High energy scattering

Ringland, Gordon A.

HIGH ENERGY SCATTERING

by

Gordon A. Ringland B.Sc. (Bristol)

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CONTENTS

Abstract

This thesis is primarily concerned with the Absorption Model for high energy scattering, and in particular the application of the Absorption Model to the processes Nucleon-Nucleon charge exchange, and Nucleon-Antinucleon charge exchange.

In the first chapter, a review of the development of the one particle exchange model of high energy scattering is given. The difficulties of the earlier applications are discussed, and the role of absorptive corrections in resolving these difficulties is outlined.

In Chapter 2, the mathematical framework for adding absorptive corrections to one particle exchange contributions is given. The original prescription due to Sopkovich is discussed, and recent approaches to the problem are reviewed.

In Chapter 3, the Absorption Model is applied to the process of neutron-proton backward scattering, or charge exchange. It is found that one-pion exchange in the Absorption Model gives a good fit to the charge exchange peak at small angles, but there is disagreement with

experiment at large angles. Rho-meson exchange is found
to be unsatisfactory. The energy dependence of one-meson
exchange in the Absorption Model is investigated.

In Chapter 4, the Absorption Model is applied to
Nucleon-Antinucleon charge exchange. Good agreement with
experiment is obtained. The sensitivity of the results
to the precise model of elastic scattering is investigated.

In Chapter 5, we discuss various refinements to our
work in Chapter 3, to see if better large angle results
can be obtained.

In our concluding chapter, the Absorption Model is
compared with the Regge pole exchange model, and the
possibility of combining the two approaches is discussed.

Acknowledgements

C H A P T E R 1

THE EVOLUTION OF THE PERIPHERAL MODEL

It was first suggested by Chew and Low (1) that the exchange of pions in inelastic pion-nucleon collisions should give dominant contributions to the cross-section for low values of momentum transfer. The standard plausibility argument for this assertion is as follows: if we consider the scattering of two spin-zero particles of equal mass, with the exchange of a scalar particle mass μ, then the contribution to the differential cross-section is proportional to $g^4/(\Delta^2 + \mu^2)^2$, where $\Delta^2 = [4\text{-momentum transfer}]^2$, and g^2 is the coupling constant of the exchange particle to the external particle. Thus, if μ is small, and we consider a region where Δ^2 is small, then the size of the denominator gives rise to the expectation that the term will be dominant.

The motivation for this suggestion was to develop a method of extrapolation for the pole terms arising from single particle exchange forces, and in this way to obtain the coupling constants from residues of the poles corresponding to the given particle exchanges.

This concept was extended to form the basis of a phenomenological calculation of strong-interaction high energy inelastic processes which are sharply peaked in the forward direction - the peripheral processes - by Drell(2), and Salzman (3).

These calculations had some success, but these successes were far from quantitative or uniform. In particular, careful calculations of one-particle exchange diagrams for realistic processes, i.e. those involving spin, demonstrated that the above plausibility argument could in certain circumstances be very misleading. Perhaps the most dramatic instance of this is the one-pion contribution to neutron-proton charge exchange. If we were to follow on naive plausibility argument, namely treating the neutron, proton and pion as scalar particles, we should find close agreement with the experimental angular distributions. A realistic calculation shows, however, gross qualitative disagreement with the observed data. Indeed, it was found as a general feature that the proper inclusion of spins and parities, internal and external, gave rise to a much broader angular distribution than that expected from the simple denominator argument.

It was suggested by Ferrari and Selleri (4) that agreement could be obtained by inserting a form factor dependent on momentum transfer. While such form factors can exist and indeed have a very important role in the electromagnetic interactions, the proposal was unsatisfactory

for the following reasons.

Firstly, because there was no attempt to obtain these
form factors theoretically - they were merely inserted to
give a good fit to the experimental data. It was found
that, to obtain a reasonable fit, the dependence of these
form factors on momentum transfer was very s trong, and
indeed completely masked the momentum transfer dependence
of the input, one-particle exchange, term. Thus it is hard
to see just what value such an approach has. There are
also further criticisms. Consider the processes

$$NN \rightarrow N^*N \quad (a) \quad \left.\right\} \quad \text{where N is the Nucleon}$$
$$\text{and} \quad N\bar{N} \rightarrow N\bar{N}^* \quad (b) \quad \left.\right\} \quad \text{and } N^* \text{ the 3,3 Resonance.}$$

Ferrari and Selleri (4) were able to obtain a good fit to
process (a) with one pion exchange and their ad hoc form
factors. But an identical form factor is required for the
vertex $\bar{n}$ n*π, and so the Ferrari-Selleri theory gives the
same predictions for processes (a) and (b). Process (b)
has, however, a smaller cross-section than (a), and is much
more sharply peaked.

So far, in discussing the evolution of the peripheral
model, we have concentrated on a comparison of its predictions
with experiment. But a theory must not only be satisfactory
in comparison with experiment, it must also conform to basic

conservation laws, for instance probability (in the form of
unitarity), and those implied by the Lorentz group.

One finds that the straight one particle exchange
model frequently gives rise to violation of unitarity
bounds for low partial waves e.g. Gottfried and Jackson find
that for the process $\pi p \to \rho p$ at 4 Gev/c a violation of the
unitarity bound for s and p waves. The unitarity bound is
exceeded for s waves by a factor approximately 200, and for
p waves by about 10. Of course, these violations and the
prediction of too flat an angular distribution are not
unconnected. Since the large angle scattering is from low
partial waves, it is clear that a model which gives a large
contribution to these will have difficulty in giving a
largely peaked angular distribution.

The answer to this dilemma was proposed by Gottfried
and Jackson (5) and independently by Durand and Chiu (6).
The intuitive basis of their proposal, a revival of an
original idea and formalism given by Sopkovich (7), was as
follows. At energies in the Gev range, inelastic channels
are open, and these channels compete both in the initial and
final states for their share of the total cross-section.
Now, it is reasonable to suppose that such inelastic
processes, particularly those complex processes involving

genuine many particle final states, i.e. not quasi 2-body processes, should be initiated by collisions involving low partial waves. And conversely, we should expect that this competition in low partial waves due to many competing open channels will imply a reduction in the low partial wave interaction amplitude to any given channel, while leaving higher partial waves largely unchanged.

Thus, an effect has been introduced which is intuitively reasonable, and which offers hope of reducing the excess over unitarity bounds of low partial waves and of producing a change in the angular distributions, collimating them in the forward direction. It is now necessary to translate this idea into mathematical formalism permitting us to make calculations to test the idea. A prescription for taking into account the effects of competing channels, and parameterising them in terms of elastic scattering was given by Sopkovich, and rederived by Gottfried and Jackson, and Durand and Chiu. We shall give details in the following chapter.

C H A P T E R 2

THE ABSORPTION MODEL

We shall first discuss the original construction of a prescription to take into account the absorptive effects of many competing channels on the one-particle exchange contribution, following the treatment due to Jackson (5). Such a modified peripheral model is now usually known as the Absorption Model. The terms prescription and construction are used advisedly, for the Absorption Model is dependent on many assumptions and approximations, and few of them are to be regarded as altogether well-founded. However, since an exact solution would require the solution of coupled multi-particle, multi-channel equations, and this is a very distant goal, we must expect that any useful result will involve crude approximations in its derivation.

The first version of the Absorption Model is essentially a distorted wave Born approximation, more familiar to Nuclear and Atomic Physics than to High Energy Physics. Consider the scattering of spinless particles $a + b \rightarrow c + d$, where the force is approximated by one-particle exchange. In potential language, the potential V giving rise to the transition is well represented by its first Born approximation. Let $U^{(+)}$ and $U^{(-)}$ be the remaining interactions between a and b, and c and d, respectively.

Thus $U^{(+)}$ and $U^{(-)}$ are the potentials which give rise to the elastic scattering $a + b \rightarrow a + b$, and $c + d \rightarrow c + d$. Since in any application of the peripheral model, the cross-sections considered are much smaller than the elastic scattering cross-sections, the appropriate case to consider is $V \ll U^{(\pm)}$.

From Gell-Mann - Goldberger scattering theory (8), the scattering amplitude to all orders in $U^{(\pm)}$ and first order in V is given by

$$M_{fi} = \langle \psi_f^{(-)} |V| \psi_i^{(+)} \rangle \qquad (2.1)$$

where $\psi_f^{(-)}$ and $\psi_i^{(+)}$ are the eigenstates of the Hamiltonian corresponding to $U^{(-)}$ and $U^{(+)}$ respectively. That is

$$\psi_i^{(+)} = \phi_i + \left(\frac{1}{E - (H-V) + i\eta} \right) U^{(+)} \phi_i$$

$$\qquad (2.2)$$

$$\psi_f^{(-)} = \phi_f + \left(\frac{1}{E - (H-V) - i\eta} \right) U^{(-)} \phi_f$$

where ϕ_f, ϕ_i are plane wave eigenstates of the non-interacting Hamiltonian $H_0 = H - u - v$.

and E is the energy , H the total Hamiltonian; and η is a small quantity, defining by its sign whether the wave

function has ingoing or outgoing boundary conditions;
+ for outgoing, - for ingoing.

At high energies and small momentum transfer one can
use the approximation for the wave functions developed by
Glauber (9) from the W.K.B. approximation.   Since this
involves the use of the impact parameter formalism, it is
natural to convert the partial wave sum over all angular
momenta to an integral over all impact parameters.   This
may be done at small scattering angles by using the
asymptotic formula $P_\ell(\cos\ \theta)\ \simeq\ J_0((2\ell + 1)\ \sin \frac{1}{2}\theta)$,
where P_ℓ is the Legendre Polynomial of order ℓ, and J_0 is
the cylindrical Bessel function.   Hence the partial
wave sum $M_{fi} = \Sigma(2\ell + 1)A_\ell\ P_\ell(\cos\ \theta)$ (2.3)

becomes $M_{fi} \simeq 2q^2 \int_0^\infty bdb\ A(x)\,J_0(wx)$ (2.4)

where b is the impact parameter, q the centre of mass
momentum, $w = 2\sin\frac{1}{2}\theta$ and $x = qb$.   After manipulation
we may retrieve the partial wave sum by inspection.

In order to relate $U^{(\pm)}$ to high energy elastic
scattering we must allow them to be complex - and in
constructing the Bra (final state) we must use the complex
conjugate of the potential.   The wave functions in the
centre of mass frame are then given by

$$\psi_{q_+}^{\mp}(\underline{b},z) \simeq \exp[i\underline{q}\cdot x]\, \exp[-\frac{i}{v_{\pm}}\int_{\pm\infty}^{z} U^{\pm}(\underline{b}+\hat{\underline{k}}z')dz']\qquad(2.5)$$

where $v_{\pm}$ is the relative velocity of the particles,

q_+ and q_- are the initial and final 3-momenta,

and we have assumed one 3-dimensional degree of freedom z

chosen along $\underline{k} = \underline{q}_+ + \underline{q}_-$, $\hat{\underline{k}} = \dfrac{\underline{q}_+ + \underline{q}_-}{|\underline{q}_+ + \underline{q}_-|}$. The impact

parameter vector $\underline{b}$ is perpendicular to $\hat{\underline{k}}$,

and the position vector $\underline{r} = \underline{b} + \hat{\underline{k}}z$.

In this approximation the scattering amplitude becomes:

$$M_{fi} = \int_0^\infty d^2b \int_{-\infty}^\infty dz\, \exp[i\underline{\Delta}\cdot\underline{b}]\, V(\underline{b}+\hat{\underline{k}}z) \times \exp[-\frac{i}{v_-}\int_z^\infty U^{(-)}(\underline{b}+\hat{\underline{k}}z')^*dz']$$

$$\times \exp[-\frac{i}{v_+}\int_{-\infty}^z U^{(+)}(\underline{b}+\hat{\underline{k}}z'')dz'']\qquad(2.6)$$

where $\underline{\Delta}$ is the 3-momentum transfer. Now we note that

$$-\frac{1}{v}\int_{-\infty}^\infty U(\underline{b}+\hat{\underline{k}}z)dz = 2\delta(b)\qquad(2.7)$$

where $\delta(b)$ is the phase shift of a wave packet travelling

through a potential U at impact parameter $\underline{b}$, and

$$B(\underline{b}) = \int_{-\infty}^\infty V(\underline{b}+\underline{k}z)dz\qquad(2.8)$$

is the unmodified Born approximation for the partial wave amplitude corresponding to the potential V.

To obtain some simple relation from (2.6) for the distorted wave Born approximation, the unmodified Born approximation, and the elastic scattering phase shifts, further assumptions must be made. Two possible assumptions will give the desired simplification.

Firstly, if we assume that $U^{(+)} = U^{(-)}$, and $v_+ = v_-$, then (2.6) reduces to

$$M_{fi} = 2\pi \int_0^\infty J_0(\Delta b)e^{2i\delta(b)}B(b)b\,db \qquad (2.9)$$

where $\Delta = 2q(1 - \cos\theta)^{\frac{1}{2}} = 2q\sin\tfrac{1}{2}\theta$.

Thus, by comparison with the partial wave sum and its integral representation, we find that the effect of the initial and final state interaction is to multiply the partial wave Born amplitude B_ℓ for the transition by the partial wave s-matrix for elastic scattering, S_ℓ, of the particles in the initial (final) state. The modified partial wave amplitude A_ℓ is given by $A_\ell = S_\ell B_\ell$, which we rewrite as

$$A_\ell = S_\ell^{\frac{1}{2}} B_\ell S_\ell^{\frac{1}{2}} \qquad (2.10)$$

to compare with the case of non-identical initial and final states.

When $U^{(+)} \neq U^{(-)}$, one can only simplify (2.6) if the range of V is much less than that of $U^{(+)}$ or $U^{(-)}$. In this case, (2.6) becomes

$$M_{fi} = 2\pi \int_0^\infty b\,db \ J_0(\Delta b) B(b) \exp\{\tfrac{i}{2}[2\delta^{(+)}(b)+2\delta^{(-)}(b)]\} \quad (2.11)$$

where
$$2\delta^{(+)}(b) = -\frac{1}{v_+} \int_{-\infty}^\infty U^{(+)}(\underline{b}+\underline{\hat{k}}z)\,dz$$

and
$$2\delta^{(-)}(b) = -\frac{1}{v_-} \int_{-\infty}^\infty U^{(-)}(\underline{b}+\underline{\hat{k}}z)\,dz,$$

i.e. $\delta^{(+)}(b)$ and $\delta^{(-)}(b)$ are the phase shifts for the elastic scattering of the particles in the initial and final states respectively. The modified Born term is then given by

$$A_\ell = (S_\ell^i)^{\frac{1}{2}} B_\ell (S_\ell^f)^{\frac{1}{2}}, \quad (2.12)$$

which is the original prescription given by Sopkovich (7). S_ℓ^i and S_ℓ^f are the partial wave S matrix elements for elastic scattering in the initial and final states.

Now for most applications - though not our own work on charge exchange processes - there is no reason to believe the near equality of $U^{(+)}$ and $U^{(-)}$. Thus in using equation

(2.12), one is assuming the range of V is much smaller than that of $U^{(+)}$ and $U^{(-)}$. This is in general very far from the case. Characterizing the range of an exchange particle by its Compton wavelength, the range of V for one pion exchange is 1.4F, whereas optical models for N-N elastic scattering give a range for U of the order of 1F.

However, it is possible that this criticism may not be too important in practice; Durand (private communication) has carried out a number of numerical computations in potential theory, using potentials V and U, for V less than U, but violating the range assumption required for equation (2.12). He finds that, though in these circumstances equation (2.12) is not a good approximation for the modified partial waves, on summing up the partial waves given by (2.12) there is a strong cancellation of errors and a very good approximation to the exact amplitude is obtained for small angle scattering.

Another criticism of the preceeding derivation is the use of the Schrödinger equation with an optical potential which, though it may give a good representation of the angular distribution for scattering, does not necessarily give good information on the wave functions - a situation known to obtain in Nuclear Physics (C.F.Clement, private

communication). Furthermore, one can object to using the Schrödinger equation to obtain results which will be used in a highly relativistic situation.

We now consider briefly the result of the absorptive modifications given by equation (2.12). To do this, we must know the elastic scattering phase shifts in the initial and final states. However we may note immediately that since $\text{Im} \delta_\ell \geqslant 0$, any inelasticity in the elastic scattering must reduce the Born terms. It must be emphasized that these cannot be predicted, but must be analyzed from elastic scattering data. This immediately raises a problem for inelastic processes, since it is impossible to devise an elastic scattering experiment for the final states of any inelastic interaction – this point will be discussed further when we review work on inelastic processes. In any case, thorough and reliable phase shift analyses are difficult and complicated matters, the complexity increasing rapidly with energy and hence the number of significant partial waves. In practice, we find it necessary to rely on very crude models to obtain results for elastic phase shifts above about 1 GeV.

To be specific, we sketch one such model below.

The Gaussian model

The elastic scattering diffraction peak above 1 or 2 GeV is well parametrized by the exponential form

$$\frac{d\sigma}{dt} = \frac{d\sigma}{dt}\Bigg|_{t=0} e^{at} \qquad (2.13)$$

where t is the (4-momentum transfer)^2, negative in the physical region, and a positive function of S.   We then make the following approximations:

a)   assume that equation (2.13) holds for all t, not just in the diffraction peak.

b)   suppose that the forward amplitude (which is largely imaginary) is exactly imaginary;   and further, that this is true of the amplitude at all angles.

c)   neglect spin.

It is then possible to obtain an analytic expression for the partial wave S-Matrix by using the impact parameter formalism.   From the above assumptions and the Optical Theorem, the amplitude $M(t,q)$, where q is the centre of mass momentum, is given by

$$M(t,q) = \frac{i\sigma_{tot}}{4\pi} q \ e^{\frac{1}{2}at} . \tag{2.14}$$

But $\quad M(t,q) = - iq \int_0^\infty b \, db \, J_0(b\sqrt{-t})(e^{2i\delta(b)}-1), \tag{2.15}$

where, by comparison with equation (2.14), $\delta(b)$ must be pure imaginary. Now, equation (2.15) is a Fourier-Bessel transform and may be inverted. If

$$b(y) = \int_0^\infty a(x) \ J_0(xy) dx,$$

then the inverse is

$$a(x) = \int_0^\infty b(y) \ J_0(xy) dy.$$

Hence, from (2.15)

$$e^{2i\delta(b)}-1 = \frac{i}{q}\int_0^\infty d\sqrt{-t} \ \sqrt{-t} \ i \ \sigma_{tot} \ \frac{q}{4\pi} \ e^{-a(\sqrt{-t})^2} J_0(\sqrt{-t} \, b) \tag{2.16}$$

But the Fourier-Bessel transform of a Gaussian is itself a Gaussian:

$$\int_0^\infty e^{-ax^2} J_0(xy) \ x \ dx = \frac{1}{2a} \ e^{-y^2/4a} ,$$

giving $$e^{2i\delta(b)} = 1 - \frac{\sigma_{tot}}{4\pi a} e^{-b^2/2a} .\qquad(2.17)$$

Remembering $\ell = qb$, we have:

$$S_\ell = e^{2i\delta_\ell} = 1 - \frac{\sigma_{tot}}{4\pi a} e^{-\frac{\ell^2}{2aq^2}} ,\qquad(2.18)$$

which is the Gaussian model for elastic diffraction scattering.

Re-writing more simply, we have

$$S_\ell = 1 - Ce^{-\gamma\ell^2}\qquad(2.19)$$

where $C \leqslant 1$ for unitarity.

