The dual model of scattering

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THE DUAL MODEL OF SCATTERING

Thesis submitted towards the degree of

Doctor of Philosophy

by

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August 1971

Abstract

The bootstrap idea in the sense of finite energy sum rules and the saturation with zero-width resonances are developed further in the thesis for the reaction $\rho\rho \to \rho\sigma$ and $\rho\rho \to \rho\eta$ which are identical in all the three channels and therefore provide us with a genuine bootstrap of the Regge trajectories, contrary to, say, the reaction $\pi\rho \to \pi\rho$. A set of fourteen FESR,s for all the invariant amplitudes of the reaction $\rho\rho \to \rho\sigma$ and also a set of thirteen FESR,s for $\rho\rho \to \rho\eta$ in different steps of approximation have been written down which can be studied further in different aspects. Notice that a previously considered by other authors reaction $\pi\pi \to \pi\omega$ finally led to a special representation, namely the Veneziano model which has many attractive features. With the appearance of this model and the concept of duality we devoted ourselves to the idea of mass extrapolation along the Regge trajectory, a show-case of which is the annihilation $\bar{p}n \to 3\pi$ at rest. The Dalitz plot and the overall normalization for this process were obtained with a certain degree of success. An analogous attempt was done for the overall normalization of the annihilation process $\bar{p}p \to \gamma\pi$ at rest. The new experimental data on the annihilation process for low laboratory momenta of antiproton P_{lab} = 100 - 700 MeV give a further support to this kind of extrapolation along the Regge trajectory. These data indicate the existence of angular momenta upto $\ell = 3$ at these near-threshold energies. An impact parameter picture with the reasonable radius of interaction gives $\ell \lesssim 1$, while the explanation within the above model ℓ is very natural since the Regge trajectory α is very near to 3 .

A step has also been done towards the construction of physical dual resonance models (DRM) with unnatural parity couplings and without the tachyon states. One of the motivations has been to see whether these physical requirements give a natural way to get a double or more degenerate $(\omega - A_2)$-trajectory in 3π-channels when one factorizes the states analogous to the degeneray of the daughters levels.

One has to admit that the unitarization of DRM still remains a problem. However, from a theoretical point of view the KSV-program to consider the model as a Born term of a field-theoretic expansion is the most attractive one. In admitting this

program, the best place to look for its predictions is the field of inclusive experiments both in purely hadronic and photonic processes where one measures the discontinuities. For purely hadronic reactions in DRM Feynman scaling is obtained provided the trajectory exchanged is associated with the Pomeron with unit intercept. For photonic processes in DRM the Bjorken scaling is obtained due to the existence of current algebra fixed pole. The latter question is studied in the thesis in a model where the currents are included into DRM through a minimal gauge interaction prescription, in which one has the minimum amount of freedom, and the dual renormalization has also been used. With the use of Muellerism the generalized Bjorken scaling for some quasi-inclusive reactions has also been obtained. The motivation for the above analysis has been the experimental indication that the nondiffractive part of the electroproduction structure functions do show scaling. Without the fixed pole responsibility it would be hard to understand the scaling of the resonant part. Also notice that with the exception of $\lambda \varphi^3$-theory all the other field-theoretic models have failed to produce Bjorken scaling unless one introduces an unjustified cutoff and therefore one would like to argue that in the DRM the renormalization term as if replaces this cutoff in a more natural way.

The above and many other points studied in the young but vast literature may indicate that the DRM might be "not that far" from the real world of hadron physics.

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Acknowledgements

I wish to express my deepest gratitude to my supervisor Professor E.J. Squires for constant guidance, encouragement and patience at all stages of the work.

I am much grateful to Professor H.R. Rubinstein for illuminating discussions and friendly advice.

The tenure of a Weizmann Institute Fellowship during the academic year 1969-70 and a Durham University Research Assistantship are highly appreciated.

Chapter I

Introduction

The use of the general principles of S-matrix theory, embodied in the analyticity, crossing and unitarity, together with the dynamical elements contained in the Regge pole theory seems to be a promising attempt in the theory of elementary particles. The resulting scheme may hopefully put strong enough restrictions on the scattering amplitudes, so that the Regge trajectories and their residue functions will be uniquely determined. As a consequence, the spectrum of particles and their couplings may be completely determined and their footstrap accomplished.

The first approach of this kind was started by De Alfaro et al.[1] when they discovered superconvergence and proposed their use in particle physics. Their observation was as follows: if for a given process the t-channel helicity flip is sufficiently large and the quantum numbers exchanged are such that the leading Regge trajectory is below a certain value, then the corresponding invariant amplitude A obeys a sumrule of the form

$$\int_{-\infty}^{\infty} Im\, A(\nu, t)\, d\nu = 0 \quad , \tag{1.1}$$

where $\nu = \frac{1}{4}(s - u)$ and s, t, u are the Mandelstam variables. Equation (1.1) is a consequence of analyticity and Regge asymptotic behaviour and the evaluation of the integral can be performed by means of unitarity. In Ref. 1 and in the subsequent papers 2) ImA was approximated by a few low-lying resonances, so that the equations result in relations among the parameters of s- and u-channel resonances. The above equation only becomes physically relevant when some prescription, like the above-mentioned one, is given for calculating the integral. This is called in the literature the saturation problem. Saturation in terms of a finite number of resonances has been shown to lead to difficulties [2] when the equations are asked to be exactly satisfied in a certain range of t. To avoid this problem a different type of saturation philosophy was used in which the high energy part of (1.1) was explicitly taken into account by the use of Regge theory. Such a philosophy provides for a method of analytic continuation of equation (1.1) to the values of t where the integral is meaningless. This analytic continuation has been derived by subtracting the asymptotic limit from the amplitude and writing equation (1.1) for the difference and has been first proposed by Igi[3] and rediscovered by many authors [4] and fully exploited by Dolen, Horn and Schmid [5] in their "finite energy sum rules". These new sum rules have the form

$$\int_0^{\bar{\nu}} \nu^n \, \mathrm{Im}\, A(\nu,t)\, d\nu = \sum_i \beta_i(t) \frac{\bar{\nu}^{\alpha_i(t)+n+1}}{\alpha_i(t)+n+1} , \qquad (1.2)$$

where the Regge behaviour

$$\mathrm{Im}\, A(\nu,t) \xrightarrow[\nu\to\infty]{} \sum_i \beta_i(t)\, \nu^{\alpha_i(t)} \qquad (1.3)$$

has been used.

In a few cases like π N-scattering, the experimental data from both low energy and high energy fits can be used directly to check the equations. A more interesting application is to use the low energy data as an input to predict the relevant parameters of high energy scattering, like the leading Regge trajectory. This has been done in Ref. 5 for π N-charge exchange scattering. The results of the thorough analysis of Dolen, Horn and Schmid, not only showed an excellent agreement with experiment but also made evident a rather surprising property of the Regge representation, i.e. while the physical amplitude differs from the Regge term in the region of prominent resonances of the direct channel, the local avarage of the amplitude coincides with the extrapolation of the Regge term up to much lower energies. This fact nowadays is referred to as the "duality" property.

Since what we need in the sum rule (for low moments) is exactly the average, this property permits us to cut the integral rather low, thus opening a number of possible applications.

Another way of exploiting the above equations which is more theoretical and attractive is the one which was proposed by the authors of [6] and was called there the bootstrap of Regge trajectories. The general idea of this approach is that, for some particular reaction for instance $\pi\pi \to \pi\omega$, the amplitude in the resonance region of the direct channel can be obtained by use of crossing as the analytic continuation of the Regge amplitude describing scattering at high energy in the crossed channel. The essence of the problem lies therfore in finding a trajectory and residue function which when introduced as input reproduces itself consistently. Also one must find a parametrization of the scattering amplitude which obeys the constraints of analyticity, unitarity and crossing. The simplest model of such a theory is the one based on the narrow resonances approximation and, consistently, on real rising Regge trajectories. In this frame we are provided with a set of algebraic relations in terms of the Regge parameters only. This model has also been proposed by Mandelstam [7] and fully exploited in a series of papers [6), 8)-12)].

For a bootstrap program it turns out that the meson systems are more adventageous than baryonic ones since the mesonic systems can possess strong symmetries in all the three channels - a situation that cannot occur with baryons and also that by an appropriate choice of the reaction one can suppress the type and the number of intermediate states that can contribute due to charge conjugation etc. The thorough study of such simple reactions as $\pi\pi \to \pi\omega$ etc. in 6), 8)-12) finally led Veneziano [13] to write down a representation for the scattering amplitude which possesses the requirements of S-matrix theory in a narrow resonances and a real linearly rising Regge trajectory approximation. This representation is the so-called

"Veneziano Model" which has been generalized to any number of external particles, has been given a treatment analogous to the field-theoretic expansion, for which has been developed an operator formalism, functional integral technique and has extensively been studied in many aspects - both theoretically and phenomenologically [14].

The investigation of the present thesis started when by the use of finite energy sum rules (FESR) of equation (1.2) for the theoretical and very simple systems such as $\pi\pi \to \pi\omega$, $\pi\pi \to \pi\chi_J$ ($\chi_J = A_2(2^+)$, $\omega(3^-)$) interesting and beautiful results were obtained [6]. In chapter II the case of $\pi\pi \to \pi\omega$ is very briefly discussed, in particular the interesting result that for a certain value of $t \simeq -0.6$ both the left and the right hand sides of eq. (1.2) which are proportional to $(2\,m_\rho^2 + t - 3\,m_\pi^2 - m_\omega^2)$ and $\alpha(t)$ respectively, vanish.

The natural idea which comes to mind is to follow the same line as in Ref. 6 and generalize the same considerations for some other reactions as $\rho\rho \to \rho\sigma$ or/and $\rho\rho \to \rho\eta$. These reactions are much richer in the sense that a bigger number of invariant amplitudes and, correspondingly, finite energy sum rules are at disposal, but they are still the simplest ones in the sense that all the three channels are identical like the case, say, in $\pi\pi \to \pi\omega$. The hope would be that a thorough study of the whole set of FESR's for all the invariant amplitudes of such reactions simultaneously might bring further information towards the accomplishment of a reciprocal bootstrap for mesonic systems.

By the time the above reactions were under study the Veneziano representation [13] was proposed and its generalization for n external scalar particles [14] makes it, in principle, trivial to write down the amplitudes for the reactions with spinful external particles starting from the expressions for the reactions with spinless external particles. Because of the latter we believe that the study of the above-mentioned reactions at the moment perhaps carries only an academic interest, although some kinematical aspects and also the absence of tachyon problems, contrary to the generalizations of Veneziano model, remain interesting. In chapters III and IV the results of these studies for the reactions $\rho\rho \to \rho\sigma$ and $\rho\rho \to \rho\eta$ are very briefly written down, although they are left incompleted and some further study can still be carried out.

Appendix A contains some application of the generalized Veneziano model to hadronic reactions and the construction of some of them.

Chap. 5 and 6 are devoted to the study of some current amplitudes, where the currents are included in the dual resonance model for the hadronic amplitudes through the minimal gauge interaction prescription. The sealing property of such amplitudes in the Bjorken [15] limit are considered and it comes out that these amplitudes satisfy the Bjorken scaling due to the existence of fixed pole in the amplitude with currents which in turn is responsible for the validity of Fubini-Dashen-Gell-Mann sum rule. The quasi-inclusive processes with one final hadron detected are also considered through the use of Muellerism. A vague analogy with the parton model results is obtained.

Appendices A and B consist of the materials of references 16)-22).

Chapter II

The reaction $\pi\pi \to \pi\omega$

The most suitable reaction providing for a bootstrap of the rho trajectory is $\pi\pi \to \pi\omega$. The T-matrix is described in terms of a single invariant amplitude $A(\nu,t)$ through

$$T^{\alpha\beta\gamma} = \epsilon_{\alpha\beta\gamma}\, \epsilon_{\mu\nu\rho\sigma}\, e_{\mu}\, p_{1\nu}\, p_{2\rho}\, p_{3\sigma}\, A(\nu,t) \quad , \tag{2.1}$$

where the momenta and isospin indices are taken as in Fig. 1

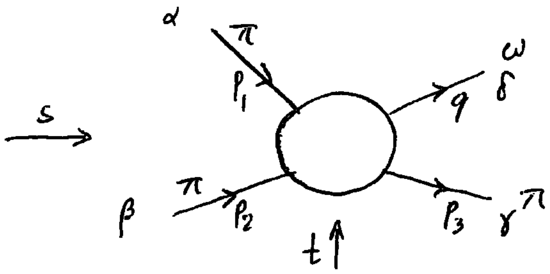


Fig. 1

$$s = (p_1+p_2)^2, \; t = (p_2-p_3)^2, \; \nu = \tfrac{1}{4}(s-u)$$

The remarkable property of this reaction is that it selects among all the particles and Regge poles which can be exchanged in all the three channels those corresponding to I = 1, G = + and normal parity trajectories.

The contribution of the leading Regge pole to the amplitude will be parametrized for fixed t and large ν as

$$A(\nu,t) \longrightarrow \beta(t)\, \xi_\alpha \left(\frac{\nu}{\nu_1}\right)^{\alpha(t)-1} \quad , \tag{2.2}$$

with

$$\xi_\alpha = \frac{1 - e^{-i\pi\alpha(t)}}{\sin \pi\alpha(t)} \quad . \tag{2.3}$$

We also parametrize the residue function $\beta(t)$ as

$$\beta(t) = \frac{\bar{\beta}(t)}{\Gamma(\alpha(t))} \quad , \tag{2.4}$$

where $\bar{\beta}(t)$ is an entire function of t.

Regge behaviour and analyticity requirement allow one to write the following family of sum rules

$$\int_0^{\bar{\nu}} \nu^n \, Im \, A(\nu,t)\, d\nu = \frac{\beta(t)}{\alpha(t)+n} \left(\frac{\bar{\nu}}{\nu_1}\right)^{\alpha(t)-1} \bar{\nu}^{\,n+1} \, . \qquad (2.5)$$

Consider the lowest moment sum rule corresponding to n = 1 in (2.5). Saturating the left hand side of (2.5) with a single ρ-pole in a narrow resonance approximation and with the definition of couplings as

$$\mathcal{L}_{\rho\pi\pi} = f_1 e_\mu (p_1+p_2) \frac{(p_2-p_1)_\mu}{2} \, ,$$

$$\mathcal{L}_{\rho\pi\omega} = g_1 \, \epsilon_{\mu\nu\rho\sigma} \, e_\mu(q) \, e_\nu(p_1+p_2) \frac{(p_2-p_1)_\rho}{2} \frac{(p_3-q)_\sigma}{2} \, ,$$

and therefore

$$Im \, A(\nu,t) = -g_1 f_1 \pi \, \delta(s-m_\rho^2) = -g_1 f_1 \pi \, \delta\left[\tfrac{1}{2}(4\nu+\Sigma-t) - m_\rho^2\right] ,$$

$$\Sigma = 3m_\pi^2 + m_\omega^2 \, ,$$

we get from (2.5) for n = 1 the following expression

$$-g_1 f_1 \pi \, \frac{1}{2} \, \frac{2m_\rho^2 + t - \Sigma}{4} = \frac{\bar{\beta}(t)\,\alpha(t)}{\Gamma(\alpha+2)} \left(\frac{\bar{\nu}}{\nu_1}\right)^{\alpha(t)-1} \bar{\nu}^{\,2} \, . \qquad (2.6)$$

Equation (2.6) predicts a zero at

$$t = -2m_\rho^2 + \Sigma = -\,0.53 \; (BeV)^2 \qquad (2.7)$$

for the right hand side and the natural explanation is, of course, the zero of $\alpha(t)$ at that value.

Using crossing and the fact that the ρ-pole should lie on the Regge trajectory at the value of $t = m_\rho^2$ and using

$$\frac{1-e^{-i\pi\alpha}}{\sin \pi\alpha} \underset{\alpha\approx 1}{\approx} \frac{2}{-\pi(\alpha-1)} \underset{s\approx m_\rho^2}{\approx} \left[\frac{2}{-\pi\alpha'(s-m_\rho^2)}\right] \, , \qquad (2.8)$$

where α' is the slope of the Regge trajectory near $s = m_\rho^2$,

we get

$$\frac{g_1 f_1 \pi}{2} = \frac{-\beta(m_\rho^2)}{\alpha'} \, , \qquad (2.9)$$

which accomplishes the bootstrap of the ρ-trajectory in a first stage approximation.

For the requirement of (2.6) being valid in a certain rnage of variable t and the following steps of approximation we refer to 6).

Chapter III

The reaction $\rho\rho \to \rho\sigma$

The T-matrix for this reaction can be written in terms of 14 invariant amplitudes as the following

$$
\begin{aligned}
T = (\epsilon_2 \cdot P)\Big\{&(\epsilon_1 \cdot P)(\epsilon_3 P)A_1(\nu,t) + [(\epsilon_3 P)(\epsilon_1 Q) - (\epsilon_1 P)(\epsilon_3 Q)]\, A_2(\nu,t) \\
&+ [(\epsilon_3 P)(\epsilon_1 Q) + (\epsilon_1 P)(\epsilon_3 Q)]A_3 + (\epsilon_1 Q)(\epsilon_3 Q)A_4 + (\epsilon_1 \cdot \epsilon_3)A_5\Big\} \\
+(\epsilon_2 \cdot Q)\Big\{&(\epsilon_1 Q)(\epsilon_3 Q)A_6 + [(\epsilon_3 P)(\epsilon_1 Q) - (\epsilon_1 P)(\epsilon_3 Q)]\, A_7 \\
&+ [(\epsilon_3 P)(\epsilon_1 Q) + (\epsilon_1 P)(\epsilon_3 Q)]A_8 + (\epsilon_1 P)(\epsilon_3 P)A_9 + (\epsilon_1 \cdot \epsilon_3)A_{10}\Big\} \\
+[(\epsilon_2 \epsilon_3)&(\epsilon_1 P) - (\epsilon_2 \epsilon_1)(\epsilon_3 P)]\, A_{11} + [(\epsilon_2 \epsilon_3)(\epsilon_1 P) + (\epsilon_2 \epsilon_1)(\epsilon_3 P)]A_{12} \\
+[(\epsilon_2 \epsilon_3)&(\epsilon_1 Q) - (\epsilon_2 \epsilon_1)(\epsilon_3 Q)]A_{13} + [(\epsilon_2 \epsilon_3)(\epsilon_1 Q) + (\epsilon_2 \epsilon_1)(\epsilon_3 Q)]\, A_{14}\,, \qquad (3.1)
\end{aligned}
$$

where

$$
\begin{aligned}
P &= \tfrac{1}{2}(p_2 + p_4)\,, \\
Q &= \tfrac{1}{2}(p_1 + p_3)\,, \\
s &= (p_1 + p_2)^2\,, \\
t &= (p_1 - p_3)^2\,, \\
\nu &= \tfrac{1}{4}(s - u) = P \cdot Q\,,
\end{aligned}
$$

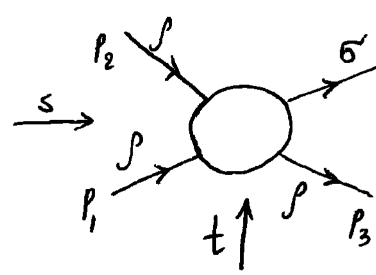


Fig. 2

$\epsilon_1(p_1)$, $\epsilon_2(p_2)$, $\epsilon_3(p_3)$ are the polarization vectors of the ρ-mesons and the isospin indices of the particles are dropped.

These 14 invariant amplitudes are free of kinematical singularities and therefore appropriate for being used in the sum rules. Notice, that they are no other (redundant) invariant amplitudes which can be written in terms of the above 14 amplitudes and therefore the question of which ones of them are kinematical singularity free does not arise for this reaction (for the reaction $\rho\rho \to \rho\eta$ this question arises. See chapter IV).

The particles and the Regge trajectories which can be exchanged in all the three channels should have I = 1, G = + and can have natural or unnatural parity.

In order to write the FESR's (1.2) for this reaction we have to know the Regge behaviour for each of the invariant amplitudes. One way of finding the Regge behaviour is i) to write down all the 14 helicity amplitudes $f_{\lambda_1\lambda_3;\lambda_2 0}$ in terms of these invariant amplitudes ii) find all the parity conserving kinematical singularity free helicity amplitudes $\bar{f}^{+}$ and $\bar{f}^{-}$ where + and - indices correspond to natural and unnatural parity trajectories and iii) to solve all the invariant amplitudes in terms of these helicity amplitudes and then reggeize them [23]. In our case we proceed in this way, but since the system of fourteen equations for the invariant amplitudes is very involved we do not solve them and leave them simply for the check of their behaviour which we have found from another method - covariant method of reggeization.

By going to the t-channel centre of mass system, where

$$p_1^\mu = (\omega, 0, 0, p) \quad , \quad |\vec{p}_1| = |\vec{p}_3| = p \quad ,$$
$$p_3^\mu = (\omega, 0, 0, -p) \quad , \quad |\vec{p}_2| = |\vec{q}| = q \quad ,$$
$$p_2^\mu = (\omega', q\sin\theta_t, 0, q\cos\theta_t),$$

and for the helicity states of the polarization vectors we have

$$\begin{aligned}
\epsilon_\mu^{(\pm)}(\vec{p}_1) &= \frac{1}{\sqrt{2}}(0, \pm 1, i, 0) \\
\epsilon_\mu^{(0)}(\vec{p}_1) &= \left(\frac{p}{m}, 0, 0, -\frac{\omega}{m}\right) \\
\epsilon_\mu^{(\pm)}(\vec{p}_2) &= \frac{1}{\sqrt{2}}(0, \pm\cos\theta_t, i, \mp\sin\theta_t) \\
\epsilon_\mu^{(0)}(\vec{p}_2) &= \left(\frac{q}{m}, -\frac{\omega'}{m}\sin\theta_t, 0, -\frac{\omega'}{m}\cos\theta_t\right) \\
\epsilon_\mu^{(\pm)}(\vec{p}_3) &= \frac{1}{\sqrt{2}}(0, \mp 1, i, 0) \\
\epsilon_\mu^{(0)}(\vec{p}_3) &= \left(\frac{p}{m}, 0, 0, \frac{\omega}{m}\right) \\
m &\equiv m_\rho \quad ,
\end{aligned} \tag{3.2}$$

after some algebra we get for the parity conserving kinematical singularity free helicity amplitudes $\bar{f}^{\pm}_{\lambda_1\lambda_3;\lambda_2 0}$ the following expressions

$$\bar{f}^{+}_{11;10} \equiv \bar{f}^{t}_{1} = -\frac{p}{\sqrt{2}}\left\{-q^2 \sin^2\theta\, A_9 + 2A_{10} + 2\frac{q}{p}\cos\theta\, A_{12}\right\}$$

$$\bar{f}^{-}_{11;10} \equiv \bar{f}^{t}_{2} = \frac{2q}{\sqrt{2}} A_{11}$$

$$\bar{f}^{+}_{11;00} \equiv \bar{f}^{t}_{3} = \frac{q\omega}{m}\left[-\frac{q^2}{2}\sin^2\theta\, A_1 + A_5\right] - \frac{p\omega'}{m}\cos\theta\left[-\frac{q^2}{2}\sin^2\theta\, A_9 + A_{10}\right] + \frac{q\omega'}{m}\sin^2\theta\, A_{12}$$

$$\bar{f}^{+}_{1-1;10} \equiv \bar{f}^{t}_{4} = \frac{2q}{\sqrt{2}}\left\{pq\cos\theta\, A_9 + 2A_{12}\right\}$$

$$\bar{f}^{-}_{1-1;10} \equiv \bar{f}^{t}_{5} = -\frac{2pq^2}{\sqrt{2}} A_9$$

$$\bar{f}^{+}_{1-1;00} \equiv \bar{f}^{t}_{6} = \frac{q}{2m}\left\{q^2\omega A_1 - pq\omega'\cos\theta\, A_9 - 2\omega' A_{12}\right\} \qquad (3.3)$$

$$\bar{f}^{+}_{10;10} \equiv \bar{f}^{t}_{7} = \Big\{ \frac{2p^2q\omega}{m}\cos\theta\, A_7 - \frac{2p^2q\omega}{m}\cos\theta\, A_8 - 2qp\cos\theta\left[\frac{p}{m}(\omega-\omega') - \frac{q\omega}{m}\cos\theta\right] A_9$$
$$+ \frac{2p}{m}(\omega-\omega')A_{11} + \left[\frac{3q\omega}{m}\cos\theta - \frac{2p}{m}(\omega-\omega')\right] A_{12} + \frac{2p\omega}{m} A_{13} - \frac{2p\omega}{m} A_{14}\Big\}$$

$$\bar{f}^{-}_{10;10} \equiv \bar{f}^{t}_{8} = -\frac{2p^2q\omega}{m} A_7 + \frac{2p^2q\omega}{m} A_8 + 2pq\left[\frac{p}{m}(\omega-\omega') - \frac{q\omega}{m}\cos\theta\right] A_9 - \frac{2q\omega}{m} A_{11} - \frac{2q\omega}{m} A_{12}$$

$$\bar{f}^{+}_{10;00} \equiv \bar{f}^{t}_{9} = \Big\{-\frac{q^2\omega}{\sqrt{2}m}\left[\frac{p}{m}(\omega-\omega') - \frac{q\omega}{m}\cos\theta\right] A_1 + \frac{pq^2\omega^2}{2m^2} A_2 - \frac{pq^2\omega^2}{2m^2} A_3 - \frac{p^2q\omega\omega'}{\sqrt{2}m^2}\cos\theta\, A_7$$
$$+ \frac{p^2q\omega\omega'}{\sqrt{2}m^2}\cos\theta\, A_8 + \frac{pq\omega'}{\sqrt{2}m}\cos\theta\left[\frac{p}{m}(\omega-\omega') - \frac{q\omega}{m}\cos\theta\right] A_9 - \left[\frac{pq^2}{\sqrt{2}m^2} + \frac{p\omega'}{\sqrt{2}m^2}(\omega-\omega')\right] A_{11}$$
$$- \left[\frac{pq^2}{\sqrt{2}m^2} - \frac{p\omega'}{\sqrt{2}m^2}(\omega-\omega') + \frac{2q\omega\omega'}{\sqrt{2}m^2}\cos\theta\right] A_{12} - \frac{p\omega\omega'}{\sqrt{2}m^2} A_{13} + \frac{p\omega\omega'}{\sqrt{2}m^2} A_{14}\Big\}$$

$$
\begin{aligned}
\bar{f}^{+}_{01;10} \equiv \bar{f}^{t}_{10} = \Big\{ & \frac{2pq^2\omega}{m}\cos\theta A_7 + \frac{2pq^2\omega}{m}\cos\theta A_8 - 2qp\Big[\frac{p}{m}(\omega-\omega') + \frac{q\omega}{m}\cos\theta\Big]A_9 - \frac{2p}{m}(\omega-\omega')A_{11} \\
& - \Big[\frac{2p}{m}(\omega-\omega') + \frac{3q\omega}{m}\cos\theta\Big]A_{12} + \frac{2p\omega}{m}A_{13} + \frac{2p\omega}{m}A_{14}\Big\} \\
\bar{f}^{-}_{01;10} \equiv \bar{f}^{t}_{11} = {} & \frac{2pq^2\omega}{m}A_7 + \frac{2pq^2\omega}{m}A_8 - 2qp\Big[\frac{p}{m}(\omega-\omega') + \frac{q\omega}{m}\cos\theta\Big]A_9 + \frac{2q\omega}{m}A_{11} - \frac{2q\omega}{m}A_{12} \\
\bar{f}^{+}_{01;00} \equiv \bar{f}^{t}_{12} = \Big\{ & \frac{q^2\omega}{\sqrt{2}m}\Big[\frac{p}{m}(\omega-\omega') + \frac{q\omega}{m}\cos\theta\Big]A_1 - \frac{pq^2\omega^2}{\sqrt{2}m^2}A_2 - \frac{pq^2\omega^2}{\sqrt{2}m^2}A_3 + \frac{p^2q\omega\omega'}{\sqrt{2}m^2}\cos\theta A_7 \\
& + \frac{p^2q\omega\omega'}{\sqrt{2}m^2}\cos\theta A_8 - \frac{pq\omega'}{\sqrt{2}m}\cos\theta\Big[\frac{p}{m}(\omega-\omega') + \frac{q\omega}{m}\cos\theta\Big]A_9 - \Big[\frac{p\omega'}{\sqrt{2}m^2}(\omega-\omega') + \frac{pq^2}{\sqrt{2}m^2}\Big]A_{11} \\
& - \Big[\frac{p\omega'}{\sqrt{2}m^2}(\omega-\omega') - \frac{pq^2}{\sqrt{2}m^2} + \frac{2q\omega\omega'}{\sqrt{2}m^2}\cos\theta\Big]A_{12} + \frac{p\omega\omega'}{\sqrt{2}m^2}A_{13} + \frac{p\omega\omega'}{\sqrt{2}m^2}A_{14}\Big\} \\
\bar{f}^{+}_{00;10} \equiv \bar{f}^{t}_{13} = \Big\{ & \frac{p^3\omega^2}{\sqrt{2}m^2}A_6 + \frac{p^2\omega}{\sqrt{2}m}\frac{2p}{m}(\omega-\omega')A_7 - \frac{2p^2q\omega^2}{\sqrt{2}m^2}\cos\theta A_8 - \frac{p}{\sqrt{2}}\Big[\frac{p^2}{m^2}(\omega-\omega')^2 - \frac{q^2\omega^2}{m^2}\cos^2\theta\Big]A_9 \\
& - \frac{p}{\sqrt{2}}\Big(\frac{p^2}{m^2} + \frac{\omega^2}{m^2}\Big)A_{10} + \Big[\frac{2p\omega}{\sqrt{2}m^2}(\omega-\omega')\Big]A_{11} + \frac{2q\omega^2}{\sqrt{2}m^2}\cos\theta A_{12} - \frac{2p\omega^2}{\sqrt{2}m^2}A_{14}\Big\} \qquad (3.3) \\
\bar{f}^{+}_{00;00} \equiv \bar{f}^{t}_{14} = \Big\{ & \frac{q\omega}{m}\Big[\frac{p^2}{m^2}(\omega-\omega')^2 - \frac{q^2\omega^2}{m^2}\cos^2\theta\Big]A_1 - \frac{2p^2q\omega^2}{m^3}(\omega-\omega')A_2 + \frac{2pq^2\omega^3}{m^3}\cos\theta A_3 \\
& - \frac{p^2q\omega^3}{m^3}A_4 + \frac{q\omega}{m}\Big(\frac{p^2}{m^2} + \frac{\omega^2}{m^2}\Big)A_5 + \frac{p^3\omega^2\omega'}{m^3}\cos\theta A_6 + \frac{2p^3\omega\omega'}{m^3}(\omega-\omega')\cos\theta A_7 \\
& - \frac{2p^2\omega^2\omega'}{m^3}\cos^2\theta A_8 - \frac{p\omega'}{m}\cos\theta\Big[\frac{p^2}{m^2}(\omega-\omega')^2 - \frac{q^2\omega^2}{m^2}\cos^2\theta\Big]A_9 - \frac{p\omega'}{m}\Big(\frac{p^2}{m^2} + \frac{\omega^2}{m^2}\Big)\cos\theta A_{10} \\
& + \Big[\frac{2p\omega\omega'}{m^3}(\omega-\omega') + \frac{2pq^2\omega}{m^3}\Big]\cos\theta A_{11} + \Big[\frac{2p^2q}{m^3}(\omega-\omega') + \frac{2q\omega^2\omega'}{m^3}\cos^2\theta\Big]A_{12} \\
& - \frac{2p^2q\omega}{m^3}A_{13} - \frac{2p\omega^2\omega'}{m^3}\cos\theta A_{14}\Big\}
\end{aligned}
$$

The covariant method of reggeization is the following: we exchange a spin-J-particle in the t-channel of the reaction with all its possible couplings at the two vertices, find the contribution of this exchange to each one of the invariant amplitudes and then change everywhere J to $\alpha(t)$ and $\frac{1}{t - m^2}$ to $\frac{-\pi\alpha}{2} \frac{1 - e^{-i\pi\alpha(t)}}{\sin\pi\alpha(t)}$ a la Van Hove [24]. The expression for general vertices and propagators are given in 25). In our case the use of covariant reggeization is more advantageous since in order to accomplish the bootstrap we have finally to connect the two sets of residue functions and the coupling constants as in (2.9). But the latter has been as an ingredient in this method of reggeization and therefore the crossing between the t- and s-channels becomes almost trivial.

