Statistical analysis op earthquake data

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STATISTICAL ANALYSIS OF EARTHQUAKE DATA

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ABSTRACT

The Statistical properties of earthquake data from I4 different areas have been studied by considering the earthquake occurrence as a one-dimensional stationary point process.

A review of the main properties of the Point processes is given and some counting and interval properties of the mutually exciting processes are derived. As a result of an exploratory analysis the Poisson and renewal models are not found adequate to describe the earthquake occurrence and any kind of periodicity is not well established. The Neymann-Scott model with mixed exponential decay is found more suitable than the one with single exponential for describing the earthquake occurrence. The fit of the mutually exciting process is as satisfactory as the fit of the Neymann-Scott with mixed exponential from the spectral analysis viewpoint but not from the viewpoint of the interval analysis.

As a result of the interval analysis a four-variate mutually exciting process is proposed for describing the earthquake occurrence, which also takes into account the differences according to depth.

Finally an attempt to classify the areas under investigation is made and some ideas about the study of the earthquake phenomenon as a multidimensional point process are put forward.

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C O N T E N T S

CHAPTER 1

INTRODUCTION

1.1. The stochastic approach to the study of earthquakes

During the past few years there has been strong interest in the statistical nature of earthquake occurrences. The main reasons for this have been the introduction of new methods in detecting earthquakes and therefore the possibility of collecting more reliable earthquake records, the need for discriminating earthquakes from nuclear explosions and the economic interest in earthquake risk analysis. Vere-Jones (1970) and Schlien and Toksöz (1970) give in detail references of relevant work. The main general results of the above studies are, the inadequacy of the simple Poisson model to explain the time distribution of low-magnitude shocks and the failure for establishing any kind of periodicity in the earthquake activity.

The most important characteristic of the earthquake time series, which the simple Poisson model has not taken into account, is the tendency of earthquakes to occur in clusters or, the triggering characteristic of the earthquake occurrence. These clusters or aftershocks, usually start immediately after a main shock, which can be considered as an earthquake itself or as a kind of triggering mechanism. The duration of the aftershocks ranges from several days to a year and their frequency distribution appears to follow the inverse power or the exponential decay law and depends on the magnitude, depth and location of the main event; In other words the aftershock phenomenon seems to be the natural result of the fracturing characteristics of materials. (Mogi, 1962 and Solo'ev and Solov'eva 1962). A special kind of clustering, which has mainly observed in Japan, is the one where the cluster centres too have a tendency of clustering in a different manner from ordinary aftershock sequences. (Discussion in Vere-Jones,1970 paper).

In view of the above characteristics of earthquake activity it would be reasonable to describe the earthquake occurrence by using a clustering model or a self-exciting one in which the rate of occurrence in some particular instance is influenced by the activity in the past. Thus, Vere-Jones (1970) considering the sequence of occurrence of earthquakes as one-dimensional stationary point process examines the adequacy of the Neymann-Scott cluster model with data from the main earthquake area of New-Zealand. He considers the main process as stationary Poisson and the members of the subsidiary processes located in distances from the cluster centre, which are distributed according to the inverse law or the exponential one. According to his findings the Neymann-Scott model with the inverse power decay law is more suitable in describing the New-Zealand earthquake data. Shlien and Toksöz (1970) examine the adequacy of a cluster model, which is a generalization of the triggering one considered by Vere-Jones and Davies (1966), with data from different areas of the Earth. In this model the cluster centres follow a simple Poisson one and the frequency of subsidiary events around the cluster centre follows an inverse power law.

The work done up to now has been a great contribution to the study of earthquakes from the stochastic viewpoint. However much more work has to be done in improving the methods of statistical analysis of earthquake data and in developing more suitable stochastic models in describing earthquake occurrence and energy. Since undoubtedly the stochastic element is an essential part of the earthquake nature the application of statistical ideas in this field holds considerable interest for the geophysicist. A detailed statistical analysis might show up new aspects of the pattern of earthquake occurrence while the search for a suitable stochastic model might suggest important ideas about the mechanism of earthquakes. The joint study of activity and occurrence of earthquakes by using recent theoretical results on marked point processes, (Hawkes, 1971 and Bartlett, 1967 b) and the application of the prediction theory of point processes in the case of earthquakes, (Jowett and Vere-Jones, 1972), will open up new ways of studying the earthquake phenomenon and of determining more effective safety procedures to face its disastrous consequences.

In the following two other preliminary sections the main characteristics of earthquakes are described, a brief outline of the continental drift theory, which quite satisfactorily explains the earthquake phenomenon, is given, and the main objectives of the present thesis are defined.

1.2. Earthquakes and the continental drift theory

The usual earthquake lists give the main characteristics of a shock which are: The geographical coordinates of the epicentre (its latitude and longitude) focal depth (depth of the focus beneath the Earth's surface), origin time (the time of occurrence of the shock at its source) and instrumental magnitude which is an estimate of the released energy. The dimensions of the source are very small compared with the dimensions of the area under investigation. The duration of the shock is of the order of a few seconds while the period of the survey is usually of the order of several years. (In the present survey the period is 20 years). Therefore each earthquake can be considered as a point event and this view will be adopted throughout the present investigation.

The shocks are classified by Gutenberg and Richter (1954) as shallow when the depth does not exceed 70 Km, intermediate when the depth is from 70 Km - 300 Km and deep when it exceeds 300 Km. Symbols for classification by magnitude are:

Class:	a	b	c	d	e
Magnitude:	$7^3/4-8^1/2$	7.0-7.7	6.0-6.9	5.3-5.9	below 5.3

Classes a and b are recorded by all the stations and the relation of magnitude and released energy is,

$$\lg E = a + bM \qquad (1.2.1)$$

where a and b are constants, E is the energy and M is the instrumental magnitude.

One of the most succesful theories in explaining the earthquake phenomenon from the geophysical viewpoint is the continental drift theory, which was originally derived in 1912 by the German Scientist Wegener (1924). The main points of the above theory have as follows:

The outer shell of the Earth is divided into a small number of "plates" and major changes in the Earth's surface occur only at the boundaries between plates. The plates fit closely together but they can move around by internal forces which extend the plates in some regions and destroy them in others. The outlines of the plates are deduced from the zones at which the earthquakes most frequently occur. Two plates can move backwards or forwards relative to each other or they can slip alongside one another without either parting or approaching to any great extent. In the backwards motion, Fig. 1.1.(b), as no gap can exist between the plates, one plate can ease itself away from another but instantly hot material rises from below to fill the gap creating volcanoes accompanied by earthquakes. For example this is the situation in the Mid-Atlantic ridge.

If plates are moving towards each other, Fig. 1.1(c), since they can not overlap to any great extent, one of them dips and the material of that plate passes under the edge of the other re-entering the interior of the earth. The bend of the down moving one creates an ocean trench. The sinking plate again causes earthquakes and the friction which it generates as it drives down into the Earth causes volcanoes. In this kind of situation two special cases can be distinguished:

(i) One oceanic plate is driving under another resulting in an Island arc, like the Aleoutian arc, Tonga Salient, Sunda arc e.t.c.

(ii) The oceanic plate which is going down under a continent or Island arc carries another continent. In this case the two continents collide and create a mountain range. For example the Indian subcontinent was a part of Africa many millions of years ago. The Indian ocean plate carrying India moved towards what is now Asia and from the collision of the two continents the Himalayas were created. The Indian plate unable to push Indian further must now dive out of its way probably creating the present earthquakes in the area. The same phenomenon occured in the Mediterranean. Italy had been carried by the Mediterranean plate, which, being a part of the African one, collided with that of Europe, and thus the Alps have been created. Because of the continuing movement of the Africa plate towards Europe the Mediterranean sea is beeing squeezed out of existence, and that causes the earthquakes in the area. Similarly Southern Greece, Crete and the Aegean Islands are riding on a small plate which is travelling South-Eastwards towards Africa. The plate on its way overrides the Mediterranean floor which bends downwards creating a pronounced trench of deeper water off the coast of Crete. That is probably again an explanation for the volcanoes and the earthquakes in the area.

In the third kind of plate motion, Fig. 1.1(d), in which the two plates slip alongside one another, the plate edges are called transform faults. This type of fault occurs in the west coast of North America where the famus San Andreas fault of California, which quite often causes disastrous earthquakes, is found (Calder, 1972).

Summarising, each plate of the Earth's shell is marked out by a ring of earthquakes around ridges, faults and trenches. Figure 1.2 shows a map of the main earthquake activity of the present century. It is easy to notice that most earthquakes originate within two well defined zones. Gutenberg and Richter (1954) describe them as the circum - Pacific zone and the Alpine one. Most of the remaining shocks occur in the rift valley regions of Eastern and Central Africa and along the Mid-Atlantic rift. The circum-Pacific zone includes the Island arcs from Alaska to New-Zealand, together with the Sunda arc which is clearly of the Pacific type. On the American side the Caribbean loop invades the Atlantic while a similar loop in the extreme South links South America to Graham land in Antarctica. The circum-Pacific zone with its extensions into the neighbouring oceans in credited with about 80 percent of the world's most important shallow shocks, 90 per cent of the intermediate and nearly all the deep shocks. The Alpine zone extends from the Azores and the Alpine mountain arcs of Europe and North Africa, through Asia Minor and the Caucasus, Persia and Baluchistan to the Pamirs, Himalayas, Tibet and China. In this zone, occurs most of the remaining larger shocks of shallow origin and nearly all the remaining intermediate shocks.

In the present investigation the list of earthquakes edited by the Institute of Geological Sciences in Edinburgh is used, which covers the period from 1800 to 1971. The series of intervals of successive shallow earthquakes in the period 1950-1971, which have occurred in the following areas, are analysed.

1. North Atlantic, Latitude (+30,+60), Longitude (-43,-27)
2. South Atlantic, Latitude (-60,-25), Longitude (-15,+15)
3. Central Atlantic, Lat. (-2,+10), Long. (-40,+19)
4. Aleoutian Islands, Latitude (50,56), Longitude (-180,-172)
5. Fox Islands, Latitude (50,54), Longitude (-172,-165)
6. Sunda-arc,Latitude (-10,0), Longitude (101,112)

7. Bandasea, Latitude (-8,-5), Longitude (122,132)
8. Fiji Islands, Latitude (-27,22), Longitude (-180,-176)
9. Tonga Islands, Latitude (-22,-16), Longitude (-180,-176)
10. Himalayas, Latitude (26,40), Longitude (70,90)
11. Greece, Latitude (34,41), Longitude (21,31)
12. Spain, Latitude (32,40), Longitude (-7,-1)
13. Yugoslavia, Latitude (39,46), Longitude (15,22)
14. Italy, Latitude (39,44), Longitude (9,17).

The above areas have been chosen, after consulting the department of Geophysics of the University of Durham, with careful consideration of the homogeneity of instrumental coverage and the best representation of the different types of earthquakes according to the continental drift theory.

1.3. Plan of the Thesis

The objective of this study is the fit of the most suitable stochastic model for the occurrence of earthquakes and, if it is possible, by looking at the main characteristics of the model, the investigation of similarities and differences among the world's most active earthquake areas predicted by the continental drift theory. This objective is approached by considering the earthquake occurrence as a realization of a stationary one-dimensional point process and analysing the series of intervals between successive shocks in the specially chosen areas given in § 1.2.

An outline of the main theoretical results of the point processes and their estimation procedures are given in chapters 2 and 3. Some counting and interval properties of the mutually exciting processes are derived in § 2.8.

In chapter 4, by using standard tests based on the statistical properties of the second order moments the Poisson and renewal hypotheses are examined. Further the fit of Neymann-Scott model with single and mixed exponential decay, and the fit of some special cases of the mutually exciting processes are attempted. A classification of the areas under investigation is given in § 4.6.

In chapter 5 some other approaches to the statistical study of the earthquake problem are suggested and in view of the results of the interval analysis some extensions to the two variate mutually exciting processes are proposed.

Finally in chapter 6 the general conclusions of the overall analysis are stated.

The motivation of this study arose from the work of Vere-Jones (1970) dealing with the statistical analysis of earthquakes occurring in the main active earthquake area of New-Zealand and the interest of the Department of Geophysics of the University of Durham, in the origin of earthquakes in East Africa and the Mediterranean.

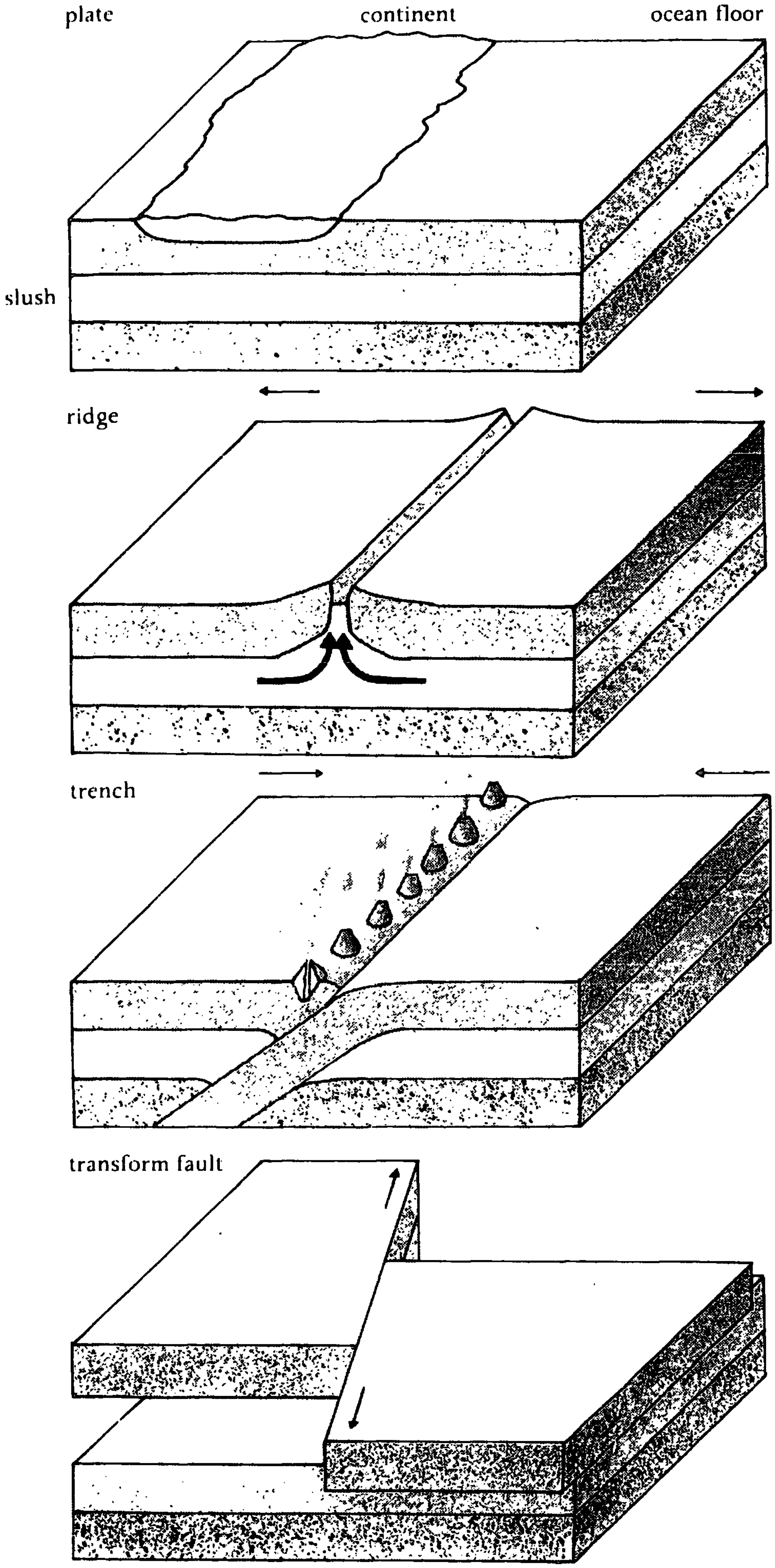


Fig. I.I.- A plate of the Earth's shell, a, carrying a continent, and the three kinds of boundaries between plates b, c and d. (Calder, 1972, p.48)

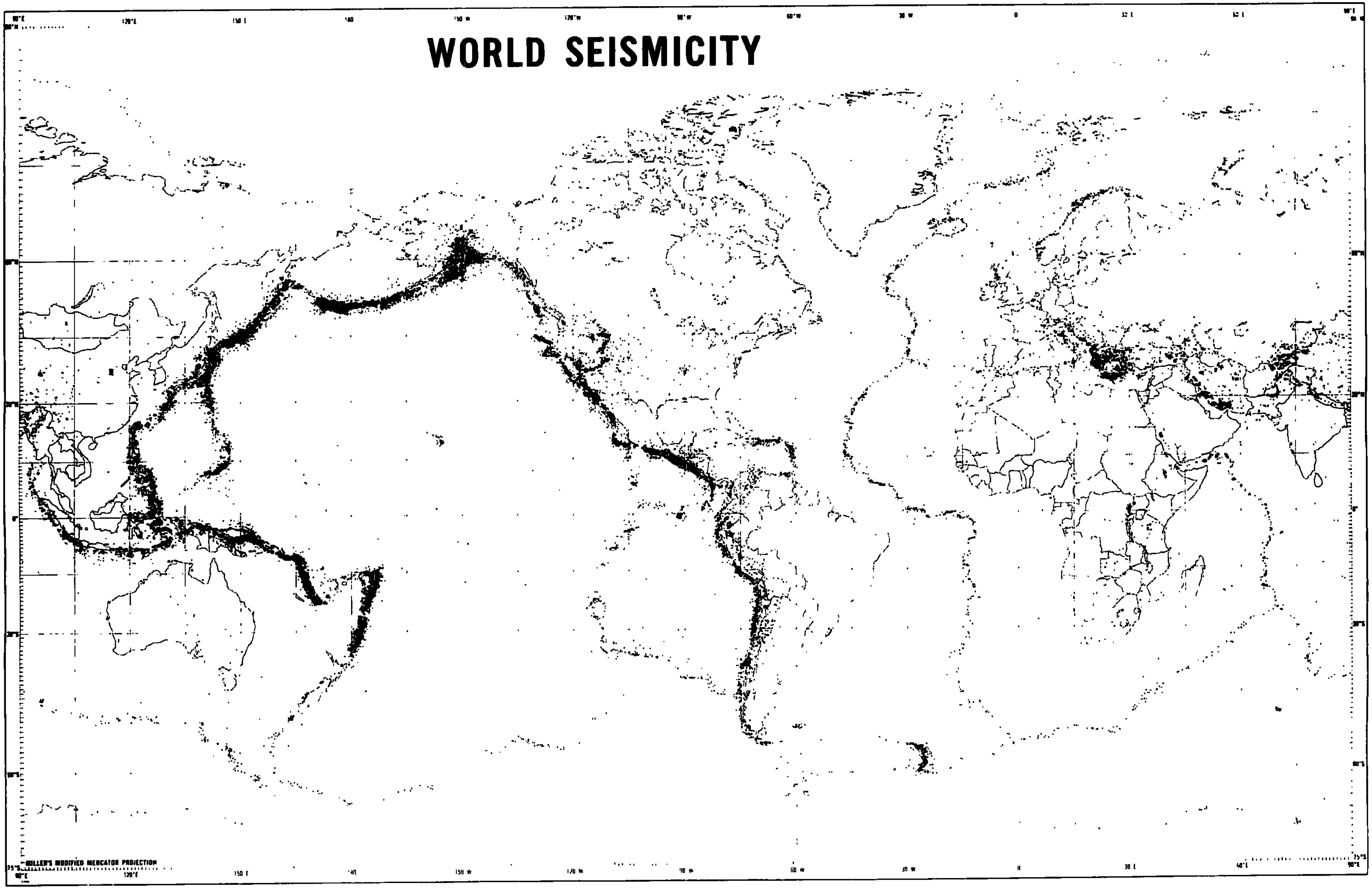


Fig. I.2.- Map of the main earthquake activity of the present century.

CHAPTER 2

STOCHASTIC POINT PROCESSES

2.1. Definition and general properties

A stochastic point process is the mathematical abstraction which arises from considering such phenomena as a randomly located population or a random sequence of events. Since a realization of any of these phenomena is just a set of points in time or space, a family of such realization has come to be called a point processes.

Formally, there is a state space, T, and a set of points $\{t_n\}$ from T which represents the locations of the different members of the population or the times at which the events occur. For example if T is the time axis, $\{t_n\}$ might be the times of occurrence of earthquakes in some area (One-dimensional point process) or if T is the surface of a sphere, $\{t_n\}$ might be the position of a certain kind of stars expressed by their spatial coordinates (Multi-dimensional point process). In the case of multi-dimensional point process the dimensions can either correspond to the spatial coordinates or one to the time and the others to the spatial coordinates. One such example is the occurrence of cosmic ray showers which can be considered as a multi-dimensional point process in time and space.

Another kind of point process is the one called by Bartlett (1967) a point process with ancillary variable and also by Hawkes (1971) Marked process. In this process with each point t_i in the basic point process there is associated an auxiliary variable y_i lying in a mark space Y. In general these $\{y_i\}$ need not to be independent either amongst themselves or of the $\{t_i\}$. As examples, y_i may represent the speed of vehicles passing a certain point at time t_i, or the energy of earthquakes occurring at time t_i in some well defined area, or more generally the history of the point process up to time t_i. Thus the Marked process is a point process defined on the product space $T \times Y$.

In the following the one-dimensional point processes are considered and for convenience as state space the time axis is taken. The properties of these point processes can be classified in two categories. In the first one belong the properties which are related with the numbers of points falling within specified time intervals and which are expressed in terms of the counting process, $N(t)$ = Number of events in the interval $(0,t]$, where $t>0$ or the differential point process,

$dN(t)$ = Number of events in $(t, t+dt)$.

In the second one belong the properties relating to the intervals between events expressed in terms of the process of intervals,

$\{X_n\} = \{t_n - t_{n-1}\}$, $n \in Z_+$, where Z_+ is the set of positive integers. The relation between the two sequences, $\{X_n,\ n \in Z_+\}$ and $\{N(t),\ t>0\}$ is expressed by the formula,

$$\text{Prob}(N(t)<r) = \text{Prob}\left(\sum_{k=1}^{r} X_k > t\right), \quad r = 1,2,3,\ldots \qquad (2.1.1)$$

The meaning of (2.1.1) is that the two processes are equivalent only through their complete distributions. However if a statistical analysis of a point process is based only on first and second order moments, then the first and second order properties of both the counting and interval processes are informative, i.e they are not equivalent.

A point process is stationary if the probabilistic structure of the process remains unchanged by a translation of the time axis or more formally.

Definition 2.1. A point process defined in the state space T is stationary if the joint distributions,

$$\text{Prob}\,\{N(A_i + y) = k_i,\ \ k_i \in Z_+,\ \ i = 1,\ldots,r\}$$

are independent of $y \in T$ where ~~R the set of real numbers.~~ A_i are Borel sets in T.

Immediate consenquence of the above definition is that the distribution of the number of events in an interval depends only on the length of the interval.

Definition 2.2. A point process has stationary intervals if the joint distributions,

$$P\left\{X_{i+n} \leqslant y_i, \quad y_i \in R, \quad i = 0, \pm 1, \ldots, \pm r\right\}$$

are independent of $n \in Z$. For example a renewal process on T has stationary intervals. One common question is to ask whether a stationary point process has also stationary intervals. Daley and Vere-Jones (1971) give an answer to this question which is "almost no". They prove that the only stationary point process in T which also has stationary intervals is the deterministic one which is a delayed renewal process where the interval between successive events are equal. More generally they show that any stationary point process which starts from an arbitrary event has also stationary intervals. Finally the definition of orderliness of point processes is given.

Definition 2.3. A point process is called orderly if,

$$\Pr(dN(t) \geqslant 2) = o(dt)$$

In what follows the first and second moments of the counting and interval processes are given and some specific examples of point processes are examined.

2.2. Moments of the counting and interval processes

Providing that the point process is orderly the first two factorial measures, $M_k(\cdot)$, k = 1,2 and the first two cumulant measures $C_k(\cdot)$, k = 1,2 are defined as follows:

(i) $M_1(dt) = E(dN(t))$

$= \text{Prob(one event in } (t, t+dt))$

$= m_1(t)dt$ (2.2.1)

The quantity $m_1(t)$ is called the intensity function or the mean rate of occurrence.

(ii) $M_2(dt_1 X dt_2) = E(dN(t_1)dN(t_2))$

$= \text{Prob}(\text{one event in } (t_1, t_1+dt_1) \text{ and one in } (t_2, t_2+dt_2))$

$= m_2(t_1, t_2)dt_1dt_2$ (2.2.2)

(iii) $C_1(dt) = M_1(dt)$ (2.2.3)

(iv) $C_2(dt_1 X dt_2) = M_2(dt_1 X dt_2) - M_1(dt_1)M_1(dt_2)$

$= (m_2(t_1, t_2) - m_1(t_1)m_1(t_2))dt_1dt_2$

$= \mu(t_1, t_2)dt_1dt_2$ (2.2.4)

The quantity $\mu(t_1,t_2)$ is called the covariance density of the process. In the case of stationary point processes if $t_1 = t$ and $t_2 = t+\tau$, then $\mu(t_1, t_2) = \mu(\tau)$, $m_1(t) = m_1(t+\tau) = \lambda$ and therefore the covariance density becomes,

$$\mu(\tau) = \frac{E(dN(t)dN(t+\tau))}{(dt)^2} - \lambda^2 \qquad (2.2.5)$$

It is clear from (2.2.5) that for real processes $\mu(-\tau) = \mu(\tau)$. In order to extend the covariance density at $\tau = 0$ and in the same time be consistent with the fact that the process is orderly, $\lambda\delta(\tau)$ is added to the $\mu(\tau)$, where $\delta(\tau)$ is the Dirac delta function, and thus the complete covariance density is obtained, which is,

$$\mu^{(c)}(\tau) = \lambda\delta(\tau) + \mu(\tau)$$

For the mean and the variance of the number of events in any interval $(0,t)$, providing again that the process is stationary and orderly, the following results are obtained:

(i) $E(N(0,t)) = \int_0^t E(dN(t)) = \lambda t$ (2.2.6)

(ii) $\text{Var}(N(0,t)) = V(t) = \lambda t + 2\int_0^t (t-u)\mu(u)du$ (2.2.7)

The quantity, $R = \lim_{t\to\infty} \frac{V(t)}{\lambda t} = 1 + \frac{2}{\lambda}\int_0^\infty \mu(u)du$ (2.2.8)

is some measure of the deviation of the process from the Poisson model. If $R>1$ the process may be said to be over-dispersed relative to the Poisson and exhibits some degree of clustering; If $R<1$, the process may be said to be underdispersed relative to the Poisson, and shows some tendency towards regular occurrence of events.

Bartlett (1963 b) defines the complete spectral density of the stationary point process as

$$g(\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{-i\omega\tau}\mu^{(c)}(\tau)d\tau$$

$$= \frac{1}{2\pi}\left\{\lambda + \int_{-\infty}^{+\infty} e^{-i\omega\tau}\mu(\tau)d\tau\right\}$$

$$= \frac{1}{2\pi}\left\{\lambda + \mu^*(i\omega) + \mu^*(-i\omega)\right\} \qquad (2.2.9)$$

where $\mu^*(s) = \int_0^{\infty} e^{-\tau s}\,\mu(\tau)d\tau$ is the Laplace transform of the covariance density $\mu(\tau)$. Since $g(-\omega) = g(\omega)$ we need consider only positive ω and therefore the spectral density, $g_+(\omega)$, for positive ω is defined as:

$$g_+(\omega) = 2g(\omega) = \frac{1}{\pi}\left\{\lambda + \mu^*(i\omega) + \mu^*(-i\omega)\right\}, \qquad \omega > 0$$

The function $\mu(\tau)$ is continuous in the time domain which makes the spectral density, $g_+(\omega)$, not a periodic function. Therefore it has to be decided over what range of ω to estimate $g_+(\omega)$.

Providing that the function $g_+(\omega)$ has no jump at $\omega = 0$, by (2.2.8) and (2.2.9) the following immediate results are obtained:

$$\pi g_+(0)/\lambda = R \quad \text{or} \quad R = \frac{g_+(0)}{g_+(\infty)} \qquad (2.2.10)$$

where $g_+(\infty) = \lim_{u\to\infty} g_+(\omega)$ and $g_+(0) = \lim_{u\to 0_+} g_+(\omega)$

The correlation properties of the interval process $\{X_t\}$, that is the second order joint moments, are given by standard functions in the theory of time series. The first of them is the autocovariance function,

$$\gamma(k) = \mathrm{Cov}(X_i, X_{i+k}), \quad k = 0, \pm 1, \pm 2, \ldots \qquad (2.2.11)$$

or the autocorrelation function,

$$\rho(k) = \frac{\gamma(k)}{\gamma(0)} = \frac{Cov(X_i,\ X_{i+k})}{Var(X)}\ ,\quad k = 0, \pm 1, \pm 2, \ldots. \qquad (2.2.12)$$

The second one is the power spectrum, $f(\omega)$, $-\pi \leqslant \omega < \pi$, which is the Fourier transform of the autocovariance function,

$$\text{i.e} \qquad f(\omega) = \frac{1}{2\pi} \sum_{-\infty}^{+\infty} \gamma(s) e^{-i\omega s}\ , \quad -\pi \leqslant \omega \leqslant \pi \ .$$

The Fourier transform of the autocorrelation function $\rho(s)$ is called the spectrum density and it is equal to $\frac{f(\omega)}{\sigma^2}$. In the case of real processes, for which $\gamma(-s) = \gamma(s)$ and $\rho(-s) = \rho(s)$, the power spectrum and the spectrum density become,

$$f(\omega) = \frac{1}{2\pi} \sum_{-\infty}^{+\infty} \gamma(s)\cos(s\omega) \quad \text{and} \quad \frac{f(\omega)}{\sigma^2} = \frac{1}{2\pi} \sum_{-\infty}^{+\infty} \rho(s)\cos(s\omega) \qquad (2.2.13)$$

and by inverting them,

$$\gamma(s) = \int_{-\pi}^{\pi} f(\omega)\cos(s\omega)d\omega \quad \text{and} \quad \rho(s) = \frac{1}{\sigma^2} \int_{-\pi}^{\pi} f(\omega)\cos(s\omega)d\omega \qquad (2.2.14)$$

It is well known that the sequence of autocovariances $\gamma(\tau)$ for a starionary process can be always represented in the form,

$$\gamma(\tau) = \int_{-\pi}^{\pi} e^{i\tau\omega} dF(\omega),$$

where $\frac{F(\omega)}{\sigma^2}$ is a distribution function, i.e it is monotonically increasing (or, at least non-decreasing) and bounded with $F(-\pi) = 0$, and $F(\pi) = \gamma(0) = \sigma^2$. If $F(\omega)$ is differentiable then $dF(\omega) = f(\omega)d\omega$.

The first of (2.2.14) gives, $\sigma^2 = \gamma(0) = \int_{-\pi}^{\pi} f(\omega)d\omega$ and therefore the power spectrum shows how the variance of the process is distributed over the angular frequency. Specifically the variance of the process which is due to the angular frequency in the range $(\omega, \omega + d\omega)$ is approximately $f(\omega)d\omega$. Thus the introduction of the power spectrum accomplishes a mapping of the properties of the stochastic process from the time domain to the corresponding properties in the frequency domain. For example: If the stochastic process is considered as an electrical signal then it can be said that the spectral distribution function, $\frac{F(\omega)}{\sigma^2}$ gives the "proportion" of energy in the stochastic process at or below the frequency ω. Hence a peak in the spectral density function suggests a possible periodicity in the stochastic process being studied. Moreover since the spectrum characterizes the autocovariance function of the process it also characterizes the method by which the process has been generated. For example if the process is periodic, the spectrum has jumps at the periods, if it is autoregressive the spectrum is of rational type and if the process is white noise the spectrum is constant. Hence it seems rational that the use of the spectrum should play a natural role in statistical inference for stochastic process.

Finally another function which is useful in the studying of the sequence of times between events is the hazard function, which is defined as,

$$Z_X(x) = \frac{f_X(x)}{1-F_X(x)} = \frac{f_X(x)}{R_X(x)} \qquad (2.2.15)$$

where $f_X(x)$, $F_X(x)$ and $R_X(x)$ are respectively the density function, the distribution function and the survival function of the time between successive events.

By (2.2.15), $R_X(x) = \exp\left[-\int_0^x Z_X(u)du\right]$ or $\lg R_X(x) = -\int_0^x Z_X(u)du$ with first and second order derivatives:

$$\frac{d\log R_X(x)}{dx} = -Z_X(x) \qquad \text{and} \qquad \frac{d^2\log R_X(x)}{dx^2} = -\frac{dZ_X(x)}{dx}$$

Therefore a monotone non-increasing hazard is equivalent to the logarithm of the survival function being concave and a monotone non-decreasing hazard is equivalent to the logarithm of the survival being convex.

If now $P(z,\tau)$ is the probability generating function of the $N(t, t+\tau)$, which is the number of events in the interval $(t, t+\tau)$, then $P(0,\tau)$ is the survival function for the forward recurrence time τ and consequently,

$$P(0,\tau) = P(0, \tau+\Delta\tau)+\lambda\Delta\tau(1-F_X(\tau)) \quad \text{or}$$

$$1-F_X(x) = -\frac{1}{\lambda}\frac{d\,P(0,\tau)}{d\tau} \tag{2.2.16}$$

where λ is the intensity function of the process and $F_X(x)$ the distribution function of the time between successive events. By using (2.2.16), $Z_X(\tau)$, the hazard of the time between successive events becomes:

$$Z_X(\tau) = \frac{f_X(\tau)}{1-F_X(\tau)}$$

$$= \frac{1/\lambda \dfrac{d^2P(0,\tau)}{d\tau^2}}{-1/\lambda \dfrac{dP(0,\tau)}{d\tau}}$$

$$= \frac{\left[h_R^2(\tau)-h_R'(\tau)\right]P(0,\tau)}{h_R(\tau)\,P(0,\tau)}$$

$$= h_R(\tau) - \frac{h_R'(\tau)}{h_R(\tau)} \tag{2.2.17}$$

where $h_R(\tau)$ is the hazard of the forward recurrence time.

2.3. Poisson processes

The natural starting point for a discussion of models of point processes is the stationary Poisson process. Assuming that $N(t, t+h)$ is the number of events in the interval $(t, t+h)$ and $h \longrightarrow 0$, the conditions for a stationary Poisson process are:

(i) $\text{Prob}(N(t, t+h) = 0) = 1-\lambda h+o(h)$

(ii) $\text{Prob}(N(t, t+h) = 1) = \lambda h+o(h)$

(iii) The random variables $N(t, t+h)$ for different h, are statistically independent of the number and position of the events in $(0,t)$.

The immediate consequences of the above statements are the following:

(i) The chance of two or more events occuring simultaneously is negligible.

(ii) The distribution of the number of events occurring in the interval $(0,t)$ is Poisson with parameter λt.

Hence:

$E(N(0,t)) = Var(N(0,t)) = \lambda t$ and

$$g(\omega) = \lambda/2\pi \qquad (2.3.1)$$

(iii) The time between successive events or the time starting from an arbitrary point to the next event is exponentially distributed with parameter λ. Thus according to (2.2.13) the power spectrum is: $f(\omega) = \frac{\sigma^2}{2\pi}$ where σ^2 is the variance of the intervals.

A generalization of the above process is the non-stationary Poisson process, whose mean rate of occurrence is a function of the time. In that case the distribution of the events in the interval (0,t) is again Poisson but with parameter $\int_0^t \lambda(t)dt$.

Many processes approximate quite well to the Poisson Process in spite of its restrictive definition. One reason for this, is that a process built up of a number of small independent components will resemble a Poisson Process more and more closely as the number of components increases and the contribution from each decreases (Vere-Jones and Davies, 1966).

2.4. Renewal Processes

Renewal process is a series of events in which the times between successive events are idependently and identically distributed. If $F_X(x) = \int_{-\infty}^{x} f_X(x)dx$ the life time distribution, $F_n(t) = Pr(X_1+X_2 \ldots X_n<t)$, $dF_n(t) = f_n(t)dt$ and $K^*(s) = \int_0^{\infty} K(\tau)e^{-\tau s}\, d\tau$ then the following results are obtained:

(i) $$M_f(t) = E(N(0,t)) = \sum_{n=1}^{\infty} F_n(t) \qquad (2.4.1)$$

(ii) $$m_f(t) = \frac{d\, M_f(t)}{dt} = \sum_{n=1}^{\infty} f_n(t), \quad m_f^*(s) = \frac{f_X^*(s)}{1-f_X^*(s)} \qquad (2.4.2)$$

The quantity $m_f(t)$ is called renewal density and has the following probabilistic meaning:

$m_f(t)dt$ = Prob(renewal in (t, t+dt) / renewal in the origin).

(iii) If $m = \frac{1}{E(X)}$ then the Laplace transform of the variance of the number of events in the interval (0,t) is:

$$V^*(s) = \frac{m}{s^2} \cdot \frac{2f_X^*(s)}{s^2(1-f_X^*(s))} - \frac{2m^2}{s^3} \qquad (2.4.3)$$

(iv) The spectrum of the counts is

$$g_+(\omega) = \frac{m}{\pi}\left[1+\frac{f_X^*(i\omega)}{1-f_X^*(i\omega)}+\frac{f_X^*(-i\omega)}{1-f_X^*(-i\omega)}\right] \qquad (2.4.4)$$

and the power spectrum:

$$f(\omega) = \frac{\sigma^2}{2\pi} \qquad (2.4.5)$$

where σ^2 is the variance of the distribution of the intervals (Cox and Lewis, 1966, p. 79).

2.5. Doubly Stochastic Poisson Processes

A Doubly Stochastic Poisson process is a non-stationary Poisson Process in which the rate of occurrence of events is itself a realization of a stationary, continuous time stochastic process $\{\Lambda(t)\}$. If $\bar{\lambda}$ and $\gamma_\Lambda(\tau)$ are respectively the mean and the autocovariance function of the stochastic process $\{\Lambda(t)\}$ then:

(i) $E\{N(0,t)/\lambda(u),\ 0\leqslant u\leqslant t\} = \int_0^t E(dN(t)) = \int_0^t \lambda(u)du,$

$$\begin{aligned} E\{N(0,t)\} &= E_\Lambda E\{N(0,t)/\lambda(u),\quad 0\leqslant u\leqslant t\} \\ &= \int_\Lambda \left(\int_0^t \lambda(u)du\right) f_\Lambda(\lambda)d\lambda \\ &= \int_0^t \bar{\lambda}du = \bar{\lambda}t \end{aligned} \qquad (2.5.1)$$

(ii)
$$\begin{aligned} E(dN(t)dN(t+\tau)) &= E_\Lambda E(dN(t)dN(t+\tau)) \\ &= E_\Lambda\left[Pr(dN(t) = dN(t+\tau) = 1)\right] \\ &= E_\Lambda(Pr(dN(t+\tau) = 1/dN(t) = 1)\ Pr(dN(t) = 1)) \\ &= E_\Lambda(\lambda(t+\tau)\lambda(t))(dt)^2 \\ &= (\gamma_\Lambda(\tau)+\bar{\lambda}^2)(dt)^2 \end{aligned}$$

consequently, $\gamma_\Lambda(\tau) = \frac{E(dN(t)dN(t+\tau))}{dt^2} - \bar{\lambda}^2$

and by (2.2.5),

$$\gamma_{\Lambda}(\tau) = \mu(\tau) \tag{2.5.2}$$

where $\mu(\tau)$ is the covariance density of the process.

(iii) By (2.2.7), (2.5.1) and (2.5.2),

$$V(t) = \int_0^t \bar{\lambda}\,dt + 2\int_0^t (t-u)\gamma_{\lambda}(u)du = \bar{\lambda}t + 2\int_0^t (t-u)\gamma_{\Lambda}(u)du \tag{2.5.3}$$

and $\mathcal{R} = \lim_{t\to\infty} \frac{V(t)}{\bar{\lambda}t} = 1 + \frac{2}{\bar{\lambda}}\int_0^{\infty}\gamma(u)du$ (Cox and Lewis, 1966, p. 180-181).

2.6. The mutually and self-exciting point processes

The mutually exciting point processes, which have been introduced by Hawkes (1971 a,b) are k-variate one-dimensional point processes with counting process $\underset{\sim}{N}^T(t) = (N_r(t),\ r = 1,2,\dots k)$ such that:

$Pr(\Delta N_r(t) = 1/\underset{\sim}{N}(s),\ s\leqslant t) = \Lambda_r(t)\Delta t + o(\Delta t)$

$Pr(\Delta N_r(t) > 1/\underset{\sim}{N}(s),\ s\leqslant t) = o(\Delta t)$, independently for each r, where

$\Lambda_r(t) = \nu_r + \sum_{s=1}^{k}\int_{-\infty}^{t} \beta_{rs}(t-u)dN_s(u)$ or in matrix notation,

$$\underset{\sim}{\Lambda}(t) = \underset{\sim}{\nu} + \int_{-\infty}^{t} \underset{\sim}{\beta}(t-u)d\underset{\sim}{N}(u) \tag{2.6.1}$$

It is assumed that $\beta_{ij}(u)$, $i,j = 1,2,\dots k$ are positive for $u\geqslant 0$ and zero otherwise.

In the case of a stationary process if $\underset{\sim}{I}$ is the ~~diagonal~~ unit matrix,

$\underset{\sim}{D} = \begin{pmatrix} \lambda_1 & 0 & \dots & 0 \\ 0 & \lambda_2 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \lambda_k \end{pmatrix}$ and $\underset{\sim}{B}(\omega) = \int_0^{\infty} \underset{\sim}{\beta}(u)e^{-iu\omega}du$ the following general results have been derived by Hawkes (1971 b).

(i) The stationary mean rates of occurrence are,

$$\underset{\sim}{\lambda} = (\underset{\sim}{I} - \underset{\sim}{B}(0))^{-1}\underset{\sim}{\nu} \tag{2.6.2}$$

(ii) Necessary condition for the existence of the precess is that the elements of the matrix $(\underset{\sim}{I}-B(\underset{\sim}{0}))^{-1}$ must be positive.

(iii) The Fourier transform of the covariance density $\underset{\sim}{\mu}(\tau)$ is

$$\underset{\sim}{M}(\omega) = \left[\underset{\sim}{I}-\underset{\sim}{B}(\omega)\right]^{-1}\left[\underset{\sim}{B}(\omega)\underset{\sim}{D}+\underset{\sim}{D}\underset{\sim}{B}^T(-\omega)-\underset{\sim}{B}(\omega)\underset{\sim}{D}\underset{\sim}{B}^T(-\omega)\right]\left[\underset{\sim}{I}-\underset{\sim}{B}^T(-\omega)\right]^{-1} \tag{2.6.3}$$

(iv) The spectral density matrix is,

$$\underset{\sim}{G}(\omega) = \frac{1}{2\pi}\left[\underset{\sim}{I}-\underset{\sim}{B}(\omega)\right]^{-1}\underset{\sim}{D}\left[\underset{\sim}{I}-B^{T}(-\omega)\right]^{-1} \qquad (2.6.4)$$

In the univariate case, the so-called self-exciting process, (2.6.4) and (2.6.3) can be written as,

$$M(\omega) = \frac{\lambda\left[B(\omega)+B(-\omega)-B(\omega)B(-\omega)\right]}{\left[1-B(\omega)\right]\left[1-B(-\omega)\right]} \qquad (2.6.5)$$

$$g(\omega) = \frac{\lambda}{2\pi(1-B(\omega))(1-B(-\omega))} \qquad \text{or for real } \omega$$

$$g(\omega) = \frac{\lambda}{2\pi\left|1-B(\omega)\right|^{2}} \qquad (2.6.6)$$

The self-exciting point process can be considered as a self-exciting shot process in which the current intensity of events is determined by events occurring in the past. We apply the above general results in the following special cases:

a) Self-exciting w ith,

$$\beta(u) = \begin{cases} \alpha\rho\, e^{-\rho u} & \text{if } u>0 \\ 0 & \text{otherwise} \end{cases} \qquad \text{with } \rho>0 \text{ and } 0<\alpha\leqslant 1$$

By virtue of (2.2.7), (2.2.9), (2.6.5) and (2.6.6) the following results for the covariance density, variance time curve and the spectrum of counts are obtained respectively:

$$\mu(\tau) = \frac{\alpha\rho\lambda(2-\alpha)}{2(1-\alpha)} \qquad (2.6.7)$$

$$V(\tau) = \lambda\left(\frac{1}{1-\alpha}\right)^{2}\tau - \frac{\alpha\lambda(2-\alpha)}{\rho(1-\alpha)^{3}}\left(1-e^{-\rho(1-\alpha)\tau}\right) \qquad (2.6.8)$$

$$g_{+}(\omega) = \frac{\lambda}{\pi}\left[1+\frac{\alpha\rho^{2}(2-\alpha)}{\rho^{2}(1-\alpha)^{2}+\omega^{2}}\right] \qquad (2.6.9)$$

Since from (2.6.8), $R = \lim_{\tau\to\infty}\frac{V(\tau)}{\lambda\tau} = \left(\frac{1}{1-\alpha}\right)^{2} > 1$, the process is overdispersed.

b) Self-exciting with,

$$\beta(u) = \begin{cases} \alpha_1 \rho_1 e^{-\rho_1 u} + \alpha_2 \rho_2 e^{-\rho_2 u} & \text{if } u > 0 \\ 0 & \text{otherwise} \end{cases}$$

where $\rho_1, \rho_2 > 0$ and $0 < \alpha_1 + \alpha_2 < 1$

By virtue of (2.6.6) the spectrum of counts is,

$$g_+(\omega) = \frac{\lambda}{\pi} \frac{(\rho_1^2+\omega^2)\ (\rho_2^2+\omega^2)}{\left[\rho_1\rho_2(1-\alpha_1-\alpha_2)-\omega^2\right]^2+\omega^2\left[\rho_2(1-\alpha_2)+\rho_1(1-\alpha_1)\right]^2} \qquad (2.6.10)$$

c) Two-variate mutually exciting with

$$\underset{\sim}{\beta}(u) = \begin{pmatrix} 0 & \beta_{12}(u) \\ 0 & \beta_{22}(u) \end{pmatrix} \quad \text{and} \quad \underset{\sim}{\nu} = \begin{pmatrix} 0 \\ \nu \end{pmatrix}.$$

If more specially $\beta_{12}(u) = \alpha_{12}\rho_1 e^{-\rho_1 u}$ and $\beta_{22}(u) = \alpha_{22}\rho_2 e^{-\rho_2 u}$ then

$$\underset{\sim}{B}(\omega) = \begin{pmatrix} 0 & B_{12}(\omega) \\ 0 & B_{22}(\omega) \end{pmatrix} \quad \text{where}$$

$$B_{i2}(\omega) = \frac{\alpha_{i2}\rho_i}{\rho_i + i\omega}, \quad i = 1,2 \qquad (2.6.11)$$

By virtue of (2.6.2) and (2.6.11) the rate of occurrence, λ, is

$$\underset{\sim}{\lambda} = \frac{1}{1-\alpha_{22}} \begin{pmatrix} \alpha_{12}\nu \\ \nu \end{pmatrix}$$

Thus $\lambda_1 = \dfrac{\alpha_{12}\,\nu}{1-\alpha_{22}}$, $\lambda_2 = \dfrac{\nu}{1-\alpha_{22}}$ and $\lambda = \lambda_1+\lambda_2 = \dfrac{\nu\,(1+\alpha_{12})}{1-\alpha_{22}}$ (2.6.12)

By using (2.6.4) the spectral matrix is obtained, which is,

$$\underset{\sim}{G}(\omega) = \left[(\underset{\sim}{I}-\underset{\sim}{B}^T(-\omega))\underset{\sim}{D}^{-1}(\underset{\sim}{I}-\underset{\sim}{B}(\omega))\right]^{-1}$$

$$= \frac{1}{|1-B_{22}(\omega)|^2} \begin{pmatrix} \lambda_2 B_{12}(\omega)B_{12}(-\omega)+\lambda_1\,|1-B_{22}(\omega)|^2 & \lambda_2 B_{12}(\omega) \\ \lambda_2 B_{12}(-\omega) & \lambda_2 \end{pmatrix}$$

Therefore by virtue of (2.6.11) the spectral density of the overall process, which is the sum of the elements of the spectral matrix, is,

$$g(\omega) = G_{11}(\omega)+G_{12}(\omega)+G_{21}(\omega)+G_{22}(\omega)$$

$$= \lambda_1+\lambda_2 \frac{(\rho_2^2+\omega^2)\left[\rho_1^2(1+\alpha_{12})^2+\omega^2\right]}{(\rho_1^2+\omega^2)\left[\rho_2^2(1-\alpha_{22})^2+\omega^2\right]} \qquad (2.6.13)$$

2.7. Cluster processes

Here the process is defined in terms of two components:

(i) A process of cluster centres,

(ii) A subsidiary process which is developed in each cluster in such a way that the occurrence of events in one cluster does not effect the probability of occurrence of events from another cluster.

