simple model would be :-

Data Preparation Error	X
Quality of Model Error	Y
Overall Error	Z

$$Z = X + Y$$

$$V(Z) = V(X) + V(Y) + 2\,\mathrm{COV.}\,(X, Y)$$

if the covariance term is negative then V(Z) is smaller than the sum of the variances of X and Y. This effect occurring will produce the result discussed.

Of course the combination of the data preparation error with the quality of model error to produce the overall error in not linear, even though their expectations are i.e. E (overall) = E (Data Prep.) + E (Quality of Model)

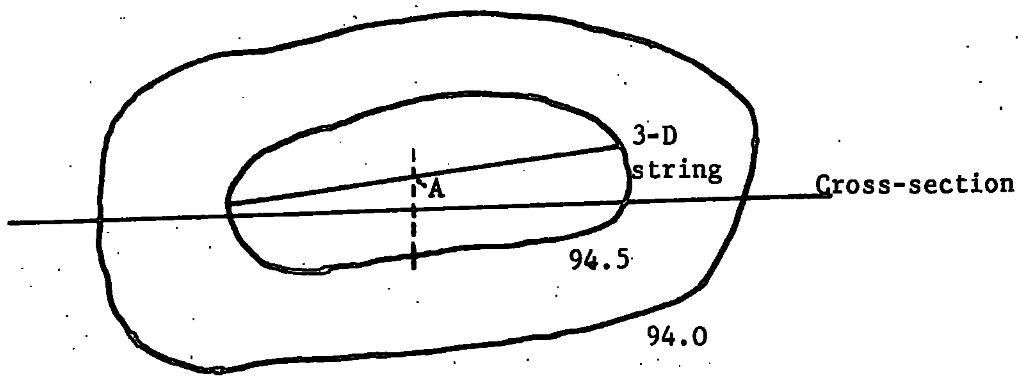
There are many interaction terms, such as length of interpolation to be considered in producing an analysis of the overall variance. The analysis of the variance and covariance terms for the factors involved is both complex and unnecessary at this stage of development.

5.4.4(b) 3-D strings need always be included across the innermost closed contour of any set of nested contours.

In physical terms a set of nested contours describe hills or hollows (valleys) and because they are lines of equal height the crest of the hill or the bottom of a hollow will never be adequately shown without the use of 3-D strings. A spot height will not remove this difficulty since the model relies wholly on the intersection of strings for the extraction of results. If no model detail is stored then the program can never detect it and even mathematical interpolation techniques will not aid the problem.

If one 3-D string is placed across the contour passing through the peak or the lowest point then no further problems ensue. If on cross-section extraction the 3-D string is not intersected then the secondary interpolation process will determine two adjacent points of equal level and infill a necessary point interpolated from the 3-D string and contour, e.g. FIG. 5.4.4.(ii)

FIG. 5.4.4.(ii)



two adjacent levels equal detected.
Point A infilled by secondary interpolation.

One can decide from this that only one 3-D string is required since the detail will be picked up by either primary or secondary interpolation.

5.4.4.(c) where spacing between contours become excessive (i.e. over flat ground) 3-D strings should replace them and be placed in a systematic manner.

It required only a cursory glance at table 8.5(b), 8.5(c), 8.6(b), 8.7(b) and 8.7(c) to realise that the scale of errors for Block B of the Heddon Test Area are markedly different to those for Block A. The confidence intervals drawn on graphs 8.12(b), 8.12(c), 8.13(b), 8.13(c), 8.14(b) and 8.14(c) display this to a greater extent.

It ought to be stressed that the Heddon Test Area was specially digitised with a knowledge of the purpose of the tests. The area of the tests had previously been partially covered by a square grid model. In supplying a square grid model the contractor provides the model in card form and also a contoured plan in map form. Investigation of the map for the square grid and the plot for the string ground model revealed some interesting differences.

The test area could be split into two halves, Block A and Block B the first being sloping ground, and the second being flat. Over the sloping ground the contours of the two plans followed a similar pattern even though they were slightly displaced. Over the second flat area, however, the contours were widely different. In the tests the sloping ground (Block A) had very little data preparation error (see Table 8.5(d)) whereas this error was substantial in both size and variance for the flatter Block B (Table 8.5(c)) The above facts seem to imply the difficulty of the photogrammetist in defining contours over flat regions.

Even when contours are included over flat areas the specifications lay down the need to infill 3-D strings when contours become too sparse. The quality of model error as tabulated in table 8.6(c) does not reflect the anticipated improvement to the model by having this specification.

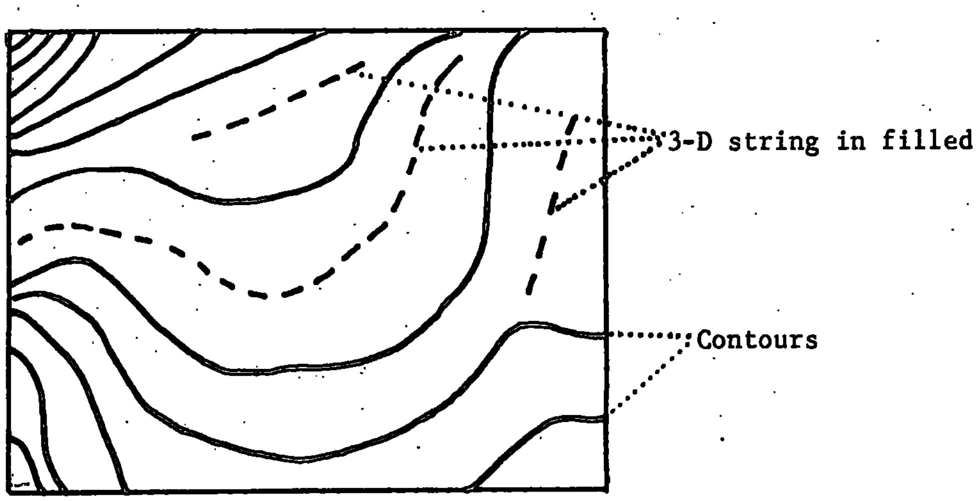
Obviously with faults both in the data preparation error and in the quality of model error one cannot expect to obtain a satisfactory overall error and table 8.7(c) is a reminder of this. The result is that a regular sampling of the ground fails to be achieved and depending on the operator tends to be systematically too high or too low.

It is accepted that the operator of the stereoscopic equipment can define 3-D strings more accurately than 2-D strings. This is because the 3-D strings are defined by a "discrete process" - the operator picks up the grid coordinates and sets the floating mark over the point. The contours are digitised by keeping the level of the floating mark constant and moving across the photographs keeping the mark as close to the surface as possible. The grid coordinates are picked up at a certain time interval, and obviously longitudinal and lateral inaccuracies may play a part in the final error for the contours whereas they do not for the 3-D strings.

This difference between the methods will be exaggerated over flat ground where the small difference in level provides little contrast to the operator. Having examined the mediocrity of the results and their causes a different specification needs to be used.

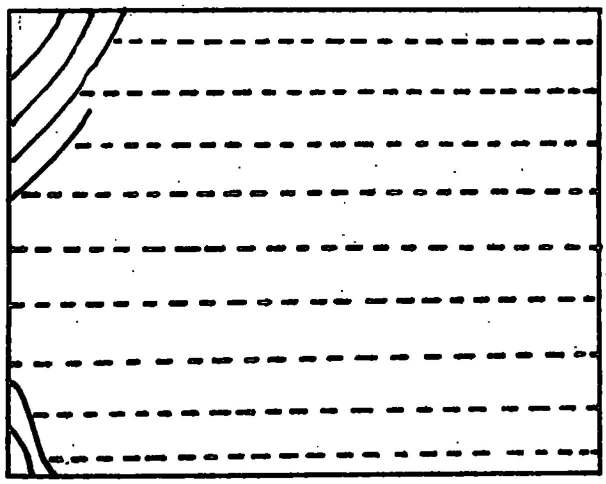
An improved specification might well be to ask for 3-D strings only across an area which drops below a specified minimum slope. This would require the stopping of contours when their spacing between them become excessive and obviate the need for infilling (usually) meaningless 3-D strings. The definition given would be by 3-D strings which either followed the general pattern of contours or more preferably near parallel straights across the area. The straights need not be perfectly parallel provided the 3-D coordinates were collected accurately. It is not necessary for them to be in the form of a square grid or even strings crossed roughly perpendicularly. From the earlier results (secondary interpolation), if on cross-section extraction the 3-D strings were not cut on primary interpolation they certainly would under secondary interpolation.

Diagrams FIG. (5.4.4.(c)) demonstrate the suggested specification.



Present specification.

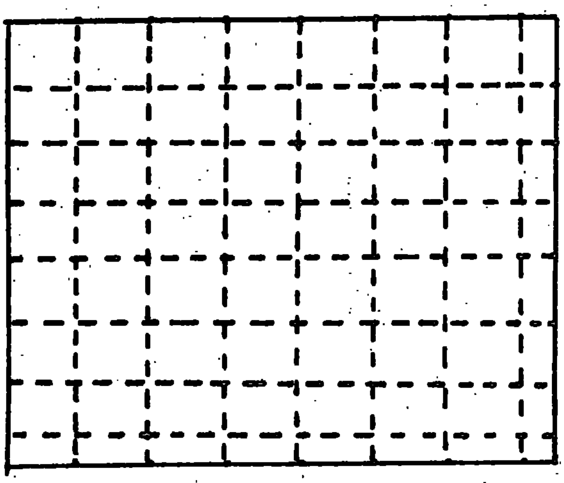
FIG. 5.4.4.(c)



perfectly parallel 3-D strings

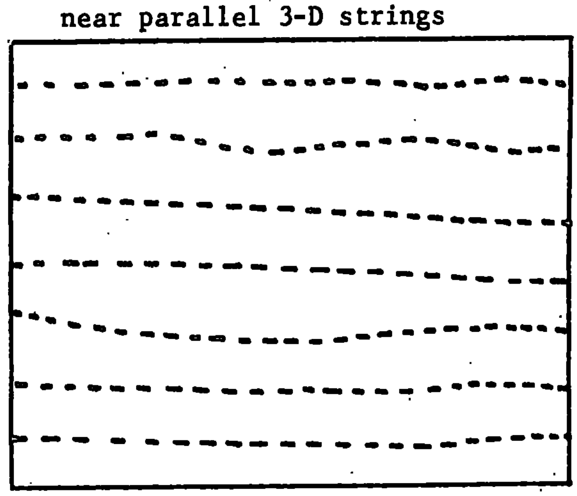
alternative specification (not necessary)

near parallel 3-D strings



Square Grid Mesh 3-D strings.

alternative specification (not necessary)



SUGGESTED SPECIFICATION.

5.5 Summary

The present chapter has investigated the methods of storage of, and retrieval of information from the string ground model. Errors have been shown to be noticeably reduced by the inclusion of some easy to apply techniques both to the collection methods and the retrieval of cross-sections.

Taking cross-sections at a 10 metre intervals for detailed 500 scale design is much better than 20 metre intervals. For complicated interchanges and where there is a predominant curvature in the horizontal alignment, a "length factor" different to the chainage interval may be preferable.

One of the great advantages of an aerial string ground model is its visual effect. This helps in substantially reducing the grave hazards of gross errors and blunders.

Consideration of the cross-sectional area enabled some inadequacies of the present techniques to be highlighted. Using the same stored information better results can be gained by using secondary interpolation. In certain areas there also appears a need to give a clearer indication as to how the model should be stored.

Overall the model is shown to be a reliable and accurate representation of the land surface and the modifications suggested help make the use of ground models totally justified.

CHAPTER 6. VERIFICATION OF SUBMITTED CONTRACTS

6.1 Introduction

In the initial stages of aerial survey development the various authorities prepared their own contract specification for the provision of a survey. The specification was a "method" specification and was primarily concerned with the complete air survey, the production of a ground model being secondary. It was a "method" specification in the sense that the procedures for setting up the stereomodel and the taking off of measurements and the drawing of plans were laid down. No testing procedure applicable to the data was defined.

In an attempt to ensure a consistency of specification throughout the country the Department of the Environment (D.O.E.) prepared Technical Memorandum H9/70 which recommends a standard practice. Again this memorandum is primarily a "method" specification for the complete aerial survey and digital ground models are only one part of it and even then square grid models only are referred to.

At the present time the majority of specifications for Aerial Surveys either quote Technical Memo. H.9/70 or are based on it. Durham County Council have modified the memo for their own use and have drawn up a specification for the creation of String Digital Ground Models which is also widely used by other authorities. This first attempt to extend Technical Memo. H9/70 is once again completely of the "method" type and since there is no recognised testing procedure there can be no guarantee of good results.

The results of Chapter 5 have provided clearer guide line for the Specification of a greatly improved model that may be achieved with little extra effort. The complete model then allows for the testing procedure described in 3.4 to be implemented thus ensuring the terms of reference of a contract are properly fulfilled.

The following sections will make a critical appraisal of present specifications suggesting amendments and including a general proposed testing procedure by which the client may accept or reject the contracted ground model. It ought to be stressed that at this stage only a broad framework for a testing procedure can be proposed because investigations need to be much more extensive before specific parameters are included in a contract document. Nevertheless, the framework should provide a basis for further study and discussion, enabling a definite specification to be produced.

To avoid the need for duplication relevant sections of a typical specification are referred to by use of the reference in that specification. The relevant sections are reproduced in Appendix 1. Where amendments are suggested the revised specification is given in upper case letters followed by a cross-reference so that the reasons for the change may be understood.

6.2 Revised Specifications

c.f. 3.6 Contours

Contours, where required, shall be shown at vertical intervals of :

(a) 0.5 metres at 1/500 mapping scale.

(b) 1.0 metres at 1/1000 and 1/1250 scales or at 2 metre intervals when a digital ground model is specified.

(c) 2.0 metres at 1/2500 scale.

Where steep slopes are encountered and it is not practicable on the plan to represent each contour fully throughout its length, the Contractor may with the Engineer's approval terminate certain intermediate contours. In flat areas where the horizontal distance between contours exceeds 30 metres, the Contractor shall, DISCONTINUE CONTOURS AND SATURATE THE AREA WITH 3-D STRINGS OF SPOT LEVELS AT A MAXIMUM SPACING OF 20 METRES, paying particular attention to local high and low points in the area.

Cross-Reference 5.4.5(c)

c.f. <u>3.10 Accuracy of Contours</u>

WITHIN ANY AREA OF THE SURVEY ALL CONTOURS, WHEN CHECKED BY PRECISE LEVELLING FROM THE AGREED ORDNANCE DATUM SHALL BE ACCEPTABLE TO THE AGREED TESTING PROCEDURES AS SET OUT IN APPENDIX 6.

Cross-Reference 3.3 and 6.3.

c.f. 5.4.2. <u>Strings</u>

The topography is described by the use of both 3-D and 2-D strings (break lines and contours).

c.f. 5.4.2.1. <u>3-D String Definition</u>

A string shall be placed along EVERY sharp feature or change of ground slope. ONE 3-D STRING SHALL ALSO BE PLACED ACROSS THE BROW OF EVERY HILL AND DIP OF EVERY HOLLOW, THE HIGHEST AND LOWEST POINT RESPECTIVELY BEING DEFINED. 3-D STRINGS SHALL ALSO REPLACE

THE USE OF CONTOURS WHERE CONTOURS ARE SEPARATED IN EXCESS OF 30 METRES. ACROSS SUCH FLAT AREAS 3-D STRINGS SHALL BE INCLUDED WITH A MAXIMUM SEPARATION OF 20 METRES.

PARTICULAR ATTENTION SHOULD BE PAID TO THE NOTES DETAILED IN APPENDIX 6.

Cross-Reference 5.4.5. and 6.3.

c.f. 5.4.2.2. <u>3-D Strings</u> - Accuracy of Measured Points.

POINTS SHALL BE RECORDED ALONG 3-D STRINGS AT A FREQUENCY AND ACCURACY SUFFICIENT TO CONFORM TO THE TESTING PROCEDURES AS SET OUT IN APPENDIX 6.

Cross-Reference 6.3.

c.f. 5.4.2.5. <u>Density of 2-D Strings</u>

(a) Flat Areas - where contours are sparse - greater than 30 metres apart -THE CONTOURS SHALL BE DISCONTINUED AND THE AREA SATURATED WITH 3-D STRINGS SO THAT THE DISTANCE BETWEEN ADJACENT 3-D STRINGS DOES NOT EXCEED 20 METRES.