We start first with the contribution of the natural parity exchanges, corresponding to $\rho(1^-)$ and its recurrences. Everywhere the notation (+) is used for natural parity exchanges and the notation (-) for the unnatural ones, corresponding to $B(1^+)$ and its recurrences.

Exchanging a spin-J natural parity particle in the t-channel of our reaction we get after some lengthy manipulations the following Regge behaviour

$$
\begin{aligned}
A_1^+(\nu, t) &= \beta_1(t)\, \xi_\alpha\, \alpha_t(\alpha_t - 1) \left(\frac{\nu}{\nu_1}\right)^{\alpha_t - 2}, \\
A_2^+ &= \beta_2(t)\, \xi_\alpha\, \alpha(\alpha - 1) \left(\frac{\nu}{\nu_1}\right)^{\alpha - 2}, \\
A_3^+ &= \beta_3(t)\, \xi_\alpha\, \alpha \left(\frac{\nu}{\nu_1}\right)^{\alpha - 1}, \\
A_4^+ &= \beta_4(t)\, \xi_\alpha \left(\frac{\nu}{\nu_1}\right)^{\alpha}, \\
A_5^+ &= \beta_5(t)\, \xi_\alpha \left(\frac{\nu}{\nu_1}\right)^{\alpha}, \\
A_6^+ &= \beta_6(t)\, \xi_\alpha\, \alpha \left(\frac{\nu}{\nu_1}\right)^{\alpha - 1}, \\
A_7^+ &= \beta_7(t)\, \xi_\alpha\, \alpha(\alpha - 1)(\alpha - 2) \left(\frac{\nu}{\nu_1}\right)^{\alpha - 3}, \\
A_8^+ &= \beta_8(t)\, \xi_\alpha\, \alpha(\alpha - 1) \left(\frac{\nu}{\nu_1}\right)^{\alpha - 2}, \\
A_9^+ &= \frac{1}{4}(t\beta_2 - x\beta_1(t))\, \xi_\alpha\, \alpha(\alpha - 1)(\alpha - 2) \left(\frac{\nu}{\nu_1}\right)^{\alpha - 3}, \\
A_{10}^+ &= \beta_9(t)\, \xi_\alpha\, \alpha \left(\frac{\nu}{\nu_1}\right)^{\alpha - 1}, \\
A_{11}^+ &= 0 \\
A_{12}^+ &= \frac{1}{4}(t\beta_2(t) - x\beta_1(t))\, \xi_\alpha\, \alpha(\alpha - 1) \left(\frac{\nu}{\nu_1}\right)^{\alpha - 2}, \\
A_{13}^+ &= \beta_7(t)\, \xi_\alpha\, \alpha(\alpha - 1) \left(\frac{\nu}{\nu_1}\right)^{\alpha - 2}, \\
A_{14}^+ &= \beta_8(t)\, \xi_\alpha\, \alpha \left(\frac{\nu}{\nu_1}\right)^{\alpha - 1}
\end{aligned}
\qquad (3.4)
$$

where $\xi = \frac{1 - e^{-i\pi\alpha(t)}}{\sin\pi\alpha(t)}$, $x = \frac{1}{2}(m_\rho^2 - m_\sigma^2)$ and the connection of the residue functions and the coupling constants, such as $\beta_1 = f_2 g_1 \frac{\pi a}{2}$, $\beta_6 = f_1 g_5 \frac{\pi a}{2}$ etc., have already been used. f_1 and f_2 are the couplings of spin J particle to $\rho\sigma$-vertex and g_1, $g_2 = g_3$, g_4, g_5 are the couplings to $\rho\rho$-vertex [26].

An unnatural parity exchange in the t-channel gives the following Regge behaviour

$$\begin{aligned} &A_1^- = A_5^- = A_8^- = A_9^- = A_{10}^- = A_{12}^- = 0 \quad , \\ &A_4^- \; , A_{13}^- \sim \left(\frac{\nu}{\nu_2}\right)^{\alpha} \quad , \\ &A_3^- , A_6^- , A_7^- , A_{11}^- , A_{14}^- \sim \left(\frac{\nu}{\nu_2}\right)^{\alpha-1} \quad , \\ &A_2^- \sim \left(\frac{\nu}{\nu_2}\right)^{\alpha-2} . \end{aligned} \tag{3.5}$$

The crossing property of invariant amplitudes under $\nu \leftrightarrow -\nu$ is :

$$\begin{aligned} &A_3, A_6, A_7, A_9, A_{10}, A_{11}, A_{14} = \text{even under s-u crossing} , \\ &A_1, A_2, A_4, A_5, A_8, A_{12}, A_{13} = \text{odd.} \end{aligned} \tag{3.6}$$

To write the FESR's for the invariant amplitudes A_i, i = 1,2,..., 14 we will calculate the contributions of ρ , $B(J^p = 1^+)$ and $g(J^p = 3^-)$ exchanges in the s- and u-channels of the reaction to the imaginary part of these amplitudes. We need the propagator of a spin 3-particle which is

$$\begin{aligned} P_{ikl,i'k'l'} = \frac{1}{3!}\Big[& R_{ii'}R_{kk'}R_{ll'} + R_{ii'}R_{kl'}R_{lk'} + R_{ik}R_{ki'}R_{ll'} \\ & + R_{ik'}R_{kl'}R_{li'} + R_{il'}R_{ki'}R_{lk'} + R_{il'}R_{kk'}R_{li'} \\ & - \frac{2}{5}\big(R_{ik}R_{i'k'}R_{ll'} + R_{il}R_{i'k'}R_{kl'} + R_{ik}R_{i'l'}R_{lk'} \\ & + R_{il}R_{i'l'}R_{kk'} + R_{kl}R_{i'k'}R_{il'} + R_{kl}R_{i'l'}R_{ik'} \\ & + R_{ik}R_{k'l'}R_{li'} + R_{il}R_{k'l'}R_{ki'} + R_{kl}R_{k'l'}R_{ii'} \big)\Big] , \end{aligned} \tag{3.7}$$

where $R_{ik} = g_{ik} - \frac{k_i k_k}{m_g^2}$.

The contribution of ρ-exchange in the s-channel to the invariant amplitudes is

$$
\begin{aligned}
A_1^\rho &= \frac{2}{\pi a}\left[\tfrac{1}{8}\beta_6(m_\rho^2) - \tfrac{1}{4}\beta_3(m_\rho^2) + \tfrac{y}{8}\beta_4(m_\rho^2)\right]\frac{-1}{s-m_\rho^2}\,,\\
A_2^\rho &= \frac{2}{\pi a}\left[\tfrac{1}{4}\beta_6 - \tfrac{1}{4}\beta_3\right]\frac{-1}{s-m_\rho^2}\,,\\
A_3^\rho &= \frac{2}{\pi a}\left[-\tfrac{1}{8}\beta_6 - \tfrac{1}{2}\beta_3 + \tfrac{y}{8}\beta_4\right]\frac{-1}{s-m_\rho^2}\,,\\
A_4^\rho &= \frac{2}{\pi a}\left[-\tfrac{3}{8}\beta_6 - \tfrac{3}{4}\beta_3 + \tfrac{y}{8}\beta_4\right]\frac{-1}{s-m_\rho^2}\,,\\
A_5^\rho &= \frac{2}{\pi a}\left[\tfrac{1}{2}\beta_8\right]\frac{-1}{s-m_\rho^2}\,,\\
A_6^\rho &= \frac{2}{\pi a}\left[-\tfrac{3}{8}\beta_6 - \tfrac{1}{4}\beta_3 + \tfrac{y}{8}\beta_4\right]\frac{-1}{s-m_\rho^2}\,,\\
A_7^\rho &= \frac{2}{\pi a}\left[\tfrac{1}{4}\beta_6 - \tfrac{1}{4}\beta_3\right]\frac{-1}{s-m_\rho^2}\,,\\
A_8^\rho &= \frac{2}{\pi a}\left[-\tfrac{1}{8}\beta_6 + \tfrac{y}{8}\beta_4\right]\frac{-1}{s-m_\rho^2}\,,\\
A_9^\rho &= \frac{2}{\pi a}\left[\tfrac{1}{8}\beta_6 + \tfrac{1}{4}\beta_3 + \tfrac{y}{8}\beta_4\right]\frac{-1}{s-m_\rho^2}\,,\\
A_{10}^\rho &= \frac{2}{\pi a}\left[\tfrac{1}{2}\beta_8\right]\frac{-1}{s-m_\rho^2}\,,\\
A_{11}^\rho &= \frac{2}{\pi a}\left[-\tfrac{1}{4}\beta_8 + \tfrac{1}{4}\beta_9 + \tfrac{y}{4}\beta_5\right]\frac{-1}{s-m_\rho^2}\,,\\
A_{12}^\rho &= \frac{2}{\pi a}\left[-\tfrac{1}{4}\beta_8 - \tfrac{1}{4}\beta_9 - \tfrac{y}{4}\beta_5\right]\frac{-1}{s-m_\rho^2}\,,\\
A_{13}^\rho &= \frac{2}{\pi a}\left[-\tfrac{1}{4}\beta_8 - \tfrac{3}{4}\beta_9 + \tfrac{y}{4}\beta_5\right]\frac{-1}{s-m_\rho^2}\,,\\
A_{14}^\rho &= \frac{2}{\pi a}\left[-\tfrac{1}{4}\beta_8 + \tfrac{3}{4}\beta_9 - \tfrac{y}{4}\beta_5\right]\frac{-1}{s-m_\rho^2}\,,
\end{aligned}
\tag{3.8}
$$

where a is the slope of the ρ-trajectory near the ρ-pole: $\alpha(s) - 1 \approx a(s-m_\rho^2)$, $y = \frac{\Sigma - 2t - s}{4}$, $\Sigma = 3\,m_\rho^2 + m_\sigma^2$ and everywhere one has to put $s = m_\rho^2$ (3.9).

The contribution of $B(J^P = 1^+)$-exchange in the s-channel is

$$
\begin{aligned}
A_1^B &= \frac{2}{\pi a'}\left[5\beta_1'(m_B^2) + \left(y + \tfrac{x}{2} + z + \tfrac{s}{2}\right)\beta_2'(m_B^2)\right]\frac{1}{s-m_B^2}\,,\\
A_2^B &= \frac{2}{\pi a'}\left[5\beta_1' + \left(x + z + \tfrac{3}{4}s\right)\beta_2'\right]\frac{1}{s-m_B^2}\,,\\
A_3^B &= \frac{2}{\pi a'}\left[10\beta_1' + \left(y - \tfrac{x}{2} + 2z - \tfrac{3}{4}s\right)\beta_2'\right]\frac{1}{s-m_B^2}\,,\\
A_4^B &= \frac{2}{\pi a'}\left[15\beta_1' + \left(y - \tfrac{3}{2}x + 3z\right)\beta_2'\right]\frac{1}{s-m_B^2}\,,\\
A_5^B &= \frac{2}{\pi a'}\left[-\left(5x + \tfrac{15}{2}s\right)\beta_1' - \left(-\tfrac{1}{2}sy - xz - \tfrac{3}{2}sz\right)\beta_2'\right]\frac{1}{s-m_B^2}\,,\\
A_6^B &= \frac{2}{\pi a'}\left[5\beta_1' + \left(y - \tfrac{3}{2}x + \tfrac{3}{2}s + z\right)\beta_2'\right]\frac{1}{s-m_B^2}\,,\\
A_7^B &= \frac{2}{\pi a'}\left[5\beta_1' + \left(x - \tfrac{s}{4} + z\right)\beta_2'\right]\frac{1}{s-m_B^2}\,,\\
A_8^B &= \frac{2}{\pi a'}\left[\left(y - \tfrac{x}{2} - \tfrac{s}{4}\right)\beta_2'\right]\frac{1}{s-m_B^2}\,,
\end{aligned}
\tag{3.10}
$$

$$A_9^B = \frac{2}{\pi a'}\left[-s\beta_1' + \left(y + \frac{x}{2} - z\right)\beta_2'\right]\frac{1}{s-m_B^2},$$

$$A_{10}^B = \frac{2}{\pi a'}\left[\left(-5x + \frac{5}{2}s\right)\beta_1' + \left(-\frac{1}{2}sy - xz + \frac{1}{2}sz\right)\beta_2'\right]\frac{1}{s-m_B^2},$$

$$A_{11}^B = \frac{2}{\pi a'}\left[\left(\frac{5}{2}x + \frac{5}{4}s\right)\beta_1' + \left(-\frac{1}{4}sy + \frac{1}{2}xz + \frac{1}{4}sz\right)\beta_2'\right]\frac{1}{s-m_B^2},$$

$$A_{12}^B = \frac{2}{\pi a'}\left[\left(\frac{5}{2}x + \frac{5}{4}s\right)\beta_1' + \left(-\frac{1}{4}sy + \frac{1}{2}xz + \frac{1}{4}sz\right)\beta_2'\right]\frac{1}{s-m_B^2},$$

$$A_{13}^B = \frac{2}{\pi a'}\left[\left(\frac{5}{2}x - \frac{15}{4}s\right)\beta_1' + \left(-\frac{1}{4}sy + \frac{1}{2}xz - \frac{3}{4}sz\right)\beta_2'\right]\frac{1}{s-m_B^2}, \qquad (3.10)$$

$$A_{14}^B = \frac{2}{\pi a'}\left[\left(\frac{5}{2}x - \frac{15}{4}s\right)\beta_1' + \left(-\frac{1}{4}sy + \frac{1}{2}xz - \frac{3}{4}sz\right)\beta_2'\right]\frac{1}{s-m_B^2},$$

where a' is the slope of the B-trajectory near the B-pole, β_1' and β_2' are the residue functions of the B-trajectory,

$$x = \frac{1}{2}(m_\rho^2 - m_\sigma^2)$$

$$y = \frac{\Sigma - 2t - s}{4} \qquad (3.11)$$

$$z = \frac{1}{4}(4m_\rho^2 - s)$$

and everywhere s has to be put equal to m_B^2.

The contribution of $g(3^-)$-exchange to the invariant amplitudes has also been calculated [26].

Now let us write down the set of the lowest moment FESR's i.e. for n = 0 or 1 in (2.5):

$$\int_0^{\bar{\nu}} \mathrm{Im}\left[A_1^{\rho} + A_1^{B} + A_1^{g(=3^-)}\right] d\nu = \beta_1(t)\, \frac{\alpha(t)(\alpha(t)-1)}{\alpha(t)-1} \left(\frac{\bar{\nu}}{\nu_1}\right)^{\alpha(t)-2} \bar{\nu}$$

$$\int_0^{\bar{\nu}} \mathrm{Im}\left[A_2^{\rho} + A_2^{B} + A_2^{g}\right] d\nu = \beta_2(t)\, \frac{\alpha(\alpha-1)}{\alpha-1} \left(\frac{\bar{\nu}}{\nu_1}\right)^{\alpha-2} \bar{\nu} + \beta_1'(t)(t+2x)\, \frac{\alpha'(\alpha'-1)}{\alpha'-1} \left(\frac{\bar{\nu}}{\nu_2}\right)^{\alpha'-2} \bar{\nu}$$

$$\int_0^{\bar{\nu}} \nu\, \mathrm{Im}\left[A_3^{\rho} + A_3^{B} + A_3^{g}\right] d\nu = \beta_3\, \frac{\alpha}{\alpha+1} \left(\frac{\bar{\nu}}{\nu_1}\right)^{\alpha-1} \bar{\nu}^2 + \left[\beta_2'(4m_\rho^2 - t) - 4\beta_1'\alpha'\right] \frac{\alpha'}{\alpha'+1} \left(\frac{\bar{\nu}}{\nu_2}\right)^{\alpha'-1} \bar{\nu}^2$$

$$\int_0^{\bar{\nu}} \mathrm{Im}\left[A_4^{\rho} + A_4^{B} + A_4^{g}\right] d\nu = \beta_4\, \frac{1}{\alpha+1} \left(\frac{\bar{\nu}}{\nu_1}\right)^{\alpha} \bar{\nu} - 8\beta_2'\, \frac{\alpha'}{\alpha'+1} \left(\frac{\bar{\nu}}{\nu_2}\right)^{\alpha'} \bar{\nu}$$

$$\int_0^{\bar{\nu}} \mathrm{Im}\left[A_5^{\rho} + A_5^{B} + A_5^{g}\right] d\nu = \beta_5\, \frac{1}{\alpha+1} \left(\frac{\bar{\nu}}{\nu_1}\right)^{\alpha} \bar{\nu}$$

$$\int_0^{\bar{\nu}} \nu\, \mathrm{Im}\left[A_6^{\rho} + A_6^{B} + A_6^{g}\right] d\nu = \beta_6\, \frac{\alpha}{\alpha+1} \left(\frac{\bar{\nu}}{\nu_1}\right)^{\alpha-1} \bar{\nu}^2 - 4x\beta_2'\, \frac{\alpha'}{\alpha'+1} \left(\frac{\bar{\nu}}{\nu_2}\right)^{\alpha'-1} \bar{\nu}^2$$

$$\int_0^{\bar{\nu}} \nu\, \mathrm{Im}\left[A_7^{\rho} + A_7^{B} + A_7^{g}\right] d\nu = \beta_7\, \alpha(\alpha-2) \left(\frac{\bar{\nu}}{\nu_1}\right)^{\alpha-3} \bar{\nu}^2 - \beta_2' t\, \frac{\alpha'}{\alpha'+1} \left(\frac{\bar{\nu}}{\nu_1}\right)^{\alpha'-1} \bar{\nu}^2$$

$$\int_0^{\bar{\nu}} \mathrm{Im}\left[A_8^{\rho} + A_8^{B} + A_8^{g}\right] d\nu = \beta_8\, \alpha(t) \left(\frac{\bar{\nu}}{\nu_1}\right)^{\alpha-2} \bar{\nu}$$

$$\int_0^{\bar{\nu}} \nu\, \mathrm{Im}\left[A_9^{\rho} + A_9^{B} + A_9^{g}\right] d\nu = \frac{1}{4}(t\beta_2 + 2x\beta_1)\, \alpha(\alpha-2) \left(\frac{\bar{\nu}}{\nu_1}\right)^{\alpha-3} \bar{\nu}^2 \qquad (3.12)$$

$$\int_0^{\bar{\nu}} \nu\, \mathrm{Im}\left[A_{10}^{\rho} + A_{10}^{B} + A_{10}^{g}\right] d\nu = \beta_9\, \frac{\alpha}{\alpha+1} \left(\frac{\bar{\nu}}{\nu_1}\right)^{\alpha-1} \bar{\nu}^2$$

$$\int_0^{\bar{\nu}} \nu\, \mathrm{Im}\left[A_{11}^{\rho} + A_{11}^{B} + A_{11}^{g}\right] d\nu = \left[-\beta_2'\, \frac{4m_\rho^2 - t}{4} + \beta_1'\alpha'\right] t\, \frac{\alpha'}{\alpha'+1} \left(\frac{\bar{\nu}}{\nu_2}\right)^{\alpha'-1} \bar{\nu}^2$$

$$\int_0^{\bar{\nu}} \mathrm{Im}\left[A_{12}^{\rho} + A_{12}^{B} + A_{12}^{g}\right] d\nu = \frac{1}{4}(t\beta_2 + 2x\beta_1)\, \alpha \left(\frac{\bar{\nu}}{\nu_1}\right)^{\alpha-2} \bar{\nu}$$

$$\int_0^{\bar{\nu}} \mathrm{Im}\left[A_{13}^{\rho} + A_{13}^{B} + A_{13}^{g}\right] d\nu = \beta_7\, \alpha \left(\frac{\bar{\nu}}{\nu_1}\right)^{\alpha-2} \bar{\nu} + \beta_2' t\, \frac{\alpha'}{\alpha'+1} \left(\frac{\bar{\nu}}{\nu_2}\right)^{\alpha'} \bar{\nu}$$

$$\int_0^{\bar{\nu}} \nu\, \mathrm{Im}\left[A_{14}^{\rho} + A_{14}^{B} + A_{14}^{g}\right] d\nu = \beta_8\, \frac{\alpha}{\alpha+1} \left(\frac{\bar{\nu}}{\nu_1}\right)^{\alpha-1} \bar{\nu}^2 - 2x\beta_1'\, \frac{\alpha'^2}{\alpha'+1} \left(\frac{\bar{\nu}}{\nu_2}\right)^{\alpha'-1} \bar{\nu}^2 ,$$

$$\alpha' \equiv \alpha_B(t) ,$$
$$\alpha \equiv \alpha_\rho(t) .$$

Limiting ourselves to the ρ - and B-contributions only we get the final expression for the set of the lowest moment FESR's for the reaction $\rho\rho \to \rho\sigma$:

$$
\left\{
\begin{aligned}
&-\frac{1}{a}\Big[\tfrac{1}{8}\beta_6(m_\rho^2)-\tfrac{1}{4}\beta_3(m_\rho^2)+\frac{y}{8\nu_1}\beta_4(m_\rho^2)\Big]_\rho+\frac{1}{a'}\Big[5\beta_1'(m_B^2)+\big(y-\tfrac{x}{2}+z+\tfrac{s}{2}\big)\tfrac{1}{\nu_2}\beta_2'(m_B^2)\Big]_B=\beta_1(t)\,\alpha\Big(\frac{\bar\nu}{\nu_1}\Big)^{\alpha-2}\bar\nu &(1)\\
&-\frac{1}{a}\Big[\tfrac{1}{4}\beta_6-\tfrac{1}{4}\beta_3\Big]_\rho+\frac{1}{a'}\Big[5\beta_1'+\big(x+z+\tfrac{3}{4}s\big)\tfrac{1}{\nu_2}\beta_2'\Big]_B=\beta_2\,\alpha\Big(\frac{\bar\nu}{\nu_1}\Big)^{\alpha-2}\bar\nu+\beta_1'(t+2x)\,\alpha'\Big(\frac{\bar\nu}{\nu_2}\Big)^{\alpha'-2}\frac{\bar\nu}{\nu_2} &(2)\\
&-\frac{\nu_\rho}{a}\Big[-\tfrac{1}{8}\beta_6-\tfrac{1}{2}\beta_3+\frac{y}{8\nu_1}\beta_4\Big]_\rho+\frac{\nu_B}{a'}\Big[10\beta_1'+\big(y-\tfrac{1}{2}x+2z-\tfrac{3}{4}s\big)\tfrac{1}{\nu_2}\beta_2'\Big]_B=\beta_3\frac{\alpha}{\alpha+1}\Big(\frac{\bar\nu}{\nu_1}\Big)^{\alpha-1}\bar\nu^2+\Big[\beta_2'\frac{4m_\rho^2-t}{\nu_2}-4\beta_1'\alpha'\Big]\frac{\alpha'}{\alpha'+1}\Big(\frac{\bar\nu}{\nu_2}\Big)^{\alpha'-1}\bar\nu^2 &(3)\\
&-\frac{1}{a}\Big[-\tfrac{3}{8}\beta_6-\tfrac{3}{4}\beta_3+\frac{y}{8\nu_1}\beta_4\Big]_\rho+\frac{1}{a'}\Big[15\beta_1'+\big(y-\tfrac{3}{2}x+3z\big)\tfrac{1}{\nu_2}\beta_2'\Big]_B=\beta_4\frac{1}{\alpha+1}\Big(\frac{\bar\nu}{\nu_1}\Big)^{\alpha}\bar\nu-8\beta_2'\frac{\alpha'}{\alpha'+1}\Big(\frac{\bar\nu}{\nu_2}\Big)^{\alpha'}\bar\nu &(4)\\
&-\frac{1}{a}\Big[\tfrac{1}{2}\beta_8\Big]_\rho+\frac{1}{a'}\Big[-\big(5x+\tfrac{15}{2}s\big)\tfrac{1}{\nu_2}\beta_1'-\big(-\tfrac{s}{2}y-xz-\tfrac{3}{2}sz\big)\tfrac{1}{\nu_2^2}\beta_2'\Big]_B=\beta_5\frac{1}{\alpha+1}\Big(\frac{\bar\nu}{\nu_1}\Big)^{\alpha}\bar\nu &(5)\\
&-\frac{\nu_\rho}{a}\Big[-\tfrac{3}{8}\beta_6-\tfrac{1}{4}\beta_3+\frac{y}{8\nu_1}\beta_4\Big]_\rho+\frac{\nu_B}{a'}\Big[5\beta_1'+\frac{y-\frac{3}{2}x+z+\frac{3}{2}s}{\nu_2}\beta_2'\Big]_B=\beta_6\frac{\alpha}{\alpha+1}\Big(\frac{\bar\nu}{\nu_1}\Big)^{\alpha-1}\bar\nu^2-4x\beta_2'\frac{\alpha'}{\alpha'+1}\Big(\frac{\bar\nu}{\nu_2}\Big)^{\alpha'-1}\frac{\bar\nu^2}{\nu_2} &(6)\\
&-\frac{\nu_\rho}{a}\Big[\tfrac{1}{4}\beta_6-\tfrac{1}{4}\beta_3\Big]_\rho+\frac{\nu_B}{a'}\Big[5\beta_1'+\big(x+z-\tfrac{s}{4}\big)\tfrac{1}{\nu_2}\beta_2'\Big]_B=\beta_7\,\alpha(\alpha-2)\Big(\frac{\bar\nu}{\nu_1}\Big)^{\alpha-3}\bar\nu^2-\beta_2' t\frac{\alpha'}{\alpha'+1}\Big(\frac{\bar\nu}{\nu_2}\Big)^{\alpha'-1}\frac{\bar\nu^2}{\nu_2} &(7)\\
&-\frac{1}{a}\Big[-\tfrac{1}{8}\beta_6+\frac{y}{8\nu_1}\beta_4\Big]_\rho+\frac{1}{a'}\Big[\frac{y-\frac{x}{2}-\frac{s}{4}}{\nu_2}\beta_2'\Big]_B=\beta_8\,\alpha\Big(\frac{\bar\nu}{\nu_1}\Big)^{\alpha-2}\bar\nu &(8)\\
&-\frac{\nu_\rho}{a}\Big[\tfrac{1}{8}\beta_6+\tfrac{1}{4}\beta_3+\frac{y}{8\nu_1}\beta_4\Big]_\rho+\frac{\nu_B}{a'}\Big[-5\beta_1'+\frac{y+\frac{x}{2}-z}{\nu_2}\beta_2'\Big]_B=\frac{1}{4}(t\beta_2+2x\beta_1)\,\alpha(\alpha-2)\Big(\frac{\bar\nu}{\nu_1}\Big)^{\alpha-3}\frac{\bar\nu^2}{\nu_1} &(9)\\
&-\frac{\nu_\rho}{a}\Big[\tfrac{1}{2}\beta_8\Big]_\rho+\frac{\nu_B}{a'}\Big[\frac{-5x+\frac{5}{2}s}{\nu_2}\beta_1'+\frac{-\frac{sy}{2}-xz+\frac{sz}{2}}{\nu_2^2}\beta_2'\Big]_B=\beta_9\frac{\alpha}{\alpha+1}\Big(\frac{\bar\nu}{\nu_1}\Big)^{\alpha-1}\bar\nu^2 &(10)
\end{aligned}
\right.
\qquad (3.13)
$$

$$
\left\{
\begin{aligned}
&-\frac{\nu_\rho}{a}\left[-\frac{1}{4}\beta_8+\frac{1}{4}\beta_9+\frac{y}{4\nu_1}\beta_5\right]_\rho+\frac{\nu_B}{a'}\left[\frac{\frac{5}{2}x+\frac{5}{4}s}{\nu_2}\beta_1'+\frac{-\frac{sy}{4}+\frac{xz}{2}+\frac{sz}{4}}{\nu_2^2}\beta_2'\right]_B=\left[-\beta_2'\frac{4m_\rho^2-t}{4\nu_2}+\beta_1'\cdot\alpha'\right]t\cdot \\
&\qquad\cdot\frac{\alpha'}{\alpha'+1}\left(\frac{\bar\nu}{\nu_2}\right)^{\alpha'-1}\frac{\bar\nu^2}{\nu_2} \qquad (11) \\
&-\frac{1}{a}\left[-\frac{1}{4}\beta_8-\frac{1}{4}\beta_9-\frac{y}{4\nu_1}\beta_5\right]_\rho+\frac{1}{a'}\left[\frac{\frac{5}{2}x+\frac{5}{4}s}{\nu_2}\beta_1'+\frac{-\frac{sy}{4}+\frac{xz}{2}+\frac{sz}{4}}{\nu_2^2}\beta_2'\right]_B=\frac{1}{4}(t\beta_2+2x\beta_1)\,\alpha\left(\frac{\bar\nu}{\nu_1}\right)^{\alpha-2}\frac{\bar\nu}{\nu_1} \qquad (12) \\
&-\frac{1}{a}\left[-\frac{1}{4}\beta_8+\frac{3}{4}\beta_9+\frac{y}{4\nu_1}\beta_5\right]_\rho+\frac{1}{a'}\left[\frac{\frac{5}{2}x-\frac{15}{4}s}{\nu_2}\beta_1'+\frac{-\frac{sy}{4}+\frac{xz}{2}-\frac{3}{4}sz}{\nu_2^2}\beta_2'\right]_B=\beta_7\,\alpha\left(\frac{\bar\nu}{\nu_1}\right)^{\alpha-2}\bar\nu+\beta_2'\cdot t\,\frac{\alpha'}{\alpha'+1}\left(\frac{\bar\nu}{\nu_2}\right)^{\alpha'}\frac{\bar\nu}{\nu_2} \qquad (13) \\
&-\frac{\nu_\rho}{a}\left[-\frac{1}{4}\beta_8+\frac{3}{4}\beta_9-\frac{y}{4\nu_1}\beta_5\right]_\rho+\frac{\nu_B}{a'}\left[\frac{\frac{5}{2}x-\frac{15}{4}s}{\nu_2}\beta_1'+\frac{-\frac{sy}{4}+\frac{xz}{2}-\frac{3}{4}sz}{\nu_2^2}\beta_2'\right]_B=\beta_8\frac{\alpha}{\alpha+1}\left(\frac{\bar\nu}{\nu_1}\right)^{\alpha-1}\bar\nu^2-2x\beta_1'\frac{\alpha'^2}{\alpha'+1}\left(\frac{\bar\nu}{\nu_2}\right)^{\alpha'-1}\frac{\bar\nu^2}{\nu_2} \qquad (14)
\end{aligned}
\right.
\qquad (3.13)
$$

where $x=\frac{1}{2}(m_\rho^2-m_\sigma^2)$, $y=\frac{\Sigma-2t-s}{4}$, $z=\frac{1}{4}(4m_\rho^2-s)$,

$\nu_\rho=\frac{2m_\rho^2+t-\Sigma}{4}$, $\nu_B=\frac{2m_B^2+t-\Sigma}{4}$,

$\alpha\equiv\alpha_\rho(t)$, $\alpha'\equiv\alpha_B(t)$, and subindices ρ and B mean that inside the square brackets S should be put equal to m_ρ^2 and m_B^2 respectively.