To revert to our original question, how are the Born partial waves modified by the initial and final state effects?   For p-p scattering at 10 GeV, with this crude model of elastic scattering, $C \simeq 0.9$:   thus the S-wave Born term for an initial and final Nucleon-Nucleon state is diminished to $\frac{1}{10}$ of its original value.   All the partial waves are reduced, but the higher partial waves are reduced correspondingly less, since $S_\ell \to 1$ as $\ell \to \infty$. Thus, the effect suggested at the end of Chapter 1, the strong suppression of low partial waves by the initial

and final state interactions, is a consequence of the Absorption Model. This effect of reducing low partial waves can be thought of as giving a longer range to the "centre of gravity" of the exchange forces, and the effectively longer range forces then giving rise to a more sharply collimated scattering peak.

At this point we shall reverse the historical order for the sake of completeness, and discuss some recent approaches to the derivation of absorptive corrections to one-particle exchange terms. Though the absorption model has had considerable success, the derivation of a model for high energy processes by the use of potential-theory is somewhat unsatisfactory, as pointed out above. For this reason a number of authors - Squires (10, 11, 12), Ball and Frazer (13), Omnes (14), Dietz and Pilkuhn (15,16), Trefil (17) and Watson (18) - have sought to give a derivation without recourse to potential models.

The most significant ingredient of these approaches is the attempt to include absorptive corrections by considering the Unitarity equation with inelastic inter-mediate states. In some approaches, the one-particle exchange, or pole, driving terms are also modified by being cast into a unitarized form, a device we shall discuss in

the context of the K-matrix formalism.   A natural
framework for the problem is the N/D formalism of Chew
and Mandelstam (19).   This is used by Squires, and Ball
and Frazer.   There is a correspondence, which we shall
discuss, between the N/D and the K-matrix methods.   The
latter is used by Dietz and Pilkuhn, and by Watson.
Trefil uses both formalisms.

We will consider the many-channel case, and denote
the scattering amplitude in the partial wave ℓ by $\underset{\sim}{A}^{\ell}(s)$;
Writing

$$\underset{\sim}{A}^{\ell}(s) = \underset{\sim}{N}^{\ell}(s) \; \underset{\sim}{D}^{\ell}(s)^{-1} \; , \qquad (2.20)$$

where for the n channel case, $\underset{\sim}{A}$, $\underset{\sim}{N}$ and $\underset{\sim}{D}$ are n × n matrices,
s being (centre of mass energy)^2. $D^{\ell}(s)$ is a real
analytic function in s, except for the unitarity cut on the
positive real axis extending from the lowest threshold to
infinity.   Similarly, $N^{\ell}(s)$ is real analytic except for a
left hand cut and, pessimistically, possible poles on the
left.

To obtain a tractable form for the Unitarity equation,
and hence avoid complicated integral equations, the analysis
has to be restricted to 2-particle intermediate states.
The Unitarity condition on $\underset{\sim}{A}^{\ell}$ is

$$\text{Im} \; \underset{\sim}{A}^{\ell} = \underset{\sim}{A}^{\ell} \; \underset{\sim}{P} \; \underset{\sim}{A}^{\ell} \qquad (2.21)$$

where $\qquad (\underset{\sim}{P})_{ij} = \delta_{ij} \; \theta(s - s_i) \rho_i \rho_j \; ,$

s_i is the threshold for the ith channel,

$\rho_i = \left(2 \dfrac{k_i}{\sqrt{s}} \right)^{\frac{1}{2}}$ is the phase space factor in the ith channel,

k_i is the modulus of the 3-momentum in the centre of mass frame.

Combining (2.20) and (2.21) we have

$$\text{Im} \; \underset{\sim}{D}^{\ell} = \underset{\sim}{P} \; \underset{\sim}{N}^{\ell} \; . \qquad (2.22)$$

Writing a dispersion relation for $\underset{\sim}{D}$ in terms of Im $\underset{\sim}{D}$, and assuming that one subtraction is sufficient, we have

$$\underset{\sim}{D} = \underset{\sim}{I} - \frac{s - s_0}{\pi} \int_{\text{Threshold}}^{\infty} \frac{ds' \; \underset{\sim}{P} \; \underset{\sim}{N}(s')}{(s' - s)(s' - s_0)} \qquad (2.23)$$

where we have normalised $\underset{\sim}{D}(s_0) = 1$, s_0 below threshold.

Following Dalitz (20), we obtain the connection with the K-matrix method. Using (2.22), $\underset{\sim}{D} = \text{Re} \; \underset{\sim}{D} - i \underset{\sim}{P} \underset{\sim}{N}$

$$\therefore \quad \underset{\sim}{A} = \underset{\sim}{N} (\text{Re} \; \underset{\sim}{D} - i \underset{\sim}{P} \underset{\sim}{N})^{-1} \; .$$

Define $\quad \underset{\sim}{K} = \underset{\sim}{N} (\text{Re} \; \underset{\sim}{D})^{-1} \; ,$

then $\quad \underset{\sim}{A} = \underset{\sim}{K} (\underset{\sim}{I} - i \underset{\sim}{P} \underset{\sim}{K})^{-1}$

$$(2.24)$$

This is just the multi-channel K-matrix equation (20, 21).

The Unitarity of the S-matrix corresponding to $\underline{A}$ is assured provided that $\underline{K}$ is real hermitean.

We must now consider what approximations to introduce. In the spirit of peripheralism - or, in analyticity language, the dominance of the near-by singularities in the left - the singularities of N are approximated by one particle exchange forces. Now it is necessary to make some further assumptions to simplify equation (2.24).

Consider some transition A_{12}. Then we have

$$\text{Im } A_{12}^{\ell} = (A_{11}^{\ell})^{*}P_{11}A_{12}^{\ell} + (A_{12}^{\ell})^{*}P_{22}A_{22}^{\ell} + \sum_{n>2} (A_{1n}^{\ell})^{*}P_{nn}A_{n2}^{\ell}$$

$$(2.25)$$

Ball and Frazer truncate this already approximate Unitarity equation by neglecting $\sum_{n>2}$, i.e. all the "indirect" couplings between channels 1 and 2. They are then able to obtain a form resembling (2.12).

$$A_{12}^{\ell} = (S_{11}^{\ell})^{\frac{1}{2}} \tilde{B}^{\ell} (S_{22}^{\ell})^{\frac{1}{2}} \tag{2.26}$$

where

$$\tilde{B}^{\ell} = \frac{1}{2\pi i} \int_{\text{Left}} \frac{dW' D_{11}(W') D_{22}(W') \text{ disc } [B^{\ell}(W')]}{W' - W} \tag{2.27}$$

and W is the total energy in the centre of mass frame,

$$B^{\ell}(W) = \frac{1}{2\pi i} \int_{Left} \frac{dW' \; disc \; [B^{\ell}(W')]}{W' - W}$$

= one particle exchange term.

Now we must ask in what circumstances will (2.26) yield the Sopkovich prescription (2.12). Clearly we require $\tilde{B}^{\ell} \equiv B^{\ell}$. By inspection of (2.27) we see that this will be the case for $D_{ii}^{\ell}(W')$ not significantly different from 1. This will be true in high partial waves, where we shall only see the tail of the strong interaction forces, and in any given partial wave if the forces are weak. So the unfortunate conclusion is that the work of Ball and Frazer can only give a simple prescription for the absorptive modifications where these are small, which is not, of course, the case of interest. Omnes obtains a similar result, which gives a simple expression (in fact the Sopkovich prescription) only in the case of weak absorption.

We noted above that Ball and Frazer truncated the Unitarity equation to the two-channel case. But even at

energies about 2 GeV, elastic scattering plus one inelastic channel is a poor approximation. The approximation becomes rapidly worse as the energy increases past the threshold for higher inelastic channels. Indeed, truncating the system to a two-channel one is in direct contradiction to the effect we are seeking to describe, i.e. the effects of many competing channels. However, even if we restrict ourselves to two particle channels, for most processes of interest the large number of channels open at energies greater than 2 GeV becomes an embarrassment. For instance, consider the k^+p reaction at 3 GeV/c laboratory momentum. One finds that the elastic scattering, plus the following three quasi two-particle processes:- $k^+p \rightarrow kn^*$, $k^+p \rightarrow k^*n$, $k^+p \rightarrow k^*n^*$, account for only 50% of the total cross-section (16).

To include a large number of such channels explicitly requires a knowledge of the coupling constants involved, and of course assumptions about the particle exchange terms dominating in each case. There is thus a danger of introducing so many unknowns that the problem becomes a complicated and devious form of curve-fitting.

Squires (10, 11) suggested a way out of this dilemna. His proposal was essentially that we should assume the

elements of $\underset{\sim}{N}$ to be of equal modulus but random in sign, except that $N_{ij} = N_{ji}$. The assumption of random distribution of sign for the matrix elements of $\underset{\sim}{N}$ gave him a method for estimating their total effect. He was then able to obtain an expression for the off-diagonal amplitude in terms of the elastic scattering amplitude, and the off-diagonal N-matrix element, which was approximated by a one-particle exchange term. Essentially, the relationship between this and the uncorrected peripheral model is as follows - in the latter, $\underset{\sim}{N}$ is given by cross-channel poles, whilst $\underset{\sim}{D}$ is put equal to 1. By making the random phase approximation, one is able to parameterize $\underset{\sim}{D}$ in terms of the elastic scattering amplitude, while using the same approximation for $\underset{\sim}{N}$.

Assuming the phase shift for elastic scattering to be purely imaginary, we have for the partial wave amplitude A_{11}^{ℓ}

$$A_{11}^{\ell} = \frac{\exp(-2\gamma^{\ell}) - 1}{2i} \tag{2.28}$$

where γ^{ℓ} is the (imaginary) phase shift. Squires then shows that the off-diagonal element of the scattering amplitude is given by

$$A_{12}^{\ell} = \left\{ \frac{(1 + \exp(-2\gamma^{\ell}))^2}{4} \right\} B_{12}^{\ell}(\text{pole}) \qquad (2.29)$$

where $B_{12}^{\ell}(\text{pole})$ is the one-particle exchange term.

Thus, once again, a correction formula has been obtained which agrees with the Sopkovich prescription in the limit of weak absorption, i.e. small γ. We note, however, that the maximum absorption possible, as $\gamma \to \infty$, is $\frac{1}{4}$. This is most unfortunate since, as we remarked in Chapter 1, it is quite possible for the S wave to exceed unitarity bounds by factors as large as 200. We must therefore examine the assumptions further.

The random sign hypothesis and the assumption of existence of sufficient numbers of channels required to validate the above statistical arguments will certainly be incorrect at low energies, when only a few inelastic channels are open. However, even at quite moderate energies (5 to 10 GeV), these seem quite plausible assumptions, the plausibility of the random phase assumption being enhanced by crossing-matrix arguments (Squires, private communication). But at presently attainable energies the assumption of equal magnitude for the matrix elements of $\underset{\sim}{N}$ is clearly false. For example, in nucleon-nucleon scattering a small number

of inelastic channels seem to dominate up to 30 GeV,
these channels being characterized by energy independent
"total" cross-sections and vacuum quantum numbers in the
t-channel.   These circumstances are perhaps connected by,
for example, Pomeranchuk dominance.

Squires (12) has since suggested that the failure of
formula (2.29) to give strong absorptive effects is due
to making the unphysical assumption about equal magnitude
for all $\underset{\sim}{N}$ matrix elements, and then analysing the experimental
situation, to which these assumptions do not apply, to
obtain the absorptive corrections.   He has further suggested
that this may be improved in cases where we know a small
number of known channels to be important, and where we can
have some hope of calculating these processes with the
peripheral model as a basis.   The remaining inelastic
channels are treated in the random phase approximation.
This method ensures in practice that the final result
obeys unitarity but still has the vital property that it
enables inelastic processes to be calculated from a
knowledge of the peripheral terms, and of the elastic
scattering phase shift.

Squires has also suggested that the input pole terms
should be approximately unitarized before taking into

account absorptive corrections, and that the N/D
determinantal method should be used for this purpose.   It
is perhaps simpler to look at this problem in the K-matrix
formalism, which is, as we have pointed out, equivalent
to setting $\underset{\sim}{K} = \underset{\sim}{N}/\text{Re } \underset{\sim}{D}$.   Since we are considering
unitarisation in the absence of competing channels, let us,
for simplicity, consider the one-channel case.
Then

$$A_\ell = \frac{K_\ell}{1 - iK_\ell}$$

and in the limit of small A_ℓ, $A_\ell = K_\ell = B_\ell$.   We make the
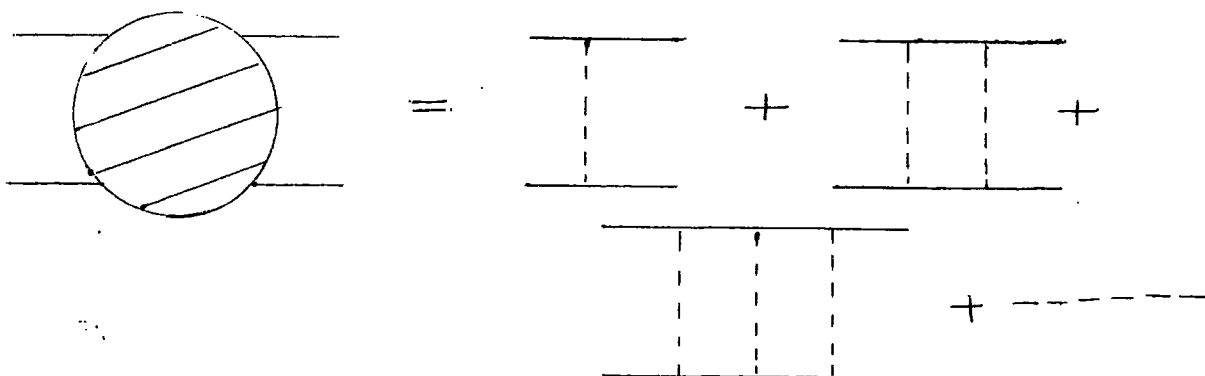identification $K_\ell = B_\ell$ and obtain

$$A_\ell = B_\ell + i\, B_\ell\, A_\ell \qquad\qquad (2.30)$$

which, given B_ℓ real, ensures the unitarity of A_ℓ.   We
then consider iterating (2.30), and see that on unitarisation
of the Born term is equivalent to taking a new set of
perturbation diagrams, essentially a set of ladder graphs.

Consider equation (2.30) diagrammatically;   we have

$$A_\ell \quad = \quad B_\ell \quad + \quad i\, B_\ell\, A_\ell.$$

Iterating,

Because equation (2.30) gives an output within unitarity bounds, even with input Born terms B_ℓ that far exceed the unitarity bound, this treatment of the Born term is often referred to as self-damping.

CHAPTER 3

THE ABSORPTION MODEL AND

NEUTRON-PROTON CHARGE EXCHANGE

In this chapter, we present part of our work on the Absorption Model for the neutron-proton backward (or charge exchange) peak. The work presented should not be thought of only in connection with this very interesting process but also as an investigation of the Absorption Model, e.g. we consider the question of whether the Absorption Model can change the energy dependence of the pole terms. We first briefly review the experimental situation. Then we consider the effect of absorptive modification on the one pion exchange contribution, and compare these results with experiment. We also discuss the effect of absorption on rho-meson exchange (R.M.E.), and investigate the effect of absorptive damping on the energy dependence of the R.M.E.

The experimental situation up to 1962 has been reviewed by Wilson (22), subsequent experiments on n-p charge exchange have been performed by Friedes, Palevsky et al (23) at 3.0 GeV/c lab. momentum, and Manning et al (24) at 8.0 GeV/c. We are here only concerned with relativistic energies, and will discuss the data in the GeV range.

It is found that the charge-exchange peak is small compared with forward elastic scattering, e.g. at 3.7 GeV/c.

$$\frac{d\sigma}{d\Omega} (\theta = 0) \text{ charge exchange} \sim 2.8 \text{ millibarns/Steradian}$$

whereas
$$\frac{d\sigma}{d\Omega} pp^{(\theta=0)} \sim 45 \text{ mb/St.}$$

The most interesting feature of the charge-exchange peak
is the extreme sharpness of the angular distribution.
A measure of the sharpness of this peak is the width of
the differential cross-section at half height, which is
$\simeq 0.02$ $(\text{GeV/c})^2$ at 3.7 GeV/c, and varies little with energy.
This should be compared with a half width of 0.1 $(\text{GeV/c})^2$
at 3.7 GeV/c for pp scattering.

To orientate ourselves about the ranges of force
corresponding to this sharp distribution, consider the
naive model of spinless one-particle exchange discussed
in Chapter 1.   In this crude model, we find that 1 pion
exchange gives a fairly close fit to the shape: it has
a half width of 0.008 $(\text{GeV/c})^2$, whereas ρ exchange gives
a half width of approximately .25 $(\text{GeV/c})^2$.

Finally, to gain some insight into the exchange
mechanism, we must consider the energy dependence of the
process, and also what information the optical Theorem

can furnish on the phase of amplitude. It is found (24)
that the peak is fitted by

$$\frac{d\sigma \ (t=0)}{dt} \sim S^{-2} ,$$

for $\qquad$ 1.26 GeV/c $< p_{1ab} <$ 8 GeV/c ,

i.e. over the whole range of relativistic data. Now, for
the exchange of a particle, spin, J, we have the asymptotic
formula

$$\frac{d\sigma}{dt} \sim S^{2J-2} . \qquad\qquad (3.1)$$

Thus the observed dependence implies that, if single
particle exchange forces are responsible, the significant
exchange is of spin zero.

We must now enquire what information is available on
the phase of the amplitude. This is important because
an unmodified O.P.E. gives a real contribution to the
charge exchange (C.E.) amplitude, and the simple Gaussian
model for the plane wave elastic scattering S-matrix
leaves the phase real after absorptive modification.

Consider isotopic (I)-spin invariance for Nucleon-
Nucleon scattering. This relates the physical N-N
scattering amplitudes to M_0 and M_1, which are the amplitudes

for total I-spin $I = 0$ and $I = 1$, in the direct channel:

$$M_{pp}(\theta, \; p+p \to p+p) = M_1 \; ; \qquad\qquad (3.2)$$

$$M_{np}(\theta, \; n+p \to n+p) = \tfrac{1}{2}(M_1 + M_0) \; ; \qquad (3.3)$$

$$M_{ex}(\theta, \; n+p \to p+n) \equiv M_{np}(\pi - \theta)$$

$$= \tfrac{1}{2}(M_1 - M_0) \; ; \qquad (3.4)$$

θ is the centre of mass scattering angle. Thus by knowing $\sigma_{tot}(np)$ and $\sigma_{tot}(pp)$, the optical Theorem gives us the imaginary part of the spin–averaged forward amplitude for n-p charge exchange, and its contribution to the forward differential cross-section;

$$\frac{d\sigma}{d\Omega} \, ex \, \Bigg|^{(\theta=0)}_{opt} = \left(\frac{q(\sigma_{tot}(pp) - \sigma_{tot}(np))}{4\pi} \right)^2 .$$

Inserting the experimental values for σ_{tot} at 3.7 GeV/c lab. momentum gives

$$\frac{d\sigma}{d\Omega} \, ex \, \Bigg|^{(\theta=0)}_{opt} = 0.27 \; mb/St.$$

But the experimental value is

$$\frac{d\sigma}{d\Omega}^{(\theta=0)}_{ex} = 2.8 \text{ mb/St.}$$

In the absence of strong spin dependence, this indicates that the forward amplitude for n-p charge exchange is mainly real.

