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THE COMPENSATION OF THE EFFECTS OF POTENTIAL GRADIENT VARIATIONS IN THE MEASUREMENT OF THE ATMOSPHERIC AIR-EARTH CURRENT

by

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CHAPTER 1. INTRODUCTION

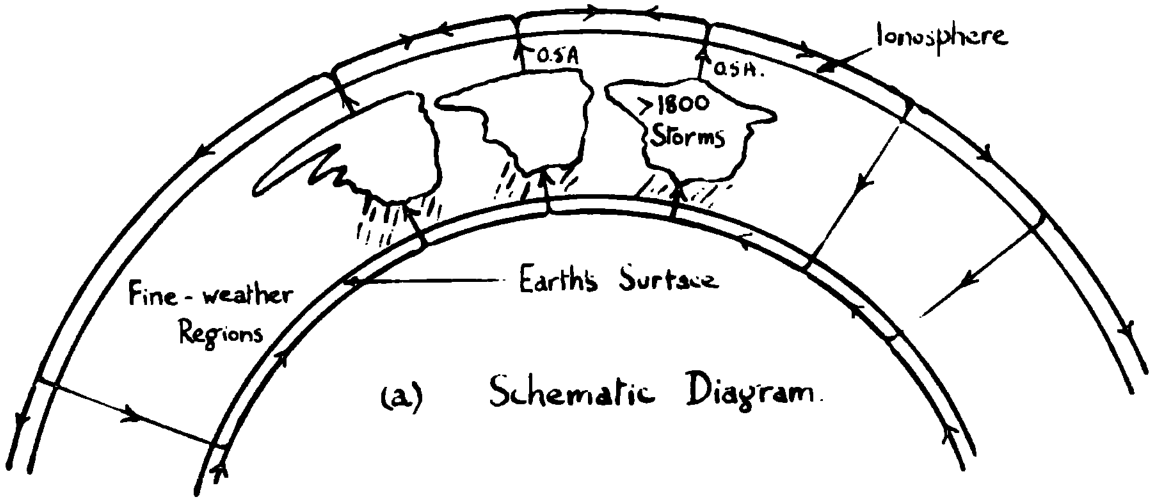
1.1 Resume of electrical phenomena in the atmosphere

In order to provide a brief explanation of the work which has been undertaken, the earth may be likened to the inner sphere of a concentric shell condenser. The outer conductor is the ionosphere; itself a complex structure of ion concentrations which exist in two, sometimes three layers. Although these layers have been identified as being of most pronounced concentration at heights above 100km., ionization takes place to such an extent even below this height, that, in considering electrical effects within the earth's atmosphere, the region above an altitude of about 60km. may be regarded as a perfect conductor. The dielectric between the two conductors is, of course, the atmosphere itself, and since this system is observed to be electrically active, with measurable fields and currents, some generator must be responsible for these phenomena. Moreover, we must expect this generator to be in the terrestrial system itself, because the ionosphere is such a good conductor that the earth must be well screened, electrostatically, from any electrical disturbances such as are known to take place, for instance, on the surface of the sun. Cosmic rays, it is true, penetrate even the lowest regions of the atmosphere, not to mention the oceans, but the ionization which they produce, although it governs the conductivity of the atmosphere, cannot be responsible for the driving force which circulates some

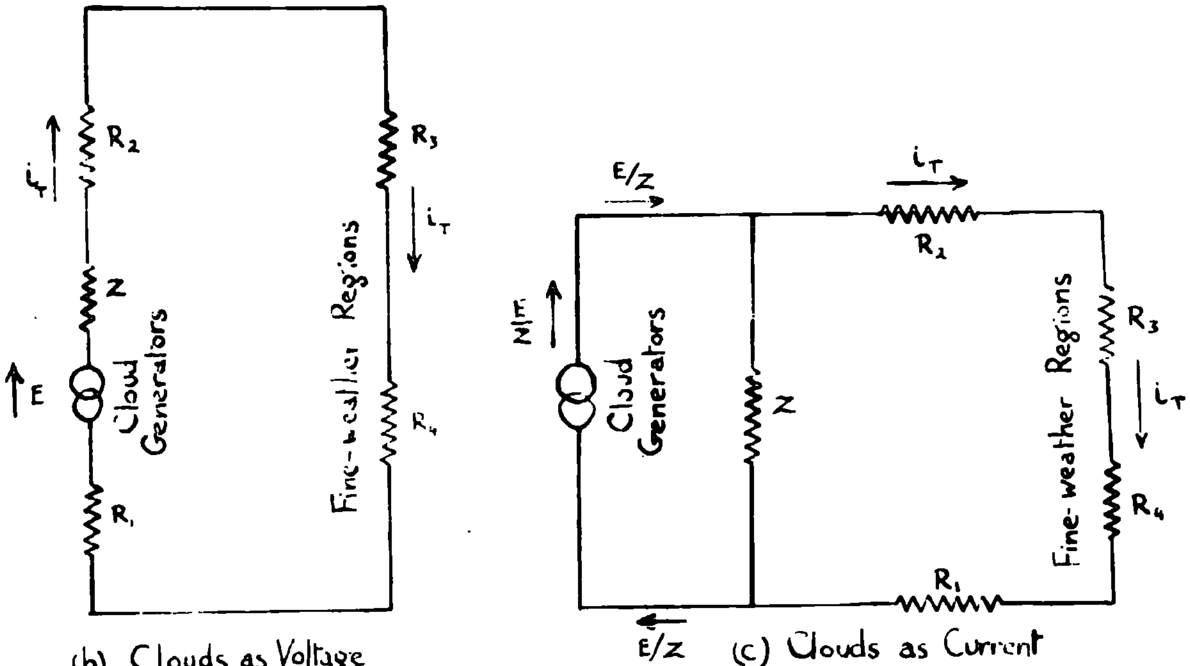
200A. from the ionosphere to the earth's crust. It is now known beyond reasonable doubt that the main process contributing toward the generation of electricity in the atmosphere is the development of thunderstorms. In some way, the exact nature of which has not yet been ascertained, the same process by which heavier cloud particles, ultimately forming precipitation, fall more rapidly against the updraughts than the small cloud droplets, also gives rise to a separation of electric charge in such a manner that the top of the thundercloud becomes positively charged; the base negatively.

1.2 The 'equivalent circuit' of the atmosphere

To understand how the separation of charges in thunderstorms is responsible for some of the more fundamental effects observed in the atmosphere, an 'equivalent circuit' of the system is useful. A simple form of this (Fig. 1) illustrates the principal features which control the circulation of current in the atmosphere. In the 'schematic diagram' of the concentric shell condenser, it should be noted that the outer conductor should be represented as being much closer to the inner one than appears here; it ought to be 0.02 in. away on the scale shown. In other words, the ionosphere is so close to the earth (relatively speaking) that the mathematics of a parallel plate condenser from which the edge effects are missing, is quite appropriate to it. For convenience, all thunderstorms taking place simultaneously over the earth have been grouped together, and the rest of the earth is supposed to be enjoying fine weather. The development of positive polarity at the tops of the clouds, and negative at the



(a) Schematic Diagram.



(b) Clouds as Voltage Generators.

(c) Clouds as Current Generators.

Fig. 1. Schematic Diagram and Equivalent Circuits of the Current Circulation in the Atmosphere.

bases, causes a positive current to circulate through the atmosphere above the clouds to the ionosphere, which then distributes the flow to the fine-weather regions. Parts of the earth which are experiencing fine weather will therefore receive a current of negative charge, which is then returned to the stormy regions, where it is transmitted back to the cloud bases through the atmosphere below them. Although it must not be forgotten that this electrical system is also capacitative, not merely resistive, it will be useful for the present argument to consider circumstances in which the former aspect can be ignored, by supposing that changes in electrical activity occur over periods which are long compared to the time constant of the atmosphere. There are in fact some important observations which have been made in conditions which fall into this category, of which further mention will be made. In practice it means that changes must take place in approximately an hour or more, to fulfil the condition. The equivalent circuit then represents the clouds as a generator of electromotive force E, driving current i_T through an internal impedance Z and the atmosphere above the clouds, here represented by a resistance R_2. The ionosphere and earth act as the low resistance 'leads' carrying the current to and from the fine-weather regions; the upper and lower atmospheres here being represented by R_3 and R_4 ; respectively. The current is returned to the negative pole of the generator through the resistance R_1, although it must be stated that the atmosphere below storms does not behave to any extent as an ohmic element. Similarly, the designation of an internal impedance Z is rather misleading, since the limitation of the mechanical rate of separation of the charged particles must affect the current delivered to the external circuit at least as much as the conductivity within the cloud. However, according to the well-known theorem, the voltage generator may be replaced by a current generator delivering a current E/Z to the external circuit with an impedance Z in parallel. This is definitely a more suitable representation in view of the fact that measurements have been made of the currents above thunderstorms (Gish and Wait, 1950). In terms of the equivalent circuit, these are actually the contributions which, summed for all storms taking place simultaneously, constitute the current which flows through R_1 into the ionosphere, that is, the current to the right of Z in Fig. 1(c). Using the knowledge we have of these currents, we do not therefore need to consider the actual mechanism of the storms unless we are interested in events taking place within the clouds themselves. Gish and Wait, making measurements above storms in an aircraft, found that the current is always positive upwards, a fact which supports the idea underlying the equivalent circuit, and, integrating over the cloud formation for each storm, obtained figures between zero and 1.4A., with an average of 0.5A. Brooks (1925) estimated that about 1800 storms, on the average, are in progress at the same time so that these would provide a total current of 900A. On the basis of measurements of the fine-weather conduction over land and over the oceans, it is believed that the total fine weather current i_T must be approximately 1800A. Thus it is seen that storms provide one half of the total current circulating in the system. Further, Brooks's estimate is a very conservative one in so far as his figures do not take into account more than one storm per day in each locality, even when there may have been a number of them in that period. Also, it is an established fact that the shower cloud develops the same polarity as the thunder cloud, but to a lesser degree, so that if we take these, and perhaps a large number of storms that are not accounted for by Brooks's estimate, it is probable that the two cloud types between them would account for a very large fraction of the total current.

Measurements of the conductivity at different altitudes lead to the conclusion that the total columnar resistance between the earth and the ionosphere is approximately 200ohm. This means that the ionosphere has a potential of 3.6×10^5V. above earth. It can also be seen from the equivalent circuit that the potential at the top of the thunder cloud will be greater than this by an average voltage of $0.5 \times r_2$, where r_2 represents the columnar resistance between the top of the cloud and the ionosphere. Similarly, the base of the cloud will be at a negative potential with respect to the earth.

The consequence of a current of density i per unit area flowing through a fine-weather region of the atmosphere which has a specific conductivity λ is that a vertical potential gradient $F = i/\lambda$ is set up in the region. The potential gradient is much easier to measure than the current, and if it can be assumed that λ is constant, its value is proportional to the current. In this connection, striking support of the idea that thunder clouds are the generators of the atmospheric current has been afforded by the fact that wherever measurements have been made under conditions where there is no local pollution and λ therefore remains constant, it has been found that a diurnal maximum value of F occurs at 1900 hr. Greenwich Mean Time. Whipple (1929), analysing the thunderstorm statistics of Brooks, found that the earth's thunderstorm activity has a maximum also at 1900 hr. G.M.T. That these two observations are to be correlated is also suggested by another common property: not only do the maxima occur at the same time, but they also have the same shape; they both show a gradual increase towards the peak followed by a more rapid fall. Such a cause and effect is obviously to be expected provided that the variation in thunderstorm activity takes place in a time which is not shorter than the time constant of the atmosphere. Of course, the observations do not prove that the thunderstorms are responsible for the entire circulation of charge, but they do indicate that their contribution is significant.

It will be necessary to consider more specifically some of the electrical properties of the atmosphere in the next chapter. At present it is sufficient to add that measurements of the potential gradient do not give a direct indication of the current density unless two conditions are fulfilled: the current must be carried entirely by ions; that is, i is a true conduction current density, and secondly, there must be no local pollution which would cause variations in the specific conductivity, λ . Conditions where F is a measure of i are to be found in fine weather when there is no smoke or dust, and above cloud level in disturbed weather regions.

1.3 Description of the apparatus

In this volume, the development of an electronic method of measuring the current collected by a receiver in various weather conditions, is described. This is equivalent to isolating a portion of the earth's surface and measuring the current flowing into it. The principle underlying the method is very simple: the current is allowed to flow through a known resistance and the voltage drop across it is measured. However, the current is so small, approximately 10^{-13} – 10^{-10}A., that a very high resistance must be used to produce a measurable voltage, to which a conventional meter would present such a low impedance that it would be effectively shorted out. A D.C. amplifier which acts as an impedance matching circuit has been constructed and described by Kay (1950), and causes the same voltage to be produced across a low impedance so that a galvanometer can be used for recording. Between 1950 and 1953, this amplifier was used to measure the currents brought to earth by precipitation, a purpose for which it could be employed more or less directly, but it became necessary to extend the use of the apparatus to measurements in which an awkward complication presents itself. When such a collector is used to measure conduction current, that is a current carried by ions alone, it must also be exposed to the lines of force depicting the potential gradient. The 'bound' charge σ on the collector in which these lines of force terminate does not affect the measurement of the current so long as the potential gradient remains constant, but a change of potential gradient gives rise to a change of bound charge and the current $d\sigma/dt$ takes the same path, through the input resistor of the D.C. amplifier, as the conduction current. Unfortunately, the 'displacement current', as it is called, may be many times larger than the current it is desired to measure. A recording taken by Kay shows the large excursions of displacement current completely masking the conduction current, and the same effect has been found by the present author. Although there are fairly simple ways of either taking into account or else eliminating the effect of potential gradient variations in the measurement of conduction currents, these have so far only been applicable to discontinuous methods of recording. Such methods do not lend themselves to observations of short period changes, such as it was hoped to measure with the present apparatus.

At this juncture it will be useful to consider briefly two items of terminology. The state of electrical stress in the atmosphere has already been referred to as the potential gradient. This term is used, rather than 'field', because, although atmospheric electricians commonly describe the normal fine-weather field as positive, it is the gradient of potential which is positive; that is, if the upward sense of the vertical is assigned to be positive. Since by strict definition, the field is the negative gradient of potential, the fine-weather field is really negative. However, because the burden of mentally reversing the sign to conform with strict usage seems a rather unnecessary encumbrance, the term 'potential gradient' is a convenient alternative to 'field', and at the same time removes the ambiguity (Chalmers, 1957). Secondly, since in rain or snow, charge is brought to earth by the precipitation as well as by conduction, a term is needed to denote the total current which an area of the earth's surface receives under these conditions. Accordingly, the sum of the conduction and precipitation current densities will be described as the 'air-earth current'.

The relative effect of the displacement current depends on the type of weather in which the apparatus is working. In fine weather, the effect is least, but even then, the displacement current is likely to reach, occasionally, values of a few times the conduction current. In conditions where there is a cover of nimbo-stratus, associated with continuous rain or snow, typical displacement currents may well be less than in fine weather, because the potential gradient itself is frequently not as large. Sometimes, however, particularly when the precipitation is rain, the air earth current comprises a conduction current and precipitation current of opposite sign; if these components should happen to be approximately equal in magnitude, the displacement current, relative to their sum, would still be large. In showery or stormy weather, there is no predictable correlation between potential current variations and air-earth current, and because excursions in potential gradient can be very violent, the displacement current may easily attain a value of ten times the maximum air-earth current expected. It should be noted that, to make matters more difficult, the maximum values of displacement current do not necessarily occur at times of maximum air-earth current, because in this type of weather, variations in the electrical phenomena may take place in times of the order of a few seconds, much less than the time constant of the atmosphere, so that the condition upon which the discussion has been based so far is not operative. A complete discussion of the thunderstorm would involve the capacitative as well as the resistive, impedance of the atmosphere.

The design of the apparatus has been based on a method of compensating for the displacement current, rather than eliminating it. The air-earth current, together with the unwanted component is received by the collector as in the unmodified arrangement, but at another terminal in the amplifier a signal proportional to the displacement current alone is impressed in such a manner that the output is proportional to the difference in the two signals. The apparatus used to provide the compensating voltage is the field mill described in Chapter 5, the output of which is fed to the D.C. amplifier through a differentiating circuit. Its most important feature is the application of overall negative feedback to secure improved gain stabilityand linearity of response over a wide range of potential gradient. A simpler method than this has in fact been used previously (Stockill and Chalmers, 1958), in the case when the current carried on precipitation particles is much greater than the conduction current, as, for example, in the thunderstorm, but it was open to certain other objections to which the present compensation method is not liable, (§ 2.4). Accordingly, although it was expected that the equipment would be used mainly to investigate the current in steady rainfall, it was desirable that it should also be able to deal with the more disturbed phenomena of the shower cloud if possible. The method has involved considerable modification of the original D.C. amplifier as well as the design of new apparatus. The outcome is a system in which a few of the features which it was hoped to incorporate have had to be abandoned. Although only a few recordings have been taken with it, these show that satisfactory results will be obtained in fine weather and steady rainfall. In very disturbed conditions, however, the apparatus will probably not function with great accuracy. It is to be expected that the response will be more nearly equal to the true air-earth current than without any attempts at modification, because partial compensation of the displacement current will be achieved, so that average values of current in such conditions will be closer to the true averages. The equipment will not give satisfactory recordings of short term variations however, partly because its time constant is too long, approximately 20 sec., and also because of spurious short period fluctuations produced whenever there is a violent change in potential gradient. These arise because although the method of compensation operates satisfactorily with the first differential of the potential gradient, it copes with higher differentials to only a limited extent. This is a property of the circuit used, as has also been verified theoretically. Thus although a long, steady change of potential gradient is compensated accurately after a certain time has elapsed, there is a spurious transient when the period of change begins and ends. The transient is hardly appreciable even at the onset of quite large displacement currents, but if the potential gradient undergoes a sudden increase, corresponding to an impulsive displacement current, the resulting transient is not negligible. There has been no time to investigate extensively the responses to potential gradient changes of varying rapidity, but these may not be satisfactory in compensating for the most vigorous changes that take place in thunderstorms. Unfortunately, as might be expected, the shorter the period of response of the system to changes in the electrical quantities, the larger the transients that result from the higher differentials of these variations. Hence a compromise has been arrived at between achieving a sufficiently short time of response and having negligible transients. Even so, the apparatus has a sufficiently short time constant to be probably of wider scope than the discontinuous methods of recording when the times between readings are of the order of a few minutes.

With the experience gained, an alternative method has suggested itself which may be free from some of the disadvantages attending the present system. The alternative design, which is described in Chapter 8, would probably permit a considerable reduction in the time constant of response, and at the same time, spurious responses to sudden changes in potential gradient would become smaller.

The equations and formulae in subsequent chapters are in rationalized M.K.S. units, unless otherwise stated.

CHAPTER 2. INTENTIONS REGARDING THE USE OF THE APPARATUS

2.1 Fine-weather phenomena

In fine weather a current of approximately + 2 x 10^{-12} A/m^2 flows vertically through the atmosphere, and at ground level, when there is no pollution, the potential gradient is of the order of + 100 V/m. If there is pollution, the potential gradient F will in general be greater than this as a result of a decrease in the conductivity λ, since $F = i/\lambda$. This follows from a reduction in the average mobility of the ions in the air because some of the small ions become attached to pollution particles. In C.G.S. units the mobility of small ions is approximately 1 cm. $sec.^{-1}$ $V.^{-1}$ while it may be 500 times less for the large ions. At heights above a few kilometres, measurements have shown that the conductivity is what would be expected if all the ionization were produced by cosmic rays, (Stergis, Coroniti, Nazarek, Kotas, Seymour and Werme, 1955). The conductivity in fact increases with height so that although the current is the same at all levels, the potential gradient decreases with increasing height. The investigators just mentioned also found that the conductivity above clouds was the same as at the same altitude in fine weather. This had also been suggested earlier by the measurements of Gish and Wait (1950). Since the cosmic ray activity does not vary with time, it appears that the impedances R_2 and R_3 in the equivalent circuit (Fig. 1) behave as fixed resistances. In fact the phenomena in these two regions of the atmosphere are very well accounted for. Nearer the earth, the columnar resistance up to the level of a few kilometres will also be constant unless there is pollution of the atmosphere. When the conductivity can be assumed to be constant in time, it is evident that the potential gradient is proportional to the current, If, however, measurements of potential gradient are made in inhabited areas, smoke from domestic fires and industries will cause large variations in conductivity near the ground, and the potential gradient will have no more significance than of being an indicator of local disturbances, because any fundamental variation of current density due to a change in the thunderstorm and shower 'supply' will be masked by completely uncorrelated changes of λ. On the other hand, local variations in conductivity such as commonly occur near the ground, will hardly affect the current density in a fine-weather region because the variation only concerns a small fraction of the total columnar resistance, by far the greater part of which remains constant. Thus the current density, unaffected as it is by local disturbances, is a better indicator of the salient electrical properties of the earth and its atmosphere than the potential gradient. It is to be noted that this statement is not tautologous with the thesis that the total circulating current i_T is of great significance as a measure of the activity of the supply. If it is true that the supply approximates to a constant current generator, (using the term in the conventional sense, not to mean that the generator delivers a fixed current for all time, but that the current it does give, although it may change owing to the varying activity of the generator, is unaffected by the impedance across its terminals), then even wholesale variations in conductivity throughout the atmosphere would not effect a change in the total circulating current, which would still be of paramount significance. But in order that the current density at any place, as opposed to the total circulating current, shall have a similar importance, it must be a constant fraction of the total current. Thus the lateral distribution of the total current over the fine-weather regions of the earth must be constant, and this requires that the columnar resistance over the region where the current density is being measured shall bear a constant ratio to the whole impedance of the fine-weather regions.

2.2 General features of thunderstorms and showers

When thunderstorms or showers, both characterized by the attendance of cumulo-nimbus, prevail in a region, the simplicity of the ohmic relation, $F = i/\lambda$, between potential gradient and current density, is lost amidst a host of other factors which come into operation. Three additional current-carrying mechanisms are introduced besides the ions which transmit the true conduction current; indeed, during the time that any of these other modes is operating, the conduction current is negligible by comparison. The most obvious and familiar of these is the lightning flash, although this is of course not included in phenomena to be ascribed to shower clouds. Of the discharges which do reach the ground, by far the larger proportion bring down negative charge. The assessments of most authorities, e.g. Wait (1950), give the current density due to fine-weather conduction current, averaged over the whole surface of the earth, as about 100 C. $km.^{-2}$ $yr.^{-1}$, while that due to lightning is - 20 C. $km.^{-2}$ $yr.^{-1}$ Thus lightning electricity constitutes about 20% of the returning circulating current from the earth to the cloud 'generators'.

At the approach of a thunderstorm or shower, potential gradients of at least ten times those commonly occurring in fine weather, may be produced because of the high concentration of charge in the bases of the clouds. Above a certain 'onset' value of the potential gradient, the potential of the atmosphere at the level of the tops of trees and other tall objects, may become so much different from the earth potential of the objects themselves that intense local fields are created near sharp angles and points on these objects, sufficient to form avalanches of ions. The multiplication that results from the ionization by collision appears as a current flowing into the object which is of the order of 1 μA., much greater than any conduction current flowing into the same area. The occurrence of this 'point discharge' current leaves a space charge of opposite sign in the air above, which under the action of the potential gradient, moves slowly up into the cloud, although it will also be carried and dispersed horizontally by the wind; or on the other hand, some or all of it may be returned to the earth if precipitation falls through the space charge 'blanket' (Simpson, 1949). Although point-discharge will occur for either sign of potential gradient, the potential gradient is usually negative when it is large enough to produce the phenomenon, consequently the net effect is to carry negative charge to the earth. It appears from assessments that have been made, based upon assumptions concerning the number and separation of objects which could cause point-discharge, that a very large proportion of the total circulating current is returned to the clouds by this process.

Electric charges are also observed to be carried to earth on precipitation particles, the average charge on a particle in any period of precipitation, together with the rate of fall, determining a current. Precipitation current is ordinarily measured with conduction current in the usual type of collector, and the sum of these is termed here the 'air-earth' current, although the charge brought down by conduction in a thunderstorm or shower is probably negligible compared with that brought down by precipitation. The net current carried by precipitation is believed to bring excess of positive charge to the earth and is therefore in the opposite direction to the return current to the clouds. Many results need to be obtained before any general conclusions can be drawn about the electrical properties of the rainfall in a particular area, because the currents are likely to vary markedly over different periods occurring even in the same storm or shower. Consequently any statement made about the contribution of precipitation to the total circulating current must be tentative. It is nevertheless probably true to say that the current brought to earth in a storm by conduction is much less than that brought by precipitation. The value of the precipitation current in these conditions is usually of the order of 10^{-10} A/m.^{2} as against 10^{-12} A/m.^{2} for the fine-weather conduction current, but it may rise to 10^{-9} or 10^{-8} A/m.^{2} on occasions.

The potential gradients observed in thundery or showery conditions may attain very large magnitudes of either sign. 1000 V/m. is an expected value, but it may rise to 10,000 V/m. for short periods. The potential gradient will depend on the amount and distance of any space charge overhead, such as may result from point-discharge (Davis and Standring, 1947; Whitlock and Chalmers, 1956), or on the space charge of falling precipitation itself, as well as on the distribution of charges in the cloud bases. Results for potential gradient are very difficult to interpret and do not lead readily to information about the separation of charge which proceeds within the clouds themselves. In the case of the currents, which occur in the various forms just described, the situation is more amenable to interpretation. Taken moment by moment, the current within the cloud is probably not the same as that which flows to earth. This is because the shower or storm cloud has a dynamic nature characterized by violent vertical air-currents, and during the early part of development, part of the separation process goes to building up intense concentrations of charge. Clearly, while there is building up of charge concentrations, there cannot be continuity of current, and the current above and below the cloud will be less than that within, the difference being the rate of accumulation in the charge centres in the cloud. When the cloud is in its degenerating stage however, the balance will be disturbed the other way: The current within the cloud will be less than the currents outside, on account of the net loss of charge from the poles of the cloud when the mechanical process of separation has ceased. If an average is taken of the total current below or above the cloud, this will probably be close to the value of the generating current caused by the mechanical separation of charged particles within the cloud. This will be the more true, the more quickly after the end of the shower or storm that space charges find their way to the earth or the ionosphere, and the more closely they remain within the boundaries of the disturbed region during their journey.

It must be realized that precipitation and point-discharge currents are not in themselves sources of the electricity in the atmosphere. They are effects, not causes, and they must be regarded as the agents which carry the circulating current below shower and thunder clouds, in the same way that the conduction current is the agent in fine-weather regions. Obviously, in estimating the current in disturbed regions, every mode of transport of this current must be taken into account.

2.3 General features of continuous precipitation

The mechanical, thermal and electrical conditions in the type of weather characterized by nimbo-stratus are undoubtedly much simpler than in the thunderstorm. This is because, once this cloud has developed, it usually remains in a state of dynamic equilibrium for a considerable time, of the order of a few hours, in which no vertical development of the extent of the cloud takes place, and it is probably safe to assume continuity of current on the grounds that there will be no building up of charge concentrations, after the initial development. Further, the potential gradient in this kind of weather is of the same order as that in fine weather, and rarely increases to a value likely to promote point-discharge. The only carriers will therefore be the conduction and precipitation currents. In spite of its apparent simplicity, very little has been known about this type of cloud until fairly recently, largely on account of the smallness of the potential gradient and current as compared with the electrical effects of the shower or storm. Chalmers(1956) has made measurements on a large number of individual periods of steady rain and snow and has observed that in about 80% of these the potential gradient was negative. In the case of rain, however, the current (air-earth current) brought to earth in nearly all the periods was positive, while in snow it was nearly always negative. The main point at issue is to discover whether the same process of charge separation could obtain in both the cumulonimbus and nimbo-stratus types of cloud. One factor which militates against the probability that they are the same process is the very large difference in the magnitudes of the effects. Thus the potential gradients observed in continuous rain and snow were only of the order of the fine-weather values, while the current densities, on the average, were $+ 3.8 \times 10^{-12}$ A/m^2 for rain and $- 3.5 \times 10^{-12}$ A/m^2 for snow. Incidentally, the fact that these currents are of the same order as the conduction currents measured in fine weather suggests that, in contradistinction to the conditions in storms and showers, the conduction current in continuous rain might be a considerable fraction of the total air-earth current. The sign of the charge on continuous rain is the same as that predominating in the shower and storm, and the negative potential gradient indicates that the base of the cloud is negatively charged: another point of similarity to the shower cloud. If it is assumed that in all the clouds investigated, precipitation started as ice particles, it is easy to see that these particles, acquiring a negative charge by some means as yet not ascertained, and falling through the cloud, would provide it with negative charge in its lower layers, thus giving rise to a negative potential gradient, and when the precipitation fell as snow, it would have the observed negative charge. What is not clear is why the precipitation should have a positive charge when it falls as continuous rain. In order to elucidate this problem, measurements would have to be taken in the clouds themselves to find out if the rain does in fact begin its life as negatively charged ice particles, and at what level the sign of the charge is reversed, for example: whether it changes at the freezing level in the clouds. At present there is not sufficient information to enable an estimate to be made of the contribution to the total circulating current of charges on continuous rain and snow. Generally speaking, the contribution of all kinds of precipitation remains one of the greatest uncertainties in the elucidation of the balance of currents.

2.4 Applications of the proposed apparatus to measurements in storms and showers.

Superficially, the solution to the problem of eliminating the effect of displacement current in a continuously recording apparatus is not difficult: it is to provide the collector with an earthed cylindrical shield, which extends a distance above it of the order of its own diameter. This has been done by Stockill and Chalmers (1958). The lines of force depicting the potential gradient then nearly all end on this shield and a change in their concentration does not affect the collector itself. However, it is also evident that the conduction current will terminate on the shield rather than on the collector, which will then measure only precipitation current. This may be no disadvantage when the main interest is in thunderstorms, since it can be assumed that the precipitation current then makes a far larger contribution than the conduction current. There is, however, another inherent drawback in this method, particularly in view of the results of Smith (1955), who found that while small raindrops carried charges of sign opposite to that of the potential gradient, large drops often carried the same sign. It is therefore desirable to dispense with the screening cylinder because, if there is any wind, a smaller proportion of the small drops than of the large ones, will fall into the collector if this is screened, and a misleading record of the current will be obtained. In addition, of course, a false picture will also be given when there is rain and a gusty wind, because more of the rain will be collected when there is a lull, than during a squall.

Given a method such as that which has been developed at Durham, not subject to these objections, it would be interesting to see if the results obtained compared with those of Simpson (1949), who drew the conclusion that there is an inverse relation between rain current and potential gradient, not limited only to steady conditions, but also exhibited when the field is changing, the rain current following the changes in magnitude, but having the opposite sign, ('mirror-image effect'). Simpson used a partially shielded collector, so a fair sample of all the drops may not have been received by the collector, and in fact the absence of the mirror-image effect has been noted by other workers. The inverse relation between potential gradient and rain current was explained by Simpson to be the result of the capture, by the drops, of the ions left in the atmosphere when point-discharge takes place. It has previously been mentioned that an effect of point-discharge is to leave a space charge in the air which will drift due to wind, and will of course also move in the potential gradient. Elementary considerations show that the space charge has a sign opposite to that of the potential gradient. In this connection, it was believed that useful results might be obtained from an investigation of the relation between precipitation currents and point-discharge, since apparatus has already been in use in Durham for measuring the latter, (Kirkman and Chalmers, 1957).

The use of a rapidly responding recorder in which compensation for displacement current is effected without resorting to an earthed shield ought in any case to be ideal for use in thundery and showery weather, in which large and rapid changes of potential gradient, and undoubtedly of air-earth current, occur. Unfortunately the apparatus as it now stands has neither a sufficiently rapid nor accurate response, to give unambiguous recording when sudden changes of potential gradient occur. However, it would be only in thunderstorms, as distinct from showers, that the apparatus would fail to any serious extent; the indications are that in the less violent conditions of showery weather, satisfactory results should be obtained nearly always.

2.5 Application to measurements in continuous rain and snow

It is by no means certain that when the cloud cover is nimbo-stratus, the conduction current is only a small fraction of the air-earth current, as it probably is in showers and storms. The results of Chalmers (1956) indicate rather, that the conduction current represents a large fraction, perhaps half of the total current, and moreover may be opposite in sign to the precipitation current itself. If the air-earth current density is to be measured therefore, it is imperative to employ a method other than screening the collector, to avoid recording the displacement current. Provided no point-discharge occurred during a period of continuous precipitation, it would be correct to say, as has been described, that the current measured by the collector, the current within the cloud and that above the cloud, would all be the same. Measurements of the air-earth current at the ground would therefore furnish more evidence to enable an attempt to be made to explain how the charge separation might occur within the cloud. This would probably be much more feasible than similar calculations for showers and storms, because it is supposed that the structure of nimbo-stratus is much simpler, and it would be easier to calculate the current generated by any proposed mechanism of charge separation, to see if this were in agreement with observation. Nevertheless, a knowledge of the air-earth current below the cloud does not lead directly to a figure for the amount of charge separated in the cloud per unit time, because the net current within the cloud is the difference between the current produced by the mechanical separation of the charges of opposite signs, and a dissipating current in the reverse direction. This dissipating current arises by virtue of the potential difference between the poles of the cloud layer, and the conductivity within it. Measurements of potential gradient and conductivity within the clouds would therefore eventually have to be made for a complete study.

Apart from the smallness of the continuous precipitation currents requiring the use of more sensitive apparatus than for thunderstorms, the elimination of the effects of displacement current is a major problem. It has been found that the equipment which has been developed ought to be very suitable for measurements in continuous rain and snow, since it can easily deal with potential gradient changes that accompany such weather. In addition, it is sufficiently reliable to be run for several hours without attention, so that a representative sample of the rain in a prolonged spell of such conditions could be measured.

2.6 Application to measurements in fine weather

In measuring the conduction current in fine weather the use of some device for eliminating displacement current effects is essential, unless a simultaneous recording is made of the potential gradient, so that a calculated correction can be made (Simpson, 1910; Chalmers, 1956).

While the present apparatus should certainly function well in this type of weather, it is probably not a great improvement on other types of apparatus using discontinuous recording to eliminate the effects of potential gradient variation (Chapter 3). If, however, the time constant could be reduced, it might lead to interesting results. A short period continuous recorder with compensation for displacement current has never been used before in measuring conduction currents, and so it might be that in fact short period variations do occur in the conduction current, which have not hitherto been observed.

CHAPTER 3. THE COMPENSATION OF FIELD CHANGES

3.1 The magnitude of the displacement current effect

Fig. 17 shows schematically the connection of the hemispherical collector to one grid of the differential electrometer stage which feeds into the D.C. amplifier. Assuming an 'effective' area A for the collector, (in fact there is a difference in the effective areas for conduction and displacement currents as discussed in § 4.2) a current Ai due to an air-earth current density i will be received, and the voltage this produces across the resistance R is measured by the amplifier. The total displacement current $I(t)$ is the rate of change of bound charge σ on the collector and this is related to the potential gradient F by the simple equation:

$$I = d\sigma/dt = A\varepsilon_0\, dF/dt$$

where ε_0, the permittivity of free space, has the value 8.8×10^{-12} farad/m.

A large value of air-earth current density, such as might easily occur in a shower or thunderstorm, is 10^{-10} A/m^2. The bound charge density on the collector due to the potential gradient is $\varepsilon_0 F$ C/m^2 and therefore this air-earth current would be almost equalled by a potential gradient change of 10 V/m. per sec. However, rates of change of the order of 100 V/m. per sec. are not uncommon in disturbed weather. To give added point to the comparison, it will be remembered that in the condition under consideration, the potential gradient is not a function of the air-earth current only, and that large potential gradients and large currents do not necessarily occur simultaneously, so that the factor by which the displacement current exceeds the air-earth current might be considerably greater than 10.

