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THE MEASUREMENT OF THE EARTH TIDE

and

REGIONAL HETEROGENEITY DUE TO THE OCEAN TIDE

A thesis submitted for the degree of Doctor of Philosophy in the University of Durham

Donald R. Bower

The Graduate Society

July 1971

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SUMMARY

The tide-producing force is derived as a sum of harmonic terms and directly as a function of time. The Love Numbers are then introduced to relate the tide-producing forces to the earth tide and in particular to changes in the amplitude and direction (tilt) of gravity as measured at the surface of the earth. The classical problem of earth-tide research, the determination of the Love Numbers, is complicated by the surface and body-force effects of the ocean tide. The present thesis is concerned with evaluating these effects with a view to: 1. Determining true values of the earth tide uncontaminated by ocean-tide effects. 2. Investigating the use of the ocean tide as a probe for determining the mechanical characteristics of the crust and mantle.

A method is introduced for representing the ocean tide by n-sided surface polygons of constant tidal amplitude and phase, and for determining the gravitational attraction of these polygons at any point on a spherical earth. The Boussinesq solution for elastic yielding of an infinite halfspace is then used with this ocean-tide representation to predict the ocean tide loading effect on tilt throughout Western Europe. These predictions confirm the main features of observed spatial variations and encourage a more accurate treatment. The Slichter-Caputo solution for a 2-shell earth is then adapted to the problem and preliminary results are obtained for the effect on tilt of distant ocean tides including the body-force components of the effect. The

Longman solution for a Gutenberg earth is dealt with next and a method of extrapolation for determining results in the range 0-30° is introduced. A computer program is described for implementing the prediction of world-wide tilt and gravity tides when provided with the Longman solution, or any other solution expressed as a Green's function, and ocean-tide data in the form of polygons. In general, predictions are found to be consistent with world-wide coastal observations and with inland observations in North America. Predictions for the gravity effect at mid-European and mid-Asian sites are not consistent with observations; likely due to inaccurate ocean tide data for the Arctic and Indian Oceans. Observations of tilt throughout the same area show marked regional variations which can not be due to ocean-tide effect and may reflect local hydrological effects.

The establishment of a tiltmeter site is described and several computer programs used in the analyses are presented. The development and testing of a new type, sensitive hydrostatic tiltmeter is described.

ACKNOWLEDGEMENTS

I wish to thank Professor G. M. Brown for providing facilities for this research at the University. I would also like to acknowledge the support of the Earth Physics Branch of the Dept of Energy, Mines and Resources (Canada) and in particular that of Dr M.J.S. Innes, Chief of the Gravity Division, who encouraged me to undertake this study.

I am especially grateful to my supervisor Dr M.H.P. Bott for his expert advice and his kind cooperation in all matters. His method of evaluating magnetic anomalies due to finite three dimensional bodies is the basis of the method used here for determining the gravitational attraction of the ocean tide.

I am indebted to the Weardale Lead Co. Ltd. for the underground facilities in the Redburn Mine at Rookhope. The manager Mr H. Green facilitated the preparation of a suitable tiltmeter site and tolerated the inconveniences brought about because of its presence. The tiltmeter installation could not have been kept operating throughout the two year period of use, without the active assistance and cooperation of individual miners, too numerous to mention, and in particular of the foreman Mr T.I. Martin.

Several members of the technical staff of the department were particularly helpful. Mr Dave Asbery carried out repairs on much of the electronic equipment. Mr W. Teasdale did the machine work in adapting the water-tube tiltmeters to transducers. I am very much indebted to Mr M.S. Macconachie, who worked closely with me during my

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Finally, I would like to express my appreciation for the stimulation provided by close association with Dr A. Lambert and Mr C.R. Beaumont during the last few months of this study. Chris Beaumont kindly made available to me his results for the Green's function near the load calculated on the basis of a finite-element model of the crust.

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CHAPTER 1

INTRODUCTION

The classical aim of earth-tide research is to verify the theory of earth tides proposed by Love (1909). According to this theory there are two numbers which can be determined by measurements of the earth tide and which characterize the elastic behaviour of a spherically-symmetric earth without regard to any assumed density law or physical state of the earth. The significance of this theory is best appreciated when theories prior to Love's are considered: Kelvin (1890) for example assumed a uniform earth and obtained an expression connecting the deformation of the surface due to tidal forces with 'the rigidity of the Earth'. Kelvin's 'rigidity' however is not real but rather depends on the law of rigidity assumed and on the particular earth-tide phenomenon observed. Love's numbers on the other hand, for a spherically-symmetric and oceanless earth, are physical constants independent of any hypothesis on the interior of the earth. Given the tide-producing forces they are sufficient to determine such earth-tide phenomena as: deviations of the vertical with respect to the crust; deviations of the vertical with respect to the axis of the earth; variations in the speed of rotation of the earth; variations in gravity, elastic linear strain and cubical expansions.

The two principal means of observing the earth-tide are by recording gravimeter and by horizontal pendulum. The

latter measures the deviation of the gravity vertical with respect to the local surface. Love numbers determined during the last few decades by both these methods however have been found to exhibit regional variations far beyond those which might be due to the Earth not being spherically symmetric. Some of these apparent regional variations have been due simply to measurement errors but this cause has become steadily less important as better instruments, better methods of installations and, most important, better means of calibration have been developed. Regional differences have persisted in spite of these improvements however and as a result the Love numbers determined from observations are uncertain within ± 1 or 2 percent. Since, in order to be useful for evaluating various present-day earth models, the Love numbers should be known with an uncertainty of less than ± 0.1 percent it is important to account for regional variations. This is the purpose of the present thesis.

The problem is approached here on a broad front because many of the methods in use for investigating the earth tide when this work began are ill-suited to the study of regional variations and required some attention. This includes methods of data analysis and methods of making the measurements. The principal 'legitimate' cause of regional variations however is surface loading effects of the ocean tide and the main research effort is concentrated on the problem of predicting this effect in terms of earth structure. Western Europe is the logical place to begin a

study of this type because of the concentration of earth-tide measurements in Belgium and France and because the neighboring ocean tides are particularly well known. This data is supplemented here by British measurements from Liverpool and from a new site which we established at Rookhope. For treating global effects data from diverse sources is considered. Original data from Ottawa, including tiltmeter data from nearby Holland Mills, is particularly useful as being representative of midcontinental measurements.

CHAPTER 2

THE EARTH TIDE

2-1 The Tide-producing Force

Tide-producing stresses are set up in the Earth because the gravitational attraction of the Moon (Sun) is not the same everywhere throughout the body of the Earth. At the center of the Earth the gravitational attraction must provide the centripedal force necessary because of rotation of the Earth-Moon system about its center of mass. Elsewhere within the Earth, and at its surface, the gravitational attraction will be different and the consequent stresses will cause a deformation which we call the Earth tide.

The tide-producing potential is the difference between the potential due to rotation of the Earth-Moon system and the potential due to gravitational attraction. This is given by:

$$V = GM[\,(1/\rho) - (1/R) - (r\cos\theta)/R^2\,] \qquad (2\text{-}1)$$

where G is the gravitational constant; M is the mass of the Moon; ρ is the distance from the observing point to the Moon; R is the distance between the center of the Earth and the center of the Moon ; r is the distance from the center of the Earth to the observing point and θ is the geocentral zenith distance to the Moon. The distance ρ can be written as:

$$(1/\rho) = (1/R)[\,1 - (2r\cos\theta/R) + (r^2/R^2)\,]^{-1/2} \qquad (2\text{-}2)$$

Since $r < R$ this can be represented by an expansion in Legendre polynomials:

$$(1/\rho) = (1/R)[Po(\cos\theta) + P1(\cos\theta) + P2(\cos\theta) + \ldots] \quad (2-3)$$

Substituting this in (2-1) yields the tide-producing potential:

$$V = (GMr^2/R^3)\,[P2(\cos\theta) + P3(\cos\theta)\,(r/R) + \ldots] \quad (2-4)$$

The third order and higher terms are generally ignored.

It is necessary next to express the geocentral distance θ in terms of a coordinate system which is fixed with respect to the celestial sphere.

2-1-1 Calculation Of The Theoretical Tide, Harmonic Method.

Doodson (1921) shows that the tide-producing potential due to both the Moon and the Sun can be expressed as a sum of harmonic terms each of whose arguments is a linear function of six astronomical coordinates with integer coefficients. The astronomical coordinates, with their periodicities are:

T = The mean lunar time. (1 day)

S = The mean longitude of the Moon. (1 month)

H = Mean longitude of the Sun.. (1 year)

P = Longitude of lunar perigee. (8.85 years)

N = -n, where n is the longitude of the ascending node of the Moon. (18.61 y)

Ps = longitude of the perihelion. (20900 years)

The mean lunar time expressed in terms of Greenwich Mean Time is:

$$T = 15\,t + H - S - L$$

where L is the longitude west of Greenwich, and t is Greenwich Mean Time. A convenient notation for each of the

possible harmonic terms results if the number five is added to each of the coefficients in the linear function of astronomic coordinates, except the first. Each term can then be written in the form:

$$V_{abcdef} = C_{abcdef}\, G_j \cos[aT+(b-5)S+(c-5)H+(d-5)p+(e-5)N+(f-5)P_s]$$

where a, b, c, d, e and f are integers, Cabcdef is a numerical coefficient and Gj is a geodetic coefficient involving the radius of the Earth at the point in question and the latitude.

The most fundamental classification of the harmonic terms is on the basis of the long period, diurnal, semi-diurnal and the ter-diurnal species formed when a=0, 1, 2 and 3 respectively. Within each species the terms are classified according to the group number b with neighboring groups differing in frequency by one cycle per month. The waves within each group can be further subdivided according to the constituent member with neighboring constituents differing by one cycle per year. In Doodson's four schedules values for Cabcdef and for Gj are given for 99 long period terms, 158 diurnal terms, 115 semi-diurnal terms and 14 ter-diurnal terms. Also noted in the schedules is whether the cosine or the sine term is to be used.

To facilitate the determination of horizontal tidal forces from the potential given in the schedules it is best to retain in the formula for G the radius ρ which Doodson has replaced by the mean radius a. For the unprimed values

G0, G1 and G2 found in the schedules substitute $G(\rho/a)^2$ and for the primed values substitute $G(\rho/a)^3$. For G itself use: (Melchior, 1965, p 19)

$$G = 3Mga^4/4Ec^3 = 26.206 \text{ cm}^2/\text{sec}^2$$

The numerical coefficients have been adjusted by Doodson so that the same G factor can be used for all.

To illustrate the notational scheme consider the principal component of the N2 group given in Doodson's schedule 2. This is:

$$V^{2}{}_{45655} = .17387\ G\rho^2\ \text{Cos}^2\lambda\ \text{Cos}(2T-S+P)/a^2$$

A minor component of this same group involves the primed geodetic coefficient G2' and it becomes:

$$V^{2}{}_{45555} = .00567(2.59808)\ G\rho^3\ \text{Sin}\lambda\ \text{Cos}^2\lambda\ \text{Sin}(2T-S+P+2n)/a^3$$

The vertical and horizontal components of the tide producing force are determined by differentiating as follows:

$$Fv = -\delta V/\delta\rho \qquad Fn = -\delta V/\rho\delta\lambda \qquad Fw = -\delta V/\rho\text{Cos}\lambda\delta L$$

When this is done it is found that the tidal gravity, and the north and west components of tilt can each be determined from the schedule of numerical coefficients by determining a new series of geodetic coefficients for use as multiplying factors. These new geodetic coefficients can be found in quite a straight forward manner by differentiating the several kinds of terms found in the schedules. This has been done by Melchior (1965, p64) and his results have been presented together with a shorter and easier to use version

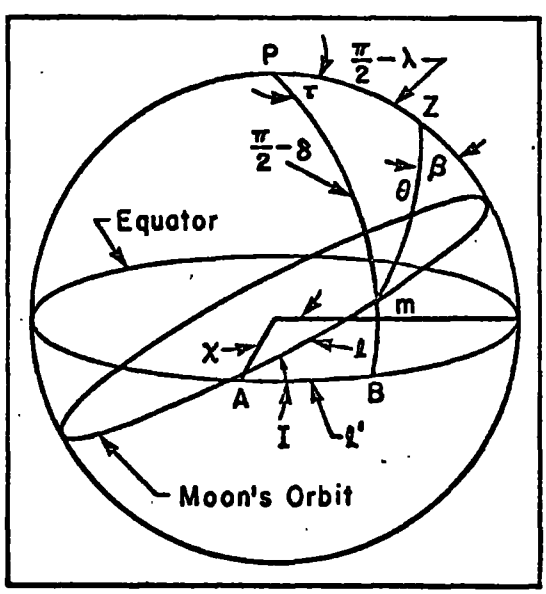

FIGURE (2-1) Construction for deriving the expression for the attraction of the Moon. (See text for definition of the symbols)

of Doodson's schedules. Melchior omits from his table terms whose amplitude is less than .001 relative to the amplitude of the largest term leaving a total of 79 terms. Melchior's table has been used in the analysis program JANET, which is documented in the appendix and will be described later.

2-1-2 Calculation Of The Theoretical Tide, Direct Method. Longman (1959) has presented a summary of the formulas used in developing a closed expression for the vertical component of tidal acceleration as a function of time for any given place on the Earth's surface. His summary does not include formulas for determining the north-south and east-west components of the horizontal tidal acceleration therefore these formulas are developed here. In addition to the symbols used by Longman we will use the symbols β, δ and t respectively for the azimuth, declination and hour angle of the attracting body. Referring to Figure (2-1), the letters P, Z and m denote the position on the celestial sphere of the north pole, the zenith and the attracting body. The angular distance l' is the projection of l on the equator. Expressions derived for the Moon are immediately applicable to the Sun after the obvious substitutions. According to Longman the horizontal tidal acceleration due to the Moon is given by:

$$H = C \sin 2\theta + [Cr(5\cos^2\theta)\sin\theta]/d \qquad (2\text{-}5)$$

where: $C = 3\omega Mr/2d^3$

From the figure, the south and west components of H are given by:

$$Hs = 2C\,\cos\theta\sin\theta\cos\beta + (Cr/d)(5\cos^2\theta - 1)\sin\theta\cos\beta \quad (2\text{-}6)$$

$$Hw = 2C\,\cos\theta\sin\theta\sin\beta + (Cr/d)(5\cos^2\theta - 1)\sin\theta\cos\beta \quad (2\text{-}7)$$

using the following relations for Δmpz:

$$\cos\theta = \sin\lambda\sin\delta + \cos\lambda\cos\delta\cos T \quad (2\text{-}8)$$

$$\sin\theta\cos\beta = \cos\lambda\sin\delta + \sin\lambda\cos\delta\cos T \quad (2\text{-}9)$$

$$\sin\theta\sin\beta = \cos\delta\sin T \quad (2\text{-}10)$$

and from amb

$$\cos\delta = \cos I/\cos I' \quad (2\text{-}11)$$

$$\sin\delta = \sin I\sin I \quad (2\text{-}12)$$

$$\tan I' = \tan I\cos I \quad (2\text{-}13)$$

Now $T = x - l'$ and from (2-11), (12) and (13):

$$\cos\delta\cos T = \cos x\cos l + \sin x\sin l\cos I \quad (2\text{-}14)$$

$$\cos\theta\sin T = \sin x\cos l - \cos x\sin l\cos I \quad (2\text{-}15)$$

Substituting (14) and (15) in (8), (9) and (10) we obtain finally:

$$\cos\theta = \sin\lambda\sin l\sin I + \cos\lambda(\cos x\cos l + \sin x\sin l\cos I) \quad (2\text{-}16)$$

$$\sin\theta\cos\beta = -\cos\lambda\sin l\sin I + \sin\lambda(\cos x\cos l + \sin x\sin l\cos I) \quad (2\text{-}17)$$

$$\sin\theta\sin\beta = \sin x\cos l - \cos x\sin l\cos I \quad (2\text{-}18)$$

These are the three expressions needed to determine the three components of the tidal acceleration from equations (6) and (7) and from Longman's formulas for x, l and I.

A computer program (ROBIN) based on these equations was written to determine the vertical tidal acceleration and the horizontal tidal acceleration along any specified azimuth. This direct method yielded more accurate results at less computing cost than the harmonic method and was used to check the latter method and to provide test data for the

data analysis program JANET. The program is documented in the appendix.

2-2 The Love Numbers and the Gravimetric and Diminishing Factors.

An ocean completely covering the Earth would everywhere define an equipotential surface. The tide-producing potential would displace such a universal sea radially a distance V/g where V is the tidal potential. This is called the height of the equilibrium tide. It is a useful concept in ocean tide theory although tidal heights actually experienced in the ocean are generally much altered from the equilibrium value because of the dynamic effects of friction and inertia. The equilibrium height is of greater use in connection with Earth tides where dynamic effects are negligible. Love (1911) has shown that both the surface displacement and the newly created potential due to the displacement are proportional to the equilibrium tidal height. He has introduced the constants h and k to describe the proportionality and it is these numbers that have become known as the Love Numbers.

The Love Numbers have been evaluated theoretically on the basis of different Earth models by many early workers (see Jeffreys Sec 7.05, The Earth) and more recently by Takeuchi (1950), Alterman Jarosch and Pekeris (1959) and Alsop and Kuo (1964). Experimentally the Love Numbers have been determined from measurements of various physical phenomena;those of interest here are tidal variations in

gravity and tidal deflections of the vertical with respect to the local crust. It is convenient to relate these phenomena to corresponding phenomena which would be observed on a perfectly rigid Earth. Thus the tidal variation in gravity can be expressed as:

$$G = [1 + h - (3k/2)]\, G'$$
$$= \delta G'$$

where G' is the rigid-Earth tidal gravity. The tidal tilt can be written:

$$T = [1 + k - h]T'$$
$$= DT'$$

where T' is the rigid-Earth tilt. The factors δ and D defined in this way are called gravimetric and diminishing factors respectively. Typical observed values for δ and D fall within the ranges of 1.135 to 1.230 and 0.550 to 0.850 respectively.

Alsop and Kuo (1964) have calculated values for the Love Numbers for a wide range of spherically-symmetric Earth models. They find that differences in the structure of the crust and upper mantle or the presence of a solid inner core have little influence on the Love Numbers. The presence of a universal ocean of thickness 5 km however is found to have a very large effect. This is shown in their Table 3 which is reproduced here as Table (2-1). The Bullen B modified model is the Jefferies-Bullen B with an oceanic crust. The Bullen-B Oceanic is the Bullen-B modified with the top 5 kms of the 10 km oceanic crust replaced with water. This can

(Tidal Period = 12 hr.)
(Inner core $\mu = 4.0 \times 10^{12}$ cgs)

		Jeffreys-Bullen B						Bullen B Modified						Bullen B Oceanic					
Average Core $\mu(\times 10^{11})$	Outer Core $\mu(\times 10^{11})$	h	l	k	G $1+h-3/2\,k$	D $1+k-h$	L $1+k-l$	h	l	k	G	D	L	h	l	k	G	D	L
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1.9	0.0367	.617	.088	.304	1.161	.687	1.216	.630	.088	.310	1.165	.680	1.222	.641	.088	.285	1.214	.644	1.197
2.0	0.142	.613	.088	.302	1.160	.689	1.214	.623	.089	.307	1.162	.684	1.218	.636	.088	.283	1.212	.647	1.195
2.1	0.247	.609	.088	.300	1.159	.691	1.212	.620	.089	.305	1.162	.685	1.216	.633	.088	.281	1.212	.648	1.193
2.2	0.351	.604	.088	.297	1.158	.693	1.209	.615	.089	.303	1.160	.688	1.214	.629	.088	.279	1.210	.650	1.191
2.3	0.456	.600	.088	.295	1.157	.695	1.207	.610	.089	.301	1.158	.691	1.212	.625	.088	.277	1.210	.652	1.189
2.4	0.561	.597	.088	.294	1.156	.697	1.206	.607	.089	.299	1.158	.692	1.210	.621	.088	.275	1.208	.654	1.187
2.5	0.666	.593	.088	.292	1.155	.699	1.204	.603	.089	.297	1.158	.694	1.208	.617	.088	.273	1.208	.656	1.185
2.75	0.928	.584	.088	.287	1.154	.703	1.199	.593	.089	.292	1.155	.699	1.203	.609	.088	.269	1.206	.660	1.181
3.00	1.19	.575	.088	.283	1.150	.708	1.195	.584	.089	.288	1.152	.704	1.199	.599	.088	.264	1.203	.665	1.176
3.25	1.45	.567	.088	.279	1.148	.712	1.191	.576	.089	.284	1.150	.708	1.195	.591	.088	.260	1.201	.669	1.172
3.50	1.71	.559	.088	.275	1.146	.716	1.187	.567	.089	.279	1.148	.712	1.190	.583	.088	.256	1.199	.673	1.168
3.75	1.98	.552	.088	.271	1.145	.719	1.183	.559	.089	.276	1.145	.717	1.187	.575	.088	.252	1.197	.677	1.164
4.00	2.24	.545	.088	.268	1.143	.723	1.180	.552	.089	.272	1.144	.720	1.183	.568	.088	.248	1.196	.680	1.160
4.25	2.50	.538	.089	.265	1.140	.727	1.176	.545	.090	.268	1.143	.723	1.178	.561	.088	.245	1.194	.684	1.157
4.50	2.76	.531	.089	.261	1.139	.730	1.172	.538	.090	.265	1.140	.727	1.175	.554	.088	.241	1.192	.687	1.153
4.75	3.03	.525	.089	.258	1.138	.733	1.169	.531	.090	.262	1.138	.731	1.172	.547	.088	.238	1.190	.691	1.150
5.00	3.29	.518	.089	.255	1.137	.737	1.166	.525	.090	.259	1.136	.734	1.169	.541	.088	.235	1.188	.694	1.147
6.0	4.34	.496	.089	.244	1.130	.748	1.155	.501	.090	.247	1.130	.746	1.157	.518	.088	.223	1.184	.705	1.145
7.0	5.39	.476	.089	.234	1.125	.758	1.146	.481	.090	.237	1.126	.756	1.147	.498	.088	.213	1.178	.715	1.135
8.0	6.44	.458	.089	.226	1.119	.768	1.137	.463	.090	.226	1.121	.765	1.138	.480	.088	.204	1.171	.724	1.116
9.0	7.48	.443	.089	.216	1.116	.775	1.129	.446	.090	.220	1.116	.774	1.130	.464	.088	.196	1.170	.732	1.108
10.0	8.53	.428	.089	.211	1.112	.783	1.122	.432	.090	.213	1.112	.781	1.123	.450	.088	.189	1.166	.739	1.101

TABLE (2-1) The Love Numbers (Alsop and Kuo)

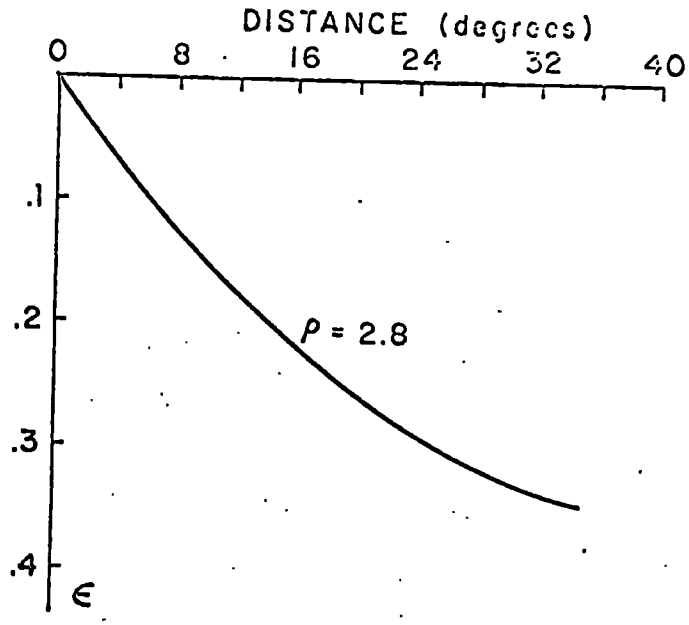

FIGURE (3-1) The function ϵ near the load.

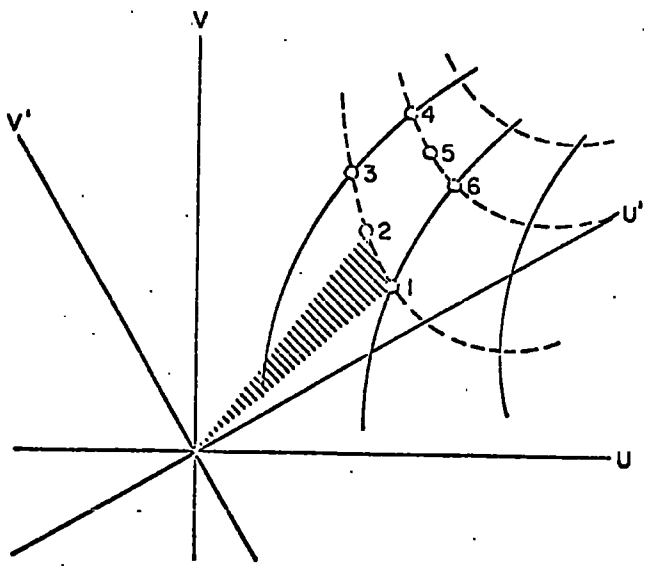

FIGURE (3-3) Construction for determining the attraction of n-sided polygon.

only be a rough approximation to reality but the result indicates that Love Numbers observed near oceans should be significantly larger than continental values.

CHAPTER 3

THE OCEAN-TIDE EFFECT*

3-1 Introduction

An essential first step in the interpretation of measurements of the earth tide by tiltmeter or gravimeter is to estimate the effect of the ocean tide. This indirect effect is due to the gravitational attraction of the water and of the land masses which have elastically yielded to the weight of the water, as well as to the elastic displacement and possible tilting of the measuring site due to the loading of the surface. The ocean tide is not known generally and estimates of it for mid-ocean areas in particular are poor since they are inferred from coastal measurements made perhaps many hundreds of miles distant. In spite of these difficulties estimates of the indirect effect may be sufficient to correct observations of the bodily tide made at continental sites for the effects of the ocean tide. In the case of coastal sites where the indirect effect is large an estimate of its value may be useful in determining, through an iterative process incorporating the earth-tide observations, the elastic properties of the regional crust and uppermantle.

In what follows I describe the development of the various techniques which I have used to estimate the indirect effect throughout Britain and northwestern Europe and throughout the world generally. I describe first a method of determining the regional effects of the ocean tide

* The ocean-tide effect referred to is that due to the M_2 constituent

assuming an infinite, half-space earth. The particular method described differs from earlier ones in the manner in which the ocean tide is described and its attraction computed. It is a computer-oriented (rather than site oriented) method and permits working directly from cotidal-corange charts. This method is then applied to the case of a 2-shell Earth and the effects of ocean tides throughout the world are considered. The attraction computations for the global case are immediately usable in equations presented by Longman (1962, 1963) for determining the response of a Gutenberg-model Earth to surface forces. The series appearing in Longman's solutions are not convergent in the form presented by Longman for distances less than about 30 degrees but an extrapolation procedure permits solutions for distances down to a few degrees. The Longman results are tested by comparison with the results for the 2-shell Earth and by comparision with recent results determined by Beaumont (1971) using a finite-element method. The Beaumont results for surface displacement and tilt are adopted throughout the distance 0-8° where the Longman extrapolated results are likely to be in error. In this way a method of calculating the indirect effect from the computed attraction of the ocean tide has been determined which is valid over the globe.

3-2 The Effect On An Infinite Half-Space Earth.

In general the ocean tide throughout the world and the deformation of the earth due to the presence of the ocean

tide must be known before its influence on earth-tide measurements can be determined for a particular site. In the particular case of earth-tide tilt measurements however the influence of the ocean tide falls off rapidly enough with increasing distance to justify assuming, as a first approximation, a plane earth and considering only those waters within 20 or 30 degrees of the site. We will show later that the effect of sphericity and other real-Earth characteristics is to cause surface tilt to diminish even more quickly with increasing distance.

Assume that the earth can be represented by an homogeneous medium in the region $z \geq 0$. The normal deformation of the surface by the distributed normal load $p(x,y)$ acting at the point x,y in the X, Y plane is given by: (Sokolnikoff, 1956)

$$w = [(\lambda+2\mu)/(4\pi\mu(\lambda+\mu))] \iint [p(x,y)/r \; dx \; dy] \qquad (3\text{-}1)$$

where $r^2=(X-x)^2+(Y-y)^2$ and μ and λ are the Lamé constants. It can be expressed too in terms of the gravitational potential of the load, which is:

$$E = \iint [Gs(x,y)/r \; dx \; dy] \qquad (3\text{-}2)$$

when there is a surface mass density which can be expressed by:

$$p(x,y) = s(x,y)\,g \qquad (3\text{-}3)$$

where g is the acceleration due to gravity. Equation 3-1 then becomes:

$$w = (\lambda+2\mu)\, gE/[\, 4\pi\mu(\lambda+\mu)\, G\,] \qquad (3\text{-}4)$$

where G is the gravitational constant. Adopting the terminology used by Melchior (1965, p192) we define the tilt due to the gravitational attraction of the load by:

$$A = \delta E/g\delta r \qquad (3\text{-}5)$$

The tilt of the surface then is:

$$\delta w/\delta r = (\lambda+2\mu)\, g^2 A/[\, 4\pi\mu(\lambda+\mu)\, G\,] \qquad (3\text{-}6)$$

$$= \beta A$$

The horizontal attraction due to the displaced masses of the medium is written in the form $-\epsilon A$ where ϵ is a function to be determined. The sum of the indirect effects on tilt then is:

$$I = (1 + \beta - \epsilon)\, A \qquad (3\text{-}7)$$

An estimate of the horizontal attraction ϵA was obtained using the global deformation given by the solution of Slichter and Caputo (1960) for a homogeneous shell of rigidity $\mu=1.2\times10^{12}$ (dynes/cm^2) enclosing a fluid core. For the purpose of computing the horizontal attraction the deformation was represented by a global sheet of mass surface density 2.8h where h is the deformation and 2.8 is the volume density assumed for the displaced masses. Thus all the displacement was assumed to take place on the surface. The function ϵ then was determined for a range of distances from the loading mass by dividing the horizontal attraction by the horizontal attraction of the loading mass. The resulting function is reproduced in Figure (3-1). The function ϵ is the same as the function T2/T3 introduced

later and shown for further models in Fig.3-5.

Equation (3-7) demonstrates the three principal effects of the ocean tide on the measurement of tilt and expresses these effects in terms of the gravitational attraction of the tide. A fourth effect is the elastic uplift of ground caused by the gravitational attraction of the water masses. This effect is significant in the case of gravity measurements at great distances from the load but the associated tilting is negligible. Later we will compare the results obtained here assuming an infinite-plane earth with the results obtained using more realistic models.

3-2-1. The Attraction Of The Ocean Tide.

We will introduce now a method of determining the potential of the M2 component of the ocean tide, from which the attraction A can be found for use in Eqn. (3-7). Consider first the cotidal-corange chart shown in Fig. (3-2). This chart describes the behaviour of the M2 component of the ocean tide in the waters surrounding the British Isles. It is essentially British Admiralty Chart 5058 (1937) supplemented by material from the recent German North Atlantic Chart (W.Hansen, Institute fur Meereskunde der Universitat Hamburg, 1969). The contours are drawn through points of equal amplitude (solid lines) and phase (dotted lines). The range (double amplitude) is given in feet and the phase is in degrees lagging the lunar transit of Greenwich. The classical method of determining the gravitational attraction due to this distribution of mass is

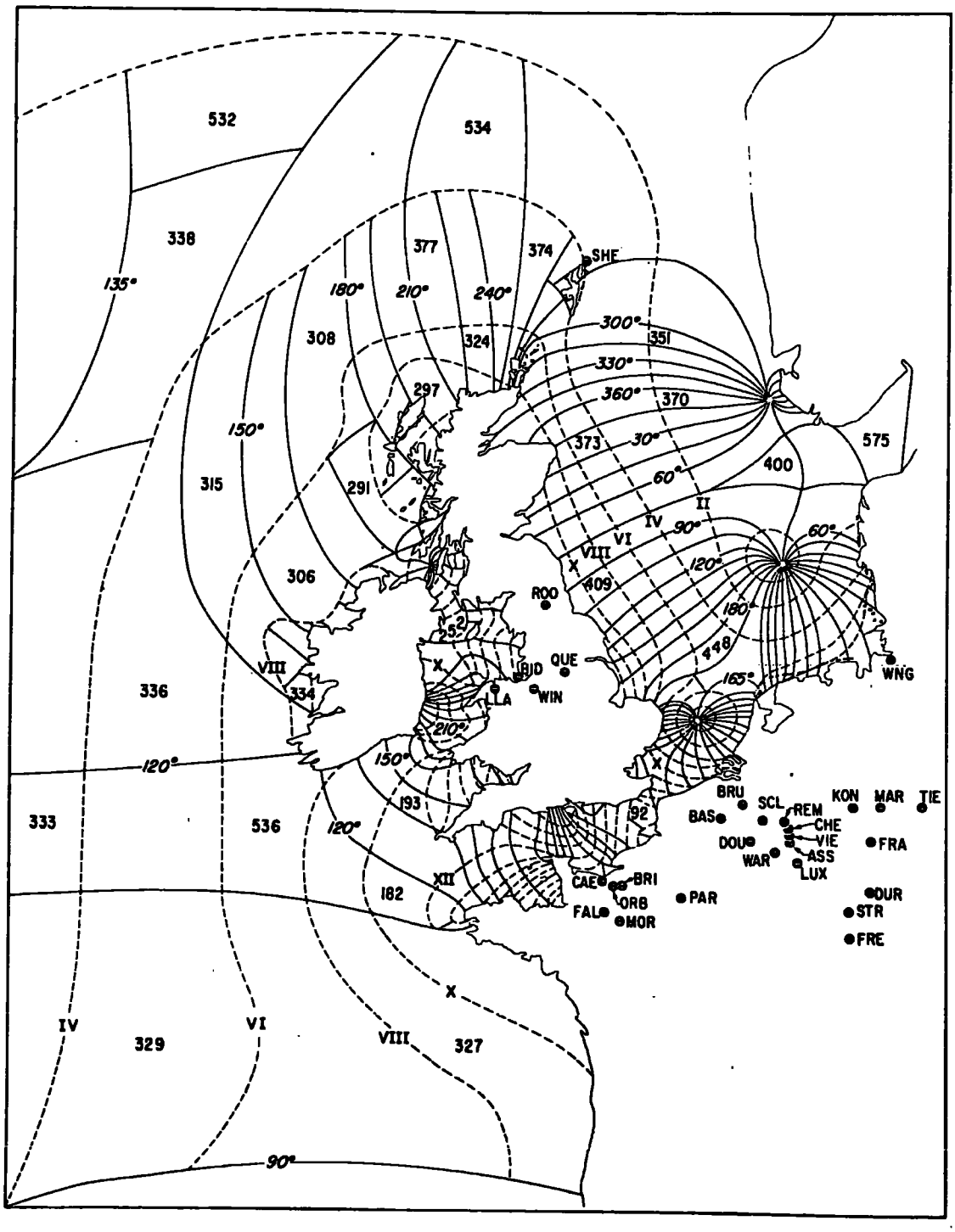

FIGURE (3-2) Ocean-tide contours near Britain (Region 0). Earth-tide measuring sites are identified by the first 3 figures in the name.

that using Hayford zones of constant amplitude and phase centered on the site where the attraction is to be determined. There are two objections to this method: It is difficult to fit the Hayford zones to the contour pattern and thus a large number of zones is required for an acceptable representation. The second objection is that a new system of Hayford zones must be set up for each site. An alternative method of representing the contour pattern is by a series of spherical harmonics. This harmonic method has been used by Pertsev (1966) in considering the effect of a world-wide distribution of ocean tides. Zones of equal amplitude and phase in this case are spherical trapezoids of equal area bounded by parallels and meridians. The zones of course are independent of site location but like the Hayford zones they are difficult to fit to the tidal contours and in addition they cannot be adjusted in size to fit requirements. This method then is practical only for first approximation global effects and not for determining the effects of regional tides. Since the principal aim of this investigation is to explain the spatial variations in earth-tide measurements the effect of regional tides is important and must be estimated for a dense distribution of sites. Consequently neither of the methods described was applicable.

It will be noticed that the corange and cotidal lines defining the tides tend to intersect orthogonally to form rectangles of approximately uniform amplitude and phase. In

fact it can be shown (Defant, 1961, p356) that in the absence of friction these lines would be everywhere perpendicular. To take advantage of this feature a general method of determining the attraction of such rectangles was developed but was subsequently abandoned in favour of a method suggested by Bott which was usable with polygons. This method is derived from one used by Bott (1963) for determining the magnetic field due to finite 3-dimensional bodies. We will use the method again later in three dimensions (program GLOBL) but it is best described in the simple 2-dimensional form required here.

Consider a n-sided polygon formed by the intersection of cotidal and corange lines drawn in the x,y plane (Fig. 3-3). The behaviour of the water within the polygon can be described by a single set of harmonic constants (contrast this with the case when the shape of the polygon is arbitrarily assigned). Number the polygon points clockwise from j=1 to j=n and assume a right-handed coordinate frame u,v centered on the site o, where the potential is to be determined. Rotate the u,v axes to new positions u', v' so that the positive v' axes is in the same direction as the side j+1 of the polygon. Denoting the coordinates of the j-th point with respect to the two positions of the axes by u(j), v(j) and u'(j), v'(j) the potential of the lamina j, 0, j+1 then is:

$$\Delta Ej = C\ u(j)\ \log(M) \qquad (3\text{-}8)$$

where:

$$M = [v'(j+1) + (v(j+1)^2 + u'(j)^2)^{1/2}]/$$
$$[v'(j) + (v(j)^2 + u'(j)^2)^{1/2}]$$

and:

$$v'(j+1) = [u(j+1)(u(j+1)-u(j))-v(j+1)(v(j+1)-v(j))]/L$$
$$v'(j) = [u(j)(u(j+1)-u(j))+v(j)(v(j+1)-u(j))]/L$$
$$u'(j) = [u(j)(v(j+1)-v(j))-v(j)(u(j+1)-u(j))]/L$$
$$u'(j+1) = u'(j)$$

L is the distance from point j to point j+1, and C is a constant incorporating the amplitude and phase factors, the density of water, and the gravitational constant. The potential of the entire polygon then is the algebraic sum of the potentials found for the n laminae:

$$E = \Sigma \Delta Ej \qquad (3-9)$$

3-2-2 Program PRESS

A computer program (PRESS) has been written to compute the surface deformation on the basis of the foregoing assumptions. The region of W. Europe, particularly Britain, has been treated in detail using the cotidal-corange chart described earlier (Fig. 3-2). The ocean tide considered is that for the region east of longitude 25° west, south of latitude 62° north, and north of latitude 42° north. The cotidal charts were digitized using a electronic line follower to input the points defining each polygon. About 500 polygons were used initially; their general arrangement is indicated in Fig. (3-2). The number of every 25th polygon is shown. The amplitude and phase of the ocean tide throughout each polygon are given in Appendix VII. For this

tidal data, and for assigned values of rigidity and compressibility, program PRESS determines: 1. The amplitude and phase of the normal displacement of the surface. 2. The amplitude and the azimuth of tilt for 30 degree intervals of phase. The latter provides data for drawing ellipses describing the behaviour of the tilt vector (horizontal force vector/g) as a function of time.A copy of program PRESS is reproduced in Appendix III.

3-2-3 Discussion And Results.

It was originally intended to contour the amplitude and phase of the surface deformation in the same way as that of the ocean tide. The phase is fairly constant over the entire region however, lagging the transit of the moon at Greenwich by 120 to 140 degrees, and the corange lines alone are sufficient to represent the deformation. A map showing corange lines for the deformation due to loading by the ocean tide has been determined by PRESS and is presented in Fig. (3-4). The data from which this map was drawn was determined on the basis of an infinite half space with $\mu = 1.0 \times 10^{12}$ dynes/cm^2 and Poisson's ratio equal to 0.5.

Results for 24 earth-tide observing sites are presented in Table 3-1. The location of each of these sites is shown in Fig. (3-2). These results represent a preliminary estimate of the ocean-tide effect in this important region where 90 percent of the world total of earth-tide observations have been made. Striking variations in the results of observations throughout this region have been reported and

SITE	LONG	LAT	A_s	A_w	I_s	I_w	U
LLA	-3.80	53.18	14.79(141°)	2.50(-166)	51.49(142°)	8.08(-157)	2.35(144°)
BID	-3.07	53.40	15.96(139°)	13.98(-56°)	56.36(140°)	54.08(-56°)	1.92(140°)
WIN	-2.50	53.20	5.15(150°)	2.48(-56°)	16.33(154°)	11.58(-52°)	2.55(140°)
ROO	-2.21	54.78	2.28(-109)	4.29(-74°)	9.47(-98°)	17.41(-69°)	2.46(130°)
CAE	-0.22	49.11	7.82(103°)	4.78(176°)	23.44 (99°)	24.25(160°)	2.85(146°)
FAL	-0.12	48.53	4.06(098°)	4.75(153°)	9.10(085°)	19.61(151°)	3.06(140°)
ORD	0.24	49.10	6.29(109°)	4.89(170°)	18.67(106°)	17.58(172°)	2.60(144°)
MOR	0.33	48.32	3.17(101°)	5.05(143°)	6.70(088°)	15.35(148°)	2.87(139°)
BRI	0.43	49.12	5.97(113°)	4.62(172°)	17.76(112°)	15.85(176°)	2.51(143°)
VIL	2.19	49.07	2.20(124°)	2.39(149°)	5.86(127°)	5.50(163°)	2.06(138°)
PAR	2.33	48.83	1.86(120°)	2.69(144°)	4.66(122°)	6.21(155°)	2.09(137°)
BRU	4.35	50.80	0.40(141°)	0.94(106°)	0.68(-172)	1.36(57°)	1.57(136°)
DUR	4.63	50.10	0.64(125°)	1.35(131°)	1.31(134°)	2.05(136°)	1.59(137°)
SLA	5.02	50.50	0.47(117°)	1.30(128°)	0.72(121°)	2.01(127°)	1.52(137°)
WAR	5.38	49.83	0.50(115°)	1.52(133°)	0.93(118°)	2.74(139°)	1.52(136°)
REM	5.70	50.48	0.44(105°)	1.42(131°)	0.74(95°)	2.54(134°)	1.45(137°)
VIE	5.90	50.27	0.43(101°)	1.44(132°)	0.98(089°)	2.61(135°)	1.44(137°)
LUX	6.13	49.62	0.41(112°)	1.55(133°)	0.77(112°)	2.93(137°)	1.44(136°)
STR	7.77	48.58	0.23(096°)	1.46(133°)	0.44(091°)	2.78(138°)	1.28(135°)
FRI	7.87	48.00	0.18(082°)	1.50(132°)	0.36(074°)	2.87(137°)	1.28(135°)
NEU	7.93	50.78	0.40(104°)	1.36(137°)	0.83(097°)	2.69(144°)	1.20(137°)
DUR	8.48	49.00	0.21(099°)	1.37(134°)	0.40(094°)	2.59(139°)	1.20(136°)
MAR	8.80	50.80	0.35(110°)	1.25(138°)	0.73(108°)	2.41(146°)	1.11(137°)
TIE	10.17	50.72	0.28(116°)	1.09(137°)	0.57(118°)	1.99(145°)	0.99(137°)

TABLE (3-1) Elastic deformation of Western Europe due to ocean-tide loading. A is attraction tilt in msecs and degrees lag relative to transit of the Moon at Greenwich. I is attraction plus elastic tilt; U is depression in cms; the subscripts s and w denote the directions south and west respectively.
(Boussinesq solution with $\mu = 10^{12}$ cgs)

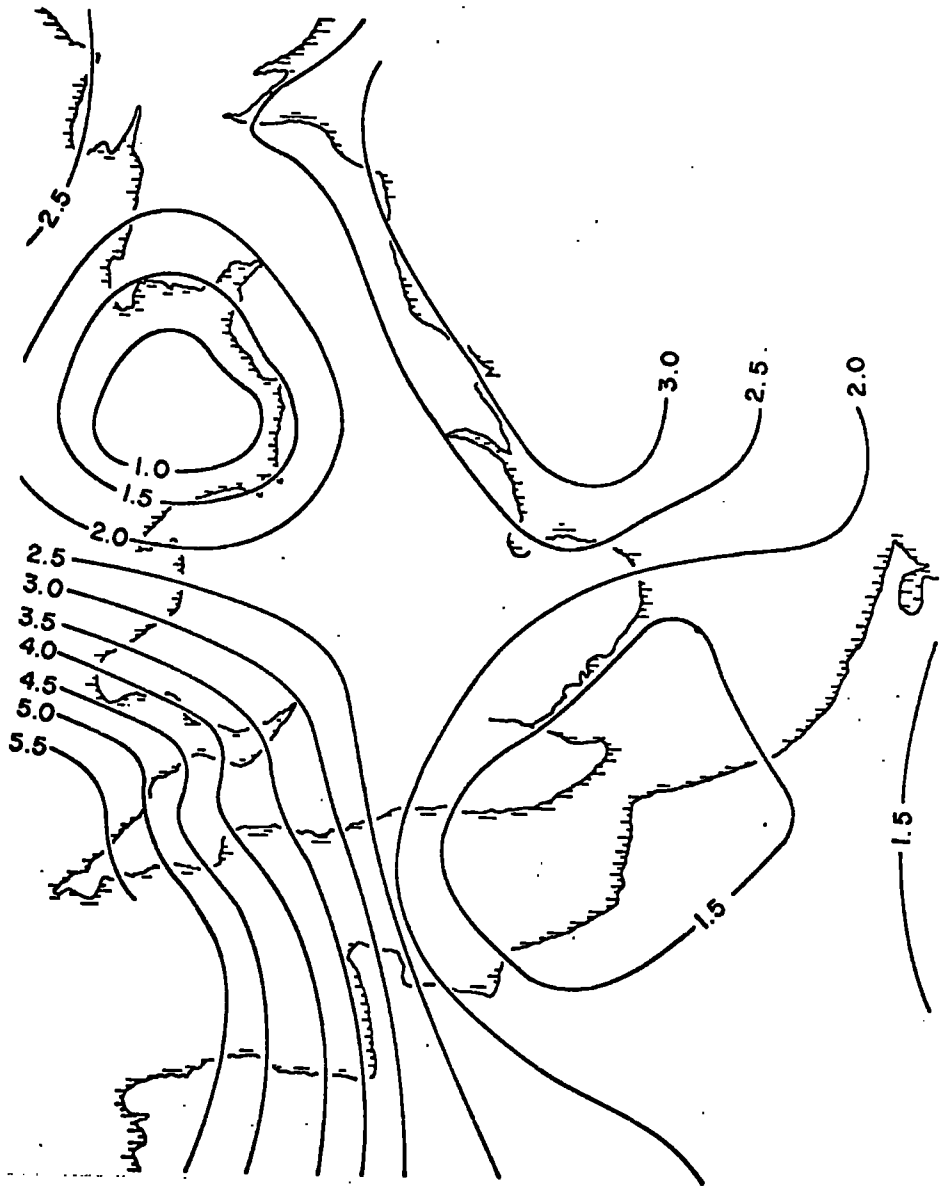

FIGURE (3-4) Elastic depression in cms due to ocean-tide loading (Boussinesq solution for plane). For the case: $\mu = 1.0 \times 10^{12}$, $\lambda = \infty$

an early objective of this research was to determine whether such variations could possibly be due to ocean-tide effect. Program PRESS provides an economical means of rapidly evaluating the spatial variations in ocean-tide effect. Its use demonstrated that some of the observations must be affected by some anomalous local condition since they could not be explained on the basis of any reasonable ocean-tide loading effect. Most of the observations however reflected the predicted ocean-tide loading and this encouraged the use of a better model and more complete ocean-tide data. This is pursued in the sections which follow and the present PRESS results will not be discussed further.

3-3. The Effect On A 2-Shell Earth.

Slichter and Caputo (1960) and Caputo (1961) have considered the deformation of a layered earth by an axially symmetric surface mass distribution. I have used their method of solution for the non gravitating, homogeneous shell, fluid core case (Slichter and Caputo, 1960) to determine the radial deformation when the load is a single circular cap approximating a point mass (half angle of one degree). Using this result I have determined the tilting effect and the perturbation of gravity with an approximate correction for gravitation effects. The computation was implemented by program CAPS, which is documented in Appendix IV, and the results are presented in Tables 3-4 and 3-5 expressed in a convenient non-dimensional form introduced by Longman (1963). Longman's variables are u/(am'/m) and

Table 3-2a **Influence values for surface displacements (Taylor): point load** $\rho = I\rho P/2\pi hE$

Vertical displacement ρ_z

r/h \ ν	0	0·2	0·4	0·5
0·05	37·580	35·921	31·052	27·351
0·1	17·586	16·728	14·260	13·360
0·2	7·624	7·162	5·897	4·914
0·3	4·327	4·016	3·154	2·480
0·4	2·720	2·478	1·827	1·320
0·5	1·792	1·599	1·092	0·699
0·6	1·212	1·048	0·635	0·290
0·7	0·823	0·690	0·352	0·051
0·8	0·560	0·450	0·168	−0·079
0·9	0·373	0·286	0·053	−0·160
1·0	0·250	0·182	−0·011	−0·183
1·25	0·080	0·031	−0·085	−0·194
1·5	0·013	−0·002	−0·077	−0·156
1·75	−0·007	−0·007	−0·048	−0·123
2·0	−0·012	−0·011	−0·039	−0·083
2·5	−0·004	−0·017	−0·025	−0·036
3·0	−0·003	0·001	−0·008	−0·025
3·5	−0·003	0·000	−0·004	−0·018
4·0	−0·001	0·000	−0·003	−0·012
6·0	−0·000	−0·000	−0·001	−0·002
8·0	−0·000	−0·000	−0·000	−0·000
10·0	−0·000	−0·000	−0·000	−0·000

Table 3-2b **Influence values for surface displacements (Taylor): point load** $\rho = I\rho P/2\pi hE$

Radial displacement ρ_r

r/h \ ν	0	0·2	0·4	0·5
0·05	19·959	14·362	5·559	−0·041
0·1	9·948	7·124	2·723	−0·078
0·2	4·896	3·455	1·250	−0·156
0·3	3·183	2·184	0·716	−0·225
0·4	2·308	1·523	0·426	−0·288
0·5	1·773	1·064	0·232	−0·326
0·6	1·277	0·824	0·102	−0·376
0·7	1·000	0·620	0·008	−0·405
0·8	0·789	0·465	−0·063	−0·420
0·9	0·627	0·349	−0·011	−0·421
1·0	0·499	0·259	−0·141	−0·417
1·25	0·292	0·150	−0·175	−0·380
1·5	0·167	0·048	−0·163	−0·315
1·75	0·097	0·012	−0·134	−0·250
2·0	0·060	0·002	−0·109	−0·195
2·5	0·027	0·003	−0·070	−0·118
3·0	0·010	−0·002	−0·038	−0·072
3·5	0·003	−0·008	−0·022	−0·046
4·0	0·002	−0·000	−0·014	−0·029
6·0	0·000	−0·000	−0·002	−0·002
8·0	0·000	−0·000	−0·000	−0·001
10·0	0·000	−0·000	−0·000	−0·000

Table 3-3

Properties of the Gutenberg Earth Model

$a-r$ km	ρ g/cm³	C_p km/s	C_s km/s	μ 10^{11} dyn/cm²	λ 10^{11} dyn/cm²	g cm/s²
0	2·74	6·14	3·55	3·45	3·42	982
19	3·00	6·58	3·80	4·33	4·32	983
38	3·32	8·20	4·65	7·18	7·97	984
50	3·34	8·17	4·62	7·13	8·04	985
60	3·35	8·14	4·57	7·00	8·20	985
70	3·36	8·10	4·51	6·83	8·38	986
80	3·37	8·07	4·46	6·70	8·54	986
90	3·38	8·02	4·41	6·57	8·59	986
100	3·39	7·93	4·37	6·47	8·37	986
125	3·41	7·85	4·35	6·45	8·11	987
150	3·43	7·89	4·36	6·52	8·31	988
175	3·46	7·98	4·38	6·64	8·76	989
200	3·48	8·10	4·42	6·80	9·23	989
225	3·50	8·21	4·46	6·96	9·67	990
250	3·53	8·38	4·54	7·28	10·24	991
300	3·58	8·62	4·68	7·84	10·92	992
350	3·62	8·87	4·85	8·52	11·45	993
400	3·69	9·15	5·04	9·37	12·15	995
450	3·82	9·45	5·21	10·37	13·38	996
500	4·01	9·88	5·45	11·91	15·32	997
600	4·21	10·30	5·76	13·97	16·73	998
700	4·40	10·71	6·03	16·00	18·47	998
800	4·56	11·10	6·23	17·70	20·79	997
900	4·63	11·35	6·32	18·49	22·66	995
1 000	4·74	11·60	6·42	19·54	24·71	993
1 200	4·85	11·93	6·55	20·81	27·41	990
1 400	4·96	12·17	6·69	22·20	29·06	986

DST	U1	U3	U4	G1	G2	G4	G4/G3	T1	T2	T3
1	-63.23	3.45	-59.87	-120.11	46.56	-102.19	3.57	3040.662	-32.28	3282.76
2	-24.71	2.81	-21.99	-44.13	17.54	-40.91	2.86	1300.031	-19.06	820.66
3	-16.84	2.43	-14.50	-29.08	11.71	-26.92	2.82	344.749	-12.91	364.71
4	-12.03	2.16	-9.96	-19.99	8.17	-18.98	2.65	199.547	-9.21	205.13
5	-9.39	1.95	-7.53	-15.11	6.25	-14.59	2.55	110.902	-7.59	131.27
6	-7.78	1.78	-6.09	-12.22	5.10	-11.90	2.49	78.156	-6.10	91.15
7	-6.35	1.63	-4.80	-9.64	4.07	-9.66	2.36	55.217	-5.27	66.95
8	-5.58	1.50	-4.17	-8.36	3.55	-8.39	2.34	40.239	-4.56	51.25
9	-4.70	1.39	-3.40	-6.82	2.93	-7.08	2.22	32.999	-4.00	40.49
10	-4.22	1.29	-3.01	-6.05	2.61	-6.31	2.20	24.033	-3.63	32.79
11	-3.67	1.20	-2.56	-5.14	2.23	-5.51	2.11	21.626	-3.21	27.09
12	-3.29	1.12	-2.26	-4.53	1.98	-4.94	2.07	16.001	-2.99	22.76
13	-2.96	1.04	-2.00	-4.02	1.76	-4.46	2.02	14.858	-2.69	19.38
14	-2.62	0.97	-1.74	-3.49	1.54	-4.00	1.95	11.631	-2.53	16.71
15	-2.41	0.91	-1.60	-3.20	1.41	-3.70	1.93	10.478	-2.32	14.55
16	-2.13	0.84	-1.37	-2.76	1.23	-3.33	1.85	9.015	-2.17	12.78
17	-1.98	0.79	-1.28	-2.57	1.14	-3.12	1.85	7.574	-2.04	11.32
18	-1.75	0.73	-1.11	-2.23	1.00	-2.83	1.77	7.233	-1.90	10.09
19	-1.62	0.68	-1.03	-2.07	0.92	-2.66	1.76	5.684	-1.82	9.05
20	-1.46	0.63	-0.91	-1.83	0.82	-2.45	1.70	5.850	-1.69	8.16
21	-1.33	0.59	-0.83	-1.66	0.74	-2.29	1.67	4.509	-1.63	7.40
22	-1.21	0.55	-0.75	-1.51	0.67	-2.15	1.64	4.684	-1.52	6.74
23	-1.08	0.50	-0.66	-1.33	0.59	-1.99	1.59	3.798	-1.47	6.16
24	-1.00	0.47	-0.62	-1.25	0.55	-1.90	1.58	3.697	-1.39	5.66
25	-0.87	0.43	-0.53	-1.07	0.47	-1.75	1.51	3.341	-1.34	5.21
26	-0.81	0.39	-0.51	-1.01	0.44	-1.68	1.51	2.912	-1.28	4.81
27	-0.70	0.36	-0.43	-0.87	0.38	-1.56	1.45	2.978	-1.22	4.46
28	-0.64	0.33	-0.40	-0.81	0.35	-1.49	1.44	2.349	-1.19	4.14
29	-0.56	0.30	-0.35	-0.70	0.30	-1.40	1.40	2.616	-1.12	3.86
30	-0.49	0.27	-0.31	-0.62	0.26	-1.32	1.37	1.888	-1.10	3.60

TABLE (3-4) DATA FOR HOMOGENEOUS-SHELL EARTH (MU2= 1.86E+12)

DST	U1	U3	U4	G1	G2	G4	G4/G3	T1	T2	T3
35	-0.21	0.14	-0.16	-0.32	0.11	-1.03	1.24	1.551	-0.91	2.64
40	0.02	0.03	-0.04	-0.07	-0.01	-0.81	1.11	1.246	-0.76	2.01
45	0.20	-0.05	0.06	0.12	-0.10	-0.63	0.97	0.914	-0.63	1.58
50	0.33	-0.12	0.12	0.25	-0.17	-0.51	0.86	0.598	-0.53	1.27
55	0.42	-0.17	0.16	0.33	-0.21	-0.42	0.78	0.405	-0.42	1.04
60	0.48	-0.20	0.19	0.39	-0.25	-0.35	0.71	0.314	-0.32	0.87
65	0.53	-0.22	0.22	0.44	-0.27	-0.30	0.64	0.208	-0.22	0.73
70	0.56	-0.24	0.23	0.46	-0.29	-0.26	0.60	0.046	-0.13	0.62
75	0.55	-0.24	0.23	0.46	-0.29	-0.24	0.59	-0.098	-0.04	0.54
80	0.53	-0.23	0.21	0.43	-0.28	-0.24	0.62	-0.167	0.05	0.46
85	0.50	-0.22	0.20	0.40	-0.27	-0.24	0.65	-0.206	0.13	0.40
90	0.46	-0.20	0.18	0.36	-0.25	-0.25	0.70	-0.283	0.20	0.35
95	0.41	-0.17	0.15	0.30	-0.23	-0.27	0.79	-0.383	0.27	0.31
100	0.34	-0.14	0.11	0.22	-0.19	-0.30	0.91	-0.440	0.32	0.27
105	0.26	-0.10	0.07	.14	-0.16	-0.32	1.05	-0.443	0.37	0.24
110	0.19	-0.07	0.03	0.07	-0.12	-0.36	1.18	-0.450	0.40	0.21
115	0.11	-0.03	-0.01	-0.01	-0.09	-0.40	1.34	-0.494	0.41	0.19
120	0.02	0.01	-0.05	-0.11	-0.04	-0.44	1.52	-0.530	0.41	0.17
125	-0.06	0.05	-0.10	-0.20	-0.00	-0.48	1.71	-0.512	0.40	0.15
130	-0.15	0.09	-0.14	-0.28	0.04	-0.52	1.90	-0.464	0.37	0.13
135	-0.22	0.13	-0.18	-0.36	0.07	-0.56	2.06	-0.446	0.33	0.11
140	-0.30	0.17	-0.22	-0.44	0.11	-0.60	2.24	-0.455	0.27	0.10
145	-0.37	0.20	-0.26	-0.52	0.15	-0.64	2.43	-0.429	0.20	0.08
150	-0.44	0.23	-0.29	-0.59	0.18	-0.67	2.59	-0.348	0.12	0.07
155	-0.49	0.26	-0.32	-0.64	0.20	-0.70	2.71	-0.270	0.04	0.06
160	-0.54	0.29	-0.34	-0.68	0.22	-0.72	2.82	-0.247	-0.06	0.04
165	-0.58	0.30	-0.36	-0.73	0.24	-0.74	2.92	-0.238	-0.16	0.03
170	-0.61	0.32	-0.38	-0.77	0.26	-0.76	3.02	-0.159	-0.26	0.02
175	-0.63	0.32	-0.39	-0.78	0.27	-0.77	3.3 6	-0.027	-0.35	0.01

TABLE (3-5) DATA FOR HOMOGENEOUS-SHELL EARTH (MU2= 1.86D+12)

$\gamma/(gm'/m)$ rather than the radial displacement u and the perturbation of gravity γ, where m' is the mass of the load, m is the mass of the earth and a is the radius of the earth. In addition to these variables I have used the variable $\phi/(m'/m)$ to represent tilt where ϕ is the actual tilt in milliseconds of arc. The letters U, G and T refer to non-dimensional displacement, gravity change and tilt respectively. In the description that follows, the symbols P(n) and Q(n) refer to the Legendre polynomials of degree n in ω and θ respectively where ω is the half angle of the cap load and θ is a variable denoting the angular distance to the load axis.