Cross-Reference 5.4.5.(c)

6.3 <u>Suggested Testing Procedure (Specification)</u>

(<u>NOTE</u> - This section will comprise APPENDIX 6 as referred to in 6.2).

The digitised ground model shall satisfy the following criteria and be subject to tests by an independent contractor or the Engineer.

The tests shall be sufficient cause for Clauses 21, 22 and 35 of the conditions of Contract to be implemented.

1. The plot of the model will be visually inspected for :

   (i) absence or partial absence of 2-D strings (contours)

   (ii) absence or partial absnece of 3-D strings (features)

(iii) inaccurate overlapping of closed contours.

(iv) double digitising of sections of contours.

(v) crossing of contours.

(vi) unintelligible 3-D strings.

<u>NOTES</u> : (a) 3-D strings are required at every change of curvature and especially across contours where regular vee-shapes occur in them. They are also necessary across the brow of a hill and the bottom of a hollow i.e. across the innermost closed contour of a set of nested contours.

(b) 3-D strings should be given in such a fashion as to make them meaningful. This improves the visual aspect of the model and reduces the probability of blunder.

(c) the plotting of the string model is an efficient means of reducing "blunders" in the model. Discussion of these details are found in M.O.S.S. R. & D. Note 1 : 1/74 (*).

2. Model tests have shown that the systematic error in the aerial survey tends to be the most significant. In the photogrammetric evaluation the systematic errors for each photograph or stereopair may be different and is not stable. Furthermore along a "straight strip" of photographs there may again be a systematic error. For these reasons a small sample of information through each stereomodel need s to be collected by precise measurement.

(i) consider 3, 200 metre straight cross-sections in each stereomodel.

(ii) level each cross-section with a precise land instrument.

* M.O.S.S. R. & D. Note 1:1/74 is effectively Chapter 5.3.

(iii) remove the model representation of the cross-sections from the stored data.

(iv) evaluate the Data Preparation Differences and the Overall difference (i.e. errors)

(v) compute means, variances and sizes of sample for each cross-section, and for all combined.

(vi) apply the following criteria to the data.

(a) $|\bar{x}| \leqslant b$

(b) $s^2 \leqslant w^2$

where $\bar{x} = \frac{1}{n} \sum_{i=1}^{n} X_i$ X_i is the difference in level of a point by land survey against model, n is the size of sample.

$$s^2 = \frac{1}{(n-1)} \sum_{i=1}^{n} (X_i - \bar{x})^2$$

b and w^2 are critical values set for the particular scale mapping undertaken, and for 1/500 scale mapping are both 0.125.

(vii) should the above criteria hold the data should be deemed satisfactory. Otherwise the discrepancy should be investigated.

<u>NOTES</u> (a) Justification and discussion of this criteria is contained in M.O.S.S. R. & D. Note 2:1/74 (**)

(b) The field survey should be done using a precise levelling instrument and measurement made so as to accurately define every point on the cross-section by linear interpolation and at least one

(**) M.O.S.S. R. & D. Note 2:1/74 is effectively Chapter 3.4.

point per 10 metre length of cross-section should be taken off.

6.4 Examples

The use of the testing procedure as set out in 6.3 may be demonstrated by reference to the test data collected.

6.4.1 Bowburn East-West

The Bowburn East-West model was a 1:2500 scale mapping model, having a two metre contour interval. Equivalent to the critical values of b and w^2 being both 0.125 for 1:500 mapping the values for 1:2500 scale mapping are :-

$$b = 0.5 \;:\; w^2 = 2.0$$

The values are derived from

1:500 $\frac{N_\alpha}{\sqrt{n}} \; w \leqslant 0.25$ (half the contour interval)

max $w = (0.125)^{\frac{1}{2}}$ $\quad \therefore \; \frac{N_\alpha}{\sqrt{n}} = \frac{0.25}{(0.125)^{\frac{1}{2}}}$

$\frac{N_\alpha}{\sqrt{n}}$ is constant for different scales.

$\therefore$ for :2500 $\frac{N}{\sqrt{n}} \; w' \leqslant 01.0$ (half the contour interval)

$\therefore \; w' \leqslant 1.0 \frac{\sqrt{n}}{N_\alpha} \quad \therefore \; W'^2 \leqslant 2.0$

for further explanation See 3.3.

Table 8.5(a) shows that for the Data Preparation Error the largest absolute value of the mean (under secondary interpolation) is 0.486 and for the variance is 0.157466. Both of these lie within the critical limits especially the variance and so the Bowburn Area may be deemed satisfactory.

The criteria also hold for the overall error with (secondary interpolation) except for the first cross-section. (See Table 8.7(a)). The way in which secondary interpolation has improved the results is noticeable (e.g. Table 8.11(a)) and the inclusion of the other modifications (for example more 3-D strings because of the rapidly changing nature of the ground) would no doubt improve the model even further.

6.4.2. Horsley Heddon

The values in Table 8.5(b) show that for Block A, after the secondary process, the data is more than acceptable (largest absolute values are mean: 0.044, variance : 0.002888, both substantially less than 0.125).

The overall error for Block A (Table 8.7(b)) is also acceptable. In fact, if one works backwards from the criteria that $\frac{N_\alpha w}{\sqrt{n}} = 0.125$ and substitutes the largest absolute overall error variance (0.00346) for the stated number of observations (26), then the calculated value of N_α is 10.82 which implies that 100α is less than 1.0×10^{-10}. Thus in this instance the probability that the data is unacceptable is less than 1.0×10^{-12}.

For Block B the criteria do not hold for the mean. One would expect the variance to be large (which it is in comparison to Block A) when it is noted that when flown the area was under plough but when land levelled was under pasture. However this cannot account for the bias in mean. The explanation for this apparent anomaly is probably two fold.

First the definition of the model over flat areas is difficult for the photogrammetist in that there is no contrast in light shade and, in leading the floating mark around a contour he has some problems in distinguishing a level difference. Secondly, different operators have different techniques for dealing with ploughed or furrowed ground; some pick up the tops of the furrows, and others the bottoms. Taken together, the bias being large and the variance being large in comparison to Block A the results are understandable. A discussion of how these problems may be overcome is detailed in 5.4.5(c), where it is proposed that 3-D strings ought completely to replace contours and should be given in a systematic manner. Table 8.6(c) adds weight to the argument that insufficient detail has been given over flat areas, in that after secondary interpolation the quality of model error is good with respect to bias but poor (in comparison with Block A (Table 8.6(b)) with respect to variance.

6.4.3 Summary

The most significant result of the above examples is the slackness of the criteria. For both 1:2500 scale mapping and 1:500 the worst variance was less than one eighth of the critical value. The effect is to make the mean error more significant. Although the Heddon Horsley area was specially digitised the Bowburn East-West was not and the tests were done in a poor area, of quarry and bracken. Even though the spirit of the present specification was followed the test criteria was made slightly tighter. The overwhelming fact to emerge, therefore, is that under the present conditions of contract it is virtually impossible for the aerial survey contractor not to provide an acceptable ground model, and the critical values need to be reassessed, in the light of further research.

6.5 Practical Application of the Testing Procedure

It has been proposed that three or more cross-sections be taken in each stereomodel and precisely levelled. The levels given by the Aerial Survey are based on the levels of permanent ground markers (P.G.M.) distributed throughout the length of the whole model. The P.G.M's are set up either by the contractor or the client and are tied into the Ordnance Survey. They serve two purposes. First to provide ground control for the aerial survey and secondly for eventual setting out of the designed road and because of the dual purpose they need not necessarily be sited ideally for the aerial survey.

The smallest errors may be expected to occur around the P.G.M's, and it would seem logical to set out and level cross-sections close to them. In each stereomodel it is improbable that there will be at least one P.G.M. For such stereomodels, supplementary control should be given from the closest P.G.M. The test cross-sections should then be levelled.

Over different parts of the model and especially over differing types of terrain, it is important to realise that inaccuracies may vary considerably. This is especially true, for example, in wooded valleys, where the aerial survey contractors readily accept the difficulties involved in following contours. The only real solution in this instance is for a field party to do a supplementary terrestial survey. However from engineering considerations the ground shape is only realistic for earthworks calculations once the site has been cleared of trees and scrub, and importance should not be placed on high accuracy in such areas. It may indeed be advantageous in such situations to double the critical values.

Ideally the cross-sections should be taken right across the model and perpendicular to the anticipated design line (typical model width at 1:500 scale is 200 metres). At least twenty points at ten metre intervals should be levelled and supplemented by the addition of levels at points of angular change so that there will usually be between twenty five and thirty points per cross-section.

Following the theory of 3.4 the number of points levelled then enables a satisfactory analysis to be carried out. If each of the cross-sections agree to the criteria laid down then the model may be deemed satisfactory. Otherwise by taking all the cross-sections together more confidence may be placed in the results and if the criteria are still not satisfied then a closer examination of the source and cause of error is required.

The analysis of the level information may be accomplished using computer programs specifically written, which consider both the land survey and aerial survey representations of the cross-sections as a series of offsets and levels. These representations are compatible with those used in program "HREACS" in the B.I.P.S. suite of programs (See Ref. 4) Fig. 6.5.1 shows a flowchart of the processing and analysis of the data for the testing procedure. The relevant computer programs are indicated in parenthesis.

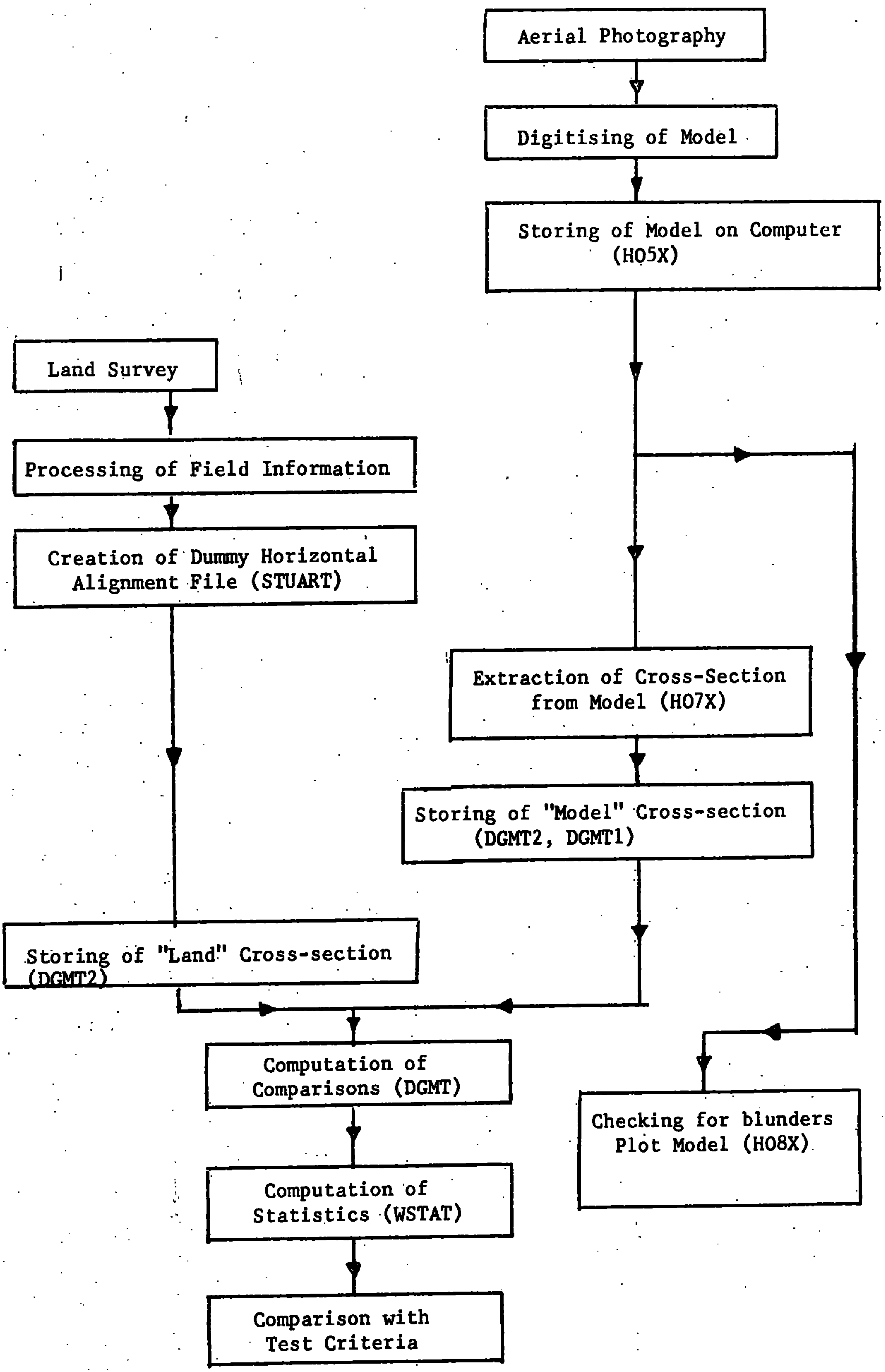

Aerial Photography
Digitising of Model
Storing of Model on Computer (H05X)
Land Survey
Processing of Field Information
Creation of Dummy Horizontal Alignment File (STUART)
Extraction of Cross-Section from Model (H07X)
Storing of "Model" Cross-section (DGMT2, DGMT1)
Storing of "Land" Cross-section (DGMT2)
Computation of Comparisons (DGMT)
Checking for blunders Plot Model (H08X)
Computation of Statistics (WSTAT)
Comparison with Test Criteria

FIG. 6.5.1.

CHAPTER 7. CONCLUSIONS

7.1 Introduction

The terms of reference and the purpose of the project were set out in Chapter 1 as :-

1. Justify the use of a ground model in calculating earthworks quantities.
2. Investigate the methods of storage and retrieval of information and suggest additions or amendments to the present techniques wherever necessary.
3. Investigate testing procedures for acceptance of contracted ground models by aerial survey and suggest an appropriate test on which to base the acceptance.

The conclusions may be conveniently categorised into these sections. The project, however, cannot assume that every question has been answered and further topics of research have been revealed in the study.

7.2 Justification of the use of ground models

The aerial surveyed string digital ground model has been shown to be a very sound concept and its use in calculating earthworks quantities is justified. The models can have an important role in both the preliminary and final stages of highway design. Their use in preliminary design is often decided on economics rather than a combination of economics and potential for thorough scheme evaluation.

Model tests have shown that the systematic error in the aerial survey tends to be most significant, which is certainly the case at 1:2500 scale mapping, but if the levels of the vertical alignment tie points are taken from the same survey then the vertical offsets are relative and the earthworks volumes are realistic. When designing the final scheme utilising a 1:500 scale survey and model, the tie points will be hand levelled but because of the greater accuracy of the aerial survey the earthworks volumes extracted from the models at different scales will compare favourably.

The visual impact of the plotted string ground model also helps the design engineer to fully appreciate the terrain surface.

7.3 Investigation of Methods of Storage and Retrieval

In making any criticism of a new technique it is necessary to understand the implications of those presently available. The accuracy of using the end area method of calculating earthworks quantities is dependent on the spacing of the end cross-sections, and the "length factor", to much the same extent as the errors implicit in the model data.

Although the spacing of cross-sections and the "length factor" have not been fully investigated, preliminary analysis and results (3.2 and 5.2) have shown them to be more than comparable with the errors inherent in the string ground model concept.

Some inadequacies in the present practise of storing and retrieving information from the string model have been studied in Chapter 5. Perhaps the most important inadequacy in storing the information has been the use of three-dimensional strings both for angular features and for describing flat ground.

The incorporation of a secondary interpolation procedure has dramatically improved the retrieval techniques. Empirical values have been introduced for this secondary process and these may demand further investigation. The minimum distance between offsets has been suggested as 20 metres. This seems reasonable for a 1:500 model but may produce too many cross-section points in certain circumstances. The lateral interrogation for the secondary process is also suggested as 20 metres. This ensures the secondary process is local but perhaps this also may need to be reconsidered.

7.4 Investigation of Testing Procedures

There are three important requirements to be borne in mind when developing a testing procedure. First it is useless unless it may be practically applied; secondly both the contractor and client must be satisfied that it is fair; and thirdly it must be theoretically sound. The testing procedure described in 3.4 and explained and documented further in Chapter 6 would seem to serve these requirements.

The plotting of the model is invaluable in the rectification of blunders and also for its visual impact on the design engineer.

The taking of test cross-sections in the field is practical to the engineer and gives a good indication of the inherent inaccuracy of each individual model.