3.2 Previous methods of avoiding displacement current effects

Most of the successful schemes which have been devised in the past have overcome the displacement current obstacle by employing a discontinuous form of recording. Wilson (1906, 1916) used an electrometer to measure the charge received by the collector in a specified time interval. At the beginning and end of an exposure the collector was screened by an earthed plate so that the bound charge did not affect the reading. Although the method is strictly discontinuous, the duration of a cycle could in theory be reduced up to a point determined by the maximum possible speed of operation of the earthed plate and the period of response of the measuring instrument. Clearly, if a D.C. amplifier were used instead of an electrometer, a very small limit to the cycle would be imposed, but on the other hand, a very rapid movement of the earthed plate would result in a disturbance of the air which would falsify the current measurement. An additional departure from the true current would probably result in the event of precipitation, on account of the fracture of drops by the edge of the plate.

As the time of exposure of the collector is made longer, so the total charge collected in the interval due to the air-earth current increases relatively to any probable difference in the bound charge at the beginning and end of the exposure, and eventually the need for screening the collector is obviated. Chalmers and Little (1947) dispensed with the earthed plate, collecting the charge over periods of 10 min. The logical extension of the method is, of course, to use a continuous recorder with a long time constant; changes of potential gradient in times short compared with the time constant will not register, while longer period changes will be equivalent to only small displacement currents. Kasemir (1951) employed this last method, using an amplifier with a time constant of 3000 sec. Kasemir was interested in long term variations of the air-earth current with time of day so the large time constant was no disadvantage; as was indicated in § 1.3, methods of the type described above will not record the rapid changes in air-earth current such as undoubtedly accompany disturbed weather.

Even when the exposure time is of the order of a few minutes, the change in bound charge during the period may not be negligible compared with the charge received from the air-earth current, unless measures are taken to screen the collector intermittently as described above. For instance, if the potential gradient changed by 1000 V/m. over a 10 min. exposure, the effect would be the same as if a uniform air-earth current of 1.5×10^{-11} A/m.^{2} had flowed throughout the interval. This postulated change in potential gradient is of an order which it is reasonable to expect in disturbed weather, in which the air-earth current may have any value between $\pm 10^{-12}$ and 10^{-10} A/m.^{2}, so that even with this long exposure time, the potential gradient has an appreciable effect. Chalmers (1956) has improved on this method in a scheme whereby the charge is collected for periods of $4\frac{1}{2}$ min. and then measured by an electronic device, while, concurrently, a recording of the potential gradient is made, from which the appropriate correction can be calculated. Most of the tedium of applying the corrections by otherwise pedestrian arithmetic is alleviated by having both recordings taken on the same rotating drum carrying photographic paper, using two mirror galvanometers. The sensitivities are adjusted so that the correction can be transferred directly from the potential gradient trace to that representing the air-earth current, with a pair of dividers.

A method, which, though ingenious, has very severe limitations, is described by Kasemir (1955) and allows of continuous recording; it is considered in some detail by Israel (1955). The collecting plate is connected to earth through a resistance R and capacitance C in parallel. It can be shown that, if the product $RC = \varepsilon_0/\lambda$, where λ is the conductivity of the air near the instrument, a meter in the resistive branch will record the air-earth current without there being any effect due to the displacement of bound charge. This is provided that the potential gradient is determined entirely by the relation $F = i/\lambda$; so that, apart from the difficulty that λ is not constant, the system would only behave accurately if there were no space charge to affect the potential gradient, and if there were no precipitation current, which is not related to F by Ohm's law. The method would doubtless give a more reliable recording than with no compensation, under all conditions, but the degree of improvement would at least be difficult to calculate. This makes the method quite unsuitable for the present purpose, where it was envisaged that most of the recordings would be made in disturbed weather.

Mention has already been made of a partial solution to the problem effected by standing over the collector an earthed cylindrical shield, and some reasons have been given why it is unsuitable for the present purpose (§ 2.4). In addition, any effects due to splashing would be lost as a result of the action of a shield. It is possible that the splashing of drops at the ground contributes to the current received at the earth's surface, by a mechanism which leaves the drop, and subsequently the earth, with a certain charge, while an ion of the opposite polarity is left in the air. In the presence of an earthed screen the ion would be quickly lost from the air, and in any case the separation of charge by splashing might require an electric field to initiate it, which, for other reasons, the screen is deliberately put there to remove.

The prototype of methods in which the displacement current is compensated for, rather than eliminated, is due to Scrase (1933). Using a quadrant electrometer as a measuring instrument, Scrase connected one pair of quadrants to the air-earth current collector, and the other pair, not to earth, but through a condenser to a polonium collector which registered the potential gradient. The purpose of the condenser was to differentiate the output from the polonium collector, since the displacement current is proportional to the differential of the potential gradient. Since the deflection of an electrometer needle is proportional to the potential difference between the sets of quadrants, it was possible in principle at least, to adjust the condenser until the effect of the potential gradient variation was the same on both sets of quadrants, so that the deflection would be proportional to the air-earth current alone. Scrase found, however, that the polonium collector was too sluggish to follow the potential gradient accurately, and this difficulty of adjusting response times is the whole crux of the problem of compensating for the displacement current, in the present author's experience. Scrase partly overcame the obstacle by connecting to the radioactive collector a grid, placed at a distance above the air-earth current receiver such that in steady conditions its potential was the same as that of the atmosphere at the same level. Then, even allowing for the time lag of the polonium collector, the displacement current effects on the two sets of quadrants would be the same. However, when the potential gradient was varying, the potential of the grid would not follow the potential of the surrounding air accurately, so that the current measured would not be the true air-earth current which would obtain in natural conditions. Goto (1951) describes a method using a differential quadrant electrometer, in which an agrimeter type of field registering device would be substituted for the polonium collector. An agrimeter can be made to have a time constant of a few milliseconds, as opposed to approximately 30 sec. for the polonium collector, but it is probable that with the alteration, difficulties would arise because of transients which would be introduced by the capacity of the cables connecting the electrometer with the collector.

3.3 Compensation using the D.C. amplifier

The present method involves the use of a D.C. amplifier with an electrometer double tetrode (Ferranti DBM4A), having balanced characteristics, as the input stage. With the mode of connection of the electrometer valve shown in the basic circuit of Fig. 7, it will be shown (§ 4.5) that the output voltage E_0 is proportional to $(V_2 - V_1)$ to the same degree as the characteristics of the two halves of the tube in the chosen operating region are similar. It is also true that the presence of the feedback loop in Fig. 7 preserves the nature of this in the form $V_0 = a(V_2 - V_1')$, where V_0 is the final output measured by the galvanometer and V_1' is the voltage across the input resistor R.

In the light of the upper diagram in Fig. 17, which shows how the signals from the current collector and the field mill registering the potential gradient are fed into the amplifier, a more detailed discussion of the general problems that arise in this type of compensation can now be given. Assuming for a moment that all capacitance effects can be eliminated in the current collecting part of the circuit, the voltage across the input resistor R connected to grid G_1 (not the same as the voltage on G_1 itself) will be accurately $V_1' = RI(t)$, whatever the form of $I(t)$. In order to obtain a voltage on G_2 corresponding to the rate of change of potential gradient, the output from the field mill must be differentiated, and a capacitor-resistor circuit D, S has been chosen to do this. A simple calculation (Eq. 6.1) gives the condition for compensation of a steady displacement current, i.e., a linear rate of change of potential gradient:

$$DS = \gamma \text{ (constant)},$$

where γ is determined by the field mill sensitivity and R. It is clear that this equation has determined also the time constant of the input to G_2, and that V_2, the voltage on that grid, will only be accurately proportional to dF/dt for displacement currents which show variations only in times long compared with γ. In the event of a step change in $I(t)$ from $I(t) = 0$ when $t < 0$, to $I(t) = I_i$ when $t \geqslant 0$, the voltage on G_2 is given by

$$V_2 = R I_i (1 - e^{-t/\gamma}) \qquad \text{..... Eq.3.1}$$

Thus, for such a displacement current a resultant output, $V_0 = R I_i e^{-t/\gamma}$ would be observed, which would only approach zero after an interval equal to a few times γ. It is obviously necessary to arrange for a time constant circuit on G_1 so that an exponential term will appear in V_1' which will cancel that in Eq. 3.1. The question now arises as to whether it is possible to adjust the circuit for steady displacement currents and for equal time constants independently. At first it looked as if the introduction of a variable condenser X, across R would be the solution. By making $RX = \gamma$, the exponential terms in the equation for the net response,

$$V_0 = RI_i \left[\left(1 - e^{-t/RX}\right) - \left(1 - e^{-t/SD}\right) \right]$$

would cancel out, as well as the terms RI_i representing the effects of a uniform displacement current of infinite duration. It is of course evident that the partial failure of Scrase's apparatus was simply a failure to obtain equal time constants for the inputs to the two pairs of quadrants of his electrometer.

However, even the assumption that *only* one time constant makes its appearance in the input circuits to either grid, is a gross over-simplification, for the cable capacitance from the collector to the electrometer valve has been neglected; also, it has been assumed that the field mill has an instantaneous response to fluctuations in potential gradient. In general therefore, the response on one of the grids to a step function of displacement current, is more likely to be represented by an equation

$$V = RI_i \left(1 - B_1 e^{-t/\gamma_1} - B_2 e^{-t/\gamma_2} - B_3 e^{-t/\gamma_3} \ldots\right)$$

Obviously, it would be a hopelessly complex problem to try and match every time constant appearing in the equation for one grid to a time constant in the equation for the other. The logical procedure is to see what can be discovered about the relative effects of the time constants appearing in one of the equations. As will be shown in

Chapter 7, the coefficients $B_{1,2}\dots$ are functions of the γ's only, and have the property that if one time constant, γ_n , is made much larger than all the rest, then the coefficient B_n approaches unity, while all the others tend to zero; see, for example, Eq. 7.19. Thus an approach would be made to Eq. 3.1.

Although this procedure sounds easy enough, it is not so in practice, particularly if the largest γ is still to have an absolute value of not more than a few seconds, because in general, the γ's are not independent, even considering either grid separately. It is evident that they cannot be so unless no two of them comprise the same resistive or capacitative component. This means that there is a severely limited number of variations that can be made on the circuit elements; and in the apparatus here described, there is the additional limitation that, owing to the negative feedback, one of the time constants appears in the equation for either grid.

To make the idea more concrete, the problem is that there are two equations representing the responses to the signals on the grids:

$$V_1' = RI_i\left(1 - A_1 e^{-t/\alpha_1} - A_2 e^{-t/\alpha_2} - A_3 e^{-t/\alpha_3} \dots\right)$$

$$V_2 = RI_i\left(1 - B_1 e^{-t/\gamma_1} - B_2 e^{-t/\gamma_2} - B_3 e^{-t/\gamma_3} \dots\right)$$

It is known that, if it were possible to deal with the time constants separately, so that, say, $\alpha_n \gg \alpha_1, \alpha_2 \alpha_3$ and $\gamma_n \gg \gamma_1\ \gamma_2\ \gamma_3$ these equations would become:

$$V_1' = RI_i\left(1 - e^{-t/\alpha_n}\right)$$
$$V_2 = RI_i\left(1 - e^{-t/\gamma_n}\right)$$

and all that would remain to be done would be to make $\alpha_n = \gamma_n$. It is impossible to satisfy such conditions exactly in practice: first, because all the α's are not independent, nor are all the γ's; and secondly, one of the α's is identical with one of the γ's, but without the corresponding A and B being equal. In the course of optimizing the circuit parameters, calculations were made to see what would be the effect of making some of the α's or γ's equal; since, with the limitations imposed by the interdependence of these quantities, it was likely to be easier to equalize them than make them very different. Nothing was gained by this, however: the result is the introduction of a term of the form $C\, t/\alpha\, e^{-t/\alpha}$, where α is either of the equalized time constants.

The treatment of the problem as it applies specifically to the present equipment, will be given in Chapter 7.

3.4 Stability requirements

Usually, when corrections have to be applied to experimental data, the corrections are only a small percentage of the desired quantities, so that they themselves need not be known to great accuracy. However, in the measurement of the air-earth current, the effect of bound charge may be of several times greater account than the true current, so that the accuracy of the compensating voltage must be much higher than that to which the air-earth current is required to be known. Suppose for instance that the displacement current may rise to 10 times the air-earth current; then if it is necessary to know this current to within 10%, the compensation must be accurate to within 1%. This immediately implies that the gain of the field mill producing the compensating voltage must be constant to within 1%. The requirements as to the linearity of the compensating apparatus are not, however, so easy to define. Since the displacement current is the differential of the potential gradient, a spurious change of, say, 0.1 V. in the output of the field mill when this was recording a field of 100 V/m. would have the same effect as the same spurious change when the field was 1 V/m. Thus a certain absolute accuracy, rather than a specified relative accuracy, is required, but this has to be compared with the air-earth current at the time, which, to all intents and purposes, is a completely independent quantity. Probably the most that can be got out of the argument is an air-earth current equivalent to the maximum absolute deviation in field mill output expected, on the following lines. Suppose the output of the field mill corresponds with the value upon which the adjustments for compensation are based, to within an equivalent input of ± 0.5 V/m. and that the time constant of the D.C. amplifier is 5 sec. The maximum error in the difference of the outputs at the beginning and end of an interval t sec. corresponds to ± 1 V/m. and this means that an error of $\pm 1/t$ V/m. per sec. represents the amount of failure of the compensation. However, the apparatus does not register changes taking place in much less than 5 sec. as progressively increasing displacement currents[*], and the longer t becomes, the less is the effective displacement current error anyway, (§ 3.2) so that the maximum error obtainable is of the order 0.2 V/m. per sec. This would correspond to a displacement current of 2×10^{-12} A/m^2, which is about the average value of the

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[*] See p. 39.

air-earth current in fine weather. A similar criterion applies to the short term stability of the output. Noise on the field mill output will have negligible effect except at frequencies comparable with the reciprocal of the time constant of the differentiating circuit.

The demands upon the stability and linearity of the compensating equipment are seen to be quite stringent, becoming less so as the time constant of the differentiating circuit or the D.C. amplifier (assumed to have equal time constants) increases. A description of the actual performance of the apparatus is given in Chapter 7.

[*] This requires a little further explanation. By the use of the Laplace Transform, it can easily be shown that the amplitude of the response of the differentiating circuit with time constant T, to a sinusoidal potential gradient of angular frequency ω, is equal to $K\omega/(1+\omega^2T^2)^{1/2}$, where K is the response for unit rate of change of potential gradient. In the limit, as ω increases, the response has an amplitude K/T, which is the same as the response to a potential gradient rate of change of $1/T$ V/m. per sec. If the time constant were zero, the effective displacement current amplitude would rise with increasing frequency, as can be seen also by differentiating $F = F_0 \cos \omega t$.

CHAPTER 4. THE DIRECT COUPLED AMPLIFIER

4.1 The original form of the amplifier

Only a brief description of the original amplifier of Kay is given here, as a complete discussion appears in his thesis (1950). The complete circuit diagram is given in Figs. 2 and 3. In the main amplifier (Fig. 3), the coupling is straightforward from stage to stage, and the last stage, being a cathode follower, presents a low impedance output to operate a robust galvanometer. The output impedance is further reduced by the negative feedback loop. The use of the cathode-coupled double triode in the remainder of the circuit is now a familiar feature of D.C. amplifiers. The circuit was first used by Miller (1941) and its main properties are described by Williams (1944), who also gives the derivation of the expression for the gain of such a stage. The output voltage E_o for voltages V_1 and V_2 on the grids (Fig. 2, basic circuit) is given by

$$E_o \;=\; \frac{-\mu R_a(V_1 - V_2) + V_1 r_a / R_c(1+\mu)}{2r_a + R_a + r_a(r_a + R_a)/R_c(1+\mu)}$$

where R_a and R_c are the anode and common cathode resistors respectively and V_1 is the voltage applied to the grid of the amplifying section of the valve. The other symbols are conventional and all the voltages are of course incremental. Under the conditions in which the circuit is always used, $R_c >> 1/g$, where g is the transconductance from control grid voltage to anode current, and the equation approximates

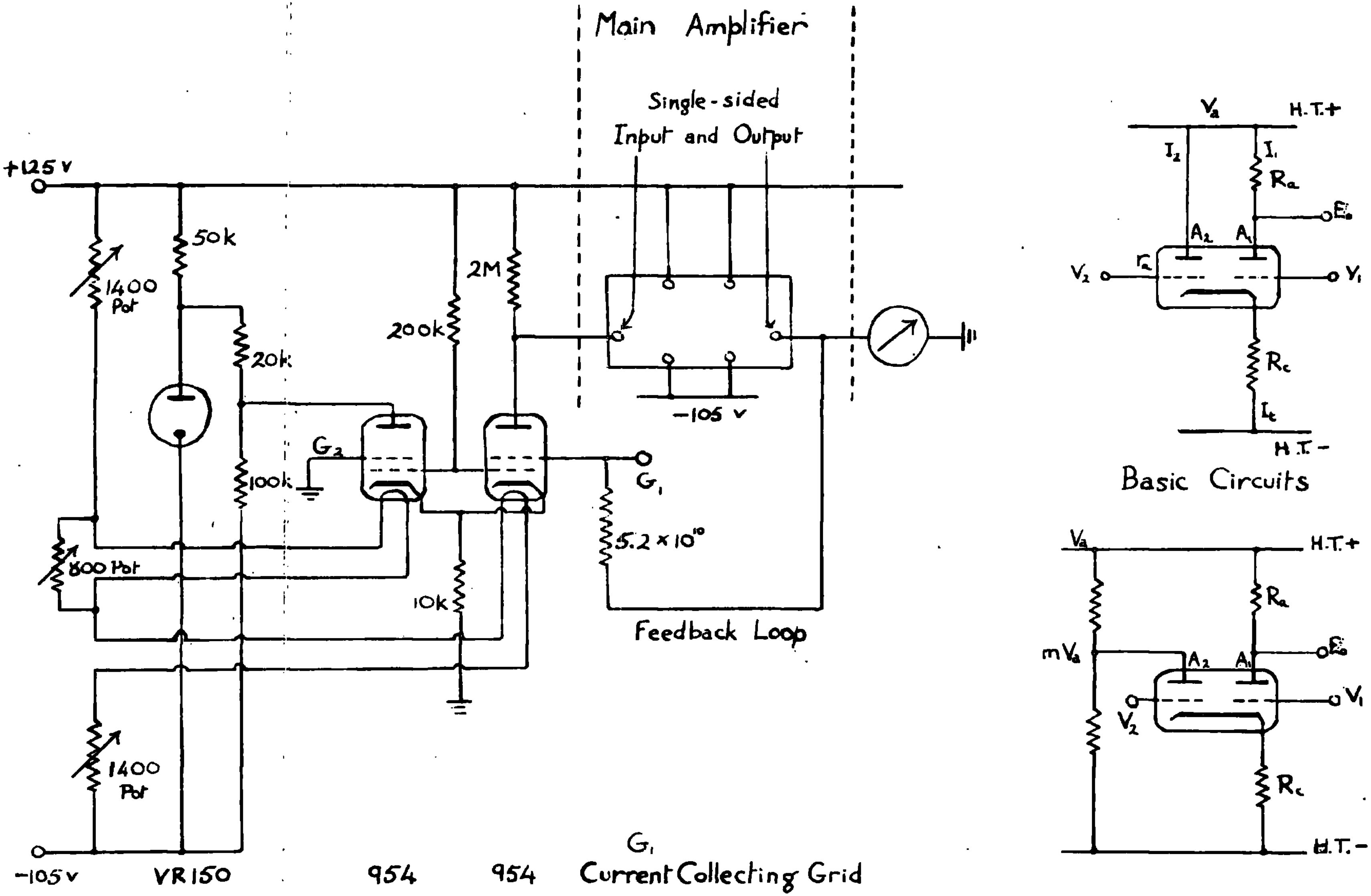


Fig. 2 Electrometer Stage (Original Form)

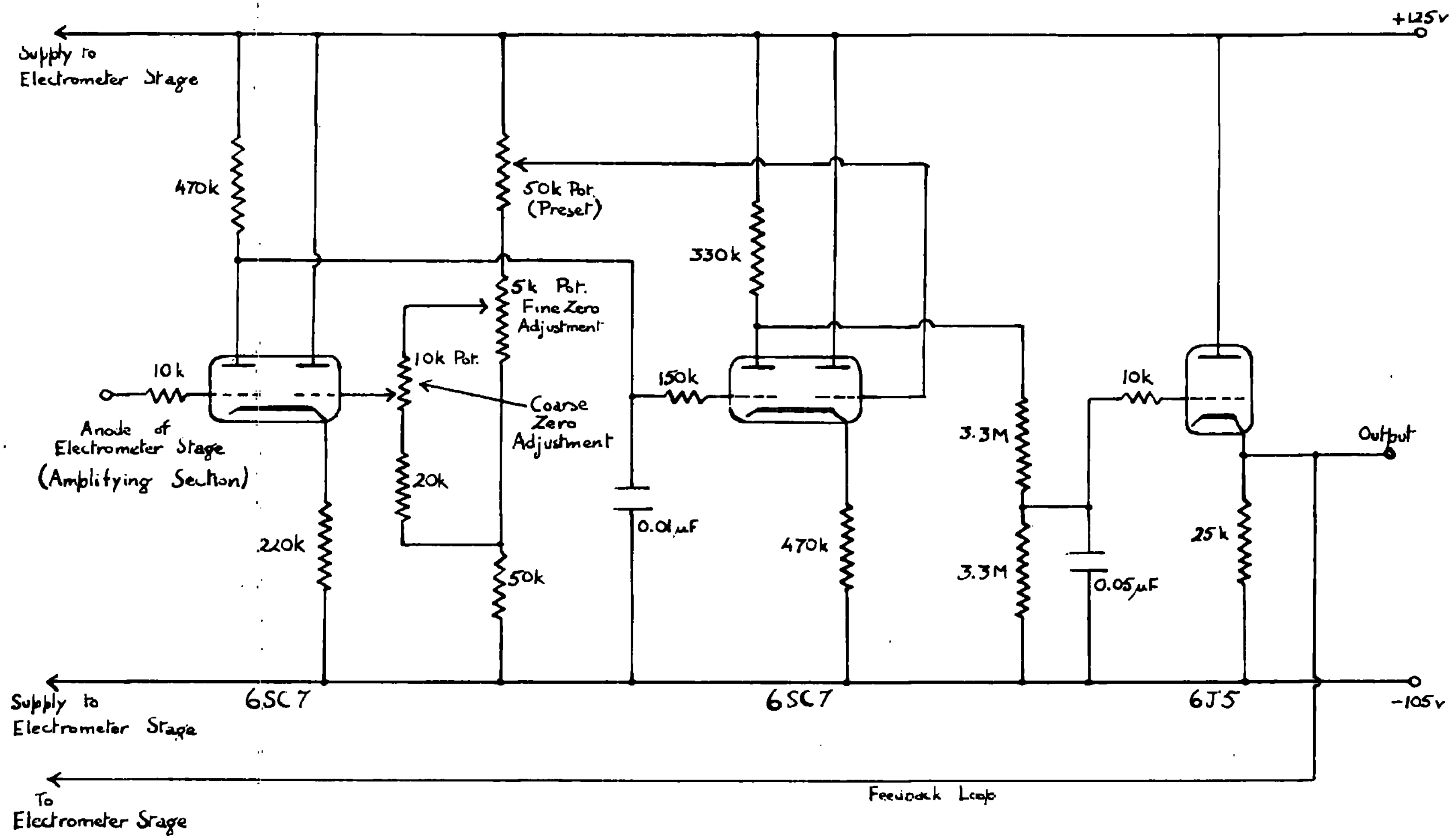


Fig. 3 D.C. Amplifier (Original Form)

to:-

$$E_0 \simeq \frac{-\mu R_a}{2r_a + R_a}(V_1 - V_2) \qquad \text{...... Eq. 4.1}$$

showing that the stage behaves as a differential amplifier. It should be noted that the equation implies that the characteristics of both triode sections are the same. The valves used were the type 6SC7, although these are now obsolescent. The two sections are in the same envelope with a common cathode and the heaters are connected in series. In Kay's amplifier the second grid of each stage is held at earth so that

$$E_0 \simeq \frac{-\mu R_a V_1}{2r_a + R_a}$$

The gain of the stage is thus approximately the same as that of an ordinary triode. The cathode-coupled double triode however, has the advantage over a single triode when direct coupling is involved, in two respects. First, since the anode of one stage is direct-coupled to the grid of the next, it is necessary either to drop the anode voltage across a stabilising valve or a battery, or else to arrange that the cathodes of successive stages shall operate at progressively higher potentials, about 100-150 V. per stage. The only way to do this using simple triodes or pentodes is to have several supply voltages. If a large cathode resistor R_c were used to obtain the correct potential, the feedback across it when a signal was applied would reduce the gain to R_a/R_c. If however a constant current element could be used in the cathode circuit, there would be no feedback. This is, in effect, what is achieved by using the cathode-coupled double triode, one section acting as the constant current device for the other. Any tendency for the current through the cathode resistor to change, owing to the application of a signal to either grid, results in a change in bias voltage of the other section, in such a direction that the current is restored to its original value. Thus any increase of current passing through, say, the amplifying section (this current producing the output voltage across the anode resistor) is compensated by a decrease in the current through the other section, the sum remaining constant.

The other advantage of the cathode-coupled double triode is that the effects of changes in heater current are minimised. A variation in cathode temperature alters the grid to cathode contact potential (Yarwood and Le Croisette, Feb., 1954), but since, owing to this cause, equal increments will be provided to both grids, the effect on the output voltage, as appears from Eq. 4.1, will be zero.

The electrometer stage in its original form was not all that could be desired because suitable valves were not available at that time. Kay selected two R.A. 954 acorn pentodes which had similar characteristics, and operated them at low H.T. and heater currents so that grid current would be reduced, thereby simulating an electrometer valve. However, no matching was attempted for emission changes accompanying heater current variations, and there is an obvious drawback here to the use of separate valves. The heaters were however connected in series, and one had a potentiometer connected across it so that the emissions could be equalized in the first instance. Additional coupling between the valves was achieved by connecting together the space charge grids: normally the control grids in this type of valve. The space charge grid is a common feature of the electrometer valve; it is normally held at a few volts positive with respect to the cathode and prevents the small number of positive ions emitted by the cathode from contributing to the control grid current (normally the screen grid in the 954 pentode). The effect of connecting the idle anode to a separate stabilizing valve (type VR.150) will be discussed in § 4.5. Although the mode of connection was superficially similar to that of the cathode-coupled double triode, the condition that R_c should be large compared with $1/g$ was not fulfilled. Kay's determination for either valve gave the transconductance as 50μA/V., which would mean that a cathode resistor of at least 200 K., instead of 10 K., would be required. Thus the electrometer stage could not be truly differential. However, since a signal appeared at only one of the grids, this did not matter, and at least the earthing of the other would provide, to some extent, compensation against drift. The 954 pentrodes were housed in light-tight boxes and thermally insulated with asbestos. The input resistor to the electrometer valve, and the valve itself, were housed in the same compartment and this was evacuated to reduce ionic currents external to the valve which might affect the grid, although the present author discontinued this practice.

Total negative feedback results from connecting the output from the main amplifier to the other end of the input resistor. It is very easy to show that the behaviour of the complete system is given by the following equation for the output V_A

$$V_A = \frac{-GRi}{1+G}$$

where i is the current through the input resistor R, and G is the loop gain of the amplifier, i.e. the gain without feedback. Since G is large, the gain with feedback is very close to -1. Physically, what happens is that when the current i flows, the grid hardly changes in potential and the change in voltage $-Ri$ appears at the other end of the resistor. The grid potential does in fact change by an amount $Ri/(1+G)$, which is just sufficient to produce the output voltage when it is amplified by G. The fact that the grid potential, and therefore that of the collector, does not vary much from earth is obviously a desirable feature.

4.2 The collector

The disposition of the collector, housed in a pit a few yards outside the recording room, is shown in Plate 1. It is a copper hemispherical bowl about 50 cm. in diameter, surrounded by an earthed guard ring of aluminium, the overturned rim of which, shielding but not touching the edge of the collector, can be seen in the plate.

The guard ring prevents fields from being set up on the outside surface of the collector. The whole assembly is very easily dismantled for cleaning. Arrangements are made for keeping warm the polystyrene insulator supporting the collector, to prevent condensation on it. A complete description of the collector is given by Kay in his thesis. The only improvement that has been made is that when the pit was extended to receive the field mill, a cement floor was laid in both compartments and the walls were lined with bricks, cemented in position. The housing can therefore be kept in a clean condition. Plate 1 shows also the access pit which allows of limited maintenance 'in situ'.

An interesting detail concerning the collector is that the effective areas for conduction (or displacement) and precipitation currents are different. Since the paths of precipitation particles will be affected hardly at all by the field distribution over the hollow surface, the effective area for the current carried on them is simply equal to the area of the aperture, which is 1900 cm^2. The conduction current is carried along the lines of force, however, and since these are obviously concentrated near the rim of the collector, most current will be collected at the apex formed by the lip of the guard ring and the rim of the collector. Exactly how much of this marginal current is collected by the guard ring is impossible to determine, and in any case it has not been found possible to produce a theoretical account of the field distribution over the inner surface of the hemisphere. However, on the basis of experiments in which a steadily increasing potential gradient of known magnitude was applied to the collector (§ 6.3), the exposure factor for conduction currents appears to be almost exactly $\frac{1}{2}$. Hitherto, when measurements have been made only of precipitation currents, this fact has been of no importance, but it will obviously need to be corrected when measurements are made of the total air-earth current. This could easily be achieved by placing a false bottom in the collector almost flush with the rim. This could be filled with a shallow layer of earth, more nearly simulating natural conditions, and if there are any electrical effects due to the splashing of precipitation particles on the ground (Israel and Lahmeyer, 1948; Smith, 1955), these would be more reliably accounted for with the apparatus so modified.

4.3 Ancillary equipment

Kay furnished the amplifier with specially constructed H.T. and heater supplies and a range selecting unit incorporating Schmitt trigger circuits, which automatically changed the shunt across the recording galvanometer if the current received by the collector became too great. Slight modifications to the power supplies have been made by the present author. Originally the reference voltage for the H.T. power supply was set by a VR105 regulator, but after a rather short time, this voltage began to jump at fairly low frequency, having a disastrous effect upon the amplifier's performance. This defect of the VR105 has been reported by Kirkpatrick (1947). Stockill and Chalmers (1958) replaced the regulating valve by an H.T. dry battery of conventional type giving 120 volts, but the method of applying the reference voltage to the power supply necessitated the flow of a few microamperes into the battery in such a direction as to charge it. After a few days of continuous operation the voltage delivered used to increase by as much as 5 to 10 volts. In this connection it is worth noting that the temperature coefficient of voltage of a dry battery may be much greater than for a gas discharge regulator: Harris and Bishop (Sept., 1949) give the temperature coefficient as + 100 mV/°C., for a 120 volt battery as against 5-20 mV/°C for neon regulators. The present author found a most satisfactory solution in the use of a reference voltage valve, type 85A1. The properties of this valve have been reported by Benson (1952), who found it to retain a remarkably constant running voltage, after a short period of initial drift. In addition, the valve requires no shielding from light, and it has a low temperature coefficient (- 2.5 mV/°C). It is believed from measurements described in § 4.5 that with this valve the stabilized supplies of + 125 volts and - 105 volts are held constant to one millivolt over an hour or so.

A clockwork drum camera carrying a strip of photographic recording paper in conjunction with a mirror galvanometer, is used for recording. Stockill added timing circuits actuated by Post Office uniselectors, which are triggered by the half minute pulses from a clock. The uniselectors switch on a fogging lamp or switch off the galvanometer lamps at regular intervals thus superimposing a time scale on the record. The camera can be run at the rate of one revolution, equivalent to approximately 14 inches of record, in 1, 5, or 20 hours, after which the paper must be changed.

4.4 A differential electrometer stage

Referring again to Fig. 2, showing the form of the electrometer stage used by Stockill and Chalmers (1958) for the measurement of precipitation currents, it was hoped that it might be possible to free the grid G_2 and apply to it the compensating voltage without any further alteration in the circuit. An equation will now be derived for the final output from the main amplifier since the way the feedback loop affects the response to a signal on G_2 is not self-evident. It will be assumed that for voltages V_1 and V_2 applied to the grids (see Fig. 4), the output E_o of the electrometer stage is not truly differential but is given by the equation:

$$E_o = -M.V_1 + NV_2 \qquad \text{...... Eq. 4.2}$$

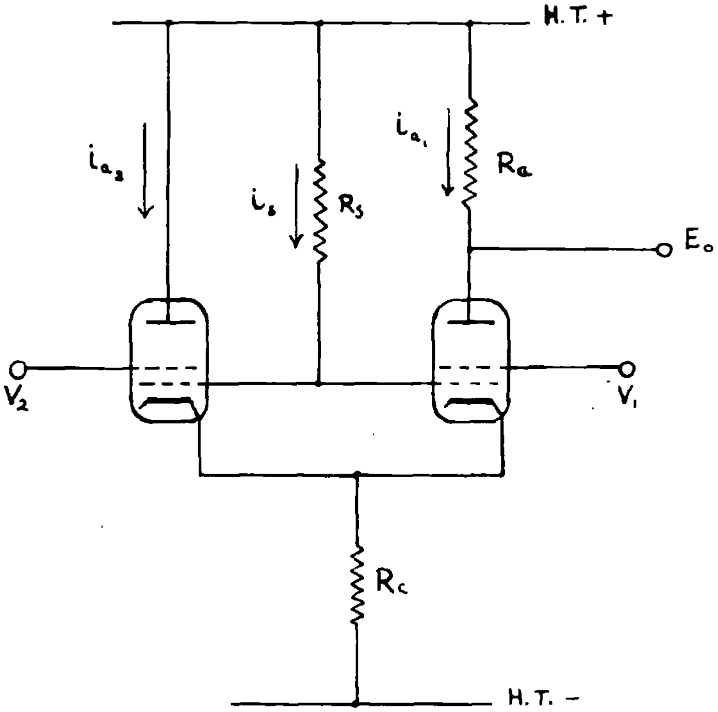
This is a more generalized equation for the behaviour and also gives a little more information when the result is obtained than if $E_o = K(V_1 - V_2)$ had been assumed. If i is the current flowing into the input resistor R, and G' is the gain of the main amplifier only, essentially a positive quantity,

$$V_1 = Ri + E_o G'$$

Therefore, $E_o = -(MRi + ME_oG') + NV_2$

and the output voltage is expressed by:-

$$E_o G' = \frac{-MG'(Ri - N/M \cdot V_2)}{1 + MG'}$$



$$E_0 = MV_1 - NV_2$$

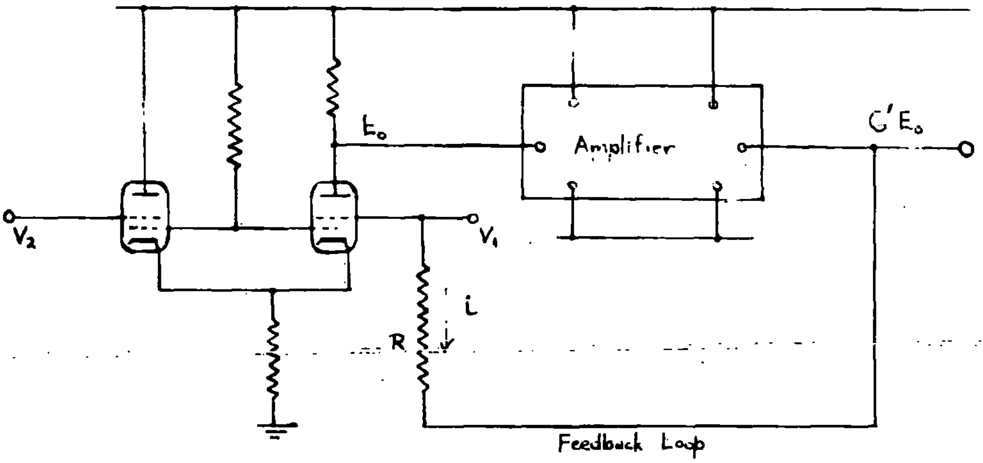


Fig. 4 Illustrating Behaviour of D.C. Amplifier with Unbalanced Electrometer Stage

With the usual feedback condition that $MG' >> 1$

$$E_o G' \simeq -(Ri - V_2 . N/M) \qquad \text{...... Eq. 4.3}$$

It is interesting to note that the ratio of the terms corresponding to the two signals is preserved in the output, allowing for the fact that one input is a current input and the other a voltage input. However, it is evident that while the part of the output corresponding to the current i is made independent of valve characteristics, the output corresponding to V_2 is dependent on the ratio N/M , which is a function of the electrometer stage characteristics. This is also true if N/M is nominally unity, although it would not have been evident if this had been written down implicitly in Eq. 4.2. It is more useful to say that the stability of the electrometer stage determines how closely the output can be maintained truly differential. The present author tested Kay's amplifier by applying known voltages across R and between G_2 and earth. Both terminals of the source supplying the first voltage had to be 'floating', of course, to represent a current input. It was found that the output from the main amplifier corresponding to a voltage across R was approximately ten times as great as that due to the same voltage on G_2. The reason for this was that the cathode resistor was too low. Kay (1950) derives the following expression for the output voltage E_o from the electrometer stage.

$$E_0 \propto \left[(k+q)(V_1-V_2)+\frac{1+3pR_s+4pR_c}{4pR_cR_s}V_1+\frac{V_2}{4R_c}\right] \quad \text{..... Eq.4.4}$$

k is the reciprocal of the anode characteristic resistance and p and q are respectively the transconductances of the space charge and control grids, assumed to be the same for each of the pair of valves. The significance of the other symbols is shown in Fig. 4. It can be seen that with R_c sufficiently great, the second and third terms in the square bracket become negligible compared with the first, which represents a truly differential behaviour. The requirement is that R_c shall be considerably greater than k or q. In fact k, p and q were each approximately 50 K. whereas R_c was only 10 K. Substitution of these numbers into Eq. 4.4 together with 200 K. for R_s shows that $E_0 \propto [9.4 V_1 - V_2]$. Thus the behaviour found in practice is explained. Since it was obvious that the electrometer stage would have to be modified at least for this reason, it was decided to investigate some of the characteristics of cathode-coupled twin stages, particularly with regard to their stability against changes of supply voltage.