The inadequacy of unmodified one pion exchange (O.P.E)

The analysis of the experimental data given above is suggestive of an O.P.E. mechanism for charge exchange — charge exchange requiring an $I = 1$ meson to carry charge in the t-channel. But in our brief discussion of the range of the forces required to give the observed angular distribution we considered only a scalar system. A realistic calculation shows that O.P.E. gives rise to a charge exchange amplitude which is identically zero in the forward direction.

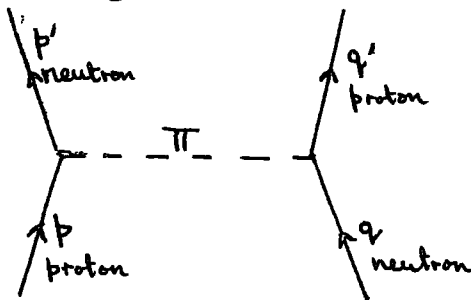The Feynman diagram we consider is

Fig 3.1

where p and q are the initial 4 momenta of the incident proton and neutron, and q' and p' are the final 4 momenta of the outgoing proton and neutron respectively - all these being defined in the centre of mass frame.

The above diagram helps to clarify the somewhat confusing terminology for this process. One may speak of the backward n-p amplitude, or as in equation (3.4), the charge exchange amplitude. The relationship of the angles corresponding to the two ways of looking at the phenomenon is then clear: backward n-p scattering becomes forward n-p charge exchange scattering. So, in the centre of mass frame, an elastic scattering angle $(\pi - \theta)$ corresponds to a charge exchange scattering angle of θ. In future we shall use the charge exchange angle. Thus, in fig (3.1), $p = p'$ corresponds to forward charge exchange, or backward elastic scattering.

Now the above diagram gives the following contribution to the centre of mass amplitude, using the notation of Cziffra, Macgregor, Moravcsik and Stapp (26): -

$$M_{rs,r's'}(p,q;p'q') = - \frac{C g^2 m^2}{2k^2 E} \left[\frac{\overline{u}_r(p) \gamma_5 u_{r'}(p') \overline{u}_s(q) \gamma_5 u_{s'}(q')}{x_0 - x} \right]$$

$$(3.5)$$

where $u_r(p)$ etc. are the Dirac 4 spinors, spin index r;

$$x = \cos \theta_{c\ m}\ , \quad x_0 = 1 + \left(\frac{\mu^2}{2k^2}\right)\ ,$$

μ is the pion mass, k the centre of mass 3-momentum, m is the nucleon mass, E the total energy of one nucleon, g^2 the rationalised π-N coupling constant, = 14.4, and C is an isospin factor.

To show that this term vanishes in the forward direction it is only necessary to consider the expression for the vertex,

$$\bar{u}_r(p)\gamma_5\ u_{r'}(p') \qquad\qquad (3.6)$$

where $\bar{u}_r$ and $u_{r'}$ satisfy their respective Dirac equations,

$$\bar{u}_r(p)\ (\not{p} - m) = 0 \qquad\qquad (3.7)$$

and

$$(\not{p}' - m)\ u_{r'}(p') = 0 \qquad\qquad (3.8)$$

$$\not{p} = \underline{p}_0\underline{\chi} - p_0\gamma_0$$

Multiplying (3.7) on the right by $\gamma_5\ u_{r'}(p')$ and (3.8) on the left by $\bar{u}_r(p)\gamma_5$, we obtain

$$\bar{u}_r(p)(\not{p} - m)\gamma_5\ u_{r'}(p') = 0 \qquad\qquad (3.9)$$

and $$\bar{u}_r(p)\gamma_5(\not{p}' - m)u_{r'}(p') = 0 \qquad (3.10)$$

Adding and rearranging equations (3.9) and (3.10):

$$\bar{u}_r(p)\gamma_5 u_{r'}(p') = \frac{1}{2m}\bar{u}_r(p)(\not{p}\gamma_5 + \gamma_5\not{p}')u_{r'}(p') \qquad (3.11)$$

Since γ_5 anticommutes with γ_0, γ_1, γ_2, γ_3, we have

$$\gamma_5\not{p} = -\not{p}\gamma_5 .$$

Substituting this into equation (3.11),

$$u_r(p)\gamma_5 u_{r'}(p') = \frac{1}{2m}\bar{u}_r(p)\gamma_5(\not{p}' - \not{p})u_{r'}(p') . \qquad (3.12)$$

To simplify (3.12) further, we require an explicit representation of the γ matrices and 4-spinors. This is not necessary for our present purpose, since we see that in the forward direction $p = p'$ and therefore $\not{p}' - \not{p} = 0$. Thus we find that both the vertex factors of equation (3.5) vanish in the forward direction, establishing our assertion that the O.P.E contribution vanishes at $\theta = 0$. The above analysis also demonstrates a very pertinent point:- the vanishing of the amplitude for $\theta = 0$ cannot be changed by any allowed (i.e. non-singular) form factor.

Absorptive corrections to O.P.E.

We now discuss the effect of the initial and final state corrections to the O.P.E. model for n-p charge exchange.

These corrections are introduced via the Sopkovich prescription (2.12). We must first obtain the partial wave amplitudes for this process. We use the conventional singlet-triplet notation of Stapp (27); this has the advantage that the angular momentum decomposition is naturally in terms of the orbital angular momentum ℓ, which is in the spirit of the derivation of (2.12). The elegant helicity formalism of Jacob and Wick (28) gives rise to an angular momentum decomposition in terms of J, the total angular momentum.

We give a brief account of the notation here, leaving the details to Appendix 1. To describe the spin of the system, the scattering matrix M is sandwiched between states of total intrinsic spin and the projection of this spin along some arbitrary direction –

$$< S, S_z | M | S', S_z' > \ .$$

Now for N-N scattering S can only be 0 or 1. Furthermore, for p-p scattering, statistics and conservation of parity forbid transitions between singlet and triplet spin states. The same rule applies to n-p scattering in the limit of I-spin conservation.

Thus we can write the amplitude in spin space as a 4×4 matrix,

$$
\underline{M} = \begin{pmatrix}
M_{ss} & 0 & 0 & 0 \\
0 & M_{11} & M_{10} & M_{1-1} \\
0 & M_{01} & M_{00} & M_{0-1} \\
0 & M_{-11} & M_{-10} & M_{-1-1}
\end{pmatrix}
\tag{3.13}
$$

where the indices on the 3×3 sub-matrix refer to the projection of the spin in the final and initial states respectively, and M_{ss} is the singlet-singlet amplitude. Of these, only 5 are independent - the number is reduced by the following relations from rotational invariance.

$$
M_{-1-1} = M_{11}, \quad M_{01} = -M_{0-1},
$$
$$
M_{-11} = M_{1-1}, \quad M_{10} = -M_{-10}
\tag{3.14}
$$

From time reversal we have (29)

$$
M_{11} - M_{1-1} - M_{00} = \sqrt{2} \cot \theta (M_{10} + M_{01})
\tag{3.15}
$$

The differential cross-section is given by

$$
\frac{d\sigma}{d\Omega} = \tfrac{1}{2}|M_{11}|^2 + \tfrac{1}{4}|M_{00}|^2 + \tfrac{1}{4}|M_{ss}|^2 + \tfrac{1}{2}|M_{10}|^2
$$
$$
+ \tfrac{1}{2}|M_{01}|^2 + \tfrac{1}{2}|M_{1-1}|^2
\tag{3.16}
$$

Following C.M.M.S. (see also Grashin (30)), equation (3.5) becomes, in the singlet-triplet notation and taking the $(I = 1) - (I = 0)$ combination:

$$M_{ss} = -(g^2/4\pi)\alpha/2E$$

$$M_{11} = -(g^2/4\pi)\alpha(1 - x)/4E$$

$$M_{oo} = -(g^2/4\pi)\alpha x/2E \qquad (3.17)$$

$$M_{10} = M_{01} = (g^2/4\pi)\sqrt{2}\sin\theta\ \alpha/4E$$

$$M_{1-1} = -(g^2/4\pi)\alpha(1 + x)/4E$$

So
$$\frac{d\sigma}{d\Omega} = (g^2/4\pi)^2(\alpha/2E)^2 , \qquad (3.18)$$

where recalling the previous definitions for convenience

$$x = \cos\theta, \qquad x_0 = 1 + \frac{\mu^2}{2k^2} ,$$

θ the centre of mass charge exchange angle,

k the centre of mass momentum,

$$\alpha = \frac{1 - x}{x_0 - x} , \qquad E = \sqrt{m^2 + k^2} .$$

We now require the partial wave decomposition of the $\underline{M}$ matrix, and the partial waves corresponding to (3.17). The diagonal terms of $\underline{M}$ are expanded in terms of $P_\ell(x)$; the terms with one unit of spin-flip, e.g. M_{01}, in terms of $P_\ell{}^1(x)$; and those terms representing double spin-flip, e.g. M_{1-1}, in terms of $P_\ell{}^2(x)$. $P_\ell{}^m(x)$ are the associated Legendre functions, as defined in Blatt and Weisskopf (31). For a fixed value of total angular momentum J there are two possible values for the orbital angular momentum ℓ (in either initial or final state): $\ell = J$, and $\ell = J \pm 1$. This second class can only occur in the triplet state, and by conservation of parity the two classes cannot be coupled.

By conservation of total angular momentum J and its projection M, the partial waves have the angular momentum structure $< \ell,\ S,\ J,\ M |R| \ell',\ S',\ J,\ M >$, where $s = s'$ since for the case of interest singlet-triplet transitions are forbidden. Then for the class $\ell = J$ we have

$$< J,\ 0,\ J,\ M\ |R|\ J,\ 0,\ J,\ M > = \alpha_J \qquad (3.19)$$

$$< J,\ 1,\ J,\ M\ |R|\ J,\ 1,\ J,\ M > = \alpha_{\ell J} = \alpha_{\ell\ell}, \qquad (3.20)$$

and for $\ell = J \pm 1$ we have

$$< J \pm 1,\ 1,\ J,\ M\ |R|\ J \mp 1,\ 1,\ J,\ M > = \alpha_{\pm}{}^J \qquad (3.21)$$

$$\text{by time reversal} \qquad = \alpha^J \qquad (3.22)$$

$$< J + 1, 1, J, M \ |R| \ J + 1, 1, J, M > = \alpha_{\ell J} = \alpha_{\ell, \ell-1}$$

$$(3.23)$$

$$< J - 1, 1, J, M \ |R| \ J - 1, 1, J, M > = \alpha_{\ell J} = \alpha_{\ell, \ell+1}$$

$$(3.24)$$

We note that all the partial waves are diagonal in ℓ, except for α^J, where the initial and final states differ by 2 units of orbital angular momentum.

The rather complicated partial wave decomposition of $\underset{\sim}{M}$ is then (27)

$$M_{ss}(\theta) = (ik)^{-1} \sum_{\ell=0}^{\infty} P_\ell(\theta) \ \frac{2\ell+1}{2} \ \alpha_\ell \qquad (3.25)$$

$$M_{11}(\theta) = (ik)^{-1} \sum_{\ell=0}^{\infty} P_\ell(\theta) \left\{ \left(\frac{\ell+2}{4} \right) \alpha_{\ell, \ell+1} + \left(\frac{2\ell+1}{4} \right) \alpha_{\ell, \ell} \right.$$

$$+ \left(\frac{\ell-1}{4} \right) \alpha_{\ell, \ell-1} - \tfrac{1}{4}[(\ell+1)(\ell+2)]^{\frac{1}{2}} \alpha^{\ell+1} - \tfrac{1}{4}[(\ell-1)\ell]^{\frac{1}{2}} \alpha^{\ell-1} \left. \right\},$$

$$(3.26)$$

$$M_{oo}(\theta) = (ik)^{-1} \sum_{\ell=0}^{\infty} P_\ell(\theta) \left\{ \left(\frac{\ell+1}{2}\right) \alpha_{\ell,\ell+1} + \left(\frac{\ell}{2}\right) \alpha_{\ell,\ell-1} + \right.$$

$$\left. + \tfrac{1}{2}[(\ell+1)(\ell+2)]^{\frac{1}{2}} \alpha^{\ell+1} + \tfrac{1}{2}[(\ell-1)\ell]^{\frac{1}{2}} \alpha^{\ell-1} \right\} , \qquad (3.27)$$

$$M_{o1}(\theta) = (ik)^{-1} \sum_{\ell=1}^{\infty} P_\ell^{\ 1}(\theta) \left\{ -\frac{\sqrt{2}}{4}\left(\frac{\ell+2}{\ell+1}\right) \alpha_{\ell,\ell+1} + \right.$$

$$+ \frac{\sqrt{2}}{4}\left(\frac{2\ell+1}{\ell(\ell+1)}\right) \alpha_{\ell,\ell} + \frac{\sqrt{2}}{4}\left(\frac{\ell-1}{\ell}\right) \alpha_{\ell,\ell-1} + \frac{\sqrt{2}}{4}\left(\frac{\ell+2}{\ell+1}\right)^{\frac{1}{2}} \alpha^{\ell+1} -$$

$$\left. - \frac{\sqrt{2}}{4}\left(\frac{\ell-1}{\ell}\right)^{\frac{1}{2}} \alpha^{\ell-1} \right\} , \qquad (3.28)$$

$$M_{1o}(\theta) = (ik)^{-1} \sum_{\ell=1}^{\infty} P_\ell^{\ 1}(\theta) \left\{ \frac{\sqrt{2}}{4} \alpha_{\ell,\ell+1} - \frac{\sqrt{2}}{4} \alpha_{\ell,\ell-1} + \right.$$

$$\left. + \frac{\sqrt{2}}{4}\left(\frac{\ell+2}{\ell+1}\right)^{\frac{1}{2}} \alpha^{\ell+1} - \frac{\sqrt{2}}{4}\left(\frac{\ell-1}{\ell}\right)^{\frac{1}{2}} \alpha^{\ell-1} \right\} , \qquad (3.29)$$

$$M_{1-1}(\theta) = (ik)^{-1} \sum_{\ell=2}^{\infty} P_{\ell}^{2}(\theta)\left\{\left(\frac{1}{4(\ell+1)}\right)\alpha_{\ell,\ell+1}\right.$$

$$-\left(\frac{2\ell+1}{4\ell(\ell+1)}\right)\alpha_{\ell,\ell} + \frac{1}{4\ell}\alpha_{\ell,\ell-1} - \tfrac{1}{4}[(\ell+1)(\ell+2)]^{-\frac{1}{2}}\alpha^{\ell+1}$$

$$\left. - \tfrac{1}{4}[(\ell-1)\ell]^{-\frac{1}{2}}\alpha^{\ell-1}\right\} \text{,} \tag{3.30}$$

the above summations extend over both even and odd ℓ, since the charge exchange amplitude is a combination of $I = 1$ and $I = 0$.

From C.M.M.S. (26) for one pion exchange, the $(I = 1) - (I = 0)\alpha$'s are:-

$$\alpha_{\ell,\ell+1}^{P} = -\frac{ik(g^{2}/4\pi)}{E(2\ell+3)}[Q_{\ell+1}(x_{0}) - Q_{\ell}(x_{0})] \tag{3.31}$$

$$\alpha_{\ell,\ell-1}^{P} = -\frac{ik(g^{2}/4\pi)}{E(2\ell-1)}[Q_{\ell}(x_{0}) - Q_{\ell-1}(x_{0})] \tag{3.32}$$

$$\alpha_{\ell,\ell}^{P} = -\frac{ik(g^{2}/4\pi)}{E(2\ell+1)}[\ell Q_{\ell+1}(x_{0}) + (\ell+1)Q_{\ell-1}(x_{0}) -$$

$$- (2\ell+1)Q_{\ell}(x_{0})] \tag{3.33}$$

$$\alpha^{JP} = -\frac{ik(g^{2}/4\pi)}{E(2\ell+1)}[J(J+1)]^{\frac{1}{2}} \times [Q_{J+1}(x_{0}) + Q_{J-1}(x_{0})$$

$$- 2Q_{J}(x_{0}) \tag{3.34}$$

$$\alpha_\ell^P = \frac{ik(g^2/4\pi)}{E}[(x_0 - 1)Q_\ell(x_0) - \delta_{\ell_0}] \tag{3.35}$$

δ_{ℓ_0} is the Kronecker delta,

and the $Q_\ell(x_0)$ are the Legendre functions of the 2nd kind as defined in Morse and Feshbach (32).

To introduce absorptive corrections using (2.12), we use empirical studies of p-p diffraction scattering (expecting n-p to be similar), in which the phase shift at a given energy is assumed to depend on the orbital angular momentum ℓ only - i.e. the assumption of spin independence. The unmodified and modified partial waves diagonal in ℓ - α_ℓ; $\alpha_{\ell,\ell}$; $\alpha_{\ell,\ell+1}$; $\alpha_{\ell,\ell-1}$ - which we represent as b_ℓ and A_ℓ respectively are then simply related by (2.12):

$$A_\ell = \sqrt{S_\ell}\, b_\ell\, \sqrt{S_\ell}, \tag{3.36}$$

whereas α^J is modified by

$$A^J = \sqrt{S_{J-1}}\, \alpha^J\, \sqrt{S_{J+1}}. \tag{3.37}$$

We stress that it would not be correct simply to expand $\underline{M}$ in terms of Legendre functions P_ℓ, P_ℓ^1, and P_ℓ^2 and damp the coefficients according to ℓ, since some

coefficients contain coupling from $\ell \rightarrow \ell \pm 2$. Thus mis-treating the coupling terms would violate the time-reversal symmetry of the S-matrix, and equation (3.15) would not be satisfied.

The modification of the partial wave amplitudes and their resummation is not possible by analytic means, and it was therefore necessary to do this by computer. Details will not be given here, except to remark/the basic on techniques for minimising the complexity of these summations when the exchange force has a longer range than the absorptive corrections.

Writing the unmodified, or pole, amplitude $M^P = \sum_\ell M_\ell^P$,

and the final amplitude M,

$$M = M^P - \sum_{\ell=0}^{\ell_{max}} M_\ell^P + \sum_{\ell=0}^{\ell_{max}} \sqrt{S_\ell}\, M_\ell^P \sqrt{S_\ell} \, , \qquad (3.38)$$

where ℓ_{max} is the partial wave where the absorption has become negligible. Then it is not necessary to sum over all significantly large partial waves, but only over those which are significantly modified. A very slight error – an end effect – is introduced in this way, since in $\sum_{\ell_{max}+1}^{\infty} M_\ell$

there will be terms connecting to ℓ_{max} and $\ell_{max}-1$. The

programme was constructed so that the phenomenological

fits to S_ℓ were input data, which allowed us to investigate

the sensitivity of the output to the exact form of S_ℓ.

The phenomenological model first used is due to

Serber (33), who constructed an optical potential - imaginary

Yukawa at short range and imaginary Gaussian at long range -

which gives excellent fit to the p-p data at a wide range

of angles and energies. This gives, of course, purely

imaginary phase shifts. In the impact parameter (b)

formalism, the phase shift

$$\delta(b) = i\chi(b), \quad \chi(b) \text{ real}$$

In the region $0 < b < .33F$

$$\chi(b) = -(1 + \tfrac{1}{4}\Lambda^2 b^2)\ell n \,\tfrac{1}{2}\gamma\Lambda b + \tfrac{1}{4}\Lambda^2 b^2 ,$$

where Λ, the inverse range of the Yukawa potential is

$1.34\ F^{-1}$, and γ is Euler's constant ~ 0.557.

$$\therefore \quad e^{2i\delta(b)} = S_b = e^{2(1 + \tfrac{1}{4}\Lambda^2 b^2)\ell n \tfrac{1}{2}\gamma\Lambda b}\ e^{-2\Lambda^2 b^2} \qquad (3.39)$$

The first factor $\to 0$ as $b \to 0$, and the second $\to 1$ as $b \to 0$;

so this model gives full absorption of the S-wave.

