The kaon nucleon interaction

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THESIS SUBMITTED TO THE

UNIVERSITY OF DURHAM

BY

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FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

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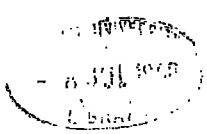
Date: January 1968



ABSTRACT

A new forward dispersion relation is developed to describe the kaon nucleon interaction by observing that the real part of a resonant partial wave amplitude goes through a zero at the resonance position. This relation eliminates some of the practical deficiencies inherent in the conventional forward dispersion relations.

The ΛKN and ΣKN coupling constants are determined by using the combinations of dispersion relations suggested by Lusignoli et al. evaluated at the kaon nucleon threshold. Initially the energy independent scattering lengths of Kim are used to parametrise the K^-N amplitudes in the low energy region in terms of a single channel s-wave zero-range approximation. Just above threshold the K^+N amplitudes are parametrised in terms of the constant s-wave scattering lengths and effective range terms found by Goldhaber et al. and Stenger et al. Subsequently, the K^-N s-wave scattering lengths are given an energy dependence in the unphysical region through the multi-channel K matrix formalism. Various constraints are placed on these I = 0 K^-N scattering lengths such that the constant elements α_0, β_0, γ_0 of the corresponding R-matrix should reproduce the $Y_0^*(1405)$ resonance with its correct position and width in the $\Sigma\pi - \Sigma\pi$ channel, and also the values of the energy independent scattering lengths at the K^-N threshold. Furthermore, an energy dependence is then introduced into α_0, β_0, γ_0.

Similarly, using the I = 1 $\bar{K}N$ energy independent and dependent scattering lengths in the appropriate dispersion relation gives an equation for the ΣKN coupling constant which involves the p-wave $Y_1^*(1385)$ resonance. The effects of this resonance are approximated in terms of its position and width and the Y_1^*KN coupling constant. A brief survey of previous determinations of the Λ, Σ coupling constants shows that these predictions are consistent within the large errors, except for a very recent calculation performed by Kim.

By differentiating specific forward dispersion relations an attempt is made to calculate the ΛKN, ΣKN and Y_1^*KN coupling constants explicitly. However, the results indicate a slight inconsistency in the values obtained from conventional forward dispersion relations, while the predictions of the new relations are reasonable within the large errors.

Finally, the predictions of the new relations are compared with the experimental data for the $K^{\pm}p$ interactions and the charge exchange processes. The results are found to be in good agreement.

ACKNOWLEDGEMENTS

I should like to express my thanks to my supervisor, Doctor A.D. Martin, for his encouragement, continued assistance and many helpful suggestions throughout the course of this work.

I am grateful to Professor B.H. Bransden for his support and also for the receit of travelling expenses enabling me to commute between Durham and Newcastle.

I should also like to thank the respective staffs of both the University of Durham and Newcastle University Computer Units and for the hospitality extended to me whilst in Newcastle.

I am also indebted to Miss Judy Law for interpreting my original manuscript and typing this thesis.

I should like to thank the Science Research Council for the award of a research studentship enabling me to carry out this research.

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CHAPTER I

AN INTRODUCTION TO STRONG INTERACTIONS

1. THE FUNDAMENTAL PARTICLES

With the discovery of the neutron by Chadwick[1] in 1932 and the subsequent proposal that an atomic nucleus is composed of protons and neutrons, arose a dilemma concerning the nature of the force which binds the nucleon constituents together, as the electrostatic force only exists between protons and is repulsive.

In 1935 Yukawa[2] proposed the existence of heavy quanta, with a mass some two hundred times greater than that of the electron, to account for these strong short-range nuclear forces. Assuming such a Yukawa coupling between the nucleons, conservation of angular momentum requires that the spin of the quanta must be an integer, and consequently they are Bose particles. Also, from a study of the nucleon-nucleon potential and the empirical evidence that nuclear forces are charge independent, it was concluded[3] that a triplet of such particles should exist with positive, negative and zero charge states.

This hypothesis was upheld through the discovery of pi-mesons in 1947 by Lattes et al.[4] who bombarded a very sensitive emulsion plate with cosmic rays. These particles were identified with the heavy quanta postulated by Yukawa.

At about the same time Butler and Rochester[5] discovered the first of a series of particles with a curious anomaly in their decay

rates, that is, some of the decays were very much slower than expected. This characteristic led Pais[6] to suggest that these particles should contain some internal degree of freedom specified by a quantum number called strangeness. Since then, bubble chambers and particle accelerators have provided evidence for many more particles which are identified in terms of their mass and spin and various internal quantum numbers such as isospin and strangeness[7].

Apart from the photon, electron, muon and neutrino, the list of known particles may be divided into two categories, mesons and baryons. The meson class contains the pion triplet (π^+, π^o, π^-) and the strangeness +1 kaon doublet (K^+, K^o) with its antikaon counterpart (K^-, $\bar{K}^o$) with strangeness -1, plus the various meson resonances such as ρ and K^* etc. The baryon class consists of the nucleons (N), that is the neutron (n) and the proton (p), a hyperon singlet, doublet and triplet with strangeness -1, -2, -1 respectively, as well as the nucleon and hyperon resonances.

At the present time the rapid growth in the number of particles observed is primarily due to the discovery of new resonances. However, whereas the baryon resonances are comparatively easily established by using a particle accelerator to impact a meson beam with a nuclear target, no such mesonic targets exist as the meson lifetime is about 10^{-8} seconds. Thus the existence of many meson resonances, which occur through meson-meson interactions, is question-

able.

All these particles are not independent objects in the sense that they interact with each other and are therefore transformed in various ways. It is customary to divide the elementary particle interactions into three distinct classes according to their strengths (that is the coupling of the interaction). These are, the electromagnetic interactions, the weak interactions and the strong interactions.

Electromagnetic interactions are responsible for electromagnetic processes such as the Compton effect, and for other processes based on the emision or absorption of virtual photons by a charged particle, and for mass differences within the various particle multiplets (e.g. the n-p mass difference). The coupling constant is e, the electric charge, and in rationalized units

$$e^2 = \frac{1}{137}$$

Weak interactions include β decay and the decay of strange particles. The order of magnitude of the coupling constant g_w is approximately

$$g_w{}^2 \sim 10^{-14}$$

Strong interactions occur between mesons and baryons and are responsible for nuclear forces, the production and absorption of pions, the production of strange particles and the binding of hyperons in nuclei. The strength of such interactions is measured by a coupling constant g, say, and is of the order

$$g^2 \sim 15$$

To gain insight into the forces acting between the particles during a strong interaction process it is necessary to introduce the concept of the scattering matrix.

2. THE S-MATRIX

In an attempt to formulate a theory for strong interactions, Heisenberg[8a] regarded three criteria[8b] as observable quantities, which must necessarily be described in any theory. These conditions led him to investigate the properties of the scattering matrix, or S-matrix, first introduced by Wheeler[9] in 1937, which was defined as the operator which transforms the incoming state into the outgoing state.

Subsequent attempts to describe the effects of strong interaction processes were analogous with the methods used to obtain a successful description of electromagnetic interactions in which the form of the Lagrangian was surmised from classical physics and the dynamical equations obtained were formulated as a convergent perturbation expansion in e^2. Note that it was possible to express the S-matrix in operator form as an integral of a time-ordered product of interaction Hamiltonian densities $H_I(x_i)$ at the space-time points x_i[10]. However this approach fails to account for the dynamics of strong interactions, even if it were possible to guess the correct form of the Lagrangian, because the resulting perturbation series is non-convergent as the expansion is made in powers of the coupling constant which is very large.

More recently a theory has been developed which, instead of probing into the detailed mechanism of the interaction, is based on a study of Heisenberg's[8a] S-matrix whose matrix elements are, in

principle, directly observable transition amplitudes.

If the forces involved in strong interactions have a sufficiently short range than the incoming and outgoing particles may be assumed to be non-interacting a sufficient time before and after the collision to allow the initial and final states to be thought of as consisting of free particles. Consequently, these states are specified by the aggregate of the individual particle momenta and quantum numbers. For such an initial state $|i\rangle$ the superposition principle of quantum mechanics allows one to write the final state as $S|i\rangle$, where S is a linear operator. Thus the S-matrix element $\langle f|S|i\rangle$ may be defined such that for an initial state $|i\rangle$ the probability of $|f\rangle$ being a final state is

$$|\langle f|S|i\rangle|^2$$

Similarly, if S^+ is the adjoint of S, the probability is

$$|\langle i|S^+|f\rangle|^2$$

Thus, assuming that the states $|i\rangle$ form a complete orthonormal set, conservation of probability implies

$$S^+S = SS^+ = 1$$

Hence S is unitary. Moreover, if a proper Lorentz transformation L transforms the state $|i\rangle$ into $|i_L\rangle$, then relativistic invariance requires that

$$|\langle f|S|i\rangle|^2 = |\langle f_L|S|i_L\rangle|^2$$

and the phase of the matrix element can be chosen so that

$$\langle f|S|i\rangle = \langle f_L|S|i_L\rangle^*$$

It follows that for spinless particles the S-matrix elements depend on the four-momenta of the particles only through their invariant scalar products, and for particles with spin the matrix element is composed of a number of such invariant functi ns multiplied by certain vector or spinor terms.

It is convenient to separate the S-matrix into two parts by subtracting off the term when the particles do not interact at al. Thus we may write a 'two-by-two' S-matrix element for spinless particles, that is the matrix element which describes the scattering of two particles with four momenta p_1 q_1 into a final state of two particles with four momenta p_2, q_2, as

$$\langle p_2q_2|S|p_1q_1\rangle = \langle p_2q_2|1|p_1q_1\rangle + i(2\pi)^4\delta^4(p_1+q_1-p_2-q_2)\langle p_2q_2|T|p_1q_1\rangle \qquad (1.1)$$

where 1 is the identity operator and the delta functions arising from translational invariance specify total energy momentum conservation. The scattering amplitude $F(p_1q_1p_2q_2)$, where

$$F = \langle p_2q_2|T|p_1q_1\rangle \qquad (1.2)$$

is related to the experimentally observable scattering cross section σ by

$$\sigma = \frac{1}{(8\pi)^2}\frac{1}{W p_i}\int \frac{p_f}{W}|F|^2\, d\Omega \qquad (1.3)$$

where p_i, p_f are the centre of mass momenta of particles in the initial and final states, W is the centre of mass energy and Ω is the solid angle in the final state.

It is convenient to define new variables s, t, u by

$$\begin{aligned} s &= -(p_1+q_1)^2 \\ t &= -(q_1-q_2)^2 \\ u &= -(p_1-q_2)^2 \end{aligned} \tag{1.4}$$

However $p_j^2 = -m_j^2$ for the j^{th} particle, where j = 1, 2, 3, 4 and the overall energy momentum conservation condition

$$p_1+q_1 = p_2+q_2 \tag{1.5}$$

implies that

$$s+t+u = \sum_{j=1}^{4} m_j^2 \tag{1.6}$$

and so only two of s,t,u are independent. For more than four particles involved in a scattering process the number of independent varibles rises sharply. Thus the restriction of considering only two particles in the initial state and two in the final state is necessary for most practical calculations of strong interaction processes.

In addition to the previous assumptions we suppose that F(s,t,u) has no kinematical singularities and furthermore, apart from the pole terms discussed later, the only singularities of the S-matrix elements are those demanded by the unitarity equation. The presence of the latter singularities is best illustrated by combining equation (1.1) and the unitary condition for S which gives

$$\langle p_2q_2|\mathcal{T}|p_1q_1\rangle - \langle p_1q_1|\mathcal{T}|p_2q_2\rangle^* \tag{1.7}$$
$$= \frac{i}{(2\pi)^2}\int \frac{\vec{dk}_1\,\vec{dk}_2}{W^2}\,\delta^4(p_1+q_1-k_1-k_2)\langle p_2q_2|\mathcal{T}|k_1k_2\rangle\langle p_1q_1|\mathcal{T}|k_1k_2\rangle^*$$

where the asterisk denotes the complex conjugate. Above the energy threshold for inelastic scattering additional terms arise on the right hand side of equation (1.7) since all intermediate states will occur which are allowed by energy conservation and quantum number selection rules. This implies that the scattering matrix has a singularity at each energy corresponding to a threshold for a new allowed physical process. These thresholds are the branch points of the amplitude $F(s,t,u)$ from which the branch cuts are usually drawn parallel to the real axis in the complex energy squared plane, or the s-plane.

The physical sheet is one particular Riemann surface on which the amplitude is single valued and is defined by any simple closed contour in the s-plane which coes not cross any of these cuts.

The physical amplitude is defined as the boundary value of the amplitude when s tends to the real s axis from above, and is for

$$\lim_{\epsilon \to 0} \quad F(s + i\epsilon, t, u) \tag{1.8}$$

Similarly the amplitude $\langle p_1 q_1 | \mathcal{H} | p_2 q_2 \rangle$ is defined as a limiting procedure from below and can be related to the physical amplitude by analytic continuation. Thus the left hand side of equation (1.7) involves the discontinuity of the amplitude across the branch cuts. The symmetry condition for the 'two-to-two' scattering of spinless particles

$$\langle i | S | f \rangle = \langle f | S | i \rangle \tag{1.9}$$

implies that this discontinuity is twice the imaginary part of the

amplitude.

In particular if $|f> = |i>$, and hence $t = 0$, equation (1.7) for four equal mass particles gives

$$\text{Im } F(s,u,0) = 2k W \sigma_{tot} \tag{1.10}$$

where k, W are the centre of mass momentum and energy respectively of the initial state, and σ_{tot} is the total scattering cross-section. This relation is known as the optical theorem.

If F(stu) is the amplitude for the physical scattering process

$$a_1+b_1 \longrightarrow a_2+b_2 \tag{1.11}$$

then the energies and momenta of the four particles are real. If the particles have equal masses this implies

$$s \geq 4m^2, \quad t \leq 0, \quad u \leq 0 \tag{1.12}$$

If s, u and consequently t are considered as complex variables then by analytic continuation to the region

$$u \geq 4m^2, \quad s \leq 0, \quad t \leq 0 \tag{1.13}$$

the previous assumption that transition amplitudes are the values of analytic functions on real boundaries, implies that the resulting function F(s,t,u), evaluated in a suitable limit, now gives the physical scattering amplitude for the process

$$a_1 + \bar{b}_2 \longrightarrow a_2 + \bar{b}_1 \tag{1.14}$$

where the bar denotes the anti-particle. Similarly by analytic continuation to the region

$$t \geq 4m^2, \quad s \leq 0, \quad u \leq 0 \tag{1.15}$$

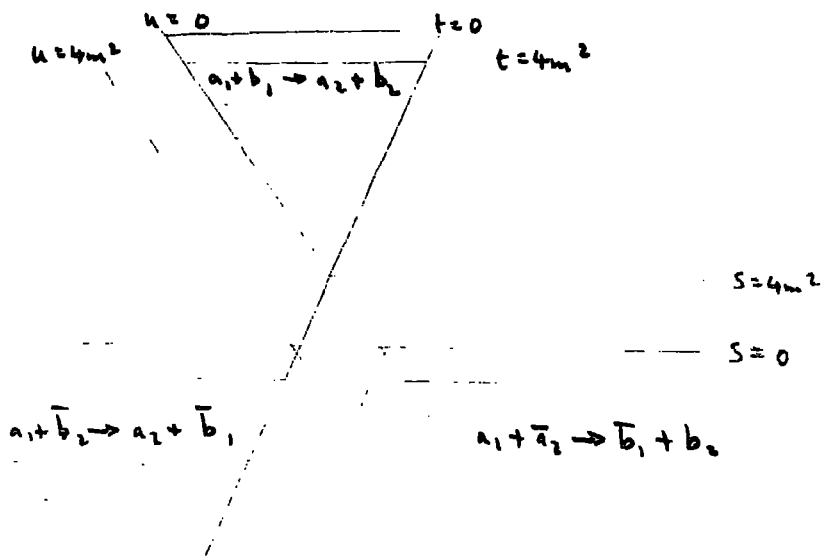
the function F, again evaluated in a suitable limit, now gives the

physical scattering amplitude for the process

$$a_1 + \bar{a}_2 \longrightarrow \bar{b}_1 + b_2 \qquad (1.16)$$

These results are known as 'crossing relations' and imply that the same analytic function can be used to describe three different physical processes by a suitable choice of s, t, u. This is illustrated by the following diagram in which the physical regions for the s, t, u 'channels' are shaded.

Figure 1: The three physical regions for an interaction involving four equal mass particles.



If the quantum numbers and selection rules allow the possibility of a single particle intermediate state less massive than the two initial particles then the amplitude F(s,t,u) has a pole singularity at an unphysical value of the variable $s = m_s^2$ for scattering in the

s channel. These poles represent stable particle. If the mass of the intermediate single particle is greater than the combined masses of the two initial particles then the singularities, which are off the real s axis, are said to represent unstable particles, or resonances.

Thus for the equal mass case there are branch points at

$$\begin{aligned} s &= 4m^2, (I_s)^2, \ldots. \\ u &= 4m^2, (I_u)^2, \ldots. \\ t &= 4m^2, (I_t)^2, \ldots. \end{aligned} \tag{1.17}$$

where I_s, I_u, I_t denote the first inelastic thresholds for processes s, u, t respectively. There may also be poles at $s = m_s^2$, $u = m_u^2$ and $t = m_t^2$.

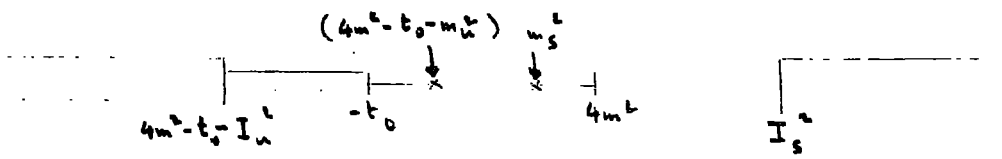
For a fixed value $t = t_o$, say, the branch points in the s plane are at

$$s = -t_o, \; 4m^2 - t_o - (I_u)^2 \tag{1.18}$$

and the pole is at

$$s = 4m^2 - t_o - m_u^2 \tag{1.19}$$

Figure 2: The singularites in the s-plane when t is fixed.



Thus the only singularities of the S-matrix are the poles corresponding to stable and unstable particles and the further singularities generated by unitarity. This is the postulate of 'maximal analyticity'. A complete set of assumptions involved in the formulation of S-matrix theory may be written down

(a) The superposition principle

(b) The existence of a unitary S-matrix

(c) Lorentz invariance of the S-matrix

(d) The disconnectness of the S-matrix due to the short range forces (i.e. the presence of the identity 1 in equation (1.1).

(e) Maximal analyticity.

To exploit the potential of the theory developed so far it is necessary to introduce the concept of 'dispersion relations'.

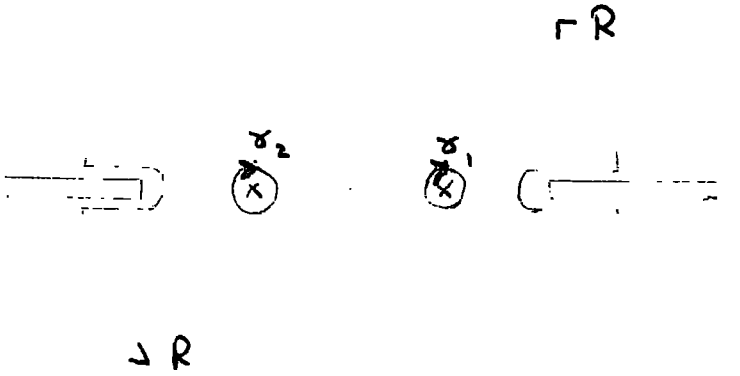
3 DISPERSION RELATIONS

If figure 2 represents all the singularities of F(s,t,u) on the physical sheet, then for $s = \sigma$, where σ is complex and t fixed, F(s,t) may be written by a Cauchy formula as

$$F(\sigma, t) = \frac{1}{2\pi i} \oint_{R, \gamma_1, \gamma_2} \frac{F(s', t)}{s' - \sigma} \, ds' \qquad (1.20)$$

The contours R, γ_1, γ_2 enclose all the singularities and R is closed by two semi-circles of infinite radii.

Figure 3: The contours of integration on the physical sheet.



Around γ_1 the contribution to equ(1.20) is

$$g^2 / (\sigma - m^2) \qquad (1.21)$$

where g^2, the residue of the pole at m^2, is usually written in terms of the renormalized coupling constants defined in field theory.

Secondly, around γ_2 replace $s'-\sigma$ by $-(u'-U)$ and ds' by $-du'$ so we obtain a contribution

$$g^2 / (u - m^2) \tag{1.22}$$

where $U = \sum_{i=1}^{4} m_i^2 - \sigma - t$. (1.23)

The integration on the contour enclosing the right hand cut gives

$$\frac{1}{2\pi i} \int_{s_0}^{\infty} \frac{[F(s'+i\epsilon, t) - F(s'-i\epsilon, t)]}{s' - \sigma} ds' \tag{1.24}$$

~~However the symmetry condition, equ(1.9) implies~~

~~$$\lim_{\epsilon \to 0} \frac{1}{2i} [F(s'+i\epsilon, t) - F(s'-i\epsilon, t)] = \text{Im } F(s', t)$$~~

However hermitian analyticity implies

$$F(s^*, t) = F^*(s, t) \tag{1.25}$$

where the physical amplitude is written on the right hand side of the above relation.

Therefore the contribution is

$$\frac{1}{\pi} \int_{s_0}^{\infty} \frac{\text{Im } F(s' t)}{s' - \sigma} ds' \tag{1.26}$$

Similarly, on the left hand cut the contribution is

$$+\frac{1}{\pi} \int_{-\infty}^{s_1} \frac{\text{Im } F(s' t)}{s' - \sigma} ds' \tag{1.27}$$

Hence,

$$F(\sigma, t) = \frac{g^2}{\sigma - m^2} + \frac{g^2}{u - m^2} + \frac{1}{\pi} \int_{s_0}^{\infty} \frac{\text{Im } F(s' t)}{s' - \sigma} ds' + \frac{1}{\pi} \int_{s_0}^{\infty} \frac{\text{Im } F(u' t)}{u' - u} du' \tag{1.28}$$

Let $\sigma \to s + i\epsilon$, therefore $F(\sigma\, t) \xrightarrow[\epsilon \to 0]{}$ the physical amplitude. Simply take the real parts of both sides and write

$$\lim_{\epsilon \to 0} \int \frac{\mathrm{Im}\, F(s',t)(s'-s)\,ds'}{(s'-s)^2 + \epsilon^2} = \frac{1}{\pi} P \int \frac{\mathrm{Im}\, F(s',t)}{s'-s}\, ds' \tag{1.29}$$

Thus

$$\mathrm{Re}\, F(s,t) = \frac{g^2}{s-m^2} + \frac{g^2}{u-m^2} + \frac{1}{\pi} P \int_{s_0}^{\infty} \frac{\mathrm{Im}\, F(s',t)}{s'-s}\, ds' + \frac{1}{\pi} \int_{s_0}^{\infty} \frac{\mathrm{Im}\, F(u',t)}{u'-u}\, du' \tag{1.30}$$

This integration equation is called a 'dispersion relation' (the terminology is due to a previous application of similar equations to the theory of the dispersion of light in optics). Similar relations can be deduced by keeping u or s fixed instead of t. If t = 0 equ(1.30) is known as a 'forward disperison relation'. These relations (1.30) are in fact special cases of a more general relation first written down by Mandelstam[11] to satisfy all the reasonable requirements imposed on F(s,t,u) by ~~elastic~~ unitarity and crossing, and therefore his 'double' dispersion relation gives the most analytic form of a two body amplitude compatiable with these conditions.

So far we have been concerned with spinless particles. To describe the kaon-nucleon interaction it is necessary to generalize our arguments to include the spin and charges of the particles involved.

4 THE EFFECTS OF CHARGE AND SPIN.

To describe a scattering process between charged particles which involves spin, it is necessary to consider the invariant amplitude as an operator in spin and isospin space. In particular for the pseudoscalar meson nucleon interaction there are two independent spin scalar operators 1 and $i\gamma_\mu (q_1+q_2)^\mu/2$ where q_1, q_2 are the initial and final meson four momenta. Thus the transition amplitudes may be written in terms of two invariant functions A and B such that

$$F(s,t,u) = \bar{u}(p_2)\left[A(s,t,u) - \tfrac{1}{2} i\gamma_\mu (q_1+q_2)^\mu B(s,t,u)\right] u(p_1) \qquad (1.31)$$

where $u(p_1)$ and $u(p_2)$ are four-spinors representing the initial and final state nucleons with four momenta p_1 and p_2, and γ_μ are the well known matrices.

If the meson is a pion, and the amplitudes for $\pi^+ p$ and $\pi^- p$ elastic scattering are denoted by the subscripts (+) and (-) representively, then crossing symmetry implies

$$\langle p_2 q_2 | A_+ | p_1 q_1 \rangle = \langle -q_1 p_2 | A_- | p_1 -q_2 \rangle \qquad (1.32)$$

and it is convenient to define new invariant functions

$$A^+ = \tfrac{1}{2}(A_- + A_+) \qquad A^- = \tfrac{1}{2}(A_- - A_+) \qquad (1.33)$$

and similarly for $B^\pm$, for which it may be shown that A^+, B^- are symmetric under crossing and A^-, B^+ are antisymmetric. These properties are knownas the 'crossing relations' for $A^\pm$ and $B^\pm$.

Other useful quantities are the amplitudes defined in terms of the eigenstates of isotopic spin. The πN system has values of isospin $I=\frac{1}{2}, {}^{3}/_{2}$ and using (1.33) it may be shown that these pion-nucleon isospin amplitudes can be expressed in terms of the above crossing-symmetric and anti-symmetric amplitudes as

$$A^{\frac{1}{2}} = A^{+} + 2A^{-}, \quad A^{3/2} = A^{+} - A^{-} \tag{1.34}$$

and identical relations hold for $B^{\frac{1}{2}}$, $B^{3/2}$.

The crossing relation (1.32) can also be used to define four invariant amplitudes $A_{\pm}$, $B_{\pm}$ for the kaon nucleon interaction, where $\pm$ now refers to $K^{\pm}$, which may be expressed in terms of $A^{\pm}$ $B^{\pm}$ as in equ(1.33). These amplitudes may be decomposed into isospin amplitudes as follows

$$\begin{aligned} A_{K^+p} &= A^{I=0} \\ A_{K^-p} &= \tfrac{1}{2}\left(\bar{A}^{I=0} + \bar{A}^{I=1} \right) \end{aligned} \tag{1.35}$$

Correspondingly, if the nucleon involved in the scattering is a neutron then

$$\begin{aligned} A_{K^+n} &= \tfrac{1}{2}\left(A^{I=0} + A^{I=1} \right) \\ A_{K^-n} &= \bar{A}^{I=1} \end{aligned} \tag{1.36}$$

The isospin amplitudes A^{0}, A^{1} refer to $K^{+}N$ scattering and the amplitudes $\bar{A}^{0}$ $\bar{A}^{1}$ refer to $K^{-}N$ scattering. The additional complication here, as apposed to πN, is due to the fact that after collision the $K^{-}N$ state may decay into a pion hyperon state which has the same

quantum numbers. Note that identical results to (1.35)-(1.36) hold for the $B^{\pm}$ amplitudes.