4.5 Stability considerations

Although there is substantial literature on the properties of various D.C. amplifying stages, and in particular two series of articles were found to be of value (Yarwood and Le Croisette, 1954;

Harris and Bishop, 1949), no estimate seems ever to have been made of the behaviour of the cathode-coupled double triode in response to changes in line voltage. It is quite easy to obtain an expression for this and it has a bearing on the way the supplies should be brought to the two anodes. In the first place it is evident that to obtain a truly differential output from the electrometer stage the characteristics of each section must be the same. Since only one section has an anode load R_a (Fig. 2), this suggests using a higher supply voltage for this section than for the other in order to keep the quiescent voltages at the anodes the same. An expression has been derived for the output of the electrometer stage when the characteristics are unbalanced:

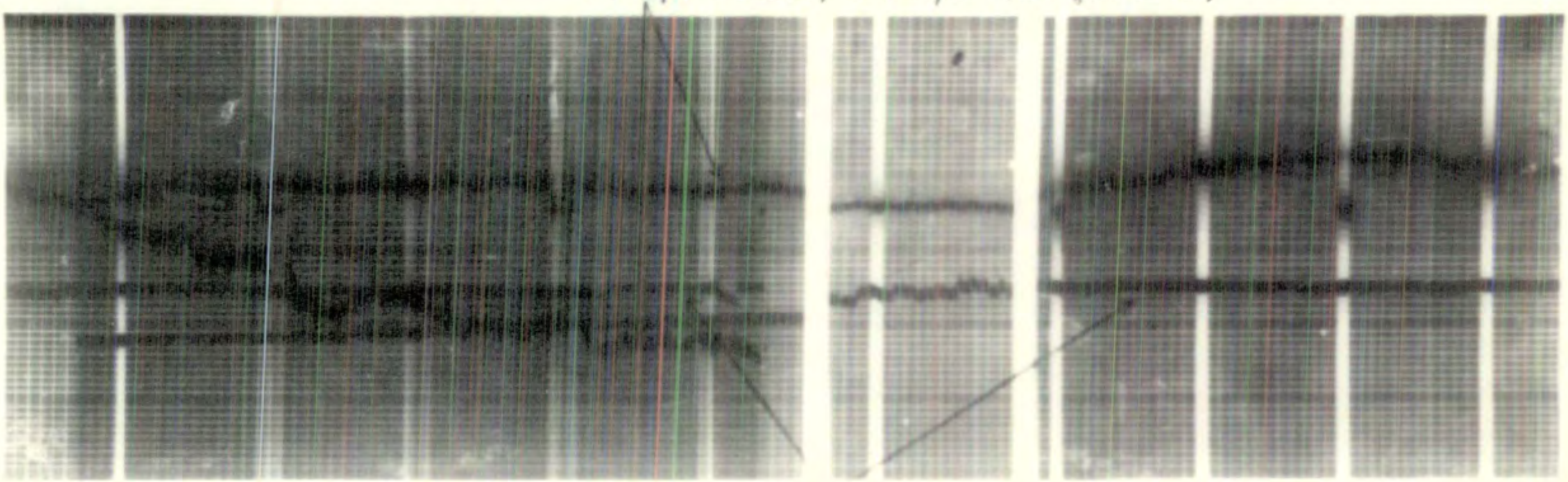
$$E_0 \propto q_1\left[1+\{p_1+p_2\}\{2R_cR_s(k_2+q_2)+R_s+2R_c\}+p_1R_s\right]V_1$$
$$-q_2\left[2R_cR_s\{p_1+p_2\}\{k_1+q_1\}-p_1R_s\right]V_2$$

The symbols have the same meaning as in Eq. 4.4, and the suffices 1 and 2 refer to the amplifying and non-amplifying sections respectively. If there is no difference between the characteristics for either valve then the expression can be shown to be identical with Eq. 4.4. If however there is a difference, and if R_c is made very large, it is found that

$$E_0 \propto \left[q_1(k_2+q_2)V_1-q_2(k_1+q_1)V_2\right]\left[p_1+p_2\right]$$

which is an output of the type discussed in § 4.4. It will be noticed that the p's, the transconductances of the space charge grids, do not appear except in a constant of proportion for signals on either grid, when the usual cathode follower condition for R_c is applied. That grid will therefore not be taken into account in the further discussion: there is no rigorous justification for this omission, but in fact the physics of the problems to be discussed can be understood adequately using the theoretical results as a guide and not to furnish precise answers. Actually the space charge grid circuit passes the same order of current as the anode circuit.

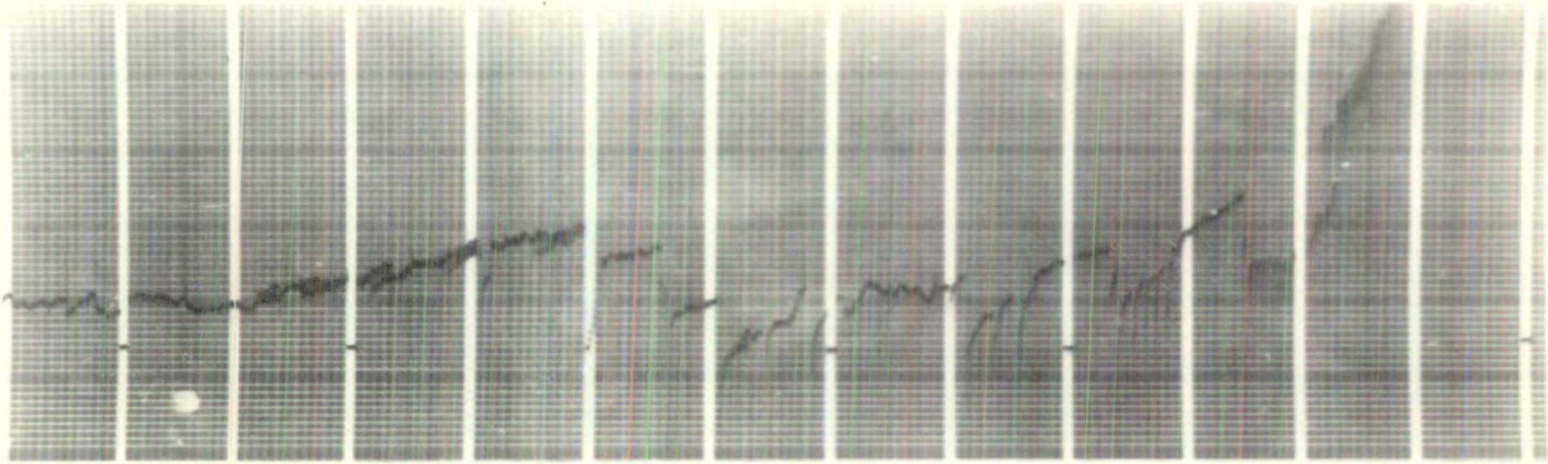
The question of the stability of the electrometer stage was raised initially when 'zero' traces, i.e. records of the output with the collector disconnected from its grid and the other grid earthed were obtained, showing large fluctuations. Two such records are shown in Fig. 5. This diagram, and others which are similar, were prepared by mounting the original records on card and photographing them. The details in some are not very clear because of difficulties in obtaining the right contrast in the reproduction. The time markings are easily seen: the broad vertical white lines on Record 25 are separated by one hour, and between them, much fainter, are the five minute interval marks. This is one of the most erratic of the 'zero' traces. In using the figure shown on the record for the sensitivity: 1.9 mm/mV., it should be remembered that the original record has twice the linear dimensions of this reproduction. The separation between the darker lines parallel with the length of the grace is 2 cm. on the original. These graduations were obtained by means of a graticule on the cylindrical lens of the camera. To gauge the importance of the fluctuations, 2 cm. on the original record corresponds to the normal fine-weather



Zero Output : Sensitivity 0.45 mm/mV (on Original Record)

H. T. Variations : Sensitivity 4.9 mm/100 mV.
(on Original Record)

Record 21



Sensitivity 1.9 mm/mV (on Original Record)

Record 25

Fig. 5 Zero Drift of D.C. Amplifier (Two Separate Pentodes in Electrometer Stage) value of the air-earth current. Apart from the slow drift observed, Record 25 is interesting in that it shows in the last three hours rapid fluctuations of large magnitude accompanied by a considerable change in the general level of the output. The jumps were observed to correspond with sudden changes in the glow of the VR105 regulator stabilizing the H.T. supply (§ 4.3). Other transients on the same record were probably strays picked up from the relays in the timing circuit.

To discover whether the slower variations were caused by changes in the supply, the line voltage and a suitably adjusted steady voltage from a 120 volt dry battery were applied to the terminals of a galvanometer, and the variations recorded at high sensitivity together with the zero trace. This is exemplified by Record 21, Fig.5. It was felt that the battery might not provide a sufficiently constant backing-off voltage for this purpose, having regard to its high temperature coefficient, so an attempt was made to compare, in a similar way, a fraction of the supply voltage with a standard cell. This met with failure on account of the temperature coefficient of the resistors used. Finally it was decided to calculate what ought to be the effect on the amplifier of variations in the supply.

Returning to Fig. 2, inset, the voltage for the anode A_2 of the non-amplifying section is supposedly tapped off a resistor chain at a fraction m of the supply voltage to bring the anodes to the same potential. There is imagined to be a change V_a in the supply voltage; hence a change mV_a in the voltage applied to A_2 , and a corresponding output voltage E_o from the electrometer stage. If the grids are held at fixed potentials and r_a and g are the anode resistance and control grid transconductance, the equation for the current in the amplifying section is:

$$I_1 = \frac{E_o - R_c(I_1 + I_2)}{r_a} - gR_c(I_1 + I_2)$$

and for the other section:

$$I_2 = \frac{mV_a - R_c(I_1 + I_2)}{r_a} - gR_c(I_1 + I_2)$$

Substituting $E_o = V_a - R_a I_1$ into the first of these and eliminating I_2 between them gives an expression for I_1 and hence for E_o in terms of V_a :

$$E_o = \frac{V_a\left[r_a + R_c(1/r_a + g)(2r_a + mR_a)\right]}{r_a + R_a + R_c(1/r_a + g)(2r_a + R_a)} \qquad \text{..... Eq.4.5}$$

If the cathode follower condition $R_c g >> 1$ holds, this approximates to

$$E_o \simeq \frac{V_a(2r_a + mR_a)}{2r_a + R_a} \qquad \text{..... Eq.4.6}$$

This shows that when A_2 is connected directly to the supply, the full change in supply voltage appears at the output of the electrometer stage, but that some reduction is effected when $m < 1$. In the original form of Kay's amplifier, A_2 was supplied from a VR150 regulator across the stabilised supply. Since the supply to this valve is itself stabilised, it would normally be reasonable to expect that variations of voltage across the regulator would be entirely negligible, as when such valves are used in cascade. This would be equivalent to $m = 0$, and allowing also for the fact that $R_c g \gg 1$ in the actual arrangement, by substituting the values directly into Eq. 4.5, E_o would be about 0.3% of V_a . However it has been shown that the VR105 regulator can develop instabilities even when the applied voltage is constant (Kirkpatrick, 1947), and although the VR150 is undoubtedly better in this respect, it is of similar construction to the VR105, and it might be more realistic to give m a value between 0 and 1. If $m = 1$, the variations in E_o would be 30% of those in the line.

However, examination of Record 21, Fig. 5, shows that anode supply variations cannot be the only cause of zero drift. At the left hand end of the record relatively small variations in the H.T. voltage were accompanied by a large excursion of the 'zero', while in the central section, fluctuations in H.T. produced no sensible zero drift. (The apparent discontinuity of the traces in the plate is owing to the fact that the whole of the record is not presented, but only the interesting parts of it). Attention was therefore turned to the effect of variation of heater current. Harris and Bishop (Sept., 1949) illustrate the order of stability that can be achieved in this direction as that of a typical balanced amplifier in which 0.2% change in heater voltage produced an output equivalent to 1 mV. at the grid. An experiment on Kay's amplifier showed that the same relative change in heater voltage was equivalent to 16 mV.

at the grid. Obviously the stability against heater voltage was seriously deficient and it was decided that the existing electrometer stage must be replaced by a valve with a common heater and cathode for both sections.

Some revision of the circuit was evidently necessary, and as a first step in deciding what alternative would be most effective, calculations were made of the amount of H.T. variation that could be tolerated. The smallest air-earth current required to be measurable was arbitrarily fixed at 10% of 2 x 10^{-12} A/m.^{2}, the fine-weather value; this corresponds to an output of 1 mV. The result of a change V_a in line voltage affecting only the electrometer will first be considered. Referring to Fig. 6, the change E_0 in the anode voltage of the electrometer stage in the absence of feedback (i.e. with the switch S connected to earth) is supposed to be aV_a. On account of the feedback, however, a voltage e will be impressed on the grid; this will of course also be the output voltage. If K_1 is the amplification of the electrometer stage, K_2 that of the rest of the amplifier, the value of E_0 when S is connected to the feedback loop is simply $E_0 = aV_a - K_1 e$. Since $e = K_2 E_0$

$$e = \frac{a K_2 V_a}{1 + K_1 K_2}$$

It has already been shown that $a \simeq 1/3$ and it can also be shown that $K_1 \simeq 1$. Since $K_2 >> 1$, $e \simeq 1/3\, V_a$. The condition on e is that it shall be less than 1 mV. and therefore the supply voltage must be constant to within 3 mV.

Because the gain of the electrometer stage is approximately

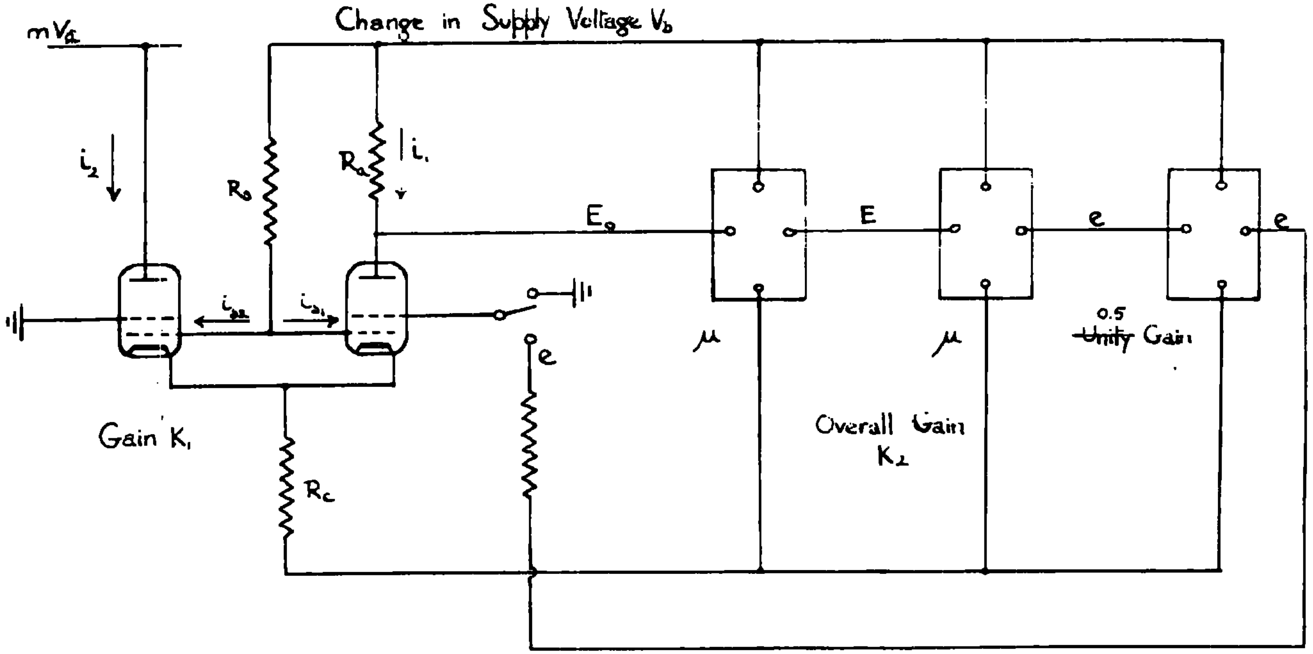


Fig. 6 Circuit Diagram for Use in Calculating Effect of Changes in Supply Voltage

unity, it was also important to consider the effect of line voltage changes on the next stage, i.e. the first stage of the main amplifier. Supposing now the electrometer stage to be perfectly compensated for supply variations, and referring again to Fig. 6 let the zero drift in the final output be e. This voltage is applied to the grid of the electrometer stage so the input at the next stage is $-K_1 e$. If this stage and the next each have a gain μ, the output from the first of them is $E_1 = V_a + \mu K_1 e$, there being no compensation for H.T. variations within this stage, because the non-amplifying anode is connected directly to the supply. The output of the next stage is divided by approximately 2, because of a resistor chain to which the grid of the cathode follower is tapped, so:

$$e = -\tfrac{1}{2}\mu\left(V_a + \mu K_1 e\right)$$

Therefore, $e = -\frac{1}{2} \cdot \frac{\mu V_a}{1 + \mu^2 K_1/2} \simeq -\frac{V_a}{\mu K_1}$

With a limit of 1 mV. for e and $\mu = 25$, V_a is limited to 25 mV. This takes no account of the fact that the fixed grid of the stage in question is tapped off a resistance across the supply. The change in potential of this grid will produce a zero drift in the same sense as the supply fluctuation causing it. The grid voltage will undergo approximately half the change in line voltage, but since it is 25 times as effective as the anode, the line voltage would have to be stable to 2 mV. unless the grid were kept at a constant level by other means.

The stability of the supply was eventually tested by opening the feedback loop, disconnecting the electrometer stage and strapping together the grids of the first stage of the main amplifier so that they would not be affected by the line variations. A record of the zero was taken which showed no variation greater than 250 mV. Since the full gain of 320 was employed, the supply voltage variations must not have been greater than 0.8 mV. This showed that the H.T. supply, which had by then had its VR105 regulator supplanted by an 85A1 (§ 4.3), was functioning perfectly satisfactorily.

4.6 Modifications to the D.C. amplifier

The circuit finally adopted for the electrometer stage is shown in Fig. 7, in which the valve employed is the Ferranti DBM4A twin electrometer tetrode. One of a pair of such valves obtained was chosen as having the most similar dynamic characteristics for each section, and the other curves, anode current against anode voltage, and cathode current against space charge grid voltage, were also plotted so that accurate values of the necessary resistors could be calculated. It will be observed that the connection is no longer that of the cathode-coupled double triode, but that the stage is completely balanced and the output two-sided. It is easy to show that the output is differential and that the gain is expressed by the equation:

$$E_0 = \frac{-\mu R_a (V_1 - V_2)}{R_a + r_a} \qquad \text{..... Eq. 4.7}$$

In theory at least, changes in supply voltage have no effect on E_0

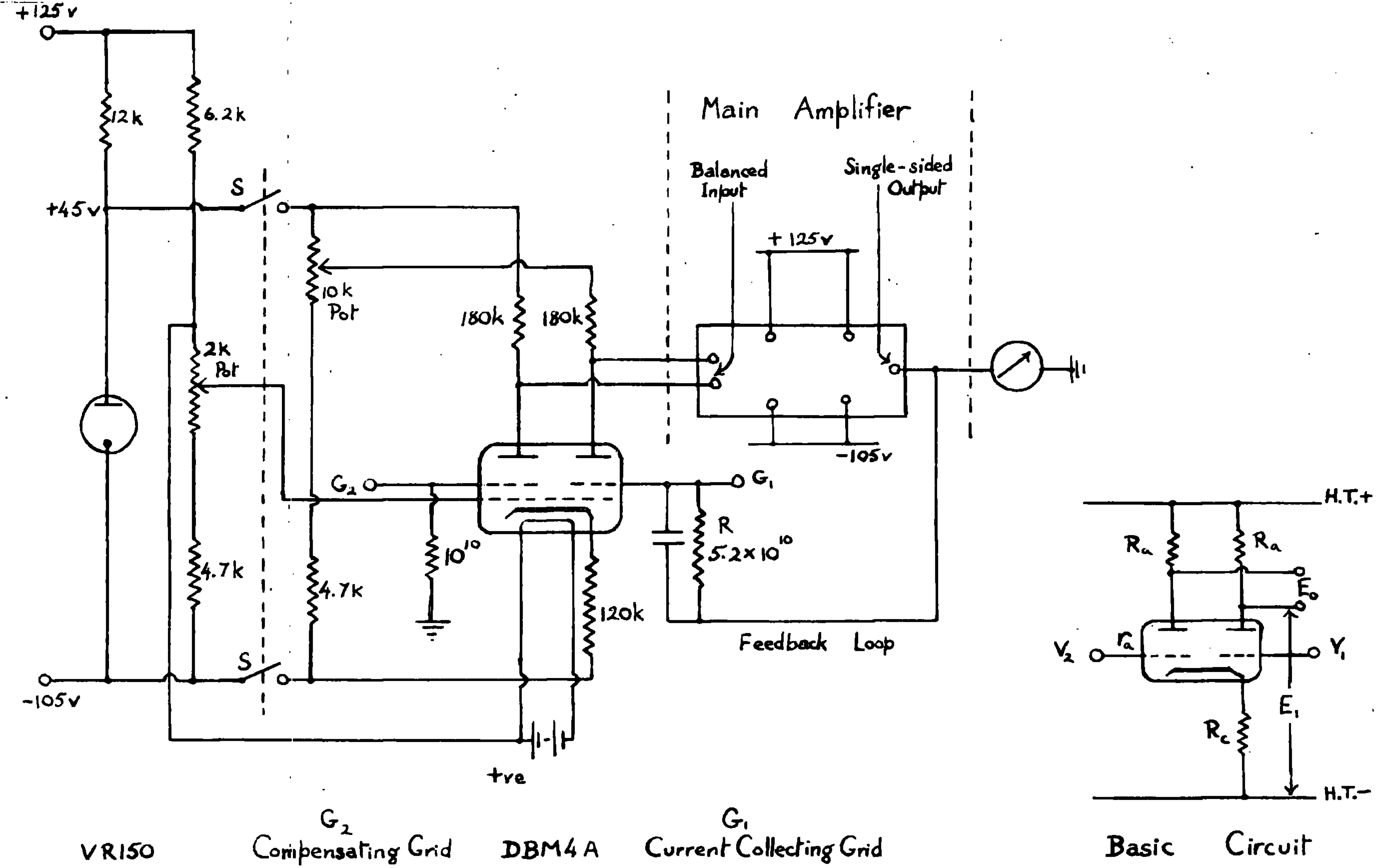


Fig. 7 Electrometer Stage (Modified Form)

and there is also stability against variations in emission, the more so as both sections have a common cathode and heater. As an extra precaution both anodes are supplied from an extra stage of stabilising: a VR150 across the supply, and one anode has a variable voltage which besides enabling final attainment of balance, affords a measure of zero output adjustment. The space charge grid voltage is also adjustable. In practice this was first adjusted to its correct value and then it was found that all the other voltages were within 0.25 V. of the values chosen on the curves as constituting the most favourable operating point.

It should be observed that no approximation, apart from the assumption of linear characteristics, is involved in Eq. 4.7, which is quite independent of the cathode resistor R_c. However, the value of this is important for another reason (Yarwood and Le Croisette, Feb., 1954). When equal signals V are applied to the two grids, $E_0 = 0$, but each anode falls equally by a voltage E_1 (Fig. 7), where

$$E_1 = \frac{-\mu R_a V}{r_a + R_a + 2R_c(\mu+1)}$$

The disadvantage of having a large E_1 is seen in considering the subsequent stage, assumed to be a similar one; a signal E_1 now appears at each grid. Both anode voltages fall and the cathode voltage rises, perhaps to such an extent that the stage is driven off the linear part of its anode characteristic. For out-of-phase signals, $+V$, $-V$

$$E_1 = \pm \frac{\mu R_a V}{2(r_a + R_a)}$$

Although a large E_1 may still throw the two halves of the next stage off the linear part of the characteristic, this is less likely, in that only the anode voltages of that stage will vary, but not the cathode voltage, since the signals on the grids of this stage are out of phase.

The ratio: $$\frac{E_1 \text{ for in-phase signals}}{E_1 \text{ for out-of-phase signals}} = \frac{2(r_a + R_a)}{r_a + R_a + 2R_c(\mu+1)}$$

is called the gain ratio and it should be kept as small as possible. This is obviously to be achieved by increasing R_c. No increase on Kay's value of 10 K. would have been possible if the connection of this resistor to earth had been maintained, so it was decided to connect it to the - 105 V. supply, enabling 120 K. to be employed.

The other important modification to the electrometer stage was that the connection of the heater to tapping points on the power supply was dispensed with, mainly on the ground that the latter would not deliver the required current without moving off the plateau of its regulating curve, and two lead-acid accumulators, connected in series, were used instead. It is not generally known that the voltage from lead-acid accumulators remains extraordinarily constant; the temperature coefficient of the open circuit voltage is stated to be + 0.24 mV/°C., (Standard Handbook for Electrical Engineers, 1941). The current through the filament is monitored by a milliammeter, and as soon as the current falls by more than $2\mu A$ below its correct value of 250μA the accumulators are changed. A dual set of terminals connected to the filament facilitates this without having to switch off completely. It is found that the accumulators have to be changed every four to six days. When this is done an opportunity is presented of checking the stability of the stage against heater current variations. It was found that a zero drift equivalent to 1 mV. across the input resistor was produced by 1% variation in heater current. This compares favourably with the figures referred to in § 4.5, quoted by Harris and Bishop.

It can be seen from Fig. 7 that there is a large voltage drop across the cathode resistor when the valve is hot. If the H.T. and heaters were switched on simultaneously, the full 150 volts given by the VR150 regulator would appear across the valve until it warmed up, thus shortening its life. The switch S was therefore incorporated, which connects both positive and negative supplies after time for warming up has been allowed.

It will be apparent, later (Eq. 6.1), that effective compensation for displacement current depends on the constancy of the ratio of the resistors connected to the two grids. At the time it was considered that long term stability would be of paramount importance, although, the equipment now complete, it is the practice to test the compensation from day to day. Victoreen resistors were considered to be best in this respect, a 'shelf-life" stability of 3% being quoted. They are housed with the electrometer valve in a metal cylindrical case. The valve is mounted horizontally and the cylinder is closed by a thick rubber disc which holds the valve base and helps to reduce microphonics. The grids are led to top-cap connections and leads from them are taken out of the cylinder through polystyrene insulators which are held in the rubber. The insulators were carefully cleaned and polished, and were then tested by measuring the rate of loss of charge from an air condenser connected to each lead in turn and charged. In this way it was found that the insulation of the grid connections was better than 8 x 10^{-13} ohm.

Since the output of the electrometer stage was no longer single-sided, it was necessary to make minor modifications to the main amplifier (Fig. 8). The grid of the first stage which formerly was held at a fixed potential by a resistor chain across the supply has been disconnected and both grids are now connected to the anodes of the electrometer stage. This has meant transferring the coarse and fine zero adjustment controls to the next stage; otherwise nothing has been altered. The complete amplifier is similar to one described by Peirson (1950). In his amplifier, however, one of the grids of the electrometer stage was earthed, although this stage had a load at each anode so that the connection to the first stage of the main amplifier was push-pull.

Plate 2 shows a general view of the recording room. In the lower right hand corner part of the chassis of the main amplifier can be seen. The cylindrical box immediately behind this contains the electrometer valve and grid resistors, and the leads from these pass into the rectangular compartment beyond, which contains all the circuit

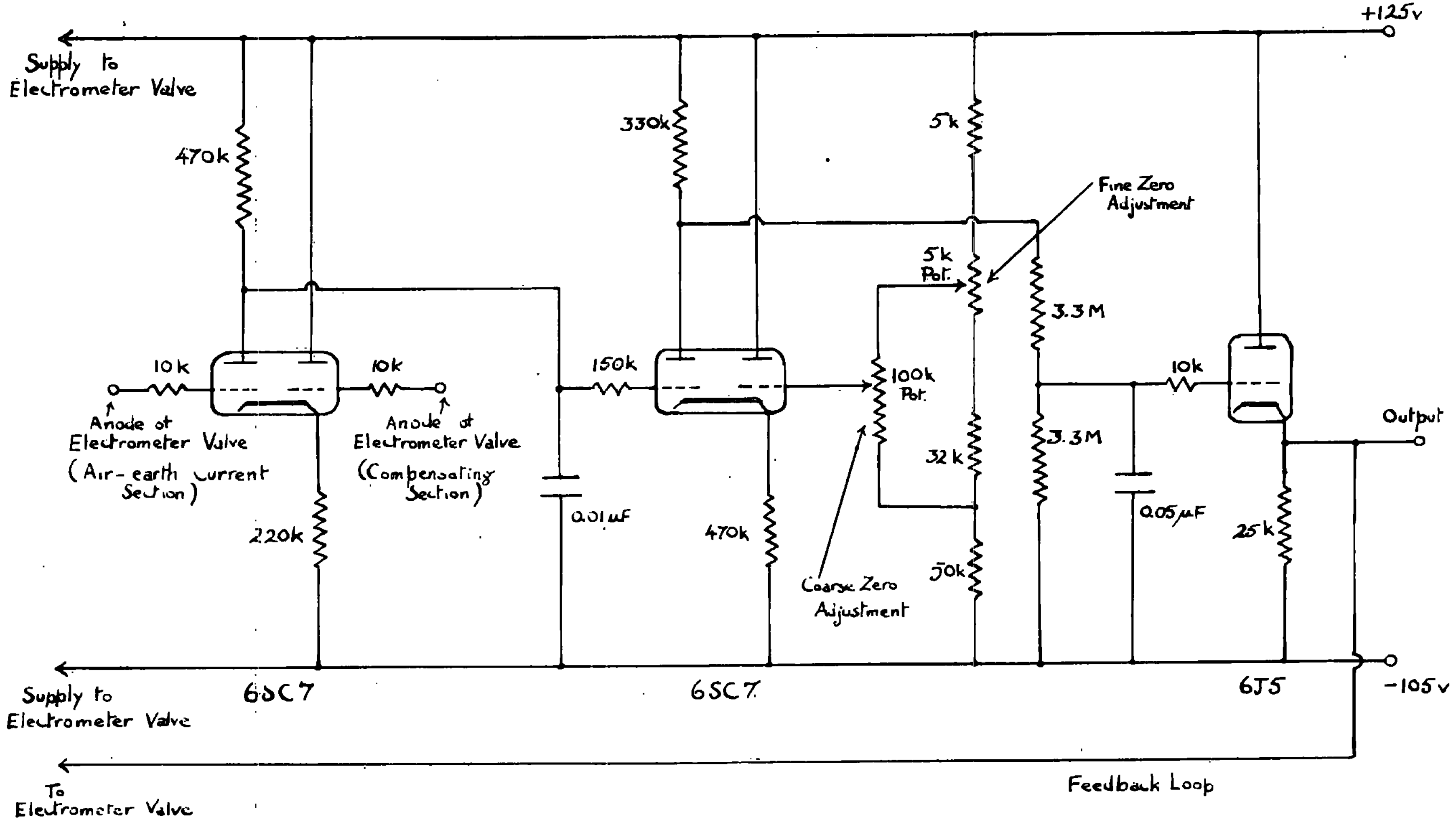


Fig. 8. D.C. Amplifier (Modified Form)

components for the electrometer stage. Next to this is the housing where the cable from the collector is connected to a highly insulated switch by means of which the collector can be earthed or connected to the amplifier at will. The same container also houses the variable air condenser which shunts the air-earth current input resistor: the purpose of this condenser is described in § 6.2. The box nearest to the wall in the background of Plate 2 contains the galvanometer shunts which are switched over manually. On the galvanometer pillar can be seen the meter indicating the electrometer heater current, but the accumulators supplying this are below the level of the shelf supporting the apparatus. The units on the extension to the shelf at the left of the plate are to do with the field mill and test equipment and will be described in due course. Normally, all the boxes are fitted with lids which complete the electrostatic shielding.

4.7 The performance of the amplifier

The modified amplifier has fulfilled all the requirements desired of it; in particular its long term stability and its differential response to signals applied to both grids, are very satisfactory. In Fig. 19, Record 42 shows the zero output when an earthed plate was placed immediately over the collector. The stability was very good except for occasional small excursions which were equivalent in magnitude to a typical fine-weather current. It is very likely that the spurious signals were due to wind causing slight movement of the collector, since when this was disconnected from the grid, the drift in zero output was entirely negligible.

The response to voltages applied to the two grids was tested in the manner indicated in § 4.4, using 9 volt dry batteries and decade potentiometers. First, the output from the amplifier corresponding to a given voltage V' across the air-earth current input resistor was measured; then the voltage it was necessary to apply to the compensating grid in order to restore zero output. The results are plotted in Fig. 9. Strictly speaking the output for a positive signal V' is negative but since in actual use the output is always designated positive or negative according to whether the signal is positive or negative, this convention has been adhered to in the diagram. Not much significance is to be attached to the amplifier having an apparent gain greater than unity. The output was not measured with a voltmeter, but in terms of the galvanometer scale, and it is possible that the calibration of the galvanometer was not very accurate at that time.