At long range, $b \geqslant 1.1\ F$,

$$\chi(b) = Ae^{-\lambda^2 b^2} ,$$

where A = .454, and λ^2 = 1.224. Serber gives numerical values for interpolation in the region $0.33F \leqslant b \leqslant 1.1F$. It is not surprising, since the above rather complicated form fits the exponential peak, that the result resembles the Gaussian form (2.19) with C = 1.

The cross-sections for n-p charge exchange predicted from O.P.E. with Serber damping are shown by the solid lines in figures (3.2) and (3.3) for 3.7 and 8.0 GeV/c respectively. Also shown are the experimental results of Friedes (23) and Manning (24), and the unmodified O.P.E. cross-sections (dashed line). It can be seen that the absorptive damping of O.P.E. produces remarkable changes. The dip at θ_{ex} = 0 is converted into a very narrow peak — which gives a good fit to the experimental angular distribution out to the half-width. The magnitude of the unmodified O.P.E. is greatly reduced at wider angles, but leaving a broad secondary maximum which is not seen experimentally.

We must now consider in a little more detail the origin of the results shown. Unfortunately this must be done numerically, and we here summarize and discuss our results. The narrow forward peak comes dominantly from the M_{ss} and M_{oo} amplitudes. If we consider these amplitudes in terms
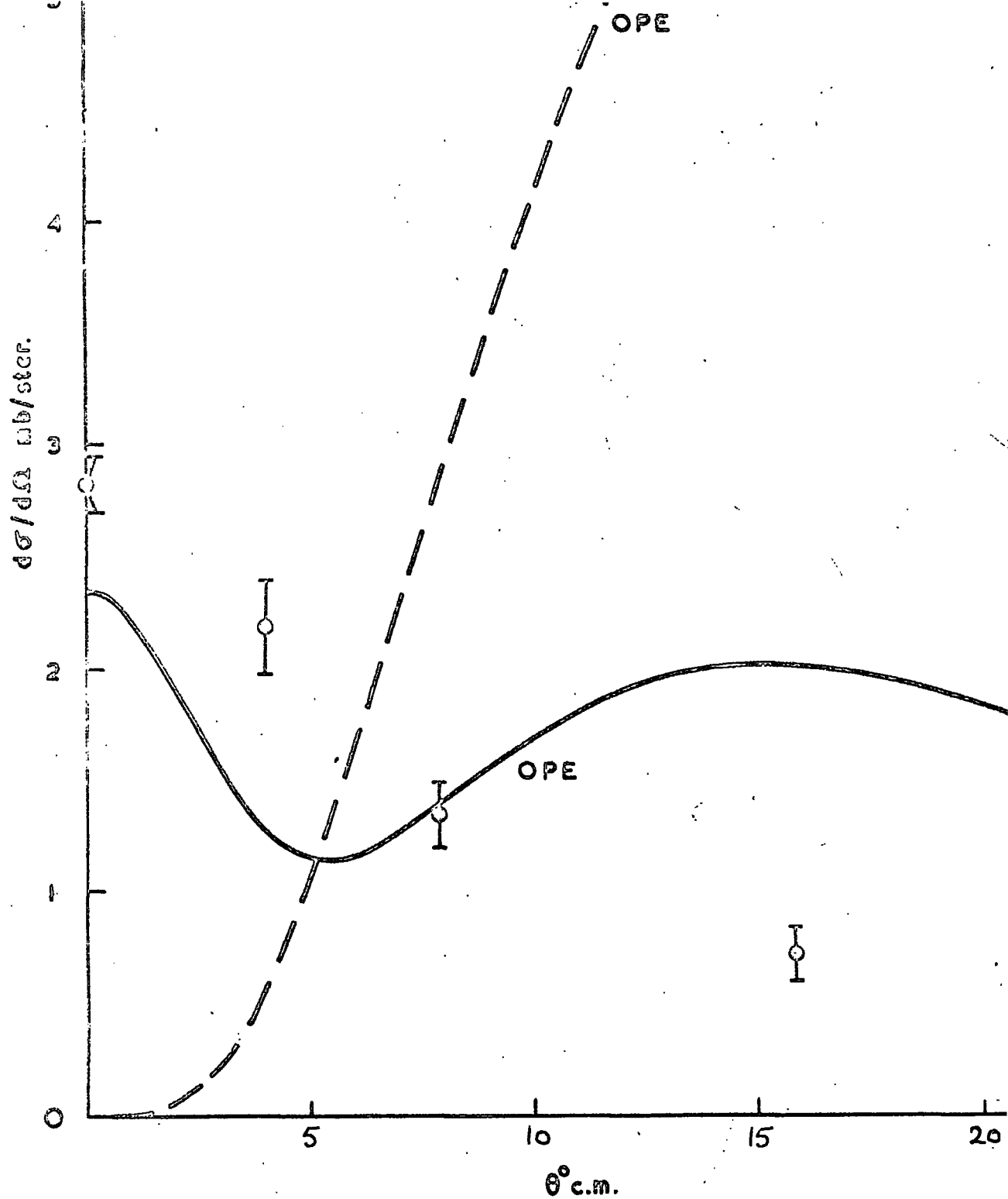
Fig. (3.2)

n-p Charge exchange at 3.7 GeV/c. The solid line refers
to the Absorptively modified O.P.E. exchange, the dashed
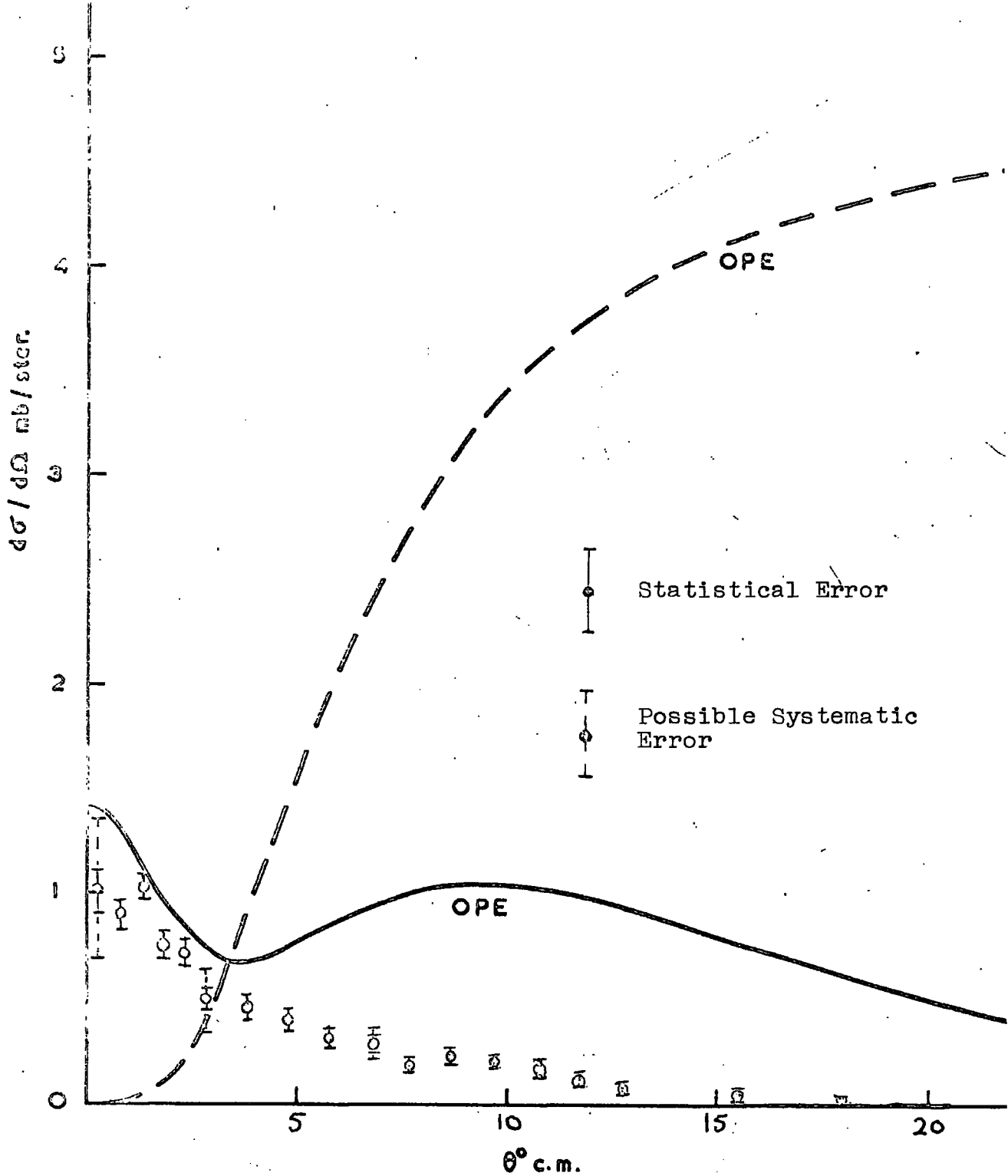line to the unmodified O.P.E. exchange.

Fig. (3.3)

n-p charge exchange at 8.0 GeV/c . The solid line refers to the Absorptively modified O.P.E. exchange, the dashed line to the unmodified O.P.E. exchange.

of partial waves, we see that the vanishing of both these from unmodified O.P.E. (as demonstrated earlier in this Chapter) is due to the S-wave contribution being of opposite sign to the other partial waves, which sum to cancel the S-wave contribution identically in the forward direction. The absorptive corrections remove the S-wave terms, but the higher partial waves still sum to give a non-vanishing contribution. This can be seen from equation (3.39) or the simpler (2.19), since absorption decreases steadily as ℓ increases. Thus, the absorptive corrections destroy the balance of terms which caused M_{ss} and M_{00} to vanish, and these terms now give a sharp peak in good agreement with the small angle experimental data.

The secondary bump comes mainly from the M_{1-1} amplitude. M_{1-1}, damped or undamped, vanishes at $\theta = 0$ because of rotational invariance. One need only consider the partial sum (3.17) for M_{1-1}, which is in terms of $P_\ell^2(\theta)$: now

$$P_\ell^m(\theta) = \left[\frac{2\ell+1}{4\pi} \frac{(\ell-m)!}{(\ell+m)!} \right]^{\frac{1}{2}} \sin^m\theta \frac{d^m}{d(\cos\theta)^m} P_\ell(\cos\theta): \quad \text{thus}$$

the $\sin^2\theta$ factor in all terms of M_{1-1} gives a vanishing amplitude at $\theta = 0$, independent of the dynamics. The M_{1-1} amplitude from undamped O.P.E. gives a large contribution to $\frac{d\sigma}{d\Omega}$, and after damping still gives a significant

contribution at large angles. It is worth pointing out
that the partial wave sum for M_{1-1} is for $\ell \geqslant 2$, and at
3.7 GeV/c the absorption, complete in the S-wave, has dropped
to a factor of 75% for the D wave. It does not appear
possible to suppress the M_{1-1} contribution with any spin
independent set of S_ℓ consistent with elastic scattering.

To summarize the results of our calculation, we may
conclude: -

1) Absorptive modification (damping) goes far to reconcile
   O.P.E. with experiment. We have good agreement in
   angular distribution and magnitude down to the half-
   width. This is true at 3.7 GeV/c and 8.0 GeV/c,
   indicating a good fit to the experimental energy
   dependence. Since unmodified O.P.E. fits the experi-
   mental E-dependence, this means that absorptive
   modifications have not significantly changed the E-
   dependence. A secondary peak is, however, predicted
   and not seen experimentally.

2) The secondary peak is a spin-flip effect, coming mainly
   from the M_{1-1} (double spin-flip) amplitude.

3) The M_{1-1} terms follow directly from the spin dependence
   of O.P.E., and it cannot consistently be removed by spin

independent absorptive modifications.

4) Finally, we stress that such agreement as we obtain is in no sense due to curve fitting with arbitrary parameters.

We now consider rho meson exchange (R.M.E.) separately. Many R.M.E. models use a Reggeized ρ; but it is far from clear how far the effects of initial and final state absorption are implicitly included in a Regge pole. This is a point we shall discuss later, and we therefore only consider here "elementary R.M.E." Undamped R.M.E. is known to have unacceptable energy dpeendence, and would give

$$\frac{d\sigma_{ex}}{dt} \sim \text{constant}, (\text{equation } (3.1)); \quad \text{whereas experimentally}$$

$$\frac{d\sigma_{ex}}{dt} \sim \frac{1}{S^2}. \quad \text{It is important to investigate whether the}$$

absorption model can change this behaviour. Furthermore, at any particular energy, R.M.E. illustrates the general case of short range, spin independent charge exchange; it is therefore interesting to see how strongly the angular distribution is altered by damping.

Near $\theta = 0$, the elementary R.M.E. amplitude for charge exchange is, to a good approximation (exactly at $\theta = 0$), both

spin independent and determined by the vector $\overline{N}N\rho$ coupling. It has the form (Perring and Phillips (34))

$$a = \frac{f^2}{4\pi}\frac{1}{2E}\left\{\frac{(m+E)^2}{2k^2} + 1 + 2x + \frac{k^2 x^2}{2(E+m)^2}\right\}\bigg/(x_1 - x),$$

$$(3.40)$$

where $\frac{f^2}{4\pi}$ is the $\overline{N}N\rho$ vector coupling constant,

and $\quad x_1 = 1 + {}^m\rho^2\big/_{2k^2}$.

In terms of the $\underset{\sim}{M}$-matrix, $M_{11} = M_{00} = -M_{ss} = a$, and the spin-flip terms are zero. This is therefore spin independent in the sense of forward charge exchange.

In the small angle approximation considered here, the coupling between different orbital angular momenta is negligible. We set $\alpha^J = 0$. From Perring and Phillips, the $(I = 1) - (I = 0)$ combination for the unmodified R.M.E. partial waves is then:-

$$\alpha_\ell = i.2\left(\frac{f^2}{4\pi}\right)\frac{k^2 + E^2}{Ek}Q_\ell(x_1)$$

$$\alpha_{\ell,\ell} = i.2\left(\frac{f^2}{4\pi}\right)\frac{k}{E}\left\{\frac{E^2}{k^2}Q_\ell(x_1) + \frac{(\ell+1)Q_{\ell-1}(x_1) + \ell Q_{\ell+1}(x_1)}{2\ell + 1}\right\}$$

$$\alpha_{\ell,\ell+1} = i.2\left(\frac{f^2}{4\pi}\right)\frac{1}{Ek(2\ell+3)}\left\{[(2\ell+1)mE+E^2]Q_\ell(x_1) + [(3\ell+4)k^2\right.$$

$$\left. + (\ell+1)(E-m)^2 x_1]Q_{\ell+1}(x_1)\right\}$$

$$\alpha_{\ell,\ell-1} = i \cdot 2 \left(\frac{f^2}{4\pi} \right) \frac{1}{Ek(2\ell-1)} \left\{ [2\ell mE - E^2]Q_\ell(x_1) + [(3\ell-1)k^2 \right.$$

$$\left. + \ell(E-m)^2 x_1]Q_{\ell-1}(x_1) - (E-m)^2 \delta_{\ell 1} \right\}$$

The partial waves are then modified as for O.P.E., and resummed to give the Absorption Model amplitude for R.M.E. The resultant unmodified and modified cross-sections for n-p charge exchange are shown in figures (3.4) and (3.5) for $p_{\ell ab}$ = 3.7 and 8.0 GeV/c respectively. We have taken $\frac{f^2}{4\pi}$ = 2.0, but little confidence should be attached to this value as there is considerable disagreement in the literature. Our conclusions are, however, unaffected by the exact numerical value for $\frac{f^2}{4\pi}$.

As expected, absorptive corrections narrow the R.M.E. peak. For instance, at 8 GeV/c the unmodified half-width is ~ 15°, which is reduced by absorptive modifications to ~ 10°. This modified value is still far from the experimental half-width of ~ 3°.

Absorption greatly reduces the magnitude of the R.M.E. contribution, decreasing $\frac{d\sigma}{d\Omega}(\theta=0)$ by a factor of about 14 in this case. This is much greater than for O.P.E., where $\left| M_{1-1} \right|^2$ is reduced by factors of less than 2 up to 10°.
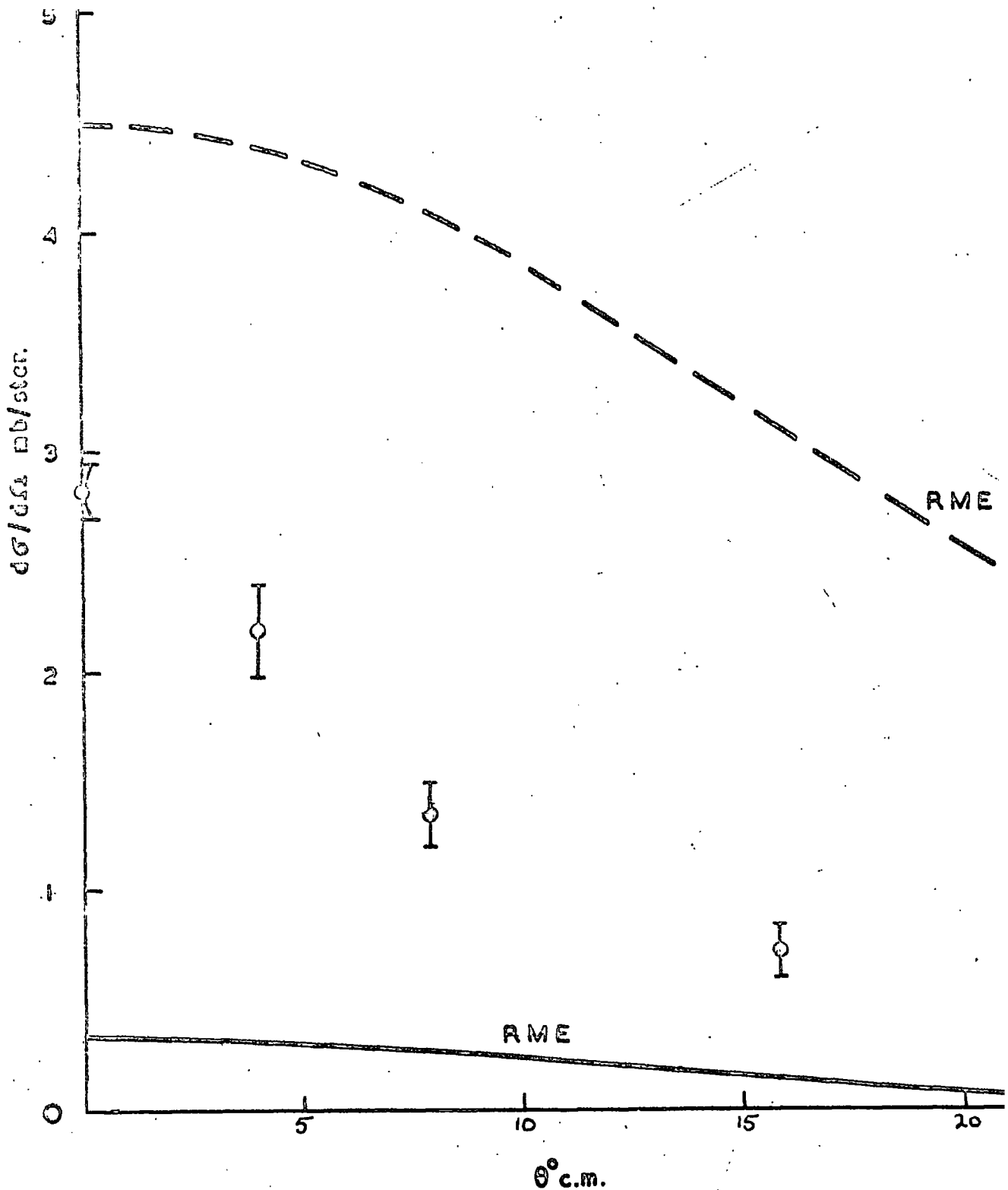
Fig. (3.4)