Using the sample mean and sample variance as criteria in the testing procedure should be readily acceptable. However the values used in the criteria have been shown to err on the side of slackness. This is inevitable in preliminary acceptance of a testing procedure, but based on further research might well be tightened up. This would be to the mutual advantage of contractor and client since the quality of the model would be known.

7.5 Suggested Further Research

The research has highlighted a number of topics which could be usefully investigated further.

1. The problems of cross-section spacing and "length factor" have been dealt with superficially because they were not directly applicable to aerial survey. Contributory factors such as road curvature (both laterally and longitudinally) and the cumulative effects of the ground error in cutting and embankment on the overall earthworks value, need to be considered.

2. The need for more definition of three-dimensional strings for angular features and over flat areas is unquestionable. However, to achieve an economic and viable model, the extent to which they need to be included, especially over flat ground, needs further investigation.

   For this purpose it has been agreed, following a discussion of the project with an aerial survey company, that Block B of the Horsley Test Area be saturated with three-dimensional strings so that further comparisons may be made.

3. Various empirical values have been postulated for the secondary process to demonstrate its usefulness. Although they seem sensible further investigation should reveal whether they need to be changed.

4. Resources have dictated that tests were confined to small areas. Obviously a more comprehensive range of types of terrain needs to be investigated. These will be considered as contracts are awarded and not as an exercise in itself.

5. Following the research detailed in (4) above, the criteria used in the testing procedure will be reconsidered. This

would entail the compilation of new probability of rejection tables explained in 3.4 (for example, if at 1/500 scale the critical values are changed from 0.125 to 0.1 then new probability of rejection tables would indicate the size of sample required.)

7.6 Summary

The investigations have confirmed the model concept is sound and the amendments to the procedures, highlighted by the research, have improved the quality of the model and results. The culmination of the research has been the testing procedures. To the author's knowledge this is the first practical method of testing ground models to be produced, which has a theoretical basis, and is available to assess the models giving confidence to both the contractor and the client.

CHAPTER 8. HISTOGRAMS, TABLES AND GRAPHS

The following diagrams and tables present the results found in the three test areas considered. Each section is prefaced by an explanation.

Positive errors indicate that the true value is higher than the value given by the model.

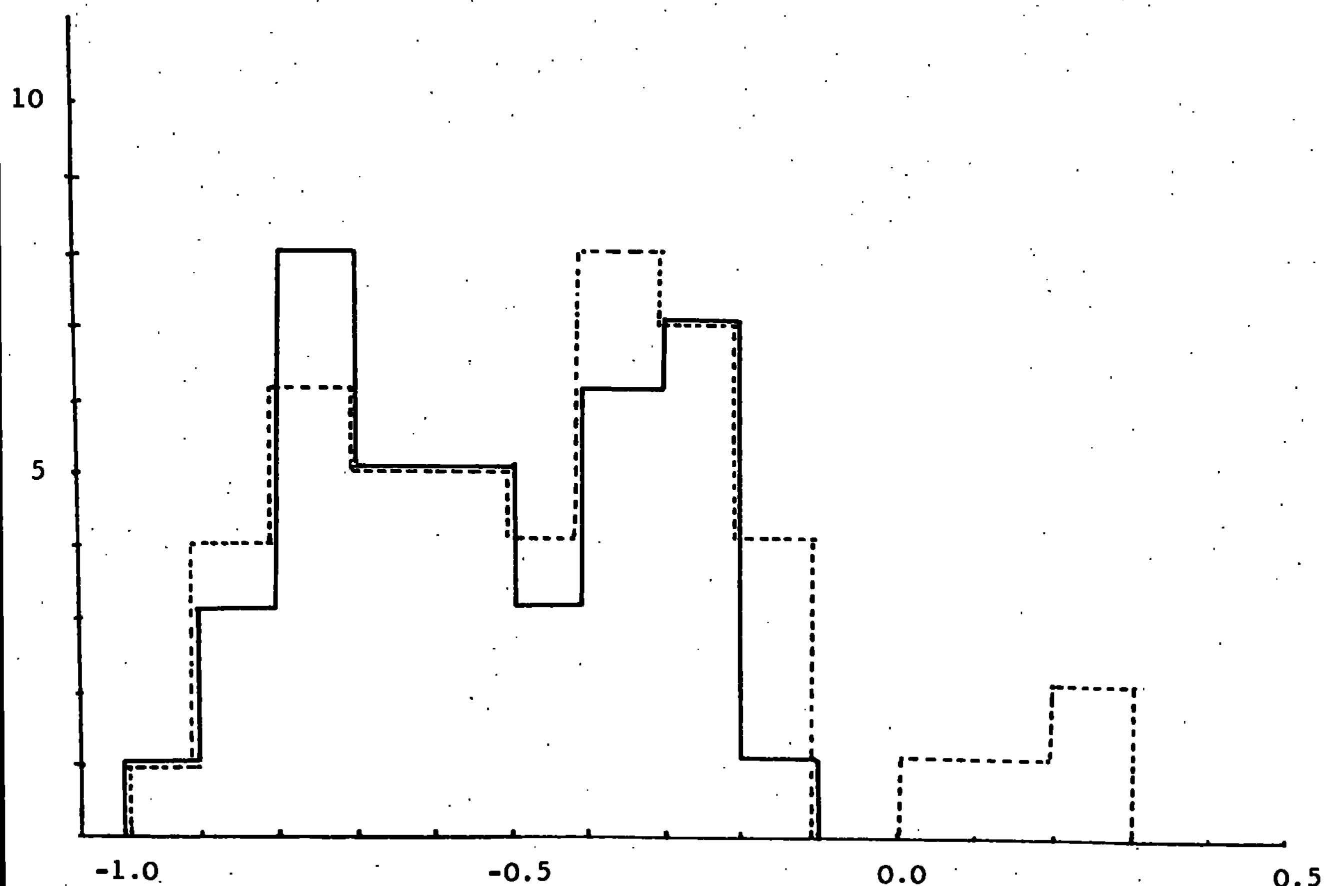
There is no Table 8.4.

In those tables where a revised model Block A is enumerated the revised model consisted of the basic model plus some 3-D strings which were added in across the necks of vee-shapes. For further explanation see 5.4.4.(a).

Tables 8.1, 8.2, 8.3 show histograms of the errors for the three test areas considered. Each table has the errors with primary interpolation only shown by a continuous line. The results after the secondary interpolation process are shown by a dotted line. Where errors lie outside the range shown the number of such "outliers" are given.

8.1(a) BOWBURN EAST-WEST.

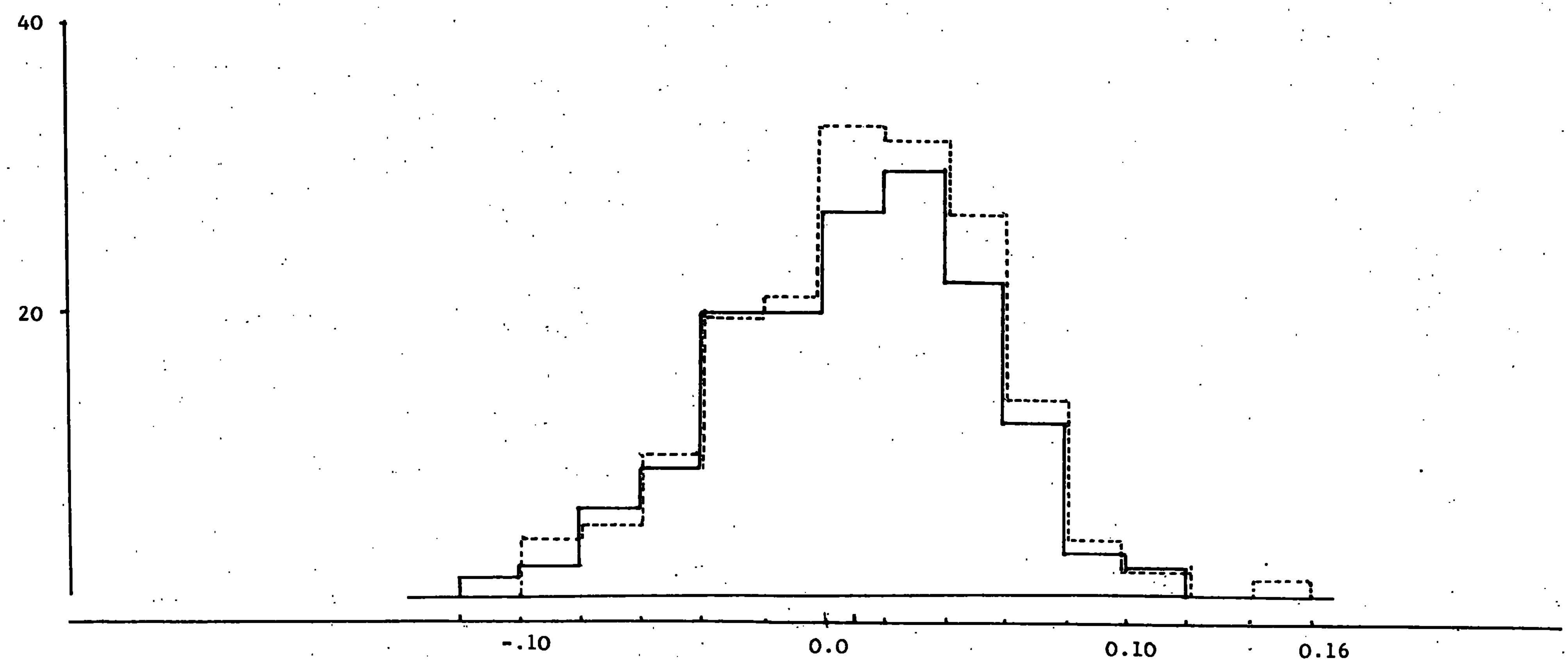
HISTOGRAM OF DATA PREPARATION ERROR.



8.1(b) HORSLEY BLOCK A.

HISTOGRAM OF DATA PREPARATION ERROR.

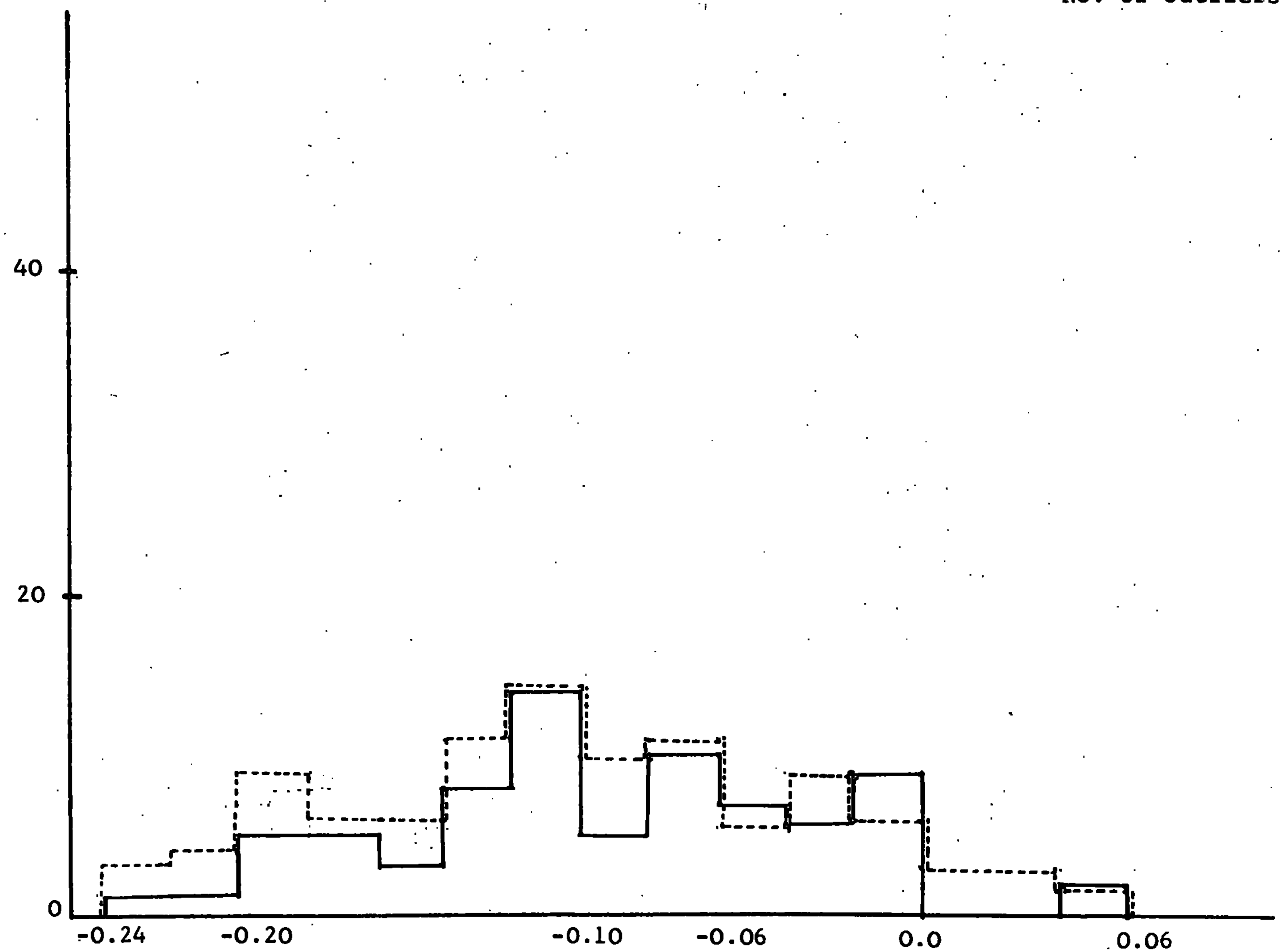
No. of outliers on primary = 3.



8.1(c) HORSLEY BLOCK B.

HISTOGRAM OF DATA PREPARATION ERROR.

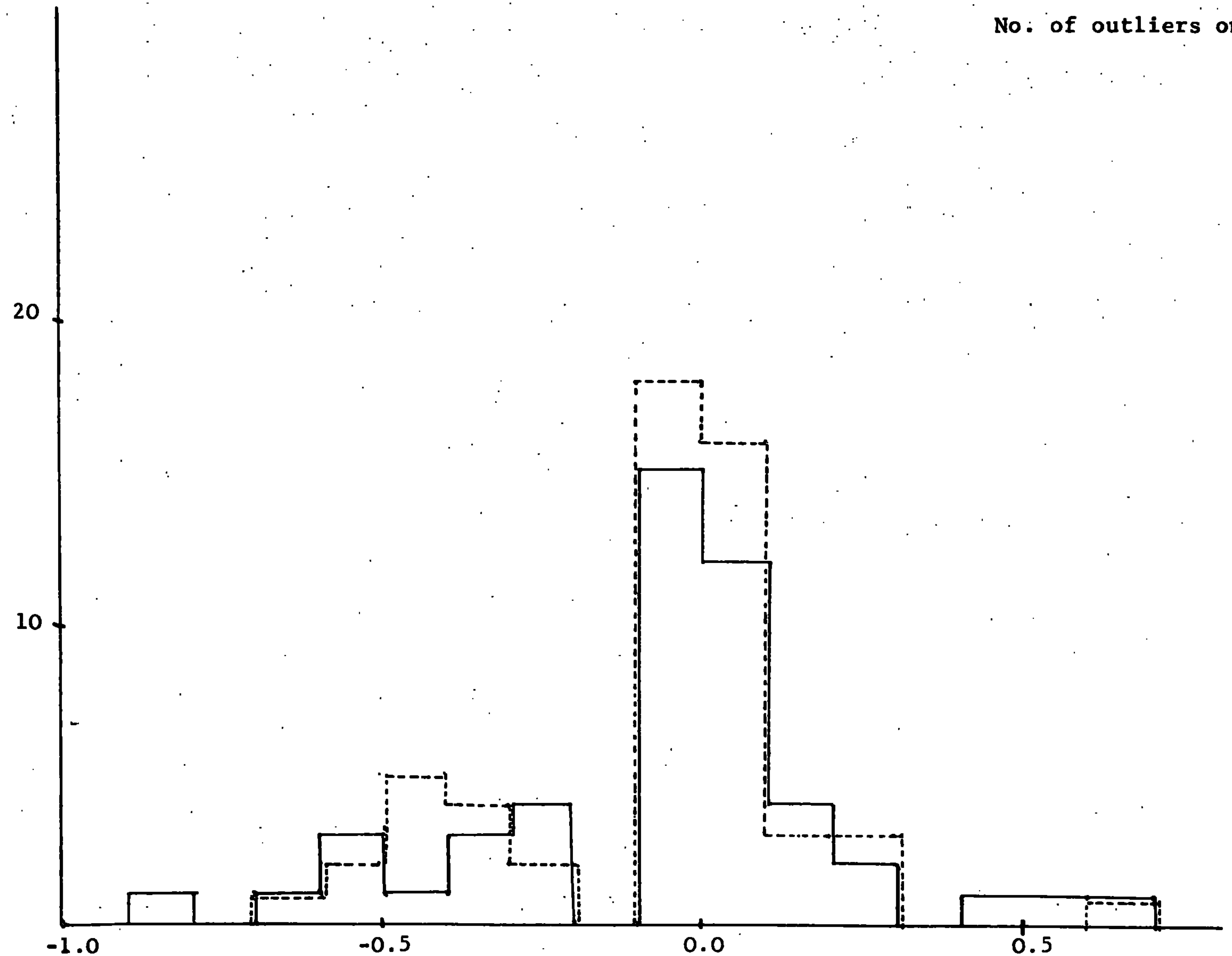
No. of outliers on primary = 12
No. of outliers on secondary = 15.