To complete this introduction to strong interactions let us briefly consider the success of previous attempts to apply forward dispersion relations to the pion nucleon interaction.

5 THE PION NUCLEON FORWARD DISPERSION RELATIONS.

Since the original derivation of the charged pion nucleon forward dispersion relations[12], the subsequent improved proofs[13] have increased their importance with the realization that experimental verification of these relations, in turn, provides a check on the assumptions of unitarity, relativistic invariance and local commutativity used in their formulation.

As before

$$s = -(p_1 + q_1)^2 \tag{1.37}$$

and if w_L is the total incident pion energy in the laboratory system then

$$s = M^2 + \mu^2 + 2Mw_L \tag{1.38}$$

where M, μ refer to the nucleon and pion masses.

The dispersion relations are sometimes written in terms of another invariant ν , where

$$\begin{aligned} \nu &= \frac{1}{4M}(s - u) \\ &= w_L + t/4M \end{aligned} \tag{1.39}$$

and for $t \neq 0$ are known as fixed momentum transfer dispersion relations In analogy with equ(1.30) the dispersion relations for the invariant amplitudes A, B[21] are

$$\mathrm{Re}\, A^{\pm}(\nu,t) = \frac{1}{\pi} P \int_{\mu + t/4M}^{\infty} d\nu' \, \mathrm{Im}\, A^{\pm}(\nu' t) \left[\frac{1}{\nu' - \nu} \pm \frac{1}{\nu' + \nu} \right] \tag{1.40}$$

$$\mathrm{Re}\, B^{\pm}(\nu,t) = \frac{G^2}{2M} \left[\frac{1}{\nu_p - \nu} \mp \frac{1}{\nu_p + \nu} \right] + \frac{1}{\pi} P \int_{\mu + \frac{t}{4M}}^{\infty} d\nu' \, \mathrm{Im}\, B^{\pm}(\nu' t) \left[\frac{1}{\nu' - \nu} \mp \frac{1}{\nu' + \nu} \right] \tag{1.41}$$

where ν_p denotes the position of the nucleon pole term. The crossing relations

$$\begin{aligned} A^{\pm}(-\nu, t) &= \pm A^{\pm}(\nu, t) \\ B_{\pm}^{+}(-\nu, t) &= \mp B^{\pm}(\nu, t) \end{aligned} \tag{1.42}$$

have been used to yield integrations over physical amplitudes. G^2 is the rationalized renormalized pseudoscalar coupling constant.

Defining

$$T^{\pm}(\nu, t) = A^{\pm}(\nu, t) + \nu\, B^{\pm}(\nu, t) \tag{1.43}$$

equations (1.42) imply

$$T^{\pm}(-\nu, t) = \pm T^{\pm}(\nu, t) \tag{1.44}$$

From (1.33)

$$T_{\pm}(\nu, t) = T^{+}(\nu, t) \mp T^{-}(\nu, t) \tag{1.45}$$

The dispersion relations for $T^{\pm}(\nu\, t)$ become

$$\begin{aligned} \operatorname{Re} T^{\pm}(\nu\, t) = {} & \frac{G^2}{2m}\,\nu\left[\frac{1}{\nu_p - \nu} \mp \frac{1}{\nu_p + \nu}\right] \\ & + \frac{1}{\pi} P \int_{A + \frac{t}{4m}}^{\infty} d\nu'\, \operatorname{Im} T^{\pm}(\nu' t)\left[\frac{1}{\nu' - \nu} \pm \frac{1}{\nu' + \nu}\right] \end{aligned} \tag{1.46}$$

To apply the relations (1.46) to actual scattering problems it is necessary that the integrals should be asymptotically convergent For instance if Im T behaved like an $n^{\underline{th}}$ order polynomial in ν then one way to ensure that the dispersive integrals were at worst only logarithmically divergent would be to consider the dispersion

relation for

$$\mathrm{Re}\ T(\nu,t) - \sum_{i=1}^{n} \mathrm{Re}\ T(\nu_i, t_i) \qquad (1.47)$$

where the values of Re $T(\nu,t)$ are known for $\nu = \nu_i$ and $t = t_i$. The application of this technique is known as 'making subtractions'. However, for a forward scattering amplitude $f(s,t=0)$, Froissart[14] obtained the following bound

$$|f(s,t=0)| \leq \text{constant} \times s(\log s)^2 \quad \text{as} \quad s \to \infty \qquad (1.48)$$

Hence the relation for $T^+(\omega, t=0)$ requires a subtraction to obtain convergence at high energies. The Pomeranchuk theorem[15] states

$$\sigma_-(\omega) - \sigma_+(\omega) \underset{\omega\to\infty}{\longrightarrow} 0 \qquad (1.49)$$

where $\sigma_\pm(\omega)$ are related to $\mathrm{Im}\ T_\pm(\omega,0)$ by the optical theorem . With the additional assumption

$$[\sigma_-(\omega) - \sigma_+(\omega)] \log\omega \underset{\omega\to\infty}{\longrightarrow} 0 \qquad (1.50)$$

the relation for $T^-(\omega,0)$ does not need a subtraction. The non-forward fixed t dispersion relations suffer from the fact that a subtraction introduces an arbitrary function of momentum transfer (i.e. the t_i in (1.47)) instead of a constant as is the case for the forward dispersion relations. Therefore, it is difficult to check their predictions.

The most obvious application of forward dispersion relations is to predict a value for G^2, or the pseudo vector coupling

constant f^2, which are related by the equivalence theorem

$$f^2 = \frac{1}{4\pi}\left(\frac{G_\mu}{2M}\right)^2 \tag{1.51}$$

This has been done in several ways which are explained briefly below.

With the notation

$$T^{\pm}(w,0) = D^{\pm}(w,0) + i\,A^{\pm}(w,0) \tag{1.52}$$

where $D^{\pm}$, $A^{\pm}$ are real functions, consider the subtracted forward dispersion relations

$$D^{+}(w) - D^{+}(\mu) \tag{1.53}$$

and

$$D^{-}(w) - \frac{w}{\mu} D^{-}(\mu)$$

(i.e. the subtraction is made at threshold, where $w = \mu$). Using the relations (1.45) and (1.46) the following forward dispersion relation was first obtained by Goldberger et al.[12]

$$\begin{aligned} D_{\pm}(w,0) = {} & \tfrac{1}{2}\left(1 + \tfrac{w}{\mu}\right) D_{\pm}(\mu) + \tfrac{1}{2}\left(1 - \tfrac{w}{\mu}\right) D_{\mp}(\mu) \\ & \pm \frac{2f^2}{\mu^2} \frac{k_L^2}{\left(w \mp \frac{\mu^2}{2M}\right)} \frac{1}{\left(1 - \mu^2/4M^2\right)} \\ & + \frac{k_L^2}{4\pi^2} P \int_{\mu}^{\infty} \frac{dw'}{k_L'} \left[\frac{\sigma_{\pm}(w')}{(w' - w)} + \frac{\sigma_{\mp}(w')}{(w' + w)} \right] \end{aligned} \tag{1.54}$$

where k_L is the incident pion momentum in the laboratory system and the suffix L on w has been dropped.

The earlier attempt to use (1.54), by Puppi and Stanghellini[16], suffered from a lack of precision in the data measurements from

which the dispersive integrands were calculated. With corrected data measurements Spearman[17] computed the curve for $D_{+}(\omega)$ for several values of f^2 and compared the result with the $\pi^{\pm}p$ data. He found

$$f^2 = 0.08 \pm 0.005$$

This value is in good agreement with the value of f^2 found from the effective range thoery suggested by Chew and Low[18], for the isospin I=3/2, angular momentum J = 3/2 pion-nucleon resonance.

Schnitzer and Salzmann[19] re-expressed (1.54) in a form with a linear dependence on ω. Plotting the appropriate dispersive contributions against ω and comparing with the $\pi^{\pm}p$ data produced

$$f^2 = 0.08 \pm 0.01$$

Alternatively, by using the identity

$$\frac{1}{\omega'^2 - \omega^2} = \frac{1}{\omega'^2} + \frac{\omega^2}{\omega'^2(\omega'^2 - \omega^2)} \qquad (1.55)$$

to increase the asymptotic convergence of the dispersion integrals, Haber-Schaim[20] re-expressed the dispersion relation for $D^{-}(\omega, t=0)$, from (1.46), (1.52), as a linear function of ω^2. The appropriate dispersive contributions plotted against ω^2 gave

$$f^2 = 0.082 \pm 0.015.$$

All the dispersion relations used so far are discussed in greater detail in Chapter V when they are used to determine the KN coupling constants.

The dispersion relation for Re $B_{+}(\omega, t=0)$ given by (1.33), (1.41), provides a very accurate method of calculating f^2 as the

the major contribution to the principal valued integral arises from the well known $I=3/2$, $J=3/2$ pion-nucleon resonance, and Im B_- (ω, t=0) is not involved in a principle valued integral and so need not be so accurately determined. The contributions of the dispersion integrals were estimated from the results of semi-phenomenological fits to the experimental data, as the optical theorem only relates the total cross section to the imaginary part of the whole forward amplitude. This procedure enabled Woolcock [21] to obtain

$$f^2 = 0.081 \pm 0.003.$$

All these results depend on the use of forward dispersion relations. However, f^2 can be determined from photomeson production without the use of dispersion relations if the photomeson amplitude is assumed analytic in the region containing the physical region $|\cos\theta| < |$, and including the crossed pion pole at $\cos\theta = 1/q_\pi$ as an isolated singularity (where q_π is the pion velocity in the centre of mass system). With these assumptions Taylor et al.[22] found f^2 to be

$$f^2 = 0.064 \pm 0.041$$

Within the errors this is in good agreement with the value of f^2 predicted by forward dispersion relations.

Anderson et al.[23] evaluated the $\pi^\pm$ p forward dispersion relations with $f^2 \sim 0.08$ and found good agreement with the low energy experimental data. This successful agreement was extended by Amblard et al.[24], throughout the region $\omega < 1.98$ BeV.

More recently the $\pi^{\pm}p$ comparison with experiment has been made in the energy region 8 BeV/$c < k_L <$ 29 BeV/c[25]. The results at these high energies are virtually independent of f^2. Defining

$$\alpha_{\pm} = \mathrm{Re}\, T_{\pm}(\omega, 0) \,/\, \mathrm{Im}\, T_{\pm}(\omega, 0) \tag{1.58}$$

the closeness of the forward dispersion relation predictions to the experimental results is very impressive for the quantity $(\alpha_+ + \alpha_-)$. The forward dispersion relations predictions for the quantity $(\alpha_- - \alpha_+)$ are of the right shape but differ systematically from the experimental results. Nevertheless, this does imply a verification of the Pomeranchuk theorem as $(\alpha_- - \alpha_+)$ is approximately $2T^-$. Thus we may conclude that forward dispersion relations do provide a means of making 'real' theoretical predictions.

The general kinematics for nucleon pseudoscalar meson interactions are formulated in Chapter II, which also contains a discussion of the complications induced by the presence of an 'unphysical' region in the kaon nucleon forward dispersion relations.

The values of the coupling constants g^2_{Λ}, g^2_{Σ} due to the Λ and Σ poles, are calculated in Chapter III by using a forward dispersion relation method suggested by Lusognoli et al.[26] with which our results are compared.

To overcome the inherent disadvantages of this method a new forward dispersion relation is suggested in Chapter IV and the predictions are compared with those obtained by using the previous forward dispersion relation.

The various methods used to predict values for g^2_Λ, g^2_Σ are summarised in Chapter V. Chapter VI contains the predictions obtained by differentiating two dispersion relations with respect to energy.

Finally, Chapter VII contains the results of a comparison of the new forward dispersion relation with experiment, and some general conclusions which may be deduced.

CHAPTER II

THE KAON NUCLEON LOW ENERGY REGION AND KINEMATICS.

1 INTRODUCTION

Although the forward dispersion relations for the kaon nucleon process are not on the same firm theoretical basis as those for the pion nucleon interactions, indeed for K mesons strangeness upsets the plausibility arguments based on the inherent symmetry of the pion[27], the successful prediction of the pion nucleon coupling constant indicated that appropriate forward dispersion relations, if valid, could provide an accurate determination of the kaon nucleon coupling constants due to the lambda and sigma poles. To evaluate the latter relations the effects of the $K^{-}N$ absorption channels have previously been taken into account by Dalitz and Tuan[28] and independently by Jackson and Wyld[29] using a multi-channel formalism, with the assumptions that the open three-particle channels are either weak, in particular the $\Lambda\pi\pi$ channel is neglected, or have thresholds outside the range of interest. Both methods depend upon re-expressing the S-matrix in terms of another matrix, the K-matrix.

2. THE K MATRIX

In general, the S-matrix may be expressed in operator notation as

$$S = 1 + 2iT \tag{2.1}$$

where T is the operator which refers to the connected parts of the S-matrix.

It is useful to define another operator, usually denoted by K, which satisfies

$$S = \frac{1 + iK}{1 - iK} \tag{2.2}$$

The unitarity of S corresponds to K being a hermitian operator

$$K^{+} = K \tag{2.3}$$

The relationship between K and T may be expressed through the following equation[30]

$$T = K + iKT \tag{2.4}$$

The unitarity condition takes a particularly simple form when expressed in terms of states with a definite total angular momentum J for which

$$T_J - T_J^{+} = 2i\, T_J^{+} T_J \tag{2.5}$$

Formally this is exactly the same as the relation obtained by using equation (2.1) and the condition $S^{+}S = 1$.

The counterpart of (2.1) for the matrix S_J defined by

$$S_J = 1 + 2i\, T_J \tag{2.6}$$

is often useful. This is

$$S_J^+ S_J = 1 \tag{2.7}$$

We can also define K matrix elements in terms of a matrix K_J where

$$K_J = K_J^+ \tag{2.8}$$

and

$$T_J = K_J + iK_J T_J \tag{2.9}$$

This relation (2.9) may be rewritten as

$$(T_J)^{-1} = (K_J)^{-1} - i \tag{2.10}$$

The importance of this equation arises from the fact the invariance under time reversal combined with the hermitian property for K implies that K_J, and similarly $(K_J)^{-1}$, is a real symmetric matrix. Thus equation (2.10) represents the separation of $(T_J)^{-1}$ into its real and imaginary parts.

The usefulness of these relations may be illustrated by considering a single channel process involving the scattering of particles with zero spin. In this case T_J and S_J are just scalar amplitudes and equs(2.5)-(2.7) become

$$\text{Im } T_J = |T_J|^2 \tag{2.11}$$

$$|S_J|^2 = 1 \tag{2.12}$$

Equations (2.8), (2.10) give

$$\text{Im } (T_J)^{-1} = -1 \tag{2.13}$$

Equation (2.12) allows one to write

$$S_J = e^{2i\delta_J} \tag{2.14}$$

where the factor 2 in the exponential is introduced to agree with

standard convention, and δ_J is a real scalar function of the centre of mass momentum k known as the phase-shift for scattering in the partial wave J.

From equs (2.10)-(2.14) we obtain

$$(K_J)^{-1} = \cot \delta_J \tag{2.15}$$

If instead of T_J we use the partial-wave amplitude f_J defined by

$$f_J = \frac{1}{k} T_J \tag{2.16}$$

then

$$f_J = \frac{e^{i\delta_J} \sin \delta_J}{k} \tag{2.17}$$

Thus, so long as the energy is below the threshold for any inelastic processes each partial wave amplitude may be expressed in terms of a real function of momentum.

In addition, the l^{TH} partial wave (where l is the arbital angular momentum, so $J = l$ for spinless particles whilst $J = l \pm \frac{1}{2}$ for a spin $\frac{1}{2}$ -spin 0 interaction), has a further momentum dependence of k^{2l} when $k \sim 0$[31](this point is discussed in appendix A). This suggests that in general we should define the K matrix elements so that the i, j^{th} element is

$$K_{ij} = k_i^{l_i + \frac{1}{2}} R_{ij} k_j^{l_j + \frac{1}{2}} \tag{2.18}$$

where k_i, l_i and k_j, l_j are the centre of mass momentum, orbital angular momentum for scattering from the i^{th} to the j^{th} channel.

Thus the elements of the R-matrix are real and symmetric and do not contain the threshold branch points associated with the K-matrix.

For spinless particles $\ell = J$ and so equ(2.15) becomes

$$(R_\ell)^{-1} = k^{2\ell+1} \cot \delta_\ell \qquad (2.19)$$

Moreover, because t he R matrix elements are analytic in k^2 (32) we may express them as a power series in k^2. For a single channel this procedure simply gives the effective range formula suggested to describe the low energy phase shifts, that is

$$k^{2\ell+1} \cot \delta_\ell = \frac{1}{a_\ell} + \frac{1}{2} r_\ell k^2 + O(k^4) \qquad (2.20)$$

where the constants a_ℓ, r_ℓ are called the 'scattering length' and 'effective range'. If $r_\ell = 0$ the relation (2.20) is known as the'zero-range approximation'.

For $\bar{K}N$ scattering, the isospin I = 0 state involves the $\Sigma\pi$ K^-N channels, and the isospin I = 1 state involves the $\Sigma\pi$, $\Lambda\pi$, K^-N channels. At the present time the lack of knowledge of the values of all the parameters involved necessitates a further simplification that is, that the isospin I = 1 absorptive effects are given entirely by a single pion hyperon channel whose threshold coincides with that for $\Lambda\pi$. Thus for given J the R-matrix becomes

$$\underline{\underline{R}}_J = \begin{pmatrix} \alpha_0 & \beta_0 & \underline{\underline{0}} \\ \beta_0 & \gamma_0 & \\ & & \alpha_1 \quad \beta_1 \\ \underline{\underline{0}} & & \beta_1 \quad \gamma_1 \end{pmatrix} \qquad (2.21)$$

Written in terms of its two disjoint submatrices which refer to the isospin states I = 0, 1. Note that the J suffix on $\alpha_i, \beta_i, \gamma_i$ has been suppressed for clarity. From the lack of evidence to the contrary these elements $(\alpha_i \beta_i \gamma_i)$ are normally assumed to be constant, although section (IV-3) illustrates the results obtained when an explicit k^2 dependence is included in the R-matrix elements.

Furthermore, let us assume that for a given isospin state, in which case we are dealing with 2 x 2 matrices, the T-matrix elements refer to the various channels as follows -

$$\begin{pmatrix} \bar{K}N \to \bar{K}N & \bar{K}N \to Y\pi \\ Y\pi \to \bar{K}N & Y\pi \to Y\pi \end{pmatrix} \tag{2.22}$$

with the hyperon Y = $\binom{\Sigma}{\Lambda}$ for the isospin state I = $(\frac{0}{1})$. For a given isospin state equ(2.9) becomes

$$\underline{\underline{T}}_J = \frac{\begin{pmatrix} k^{2l+1}\alpha & (qk)^{l+\frac{1}{2}}\beta \\ (qk)^{l+\frac{1}{2}}\beta & q^{2l+1}\gamma \end{pmatrix}\begin{pmatrix} 1-iq^{2l+1}\gamma & i(qk)^{l+\frac{1}{2}}\beta \\ i(qk)^{l+\frac{1}{2}}\beta & 1-ik^{2l+1}\alpha \end{pmatrix}}{(1-ik^{2l+1}\alpha)(1-iq^{2l+1}\gamma) + (qk)^{2l+1}\beta^2} \tag{2.23}$$

where k, q are the KN, Yπ centre of mass momenta. The denominator of this expression may be written as

$$\Delta = (1-iq^{2l+1}\gamma)\left[1 - ik^{2l+1}\alpha + \frac{(kq)^{2l+1}\beta^2}{1-iq^{2l+1}\gamma}\right] \tag{2.24}$$

Thus the generalization of equ(2.16) to

$$\left(f^{I}_{J}\right)_{ij} = \frac{\left(T^{I}_{J}\right)_{ij}}{\sqrt{k_i k_j}} \tag{2.25}$$

where

$$T^{I}_{J} = e^{i\delta^{I}_{J}} \sin \delta^{I}_{J} \tag{2.26}$$

gives the $\bar{K}N \rightarrow \bar{K}N$ partial wave amplitude

$$\left(\bar{f}_{J}\right)_{11} = \frac{k^{2l}}{\Delta}\left(\alpha + i q^{2l+1}(\beta^2 - \alpha\gamma)\right) \tag{2.27}$$

Similarly the $Y\pi \rightarrow Y\pi$ partial wave amplitude is

$$\left(\bar{f}_{J}\right)_{22} = \frac{q^{2l}}{\Delta}\left(\gamma + i k^{2l+1}(\beta^2 - \alpha\gamma)\right) \tag{2.28}$$

Below the $\bar{K}N$ threshold k becomes imaginary

$$\therefore \quad k = i\kappa \tag{2.29}$$

where $\kappa = |k|$. (2.30)

For s waves ($l = 0$) we may write

$$\begin{aligned} a &= \alpha - q^2\gamma\beta^2/(1 + q^2\gamma^2) \\ b &= q\beta^2/(1 + q^2\gamma^2) \end{aligned} \tag{2.31}$$

In the $\bar{K}N \rightarrow \bar{K}N$ channel we obtain the following relation from (2.19) (2.25) (2.27) (2.31) for the s-wave phase shift

$$k \cot \delta_{l=0} = \frac{1}{a+ib} \tag{2.32}$$

Thus the absorptive effects may be incorporated into the single channel s-wave zero ranges formula for the $\bar{K}N$ channel by ~~allowing~~ taking

the scattering length to become complex constant.

In the zero range approximation the $\bar{K}N$ s-wave centre of mass amplitude is given by (2.27), and is for isospin I

$$\bar{f}_0^I = (a_I + ib_I) / (1 - ik(a_I + ib_I)) \tag{2.33}$$

from (2.31).

Above the $\bar{K}N$ threshold (dropping the I suffix for clarity)

$$\mathrm{Im}\,\bar{f}_0 = \frac{b + k(a^2+b^2)}{(1+kb)^2 + (ka)^2} \tag{2.34}$$

$$\mathrm{Re}\,\bar{f}_0 = \frac{a}{(1+kb)^2 + (ka)^2} \tag{2.35}$$

Below the $\bar{K}N$ threshold

$$\mathrm{Im}\,\bar{f}_0 = \frac{b}{(1+\pi a)^2 + (\pi b)^2} \tag{2.36}$$

$$\mathrm{Re}\,\bar{f}_0 = \frac{a + \pi(a^2+b^2)}{(1+\pi a)^2 + (\pi b)^2} \tag{2.37}$$

Equations (1.35) and (1.36) give the decomposition of the K^-p, K^-n centre of mass s-wave amplitudes in terms of the isospin amplitudes.

Without the complications due to absorption channels the K^+N s-wave amplitudes may be written in terms of the usual effective range approximation (2.20) as

$$f_0^I = B_I / (1 - i k B_I) \tag{2.38}$$

where

$$B_I = a_+^I / (1 + \tfrac{1}{2} a_+^I r_+^I k^2) \tag{2.39}$$

The s-wave centre of mass amplitude for K^+p which is pure isospin I=1 from (1.35), is given by equs(2.38)-(2.39) with
$a_+^I = a_+^1$ the s-wave I = 1 scattering length and
$r_+^I = r_+^1$ the s-wave I = 1 effective range. The K^+n s-wave centre of mass amplitude involves both isospin I = 1 and I = 0 from (1.36) and thus requires a knowledge of a_+^0 , the K^+N I = 0 s-wave scattering length.

3 KN KINEMATICS

In addition to the modifications due to charge and spin it is convenient to introduce an extra energy factor into the relation for the elastic S-matrix which then becomes[33]

$$S = S_{fi} - i(2\pi)^4 \delta^4(q_1+q_1-p_2-q_2)\left(\frac{M^2}{4E_1E_2\omega_1\omega_1}\right)^{\frac{1}{2}} \bar{u}_2 T u_1 \quad (2.40)$$

and $S_{fi} = 0$ unless the initial and final states are identical when $S_{fi} = 1$. Also $\omega_i = (\mu^2+\vec{q}_i^2)^{\frac{1}{2}}$ and $E_i = (M^2+\vec{p}_i^2)^{\frac{1}{2}}$ where μ, M are the kaon and nucleon masses and p_1, p_2 and q_1, q_2 refer to the initial, final four momenta of the nucleons and mesons respectively. As before u_1 and u_2 are the Dirac spinors for the initial and final nucleon states and $\bar{u}\,u = 1$.

The re-defined invariant amplitudes are then expressed as

$$T = -A + \frac{i}{2}\gamma_\mu (q_1+q_2)^\mu B \quad (2.41)$$

If k_L, w_L are the incident kaon momenta and energy in the laboratory system then

$$s = M^2 + \mu^2 + 2M\omega_L \quad (2.42)$$

with

$$\omega_L = (\mu^2 + \vec{k}_L^2)^{\frac{1}{2}} \quad (2.43)$$

using our previous definition of s, t, u (i.e. equ.(1.4)).

The total energy in the centre of mass (c.m.) system is

$$W = (M^2+\vec{k}^2)^{\frac{1}{2}} + (\mu^2+\vec{k}^2)^{\frac{1}{2}} \quad (2.44)$$

where k is the c.m. momentum and $s = W^2$ for scattering in the s channel.

Therefore (2.44) implies

$$k^2 = (s - (M+\mu)^2)(s - (M-\mu)^2) / 4s \qquad (2.45)$$

Equs(2.42), (2.43) (2.45) give

$$k_2 = \frac{W}{M} k \qquad (2.46)$$

With the definitions (1.4), t is the invariant momentum transfer, and for a c.m. scattering angle θ

$$t = -2k^2(1 - \cos\theta) \qquad (2.47)$$

It is helpful to express the invariant amplitude, defined by equ (2.41), in terms of the amplitudes $f_1(\theta)$, $f_2(\theta)$ which are related in a simple way to the helicity amplitudes of Jacob and Wick[34]. Thus, if $|1\rangle$ and $|2\rangle$ are the Pauli spinors for the initial and final nucleon spin states then

$$\frac{M}{4\pi W} \bar{u}_2 T u_1 = -\langle 2 | f_1(\theta) + \frac{(\sigma\cdot\bar{q}_2)(\sigma\cdot\bar{q}_1)}{q^2} f_2(\theta) | 1 \rangle \qquad (2.48)$$

The negative sign is conventional; $\frac{1}{2}\sigma$ is the nucleon spin and q_1, q_2 the initial and final c.m. meson momenta. From equations (2.41)-(2.48) we deduce

$$f_1 = \frac{E+M}{8\pi W}\left[A + (W-M)B\right] \qquad (2.49)$$

$$f_2 = \frac{E-M}{8\pi W}\left[-A + (W+M)B\right] \qquad (2.50)$$

where $E = (M^2+k^2)^{1/2}$ is the energy of the nucleon in the c.m. system.