Now if $M_k(\cdot/t)$ is the k^{th} factorial moment measure of the process of the cluster members given that the cluster centre is at t, $\tilde{C}_k(\cdot)$ the k^{th} factorial cumulant measure for the process of the cluster centres, and

$f_1(x)dx \doteq \text{Prob(one cluster in } (x, x+dx)),$

$f_2(x_1, x_2)dx_1 dx_2 = \text{Prob(one cluster in } (x_1, x_1+dx_1) \text{ and one in } (x_2, x_2+dx_2)$

then the two first factorial cumulant measures of the overall process are:

$$C_1(dt) = E(dN(t))$$

$$= \int_x M_1(dt/x)f_1(x)dx \qquad (2.7.1)$$

$$= \int_x M_1(dt/x)\tilde{C}_1(dx)$$

$$C_2(dt_1 x dt_2) = E(dN(t_1)dN(t_2)) - E(dN(t_1))E(dN(t_2))$$

$$= \int_x M_2(dt_1 x dt_2/x)f_1(x)dx + \int_{x_1}\int_{x_2} M_1(dt_1/x_1)M_1(dt_2/x_2)f_2(x_1, x_2)dx_1 dx_2 +$$

$$- \int_{x_1} M_1(dt_1/x_1)f_1(x_1)dx_1 \int_{x_2} M_1(dt_2/x_2)f_1(x_2)dx_2$$

$$= \int_x M_2(dt_1 x dt_2/x)\tilde{C}_1(dx) + \int_{x_1}\int_{x_2} M_1(dt_1/x_1)M_1(dt_2/x_2)\tilde{C}_2(dx_1 x dx_2)$$

$$(2.7.2)$$

The above results are derived by Vere-Jones (1970) by using methods based on properties of the generating probability functional.

If now $\lambda(t)$ and $\mu(t_1, t_2)$ are respectively the intensity and covariance density of the overall process and similarly $\tilde{\lambda}(t)$ and $\tilde{\mu}(t_1, t_2)$ the intensity and the covariance density of the main process by virtue of (2.7.1) and (2.7.2),

$$\left.\begin{aligned}\lambda(t) &= \int_x m_1(t/x)\tilde{\lambda}(x)dx \qquad \text{and} \\ \mu(t_1,t_2) &= \int_x m_2(t_1,t_2/x)\tilde{\lambda}(x)dx + \int_{x_1}\int_{x_2} m_1(t_1/x_1)m_1(t_2/x_2)\tilde{\mu}(x_1,x_2)dx_1 dx_2\end{aligned}\right\} \quad (2.7.3)$$

The following two types of cluster models are further examined in detail.

A.- The Neymann-Scott cluster model

This model, proposed by Neymann and Scott (1958) for the distribution of the galaxies and Vere-Jones (1970) in describing the series of the time origin of earthquakes in New Zealand. In this model the process of the cluster centers is stationary and Poisson with parameter ν, while conditional on a given member size with probability generating function $\Pi(z) = \sum_{n=0}^{\infty} \pi_n z^n$, the cluster members are independently and identically distributed about the cluster centre with common distribution function $L(x)$, where x is the distance of the member from the cluster centre.

Since the process of the cluster centres is stationary and the probability structure of the process of the cluster members depends only in the distances from the cluster centre, and not on the location of the cluster centre, the sum process will be stationary. Therefore, if λ the mean rate of occurrence of the sum process, $\mu(u)$ its covariance density and the cluster centre is taken as a cluster member then by using direct probabilistic arguments:

$$\lambda = \nu \cdot 1 + \nu \sum n\pi_n = \nu(1+\alpha) \qquad (2.7.4)$$

$$\begin{aligned}\mu(u) &= \nu\, l(u) \sum n\pi_n + \nu \int_x l(x)l(x+u) \sum n(n-1)\pi_n \\ &= \nu\, \alpha l(u) + \nu\beta \int_x l(x)l(x+u)dx \qquad (2.7.5)\end{aligned}$$

where $\alpha = E(N)$, $\beta = E(N(N-1))$, $l(x) = \dfrac{dL(x)}{dx}$ and N represents the numbers of subsidiary events in a cluster.

If the cluster centre is not considered (2.7.4) and (2.7.5) become,

$$\lambda = \nu \sum n\pi_n = \nu\alpha$$

$$\mu(u) = \nu\beta \int_x l(x)l(x+u)dx.$$

Thus by (2.2.7), (2.2.9), (2.7.4) and (2.7.5) the variance time curve and the spectrum of counts for a general Neymann-Scott model are:

(i) The Variance time curve,

$$V(t) = \lambda t + 2\int_0^t (t-u)\mu(u)du$$

$$= \lambda t + 2\nu\alpha\int_0^t l(u)(t-u)du + 2\nu\beta\int_0^t\int_x (t-u)l(x)l(x+u)dxdu \qquad (2.7.6)$$

Hence the ratio,

$$R = \lim_{t\to\infty}\frac{V(t)}{\lambda t} = 1 + \frac{2\nu\alpha}{\nu(1+\alpha)} + \frac{\nu\beta}{\nu(1+\alpha)} > 1 ,$$ i.e the process is over-dispersed.

(ii) The spectrum of counts,

$$\pi g_+(u) = \lambda + \nu\alpha\int_{-\infty}^{+\infty} l(u)e^{-i\omega u}du + \nu\beta\int_{-\infty}^{+\infty}\int_x l(x)l(x+u)e^{-i\omega u}dxdu$$

$$= \lambda + \nu\alpha\left[\int_{-\infty}^{0} l(-u)e^{-i\omega u}du + \int_0^{\infty} l(u)e^{-i\omega u}du\right] +$$

$$+ \nu\beta\int_0^{+\infty} l(x+u)e^{-i(x+u)\omega}d(x+u)\int_0^{\infty} l(x)e^{i\omega x}dx$$

$$= \lambda + \nu\alpha\left[\hat{l}(\omega) + \overline{\hat{l}(\omega)}\right] + \nu\beta\left|\hat{l}(\omega)\right|^2 \qquad (2.7.7)$$

where $\hat{l}(\omega) = \int_x e^{i\omega x}\, l(x)dx$, and $\overline{\hat{l}(\omega)} = \int_x e^{-i\omega x}\, l(x)dx$

If the cluster centre is not considered as a cluster member then the spectrum of counts,

$$\pi g_+(\omega) = \lambda + \nu\beta\left|\hat{l}(\omega)\right|^2 \qquad (2.7.8)$$

In the following the above obtained general results are applied in some special cases of the Neymann-Scott model.

A1) The exponential decay model with $l(x) = \rho e^{-\rho x}$, $\rho > 0$ for which;

(i) Covariance density,

$$\mu(u) = \frac{1}{2}\nu(2\alpha+\beta)\rho e^{-\rho u} = \frac{1}{2}\beta'\rho e^{-\rho u} \qquad (2.7.9)$$

where $\beta' = E(N(N+1))$.

(ii) Variance time curve,

$$V(t) = \nu(1+\alpha)t+\nu\beta' t - \frac{\nu\beta'}{\rho}(1-e^{-\rho t}) \qquad (2.7.10)$$

(iii) Spectrum of counts,

$$\pi g_+(\omega) = \nu(1+\alpha)+\nu\alpha\left(\frac{\rho}{\rho+i\omega}+\frac{\rho}{\rho-i\omega}\right)+\nu\beta\frac{\rho^2}{\rho^2+\omega^2} = \nu(1+\alpha)+\nu\beta'\frac{\rho^2}{\rho^2+\omega^2}$$

If the spectrum centre is not considered then by (2.7.8),

$$\pi g_+(\omega) = \nu\alpha+\nu\beta\frac{\rho^2}{\rho^2+\omega^2}, \text{ where } \beta = E(N(N-1)) \qquad (2.7.11)$$

A2) The mixed exponential decay model with $l(x) = \gamma\rho_1 e^{-\rho_1 x}+(1-\gamma)\rho_2 e^{-\rho_2 x}$,

$0 \leqslant \gamma \leqslant 1$, $\rho_1 > 0$ and $\rho_2 > 0$ for which the spectrum of counts is,

$$\pi g_+(\omega) = \nu(1+\alpha) + \frac{\nu\beta'\left[\rho_1^2(\rho_2^2+\omega^2)\gamma+\rho_2^2(\rho_1^2+\omega^2)(1-\gamma)\right]-\nu\beta\gamma(1-\gamma)(\rho_1-\rho_2)^2}{(\rho_1^2+\omega^2)(\rho_2^2+\omega^2)} \qquad (2.7.12)$$

If again the cluster centre is not considered the above formula becomes,

$$\pi g_+(\omega) = \nu\alpha+\nu\beta\frac{\rho_1^2\rho_2^2+\left[\gamma\rho_1+(1-\gamma)\rho_2\right]^2\omega^2}{(\rho_1^2+\omega^2)(\rho_2^2+\omega^2)} \qquad (2.7.13)$$

A3) The inverse power decay model with $l(x) = \rho c^{\rho}/(c+x)^{\rho+1}$, $c>0$, $\rho>0$ for which:

(i) Covariance density,

$$\mu(u) = \nu\alpha\frac{\rho c^{\rho}}{(c+u)^{\rho+1}} + \nu\beta\rho^2 c^{2\rho}\int_0^{\infty}\frac{dx}{(c+x)^{1+\rho}(c+x+u)^{1+\rho}} \qquad (2.7.14)$$

(ii) Variance time curve,

$$V(t) = \nu(1+\alpha)t+2\nu\alpha\rho c^{\rho}\int_0^t\frac{(t-u)du}{(c+u)^{\rho+1}}+2\nu\beta\rho^2c^{2\rho}\int_0^t\int_0^{\infty}\frac{(t-u)dxdu}{\left[(c+x)(c+x+u)\right]^{\rho+1}}$$

and by changing the order of integration and integrating by parts,

$$V(t) = \varphi(t)+k\int_0^{\infty}\left[\frac{1-(1+t/(c+x))^{1-\rho}}{(c+x)^{2\rho}}\right]dx \qquad (2.7.15)$$

where,

$$\left.\begin{aligned}\varphi(t) &= \nu(1+\alpha)t-2\nu\alpha t^2 - \frac{2\nu\alpha c^{\rho}t}{(1-\rho)(c+t)^{\rho-1}}+\frac{2\nu\alpha ct}{1-\rho},\\ \text{and } K &= \frac{2\nu\beta\rho c^{2\rho}}{1-\rho}\end{aligned}\right\} \qquad (2.7.16)$$

Providing that $\rho \neq {}^1/_2$ the evaluation of the integral in (2.7.15) can be done as follows:

Firstly by substituting $y = {}^t/x{+}c$ the integral becomes,

$$F_C(t) = \int_0^{\infty} \frac{1-(1+\frac{t}{x+c})^{1-\rho}}{(x+c)^{2\rho}} dx$$

$$= t^{1-2\rho} \int_0^{t/c} y^{2\rho-2} \left[1-(1+y)^{1-\rho}\right] dy$$

$$= c^{1-2\rho} \upsilon^{1-2\rho} G(\upsilon)$$

where $\upsilon = t/c$ and $G(\upsilon) = \int_0^{\upsilon} y^{2\rho-2} \left[1-(1+y)^{1-\rho}\right] dy$ (2.7.17)

Integrating by parts repeatedly (2.6.17) gives,

$$G(\upsilon) = \frac{\upsilon^{2\rho-1}}{2\rho-1}\left[1-(1+\upsilon)^{1-\rho}\right] + \frac{1-\rho}{2(2\rho-1)\rho} \frac{\upsilon^{2\rho}}{(1+\upsilon)^{\rho}} B(\upsilon) \text{ where,}$$

$B(\upsilon) = \sum_{r=0}^{\infty} \beta_r (\frac{\upsilon}{1+\upsilon})^r$ which $\beta_0 = 1$ and $\beta_r = \frac{\rho(\rho+1)(\rho+2)\ldots(\rho+r-1)}{(2\rho+1)(2\rho+2)\ldots(2\rho+r)} =$

$= (1 - \frac{1+\rho}{2\rho+r})\beta_{r-1}$, $r \geqslant 1$. For υ not too large, say $\upsilon < \upsilon_1$ the series $B(\upsilon)$ converge quite rapidly. For large $\upsilon > \upsilon_1 > 1$ (2.7.17) can be written as,

$$G(\upsilon) = G(\upsilon_1) + \int_{\upsilon_1}^{\upsilon} y^{2\rho-2} \left[1-(1+y)^{1-\rho}\right] dy$$

$$= G(\upsilon_1) + \frac{1}{2\rho-1}\left[\upsilon^{2\rho-1} - \upsilon_1^{2\rho-1}\right] + A(\upsilon_1, \upsilon) \qquad (2.7.18)$$

where,

$A(\upsilon_1,\upsilon) = -\int_{\upsilon_1}^{\upsilon} y^{2\rho-2}(1+y)^{1-\rho} dy$ and since $y > \upsilon_1 > 1$,

$A(\upsilon_1,\upsilon) = -\int_{\upsilon_1}^{\upsilon} y^{\rho-1} (\sum_{r=0}^{\infty} a_r y^{-r}) dy$ with $a_0 = 0$

and $a_r = \frac{(1-\rho)(-\rho)(-\rho-1)\ldots(2-\rho-r)}{r!} = -a_{r-1}(1 - \frac{2-\rho}{r})$

Therefore $A(\upsilon_1,\upsilon) = \sum_{r=0}^{\infty} a_r \int_{\upsilon_1}^{\upsilon} y^{-\rho-1-r} dy$

$$= \sum_{r=0}^{\infty} a_r \frac{1}{r-\rho}\left[(\frac{1}{\upsilon})^{r-\rho} - (\frac{1}{\upsilon_1})^{r-\rho}\right]$$

and (2.7.18) for $\upsilon > \upsilon_1 > 1$ becomes,

$$G(\upsilon) = G(\upsilon_1) + \frac{1}{2\rho-1}\left\{\upsilon^{2\rho-1} - \upsilon_1^{2\rho-1}\right\} + A(\upsilon) - A(\upsilon_1) \qquad \text{where}$$

$$A(\upsilon) = \upsilon^{\rho} \sum_{r=0}^{\infty} \frac{a_r}{r-\rho}\left(\frac{1}{\upsilon}\right)^r \qquad (2.7.19)$$

If $\upsilon_1 \approx 5/3$ then $\frac{1}{\upsilon_1} = 0{\cdot}6$ and $\frac{\upsilon_1}{1+\upsilon_1} = 5/8 = 0{\cdot}625$; hence $G(\upsilon_1)$ and $A(\upsilon_1)$ can both converge simultaneously quite rapidly for $\upsilon > 5/3$. Finally the truncation error for the series $B(\upsilon)$ since $\beta_r < \beta_{r-1}$ is:

$$R = \sum_{n+1}^{\infty} \beta_r \left(\frac{\upsilon}{1+\upsilon}\right)^r < \beta_N \sum_{N+1} \left(\frac{\upsilon}{1+\upsilon}\right)^r = \upsilon\beta_N \left(\frac{\upsilon}{1+\upsilon}\right)^N$$

B.- The Bartlett-Lewis cluster model

Bartlett (1963 b) derived the above model to describe a clustering effect in observed series of times at which vehicles pass a point on a road and Lewis (1964) used the same model to describe series of computer failures. In this model the cluster centres have a Poisson process with parameter ν, the times between events in the subsidiary process Y_i, $i = 1,2,\ldots$ are independent and identically distributed and the cluster size N has a distribution $p(n)$, $n = 0,1,\ldots$. Therefore if $f(y)$ is the density function of Y_i, $i = 1,2,\ldots$ with characteristic function $M(\theta) = \int_0^{\infty} e^{i\theta x} f(x)dx$, the distribution of $Y_1 + Y_2 + \ldots + Y_n$ will be the n-fold convolution of the distribution of Y with density function $f^{(n)}(y)$ and characteristic function $M^n(\theta)$. Considering the cluster centres as members of the subsidiary process and following a similar method as the one in the case of the Neymann-Scott model the following results for the intensity and covariance densities of the whole process are obtained:

$$\lambda = \nu\left\{1{\cdot}p(0) + 2{\cdot}p(1) + \ldots + (n+1)p(n) + \ldots\right] = \nu E(n+1) \qquad (2.7.20)$$

$$\mu(\tau) = \nu\left\{f(\tau)(p(1) + 2p(2) + 3p(3) + \ldots) + f^{(2)}(\tau)(p(2) + 2p(3) + 3p(4) + \ldots) + f^{(3)}(\tau)(p(3) + 2p(4) + \ldots) + \ldots\right]$$

$$= \nu\left[f(\tau)E(N) + f^{(2)}(\tau)E(N-1) + f^{(3)}(\tau)E(N-2) + \ldots\right] \qquad (2.7.21)$$

consequently the Fourier transform of the $\mu(\tau)$ is,

$$\mu^*(\omega) = \int_{-\infty}^{+\infty} \mu(\tau)e^{-i\omega\tau}d\tau = \nu\left[(M(\omega) + M(-\omega))E(N) + (M^2(\omega) + M^2(-\omega))E(N-1) + \ldots\right]$$

and therefore the spectrum of counts,

$$g_+(\omega) = \frac{1}{\pi}\left\{\lambda + \nu \sum_{s=1}^{\infty} \left\{M^s(\omega) + M^s(-\omega)\right\} E(N-s+1)\right\} \qquad (2.7.22)$$

where $E(k) = 0$ for $k<0$. Since $\lim_{\omega\to 0} M(\omega) = 1$ and $\lim_{\omega\to\infty} M(\omega) = 0$, see Feller (1966) Vol. II, the limiting values of $g_+(\omega)$ are:

$$\lim_{\omega\to 0} \pi g_+(\omega) = \lambda\left[1 + \frac{E(N(N+1))}{E(N+1)}\right] \quad \text{and} \quad \lim_{\omega\to\infty} \pi g_+(\omega) = \lambda \qquad (2.7.23)(a)$$

In the special case where the cluster size has a geometric distribution with parameter p and r is the probability that the subsidiary process starts,

$$p(n) = \text{Prob}(n \text{ events in the subsidiary process})$$

$$= \begin{cases} 1-r & n = 0 \\ rp^{n-1}(1-p) & n = 1,2,\ldots \end{cases}$$

and $E(N-S) = \dfrac{rp^s}{1-p}$, $s = 0,1,\ldots$ Consequently if the times between successive events in the subsidiary process have Gamma with index 2 distribution, with $M(\omega) = (1 - \frac{i\omega}{2k})^{-2}$ (2.7.22) becomes,

$$g_+(\omega) = \frac{1}{\pi}\left\{\lambda + \nu \sum_{s=1}^{\infty} \left[\frac{1}{(1-\frac{i\omega}{2k})^{2s}} + \frac{1}{(1+\frac{i\omega}{2k})^{2s}}\right] \frac{rp^{s-1}}{1-p}\right.$$

$$= \frac{1}{\pi}\left\{\lambda + \frac{\nu r}{1-p} \frac{2(1-\omega^2/4k^2-p)}{\frac{\omega^2}{k^2}(1+\omega^2/4k^2) - 2p(1-\omega^2/4k^2) + p^2}\right\}$$

$$= \frac{1}{\pi} \frac{P_1(\omega)}{P_2(\omega)} \qquad (2.7.23)(b)$$

where $P_1(\omega)$ and $P_2(\omega)$ are two fourth degree polynomials.

Lewis (1964) gives the spectrum of the intervals of the whole process when the size N of the subsidiary process is geometrically distributed with parameter p and $\beta = \dfrac{E(X)}{E(Y)} \to \infty$, where X is the time between successive events in the main process and Y the time interval between successive events in the subsidiary process. The exact formula is,

$$f_+(\omega) = \frac{2(1+E(N))}{\pi(1+2E(N))} \frac{e}{e^2+d^2}$$

where $$e = 1-\cos\omega + \frac{r\{\cos(2\omega)-(1+p)\cos\omega+p\}}{1+p^2-2p\cos\omega}$$

$$d = \sin\omega + \frac{r\{\sin(2\omega)-(1+p)\sin\omega\}}{1+p^2-2p\cos\omega} \qquad (2.7.24)$$

2.8. Cluster representation of the mutually exciting process

Hawkes and Oakes (1972) by considering the self-exciting process as an immigration-birth process derive some counting and interval properties of the process. The mutually exciting processes, $N_i(t)$, $i = 1,2,\ldots,k$, which have been defined in § 2.6., can be also interpreted as a k immigration birth process of Poisson type forming cluster centres at rate ν_i. Each individual of type j and age x then has probability $\beta_{ij}(x)dt+o(dt)$ of giving birth to a type i individual in the next interval dt. There will be k different types of clusters each containing all the descendants of various types of an immigrant of type i. In what follows by using the concept of the probability Generating functional (p.g. fl) of a k-variate point process we derive some counting and interval properties of the mutually exciting processes which can be considered as a generalization of the ones obtained by Hawkes and Oakes (1972) in the case of self-exciting processes, which is a mutually exciting processes with k = 1.

Definition 1: The p.g. fl of a k-variate point process, $N_i(t)$, $i = 1,2,\ldots,k$, is defined as,

$$G(\underset{\sim}{s}(\cdot)) = E\left\{\exp\int\sum_{i=1}^{k} \lg s_i(t)dN_i(t)\right\}$$

If $$s_i(x) = \begin{cases} z_i & \text{for } 0 \leq x \leq t \\ 1 & \text{otherwise} \end{cases} \qquad \text{for } i = 1,2,\ldots k$$

then the p.g. fl reduces to the ordinary p.g. f,

$\Pi(\underset{\sim}{z}) = E(z_1^{r_1} z_2^{r_2} \ldots z_k^{r_k})$ where r_1 is the number of objects of the 1st kind,

r_2 is the number of objects of the second kind ..., r_k is the number of objects of k^{th} kind in the interval (0,t).

Theorem 1: The p.g. fl of a k-variate point processes where the individual processes are independent non-stationary Poisson with indensities $\lambda_i(t)$, $i = 1,2,\ldots,k$ is,

$$G(\underset{\sim}{s}(\cdot)) = \exp\left\{\sum_{i=1}^{k} \int_{-\infty}^{+\infty}(s_i(t)-1)\lambda_i(t)dt\right\} \qquad (2.8.1)$$

In the case of stationary poisson processes the above formula becomes,

$$G(\underset{\sim}{s}(\cdot)) = \exp\left\{\sum_{i=1}^{k} \lambda_i \int_{-\infty}^{+\infty}(s_i(t)-1)dt\right\} \qquad (2.8.2)$$

Proof: The theorem can be proved either directly from the definition of the p.g. fl by replacing the integrals with the corresponding sums or by considering the k-variate point process as the superposition of k independent univariate Poisson processes and applying well known results about the p.g. fl of the non-stationary Poisson processes and the superposition of independent point processes (Vere-Jones, 1970).

Returning now in the case of mutually exciting point processes, in each cluster of type i, which is originated at $t = 0$ by an individual of the same type, the offspring in the first generation can be considered as generated by a non-stationary Poisson process which is the superposition of k independent non-stationary Poisson processes each with intensity $\beta_{ji}(t)$, $j = 1,2,\ldots k$. Hence by virtue of Theorem 1 the p.g. fl for the offspring in the first generation is,

$$f^{(i)}(\underset{\sim}{s}(\cdot)) = \exp\left\{\sum_{j=1}^{k} \int_{0}^{\infty}(s_j(t)-1)\beta_{ji}(t)dt\right\} \quad i = 1,2,\ldots k \qquad (2.8.3)$$

where $\underset{\sim}{s}(\cdot) = (s_1(\cdot), s_2(\cdot), \ldots s_k(\cdot))$.

Now the representation of the mutually exciting processes as a multitype cluster processes and the use of the previously derived general results about the p.g. fl of a multivariate point process enables us to derive the p.g.fl of the overall process which is given by the following theorem.

Theorem 3: The p.g.fl of the mutually-exciting processes has the form,

$$G(\underset{\sim}{s}(\cdot)) = \exp\left\{ \sum_{i=1}^{k} \int_{-\infty}^{+\infty} \nu_i (F^{(i)}(\underset{\sim}{s}_t(\cdot)) - 1) dt \right\} \tag{2.8.3}$$

where $F^{(i)}(s(\cdot))$ is the p.g.fl of the i^{th} cluster generated by an immigrant of type i arriving at time zero and including that immigrant, while $\underset{\sim}{s}_t(\cdot) = \underset{\sim}{s}(t+\cdot)$ is simply the translation of $\underset{\sim}{s}(\cdot)$. The functionals $F^{(i)}(\underset{\sim}{s}(\cdot))$, $i = 1,2,\ldots k$ satisfy the equations,

$$F^{(i)}(\underset{\sim}{s}(\cdot)) = s_i(0)\exp\left\{ \sum_{j=1}^{k} \int_{0}^{\infty} \left[F^{(j)}(\underset{\sim}{s}_t(\cdot)) - 1\right] \beta_{ji}(t)dt \right\}, \quad i = 1,2,\ldots k \tag{2.8.4}$$

Proof: Vere-Jones (1970) gives the p.g.fl of the overall process in the univariate cluster process which by a direct generalization becomes in the case of multivariate cluster processes,

$$G(\underset{\sim}{s}(\cdot)) = G_0(F^{(1)}(\underset{\sim}{s}(\cdot)/t), \; F^{(2)}(\underset{\sim}{s}(\cdot)/t), \ldots F^{(k)}(\underset{\sim}{s}(\cdot)/t)) \tag{2.8.5}$$

where $G_0(\underset{\sim}{s}(\cdot))$ is the p.g.fl of the process of the cluster centres and $F^{(i)}(\underset{\sim}{s}(\cdot)/t)$ is the p.g.fl of the i^{th} cluster given that the cluster centre is at time t. If the process is time homogeneous then $F^{(i)}(\underset{\sim}{s}(\cdot)/t) = F^{(i)}(\underset{\sim}{s}_t(\cdot))$, where $F^{(i)}(\underset{\sim}{s}(\cdot))$ corresponds to a cluster centre at time 0.

In the present case $G_0(\underset{\sim}{s}(\cdot))$ is given by (2.8.2) and so,

$$G_0(\underset{\sim}{s}(\cdot)) = \exp\left\{ \sum_{i=1}^{k} \int_{-\infty}^{+\infty} \nu_i \left[s_i(t) - 1\right] dt \right\} \tag{2.8.6}$$

Let now $F_n^{(i)}(\underset{\sim}{s}(\cdot))$ be the p.g.fl of the process which consists of all births of different types of individuals in all generations up to and including the n^{th} generation, and descended from an individual of type i which originates the cluster at t = 0 and belongs in the cluster. Then by treating the first generation as the process of the cluster centres each of which generates further sub-clusters and by using (2.8.1) the following backward equations for the multitype branching process are obtained,

$$F_n^{(i)}(\underset{\sim}{s}(\cdot)) = s_i(0)f^{(i)}(\underset{\sim}{F}_{n-1}(\underset{\sim}{s}_t(\cdot)))$$

$$= s_i(0)\exp\left\{\sum_{j=1}^{k}\int_0^{\infty}\left[F_{n-1}^{(j)}(\underset{\sim}{s}_t(\cdot))-1\right]\beta_{ji}(t)dt\right\} \qquad (2.8.7)$$

for $i = 1,2,\ldots k$.

In the limit as $n \longrightarrow \infty$, the cluster consists of all generations of the family tree of type i and therefore the theorem follows by using (2.8.5) and (2.8.6). If $\underset{\sim}{s}(x) = \underset{\sim}{s}$ for all x (2.8.7) becomes,

$$\Pi_n^{(i)}(\underset{\sim}{s}) = s_i \exp\left\{\sum_{j=1}^{k}(\Pi_{n-1}^{(j)}(\underset{\sim}{s})-1)a_{ji}\right\} \qquad i = 1,2,\ldots n$$

and in the limit as $n \longrightarrow \infty$

$$\Pi^{(i)}(\underset{\sim}{s}) = s_i \exp\left\{\sum_{j=1}^{k}(\Pi^{(j)}(\underset{\sim}{s})-1)a_{ji}\right\} \qquad i = 1,2,\ldots k \qquad (2.8.8)$$

where $\Pi^{(i)}(\underset{\sim}{s})$ is the p.g.f of the total size of cluster of type i, and $a_{ji} = \int_0^{\infty}\beta_{ji}(u)du$ is the mean number of offspring of type j in the first generation generated in a cluster of type i.

In the special case considered in § 2.6. where $k = 2$, $\beta_{11}(u) = \beta_{21}(u) = 0$ and $\underset{\sim}{\nu} = \binom{0}{\nu}$, the complete intensities of the two processes are respectevely,

$$\Lambda_1(t) = \int_{-\infty}^{t}\beta_{12}(t-u)dN_2(u)$$

$$\Lambda_2(t) = \nu + \int_{-\infty}^{t}\beta_{22}(t-u)dN_2(u)$$

and so the $N_2(t)$ is a self-exciting point process not influenced by the $N_1(t)$ which is simply excited by the $N_2(t)$. Therefore the overall process can be considered as a cluster process of the Neymann-Scott type where the process of the cluster centres is a self-exciting one (instead of a ~~renewal~~ Poisson type) with complete intensity $\Lambda_2(t)$ and with the cluster members identically and independently distributed with common distribution function,

$B(x) = \int_0^x \frac{\beta_{12}(x)}{a_{12}}dx$, where x is the distance of the cluster member from the cluster centre and $a_{12} = \int_0^{\infty}\beta_{12}(x)dx$ is the mean cluster size (Vere-Jones 1970).

The above special case of the mutually exciting processes applied in the case of earthquake occurrence takes into account the fact that the cluster centres (main shocks) too have the tendency of clustering in a different manner from the cluster members (aftershock sequence). This very important feature of the earthquake occurrence has been observed by Utsu in Japan and other seismic regions. See discussion in Vere-Jones (1970).

By a direct application of the Theorem 3 in the case of the above considering special case the following results can be obtained for the p.g.fl's of the process of cluster members and overall process,

$$F^{(1)}(s_1(\cdot),\ s_2(\cdot)) = s_1(0)$$

$$F^{(2)}(s_1(\cdot),\ s_2(\cdot)) = s_2(0)\exp\left\{\int_0^\infty (s_1(t)-1)\,\beta_{12}(t)dt + \int_0^\infty \left[F^{(2)}(\underset{\sim}{s}_t(\cdot))-1\right]\beta_{22}(t)dt\right\} \qquad (2.8.9)$$

$$G(s_1(\cdot),\ s_2(\cdot)) = \exp\left\{\int_{-\infty}^{+\infty} \nu\left[F^{(2)}(\underset{\sim}{s}_t(\cdot))-1\right] dt\right\} \qquad (2.8.10)$$

No explicit solutions of (2.8.9) and (2.8.10) are known but useful equations may be obtained by choosing particular functions $\underset{\sim}{s}(\cdot)$. For example if $\underset{\sim}{s}(x) = \underset{\sim}{s}$ for all x then $F^{(2)}(\underset{\sim}{s}(\cdot)) = \Pi^{(2)}(\underset{\sim}{s})$ is the p.g.function of the number of events in a cluster originated with an individual of type II and (2.8.9) becomes,

$$\Pi^{(2)}(s_1,s_2) = s_2\ \exp\left\{(s_1-1)a_{12} + \left[\Pi^{(2)}(s_1,s_2)-1\right]a_{22}\right\} \qquad (2.8.11)$$

From (2.8.11) the mean sizes of offspring of first and second kind in one cluster can be obtained which are respectively,

$$m_{12} = \left.\frac{\partial\,\Pi^{(2)}(s_1,s_2)}{\partial s_1}\right|_{\substack{s_1=1\\ s_2=1}} = \frac{a_{12}}{1-a_{22}}$$

$$m_{22} = \left.\frac{\partial\,\Pi^{(2)}(s_1,s_2)}{\partial s_2}\right|_{\substack{s_1=1\\ s_2=1}} = \frac{1}{1-a_{22}}$$

Therefore the intensity of the overall process is

$$\lambda = \nu\left\{\frac{1}{1-a_{22}} + \frac{a_{12}}{1-a_{22}}\right\} = \frac{\nu(1+a_{12})}{1-a_{22}}$$ which is the same with the one obtained in § 3.6.

Returning now in the general case let

$$\underset{\sim}{s}(x) = \begin{cases} \underset{\sim}{s} & \text{for } y \leq x \leq y+1 \\ \underset{\sim}{1} & \text{elsewhere} \end{cases}$$

Then $F^{(i)}(\underset{\sim}{s}_t(\cdot)) = \Pi^{(i)}(y-t, 1, \underset{\sim}{s})$ is the p.g.f of the number of offspring of all kinds in the interval $[y-t, y-t+1]$ in a cluster originated at $t = 0$ with an individual of i^{th} type. Then (2.8.7) becomes,

$$\Pi^{(i)}(y,1,\underset{\sim}{s}) = \begin{cases} \exp\left\{\sum_{j=1}^{k} \int_0^{y+1} [\Pi^{(j)}(y-t,1,\underset{\sim}{s}) - 1]\, \beta_{ji}(t)dt\right\} & y > 0 \\ s_i \exp\left\{\sum_{j=1}^{k} \int_0^{y+1} [\Pi^{(j)}(y-t,1,\underset{\sim}{s}) - 1]\, \beta_{ji}(t)dt\right\} & -1 \leq y \leq 0 \\ 1 & y < -1 \end{cases} \tag{2.8.12}$$

The last result arises since if $y < -1$ the considering interval proceeds the cluster centre and hence there are not any events in it.

Now let us put $\underset{\sim}{s}(x) = \begin{cases} \underset{\sim}{s} & 0 \leq x \leq 1 \\ \underset{\sim}{1} & \text{elsewhere} \end{cases}$

in the equation (2.8.3). Then $G(\underset{\sim}{s}(\cdot)) = Q_1(\underset{\sim}{s})$ is the p.g.f for the number of events in $(0,1]$ of the equilibrium mutually exciting process and (2.8.3) becomes

$$Q_1(\underset{\sim}{s}) = \exp\left\{\sum_{i=1}^{k} \int_{-\infty}^{1} \nu_i\, [\Pi^{(i)}(-t,1,\underset{\sim}{s}) - 1]\, dt\right\} \tag{2.8.13}$$

Equations (2.8.12) and (2.8.13) determine the distribution of counts. However they can be used to establish some interval properties of the process.

Theorem 4: (a) The forward recurrence time survivor function is,

$$R_L(1) = P(L > 1) = \exp\left\{-1 \sum_{i=1}^{k} \nu_i - \sum_{i=1}^{k} \nu_i \int_0^{\infty} [1 - \Phi_Y^{(i)}(t,1)]\, dt\right\} \tag{2.8.14}$$

where $\Phi_Y^{(i)}(t,1)$ satisfies the equation

$$\Phi_Y^{(i)}(y,1) = \begin{cases} \exp\left\{\sum_{j=1}^{k} \int_0^{y+1} [\Phi_Y^{(j)}(y-t,1) - 1]\, \beta_{ji}(t)dt\right\} & y > 0 \\ 0 & -1 \leq y \leq 0 \\ 1 & y < -1 \end{cases} \tag{2.8.15}$$

(b) The equilibrium distribution of the interval between successive events has survivor function,

$$R_T(t) = P(T>t) = \frac{-1}{\lambda}\frac{dR_L(t)}{dt}$$

Proof: a) (2.8.13) can be written as,

$$Q_1(\underset{\sim}{s}) = \exp\left\{\sum_{i=1}^{k}\int_{-1}^{\infty}\nu_i\left[\Pi^{(i)}(y,1,\underset{\sim}{s})-1\right]dy\right\}$$

$$= \exp\left\{\sum_{i=1}^{k}\int_{-1}^{0}\nu_i\left[\Pi^{(i)}(y,1,\underset{\sim}{s})-1\right]dy + \sum_{i=1}^{k}\int_{0}^{\infty}\nu_i\left[\Pi^{(i)}(y,1,\underset{\sim}{s})-1\right]dy\right\}$$

Hence by virtue of (2.8.12),

$$R_L(1) = Q_1(\underset{\sim}{0}) = \Pr\text{ (No events of any kind in }(0,1))$$

$$= \exp\left\{-1\sum_{i=1}^{k}\nu_i - \sum_{i=1}^{k}\int_{0}^{\infty}\nu_i\left[1-\phi_Y^{(i)}(t,1)\right]dt\right\}$$

where $\phi_Y^{(i)}(y,1) = \Pi^{(i)}(y,1,\underset{\sim}{0})$ is the probability of no events of any kind in the interval $(y, y+1)$ for a cluster of type i^{th} originated at $t=0$. The equations (2.8.15) follow simply by taking $\underset{\sim}{s} = \underset{\sim}{0}$ in (2.8.12).

(b) This is immediate consequence of the relationship (2.2.16).

In the previously considering special case $\nu_1 = 0$, $\nu_2 = \nu$, $k = 2$ and by (2.8.9) $\phi_Y^{(1)}(y,1) = \Pi^{(1)}(y,1,\underset{\sim}{0}) = 0$. Hence the forward recurrence time survivor function becomes, $R_L(1) = P(L>1) = \exp\left\{-1\nu-\nu\int_0^{\infty}\left\{1-\phi_Y^{(2)}(t,1)\right\}dt\right\}$ where

$$\phi_Y^{(2)}(y,1) = \begin{cases} \exp\left\{-\int_y^{y+1}\beta_{12}(t)dt + \int_0^{y+1}\left[\phi_Y^{(2)}(y-t,1)-1\right]\beta_{22}(t)dt\right\} & y>0 \\ 0 & -1\leqslant y\leqslant 0 \\ 1 & y<-1 \end{cases} \quad (2.8.16)$$

Equation (2.8.16) may be solved in principle by repeated numerical integration using the recurrence relation (2.8.7) which in this case takes the form,

$$\phi^{(2)}_{Y,n}(y,1) = \begin{cases} \exp\left\{-\int_y^{y+1} \beta_{12}(t)dy + \int_0^{y+1} \left[\phi^{(2)}_{Y,n-1}(y-t,1)-1\right] \beta_{22}(t)dt\right\} & y>0 \\ 0 & -1\leq y\leq 0 \\ 1 & y<-1 \end{cases}$$

and with the initial condition

$$\phi^{(2)}_{Y,0}(y,1) = 0 \qquad -1\leq y\leq 0$$
$$= 1 \qquad \text{elsewhere.}$$

Continuing now with further studying of the structure of the cluster of the mutually exciting processes,

let $\underset{\sim}{s}(x) = \begin{cases} \underset{\sim}{s} & 0\leq x\leq u \\ \underset{\sim}{1} & \text{otherwise} \end{cases}$

Then $F^{(i)}(\underset{\sim}{s}(\cdot)) = \Pi_u^{(i)}(\underset{\sim}{s})$ is the p.g.f of events in (0,u) of a cluster originated at time zero with an individual of i^{th} type, and (2.8.4) becomes,

$$\Pi_u^{(i)}(\underset{\sim}{s}) = s_i \exp\left\{\sum_{j=1}^{k} \int_0^u \left[\Pi^{(j)}_{u-t}(\underset{\sim}{s})-1\right] \beta_{ji}(t)dt\right\} \qquad (2.8.17)$$

Let now $M_{ji}(u)$ be the number of j^{th} type subsidiary events in the interval (0,u) for a cluster starting with an i^{th} immigrant, and $H_{ji}(u) = E(M_{ji}(u))$, $i,j = 1,2,\dots k$. Then,

$$\delta_{ji}+H_{ji}(u) = \left.\frac{\partial \Pi_u^{(i)}(\underset{\sim}{s})}{\partial s_j}\right|_{\underset{\sim}{s}=\underset{\sim}{1}} = \delta_{ji} + \sum_{r=1}^{k} \int_0^u \left[\delta_{jr}+H_{jr}(u-t)\right]\beta_{ri}(t)dt$$

where $\delta_{ji} = \begin{cases} 1 & j=i \\ 0 & j\neq i \end{cases}$

and by taking Laplace transforms

$$H^*_{ji}(s) = \frac{1}{s} \sum_{r=1}^{k} \delta_{jr}\beta^*_{ri}(s) + \sum_{r=1}^{k} H^*_{jr}(s)\beta^*_{ri}(s) \quad \text{or in matrix notation}$$

$$\underset{\sim}{H}^*(s) = \frac{1}{s}\underset{\sim}{\beta}^*(s) + \underset{\sim}{H}^*(s)\underset{\sim}{\beta}^*(s) \quad \text{and therefore}$$

$$\underset{\sim}{H}^*(s) = \frac{1}{s}\underset{\sim}{\beta}^*(s)\left[\underset{\sim}{I}-\underset{\sim}{\beta}^*(s)\right]^{-1} \qquad (2.8.18)$$

In the previously examined special case,

$$\beta^*(s) = \begin{pmatrix} 0 & \beta^*_{12}(s) \\ 0 & \beta^*_{22}(s) \end{pmatrix} \qquad I-\beta^*(s) = \begin{pmatrix} 1 & -\beta^*_{12}(s) \\ 0 & 1-\beta^*_{22}(s) \end{pmatrix}$$

and $H^*(s) = \frac{1}{s}\, \beta^*(s)\left[I-\beta^*(s)\right]^{-1}$

$$= \frac{1}{s(1-\beta^*_{12}(s))} \begin{pmatrix} 0 & \beta^*_{12}(s) \\ 0 & \beta^*_{22}(s) \end{pmatrix}$$

Hence, $$H^*_{12}(s) = \frac{\beta^*_{12}(s)}{s(1-\beta^*_{22}(s))}$$

$$H^*_{22}(s) = \frac{\beta^*_{22}(s)}{s(1-\beta^*_{22}(s))}$$

$$\lim_{u\to\infty} M_{12}(u) = \lim_{s\to 0} s\, H^*_{12}(s) = \frac{a_{12}}{1-a_{22}}$$

$$\lim_{u\to\infty} M_{22}(u) = \lim_{s\to 0} s\, H^*_{22}(s) = \frac{a_{22}}{1-a_{22}} = \frac{1}{1-a_{22}} - 1$$

which are consistent with the results in § 2.6.

Finally some results for the length Y of a cluster of i^{th} type, i.e the time between the first and last events of the cluster, are derived.

Theorem 5: The distribution $D_Y(y) = P(Y \leq y)$ of the length of a cluster satisfies the equation,

$$D_Y^{(j)}(y) = \exp\left\{ -\sum_{j=1}^{k} a_{ji} + \sum_{j=1}^{k} \int_0^y D_Y^{(j)}(y-t)\beta_{ji}(t)dt \right\} \qquad y \geq 0$$
$$= 0 \qquad y < 0 \tag{2.8.19}$$

Proof: This follows once again from (2.8.4) by taking

$$s(x) = \begin{cases} 1 & \text{for } x \leq y \\ 0 & x > y \end{cases}$$

Then $F^{(i)}(s(\cdot))$ is the probability of no events in the i^{th} cluster after time y, i.e $P(Y \leq y)$ because $F^{(i)}(s(\cdot)) = E(\exp \int \sum_{i=1}^{k} \lg s_i(t)dN_i(t))$ and the random variable $\exp \int \sum_{i=1}^{k} \lg s_i(t)dN_i(t)$ takes the value 1 if there are no any events after y and the value 0 otherwise. The probability of no subsidiary events is $D_Y^{(i)}(0) = e^{-\sum_{j=1}^{k} \alpha_{ji}}$

The above Theorem applied in the case of the 2-variate mutually exciting processes with $\beta_{11}(u) = \beta_{21}(u) = 0$ gives,

$$D_Y^{(1)}(y) = \begin{cases} 1 & y \geqslant 0 \\ 0 & y < 0 \end{cases}$$

$$D_Y^{(2)}(y) = \begin{cases} \exp\left\{-a_{12} - a_{22} + \int_0^y D_Y^{(2)}(y-t)\,\beta_{22}(t)dt + \int_0^y \beta_{12}(t)dt\right\} & y \geqslant 0 \\ 0 & y < 0 \end{cases} \qquad (2.8.20)$$

and therefore $D_Y^{(2)}(0) = \Pr$ (No subsidiary events) $= e^{-a_{22}}$.

Equation (2.8.20) may also be solved by repeated numerical integration since (2.8.7) takes the form,

$$D_{Y,n}^{(2)}(y) = \begin{cases} \exp\left\{-a_{12} - a_{22} + \int_0^y D_{Y,n-1}^{(2)}(y-t)\,\beta_{22}(t)dt + \int_0^y \beta_{12}(t)dt\right\} & y \geqslant 0 \\ 0 & y < 0 \end{cases}$$

with initial conditions $D_{Y,0}^{(2)}(y) = \begin{cases} 1 & y \geqslant 0 \\ 0 & \text{otherwise} \end{cases}$

CHAPTER 3

ESTIMATION OF THE SECOND ORDER PROPERTIES OF THE STATIONARY POINT PROCESSES

3.1. Introduction

Data which come from a point process can be analysed by using the estimates of the first and second order moments and their distributional properties. There are two general ways of statistical analysis.

(i) Exploratory analysis, in which no particular model is considered and the gross characteristics of the data being examined,

(ii) Analysis in which specific models are to be tested against the data and their parameters are to be estimated.

In the first case the exploratory analysis may be used to suggest a relevant model or to discover the physical mechanism which generates the data. In this kind of analysis the investigation usually starts with an examination of the existence of trends in the data.

A plot, for instance, of the cumulative number of events against the time can give evidence of trends. The slope of the line which joins two points A and B in the cumulative plot, is the mean rate of occurrence for the period t_B-t_A.

If there is no evidence of trends it can be assumed that the process is stationary and the next step is to examine if a Poisson or a renewal model is consistent with the data. The Poisson hypothesis is tested by using the Anderson-Darling statistic, which is,

$$W_n^2 = -n - \frac{1}{n} \sum_{i=1}^{n} \left[(2i-1)\lg \frac{t_i}{t_o} + (2(n-i)+1)\lg(1-t_i/t_o)\right] \qquad (3.1.1)$$

where t_i is the time to the i^{th} event and t_o the whole interval of the observation. The above test is not consistent against a certain alternative and is most sensitive to trend alternatives.

Another test for the Poisson hypothesis against renewal alternative in which the intervals X_i, $i = 1,2,...n$ between successive events have Gamma distribution with index ≥ 2, is the Moran one whose statistic is,

$$L_n = \frac{2n(\lg \bar{X} - \frac{1}{n} \sum_{i=1}^{n} \lg X_i)}{1 + \frac{n+1}{6n}}, \qquad (3.1.2)$$

and which is distributed as a chi-square variate with n- 1 d.f. (Cox and Lewis, 1966, p.150-161).