3-3-1. Program CAPS.

The solution obtained by Slichter and Caputo (1960) is solved in program CAPS for the case of a single cap load. The cap load is defined by the surface density:

$$f(\theta) = T/g \quad 0 < \theta \leq \omega \qquad (3\text{-}10)$$
$$= 0 \quad \text{otherwise}$$

where T is the assigned surface pressure and g is the acceleration due to gravity. This can be represented by the infinite series:

$$f(\theta) = \Sigma\, n = \Sigma T[P(n-1)-P(n+1)]Q(n)/2g + T(1-\cos\omega)/2g \qquad (3\text{-}11)$$

It will be shown later that the term n=1 of this series requires special treatment. Slichter and Caputo (1960) avoid using this term by assuming instead of a single cap load two

identical anti-podal caps. In our implementation of their method we assume a single cap load but omit the term n=1 from the computation. We calculate the effect of this term separately by a method described in Appendix IX and add it to the results determined for the other terms. The non-dimensional displacement due to this term is denoted by U5 in program CAPS. It is shown in the appendix to be, for a solid homogeneous earth:

$$U5 = -(3g^2/(3\gamma+2))(\cos\theta/8G\pi\mu) \qquad (3\text{-}12)$$

where $\gamma = \lambda/\mu$. (The displacement for a homogeneous shell was determined also by this technique but was found to differ only slightly from the homogeneous earth case).

The elastic equilibrium considered by Slichter and Caputo is that of a spherical shell with the applied stress independent of longitude and the elastic parameters λ and μ constant. A solution in the form:

$$u = \Sigma R(r,n)\ Q(n) \qquad (3\text{-}13)$$

is assumed, where $R(r,n)$ is determined by their Eqns. (21) and (22) in terms of the elastic parameters of the shell and the boundary conditions. The conditions appropiate to a fluid core are assumed on the inner boundary. On the outer boundary a surface-load function $S(n,\omega)$ is assumed which, for the single-cap load considered here, becomes:

$$S(n,\omega) = -rT(P(n-1)-P(n+1))/2g \qquad (n>0)$$

$$= -rT(1-\cos\omega)/2g \qquad (n=0)$$

After determining $R(r,n)$ the radial displacement U1 is determined using Eqn. (3-13), assuming ω=1 degree and summing

to n=200. There is in addition to this yielding due to the surface force an uplift due to the body force exerted by the load. Love (1927, Section 174) gives for this uplift on an homogeneous earth:

$$\delta Un = Vn\rho n[\, n((n+2)\lambda+(n+1)\mu)a^2-(n-1)((n+1)\lambda+n\mu)r^2\,] \\ /[\,2(n-1)\mu((2n^2+4n+3)\lambda+2(n^2+n+1)\lambda)\,] \qquad (3\text{-}14)$$

where ρ is the density of the homogeneous earth and Vn is the gravitational potential due to the loading mass. At a point on the surface Vn is given by:

$$Vn = (\Delta n)4\pi Ga/(2n+1) \qquad (3\text{-}15)$$

where a is the radius of the Earth. This uplift has been computed by CAPS for a mean density of 4.47 and is tabulated as U3 in the Tables. The total radial displacement is taken as the algebraic sum of yielding and uplift and is shown as U4 in the tables. This includes too a small constant term Uc due to the displacement when n=0 and the term U5 described above.

There are three tilting effects to consider. The principal one T1 is due to the radial displacement of the surface. This is the surface tilt and is found by differentiating the expression for the total radial displacement with respect to $a\theta$. The next largest tilting effect is called T3 and is due to the horizontal component of the gravitational attraction of the load; that referred to as A in Eqn. (3-7). This has the effect of deflecting the vertical which, with the pendulum-type tiltmeters used here, is indistinguisable from a tilt of the surface. In the

non-dimensional form it has the simple form:

$$T3 = [\cos(\theta/2)]/[4\sin^2(\theta/2)] \qquad (3\text{-}16)$$

Finally there is a negative attraction due to the masses of the earth which have been displaced; the attraction denoted by $-\epsilon A$ in Eqn. (3-7) and which will be called T2 here. Nishimura (1950) assumed as a first approximation for this attraction the constant ratio -.5A. We obtain an expression for this factor by differentiating the gravitational potential of the radial displacement, Eqn. (3-19) below. The result expressed as T2/T3 has been computed as a function of distance assuming a density for the displaced masses of 2.8 and is shown in Fig. (3-5) for two values of rigidity. Also shown in this figure is the ratio T2/T3 determined by the Longman method discussed in Section (3-6) below.

There are also three parts in the perturbation of gravity. The principal part is called G1 and is due to simple radial displacement in the Earth's exterior gravitational field. The part due to the vertical component of the gravitational attraction of the load is:

$$G3 = -1/4\sin(\theta/2) \qquad (3\text{-}17)$$

G2 is the vertical attraction due to the displaced masses. To estimate it we proceed as follows: We assume the Earth is incompressible and that the displacement of the surface is given by U4, determined above, which can be represented as a series of spherical harmonics in the form:

$$U4 = \Sigma u\ P\ (\cos\theta) \qquad (3\text{-}18)$$

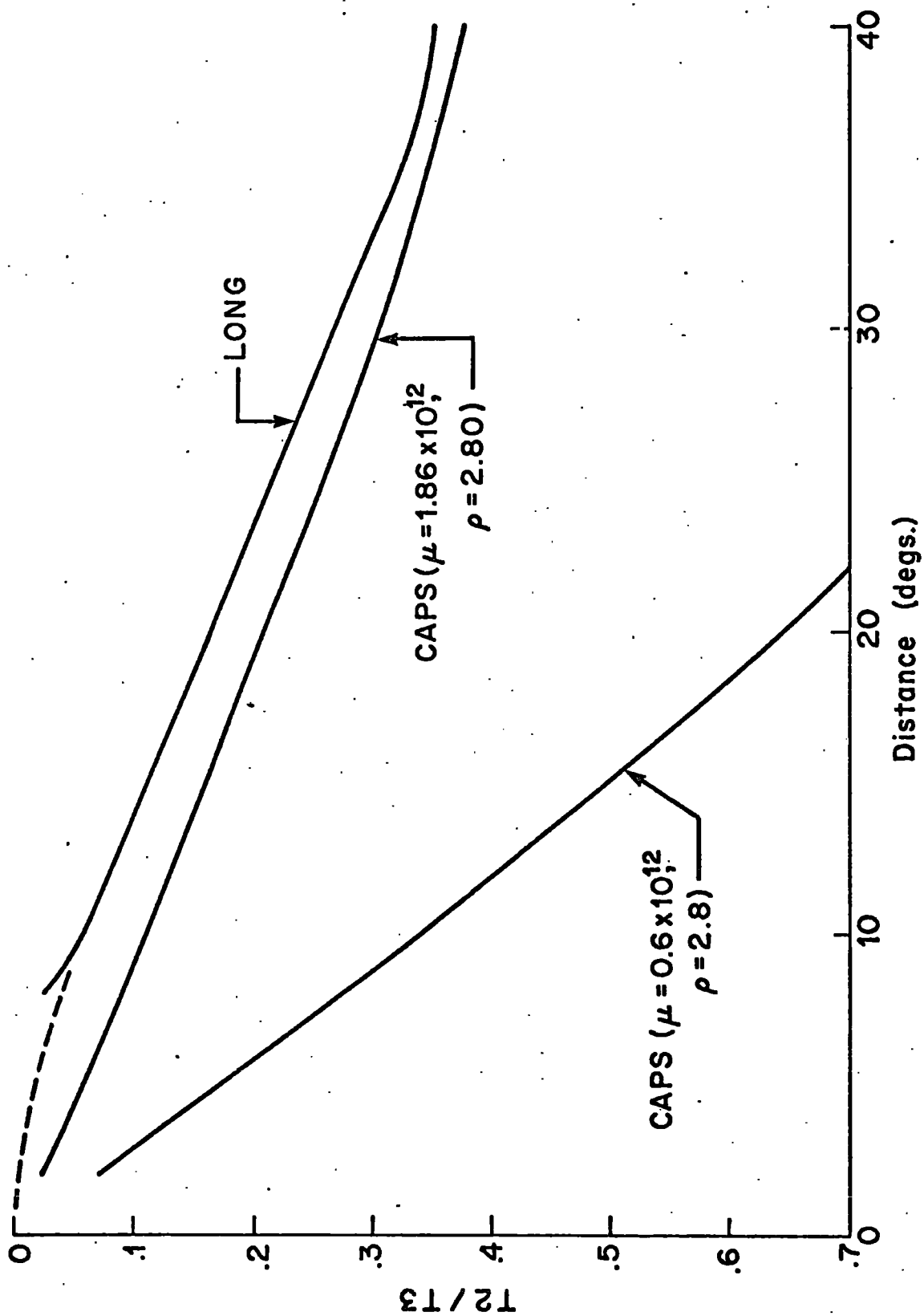

FIGURE (3-5) The function T2/T3 for three models.

The gravitational potential due to this redistribution of mass is approximately:

$$V = \Sigma (4\pi Ga)(a/r)(\rho u\ P_n(\cos\theta)) \qquad (3\text{-}19)$$

This is the exterior potential of a sphere coated with the mass density $s=u\rho$ where ρ is the density assumed for the displaced masses. The vertical component of attraction at the surface then is:

$$G2 = \Sigma (n+1)(4\pi G\rho u\ P_n(\cos\theta))/(2n+1) \qquad (3\text{-}20)$$

In this estimate we assume for the density the mean crustal value of 2.8. Note that the term n=0 is not included in Eqn. (3-20) since it would not be present on an incompressible Earth.

The total gravity perturbation G4 is the sum of G1, G2 and G3.

3-3-2 Discussion.

The displacement determined by program CAPS may be compared to that determined by Eqn. (3-1) for the infinite half space. As the distance is made small the radial displacement U1 should approach and merge with the displacement determined by Eqn. (3-1). This result is demonstrated in Fig. (3-6) where displacement has been plotted for the two cases for a coefficient of rigidity of 0.6×10^{12} dynes/cm^2. The displacement is less for the spherical model and less still when the uplift due to body forces (U3) is added to U1.

The attraction tilt T3 is of course identical in the

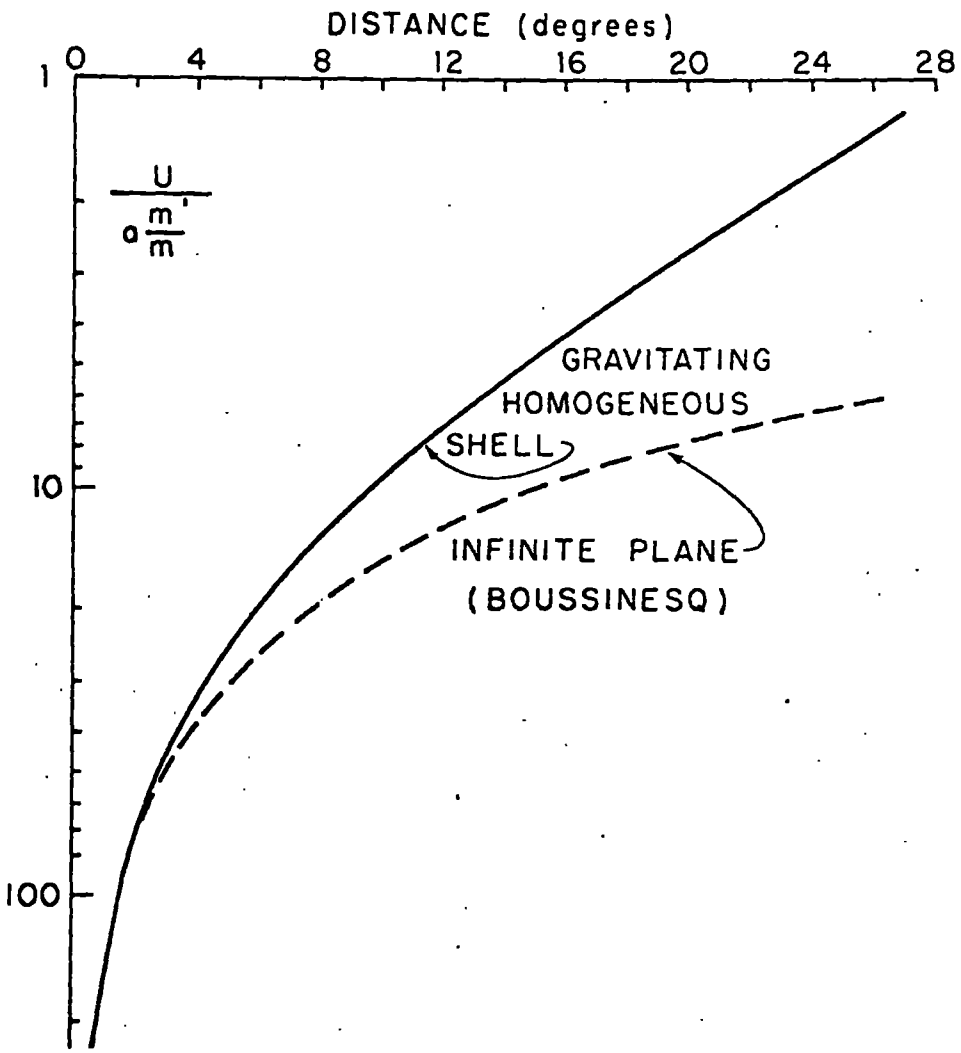

FIGURE (3-6) Comparison of elastic depression on Boussinesq and CAPS models.

two cases. The tilt T2 is too small to be significant at distances where the plane-Earth model is applicable.

3-4 The Effect On A Gutenberg-Model Earth.

3-4-1 Program LONG.

Longman (1962,1963) has derived the equations of equilibrium for an Earth model under the stress of a surface load when the three elastic parameters λ, μ and ρ are arbitrary functions of the depth. He has solved these equations for the case where the elastic parameters take on the values specified for the Gutenberg-model Earth (Alterman et al, 1961). Program LONG is an implementation based on these solutions but including additions and modifications which will be described later.

For the vertical displacement of the surface, the horizontal attraction (tilt) of the loading mass and of the displaced masses, and the change in the gravity field at the deformed surface Longman obtains the solutions:

$$Un = H'n\ Pn(\cos\theta) \qquad (3\text{-}21)$$

$$Tn = (1+K'n)P'n(\cos\theta) \qquad (3\text{-}22)$$

$$Gn = K'n[n+2H'n-(n+1)\ \ ]Pn(\cos\theta) \qquad (3\text{-}23)$$

where $Pn(\cos\theta)$ is the Legendre polynomial, $P'n(\cos\theta)$ is its derivative with respect to θ; U_n, Gn and Tn are the non-dimensional forms introduced earlier for the displacement, tilt and gravity perturbation respectively. H'n and K'n are the load-deformation coefficients (Munk and Macdonald, 1960) which Longman determines for n up to 40 from the equations of equilibrium.

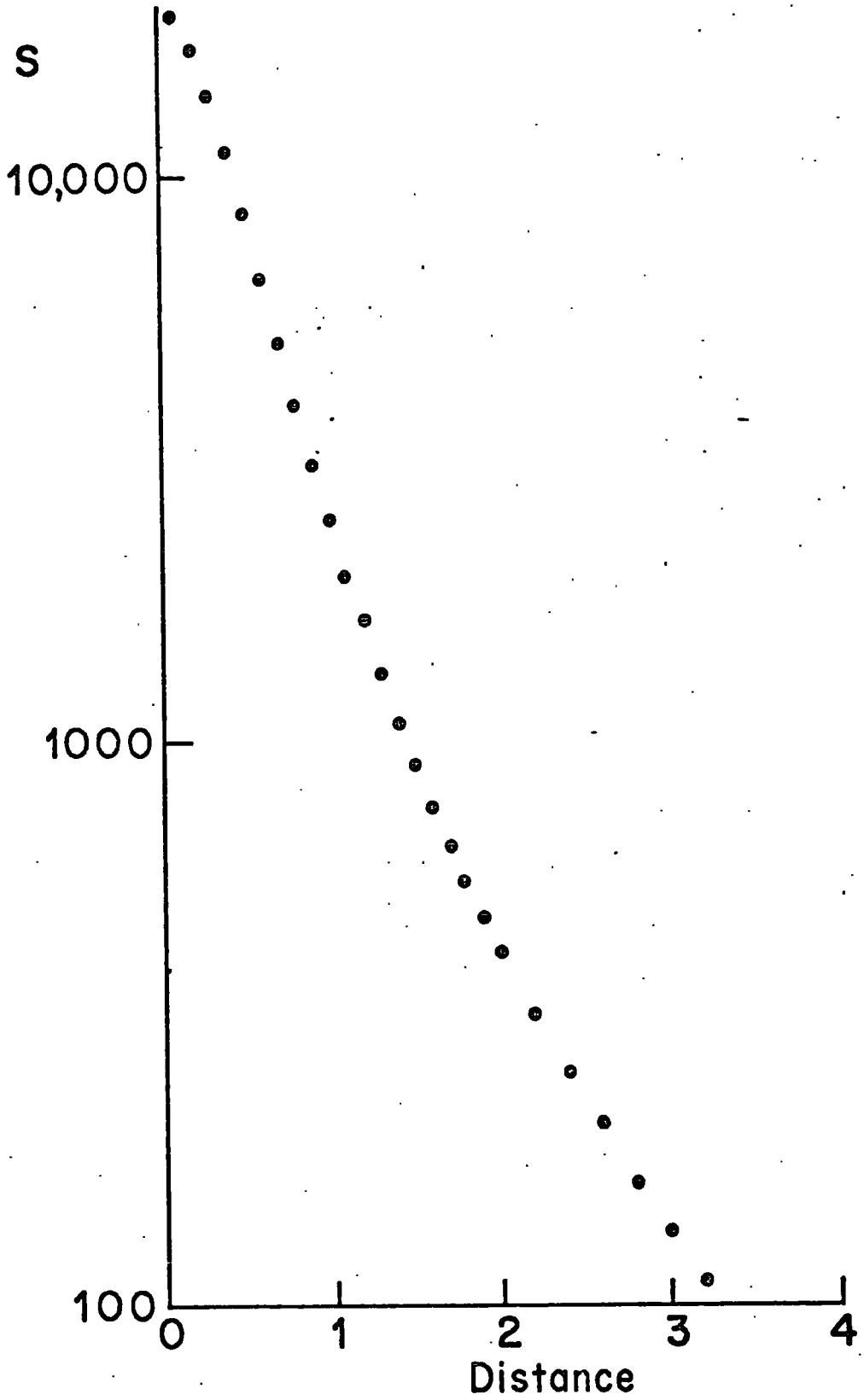

FIGURE (3-7) The surface-load function for p = 0.99 (from Eqn. 3-24).

The numerically simple form of the equations is a consequence of the definition of the load deformation coefficients and the choice of a particularly convenient point-mass surface load equal to the mass of the Earth. The solution may be considered as a Green's function from which the response due to arbitrary surface loads can be obtained by convolution. The surface load assumed has the Legendre expansion:

$$S = (g/4\pi G) \lim \Sigma (2n+1) \quad Pn(\cos\theta) \qquad (3\text{-}24)$$

and the interior potential:

$$Vn = ga(r/a) \qquad (3\text{-}25)$$

For the point-mass load assumed summations of the terms represented by Eqns. (3-21,23,24) diverge but the Poisson series obtained by multiplying each term by p^n does converge to the right value for $p<1$. This has the effect however of broadening the representation of the load so that the assumed point mass may be distributed over several degrees. The expansion representing the load, Eqn. (3-24), is shown for $p=0.99$ in Fig. (3-7). Longman finds for summation to $n=40$ (the maximum for which he has determined n) that $p=0.85$ is the maximum value which can be used. Even for this value of p the effect of terms with $n>40$ becomes large when θ is less than about 30^0 and consequently results within this limit are not correct. To permit p to approach as close as possible to 1.0 and computations to be made as close as possible to the load Longman suggests summing the series for very large values of n; using the asymptotic behaviour of

H'n and K'n as n tends to infinity to determine H'n and K'n. This is the procedure we have followed here to determine the displacement and gravity perturbation. Fig. (3-8) is a plot on a semi-logarithmic scale of the values given by Longman for H'n and K'n. The asymptotes for each of the coefficients are given by:

$$H'n = -0.86 \log n + 0.58 \qquad (3\text{-}26)$$

$$K'n = 0.025 \log n - 0.127 \qquad (3\text{-}27)$$

Using this procedure, summing to n=1000 and putting p=0.99, produces reasonable looking results to within one degree of the load while for greater distances it reproduces closely the results obtained by Longman.

Longman omits the term n=1 in the summation for total displacement and gravity perturbation because he considers the load-deformation coefficients for this term to imply displacement of the earth as a whole. This appears to be incorrect; it is shown in Appendix IX that mass loading represented by this term results in an equilibrating system of body and surface forces and significant elastic deformation. The term is neglected too by others: Munk and MacDonald (1960, p 31) find, that the opposing effects of yield due to surface force and uplift due to body force cancel for this term with the net result that the term is equivalent to a rigid body displacement of the earth with no effect on surface gravity. This result is true for the incompressible case treated by them but it is not generally true (see Eqn. (3-12)). An erroneous interpretation of the

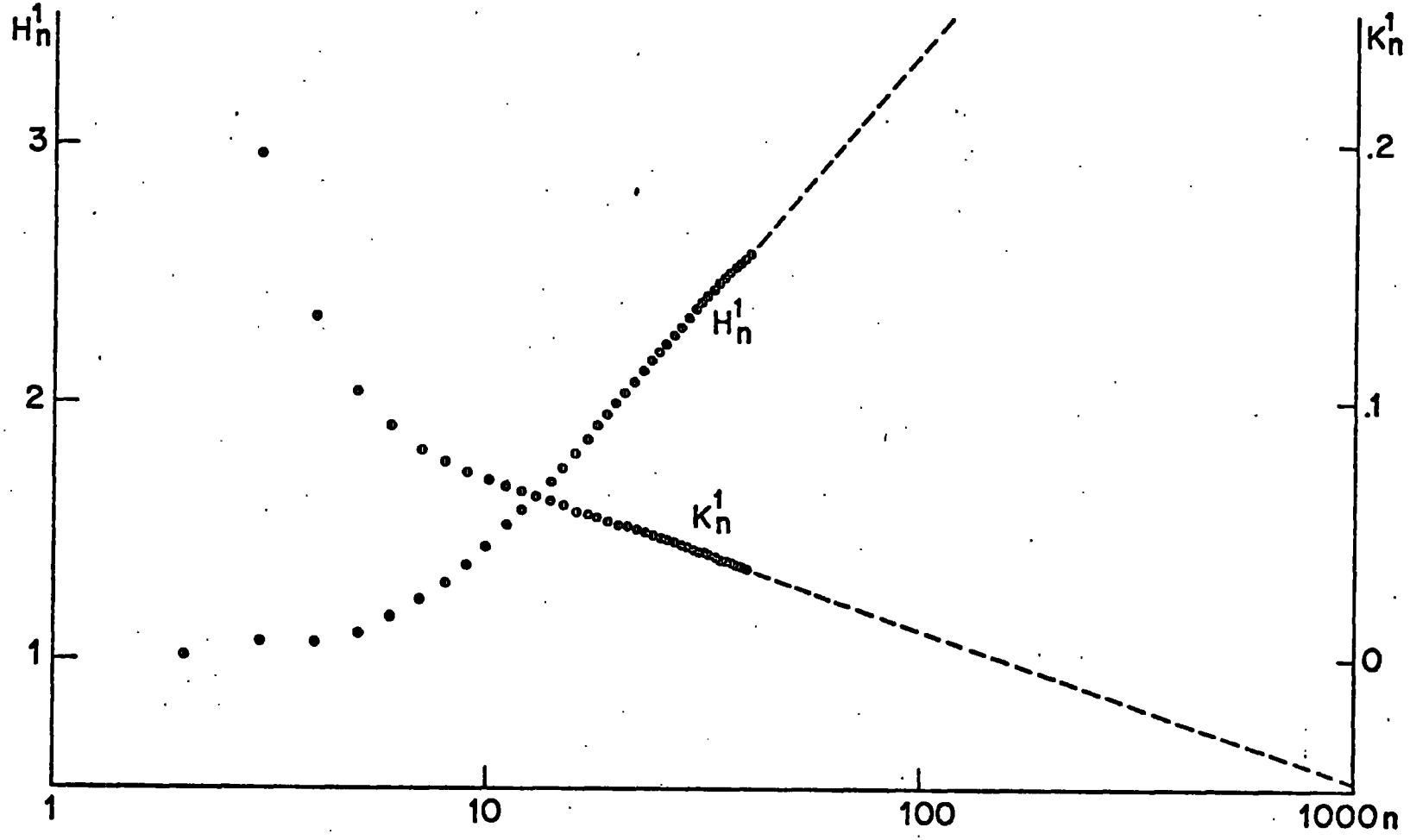

FIGURE (3-8) The load-deformation coefficients H^1n and K^1n showing the extrapolation used for n>40.

effect of the n=1 term might be made also from Jefferies' discussion of The Straining of a Sphere (1962, Appendix B) in which he deals separately with the effects of surface and body forces. The term n=1 in the series representation of either of these forces taken separately does imply displacement of the earth as a whole because static equilibrium is impossible. This is reflected in the appearance of the factor n=1 in the denominator of the expressions derived by Jeffreys.

The effect of omitting the n=1 term in the series representation of the cap load is to distribute a loading mass over the entire hemisphere opposite the cap (and a negative mass over the hemisphere on the same side as the cap).

The effect of the term n=1 on surface displacement is calculated in Appendix IX and added to the results determined below using Longman's method for the terms n≠1. The effect of the same term on surface gravity is calculated according to the method described in Section 3-3-1 and added to the Longman result. The tilt effect due to this term is negligible because of its large wavelength and relatively small amplitude.

To obtain the total Green's function for tilt the elastic tilt at the surface must be added to the solution given by Eqn. (3-22) which includes only the tilt due to attraction effects. This is done in program LONG by locally fitting the tabulated results for displacement to a

third-order polynomial and differentiating this approximating function.

3-5 Discussion.

The principal results obtained using programs CAPS and LONG are presented in Tables 3-4,7. Consider first the non-dimensional displacement determined for the distance range from 30 degrees to 180 degrees. The LONG results for this range and the CAPS results for rigidities of 1.2×10^{12} and 1.86×10^{12} and the same range are plotted in Fig. (3-9). This demonstrates that the displacement of a Gutenberg-model Earth due to surface loading by a point mass is approximately the same as that of an Earth consisting of a single homogeneous shell enclosing a fluid core; at least for loading at distances greater than 30 degrees. This corresponds fairly well with the figure of 1.7×10^{12} found for the 'mean' rigidity of the Earth's mantle to a depth of 2900 kilometers when Bullen's density law is assumed. (Melchior, p324, 1965).

The total gravity perturbation for the same three cases is shown in Fig. (3-10). The agreement between the LONG and CAPS results is not as good here. There is a systematic difference which is apparently due to the assumption made in the CAPS implementation that the displaced masses are simply redistributed on the surface.

3-6 Effects Within 30° Of The Load.

The results for the distance range 0-60° are shown in greater detail in Figs. (3-11,12). The total displacement U4

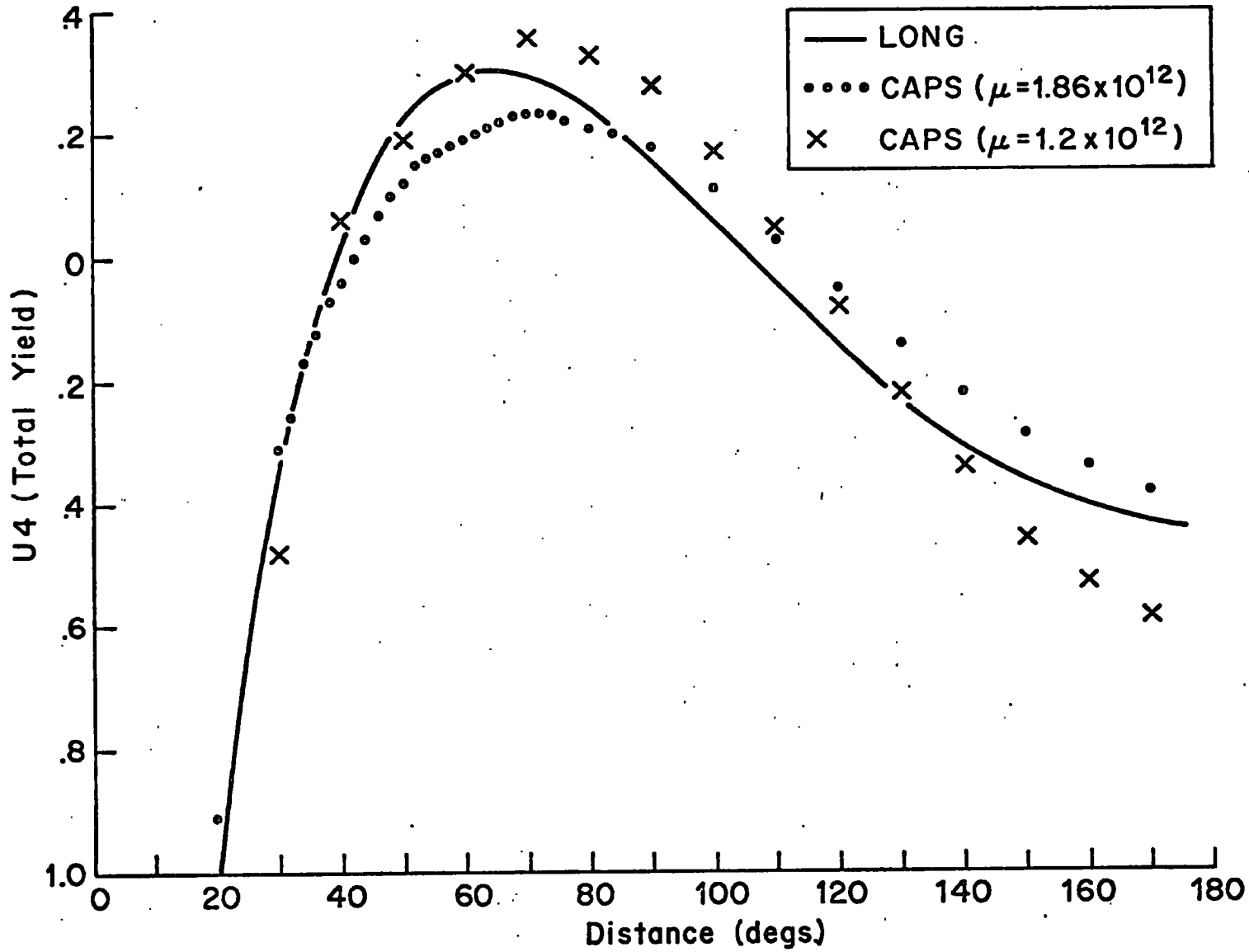

FIGURE (3-9) Comparison of elastic depression for 3 models.

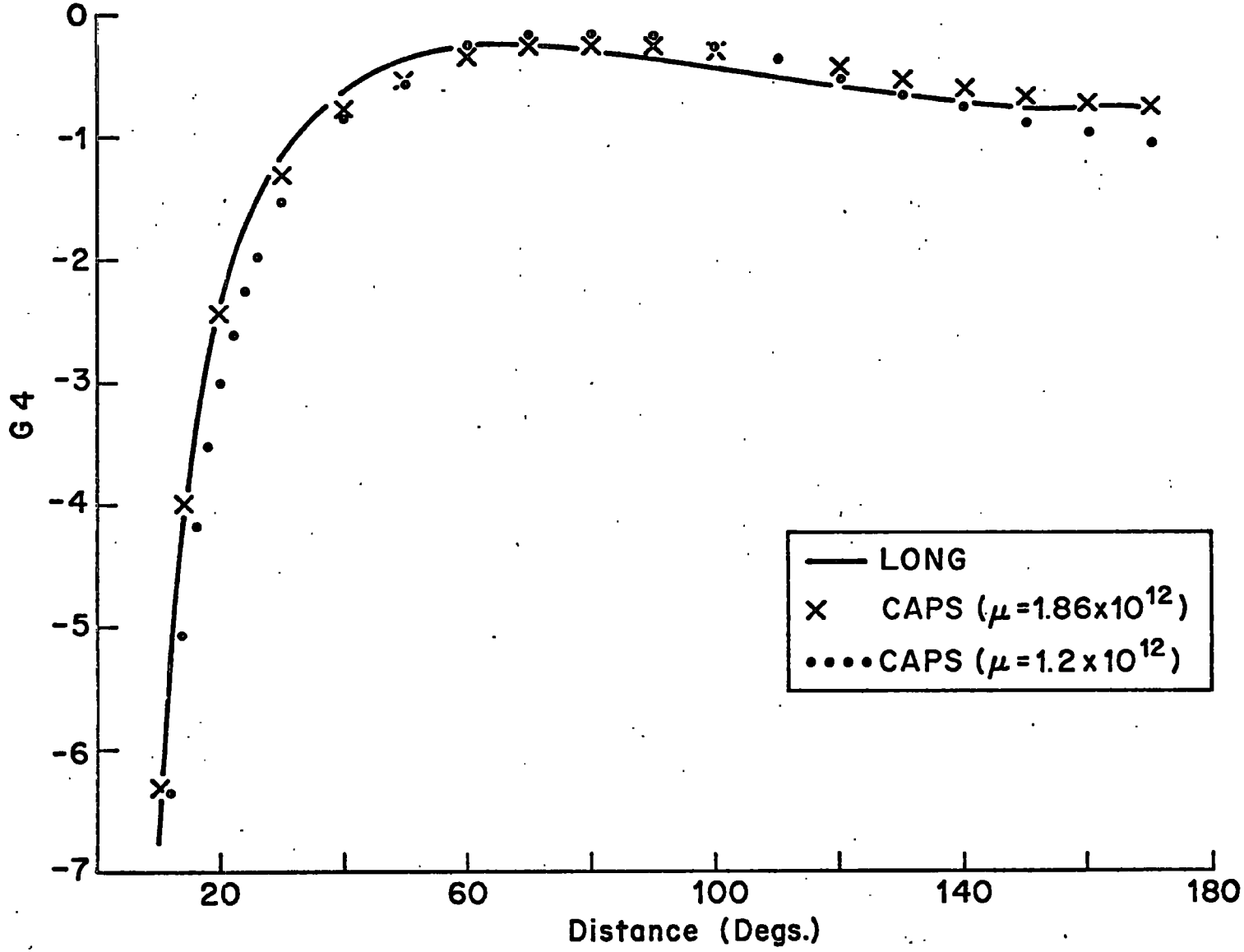

FIGURE (3-10) Comparison of gravity perturbation for 3 models.

DIST	U4	T1	T2	T3	T4	T4/T3	G4	G4/G3
1.0	-105.752	-5970.47	39.317	3282.765	-9213.92	2.807	-177.818	6.207
2.0	-41.945	-1926.74	28.577	-820.660	-2718.82	3.313	-72.421	5.056
3.0	-22.679	-604.42	18.530	-364.714	-950.60	2.606	-42.426	4.442
4.0	-14.132	-325.56	12.133	-205.134	-518.56	2.528	-27.303	3.812
5.0	-9.814	-183.48	8.853	-131.271	-305.90	2.330	-19.811	3.457
6.0	-7.193	-115.23	6.734	-91.147	-199.65	2.190	-14.664	3.070
7.0	-5.590	-74.89	5.422	-66.954	-136.42	2.037	-11.714	2.860
8.0	-4.469	-52.64	4.466	-51.252	-99.43	1.940	-9.416	2.627
9.0	-3.704	-37.26	3.803	-40.487	-73.95	1.826	-7.966	2.500
10.0	-3.134	-28.05	3.265	-32.786	-57.58	1.756	-6.787	2.366
11.0	-2.709	-21.17	2.862	-27.089	-45.40	1.676	-5.935	2.275
12.0	-2.381	-16.90	2.527	-22.755	-37.13	1.632	-5.242	2.192
13.0	-2.111	-13.78	2.275	-19.383	-30.89	1.594	-4.653	2.107
14.0	-1.893	-11.76	2.078	-16.707	-26.39	1.580	-4.187	2.041
15.0	-1.697	-10.37	1.931	-14.548	-22.99	1.580	-3.746	1.956
16.0	-1.530	-9.23	1.816	-12.782	-20.20	1.580	-3.410	1.899
17.0	-1.373	-8.38	1.719	-11.317	-17.98	1.589	-3.083	1.823
18.0	-1.238	-7.39	1.633	-10.090	-15.85	1.571	-2.835	1.774
19.0	-1.115	-6.58	1.545	-9.052	-14.09	1.557	-2.597	1.714
20.0	-1.008	-5.69	1.465	-8.165	-12.39	1.517	-2.404	1.670
21.0	-0.915	-5.04	1.386	-7.402	-11.05	1.493	-2.224	1.621
22.0	-0.831	-4.48	1.320	-6.740	-9.90	1.469	-2.061	1.573
23.0	-0.757	-4.16	1.263	-6.163	-9.06	1.469	-1.918	1.529
24.0	-0.686	-3.96	1.218	-5.657	-8.40	1.485	-1.774	1.476
25.0	-0.619	-3.83	1.180	-5.210	-7.86	1.509	-1.656	1.434
26.0	-0.552	-3.73	1.145	-4.814	-7.40	1.537	-1.534	1.380
27.0	-0.489	-3.51	1.109	-4.461	-6.86	1.538	-1.434	1.339
28.0	-0.429	-3.27	1.070	-4.145	-6.35	1.532	-1.331	1.288
29.0	-0.375	-2.93	1.034	-3.861	-5.76	1.492	-1.244	1.246
30.0	-0.327	-2.64	0.989	-3.605	-5.26	1.458	-1.160	1.201

TABLE (3-6) Data for Gutenberg-model Earth

DIST	U4	T1	T2	T3	T4	T4/T3	G4	G4/G3
35.0	-0.130	-2.27	0.871	-2.637	-4.03	1.530	-0.848	1.021
40.0	0.031	-1.50	0.728	-2.008	-2.78	1.383	-0.623	0.853
45.0	0.141	-1.06	0.606	-1.577	-2.04	1.290	-0.469	0.718
50.0	0.220	-0.74	0.494	-1.269	-1.51	1.194	-0.363	0.613
55.0	0.271	-0.40	0.386	-1.040	-1.06	1.017	-0.297	0.549
60.0	0.294	-0.18	0.292	-0.866	-0.76	0.873	-0.260	0.520
65.0	0.303	0.02	0.203	-0.730	-0.51	0.700	-0.252	0.541
70.0	0.291	0.21	0.121	-0.622	-0.29	0.469	-0.255	0.585
75.0	0.269	0.32	0.045	-0.535	-0.17	0.321	-0.278	0.677
80.0	0.226	0.42	-0.015	-0.464	-0.06	0.134	-0.290	0.745
85.0	0.197	0.49	-0.064	-0.404	0.02	-0.058	-0.331	0.895
90.0	0.151	0.54	-0.109	-0.354	0.07	-0.208	-0.365	1.033
95.0	0.104	0.57	-0.148	-0.311	0.11	-0.364	-0.405	1.195
100.0	0.052	0.59	-0.174	-0.274	0.14	-0.506	-0.436	1.336
105.0	0.002	0.56	-0.189	-0.242	0.13	-0.552	-0.477	1.515
110.0	-0.047	0.58	-0.202	-0.214	0.16	-0.762	-0.514	1.683
115.0	-0.098	0.57	-0.211	-0.189	0.17	-0.913	-0.556	1.876
120.0	-0.146	0.53	-0.211	-0.167	0.15	-0.888	-0.587	2.033
125.0	-0.190	0.50	-0.205	-0.147	0.15	-0.999	-0.624	2.213
130.0	-0.232	0.45	-0.2#1	-0.129	0.12	-0.967	-0.657	2.381
135.0	-0.270	0.43	-0.193	-0.112	0.12	-1.079	-0.681	2.517
140.0	-0.307	0.41	-0.178	-0.097	0.14	-1.424	-0.704	2.646
145.0	-0.341	0.34	-0.158	-0.083	0.10	-1.172	-0.733	2.795
150.0	-0.366	0.29	-0.139	-0.069	0.08	-1.175	-0.751	2.900
155.0	-0.391	0.24	-0.120	-0.057	0.06	-1.070	-0.771	3.011
160.0	-0.408	0.20	-0.097	-0.045	0.06	-1.268	-0.776	3.058
165.0	-0.426	0.16	-0.073	-0.033	0.05	-1.506	-0.797	3.161
170.0	-0.435	0.07	-0.052	-0.022	0.00	-0.014	-0.798	3.179
175.0	-0.444	0.19	-0.020	-0.3 11	0.16	-14.217	-0.813	3.249

TABLE (3-7) Data for Gutenberg-model Earth

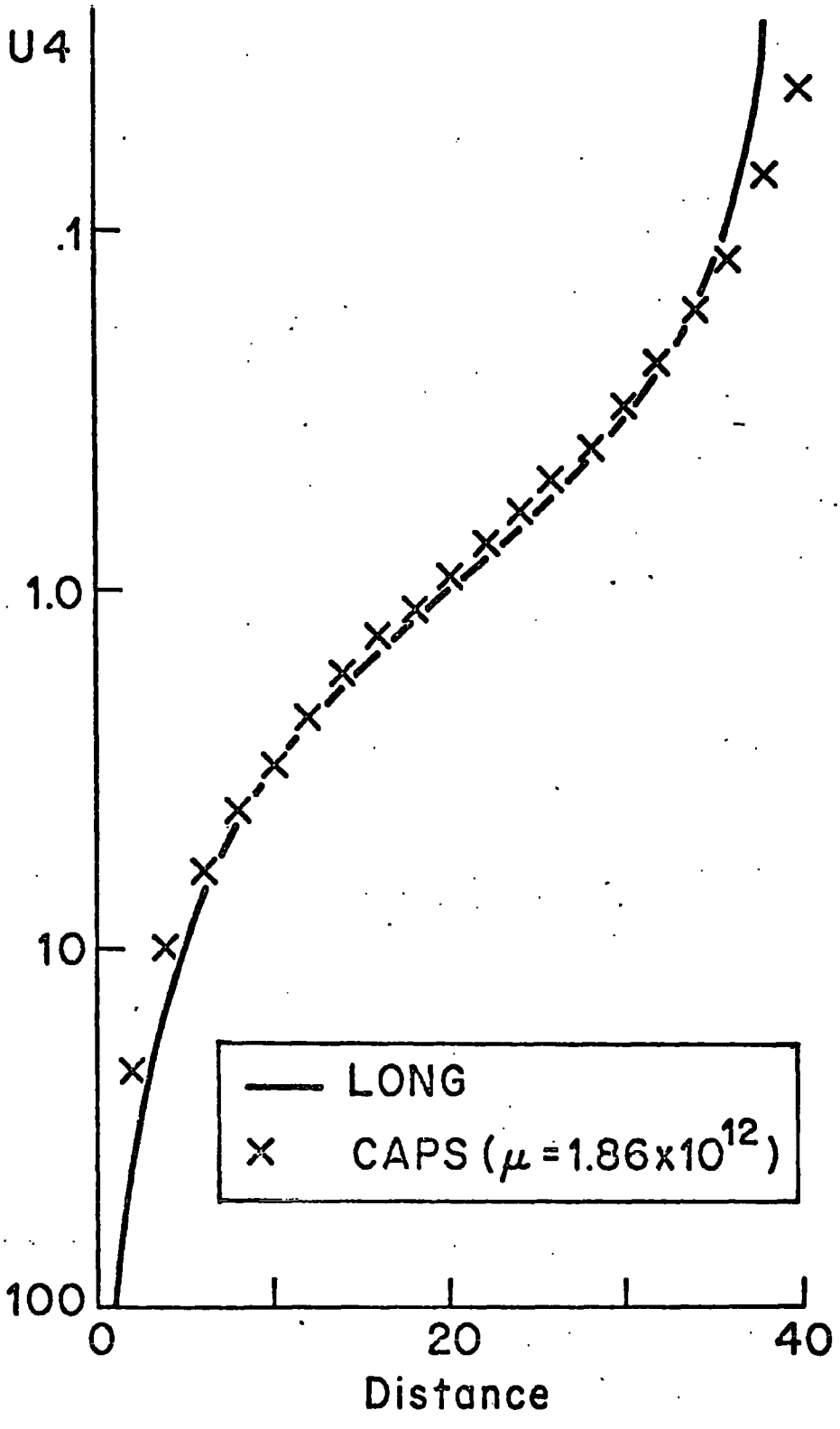

FIGURE (3-11) Elastic depression near the load- results for CAPS and LONG.

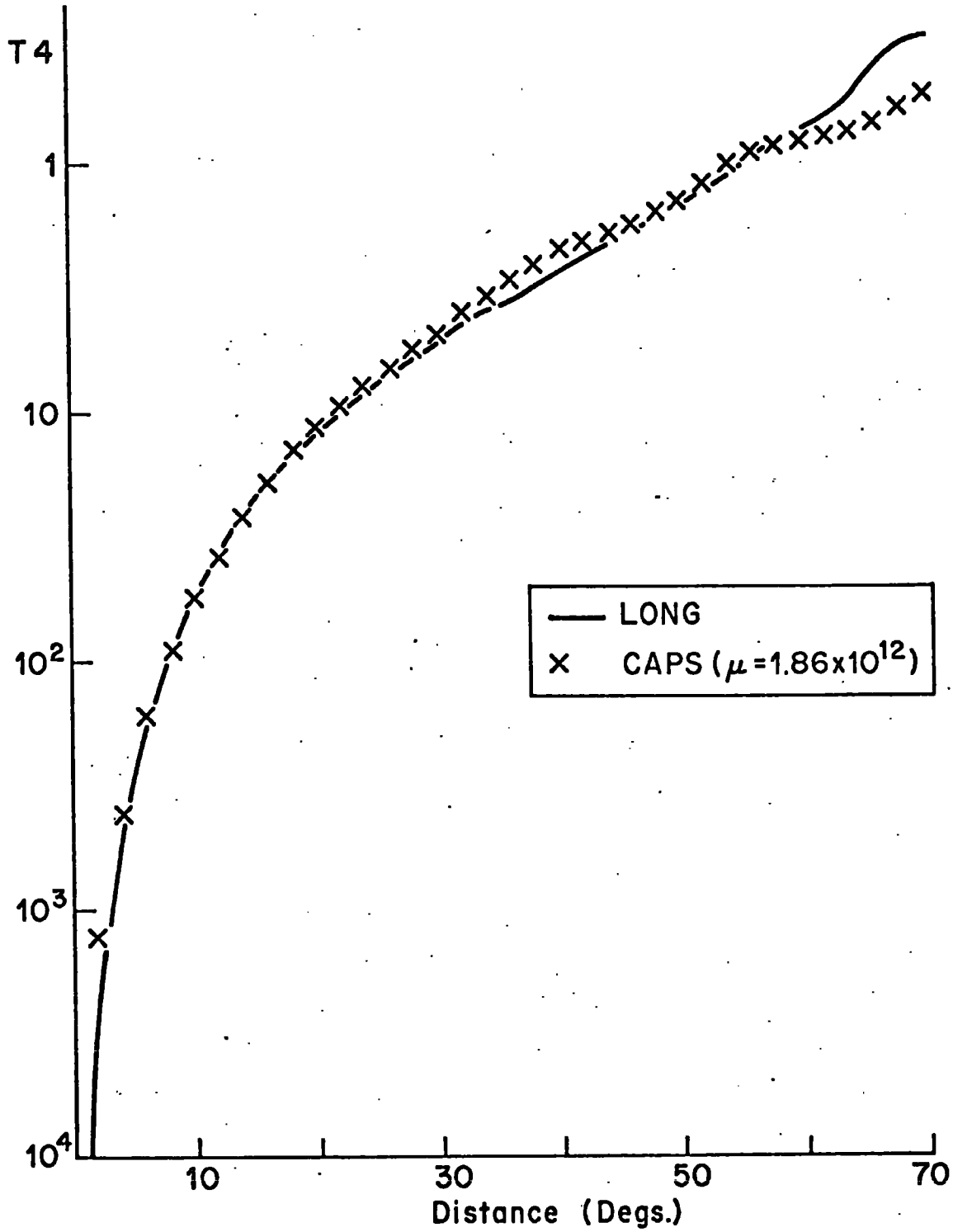

FIGURE (3-12) Comparison of total tilts near the load-results for CAPS and LONG.

and the total tilt T4 vary rapidly in this range and are reproduced here on a logarithmic scale. The agreement between LONG and CAPS results is very close down to within a few degrees of the load. We have previously seen that the elastic yield and tilt determined by CAPS agrees with that determined on the basis of the Boussinesq solution very close to the load. This result indicates that the LONG result too is valid in this range. We will look at this data in more detail later but before we do it is useful to consider the Burmister problem.

3-6-1 The Burmister Problem.

Burmister (1956) has treated the case of a point mass loading the surface of an elastic layer of thickness h, underlain by a rigid half space. We have reproduced in Table 3-2 numerical results presented by Poulos (1967) for the displacement of the first surface assuming several different values for Poisson's ratio. The 'influence factor' used in the tables is defined by the equation:

$$I = \rho 2\pi hE/P \qquad (3\text{-}28)$$

where I is the influence factor, ρ is the vertical displacement of the upper surface, h is the thickness of the upper layer, E is Young's Modulus and P is the force exerted by the point load at r=0 .

It can be seen that the displacement falls to zero for $r/h > 1.5$ (Table 3-2). Similar results are found for the stress exerted on the surface of the underlying layer (Poulos Table 1, 1967); the stress is zero for radii greater

than 1.5 h. The present problem is to calculate the surface deformation due to point-mass loading of a shallow layer of low mean rigidity underlain by a non-rigid half space. We want to determine the distance beyond which the effect of the first layer is negligible. If we assume that the underlying layer does not yield, then Burmister's results indicate that this distance is 1.5 h. The actual yielding of the underlying layer can be calculated using the solution given in the previous section for the infinite half space, assuming for the load a mass distribution determined by the stress distribution given by the Burmister result. We can approximate this load by a square surface mass of side 2 h. It can be shown that the horizontal attraction of this mass distribution beyond a distance of 3 h is approximately the same as a point distribution of equal mass. Thus, this is the distance beyond which the effect of layers to a depth of h is negligible.

In Table 3-3 I have reproduced part of the table of physical properties presented by Alterman et al (1961) for the Gutenberg Earth Model. It will be seen that the rigidity of layers deeper than 38 kilometers is uniformly near 0.7×10^{12} throughout the first 300 kilometers. From the Burmister results the influence of layers less than 38 kilometers in depth is negligible at distances greater than 114 kilometers, or approximately one degree. Thus the minimum effective rigidity for distances from the load greater than one degree would be expected to be about 0.7×10^{12}.

3-6-2 Comparision With Other Results.

We have seen that the results determined by program LONG become increasingly uncertain as the distance to the load is decreased. The extrapolation method described in Section 3-4-1 produces well-behaved results to within one degree of the load but it remains to be shown that these extrapolated results are accurate. In Fig. (3-13) I have reproduced the ratio T4/T3 from both the LONG and CAPS results for the distance range 0-28°. The same ratio has been determined for the infinite half-space Earth, computed from the Boussinesq solution assuming the elastic constants shown in the figure. The LONG results appear reasonable; a straight line extrapolation from the LONG data intersects the zero-distance axis near 3.5 which, as pointed out in the last section, " is the largest value expected for distances greater than 1 degree. The CAPS results are determined on the basis of a homogeneous shell of rigidity 1.86×10^{12} (dynes/cm^2) which is much too hard to be representative of that part of the mantle concerned when the site is within a few degrees of the load. As expected therefore the CAPS ratios appear too small.

Until recently the only additional evidence on the response of the Earth to surface loads in the distance range 0-10 was the empirical determinations of Nishimura (1950) and the theoretical results of Takeuchi (1950) who considered the problem of surface loading on a semi-infinite elastic body with elastic constants which were a particular

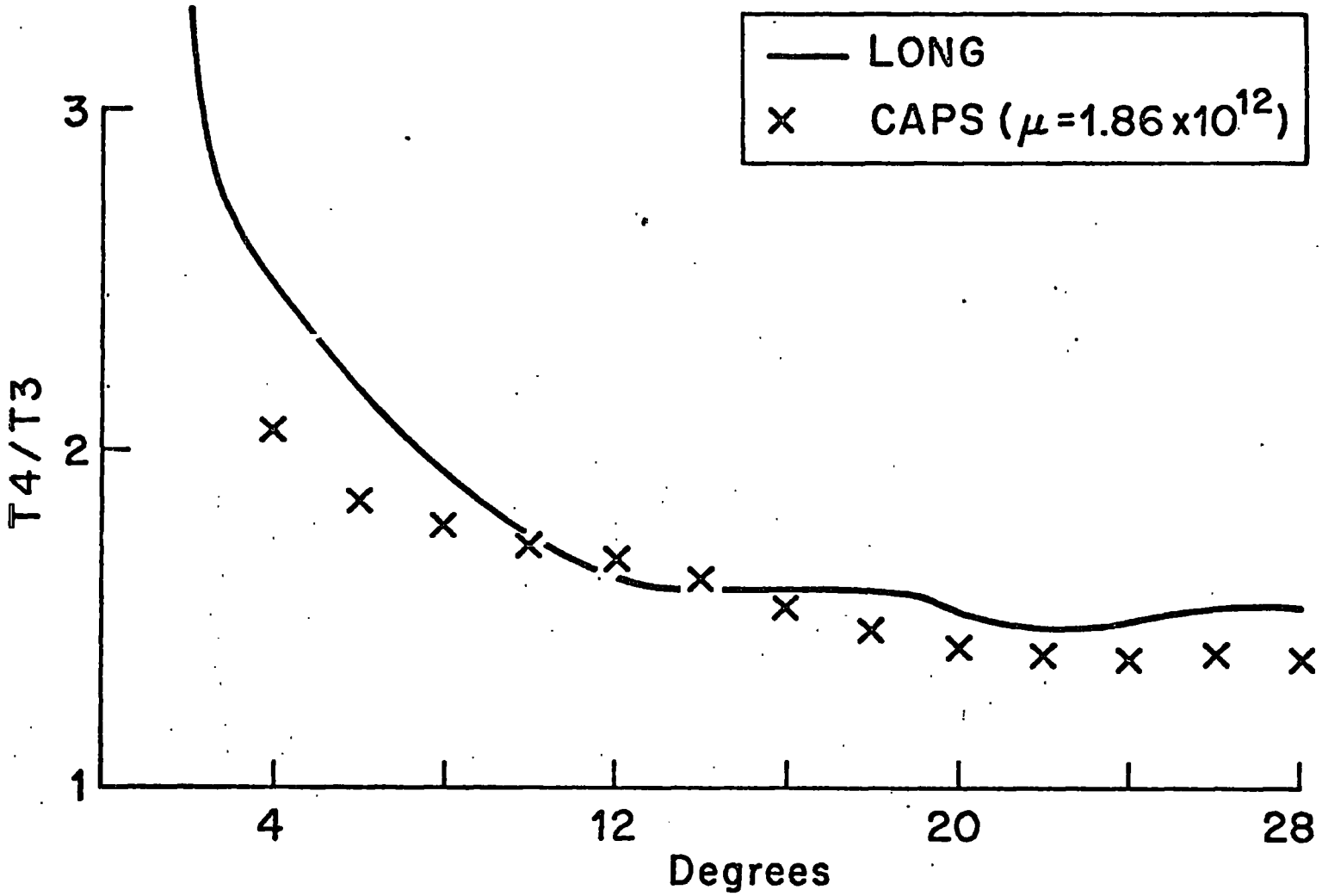

FIGURE (3-13) Comparison of tilt ratios near the load-results for CAPS and LONG. (The ratio T4/T3 is used in the analysis as a Green's function for determining tilt).

exponential function of depth. Recently however the finite-element method (Zienkiewicz, 1967) has been applied with great success to the general problem and preliminary results have been obtained by Beaumont (1971) for the surface displacement near a load on a Gutenberg earth. Beaumont's results are shown in Fig. (3-14) together with results determined by CAPS and LONG. From the displacement we have computed the tilt ratio T4/T3 and this is shown in Fig. (3-15) together with tilt ratios determined by Nishimura and by Takeuchi. The tilt ratios shown previously for LONG are reproduced here also. The gravity perturbation ratio G4/G1 is shown for the 0 -30° region of the LONG result in Fig. (3-16).

It can be seen that the LONG and Beaumont results for displacement agree fairly well throughout the range 1°-14° shown in Fig. (3-14). The Beaumont results would be expected to fall below (show a smaller displacement than) those of LONG where the effect of uplift, which is not considered by Beaumont, becomes important. This uplift is not determined separately in the LONG implementation since the load deformation coefficient contains both the uplift effect and the elastic yield. It is determined separately in program CAPS although subject to the assumption that the Earth is homogeneous and of mean rigidity 4.47 (see Eqn. 3-14). This is tabulated as U3 in the tables. The displacements shown for Beaumont's results in Fig. (3-14) should be decreased for the uplift effect which is approximated by U3. The

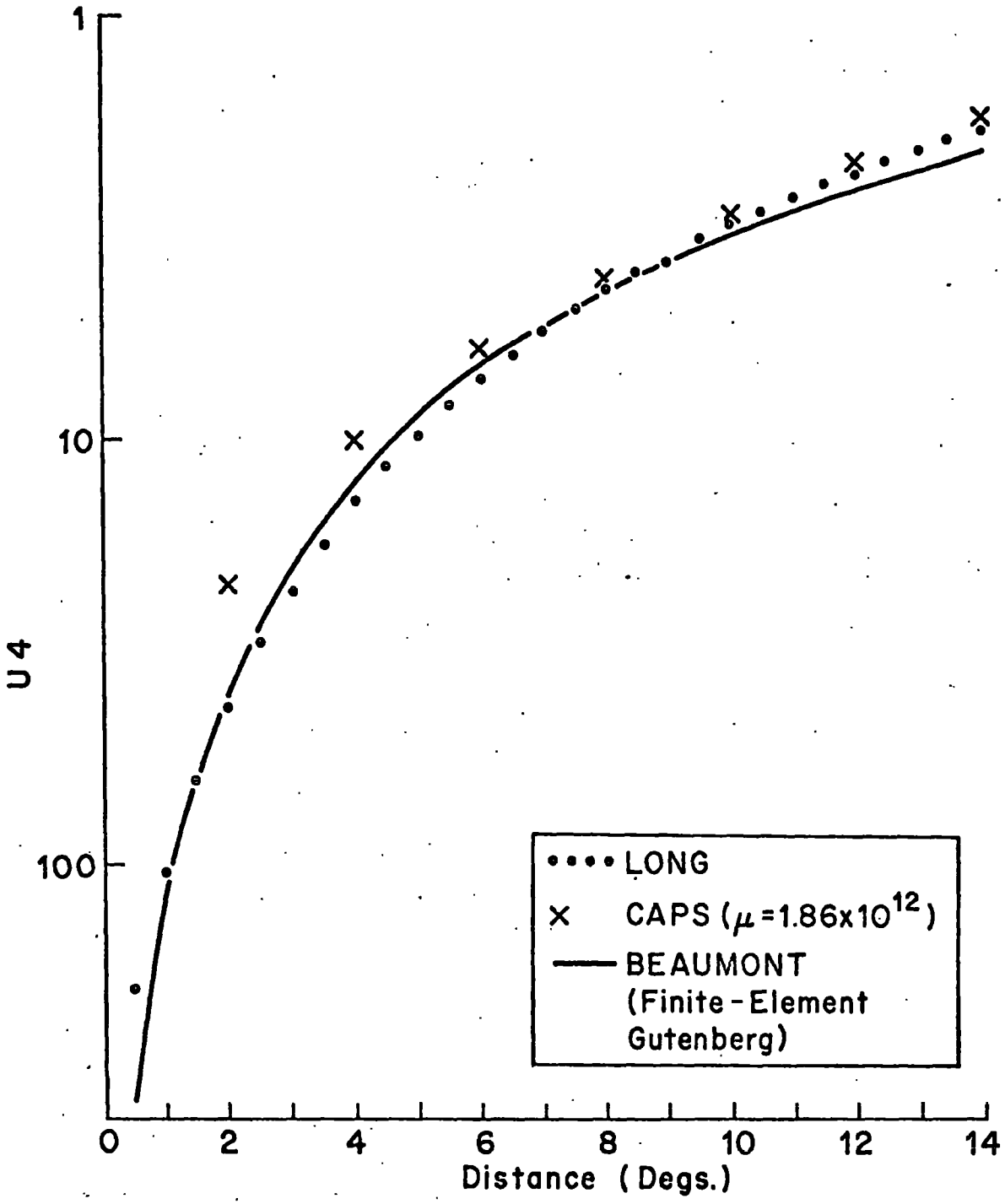

FIGURE (3-14) Depression near the load- results for CAPS, LONG and Beaumont.