Inverting these relations gives

$$\frac{1}{4\pi} A = \frac{W+M}{E+M} f_1 - \frac{W-M}{E-M} f_2 \qquad (2.51)$$

$$\frac{1}{4\pi} B = \frac{1}{E+M} f_1 + \frac{1}{E-M} f_2 \qquad (2.52)$$

The partial wave amplitudes $f_{l\pm}$ corresponding to total angular momentum $J = l \pm \frac{1}{2}$ are expressed in terms of the phase shifts $\delta_{l\pm}$ by equ(2.17) i.e.

$$f_{l\pm} = \frac{e^{2i\delta_{l\pm}} - 1}{2ik} \qquad (2.53)$$

From reference (34) the following relations are obtained

$$f_1(\theta) = \sum_{l=0}^{\infty} \left(f_{l+} P'_{l+1}(x) - f_{l-} P'_{l-1}(x) \right) \qquad (2.54)$$

$$f_2(\theta) = \sum_{l=1}^{\infty} \left(f_{l-} - f_{l+} \right) P_l'(x) \qquad (2.55)$$

where $x = \cos\theta$; $(')$ denotes differentiation with respect to x, and P_l are Legendre polynomials.

Using the orthogonality relation

$$\delta_{kl} = \int_{-1}^{1} dx\, P'_k(x) \left(P_{l+1}(x) - P_{l-1}(x) \right) \qquad (2.56)$$

the inverse relations become

$$f_{l\pm} = \frac{1}{2} \int_{-1}^{1} dx \left(P_l(x) f_1 + P_{l\pm1} f_2 \right) \qquad (2.57)$$

This relation makes it possible to express the partial wave amplitudes in terms of the A and B amplitudes and thus gives the analytic properties of the partial waves. As the behaviour of f_l is like k^l for small k, in general only a few partial waves are necessary to describe the behaviour of the A, B amplitudes at low

energies. For this region the low energy KN, $\bar{K}N$ contributions to the forward dispersion relations are assumed to arise wholly from the $\ell = 0$ partial waves apart from the special case of the $Y_1^*(1385)$ (see section (IV-4).

The forward scattering amplitude f(s,t) in the c.m. system is

$$f(k, \theta=0) = f_1(k, \theta=0) + f_2(k, \theta=0) \tag{2.58}$$

from equ(2.48). Note that t = 0 when $\theta = 0$ from (2.47).

By (2.49), (2.50) and (2.42)

$$f(k, 0) = \frac{M}{4\pi W}(A + w_L B) \tag{2.59}$$

The optical theorem (1.10) becomes

$$Im\, f(k, 0) = \frac{k}{4\pi} \sigma_{tot} \tag{2.60}$$

However if the nucleon is at rest (i.e.in the laboratory frame) the optical theorem assumes the form[35]

$$Im\, f_L(k, 0) = \frac{k_L}{4\pi} \sigma_{tot} \tag{2.61}$$

where

$$f_L(k_L, 0) = \frac{1}{4\pi}(A + w_L B) \tag{2.62}$$

is the forward laboratory amplitude.

Because of the presence of W in equation (2.59) the forward dispersion relations must be written in terms of the analytic

laboratory forward scattering amplitude $f_L(w,t)$ where

$$f_L(w,0) = \frac{W}{M} f(w,0) \tag{2.63}$$

from (2.59) and (2.62).

In keeping with our previous notation we define

$$T^{\pm}(w,0) = A^{\pm}(w,0) + w B^{\pm}(w,0) \tag{2.64}$$

which satisfies the crossing relations (1.44) when $t = 0$.

The $K^{\pm}N$ forward amplitudes are defined by

$$T_{\pm}(w,0) = T^{+}(w,0) \mp T^{-}(w,0) \tag{2.65}$$

With these definitions the kaon nucleon forward dispersion relations may be written down in analogy with equation (1.46).

CHAPTER III

THE CONVENTIONAL FORWARD DISPERSION RELATIONS FOR KAON NUCLEON SCATTERING

1. THE FORM OF THE RELATIONS.

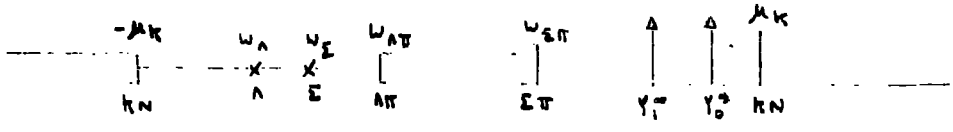
The relative simplicity of the form of the forward dispersion relations (1.46) depends on the symmetry under crossing of the pion - no new competing channels are opened up. However the prescence of absorption channels in the $\bar{K}N$ process means that Im $T_-(w,t)$ is non-zero in the region

$$\mu_Y + \mu_\pi < W < m_N + \mu_K \qquad (3.1)$$

that is, below the kaon nucleon threshold but above the pion hyperon threshold, where the centre of mass energy W is given by equation (2.44).

The cut diagram for the KN interaction in the w plane is shown below.

Figure 4:



Taking account of the 'unphysical' region (3.1) leads to a forward dispersion relation for KN which is

$$D_{\pm}(w, t=0) = \sum_{\gamma} \frac{g_{\gamma}^2 \chi(\gamma)}{w_{\gamma} \pm w} + \frac{1}{\pi} \int_{w_{\gamma\pi}}^{\mu} \frac{A_{-}(w'0)\, dw'}{w' \pm w} \qquad (3.2)$$
$$+ \frac{1}{\pi} \int_{\mu}^{\infty} \frac{A_{\mp}(w'0)\, dw'}{w' + w} + \frac{1}{\pi} \int_{\mu}^{\infty} \frac{A_{\pm}(w'0)\, dw'}{w' \mp w}$$

in unsubtracted form. Where the $\pm$ refer to the $K^{\pm}N$ scattering amplitudes in the laboratory frame, and the numerators of the pole terms in (3.2) may be rewritten in terms of the field theoretic unrationalized renormalized coupling constants by using Feynman[36] rules. Thus

$$\chi(\gamma) = \frac{(m_{\gamma} - m)^2 - \mu^2}{4 m m_{\mu}} \qquad (3.3)$$

The signs in equ(3.3) rely on the relative parities assigned to the strange particles. The relation (3.3) is discussed in appendix D.

Alternatively, equation (3.2) may be rewritten using (2.65).

$$D^{\pm}(w,0) = \sum_{\gamma} g_{\gamma}^2 \frac{\chi(\gamma)}{2} \left[\frac{1}{w_{\gamma} - w} \pm \frac{1}{w_{\gamma} + w} \right]$$
$$+ \frac{1}{2\pi} \int_{w_{\gamma\pi}}^{\mu} \frac{A_{-}(w'0)}{w'^2 - w^2} \cdot (w' + w \pm w' \mp w)\, dw'$$
$$+ \frac{1}{2\pi} \int_{\mu}^{\infty} \frac{dw'}{w'^2 - w^2} \left[\begin{array}{l} A_{-}(w'0)\cdot(w' + w \pm w' \mp w) \\ + A_{+}(w'0)(w' - w \pm w' \pm w) \end{array} \right] \qquad (3.4)$$

The asymptotic behaviour of equs(3.2), (3.4) is fixed by the Froissart[14] bound, equ(1.48), and the assumption of the Pomeranchuk theorem[15], equ(1.49), so that only the relation for $D^{-}(w)$ is useful in unsubtracted form. Consequently we shall in future mean the dispersion relation for $D^{-}(w,0)$ when we refer to equation (3.4). $D_{\pm}(w)$ are given by the s-wave scattering lengths whenwhen equ(3.4) is evaluated at threshold, $w = \mu$, to determine the coupling constants g^2_{γ}. The values of g^2_{γ} may then be compared

with previous results and in particular with the predictions of the SU(3) symmetry scheme discussed in chapter V.

Note that the contribution from the unphysical region is complicated further by the presence of two experimentally observed resonances with the KN quantum numbers - the s-wave (ℓ=0) $Y_0^*(1405)$ and the p-wave (ℓ=1) $Y_1^*(1385)$

2 THE CALCULATION OF THE COUPLING CONSTANTS

In this section the values of the rationalized renormalized coupling constants $g^2_{\Lambda\bar{K}N}$ and $g^2_{\Sigma\bar{K}N}$ are redetermined using the method of Lusognoli et al.[26]. Firstly, g^2_{Λ} was found by considering the $K^{\pm}p - \frac{1}{2} K^{\pm}n$ combination of the dispersion relations of the type (3.4), which eliminates both the Σ pole and the $Y_1^*(1385)$ resonance contribution through equations (1.35)-(1.36), and assuming that the integral over the unphysical region is correctly given by extrapolation using the I = 0 s-wave $\bar{K}N$ scattering length a_0+ib_0 with the values determined by Kim[37].

Similarly, the $K^{\pm}n$ dispersion relations of the type (3.4) gave an upper bound for g^2_{Σ} when the s-wave contribution to the integral over the unphysical region was evaluated by extrapolation using the values given in reference (37) for the I = 1 $\bar{K}N$ scattering length a_1+ib_1. This inequality arises because the sign of the Y_1^* contribution is fixed by the imaginary part of the p-wave amplitude, which gives an effect of constant sign in the resonance region, and in addition remembering the p wave k^{2l} factor which is $-K^2$ in this case.

Rewriting equ(3.4) with the pole terms on the left hand side of the equation, the contributions on the right hand side may be denoted by

$$c_1 + c_2 + c_3 + c_4 + c_5 - 2(Y_1^*) \qquad (3.5)$$

where

c_1 = term involving $D_{\pm}(\mu)$

c_2 = K^- (s-wave) contribution from wY_π to w_2 = 574.2 MeV.

c_3 = K^- cross section integral from w_2 to 20 GeV.

c_4 = K^+ cross section integral from μ to 20 GeV.

c_5 = the integral involving σ_t above 20 GeV.

$Z(Y_1^*)$ is a positive quantity representing the effect of the $Y_1^*(1385)$ contribution. Note that $Z_n(Y_1^*) = 2Z_p(Y_1^*)$ from charge independence where the n, p subscript denotes the nucleon involved, and $g^2_{K^-\Sigma^-n} = 2g^2_{p\Sigma^0\bar{K}}$ - neglecting the very small mass difference effect.

The contributions c_1 and c_2 were calculated using the energy independent scattering lengths of Kim[37] for $\bar{K}N$, the I =1 scattering length and effective range of Goldhaber et.al.[38] for K^+p, and in addition the I = 0 scattering length of Stenger et.al.[39] for K^+n. The K^+p s-wave scattering length and effective range and hence the s-wave amplitude are fairly well determined near threshold, while although, not so well known, the I = 0 s-wave scattering length gives a very small contribution to the K^+n s-wave amplitude. Therefore, up to energy w_1 = 514 MeV where the K^+N total cross section data commences, the contributions from the dispersive integrals are well determined and comparatively small. In the range $w_1 < w' < 20$ GeV the K^+N total cross section data[40] was used to give the contribution c_4. Above w_2 the $\bar{K}N$ total cross section data[41] gives c_3. Above w' = 20 GeV the combined effects of the KN and $\bar{K}N$ dispersive integrals

were estimated by using the formalism of Phillips and Rarita[42]. An excellent feature of this relation is the way the total cross sections subtract so that the only contributions from the asymptotic region come from the ρ and ω meson trajectories. Further details are given in appendix B. The results of this calculation for $w = \mu$ are compared in table 2 with those of Lusognoli et.al.[26]. The values of the parameters used to determine the low energy contributions are shown in the following table.

Table 1: The input value (in fermis) of the low energy parameters for the present calculation.

$\bar{K}N$	KN
$a^0 = -1.67 \pm 0.04$	$a_+^1 = -0.29 \pm 0.015$
$b^0 = 0.71 \pm 0.04$	$r_+^1 = 0.5 \pm 0.5$
$a^1 = -0.07 \pm 0.06$	$a_+^0 = 0.04 \pm 0.04$
$b^1 = 0.68 \pm 0.03$	$r_+^0 = 0$

The nucleon, lambda, sigma, kaon and pion masses were taken to be 938.2, 1115.4, 1195, 493.8 and 138 MeV respectively.

Table 2: The contributions to the ($K^{\pm}p$) and ($K^{\pm}n$) dispersion relations of the type equ(3.4)in units 10^{-7} MeV^{-2}. Columns 2,3 contain the results of references (26) and the present investigation respectively. (Only the combination $c_1+c_2+c_4$ is known for column (2)).

	$K^{\pm}p$ (2)	$K^{\pm}n$	$K^{\pm}p$ (3)	$K^{\pm}n$
c_1			-45.5	4.3
c_2			102.2	29.8
c_3	-81.3	-63.5	-81.2	-63.5
c_4			44.5	43.8
c_5	-5.3	-3.3	-5.1	-3.1
$c_1+c_2+c_4$	101.0	75.6	(101.2)	(77.9)
Total	14.4	8.8	14.9	11.3

Note that Luis**o**gnoli et.al.[26] used the $\bar{K}N$ scattering lengths given by Kim[43] as opposed to those of Kim in reference (37) shown in table 1. The value of a_1 differs appreciably between these two references.

The values of the coupling constants were calculated from

$$2.12g^2_{\Lambda} = K^{\pm}p - \tfrac{1}{2}K^{\pm}n \qquad (3.6)$$

and

$$3.66g^2_{\Sigma^0} \leq K^{\pm}n \qquad (3.7)$$

Thus, Luis**o**gnoli et.al.[26] obtained

$$g^2_{\Lambda} = 4.8 \qquad g^2_{\Sigma^0} \leq 3.2 \qquad (3.8)$$

The results in column 3 of table 2 imply

$$g^2_{\Lambda} = 4.3 \qquad g^2_{\Sigma^0} \leq 3.1 \qquad (3.9)$$

Apart from the differences in the $\bar{K}N$ scattering length a_1, which contributes to c_1 for K^-n, a slight discrepancy in the values predicted for the coupling constants (3.8), (3.9) could arise from the small scattering length $a_+^o = 0.04$ fermi[39] which Lusignoli et.al.[26] possibly neglected. This point is discussed by Rood[44].

The previous table shows the acute dependence of the equ(3.4) on the cancellation of the contributions arising from the integrations over the unphysical region $w_{Y\pi} < w' < \mu$ and the term containing the real part of the amplitudes, both of which are large and have large errors associated with them. When evaluated in terms of the energy independent scattering lengths an estimate of these errors may be ascertained from the uncertainty in the scattering lengths in table 1. The size of the contribution of the unphysical region depends, to some extent, on the fact that the dispersive integrand contains the imaginary part of a resonant amplitude in the appropriate isospin and orbital angular momentum states - in the calculation of g_Λ^2 the Y_o^* contributions in the s-wave isospin I = 0 $\bar{K}N$ amplitude. The contribution from this effect is opposite in sign to that from the $D_\pm(\mu)$ term, so the value of their sum is much smaller. The second disadvantageous feacture of equ(3.4) is the principle-valuedness of the integrals containing the $\bar{K}N$ amplitudes. Under such an integration it is clear that the errors associated with the $\bar{K}N$ amplitudes due to the uncertainties in the scattering lengths in table 1, will be stressed when $w = \mu$. The quoted erros on a_o, a_1, b_o, b_1 in reference[37] enable one to conclude that g_Λ^2 is more sensitive to variations in a_o

than b_0 and $3.39 < g^2_{\Lambda} < 4.34$, whilst the upper bound on $g^2_{\Sigma 0}$ is more dependent on b_1 than the rather small a_1 scattering length and $2.77 < g^2_{\Sigma 0} < 3.42$. Note that the $\bar{K}N$ energy independent scattering lengths were varied in both the ranges $w_{Y\pi} < w' < \mu$ and $\mu < w' < w_2$ to give these results.

The calculation involving the use of the $\bar{K}N$ energy dependent scattering lengths given by equ(2.31), has been performed by Rood[44] using various sets of $\alpha_0, \beta_0, \gamma_0$ throughout the low energy region $w_{Y\pi} < w' < w_2$ for the $K^{\pm}p - \frac{1}{2} K^{\pm}n$ combination of equ(3.4). The corresponding variations in the position and width of the Y_0^* in the $\Sigma\pi - \Sigma\pi$ channel are reproduced below.

Table 3: Some results of reference (44).

α_0	$(\beta_0)^2$	γ_0	w_R	Γ_R
-1.41	0.3	0	1401	22
	0.4		1460	32
	0.5		1399	49
-1.51	0.4	0	1404	26
	0.5		1404	37
	0.6		1403	56
-1.61	0.4	0	1408	21
	0.5		1408	30
	0.6		1407	41
-1.51	0.3	0.1	1405	19
	0.4		1404	29
	0.5		1404	45
	0.3	0.2	1405	21
	0.4		1404	35
	0.5		1404	58
	0.3	0.3	1405	24
	0.4		1404	43

The conclusion of this investigation was that $g_{\Lambda}^2 = 7.4 \pm 1.2$. However, it does seem feasible that since the energy independent scattering lengths have been fitted to the experimental data for $\mu < \omega' < \omega_2$, the $\bar{K}N$ amplitudes may be assumed to be given by these constant scattering lengths, in this region apart from the uncertainty induced by the errors in table 2. Moreover, the effects of the absorption channels on the $\bar{K}N$ amplitudes must vanish at the pion-hyperon threshold. However, energy independent scattering lengths do not fulfull this condition as b_I is a non-zero constant (see equ(2.31)). Consequently, if only s-waves are important, the region $\omega_{\pi\Sigma} < \omega' < \mu$ may be parameterised in terms of the s-wave α, β, γ which satisfy this requirement using equs(2.31),(2.34) -(2.37). In addition, the scattering lengths must be continuous through the kaon nucleon threshold and for the I = 0 state can be adjusted until $\bar{f}_{22}$ reproduces the experimentally observed position and width of the Y_0^* (1405) resonance, and thence $\bar{f}_{11}$ may be evaluated in the dispersion relations with these values of $\alpha_0, \beta_0, \gamma_0$.

An alternative treatment of the $\bar{K}N$ I = 0 low energy region, which combines the methods of Luisognoli et.al.[26] and Rood[44] used to obtain the results in table 2 and table 3 respectively, is presented below.

Fixing the contribution of the region $\mu < \omega' < \omega_2$ by using the energy independent scattering lengths (this is done in all our

subsequent calculations), the unphysical region $w_{Y\pi} < w' < \mu$ was parameterized by keeping γ_o as an input parameter and fitting α_o and β_o to the energy independent scattering lengths a_o, b_o at threshold $w = \mu$ through equ(2.31). The position and width of the Y_o^* resonance now output may be compared with their experimental values in the $\Sigma\pi - \Sigma\pi$ channel. The results of this calculation are shown in the following table.

Table 4: The variation in g_Λ^2 and the c_2 contribution to the $K^\pm p - \frac{1}{2}K^\pm n$ relation. W_R and Γ are the position and width of the peak in the imaginary part of the I=0, $\Sigma\pi$-$\Sigma\pi$ amplitude (The first line of the table corresponds to the energy indepdenent a^o, b^o).

γ_o	W_R	Γ	c_2	g_Λ^2
—	—	—	87.3	4.34
0.4	1396	$\geq$70	95.50	8.2
0.2	1404	65	95.4	8.2
0	1409	41	94.8	7.9
-0.2	1413	31	93.6	7.4
-0.4	1415	25	91.9	6.6

The shape of the resonance in the $\Sigma\pi$-$\Sigma\pi$ channel was found to be very asymmetrical. Note that this procedure increases the value of g_Λ^2 significantly.

The disparity between this calculation and one using the energy dependent scattering lengths throughout the region $w_{\Sigma\pi} < w' < w_2$ may

be estimated by using $\alpha_0, \beta_0, \gamma_0$ to calculate the energy dependent a_o, b_o given by (2.31) at $w' = w_2$.

Table 5: The values of the energy dependent a_o, b_o at $w' = \mu$, w_2 using the $\alpha_0, \beta_0, \gamma_0$ shown.

α_0	$(\beta_0)^2$	γ_0	$a_0(\mu)$	$b_0(\mu)$	$a_0(w_2)$	$b_0(w_2)$
-1.41	0.887	0.4	-1.67	-0.71	-1.81	0.85
-1.54	0.810	0.2	-1.67	-0.71	-1.75	0.90
-1.67	0.784	0	-1.67	-0.71	-1.67	0.92
-1.80	0.810	-0.2	-1.67	-0.71	-1.59	0.90
-1.93	0.887	-0.4	-1.67	-0.71	-1.53	0.85

As g_Λ^2 is more sensitive to variations in a_o than in b_o, it appears that $|\gamma_0| \leq 0.3$ is required to give reasonable agreement between the calculations illustrated in tables 2, 4. However table 4 indicates that $\gamma_0 > 0$ to obtain a width for the Y_o^* resonance in reasonable agreement with experiment. This is in agreement with the values of γ_0 chosen by Rood and shown in table 3, that is $0 < \gamma_o < 0.3$.

We may conclude that it does seem necessary to reduce the variations in g_Λ^2 shown in equ(3.9) and tables 3, 4, due to the contribution of the unphysical region to the forward dispersion relations. Equivalently, the contribution of the Y_o^* resonance to the I = 0 unphysical region must somehow be suppressed. A suggestion to obtain this effect is ...

outlined in the following Chapter and the results of the subsequent forward dispersion relations are compared with those of equation (3.4).

CHAPTER IV

THE NEW FORWARD DISPERSION RELATIONS FOR KAON-NUCLEON SCATTERING

1. INTRODUCTION

The disadvantages of the previous forward dispersion relations equs(3.2)-(3.4) may be summarized as follows. For $w = \mu$

a) Only equ(3.4) is useful in unsubtracted form at $w = \mu$.

b) The contribution c_1 depends on both $D_-(\mu)$, with its corresponding errors, and $D_+(\mu)$.

c) The contribution from the unphysical region arises from an integration over the imaginary part of a resonant amplitude.

d) The errors on the previous two contributions are emphasized by the fact that they subtract to give a comparatively small total.

e) The low energy region is principal-valued when the coupling constants are determined.

For general w

f) An elaborate subtraction is necessary for equ(3.2) (see reference (4.5)).

To reduce the errors associated with this list, we note that the real part of the resonant amplitude goes through zero at the resonance position. This gives a clue for an alternative treatment of the unphysical region.

2 THE NEW FORWARD DISPERSION RELATIONS

Keeping the notation of section (I.5), viz

$$T_{\pm}(\omega) = D_{\pm}(\omega) + i A_{\pm}(\omega) \tag{4.1}$$

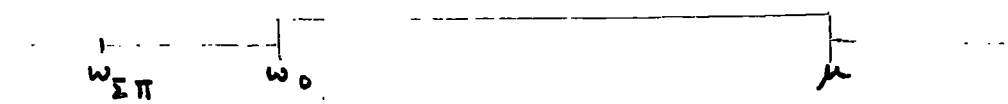
refer to the laboratory amplitudes for $K^{\pm}N$ scattering, consider the amplitude

$$\frac{T_{-}(\omega, 0)}{\sqrt{(\omega-\mu)(\omega-\omega_0)}} \tag{4.2}$$

in the complex laboratory energy plane w, where $\omega_{\Sigma\pi} < \omega_0 < \mu$

The extra cut due to $\sqrt{\xi(\omega)} = \sqrt{(\omega-\mu)(\omega-\omega_0)}$ is shown below.

Figure 5.



In the complex energy (ω) plane we may write

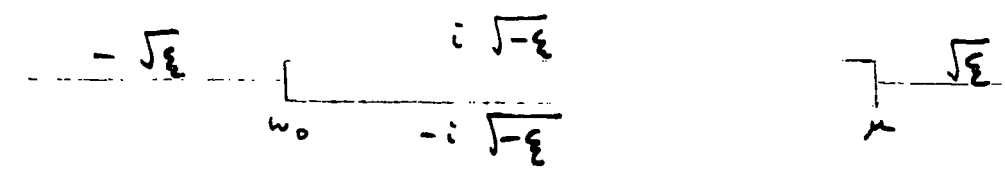
$$z = |\omega-\mu| e^{i\theta} \tag{4.3}$$

$$z' = |\omega-\omega_0| e^{i\theta'} \tag{4.4}$$

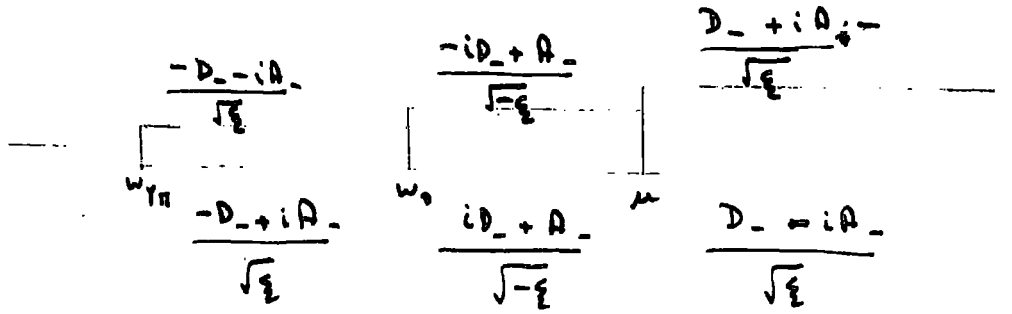
where θ, θ' are the arguments of z, z' in the Argand diagram.

Defining $\sqrt{\xi}$ to be positive for $w > \mu$, $\sqrt{z z'}$ becomes

Figure 6.



Therefore $T_-(\omega;0)/\sqrt{\xi(\omega)}$ is defined above and below the cut as follows. Figure 7.