The existence of serial correlation can be also examined by using tests based on the statistical properties of the estimates of the autocovariance or autocorrelation function and of the spectrum of which a brief outline is given later. If again there is no evidence of serial correlation, as a next step the empirical distribution function of intervals and its main characteristics can be estimated.

In the second case of statistical analysis a special model, which comes as a result of an exploratory analysis or from previous knowledge is considered. The adequacy of this particular model can be examined again by tests based on the second order properties of the interval and the counting processes.

In what follows the stationary discrete time series X_i, $i = 1,2,...,n$, which consists of the times between n+1 successive events, and the counting process $\{N(t)\}$, which is continuous and consists of the number of events up to time t, are always considered.

For the computing the SASEIV program developed by Lewis, Katcher and Weiss (1969) is used. The progrem is written entirely in Fortran IV, it is designed to run in an I.B.M. 360 system under O.S 360 provided the system has 512K bytes, and has been run for the present investigation under M.T.S.

system at the computer unit of University of Durham. Firstly the estimation of the autocovariance function is examined.

3.2. Estimation of the autocovariance and autocorrelation functions

Given the process X_t, $t = 1,2,\ldots,n$ the usual estimates of the autocovariance function are,

$$\text{(i)}\quad \hat{\gamma}(s) = \begin{cases} \dfrac{1}{n}\sum\limits_{i=1}^{n-s}(X_i-\bar{X})(X_{i+s}-\bar{X}) & s \leq n \\ 0 & s > n \end{cases} \qquad (3.2.1)$$

$$\text{(ii)}\quad \hat{c}(s) = \begin{cases} \dfrac{1}{n-s}\sum\limits_{i=1}^{n-s}(X_i-\bar{X})(X_{i+s}-\bar{X}) & s \leq n \\ 0 & s > n \end{cases} \qquad (3.2.2)$$

where $\bar{X} = \sum\limits_{i=1}^{n} X_i/n$. If the process X_t, $t = 1,2,\ldots$ has zero mean then the expectations of the above estimates are,

$$E(\hat{c}(s)) = \gamma(s) \quad \text{and} \quad E(\hat{\gamma}(s)) = (1-\frac{s}{n})\gamma(s) \qquad (3.2.3)$$

Therefore in this case, the estimate $\hat{c}(s)$ is umbiased while the $\hat{\gamma}(s)$ is asymptotically umbiased.

Estimates of the autocorrelation function are obtained by dividing the estimates of the autocovariance function by the estimate of the common variance of the process, thus,

$$\hat{\rho}(s) = \frac{\hat{c}(s)}{\hat{c}(0)} \qquad (3.2.4)$$

The plot of the function $\hat{\rho}(s)$, $s = 0,1,\ldots,$ is called correlogram, which specifies the process to the same extent as the periodogram does. Whether it is preferable to think in terms of the periodogram or the correlogram depends upon a number of considerations among which the physical context is of prime importance (Hannan, 1960).

The variance of the estimate (3.2.4) is given by Cox and Lewis (1966) p. 92, and is,

$$V(\hat{\rho}(s)) \simeq \frac{1}{n-s}\left(1+2\sum_{i=1}^{\infty}\rho^2(i)\right) \qquad (3.2.5)$$

As far as the distribution properties of $\hat{\rho}(s)$ are concerned no general theorems about the asymptotic normality of $\hat{\rho}(s)$ appear to be available. The problem according to Hannan (1960), can be reduced to that of the joint asymptotic normality of $\hat{c}(s)$ and $\hat{c}(0)$, since if the distribution of the $\sqrt{n-t}(\hat{c}(t)-\gamma(t))$ tends to the normal distribution that of $\sqrt{n-t}(\hat{\rho}(t)-\rho(t))$ will also tend to normality. In testing the lack of serial dependence the statistic $\sqrt{n-1}\ \hat{\rho}(1)$ can be used which for $\rho(1) = 0$ and large n has normal distribution with mean zero and variance one. Moran (1970) showed that $n^{1/2}\ \hat{\rho}(1)$ converges in distribution to a unit normal variate if the first four moments of X_i, $i = 1,2,\ldots$ exist.

The program SASE IV gives in separate columns the s, $\hat{\rho}(s)$ and $\sqrt{n-s}\ \hat{\rho}(s)$. It also plots the normalized serial correlation coefficient $\sqrt{n-s}\ \hat{\rho}(s)$ versus the lag s.

3.3. Periodogram of the intervals and the estimation of the Power spectrum

One approach to the estimation of the power spectrum is by using the periodogram of intervals which is defined as the quantity

$$I_X(\omega) = \frac{1}{2\pi n}\left\{A_X^2(\omega)+B_X^2(\omega)\right\} \qquad (3.3.1)$$

where $A_X(\omega) = \sum_1^n X_t\cos(\omega t)$, $B_X(\omega) = \sum_1^n X_t\sin(\omega t)$ and $-\pi \leq \omega \leq \pi$

If the process under consideration, $\{X_t\}$, has zero mean, then the following immediate results are obtained:

(i) The periodogram is the Fourier transform of the sample autocovariance function $\hat{\gamma}(s)$.

Since, $I_X(\omega) = \frac{1}{2\pi n}\left\{A_X^2(\omega)+B_X^2(\omega)\right\}$

$$= \frac{1}{2\pi n}\left\{\sum_1^n X_u e^{i\omega u}\sum_1^n X_u e^{-i\omega u}\right\}$$

$$= \frac{1}{2\pi n} \sum_{1}^{n} \sum_{1}^{n} X_u X_v e^{-i\omega(v-u)}$$

$$= \frac{1}{2\pi n} \sum_{s=-n+1}^{n-1} \sum_{u} X_u X_{u+s} e^{-i\omega s}$$

$$= \frac{1}{2\pi} \sum_{-n+1}^{n-1} \hat{\gamma}(s) e^{-i\omega s} \qquad (3.3.2)\ (a)$$

(ii) The periodogram is asymptotically umbiased estimate of the power spectrum.

Since, $E(I_X(\omega)) = \frac{1}{2\pi} \sum_{-n+1}^{n-1} E(\hat{\gamma}(s)) e^{-i\omega s}$ by (3.3.2)

$$= \frac{1}{2\pi} \sum_{-n+1}^{n-1} \left(1 - \frac{|s|}{n}\right) \gamma(s) e^{-i\omega s} \qquad \text{by (3.2.3)}$$

and therefore, $\lim_{n \to \infty} E(I_X(\omega)) = \frac{1}{2\pi} \sum_{-\infty}^{+\infty} \gamma(s) e^{-i\omega s} = f(\omega)$

(iii) If the data contains a periodic term of period ω_0 the periodogram will have a peak at ω_0 and subsidiary peaks at $\omega = \omega_0 + \frac{2\omega_0}{n}$, (Granger and Hatanaka, 1964). This property makes the periodogram a very efficient tool in discovering hidden periodicities in the data under analysis. However the main disadvantage of the periodogram is its inconsistence as an estimate of the power spectrum. This can be demonstrated in a special case where the X_i's are independent and normally distributed with variance σ^2 and zero mean. In this case and for $\omega_k = \frac{2\pi k}{n}$, $k = 0,1,2,\ldots n/2$ the following results are obtained:

$$E(A_X(\omega_k)) = E(B_X(\omega_k)) = 0$$

$$Var(A_X(\omega_k)) = E(A_X^2(\omega_k)) = \sigma^2 \sum_{1}^{n} \cos^2(\omega_k t) = \begin{cases} \sigma^2 n/2 & \text{for } k \neq 0, n/2 \\ \sigma^2 n & k = 0, n/2 \end{cases}$$

$$Var(B_X(\omega_k)) = E(B_X^2(\omega_k)) = \sigma^2 \sum_{1}^{n} \sin^2(\omega_k t) = \begin{cases} \sigma^2 n/2 & k \neq 0, n/2 \\ 0 & k = 0, n/2 \end{cases}$$

$$Cov(A_X(\omega_k), A_X(\omega_\lambda)) = Cov(A_X(\omega_k), B_X(\omega_\lambda)) = Cov(B_X(\omega_k), B_X(\omega_\lambda)) = 0$$

and therefore:

(i) For $k \neq 0$, $n/2$ the r.v $Y(\omega_k) = \frac{A_X^2(\omega_k)}{Var(A_X(\omega_k))} + \frac{B_X^2(\omega_k)}{Var(B_X(\omega_k))} \doteq \frac{4\pi I_X(\omega_k)}{\sigma^2}$

is distributed as a chi-square variate with 2 d.f. Consequently

$E(\frac{4\pi I_X(\omega_k)}{\sigma^2}) = 2$ and $E(I_X(\omega_k)) = \frac{\sigma^2}{2\pi} = f(\omega_k)$

which means that $I_X(\omega_k)$ provides an umbiased estimate of the spectrum of the process under consideration at the points $\omega_k = \frac{2\pi k}{n}$, $k = 0,1,\ldots,n/2$.

Further, $Var(\frac{4\pi I_X(\omega_k)}{\sigma^2}) = 4$ and $Var(I_X(\omega_k)) = \frac{\sigma^4}{4\pi^2}$ which simply means that the $I(\omega_k)$ is an inconsistent estimate of $f(\omega_k)$, since $E|I_X(\omega_k)-f(\omega_k)|^2 \not\to 0$ as $n \longrightarrow \infty$.

(ii) For $k = 0$, $n/2$ $B(\omega_k) = 0$, and therefore the r.v $Y(\omega_k) = \frac{2\pi I_X(\omega_k)}{\sigma^2}$ is distributed as a chi-square with 1 d.f.

(iii) The r.v's $Y(\omega_k) = \frac{4\pi I_X(\omega_k)}{\sigma^2}$, $k = 0,1,\ldots,n/2$ are mutually independent which explains the erratic behaviour of the plot of the periodogram.

The above results were derived for the angular frequencies $\omega_k = \frac{2\pi k}{n}$ under the assumption that the process $\{X_t\}$ is normal white noise. We can generalize them for all the frequencies and for non-Normal processes.

Thus the following results are obtained by Jenkins and Watts (1969), p. 239:

(i) If the X_i's are independent and have normal distribution the r.v $\frac{4\pi I_X(\omega)}{\sigma^2}$ is exactly distributed as a chi-square with 2 d.f for all ω.

(ii) If the X_i's are independent but not normally distributed the r.v $\frac{4\pi I_X(\omega)}{\sigma^2}$ is approximately distributed as a chi-square with 2 d.f, for all ω as $n \longrightarrow \infty$.

(iii) The variance of $I_X(\omega)$ is always dominated by a constant term which remains finite as $n \longrightarrow \infty$. Therefore the periodogram $I_X(\omega)$ is always an inconsistent estimate of the power spectrum $f(\omega)$. However the variance of the estimate can be reduced by using smoothing procedures as follows:

If $w(u)$ is a symmetrical function, which is zero for $|u|>M$ where $M<n$, and $w^*(s) = \frac{1}{2\pi}\sum_{-\infty}^{+\infty} w(u)e^{-ius}$ its Fourier transform, the smoothed estimate of the spectrum is defined as:

$$\hat{f}(\omega) = \frac{1}{2\pi}\sum_{-M}^{M} w(k)\hat{\gamma}(k)e^{-ik\omega}$$

$$= \int_{-\infty}^{+\infty} w^*(s)I_X(\omega-s)ds \qquad (3.3.2)\ (b)$$

where the function $\hat{\gamma}(k)$ is defined in (3.2.1) and (3.3.2)(a).

The function $w(u)$ is called the lag window and its Fourier transform $w^*(s)$ the spectral window.

Examples of lag windows which are widely used in spectral analysis are given in Table 3.1 and their plots in Figure 3.1.

The main distributional properties of the smoothed estimate of the power spectrum are given by Jenkins and Watts (1969) p. 246-254, which briefly are:

(i) The mean value of the estimate for n large is:

$$E(\hat{f}(\omega)) = \int_{-\infty}^{+\infty} w^*(s)E(I_X(\omega-s))ds \simeq \int_{-\infty}^{+\infty} w^*(s)f(\omega-s)ds \qquad (3.3.3)$$

The bias $B(\omega) = E(\hat{f}(\omega))-f(\omega)$ decreases as the truncation point M increases.

(ii) The variance of the estimate is:

$$Var(\hat{f}(\omega)) \simeq \frac{f^2(\omega)}{n}\int_{-\infty}^{+\infty} w^2(u)du = f^2(\omega)\,\frac{I}{n} \qquad (3.3.4)$$

where $I = \int_{-\infty}^{+\infty} w^2(u)du$.

From the Table 3.2 can be seen that for the most of the main windows the ratio $\frac{I}{n}$ is linear function of the truncation point M. Hence the variance decreases as the M decreases.

(iii) The quantity $b = \frac{1}{I}$ is called the bandwidth of the lag window and from (3.3.3),

$$\mathrm{Var}(\hat{f}(\omega) \times b) = \text{constant}.$$

Hence small variance is associated with large bandwidth and large variance with small bandwidths. The values of b for different windows are given in Table 3.2.

(iv) The random variable $\frac{\nu \hat{f}(\omega)}{f(\omega)}$ is distributed approximately as chi-square with $\nu = 2nb$ degrees of freedom. Therefore the 100(1-a)% confidence interval will be $\left(\frac{\nu \hat{f}(\omega)}{X^2_\nu(1-a/2)}, \frac{\nu \hat{f}(\omega)}{X^2_\nu(a/2)}\right)$. If the spectral estimates are plotted on logarithmic scale then the confidence interval for the spectrum is simply represented by a constant interval about the spectral estimate.

Now according to the second of the above properties the variance of the estimate can be reduced by reducing the truncation point M.

However according to the first one by reducing M the bias of the estimate is increased while the resolvability of the spectrum is decreased, i.e ability to discriminate between the values of $f(\omega)$ for different frequencies for a given sample size n; which is the main objective in the estimation of the spectrum.

Much work has been done by Parzen (1961a), Priestley (1962), Daniels (1962) and others aimed at choosing the best lag-window which will effect a suitable compromise between the variance and the bias of the estimate of the spectrum.

The above theoretical results assisted by empirical ones show that the important question in spectral analysis is not the choise of the window lag, but the choice of the truncation point M. A compromise can be achieved by trying different values of M usually in the range $\frac{1}{20} < \frac{M}{n} < \frac{1}{3}$. If it is required to detect detail of width α or more in the spectrum the bandwidth of the lag window has to be chosen such that $b \leqslant \alpha$. In the case, for example, of the Parzen window, Table 3.2, $b = \frac{1.86}{M}$ and therefore must take $M \geqslant \frac{1.86}{\alpha}$.

Another point which has to be stressed is the fact that it is sufficient to estimate the spectrum at the points $\omega_j = \frac{\pi j}{M}$, $j = 0, 1, 2, \ldots, M$, since there are only M+1 mathematically independent values of $\hat{f}(\omega)$ in $(0, \pi)$ and any other values of $\hat{f}(\omega)$ serving only to connect these M+1 values. (Cox and Lewis, 1966, p. 107).

Hence if someone is interested in some particular frequency, say $\frac{2\pi}{t}$, then he has to look at the frequency point and its neighborhood for which

$$\omega_j = \frac{\pi j}{M} \simeq \frac{2\pi}{t} \quad \text{or} \quad j = \frac{2M}{t}.$$

Finally the quantity $s_p = \sum_{j=1}^{p} I'_X(j)$, $I'_X(j) = \frac{I_X(j)}{f(j)}$, or equivalently the quantity,

$T_p = \sum_{j=0}^{p} I'_X(j) \Big/ \sum_{j=0}^{n/2} I'_X(j) = \frac{s_p}{s_{n/2}}$, where $s_{n/2}$ is given equal to its expectation $\left(n \cdot \frac{\sigma^2}{4\pi} \right)$ may be tested against its expectation $E(T_p) \simeq \frac{2p}{n}$ by checking that the deviation of T_p does not exceed the $\pm \lambda(a)/\sqrt{n/2}$ boundary obtained by the two-sided Kolmogorov-Smirnov test. The probability of remaining within the boundary is, $\sum_{-\infty}^{+\infty} (-1)^s \, e^{-2\lambda^2 s^2}$ with $\lambda(0.05) = 1.36$ and $\lambda(0.01) = 1.63$. These results can be used for testing the adequacy of the proposed models (Bartlett, 1966, p. 329).

The SASE IV program by using the Parzen lag window gives in separate columns:

The smoothed estimates of the density spectrum for three different values of the truncation point M, the unsmoothed periodogram divided by $\hat{\sigma}_X^2$ and the normalized cumulative periodogram. It also gives a simultaneous plot of the estimated spectral density, smoothed over three different values of M versus the index J and the plot of the normalized cumulative periodogram again versus J.

Table 3.1 Lag and spectral windows

Description	Lag window	Spectral window
Rectangular	$W_R(u) = \begin{cases} 1 & \lvert u\rvert \leq M \\ 0 & \lvert u\rvert > M \end{cases}$	$W_R^*(\omega) = 2M\left(\frac{\sin(\omega M)}{\omega M}\right), \ -\pi \leq \omega \leq \pi$
Bartlett	$W_B(u) = \begin{cases} 1 - \frac{\lvert u\rvert}{M} & \lvert u\rvert \leq M \\ 0 & \lvert u\rvert > M \end{cases}$	$W_B^*(\omega) = M\left(\frac{\sin\frac{\omega M}{2}}{\frac{\omega M}{2}}\right), \ -\pi \leq \omega \leq \pi$
Parzen	$W_P(u) = \begin{cases} 1-6\left(\frac{u}{M}\right)^2 + 6\left(\frac{\lvert u\rvert}{M}\right)^3 & \lvert u\rvert \leq \frac{M}{2} \\ 2\left(1 - \frac{\lvert u\rvert}{M}\right)^3 & \frac{M}{2} < \lvert u\rvert \leq M \\ 0 & \lvert u\rvert > M \end{cases}$	$W_P^*(\omega) = \frac{3}{4} M \left(\frac{\sin(\frac{\omega M}{4})}{\frac{\omega M}{4}}\right)$ $-\pi \leq \omega \leq \pi$

Table 3.2 Properties of spectral windows

Description	Variance ratio I/n	Degrees of freedom	Bandwidth
Rectangular	2 M/n	n/M	0.5/M
Bartlett	0.667 M/n	3 n/M	1.5/M
Parzen	0.539 M/n	3.71 n/M	1.86/M

3.4. Estimation of the mean rate of occurrence of events and of the Variance-time curve

If n is the number of events observed in a period of length t_o, an umbiased estimate of the mean rate of occurrence of events is

$$\hat{\lambda} = \frac{n}{t_o} \tag{3.4.1}$$

The variance of the estimate is

$$\mathrm{Var}(\hat{\lambda}) = \frac{V(t_o)}{t_o^2} \tag{3.4.2}$$

which tends to $\frac{V'(\infty)}{t_o}$ as $t_o \longrightarrow \infty$. Since for long series the precision of $\hat{\lambda}$ can be estimated from the limiting slope of the Variance-time curve.

The Variance of the number of events in the interval (0,t) or the Variance-time curve, $V(t)$, can be estimated in two different ways:

(i) By using the estimates of the covariance between the numbers of events in two intervals of equal length,

(ii) By using a moving average over a number of possible intervals of length t.

Let the whole interval $(0, t_0)$ be divided in s equal intervals of length τ, such that $s\tau \leqslant t_0$, and let n_i be the number of events in the i^{th} interval. Then in the first case, (i), if $C_i(\tau)$ is the covariance between the numbers of events in two intervals of length τ, seperated by i- 1 other similar intervals, an estimate of $C_i(\tau)$ is,

$$\hat{C}_i(\tau) = \frac{1}{k-i}\sum_{j=1}^{k-i} n_j n_{j+i} - \frac{1}{(k-i)^2}\left(\sum_{j=1}^{k-i} n_j\right)\left(\sum_{j=1}^{k-i} n_{j+i}\right), \quad i = 1,2,\ldots,k-1 \qquad (3.4.3)$$

and therefore, according to the result,

$$V(t) = V(r\tau) = rC_0(\tau)+2\sum_{j=1}^{r-1}(r-j)C_j(\tau)$$

an estimate of $V(t)$ is

$$\hat{V}(t) = r\hat{C}_0(\tau)+2\sum_{j=1}^{r-1}(r-j)\hat{C}_j(\tau) \qquad (3.4.4)$$

(Cox and Lewis, 1966, p. 115).

In the second procedure, which has been suggested by Cox and Smith (1953), in order to estimate the variance-time curve, $V(t)$, at the point $t = r\tau$ where $r \leqslant s$, the following random variables are firstly defined:

$$V_1^{(r)} = n_1+n_2+\ldots+n_r$$

$$V_2^{(r)} = n_2+n_3+\ldots+n_{r+1}$$

$$\ldots\ldots\ldots\ldots$$

$$\ldots\ldots\ldots\ldots$$

$$V_{rk-r+1}^{(r)} = n_{rk-r+1}+\ldots+n_{rk}$$

where $k = \frac{t_o}{t}$ and each of the $V^{(r)}$ r.v's is the number of events occurring in an interval of length $t = r\tau$. Hence the estimate of V(t) can be formed by the corrected sum of squares of the r.v's $V_i^{(r)}$, $i = 1,2,\ldots,rk-r+1$, i.e

$$\hat{\hat{V}}(t) = A\left[\sum_{i=1} (V_i^{(r)})^2 - \frac{1}{M}\left(\sum_{i=1}^{M} V_i^{(r)}\right)^2\right] \qquad (3.4.5)$$

where A is a constant and $M = rk-r+1$. In the case of a Poisson process, if $A = \frac{3M}{3M^2-3Mr+r^2-1}$, the estimate $\hat{\hat{V}}(t)$ is umbiased. If $r = 1$ then $t = \tau$ and the coefficient A takes the usual form $A = \frac{1}{k-1}$.

The interval τ has to be chosen in such a way that the average number of events in an interval of length τ is less or equal to 2. The moments of the two estimates $\hat{V}(t)$ and $\hat{\hat{V}}(t)$ and also a general comparison between them are given by Cox and Lewis (1966), p. 117-118.

The SASEIV program gives the estimates of the Variance-time curve V(t) and of the ratio $V(t)/\lambda t$ for $t = r\tau$, $r=1,2,\ldots,k<8000$, and also plots the V(t) versus the time t.

3.5. The periodogram of the counts and the estimation of the Bartlett's spectrum

One of the ways in which the periodogram of the counts arises is as the modulus squared of the Fourier-Stieltjes transform of the sample function N(t) of an observed series of events.

Assumming that the series of events has observed for a period of length t_o during which n events occur at times $t_1, t_2, \ldots, t_n$ the Fourier-Stieltjes transform of the function N(t), is,

$$J_{t_o}(\omega) = \frac{1}{(\pi t_o)^{1/2}} \int_0^{t_o} e^{it\omega} dN(t)$$

$$= \frac{1}{(\pi t_o)^{1/2}} \sum_{j=1}^{n} e^{itj\omega}$$

$$= \frac{1}{(\pi t_o)^{1/2}} \left\{ \sum_{j=1}^{n} \cos(t_j\omega) + i \sum_{j=1}^{n} \sin(t_j\omega) \right\}$$

$$= \frac{1}{(\pi t_o)^{1/2}} \left\{ A_{t_o}(\omega) + i\, B_{t_o}(\omega) \right\} \qquad (3.5.1)$$

where $A_{t_o}(\omega) = \sum_{j=1}^{n} \cos(t_j\omega)$, $B_{t_o}(\omega) = \sum_{j=1}^{n} \sin(t_j\omega)$ and n is the observed value of the random variable $N(t_o)$. The divisor $(\pi t_o)^{1/2}$ in (3.5.1) is arbitrary, as far as the Fourier theory is concerned, but is essential to give the periodogram the right properties as an estimator of $g_+(\omega)$.

Consequently according to the definition in the begining of this section the periodogram of series of events is,

$$I_{t_o}(\omega) = J_{t_o}(\omega)\ \overline{J_{t_o}(\omega)}$$

$$= \frac{1}{\pi t_o} \left[A^2_{t_o}(\omega) + B^2_{t_o}(\omega) \right]$$

$$= \frac{1}{\pi t_o} \sum_{k=1}^{n} \sum_{\lambda=1}^{n} e^{-i(t_\lambda - t_k)\omega}$$

$$= \frac{1}{\pi} \left[\frac{n}{t_o} + \frac{2}{t_o} \sum_{s=1}^{n-1} \sum_{j=1}^{n-s} \cos \omega (t_{s+j} - t_j) \right] \qquad (3.5.2)$$

The periodogram of counts $I_{t_o}(\omega)$ is an asymptotically umbiased estimate of the spectrum of counts $g_+(\omega)$, since from (3.5.1)

$$I_{t_o}(\omega) = \frac{1}{\pi t_o} \int_0^{t_o}\int_0^{t_o} e^{-i(t_2-t_1)\omega}\, dN(t_1)dN(t_2) \quad \text{and by (2.2.5)}$$

$$E(I_{t_o}(\omega)) = \frac{1}{\pi t_o} \int_0^{t_o}\int_0^{t_o} e^{-i(t_2-t_1)\omega} (\mu(t_1-t_2) + \lambda\delta(t_1-t_2) + \lambda^2)\, dt_1\, dt_2$$

$$= \frac{1}{\pi t_o} \left[\int_{-t_o}^{0} e^{-iu\omega}(\mu(u)+\lambda\delta(u)+\lambda^2)\,du \int_0^{u+t_o} dt_1 + \int_0^{t_o}\int_u^{t_o} e^{-iu\omega}(\mu(u)+\lambda\delta(u)+\lambda^2)\,du\,dt_1 \right]$$

$$= \frac{1}{\pi t_o} \int_{-t_o}^{t_o} (t_o - |u|)(\mu(u)+\lambda\delta(u)+\lambda^2)\, e^{-iu\omega}\, du$$

Therefore,

$$\lim_{t_o\to\infty} E(I_{t_o}(\omega)) = \frac{1}{\pi} \left[\int_{-\infty}^{+\infty} \mu(u) e^{-iu\omega} du + \lambda + \lambda^2 \int_{-\infty}^{+\infty} e^{-iu\omega} du \right] =$$

$$\simeq \frac{1}{\pi}\left[\int_{-\infty}^{+\infty}\mu(u)e^{-iu\omega}\,du+\lambda\right] = g_+(\omega)$$

The Poisson process plays the role in the spectral analysis of Point processes which a normal white noise plays in ordinary time series. For the Poisson process with rate λ the following results are obtained:

(i) If ω is a multiple of $2\pi/t_o$, then $A_{t_o}(\omega)$ and $B_{t_o}(\omega)$ are uncorrelated random variables with a distribution which tends to a normal distribution with mean zero and variance $\lambda/2\pi$ as $t_o \longrightarrow \infty$.

(ii) As a consequence $I_{t_o}(\omega)$ has asymptotically an exponential distribution with mean λ/π and standard deviation λ/π (i.e $I_{t_o}(\omega)$ is proportional to a chi-squared variable with two degrees of freedom).

This property can be extended for processes which are derived from Poisson processes. Thus Bartlett (1963b) has shown for the doubly stochastic Poisson process and branching Poisson process, that $I_{t_o}(\omega)$ has asymptotically an exponential distribution with mean $g_+(\omega)$. Therefore the quantity $I'_{t_o}(\omega) = \dfrac{I_{t_o}(\omega)}{g_+(\omega)}$ is a random variable asymptotically proportional to a chi-square one with two degrees of freedom. Hence under the assumption, $E(I_{t_o}(\omega)) = g_+(\omega)$, the r.v $I'_{t_o}(\omega)$ becomes,

$$I'_{t_o}(\omega) \simeq \frac{1}{2}\chi^2_2 \qquad (3.5.3)$$

For general values of ω, the expectation of the periodogram of counts is,

$$E(I_{t_o}(\omega)) \simeq \frac{\lambda}{\pi}+\lambda^2 t_o \left(\frac{\sin(\frac{\omega t_o}{2})}{\omega t_o/2}\right) \qquad (3.5.4)$$

Thus the bias of the estimate is zero for non-zero multiples of $\frac{2\pi}{t_o}$ and has its maximum value at $\omega = 0$. That is why the estimation of $g_+(\omega)$ is difficult near $\omega \simeq 0$.

(iii) The exact variance of $I_{t_o}(\omega)$ for a Poisson process and for ω multiple of $\frac{2\pi}{t_o}$ is,

$$\mathrm{Var}(I_{t_o}(\omega)) = \frac{\lambda^2}{\pi^2}\left(1+\frac{1}{\lambda t_o}\right) \qquad (3.5.5)$$

which simply says that $I_{t_o}(\omega)$ is an inconsistent estimate of the spectrum of counts.

(iv) The exact correlation between periodogram values in two different values of ω, which are multiples of $\frac{2\pi}{t_o}$ is,

$$\mathrm{Corr}(I_{t_o}(\omega_1),\ I_{t_o}(\omega_2)) = \frac{1}{1+\lambda t_o} \qquad (3.5.6)$$

(Cox and Lewis, 1966, p. 127).

Thus the situation for the Poisson process is not quite the same as for the normal white noise in the case of time series. The $I_{t_o}(\omega_1)$ and $I_{t_o}(\omega_2)$ behave like the variables X_1+Z and X_2+Z where the X's are mutually uncorrelated with mean $\lambda/2\pi$ and variance λ^2/π^2 and are uncorrelated with the variable Z, which has mean zero and variance $\frac{\lambda}{\pi^2 t_o}$.

In the applications, the normalized periodogram of counts is usually computed, which is,

$$I'_{t_o}(\omega) = \frac{I_{t_o}(\omega)}{\hat{\lambda}} \quad \text{and by (3.5.2)}$$

$$I'_{t_o}(\omega) = \frac{t_o}{n} \cdot \frac{1}{\pi t_o} \sum_{k=1}^{n} \sum_{\lambda=1}^{n} e^{-i(t_\lambda - t_k)\omega}$$

$$= \frac{1}{n\pi} \sum_{k=1}^{n} \sum_{\lambda=1}^{n} \exp\left\{-i(t_\lambda - t_k) \cdot \frac{2\pi}{t_o}\right\}$$

$$= \frac{1}{n\pi} \sum_{k=1}^{n} \sum_{\lambda=1}^{n} \exp\left\{-i\,\frac{2\pi}{n} \cdot \frac{n}{t_o}(t_\lambda - t_k)\right\}$$

$$= \frac{1}{n\pi} \sum_{k=1}^{n} \sum_{\lambda=1}^{n} \exp\left\{-i\,\frac{2\pi}{n}(t'_\lambda - t'_k)\right\} \qquad (3.5.6)$$

where $t'_\lambda = \frac{n}{t_o} t_\lambda$ and $t'_k = \frac{n}{t_o} t_k$ and the $I'_{t_o}(\omega)$ is computed at the usual points $\omega_j = \frac{2\pi j}{n}$, $j = 1,2,\ldots,n/2$.

According to the property (iii), the $I'_{t_0}(\omega)$ is an inconsistent estimate of $g(\omega)$. Therefore the estimate has to be smoothed. The object of smoothing is, as in the case of the periodogram of time series, to produce an estimate of the spectrum whose variance and bias at any point ω decrease as t_0 increases, and whose values at separate frequencies ω_1 and ω_2 are correlated. The last requirement is the reason for the use of the name "smoothing".

Intuitively one feels that the true spectrum does not vary without constraint from one frequency to another and it is reasonable for the estimate to behave similarly. All the spectral windows of Table 3.2 may be used. The uniform lag window is usually used, because of its simplicity in computation, or a quadratic weighting scheme which has put forward by Bartlett (1963).

In the case of a uniform lag window the smoothed estimate of $g_+(\omega)$ is given by the formula

$$I''_{t_0}(\omega_c) = \sum_{p=c-k/2+1/2}^{c+k/2+1/2} \frac{I'_{t_0}(\omega_p)}{k} \tag{3.5.7}$$

where $\omega_p = \frac{2\pi p}{n}$, k the number of points or frequencies ω_p over which the smoothing is performed and c is an integer if k is odd, and integer plus a half if k is even. (Cox and Lewis, 1966, p. 130).

The main properties of the smoothed spectrum are for n large and constant k,

(i) $E(I''_{t_0}(\omega_c) \simeq g_+(\omega_c)$

(ii) $Var(I''_{t_0}(\omega_c)) \simeq \frac{g_+^2(\omega_c)}{k}$

(iii) The $I''_{t_0}(\omega_c)$ approximately is proportional to a chi-square variate with 2k degrees of freedom.

There are certain difficult points which arise with the estimation of the spectrum of counts. The main ones are:

(i) It is not possible to use the fast Fourier transform algorithm, introduced by J.W.Cooley and J.W.Tukey (1965), to compute the periodogram of counts, since the random variable t_j occurs in the exponent in (3.5.2), while in the case of periodogram of intervals it appears as a multiplier of a power of the n^{th} root of unity. Therefore the computation time is proportional to n^2 where n is the number of events in the interval $(0, t_0)$. This can be a severe limitation on the use of this type of spectral analysis.

(ii) A problem arises in using the spectrum of the point process, particularly in tests of fit of models to data, because while the spectrum of counts is unlimited in extent, compared with the spectrum of intervals which is a periodic function, there are only n computationally independent values in the estimated spectrum. Statistically speaking there are, in a rough sense, no more than n degrees of freedom available for tests. Consequently if more than n/2 of the estimated points are used in a test then the distribution theory of the tests, if it is based on the points being approximately chi-square with two degrees of freedom variates, is no longer valid (Lewis, 1970).

It is easy to see that the spectrum of counts, the variance-time curve and the covariance density are mathematically equivalent functions. The main advantage of the spectrum is that its sampling theory is easier than that of the others. Unfortunately its physical meaning is not so obvious as in the case of the spectrum of intervals. The only point process in which the spectrum of counts has the analogue interpretation of the power spectrum is possibly the doubly Poisson process.

Finally the Program SASEIV computes and prints out in adjacent columns the quantities: J, $A_{t_0}(J)$, $B_{t_0}(J)$, $I'_{t_0}(J)$ and the average of successive sets of two, three, four, five, ten and twenty $I'_{t_0}(J)$'s. It also gives a simultaneous plot of the estimated spectrum smoothed over k = 5, 10 and 20, versus the index J.

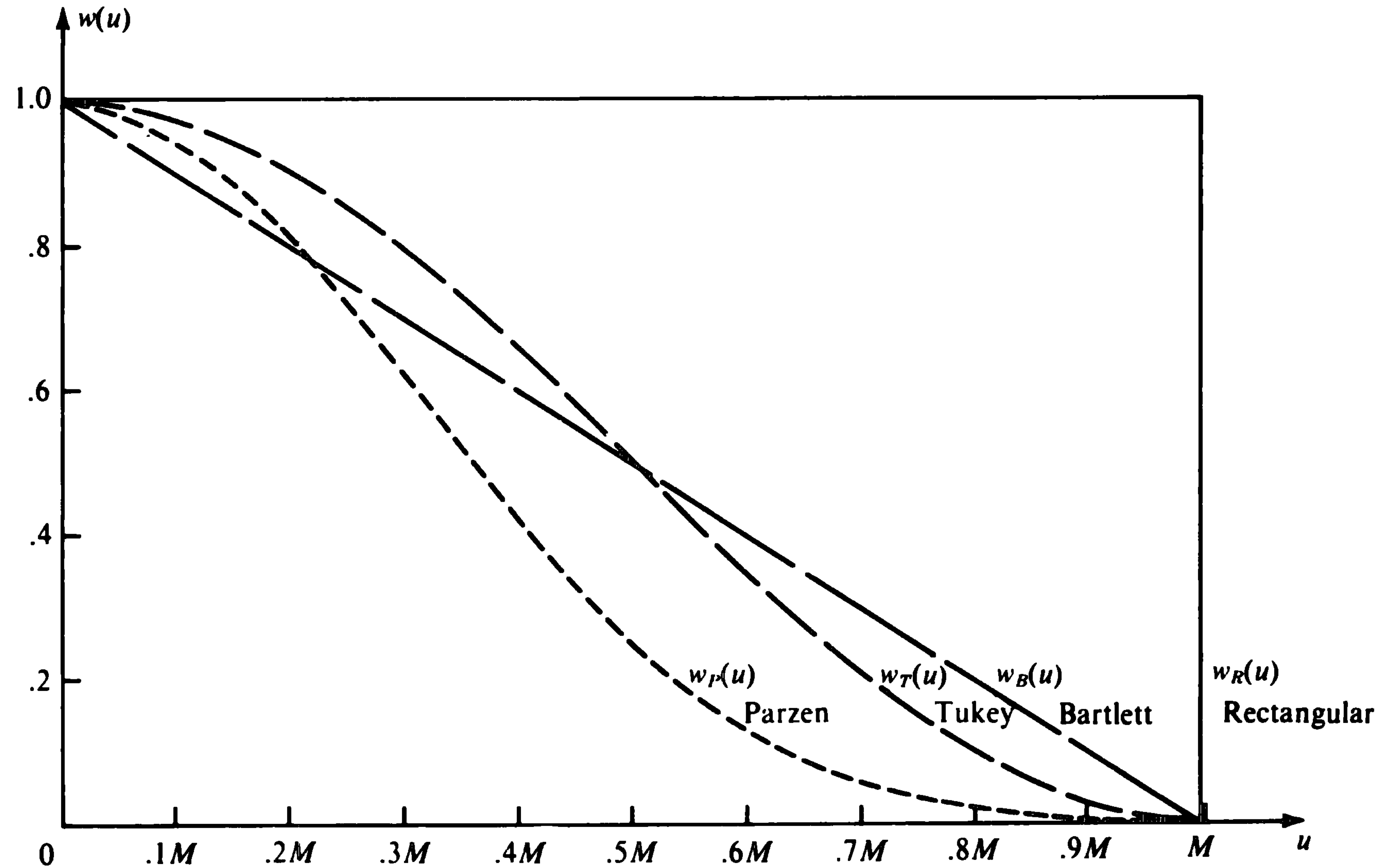


Fig. 3.1. Some of the main Lag-Windows.

CHAPTER 4

ANALYSIS OF THE DATA

4.1. Origin of the data

The data under investigation are sets of time intervals between successive earthquakes in units of 1000 minutes, which occured during the period from January 1950 until February 1971 in 14 different areas. Appendix gives the 14 sets of data with the geographical coordinates of each area. These data have been extracted from a list of earthquakes which has been provided by the Institute of Geological Sciences of Edinburgh. This list which contains earthquake records from the whole of the Earth and for the period 1800-1971 was mounted on a magnetic tape with Data control block: FB, LRECI = 80, BLCSIZE = 4,000.

The first problem which one is faced with in attempting a general statistical survey of earthquakes from different areas is to decide on the boundaries of the areas. In order to obtain a complete homogeneous set of data, questions of instrumental coverage have to be considered very carefully. Vere-Jones (1970) has taken only earthquakes with $M \geqslant 4.5$ in order to ensure uniform coverage. In the present study, since the local magnitude of the earthquakes was not available, the decision about the boundaries of the areas under investigation was made in consultation with the department of Geophysics of the University of Durham for the best representation of the main active earthquake areas and by studying scatter diagrams of shocks with their longitude and latitude as coordinates.

Another problem was the fact that the list under consideration contained more than one record of the same shock from different recording stations without any discriminating character. The problem was resolved by deciding that records in which the origin times differed by less than 1 minute came from the same shock. A random sample indicates that the records under question are originated from the same position.

The last main problem was the possibility of non-stationarity. Fifure 4.1 shows a plot of the numbers of shocks in a time period of 3 months for the area of North Atlantic; the situation is almost the same for the rest of the areas. The extremely complex structure of the records which is revealed in the above plot is not unrelated with the problem of aftershocks, since periods of increased activity are often marked by the occurrence of very large aftershock sequences or swarms. Therefore in the present study the apparent short term non-stationary character of the earthquake records is attributed to the clustering effect which is considered as an essential feature of the earthquake activity. There is also some indication of a slight long term trend, probably due in part to increased instrumental coverage and therefore better detection. However, the magnitude of this is small compared to the local fluctuations which will dominate the analysis so that no attempt will be made to eliminate trend.

4.2. Exploratory analysis

As has been mentioned earlier the first objective of the present study is the fitting of the most suitable stochastic model describing the earthquake occurrences for each of the areas under consideration. By regarding the sequence of occurrences of earthquakes as a realization of a one-dimensional stationary process and by using the results in chapter 3, as a starting point, an exploratory analysis has been done.

Table 4.1 shows for each of the areas the results obtained for the moments of the distribution of the intervals, the results of the MORAN and ANDERSON-DARLING tests and the estimate of the normalized autocorrelation function. The high value of the estimate of the coefficient of variation, C, indicates divergence from the Poisson hypothesis, for which $C = 1$, and shows up clustering effect. The results of the MORAN and ANDERSON-DARLING tests support the above conclusion since they are highly significant for all the areas under consideration at a very low significance level. Another argument in favour of rejection of the Poisson hypothesis is the shape of the smoothed periodogram of counts. Figures 4.11-4.14 give the smoothed periodograms of counts for all the areas which have been obtained by using weighting lag window with k = 10, see § 3.5. By comparing these graphs with the theoretical spectrum of the Poisson process, which is $g_{+}(\omega) = \frac{\lambda}{\pi}$, again the rejection of the Poisson hypothesis is supported.

Next the adequacy of a renewal model is examined. None of the first ten estimates of the autocorrelation function, for all the areas, is consistent with the renewal hypothesis. For example Table 4.1 gives the values of the statistic $\hat{\rho}(1)(n-1)^{1/2}$ which under certain assumptions has normal distribution with zero mean and variance one (§ 3.2). The value of the normalized autocorrelation coefficient with lag 1 is for all the areas higher than 1.96, which is the upper two-sided five per-cent significance point of the standard normal distribution. Another indication against the renewal hypothesis is the shape of the smoothed periodogram of intervals which has been obtained by using the Parzen window.

Figure 4.2 shows the smoothed periodogram of intervals for the area of Spain with truncation point M = 60 and the 95% confidence limits. Again the shape of the smoothed periodogram for the above area, favours the rejection of the renewal hypothesis for which the normalized spectrum of intervals is $\frac{1}{\pi}$. The situation appears to be the same for the other areas. The main characteristic of the periodogram of intervals is a high peak near the origin, which indicates either a trend in the series or long term persistence due to low frequencies variations at periods above the length of the series (20 years). The periodogram of the counts has also a high peak at the origin which is consistent with the shape of the periodogram of intervals. The shape of the periodogram of intervals does not reveal any kind of periodicity for any of the areas under investigation.

4.3. Fitting of the Neymann-Scott model

The rejection of the Poisson and renewal models and the way in which most of the earthquakes occur, a main shock followed by a sequence of aftershocks, suggests a clustering model. In this model the origin times of earthquakes are considered as a realization of a one-dimensional stationary point process in which the main earthquakes correspond to the cluster centres and the aftershocks or swarms to the members of the subsidiary process. The fitting of the Neymann-Scott model with exponential decay function, which has been used by Vere-Jones (1970) to describe the occurrence of earthquakes in New-Zealand, is firstly examined. A major problem with fitting any kind of clustering model is the estimation of its parameters. Vere-Jones and Davies (1966) use the Least-squares method to fit the spectrum to the periodogram ordinates. The main disadvantage of this method is the fact that the least-squares method has optimal properties for normal Variates while the periodogram ordinates, although approximately independent, are exponentially rather than normally distributed. Lewis (1969) and others estimate the parameters of their models by equating the theoretical and estimated second order moments. A more satisfactory method of estimating the parameters of the model, when the fitting of the spectrum is considered, is the maximum likelihood one, which unfortunately has two main disadvantages. Firstly the expression of the likelihood function is too complicated in the case of the cluster models and secondly, since only the approximate likelihood can be evaluated, a sufficient ammount of information from the data cannot be utilised. Whittle (1952) has developed a general theory of maximum-likelihood estimation of the spectrum but the obtained formulae are not easy to apply. Generally if $\underset{\sim}{\theta}^T = (\theta_1, \ldots, \theta_k)$ is the parametric vector, $\underset{\sim}{y}$ the observed sample and $L(\underset{\sim}{\theta}; \underset{\sim}{y})$ the likelihood function, the estimation of the parameters by the maximum likelihood method is converted to the solution of the simultaneous likelihood equations,

$$\frac{\partial L}{\partial \vartheta_i} = 0, \quad i = 1,2,\ldots,k \qquad (4.3.1)$$

which are usually non-linear. The solution of the (4.3.1) can be done either by applying iteration processes, (e.g. Kale, 1962) or by minimizing the function,

$$\varphi(\underset{\sim}{\vartheta}) = \sum_{i=1}^{k} \left\{ \frac{\partial L}{\partial \vartheta_i} \right\}^2 \qquad (4.3.2)$$

However the main problem in the solution of simultaneous non-linear equations by iteration procedure is the difficulty in succeeding convergence. According to Vere-Jones and Davies (1966), with as many as 100 ordinates of the spectrum it seems likely that any two reasonable methods of estimation of the parameters of the model will lead to similar estimates although the least-squares method will tend to give more weight to points for which the spectrum is large, than would the maximum likelihood method.

In the present study the estimates of the parameters are obtained by maximization of the approximate likelihood function of the periodogram of counts. The normalized theoretical spectrum of counts in the case of the Neymann-Scott model with exponential decay function is, (§2.7(A1))

$$F(j) = \pi g_+(j)/\lambda = 1 + \frac{\beta\rho^2}{\rho^2 + w(j)} \qquad (4.3.3)$$

where $\beta = \frac{E(N(N+1))}{1+E(N)}$, N is the cluster size, ρ is the parameter of the exponential decay function, $w(j) = \frac{4\pi^2 j^2}{M^2}$, $j = 1,2,\ldots,\frac{M}{2}$ and M is the number of the observations. Now given that, the unsmoothed estimate of the normalized spectrum of counts, $I_{t_0}(j)$, is asymptotically exponentially distributed with mean $F(j)$, and independent for frequencies of the above form (§ 3.5), the likelihood for M/2 ordinates of the periodogram is,

$$L(\beta,\rho;\ I_{t_0}(j),\ j = 1,2,\ldots,M/2) = \prod_{j=1}^{M/2} \frac{1}{F(j)}\, e^{-I_{t_0}(j)/F(j)} \qquad \text{or}$$

$$l = -\lg_e L = \sum_{1}^{M/2} \left\{ -\lg \frac{1}{F(j)} + I_{t_0}(j) \cdot \frac{1}{F(j)} \right\} \qquad (4.3.4)$$

If $\frac{1}{F(j)} = G(j)$ the first and second degree derivatives of l are given by the formulae,

$$\frac{\partial l}{\partial x_i} = \sum_{1}^{M/2} (I_{t_o}(j)-F(j)) \frac{\partial G(j)}{\partial x_i}, \quad i = 1,2$$

$$\frac{\partial^2 l}{\partial x_i \partial x_k} = \sum_{1}^{M/2} \left\{ F^2(j) \frac{\partial G(j)}{\partial x_i} \frac{\partial G(j)}{\partial x_k} - (F(j)-I_{t_o}(j)) \frac{\partial^2 G}{\partial x_i \partial x_k} \right\} \quad i,k = 1,2 \qquad (4.3.5)$$

where $x_1 = \beta$ and $x_2 = \rho$. From (4.3.5) the observed information matrix can be obtained and consequently the errors of the estimates. The maximum likelihood estimates of β and ρ are obtained by minimizing numerically the quantity $l = -\log_e L$ using Rosenbrock's (1960) optimization technique. Initial set of estimates for the parameters is obtained by using the Monte-Carlo method. By applying the normal theory results of the maximum likelihood estimates the errors are estimated. The mean rate of occurrence of the sum process, λ, is estimated by $\hat{\lambda} = \frac{M}{t_o}$ where M is the total number of events occurring in the fixed time interval t_o. Table 4.2 gives the estimates of β, ρ and λ. The ~~time-life~~ life time parameter $1/\rho$ has been rescaled by λ to convert to real time (§ 3.5 p. 50).