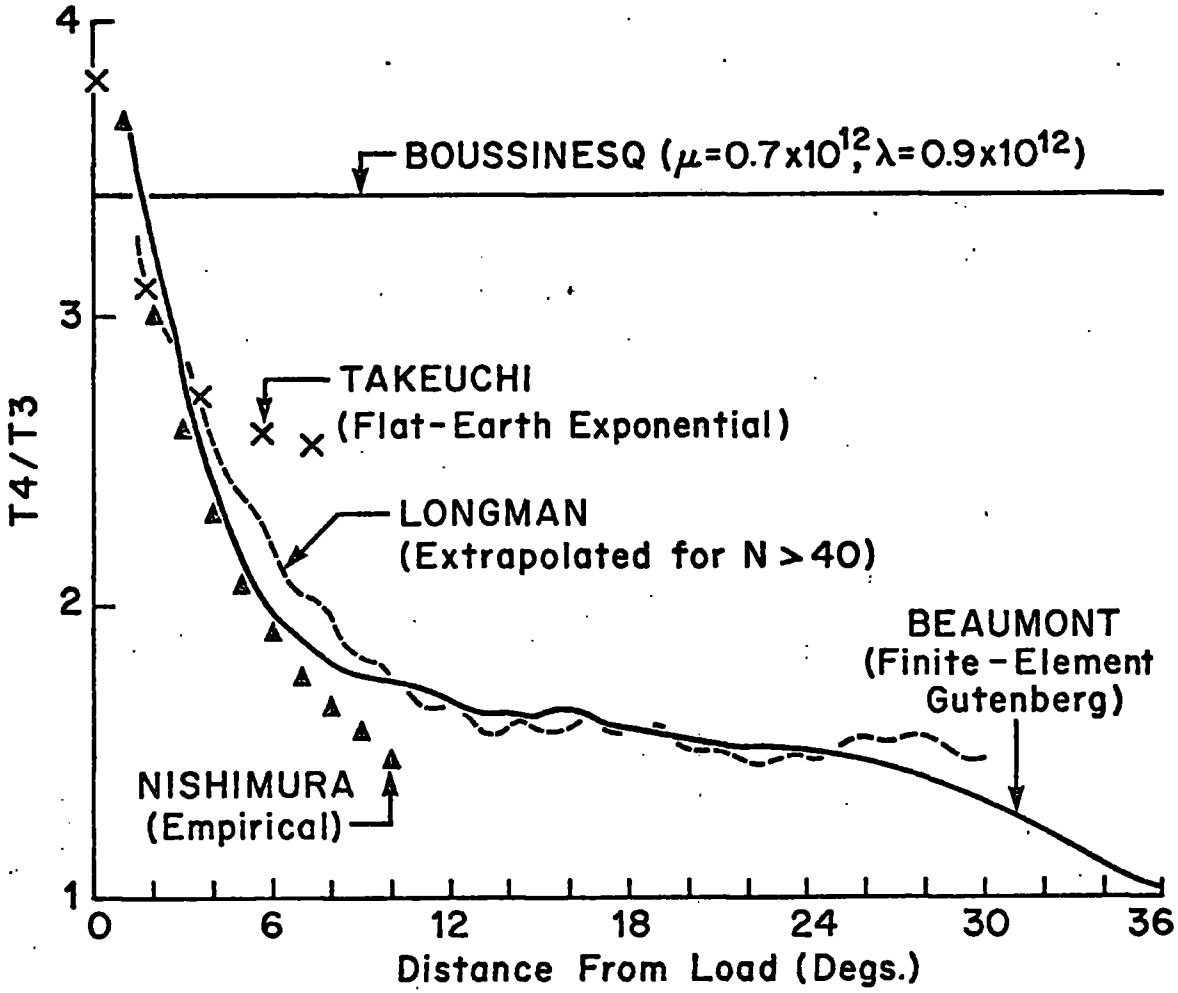

FIGURE (3-15) Tilt ratio(Green's function) near the load-comparison of several results.

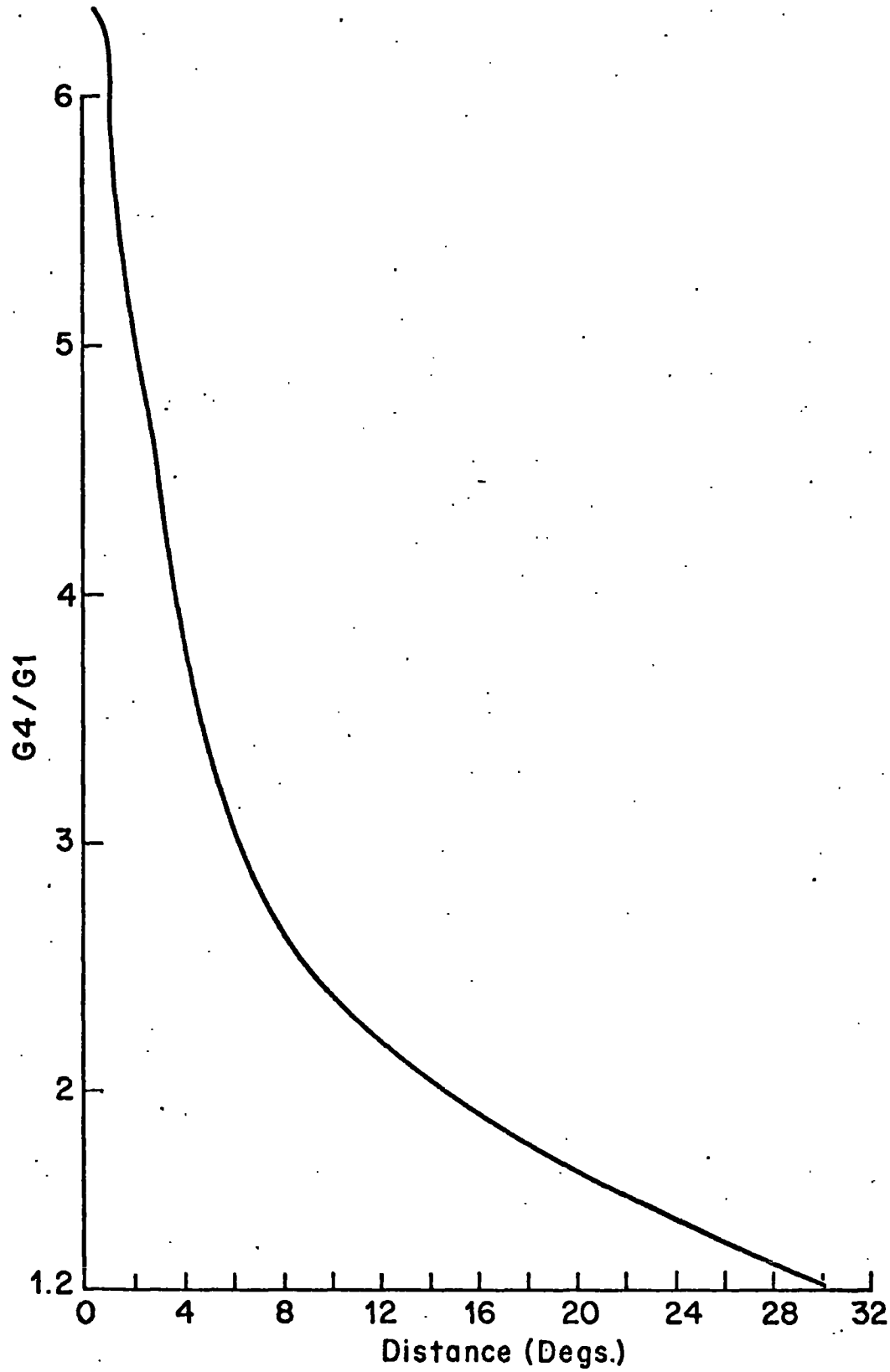

FIGURE (3-16) Gravity perturbation ratio (Green's function) near the load.(extrapolated LONG result).

displacements determined by Beaumont then appear too small with respect to the LONG results for distances greater than about 7°. In this particular Beaumont model however the Earth is assumed to be held fixed at the the distance θ=90°. This may explain the result here. The effect should be negligible closer to the load than 7°. The CAPS result shows the effect of a rigidity which is increasingly too large as the distance is decreased.

Tilt factors are compared in Fig.(3-15) for five results. The Beaumont and LONG results again agree quite well although, as previously mentioned, this may be fortuitous because gravitational uplift has been neglected by Beaumont. A valid comparison of the various results can be made best in the region between 1° and 5°; that is, beyond that distance where the effect of the crust is shown by Beaumont's method. In the range 1°-4° the Takeuchi and LONG results are virtually identical. This is convincing evidence for the validity of the LONG result since the Takeuchi function was determined theoretically on the basis of a flat Earth and a distribution of elastic constants which approximates that of Gutenberg's model to a depth of 1200 kilometers but omits the crust. The LONG implementation too would be expected to leave out the effect of the crust because the load deformation coefficients for large n were obtained by simple extrapolation from mantle-effective values. The flat-Earth assumption and the use of fictious elastic constants for depths greater than 1200 kilometers

are the reasons for the failure of the Takeuchi function beyond about 4°. The Beaumont function, which includes the effect of a crust, does not disagree with Takeuchi and LONG results in this range. The result determined by Nishimura is of interest. This function was determined experimentally by Nishimura as that which best fitted more than two years of differential tilt measurements carried out in Japan. This differential method of treating a series of tilt measurements made throughout a small region does not seem to have been repeated by others. The basis for the method is that the difference between the tilt measured at two or more neighboring sites is relatively free of distant ocean-tide loading effects. Nishimura assumed that T2/T3 was a constant equal to 0.5 and obtained the best fit to T4/T3 to be:

$$T4/T3 = 0.5 + (12.6/[r+3]) \qquad (3\text{-}30)$$

where r is the distance to the load in degrees. We have seen that the function T2/T3 is in fact very different from 0.5, especially near the load, but this does not alter the validity of Nishimura's result for T4/T3.

Finally, in Fig.(3-16) the behaviour of the gravity perturbation (G4/G1) is shown. I know of no previous results with which this could be compared, especially with regard to the region within 10 of the load.

CHAPTER 4

PREDICTION OF THE OCEAN-TIDE EFFECT

4-1 Introduction

In the previous chapter I described a method of determining the gravitational attraction of the ocean tide on a horizontal plane. I then used the Boussinesq solution for the elastic displacement of a plane due to a point-mass surface load to determine the displacements due to the load of the ocean tide. The Boussinesq solution in this way is used as a Green's Function and the displacement is determined by convolution with the gravitational attraction over the surface. Later in the same chapter the loading effects of a point mass located on a spherical, Gutenberg-model Earth are derived and these too may be considered as Green's Functions determining the effects of an arbitrary distribution of loading masses on the spherical Earth. Values determined for these Green's Functions are summarized in Table (4-1) . The tilt-ratio function varies rapidly when the distance is less than 1°, reflecting the influence of the crust, and in this region the function has been determined at 5 km intervals and is shown in Figure (4-1). This latter data has not previously been presented; it was obtained from Beaumont's solution for a finite-element Gutenberg Earth using a 10 km mesh interval and a load of radius of 10 km. This is not quite the same Beaumont solution discussed in the previous chapter; a smaller mesh interval and load radius were assumed for a

(000-036)	(037-074)	(075-112)	(113-150)	(151-180)
630	91	64	172	286
620	87	67	175	288
505	84	68	178	290
444	82	70	182	293
381	79	71	185	295
345	76	72	189	297
307	73	74	193	298
286	71	76	196	300
263	69	79	200	301
250	67	82	203	302
237	64	84	207	304
227	62	86	210	305
220	60	90	214	306
211	59	94	218	308
204	57	96	221	311
195	56	99	225	313
189	55	102	228	314
183	54	105	232	316
177	53	109	235	316
171	52	112	238	317
167	52	115	241	317
162	51	119	244	318
157	51	121	247	318
152	50	125	249	320
137	51	127	252	320
143	51	130	255	321
138	52	133	257	323
134	52	137	260	325
130	53	140	263	326
125	54	144	265	327
120	54	148	268	327
109	55	151	271	328
105	56	155	274	329
103	57	159	276	
100	59	162	279	
97	61	166	281	
93	62	169	284	

TABLE (4-1a) Values of the Green's function for gravity.
(Value x100 for each degree 0-180°)
(Determined by LONG)

(000-037)	(038-074)	(075-112)	(113-150)	(151-180)
632	144	35	-74	-118
334	143	31	-77	-118
305	142	28	-79	-118
287	141	25	-81	-118
255	139	21	-82	-118
237	137	18	-84	-119
219	136	15	-86	-120
204	134	11	-87	-121
195	132	8	-89	-121
181	130	5	-91	-122
177	127	1	-92	-124
165	125	-5	-92	-126
165	123	-6	-93	-129
158	121	-8	-94	-131
158	117	-13	-96	-134
157	115	-16	-96	-135
158	111	-18	-98	-139
159	107	-22	-99	-143
156	104	-25	-100	-146
156	100	-28	-101	-149
150	97	-32	-102	-153
150	93	-35	-103	-155
146	90	-37	-104	-157
147	87	-40	-106	-159
147	84	-43	-108	-161
151	81	-45	-109	-163
154	78	-48	-110	-166
154	75	-50	-111	-167
155	70	-52	-113	-170
149	66	-57	-113	-173
148	61	-58	-115	-176
151	57	-61	-116	-179
150	52	-64	-117	-179
150	49	-66	-118	
149	45	-68	-119	
148	41	-70	-119	
146	38	-73	-118	

TABLE (4-1b) Values of the Green's function for tilt.
(Value x100 for each degree 0-180°)
(Determined by LONG)

0-100 kms	105-200 kms
625	339
620	338
610	336
594	336
578	335
558	335
533	334
506	334
476	334
445	333
416	333
393	332
375	330
361	331
350	330
344	330
340	330
340	328
340	327
340	326
339	

TABLE (4-1c) Values of the Green's function for tilt at 5 km intervals in the range 0-200 kms. (Values given are x100) (Beaumont preliminary solution for a Gutenberg model earth. (Beaumont, 1971)).

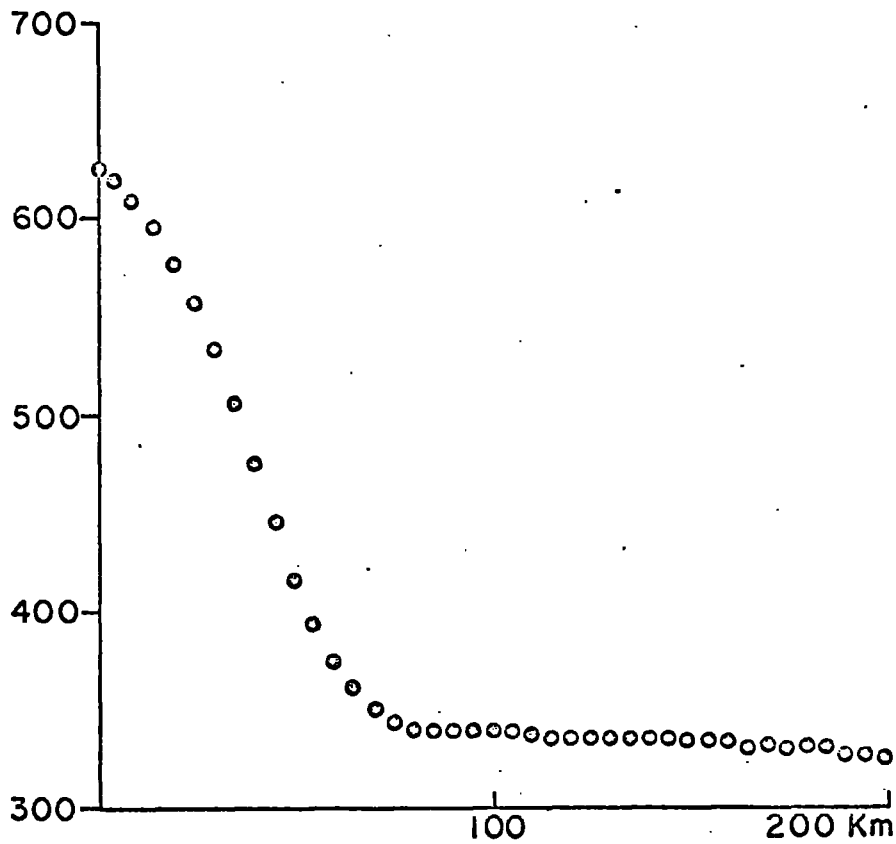

FIGURE (4-1) The Green's Function for Tilt. (Beaumont's preliminary solution for a Gutenberg earth)

better behaved solution in the critical region 0 to 200 km.

In this chapter I describe the method I have used to compute the gravitational attraction of the world-wide distribution of ocean tides and then, through the use of the Green's Functions, the calculation of the various loading effects.

4-2 The Ocean-Tide Data.

Ocean-tide data is normally published in the form of contours representing lines of equal amplitude and phase. This representation can be conveniently digitized as a network of surface polygons containing regions of approximately constant amplitude and phase. This is the procedure I have followed here in digitizing world-wide data directly from published tidal maps. Each polygon was digitized and recorded according to the following format: Number of the polygon (3 figures), amplitude of the tide in centimetres (3 figures), phase lag of the tide in degrees relative to transit of the Moon at Greenwich (3 figures), the region of the Earth (1 figure), and the latitude and the longitude of each of the polygon points taken in clockwise order when viewed from above (up to 7 points at 10 figures per point). Each polygon can therefore be fully described on one 80 column card. The polygons used are listed in Appendix VIII. A discussion of the original ocean-tide data used together with figures showing the location of most of the polygons is presented below.

The polygons of region 0 were described and illustrated

(Fig.3-2) in the last chapter. This region is that affecting the many Earth-tide stations in Western Europe and for this reason it has been digitized in fine detail. Some small-scale polygons near Bidston (BID) are shown in Figure (7-6). These are polygon numbers 1 through 12 and were drawn using data due to Lennon (1961). The tide assumed for the northern part of region 0 is of doubtful validity.

The main part of the North Atlantic is called region 1. The polygon map is reproduced in Figure (4-2). The original data from which the polygons were drawn is due to Hansen (1969) and is reproduced in Figure (4-3). The polygon map for region 3, the South Atlantic, is reproduced in Figure (4-4); the original data is due to Hansen (1969). Regions 3 and 4 are the North and South Pacific Ocean and are shown in Figure (4-5). The original data is due to Bogdanov (1961). Some polygons in region 3 near the west coast of North America were subsequently removed when this coastal region was included within region 6 which is described below.

Region 5 is taken as that water east of the southern tip of Africa, west of the east coast of Australia and south of Indonesia. The only ocean-tide data found for this region is that of Uneo (1964). Uneo solved the hydrodynamic equations for this region by finite difference methods using 5 meshes and neglecting bottom friction. The resulting data is not suitable for determining regional loading effects but should be adequate for determining global effects. The polygons for this region are simple 5° meshes and are not

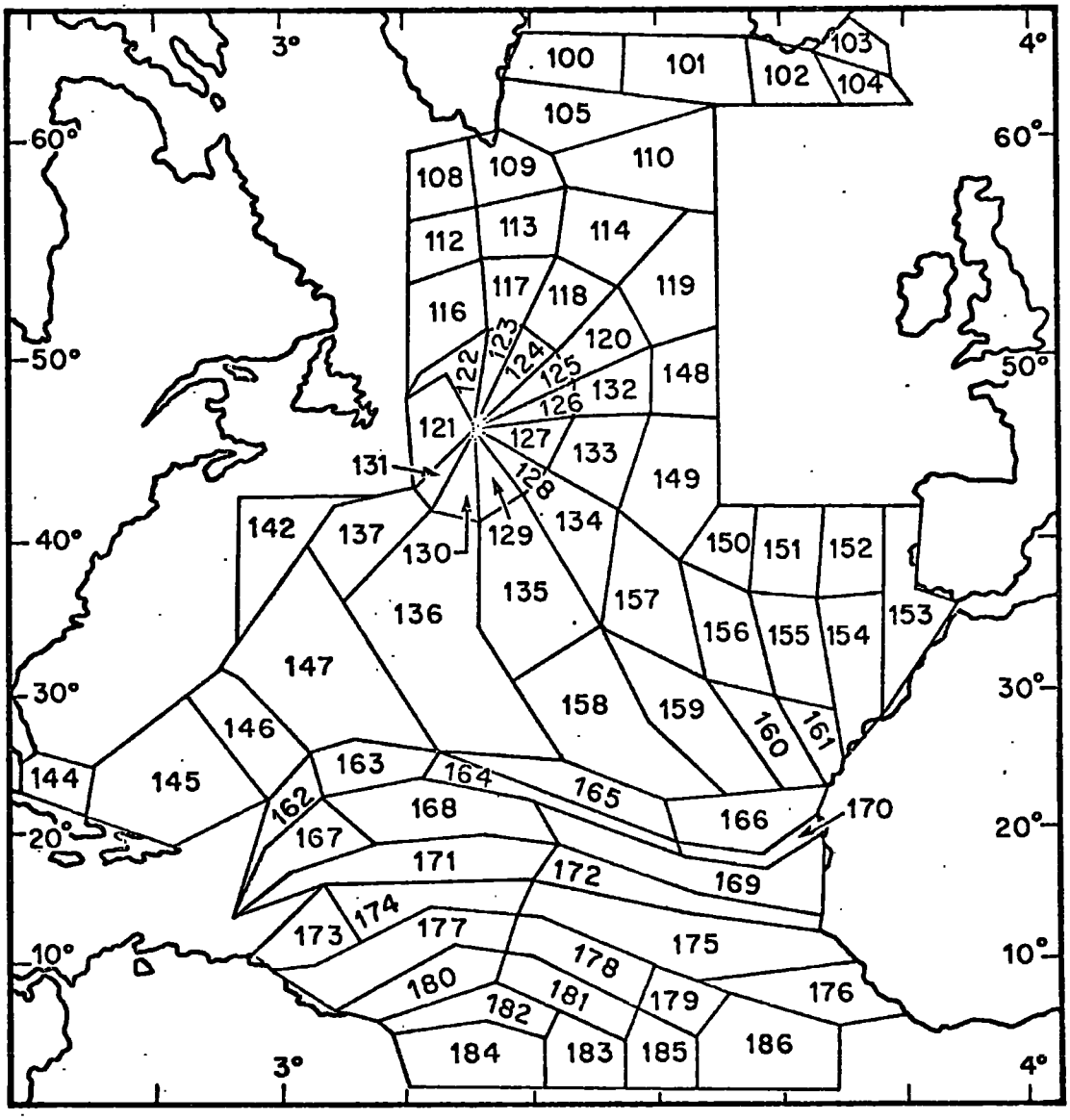

FIGURE (4-2) Ocean-tide polygons for the North Atlantic. (Region 1)

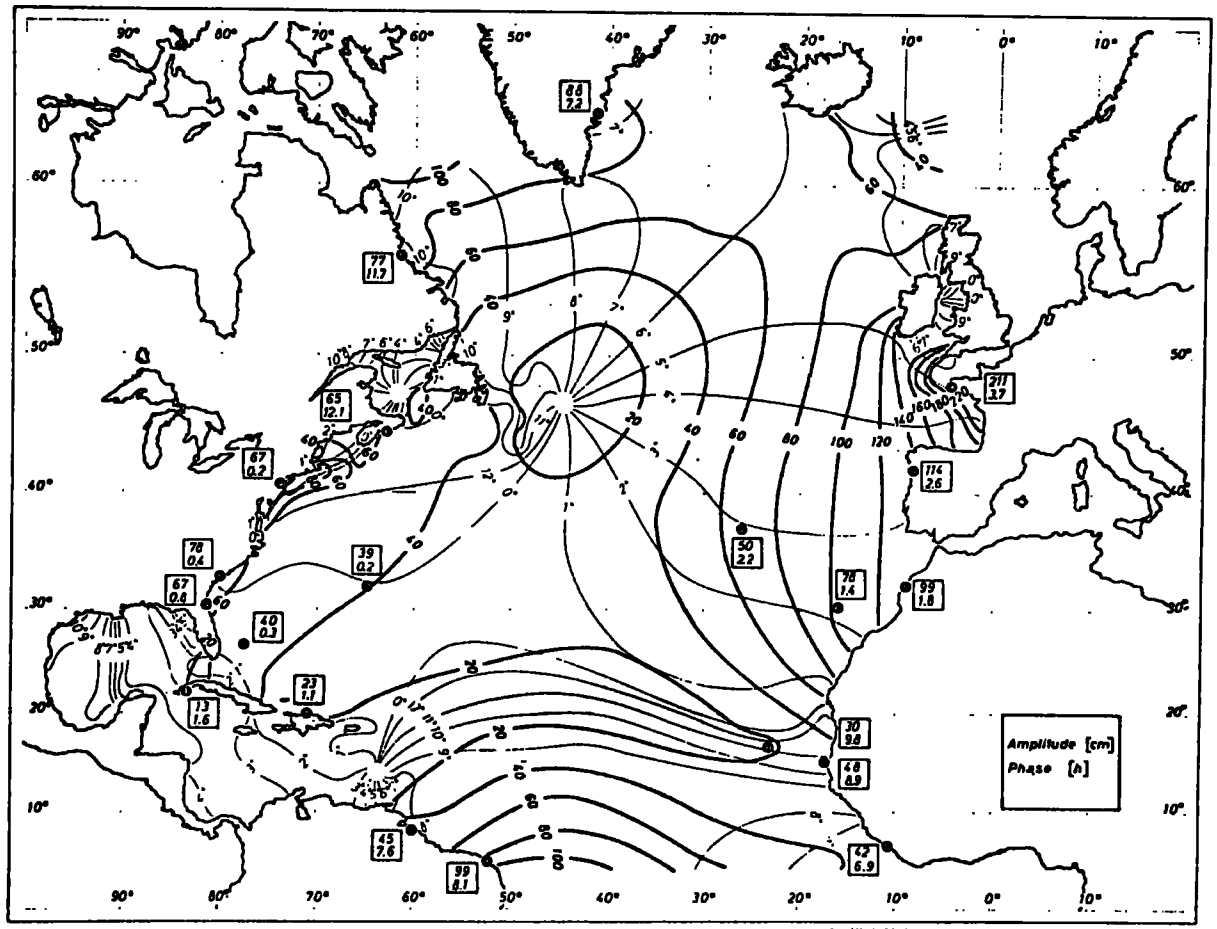

FIGURE (4-3) The ocean tide of the North Atlantic. (Hansen 1969)

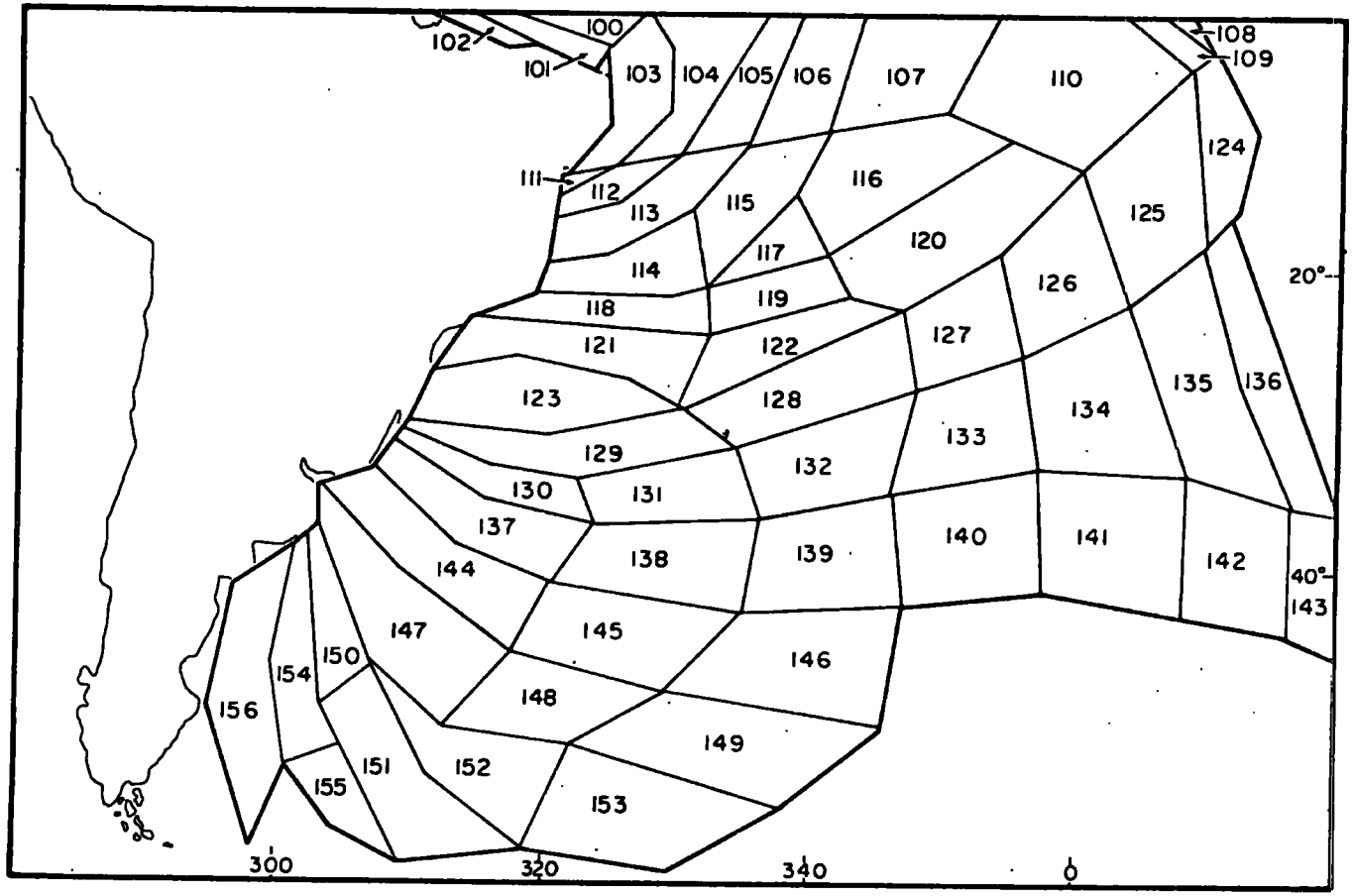

FIGURE (4-4) Ocean-tide polygons for the South Atlantic. (Region 2)

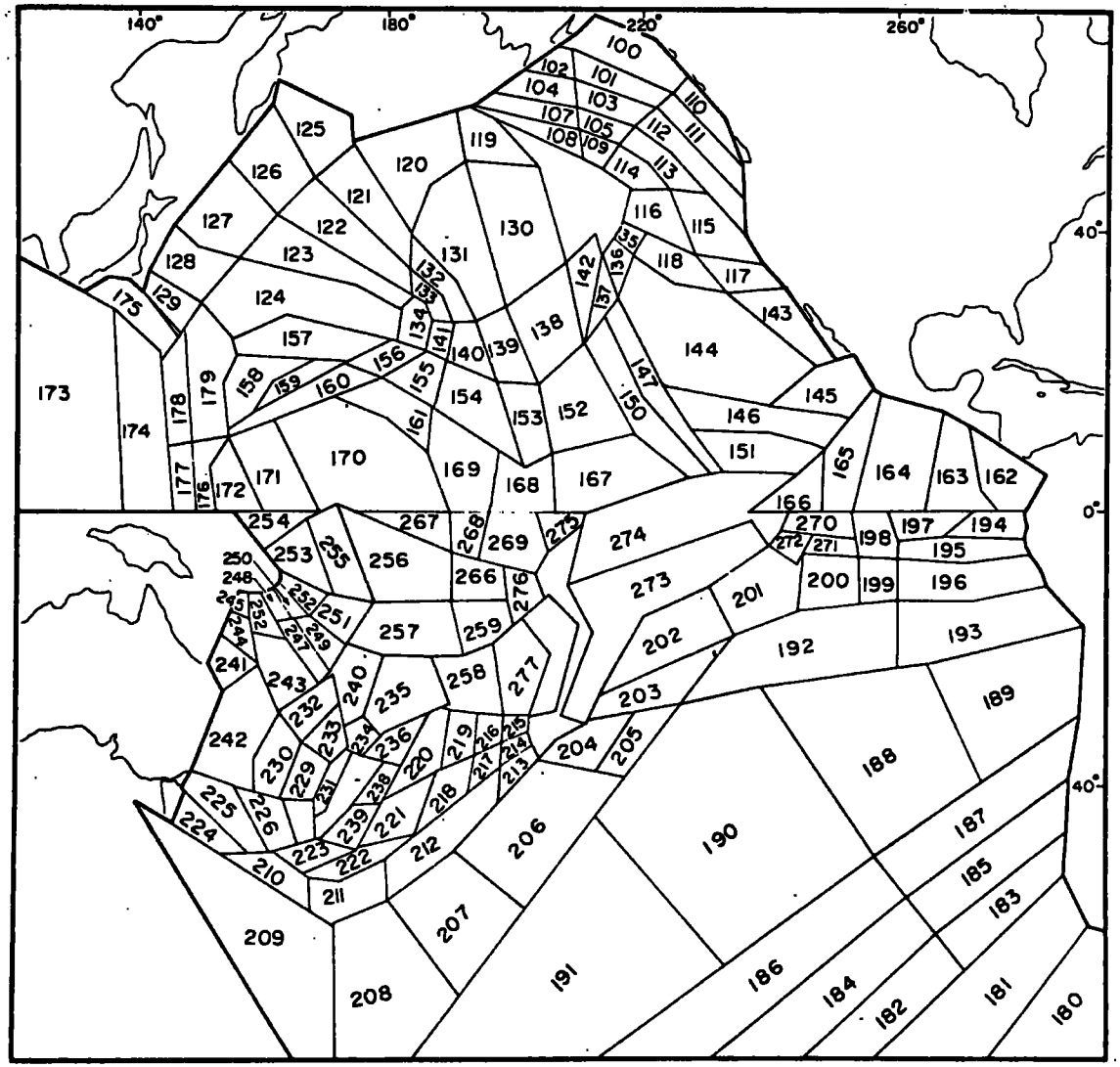

FIGURE (4-5) Ocean-tide polygons for the Pacific. (Regions 3, 4)

illustrated.

Region 6 includes much fine detail about the North American continent. Figures (4-6,7,8) illustrate the polygons assumed for the waters of off-shore eastern Canada, Hudson Bay and Straits, and Davis Strait respectively. The original data for these results is due to Dohler (1964). Figures (4-9,10) show the part of region 6 bordering the east coast of the United States and the Gulf of Mexico. Original data is that of Hansen (1969) with near-shore detail due to Emery (1970).

There are many tidal charts for the Pacific Ocean. Munk et al (1970) have summarized the most recent ones in charts which I have reproduced in Figures (4-11,12). The tidal distribution which we have assumed previously for the Pacific Ocean is that of Bogdanov (1962). It is not shown but it is very like that of Bogdanov et al (1964), shown in Figure (4-12), with the amphidromic point situated near 42 north latitude. The tidal data shown by Munk et al in Figure (4-12) is the first determination made on the basis of deep-sea tide measurements. The evidence for the existence of the amphidromic point shown by them appears strong and consequently I have removed the polygons determined previously for this area and replaced them with new polygons designed to fit Munk's results. The resulting polygon map is incorporated in region 6 and is illustrated in Figure (4-13).

Region 7 includes the Mediterranean, Adriatic and Red

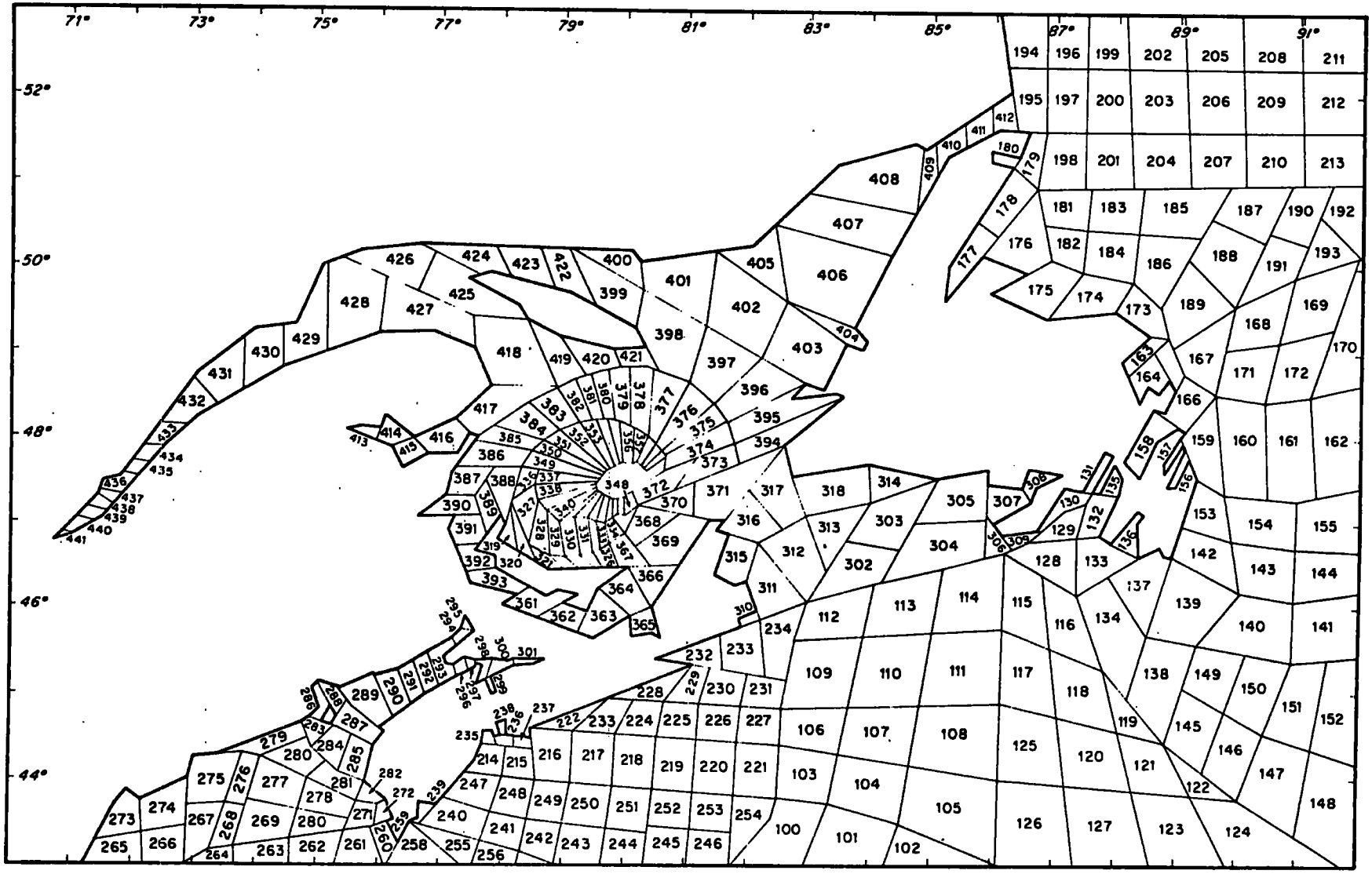

FIGURE (4-6) Ocean-tide polygons for Eastern Canada. (Part of Region 6)

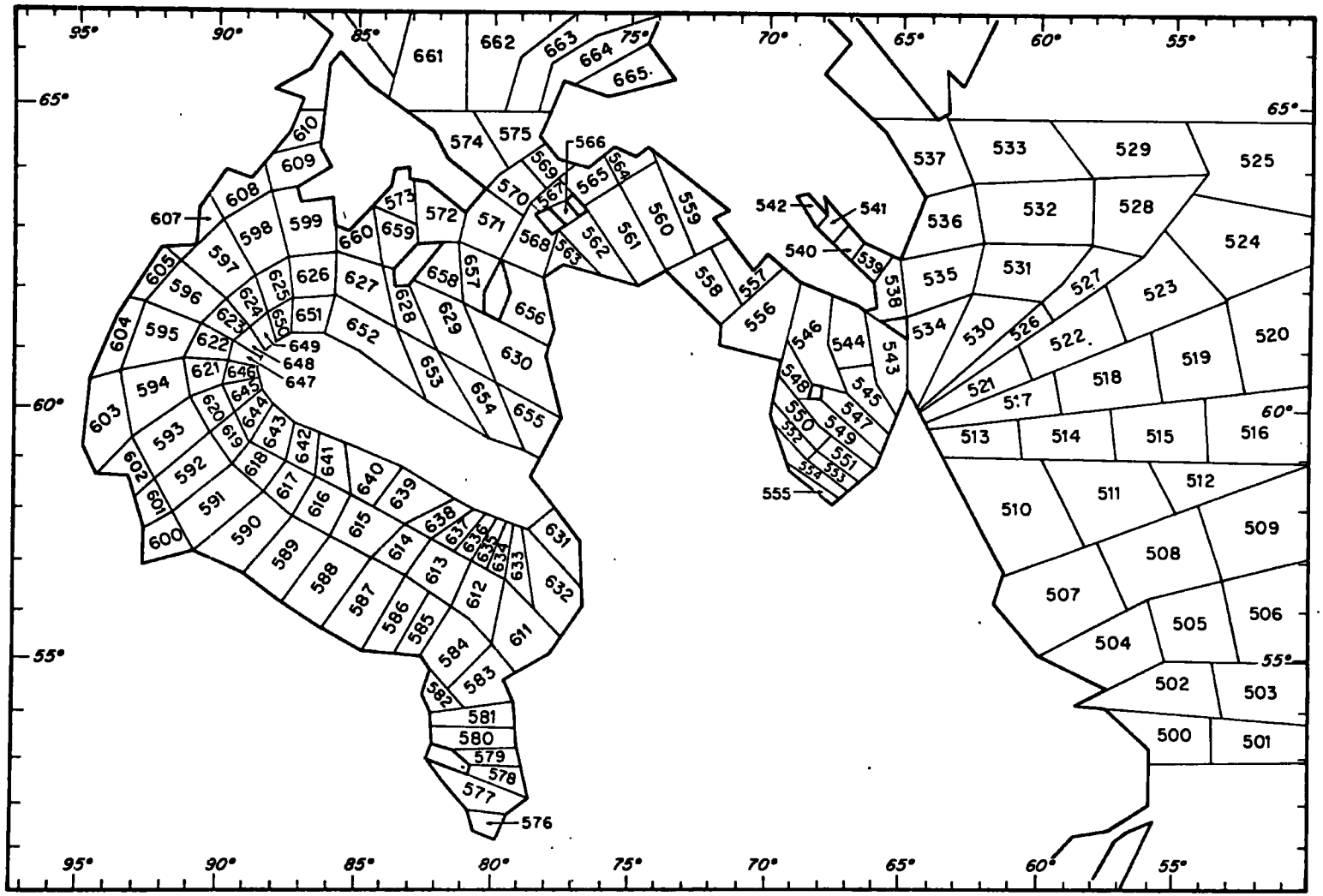

FIGURE (4-7) Ocean-tide polygons for Hudson Bay and Straits. (Part of Region 6)

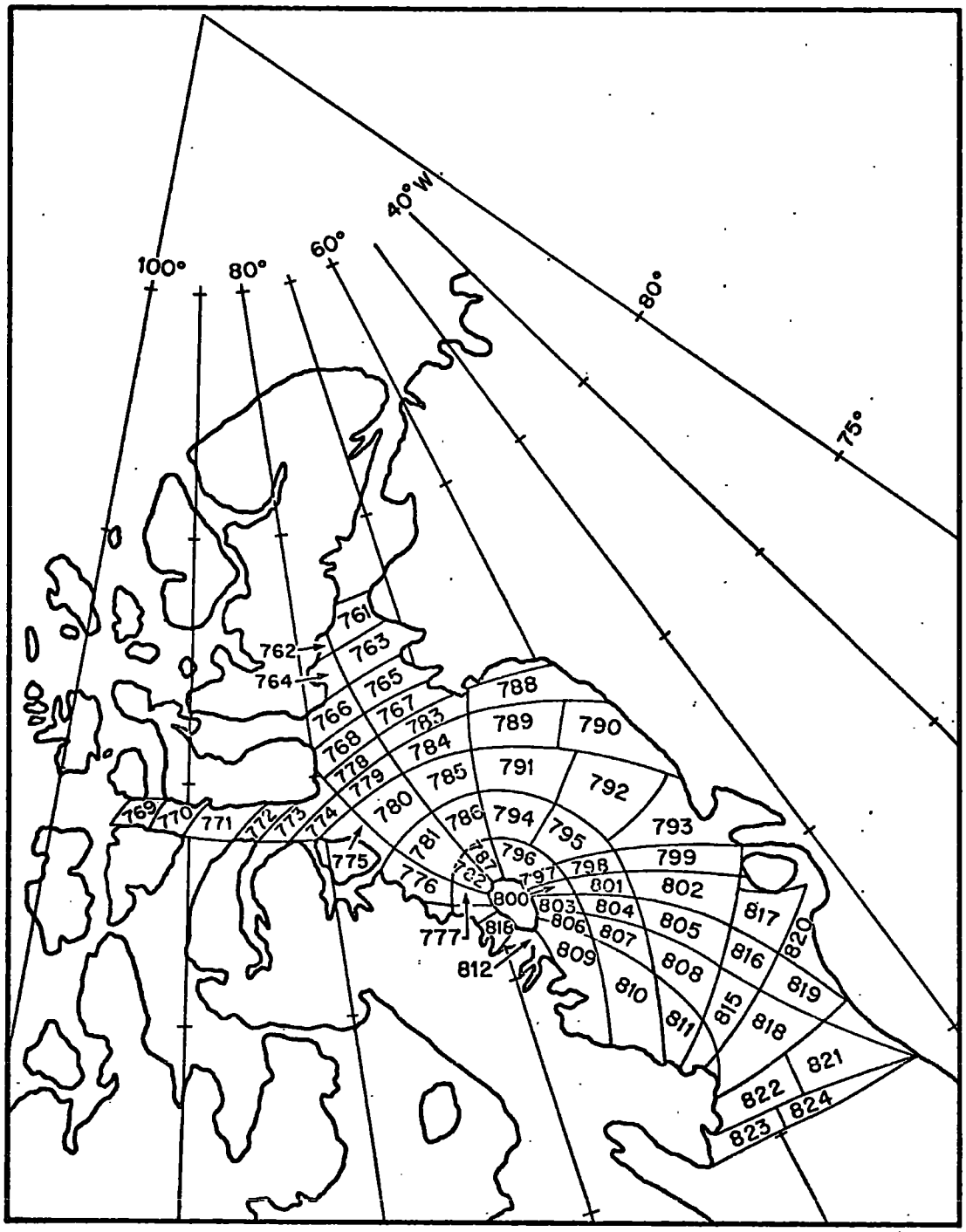

FIGURE (4-8) Ocean-tide polygons for Davis Strait.

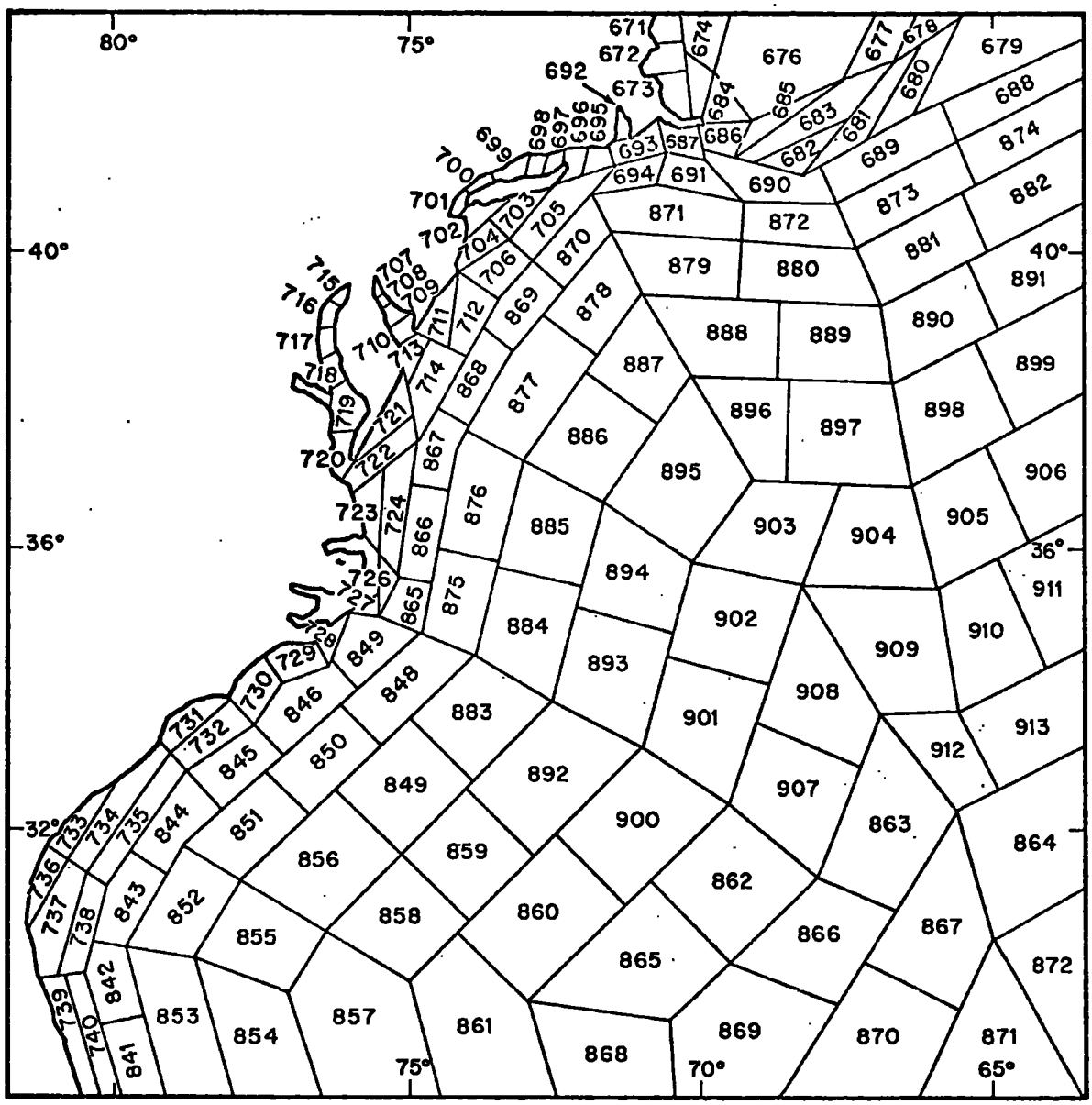

FIGURE (4-9) Ocean-tide polygons for U.S.East Coast.

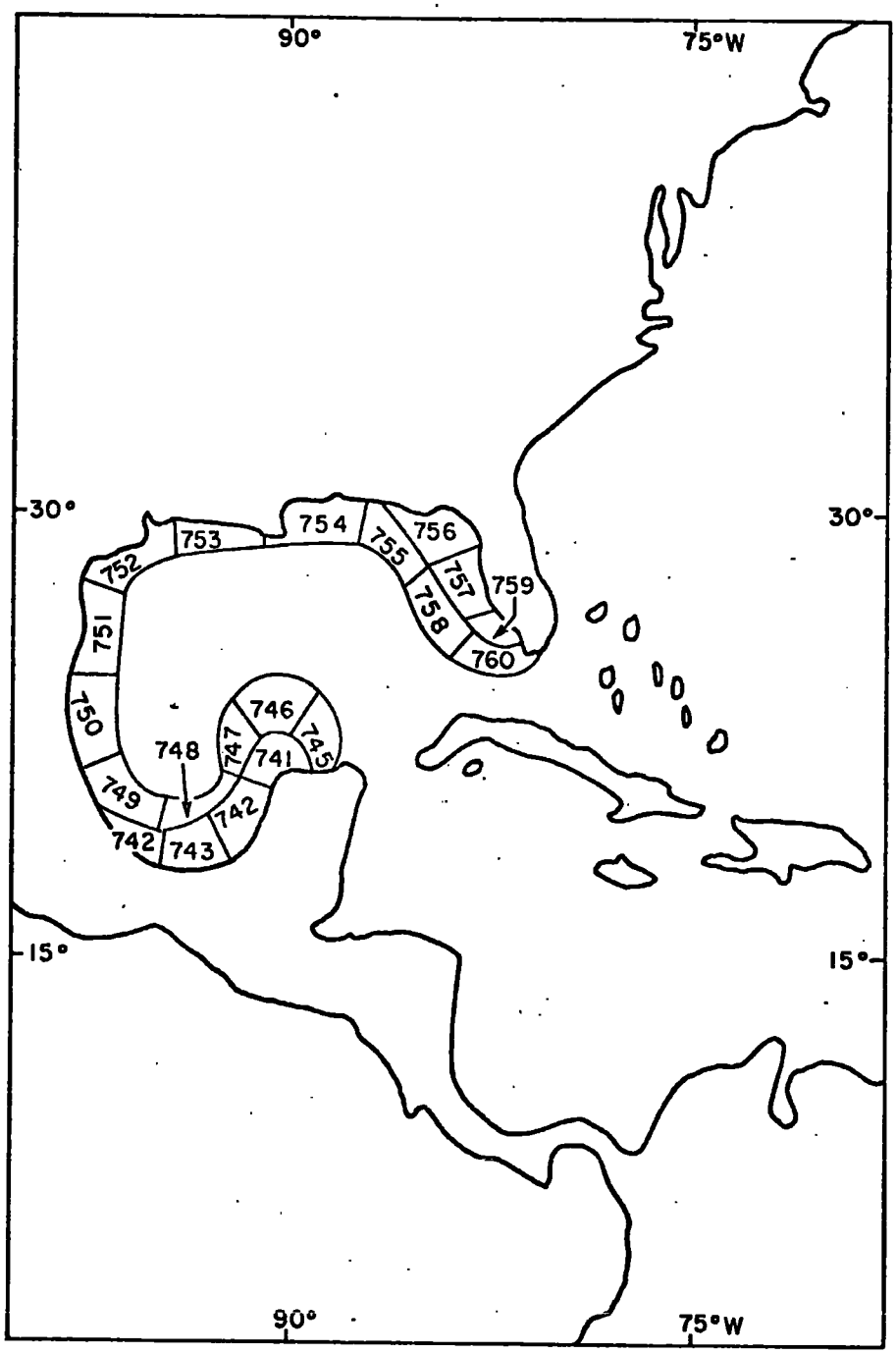

FIGURE (4-10) Ocean-tide polygons for Gulf of Mexico.
(Part of region 6.)