n-p charge exchange a t 3.7 GeV/c.   The theoretical curves
are scaled down by 1/25.   Damped and undamped cases are
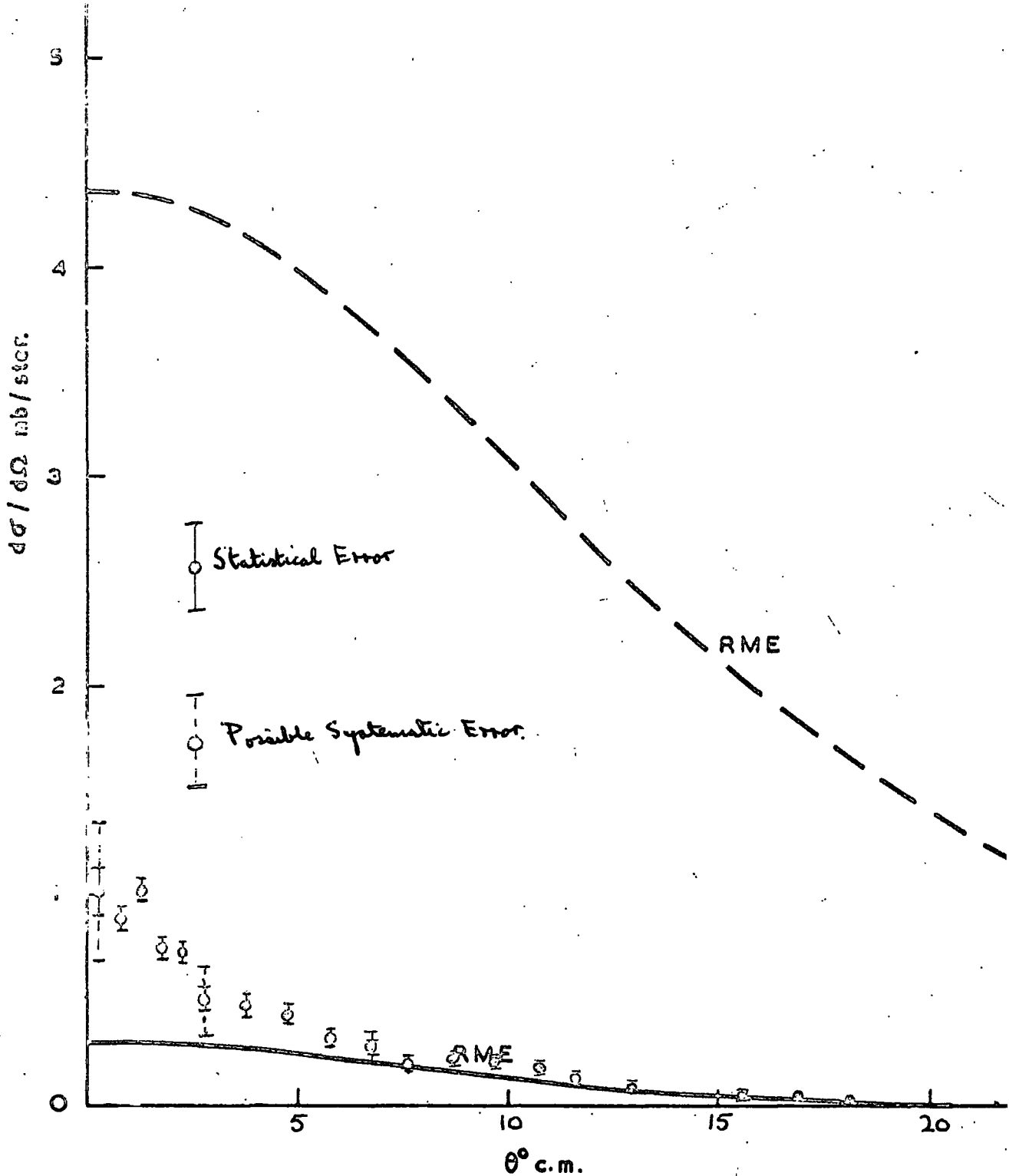distinguished by solid and dashed curves respectively.

Fig. (3.5)

n-p charge exchange at 8.0 GeV/c.   The theoretical curves
are scaled down by a factor of 1/60.   Damped and undamped
cases are distinguished by solid and dashed curves
respectively.

The reason for the the much more severe reduction for R.M.E. is that the ρ has a much shorter range than the π (approximately .28 F compared with 1.4 F). Since absorption is stronger in low partial waves, most of the significant partial waves for ρ exchange are severely damped. However, for π exchange at 3.7 GeV/c, at an impact parameter of 1.4 F corresponding to $\ell = 7$, 90% of the undamped partial wave survives after damping. Thus we see that the contribution of high mass exchange particles is much more reduced by absorption than the contribution of low mass exchange.

Energy dependence in the Absorption Model

The most significant conclusion to be drawn from the R.M.E. results concerns the effect of absorptive modifications on the energy dependence of the ρ exchange contribution. We observe from ρ exchange that

$$\frac{d\sigma}{d\Omega}(\theta=0)\bigg|_{\text{unmodified}} \bigg/ \frac{d\sigma}{d\Omega}(\theta=0)\bigg|_{\text{modified}} \simeq 14.0 \text{ at } 3.7 \text{ GeV/c },$$

$$\frac{d\sigma}{d\Omega}(\theta=0)\bigg|_{\text{unmodified}} \bigg/ \frac{d\sigma}{d\Omega}(\theta=0)\bigg|_{\text{modified}} \simeq 14.4 \text{ at } 8.0 \text{ GeV/c }.$$

Thus we find, numerically, that to a very good approximation

(i.e. within 3% for the cross-section and 1.5% for the amplitude) the absorptive corrections just scale the amplitude down by the same amount at the two different energies. <u>This means that the modified terms have the same energy dependence as the unmodified pole terms.</u>

We can investigate how this comes about using the impact parameter formalism discussed in Chapter 2. Consider first equation (3.40):

$$a = \frac{f^2}{4\pi} \frac{1}{2E} \left\{ \frac{(m+E)^2}{2k^2} + 1 + 2x + \frac{k^2 x^2}{2(E+m)^2} \right\} \Big/ (x_1 - x)$$

Rearranging we have

$$a(t,k) = \frac{f^2}{4\pi} \frac{1}{2E} \left\{ \left[\frac{(m+E)^2 + 2k^2}{2k^2} + 2x_1 + \frac{k^2 x_1^2}{2(m+E)^2} \right] \Big/ (x_1 - x) \right.$$
$$\left. + \left[-2 - \frac{k^2 x_1}{2(m+E)^2} - \frac{k^2 x}{2(m+E)^2} \right] \right\} \tag{3.41}$$

We note that the second term of equation (3.41) simply represents additional S and P-wave terms, which vanish as $k \to \infty$. The neglect of these terms gives a good approximation in the GeV range; e.g. at 8 GeV/c the approximation introduces an error of only 4% in the undamped

amplitude. We therefore restrict ourselves to considering the energy dependence of

$$a(t,k) = \frac{f^2}{4\pi} \frac{1}{2E} \left\{ \frac{(m+E)^2+2k^2}{2k^2} + 2x_1 + \frac{k^2 x_1^2}{2(m+E)^2} \right\} \bigg/ (x_1 - x)$$

(3.42)

which we write more simply as $a(t,k) = F(k)/(x_1-x)$.

We rewrite $\dfrac{1}{x_1-x}$ as $\dfrac{1}{\varepsilon^2+\omega^2}$, where $\omega = 2 \sin \frac{1}{2}\theta$, $\varepsilon^2 = \dfrac{m_\rho^2}{2k^2}$.

Now, the essential tool for our analysis is the identity (35)

$$\frac{1}{\varepsilon^2+\omega^2} = \int_0^\infty \ell \, d\ell \, J_0(\omega\ell) K_0(\varepsilon\ell) \qquad (3.43)$$

Where K_0 is the modified Hankel function of zero order. Comparing (3.43) with the partial wave integral (3.4)

$$M_{fi} \sim 2\int_0^\infty \ell \, d\ell \, J_0(\omega\ell) A(\ell),$$

where $\ell = kb$, ℓ is the orbital angular momentum, k the centre of mass momentum and b the impact parameter, we find the partial wave amplitude $A(\ell)$ corresponding to

an amplitude $\dfrac{1}{\varepsilon^2+\omega^2}$ is $\dfrac{K_0(\varepsilon\ell)}{2}$. Since $F(k)$ is not a

function of ω it can be treated as a multiplicative factor

which can then be ignored for the moment, remembering that

the partial amplitude corresponding to $\dfrac{F(k)}{\varepsilon^2+\omega^2}$ is then

$\dfrac{F(k)K_0(\varepsilon\ell)}{2}$.

Writing (3.43) in terms of the impact parameter b,

$$\frac{1}{\varepsilon^2+\omega^2} = k^2 \int_0^\infty b\ ab\ J_0(\omega k\ b)\ K_0\left(\frac{m_\rho}{\sqrt{2}}\ b\right)$$

where $\omega k = \sqrt{-t}$.

Now we can obtain the modified amplitude by absorptive

corrections to the partial wave amplitude introduced via

(2.12).

$$K_0\left(\frac{m_\rho}{\sqrt{2}}\ b\right) \rightarrow \sqrt{S_i(b)}\ K_0\left(\frac{m_\rho}{\sqrt{2}}\ b\right)\sqrt{S_f(b)} = K_0\left(\frac{m_\rho}{\sqrt{2}}\ b\right)S(b)\ .$$

Thus

$$k^2\int_0^\infty b\ db\ J_0(b\sqrt{-t})K_0\left(\frac{m_\rho}{\sqrt{2}}\ b\right) \rightarrow k^2\int_0^\infty b\ db\ J_0(b\sqrt{-t})K_0\left(\frac{m_\rho}{\sqrt{2}}\ b\right)S(b)\ ,$$

and the ρ amplitude (3.42) is given by

$$a(t,k) = F(k)k^2 \int_0^\infty b \, db \, J_0(b\sqrt{-t}) K_0\left(\frac{m_\rho}{\sqrt{2}} b\right) . \qquad (3.44)$$

The amplitude after corrections, $a^{mod}(t,k)$ is:-

$$a^{mod}(t,k) = F(k)k^2 \int_0^\infty b \, db \, J_0(b\sqrt{-t}) K_0\left(\frac{m_\rho}{\sqrt{2}} b\right) S(b) \qquad (3.45)$$

Let us consider the energy dependence of (3.44) and (3.45) at fixed t. $\int_0^\infty b \, db \, J_0(b\sqrt{-t}) K_0\left(\frac{m_\rho}{\sqrt{2}} b\right)$ is manifestly

independent of k, and therefore the energy dependence of $a(t,k)$ is entirely contained in $F(k)k^2$.

We can now see clearly under what circumstances absorptive modifications can leave the energy dependence of the exchange terms unaltered. Consider (3.45), the unmodified part of the integrand $b \, J_0(b\sqrt{-t}) K_0\left(\frac{m_\rho}{\sqrt{2}} b\right)$ is

independent of k for fixed t. Thus if $S(b)$, the S-matrix in impact parameter representation, is independent of k,

then the integral $\int_0^\infty b \, db \, J_0(b\sqrt{-t}) K_0\left(\frac{m_\rho}{\sqrt{2}} b\right) S(b)$ is

independent of k, and the energy dependence of $a^{mod}(t,k)$ is

identical with the unmodified amplitude, the E-dependence being in both cases contained entirely in $F(k)k^2$.

We now see the reason for the numerical result for R.M.E. Serber's model for pp elastic scattering gives $S(b)$ independent energy, and we see from the above analysis that this is the crucial factor which gives an unmodified energy dependence.

The assumption of energy independence for high energy scattering is certainly in the spirit of the crude models used to obtain S_ℓ. However, even if we take into account the energy dependence of the angular distribution in p-p scattering, our conclusions are not materially altered. For simplicity, consider the Gaussian model (2.17) $S(b) = 1 - \frac{\sigma_{Tot}}{4\pi a} e^{-b^2/2a}$. A logarithmic shrinkage of $\frac{d\sigma}{dt}$ corresponds to an E-dependence of a of the form $a \sim \ell n\, E$. This logarithmic modification will not significantly affect a power dependence on E, from the exchange particle.

We conclude that elementary R.M.E., damped or undamped, is an untenable model for n-p charge exchange.

Now, the above analysis is specific to the case of the ρ exchange contribution to n-p charge exchange, and we must

ask whether the result may be generalised.   The first

observation to make is that the result may be generalised

to a more complex amplitude than the spin independent ρ

case, which has no angular dependence in the numerator.

The analysis given above goes through with no

modification, for an amplitude of the form

$$a(t,k) = F(k) \frac{\omega^{\nu}}{\varepsilon^2 + \omega^2}, \quad \nu \text{ integer} \tag{3.46}$$

This follows from the generalised version of identity (3.43),

namely

$$\frac{\omega^{\nu}}{\varepsilon^2 + \omega^2} = \varepsilon^{\nu} \int_0^{\infty} \ell \, d\ell \, J_{\nu}(\omega\ell) K_{\nu}(\varepsilon\ell) \tag{3.47}$$

which is the Weber-Schafheitlin equality (35).

That our result for the energy dependence of (3.42)

may be generalised to amplitudes of the form (3.46) need not

surprise us.   We have already seen numerically that the

energy dependence of the O.P.E. terms is not changed by

absorptive modification;   and recalling the undamped O.P.E.

amplitudes (3.17), we observe that all these may be expressed

in terms of (3.46).

This still does not constitute a complete generalisation -

in particular it is necessary to know if any given amplitude

has exceptional low partial wave terms, and whether these can be neglected to a good approximation, exact in the high energy limit. It does not seem possible to come to any general conclusion on this point.

However, for a number of particular processes with given exchange, (e.g. the ρ contribution to πp charge exchange and $\pi N \to \pi N^*$) it has been seen numerically that the E-dependence is not significantly changed by absorptive corrections. The exceptional terms of these amplitudes can be neglected, and our analysis explains how these results come about. Even through this result may not hold for a general case, the fact that it holds in a number of specific instances demonstrates that the Absorption Model cannot, in general, work for high spin exchanges.

Essentially, one is saying that the unacceptable E-dependence of high spin exchanges cannot be put right by absorptive corrections. A resolution of this could be achieved by treating high spin exchanges as Regge poles; though the relation between absorptive corrections and Reggeization is not yet understood.

CHAPTER 4

NUCLEON–ANTINUCLEON CHARGE EXCHANGE

IN THE ABSORPTION MODEL

Having considered nucleon-nucleon charge exchange, it is of interest to consider the cross-channel process, nucleon-antinucleon charge exchange, in the Absorption Model. We can make clear the significance of this comparison as follows:- any given one meson exchange gives, up to a phase, the same contribution to the process $np \to pn$ as to the process $p\bar{p} \to n\bar{n}$. As remarked in Chapter 1, it is precisely for this reason that a one meson exchange model, even with arbitrary form factors does not satisfactorily account for both the direct and cross-channel processes in nucleon-nucleon scattering.

However, in the case of the Absorption Model, it is inevitable that different results will be obtained for the two channels, since elastic N-N and N-$\bar{N}$ scattering are different. It is therefore of considerable interest to see how far O.P.E. in the Absorption Model will account for the reaction $p\bar{p} \to n\bar{n}$, and to compare this with the results discussed in the previous chapter for $np \to pn$. Of course, any $I = 1$ exchange in the t-channel can contribute to both these processes; and by restricting ourselves to O.P.E. we are not, even in the context of the "nearest singularity" philosophy, telling the whole story.

However, the conclusion we draw from our discussion of the energy dependence of R.M.E. in Chapter 3 is that the inclusion of any of the known I = 1 mesons other than the pion will give rise to an energy dependence which cannot fit experiment.

The experimental situation with regard to $p\bar{p} \rightarrow n\bar{n}$ is much less satisfactory than for $pn \rightarrow np$. The only published data in the GeV region is by the C.E.R.N. group (36) at 3.0 and 3.6 GeV/c. This allows a comparison with the $pn \rightarrow np$ case, for which data exists at these energies. It is found that the forward differential cross-section is of the same order as that for forward n-p charge-exchange, but the angular distribution is much wider in the $p\bar{p} \rightarrow n\bar{n}$ case.

The C.E.R.N. results, taken together with unpublished data at 1 GeV/c and 7 GeV/c, show an energy dependence consistent with one pion exchange, i.e. $\frac{d\sigma}{dt} \curvearrowleft S^{-2}$ (D.R.O. Morrison, private communication). It is unfortunately not possible to say anything very definite about the phase of the spin averaged forward $p\bar{p} \rightarrow n\bar{n}$ amplitude. The errors on both $\sigma_{tot}(p\bar{p})$ and $\sigma_{tot}(p\bar{n})$ are of the same order as the difference, and it would be equally consistent with present data for the amplitude to be purely real or purely imaginary.

We will now discuss in a little more detail the assertion that the contribution from a given meson exchange to np and $\bar{p}p$ charge exchange is up to a phase the same. We shall apply the argument given by Leader and Slansky (37) relating pp → pp and $\bar{p}n \rightarrow \bar{p}n$ to the charge exchange case. Consider in the S channel

$$pn \rightarrow np. \qquad (4.1)$$

In the t channel we have (order important)

$$\bar{n}p \rightarrow p\bar{n}. \qquad (4.2)$$

Now consider in the S channel

$$p\bar{p} \rightarrow n\bar{n}. \qquad (4.3)$$

In the t channel we have

$$\bar{n}p \rightarrow \bar{n}p. \qquad (4.4)$$

Now by operating on the final state of (4.2) to exchange the particles e.g. by G-parity, we have (4.4); and by crossing back to the S cahnnel we obtain $p\bar{p} \rightarrow n\bar{n}$, times the phase picked up in transforming $|p\bar{n}>$ to $|\bar{n}p>$. Remember that we are allowed to carry out such operations as G-conjugation on the $\bar{n}p$ state in the t channel if this is a well defined state - this is of course the case when we have one given meson exchange in the t channel. Now the question is, what is the phase we pick up in going from $|\bar{p}n>$ to $|n\bar{p}>$? This transformation can be accomplished by space exchange and spin exchange. Now for π in the t channel, the space and spin angular momentum state of the

$N\bar{N}$ is $^1S_0$. Remembering N and $\bar{N}$ have opposite parity, the total parity of the system is $(-1)^{\ell+1}$; and the singlet state is antisymmetric under spin exchange. Thus for $N\bar{N}$ coupled to a pion, (space exchange) × (spin exchange) gives a phase of $(-1) \times (-1) = +1$. Consider now the $N\bar{N}$ system coupled to a ρ; here the angular momentum state of $N\bar{N}$ is $^3S_1 + {}^3D_1$. This gives the same parity but we are now in a triplet spin state, i.e. symmetric under spin exchange. So the phase relating $pn \rightarrow np$ and $p\bar{p} \rightarrow n\bar{n}$ for ρ exchange is $(-1) \times (+1) = -1$.