The contribution of $g(3^-)$-exchange can also be included 26) and a typical contribution to, say, the eighth sum rule of (3.13) is the following:

$$
\begin{aligned}
\int_0^{\bar{\nu}} \mathrm{Im}\, A_8^g(\nu,t)\,d\nu = {} & \frac{m_g^2}{4a\nu_1}\left\{6\left(x'^2-1+\frac{1}{5}\frac{p'^2}{m_g^2}\right)+\left[\frac{2}{m_g^2}(2-3x')+\frac{14}{5}\frac{x'}{m_g^2}\right]6Z_s\right\}\beta_2 \\
& -\frac{m_g^2}{8a\nu_1^2}\left\{2(1-x')^2\,6Z_s+2\frac{p'^2}{m_g^2}\left(\frac{6}{5}\right)Z_s\right\}\beta_1 \\
& +\frac{1}{16a\nu_1^2}\left\{2p'^2\left(\frac{6}{5}\right)Z_s\right\}\beta_3+\frac{1}{16a\nu_1^3}\left\{Z_s^3-\frac{3}{5}Z_0^2Z_s\right\}\beta_4 \\
& -\frac{1}{16a\nu_1^2}\left\{3Z_s^2-\frac{3}{5}Z_0^2-2p^2x'\frac{6}{5}Z_s\right\}\beta_6 \\
& +\frac{1}{8a\nu_1}\left\{\left[2(x'-1)-\frac{14}{5}x'\right]6Z_s-\frac{6}{5}p'^2\right\}\beta_8 ,
\end{aligned}
\tag{3.14}
$$

where $x' = \frac{1}{2}+\frac{x}{s}$, $p^2 = m_\rho^2-\frac{s}{4}$, $p'^2 = p^2-\frac{(s+2x)^2}{4s}$,

$Z_s = \frac{2t+s-\Sigma}{2}$, $Z_0 = (m_\rho^2-\frac{s}{4})(m_\rho^2-x'^2s)$,

$s \equiv m_g^2$.

The set of FESR's (3.13) can be studied in different stages of approximation. For the first step one can ignore the unnatural parity contribution altogether and put only the ρ-contribution in both sides of the sum rules. One then takes the cut off $\bar{\nu}$ to be somewhere between the ρ- and the g-poles. At this stage one can choose a special subset from these equations, go to special values (but not necessarily the same for different equations) of t so that the right hand sides of them vanish, say, due to sense-nonsence couplings. One then obtains a set of linear homogeneous equations for the residues β_i, determinant of which is a function of the external masses m_ρ^2 and m_σ^2, the exchanged particle mass m_ρ^2 and t's. Having chosen special values for the t's the determinant is only a function of m_σ^2 and its zeros as a function of external mass m_σ^2 can be studied x).

Further study of these sum rules has been left incompleted (see the introduction).

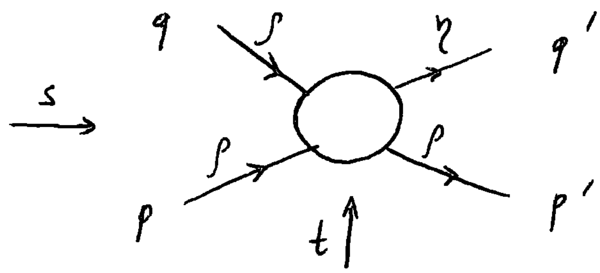
x) As an example, if one puts in the equations (3) and (6) of (3.13) $t \simeq -0.6$ so that $\alpha(t) = 0$ and in the equation (9) puts $t \simeq 1.6$ so that $\alpha(t) = 2$ then the condition det = 0 gives $m_\sigma \simeq 800$ MeV. Also besides the set (3.13), where the lowest moment sum rules with n = 0 for A_i, i = 1,2,4,5,8,12 and 13 have been used, one can write down the next set of sum rules with n = 2 for the above amplitudes and then all the left hand sides have the kinematical factor $\frac{2s+t-\Sigma}{4}$ analogous to the one in (2.6) and the right hand side factors $\alpha, (\alpha-1)$ etc. also change.

Chapter IV

The reaction $\rho\rho \rightarrow \rho\,\eta$

The kinematics is as

$$
\begin{aligned}
Q &= \tfrac{1}{2}\,(p+p') \\
P &= \tfrac{1}{2}\,(q+q') \\
\Delta &= p-p' = q'-q \\
s &= (p+q)^2 \\
t &= (p-p')^2 \\
\nu &= \tfrac{1}{4}\,(s-u) = Q\cdot P
\end{aligned}
$$



The objects exchanged in all three channels again have I = 1, G = + and can be of both natural and unnatural parity. For this reaction there exist 13 helicity amplitudes which can be most easily understood by comparing with the reaction $\rho\rho \rightarrow \rho\,\sigma$, for which there are 14 helicity amplitudes but for the present reaction the $< 00|00 >$ helicity amplitude vanishes because of parity conservation since $\rho\rho \rightarrow \rho\,\eta$ is an unnatural parity reaction.

The T-matrix for the reaction is

$$T = \epsilon_\mu(q)\,\epsilon_\nu(p)\,\epsilon^*_{\nu'}(p')\,M_{\mu\nu\nu'}\,, \tag{4.1}$$

for which $M_{\mu\nu\nu'}$ can therefore be written in terms of 13 independent invariant amplitudes:

$$M_{\mu\nu\nu'} = \sum_{i=1}^{13} K^{i}_{\mu\nu\nu'}\,A_i(\nu,t)\,. \tag{4.2}$$

However, for this reaction one can write 36 different Lorentz factors $K^{i}_{\mu\nu\nu'}$ among which, of course, 23 are linearly dependent through the so-called "equivalence relations". The question arises: which one of these invariant amplitudes to Choose? The prescription is the following [27] : in order to choose the invariant amplitudes which are free of kinematical singularities one has to write down all the equivalence relations for the Lorentz factors

$K^{i}_{\mu\nu\nu'}$ and choose among them the ones which can be expressed in terms of the others without introducing any kinematic factors which can become singular as the redundant ones and the rest as the Lorentz factors in front of the corresponding invariant amplitudes.

For our case this Lorentz expansion will be

$$\begin{aligned} M_{\mu\nu\nu'} = & (\epsilon_{\nu\mu} Q P P_{\nu'} + \epsilon_{\nu'\mu} Q P P_{\nu}) A_1(\nu,t) + (\epsilon_{\nu\mu} Q P P_{\nu'} - \epsilon_{\nu'\mu} Q P P_{\nu}) A_2 \\ & + (\epsilon_{\nu\mu} Q \Delta P_{\nu'} + \epsilon_{\nu'\mu} Q \Delta P_{\nu}) A_3 + (\epsilon_{\nu\mu} Q \Delta P_{\nu'} - \epsilon_{\nu'\mu} Q \Delta P_{\nu}) A_4 \\ & + (\epsilon_{\nu\mu} P \Delta P_{\nu'} + \epsilon_{\nu'\mu} P \Delta P_{\nu}) A_5 + (\epsilon_{\nu\mu} P \Delta P_{\nu'} - \epsilon_{\nu'\mu} P \Delta P_{\nu}) A_6 \\ & + \epsilon_{\nu'\nu} Q P Q_{\mu} A_7 + \epsilon_{\nu'\nu} Q \Delta Q_{\mu} A_8 + \epsilon_{\nu'\nu} P \Delta Q_{\mu} A_9 \\ & + \epsilon_{\nu'\nu\mu} Q A_{10} + \epsilon_{\nu'\nu\mu} P A_{11} + \epsilon_{\nu'\nu\mu} \Delta A_{12} + N_{\mu} g_{\nu\nu'} A_{13} \ , \end{aligned} \tag{4.3}$$

where
$$\begin{aligned} N_{\mu} &= \epsilon_{\mu\nu\lambda\tau} Q_{\nu} P_{\lambda} \Delta_{\tau} \equiv \epsilon_{\mu} Q P \Delta \ , \\ \epsilon_{\nu\mu} Q \Delta &\equiv \epsilon_{\nu\mu\lambda\tau} Q_{\lambda} \Delta_{\tau} \ \text{etc.} \end{aligned} \tag{4.4}$$

The equivalence relations are obtained from the identities like

$$\epsilon_{\mu\nu\sigma\tau} g_{\alpha\beta} = \epsilon_{\alpha\nu\sigma\tau} g_{\mu\beta} + \epsilon_{\mu\alpha\sigma\tau} g_{\nu\beta} + \epsilon_{\mu\nu\alpha\tau} g_{\sigma\beta} + \epsilon_{\mu\nu\sigma\alpha} g_{\tau\beta} \ , \tag{4.5}$$

by contracting them with different vectors.

The equivalence relations are the following:

$$\epsilon_{\nu'\nu} Q P P_\mu = -P^2 \epsilon_{\nu'\nu\mu} Q + \epsilon_{\nu'\mu} Q P P_\nu - \epsilon_{\nu\mu} Q P P_{\nu'} + (Q\cdot P)\epsilon_{\nu'\nu\mu} P ,$$

$$\epsilon_{\nu'\nu} P \Delta P_\mu = P^2 \epsilon_{\nu'\nu\mu} \Delta + \epsilon_{\nu'\mu} P \Delta P_\nu - \epsilon_{\nu\mu} P \Delta P_{\nu'} - (\Delta\cdot P)\epsilon_{\nu'\nu\mu} P ,$$

$$\epsilon_{\nu'\nu} Q \Delta P_\mu = -(\Delta\cdot P)\epsilon_{\nu'\nu\mu} Q + \epsilon_{\nu'\mu} Q \Delta P_\nu - \epsilon_{\nu\mu} Q \Delta P_{\nu'} + (Q\cdot P)\epsilon_{\nu'\nu\mu} \Delta ,$$

$$\begin{aligned} 4\epsilon_{\nu'\mu} Q \Delta Q_\nu = {} & -2\left[Q^2 + (Q\cdot P)\right]\epsilon_{\nu'\nu\mu}\Delta + \left[-\Delta^2 + 2(\Delta\cdot P)\right]\epsilon_{\nu'\nu\mu} Q \\ & + 2\epsilon_{\nu'\nu} Q\Delta Q_\mu - 2\epsilon_{\nu'\mu} Q \Delta P_\nu + 2\epsilon_{\nu\mu} Q \Delta P_{\nu'} , \end{aligned}$$

$$\begin{aligned} 4\epsilon_{\nu\mu} Q \Delta Q_{\nu'} = {} & 2\left[Q^2 - (QP)\right]\epsilon_{\nu'\nu\mu}\Delta + \left[-\Delta^2 + 2(\Delta\cdot P)\right]\epsilon_{\nu'\nu\mu} Q \\ & - 2\epsilon_{\nu'\nu} Q\Delta Q_\mu - 2\epsilon_{\nu'\mu} Q \Delta P_\nu + 2\epsilon_{\nu\mu} Q \Delta P_{\nu'} , \end{aligned}$$

$$\begin{aligned} 4\epsilon_{\nu'\mu} Q P Q_\nu = {} & -2\left[Q^2 + (Q\cdot P)\right]\epsilon_{\nu'\nu\mu} P + \left[2P^2 - (P\cdot\Delta)\right]\epsilon_{\nu'\nu\mu} Q \\ & + 2\epsilon_{\nu'\nu} Q P Q_\mu - 2\epsilon_{\nu'\mu} Q P P_\nu + 2\epsilon_{\nu\mu} Q P P_{\nu'} , \end{aligned}$$

$$\begin{aligned} 4\epsilon_{\nu\mu} Q P Q_{\nu'} = {} & 2\left[Q^2 - (Q\cdot P)\right]\epsilon_{\nu'\nu\mu} P + \left[2P^2 - (P\cdot\Delta)\right]\epsilon_{\nu'\nu\mu} Q \\ & - 2\epsilon_{\nu'\nu} Q P Q_\mu - 2\epsilon_{\nu'\mu} Q P P_\nu + 2\epsilon_{\nu\mu} Q P P_{\nu'} , \end{aligned} \tag{4.6}$$

$$\begin{aligned} 4\epsilon_{\nu'\mu} P \Delta Q_\nu = {} & -\left[2P^2 + 2(Q\cdot P) - (P\cdot\Delta)\right]\epsilon_{\nu'\nu\mu}\Delta + \left[-\Delta^2 + 2(\Delta\cdot P)\right]\epsilon_{\nu'\nu\mu} P \\ & + 2\epsilon_{\nu'\nu} P\Delta Q_\mu - 2\epsilon_{\nu'\mu} P \Delta P_\nu + 2\epsilon_{\nu\mu} P \Delta P_{\nu'} , \end{aligned}$$

$$\begin{aligned} 4\epsilon_{\nu\mu} P \Delta Q_{\nu'} = {} & \left[-2P^2 + 2(Q\cdot P) + (P\Delta)\right]\epsilon_{\nu'\nu\mu}\Delta + \left[-\Delta^2 + 2(\Delta\cdot P)\right]\epsilon_{\nu'\nu\mu} P \\ & - 2\epsilon_{\nu'\nu} P\Delta Q_\mu - 2\epsilon_{\nu'\mu} P \Delta P_\nu + 2\epsilon_{\nu\mu} P \Delta P_{\nu'} , \end{aligned}$$

$$\begin{aligned} 4N_\nu g_{\nu'\mu} = {} & 4N_\mu g_{\nu\nu'} + \left[4Q^2 - 4(Q\cdot P) - \Delta^2 + 2(\Delta P)\right]\epsilon_{\nu'\nu\mu} P + 2\left[2P^2 - (P\Delta)\right]\epsilon_{\nu'\nu\mu} Q \\ & + \left[-2P^2 + 2(Q\cdot P) + (P\Delta)\right]\epsilon_{\nu'\nu\mu}\Delta - 4\epsilon_{\nu'\nu} Q P Q_\mu - 4\epsilon_{\nu'\mu} Q P P_\nu + 4\epsilon_{\nu\mu} Q P P_{\nu'} \\ & - 4\epsilon_{\nu\mu} Q\Delta P_{\nu'} - 2\epsilon_{\nu'\nu} P\Delta Q_\mu - 2\epsilon_{\nu'\mu} P\Delta P_\nu + 2\epsilon_{\nu\mu} P\Delta P_{\nu'} , \end{aligned}$$

$$\begin{aligned} 4N_{\nu'} g_{\nu\mu} = {} & 4N_\mu g_{\nu\nu'} - \left[4Q^2 + 4(QP) + \Delta^2 - 2(\Delta P)\right]\epsilon_{\nu'\nu\mu} P \\ & + 2\left[2P^2 - (P\Delta)\right]\epsilon_{\nu'\nu\mu} Q - \left[2P^2 + 2(Q\cdot P) - (P\cdot\Delta)\right]\epsilon_{\nu'\nu\mu}\Delta \\ & + 4\epsilon_{\nu'\nu} Q P Q_\mu - 4\epsilon_{\nu'\mu} Q P P_\nu + 4\epsilon_{\nu\mu} Q P P_{\nu'} \\ & - 4\epsilon_{\nu'\mu} Q\Delta P_\nu + 2\epsilon_{\nu'\nu} P\Delta Q_\mu - 2\epsilon_{\nu'\mu} P\Delta P_\nu + 2\epsilon_{\nu\mu} P\Delta P_{\nu'} , \end{aligned}$$

$$N_\nu P_\mu - N_\mu P_\nu = -P^2 \epsilon_{\nu\mu} Q\Delta + (Q\cdot P)\epsilon_{\nu\mu} P\Delta + (P\Delta)\epsilon_{\nu\mu} QP ,$$

$$N_\nu Q_\mu - N_\mu Q_\nu = Q^2 \epsilon_{\nu\mu} P\Delta - (QP)\epsilon_{\nu\mu} Q\Delta ,$$

$$N_\nu P_\mu + N_\mu Q_\nu = \frac{1}{2}\left[\Delta^2 \epsilon_{\nu\mu} QP - (P\Delta)\epsilon_{\nu\mu} Q\Delta \right] ,$$

$$N_\nu P_{\nu'} - N_{\nu'} P_\nu = P^2 \epsilon_{\nu'\nu} Q\Delta - (QP)\epsilon_{\nu'\nu} P\Delta - (P\Delta)\epsilon_{\nu'\nu} QP ,$$

$$N_{\nu'} P_\mu - N_\mu P_{\nu'} = -P^2 \epsilon_{\nu'\mu} Q\Delta + (QP)\epsilon_{\nu'\mu} P\Delta + (P\Delta)\epsilon_{\nu'\mu} QP ,$$

$$N_\nu Q_{\nu'} - N_{\nu'} Q_\nu = -Q^2 \epsilon_{\nu'\nu} P\Delta + (QP)\epsilon_{\nu'\nu} Q\Delta ,$$

$$N_{\nu'} Q_\mu - N_\mu Q_{\nu'} = Q^2 \epsilon_{\nu'\mu} P\Delta - (QP)\epsilon_{\nu'\mu} Q\Delta ,$$

$$N_\nu Q_{\nu'} + N_{\nu'} Q_\nu = \frac{1}{2}\left[-\Delta^2 \epsilon_{\nu'\nu} QP + (P\Delta)\epsilon_{\nu'\nu} Q\Delta \right] ,$$

$$N_{\nu'} P_\mu - N_\mu Q_{\nu'} = \frac{1}{2}\left[\Delta^2 \epsilon_{\nu'\mu} QP - (P\Delta)\epsilon_{\nu'\mu} Q\Delta \right] , \qquad (4.6)$$

$$\begin{aligned} 8 N_\mu P_\nu P_{\nu'} = {} & [-4(QP)(\Delta P)]\epsilon_{\nu'\nu\mu} Q + [2\Delta^2 - 4(P\Delta)]\epsilon_{\nu\mu} QP P_{\nu'} \\ & + [4P^2 - 2(P\Delta) - 4(QP)]\epsilon_{\nu\mu} Q\Delta P_{\nu'} + [4(P\cdot\Delta)]\epsilon_{\nu'\nu} QP Q_\mu \\ & + 2[\Delta^2 - 4(P\Delta)]\epsilon_{\nu'\mu} QP P_\nu + [-4P^2]\epsilon_{\nu'\nu} Q\Delta Q_\mu \\ & + [4P^2 - 2(P\Delta) + 4(QP)]\epsilon_{\nu'\mu} Q\Delta P_\nu + [-4Q^2 - 4(QP)]\epsilon_{\nu'\mu} P\Delta P_\nu \\ & + [4Q^2 - 4(QP)]\epsilon_{\nu\mu} P\Delta P_{\nu'} + [4(Q\cdot P)]\epsilon_{\nu'\nu} P\Delta Q_\mu , \end{aligned}$$

$$\begin{aligned} 8 N_\mu P_\nu Q_{\nu'} = {} & [-4(QP)(P\Delta)]\epsilon_{\nu'\nu\mu} Q + [4(P\Delta)]\epsilon_{\nu'\nu} QP Q_\mu + [-2\Delta^2 + 4(P\Delta)] \\ & \epsilon_{\nu'\mu} QP P_\nu + [2\Delta^2 - 4(P\Delta)]\epsilon_{\nu\mu} QP P_{\nu'} + [-4P^2]\epsilon_{\nu'\nu} Q\Delta Q_\mu \\ & + [-4P^2 + 2(P\Delta) + 4(QP)]\epsilon_{\nu'\mu} Q\Delta P_\nu \\ & + [4P^2 - 2(P\Delta) - 4(QP)]\epsilon_{\nu\mu} Q\Delta P_{\nu'} + [-4Q^2 + 4(QP)]\epsilon_{\nu'\mu} P\Delta P_\nu \\ & + [4Q^2 - 4(QP)]\epsilon_{\nu\mu} P\Delta P_{\nu'} + [4(QP)]\epsilon_{\nu'\nu} P\Delta Q_\mu , \end{aligned}$$

$$
\begin{aligned}
8 N_\mu Q_\nu P_{\nu'} = {} & [4(QP)(P\Delta)]\, \epsilon_{\nu'\nu\mu} Q + [-4(P\Delta)]\, \epsilon_{\nu'\nu} Q P Q_\mu \\
& + [-2\Delta^2 + 4(P\Delta)]\, \epsilon_{\nu'\mu} Q P P_\nu + [2\Delta^2 - 4(P\Delta)]\, \epsilon_{\nu\mu} Q P P_{\nu'} \\
& + [4P^2]\, \epsilon_{\nu'\nu} Q \Delta Q_\mu + [2(P\Delta) - 4P^2 - 4(QP)]\, \epsilon_{\nu'\mu} Q \Delta P_\nu \\
& + [-2(P\Delta) + 4P^2 + 4(QP)]\, \epsilon_{\nu\mu} Q \Delta P_{\nu'} + [4Q^2 + 4(QP)]\, \epsilon_{\nu'\mu} P \Delta P_\nu \\
& + [-4Q^2 - 4(QP)]\, \epsilon_{\nu\mu} P \Delta P_{\nu'} + [-4(QP)]\, \epsilon_{\nu'\nu} P \Delta Q_\mu \,, \\
8 N_\mu Q_\nu Q_{\nu'} = {} & [2Q^2\Delta^2 - 4Q^2(\Delta P)]\, \epsilon_{\nu'\nu\mu} P + [4(QP)(P\Delta)]\, \epsilon_{\nu'\nu\mu} Q \\
& + [-2Q^2(P\Delta) + 4Q^2P^2 - 4(QP)^2]\, \epsilon_{\nu'\nu\mu} \Delta \qquad (4.6) \\
& + [-2\Delta^2]\, \epsilon_{\nu'\nu} Q P Q_\mu + [2(P\Delta)]\, \epsilon_{\nu'\nu} Q \Delta Q_\mu \\
& + [4Q^2]\, \epsilon_{\nu'\mu} P \Delta P_\nu + [-4Q^2]\, \epsilon_{\nu\mu} P \Delta P_{\nu'} \\
& + [-4(QP)]\, \epsilon_{\nu'\mu} Q \Delta P_\nu + [4(QP)]\, \epsilon_{\nu\mu} Q \Delta P_{\nu'} \,.
\end{aligned}
$$

By looking at the relations (4.6) it is easy to understand the choice of invariant amplitudes as in (4.3). Besides, by writing the helicity amplitudes in terms of the expansion (4.3) one can ensure oneself that all the kinematical factors $(\sin \frac{\theta}{2})^{|\lambda-\mu|}$ $(\cos \frac{\theta}{2})^{|\lambda+\mu|}$ are correctly reproduced; another choice of invariant amplitudes would not have these factors which would be a sign of presence of kinematic singularities in them.

The (s $\leftrightarrow$ u) crossing property of these amplitudes is the following: A_1, A_3, A_6, A_7, A_8, A_{11}, A_{12} and A_{13} are even under $\nu \leftrightarrow -\nu$, A_2, A_4, A_5, A_9 and A_{10} are odd.

We proceed further exactly as in chapter III by writing all the helicity amplitudes in the t-channel, exchange a particle of spin J in the t-channel and then find its contribution to each invariant amplitude. The calculations are lengthy and the equivalence relations (4.6) are to be used extensively in order to reduce all the redundant $K^{2}_{\mu\nu\lambda}$'which appear at intermediate stages to the ones chosen in the expansion (4.3).

We shall not write down these lenghty expressions and the final sum rules analogous to (3.13) [26)] and close this chapter by writing the Regge behaviour of the natural parity (ρ) - trajectory:

$$\begin{aligned}
&A_1\,,\; A_3 \sim \nu^{\alpha_\rho(t)-3}\,, \\
&A_2\,,\; A_5\,,\; A_9 \sim \nu^{\alpha_\rho(t)-2}\,, \\
&A_6\,,\; A_7\,,\; A_8\,,\; A_{11}\,,\; A_{13} \sim \nu^{\alpha_\rho(t)-1}\,, \\
&A_4\,,\; A_{10} \sim \nu^{\alpha_\rho(t)} \\
&A_{12} \sim \nu^{\alpha_\rho(t)+1}\,.
\end{aligned} \tag{4.7}$$

The behaviour of the invariant amplitude $A_{12} \sim \nu^{\alpha+1}$ may at first glance seem surprising, but one can check that in the expressions for helicity amplitudes, A_{12} appears always in the combination of $A_{12} + \nu\, A_4$ which has no $\nu^{\alpha+1}$ behaviour anymore. The same situation, i.e. $A_{12} \sim \nu^{\alpha_B+1}$, happens for the unnatural parity trajectory.

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27) Moussa and R. Stora, CERN preprint (1968).

Chapter 5

DUAL CURRENT AMPLITUDES

In this and the next chapter we present some progress which has been obtained in the construction of dual amplitudes for currents.

One of the most challenging subjects of the dual resonance model is the construction of dual amplitudes of currents which are consistent with current algebra. During the last few years many attempts have been done in order to extend the dual resonance model (DRM) of strong interactions in order to include the electromagnetic and weak interactions of hadrons. [1) - 7)]

The construction of a dual current amplitude may be suggested by the following considerations: First of all, in a world where hadronic amplitudes are dominated by narrow resonances, it is natural to assume that currents are also dominated by narrow resonances. The existence in the hadronic spectrum of an infinite sequence of vector mesons, provides us with the frame for such a dominance. Furthermore, since the form factors are rapidly convergent, we expect that the current amplitudes can be expressed as sum of poles in the current mass q^2, as well as in the energy variables of the hadronic channels.

Another quite suggestive argument is given by Bloom and Gilman [8)], who have analyzed the structure function $\nu W_2 (\nu , q^2)$ for inelastic electron-proton scattering (for notations cf. Appendix B). Considering νW_2 as a function of scaling variable $\omega' = \frac{2M\nu+M^2}{-q^2}$, they compared in the same range of ω' the scaling limit behaviour (i.e. large values of ν and $-q^2$) with the resonance region behaviour, showing that the smooth curve of the scaling limit averages the wavy curve of the resonance region (cf. also Rittenberg and Rubinstein [9] in this connection). This kind of behaviour is quite analogous to that of finite energy sum rules for strong interactions and suggests that current amplitudes actually obey duality.

In attempting to construct a current amplitude in the framework of DRM one would like to satisfy the following properties:

(i) absence of moving poles, i.e. no dependence of the pole positions of the amplitude on the external momenta,

(ii) planar duality,

(iii) generalized Ward-Takahashi identities,

(iv) good asymptotic behaviour in q^2 and in energy variables and

(v) factorization in the sense that the hadronic spectrum in any channel be consistent with that of the DRM.

Most of the difficulties lie in the construction of the off-mass-shell amplitudes and the conserved current (see chapter VI).

Up to now a model satisfying all the above requirements has not been given in the literature - each particular model is able to reproduce only part of those properties. According to this fact, the models which havebeen proposed until now can be essentially grouped into two classes: phenomenological models which are mainly connected with the properties (i), (ii) and (iv) (or even (iii)) and factorizable models which enforce the properties (iii) and (v).

Some of the phenomenological models [1)] are based on suitable modifications of the hadronic N-point function, where some fictitious constant trajectories were introduced. This automatically insures the correct pole structure in all channels, asymptotic behaviour and duality. Of course, the constant trajectories give rise to fixed power behaviour in all channels, in addition to Regge behaviour. This feature is not satisfactory, since one would like the absence of fixed poles in all channels, except for the channel of two adjacent currents where the fixed pole is required by current algebra. Some of these models will be described in more details in chapter VI.

A phenomenological model has been given by Landshoff and Polkinghorne [2)] for the two currents-two hadrons amplitude which incorporates many of the features one usually needs of a dual current-hadron amplitude. It has infinite number of poles in the current mass variables; it satisfies the Fubini-Dashen-Gell-Mann sum rule and gives rise to the desired Bjorken scaling [15)].

This amplitude for the double spin-flip part of the scattering of a vector current on hadron is given by the following expression

$$A(s,t,q_1^2,q_2^2)=\alpha' N^2 \iiint_0^\infty \frac{du_1\,du_2\,dz}{u_1 u_2 z}\left(1+\frac{1}{u_1}\right)^{\alpha_V(q_1^2)-1}$$

$$\left(1+\frac{1}{u_2}\right)^{\alpha_V(q_2^2)-1}\left(1+\frac{1}{z}\right)^{\alpha(t)-2}\left(1+zX(u_1,u_2,z)\right)^{\beta(s)} Y(u_1,u_2,z), \tag{5.1}$$

where

$$X(u_1,u_2,z)=\left[1+\frac{u_1 u_2}{e^z+u_1+u_2}\right]^{-1}$$

and (5.2)

$$Y=\left[1+\frac{z(u_1+u_2+u_1u_2)}{1+z+u_1u_2}\right]^{-m}\left[1+\frac{N(u_1+u_2)}{1+z}\right]^{-1}.$$

The expression (5.1) is reminiscent of a hadronic six-point function which is natural, because of its pole-structure, namely three simultaneous tree-graph poles.