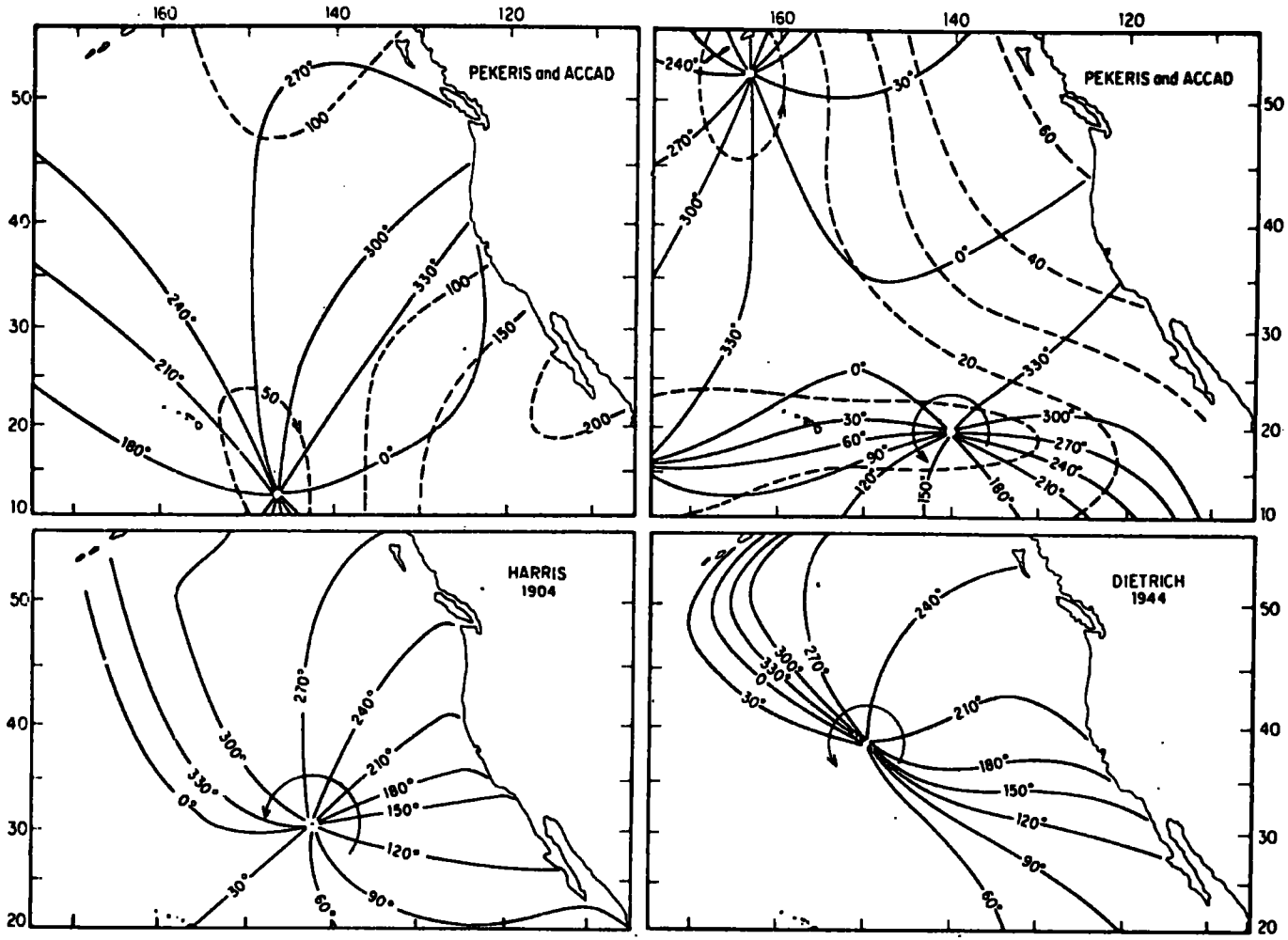

FIGURE (4-11) Tidal charts for the Pacific. (Summarized by Munk et al 1970)

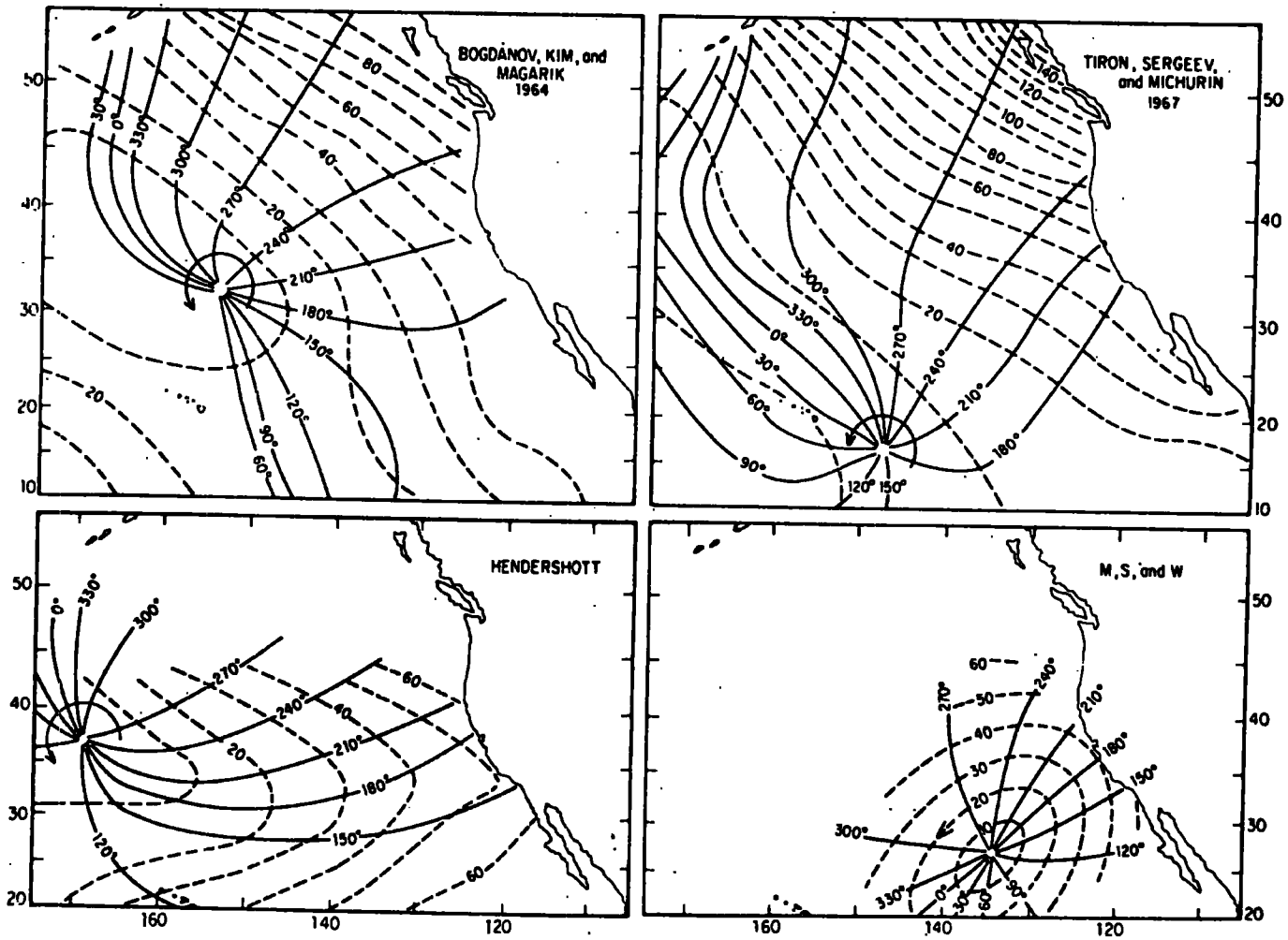

FIGURE (4-12) Tidal charts for the Pacific. (Summarized by Munk et al 1970)

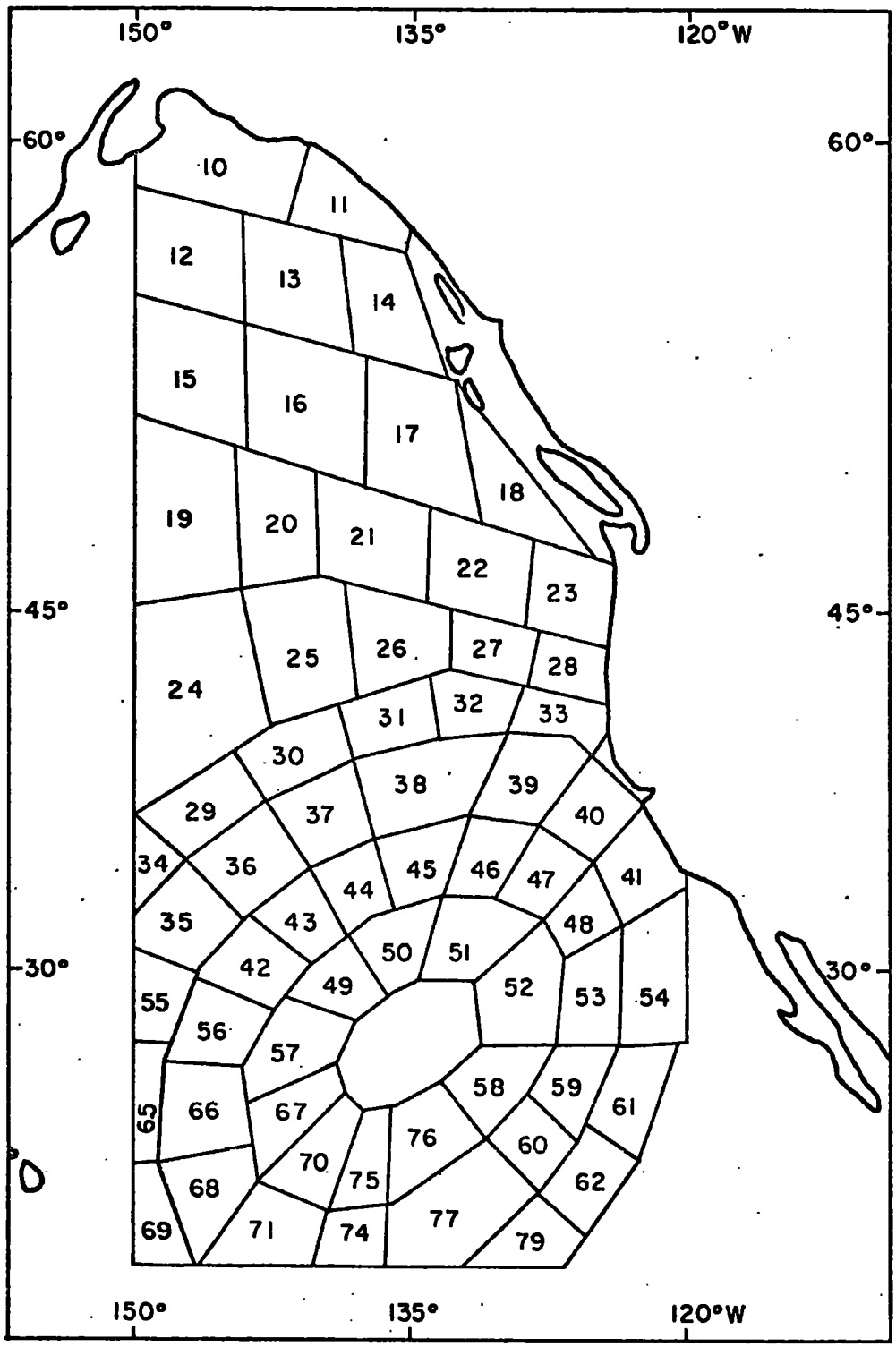

FIGURE (4-13) Ocean-tide polygons for the Pacific to fit M,S and W results of Fig. (4-12)

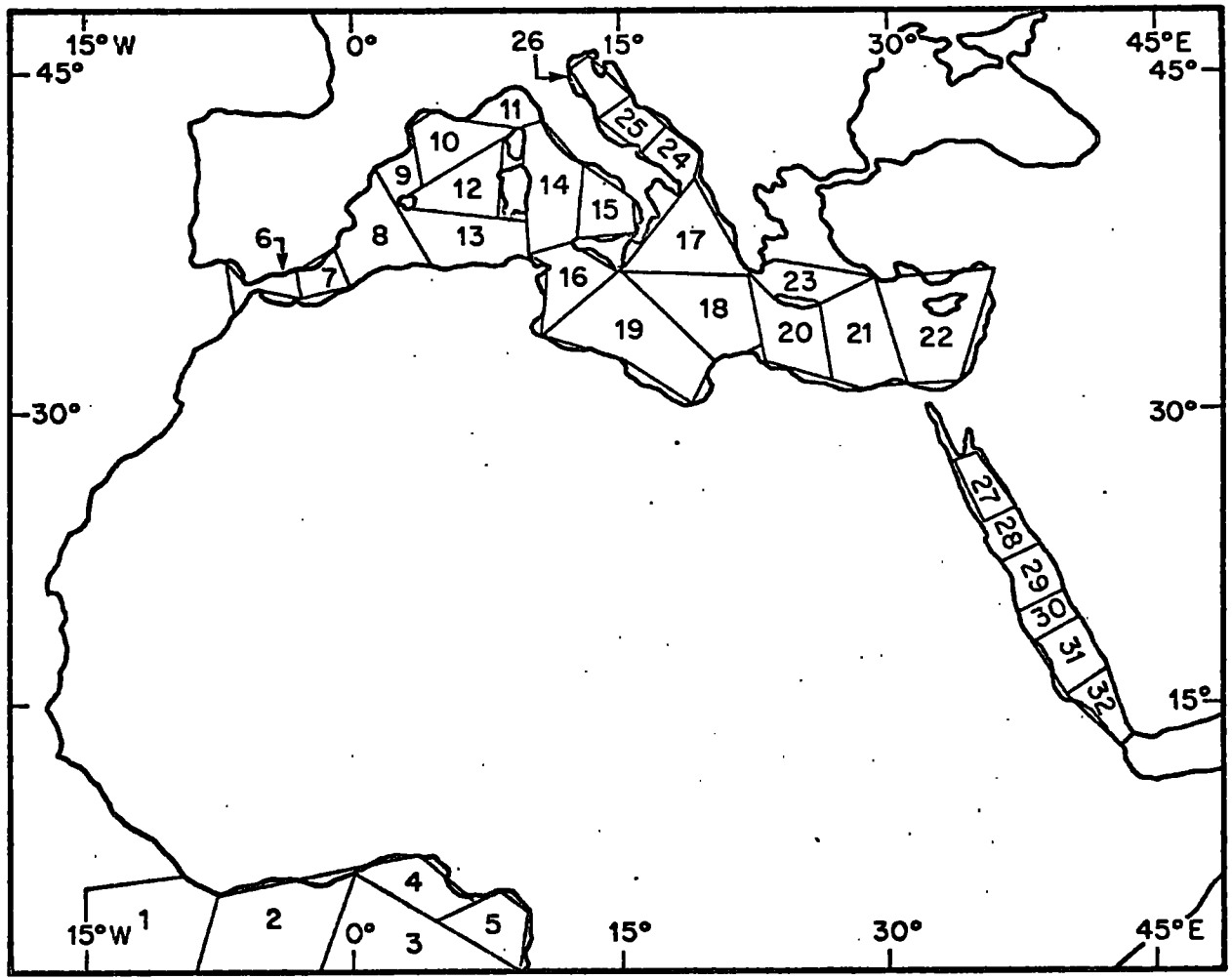

FIGURE (4-14) Ocean-tide polygons for the Mediterranean area. (Region 7)

Seas. The original data was drawn from Defant (1961). The polygons are shown in Fig. (4-14).

Region 9 is an alternative interpretation of that part of region 6 in the Pacific Ocean just discussed. The polygon map for this region is based on Dietrich's (1944) data shown in Figure (4-11) with amplitude contours taken from Kuo (1970). This will be referred to again in a later chapter. Region 8 has not been assigned.

This completes the description of the ocean-tide data used in the analysis to be discussed later. The most significant lack of data is that for the Arctic Ocean.

4-3 Program GLOBL

This program accepts as a specification the latitude and longitude of the site where the loading effects are to be calculated. An ocean-tide polygon is then read in from a tape and the following computations performed:

1. A crude estimate of the attraction of the polygon is made by assuming it's area to be equal to the product of the first and second sides and taking the distance from the site as that to the first polygon point. If the resulting attraction is less than a certain limit, assignable as the specification: PULLSIZE, the polygon is omitted and another polygon is read from the input tape.

2. If the polygon is accepted then the direction of the radius vector drawn from the center of the earth to the center of the line joining the first point of the polygon to the third point is determined. This is an attempt to

approximate the center of the polygon as economically as possible. In the case of a rectangular-like polygon it is a good approximation. It is a poor approximation for a 3-sided polygon but there are very few such polygons and they will be avoided completely in future. This vector defines the w axis which remains fixed for all computations on this polygon.

3. A u,v plane is determined next passing through the site and perpendicular to the w axis. This now is the problem discussed in detail by Bott (1963) and I have implemented his results. The u,v axes are rotated for each lamina so that it is necessary to resolve the components of attraction of each lamina along x,y,z axes which remain fixed in the Earth. The net attraction due to the polygon is then determined and resolved along vertical, east and north axes through the site. Before storing these components of attractions for accumulation they are multiplied by the appropiate Green's Function (for gravity or tilt effects). A new Green's Function may be read in as input data by specifying on the header card: NEWTON=2. The default is to NEWTON=0 which causes the Green's Function listed in the source program to be used. If NEWTON=1 is specified the Green's Function for tilt is put equal to the constant 1.0. This is necessary when the height effect at a site is to be determined. The tilts computed then are due only to the Newtonian attraction and the height effect is the difference between the tilts found for HEIGHT=0 and HEIGHT equal to the

actual site height.

A single line of output is listed for each polygon. This line includes the 3 loading effects (gravity in microgals, north and east tilt in millisecs) due to the polygon and the accumulated effect due to all the polygons considered to that point.

4. When the final polygon has been processed the direct attraction of the Moon is calculated and resolved along the vertical and horizontal directions to determine the direct gravity and tilt effects. The accumulated gravity and tilt effects due to the polygons is then divided by these direct attraction effects and the resulting ratio is listed together with the phase difference. Finally, the gravity and tilt which would be expected on an 'average' Earth without oceans is estimated by multiplying the direct attraction effects by the gravimetric factor 1.16 and the diminishing factor 0.700. The calculated loading effect of the polygons is then added to this result to yield an estimate of the Earth-tide gravity and tilt which should actually be observed on such an Earth at the site. It is this final result which is later compared to Earth-tide observations made throughout the world.

4-3-1 Refinement For Computing Local Effects.

When the distance to the polygon becomes comparable to its dimensions some defects which previously could be neglected become important. The normal to each polygon is determined by the radius vector to the center of the

polygon; if this point is assumed to lie in the plane of the polygon then the edges of the polygon will project out above the surface of the Earth. If the polygon is located near the site this projection may cause a large anomalous gravity effect. To minimize this type of error the polygon is adjusted along the w axis to pass through the first point of the polygon. The polygon therefore falls below the surface of the Earth near the center but the edges, nearer a potential site, are represented more realistically.

We have assumed that the Green's Functions vary sufficiently slowly with distance from the site that they can be evaluated at the closest degree distance and taken outside the attraction integral. Where the distance is less than one degree however the tilt Green's Function varies rapidly, as can be seen in Figure (4-1). In this region therefore it is necessary to proceed in a different way. The integrals concerned are: (See Bott, 1963)

$$DFU = \iint u\ K[R](u^2+v^2+w^2)^{3/2}\,du\,dv$$

$$DFV = \rho\rho v\ K[R](u^2+v^2+w^2)^{3/2}\,du\,dv$$

where DFU and DFV are the u and v components of tilt due to the attraction of a single lamina of a polygon. K[R] is the Green's function for tilt evaluated at the distance R. The constant K[R] is taken outside the integral sign and applied as a final step to the tilts resolved along the north and east directions.

We can take into account the first order variation of the Green's Function over each lamina by writing: (Square

brackets are used here to mean 'function of')

$$K[r] = K[R] + \beta(r-R)$$

where β is the first derivative of K with respect to the distance r evaluated at R. Since we restrict this integral to distances of the order of one degree, W can be neglected and we can write:

$$DFU = \int\int K[r]\, u\, (u^2+v^2+w^2)^{3/2}\, du\, dv$$

$$= (K[R]-\beta R) \int\int u\, (u^2+v^2+w^2)^{3/2}\, du\, dv$$

$$+ \int\int \beta u\, (u^2+v^2)^{1/2}\, du\, dv$$

with a similar expression for DFV. The second integral introduced here is an elementary form. This method of solution is implemented by the program when the distance is less than 200 km and the specification 'DETAIL' has been set equal to 1. Both the Green's Function for tilt and its derivatives have been set within the program to values at 5 km intervals from 0 to 200 kms.

CHAPTER 5

THE MEASUREMENTS

5-1 The Horizontal Pendulums.

In this device the pendulum is supported by wires or fibers so as to swing freely in a horizontal plane, much like a gate on it's hinges. Like a gate too, it is very sensitive to tilting of the hinge axis in a direction away from the plane defined by the pendulum and the hinge. It is this sensitivity, which can be increased to the point of instability by setting the hinge axis closer to the vertical, which makes the horizontal pendulum a useful device for the measurement of tidal tilts. The horizontal pendulums used both at Rookhope and at Holland Mills are of Verbaandert-Melchior (1958) design and are constructed entirely of fused quartz so as to be as friction free and dimensionally stable as possible. They are mounted on right-angled shaped aluminium bases supported on 3 legs, two of which are adjustable for independently controlling the sensitivity and the azimuth of the pendulum. The fixed leg is located directly under the near-vertical hinge axis of the pendulum. The sensitivity is increased by increasing the height of the period leg which is located in the plane of the pendulum and hinge beneath the pendulum bob. The pendulum is caused to rotate about the hinge axis by changing the height of the third leg which supports the third corner of the right-angled base; this is called the azimuth or drift leg.

Melchior (1965, p65) shows that the sensitivity of the pendulum to tilting depends upon the angle of the hinge axis to the vertical and that this determines too the period of natural oscillations of the pendulum. The sensitivity in fact varies as the inverse square of the natural period and thus is determined when both the period and proportionality constant are known. The proportionality K is measured when the pendulum is constructed by means of an independent method of tilt measurement. This second method has been devised by Verbaandert (1959) and utilizes a stainless-steel disk containing a small cavity which communicates with a reservoir of mercury by means of a connecting tube. The thick upper surface of the disk is caused to deflect slightly by raising or lowering the height of the mercury reservoir. The deflection is approximately linear and can be measured precisely using metrology techniques which Verbaandert describes. To determine the value of K the horizontal pendulum is tilted a precise amount by placing this disk under the drift leg of the pendulum and changing the height of the mercury reservoir.

Both the Holland Mills and Rookhope sites are equipped with disks permanently installed under the drift legs. The sensitivity of the pendulums can therefore be measured either from a period measurement or directly from the response to known deflections of the disk.

Motion of the pendulum is measured in the classic way by the deflection of a beam of light which is reflected from

a mirror mounted on the pendulum near its axis of rotation. The sensitivity of the pendulum to tilts is given for an optical lever length of 5 m by:

$$S = K/T^2 \qquad (5\text{-}1)$$

where K is the proportionality constant and T is the period of free oscillations of the pendulum. The sensitivity S is expressed as the pendulum tilt in arc seconds required to produce a deflection of the reflected image of 1.0 mm. The pendulums are normally operated with natural periods in the range 70-85 seconds which yield, for a typical value for K of 6.000, a sensitivity of .0012 to .0008 arc seconds per mm.

The dynamic response of the pendulums it was found could be approximated with sufficient accuracy by the second-order differential equation describing a simple oscillator with linear damping. The static sensitivity of the pendulum expressed as the rotation of the pendulum in arc seconds per arc second tilt of the pendulum base is a useful term to isolate in this equation. It is related to the period of the pendulum and the proportionality constant K by:

$$s = 20.6\ T^2/K \qquad (5\text{-}2)$$

The pendulum equation then can be written:

$$(p^2 + p2\delta\omega' + \omega'^2)\theta = s\omega'^2 d \qquad (5\text{-}3)$$

where p is the differentiating operator of Laplace transformation notation, ω' is the angular velocity of natural oscillations of the pendulum, θ and d represent

rotation of the pendulum and tilt of the pendulum base respectively. δ is the damping coefficient expressed as the fraction of crictical damping and is typically near 0.01. The response of the pendulum to periodic tilting of the base with amplitude d and angular velocity ω is:

$$\theta = s\ d\ \mathrm{Sin}\omega t/[\ (1-\omega^2\omega'^2)^2 + (2\delta\omega/\omega')^2\ ]^{1/2} \quad (5-4)$$

For frequencies not too close to the natural frequency of the pendulum the damping term is negligible and this equation can be written:

$$\theta = sd\omega'^2/(\omega'^2-\omega^2) \quad (5-5)$$

5-2 Recording the Pendulum Measurements

The tilts detected by the pendulums are recorded by a two-channel light-spot follower (Sefram). The pens of this recorder are mounted on trolleys which are servo-driven to follow the light spot reflected from the pendulums. In order to use this device it was necessary to replace the optical system supplied with the pendulums with a collimated light source which placed an adequately bright slit image on the photcell for an absolute minimum of source power. A standard automobile bulb rated at 12 watts but used at half power gave several months of continuous operation. In addition to the analog record, made directly on metallic paper by pens mounted on the trolleys, an electrical signal was obtained from two linear potentiometers which were driven by the trolleys. These signals were used to detect when a light spot was approaching the limit of travel of a trolley. This condition caused a low-speed motor coupled to the drift

screw of the appropiate pendulum to be energized briefly to bring the pendulum back into the operating region. This action was more difficult to achieve than expected. It was necessary to contact and turn the drift screw without at the same time jarring the pendulum; a pure rotational motion was required at the coupling. The desired behaviour was finally achieved with a flexible fork and bar coupling suggested by J. Geuer (Dominion Observatory Drafting Office). The fork and bar contact only during driving and when the desired rotation has been achieved the motor is reversed to back the fork off slightly from the bar.

When the original spot follower recorder used at Rookhope was lost in a mine flood in December 1968 it was replaced by a new all-transistor model. This cooler running instrument was found to require supplementary heating to avoid deterioration of the metallic chart paper by moisture. The new recorder too was found to sometimes lose servo lock after the power had been interrupted briefly, a common occurrence in a working mine. This is a difficult fault to recognize because brief power failures are not obvious on the chart record. The trouble was overcome by introducing relays which delayed the restoration of power to the spot follower until the light sources had reached full brilliance.

5-3 Preparation of the Rookhope Site.

Rookhope is located in County Durham approximately mid-way between the east and west coasts. Geographical

coordinates are 54° 47' North, 2° 6.3' West and height above sea level is 350 m. The site used was in an operating fluorspar mine owned by the Weardale Lead Co., Ltd. A plan of part of the mine showing the pendulum site is reproduced in Figure (5-1).

An operating mine was selected as a site as a last resort after many non-operating properties had been investigated and found unsuitable because of flooding, poor rock conditions, remoteness or other reasons. The concern about an operating mine was that the movement of ore trains, the frequent dynamite blasts and the operation of pumps would produce an intolerable level of noise in the tiltmeter measurements. The advantages of an operating mine however, particularly for short-term measurements, proved irresistable and so it was decided to attempt to make tilt measurements in the Rookhope mine.

The site made available to us was a large room used originally as a chamber from which exploratory drilling could be made. It was located at the end of a 200m long cross-cut. This level was 17 fathoms (31 m) beneath the surface and near the base of the Great Limestone. The floor here was dry and sound under approximately 30 cm of rubble so it was decided to install the tiltmeters directly on the floor rather than in a large niche cut in the wall which is the normal practise. Niche cutting had proven expensive and time consuming at Holland Mills and it was desirable to find an alternative technique. The installation required for each

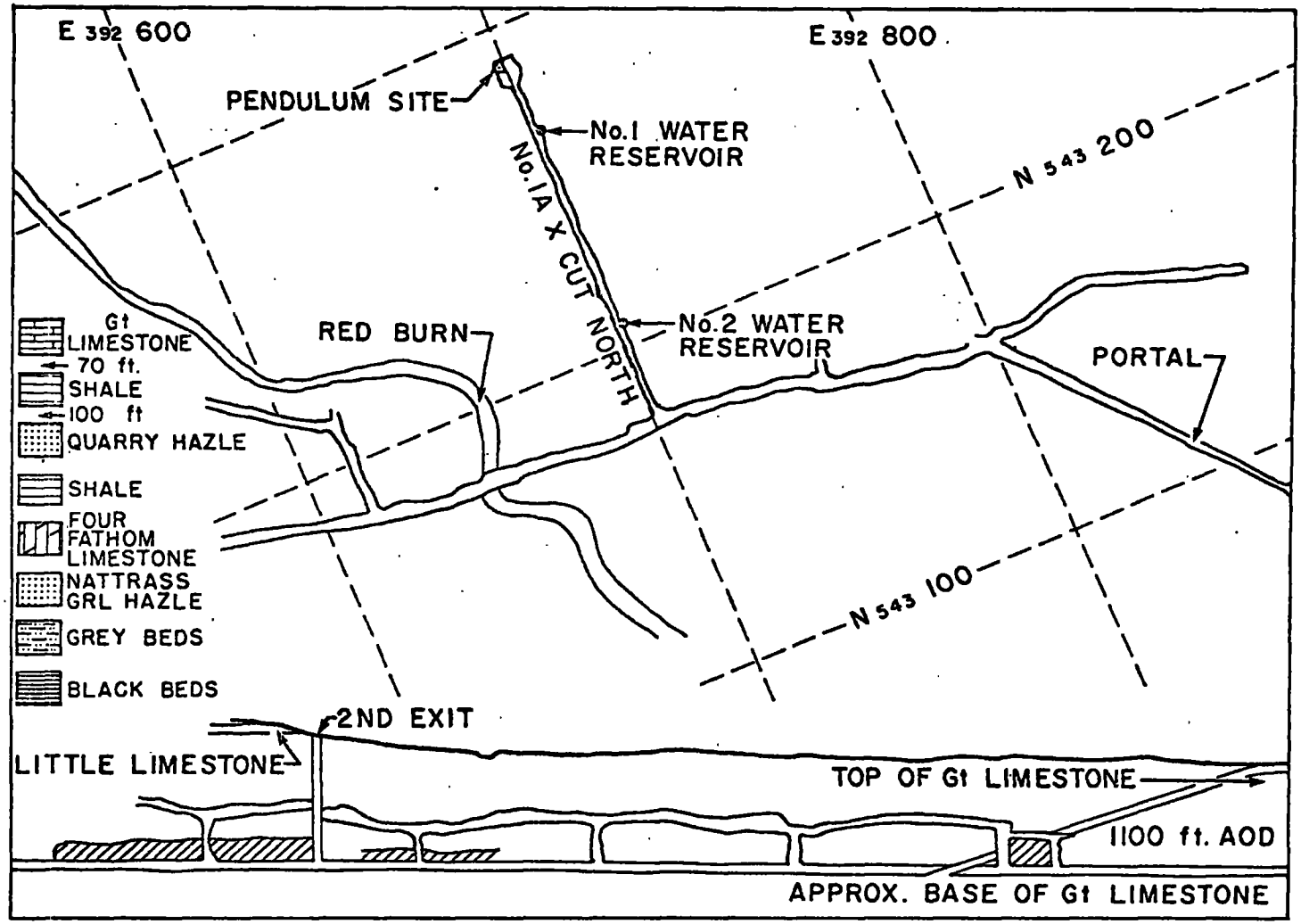

FIGURE (5-1) Plan of the Rookhope mine showing the pendulum and water-tube sites.

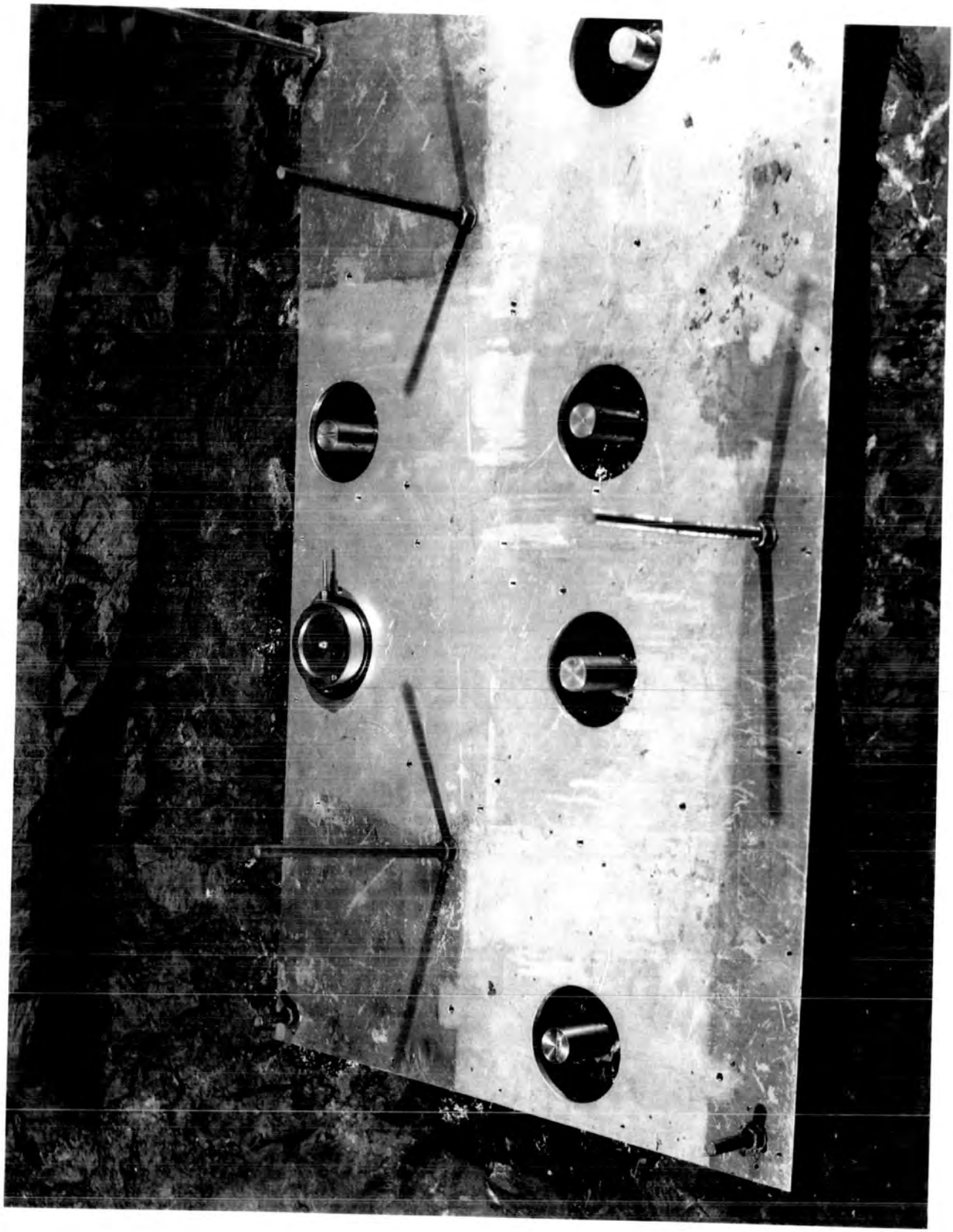

PLATE (5-1) Preparation of the Rookhope Site- The leveled base plate from which the holes for the support pins were drilled.

PLATE (5-2) Preparation of the Rookhope Site- Drilling the holes for the support pins. (Some of the support pins are shown in place.)

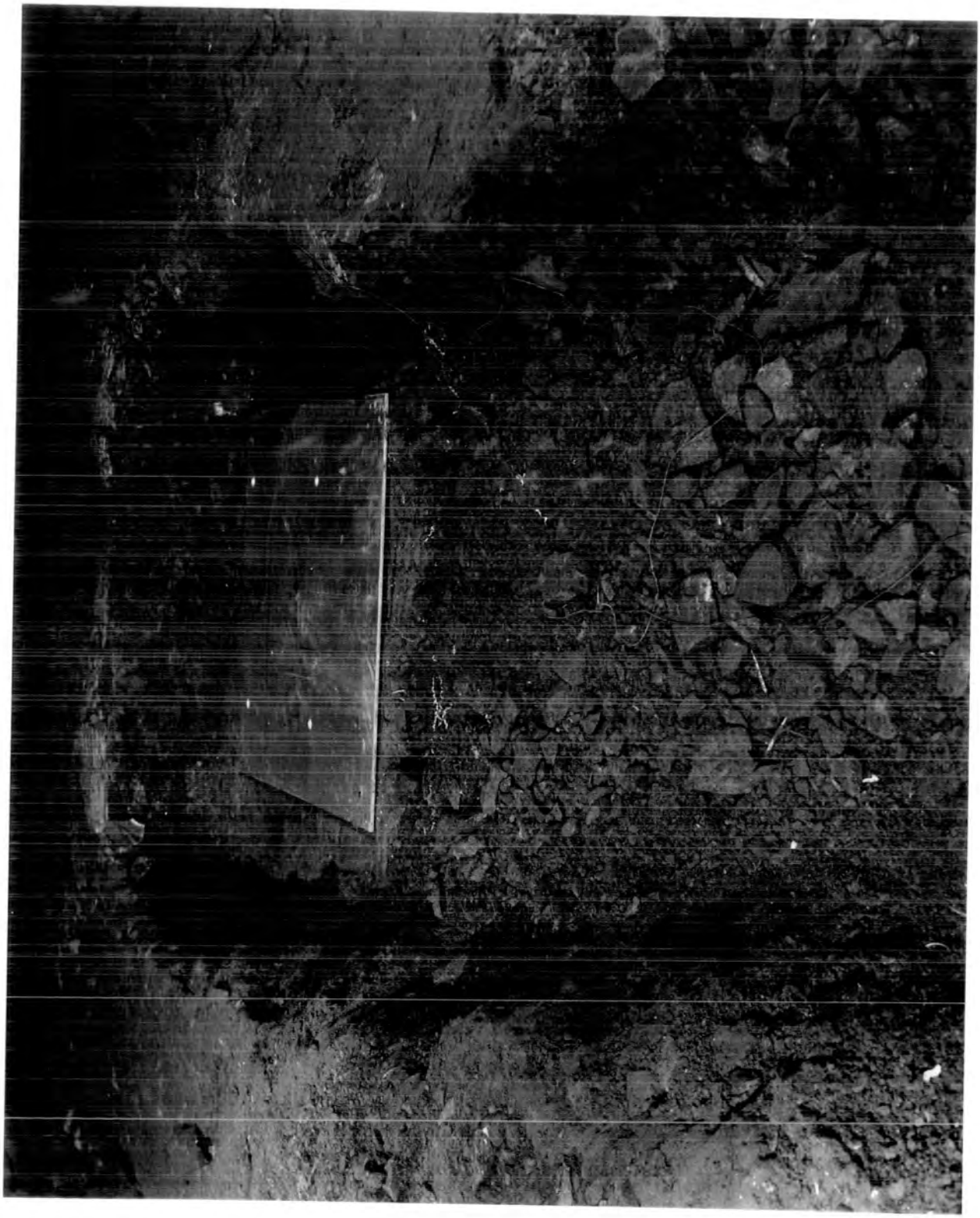

PLATE (5-3) Preparation of the Rookhope Site- The base plate for the Spot Follower installed on solid rock.

PLATE (5-4) Preparation of the Rookhope Site- The completed moisture-sealed pendulum case. The drift-compensating motors are mounted on the front of the case. The calibrating device is on top of the case.

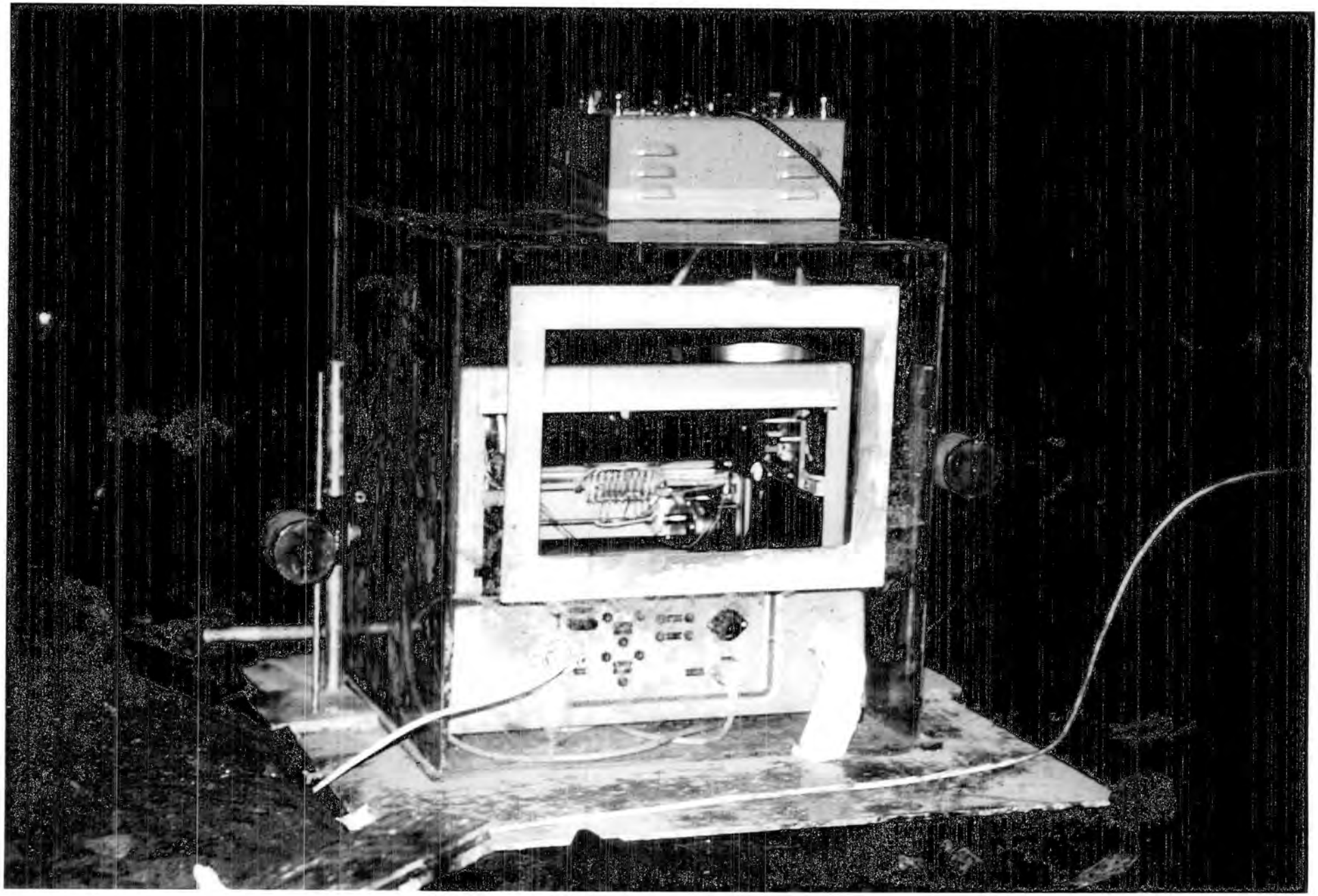

PLATE (5-5) The Sefram 2-channel spot follower and the light sources

pendulum consisted of installing securely in the rock three stainless-steel pins on which the three legs of the pendulum would rest. Plates (5-1) and (5-2) illutstrate how this was done. Long steel studs were first set in the floor by means of expansion plugs (Rawlplug). An aluminum plate was predrilled to mark the locations of the six holes to be drilled in the rock. This plate was mounted on the steel studs and washers and nuts on the studs were adjusted until the plate was supported precisely horizontal and as close to the floor as possible. (The floor had previously been cleaned to sound rock and leveled by chipping). A drill press was then clamped on the level plate and holes for the leg support pins were drilled to the required depth, about 15 cms. This technique ensured that the pins were all at the correct height and the upper surfaces level. Important too was the perfect roundness of the holes which was the result of clamping the drill press firmly to the plate during drilling. This was desirable in order to maintain a uniform clearance of about 0.25 mm around the pins for the adhesive, which was applied next. The adhesive is a two-component polyester resin of the type used for bonding strain-gauges to rock surfaces at low temperatures. This is generally considered to have better creep characteristics than an expoxy adhesive, particularly in moist areas.

The spot follower site was prepared next. This is shown in Plate (5-3); it consists simply of an aluminium base plate pre-drilled to accept the four feet of the spot

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follower. It is secured to the floor at the proper horizontal level, distance and direction relative to the pendulum. This is a convenient stage at which to connect by surveying the pendulum azimuth to some known azimuth line in the mine. To do this it is generally assumed that the center of the fixed leg and the center of the period leg define the azimuth of the pendulum. A template was made which fitted the position constraints (the classic hole, slot and plane) on the surfaces of the three pins and at the same time provided marking pins on which a transit could be sighted. In this way azimuths of north 29° west and east 29° north were determined for the two pendulums.

The completed installation is shown in Plates (5-4) and (5-5). A large aluminum case is fitted to the base plate and sealed against moisture. Fiber glass and epoxy has been used to seal the holes where the pins enter the box. This glass cloth is thin enough not to transmit to the pendulum any motion of the box. The two motors which operate the drift screws can be seen mounted on the front of the box at the lower right. Mounted on top of the box is a clock-controlled mechanism for varying the height of the mercury reservoir which can be seen connected to the pendulums by long tubes. This calibrating device is energized once a week. Plate (5-5) shows the spot follower and its plastic cover which is essential for moisture protection. On either side of the spot follower a light source can be seen mounted on a vertical post. A slit of the proper dimensions is formed by

a mask fastened to the light source and an image of this slit is focused on the photocell by a 5 m focal length lens after reflection at the pendulum mirror. The 5 m lenses were mounted on the small plastic covers which can be seen enclosing each pendulum. It is worth noting here that these plastic covers, which are supplied by the manufacturer, should be modified by having a panel cut in the front so the pendulum can be accessed without having to move the cover over the top of the pendulum.

5-4 Operation of the Rookhope Site.

5-4-1 Initial Setting Up

The setting up procedure for the horizontal pendulums is described by Melchior in his book and in detailed instructions accompanying each Verbaandert-Melchior pendulum. The sensitivity is increased in small steps until a natural period of about 75 seconds is obtained. After each step change in sensitivity a trimming adjustment of the drift screw is required to keep the pendulum in its operating range. When the proper period range is reached the sensitivity is decreased, without changing the drift screw adjustment, to give a period of about 10 seconds. The position of the reflected slit image at this time is taken to represent zero tilt of the pendulum base; that is, the pendulum is assumed to lie parallel to the line joining the period screw and the fixed screw. This is the reference direction mentioned in the last section. The sensitivity is restored to the 75 second region for normal operation.

The difficulties in this technique are due mainly to the almost total lack of damping in the pendulum motion. At high periods a disturbance is introduced each time the period or drift screws is adjusted, however carefully, and this causes large oscillations of the pendulum which take several minutes to damp out. Moreover these adjustments must be carried out with the observer's weight immediately next to the pendulum so that even when stable operation is achieved it may be upset when the observer moves away. With patience, successful operation is eventually attained but the difficulties are a serious hindrance to the application of the tiltmeter. Some consideration was given to the possiblility of achieving critical damping of the pendulum by causing the mercury reservoir to move up and down as part of a servo loop with controlling signals derived from the potentiometers of the spot follower. Analysis shows that this could be implemented by moving the reservoir at speeds less than 7.0 mm/sec and through a vertical range equal to three times the amplitude of expected tilt steps measured at the recorder. This was demonstrated by controlling the reservoir height manually but lack of time prevented construction of an operating system.

5-4-2 Summary

The first tilt observations were made at the Rookhope site in April 1968. Many interruptions were experienced during the first few months due to failures of the spot follower or due to prolonged power interruptions. The site

could be visited only once or twice per week so that failures of any kind caused a data loss of a few days. Many of these early records are being salvaged now by identifying power interrruptions and correcting timing errors. Continuously good records began to appear in August and continued until mid-December when a major rock fall closed the mine for several weeks. The pumps were without power after the rock fall and the tilt recording equipment was soon submerged under six feet of water. The spot follower was a total loss, mainly because of the corrosive effects of the water. The pendulums although unclamped throughout the the flood were found undamaged but very dirty. They were later scrubbed clean and the 40-micron suspension fibers broken off and replaced with new ones. A replacement spot follower was borrowed from the Dominion Observatory and the site reoccupied in February 1969. Regular recording began in May and continued until the project ended in December 1969.

In spite of the fact that the Rookhope site is a working mine the tilt records were generally quieter than the Holland Mills records. This is believed to be due to the difference in topography. The Holland Mills site is located in an abandoned asbestos mine in the midst of rough, irregular local topography. This topography is likely reflected in a complex distribution of underground water courses which cause anomalous tilting effects during rain storms and possibly during seasonal changes as well. The

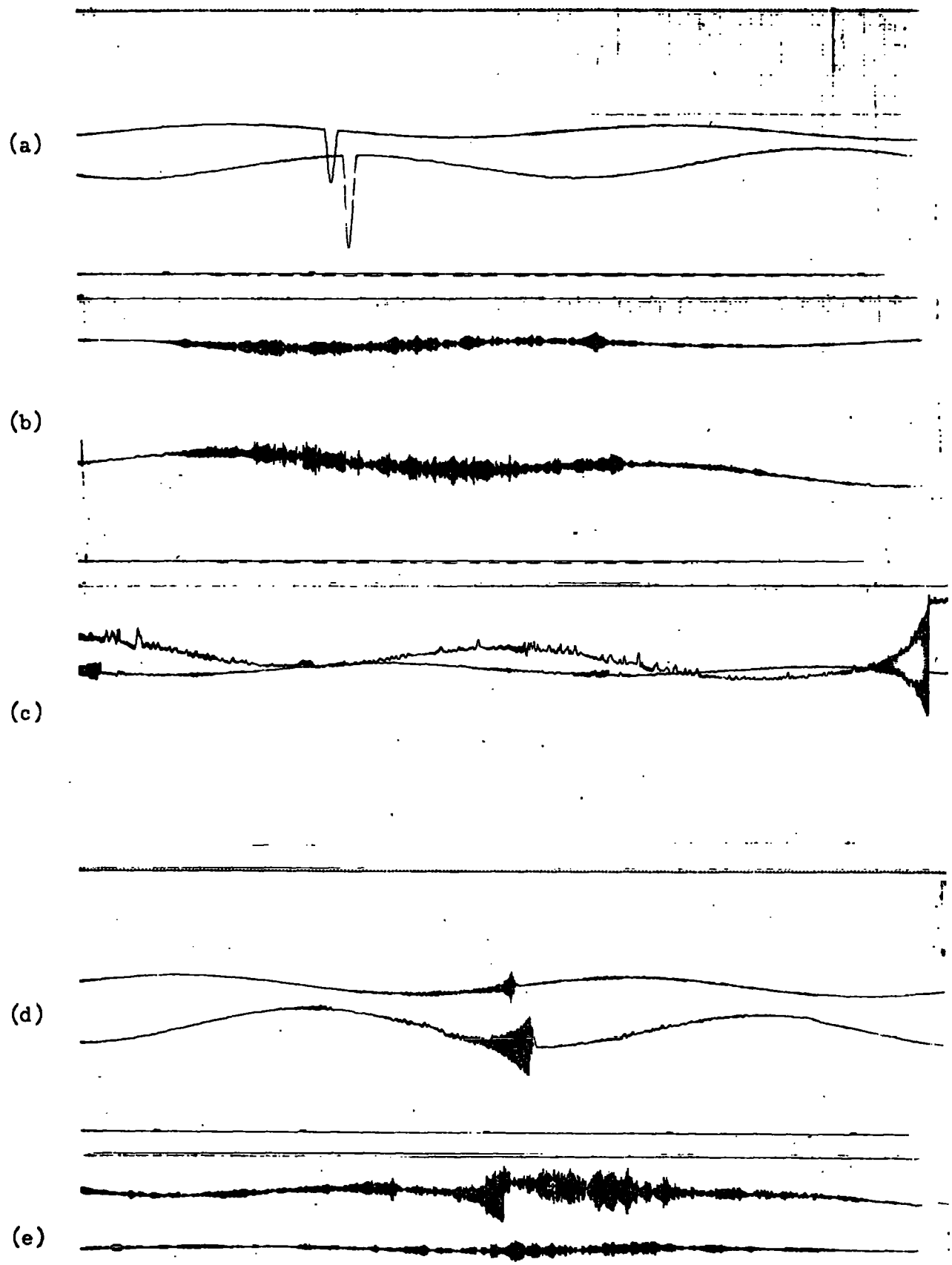

PLATE (5-6) Representative Records Of Tilt From The Holland Mills Site.
(Horizontal scale:1.3cm/hr. Vertical scale:1.0cm/.05 arc secs.)

topography too results in a greater wind exposure that that at Rookhope.

Reproduced in Plate (5-6) are several records from the Holland Mills site which illustrate the behaviour of the horizontal pendulums. Frame (a) shows a typically 'quiet' record and a calibrating signal. This calibration was produced by a single rotation of the lever arm carrying the mercury reservoir. Normally several rotations are made so that the effect of noise can be averaged out. Frames (b) and (c) are typical records made during high winds and during heavy rains respectively. The low frequency noise in Frame (c) is principally on the east-west record; it is unsymmetrical with respect to the tidal curve and usually begins and ends with the rain. This same pendulum in the spring shows frequent permanent 'tares' in the same direction as this noise. An automatic scale shift appears at the right of Frame (c). The disturbance shown in Frame (d) resembles a seismic event but was not detected at the Ottawa seismological station. Frame (e) illustrates a particularly noisy record and a 'tare' of the type that seems to be observed by all horizontal pendulums occasionally.

Wind effects at Rookhope are much less noticeable than at Holland Mills, presumably because the smooth rolling topography near Rookhope presents little wind exposure. Low frequency noise of the type shown in Frame (c) is sometimes experienced at Rookhope but it does not seem to be related to rainfall.

Long term tilting measured at the Rookhope site was generally smaller than at Holland Mills. The direction of tilt measured at both sites was the same throughout the year and remained the same after the suspension fiber of a pendulum was changed. This indicates that the long term tilts measured were real although they are probably too large to represent tilting of more than a very local area about the pendulums. At Rookhope the tilting during the summer of 1968 was towards the direction west 49° south at the rate of 4.5 millisecs/day. The following summer, after the rock fall in December 1968 and repairs to the tiltmeters, the direction was west 29° south at the rate of 8.0 millisecs/day. These directions tend to be toward rather than away from the working area of the mine where approximately 500 tons of rock was being removed each week.

The effects of blasting and the movements of ore trains at Rookhope were of no importance in the measurement of tidal tilt at diurnal and semi-diurnal frequencies. The closest traffic and the closest blasting were located 1000 m and 7000 m respectively from the pendulums. The temperature in the pendulum room, which was not sealed off from the rest of the mine, remained at 10.35°C ±0.03° throughout 1968. Throughout 1969, using a a different thermometer, the temperature was 10.27°C ±0.03°.

CHAPTER 6

DATA ANALYSIS

We will be concerned here with the problem of determining the amplitude and phase of the constituent waves in earth tide gravimeter and tiltmeter records.

6-1 Editing The Data.

The chart records were digitized using a line-follower device recording directly on magnetic tape. The output of this preliminary phase is put on cards. The chart values were scaled to a maximum value of 999 so that a single card carries 24 hourly values of 3 figures in addition to necessary identifying information. It will be shown later that the error associated with this level of resolution is negligible in comparision with the errors due to other causes. The data cards were next edited for isolated errors or level changes, missing data was estimated by interpolation where possible, the non-tidal noise level was estimated and finally, calibration constants were applied and corrected cards punched out. These editing functions were accomplished with the aid of program BETTY which will now be described.

Errors or level changes in the data are detected in a smoothness test (Cartwright, 1966) which compares each hourly data with a value found for that hour by interpolation according to the formula:

$$X'(n) = .75[X(n+1)+X(n-1)] - .30[(X(n+2)+X(n-2)] + .05[X(n+3)+X(n-3)] \qquad (1)$$

The difference (Xn-X'n) is zero for any 5-th order

polynomial. An isolated error produces a recognizable symmetrical sequence of differences and a general shift in level produces an anti-symmetrical sequence. The original data is corrected for isolated errors revealed by this test and level changes are subtracted from all subsequent data by adding to the program input assignment cards of the form FST(J,M)=C, where J is the day and M is the hour of the level change of magnitude C. A second running of the program will reveal whether the corrections have been applied correctly. Short sections of missing data, not exceeding a day in length, may sometimes be estimated at this stage. Equation 1 is used again for this interpolation with consecutive terms separated by 1, 2 and 3 days rather than 1, 2 and 3 hours. It can be shown that an interpolation made with this linear combination is not affected by the presence of periods of 12 hours or 24 hours. (Melchior, 1965, p71).

After the data has been proof-read and corrected in this way the computer is caused to apply calibration constants, estimate noise level and output data cards usable by the analysis program JANET, which will be described later. Daily calibrations are determined by interpolation between whatever calibration constants are entered as input. Noise is estimated by applying to the data for each day, a linear combination which discriminates against tidal periodicities. This is described by:

$$Y(t) = \Sigma Wi\ y(t-i)$$

where $y(t)$ is the hourly data, Wi is a term of the linear

combination and Y(t) is the result of the linear combination and represents in this case the effect of noise only. If the noise in the original data is random with an rms value of ϵ then:

$$Y^2(t) = \epsilon^2 \Sigma Wi^2$$

In the case of the Lecolazet (1959) filter used here: $\Sigma Wi^2 = 1872$. If there are n days of data then:

$$\epsilon^2 = [Y^2(t) + Y^2(t+24) \ldots\ldots Y^2(t+n24)]/m\Sigma Wi^2$$

A typical value for tiltmeter records is given by Melchior as ϵ=0.3 mm. We can compare this to the error due to quantization when the resolution of a 250mm record is 0.001. The scale unit then is 0.25mm and the standard deviation of the quantization error is (Bendat, 1966), 0.29 x .25 = 0.075 mm. This is negligible in comparision to the noise present in the record due to other causes. Program BETTY is described in more detail in the documentation presented in the appendix.

6-2 The Fourier Method.

It was shown earlier that the potential of the tide-producing force can be written as a series of terms involving six fundamental frequencies. The six frequencies are not commensurable and as a result the tidal potential is not periodic in the sense that the function X(t) is periodic when for any integer n: X(t) = X(t + nT) where T is some finite time. On the other hand the tidal potential is perfectly predictable and its spectrum consists of a finite number of discrete lines. It is therefore fundamentally

different from non-periodic phenomena in general which is usually transitory and is represented in the frequency domain by a continuum of energies. The term 'almost-periodic' has been used (Bendat, 1966) to describe the predictable non-periodic function. The distinction is important because it is on these considerations that an optimum means of analysis is determined. If the function to be analysed is periodic then it can be precisely represented by a Fourier series. The series will contain all possible constituent waves; the complete orthonormal set of functions for the specified interval. The coefficients of the series are found term by term by multiplying both the series and the given function by a Fourier function and integrating throughout the interval. This turns out to be a simple operation because of the orthogonality of the Fourier functions. If the function to be analysed is not periodic it may be assumed to have a period equal to infinity and a Fourier representation as before. In this case however the Fourier coefficients must be determined by integration over an infinite interval. Since the given function will generally not be known for all times it must be replaced by a truncated function equal to the given function throughout the period in which it is defined and vanishing beyond this interval. We can in this way find the Fourier representation of the truncated function. This may or may not be a good representation of the real function. Truncation can be considered as multiplication of the given function by

another function of time. In the frequency domain this is represented by convolution of the two corresponding frequency functions which, in this case, results in a 'smearing' of the line spectra of the given function. Adjacent lines will tend to widen and merge. When the given spectrum is mainly a band limited continum with very little line content truncation effects are not so serious. The Fourier transform and the power spectral density function, which are both developed from the Fourier representation described here, are useful in cases where the chief aim is to establish the frequency content of the given function. Here the situation is different; we are concerned with determining the amplitude and phase of a limited number of waves of known frequency. If there are p frequencies present we know that the function can be represented by the series:

$$y(t) = B1e^{j\omega t} + B^*1e^{-j\omega t} + \ldots\ldots\ldots\ldots Bpe^{j\omega t} + B^*pe^{-j\omega t}$$

The conjugate coefficients Bj and B*j must be determined by comparision with the given function. If there is any noise at all in the given function the determination can not be precise and some criterion of 'goodness of fit' must be assumed. It can be shown that if the interfering noise is normally distributed then the most probable value for the coefficient is obtained by minimizing the squares of the residuals (Murray, 1965). In this way we are led to the least-squares method of approximating almost periodic functions in terms of the known constituent waves.

6-3 The Least Square Method

There is a connection between the Fourier series method and the method of least squares. It can be shown that the Fourier coefficients give the best least-squares fit when a function is expanded in terms of an orthonormal set of functions. This is a special case of least-squares approximation; the orthogonality of the functions has the effect of 'decoupling' the normal equations reducing each to an equation in one unknown.

In general the functions of the least-squares approximation will not be an orthogonal set and consequently they will be, at least to some extent, linearly dependent. The solutions to the normal equations will become less and less well defined as this interdependence grows until finally when the determinant vanishes a solution becomes impossible. Since there will be a greater degree of interdependence between the waves of neighboring frequency than between waves having widely different frequency the question arises whether a 'smearing' effect similar to that encountered with the Fourier transform will be met with too in a least-squares approximation to a non-periodic function. This question has been dealt with by Munk and Hasselmann (1964), who compare the results obtained by the least-squares method with that obtained using the Fourier transform in the case of a data spectrum containing two neighboring lines plus a broad-band noise. A truncation effect is found with both methods but the 'smearing effect' of the Fourier method is found to

depend on the record length while smearing in the least-squares approximation is shown to depend on record length only through the accompanying noise and will be absent if there is no noise. Munk and Hasselmann demonstrate that resolution significantly better ('super resolution') than the inverse of the record length is therefore possible in a least-sqares approximation; this is a compelling argument for the choice of the least-squares method for the analysis of almost-periodic functions. Important too is the convenience of the least-squares method in being able to use data in the form of arbitrary-time samples of the given function. Gaps in earth-tide records are common, particularly in tiltmeter records, because of the difficult operating conditions met with in underground sites. The least-squares method is the only feasible method of data analysis when the record gaps are too large to be filled in by interpolation and the length of any continuous section is too short for a Fourier analysis. Beyond this, it may be more economical and just as reliable in many cases to obtain separation of neighboring constituents by a series of short recording intervals separated by long gaps than by a single long recording.

For these reasons I chose to use a least-squares method for analysing the earth-tide data. This analysis is implemented by program JANET.

