Durham University

# Durham E-Theses

## *Analysis of polyhedral domed sandwich structures*

Manos, G. C.

**How to cite:**

Manos, G. C. (1975) *Analysis of polyhedral domed sandwich structures*, Durham theses, Durham University. Available at Durham E-Theses Online: http://etheses.dur.ac.uk/8427/

**Use policy**

The full-text may be used and/or reproduced, and given to third parties in any format or medium, without prior permission or charge, for personal research or study, educational, or not-for-profit purposes provided that:

- a full bibliographic reference is made to the original source
- a link is made to the metadata record in Durham E-Theses
- the full-text is not changed in any way

The full-text must not be sold in any format or medium without the formal permission of the copyright holders.

Please consult the full Durham E-Theses policy for further details.

Academic Support Office, The Palatine Centre, Durham University, Stockton Road, Durham, DH1 3LE
e-mail: e-theses.admin@durham.ac.uk Tel: +44 0191 334 6107
http://etheses.dur.ac.uk

# ANALYSIS
# OF
# POLYHEDRAL DOMED SANDWICH STRUCTURES

by

G.C. Manos

A thesis submitted to satisfy the requirements
for the degree of Ph.D.

The copyright of this thesis rests with the author.
No quotation from it should be published without
his prior written consent and information derived
from it should be acknowledged.

Department of Engineering Science
University of Durham

1975

13 APR 1977

This work is dedicated to my father

"Τὰ πάντα ῥεῖ"

Ἡράκλειτος

## ABSTRACT

The finite element method was employed for the analysis of the polyhedral domed sandwich structures.

Two different variational approaches were used for comparison reasons. These are the "displacement formulation" and the "mixed formulation" as they are commonly known.

Initially seven sandwich plate bending models were developed. These models were used to solve a number of problems where a numerical or experimental solution existed and comparisons were made.

The agreement varied from fair to excellent depending on the nature of the model and the type of the solved problem.

As a result of this comparative study four of these models were consequently selected to be extended for the development of the sandwich dome models.

The accuracy of these four sandwich dome models was tested by modelling five polyhedral dome structures. The results derived from each individual model were compared with experimental results obtained by other researchers and by the author himself.

The author's contribution to the experimental work was the design, construction and subsequent testing of two full scale prototypes, namely, the 24 faced and the 36 faced domes.

From the whole analysis it was established that the developed numerical models, when selectively applied in the most appropriate way with regard to their special characteristics and the nature of the problem, produce reliable results.

Special problems were investigated arising from the boundary conditions as well as structural details of the joint-lines of the plates forming the polyhedron, and thus a solution was suggested.

Finally, a data generation routine is also described in order to facilitate further application of the various developed models by future users or researchers.

ACKNOWLEDGEMENTS

This work was carried out under the supervision of Dr. G.M. Parton, lecturer in Structural Engineering at the Department of Engineering Science, University of Durham, to whom the author is greatly indebted for his constant guidance and assistance as well as his personal encouragement.

The author is also deeply grateful to Professor G.R. Higginson, Department of Engineering Science, University of Durham, for all his encouragment and help.

The author takes the opportunity of acknowledging his gratitude to Dr. P. Bettes lecturer at the Department of Civil Engineering, University College of Swansea, for all his help.

For their contribution to the experimental work the author would like to thank the technical staff of the Department of Engineering Science, University of Durham.

For their contribution to the various problems relevant to the development of the numerous computer programs, the author wishes to express his thanks to members of staff of the Computer Unit Department, University of Durham.

Thanks must also be expressed to the staff of Durham University Library.

Finally, the author thanks Mrs. J. Henderson and Miss J. Campbell for typing the manuscript.