By Cauchy's theorem (in analogy with section (I.3)).

$$\frac{T_-(\omega+i\varepsilon,\,0)}{\sqrt{\xi(\omega+i\varepsilon)}} = \text{Pole terms} + \frac{1}{2\pi i}\int_{\omega_{\gamma\pi}}^{\infty} \frac{d\omega'\ \mathrm{Disc}\left[T_-(\omega'0)/\sqrt{\xi(\omega')}\right]}{\omega'-\omega-i\varepsilon} + \frac{1}{2\pi i}\int_{-\omega}^{\omega_\ell} \frac{d\omega'\ \mathrm{Disc}\left[T_-(\omega'0)/\sqrt{\xi(\omega')}\right]}{\omega'-\omega-i\varepsilon} \tag{4.5}$$

where $0 < \varepsilon << 1$, and ω_ℓ denotes the end of the left-hand discontinuity. Using the properties of crossing, equ(1.44), we obtain the forward dispersion relation.

$$\frac{D_-(\omega 0)}{\sqrt{\xi(\omega)}} = -\sum_\gamma \frac{X(\gamma)\, g_\gamma^2}{\sqrt{\xi(\omega_\gamma)}\,(\omega_\gamma-\omega)} - \frac{1}{\pi}\int_{\omega_{\gamma\pi}}^{\omega_0} \frac{A_-(\omega')\,d\omega'}{\sqrt{\xi(\omega')}\,(\omega'-\omega)} \tag{4.6}$$

$$-\frac{1}{\pi}\mathcal{P}\int_{\omega_0}^{\mu} \frac{D_-(\omega')\,d\omega'}{\sqrt{-\xi(\omega')}\,(\omega'-\omega)} + \frac{1}{\pi}\mathcal{P}\int_{\mu}^{\infty} \frac{A_-(\omega')\,d\omega'}{\sqrt{\xi(\omega')}\,(\omega'-\omega)}$$

$$-\frac{1}{\pi}\int_{\mu}^{\infty} \frac{A_+(\omega')\,d\omega'}{\sqrt{\xi(-\omega')}\,(\omega'+\omega)}$$

where X(Y) is defined as in section (III-1). Note the subtle change in the contribution from the unphysical region compared with equ(3.2). The third term on the right hand side of equ(4.6) involves an integration over the real part of the $\bar{K}N$ forward amplitude.

Likewise, or by the crossing relation when $\omega \to -\omega$

$$\frac{-D_+(\omega)}{\sqrt{\xi(-\omega)}} = -\sum_Y \frac{g_Y^2\, X(Y)}{\sqrt{\xi(\omega_Y)}\,(\omega_Y+\omega)} - \frac{1}{\pi}\int_{\omega_{Y\pi}}^{\omega_0} \frac{A_-(\omega')\,d\omega'}{\sqrt{\xi(\omega')}\,(\omega'+\omega)}$$

$$- \frac{1}{\pi}\int_{\omega_0}^{\mu} \frac{D_-(\omega')\,d\omega'}{\sqrt{-\xi(\omega')}\,(\omega'+\omega)} + \frac{1}{\pi}\int_{\mu}^{\infty} \frac{A_-(\omega')\,d\omega'}{\sqrt{\xi(\omega')}\,(\omega'+\omega)} \qquad (4.7)$$

$$- \frac{1}{\pi}\int_{\mu}^{\infty} \frac{A_+(\omega')\,d\omega'}{\sqrt{\xi(-\omega')}\,(\omega'-\omega)}$$

It is clear that the convergence of the asymptotic integrals is inherent in equs(4.6)-(4.7) so that no subtractions are necessary. Also the usefulness of the optical theorem is retained as opposed to the method adopted by Gilbert[46] for pion nucleon, which is illustrated in appendix C.

Unfortunately, when evaluated at threshold $\omega = \mu$, the dispersion relation for $D_-(\omega,0)$, (4.6), requires a knowledge of the s-wave effective range terms and the p-wave scattering lengths for the $\bar{K}N$ scattering amplitudes. This is easily seen by the following argument.

For simplicity, consider the case when the $\bar{K}N$ scattering amplitude is given by the energy independent scattering lengths. Remembering

that the dispersion relations equs(4.6)-(4.7) have been written in terms of laboratory variables, equ(2.63) and equ(2.37) give, just below threshold

$$\mathrm{Re}\,\bar{T}^I = D^I = \frac{W}{\eta}\left[\frac{a_I + k(a_I^2+b_I^2)}{1+2ka_I+k^2(a_I^2+b_I^2)} - G k^2\right] \tag{4.8}$$

for the appropriate isospin (I) amplitude in the laboratory system, where G represents the s-wave effective range and p-wave scattering length terms.

Writing $x = \sqrt{\mu - w}$, we obtain

$$D^I \xrightarrow[x\to 0]{} \left(\frac{\eta+\mu}{\eta}\right)a_I + (b_I^2 - a_I^2)\sqrt{2\mu}\,x + x^2\left[\frac{2\mu\eta}{\eta+\mu}(a_I^3 - 3a_I b_I^2 - G) - \frac{a_I}{\eta+\mu}\right] + O(x^3) \tag{4.9}$$

In an obvious notation

$$D^I = R^I + U^I x + V^I x^2 + O(x^3) \tag{4.10}$$

Just above threshold equs(2.63)-(2.34) yield

$$\mathrm{Im}\,\bar{T}^I = A^I = \frac{W}{\eta}\left[\frac{b_I + k(a_I^2+b_I^2)}{1+2kb_I+k^2(a_I^2+b_I^2)} + H k^2\right] \tag{4.11}$$

where H represents the s-wave effective range and p-wave scattering length terms in this case.

If $y = \sqrt{w - \mu}$

$$A^I \underset{y\to 0}{\longrightarrow} \left(\frac{M+\mu}{M}\right) b_I + \left(a_I^2 - b_I^2\right)\sqrt{2\mu}\, y + y^2\left[\frac{2\mu M}{M+\mu}\left(b_I^3 - 3a_I^2 b_I + M\right) + \frac{b_I}{M+\mu}\right] + O(y^3) \tag{4.12}$$

Again, write

$$A^I = \mathcal{R}^I + \mathcal{U}^I y + \mathcal{V}^I y^2 + O(y^3) \tag{4.13}$$

The troublesome part of equ(4.6) as $w \to \mu+$ is

$$\frac{-D_-(w)}{\sqrt{\xi(w)}} - \frac{1}{\pi} P\int_{w_0}^{\mu} \frac{D_-(w')\,dw'}{\sqrt{\xi(w')}\,(w'-w)} + \frac{1}{\pi} P\int_{\mu}^{w_2} \frac{A_-(w')\,dw'}{\sqrt{\xi(w')}\,(w'-w)} \tag{4.14}$$

This may be evaluated by rewriting the integrals in the form

$$\frac{2}{\pi}\int_0^{\sqrt{\mu-w_0}} \frac{D_-(w') - R - U x}{x^2\sqrt{\mu - w_0 + x^2}}\, dx \tag{4.15}$$

where $w_0 \leq \beta < \mu$ and

$$\frac{2}{\pi}\int_0^{\sqrt{w_2-\mu}} \frac{A_-(w') - \mathcal{R} - \mathcal{U} y}{y^2\sqrt{\mu - w_0 + y^2}}\, dy \tag{4.16}$$

where R, U, V, $\mathcal{R}, \mathcal{U}, \mathcal{V}$ refers to the K^-N combinations of the isospin states, and x,y are as before.

Considered in this manner the mutual cancellation of apparently non-existent terms at $w = \mu$ becomes evident. At $w = \mu$ equ(4.14) reduces to

$$\frac{2}{\pi}\int_0^{\sqrt{\beta-w_0}} \frac{D_-(z)\,dz}{(\mu-w_0-z)^{3/2}} - \frac{2R}{\pi}\frac{1}{\mu-w_0}\sqrt{\frac{\beta-w_0}{\mu-\beta}}$$

$$+\frac{2}{\pi}\int_0^{\sqrt{\mu-\beta}} \frac{D_-(x)-R-Hx}{x^2\sqrt{\mu-w_0-x^2}}\,dx$$

$$+\frac{2}{\pi}\int_0^{\sqrt{w_2-\mu}} \frac{A_-(y)-R-Hy}{y^2\sqrt{\mu-w_0+y^2}}\,dy$$

$$+\frac{H}{\pi}\frac{1}{\sqrt{\mu-w_0}}\log\left[\frac{\sqrt{w_2-w_0}-\sqrt{\mu-w_0}}{\sqrt{w_2-w_0}+\sqrt{\mu-w_0}}\right]$$

$$-\frac{R}{\pi}\frac{2}{(\mu-w_0)}\sqrt{\frac{w_2-w_0}{w_2-\mu}} \qquad (4.17)$$

where $z = \sqrt{w' - w_0}$.

Thus for $w' \sim \mu$ the integrands of equ(4.15) and equ(4.16) which contribute to equ(4.17), contain a dependence on V, $\mathcal{V}$ respectively, which in turn depend on G, H the s-wave effective range and p-wave scattering length terms.

For the $\bar{K}N$ amplitude given by the energy dependent scattering lengths of section (II-2), the above reasoning still applies. Here, the energy dependence of a_I, b_I is given entirely in terms of q, the c.m. momentum of the pion-hyperon channel appropriate to

the isospin state, whose only effect is to change the coefficients of x^2 y^2 in the expansions equ(4.9) and equ(4.12). Therefore it is possible to conclude that the principal-valuedness of equ(4.6), coupled with the square-root in the dispersive denominators in equ(4.14) emphasizes the $\bar{K}N$ s-wave effective range terms and the p-wave scattering lengths. Hence equ(4.7) must be used for the initial determination of the coupling constants $g^2_{\bar{K}p\Lambda}$ and $g^2_{\bar{K}p\Sigma^0}$. However, having done this, an interesting point is that it should be possible, in principle, to return to equ(4.6) and calculate G, H, although in practice the accuracy of the calculation does not permit this.

For w in the range $w_2 < w < 20$ GeV equ(4.6) can be used to determine $D_-(w,0)$ which can then be compared with the experimental data points. This is done in Chapter VII.

The attractive features of the $D_+(w\ 0)$, (4.7), forward dispersion relation when evaluated at $w = \mu$ are as follows. Firstly, the term $D_+/\sqrt{\xi(-w)}$ depends solely on the K^+N scattering lengths, thus eliminating an important source of error in equ(3.4) due to the errors on the $\bar{K}N$ parameters in table 1. Secondly, the integration over D_- in the resonance region $w_0 < w < \mu$ contains a self-cancelling effect as Re $\bar{f}^I_l = 0$ at the resonance position. The overall size of this contribution is also diminished as the integral is not principal valued. Note that the previous two contributions are additive, thus lessening the uncertainties in equ(3.4) due to IV-1(d). Thirdly the principal-valued integrals

only emphasize the imaginary parts of the K^+N forward amplitudes. The curves for the K^+N total cross sections plotted against energy are very smooth and known within very fine limits from the measurements of Cool et al.[40]. Accordingly the corresponding principal-valued integrals should not induce any serious inaccuracies.

It is again possible to rewrite equ(4.7) with the pole terms on the left hand side, and the right hand side in the form of equ(3.5) where the c_i are defined in exactly the same way. However, because the real part of the p-wave amplitude changes sign at the $Y_1^*(1385)$ resonance and the magnitude of this contribution also depends on w_o, it is impossible to give prior comment concerning the sign of $\alpha(Y_1^*)$. Neglecting the Y_1^* effect the results are tabulated below when the KN and the $\bar{K}N$ low energy regions are determined from the energy independent scattering lengths in table 1. c_5 was determined from the appropriate formula in appendix B.

Table 6: The conditions to the $(K^\pm p)$ and $(K^\pm n)$ dispersion relations of the type equ(4.7) (in units 10^{-7} MeV^{-2}). The energy w_o in equ(4.7) is chosen to be 401-2 MeV corresponding to a total c.m. energy W_o=1370MeV.

	$K^\pm p$	$K^\pm n$
c_1	23.9	10.3
c_2	28.9	20.3
c_3	-75.1	-59.4
c_4	45.4	44.7
c_5	-5.1	-3.1
Total	18.0	12.8

These results are to be compared with those obtained in table 2 for the conventional forward dispersion relation of the type equ(3.4) which yield.

$$2.12g^2_{\Lambda} = 14.9 - \tfrac{1}{2}(11.3) \qquad (4.18)$$

$$3.66g^2_{\Sigma^o} = 11.3 \qquad (4.19)$$

whereas the relations of the type equ(4.7) predict

$$2.40g^2_{\Lambda o} = 18.0 - \tfrac{1}{2}(12.8) \qquad (4.20)$$

$$4.24g^2_{\Sigma^o} = 12.8 \qquad (4.21)$$

that is

$$g^2_{\Lambda k^- p} = 4.8 \; ; \qquad g^2_{\Sigma o K^- p} = 3.0 \qquad (4.22)$$

if $Z(Y_1^*) = 0$. These results are in close agreement.

To test the sensitivity of equ(4.7) to the value chosen for w_o, the calculation was repeated with different values of w_o covering its acceptable range, that is, for 1340 MeV $< W_o <$ 1395MeV, where W_o, w_o are related by equ(2.42). These results are shown below.

Table 7: The effect of w_o on g^2_{Λ} and $g^2_{\Sigma o}$.

W_o	g^2_{Λ}	$g^2_{\Sigma o}$
1340	4.54	2.78
1355	4.74	2.89
1370	4.82	2.97
1380	4.80	3.01
1395	4.74	3.05

Therefore the variations in w_o play a relatively unimportant role in the predictions for g^2_Λ and $g^2_{\Sigma o}$. The more important possible sources of error are threefold. In the first instance any ill-determined points for the KN, $\bar{K}N$ total cross-sections may affect the accuracy with which g^2_Λ and $g^2_{\Sigma o}$ are determined. The data for K^+N is especially vulnerable as the corresponding dispersive integral is principal-valued for $w > w_1$. However, very accurate experimental measurements[40] have recently become available for just this process. If the high energy ($w > 5$ BeV) values of $\sigma_{tot}(KN)$ and $\sigma_{tot}(\bar{K}N)$ are not changed radically then an estimate of ± 0.3 would probably cover the range of variation of g^2_Λ for small changes in the structure of the input cross-sections.

Secondly the coupling constants depend on the particular model chosen to reproduce the correct asymptotic behaviour of the KN, $\bar{K}N$ amplitudes. The model of Phillips and Rarita[42] (see appendix B) gives a contribution ~ -1.5 to g^2_Λ and ~ -1.1 to $g^2_{\Sigma o}$.

Thirdly there remains the question of the effects of the unphysical region $w_{Y\pi} < w' < \mu$ and its associated errors. For the reasons discussed earlier the $\bar{K}N$ amplitudes in the region $\mu < w < w_2$ are assumed to be given by the energy independent a_I b_I in table 1.

3 THE EXTRAPOLATION OVER THE UNPHYSICAL REGION.

As emphasized by Rood[44] and section (III-2), a_I and b_I must surely contain some energy dependence, even in the zero range approximation. Consider first the determination of g^2_Λ. Here the zero range approximation gives the I = 0 $\bar{K}N$ extrapolation with

$$a_o = \alpha_o - \frac{\gamma_o \beta_o^2 q^2}{1+q^2\gamma_o^2} \quad ; \quad b_o = \frac{\beta_o^2 q^2}{1+q^2\gamma_o^2} \qquad (4.23)$$

from equ(2.31). Thus unless $|\gamma_o| \gtrsim 1$ fermi, and tables (2)-(4) indicate that $|\gamma_o|$ is smaller than this limit, b_o is expected to be much more energy dependent than a_o in the unphysical region. Now in equ(2.37) b_o only occurs in the form $a_o^2+b_o^2$ where $a_o \sim -1.7$ fermi and $b_o \sim 0.7$ fermi, and so in comparison to equ(2.36), equ(2.37) is much less susceptible to uncertainties in the energy dependence of b_o. Consequently the error associated with the extrapolation into the unphysical region should be much less in equ(4.7) than in equ(3.4). This conclusion is borne out by computation. Similar reasoning also applies to the uncertainties in the energy independent scattering lengths where the errors on a_o, b_o in table 1 give $4.63 < g^2_\Lambda < 5.01$ (c.f. $3.39 < g^2_\Lambda < 4.34$ from equ(3.4)). Note that this reasoning does not apply to the calculation of $g^2_{\Sigma o}$. Using (4.7), and the errors of a_1 b_1 shown in table 1 gives

$$2.67 < g^2_{\Sigma o} < 3.26 \quad \text{(c.f. } 2.77 < g^2_{\Sigma o} < 3.42\text{)}$$

Under the assumption that only s-waves contribute to the I = 0 $\bar{K}N$ unphysical region there are four constraints which one may impose. These are the values of the energy independent a_o b_o at $w = \mu$ and the position (W_R) and width (Γ) of the Y_o^* resonance in the $\Sigma\pi - \Sigma\pi$ channel. Some, or all, of these restrictions have been used in the following attempts to parametrize this region.

If W_R is known then α_o c an be found from equ(2.28) as Re $\bar{f}_{2\,2\,\substack{I=0 \\ \ell=0}} = 0$ at the resonance position. Therefore

$$1 + K_R \alpha_o = 0 \qquad (4.24)$$

Furthermore β_o and γ_o can now be determined explicitly by equ(2.31) when $w = \mu$ if the values of a_o b_o given in table 1 are correct. The results are shown below.

Table 8 The value of g_Λ^2 when $\alpha_o\, \beta_o\, \gamma_o$ are determined explicitly from the resonance position and the energy independent scattering lengths at threshold.

α_o	β_o^2	γ_o	E_R	Γ_R	g_Λ^2
-1.51	-0.824	0.249	1403	60	7.14

Alternatively, if γ_o is allowed to vary freely then the energy independent a_o b_o and (2,31) at $w = \mu$ fix the values of α_o, β_o. The results from this procedure are comparable to those in table 4. c_2 is in units 10^{-7} MeV^{-2} and is given by the $K^+p - \frac{1}{2}K^+n$

combination of (4.7).

Table 9: The values of g^2_Λ and c_2 from a threshold fit of the energy dependent scattering lengths to a_o and b_o. (The first line corresponds to the energy independent a_o, b_o).

γ_o	c_2	g^2_Λ
	18.8	4.82
0.4	24.6	7.25
0.2	24.2	7.09
0	23.6	6.84
-0.2	22.8	6.50
-0.4	21.8	6.08

Note that the variations of g^2_Λ are much less in Table 9 than in Table 4.

Clearly the trial values of $\alpha_o, \beta_o, \gamma_o$ in Tables, 4, 9 are not in close agreement with the observed parameters W_R = 1405 MeV, M = 35 MeV, of a symmetric Y_o^* resonance. Therefore the integral over the unphysical region was calculated using various sets of the reaction matrix parameters, each consistent with the Y_o^* and each in tolerable agreement with a_o, b_o at threshold $w = \mu_K$. The results are shown in the following table.

Table 10: Predictions for g^2_Λ obtained using the reaction matrix parameters consistent with a symmetric Y_o^*. The threshold values of a_o, b_o calculated from these parameters are also shown.

a) $\gamma_o < 0$

α_o	β_o^2	γ_o	E_R	Γ	g^2_Λ	$a_o(\mu)$	$b_o(\mu)$
-1.51	0.5	-0.4	1405	27	4.02	-1.37	0.40
	0.7		1404	38	4.37	-1.31	0.56
	0.9		1403	51	4.85	-1.25	0.72
	0.5	-0.3	1405	27	4.46	-1.40	0.42
	0.6		1404	33	4.65	-1.37	0.51
	0.6	-0.2	1404	34	5.10	-1.42	0.53
	0.65		1404	38	5.23	-1.41	0.57
	0.7		1404	42	5.36	-1.40	0.61
	0.8		1403	49	5.63	-1.38	0.70
	0.5	-0.1	1405	28	5.23	-1.47	0.45
	0.6		1404	36	5.50	-1.46	0.54
-1.41	0.6	-0.1	1400	43	5.18	-1.36	0.54
-1.61	0.6	-0.1	1408	30	5.82	-1.56	0.54

b) $\gamma_0 \geq 0$

α_0	β_0^2	γ_0	E_R	Γ	g_Λ^2	$a_0(\mu)$	$b_0(\mu)$
-1.51	0.5	0	1405	30	5.55	-1.51	0.45
	0.6		1404	38	5.84	-1.51	0.54
-1.41	0.4		1401	28	4.91	-1.41	0.36
-1.61	0.5		1408	26	5.88	-1.61	0.45
-1.51	0.5	0.1	1405	32	5.81	-1.55	0.45
	0.6		1404	41	6.11	-1.56	0.54
	0.5	0.2	1404	35	6.01	-1.59	0.44
	0.6		1404	44	6.33	-1.61	0.53
	0.4	0.3	1405	28	5.83	-1.60	0.34
	0.5		1405	39	6.16	-1.63	0.42
	0.6		1404	49	6.49	-1.65	0.51
	0.3	0.4	1405	23	5.59	-1.60	0.24
	0.45		1405	37	6.09	-1.64	0.36
	0.5		1405	42	6.26	-1.66	0.40

From the above table it is clear that increasing either W_R or Γ increases g^2_Λ. Also with increasing γ_0, the values of β_0^2 for the correct width decreases and the value obtained for g_Λ^2 increases. α_0 is essentially fixed by W_R. This prodecure cannot be applied to the conventional forward dispersion relation (3.4), because the principal valued integral at $w = \mu$ demands that b_I is continuous through threshold.

The parameterization of the I = 0 $\bar{K}N$ amplitudes can be made more stringent by the inclusion of R_α, R_β and R_γ, the effective range terms corresponding to the R matrix elements α, β, γ. In this way it is possible to fit W_R and the I = 0 threshold scattering lengths exactly. Although theoretically it is possible to reduce the input parameters to two, it is easier in practice to input γ_0, R_β, R_γ and use the value of Γ as an output constraint. Note that it is impossible to expand the R-matrix elements in a power series about threshold, when the scattering lengths a_0, b_0 are fitted there, otherwise no useful information is obtained. Therefore the expansion was made about the resonance position. Thus,

$$\frac{1}{\alpha} = \frac{1}{\alpha_0} + \frac{1}{2} R_\alpha \left(k_R^2 - k^2 \right) \qquad (4.25)$$

$$\frac{1}{\beta} = \frac{1}{\beta_0} + \frac{1}{2} R_\beta \left(k_R^2 - k^2 \right) \qquad (4.26)$$

$$\frac{1}{\gamma} = \frac{1}{\gamma_0} + \frac{1}{2} R_\gamma \left(k_R^2 - k^2 \right) \qquad (4.27)$$

From equ(4.24)

$$\alpha_0 = -\frac{1}{k_R} \qquad (4.28)$$

From equs(4.23), (4.25), (4.27)

$$R_\alpha = -\frac{2}{k_R} \left\{ \frac{\alpha_0}{\left[a_0 + \frac{q_T \gamma_0 b_0}{(1 + \frac{1}{2} R_\beta \gamma_0 k_T^2)} \right]} - 1 \right\} \qquad (4.29)$$

where q_T is the $\bar{\Sigma}\pi - \Sigma\pi$ momentum and the suffix T refers to the threshold $w = \mu$.

Also equ(4.23) gives

$$\beta_T^2 = \frac{b_o}{q_T}\left(1 + (q_T \gamma_T)^2\right) \qquad (4.30)$$

$$= X^2 \quad \text{(say)}$$

Therefore

$$\beta_o = \frac{R_\beta X^2 \kappa_R^2 \pm 2X}{2\left(1 - R_\beta^2 \frac{\beta^2 \kappa_R^4}{4}\right)} \qquad (4.31)$$

Hence α_o, β_o, R_α are given in terms of γ_o, R_γ, R_β. The results for this parametrization of the unphysical region are given below.

Table 11: Predictions for g_A^2 obtained from equ(4.7) by fitting W_R and a_o, b_o at threshold with effective rnage terms included in the reaction matrix elements. The values of the energy dependent a_o, b_o are calculated at $w = w_2$ and listed in the last two columns.

Table 11

α_0	β_0	γ_0	R_α	R_β	R_σ	W_R	Γ_Σ	g_n^2	$a_0(\omega_2)$	$b_0(\omega_2)$
-1.503	-1.261	1	-1.391	-0.2	0	1402	54	2.05	-1.31	0.58
	-1.20			0		1403	49	2.09	-1.45	0.71
	-1.08			0.4		1404	43	1.99	-1.95	1.13
	-1.03			0.6			39	1.86	-2.38	1.50
	-0.99			0.8			37	1.69	-3.07	2.09
	-0.96		-1.178		0.4		46	2.65	-2.87	2.36
	-1.02		-1.675		-0.4		32	0.64	-3.16	1.64
	-1.15	0.8	-0.902	-0.2	0	1402	60	3.19	-1.37	0.62
	-1.09			0		1403	54	3.40	-1.49	0.75
	-1.04			0.2			48	3.54	-1.65	0.91
	-1.00			0.4			43	3.63	-1.86	1.41
	-0.96			0.6		1404	40	3.66	-2.17	1.47
	-1.01	0.6	-0.511	0	0	1403	58	4.77	-1.55	0.80
	-0.97		-0.511	0.2			49	5.09	-1.67	0.96
	-0.93		-0.555	0.4	-0.3	1404	42	5.01	-1.84	1.10
	-0.93		-0.511		0		44	5.34	-1.82	1.17
	-0.92		-0.471		0.3		46	5.63	-1.80	1.23
	-0.89		-0.511	0.6	0		39	5.53	-2.03	1.47
	-0.90	0.4	-0.19	0.2	0	1403	52	6.71	-1.74	1.01
	-0.87			0.4			45	7.14	-1.83	1.22

Table 11 (cont.)

α_0	β_0	γ_0	R_α	R_β	R_γ	W_R	Γ_E	g^2_{π}	$a_0(w_2)$	$b_0(w_0)$
-1.503	-0.90	0.2	0.076	0	0	1402	60	7.53	-1.83	0.90
	-0.85			0.3		1403	54	8.68	-1.89	1.15
	-0.82			0.4		1403	50	9.10	-1.92	1.27
	-0.85	0	0.301	0.2	0	1402	—	9.69	-2.14	1.08
	-0.89			0		1401	—	8.45	-2.14	0.92
	-0.92			-0.2		1399	—	7.01	-2.14	0.79
	-0.96			-0.2		—	—	5.09	-3.29	0.75
-1.66	-0.98	0.4	-0.59	-0.2	0	1409	40	3.99	-1.37	0.73
	-0.88			0.4			28	4.66	-1.61	1.22
	-1.14	0.8	-1.46	-0.2			38	1.60	-1.24	0.62
	-1.01			0.4			40	1.50	-1.73	1.14

These results can be summarized as follows.

(a) For γ_0 increasing with R_γ and R_β fixed:

$|a_0|$ and $|b_0|$ both decrease to a minimum which depends on the fixed values of R_γ, R_β. Γ and g_π^2 both decrease.

(b) For R_γ increasing with γ_0, R_β fixed:

$|a_0|$ decreases whilst $|b_0|$, Γ, g_π^2 all increase.

(c) For R_β increasing with γ_0, R_γ fixed:

$|a_0|$ and $|b_0|$ increase whilst Γ decreases. The value of g_π^2 increases when γ_0 is fixed to the less than 0.8 fermi and decreases when γ_0 =1 fermi.

(d) If $\gamma_0 < 0$, the resonance width, and for more negative γ_0 the position, are undefined.

In these conclusions (a), (b), (c), (d) the position of the Y_0^* resonance was input as W_R = 1405 MeV. However, the last four lines of table 11 contain the results for an asymmetric resonance with an input position W_R = 1410 MeV. This produces better agreement between the output Γ, $a_0(w_2)$ and $b_0(w_2)$. The value α_0 is essentially determined from the resonance position W_R by equ(4.28).