Figure 4.3 gives a contour graph of the function $l = -\log_e L$ for the area of South Atlantic, which is consistent with the results obtained by Rosenbrock's optimization method. As a check of the adequacy of the fit the values of $I_{t_o}(j)$ for each area were rescaled by dividing with $F(j)$. According to § 3.5 the ratio $I'_{t_o}(j) = \frac{I_{t_o}(j)}{F(j)}$ is distributed as a half chi-squared variate with two degrees of freedom. Table 4.7 gives for each area the percentage of the ratios $\frac{I_{t_o}(j)}{F(j)}$ which are less than 0.05 and larger than 3.0 and which are respectively the one half of the lower and of the upper 5% point of the Chi-squared variate with two degrees of freedom. The results do not indicate diversions from the hypothesis $E(I_{t_o}(j)) \simeq F(j)$. By plotting the cumulative periodograms,

$$S(q) = \sum_{j=1}^{q} I'_{t_o}(j), \qquad q = 1,2,\ldots,M/2$$

and the P = 0.01 and 0.05 boundaries it may be checked whether the spectrum has been made uniform, (§ 3.5). It can be seen from the cumulative periodograms that the fit of the Neymann-Scott model with exponential decay function is satisfactory for all the areas except those of Aleoutian Islands, Fox Islands, Sundarc, Bandasea, Tonga Islands, Greece and Himalayas.

In searching for a more adequate model for the data, since both long and short term effects were suspected from the shape of the periodograms of intervals and counts, the fit of Neymann-Scott model with the mixed exponential decay function is examined next. In order to make easier the optimization problem the cluster centre is not considered in the evaluation of the normalized theoretical spectrum of counts which in this case is,

$$F(j) = \frac{\pi g_+(j)}{\lambda} = 1 + \beta \frac{\rho_1^2 \rho_2^2 + [\gamma\rho_1 + (1-\gamma)\rho_2]^2 w(j)}{(\rho_1^2 + w(j))(\rho_2^2 + w(j))}$$

where $\beta = \frac{E(N(N-1))}{E(N)}$, N is again the cluster size, γ, ρ_1 and ρ_2 are the parameters of the exponential decay function and $w(j) = \frac{4\pi^2 j^2}{M^2}$ for $j = 1,2,\ldots,\frac{M}{2}$ (§ 2.7 A2). Therefore,

$$l = -\log_e L(\beta,\gamma,\rho_1,\rho_2;\ I_{t_o}(j),\ j = 1,2,\ldots,M/2) = \sum_1^{M/2} \left[\lg F(j) + \frac{I_{t_o}(j)}{F(j)} \right] \quad (4.3.6)$$

and the maximum likelihood estimates of the parameters of the model are obtained by using the same procedure as in the previous case. The elements of the observed information matrix are given by,

$$\frac{\partial^2 \ell}{\partial x_i \partial x_k} = \sum_1^{M/2} \left[F^2(j) \frac{\partial G(j)}{\partial x_i} \frac{\partial G(j)}{\partial x_k} - (F(j) - I_{t_o}(j)) \frac{\partial^2 G(j)}{\partial x_i \partial x_k} \right] \quad i,k = 1,2,3,4. \quad (4.3.7)$$

where $x_1 = \gamma$, $x_2 = \beta$, $x_3 = \rho_1$, $x_4 = \rho_2$ and $G(j) = \frac{1}{F(j)}$. The errors of the estimates are obtained from the observed information matrix.

In order to get more reliable confidence intervals for the parameters under consideration, we firstly calculate confidence intervals for the transformed ones for which the likelihood is more symmetrical. A sensible transformation which makes the likelihood more symmetrical is,

$$\gamma \rightarrow \phi(\gamma) = \log\left[\frac{\gamma}{1-\gamma}\right],\ \beta \rightarrow \phi(\beta) = \beta,\ \rho_1 \rightarrow \phi(\rho_1) = \rho_1^{1/3} \text{ and } \rho_2 \rightarrow \phi(\rho_2) = \rho_2^{1/3}$$

(Edwards, 1972, p.80).

Now by using the well known result,

$$\text{s.e } \hat{\phi}(x) = (\text{s.e } \hat{x})\frac{d\phi}{dx} \qquad \text{(Jenkins and Watts, 1969),}$$

we get the 95% confidence intervals, $\phi_1(\cdot),\ \phi_2(\cdot) = \hat{\phi}(\cdot) \pm 1.96$ s.e $\hat{\phi}(\cdot)$ for the transformed variables, and by inverting the so obtained confidence intervals, the 95% ones for the original variables are obtained. Table 4.3 gives the estimates of the original variables with the 95% confidence intervals.

The results in Table 4.7, as in the case of the Neymann-Scott model with exponential decay do not indicate serious diversions from the hypothesis $E(I_{t_o}(j)) \simeq F(j)$. Figures 4.4 - 4.10 show the cumulative periodograms of counts with the $p = 0.01$ and 0.05 boundaries for the areas for which the fit of the Neymann-Scott model with exponential decay is not satisfactory; the periodogram ordinates have been rescaled by the theoretical spectra. These graphs demonstrate that the Neymann-Scott model with mixed exponential decay is adequate for the areas under question while the one with single exponential decay is not. The cumulative periodograms for the rest of the areas also give evidence of a satisfactory fit of the Neymann-Scott model with mixed exponential as well as in the case of Neymann-Scott with single exponential.

Another element in favour of the Neymann-Scott model with mixed exponential is the shape of the spectrum of counts. The smoothed periodograms of counts with the theoretical spectra of the models under consideration for all the areas are shown in figures 4.11 - 4.14. It can be seen from these graphs that while the spectrum of the Neymann-Scott model with single exponential is flat near the origin the spectrum of Neymann-

Scott with mixed exponential has a peak similar to the one of the periodogram of counts. That again proves that the Neymann-Scott model with mixed exponential is consistent with the main characteristic of the data which is the presence of a long term clustering effect for all the areas under investigation. This is related to the high values of the estimates of $R = 1+\beta$ obtained by fitting the Neymann-Scott model with the mixed exponential compared with the low ones obtained by fitting the Neymann-Scott one with the single exponential. Table 4.6 gives the predicted values of R by the two models and also the estimated asymptotic slope of the Variance-time curve and estimates of the normalized periodogram of counts at the origin.

The Variance-time curve is estimated by using the method of Cox and Smith (1953) which is described in § 3.4. According to Cox and Lewis (1966, p.118) the estimated Variance-time curve for $t > t_0/4$, where t_0 is the length of the observation interval, will start to decrease or increase rapidly. For this reason the Variance-time curve is estimated on the interval $(0, t_0/4)$ and therefore the estimates of its asymptotic slope for the different areas, which are given in Table 4.6, must be considered with caution. However, since $R = \lim_{t \to \infty} \frac{V(t)}{\lambda t} = \pi g_+(0)/\lambda$, (§ 2.2.[illegible]), the results of Table 4.6 give a reasonable justification for the high values of β obtained by fitting the Neymann-Scott model with mixed exponential rather than for the low ones obtained by fitting the Neymann-Scott with single exponential. Finally, Table 4.8 gives the values of the quantity $l = -\log_e L$ for all the areas and for the models under investigation. It can be seen from the results of Table 4.8 that the values of $-\log_e L$ for all the areas, which are obtained by fitting the Neymann-Scott model with mixed exponential are lower (consequently the values of L higher) than the ones obtained by fitting the Neymann-Scott with single exponential. These differences are larger in the case of the areas for which the Neymann-Scott with single exponential was not adequate. Since there are two extra parameters a difference of

3.00 (5%) and 4.6 (1%) in $-\lg_e L$ would be regarded as significant limit on X_2^2. These are exceeded in all areas, in most areas very substantially.

To summarize, by considering the above results, it appears that the fit of Neymann-Scott model with mixed exponential is more satisfactory than the fit of Neymann-Scott one with single exponential.

The fit of the Neymann-Scott model with mixed exponential and Gamma gave similar results as the one with mixed exponential. So that the exact form of the decay function does not seem important for the spectrum as long as it has short and long term components. It has not been attempted to use power law, advocated by Vere-Jones, because there is not a satisfactory way of estimating the parameters. However, note that this distribution has roughly similar characteristics of a high initial value and a long tail.

4.4. Fitting of some special mutually exciting processes

The first model which is examined after the Neymann-Scott is the Self-exciting point process which is defined in § 2.6. If a simple exponential decay function is used in this model, the Bartlett spectrum has the same form as the Neymann-Scott process with exponential distribution. Such a self-exciting process will therefore also be an unsatisfactory model. Hence in order to take into account the presence of both short and long term effects the function

$$\beta(u) = \begin{cases} \alpha_1 \rho_1 e^{-\rho_1 u} + \alpha_2 \rho_2 e^{-\rho_2 u}, & u > 0 \\ 0 & \text{otherwise} \end{cases}$$

has been chosen. The normalized spectrum of counts is by (2.6.10),

$$F(j) = \frac{\pi g_+(j)}{\lambda} = \frac{(\rho_1^2+w(j))(\rho_2^2+w(j))}{\left[\rho_1\rho_2(1-\alpha_1-\alpha_2)-w(j)\right]^2 + w(j)\left[\rho_2(1-\alpha_2)+\rho_1(1-\alpha_1)\right]^2}$$

where ρ_1, ρ_2, α_1 and α_2 are the parameters of the model and $w(j) = \dfrac{4\pi^2 j^2}{M^2}$, $j = 1,2,\ldots,M/2$. The parameters of the model are estimated by maximizing the likelihood function for a sample of $M/2$ ordinates of the periodogram of counts as in § 4.3.

Table 4.4 gives the obtained estimates. The values of the quantity $l = -\log_e \mathrm{Lik}$ at the maximum likelihood estimates of the parameters are given in Table 4.8. The values of l for most of the areas lie between the ones obtained for the Neymann-Scott model with single and mixed exponential decay functions. The results regarding the cumulative periodogram of counts are almost the same as in the case of the Neymann-Scott model with single exponential decay function. We therefore conclude that such a self-exciting process is not a good model.

Finally the fit of a special case of the mutually exciting processes, which is defined in § 2.6 and some of its counting and interval properties have been derived in § 2.8, is examined. This model can be also considered as a clustering one of the Neymann-Scott type, with Poisson distribution of cluster size where the process of the cluster centres is self-exciting instead of Poisson, therefore again clustering. Thus we consider main events of second kind, II, and each of the main events generates a subsidiary process consisting of events of the first kind, I, with

$$l(x) = \frac{\beta_{12}(x)}{\int_0^{\infty} \beta_{12}(x)\,dx}, \qquad (\S\ 2.8)$$

Thus the latter model is consistent with the observation made by Utsu and reported by him during the discussion of Vere-Jones paper (1970), according to which there are many instances in Japan and probably in other seismic regions to indicate that the cluster centres also have a tendency of clustering perhaps in a different manner than the aftershocks.

Returning now in the estimation of the parameters of this model, the normalized spectrum is, (§ 2.6)

$$F(j) = \frac{\pi g(j)}{\lambda} = \frac{\alpha_{12}}{1+\alpha_{12}} + \frac{1}{1+\alpha_{12}} \frac{(\rho_2^2 + w(j))\left[\rho_1^2 (1+\alpha_{12})^2 + w(j)\right]}{(\rho_1^2 + w(j))\left[\rho_2^2 (1-\alpha_{22})^2 + w(j)\right]}$$

where α_{12}, α_{22}, ρ_1 and ρ_2 are the parameters of the model and $W(j) = \frac{4\pi^2 j^2}{M^2}$, $j = 1, 2, \ldots, \frac{M}{2}$. The likelihood function and the information matrix are given by (4.3.6) and (4.3.7) respectively. The estimates of α_{12}, α_{22}, ρ_1 and ρ_2 are obtained in the same way as in the previous cases while ν is estimated by solving (2.6.I2) with respect to ν and substituting the other parameters with their estimates. Table 4.5 gives the obtained estimates. From the values of ρ_1 and ρ_2 in Table 4.5 is clear that the cluster centres (large earthquakes) are resulted by the long term effect while the subsidiary events (aftershocks) by the short term effect.

The results in Tables 4.7 and 4.8 and the cumulative periodograms in Fig. 4.4-4.IO show that the behaviour of the mutually exciting process is similar to the one of the Neymann-Scott with mixed exponential. There is not any well established evidence of existence of any significant difference between the two models, as far as the results of the above analysis are concerned. However the mutually exciting process gives more plausible description of the mechanism which generates the earthquakes and this is its main advantage comparing with the Neymann-Scott with mixed exponential. Both the models take into account the existence of short and long term effects.

4.5. Analysis of the intervals

In this section we apply the theory of § 2.8 in order to obtain some results for the distribution of the intervals in the case of the mutually exciting processes. The interval properties of the Neymann-Scott model are not easily derivable and therefore are not used in detailed analysis. The correlation properties of the intervals for both the Neymann-Scott and mutually exciting processes are not available either.

We now observe that,

$R_L(t;x)$ = Prob$(L(x)>t)$ = Pr(No events in $(x,\ x+t)$)

= Prob(No primary events in $(x,\ x+t)$) Prob(No subsidiary processes generated in $(0,x)$ survive past x),

and in the case of equilibrium distribution,

$$R_L(t) = \lim_{x\to\infty} R_L(t;x) = e^{-\nu t}A \qquad (4.5.1)$$

where A is a constant less than one and ν is the rate of occurrence of the primary events. Hence by using (4.5.1) and the results of § 2.8 the survivor function of the time interval between successive events is,

$$P_T(t) = Pr(T>t)$$

$$= -\frac{1}{\lambda}\frac{dR_L(t)}{dt}$$

$$= \frac{1}{\lambda} A\nu e^{-\nu t} \qquad \text{or}$$

$$\lg P_T(t) = \text{constant}-\nu t \qquad (4.5.2)$$

Thus the slope of the tail of the logsurvivor of the intervals is $-\nu$.

By using the last result it is possible to check the fit of the model by comparing the values of ν obtained by using the theoretical survivor with the ones obtained by using the empirical survivor. Table 4.9 gives for all the areas under investigation the values of ν obtained by using the theoretical survivor, the empirical one, and the formula (2.6.12). Table 4.10 gives the values of the survivor of the intervals for the area of Greece and Fig. 4.15 the corresponding curves. The situation is almost the same for the rest of the areas.

The above results of the interval analysis do not favour the mutually exciting processes since the predicted value of ν by the model, is substantially lower than the one obtained by the empirical survivor.

If ν is now estimated by using the tail of the empirical survivor then, by virtue of (2.6.12), the number of parameters of the model is reduced from four to three.

If the same estimation procedure, as in § 4.4, is carried out, then the estimates of the parameters of the model, which are shown in Table 4.II, are obtained. By using these estimates the cumulative periodograms for the areas under investigation can be obtained. Fig. 4.16 shows the cumulative periodograms for the areas Aleoutian Islands, Fox Islands and Greece. The cumulative periodograms for the above areas give reasonable evidence which also does not support the fit of the two variate mutually exciting processes from the spectrum point of view, when the results of interval analysis are taken into account.

Therefore, it can be deduced that although the mutually exciting processes fit the data quite satisfactorily from the viewpoint of the spectrum analysis, the situation is not the same from the viewpoint of the interval analysis.

In the next section, by using the values of the parameters of the Neymann-Scott model with mixed exponential decay, a classification of the areas under investigation is attempted.

4.6 Classification of the areas

The areas under investigation, according to the continental drift theory (§ 1.2), can be considered as representing the following geological types:

A: Atlantic ridge, B: North island arcs, C: South island arcs, D: Himalayas and E: Mediterranean.

One sensible way of examining the existence of similarities or differences within earthquake regions of similar geological type and between different types of earthquake regions is by fitting the same stochastic model in all areas under investigation and comparing the parameters of the model.

Since the evidence so far in the present analysis is in favour of the Neymann-Scott model with mixed exponential decay, this model is employed. Table 4.3 gives for all the areas the estimates of the parameters of the above model and their 95% C.I's. It also gives for each parameter x_{ij}, $i = 1,\dots,5$, $j = 1,\dots,14$ its mean

$$m_i = \frac{\sum_{j=1}^{14} x_{ij}}{14} \text{ and its standard error } \sigma_i = \sqrt{\frac{1}{13}\sum_{j=1}^{14} (x_{ij}-m_i)^2},$$

$i = 1,2,3,4,5$. It can be seen from the results shown in Table 4.3 that the errors of the estimates in some areas are too large relative to the corresponding estimates to allow a safe discrimination. However by looking at the estimates themselves and inspecting some relevant graphs, Fig. 4.I7, we note the following points:

(1) Atlantic ridge, type A, has a very consistent long-term effect of about $3\,^1/2$ years and below average values of the other parameters (except for $1/\rho_1$), the variability coefficient, β, being particularly low and consistent.

(2) Nothern Island arcs, type B, are very consistent for all parameters. The long-term effect $(1/\rho_2)$ is very short, about $1\,^1/2$ months while β and γ are also low.

(3) Southern island arcs, type C, are more complex and show no dissimilarity from the Himalayas. There is high consistency in $1/\rho_1$, (except for 9) being about 2 days, some consistency in $1/\rho_2$ (except for 7) being between 1 and 2 years. They have high variability β (except for 6) while γ is average or above in each area. In all these respects they differ from B.

(4) Mediterranean, group E, is very variable. E(N) is consistently small. There may be some consistency in β which divides them into the eastern pair (areas 11 and 13) and the western one (areas 12 and 14). Otherwise there is no clear pattern. There is not marked similarity with D which would be in agreement with the continental drift theory.

We conclude that there is some evidence of consistency of seismic activity, as reflected in the parameters of the model, within regions of similar geological type while differences between types are observed.

Table 4.1 : Results of the analysis of the series of intervals.

Area	N	E(x)	$\sqrt{V(x)}$	C(x)	$\gamma_1(x)$	$\gamma_2(x)$	l_n	$W_n^2(x)$	$\sqrt{(n-1)}\rho(1)$
1. North Atlantic	506	20.33	33.99	1.67	3.09	16.81	2030.50	39.94	4.30
2. South Atlantic	142	71.56	102.58	1.43	3.41	19.81	392.44	21.24	3.01
3. Central Atlantic	253	40.76	65.91	1.62	3.28	17.04	842.39	47.50	3.37
4. Aleoutian Islands	1260	8.28	17.38	2.09	4.90	37.88	4287.19	91.95	10.74
5. Fox Islands	1035	9.99	20.13	2.01	4.51	30.14	3442.70	53.75	10.79
6. Sunda-arc	602	17.43	29.66	1.70	5.24	40.93	1390.52	64.65	5.29
7. Banda-sea	715	14.56	26.38	1.81	4.60	30.14	1415.90	124.52	10.91
8. Fiji Islands	996	10.48	18.69	1.78	4.60	32.25	2551.46	102.59	9.06
9. Tonga Islands	1877	5.57	10.07	1.81	6.81	74.68	3254.57	251.07	10.82
10. Himalayas	1737	6.08	9.94	1.63	4.58	35.05	3541.73	62.95	7.86
11. Greece	1820	4.18	6.68	1.60	3.55	22.66	5312.43	208.01	9.44
12. Spain	291	34.86	79.57	2.28	5.11	36.80	1119.97	32.76	4.69
13. Yugoslavia	860	12.12	19.61	1.60	3.50	22.24	2474.67	27.71	6.11
14. Italy	584	17.81	30.88	1.73	3.61	21.22	2024.86	32.75	3.08

Notes on Table 4.1

N: Observed number of events.

E(x): Estimate of the mean of the intervals.

$\sqrt{V(x)}$: Estimate of the standard deviation.

C(x): Estimate of the coefficient of Variation.

$\gamma_1(x)$: Estimate of the coefficient of skewness.

$\gamma_2(x)$: Estimate of the coefficient of kurtosis.

$l_n(x)$: Moran Statistic (§ 3.1)

$W_n^2(x)$: Anderson-Darling Statistic (§ 3.1)

$\sqrt{(n-1)}\rho(1)$: Normalized estimate of the first autocorrelation function.

Table 4.2: Estimates of the parameters of the Neymann-Scott model with single exponential decay.

Area	λ	β	ρ	1/ρ In days
1.	0.049	10.196	4.194	3.36
2.	0.014	3.389	5.725	8.68
3.	0.024	7.162	2.988	9.48
4.	0.121	29.383	1.217	4.73
5.	0.100	22.523	1.580	4.52
6.	0.057	4.119	1.533	7.88
7.	0.069	5.237	2.100	4.82
8.	0.095	13.435	3.070	2.37
9.	0.179	6.978	0.680	5.72
10.	0.164	5.090	1.355	3.12
11.	0.239	9.974	1.487	1.96
12.	0.029	20.083	1.163	20.83
13.	0.082	15.506	2.070	4.05
14.	0.056	16.351	3.829	3.22

Table 4.3: Estimates of the parameters of the Neymann-Scott model with mixed exponential decay and their 95% confidence intervals.

Area	Type	β	γ	ρ_1	ρ_2	$1/\rho_1$ In days	$1/\rho_2$ In months	E(N)
1	A	73.136 63.187 83.085	0.647 0.623 0.670	5.605 2.860 9.431	0.011 0.005 0.021	2.52	40.32	3.28
2	A	36.974 28.348 45.600	0.750 0.717 0.780	12.898 1.602 42.160	0.042 0.019 0.083	3.84	39.25	2.18
3	A	61.271 48.696 73.846	0.714 0.682 0.744	5.150 2.147 9.869	0.023 0.013 0.040	5.50	40.66	2.17
4	B	75.440 68.897 81.982	0.613 0.589 0.636	2.310 1.959 2.658	0.157 0.129 0.196	2.49	1.22	3.77
5	B	83.279 75.280 91.278	0.678 0.658 0.697	4.068 2.767 5.582	0.125 0.100 0.159	1.70	1.85	3.81
6	C	53.990 44.598 63.382	0.830 0.814 0.845	5.950 2.322 11.829	0.022 0.014 0.034	2.03	18.67	5.91
7	C	279.560 238.619 320.500	0.898 0.890 0.905	6.700 2.233 14.517	0.007 0.006 0.009	1.42	51.06	2.81
8	C	163.098 146.903 179.293	0.744 0.730 0.757	4.449 3.074 6.019	0.011 0.007 0.018	1.64	21.46	4.22
9	C	364.110 318.552 409.668	0.940 0.936 0.944	4.920 2.341 8.689	0.010 0.007 0.015	0.80	12.68	3.49
10	D	133.180 119.382 146.978	0.894 0.888 0.900	5.650 2.952 9.355	0.011 0.010 0.013	0.79	13.03	4.45
11	E	91.922 84.354 99.489	0.755 0.743 0.766	2.472 2.098 2.839	0.032 0.026 0.041	1.17	2.97	2.97
12	E	140.710 115.665 165.755	0.805 0.783 0.826	3.370 2.143 4.888	0.054 0.040 0.075	7.18	15.02	2.65
13	E	83.534 74.824 92.244	0.588 0.566 0.609	2.250 1.934 2.558	0.010 0.003 0.027	3.74	29.22	2.50
14	E	144.710 126.570 162.850	0.693 0.673 0.712	6.219 1.903 14.110	0.012 0.006 0.022	2.01	34.31	1.94

...(continued)

Table 4.3 (continued):

	β	γ	ρ_1	ρ_2	$1/\rho_1$	$1/\rho_2$	E(N)
Mean	104.637	0.755	5.144	0.038	2.63	22.98	3.25
Variance	10610.527	0.011	7.170	0.002	3.45	264.88	1.24
S. Error	103.007	0.107	2.678	0.046	1.85	16.27	1.11

Tables 4.4: Estimates of the parameters of the Self-exciting process.

Area	α_1	α_2	$1/\rho_1$ In Days	$1/\rho_2$ In Hours
1.	0.331	0.369	0.087	0.216
2.	0.499	0.461	5.730	0.708
3.	0.422	0.232	0.121	0.051
4.	0.338	0.499	0.304	0.027
5.	0.288	0.499	0.256	0.216
6.	0.499	0.382	5.821	1.109
7.	0.499	0.464	13.643	0.042
8.	0.247	0.496	0.146	0.379
9.	0.500	0.360	14.115	1.642
10.	0.500	0.396	8.199	0.306
11.	0.299	0.500	1.256	0.174
12.	0.383	0.500	1.015	0.011
13.	0.462	0.293	0.127	0.052
14.	0.269	0.492	0.102	2.100

Table 4.5: Estimates of the parameters of the mutually exciting process.

Area	α_{12}	α_{22}	ρ_2	ρ_1	$1/\rho_2$ In Months	$1/\rho_1$ In days
1.	8.132	0.992	0.027	5.764	17.17	2.44
2.	1.801	0.990	0.131	10.785	12.66	4.59
3.	3.802	0.986	0.076	47.617	12.44	0.59
4.	14.220	0.527	0.329	2.068	0.58	2.78
5.	8.246	0.624	0.434	4.887	0.53	1.44
6.	1.057	0.860	0.126	12.824	3.20	0.94
7.	1.847	0.963	0.055	20.317	6.08	0.49
8.	9.770	0.897	0.037	4.687	6.53	1.55
9.	0.750	0.999	0.051	4.208	2.54	0.92
10.	1.214	0.907	0.086	9.273	1.64	0.45
11.	5.039	0.666	0.145	2.517	0.67	1.15
12.	6.088	0.816	0.234	3.197	3.44	7.57
13.	13.798	0.651	0.021	2.181	13.48	3.85
14.	13.118	0.966	0.032	5.224	12.83	2.36

Table 4.6: Predicted values of R by the Neymann-Scott model, the estimate of the asymptotic slope of the Variance-time curve, the first normalized ordinate, and the average of the first two ones of the periodogram of counts.

Area	$R = 1+\beta$ Single Exponential	Mixed Exponential	$\lim_{t\to\infty} \frac{V(t)}{\lambda t}$	$\pi I_{t_0}(1)/\lambda$	$(\pi I_{t_0}(1)+\pi I_{t_0}(2))/2$
1.	11.196	74.136	34.141	104.375	58.001
2.	3.389	37.974	15.430	41.822	24.557
3.	8.162	62.271	34.434	96.847	58.598
4.	30.383	76.440	58.006	49.584	131.751
5.	23.523	84.279	54.003	84.008	160.193
6.	5.119	54.990	37.004	142.177	73.088
7.	6.237	280.560	101.701	318.386	159.651
8.	14.435	164.098	100.001	347.260	182.650
9.	7.978	365.110	203.806	754.345	397.307
10.	6.090	134.180	131.002	275.248	197.452
11.	10.974	92.922	72.004	132.787	114.533
12.	21.083	141.710	55.587	139.676	106.630
13.	16.506	84.534	32.769	134.607	71.757
14.	17.351	145.710	62.644	130.175	130.175

Table 4.7: Percentages of the rescaled estimates of the spectrum of counts outside the 95% confidence interval.

Area	< 0.05				> 3.00			
	Single Expo-nential	Mixed Exponen-tial	Self Excit-ing	Mutually Exciting	Single Exponen-tial	Mixed Exponen-tial	Self Excit-ing	Mutually Exciting
1.	5.20	5.20	5.20	5.20	3.60	4.00	3.60	3.60
2.	5.63	5.00	5.00	5.00	6.34	6.43	4.28	6.43
3.	6.15	6.92	6.15	6.15	4.61	4.61	4.61	4.61
4.	3.65	3.97	3.65	3.65	4.13	3.65	4.13	3.65
5.	4.23	3.65	4.23	3.65	5.77	4.61	5.77	4.81
6.	4.00	3.00	3.00	2.67	4.66	6.00	5.66	6.00
7.	6.39	6.66	6.11	6.11	3.89	3.61	4.72	4.17
8.	3.00	2.80	3.00	2.80	2.00	1.60	2.20	2.60
9.	6.91	5.53	5.74	5.21	6.28	4.57	3.40	4.89
10.	6.78	4.71	4.60	4.60	4.48	4.94	5.40	5.17
11.	4.39	4.50	4.28	4.39	4.50	4.94	5.82	5.38
12.	6.00	3.33	4.00	4.00	6.00	5.33	7.33	4.00
13.	3.95	3.72	3.72	3.72	3.48	3.95	4.19	3.49
14.	3.10	3.10	3.10	3.10	3.45	3.10	3.79	3.45

Table 4.8: Values of the quantity $l = -\log_e \text{Lik}$ at the maximum likelihood estimates of the parameters.

Area	Single Exponential	Mixed Exponential	Self-Exciting	Mutually Exciting
1.	818.44	813.40	818.44	812.93
2.	320.24	313.38	313.07	313.05
3.	369.72	360.65	369.20	360.21
4.	2216.54	2197.26	2207.35	2197.33
5.	1819.82	1790.06	1793.29	1789.71
6.	640.05	605.59	605.57	605.54
7.	882.57	822.03	821.63	821.52
8.	1707.68	1682.26	1706.68	1681.72
9.	1897.06	1710.23	1720.48	1708.13
10.	1923.41	1787.99	1789.06	1787.77
11.	2507.22	2463.67	2480.39	2463.01
12.	466.62	448.90	457.97	448.75
13.	1440.36	1435.67	1440.35	1435.73
14.	1064.16	1055.45	1064.14	1054.93

Table 4.9: Estimates of the rate of occurrence of the primary events by using the empirical and theoretical survivor in the case of the mutually exciting processes.

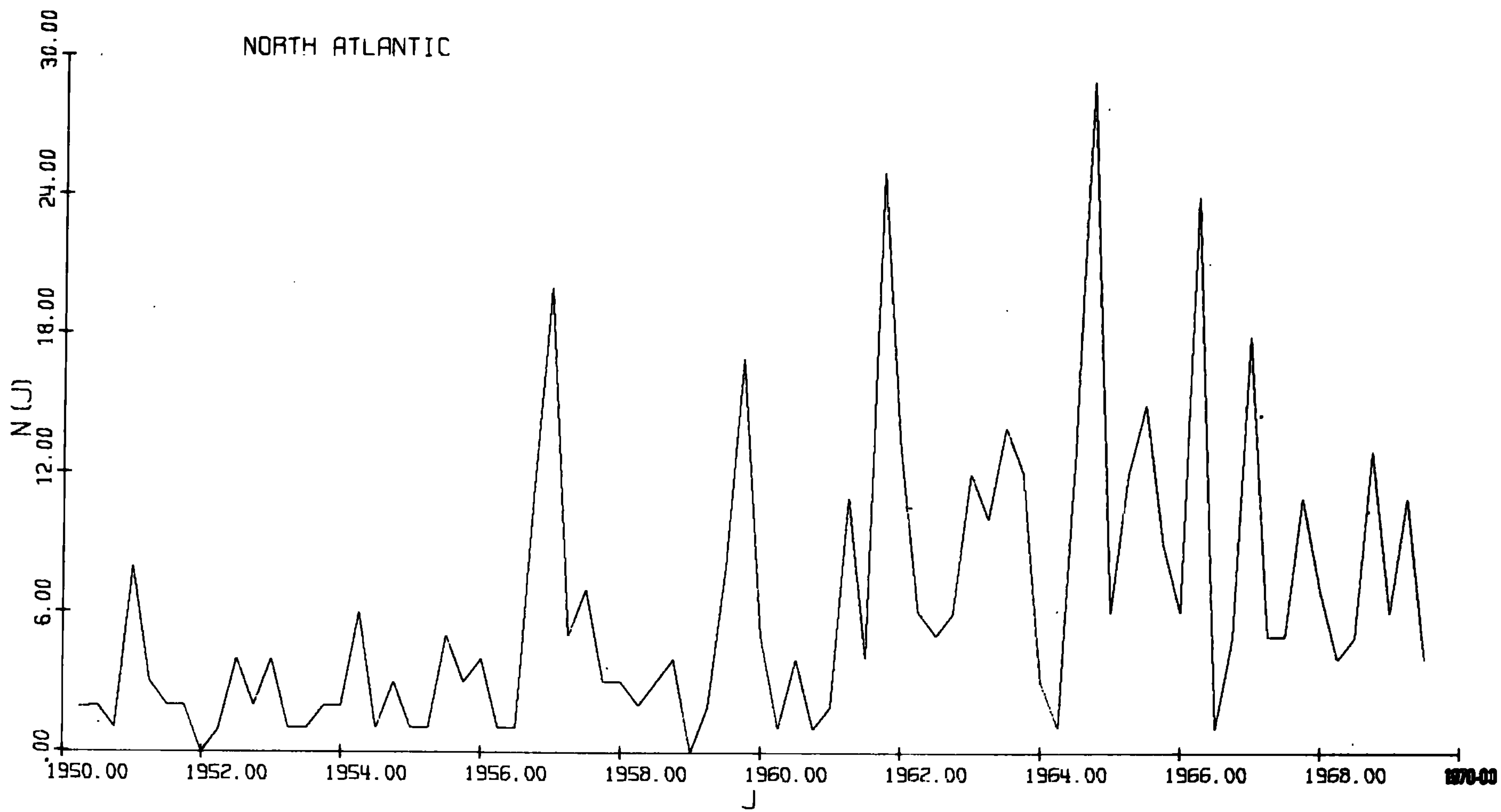
Area	$\nu = \frac{\lambda(1-\alpha_{22})}{1+\alpha_{12}}$	Slope of the tail of the Theoretical survivor	Slope of the tail of the Empirical survivor
1.	0.00005	-0.0002	-0.0153
2.	0.00004	-0.00004	-0.0034
3.	0.00006	-0.00008	-0.0113
4.	0.00373	-0.00372	-0.0320
5.	0.00407	-0.00407	-0.0260
6.	0.00408	-0.00408	-0.0097
7.	0.00092	-0.00092	-0.0237
8.	0.00086	-0.00094	-0.0226
9.	0.00003	-0.0016	-0.0513
10.	0.00670	-0.00670	-0.0370
11.	0.0132	-0.01046	-0.0804
12.	0.00078	-0.00077	-0.0108
13.	0.00188	-0.00227	-0.0327
14.	0.00014	-0.00028	-0.0288

Table 4.IO: Results of the survivor of intervals for the area of Greece.

l	Empirical		Theoretical	
	$P_L(l)$	$Log_e P_L(l)$	$P_L(l)$	$Log_e P_L(l)$
0.02	0.935	-0.068	0.988	-0.010
0.04	0.909	-0.096	0.846	-0.165
0.06	0.889	-0.118	0.735	-0.305
0.08	0.874	-0.135	0.648	-0.432
0.10	0.855	-0.156	0.578	-0.547
0.30	0.764	-0.269	0.315	-1.153
0.50	0.712	-0.340	0.208	-1.567
0.70	0.659	-0.416	0.166	-1.792
0.90	0.626	-0.469	0.145	-1.930
1.00	0.609	-0.496	0.138	-1.980
3.00	0.366	-1.005	0.099	-2.311
5.00	0.243	-1.415	0.079	-2.532
7.00	0.184	-1.692	0.068	-2.692
9.00	0.140	-1.969	0.060	-2.814
10.00	0.122	-2.105	0.057	-2.865
15.00	0.065	-2.744	0.051	-2.980
20.00	0.035	-3.379	0.044	-3.117
25.00	0.022	-3.793	0.040	-3.210
30.00	0.012	-4.416	0.037	-3.282
35.00	0.008	-4.799	0.035	-3.343
40.00	0.004	-5.428	0.033	-3.400
45.00	0.003	-5.898	0.032	-3.454
50.00	0.002	-6.408	0.029	-3.508
55.00	0.001	-6.814	0.028	-3.561
60.00	0.0005	-7.207	0.027	-3.613

Table 4.11 : Estimates of the parameters of the mutually exciting processes when the rate of occurrence of primary events is estimated by the empirical survivor function.

Area	Likelihood	α_{12}	α_{22}	ρ_1	ρ_2
1.	824.003	0.286	0.600	131.923	99.933
2.	318.164	0.102	0.449	22.999	16.332
3.	387.761	0.192	0.450	192.229	99.102
4.	2298.178	0.132	0.700	5.101	10.317
5.	1851.557	0.154	0.700	89.138	10.335
6.	608.790	1.057	0.652	6.088	0.170
7.	880.520	0.400	0.516	34.283	3.350
8.	1707.580	0.247	0.704	21.534	9.920
9.	1801.512	0.589	0.546	98.944	0.128
10.	1830.420	0.558	0.964	97.418	0.200
11.	2620.442	0.500	0.173	168.975	10.743
12.	503.124	0.063	0.600	99.974	11.746
13.	1586.231	0.126	0.550	174.361	99.999
14.	1546.194	0.071	0.450	132.885	98.720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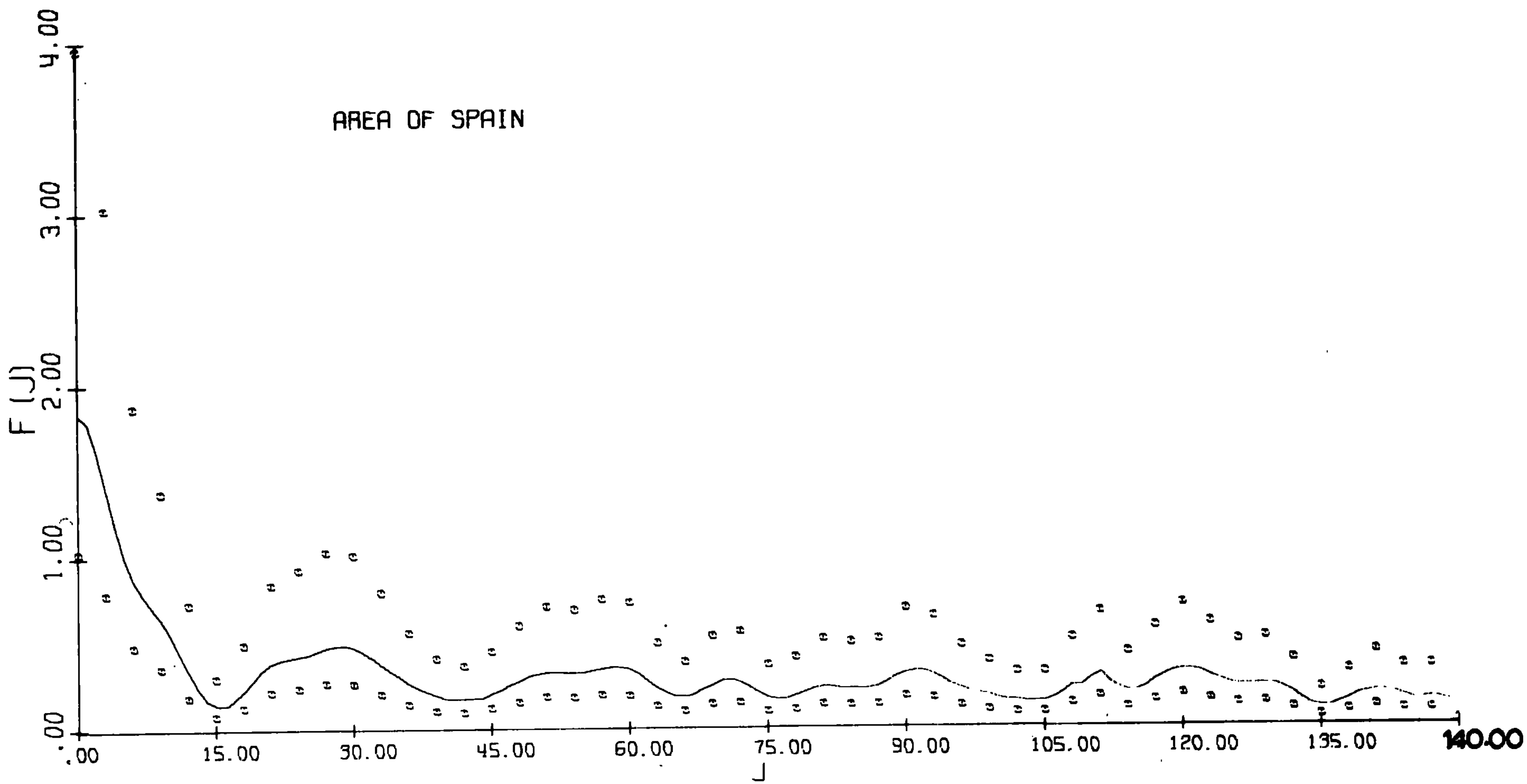


Fig. 4.2.- Smoothed periodograms of intervals with M = 60 and the 95% confidence intervals

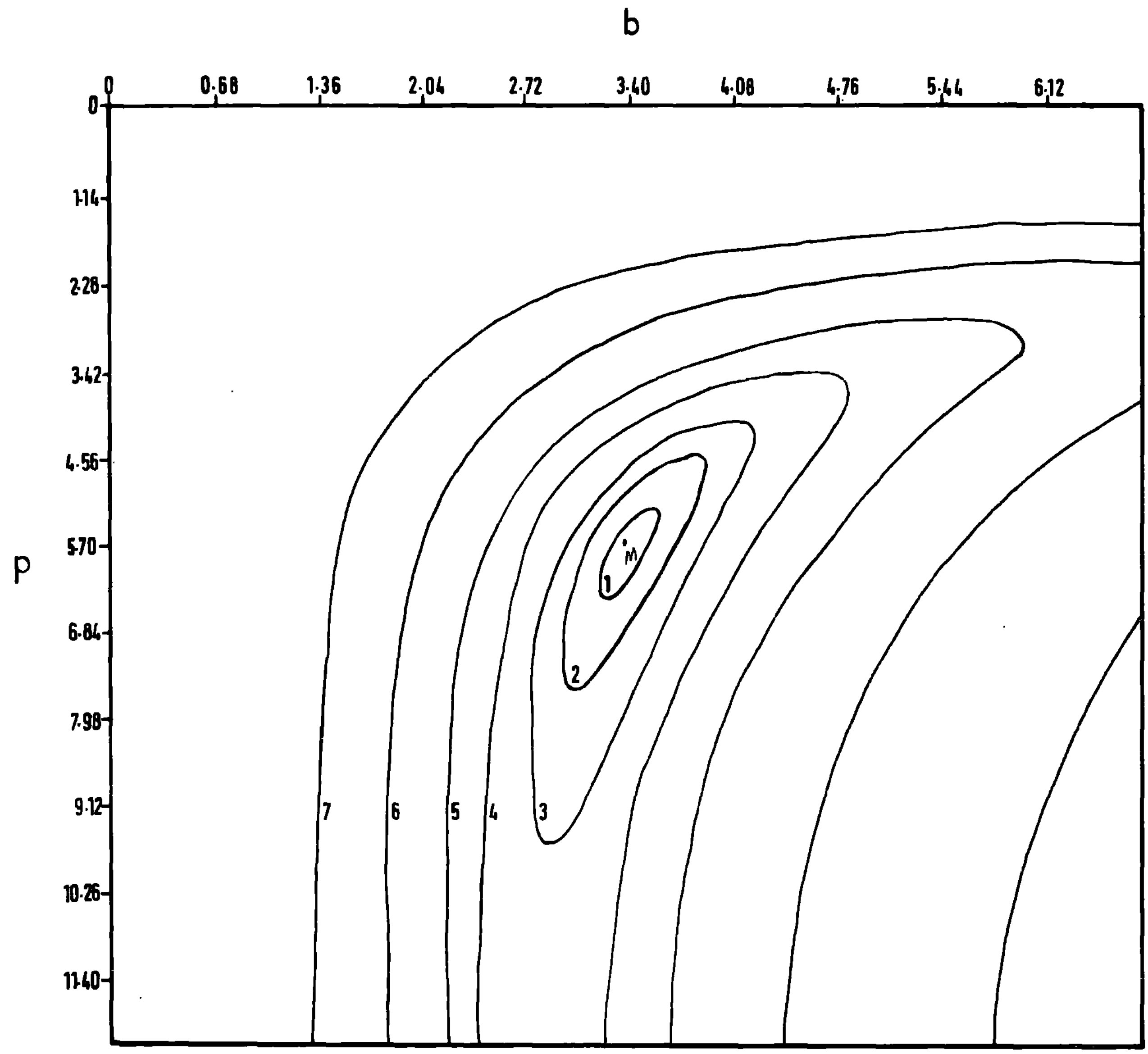


Fig. 4.3.- Contour graph of the function $l = -\log_e Lik(b,p)$ in the case of Neymann-Scott model with single exponential decay applied in the area of South Atlantic. The estimate of (p,b) is at M, the countours from I-4 are at intervals of 0•2 units of I and from 4-7 at intervals of 2•0 units of I.

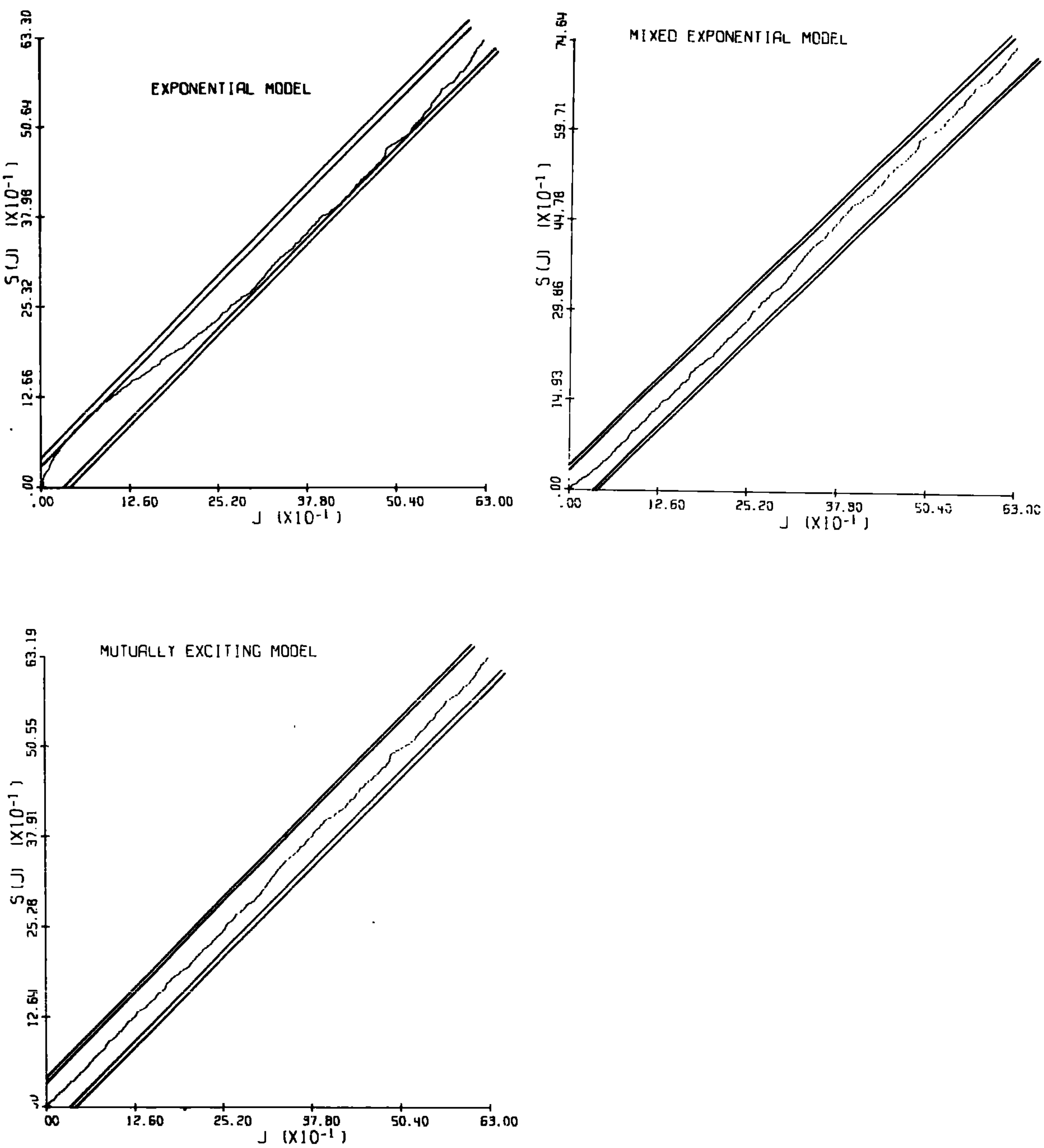


Fig. 4.4.- Cumulative periodograms of counts for the area of Aleoution Islands with P = 0.05 and P = 0.0I significance bands.