8.2(a) BOWBURN EAST-WEST.

HISTOGRAM OF QUALITY OF MODEL ERROR.

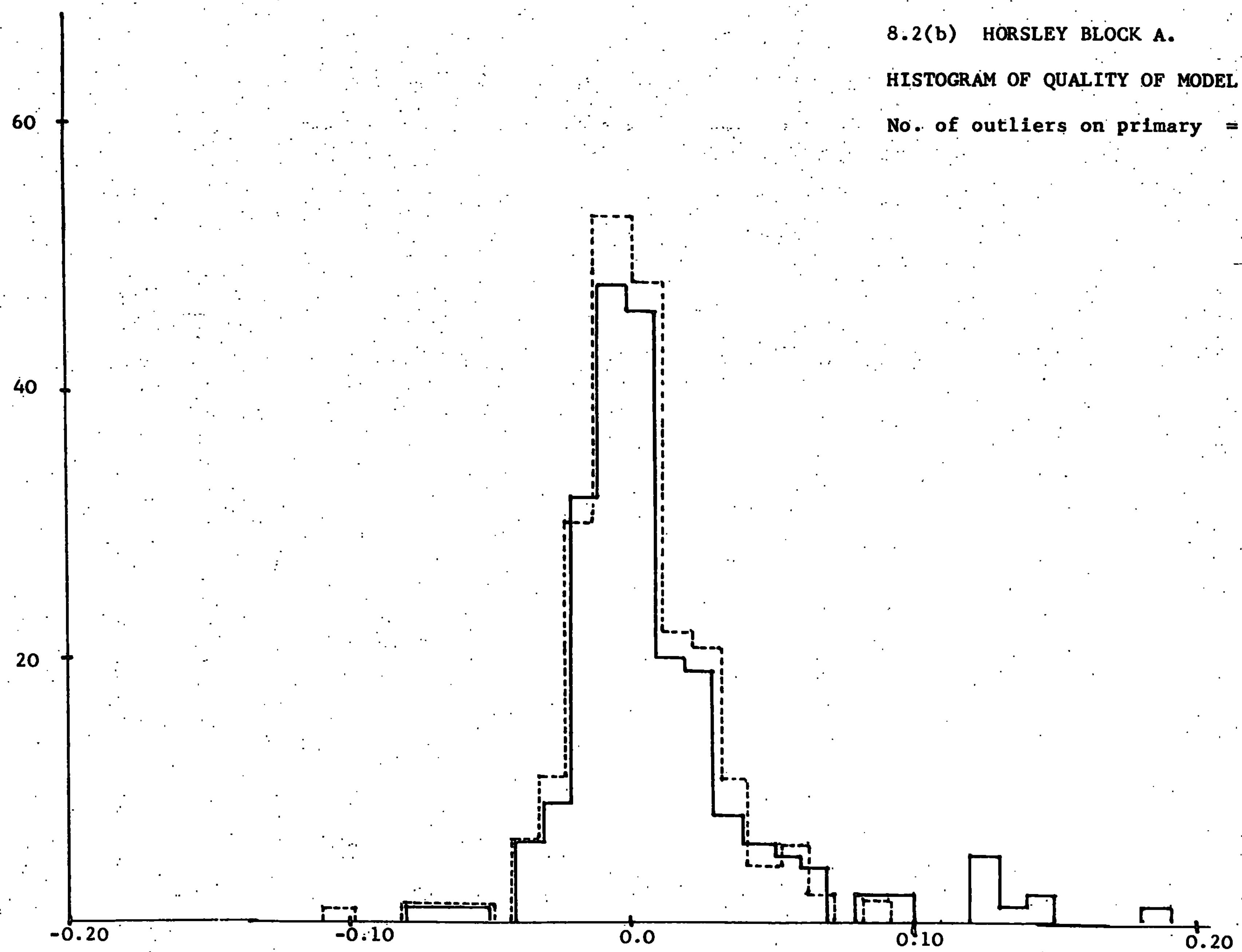
No. of outliers on primary - 3.



8.2(b) HORSLEY BLOCK A.

HISTOGRAM OF QUALITY OF MODEL ERROR.

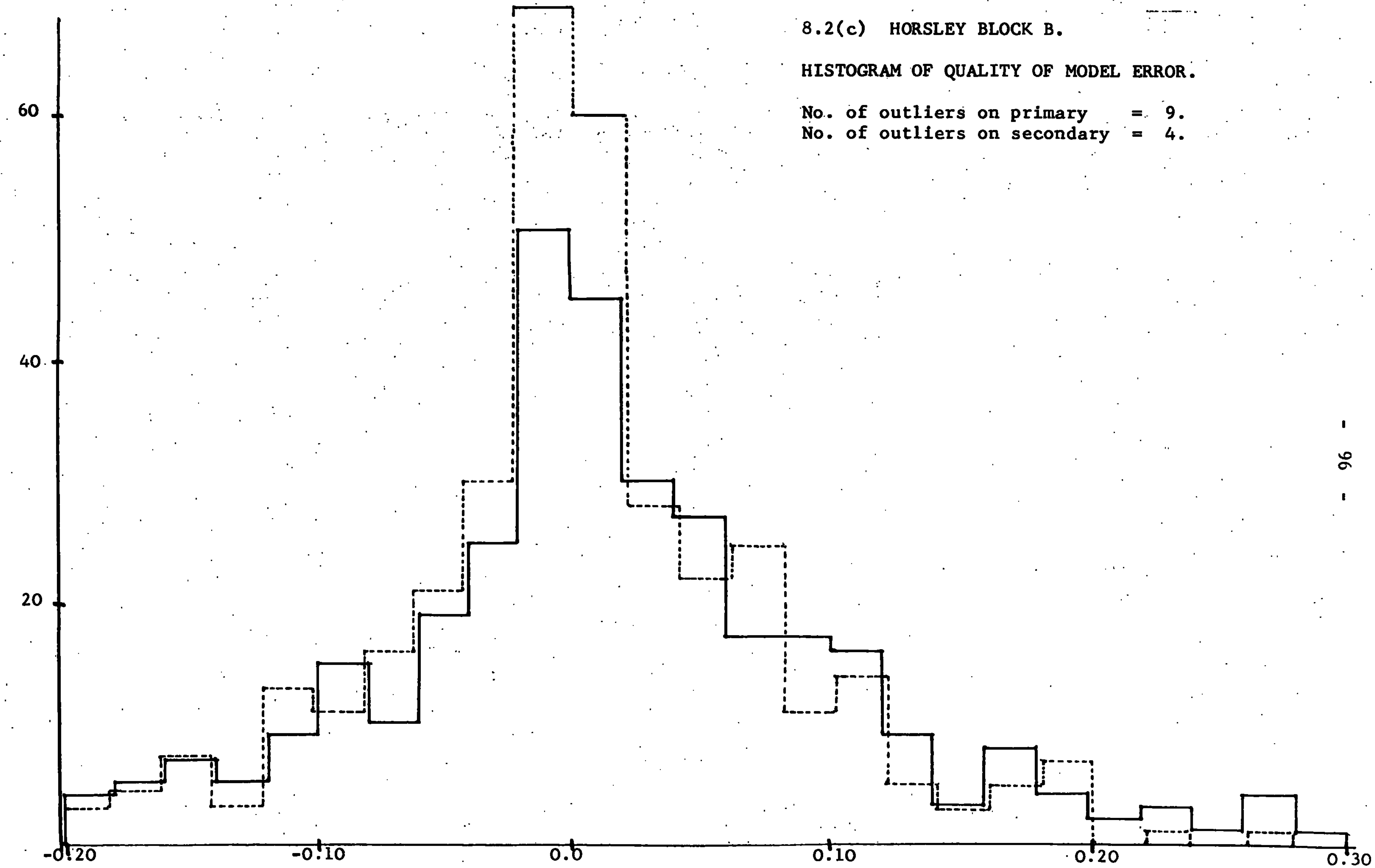
No. of outliers on primary = 2.



8.2(c) HORSLEY BLOCK B.

HISTOGRAM OF QUALITY OF MODEL ERROR.

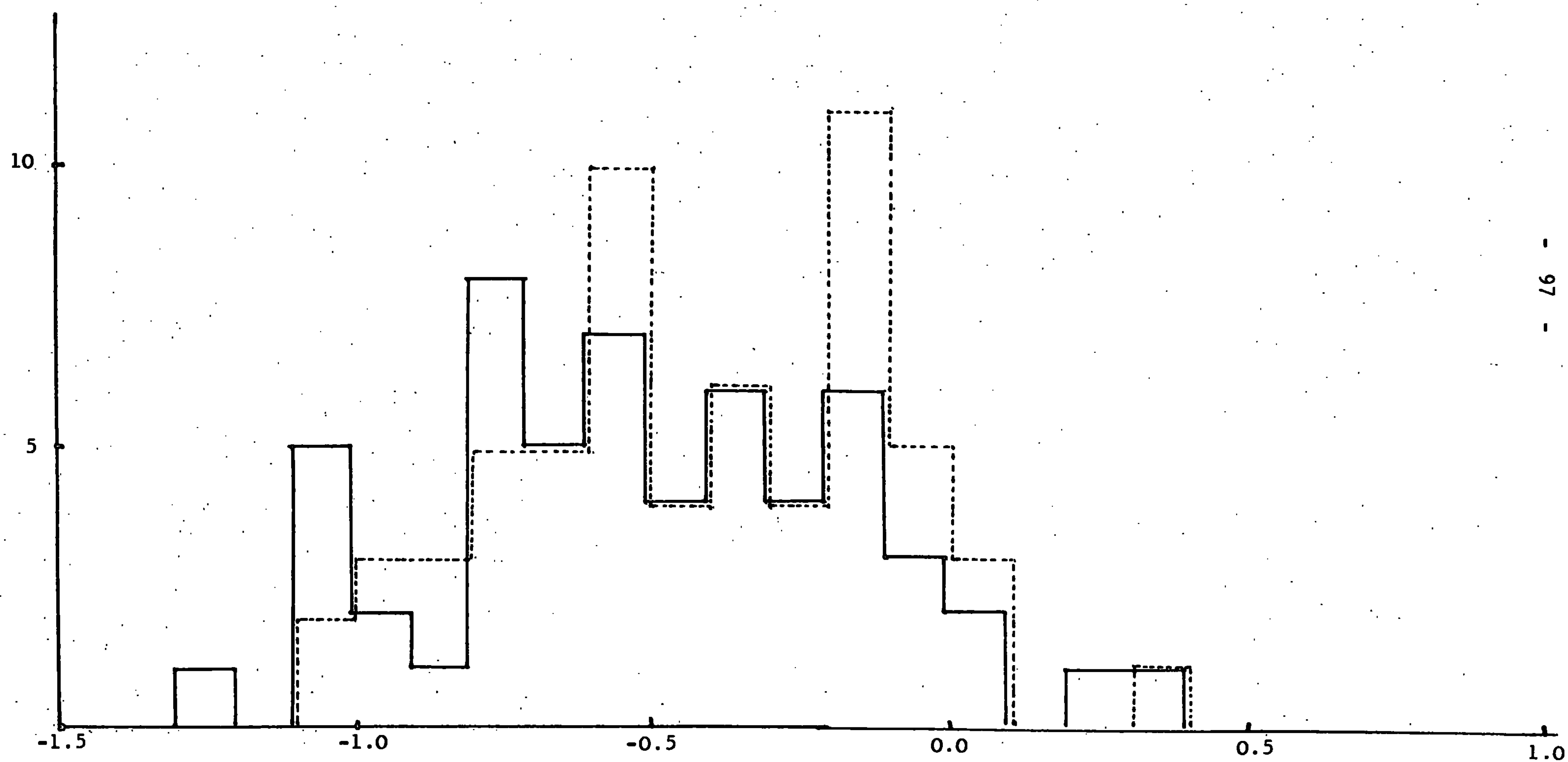
No. of outliers on primary = 9.
No. of outliers on secondary = 4.



8.3(a) BOWBURN EAST-WEST.

HISTOGRAM OF OVERALL ERRORS.

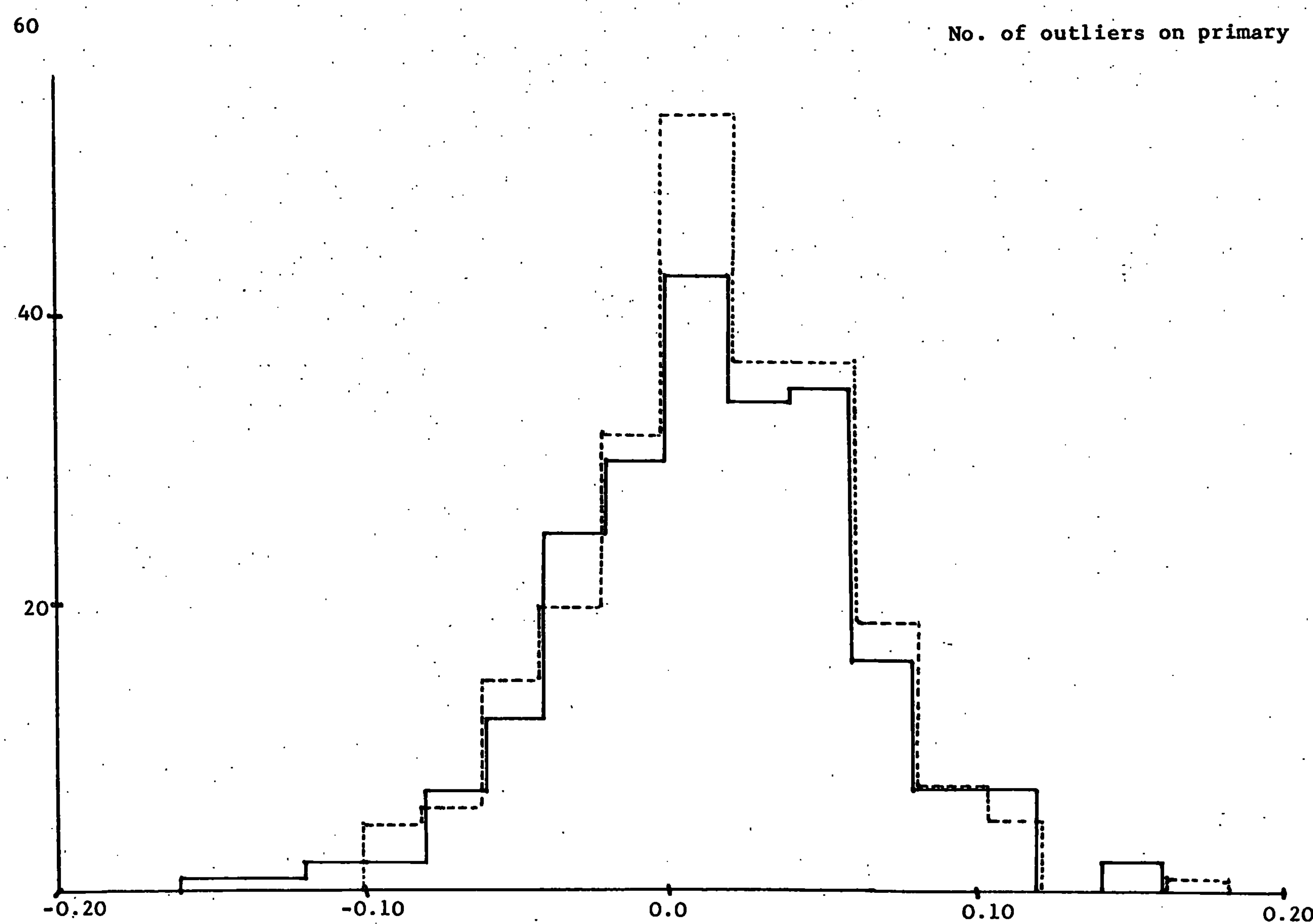
No. of outliers on primary = 2.