CONTENTS

ABSTRACT

ACKNOWLEDGEMENTS

Chapter 1. Introduction 1

1.1 Polyhedral Domed Structures 1

1.2 Sandwich Panels 2

1.3 The Finite Element Approach 3

Chapter 2. Constitutive Equations for a Sandwich Plate 5

2.1 Introduction 5

2.2 Mathematical Formulation 5

Chapter 3. Variational Approach 12

Chapter 4. Variational Principles (Small Deflections) 15

4.1 Definition of symbols and fundamental relationships 17

4.2 Displacement Models 21

4.3 Mixed Models 22

Chapter 5. Models Employed in the Formation of the Sandwich Plate and Sandwich Dome Models 28

5.1 Transverse displacement approximating models 28

5.1.1 Non-conforming triangular finite element in plate bending 29

5.1.2 Refined triangular plate bending (eighteen-degrees-of-freedom) finite element 30

5.1.3 Linear variation of the transverse displacement 31

5.1.4 Quadratic variation of the transverse displacement 31

5.1.5 Cubic variation of the transverse displacement 32

5.2 Transverse Shear Deformation approximating models 33

5.2.1 Linear variation of transverse shear deformation 33

5.2.2 Quadratic variation of transverse shear deformation 33

5.2.3 Cubic variation of transverse shear deformation 34

5.3 In plane displacement approximating models 35

5.3.1 Linear variation of the in plane displacements 35

5.3.2 Quadratic variation of the in plane displacements 35

5.3.3 Cubic variation of the in plane displacements 36

5.3.4 Cubic variation of the in plane displacements 37

5.4 Moments approximating models 38

5.4.1 Linear variation of moments 38
5.4.2 Quadratic variation of moments 39
5.5 Total rotation approximating models 40

Chapter 6. Sandwich Plate Bending Models 41
6.1 Displacement models 41
6.1.1 Deflection-shear model with 15 degrees of freedom 42
6.1.2 Deflection - shear model with 21 degrees of freedom 43
6.1.3 Deflection - shear model with 24 degrees of freedom 44
6.1.4 Deflection - shear model with 30 degrees of freedom 45
6.1.5 Total rotation model with 18 degrees of freedom 46
6.2 Mixed models 47
6.2.1 Mixed model with 12 degrees of freedom 47
6.2.2 Mixed model with 24 degrees of freedom 48
6.3 Symbols for the different elements 49

Chapter 7. Dome Models 51
7.1 Displacement models 52
7.1.1 Deflection shear model with 21 degrees of freedom 52
7.1.2 Deflection shear model with 33 degrees of freedom 53
7.1.3 Total rotation model with 30 degrees of freedom 53
7.2 Mixed models 54
7.2.1 Mixed model with 36 degrees of freedom 55

Chapter 8. Transformation - condensation 56
8.1 Transformation formulae 56
8.2 Transformation matrices for a plate 58
8.2.1 Displacement models 58
8.2.2 Mixed models 59
8.2.3 Rotation of an element 60
8.3 Transformation matrices for a dome model 62
8.4 Condensation 66

Chapter 9. Experimental Work 67
9.1 Introduction 67
9.2 Construction and Materials 67
9.3 Testing of the Domes 70
9.4 Displacement Measurements 72

Chapter 10. Behaviour of the joint 75
10.1 Introduction 75

10.2 Tests 75
10.3 Analysis of the function of the joint 76
10.3.1 Axial tensile strength 76
10.3.2 Strength under bending 76
10.4 Angular function of the joint under bending 78
10.5 Conclusion from the behaviour of the joint 79

Chapter 11. Computer programs 80
11.1 Introduction 80
11.2 Data generation routine input 82

Chapter 12. Sandwich plate bending results 91
12.1 Theoretical results
12.1.1 Comparison with Dynamic Relaxation Method 91
12.1.2 Comparison with Finite Difference Method 92
12.1.3 Comparison with Finite Difference Method 92
12.1.4 Comparison with Finite Element Models 92
12.1.5 Comparison with Fourier Series Method 93
12.2 Experimental Results 93
12.2.1 Comparison with Sandwich Plates tested by Bettes 93
12.2.2 Comparison with triangular sandwich plates under concentrated load tested by Collins 94
12.3 Conclusions 94
12.4 The extension from plate to dome models 95

Chapter 13. Results from the sandwich dome models 97
13.1 Tetrahedral dome 98
13.2 Square pyramid 98
13.3 16 faced dome 99
13.4 24 faced dome 100
13.5 36 faced dome 101

Chapter 14. Conclusions 103

Notation

References

# APPENDICES


I. Geometrical relationships for a triangle

II. Formation of matrices (reference to PDS15, DDS21)

III. Formation of matrices (reference to PDS24)

IV. Formation of Stiffness Matrix for the model PMX24

V. Formation of matrices (reference to PRO18, DRO30)

VI. Numerical Integration

VII. Listings of computer programs

# 1. INTRODUCTION

The present work is the product of the combination of three factors with a contemporary approach in the field of Structural Engineering.