It can be seen that at points A and B the amplifier saturates. Why it should do this at such low voltages is not clear; but the amplifier is closely linear between these points. Calculation from the other curve in Fig. 9 shows that the ratio of the compensating voltage V_2 to the input V', necessary to maintain zero output, is accurately constant up to a point C, and has the value

$$V_2/V' = 1.1$$

At point C the relation ceases to hold and no compensation can be obtained. The reason for this is not difficult to find. When a voltage V' is applied across the air-earth current resistor, the grid

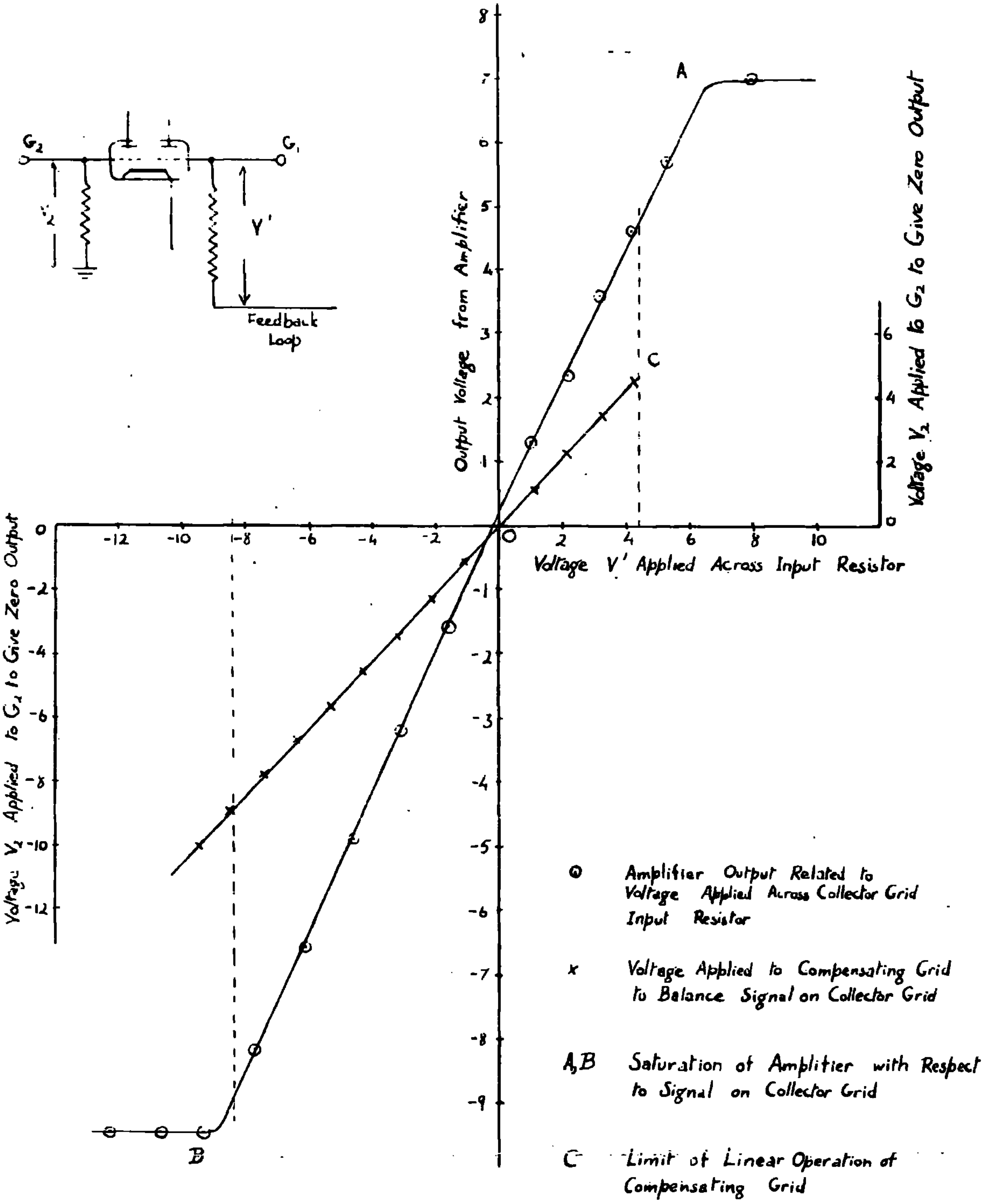


Fig. 9 Calibration of D.C. Amplifier with Respect to Voltages on Either Grid

changes by only a factor of approximately $1/G$ of the signal voltage (§ 4.1), most of the input appearing as a voltage change in the feedback loop. Thus this grid will not move off its proper operating point. When the voltage V_2 is applied to the other grid however, this grid actually rises by this amount and the feedback is negative only in so far as it raises the first grid to almost the same level. The output, proportional to the difference between the grid voltages, still corresponds to a gain of unity, but the operating points of both sections of the valve have moved considerably nevertheless. The effect is enhanced by the small anode-to-cathode potential and because the anode resistors (182 K.) are not small compared with the cathode resistor (120 K.). When either grid potential changes, the tendency is for the cathode to follow the movement by the same amount. The anode resistors, however, are large too, and the anode potentials change in the opposite sense by a fraction $182/(2 \times 120)$ of the signal, assuming the space charge grid current does not change. Thus the total change in the anode-to-cathode potential is approximately $1\frac{3}{4}$ times the signal applied to the compensating grid. For a quiescent operating anode-to-cathode potential of 7.5 V., a 4 V. signal is the most that could be tolerated. In practice the failure of this grid to function does not occur until the signal is greater than 4 V.: this suggests that the calculation ought to have taken into account the space charge grid. For negative signals saturation of the amplifier at B begins before compensation fails. The + 4 V. limit at C means that there would be no compensation for potential gradient changes greater than + 100 V/m. per sec., but it was anticipated that only rarely would variations of this magnitude occur.

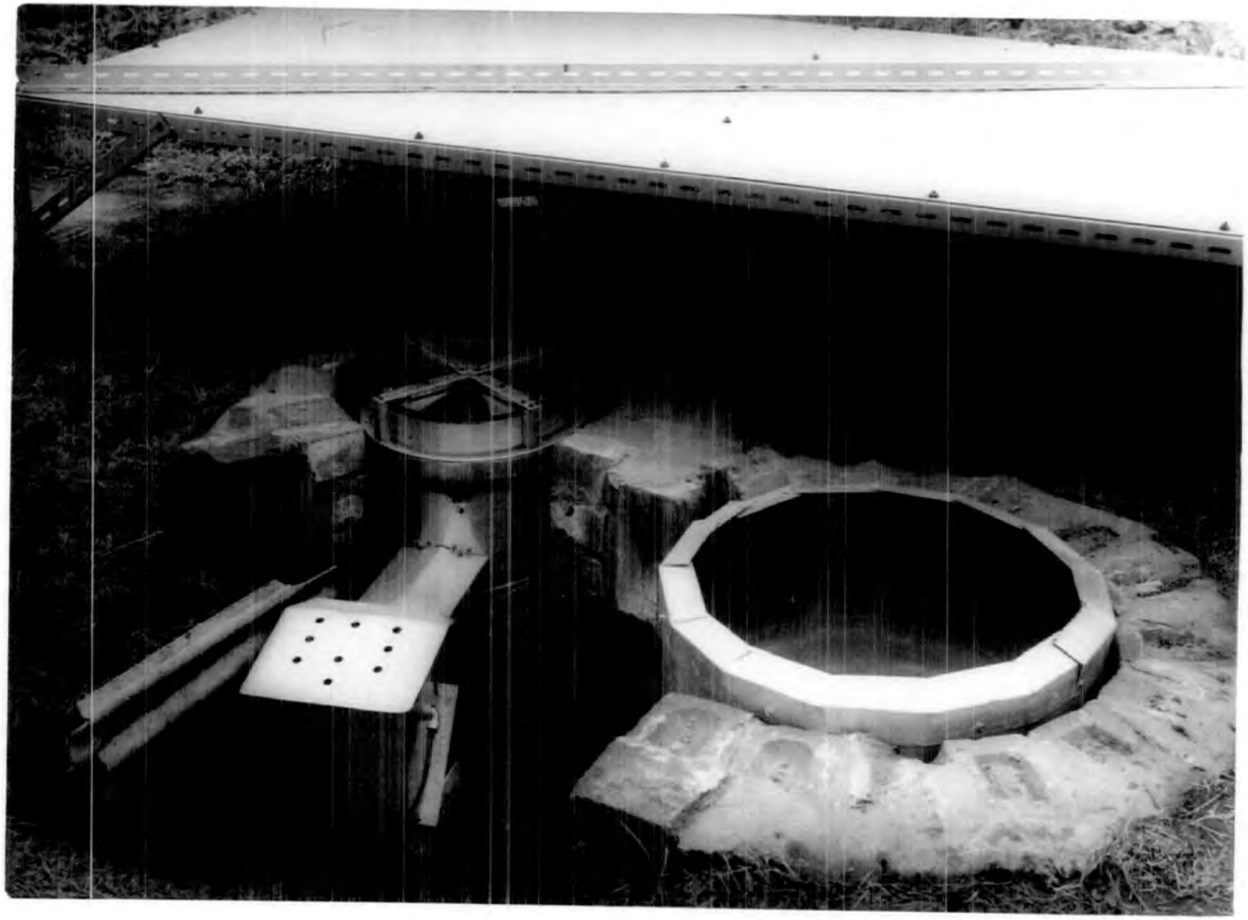


Plate 1. General View of the Field Mill and the Air-earth Current Collector

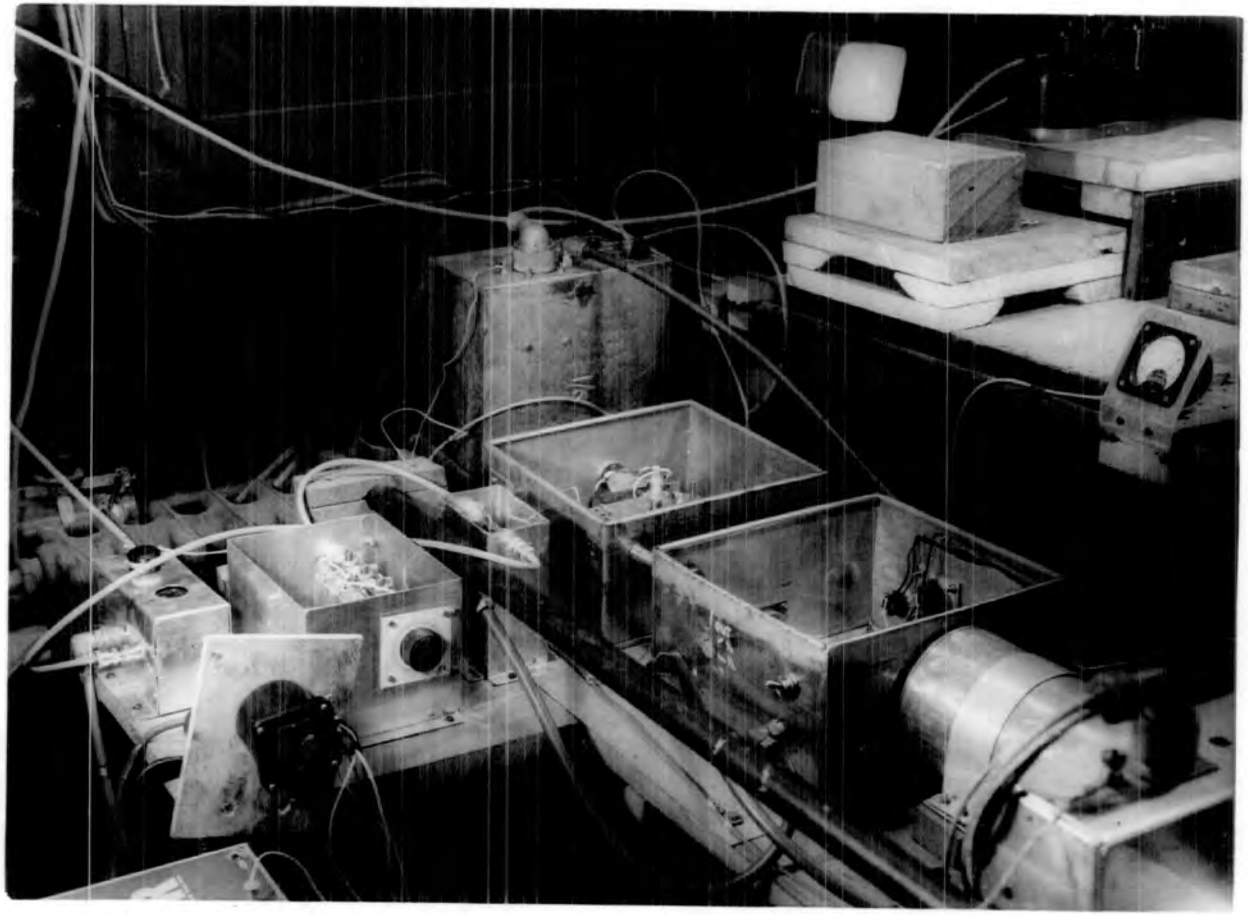


Plate 2. General View of the Recording Room

CHAPTER 5. THE FIELD MILL

5.1 The choice of a compensating apparatus

The choice of a device for giving an output proportional to the atmospheric potential gradient, which rests on the criterion of a reasonably rapid response, i.e. having a time constant less than 10 sec., eliminates all recorders except those of the 'agrimeter' or 'field mill' type. In deciding between these two alternatives, it was borne in mind that the agrimeter designed by Chalmers and used in the Research Department at Durham had been employed without success in a pilot experiment to compensate for displacement currents (Chalmers, 1953; Kay, 1950). The reason quoted for the failure was that the output was not sufficiently steady, possibly owing to a defect in the commutating system. Although at the beginning of the present work this was the main factor influencing the choice in favour of the field mill, it seems probable, in the light of what is to be discussed in Chapter 7, that fluctuating gain in the agrimeter was only partly responsible for Chalmers' experiment being ineffectual, and the failure may have been caused by the capacity to earth of the air-earth current collector.

The behaviour and theory of the agrimeter are fully explained by Chalmers (1953), although a machine working in the same way was first constructed by Russeltvedt (1925). In both agrimeter and field mill the fundamental principle is the same, - the screening and exposing of a collecting electrode so that the bound charge induced by the potential gradient is modulated. It is in the method of measuring the bound charge that the two types differ. In the agrimeter, the collecting electrode, while exposed, is earthed and then insulated, so that a charge proportional to the potential gradient is induced on it, and then, in the screened position, the charge is passed through a galvanometer. The complete cycle takes place very quickly, the drive being from an electric motor, and the various contacts to earth and to the meter are made via a commutator or cam, and collecting brushes. It is customary to employ multiple electrodes, thereby increasing the repetition frequency, and they may take the form of horizontal vanes on a vertical shaft, or conducting strips mounted axially on a horizontal cylinder.

The agrimeter has two drawbacks, however. First, there is the question as to whether the intermittent mechanical contact which causes the charge to pass from the vanes to the measuring equipment would be reliable enough for the present purpose. A few workers, dealing with the field mill type of apparatus, have preferred to use an electronic phase-sensitive detector instead of a commutator, but it is questionable whether such a method could be used successfully to interrupt the very small currents coming from the vanes of an agrimeter. Secondly, if it should be necessary to increase the output from the agrimeter, there would be the difficulty that D.C. amplification would have to be used. In these respects the field mill has the advantage over the agrimeter. The modulation of the bound charge on the collecting electrode is made to give rise to an alternating voltage across a suitable impedance.

The problem still arises of rectifying this with a phase-sensitive device, such as a commutator, if the sign of the potential gradient is required, but if necessary the output can be increased before rectification by means of a comparatively simple A.C. amplifier, to obviate the need for delicate operations with minute currents.

A unique feature of the equipment about to be described is the incorporation of negative feedback over the whole system, instead of its conventional application only to the amplifying stages.

5.2 General design considerations

Recordings of the precipitation current with Kay's shielded collector used to be made continuously over periods of about 20 hours together. Several such recordings have often been taken one after the other when weather conditions have remained sufficiently disturbed to promise interesting results: so that it seemed desirable for the mill to be capable of running for periods of several days without any attention. On this account the machine has been made much more robust than is necessitated by its other functional requirements. The motor driving the vane is a 1/6 h.p. mains operated induction motor, running at 3,000 r.p.m., nominally. Originally the motor was housed inside the mill casing, the vane and commutator being fixed directly to the shaft, which projects at both ends of the motor. For reasons which will be discussed in § 5.7, the motor later had to be housed outside the mill, and the drive transmitted by a small V-belt.

Protection from the weather was another important consideration. The materials directly exposed to the elements are mainly aluminium and brass, which do not corrode seriously, and the vanes themselves are of stainless steel, though there is a more important reason for the use of this material (§5.3 (c)). Unfortunately, to obtain the necessary strength, some of the interior components of the mill were made of steel, but these have been heavily painted.

The insulators supporting the vanes are made of polystyrene. This material has water-repelling properties and so maintains its excellent insulation even in a damp atmosphere. Waddel (1948) used it in the construction of a field mill for use on aircraft, and reports that the insulation never failed during flights, even when passing through clouds. Attention was paid mainly to protecting the insulators from direct rain. Referring to Fig. 10, it can be seen that the insulators supporting the collecting electrode are almost completely screened, even from driving rain, both by the electrode itself, and also by the aluminium guard surrounding the entire plate system. The insulators holding the upper vane are not so critical and behave satisfactorily as long as there is not a continuous film of water connecting that vane with its supporting cross-piece. The shields placed over these insulators (seen best in Plate 1) appear to be sufficient to prevent this happening, even in a heavy downpour. To assist in shedding the water collected during a rainy spell, the platform supporting the electrode system is gently sloped down from the centre. It was turned from 'Weyroc' to keep the weight small, but its upper surface is lined with tin plate which is in turn coated with aluminium paint. The 'Plessey' connectors used at the cable terminations are waterproof.

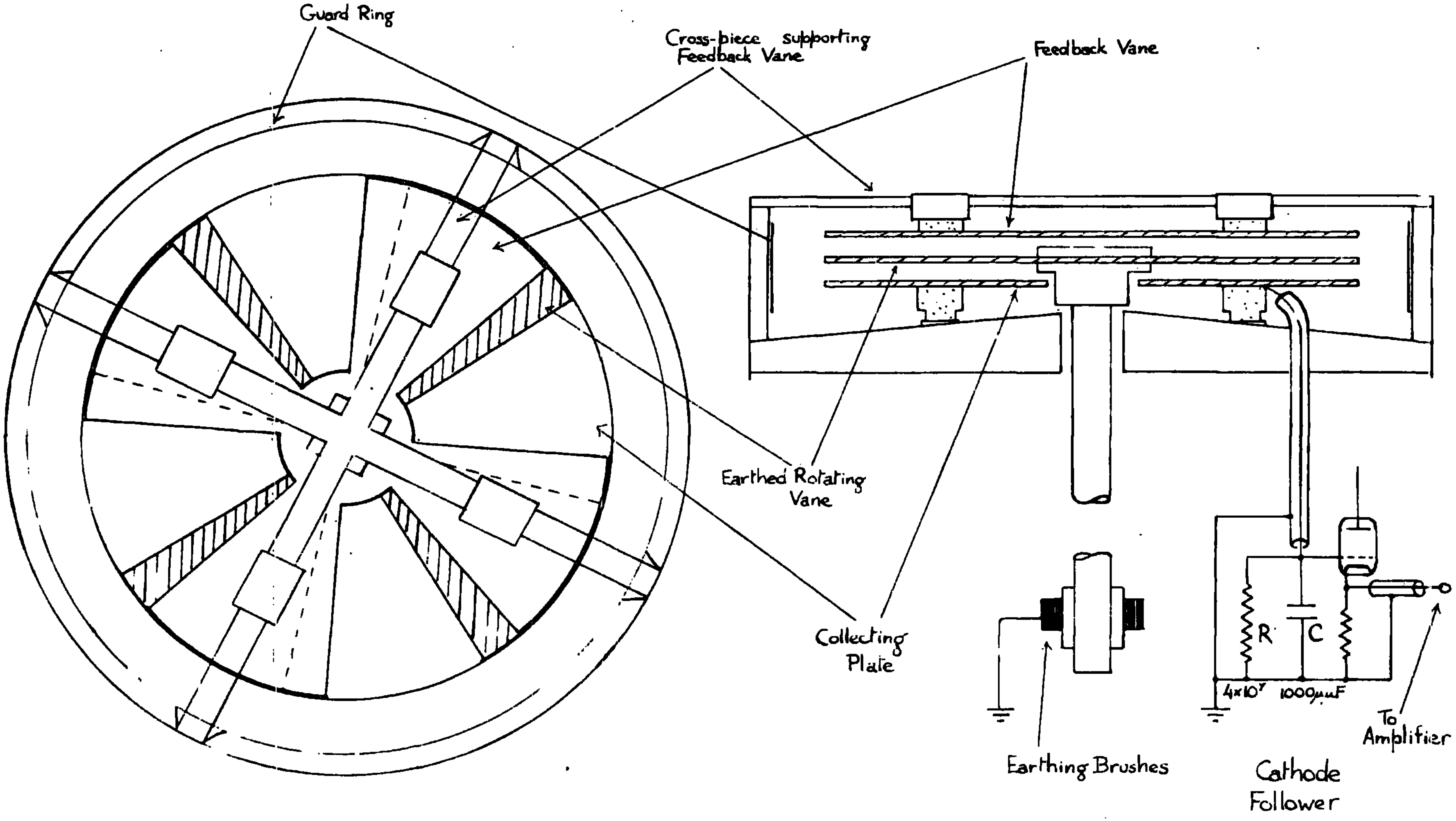


Fig. 10 Plate System of the Field Mill

Apart from the desirability of having a very stable gain, particularly over short periods, the magnitude of this gain has also to be selected with reference to maintaining other desirable characteristics in the complete system. It was shown in § 3.3 that the time constant of the D.C. amplifier must be made equal to the product γ of the differentiating circuit parameters. In Eq. 6.1 this product is shown to be

$$\gamma = \varepsilon_0 A R \frac{F}{V_m}$$

where A is the area of the air-earth current collector, R is the input resistor at the current measuring side of the D.C. amplifier ($R = 5 \times 10^{10}$ ohm.), and V_m is the output voltage of the mill for a given potential gradient F. It is obviously necessary, in order to keep γ small, that the mill system should have as high a sensitivity as possible. The maximum value of V_m/F would normally be limited by the requirement of operating the associated A.C. amplifier with a reasonable supply voltage but in the present work the limit is set by a purely geometrical factor arising from the nature of the feedback employed, viz. the greatest convenient space between the upper vane of the plate assembly and the collecting electrode. This is mentioned in § 5.5. A first calculation showed that $\gamma \simeq 5.5$ sec., which was considered satisfactory. Later, a revision in some of the parameters affecting γ, will be explained (§ 6.4). The experimental value is $\gamma = 3.4$ sec.

5.3 Design details of the vane system

There is a choice of several methods of providing for the alternate screening and exposing of the collecting electrode; a comprehensive review of these is given by Mapleson and Whitlock (1955). The most popular configuration is undoubtedly the system of coaxial vanes, and this takes its simplest form in the case of two vanes shaped like a Maltese Cross, one of which acts as the collecting electrode, and the other, being earthed, rotates above it. An important advantage of this construction is that it enables the insulators to be well concealed beneath the collecting plate (see Fig. 10). There is more purpose in this than merely to shelter them from the elements, for unless the insulators are adequately screened electrically from the rotating vane, a spurious output can be produced by the charges on their surfaces in a manner described later in this section. Typical examples of the vane type of electrode assembly were employed by Harnwell and van Voorhis (1933); Lueder (1943); Waddel (1948); and also by Mapleson and Whitlock (1955). A number of such machines, based on the design of Whitlock are at present in use in the Research Department at Durham, and are conveniently portable. That described by Waddel was also of light construction, being required for use in aircraft. The design of Harnwell and van Voorhis is very interesting: not only was it probably the first example of the use of vanes in its electrode system, but its mode of operation is described as a null method. It is in fact a type of feedback device, but whereas for the performance required of it, - it was designed to measure voltages generated by a van de Graaff machine, - it undoubtedly served its purpose well, it must have been impossible to extend its use beyond the measurement of steady fields of one sign. This is because the return voltage was derived from the mill output by means of a rectifier which was not phase-sensitive, and the feedback must therefore have been degenerative for only one sign of potential gradient. The design of the present field mill is quite similar to that of Harnwell and van Voorhis, especially in the electrode design, but differs from it in that it includes phase-sensitive rectification.

With vanes shaped as ordinary sectors of a circle, the output wave-form with which the amplifier has to deal is triangular. Some workers have considered this a disadvantage and designed vanes shaped to give sinusoidal outputs, but in the present design this elaboration was considered unnecessary. The properties of the field mill will be dealt with under five headings, as follows:

(a) Theory of operation

Referring to Fig.10, the upper electrode is in the form of a vane; it was made by cutting away alternate portions of a disc divided into eight equal sectors. An exactly similar vane beneath it is bolted to a brass bush keyed to the rotating shaft, which is earthed by means of a copper ring and graphite brushes. The collecting electrode is an unbroken disc, except for the central hole, and is mounted on four insulators to the platform described in § 5.2. All three electrodes are of stainless steel, the surface of which has been ground to a mirror finish, and are 30 cm. in diameter. For the moment it should be assumed that the upper vane is earthed, although it is in fact insulated from its supporting cross-piece; it is then easy to see how the rotation of the middle vane screens and exposes the collector so that the excursions in bound charge are proportional to the atmospheric potential gradient. Neglecting edge effects, the magnitude of the variation must obviously be the same as in the more conventional assembly of a vane-shaped collector with an earthed vane rotating above it, and no third electrode. The collecting plate is connected to the grid of a cathode follower through a network consisting of a resistance R and capacitance C in parallel; the purpose of the cathode follower being the usual one of matching the high output impedance of the collector and network to the long cable leading to the amplifier. An equivalent circuit would show that C and the mean capacitance of the collector to earth over one cycle are effectively in parallel, but measurements have shown that the latter is small compared with C. In so far as the upper vane and rotor do not screen the collector appreciably from the earth's field, Mapleson and Whitlock (1955) have shown that the peak value of the triangular wave is given by:

$$V = \varepsilon_0 F R f \pi (r_2^2 - r_1^2) \left[\frac{1 - e^{-\frac{1}{2fRC}}}{1 + e^{-\frac{1}{2fRC}}} \right] \qquad \text{..... Eq. 5.1}$$

where F is the potential gradient, r_1 and r_2 are the inner and outer radii of the sectors of the vanes, and f is the frequency of the exposure-screening cycle.

If

$$2fRC \gg 1 \qquad \text{..... Eq. 5.2}$$

the factor in square brackets approximates to $1/4fRC$ and the output becomes independent of small changes in the frequency of operation. Although to fulfil this condition was not absolutely essential, both an account of the intention to enclose the mill within the feedback loop and because the motor is an induction motor running at constant speed (mains frequency), the maximum possible output is obtainable this way, so it was expedient to have Eq. 5.2 satisfied.

In choosing R and C the magnitude of C must be kept low to maintain a high sensitivity, and the maximum value of R is limited by the grid current expected from the cathode follower (see (b) of this section). A suitable arrangement has been found by making $R = 4 \times 10^7$ ohm and $C = 1000\mu\mu F$. The frequency f is not quite four times the speed of the motor, i.e. it is just less than 200 c.p.s. It is therefore found that $1/2fRC = 0.06$, in satisfactory accord with Eq. 5.2. According to Eq. 5.1 the sensitivity would be 0.14mV per V/m.; but experiment shows it to be actually 0.04 mV per V/m. It is not clear why in practice there is such a large reduction on the theoretical estimate.

In § 3.4, the general requirements for the stability of the apparatus were considered and a slight extension of the discussion will be convenient at this stage. With the sensitivity stated above, the input at the cathode follower corresponding to a potential gradient of 1 V/m. is less than the flicker noise of the first valve of the amplifier, if the rather high value of $100\mu V$ is assumed for the equivalent grid voltage from this cause. But unless the noise were generated at a very low frequency, no output would be obtained past the differentiating circuit leading to the compensating grid of the D.C. amplifier, because of its comparatively long time constant, expected to be 5.5 sec. The high frequency end of the noise spectrum would have no effect in any case, because the frequency of the signal and the rectifier is much lower than this and the D.C. level of the output would not be affected, although of course the noise would still appear at the mill output.

(b) <u>The effect of grid current in the cathode follower</u>

A steady spurious output from the field mill would not vitiate its performance as a compensating device, but this of course depends on the constancy of the effect producing it, and it is obviously best to reduce the consequences of all phenomena other than that which it is intended to measure. The appearance of a D.C. voltage between the collecting electrode and the rotating vane will result in a signal indistinguishable from that produced by the atmospheric potential gradient, for the rotation of the earthed vane varies the capacity to earth of the collector, and a corresponding A.C. voltage appears across R and C. If the collector is at a potential v from earth and the variation in capacity has a maximum of ΔC, the alternating voltage will have a peak-to-peak value $\Delta c . v/C$. One way in which the voltage v can arise is by the flow of grid current from the cathode follower through the resistance R. In order to keep the potential gradient equivalent to this output below 1 V/m., $\Delta c . v/C < 0.08$ mV. Δc has been measured to be $5\,\mu\mu$F., therefore $v < 15$ mV. Since $R = 4 \times 10^7$ ohm, the grid current must be kept below 4×10^{-10} A.

The characteristics of the cathode follower have been measured and the grid current has been found to be less than 10^{-10}A., so that it can be entirely neglected.

(c) The effect of contact potentials

It has been found by various workers, e.g. Lueder (1943), that even when grid current has been quite low enough not to give trouble, large spurious outputs, up to the equivalent of even 100 V/m. have been obtained using plates and vanes made of certain materials. The quantum theory of metals shows that when two different metals are connected electrically it is only the levels corresponding to the mean energy of the conduction electrons, - the Fermi levels - which are at the same potential. The surfaces are not at the same potential, however, owing to the difference in the energies required to release electrons from the surfaces. These differences in work function can amount to several tenths of a volt. In the field mill, the collector and rotor are connected together through the resistance R and the earthing contact of the rotor. Neglecting any effects due to these, it is quite evident that the consequence of such a difference in potential is the same as that of grid current, except that it may be more difficult to remove. Why large spurious effects should result sometimes, even when the electrodes are of the same metal, e.g. aluminium or brass, is not clear, but it can be presumed that much may depend on the condition of the surface rather than on the actual material. Chromium plating has seemed a certain cure. The present mill, however, has a plate system of stainless steel, the surface of which has been polished to a mirror finish. The calibration curve

(Fig. 15), indicates that contact potential is responsible for a "zero" output equivalent to some -50 V/m., but perhaps it would be more accurate to say that, so far, no other cause for it has been discovered.

(d) The insulators

Insulators on measuring equipment of this sort are liable to give trouble because of the local fields arising from free charges on their surfaces. If the field, directed towards the collecting plate, is intercepted by the rotating vane, a spurious signal will be the result as the bound charge on the collector due to this source is modulated. In a mill of the vane type, the insulators are usually well screened so that few lines of force lie in the region where they can be cut by the rotor. If anything, the present design is an improvement, as the insulators in question are screened, not by a vane, but by a complete disc.

(e) Earthing of the motor shaft

It has been found with this type of machine that earthing the shaft carrying the vane directly through the bearing is not satisfactory, presumably because of the insulating effect of the lubricating oil (Lueder, 1943). In the original design, the rotating vane being fixed directly to the motor shaft, this would have been extremely unsatisfactory because of electromagnetic induction in the shaft by the current in the stator coils: this was in fact observed to be the case, the output being quite unintelligible, with the only earth being that through the bearings. It was found that the resistance of the shaft to the casing when the motor was running was approximately 100 ohm.

Completely satisfactory earthing has been achieved by a pair of graphite brushes bearing on a copper slip ring fixed to the shaft. Once the brushes were run in, the earthing resistance was found to be of the order of 0.1 ohm. If the resistance is below a certain value, the exact size seems to be immaterial: for in order to locate the cause of a fault on one occasion, a resistance of 3 ohm was inserted between the brushes and earth, but this did not affect the output to any measurable extent.

(f) Effect of air-earth currents and precipitation

Mapleson and Whitlock (1955), have analysed the effect of precipitation current on the output from a field mill. The current, flowing into the collecting plate, is interrupted by the movement of the rotating vane and an alternating voltage is impressed on the grid of the cathode follower. This is shown to be equivalent to a potential gradient F_j , where

$$F_j = j/4f\varepsilon_0$$

The current density j rarely exceeds 10^{-9} A/m^2, and the consequent value of F is less than 0.2 V/m. Since the current into the collector is unidirectional, the mean D.C. voltage of the collector will also change from zero, and the variation Δc in the capacity of the collector to earth, as it moves through the exposure-screening cycle will modulate this in exactly the same manner as when there is grid current in the cathode follower. This contribution of the precipitation current can easily be shown to be negligible, however.

Single drops affect the mill rather differently: each produces a pulse of rise and decay time $RC \simeq 0.04$ sec. The height of the pulse can be shown to be equivalent to less than 1 V/m., but in any case it would be greatly attenuated on passing through the differentiating circuit, the time constant of which is 100 times that of the pulse.

5.4 The choice of a rectifying system

Although a comparison of the merits and demerits of the mechanical rectifier and the electronic phase-sensitive detector, are certainly relevant to the design of the more conventional type of field mill, the writer's choice of a commutator and brushes was based chiefly on simplicity of design, since it was hoped that any detracting features of such a system would be automatically corrected by the feedback loop (§ 5.5). Although the objection can be raised that mechanical commutators may run into contact trouble at high speed, electronic rectifiers used under the present circumstances would have some disadvantages. Two types of phase-sensitive detector were considered initially: one is described by James, Nicholls and Phillips (1947), and has as its operating principle the addition of the A.C. signal to a larger reference voltage, which overrides the signal in determining whether the diode detectors shall pass current or not. There are some rather conflicting requirements for the successful operation of this circuit, however. There is a more satisfactory circuit (Schuster, 1951), in which the signal takes its path through either of two triodes, which one being determined by the reference voltage, of the same frequency as the signal, which triggers each of them in turn. An important consideration, however, was that large output voltages, up to 80 volts D.C., would be required, and that it might not be satisfactory to use electronic circuits to deal with the corresponding R.M.S. input voltages, particularly with only a modest supply voltage.