8.3(b) HORSLEY BLOCK A.

HISTOGRAM OF OVERALL ERRORS.

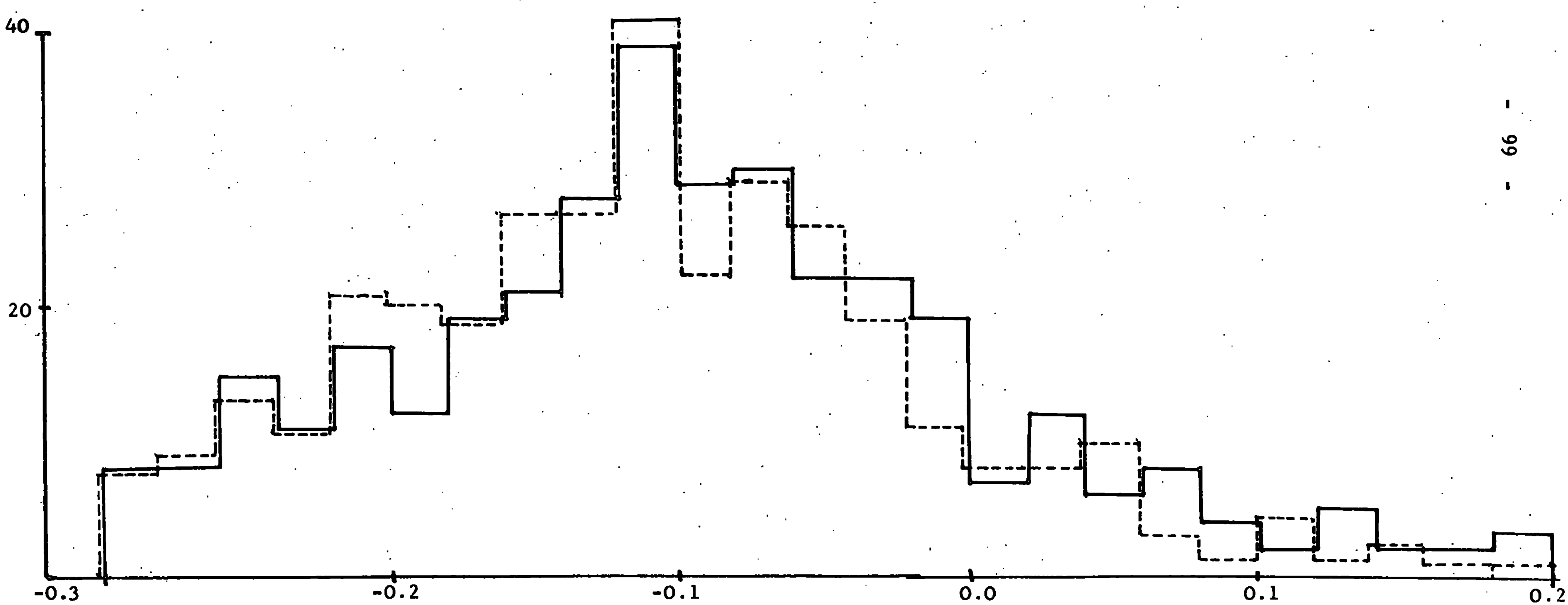
No. of outliers on primary = 17.



8.3(c) HORSLEY BLOCK B.

HISTOGRAM OF OVERALL ERROR.

No. of outliers on primary = 23.
No. of outliers on secondary = 23.



8.5 Data Preparation Error

The following tables show the data preparation errors for the test areas. They consider the means and variances of each cross-section in a set of data before and after the secondary interpolation process.

8.5(a) Bowburn East-West

Chainage	Before			After		
	No. of data points.	Mean	Variance	No. of data points.	Mean	Variance
(o)SHS5	14	-0.50	0.054838	15	-0.481	0.056466
S4 + S6	11	-0.317	0.10297	16	-0.246	0.083876
S1	10	-0.464	0.067233	10	-0.486	0.0563
S2	7	-0.456	0.032017	9	-0.376	0.045214
S3	6	-0.401	0.04716	8	-0.201	0.135485
S4	5	-0.312	0.1613	7	-0.203	0.157466

8.5(b) Horsley Heddon Block A.

Chainage	Before			After		
	No. of data points.	Mean	Variance	No. of data points.	Mean	Variance
0	25	0.024	0.001242	26	0.024	0.00194
10	23	0.048	0.002696	23	0.044	0.001096
20	21	0.007	0.00151	22	0.013	0.001676
30	19	0.010	0.0008	20	0.014	0.000705
40	17	0.022	0.01545	18	0.000	0.002888
50	16	0.019	0.01498	18	-0.002	0.002165
60	14	0.004	0.001862	18	0.005	0.001536
70	12	0.037	0.019282	15	0.005	0.001564
80	11	0.000	0.00349	13	-0.010	0.002158

8.5(c) Horsley Heddon Block B.

Chainage	Before			After		
	No. of data points	Mean	Variance	No. of data points	Mean	Variance
0	11	-0.103	0.00366	16	-0.092	0.003646
10	12	-0.059	0.0027	15	-0.056	0.002443
20	12	-0.126	0.015573	14	-0.123	0.0164
30	11	-0.128	0.0097	13	-0.125	0.00945
40	14	-0.129	0.016731	20	-0.139	0.016221
50	8	-0.105	0.002714	11	-0.101	0.0095
60	9	-0.165	0.019975	14	-0.190	0.014977
70	11	-0.149	0.00839	16	-0.167	0.006933

8.5(d) Horsley Heddon Block A Revised Model

Chainage	Before			After		
	No. of data points	Mean	Variance	No. of data points	Mean	Variance
0	25	0.024	0.001237	26	0.024	0.00126
10	23	0.048	0.002695	23	0.045	0.001418
20	21	0.007	0.001438	24	0.010	0.001478
30	19	0.010	0.000794	22	0.008	0.001024
40	17	0.001	0.0031	20	0.003	0.002505
50	16	-0.003	0.002406	20	-0.007	0.001884
60	14	0.005	0.001862	18	0.003	0.001565
70	12	0.002	0.001791	15	0.004	0.00145
80	11	0.001	0.0036	13	-0.005	0.003242

8.6 Quality of Model Error

The quality of model errors are shown for the various test areas. The means and variances are considered for each cross-section of each set of data both before and after the secondary interpolation process.

8.6(a) Bowburn East West

Chainage	Before			After		
	No. of data points	Mean	Variance	No. of data points	Mean	Variance
S1 + S5	19	0.027	0.036388	19	0.021	0.035528
S4 + S6	19	-0.185	0.196546	19	-0.050	0.066715
S1	11	0.067	0.04516	11	0.069	0.0448
S2	11	-0.120	0.10705	11	-0.070	0.08340
S3	9	-0.153	0.1565	9	-0.084	0.076444
S4	9	-0.364	0.287137	9	-0.099	0.084525

8.6(b) Horsley Heddon Block A

Chainage	Before			After		
	No. of data points	Mean	Variance	No. of data points	Mean	Variance
0	27	0.017	0.002958	27	0.004	0.000492
10	25	0.002	0.000183	25	0.002	0.000183
20	28	0.015	0.0033	28	0.003	0.001963
30	28	0.010	0.001667	28	0.002	0.000844
40	26	0.014	0.001528	26	0.006	0.000756
50	26	0.012	0.001096	26	0.006	0.000464
60	26	0.007	0.000364	26	0.002	0.000248
70	28	0.004	0.000518	28	0.001	0.000296
80	26	0.021	0.003244	26	0.006	0.001048

8.6(c) Horsley Heddon Block B

Chainage	Before			After		
	No. of data points	Mean	Variance	No. of data points	Mean	Variance
0	48	0.060	0.006558	48	0.026	0.004406
10	50	-0.014	0.004476	50	-0.016	0.004235
20	50	-0.013	0.006098	50	-0.021	0.005051
30	49	0.003	0.00655	49	0.002	0.006519
40	49	0.013	0.009875	49	0.008	0.009487
50	28	0.045	0.008559	28	0.028	0.008122
60	51	0.094	0.013214	51	0.027	0.003462
70	55	-0.014	0.010733	55	-0.017	0.009054

8.6(d) Horsley Heddon Block A Revised Model

Chainage	Before			After		
	No. of data points	Mean	Variance	No. of data points	Mean	Variance
0	27	0.017	0.002958	27	0.005	0.0005808
10	25	0.002	0.000183	25	0.002	0.000183
20	28	0.015	0.0033	28	0.001	0.000296
30	28	0.010	0.001667	28	0.003	0.000315
40	26	0.014	0.001528	26	0.004	0.00072
50	26	0.012	0.001096	26	0.005	0.000384
60	26	0.007	0.000364	26	0.002	0.000272
70	28	0.004	0.000518	28	0.001	0.0003
80	26	0.021	0.003244	26	0.008	0.00128

8.7 Overall Error

For each cross-section of each test area the overall error statistics of mean and variance are enumerated both before and after secondary interpolation.

8.7(a) Bowburn East-West

Chainage	Before			After		
	No. of data points	Mean	Variance	No. of data points	Mean	Variance
S4S5	19	-0.509	0.045844	19	-0.508	0.046811
S4 + S6	19	-0.452	0.250317	19	-0.3	0.181894
S1	11	-0.450	0.06421	11	-0.466	0.05781
S2	11	-0.592	0.12691	11	-0.490	0.11951
S3	9	-0.540	0.16595	9	-0.291	0.215489
S4	9	-0.554	0.427663	9	-0.269	0.277011

8.7(b) Horsley Heddon Block A

Chainage	Before			After		
	No. of data points	Mean	Variance	No. of data points	Mean	Variance
0	27	0.036	0.002369	27	0.027	0.001623
10	25	0.050	0.002325	25	0.046	0.000862
20	28	0.022	0.004152	28	0.019	0.002274
30	28	0.019	0.002515	28	0.017	0.001444
40	26	0.060	0.023164	26	0.008	0.00346
50	26	0.058	0.021368	26	0.010	0.00292
60	26	0.010	0.001808	26	0.010	0.001736
70	28	0.065	0.020356	28	0.004	0.000981
80	26	0.004	0.003852	26	-0.007	0.00222

8.7(c) Horsley Heddon Block B

Chainage	Before			After		
	No. of data points	Mean	Variance	No. of data points	Mean	Variance
0	48	-0.014	0.015268	48	-0.052	0.007419
10	50	-0.075	0.006578	50	-0.080	0.006143
20	50	-0.136	0.01019	50	-0.120	0.012122
30	49	-0.151	0.009758	49	-0.146	0.010196
40	49	-0.142	0.015381	49	-0.153	0.01325
50	28	-0.054	0.008659	28	-0.063	0.015022
60	51	-0.103	0.021518	51	-0.178	0.013452
70	55	-0.163	0.015369	55	-0.173	0.013561

8.7(d) Horsley Heddon Block A Revised Model

Chainage	Before			After		
	No. of data points	Mean	Variance	No. of data points	Mean	Variance
0	27	0.036	0.002365	27	0.027	0.00135
10	25	0.050	0.002325	25	0.047	0.001267
20	28	0.022	0.004155	28	0.012	0.001593
30	28	0.015	0.0033	28	0.011	0.001263
40	26	0.020	0.005004	26	0.006	0.00312
50	26	0.014	0.003375	26	0.002	0.002056
60	26	0.010	0.001804	26	0.008	0.001836
70	28	0.009	0.001478	28	0.004	0.000926
80	26	0.005	0.00388	26	-0.001	0.002772

8.8 Areas versus Interpolated Values - Data Preparation Error.

A comparison is made between the area of a cross-section and the interpolated values. The area error is normalised so that the units are square metres per metre. In this table data preparation errors are considered; in all cases secondary interpolation only is considered.

8.8(a) Bowburn East-West

Chainage	No. of data points	Interpolation Error	Normalised Area Error	Difference
S1 + S5	15	-0.481	-0.533	0.052
S4 + S6	16	-0.246	-0.234	-0.012
S1	10	-0.486	-0.571	0.085
S2	9	-0.376	-0.470	0.094
S3	8	-0.201	-0.254	0.053
S4	7	-0.203	-0.238	0.035

8.8(b) Horsley Heddon Block A

Chainage	No. of data points	Interpolation Error	Normalised Area Error	Difference
0	26	0.024	0.021	0.003
10	23	0.044	0.041	0.003
20	22	0.013	0.012	0.001
30	20	0.014	0.013	0.001
40	18	0.000	-0.002	0.002
50	18	-0.002	0.001	-0.003
60	18	0.005	0.009	-0.004
70	15	0.005	0.000	0.005
80	13	-0.010	-0.011	+0.001

8.8(c) Horsley Heddon Block B

Chainage	No. of data points	Interpolation Error	Normalised Area Error.	Difference
0	16	-0.092	-0.075	-0.017
10	15	-0.056	-0.063	+0.007
20	14	-0.123	-0.095	-0.028
30	13	-0.125	-0.149	0.024
40	20	-0.139	-0.165	0.026
50	11	-0.101	-0.093	-0.008
60	14	-0.190	-0.223	0.033
70	16	-0.167	-0.150	-0.017

8.9 Areas versus Interpolated Values - Quality of Model Error

The comparison between the two methods of data collection is continued for the quality of model error. Once again secondary interpolation only is considered.

8.9(a) Bowburn East-West.

Chainage	No. of data points	Interpolation Error	Normalised Area Error	Difference
S1 + S5	19	0.021	0.006	0.015
S4 + S6	19	-0.050	-0.043	-0.007
S1	11	0.069	0.046	0.023
S2	11	-0.070	-0.068	-0.002
S3	10	-0.084	-0.077	-0.007
S4	9	-0.099	-0.101	0.002

8.9(b) Horsley Heddon Block A

Chainage	No. of data points	Interpolation Error	Normalised Area Error	Difference
0	27	0.004	0.003	0.001
10	25	0.002	0.001	0.001
20	28	0.003	0.003	0.000
30	28	0.002	0.002	0.000
40	26	0.006	0.004	0.002
50	26	0.006	0.005	0.001
60	26	0.002	0.002	0.000
70	28	0.001	0.001	0.000
80	26	0.006	0.007	-0.001

8.9(c) Horsley Heddon Block B

Chainage	No. of data points	Interpolation Error	Normalised Area Error	Difference
0	48	0.026	0.024	0.00
10	50	-0.016	-0.018	0.002
20	50	-0.021	-0.023	0.002
30	49	0.002	0.004	-0.002
40	49	0.008	0.010	-0.002
50	28	0.028	0.028	0.000
60	51	0.027	0.026	0.001
70	55	-0.017	-0.026	0.009

8.10 Areas Versus Interpolation Values - Overall Error

This table is similar to table 8.8 and 8.9 but differs in that the overall errors are considered.

8.10(a) Bowburn East-West

Chainage	No. of data points	Interpolation error	Normalised area error.	Difference
S1 + S5	19	-0.508	-0.527	0.019
S4 + S6	19	-0.300	-0.276	-0.024
S1	11	-0.466	-0.525	0.059
S2	11	-0.490	-0.538	0.048
S3	10	-0.291	-0.331	0.04
S4	9	-0.269	-0.339	0.07

8.10(b) Horsley Heddon Block A

Chainage	No. of data points	Interpolation error	Normalised area error	Difference
0	27	0.027	0.025	0.002
10	25	0.046	0.042	0.004
20	28	0.019	0.015	0.004
30	28	0.017	0.015	0.002
40	26	0.008	0.002	0.006
50	26	0.010	0.006	0.004
60	26	0.010	0.011	-0.001
70	28	0.004	0.002	0.002
80	26	-0.007	-0.004	-0.003

8.10(c) Horsley Heddon Block B

Chainage	No. of data points	Interpolation Errors	Normalised Area Error	Difference
0	48	-0.052	-0.052	0.000
10	50	-0.080	-0.081	0.001
20	50	-0.120	-0.118	-0.002
30	49	-0.146	-0.145	-0.001
40	49	-0.153	-0.154	-0.001
50	28	-0.063	-0.064	-0.001
60	51	-0.178	-0.198	0.020
70	55	-0.173	-0.176	0.003

8.11 Cumulative Effect of Errors

The relationship between the three types of error, data preparation, quality of model, and overall error, is demonstrated by consideration of the means. The effect of secondary interpolation is also shown.