First there is the cost factor which can be countered by mass-production. At the same time we are trying to fulfil two principles of modern Architecture. The first is that the Structure must be functioning in an optimum co-existence with the Environment and the Human (Functionalism). The second is that the Structure must have the flexibility to adapt to new developments and to continuously changing economical and social conditions.(Metabolism). [13,22,50,70,100]

The second factor is the formation and investigation of new structural materials. The main aim in a given Structural application is the optimisation of the use of the material achieved by improving the properties. [6,19,40,46,55,79]

Finally the third and last factor is the mathematical analysis of the problem which involves the modelling of the Structure by using new powerful computerised methods of analysis. [39,59,72,83,84,115]

## 1.1 Polyhedral domed structures

The polyhedral domed structures approximate to structurally efficient double-curvature surfaces by using flat plates, having at the same time the advantage of easy construction in comparison with the formation of the double-curvature surfaces themselves.

The domed structure is composed from as few types of flat plates as possible, in as far as the dimensions of the plates are concerned, so that the mass-production of the simplest construction-element can be employed.

13 APR 1977
LIBRARY

The solution to a specific problem of space-coverage can be reached by various types of polyhedral domes. This provides considerable flexibility as regards the economic factors as well as aesthetic ones for the final choice.

For the above mentioned reasons it is believed that by the polyhedral domed Structures the Architectural principle of "Functionalism" is well treated.

On the other hand the assemplage of a number of plates (or group of plates) to form the whole structure includes the potentiality of an easy expansion or alteration. This presents an advantageous adaptability to new conditions so that the Architectural principle of "Metabolism" is also well preserved.

More detailed and extensive information about the geometry and construction of the polyedral domed structures is presented in reference [85].

1.2 Sandwich Panels

The present work is exclusively concerned with sandwich panels as construction elements for the polyhedral domed structures.

We define a sandwich panel as one which is a three-layer type of construction. It consists of two thin sheets of high stiffness material which are called the faces of the panel and between them is a thick layer of low average stiffness and density which is called the core of the panel. [3,20,46,85,89]

The most important advantages of sandwich construction are, firstly, that the ratio of high rigidity which can be achieved by the sandwich panels over the total dead weight of the construction is higher in comparison with conventional types of construction; secondly, the panels employed for the structure can easily be made and supplied by the industry in various types and dimensions and thirdly, the structure appears to have good thermal and acoustical insulation.

The materials used for the construction of the experimental prototypes and the work involved is to be presented in chapters 9, 10.

1.3 The Finite element approach

The method which will be used to analyse the behaviour of the polyhedral domed sandwich structures is known as the finite element method. Its basic principle is the idea of piecewise approximating continuous fields.

The method is outlined by reference [84] to Professor Oden's presentation of the differences between the classical and the Finite element approach.

"..... Classically the analysis of continuous systems began with investigations of the properties of small differential elements of the continuum under investigation. Relationships were established among mean values of various quantities associated with the infinitesmal elements and partial differential equations governing the behaviour of the entire domain were obtained by allowing the dimensions of the elements to approach zero as the number of elements become infinitely larger.

In contrast to this classical approach the finite element method begins with investigations of the properties of elements of finite dimensions.

The equations describing the continuum may be employed in order to arrive at the properties of these elements, but the dimensions of the elements remain finite in the analysis, integrations are replaced by finite summations and the partial differential equations of the continuous media are replaced for example by systems of algebraic or ordinary differential equations.

The continuum with infinite degrees of freedom is thus represented by a discrete model which has finite degrees of freedom.

Moreover if certain conditions (to be outlined in Chapter 3) are satisfied, then as the number of elements is increased and their dimensions are decreased the behaviour of the discrete system converges to that of the continuous system.

Many numerical methods were developed before the era of electronic computers and are now adapted for use with these machines.

In contrast, the finite element method is a complete product of the electronic computer age. This is due to the fact that the method possesses certain characteristics that take full advantage of the facilities offered by the high-speed computers so that it can be systematically programmed to accommodate such complex and difficult problems as non-homogeneous materials, non-linear stress-strain behaviour and complicated boundary conditions.