6-3-1 Program JANET

This program is based on a least-squares method

described by Venedikov (1966). In principal a least-squares method could be made to work for data observed at arbitrary times but the presence of drift would greatly complicate such a general method. To avoid considering drift, Venedikov's method requires a basic set of 48 continuous hours of data, although many such sets can be used in the analysis and they may be arbitrarily spaced in time. Each 48 hour set is filtered digitally to obtain two quadrature related numbers proportional to the diurnal wave amplitude in the data and two numbers proportional to the semidiurnal wave amplitude. The same four filters are applied to data determined theoretically for the site assuming a rigid earth. In this way four independent condition equations are determined by each 48 hour set. If there are a great number of such sets we could assume correction factors for the amplitude of each of the theoretical constituents and determine the most probable value for the factors on the basis of the least-squared residual of the condition equations. The resulting normal equations however would be ill defined unless the data span was very long relative to the reciprocal of the frequency difference between adjacent constituents. The customary remedy for this difficulty, and the one followed by Venedikov, is to assume that the amplitude factors (complex) are the same for all the constituents contained within certain narrow frequency bands. For data spans of less than six months Venedikov assumes 6 such bands enclosing 52 diurnal constituents and 5

bands enclosing 27 semidiurnal constituents, for a total of 22 unknowns. This assumption is valid for describing the effect of the tide-producing forces in the solid earth. It may not be valid for describing the behaviour of the ocean tide however and in areas where the ocean tide has a significant effect on measurements of the earth tide the assumption must be examined critically. Frequently the amplitude of the ocean tide is small for all but the main constituent within a frequency band (Murray, 1964) so that any error in assuming a constant factor must be small. In addition, Munk and Cartwright (1967) find that the function describing the response of the ocean to the tide-producing forces (their 'admittance' function) is smooth and can be considered constant throughout frequency bands of width 1/6 cycles per day. This is much wider than the frequency band assumed by Venedikov.

6-4 Results

6-4-1 Rookhope Results

The results of the analysis of horizontal pendulum measurements made at this site during periods in 1968 and 1969 are presented in Table 6-2. Results for hydrostatic tiltmeter measurements made at the same site are shown in Table 6-4.

6-4-2 Ottawa Results

Results are presented in Table 6-1 for the analysis of horizontal pendulum tiltmeter measurements made at Holland Mills, Quebec. This site is located approximately 50 km from

Ottawa. In Table 6-3 results are shown for gravimeter measurements made at Ottawa.

ORB73-090 HOLLAND MILLS 237-277
1967

WAVE	RATIO	ERROR	PHASE	ERROR
2N2	0.933	0.019	1.386	0.021
N2	0.915	0.005	1.089	0.006
M2	0.900	0.005	-0.423	0.005
L2	1.464	0.025	-17.672	0.017
S2	0.831	0.002	-2.850	0.002
Q1	0.789	0.080	27.500	0.102
O1	0.612	0.051	-7.559	0.084
M1	1.366	0.161	-30.525	0.118
K1	0.792	0.053	-18.503	0.066
J1	0.475	0.243	86.828	0.512
001	0.848	0.171	-10.540	0.201

ORB74-180 HOLLAND MILLS 238-275
1967

WAVE	RATIO	ERROR	PHASE	ERROR
2N2	1.306	0.015	-22.750	0.011
N2	0.734	0.005	4.764	0.007
M2	0.790	0.004	6.866	0.005
L2	1.492	0.029	-51.263	0.019
S2	0.832	0.001	1.325	0.002
Q1	9.535	2.153	-67.674	0.226
O1	13.335	2.694	-30.873	0.202
M1	8.596	2.075	134.543	0.241
K1	11.319	2.627	-19.561	0.232
J1	14.910	6.785	-115.227	0.455
001	17.402	4.645	5.613	0.267

ORB73-090 HOLLAND MILLS 272-340
1968

WAVE	RATIO	ERROR	PHASE	ERROR
2N2	0.157	0.012	27.667	0.078
N2	0.907	0.003	4.817	0.004
M2	0.889	0.005	-0.786	0.006
L2	0.524	0.023	-42.413	0.043
S2	0.811	0.003	-1.842	0.004
Q1	0.537	0.071	-38.679	0.132
O1	0.640	0.036	-13.923	0.056
M1	1.096	0.194	-32.314	0.177
K1	0.720	0.054	-14.462	0.075
J1	1.480	0.259	-17.731	0.175
001	0.448	0.701	-95.073	1.567

ORB74-180 HOLLAND MILLS 272-340
1968

WAVE	RATIO	ERROR	PHASE	ERROR
2N2	0.401	0.010	-49.070	0.024
N2	0.639	0.005	-4.420	0.008
M2	0.760	0.003	6.190	0.003
L2	0.466	0.017	39.127	0.036
S2	0.861	0.002	2.020	0.002
Q1	5.518	1.888	-94.545	0.342
O1	14.334	0.928	8.672	0.065
M1	1.737	2.500	56.605	1.440
K1	10.954	2.988	6.362	0.273
J1	5.594	5.963	13.068	1.066
001	40.232	11.324	17.074	0.281

TABLE (6-1) Diminishing factors and phase lag (E.C.S) for 1967 east and south tilts (top) and 1968 east and south tilts (bottom). Multiply RATIO by 0.9 to obtain diminishing factor.

COMB188-270 ROOKHOPE 238-285 1968

WAVE	RATIO	ERROR	PHASE	ERROR
2N2	21.462	0.539	22.228	0.025
N2	1.436	0.220	134.510	0.153
M2	1.489	0.097	143.235	0.065
L2	6.094	0.527	-38.019	0.086
S2	1.591	0.227	89.613	0.143
Q1	3.118	1.027	-171.174	0.329
O1	0.505	0.432	-41.536	0.856
M1	5.504	1.398	16.357	0.254
K1	1.446	0.707	-14.053	0.489
J1	6.577	2.003	21.232	0.304
001	6.217	4.216	-68.644	0.678

COMB137-180 ROOKHOPE 238-285 1968

WAVE	RATIO	ERROR	PHASE	ERROR
2N2	14.110	0.333	-65.684	0.024
N2	1.510	0.140	29.417	0.093
M2	1.310	0.065	40.085	0.049
L2	3.446	0.362	-128.563	0.105
S2	1.638	0.040	13.700	0.025
Q1	3.609	1.157	76.303	0.321
O1	0.678	0.701	87.760	1.034
M1	10.652	3.336	-51.589	0.313
K1	1.062	0.847	-37.430	0.798
J1	4.732	2.916	-28.724	0.616
001	7.425	0.873	-172.274	0.118

COMB188-270 ROOKHOPE 163-217 1969

WAVE	RATIO	ERROR	PHASE	ERROR
2N2	2.754	0.059	-67.686	0.021
N2	2.564	0.061	161.067	0.024
M2	1.676	0.023	140.982	0.014
L2	5.584	0.152	87.394	0.027
S2	1.841	0.105	84.913	0.057
Q1	0.292	0.208	3.270	0.713
O1	0.422	0.040	16.717	0.094
M1	1.191	0.406	20.388	0.341
K1	1.000	0.051	-16.342	0.051
J1	0.479	0.406	-124.441	0.848
001	2.788	0.943	112.003	0.338

COMB187-180 ROOKHOPE 163-217 1969

WAVE	RATIO	ERROR	PHASE	ERROR
2N2	1.259	0.063	178.798	0.050
N2	1.759	0.045	51.694	0.025
M2	1.317	0.019	33.719	0.014
L2	3.593	0.025	-3.510	0.007
S2	1.846	0.011	6.274	0.006
Q1	0.258	0.363	99.444	1.409
O1	0.836	0.100	22.947	0.120
M1	4.760	0.928	-27.548	0.195
K1	0.973	0.083	-13.081	0.085
J1	1.069	0.966	171.775	0.903
001	1.098	1.795	-68.889	1.635

TABLE (6-2) Diminishing factors and phase lag (E.C.S) for 1968 west and south tilts (top) and 1969 west and south tilts (bottom) at Rookhope. RATIO equals diminishing factor.

LR12 GRAVIMETER OTTAWA DAY 047 TO 093 1969

WAVE	RATIO	ERROR	PHASE	ERROR
2N2	1.810	0.025	27.078	0.014
N2	1.090	0.011	3.755	0.010
M2	1.196	0.004	-1.497	0.003
L2	1.843	0.022	-2.990	0.012
S2	1.199	0.001	-2.321	0.001
Q1	1.071	0.073	-3.144	0.068
O1	1.173	0.029	-0.903	0.025
M1	1.134	0.100	-6.870	0.088
K1	1.168	0.025	0.169	0.021
J1	1.171	0.091	11.228	0.078
001	1.638	0.012	-1.101	0.007

TABLE (6-3) Gravimetric factor and phase lag (E.C.S) for 1969 at Ottawa. RATIO equals gravimetric factor.

HYD01-180 ROOKHOPE 206-259 1969

WAVE	RATIO	ERROR	PHASE	ERROR
2N2	0.420	0.033	103.541	0.079
N2	0.532	0.017	60.493	0.033
M2	0.800	0.014	57.409	0.017
L2	1.037	0.084	71.013	0.081
S2	0.943	0.027	23.959	0.029
Q1	0.522	0.176	-139.852	0.338
O1	0.596	0.049	30.329	0.083
M1	3.680	0.341	-75.334	0.093
K1	0.775	0.113	-12.077	0.146
J1	0.955	0.270	148.400	0.283
001	2.067	1.348	-103.916	0.652

TABLE (6-4) Diminishing factor and phase lag (E.C.S) for 1969 at Rookhope determined by the hydrostatic tiltmeter. Multiply RATIO by 1.04 to obtain diminishing factor.

CHAPTER 7

SPATIAL VARIATIONS OF THE EARTH TIDE

7-1 Introduction

It is necessary to state clearly the definitions adopted for the discussion which follows. Tidal-gravity amplitudes are measured in microgals and referred to either in this form or in the non-dimensional form called δ obtained by normalizing to the theoretical rigid-Earth tidal gravity amplitude at the site. This latter amplitude is given by:

$$V = 74.711 \cos^2\lambda c \quad \text{(microgals)} \qquad (7\text{-}1)$$

where λc is the geocentric latitude found from the geographical latitude λ by:

$$\lambda c = \lambda - (\sin 2\lambda / 5.1826) \qquad (7\text{-}2)$$

Tidal-tilt amplitudes are measured in milliseconds of arc and referred to either in this form or in the non-dimensional form called D obtained by normalizing to the theoretical rigid-Earth tidal tilt amplitudes at the site. These latter amplitudes, for the directions north and east respectively, are given by:

$$N = 7.8706 \sin 2\lambda c\,[\,1-(\sin^2\lambda c/297)\,][\,1+(2\cos^2\lambda c/297)\,] \qquad (7\text{-}3)$$

$$E = 15.741 \cos\lambda c\,[\,1-(\sin^2\lambda c/297)\,] \qquad (7\text{-}4)$$

These three tidal components are considered positive when directed away from the center of the Earth, toward the north and toward the east respectively. When expressed in the dimensioned form the phase is a lag with respect to the phase of the equilibrium constituent at Greenwich (E.C.G).

Thus the phases of the rigid-Earth components are given by:

$$gV = -2\theta \qquad (7\text{-}5)$$

$$gN = -2\theta - 180 \qquad (7\text{-}6)$$

$$gE = -2\theta - 90 \qquad (7\text{-}7)$$

where θ is the longitude, in degrees east of Greenwich, of the site. When the amplitude is expressed in the non-dimensional form it is more appropiate to use the phase relative to the phase of the rigid-Earth component at the site. This latter phase is the same as the phase of the equilibrium constituent through the site (E.C.S). The phase used in the non-dimenional case is given by the symbol k and is called phase lag re. E.C.S.

Reference is made to the 'global gravimetric factor' and to the 'global diminishing factor'. These are nominal values for δ and D intended to approximate the factors which would be experienced on the Earth if there was no ocean loading effect. Seismological evidence and model studies indicate that these factors are approximately 1.16 and 0.700 respectively.

The gravimeter site at Bidston is located on a hill overlooking tidal waters at a distance of only 2 or 3 kms. It is necessary here to calculate the direct attraction effect due to this height. Provision is made in program GLOBL to make this computation: The Green's Function for gravity is put equal to 1.0 for all distances and two computations are made, one with the height set equal to zero

and the other using the actual height; the difference between the two results is the desired attraction effect due to height alone.

7-2 Gravity and Tilt Observations in Britain.

7-2-1 Bidston

The gravity perturbation due to ocean loading is estimated by program GLOBL to be:

$$G = 1.978 \ (-78^{\circ}) \qquad (7\text{-}8)$$

This is calculated on the basis of zero height at the site. The attraction effect alone, assuming zero height is:

$$\Delta Ga \ (h=0) = .875 \ (-53.4^{\circ}) \qquad (7\text{-}9)$$

The attraction effect for the true height of 5425 cms is:

$$\Delta Ga \ (h=5425) = .574 \ (154.5^{\circ}) \qquad (7\text{-}10)$$

The net perturbation due to height effect alone is the difference between these two attraction terms:

$$\Delta Gh = 1.408 \ (137.5^{\circ}) \qquad (7\text{-}11)$$

Adding this to the perturbation found assuming zero height yields the net perturbation due to all effects of the ocean tide; denoting this as ΔGt we have:

$$\Delta Gt = \Delta G + \Delta Gh = 1.163 \ (-122.2^{\circ}) \qquad (7\text{-}12)$$

The M2 constituent of the gravity tide observed, assuming a gravimetric factor of 1.16, then should be:

$$Go = 1.16 \ Gt + \Delta Gt \qquad (7\text{-}13)$$

where Gt is the gravity tide which would be observed on an oceanless and rigid Earth.

This predicted result is compared with the IGY earth-tide gravity observations made in 1958 by Harrison et

Site	Observed	Calculated	Ocean-tide Contribution	Theoretical Rigid Earth
Bidston	30.94 (4.4°)	30.37 (4.4°)	1.16 (-122°)	26.79 (6.1°)
Winsford	32.84 (2.2°)	32.74 (1.4°)	3.84 (-67°)	27.03 (5.0°)
Llanrwst	-	25.2 (21.3°)	6.42 (119°)	26.83 (7.6°)
Rookhope	-	28.63 (-3.6°)	4.00 (-89°)	24.65 (4.4°)

TABLE (7-1) Comparison of observed and calculated gravity tide. (Amplitudes in μgals, phase lag re E.C.G.)

0	N.E. Atlantic Ocean	0.795 (-70°)
1	N. Atlantic (remainder)	1.06 (-75°)
2	S. Atlantic	.26 (-116°)
3	N. Pacific	.23 (- 4°)
4	S. Pacific	.67 (-125°)
5.	Indian Ocean	.53 (54°)
6	N. American coastal areas	.31 (117°)
7	Mediterranean area	.07 (-106°)
8	Arctic Ocean	
	Total Effect.....	1.163(-122°)

TABLE (7-2) Bidston - Contribution to gravity tide by global zones (microgals).

Poly. Nr.	Amplitude (phase)	Cal. tilt effect North	East	Cal. grav. effect	Height effect
1	274 (315°)	27.936	17.077	1.021	1.090
2	183 (325°)	5.518	5.910	.056	.030
3	107 (350°)	1.255	.209	.009	.004
4	320 (330°)	5.001	16.857	.196	.181
5	198 (345°)	2.339	5.029	.033	.023
6	76 (020°)	.317	.450	.004	.001
7	274 (316°)	.756	9.515	.065	.031
8	290 (315°)	20.666	13.100	.184	.086
9	274 (315°)	3.503	.803	.042	.007
10	290 (310°)	3.350	3.278	.058	.009
11	290 (310°)	.784	5.340	.075	.010
12	60 (318°)	.011	.607	---	---

TABLE (7-3) Bidston - Calculated effects due to local polygons

	Observed		Calculated		Ocean-tide contr.		Theoretical	
Site	North	East	North	East	North	East	North	East
Bidston	70.80(317)	54.03(137)	73.80(314)	47.72(129)	77.16(-43)	53.45(125)	7.55(186)	9.43(-83)
Winsford	-	-	11.74(314)	5.71(167)	15.60(-31)	9.98(128)	7.56(185)	9.46(-85)
Llanrwst	-	-	54.13(311)	9.11(325)	57.15(-45)	6.77(12)	7.55(188)	9.43(-82)
Rookhope[1968]	9.38(136)	13.05(131)	8.59(118)	9.39(120)	8.01(82)	15.36(110)	7.43(184)	9.12(-86)
Rookhope[1969]	9.38(139)	14.69(133)	"	"	"	"	"	"
Rookhope(1)	-	-	9.38(136)	13.05(131)	7.05(103)	18.55(119)	"	"
Rookhope(2)	-	-	9.90(113)	11.50(120)	9.58(82)	17.46(111)	"	"
Rawdon	26.40(136)	27.40(-48)	18.00(151)	20.77(-31)	20.52(148)	19.33(-53)	3.87(-52)	11.15(37)

TABLE (7-4) Comparison of observed and calculated tidal tilts (amplitude in msecs, phase lag E.C.G)

al (1963) in Table (7-1).

It is useful to consider the gravity perturbation contributed by each of the global zones to make up the total given in Table (7-1). These are shown in Table (7-2). The 12 polygons shown in Figure (7-6), which are local to the Bidston site, have a large influence on the gravity tide measured at the site. Data for these polygons are summarized in Table (7-3). Since the ocean-tide effect is small and due principally to local loading in the distance range where the Green's function for gravity has been extrapolated this result can not be used to infer any particular crustal structure.

Tilt measurements have been made at the Bidston site for many years. Lennon and Vanicek (1969) have recently summarized the results of tilt measurements made since 1952 using several different instrument types and methods of installation. These results cover a wide range of values and there seems to be no grounds for deciding which values are the more likely. A simple average of all the numbers referred to in their Table 2 yields the 'observed' results reproduced in Table (7-4) (Bidston) below. The 'calculated' values were determined by program GLOBL. It can be seen in Table (7-3) that the tilt effect due to local polygons is relatively enormous; polygon number 1 alone contributes 36 percent of the total north effect. This illustrates the importance of precisely representing the ocean tide when the site is very close. If the reliability of the tilt

observations warranted greater prediction accuracy these local polygons could be subdivided and the variation of the tide over the surface could be estimated more accurately. Improved accuracy could be obtained too by referring to the geographical coordinates of both the site and the polygons to the nearest .001 degrees rather than to .01 degrees as used here. For this distance range all these steps would be required before the predicted results could be compared to the observations with a view to appraising the suitability of the Green's Function used for tilt. In this particular case the measurements appear to be influenced by other effects and these refinements are not warranted.

7-2-2 Winsford

Calculated effects are presented for this site for both the gravity tide and the tilt tide although only the IGY gravity observations of Harrison et al (1963) are available for this site. (A short series of tilt measurements made by Tomaschek in 1953 at this site is chiefly of historical interest.) The close agreement between calculated and observed results demonstrates the probable validity of the extrapolated values assumed for the Green's Function.

7-2-3 Llanrwst

Gravity and tilt calculations are presented in Tables (7-1) and (7-4) for this new site established by G. W. Lennon in 1968. Observations at this site have not yet been published. The gravity result will be particularly interesting when it becomes available because the calculated

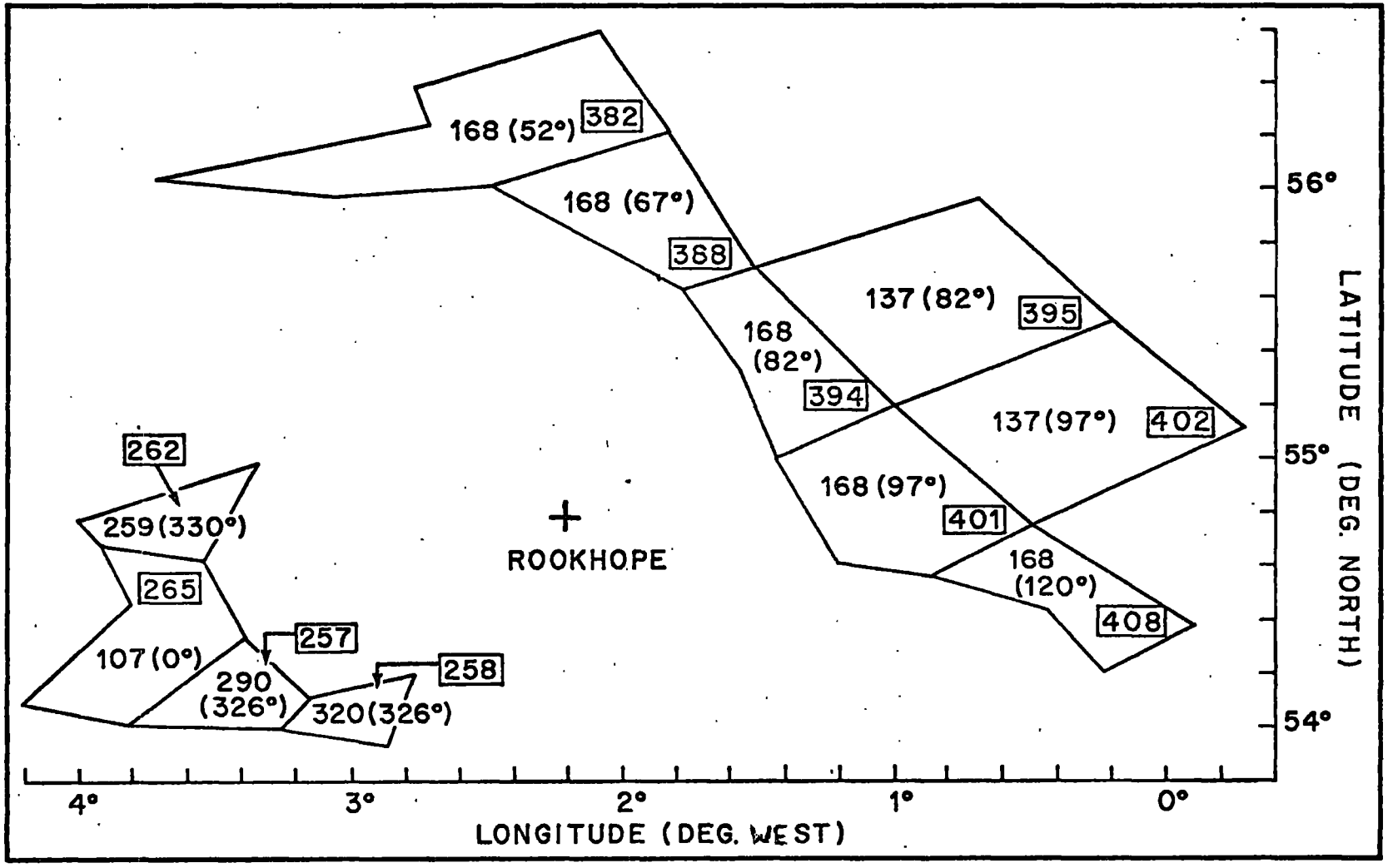

FIGURE (7-1) Ocean-tide polygons near Rookhope, England. Each polygon is numbered (blocks) and assigned an amplitude (in cms) and phase (lag relative to moon's transit of Greenwich). (Data from Admiralty Chart No. 5038).

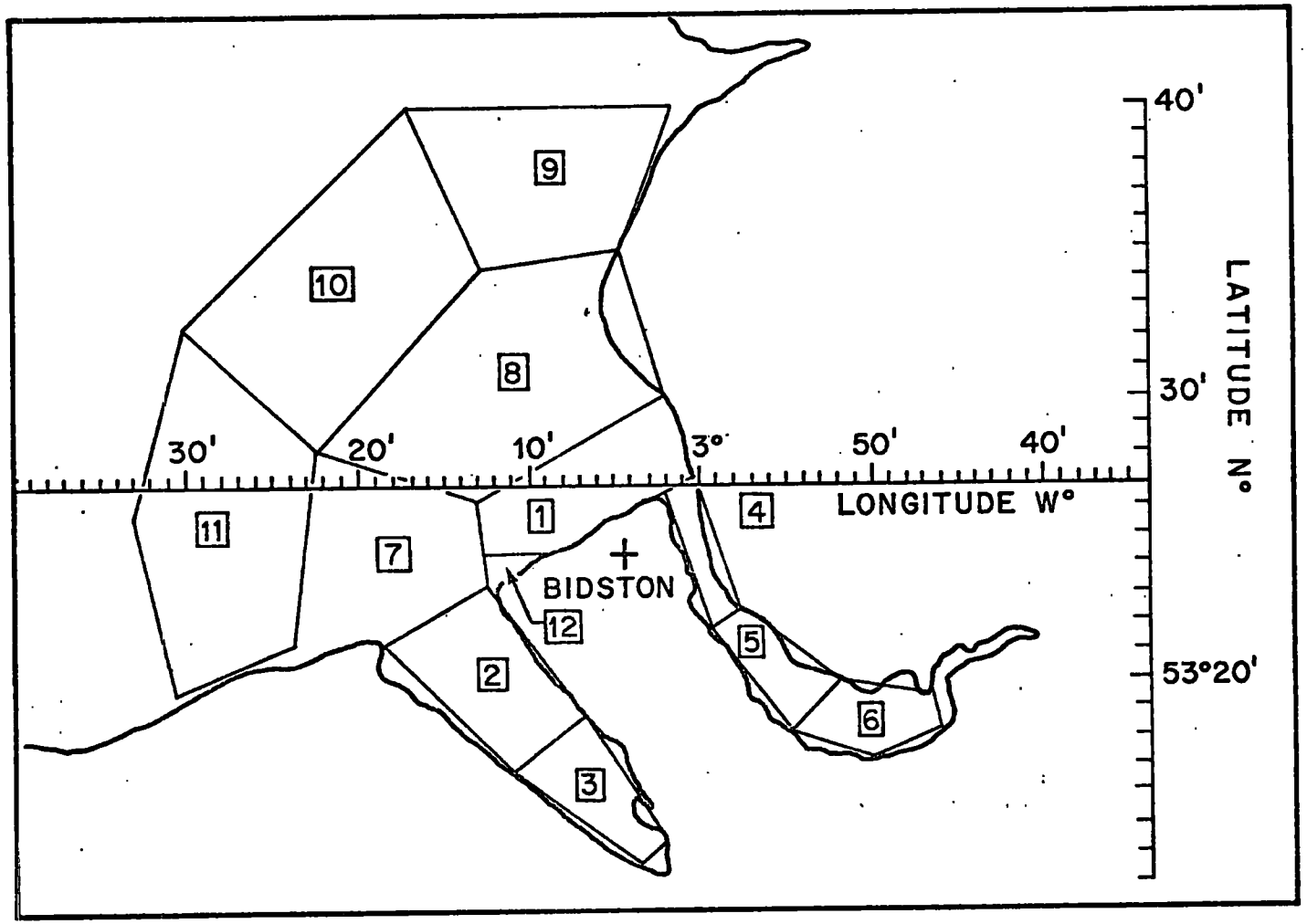

FIGURE (7-6) Ocean-tide Polygons near Bidston, England.

ocean-tide contribution is so large. Accurate observations of the gravity tide should lead to an experimental determination of the local Green's function accurate to within a few percent.

7-2-4 Rookhope

This site is ideally situated for investigating the elastic properties of the upper mantle by means of tilt measurements. Figure (7-1) shows the location of the nearest ocean-tide polygons; the site is approximately 60 kms from the nearest coast. According to Beaumont's results (1971-unpublished) the Green's Function for tilt is the same for a wide range of Earth models for distances greater than about 200 kms. In the distance range of 50-150 kms however the Green's Function for tilt is sensitive to upper mantle and deep crustal structure but not at all sensitive to the superficial layers near the surface.

Results for data obtained in the years 1968 and 1969 are presented in Table (7-4). Precisely the same north-south tilt was obtained in both years while the east-west tilt measured during the second year was about 12 percent higher than for the first. The net calculated tilt shown is the sum of the ocean-tide contribution computed by program GLOBL and the theoretical rigid-Earth tilt diminished by the factor 0.7, which is the global diminishing factor accepted throughout this thesis. There is relatively poor agreement between the calculated and observed results; particularly in the east-west direction. Working backwards from the observed

and theoretical values the ocean-tide contribution required for agreement of calculated and observed results can be found; this is shown in the line marked 1. Although the required east-west ocean effect is substantially larger than that determined by GLOBL the required north-south ocean effect is smaller. This rules out any suggestion that a simple increase of several percent in the Green's Function for distances from 50 kms to 100 kms is required. To illustrate the unsatisfactory effect the ocean-tide contribution has been recomputed for an increase in the Green's function for distances from 50 kms to 100 kms of 10 percent. (Line 2). As the Green's Function is increased to correct the east-west calculation the phase of the north-south calculation becomes increasingly in error. At least for a laterally homogeneous mantle the Green's Function assumed is probably nearly correct.

The next step in the treatment of this data should be to more precisely delineate the local tides shown in Figure (7-1); showing the large amplitudes near shore. If this better definition does not lead to better agreement between calculated and observed tilts then tilt measurements at another site in the same area should be made to confirm the peculiar features noted here. In particular if the large east-west tilt is confirmed it would constitute evidence for lateral inhomogeneities in the mantle or for possible non-elastic effects such as block-tilting. (Tomøschek 1953).

Site	Observed North	Observed East	Calculated (D = .7) North	Calculated (D = .7) East	Calculated (D = .6) North	Calculated (D = .6) East	Ocean-tide effect North	Ocean-tide effect East
Warmifontaine (1)	4.75(188)	9.65(-81)	4.67(181)	8.36(-86)	3.93(182)	7.80(-85)	.97(-58)	2.50(-51)
Warmifontaine (2)	5.01(185)	10.19(-84)	"	"	"	"	"	"
Dourbes (1)	3.32(181)	8.40(255)	4.17(186)	8.56(-86)	3.45(189)	7.60(-85)	1.54(-47)	2.26(-50)
Dourbes (2)	3.36(179)	8.04(256)	"	"	"	"	"	"
Sclaigneaux (1)	7.29(181)	8.70(252)	4.78(179)	8.42(-91)	4.04(180)	7.46(-90)	.80(-61)	1.92(-64)
Sclaigneaux (3)		7.86(250)	"	"	"	"	"	"
Remouchamps	2.86(180)	8.05(-85)	5.05(178)	8.58(-89)	4.30(179)	7.63(-88)	.72(-81)	2.32(-55)
Sclaigneaux (2)	6.19(179)	8.19(252)						
M e a n s	4.68	8.63	4.63	8.46				

TABLE (7-5) Comparison of observed and calculated tidal tilts. (Amplitude in µgals, phase re E.C.G)

7-2-5 A Suggested Site

As can be seen from Fig. (3-4) the ocean-tide loading effect in the south-west part of England should be exceptionally high. For a site located at latitude 50.50°N, longitude 356.00°E, the effects on gravity (μgals) and tilt (msecs) are computed by program GLOBL to be 10.332 (-44°) and 37.893 (-35°) north, 28.523 (-64°) east. The corresponding gravimetric and diminishing factors are 1.400 (-12°), 4.590 (130°) north, 3.618 (14°) east. These values are large and at the same time the particular site is more than 20 kms from the nearest coast so that there are no very local loads and thus no influence from superficial surface structure. For this reason this region is suggested as a site for measurement of the gravity tide for the purpose of determining empirically the near-load Green's function.

7-3 Belgium Tilt Measurements

Observed and calculated tilts are presented below (Table 7-5) for 4 sites in Belgium where anomalous results have been observed which Melchior has suggested may be due to upper mantle heterogeneites. For each series of measurements at each site I have listed the north and east components of tilt observed. For each site I have calculated and listed the ocean-tide effect, the net tilt which should be observed assuming this ocean-tide effect and a diminishing factor of 0.7, and the net tilt assuming a diminishing factor of 0.6. The small values observed for the north-south tilts at Dourbes and Remouchamps have been noted

by Melchior (1967) and similar results have been obtained at the neighboring sites of Chevron and Vielsalm. The very large north-south tilt observed at Sclaigneaux is at least as anomalous as these small values but has attracted less attention.

The difference between calculated and observed north-south results is between -2.19 and +3.01 milliseconds for the seven observations listed. This is about equal to the amplitude of the vector representing the tilt due to loading and attraction by the ocean tide. The corresponding differences for the east-west results are -.56 and +1.83.

This pattern of variations can not be explained by crustal or mantle inhomogeneities. All the sites are located more than 100 kms from the nearest coast thus the surface loading effects of the ocean tide will be determined mainly by the elastic properties of the mantle rather than the crust. Inhomogenities at this depth could not account for the small-scale regional variations evidenced by the results. The spatial change in the direct attraction of the water masses is similarly restricted; this can be seen from the Boussinesq solution for an elastic half-space which relates the attraction to the surface deformation. Thus the variation observed can not be the result of local differences in the response of the Earth to the ocean-tide surface force.

The body tide of the Earth is a function of even deeper layers (Takeuchi, 1950) and therefore, although the

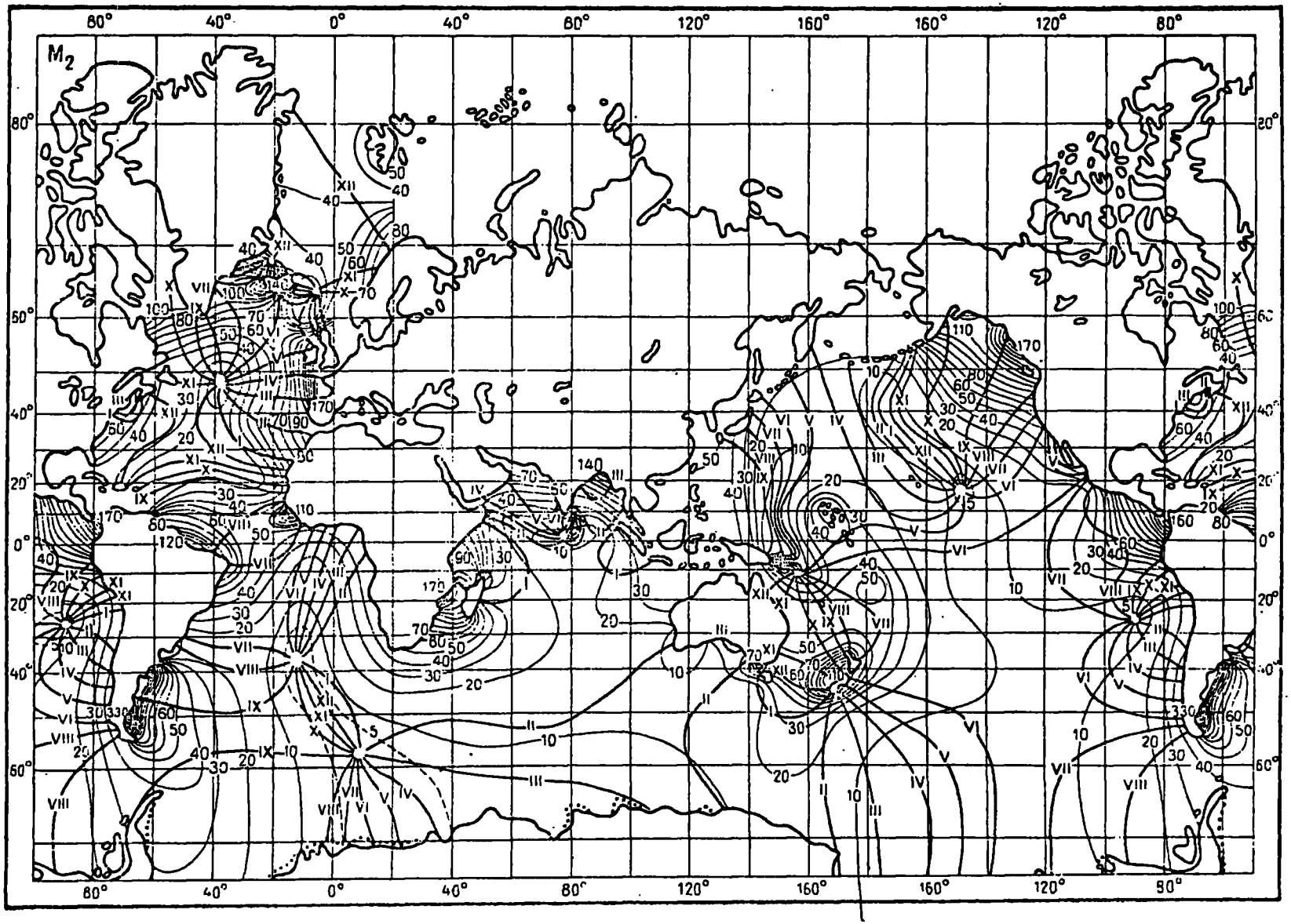

FIGURE (7-2) Coamplitude-Cophase chart of the global ocean tide. (Tiron et al 1967).

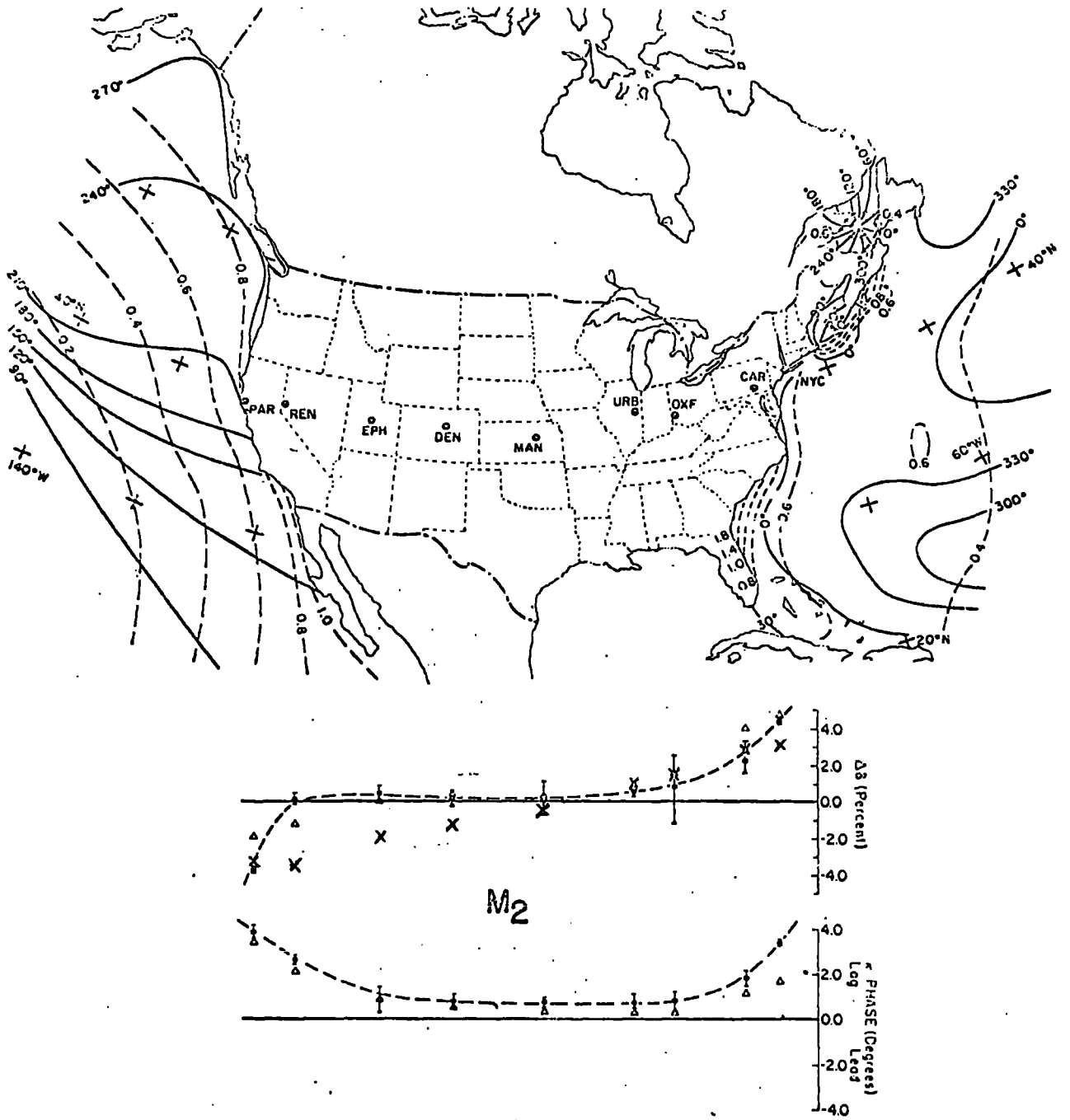

FIGURE (7-3) Trans-U.S.A earth-tide profile (Kuo et al 1969). Observed values (solid dots), calculated (triangles) by Kuo et al based on ocean-tide data shown, calculated (crosses) by GLOBL based on ocean-tide data described in text.

diminishing factor assumed may be in error by several percent, it must have the same value throughout regions having dimensions in the order of hundreds of kilometers. I have listed in Table (7-5) calculated results for each of the sites assuming a diminishing factor of 0.600 to illustrate the effect of this factor. Clearly the results can not be explained as spatial variations in the body tide.

The most likely reason for spatial variations of the diminishing factor on the scale noted here is the presence of large underground water courses. A water-saturated zone confined by impermeable walls, except for a few cracks through which water may be forced when the zone is compressed, has the ability to integrate the earth-tide body force throughout the entire zone and to concentrate certain of it's effects over a small area. This is demonstrated by the tidal variation in the level of wells. It is easy to imagine a situation where the tiltmeter is installed directly on a rock which caps a water-saturated zone and which is caused to tilt by tidal variations of the hydrostatic pressure in the zone.

Finally, it is interesting to observe the good agreement between the arithmetic means of the observed and the calculated results.

7-3-1 The Influence of Hydrology

Confined and unconfined aquifers in general are discussed by Brown (1968) and are illustrated in his Figure

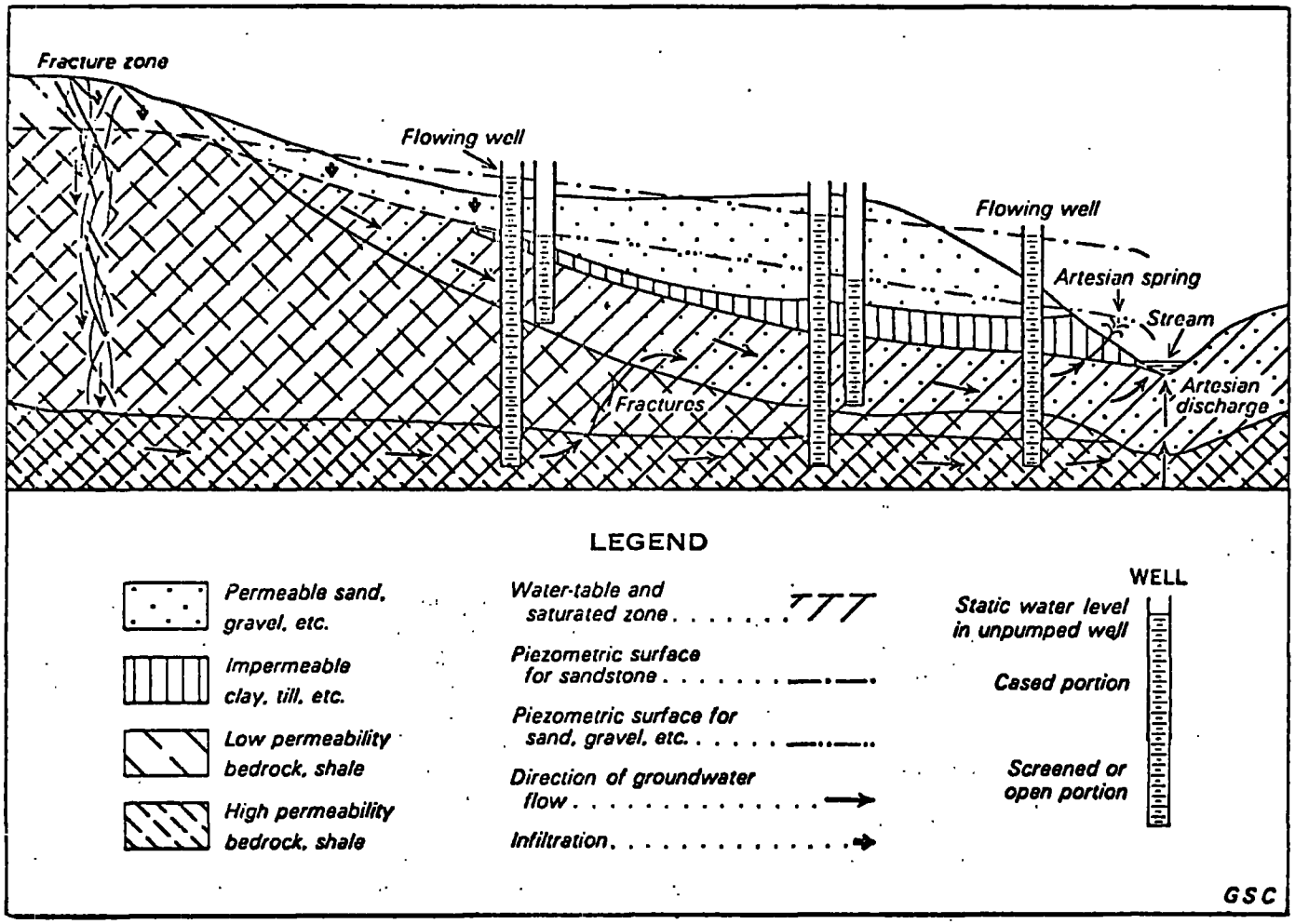

FIGURE (7-4) Confined and unconfined aquifers. (Geology and Economic Minerals of Canada, 1968).

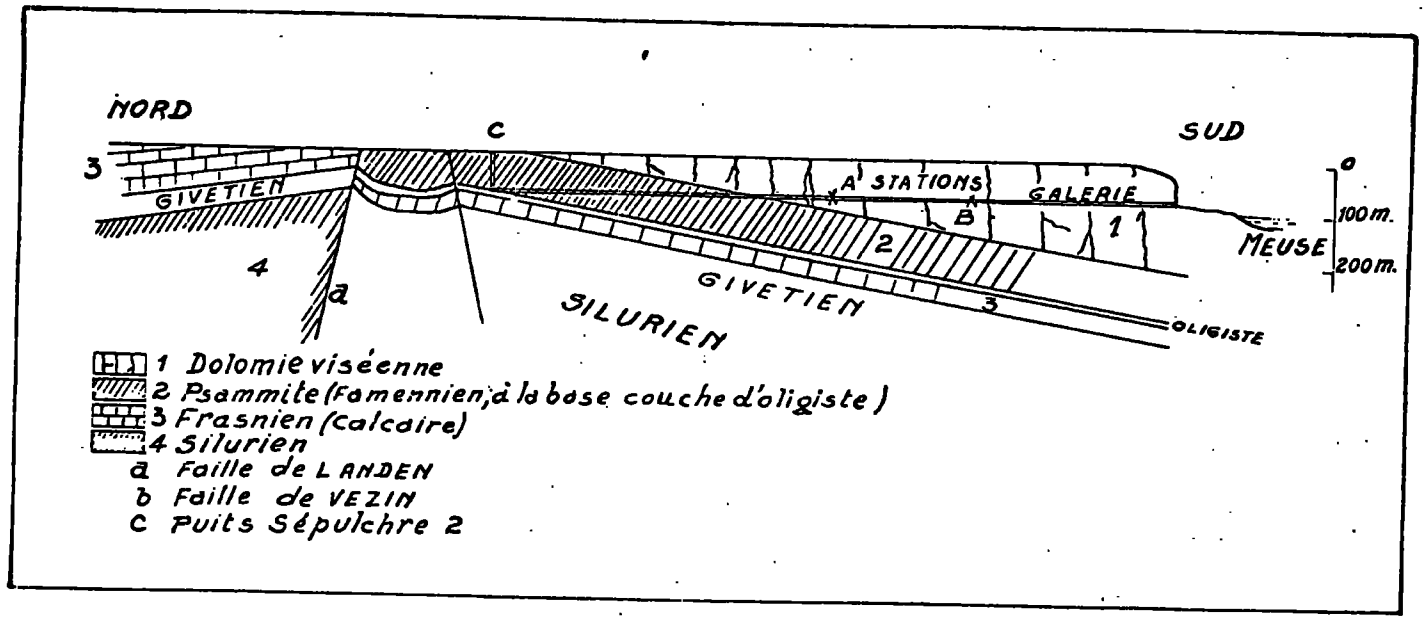

FIGURE (7-5) Geological section through the Sclaigneaux site. (Melchior and Brouet, 1964).

XIII-4 which is reproduced here in Fig. (7-4). Referring to this figure it can be seen that volume changes due to the strain of the earth tide will cause changes in the hydrostatic pressure in both the confined and unconfined aquifers. The pressure acting on the under surface of the impermeable layer will therefore vary periodically and the layer may be displaced as a consequence; perhaps differentially to cause the upper surface to tilt. In the practical case the hydrogeology is not generally known but a useful criterion for the possible influence of groundwater on earth-tide measurements may be the drift or long-term behaviour of the recorded tilt. If the measurements are influenced by groundwater then this behaviour should reflect the seasonal fluctuations due to groundwater recharge and discharge.

The site at Sclaigneaux is known to be affected by flooding of the near-by river Meuse. Melchior and Brouet (1964) have found that the observed tilt due to this flooding is about an order of magnitude greater than that due to loading of an infinite plane of rigidity equal to 10^{11} cgs units. They find too that the tilt-load relation is not linear nor is it completely reversible. A cross-section through the mine adit leading to the Sclaigneaux sites is reproduced in Figure (7-5) (Melchior and Brouet, 1964). Layer 2 is a sandstone much more porous than the dolomite of layer 1. The situation is similar to that illustrated in Fig. (7-4) and it is conceivable that the hydrostatic

SITE	δo	(κ)	LONG(E)	LAT(N)		Δδ	κ	δcorr	κ
Uccle	1.180	(-1.3)	4.21	50.48		.107	(-54)	1.118	(3)
Luxembourg	1.198	(-1.1)	6.08	49.37		.098	(-52)	1.138	(3)
Karlsruhe	1.211	(-1.0)	8.25	49.01	*	.080	(-48)	1.158	(2)
Hannover	1.179	(-1.5)	9.43	50.23	*	.080	(-48)	1.125	(1.5)
Kiesflbach	1.192	(4.9)	10.07	50.51	*	.080	(-48)	1.146	(8.0)
Bad Salzungen	1.131	(0.0)	10.14	50.48	*	.080	(-48)	1.079	(3.0)
Potsdam	1.209	(0.5)	13.04	52.23		.073	(-42)	1.156	(3.0)
Freiberg	1.127	(-0.3)	13.14	50.55	*	.073	(-42)	1.074	(2.0)
Berggieshubel	1.273	(3.6)	13.57	50.52	*	.073	(-42)	1.223	(6.0)
Dourbes	1.308	(0.0)	4.36	50.06		.106	(-53)	1.247	(4.0)
Strasbourg	1.202	(-1.0)	7.46	48.35	*	.09	(-52)	1.147	(2.5)
Frankfurt	1.180	(-1.0)	8.31	50.05	*	.08	(-48)	1.127	(2.0)
Bonn 1	1.202	(0.0)	7.5	50.44	*	.09	(-52)	1.144	(3.0)
Bonn 2	1.209	(-1.0)	7.5	50.44	*	.09	(-52)	1.150	(2.0)
Paris	1.195		2.33	48.83		.123	(-55)	1.129	(5.0)
Vedrin	1.197		4.85	50.50	*	.106	(-53)	1.136	(4.0)
M E A N S	1.200					.087		1.142	

TABLE (7-6) Comparison of observed and corrected gravity tide in western Europe. (Gravimeter factor δ, phase lags κ).

SITE	OCEAN-TIDE GRAVITY EFFECT		CALCULATED GRAV. FACTOR	
New York	2.039	(-179)	1.201	(-1)
Carlisle	1.280	(163)	1.190	(0)
Oxford	0.823	(133)	1.175	(-1)
Urbana	0.739	(119)	1.169	(-1)
Manhattan	0.719	(90)	1.156	(-1)
Denver	0.776	(80)	1.148	(-1)
Ephraim	0.927	(73)	1.141	(-1)
Reno	1.556	(61)	1.124	(0)
Pt. Arena	2.772	(43)	1.103	(1)

TABLE (7-7) Calculated gravimetric factors and ocean-tide effect (μgals) for Trans-U.S.A profile.

(Phase of ocean-tide effect is lag re E.C.G, phase of gravimetric factor is lag re E.C.S)

pressure on layer 1 is caused to vary with the earth tide. The effect of this pressure may be particularly complex because of the fractured state of the rock. There is thus sufficient reason to suspect the anomalous north-south results determined at this site.

The hydrogeological conditions at the other sites have not been discussed. Melchior has however singled out the Warmifontaine site as being particularly stable; both from the standpoint of long-term drift and of constancy in sensitivity. The observations from this station show the best agreement with the calculated results in Table (7-5).

7-4 Gravity Measurements in Western Europe.

The results of Earth-tide gravity measurements made in recent years at seventeen sites in Western Europe are listed in Table (7-6) together with calculated ocean-tide effects. The column headed $\Delta\delta$ is the perturbation of the gravimeter factor determined by program GLOBL. Values of $\Delta\delta$ marked by an asterisk were determined for neighboring sites and thus may be slightly in error. δcorr is the net corrected gravimetric factor found by subtracting $\Delta\delta$ from the observed factor δo.

It can be seen that the observed values are generally larger than the value of 1.16 ±.005 suggested by Alsop and Kuo (1964) as the most probable value of δ for the Earth as a whole. The calculated perturbation leads the phase of the equilibrium constituent at the sites by approximately 50° and has an average amplitude of .087. When these calculated

perturbations are subtracted from the observed gravimeter factors the result is generally too small relative to the probable global value. Taken individually the gravity measurements may reflect various unknown effects which have no geophysical significance. Taken as a group however this large number of measurements would be expected to yield a significant result and the low value of 1.142 for the average corrected gravimetric factor suggests a systematic error. Several possible reasons for a systematic error in the computation of the ocean-tide effect were considered and the most likely is the neglect of the ocean tide in the Norwegian and Greenland Seas. These waters were omitted because reliable tidal charts were thought not be available at the time the ocean-tide data was compiled for program GLOBL. The tidal charts of Tiron et al (1967) have become available since and these do include part of the area in question. Tiron's chart for the global M2 tide is reproduced in Figure 7-2. It will be seen that the tide in the Greenland Sea-Norwegian Sea area has a nearly uniform phase lag of 11 to 12 hours and a mean amplitude of 45 cms. This phase is in near opposition to the perturbing effect already noted and therefore will tend to yield larger values for the corrected gravimetric factors δcorr. A rough estimate of the maximum effect on the perturbing vector $\Delta\delta$ is 0.10 (.29 μgals) which, if applied uniformly to all the results, would raise the average gravimetric factor to 1.152. This result still seems low but definite conclusions must await a

detailed treatment.

7-5 North American Measurements

7-5-1 Gravity Along A Trans-U.S.A Profile

Kuo et al (1969,70) have conducted a tidal-gravity survey across the United States as a basis for investigating the spatial variations of tidal gravity. A total of nine sites were occupied, each for a minimum duration of six months with at least six sites in simultaneous operation at one time. Relative calibrations among the gravimeters was determined to within 1 percent throughout the measurements.

Their Figure 7, which is reproduced here as Figure (7-3), shows the location of the sites, the ocean-tide contours assumed, and the gravimetric factors which they observed (solid dots) and which they calculated (triangles) on the basis of a Boussinesq solution of the half-space loading problem. As a first approximation they neglected the effect of ocean tides beyond the area shown in the figure. Rather than extrapolate the Green's Function determined by Longman (1961) they chose to determine first the displacement by the Boussinesq solution and then to compute the gravitational effect of the displaced masses numerically.

The close agreement between their observed and calculated results (better than 1 percent except for one site) was taken by Kuo et al as confirmation of an earlier theoretical result (Kuo and Ewing, 1966) that the effects of

ZONE		OCEAN-TIDE EFFECT	ACCUMULATING OCEAN-TIDE EFFECT	PERCENT OF RIGID-EARTH TIDE
N.E. Atlantic	0	.077 (- 57)	.077 (- 57)	.1
N. Atlantic	1	.013 (-179)	.071 (- 66)	.0
S. Atlantic	2	.738 (-102)	.797 (- 99)	1.6
N. Pacific	3	.113 (26)	.704 (- 86)	- .3
S. Pacific	4	.363 (-134)	.983 (-102)	.8
Indian	5	.817 (75)	.170 (- 90)	-1.8
N. American Coast	6	1.796 (65)	1.643 (63)	-4.2
Mediterranean	7	.103 (- 85)	1.556 (61)	.2
			T O T A L	-3.6

TABLE (7-8) The calculated ocean-tide effect on tidal gravity measured at Reno, Nevada. (Ocean-tide effect in μgals and degrees lag re E.C.G)

ZONE		OCEAN-TIDE EFFECT	ACCUMULATING OCEAN-TIDE EFFECT	PERCENT OF RIGID-EARTH TIDE
N.E. Atlantic	0	.142 (- 55)	.142 (- 55)	-.3
N. Atlantic	1	.590 (-169)	.549 (-155)	1.1
S. Atlantic	2	.455 (-112)	.938 (-136)	.0
N. Pacific	3	.464 (50)	.479 (-142)	- .3
S. Pacific	4	.321 (-129)	.796 (-137)	.2
Indian	5	.828 (70)	.383 (142)	2.1
N. American Coast	6	.949 (168)	1.303 (161)	2.2
Mediterranean	7	.051 (- 81)	1,280 (163)	- .1
			T O T A L	+2.9

TABLE (7-9) The calculated ocean-tide effect on tidal gravity measured at Carlisle. (Ocean-tide effect in μgals and degrees lag re. E.C.G).

crustal and upper mantle structure on tidal gravity must be small. In view of the results presented earlier in this chapter this close agreement must be considered fortuitous. In the case of Bidston for example the perturbation due to loading by the tide of the South Pacific Ocean represents 2.5 percent of the rigid-Earth value and that due to the Indian Ocean represents 2.0 percent. A flat Earth approximation for determining the ocean-tide effect on tidal gravity is clearly not generally acceptable.

We have presented in Table (7-7) results calculated by program GLOBL for the ocean-tide effect on tidal gravity at each of the trans-U.S.A sites. These results have been plotted too in Figure (7-3) where they can be compared with the results of Kuo et al. We have assumed that their $\Delta\delta=0$ is with reference to the global-probable value of δ, 1.16. Their observed results for the western half of the profile now appear anomalous. This is an interesting result but is unlikely in view of the theoretical results of Kuo and Ewing (1966) and must be examined critically. In Tables (7-8) and (7-9) I have listed results calculated by program GLOBL for Reno and Carlisle; representative of western and eastern sites respectively. These tables show the influence of the various global zones on the ocean-tide effect and are presented here to demonstrate that low gravimetric factors calculated for the west and high factors calculated for the east are not due to distant ocean tides which were not considered by Kuo et al. Approximately the same result is

obtained when only the ocean tides bordering North America are considered. In this particular case the effects of distant ocean tides tend to cancel and therefore consideration of the ocean tides on the limited scale adopted by Kuo should produce fairly accurate results.