As far as the limiting speed of a mechanical commutator is concerned, the peripheral velocity of a 2 in. diameter commutator rotating at 3,000 r.p.m. is a good deal less than the highest speed at which conventional generating plant is operated (approximately 10,000 ft./min: Hayes, 1947), although the conditions are, of course, quite dissimilar. It was hoped that by careful machining, a commutator could be made to work reliably at the required speed. The assembly is shown in Fig. 11. The commutator is 2 in. in diameter and consists of two copper discs mounted on a 'Tufnol' boss. Only one of the discs is divided into eight sectors, every alternate one being connected to the other disc by the bolts holding the assembly together. The other sectors are completely insulated. With this arrangement it is practicable, in the limited space available, to have the brushes in parallel pairs, two leading in, and two collecting the current, thus reducing the probability of intermittent failure due to brush 'chatter'. The spaces between the commutator segments were filled with an "Araldite" casting resin to increase the rigidity. The commutator is keyed to the shaft, and since the rotating vane is attached in the same way, the correct relative position of these two units is always maintained after reassembling. The brush holders, including those of the brushes earthing the shaft, are mounted on four mild steel rods fixed to the lower shaft-bearing housing. The holders are all of the same robust construction, but the

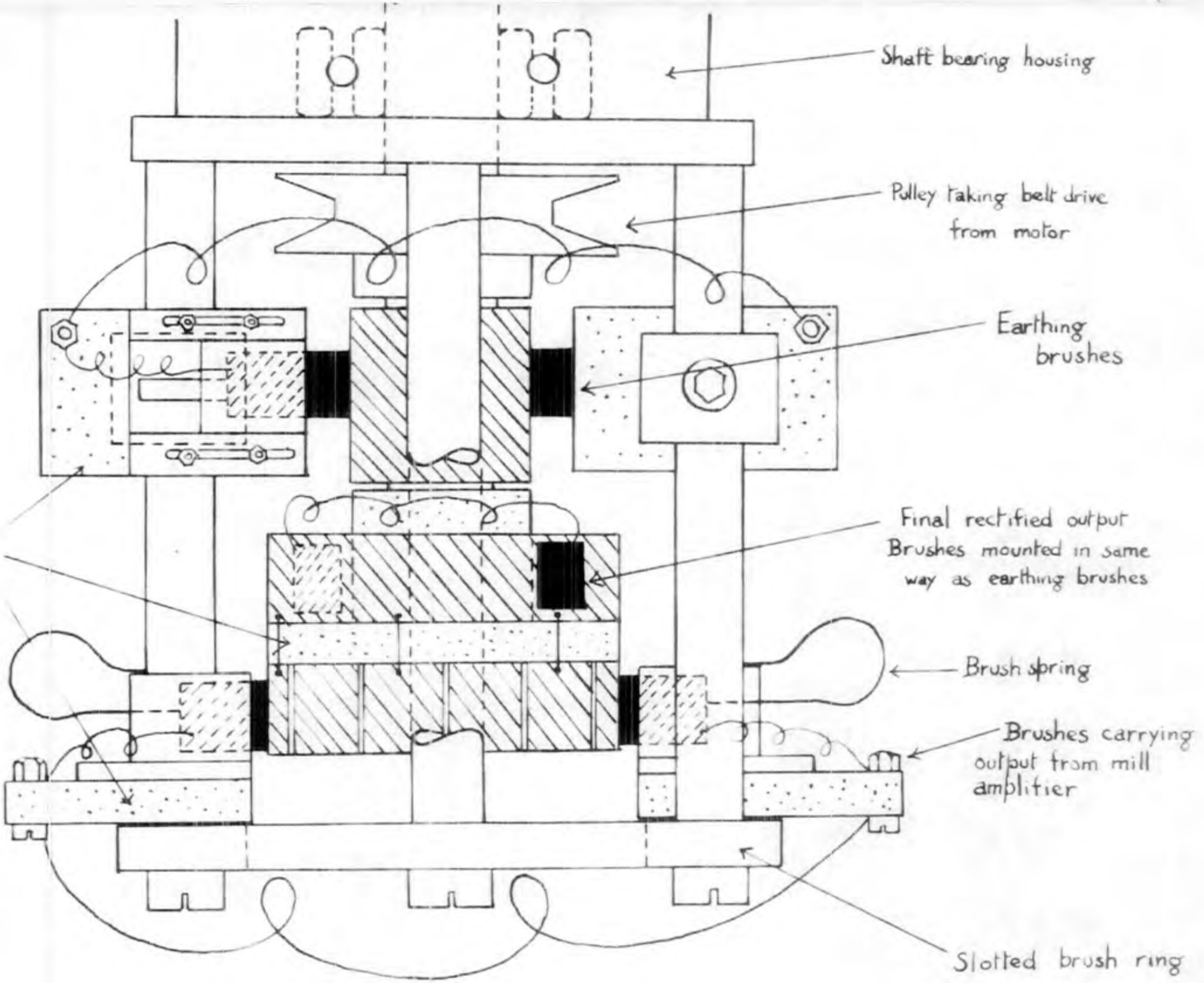


Fig. 11 Commutator and Brush Assembly

lower two, which bear upon the segmented portion of the commutator, are independently mounted on a slotted ring bolted to the rods. These brushes can therefore be moved round until they are in the position where the output is commutated correctly (§ 5.8). Their position is defined by a graduated scale marked on the ring. The brushes are a commercial type used with standard motor-cycle accessories and are mounted radially, except for the pair taking the current from the upper half of the commutator: these, because of lack of space, are arranged to have a small leading angle.

It has been found that the commutator assembly is almost completely trouble-free. A rather strong spring pressure is needed to initiate the running-in, but once this adjustment has been made, a considerable amount of wear can be tolerated before the brushes need to be changed.

5.5 The application of negative feedback

The usual course to adopt in the design of such instruments is to follow the vane system itself with an A.C. amplifier of high loop gain but with a considerable amount of negative feedback. The resultant overall gain provides a sufficient output voltage to operate a robust meter, after it has been rectified. The vane system itself is inherently linear, and by restricting the negative feedback only to the amplifier, good linearity and gain stability may still result. Because the rectifier remains outside the loop, however, the overall performance is no better than that of the rectifier.

Hitherto the upper vane of the mill has been considered as held at earth potential; if it is connected instead to the output of the commutator, it can be shown that provided this voltage is of the same sign as the potential gradient, the output is independent of the characteristics of both the amplifier and the commutator. Referring to Fig. 12, a potential gradient F gives rise to an output V_m which is fed back to the upper vane. If d is the distance between the collector and the upper vane, a potential gradient V_m/d is set up, which produces a bound charge on the collector. The rotating vane modulates this in the same way as it does the bound charge of the atmospheric potential gradient, but since one field is modulated out of phase with the other, the signal from the cathode follower can be expressed by:

$$V = a(F - V_m/d)$$

where a is the sensitivity of the plate assembly and cathode follower combined. The amplifier applies a further gain of b, and if the rectifier has a form factor c, the D.C. output voltage is:

$$V_m = abc(F - V_m/d)$$

or

$$V_m = \frac{abc\,F}{1 + abc/d} \qquad \text{..... Eq. 5.3}$$

This is completely analogous to the well-known feedback equation $V_2 = \dfrac{AV_1}{1 + A\beta}$, and provided $abc/d >> 1$, the result is:

$$V_m \simeq Fd \qquad \text{..... Eq. 5.4}$$

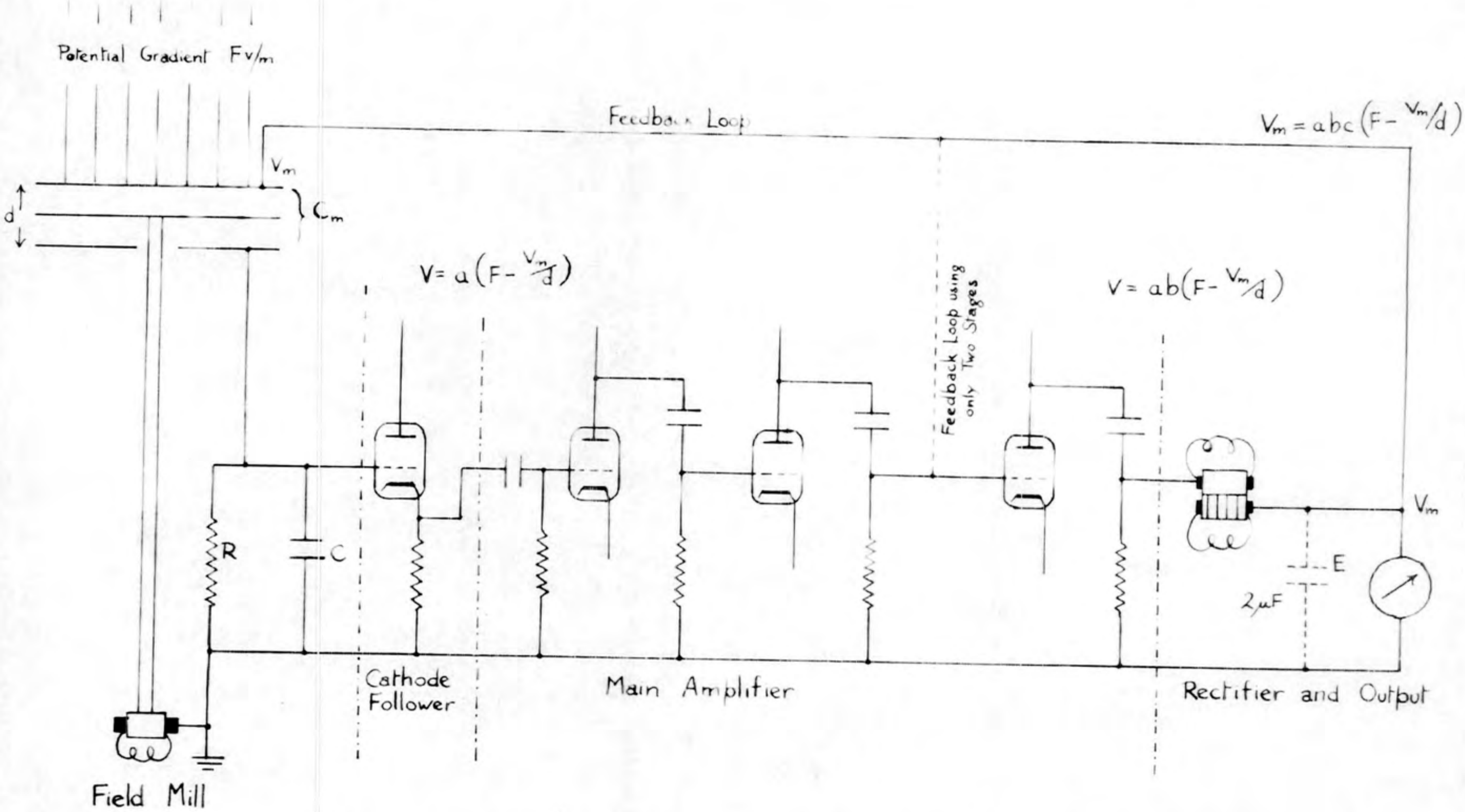


Fig. 12 Schematic Diagram of Complete Mill Circuit

The obvious advantage is that a, b and c do not occur in the equation for the output. If the feedback had been applied only over the A.C. amplifier, the result corresponding to Eq. 5.3 would have been: $V_m = acFb/(1 + b\beta)$, where β would have been the fraction of the amplifier output fed back, and the approximation: $V_m \simeq ac\beta F$ would still have been dependent on the characteristics of the vane system and rectifier.

There is a simple physical explanation of the behaviour of the system. Eq. 5.4 shows that the output voltage reaches such a value that the potential of the upper vane is the same as that of the atmosphere in the plane of this vane. It is then as if the rotating vane were moving between two unbroken circular conductors, when it could not modulate the bound charge on the lower one, the collector, even if the upper one were at a different potential. In fact the signal on the cathode follower, when considerably amplified, is just sufficient to raise the potential of the upper vane to the value necessary to produce this nominal zero-signal situation.

The effective value of d was obtained in the laboratory by applying a test field to the mill without the connection of the feedback loop. Simultaneously the upper vane was raised to a sufficient potential to produce zero output from the cathode follower. It was found in this way that $d_{eff} = 1.9$ cm., whereas the actual distance is 2.4 cm. The discrepancy presumably arises because the potential of the atmosphere at the level of the upper vane will not be as great as one would expect from the distance d , on account of the presence of the rotating vane, which is midway between the other two electrodes and earthed. Consequently the voltage to be applied to this vane for the zero-signal condition, will not be as great as the geometry predicts.

There is some case for increasing the value of d further, because as explained in § 5.2 this reduces the time constant of the system. However this alteration would require an even greater amplification in the succeeding stages; the more so as the exposure factor of the collecting plate, and hence a, would be reduced as the upper vane was raised. Thus in order to keep the feedback factor abc/d constant, b would have to be increased more than proportionally to d.

5.6 The cathode follower and A.C. amplifier

Because of the need to keep the grid current low in the cathode follower stage, the valve, a pentode EF37A, was run at reduced heater and anode voltages. Mutual characteristics were plotted at different values of these parameters, but it was found impossible to avoid working near the bend towards cut-off, and yet maintain the properties of a cathode follower. However, a compromise was reached, and the unit shown to behave in accurately linear fashion for grid swings up to ± 1 V. Fig. 13 shows how the heater voltage is obtained by using suitable resistors in the heater winding of the mains transformer in the power supply. The anode supply of 85 V. is derived by division of the stabilized voltage across a regulating valve, VR105. This also gives a measure of stability to the first pentode stage of the amplifier, which in addition has its anode supply voltage decoupled from the rest of the circuit. The other stages are triodes, the final one being a

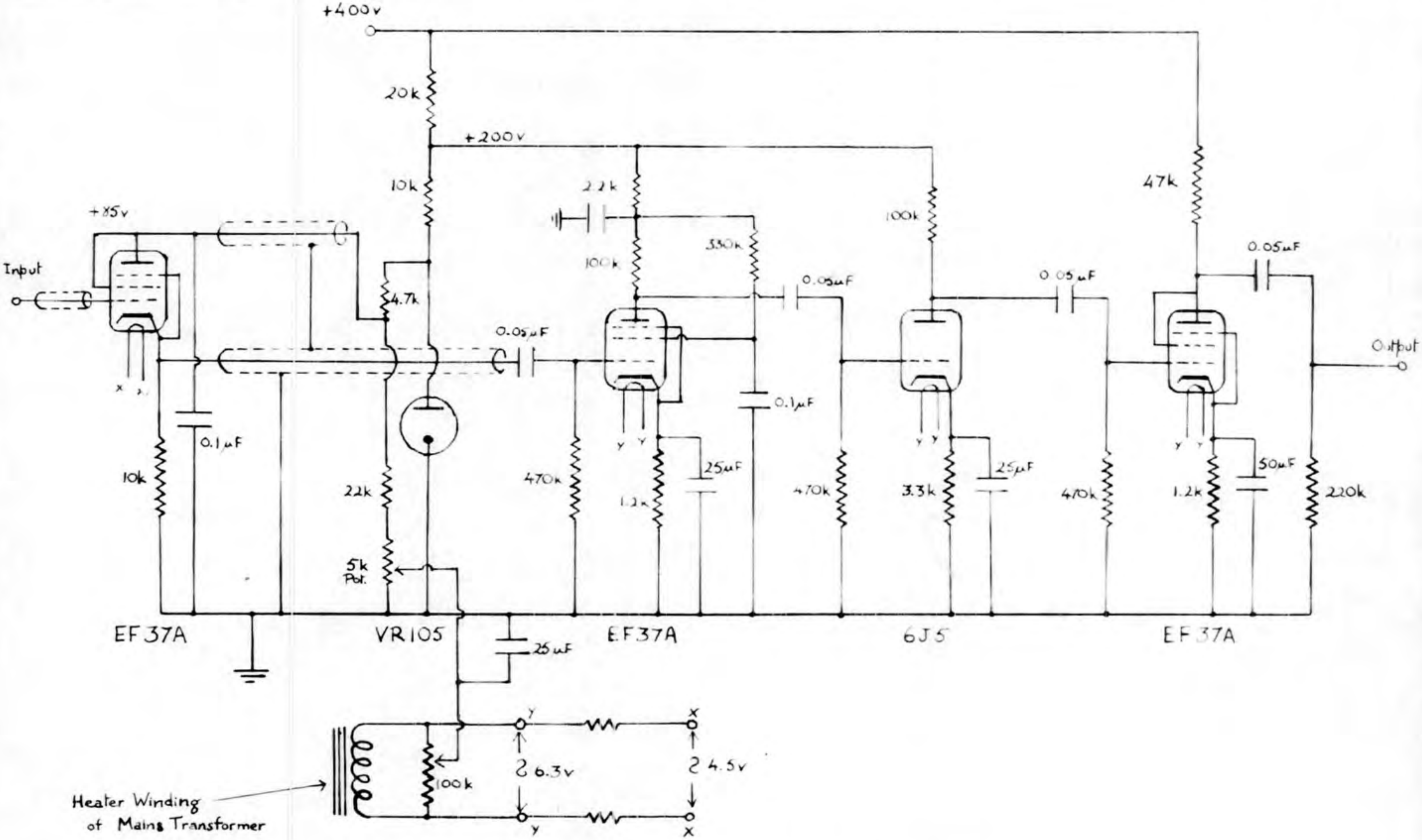


Fig. 13 Cathode Follower and Mill Amplifier.

triode-connected EF37A, normally a pentode. This was chosen rather than a 6J5, the triode used in the second stage, because its characteristics, when the valve is connected as a triode, are better suited to obtaining a large output voltage without distortion. Resistance-capacity coupling is used throughout and the design is conventional. The cathode follower unit is slung on rubber bands within the mill casing, to reduce microphonics.

5.7 Oscillations in the mill output

One of the major difficulties in operating a system which incorporates a large amount of gain and degenerative feedback, is to design within the limits of phase-shift and attenuation that can be tolerated at either extreme of frequency before the feedback reverts from negative to positive. A generalization is that if a system can oscillate at any frequency, it will do so, even if this is not the operating frequency.

Unfortunately, in addition to having to design for the D.C. feedback loop governing the ordinary operation of the mill, it quickly became apparent that there is an A.C. loop which functions as well when the motor is stationary as when it is running. This is because the upper vane and the collecting plate of the mill form a capacitative coupling between the output of the amplifier and the input grid of the cathode follower. This capacity is only $100 \mu\mu F$, but it forms a circuit of time constant 5 ms., in conjunction with the 4.7×10^7 ohm. resistor at the input of the cathode follower. The high gain of the amplifier made it very difficult to avoid relaxation oscillations, which of course did not stop when the motor was running. The commutator only interrupted the oscillations; it did not stop them.

As indicated in § 5.3, it was experimentally determined that $a = 4 \times 10^{-5}$ V. per V/m., where a represents the ratio between the peak voltage of the triangular waveform at the cathode follower output and the potential gradient. Assuming that the form factor of the (half-wave) commutator would be $1/\pi$, a gain of 10^4 across the amplifier would give a feedback factor:

$$abc/d = \frac{4 \times 10^{-5} \times 10^4}{\pi \times 0.019} = 6.7$$

This is not very large compared with unity, but 10^4 is a gain which can conveniently be obtained with a two-stage amplifier. Fig. 12 shows, however, that considering the whole system (with two-stage amplifier), as a purely A.C. network, with the upper and lower electrodes of the plate assembly acting as a coupling condenser C_m as described above, the phase of the output would be exactly the wrong one: the system would be regenerative. It was found to be so in practice.

The obvious remedy of adding an extra stage of amplification and altering the phase of the commutator by π was not, however, as simple a solution as it sounds. The rule governing the design of feedback loops is known as the Nyquist criterion of stability. A useful summary of the criterion is given by West (1950), and it is pointed out that for amplifiers with ordinary characteristics, the rule amounts to arranging matters so that, when attenuation and phase-shift are plotted against frequency, the attenuation curve is such that the loop gain of the system will have fallen to unity before the phase-shift has reached π. In an amplifier with two stages, or, more correctly, only two resistance-capacity (RC) couplings (all the others being direct), the criterion is almost automatically fulfilled except at extremely high gains, because the maximum phase shift of an RC coupling is $\pi/2$ so that the regenerative condition will not be reached, theoretically, at any frequency. When more than two such couplings are used however, and no compensating networks are included, the system is certain to oscillate at some frequency if the gain at the operating frequency is sufficiently high, because there will be some low frequency for which the total phase shift exceeds π, and it is only necessary for the gain round the loop to be still greater than unity for it to become regenerative.

With a three-stage amplifier after the cathode follower, there are no fewer than five RC couplings, counting the vane system as one; the textbooks indicate that the difficulty of avoiding oscillations increases out of all proportion to the number of stages. An endeavour was made to assess the problem quantitatively. In the first place (See Fig. 12), the vane system and cathode follower input, comprising C_m, C, and R, is not quite the same as a simple RC coupling, because it also introduces a constant attenuation factor even at frequencies for which there is no phase-shift. It is easy to show that for an RC coupling the following relation holds between the attenuating factor D and ϕ, the phase-shift:

$$D = \frac{1}{(1 + \tan^2\phi)^{1/2}}$$

so that there is no attenuation at the operating frequency for which the network is designed. For the vane system this equation becomes:

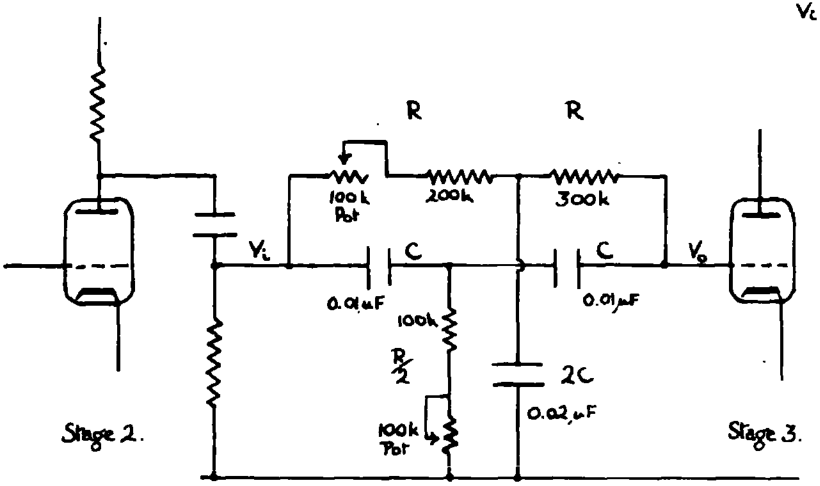
$$D = \frac{1}{(1 + C/C_m)(1 + \tan^2\phi)^{1/2}}$$

This means that, with an amplifier of gain 10^4, the effective gain for an A.C. signal even when $\phi = 0$ would be only $10^4/(1 + C/C_m)$. Since the factor $(1 + C/C_m) \simeq 10$, it is only required to ensure that the gain is reduced a thousand-fold by the time the total phase-shift becomes as great as π. The usual method of controlling the characteristics when there are more than two stages, is to design one stage to have as narrow a frequency band as permissible (F.E. Terman, 1950). Eventually, as the lower end of the frequency scale is reached, this stage will bear the whole burden of attenuating the output, only, as the initial gain is higher, so will the departure from operating frequency have to be the greater in order to do this. However, the phase shift in this stage can never be greater than $\pi/2$. The problem is then to design the rest of the amplifier with sufficiently large components so that while the narrow-band stage is attenuating as the frequency departs from normal, there is no falling-off in gain in the other stages, and consequently no phase-shift; or at most a combined phase-shift not exceeding $\pi/2$. This problem is usually difficult, because it means designing the other stages to have a flat response many octaves outside the useful range of the amplifier. As the number of stages increases, each must contribute less towards the total phase shift, $\pi/2$, which is all that can be allowed while the narrow-band stage is reducing the overall gain to unity.

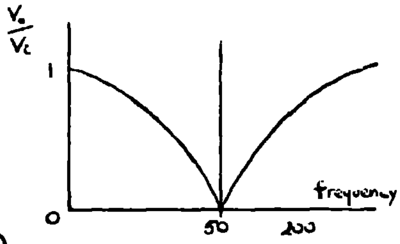
Specimen calculations based on tolerable values of capacitors and resistors yielded the conclusion that the problem was not likely to be solved this way, and experiment proved this to be so. The next resort in more conventional work would have been to design phase-advance networks which have more favourable attenuation-phase characteristics than the simple RC coupling (Valley and Wallman, 1948; Bode, 1947). However, in this case a much simpler solution was discovered. Simply by connecting a large condenser E (Fig. 12), across the output, it was found that this eliminated the oscillations. It obviously acts as a short-circuit for any A.C. component which may tend to develop, while, coming after the commutator, it does not interfere with the D.C. operation of the field mill, except to increase the time constant to approximately 0.1 sec. It also confers an additional advantage in increasing the form factor of the commutator, since the condenser holds its charge during that part of the cycle when the commutator is non-conducting. Theoretically the form factor should now be $2/\pi$, twice that assumed earlier in this section.

Even when the relaxation oscillations had been eliminated, it was found on close observation that there was, superimposed on the output produced by an artificially applied constant field, a small oscillation corresponding to 1 V/m. in amplitude and with a frequency of approximately 1 c.p.s. It was also noticeable that the oscillations were synchronous with audible beats coming from the mill motor. On injecting the mill output into the differentiating circuit the oscillation was seen to produce large excursions in the output of the D.C. amplifier. This was not surprising, because it is the rate of change of the mill output which the differentiating circuit feeds to the D.C. amplifier, and this may be large even though the oscillation amplitude is itself small. Now, however, it is believed that the disproportionate effect was due to other causes, described in Chapter 7. At the time it seemed that the apparatus would not function properly unless the oscillations were avoided.

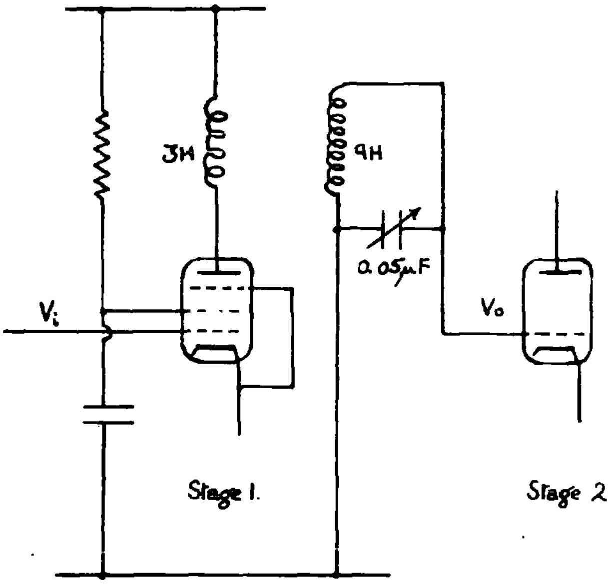
The source of the trouble was very elusive. It must be admitted that it has usually been considered undesirable to run a field mill on a mains-driven motor, because of the hum generated in the cathode follower or on the collecting plate itself. In the present case, it had been considered that any ripple in the final output would be of sufficiently high frequency not to register in the D.C. amplifier. Thus it had seemed quite reasonable to take advantage of the construction of the motor and fix the rotating vane and commutator directly to the shaft projecting at each end of the casing. Electromagnetic screening had been considered unnecessary. Improvement of the electrostatic screening, particularly round the cathode follower, and avoidance of earth loops (Zepler, 1945) were tried, unsuccessfully. Two further expedients which were used are shown in Fig. 14. The first, a parallel-T network, was designed to filter out ripple at mains frequency, but although it succeeded in this, the low frequency variation in the rectified output still remained. A tuned grid transformer coupling between the first and second stages of the main amplifier, designed to pass only the signal frequency, albeit with a fairly low Q of about 6, was also unsuccessful.



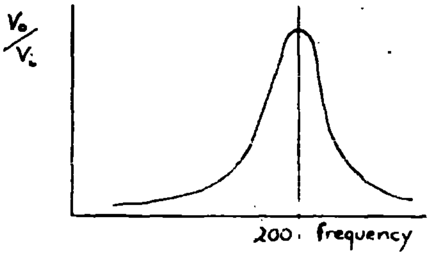
Parallel-T Network



Transmission Through Network



Tuned Grid Coupling



Interstage Amplification

Fig. 14 Expedients Designed to Eliminate Low-Frequency Oscillations in Mill Output

An experiment was then performed to see what might be the waveform of the current passing through the motor windings, by investigating the E.M.F. induced in a coil placed near it. It was evident that 50 c.p.s. pick-up was being modulated at about the same frequency as the audible beats coming from the motor, and the oscillations observed in the mill output. An induction motor revolves nominally at an integral multiple of the supply frequency, but in practice, the torque which it has to exert causes it to slip behind its nominal speed, the difference in the present case being about 1 c.p.s. It seems reasonable to expect that the current carried by the motor coils should also be modulated at this frequency. If the rotating vane was influenced by radiation from the motor, for instance, through the shaft, the 200 c.p.s. signal frequency could conceivably be modulated also at 1 c.p.s. at the cathode follower input. Clearly, if the low frequency was introduced at such an early stage, no kind of filter network in the amplifier would make any improvement.

Finally the motor was removed from inside the mill casing and a separate bearing housing constructed for the shaft, to which the drive is taken by a V-belt. To prevent static electricity produced on the belt from inducing a field on the mill plates, the belt has over it an aluminium hood which is attached to the mill case but insulated from the motor housing. As an additional precaution, the motor supply cable has been encased in an iron conduit separately from the other cables. This has resulted in a marked improvement in the steadiness of the output.

5.8 General features and calibration of the field mill

While improvements in the mill design were in progress, arrangements were made in the laboratory for applying potential gradients artificially. The mill was surrounded by an earthed aluminium sheet 1 m. square, at the level of the rotating vane, and a parallel plate of similar size slung from the walls over it, and of course insulated. To this an adjustable voltage was applied. The distance between these plates was kept below 50 cm., so that the surrounding walls would not influence the potential gradient at the mill appreciably. The commutator was correctly phased by applying a moderate voltage to the test plate and observing the output of the amplifier on a cathode ray monitor, with the feedback loop disconnected. Figs. 15 and 16 are calibration curves of the mill after this adjustment and with the feedback return operative. With low potential gradients, up to 1200 V/m., it can be seen that the points are closely linear, but that the numerical value of the slope is less for negative than for positive potential gradients by 2%. A tentative explanation of this will be given. There is also an output when the applied field is zero, equivalent to about -50 V/m. This, presumably, is due to a contact potential between the collecting plate and the rotating vane (§ 5.3, (c)). Fig. 16 shows the calibration curve up to 6,000 V/m, with the points of Fig. 15 included for reference. The most noticeable feature is the flattening off of the curves above 4,500 V/m. This is to be expected, because the amplifier characteristic is linear only up to 70 V. r.m.s. after which it quickly becomes saturated. The rectified and smoothed output corresponding to this is 46 V. The

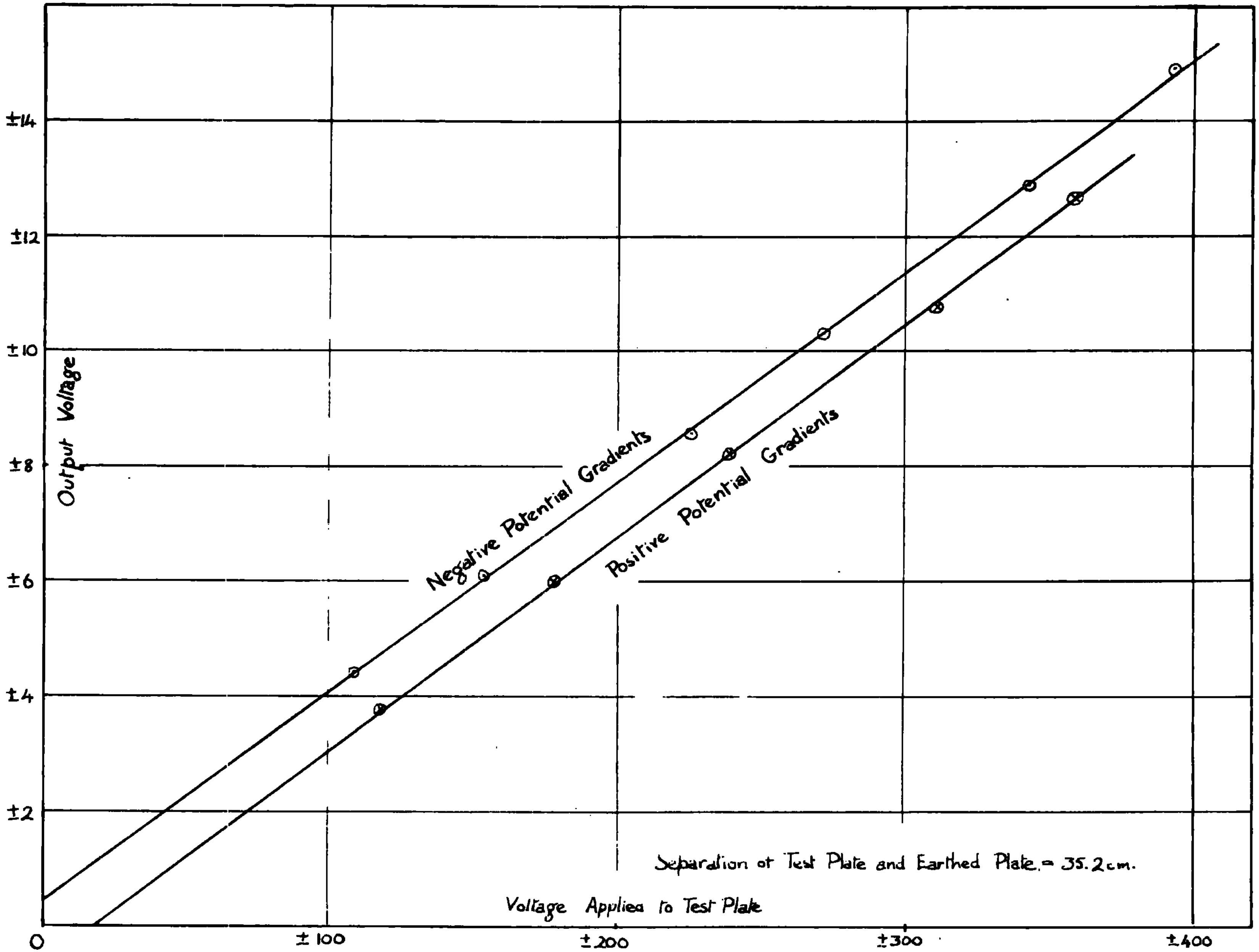


Fig. 15 Calibration of Field Mill (Low Potential Gradients)

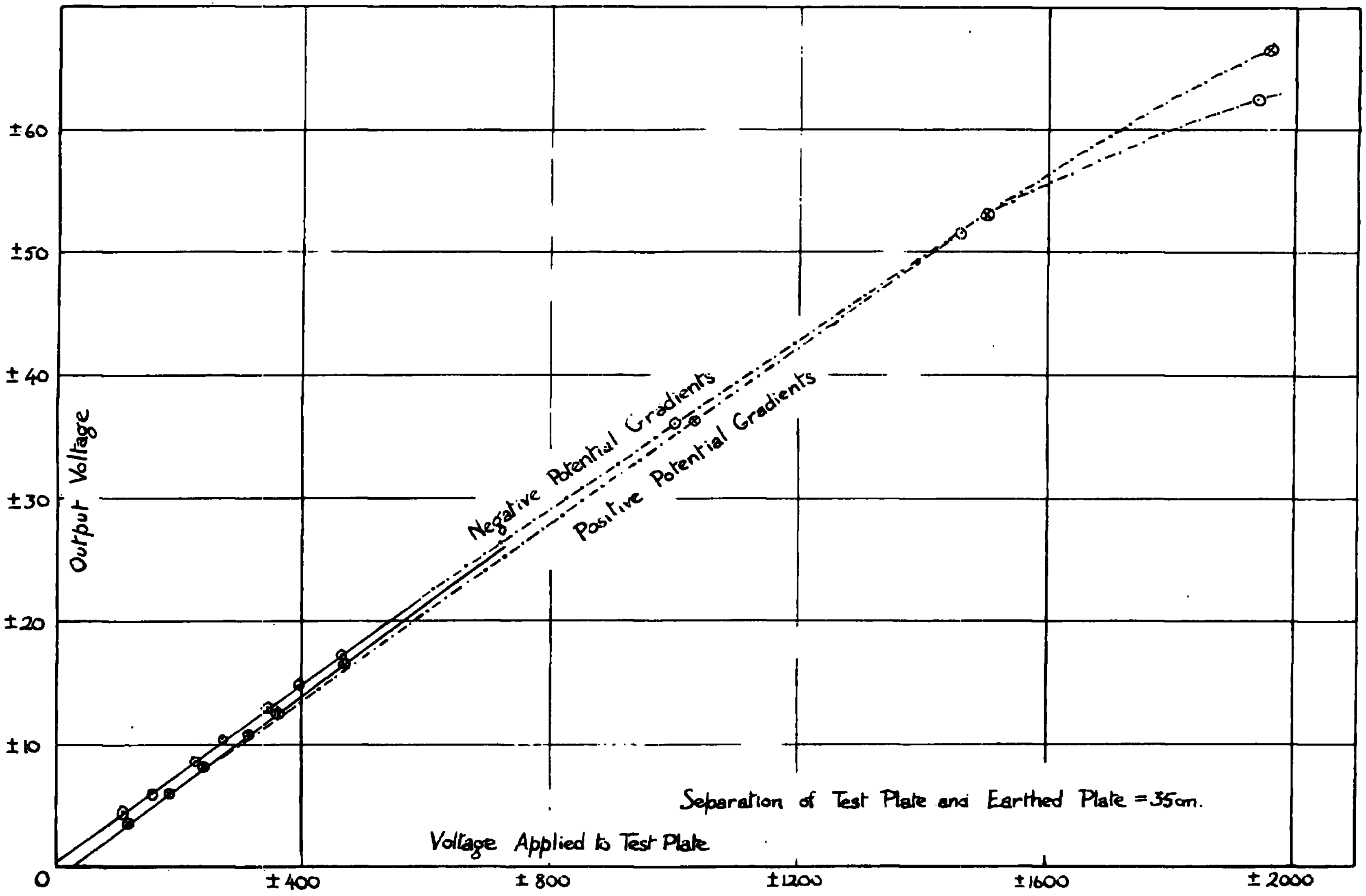


Fig. 16 Calibration of Field Mill (High Potential Gradients)

calibration curve shows that the feedback loop is performing its proper function of extending the (approximately) linear region of the overall system beyond this voltage. The apparent change of slope of the curves above the points included from Fig. 15, is of doubtful significance. To begin with, the first two points on the dotted part of the curve of positive potential gradient cannot possibly be a correct continuation of lower region, because a straight line through them intersects the straight line which can be drawn accurately through the lower points, and does not continue it smoothly. The discrepancy may be even greater than this, because, if anything, the extension of the curve will be concave downwards which would make the fit worse. The reason may be that the electrostatic voltmeter used for the high voltage measurements (dotted curve), and the moving coil meter used for the lower voltages of Fig. 15 were not correctly intercalibrated; owing to the small range overlap it was possible to make this intercalibration only at one point. The deviation from the slope at lower voltages is in any case only 2%, supposing it not to be due to a systematic error.