# 2. CONSTITUTIVE EQUATIONS FOR A SANDWICH PLATE

## 2.1 Introduction

As was mentioned in Chapter 1 (section 3), first of all the equations describing the continuum must be formed.

We start with the fundamental equations for a plate and include the effects of the sandwich form of the plate taking into account certain assumptions (to be outlined in the next section).

The basic aim is to establish the constitutive equations for a sandwich plate in such a form that together with the variational principles(to be outlined in Chapter 3) we have all that is required for the finite elements analysis

## 2.2 Mathematical formulation

We consider an infinitesimal element of a sandwich plate and we write the relations between the stress-resultants and the stress tensor (Fig. 2.1) [53,78,102]

$$N_{ij} = \int_{-h/2}^{h/2} t_{ij}\, dz \;, \quad M_{ij} = \int_{-h/2}^{h/2} t_{ij}\, z\, dz \;, \quad Q_i = \int_{-h/2}^{h/2} t_{iz}\, dz \qquad (2.1)$$

$$(i\,,j \;=\; x\,,y)$$

We introduce the following assumptions [4,20,54,71,93,94,101]

1. The displacements $u\,,v\,,w$ are constant across the thickness of the plate considering them individually.

2. For $-\frac{c}{2} \leq z \leq \frac{c}{2}$ the stresses $t_{xx}, t_{yy}, t_{xy} = 0$

   For $\frac{c}{2} \leq z \leq \frac{c}{2} + f$ the stresses $t_{xz}, t_{yz} = 0$

   and $-\frac{c}{2} - f \leq z \leq -\frac{c}{2}$

   which means that the contribution of the core to direct stresses is neglected as well as the contribution of the faces to shear stresses.

3. The following equations which relate the strain tensor with the displacements u, v, w (Fig.2.2) are valid (for r, s = x, y). The subscripts after the comma denote derivatives.

$$e_{rs} = -w_{,sr} + \frac{c}{4}\left({}_c\gamma_{r,s} + {}_c\gamma_{s,r}\right) + \varepsilon_{rs} \quad \text{for } \frac{c}{2} \leq z \leq \frac{c}{2} + f$$

$$e_{rs} = -w_{,sr} - \frac{c}{4}\left({}_c\gamma_{r,s} + {}_c\gamma_{s,r}\right) + \varepsilon_{rs} \quad \text{for } -\frac{c}{2} - f \leq z \leq -\frac{c}{2}$$

$$e_{rz} = \frac{1}{2}\ {}_c\gamma_r \qquad \text{for } -\frac{c}{2} \leq z \leq \frac{c}{2} \tag{2.2}$$

where $\varepsilon_{xx} = u_{,x} + \frac{1}{2} w_{,x}^2, \quad \varepsilon_{yy} = v_{,y} + \frac{1}{2} w_{,y}^2,$

$$\varepsilon_{xy} = u_{,y} + v_{,y} + w_{,x}\, w_{,y}$$

$$e_{zz} = 0 .$$

4. The thickness of the faces, f, is much smaller than the thickness of the core, c.

5. For both the faces and the core if we consider symmetry with respect to the x-y plane we have the following stress-strain relationship of any point in the plate in a matrix form.

$$\begin{bmatrix} t_{xx} \\ t_{yy} \\ t_{zz} \\ t_{yz} \\ t_{xz} \\ t_{xy} \end{bmatrix} = \begin{bmatrix} C^{xx}_{xx} & C^{xx}_{yy} & C^{xx}_{zz} & \phi. & \phi. & C^{xx}_{xy} \\ & C^{yy}_{yy} & C^{yy}_{zz} & \phi. & \phi. & C^{yy}_{xy} \\ & & C^{zz}_{zz} & \phi. & \phi. & C^{zz}_{xy} \\ & \text{symmetrical} & & C^{yz}_{yz} & C^{yz}_{xz} & \phi. \\ & & & & C^{xz}_{xz} & \phi. \\ & & & & & C^{xy}_{xy} \end{bmatrix} \begin{bmatrix} e_{xx} \\ e_{yy} \\ e_{zz} \\ e_{yz} \\ e_{xz} \\ e_{xy} \end{bmatrix} \tag{2.3}$$