The above amplitude is an example showing how an infinite sum of resonances with falling form factors as $-q^2 \longrightarrow +\infty$ can build up a scaling function, contrary to previously proposed diffractive mechanism [10] for the scaling. However, the Landshoff-Polkinghorne amplitude does not give a prescription how to build up in general current amplitudes in the framework of DRM. This is because the model has factorization only for the leading trajectory. Essentially on account of this lack of factorization, the construction of a current amplitude becomes non-unique, since one would like to derive any such amplitude from the basic ingredients like vertices and propagators as it is the case for the dual models for the purely hadronic reactions [18), 19)].

Further, there has been some attempt [4), 6)] to construct the factorizable models. They obtain a conserved vector current coupling to an arbitrary number of scalar mesons by making use of the Ward-like identities (see e.g. ref. 19)) of the DRM. The structure in q^2 remains here completely arbitrary, so that in principle one can construct satisfactory amplitudes for a single current. For the case of two currents, however, troubles with the divergence condition appear [7)]. There appear [7)] also unphysical singularities in the channel of two currents whose positions depend on current masses q_1^2 and q_2^2.

An entirely different approach has been proposed by Kikkawa and Sato [11)] and by Nambu [12)], who start from the infinite-component field-theoretical formulation of the dual model and introduce the conserved vector current according to the minimal gauge principle as in the ordinary field theory. However, the amplitudes of this kind contain [7)] terms analogous to the Born term of the usual field theory and therefore cannot have structure in q^2, and besides these terms are not dual. One could think that the structure in the currents will come from the higher order terms.

In the next section we will describe in details how to include currents according to the above prescription in a manner nearest to the field theory model $\lambda\varphi^3$, i.e. write the dual amplitude for hadrons in the tree-graph form in the coordinate representation and then replace the space-time derivations by the covariant ones.

The technical aspects and applications of this minimal coupling scheme to deep inelastic inclusive and semi-inclusive reactions are included in Appendix B (ref. 24) and 25)).

The minimal coupling scheme

In the electromagnetic and weak interactions the most important properties, for instance universality, conservation laws etc. are treated in a unified manner from the view point of the minimal gauge principle [13]. Here we will establish the gauge boson theory in the dual resonance formalism of the hadrons. We shall show thatthe resulting dual current amplitudes obey the divergence conditions of the conserved vector currents (CVC) and the current density algebra. In the present approach we deal only with the covariant tensor amplitudes and we do not assume the existence of the local current density operators. Thus the condition of CVC and the current density algebra are replaced by the divergence conditions and the Ward-Takahashi identities for the covariant tensor amplitudes. The proof of the current algebra conditions is presented by making use of the fact that our current amplitudes are gauge invariant, since we have introduced the non-strong interactions as the minimal gauge interactions [13]. From the high energy behaviour of the current-hadron scattering (see Appendix B) amplitude we recognize that the two-current amplitude has a fixed pole singularity in addition to the moving Regge pole in the complex angular momentum plane. Both the current algebra conditions and the existence of the fixed pole singularity make the two-current amplitude to satisfy the Fubini-Dashen-Gell-Mann sum rule [14]. The existence of this fixed pole in turn gives rise [24), 25)] to Bjorken scaling [15)] of the amplitude and due to this fact the connection between the threshold behaviour of the structure function as $\omega = \frac{2p\cdot q}{-q^2} \to 1$ and the power fall-off of the electromagnetic form factor as $-q^2 \to \infty$ is different from the one given by Drell and Yen [16)] and Bloom and Gilman [8)].

The same happens in the model of Landshoff and Polkinghorne [2), 17)], where the scaling results because of the existence of the fixed pole.

Throughout this chapter we consider all the particles as bosons belonging to the nonet representation of SU_3 and lying on the same degenerate trajectory $\alpha(s) = \alpha' s + \alpha_0 = \frac{1}{2} s + \alpha_0$ with $\alpha_0 < 0$ and the scale of mass, so that $\alpha' = \frac{1}{2}$.

Consider first the pure hadronic amplitude in the multiperipheral configuration illustrated as in Fig. 1

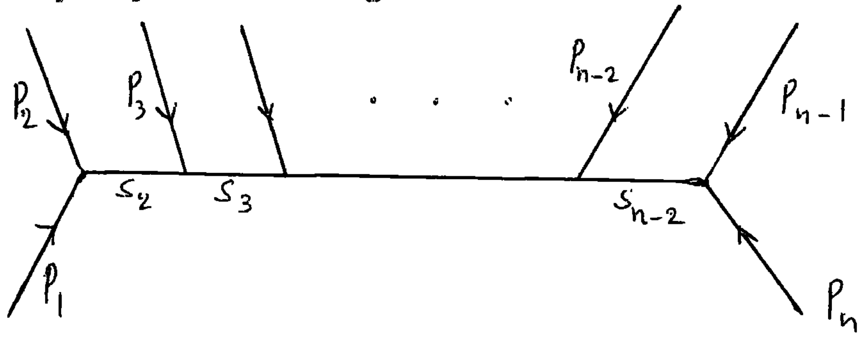

Fig. 1

By use of the operator formalism [18), 19)] the scattering amplitude in terms of vertex functions $V^{\alpha}(p)$ and propagator D(s) can be written in the following form

$$M^{\alpha_1\alpha_2\cdots\alpha_n}(p_1,\cdots,p_n) = \langle 0| \left[V^{\alpha_{n-1}}(p_{n-1})\, D(s_{n-2})\, V^{\alpha_{n-2}} \cdots D(s_2)\, V^{\alpha_2}(p_2) \right]_{\alpha_1\alpha_n} |0\rangle \tag{5.3}$$

where $V^{\alpha}(p)$ is the resonance-resonance-particle vertex defined by

$$\begin{aligned} V^{\alpha_i}(p_i) &= T_{\alpha_i}\, exp(-p_i\, a^{+})\, exp(p_i\, a) \\ &\equiv T_{\alpha_i}\, V(p_i) \end{aligned} \tag{5.4}$$

Here $a_{\mu}^{(n)+}$ and $a_{\mu}^{(n)}$ are the harmonic oscillator creation and destruction operators satisfying the commutation relation

$$\begin{aligned} \left[a_{\mu}^{(m)},\, a_{\nu}^{(n)+} \right] &= -g_{\mu\nu}\, \delta_{mn} \quad , \\ m, n &= 1, 2, \cdots \infty \quad , \end{aligned} \tag{5.5}$$

$$g_{00} = -g_{11} = -g_{22} = -g_{33} \quad ,$$

$g_{\mu\nu}$ is the metric tensor

and we use the notation

$$p_i\, a^{+} = (p_i)^{\mu} \cdot \sum_{n=1}^{\infty} \frac{a_{\mu}^{(n)+}}{\sqrt{n}} \quad ,$$
$$p_i\, a = (p_i)^{\mu} \cdot \sum_{n=1}^{\infty} \frac{a_{\mu}^{(n)}}{\sqrt{n}} \quad . \tag{5.6}$$

The indices α_i denote the internal symmetry states. Here we assume the symmetry group to be SU_3. Then the internal symmetry factor T_α in the expression for vertex function (5.4) is the 9 x 9 matrix defined by

$$(T_\alpha)_{\beta\gamma} = \frac{1}{2} Tr\,(\lambda_\alpha \lambda_\beta \lambda_\gamma) \quad , \tag{5.7}$$

where λ_α denote the Gell-Mann's 3 x 3 unitary spin matrices.

Had we assumed isospin invariance for the states we would have for T_α instead

$$(T_\alpha)_{\beta\gamma} = \frac{1}{2} Tr\,(\tau_\alpha \tau_\beta \tau_\gamma) \quad ,$$

where τ_α are the isospin matrices.

The propagator D(s) in (5.3) is given by

$$D(s_i) = \frac{1}{R - \alpha(s_i)} , \tag{5.8}$$

or in the integral form as

$$D(s_i) = \int_0^1 dx \; x^{-\alpha(s_i) - 1 + R} \tag{5.9}$$

where the Hamiltonian R of the system is defined as

$$R = -\sum_{n=1}^{\infty} n \, a^{(n)+} \cdot a^{(n)} = -\sum_{n=1}^{\infty} n \, a_\mu^{(n)+} a_\nu^{(n)} g_{\mu\nu} \tag{5.10}$$

and $\qquad s_i = (p_1 + \cdots + p_i)^2 .$

Let us define now the n-point function G by attaching the propagators of the external scalar particles to the amplitude M as:

$$G^{\alpha_1 \cdots \alpha_n}(p_1, \ldots, p_n) = \prod_{i=1}^{n} \frac{1}{-\alpha(p_i^2)} M^{\alpha_1 \cdots \alpha_n}(p_1, \ldots, p_n) . \tag{5.11}$$

This function G can be interpreted as a part of the Green's function in field theory.

We rewrite further (5.11) in the coordinate representation by introducing the functions

$$\varphi_j(x) = \exp(-i p_j \cdot x)$$

which are the space-time part of the wave function of the j-th external particle. Then the coordinate representation of (5.11) will be

$$G^{\alpha_1 \ldots \alpha_n}(x,\partial) = \langle 0 | \Big[\Big\{ \frac{-1}{\alpha(\partial^2)} \varphi_1(x) \Big\} \Big\{ \frac{-1}{\alpha(\partial^2)} V^{\alpha_2}(\partial) \varphi_2(x) \Big\} \tag{5.12}$$

$$D(\partial^2) \cdots \Big\{ \frac{-1}{\alpha(\partial^2)} V^{\alpha_{n-1}}(\partial) \varphi_{n-1}(x) \Big\} \Big\{ \frac{-1}{\alpha(\partial^2)} \varphi_n(x) \Big\} \Big]_{\alpha_1 \alpha_n} | 0 \rangle$$

where $\alpha(\partial^2)$, $V^{\alpha}(\partial)$ and $D(\partial^2)$ are obtained from $\alpha(p^2)$, $V^{\alpha}(p)$ and $D(p^2)$ respectively, by replacing the momentum p_μ by the derivative $i\partial_\mu$. The curly brackets in (5.12) indicate that the derivatives inside the bracket act upon the $\varphi_i(x)$ of the same bracket.

The relation between the momentum representation (5.11) and the coordinate representation (5.12) of G is, as usual, given by

$$\int d^4x \, G^{\alpha_1 \cdots \alpha_n}(x, \partial) = (2\pi)^4 \, \delta^4\left(\sum_{i=1}^{n} p_i\right) G^{\alpha_1 \cdots \alpha_n}(p_1, \ldots, p_n) . \tag{5.13}$$

It is straightforward now to include the non-strong interaction by replacing the space-time derivatives ∂_μ by the covariant ones according to

$$\partial_\mu \longrightarrow D_\mu = \partial_\mu - i e \, f_\beta \, A_\mu^\beta(x) \quad , \tag{5.14}$$

where e is the universal coupling constant of the gauge field A_μ^β (x) to the hadron system, and f_α is the 9 x 9 matrix of SU_3 defined as

$$(f_\alpha)_{\beta\gamma} = -i \, f_{\alpha\beta\gamma}$$

with $f_{\alpha\beta\gamma}$ being the SU_3 structure constants.

From the expression (5.12) with the change (5.14) in it, i.e. from G(x,D), one can construct the current amplitudes by power series expansion with respect to the coupling constant e.

The ℓ-current amplitude is given by

$$\lim_{\substack{p_i^2 = m^2 \\ A_\mu = 0}} \int d^4x \prod_{i=1}^{n} \left[-\alpha(\partial_i^2)\right] \frac{\delta^\ell}{\delta A_{\mu_1}^{\beta_1}(k_1) \cdots \delta A_{\mu_\ell}^{\beta_\ell}(k_\ell)} G(x, \mathcal{D}) \quad (5.15)$$

$$= (2\pi)^4 \delta^4\left(\sum_{i=1}^{n} p_i + \sum_{j=1}^{\ell} k_j\right) M_{\mu_1 \cdots \mu_\ell}^{\beta_1 \cdots \beta_\ell}(k_1, \ldots, k_\ell),$$

where $$\frac{\delta G(x, \mathcal{D})}{\delta A(k)} = \frac{\delta A(x)}{\delta A(k)} \frac{\delta G(x, \mathcal{D})}{\delta A(x)}$$

In (5.15) $A_\mu^\beta(k)$ is the Fourier transform of $A_\mu^\beta(x)$ and the internal quantum numbers and the momenta of the hadrons are suppressed. In the above, the gauge field A_μ has been treated as a c-number.

Finally, the Reggeized Feynman rules for the current amplitude from the above description can be obtained as a power series expansion in the coupling constant e of the vertex function $V(\mathcal{D})$ and the propagator $D(\mathcal{D}^2)$ through the covariant derivative (5.14). For more details we refer to 11), 24) and 25).

Instead of the above prescription one could start from the infinite-component formulation of Nambu [12] and Miyamoto [20] for the dual resonance amplitudes and include the currents according to the minimal gauge principle there. The Nambu-Miyamoto equation [12), 20] for thedual resonance amplitude is:

$$\left[-\Box + R + V(x,\partial_\mu) - i\frac{\partial}{\partial\tau}\right]\Psi(x, n^{(\lambda)}, \tau) = 0 \tag{5.16}$$

where $R = -\sum_{\lambda=1}^{\infty} \lambda\, n^{(\lambda)} = -\sum_{\lambda=1}^{\infty} \lambda\, a_\mu^{(\lambda)+} a_\nu^{(\lambda)} g_{\mu\nu}$

as in (5.10) corresponding to the free Hamiltonian of the system,

$$V(x,\partial_\mu) = :\exp\left[-i\phi_\mu(0)\partial_\mu\right]\varphi(x): \tag{5.17}$$

is the interaction potential

$$\phi_\mu(\tau) = \sum_{\lambda=1}^{\infty} \frac{1}{\sqrt{\lambda}}\left[a_\mu^{(\lambda)}\exp(i\lambda\tau) + a_\mu^{(\lambda)+}\exp(-i\lambda\tau)\right],$$

$\varphi(x)$ is the field corresponding to an external line and Ψ is the master wave function describing the hadronic system.

The gauge current is introduced then in the wave equation (5.16), according to the transformation (5.14).

The two descriptions to include the currents in DRM as given above, are identical and give the same Feynman rules. For simplicity, in the above description we have ommitted a projection operator $\mathcal{P}$ [21] in the expression for the interaction potential (5.17). This projection operator is responsible for keeping the hadronic part of the amplitude invariant under the duality transformation [21].

Current Algebra Conditions

Here we shall show that the Ward-Takahashi (W-T) identities for the current amplitudes, i.e. the following relations

$$k_1^{\mu_1} M^{\beta_1 \cdots \beta_\ell}_{\mu_1 \cdots \mu_\ell}(k_1, \ldots, k_\ell) = i \sum_{i=2}^{\ell} f_{\beta_1 \beta_i \lambda} M^{\beta_2 \cdots \lambda \cdots \beta_\ell}_{\mu_2 \cdots \mu_i \cdots \mu_\ell}(k_2, \ldots, k_i + k_1, \ldots, k_\ell) \tag{5.18}$$

are satisfied in the approach described above. As was already stated, since we have not assumed the existence of the local current density operators [22], the current algebra conditions can be replaced by the Ward-Takahashi relations (5.18).

Consider the following local gauge transformation on the gauge field

$$A'_\mu(x) = A_\mu(x) + i\,\epsilon(x) A_\mu(x) - \frac{1}{e}\partial_\mu \epsilon(x) \quad , \tag{5.19}$$

where $\epsilon(x)$ is an infinitesimal function of x. In (5.19) and in the intermediate steps the internal symmetry indices will be dropped.

The variation of the function $G(x, \mathcal{D}\{A\})$ (cf. (5.12) and (5.14)) then will be given by

$$\delta G = G(x, \mathcal{D}\{A'\}) - G(x, \mathcal{D}\{A\})$$

$$= i \int d^4k \, d^4p \, \epsilon(k) A_\mu(p) \frac{\delta}{\delta A_\mu(p+k)} G(x, \mathcal{D}\{A\}) \tag{5.20}$$

$$+ \frac{i}{e} \int d^4k \, \epsilon(k) k_\mu \frac{\delta}{\delta A_\mu(k)} G(x, \mathcal{D}\{A\}) ,$$

where $\epsilon(k)$ and $A_\mu(p)$ are the Fourier transforms of $\epsilon(x)$ and $A_\mu(x)$ respectively.

Taking the functional derivative of the eq. (5.20) with respect to $\epsilon(k)$ gives

$$\frac{\delta G}{\delta \epsilon(k)} = - \int d^4p \, A_\mu(p) \frac{\delta}{\delta A_\mu(p+k)} G(x, \mathcal{D}\{A\}) \tag{5.21}$$

$$+ \frac{i}{e} k_\mu \frac{\delta}{\delta A_\mu(k)} G(x, \mathcal{D}\{A\}) .$$

Since the theory is gauge invariant, $\frac{\delta G}{\delta \epsilon} = 0$ and from (5.21) we obtain

$$k_\mu \frac{\delta}{\delta A_\mu(k)} G(x, \mathcal{D}\{A\}) = ie \int d^4p \, A_\mu(p) \frac{\delta}{\delta A_\mu(p+k)} G(x, \mathcal{D}\{A\}) \quad . \tag{5.22}$$

This is the basic formula from which one in the usual manner can show the current conservation condition and the Ward-Takahashi identities.

From (5.22) one obtains

$$k_\mu \frac{\delta}{\delta A_\mu(k)} G(x, \mathcal{D}\{A\}) \Big|_{A_\nu = 0} = 0 \quad , \tag{5.23}$$

which when combined with (5.15) gives the current conservation condition for a single current amplitude:

$$k^\mu M_\mu^\alpha(k) = 0 \tag{5.24}$$

By applying the functional derivative of $A_\nu(q)$ to eq. (5.22) and using the definition of (5.15) for the current amplitude, we obtain the Ward-Takahashi identity for the two-current amplitude :

$$k^{\mu} M_{\mu\nu}^{\alpha\beta}(k,q) = i e f_{\alpha\beta\gamma} M_{\nu}^{\gamma}(k+q).$$

(5.25)

The identity (5.18) for ℓ-current amplitude is obtained by taking the repeated derivatives of formula (5.22).

From the W-T identity (5.25) and the definition of form factor F as

$$M_{\nu}^{\gamma}(k+q) = e\,(p_1+p_2)_{\nu}\,F^{\gamma}((k+q)^2)$$

(5.26)

it is straightforward to derive the Fubini-Dashen-Gell-Mann [14] sum rule.

Consider the two-current amplitude

$$q_1 + p_1 \longrightarrow q_2 + p_2$$

with q_1 and q_2 being the momenta of the currents, p_1 and p_2 the momenta of spinless hadrons.

The covariant amplitude $M^{\alpha\beta}_{\mu\nu}$ can be expanded in terms of invariant amplitudes as follows:

$$
\begin{aligned}
M_{\mu\nu} = {} & P_\mu P_\nu A_1(\nu, t; q_1^2, q_2^2) + P_\mu Q_\nu A_2 \\
& + P_\mu \Delta_\nu A_3 + Q_\mu P_\nu B_1 + Q_\mu Q_\nu B_2 \\
& + Q_\mu \Delta_\nu B_3 + \Delta_\mu P_\nu C_1 + \Delta_\mu Q_\nu C_2 \\
& + \Delta_\mu \Delta_\nu C_3 + g_{\mu\nu} D \quad ,
\end{aligned}
\tag{5.27}
$$

where $P = \frac{1}{2}(p_1 + p_2)$, $Q = \frac{1}{2}(q_1 + q_2)$, $\Delta = q_2 - q_1$,

$\nu = P.Q$ and $t = \Delta^2$

Then the current algebra condition (5.25) together with the definition (5.26) gives:

$$
\begin{aligned}
\nu A_1(\nu, t; q_1^2, q_2^2) + \frac{1}{4}(3q_1^2 + q_2^2 - t)\, B_1(\nu, t; q_1^2, q_2^2) \\
+ \frac{1}{2}(q_2^2 - q_1^2 - t)\, C_1 = 2i e^2 f_{\alpha\beta\gamma} F^\gamma(t) .
\end{aligned}
\tag{5.28}
$$

If now $\frac{B_1}{\nu A_1}$ and $\frac{C_1}{\nu A_1} \xrightarrow[\nu \to \infty]{} 0$, which is supported by the Regge arguments, from (5.28) we obtain

$$\lim_{\nu \to \infty} \nu A_1^{\alpha\beta}(\nu, t; q_1^2, q_2^2) = 2i\, e^2 f_{\alpha\beta\gamma} F^{\gamma}(t) \tag{5.29}$$

or from the dispersion relation for A_1 amplitude one obtains:

$$\frac{1}{\pi} \int d\nu \, \mathrm{Im}\, A_1^{\alpha\beta}(\nu, t; q_1^2, q_2^2) = i e^2 f_{\alpha\beta\gamma} F^{\gamma}(t). \tag{5.30}$$

In this way the conditions for the current amplitude to satisfy the Fubini-Dashen-Gell-Mann sum rule (5.30) are the appropriate analyticity and the fixed power behaviour $O\left(\frac{1}{\nu}\right)$ for the invariant amplitude A_1. In the language of the complex angular momentum this high energy behaviour of A_1 amplitude requires the existence of a fixed pole singularity at J = 1 in the complex angular momentum plane [23] in the channel of two charged (neutrino-antineutrino) SU_3 currents.

The high energy behaviour of the current amplitude constructed according to the prescription of this chapter shows the $\frac{1}{\nu}$ behaviour for the A_1 amplitude, and therefore it satisfies the sum rule (5.30). This is included in Appendix B.

Chapter 6

DUAL CURRENT AMPLITUDES

In this chapter for completeness we shall finally discuss some other attempts towards the construction of dual current amplitudes.

Let us consider first the selfconsistent (bootstrap) factorizable model of Brower and Weis [4]. The hadronic character of virtual photon suggests that in a current-hadron bootstrap theory, the electromagnetic interaction of hadrons should occur as a subprocess of a factorized higher point dual function, describing a system of N interacting, spinless hadrons. This has been the point of view taken in ref. 4). Strong restrictions are imposed on the current amplitudes by the requirement that the spectrum of resonances occurring as poles in q^2 and in energy variables to be the same as the spectrum of the purely hadronic amplitudes. These consistency conditions, shown diagrammatically for one current amplitude in Fig. 1,

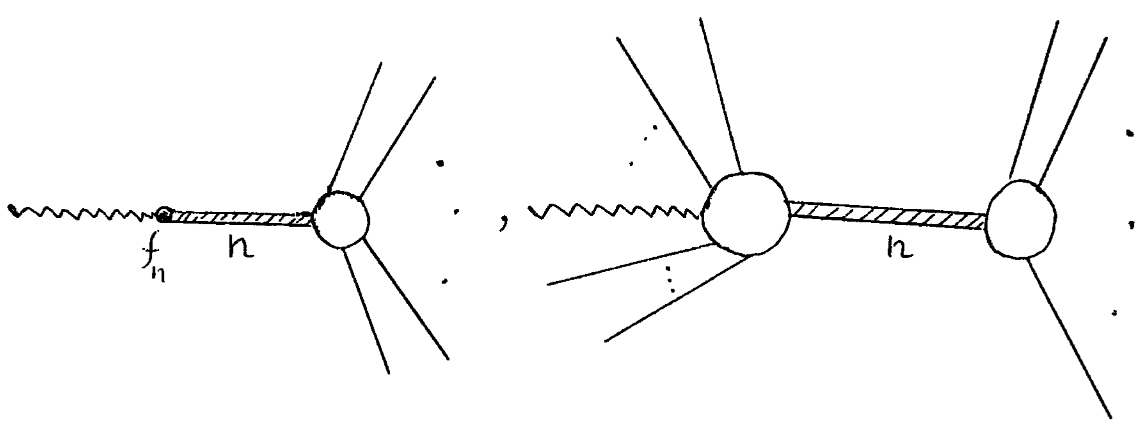

Fig. 1

include the generalized vector-meson dominance. According to this bootstrap philosophy, the current amplitudes are believed to be completely determined by these conditions. Quadratic factorization is expected to play the crucial role, hence the single-current amplitudes must be restricted so as to yield an acceptable two-current amplitude.

Despite its attractive features, this scheme only accommodates vector mesons which are universally coupled and hence fail to give Bjorken scaling [15]. The form factors fall off exponentially and, as was stated in Chapter 5, the two-current amplitudes do not satisfy the divergence conditions [7] and have unphysical singularities [7] in the channel of two currents with their position depending on current masses q_1^2 and q_2^2.

A modified version of the above-mentioned scheme has been considered by Brower, Rabl and Weis [1] in a hybrid model which leads to Bjorken scaling. However, the attractive idea of a current-hadron bootstrap has to be abandoned and the factorization is lost.

The form of a dual resonance-dominated amplitude for currents is certainly extremely non-unique if factorization and consistency with the hadronic amplitudes are not required. Nevertheless, it may be useful to put aside these requirements temporarily and study the general structure of dual resonance-dominated functions having good large-q^2 behaviour and, if possible, satisfying the requirements of current conservation and current algebra. Perhaps the most important outcome of such a study could be an improved understanding of the role of high mass vector mesons which could then help to solve the factorization problem, but such functions are also interesting and useful from a phenomenological point of view.

A number of such phenomenological approaches have been proposed [1), 2)]. Some of these models proposed by Sugawara [1)], Ohba [1)], Ademollo and Del Giudice [1)] are based on suitable modification of the hadronic N-point function [26)]

$$B_N(p_1,\dots,p_N) = \int_0^1\int_0^1\cdots\int_0^1 \frac{du_1\,du_2\cdots du_{N-3}}{J(u_i)} \cdot \prod_{i,j} u_{ij}^{-\alpha_{ij}-1}, \tag{6.1}$$

by introducing some fictitious constant trajectories(which have no physical meaning)for every channel carrying leptonic quantum numbers according to Fig. 2, for a two-current amplitude:

$l \equiv$ lepton

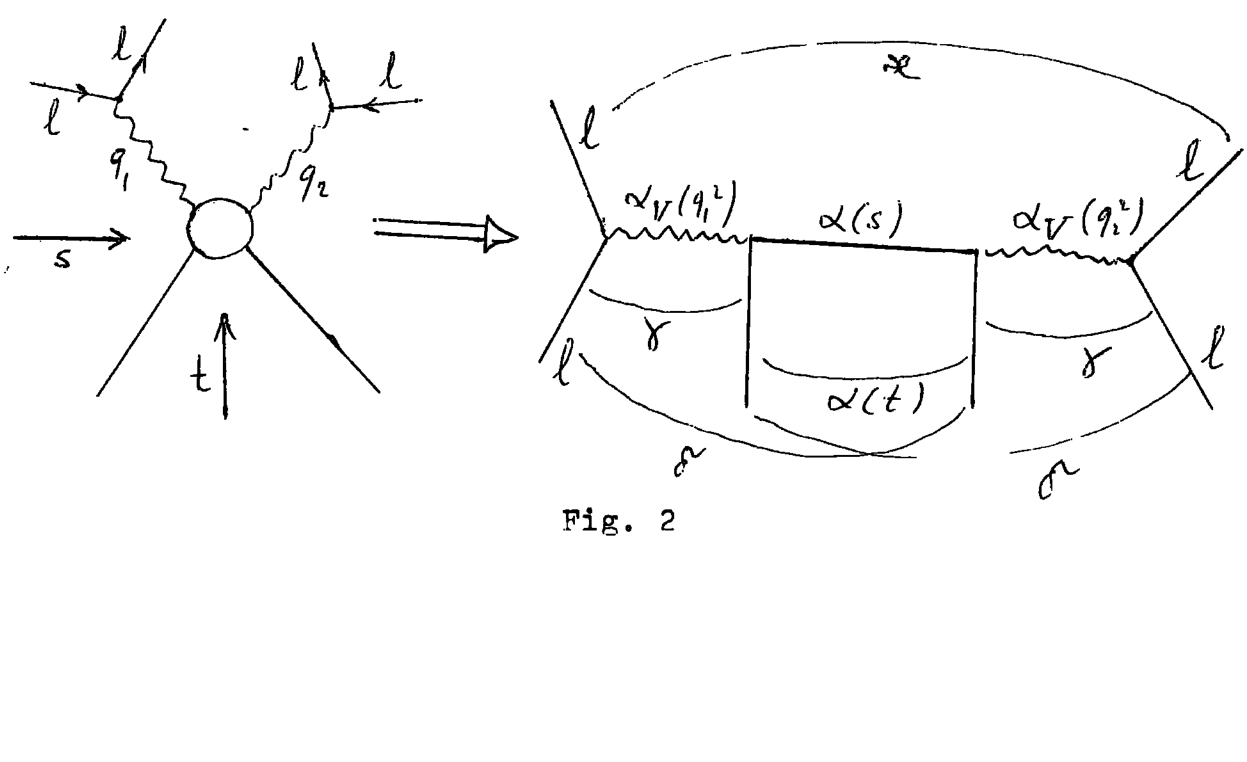

Fig. 2

which has been obtained from a hadronic six-point function.

From this kind of amplitudes one obtains fixed power as well as Regge behaviour in all the channels, e.g. $(\alpha_t)^{\alpha_s}$ and $(\alpha_t)^{\gamma}$ etc. This feature, in spite of its interesting analogies with some field theory models [27], is not satisfactory. The form factors in these models are power-behaved and we note that this power behaviour of the form factors is correlated with the presence of fixed powers in the subenergies just as is suggested by some field theory models [27]. Thus, there are two suggested ways in which the compositeness of the hadrons manifests itself in the non-strong interaction: (a) the absence of fixed poles except in the two-current channel and (b) the rapid decrease of the form factors. This is what happens in the dual amplitudes of the kind described above.

However, as discussed by Freedman [3], for the kind of models described above with power-behaved form factors, the lower trajectories will have a much greater degeneracy than the corresponding ones of the hadronic spectrum in (6.1). This by itself would not be a fatal flaw because there exists the possibility of modifying the hadronic amplitudes, since the are not yet firmly established. However, the spectrum of such current amplitudes is internally inconsistent: it is different in different channels [3].

Let us next consider the model of Bander [1] which incorporates the power fall-off of the form factors, current algebra sum rule [14], proper Regge behaviour and also the Bjorken scaling [15].