It is of interest to know the quantity abc/d in Eq. 5.3, because the extent by which this exceeds unity is a measure of the system's independence of changes in amplifier and commutator characteristics. Its value was easily obtained by observing the rectified and smoothed output for a known applied field, without having the feedback loop connected. The output for a potential gradient of 97 V/m. was observed to be 33 V. so that $abc = 33/97$. Using the effective value of d measured in the laboratory, $d_{eff} = 1.9$ cm. (§ 5.5),

$$abc/d \simeq 18$$

Inserting the values of the plate system sensitivity, $a = 10^{-4}$ V. per V/m.; the main amplifier gain $b = 2 \times 10^4$, and the theoretical form factor of the commutator $c = 2/\pi$, the feedback factor should have been 27. The reason for the lower value obtained in practice lies in the commutator not being a perfect half-wave rectifier; the form factor was in fact observed to be only 0.47. The cathode ray monitor shows that it passes more than the correct half of each cycle, on account of the width of the brushes, so that (in a positive potential gradient) the positive half of the waveform is partly cancelled by that amount of the negative half which the brush allows to pass. It is possible that such a small value of the feedback factor might give rise to the non-linearity of the calibration curve in Fig. 16. It might also account for the difference in the slopes for positive and negative potential gradients shown in Fig. 15, although it would also have to be assumed that the vane system or the commutator, or both, deal very unsymmetrically with positive and negative fields. A possible explanation of Fig. 15 is that the walls of the room exerted an influence on the field applied to the mill in the laboratory.

The mill was later calibrated outside in the position in which it is now used, for positive potential gradients only, by means of a test plate erected over it (Plate 1). It is found that an applied field of 100 V/m. produces an output of 1.64 V. The value of d_{eff} obtained from the calibration is thus 1.64 cm. notably different from the 1.9 cm obtained in the laboratory. The discrepancy is perhaps to be attributed to the slightly different exposure of the mill in the two cases.

The mill in its working position is shown in Plate 1. In the foreground are the motor housing and the hood protecting the plate system from fields induced by the driving belt. The mill chassis is surrounded by an aluminium cowling which protects it from the weather. It is held in place by leather straps so that inspection can be readily carried out. However, even minor repairs necessitate lifting out the mill altogether, as a rule. The mill and driving motor are both firmly fixed in position by rag-bolts embedded in the cement plinths on which they stand. There is provision for adjusting the position of the motor slightly to take up slack in the V-belt. The mill chassis and the rotating vane are earthed directly to a copper plate sunk beneath the floor of the pit. This earth is carried indoors by means of the cable sheaths, to the main amplifier, which is not separately connected to the mains earth via its power supply. There is no connection either between the earth lead to the motor and the chassis of the mill, as it was found that otherwise a large amount of mains ripple was induced in the cathode follower output.

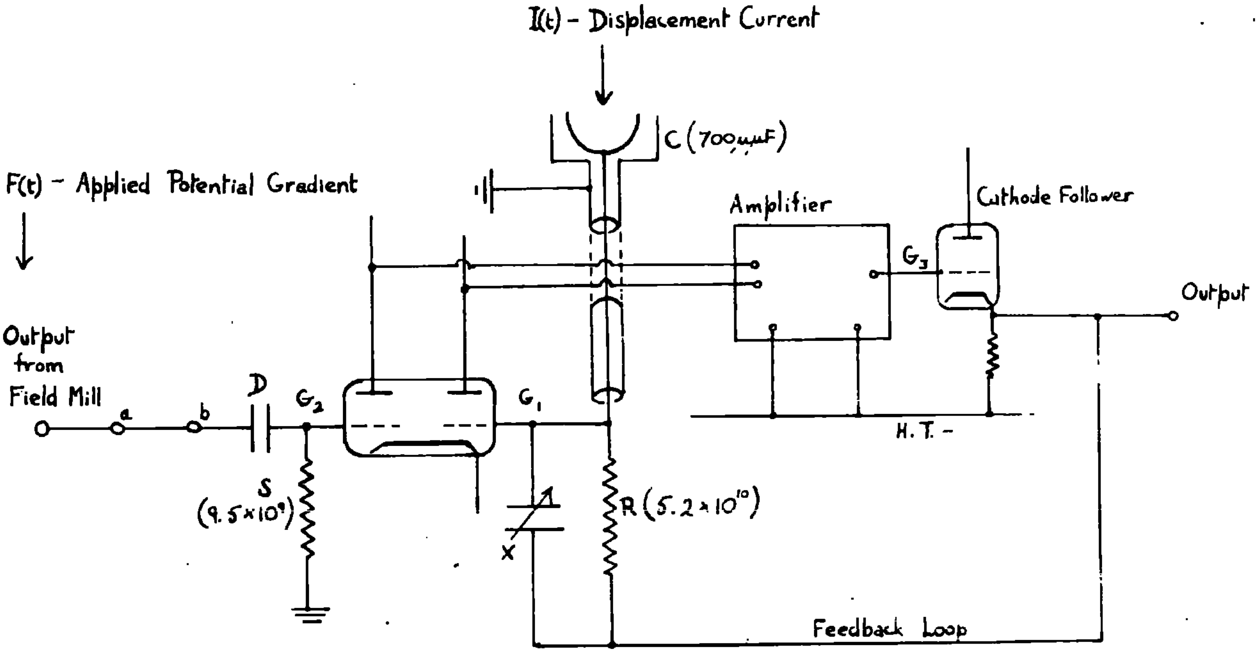
CHAPTER 6. THE FIRST TESTS ON THE COMPLETE SYSTEM

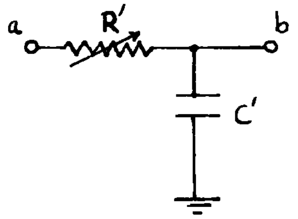
6.1 The differentiating circuit

The parts of the circuit which are relevant to this section are illustrated in Fig. 17, which shows the electrometer stage of the D.C. amplifier with grid G_1 receiving the input current from the collector, and G_2, the compensating grid. The magnitudes of the differentiating circuit parameters are very simply related to other parameters in the circuit as follows:- Suppose a potential gradient F V/m. to be created linearly in a time t. The total charge transfer from the collector to ground through the input resistor R is $\varepsilon_0 FA$, where A is the area of the collector in square metres. Since this change takes place over the time t, there will be, after any transients have died away, a current $\varepsilon_0 FA/t$ which generates across R the voltage, $V' = \varepsilon_0 RFA/t$. Let the output from the field mill for the potential gradient F be V_m. This voltage will cause a total transfer of charge DV_m to or from the differentiating condenser D, which charge, since it occurs over the time t constitutes a current, DV_m/t. This produces the compensating voltage across S: $V_2 = DSV_m/t$ The condition upon D and S is evidently:

$$DS = \varepsilon_0 \frac{F}{V_m} AR \qquad \text{...... Eq. 6.1}$$

With values of ε_0, F, V_m, and R quoted elsewhere, and a value for A of 0.2 m^2, (the geometrical area of the collector aperture), $DS = 5.5$ sec.





Smoothing Circuit:- for transients containing higher frequencies: to be connected between terminals a and b

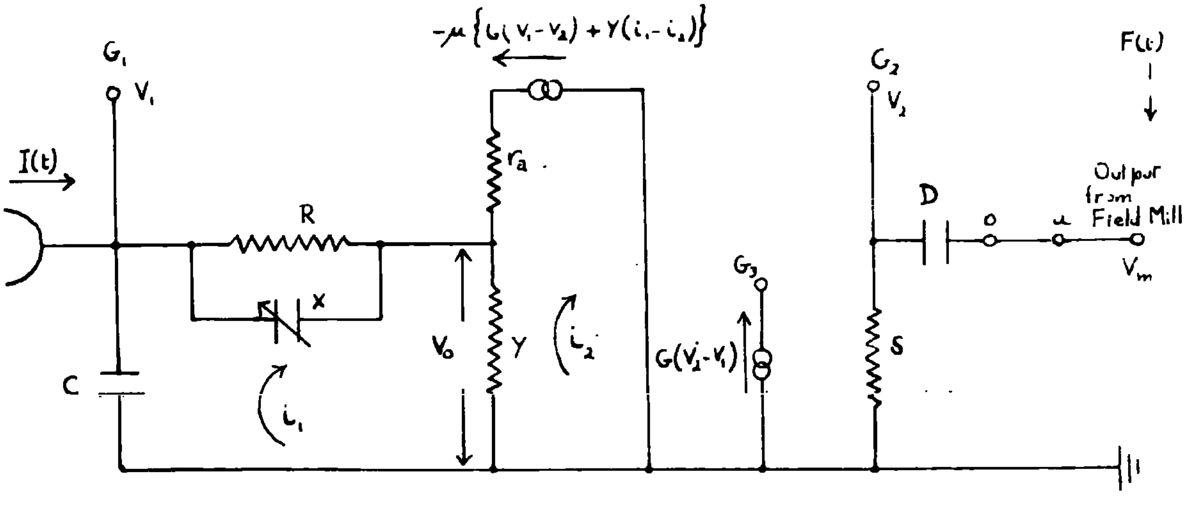


Fig. 17 Compensation Utilizing Both Grids of Input Stage with Equivalent Circuit

The equation above was derived for the steady state, after transients have disappeared. For the compensating side of the system this means after a time long compared with DS, because the steady state will be approached along an exponential $V'(1-e^{-\frac{t}{DS}})$. Further consideration will be given to this aspect in § 6.2.

It is very important that the differentiating condenser should have a high degree of insulation, so that the leakage current through it and S, caused by the application of the mill output, will not produce a signal comparable with the air-earth current on the other grid. The relative magnitudes of the leakage and air-earth currents will, of course, depend on the relation between the potential gradient and the air-earth current, but it is only necessary to enquire what is the smallest air-earth current associated with a given potential gradient. It will be taken that this is governed by the smallest observed conductivity of the atmosphere, 3×10^{-18} ohm^{-1} cm^{-1}., although this would not cover the case where a large conduction current and precipitation current of opposite sign practically cancelled each other out. The smallest current for a potential gradient F V/m. is accordingly $i_{min} = 3 \times 10^{-16} F$ A/m^2., which would produce a voltage across R, $V' = 3 \times 10^{-16} FAR$. If the differentiating condenser has a leakage resistance R_c, the output voltage V_m due to the field F will produce a signal on G_2; $V_2 = SV_m/(R_c + S)$ or, assuming $R_c >> S$: $V_2 = SV_m/R_c$. V_2 is required to be less than V', say $V_2 = V'/10$; therefore the condition is that

$$\frac{R_c}{S} \nless \frac{V_m}{3 \times 10^{-17} FAR} \qquad \text{...... Eq. 6.2}$$

Inserting the values for the various quantities, it is found that $R_c/S \nless 5 \times 10^4$; it is thus required that R_c shall be of the order 5×10^{14} ohm. The condenser used is of the air-spaced, variable type, with porcelain insulators, and has a maximum capacity of 1500$\mu\mu$F. After cleaning, it was mounted in an aluminium box which also contains a 12V. bulb to keep the whole assembly dry. The leakage resistance was determined by measuring the time constant of the condenser with a ballistic galvanometer, charging up the condenser to a small voltage and measuring the throw of the galvanometer after a known time. In this way the resistance was found to be 3.6×10^{14} ohm, which is regarded as fairly satisfactory.

6.2 Method of testing the equipment

It was indicated in § 3.3 that Eq. 6.1 is not a sufficient condition for accurate compensation because this equation neglects the fact that the differentiating circuit does not give an instantaneous response to a rate of change of potential gradient, but an exponential one with a time constant DS. It was necessary to introduce a time constant on the collector input side of the electrometer stage (grid G_1), which would match the time constant DS without of course affecting the steady state condition expressed in Eq. 6.1. At this stage in the work it was believed that none of the stray capacities in the circuit would produce a time constant comparable with DS (approximately 5 sec.), and that the time constants of the circuits on either grid could be adjusted independently. It can be seen that, with the above assumptions, the use of a condenser X across the input resistor R (Fig. 17) is a simple solution to the problem. The steady state response of this part of the circuit will still be $V' = RI$, independent of X, but V' will follow changes in current with a time constant RX, and the net response of the whole circuit to a step function I_i, assuming Eq. 6.1 is satisfied, will be

$$V_0 = RI_i\left[\left(1-e^{-\frac{t}{RX}}\right)-\left(1-e^{-\frac{t}{DS}}\right)\right] \qquad \text{..... Eq. 6.3}$$

By adjusting X, the time constant on the air-earth current side could be made equal to DS, on the compensating side, and the compensation would then be perfect.

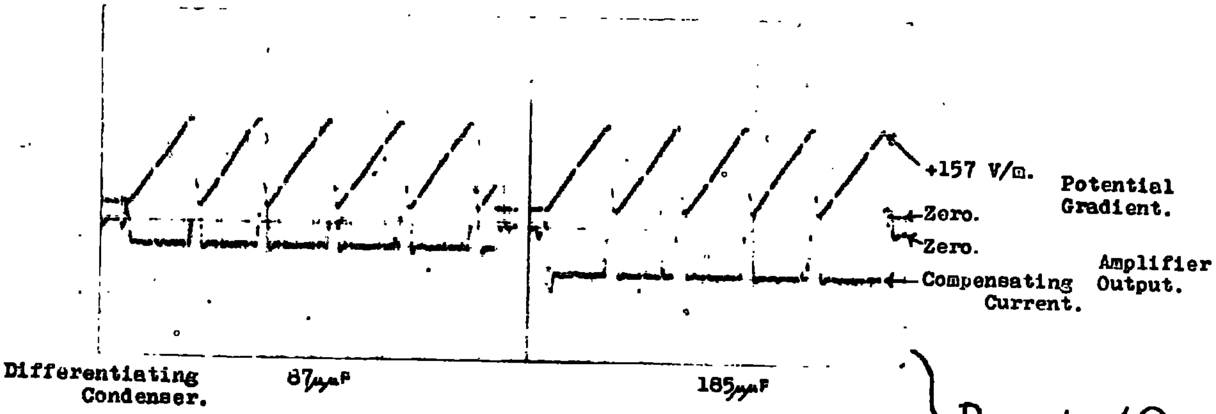
To effect this in practice, a method of applying a simple waveform of potential gradient to the air-earth current collector and the field mill has been devised. An aluminium plate 8' x 4', can be erected over the apparatus, at a height of approximately 50 cm. from the ground, with its legs standing on insulators. A rotary wirewound potentiometer is used to divide the fixed voltage from an H.T. battery and the tapped-off voltage is applied to the plate. T he potentiometer is rotated by a synchronous clock motor through a suitable gear chain. This unit is shown on the left of Plate 2. In this way a saw-tooth voltage is applied to the test plate and it was established that its sloping edge is an accurately linear function of the time. The response of the field mill to the wavefrom is shown in Figs. 18, 20, 25 and 26. In terms of displacement current an interpretation is simply obtained by differentiating. During the short 'dead' period before the steady rise in voltage, while the wiper arm is moving across the stud connected to the earthed terminal of the potentiometer, the displacement current is zero. At the onset of the steady rise, the displacement current undergoes a step function to a value which is maintained during the linear part of the waveform. When the wiper arm reaches the end of its traverse, the applied voltage remains at a steady maximum for a short time while the contact moves over the second stud, and there is a step function of displacement current back to zero again. Subsequently the voltage on the plate suddenly returns from its maximum value to zero and there is an impulsive negative displacement current, i.e. a very large negative current acting over a very short time. The cycle then begins again. It was not expected that the compensation would be good enough to maintain a zero output during the impulse just described, but it was hoped that the step functions corresponding to the beginning and end of the steady rise in voltage would be compensated satisfactorily.

It was proposed first to satisfy the condition for a steady displacement current by waiting for the transient response to the step function at the beginning of the cycle to die away, and then adjusting the differentiating condenser D so that there was zero output from the D.C. amplifier during the remainder of the period of steady displacement current. So that there would be plenty of time during the cycle to ascertain whether the compensation was accurate, the traverse of the potentiometer was arranged to last much longer than the expected time constant of the system (5.5 sec.). The linear increase of potential was accomplished in 106 sec. After the compensation for the steady state had been achieved, the condenser X shunting the collector input resistor would be adjusted until the transient at the beginning of the cycle was reduced to zero. This of course would not affect the compensation for the steady displacement current, and the adjustment would then be complete.

There is a complication in that owing to the conductivity of the atmosphere, an ionic current would flow into the collector when the test plate was not at earth. Therefore it would be strictly incorrect to adjust the differentiating condenser so that the output of the D.C. amplifier was zero. However, if the period of the saw-tooth waveform of the applied potential were short enough, the displacement current would be large compared with the conduction current, and the error introduced by neglecting the latter when the adjustments were made would be negligible. This is further discussed in § 6.3.

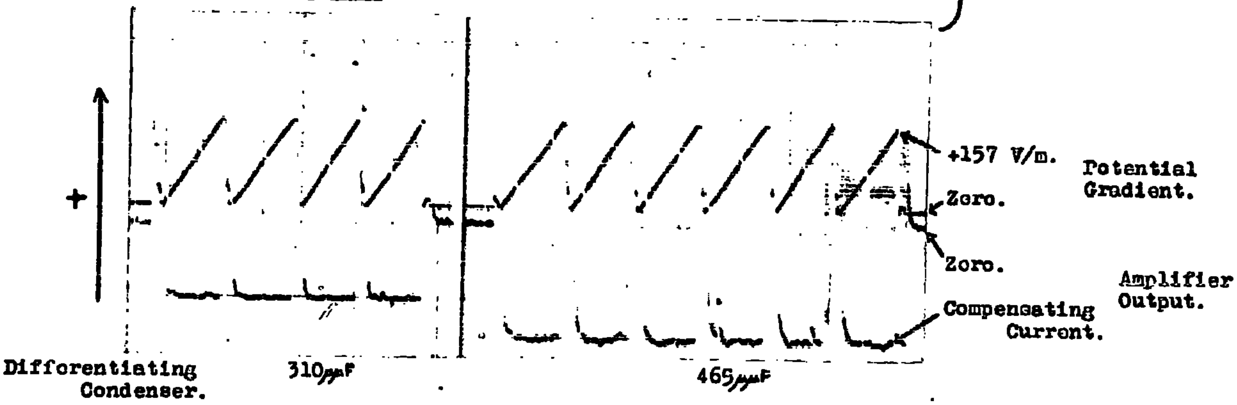
6.3 The potential gradient waveform applied to the collector alone.

As a preliminary, the varying potential gradient produced by the motor-driven potentiometer was applied to either section of the measuring equipment in turn. In the first series of experiments the differentiating circuit was disconnected from the field mill and the compensating grid of the electrometer stage held at earth. Thus the effect of the displacement current acting only on the air-earth current collecting side of the circuit, could be observed. The behaviour under these circumstances is shown in Record 38 (a), Fig. 18, where the field mill output and the response of the D.C. amplifier are recorded together. During the linear part of the sawtooth waveform, the voltage applied to the test plate increased from zero to 72 V. in 106 sec. The distance between the plate and the ground was 46 cm. The displacement



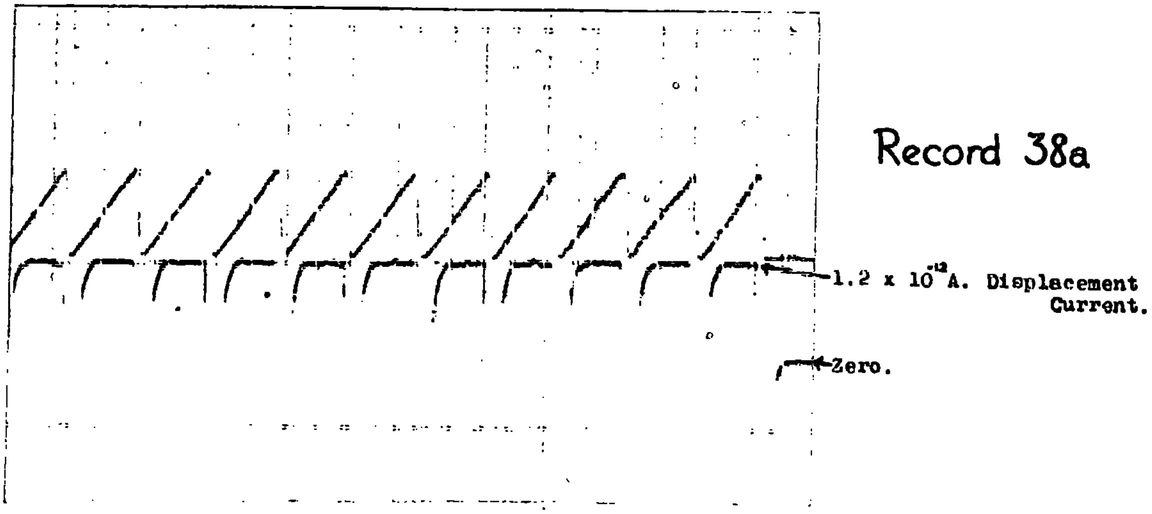
Output from Mill fed through Differentiating Circuit to Compensating Grid.

Record 40



Output from Mill fed through Differentiating Circuit to Compensating Grid.

Record 38a



Potential Gradient Applied to Collector Alone: True Displacement Current.

Fig. 18 Output of D.C. Amplifier for Saw-tooth Potential Gradient Applied to Each Section in Turn.

current effect on the D.C. amplifier is clearly seen, and the record is an experimental demonstration of how a uniformly changing potential gradient corresponds to a uniform displacement current. The transient between each period of steady deflection is easily explained in the terms set forth in § 6.2. The time constant of the current collecting side of the equipment was set to approximately 5 sec. for Record 38(a) by adjusting the condenser X. However, it was established that the final steady value of the output in any cycle was not dependent on X, which only affected the time taken to arrive at this value.

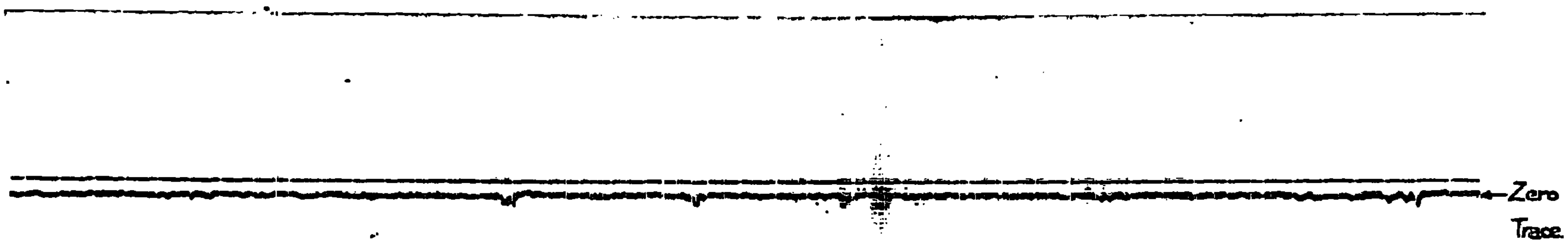
An interesting by-product of this experiment was that it enabled an estimate to be made of the effective area of the collector. From the known rate of increase of potential gradient applied to the collector, the displacement current density could be calculated. This worked out to be 1.31×10^{-11} A/m^2. On the other hand, from the output of the D.C. amplifier and its known sensitivity, the total current flowing into the collector was 1.24×10^{-12} A. as indicated in Fig. 18. From these two quantities, the effective area of the collector is obtained as 945 cm^2., whereas its geometrical aperture is approximately 1900 cm^2. The importance of this has been discussed in § 4.2.

A measurement of the response of the system to a constant applied voltage on the test plate was made, with the compensating grid still at earth potential, and the result is shown in Record 43, Fig. 19. The first $\frac{1}{8}$ in. or so at the left of the trace shows the outputs of the field mill and the D.C. amplifier with no applied voltage and the remainder demonstrates the application of a steady potential gradient of 157 V/m. (the same as the maximum of the sawtooth waveform). The amplifier output has changed by an equivalent of 1.2 x 10^{-13}A. input current. This is presumably the amount of conduction current driven across the gap by the potential gradient. It has already been stated that the displacement current induced by the sawtooth waveform is 1.24 x 10^{-12}A., so that by adjusting the differentiating condenser to give zero output in the tests proposed in § 6.2, the error in compensation must be 10% of the displacement current. However, this was regarded as being not unsatisfactory in the initial programme.

The other records in Fig. 19 are quite instructive; all three were taken on the same day. Between Record 42 and either of Records 43 and 44, there is a marked difference in the steadiness of the D.C. amplifier output. Record 42 was taken with an earthed metal plate placed over the collector and very close to it, so presumably the small fluctuations in the other two records were caused by space charge being blown into the collector. While the difference between Records 43 and 44 could have been caused by changes in the electronic equipment itself, it seems a reasonable supposition that the increase in fluctuations in Record 43 might have been due to the effect of the potential gradient, in addition to the wind, driving space charge into the collector.

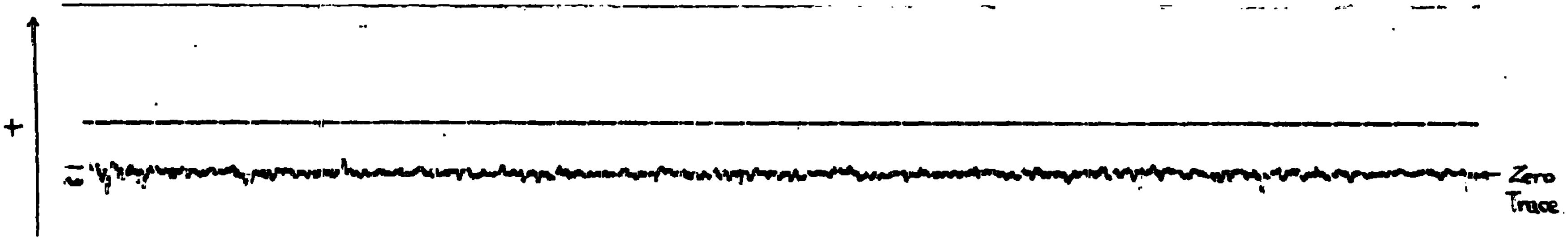
6.4 The potential gradient waveform applied to the field mill alone

In a further set of experiments, the hemispherical collector was screened by an earthed plate close over it with the test plate still in position, and the field mill was reconnected to the differentiating circuit so that the behaviour of the amplifier with the compensating signal only, could be observed, The performance when the sawtooth



Earthed Plate: Close over Collector.

Record 42



Collector Exposed to Uniform Potential Gradient (160 V/m.) from Test Plate.

Record 43

Sensitivity 1.6 mm/10^{-9}A
(on Original Records)

Collector Exposed to Zero Potential Gradient from Test Plate.

Record 44

Fig. 19 Zero Traces of D.C. Amplifier.

waveform of potential was applied to the test plate is shown in Record 40, Fig. 18, with various settings of the differentiating condenser. As far as can be judged from the record, the shape of the curves of amplifier output, for some setting of the differentiating condenser between 310$\mu\mu$F and 465$\mu\mu$F, is the negative of that observed with the potential gradient applied only to the collector. By plotting the compensating current against the setting of the condenser, it was established that the condenser must be adjusted to 360$\mu\mu$F in order to compensate the displacement current observed in Record 38 (a). The time constant of the differentiating circuit ($S = 9.5 \times 10^9$ ohm) would therefore be 3.4 sec., which permits a prediction to be made of the required setting of the condenser X, shunting the input resistor on the current collecting side. The experimental value, $\gamma = 3.4$ sec. can be compared with that deduced in § 5.2 from Eq. 6.1: $\gamma = 5.5$ sec. This was based on the values $V_m/F = 0.0195$ V per V/m., and $A = 0.19$ m^2. However, in § 6.3 it was seen that the effective value of A is 0.0945 m^2, while, in the situation in which the mill is actually used, $V_m/F = 0.0164$V per V/m., as explained in § 5.8. Making the correction, it is found that $\gamma = 3.3$ sec., in fair agreement with the experimental value.

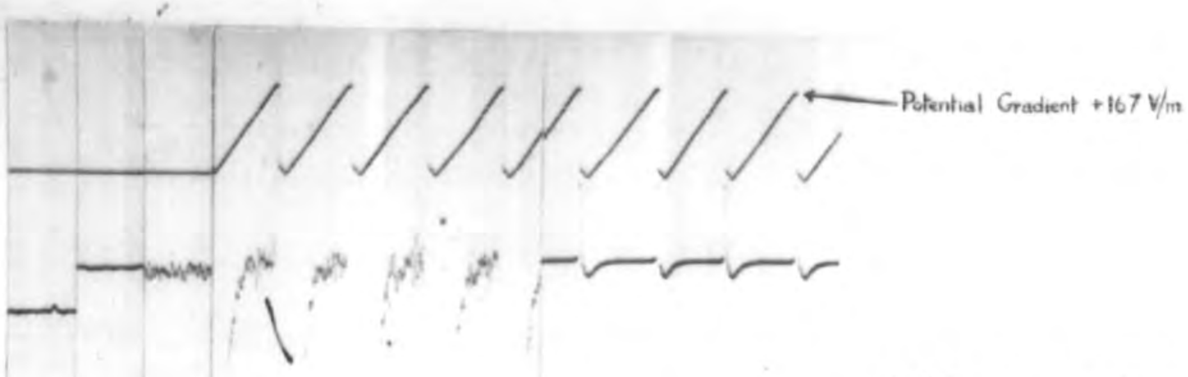
Although in general the results obtained when the effect of the varying potential gradient was transmitted through the differentiating circuit were satisfactory, a disturbing feature was that sudden fairly large deflections were observed in the amplifier output when this should have been steady. Careful examination of the field mill output revealed no spurious effects large enough to cause such deflections. The fluctuations in the mill output were never greater than an equivalent input of 0.01 V/m., and generally they were only half of this; if the theory of the apparatus which had been developed so far were correct, the effect on the amplifier output should have been negligible. It was discovered later that fluctuations in mill output could indeed have been responsible for these effects, but only because, for rapid changes at the compensating grid of the electrometer stage, the feedback loop is inoperative, and the full loop gain of 320 is applied to the signal. This is fully discussed in § 7.4 et seq.

6.5 Failure of the first attempt to achieve compensation

The differentiating condenser D and the shunt condenser X (Fig. 17), were adjusted to the approximate values deduced in § 6.4, and both grids of the electrometer stage were subjected to the sawtooth potential gradient through their respective circuits. The results of this first attempt at compensation are shown in Records 45, 46 and 47, Fig. 20. In preparing this figure, all the records had vertical sections cut from the relevant parts, and these were mounted on white card and photographed. However, the relative dispositions of the traces are preserved as in the original records, so that the D.C. amplifier zero level given in Sections 2 and 3 of each record is the level at which the output should have remained during the potential gradient cycles, if the compensation were perfect. It must be mentioned that the change in 'zero' level between Sections 1 and 2 of each record is also real, but the cause of this was never discovered. For some unknown reason, whether or not the collector was connected to the electrometer stage, even when it was receiving no current, made a considerable difference to the 'zero'. However, in practice, this was only a minor inconvenience, and a reproducible zero could be obtained, with the collector connected, by placing the earthed test plate over it. This is the zero shown in Sections 2 and 3 of all the records in Fig. 20.

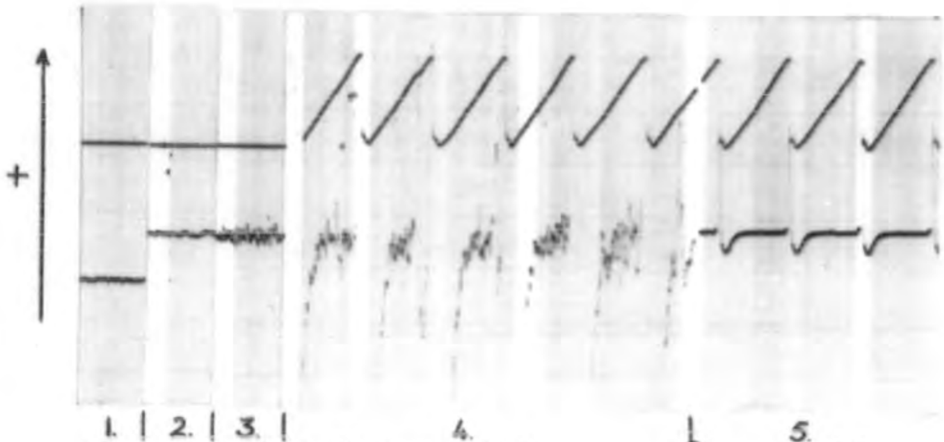
The records differ only in the value of the condenser X , shunting the input resistor, and the capacitance increases as the setting in degrees increases: the condenser was not calibrated. The differentiating condenser was set at approximately 360μμF throughout. From Sections 4 and 5 of Records 45 and 46, it can be seen that some measure of compensation was achieved for steady displacement currents, but not in any degree for the sudden changes at the beginning and end of each cycle.

The most remarkable feature is shown in Sections 5 of the records, where the low sensitivity setting of the galvanometer enables the transients to be seen clearly. These showed a large deflection in the positive direction, which decreased as the shunt condenser increased. Now when the second grid of the electrometer stage was held at earth so that there was no compensating signal, the deflection was negative at the sudden fall of potential gradient to zero, and the magnitude decreased as the shunt condenser increased, as expected. From the simple picture outlined in § 6.2, the transient should be the algebraic sum of two independent time constant effects. Since the time constant of the differentiating circuit was not altered in any of the records in Fig. 20 the contribution from this side should have remained constant,



Position of Shunt Condenser 80° Record 45

1. Mill exposed to uniform potential gradient; collector disconnected.
2. Collector exposed to uniform potential gradient; mill disconnected.
3. Mill and collector exposed to uniform potential gradient.
4. Mill and collector exposed to sawtooth potential gradient.
5. Same as 4: Low Sensitivity.