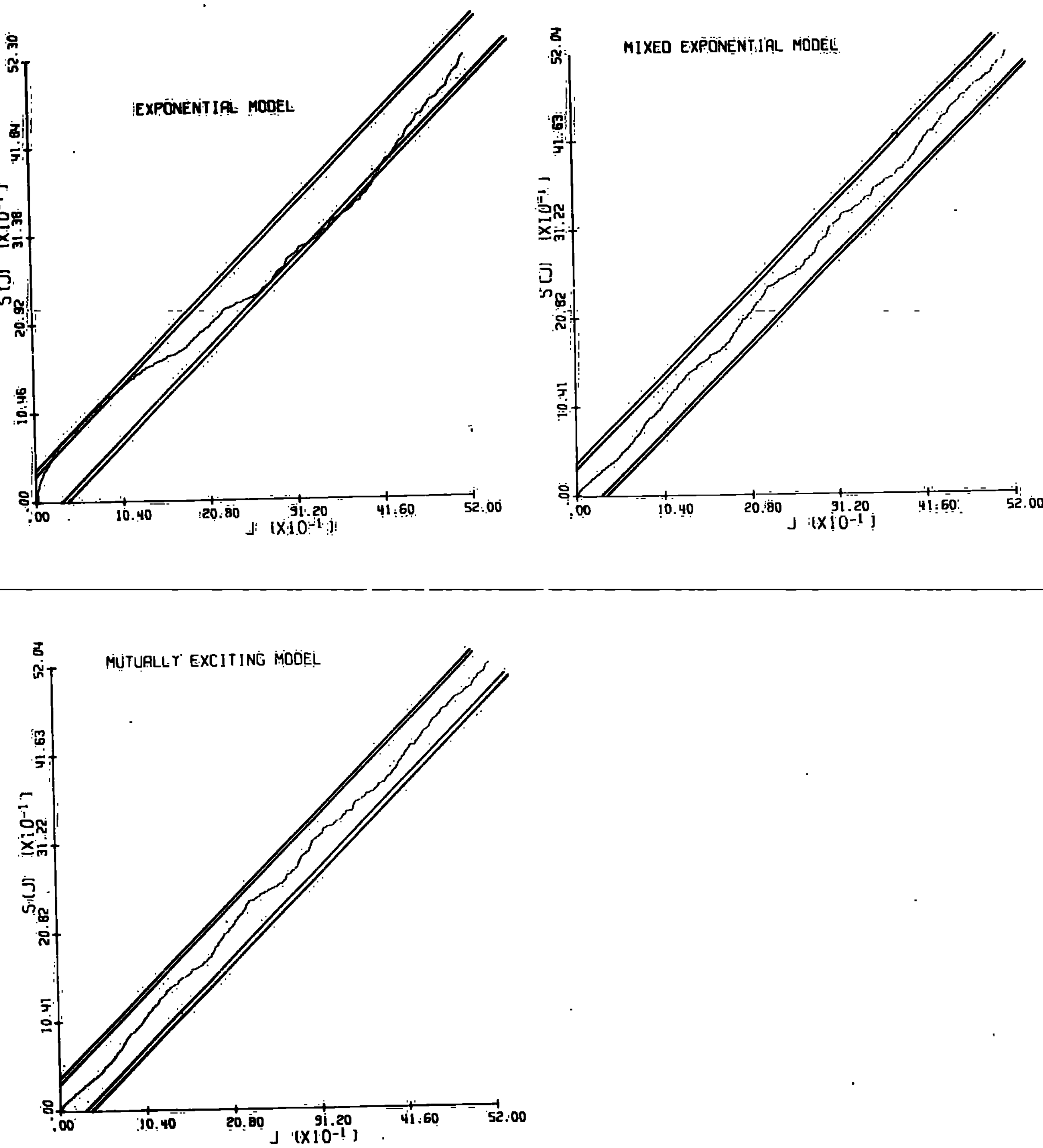


Fig. 4.5.- Cumulative periodograms of counts for the area of Fox Islands with P = 0.05 and P = 0.0I significance bands.

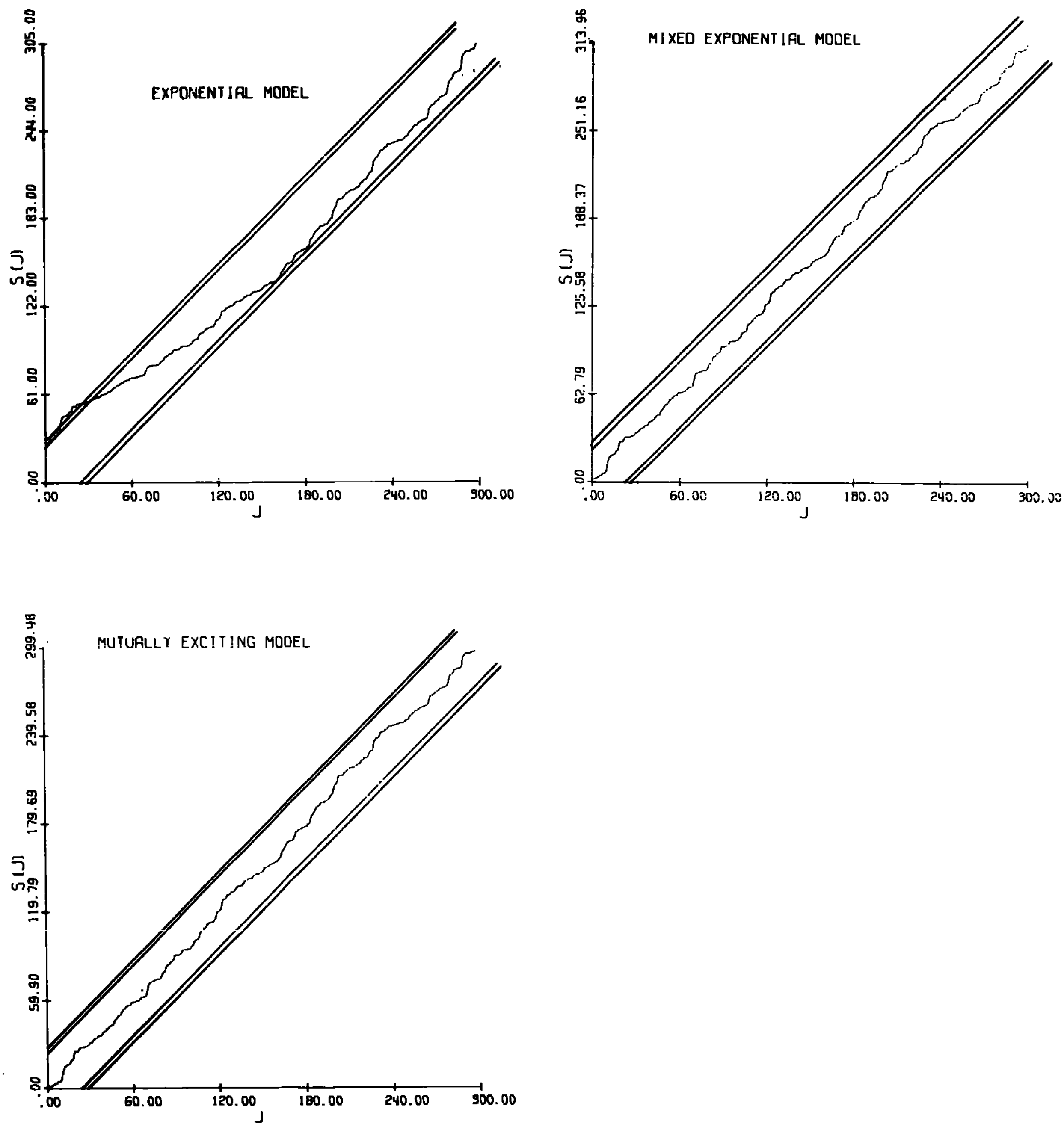


Fig. 4.6.- Cumulative periodograms of counts for the area of Sunda-arc with P = 0.05 and P = 0.01 significance bands.

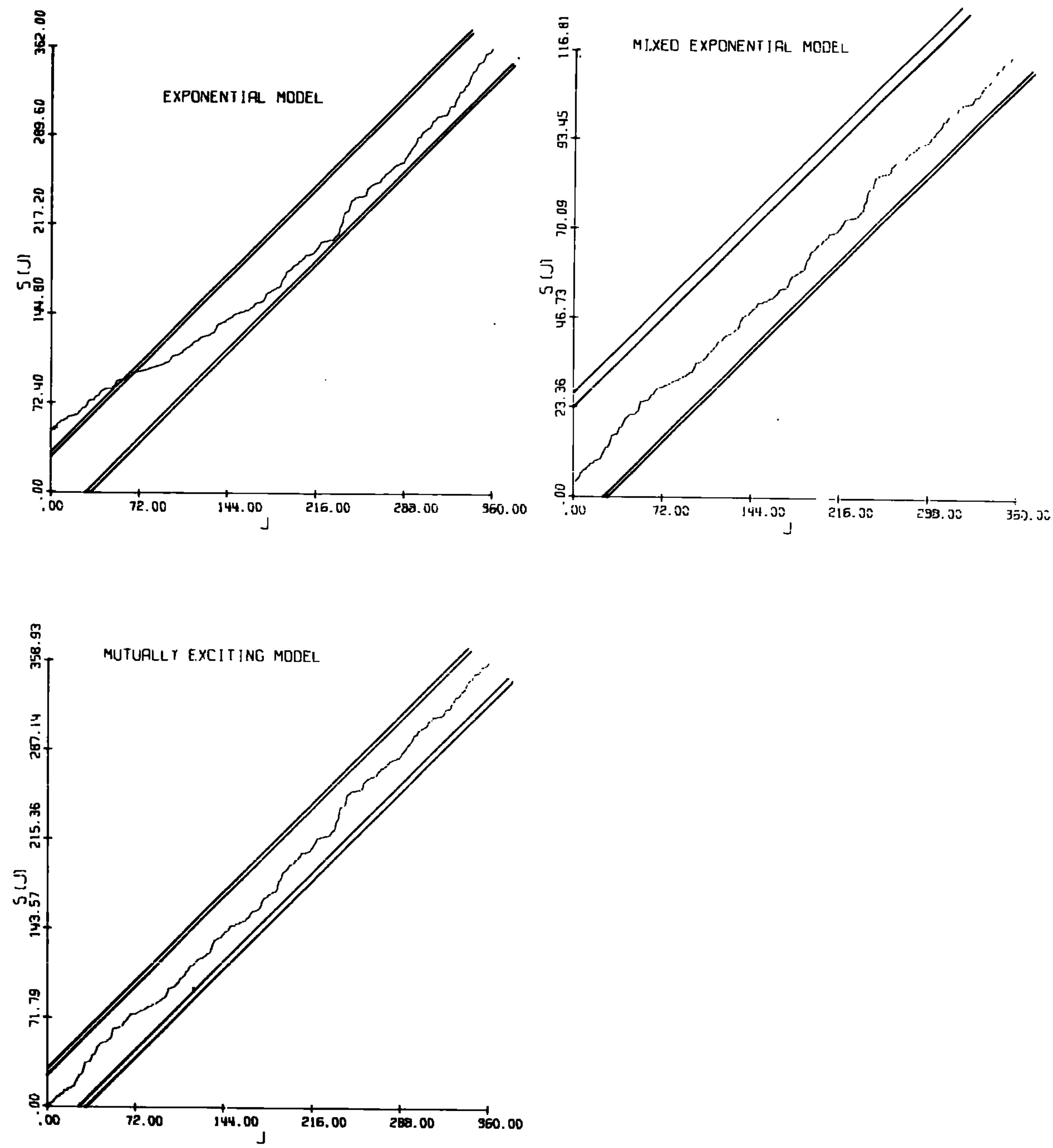


Fig. 4.7.- Cumulative periodograms of counts for the area of Banda-Sea with P = 0.05 and P = 0.0I significance bands.

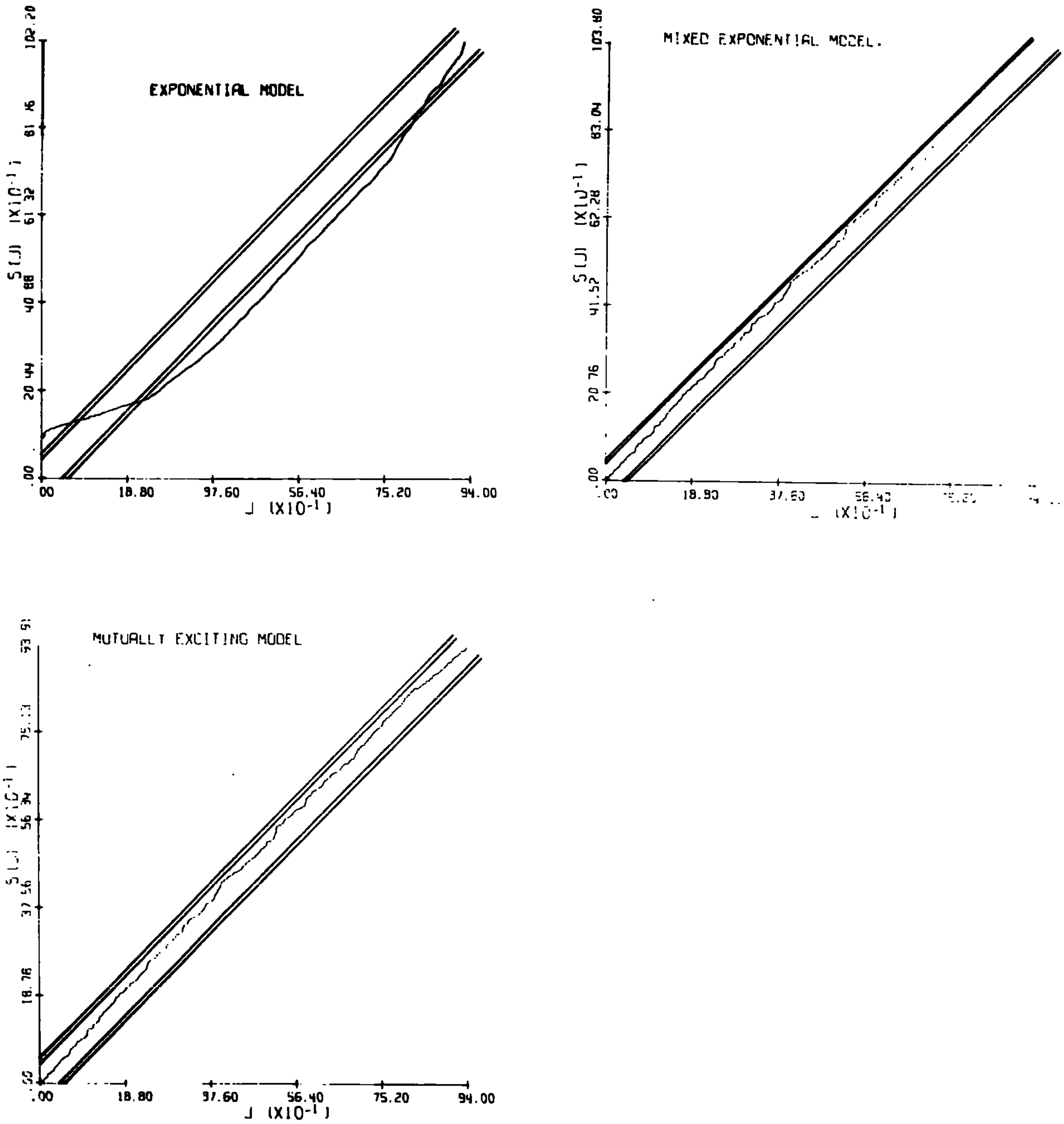


Fig. 4.3. Cumulative periodograms of counts for the area of Tonga with P = 0.05 and P = 0.01 significance bands.

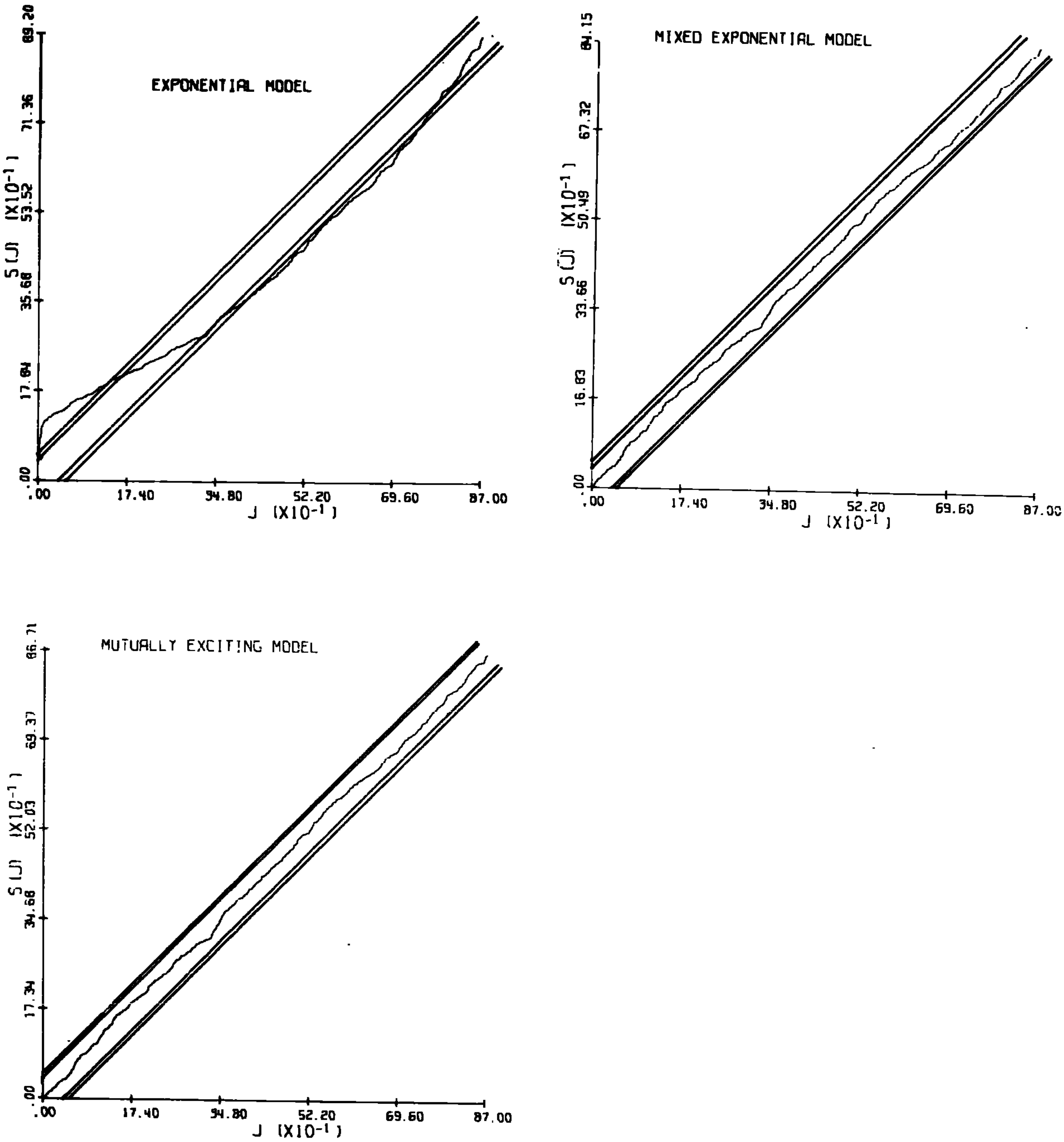


Fig. 4.9.- Cumulative periodograms of counts for the area of Himalayas with P = 0.05 and P = 0.0I significance bands.

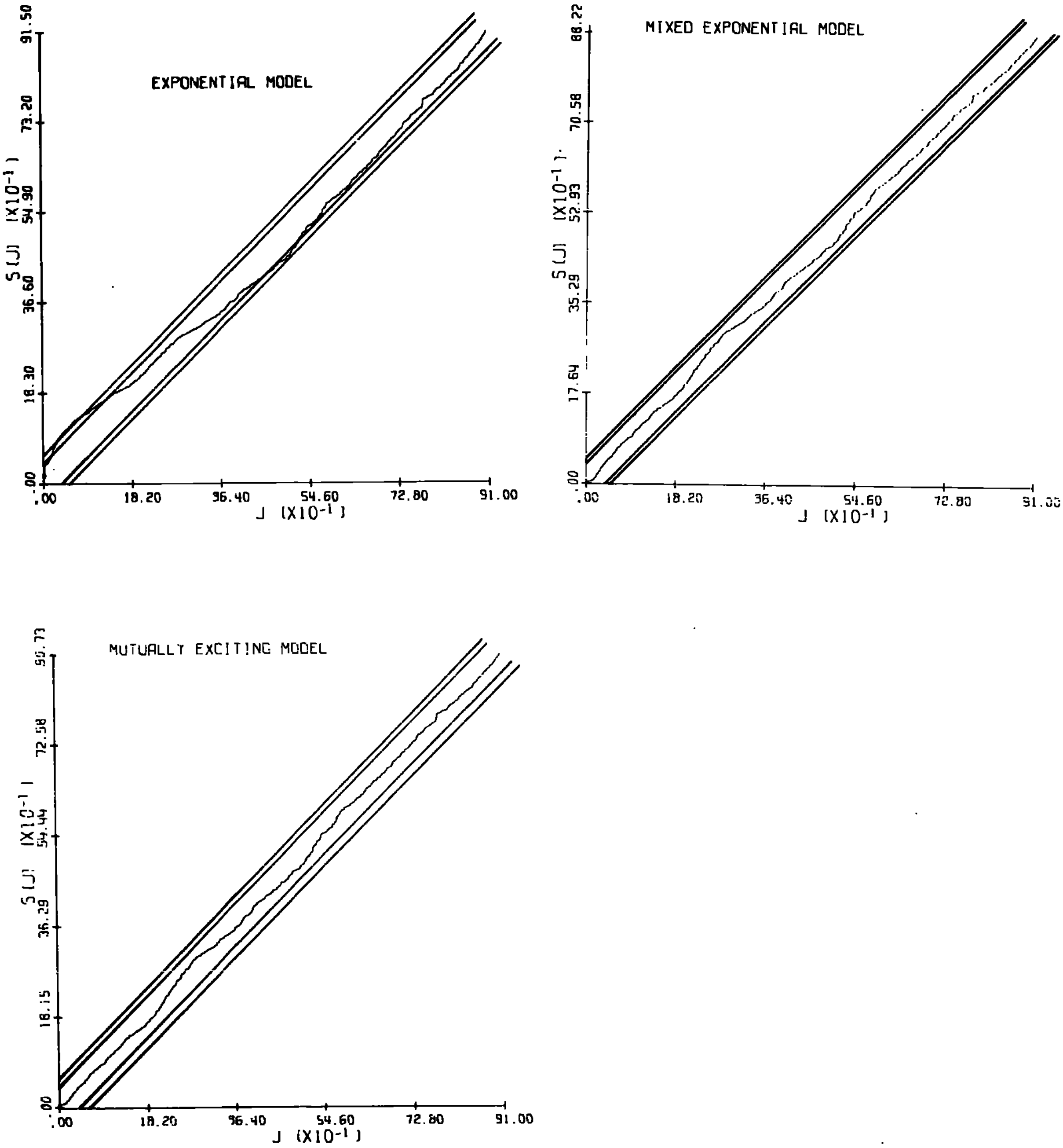


Fig. 4.IO.- Cumulative periodograms of counts for the area of Greece with P = 0.05 and P = 0.OI significance bands.

1. Exponential model
2. Smoothed spectrum with k = 10
3. Mixed exponential model

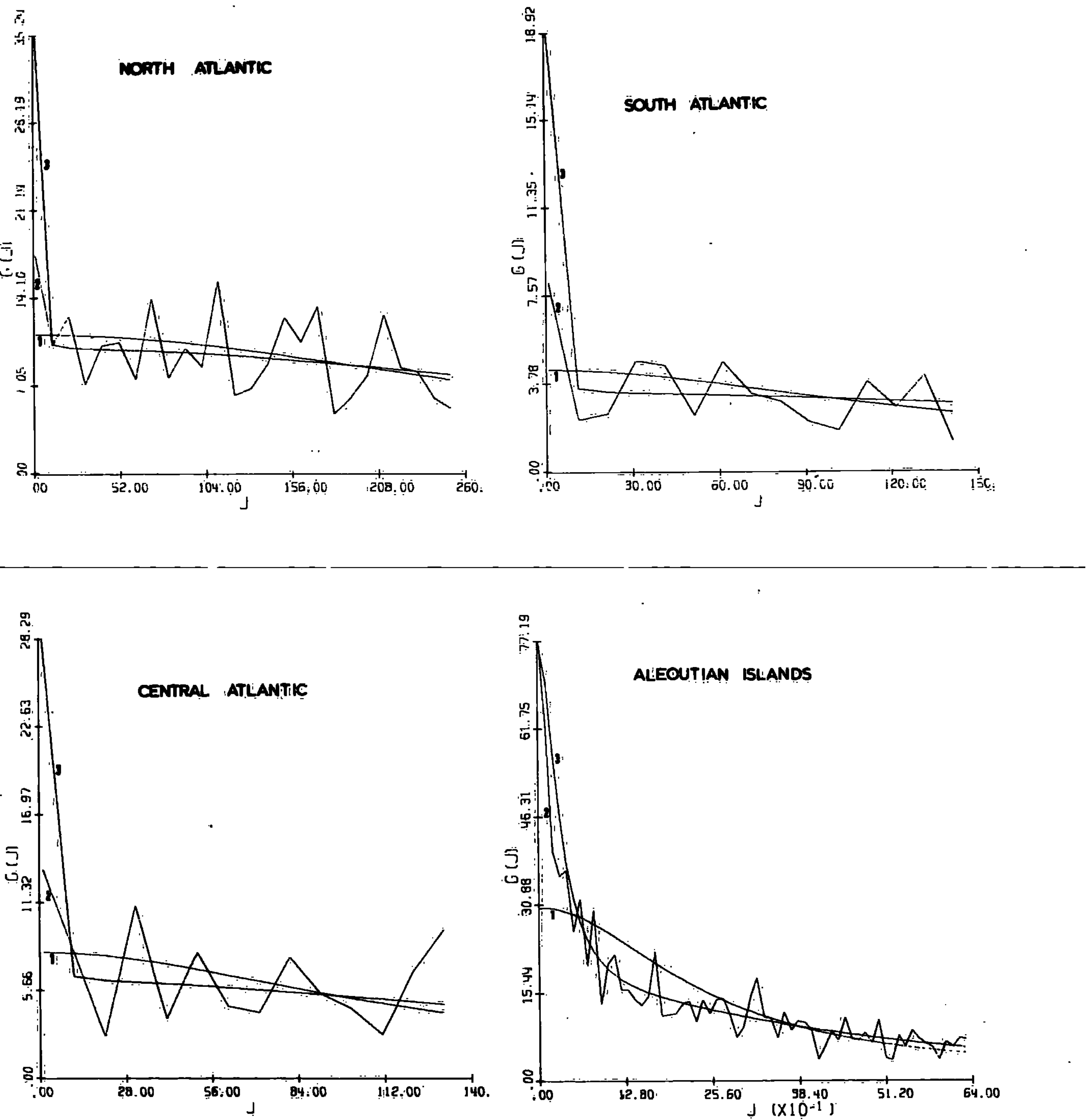


Fig. 4.II.- Smoothed spectrum of counts with the theoretical ones.

1. Exponential model

2. Smoothed spectrum with k = 10

3. Mixed exponential model

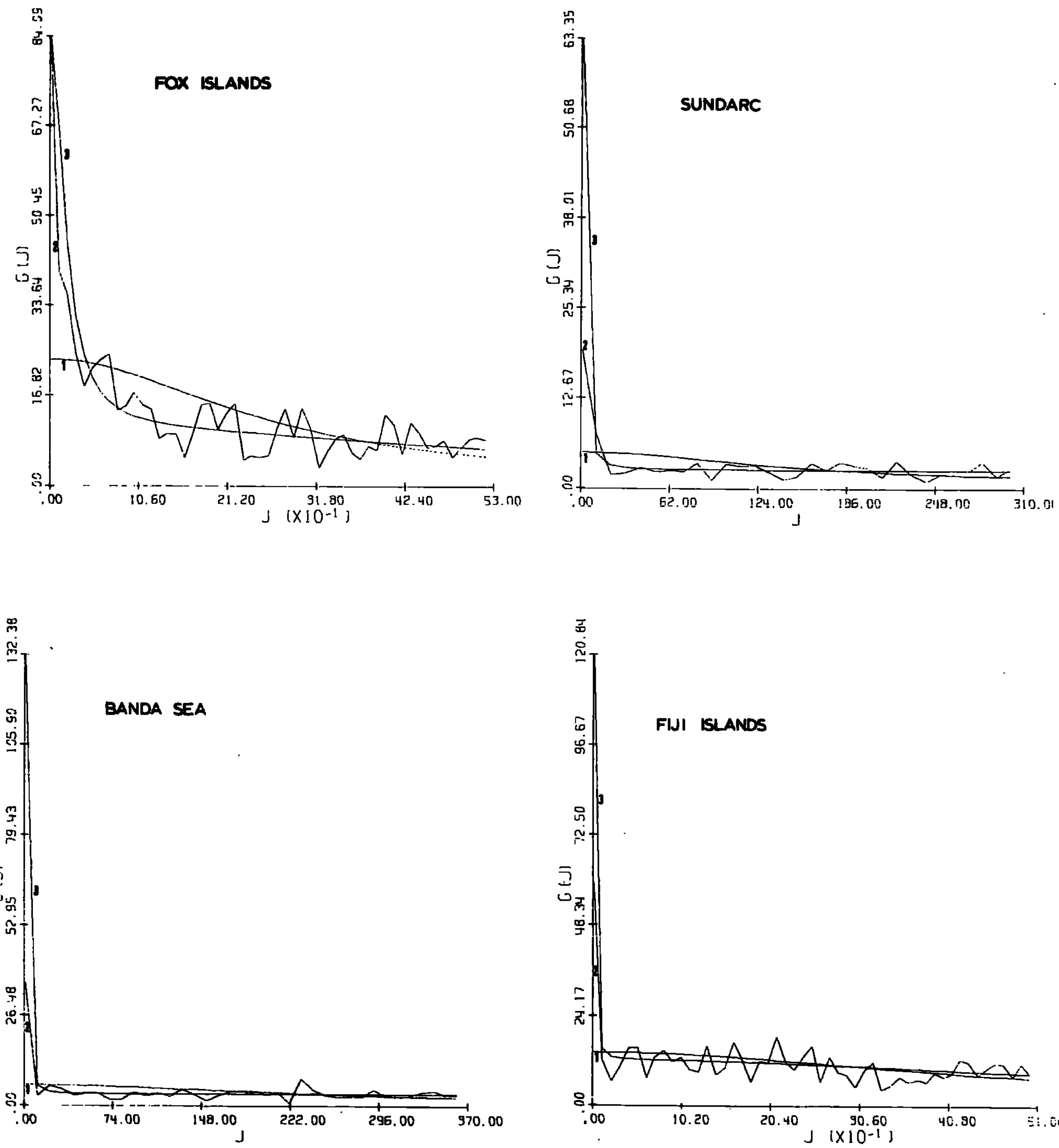


Fig. 4.I2.- Smoothed spectrum of counts with the theoretical ones.

1. Exponential model

2. Smoothed spectrum with k = 10

3. Mixed exponential model

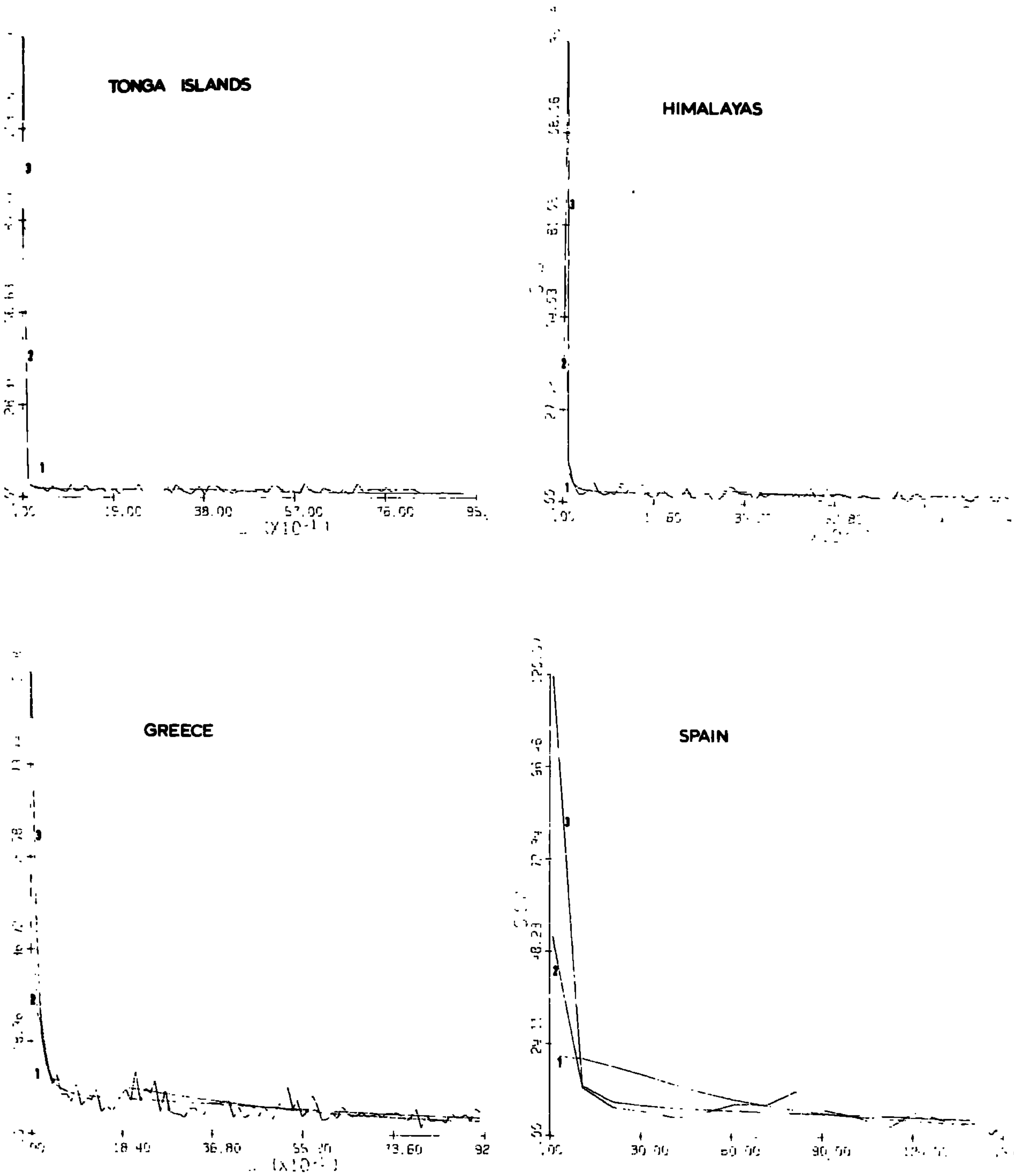
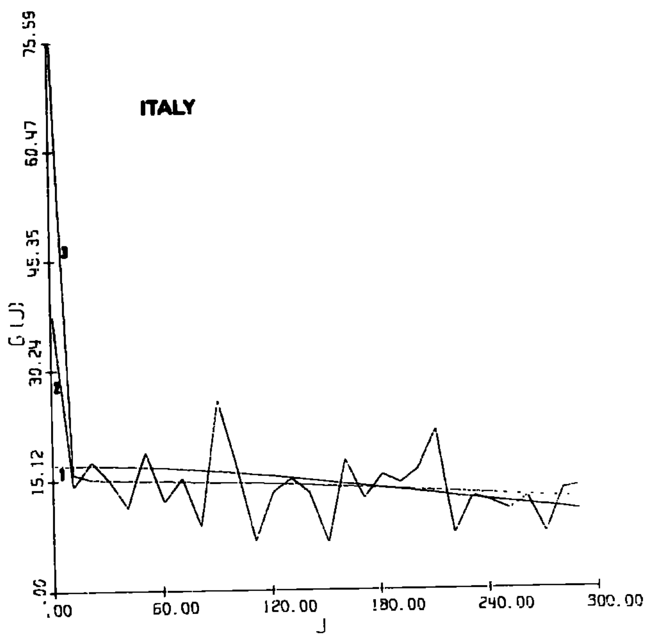


Fig. 4.I3.- Smoothed spectrum of counts with the theoretical ones.

1. Exponential model

2. Smoothed spectrum with k = 10

3. Mixed exponential model



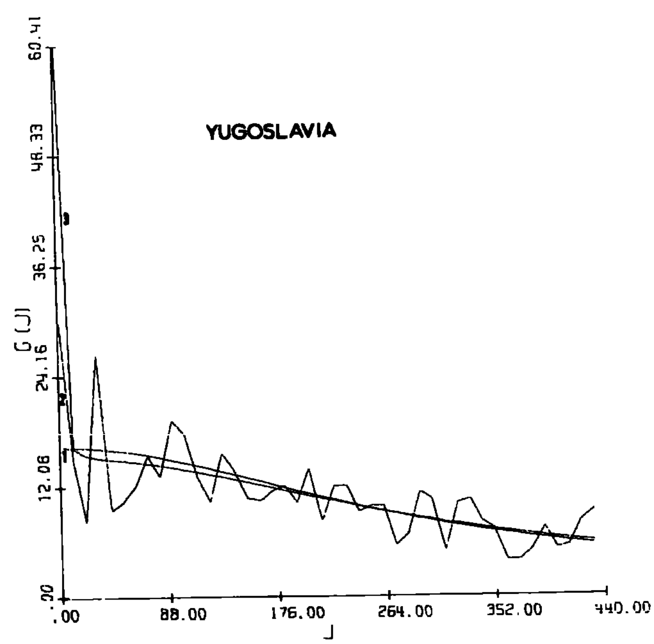
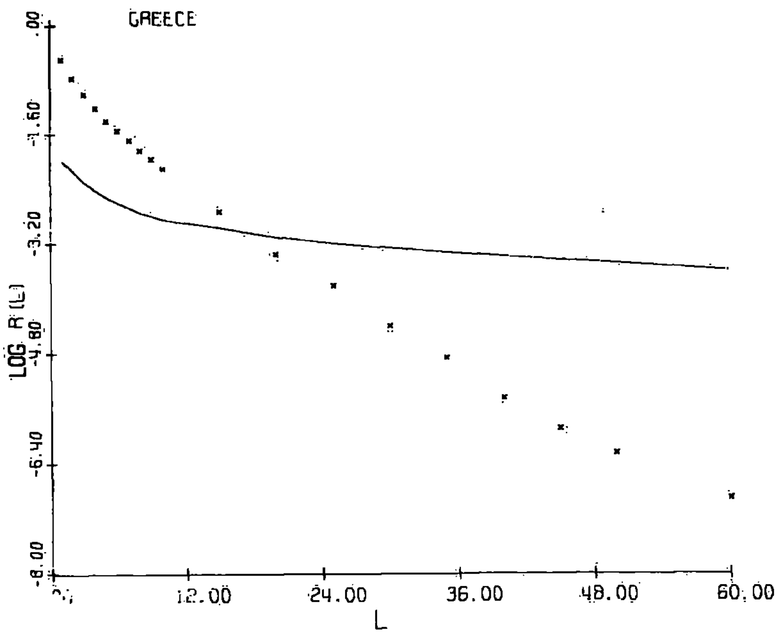


Fig. 4.I4.- Smoothed spectrum of counts with the theoretical ones.



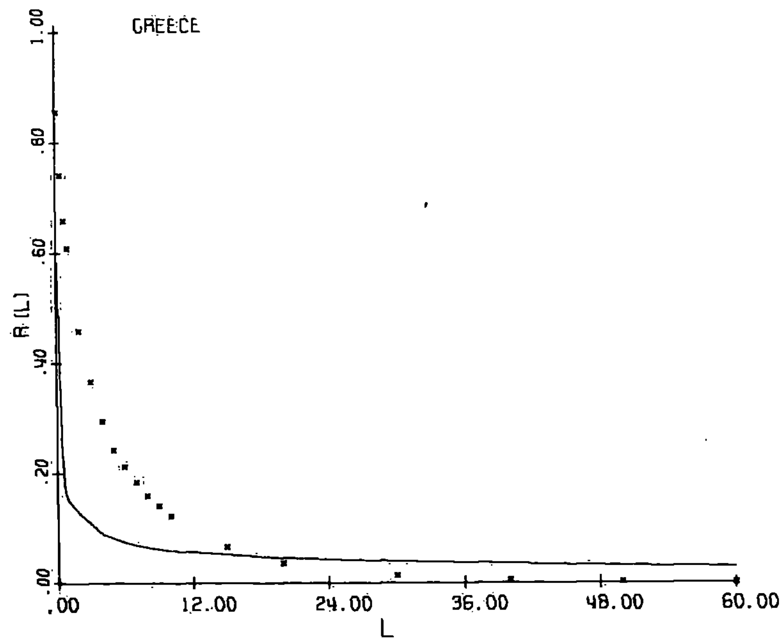


Fig. 4.I5.- Empirical survivor for the area of Greece with the corresponding theoretical one for the mutually exciting process.

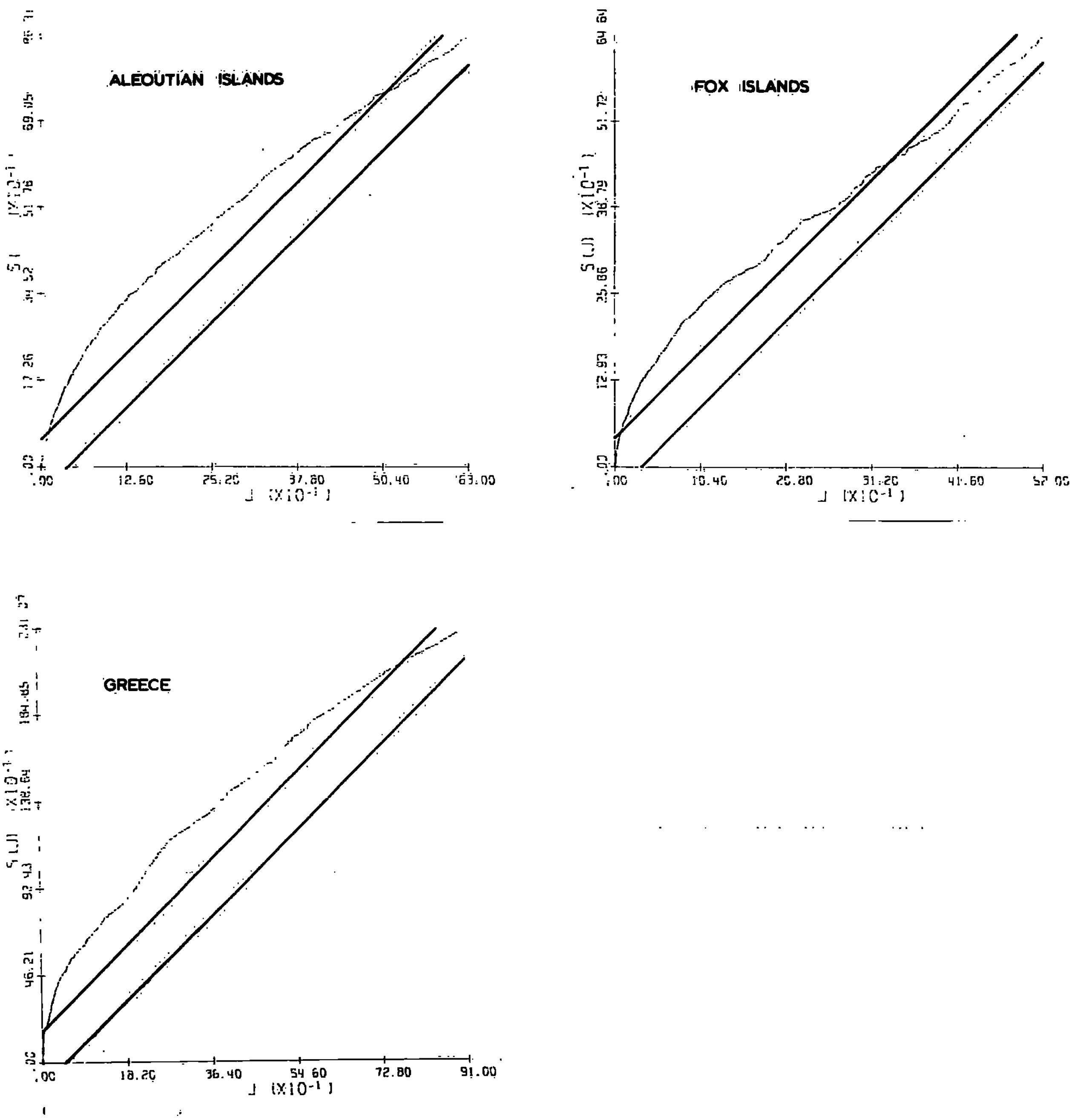


Fig.4.16 - Cumulative periodograms of counts for the modified mutually exciting model with p = 0,05 significance bands.