The expression for the double spin-flip amplitude of the virtual Compton scattering amplitude A (ν , t; q_1^2, q_2^2) (in the notation of formula (5.27) of Chapter 5, A_1-amplitude) is expressed as

$$A(\nu, t; q_1^2, q_2^2) = \alpha_s'(0)\left(1-\alpha_V(0)\right)$$

$$\cdot \Big\{ \left[(\alpha_t - 2) G_3(s, t; q_1^2, q_2^2) - (\alpha_t - 1 - \alpha_V(0)) G_2(s, t; q_1^2, q_2^2) \right] \tag{6.2}$$

$$- \left[(\alpha_t - 2) G_3(u, t; q_1^2, q_2^2) - (\alpha_t - 1 - \alpha_V(0)) G_2(u, t; q_1^2, q_2^2) \right] \Big\},$$

where

$$G_n(s, t; q_1^2, q_2^2) = \int_0^1 dv\, v^{-\alpha(s)-1} (1-v)^{-\alpha(t)+1}$$

$$\int_0^1 du_1\, du_2\, (1 - v u_1 u_2)^{\alpha(t)-n}\, u_1^{-\alpha_V(q_1^2)}\, u_2^{-\alpha_V(q_2^2)}\,.$$

(6.3)

The form factor of the first pole obtained from the expression (6.2) is given by

$$F(q^2) = \frac{1-\alpha_T(0)}{1-\alpha_t(q^2)} \underset{|q^2|\to\infty}{\sim} \left|\frac{1}{q^2}\right| . \qquad (6.4)$$

The scaling function $F_2(\omega) = \nu W_2$ has the threshold behaviour

$$F_2(\omega) \underset{\omega\to 1}{\sim} (\omega - 1) , \qquad (6.5)$$

that is in the model of Bander the connection between the threshold behaviour of the deep inelastic structure function $F_2(\omega)$ and the asymptotic fall-off of the elastic form factor $F(q^2)$ is the same as the Drell-Yan[16] and the Bloom-Gilman [8] relation. Recall that in the parton model treatment of Drell and Yan [16] of the deep inelastic electron-proton scattering and in the duality (in the sense of finite energy sum rule used in its extreme form of saturation with only one pole) approach of Bloom and Gilman [8] the following relation is satisfied:

$$p + 1 = 2n , \qquad (6.6)$$

where n is the asymptotic power fall-off of the elastic form factor

$$F(q^2) \underset{|q^2|\to\infty}{\sim} \left(\frac{1}{|q^2|}\right)^n \tag{6.7}$$

and p is the threshold behaviour of the scaling function $F_2(\ )$:

$$F_2(\omega) \underset{\omega\to 1}{\sim} (\omega - 1)^p$$

It is possible to modify (6.2) and (6.3) in order to get the following expression for the form factor:

$$F(q^2) = \frac{\Gamma(1+\ell)\,\Gamma(1-\alpha(q^2))}{\Gamma(2+\ell-\alpha(q^2))} \underset{|q^2|\to\infty}{\sim} \left(\frac{1}{|q^2|}\right)^{\ell+1} . \tag{6.8}$$

The scaling function then satisfies the threshold behaviour given by

$$F_2(\omega) \underset{\omega\to 1}{\sim} (\omega - 1)^{2\ell+1} \tag{6.9}$$

The expressions of (6.8) and (6.9) again satisfy the Drell-Yan relation (6.6).

In spite of some nice features of Bander's model mentioned above, it is completely unsatisfactory because the amplitude proposed has infinitely degenerate Regge trajectory (or resonances) even at the leading level.

In the model of Landshoff and Polkinghorne [2)] which was already mentioned in Chapter 5, the amplitude (formula (5.1)) is factorizable on the leading trajectory, satisfies the Fubini-Dashen-Gell-Mann (FDGM) sum rule and the Bjorken scaling. Furthermore, the amplitude has the important property that the form factor appearing in FDGM sum rule is the same as that obtained from the model by taking the factorized residue on the leading trajectory. Bjorken scaling arises because of the FDGM fixed pole, although it gives a non-vanishing contribution in the scaling limit even for the amplitudes, like the photon scattering, which have vanishing equal-time commutator . Because of this responsibility of FDGM fixed pole for the Bjorken scaling of the amplitude, the Drell-Yan and Bloom-Gilman type of relation (6.6) in this case takes the following form:

$$p + 1 = n \quad . \tag{6.10}$$

Recall that the Bloom and Gilman's derivation of (6.6) was based on the finite energy sum rule for the $F(\omega') = \nu W_2$ scaling function, i.e. on the following relation:

$$\frac{2M}{q^2} \int d\nu \text{ Resonances} = \int d\omega' F(\omega') . \tag{6.11}$$

With the extreme duality assumption of saturation with only one pole (elastic), the left hand side of (6.11) then gives a contribution proportional to

$$\left[F(q^2)\right]^2 \underset{|q^2| \to \infty}{\sim} \left[\frac{1}{|q^2|}\right]^{2n} . \tag{6.12}$$

The power 2n in (6.12) is exactly the right hand side of the Bloom-Gilman's relation (6.6).

For the Landshoff-Polkinghorne's amplitude the analogous to (6.11) relation should have an extra contribution due to the fixed pole which is now proportional to

$$F(q^2) \underset{|q^2| \to \infty}{\sim} \left[\frac{1}{|q^2|}\right]^{n} \tag{6.13}$$

Because now (6.13) dominates the contribution of (6.12), the power n appears on the right hand side of relation (6.10).

In the case of the minimal coupling scheme for the current amplitudes (cf. Appendix B), although the Bjorken scaling property of the virtual Compton scattering amplitude arises because of the Fubini-Dashen-Gell-Mann fixed pole, the analogous to (6.10) relation has the following form:

$$p+1 = \frac{2}{3}n + \frac{7}{3} \tag{6.14}$$

In (6.14) the form factor has been defined through the FDGM sum rule

$$A \xrightarrow[\nu \to \infty]{} -e\,\frac{F(q^2)}{\nu} \quad , \tag{6.15}$$

a logarithmic factor $\log^{-2}(q^2)$ has been neglected in its asymptotic behaviour, i.e.

$$F(q^2) \underset{|q^2| \to \infty}{\sim} \left(\frac{1}{|q^2|}\right)^4 \log^{-2}|q^2| \tag{6.16}$$

and the threshold behaviour is the usual one

$$F(\omega) \underset{\omega \to 1}{\sim} (\omega - 1)^{p} \quad . \tag{6.17}$$

The relation (6.14) does not coincide with that of (6.10) because the amplitude here has a more complicated singularity structure than single poles and also because of the effect of the renormalization (counter) term.

More recent attempts

Here we would like to go briefly through some of the recent attempts in the field of dual current amplitudes. Since up to now no solution of the problem satisfying all the requirements, stated at the beginning of Chapter 5, has been obtained, it seems necessary to try all the possible approaches with the hope that the final achievement may come as a combination of some of them.

One of such approaches was proposed by Nielsen, Susskind and Kraemmer [28)] in the context of combining the parton model idea [29)] and the continuous string picture of hadrons for the dual models [30)].

In such a picture the basic assumptions are that hadrons are bound states of some constituents called partons and that these tend to bind themselves in long chains, so that only the neighbouring partons interact. This implies that, if one constructs the interaction term in the Lagrangian from the parton fields, the important Feynman diagrams in the theory will have a surface-like structure [31)]. In other words, one can argue that the most important class of diagrams are those that can be drawn on a simply connected region like a circular disc or an infinite band and have a certain conformal invariant property. All these are equivalent to replacing the continuous string picture of dual models by an infinite but discrete chain of partons.

The physical idea in the above approach is that, while hadrons are emitted from the edge of the Koba-Nielsen [32] disc, currents emerge from its interior with an amplitude proportional to a Möbius invariant density. The form factor in this model is Gaussian:

$$F(q^2) = \left(\frac{2\lambda_0}{\pi}\right)^{\frac{q^2}{2}} \frac{\Gamma\left(-\frac{q^2}{2}+\frac{1}{4}\right)\Gamma\left(\frac{1}{2}\right)}{\Gamma\left(-\frac{q^2}{2}+\frac{3}{4}\right)}, \qquad (6.18)$$

where $\alpha' = 1$,
λ_0 is a parameter,

but is supposed not to be valid for large q^2.

The trouble with the above approach is that the arguments in it are vague and besides its intuitive character does not permit the consistency of the idea to be checked, there is no prescription how to build many current amplitudes etc.

However, the physical idea of introducing currents as attached to the interior of the Koba-Nielsen disc has given rise to further developments along this line [33) - 36)].

The model of Drummond [33] and Rebbi [34] is constructed along the line mentioned above for the scalar currents. For n off-mass-shell scalar particles the amplitude is given by the expression:

$$A(q_1, \dots, q_n) = \Omega^{-1} \int \prod_{i=1}^{n} \frac{d^2 z_i}{(\mathrm{Im}\, z_i)^2} \prod_{i,j=1}^{n} |z_i - z_j^*|^{-q_i q_j} , \tag{6.19}$$

where q_i are the momenta of the off-mass-shell external lines and z_i are the complex variables running over the upper half of the complex plane.

The Drummond-Rebbi amplitude (6.19) is free of unphysical singularities but has increasing spectrum of states already on the leading trajectory. However, the factorization is consistent between external and internal lines, but the spectrum requires two sets of harmonic oscillators. The expression (6.19) shows explicitly that the off-mass-shell amplitude is invariant with respect to all the cyclic and non-cyclic permutations of the external lines. On the mass-shell the amplitude reduces to the sum of the standard dual amplitudes. While the off-mass-shell amplitude is dual it contains wrong-signature nonsense fixed poles which, however, vanish on mass-shell. Therefore finite energy sum rules cannot be derived for off-shell process. The form factor is exponential like in 28) and the photoproduction amplitude, e.g., is given by the expression

$$A = \frac{\Gamma\left(\frac{q^2-1}{2}\right)\Gamma\left(-\frac{\alpha(s)}{2}\right)\Gamma\left(-\frac{\alpha(t)}{2}\right)\Gamma\left(-\frac{\alpha(u)}{2}\right)}{\Gamma\left(-\frac{\alpha(s)+\alpha(t)}{2}\right)\Gamma\left(-\frac{\alpha(t)+\alpha(u)}{2}\right)\Gamma\left(-\frac{\alpha(u)+\alpha(s)}{2}\right)}, \qquad (6.20)$$

$$\alpha'_V = 1 ,$$

which is simply the Virasoro amplitude off-mass-shell and in the scaling region exhibits a kind of generalized scaling behaviour for the exclusive reactions [37].

A more sophisticated, mathematically, amplitude has been given in 36) for off-shell scalar amplitudes which, unlike the Drummond-Rebbi [33), 34)] model - have good factorization property, in the sense that factorization can be obtained in the space of harmonic oscillators of the conventional dual model. It has no unphysical singularities and in the two-current channel the amplitude has Regge behaviour and seems to have no fixed pole, but instead has a singularity associated with the Pomeron appearing in non-planar loop diagrams [37]. Other properties of this model have still to be studied.

In an attempt to construct hadronic vector currents, Kikkawa and Sakita [35] combine together the method of Drummond-Rebbi [33), 34)] with the Koba-Nielsen Möbius symmetry of the integrand, and the Nambu [39] method for constructing conserved currents.

This approach gives current amplitudes with proper factorization and with no moving poles. The gauge invariance of the amplitude occurs due to the presence of seagull terms. Because of the latter terms the whole amplitude is not dual in the sense of sum of s-channel poles being equal to the sum of t-channel poles. So in this method by duality is meant the Möbius symmetry of the amplitude. The impression is, that in such models which are very close in spirit to the ordinary perturbation theory it is impossible to construct gauge invariant amplitudes without seagull terms. The form factor is Gaussian as was the case in the Drummond-Rebbi model. The Bjorken scaling property of the amplitude has not yet been studied.

To conclude, we would like to mention that, although the final solution for the dual current amplitudes seems still far from being reached, a certain amount of progress and understanding has been achieved. The elimination of certain models and the exclusion of a purely diffractive mechanism for the Bjorken scaling phenomenon are also a part of the achievements.

The study of current amplitudes provides us with a better understanding than the Regge theory for the electromagnetic processes with an arbitrary photon mass. It may also provide us with a solution to current algebra equations. Although the latter motivation emphasizes the importance of the vector currents, it may still be worthwhile as a first step to study the academic case of scalar currents, since in the dual models all the angular momentum states are on the same footing.

If off-shell dual amplitudes can be obtained for vector currents satisfying the constraints of current algebra and, eventually, also of scaling light-cone behaviour, they could be identified with physical currents [40)]. Before the latter hope can be fulfilled the axial vector currents should also be introduced and the chiral symmetry taken into account.

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Appendix A

MASS EXTRAPOLATION USING DUAL AMPLITUDES

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Received 22 August 1969

A theory of mass-extrapolation would have very interesting consequences; symmetry breaking and PCAC could be studied qualitatively and processes like $\pi\pi$ scattering and $\bar{p}n \rightarrow 3\pi$ could be related to each other. It is our purpose here to discuss a model that has some encouraging features. It is based on the use of Veneziano forms of scattering amplitudes and, though several other applications come to mind, we have started with the showcase of the Veneziano model: $\bar{p}n$ threshold annihilation into three pions.

This process was first studied by Lovelace [1], who used the fact that at threshold the $\bar{p}n$ system has the quantum numbers of the pion to relate this process to $\pi\pi$ scattering, one of the pions having mass $4M^2$. However, the method of "extrapolation" was arbitrary and no justification was given - a single $\pi\pi$ Veneziano term was used but, instead of the leading one, a satellite without the leading trajectory was taken, so the amplitude did not have correct Regge behaviour. Although some of the features of the data were partially explained, in particular the dip in the centre of the Dalitz plot, the fit was, not surprisingly, unsatisfactory.

Altarelli and Rubinstein [2] conjectured that for processes like $\bar{p}n \rightarrow \pi^-\pi^-\pi^+$ the amplitude should have the more general form:

$$A = \sum_{n,m} C_{nm} \frac{\Gamma(n-\alpha_s)\,\Gamma(n-\alpha_t)}{\Gamma(n+m-\alpha_s-\alpha_t)}\,, \qquad (1)$$

where s and t are the two $(\pi^-\pi^+)$ energies, respectively, and α is the ρ-trajectory. By fitting all the details of the experimental data, and paying attention to the constraints from $p\bar{p}$ annihilation, they found a fit with $C_{10} = 1$, $C_{11} = 1.86$, $C_{30} = 0.5$, $-0.3 < C_{20}$, $C_{21} < 0.2$. Here the ratio of the first two terms is well determined, but C_{30} is rather unimportant. Only terms with $n+m \leqslant 3$ were included in the fit, since only such terms have a zero at $\alpha_t = \alpha_s = 1.5$, where the data show a sharp dip. Although the existance of a five parameter fit is not without significance, the shortcoming of this paper is that no explanation was given for having just these terms, or of the particular coefficients multiplying them. The aim of this paper is to provide the missing theoretical justification for these coefficients.

Our basic philosophy is contained in the assumption that, when the external particles lie on leading trajectories, a good approximation to the amplitude is provided by the leading Veneziano terms or the minimum number when isospin or helicity restrictions demand more. In some sense, this principle maximizes duality and also minimizes the number of degenerate states that already over-populate the spectrum [3]. There is even some experimental support for this idea coming from the Lovelace-Shapiro-Veneziano-Yellin $\pi\pi$ amplitude [1,4], the fits of Petersen and Törnquist [5], and a posteriori the results of this paper. However, it follows from our assumption that when an external particle is a daughter, then several terms will appear with well determined coefficients.

The task of constructing physically acceptable 5-point functions is not easy. They must satisfy

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the following conditions: all desired poles, leading Regge behaviour in all channels, no spin-zero ghosts when trajectories have positive intercepts. One is tempted to attack this problem by studying purely mesonic processes, e.g. $\pi\sigma \to \pi\pi\pi$, and then, by going to $s = 4M^2$ (M = nucleon mass) in the $\pi\pi$ channel, projecting out the $J = 0$ state, and using factorisation, obtain the desired amplitude. This method is appealing since there are no major pathologies in dual models of mesonic systems. However, the presence of daughter degeneracies [3] means that factorisation does not hold, so we cannot use this method, and must consider directly the $N\bar{N} \to 3\pi$ amplitude. Factorisation is then not required, and in addition we have the considerable computational advantage that no angular momentum projection is required since we are at physical threshold. However, we must use imperfect Veneziano functions, since it is well known that there are unsolved problems even for πN elastic scattering [6].

When spin $\frac{1}{2}$ problems are involved there is arbitrariness in the choice of which invariant amplitudes are assumed to be approximated by leading Veneziano terms. We do not have a complete understanding of this problem, but we adopt what appear to be a plausible procedure for our case. We demand that the relevant piece of the 5-point function, i.e. the invariant non-flip amplitude, reduces to the leading term in each channel when we go to a pole on a leading trajectory. In particular, this gives the important restriction that our amplitude does not have the nucleon pole in both baryon channels simultaneously - since otherwise we would obtain an incorrect $\pi N \to \pi N$ non-flip amplitude.

We consider first that part of the amplitude which has poles in the $N\bar{N}$ channel. This is given by the Bardacki-Ruegg-Virasoro form [7] appropriate to the configuration of fig. 1, which

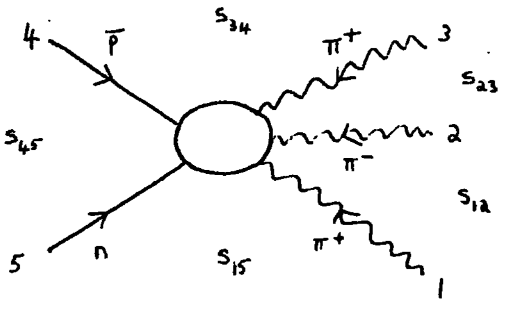

Fig. 1.

also defines our variables s_{ij}. Using the notation:

$$F(\alpha_{12}, \alpha_{23}, \alpha_{34}, \alpha_{45}, \alpha_{15}) \equiv$$

$$\equiv \int_0^1 \int_0^1 \frac{du_1 du_3}{1-u_1u_3}\, u_1^{-\alpha_{12}-1}\, u_2^{-\alpha_{23}-1} \times \tag{2}$$

$$\times\, u_3^{-\alpha_{34}-1}\, u_4^{-\alpha_{45}-1}\, u_5^{-\alpha_{15}-1} ,$$

we take as the leading term for this amplitude the expression

$$\alpha_{12}^{\rho}\, F(\alpha_{12}^{\rho},\ \alpha_{23}^{\rho}-1,\ \alpha_{34}^{B}-\tfrac{1}{2},\ \alpha_{45}^{\pi},\ \alpha_{15}^{B}-\tfrac{3}{2}) ,$$

where α^B refers to either the N or the Δ trajectory. This function has all the desired properties except that it does not have the lowest α_{15}^{B} pole. Apart from functions obtained from this by interchanging labels we have not found any other expression with these properties. Note that the absence of double poles in the baryon channels is not an input to this term, but is necessitated by the requirement of correct Regge behaviour, which would be spoilt by the replacement of $\alpha_{15}^{B} - \frac{3}{2}$ by $\alpha_{15}^{B} - \frac{1}{2}$.

Clearly Bose-statistics demands that we add to the above expression an identical term with 1 and 3 interchanged. In addition we must add a term which has spin $\frac{1}{2}$ poles in the 15 channel. The obvious way of doing this i.e. symmetrising in 4 and 5, is of course not satisfactory since it would eliminate all odd orbital angular momenta in the $\bar{p}$n channel *. Instead we add a term:

$$(\alpha_{34}^{B}-\tfrac{1}{2}) F(\alpha_{12}^{\rho}-1,\ \alpha_{23}^{\rho}-1,\ \alpha_{34}^{B}-\tfrac{1}{2},\ \alpha_{45}^{\pi}-1,\ \alpha_{15}^{B}-\tfrac{1}{2}) ,$$

where the factor $\alpha_{34}^{B} - \frac{1}{2}$ is chosen to eliminate the double nucleon poles. This does not have leading behaviour in all channels; for example, it behaves like $s_{15}^{\alpha_{\rho}-1}$ when s_{15} and s_{23} are large and their ratio is constant. However, we again believe that it is essentially unique if we demand leading behaviour in as many channels as possible. Thus, we take for our amplitude:

$$A = \alpha_{12}^{\rho} F(\alpha_{12}^{\rho},\ \alpha_{23}^{\rho}-1,\ \alpha_{34}^{B}-\tfrac{1}{2},\ \alpha_{45}^{\pi},\ \alpha_{15}^{B}-\tfrac{3}{2}) +$$

$$+\, c(\alpha_{34}^{B}-\tfrac{1}{2}) \times \tag{3}$$

$$\times\, F(\alpha_{12}^{\rho}-1,\ \alpha_{23}^{\rho}-1,\ \alpha_{34}^{B}-\tfrac{1}{2},\ \alpha_{45}^{\pi}-1,\ \alpha_{15}^{B}-\tfrac{1}{2}) + \ldots ,$$

* We could (and in general should) add terms similar to those in (3) but with 4 and 5 interchanged, and multiplied by an arbitrary coefficient. Since we only consider the $J = 0^-$ state of the $N\bar{N}$ system the inclusion of such terms would not affect our results.

where c is a constant, and the terms not written come from non-cyclic reordering of the external particles of fig. 1. Note that we have not defined the overall normalisation in eq. (3); it is not needed for our purpose.

We must now evaluate A at threshold, i.e. $s_{45} = 4M^2$. At this point, assuming linear trajectories of universal slope, it is a good approximation to take $\alpha_{45}^{\pi} = 3(\approx \alpha' \cdot 4M^2)$ so that our amplitude is given by the residue of the pole at $\alpha_{45}^{\pi} = 3$. This immediately eliminates all terms coming from different reordering in fig. 1, as the only other reordering which contains the s_{45} poles are those having exotic mesons, which we assume do not exist.

After some algebra the result can be cast in the form of (1) with

$$\begin{aligned} C_{10} &= -3c(A^2+8A+15) - (A^3+9A^2+23A+15) \\ C_{11} &= 3c(A^3+8A^2+15A) \\ C_{20} &= 3c + 6A + 21 \\ C_{21} &= 9 - 3c(2A+3) \\ C_{22} &= 3c(2A+3) - 9 \\ C_{nm} &= 0 \text{ otherwise} \end{aligned} \tag{4}$$

where

$$A = -2\alpha' M^2 + 2\alpha^{\mathrm{B}}(0) - \alpha^{\rho}(0) - 1 , \tag{5}$$

α' being the universal trajectory slope ($\approx 1/2m_{\rho}^2$). At a qualitative level we note that the non-zero C_{nm} are precisely those required in the fit for the data [2] except for the absence of C_{30} and the undesired presence of a C_{22} term. This is not trivial since if $\alpha^{\pi}(4M^2)$ had been, say, 4 then a large number of undesirable terms, not having the hole at $\alpha_t = \alpha_s = 1.5$, would have arisen.

We can determine our one free parameter, c, by requiring that $C_{22} = 0$. This completely fixes all the coefficients. For example, taking the Δ trajectory for α^{B}, we get $c = 1.25$, $C_{10} = 1$ (this is just the arbitrary normalisation), $C_{11} = 1.80$, $C_{20} = 0.26$, $C_{21} = 0$. The agreement with the fit is remarkable. If instead of Δ we use the N trajectory we obtain essentially the same results. This is not because the result is independent of $\alpha^{\mathrm{B}}(0)$ - in fact it depends sensitively on $\alpha^{\mathrm{B}}(0)$ - it just happens that both the Δ and the N gives similar answers.

We consider the extent of our agreement, using only one free parameter, as good evidence for the validity of using the leading 5-point term. In principle, if we can eliminate the $\mathrm{N\bar{N}}$ vertex this theory completely determines the total annihilation rate in terms of $\pi\pi$ scattering length parameters. We hope to study this problem.

Finally we note that, on models of this type, because of daughter degeneracy, "mass extrapolation" does not have a unique meaning. For example, as noted above, a 0^-, $s = 4M^2 (\pi\pi)$ system is not equivalent to a 0^-, $s = 4M^2$ ($\mathrm{N\bar{N}}$) system!

Two of us (H. R. R. and E. J. S.) are grateful to the Rutherford Laboratory, where this work was completed, for hospitality; H. R. R. and M. C. would like to thank Z. Koba and M. Virasoro for discussions and E. J. S. would like to thank D. Fairlie similarly.

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* * * * *

MASS EXTRAPOLATION USING DUAL AMPLITUDES

ERRATA

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There was an algebraic error in the evaluation of the coefficient C_{nm}, and eq.4 of ref.[1] should be replaced by

$$C_{10} = -3C\,(A^2 + 8A + 15) - (2A^3 + 21A^2 + 70A + 75)$$

$$C_{11} = 3C\,(2A^3 + 17A^2 + 38A + 15) - (3A^2 + 24A + 45)$$

$$C_{20} = 3C + (6A + 21)$$

$$C_{21} = -3C\,(2A + 3) + 9$$

$$C_{22} = 3C\,(2A + 3) - 9$$

The value of C required to make $C_{22} = 0$ is not altered and with this value we obtain

$$\frac{C_{11}}{C_{10}} = 3.17, \quad \frac{C_{20}}{C_{10}} = 0.42 \text{ and } \frac{C_{21}}{C_{10}} = 0 \quad \text{for the } \Delta\text{-trajectory,}$$

$$\frac{C_{11}}{C_{10}} = 2.32, \quad \frac{C_{20}}{C_{10}} = 0.77 \text{ and } \frac{C_{21}}{C_{10}} = 0 \quad \text{for the N-trajectory.}$$

The basic conclusions are not altered, although for the case of the Δ-trajectory the quantitative agreement is not as good.

We are grateful to G.H.Thomas and J.Boguta for drawing our attention to this error.

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Normalisation of the $\bar{p}n \to 3\pi$ amplitude by mass extrapolation

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Abstract

We evaluate the $\bar{p}n \to 3\pi$ rate at rest from the πNN and $\rho\pi\pi$ coupling constants. The model is consistent with the experimental rate.

In a recent letter[1] we calculated the final state distribution in the process $\bar{p}n \to \pi^+ \pi^- \pi^-$ using the Bardakci-Ruegg generalisation of the Veneziano model. The results were satisfactory and showed that the model gives good agreement with the data in a local region of the variables. A more crucial test of the model, however, is whether it extrapolates correctly over a large range of the variables. A simple example of this is provided by a comparison of the normalisation of the $\bar{p}n \to 3\pi$ amplitude evaluated at the $\bar{p}n$ threshold, i.e. $s_{45} = 4\ M^2$ in the notation of ref.[1], with the normalisation at the π-pole, i.e. $s_{45} = m_\pi^2$, see fig. 1.

In ref.[1] we write for the non-spin-flip part of the amplitude:

$$M_{\bar{p}n \to 3\pi} = \beta(\bar{v}(p_4)\gamma_5\ u(p_5))\ B_5 \tag{1}$$

where β is a constant and B_5 a combination of two five-point Veneziano functions.* We fix β by evaluating the residue at the π and ρ, see fig. 1.

This gives

$$\beta = \alpha_\pi' \sqrt{2} \quad g_{NN\pi}\ f^2_{\rho\pi\pi} \tag{2}$$

where $g_{NN\pi}$ and $f_{\rho\pi\pi}$ are the usual coupling constants given by $g^2_{NN\pi}/4\pi \simeq 15$ and $f^2_{\rho\pi\pi}/4\pi \simeq 2$.

---

*Note that the spinor factors are essential in giving our results, which would be altered by a factor $(m_\pi^2/4M^2)$ if they were ignored. This suggests that treatments of baryon amplitudes which regard the baryons as spin zero particles are likely to be seriously in error.

We can now calculate $\sigma_{\bar{p}n \to 3\pi}$ from the expression

$$\left(\frac{v}{c}\right) \sigma_{\bar{p}n \to 3\pi} = \frac{M}{\omega_{\bar{p}}} \frac{M}{\omega_n} \int |M_{\bar{p}n \to 3\pi}|^2 \frac{d^3p_1}{2\omega_1(2\pi)^3} \frac{d^3p_2}{2\omega_2(2\pi)^3} \frac{d^3p_3}{2\omega_3(2\pi)^3}$$

$$(2\pi)^4 \quad \delta^4 \; (p_1 + p_2 + p_3 + p_4 + p_5) \tag{3}$$

where v is the relative velocity of the $\bar{p}$ and n, and where we write 3π for the $\pi^+ \pi^- \pi^-$ state. On reducing B_5 to a sum of four-point functions as in ref.[1] and doing the necessary integrals we find

$$\left(\frac{v}{c}\right) \sigma_{\bar{p}n \to 3\pi} \simeq \frac{0.24}{|\mathrm{Re}\ \alpha_\pi \ (4M^2) - 3|^2 + |\mathrm{Im}\ \alpha_\pi \ (4M^2)|^2} \ \mathrm{mb} \tag{4}$$

Suppose we ignore Im α_π and take a straight line for Re α_π, then we obtain Re α_π $(4M^2) - 3 = 0.12$ and

$$\frac{v}{c} \ \sigma_{\bar{p}n \to 3\pi} \simeq 17 \ \mathrm{mb} \tag{5}$$

To compare this number with experiment we note first that $\left(\frac{v}{c}\right) \sigma_{\bar{p}p}^{\mathrm{total}}$ is constant in the range 57 to 177 MeV incident energy[2], with a value of 42 mb. The $\bar{p}n$ total cross-section is about equal to this[3], so we can take $\left(\frac{v}{c}\right) \sigma_{\bar{p}n}^{\mathrm{Total}} \approx 42$ mb. The relative rates for $\bar{p}n$ annihilation to different final states are known at threshold[4], and the $\pi^+ \pi^- \pi^-$ final state is about $\frac{1}{4}$ of the total, so we estimate a value around 10 mb for $\frac{v}{c}\ \sigma_{\bar{p}n \to 3\pi}$. The agreement between this number of the calculated number in (5) is encouraging. It suggests in particular that Re α_π $(4M^2)$ must be close to 3 and that Im α_π $(4M^2)$ must not be large, i.e. ≤ 0.15. If we take a straight line for Im α_π,

$$\mathrm{Im}\ \alpha_\pi(s) = \lambda \ (s - gm_\pi^2) \ , \tag{6}$$

then this requires $\lambda \leq .04\ (\mathrm{GeV})^{-2}$. This is somewhat smaller than best fits to the leading Baryon trajectories which require $\lambda \approx 0.15$, but not appreciably so.