The sensitivity of g_π^2 to the dispersion relation was tested by evaluating equ(3.4) for several sets of the parameters listed in table 11. The same conclusion was reached as before, that is, g_π^2 is less sensitive to the unphysical region in equ(4.7) than in equ(3.4).

Table 11 shows that the parameters behave in a maximal (or minimal) way, that is their values reach a maximum (or minimum) dependent on the other parameters, indicating that this technique should be used in conjunction with an optimisation procedure.

The results of tables 8, 9, 10 are consistent with the following value of the coupling constant

$$g^2_{\Lambda} = 5.8 \pm 1.8 \qquad (4.32)$$

If $|\gamma_0|$ in equ(4.27) is small, then table 11 indicates that g^2_{Λ} is increased. If $|\alpha_0|$ is increased then g^2_{Λ} is decreased.

If s, p waves contribute to the $\bar{K}N$ scattering amplitudes for $w' < w_2$ then the dispersive contribution to the I = 0 c_2 in the range $\mu < w' < w_2$ should still be fairly well determined as $-18.24 \pm 0.24\ 10^{-7}$ MeV^{-2} from table 6, although this means that the energy dependent s-wave scattering lengths given by (2.31) cannot be fitted at threshold to the a_0, b_0, a_1, b_1 in table 1. This fact invalidates equ(3.4) because of the principal valued integral and makes (4.7) approximate unless the procedure of Rood[44] is carried out, whereby all the partial waves are considered throughout the region $w_{\gamma\pi} < w' < w_2$ and fitted to Kim's[37] data.

4 THE $\bar{K}N$ I = 1 UNPHYSICAL REGION

It is interesting to compare the results obtained from equ (3.4) and equ(4.7) for the $\bar{K}N$ I = 1 unphysical region - here the three channel reaction matrix has been approximated by the assumption that the form of the energy dependence of a_1 and b_1 is completely represented by the $\Lambda\pi$ centre of mass momentum. The results are shown below when γ_1 was input and α_1, β_1 were fitted to a_1, b_1 at $w = \mu$ through equation (4.23).

Table 12: Predictions for $g^2_{\Sigma^0}$ (if $Z(Y_1^*) = 0$) obtained by matching the $\bar{K}N$ I = 1 energy dependent and energy independent scattering lengths at threshold. The values of $a_1(w)$ $b_1(w)$ at $w = w_2$ are listed also. The values of $g^2_{\Sigma^0}$ from (4.19), (4.22) are shown in the first row.

α_1	β_1^2	γ_1	$a_1(w_1)$	$b_1(w_1)$	(3.4) $g^2_{\Sigma^0}$	(4.7) $g^2_{\Sigma^0}$
-	-	-			3.10	2.96
0.279	0.670	0.4	-0.170	0.742	1.47	0.60
0.104	0.565	0.2	-0.133	0.783	2.02	0.50
-0.07	0.530	0	-0.07	0.802	2.84	0.85
-0.244	0.565	-0.2	-0.008	0.783	4.01	1.98
-0.419	0.670	-0.4	0.030	0.742	5.60	4.02

Note that equ(4.7) yields much smaller values for $g^2_{\Sigma^0}$, when $Z[Y_1^*] = 0$, than equ(3.4) until $\gamma_1 \sim -0.4$. As mentioned before the values quoted for $g^2_{\Sigma^0}$ are upper bounds in the case of equ(3.4). However, the values given by equ(4.7) lack such a positive

definition without a more thorough examination of the role played by the I = 1 resonant p-wave contribution.

The amplitudes for the decay of a p-wave resonance into two open channels A, B may be written[47] as

$$f_{AA} = \gamma_A (k^2 / k_R^3) / \Delta \qquad (4.33)$$

$$f_{BB} = \gamma_B (q^2 / q_R^3) / \Delta \qquad (4.34)$$

$$f_{AB} = \sqrt{\gamma_A \gamma_B}\, (qk / (k_R q_R)^{3/2}) / \Delta \qquad (4.35)$$

where

$$\Delta = W_R - W - i\left(\gamma_A \left(\frac{k}{k_R}\right)^3 \theta(W - W_0) + \gamma_B \left(\frac{q}{q_R}\right)^3 \theta(W - W_0') \right) \qquad (4.36)$$

where

k refers to the c.m. momentum of channel A

q refers to the c.m. momentum of channel B

W_0 refers to the threshold energy of channel A

W_0' refers to the threshold energy of channel B

$\theta(x)$ denotes the step function $\begin{matrix} = 0 \text{ if } X < 0 \\ = 1 \text{ if } X > 0 \end{matrix}$

The total width Γ of the resonance in terms of its partial widths Γ_A, Γ_B is

$$\Gamma = \Gamma_A + \Gamma_B \qquad (4.37)$$

where

$$\Gamma_A = \gamma_A \left(\frac{k}{k_R}\right)^3 \quad ; \quad \Gamma_B = \gamma_B \left(\frac{q}{q_R}\right)^3 \qquad (4.38)$$

These equations (4.33)-(4.35) satisfy two-channel unitarity. If A denotes the $\bar{K}N$ channel and B the $Y\pi$ channel then equ(4.33) may be extrapolated backwards to the $\bar{K}N$ unphysical region to give the $\bar{K}N$ amplitude for the decay of a resonance below the channel threshold. Thus in the unphysical region equ(4.33) becomes

$$\bar{f}^{I=1}_{\bar{K}N-\bar{K}N} = \frac{-k^2 g^2 / |k_B|^3}{(w_B - w) - i\gamma_B \left(\frac{q}{q_B}\right)^3} \qquad (4.39)$$

where g^2 represents a kinematical term with the dimensions of energy which includes the $g^2_{Y_1^* \bar{K}N}$ coupling constant. From reference (48)

$$g^2 = \frac{|k_B|^3}{12 M^3 M_{Y_1^*}} g^2_{Y_1^* \bar{K}N} \left[(M_{Y_1^*} + M)^2 - \mu^2 \right] \qquad (4.40)$$

where $M_{Y_1^*}$ is the mass of the $Y_1^*(1385)$ resonance. As g^2 is a smoothly varying function of energy the evaluation of equ(4.39) was carried out with g^2 considered as a constant given by its value at the resonance energy. From equ(4.40)

$$g^2 = 1.849 \; g^2_{Y_1^*} \quad \text{MeV} \qquad (4.41)$$

Thus the Y_1^* contribution to the dispersion relations may be determined in terms of the coupling constant $g^2_{Y_1^* \bar{K}N}$ remembering that the forward scattering amplitude is, from equ(1.51)-(1.49)-(1.50),

$$\bar{T}^{I=1}(w,0) = \bar{h}^{I=1}_{L=0}(w,0) + 2\bar{h}^{I=1}_{J=1+}(w,0) + \ldots . \qquad (4.42)$$

Where $h_{\ell=0}$ and $h_{J=1+}$ are the $\bar{K}N$ I=1 s-wave and $p(J=3/2)$ wave partial wave amplitudes which include an explicit $k^{2\ell}$ dependence and all other partial waves are assumed to give a negligible contribution in the region $w_{Y\pi} < w' < \mu_K$.

Including this effect we obtain the following results for the energy independent scattering lengths.

Equ(3.4) gives

$$3.66 g^2_{\Sigma o} + 2.94\, g^2 = 11.28 \qquad (4.43)$$

Equ(4.7) gives (with $W_o = 1370$)

$$4.25 g^2_{\Sigma o} + 6.83\, g^2 = 12.63 \qquad (4.44)$$

This means that the values quoted for $g^2_{\Sigma o}$ which are derived from equ(3.6), are also upper bounds.

Although it is possible to determine $g^2_{\Sigma o}(\sim 3.24)$ and $g^2(\sim -0.2)$ by solving equs(4.43)-(4.44) the answers are not to be taken seriously due to the discrepancy in the predictions of g^2_Λ from equ(3.9) and equ(4.22) (4.3 and 4.8 respectively). Thus we may conclude that it is necessary to obtain another relation between g^2_Λ, g^2_Σ and g^2 for both the conventional and new dispersion relations. In this way the three values of the couplings should be determined explicitly. Before attempting this procedure let us compare our predictions for the Λ, Σ coupling constants with previous determinations of their values.

CHAPTER V

PREVIOUS DETERMINATIONS OF THE KAON NUCLEON COUPLING CONSTANTS

1 FORWARD DISPERSION RELATIONS

Several alternative methods have been proposed for the determination of the kaon nucleon coupling constants, most of which are simply adaptions of the forward dispersion techniques mentioned in section (I.5). On the whole the predictions of forward dispersion relations are reasonably consistent in that $g^2_\Lambda \simeq 5.8 \pm 1.8$ agrees with the values given by Dufour[49], $5 \leq g^2_\Lambda \leq 6$, and Rimpault $g^2_\Lambda = 4.4$, who analysed ΛK productions by photons and π^- mesons respectively using specific theoretical models. A certain degree of freedom in the results is allowed by the choice of the $\bar{K}N$ s-wave energy independent scattering lengths[37] [43] [51] In the methods discussed p waves have been neglected.

Zovko[52] evaluated the dispersion relations for $D^-(w\ 0)$, (3.4) for the K_p system using the $\bar{K}N$ energy independent scattering lengths of Kim[43] to approximate the $\bar{K}N$ integrals over the region $w_{Y_\pi} < w' < w_2$, and the K^+p scattering length and effective range of Goldhaber et al.[38] to determine the K^+p contribution in the range $\mu \leq w' \leq w_1$ and obtained the following expression

$$0.654\ g^2_{\pi\bar{K}N} + 0.566\ g^2_{\Sigma o\bar{K}N} = 9.6 \qquad (5.1)$$

The $\bar{K}N$ energy independent scattering lengths of Sakitt et al.[51]

change the contribution of the unphysical region and thus the right hand side of equ(5.1) which becomes

$$0.654\, g^2_{\Lambda \bar{K}N} + 0.566\, g^2_{\Sigma^0 \bar{K}N} = 8.2 \qquad (5.2)$$

Note that above w' = 5 BeV Zovko[52] calculated the dispersive contributions from the following extrapolation through the total cross section data points

$$\sigma_-(w') - \sigma_+(w') = 20.8\, k^{-2/3} \text{ millibarnes}$$

$$\sigma_-(w') + \sigma_+(w') = (34.6+20.8k^{-2/3})\text{"}$$

where k (in BeV/c is the KN centre of mass momentum.

The $K^\pm p - \frac{1}{2} K^\pm n$ combination of equ(3.4) has been dealt with in detail in chapter III, it suffices to list the results obtained.

Table 13: The value of g^2_Λ and the upper bound on $g^2_{\Sigma^0}$ using the energy independent scattering lengths for (a) - (g). The procedure of Rood[44] gives (k). The value of w at which the determination was carried out is also listed. The references to the calculation and the $\bar{K}N$ scattering lengths used are shown in the last two columns.

	w	g^2_Λ	Upperbound on $g^2_{\Sigma^0}$	Reference to the calculation	Reference for the $\bar{K}N$ scattering lengths
a	"	4.34	3.1	Chapter III	Kim[37]
b	"	4.8±1	3.2	Lusignoli et al. (26)	Kim[44]
c	"	5.0±1.6	0.4		Sakitt et al.[51]
d	"	6.0±2.1	3.9±3.7	Davies et al. (48)	Kim[37]
e	500 MeV	6.3±2.1	4.0±2.6	"	"
f	512.5"	5.9±2.0	4.2±1.5	"	"
g	525 "	5.3±1.8	4.5±1.4	"	"
h		7.4±1.2		Rood (44)	—

The results (a), (b), (c), (h) neglect the mass differences

between the K^-p and $\bar{K}^0n$ systems as it is debateable whether or not a theory for strong interactions should contain the reult of an electromagnetic effect.

Alternatively, the technique of Haber-Schaim[20] may be applied to the $K^{\pm}p$ system by replacing the denominators of the pole terms in (3.4) by (w^2-wp^2) where $w_p = \frac{1}{2}(w_\Lambda + w_\Sigma)$. This gives a single effective pole and is expected to be a good approximation if the energy w at which the subsequent dispersion relations are evaluated, is sufficiently large compared with w_Σ . The identity (1.55) transforms equ(3.4) into

$$L(w) \equiv (w^2-w_p^2)\left[\frac{D_-(w) - D_+(w)}{2w}\right]$$
$$-\frac{w^2(w^2-w_p^2)}{4\pi^2}\int_{\mu}^{\infty}\frac{k_L'(\sigma_-(w') - \sigma_+(w'))\,dw'}{w'^2(w'^2-w^2)}$$
$$-\frac{w^2-w_p^2}{\pi}\int_{w_{\gamma\pi}}^{\infty}\frac{A_-(w')\,dw'}{w'^2-w^2} \qquad (5.4)$$
$$= F(Y) + \frac{w^2-w_p^2}{4\pi^2}\int_{\mu}^{\infty}\frac{k_L'}{w'^2}(\sigma_-(w') - \sigma_+(w'))\,dw'$$

where F(Y) represents the pole terms, and for the $K^{\pm}p$ system,

$$F(Y) = g_\Lambda^2\,X(\Lambda) + g_{\Sigma^0}^2\,X(\Sigma) \qquad (5.6)$$

Plotted versus w^2, L(w) gives F(Y) from the w = 0 intercept from equ(5.5).

Using the $\bar{K}N$ energy independent scattering lengths[43] [51] in the range $w_{\gamma\pi} < w' < w_2$, the K^+p scattering lengths and effective

range terms in table 1 and the data for the total cross sections plus equation (5.3) for $w' > 5$ BeV, Zovko[52] found the best straight line to give

$$0.654\ g^2_{\Lambda} + 0.566\ g^2_{\Sigma^0} = 8.6 \pm 5.7 \qquad (5.7)$$

Correspondingly redefine

$$H(w) = L(w) + \frac{w^2(w^2 - w_\rho^2)}{\pi} \int_{w_{\gamma\pi}}^{\infty} \frac{A_-(w')\, dw'}{w'^2 (w'^2 - w^2)} \qquad (5.8)$$

Davies et.al.[48] found that the value of H(w) dependend strongly on the unphysical region and, in particular on the $Y_1^*(1385)$ contribution. The new form of equ(5.5) was computed for five values of w in the range 0.63-3.46 BeV. A straight line

$$\tilde{H}(w) = \hat{H}(w_\rho) - A(w^2 - w_\rho^2) \qquad (5.9)$$

was fitted through the five points thus determined, by an optimization procedure. This gave

$$g^2_{\Lambda} + 0.79\ g^2_{\Sigma^0} = 9.7 \pm 5.4 \qquad (5.10)$$

when the high energy contributions were evaluated using a Regge pole model.

As the $Y_1^*(1385)$ contributes to (5.5) through the imaginary part of a p-wave amplitude, it may be included as a delta function contribution (c.f section (IV.4)). In addition to the previous points Davies et al.[48] evaluated (5.4), (5.5) at $w = \mu$ and the optimisation procedure gave

$$\text{If } g^2_{Y_1^* K^- p} = 0: \quad g^2_{\Lambda} + 0.79\ g^2_{\Sigma^0} = 11.1 \pm 1.7 \qquad (5.11)$$

$$\text{If } g^2_{Y_1^* K^- p} = 2: \quad g^2_{\Lambda} + 0.79\ g^2_{\Sigma^0} = 6.6 \pm 1.7 \qquad (5.12)$$

According to equ(5.5) and equ(5.8), the derivative of the linear Haber-Schaim function with respect to w^2 is

$$\frac{dH(w)}{dw^2} = -\frac{1}{4\pi^2}\int_{\mu}^{\infty} \frac{k_L'}{w'^2}\left[\sigma_-(w') - \sigma_+(w')\right] dw' - \frac{1}{\pi}\int_{w_{\gamma\pi}}^{\mu} \frac{A_-(w')}{w'^2}\, dw' \tag{5.13}$$

This provides a check on the consistency of the results as equ(5.9) gives

$$\frac{dH(w)}{dw^2} = -A \tag{5.14}$$

This condition (5.14) was found to be well satisfied for both (5.11) and (5.12).

Similarly, the pion nucleon forward dispersion (1.54) was extended by Zovko[52] to kaon nucleon scattering through (3.2). Write

$$J_{\pm}(w) = D^{+}(\mu) \mp \frac{w}{\mu} D^{-}(\mu) \tag{5.15}$$

where now

$$J_{\pm}(w) \equiv D_{\pm}(w) - \frac{k_L^2}{4\pi^2} P\int_{\mu}^{\infty} \frac{dw'}{k_L'(w'-w)}\sigma_{\pm}(w') - \frac{k_L^2}{4\pi^2} P\int_{\mu}^{\infty} \frac{dw'}{k_L'}\frac{\sigma_{\mp}(w')}{(w'+w)} - \frac{k_L^2}{\pi}\int_{w_{\gamma\pi}}^{\mu} \frac{dw'\, A_-(w')}{(w' \pm w)\, k_L'^2} - \sum \frac{k_L^2 g_Y^2 X(Y)}{Y(w_Y^2 - \mu^2)(w_Y \pm w)} \tag{5.16}$$

Moreover, if we define a function f(w) such that

$$\begin{aligned} f(w) &= J_+(w) && \text{for} \quad w > \mu \\ f(w) &= J_-(-w) && \text{for} \quad w < -\mu \end{aligned} \tag{5.17}$$

then

$$f(w) = C_1 + w\, C_2 \tag{5.18}$$

where C_1, C_2 are numerical constants. A plot of f(w) against w gives a straight line from (5.18), and the constants c_1, c_2 may be adjusted to give the best fit to the $K^{\pm}p$ data.

Equation (5.16) was applied to t he $K^{\pm}_p$ system by Zovko[52] and the low energy $\bar{K}N$ integrations were evaluated from the energy independent scattering lengths of Kim[43] which gave the result

$$0.654 g^2_{\Lambda} + 0.566 g^2_{\Sigma^0} = 5.2 \tag{5.19}$$

An attempt has also been made by Davies et al.[48] to determine the values of the coupling constants from the once subtracted dispersion relation for $T_+(w)$(3.2). Although relations of this type possess the advantage of suppressing the importance of the $\bar{K}N$ low energy region when evaluated for physical energies w, it was not found possible to obtain reliable values for the coupling constants for energies $w \sim \mu$ due to the very strong cancellations between the various contributions. Because of the sensitivity of this relation to the K^+p low energy region when evaluated for the $K^{\pm}p$ system, the values $a_+^1 = -0.31$ fermi, $r_+^1 = 0$[38], were found to give values of the coupling constant incompatible with previous determinations. The

alternative values for these parameters shown in table 1 produced agreement within the very large errors. At higher energies the calcellations were not so strong and neglecting the Y_1^* (1385) contribution the following results were obtianed.

Table 14: Results from the once subtracted dispersion relation for $T_+(w)$ for the $K^{\pm}p$ system using the $\bar{K}N$ energy independent scattering lengths[37].

w	Relation
1.09 GeV	$g^2_{\Lambda} + 0.62\, g^2_{\Sigma^0} = 7.7 \pm 2.5$
1.27 GeV	$g^2_{\Lambda} + 0.62\, g^2_{\Sigma^0} = 6.5 \pm 3.8$
2.03 GeV	$g^2_{\Lambda} + 0.62\, g^2_{\Sigma^0} = 9.4 \pm 3.3$

Using the values of a_+^0 and r_+^0 in table 1 to give the K^+n low energy amplitudes, the following upper bounds were obtained for $g^2_{\Sigma^0}$ (i.e. neglecting the $Y_1^*(1385)$).

Table 15: The upper bounds on $g^2_{\Sigma^0}$ from the relation (3.2) for the $K^{\pm}n$ system using the $\bar{K}N$ energy independent scattering lengths[37].

w	$g^2_{\Sigma^0}$
0.605 GeV	0.9 ± 2.1
0.724 GeV	-4.2 ± 2.0
0.810 GeV	2.2 ± 2.4
0.950 GeV	4.0 ± 2.0

Thus we may conclude that the values of the g^2_{Λ} coupling constant predicted by (4.7) and the methods just mentioned using the $\bar{K}N$ energy independent scattering lengths, are fairly consistent within the large errors, whilst the $g^2_{\Sigma^0}$ coupling constant is not so well

defined except to say that it appears to be somewhat smaller than g^2_Λ.

Recently Kim[54] has performed on inverse K matrix analysis of the low energy K^-p data involving many more parameters than his previous analysis[43], in particular allowing an effective range dependence of the diagonal elements of the s-wave inverse K-matrix. On extrapolating into the unphysical region he how predicts a $Y_0^*(1403)$ resonance of width 50 MeV and upon using equ(3.4) he finds

$$g^2_\Lambda = 16 \pm 2.5 \; ; \; g^2_{\Sigma o} = 0.3 \pm 0.5$$

The large increase in the value of g^2_Λ appears to arise from the inclusion of the s-wave effective range terms, since using a comparable constant K-matrix we find $g^2_\Lambda \sim 8$ even allowing for a Y_0^* width of 50 MeV (see table 10). Clearly the errors quoted in equ(4.32) are much too conservative. However, it is also probable that errors found by Kim are not a measure of the true uncertainty in the coupling constants since it remains to be seen how the predictions depend on the particular form of the parametrization used to analyse the low energy K^-p data. For example, in view of the importance of the s-wave effective range terms, it is essential to investigate the sensitivity of the coupling constant predictions to some small effective range dependence in the off-diagonal elements of the inverse K-matrix. The importance which may be attached to these new values of the coupling constants is realized when we discuss the SU(3) symmetry scheme in the next section.

However, a subsequent calculation by Martin and Ross (65) uses the once subtracted dispersion relation (3.2) to yield

$$g_{\Lambda}^2 + 0.79\, g_{\Sigma^0}^2 = 6.1 \pm 4.7$$

which is in accord with our result (4.32).

2 THE SU(3) SYMMETRY SCHEME

Alternatively, it is possible to use the SU(3) symmetry scheme to obtain ratios between the various meson-baryon coupling constants. The values of $g^2_{Y\bar{K}N}$ obtained from forward dispersion relations are therefore an important test of the validity of SU(3) This provides an independent check on the values of $g^2_{Y\bar{K}N}$ obtained from forward dispersion relations.

By comparing the Lagrangian for the most general interaction between baryons and pseudoscalar mesons with the SU(3) symmetric interaction, De Swart[53] was able to list the relations between the various coupling constants. In particular

$$\begin{aligned} g_{NN\pi} &= g \\ g_{N\Lambda K} &= -\frac{g}{\sqrt{3}}(1+2\alpha) \\ g_{N\Sigma K} &= g(1-2\alpha) \end{aligned} \tag{5.20}$$

where α is the F/(F+D) ratio. The pion nucleon coupling constant, g, can be written in terms of its symmetric (g_1) and anti-symmetric (g_2) couplings as

$$g = \sqrt{\frac{3}{20}}\, g_1 + \frac{1}{\sqrt{12}}\, g_2 \tag{5.21}$$

With this notation

$$\alpha = g_2 / g\sqrt{12} \tag{5.22}$$

Thus $\alpha = 0$ means pure symmetric, or D-type, coupling and $\alpha = 1$ means pure antisymmetric, or F - type, coupling.

Experimental results indicate that $\alpha \sim 0.4$[54] in which case

$$\frac{g^2_{\Sigma^0 n^- p}}{g^2_{\Lambda n^- p}} = \frac{1}{27} \qquad (5.23)$$

This corresponds to an $^F/_D$ ratio of $^2/_3$. If $g^2 = 4.8$ then (5.23) predicts $g^2_{\Sigma^0}=0.18$. These values have been used by Snow et.al.[45] to evaluate the once subtracted dispersion relations for $D_\pm(w,o)$ (see equs(5.15), (5.16)) throughout the physical energy spectrum for w, and hence to obtain a comparison with experimental measurements for $D^{\pm}_{\pm}(w)$ / $A_\pm$ (w).

However, (5.20) predicts that $g^2_\Lambda \sim 15$ which is clearly incompatible with the results of forward dispersion relations, except for the calculation performed by Kim[54]. Moreover, some of the major sources of error in the forward dispersion relation (3.4) are suppressed when we use (4.7) (from the list in section (IV.1)). Therefore, if the SU(3) predictions are valid then our basic assumption that only s-waves contribute to the low energy $\bar{K}N$ region, may be in error.

So far, the value of $g^2_{\Sigma^0}$ has been obtained dependent on $g^2_{Y_1^*}$ (see equs(4.43), (4.44), (5.11), (5.12)) for which SU(3) symmetry implies a value $g^2_{Y_1^* K^- p} = 2.4$[48] by comparing the width of the Y_1^* resonance with that of the pion nucleon $N^*(1238)$ resonance. A model which incorporates symmetry breaking, due to Wali and Warnock[55] leads to good agreement with observations for $g^2_{Y_1^* pK} = 1.9$

If we differentiate chosen forward dispersion relations with

respect to w then the resulting expressing give independent relations between the coupling constants. In principle this extra relation could enable an unambiguous determination of their values. The results of this procedure are discussed in chapter VI.

CHAPTER VI

THE DERIVATIVE DISPERSION RELATIONS

1 INTRODUCTION

If the fixed momentum transfer dispersion relations are differentiated it is possible, in principle, to use the resulting equations, together with either equation (3.4) or equation (4.7), to determine the three coupling constants g^2_{Λ}, $g^2_{\Sigma^0}$, $g^2_{Y_1^*}$. Under differentiation with respect to t, the momentum transfer variable, the form of the dispersion relations is changed as the dispersive integrals now contain a term involving the derivative of an amplitude with respect to t. This means that the dispersive contributions in the range $w_{1,2} < w' < 20$ BeV cannot be evaluated from just the optical theorem, but also require a knowledge of the complete spectrum of differential cross sections. Therefore the dispersion relations are differentiated with respect to w, the kaon laboratory energy. It remains to choose the actual relations which are best suited to this treatment.

Clearly, the errors associated with the scattering amplitudes near threshold are emphasized when the differentiated dispersion relations are evaluated at $w = \mu$. Thus the relations must ensure that the better known KN scattering amplitudes are contained in the principal-valued integrations rather than the $\bar{K}N$ amplitudes.

2 THE FORM OF THE DERIVATIVE RELATIONS

The conventional dispersion relation most suited to differentiation is equation (3.2) for $D_+(w\ 0)$, certainly it fulfills the previous requirements.

Differentiation with respect to w for $w < w_1$ yields

$$\frac{d}{dw} D_+(w,0) = -\sum_Y \frac{g_Y^2\, X(Y)}{(w_Y+w)^2} - \frac{1}{\pi}\int_{w_{Y\pi}}^{w_2} \frac{A_-(w')}{(w'+w)^2}\,dw' + \frac{d}{dw} P\frac{1}{\pi}\int_{\mu}^{w_1} \frac{A_+(w')}{(w'-w)}\,dw' + \frac{1}{4\pi^2}\int_{w_{1,2}}^{\infty} k_L' \left[\frac{\sigma_+(w')}{(w'-w)^2} - \frac{\sigma_-(w')}{(w'+w)^2}\right] dw' \qquad (6.1)$$

where k_L is the kaon laboratory momentum.