So we have shown that

$$M(pn \rightarrow np; \; \pi \text{ exchange}) = +M(p\bar{p} \rightarrow n\bar{n}; \; \pi \text{ exchange})$$

$$M(pn \rightarrow np; \; \rho \text{ exchange}) = -M(p\bar{p} \rightarrow n\bar{n}; \; \rho \text{ exchange}).$$

This checks with the line reversal arguments of Sharp and Wagner (38) who find, for an exchange meson of Isotopic Spin $= I$, G-parity $= G$, that the relation between the NN and $N\bar{N}$ one pole amplitudes is

$$M(NN \rightarrow NN; \; I,G \text{ exchange}) = [(-1)^I G] M(N\bar{N} \rightarrow N\bar{N}; \; I,G \text{ exchange})$$

$$(4.5)$$

That equation (4.5) involves G-parity should not surprise us. To obtain $p\bar{p} \rightarrow n\bar{n}$ from $pn \rightarrow np$, we had to

change $|p\bar{n}> \rightarrow |\bar{n}p> $: and the G operator can do this, since $G|p> = |\bar{n}>$ and $G|\bar{n}> = -|p>$ (39). G-parity of course plays a similar role in the $N\bar{N}$ system to the Pauli principle in the NN system. Without an analogue for the Pauli principle in the $N\bar{N}$ case, we should have a different number of independent amplitudes in the s and t channels.

After the above analysis we can consider the O.P.E. Absorption Model for $p\bar{p} \rightarrow n\bar{n}$. We already have the unmodified partial waves, and need now only consider phenomenological models for $p\bar{p}$ elastic scattering which is, by I spin invariance, identical to $n\bar{n}$ scattering. We will use two models for $p\bar{p}$ elastic scattering: the Gaussian model (2.19), and a phenomenological model due to Chretien (40) which gives a good fit to $p\bar{p}$ scattering at the energies we shall consider. Since these models, though similar, are not identical, we can investigate the sensitivity of our results to the precise form for S_{ℓ}.

Svensson (41) has determined the parameters of the Gaussian model (2.19),

$$S_{\ell} = 1 - Ce^{-\gamma\ell^2} ,$$

to fit $\bar{p}p$ scattering, he finds $C = 1$, and $\gamma = 0.0335$ and
0.0271 at 3.0 and 3.6 GeV/c respectively.

The model due to Chretien gives, for purely imaginary
S_ℓ,

$S_\ell = \varepsilon$,   for $\ell \leqslant \ell_{max} - 2\Delta$;

$$= 1 - \frac{1-\varepsilon}{4\Delta^2} (\ell - \ell_{max})^2 , \text{ for } \ell_{max} - 2\Delta \leqslant \ell \leqslant \ell_{max} ;$$

$$= 1, \text{   for } \ell \geqslant \ell_{max} ; \tag{4.6}$$

where at 3.0 GeV/c, $\varepsilon = 0.150$, $\Delta = 2.53$, $\ell_{max} = 8.03$.
This gives, in general terms, a form for S_ℓ similar to the
Gaussian model: low partial waves are strongly absorbed,
and the absorption decreases steadily over a region of
"thickness" 2Δ, going to zero for $\ell = \ell_{max}$.

This gives a good fit to $\frac{d\sigma}{d\Omega}$ over the first two
decades, but at wider angle gives diffraction minima.
Chretien then fits the wider angle data by adding a real
part to the phase shift, which he finds to be -45.1,
-37.7, -29.8, -21.5, -12.4, -2.2, $+4.7$, $+5.4$, $+2.6$, $+0.2$,
respectively, for ℓ from 0 to 9. We consider both the
pure imaginary S_ℓ model, and the inclusion of a real part for
S_ℓ .

The details of the calculation are the same as for the pn $\rightarrow$ np process discussed in Chapter 3. We now discuss the results. In figure (4.1) we show the O.P.E. results with Gaussian damping, and the experimental results (36), for $\frac{d\sigma}{dt}$. Since the experimental results for $\frac{d\sigma}{dt}$ were obtained by combining the 3.0 and 3.6 GeV/c data (to improve statistics), we simply display these points, together with the theoretical curves at 3.0 and 3.6 GeV/c. We see that, for $-t > 0.1$ $(GeV/c)^2$, we obtain good agreement with experiment in both the magnitude and angular distribution. At $0 \leqslant -t \leqslant 0.1$ $(GeV/c)^2$, we predict structure for which there is no evidence; on the other hand, it would not be inconsistent with the present data for such structure to exist.

We now investigate the sensitivity of the calculation to the precise form of S_ℓ. In figure (4.2) we show the predictions for Gaussian damping; the Chretien model with purely imaginary phase shifts, and with real and imaginary phase shifts. We note that all models for S_ℓ give very similar results. This is encouraging: since we must necessarily use a crude model for the elastic scattering phase shifts, extreme sensitivity to the exact form would not allow meaningful calculations.
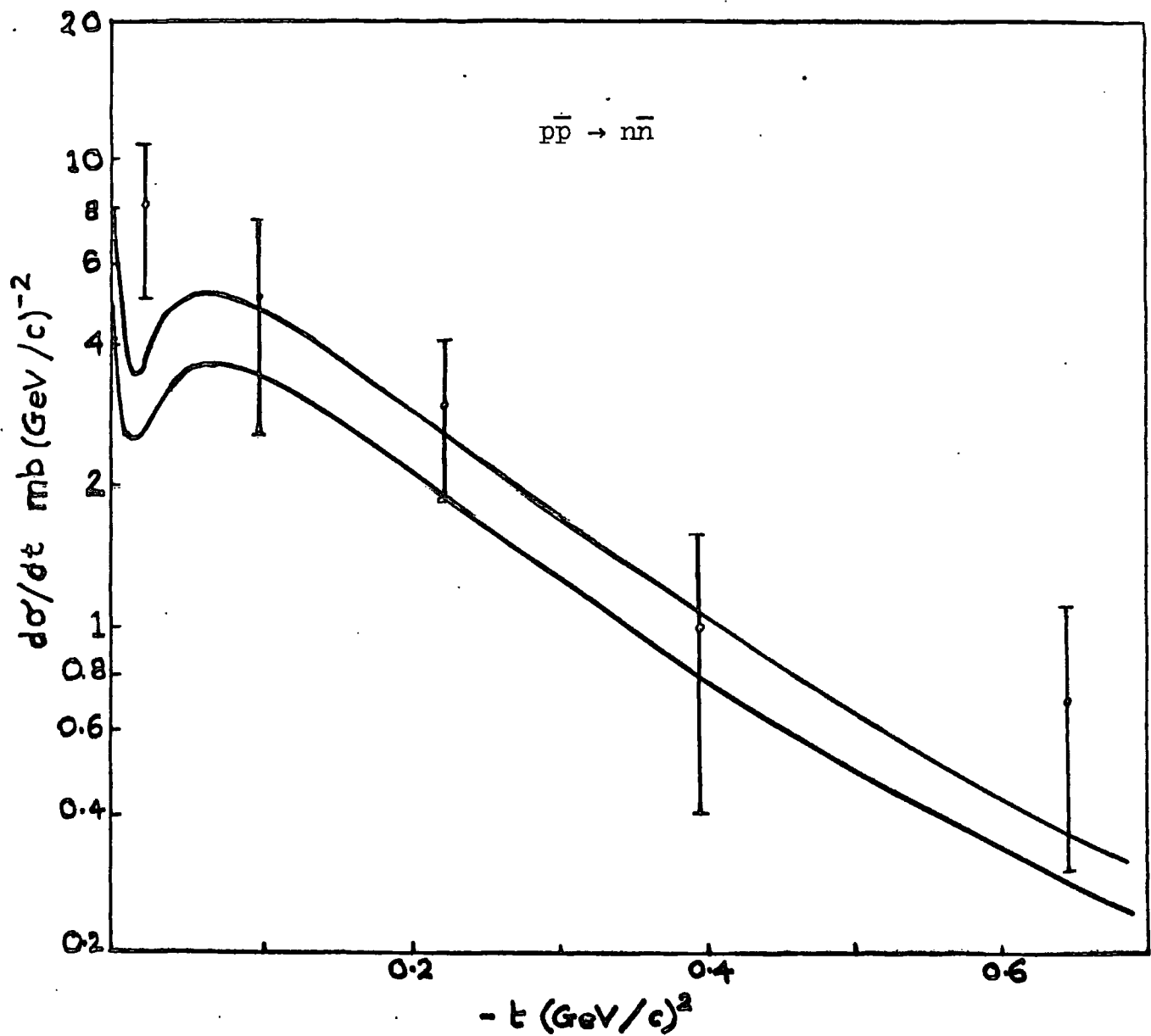
Fig. (4.1)

Cross-section $\frac{d\sigma}{dt}$ for $p+\bar{p} \to n+\bar{n}$. The upper and lower curves are the theoretical predictions of O.P.E. with Gaussian damping for 3.0 and 3.6 GeV/c, respectively. The data points, from reference 36 are combined results of runs at 3.0 and 3.6 GeV/c.

$\frac{d\sigma}{d\Omega}$ ml./sc.

p̄p → n̄n   3·0 Gev/c

Fig. (4.2) (i) Represents O.P.E. with Gaussian damping.
        (ii) Represents O.P.E. with Chretien damping,
             imaginary phase shifts only.
        (iii) Represents O.P.E. with Chretien damping,
              real and imaginary phase shifts.

$-t(Gev/c)^2$
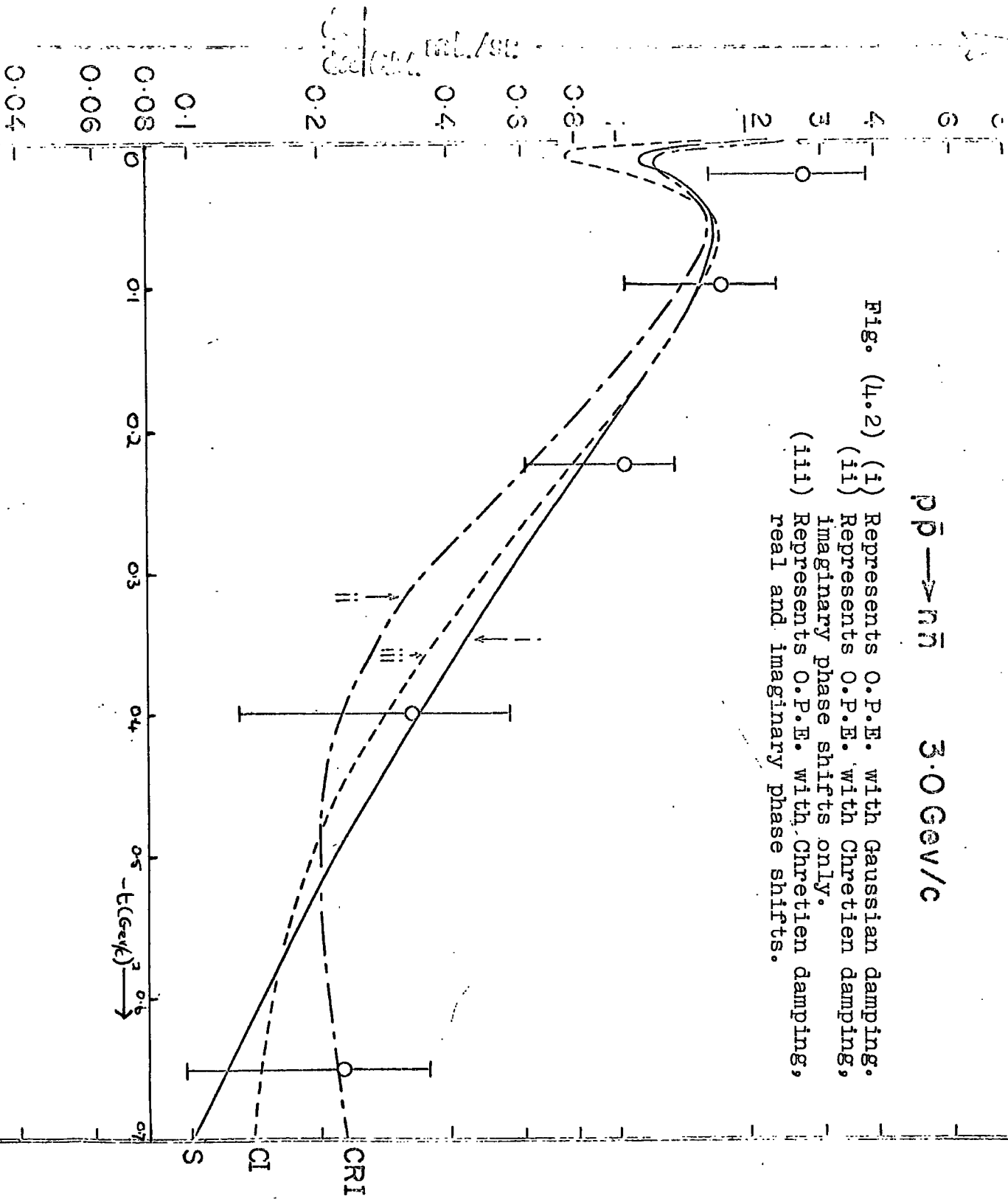
Finally, in figure (4.3), we compare the modified O.P.E. model for pn → np and for $\bar{p}p → \bar{n}n$. The solid line (i) is the exponential fit to the experimental results of Palevsky (23) et al at 3.0 GeV/c. This is compared with the O.P.E. predictions for pn → np at 3.6 GeV/c to avoid crowding on the graph at small angles. Since the absorptive corrections in the NN system are qualitatively similar to those in the $N\bar{N}$ system, it is not surprising that the result of the $\bar{p}p → \bar{n}n$ calculation has several points of similarity with that of the pn → np calculation. Both have a sharp peak for $-t < 0.02$ $(GeV/c)^2$, due mainly to the amplitudes M_{ss} and M_{oo}. Both have a wide secondary maximum, given mainly by the M_{1-1} double spin-flip amplitude. The experimental data does not, however, display the same similarities. For $\bar{p}p → \bar{n}n$, the agreement of theory and experiment is due mainly to the second maximum: the predicted forward peak is not confirmed. For pn → np, the forward peak is the one point of agreement: the second maximum conflicts strongly with experiment. We note from figure (4.3) that we could not have obtained good agreement for $\bar{p}p → \bar{n}n$ if we had used the damping parameters appropriate to pn → np. The weaker absorption in the NN

COMPARISON OF np and pp̄ CHARGE EXCHANGE

Fig. (4.3) (i) Represents an exponential fit to the pn→np data at 3.0 GeV/c.

(ii) O.P.E. absorption model calculation of pp̄→ nn̄ at 3.0 GeV/c, Gaussian damping.

(iii) O.P.E. absorption model calculation of pn→np at 3.7 GeV/c, Serber damping.
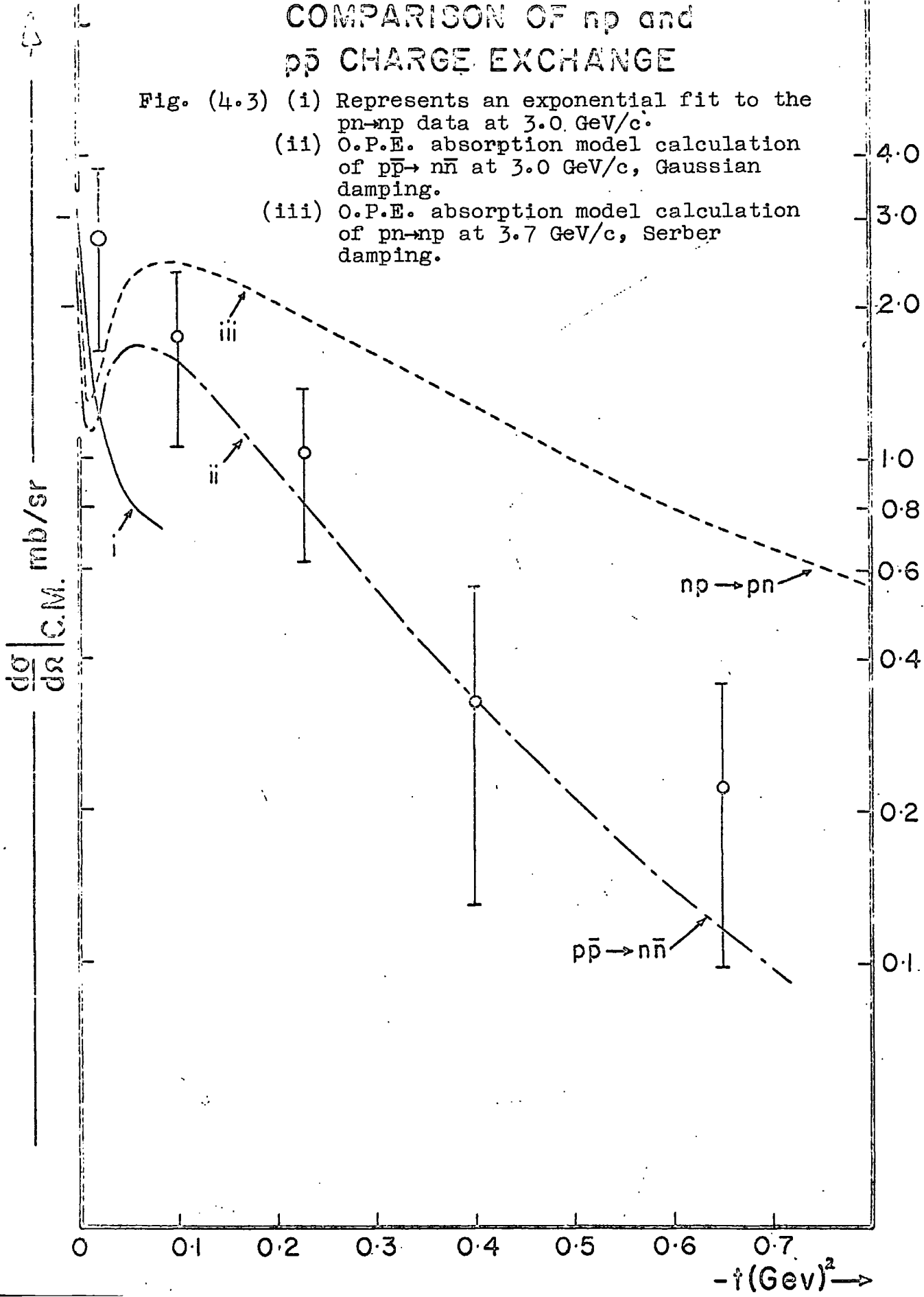
system would have given $\frac{d\sigma}{d\Omega}$ for $\bar{p}p \to \bar{n}n$ four to five times larger than experiment, for $-t > 0.2$ $(\text{GeV/c})^2$.

Thus one pion exchange in the Absorption Model does not explain the difference between $\bar{p}p$ and pn charge exchange. Some possible reasons are the following:

1) The absorptive corrections to low partial waves are much stronger for $\bar{p}p \to \bar{n}n$, at the energies considered, because of the annihilation channels open to the $N\bar{N}$ system. Thus the neglect of short-range exchange forces may be a better approximation for $\bar{p}p \to \bar{n}n$.

2) The two processes have different u channel contributions. The nearest n channel singularity for the $N\bar{N}$ system is the deuteron pole, while for the NN it is the pion pole. This could be important at larger angles.

3) As mentioned above, the one meson contribution to the two processes can differ in sign, depending in general on the G-parity of the exchange mesons. This makes it possible for other meson exchange contributions to the amplitude, neglected here, to add constructively in one case, and destructively in the other.

Finally we reiterate that O.P.E. with absorptive corrections is in good agreement in both magnitude and angular distribution with the experimental results for $\bar{p}p \to \bar{n}n$, except perhaps at very small momentum transfers. This agreement does not rely on any adjustable parameters.