We have an independent method of calculating Im α from the decay of our 0^-, $s_{45} = 4M^2$ state. In fact, for the partial-width $0^- \to 3\pi$ we have, using the value of β given above,

$$\Gamma_{0^- \to 3\pi} \simeq 2.10^{-5} \left(\frac{g_{NN\pi}}{g_{NN0^-}}\right)^2 \ \mathrm{GeV} \tag{7}$$

(assuming that all the couplings of degenerate 0^- daughters to NN are equal) where 0^- refers to the 0^-, $s = 4M^2$ meson on the third daughter trajectory of the pion. Now Im α_π is proportional to the total width but from the known relative rates[4)] we can estimate

$$\begin{aligned}\Gamma_{0^-}^{\mathrm{Total}} &\simeq 10\ \Gamma_{0^- \to 3\pi} \\ &\simeq 2.10^{-4} \left(\frac{g_{N\bar{N}\pi}}{g_{NN0^-}}\right)^2 \ \mathrm{GeV}\end{aligned} \tag{8}$$

which corresponds to

$$\mathrm{Im}\ \alpha_\pi\ (4M^2) \approx 4.10^{-4} \left(\frac{g_{N\bar{N}\pi}}{g_{NN0^-}}\right)^2 \tag{9}$$

We have no means of knowing g_{NN0^-}, and indeed we recall that the 0^- particle at $s = 4M^2$ is in fact several degenerate mesons so this coupling is a particular and unknown linear combination of the couplings. However unless it is substantially less than the coupling of nucleons to pions, (9) is completely consistent with the limit on Im α_π given above.

To summarise, then, our model does appear to be internally consistent and to be compatible with the experimental normalisation. It predicts that the 0^- third daughter of the π at $\alpha_\pi = 3$ has a width less than 75 MeV, which is perhaps somewhat surprising.

Finally we consider the analogous problem for the $\pi N \to \pi N$ amplitude. As has been pointed out [5)6)] this has many defects. However the $p\bar{p} \to 2\pi$ annihilation rate, which here goes mainly through the third daughter of the ρ (we assume a ρ trajectory $\alpha_\rho(t) = \frac{1}{2} + 0.9t$), is given approximately correctly by the Veneziano amplitude normalised to the ρ Regge pole contribution at $t = 0$; obtained by the crossed process $\pi N \to \pi N$.

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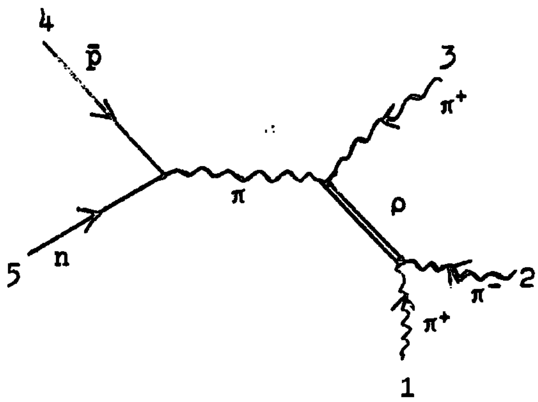

Fig. 1

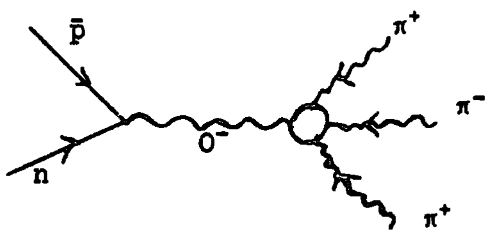

Fig. 2

Dual Amplitudes for 6 Pions with Epsilon Tensor Couplings*

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Abstract

We construct a dual amplitude for 6 pions which contains the normal parity trajectories ω-A_2 in the three-body channels and ρ-f^0 in the two-body ones. The leading trajectory is ghost-free and has no parity doubling.

Taking into account previous work, a complete solution of the construction of a 6 pion amplitude with all required physical properties in the tree approximation is now available.

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† Supported by a Royal Society Postdoctoral Fellowship.

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* This research has been supported in part by the Air Force Office of Scientific Research through the European Office of Aerospace Research, OAR, United States Air Force under Contract F-61052-68-C-0070.

In two recent papers [1], [2] a general method of constructing amplitudes which obey the restrictions required of physical trajectories, that is positive intercepts and absence of ghosts, was proposed. As has been shown in these papers, the properties of such amplitudes are basically different from those of the simplified models considered up to now [3], even in the properties of the leading trajectory. Hence the construction of these amplitudes is of considerable theoretical interest and they can also be used in phenomenological applications involving several particles in the final state.

A technique was developed which made it possible to construct an amplitude which described the 6 charged pion system, and included the ρ-f^0 trajectory in the two body channels and the π-A_1 abnormal parity trajectory in the three pion ones. Here we apply the method to build amplitudes which contain the normal parity ω-A_2 trajectory in one or more of the three body channels. At first glance the construction should follow trivially from the rules established in Ref. 1. However some interesting technical difficulties appear that make the calculations rather cumbersome.

The main difference from the results of Ref. 1 is that since B functions alone cannot provide antisymmetrical coupling of the momenta [4], we must introduce overall factors which couple the external momenta through epsilon tensors. In order to illustrate the procedure, denote the pion momenta by p_i, $i=1,\ldots,6$; and consider the 123 channel as the one in which the ω-A_2 trajectory must appear. Then the following expression has the desired properties:

$$A\ \varepsilon_{\mu\nu\rho\sigma}\, p_{1\mu} p_{2\nu} p_{3\rho}\ \varepsilon_{\alpha\beta\gamma\sigma} p_{4\alpha} p_{5\beta} p_{6\gamma}\ B\,(x_{12}, x_{123}, x_{56}, x_{23}, x_{234}, x_{61}, x_{34}, x_{345}, x_{45}) \tag{1}$$

where $\Lambda = \mathrm{Tr}\,(\tau_1\tau_2\tau_3\tau_4\tau_5\tau_6)$ is the Chan-Paton isospin factor made up of isospin Pauli matrices; $x_{ij} = m_{ij} - \alpha_{ij}$; $\alpha_{ij} = \frac{1}{2} + s_{ij}$; $s_{ij} = -(p_i + p_j)^2$; $x_{ijk} = m_{ijk} - \alpha_{ijk}$; $\alpha_{ijk} = a_{ijk} + s_{ijk}$; $s_{ijk} = -(p_i + p_j + p_k)^2$, α_{ij} being a ρ-f trajectory and α_{ijk} being a natural parity trajectory with intercept a_{ijk}. The m_{ij} and m_{ijk} are determined from the requirement of Regge behaviour in all channels, and signature and positivity for the leading trajectory, as we now explain.

In order to consider Regge behaviour we need the expansion of $\varepsilon_{\mu\nu\rho\sigma}\, p_{1\mu}p_{2\nu}p_{3\rho}\, \varepsilon_{\alpha\beta\gamma\sigma}p_{4\alpha}p_{5\beta}p_{6\sigma}$ in terms of the 9 invariants that appear in the B 's. To simplify writing we quote this in the case where we take $M_\pi = 0$ (this does not affect the argument):

$$
\begin{aligned}
8\varepsilon_{\mu\nu\rho\sigma}\, p_{1\mu}p_{2\nu}p_{3\rho}\, \varepsilon_{\alpha\beta\gamma\sigma}p_{4\alpha}p_{5\beta}p_{6\gamma} &\equiv 8\varepsilon_{123}\,\varepsilon_{456} = \\
& s_{123}^2(s_{16}-s_{234}-s_{345}+s_{34}) + s_{123}(s_{34}(s_{16}-s_{23}-s_{45}) + s_{16}(s_{34}-s_{56}-s_{12}) \\
& +s_{345}(s_{56}+s_{23}-s_{234}) + s_{234}(s_{12}+s_{45}-s_{345}) + s_{23}s_{56} + s_{12}s_{45}) \\
& +s_{16}s_{12}s_{56} + s_{34}s_{23}s_{45} + s_{234}s_{12}s_{45} + s_{345}s_{56}s_{23} \\
& -(s_{12}s_{23}(s_{45} + s_{56}) + s_{56}s_{45}(s_{23} + s_{12}))
\end{aligned}
\qquad (2)
$$

To illustrate the procedure we consider the multiperipheral diagram $61 \to 2345$ which must behave as $s^{\alpha_{123}}$ where

$$s_{34} \equiv s \to \infty \quad \text{and} \quad s_{16},\ s_{234},\ s_{345} \sim s \qquad (3)$$

From the B function we obtain $s^{\alpha_{123} - m_{123}}$ while the polynomial in eq.(2) gives a factor

$$2\, s_{123} (s_{34}\, s_{16} - s_{345}\, s_{234}) + 0(s) \qquad (4)$$

The first bracket would appear at first sight to be $0(s^2)$, which would be disastrous since it would require $m_{123} = 2$ and hence would not permit an ω or A_2 pole. However, the nine variables are not independent but are related through the condition that the Gram determinant vanishes, that is

$$\det D = 0 \qquad (5)$$

where the 5x5 matrix D is symmetric and the non-zero elements are [5]: $D_{12} = s_{12}$, $D_{13} = s_{123} - s_{23}$, $D_{14} = s_{23} + s_{56} - s_{123} - s_{234}$, $D_{15} = s_{234} - s_{16} - s_{56}$ $D_{23} = s_{23}$ $D_{24} = s_{234} - s_{23} - s_{34}$, $D_{25} = s_{34} + s_{16} - s_{345} - s_{234}$, $D_{33} = 2s_{23}$, $D_{34} = s_{234} - s_{23}$, $D_{35} = s_{16} - s_{234} - s_{45}$, $D_{45} = s_{45}$.

This condition, of course, reflects the four dimensional nature of space-time. Taking the limit (3) of the equation (5), since otherwise it cannot be solved, we obtain $s_{34}s_{16} - s_{345}s_{235} = 0(s)$ so that the second order term vanishes and we only require $m_{123} = 1$, allowing the ω and A_2 poles to exist.

Working in this way we see that, in order for the single term written in (1) to have leading Regge behaviour in all channels, we need

$$\begin{aligned} m_{12} = m_{23} = m_{45} = m_{56} &= 1 \\ m_{34} = m_{16} &= 2 \\ m_{123} &= 1 \\ m_{234} = m_{345} &= 2 \end{aligned} \qquad (6)$$

Unfortunately the resulting amplitude, when suitably symmetrized so as to have the usual $(\omega-A_2)$ trajectory, also has a trajectory with particles of (J^P,I) equal to $(2^+,0)$, $(3^-,1)$, $(4^+,0)$, etc., all of which are ghosts (i.e. have negative residue).

This trouble arises, for example, in the variable s_{234} from the $\varepsilon_{123}\,\varepsilon_{456}$ and $\varepsilon_{345}\,\varepsilon_{612}$ terms, and can therefore be easily removed, for the leading trajectory, by replacing $m_{234}=2$(and of course $m_{345}=2$) in eq. (6) by $m_{234}=3$ (and $m_{345}=3$). This has the effect that an individual term (e.g. (1)) no longer has leading Regge behaviour in all channels; the cyclically symmetric sum does of course still have leading behaviour. Here we have lowered these trajectories by the minimum amount needed. (One could have used m_{234} = any number greater than 3). This makes the individual terms as "dual" as possible.

We are thus led to write for our 6π amplitude :

$$g\,A\,\varepsilon_{\mu\nu\rho\sigma}\,p_{1\mu}p_{2\nu}p_{3\rho}\,\varepsilon_{\alpha\beta\gamma\sigma}p_{4\alpha}p_{5\beta}p_{6\gamma}$$

$$B\big(1-\alpha_{12},\,1-\alpha_{23},\,1-\alpha_{45},\,1-\alpha_{56},\,2-\alpha_{34},\,2-\alpha_{16},$$

$$1-\alpha_{123},\,3-\alpha_{234},\,3-\alpha_{345}\big)\quad+ \qquad (7)$$

cyclical and non-cyclical permutations.

The overall constant g may be determined, for example, from the $\omega\to3\pi$ decay rate. Notice that this amplitude reduces to the original Veneziano amplitude for $\omega\to3\pi$, thus ensuring bootstrap consistency.

It is now straightforward to check that the leading $G=-1$ trajectory coming from (7) has the $(J^P,\,I)$ assignment appropriate to the $(\omega,\,A_2)$ trajectory.

It is simple to deduce the $\pi\rho \to \pi\rho$ amplitude from (7). For example the $\rho^0\pi^\pm \to \rho^0\pi^\pm$ amplitude becomes:

$$T = \epsilon^\rho_\mu \bar{\epsilon}^\rho_\nu [g_{\mu\nu} A_1 - p_{3\mu} p_{4\nu} A_2 - p_{3\nu} p_{4\mu} A_3 + (p_{3\mu} p_{3\nu} + p_{4\mu} p_{4\nu}) A_4] \qquad (8)$$

where we have used the expression of Capella et al.[6] for the decomposition of the amplitude into invariants. The invariants A_i are

$$A_1 = \left(u^2 + 2su - u - \tfrac{1}{4}\right)\left\{ B(1-\alpha_u, 2-\alpha_t) - B(1-\alpha_u, 3-\alpha_s)\right\}$$
$$+ \left(s^2 + 2su - s - \tfrac{1}{4}\right)\left\{ B(1-\alpha_s, 2-\alpha_t) - B(1-\alpha_s, 3-\alpha_u)\right\}$$

$$A_2 = 2t\left\{ B(1-\alpha_s, 3-\alpha_u) - B(1-\alpha_s, 2-\alpha_t)\right\}$$
$$+ 4u\left\{ B(1-\alpha_u, 2-\alpha_t) - B(1-\alpha_u, 3-\alpha_s)\right\}$$

$$A_3 = 2t\left\{ B(1-\alpha_u, 3-\alpha_s) - B(1-\alpha_u, 2-\alpha_t)\right\}$$
$$+ 4s\left\{ B(1-\alpha_s, 2-\alpha_t) - B(1-\alpha_s, 3-\alpha_t)\right\}$$

$$A_4 = (2s-1)\left\{ B(1-\alpha_s, 3-\alpha_u) - B(1-\alpha_s, 2-\alpha_t)\right\}$$
$$+ (2u-1)\left\{ B(1-\alpha_u, 3-\alpha_s) - B(1-\alpha_u, 1-\alpha_t)\right\} \qquad (9)$$

Using the forms (9) and the connection with the helicity amplitudes given in Ref.[6], we can obtain the partial wave amplitudes. It is easily verified that to leading order in $\cos\theta_s$ $(= Z_s)$

$$t^{j-}_{11} = t^{j-}_{01} = t^{j-}_{00} = 0 \qquad (10)$$

ensuring the absence of parity doublets on the ω-A_2 trajectory. Since eq. (10) holds only for the leading power of $\cos\theta_s$, daughter levels may be parity degenerate. The residues at the poles of the helicity amplitude f^{+}_{11} are given by

$$\frac{2(j-2)\ (j-1)^2}{(j-1)!\ (2j-1)}\ [1+(-1)^j]\ \left[\frac{Z_s(j-1)^2}{2j-1}\right]^{j-1} \tag{11}$$

showing that our amplitudes have no ghosts along the leading trajectory.

Our 6π amplitude automatically has the required Adler zeros, and it follows that the same is true for the $\pi\rho \to \pi\rho$ amplitude obtained from it (eq. 9). A further important point is that it does not contain the π-A_1 trajectory, so the complete amplitude should consist of (7) plus a 6π amplitude containing the π-A_1 as its $G = -1$ trajectory. The latter problem has been solved in [1] and [2] for the charged pion case.

Either the 8-pion function may exhibit the degeneracy which is suppressed at the 6-pion level or it may induce nonleading terms in the 6-pion function through the bootstrap principle. For example the term:

$$g\,\Lambda\,\epsilon_{123}\,\epsilon_{456}\,(p_1-p_3)(p_4-p_6)\ B(2-\alpha_{12},\ 2-\alpha_{123},\ 2-\alpha_{56},\ 2-\alpha_{23},\ 4-\alpha_{234},\ 3-\alpha_{61},\ 3-\alpha_{34},\ 4-\alpha_{345},\ 1-\alpha_{45}) + \text{cyclical and non-cyclical permutations.} \tag{12}$$

doubles the A_2, leaves ω a simple pole and introduces no ghosts at the level of the leading trajectory.

J.D.D. thanks the Weizmann Institute for their hospitality. M.C. and E.J.S. thank David Fairlie for discussions.

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DNPL/P 44

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PREPRINT

NORMALISATION OF $\bar{p}p \rightarrow 4\pi$ USING A DUAL SIX-POINT FUNCTION

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To be submitted to Lett. Nuovo Cimento

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ABSTRACT

We evaluate the rate for the reaction $\bar{p}p \rightarrow 4\pi$ at rest using a six-point dual model for the singlet term, and the experimental value of 0.25 for the ratio of singlet to triplet cross-sections. The model is consistent with the experimental rate.

During the last two years there has been a great deal of interest in using the generalised Veneziano model[1] and comparing its results with experimental data. The first attempt was due to Lovelace[2] in his application of the four-point function to the process $\bar{p}p \to 3\pi$ at rest. Although this four-point function described some of experimental features, it was not able to predict the overall decay rate of the reaction. A later approach, due to Rubinstein et al.[3] used the five-point function, which, besides giving a good description of the distributions, also gave a reasonable value for the $\bar{p}n \to 3\pi$ decay rate.

Our approach to the process $\bar{p}p \to 4\pi$ at rest is in the same spirit as that of Rubinstein et al.[3], using a six-point function here as the starting point since we have one more pion. As is well known, the annihilation at rest of $\bar{p}p \to 4\pi$ proceeds from two states, the singlet and the triplet state. In either case, the initial state has the properties of an off-shell meson - the singlet mode behaves like "η" $\to 4\pi$, and the triplet mode like "ρ^0" $\to 4\pi$.

The amplitude for $\bar{p}p$ going to four charged pions, fig.1, can be written as

$$M = M_{\text{"}\eta\text{"}} + M_{\text{"}\rho\text{"}} + \text{other terms}$$

where

$$M_{\text{"}\eta\text{"}} = \beta \bar{v}(p_5)\, \gamma_5\, u(p_6)\, A_{\text{"}\eta\text{"}}$$

$$M_{\text{"}\rho\text{"}} = \beta'\, \bar{v}(p_5)\, \gamma_\mu\, u(p_6)\, A^\mu_{\text{"}\rho\text{"}} \qquad \ldots(1)$$

and at rest ($s_{56} = (p_5 + p_6)^2 = 4M^2$) only these two terms are non-zero. From the spin-parity (0^-) of the "η" we write

$$A_{\text{"}\eta\text{"}} = \sum_{\substack{1\leftrightarrow 3\\ 2\leftrightarrow 4}} \varepsilon_{\mu\nu\rho\sigma}\, p_1^{\ \mu} p_2^{\ \nu} p_3^{\ \rho} p_4^{\ \sigma}\, B_6(x_{12}, x_{23}, x_{34}, x_{45}, x_{56}, x_{61};\ x_{123}, x_{234}, x_{345})$$

$$= \varepsilon(1234)\,\{B_6 - B_6(1\leftrightarrow 3) - B_6(2\leftrightarrow 4) + B_6(1\leftrightarrow 3,\ 2\leftrightarrow 4)\} \qquad \ldots(2)$$

where the arguments are given by

$$x_{\pi^+\pi^-} = 1 - \alpha_\rho (s_{\pi^+\pi^-})$$
$$x_{3\pi} = 1 - \alpha_{A_2}(s_{3\pi})$$
$$x_B = \tfrac{3}{2} - \alpha_\Delta (s_{B\pi})$$
$$x_{56} = - \alpha_\eta (s_{56})$$

and the trajectories by

$$\alpha_\rho(s) = 0.48 + \alpha' s + i\ 0.20\ (s-4m_\pi^2)^{\frac{1}{2}}$$
$$\alpha_{A2}(s) = 0.48 + \alpha' s + i\ 0.094\ (s-9m_\pi^2)^{\frac{1}{2}}$$
$$\alpha_\Delta(s) = 0.14 + \alpha' s$$

with the common slope α' equal to 0.89 GeV^{-2}, and the imaginary parts zero below threshold. The anti-symmetry of $A_{"\eta"}$ under the interchange of particles $1\leftrightarrow3$ and of $2\leftrightarrow4$ decouples all the isospin zero particles on the degenerate ω-A_2 trajectory in the three pion channels, while the epsilon factor decouples the nucleon pole in the baryon channels. The triplet contribution can be written as

$$A_{"\rho"}^{\mu} = p_1^{\mu}\ C + p_2^{\mu}\ D + p_3^{\mu}\ E \qquad \ldots(3)$$

where C, D, E are the invariant amplitudes and can be identified with the appropriate six-point functions.

Now let us evaluate the contribution of the singlet term $M_{"\eta"}$, so that we can find the normalisation constant β. One of the important ideas in the Multi-Veneziano model is what Chan[(4)] calls bootstrap consistency, meaning that taking the residue of an amplitude at some pole gives exactly the amplitude for the reduced process, including the normalisation. Therefore we can evaluate the constant β by going to any values of the internal variables s_{ij} at which the amplitude is known. Thus, going to the poles indicated in fig.2., we obtain the relation

$$\beta = g_{\eta p\bar{p}}\ g_{A_2^+\eta\pi^-}\ g_{A_2^+\ \rho^0\pi^+}\ g_{\rho\pi\pi}\ \alpha'^2 \qquad \ldots(4)$$

The coupling constant $g_{\eta N\bar{N}}$ is known experimentally to be[5]

$$g^2_{\eta N\bar{N}} = 0.18\, g^2_{\pi N\bar{N}}$$

and $g_{A_2\eta\pi}$ and $g_{A_2\rho\pi}$ can be calculated from the corresponding branching ratios of about 12% and 85% of the total rate[6] respectively. We take the average total width of the A_2 to be[6] 90 MeV. The contribution of the singlet mode to the cross-section is given by:

$$\left[\left(\frac{v}{c}\right)\sigma_{\bar{p}p\to 4\pi}\right]_{singlet} = \frac{M}{\omega_p}\,\frac{M}{\omega_{\bar{p}}}\int_{R_4} |M_{''\eta''}|^2 .\ (2\pi)^4\delta^4(P-\Sigma p_i)\prod_{i=1,4}\frac{d^3p_i}{2\omega_i(2\pi)^3} \quad \ldots(5)$$

where $\omega_i^2 = p_i^2 + m_\pi^2$ and v is the relative velocity of the p and $\bar{p}$.

We notice that taking the common trajectory slope, $\alpha' = 0.89\ \text{GeV}^{-2}$, gives at the $\bar{p}p$ threshold $\alpha^{\eta}_{56}(4M^2)$ very close to 3, therefore to carry out the integration over phase space we can make the pole approximation for $M_{''\eta''}$ at $x_{56} = -3$, assuming also that Im α^{η}_{56} is sufficiently small (which is a posteriori justified, see below). Then the integration in eqn.(5) over a B_6 function (which is technically very difficult) is reduced to an integration over a five-point function, which can be done numerically.

For the triplet mode contribution we notice that $\alpha^{\rho}_{56}(4M^2)$ is not sufficiently close to an integer to justify using a pole approximation similar to that of the singlet. Also, since there is more than one invariant amplitude in eqn.(3) the relative residues appear as free parameters. (In principle a "ρ" $\to 4\pi$ amplitude could be found from a six-pion amplitude, but even then the computational problem remains). Therefore we prefer to use the experimental ratio[5]

$$\frac{\sigma_{singlet}}{\sigma_{triplet}} = 0.25$$

Therefore

$$\left(\frac{v}{c}\right)\sigma_{\bar{p}p\to 4\pi} = \left(\frac{v}{c}\right)\left[\sigma^{singlet}_{\bar{p}p\to 4\pi} + \sigma^{triplet}_{\bar{p}p\to 4\pi}\right] = 5\left(\frac{v}{c}\right)\sigma^{singlet}_{\bar{p}p\to 4\pi} \quad \ldots(6)$$

Performing the phase space integral over the singlet amplitude gives

$$\int_{R_4} |A_{"\eta"}|^2 \ (2\pi)^4 \ \delta^4(P-\Sigma p_i) \ \prod_{i=1,4} \frac{d^3p_i}{2\omega_i (2\pi)^3} = 0.179.10^{-7}$$

Substituting everything into eqn.(5), we get at threshold,

$$\left(\frac{v}{c}\right) \sigma_{\bar{p}p\to 4\pi} = \frac{0.5}{(\mathrm{Re}\alpha^{\eta}_{56}(4M^2) - 3)^2 + (\mathrm{Im}\ \alpha^{\eta}_{56}(4M^2))^2} \text{ mb} \qquad \ldots(7)$$

Now we observe that the experimental value of $(v/c)(\sigma_{\bar{p}p\to 4\pi})$ is approximately constant in the range 57 - 177 MeV incident energy[7] and equal to 20mb. Taking $\mathrm{Im}\ \alpha^{\eta}(4M^2)$ equal to zero, and $\mathrm{Re}\ \alpha^{\eta}(4M^2) - 3$ equal to -0.13 we get

$$\left(\frac{v}{c}\right) \sigma_{\bar{p}p\to 4\pi} = 29\text{mb}$$

This shows that within this model taking $\mathrm{Re}\ \alpha^{\eta}(4M^2)$ approximately 3 and $\mathrm{Im}\ \alpha^{\eta}(4M^2)$ near zero is compatible with the experimental value.

Giving the imaginary part of the trajectory a value of 0.08 we recover from eqn.(7) the experimental cross-section of 20mb. However there exists an independent method of calculating $\mathrm{Im}\ \alpha^{\eta}(4M^2)$; from the decay width of "η" → 4π. In fact, by extracting the residue from eqn.(7) and factorising the "η"$p\bar{p}$ vertex we get

$$\Gamma_{"\eta"\to 4\pi} = 1.5 \ 10^{-2} \left(\frac{g_{\eta N\bar{N}}}{g_{"\eta"N\bar{N}}} \right)^2 \qquad \ldots(8)$$

Here the couplings of all the degenerate daughters of "η"→4π have been assumed equal. Since $\mathrm{Im}\ \alpha(4M^2) = \alpha' \ 2M \ \Gamma^{total}_{"\eta"}$, and since

$$\Gamma^{total}_{"\eta"} = \Gamma_{"\eta"\to 4\pi} + \Gamma_{"\eta"\to 6\pi} + \ldots \simeq \Gamma_{"\eta"\to 4\pi}$$

we get for the imaginary part,

$$\mathrm{Im}\ \alpha_{\eta}(4M^2) \simeq 2.4 \ . \ 10^{-2} \left(\frac{g_{\eta N\bar{N}}}{g_{"\eta"N\bar{N}}} \right)^2 \qquad \ldots(9)$$

We do not know the coupling $g_{"\eta"N\bar{N}}$, but providing it is rather less than $g_{\eta N\bar{N}}$ this value for the impaginary part is consistent with the value given above.

We should not expect the singlet state, without the triplet, as treated here, to give perfect agreement with all possible distributions, and in fact the two-pion mass distributions, fig.3, are not too good. However, as shown by Hopkinson and Roberts(8), adding one satellite term of a particular type can give much better agreement(9). Here we have been mainly interested in the problem of normalisation.

We wish to thank Prof. E. J. Squires for many valuable comments.

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9. One of the satellite terms added in ref.(8) has the wrong parity, due to incorrectly considering the $\bar{p}p$ pair as a scalar-pseudo-scalar in the singlet state.

FIGURE CAPTIONS

Fig. 1. Diagram for the reaction $\bar{p}p \to 4\pi$.

Fig. 2. The pole at which the normalisation β is determined.

Fig. 3. The two pion mass distributions, with arbitrary normalisation. Experimental distributions from J. Diaz et. al.[5].

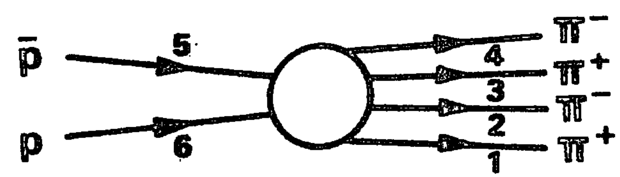


Fig. 1

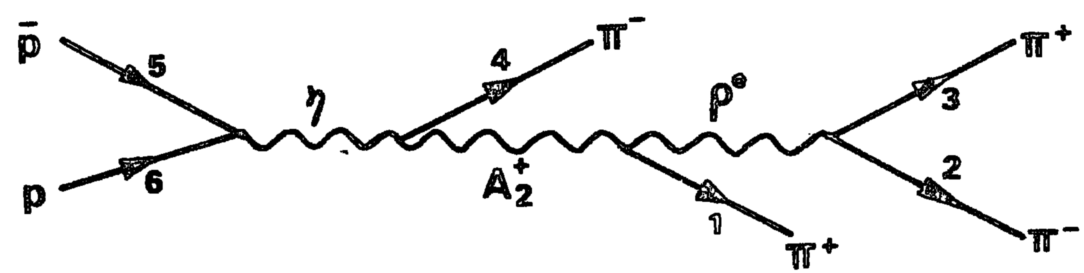
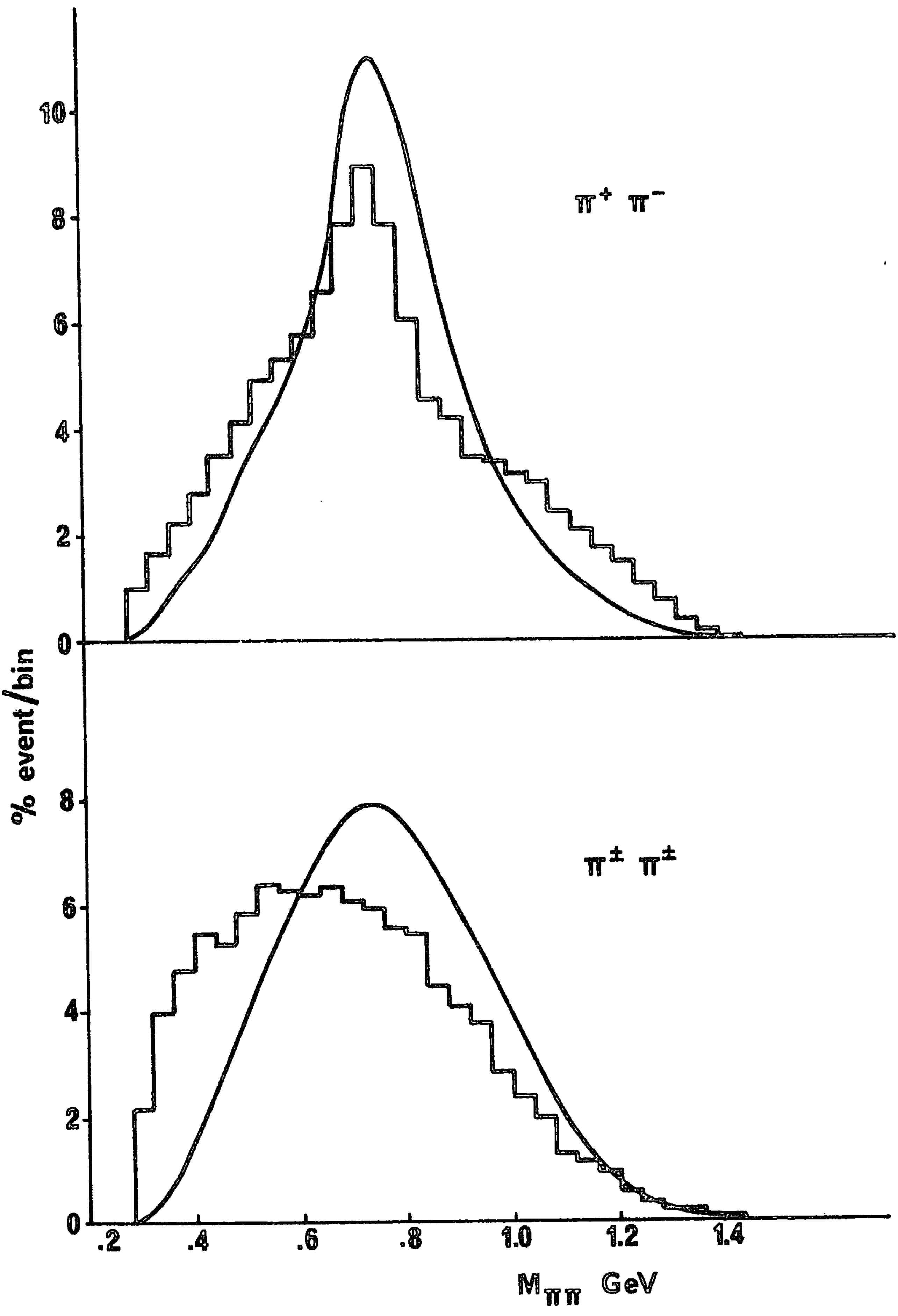


Fig.2



10
8
6
4
2
0
8
6
4
2
0
% event/bin
$\pi^+ \pi^-$
$\pi^\pm \pi^\pm$
.2
.4
.6
.8
1.0
1.2
1.4
$M_{\pi\pi}$ GeV

Fig. 3

Appendix B

SCALING IN THE DUAL RESONANCE MODEL

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Abstract

From a dual amplitude with currents included by the minimal gauge interaction we obtain, to second order in the strong coupling, scaling for νW_2, whilst W_1 vanishes in the scaling limit.