Taking into account assumptions 2 and 3 we obtain from equations 2.1 the following

$$N_{ij} = \int_{-c/2-f}^{-c/2} t_{ij}dz + \int_{c/2}^{c/2+f} t_{ij}dz$$

$$M_{ij} = \int_{-c/2-f}^{-c/2} t_{ij}zdz + \int_{c/2}^{c/2+f} t_{ij}zdz \tag{2.4}$$

$$Q_i = \int_{-c/2}^{c/2} t_{iz}dz$$

Taking into account assumption 3 and equations (2.4) the relationship (2.3) becomes

$$\begin{bmatrix} t_{xx} \\ t_{yy} \\ t_{xy} \\ t_{xz} \\ t_{yz} \end{bmatrix} = \begin{bmatrix} {}_fC^{xx}_{xx} & {}_fC^{xx}_{yy} & {}_fC^{xx}_{xy} & \phi. & \phi. \\ & {}_fC^{yy}_{yy} & {}_fC^{yy}_{xy} & \phi. & \phi. \\ & \text{symmetrical} & {}_fC^{xy}_{xy} & \phi. & \phi. \\ & & & {}_cC^{xz}_{xz} & {}_cC^{xz}_{yz} \\ & & & & {}_cC^{yz}_{yz} \end{bmatrix} \begin{Bmatrix} e_{xx} \\ e_{yy} \\ e_{xy} \\ e_{xz} \\ e_{yz} \end{Bmatrix} \qquad (2.5)$$

or

$$t_{ij} = {}_fC^{ij}_{rs} \; e_{rs}$$

$$t_{ij} = {}_cC^{iz}_{rz} \; e_{rz} \qquad (2.6)$$

(for $i, j, r, s = x, y$)

where the prefix of f or c on the elastic constants refers to the faces or the core respectively

Substituting equations (2.6) into equations (2.4) we obtain

$$N_{ij} = {}_fC^{ij}_{rs} \int_{-c/2-f}^{-c/2} e_{rs}\,dz + {}_fC^{ij}_{rs} \int_{c/2}^{c/2+f} e_{rs}\,dz$$

$$M_{ij} = {}_fC^{ij}_{rs} \int_{-c/2-f}^{-c/2} e_{rs}\,z\,dz + {}_fC^{ij}_{rs} \int_{c/2}^{c/2+f} e_{rs}\,z\,dz \qquad (2.7)$$

$$Q_i = {}_cC^{iz}_{rz} \int_{-c/2}^{c/2} e_{rz}\,dz$$

Taking into account assumption 3 with regard to equations (2.2) and evaluating of the integrals we obtain the following equations which relate the stress resultants for a sandwich plate to the displacements or the derivative of the displacements of the mid surface of the plate. These will be used in the finite elements analysis.

$$N_{ij} = {}_{c}C^{ij}_{rs}\, 2f\, \varepsilon_{rs}$$

$$M_{ij} = {}_{f}C^{ij}_{rs}\left[2w,_{rs}\left(\frac{c^2 f}{4} + \frac{cf^2}{2} + \frac{f^3}{3}\right) - (\gamma_{r,s} + \gamma_{s,r})\left(\frac{c^2 f}{4} + \frac{cf^2}{2} + \frac{f^3}{4}\right)\right] \qquad (2.8)$$

$$Q_i = {}_{c}C^{iz}_{rz}\,\frac{c+f}{2}\,\gamma_r$$

where $\gamma_r = \dfrac{c}{c+f}\,{}_{c}\gamma_r$

At this point, taking the validity of assumption 4 into account, we obtain the following equations in a matrix form