Comparison of the calculated results determined by program GLOBL with the calculated results of Kuo et al for ocean tides within a distance of 30° is subject to the criticism that the Green's Function used by GLOBL has been determined by extrapolation. To avoid this objection I will demonstrate in another way that $\Delta\delta$ for the western sites is probably substantially negative rather than near zero as assumed by Kuo et al. The gravity tide at Reno (measured positive outward from the Earth) due to body forces lags the equilibrium constituent at Greenwich by approximately 240°. High water occuring off the western coast at the same time will depress the site and cause a gravity effect opposing the body tide effect. In general, the loading effects of the regional ocean tides can be expressed as follows:

$\Delta\delta>0$ if $-30°<g<150°$ (7-7)

$\Delta\delta<0$ if $150°<g<330°$ (7-8)

where g is the phase lag of the regional ocean tide on Greenwich (E.C.G). The calculated results of Kuo et al mean that the loading effect of the ocean tide north of the 150° contour shown in Figure (7-3) is nearly cancelled by the effect of the ocean tide south of this contour. The position of the 150° cotidal line is therefore crucial in determining

SITE	GRAVIMETRIC FACTOR OBSERVED	CALCULATED	CALCULATED OCEAN-TIDE EFFECT	PERCENTAGE OF RIGID-EARTH TIDE
Ottawa	1.196 (+1.0)	1.190 (+1.0)	1.133 (172)	3.2 (21.0)

TABLE (7-10) Comparison of observed and calculated tidal gravity at Ottawa. (Ocean-tide effect in μgals and degrees lag re E.C.G)

ZONE		OCEAN-TIDE EFFECT	ACCUMULATING OCEAN-TIDE EFFECT	PERCENTAGE OF RIGID-EARTH TIDE
N.E. Atlantic	0	.189 (- 53)	.189 (- 53)	.5
N. Atlantic	1	.463 (-165)	.429 (-141)	1.2
S. Atlantic	2	.465 (-144)	.870 (-127)	1.2
N. Pacific	3	.353 (50)	.518 (-125)	1.0
S. Pacific	4	.393 (-125)	.911 (-125)	1.1
Indian	5	.805 (70)	.244 (178)	2.2
N. American Coasts	6	.908 (166)	1.148 (169)	2.4
Mediterranean etc.	7	.062 (- 85)	1.133 (172)	.2
			T O T A L	3.1

TABLE (7-11) The calculated ocean-tide effect on tidal gravity measured at Ottawa. (Ocean-tide effect is in μgals and degrees re E.C.G)

whether or not the western observations are anomalous. The particular tidal chart used by Kuo et al is one of several that have been postulated. Munk et al (1971) have recently gathered together the various charts available and have discussed them with respect to a new chart which they have derived based on off-shore tidal measurements. I have reproduced their summary of the tidal charts in Figures (4-10) and (4-11). The chart used by Kuo et al is based on that of Dietrich (1944). Amongst all the charts, it will be seen that the Dietrich chart contains the highest proportion of tidal water having a phase less than 150°; favouring a near-zero value for $\Delta\delta$. The most recent charts would all yield more negative results for $\Delta\delta$.

It is reasonable to conclude that the values of tidal gravity observed by Kuo et al along the western half of the trans-U.S.A profile are significantly larger than expected on the basis of a spherically-symmetric model of the Earth and the 'best' presently available ocean-tide data. The most likely reason for this result is large-scale errors in the ocean tide data, particularly throughout the central part of the N. Pacific Ocean.

7-5-2 Gravity Measurements At Ottawa

Observed and calculated tidal-gravity results are presented in Table (7-10). The contribution of the various zones is shown in Table (7-11).

7-5-3 Tilt Measurements

Results are presented here for Williams Bay, Wisconsin

SITE	OBSERVED NORTH	OBSERVED EAST	CALCULATED D=.7 NORTH	CALCULATED D=.7 EAST	CALCULATED D=.66 NORTH	CALCULATED D=.66 EAST
Holland Mills (1967)	.711(-7)	.811(0)	.664(-6)	.849(-3)	.624(-6)	.809(-3)
Holland Mills (1968)	.684(-6)	.800(+1)	"	"	"	"
Williams Bay	.676(-1)	.691(-6)	.720(+3)	.729(-4)	.680(+3)	.689(-4)

TABLE (7-12) Comparison of observed and calculated tidal tilts. (Diminishing factors with phase lags re. E.C.G)

(Yerkes Observatory), Holland Hills, Quebec (near Ottawa) and Rawdon Nova Scotia.

The observations referred to at Williams Bay are the classic Earth-tilt measurements of Michelson and Gale made in 1917 (Michelson and Gale, 1919). This was a water-tube type measurement using pipes 502 feet long and 6 inches in diameter buried 6 feet underground. This type of device in general offers several advantages over short-based instruments of the horizontal pendulum type: It is not sensitive to yielding at supporting feet, or to local tilting of rocks; the sensitive direction is clearly defined and the sensitivity can be readily determined in terms of fundamental units; it is inherently zero-stable and can be used to measure secular as well as periodic tilts. Because of these reasons the measurements of Michelson and Gale, which extended over a year, must be treated with a good deal of respect in spite of their antiquity. These results are shown in Table (7-12) together with results from the Holland Mills pendulum site and ocean-tide effects calculated for both sites. (Michelson and Gale seem to have misinterpreted their phase results. In their conclusion they refer to both the observed phases as lags with respect to the tide-producing forces. However, from the description given of their analysis method and from their illustrations it seems clear that the observed phases lead the theoretical phase. Phases shown in Table (7-12) therefore are shown leading). The lack of agreement between the 1967 and the

1968 Holland Mills north-south results indicates that neither is reliable. The east-west results on the other hand are quite consistent. The first calculated results shown were determined on the basis of a global diminishing factor of 0.700. This is simply the round number closest to the factor commonly observed. The second calculated results assume a factor of 0.66 and are much closer to the results observed. This is precisely the number found by Kelvin (1890) from observations of the fort-nightly tide in the Indian Ocean; the observations on which he based his now famous remark that the Earth was more rigid than steel. The number 0.66 was determined too by Schweydar (1907) using the same technique but with more frequent observations. The present result shows that the measurements of Michelson and Gale (1919) at a semi-diurnal frequency yield the same number when the measurements are corrected for the surface loading effect of the ocean tide. The number 0.66 is confirmed too by the calibrated pendulum measurements made at Holland Mills.

Rawdon is located in Nova Scotia near the large loading tides of the Bay of Fundy. Lambert (1970) has measured the two components of tilt there using Verbaandert-Melchior pendulums. His observed results are listed in Table (7-4) together with the calculated results assuming a global diminishing factor of 0.7 and an ocean tide effect computed by program GLOBL. Both calculated results are small in comparison with the results observed. The Green's Function

SITE	δo	(k)	Δδ	(k)	δc	(k)	Ocean-tide Effect	LONG °E	LONG °N
Bamako I	1.227	(-1.0)	.049	(- 58)	1.187	(-2)	3.380 (- 41)	351.99	12.39
Bamako II	1.180	(3.0)	"		"		"	"	"
New Delhi	1.153	(-2.0)	.008	(72)	1.163	(0)	0.496 (- 82)	77.18	28.43
Bunia	1.185	(0)	.017	(- 19)	1.176	(0)	1.305 (- 78)	30.18	1.53
Langchow	1.148	(3.5)	.015	(42)	1.171	(0)	0.715 (-167)	103.85	36.08
Novosibirsk	1.158	(1.1)	.030	(55)	1.178	(1)	0.715 (-113)	83.05	55.04
Tashkent	1.143	(4.3)	.015	(41)	1.171	(0)	0.590 (- 99)	69.30	41.53
Talgar	1.166	(3.0)	.017	(59)	1.169	(1)	0.637 (- 96)	77.15	43.15

TABLE (7-13) Comparison of observed and calculated gravimetric factors for sites in Africa, Asia and India (Ocean-tide effect in μgals and phase E.C.G)

used here was the same as that used to calculate the tilt at Bidston and Rookhope; the varying degrees of agreement obtained between observed and calculated results may reflect differences in crustal or (particularly in the case of Rookhope) upper mantle structure. Experimental work in this unique area is being continued by Beaumont with a view to determining an accurate finite-element model of the regional crust and mantle.

7-6 Africa, Asia and Japan

Results for seven sites are shown in Table (7-13). The ocean-tide effect determined by GLOBL for the four mid-Asian sites appears to be in error. Except for Talgar the observed values are all very low although the ocean-tide effect is computed as being additive. This may be due to major errors in the assumed distribution of ocean tides in the Indian Ocean or to omitting the effects of the tide of the Arctic Ocean. Tiron et al (1967) have recently presented a new cotidal chart for the Indian Ocean which is likely much improved over the purely theoretically-derived chart of Uneo (1965) used in program GLOBL. This new chart is being fitted to the program now. The absence of tidal charts for the Arctic basin remains a difficulty.

The first tilt measurements in Africa were made recently by Blum (1969) at Ecker, Algeria (Long 5.08 °E. Lat 24.05°N). The diminishing factors observed for the M2 constituent are D=0.668 for the east-west component and D=0.351 for the north-south component. This site is located

approximately 2000 kms from the closest ocean and thus should be little influenced by ocean tides. When the correction computed by program GLOBL is applied to Blum's factor the result is: D=0.608 (NS) and D=0.405 (EW). The only apparent explanation available for the anomalously low east-west factor is a hydrological condition such as that suggested previously to explain the Belgium results.

The results of tilt measurements made at 27 sites in the Soviet Union have been summarized by Aksentieva et al (1969). Although the ocean-tide effect for all of these sites is small the range of values determined for the diminishing factor is from 0.476 to 0.955; there is one value of 0.252. The means however are 0.653 (+0.015) for the north-south direction and 0.683 (+0.007) for the east-west. This is further support for a global diminishing factor near 0.66.

CHAPTER 8

DEVELOPMENT OF A SENSITIVE HYDROSTATIC TILTMETER

8-1 Introduction

The initial aim of the research described in this thesis was to experimentally observe tidal tilts at several points throughout Britain. The impracticability of this plan only became apparent as the difficulties of site preparation and the peculiarities of horizontal pendulum operation were realized. The basic difficulty is that the horizontal pendulum responds to the tilt of a base which is only 23 cms long. The great sensitivity required of the tiltmeter is obtained at the expense of stability of the zero or initial position of the pendulum and sensitivity to dimensional change. The short base line contributes too to the problem of placing the tiltmeter on the Earth's surface so that the tilt it experiences will represent the large-scale tilt due to the tide rather than spurious movements of the small rock or of the support pins on which the tiltmeter is placed. In practise these difficulties have the following consequences: The horizontal pendulum is difficult and expensive to set up; frequent and skilled attention is required to keep it in operation; site occupancy of one year is probably required for reliable (repeatable) results; the results obtained may not represent regional tilting but rather a spurious local effect; all pendulum tiltmeters exhibit a long-term drift and none are suitable for studying secular tilting.

Faced with these difficulties we abandoned plans to

measure tilt at more than one site and investigated alternative means of measuring tilt. Several modern tiltmeters of the short-base type were available but were rejected in favour of the classic water-tube tiltmeter used for the first time for measuring tidal tilts by Michelson and Gale in 1914 (see Section 7-7). Theoretically at least the hydrostatic tiltmeter is zero stable and it's sensitivity is determined solely by the length of the base line. Some additional advantages over the the horizontal pendulum are: The placement problem is no longer critical and installation is quick and cheap; the direction in which tilting is measured is precisely determined; a system sealed to prevent evaporation will operate for long periods of time without attention.

Extensive bibliographies on hydrostatic leveling are presented in Melchior's book (1965) and in Scheel (1956) who develops the theory and deals with the problem of eliminating errors due to temperature and pressure effects. Modern semi-portable hydrostatic tiltmeters are described by Eaton (1950), Bonchkovskiy and Skur'yat (1961), and by Eto (1966). None of these tiltmeters however provide a sensitivity near that of the horizontal pendulum, which is typically 1.0 mm of chart displacement per 1.0 millisecond of tilt. Melchior (1965, p91) quotes an rms error of 0.3 mm in reading the horizontal pendulum tiltmeter chart so that the rms error in reading the gross tilt (the sum of all harmonic constituents) is 0.3 milliseconds. To measure the

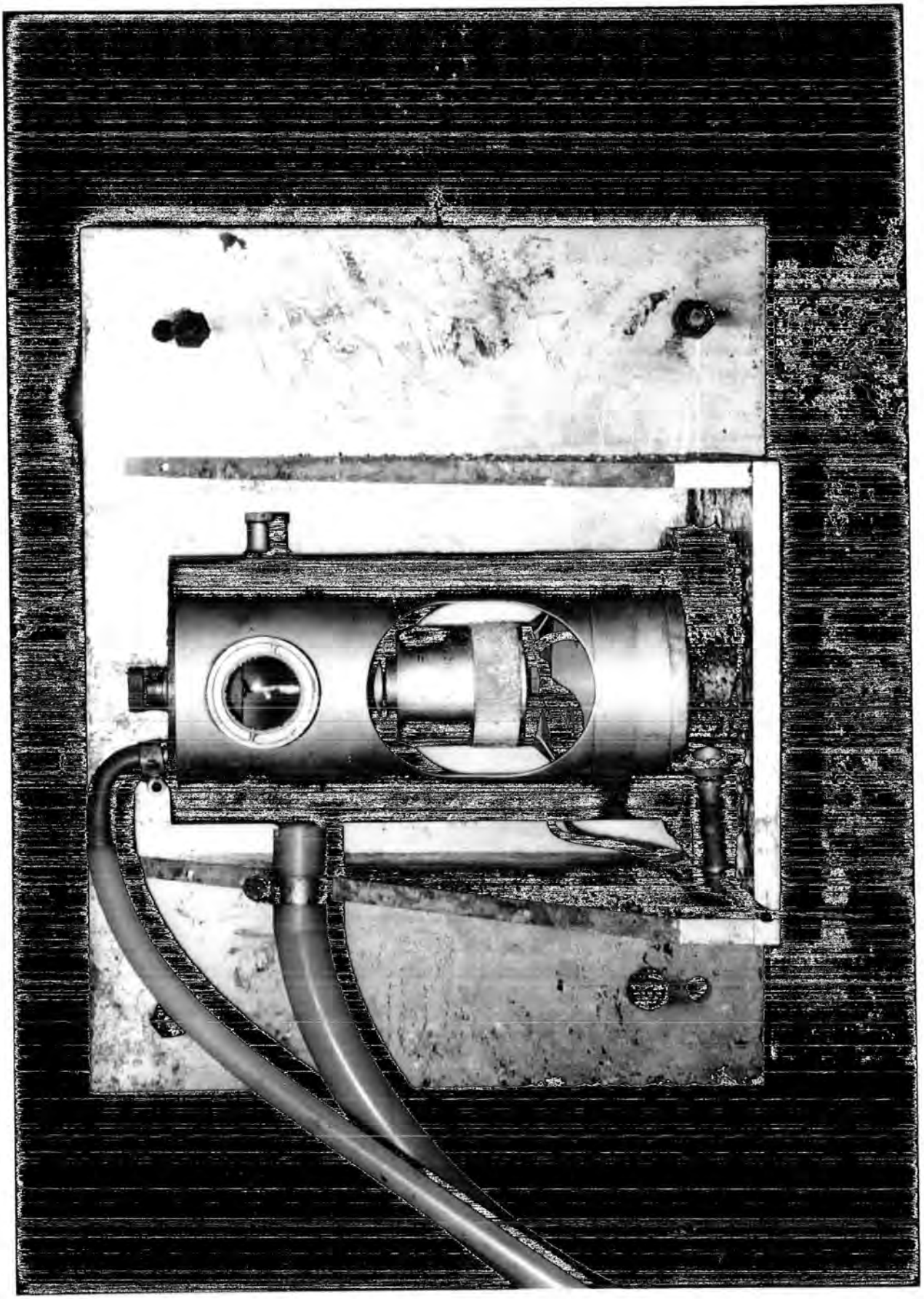

PLATE 8-1) The measuring pot of the water-tube tiltmeter mounted on a shelf bolted to the rock face.

tilt of a hydrostatic tiltmeter with a 100-meter base length to the same accuracy the water level at each end must be measured to an accuracy of ±0.065μ (±2.6 microinches). This is just within the capability of modern displacement measuring transducers of the differential transformer type. In particular a transducing system manufactured by the Boulton-Paul Aircraft Company of England measures the displacement of the armature of a differential transformer to an accuracy (at constant temperature) of 1 percent throughout a range of 250 microinches. The remainder of this chapter will deal with the design and use of a sensitive hydrostatic tiltmeter incorporating these Boulton-Paul transducers.

8-2 Description of the Apparatus.

The tiltmeter was designed originally as an exact copy of the visually-read tiltmeter described by Eaton (1959). This tiltmeter consists of two pots with connecting water and air tubes. The pots are approximately 10 cms in diameter and 20 cms in height. A manually-adjustable micrometer is fitted through the bottom of each pot through a rubber seal. This micrometer adjusts the height of a pointed-head spindle which, when viewed through a window in the wall of the pot, can be set to mark the level of the water. Variations in the water level can in this way be measured with an rms error of about 1.5μ.

To accept the Boulton-Paul transducers the upper end of the micrometer spindle was modified so that the transducer

could be inserted into it and sealed with epoxy. The transducer connecting leads were taken out through a hole drilled axially through the spindle and micrometer head. The transducer armature, which has the form of an iron rod of diameter 1.5 mm and length 1.5 cm, was fastened to a light plastic float so that when the float rested on the water surface the armature projected into the axial space of the transformer with approximately 0.2 mm clearance on all sides. Subsequent changes in the water level, within the linear range of the transducer, were then detected and outputted as proportional changes in voltage.

After laboratory tests and modifications, which will be described in the next section, the apparatus was installed in the long straight adit leading to the pendulum site. The pots were clamped to aluminum shelves secured to the rock wall with bolts and expansion nuts (Rawlplug) and located 60 m apart. The air and water lines were of diameter 0.93 and 1.27 cms respectively. The installation of one pot is shown in Plate (8-1).

8-3 Laboratory Tests and Modifications.

Laboratory tests of the water-tube tiltmeter began in October 1968 with the system filled with tap water. At first the output voltage was observed to drift off quickly in a series of steps or sudden 'tares'. Much of the drift was eliminated by rebuilding the rubber seal through which the micrometer entered the pot and by boiling the water to remove dissolved air. 'Tares' were still observed however

when the pots were tapped lightly. This was thought to be due to rubbing of the armature against the transducer body and a mechanical vibrator was constructed and fastened to the pot in an attempt to keep the armature in continuous motion. The vibrator completely removed the tares but when the sensitivity of the system was increased another trouble became apparent. On first sight this seemed to be a repetition on a smaller scale of the tareing effect noticed previously. The behaviour however was different; repeated tapping of the tiltmeter pot would cause the output to tare repeatedly in the same direction. This is consistent with the hysteresis effect of capillary forces described by Landau and Lifshitz (1959). The contact angle of a liquid on a solid surface is in general different depending on whether the liquid is advancing over a dry surface or receding from a previously wetted one. Changes in the capillary forces acting on the float are balanced by the buoyancy forces and the depth of immersion required to balance the capillary force is:

$$h = 2\gamma \mathrm{Cos}\theta/\rho g r \qquad (8\text{-}1)$$

Here γ is the surface tension of the fluid, θ is the angle between the surface contacted and the tangent to the fluid surface, ρ is the density of the fluid and r is the radius of the float. For the case here r=4.0 cm, ρ =1.0, γ=74.0 and the immersion depth assuming perfect wetting (θ=0°) is h=380μ. Clearly, variations in contact angle in general and the hysteresis effect in particular represent serious

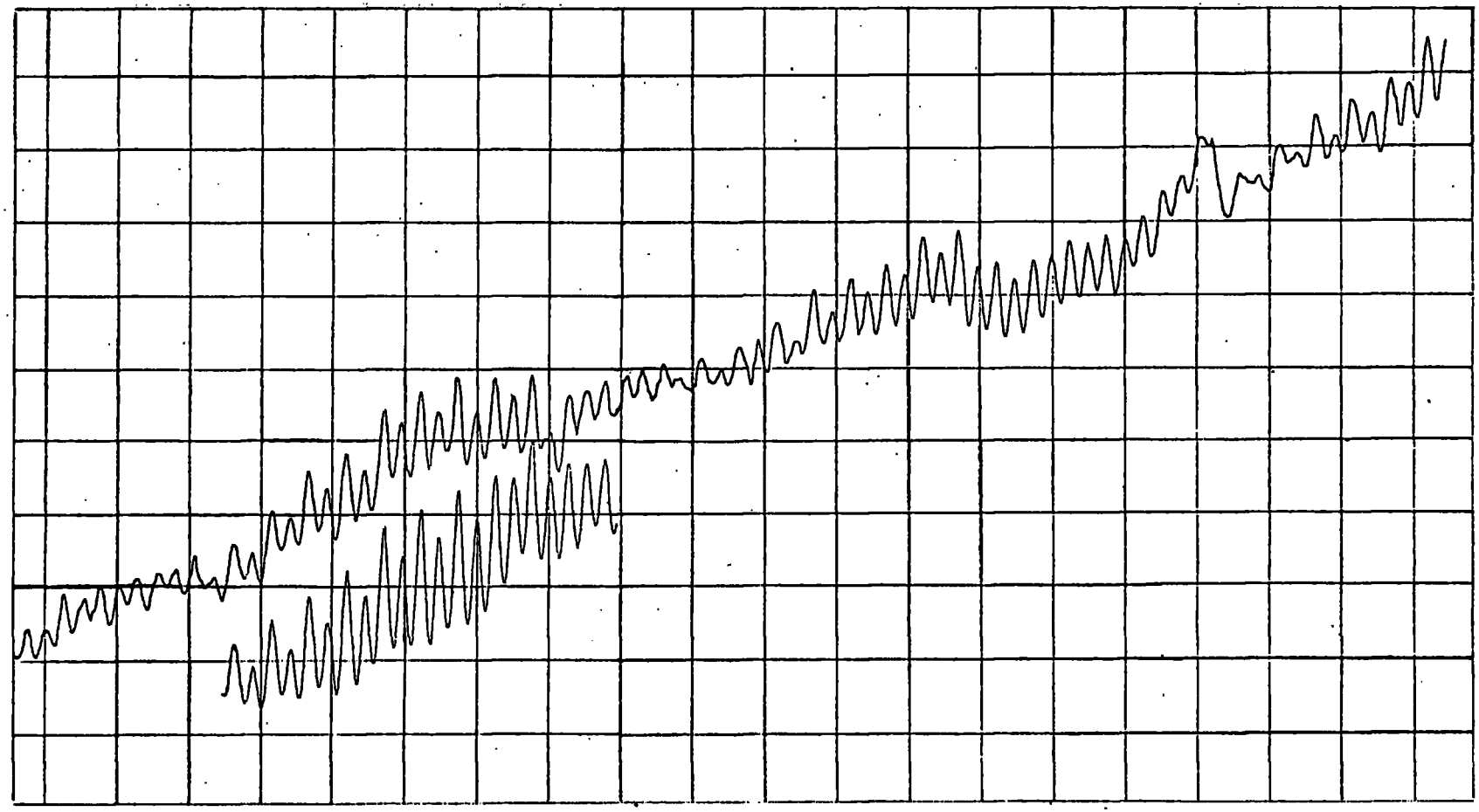

FIGURE (8-1) Records of tilt measured by the water-tube tiltmeter (upper trace) and by the horizontal pendulum (lower trace) at Rookhope, day 200 to day 240 1969. (Horizontal interval = 48 hours vertical interval = 0.100 arc seconds).

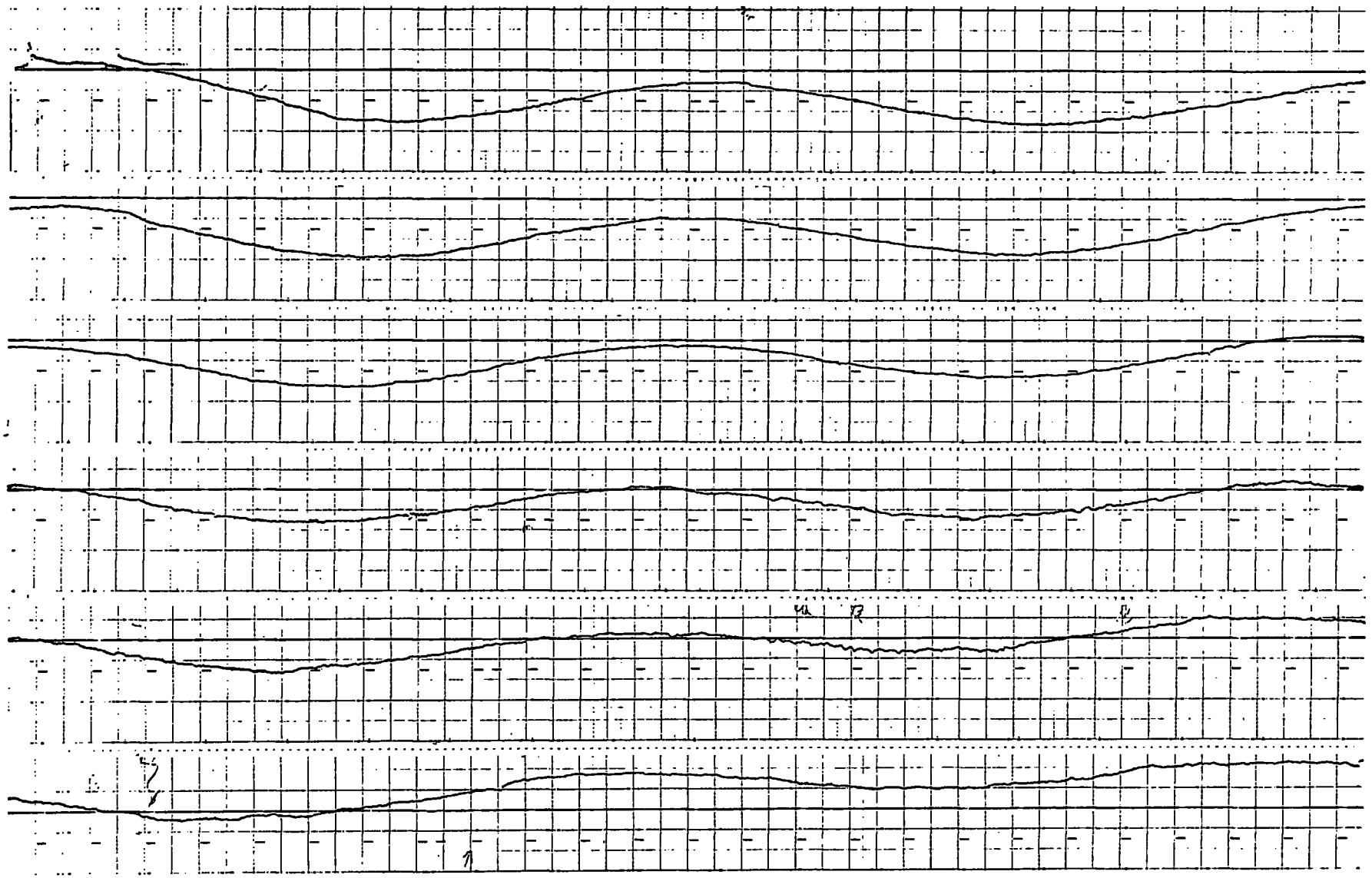

FIGURE (8-2) Water-tube tiltmeter record for the period 1200 GMT Aug 14, 1969 (top left) to 1700 GMT Aug 20, 1969 (bottom right) at Rookhope, England.
(Timing marks at intervals of one hour with double marks at midnight. Major vertical div ≈10 Msecs.)

problems since we are attempting to measure the water level to an accuracy of ±0.065μ. Given the float radius all that can be done to lessen the effect of variable forces of capillary is to choose a fluid with a low ratio of surface tension to density and ensure that all surfaces are as clean and as smooth as possible and either completely wetted (θ=0°) or completely unwetted (θ=180°) by the fluid. In the present case the surface tension was lowered by about one third by adding a detergent to the water. An examination then of the contact line of water and float revealed that the contact was very irregular. Thorough degreasing of the float improved on this situation but irregularities were still present apparently due to surface roughness. Finally the surface of the float was coated with teflon and polished smooth. This changed the contact angle drastically, from near perfect wetting to non-wetting. More important however was the fact that the contact angle now appeared uniform about the circumference. With these modifications the tiltmeter still showed occasional 'stiction' effects but they were much smaller than previously.

Before installing the tiltmeter in the mine the vibrator system was replaced with a loud speaker introduced into the air line. A speaker of diameter 18 cm was driven by approximately 10 watts of power at a frequency of 50 hertz. The configuration of the vibrating system was arbitrary except that the materials were convenient and it had the desired the effect.

8-4 Results

The water-tube tiltmeter was operated intermittently over the period April 1969 to March 1970. There were frequent breakdowns of the transducer electronics which had to be returned to the manufacturer twice for long periods. Unfortunately too, defects in the transducer electronics produced an effect which resembled the 'stiction' effect due to capillary forces. Consequently the latter effect could not be explored in a systematic way. Observations made during the period July 8, 1969 to September 16, 1969 were analysed by the methods described in Chapter 6 and the results are presented in Section 6-4-1, The tilt measured by the water-tube tiltmeter is anomalously low for all harmonic constituents. The manufacturer's calibration was used to determine the transducer sensitivity for this analysis. There was no convenient method of confirming this sensitivity on the assembled water-tube tiltmeter. In principle it would be sufficient to change the water capacity of the system by a precise amount, by dropping a ball bearing of known volume into one of the pots for example, but it was impossible to open the air seal of the system without introducing other effects. A method of calibrating which will avoid this difficulty is discussed in the next section.

In Figure (8-1) simultaneous tilt measurements by both the water-tube apparatus and the horizontal pendulums are shown for the period July 24, 1969 to August 6, 1969. The

drift behaviour of both tiltmeters is very nearly the same which suggests that it is real. The water-tube tiltmeter record is shown in more detail in Figure (8-2).

8-5 Conclusions.

The results demonstrate the feasibility of building a sensitive water-tube tiltmeter to replace the horizontal pendulum. For a system using a float to define the level of the water some error due to varying capillary forces seems inevitable. These variations can be minimized by using a large diameter, massive float and ensuring that its surface is uniformly clean and smooth and is either completely wetted or not wetted by the fluid. The hysteresis effect discussed in Section 8-3 seems unavoidable but can be kept small enough to be unimportant in tidal studies. It may be a handicap though in measuring secular tilts or in measuring residual tilts due to seismic events.

The detergent-water solution used here was selected on the basis of immediate availability. Propyl alchohol (preferred name is 1-propanol) has been suggested as being more suitable because of it's tendency to dissolve grease or other contaminants which might collect on the surface of the float.

The experiment demonstrated the need for a method for frequently measuring the sensitivity of the water-tube system without disturbing it in anyway. A means of doing this is suggested by the calibrating device of Verbaandert (1958) which was described in Chapter 5. A precise variation

in the water-carrying volume of the water-tube tiltmeter will induce an equally precise variation in water level in each of the pots. The precise variation in volume could be obtained by the elastic deformation of a Verbaandert-type device forming part of the system volume and caused to deform by the hydrostatic force exerted by a column of mercury. The height of the mercury column could be caused to vary periodically in the same way as in the Verbaandert method of pendulum calibration. The transducer outputs would be read out individually rather than as a difference and the sensitivity of each transducer determined.

The anomalously low sensitivity observed in the water-tube tiltmeter results is not the same for all constituents. This suggests that the tiltmeter is in some way distorting the harmonic constituents of the tide. The calibration apparatus described above is suitable for driving the system smoothly at any given frequency and therefore will be useful in determining both the frequency response of the system and the degree of non-linear distortion.

CHAPTER 9

CONCLUSIONS

A method of determining the effect of the ocean tide on measurements of the earth tide has been applied for the particular case of an 'extrapolated' Gutenberg-model earth. The results suggest values for the gravimetric and diminishing factors of 1.16 and 0.66 respectively although it is clear that the global ocean tides are not yet well enough known to correct accurately for their effects. The computed ocean-tide effects are consistent with observations throughout eastern North America but western observations of tidal gravity suggest a major error in the assumed ocean tide distribuiton in the North Pacific. Observations of tidal gravity throughout Asia, and to a lesser degree throughout Europe, lead to anomalously low gravimetric factors when corrected for the computed ocean-tide effect. The European measurements may be brought into line with theory when the ocean tide of the Norwegian-Greenland Seas, which has just recently become available, is considered. The anomalous Asian determinations may be due to the effect of unknown Arctic Ocean tides but are more likely due to systematic errors in the distribution assumed for the tide in the Indian Ocean.

In the above it can be seen that a gravimetric factor has been assumed and the tidal-gravity observations have then been used to appraise the accuracy of the ocean-tide data, rather than to determine the gravimetric factor. This

is the appropiate direction to take because seismological evidence limits allowable variations of the gravimetric factor to about 1 per cent which, in typical conditions, is less than the effect of assuming different but equally probable ocean-tide models. The case is different for tidal-tilt measurements because the effect of ocean tides at distances greater than about 50 degrees is negligible. This reduces the need for accurate ocean-tide data to a regional scale and a mid-continental site may require no ocean-tide correction at all. However, the large number of tidal-tilt measurements made throughout Europe and Asia during the past several years have produced a wide variety of results for the amplitude and phase of the diminishing factor. Many of the anomalous results have been shown to be reliable by parallel observations but the sites are too far inland for the discrepancies to be due to ocean-tide effects. The only apparent explanation for anomalous tilting at tidal frequencies is a differential surface displacement due to the effects of tidal strain in an underlying confined aquifer. The presence of such an aquifer effect is indicated when 'drift' tilting correlates with periods of precipitation or atmospheric pressure variations. This characteristic in the drift behaviour suggests a much-needed means of identifying inaccurate results and poor sites.

Aquifer effects may be lessened by using a tiltmeter having a longer base-line length; such as the water-tube tiltmeter described here. In any case, future emphasis on

water-tube instruments is suggested because of their inherent stability and ease of installation.

The determination of the effects on earth-tide measurements of very local ocean tides has been demonstrated and the basis has been established for using such local loads for investigating the mechanical properties of the crust and upper mantle.

APPENDIX I

Program JANET

```
1       JANET:PROCEDURE OPTIONS(MAIN);                                          00000100
        /*****************************************************************/00000001
        /*DESCRIPTION:                                                     */00000003
        /*THIS PROGRAM FINDS BY THE METHOD OF LEAST-SQUARES THE COEFFICIENTS*/00000005
        /*OF THE EXPANSION OF A GIVEN TIME SERIES IN TERMS OF THE 52*/00000007
        /*PRINCIPAL DIURNAL CONSTITUENTS OF THE TIDE PRODUCING POTENTIAL AND*/00000009
        /*THE 27 PRINCIPAL SEMI-DIRNAL CONSTITUENTS. THE GIVEN TIME SERIES*/00000011
        /*MAY BE OBSERVATIONS MADE BY EITHER GRAVIMETER (VORH=1) OR TILTMETER*/00000013
        /*(VORH=2). IN THE LATTER CASE THE AZIMUTH (AZ) FOR POSITIVE DATA*/00000015
        /*MUST BE GIVEN. AZIMUTH IS MEASURED CLOCKWISE FROM NORTH. THE TIME*/00000017
        /*SERIES MUST BE ENTERED AS AN ARBITRARY NUMBER OF COMPLETE 2-DAY*/00000019
        /*SETS. EACH SET MUST CONTAIN 48 CONSECUTIVE HOURLY OBSERVATIONS AND*/00000021
        /*BE IDENTIFIED BY COLUMNS CONTAINING THE DAY OF THE YEAR AND THE*/00000023
        /*LEAST SIGNIFICANT DIGIT OF THE YEAR. THERE MAY BE ANY NUMBER OF*/00000025
        /*SETS (N) (SEE STORAGE REQUIREMENTS), SEPARATED BY ANY NUMBER OF*/00000027
        /*DAYS, THE SINGLE PROVISO IS THAT IF THE DATA EXTENDS OVER MORE THAN*/00000029
        /*ONE YEAR THEN THE DAY NUMBERS OF SETS IN THE FOLLOWING YEARS MUST*/00000031
        /*BE INCREASED BY THE NUMBER OF DAYS IN THE PRECEDING YEARS.       */00000033
        /*INPUT REQUIRED:                                                  */00000035
        /*TWO GROUPS OF FIXED DATA ARE ENTERED FIRST. THE FIRST GROUP*/00000037
        /*CONTAINS ON 48 CARDS THE WEIGHTING COEFFICIENTS FOR THE FOUR*/00000039
        /*VENEDIKOV FILTERS. THE SECOND GROUP CONTAINS ON 79 CARDS ALL*/00000041
        /*REQUIRED DATA PERTAINING TO THE INDIVIDUAL CONSTITUENTS, INCLUDING*/00000043
        /*THE AMPLITUDE RESPONSE OF THE VENEDIKOV FILTERS TO THE*/00000045
        /*CONSTITUENT.THE N,2-CARD SETS ARE NOW ENTERED, PRECEDED BY TWO*/00000047
        /*HEADER CARDS. THE FIRST HEADER CARD (CRD1) IS NOT USED IN THE*/00000049
        /*ANALYSIS BUT SIMPLY COPIED AS A TITLE LINE FOR THE OUTPUT LISTING.*/00000051
        /*THE SECOND HEADER CARD SHOULD CONTAIN AZ, N, VORH, YEAR, DAYI, LAT,*/00000053
        /*AND LONG. THE FIRST THREE OF THESE HAVE BEEN EXPLAINED: DAYI AND*/00000055
        /*YEAR ARE THE DAY AND THE YEAR OF THE FIRST DAY OF THE FIRST SET;*/00000057
        /*LAT AND LONG ARE THE LATITUDE AND THE LONGITUDE OF THE SITE IN*/00000059
        /*DECIMAL DEGREES.                                                 */00000061
        /*AN ARBITRARY NUMBER OF THESE 2-CARD DATA SETS WITH HEADER CARDS  */00000063
        /*MAY BE USED.                                                     */00000065
        /*****************************************************************/00000066
2       ON ENDFILE(SYSIN) GO TO FIN;                                            00000190
4       DECLARE                                                                 00000200
            V FIXED,                                                            00000300
            (GRAV,YEAR,AZ,DAYI,FIRST,SECOND,                                    00000400
            VORH,LAT,LONG,SH,SP,MP,MN,MS)FLOAT                                  00000500
            INITIAL(0),                                                         00000600
            (HGR(79),HAZ(79),PHG(79),PHZ(79),Q(79),                             00000700
            D1(79),D2(79),D3(79),D4(79),D5(79),                                 00000800
            D6(79),PI(79),HO(79),CK(79),PHX(79))                                00000900
            FLOAT INITIAL((79)0),                                               00001000
            (CT1(48),ST1(48),CT2(48),ST2(48))                                   00001100
            FLOAT(12),(TEST,DLM)FIXED,                                          00001200
            (C(2,79),S(2,79),H(2,79))FLOAT,                                     00001300
            R(12)FLOAT,(Y(11),Z(11))FIXED,                                      00001400
            ANGLE ENTRY(FLOAT(12),FLOAT(12),FLOAT(12),                          00001500
            FLOAT(12),FLOAT(12))RETURNS(FLOAT(12)),                             00001600
            CTA(24)FLOAT,LIT(48)FLOAT,                                          00001700
```

```
                (I,J,JA,JB,JC,JM,JP,K,KAJ,KM,L,N,M)FIXED BIN(31),                    00001800
                (DAY2,DAY3,DAY4)FIXED BIN(31),                                       00001900
                CRD1 CHARACTER(80),T FLOAT(12),                                      00002000
                E(6) CHARACTER(3):                                                   00002100
  5             DO K=1 TO 48;                        /*READ IN COEFFICIENTS OF DIGI*/00002200
  6             GET EDIT(J,CT1(J),ST1(J),CT2(J),ST2(J),/*TAL FILTER (VENEDIKOV P467) */00002300
  7             DUM)(F(2),4 F(11),X(33),F(1));END;                                   00002400
  8             DO J= 1 TO 79;                       /*READ IN FOR 79 CONSTITUENTS:*/00002500
  9             GET EDIT(K,D1(K),D2(K),D3(K),D4(K),  /*THE DOODSON ARGUMENT NUMBER,*/00002600
                D5(K),D6(K),PI(K),Q(K),HO(K),CK(K),  /*PHASE,FREQUENCY AND AMPLITUD*/00002700
                C(1,K),S(1,K),C(2,K),S(2,K))(F(4),X(1),/*E DETAILS FROM MELCHIOR(P65)*/00002800
                6 F(1),F(3),F(11),F(6),F(2),X(4),    /* AND THE AMPLIFICATION FACTO*/00002900
  10            3 F(10),F(13));    END;              /*RS OF THE VENEDIKOV FILTER F*/00003000
  11            J=J-1;                               /*OR THESE CONSTITUENTS.      */00003100
  12            IF(J¬=K) THEN DO;                    /*     CHECK THAT LAST CONSTIT*/00003200
  14            PUT EDIT('LAST CON. CARD IS NOT 79  ') /*UENT CARD IS NO. 79. IF NOT */00003300
                (A)SKIP;                             /*THEN END PROCESSING.        */00003400
  15            GO TO FIN;END;                                                       00003500
  17            READ03:                                                            00003600
                GET EDIT(CRD1)(A(80));               /*READ TITLE CARD             */00003700
  18            GET DATA(GRAV,AZ,N,VORH,YEAR,DAYI,   /*READ DATA HEADER CARD       */00003800
                 LAT,LONG,TEST,FIRST,SECOND);                                        00003900
  19            AZ=AZ+180.0;                         /*CONVERTS EXTERNAL AZ TO     */00004000
                                                     /*INTERNAL FORM.              */00004100
  20            ANGLE:PROC(A,B,C,D,T)FLOAT(12);      /*****************************/00004200
  21               DCL(A,B,C,D,T)FLOAT(12);          /*FUNCTION SUBROUTINE FOR USE */00004300
  22               P=A+B*T+C*(T**2)+D*(T**3);        /*IN DETERMINING ORBITAL      */00004400
  23               P=P-(FLOOR(P/360))*360;           /*COORDINATES.                */00004500
  24               RETURN(P);END ANGLE;              /*****************************/00004600
  26            ORBIT:                               /*ORBITAL ELEMENTS:           */00004700
                T=(365*(YEAR-1900)+                  /*JULIAN CENTURIES TO 0000 GMT*/00004800
                FLOOR((YEAR-1901)/4)+DAYI-.5)/36525; /*OF DAYI.                    */00004900
  27            SH=ANGLE(279.6967,36000.7699,0,0,T); /* MEAN LONG. OF SUN.         */00005000
  28            SP=ANGLE(281.2208,1.7192,.00045,0,T); /* MEAN LONG. OF PERIHELION. */00005100
  29            MP=ANGLE(334.3296,4069.034,-.01032,0,T);/*MEAN LONG OF LUNAR PERIGEE */00005200
  30            MN=ANGLE(259.1833,-1934.142,.00208,0,T);/*ASCENDING NODE OF MOON.    */00005300
  31            MN=-MN;                              /*INVERTED FOR DOODSON'S NP.  */00005400
  32            MS=ANGLE(270.4366,481267.8906,.002,0,T);/*MEAN LONG OF MOON.        */00005500
                                                     /*****************************/00005600
                                                     /*  MELCHIOR   AMPLITUDE     */00005700
                                                     /*  FACTORS:                 */00005800
                                                     /*****************************/00005900
  33            AMPFAC:CST1=173854:CST2=.36309:CST3=1.29904:EE=.00335:CST4=1.0E-04;   00006000
  38                  CST5=1.0E-09:FFF=86669*980*(1-EE*(SIND(LAT)**2))/GRAV;          00006001
  40                  LAT=(LAT*1.745E-02)-EE*SIND(2*LAT):LAT=LAT*57.29577;            00006002
  42                  R(1)=8.2275*SIND(2*LAT);                                        00006003
  43                  R(2)=8.9618867E+0*COSD(LAT)*(1.0E+0-5.0E+0*(SIND(LAT)**2));     00006004
  44                  R(3)=16.455*(COSD(LAT)**2);                                     00006005
  45                  R(4)=2.59809*SIND(LAT)*(COSD(LAT)**2);                          00006006
  46                  R(1)=R(1)*CST4;R(2)=R(2)*CST4;R(3)=R(3)*CST4;R(4)=R(4)*CST4;    00006007
  50                  R(5)=-2*FFF*(COSD(2*LAT)-EE*(SIND(2*LAT)**2))*COSD(AZ);         00006008
  51                  R(6)=.72618*FFF*SIND(LAT)*(15*(COSD(LAT)**2)-4)*COSD(AZ);       00006009
```

```
52                R(7)=2*FFF*SIND(2*LAT)*(1+2*EE*(COSD(LAT)**2))*COSD(AZ);          00006010
53                R(8)=2*2.59808*FFF*COSD(LAT)*(2-3*(COSD(LAT)**2))*COSD(AZ);       00006011
54                R(5)=R(5)*CST5;R(6)=R(6)*CST5;R(7)=R(7)*CST5;R(8)=R(8)*CST5;      00006012
58                R(9)=2*FFF*SIND(LAT)*SIND(AZ);                                    00006013
59                R(10)=.72618*FFF*(1-5*(SIND(LAT)**2))*SIND(AZ);                   00006014
60                R(11)=4*FFF*COSD(LAT)*SIND(AZ);                                   00006015
61                R(12)=2*2.59908*FFF*SIND(2*LAT)*SIND(AZ);R(12)=R(12)*CST5;        00006016
63                R(9)=R(9)*CST5;R(10)=R(10)*CST5;R(11)=R(11)*CST5;                 00006017
66             KAJ=2*N;                              /*NUMBER OF OBSERVATION EQNS. */00006100
67             NBLOCK:BEGIN;                                                       00006200
68             DECLARE                                                           00006300
                (DAY(KAJ),MI(2,KAJ),TI(N),VC(2,79,N),                            00006400
                RES(KAJ))FLOAT;                                                   00006500
69             READ04:DO K=1 TO N;      /*************************************/00006600
                                        /*          READ04  LOOP             */00006700
                                        /*READ IN N, 2-DAY, 4 CARD SETS OF DATA. TEST*/00006800
                                        /*TO DETERMINE IF THERE ARE 2 CARDS PER DAY  */00006900
                                        /*AND IF THE 2 DAYS COMPOSING EACH SET ARE   */00007000
                                        /*CONSECUTIVE.                               */00007100
70                 GET EDIT(DAY(K),(LIT(J) DO J= 1 TO 12))(X(4),F(3),X(1),12 F(6));  00007200
71                 GET EDIT(DAY2  ,(LIT(J) DO J=13 TO 24))(X(4),F(3),X(1),12 F(6));  00007300
72                 GET EDIT(DAY3  ,(LIT(J) DO J=25 TO 36))(X(4),F(3),X(1),12 F(6));  00007400
73                 GET EDIT(DAY4  ,(LIT(J) DO J=37 TO 48))(X(4),F(3),X(1),12 F(6));  00007500
74                    IF((DAY(K)¬=DAY2)|(DAY3¬=DAY4)|(DAY2¬=(DAY3-1)))               00007600
75                    THEN DO;PUT EDIT('CARD PHASING IS WRONG IN THIS RUN')(A)SKIP;  00007700
77                           PUT EDIT('LOOK AT DAY ',DAY(K))(A,F(3))SKIP;           00007800
78                           GO TO FIN;END;                                         00007900
80             FILTER:                  /*APPLY VENEDIKOV'S FOUR DIGITAL FILTERS  */00008000
               MI(1,K  )=SUM(CT1*LIT); /*TO EACH 2-DAY SET AND STORE THE RESULTING  */00008100
81             MI(1,N+K)=SUM(ST1*LIT); /*4 VALUES DETERMINED FOR EACH SET.         */00008200
82             MI(2,K  )=SUM(CT2*LIT); /*                                         */00008300
83             MI(2,N+K)=SUM(ST2*LIT); /*          END OF READ04                  */00008400
84             END READ04;             /*************************************/00008500
85             TIMSET:                              /*CENTRAL HOUR OF FIRST SET   */00008600
               TI(1)=23.5;                          /*FROM 0000 Z ON DAY1 IS 23.5.*/00008700
86             DO I=1 TO N-1;                       /*TIMSET DETERMINES CENTRAL   */00008800
87             TI(I+1)=TI(I)+24.0*(DAY(I+1)-DAY(I)); /*HOUR OF SUBSEQUENT SETS.NOTE*/00008900
88             END;                                 /*THAT DAYS NEED NOT BE CONSE-*/00009000
                                                    /*CUTIVE EXCEPT WITHIN A SET. */00009100
89             AMP:DO J=1 TO 79;                    /*DETERMINATION OF AMPLITUDE  */00009200
90             JA=CK(J);JB=JA+4;JC=JA+8;            /*OF TIDAL GRAVITY AND TILT.  */00009300
93             HGR(J)=HC(J)*R(JA);                  /*BASED ON MELCHIOR TABLE 8A  */00009400
94             PHZ(J)=ATAND((HC(J)*R(JC)),-(HC(J)*R(JB)));                       00009500
95             HAZ(J)=SQRT((HC(J)*R(JB))**2+(HC(J)*R(JC))**2);                   00009600
96             END AMP;                                                          00009700
97             H(1,*)=HGR(*);H(2,*)=HAZ(*);                                      00009800
99             PHASE:                               /*DITTO FOR DETERMINATION OF */00009900
               PHC=D1*(SH-MS-LONG)+(D2-5)*MS+(D3-5)*SH /*PHASE, (PHC=GRAVITY PHASE) */00010000
               +(D4-5)*MP+(D5-5)*MN+(D6-5)*SP+PI*90;  /*(PHZ=TILT PHASE).         */00010100
100            PHZ=PHC+PHZ;                                                      00010200
101            DO J=1 TO 79; DO I=1 TO N;                                        00010210
103            VC(1,J,I)=PHC(J)+Q(J)*TI(I);                                      00010220
```

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104          VO(2,J,I)=PHZ(J)+Q(J)*TI(I);END;END;                                  00010230
107          LIMITS:                          /*Y(K) AND Z(K) DEFINE THE   */00010300
108          Y(1)=1;Z(1)=11;Y(2)=12; Z(2)=21;Y(3)=22;/*HARMONIC  CONSTITUENTS TO  */00010400
112          Z(3)=29;Y(4)=30;Z(4)=40;Y(5)=41;Z(5)=45;/*BE INCLUDED IN GROUP K.THE */00010500
117          Y(6)=46;Z(6)=52;Y(7)=53;Z(7)=57;Y(8)=58;/*LIMITS ARE THOSE OF VENEDIK*/00010600
122          Z(8)=61;Y(9)=62;Z(9)=66;Y(10)=67;Z(10)= /*OV (TABLE VI) FOR DATA SPAN*/00010700
127          71;Y(11)=72;Z(11)=79;L=VORH:             /*OF SIX MONTHS OR LESS      */00010800
130          PHX=PHG*(2-L)+PHZ*(L-1);                                               00010900
131          IF(TEST¬=1) THEN GO TO CONTI;                                          00011000
133          SPECL:DO K=0 TO KAJ-1;                   /*FOR TESTING PURPOSES SPECL */00011100
134          DAY(K+1)=DAY(1)+K;                       /*LISTS      THE TIDAL       */00011200
135          DO J=0 TO 23;                            /*GRAVITY/TILT FOR KAJ CONSE-*/00011300
136          CTA(J)=SLM(H(L,*)*COSD(PHX(*)            /*CUTIVE DAYS BEGINNING AT   */00011400
137          +Q(*)*(J+K*24)));END;                    /*DAYI. FORMAT OF LISTING IS */00011500
138          PUT EDIT('LST',DAY(K+1),'00',(CTA(I) DO /*THE SAME AS THAT OF DATA   */00011600
             I=0 TO 11))(A,F(3),A,12 F(6,0,1))SKIP;  /*INPUT.                     */00011700
139          PUT EDIT('LST',DAY(K+1),'12',(CTA(I) DO                                00011800
             I=12 TO 23))(A,F(3),A,12 F(6,0,1))SKIP;                                00011900
140          END SPECL;                                                             00012000
141          CONTI:                                                                 00012100
             PUT PAGE;                            /* PRINT TITLE CARD AT TOP   */00012200
142          PUT EDIT(CRD1,DATE)(A,A)SKIP(4);     /* OF NEW PAGE.              */00012300
143          PUT EDIT('WAVE','RATIO','ERROR','PHASE','ERROR') /*PRINT COL HEADINGS*/ 0001240
             (A,X(4),A,X(3),A,X(4),A,X(3),A)SKIP(3); /*FOR MAIN DATA OUTPUT.     */00012500
144          LOOP:DO FIRST=7,1;                   /*DIURNAL COMPONENTS PROCESSED*/00012600
145          L=VORH;                                                                00012650
146          IF(FIRST=1)THEN DO;SECOND=6;V=1;END;  /*WHEN FIRST=1, SEMIDIURNAL   */00012700
151          IF(FIRST=7)THEN DO;SECOND=11;V=2;END; /*COMPONENTS WHEN FIRST=7.     */00012800
156          JP=(SECOND+1-FIRST);                 /*NUMBER OF CONSTITUENT GROUPS*/00012900
157          JC=JP*2;                             /*TOTAL NR OF UNKNOWNS.      */00013000
158          JBLOCK:BEGIN;                                                          00013100
159          DECLARE                                                                00013200
              (GAMMA(JP),KAPPA(JP),ERRGA(JP),                                       00013300
               ERRKA(JP))FLOAT,                                                     00013400
              (F(JC,JC),F1(JC,JC))FLOAT,                                           00013500
              (W(JC),W1(JC),X(JC))FLOAT,                                           00013600
              A(KAJ,JC)FLOAT,RMS(JC)FLOAT;                                         00013700
160           F,F1=0;  W,W1,X=0;  A=0;                                             00013800
163          DCL SOLVE ENTRY((*)FLOAT,(*,*)FLOAT.,FIXED BIN(31));                  00013850
164          EQNOBS:DO I=1 TO N;                  /*EQNOBS FORMS EQUATIONS OF  */00013900
165          DO K=FIRST TO SECOND;M=K-FIRST+1;    /*OBSERVATIONS.              */00014000
167          DO J=Y(K) TO Z(K);                   /*THEORETICAL CONSTITUTENTS  */00014100
168          A(I,M)=A(I,M)+C(V,J)*H(L,J)*         /*FILTERED AND SUMMED FOR EACH*/00014200
                COSD(VO(L,J,I));                  /*GROUP AND FOR EACH OF THE 2N*/00014300
169          A(I,JP+M)=A(I,JP+M)+C(V,J)*H(L,J)*   /*CONDITIONS. THE RESULTING  */00014400
                SIND(VO(L,J,I));                  /*FORM IS:                   */00014500
170          A(N+I,M)=A(N+I,M)-S(V,J)*H(L,J)*     /* A(2N,JC)*X(JC)=M(2N)      */00014600
                SIND(VO(L,J,I));                                                    00014700
171          A(N+I,JP+M)=A(N+I,JP+M)+S(V,J)*H(L,J)*                               00014800
                COSD(VO(L,J,I));                                                    00014900
172          END EQNOBS;                                                          00015000
175          NORME:                               /*THE NORMAL EQUATIONS ARE   */00015100
```

```
176             DO I=1 TO JC;DO J=1 TO JC;DO K=1 TO KAJ;/*DEVELOPED HERE IN THE FORM */00015200
178             F(I,J)=F(I,J)+(A(K,I)*A(K,J));          /* F(JC,JC)*X(JC)=W(JC)    */00015300
179             END NORMF;                                                          00015400
182             NORMW:                                                              00015500
183             DO I=1 TO JC; DO K=1 TO KAJ;                                        00015600
184             W(I)=W(I)+A(K,I)*MI(V,K);                                           00015700
185             END NORMW;                                                          00015800
                                /*****************************************/00015900
                                /*  SUBROUTINE SOLVE                     */00016000
                                /*  SOLVES THE M SIMULTANEOUS EQNS: AY=C  */00016100
                                /*****************************************/00016200
187             SOLVE:PROC(C,A,Y,M);                                                00016300
188             DCL(C(*),A(*,*),Y(*))FLOAT,M FIXED BIN(31);                         00016400
189             DO L=1 TO M-1;DO J=L+1 TO M;CM=A(J,L)/A(L,L);                       00016500
192             DO K=L TO M;A(J,K)=A(J,K)-CM*A(L,K);END;                            00016600
195             C(J)=C(J)-CM*C(L);END;END;Y(M)=C(M)/A(M,M);                         00016700
199             DO KM=0 TO M-2;K=-KM+M-1;CC=0;DO JM=0 TO KM;J=-JM+M;                00016800
204             CC=CC+A(K,J)*Y(J);END;                                              00016900
206             Y(K)=(C(K)-CC)/A(K,K);                                              00017000
207             END SOLVE;                                                          00017100
209             W1=W;F1=F;                      /*F1 AND W1 WILL BE DESTROYED */00017200
211             CALL SOLVE(W1,F1,X,JC);         /*BY SOLVE.SOLUTION IN X(JC). */00017300
212             RESIDL:DO I=1 TO KAJ;           /*RESIDL DETERMINES RESIDUALS */00017400
213             RES(I)=SUM(A(I,*)*X(*))-MI(V,I);/*IN OBSERVATION EQUATIONS.   */00017500
214             END RESIDL;                                                         00017600
215             EVALUE:DO MN=1 TO JC;           /*EVALUE COMPUTES THE         */00017700
216             F1=F;                           /*REDUCED COFACTORS OF THE    */00017800
217             F1(JC,*)=F(MN,*);               /*DIAGONAL PART OF THE MATRIX */00017900
218             F1(MN,*)=F(JC,*);               /*F(JC,JC).THE JC TERMS ARE   */00018000
219             TRIAG:DO L=1 TO JC-1;           /*STORED IN RMS(JC) (SEE      */00018100
220             DO J=L+1 TO JC; CM=F1(J,L)/F1(L,L);  /* HILDEBRAND CHAPTER 7).  */00018200
222             DO K=L TO JC;F1(J,K)=F1(J,K)-CM*F1(L,K);                            00018300
224             END TRIAG;                                                          00018400
227             RMS(MN)=SQRT(ABS(1/F1(JC,JC)));                                     00018500
228             END EVALUE;                                                         00018600
229             WKAP=SUM(RES*RES);                                                  00018700
230             SIGMA=SQRT(WKAP/(2*N-JC));                                          00018800
231             RMS=RMS*SIGMA;                  /*RMS(JC) IS CHANGED HERE TO  */00018900
                                                /*THE ESTIMATED RMS ERROR IN  */00019000
                                                /*THE DETERMINATION OF X(JC). */00019100
232             VALOUT:DO J=1 TO JP;            /*VALOUT COMPUTES THE FACTORS */00019200
233             GAMMA(J)=SQRT((X(J)**2)+(X(J+JP)**2)); /*GAMMA AND KAPPA AND FINDS */00019300
234             KAPPA(J)=ATAND(-X(J+JP),X(J));  /*THE CORRESPONDING RMS ERRORS*/00019400
235             ERRGA(J)=SQRT(((X(J)**2)/(GAMMA(J)**2))*(RMS(J)**2)
                    +((X(J+JP)**2)/(GAMMA(J)**2))*(RMS(J+JP)**2));
236             ERRKA(J)=ERRGA(J)/GAMMA(J);                                         00019700
237             END VALOUT;                                                         00019800
238             IF(FIRST=1)THEN DO;             /*DIURNAL WAVES.              */00019900
240             E(1)='Q1 ';E(2)='O1 ';E(3)='M1 ';  /*SET LINE TITLES.         */00020000
243             E(4)='K1 ';E(5)='JI ';E(6)='001';                                   00020100
246             END;                                                                00020150
247             ELSE DO;                        /*SEMIDIURNAL WAVES.          */00020200
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248             E(1)='2N2';E(2)='N2 ';E(3)='M2 ';      /*SET LINE TITLES.          */00020300
251             E(4)='L2 ';E(5)='S2 ';END;                                          00020400
254             GAMMA=GAMMA/10.0;ERRGA=ERRGA/10.0;     /*DATA INPUT IS MSECS TIMES 10*/00020500
                                                       /*OR MICROGALS TIMES 10.    */00020600
256             DO J=1 TO JP;                          /*PRINTS OUT JP LINES LISTING:*/00020700
257             PUT EDIT(E(J),GAMMA(J),ERRGA(J),KAPPA(J),/*DOODSON SYMBOL, AMP FACTOR*/00020800
                ERRKA(J))(X(1),A,X(1),F(8,3),F(8,3),X(1),F(8,3),F(8,3))SKIP;        00020900
                                                        /*AND PHASE WITH           */00021000
258             END LOOP;                              /*ESTIMATE OF RMS ERROR.    */00021200
261             GO TO READ03;                          /*LOOKS FOR ANOTHER INPUT.  */00021300
262             FIN:END JANET;                         /*TERMINUS                  */00021400
```

APPENDIX II

Program ROBIN