The Bjorken scaling limit[1] has initiated the study of a series of field theoretical considerations[2)-6)]. A common feature for all these models to obtain scaling is to impose a cutoff on the transverse momentum. When no cutoff is used scaling breaks down. An exception is the case in [2], but this is because $\lambda\phi^3$ theory has a good convergence property. The parton model[7] also has a transverse momentum cutoff as its ingredient. Another approach[8] based on a dual resonance model point of view for the parton theory has been used for the case of deep inelastic region. Here too the normal mode expansion for the hadronic string is cutoff from the start. There exists, however, no theoretical background whatsoever for justifying this fundamental assumption of a cutoff.

In this note we study the Bjorken limit of a two currents dual amplitude where the currents are included in the dual resonance model according to the minimal gauge interactions prescribed by Kikkawa and Sato[9) and 10)]. We find that to second order in strong interaction coupling constant the structure function νW_2 scales : $\nu W_2 \to F_2(\omega)$, while W_1 vanishes : $W_1 \to (k^2)^{-1} F_1(\omega)$. No cutoff has been imposed but the exponential divergence coming up in the dual loop diagrams has been removed by the renormalization procedure of Neveu and Scherk[11]. At this stage one would like to speculate and conjecture that the strong divergence appearing in such dual models and its consequent renormalization effectively replaces the cutoff needed in the conventional field theoretical treatments in a natural way.

The reggeized Feynman rules for the current(s) interacting with the usual dual vertices and propagators are given in [10] in the operator formalism language.

We calculate the five diagrams (planar) shown in Fig. 1 with the following notations

$$M_{\mu\upsilon}(\upsilon,\ t;\ k_1^2, k_2^2) = P_\mu P_\upsilon A_1 + \ldots + g_{\mu\upsilon} A_{10}\ ,$$

$$W_1 = \frac{1}{\pi}\ \mathrm{Im}\ A_{10}\ (\upsilon,\ k_1 = k_2 = k),$$

$$W_2 = \frac{m^2}{\pi}\ \mathrm{Im}\ A_1\ (\upsilon,\ k_1 = k_2 = k), \qquad (1)$$

$$t = (k_1 - k_2)^2, \quad \upsilon = \tfrac{1}{4}(s-u), \quad P = \tfrac{1}{2}(p_1 + p_2), \quad p_1^2 = p_2^2 = m^2$$

$$\alpha(s) = \tfrac{1}{2}s + \alpha_0\ , \quad \alpha_0 < 0, \quad \alpha(m^2) = 0.$$

The contribution of diagram 1(a) after renormalization is [10]

$$M_{\mu\upsilon}^{(a)} = e^2 g^2 \int d^4Q\ \exp(-\tfrac{1}{2}\ \ell n\, w\, Q^2) \int_0^1 dxdydzw^{-\alpha_0 - 1}(1-u)^{\alpha_0 - 1}(1-v)^{\alpha_0 - 1}(1-w)$$

$$\left(\prod_{n=1}^{\infty} \frac{1}{1-w^n}\right)^4 \left\{[\psi(u,w)]^{m^2} - [\tilde{\psi}(u,w)]^{m^2}\right\}$$

$$\exp\left\{\frac{t}{2}\left[-\frac{\ell nx\ \ell nz}{\ell nw} + \frac{\ell ny\ \ell nu}{2\,\ell nw} + \sum \frac{v^n + u^n}{n(1-w^n)}\right]\right\}$$

$$\exp\left\{-\upsilon\frac{\ell ny\ \ell nu}{\ell nw} - \frac{k_1^2 + k_2}{2}\ \frac{\ell ny\ \ell nx\ zu}{2\,\ell nw} + \frac{k_1^2 - k_2^2}{2}\ \ell ny\ \frac{\ell nx^2 u - \ell nxzu}{2\,\ell nw}\right\}$$

$$\left\{\left(\frac{\ell nu}{\ell nw}\right)^2 P_\mu P_\upsilon + \ldots + \frac{1}{\ell nw}\ g_{\mu\upsilon}\right\}\ , \qquad (2)$$

where $v = xyz$, $w = xyzu$

$$\psi(u,w) = \exp\left[-\frac{\ell nu\ \ell n\frac{w}{u}}{2\,\ell nw} + \sum_{n=1}^{\infty} \frac{2w^n - u^n - (\frac{w}{u})^n}{n(1-w^n)}\right]$$

$$\tilde{\Psi}(u,w) = -\frac{\ell nw}{\pi}\ \sin\left(\pi\frac{\ell nu}{\ell nw}\right) .$$

For $\nu \to -\infty$, t = fixed behaviour of invariant amplitudes in (2) we use Mellin transform and find for A_1 amplitude a singularity at $\beta = -1$ (coming from $y \approx 1$), and $\beta = \alpha_t - 2$ (coming from $u \approx 1$). Hence

$$A_1^{(a)} \xrightarrow[\nu \to -\infty]{} \frac{eF(t)}{\nu} + \beta(t)\nu^{\alpha_t - 2}$$

The residue of the fixed pole at $J = 1$, i.e. $F(t)$ coincides with the expression for the form factor corresponding to Fig. 2 and hence Fubini-Dashen-Gell-Mann sum-rule is satisfied. Analogously $A_{10}^{(a)}$ amplitude has $\beta = -1$ and $\beta = \alpha_t$ singularities corresponding to a fixed $J = -1$ pole and the usual moving pole.

For the scaling limit we do a Mellin transform with respect to $(-k^2)$, keeping $\frac{\nu}{-k^2} = \omega$ constant. The rightmost singularity for both A_1 and A_{10} is at $\beta = -1$ and therefore we get

$$\begin{aligned} \nu W_2(\nu, k^2) &\longrightarrow F_2(\omega) \\ W_1(\nu, k^2) &\xrightarrow[\substack{k^2 \to -\infty \\ \frac{\nu}{-k^2} = \omega \text{ fixed}}]{} (k^2)^{-1} F_1(\omega) \end{aligned} \quad , \qquad (3)$$

where

$$F_2(\omega) = e^2 g^2 m^2 \omega \int d^4Q \, \exp(-\tfrac{1}{2} \ln w Q^2) \int_0^1 dx dz du \, w^{-\alpha_0 - 1} (1-u)^{\alpha_0 - 1} \left(\prod \frac{1}{1-w^n} \right)^4$$

$$\left\{ [\psi(u,w)]^{m^2} - [\tilde{\psi}(u,w)]^{m^2} \right\}$$

$$\left(\frac{\ln u}{\ln w} \right)^2 \delta \left(\omega \frac{\ln u}{\ln w} - \frac{\ln xzu}{2 \ln w} \right) , \qquad w = xzu \qquad (4)$$

$$F_1(\omega) = e^2 g^2 \int \; \ldots\ldots \left(\frac{1}{\ln w} \right) \delta \left(\omega \frac{\ln u}{\ln w} - \frac{\ln xzu}{2 \ln w} \right) .$$

There seems to be a similarity with the parton model[7] if

$x = \frac{1}{2\omega} = \frac{\ell nu}{\ell nxzu}$ is interpreted as the fraction of longitudinal momentum carried by a parton in an infinite momentum frame. $F_2(\omega)$ and $F_1(\omega)$ have a high ω behaviour of $\omega^{\alpha_0 - 1}$ and ω^{α_0} respectively.

Proceeding further we find the contribution from diagram 1(b) to give

$$A_{10}^{(b)} \equiv 0,$$

$$A_1^{(b)} \underset{\upsilon\to-\infty}{\sim} \upsilon^{-2} + \text{Regge terms.}$$

Since $\upsilon A_1^{(b)} \underset{\upsilon\to-\infty}{\longrightarrow} 0$, there exists no form factor corresponding to this amplitude. For inelastics structure functions we get

$$W_1^{(b)} \equiv 0 \tag{5}$$

$$\upsilon W_2^{(b)} \underset{\substack{k^2\to-\infty \\ \omega \text{ fixed}}}{\longrightarrow} (k^2)^{-1} \text{ x a function of } \omega + (k^2)^{2\alpha_0 - 1} \text{ x a function of } \omega + \text{lower terms.}$$

Diagram 1(c) gives the contribution

$$W_1^{(c)} = (k^2)^{\alpha_0 - 2}\left[\ell n \frac{k^2}{2}\right]^{\alpha_0 - 1} \text{ x a function of } \omega + (k^2)^{2\alpha_0 - 2}\left[\ell n \frac{k^2}{2}\right]^{2\alpha_0 - 3} \text{ x a function of } \omega + \text{lower terms}$$

$$\upsilon W_2^{(c)} = (k^2)^{\alpha_0 - 1}\left[\ell n \frac{k^2}{2}\right]^{\alpha_0} \text{ x a function of } \omega + (k^2)^{2\alpha_0 - 1}\left[\ell n \frac{k^2}{2}\right]^{2\alpha_0 - 2} \text{ x a function of } \omega + \text{lower terms.} \tag{6}$$

From 1(d) and 1(e) we get the contribution

$$W_1^{(d)} = (k^2)^{-2} \text{ x a function of } \omega + (k^2)^{2\alpha_0 - 2} \text{ x function of } \omega + \text{lower terms}$$

$$\upsilon W_2^{(d)} = (k^2)^{-1} \text{ x a function of } \omega + (k^2)^{2\alpha_0 - 1} \text{ x a function of } \omega + \text{lower terms} \tag{7}$$

and

$$W_1^{(e)} \equiv 0, \qquad \nu W_2^{(e)} \equiv 0.$$

In the light of the analysis of Bloom and Gilman[12] which indicates that the resonant component of the structure function νW_2 does show the scaling property the result of the present note seems interesting[13].

The question of whether the sum of higher order diagrams may restore scaling for W_1 or even spoil the scaling of νW_2 in the present model, certainly needs further study.

I wish to thank E.J.Squires for many helpful discussions.

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13. One would like to ask what could happen in the case of the intercept of the highest Regge trajectory to be $\approx \frac{1}{2}$, ignoring the problem of ghosts ($m^2 < 0$). Even in this case by putting $\alpha_0 = \frac{1}{2}$ in the expressions (3) - (7) one still does not get scaling for W_1 but its vanishing as $(k^2)^{-1} f(\omega)$.

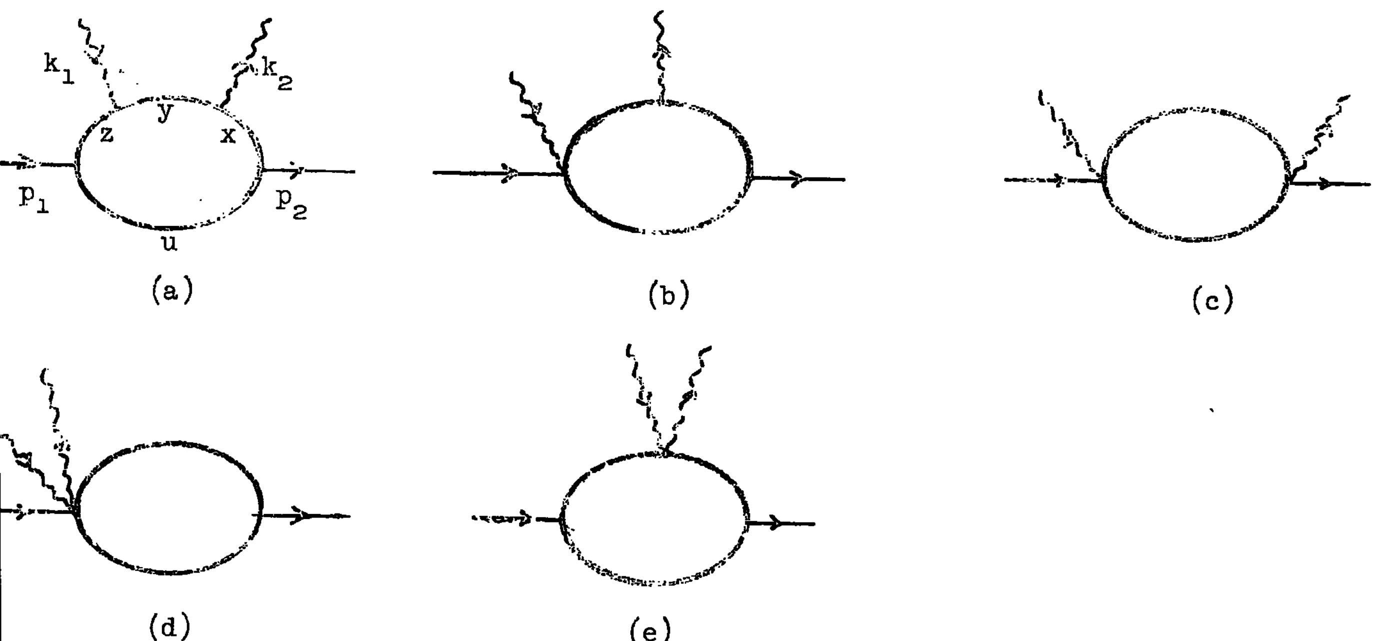

Figure 1. The set of planar diagrams used for the study of Bjorken scaling limit.

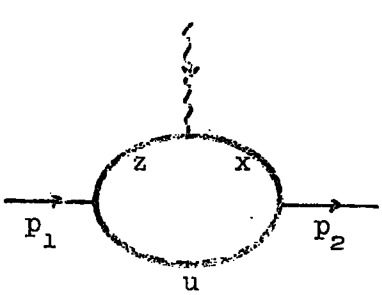

Figure 2. Form factor corresponding to diagram 1(a) through Fubini-Dashen - Gell-Mann sum rule.

BJORKEN SCALING FOR INCLUSIVE AND QUASI - INCLUSIVE PROCESSES IN THE DUAL RESONANCE MODEL

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ABSTRACT

From a dual resonance model with currents included through the minimal gauge interaction the deep inelastic electron scattering is considered. It gives the Bjorken scaling for νW_2 while W_1 vanishes, a property which is common among the models where the current is coupled to bosons. Scaling occurs because of existence of current algebra fixed pole. Deep inelastic electron scattering with one detected final state particle is also considered which following Mueller is connected with the discontinuity of a six-point amplitude. In a special kinematic region, three out of four structure functions scale because of a fixed pole, while outside this region the fixed pole cannot be responsible for scaling anymore. Then a speculation of Pomeron assignment does show the scaling. Inelastic Compton scattering is also considered which in the parton model of Bjorken and Paschos scales and is proportional to νW_2. This property is satisfied in the present model. Electron - positron annihilation into hadrons is considered without renormalization whose cross section falls off as $s^{-\frac{5}{2}}$. It is suggested that a proper dual renormalization for self-mass diagram of photon may change this result.

I. INTRODUCTION

There exists the possibility that the dual resonance model (DRM)[1,2] may finally provide us with a theory of hadronic processes. The unitarization program to treat the model as a Born term[3] puts it on the same footing as a field theory expansion. Further, besides some quantitative agreements of the model with the data, it reproduces some qualitative features of the hadronic inclusive reactions[4,5] such as Feynman scaling law[6] and the pionization, the limiting fragmentation[7] and the small transverse momentum of the produced particles etc.[8]. However, notice that these limiting distributions are obtained when one puts the intercept of the relevant Regge trajectory α_0 equal to one, i.e. Pomeron is exchanged, while in the case of usual Regge trajectories with $\alpha_0 \neq 1$ exchanged one gets scaling (generalized) only for the ratio of differential cross section to total cross section[5,8]. The above-mentioned successes of DRM in purely hadronic exclusive and inclusive reactions are certainly interested both theoretically and phenomenologically.

In the processes where currents are involved there exists a "similar" kind of scaling behaviour, namely the one originally predicted by Bjorken[9] for the deep inelastic electroproduction structure functions for inclusive reactions and its generalization for quasi-inclusive ones.

Besides the parton model[10] which has a transverse momentum cut off as its ingredient the only field - theoretical model which gives the scaling of at least the structure function νW_2 is the sum of ladder graphs in $\lambda\varphi^3$ - theory[11]. But this is because $\lambda\varphi^3$ - model is a superrenormalizable theory and has good convergence properties. All the other models[12] including the field-theoretical treatment to parton model[13] also need a transverse momentum cutoff in order to obtain scaling. When no cutoff is imposed Bjorken scaling breaks down. Even the sum of an infinite set of renormalized graphs in field theory[14] does not possess scaling. Another approach[15] based on a DRM point of view for the parton model has been used for the case of deep inelastic region. Here too, the normal mode expansion for the hadronic string is cut off from the start. There exists, however, no theoreticalbackground whatsoever for justifying the crucial assumption of a cutoff in the above-mentioned approaches.

From the other side, it is known that the DRM has a cutoff of the exponential type in the transverse momentum[4,8) seealso 16)]. This fact already gives a hint that a current amplitude which has the same property as the DRM for its strong part may have a good chance to give scaling; in this case the analysis of Bloom and Gilman[17)] which indicates that the resonant component of the structure functions does show the scaling property could be most naturally understood.

A vast number of prescriptions on how to include the currents in DRM has been proposed each having its own shortcomings. Among them we intend to use the one which has the least freedom. One of such models is the prescription of including the currents , as the minimal gauge interactions $\partial_\mu \to \partial_\mu - ie A_\mu$ proposed by Kikkawa and Sato[18)]. Once the minimal gauge prescription is accepted, in principal everything is fixed and there is no freedom left. The only ambiguity is the one due to the renormalization of dual loops[19)]. Nevertheless, if one takes the model seriously then it may restrict the ambiguities in the dual loop renormalization. Throughout the present note we renormalize *a la* Neveu and Scherk[20)].

One of the shortcomings of the present model used here is that its Born term has only one single pole, say in the s-channel and therefore has no duality property between s - and t - channels [see e.g.21)]. But since we are interested in inclusive reactions and therefore in the discontinuites of the graphs we shall ignore the above shortcoming since those graphs simply don't give contribution to the discontinuities and therefore the whole treatment of the present not is dual in this sense.

The DRM with currents is the next step towards the construction of a theoretical frame where one e.g. using Mueller's analysis[22)] can study all the relevant limits in the same manner as in 4,5,8) for purely hadronic reactions, investigate the existence or absence of fixed poles in the amplitudes with currents and in case such amplitudes satisfy the Bjorken scaling, to find the "dynamical" origin of it. It comes out, for instance, that such amplitudes satisfy Bjorken scaling and that the existence of current algebra fixed pole for the amplitudes with two currents is responsible for this scaling (see a similar situation in 12a)). This is contrary to a "similar" situation in purely hadronic processes where only Pomeranchuk exchange is responsible for Feynman scaling law and the limiting distributions 4,5,8), while the usual Regge trajectories give vanishing contributions in these limits. This fixed pole responsibility for Bjorken scaling in the language of light cone expansion would probably mean that the Regge trajectories have nothingto do with the degree of singularity on the light cone[23].

In Sec. II we consider the two structure functions W_1 and νW_2 of inelastic electronscattering. In the model νW_2 scales while W_1 vanishes. This is a property of all the other models where currents are coupled to spin zero particles. When a proper DRM for fermions is constructed we suggest the same minimal gauge interaction which now would couple the current to the tower of fermions will restore the scaling for W_1 .

In Sec. III we consider the inelastic electron scattering where a final state hadron with momentum p' is detected, i.e.

$$e + \text{hadron } (p) \longrightarrow e' + \text{Hadron } (p') + \text{anything.}$$

Following Mueller we connect this process to the discontinuity of a forward six-point function which we then study. It appears that here too in the Bjorken limit a fixed pole is responsible for the scaling behaviour of three out of the four structure functions νW_2, νW_3, νW_4 and the vanishing of W_1, reminiscent of the same situation in Sec.II which was suggested to be due to the coupling of currents to tower of bosons rather than to the tower of fermions. Notice, however, that this fixed pole can be responsible for the above scaling only in a special kinematical region of $p \cdot p'$ = fixed and not large, i.e. when the detected hadron is very near to the forward direction in the centre of mass system or is slowly moving in the lab. system. Beyond this kinematical region, i.e. where $p \cdot p'$ is large the fixed pole can not be responsible for the scaling anymore. In this case, only with the speculation of assigning the Regge trajectory to a Pomeron with the intercept $\alpha_0 = 1$ one gets the above scaling. This is very similar to the purely hadronic case [4,5] where limiting distributions are obtained by putting $\alpha_0 = 1$.

Sec. IV is to study the inelastic Compton scattering

photon (k) + hadron $(p) \longrightarrow$ photon (k') + anything.

This reaction is interesting since from the parton model of Bjorken and Paschos[10] there should be a similar scaling law for the structure functions where the scaling variable now is $\frac{(k-k')p}{k \cdot k'}$ and in addition from the same parton model one concludes that the present reaction should be proportional to the electron scattering of Sec. II, for partons of unit charge and spin 0 or $\frac{1}{2}$. Both these two results are also valid within the model of present paper which may suggest a deeper analogy with the parton model.

Finally, Sec. V is devoted to the study of high energy behaviour of electron-positron total annihilation into hadrons. It comes out that this cross section falls off like $s^{-\frac{5}{2}}$. This result is incompatible with the results of other models. In evaluating the above high energy behaviour we have not renormalized the amplitude for the self-energy of virtual photon which has the exponential divergence of the dual loops. It may happen that a dual renormaliztion changes the above high energy behaviour. We hope to study this question further.

Throughout the paper only one leading diagram for each process is written down and discussed and dots mean non leading diagrams.

II. DEEP INELASTIC ELECTRON SCATTERING

Consider the virutal Compton scattering averaged and summed over the spins of hadrons corresponding to Fig.1.

$$k_1 + p_1 \longrightarrow k_2 + p_2 \quad . \qquad (2.1)$$

The notation is the usual one [24]

$$\begin{aligned}
&M_{\mu\nu}(\nu, t; k_1^2, k_2^2) = P_\mu P_\nu A_1 + \cdots + g_{\mu\nu} A_{10} \\
&W_1 = \frac{1}{\pi} \, Im \, A_{10} \, (\nu, k_1 = k_2 = k) \\
&W_2 = \frac{m^2}{\pi} \, Im \, A_1 \, (\nu, k_1 = k_2 = k) \qquad (2.2) \\
&t = (k_1 - k_2)^2, \ \nu = \frac{1}{4}(s-u), \ P = \frac{1}{2}(p_1 + p_2), \ p_1^2 = p_2^2 = m^2 \\
&\alpha(s) = \alpha' s + \alpha_0 \ , \ \alpha_0 < 0 \ , \ \alpha(m^2) = 0 \ , \ \alpha' = \frac{1}{2} \, .
\end{aligned}$$

The contribution of the last diagram of Fig.1. after the dual loop renormalization a la Neveu and Scherk [20] is

$$\begin{aligned}
M_{\mu\nu} = {} & e^2 g^2 \int d^4 Q \exp\left(-\frac{1}{2}\ln W Q^2\right) \int_0^1 dx\,dy\,dz \, W^{-\alpha_0 - 1}(1-u)^{\alpha_0 - 1}(1-V)^{\alpha_0 - 1}(1-W) \\
& [f(W)]^{-4} \left\{ [\psi(u,W)]^{m^2} - [\tilde{\varphi}(u,W)]^{m^2} \right\} \\
& \exp\left\{ \frac{t}{2}\left[\frac{-\ln x \ln z}{\ln W} + \frac{\ln y \ln u}{2 \ln W} + \sum_{n=1}^{\infty} \frac{V^n + u^n}{n(1-W^n)} \right] \right\} \\
& \exp\left\{ -\nu \frac{\ln y \ln u}{\ln W} - \frac{k_1^2 + k_2^2}{2} \frac{\ln y \ln xzu}{2 \ln W} + \frac{k_1^2 - k_2^2}{2} \ln y \frac{\ln z^2 u - \ln xzu}{2 \ln W} \right\} \\
& \left\{ \frac{\ln^2 u}{\ln^2 W} P_\mu P_\nu + \cdots + \frac{1}{\ln W} g_{\mu\nu} \right\} \ , \qquad (2.3)
\end{aligned}$$

where $V = xyz$, $W = xyzu$,

$$\begin{aligned}
&\psi(u,W) = \exp\left[-\frac{\ln u \ln W/u}{2 \ln W} + \sum_{n=1}^{\infty} \frac{2W^n - u^n - (W/u)^n}{n(1-W^n)} \right] , \\
&\tilde{\varphi}(u,W) = -\frac{\ln W}{\pi} \sin(\pi \ln u / \ln W) \ , \qquad (2.4) \\
&f(W) = \prod_{n=1}^{\infty} (1 - W^n) \ .
\end{aligned}$$

Whevever needed we can use for the loop integrations (with the Wick rotation) the following formula

$$\int d^4 Q \exp\left(-\frac{1}{2}\ln W Q^2\right) = \left(\frac{\pi}{\frac{1}{2}\ln W} \right)^2 \ .$$

For the large values of a variable s of a function $\varphi(s, \ldots)$, we use the Mellin transform technique

$$\tilde{\varphi}(\beta, \cdots) = \int_0^\infty \varphi(s, \ldots) \, s^{-\beta - 1} ds \quad .$$

Then the rignt-most singularity of $\tilde{\varphi}(\beta, \ldots)$ in the plane of the Mellin transform variable β defines the leading high s- behaviour of the function $\varphi(s, \ldots)$.

From (2.3) for $\nu \to -\infty$, t = fixed behaviour of the invariant amplitudes defined in (2.2) after the use of Mellin transform we find for A_1 amplitude a singularity at $\beta = -1$ (coming from $y \approx 1$) corresponding to a fixed pole at $J = 1$ in the angular momentum plane, and $\beta = \alpha(t) - 2$ (coming from $u \approx 1$ region) corresponding to the usual Regge pole. Hence

$$A_1 \xrightarrow[\nu \to -\infty]{} \frac{-e F(t)}{\nu} + \beta(t)\, \nu^{\alpha(t)-2} \qquad (2.5)$$

The residue of the fixed pole at $J = 1$, i.e. $F(t)$ coincides with the expression for the form factor corresponding to Fig. 2 and hence the Fubini - Dashen - Gell - Mann sum rule is satisfied. The expression for the form factor of Fig. 2 is

$$F(t) = e g^2 \int d^4Q \exp(-\tfrac{1}{2} \ln W Q^2) \int_0^1 dx\, du\, dz\, W^{-\alpha_0-1} (1-xz)^{\alpha_0-1} (1-u)^{\alpha_0-1}$$
$$[f(W)]^{-4} \{ [\psi(u,W)]^{m^2} - [\tilde{\psi}]^{m^2} \} (1-W) \frac{\ln u}{\ln W}$$
$$\exp\left\{ t \left[-\frac{\ln x \ln z}{2 \ln W} + \frac{1}{2} \sum_{n=1}^{\infty} \frac{(xz)^n + u^n}{n(1-W^n)} \right] \right\} , \qquad (2.6)$$

with $W = xuz$

and the high t - behviour of it is $\sim (t)^{-1-\frac{3}{4}m^2} (\log^{-2} t)$ which is the singularity in Mellin transform variable coming from the region of integration $(1-x) z (1-u) \approx 1$ or $x(1-z)(1-u) \approx 1$.

Analogously, A_{10} amplitude has $\beta = -1$ and $\beta = \alpha(t)$ singularities corresponding to a fixed $J = -1$ pole and the usual moving pole.

In the Bjorken scaling limit we put $k_1 = k_2 = k$ and do a Mellin transform with respect to $(-k^2)$, keeping $\frac{2\nu}{-k^2} = \omega$ fixed. The right most singularity for both A_1 and A_{10} is at $\beta = -1$, coming from the region $y \approx 1$, i.e. exactly where the current algebra fixed pole in (2.5) came from. Therefore, we get

$$\nu A_1(\nu, k^2) \xrightarrow[B_j]{} e^2 g^2 \int d^4Q \exp(-\tfrac{1}{2} \ln W Q^2) \int_0^1 dx\, dz\, du\, W^{-\alpha_0-1} (1-u)^{\alpha_0-1}$$
$$(1-xz)^{\alpha_0-1} (1-W) [f(W)]^{-4} \{ [\psi(u,W)]^{k^2} - [\tilde{\psi}]^{m^2} \}$$
$$\frac{\ln u}{\ln W} \frac{1}{\frac{2\nu}{-k^2} - \frac{\ln W}{\ln u}} , \qquad (2.7)$$

where $W = xzu$,

and an analogous expression for A_{10}. Finally, for the structure functions we get

$$\nu W_2(\nu, k^2) \longrightarrow F_2(\omega) \quad ,$$
$$W_1(\nu, k^2) \underset{Bj}{\longrightarrow} \frac{1}{k^2} F_1(\omega) \quad , \qquad (2.8)$$

where

$$F_2(\omega) = e^2 g^2 m^2 \omega \int d^4Q \exp(-\tfrac{1}{2} \ln W Q^2) \int_0^1 dx\, dz\, du\, W^{-\alpha_0 -1} (1-u)^{\alpha_0 -1} (1-xz)^{\alpha_0 -1}$$
$$(1-W)[f(W)]^{-4} \{ [\psi(u,W)]^{m^2} - [\tilde{\psi}]^{m^2} \} \qquad (2.9)$$
$$\left(\frac{\ln u}{\ln W}\right) \delta\left(\omega - \frac{\ln W}{\ln u}\right) \quad , \quad W = xzu$$

$$F_1(\omega) = e^2 g^2 \int \cdots \left(\frac{1}{\ln u}\right) \delta\left(\omega - \frac{\ln W}{\ln u}\right) \qquad (2.10)$$

There seems to be a similarity with the parton model if $x = \frac{1}{\omega} = \ln u / \ln xzu \; (\leq 1)$ is interpreted as the fraction of longitudinal momentum carried by a parton in an infinite momentum frame. As was already mentioned in the introduction the scaling of νW_2 amd vanishing of W_1 is typical for the models where the current is attached to Bosons rather than to fermions.