8.11(a) Bowburn East-West

Chainage	Data Preparation Error		Quality of Model Error		Overall Error	
	Before	After	Before	After	Before	After
S1 + S5	-0.500	-0.481	0.027	0.021	-0.509	-0.508
S4 + S6	-0.317	-0.246	-0.185	-0.050	-0.452	-0.300
S1	-0.464	-0.486	0.067	0.069	-0.450	-0.466
S2	-0.456	-0.376	-0.120	-0.070	-0.592	-0.490
S3	-0.401	-0.201	-0.153	-0.084	-0.540	-0.291
S4	-0.312	-0.203	-0.364	-0.099	-0.554	-0.269
MEAN	-0.408	-0.332	-0.121	-0.035	-0.516	-0.387

8.11(b) Horsley Heddon Block A

Chainage	Data Preparation Error		Quality of Model Error		Overall Error	
	Before	After	Before	After	Before	After
0	0.024	0.024	0.017	0.004	0.036	0.027
10	0.048	0.044	0.002	0.002	0.050	0.046
20	0.007	0.013	0.015	0.003	0.022	0.019
30	0.010	0.014	0.010	0.002	0.019	0.017
40	0.022	0.000	0.014	0.006	0.060	0.008
50	0.019	-0.002	0.012	0.006	0.058	0.010
60	0.004	0.005	0.007	0.002	0.010	0.010
70	0.037	0.005	0.004	0.001	0.065	0.004
80	0.000	-0.010	0.021	0.006	0.004	-0.007
MEAN	0.019	0.010	0.011	0.004	0.036	0.015

8.11(c) Horsley Heddon Block B

Chainage	Data Preparation Error		Quality of Model Error		Overall Error	
	Before	After	Before	After	Before	After
0	-0.103	-0.092	0.060	0.026	-0.014	-0.052
10	-0.059	-0.056	-0.014	-0.016	-0.075	-0.080
20	-0.126	-0.123	-0.013	-0.021	-0.136	-0.120
30	-0.128	-0.125	0.003	0.002	-0.151	-0.146
40	-0.129	-0.139	0.013	0.008	-0.142	-0.153
50	-0.105	-0.101	0.045	0.028	-0.054	-0.063
60	-0.165	-0.190	0.094	0.027	-0.103	-0.178
70	-0.149	-0.167	-0.014	-0.017	-0.163	-0.173
MEAN	-0.121	-0.124	0.022	0.005	-0.105	-0.121

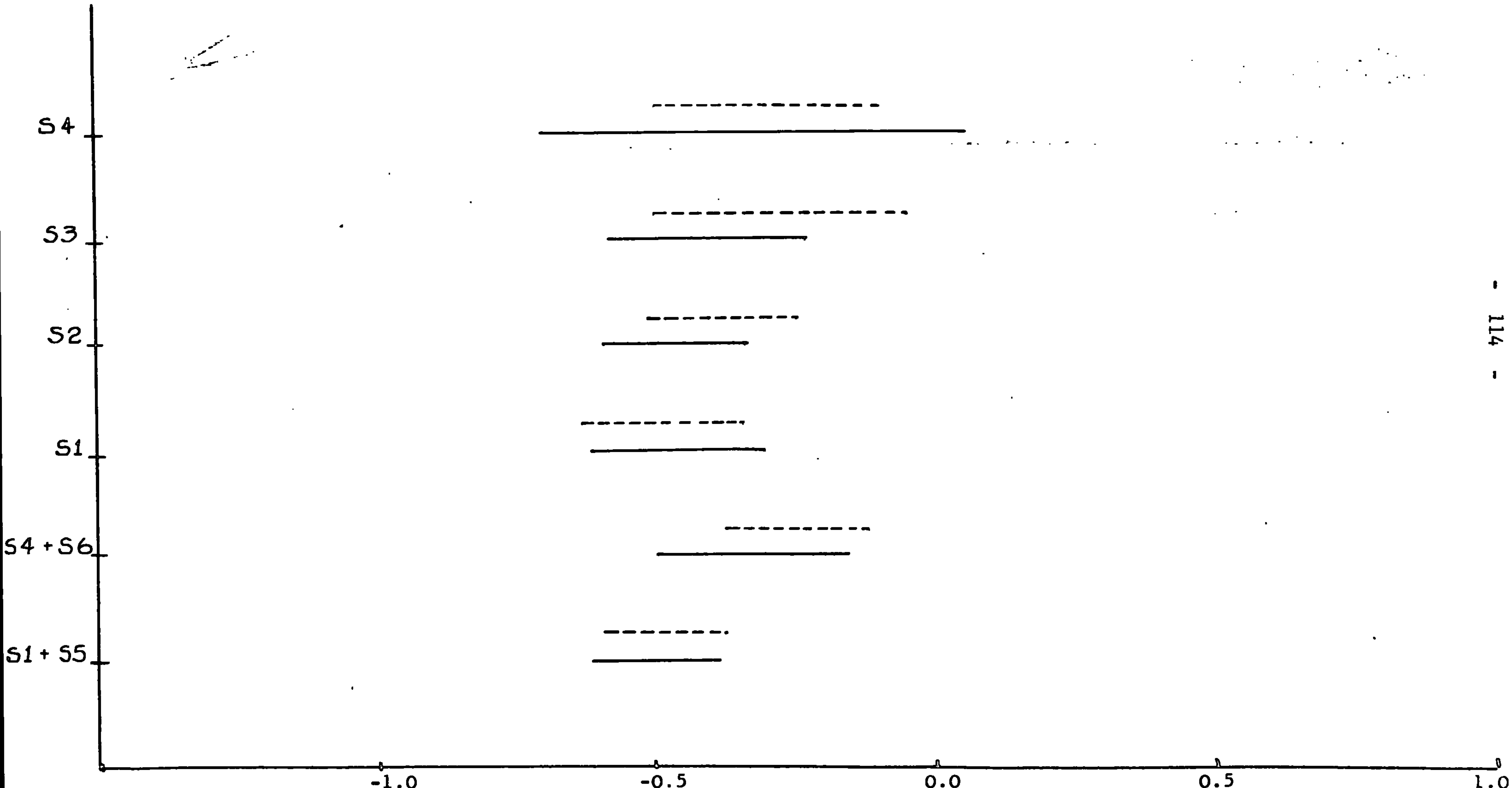
Table 8.12, 8.13, 8.14.

The following graphs show the 90% confidence intervals on the mean error be it data preparation, quality of model or overall. The intervals are based on the Students 't' distribution which is the distribution of the estimate for the mean of a normal distribution with unknown variance.

For each cross-section the interval for the error after the secondary interpolation process is shown immediately above the interval for the error before the secondary interpolation process, and is a dotted line.

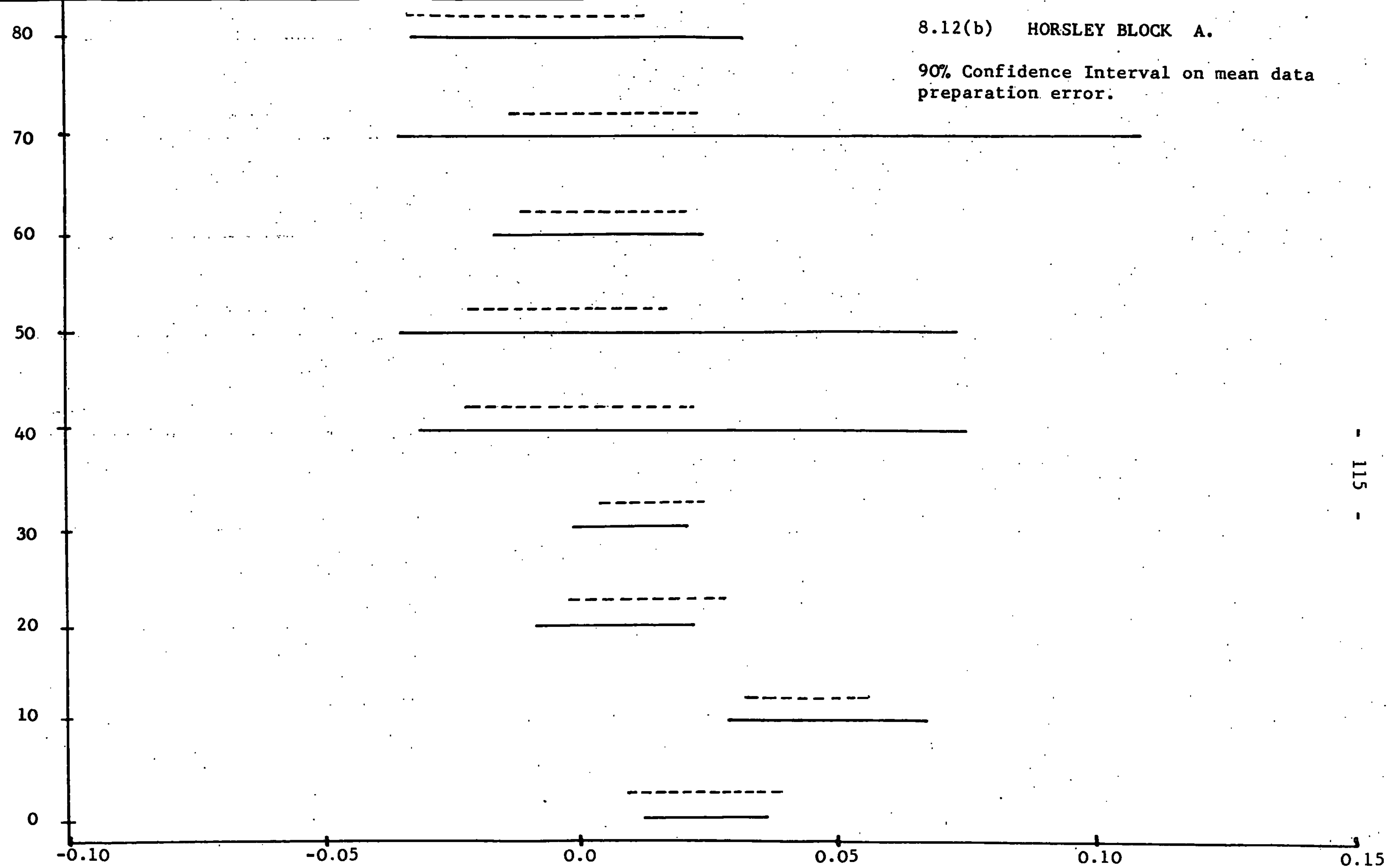
8.12(a) BOWBURN EAST-WEST.

90% Confidence Interval on mean data preparation error.



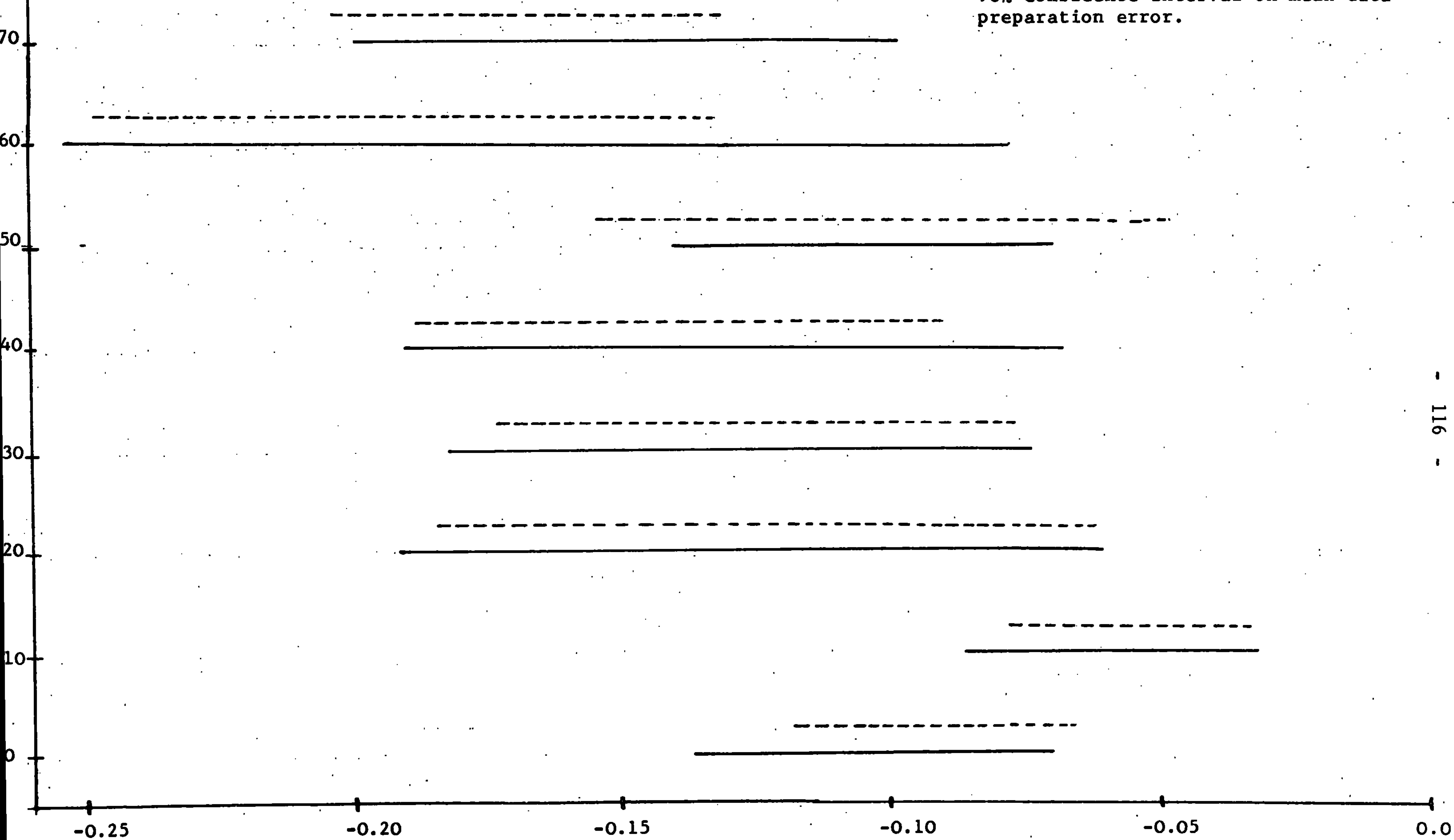
8.12(b) HORSLEY BLOCK A.

90% Confidence Interval on mean data preparation error.