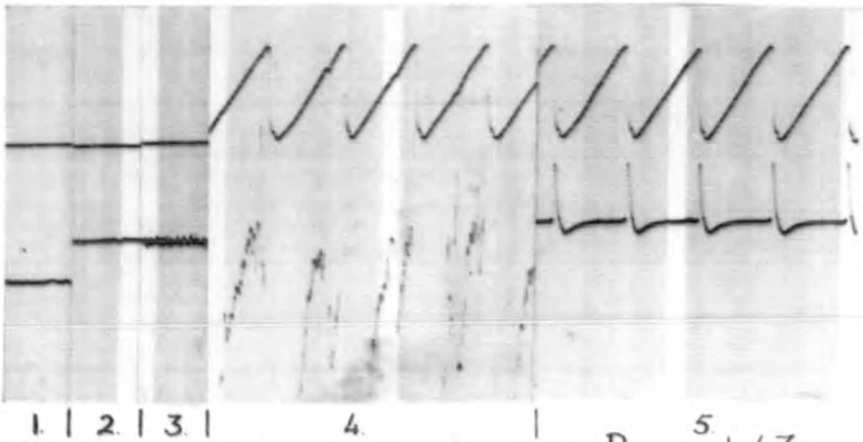


Position of Shunt Condenser 60° Record 46

High Sensitivity: 1.6 mm. / 10^{-9} A.

Low Sensitivity: 2.2 mm. / 10^{-8} A.

(on Original Records)



Position of Shunt Condenser 100° Record 47

Fig. 20 Compensation Defective Owing to Capacitance of Input Cable.

and therefore, the resultant positive deflection should have increased as the shunt condenser was increased, instead of which the reverse happened.

It was therefore concluded that the theory of independent time constants described in § 6.2 is not correct. Part of the transient caused by the compensating signal must contain a time constant which is affected by varying the shunt condenser; this is equivalent to the 'crossing over' of a parameter, as explained in § 3.3. An important clue as to where the connection might be was afforded by a result shown on closer inspection of Fig. 20. In Section 2 of each record is the expected result, that exposure of the collector to the test plate, when this has no voltage on it, and with the field mill disconnected, did not affect the output of the D.C. amplifier. In Section 1, apart from the change in zero level, it is also seen that when the mill was connected via the differentiating circuit to the compensating grid, the output remained very steady provided the collector was disconnected from the first grid of the electrometer stage. However, as Section 3 shows, connecting the collector caused the output to become much less stable, even though the collector was not receiving any signal from the test plate. The instability decreased as the shunt condenser was made larger; but the important conclusion remains, that the capacity of the collector and its cable, hitherto neglected, has a disturbing effect on the behaviour of the circuit.

It was quickly established experimentally that it would be impossible to obtain results in the smallest degree satisfactory with the existing arrangement. As it was not obvious in what respect the circuit would have to be altered, except the impractical one of reduction of the capacity of the collector and its cable, a complete theoretical study of the system was undertaken. This is described in the following chapter.

CHAPTER 7. ANALYSIS OF THE CIRCUIT BEHAVIOUR

7.1 The equivalent circuit of the apparatus.

In order to visualise the way in which the amplifier and field mill may be described mathematically, the circuit parameters are rearranged into the equivalent circuit of Fig. 17. The inset in the centre of this figure is a smoothing circuit eventually needed to follow the field mill output; this has not entered into the description so far and will be ignored for the present.

Beginning at the left of the equivalent circuit, the input current $I(t)$, assumed to be a displacement current, flows through the input resistor R on the air-earth current grid G_1 of the D.C. amplifier, producing a voltage V_1 on that grid. The other end of R is connected to the feedback loop which is the same point as the final output from the cathode follower, this output being V_0 . V_0 arises, of course, not only from the displacement current into the collector, but also from the output V_m of the field mill, the differential of which, through D, S is applied as a voltage V_2 on the other grid, G_2 . Because of the capacity to earth of the collecting hemisphere and its cable, the flow of current through R is delayed. The effect is allowed for by showing the lumped capacity in the network as C . X is the variable shunt condenser described in § 6.2. The centre section of the equivalent circuit represents the final cathode follower. Here the electrometer stage and the main amplifier have been considered not to introduce any phase change, and their effect is implicit in the voltage $G(V_2 - V_1)$ ascribed to the cathode follower grid G_3, G being the overall gain between input and cathode follower. The cathode follower is equivalent to a generator and the expression for the voltage generated indicates that it is the amplification factor multiplied by the difference between the grid voltage and the cathode feedback voltage. The anode resistance of the cathode follower is represented by r_a, and Y is the parallel combination of the cathode resistance and the resistance of the recording galvanometer. Y is closely equal to the cathode resistance alone, however, because the galvanometer resistance is much greater than this.

When the output is caused by an input on only one of the two grids, V_0 will be replaced by V_A or V_B, corresponding to whether the signal is on G_1 alone, or G_2. Thus, $V_0 = V_A + V_B$

7.2 The Laplace transform

A detailed discussion of the Laplace transform would be out of place here, but it is worth pointing out the main advantages of this mathematical device as applied to circuit theory. All the fundamentals required in this discussion are dealt with by Jaeger (1949).

It may be supposed that a differential equation in y as a function of t must be solved, where

$$A\frac{d^n y}{dt^n} + B\frac{d^{n-1} y}{dt^{n-1}} + \cdots\cdots Yy + Z = f(t) \qquad \text{...... Eq. 7.1.}$$

The purpose of the transform is to enable the operators $\frac{d^n y}{dt^n}$, etc., to be replaced by algebraic symbols of the type $\bar{y} . u(p)$, where $\bar{y}$ is called the transform of the solution for y and u is a function of p, a parameter introduced by the nature of the transform. The equation is thus altered to the form

$$\left[A u_n(p) + B u_{n-1}(p) + \cdots\cdots Y u_1(p)\right]\bar{y} + u_0(p) Z = \bar{f}(p) + \phi$$

where ϕ is a function of the boundary conditions and $\bar{f}(p)$ is the transform of $f(t)$. $\bar{y}$ is obtained as a function of p, by solving the now algebraic equation, and y determined by performing the inverse of the transform process on $\bar{y}$.

The Laplace transform is defined by the equation:

$$\bar{f}(p) \equiv \int_0^\infty e^{-pt} f(t)\,dt$$

The particular relevance to network theory of the kind in question here is twofold. First, given zero initial conditions, the transformation is equivalent to writing Kirchoff's laws using, instead of the actual currents and voltages, their transforms $\bar{I}$ and $\bar{V}$; and writing down the effects of a resistance R, a capacity C and an inductance L as the impedances R, $1/pC$ and pL respectively. It is evident that there is a close analogy here to the use of the symbol $j\omega$ instead of the operator d/dt to find the particular integral in a problem involving alternating voltages of angular frequency ω. Secondly, the transforms of square and impulsive functions of impressed currents and voltages, (such as will appear in $f(t)$ on the right hand side of Eq. 7.1 in the present problem) are particularly simple functions of p.

7.3 Types of input and solution

The main features of the sawtooth waveform of potential gradient used to test the apparatus may be described by two functions, for which references should be made to the insets in Figs. 21 and 22.

Type A: Fig. 21

First it is supposed that the potential gradient has been zero from $t \rightarrow -\infty$ to $t = 0$, and then begins to rise at a rate F_i V/m. per sec.

The transform of $F(t) = F_i t$ is $\bar{F}(p) = F_i/p^2$

Corresponding to this, the displacement current $I(t)$ will remain at zero until $t = 0$ and will then undergo a step change to a constant value $I_i = F_i \varepsilon_0$

The transform of $I(t)$ is $\bar{I}(p) = I_i/p$

Type B: Fig. 22

Next a step function of potential gradient is considered, such that $F = 0$ from $t \rightarrow -\infty$ to $t = 0$, and then rises instantaneously (ideally) to the value F_i. The displacement current then rises impulsively to a very large value and falls to zero again at the instant $t = 0$. Essentially the nature of the impulsive function is indeterminate, but for convenience it may be supposed that the potential gradient takes an infinitesimal time ϵ to rise to F_i. Then

$$I(t) = I_i \delta(t); \quad \text{where } \delta(t) = 0,\ t \leq 0 \ ; \delta(t) = 1/\epsilon,\ 0 < t < \epsilon; \ \delta(t) = 0,\ \epsilon \leq t$$

I_i is again related to F_i by $I_i = F_i \varepsilon_0$. The transforms of this function of potential gradient and the corresponding displacement

displacement current are:

$$\bar{F}(p) = F_i(p) \; ; \; \bar{I}(p) = I_i$$

It is found that the transform of the solution for the output voltage, in every case, can be split into partial fractions and expressed in the form

$$\bar{V}_0 = F_i \left[A + \frac{B}{p} + \frac{C_1}{p+\lambda_1} + \frac{C_2}{p+\lambda_2} \cdots\cdots + \frac{C_n}{p+\lambda_n} + \frac{D}{(p+\lambda)^2} \right]$$

where any of the constants A, B, $C_{1,\ldots,n}$, D may be zero. The solution of the problem, the inverse transform of $\bar{V}_0$, is

$$V_0 = F_i \left[A\,\delta(t) + B + C_1 e^{-\lambda_1 t} \cdots\cdots + C_n e^{-\lambda_n t} + D t e^{-\lambda t} \right]$$

The terms in the bracket correspond respectively to those in the bracket of the expression for $\bar{V}_0$. The last term arises when two circuits having the same time constant are connected in series, and λ will be equal to one of the λ_n's.

7.4 Preliminary calculation to show the main features of the circuit behaviour

In this section the currents in the network are solved and the output voltage obtained with the simplifying assumption that the time constant of the differentiating circuit is zero. This enables one to see very clearly in what fundamental respect the circuit misbehaves. The same irregularity is of course implicit in the equations when the differentiating time constant is taken into account, but the complexity is greatly increased by including it and somewhat masks the conclusion to be drawn.

Except in one or two cases which will serve as examples of the method, results will be quoted without the calculations leading up to them, as these are simple though tedious. Calculations are made of:

(a) The general equation for the output V_A of the D.C. amplifier for any displacement current affecting only the collector.

(b) The general equation for V_B , the output of the amplifier, when any voltage $V_2(t)$ is applied to the compensating grid; (collector screened off).

(c) The resultant output V_0 when simultaneous step functions of current and voltage are applied to G_1 and G_2 respectively.

(d) The resultant output V_0 when simultaneous impulsive functions of current and voltage are applied to G_1 and G_2.

Articles (a) and (b) will obviously be of direct application to any problem concerning the amplifier; (c) and (d) are equivalent to assuming that the compensating signal applied by the mill is treated by an ideal circuit which differentiates it without introducing a time constant.

(a) <u>General equation for V_A, displacement current $I(t)$ affecting only the collector</u>

The transforms of Kirchoff's equations for the closed loops in which the currents ι_1, ι_2 , flow (Fig. 17) are:

$$\left[Y + \frac{1}{pC} + \frac{1}{pX + 1/R}\right]\bar{\iota}_1 - Y\bar{\iota}_2 = \frac{1}{pC}\bar{I}(p) \qquad \text{...... Eq. 7.2}$$

$$-\mu\left[\bar{V}_1 G + Y(\bar{i}_1 - \bar{i}_2)\right] = -r_a \bar{i}_2 + Y(\bar{i}_1 - \bar{i}_2) \qquad \text{...... Eq.7.3}$$

Since $\bar{V}_1 = \left(\bar{I}(p) - \bar{i}_1\right)/pC$ and $\mu/r_a = g$, the transconductance of the cathode follower, Eq. 7.3 may be rewritten:

$$\left(Y - G/pC_1\right)\bar{i}_1 - \left(Y + 1/g\right)\bar{i}_2 = -\bar{I}(p)\, G/pC \qquad \text{...... Eq.7.4}$$

Equations 7.2 and 7.4 are solved for $\bar{i}_1$ and $\bar{i}_2$, making the justifiable assumptions that $Y >> 1/g$, the normal cathode follower condition, and $G >> 1$; ($G \simeq 320$, by experiment). Then the transform of the solution is

$$\bar{V}_A = Y(\bar{i}_1 - \bar{i}_2) = \frac{-I(p)}{\left[X + C/G\right]\left[p + \frac{1}{R(X + C/G)}\right]} \qquad \text{...... Eq.7.5}$$

(b) <u>General equation for V_B, voltage $V_2(t)$ on G_2 only.</u>

The equations for the two loops are of the same form as equations 7.2 and 7.3 except that there is now a voltage on G_2 and $I(t) = 0$. The solution is

$$\bar{V}_B = \bar{V}_2(p)\left[\frac{X + C}{X + C/G}\right]\left[\frac{p + \frac{1}{R(X+C)}}{p + \frac{1}{R(X + C/G)}}\right] \qquad \text{...... Eq.7.6}$$

(c) <u>Particular case when step functions of current and voltage are applied</u>

Inputs of the form described in § 7.3 are substituted into Equations 7.5 and 7.6: $\bar{I}(p) = I_i/p$; $\bar{V}_2(p) = V_i/p$.

Thus $\overline{V}_A = \dfrac{-I_i}{p\left[x + C/G\right]\left[p + \dfrac{1}{R(x + C/G)}\right]}$

$$= -RI_i\left[\frac{1}{p} - \frac{1}{p + \dfrac{1}{R(x + C/G)}}\right] \qquad \text{...... Eq.7.7}$$

The inverse transform of $\overline{V}_A$ is

$$V_A = -RI_i\left[1 - e^{\frac{-t}{R(x+C/G)}}\right] \qquad \text{...... Eq.7.8}$$

It will be noticed that this is the equation that was discussed in previous chapters, where it was assumed that $C = 0$, and that the shunt condenser X would cause the current flow into R to be delayed with a time constant RX.

Eq. 7.6 yields the solution

$$V_B = V_i\left[1 - \left(1 - \frac{X+C}{X+C/G}\right)e^{\frac{-t}{R(X+C/G)}}\right] \qquad \text{...... Eq.7.9}$$

Since it is assumed that the compensation is effective for uniform displacement currents, $V_i = RI_i$ and the resultant output is

$$V_o = V_A + V_B = \left[\frac{X+C}{X+C/G}\right]e^{\frac{-t}{R(X+C/G)}} \qquad \text{...... Eq.7.10}$$

It will be remembered that in Chapters 2 and 6 it was not expected that the time constant of the air-earth current collector would appear in the equation for V_B. It only does so on account of C, as can be seen by putting $C = 0$ in Eq. 7.9 when it is found that $V_B = V_i$.

Considering Eq. 7.10 in more detail, if an ideal differentiating circuit had been available, there would have been no need for the shunt condenser X (see § 6.2). With $X = 0$ in Eq. 7.10, the resultant output is

$$V_o = G e^{\frac{-t}{RC/G}} \qquad \text{...... Eq.7.11}$$

Thus instead of perfect compensation there would be a very large, though rapid transient, with a peak value corresponding to the full loop gain of the amplifier acting on the displacement current. It is also to be remarked that the transient of Eq. 7.10 is in the direction of the V_B component of V_o and decreases as X increases. The experimental observation of this property was commented on in § 6.5 as being quite unexpected.

(d) <u>Particular case when impulsive functions of current and voltage are applied</u>

$\bar{I}(p) = I_i$ and $\bar{V}_2(p) = V_i$ are substituted into Equations 7.5 and 7.6.

Thus $$V_A = \frac{I_i}{\left[X + C/G\right]} e^{\frac{-t}{R(X+C/G)}} \qquad \text{...... Eq.7.12}$$

and $$V_B = \left[\frac{X+C}{X+C/G}\right]\left[\delta(t) - \frac{C}{R(X+C)(X+C/G)} e^{\frac{-t}{R(X+C/G)}}\right] \qquad \text{...... Eq.7.13}$$

Therefore $$V_o = \frac{I_i}{\left[X+C/G\right]}\left[R(X+C)\delta(t) - \left(\frac{C}{X+C/G} + 1\right) e^{\frac{-t}{R(X+C/G)}}\right] \qquad \text{..... Eq.7.14}$$

In an hypothetical experiment with an ideal differentiating circuit, $X = 0$ and

$$V_0 = G I_i \left[R\delta(t) - \frac{G}{C} e^{\frac{-t}{RC/G}} \right] \qquad \text{.....Eq.7.15}$$

It will again be noticed that with $C = 0$, Eq. 7.13 gives $V_s = V_i \delta(t)$, which is the result assumed in Chapter 6.

From Equations 7.11 and 7.15 it is evident, from the G-multiplied term, that in some way the feedback loop is inoperative for sudden changes on the compensating grid, and further, that this is caused by the input cable capacitance C, since when $C = 0$ the expressions for V_s revert to the forms previously expected. In the description of the way in which a signal on G_2 produced an output, it was remarked in § 4.7 that a kind of feedback operated which made the gain unity for signals on that grid as well as for those across R. This is because a voltage V_2 on G_2 causes the output voltage to appear on G_1 and this voltage adjusts itself, on account of the differential behaviour of the electrometer stage, so that a null signal is applied between the grids. Considering, however, an instantaneous voltage applied to G_2, a similar change will occur in the potential of the feedback loop, but the transmission of this to G_1 will be delayed because the cable capacitance drains current from R, preventing an instantaneous rise in the voltage on G_1. For a short time, depending on the time constant RC/G, the voltage on G_1 corresponding to V_2 remains much smaller than V_2, so that approximately the entire loop gain, G, is applied to this signal, as is directly evident from Eq. 7.11. Transients of the same order of magnitude as the displacement current had been envisaged in setting up the apparatus, but hardly transients 300 times as large.

7.5 Detailed calculation of the circuit behaviour

The main conclusions of § 7.4 apply also to the equations in this section, although the introduction of two more time constants makes them considerably more complicated. One of these is $\gamma = SD$, the time constant of the differentiating circuit, which has so far been ignored. The other is the time constant of the field mill circuit. Now this time constant, as it has been described so far, is approximately 0.1 sec., much smaller than any of the time constants in the rest of the equipment. However, it was thought that a solution to the problem of avoiding large transients might be to use the smoothing circuit shown in the inset, Fig. 17. This would be inserted between the field mill amplifier and the input to the differentiating circuit, and would introduce a comparatively large time constant $\tau = R'C'$, so that the input to the differentiating condenser, for a step function F_i of potential gradient, would be

$$V_m' = KF_i(1 - e^{-t/\tau}) \qquad \text{...... Eq.7.16}$$

where $K = V_m/F$, the sensitivity of the field mill circuit alone. τ will be called the time constant of the field mill.

To perform the calculations, the displacement current function on grid G_1 must be used with the corresponding potential gradient function applied to the field mill, as discussed in § 7.3. The results expressed by Equations 7.8 and 7.12 are unchanged by the introduction of the time constant τ, so one only need reconsider the contribution of V_B to the resultant output.

The sequence of calculations is as follows. It can easily be shown that the relation between the transform of the voltage V_2 applied to the compensating grid, and the transform of a generalized potential gradient function $F(t)$ is

$$\bar{V}_2 = K\bar{F}(p)\,\frac{p}{\tau\left(p+\frac{1}{\tau}\right)\left(p+\frac{1}{SD}\right)} \qquad \text{..... Eq.7.17}$$

It is then only necessary to multiply this expression by the right hand side of Eq. 7.6, to obtain the complete transfer function of the compensation side:-

$$\bar{V}_2 = K\bar{F}(p)\,\frac{p\left[X+C\right]\left[p+\frac{1}{R(X+C)}\right]}{\tau\left[p+\frac{1}{\tau}\right]\left[p+\frac{1}{SD}\right]\left[X+C/G\right]\left[p+\frac{1}{R(X+C/G)}\right]} \qquad \text{..... Eq.7.18}$$

To simplify the expressions the following symbols are used for the time constants: $\alpha \equiv R(X+C)$; $\beta \equiv R(X+C/G)$; $\gamma \equiv SD$. It must be remembered that α and β are not independently variable.

The calculations give the following results for the potential gradient functions of § 7.3

<u>Type A - Ramp change of potential gradient</u>

Eq. 7.18 yields

$$V_B = KF_i\gamma\left[1 - \frac{\tau(\tau-\alpha)}{(\tau-\gamma)(\tau-\beta)}e^{-\frac{t}{\tau}} - \frac{\gamma(\gamma-\alpha)}{(\gamma-\tau)(\gamma-\beta)}e^{-\frac{t}{\gamma}} - \frac{\beta(\beta-\alpha)}{(\beta-\tau)(\beta-\gamma)}e^{-\frac{t}{\beta}}\right]$$

..... Eq.7.19

To obtain V_o, the sum $V_A + V_B$ is formed, the appropriate expression for V_A being Eq. 7.8. Since the apparatus is supposed to be compensated for a steady displacement current, γ is adjusted so that

$$KF_i\gamma = RI_i \qquad \text{...... Eq. 7.20}$$

Still considering V_B only, there are two particular cases of interest here:

(i) $\tau = 0, \beta = \gamma$, the conditions in the experiments described in § 6.5. It must be observed that the solution cannot be obtained by substituting directly into Eq. 7.19, because zero terms arise in numerator and denominator of a term on simplification. The substitution must be made in Eq.7.18. Then $V_B = KF_i\gamma\left[1 - e^{-\frac{t}{\gamma}} + \left(\frac{\alpha}{\gamma} - 1\right)\frac{t}{\gamma}e^{-\frac{t}{\gamma}}\right]$ ... Eq.7.21

With Eq.7.8 this gives

$$V_o = KF_i\gamma\left[\frac{\alpha}{\gamma} - 1\right]\frac{t}{\gamma}e^{-\frac{t}{\gamma}} \qquad \text{... Eq.7.22}$$

Taking γ as 3.42 sec. and C = 700μμF (Kay, 1950), the result is a transient four times as large in magnitude as the steady effect of the same displacement current, uncompensated. It is also interesting to add the condition $C = 0$ to Eq. 7.22. Then $\alpha = \beta = \gamma$, and $V_o = 0$

(ii) $\tau = \beta \gg \gamma$

Then

$$V_B \simeq KF_i\gamma\left[1 - e^{-\frac{t}{\beta}} - \frac{\beta - \alpha}{\beta^2}te^{-\frac{t}{\beta}}\right. \qquad \text{... Eq.7.23}$$

If β is made sufficiently large by increasing X, $\beta \rightarrow \alpha$ and

$V_B \simeq KF_i\gamma\left[1 - e^{-\frac{t}{\beta}}\right]$. With Eq. 7.8 this gives $V_0 \simeq 0$. Thus by making the mill time constant and β , equal, and both so large that $\beta >> \gamma$ and $X >> C$, fair compensation will result for suddenly applied ramp changes of potential gradient.

Type B - Step function of potential gradient

The result of putting $\bar{F}(p) = F_i$ into Eq. 7.18 is

$$V_B = KF_i\gamma\left[\frac{\beta-\alpha}{(\beta-\tau)(\beta-\gamma)}e^{-\frac{t}{\beta}} + \frac{\gamma-\alpha}{(\gamma-\beta)(\gamma-\tau)}e^{-\frac{t}{\gamma}} + \frac{\tau-\alpha}{(\tau-\gamma)(\tau-\beta)}e^{-\frac{t}{\tau}}\right]$$

.....Eq.7.24

V_0 may be obtained by adding Eq. 7.12 and using Eq. 7.20.

Particular cases are considered:

(i) $\tau = 0$, $\beta = \gamma$, the conditions of § 6.5.

$$V_B = KF_i\gamma\left[\frac{\alpha}{\gamma^2}e^{-\frac{t}{\gamma}} + \frac{\gamma-\alpha}{\gamma^2}\,\frac{t}{\gamma}e^{-\frac{t}{\gamma}}\right]$$ Eq.7.25

If the condition $C = 0$ is imposed it is easy to show that $V_0 = V_A + V_B = 0$

(ii) $\tau = \beta$

$$V_B = KF_i\gamma\left[\frac{\gamma-\alpha}{(\gamma-\beta)^2}e^{-\frac{t}{\gamma}} - \frac{\gamma-\alpha}{(\gamma-\beta)^2}e^{-\frac{t}{\beta}} + \frac{\alpha-\beta}{\beta(\gamma-\beta)}\,\frac{t}{\beta}e^{-\frac{t}{\beta}}\right]$$

.....Eq.7.26

If X is so much greater than C that $\tau = \beta >> \gamma$ and $\beta \rightarrow \alpha$, then V_0 does not tend to zero identically (as with the Type A potential gradient function) but shows a short lived transient of small magnitude:

$$V_0 \simeq \frac{KF_i\gamma}{\beta} e^{-\frac{t}{\gamma}} \qquad \text{.....Eq. 7.27}$$

The results of some of these calculations are demonstrated graphically in Figs. 21 to 24, where the resultant output V_0 is plotted against time for different combinations of the circuit parameters. It is assumed that the compensation is satisfactory for uniform displacement currents, once the transient has died away, so $V_0 \rightarrow 0$ as $t \rightarrow \infty$. The abscissa is a 'reduced' time, t/β, since $\beta = R\left(x + C/G\right)$ is the greatest time constant involved in the equations and is therefore paramount in controlling the rate of rise and decay of the transients. The unit of ordinate is $RI_i = KF_i\gamma$ and is therefore of the same magnitude as, but of opposite sign to the steady output which would have resulted if the displacement current had been completely uncompensated. The measured values of the parameters required for the calculations are $C = 700\mu\mu F$, $\gamma = 3.4$ sec.

Figs. 21 and 22 are relevant to the state of the apparatus in the experiments described in Chapter 6, with step and impulsive functions of displacement current, respectively. Of each set of curves, the condition $\beta/\gamma = 1$ is the one which was aimed for in the first attempts at compensation. It can be seen from Fig. 21 that the transient is four times as large as the actual displacement current for this condition, and the behaviour of the curves with increasing β explains the unexpected

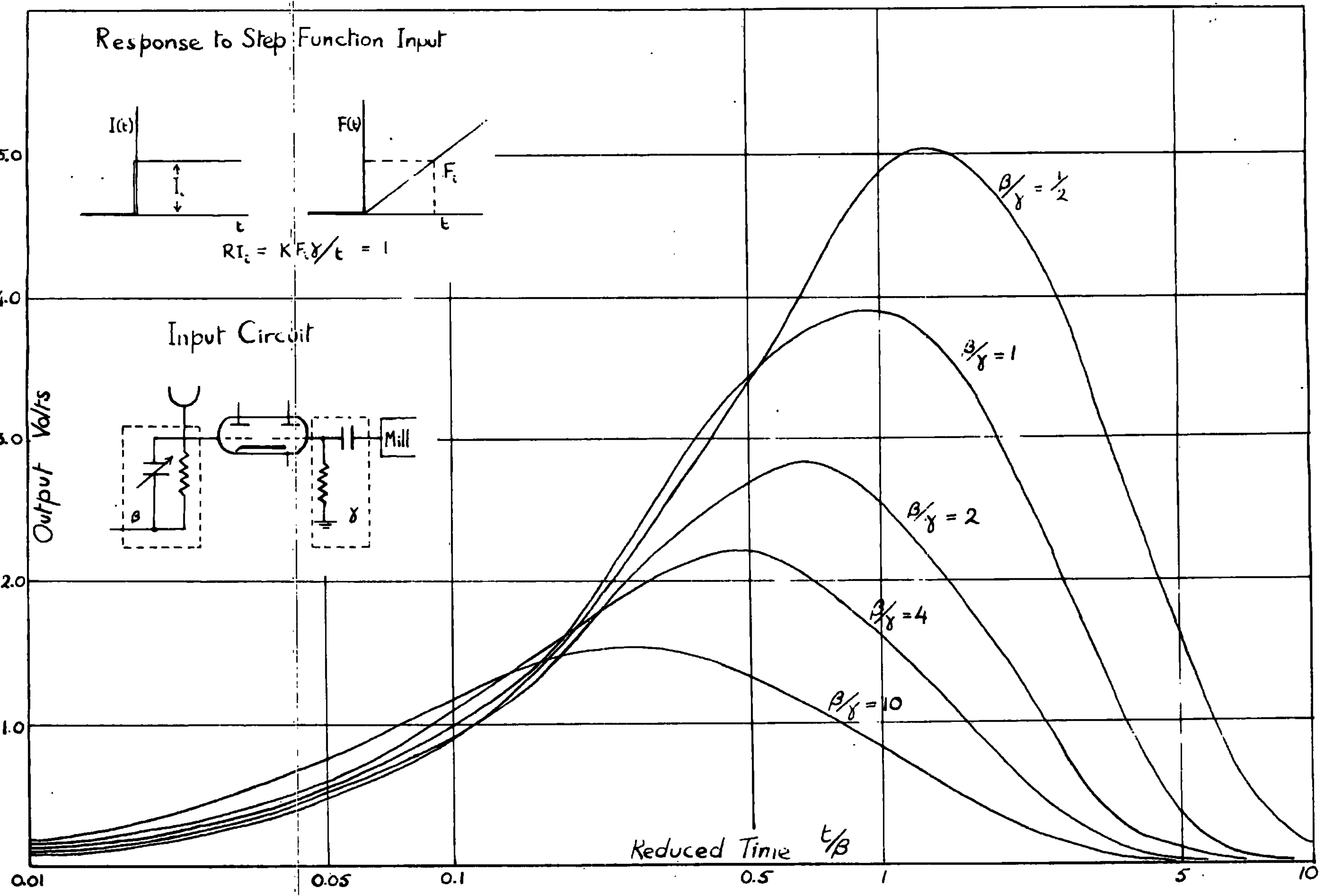


Fig. 21 Output of Amplifier Balanced for Steady Displacement Current

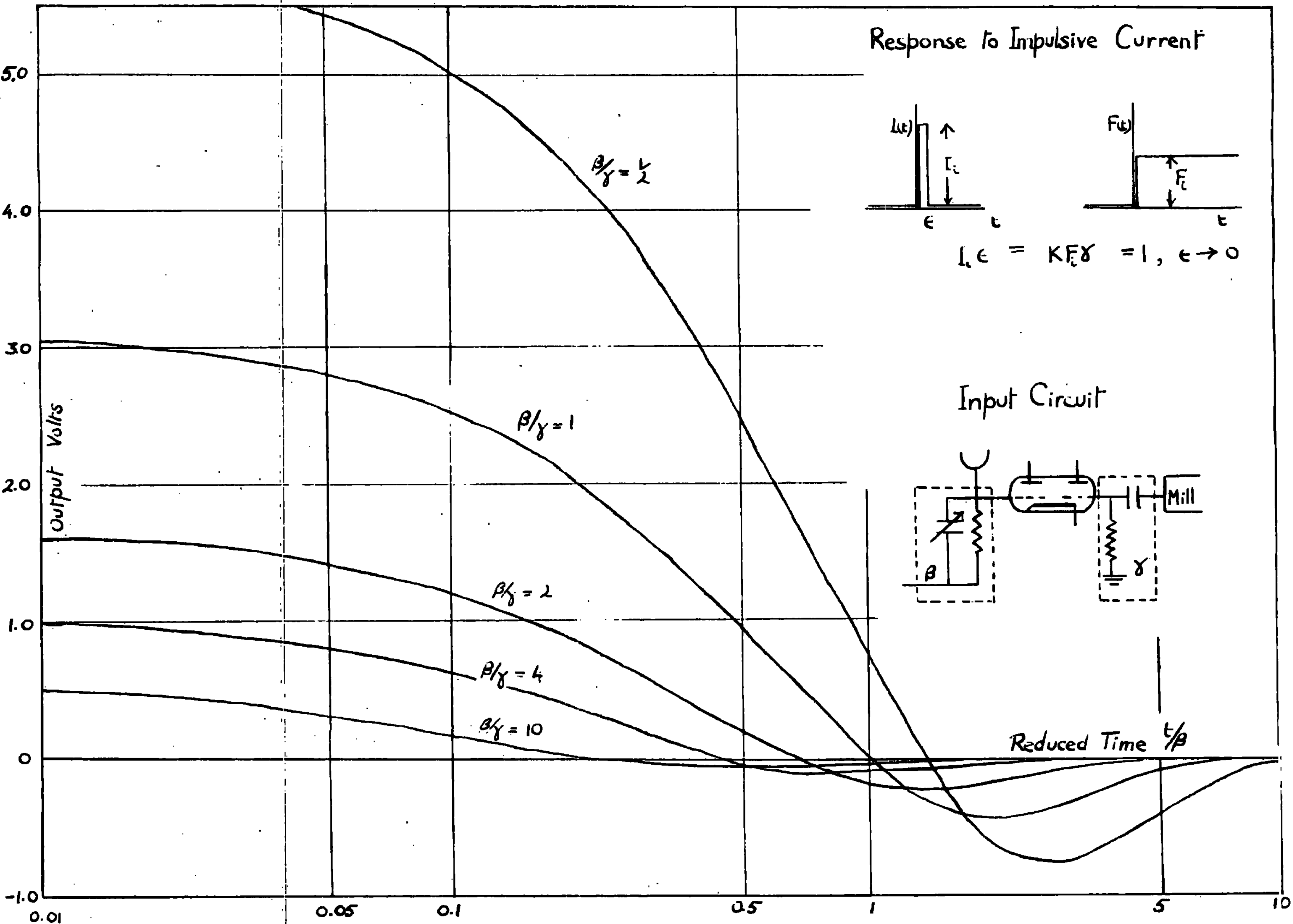


Fig. 22 Output of Amplifier Balanced for Steady Displacement Current.