C H A P T E R   5

MODIFICATIONS   TO   THE   ABSORPTION

MODEL   FOR   n-p   CHARGE   EXCHANGE

In this Chapter, we consider a number of possible additions and modifications to our work in Chapter 3. The object of the exercise is to see if any plausible modifications of the O.P.E. Absorption Model can improve the agreement with the wider angle np → pn data.

Spin-dependent damping

In Chapter 3, we remarked that no spin-independent parameterization of the elastic scattering phase shifts can remove the secondary bump from the large M_{1-1} amplitude in absorptively modified O.P.E. For the sake of completeness, we now consider spin-dependent possibilities. The analysis is in no sense exhaustive, and we have no experimental justification for supposing there to be strong spin dependence here. The spirit of the investigation is merely to see whether this is a possible resolution of the discrepancy with experiment.

We consider the partial wave undamped O.P.E. contributions to M_{1-1}. We note that the strongest contribution is from $\alpha_{\ell,\ell}$ – this is at least four times larger than any of the other partial waves. Thus, we increase the damping in $\alpha_{\ell,\ell}$ to see if this can give better agreement.

However, since our damping factors are the partial wave S-Matrix elements for the elastic scattering, we cannot change one partial wave without compensating changes in another if we want to keep agreement with elastic scattering. We choose the most advantageous case. We note that $\alpha_{\ell,\ell}$ contributes most strongly to M_{1-1} from O.P.E., and $\alpha_{\ell,\ell+1}$ gives the smallest contribution. We therefore increase damping on $\alpha_{\ell,\ell}$, while decreasing the damping on $\alpha_{\ell,\ell+1}$ by a corresponding amount. This keeps the same total cross-section for elastic scattering; it cannot, however, simultaneously preserve the original angular distribution for elastic scattering.

We check how much this is changed by reconstructing the elastic scattering from our partial wave S-Matrix elements. We show the results of a 10% increase in the Serber damping of the O.P.E. contribution to $\alpha_{\ell,\ell}$ for the process pn → np at 3.7 GeV/c.

$-t$ (GeV/c)^2	$\frac{d\sigma}{d\Omega}$ (spin independent damping) mb/st	$\frac{d\sigma}{d\Omega}$ (10% increase in $a_{\ell,\ell}$ damping) mb/st
0	2.40	2.34
.0065	1.34	1.27
.0262	1.44	1.36
.0588	1.95	1.85
.1042	2.08	1.96
.1622	1.93	1.78
.4090	1.10	0.93

The elastic scattering was little changed: out to
$-t = 0.4$ (GeV/c)^2 there is 2% difference in the spin-
independent and spin-dependent cases. We observe that,
although the secondary maximum is reduced, the change
is not significant. This conclusion is not altered
for an $\alpha_{\ell,\ell}$ damping increase of 30%, and a corresponding
decrease in $\alpha_{\ell,\ell+1}$ damping. For this larger damping,
the diffraction peak is considerably different from the
spin-independent case – the differential cross-section
rises by 50% at $-t = .35$ (GeV/c)^2.

The above discussion does not exhaust the possibilities
of spin-dependent damping. It does however, show that
it is unlikely to be the answer to the problem.

u-channel poles

In Chapter 4, we suggested that the u-channel poles
could be significant for pn → np (since the nearest
pole is the pion), but not for $\bar{p}p$ → $n\bar{n}$ (where the
nearest pole is the deuteron). We here investigate this

possibility. we shall relate the u-channel O.P.E. contribution to the already known t-channel contribution. This can be done by looking at the $I = 1$ and $I = 0$ contributions separately, and using anti-symmetry. For simplicity, we consider only M_{ss} .

Consider first an $I = 0$ scalar exchange, with g^2 chosen to give

$$M_{ss} = \frac{1}{x_0 - x} = \sum_{\ell = 0}^{\infty} (2\ell + 1) Q_{\ell}(x_0) P_{\ell}(x) .$$

This is a forward t-channel pole.

Now relate this to an $I = 1$ exchange, using a $\underset{\sim}{\tau}^{(1)} \cdot \underset{\sim}{\tau}^{(2)}$ factor. Then the forward pole in the $I = 1$ state just gives the same contribution, and (-3) times this in the $I = 0$ state. So we now have

$$M_{ss}(I = 1) = \frac{1}{x_0 - x} \tag{5.1}$$

$$M_{ss}(I = 0) = -\frac{3}{x_0 - x} \qquad (5.2)$$

Now, the backward contribution is just the term we add to make (5.1) and (5.2) anti-symmetric under interchange of $x \to -x$. So, adding the u-channel poles, we have

$$M_{ss}(I = 1) = \frac{1}{x_0 - x} + \frac{1}{x_0 + x}$$

$$M_{ss}(I = 0) = -\frac{3}{x_0 - x} + \frac{3}{x_0 + x} \qquad .$$

Taking the appropriate combinations, (3.2), (3.3), and (3.4), to obtain the physical channels, we have

$$M_{ss}(pp) = \frac{1}{x_0 - x} + \frac{1}{x_0 + x} = 2 \sum_{\text{even } \ell} (2\ell + 1) Q_\ell(x_0) P_\ell(x) ,$$

because the terms with $(-x)$ cancel in odd partial waves, and add in even partial waves, since $P_\ell(-x) = (-1)^\ell P_\ell(x)$.

Similarly,

$$M_{ss}(np \to np) = -\frac{1}{x_0 - x} + \frac{2}{x_0 + x}$$

$$= \sum_{\text{even } \ell} (2\ell + 1) P_\ell(x) Q_\ell(x_0) - 3 \sum_{\text{odd } \ell} (2\ell + 1) Q_\ell(x_0) P_\ell(x)$$

$$M_{ss}(np \to pn) = \frac{2}{x_0 - x} - \frac{1}{x_0 + x}$$

$$= \sum_{even\,\ell} (2\ell+1) P_\ell(x) Q_\ell(x_0) + 3 \sum_{odd\,\ell} (2\ell+1) Q_\ell(x_0) P_\ell(x)$$

$$(5.3)$$

Now compare (5.3) with the t-channel contribution to np → pn, i.e. with

$$\frac{2}{x_0 - x} = 2 \sum_{all\,\ell} (2\ell+1) Q_\ell(x_0) P_\ell(x) .$$

The prescription for the inclusion of the u-channel contribution is now clear. If the t-channel pole gives $\sum_{all\,\ell} a_\ell P_\ell$, then adding the u-channel contribution turns this into

$$\tfrac{1}{2} \sum_{even\,\ell} a_\ell P_\ell + \tfrac{3}{2} \sum_{odd\,\ell} a_\ell P_\ell .$$

Effectively then the u-channel contribution just adds $\tfrac{1}{2} \left(\sum_{odd} - \sum_{even} \right)$ to the t-channel contribution. This gives

a cancellation of odd and even ℓ at cos θ = 1, with a build up into a reinforcement at the backward limit cos θ —⌐—-1; as is expected from a backward pole.

Modifying the partial wave sum as shown above, we calculate the absorptively modified O.P.E. amplitude with the u-channel contribution included. We show the results in the following table, compared with the absorptively modified t-channel O.P.E. results: both for 3.7 GeV/c.

-t (GeV/c)^2	$\frac{d\sigma}{d\Omega}$ (t-channel pion)	$\frac{d\sigma}{d\Omega}$ (t- and u-channel pion)
	Millibarns/steradian	
0	2.40	2.42
.00655	1.35	1.33
.0262	1.44	1.38
.0588	1.95	1.84
.104	2.08	1.94
.162	1.93	1.78
.233	1.66	1.48
.315	1.37	1.14

As expected, the u-channel contribution becomes more important at larger angles. The secondary maximum from M_{1-1} is, however, hardly affected, and the disagreement with experiment persists.

Self-damping effects in absorptively modified O.P.E.

As remarked in Chapter 2, one possible approach to the Absorption Model is via the K-Matrix formalism.

We recall that the partial wave amplitude (which is in general many-channel) can be written, (2.24), as

$$\underset{\sim}{A}^{\ell} = \frac{\underset{\sim}{K}^{\ell}}{(\underset{\sim}{K}^{\ell} - i \underset{\sim}{P} \underset{\sim}{K}^{\ell})}$$

and if $\underset{\sim}{K}$ is real symmetric, then $\underset{\sim}{A}^{\ell}$ satisfies the full unitarity condition. Now, though (2.24) gives us a simple requirement to satisfy unitarity, we are no further on unless we have a dynamical model for $\underset{\sim}{K}^{\ell}$. The K-Matrix formalism can, however, form the framework for an approximation scheme which ensures that the input Born terms satisfy unitarity bounds, and this is the way we use it here.

We noted in Chapter 2 that, for the one-channel case, in the limit of small $\underset{\sim}{A}^{\ell}$, we can identify $\underset{\sim}{K}^{\ell}$ with the Born term $\underset{\sim}{B}^{\ell}$: and elastic unitarity is then satisfied. This identification is, of course, an assumption when used outside the small $\underset{\sim}{A}^{\ell}$ limit: we shall make this assumption, to satisfy unitarity bounds.

It must be stressed that absorptive modifications via the Sopkovich prescription (2.12) do not force the Born amplitude to satisfy unitarity, or to be within unitarity bounds. In most cases studied, however, absorptive corrections do give rise to an amplitude satisfying unitarity bounds. In this section we impose unitarity bounds by considering (2.24) in the one-channel case, and making the assumption $\underline{K}^{\ell} = \underline{B}^{\ell}$. Since we only use the one-channel case, we ignore the effects of competing channels. To take into account their effect, we apply the Sopkovich prescription (2.12). We investigate whether the identification of the O.P.E. Born term with $\underline{K}^{\ell}$ (self-damping), and the inclusion of absorptive corrections, helps to bring the wider angle predictions of O.P.E. into agreement with experiment for np → pn.

It is not clear, in combining self-damping with the absorptive effects due to competing channels, whether one should self-damp the Born term before applying absorptive corrections (which we shall call pre-K), or whether one should apply absorptive corrections to the Born term and then self-damp the modified amplitude (post-K).

We consider both the pre-K and post-K procedures, and find that in this particular case there is not much difference.

Considering only the contribution of O.P.E. to the charge exchange scattering, damped or undamped, denoted B, (i.e. ignoring the contribution to the diffractive forward scattering) we have

$$\underset{\sim}{T}^B = \begin{pmatrix} 0 & B \\ B & 0 \end{pmatrix} \qquad (5.4)$$

for the 4 partial waves diagonal in ℓ; where $T_{11}{}^B$ and $T_{22}{}^B$ correspond to pn $\to$ np and np $\to$ np respectively, and $T_{12}{}^B$ and $T_{21}{}^B$ correspond to pn $\to$ np and np $\to$ pn. Then we have, using equation (2.30),

$$\underset{\sim}{A} = \frac{\underset{\sim}{T}^B(1 + i\underset{\sim}{T}^B)}{1 + (\underset{\sim}{T}^B)^2} \qquad (5.5)$$

Now,

$$(\underset{\sim}{T}^B)^2 = \begin{pmatrix} B^2 & 0 \\ 0 & B^2 \end{pmatrix}$$

$$1 + (\underset{\sim}{T}^B)^2 = \begin{pmatrix} 1+B^2 & 0 \\ 0 & 1+B^2 \end{pmatrix}$$

$$\therefore (1 + (\underset{\sim}{T}^B)^2)^{-1} = \begin{pmatrix} (1+B^2)^{-1} & 0 \\ 0 & (1+B^2)^{-1} \end{pmatrix}$$

$$\therefore \underline{A}^\ell = \frac{\underline{T}^B(1+i\underline{T}^B)}{1+(\underline{T}^B)^2} = \begin{pmatrix} 0 & \dfrac{B}{1+B^2} \\[2ex] \dfrac{B}{1+B^2} & 0 \end{pmatrix} + i\begin{pmatrix} \dfrac{B^2}{1+B^2} & 0 \\[2ex] 0 & \dfrac{B^2}{1+B^2} \end{pmatrix}$$

$$(5.6)$$

Thus the self-damped partial wave amplitude corresponding to a Born term (5.4) — damped or undamped — is

$$T = \frac{B}{1+B^2} \ , \qquad (5.7)$$

where we note that this gives no charge-exchange imaginary part. We obtain essentially the same result for the coupled $\ell = J\pm1$ partial wave: the above analysis goes through formally, with B a 2×2 matrix.

We now present the results of the pre-K and post-K calculations, and compare them with our original O.P.E. Absorption Model (A.M.) calculation for pn - np, at 3.7 GeV/c. The damping factors are in all cases the Serber results used in Chapter 3.

$-t$ (GeV/c)^2	$\frac{d\sigma}{d\Omega}$ (A.M.)	$\frac{d\sigma}{d\Omega}$ (pre-K)	$\frac{d\sigma}{d\Omega}$ (post-K)
	(millibarns/steradian)		
0	2.40	2.89	2.44
.0065	1.34	1.63	1.36
.0262	1.44	1.48	1.43
.0588	1.95	1.83	1.92
.1042	2.08	1.86	2.03
.1622	1.93	1.66	1.87
.4090	1.10	.777	1.03

The first observation is that the secondary bump is essentially untouched. We therefore see that self-damping cannot remove the wide angle discrepancy between the predictions of O.P.E. with absorptive modifications, and experiment. From the above table it can be seen that the post-K treatment has even less effect than pre-K. The reason for this is simple - equation (5.7) shows that the smaller the input term, the less difference self-damping makes. Since absorptive corrections already reduce the Born terms considerably, subsequent self-damping makes little difference.

π-ρ interference

Finally, we remark briefly on an effect which absorptive modifications can produce. This is the

possible interference, after absorptive modification, of two different exchange contributions which do not interfere before damping. (We thank H. Høgaasen for drawing our attention to this possibility).

The undamped π and ρ contributions to n-p charge exchange do not interfere. This can best be seen in the helicity formalism - see Muzinich (42) - of the five independent helicity amplitudes ϕ_i, (i = 1,2,3,4,5), π contributes only to $\phi_2 = \phi_4$. $\phi_1 = \phi_3 = \phi_5 = 0$. ρ contributes to $\phi_2 = -\phi_4$, $\phi_1 = \phi_3$, and also to ϕ_5. The non-interference of the undamped π and ρ contributions depends on $\phi_2{}^\pi = \phi_4{}^\pi$, and $\phi_2{}^\rho = -\phi_4{}^\rho$. This balance of terms does not in general survive absorptive corrections.

Investigating this for the specific pn → np case, we found the π-ρ interference differential cross-section to be positive and sharply peaked for small angles, broad and negative for larger angles. This effect could cancel the broad secondary maximum from damped O.P.E.

The previous objections to R.M.E. remain, and agreement with experiment cannot be obtained by using elementary ρ exchange. As a basis for a phenomenological investigation, we gave the ρ-nucleon coupling constant an

energy dependence such that the R.M.E. contribution had the same energy dependence as that of O.P.E. We then found that by varying the coupling constants, excellent agreement with experiment could be obtained. However this required a coupling constant $g^2 NN\pi$ of 9.85, compared with the established value of 14.4.

The above example is not put forward as a convincing model of the process $pn \to np$, but only as an indication that interference of different exchange particles after damping could form the basis for a model. Ascribing a $'/P_{\ell ab}$ dependence to the ρ coupling constant is, of course, quite arbitrary. It might, however, form the basis for an analysis in terms of a Reggeized ρ contribution, or possibly Reggeized $\rho + A_2$.

It should be noted that, even if a justification for the above model for $pn \to np$ could be found, we would then be in difficulties with regard to our model for $p\bar{p} \to n\bar{n}$. From the analysis in Chapter 4 of the relation between meson exchanges in the NN and N$\bar{\text{N}}$ systems, we see that the sharply constructive interference of π and ρ in NN now becomes destructive in N$\bar{\text{N}}$. Applying the same model to $p\bar{p} \to n\bar{n}$, we then predict a very small differential cross-section at small angles, in strong disagreement with experiment.

C H A P T E R   6

CONCLUDING   REMARKS

We shall first comment on the applications of the Absorption Model by other workers.   There are a number of excellent reviews of these applications, by Jackson (43), and Drell and Hearn (44) amongst others.   We shall therefore give only a very brief discussion.

The Absorption Model has been applied to a large number of peripheral processes, e.g. $\pi p \to \rho p$, $\pi^- p \to \pi^0 n$, $\pi p \to \pi N^*$, $\pi p \to \rho N^*$, $\pi p \to \omega N$, $NN \to NN^*$, $N\bar{N} \to N^*\bar{N}^*$, $Kp \to K^*N$, $Kp \to KN^*$.   These calculations have been performed at a wide range of incident momenta - usually in the range 2 to 8 GeV/c.

In a number of cases, quite excellent agreement with experiment has been obtained.   The O.P.E. absorption model for $\pi p \to \rho p$ gives agreement with the experimental differential cross-sections, both in magnitude and angular distribution.   The decay correlations and energy dependence are also well predicted.   Similarly excellent results are obtained for $\pi p \to \rho N^*$, $NN \to NN^*$,   and $N\bar{N} \to N^*\bar{N}$.   In some of the peripheral reactions treated by the Absorptiom Model - $\pi^- p \to \pi^0 n$ and $\pi p \to \omega N$ - very poor results are, however, obtained.

We must ask if there is any explanation for the excellent agreement in one set of processes, and the bad agreement in the other set. One observes that in all cases where the meson exchange is taken to be a vector or higher spin meson, bad agreement results. Our analysis of the effect of absorptive damping on energy dependence explains one of the sources of disagreement. In those cases where selection rules forbid the exchange of a pseudo-scalar meson, e.g. $\pi^- p \to \pi^0 n$, and the lowest mass meson allowed is a vector, e.g. the ρ-meson, poor agreement with experiment is found.

The inference we draw is that absorptive corrections do have an important role in simple one-particle exchange models, and that the success of the one-pion exchange contribution with absorptive corrections is unlikely to be fortuitous. However the one meson exchange model does not appear to give a good representation of the dynamics when the exchange meson is of spin one or greater. This is not surprising: the divergence associated with exchanges of spin greater than one is a major topic in Dispersion Theory, and the idea of the Regge pole was suggested to overcome this problem.

The Regge pole exchange model has had considerable success of late - see for example the work of Phillips and Rarita (45) on πp charge exchange and $\pi p \to \eta n$, where the exchange considered is a Reggeized ρ or A_2. However we must ask whether a Reggeized π model would be successful for such processes as $\pi p \to \rho p$. We note that such a procedure would not be expected to change significantly the results of the elementary one pion exchange, at least at small momentum transfers. This is because we know that the pion has spin zero at $t = .02$ $(GeV/c)^2$. Even with a slope for the pion Regge trajectory of 1 $(GeV/c)^2$, $\alpha(t)$ would not be too different from 0 for small values of $-t$. Now, since Reggeization of the pion is not expected to significantly change its contribution, agreement with experiment cannot be obtained in a large number of cases, unless absorptive corrections are included.

We must therefore ask whether the Regge pole model and the Absorption Model should be combined. It has not yet been resolved whether such a procedure would be consistent. The problem is simply that it is not clear how far Reggeization already includes absorptive corrections.

Consider the Pomeranchuk pole. It is postulated that
this pole controls the high energy behaviour of elastic
scattering and total c ross-sections. To do this,
Pomeranchuk exchange must represent the effect of all open
channels. Thus the Pomeranchuk pole, asymptotically at
least, does not r equire the addition of modifications due
to competing channels. On the other hand, we have the
problem of the pion already discussed.