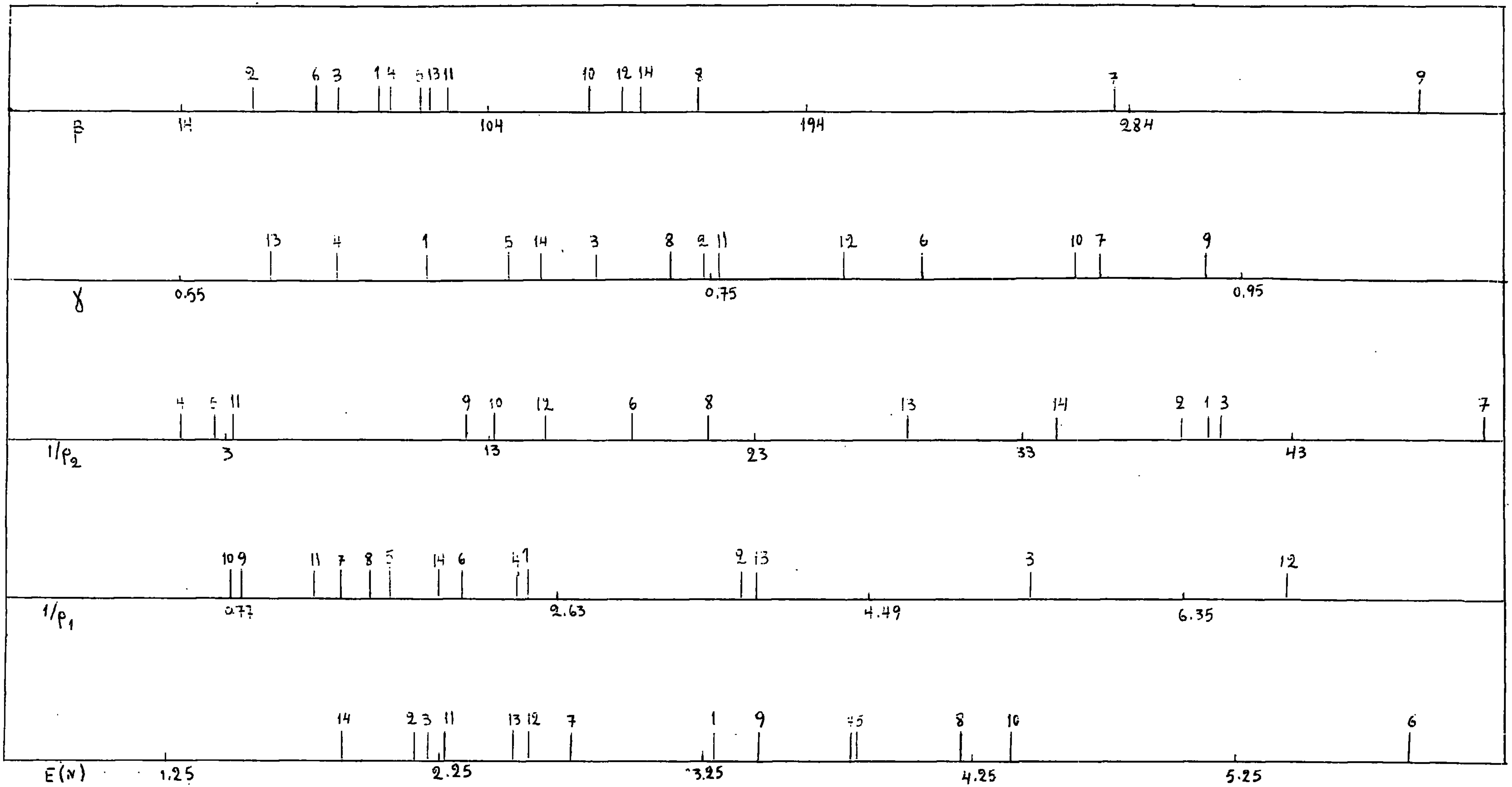


Fig. 4.17.- Linear plots of the parameters of the Neymann-Scott model with mixed exponential decay for all the areas under investigation.

CHAPTER 5

GENERALIZATIONS

5.1. Modifications of the Two-variate mutually exciting processes

The main weakness of the two-variate mutually exciting processes, in describing the earthquake data, is the small rate of occurrence of the cluster centres compared with the one obtained by the empirical survivor function. That results in a high clustering effect as can be easily seen from the theoretical spectrum of counts.

In order to cut down the clustering effect predicted by the two-variate mutually exciting model and make it more consistent with the data, we assume that there are three kinds of cluster centres:

(i) The first one, II type of events, follows a self-exciting process and each of the cluster centres generates cluster members of type I in the rate $\beta_{12}(x) = \alpha_{12}\rho_2 e^{-\rho_2 x}$.

(ii) The second one, III type of events, follows a stationary Poisson of rate, a, and is independent of the others without creating any subsidiary process.

(iii) The third one, IV type of events, follows also a stationary Poisson, of rate b, is independent of the others, and creates cluster members of the I type in the rate $\beta_{14}(x) = \alpha_{14}\rho_4 e^{-\rho_4 x}$.

It is reasonable to take $\alpha_{12}=\alpha_{14}$ and $\rho_1=\rho_2$ which at the same time reduces the number of parameters of the model.

Hence the covariance matrix is

$$\underset{\sim}{\beta}(u) = \begin{pmatrix} 0 & \beta_{12}(u) & 0 & \beta_{14}(u) \\ 0 & \beta_{22}(u) & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

and the rate of the primary events,

$\underset{\sim}{\nu} = \begin{pmatrix} 0 \\ \nu \\ a \\ b \end{pmatrix}$. Consequently by applying the results of § 2.7 the following results are obtained.

(i) The stationary rate of occurrence for the sum process,

$$\underset{\sim}{\lambda} = \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{pmatrix} \text{ where } \lambda_1 = \frac{\nu\alpha_{12}}{1-\alpha_{22}} + b\alpha_{14}, \ \lambda_2 = \frac{\nu}{1-\alpha_{22}}, \ \lambda_3 = a \text{ and } \lambda_4 = b \quad (5.1.1)$$

(ii) The spectral matrix,

$$2\pi \underset{\sim}{f}(\omega) = \begin{pmatrix} \lambda_1 + \frac{\lambda_2 |B_{12}(\omega)|^2}{|1-B_{22}(\omega)|^2} + \lambda_4 |B_{14}(\omega)|^2 & \frac{\lambda_2 B_{12}(\omega)}{|1-B_{22}(\omega)|^2} & 0 & \lambda_4 B_{14}(\omega) \\ \frac{\lambda_2 B_{12}(-\omega)}{|1-B_{22}(\omega)|^2} & \frac{\lambda_2}{|1-B_{22}(\omega)|^2} & 0 & 0 \\ 0 & 0 & \lambda_3 & 0 \\ \lambda_4 B_{14}(-\omega) & 0 & 0 & \lambda_4 \end{pmatrix}$$

where $B_{ij}(\omega) = \int_{-\infty}^{+\infty} \beta_{ij}(u) e^{-i\omega u} du = \frac{\alpha_{ij}\rho_j}{\omega^2+\rho_j^2}$

(iii) The Bartlett's spectrum

$$2\pi g(\omega) = \sum_{i,j} f_{ij}(\omega)$$

$$= \lambda_1 + \lambda_3 + \lambda_2 \frac{(\rho_2^2+\omega^2)\left[(1+\alpha_{12})^2 \rho_1^2+\omega^2\right]}{(\rho_1^2+\omega^2)\left[(1-\alpha_{22})^2 \rho_2^2+\omega^2\right]} + \lambda_4 \frac{(1+\alpha_{14})^2 \rho_3^2+\omega^2}{\rho_3^2+\omega^2} \quad (5.1.2)$$

If a=b=0 then $\lambda_3=\lambda_4=0$ and the formulae (5.1.1) and (5.1.2) reduce to (2.6.12) and (2.6.13) respectively.

The general results for the distribution of intervals obtained in § 2.8 give for the forward recurrence time survivor function,

$$R_L(l) = P_r(L>l) = \exp\left[-l(\nu+a+b) - \nu\int_0^{\infty}\left[1-\Phi_Y^{(2)}(t,l)\right] dt\right]$$

where

$$\Phi_Y^{(i)}(y,l) = \begin{cases} \exp\left[\sum_{j=1}^{4}\int_0^{y+l}\left[\Phi_Y^{(j)}(y-t,l)-1\right]\beta_{ji}(t)dt\right] & y>0 \\ 0 & -l\leq y\leq 0 \\ 1 & y<-l \end{cases} \quad i = 1,2$$

and

$$\Phi_Y^{(i)}(y,l) = \begin{cases} 0 & -l\leq y\leq 0 \\ 1 & \text{otherwise.} \end{cases} \quad i = 3,4$$

In the case of the equilibrium distribution the asymptotic survivor function ~~becomes~~, of the forward recurrence time becomes

$$R_L(y) = A^{-t(\nu+a+b)}, \quad 0 < A < 1,$$

and the Log survivor function of the time interval between successive events,

$$\mathrm{Log}P_T(t) = \text{constant}-(\nu+a+b)t.$$

The above model could describe the earthquake phenomenon by considering the deep earthquakes as events of the III type and the shallow and intermediate ones as II and IV types respectively. Thus the deep earthquakes follow a stationary Poisson without triggering any subsidiary process, while the shallow and intermediate ones trigger cluster members of type I. This view contrasts with the suggestion sometimes put forward that deep shocks show no clustering effects apart from the occurrence of "doublets" or "triplets" (that is two or three shocks very close in time and space and often of similar energies) (Vere-Jones, 1970).

The model is also more suitable for the present investigation since we are considering all the kinds of earthquakes together, without separating the shallow ones from the deep and intermediate. However, the main disadvantage of the model is again the large number of parameters which creates a lot of difficulties in the estimation procedure.

5.2. Models for the sequence of origin times and energies

A problem of great significance from the geophysical point of view is the investigation of the relation between the occurrence time and the emitted energy. In spite of the importance of the problem adequate data were not available for the statistical analysis. A mathematical model which can be employed in this case is the Marked processes developed by Hawkes (1971), where each event (origin time of earthquakes) is associated with its mark (ammount of the emitted energy). The statistical analysis can be done by using relevant theory of the multivariate time series analysis properly modified to the situation of Point processes. (Jenkins, 1969 and Bartlett, 1955).

Another approach to the problem of earthquakes, which considers all the main characteristics of a shock, is by using the concept of the multidimensional point process where the one dimension is the time axes and the rest of the dimensions is the spatial coordinates and the emitted energy. Vere-Jones (1970) gives the general lines along which the study of the mathematical models and the methods of the statistical analysis of the preceding sections may be extended in the multidimensional case. ~~The relevant theory of the multidimensional point processes is developed by Cox and Lewis (1971)~~.

5.3. The estimation problem

The main problem in the statistical analysis of Point processes, as it has previously been stressed is the estimation problem. The situation is particularly difficult when the spectral analysis is used because of the very complicated character of the spectrum itself and the large number of parameters. One way by which the estimation techniques can be improved is by using simulation methods to study the distributional properties of the parameters and to get some kind of confirmation of the results obtained by another method.

CHAPTER 6

CONCLUSIONS

The statistical properties of earthquake data from 14 different areas have been studied by considering the earthquake occurrence as a one-dimensional stationary point process. The following conclusions may be drawn from this analysis.

(i) The theory of stochastic point processes provides a satisfactory way of describing the earthquake occurrence. An exploratory analysis of the given data, by using tests based on the properties of the first and second order moments can often give an approximate idea about the most suitable model for describing the data.

(ii) The existence of any kind of periodicity regarding the earthquake occurrence is not well established. There is strong evidence of clustering and the spectrum of counts reveals a slow rate of decay of the covariance density for large values of t (small frequencies). Therefore the simple Poisson process can be rejected since it does not allow for the tendency of earthquakes to occur in groups. These groups of earthquakes (aftershocks) are often triggered by a large main shock. The renewal process is also rejected.

(iii) The class of clustering models is a suitable one for describing the earthquake occurrence. Their main disadvantage is the difficulty in the estimation of the parameters. The improvement of the estimation methods by using simulation techniques will certainly increase the value of the obtained information, by fitting the relevant model. The fit of the Neymann-Scott model with mixed exponential decay is more satisfactory than the fit of N.S. with single exponential decay. The most plausible justification for this is the apparent presence of both short and long term effects and therefore the inability of the exponential model to describe satisfactorily the data especially near the origin. Therefore the N.S. model with single exponential can be rejected in spite of the fact that the Kolmogorov-Smirnov test is not significant for some of the areas. The complicated character of the data of this study and the results of the interval analysis are in favour of a four-variate mutually exciting processes where the different behaviour of the earthquakes according to their depth is also taken into account.

(iv) The statistical analysis does not appear to be quite sufficient to predict similarities or differences among the areas under investigation. The errors of the estimates in some areas are too large relative to the corresponding estimates to allow a safe discrimination. However, there is some evidence of consistency of seismic activity, as reflected in the parameters of the stochastic model, within regions of similar geological type while differences between types are observed.

(v) Finally the problem of studying the relation between occurrence time and the emitted energy is stated and some suggestions of the way to approach it are made.

APPENDIX

FOURTEEN SERIES OF TIMES BETWEEN EARTHQUAKES IN UNITS OF 1000 MINUTES

1. NORTH ATLANTIC, LATITUDE(+30,+60), LONGITUDE(-43,-27), N=506

24.575	19.415	6.107	102.962	53.476	179.824	5.819	58.813	6.725	0.124	11.953	0.001
40.656	6.691	54.995	23.186	13.224	142.936	24.105	35.637	16.490	304.328	40.452	[illegible]
48.077	30.647	143.218	10.375	97.680	0.197	20.407	22.343	128.450	22.319	143.731	42.818
74.589	101.417	43.399	0.251	8.546	15.204	39.156	27.406	83.610	105.965	30.083	44.921
120.677	15.421	140.520	52.257	42.364	0.036	19.692	77.328	21.312	23.598	42.459	0.395
64.574	0.001	36.396	129.292	127.507	30.897	3.630	9.001	19.568	0.008	0.179	0.008
0.179	5.569	13.195	75.974	5.472	5.915	2.668	0.013	0.091	0.181	0.091	[illegible]
0.026	0.038	0.013	0.040	0.038	0.052	0.047	1.227	0.534	11.482	8.202	103.223
30.535	22.189	2.183	8.627	53.331	52.746	1.284	0.003	0.003	26.043	14.490	42.109
59.546	34.507	67.696	15.789	12.616	59.483	16.599	114.938	6.604	95.082	14.810	0.107
53.604	9.309	183.785	37.372	94.457	0.005	0.171	6.074	29.884	23.840	9.052	23.076
43.417	2.827	38.040	0.440	11.260	0.179	3.851	0.057	0.057	7.890	8.962	0.092
0.092	1.146	14.219	5.234	21.542	25.834	1.133	0.291	26.795	3.988	160.105	59.206
57.253	14.199	5.364	152.603	69.887	48.601	125.908	0.183	0.183	0.183	5.916	5.478
9.397	0.001	0.018	0.019	7.735	5.141	43.694	45.698	6.928	75.133	0.003	0.003
1.643	0.521	0.764	0.327	0.260	0.521	0.587	0.304	0.265	0.265	1.117	0.001
0.037	0.037	0.037	0.301	1.310	0.184	0.194	1.165	42.223	3.191	95.610	0.180
0.006	0.348	31.175	2.433	1.105	0.091	9.086	0.001	2.541	0.038	0.001	42.544
18.919	0.038	8.256	15.029	0.006	150.672	0.006	0.216	5.907	16.359	45.754	[illegible]
1.714	8.986	20.132	9.055	52.774	18.956	54.635	2.501	5.572	9.056	0.258	0.735
0.359	2.635	0.886	3.026	14.708	0.029	0.101	0.362	0.239	0.197	9.383	59.347
0.001	36.579	19.798	5.240	25.266	13.099	14.545	2.508	49.524	0.001	0.820	6.857
0.020	0.572	0.111	4.231	25.663	0.022	0.416	0.015	0.467	0.001	4.359	0.001
1.330	26.622	17.565	14.881	47.200	27.347	30.984	91.142	131.980	0.001	12.395	19.633
0.198	3.249	0.177	0.001	6.246	42.822	9.736	0.917	0.269	35.547	10.743	0.001
6.928	2.090	0.108	0.001	0.599	0.001	0.833	3.677	1.378	0.961	0.062	0.062
47.999	0.029	0.005	0.004	0.040	0.036	18.679	0.452	0.016	0.044	0.795	4.429
2.324	2.329	9.835	23.251	17.496	15.168	30.065	38.368	13.852	3.047	38.022	[illegible]
4.864	25.408	29.737	6.627	2.210	0.551	0.003	4.690	8.430	11.895	0.014	0.688
42.048	4.772	2.392	5.559	0.075	0.105	0.483	0.015	33.433	0.030	7.628	[illegible]
0.018	0.004	17.415	0.008	5.906	69.489	15.759	14.392	13.822	38.320	58.272	2.908
10.316	3.026	20.998	0.008	0.013	0.011	0.025	0.023	0.056	0.057	0.149	0.174
0.017	0.147	0.233	0.572	0.208	0.171	0.629	0.032	0.249	3.035	25.073	25.378
6.303	44.214	127.413	86.857	25.072	0.032	0.038	16.800	14.107	63.095	6.935	0.115
0.035	0.017	0.016	0.040	0.039	0.033	0.064	0.089	0.015	0.276	2.216	0.339
6.341	0.159	66.812	8.430	56.910	5.007	0.018	42.674	45.293	31.669	8.919	7.790
40.808	0.049	12.284	0.224	7.775	9.766	1.094	21.959	0.033	44.958	9.614	24.594
26.383	9.152	0.130	0.077	0.668	9.607	76.798	38.077	19.230	55.705	45.857	1.131
15.304	26.076	18.489	65.227	0.147	0.059	0.036	0.011	0.096	0.131	0.104	5.690
0.022	29.323	22.774	1.255	51.105	7.850	65.593	4.983	0.732	32.163	49.143	21.320
15.402	0.578	0.002	0.024	0.004	0.218	2.950	14.749	16.672	70.950	50.332	1.975
14.120	29.112	7.193	14.127	0.730	0.008	43.378	2.723	0.007	6.971	18.227	3.926
28.895	32.655										

2. SOUTH ATLANTIC, LATITUDE(-60,-25), LONGITUDE(-15,+15), N=142

191.219	221.083	757.109	[illegible]	46.914	26.489	207.228	323.419	50.772	115.293	233.557	138.322
94.954	0.001	579.705	169.389	152.312	289.293	75.818	242.306	114.490	[illegible]	127.597	[illegible]
66.976	169.194	0.001	23.482	0.035	31.495	0.071	0.071	23.278	[illegible]	[illegible]	[illegible]
1.442	9.207	180.077	47.381	66.737	1.363	240.902	139.231	[illegible]	110.947	[illegible]	[illegible]
53.360	119.206	39.654	7.168	1.087	142.897	10.963	7.411	3.826	8.799	140.781	[illegible]
126.188	9.120	143.078	40.539	19.307	32.378	55.031	[illegible]	33.394	[illegible]	[illegible]	[illegible]
25.740	4.883	6.626	87.145	33.431	49.537	1.631	0.104	4.946	0.036	74.544	[illegible]
0.001	13.467	4.089	[illegible]	53.778	[illegible]	67.937	158.374	102.579	58.070	52.872	[illegible]
54.924	31.105	15.286	11.621	108.282	61.854	0.037	0.922	51.995	18.419	2.734	12.947
0.252	90.438	2.953	22.953	25.401	2.466	4.546	0.020	46.614	72.595	122.823	6.376
30.219	6.007	43.326	26.414	4.615	0.058	0.507	5.237	[illegible]	57.831	33.868	[illegible]
42.390	29.260	1.544	[illegible]	15.732	235.598	[illegible]	0.758	3.981	24.323		

3. CENTRAL ATLANTIC, LATITUDE(-2,+10), LONGITUDE(-40,19), N=253

85.453	105.939	31.466	266.398	[illegible]	[illegible]	86.079	144.599	186.358	203.098	74.683	1.174
59.406	75.255	51.002	229.693	203.054	54.079	208.231	[illegible]	19.330	249.836	97.335	151.275
4.913	107.499	209.123	[illegible]	8.298	269.365	108.335	36.224	9.496	27.759	70.144	0.633
488.826	0.019	3.447	53.222	30.529	19.987	[illegible]	5.797	73.407	157.884	149.671	177.424
0.001	39.297	20.059	150.251	22.085	5.897	0.046	131.479	47.446	13.947	86.078	19.746
26.112	74.315	140.233	0.001	5.373	0.001	111.956	84.413	56.644	[illegible]	[illegible]	[illegible]
24.905	9.831	0.001	197.996	20.529	0.313	0.573	[illegible]	35.531	15.223	[illegible]	[illegible]
0.001	161.820	69.259	44.665	32.047	11.427	[illegible]	[illegible]	2.075	3.430	44.665	27.721
1.979	32.677	0.001	40.544	44.978	112.035	3.473	[illegible]	1.495	4.575	62.495	36.393
13.238	9.873	32.183	[illegible]	0.001	13.415	15.038	0.757	20.639	24.663	0.001	70.376
18.481	28.939	0.005	0.171	0.106	0.005	7.051	0.035	0.035	11.550	2.312	9.844
0.271	66.321	27.723	17.436	0.159	7.167	0.455	14.170	0.519	38.706	59.886	12.553
26.720	39.801	5.178	[illegible]	32.759	0.352	3.890	29.621	26.113	1.913	132.353	19.745
0.582	7.645	7.630	3.634	11.369	8.622	5.179	10.093	2.632	12.996	0.570	19.555
23.909	26.210	10.497	80.402	62.475	10.416	11.461	96.773	2.707	79.375	[illegible]	11.530
61.515	0.112	0.027	0.022	0.011	0.052	0.035	0.005	[illegible]	0.021	[illegible]	[illegible]
0.376	[illegible]	0.038	3.477	1.364	4.565	0.019	[illegible]	16.214	29.632	[illegible]	11.433
59.908	8.386	16.161	7.935	26.454	[illegible]	5.982	4.443	13.631	0.390	0.816	49.107
9.508	0.024	34.397	15.140	109.326	30.407	[illegible]	0.495	[illegible]	16.181	[illegible]	[illegible]
52.665	0.013	14.963	63.741	1.729	19.147	13.807	[illegible]	[illegible]	1.237	[illegible]	[illegible]
0.369	17.053	[illegible]	4.754	110.367	16.982	14.131	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
11.586											

4. ALEUTIAN ISLANDS ,LATITUDE(50,+55),LONGITUDE(-1[illegible],-172),N=12[illegible]

0.239	5.446	171.399	1.391	56.429	2.588	0.502	65.157	0.174	.15	0.35	.444
2.593	0.351	0.371	77.459	28.914	49.036	30.563	4.74	10.495	49.302	9.077	32.419
60.057	0.896	11.203	67.94	0.629	16.457	14.567	33.625	0.064	0.409	0.008	.032
0.282	28.621	117.534	174.27	32.512	75.699	82.421	27.137	75.953	17.955	37.891	0.144
18.057	137.143	164.76	12.99	0.414	6.039	2.593	94.419	0.774	91.021	27.774	43.712
0.001	47.833	0.716	0.89	1.434	2.677	22.214	62.379	2.366	34.972	0.001	15.475
3.871	4.655	18.954	1.577	0.327	37.770	5.921	93.954	0.662	0.984	0.117	0.019
10.277	23.037	38.502	71.553	0.415	120.052	6.987	42.395	115.14	7.693	37.073	0.706
2.414	64.595	19.041	45.965	.079	0.051	0.012	0.025	0.144	.002	0.056	0.068
0.098	0.021	0.51	0.06	0.1.9	.002	0.145	0.047	0.007	0.056	0.322	0.025
0.018	0.113	0.071	0.161	0.022	0.234	0.919	0.104	0.023	0.020	0.010	0.143
0.351	0.010	0.044	0.373	0.040	0.477	0.732	0.002	0.042	0.122	0.226	0.011
0.024	0.155	0.066	0.019	0.149	0.145	0.095	0.242	0.078	0.227	0.229	0.256
0.020	0.045	0.183	0.121	1.264	0.352	0.013	0.045	0.010	0.065	0.312	0.354
0.056	0.260	0.149	0.615	0.261	0.021	0.059	0.357	0.290	0.080	0.516	0.062
0.305	0.972	0.294	0.848	.455	.459	0.082	0.001	0.174	0.493	0.019	0.144
0.047	0.455	.470	. 97	0.491	0.677	0.113	0.957	0.606	0.310	0.752	0.015
1.079	0.384	0.677	1.544	0.979	1.859	2.947	0.068	.287	0.160	1.491	0.087
0.449	0.335	0.672	0.5 5	.023	0.349	1.162	0.389	1.560	0.267	0.560	0.326
2.011	0.993	2.033	.043	0.189	0.017	0.192	0.293	3.016	0.097	0.523	0.251
2.337	0.152	3.730	.310	0.454	3.009	0.092	3.567	3.654	1.061	0.582	4.709
0.059	0.571	2.447	0.479	1.332	5.390	1.592	4.023	0.901	6.947	0.324	4.993
0.420	1.532	0.299	2.545	2.931	3.579	4.002	3.196	6.300	0.011	6.042	0.611
0.785	5.875	1.152	. 6	0.001	1.996	1.184	4.399	3.517	10.359	3.101	5.855
0.157	2.704	25.230	0.194	0.018	2.937	0.258	3.677	2.401	4.291	9.801	2.068
14.647	7.716	14.281	1.752	12.241	4.375	11.501	14.344	4.697	0.114	6.109	21.275
1.129	4.293	1.447	2.481	12.043	0.041	2.448	3.394	2.251	7.837	1.911	1.679
0.001	9.495	1.365	15.444	9.241	75.697	3.275	0.156	3.050	8.144	0.151	0.224
0.149	0.019	0.329	1.835	21.619	2.944	19.869	1.102	0.529	3.206	13.541	0.964
4.479	0.027	28.413	5.885	0.702	13.150	1.300	2.254	4.954	3.663	0.959	10.735
26.314	0.236	6.448	7.566	8.672	5.909	0.944	0.125	2.350	0.027	0.090	0.220
2.444	2.714	0.871	7.026	5.749	0.023	7.294	5.264	3.637	1.474	0.115	0.565
0.001	1.122	0.023	2.759	1.191	0.102	0.726	3.170	4.587	1.523	67.294	31.059
2.475	103.820	7.393	28.517	3.696	0.037	2.288	3.660	3.747	3.729	4.593	19.943
8.325	49.361	13.658	18.729	10.953	3.749	5.727	0.019	0.001	10.811	31.114	6.598
18.398	20.347	1.212	0.001	3.276	38.893	87.391	50.193	49.736	29.739	54.772	0.727
6.341	0.038	0.483	9.663	3.062	3.001	16.264	0.061	12.576	0.419	8.442	6.744
11.948	7.585	1.931	0.066	39.534	44.749	0.041	0.209	7.372	13.261	1.044	1.347
0.117	6.120	0.794	0.152	1.815	4.622	0.855	3.579	18.152	11.748	3.351	9.391
0.001	31.405	3.460	5.940	24.751	7.257	19.029	69.708	19.857	26.542	0.271	0.704
33.542	0.001	3.199	5.123	1.697	2.115	72.782	0.089	33.568	3.326	41.529	5.975
19.236	3.772	2.726	1.841	19.171	29.314	20.880	5.460	4.399	16.766	16.529	1.485
5.833	2.339	0.376	0.376	55.958	2.441	12.575	5.678	20.761	4.903	75.351	41.305
27.350	3.676	1.135	0.402	0.402	7.811	43.728	38.262	14.659	21.533	13.235	13.612
39.156	6.453	16.562	25.842	5.040	18.347	0.054	0.261	0.207	0.055	0.245	0.196
0.056	0.056	1.095	13.072	11.308	3.102	0.244	0.244	36.574	37.530	20.675	16.968
0.001	33.902	19.797	5.952	17.825	2.403	7.075	4.750	0.001	59.714	1.151	1.333
2.970	0.001	18.954	2.194	0.001	8.514	18.093	4.610	0.501	29.301	23.657	7.498
2.989	10.176	12.119	17.532	6.078	6.365	6.146	0.625	0.006	0.094	4.142	3.065
8.391	8.422	4.921	4.297	5.784	10.769	1.277	14.133	11.747	2.293	3.626	20.927
2.410	19.979	7.335	0.460	1.014	14.557	1.424	1.503	13.534	12.999	12.556	0.004
0.013	0.018	0.026	0.019	3.044	0.016	0.057	12.794	4.848	1.843	14.930	3.381
21.329	30.283	75.902	17.109	9.176	12.698	2.474	0.053	2.550	44.559	22.732	14.953
5.965	18.025	7.987	24.865	44.703	3.410	7.266	2.049	0.344	17.422	11.214	23.089
24.975	42.383	0.669	2.752	3.649	0.376	0.326	0.001	8.073	0.038	0.015	0.356
0.295	0.004	0.219	22.309	11.906	6.167	3.561	0.366	3.764	2.467	0.322	0.735
1.473	0.124	0.154	0.217	4.991	0.834	0.009	0.935	6.839	1.809	0.292	4.793
1.913	7.849	7.077	1.165	6.591	0.896	0.108	0.614	5.690	2.641	2.444	0.971

0.117	2.873	0.031	7.541	1.556	3.166	3.335	0.048	0.688	7.359	0.219	0.963
15.784	3.478	2.147	3.931	1.803	0.723	3.130	5.429	8.811	1.089	11.763	4.667
3.342	1.093	3.151	10.411	11.503	0.542	6.948	1.242	14.790	23.995	22.870	5.046
5.059	2.945	0.880	8.660	3.447	0.442	1.652	4.507	4.309	6.231	9.165	0.001
17.261	0.422	0.427	6.513	7.244	0.001	5.803	9.277	1.211	0.030	0.039	0.011
1.912	2.273	0.959	0.011	11.790	5.311	1.268	1.987	16.612	0.945	2.357	7.395
10.487	0.385	0.014	0.385	0.014	0.338	0.239	1.941	0.016	0.056	1.311	0.778
4.319	5.862	1.550	3.719	7.554	2.019	16.773	1.969	10.732	19.937	9.432	5.577
8.008	11.082	8.124	10.213	13.961	6.996	10.036	3.297	10.900	33.253	1.334	0.158
19.673	0.554	1.504	0.001	14.495	1.440	2.545	12.285	2.891	1.423	18.513	1.108
6.261	4.226	0.354	0.271	9.976	0.951	1.173	2.972	4.355	0.903	0.070	0.499
0.936	0.064	0.036	0.032	0.061	0.011	0.247	0.038	0.227	0.123	1.217	3.429
17.209	1.936	0.118	0.640	1.020	0.544	1.455	0.943	0.677	0.032	6.257	0.235
0.116	0.014	0.008	0.047	0.286	0.502	3.376	1.056	0.564	5.760	0.001	12.132
5.971	12.102	12.014	11.179	9.259	0.782	21.800	7.607	4.496	0.285	0.155	0.120
0.014	3.492	5.204	3.366	39.001	2.059	7.799	0.971	0.007	0.342	0.197	0.111
0.001	0.136	0.085	2.156	0.047	0.007	0.023	0.001	0.041	0.190	0.346	0.097
0.018	0.160	1.194	1.934	0.070	0.004	0.222	0.704	2.792	21.754	5.632	3.343
6.349	3.597	0.777	7.226	4.034	11.328	0.971	8.814	1.044	0.318	2.687	22.278
15.152	0.924	14.620	11.393	42.368	20.911	19.140	0.510	4.727	1.649	6.839	7.268
0.310	0.222	4.775	0.095	4.172	15.431	5.745	26.277	5.045	2.451	1.941	19.534
5.872	16.616	14.565	9.264	35.526	10.718	7.278	4.176	22.767	42.635	9.935	8.015
4.582	6.191	7.568	0.647	4.376	0.070	2.192	6.472	7.395	1.953	17.471	62.483
3.016	0.592	2.353	0.003	3.767	0.022	0.062	0.095	0.008	0.003	0.010	0.333
0.622	0.110	0.126	0.057	0.010	0.017	0.357	0.090	0.355	0.037	0.090	0.657
2.739	0.023	0.708	23.010	4.746	0.209	2.795	13.108	4.292	25.473	15.554	14.906
5.277	4.921	70.845	6.270	2.576	22.634	7.625	16.737	14.742	4.775	21.561	1.639
9.685	6.574	4.765	11.110	45.759	8.414	9.968	25.016	5.419	1.718	0.847	2.898
16.937	2.119	6.604	2.794	0.347	47.285	38.147	16.897	1.358	1.193	4.262	2.026
16.124	8.864	23.014	14.103	15.769	0.156	0.409	0.085	0.349	11.889	21.580	2.882
3.263	1.594	4.519	7.673	0.313	35.285	9.893	14.967	35.343	19.189	0.027	0.017
0.001	0.006	0.003	0.035	0.013	0.011	0.029	0.028	0.008	0.029	0.067	0.034
0.015	0.020	0.033	0.099	0.007	0.162	0.403	0.098	1.251	0.068	0.647	2.612
19.636	9.133	2.422	14.493	3.589	3.932	1.529	1.033	0.542	2.120	4.684	0.014
0.346	4.791	0.979	2.396	1.410	0.285	4.326	0.055	0.151	6.726	14.459	6.699
4.399	16.444	7.604	14.491	0.977	17.496	4.829	1.890	19.708	7.805	3.673	20.889
5.742	0.010	0.032	0.001	0.028	0.036	0.071	0.527	0.327	0.090	0.055	0.191
0.297	5.291	6.654	1.364	0.010	3.430	1.441	2.139	2.410	7.269	6.174	3.906
6.622	0.436	2.622	0.319	7.987	3.339	0.925	24.385	0.617	6.975	13.789	2.236
7.319	10.340,	4.522	17.355	27.403	9.457	3.459	9.862	0.051	1.591	19.041	5.153
9.367	2.192	3.521	9.943	9.629	1.231	33.135	12.272	8.848	1.325	0.789	3.948
2.027	9.197	1.407	2.765	11.948	1.726	3.697	3.158	9.710	9.563	0.4[illegible]	1.979
14.497	0.020	0.305	0.340	9.082	0.105	0.052	0.009	5.172	1.555	11.574	2.200
6.696	4.342	1.821	10.565	26.719	1.644	0.013	0.127	1.231	3.669	1.932	0.137
1.011	0.071	1.085	1.302	4.914	0.013	0.009	0.007	0.007	0.003	0.003	0.006
0.001	0.001	0.019	0.022	0.067	0.101	0.915	0.011	0.019	0.059	0.086	0.091
0.043	0.116	0.762	2.326	1.469	0.139	0.981	2.173	3.948	12.345	4.159	0.641

5. FOX ISLANDS ,LATITUDE(50,+54),LONGITUDE(-172,-165),N=1035

45.651	62.376	0.002	53.486	63.453	39.837	0.048	0.900	0.448	13.976	59.642	62.394
20.671	3.392	9.304	2.667	0.001	4.651	84.061	139.069	84.354	2.537	108.766	121.951
0.626	62.198	2.475	148.754	49.977	201.015	53.122	70.531	181.242	52.433	157.667	18.151
0.032	0.009	12.214	78.469	38.395	2.543	58.533	35.498	0.109	39.776	0.577	0.256
31.263	38.981	13.598	155.466	14.846	8.095	69.482	123.604	2.228	113.697	79.697	47.909
12.539	1.693	0.008	0.024	1.618	6.877	32.840	0.189	0.098	0.055	0.018	0.011
0.007	0.015	0.406	3.118	0.089	0.215	0.410	3.874	4.334	0.863	0.001	60.877
26.289	0.047	0.076	0.021	0.026	0.007	0.015	0.017	0.077	0.063	0.143	0.060
0.092	0.047	0.099	0.071	0.229	0.076	0.170	0.331	0.179	0.012	0.590	0.180
0.037	0.075	0.009	0.106	0.081	0.598	0.262	0.046	0.078	0.170	0.454	0.174
0.260	0.007	0.140	0.261	0.019	0.362	0.587	0.063	0.353	0.637	0.863	0.093
0.733	2.306	0.266	0.221	0.066	0.132	0.004	0.251	0.220	0.001	0.325	1.462
0.002	0.453	0.077	0.438	0.020	0.619	1.530	1.550	0.012	0.156	1.040	1.476
0.001	0.030	0.296	0.487	0.814	0.717	0.054	1.509	2.395	0.542	0.135	0.051
0.873	2.180	3.701	0.039	0.794	0.726	3.194	0.218	0.656	1.077	0.649	0.256
0.764	0.422	1.340	0.503	0.364	0.088	0.001	1.031	1.101	0.166	3.694	1.219
0.935	0.237	0.652	0.288	2.475	0.395	0.952	0.223	0.013	9.152	2.169	0.531
0.291	0.007	4.011	0.174	0.547	0.009	2.864	0.015	4.172	3.392	10.814	1.409
2.778	2.368	3.386	2.920	2.381	0.322	8.601	1.949	1.229	7.106	6.614	0.479
17.441	0.654	0.956	5.199	1.117	9.338	0.004	4.093	0.049	6.186	0.980	0.005
2.079	1.896	2.543	0.850	0.053	4.060	8.138	25.312	0.921	2.300	0.460	14.609
0.920	3.223	5.176	9.249	5.627	1.167	2.005	9.041	0.328	0.029	14.005	4.113
0.116	0.914	0.001	0.327	17.615	2.801	0.422	4.598	2.036	2.825	19.008	7.594
3.618	3.083	8.477	1.047	3.041	14.320	1.177	22.843	1.580	12.871	22.085	14.044
28.301	15.146	6.485	12.382	1.638	16.643	19.918	6.912	7.344	0.628	2.880	11.189
1.470	8.136	0.998	11.091	3.302	5.087	6.188	0.065	5.480	4.929	2.361	0.491
4.122	0.001	0.040	0.001	7.205	6.880	5.978	6.409	33.502	33.030	5.800	4.569
5.868	1.449	28.734	17.309	135.819	0.639	1.103	8.692	3.971	3.737	12.417	5.745
23.373	12.475	2.734	27.242	6.963	13.288	6.032	19.774	3.607	1.417	24.591	16.925
17.212	17.392	7.236	7.299	26.078	0.691	0.521	0.001	38.220	9.076	20.408	4.937
25.223	18.949	3.781	11.158	1.800	8.214	75.033	5.424	6.266	1.522	60.225	10.779
2.201	5.769	74.760	3.888	9.078	11.777	4.446	26.751	57.620	0.001	65.056	15.618
0.001	10.340	1.470	6.338	15.076	3.792	2.168	0.001	5.306	7.257	18.443	0.586
4.502	27.229	9.310	0.185	0.059	0.004	0.024	0.037	0.059	0.111	5.907	3.412
9.367	7.877	19.998	9.427	27.211	0.001	66.307	10.433	9.028	0.351	24.401	36.884
13.893	18.196	0.020	2.837	2.735	12.938	44.020	22.062	33.994	0.079	0.204	0.079
0.612	0.691	6.390	0.335	11.581	2.101	9.630	0.465	0.465	0.229	2.761	3.979
0.027	0.027	6.598	26.050	0.480	0.007	0.006	0.001	14.850	1.139	10.332	3.396
23.489	0.003	10.046	8.467	63.539	12.031	19.540	25.562	9.387	30.793	23.071	19.557
45.063	127.062	15.146	61.308	13.664	0.168	0.168	23.799	7.367	15.292	25.432	7.698
18.429	2.950	0.004	0.004	0.036	0.089	0.006	0.008	0.018	0.028	0.350	0.370
0.388	0.687	0.009	0.001	0.330	0.030	0.018	0.299	1.133	1.168	0.256	0.228
0.256	0.228	3.516	10.185	13.259	25.118	2.634	5.364	6.782	12.650	8.609	12.130
31.260	6.427	1.028	1.959	1.485	7.149	0.001	3.901	42.496	18.460	11.594	20.681
8.455	5.342	0.008	0.027	23.249	12.445	39.991	12.267	9.439	6.533	14.055	20.005
1.486	10.919	7.385	7.213	80.460	4.063	2.832	7.826	2.171	12.409	2.686	7.034
27.107	7.087	0.469	13.284	26.876	2.484	7.592	2.932	1.774	0.707	1.912	31.221
4.480	7.898	0.029	24.556	2.524	4.985	0.183	0.035	0.107	1.070	0.637	0.310
20.715	0.844	5.269	2.461	0.279	43.984	1.024	12.468	0.112	2.108	4.083	46.395
0.540	13.289	6.970	15.076	3.303	34.878	3.304	6.120	0.036	19.841	45.926	2.318
9.753	4.459	4.408	0.656	11.137	1.658	15.692	16.087	4.860	6.370	21.798	0.351
10.417	9.681	30.316	19.935	0.732	1.781	0.001	2.642	13.944	0.007	11.174	11.407
5.689	37.023	7.293	43.589	16.758	0.089	0.250	0.006	0.089	0.250	0.006	8.770
12.258	0.401	0.022	0.041	0.096	0.242	0.168	0.022	0.041	0.008	0.033	0.055
0.005	0.067	0.035	0.013	0.007	0.039	0.021	0.053	0.002	0.038	0.020	0.051
0.076	0.835	0.777	0.021	0.501	0.255	0.005	0.317	0.084	0.663	0.402	0.128
0.062	0.672	3.668	0.045	3.378	3.058	1.428	1.238	1.625	3.717	0.602	3.006
0.265	1.877	6.336	3.657	9.845	2.613	12.537	3.636	2.791	3.772	3.163	10.462
5.577	0.947	5.013	2.491	20.063	21.414	30.247	2.016	0.287	4.847	18.367	32.494
7.923	4.141	4.176	16.008	14.757	6.401	9.901	52.678	0.409	2.072	16.979	12.786
2.022	1.268	0.995	1.131	3.122	8.668	0.796	0.013	0.224	0.001	0.436	0.012
1.519	1.096	0.823	3.506	5.141	0.977	0.120	1.021	1.985	0.577	4.686	7.896
3.444	3.871	0.769	1.464	0.016	0.001	4.921	7.240	0.975	0.002	0.184	0.177
2.888	0.501	3.571	0.127	8.195	1.742	13.849	5.469	3.695	2.192	0.716	0.150
1.947	4.162	5.652	0.836	1.408	0.262	3.053	0.933	0.333	1.195	16.855	6.272
0.005	1.421	10.132	0.032	1.368	16.132	0.214	2.100	0.001	12.041	5.583	16.018
11.376	53.939	0.184	6.205	1.276	15.700	22.207	19.100	31.375	0.047	1.776	4.069
6.277	2.565	0.013	0.018	0.007	0.011	0.008	0.045	0.011	0.015	0.031	0.028
0.014	0.006	0.007	0.016	0.071	0.077	0.038	0.014	0.174	0.276	0.291	1.231
0.758	0.862	6.929	0.697	9.727	9.322	4.086	17.124	2.677	10.268	5.283	19.885
3.681	12.213	3.622	27.888	0.293	2.859	1.553	0.059	0.031	0.004	7.723	10.567
0.462	14.473	1.056	11.692	1.281	13.758	0.542	0.196	0.055	0.075	0.003	0.243
0.044	3.430	4.217	0.465	2.232	8.203	4.570	0.004	0.148	5.147	0.287	10.120
0.144	23.505	0.100	4.289	6.477	4.024	18.510	13.388	0.479	1.350	2.121	27.345
7.222	11.471	4.398	2.267	4.784	39.700	2.711	5.304	3.715	9.344	8.708	28.570
5.695	4.228	8.731	4.935	1.437	0.303	0.992	0.632	0.795	10.444	2.596	9.067
0.122	7.541	5.961	0.078	5.511	5.423	2.893	44.442	12.626	36.860	6.590	14.483
10.792	48.115	51.955	0.583	1.620	0.072	1.475	35.623	26.544	20.364	7.516	4.184
15.895	12.620	1.759	0.966	15.103	3.181	24.974	35.259	4.848	8.406	3.048	39.509
17.725	38.077	63.880	0.326	4.073	9.879	8.676	22.773	17.748	0.997	14.900	0.302
1.307	33.278	2.571	15.472	1.598	5.432	1.519	0.009	3.843	13.680	1.105	0.127
0.267	10.192	0.292	0.125	3.193	0.059	0.079	0.159	16.130	0.234	17.753	14.613
0.216	1.180	2.469	15.491	8.106	10.325	0.541	2.195	3.625	11.599	11.799	2.459
1.120	29.938	23.614	3.313	2.193	3.657	15.099	8.819	0.137	16.495	17.076	18.160
6.961	8.579	9.483	14.820	42.803	43.187	24.164	60.098	5.956	10.600	8.161	47.159
17.093	5.534	34.176	7.958	57.140	12.406	6.033	0.383	1.191	19.762	2.971	13.800
29.004	0.137	34.504									

7. [illegible]ANDASEA,LATITUDE(-[illegible],-5),LONGITUDE(+122,+132),N=715

22.617	35.912	102.197	51.[illegible]	39.325	[illegible].284	43.456	20.825	1.967	197.990	0.523	33.414
173.599	116.815	104.680	43.192	36.399	114.497	114.267	0.092	1.138	51.147	59.254	147.441
24.343	46.549	112.420	[illegible].231	162.497	125.432	70.942	121.754	1.120	0.001	5.054	98.230
228.14	156.497	40.244	4.137	59.250	21.188	1.365	141.275	34.690	81.360	55.078	39.871
259.45	70.571	35.423	47.191	34.839	105.235	6.346	2.335	10.998	3.077	70.850	59.476
55.795	161.225	[illegible]	32.777	10.281	6.229	17.320	11.670	1.999	5.512	0.390	6.676
12.333	11.212	19.322	35.569	15.034	0.001	119.983	49.022	20.449	20.544	47.294	13.391
16.015	16.964	12.417	4.367	22.909	35.267	21.160	24.337	93.519	1.708	55.291	2.458
1.956	9.568	4.847	19.383	2.367	20.719	12.074	0.176	8.796	7.855	5.705	27.097
1.345	9.095	7.966	27.699	7.314	12.313	39.417	13.854	4.522	10.798	23.023	2.391
0.416	0.001	0.038	1.391	8.036	7.120	19.706	21.119	5.720	20.050	21.452	0.146
0.276	0.703	1.199	0.731	42.965	2.071	1.595	28.681	4.994	1.488	27.099	0.114
6.179	0.043	2.400	14.313	13.094	2.361	13.763	9.749	10.912	16.532	1.120	6.835
0.613	17.801	5.118	0.407	0.497	9.536	3.165	44.390	1.492	7.794	9.252	1.353
15.381	2.596	4.827	24.770	1.567	2.782	29.054	0.727	2.327	15.310	4.654	9.799
4.622	75.573	21.871	9.307	9.758	2.158	11.901	8.616	11.223	19.889	0.128	0.452
0.079	0.193	0.400	0.611	0.840	0.180	0.027	0.211	0.001	0.277	1.024	1.125
8.510	8.075	3.554	6.222	20.973	2.436	10.977	11.398	19.424	29.921	30.584	6.225
41.178	20.263	0.349	28.112	43.175	14.903	24.105	0.001	42.164	4.519	7.274	7.704
17.996	1.216	17.413	0.061	9.074	9.194	14.384	1.849	11.225	0.230	8.429	32.480
17.786	10.301	13.399	67.026	1.430	4.944	11.536	1.123	4.207	5.289	10.343	14.677
23.129	9.815	14.308	24.195	10.462	0.057	0.022	0.067	0.077	0.022	0.591	0.279
0.145	0.419	0.050	0.157	0.996	7.462	0.894	4.015	0.581	1.184	2.679	4.906
2.243	12.169	0.115	1.706	1.844	10.243	6.051	1.463	1.458	11.529	0.277	1.417
5.425	1.595	8.003	17.391	3.230	1.450	9.763	14.391	1.571	6.774	47.755	0.679
0.430	5.043	3.549	0.084	2.024	12.648	7.713	8.319	1.971	3.853	10.297	9.250
0.969	5.452	1.398	14.451	4.418	13.337	8.390	8.505	11.994	12.652	0.507	17.077
5.327	0.886	0.001	9.091	2.736	5.623	4.637	18.230	4.507	2.564	14.392	17.189
12.683	0.109	7.484	0.103	5.652	11.579	18.934	6.916	2.133	8.542	1.479	1.291
19.711	4.062	5.999	0.789	4.785	0.719	10.252	0.165	4.967	2.247	17.276	4.374
1.577	0.001	16.373	15.705	2.614	7.694	12.049	3.495	0.001	8.251	3.751	13.177
5.724	3.323	4.315	9.294	12.153	5.234	21.797	1.172	9.301	3.263	0.597	36.844
1.652	9.293	11.634	6.346	7.161	14.219	11.966	3.473	11.594	0.096	1.961	2.449
0.137	6.915	15.160	33.034	4.008	6.593	13.479	2.561	1.399	7.797	13.387	17.214
8.513	16.199	3.959	0.755	2.257	11.856	3.356	1.324	1.521	5.394	22.531	0.189
9.259	2.748	1.166	13.058	11.195	8.699	19.327	0.256	27.060	10.573	1.484	16.634
9.816	4.289	4.031	7.652	4.753	13.450	17.867	9.651	4.391	23.845	8.884	10.591
27.287	1.544	11.759	7.216	25.645	11.750	9.150	6.389	7.369	13.490	9.359	4.078
8.242	18.760	3.504	0.342	0.482	5.401	5.541	27.396	36.191	13.319	0.831	2.412
2.795	1.899	9.494	[illegible]	15.922	13.150	5.497	25.306	1.995	2.318	0.492	5.552
0.704	49.910	21.095	4.47[illegible]	4.5[illegible]7	2.267	8.879	2.633	3.199	16.380	13.216	15.572
5.465	8.752	4.345	2.363	1.207	2.409	4.765	0.902	1.933	1.463	0.197	9.467
3.404	45.652	17.774	19.156	1.939	3.266	5.765	21.150	16.936	3.949	6.052	9.027
21.959	4.504	12.201	[illegible]	3.732	13.123	1.974	0.962	0.317	5.764	0.665	4.926
2.160	9.782	11.346	6.912	3.524	0.815	9.326	0.040	1.509	11.191	5.600	2.495
2.474	14.937	2.092	7.443	17.345	31.394	0.764	10.074	9.039	1.745	22.169	1.841
15.393	7.637	2.341	2.054	3.208	11.221	24.901	17.225	0.034	12.499	5.836	5.244
19.330	4.505	4.330	2.467	3.134	1.526	5.241	13.779	3.737	7.869	2.496	3.554
1.221	0.012	1.879	0.671	4.487	2.049	36.857	7.923	1.606	9.945	1.907	27.801
29.952	4.699	3.247	1.071	4.920	6.555	19.542	3.923	6.420	0.337	33.972	2.444
2.915	2.027	7.428	0.599	0.799	3.332	9.606	13.881	15.611	1.553	0.931	4.221
16.616	13.434	1.789	1.094	21.572	0.565	0.655	7.191	22.183	14.993	0.899	1.519
1.239	23.863	5.012	23.225	3.351	0.654	2.697	12.214	26.252	7.723	2.861	9.584
14.172	2.344	4.730	5.362	1.098	1.951	12.055	17.182	17.612	0.673	11.836	1.285
22.546	5.965	22.442	1.941	12.802	4.294	28.532	3.165	3.022	33.302	7.978	5.355
23.324	3.561	8.565	0.872	2.384	9.591	11.225	5.259	2.210	32.012	40.377	15.673
24.641	2.347	33.775	0.944	20.458	5.389	6.814	9.514	29.639	1.711	32.174	15.453
0.313	67.361	35.329	14.467	1.617	25.079	4.186	23.817	11.165	7.033	16.276	14.481
3.487	5.729	0.340	3.671	10.340	15.449	3.622	0.964	4.673	5.904	15.764	1.121
4.101	4.364	3.411	1.263	13.154	12.642	17.424					