Any error in equ(6.1) due to the uncertainty in the K+n scattering length is removed if the relation is considered for the $K^{\pm}p$ system. This has the added advantage that the s-wave effective range term is known[38] so that any additional p-wave effects are very small. This eliminates another possible source of error. Furhtermore, the integration over the unphysical region involves K^-p amplitudes, and therefore gives a relation containing the three coupling constants. This provides a check on the value of g_Λ^2, obtained from equ(3.4).

There is no need for a subtraction in (6.1) as the asymptotic convergence is guaranteed. With $w=\mu \ll 20$ GeV the asymptotic cross sections subtract approximately to give a small contribution from the high energy region. More comprehensive details

are contained in appendix B.

The term involving the ~~differential~~ derivative of a principal-valued integral must be treated with great care. As $w < w_1$ this term can be written as

$$\frac{d}{dw}\left\{ \frac{1}{\pi}\int_{\mu}^{w_1} \frac{A_+(w') - \frac{k_L'}{k_L} A_+(w)}{w'-w}\, dw' + \frac{1}{\pi}\int_{\mu}^{w_1} \frac{A_+(w)}{k_L}\,\frac{k_L'\, dw'}{w'-w} \right\} \tag{6.2}$$

Equations (2.38), (2.39) show that $(A_+(w)/k_L)\big|_{w=\mu}$ exists. The first integral could be evaluated for two close values of w and its gradient with respect to w found, whilst the second integral could be solved analytically and then differentiated with respect to w. This method agrees with the following procedure. Equation (6.2) becomes

$$\frac{1}{\pi}\int_{\mu}^{w_1} \frac{k_L'}{(w'-w)^2}\left[\frac{A_+(w')}{k_L'} - \frac{A_+(w)}{k_L} - (w'-w)\frac{d}{dw}\left(\frac{A_+(w)}{k_L}\right)\right] dw'$$

$$+ \frac{A_+(w)}{\pi k_L}\left[\int_{\mu}^{w_1} \frac{k_L'}{(w'-w)^2}\, dw'\right] + \frac{1}{\pi}\frac{d}{dw}\left(\frac{A_+(w)}{k_L}\right)\int_{\mu}^{w_1} \frac{k_L'\, dw'}{(w'-w)} \tag{6.3}$$

The first integral was computed numerically for $w = \mu$ and the second integral gives (at $w = \mu$)

$$\frac{1}{\pi} a_+'^2 \left[\log\left(\frac{w_1 + \sqrt{w_1^2 - \mu^2}}{\mu}\right) - 2\sqrt{\frac{w_1+\mu}{w_1-\mu}}\right] \tag{6.4}$$

The third integral gives

$$\frac{1}{\pi}\frac{d}{dw}\left(\frac{A_+(w)}{k_-}\right)_{w=\mu}\left[\sqrt{w_1^2-\mu^2}+\mu\log\left(\frac{w_1+\sqrt{w_1^2-\mu^2}}{\mu}\right)\right] \qquad (6.5)$$

Therefore we are able to conclude that (6.1) is well behaved when evaluated at $w = \mu$.

For the reasons discussed previously the derivative of equation (4.7) is considered f or the K^+p system, and if $w < w_1$ is

$$-\frac{d}{dw}\left(\frac{D_+(w)}{\sqrt{\xi(-w)}}\right) = \sum\frac{g_\gamma^2\,X(\gamma)}{(w_\gamma+w)^2\sqrt{\xi(w_\gamma)}} + \frac{1}{\pi}\int_{w_{\gamma\pi}}^{w_0}\frac{A_-(w')\,dw'}{(w'+w)^2\sqrt{\xi(w')}}$$
$$+\frac{1}{\pi}\int_{w_0}^{\mu}\frac{D_-(w')\,dw'}{(w'+w)^2\sqrt{-\xi(w')}} - \frac{d}{dw}\mathcal{P}\frac{1}{\pi}\int_{\mu}^{w_1}\frac{A_+(w')\,dw'}{(w'-w)\sqrt{\xi(-w')}} \qquad (6.6)$$
$$-\frac{1}{4\pi^2}\int_{w_{\pi\pi}}^{\infty}k_+'\left[\frac{\sigma_-(w')}{(w'+w)^2\sqrt{\xi(w')}}+\frac{\sigma_+(w')}{(w'-w)^2\sqrt{\xi(-w')}}\right]dw'$$

Again, the principal-valued integral was written in the form

$$\frac{d}{dw}\left\{\frac{1}{\pi}\int_{\mu}^{w_1}\frac{A_+(w')-k_+'/k_-A_+(w)\;dw'}{(w'-w)\sqrt{(w'+\mu)(w'+w_0)}}\right.$$
$$\left.+\frac{1}{\pi}\int_{\mu}^{w_1}\frac{A_+(w)}{k_-}\frac{k_+'\,dw'}{(w'-w)\sqrt{(w'+\mu)(w'+w_0)}}\right\} \qquad (6.7)$$

The first integral was calculated numerically at $w = \mu$, as before, and the second integral becomes

$$\frac{1}{\pi}\frac{d}{dw}\left\{\frac{A_+(w)}{t_1}\int_{\mu}^{w_1}\left(1+\frac{w-\mu}{w'-w}\right)\frac{dw'}{\sqrt{(w'-\mu)(w'+w_0)}}\right. \tag{6.8}$$

This reduces to

$$-\frac{2a_+'^2}{\pi(\mu+w_0)}\sqrt{\frac{w_1+w_0}{w_1-\mu}}$$

$$\left.+2\left[\frac{d}{dw}\left(\frac{A_+(w)}{t_1}\right)\right]_{w=\mu}\log\left(\frac{\sqrt{w_1-\mu}+\sqrt{w_1+w_0}}{\sqrt{\mu+w_0}}\right)\right. \tag{6.9}$$

The interchange of differentiation and integration gives an identical result at $w = \mu$.

If w_0, μ, w are neglected in comparison to $w' > 20$ GeV then the assymptotic cross sections add in equ(6.6). This is compensated for by the extra power of w' in the denominators of the dispersive integrals. The overall effect is to retain a comparatively small high energy ($w' > 20$ GeV) contribution. The exact form of these integrals is given in appendix B.

3 THE EVALUATION OF THE DERIVATIVE DISPERSION RELATIONS

Both equation (6.1) and equation(6.6) were evaluated at $w = \mu$, in the first instance by using the energy independent scattering lengths of Kim[37] to determine the $\bar{K}N$ s-wave amplitudes for $w_{Y\pi} < w' < w_2$. The contributions to the two equations are tabulated below following the procedure of section (III.2) but in addition c_6 denotes the contribution of the $Y_1^*(1385)$ resonance in the unphysical region. Equation (3.5) becomes

$$\sum_{i=1}^{5} c_i - g^2 c_6 \qquad (6.10)$$

where g^2 is defined in section (IV.4) and $g^2 = 1.85\, g^2_{Y_1^*}$ from (4.41).

Table 16: The contributions in units 10^{-7} MeV^{-2} to equ(6.1)-(6.6) at $w = \mu$ using the energy independent scattering lengths.

c_i	equ(6.1)	equ(6.6)
c_1	7.43	17.51
c_2	-6.54	37.51
c_3	-43.64	-33.31
c_4	41.88	3.71
c_5	-3.93	-1.22
total	$t_1 = 4.80 \pm 0.25$	$t_2 = 24.20 \pm 0.23$
c_6	0.116 ± 0.01	1.97 ± 0.14

The errors on t_1, t_2 are determined from the errors on the $\bar{K}N$ scattering lengths, and on c_6 from the position ($W_R = 1385 \pm 2$ MeV)

and width (Γ = 37±3 MeV) of the Y_1^* resonance[7].

Equ(6.1) gives

$$0.937\, g^2_{\Sigma o} + 1.632 g^2_{\Lambda} + 0.116 g^2 = t_1 \tag{6.11}$$

Equ(6.6) gives

$$3.29\, g^2_{\Sigma o} + 4.2\, g^2_{\Lambda} + 1.97 g^2 = t_2 \tag{6.12}$$

Eliminating the Y_1^* contribution between equ(4.43) and equ(6.11)

$$3.48\, g^2_{\Sigma o} + 7.15\, g^2_{\Lambda} = 19.08 \tag{6.13}$$

This gives an upper bound ($g^2_{\Sigma o} = 0$) of

$$g^2_{\Lambda} = 2.67 \tag{6.14}$$

which is to be compared with the value obtained from equ(3.4), $g^2_{\Lambda} = 4.34$.

Similarly equ(4.44) and equ(6.6) give

$$0.533\, g^2_{\Sigma o} + 1.10\, g^2_{\Lambda} = 5.21 \tag{6.15}$$

This gives an upper bound of

$$g^2_{\Lambda} = 4.74 \tag{6.16}$$

This value may be compared with the value obtained from equation (4.7) $g^2_{\Lambda} = 4.87$. Thus the new dispersion relation (4.7) gives a more consistent result for g^2_{Λ}. With the assumption that g^2 is positive equ(6.13) gives the following ranges of values for the three couplings

$2.96 \geq g^2_{\Lambda} \geq 1.16$, $0 \leq g^2_{\Sigma o} \leq 3.10$, $3.80 \geq g^2 \geq 0$

Similarly, equ(6.15) gives

$4.74 \geq g^2_{\Lambda} \geq 3.29$, $0 \leq g^2_{\Sigma o} \leq 2.97$, $1.84 \geq g^2 \geq 0$.

If the additional constraint of SU(3) is imposed, $g^2_{\Sigma o}/g^2_{\Lambda} = 1/27$, equ(6.13) implies

$$g^2_\Lambda = 2.62; \quad g^2_{\Sigma^0} = 0.10; \quad g^2 = 3.71 \tag{6.17}$$

Equation (6.16) imples

$$g^2_\Lambda = 4.64; \quad g^2_{\Sigma^0} = 0.17; \quad g^2 = 1.74 \tag{6.18}$$

Alternatively the $\bar{K}N$ amplitudes in the unphysical region may be parameterized by using (4.23) with γ_I as input parameters and α_I, β_I determined by matching the energy dependent and independent scattering lengths at $w = \mu$. The results of this investigation are shown in table 18. Also tabulated are the values of g^2_Λ using this procedure for the undifferentiated dispresion relations (c.f. table 4 and table 9), and the upper bounds on g^2_Λ obtained by the equivalent method to that which gave equation (6.13) and equation (6.16) (i.e. by using the tabulated t_1 and t_2). The corresponding α_0, β_0 are shown in table 5.

Table 18: The results of (6.1), (6.6) by matching the energy dependent and independent scattering lengths at threshold. Columns (5) and (7) give the upper bounds on g^2_Λ from (6.1), (6.6) respectively. Columns (6), (8) give the values of g^2_Λ from tables 4, 9 respectively.

				Conventional Relation		New Relation	
γ_0	γ_1	t_1	t_2	Bound on g^2_Λ	g^2_Λ	Bound on g^2_Λ	g^2_Λ
0.4	0.2	4.67	24.68	2.69	8.24	5.62	7.25
	0	4.88	25.68	2.74		5.75	
	-0.2	5.28	28.70	2.88		6.13	
0	0.2	4.34	23.26	2.48	7.94	5.28	6.84
	0	4.54	24.27	2.54		5.42	
	-0.2	4.94	27.28	2.68		5.80	
-0.4	0.2	4.06	21.06	2.31	6.58	4.77	6.08
	0	4.27	22.06	2.37		4.90	
	-0.2	4.66	25.08	2.51		5.28	

From table 18 we see that the agreement between the conventional forward dispersion relations (3.4), (6.1) is very poor. However the new dispersion relations (4.7), (6.6) indicate that γ_1 is negative and $\gamma_1 \leq -0.4$. Also, as γ_0 decreases the general trend is towards a small discrepancy between the predictions of (4.7), and (6.6). Unfortunately this disagrees with the Y_0^* parameters which require $0 < \gamma_0 < 0.3$. If we use values $\alpha_0 = -1.51$, $\beta_0^2 = 0.45$, $\gamma_0 = 0.4$ for equ (6.6), which are given toberable agreement with the Y_0^* position and width, then $a_0(w^1)$ is consistent with the energy independent scattering length a_0 in the range $\mu < w' < w_2$, and the upper bound is $g_\Lambda^2 = 5.22$. This compares with the value $g_\Lambda^2 = 6.09$ shown in table 10. Similarly, taking $\alpha_0 = -1.51$, $\beta_0^2 = 0.6$, $\gamma_0 = -0.2$, for which $b_0(w^1)$ is consistent with the energy independent scattering length b_0 in the range $\mu < w' < w_2$ then the upper bound is $g_\Lambda^2 = 4.55$. This compares with the value $g_\Lambda^2 = 5.10$ in table 10. Even now, the differential dispersion relations still appear to underestimate the value of g_Λ^2. This inconsistency could be attributed to either the way in which the Y_1^* contribution is evaluated, or a defect in our assumption that only zero range s waves contribute to the low energy $\bar{K}N$ amplitudes, if the cross-section data measurements and asymptotic region parametrization are accurate. However this fault is more obvious in the case of the conventional forward dispersion relations than our new dispersion relation. Nevertheless, the fact that we have obtained a value for the sum of the pole terms and the unphysical region contribution, albeit

at $w = \mu$, enables us to evaluate $D_+(w,t=0)$, (4.7), for general values of w and compare with the experimental data. The comparison with experiment of $D_-(w,t=0)$, (4.6), is also discussed in the subsequent and final chapter.

CHAPTER VII

THE COMPARISON OF THE NEW RELATIONS WITH EXPERIMENT

1 THE RELATIVE MERITS OF THE VARIOUS RELATIONS.

So far, the forward dispersion relations have been used to predict values of the ΛKN and ΣKN coupling constants. Having done this it is possible to predict the real parts of the $K^{\pm}p$ and $K^{\pm}n$ forward scattering amplitudes and compare the results with experiment.

As discussed in section (IV-3) the most serious problem is the evaluation of the forward dispersion relations for $\bar{K}N$ scattering is due to the uncertainties associated with the Λ and Σ pole term contributions and the computation of the dispersion integrals over the unphysical region below the $\bar{K}N$ threshold. Let us consider the two methods adopted by Lusignoli et al[26] and Queen[56], and compare these relations with our new dispersion relations (4.6), (4.7). The sum of the pole terms and the unphysical region contribution in (5.16) for $D_+(w)$ used in Lusignoli et al.[26] may be written as

$$-I_L(w) = k^2\left[\frac{1}{\pi}\int_{w_{\gamma\pi}}^{\infty}\frac{A_-(w')\,dw'}{k'^2(w'+w)} + \sum_{\gamma}\frac{X(\gamma)\cdot g^2_{\gamma}}{(w_{\gamma}+w)(w_{\gamma}^2-\mu^2)}\right] \quad (7.1)$$

Note that equ(5.16) is essentially a twice subtracted dispersion relation. An alternative form is the once subtracted relation for $D_+(w)$, equ(3.2), which has been evaluated by Queen[56], where now the unphysical region contribution is given by

$$I_q(w) = (w-\mu)\left[\frac{1}{\pi}\int_{w_{\gamma\pi}}^{\infty}\frac{A_-(w')\,dw'}{(w'-\mu)(w'+w)} + \sum_{\gamma}\frac{X(\gamma)\cdot g^2_{\gamma}}{(w_{\gamma}+\mu)(w_{\gamma}+w)}\right] \quad (7.2)$$

The corresponding relations for the real part of the $\bar{K}N$ amplitudes are obtained by using the crossing relation

$$D_-(w) = D_+(-w) \qquad (7.3)$$

However, as table 13 shows, the fluctuations in the predicted value of the coupling constants arise mainly from the particular way in which the unphysical region is parameterized because of the large canelllations which occur between the various terms. It is therefore desirable to calculate $D_\pm(w)$ from a relation in which the errors compensate each other as much as possible. For example, for sufficiently large $|w|$ the errors on some of the individual contributions to equation (5.16) are correlated since using the unsubtracted relation (3.4) and (7.1) we find

$$\frac{w}{2\mu}\left[D_-(w) - D_+(w)\right] \sim I_L(w) \sim \frac{w}{4\pi^2}\int_\mu^\infty \frac{\sigma_- - \sigma_+}{t'}\, dw' \qquad (7.4)$$

as $|w| \to \infty$. That is, at high energies the error due to the combined contribution to equ (5.16) of the second subtraction constant, the pole terms and the integral over the unphysical region, is simply equal to the error on the right hand side of equ(7.4). Although the relation used by Queen[56] does not enjoy a similar advantage, this is offset to some extent as it only involves one subtraction as opposed to the two subtractions in equ(5.16).

Consider now the unsubtracted dispersion relation (4.7)[57],

with $w_o \gtrsim 0.4$ GeV, and write

$$I(w) = |\sqrt{\xi(-w)}| \left\{ \frac{1}{\pi} \int_{w_{th}}^{w_o} \frac{A_-(w')\,dw'}{|\sqrt{\xi(w')}|\,(w'+w)} + \frac{1}{\pi} \int_{w_o}^{\mu} \frac{D_-(w')\,dw'}{|\sqrt{-\xi(w')}|\,(w'+w)} + \sum_{\gamma} \frac{X(\gamma)\, g^2_\gamma}{|\sqrt{\xi(w_\gamma)}|\,(w_\gamma + w)} \right\} \qquad (7.5)$$

Since equ(4.7) is convergent in unsubtracted form we can calculate $I^p(\mu)$ and $I^n(\mu)$ directly from $\sigma_\pm$ and $D_+(\mu)$. Further, from the form of equ(7.5) the computed values of I(w) are expected to be almost independent of w except when w is in the neighbourhood of the $\bar{K}N$ unphysical region. The important point is that if equ(4.7) is used to calculate $D_\pm(w)$, the large errors associated with the extrapolation and the coupling constants are correlated in such a way that the total error on I(w) is expected to be comparable to that for $I(\mu)$, except near $w = -\mu$. In the next section we shall describe how the values of $D_\pm(w)$ are calculated from equ(4.7).

2. THE EVALUATION OF THE NEW RELATIONS

It is convenient to write equation (4.7) in the form

$$D_+(w) = \sum_{i=1}^{5} c_i(w) \qquad (7.6)$$

where the c_i in (3.5) have been redefined as $c_i(w)\sqrt{q(w)}$ and c_1 now refers to the contribution of the pole term, so that

$$\bar{I}(w) = c_1(w) + c_2(w) \qquad (7.7)$$

From table 6 we find

$$\begin{aligned} \bar{I}^p(\mu) &= 1.04 \pm 0.09 \ \text{GeV}^{-1} \\ \bar{I}^n(\mu) &= 0.73 \pm 0.20 \ \text{GeV}^{-1} \end{aligned} \qquad (7.8)$$

In tables 19, 20 we show the various contributions to $D_\pm^p(w)$ and $D_\pm^n(w)$ respectively at a few selected momenta. The tabulated value of $\bar{I}(w)$ corresponds to the constant scattering length extrapolation. The calculation of $\bar{I}(w)$ was repeated using various K-matrix extrapolations, each consistent with the Kim[37] $\bar{K}N$ scattering lengths and a Y_0^* resonance, but they produced a negligible change in the value of $\bar{I}(w)$ except for w in the low energy region. As the calculation was completed before Kim's[54] new parameters were available we did not explicitly verify that the extrapolation with these parameters also produces no change in $\bar{I}(w)$. Notice from tables 19, 20 that, as predicted, the value of $\bar{I}^N(\pm w) \simeq \bar{I}^N(\mu)$ except for w in the low energy K^-N region. The tabulated errors on $\bar{I}(w)$ reflect the errors shown in equ(7.8)

Therefore equ(4.7) is particularly suitable to the calculation of $D_{\pm}(w)$ as the total error on $\bar{I}(w)$ is comparable to that for $\bar{I}(\mu)$ except for $w \simeq -\mu$.

Independent confirmation of the constancy of $\bar{I}^{p}(w)$ with respect to w is obtained by evaluation the derivative form of equ(4.7), that is equ(6.6), at the $K^{+}p$ threshold. Using the known parameters of the $K^{+}p$ s-wave effective range expansion[38] and the evidence that the $K^{+}p$ p-wave scattering lengths are small, gives

$$\left.\frac{d\bar{I}^{p}}{dw}\right|_{w=\mu} = -0.15 \pm 0.25 \quad G_F v^{-2} \qquad (7.9)$$

Table 19: The contributions (in units of GeV^{-1}) to $D_{\pm}^{p}(w)$ from different parts of the dispersion relation of equ(4.7). The notation is that of equ(7.6).

	k (Gev/o)	Poles + Continuum = $\bar{I}$ $C_1(w)$	$C_2(w)$	$C_3(w)$	$C_4(w)$	$C_5(w)$	D^p (Gev^{-1})
K^+p	0.52	-1.51+2.60	1.09 (±0.1)	-7.98 (±0.07)	5.20 (±0.1)	-0.56 (±0.01)	-2.25 (±0.16)
	0.97	-1.37+2.48	1.11 (±0.1)	-9.29 (±0.08)	6.82 (±0.1)	-0.67 (±0.02)	-2.03 (±0.16)
	1.97	-1.23+2.34	1.11 (±0.1)	-11.89 (±0.10)	7.45 (±0.1)	-0.79 (±0.02)	-4.12 (±0.17)
	3.50	-1.16+2.26	1.10 (±0.1)	-14.83 (±0.16)	8.42 (±0.2)	-0.54 (±0.03)	-5.85 (±0.27)
	9.80	-1.10+2.17	1.07 (±0.1)	-21.33 (±0.36)	2.98 (±0.5)	7.28 (±0.32)	-10.0 (±0.7)
K^-p	0.415	-0.38-1.22	-1.60 (±0.87)	1.62 (±0.18)	-0.51 (±0.04)	0.12 (±0.002)	-0.36 (±0.9)
	0.95	-0.68+1.20	0.52 (±0.32)	3.42 (±0.21)	-1.50 (±0.04)	0.43 (±0.01)	2.87 (±0.4)
	1.975	-0.87+1.76	0.89 (±0.12)	1.57 (±0.29)	-3.27 (±0.05)	1.30 (±0.04)	0.48 (±0.3)
	3.46	-0.95+1.93	0.98 (±0.11)	2.20 (±0.46)	-5.25 (±0.07)	3.05 (±0.11)	0.98 (±0.5)
	5.0	-0.99+1.99	1.00 (±0.10)	1.53 (±0.58)	-6.79 (±0.09)	5.49 (±0.25)	1.23 (±0.7)
	9.0	-1.02+2.02	1.00 (±0.10)	-4.82 (±0.86)	-9.53 (±0.13)	15.23 (±0.72)	1.88 (±1.7)

Table 20: The contributions (in units of Gev^{-1}) to $D_{\pm}^{n}(\omega)$ from different parts of the dispersion relation of equ(4.7). The notation is that of equ(7.6).

	k (Gev/c)	$c_1(\omega)$ $c_2(\omega)$ Poles + Continuum = $\bar{I}$		$c_3(\omega)$	$c_4(\omega)$	$c_5(\omega)$	D^n (Gev^{-1})
	0.52	-1.09+1.81	0.72 (±0.2)	-6.35 (±0.1)	5.64 (±0.4)	-0.32 (±0.01)	-0.3 (±0.5)
	0.97	-1.01+1.73	0.72 (±0.2)	-7.45 (±0.1)	7.18 (±0.3)	-0.36 (±0.02)	+0.1 (±0.4)
K^+n	1.97	-0.93+1.63	0.70 (±0.2)	-9.65 (±0.2)	7.46 (±0.2)	-0.30 (±0.02)	-1.8 (±0.3)
	3.50	-0.88+1.57	0.69 (±0.2)	-12.2 (±0.3)	8.50 (±0.5)	0.28 (±0.03)	-2.7 (±0.6)
	9.80	-0.84+1.51	0.67 (±0.2)	-17.9 (±1.0)	2.76 (±1.5)	9.27 (±0.32)	-5.2 (±1.8)
	0.415	-0.32-0.57	-0.89 (±0.8)	1.43 (±0.3)	-0.52 (±0.09)	0.08 (±0.002)	0.1 (±0.9)
	0.95	-0.65+0.89	0.33 (±0.4)	2.24 (±0.3)	-1.54 (±0.14)	0.31 (±0.01)	1.3 (±0.5)
	1.975	-0.69+1.24	0.55 (±0.25)	2.30 (±0.5)	-3.35 (±0.15)	0.97 (±0.04	0.5 (±0.6)
K^-n	3.46	-0.75+1.35	0.60 (±0.2)	3.94 (±0.8)	-5.37 (±0.2)	2.41 (±0.11)	1.6 (±0.8)
	5.0	-0.77+1.40	0.63 (±0.2)	3.73 (±0.8)	-6.95 (±0.3)	4.51 (±0.25)	1.9 (±0.9)
	9.0	-0.79+1.43	0.64 (±0.2)	-1.51 (±1.7)	-9.74 (±0.4)	13.3 (±0.7)	2.7 (±1.9)

3 THE COMPARISON WITH EXPERIMENT

In figures 1, 2 the disperison relations predictions for $\alpha_{\pm}^{p} = D_{\pm}^{p} / A_{\pm}^{p}$ are shown, together with the values that are obtained from extrapolating the differential cross sections for $K^{\pm}p$ elastic scattering[58][59] to the forward direction, where

$$\left(\frac{d\sigma}{d\Omega}\right)_{t=0}^{lab} = |D|^2 + |A|^2 \qquad (7.10)$$

The uncertainty in the disperison relation predictions for α can be estimated from the tabulated errors on the predictions for D(w). The signs of the experimental values of $\alpha_{\pm}$ are undetermined and in figures 1, 2 we have taken the sign which is in better agreement with the calculated curves. Above w = 5 GeV we also show the predictions fro $\alpha_{\pm}$ that are obtained from the Phillips and Rarita Regge-pole parameters (see appendix B). Since these Regge parameters are used to calculate the asymptotic contribution to the dispersion relations the two predictions must agree asymptotically. For completeness figures 4, 5, 6, 7 show the values obtained for $D_{+}^{p}(w)$, $D_{-}^{p}(w)$, $D_{+}^{n}(w)$ and $D_{-}^{n}(w)$ respectively plotted against w.