III. BJORKEN SCALING OF QUASI-INCLUSIVE PROCESSES

Consider reactions like

hadron + hadron $\longrightarrow \mu^- \mu^+$ + anything (3.1)

or

e + hadron $\longrightarrow$ e$'$ + hadron + anything . (3.2)

Using the Mueller's analysis we connect these reactions to the discontinuity of a forward six-point reaction as shpwn in Fig. 3. For these processes there are four structure functions W_1, W_2, W_3, W_4 analogous to the two functions in deep inelastic electron scattering. The differnetial cross sections for the above reactions are proportional to the tensor $W_{\mu\nu}$ [25], where

$$W_{\mu\nu} = -\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) W_1 + \frac{1}{m^2}\left(p - \frac{p.k}{k^2} k\right)_\mu \left(p - \frac{p.k}{k^2} k\right)_\nu W_2$$
$$+ \frac{1}{m^2}\left(p' - \frac{p'.k}{k^2} k\right)_\mu \left(p' - \frac{p'.k}{k^2} k\right)_\nu W_3 + \frac{1}{m^2}\left\{\left(p - \frac{p.k}{k^2} k\right)_\mu \left(p' - \frac{p'.k}{k^2}\right)_\nu + p \leftrightarrow p'\right\} W_4 , \qquad (3.3)$$

and $W_{\mu\nu}$ = Disc $M_{\mu\nu}$, where $M_{\mu\nu}$ is the forward six-point amplitude of Fig. 3 .

In a certain region of kinematical variables

$$k\cdot p \to \infty\ , \ \frac{-k^2}{k\cdot p}\ , \ \frac{k\cdot p'}{k\cdot p}\ , \ \frac{p\cdot p'}{k\cdot p} = \text{fixed} \tag{3.4}$$

from the analogous considerations of the original Bjorken [25], one would expect to having scaling for all the four structure functions, namely that W_1, $(k\cdot p)\ W_2$, $(k\cdot p')\ W_3$ and $((k\cdot p)(k\cdot p'))^{\frac{1}{2}}\ W_4$ should all become functions of the ratios $\frac{-k^2}{k\cdot p}$, $\frac{k\cdot p'}{k\cdot p}$ and $\frac{p\cdot p'}{k\cdot p}$. The purpose of this Sec. is study of the above scaling in the aspect of DRM of the present paper.

The dual amplitude of Fig. 3(a) gives the following contribution

$$\begin{aligned}
M^{(a)}_{\mu\nu}(k^2, k\cdot p, k\cdot p', p\cdot p') = {} & \frac{e^2}{4} g^4 \int d^4Q \int_0^1 dx\,dy\,dz\,du\,dv\,dw \exp\left(-\tfrac{1}{2}\ln W Q^2\right) W^{-\alpha_0-1} \\
& [(1-xyz)(1-u)(1-v)(1-w)]^{\alpha_0-1}(1-W)[f(W)]^{-4}\left\{[\psi(uvw, W)]^{p^2}-[\tilde{\psi}]^{p^2}\right\} \\
& \left\{[\psi(v, W)]^{p'^2}-[\tilde{\psi}(v, W)]^{p'^2}\right\} \\
& \exp\left\{k^2\left[-\frac{\ln y \ln \frac{W}{y}}{2\ln W}\right]+k\cdot p\left[-\frac{\ln y \ln uvw}{\ln W}\right]+k\cdot p'\left[\frac{\ln y \ln v}{\ln W}\right]\right. \\
& \left.+p\cdot p'\left[\frac{\ln v \ln \frac{W}{uvw}}{\ln W}-\sum_{n=1}^{\infty}\frac{(1-v^n)(1-(xyz)^n)(u^n+w^n)}{n(1-W^n)}\right]\right\} \\
& \left\{\left(\frac{-4}{\ln W}\right)g_{\mu\nu}+\left[\frac{4\ln^2(uvw)}{-\ln^2 W}\right]p_\mu p_\nu+\left[\frac{4\ln^2 v}{-\ln^2 W}\right]p'_\mu p'_\nu+\left[\frac{4\ln v \ln(uvw)}{\ln^2 W}\right](p_\mu p'_\nu+p_\nu p'_\mu)\right. \\
& \left.+\left[1+\frac{4\ln y}{\ln W}+4\frac{\ln^2 y}{\ln^2 W}\right]k_\mu k_\nu+\cdots\right\},
\end{aligned} \tag{3.5}$$

Where $W = xyzuvw$.

Consider the Regge limit $\nu = k\cdot p$, $k\cdot p' \to -\infty$ and the other variables fixed. After Mellin transform we get $\beta = -1$ singularity coming from the region $y \approx 1$ of integration for all the four invariant amplitudes

$$M^{(a)}_{\mu\nu} \xrightarrow[\text{Regge}]{} g_{\mu\nu}\,\nu^{-1}+p_\mu p_\nu\,\nu^{-1}+p'_\mu p'_\nu\,\nu^{-1}+(p_\mu p'_\nu+p_\nu p'_\mu)\,\nu^{-1}, \tag{3.6}$$

i.e. There is a fixed pole in the angular momentum plane J .

For a special case of scaling limit, namely

$$B_j' : -k^2 \to \infty\ ; \ \frac{k\cdot p}{-k^2}\ , \ \frac{k\cdot p'}{-k^2}\ , \ \frac{p\cdot p'}{} = \text{fixed}\ , \tag{3.7}$$

again the same region $y \approx 1$ as in (3.6) gives the singularity $\beta = -1$ and one gets the behaviour

$$M^{(a)}_{\mu\nu} \xrightarrow[B_j]{} g_{\mu\nu}(k^2)^{-1} + p_\mu p_\nu (k^2)^{-1} + p'_\mu p'_\nu (k^2)^{-1} + (p_\mu p'_\nu + p_\nu p'_\mu)(k^2)^{-1} \quad , \qquad (3.8)$$

exactly because of the same fixed pole as in the Regge limit.

Therefore in the limit of (3.7)

$$\begin{aligned} W_1 &\longrightarrow (k^2)^{-1} \quad F_1(\omega, \omega') \\ \nu W_2 &\longrightarrow F_2(\omega, \omega') \\ \nu' W_3 &\longrightarrow F_3(\omega, \omega') \\ (\nu\nu')^{\frac{1}{2}} W_4 &\longrightarrow F_4(\omega, \omega') \quad , \end{aligned} \qquad (3.9)$$

where $\nu = k.p$, $\nu' = k.p'$, $\omega = \frac{2\nu}{-k^2}$, $\omega' = \frac{2\nu'}{-k^2}$.

In the scaling limit of (3.4) or

$$B_j : -k^2 \to \infty \, , \; \omega, \omega' \text{ and } \frac{p.p'}{-k^2} = x \text{ fixed} \, , \qquad (3.10)$$

the above fixed pole is not responsible any more for the scaling and the region of integration $xyz \approx 1$ becomes important and one gets

$$M^{(a)}_{\mu\nu} \xrightarrow[B_j]{} g_{\mu\nu}(k^2)^{\alpha_0 - 2} M_1 + p_\mu p_\nu (k^2)^{\alpha_0 - 2} M_2 + p'_\mu p'_\nu (k^2)^{\alpha_0 - 2} M_3 + (p_\mu p'_\nu + p_\nu p'_\mu)(k^2)^{\alpha_0 - 2} M_4 . \qquad (3.11)$$

Expression (3.11) shows that only by putting $\alpha_0 = 1$ i.e. the assignment of the Regge trajectory with a Pomerson can give the scaling of the structure functions.

The responsibility of the fixed pole for scaling in the region (3.7) and disappearance of its effect in the region (3.10) is interesting both theoretically and may be experimentally. First of all, since the fixed pole is due to existence of the currents in the amplitude it is not clear why its effect should depend on the variable p.p′ which is entirely in the strong part of the amplitude. Other more detailed models such as parton model or even light cone expansion technique may shed some light on this question and then perhaps the origin of fixed poles becomes more clear. Experimentally it may be interesting since it predicts that if some analysis similar to the one done by Bloom and Gilman [17] will become possible in future then in the region (3.10) the resonance component of the structure functions will not show the scaling property, contrary to the case of deep inelastic electron scattering.

The diagram of Fig. 3(b) has no fixed pole in the Regge limit and, corresondingly, no scaling property coming from the fixed pole. In the Regge limit, say $\nu = k\cdot p \longrightarrow -\infty$, other variables fixed we get

$$M^{(b)}_{\mu\nu} \xrightarrow[\text{Regge}]{} g_{\mu\nu}\nu^{\alpha_0} + p_\mu p_\nu \nu^{\alpha_0-2} + p'_\mu p'_\nu \nu^{\alpha_0} + (p_\mu p'_\nu + p_\nu p'_\mu)\nu^{\alpha_0-1} \quad . \qquad (3.12)$$

In the scaling limit of both (3.7) and (3.10) we get

$$M^{(b)}_{\mu\nu} \xrightarrow[B_j \text{ or } B_j']{} g_{\mu\nu}(k^2)^{\alpha_0-2}M_1 + p_\mu p_\nu (k^2)^{\alpha_0-2}M_2 + p'_\mu p'_\nu (k^2)^{\alpha_0-2} + (p_\mu p'_\nu + p_\nu p'_\mu)(k^2)^{\alpha_0-3}M_4 . \qquad (3.13)$$

Again with $\alpha_0 = 1$ it gives scaling for νW_2 , $\nu' W_3$ and vanishing of W_1 and $(\nu\nu')^{\frac{1}{2}} W_4$.

IV. INELASTIC COMPTON SCATTERING

Consider the reaction

photon (k) + Hadron (p) $\longrightarrow$ photon (k') + anything. (4.1)

The expression from the dual diagram of fig. 4 in the general case of $k^2, k'^2 \neq 0$ is the following

$$M_{\mu\mu'\nu\nu'}(k^2,k'^2,\ldots) = e^4 g^2 \int d^4Q \exp\left(-\tfrac{1}{2}\ln W Q^2\right)\int_0^1 dx\,dy\,dz\,du\,dv\,dw\; W^{-\alpha_0-1}$$
$$(1-\tfrac{W}{w})^{\alpha_0-1}(1-w)^{\alpha_0-1}(1-W)[f(W)]^{-4}\left\{[\psi(w,W)]^{m^2} - [\tilde\psi]^{m^2}\right\}$$
$$\exp\left\{k^2\left[-\frac{\ln yzu \ln\frac{W}{yzu}}{2\ln W}\right] + k'^2\left[-\frac{\ln z \ln\frac{W}{z}}{2\ln W}\right] + (k\cdot p)\left[-\frac{\ln w \ln yzu}{\ln W}\right]\right.$$
$$\left. + (k'\cdot p)\left[\frac{\ln z \ln w}{\ln W}\right] + (k\cdot k')\left[\frac{\ln z \ln\frac{W}{yzu}}{\ln W}\right]\right\} \qquad (4.2)$$
$$\frac{1}{\ln^2 W}\left\{\frac{\ln^4 w}{\ln^2 W} p_\mu p_\nu p_{\mu'} p_{\nu'} + (g_{\mu\nu}g_{\mu'\nu'} + g_{\nu\mu'}g_{\mu\nu'} + g_{\mu\mu'}g_{\nu\nu'}) + \cdots\right\} ,$$

where $W = xyzuvw$.

If we define the decomposition of the amplitudes as

$$M_{\mu\mu'\nu\nu'} = p_\mu p_\nu p_{\mu'} p_{\nu'} \widetilde{M}_2 + \ldots \qquad (4.3)$$

and $\widetilde{W}_2 = \mathrm{Disc}\ \widetilde{M}_2$,

from the parton model of Bjorken and Paschos[10] it comes out that (for the case of real photons $k^2 = k'^2 = 0$)

$$\tilde{\nu}^3 \widetilde{W}_2 = (\text{a factor not depending on } \tilde{x}) \sum_N P(N) \tilde{x} f_N(\tilde{x}) \langle \sum_i Q_i^4 \rangle_N , \qquad (4.4)$$

where

$$\tilde{x} = \frac{-t}{2\tilde{\nu}}, \quad t = (k-k')^2 = -2k.k', \quad \tilde{\nu} = (k-k')\cdot p, \qquad (4.5)$$

is satisfied for large values of t, $\tilde{\nu}$ and $k\cdot p$ with their ratios fixed. In other words for large values a scaling law is satisfied for the structure function $\tilde{\nu}^3 \widetilde{W}_2$ with the scaling variable $\tilde{x} = \frac{-t}{2\tilde{\nu}}$, where $\tilde{x}$ is the longitudinal fraction of the parton momentum in the infinite momentum frame. From the same parton model[10] one gets for νW_2 structure function of deep inelastic electrons scattering an expression

$$\nu W_2(\nu, k^2) = \sum_N P(N) x f_N(x) \langle \sum_i Q_i^2 \rangle_N \qquad (4.6)$$

with the scaling variable $x = \frac{1}{\omega} = \frac{-k^2}{2\nu}$. By looking at the expressions (4.4) and (4.6) one sees that the structure functions of inelastic Compton scattering has the same analytic dependence as a function of $\tilde{x}$ as the structure function of deep inelastic electron scattering does as a function of x in the case of partone charge $Q_i = 0, 1$.

It would be interesting to study the same question in the DRM of the present paper in order to reveal deeper analogy with the parton model. By putting $k^2 = k'^2 = 0$ in (4.2) and finding $\widetilde{M}_2$ from the definition (4.3) we do a Mellin transform with respect to $k.k'$ keeping $\frac{k\cdot p}{k\cdot k'}$ and $\frac{k'\cdot p}{k\cdot k'}$ fixed. There is a pole at $\beta = -3$ coming from $yzu \approx 1$ and we get the limit

$$\widetilde{M}_2 \longrightarrow e^4 g^2 (k.k')^{-3} \int d^4Q \exp(-\tfrac{1}{2} \ln W Q^2) \int_0^1 dx\, dv\, dw\, W^{-\alpha_0 -1} (1-xv)^{\alpha_0 -1} (1-w)^{\alpha_0 -1} (1-W) [f(W)]^{-4} \{ [\varphi(w, W)]^{m^2} - [\varphi]^{m^2} \} \frac{\ln^4 w}{\ln^4 W} \left[\frac{2\tilde{\nu}}{t} \frac{\ln w}{\ln W} + 1 \right]^{-3}, \qquad (4.7)$$

where $W = xvw$.

Surprisingly enough, the variables $k \cdot p$ and $k' \cdot p$ in the complicated expression (4.2) combine just in a manner to give (4.7) depending only on $\frac{2\tilde{\nu}}{-t}$ with $\tilde{\nu} = (k-k')p$. By looking at (2.7) and (4.7) one sees that the two expressions are indeed very similar, provided one changes $v \leftrightarrow z$ and $w \leftrightarrow u$. Furthermore, assuming the analyticity of the discontinuity of $\tilde{M}_2$ in a proper region of variable $\tilde{\nu}$ and excluding the end points of the scaling variable $\tilde{x} = \frac{-t}{2\tilde{\nu}} = \frac{\ln w}{\ln xvw}$ i.e. $\tilde{x} \neq 0$ and 1 (where δ- function of these end point values and their derivatives appear) and with the use of $\tilde{W}_2 = \text{Disc } \tilde{M}_2 = \left[\text{Disc } \frac{\partial^2}{\partial \nu^2} A_1\right] = \left[\frac{\partial^2}{\partial \nu^2} \text{Disc } A_1\right]_{\substack{x \to \tilde{x} \\ \nu \to \tilde{\nu}}} = \left[\frac{\partial^2}{\partial \nu^2} W_2\right]_{\substack{x \to \tilde{x} \\ \nu \to \tilde{\nu}}}$, one can convince himself that the two structure functions are indeed proportional to each other.

V. $e^- e^+$ - ANNIHILATION INTO HADRONS

The total cross section of $e^- e^+$- annaihilation into hadrons is [26)]

$$\sigma(k^2) = \frac{1}{2} \frac{16\pi^2\alpha^2}{(k^2)^2} \rho(k^2) , \tag{5.1}$$

where $$M_{\mu\nu}(k^2) = \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\rho(k^2) .$$

In the dual model of the present paper the calculation of $M_{\mu\nu}(k^2)$ is approximated by the discontinuity of the self energy digram of the virtual phton, as illustrated in Fig. 5, and its contribution witnout renormalization is

$$M_{\mu\nu}(k^2) = e^2 \int d^4Q \exp\left(-\frac{1}{2} \ln W Q^2\right) \int_0^1 dx\, dy\, W^{-\alpha_0-1} (1-W)^{\alpha_0-1}$$
$$[f(W)]^{-4} \exp\left[k^2 \frac{\ln^2 x}{2\ln W}\right] \left\{\frac{1}{\ln W} g_{\mu\nu} + \cdots\right\} , \tag{5.2}$$

$W = xy$.

The singularity of Mellin variable is at $\beta = -\frac{1}{2}$ and therefore

$$\rho(k^2) \longrightarrow (k^2)^{-\frac{1}{2}} , \tag{5.3}$$

$$\sigma(k^2) \longrightarrow \frac{1}{(k^2)^{\frac{5}{2}}} \quad . \tag{5.3}$$

This is a kind of fixed pole behaviour independent of the trajectories exchanged. The result (5.3) is incompatible with the results of the other models[26] . Notice, however, that the expression (5.2) has the exponential divergence of the dual loops and has not been renormalized In evaluation of (5.3) we have done in fact a simple cutoff of the type $\Theta(2 - \epsilon - x - y)$. We suggest that a prpper dual renormalization of the self-mass diagram of photon may change the result (5.3). We hope to study this question further.

ACKNOWLEDGEMENT

It is a pleasure to thank E.J. Squires and A.M.S. Amatya for discussions and useful comments.

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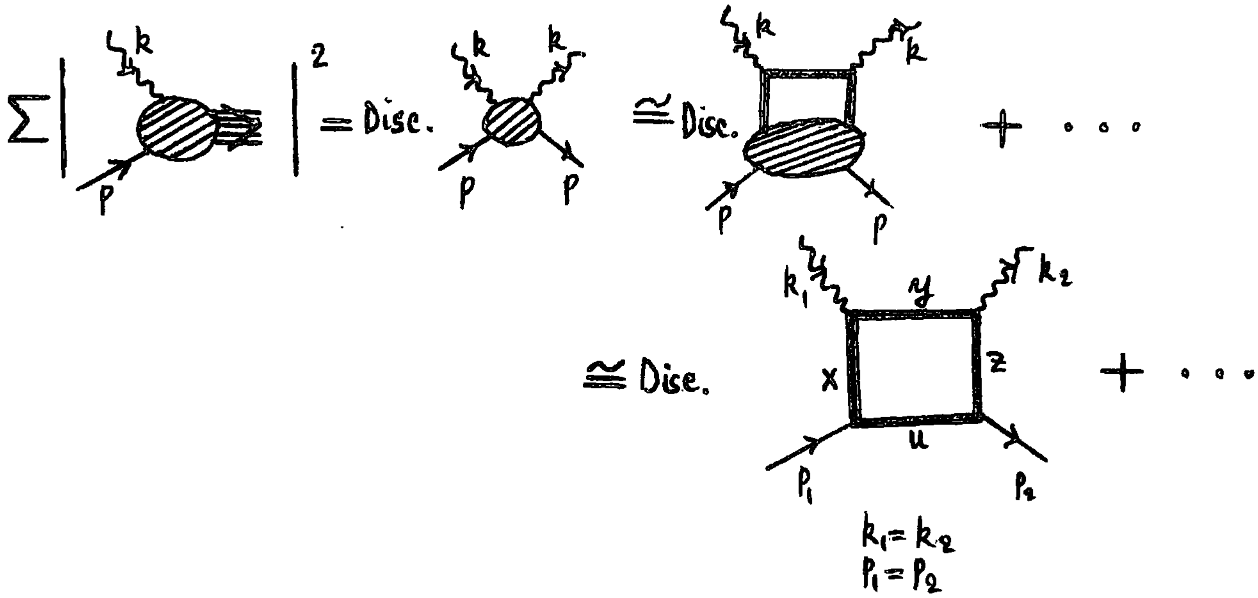

Fig. 1. The dual diagram used for the study of Bjorken scaling. Thick lines are the tower of boson resonances, dots mean non-leading diagrams.

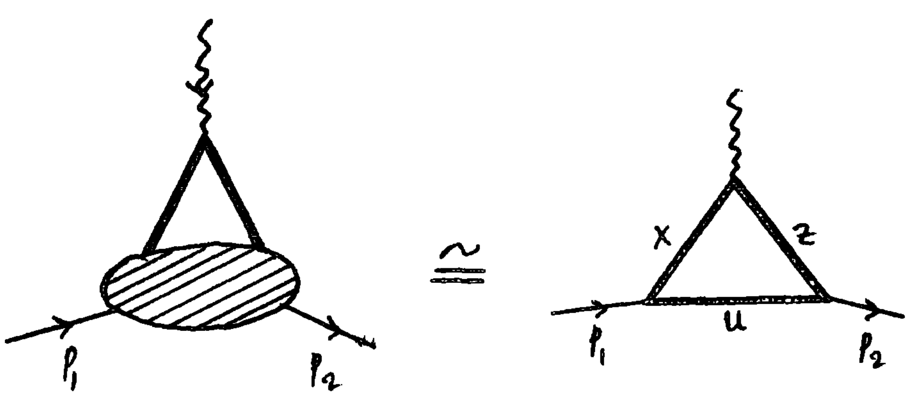

Fig. 2. Form factor corresponding to the last diagram of Fig. 1 through The Fubini - Dashen - Gell - Mann sum rule.

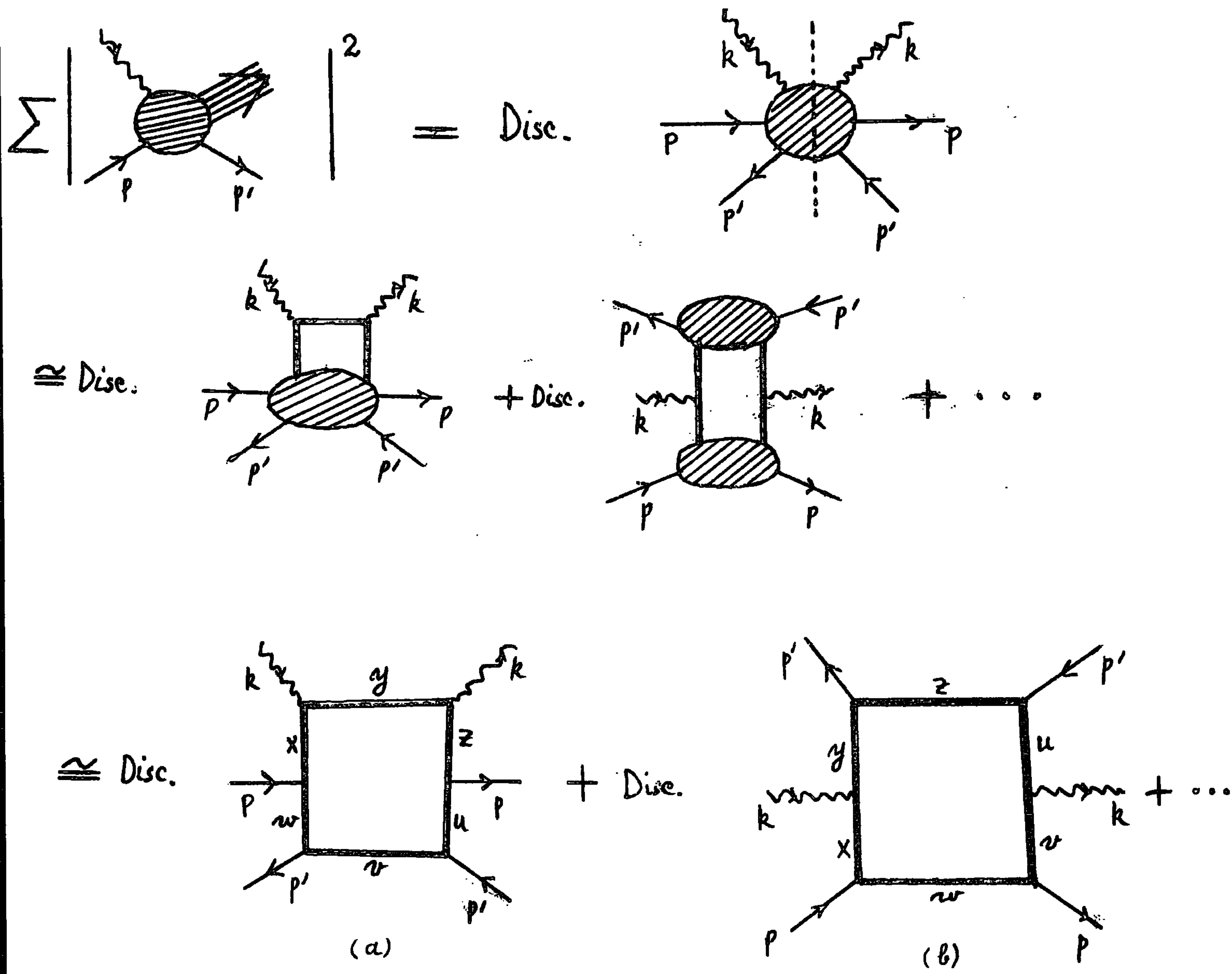

Fig. 3. Diagrams considered for the study of Bjorken scaling in deep inel. electron scattering with one final state hadron detected.

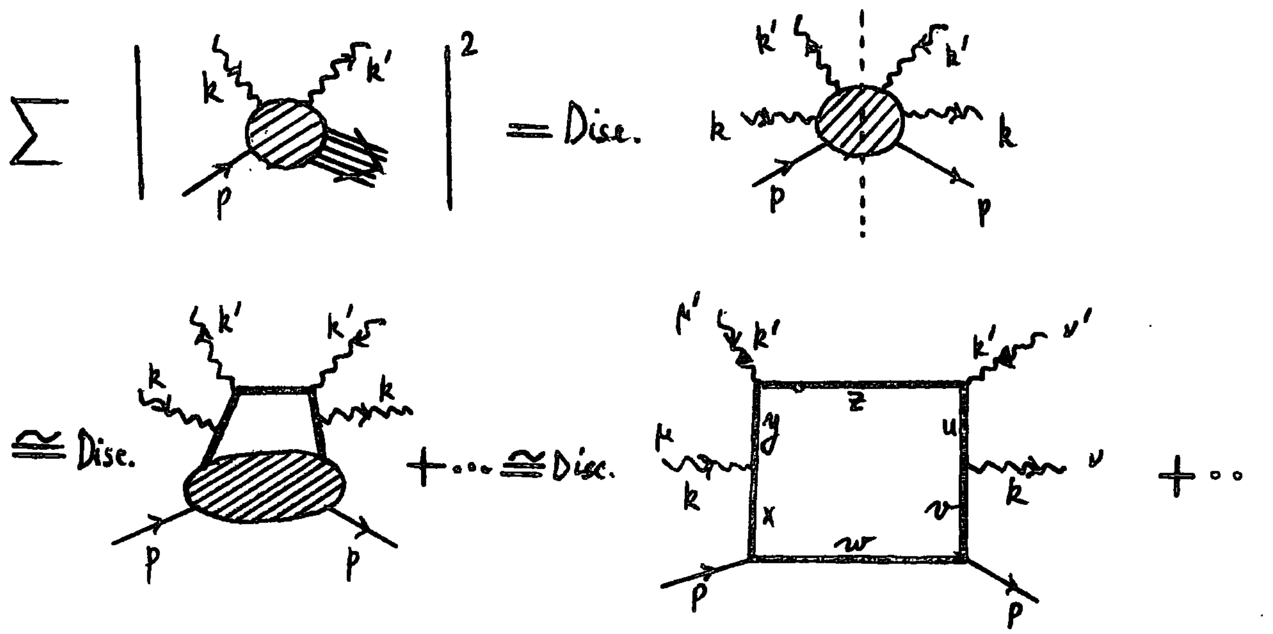

Fig. 4. Diagram for inelastic Compton Scattering.

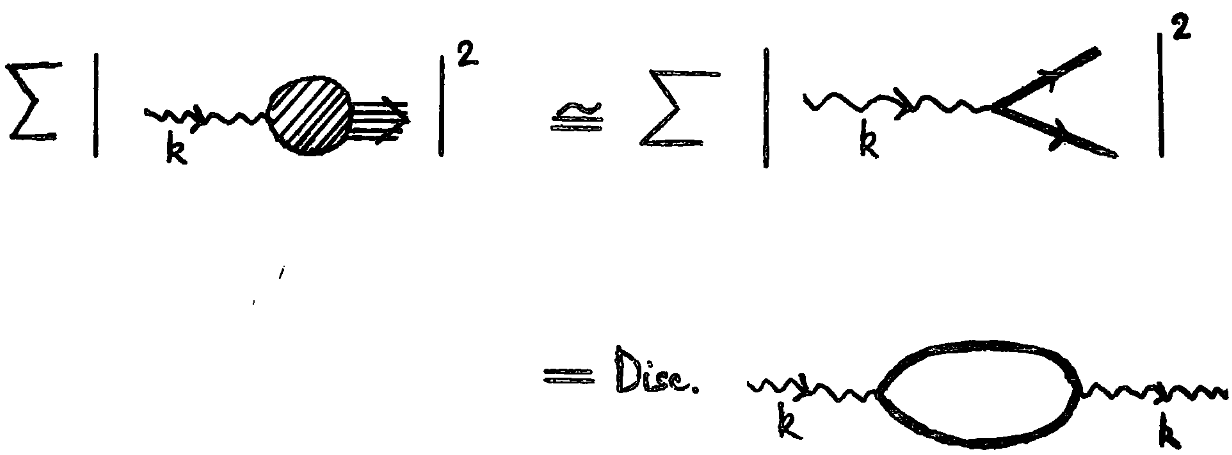

Fig. 5. Diagram for the electron-position annihilation into hadrons.