```
MEMBER NAME  ROBIN
 ROBIN:PROCEDURE OPTIONS(MAIN);                                                  00000100
 /*************************************************************************** 00000200
 BEARINGS GIVEN AT START OF EACH LINE ARE CORRECT MEASURED CLOCKWISE FR 00000300
 NORTH. AZIMUTH IS THAT OF NOMINAL NORTH AT SITE MEASURED EASTWARD       00000400
 FROM NORTH.                                                            00000500
*************************************************************************/00000600
 DCL SYSPNCH FILE STREAM OUTPUT;                                        00000700
 DECLARE(GM(0:24),GS(0:24),HZS(0:24),HZW(0:24),HZSE(0:24),HZWE(0:24))   00000800
 FLOAT(12);                                                             00000900
 GM,GS,HZS,HZW,HZSE,HZWE=0;                                             00001000
 DECLARE ANGLE ENTRY(FLOAT(12),FLOAT(12),FLOAT(12),FLOAT(12),FLOAT(12))R00001100
 RETURNS(FLOAT(12));                                                    00001200
 DCL (TIM,DAT)CHAR(9),PLACE CHAR(20);                                   00001300
 DECLARE (MI,MN,MV,ME,ML,MP,MS,MZC,MZS,MD,SH,SL,SP,SZC,SZS,SD,EE,ER,RT, 00001400
 RJ,RX,RXA,RH,RD,RY,T)FLOAT(12);                                        00001500
 MI,MN,MV,ME,ML,MP,MS,MZC,MZS,MD,SH,SL,SD,SZC,SZS,SD,EE,ER,RT,RJ,RX,RXA,00001600
 RH,RD,RY=0;                                                            00001700
 /*ASSIGN CONSTANTS*/                                                   00001800
 /*EARTHS EQUATORIAL RADIUS*/                  CR=6.37827E+8;           00001900
 /*ECCENTRICITY OF MOONS ORBIT*/               CE=5.49E-2;              00002000
 /*RATIO OF MEAN MOTIONS,SUN TO MOON*/         CF=7.4804E-2;            00002100
 /*INCLINATION OF EARTHS EQUATOR TO ECLIPTIC*/CO=23.452;                00002200
 /*INCLINATION OF MOONS ORBIT TO ECLIPTIC*/    CI=5.145;                00002300
 /*NEWTONS GRAVITATIONAL CONSTANT*/            CU=6.67E-8;              00002400
 /*MASS OF MOON*/                              CM=7.3537E+25;           00002500
 /*MASS OF SUN*/                               CS=1.993E+33;            00002600
 /*MEAN DISTANCE EARTH MOON*/                  CC=3.84402E+10;          00002700
 /*MEAN DISTANCE EARTH SUN*/                   CD=1.495E+13;            00002800
 /*ASSIGN INPUT: IDENTIFICATION AS FOLLOWS:                             00002900
 HEIGHT OF SITE ABOVE SEA LEVEL                                 AH      00003000
 LONGITUDE OF SITE                                              AG      00003100
 LATITUDE OF SITE                                               AL      00003200
 FIRST RESULT REQUIRED:YEAR                                     AYA     00003300
                       DAY OF YEAR                              ADA     00003400
 NUMBER OF DAYS TO DETERMINED                                   JNDR    00003500
 INCLINATION OF LUNAR ORBIT TO EQUATOR                          MI      00003600
 LONGITUDE OF LUNAR ASCENDING NODE                              MN      00003700
 LONGITUDE IN CELESTIAL EQUATOR OF ASC. INTERSECTION      MV            00003800
 LONGITUDE IN ORBIT OF ASC. INT. WITH CELESTIAL EQ.       ME            00003900
                                                                      */00004000
 /*IDENTIFICATION OF PROCESS VARIABLES:                                 00004100
 MOON LONGITUDE IN ORBIT FROM A SCENDING INTERSECTION           ML      00004200
      MEAN LONGITUDE OF LUNAR PERIGEE                           MP      00004300
      MEAN LONGITUDE IN ORBIT FROM REFERED EQUINOX (ARRAY)      MS      00004400
      ZENITH ANGLE (ADD C FOR COSINE,S FOR SINE)   (ARRAYS)     MZ      00004500
      DISTANCE EARTH CENTER TO MOON CENTER                      MD      00004600
 SUN :MEAN LONGITUDE                                            SH      00004700
      LONGITUDE IN ECLIPTIC FROM VERNAL EQUINOX                 SL      00004800
      MEAN LONGITUDE OF PERIGEE                                 SP      00004900
      ZENITH ANGLE (ADD C FOR COSINE,S FOR SINE)   (ARRAYS)     SZ      00005000
      DISTANCE EARTH CENTER TO SUN-CENTER                       SD      00005100
 EARTH:ECCENTRICITY OF EARTHS ORBIT                             EE      00005200
       DISTANCE SITE TO CENTER OF EARTH                         ER      00005300
 REMAINDER:GREENWICH CIVIL TIME MEASURED IN HOURS               RT      00005400
           NUMBER OF JULIAN CENTURIES SINCE 1200 GCT DEC31,1899RJ       00005500
           RIGHT ASCENSION OF MERIDIAN OF SITE FROM MA          RX      00005600
           RIGHT ASCENSION OF MERIDIAN FROM VERNAL EQUINOX      RXA     00005700
           GCT HOUR                                             RH      00005800
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MEMBER NAME  ROBIN
          GCT DAY OF THE YEAR                                   RD        00005900
          GCT YEAR                                              RY        00006000
       CONSTANT FROM EQ 2.                                      XM        00006100
        CONSTANT FROM EQ 3.                                     XS        00006200
        DECLINATION MOON SINE                                   DMS       00006300
        DECLINATION MOON COS                                    DMC       00006400
        DECLINATION SUN SINE                                    DSC       00006500
        HORIZONTAL ACC MOON SOUTH                               HMS       00006600
             "          "  SUN     "                           HSS       00006700
            "           "  MOON WEST                           HMW       00006800
              "         "  SUN      "                          HSW       00006900
        HOUR ANGLE MOON (WESTERLY)                              HM        00007000
        HOUR ANGLE SUN (WESTERLY)                               HS        00007100
        AZIMUTH MOON COS (CW FROM SOUTH)                        AMC       00007200
        AZIMUTH SUN COS (CW FROM SOUTH)                         ASC       00007300
        TOTAL SOUTH ACC.                                        HTS       00007400
        TOTAL WEST ACC.,                                        HTW       00007500
        TOTAL HORIZ MOON FROM EQ 2                              HMY       00007600
        TOTAL HORIZ SUN FROM EQ 3                               HST       00007700
                                                                          */00007800
        ANGLE:PROC(A,B,C,D,T)FLOAT(12);DCL(A,B,C,D,T,Y)FLOAT(12);         00007900
        Y=A+B*T+C*(T**2)+D*(T**3);Y=Y-(FLOOR(Y/360))*360;                  00008000
        RETURN(Y);END ANGLE;                                              00008100
DCL(AH,AG,AL,AYA,ADA,ALPHA)FLOAT;   DCL (JNBR)FIXED;                      00008200
ON ENDFILE(SYSIN) GO TO FIN;                                              00008300
START:                                                                    00008400
GET DATA;                                                                 00008500
GAMMA=180+ALPHA;   BETA=270+ALPHA;                                        00008600
T=(365*(AYA-1900)+FLOOR((AYA-1901)/4)+ADA-.5)/36525;                      00008700
AYA=AYA-1900;                                                             00008800
JNBR=JNBR-1;                                                              00008900
   ER=(SQRT(1/(1+.006738*(SIND(AL)**2))))*CR+AH;          /*EQ 33*/      00009000
ARR=1/(CC*(1-(CE**2)));                                                   00009100
DAT=DATE;                                                                 00009200
GROUP:DO JBR=6 TO JNBR BY 7;                                              00009300
PUT EDIT(PLACE,'ROBIN ',DAT,'LAT=',AL,' LONG=',AG,' AZIMUTH=',ALPHA)      00009400
(PAGE,LINE(4),4(A),F(6,2),A,F(7,2),A,F(7,2));                             00009500
DATA:DO JC=JBR-6 TO JBR;JA=24*JC;JB=JA+23;                                00009600
   MP=ANGLE(334.3296,4069.034,-.01032,-.00001,T);                         00009700
   SP=ANGLE(281.2208,1.71902,.00045,.000003,T);        /*EQ 26 */      00009800
   FE=ANGLE(.01675104,-.0000418,-.000000126,0,T);      /*EQ 27 */      00009900
   MN=ANGLE(259.1833,-1934.142,.00208,.000002,T);                         00010000
   MI=(COSD(CL)*COSD(CI))-(SIND(CO)*SIND(CI)*COSD(MN));  /*EQ 20 */      00010100
    MI=ATAND(SQRT(1-(MI**2))/MI);                                         00010200
   MV=SIND(CI)*SIND(MN)/SIND(MI);                      /*EQ 21 */      00010300
      MV=ATAND(MV/SQRT(1-(MV**2)));                                       00010400
RSA=SIND(CO)*SIND(MN)/SIND(MI);                                           00010500
RSC=COSD(MN)*COSD(MV)+SIND(MN)*SIND(MV)*COSD(CO);                         00010600
RSC=RSC+1;                                                                00010700
MF=2*ATAND(RSA,RSC);                                                      00010800
MF=MN-MF;                                                                 00010900
   MS=ANGLE(270.4366,481267.8906,+.002,.000002,T);                        00011000
   SH=ANGLE(279.6967,36000.7689,.0003,0,T);                               00011100
PUT DATA(T,MS,SH)SKIP;                                                    00011200
PUT DATA(MP,SP,MN)SKIP;                                                   00011300
DATB:DO J=JA TO JB;                                                       00011400
   SH=ANGLE(279.6967,36000.7689,.0003,0,T);            /*EQ 12 */      00011500
   SL=  +2*E*SIND(SH-SP);                              /*EQ 25 */      00011600
```

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MEMBER NAME  RDRIN
 SL=57.29577*SL+SH;                                                                    00011700
    SD=(1/CD)+(1/(CD*(1-(EE**2))))*EE*COSD(SH-SP);                  /*EQ 30 */00011800
    SD=1/SD;                                                                           00011900
    MS=ANGLE(270.4366,.481267.8906,+.002,.000002,T);                                   00012000
    ML=      +2*CE*SIND(MS-MP)+1.25*(CE**2)*SIND(2*(MS-MP))+    /*EQ  9 */00012100
       3.75*CE*CE*SIND(MS-(2*SH)+MP)+1.375*(CE**2)*SIND(2*(MS-SH));                    00012200
 ML=57.29577*ML+MS-ME;                                                                 00012300
 MD=(1/CC)+APR*CE*COSD(MS-MP)+APR*(CE**2)*COSD(2*(MS-MP))+1.875*APR*CE*  00012400
  CE*COSD(MS-(2*SH)+MP)+APR*(CF**2)*COSD(2*(MS-SH));                                   00012500
    MD=1/MD;                                                                           00012600
 XM=3*CU*CM*ER/(MD**3);                                                                00012700
 XS=3*CU*CS*ER/(SD**3);                                                                00012800
 AHA=0;                                                                                00012900
    RD=ADA+FLOOR((AHA+J)/24);                                                          00013000
    RH=AHA+J-24*(RD-ADA);                                                              00013100
    RX=15*(RH-12)-AG+SH-MV;                                         /*EQ 23 */00013200
    RXA=15*(RH-12)-AG+SH;                                           /*EQ 28 */00013300
    MZC=SIND(AL)*SIND(MI)*SIND(ML)+COSD(AL)*((COSD(MI/2)**2)                           00013400
       *COSD(ML-RX)+(SIND(MI/2)**2)*COSD(ML+RX));                                      00013500
    SZC=SIND(AL)*SIND(CO)*SIND(SL)+COSD(AL)*((COSD(CO/2)**2)  /*EQ  8 */00013600
       *COSD(SL-RXA)+(SIND(CO/2)**2)*COSD(SL+RXA));                                    00013700
 JN=J-JA;                                                                              00013800
 GM(JN)=(CU*CM*ER/(MD**3))*(3*(MZC**2)-1);                                             00013900
 GM(JN)=GM(JN)+(1.5)*(CU*CM*(ER**2)/(MD**4))*(5*(MZC**3)-3*MZC);                       00014000
 GS(JN)=(CU*CS*ER/(SD**3))*(3*(SZC**2)-1);                                             00014100
 GM(JN)=GM(JN)+GS(JN);                                                                 00014200
 DMS=SIND(MI)*SIND(ML);DMC=SQRT(1-(DMS**2));                                           00014300
 DSS=SIND(CO)*SIND(SL);DSC=SQRT(1-(DSS**2));                                           00014400
 HRMC=(COSD(RX)*COSD(ML)+SIND(RX)*SIND(ML)*COSD(MI))/DMC;                              00014500
 HRMS=(SIND(RX)*COSD(ML)-COSD(RX)*SIND(ML)*COSD(MI))/DMC;                              00014600
 HRSC=(COSD(RXA)*COSD(SL)+SIND(RXA)*SIND(SL)*COSD(CO))/DSC;                            00014700
 HRSS=(SIND(RXA)*COSD(SL)-COSD(RXA)*SIND(SL)*COSD(CO))/DSC;                            00014800
 HMS=(XM*MZC)*(-(COSD(AL)*DMS)+(SIND(AL)*DMC*HRMC));                                   00014900
 HMS=HMS+.5*XM*(ER/(MD**3))*(5*(MZC**2)-1)*(-(COSD(AL)*DMS)+(SIND(AL)    00015000
 *DMC*HRMC));                                                                          00015100
 HSS=(XS*SZC)*(-(COSD(AL)*DSS)+(SIND(AL)*DSC*HRSC));                                   00015200
 HMW=(XM*MZC)*DMC*HRMS;                                                                00015300
 HMW=HMW+.5*XM*(ER/(MD**3))*(5*(MZC**2)-1)*DMC*HRMS;                                   00015400
 HSW=(XS*SZC)*DSC*HRSS;                                                                00015500
 HTS=HMS+HSS;                                                                          00015600
 HTW=HMW+HSW;                                                                          00015700
 HMT=XM*MZC*SQRT(1-(MZC**2));                                                          00015800
 HST=XS*SZC*SQRT(1-(SZC**2));                                                          00015900
 HZS(JN)=(HMS+HSS)/(9.8204*4.8481E-7);                                                 00016000
 HZW(JN)=(HMW+HSW)/(9.8204*4.8481E-7);                                                 00016100
 T=T+(1/24)*(1/3.652500E4);                                                            00016200
 END PATH;                                                                             00016300
 GM=GM+(.05E-6)*SIGN(GM);                                                              00016400
 HZSE=HZS*COSD(ALPHA)+HZW*SIND(ALPHA);                                                 00016500
 HZWE=HZW*COSD(ALPHA)-HZS*SIND(ALPHA);                                                 00016600
 HZS=HZSE;HZW=HZWE;                                                                    00016700
 HZS=HZS+(.05)*SIGN(HZS);                                                              00016800
 HZW=HZW+(.05)*SIGN(HZW);                                                              00016900
 PUT EDIT        ('VERT',RD,AYA,AL,AG,'00',(GM(I)DO I=0 TO 11))(A,F(3),  00017000
  F(2),F(3),F(4),A,(12)F(5,0,7))SKIP(2);                                               00017100
 PUT EDIT      ('VERT',RD,AYA,AL,AG,'12',(GM(I) DO I=12 TO 23))(A,F(3),  00017200
  F(2),F(3),F(4),A,(12)F(5,0,7))SKIP;                                                  00017300
 PUT EDIT('H',GAMMA,RD,AYA,AL,AG,'00',(HZS(I)DO I=0 TO 11))(A,2 F(3),     00017400
```

```
MEMBER NAME  ROBIN
  F(2),F(3),F(4),A,(12)(X(1),F(4,0,1)))SKIP;                                    00017500
 PUT EDIT('H',GAMMA,RD,AYA,AL,AG,'12',(HZS(I)DO I=12 TO 23))(A,2 F(3),          00017600
  F(2),F(3),F(4),A,(12)(X(1),F(4,0,1)))SKIP;                                    00017700
 PUT EDIT('H', BETA,RD,AYA,AL,AG,'00',(HZW(I)DO I=0 TO 11))(A,2 F(3),           00017800
  F(2),F(3),F(4),A,(12)(X(1),F(4,0,1)))SKIP;                                    00017900
 PUT EDIT('H', BETA,RD,AYA,AL,AG,'12',(HZW(I)DO I=12 TO 23))(A,2 F(3),          00018000
  F(2),F(3),F(4),A,(12)(X(1),F(4,0,1)))SKIP;                                    00018100
 IF (CARDS=1)THEN DO;                                                           00018200
 PUT FILE(SYSPNCH)EDIT('1',RD,'00',(HZS(I)DO I= 0 TO 11))(A,F(5),A,12           00018300
 F(6,0,1));                                                                     00018400
 PUT FILE(SYSPNCH)EDIT('1',RD,'12',(HZS(I)DO I=12 TO 23))(A,F(5),A,12           00018500
 F(6,0,1));                                                                     00018600
 END;ELSE IF (CARDS=2) THEN DO;                                                 00018700
 PUT FILE(SYSPNCH)EDIT('2',RD,'00',(HZW(I)DO I= 0 TO 11))(A,F(5),A,12           00018800
 F(6,0,1));                                                                     00018900
 PUT FILE(SYSPNCH)EDIT('2',RD,'12',(HZW(I)DO I=12 TO 23))(A,F(5),A,12           00019000
 F(6,0,1));                                                                     00019100
 END;ELSE IF (CARDS=3) THEN DO;                                                 00019200
 PUT FILE(SYSPNCH)EDIT('3',RD,'00',(GM(I) DO I= 0 TO 11))(A,F(5),A,12           00019300
 F(6,0,7));                                                                     00019400
 PUT FILE(SYSPNCH)EDIT('3',RD,'12',(GV(I) DO I=12 TO 23))(A,F(5),A,12           00019500
 F(6,0,7));                                                                     00019600
 END;                                                                           00019700
 END DATA;                                                                      00019800
 END GROUP;                                                                     00019900
 GO TO START;                                                                   00020000
 FIN:END ROBIN;                                                                 00020100
```

APPENDIX III

Program PRESS

```
1              PRESS:PROCEDURE OPTIONS(MAIN);
2              MU2=3.6E12;                    /*THIS ASSIGNS RIGIDITY  */
3              DCL MU2 FLOAT;
4              DECLARE(RFAC(9),CST(9),CEC(9),NABLA(9))FLOAT(12);
5              DCL NBR(9);
6              DECLARE(DOP(9),DOQ(9),RAO(9),RPO(9))FLOAT(12);
7              DCL (TIM,DAT)CHAR(9);
8              DECLARE (AMP(9),PHS(9))FLOAT(12);
9              DECLARE (DELX,DELY,DO,DS,DW,VC,VS,VW,WC,WS,WW,PCTC,PCTS,PCTW)FLOAT(12);
10             DCL(RN(530),FN(530),YN(530,9),XN(530,9),QN(530))FLOAT(12);
11             RN,FN,QN=0;XN,YN=0;
13             DECLARE(XS(9),YS(9),Y(9),X(9),YY(9),XX(9),LC,PC(9),PS(9),PW(9),TSP(9),
               TSQ(9),TWP(9),TWQ(9),RAS(9),RPS(9),RAW(9),RPW(9))FLOAT;
14             JOT=1;
15             CE=2.063*.001;
               /*  CONSTANT CE CONVERTS DIFFERENCES OF COMPUTED DISPLACEMENTS (IN CMS
                X E5) AT SITES SEPARATED BY 1) KM TO TILTS (IN MSECS)          */
16             CD=(6.67E-8)*(30.48)/(2.*981.0);  CD=CD*1.9E5;
                     /* FACTOR 2 IN DEN CONVERTS INPUT RANGE TO AMPLITUDE      */
18             CA=6.3733E+8;CB=1.7453E-9;CC=CA*CB;
21             RNC=0;
22             DO I=1 TO 530 WHILE(RNC¬=535);/*LAST CARD MUST BE RUN 535  */
23             GET EDIT(RN(I),FN(I),QN(I),(YN(I,J),XN(I,J)DO J=1 TO 9))(F(3),F(2),F(3)
               ,18 F(4));RNC=RN(I);END;ILAST=I-1;
27             ON ENDFILE(SYSIN) GO TO FINAL;
29             ORIGIN:
               XS,YS,Y,X,YY,XX,PC,PW,PS,TSP,TSQ,TWP,TWQ,RAS,RPS,RAW,RPW=0;
30             DOP,DOQ=0;
31             GET LIST((NBR(J),XS(J),YS(J) DO J=1 TO 9));
32             PUT PAGE;
33             DAT=DATE;
34             PUT SKIP(3);
35             PUT EDIT('(PRESS ',DAT,'MODIFIED JUNE 11/69 TO INCLUDE TAKEUCHI RATIO)'
               )(3(A));
36             PUT SKIP(2);
37             PUT EDIT('SITES REFERED TO FOR THIS PRODUCTION')(A);
38             PUT DATA(XS,YS)SKIP(2);
39             PUT SKIP(2);
40             START:DO I=1 TO ILAST;
41                   RUN=RN(I);F=FN(I);Q=QN(I);Y(*)=YN(I,*);X(*)=XN(I,*);
46             ON ZERODIVIDE BEGIN;
48             PUT EDIT('ZERODIVIDE ERROR')(SKIP(2),A);
49               PUT DATA(RUN,J,K,DELY,DELX,LO,WO,VC,UO,US,WS,VS,UW,VW,WW)SKIP;
50               PUT SKIP(3);
51               PO,PS,PW=0;
52             GO TO SHUT;END;
54                   DO J=1 TO 9 WHILE(X(J)¬=0&Y(J)¬=0);JMAX=J;END;
57                   X(JMAX+1)=X(1);Y(JMAX+1)=Y(1);
59                   DO K=1 TO 9;
61                   YY=(Y-YS(K))*CC;XX=(X-XS(K))*CC*COSD(Y(1)/100);
               /*  YY AND XX ARE IN KILOMETERS  */                          NOTE
               /* DST IS DISTANCE IN CMS FROM SITE K TO FIRST PT. OF POLYGON I.
               RFAC WILL BE TAKEUCHIS RATIO (HIS EQN 578) OR NISHIMURAS FACTOR. ONE OF
               THE TWO DEFINING CARDS MUST BE OMITTED BY ASTERISK CARDS. DEG IS
               ANGLE IN DEGREES FROM POLYGON 1 TO SITE K   */
62             DST(K)=SQRT(XX(1)**2+YY(1)**2)*(1.0E+05);
63             DEG(K)=2*ATAND(DST(K),SQRT(4*((6.37E+8)**2)-DST(K)**2));
```

```
                    /*BOUSSINESQ FORM WITH LAMBDA=1.44 X MU2  */
      RFAC(K)=((9.81E2)**2)*1.44/(12.56*6.67E-8*MU2);
            DO J=1 TO JMAX;
      DELY=YY(J+1)-YY(J);  DELX=XX(J+1)-XX(J);
      LO=SQRT(DELY**2+DELX**2);
      WO=(XX(J+1)*DELX+YY(J+1)*DELY)/LO;
      VO=(XX(J)*DELX+YY(J)*DELY)/LO;
      UO=(XX(J)*YY(J+1)-XX(J+1)*YY(J))/LO;
      IF (UO**2)<=1 THEN DO;PDTO=0;GO TO PA;END;
      ARGO=(WO+SQRT((WO**2)+(UO**2)))/(VO+SQRT((VO**2)+(UO**2)));
      IF ARGO<=0 THEN DO;PUT EDIT('LOG ARG ZERO OR NEGATIVE')(SKIP(2),A);
        PUT DATA(RUN,J,K,DELY,DELX,LO,WO,VO,UO,US,WS,VS,UW,VW,WW)SKIP;
      PUT SKIP(3);I=ILAST;GO TO SHUT;END;
      PDTO=-UO*LOG(ARGO);
      PA:
      DELX=10*DELX; DELY=10*DELY;  /*THIS SETS DISTANCE OVER WHICH TILT IS
      DETERMINED TO 10 KMS  */
      US=UO-(DELX/LO);VS=VO+(DELY/LO);WS=WO+(DELY/LO);
      IF (US**2)<=1 THEN DO;PDTS=0;GO TO PB;END;
      ARGO=(WS+SQRT((WS**2)+(US**2)))/(VS+SQRT((VS**2)+(US**2)));
      IF ARGO<=0 THEN DO;PUT EDIT('LOG ARG ZERO OR NEGATIVE')(SKIP(2),A);
        PUT DATA(RUN,J,K,DELY,DELX,LO,WO,VO,UO,US,WS,VS,UW,VW,WW)SKIP;
      PUT SKIP(3);I=ILAST;GO TO SHUT;END;
      PDTS=-US*LOG(ARGO);
      PB:
      UW=UO+(DELY/LO);VW=VO+(DELX/LO);WW=WO+(DELX/LO);
      IF (UW**2)<=1 THEN DO;PDTW=0;GO TO PO;END;
      ARGO=(WW+SQRT((WW**2)+(UW**2)))/(VW+SQRT((VW**2)+(UW**2)));
      IF ARGO<=0 THEN DO;PUT EDIT('LOG ARG ZERO OR NEGATIVE')(SKIP(2),A);
        PUT DATA(RUN,J,K,DELY,DELX,LO,WO,VO,UO,US,WS,VS,UW,VW,WW)SKIP;
      PUT SKIP(3);I=ILAST;GO TO SHUT;END;
      PDTW=-UW*LOG(ARGO);
      PO:
      FINA:PO(K)=PO(K)+PDTO;PS(K)=PS(K)+PDTS;PW(K)=PW(K)+PDTW;END;END;
      /* TO THIS POINT PO,PS,PW HAVE DIMENSIONS OF KM.                    */
      RFAC=RFAC*(1-2.435*(DST/CA)+1.59*(DST**2/CA**2));
            /*CORRECTION TO RFAC DETERMINED BY BOWER FROM CAPS DATA FOR
              SPHERICAL, HOMOGENEOUS SHELL.                              */
      NABLA=1.01*(DST/CA)-.71*((DST/CA)**2);
             /* FACTOR DETERMINED AS ABOVE TO ACCOUNT FOR ATTRACTION OF
              DISPLACED EARTH MASSES.FORM BEGINNING 1.01 USES DENSITY 2.8 */
      PO=H*CO*PO*RFAC;PS=H*CO*PS*RFAC;PW=H*CO*PW*RFAC;
      /*PO(K),PS(K),PW(K) NOW ARE VERTICAL DISPLACEMENTS IN CMS E+5 POSITIVE
      TOWARDS CENTER OF EARTH) AT THE K TH SITE (PO), 10 KM SOUTH OF K TH SITE
      (PS) AND 10 KM WEST OF KTH SITE,WITH COEFFICIENT INVOLVING ELASTIC
      CONSTANTS REPLACED BY APPROPIATE EXPRESSION IN RFAC    */
      /* TSP AND TWP ARE POSITIVE FOR TILTS TOWARDS THE SOUTH AND THE WEST RE
      SPECTIVELY                                                         */
      COP=COP+PO*COSD(Q);  DOQ=DOQ+PO*SIND(Q);
      RFAC=(1+RFAC-NABLA)/RFAC;
      TSP=TSP-(PO-PS)*CE*COSD(Q)*RFAC;
      TSQ=TSQ-(PO-PS)*CE*SIND(Q)*RFAC;
      TWP=TWP-(PO-PW)*CE*COSD(Q)*RFAC;
      TWQ=TWQ-(PO-PW)*CE*SIND(Q)*RFAC;
      /*TSP AND TSQ ARE IN-PHASE AND QUADRATURE COMPONENTS OF SOUTHERLY TILT   NOTE
      WITH RESPECT TO TIDAL REFERENCE PHASE. THEY ARE TOTAL TILTS DUE TO
      SUM OF BOTH DIRECT AND INDIRECT EFFECTS.  UNITS ARE MSECS.    */
```

```
142             PC,PS,PW=0;
143             SHUT:
                END START;
144             FINISH:RAS=SQRT(TSP**2+TSQ**2);RPS=ATAND(TSQ,TSP);
146                    RAW=SQRT(TWP**2+TWC**2);RPW=ATAND(TWQ,TWP);
148                    RAC=SQRT(DCP**2+DCC**2);RPC=ATAND(DCQ,DCP);
                /* THEN TILT TOWARDS SOUTH =RAS COS(WT-RPS),TOWARDS WEST=RAW COS(WT-RPW).
                                                                                */
150             PUT SKIP;
151             PUT EDIT('RAS AND RPS ARE AMP AND PHASE WITH RESPECT TO LUNAR TRANSIT
                OF GREENWICH OF SOUTHERLY TILT IN MSECS,RAW AND RPW SAME FOR WEST')(A);
152             PUT DATA(RAS,RPS,RAW,RPW)SKIP(1);
153             PUT SKIP(2);
154             PUT EDIT('RAD AND RPD ARE AMP AND PHASE WITH RESPECT TO LUNAR TRANSIT
                OF GREENWICH OF DISPLACEMENT IN MICRONS ASSUMING LAMBDA =1.44 X MU2 AND
                RIGIDITY EQUAL TO MU2')(A);
155             PUT DATA(RAD,RPD)SKIP(2);
156             PUT SKIP(3);
157             PUT EDIT('AMPLITUDE AND AZIMUTH FROM SOUTH FOR 2T=0 TO 330 DEGS FOLLOW
                IN 9 COLUMNS FOR 9 SITES')(A);
                /* NOTE T HERE REFERS TO LUNAR TIME                              */
158             DO J=0 TO 330 BY 30;
159             AMP=SQRT((RAS**2)*(COSD(J-RPS)**2)+(RAW**2)*(COSD(J-RPW)**2));
161             PHS=ATAND((RAW*COSD(J-RPW)),(RAS*COSD(J-RPS)));
161             PUT EDIT(AMP)(SKIP(2),9 F(8,2));
162             PUT EDIT(PHS)(SKIP,9 F(6,1));END;
                /*FOLLOWING COMPUTES PHASE 2T FOR MAX OR MIN AMPLITUDE SQUARED (ARGN)   T,LUNAR
                  AZIMUTH COMPUTED THEN FOR THIS PHASE CW FROM SOUTH            (ARGA)
                  AND FOR THIS PHASE + 90 DEGS                                 (ARGB)
                  THE AMPLITUDE CORRESPONDING TO ARGA IS COMPUTED              (AMPA)
                  AND ALSO THAT CORRESPONDING TO ARGB                          (AMPB)*/
164             DCL (ARGA(9),ARGN(9),ARGB(9),AMPA(9),AMPB(9))FLOAT;
165             ARGN=.5*ATAND(((RAS**2)*SIND(2*RPS)+(RAW**2)*SIND(2*RPW)),
                              ((RAW**2)*COSD(2*RPW)+(RAS**2)*COSD(2*RPS)));
166             ARGA=ATAND((RAW*COSD(ARGN-RPW)),(RAS*COSD(ARGN-RPS)));
167             ARGN=ARGN+90;
168             ARGB=ATAND((RAW*COSD(ARGN-RPW)),(RAS*COSD(ARGN-RPS)));
169             ARGN=ARGN-90;
171             AMPA=RAS*COSD(ARGN-RPS)*COSD(ARGA)+RAW*COSD(ARGN-RPW)*SIND(ARGA);
171             ARGN=ARGN+90;
172             AMPB=RAS*COSD(ARGN-RPS)*COSD(ARGB)+RAW*COSD(ARGN-RPW)*SIND(ARGB);
173             PUT DATA(ARGN,ARGA,ARGB,AMPA,AMPB)SKIP(3);
174             ARGA=ARGA+180; ARGB=ARGB+180; ARGN=ARGN+90;XS=XS-2000;
176             RAD=RAD/(1.0E+05);
179             XS=XS/100;  YS=YS/100;
181             PUT SKIP(2);
182             PUT EDIT('NBR    LONG    LAT   RAC    RPC   AMPA  AMPB  ARGN   RAS   RA
                W    RPS    RPW')(A);
183             DO J=1 TO 9;
184             PUT SKIP;
185             PUT EDIT(NBR(J),XS(J),YS(J),RAC(J),RPC(J),AMPA(J),AMPB(J),ARGN(J),RAS(J
                ),RAW(J),RPS(J),RPW(J))(F(3),F(8,2),F(7,2),F(6,2),F(7,1),F(7,2),F(6,2),
186             F(6,1),F(6,2),F(6,2),F(7,1),F(7,1));END;
187             PUT EDIT('(PRESS ',DAT,')')(3 A)SKIP(2);
188             PUT EDIT('CHANGES JULY 10   POLYGONS ABOUT BORDERS ALTERED TO LIMIT THE
                REGION TO THAT WITHIN LAT 42N,LAT 62N,AND LONG 25W.THE SHAPE OF THE SOU
                THERN POLYGONS WAS ALTERED TOO TO CONFORM WITH LATEST HANSEN CHART,AMP
```

```
                AND PHASE CONSTANTS WERE CHANGED SLIGHTLY. ON AUG 25,1969 PROGRAM
                 WAS MODIFIED TO TAKE REAC AND NABLA COMPUTED NUMERICALLY FROM CAPUTO
                 AND LONGMAN PAPERS')(A)SKIP(2);
  189            PUT EDIT('FOLLOWING POLYGONS HAVE BEEN REMOVED FOR OKING')(A)SKIP;
  190            PUT EDIT('112,527,528,376,372,367')(A)SKIP;
  191            GO TO ORIGIN;
  192            FINAL:
                 END PRESS;
```

APPENDIX IV

Program CAPS

```
MEMBER NAME  CAPS
 (NOUNDERFLOW):                                                                  00000001
 CAPS:PROCEDURE OPTIONS(MAIN);                                                   00000200
 /*****************************************************************************/00000300
 /*                                                                           */00000400
 /*  COMPUTES RADIAL DISPLACEMENT, GRAVITY PERTURBATION AND TILT ON           */00000500
 /*  HOMOGENEOUS SHELL OF ARBITRARY RIGIDITY AND THICKNESS ENCLOS-            */00000600
 /*  ING A FLUID CORE WHEN SURFACE STRESS IS UNIFORM PRESSURE OVER            */00000700
 /*  A SINGLE AXIALLY-SYMMETRIC  CAP OF ARBITRARY DIAMETER. (SOME            */00000800
 /*  LESSER CONTRIBUTIONS ARE DETERMINED ON THE BASIS OF A HOMO-              */00000900
 /*  GENEOUS EARTH).                                                          */00001000
 /*                                                                           */00001100
 /*****************************************************************************/00001200
 R2=6.371E+08;                          /*ASSIGN OUTER RADIUS OF SHELL*/      00001300
 R1=0.545*R2;                           /*ASSIGN RADIUS OF CORE.      */      00001400
 TETAF=1.0;                             /*DIST TO FIRST SITE-DEGREES. */      00001500
 TETAL=30.0;                            /*DIST TO LAST SITE-DEGREES.  */      00001600
 TETAINC=1.0;                           /*DIST INCREMENT-DEGREES.     */      00001700
 MU2=  0.6E+12;                        /*ASSIGN RIGIDITY OF SHELL*/           00001800
 RHO=2.80;                          /*ASSIGN DENSITY OF DISPLACED MASSES*/     00001900
 RHO2=4.47;                         /*ASSIGN DENSITY FOR BODY REACTION. */     00002000
 DIM=200;                               /*ASSIGN HIGHEST POLY ORDER.  */      00002100
 MEGA=1.0000;                           /*ASSIGN DIAM OF CAP-DEGREES. */      00002200
 ON ENDFILE(SYSIN) GO TO FIN;                                                    00002300
 PO=1.0E+6;                          /* ASSIGN PRESSURE OF CAP*/                00002400
 DECLARE                                                                         00002500
       (R1,R2,MU2,LAMBDA2,LAMBDA1,PO,KST1,KST2)FLOAT,                            00002600
       DIM FIXED,                                                                00002700
       (K1,K2)FLOAT(12),                                                         00002800
       (G1,G2)FLOAT(12),                                                         00002900
       (C11,C12,C21,C22)FLOAT(12),                                               00003000
       (A,D,S)FLOAT(12),                                                         00003100
       HZERO FLOAT(12),                                                          00003200
       (DGL1,DGL2,DGL3,DELT,DTL1,DTL3)FLOAT,                                     00003300
       DLN FLOAT(12),                                                            00003400
       (A1,A2,A3,A4)FLOAT(12),                                                   00003500
       (UR,UL,X1,X2,X3)FLOAT,                                                    00003600
       (MEGA,TETA,TMN,TLN,TILTL,TILTM)FLOAT(12),                                 00003700
            (ARGD) FLOAT BIN(53),ERROR EXTERNAL CHAR(1),                         00003710
             ANGNR BINARY FIXED,                                                 00003712
       (CST1,CST2) FLOAT;                                                        00003800
 DECLARE (J,N)FIXED BIN(31);                                                     00003900
 START:                                                                          00004000
 GET DATA(MU2,    RHO,RHO2,MEGA,TETAF,TETAL,TETAINC);                            00004100
 ARGD=.01745*TETAINC;                                                            00004110
 LAMBDA2=0.1273E+1*MU2;                        /*COMPRESSIBILITY OF SHELL.   */00004200
 LAMBDA1=8.0*MU2;                              /*COMPRESSIBILITY OF CORE.    */00004300
 TOP=DIM;                                                                        00004400
 LEN=((TETAL-TETAF)/TETAINC)+1;                                                  00004500
 ANGNR=LEN;                                                                      00004510
 SCENE:BEGIN;                                                                    00004600
 DECLARE                                                                         00004610
     SHAPE(LEN),(DZ1(LEN),DZ2(LEN),DZ3(LEN),DZ4(LEN))BIN FLOAT(53);              00004620
 SHAPE=0;                                                                        00004700
 DECLARE RRN(DIM) FLOAT(12);                                                     00004800
 DECLARE(CST3,CST4,CST5,CST6,CST7,CST8)FLOAT(12);                                00005000
 DECLARE                                                                         00005100
     P(0:TOP)FLOAT(12),                                                          00005200
     Q(0:TOP)FLOAT(12);                                                          00005300
```

```
MEMBER NAME  CAPS
 CST2=CST2/2;                                                                    00010600
 CST3=PO*4.0*3.14159*6.67*6.371/(980.616);                                       00010700
 ACT:DO J=1 TO DIM-2 BY 1;                                                       00010800
    P(J+1)=((2*J+1)*COSD(MEGA)*P(J)-(J)*P(J-1))/(J+1);                           00010900
    P(J+2)=((2*J+3)*COSD(MEGA)*P(J+1)-(J+1)*P(J))/(J+2);                         00011000
    Q(J+1)=((2*J+1)*COSD(TETA)*Q(J)-(J)*Q(J-1))/(J+1);                           00011100
    Q(J+2)=((2*J+3)*COSD(TETA)*Q(J+1)-(J+1)*Q(J))/(J+2);                         00011200
 N=J+1;  J=J+1;                                                                  00011300
 S=CST1*(P(J-1)-P(J+1))*PO;                                                      00011400
 S=S/2;                                                                          00011500
 XLOAD=XLOAD+S*Q(J);                                                             00011600
 VN=CST3*(P(J-1)-P(J+1))*Q(J)/(2*(2*J+1));                                       00011700
 VM=CST3*(P(J-1)-P(J+1))/(2*(2*J+1));                                            00011800
 CST4=J*((J+2)*LAMBDA2+(J+1)*MU2)*R2;                                            00011900
 CST5=(J-1)*((J+1)*LAMBDA2+J*MU2)*R2;                                            00012000
 CST6=(2*(J**2)+(4*J)+3)*LAMBDA2;                                                00012100
 CST7=2*((J**2)+J+1)*MU2;                                                        00012200
 CST8=5.51*J*VN/(2*(J-1)*MU2);                                                   00012300
 CST9=5.51*J*VM/(2*(J-1)*MU2);                                                   00012400
 UC=UC+((CST4-CST5)*CST8/(CST6+CST7));                                           00012500
 UR=UR+Q(J)*RHO(J)*S;                                                            00012600
 DELU=Q(J)*RHO(J)*S+((CST4-CST5)*CST8/(CST6+CST7));                              00012700
 DELV=     RHO(J)*S+((CST4-CST5)*CST8/(CST6+CST7));                              00012800
 DELT=J*(J+1)*(Q(J+1)-Q(J-1))/((2*J+1)*SIND(TETA));/*P'(COS TETA).     */00012900
 TILTS=TILTS+(DELV*DELT/(R2));              /*TILT OF SURFACE.          */00013000
 TILTX=4*3.14*5.67E-8*RHO*DELV*DELT;                                             00013100
 TILTM=TILTM-(TILTX/(980.6*(2*J+1)));     /*TILT DUE TO DISPLACED MASS.*/00013200
 DGL2=DGL2+(3.086*DELU);                                                         00013300
 DGL3=DGL3+((J+1)*4*3.14*DELU*6.67E-8*RHO /(2*J+1));                             00013400
 J=J-1;                                                                          00013500
    END ACT;                                                                     00013600
 JETA=((TETA-TETAF)/TETAINC)+1;                                                  00013700
 UR1=-((3*R2*PO*(SIND(MEGA)**2)*COSD(TETA))/(8*MU2*(3*G2+2)));                   00013800
 UR=UR+UR1;                                                                      00013900
 DGL2=DGL2+3.086*UR1;                                                            00014000
 DGL3=DGL3+((2)*4*3.14*UR1*5.67E-8*RHO/(3));                                     00014100
 SHAPE(JETA)=XLOAD+(CST1*PO*(1-COSD(MEGA)))/2;                                   00014200
 DGL2=DGL2+3.086*CST2;                                                           00014300
 DGL2=DGL2*1.00E-6;                                                              00014400
 DEN=(PU/980.616)*(R2/6.0E+27)*2*3.14159*(R2**2)*(1-P(1));                       00014500
 U1=UR/DEN; U2=CST2/DEN;                  /*U1 IS RADIAL DISPLCMENT.    */00014600
                                          /*U2 IS CONSTNT TERM. U3 IS   */00014700
 U3=UC/DEN; U4=U1+U2+U3;                  /*DUE TO BODY FORCE OF LOAD.  */00014800
 DGL1=-1/(4*SIND(TETA/2));                                                       00014900
 DGL2=DGL2*R2/(DEN*980.0);                                                       00015000
 DGL3=-DGL3/(DEN*980.616*R2);                                                    00015100
 DTL1=(U4-PREVU)*57.4/TETAINC; /*SERIES SOLN IS:DTL1=TILTS*R2/DEN */           00015200
                               /*THIS FORM APPLIES TO DISTANCE TETA*/           00015300
                               /*MINUS TETAINC/2.            */                00015400
 DTL3=(COSD(TETA/2)/(4*(SIND(TETA/2)**2)));/*TILT DUE TO ATTRACTION.  */00015500
 DTL2=TILTM*R2/DEN;     /*ATTRACTION OF DISPLACED MASSES   */                    00015600
 DTL4=(TILTM*R2/DEN)+DTL3;                /*PARTIAL SUM OF TILTS.       */00015700
 DGL4=DGL1+DGL2+DGL3;                                                            00015800
 RATIO=DGL4/DGL1;                                                                00015900
 PEGA=MEGA;PETA=TETA;                                                            00016000
 /*                                                                              00016010
 PUT EDIT('|',TETA,'|',U1,'|',U3,'|',U4,'|',DGL2,'|',DGL3,'|', DGL4,'|',00016100
 RATIO,'|',                                                                      00016200
```

```
MEMBER NAME  CAPS                                                           00016300
 DTL1,'|',DTL2,'|',DTL3,'|')(A,F(3),A,F(7,2),A,F(5,2),A,F(7,2),A,F(7,2),  00016400
 A,F(6,2),A,F(7,2),A,F(5,2),A,F(8,2),A,F(6,2),A,F(7,2),A)SKIP;             00016410
 */                                                                         00016500
 PREVU=U4;                                                                  00016510
 DZ1(JJ)=U4;                                                                00016520
 DZ2(JJ)=DTL2;                                                              00016530
 DZ3(JJ)=DTL3;                                                              00016540
 DZ4(JJ)=TETA;                                                              00016600
 END RUN2;                                                                  00016700
 PUT EDIT('|___|_____|_____|_____|_____|_____|_____|_____|_____|           00016800
 ___|_____|_____|')(A)SKIP;                                                 00016900
 PUT SKIP(4);                                                               00017000
 /*                                                                         00017100
 PUT DATA(DEN,MU2,LAMBDA2)SKIP;                                             00017200
 PUT DATA(RHO,PO,R1)SKIP;                                                   00017300
 PUT DATA(P2,A,MEGA)SKIP;                                                   00017400
 PUT DATA(U2,DIM,RHO2)SKIP;                                                 00017500
 */                                                                         00017600
 PUT EDIT('              TABLE  o DATA FOR HOMOGENEOUS-SHELL EARTH (MU2=',  00017700
 MU2,')')(A,E(9,2),A)SKIP;                                                  00017800
 PUT EDIT('                                                  DRR')(A)SKIP;  00017900
*CALL OPT3(ARGD,DZ1,DZ1,ANGNR);                                             00017910
 PUT EDIT((DZ1(J),DZ2(J),DZ3(J),DZ4(J) DO J=1 TO ANGNR))                    00017920
        (SKIP, 4(X(3),F(8,3)));                                             00017930
 DZ1=DZ1+DZ2+DZ3;                                                           00017940
 DZ1=DZ1/DZ3;  DZ2=DZ2/DZ3;                                                 00017950
 PUT EDIT((DZ4(J),DZ2(J),DZ1(J),DZ3(J)DO J=1 TO ANGNR))                     00017960
        (SKIP, 4(X(3),F(8,3)));                                             00017970
 GO TO START;                                                               00017980
 END SCENE;                                                                 00017990
 FIN:                                                                       00017950
 END CAPS;                                                                  00018100
```

APPENDIX V

Program LONG

```
MEMBER NAME  LONG
 (NOUNDERFLOW):                                                                  00000100
 LONG:PROCEDURE OPTIONS(MAIN);                                                   00000200
 /*****************************************************************************/00000300
 /*                                                                           */00000400
 /*     TO SUM LONGMAN'S SERIES FOR VERTICAL DISPLACEMENT (EQN.37)            */00000500
 /*     AND FOR THE INCREASE IN THE VERTICAL COMPONENT OF GRAVITY (41)        */00000600
 /*                                                                           */00000700
 /*****************************************************************************/00000800
 DECLARE                                                                         00000900
        DIM BINARY FIXED,                                                        00001000
        PP BINARY FLOAT(53);                                                     00001100
 DIM=1000;                                                                       00001200
 RESU1=0;                                                                        00001300
 BEGIN:                                                                          00001400
 DECLARE POSV ENTRY(BINARY FLOAT(53),(*)BINARY FLOAT(53),BINARY FIXED,           00001500
        CHARACTER(1),BINARY FLOAT(53));                                          00001600
 DECLARE                                                                         00001700
        (H( 41),K( 41),C(DIM),E(DIM),D(DIM))BINARY FLOAT(53),                    00001800
        (HH,KK)BINARY FLOAT(53),                                                 00001900
        PUP CHARACTER(1),                                                        00002000
        (ARG,ANG,P,RESU,RESG,RESS)BINARY FLOAT(53);                              00002100
 PUP='P';                                                                        00002200
        H( 1)=0.134;H( 2)=0.000;H( 3)=1.007;H( 4)=1.059;H( 5)=1.059;             00002300
        H( 6)=1.093;H( 7)=1.152;H( 8)=1.223;H( 9)=1.296;H(10)=1.369;             00002400
        H(11)=1.439;H(12)=1.506;H(13)=1.572;H(14)=1.631;H(15)=1.691;             00002500
        H(16)=1.747;H(17)=1.798;H(18)=1.852;H(19)=1.902;H(20)=1.949;             00002600
        H(21)=1.994;H(22)=2.037;H(23)=2.078;H(24)=2.117;H(25)=2.156;             00002700
        H(26)=2.194;H(27)=2.223;H(28)=2.257;H(29)=2.291;H(30)=2.322;             00002800
        H(31)=2.351;H(32)=2.380;H(33)=2.408;H(34)=2.429;H(35)=2.455;             00002900
        H(36)=2.470;H(37)=2.497;H(38)=2.521;H(39)=2.535;H(40)=2.562;             00003000
        H(41)=2.581;                                                             00003100
        K( 1)=0.000;K( 2)=0.000;K( 3)=0.310;K( 4)=0.197;K( 5)=0.133;             00003200
        K( 6)=0.104;K( 7)= .090;K( 8)= .082;K( 9)= .075;K(10)= .072;             00003300
        K(11)=0.069;K(12)= .066;K(13)= .064;K(14)= .062;K(15)= .060;             00003400
        K(16)=0.058;K(17)= .056;K(18)= .055;K(19)= .054;K(20)= .052;             00003500
        K(21)=0.051;K(22)= .050;K(23)= .049;K(24)= .048;K(25)= .047;             00003600
        K(26)=0.046;K(27)= .045;K(28)= .044;K(29)= .043;K(30)= .042;             00003700
        K(31)=0.041;K(32)= .040;K(33)= .040;K(34)= .039;K(35)= .038;             00003800
        K(36)=0.037;K(37)= .037;K(38)= .036;K(39)= .035;K(40)= .034;             00003900
        K(41)=0.034;                                                             00004000
 ON ENDFILE(SYSIN) GO TO FIN;                                                    00004100
 START:                                                                          00004200
   GET DATA(ANG1,ANG2, ANGINC);                                                  00004300
 PUT PAGE;                                                                       00004400
 PUT EDIT(' _________ _________ _________ _________ _________')(A)SKIP;         00004500
 PUT EDIT('|DISTANCE|   TILT   |DISPLACE| GRAVITY |  RATIO  |')(A)SKIP;         00004600
 PUT EDIT('|_________|_________|_________|_________|_________|')(A)SKIP;         00004700
 ANGLE:DO ANG=ANG1 TO ANG2 BY ANGINC;                                            00004800
       ARG=COSD(ANG);                                                            00004900
       LIMIT:DO P= .99;                                                          00005000
                PP=1/P;                                                          00005100
             FORM:DO J=1 TO 41 ;                                                 00005200
                PP=PP*P;                                                         00005300
                C(J)=-H(J)*PP;                                                   00005400
                D(J)=(2*(J-1)+1)*PP;                                             00005500
                E(J)=PP*(J-1-(2*H(J))+(J)*K(J));                                 00005600
                END FORM;                                                        00005700
                EXTR:DO J=42 TO DIM;                                             00005800
```

```
MEMBER NAME  LONG
                    HH=+.86*LOG(J-1)-.54;                                          00005900
                    KK=-.025*LOG(J-1)+.127;                                        00006000
                    PP=PP*P;                                                       00006100
                    C(J)=-HH*PP;                                                   00006200
                    D(J)=(2*(J-1)+1)*PP;                                           00006300
                    E(J)=PP*(J-1-(2*HH)+J*KK);                                     00006400
                    END EXTR;                                                      00006500
 THEORG=-1/(4*SIND(ANG/2));                                                        00006600
               CALL POSV(ARG,C,DIM,PUR,RESU);                                      00006700
               CALL POSV(ARG,E,DIM,PUR,RESG);                                      00006800
 TILT=(RESU-RESU1)*57.4;                                                           00006900
 RATO=RESG/THEORG;                                                                 00007000
 PUT EDIT('|',ANG,'|',TILT,'|',RESU,'|',RESG,'|',RATO,'|')                        00007100
 (A,F(6,1),X(2),A,F(8,2),X(1),A,F(7,2),X(1),A,F(7,2),X(1),A,F(6,3),               00007200
 X(1),A)SKIP;                                                                      00007300
 RESU1=RESU;                                                                       00007400
               END LIMIT;                                                          00007500
       END ANGLE;                                                                  00007600
 PUT EDIT('|________|________|_______|_______|______|')(A)SKIP;                   00007700
 PUT SKIP(4);                                                                      00007800
 PUT EDIT('    TABLE    . DATA FOR GUTENBERG-MODEL EARTH    ')(A)SKIP;           00007900
 PUT EDIT('                                              .')(A)SKIP;              00008000
 GO TO START;                                                                      00008100
 POSV:                                                                             00008200
 /*****************************************************************************/ 00008300
 /*                                                                           */ 00008400
 /*      EVALUATE N-TERM SERIES EXPANSION IN ORTHOGONAL POLYNOMIALS           */ 00008500
 /*                                                                           */ 00008600
 /*****************************************************************************/ 00008700
    PROCEDURE(X,C,N,OPT,SUM);                                                      00008800
    DECLARE                                                                        00008900
         (LX,H,H0,H1,H2,FN) BINARY FLOAT(53),                                      00009000
         (X,C(*),SUM)                                                              00009100
         BINARY FLOAT,                    /*SINGLE PRECISION VERSION  /*S*/00009200
         BINARY FLOAT(53),                /*DOUBLE PRECISION VERSION  /*D*/00009300
         (N,I) BINARY FIXED,                                                       00009400
         OPT CHARACTER(1);                                                         00009500
    I    =N;                                                                       00009600
    IF I >= 1                           /*BYPASS OPERATION IF N LE 0   */00009700
    THEN DO;                                                                       00009800
         LX   =X;                                                                  00009900
         IF OPT='L'                     /*LAGUERRE POLYNOMIALS L(X)    */00010000
         THEN LX   =1-LX;                                                          00010100
         H2,H1=0;                       /*ZERO U(N+1), U(N+2) OR V(N+2)*/00010200
         FN   =I;                                                                  00010300
ITER:                                   /*LOOP OVER I = N TO 1 BY -1   */00010400
         IF OPT='T'                     /*CHEBYSHEV POLYNOMIALS T(X)   */00010500
         THEN DO;                                                                  00010600
              H0   =LX*H1;                                                         00010700
              H    =H0-H2+H0;            /*H = 2*X*U(I+1)-U(I+2)        */00010800
              END;                                                                 00010900
         ELSE DO;                                                                  00011000
              IF OPT='H'                /*HERMITE POLYNOMIALS H(X)     */00011100
              THEN DO;                                                             00011200
                   H    =LX*H1-FN*H2;                                           00011300
                   H    =H1+H;           /*H = 2*(X*U(I+1)-I*U(I+2))    */00011400
                   END;                                                          00011500
              ELSE DO;                  /*LAGUERRE OR LEGENDRE POLYNOM.*/00011600
```

```
 MEMBER NAME  LONG
                    H0    =H1;                /*SAVE U(I+1)                    */00011700
                    H     =H1/FN;                                                 00011800
                    H1    =H1-H;              /* COMPUTE V(I+1)                 */00011900
                    IF OPT='L'                /*LAGUERRE POLYNOMIALS L(X)      */00012000
                    THEN H =H1+LX*H+H1;       /*H = 2*V(I+1)+(1-X)*U(I+1)      */00012100
                    ELSE H =LX*(H1+H0);       /*LEGENDRE POLYNOMIALS L(X)      */00012200
                         H     =H-H2;         /*H = X*(V(I+1)+U(I+1))          */00012300
                         END;                 /*FOR BOTH H = H-V(I+2)          */00012400
               FN    =FN-1;                   /*DECREASE INTEGER FACTOR        */00012500
               END;                                                              00012600
          H2   =H1;                          /*SAVE U(I+1) RESP. V(I+1)       */00012700
          H1   =H+C(I);                      /*COMP. U(I) = H+C(I)            */00012800
          I    =I-1;                         /*DECREASE COUNTER I             */00012900
          IF I > 0                                                             00013000
          THEN GO TO ITER;                   /*END OF LOOP OVER I             */00013100
          IF OPT='T'                                                           00013200
          THEN H1    =H1-H0;                 /*MODIFY U(1) IN CHEBYSHEV CASE*/00013300
          SUM  =H1;                          /*RETURN VALUE OF SERIES         */00013400
          END;                                                                 00013500
   END POSV;                                                                   00013600
   FIN:                                                                        00013700
   END LONG;                                                                   00013800
```