If the combination of Regge exchange and absorptive
damping is shown to be a consistent procedure, then this
could afford some hope of resolving the problem of the
secondary bump, predicted by O.P.E. in the Absorption Model
for pn → np. As remarked in the final section of
Chapter 5, the interference, after damping, of another
exchange can remove the secondary maximum due to the O.P.E.
contribution from M_{1-1}. It is possible that by combining
a Reggeized ρ with the O.P.E. contribution, and by
including absorptive correction, a full explanation of n-p
charge exchange could be obtained.

Appendix

We outline the derivation of the partial wave
decomposition of the singlet-triplet amplitudes, (3.25 -
3.30), following the treatment given by MacGregor,
Moravcsik and Stapp (46).

The S-Matrix, by definition, depends only on the
asymptotic form of the wave function; it is a transformation
in the spin-angle variables, the radial dependence being
essentially known. To simplify matters, the radial part
$i^{\ell}j_{\ell}(kr)$ is suppressed, where $j_{\ell}(kr)$ is the spherical
Bessel function. Specifically $|\ell, m; S_1, m_1; S_2, m_2 >$
represents the state with spin quantum numbers (S_1, m_1)
and (S_2, m_2) for the first and second particles respectively,
and a spatial dependence $Y_{\ell}^{m}(\theta, \phi)(i^{\ell}j_{\ell}(kr))$, Y_{ℓ}^{m} being the
spherical Bessel function defined in (31). The symbol
$< \ell, m; S_1, m_1; S_2, m_2 |\psi>$ will represent the amplitude of
this state.

Since the asymptotic out-going part of $i^{\ell}j_{\ell}(kr)$ is
$e^{ikr}/2ikr$, the scattering amplitude becomes in this
convention:

$$f^{m_1 m_2}_{S_1 S_2}(\theta,\phi) = \frac{1}{2ik} \sum_{\ell,m} Y^m_\ell(\theta,\phi) < \ell,m; \; S_1,m_1; \; S_2,m_2 | \underset{\sim}{R} | in >$$

$$\equiv \frac{1}{2ik} <\theta,\phi| \ell,m >< \ell,m; \; S_1,m_1; \; S_2, \; m_2 | \underset{\sim}{R} | in >$$

$$\equiv \frac{1}{2ik} < \theta,\phi; \; S_1,m_1; \; S_2,m_2 | \underset{\sim}{R} | in > , \qquad (A.1)$$

where we have set $< \theta,\phi | \ell,m > \equiv Y^m_\ell(\theta,\phi)$; and the Einstein summation convention, and the completeness relation $| \ell,m ><\ell,m | = 1$, have been used. In the above convention, the state represented by $| \theta',\phi' >$ is $(4\pi)^{-1}$ times a plane wave moving in the direction $\theta'\phi'$ — which can be seen from the Gegenbauer expansion (31),

$$\sum_{\ell,m} i^\ell j_\ell(kr) \; Y^m_\ell(\theta,\phi) \; Y^m_\ell(\theta',\phi') = \frac{\exp(i \, \underline{k}' \cdot \underline{r}')}{4\pi} . \qquad (A.2)$$

$\underline{R}$ is the operator in spin-space having matrix elements which, except for a normalisation factor, are the scattering amplitudes for individual initial and final spin states. $\underset{\sim}{M}$ is a spin matrix, having matrix elements which are exactly

the scattering amplitude in various final spin states for fixed initial spin states.

Combining these definitions with (A.1) and (A.2), we have

$$\underset{\sim}{M} = \frac{4\pi}{2ik} < \theta \; \phi | \underset{\sim}{R} | \theta' \; \phi' > \qquad (A.3)$$

We consider $\underset{\sim}{M}$ in the singlet-triplet representation, and take matrix elements of $\underset{\sim}{R}$ with respect to states characterized by quantum numbers J, ℓ, S and M. M is the z-component of total angular momentum J, and S is 0 or 1 for singlet or triplet states respectively. This gives

$$< S,m_s | \underset{\sim}{M} | S',m'_s > = \frac{4\pi}{2ik} < \theta,\phi; \; S,m_s | \underset{\sim}{R} | \theta',\phi'; \; S',m'_s >$$

$$= \frac{4\pi}{2ik} < \theta,\phi; \; S,m_s | \ell,S'',J,M >< \ell,S'',J,M | \underset{\sim}{R} | \ell',S''',J',M' >$$

$$\times \quad < \ell',S''',J',M' | \theta',\phi'; \; S',m'_s > \qquad (A.4)$$

The transformation functions in (A.4) are sums of products of spherical harmonics and Clebsch-Gordon coefficients,

$$< \theta, \phi; \, S, m_S \,|\, \ell, S'', J, M > \; = \; < \theta, \phi \,|\, \ell, m > < \ell, m; \, S, m_S \,|\, \ell, S'', J, M >$$

$$\equiv \sum_m Y_\ell^m(\theta, \phi) \; C_{\ell S}(J, M; \, m, m_S) \delta_{SS'}$$

$$(A.5)$$

where $C_{\ell S}(J, M; \, m, m_S) \equiv < \ell, m; \, S, m_S \,|\, \ell, S, J, M >$ is the Clebsch-Gordon coefficients as defined in (31).

Equations (A.4) and (A.5) allow the $\underset{\sim}{M}$ matrix elements to be expressed as $\underset{\sim}{R}$ matrix elements $< \ell, S, J, M \,|\, \underset{\sim}{R} \,|\, \ell', S', J', M >$. Total angular momentum J and its projection M are conserved, so these matrix elements vanish unless $J = J'$, $M = M'$. The matrix elements are also independent of M, by rotational invariance. For a fixed total angular momentum J, we then have the following possible values of ℓ;

$$\ell = J, \quad \text{and} \quad \ell = J \pm 1 \; .$$

The second class can only occur in the triplet case. The two classes cannot be coupled (conservation of parity). Anti-symmetry for the p-p case precludes coupling between singlet and triplet states: the addition of isospin invariance gives this condition for the n-p case.

We then have the set of matrix elements $< \ell,S,J,M|\underset{\sim}{R}|\ell',S,J,M >$ given in equations (3.19) to (3.24). For the class $\ell = J$,

$$< J,0,J,M|\underset{\sim}{R}|J,0,J,M > = \alpha_J \qquad (3.19)$$

$$< J,1,J,M|\underset{\sim}{R}|J,1,J,M > = \alpha_{\ell J} = \alpha_{\ell\ell} \ . \qquad (3.20)$$

For the class $\ell = J \pm 1$,

$$< J \pm 1,1,J,M|\underset{\sim}{R}|J \mp 1,1,J,M > = \alpha^J \ ; \qquad (3.22)$$

$$< J + 1,1,J,M|\underset{\sim}{R}|J + 1,1,J,M > = \alpha_{\ell J} = \alpha_{\ell,\ell-1} \ ; \qquad (3.23)$$

$$< J - 1,1,J,M|\underset{\sim}{R}|J - 1,1,J,M > = \alpha_{\ell J} = \alpha_{\ell,\ell+1} \ . \qquad (3.24)$$

By carrying out the arithmetic implied in (A.4) and (A.5), and using the notation (3.19 - 3.24), the partial wave expansion of the $\underset{\sim}{M}$ matrix, equations (3.25) to (3.30), is obtained.

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ABSORPTIVE DAMPING IN n-p CHARGE-EXCHANGE

G. A. RINGLAND *

Tait Institute of Mathematical Physics, The University, Edinburgh

and

R. J. N. PHILLIPS

A.E.R.A., Harwell, England

Recently it has been realised that one-particle exchange reactions can be severely modified by absorption in the initial and final states [1-5]. Attempts have been made to explain the backward peak in high-energy n-p scattering [6] by one-meson exchange processes [7-11], but none have considered the effect of absorption. The latter is a particularly favourable process in which to consider damping, since both the initial and final state interactions are quite well known, (in contrast to other situations that have been studied [1-5]).

In the present letter we calculate this damping effect for one-pion exchange (OPE) and ρ-meson exchange (RME), with absorption deduced from p-p diffraction scattering [12,13]. We find that damping goes a long way toward reconciling OPE with experiment, though a significant discrepancy remains. For elementary RME, damping does not

change the unacceptable energy-dependence. Setting this aside, RME illustrates short-range spin-independent charge-exchange; damping narrows the corresponding peak but not nearly enough to fit the data.

The problem is to combine two interactions, the absorption U that gives diffraction scattering and the exchange interaction V. To first order in V, the correct amplitude is

$$T_{fi} = \langle \chi_f^{(-)} | V | \chi_i^{(+)} \rangle + \langle \varphi_f | U | \chi_i^{(+)} \rangle \quad (1)$$

in the notation of ref. 14, where $\chi_i^{(+)}$ and $\chi_f^{(-)}$ are appropriate initial and final eigenstates of scattering with U. We shall ignore the second term in eq. (1), which describes backward scattering from the absorptive potential itself (see, however ref. 15). The other term we calculate by the Sopkovich [1] prescription: the Born amplitude for V between initial partial wave a and final partial wave b is multiplied by $\sqrt{(S_b^f S_a^i)}$, where S_a^i and S_b^f are
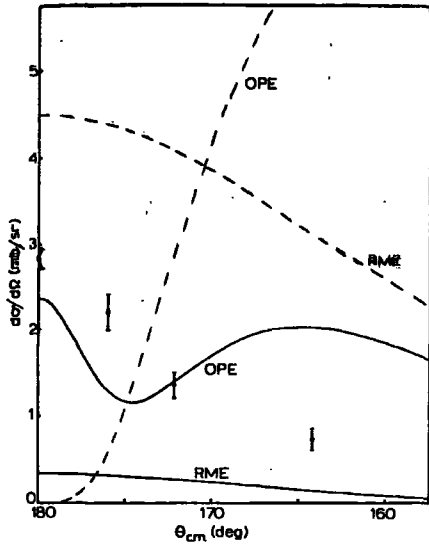
Fig. 1. n-p backward scattering at 2.85 GeV. The experimental points are from ref. 6. Damped and undamped cases are distinguished by solid and dashed curves respectively. The RME cases are scaled down by $\frac{1}{25}$.
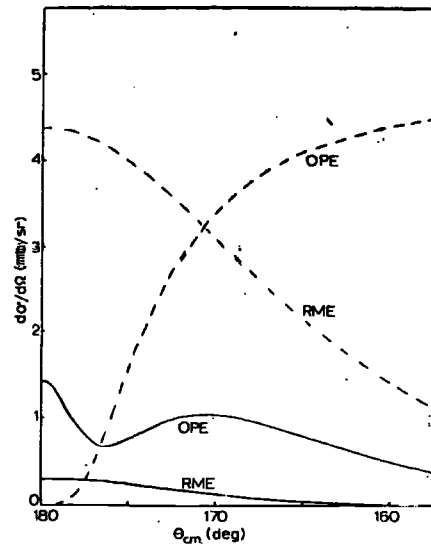

Fig. 2. n-p backward scattering at 7 GeV. Solid and dashed curves denote damped and undamped cases respectively. The RME curves are scaled down by $\frac{1}{50}$.

the S-matrix elements for scattering with U in the initial and final channels. For pure diffraction scattering, each of the latter has the form $S = \exp(-2\chi)$, where $i\chi$ is the imaginary phase shift. This prescription has not been formally justified for the situation of interest, where V has longer range than U, but is claimed to be a good approximation at small angles [3]. Furthermore, an argument [16] based on the coupled-channel N/D method and a random phase approximation also gives this prescription, in the case of pure imaginary phase shifts.

Consider first OPE. The contribution to the n-p scattering amplitude from the backward pole has the form

$$M_{ss} = -(g^2/4\pi)\, \alpha/2E \ ,$$
$$M_{11} = -(g^2/4\pi)\, \alpha(1+x)/4E \ ,$$
$$M_{00} = (g^2/4\pi)\, \alpha x/2E \ , \qquad\qquad (2)$$
$$M_{10} = M_{01} = -(g^2/4\pi)\sqrt{2}\,\sin\theta\ \alpha/4E \ ,$$
$$M_{1-1} = -(g^2/4\pi)\, \alpha(1-x)/4E \ ,$$

in the conventional singlet-triplet notation [17]. Here $x = \cos\theta$, $\alpha = (1+x)/(x_0+x)$, $x_0' = 1 + m_\pi^2/2p^2$, θ is the c.m. angle, p is the relative momentum, $g^2/4\pi = 14.4$ is the pion-nucleon coupling constant and $E = \sqrt{(m_N^2+p^2)}$. The corresponding differential cross-section is:

$$d\sigma/d\Omega = (g^2/4\pi)^2\, (\alpha/2E)^2 \ . \qquad (3)$$

Reggeized OPE would give almost exactly the same, in the region of interest close to the pole. There are several remarkable things about OPE. The cross section vanishes at $\theta = 180°$ where experimentally there is a peak; at other angles it becomes much bigger than this peak; it is strongly spin-dependent (the double-spin-flip term M_{1-1} accounts for half of $d\sigma/d\Omega$ near $180°$).

To introduce damping we use empirical studies of p-p diffraction scattering (expecting n-p to be similar), in which the phase shift is assumed to be pure imaginary and to depend only on orbital angular momentum L. We decompose the Born amplitudes of eq. (2) into partial waves: terms corresponding to scattering with L are modified by factors $\exp(-2\chi_L)$: terms which couple L to $L\pm 2$ have factors $\exp(-\chi_L - \chi_{L\pm 2})$. It would not be correct simply to expand M in terms of Legendre functions P_L, P_L^1 and P_L^2 and to damp the coefficients according to L, since some coefficients contain couplings to $L\pm 2$. (Thus mistreating the coupling terms would violate the time-reversal symmetry of the S-matrix).

The resulting cross section is shown in figs. 1 and 2, for 2.85 and 7 GeV respectively. The damping factors are from Serber's model [12] (ref. 13 gives similar results); this model was not strictly designed to match diffraction scattering at the lower energy, but we expect it to in-

troduce little error. It gives almost 100% absorption for $L = 0$, decreasing steadily as L increases. Bare OPE cross sections are also shown.

It can be seen that absorptive damping produces remarkable changes. It converts the dip at $\theta = 180^O$ into a narrow peak, similar to experiment out to the half-width, and greatly reduces the cross section at wider angles; but it leaves a broad second maximum that is not observed. The first peak comes mainly from M_{SS} and M_{00}; damping destroys the balance of terms which previously made them vanish at 180°. The second peak is in fact not surprising: it comes from M_{1-1}, which has to vanish at $\theta = 180^O$ for invariance reasons and is large in bare OPE. It cannot be removed by any set of χ_L consistent with total cross sections: its presence follows directly from the spin-dependence of OPE. Any model which neglects spin [18] will not have this term, but will be misleading.

We have also considered a form factor in bare OPE; plausible form factors depress the second peak only a little. The second peak may however be cancelled by some unknown background effect, as in ref. 7.

Consider now RME. Many RME models use a Reggeized ρ, but it has not been clarified how far the effects of damping are implicitly included in a Regge pole. We therefore apply damping only to "elementary" RME. The latter is known to have unacceptable energy dependence; however, it is interesting to consider whether damping affects this behaviour. Furthermore, at any particular energy, RME illustrates the general case of a short-range spin-independent charge-exchange; it is interesting to see how strongly the angular distribution is altered by damping.

Near $\theta = 180^O$, the elementary RME amplitude from the backward pole is, to a good approximation (exact at 180°), both spin-independent and determined by the vector $\bar{N}N\rho$ coupling. It has the form

$$a = \frac{f^2}{4\pi} \frac{1}{2E} \left(\frac{(m_N + E)^2}{2p^2} + 1 - 2x + \frac{p^2 x^2}{2(E + m_N)^2} \right) / (x_1 + x) , \tag{4}$$

where $f^2/4\pi$ is the $\bar{N}N\rho$ vector coupling constant, which we take to be $=2$; $x_1 = 1 + m_\rho^2/2p^2$; other notation as in eq. (2). In terms of the M-matrix, $M_{11} = M_{00} = -M_{SS} = a$ and spin-flip terms $= 0$; this is spin-independent in the sense of a forward charge-exchange.

We introduce damping as before (though now there are no $L \to L \pm 2$ couplings); the resultant bare and damped RME cross sections are illustrated in figs. 1 and 2. Note that damping does not change the energy-dependence at $\theta = 180^O$ for the examples shown. This result can easily be shown in general, by using an integral representation for the partial wave sum as in refs. 3 and 5. Thus elementary RME remains physically untenable.

The OPE and RME amplitudes, bare or damped, are real and do not interfere. Of course we may expect other contributions too. In particular, the difference of total cross sections $\sigma_T(pp) - \sigma_T(np)$ implies a small imaginary amplitude, spin independent in the same sense as RME, which does not interfere with either OPE or RME. At 2.85 GeV, this last contribution gives about 0.3 mb/sr at $\theta = 180^O$, which would bring OPE closer to experiment at this angle.

Our results indicate these conclusions:
1. Damping goes far to reconcile OPE with experiment, but leaves a secondary peak which is not observed.
2. This second peak is a spin-flip effect.
3. Damping does not change the energy-dependence of OPE or RME.
4. The overall magnitude of RME is more strongly reduced than OPE, because of its shorter range.
5. Damping narrows the elementary RME peak, but not enough to fit experiment.

Finally, we stress that our treatment of damping depends on no arbitrary parameters.

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NUCLEON-ANTINUCLEON CHARGE EXCHANGE IN THE ABSORPTION MODEL

R. J. N. PHILLIPS
A.E.R.E., Harwell

and

G. A. RINGLAND
Department of Mathematics, University of Durham

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The results of an absorptive model calculation of the reaction $p + \bar{p} \rightarrow n + \bar{n}$ at 3.0 and 3.6 GeV/c are presented. Good agreement with experiment is obtained for $-t \geqslant 0.1$ (GeV)^2; at smaller angles there is a possible discrepancy.

In this letter we report a calculation of nucleon-antinucleon charge exchange scattering, $p + \bar{p} \rightarrow n + \bar{n}$, assuming one-particle exchange plus absorptive corrections [1].

We consider only one pion exchange. Though other mesons such as ρ and A_2 can contribute to this process, those with spin greater than zero give an unacceptable energy dependence if treated as elementary particles. It has been found that elementary ρ exchange plus absorptive corrections gives the wrong magnitude and angular distribution, as well as the wrong energy dependence, for np [2] and πp [3] charge exchange. Reggeizing ρ and A_2 can correct the energy dependence, but the consistency of adding absorption corrections to a Regge pole is still uncertain.

After decomposing one pion exchange into particle waves [4] we introduce absorptive corrections using the original Sopkovich prescription [5]

$$T = \exp(i\delta_a) \, T_B \, \exp(i\delta_b) \, ,$$

where T is the corrected partial wave amplitude, T_B is the unmodified (Born) amplitude, δ_a and δ_b are the phase shifts for elastic scattering in the initial and final states. To obtain elastic phase shifts in the GeV region, one must resort to phenomenological models. We assume a Gaussian model which gives

$$\exp(2i\,\delta_L) = 1 - A \exp(-\gamma L^2)$$

with L the orbital angular momentum. Svensson [6] has determined the parameters from p$\bar{\text{p}}$ elastic scattering: $A = 1$, $\gamma = 0.0335$ and 0.0271 at 3.0 GeV/c and 3.6 GeV/c respectively. Charge-independence gives the same phase shifts for n$\bar{\text{n}}$ scattering.

The results of our calculation, using the Gaussian model phase shifts and a pion-nucleon coupling $g^2 = 14.4$, are shown in fig. 1. The experimental points are from ref. 7, and represent an average over the data at 3.0 and 3.6 GeV/c. It can be seen that, for $-t > 0.1$ (GeV/c)^2, there is good agreement with experiment in both magnitude and angular distribution. At small momentum transfers, however, the theory predicts structure for which there is no experimental evidence.

We have tried other models for the elastic