6. SØNDAARC ,LATITUDE(-10,-00),LONGITUDE(+100,+112),N=602

101.324	0.001	246.131	56.500	1.440	36.761	85.960	82.372	33.122	16.092	18.399	4.934
2.965	32.924	236.682	312.002	1.144	10.343	59.969	4.983	114.594	24.271	29.498	64.486
72.813	160.639	20.593	21.477	277.379	10.611	46.210	14.746	0.001	120.075	0.001	42.554
8.860	4.955	87.279	0.001	159.984	144.070	8.490	21.116	40.547	9.590	19.969	0.001
30.015	3.131	46.827	0.998	36.431	34.924	50.598	7.821	60.953	11.445	85.792	11.395
0.876	7.460	18.154	32.209	5.821	7.142	17.022	8.040	4.445	0.001	7.299	0.001
31.533	2.831	29.769	2.780	13.959	13.832	39.755	119.344	2.070	49.955	12.679	0.019
67.213	22.240	31.261	5.326	4.434	24.414	23.937	49.590	19.307	21.254	12.961	0.301
0.006	0.056	5.155	4.896	13.995	7.236	90.644	0.011	0.002	49.763	3.572	9.645
49.376	20.381	9.447	7.655	26.042	75.191	29.839	46.450	12.229	1.379	20.775	51.814
2.152	43.963	9.260	20.775	32.966	0.963	1.182	6.518	0.031	15.729	5.774	19.202
4.122	47.913	0.543	2.369	13.123	13.721	13.983	1.254	21.490	11.099	32.911	24.516
1.584	28.238	10.565	12.599	1.409	2.178	53.106	95.694	4.135	6.059	5.192	18.870
5.497	51.071	15.183	6.558	2.026	32.332	0.001	13.220	17.537	3.790	9.829	21.424
3.302	17.940	4.184	19.201	17.637	16.284	6.011	9.960	3.316	2.223	4.453	16.829
3.386	0.001	20.334	13.770	9.651	2.467	31.894	4.800	6.275	1.304	0.001	45.696
15.514	0.098	0.098	3.329	17.519	0.001	6.058	19.366	0.001	16.982	5.329	19.555
0.668	11.165	6.242	17.883	0.377	23.390	0.359	3.301	0.128	5.309	1.465	3.475
2.209	0.105	0.001	0.107	64.979	20.797	15.451	5.744	3.013	0.013	0.069	0.018
19.773	9.670	0.001	109.437	66.757	61.502	18.757	33.451	41.973	11.547	14.500	1.515
27.014	19.516	39.494	0.009	0.300	4.311	0.115	35.441	16.840	1.536	1.449	6.412
7.429	31.929	1.565	3.552	15.238	15.645	4.809	0.001	20.659	1.440	8.394	1.176
2.714	4.019	2.254	0.807	14.152	7.283	17.190	0.677	17.546	12.099	1.644	6.925
5.434	22.133	3.604	15.689	23.030	5.944	7.790	21.491	15.159	0.001	3.241	16.079
17.928	0.269	0.377	0.015	0.039	0.091	0.710	13.524	1.683	0.268	3.002	7.842
4.206	0.001	9.332	15.259	0.001	8.035	6.372	0.001	8.315	12.468	3.159	5.075
7.041	14.833	18.360	4.035	36.133	8.129	8.116	0.946	23.666	30.981	32.015	6.081
8.986	45.544	0.036	17.967	24.319	5.769	0.001	2.299	5.347	36.200	19.286	7.759
20.403	1.393	0.384	1.824	36.457	21.993	3.013	8.266	7.207	2.804	9.284	2.649
17.379	2.829	20.493	7.657	2.946	10.925	31.670	1.352	23.457	0.001	13.225	6.252
21.963	0.001	26.174	0.001	7.655	0.001	14.582	3.529	2.936	13.424	0.371	1.834
0.298	6.641	0.420	0.924	22.346	4.097	11.001	17.577	13.145	45.502	9.950	22.047
12.539	2.598	9.103	14.238	2.084	2.261	0.569	7.500	6.092	17.003	23.281	0.001
10.830	25.282	31.225	5.198	12.648	13.982	15.047	9.691	2.914	11.211	1.497	26.319
3.029	50.979	5.978	0.473	12.164	8.570	5.880	71.518	2.302	3.566	3.799	20.971
2.632	2.821	26.149	49.337	6.865	27.301	31.066	30.999	5.499	0.765	19.331	5.542
2.711	0.018	32.528	10.805	28.799	0.656	0.424	0.166	6.561	9.950	42.881	4.665
18.442	2.269	27.033	1.090	0.233	9.657	10.240	3.339	31.836	16.515	5.693	2.438
8.203	6.741	11.943	26.194	6.512	18.623	4.335	6.533	20.419	1.339	44.263	8.023
6.485	8.272	14.879	26.130	13.210	32.354	69.893	1.612	13.585	44.248	0.977	9.817
19.414	2.549	9.803	16.728	10.897	16.943	17.289	16.089	17.349	55.767	5.666	21.338
12.195	4.113	5.204	15.642	16.373	16.629	9.306	33.878	17.501	13.821	4.598	19.958
0.634	5.078	39.200	26.554	12.393	4.047	0.816	4.313	0.006	1.394	1.592	10.238
0.534	19.163	10.376	8.034	14.156	18.630	8.536	9.667	10.596	12.239	25.909	23.792
0.504	2.083	10.141	2.735	0.264	0.004	2.739	10.577	18.231	4.060	4.366	1.355
0.004	3.689	5.455	2.483	4.623	0.394	23.451	0.060	6.277	4.737	55.139	4.126
13.698	8.397	0.867	12.274	3.246	6.995	27.387	17.187	59.530	12.438	0.257	0.040
5.703	29.077	18.577	74.076	9.790	16.427	4.619	8.727	4.665	25.450	9.473	9.027
32.122	0.961	36.567	8.501	11.068	8.394	8.513	12.420	1.604	17.700	12.069	10.817
4.935	1.919	6.048	4.155	3.378	0.269	45.258	2.039	0.014	5.399	7.783	3.244
8.347	2.763										

8. FIJI ISLANDS,LATITUDE([illegible]),LONGITUDE(-180,-176),N=966

[illegible]	5.622	34.570	[illegible]	74.899	73.634	22.684	1.402	129.234	11.543	18.272	46.215
[illegible]	14.619	63.889	[illegible]	9.770	110.594	5.910	7.428	0.270	0.073	10.099	6.554
11.21	13.247	29.912	9.634	6.754	39.743	19.366	46.385	16.227	7.421	49.981	1.096
14.1	0.824	1.163	42.422	145.813	99.274	48.486	173.576	170.597	4.238	0.169	192.690
63.092	20.298	2.783	54.387	0.001	79.452	44.071	36.801	0.213	30.308	89.940	0.020
13.0 6	38.698	0.074	3.536	9.431	94.481	16.665	21.099	5.062	13.797	24.101	8.023
6.680	22.858	47.922	10.105	0.086	1.467	1.165	1.802	2.764	8.766	9.716	4.016
27.166	0.544	2.301	0.923	27.301	8.319	20.643	10.817	75.189	0.001	21.593	1.681
25.714	64.147	13.272	6.035	29.349	51.602	2.122	0.496	14.123	12.470	0.510	37.509
8.144	9.435	25.376	14.711	11.519	80.008	2.214	23.977	17.332	3.100	15.861	0.001
30.125	94.455	15.140	29.835	1.012	84.736	0.001	36.161	78.156	38.016	36.327	1.569
8.274	22.795	69.749	13.334	2.011	4.970	5.708	0.174	14.214	50.468	9.120	157.995
3.195	5.591	5.415	1.544	7.212	41.531	5.826	36.038	66.451	0.009	0.020	0.060
0.098	0.017	0.004	0.532	12.191	0.075	5.513	12.517	15.315	8.479	0.001	5.110
8.540	2.664	3.168	40.424	21.686	11.229	0.354	2.173	21.062	26.645	0.709	0.422
0.233	0.325	12.689	9.182	4.553	14.831	1.439	7.636	4.201	2.584	25.750	4.529
1.739	0.054	2.477	0.568	0.001	1.155	1.114	2.197	32.700	31.364	12.735	6.781
20.293	28.390	3.738	9.292	8.648	2.559	20.532	5.350	0.012	20.616	7.724	0.001
3.751	0.006	0.036	14.345	15.315	38.640	20.058	16.303	25.685	0.692	4.003	0.748
0.0 1	0.242	28.528	3.390	0.563	85.719	0.224	0.359	0.196	0.028	0.135	0.063
0.578	0.084	0.094	0.156	0.002	0.018	0.026	0.010	0.011	0.039	0.012	0.636
2.550	1.004	0.988	0.655	0.721	0.611	1.248	1.838	9.914	25.502	32.533	12.962
3.417	10.299	1.487	10.193	2.008	12.509	25.879	5.062	1.081	0.139	3.137	1.629
1.852	58.570	0.225	0.071	0.154	1.060	0.117	2.264	0.618	0.105	0.318	0.105
0.214	0.095	5.019	4.139	2.014	1.400	3.159	3.377	3.165	2.103	0.374	0.374
3.716	0.231	0.067	4.368	29.526	31.978	10.171	5.029	0.457	10.905	7.625	3.925
7.852	0.001	7.632	3.785	19.409	5.709	3.206	2.508	0.001	3.286	21.160	20.524
34.247	7.557	8.224	6.323	1.219	19.517	0.002	4.804	2.664	6.671	31.467	2.532
23.414	1.791	0.018	3.471	4.041	14.764	3.835	0.327	0.327	7.390	6.154	20.809
11.997	23.522	3.536	0.371	37.459	29.496	14.766	14.683	15.408	9.671	7.700	2.795
2.934	29.693	0.292	0.446	7.684	0.001	21.614	23.191	9.706	0.054	15.193	1.754
19.956	3.558	26.700	5.000	3.435	10.141	12.512	5.283	18.617	1.601	1.495	9.642
2.022	15.995	15.439	17.190	7.852	3.770	2.007	5.680	0.561	2.672	7.693	3.400
0.212	29.913	0.769	13.032	22.494	4.863	16.006	20.316	0.001	9.791	12.609	4.116
1.355	4.501	4.697	2.053	1.968	8.833	2.987	3.823	6.382	6.911	1.136	7.149
0.872	1.876	8.419	28.708	5.095	5.399	4.259	0.982	6.953	14.746	23.096	1.740
0.285	1.698	14.920	31.142	7.631	6.544	0.782	2.715	0.821	5.650	0.811	1.980
0.207	0.120	0.429	1.374	0.303	0.691	4.163	0.911	3.260	0.101	6.513	0.941
0.810	18.220	15.909	7.540	8.083	1.914	3.700	2.634	13.519	0.001	20.696	1.830
9.380	0.882	6.350	0.001	5.068	18.447	2.304	0.793	5.108	2.685	2.549	3.819
15.661	3.859	0.367	13.310	13.095	3.725	3.875	10.180	4.843	15.032	8.296	13.812
7.813	13.919	6.059	4.987	4.403	7.728	0.677	0.168	0.234	5.344	6.325	0.358
21.239	5.654	3.130	6.306	4.839	7.360	7.198	7.837	16.139	12.439	2.341	14.959
0.028	0.003	0.918	3.214	8.395	3.755	12.439	2.334	2.924	2.181	9.579	8.504
4.475	2.114	3.422	6.654	4.736	0.827	14.076	3.398	5.226	6.038	4.267	0.543
1.864	3.125	2.682	8.670	23.266	1.338	2.252	5.308	3.194	2.166	3.576	8.367
2.455	5.957	16.282	35.609	8.558	4.687	0.180	0.145	0.178	0.110	0.030	0.003
0.027	0.017	0.030	0.058	0.103	0.001	0.010	0.064	0.006	0.036	0.081	0.027
0.094	0.086	0.009	0.040	0.008	0.003	0.025	0.037	0.009	0.149	0.026	0.022
0.033	0.043	0.015	0.080	0.048	0.003	0.712	0.094	0.041	0.066	0.048	0.041
0.069	0.189	0.075	0.011	0.183	0.316	0.077	0.267	0.148	0.257	0.884	0.716
0.512	0.084	0.056	0.370	0.343	1.819	2.542	0.006	4.735	0.924	1.422	1.287
1.356	4.974	7.369	11.361	0.705	0.310	0.592	1.265	0.005	0.011	0.018	0.022
0.564	0.757	0.826	0.787	0.437	11.750	4.201	0.694	12.999	1.555	0.376	16.269
5.877	7.179	0.033	0.141	0.017	12.027	1.007	4.169	1.551	11.132	10.432	4.438
6.208	11.449	1.228	10.098	7.850	2.427	1.472	2.062	0.839	0.638	0.493	0.001
6.275	0.363	0.167	0.613	5.893	2.263	0.001	12.677	6.658	4.918	1.330	1.890
0.001	0.356	5.465	5.529	6.232	7.762	7.420	0.955	14.145	0.013	1.991	0.119
1.201	0.285	0.001	2.390	2.826	6.453	3.793	2.341	4.091	0.301	4.380	10.865
0.595	2.625	10.433	6.180	21.544	4.318	9.579	0.582	0.390	9.087	6.894	1.194
5.402	1.443	7.854	1.435	1.656	0.066	1.821	6.756	1.920	7.766	17.192	0.504
4.561	13.793	4.887	4.049	8.643	3.384	4.424	19.183	0.822	2.912	8.494	4.355
5.779	3.947	15.052	8.575	1.100	5.164	8.811	4.369	1.298	10.898	0.699	1.745
7.141	24.741	1.350	1.347	2.263	9.062	11.962	5.278	11.506	6.862	0.001	3.880
3.602	8.308	11.457	0.409	12.423	2.922	10.803	1.578	6.617	0.638	3.204	2.746
6.068	0.199	0.230	0.099	6.788	6.613	0.110	1.004	18.439	0.966	4.274	0.165
4.029	15.354	15.993	2.276	0.578	21.179	33.527	7.021	16.794	6.923	0.900	7.888
48.265	18.129	0.112	1.889	12.053	10.725	12.941	5.893	5.662	44.070	51.968	13.177
3.101	34.871	8.270	46.058	17.551	10.002	7.513	35.231	6.174	17.928	9.229	13.823
10.836	3.344	13.900	2.360	8.622	15.343	1.335	12.589	8.099	39.278	3.176	5.396
14.983	10.094	20.036	24.862	10.805	0.822	0.046	5.078	2.154	22.382	7.002	11.493
0.324	29.627	5.948	1.604	2.418	57.649	18.085	0.259	10.872	19.294	1.980	0.518
10.366	2.727	6.945	7.273	6.130	8.539	18.375	32.140	17.741	15.690	34.326	22.021
4.566	1.123	0.056	12.611	10.697	7.040	0.812	18.556	24.632	6.983	6.279	3.723
4.111	1.327	4.853	0.357	3.864	43.358	1.211	35.221	6.067	4.673	18.040	7.858
8.958	12.742	4.832	6.025	48.012	10.804	4.439	11.757	0.733	3.771	7.628	5.439
3.741	41.483	2.047	5.029	12.834	4.933	11.219	2.623	3.891	0.449	0.747	0.069
2.492	2.288	6.962	3.201	1.053	0.094	0.962	0.005	7.508	4.147	5.600	1.349
19.240	7.839	14.127	2.264	10.624	1.266	15.316	1.584	10.316	9.767	9.167	0.078
0.027	0.067	0.125	0.090	0.201	0.438	0.154	1.245	0.223	0.733	1.071	1.372
1.129	11.995	21.916	20.103	2.000	3.986	6.008	8.757	33.892	15.414	16.943	0.304
12.632	35.374	12.440	2.158	8.127	34.457	0.485	6.046	31.913	3.278	13.112	38.379
39.783	20.658	17.440	32.127	36.144	7.205	32.470	1.676	4.766	3.147	20.713	8.446

[illegible]

15.7[illegible]	4.[illegible]	12.[illegible]	[illegible].343	1[illegible]	28.7[illegible]	.5[illegible]	[illegible]	24.[illegible]	[illegible]	[illegible]	[illegible]
7.7[illegible]4	5.11	7.[illegible]1	12.[illegible]	25.324	10.[illegible]	15.175	17.[illegible]	1.[illegible]	[illegible]	[illegible]	[illegible]
20.[illegible]	4.177	47.351	[illegible].065	27.177	1.03	3.38	13.5[illegible]	.73	27.[illegible]	[illegible]	[illegible]
3.355	5.694	15.470	[illegible].414	2.955	16.857	7.181	23.141	[illegible]	4.521	[illegible]	[illegible]
[illegible].728	.1	4.3[illegible]2	1[illegible].016	4.796	17.521	36.449	[illegible].006	41.6[illegible]	11.170	6.471	.675
1.147	32.58[illegible]	172.0[illegible]6	17.922	60.972	11.491	20.1[illegible]1	[illegible].563	7.050	24.15[illegible]	[illegible]	[illegible]
34.373	35.982	7.1[illegible]7	36.011	64.967	25.952	0.975	14.717	1.[illegible]	[illegible]	[illegible]	[illegible]
2.236	3[illegible]	9.654	2.467	4.143	[illegible].539	47.583	[illegible].257	11.535	[illegible].345	[illegible]	[illegible]
9.641	7.329	21.319	2.155	6.155	2.482	4.565	6.524	6.592	23.[illegible]17	51.[illegible]	17.1[illegible]
12.146	56.955	2.562	[illegible]	6.713	1.780	3.707	3.0[illegible]7	38.7[illegible]	.10	[illegible].474	33.[illegible]
20.796	4.809	20.997	11.1[illegible]8	4.314	13.627	7.594	2.[illegible]65	3.310	6.033	67.[illegible]	2.2[illegible]
5.[illegible]	4.177	17.[illegible]84	26.[illegible]64	1.433	26.301	3.224	27.301	14.259	7.[illegible]	7.[illegible]4	12.711
17.226	38.347	12.242	5.481	48.757	52.725	6.25[illegible]	6.1[illegible]	[illegible].211	19.[illegible]56	2.741	[illegible]
10.573	11.296	20.767	1.594	16.826	9.896	4.641	0.496	1[illegible].753	1.564	[illegible].1[illegible]6	1.[illegible]7
16.584	31.928	14.944	27.6[illegible]	0.271	6.656	8.[illegible]49	6.981	7.936	46.455	7.534	[illegible]47
0.[illegible]43	8.659	12.241	28.836	27.570	15.305	19.816	5.527	[illegible].174	37.92[illegible]	4[illegible].720	13.14[illegible]
3.278	3.356	3.362	38.595	11.721	12.798	0.[illegible]24	3.982	6.835	5.267	19.574	4.643
2.[illegible]39	23.556	104.091	7.335	54.202	4.561	28.965	9.153	1.812	6.904	7.147	13.[illegible]
9.531	32.715	3.561	15.701	7.281	[illegible].276	0.457	1.677	5.691	4.4[illegible]1	[illegible].24	12.[illegible]1
10.626	14.694	6.205	4.2[illegible]9	2.462	[illegible].239	3.374	[illegible].530	2.[illegible]79	15.223	11.[illegible]	18.851
42.671	18.482	0.[illegible]01	16.4[illegible]5	[illegible].023	3.355	4.361	5.674	[illegible]	5.[illegible]57	[illegible]	12.06[illegible]
5.[illegible]63	0.174	4.316	13.354	9.8[illegible]2	4.71[illegible]	2.368	4.129	16.114	6.[illegible]76	1[illegible].474	.313
0.920	0.919	4.574	5.5[illegible]7	2[illegible].833	5.971	2.913	22.301	2.564	6.[illegible]	10.[illegible]61	1.464
[illegible]1	7.331	1.923	4.693	[illegible].613	7.384	14.354	3.174	1.413	4.945	1.0[illegible]	6.[illegible]
9.[illegible]7	0.[illegible]97	13.730	3.95[illegible]	[illegible]	3.674	2.156	.749	14.7[illegible]1	7.652	11.636	5.[illegible]12
0.744	0.908	23.138	4.1[illegible]6	7.407	4.273	[illegible].11[illegible]	[illegible].1[illegible]	9.[illegible]25	34.953	5.593	.557
0.558	1.889	10.8[illegible]	3.616	[illegible].873	[illegible].41[illegible]	28.162	1.4[illegible]	1.583	2.550	[illegible].456	4.119
2.329	6.456	1.2[illegible]8	3.937	3.543	10.241	2.472	1.392	21.675	6.962	1[illegible].544	5.179
0.[illegible]1	4.534	1.614	1.539	[illegible].029	[illegible].307	0.817	5.644	6.749	1[illegible].513	4.3[illegible]5	7.25
1.654	[illegible].915	[illegible].460	[illegible].46[illegible]	1.617	11.035	14.889	6.245	13.377	1.942	[illegible].352	2.736
3.738	1.207	9.9[illegible]	1.25[illegible]	14.274	21.362	0.796	1.806	1.5[illegible]6	4.567	[illegible].946	2.242
10.4[illegible]5	5.185	1.994	4.297	[illegible]6	[illegible].331	0.006	1[illegible].353	[illegible].645	.[illegible]49	1.[illegible]55	.5[illegible]
0.[illegible]9	5.764	8.[illegible]32	5.725	[illegible].138	6.547	8.700	1.258	21.656	[illegible].1[illegible]3	13.828	3.[illegible]56
4.236	2.211	4.789	1.974	2.381	5.839	13.327	5.826	11.933	[illegible].676	15.[illegible]7	2[illegible].23[illegible]
12.575	15.094	1.956	9.532	1.173	1[illegible].[illegible]76	1.[illegible]	9.754	1.447	3.572	6.551	6.61
16.[illegible]1	2.192	2.227	1.392	9.717	6.789	18.935	5.2[illegible]1	4.127	39.294	4.[illegible]4	[illegible].97[illegible]
4.931	0.[illegible]53	4.256	19.2[illegible]4	[illegible].245	8.746	1.9[illegible]8	4.[illegible]	[illegible].647	[illegible].447	3.542	7.969
0.[illegible]01	[illegible].235	6.258	4.313	1.715	4.339	12.572	4.91[illegible]	2.349	1.974	[illegible].4[illegible]7	8.83[illegible]
[illegible].971	12.154	3.674	4.614	3.[illegible]3	1[illegible].193	3.446	1.316	1.925	4.511	3.165	2.1[illegible]3
0.374	3.947	[illegible].[illegible]67	1.[illegible]76	12.429	9.757	2.835	5.772	1.139	[illegible].251	[illegible].48[illegible]	3.[illegible]
4.134	1.809	6.908	2.312	[illegible].317	[illegible].317	12.566	1.387	3.990	2.411	1.59[illegible]	5.267
1.675	0.616	2.649	4.737	8.111	1.015	2.383	7.101	5.75[illegible]	8.547	1.262	9.917
1.621	4.732	[illegible].136	[illegible].136	4.320	5.454	3.467	1.801	[illegible].70[illegible]	1[illegible].455	4.179	4.3[illegible]9
2.320	0.001	9.941	[illegible].173	[illegible].173	9.336	4.244	3.479	3.124	[illegible].215	4.572	1.618
1.720	6.134	0.314	[illegible].314	4.554	[illegible].568	0.552	3.974	2.944	5.360	5.6[illegible]0	3.85[illegible]
11.405	3.979	4.046	7.356	5.590	7.140	0.001	18.029	2.1[illegible]0	16.177	4.539	6.266
1.246	4.595	6.345	0.453	1.792	[illegible].017	4.483	3.030	18.521	13.949	12.091	8.096
9.010	3.170	7.579	6.635	9.758	2.578	5.269	18.468	5.243	6.290	[illegible].135	[illegible].145
5.454	2.803	2.629	[illegible].613	5.9[illegible]5	[illegible].088	[illegible].088	7.486	3.2[illegible]	2.226	6.[illegible]58	6.644
5.549	5.045	[illegible].[illegible]1	4.5[illegible]4	12.244	2.5[illegible]	1.898	1.906	1.429	19.351	2.47[illegible]	[illegible].464
2.2[illegible]4	0.473	6.130	[illegible].988	19.546	9.615	1.620	3.868	6.868	2.644	[illegible].7[illegible]	3.321
2.750	1.671	0.225	1.527	[illegible].856	4.430	13.395	2.2[illegible]7	5.914	3.522	[illegible].776	5.[illegible]
0.533	0.370	5.551	[illegible].853	4.341	1.719	9.198	2.367	2.968	[illegible].243	[illegible].755	[illegible].188
10.844	1.722	1.012	2.345	1[illegible].025	9.472	3.074	3.[illegible]91	5.733	[illegible].989	8.241	.777
2.[illegible]8	11.397	4.9[illegible]9	[illegible].433	5.971	2.859	5.083	[illegible].771	4.649	19.991	9.752	19.444
0.106	0.[illegible]1	8.060	2.[illegible]98	1.[illegible]75	2.975	0.[illegible]42	2.[illegible]11	2.123	[illegible].7[illegible]1	3.77	[illegible].381
3.515	2.676	1.170	4.178	0.516	1.419	2.390	[illegible].[illegible]01	4.884	[illegible].291	[illegible].272	1.336
3.696	0.448	2.504	2.798	[illegible].809	9.065	6.023	4.697	1.944	1.724	[illegible].335	9.[illegible]7
0.347	2.367	0.[illegible]2	12.213	1[illegible].965	4.018	[illegible].[illegible]1	2.[illegible]65	4.625	3.599	2.66[illegible]	[illegible].292
3.974	3.755	1.810	1.2[illegible]8	4.106	9.453	7.60[illegible]	1.28[illegible]	7.891	7.111	2.33[illegible]	2.171
2.2[illegible]4	2.724	1.082	[illegible].445	[illegible].152	[illegible].965	1.046	0.773	2.601	3.126	0.317	4.6[illegible]5
8.438	2.105	1.349	.933	1[illegible].612	[illegible].815	1.922	16.534	4.956	[illegible].579	3.88[illegible]	[illegible].221
13.[illegible]20	0.339	1.486	2.653	4.[illegible]47	1.764	3.449	[illegible].273	4.531	3.561	0.[illegible]1	[illegible].16[illegible]
0.102	5.146	2.945	2.352	1.[illegible]29	[illegible].452	4.163	4.5[illegible]7	5.119	0.[illegible]68	[illegible]8	[illegible].915
8.197	3.002	1.527	1.495	2.756	[illegible].865	[illegible].177	1.001	3.566	11.354	[illegible].596	9.509
1.676	0.876	[illegible].341	[illegible].843	0.618	12.398	4.747	[illegible].444	6.639	2.615	2.893	1[illegible].220
0.235	3.011	2.755	[illegible].[illegible]63	[illegible].691	9.126	13.449	5.321	[illegible].194	3.5[illegible]2	[illegible].646	7.959
0.3[illegible]3	5.195	1.141	2.352	1.631	0.[illegible]6[illegible]	3.226	2.378	1.876	2.398	1.34[illegible]	[illegible].332
4.381	4.175	5.827	4.275	2.[illegible]65	[illegible].732	1.339	[illegible].533	3.543	1.791	2.83[illegible]	4.[illegible]
9.531	3.734	1.284	[illegible].[illegible]1	[illegible].184	3.934	[illegible].408	2.464	[illegible].[illegible]79	[illegible].592	[illegible].882	.[illegible]81
3.310	[illegible].[illegible]72	5.479	4.847	[illegible].[illegible]73	1.331	2.25[illegible]	1.531	1.916	3.552	3.492	1.532
5.374	0.556	17.792	5.782	[illegible].1[illegible]8	1.179	2.218	3.859	[illegible].367	1.182	1.765	[illegible]
3.443	1.898	1.646	1.[illegible]12	1.923	2.347	1.42[illegible]	11.[illegible]77	1[illegible].486	5.569	[illegible].594	2.423
0.132	13.1[illegible]	3.574	5.157	4.314	5.511	5.898	1.673	1.7[illegible]1	1.375	3.794	1[illegible].637
4.765	1.948	2.971	9.39	[illegible].196	2.[illegible]43	4.241	3.49	5.915	1[illegible].[illegible]32	14.552	.475
1.[illegible]73	6.35[illegible]	4.933	13.[illegible]13	3.4[illegible]	3.142	1.505	5.72[illegible]	2.356	2.979	[illegible].424	3.121
0.[illegible]01	3.178	5.817	5.721	6.357	1.348	1.971	[illegible].[illegible]13	3.[illegible]	3.792	[illegible].225	[illegible]
2.674	0.67[illegible]	2.279	3.375	1.184	[illegible].099	4.906	3.679	[illegible].688	[illegible].674	1.175	[illegible].754
0.379	7.640	1.817	2.334	4.208	[illegible].907	0.339	[illegible].826	1.[illegible]77	2.59[illegible]	7.224	2.097
6.986	1.363	8.[illegible]94	3.94[illegible]	6.529	3.117	4.387	[illegible].289	5.823	1.142	[illegible].88	[illegible].4[illegible]7
9.796	3.117	10.[illegible]	2.[illegible]36	3.714	1.467	4.358	2.83	13.143	[illegible].174	2.24	[illegible].1[illegible]4
0.568	5.339	3.559	1.843	[illegible].517	[illegible].731	[illegible].845	[illegible].5[illegible]4	1.619	3.655	1.[illegible]	[illegible].207
4.168	3.659	1.[illegible]61	3.529	2.556	2.[illegible]11	1.875	[illegible].444	2.555	[illegible].184	1.1[illegible]	.365
4.183	4.[illegible]16	[illegible].1[illegible]	3.616	9.272	2.692	7.531	2.234	[illegible].[illegible]54	13.717	[illegible].943	[illegible].163

0.632	1.271	5.127	1.337	.098	4.712	0.503	•1.522	0.255	0.[illegible]1	3.578	4.228
0.871	4.726	0.020	3.453	0.262	1.282	3.176	2.183	1.210	0.576	0.[illegible]1	1.142
3.461	2.792	2.677	4.045	2.272	.091	14.125	0.147	4.001	1.702	2.304	1.113
2.065	3.207	3.671	2.286	1.279	0.318	5.349	0.714	0.017	1.134	3.530	4.[illegible]7
3.995	3.335	1.655	1.274	.320	.012	.127	2.300	.383	3.688	0.117	17.200
0.019	4.744	0.415	5.965	.050	.214	5.255	1.435	0.1[illegible]2	.613	0.7[illegible]1	.912
6.685	1.075	4.932	1.833	1.740	13.368	4.187	3.963	0.570	0.001	3.008	3.021
2.700	5.827	0.215	1.227	2.340	0.511	0.06	0.817	1.015	0.134	1.222	4.867
0.001	0.356	0.933	0.351	2.156	1.460	1.194	1.856	4.561	0.208	.316	0.2[illegible]8
1.432	1.394	0.441	0.441	5.019	0.591	0.591	2.272	3.730	0.361	0.404	3.674
0.872	0.201	4.125	0.384	0.293	3.135	5.481	0.172	4.155	2.580	2.227	0.565
0.071	.001	6.278	8.384	2.142	7.222	0.599	4.625	3.990	2.758	4.124	0.314
2.891	0.370	2.324	1.791	3.794	5.826	1.673	2.917	.450	0.019	0.016	2.397
0.364	7.420	0.555	.555	0.555	3.848	0.315	4.244	4.175	2.661	2.501	3.158
0.813	2.883	4.433	1.086	0.391	0.013	0.013	10.661	3.051	0.024	0.335	0.076
3.773	2.393	2.547	0.92	0.063	0.086	0.263	0.086	4.046	2.295	0.311	0.311
4.431	2.535	4.413	5.9[illegible]8	1.084	3.768	1.504	1.204	0.001	0.6[illegible]6	0.103	.[illegible]48
1.762	9.117	4.155	6.660	1.886	6.223	0.541	5.053	2.618	5.685	0.274	1.942
2.213	0.129	2.227	0.057	1.609	2.953	4.615	5.651	8.372	0.252	4.374	3.729
1.161	3.081	4.355	.454	2.406	1.049	3.437	0.798	0.465	0.351	1.146	2.128
1.084	7.406	0.595	1.763	0.196	1.034	0.517	0.324	0.802	0.151	5.365	6.38[illegible]
6.792	0.552	0.099	1.940	2.144	2.514	0.955	.528	2.237	1.559	1.072	0.213
1.510	4.026	2.370	0.074	4.510	.696	1.005	0.224	1.449	4.902	0.250	3.347
0.451	10.513	5.788	0.720	0.369	1.805	2.135	1.771	3.994	5.325	0.251	0.607
1.829	4.592	2.045	2.233	0.378	0.001	0.277	3.693	0.194	1.792	1.706	2.355
9.297	1.705	0.001	1.515	0.121	1.416	4.141	0.270	0.001	.038	1.226	6.396
0.113	0.555	7.625	3.343	1.450	4.440	6.434	0.866	2.103	0.432	6.060	0.150
2.275	3.700	6.194	.016	0.410	0.734	0.632	3.291	0.190	0.328	0.218	1.947
3.872	6.503	4.807	5.344	3.854	1.703	0.661	4.028	0.101	3.417	0.844	9.433
0.133	1.393	0.649	8.844	2.800	2.498	0.163	2.042	3.634	6.969	0.142	1.412
0.377	0.327	4.029	0.030	4.252	4.469	15.493	1.131	0.442	0.144	2.048	0.577
1.896	4.422	0.303	4.992	0.907	1.192	5.766	0.715	1.115	2.909	6.705	7.714
5.923	6.492	2.034	0.615	3.450	0.453	3.20	12.384	11.974	22.716	6.475	3.573
0.306	3.217	0.199	0.193	2.903	12.298	0.905	1.889	0.790	2.667	10.321	3.440

4.121	11.244	3.776	1.333	0.768	4.164	7.310	4.265	2.4[illegible]3	2.784	6.950	1.795
4.455	2.064	0.587	3.365	1.192	3.333	3.633	2.770	2.736	3.056	2.843	2.724
1.884	16.575	14.736	0.634	2.680	7.980	10.021	9.902	1.050	3.305	0.032	0.37
1.253	10.973	5.162	0.192	2.433	1.370	2.621	2.661	13.212	1.317	0.334	2.476
0.220	1.873	5.733	5.407	2.884	10.180	1.800	1.160	2.413	1.010	4.459	0.5[illegible]8
2.519	17.652	3.438	0.012	1.935	0.494	4.836	3.694	0.219	3.770	8.856	13.515
6.249	0.214	0.043	2.709	0.757	1.838	7.079	2.314	3.101	2.406	3.295	0.072
3.602	6.701	0.952	1.239	1.591	.194	1.414	9.306	4.043	1.374	1.611	3.897
3.555	5.259	8.840	3.10	1.030	6.601	9.245	1.506	6.705	0.605	4.345	6.727
10.927	3.762	7.635	3.360	3.839	.243	13.551	5.306	3.817	11.4[illegible]9	2.051	1.4[illegible]7
1.404	1.082	10.723	13.702	.519	3.555	10.904	12.612	2.535	5.629	0.472	17.897
1.318	3.612	.133	7.143	0.050	0.782	1.113	3.476	4.0[illegible]9	0.619	0.103	2.669
9.114	5.754	3.802	2.515	4.531	1.012	2.116	3.992	0.270	.393	4.232	5.391
13.072	3.168	8.152	7.455	.664	0.112	1.934	13.206	5.946	0.773	4.423	1.638
5.376	6.464	4.607	0.119	1.990	7.495	9.478	7.543	5.497	2.067	5.217	15.698
0.492	0.214	4.459	5.007	3.251	0.629	1.675	4.966	7.557	2.627	5.810	5.247
8.269	3.855	1.584	0.367	0.356	0.362	8.202	0.474	14.292	1.472	10.358	2.251
0.314	2.449	4.333	2.328	0.258	6.259	5.103	1.784	0.300	0.18	1.1[illegible]3	2.2[illegible]5
1.709	2.[illegible]2	0.344	0.840	1.505	2.325	6.956	5.939	1.345	4.179	1.670	1.124
3.230	15.044	1.164	1.007	2.994	4.381	0.937	3.505	2.207	2.255	2.465	0.257
3.901	12.336	14.993	3.455	6.256	1.566	2.323	4.750	10.047	0.310	1.605	3.5[illegible]
1.722	3.461	0.308	.005	2.016	2.932	24.496	14.781	12.432	0.92	1.093	7.135
3.823	5.292	0.019	9.553	7.717	0.061	1.056	4.673	4.673	1.297	2.053	14.868
21.326	0.060	0.674	1.047	6.155	4.634	4.451	2.601	9.342	11.635	2.426	7.132
1.399	1.064	4.218	3.107	2.633	2.318	0.303	1.350	0.75	6.327	1.755	12.106
0.503	2.447	0.177	0.297	11.883	1.975	0.408	3.560	0.730	5.365	7.682	14.73
3.400	2.[illegible]47	4.333	0.262	0.036	.298	5.860	.044	0.065	4.330	6.2[illegible]5	1.794
3.076	3.413	1.058	2.322	2.441	12.115	2.843	0.[illegible]32	5.411	9.502	4.147	5.6
4.550	18.521	.022	.212	6.2[illegible]1	0.420	1.363	12.822	11.617	15.590	4.325	3.506
17.450	7.515	7.104	3.411	0.831	5.250	10.868	7.142	3.731	.504	.0[illegible]6	6.761
0.517	12.825	2.345	.113	3.677	2.660	5.663	3.132	0.033	1.632	3.141	4.[illegible]33
6.199	4.759	1.663	10.373	2.058	6.790	.142	1.038	2.358	4.030	0.833	0.120
2.575	3.393	4.665	0.373	0.770	3.403	2.001	2.455	5.591	12.714	.864	3.516
5.533	19.341	4.571	7.374	5.520	47.800	2.543	1.724	.263	3.917	5.364	.191
11.536	5.116	3.127	13.331	1.023	2.302	7.579	6.080	19.840	0.[illegible]46	.104	3.373
8.847	1.533	4.541	11.034	3.735	2.180	17.454	3.495	27.625	7.575	4.477	4.331
0.554	6.393	1.[illegible]85	4.143	2.316	3.175	2.32[illegible]	0.47	1.197	5.644	4.332	0.23[illegible]
2.632	13.990	0.853	.[illegible]8	3.765	3.352	1.413	0.050	7.378	10.465	.403	2.[illegible]4
6.371	6.137	.273	.532	1.304							

10. HIMALAYAS,LATITUDE(26,4[illegible]),LONGITUDE(7[illegible],2[illegible]),N=1[illegible]

48.538	1.539	1.440	14.571	17.412	12.724	60.747	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
9.101	0.042	0.473	9.243	7.678	7.934	1.[illegible]	4.721	5.247	.[illegible]	[illegible]	0.777
0.438	0.115	7.552	7.417	5.758	2[illegible].561	1.443	21.[illegible]	2.574	.112	7.433	.411
12.213	1.873	6.544	1[illegible].[illegible]32	.741	.784	0.327	3.741	13.[illegible]	.348	9.[illegible]7	.2[illegible]
3.943	12.098	5.514	0.152	1.441	3.135	[illegible].713	7.[illegible]	14.[illegible]7	3.[illegible]47	.14[illegible]	.[illegible]14
1.817	5.437	2.967	6.[illegible]1[illegible]	28.995	15.483	1.72	.[illegible]9	0.152	1[illegible].105	12.422	4.358
5.768	1.[illegible]37	.010	0.783	3.061	.[illegible]39	0.106	3.256	.847	14.[illegible]47	1.245	4.[illegible]47
1.947	3.666	1[illegible].570	1.394	1.989	12.229	1.613	2.474	.419	[illegible].[illegible]84	[illegible]	.732
1.448	[illegible].724	1.789	4.173	1.960	.224	4.592	.823	0.5[illegible]4	[illegible].[illegible]16	.372	.7[illegible]
0.787	0.653	6.058	1.354	[illegible].447	3.83	4.004	8.286	[illegible].814	14.[illegible]48	.253	[illegible].729
0.041	5.503	2.874	2.754	[illegible].474	3.[illegible]61	4.365	2.434	3.132	1.5[illegible]0	6.[illegible]20	1.74[illegible]
0.860	0.729	0.173	9.[illegible]1[illegible]	1.382	[illegible].[illegible]66	1.328	.[illegible]1	1.[illegible]29	[illegible].115	3.270	.13[illegible]
0.151	4.739	3.34	1.267	.[illegible]19	2.385	2.822	1.[illegible]19	5.545	1.867	15.[illegible]7	3.[illegible]7
0.517	9.701	3.366	3.284	14.954	0.744	2.[illegible]26	5.342	1.[illegible]92	.237	[illegible].41	.373
0.723	0.415	4.902	0.127	1.452	1.710	4.624	.[illegible]41	2.920	4.492	3.413	2.2[illegible]
2.737	0.97[illegible]	0.261	1.671	0.419	2.294	1.153	4.10[illegible]	5.720	5.178	0.245	5.634
3.776	3.[illegible]09	0.758	0.382	3.508	0.595	0.284	0.283	3.499	3.934	1.[illegible]	[illegible].[illegible]7
3.677	2.043	2.028	0.[illegible]46	3.803	.1[illegible]6	.[illegible]54	1.13[illegible]	3.70	.782	4.451	6.064
1.308	0.373	1.954	1.874	1.951	[illegible].[illegible]63	4.015	0.644	7.282	2.206	1.3[illegible]6	2.7[illegible]5
6.422	4.071	5.160	0.743	2.21[illegible]	2.307	[illegible]	1.803	12.720	5.542	2.321	1.954
0.587	1.054	0.753	0.627	0.253	1.375	2.11[illegible]	8.424	1.488	1.767	[illegible].703	4.443
3.468	1.602	0.971	2.960	5.641	3.829	3.921	4.666	1.722	[illegible].215	4.62[illegible]	[illegible].5
2.880	3.019	1.490	1.[illegible]62	2.277	2.921	0.145	0.890	6.883	.[illegible]17	.[illegible]19	3.520
3.724	3.324	0.794	2.39[illegible]	2.216	12.420	0.477	4.[illegible]34	2.232	3.[illegible]	3.124	1.456
0.787	1.245	[illegible].546	2.117	5.124	58.513	15.924	31.323	2.[illegible]6	65.[illegible]23	37.794	.1[illegible]1
3.130	67.745	9.894	47.299	[illegible].[illegible]43	4.155	28.816	5.217	4[illegible].713	1.[illegible]16	9.457	14.654
46.945	3.065	14.325	17.9[illegible]7	15.383	64.327	11.880	39.[illegible]77	6[illegible].561	17.365	35.181	3.236
16.339	22.031	[illegible].[illegible]72	57.386	2[illegible].137	1.[illegible]09	31.593	.357	71.119	26.[illegible]77	11.38[illegible]	4.[illegible]4
20.918	0.962	17.620	5.781	2.606	5.997	15.479	23.662	1.6[illegible]	[illegible].[illegible]01	.[illegible]32	2.956
14.880	24.586	30.368	13.52	17.831	[illegible].212	8.655	19.818	24.381	27.8[illegible]	8.755	3[illegible].457
89.652	17.574	8.535	8.794	2.261	[illegible].[illegible]40	14.989	29.485	32.981	32.5[illegible]1	44.639	[illegible].874
6.421	33.544	6.480	5.297	5.254	20.646	36.201	4.749	[illegible].789	1[illegible].[illegible]2	4.445	98.749
26.039	3.890	5.333	35.262	9.[illegible]06	3.585	6.[illegible]28	24.831	3[illegible].593	[illegible].[illegible]1	22.154	1[illegible].474
58.888	21.402	11.503	14.371	14.135	[illegible].565	10.542	[illegible].[illegible]84	5.789	5.27[illegible]	1[illegible].843	8.195
0.001	14.394	23.263	43.937	6.114	4.576	5.081	5.860	6.704	8.9[illegible]0	7.574	1[illegible].329
0.811	3.066	11.390	9.[illegible]1	[illegible].569	0.662	0.001	1.957	14.159	8.725	2.35	[illegible].[illegible]1
15.726	6.632	6.929	3[illegible].013	19.379	8.458	2.801	4.542	3.863	16.[illegible]93	5.[illegible]	7.243
25.444	1.691	9.503	.054	13.448	34.549	[illegible].18[illegible]	2.594	[illegible].[illegible]17	[illegible].413	[illegible].[illegible]1	2[illegible].543
5.160	19.084	0.564	10.425	0.477	3.816	14.588	18.862	6.594	[illegible].[illegible]77	4.854	5.54
30.988	9.001	1.314	4.779	29.999	58.559	5.285	5.731	[illegible].617	4.52[illegible]	47.813	9.241
26.044	0.073	127.112	1.165	26.016	1.918	45.398	21.796	10.736	[illegible].622	1.119	3.338
33.654	19.204	9.415	17.933	2.588	0.014	15.402	5.291	13.313	6.536	3.839	25.771
14.858	6.745	0.001	1.627	9.711	17.727	3.388	8.632	14.413	5.255	[illegible].[illegible]1	1.215
11.1[illegible]4	23.082	6.517	7.748	15.194	6.482	1.707	13.347	6.569	33.235	1.804	7.955
0.218	18.205	8.745	7.4[illegible]7	28.809	7.367	6.274	12.311	1.074	1.957	0.0[illegible]1	2.213
1.435	1.604	3.598	11.528	2.71[illegible]	.744	0.0[illegible]1	18.175	19.9[illegible]3	26.454	.[illegible]26	.[illegible]27
16.147	9.220	15.223	[illegible].[illegible]1	5.778	0.035	1.769	19.749	12.391	28.147	1.[illegible]78	11.457
3.303	2.137	15.471	31.247	7.576	1.606	0.2[illegible]7	2.9[illegible]5	5.956	1.975	[illegible].[illegible]2	6.[illegible]89
64.542	12.357	10.45[illegible]	[illegible].[illegible]01	9.9[illegible]4	3.734	30.954	11.454	11.322	5.31	33.[illegible]52	16.[illegible]55
21.890	2.598	0.001	2.979	14.655	32.257	[illegible].63[illegible]	1.168	8.[illegible]23	6.727	3.49[illegible]	.[illegible]9
0.509	0.242	1.019	0.690	[illegible].436	0.571	1.056	[illegible].[illegible]12	0.985	4.045	[illegible].45[illegible]	3.242
0.219	1.616	0.226	6.[illegible]2	5.048	4.749	0.189	1.161	32.926	[illegible].[illegible]3	7.973	2.381
0.323	9.895	0.001	9.01[illegible]	0.215	0.153	0.584	2.721	1.853	1.434	9.654	3.467
2.378	22.398	0.012	9.289	6.7[illegible]1	1.789	3.475	4.[illegible]85	[illegible].[illegible]1	1.419	22.92	11.13
6.877	13.431	6.681	3.001	1.882	6.705	17.661	6.803	[illegible].[illegible]1	2.495	2.381	3.044
42.248	5.507	4.376	6.751	4.517	10.394	14.405	[illegible].167	21.9[illegible]3	[illegible].784	[illegible].[illegible]2	[illegible].316
0.631	0.001	6.317	7.[illegible]39	5.398	3.156	6.321	1.759	8.951	11.743	34.201	3.574
0.220	0.220	14.336	[illegible].664	[illegible].664	6.171	8.515	11.887	1.440	1.117	.001	3.409