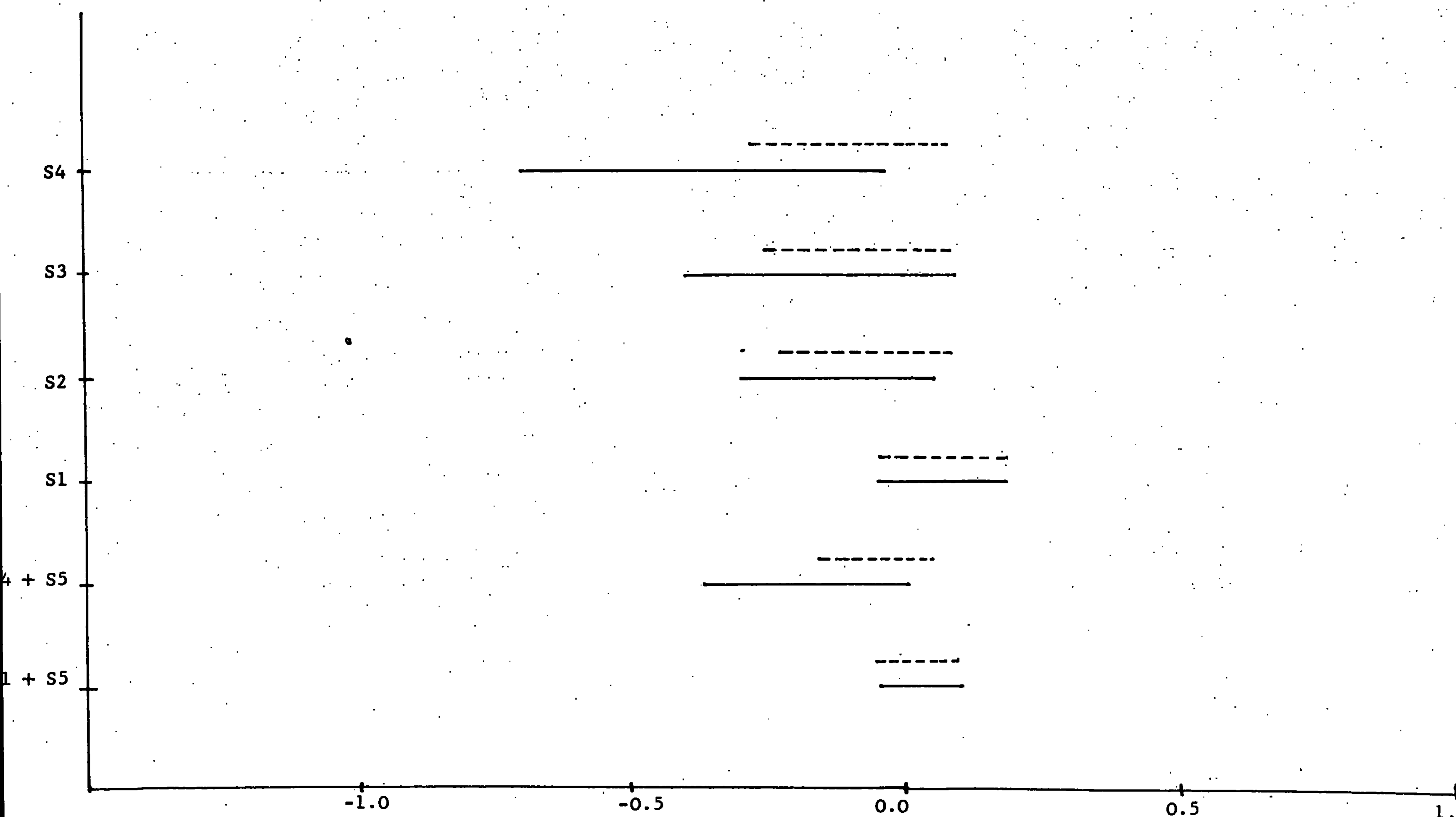
8.12(c) HORSLEY BLOCK B.

90% Confidence interval on mean data preparation error.



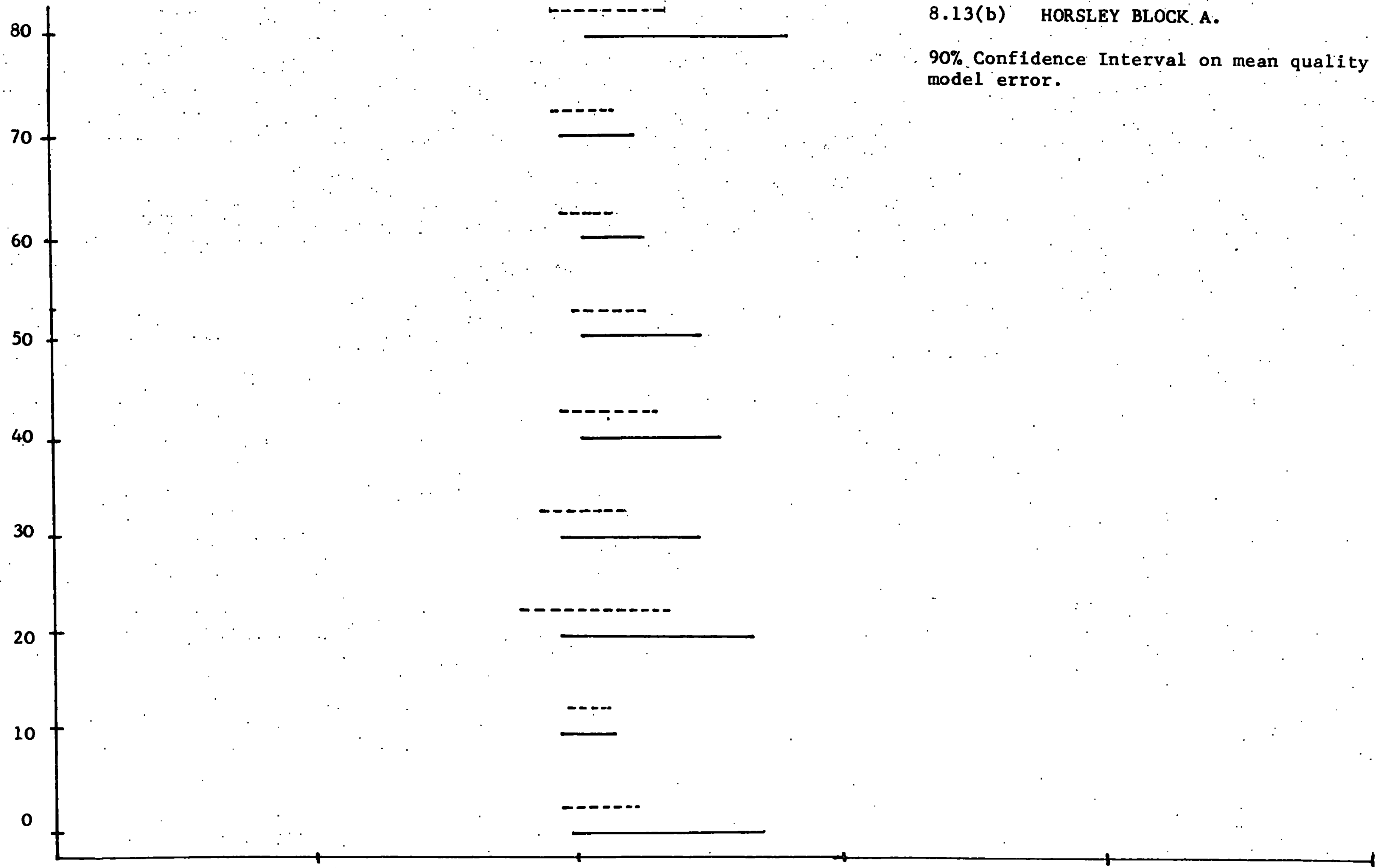
8.13(a) BOWBURN EAST-WEST

90%Confidence Interval on mean quality of model error.



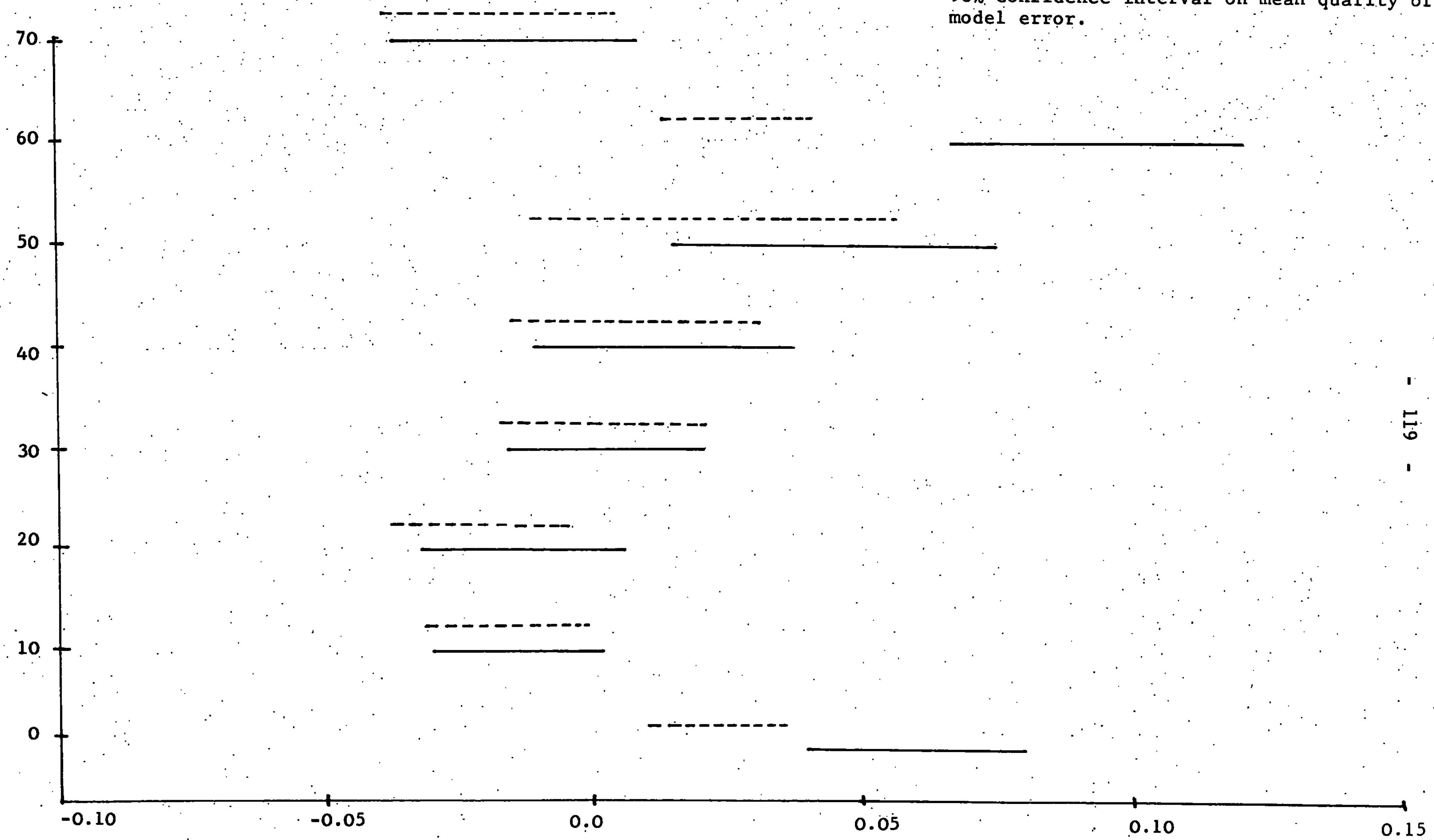
8.13(b) HORSLEY BLOCK A.

90% Confidence Interval on mean quality of model error.



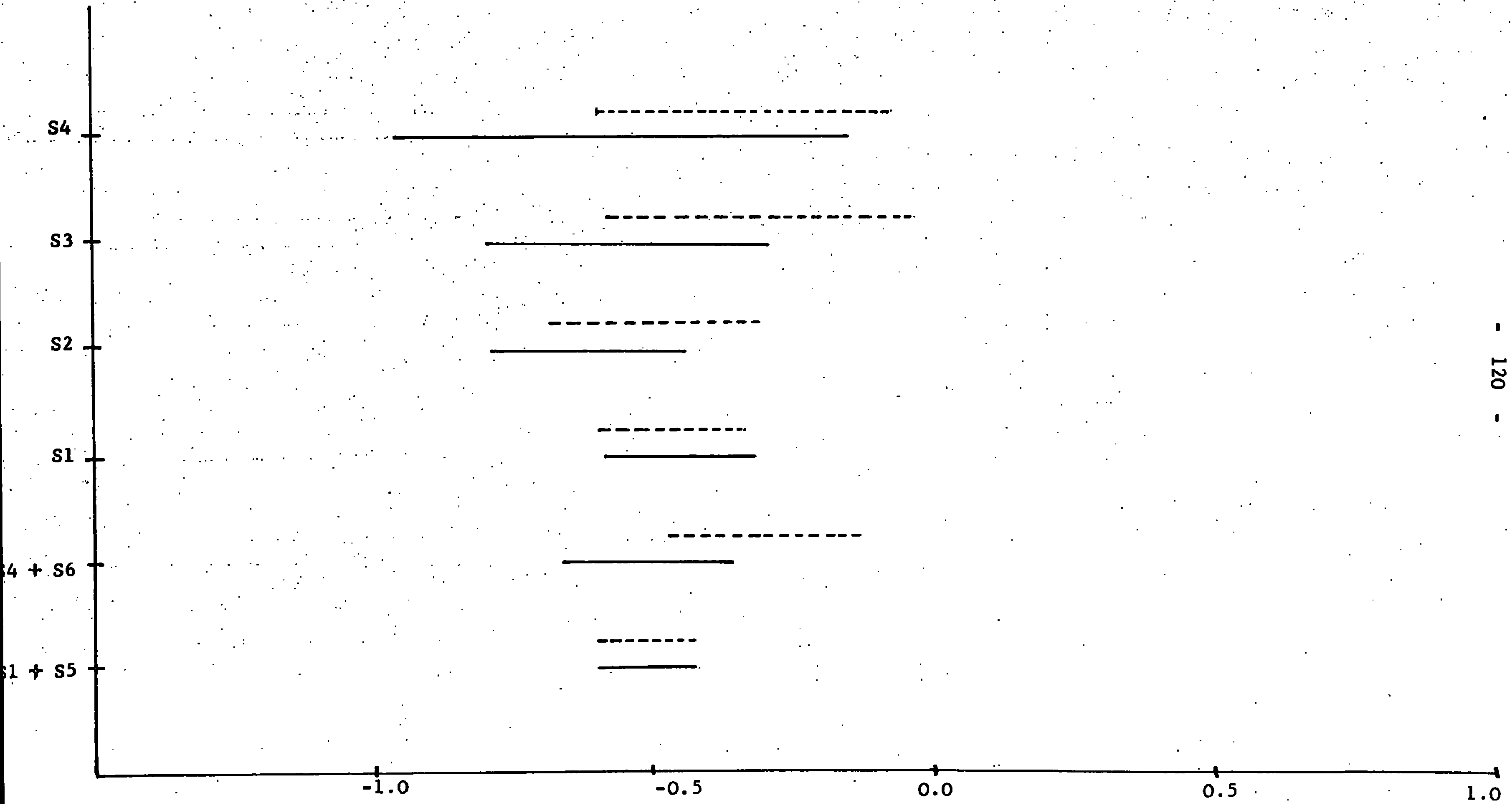
8.13(c) HORSLEY BLOCK B.

90% Confidence Interval on mean quality of model error.



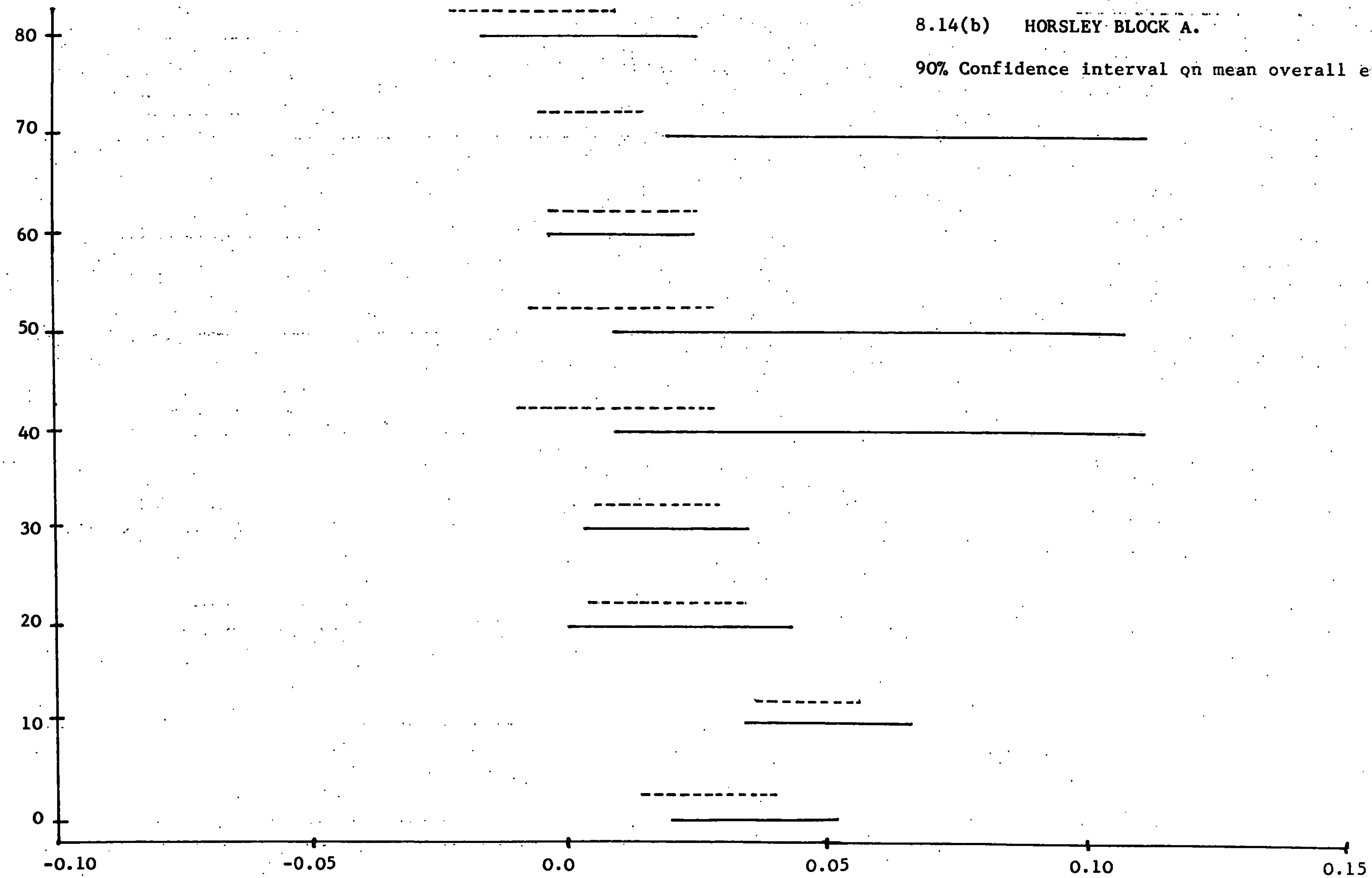
8.14(a) BOWBURN EAST-WEST.