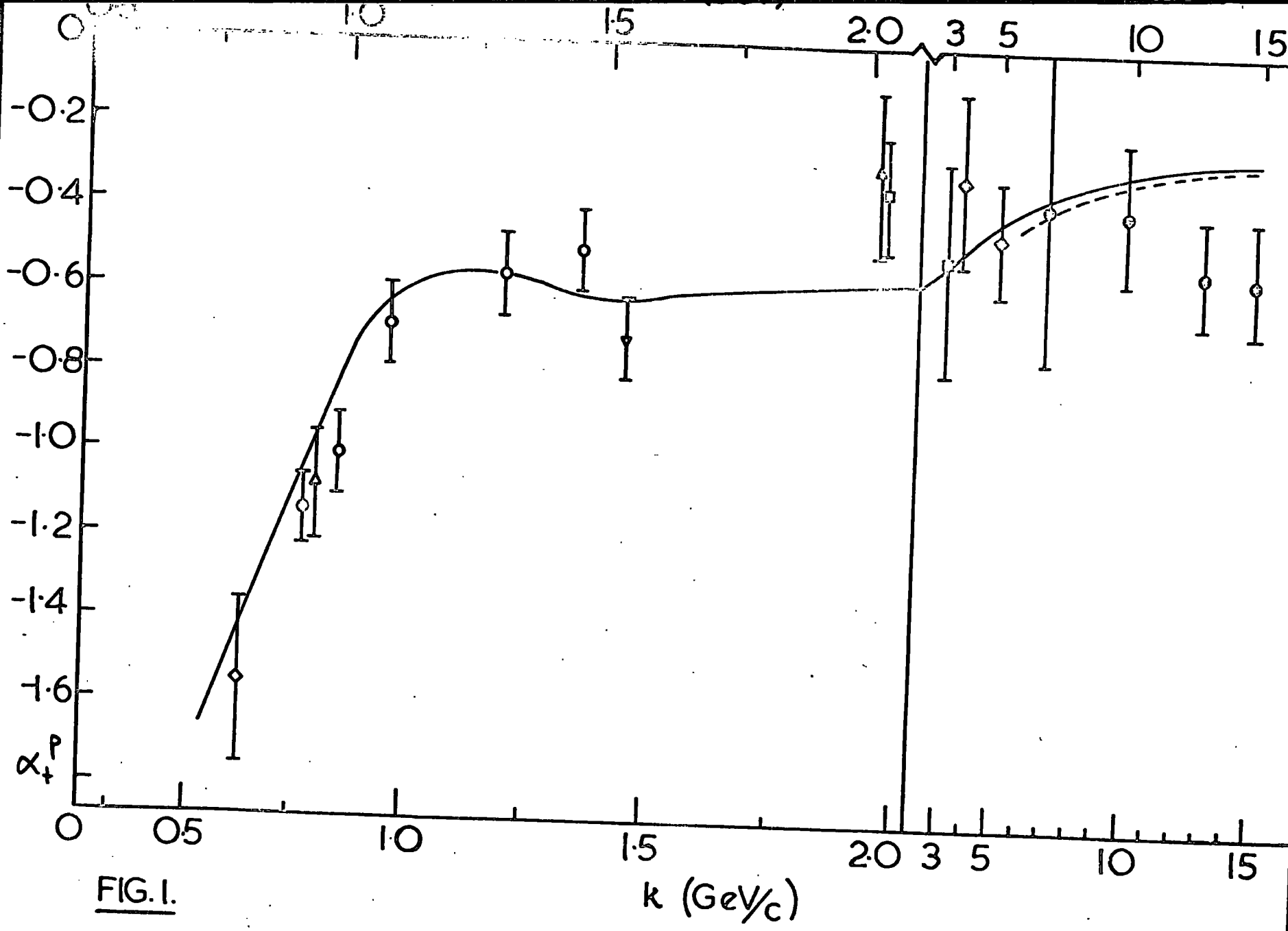


FIG. 1.

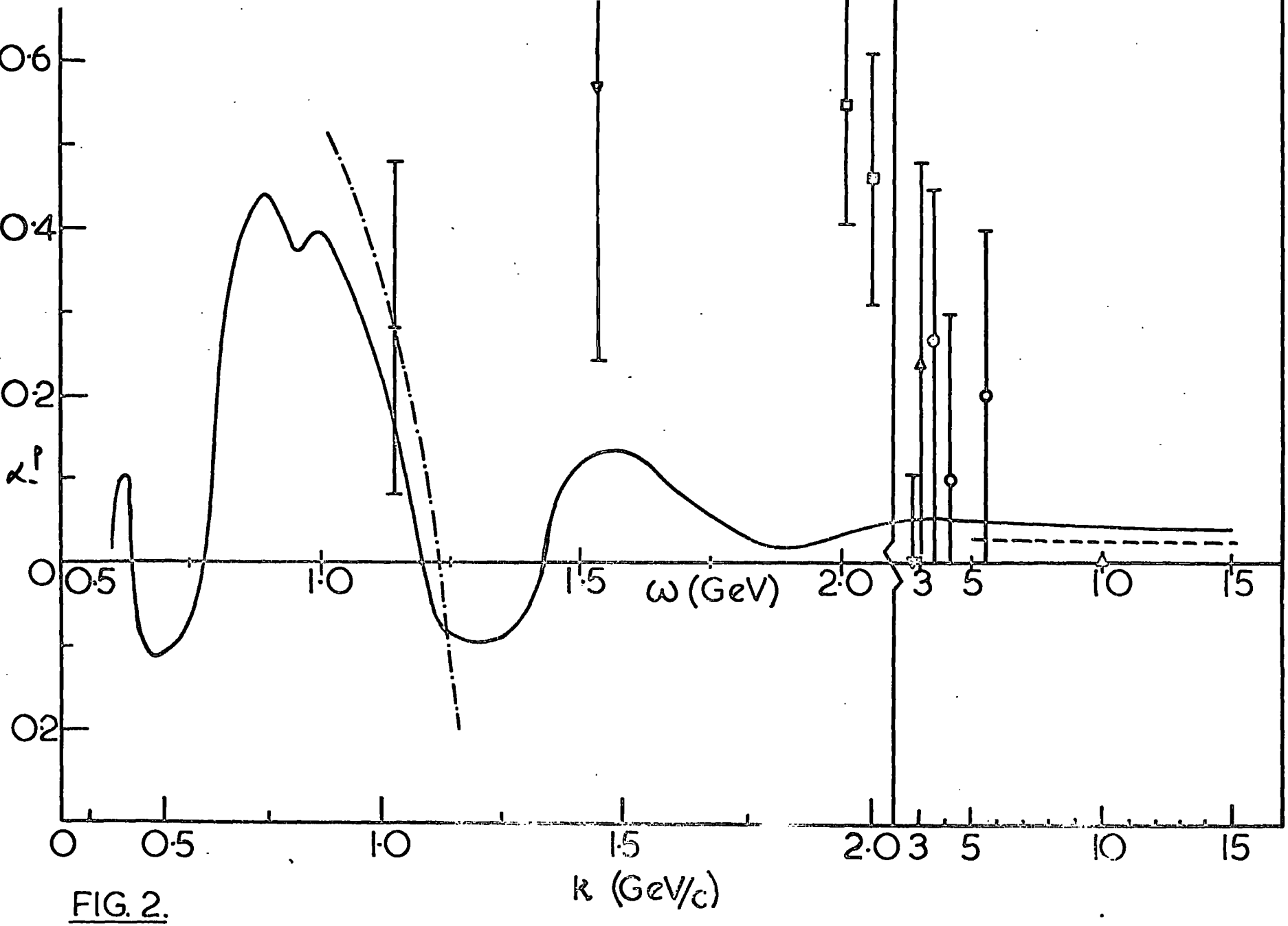


FIG. 2.

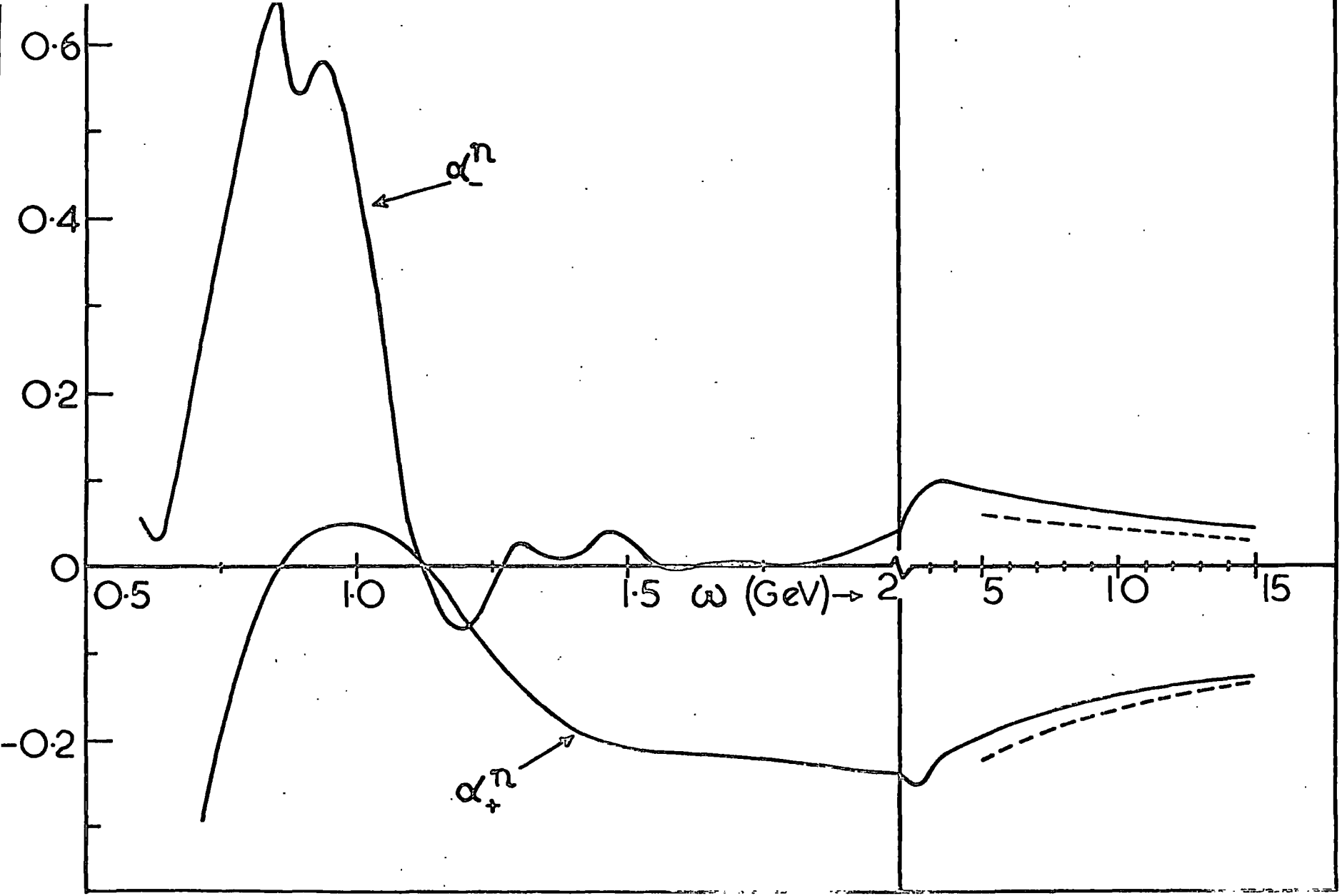


FIG. 3.

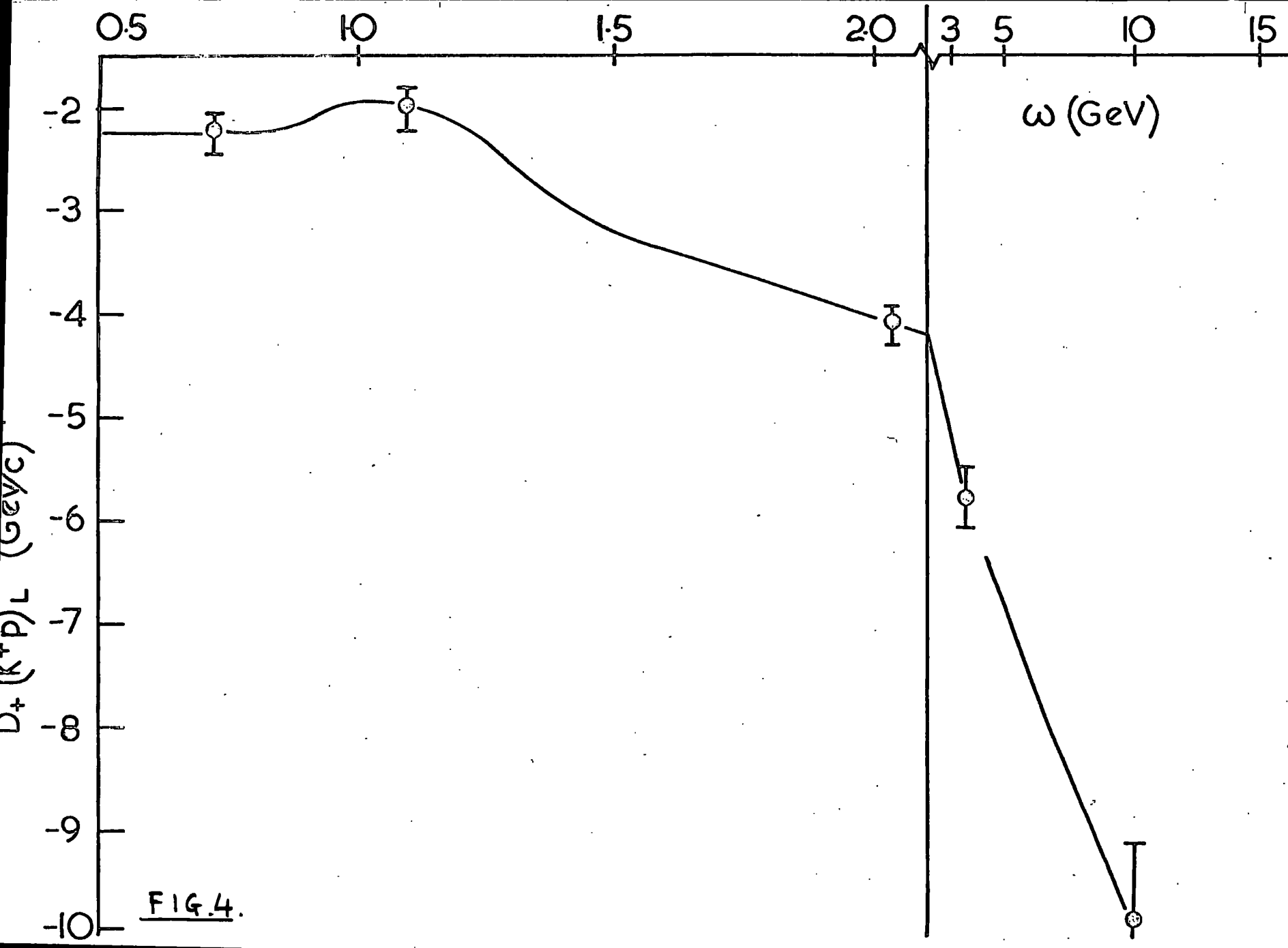


FIG.4.

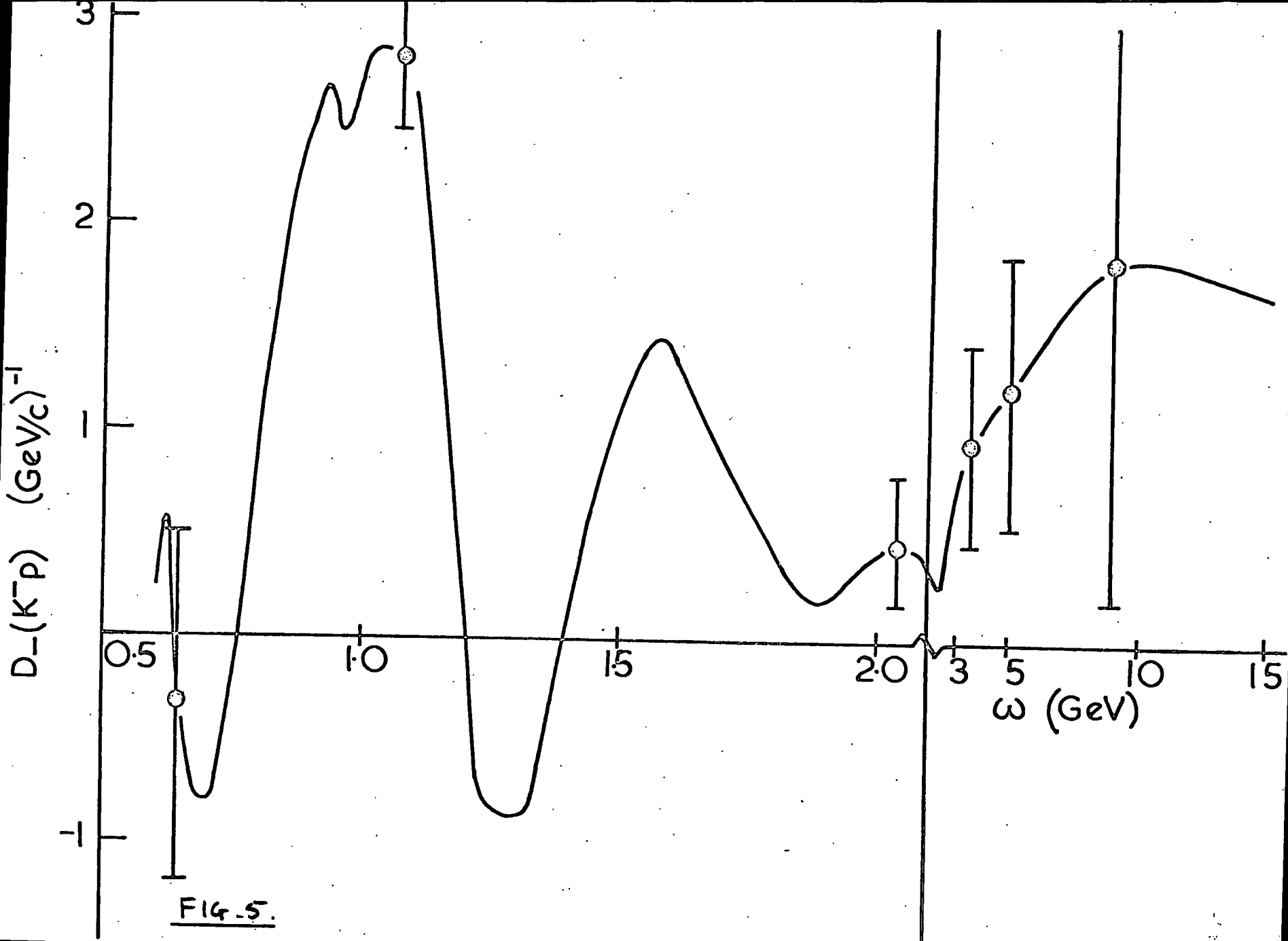


FIG. 5.

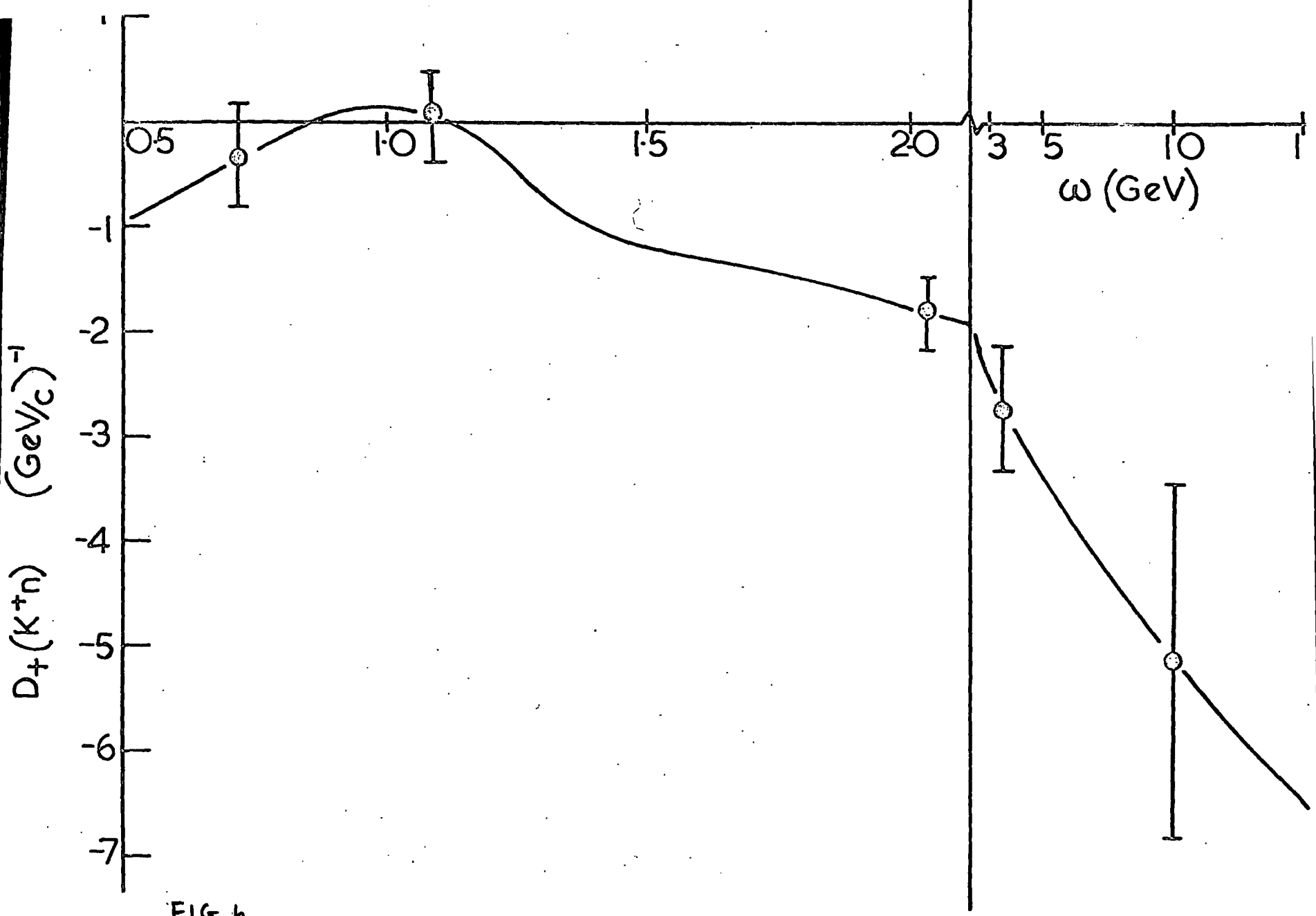


FIG. 6.

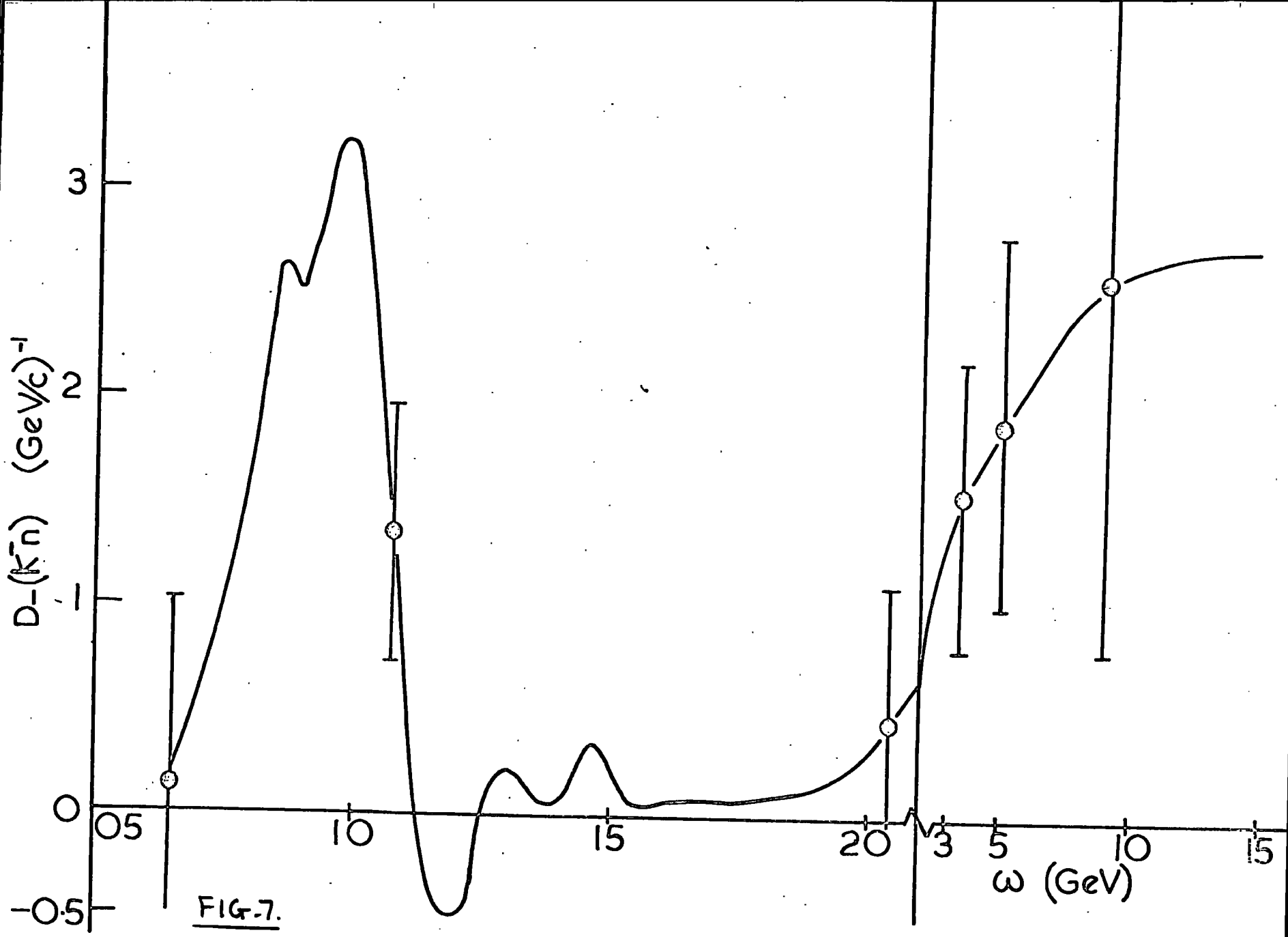


FIG-7.

Figure Captions

Figure 1: The energy dependence of the ratio α for K^+p. The solid curve is the prediction of the dispersion relation, eq(4.7), and the dashed curve is the Regge -pole prediction. The experimental points are obtained from the following references: ◆ (38); ▼ (58a); ▲ (58b); ◊ (58c); ▽ (58d); ▲ (58e); □ (58f); ■ (58g); ◆ (58h); ● (58i).

Figure 2: The ratio α for K^-p. The solid (dashed) curve is the dispersion relation (Regge-pole) prediction. The experimental points are obtained from the following references:-.-.- (59a); ▼ (59b); □ (59c); ■ (59d); ▼ (59e); ▲ (59f); ● (59g) ○ (59h); ▲ (59i).