10.813	9.981	4.698	[illegible].67	7.815	0.018	[illegible].063	4.303	0.751	[illegible].625	3.39[illegible]	1.391
0.001	1.003	0.318	[illegible].813	4.952	9.763	0.945	2.684	5.638	4.038	6.236	[illegible].[illegible]1
13.639	2.96[illegible]	63.605	3.723	[illegible].[illegible]84	[illegible].865	1.170	5.137	[illegible].616	1.517	7.220	1.385
1.033	0.001	3.390	4.775	1.601	4.885	6.512	1.703	5.5[illegible]5	[illegible].235	8.741	2.[illegible]7
3.188	6.992	5.179	8.123	1.173	1.249	1.59[illegible]	.413	3.838	1.746	9.19[illegible]	6.[illegible]3
0.045	12.798	2.418	6.384	11.126	[illegible].373	7.455	1[illegible].384	3.800	1.324	3.267	[illegible].[illegible]1
1.382	7.091	1.614	3.768	6.788	2.631	3.978	[illegible].[illegible]1	.861	6.363	5.511	2.[illegible]6
0.609	9.855	0.089	2.[illegible]34	[illegible].1[illegible]	.10[illegible]	1.317	[illegible].734	.440	3.700	9.103	6.525
5.336	2.318	4.495	0.229	[illegible].228	5.362	3.083	.645	9.611	3.563	.373	7.[illegible]
2.612	0.764	6.698	4.83[illegible]	5.787	[illegible].9[illegible]	1.752	5.474	[illegible].551	3.196	[illegible].49[illegible]	1.[illegible]43
1.119	2.886	2.205	1.213	.301	3.244	3.387	[illegible].473	[illegible].927	1.754	1.177	6.527
3.652	4.655	7.434	[illegible].873	1.039	0.211	1.756	4.387	1.5[illegible]9	4.[illegible]	[illegible].4[illegible]4	7.7[illegible]1
0.916	0.031	2.01[illegible]	1.8[illegible]	2.234	5.378	3.3[illegible]	4.423	[illegible].153	1.[illegible]25	[illegible].552	9.2[illegible]3
0.661	3.220	1.456	[illegible].727	0.279	1.333	1.285	.[illegible]1	[illegible].439	.94[illegible]	[illegible].125	7.153
0.001	0.895	6.717	2.288	5.[illegible]45	.471	5.[illegible]39	1.567	4.[illegible]	4.[illegible]1	[illegible].[illegible]1	.317
2.090	1.520	0.703	7.585	[illegible].[illegible]59	[illegible].[illegible]94	11.532	.[illegible]32	[illegible].[illegible]22	1.416	.[illegible]6	1.1[illegible]

1.897	6.795	0.581	2.515	7.428	0.461	0.151	1.344	1.641	0.917	2.641	4.559
2.340	0.515	0.882	9.517	0.001	1.320	10.531	4.039	1.111	7.752	0.196	6.344
0.001	6.279	0.237	39.172	0.441	1.692	23.150	2.441	1.440	1.251	10.036	17.607
10.511	0.292	21.941	0.113	2.437	0.001	0.289	0.782	6.076	4.574	10.147	4.392
11.176	21.942	15.498	1.603	8.012	3.085	13.552	1.734	4.734	2.554	4.302	5.905
3.070	4.182	0.243	0.001	9.354	1.121	0.224	1.049	4.736	0.936	2.549	0.674
8.723	1.383	2.064	0.330	0.958	0.309	2.135	0.523	1.114	2.979	1.328	0.605
1.012	0.357	5.259	4.609	1.125	0.781	4.143	4.346	1.499	8.077	0.554	9.107
4.984	0.710	4.634	0.364	6.092	0.231	14.210	5.509	0.294	1.843	0.001	3.607
6.146	7.961	4.793	3.942	4.753	0.604	1.812	0.720	1.083	2.180	0.001	10.236
1.627	1.396	0.819	1.056	2.036	4.068	0.001	0.994	0.001	2.698	0.429	2.455
7.586	3.611	8.494	6.514	2.045	0.655	6.000	0.173	6.022	9.342	4.093	2.399
4.187	0.100	0.836	3.564	15.228	6.494	1.926	3.457	0.070	3.654	2.590	0.711
6.409	1.025	5.810	5.107	0.592	1.883	10.561	0.157	0.499	1.679	2.661	5.464
1.827	3.272	0.822	4.248	3.196	0.291	1.692	0.001	1.019	.309	6.125	3.654
3.245	5.514	1.044	1.933	2.837	1.189	0.416	1.373	1.850	0.703	4.712	7.393
0.001	3.605	0.182	0.798	0.192	0.367	1.645	0.268	0.319	1.969	0.436	1.499
2.631	4.024	0.912	4.374	1.005	3.452	2.529	8.700	0.695	11.035	1.964	0.469
2.522	0.316	2.491	2.107	2.440	0.250	1.254	0.127	4.630	13.137	6.262	1.299
1.195	3.134	0.440	1.916	3.611	0.129	1.885	2.008	0.524	4.142	3.221	8.161
0.626	0.023	2.935	0.584	3.084	1.410	2.536	2.387	0.265	3.126	0.001	3.712
4.810	0.001	1.835	2.235	0.660	7.867	11.923	2.455	3.704	0.358	7.529	21.155
3.734	0.642	1.161	3.807	0.117	0.713	4.238	5.236	2.740	4.426	0.731	2.472
1.812	6.669	2.216	0.387	0.043	2.162	0.727	0.544	2.401	4.599	3.001	4.548
0.927	1.557	1.497	1.289	0.381	4.442	0.155	0.445	17.834	0.391	0.744	1.425
6.007	1.420	2.924	2.231	0.455	0.321	0.966	0.024	0.224	7.957	0.001	1.405
2.942	0.531	0.001	0.384	0.111	5.712	0.001	3.512	1.225	0.001	1.303	3.610
0.283	2.138	12.706	2.894	0.921	4.091	0.001	0.229	0.702	7.741	1.997	0.606
8.121	1.033	2.337	4.037	5.166	9.340	3.929	7.547	1.067	1.808	1.601	0.836
3.173	12.726	18.196	0.691	0.681	2.142	0.296	2.642	2.143	3.180	3.963	8.619
2.786	0.161	0.501	2.999	5.493	4.364	0.068	0.068	0.358	3.145	0.088	0.194
1.174	2.647	5.091	1.486	1.098	2.255	1.931	1.667	3.752	0.001	2.824	4.603
2.176	1.271	5.706	0.998	0.060	3.207	3.028	5.188	2.173	0.457	0.849	7.593
0.157	9.506	12.101	0.861	6.020	0.916	0.001	12.340	6.817	5.550	9.413	1.791
6.785	0.001	0.209	10.052	1.190	5.322	4.772	2.914	3.646	2.142	7.057	0.031
0.070	0.933	0.342	3.424	1.906	0.561	0.510	1.273	8.395	0.001	0.187	13.393
4.799	2.999	2.649	0.005	1.014	2.172	1.905	2.920	6.262	1.776	19.901	1.333
4.660	1.233	4.104	3.987	2.058	2.095	0.431	2.554	8.296	1.091	3.075	0.639
6.730	10.567	4.529	2.265	2.130	1.621	0.903	2.914	2.168	0.413	1.878	3.542
1.284	0.104	14.332	3.035	2.400	1.590	3.112	0.981	2.219	5.779	4.253	0.066
1.804	2.756	0.803	1.740	2.796	0.006	0.002	0.010	0.022	0.154	1.549	0.533
6.485	1.198	3.072	1.863	4.798	2.106	6.099	3.345	4.504	2.112	4.171	6.552
0.122	3.824	1.159	0.451	13.205	1.267	1.441	7.767	3.793	0.001	0.105	6.131
3.756	5.927	4.812	7.140	5.602	6.971	4.871	1.287	3.405	2.564	5.779	2.711

9.094	0.742	0.007	4.221	0.536	3.865	8.178	5.615	6.854	0.797	1.236	0.395
1.600	3.243	0.037	11.913	2.984	0.084	13.782	3.280	6.339	1.259	1.436	0.162
7.814	4.831	0.080	2.910	3.565	0.723	1.681	3.775	0.001	10.390	3.590	13.012
11.832	3.459	10.048	0.379	11.409	1.579	1.921	11.417	0.055	1.279	15.790	12.644
0.734	4.070	14.196	8.759	34.302	2.648	4.317	4.565	9.227	0.108	3.974	0.871
3.099	5.911	2.716	9.655	0.660	0.383	1.018	3.293	1.276	22.087	1.501	0.919
7.331	8.936	3.406	8.117	16.899	4.788	9.416	6.834	3.025	1.158	8.225	3.406
5.394	7.183	4.532	2.416	13.161	0.297	14.583	3.599	2.986	14.138	14.975	29.596
5.957	6.643	24.159	7.816	6.561	11.249	2.066	0.992	13.972	0.609	1.812	6.846
24.713	9.745	1.637	10.326	4.600	0.562	1.093	0.160	14.414	16.439	24.383	1.131
7.938	4.557	13.334	12.036	0.201	8.254	19.564	2.996	7.434	10.674	4.826	4.449
9.386	4.622	0.324	4.464	16.633	7.207	2.566	15.641	5.599	9.386	11.873	19.417
11.765	20.095	2.778	4.240	3.921	13.140	22.363	8.833	0.268	0.171	13.436	2.059
2.442	4.633	0.065	18.838	4.199	14.287	11.125	10.433	4.608	2.540	4.303	16.038
0.438	5.629	20.896	0.225	3.493	1.500	51.561	0.463	2.712	0.498	2.274	4.893
0.014	16.428	7.587	0.498	4.932	2.534	16.924	15.209	5.879	6.827	6.627	2.598
1.449	2.688	0.143	3.372	5.122	3.210	14.788	1.194	3.607	8.212	0.038	12.648
2.834	0.481	10.751	29.750	-0.153	25.883	5.423	20.310	2.592	0.078	0.245	0.009
8.571	11.651	3.458	0.089	3.184	15.577	1.557	4.555	3.907	33.274	1.456	9.444
0.736	17.343	9.835	11.574	1.347	3.001	0.588	4.869	11.095	36.453	1.519	12.323
0.828	1.891	14.054	4.731	0.367	0.890	3.706	0.919	6.526	9.449	5.053	0.206
14.705	2.381	4.133	12.904	2.545	11.855	0.553	16.193	2.529	16.114	7.043	5.749
0.196	6.313	7.241	1.961	14.948	44.941	0.107	1.340	10.029	32.916	12.434	0.461
0.398	7.493	3.440	10.074	4.948	1.974	31.375	2.471	1.914	6.196	6.181	0.389
2.931	26.980	2.124	3.072	9.750	0.302	6.106	0.413	1.226	0.047	16.164	13.364
2.970	8.590	22.302	0.221	13.140	0.657	24.109	0.919	5.857	1.946	11.549	1.257
1.286	0.448	2.971	24.383	9.419	6.177	6.725	5.437	2.972			

11. GREECE ,LATITUDE(34,41),LONGITUDE(21,31),N=182

[illegible]	[illegible]	3. 76	3.644	1 .567	9.351	2.301	.214	[illegible]	[illegible]	[illegible]	[illegible]
0. 78	12.533	9.3 1	[illegible]	13.549	4.742	9.231	3.906	0. 72	14.759	41.127	0.125
0.135	4.230	26.356	17. 61	32.	3.427	4.816	3.672	29.320	17.274	2.619	[illegible]
12.141	2.623	25. 71	2.56	0. 7	0.681	0.767	0.033	0.456	0. 7	[illegible]	.177
0.171	0.195	0.025	1.363	[illegible]	2.869	0.132	0.235	0.013	2.741	15.544	3.893
2.823	0.031	3.816	3.266	26.175	3.987	0.435	1.132	0.048	1.508	0.576	7.487
13.192	1.674	1.362	11.331	2.318	4.481	0.141	0.183	0.141	0.319	0.013	[illegible]
1.244	0.281	0.573	.157	0.798	3.324	2.226	0.091	0.592	1.108	.182	.937
3.342	0.243	31.887	4.473	19.707	3.114	1.459	1.011	1.309	5.197	2.007	28.377
4.094	4.628	1.169	3.785	3.213	12.137	22.896	0.693	3.199	2.311	14.875	4.971
0.355	1.115	7.024	9.737	0.061	8.622	1.212	0.213	7.010	0.154	0.001	0.347
0. 72	0.275	0.452	1.595	7.263	2.656	1.779	11.309	2.964	9.367	0.215	0.215
0.019	12.157	11.294	26.899	2 .790	12.620	4.612	3.823	22.877	7.539	18.083	2.529
0.153	12.953	12.098	4.739	3.126	0.258	0.137	2.549	3.074	27.524	2.758	2.005
8.550	2.823	7.582	9.366	3.252	9.571	5.678	1.410	1.614	1. 04	3.986	3.2 8
7.344	5.447	2.360	19.072	1.490	21.557	2.193	0.548	14.952	9.580	21.205	1.8 4
12.156	4.839	0.115	1.430	1.264	0.001	0.167	0.057	43.131	7.853	19.750	31.621
48.925	17.541	0.489	1.043	0.157	1.202	2.355	1.732	0.622	1.913	1.654	0.973
0.426	0.021	1.612	4.320	4.226	0.300	4.522	0.565	1.917	3.505	0.052	2.624
0.277	8.825	7.248	1 .660	9.750	6.599	7.538	10.761	4.191	1.189	1.102	23.279
3.207	9.598	0.505	12.975	2.173	0.103	4.567	3.041	0.039	0.506	7.993	16.465
2.727	0.766	0.332	0.309	0.290	0.476	2.216	6.258	8.725	0.973	12.787	1.856
2.300	0.404	1.015	0.146	25.980	4.194	16.616	2.960	4.505	12.810	7.040	0.061
7.253	4.815	1.029	1.783	0.001	6.914	0.001	31.092	2.579	8.720	0.729	5.426
8.172	3.256	3.981	5.84	0.719	11.001	25.317	6.703	1.367	5.160	6.223	0.378
0.531	5.444	10.887	4.102	2.209	0.013	4.090	1.962	3.932	16.644	5.690	4.469
0.314	17.195	6.642	2.40	2.411	3.995	1.245	1.015	0.023	6.110	1.186	0.594
0. 8	0.021	0.393	0.381	1.563	.695	1.477	3.263	7.822	1.598	4.402	9.233
3.442	4.283	2.873	4.562	12.498	3.597	12.429	3.134	1.643	2.892	1.187	11.010
9.116	4. 41	1.817	1.365	0. 1	17.580	3.998	0.327	2.499	3.385	3.700	2.335
0.741	0.487	1.309	17.573	1 .274	5.330	1.326	1.323	3.280	16.473	6.705	3.092
4.856	15.925	5.179	0.161	4.958	4.899	0.490	17.352	0.039	0.428	6.285	2.005
8.176	10.961	3.11	4.444	3.343	1.297	0.298	3.765	4.419	0.024	4.997	5.491
0.309	2.998	0.001	11.2 2	1.849	3.719	2.503	2.563	21.589	22.254	2.344	9.370
3. 44	3.384	3.753	2.975	14.349	4.250	0.562	34.397	3.819	0.200	2.200	11.051
0.222	0.323	29.725	2.121	16.714	1 .195	3.943	4.064	0.024	1.102	0.001	0.010
5.744	0.014	12.146	. 1	10.727	15.651	4.936	6.674	4.994	1.752	10.714	18.756
19.612	11.093	5.650	3.506	9.207	17.555	0.352	3.294	7.265	.737	11.564	1.763
1.230	7.953	6.197	2.369	7.693	7.965	4.031	1.545	2.654	8.190	1.109	12.907
12.269	4.839	. 37	3.187	3.444	3.503	0.037	0.037	1.366	0.494	10.038	6.562
0.492	3.275	11.152	3.412	2.448	1.060	4.046	0. 1	4.625	9.150	25.156	28.244
5.494	0.697	1.534	1.662	7.214	1 .257	1.934	2.922	39.591	0.625	13.382	0.160
0.160	9.239	36.247	14.245	14.318	10.620	23.244	0.008	0.854	. 56	0.008	16.459
1.996	2.012	0.476	2.374	6.353	4.409	0.385	0.085	0.493	0.749	0.516	2.513
7.796	0. 47	0.069	1.692	1.205	9.651	0.507	6.407	0.232	12.857	[illegible]	0. 77
0. 71	5.243	9.713	9.073	15.899	6.962	3.695	3.177	4.822	0.388	0.001	8.457
1.198	16.396	3.669	9.372	7.174	2.137	16.722	4.541	17.107	5.217	14.149	1.606
0.073	6.171	0.122	7.983	5.440	1.045	0.045	79.632	14.422	6.276	0.001	15.369
6.658	17.317	0.381	15.900	6.762	5.428	15.646	12.490	9.379	4.660	3.738	0.265
8.614	1.535	3.691	3.824	1.302	0.136	6.252	8.922	1.122	0.001	1.173	9.181
5.205	4.297	0.001	2.836	6.255	12.551	0.001	3.159	27.254	6.160	0.001	6.423
2.233	38.597	5.417	8.056	4.421	2.357	23.787	1.079	61.885	11.749	7.752	7.885
3.106	1.664	2.636	0.698	40.916	4.713	1.307	3.363	2.628	15.200	0.075	5.438
28.649	4.616	17.184	1.329	10.818	24.639	3.690	.167	1.157	0.027	.931	0.063
19.359	5.072	.271	7.775	1.465	0.001	0.014	. 02	3.007	3.553	3.112	4.502
12. 46	2.333	2.702	11.230	2.229	5.919	1.281	0.525	1 .359	4.428	0.426	0.532
0.536	4.560	0.522	3.856	2.049	0.636	0.817	4.055	1.282	0.693	0.767	4.490
0.759	0.029	1.482	1.632	1.561	2.478	5.618	7.366	7.297	0.080	10.100	5.832

7.087	5.587	5.000	0.217	2.123	1.782	16.412	10.165	1.403	1.062	4.119	2.229
4.851	0.873	0.115	4.420	0.001	0.534	0.354	0.824	0.328	6.826	4.393	9.239
6.394	9.566	2.779	2.360	1.333	2.567	3.897	4.377	0.056	0.001	0.186	0.011
0.020	0.175	0.009	0.189	0.085	0.093	1.386	1.413	0.571	0.001	0.035	0.143
0.017	2.690	0.001	1.214	1.918	2.572	3.947	0.067	0.994	0.010	0.497	5.549
0.234	7.917	0.099	0.05	. 77	0.094	0.136	1.190	1.5 9	0.067	0.708	0.923
0.675	5.525	1.967	0.490	2.212	.195	1.200	.230	0.360	0.337	3.941	0.347
4.507	0.031	0. 1	0.095	0.113	.555	0.259	0.791	0.096	0.158	1.281	.391
0.541	3.637	0.827	1.214	.636	.135	1.27	.775	1.325	.131	1.348	.219
0.370	0.437	2.110	1.121	0.656	2. 0	3.944	1.	1.475	.649	6.892	5.861
14.568	0.775	0.477	4.951	2.5 9	5.671	0.972	1.611	.3 5	4.374	12. 7	0.012
0. 10	0.015	3.159	0.9 6	.074	.824	2.202	1. 92	1.720	1.9 9	1.047	2.911
4.093	3.002	5.112	3.121	1.299	0.526	6.469	2.5 9	0.545	1.126	3.444	4.170
2.203	2.936	9.814	0.285	.366	.964	2.612	1.8 3	0.251	.719	3.566	4.542
1.769	6.584	0.815	1.397	1.180	2.055	.646	1.515	0.4 2	. 13	.735	1.429
0.534	0.161	3.749	0.024	4.122	4.329	1.400	7.292	0.217	0.204	2.575	5.714
1.135	0.109	0.094	. 22	. 19	.014	0. 01	0.07	0.017	0.003	0.027	.028
0.098	0.004	0.034	0. 59	0.064	0.011	0.001	0.90	0.092	0. 92	1.214	1.840
1.418	0.144	0. 1	0.300	.1 6	.036	.249	1.252	2. 74	2.337	0.310	0.274
0.622	0.064	0.242	0.579	2.526	2.256	0.874	2.211	1.440	1. 20	2.548	2.981
0.134	0.134	0.497	1.091	0.753	1.160	0.001	.672	0.444	.104	0.447	0.152
0. 61	0.944	0.1 0	1.316	1.479	.149	0. 22	0. 01	.154	.041	6.422	.641
3.642	1.313	0.13	3.242	7.444	1.077	2.418	. 1	0.412	.443	.722	0.452

1.057	0.043	0.480	2.790	0.001	3.258	0.167	11.452	0.067	0.654	0.096	1.055
1.744	9.103	5.835	0.998	0.147	3.461	3.156	4.155	5.738	4.597	0.564	0.001
0.014	1.805	2.758	0.159	1.077	1.338	1.170	0.047	0.550	0.001	7.125	2.582
2.972	0.015	1.155	9.338	0.516	0.184	0.432	2.032	0.091	3.312	1.797	1.185
1.707	0.605	1.527	2.034	0.734	1.568	16.427	8.246	13.532	13.521	0.091	0.574
0.188	1.366	1.366	0.300	1.682	4.600	2.712	1.118	3.830	1.952	0.092	1.832
1.768	1.312	1.361	3.166	4.526	2.136	5.541	0.005	0.002	0.011	0.232	0.225
0.305	0.109	3.834	0.440	2.217	1.974	0.860	2.597	4.231	0.001	1.804	1.056
1.933	0.664	3.846	3.946	1.231	3.104	1.384	0.422	3.786	1.387	1.839	1.596
0.607	2.354	2.943	1.603	1.590	3.096	2.839	0.061	1.750	1.050	0.582	4.912
0.013	1.811	2.268	0.229	1.596	1.257	8.540	5.833	2.005	5.980	0.250	0.943
0.190	0.176	0.716	0.388	0.931	1.988	1.220	0.596	1.083	0.989	2.723	0.751
0.001	0.001	2.213	0.956	0.211	0.445	0.020	1.597	2.567	2.755	1.203	0.672
2.457	1.936	0.975	4.279	0.022	0.022	0.035	0.145	0.159	0.055	0.333	0.449
1.410	0.787	2.426	2.106	0.175	0.176	0.175	2.041	1.895	2.958	3.668	14.552
2.455	1.408	1.563	1.429	0.419	1.097	1.638	0.001	2.864	6.547	5.131	0.226
1.303	4.568	1.043	0.010	0.047	2.066	3.284	0.412	0.568	1.905	1.048	1.847
3.141	0.140	3.560	0.184	2.861	27.869	4.390	2.854	1.113	16.389	1.260	0.332
1.575	1.204	1.394	2.273	2.781	2.671	2.433	0.165	2.061	5.122	0.857	2.966
5.228	1.587	1.995	0.544	3.143	2.284	0.097	1.239	0.354	2.692	0.593	8.330
0.911	0.062	0.852	3.800	0.880	1.254	0.189	0.137	1.239	0.957	0.398	0.539
0.285	1.195	0.570	0.994	0.306	2.029	0.569	0.102	1.879	5.269	2.215	0.563
0.218	0.541	1.544	0.499	0.090	0.217	0.177	2.451	9.377	1.885	1.618	0.698
0.002	8.007	0.104	10.865	1.005	7.479	2.054	0.080	3.293	11.129	3.898	0.466
3.822	0.019	5.556	4.483	1.903	4.032	1.199	6.463	3.792	8.050	1.198	1.273
3.099	0.541	0.251	4.205	3.982	2.948	3.081	4.445	6.267	4.460	0.107	1.490
4.762	0.837	0.610	5.054	7.255	0.452	4.976	5.156	0.692	0.784	0.187	0.001
0.201	1.032	3.753	9.405	9.257	3.907	12.414	0.801	0.237	6.356	3.308	0.574
0.837	2.652	2.335	14.516	8.169	3.654	11.894	26.242	10.809	5.030	7.097	0.072
2.030	16.020	48.311	11.830	8.957	0.040	12.905	23.541	9.655	0.792	0.056	38.234
0.161	0.066	0.013	0.179	0.003	0.289	0.122	0.527	0.404	0.678	1.392	0.605
0.507	0.018	1.015	0.504	3.813	1.302	0.235	6.975	0.001	1.676	0.020	10.754
11.767	0.645	8.955	5.141	1.276	0.082	0.489	0.432	1.122	23.970	1.389	5.609
0.956	0.932	18.765	0.432	4.358	0.052	0.021	0.098	0.048	0.046	0.047	0.093
0.262	0.098	0.045	0.174	0.495	0.442	0.261	1.737	1.479	1.178	4.105	0.012
1.047	0.734	0.052	2.308	9.180	8.376	16.928	2.242	0.837	6.610	0.093	2.832
1.228	3.172	6.055	2.866	6.307	10.406	1.579	20.184	3.580	15.046	2.801	19.313

5.282	6.941	0.062	24.587	12.814	0.341	23.812	13.068	40.267	17.303	0.068	0.046
0.049	0.053	0.234	0.200	0.006	0.429	0.255	0.192	1.559	0.161	0.117	7.586
0.017	2.830	9.627	1.999	3.855	9.323	7.078	2.956	0.228	4.076	2.883	3.675
0.311	6.014	2.149	1.757	10.643	12.356	1.576	15.784	35.042	3.901	12.864	8.771
2.937	19.574	1.150	0.090	5.424	0.037	0.016	1.226	10.422	11.534	3.229	6.156
0.679	3.203	1.046	14.891	35.714	1.325	16.307	3.859	0.408	2.421	6.029	1.885
3.977	11.507	0.866	6.476	12.272	3.042	8.198	1.560	0.190	3.568	3.797	0.005
1.521	1.987	4.766	1.452	0.099	1.985	0.019	0.224	0.151	1.988	18.074	12.373
0.009	0.045	0.053	0.692	0.575	5.935	0.906	15.188	0.198	6.263	9.204	1.056
7.907	7.342	0.131	2.355	23.968	5.440	11.869	11.730	6.836	9.384	1.498	2.214
10.093	0.691	13.895	0.375	1.253	0.291	0.059	0.315	0.201	0.039	1.508	0.007
0.050	0.022	0.093	0.678	0.822	1.494	0.461	0.232	0.262	11.147	0.541	0.006
11.055	2.696	0.167	1.081	0.026	0.093	0.704	6.238	0.021	2.902	1.046	2.500
1.593	4.982	1.426	0.039	1.476	1.553	0.762	2.772	1.920	0.015	5.888	0.890
0.051	1.454	0.161	1.350	0.077	4.932	1.673	0.110	0.914	0.137	18.147	19.760
0.167	0.443	1.416	0.769	0.045	0.926	5.914	8.590	11.731	1.471	9.572	1.354
18.662	0.012	26.193	2.505	9.392	10.762	2.945	13.193	9.114	9.517	0.798	0.867
2.697	1.422	6.116	0.820	7.715	14.008	32.042	11.140	12.944	26.136	1.066	1.701
1.692	6.078	5.617	0.029	22.624	7.143	3.959	9.223	15.455	7.790	2.339	2.297
0.613	2.401	0.213	0.004	2.032	14.407	35.711	0.039	0.018	0.006	0.035	0.031
0.017	0.015	0.105	0.037	0.036	0.005	0.009	0.016	0.226	0.062	0.114	0.285
0.274	0.695	0.003	0.170	0.036	0.051	0.121	0.141	0.164	0.246	0.021	0.232
0.016	0.159	0.024	0.260	0.207	0.563	0.119	0.393	0.757	0.450	1.604	1.776
0.052	2.785	0.306	0.467	0.353	0.892	1.222	0.631	1.700	2.572	0.560	2.601
0.951	0.002	0.518	0.575	0.061	0.656	0.032	0.140	1.136	0.550	0.414	0.635
0.963	0.218	0.498	0.258	0.296	0.500	0.141	0.033	0.420	0.363	0.628	0.680
0.794	0.591	0.169	0.193	0.999	0.120	0.717	3.398	0.159	0.460	0.001	0.769
3.874	0.105	0.435	8.463	1.787	0.249	0.422	0.270	4.198	1.321	2.805	13.769
1.871	3.399	2.686	7.824	0.397	3.514	3.747	3.117	4.637	4.621	2.932	1.393
16.426	14.263	33.359	3.561	1.900	14.727	36.910	6.343	4.455	4.457	1.969	3.700
4.467	0.017	24.654	8.003	0.279	24.564	7.406	0.363	5.506	7.184	6.232	24.028
4.944	0.901	10.283	0.798	0.049	0.421	0.369	0.349	0.467	0.029	1.311	1.517
0.275	0.153	2.001	.632	6.171	0.961	11.099	4.924	16.204	3.916	4.951	3.861
7.959	7.136	3.312	1.754	0.393	1.592	1.968	3.557				

12. SPAIN ,LATITUDE(37,40),LONGITUDE(-7,-1),N=291

0.001	95.524	14.152	3.701	196.414	90.207	13.257	81.497	99.736	43.728	11.144	27.455
15.940	2.111	10.727	1.328	56.651	59.373	35.378	92.399	65.487	105.801	106.115	4.874
109.329	23.721	16.779	51.99	167.731	86.398	0.002	129.128	45.627	29.613	165.630	13.095
143.395	35.532	13.913	120.642	127.308	22.190	15.426	42.017	406.459	15.906	141.484	25.075
69.973	168.776	28.269	0.516	0.132	7.930	0.048	0.012	0.009	0.097	0.005	0.375
0.237	0.098	0.311	0.033	0.027	0.014	0.025	0.259	0.430	24.930	0.006	0.256
18.172	346.401	135.447	1.693	46.929	15.817	35.670	30.311	10.004	10.329	0.213	14.539
214.521	90.391	34.411	47.666	0.001	100.107	29.956	23.680	3.304	7.417	7.194	19.329
4.336	24.991	0.001	0.001	26.021	38.017	13.675	51.746	0.099	1.441	181.272	16.911
36.097	102.318	55.451	122.396	29.873	8.079	0.001	181.578	94.356	6.963	23.303	0.015
28.842	17.735	5.673	15.219	2.880	0.572	20.640	0.625	0.961	8.981	2.667	3.805
1.339	3.493	12.442	0.613	0.366	4.229	5.667	7.549	6.267	0.226	36.541	0.029
0.001	0.018	0.013	0.025	0.057	1.356	0.101	0.281	1.579	0.637	0.653	1.817
6.926	2.969	0.829	8.466	0.055	0.110	0.011	0.851	0.922	0.972	7.024	13.850
8.022	40.893	20.354	0.070	0.014	0.002	1.460	4.914	0.008	0.297	1.620	0.913
1.917	0.166	0.019	0.912	1.757	4.755	20.931	5.764	9.896	9.664	0.306	22.270
5.359	13.865	0.092	0.293	0.099	0.703	5.792	3.495	31.104	0.710	3.139	8.497
0.074	1.658	6.561	0.564	9.344	4.942	26.423	13.625	18.180	0.597	0.914	10.249
21.513	9.291	0.787	0.018	22.109	2.123	11.546	3.827	2.475	0.030	0.148	0.649
1.419	15.435	7.770	20.258	11.263	2.601	6.622	27.427	7.919	5.530	2.997	14.044
4.090	1.149	20.745	6.529	21.254	20.661	1.706	1.989	0.780	1.177	21.777	13.319
4.616	0.075	0.048	0.886	17.792	2.168	15.216	8.590	20.168	2.703	1.594	1.980
0.961	4.593	4.899	7.630	0.957	2.143	1.434	1.281	4.266	5.763	6.955	97.894
127.544	679.591	169.331	30.938	91.575	50.689	96.729	45.818	84.833	96.961	210.644	389.939
46.791	254.554	1.836									

14. ITALY,LATITUDE(39,44),LONGITUDE(9,17),N=584

0.412	9.767	4.936	45.804	0.001	2.844	20.602	117.680	22.816	18.942	7.089	47.161
0.001	9.125	5.754	0.001	0.028	0.028	26.497	84.102	0.370	0.370	54.671	124.278
22.723	127.722	23.273	0.055	0.055	94.007	55.548	56.956	0.001	5.692	35.622	792.011
15.426	24.197	15.307	4.792	48.600	12.366	8.706	78.711	1.730	140.759	10.026	21.821
39.872	1.250	50.970	68.382	90.574	33.610	67.227	12.729	68.825	12.085	24.365	10.881
8.632	153.666	3.869	194.648	6.651	36.460	32.546	63.198	58.472	44.139	0.083	4.182
3.010	1.624	0.250	2.752	76.718	24.827	1.232	43.268	8.257	5.368	17.184	52.956
16.686	159.063	36.083	7.803	0.378	18.565	23.987	1.268	37.177	51.507	48.721	6.169
57.537	10.240	5.323	11.088	10.144	12.813	0.244	0.457	34.294	185.819	16.139	5.280
11.361	31.863	0.190	34.541	13.567	14.673	39.667	38.553	11.596	0.305	59.001	92.987
34.899	20.294	44.127	19.388	61.696	5.256	30.945	70.853	21.676	0.584	64.074	5.331
12.620	10.933	15.014	3.975	0.046	37.927	14.666	15.899	3.102	25.758	5.552	5.435
13.050	16.567	13.669	2.382	1.686	8.400	31.430	3.134	2.708	0.760	6.606	34.447
8.905	3.153	1.461	0.800	0.064	0.039	7.065	0.002	0.044	0.023	16.524	17.971
9.869	0.703	36.068	23.271	7.914	0.126	0.282	2.320	0.034	1.386	4.287	0.941
0.022	0.412	3.782	13.774	4.975	16.308	8.787	6.744	9.441	0.341	0.645	25.387
1.728	0.203	7.875	11.450	24.077	16.763	19.194	14.705	0.689	4.603	0.536	6.741
9.635	22.708	35.370	31.661	26.140	18.668	0.001	19.412	5.421	2.508	9.122	30.640
27.974	1.154	4.559	4.557	18.573	62.309	3.220	15.128	8.010	0.649	2.946	10.862
45.342	2.325	41.649	1.664	0.631	17.977	19.486	51.419	3.649	0.225	0.143	0.066
0.050	0.077	4.901	0.012	0.016	0.009	0.008	0.024	0.017	0.015	0.004	0.067
0.048	0.035	0.042	0.036	0.025	0.199	0.148	0.051	0.019	0.104	0.264	0.178
1.062	0.061	0.045	0.272	0.027	0.162	0.027	0.020	0.030	0.061	0.012	0.850
0.233	0.269	0.224	0.049	8.111	0.708	0.204	1.838	0.056	0.956	1.164	1.584
73.695	12.353	3.479	7.451	2.030	13.209	8.295	13.145	13.847	7.030	31.828	1.558
9.930	5.046	1.454	14.474	8.884	52.018	19.832	0.010	0.025	0.132	0.002	0.010
0.025	0.084	0.080	0.128	3.648	8.207	0.419	0.343	0.015	0.187	0.341	22.858
4.595	15.687	7.569	10.094	10.692	0.578	29.875	3.710	1.594	1.468	32.028	33.364
9.967	23.977	2.674	8.252	8.168	5.346	1.918	1.473	4.734	14.654	15.128	15.763
2.789	0.778	1.831	0.711	1.388	4.691	1.628	2.005	7.372	13.969	6.556	5.408
37.802	0.136	0.017	17.159	6.680	0.019	9.862	11.365	8.907	3.376	1.417	33.606
3.298	13.951	4.568	9.432	23.347	90.201	35.969	62.100	33.329	4.809	11.360	1.666
31.975	10.663	19.814	18.165	11.075	0.002	0.006	33.326	8.925	1.298	17.004	0.001
15.378	12.348	0.218	9.561	10.821	35.802	0.001	0.104	1.596	0.002	0.090	0.015
0.274	0.283	0.006	0.037	0.057	0.026	0.003	0.438	0.009	0.002	0.654	0.097
0.089	0.004	0.227	0.019	0.011	0.057	0.057	0.268	0.180	0.088	1.279	0.178
16.387	19.800	4.382	1.533	0.420	0.002	0.011	0.023	0.009	0.029	0.006	3.656
10.464	8.393	4.267	13.748	41.593	2.543	7.992	30.531	31.005	25.680	17.669	0.048
14.271	28.873	34.561	8.649	27.985	17.252	22.548	0.015	0.014	0.008	2.133	0.485
0.678	14.888	9.213	0.007	0.044	0.410	0.014	0.004	0.062	0.146	0.001	0.085
0.103	0.046	0.075	1.340	0.765	0.113	10.720	0.001	29.066	31.034	19.933	0.448
37.831	15.779	0.265	16.120	12.046	53.097	1.440	0.266	3.821	51.966	1.432	6.135
127.090	20.044	1.504	19.222	42.792	49.965	3.730	12.908	34.615	56.559	58.620	0.280
177.900	19.282	37.956	56.237	50.269	67.632	65.715	16.836	1.288	27.161	60.655	0.100
0.101	7.539	30.310	7.308	153.542	65.545	6.253	114.213	44.111	105.194	12.589	19.349
5.414	112.886	4.315	18.678	3.376	96.733	11.190	0.007	0.149	53.210	2.155	0.999
88.139	19.962	0.679	0.315	0.043	24.355	68.070	1.927	52.554	0.072	11.441	155.628
45.984	40.223	4.320	3.611	12.701	25.955	0.068	0.416	0.147	6.314	22.239	11.766
21.021	0.394	152.510	7.240	0.605	0.037	0.137	2.416				

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4.936	37.873	7.931	0.001	2.944	20.602	2.991	10.667	53.884	25.565	23.164	71.671
1.614	40.612	12.414	7.513	37.922	33.520	17.669	55.041	21.615	125.386	97.303	0.025
40.854	34.967	4.010	55.520	22.378	12.953	35.412	7.283	34.048	34.002	30.220	14.483
32.998	74.736	13.280	5.906	4.829	7.261	138.595	15.426	44.296	43.939	4.661	12.366
47.603	2.214	0.342	17.463	0.001	156.293	37.933	15.076	6.779	0.391	0.391	0.428
0.437	1.253	2.095	1.377	0.002	0.419	0.002	0.419	0.074	0.120	2.215	2.252
0.022	0.022	0.611	0.695	0.240	4.626	1.250	3.949	1.159	12.166	3.114	0.346
8.289	2.173	16.323	3.451	8.570	21.168	4.291	31.980	2.373	36.393	77.791	12.567
25.527	14.710	14.423	24.915	4.260	7.605	0.001	44.773	16.462	14.973	5.015	19.513
90.803	32.539	20.683	9.641	2.052	4.727	0.419	0.419	0.264	0.440	94.684	15.761
99.112	9.861	85.675	36.607	6.504	17.826	14.720	60.791	2.407	16.463	2.208	38.662
15.095	18.872	53.779	3.555	0.139	13.839	0.300	53.052	44.500	8.257	5.368	203.953
9.253	21.001	0.007	0.641	0.033	0.456	0.008	0.177	0.171	0.095	0.025	0.363
0.013	2.809	0.132	0.235	0.013	2.741	15.546	6.722	0.001	12.213	48.399	1.874
1.362	8.446	0.623	2.861	2.308	49.158	14.947	7.974	2.509	6.506	68.316	0.002
15.188	16.119	3.611	10.358	14.354	9.631	28.291	5.807	11.112	0.215	0.215	0.918
27.461	0.017	25.355	21.079	1.012	21.960	15.156	43.679	25.556	0.120	0.209	1.332
0.314	0.001	31.773	5.280	2.232	0.311	55.393	33.586	5.623	0.050	15.144	14.238
7.385	1.623	40.613	10.813	0.305	59.001	91.708	33.865	36.971	12.071	14.093	15.172
24.124	20.022	8.256	10.237	12.753	1.708	0.511	21.538	0.068	0.255	0.249	5.991
19.737	0.030	0.348	0.807	0.829	35.566	3.554	0.233	9.749	14.434	7.242	0.584
7.054	5.433	1.892	0.176	2.495	1.048	0.699	7.726	6.217	2.403	2.425	1.856
0.318	44.283	0.615	2.156	0.064	0.705	0.601	3.411	22.370	0.046	14.510	38.083
1.531	43.228	3.920	0.773	0.859	8.749	15.813	27.780	8.847	1.867	1.997	0.206
0.424	0.001	0.423	0.013	0.001	15.841	11.765	0.008	5.910	6.606	0.804	62.775
2.338	0.738	26.977	0.006	0.256	0.027	0.775	3.334	2.599	8.286	3.892	0.001
29.053	0.065	14.940	15.526	23.559	17.129	7.667	4.550	18.810	1.556	22.916	1.728
0.300	13.890	0.001	5.337	5.934	9.212	2.669	2.836	0.473	6.798	1.123	2.990
1.894	7.751	7.137	22.136	23.789	0.004	0.001	5.664	52.256	25.706	66.974	3.797
2.849	11.154	31.933	0.059	25.588	2.691	12.420	12.424	11.382	0.001	0.017	6.242
3.933	3.340	1.230	1.298	7.546	4.225	6.520	22.039	0.460	8.107	17.791	3.595
33.985	7.039	34.918	26.531	10.483	7.494	11.969	2.417	5.100	18.741	36.327	0.225
0.143	0.066	0.050	0.077	4.901	0.012	0.016	0.009	0.008	0.024	0.017	0.015
0.004	0.067	0.048	0.035	0.042	0.036	0.025	0.199	0.148	0.051	0.019	0.104
0.264	0.178	1.062	0.061	0.045	0.272	0.027	0.162	0.027	0.020	0.030	0.061
0.012	0.850	0.233	0.269	0.224	0.049	0.199	0.039	0.561	0.067	0.545	0.010
0.192	0.071	0.880	0.002	2.410	0.148	0.041	0.945	2.001	0.708	0.234	1.838
0.056	0.956	1.164	21.515	12.963	15.323	25.478	5.384	0.988	1.022	0.521	1.363
10.014	12.210	1.607	3.047	2.366	9.295	9.235	2.271	1.639	20.482	32.223	1.558
10.184	0.096	0.254	0.027	0.067	0.002	4.696	1.454	2.802	13.239	2.169	4.793
0.355	3.340	7.999	16.717	8.059	1.309	14.594	4.754	15.078	0.010	0.025	0.132
0.002	0.010	0.025	0.084	0.080	0.128	4.782	7.492	24.934	10.306	5.270	0.901
23.636	14.545	22.697	6.772	12.692	7.869	20.065	12.346	17.164	24.203	3.468	20.623
4.255	1.091	1.918	1.473	0.030	1.260	3.474	5.176	0.303	1.361	3.369	11.211
13.910	2.448	1.243	3.824	1.302	4.964	0.978	0.446	10.830	8.569	5.205	4.287
1.133	8.574	5.537	0.003	2.400	7.155	5.396	4.094	0.944	7.238	4.283	14.075
3.808	5.718	4.415	23.511	3.091	4.288	6.321	6.959	11.118	18.518	0.309	0.036
0.273	103.454	36.535	95.819	53.258	1.666	8.904	3.735	12.550	6.786	29.514	0.061
0.902	16.840	0.001	3.730	5.093	0.341	36.570	10.447	0.197	0.004	3.685	0.195
0.674	12.025	0.001	4.096	9.549	14.299	9.561	10.821	58.235	9.909	9.891	17.540
10.453	18.520	1.417	29.323	5.429	0.001	58.432	0.012	0.010	0.016	7.994	12.283
23.699	0.728	17.669	0.048	4.473	0.356	9.442	12.063	16.810	30.453	7.111	0.274
0.622	0.004	0.242	4.504	11.866	0.080	2.804	55.205	1.163	1.990	25.760	33.427
86.154	17.496	6.103	29.629	13.679	8.835	3.611	7.218	5.298	31.999	9.714	2.922
5.660	14.690	8.130	3.291	5.413	0.001	9.385	22.196	13.251	6.660	5.611	0.010
0.047	2.066	5.757	1.048	5.128	3.560	0.184	2.961	36.226	10.531	7.118	1.907
7.652	2.671	0.696	1.504	20.793	3.108	3.610	3.036	0.593	8.330	0.062	5.530
10.361	10.246	4.191	48.463	2.552	7.743	14.126	11.617	8.872	1.784	6.164	0.524

0.201	3.232	0.951	0.149	10.099	6.252	0.063	0.360	1.002	9.628	0.503	5.863
5.655	4.585	3.873	45.754	8.687	2.119	10.702	1.411	9.641	4.101	2.944	38.595
39.209	48.788	20.670	19.282	33.469	44.240	0.161	0.066	0.013	0.079	0.003	0.288
0.122	0.527	0.404	0.678	1.392	0.605	0.507	0.018	1.015	0.504	5.350	48.132
7.050	19.393	1.339	1.539	15.105	10.182	0.163	19.911	46.658	1.532	0.467	6.055
11.003	6.722	1.854	36.709	21.218	0.001	18.177	10.194	5.999	0.019	0.011	0.020
0.120	2.034	0.257	0.740	0.197	0.094	2.070	2.833	2.522	1.986	14.723	2.377
12.700	1.538	45.708	62.381	18.341	27.991	61.387	15.812	5.564	58.054	50.595	44.111
45.378	44.169	28.237	24.059	30.902	28.985	53.704	4.315	2.674	0.093	0.275	6.448
8.003	38.399	7.816	66.425	7.661	2.835	43.138	0.036	23.594	66.919	0.320	0.368
19.594	0.679	0.315	0.032	0.011	0.135	24.220	46.379	5.047	12.848	3.796	1.827
52.219	11.412	0.436	43.768	8.876	44.061	52.679	6.245	4.171	28.086	53.850	7.931
11.582	0.501	8.619	6.302	2.500	5.467	3.685	0.484	0.147	0.484	5.830	22.037
11.968	21.021	0.394	15.588	41.382	26.950	55.506	21.103				

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