APPENDIX VI

Program GLOBL

```
:PROCEDURE OPTIONS(MAIN);                                                00000700                    PAGE      2

     GLOBL:PROCEDURE OPTIONS(MAIN);                                                                  00000700
     DECLARE DRB FLOAT(12)INITIAL(0);                                                                00000800
     DECLARE Z(0:180) INITIAL                                                                        00000900
      (630, 620, 505, 444, 381, 345, 307, 286, 263, 250, 237, 227, 220, 211,00001000
     204,195,189,183, 177, 171, 167, 162, 157, 152, 137, 143, 138, 134, 130,00001100
     125,120,109,105, 103, 100,  97,  93,  91,  87,  84,  82,  79,  76,  73,00001200
     71,69,  67,  64,  62,  60,  59,  57,  56,  55,  54,  53,  52,  52,  51,00001300
     51,50,  51,  51,  52,  52,  53,  54,  54,  55,  56,  57,  59,  61,  62,00001400
     64,67,  68,  70,  71,  72,  74,  76,  79,  82,  84,  86,  90,  94,  96,00001500
     99,102, 105, 109, 112, 115, 119, 121, 125, 127, 130, 133, 137, 140,144,00001600
     148,151,155, 159, 162, 166, 169, 172, 175, 178, 182, 185, 189, 193,196,00001700
     200,203,207, 210, 214, 218, 221, 225, 228, 232, 235, 238, 241, 244,247,00001800
     249,252,255, 257, 260, 263, 265, 268, 271, 274, 276, 279, 281, 284,286,00001900
     288,290,293 ,295, 297, 298, 300, 301, 302, 304, 305, 306, 308, 311,313,00002000
     314,316,316, 317, 317, 318, 318, 320, 320, 321, 323, 325, 326, 327,327,00002100
     328,329);                                                                                       00002200
     DECLARE INCC BINARY FLOAT, IDEG BINARY FIXED;
     DECLARE XAMP(41)    BINARY FLOAT INITIAL(
     930,740,605,530,462,425,382,350,323,299,279,263,251,242,236,231,229,
     229,228,226,224,223,222,221,220,219,216,214,211,209,206,204,202,200,
     198,196,195,193,192,191,190);
     DECLARE ZH(0:180) INITIAL                                                                       00002300
      (382, 334, 305, 287, 255, 237, 219, 204, 195, 181, 177, 165, 165,158, 00002400
     158,157,158,159, 156, 156, 150, 150, 146, 147, 147, 151, 154, 153,155, 00002500
     149,148,151,150, 150, 149, 148, 146, 144, 143, 142, 141, 139, 137,136, 00002600
     134,132,130,127, 125, 123, 121, 117, 115, 111, 107, 104, 100,  97,93,  00002700
     90,87,  84,  81,  78,  75,  70,  66,  61,  57,  52,  49,  45,  41,38,  00002800
     35,31,  28,  25,  21,  18,  15,  11,   8,   5,   1,  -5,  -6,  -8,-13, 00002900
     -16,-18,-22,-25, -28, -32, -35, -37, -40, -43, -45, -48, -50, -52,-57, 00003000
     -58,-61,-64,-66, -68, -70, -73, -74, -77, -79, -81, -82, -84, -86,-87, 00003100
     -89,-91,-92,-92, -93, -94, -96, -96, -98, -99,-100,-101,-102,-103,-104,00003200
      -106,-108,-109,-110,-111,-113,-113,-115,-116,-117,-118,-119,-119,-118,00003300
      -118,-118,-118,-118,-118,-119,-120,-121,-121,-122,-124,-126,-129,-131,00003400
      -134,-135,-139,-143,-146,-149,-153,-155,-157,-159,-161,-163,-166,-167,00003500
      -170,-173,-176,-179);                                                                          00003600
     DECLARE XRAT(41)    BINARY FLOAT INITIAL(
     000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,
     000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,000,
     000,000,000,000,000,000,000,000,000);
     XAMP=XAMP/100;    XRAT=-XRAT/100;
     XAMP=(XAMP*0.9)+1.0;
     DECLARE (CODF)FLOAT;                                                                            00003700
     DECLARE(HH(0:100))FLOAT;  HH=100;                                                               00003800
     DCL(FV,FN,FE,FVQ,FNQ,FEQ,XT,YT,H,Q,LV,MV,NV,LN,MN,NN,LE,ME,NE,L3,M3,N3,00003900
      L6,L2,M2,N2,L1,M1,N1,XJ,YJ,ZJ,XJJ,YJJ,ZJJ,XS,YS,ZS,LL,UJ,VJ,WJ,UJJ,VJJ00004000
      ,M6,WJJ,AJ,PJ,RJ,FJ,CJ,DFU,DFV,DFW,DFX,DFY,DFZ,DBV,DBN,DBE,FVC,FNC,FEC00004100
     ,N6,L7,M7,N7,AV,AN,AE,PV,PN,PE,AJJ,PJJ,RJJ,FJJ)FLOAT(12)INITIAL(0);    00004200
     DECLARE (UNIT,DELX,DELY,LO,UO,WO,VO,ARGO)FLOAT(12)INITIAL(0);                                   00004300
     DECLARE                                                                                         00004400
          LOG1 FILE RECORD,                                                                          00004500
          BUFF1 CHAR(80),                                                                            00004600
          CRBE FILE PRINT;                                                                           00004700
     ON ENDFILE(LOG1) GO TO FIN;                                                                     00004800
      ON ENDFILE(SYSIN)  GOTO FIN2;                                                                  00005300
     DCL(YD(8),XD(8),X(8),Y(8));                                                                     00005400
     CA=6.3783E+8; CB=1.7453E-9; CC=CA*CB;                                                           00005500
     /*THIS PROGRAM DETERMINES THE GRAVITATIONAL ATTRACTION IN MICROGALS AT                          00005600
```

```
PROCEDURE OPTIONS(MAIN);                                        00000700                    PAGE     3

     SITE XT(LONG EAST TIMES 100),YT (LAT NORTH TIMES 100) DUE TO TIDAL WA00005700
     DEFINED BY SURFACE  POLYGONS WITH ASSIGNED AMPLITUDE (H IN CMS) AND P00005800
     (Q IN DEGS).THE POLYGONS MAY BE LOCATED ANYWHERE ON THE GLOBE BUT MUS00005900
     DEFINED BY NOT MORE THAN 7 POINTS. BEFORE COMPUTATION EACH            00006000
     POLYGON IS PROJECTED ONTO A PLANE WHOSE NORMAL IS THE VERTICAL        00006100
     THROUGH THE FIRST POINT OF THE POLYGON. THE METHOD AND MANY OF THE    00006200
     SYMBOLS USED ARE THOSE OF BOTT (GEO. PROS. VOL XI,3,1963.)          */00006300
          /*SYMBOL LIST FOLLOWS                                          00006400
          XT,YT          POSITION OF SITE. X IS LONG (EAST DEGS*100),Y IS LAT00006500
          XD(8),YD(8)    POSITION OF POLYGON POINTS. ONLY 7 MAY BE ASSIGNED(A00006600
          H              AMPLITUDE OF TIDE(IN CMS).LATER MADE H*6.664E-8*E*6.00006700
          Q              PHASE ANGLE WITH RESPECT TO LUNAR TRANSIT OF GREENWI00006800
          X,Y(8)         POSITION OF POLYGON POINTS IN KMS FROM SITE (EAST AN00006900
          L,M,N          ALL OF THIS TYPE ARE DIRECTION COSINES WITH RESPECT 00007000
                         AXES DIRECTED TO LAT.LONG (0,0),(0,90),(90,0).      00007100
          LV,MV,NV       DIR. COS. OF VERTICAL THROUGH SITE.                 00007200
          LN,MN,NN       DIR. COS. OF NORTH THROUGH SITE.                    00007300
          LE,ME,NE       DIR. COS. OF EAST THROUGH SITE.                     00007400
          L7,M7,N7       EQUAL TO 3X6.                                       00007500
          L6,M6,N6       EQUAL TO 3X2 WHERE 2 IS FIRST VECTOR 2.             00007600
          L3,M3,N3,W     DIR. COS. OF VERTICAL THROUGH PT. J OF POLYGON. (W A00007700
          L2,M2,N2,V     DIR. COS. OF VECTOR PT. J TO PT. J+1 OF POLYGON.(V A00007800
                         (SEE ALSO PROGRAM NOTES CONCERNING THIS VECTOR)     00007900
          L1,M1,N1,U     DIR. COS. OF VECTOR ORTHOGONAL TO BOTH 2 ABOVE. (U A00008000
          XJ,XJJ,XS      X COORD OF PT. J,J+1 AND SITE. (CMS)                00008100
          YJ,YJJ,YS      Y COORD OF PT. J,J+1 AND SITE. (CMS)                00008200
          ZJ,ZJJ,ZS      Z COORD OF PT. J,J+1,AND SITE. (CMS)                00008300
          LL             DISTANCE PT. J TO PT. J+1.      (CMS)               00008400
          UJ,VJ,WJ       DISTANCE SITE TO PT. J ALONG U,V AND W AXES.         00008500
          UJJ,VJJ,WJJ    DISTANCE SITE TO PT. J+1 ALONG U,V, AND W AXES.      00008600
          AJ,PJ,RJ,FJ    BOTT'S SYMBOLS ALPHA,RHO,R,F FOR PT. J. ADD J FOR J+00008700
          CJ             DETERMINES WHETHER LAMINA IS ADDED OR SUBTRACTED.SEE00008800
          DFU,DFV,DFW    FORCE COMPONENTS ALONG U,V,W AXES AT SITE.NON-ACCUMU00008900
          DFX,DFY,DFZ    FORCE COMPONENTS ALONG X,Y,Z AXES AT SITE.ACCUMULATI00009000
          DBV,DBN,DBE    FORCE COMPONENTS ALONG VERTICAL,NORTH,EAST AT SITE.  00009100
          FVC,FNC,FEC    ACCUMULATING  IN-PHASE COMPONENT OF ABOVE.           00009200
          FVQ,FNQ,FEQ    ACCUMULATING QUADRATURE COMPONENT OF ABOVE.          00009300
          AV,AN,AE       AMPLITUDE OF VERTICAL,NORTH AND EAST ATTRACTION.(MIC00009400
          PV,PN,PE       PHASE     OF VERTICAL,NORTH AND EAST ATTRACTION.(DEG00009500
          HH,V1,V2,V3,W1,W2,W3     ADDED TEMPORARILY                         00009600
          END OF SYMBOL LIST   */                                           00009700
     /* READ SITE POSITION, CHANGE TO DEGREES. COMPUTE DIR. COSINES OF VERTI00009800
        NORTHWARD, AND EASTWARD VECTORS THROUGH SITE. COMPUTE X,Y,Z COORDS O00009900
         SITE (XS,YS,ZS). PRINT OUT ALL SITE DATA.     */                   00010000
     SKIPZONE=9;                                                          00010100
     PULLSIZE=.002;                                                       00010200
     ACT1:GET DATA;  /*REQUIRES SITE COORDS:XT AND YT. ACCEPTS CHANGES IN*/ 00010300
                     /*Z, ZH, SKIPZONE OR PULLSIZE.                   */ 00010400
     Z=Z/100.0;    ZH=ZH/100.0;                                           00010500
     SZ0=ZH(0); SZ1=ZH(1); SZ2=ZH(2); SZ3=ZH(3);
     SG0=Z(0);  SG1=Z(1);  SG2=Z(2);  SG3=Z(3);
     OPEN FILE(LOG1) INPUT;                                               00010600
     LV=COSD(YT)*COSD(XT);MV=COSD(YT)*SIND(XT);NV=SIND(YT);               00010700
     LN=-SIND(YT)*COSD(XT);MN=-SIND(YT)*SIND(XT);NN=COSD(YT);             00010800
     LE=MN*NV-MV*NN;ME=-LN*NV+LV*NN;NE=LN*MV-LV*MN;                       00010900
     XS=LV*CA;   YS=MV*CA;   ZS=NV*CA;                                    00011000
     /* START READS DATA AND COMPUTES RESULTS FOR ONE POLYGON WITH STORAGE  00011100
```

```
     FOR CONTINUED ACCUMULATION WITHOUT LIMIT.     */                    00011200
   /*
   PUT PAGE;                                                             00011300
   PUT EDIT('NBR AREA   VERTICAL        NORTH          EAST')(A)SKIP(3); 00011400
   PUT EDIT('CORRECTED GRAVITATIONAL ATTRACTION IN MICROGALS DUE TO')(A)  00011500
   SKIP(2);                                                              00011600
   PUT EDIT('GLOBAL TIDAL WATERS')(A)SKIP;                               00011700
   PUT EDIT('AREAS ARE:(1)-N.ATLANTIC.(2)-S.ATLANTIC.(3)-N.PACIFIC.(4)-S.P00011800
   ACIFIC.(0)-W.EUROPEAN DETAIL.')(A)SKIP;                               00011900
   PUT EDIT('(5)-INDIAN OCEAN. (6)-E AND NE CANADA DETAIL.')(A)SKIP;     00012000
   PUT EDIT('REGIONAL DETAIL WILL BE REQUIRED FOR COASTAL SITES')(A)SKIP; 00012100
   PUT EDIT('NOTE 1. LAST PROGRAM CHANGE- FEB  2/1971')(A)SKIP;          00012200
   PUT EDIT('SITE COORDINATES FOLLOW')(A)SKIP(2);                        00012300
   PUT EDIT('LAT=',YT,'LONG=',XT)(A,F(6,2),X(2),A,F(6,2))SKIP;           00012400
   PUT EDIT('Z(0)=',Z(0),'Z(1)=',Z(1),'Z(2)=',Z(2),'Z(3)=',Z(3))         00012410
         (4(X(3),A,F(5,3)))SKIP;                                         00012420
   PUT EDIT('ZH(0)=',ZH(0),'ZH(1)=',ZH(1),'ZH(2)=',ZH(2),'ZH(3)=',ZH(3)) 00012430
         (4(X(3),A,F(5,3)))SKIP;                                         00012440
   PUT EDIT('SKIPZONE',SKIPZONE,'PULLSIZE',PULLSIZE)                     00012450
         (X(3),A,F(2),X(3),A,F(9,4));                                    00012460
   */
   PUT SKIP(2);                                                          00012500
   INCC=5;  IDEG=41;  DDFU,DDFV,DDFW=0;
   CALL DET5(INCC,XAMP,XRAT,IDEG);
   XRAT=XRAT*1.0E-5;
   START:                                                                00012600
   ZH(0)=SZ0; ZH(1)=SZ1; ZH(2)=SZ2; ZH(3)=SZ3;
    Z(0)=SG0;  Z(1)=SG1;  Z(2)=SZ2;  Z(3)=SG3;
   /*READ FILE(LOG1) INTO (BUFF1); */                                    00012700
    GET EDIT(BUFF1)(A(80));
   GET STRING(BUFF1) EDIT(RUN,H,Q,CODE,(XD(J),YD(J) DO J=1 TO 7))        00012800
                          (3 F(3),F(1),14 F(5));                         00012900
   /*MULTIPLY TIDAL HALF RANGE BY GRAVITATIONAL CONSTANT(6.6E-8) AND     00013000
     ALSO BY (1.0E+6) TO YIELD ATTRACTION IN MICROGALS */                00013100
   H=H*.06664;                                                           00013200
   IF(CODE=SKIPZONE)THEN GO TO START;                                    00013300
   /*DETERMINE GREAT CIRCLE DISTANCE TO FIRST POLYGON PT */
   AA=90-(YD(1)/100);
   BB=90-YT;
   CC=XT-(XD(1)/100);
   /* CSC IS COS C WHERE C IS ANGLE SUBTENDED AT CENTER OF EARTH BY
      GREATE CIRCLE DISTANCE GCD. */
   CSC=COSD(AA-BB)-SIND(AA)*SIND(BB)*(1-COSD(CC));
   /* LL IS DISTANCE TO FIRST POLYGON PT IN PLANE OF POLYGON */
   LL=CA*SQRT(1-CSC**2);
   DST2=LL**2;
         IF(DST2<4.0E+14) THEN DO;
         IDST=((SQRT(DST2)*1.0E-5)+2.5)/5; IDST=IDST+1;
         XP=XAMP(IDST)-XRAT(IDST)*SQRT(DST2);
         XR=XRAT(IDST); ZH(0),ZH(1),ZH(2),ZH(3),Z(0),Z(1),Z(2),Z(3)=1;
    END;
    ELSE DO;  XP=1;  XR=0; END;
   PUT DATA(DST2,XP,XR);
   SIDE1=SQRT((XD(1)-XD(2))**2+(YD(1)-YD(2))**2);                        00013700
   SIDE2=SQRT((XD(3)-XD(2))**2+(YD(3)-YD(2))**2);                        00013800
   AREA=SIDE1*SIDE2*1.0E10;                                              00013900
   PULL=(AREA*H)/DST2;       /*ATTRACTION IN MICROGALS */              00014000
```

```
   IF(PULL<PULLSIZE) THEN GO TO START;                                    00014100
   XD(8)=0;YD(8)=0;                                                       00014200
   /* DETERMINE NUMBER OF POLYGON SIDES (JMAX)      */                   00014300
   DO J=1 TO 7 WHILE(YD(J)¬=0|XD(J)¬=0);JMAX=J;END;                       00014400
   YD(JMAX+1)=YD(1);XD(JMAX+1)=XD(1);                                     00014500
   YD=YD/100;  XD=XD/100;                                                 00014600
   DFX,DFY,DFZ=0;                                                         00014700
   /*COMPUTE DIRECTION COSINES OF OUTWARD NORMAL TO POLYGON CALLED RUN AND00014800
     DESIGNATE BY L3,M3,N3. THIS NOW IS DIRECTION OF W AXIS  */           00014900
   J=1;                                                                   00015000
   XJJ=CA*COSD(YD(J+1))*COSD(XD(J+1));XJ=CA*COSD(YD(J))*COSD(XD(J));      00015100
   YJJ=CA*COSD(YD(J+1))*SIND(XD(J+1));YJ=CA*COSD(YD(J))*SIND(XD(J));      00015200
   ZJJ=CA*SIND(YD(J+1));                ZJ=CA*SIND(YD(J));                00015300
   L3=XJ/CA;  M3=YJ/CA;  N3=ZJ/CA;                                        00015400
   WJ=L3*(XJ-XS)+M3*(YJ-YS)+N3*(ZJ-ZS);     WJ=-WJ;                       00015500
   /* LAT AND LONG OF NORMAL AT SURFACE */
   CLONG=ATAND(L3,M3);
   CBASE=SQRT(L3**2+M3**2);
   CLAT=ATAND(CBASE,N3);
   /* STRAIGHT-LINE DISTANCE SITE TO NORMAL AT SURFACE */
   DSTSC=SQRT((L3*CA-XS)**2+(M3*CA-YS)**2+(N3*CA-ZS)**2);
   /* TEST OUTPUT */
   PUT DATA(L3,M3,N3,CLONG)SKIP;
   PUT DATA(CBASE,CLAT,DSTSC)SKIP;
   /* JLOOP COMPUTES THE ATTRACTION DUE TO EACH OF THE JMAX LAMINAS AND  00015600
      ACCUMULATES THE X,Y AND Z COMPONENTS OF THIS ATTRACTION IN          00015700
      DFX, DFY AND DFZ.          */                                      00015800
   JLOOP:DO J= 1 TO JMAX;                                                 00015900
   XJJ=CA*COSD(YD(J+1))*COSD(XD(J+1));XJ=CA*COSD(YD(J))*COSD(XD(J));      00016000
   YJJ=CA*COSD(YD(J+1))*SIND(XD(J+1));YJ=CA*COSD(YD(J))*SIND(XD(J));      00016100
   ZJJ=CA*SIND(YD(J+1));                ZJ=CA*SIND(YD(J));                00016200
   LL=SQRT((XJJ-XJ)**2+(YJJ-YJ)**2+(ZJJ-ZJ)**2);                          00016300
   L2=(XJJ-XJ)/LL;  M2=(YJJ-YJ)/LL;  N2=(ZJJ-ZJ)/LL;                      00016400
   /*THE COMPONENT OF VECTOR 2 IN THE PLANE PERPENDICULAR TO THE W AXIS  00016500
   (VECTOR 3) IS NEXT DETERMINED AND CALLED VECTOR 7.  VECTOR 7 IS THEN  00016600
   MADE A UNIT VECTOR AND CALLED THE NEW VECTOR 2 WHICH THEN BECOMES THE 00016700
   V AXIS. THUS EACH LAMINA OF THE POLYGON IS PROJECTED ON TO THE SAME   00016800
   PLANE, PERPENDICULAR TO THE SINGLE w AXIS. THE W AXIS IS DETERMINED BY 00016900
   THE EARTH'S RADIUS VECTOR THROUGH THE FIRST POINT OF THE POLYGON.    */00017000
   L6=(M2*N3-N2*M3);  M6=-(L2*N3-N2*L3);  N6=(L2*M3-M2*L3);               00017100
   L7=(M3*N6-M6*N3);  M7=-(L3*N6-L6*N3);  N7=(L3*M6-M3*L6);               00017200
   UNIT=L7*L7+M7*M7+N7*N7;  UNIT=SQRT(UNIT);                              00017300
   L2=L7/UNIT;  M2=M7/UNIT;  N2=N7/UNIT;                                  00017400
   L1=(M2*N3-N2*M3);M1=(L3*N2-L2*N3);N1=(L2*M3-L3*M2);                    00017500
   UJ=L1*(XJ-XS)+M1*(YJ-YS)+N1*(ZJ-ZS);                                   00017600
   VJ=L2*(XJ-XS)+M2*(YJ-YS)+N2*(ZJ-ZS);                                   00017700
   VJJ=L2*(XJJ-XS)+M2*(YJJ-YS)+N2*(ZJJ-ZS);                               00017800
   IF UJ>0 THEN CJ=1; IF UJ=0 THEN CJ=0; IF UJ<0 THEN DO;CJ=-1;UJ=-UJ;    00017900
      L1=-L1;M1=-M1;N1=-N1;END;                                           00018000
   AJ=VJ/UJ;AJJ=VJJ/UJ ;PJ=SQRT(UJ**2+VJ**2);PJJ=SQRT(UJ**2+VJJ**2);     00018100
   RJ=SQRT(UJ**2+VJ**2+WJ**2);RJJ=SQRT(UJ**2+VJJ**2+WJ**2);               00018200
   FJ=SQRT(1+AJ**2);FJJ=SQRT(1+AJJ**2);                                   00018300
   DFU=(AJJ*LOG((RJJ+PJJ)/ABS(WJ))/FJJ)-.5*LOG(RJJ+VJJ)+.5*LOG(RJJ-VJJ)  00018400
       -AJ*LOG((PJ+PJJ)/ABS(WJ))/FJ+.5*LOG(RJ+VJ)-.5*LOG(RJ-VJ);          00018500
   DFV=-1*LOG((RJJ+PJJ)/ABS(WJ))/FJJ+1*LOG((RJ+PJ)/ABS(WJ))/FJ;           00018600
   DFW=-(WJ/ABS(WJ))*ATAN(AJJ*(RJJ-ABS(WJ)),(RJJ+(AJJ**2)*ABS(WJ)))      00018700
       +(WJ/ABS(WJ))*ATAN(AJ *(RJ -ABS(WJ)),(RJ +(AJ**2) *ABS(WJ)));      00018800
```

```
   PUT DATA(DFU,DFV)SKIP;
   DO;
   DDFU=UJ*(ATAN(VJJ,UJ)-ATAN(VJ,UJ));
   DDFV=UJ*(LOG(PJJ)-LOG(PJ));
   DFU=(DFU*XP)+(DDFU*XR);
   DFV=(DFV*XP)+(DDFV*XR);
   END;
   PUT DATA(DFW)SKIP;
   PUT DATA(DDFU,DDFV,DFU,DFV)SKIP;
   PUT DATA(UJ,VJ,VJJ,WJ)SKIP;
   DFW=-DFW;DFU=-DFU;DFV=-DFV;    /*ORIGINAL EQNS ASSUME NEG GRADIENTS */  00018900
   DFX=DFX+CJ*(L1*DFU+L2*DFV+L3*DFW);                                      00019000
   DFY=DFY+CJ*(M1*DFU+M2*DFV+M3*DFW);                                      00019100
   DFZ=DFZ+CJ*(N1*DFU+N2*DFV+N3*DFW);                                      00019200
   END JLOOP;                                                              00019300
   /* RESOLVE THE X,Y,Z COMPONENTS OF ATTRACTION DUE TO THE ENTIRE PO_YGON00019400
      ALONG VERTICAL NORTHWARD AND EASTWARD AXES THROUGH THE SITE AND ACCU00019500
      SEPARATELY BOTH THE IN-PHASE COMPONENTS (FV,FN,FE) AND THE QUADRATUR00019600
      COMPONENTS (FVQ,FNQ, FEQ) FOR ALL POLYGONS     */                   00019700
   DBV=DFX*LV+DFY*MV+DFZ*NV;                                               00019800
   DRB=(4*CA*CA/(RJ*RJ))-1;                                                00019900
   DRB=ABS(DRB)+.0001;                                                     00020000
   KR=ABS(2*ATAND(1/SQRT(DRB)))+0.5;  /*KR IS ANGULAR DISTANCE IN ROUNDED 00020100
      DEGREES   */                                                         00020200
   DBV=DBV*Z(KR);                                                          00020300
    FV=DBV*H*COSD(Q)+FV;FVQ=DBV*H*SIND(Q)+FVQ;                             00020400
   DBN=DFX*LN+DFY*MN+DFZ*NN;   DBN=DBN*ZH(KR)/(.98*4.848);                 00020500
                               FN=DBN*H*COSD(Q)+FN;FNQ=DBN*H*SIND(Q)+FNQ;  00020600
   DBE=DFX*LE+DFY*ME+DFZ*NE;   DBE=DBE*ZH(KR)/(.98*4.848);                 00020700
                               FE=DBE*H*COSD(Q)+FE;FEQ=DBE*H*SIND(Q)+FEQ;  00020800
   /* OUTPUT NET AMPLITUDE AND PHASE AT SITE IN MICROGALS AND DEGREES */   00020900
   AV=SQRT(FV**2+FVQ**2);IF AV¬=0 THEN PV=ATAND(FVQ,FV);                   00021000
   AN=SQRT(FN**2+FNQ**2);IF AN¬=0 THEN PN=ATAND(FNQ,FN);                   00021100
   AE=SQRT(FE**2+FEQ**2);IF AE¬=0 THEN PE=ATAND(FEQ,FE);                   00021200
     /*PV, PN, AND PE ARE PHASE LAGS OF UP,NORTH AND EAST EFFECTS       */ 00021300
     /*RELATIVE TO TRANSIT OF GREENWICH BY MOON.                        */ 00021400
   DBV=DBV*H;  DBN=DBN*H;  DBE=DBE*H;                                      00021500
            /*DBV, DBN, AND DBE ARE POSITIVE FOR UP, NORTH AND EAST.*/     00021600
            /*DBV IN MICROGALS, DBN AND DBE IN MILLISECS.          */     00021700
   PUT EDIT(RUN,CODE,AV,'(',PV,')',AN,'(',PN,')',AE,'(',PE,')',DBV,DBN,    00021800
   DBE,KR,Z(KR),ZH(KR))(F(3),F(2),3(F(9,3),A,F(4),A),X(3),3 F(7,4),F(4),   00021900
   2 F(4,0,2))SKIP;                                                        00022000
   PUT DATA(DBV,DBN,DBE)SKIP;
   /* GO TO START;  */                                                     00022200
   FIN:                                                                    00022300
   PUT FILE(CRBE) DATA(XT,YT,AV,PV,AN,PN,AE,PE)SKIP(3);                    00022400
   CLOSE FILE(LOG1);                                                       00022500
   TV=74.702*(COSD(YT)**2);   /*EARTH TIDE GRAVITY */                     00022600
   TN=7.8630*(SIND(2*YT));    /*EARTH TIDE TILT NORTH  */                  00022700
   TE=15.7261*COSD(YT);       /*EARTH TIDE TILT EAST   */                  00022800
   AV=AV/TV;  AN=AN/TN;  AE=AE/TE;                                         00022900
   TV=1.16;                          /*AMP FACTOR ASSUMED              */ 00023000
   TN=0.700;                         /*TILT FACTOR ASSUMED             */ 00023100
   TE=0.700;                         /*TILT FACTOR ASSUMED             */ 00023200
   TVP=-2*XT;                     /*THEORETICAL PHASE LAG, UP POSITIVE.  */ 00023300
   TNP=(-2*XT)-180;               /*THEORETICAL PHASE LAG, NORTH POSITIVE*/ 00023400
   TEP=(-2*XT)-90;                /*THEORETICAL PHASE LAG, EAST POSITIVE.*/ 00023500
```

```
   QV=PV-TVP;  IQV=(QV+180)/360;  QV=QV-(IQV*360);                    00023600
   QN=PN-TNP;  IQN=(QN+180)/360;  QN=QN-(IQN*360);                    00023700
   RVX=TV*COSD(TVP);  RVY=TV*SIND(TVP);                               00023800
   QE=PE-TEP; IQE=(QE+180)/360;  QE=QE-(IQE*360);                     00023900
   RAVX=AV*COSD(PV);  RAVY=AV*SIND(PV);                               00024000
   RVX=RAVX+RVX;  RVY=RAVY+RVY;                                       00024100
   PUT EDIT(RUN,CODE,AV,'(',QV,')',AN,'(',QN,')',AE,'(',QE,')',DBV,DBN,  00024200
   DBE,KR,Z(KR),ZH(KR))(F(3),F(2),3(F(9,3),A,F(4),A),X(3),3 F(7,4),F(4),  00024300
   2 F(4,0,2))SKIP(3);                                                00024400
   AV=SQRT(RVX**2+RVY**2);                                            00024500
   PV=ATAND(RVY,RVX);                                                 00024600
   RNX=TN*COSD(TNP);  RNY=TN*SIND(TNP);                               00024700
   RANX=AN*COSD(PN);  RANY=AN*SIND(PN);                               00024800
   RNX=RANX+RNX;  RNY=RANY+RNY;                                       00024900
   AN=SQRT(RNX**2+RNY**2);                                            00025000
   PN=ATAND(RNY,RNX);                                                 00025100
   REX=TE*COSD(TEP);  REY=TE*SIND(TEP);                               00025200
   RAEX=AE*COSD(PE);  RAEY=AE*SIND(PE);                               00025300
   REX=RAEX+REX;  REY=RAEY+REY;                                       00025400
   AE=SQRT(REX**2+REY**2);                                            00025500
   PE=ATAND(REY,REX);                                                 00025600
   PV=PV-TVP; IPV=(PV+180)/360;  PV=PV-(IPV*360);                     00025700
   PN=PN-TNP; IPN=(PN+180)/360;  PN=PN-(IPN*360);                     00025800
   PE=PE-TEP; IPE=(PE+180)/360;  PE=PE-(IPE*360);                     00025900
    /*PV, PN AND PE NOW ARE PHASE LAGS OF CALCULATED RESULTANT DIMINISHING00026000
      (TILT) OR AMPLIFICATION (GRAVITY) FACTORS RELATIVE TO THAT WHICH   00026100
      WOULD BE OBSERVED ON A YIELDING BUT OCEANLESS EARTH WHERE THESE    00026200
      FACTORS ARE 0.7 AND 1.16 RESPECTIVELY.                    */       00026300
   PUT EDIT(RUN,CODE,AV,'(',PV,')',AN,'(',PN,')',AE,'(',PE,')',DBV,DBN,  00026400
   DBE,KR,Z(KR),ZH(KR))(F(3),F(2),3(F(9,3),A,F(4),A),X(3),3 F(7,4),F(4),  00026500
   2 F(4,0,2))SKIP(3);                                                00026600
   PUT FILE(CRBE) DATA(XT,YT,AV,PV,AN,PN,AE,PE)SKIP(3);               00026700
   FV,FVQ,FN,FNQ,FE,FEQ=0;                                            00026800
   GO TO ACT1;                                                        00027100
   FIN2:                                                              00027200
   END GLOBL;                                                         00027300
```

APPENDIX VII

Program BETTY

```
MEMBER NAME  BETTY
 BETTY:PROCEDURE OPTIONS(MAIN);                                          00000100
 /*************************************************************************/00000200
 /*                                                                       */00000300
 /*      THIS PROGRAM ACCEPTS AS INPUT HOURLY SAMPLES OF TILTMETER        */00000400
 /*      OR GRAVIMETER OBSERVATIONS. THE PROGRAM DOES THE FOLLOWING:      */00000500
 /*          (1) A 6-POINT SMOOTHNESS TEST IS MADE AND HOUR BY HOUR       */00000600
 /*              RESIDUALS ARE PRINTED OUT. A DISTINCTIVE PATTERN         */00000700
 /*              OF RESIDUALS RESULT ACCORDING TO WHETHER THERE IS A      */00000800
 /*              SINGLE ISOLATED ERROR OR A GENERAL LEVEL CHANGE.         */00000900
 /*          (2) LEVEL CHANGES DETECTED AS ABOVE ARE REMOVED BY INSERT-   */00001000
 /*              ING THE APPROPIATE OFFSET INTO THE DATA WITH AN          */00001100
 /*              ASSIGNMENT OF THE FORM: FST(202,23)=+130 ON A            */00001200
 /*              HEADER CARD.                                             */00001300
 /*          (3) AVAILABLE DAILY CALIBRATION FACTORS MAY BE ENTERED       */00001400
 /*              ON A HEADER CARD IN THE FORM: RM(202)=1.234. THE         */00001500
 /*              PROGRAM LINEARLY INTERPOLATES BETWEEN CALIBRATION        */00001600
 /*              POINTS. IF NO CALIBRATION FACTORS ARE ENTERED THE        */00001700
 /*              VALUE 1.0 IS ASSUMED THROUGHOUT.                         */00001800
 /*          (4) A LACOLAZET FILTER IS APPLIED TO EACH DAYS RESULTS TO    */00001900
 /*              DETERMINE THE NON-TIDAL CONTENT OF THE RECORD. TWO       */00002000
 /*              RESULTS ARE GIVEN, THE FIRST CENTERED ON 1200 HOURS      */00002100
 /*              THE SECOND CENTERED ON 1600 HOURS.                       */00002200
 /*          (5) WHEN CARDS=0 IS SPECIFIED CARD OUTPUT IS NOT OBTAINED    */00002300
 /*              AND THE CALIBRATION FACTORS ARE NOT APPLIED. WHEN        */00002400
 /*              CARDS=1 IS SPECIFIED CARD OUTPUT DESIGNED FOR LST        */00002500
 /*              INPUT IS OBTAINED WITH CALIBRATION FACTORS APPLIED       */00002600
 /*              TO BOTH CARD OUTPUT AND PRINTER OUTPUT.                  */00002700
 /*          (6) DAYS FOR WHICH DATA IS COMPLETELY MISSING ARE            */00002800
 /*              DETECTED AS MISSING BY THE PROGRAM AND NO OUTPUT         */00002900
 /*              CARDS FOR THESE DAYS IS OBTAINED. DAYS WITH PARTIAL      */00003000
 /*              DATA ARE NOT SO DETECTED AND MUST BE REMOVED BY OTHER    */00003100
 /*              MEANS BEFORE BEING INPUT TO LST.                         */00003200
 /*                                                                       */00003300
 /*     SYMBOL LIST:                                                      */00003400
 /*       FST(*:*,0:23)  CHART DISPLACEMENTS. ASSIGNED VALUES ARE         */00003500
 /*                      ACCUMULATED WITHIN PROGRAM.                      */00003600
 /*          R(*:*,0:23) CHART READING, BOTH INPUT AND OUTPUT.            */00003700
 /*                      DIMENSIONS ASSIGNED AT EXECUTION TIME.           */00003800
 /*              RM(*,*) DAILY CALIBRATION FACTOR.                        */00003900
 /*                   J1 FIRST DAY OF DATA                                */00004000
 /*                   J2 LAST DAY OF DATA                                 */00004100
 /*                 CRD1 CONTENTS OF FIRST CARDS (TITLE)                  */00004200
 /*                 CRD2 CONTENTS OF SECOND CARD (DATA)                   */00004300
 /*               TRY(K) DATA FOR THE KTH HOUR DETERMINED BY SMOOTHING    */00004400
 /*                      FROM THE THREE PRECEDING HOURS AND THE THREE     */00004500
 /*                      FOLLOWING HOURS. LATER USED FOR THE RESIDUAL     */00004600
 /*                      FROM THE DATA ACTUALLY OBTAINED.                 */00004700
 /*                CARDS OPTION ON CRD2. SET EQUAL TO ONE FOR CARD        */00004800
 /*                      OUTPUT.                                          */00004900
 /*                  ERA DAILY LINEAR COMBINATION YIELDING NON-TIDAL      */00005000
 /*                      CONTENT. CENTERED ON 1200 HOURS.                 */00005100
 /*                  ERB AS ABOVE CENTERED ON 1600 HOURS.                 */00005200
 /*                                                                       */00005300
 /*************************************************************************/00005400
 DECLARE                                                                  00005500
        (X,Y,DEL,CARDS,JAD,YR,ERA,ERB)FLOAT BINARY STATIC,                00005600
        (J1,J2,K,M)FIXED BINARY STATIC,                                   00005700
        TRY(0:23)FLOAT BINARY STATIC,                                     00005800
```

```
MEMBER NAME  BETTY
        IDENT CHARACTER(2),                                                    00005900
        EVENT CHARACTER(1),                                                    00006000
        (CRD1,CRD2)CHARACTER(80);                                              00006100
 ON ENDFILE(SYSIN) GO TO TERM;                                                 00006200
 START:                                                                        00006300
 CARDS=0;                                                                      00006400
 GET EDIT(CRD1)(A(80));                    /*  READ FIRST CARD (TITLE)    */ 00006500
 GET EDIT(CRD2)(A(80));                    /*  READ SECOND CARD (DATA)    */ 00006600
 GET STRING(CRD2)DATA;                     /*  READ DATA                  */ 00006700
 PUT PAGE;                                                                     00006800
 PUT EDIT(CRD1,DATE)(A,A)SKIP(4);          /* PRINT TITLE AND DATE        */ 00006900
 PUT EDIT('SMOOTHNESS RESIDUALS FROM 0000 TO 2300 HOURS FOLLOW')(A)            00007000
 SKIP(3);                                                                      00007100
 PUT EDIT('       0   1   2   3   4   5   6   7   8   9  10')(A)SKIP;         00007200
 PUT EDIT('  11   12  13  14  15  16  17  18  19  20  21  22  23')(A);         00007300
 J1=J1-1;                                  /* THIS ADDS EXTRA CARDS TO    */ 00007400
 J2=J2+1;                                  /* START AND END TO OVERCOME   */ 00007500
                                           /* INDEXING PROBLEM IN SMOOTH.*/ 00007600
 T1:  BEGIN;                               /*MAIN STORAGE ASSIGNED HERE   */ 00007700
 DECLARE                                                                       00007800
        RM(J1:J2) FLOAT BINARY,                                                00007900
        (FST(J1:J2,0:23),R(J1:J2,0:23))FLOAT BINARY;                           00008000
        RM=0;  FST,R=0;  JAD=0;                                                00008100
        RM(J1+1)=1;  RM(J2-1)=1;                                               00008200
 GET DATA;                                 /* READ FST AND RM DATA IN.    */ 00008300
 DO J=J1 TO J2;                            /*ACCUMULATE LEVEL CORRECTIONS*/ 00008400
                DO K=0 TO 23;                                                  00008500
                JAD=JAD+FST(J,K);                                              00008600
                FST(J,K)=JAD;                                                  00008700
                END;                                                           00008800
 END;                                                                          00008900
 DATA:                                     /*READ HOURLY DATA CARDS       */ 00009000
    GET EDIT(J,YR,IDENT,EVENT,(R(J,M) DO M=0 TO 23))                           00009100
            (F(3),X(1),F(1),A(2),A(1),24 F(3));                                00009200
    IF(J=J2-1) THEN GO TO FIN;                                                 00009300
               ELSE GO TO DATA;                                                00009400
 (NOUNDERFLOW):                                                                00009500
 FIN:                                      /* APPLY LEVEL CORRECTIONS      */ 00009600
 DO J=J1 TO J2; DO K=0 TO 23;                                                  00009700
                R(J,K)=R(J,K)+FST(J,K);                                        00009800
                END;END;                                                       00009900
 J1=J1+1; J2=J2-1;                         /*PREPARE FOR SMOOTH           */ 00010000
 SMOOTH:                                                                       00010100
 DO J=J1 TO J2;        TRY=0;                                                  00010200
 DO K=0 TO 23;                             /* COMPUTE AN INTERPOLATED     */ 00010300
                                           /*VALUE FOR EACH HOUR.         */ 00010400
 TRY(K)=.75*(R(J+(K=23),(K+1)*(K<23))+R(J-(K<1),23*(K=0)+(K-1)*(K¬=0)))        00010500
       -.3*(R(J+(K>21),(K+2)*(K<22)+(K-22)*(K>21))                             00010600
            +R(J-(K<2),(K+22)*(K<2)+(K-2)*(K>1)))                              00010700
       +.05*(R(J+(K>20),(K+3)*(K<21)+(K-21)*(K>20))                            00010800
            +R((J-1)*(K<3)+J*(K>2),(K+21)*(K<3)+(K-3)*(K>2)));                 00010900
 END;                                                                          00011000
 TRY(*)=R(J,*)-TRY(*);                     /*DETERMINE RESIDUAL FOR EACH */ 00011100
                                           /* HOUR AND PRINT IT.          */ 00011200
 PUT EDIT(J,(TRY(K) DO K=0 TO 23))(F(3),24 F(4))SKIP;                          00011300
 END SMOOTH;                                                                   00011400
 PUT SKIP;                                                                     00011500
 PUT EDIT('CORRECTED DATA FOLLOWS')(A)SKIP;                                    00011600
```

```
MEMBER NAME  BETTY
 IF(CARDS¬=0)THEN DO;                          /* CARD OUTPUT                    */ 00011700
 PUT EDIT('  (CALIBRATION FACTORS HAVE BEEN APPLIED--CARD OUTPUT')(A);           00011800
 END;                                                                            00011900
 ELSE DO;                                      /* NO CARD OUTPUT                 */ 00012000
 PUT EDIT(' (CALIBRATION FACTORS NOT USED)')(A); END;                            00012100
 PUT EDIT('           0/12  1/13  2/14  3/15  4/16  5/17  6/18')(A)SKIP;         00012200
 PUT EDIT('  7/19  8/20  9/21 10/22 11/23')(A);                                   00012300
                                               /*ASSIGN DAILY CALIBRATION     */ 00012400
                                               /*FACTORS. INTERPOLATING BETW  */ 00012500
                                               /*EEN GIVEN VALUES WHERE       */ 00012600
                                               /*NECESSARY.                   */ 00012700
 LAST=J1;                                                                        00012800
 DO J=J1+1 TO J2;                                                                00012900
 IF(RM(J)=0) THEN GO TO DER;                                                     00013000
      Y=RM(J)-RM(LAST);                                                          00013100
       X=J-LAST;                                                                 00013200
      DEL=Y/X;                                                                   00013300
       DO K=LAST TO J;                                                           00013400
            RM(K)=RM(LAST)+DEL*(K-LAST);END;                                     00013500
       LAST=J;                                                                   00013600
      DEL=Y/X;                                                                   00013700
      DEL=Y/X;                                                                   00013800
       DO K=LAST TO J;                                                           00013900
            RM(K)=RM(LAST)+DEL*(K-LAST);END;                                     00014000
       LAST=J;                                                                   00014100
       DER:END;                                                                  00014200
 IF(CARDS¬=0) THEN DO;                         /* PUNCH TITLE CARD               */ 00014300
                   PUT FILE(SYSPNCH)EDIT(CRD1)(A)SKIP;                           00014400
                   END;                                                          00014500
 T2:DO J=J1 TO J2;                             /* OUTPUT LOOP                    */ 00014600
 IF(SUM(R(J,*)-FST(J,*))=0) THEN DO;           /*TEST FOR MISSING DAYS           */ 00014700
    PUT EDIT('DAY NO. ',J,' IS MISSING')(A,F(3),A)SKIP(2);                       00014800
 PUT SKIP(2);                                                                    00014900
     GO TO ZERO4;END;                                                            00015000
 IF(CARDS¬=0) THEN GO TO ZERO8;                /*PRINT RESULTS WITHOUT APPLY-*/ 00015100
                                               /*ING CALIBRATION FACTORS IF   */ 00015200
                                               /* NO CARD OUTPUT REQUIRED.    */ 00015300
              PUT EDIT(J,YR,IDENT,EVENT,'1',(R(J,M) DO M=0 TO 11))               00015400
                   (F(3),F(1),3(A),12 F(6))SKIP;                                 00015500
              PUT EDIT(J,YR,IDENT,EVENT,'2',(R(J,M) DO M=12 TO 23))              00015600
                   (F(3),F(1),3(A),12 F(6))SKIP;                                 00015700
 ZERO8:                                                                          00015800
 IF(CARDS=0) THEN GO TO ZERO3;                                                   00015900
 R(J,*)=R(J,*)*RM(J);                          /*APPLY CALIBRATION FACTORS.   */ 00016000
                                               /*PUNCH CARDS AND PRINT        */ 00016100
                                               /*DUPLICATE OUTPUT.            */ 00016200
              PUT EDIT(J,YR,IDENT,EVENT,'1',(R(J,M) DO M=0 TO 11))               00016300
                   (F(3),F(1),3(A),12 F(6))SKIP;                                 00016400
              PUT EDIT(J,YR,IDENT,EVENT,'2',(R(J,M) DO M=12 TO 23))              00016500
                   (F(3),F(1),3(A),12 F(6))SKIP;                                 00016600
 PUT FILE(SYSPNCH)EDIT(J,YR,IDENT,EVENT,'1',(R(J,M) DO M=0 TO 11))               00016700
                   (F(3),F(1),3(A),12 F(6));                                     00016800
 PUT FILE(SYSPNCH)EDIT(J,YR,IDENT,EVENT,'2',(R(J,M) DO M=12 TO 23))              00016900
                   (F(3),F(1),3(A),12 F(6));                                     00017000
 GO TO ZERO4;                                                                    00017100
 ZERO3:                                        /*APPLY LINEAR COMBINATIONS    */ 00017200
 ERA=-R(J,0)+5*R(J,1)-10*R(J,2)+11*R(J,3)-10*R(J,4)+11*R(J,5)-11*R(J,6)          00017300
 +10*R(J,7)-11*R(J,8)+10*R(J,9)-5*R(J,10)+R(J,11)+R(J,12)-5*R(J,13)              00017400
```

```
MEMBER NAME  BETTY
  +10*R(J,14)-11*R(J,15)+10*R(J,16)-11*R(J,17)+11*R(J,18)-10*R(J,19)      00017500
  +11*R(J,20)-10*R(J,21)+5*R(J,22)-R(J,23);                               00017600
  IF(SUM(R(J+1,*)-FST(J+1,*))=0) THEN DO;ERB=999.99;GO TO ZERO5;END;      00017700
  ERB=-1*R(J,4)+5*R(J,5)-10*R(J,6)+11*R(J,7)-10*R(J,8)+11*R(J,9)          00017800
  -11*R(J,10)+10*R(J,11)-11*R(J,12)+10*R(J,13)-5*R(J,14)+R(J,15)          00017900
  +R(J,16)-5*R(J,17)+10*R(J,18)-11*R(J,19)+10*R(J,20)-11*R(J,21)          00018000
  +11*R(J,22)-10*R(J,23)+11*R(J+1,0)-10*R(J+1,1)+5*R(J+1,2)-R(J+1,3);     00018100
  ZERO5:                              /*PRINT COMBINATIONS RESULTS  */   00018200
  PUT EDIT(ERA,ERB)(2 F(10));                                             00018300
  ZERO4:                                                                  00018400
  END T2;                             /*END OF OUTPUT LOOP          */   00018500
  END T1;                             /*END OF BEGIN BLOCK          */   00018600
  GO TO START;                        /*LOOK FOR MORE DATA          */   00018700
  TERM:                               /*TERMINUS                    */   00018800
  END BETTY;                                                              00018900
```

APPENDIX VIII

World-wide Tidal Data File

no.	amp.	phase	zone	long. °E x100	lat °N x100
12743150	-321	5340	21833250	-318	5327
51983450	-292	5330	5 76 20	-284	5328
92743150	-322	5357	102903080	-338	5350
25 46 220	300	5234	26 46 70	287	5237
29 463220	267	5247	30 463070	266	5250
33 462620	267	5262	34 462470	225	5268
37 462120	247	5292	38 461870	299	5283
41 461420	344	5272	42 461270	348	5268
45 46 820	346	5249	46 46 670	338	5244
49 76 220	317	5205	50 76 70	287	5207
53 763920	198	5209	54 763070	185	5223
57 762620	177	5256	58 762470	176	5263
61 762020	198	5294	62 761950	278	5346
65 761570	398	5296	66 761420	415	5275
69 76 970	415	5253	70 76 820	406	5237
73 76 370	345	5207	74107 220	340	5177
7710773370	216	5197	7810773220	196	5209
8113773520	241	5165	8213773370	199	5177
8516873370	192	5128	8619833520	267	5119
8922933370	203	5105	9022933300	135	5114
9332033220	151	5046	9438033220	103	5025
9725933070	61	5004	9822933220	22	5016
10119833220	-13	5085	10219833070	-13	5026
10522932770	10	4969	10619832770	-13	4962
10919826320	-22	4966	11016833070	-49	5064
11316829220	-45	5014	11416827770	-47	4997
11713732220	-85	5067	11813732070	-79	5048
12113726220	-98	5003	12210732220	-109	5059
12510727770	-129	5041	12610726220	-137	5038
129 762720	-129	5041	130 762620	-153	5038
13313724 70	-138	5017	13416824 70	-128	4998
13713733220	-162	5017	13817332320	-153	4997
14113726170	-179	5019	14216821170	-174	4997
14513726220	-204	5023	14616826220	-207	5001
14926926220	-189	4969	15010726220	-230	5059
15319818720	-244	4987	15424418720	-228	4972
15738118720	-240	4908	15838118720	-220	4895
16113718720	-324	5064	16236817220	-328	5021
16525917220	-336	4923	16629017220	-320	4907
16938118720	-224	4895	17016821570	-450	5036
17325915720	-378	4897	17429015720	-341	4838
17722914220	-449	4902	17825914220	-409	4875
18122912720	-499	4871	18218811220	-764	4839
18618315000	-300	4750	18719810500	-316	4762
19013714220	-577	5128	19113712720	-723	5078
19419814220	-599	5098	19522914220	-524	5083
19822915720	-523	5083	19925915720	-456	5081
20238119500	-316	5123	20336817220	-628	5161
206 761870	-604	5233	20740718 70	-621	5219
21019818 70	-539	5174	211 661020	-625	5248
21413720220	-576	5204	215 662170	-625	5264
21813721720	-496	5219	219 662320	-617	5274
22213723220	-459	5256	223 662470	-566	5273
22613724720	-485	5275	227 762620	-518	5252
230 762770	-604	5291	23110727720	-530	5294
23410729220	-543	5301	23513729220	-505	5309
23813730720	-561	5321	23916833150	-499	5327
24225930720	-404	5330	24320930720	-365	5331
24616833180	-532	5343	24719833180	-485	5371
31073500	-305	5322	43203300	-300	5335
72743160	-340	5338	84903150	-322	5343
112903100	-352	5332	120603180	-321	5338
27 463520	280	5238	2 463370	290	5254
31 462920	229	5242	32 462770	223	5257
35 462320	231	5277	36 462170	238	5286
39 461720	325	5284	40 461570	335	5279
43 461120	350	5262	44 46 970	348	5258
47 46 520	324	5236	48 46 370	311	5235
51 763520	257	5213	52 763370	216	5199
55 762920	175	5234	56 762770	175	5247
59 762320	179	5271	60 762170	197	5282
63 761870	320	5313	64 761720	381	5304
67 761270	459	5276	68 761120	416	5265
71 76 670	392	5226	72 76 520	364	5212
75107 70	289	5179	76107 3520	244	5185
79137 220	349	5165	80137 70	298	5159
83169 70	251	5141	84168 3520	269	5122
87198 3370	193	5127	88198 3220	133	5147
91259 3300	145	5097	92296 3220	155	5077
95290 3220	114	5035	96259 3220	75	5052
99229 3070	23	5017	100198 3300	-21	5086
103229 2920	22	4979	104198 0920	-12	4999
107229 2770	50	4954	108229 2620	-20	4966
111168 3220	-53	5063	112156 3070	-47	5037
115168 2620	-59	4981	116137 5300	-87	5069
119137 2920	-73	5026	120137 2770	-78	5011
123107 3070	-107	5057	124107 2920	-105	5039
127 763070	-123	5062	128 762920	-121	5056
131 762470	-155	5037	132107 2470	-154	5037
135 762320	-173	5041	136107 2320	-173	5040
139 762470	-195	5065	140107 2170	-184	5044
143 762620	-202	5055	144107 2020	-202	5049
147198 2020	-194	4981	148229 2020	-192	4974
151137 1670	-237	5027	152168 1870	-244	5006
155290 1870	-261	4920	156320 1870	-258	4920
159411 1870	-199	4881	160442 1870	-182	4874
163198 1720	-382	4989	164229 1720	-359	4946
167320 1720	-303	4892	168351 1720	-292	4880
171198 1570	-431	4973	172229 1570	-406	4928
175158 1420	-475	4952	176198 1420	-478	4952
179168 1270	-787	4976	180198 1270	-570	4903
183198 1120	-566	4815	184229 1120	-471	4835
188137 1720	-590	5177	189137 1570	-633	5162
192137 1420	-770	4881	193168 1420	-675	5128
196168 1570	-673	5129	197198 1570	-599	5097
200305 1720	-435	5177	203511 1870	-356	5123
204198 1720	-571	5151	205 461870	-637	5233
208137 1870	-613	5206	209168 1870	-584	5177
212 762020	-671	5233	213107 2020	-590	5221
216 762170	-579	5242	217107 2170	-539	5229
220 762320	-561	5257	221107 2320	-515	5253
224 762470	-566	5283	225107 2470	-525	5270
228107 2620	-523	5265	229137 2620	-480	5293
232137 2770	-491	5302	233 762920	-602	5295
236168 2920	-464	5319	237107 3070	-599	5299
240198 3070	-462	5301	241229 3070	-434	5304
244107 3180	-604	5306	245137 3180	-567	5320
248229 3180	-456	5380	249259 3180	-415	5374

2502903180 -363 5353	2513203180 -314 5339	2521683260 -592 5403	2531983260 -469 5460
2541133260 -513 5401	2552293260 -449 5413	2562593260 -422 5409	2572903260 -381 5403
2583203260 -327 5400	2591223300 -544 5446	2601983300 -474 5463	2612293300 -435 5465
2622593300 -392 5467	263 763000 -577 5484	264 76 00 -534 5514	265107 00 -529 5513
266 76 600 -546 5548	267107 600 -509 5543	268 91 900 -530 5567	269123 300 -488 5526
270 463150 -601 5503	271 462950 -606 5503	272 302550 -502 5513	273 302250 -606 5515
274 461950 -614 5519	275 461650 -596 5544	276 301350 -626 5554	277 461050 -591 5545
278 30 750 -591 5545	279 30 450 -592 5545	280 15 150 -568 5548	281 152850 -591 5544
282 761500 -652 5557	283 911500 -657 5556	284107 1500 -765 5568	285107 1500 -540 5592
286 761800 -604 5566	287 761800 -669 5531	288107 1720 -667 5531	289122 1650 -863 5514
290107 1680 -865 5514	291107 1770 -926 5727	292122 1800 -761 5675	293122 1860 -758 5673
294152 1870 -637 5766	295122 1870 -775 5804	296137 1870 -711 5758	297137 2020 -586 5796
298137 2170 -510 5842	299137 2320 -476 5857	300122 2470 -398 5856	301122 2620 -359 5862
302122 1620 -889 5474	303122 1570 -890 5474	304122 1530 -947 5437	305122 1530 -994 5418
306107 1570-1041 5439	307 761570-1168 5566	308 761720-1031 5670	309 761870 -843 5909
310 762020 -745 5943	311 762170 -629 5957	312 762320 -500 5965	313 762470 -402 5972
314 0 00 -292 6169	315 761420-1252 5397	316107 1420-1250 5396	317 762620-1127 5422
318122 1380-1125 5423	319122 1380-1073 5313	320107 1770 -925 5738	321107 1870 -775 5806
322107 2020 -702 5986	323107 2170 -596 5911	324107 2320 -484 5902	325107 2470 -388 5882
326107 2620 -357 5872	327152 2200 -880 4330	328137 2200-1170 4200	329107 2250-1625 4200
330 911200-2200 4200	331 761200-2500 4200	333 461350-2500 4750	334122 1270-1035 5212
335107 1270-1041 5184	336 761270-1257 5160	337 461350-2500 5093	338 611350-1977 5687
339 762770 -216 6043	340 762770 -292 5962	341 912770 -261 5955	342 762770 -193 6036
343 912920 -281 5939	344 912920 -257 5956	345 762920 -150 6012	346 462920 123 5967
347 302920 521 5848	348107 3070 -295 5911	349 763070 -241 5949	350 463070 -147 5965
351 153070 427 5967	352107 3220 -308 5862	353 763220 -217 5919	354 463220 -148 5930
355 153220 130 5930	356122 3370 -398 5796	357107 3370 -295 5852	358 763370 -206 5878
359 463370 -141 5895	360 153370 131 5912	361137 3520 -407 5761	362107 3520 -257 5823
363 763520 -196 5845	364 463520 -122 5862	365 153520 129 5899	366122 70 -266 5770
367107 70 -229 5797	368 76 70 -178 5819	369 46 70 -95 5840	370 15 70 136 5866
371122 220 -180 5747	372107 220 -184 5762	373 76 220 -143 5775	374 46 220 -47 5802
375 15 220 163 5829	376152 370 -283 5637	377137 370 -205 5661	378107 370 -138 5681
379 76 370 -64 5701	380 46 370 45 5726	381 15 370 222 5762	382168 520 -373 5603
383137 520 -207 5659	384107 520 -141 5680	385 76 520 -69 5699	386 46 520 43 5726
387 15 520 224 5760	388168 670 -249 5601	389137 670 -183 5621	390107 670 -107 5641
391 76 670 -24 5663	392 46 670 84 5691	394168 820 -178 5563	393 15 670 259 5728
395137 820 -147 5572	396107 820 -69 5596	397 76 820 24 5620	398 46 820 133 5648
399 30 820 298 5683	400 15 820 329 5690	401168 970 -143 5500	402137 970 -102 5519
403107 970 -15 5553	404 76 970 73 5580	405 46 970 179 5608	406 15 970 332 5637
407 15 970 465 5604	408168 1120 -85 5457	409137 1120 -46 5477	410107 1120 31 5512
411 761120 114 5544	412 461120 218 5575	413 301120 351 5605	414 151120 466 5603
4151681270 -21 5422	4161371270 11 5436	4171071270 84 5475	418 761270 158 5508
419 461270 255 5541	420 301270 365 5573	421 151270 447 5585	4221981420 -5 5391
4231681420 11 5397	4241371420 45 5415	4251071420 116 5446	426 761420 187 5479
427 461420 279 5517	428 301420 379 5552	429 151420 449 5568	4301981570 -6 5359
4311681570 40 5376	4321371570 82 5390	4331071570 143 5417	434 761570 221 5451
435 461570 307 5491	436 151570 391 5531	437 151570 450 5554	4381981720 4 5292
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APPENDIX IX

DEFORMATION DUE TO THE N=1 TERM

Equilibrium of an isotropic elastic solid requires that: (Symbols are those of Caputo [1961])

$$\gamma_{ij,j} = -F_i \qquad (1)$$

in the interior of the medium and on the surface bounding the medium:

$$\gamma_{ij} V_j = T_i \qquad (2)$$

The stress-strain relation is:

$$\gamma_{ij} = \lambda \delta_{ij} V + 2\mu e_{ij} \qquad (3)$$

where $V=e_{ii}$ Is the dilatation or the divergence of the displacement vector, e_{ij} is the strain. Vanishing of the moments of external forces on the body requires $\gamma_{ij} = \gamma_{ji}$. The displacement u_j is determined by:

$$u_{i,j} + u_{j,i} = 2e_{ij}$$

Equation 3 can be written also in the form:

$$\gamma_{ij} = \lambda \delta_{ij} u_{k,k} + \mu(u_{i,j} + u_{j,i}) \qquad (4)$$

Substituting this equation (1) for the stress gives:

$$\mu u_{i,jj} + (\lambda+\mu) u_{j,ji} + F_i = 0 \qquad (5)$$

which can also be written:

$$(\lambda+2\mu) \text{ grad div } u - \mu \text{ curl curl } u + F_i = 0 \qquad (6)$$

The solution to this equation is found by first finding a particular integral of (6), adding to it the complimentary function of equation (6) with $F_i = 0$, then adjusting the arbitrary constants available so that the prescribed boundary conditions are satisfied. (Love, 1892, p 252-253).

Deformation due to mass loading represented by the n=1 term can be found using Caputo's (1961) equations as follows: A particular integral of equation 1 is given by Love (1927) (p253) for a body force which is the gradient of a potential Vn. Thus a particular integral can be found for each shell. For each shell this particular integral can be added to the complementary function for the same shell and the sum used in Caputo's equations 5 and 6 (and the equations derived from them) to determine the arbitrary constants.

In the special case n=1, and assuming the single shell case m=1, A2 and A4in Caputo's equation 3 must be zero and therefore there are only two arbitrary constants; A1 and A3. When equations 3 are substituted in equations 8 the coefficient of A3 turns out to be zero and thus we can put A3=0. (A3 corresponds to translation of the earth as a whole). The stresses due to this complementary function now are:

$$P_{rr}(s_{\cdot}) = 6\mu r A_1 \cos\theta[\,1+(\lambda/\mu)-(\lambda T/2\mu)\,]$$

$$P_{r\theta}(s_{\cdot}) = -\mu A_1 (3/2) r \sin\theta[\,1+T\,]$$

where $T = (2\gamma+3)/(\gamma-1)$ and $\gamma=\lambda/\mu$

The surface stress due to a uniform pressure P_o ($f_r = -P_o$) throughout the cap whose semi-angle is ω is given by:

$$f_r = \Sigma\,(P_{n+1}(\cos\omega) - P_{n-1}(\cos\omega))P_n\cos\theta\ (P_o/2) - (P_o/2)(1-\cos\omega)$$

thus $f_{r1} = -(3/4) P_o \sin^2 \cos\theta$

Finding the potential due to this mass distribution (P_o/g) (eqn 20) and substituting the particular integral (eqn. 21)

we obtain:

$$P_{rr}\ (s\cdot) = C_1 r \cos\theta[-(6\mu/5)-\lambda]$$

$$P_{r\theta}\ (s\ ) = (2/5)\mu C_1 r \sin\theta$$

where $C_1 = (K\pi dP, \sin^2\omega)/(\lambda+2\mu)\ g$

adding the stresses due to both the particular integral and the complementary function and equating to the boundary conditions we obtain:

$$A_1 = 4\ C_\gamma/15(1+T)$$

and thus the displacement is: (from Caputo's equations . and 2)

$$U_{r1} = -(3aP,\sin^2\omega\ \cos\theta)/(8\mu(3\gamma+2))$$

Converting to Longman's dimensionless form- $[u/(am'/m)]$ yields:

$$U_{r1} = -(3g^2\ \cos\theta)/[8K\pi\mu(3\gamma+2)]$$

For $\mu=1.86\times10^{12}$ and $\gamma=1.27$ this becomes:

$$Ur1 = -.159\ Cos\theta$$

I have also obtained a solution for the case n=1 assuming an earth consisting of an outer shell enclosing a fluid core. The result for this case is:

$$Ur1 = -.157\ Cos\theta$$

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