90% Confidence Interval on mean overall error.



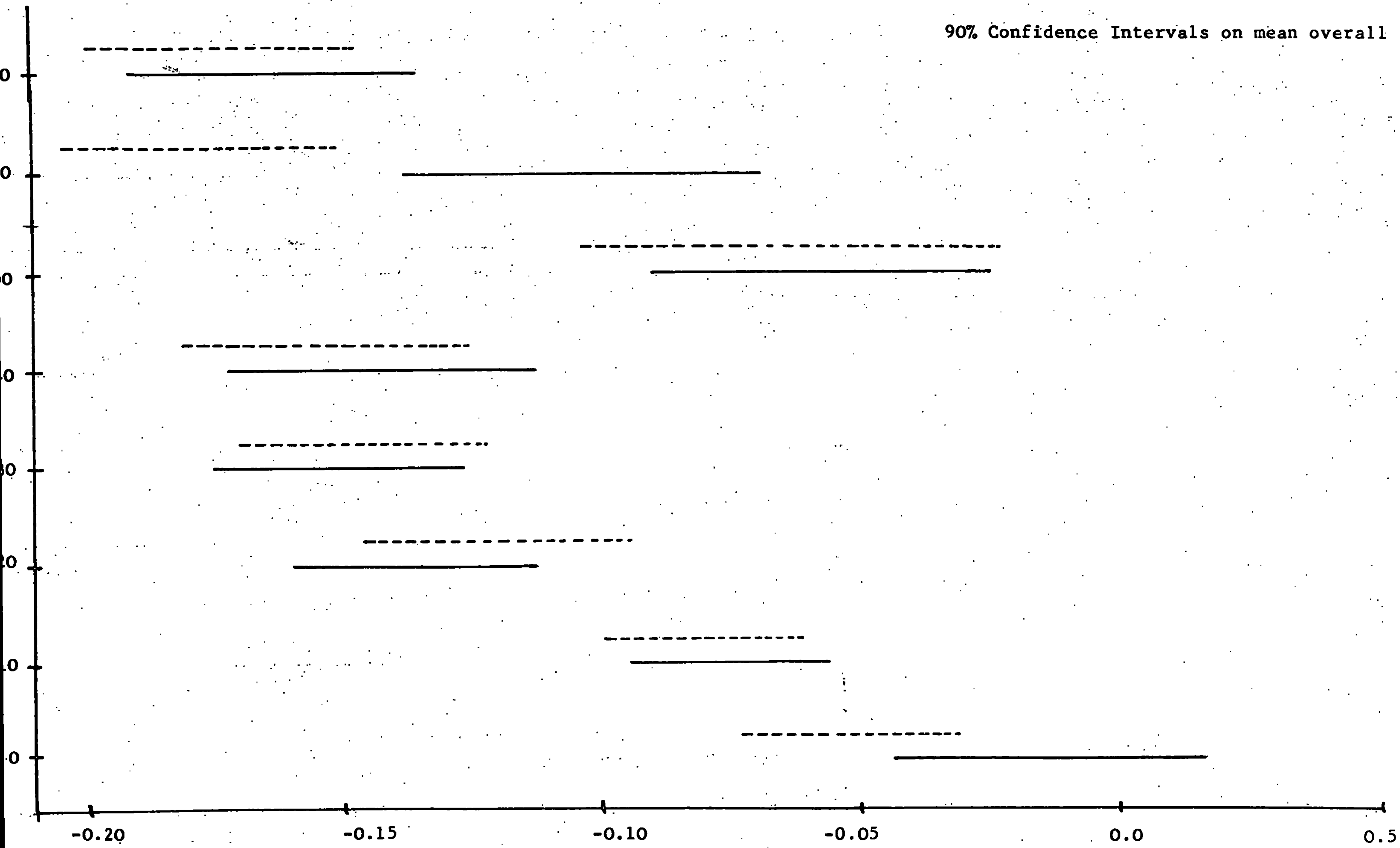
8.14(b) HORSLEY BLOCK A.

90% Confidence interval on mean overall error.



8.14(c) HORSLEY BLOCK B.

90% Confidence Intervals on mean overall error.



8.15 Difference between Interpolated Values and Areas.

The following tables show the 90% confidence interval for the mean difference between point interpolation errors and normalised area errors. In all cases secondary interpolation only is considered.

8.15(a) Data Preparation Error.

Test Area	Mean Difference	Variance	Sample Size	Confidence Limits Lower	Upper
Bowburn East-West	0.051	.001467	6	0.019	0.083
Heddon Block A.	0.001	.000008	9	-0.001	0.003
Heddon Block B.	0.002(5)	.00007	8	-0.014	0.018

8.15(b) Quality of Model Error

Test Area	Mean Difference	Variance	Sample Size	Confidence Limits Lower	Upper
Bowburn East-West	0.004	0.00075	6	-0.006	0.014
Heddon Block A.	0.000(4)	0.000002	9	-0.000(2)	0.000(6)
Heddon Block B.	0.002	0.000012	8	-0.001	0.004

8.15(c) Overall Error

Test Area	Mean Difference	Variance	Sample Size	Confidence Limits Lower	Upper
Bowburn East-West	0.035	0.001463	6	0.004	0.066
Heddon Block A.	0.002	0.000026	9	-0.008	0.012
Heddon Block B.	0.003	0.000050	8	-0.002	0.008

8.16 Non Parametric Tests

In some situations there is difficulty in distinguishing the factors affecting different methods and parametric analyses are little help in deciding which is the better. The argument as to whether primary interpolation only should be used as opposed to being supplemented by secondary interpolation is such a situation. A non parametric test was therefore devised using the binomial distribution, with the means and variance.

1 = Absolute value greater under primary interpolation than secondary.

2 = Otherwise.

8.16(a) Data Preparation Error.

Test Area	Type	Mean	Variance
Bowburn East-West	1	5	3
	0	1	3
Horsley Block A	1	5	7
	0	4	2
Horsley Block B.	1	5	6
	0	3	2

For mean pr (1) = 15/23 = 0.652

pr (0) = 8/23 = 0.348

∴ secondary interpolation improves the data preparation error for mean

For variance Pr (1) = 16/23 = 0.696.

Pr (0) = 7/23 = 0.304.

∴ secondary interpolation improves the data preparation error for mean.

NOTE : These results demonstrate that the data preparation error is not only representative of the error in actually digitising a point but also of the error in assuming linearity between digitised points.

8.16(b) Quality of Model Error.

Test Area	Type	Mean	Variance
Bowburn East-West.	1	5	6
	0	1	0
Horsley Block A.	1	9	9
	0	0	0
Horsley Block B.	1	5	8
	0	3	0

For Mean pr (1) = 19/23 = 0.826.

pr (0) = 4/23 = 0.174.

For variance Pr (1) = 23/23 = 1.0

Pr (0) = 0/23 = 0.0

Secondary Interpolation radically improves the quality of model error.

8.16(c) <u>Overall Error</u>

Test Area	Type	Mean	Variance
Bowburn East-West.	1	5	4
	0	1	2
Horsley Block A.	1	8	9
	0	1	0
Horsley Block B.	1	2	5
	0	6	3

For Mean Pr (1) = 15/23 = 0.652

Pr (0) = 8/23 = 0.348

For Variance Pr (1) = 18/23 = 0.783.

Pr (0) = 5/23 = 0.217.

i.e. Secondary Interpolation improves the overall error.

Note : Block B was over flat ground and difficulties in defining strings are apparent. These are discussed in 5.4.5(c). If the results for Block B are ignored the non-parametric test shows :-

For Mean Pr (1) = 13/15 = 0.867.

Pr (0) = 2/15 = 0.133.

For Variance Pr (1) = 13/15 = 0.867.

Pr (0) = 2/15 = 0.133.

i.e. Secondary Interpolation radically improves the overall error.

8.17 Longitudinal Spacing

The table shows the earthworks volumes calculated for a hypothetical straight length of 200m of road completely in cutting. The volumes were taken out at differing chainage intervals of 20 metres, 10 metres, 5 metres and 1 metre intervals. The test was taken in the Horsley Area which was a specially digitised model (See Chapter 4). The results are discussed in 5.2.1.

Chainage	Interval			
	20 m	10 m	5 m	1 m
0 - 10	8333	8638	8636	8633
10 - 30	18766	18715	18735	18742
30 - 50	20022	19995	20003	20007
50 - 70	20893	20908	20911	20913
70 - 90	21630	21633	21631	21632
90 - 110	22160	22152	22151	22152
110 - 130	22553	22526	22533	22535
130 - 150	22714	22771	22773	22817
150 - 170	22860	22851	22891	22848
170 - 190	22137	22229	22251	22267
190 - 200	10979	10986	10990	10990

8.18 Longitudinal Spacing(Cotinued)

The table below is derived from 8.17. It is assumed that a 1 metre chainage is sufficiently small to give near perfect results and errors are related to this. The errors are normalised to give a volumetric error per unit area dimension. Thus where X_{ij} is a value in table 8.17 and Y_{ij} is a value in Table 8.18 then

$$Y_{ij} = (X_{ij} - X_{i4})/C$$

C = length x breadth of cross-section.

i.e. for chainage 50 - 70 (i = 4) interval 10 m (j = 2)

then C = 20 x 100 = 2000

$$Y_{42} = -0.003.$$

Chainage	Interval		
	20 m	10 m	5 m
0 - 10	-0.3	0.005	0.003
10 - 30	0.012	-0.014	-0.003
30 - 50	0.007	-0.006	-0.002
50 - 70	-0.01	-0.003	-0.001
70 - 90	-0.001	0.000(5)	-0.000(5)
90 - 110	0.004	0.000	-0.000(5)
110 - 130	0.009	-0.005	-0.001
130 - 150	-0.052	-0.023	-0.022
150 - 170	0.006	0.002	0.022
170 - 190	-0.065	-0.019	-0.008
190 - 200	-0.011	-0.004	0.000
MEAN	-0.036	-0.006	-0.001

APPENDIX

The relevant sections from a typical topographical survey contract specification are reproduced. The specification is a modified version of Tech Memo H9/70 (Ref. 6) with an additional section for string digital ground models. For the full specification See Ref. 7

3.6 Contours

Contours, where required, shall be shown at vertical intervals of :-

(a) 0.5 metres at 1/500 scale.

(b) 1.0 metre at 1/1000 and 1/1250 scales or at 2 metre intervals when a digital ground model is specified.

(c) 2.0 metres at 1/2500 scale.

Where steep slopes are encountered and it is not practicable on the plan to represent each contour fully throughout its length, the Contractor may with the Engineer's approval terminate certain intermediate contours. In flat areas where the horizontal distance between contours exceeds 30 metres, the Contractor shall supply supplementary spot levels at a minimum density of 10 per hectare, paying particular attention to local high and low points in the area.

3.10 Accuracy of Contours

Within any square of 100 metres side in the survey area all contours, when checked by precise levelling from the agreed Ordnance Datum shall be correct to within the tolerance given in Column A and 85% of all contours shall be correct to within the tolerance given in Column B.

Scale	A (100%)	B (85%)
1/500	0.4 m	0.2 m
1/1000	0.8 m	0.4 m
1/1250	1.0 m	0.5 m
1/2500	2.0 m	1.0 m

Any contours which can be brought within the foregoing vertical tolerance by moving its plotted position by an amount not greater than 1 mm at mapping scale, in any direction, shall be considered as correctly plotted. Levels supplementing contours in flat areas shall be correct to within half the tolerance given in column B when checked by precise levelling from the agreed Ordnance Datum.

5.4.2 Strings

The topography is described by the use of both 3D and 2D strings (break lines and contours).

5.4.2.1. 3D String Definition - A string shall be placed along every sharp feature or change of ground slope. Points shall be recorded along the feature to define the string such that the maximum vertical and horizontal distances between the straight line joining adjacent points on the string and the actual ground feature shall not exceed the tolerances shown in the table below. The vertical tolerances is expressed in metres but the horizontal tolerance is millimetres at mapping scale and the values in Column A of the table shall apply to carriageways and hardstandings and the values in Column B to all other detail.

Scale	Vertical Tolerance	Horizontal Tolerance	
		A	B
1/500	0.20 m	0.50 mm	1.0 mm

5.4.2.2. 3D Strings - Accuracy of Measured Points - Levels shall be correct to within the following tolerance of the ground level :

1/500 scale - 0.1 m.

The plan position of the levels shall be such that it shall not contain a co-ordinate error of more than 0.5 mm at mapping scale when measured from the nearest grid line, permanent ground marker or ground control point for the edges of carriageways and hardstandings and 1.0 mm at mapping scale for all other features such as earthworks etc.

5.4.2.3. 2D String Definition and Accuracy. The number of points recorded to define the string (contour) is governed by the method of digitising detailed in Clause 5.4.2.4. The string interval shall be as follows :

1/500 scale - 0.5m.

and the accuracy of the strings shall be as detailed in Clause 3.10 above.

5.4.2.4. Method of Digitising. The method used to dictate the number of points recorded on 2D strings shall be a specified time interval. This interval is such as to ensure sufficient points are recorded to adequately define the contours and will allow optimising of the recorded points by latter processing if desired. The time interval for 1/500 scale shall be 0.7 seconds per point (or as agreed with the Engineer).

5.4.2.5. Density of 2D Strings

(a) Flat Areas - Where contours are sparse - greater than 30 metres apart - 3D strings are utilised to cover the intermediate area so that the distance between adjacent strings does not exceed the above value.

(b) Steep Areas - Where steep slopes are encountered and it is not practicable on the plan to represent each contour fully throughout its length, the Contractor may with the Engineer's approval terminate the contours through the problem area. In this instance the slope shall be bounded by a 3D string and any intermediate changes of slope or berms within this area shall also be depicted by 3D strings. Such treatment shall be applied to quarries etc.

REFERENCES

1. The British Integrated Program System for Highway Design - used by the majority of Local Authorities engaged on highway design work. Details available from the Highways Engineering Computer Branch, Department of the Environment.
2. Baussart, M. 1967, "La Photographe Aérienne dans les E'tudes d'autoroutes", Bulletin de la Société Francaise de Photogrammétrie, No. 27.
3. Deligny, J. L., 1967, "La méthode du senus de points et les programmes francais de calcul e'lectronique de tracés", Bulletin de la Société Francaise de Photogrammétrie, No. 27.
4. Volume 3, B.I.P.S. User Manual, "Ground Section". Management Information Section, Surveyor's Department, Durham.
5. Kirk, S.W.A. "The B.K.S. Procedures for Data Capture from Stereo Viewers and Storage in Machine Readable Form with Special Reference to their use in the String D.G.M. developed in Northamptonshire". Seminar Proceedings Digital Terrain Models 26th - 28th June, 1972 London P.T.R.C.
6. Technical Memorandum H.9/70 Department of the Environment.
7. 1/500 Topographical Survey for A.66 Stockton - Thornaby By-Pass (Instructions for Tendering, Form of Tender, Conditions of Contract, Specification, Bill of Quantities). July, 1973 North Eastern Road Construction Unit, Durham Sub-Unit.

   The following book has a comprehensive discussion of computerised earthworks calculations.
8. Computer Systems in Highway Design, Proceedings NATO Advanced Study Institute 3rd - 9th September, 1972. Copenhagen, Denmark.