Figure 3: The dispersion relations predictions of α for $K^{\pm}n$. (The dashed curve represents the Regge-pole predictions).

Figure 4: The dispersion relation predictions for $D_+(K^+p)$.

Figure 5: The predictions for $D_-(K^-p)$.

Figure 6: The predictions for $D_+(K^+n)$.

Figure 7: The predictions for $D_-(K^-n)$.

From figure 1 we see that for K^+p there is good agreement between the dispersion relation predictions and the experimental value of α, except at the two high energy points and the two points near 2 GeV. It is interesting to note that in the experiment

performed at 1.96 GeV/c[58e] the measured total K^+p cross section was found to be 2 millibarnes greater than the more recent measurements. Using the new value for σ_+^P and scaling the differential cross section accordingly we find $\alpha_+^P = -0.47 \pm 0.17$, which is in much better agreement with the dispersion relation prediction. The origin of the discrepancy with the somewhat earlier experimental result at 1.97 GeV/c[58f] is probably simply that the error associated with the extrapolation of the differential cross section to the forward direction was underestimated.

Now consider K^-p and the comparison in figure 2. Gelfand et al.[59a] have measured the K^-p angular distribution at twenty two values of momenta in the range 0.8 to 1.2 GeV/c. The resulting predictions for α_-^P are smoothed as a function of momentum and are shown as a dot-dash curve on figure 2 together with a typical error bar. There is reasonable agreement with the dispersion relation prediction for α_-^P. However, as pointed out previously by Lusignoli et al.[45] a discrepancy exists between the dispersion relation prediction and the experimental points in the region 1.5 to 2 GeV/c. Above 2 GeV/c α_-^P is predicted to be very small and the experimental value for α^2 is then the small difference between two large quantities. In view of the large experimental errors it is not surprising that disagreement is observed in this region.

In figure 3 we show the dispersion relation predictions for α_+^n.

At present the only experimental information concerning $\alpha_{\pm}^{n}$ is that obtained indirectly via the observation of the charge exchange $K^{+}n \rightarrow K^{o}p$ (or to be precise $K^{+}d \rightarrow K^{o}pp$) and $K^{-}p \rightarrow \bar{K}^{o}n$. The forward amplitudes for these processes are

$$\begin{aligned} T(K^{+}n \rightarrow K^{o}p) &= T_{+}^{p} - T_{+}^{n} \\ T(K^{-}p \rightarrow \bar{K}^{o}n) &= T_{-}^{p} - T_{-}^{n} \end{aligned} \qquad (7.11)$$

and so given the value of α^{p} we can, in principle, determine α^{n} from the data for small angle charge exchange scattering. It is useful to define the ratio of the real to the imaginary parts of the charge exchange amplitudes to be

$$\alpha_{\pm}^{ce} = \frac{D_{\pm}^{p} - D_{\pm}^{n}}{A_{\pm}^{p} - A_{\pm}^{n}} \qquad (7.12)$$

Data exists for the process $K^{+}d \rightarrow K^{o}pp$ at 2.3 GeV/c[60]. Glauber and Franco[61] have analysed this data, carefully allowing for the deuteron effects, and estimate $\alpha_{+}^{n} \sim -0.76$ and $\alpha_{+}^{ce} \sim -6.8$, which are to be compared with the dispersion relation predictions of $\alpha_{+}^{n} \sim -0.23$ and $\alpha_{+}^{ce} \sim -8.4$. Unfortunately in their analysis of the data they assume $\alpha_{+}^{p} \sim -0.34$ whereas it now seems probable that $\alpha_{+}^{p} \sim -0.55$ at this energy. With the latter value, a rough calculation[62] shows that $\alpha_{+}^{n} \sim -0.13$ or -0.97.

Turning now to the other charge exchange process $K^{-}p \rightarrow \bar{K}^{o}n$ we find that angular distribution data exists at many energies[63] However, in this case, the forward scattering amplitude is found to

be predominantly imaginary at most momenta and so prohibits an accurate experimental prediction of α_-^{ca}. In table 21 we list the experimental values of the forward differential cross section for $K^-p \rightarrow \bar{K}^o n$ for momenta above 1 GeV/c together with the dispersion relation predictions. Only for momenta at which $(\alpha_-^{ca})^2$ is appreciably different from zero (for example in the region of 1.5 GeV/c) is the comparison a significant test of the dispersion relation predictions for $D_-^n(w)$ and $D_-^p(w)$. The agreement with experiment is good and in particular the discrepancy noted by Lusignoli et al.[64], in the region around 1.5 GeV/c, is resolved.

Table 21: Comparison between the dispersion relation predictions and the experimental values of the forward differential cross section for $K^- p \rightarrow \bar{K}^o n$ in the c.m. frame.

k Gev/c	Dispersion relation		Experiment	
	$(\alpha^{ce})^2$	$\left(\frac{d\sigma}{d\Omega}\right)_0$ mb./ster.	$\left(\frac{d\sigma}{d\Omega}\right)_0$ mb./ster.	Reference
1.022	0.56	1.22 ± 0.40	1.58 ± 0.28	
1.08	0.04	1.54 ± 0.28	1.40 ± 0.25	63a
1.13	0.01	1.02 ± 0.22	0.94 ± 0.24	
1.18	0.17	0.70 ± 0.22	0.66 ± 0.22	
1.22	0.31	0.33 ± 0.21	0.27 ± 0.05	
1.42	0.76	0.18 ± 0.15	0.21 ± 0.04	
1.51	0.81	0.39 ± 0.18	0.38 ± 0.03	63b
1.6	0.18	0.47 ± 0.12	0.43 ± 0.09	
1.7	0.04	0.52 ± 0.10	0.49 ± 0.08	
1.8	0	0.35 ± 0.10	0.33 ± 0.07	63c
2.24	0.05	0.39 ± 0.10	0.29 ± 0.06	63d
3.5	0.04	0.42 ± 0.15	0.32 ± 0.06	63e
5.0	0.07	0.23 ± 0.16	0.29 ± 0.07	
7.1	0.07	0.22 ± 0.20	0.25 ± 0.04	63f
9.5	0.06	0.20 ± 0.12	0.26 ± 0.05	

4. CONCLUSIONS.

Except for the analysis performed by Kim[54], all the calculations of the lambda coupling constant discussed in Chapter V support the value deduced in Chapter IV, that is

$$g^2_\Lambda = 5.8 \pm 1.8 \tag{7.13}$$

The results shown in table 11 indicate that this error may be rather conservative and that a more realistic value may be

$$g^2_\Lambda = 6.0 \pm 2.5 \tag{7.14}$$

Note that the inclusion of the correct pole term factor (as discussed in appendix D) decreases these values slightly. In any case the new forward dispersion relations do seem to diminish the importance of many of the undesirable features in the conventional forward dispersion relations. This certainly appears to be so from the discussion presented in Chapter IV. Unfortunately there is still a significant contribution from the unphysical region when equation (4.7) is evaluated at the kaon nucleon threshold to determine the lambda coupling constant. Consequently

there is still an ambiguity in the exact value of g^2_Λ, as shown in the above results (7.13) and (7.14). This should be resolved by obtaining more precise values of the isospin I = 0 R-matrix elements in the low energy region to fit the known data. At the present time efforts are being made[65] to resolve this dilemma.

As suggested in Chapter V the situation regarding the sigma coupling constant is even less transparent, as very little is known about the $Y_1^*(1385)$ resonance. Attempts have been made by Davies et al.[48], and in Chapter IV, to resolve the isospin I = 1 s-wave and p-wave contributions to the $\bar{K}N$ low energy region. Success in this direction could well produce a very low value for g^2_Σ which may satisfy the SU(3) prediction of $g^2_{\Sigma 0} \sim 0.5$. Due to the signs of the Y_1^* and Σ pole term effects in equation (4.44), a decrease in the value of g^2_Σ entails a compensating increase in $g^2 Y_1^*$.

The use of derivative dispersion relations in Chapter VI appears to show the superiority of our new approach in that more consistent results are obtained from relations of the type (4.7) than from the conventional forward dispersion relations. Taken to their logical conclusion they indicate that $|g^2_\Sigma|$ is very small.

This fact is deduced from the arguments concerning the consistency of the predictions for g^2_{Λ} . From equation (6.16) one may also deduce that $g^2_{\bar{K}NY_1^*} \sim 1$, if $g^2_{\Sigma^0}$ is neglected.

Even though the exact values of the coupling constants may still be slightly suspect, the results of Chapter VII enable one to assert that the stage has almost been reached where the dispersion relation predictions for the real parts of the forward amplitudes can be used to normalize the experimental angular distributions for kaon nucleon scattering. This is true simply because the results indicate that the sum of the pole term and unphysical region contributions influences the real parts of the amplitude in a fixed way.

The conclusions reached in this thesis are, we believe, the extent of present knowledge in this rapidly growing field.

APPENDIX A

PARTIAL WAVE AMPLITUDES

For equal mass (m) spin-zero particles the partial wave amplitude $f_\ell(k^2)$ may be defined in terms of the invariant amplitude $F(st) \equiv F(k^2, \cos\theta)$ by the formula

$$f_\ell(k^2) = \frac{1}{2}\int_{-1}^{1} d(\cos\theta)\, P_\ell(\cos\theta)\, F(k^2, \cos\theta) \tag{A1}$$

and in this case (1.4) becomes

$$\begin{aligned} s &= 4(k^2 + m^2) \\ t &= -2k^2(1 - \cos\theta) \\ u &= -2k^2(1 + \cos\theta) \end{aligned} \tag{A2}$$

where k is the centre of mass momentum. By using the orthogonality property of the Legendre functions the total amplitude may be expressed in terms of the partial wave amplitudes by

$$F(k^2, \cos\theta) = \sum_{\ell=0}^{\infty} (2\ell+1)\, f_\ell(k^2)\, P_\ell(\cos\theta) \tag{A3}$$

Hence, the integrals in the dispersion relations give rise to terms of the form

$$Q_{\ell\pm}(k^2 s') = \frac{1}{2}\int_{-1}^{1} d(\cos\theta)\, P_\ell(\cos\theta)\, \frac{1}{s' + 2k^2(1 \pm \cos\theta)} \tag{A4}$$

which are simply Lengendre functions of the second kind. Certain important properties of these integrals may be seen by inspection.

(a) For small k^2 they behave like $(k^2)^\ell$

(b) For large s' they behave like $(s')^{-\ell-1}$

Condition (a) shows that we should really consider the quantities defined by

$$h_l = \frac{f_l}{f_k^{2l}} \qquad \text{(A5)}$$

APPENDIX B

THE HIGH-ENERGY CONTRIBUTIONS

In this appendix we derive the exact forms of the high-energy contributions to the ordinary and differentiated, conventional and new dispersion relations. The way in which this region, $w' > 20GeV$, is treated is in accord with the formalism of Phillips and Rarita [42] and the values of the parameters they obtained are used as input numbers. By writing the kaon nucleon scattering amplitudes in terms of five crossed channel poles, Phillips and Rarita fitted the experimental data for the total and differential cross sections. The various amplitudes of interest have the form

$$\begin{aligned} K^-p \to K^-p &= T_p + T_{p'} + T_R + T_\omega + T_\rho \\ K^+p \to K^+p &= T_p + T_{p'} + T_R - T_\omega - T_\rho \\ K^-n \to K^-n &= T_p + T_{p'} - T_R + T_\omega - T_\rho \\ K^+n \to K^+n &= T_p + T_{p'} - T_R - T_\omega - T_\rho \end{aligned} \qquad \text{(B1)}$$

where the suffices P, P', R, ω, ρ refer to the appropriate Regge trajectories.

The amplitudes T_i associated with each trajectory (i) are given by

$$\text{Im } T_i = \frac{c_i}{4\pi} \alpha_i (2\alpha_i + 1) \left(\frac{w}{w_s}\right)^{\alpha_i} \qquad \text{(B2)}$$

$$\text{Re } T_i = -\left[\frac{\cos \pi\alpha_i \pm 1}{\sin \pi\alpha_i}\right] \text{Im } T_i \qquad \text{(B3)}$$

The (±) sign is the signature factor and is (-) for the ρ, ω trajectories and (+) for the R, P, P' trajectories.

The C_i are constants given in reference(42). The α_i are parameters associated with each trajectory (i) and w_s is a scaling factor taken to be 1 GeV.

Table I: Values of the constants in equ(B2)

i	$C_i \alpha_i(2\alpha_i+1) / w_s^{\alpha_i}$	α_i
ρ	0.0183 famis	0.54
ω	0.09476 "	0.52
P	0.00899 "	1
P'	0.09018 "	0.5
R	0.1069 "	0.31

(a) The equ(3.4): For this relation the relevant dispersive integral to consider is

$$\frac{1}{\pi}\int_{20\,GeV}^{\infty} \frac{A_-(w') - A_+(w')}{w'^2 - w^2}\, dw' \qquad \text{(B4)}$$

Writing

$$A_\pm(w) = w^\alpha B_\pm(w) \qquad \text{(B5)}$$

gives

$$\frac{1}{4\pi^2}\int \frac{B_-(w') - B_+(w')}{w'^2 - \alpha\left(1 - w^2/w'^2\right)}\, dw' \qquad \text{(B6)}$$

Using (B5) to extend the notation to B_i gives for $w < 20$ GeV.

$$\frac{1}{2\pi^2}\int_x^\infty \frac{(B_\omega \pm B_\rho)}{w'^2 - \alpha}\left(1 + \frac{w^2}{w'^2} + \dots\right) dw' \qquad \text{(B7)}$$

where α refers to the appropriate α_i, and x = 20GeV. This integration can be done analytically to yield

$$\frac{1}{2\pi^2}(B_\omega \pm B_\rho)\sum_{i=\omega,\rho}\sum_{l=1}^{\infty} \frac{w^{2l-2}}{(2l-1-\alpha_i)\left(x^{2l-1-\alpha_i}\right)} \qquad \text{(B8)}$$

The $\pm$ sign refers to the fact that the dispersion relation was written for a nucleon N where N = (P_n) and this changes the sign of the B_ρ contribution accordingly, to $(\pm)$ respectively.

(b) The equ(4.7): Here the relevant integrals to consider one

$$\frac{1}{\pi}\int_{x}^{\infty}\frac{A_-(\omega')\,d\omega'}{(\omega'+\omega)\sqrt{(\omega'-\mu)(\omega'-\omega_0)}} - \frac{1}{\pi}\int_{x}^{\infty}\frac{A_+(\omega')\,d\omega'}{(\omega'-\omega)\sqrt{(\omega'+\mu)(\omega'+\omega_0)}} \tag{B.9}$$

The integrals may be expressed in the form (as ω_0, $\mu \ll \omega'$),

$$\frac{A_-}{\omega'^2\left(1+\frac{\omega}{\omega'}\right)} - \frac{A_+}{\omega'^2(1-\omega/\omega')} + \frac{1}{2}\left(\frac{\mu+\omega_0}{\omega'}\right)\left[\frac{A_-}{\omega'^2(1+\omega/\omega')} + \frac{A_+}{\omega'^2(1-\omega/\omega')}\right] \tag{B.10}$$

Integration yields

$$\frac{1}{2\pi^2}\left[(B_\omega \pm B_\rho)\sum_i\sum_1^\infty \frac{\omega^{2\ell-2}}{(2\ell-\alpha_i-1)}x^{2\ell-\alpha_i-1} - (B_\rho+B_{\rho'}\pm B_R)\sum_i\sum_1^\infty\frac{\omega^{2\ell-1}}{(2\ell-\alpha_i)}x^{2\ell-\alpha_i}\right] + \left(\frac{\mu+\omega_0}{4\pi^2}\right)\left[(B_\rho+B_{\rho'}\pm B_R)\sum_i\sum_0^\infty\frac{\omega^{2n}}{(2n+2-\alpha_i)}x^{2n+2-\alpha_i} - (B_\omega \pm B_\rho)\sum_i\sum_1^\infty\frac{\omega^{2\ell+1}}{(2\ell+3-\alpha_i)}x^{2\ell+3-\alpha_i}\right] \tag{B.11}$$

This is true for all $\omega < 20$ GeV. The $\pm$ sign refers to N = (P_n).

(c) The derivative equ(6.1): The relevant integrals here are

$$-\frac{1}{\pi}\int_{x}^{\infty}\frac{A_{-}(w')\,dw'}{(w'+w)^{2}} \quad + \quad \frac{1}{\pi}\int_{x}^{\infty}\frac{A_{+}\,dw'}{(w'-w)^{2}} \tag{B.12}$$

The integrands can be expressed in the form

$$-\left[\frac{A_{-}}{w'^{2}(1+w/w')^{2}} - \frac{A_{+}}{w'^{2}(1-w/w')^{2}}\right] \tag{B.13}$$

Thus after expanding in terms of $(w/_{w'})$ and integrating we obtain

$$-\frac{1}{2\pi^{2}}(B_{w}\pm B_{\rho})\sum_{i}\sum_{1}^{\infty}\frac{(2l-1)\;w^{2l-2}}{(2l-\alpha_{i}-1)\;x^{2l-\alpha_{i}-1}}$$

$$+\frac{1}{2\pi^{2}}(B_{\rho}+B_{\rho'}\pm B_{R})\sum_{i}\sum_{0}^{\infty}\frac{(2l+2)\;w^{2l+1}}{(2l+2-\alpha_{i})\;x^{2l+2-\alpha_{i}}} \tag{B.14}$$

This is true for $w < 20$ GeV.

(d) The derivative equ(6.6): The appropriate integrals to consider are

$$-\frac{1}{\pi}\int_{x}^{\infty}\frac{A_{-}(w')\,dw'}{(w'+w)^{2}\sqrt{(w'-\mu)(w'-w_{0})}} \qquad -\frac{1}{\pi}\int_{x}^{\infty}\frac{A_{+}(w')\,dw'}{(w'-w)^{2}\sqrt{(w'+\mu)(w'+w_{0})}} \tag{B.15}$$

The integrands can be expanded for $w < w'$ and re-expressed, as w_{0}, $\mu << w'$, as

$$-\frac{1}{\pi}\left[\frac{A_-}{w'^3(1+w/w_1)^2} + \frac{A_+}{w'^3(1-w/w_1)^2}\right]$$

$$-\frac{1}{2\pi}(\mu+w_0)\left[\frac{A_-}{w'^4(1+\frac{w}{w_1})^2} - \frac{A_+}{w'^4(1-\frac{w}{w_1})^2}\right] \qquad \text{(B.16)}$$

Integration gives

$$\frac{1}{2\pi^2}\left[\begin{array}{l}(B_w \pm B_\rho)\sum_i \sum_1^\infty \frac{2\ell\, w^{2\ell-1}}{(2\ell-\alpha_i+1)\, x^{2\ell+1-\alpha_i}} \\ -(B_\rho + B_{\rho 1} \pm B_{\rho 2})\sum_i \sum_1^\infty \frac{(2\ell-1)\, w^{2\ell-2}}{(2\ell-\alpha_i)\, x^{2\ell-\alpha_i}}\end{array}\right] \qquad \text{(B.17)}$$

$$-\frac{1}{4\pi^2}(\mu+w_0)\left[\begin{array}{l}(B_w \pm B_\rho)\sum_i \sum_1^\infty \frac{w^{2\ell-2}(2\ell-1)}{(2\ell+1-\alpha_i)\, x^{2\ell+1-\alpha_i}} \\ -(B_\rho + B_{\rho 1} \pm B_{\rho 2})\sum_i \sum_1^\infty \frac{2\ell\, w^{2\ell-1}}{(2\ell+2-\alpha_i)\, x^{2\ell+2-\alpha_i}}\end{array}\right]$$

True for w $<$ x.

(e) The equ(4.6): The relevant integrals to consider are

$$\frac{1}{\pi}\int \frac{A_-(w')\, dw'}{(w'-w)\sqrt{(w'-\mu)(w'-w_0)}} - \frac{1}{\pi}\int \frac{A_+(w')\, dw'}{(w'+w)\sqrt{(w'+\mu)(w'+w_0)}} \qquad \text{(B.18)}$$

The integrands may be expressed as

$$\frac{A_-}{w'^2(1-w/w_1)} - \frac{A_+}{w'^2(1+w/w_1)} \qquad \text{(B.19)}$$

$$+\left[(\mu+w_0)/2w'^3\right]\left[A_-/(1-w/w_1) + A_+/(1+w/w_1)\right]$$

Integration yields

$$\frac{1}{2\pi^2}\left[\begin{array}{l}(B_w \pm B_\rho)\sum_i \sum_0^\infty \frac{\omega^{2l}}{(2l+1-\alpha_i)}\, x^{2l+1-\alpha_i} \\ \\ +(B_\rho + B_{\rho_1} \pm B_\beta)\sum_i \sum_0^\infty \frac{\omega^{2l+1}}{(2l+2-\alpha_i)}\, x^{2l+2-\alpha_i}\end{array}\right] \qquad (B.20)$$

$$+\frac{1}{4\pi^2}(\mu+w_0)\left[\begin{array}{l}(B_w \pm B_\rho)\sum_i \sum_1^\infty \frac{\omega^{2l-2}}{(2l+1-\alpha_i)}\, x^{2l+1-\alpha_i} \\ \\ +(B_\rho + B_{\rho_1} \pm B_\beta)\sum_i \sum_1^\infty \frac{\omega^{2l-2}}{(2l-\alpha_i)}\, x^{2l-\alpha_i}\end{array}\right]$$

For $w < x$.

APPENDIX C

THE 'NEW' PION NUCLEON DISPERSION RELATION

Consider a function G(z) of z which satisfies the following conditions

(a) G(z) is analytic in the upper half of the complex z plane.

(b) G(z) is less than z at infinity in this plane

(c) The real part of G(z) is an odd function on the real axis, the imaginary part is an even function on the real axis.

(d) Im G(z) is zero on the real axis for $-1 < z < 1$, with the exception of a delta-function contribution

$$\frac{\pi G^2}{2M}\left[\delta(z-b) + \delta(z+b)\right] \qquad (C1)$$

We may write $G(z) \equiv B^+(s,t=0)$ as B^+ (given by (1.41)) satisfies the conditions (a)-(d), and $z \equiv w/\mu$ where w is given by (1.38) and μ is the pion mass.

Gilbert[46] suggested that one should look at the function

$$F(w) = \frac{B^+(w,0)}{\sqrt{w^2-\mu^2}} \qquad (C2)$$

where $\sqrt{z^2-\mu^2}$ is that branch of the function that is analytic in the upper half plane and is positive for w real $w > \mu$, and negative for w negative and $w < -\mu$ and is positive imaginary for w real and $-\mu < w < \mu$.

The function F(w) has the following properties

(a) It is analytic in the upper half plane

(b) It goes to zero at infinity

(c) The limit of the function to the real axis from above has an even real part and an odd imaginary part.

(e) The real part of this function is zero for $-\mu < w < \mu$ and w real with the exception of the two delta functions, and

$$\mathrm{Re}\, F(w) = \frac{\mathrm{Im}\, B^{+}(w)}{\sqrt{\mu^2 - w^2}} \qquad \text{for } -\mu < w < \mu \qquad \text{(C3)}$$

Using Cauchy's theorem

$$\mathrm{Im}\left[\frac{B^{+}(w)}{\sqrt{w^2-\mu^2}}\right] = -\frac{2}{\pi} wP \int_{\mu}^{\infty} \frac{dw'}{(w'^2 - w^2)} \frac{\mathrm{Re}\, B^{+}(w',0)}{\sqrt{w'^2-\mu^2}} + \frac{w}{w^2 - w_p^2} \frac{G^2}{M\sqrt{\mu^2 - w_p^2}} \qquad \text{(C4)}$$

The left hand side of this relation is

$$\frac{\mathrm{Im}\, B^{+}(w,0)}{\sqrt{w^2-\mu^2}} \qquad \text{for} \quad w > \mu \qquad \text{(C5)}$$

$$-\frac{\mathrm{Re}\, B^{+}(w,0)}{\sqrt{\mu^2 - w^2}} \qquad \text{for} \quad 0 < w < \mu$$

There is an infinite discontinuity in this relation as the threshold, $w = \mu$, is approached from below, since $\mathrm{Im}B^{+}$ goes to zero at threshold while $\mathrm{Re}\, B^{+}$ is like a constant in the neighbourhood of $w = \mu$. Similar relations can be written for $A^{\pm}$ and B^{-}.

Having taken care of the singularities in the integrals, and the convergence of these relations, the integrands must be evaluated from the known phase shifts, as the optical theorem is inapplicable.

Clearly this method cannot be applied to the KN problem in its present form as the only facts known with any degree of certainty

are the total cross sections, although a recent phase shift analysis[66] has been applied to K^+p.

APPENDIX D

THE POLE TERM CONTRIBUTIONS

Until recently the pole terms have been evaluated with the factor M_u in equation (3.3) taken to be M_Y, where Y denotes either the lambda or sigma particle. However, a recent suggestion has been made* that this is erroneous, and in fact M_u should be replaced by the nucleon mass M_N. In order to make a meaningful comparison with previous calculations the former method has been adopted throughout this thesis. This means that the values of the Λ and Σ coupling constants should be multiplied by (M_N/M_Λ) and (M_N/M_Σ) respectively. The derivation of this result is sketched below.

Using the normalisation and notation of Gasiorowicz**, the Λ pole term gives a contribution

$$T^+T = g_\Lambda^2\ \bar{u}\left(\frac{M_\Lambda + i\gamma(p+q)}{2M_\Lambda}\right)u$$

$$= g_\Lambda^2\ \bar{u}\left(\frac{M_\Lambda M_N + \frac{1}{2}i\gamma.(q+q')}{2M_\Lambda}\right)u \qquad (D.1)$$

where

$$p + \bar{N}q \rightarrow p + \bar{N}q' \qquad (D.2)$$

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*A.D. Martin - private communication

**W. Gasiorowicz - Fort. d. Physik 8 665 (1960)

and from (1.3)

$$\left[\frac{d\sigma}{d\Omega}\right]_{CM} = \left| -\frac{T}{8\pi W} \right|^2 \qquad (d.3)$$

Now

$$i(T^+ - T) = -4\pi M_A \delta(s - m_A^2)\, T^+ T \qquad (D.4)$$

Hence

$$A^{Pole} = g^2 \frac{m_A - m_N}{s - m_A^2} \quad ; \quad B^{Pole} = \frac{-g^2}{s - m_A^2} \qquad (D.5)$$

Therefore from (2.62)

$$f_L^{Pole} = \frac{1}{4\pi}(A + \omega B)^{Pole} = \frac{X(A)\, g_A^2}{\omega_A - \omega} \qquad (D.6)$$

where

$$X(A) = \frac{(m_A - m_N)^2 - \mu^2}{4 m_N^2} \qquad (D.7)$$

The last relation shows that $M_A = M_N$.

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