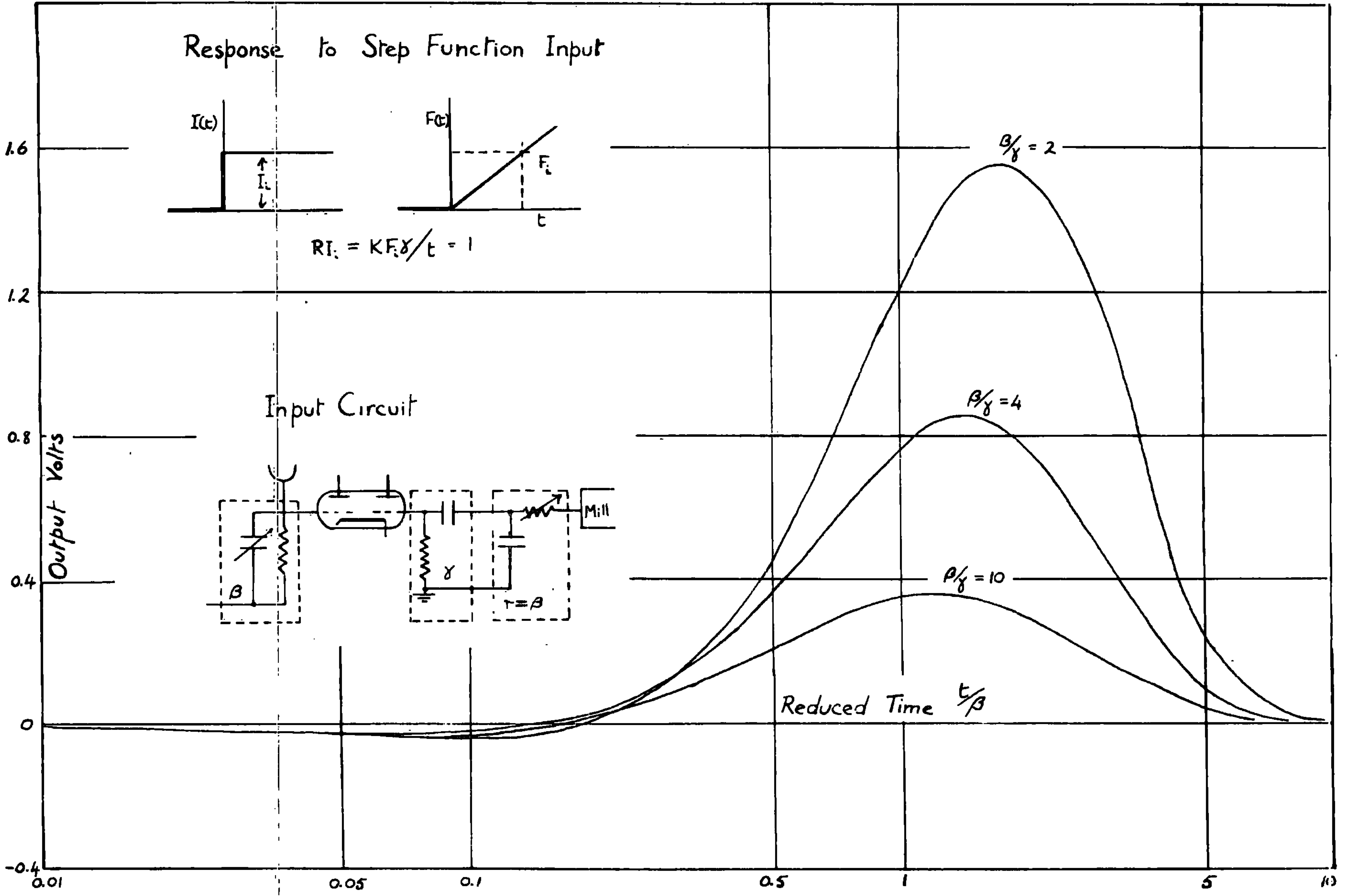


Fig. 23 Output of Amplifier Balanced for Steady Displacement Current

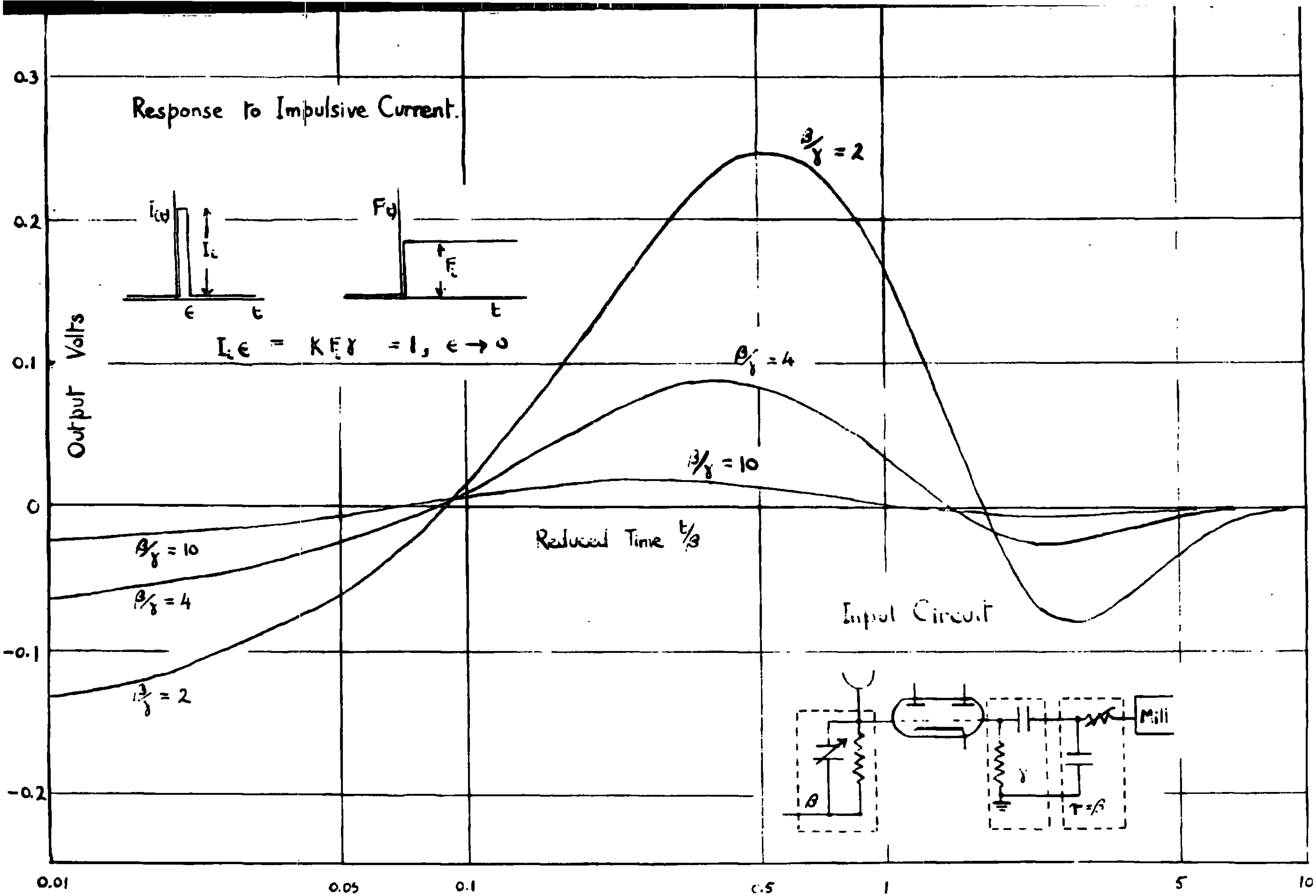


Fig. 24. Output of Amplifier Balanced for Steady Displacement Current.

observation of § 6.5, that the transient is in the direction of the compensating signal and decreases as the shunt condenser X is increased. Figs. 23 and 24 show the effect of connecting a smoothing circuit, of time constant $\tau = \beta$, after the field mill. For a given value of β/γ , the transient is considerably reduced in this way, and continues to decrease as β is made larger, although the time taken for the transient to die away is also lengthened. The reduction of the transient on introducing the smoothing circuit is particularly marked in Fig. 24, where the curves are drawn for an impulsive displacement current. However, the initial displacement at $t = 0$ is the contribution from the air-earth current collector acting alone, because of the delay produced by the smoothing circuit in the transmission of the signal from the mill to the differentiating circuit, and is given by writing $t = 0$ in Eq. 7.12.

7.6 Modifications to the apparatus

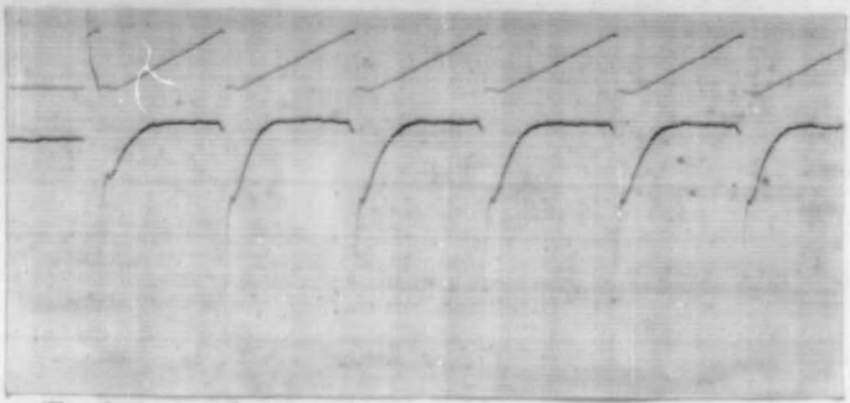
It is evident from all the above equations that if the capacity of the collector and its cable could be reduced, better compensation for sudden changes in potential gradient would result. As it was not practicable to lay a lead of very low capacitance to the collector, the expedient was tried, to good effect, of connecting the cable sheath to the feedback loop instead of to earth. A similar connection was used by Brewer (1953) to reduce the effective input capacitance of an electrometer valve voltmeter.

Since the cable sheath does not constitute the whole of the capacitance C, there is still a residual equivalent to the capacity of the collecting electrode itself, approximately 260$\mu\mu$F. The cable capacitance now appears across the input resistor R, and contributes to X. In fact, the cable capacitance was considered quite large enough to obviate the need for the variable condenser which had been used there, and the time constant β is now fixed at 23 sec. As it was not practicable to reduce C further by connecting the screen round the collector to the feedback loop, and also because, with the cable capacitance across R, there was no longer any question of equalizing β and γ, a smoothing circuit, consisting of a 4μF condenser with a variable resistor was connected between the field mill and the differentiating condenser. The integrating circuit is shown in its screening box in Plate 2.

The behaviour of the D.C. amplifier when the sawtooth potential gradient waveform was applied to the test plate, is illustrated by Fig. 25. For some reason, which there was no opportunity to investigate, the modifications described resulted in throwing out of adjustment the compensation already achieved for uniform displacement currents of long duration. The record considered to show the optimum performance of the system is reproduced in Fig. 26, where the parameters are now $\gamma = 4.9$ sec., $\beta = 23$ sec., $\tau = 14$ sec. The rate of change of potential gradient over a cycle is equivalent to 4 times a fine weather air-earth current of 2×10^{-12} A/m^2, while on the scale of the figure, the D.C. amplifier output for such a current would be 3 mm. It can be seen that a fair degree of compensation has been attained, especially when it is realized that the transients are due almost entirely to the sudden fall

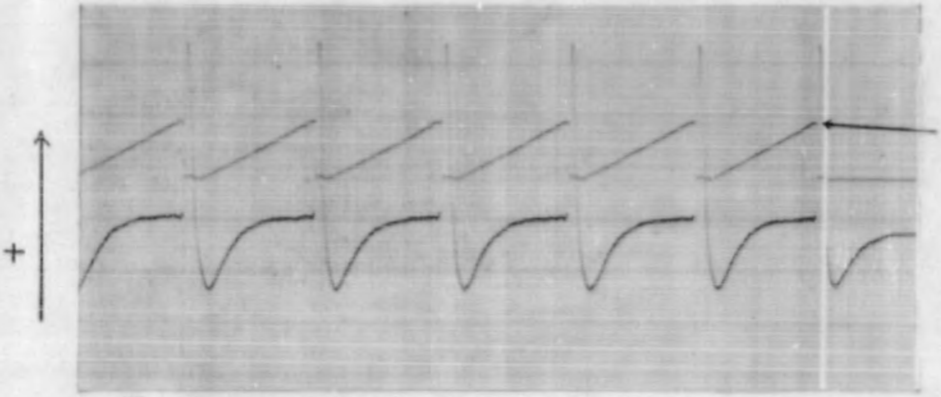
Time Constant of Differentiating Circuit : 3.4 sec.

Time Constant of Collector Circuit Estimated at approx 20 sec.



Record 61

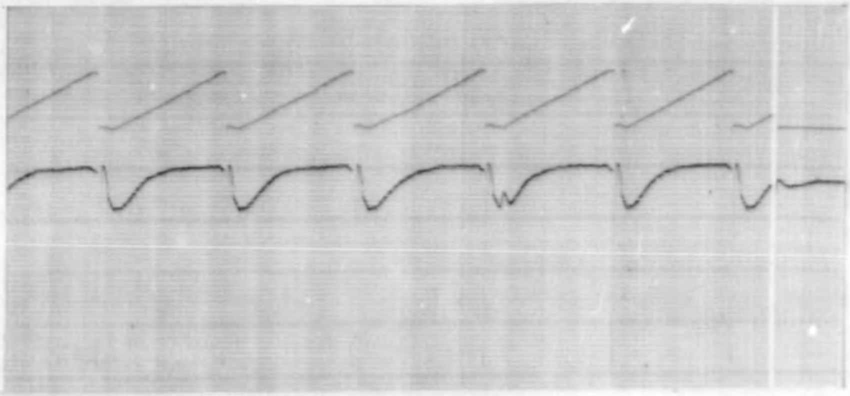
Time Constant of Smoothing Circuit : 14.1 sec.



Potential Gradient +103 V/m.

Record 62

Time Constant of Smoothing Circuit : 6.6 sec.

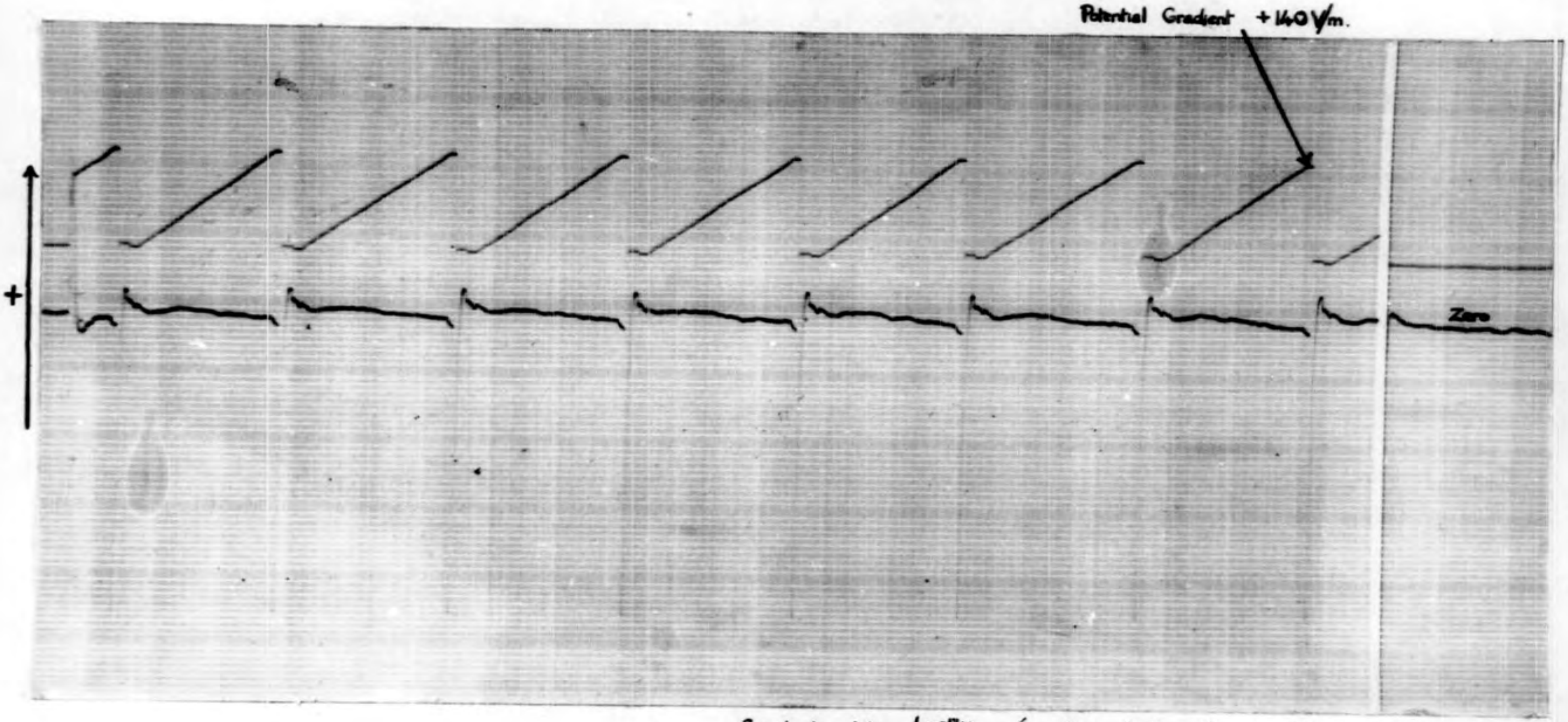


Sensitivity 1.6 mm / 10^{-13} A (on Original Records)

Record 63

Time Constant of Smoothing Circuit : 9.9 sec.

Fig. 25 Compensation Records : Adjustment of Transient Response.



Sensitivity 1.6 mm/10^{-13} A (on Original Record)

Record 67

Fig. 26 Compensation Record: Optimum Transient Response.

of potential gradient at the end of each cycle, a condition not likely to be met with in atmospheric electrical phenomena.

CHAPTER 8. CONCLUSION

8.1 Summary of the behaviour and use of the equipment

Although there was the opportunity to take only about a dozen records in natural conditions, before the termination of this research, these records and subsequent measurements made with the apparatus by Ramsay, at Durham, have demonstrated that the method of compensation is satisfactory in dealing with both the fairly steady conditions of continuous rain, and the more disturbed phenomena of showers. The apparatus has two disadvantages, neither of which is very serious. First, there is the rather long response time constant of 20 sec., which is much greater than was anticipated at the commencement of the work, and certainly much greater than one expects to achieve with conventional electronic equipment. However, this still compares favourably with apparatus which measures the current discontinuously by integrating the charge collected over a few minutes (§ 3.2). Secondly, there is a limit to the rate of change of potential gradient for which accurate compensation can be achieved, and in the most violent fluctuations in storms, there must be some doubt as to the meaning of the results. This is certainly true when the potential gradient is numerically greater than 4500 V/m., not because of capacitance effects, but because the field mill amplifier saturates beyond this level.

Referring to Fig. 9, the positive total current entering the collector, at which the amplifier saturates, is 1.4×10^{-10}A., corresponding to a very large air-earth current, only one order of magnitude less than the largest current expected in showers and storms. Also on account of saturation, this time for signals on the other grid, the compensation is effective up to a displacement current density of 8×10^{-10}A/m^2., which is equivalent to a rate of change of potential gradient of 100 V/m. per sec. It is thought that there will be comparatively few occurrences of such large rates of change. For negative air-earth and displacement surrents, the amplifier does not saturate until even greater magnitudes are attained.

The equipment is stable and reliable, and the field mill needs only infrequent attention. The brushes require changing after about a month of continuous day-time operation. The procedure which was devised for taking a record includes a daily check on the compensation. This adjustment applies only to the differentiating condenser, since the time constant of the smoothing circuit following the field mill need be only coarsely set for good results. To make the adjustment, the test plate is set up over the apparatus and earthed, and the amplifier controls are set so that there is no deflection when the galvanometer is switched in and out of the circuit. The sawtooth potential gradient waveform is then applied to the plate, and, after the transient at the beginning of a cycle has died away, the steady deflection from the zero is noted, and corrected to zero by adjusting the differentiating condenser. Usually only three cycles, each of three minutes' duration, are required to achieve satisfactory compensation. The test plate is then removed, and the apparatus is ready for use. At the end of a run, the test plate is replaced over the apparatus and earthed, and the 'zero' checked against the position of the spot when the galvanometer is switched off. The amplifier is now so stable that the change in 'zero' over a whole day is usually only of the order of an equivalent input of 5×10^{-13} A/m^2.

8.2 Results

The most interesting records that were taken after the apparatus had been developed to the required extent are shown in Figs. 27 and 28.

In Fig. 27 are reproduced two records that were made, one after the other, except for the delay necessary to change the recording paper, in a four hour period of continuous driving rain, with an easterly wind. In a series of measurements in similar conditions, with a discontinuous method of recording, Chalmers (1956) found that most often the potential gradient is negative while the air-earth current is positive. Fig. 27 demonstrates an occasion when the current was of the reverse sign, at least for three hours out of the four. Although little can be gained towards a generalization from an isolated example, it is interesting to consider how these particular observations could be explained. With the small potential gradient observed, it is almost certain that point-discharge must be ruled out as a contributor to the current arriving at the earth. Chalmers explains the more usual phenomenon of negative potential gradient and positive current in continuous rain on the basis of two charge separations. The first of

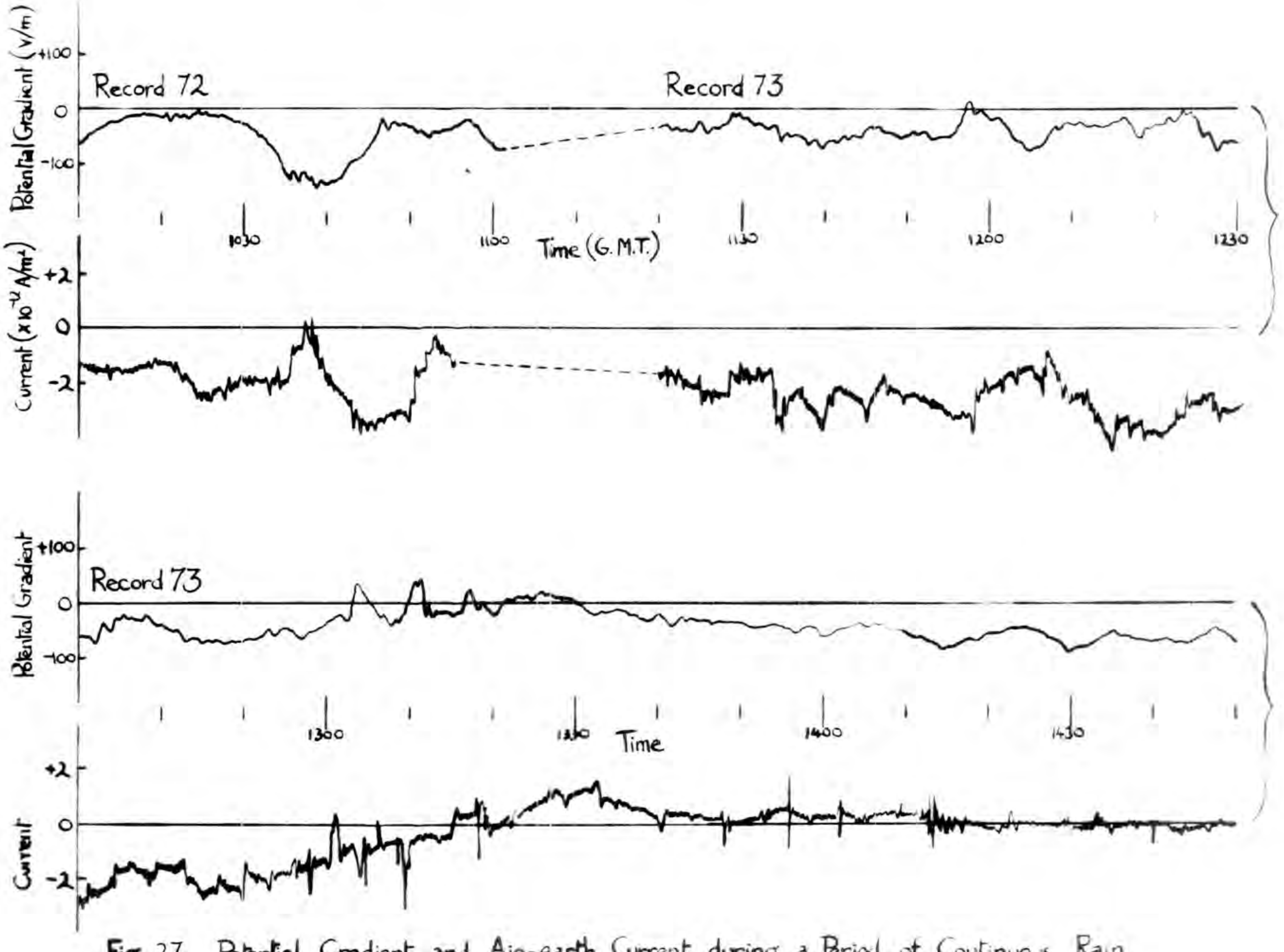


Fig. 27 Potential Gradient and Air-earth Current during a Period of Continuous Rain.

these would occur in the cloud at temperatures below the freezing-point of water and give the solid precipitation particles a negative charge, positive charges remaining behind in the cloud. This first separation explains the predominance of negative current in continuous snow. To explain the positive charge on continuous rain, Chalmers supposes a second separation process to occur either on melting, or near the ground. In the particular case of Fig. 27, it would be necessary to assume that this second process of charge separation did not occur in the conditions obtaining at the time. The variations of potential gradient and current at 1030 hr. and 1300 hr. suggest that the current variation followed the potential gradient with a delay of about 10 min. This indicates that the precipitation probably obtained its charge at a high level, the time taken to fall accounting for the delay. Remembering that a positive charge would be left behind in the cloud, it would also have to be supposed that the rain, in falling, was sufficiently highly charged for the space charge effect produced to reverse the potential gradient observed at the ground.

It is possible, particularly since the current was hardly greater in magnitude than the normal fine-weather conduction current, to go a little way towards explaining the result by assuming that the ion-capture process of Wilson (1929) was occurring. The mechanism of this is that a falling drop, polarised in the potential gradient, collects a surplus of atmospheric ions of opposite sign to the induced charge on its lower face. Chalmers (1956) shows that if this were the sole cause of the charge on precipitation in this kind of weather, the total current must be of the same sign as the conduction current, and not of such great magnitude. The difficulty encountered with Wilson's theory, as usual, is to explain how the potential gradient arises in the first place, without postulating some kind of charge separation which would inevitably give the precipitation a charge before it left the cloud.

Fig. 28 shows the potential gradient and current in three hours of showery weather. At the beginning of the record, cumulonimbus was developing, and the onset of precipitation can be clearly seen with a large precipitation current and a reduction of the potential gradient. It is remarkable that the potential gradient remained so low while there was such a large air-earth current. It is suggested that the effect of the charge at the cloud base, presumably negative, was masked by the space charge of the falling precipitation. Many workers have remarked upon the frequent occurrence of an inverse relation between the signs of the potential gradient and the precipitation current, but this is not in evidence here. It must of course be remembered that it is the total current, not just that carried by precipitation, which was measured in Fig. 28; nevertheless the contribution of precipitation in such weather conditions is probably far in excess of the conduction current. There is, however, a tendency in most of the record for the potential gradient and current to vary in opposite directions.

A large current pulse at 1230 hr., presumably due to precipitation, was unaccompanied by any significant change in potential gradient,

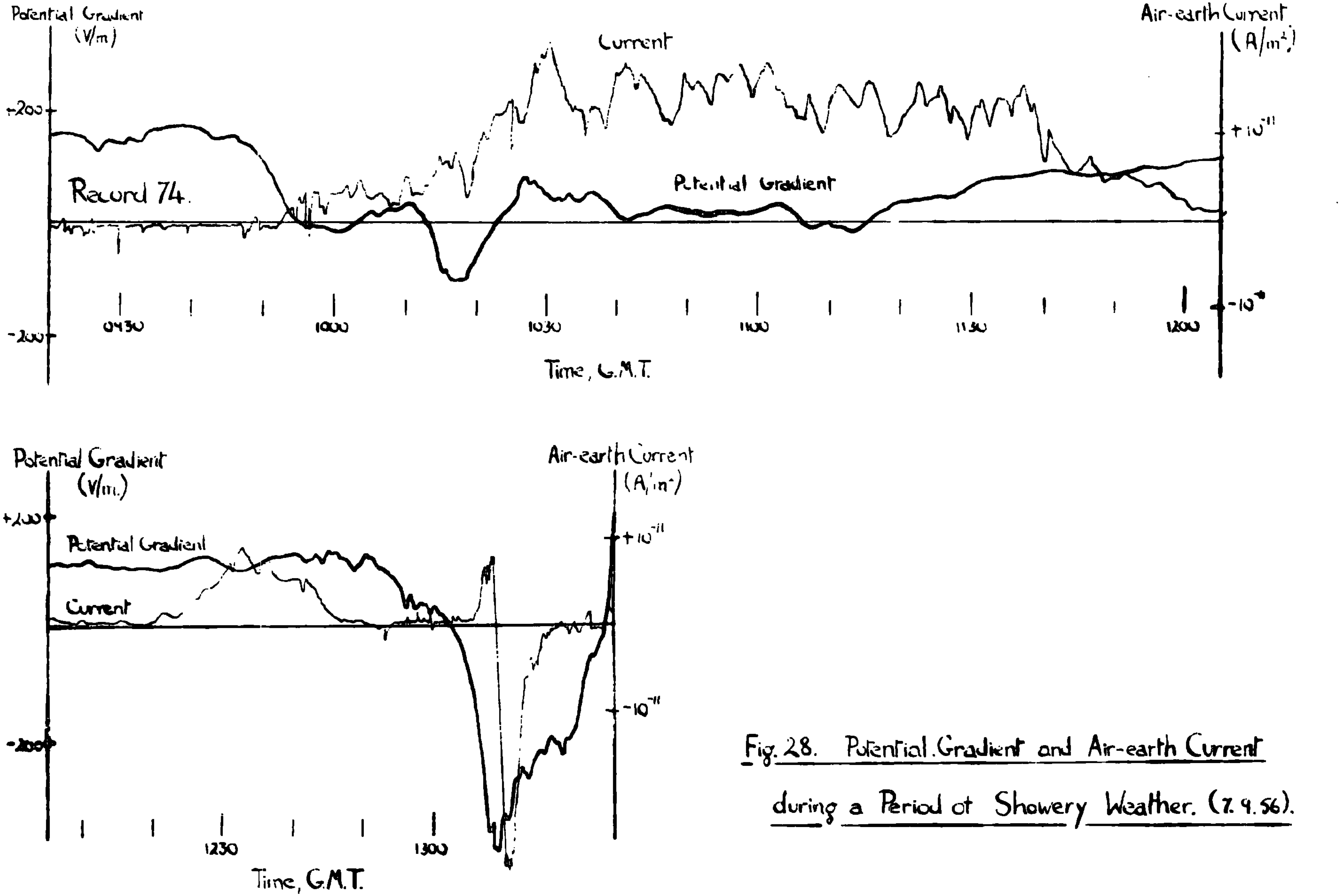


Fig. 28. Potential Gradient and Air-earth Current during a Period of Showery Weather. (7.9.56).

and might again have indicated the passage of a raining cloud, the charged base however, having had its effect at the ground cancelled, by the space charge of the falling rain. At the end of the record is a remarkable pattern of both potential gradient and current which is quite different from the general behaviour of the electrical phenomena in the remainder of the record.

Unfortunately, the proper understanding of such effects requires detailed observation of the weather at the time, and these records were only preliminary studies, for the purpose of discovering whether the compensation worked satisfactorily. They indicate that in two widely different types of weather, accurate recordings of the air-earth current can be made, free from disturbances due to the displacement of bound charge.

8.3 Suggested modification

It is evident from Chapter 7, that the capacity to earth of the air-earth current collector prevents satisfactory compensation being attained except by introducing large time constants into the circuit.

In an alternative suggestion for feeding in the compensating signal, deliberate use could be made of this capacity to match the inherent time constant of the field mill and its amplifier. The proposed circuit diagram, and its equivalent circuit, are shown in Fig. 29. The symbols are similar to those in Fig. 17, and are explained fully in Chapter 7. The second grid of the electrometer valve would be earthed, so that no use would be made of the differential characteristics of this stage. A resistor R', of much lower value than the input resistor R, would be placed in series between R and the collector, and the compensating signal, via the differentiating condenser, D, fed to a point on R', tapping off a resistance R_0 such that $R_0C = \tau$, the time constant of the field mill. Using the Laplace transform in a similar manner to that described in Chapter 7, it can be shown, in theory at least, that transient as well as uniform displacement currents could be accurately compensated. It would be necessary for the differentiating condenser to be much smaller than C, in order that it would not appreciably divert the displacement current from its path through R. C', the grid capacity of the electrometer valve, has been entered in Fig. 29, but provided $GX \gg C'$, a condition amply fulfilled, it can be neglected from consideration. G is the loop gain (approximately 320) of the D.C. amplifier. Although a small value of D would be required, this would not necessarily entail increasing the field mill sensitivity in proportion; it can easily be shown that the value of D needed to give compensation for uniform displacement currents would be reduced from that required in the existing apparatus in the ratio S/R, even with the same field mill sensitivity. Here, S is the resistance in the differentiating circuit of the equipment as it stands now.

This reduction, however, is only by a factor of five, and so it would probably be desirable to double the mill sensitivity in order to permit of an even smaller value of D. Furthermore, it would of

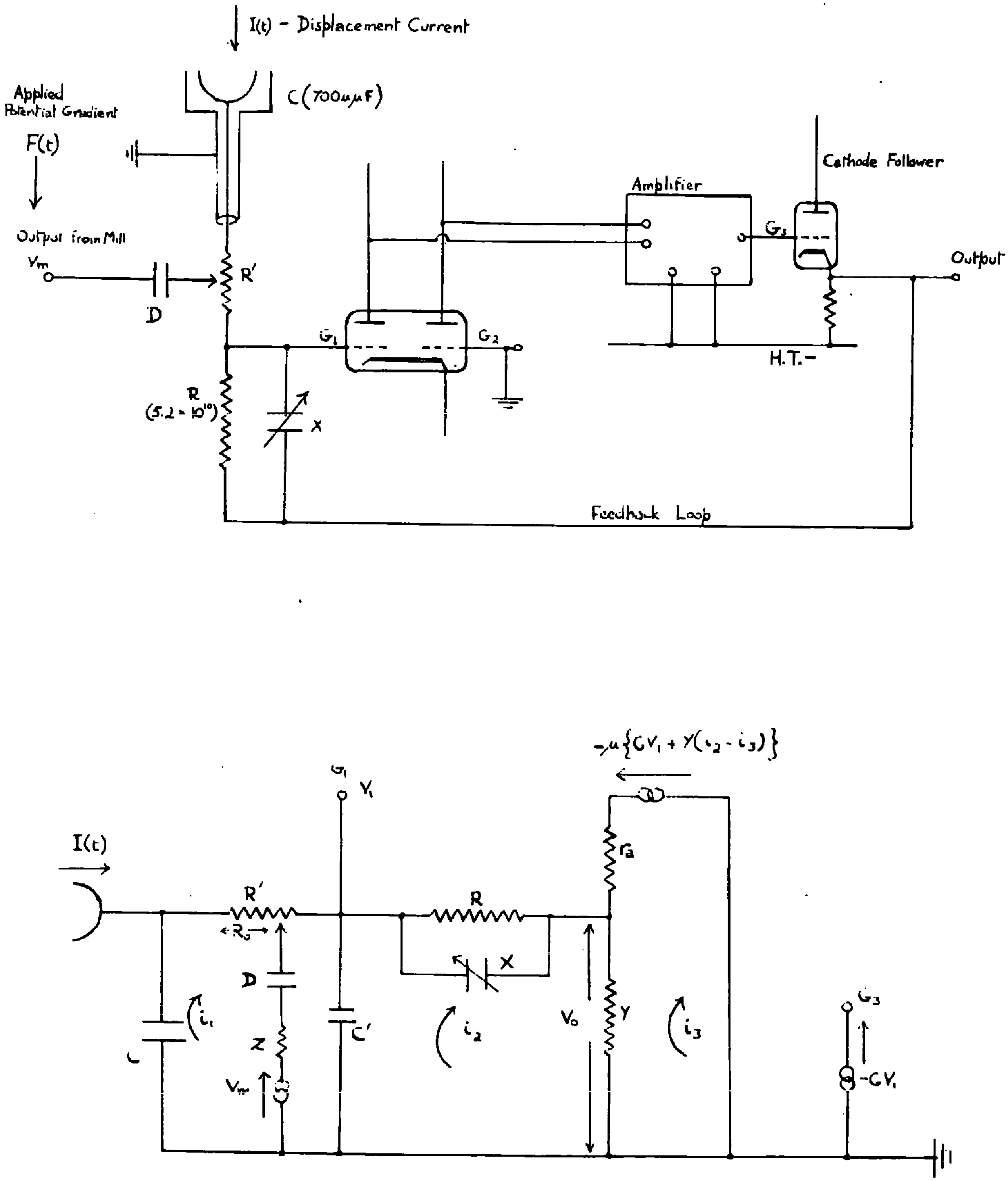


Fig. 29 Compensation Current Feeding into Same Grid as Collector Current, with Equivalent Circuit.

course be necessary to reverse the sign of the mill output for a given change of potential gradient. It is evident that this would mean a considerable modification of the field mill, for, on account of the overall feedback incorporated in it, the problem could not be solved by changing the commutator phase. A way round the difficulty, involved but yet retaining the desirable advantages of feedback in the mill, would be to follow the mill with an inverter amplifier, similar, perhaps, to the D.C. amplifier. However, both the required change of phase, and an increased sensitivity would be made much more easily possible by relinquishing the present system of feedback, and reverting to the more conventional method in which this is applied only in the amplifier (see § 5.5). This, of course, would mean that the final output would be largely dependent on the characteristic of the commutator, but it is probable that the departure from linearity would not be serious.

The method of testing the equipment would be very similar to that described in § 8.1, the resistor R' requiring probably only a coarse setting, while D would be adjusted slightly from day to day. If this form of the circuit could be made to work, it would result in a minimum permissible response time of about 1 sec., thus employing the high speeds of response obtainable from the D.C. amplifier and the field mill to much better advantage.

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