$$\begin{bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \\ Q_x \\ Q_y \\ N_{xx} \\ N_{yy} \\ N_{xy} \end{bmatrix} = \begin{bmatrix} D^{xx}_{xx} & D^{xx}_{yy} & D^{xx}_{xy}/2 & & & & & \\ D^{yy}_{xx} & D^{yy}_{yy} & D^{yy}_{xy}/2 & & & & & \\ D^{xy}_{xx} & D^{xy}_{yy} & D^{xy}_{xy}/2 & & & & & \\ & & & S^{xz}_{xz} & S^{xz}_{yz} & & & \\ & & & S^{yz}_{xz} & S^{yz}_{yz} & & & \\ & & & & & E^{xx}_{xx} & E^{xx}_{yy} & E^{xx}_{xy} \\ & & & & & E^{yy}_{xx} & E^{yy}_{yy} & E^{yy}_{xy} \\ & & & & & E^{xy}_{xx} & E^{xy}_{yy} & E^{xy}_{xy} \end{bmatrix} \begin{bmatrix} a_{xx} \\ a_{yy} \\ a_{xy} \\ \gamma_x \\ \gamma_y \\ \varepsilon_{xx} \\ \theta_{yy} \\ \varepsilon_{xy} \end{bmatrix} \qquad (2.9)$$

$$a_{xx} = \frac{\partial}{\partial_x}(w,_x - \gamma_x), \qquad a_{yy} = \frac{\partial}{\partial_y}(w,_y - \gamma_y), \qquad a_{xy} = \frac{\partial}{\partial_y}(w,_x - \gamma_x) + \frac{\partial}{\partial_x}(w,_y - \gamma_y$$

(2.10)

$$D^{ij}_{rs} = -{}_fC^{ij}_{rs}\,\frac{f}{2}\,(c+f)^2, \qquad S^{iz}_{rz} = {}_cC^{iz}_{rz}\,\frac{c+f}{2}, \qquad E^{ij}_{rs} = {}_fC^{ij}_{rs}\,2f$$

(2.11)

The constants $D^{ij}_{rs}$, $S^{iz}_{rz}$, $E^{ij}_{rs}$ can be evaluated using suitable experimental methods. [20,46]

For orthotropic faces and core the relationship (2.9) becomes

$$\begin{vmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \\ Q_x \\ Q_y \\ N_{xx} \\ N_{yy} \\ N_{xy} \end{vmatrix} = \begin{vmatrix} D^{xx}_{xx} & D^{xx}_{yy} & \phi. & & & & & \\ D^{yy}_{xx} & D^{yy}_{yy} & \phi. & & & & & \\ \phi. & \phi. & D^{xy}_{xy}/2 & & & & & \\ & & & S^{xz}_{xz} & & & & \\ & & & & S^{yz}_{yz} & & & \\ & & & & & E^{xx}_{xx} & E^{xx}_{yy} & \phi. \\ & & & & & E^{yy}_{xx} & E^{yy}_{yy} & \phi. \\ & & & & & \phi. & \phi. & E^{xy}_{xy} \end{vmatrix} \begin{vmatrix} a_{xx} \\ a_{yy} \\ a_{xy} \\ \gamma_x \\ \gamma_y \\ \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \end{vmatrix} \qquad (2.12)$$

So by equations (2.12) we can relate the stress resultants with the strains through an operator which can be written in a shorter matrix form as

$$\{\sigma\} = [D] \quad \{\varepsilon\} \qquad (2.13)$$

$\{\sigma\}$ is the stress resultants vector

$\{\varepsilon\}$ the strain vector which is related through equations (2.2), (2.10) with the displacements of the midsurface $u, v, w$ and their derivatives.

$[D]$ is an operator called the elasticity matrix. The terms of this matrix, as we have already mentioned, can be determined by certain experimental methods

FIG. 2.1.1. SIGN CONVENTION, DISPLACEMENT MODELS

FIG. 2.1.2. SIGN CONVENTION, MIXED MODELS

FIG. 2.2. KINEMATICS

## 3. VARIATIONAL APPROACH

In this chapter some elements of variational calculus are presented. These principles are used in the subsequent analysis.

Consider an expression of the form:-

$$I = \int_{x_1}^{x_2} F(x, w, \frac{dw}{dx}, \frac{d^2w}{dx^2}) \, dx \tag{3.1}$$

This expression is generally known as a "functional" and in the analysis of solid continua is an expression with regard to a specific physical state (potential energy, complementary energy etc. to be outlined in Chapter 4). [39,72,80,88,115]

The basic aim is to find a function $w(x)$ satisfying the boundary conditions and being such that the functional is rendered stationary.

This is expressed as follows:

$$\delta I = \phi. \tag{3.2}$$

(where $\delta$ is the variation operator)

Following a certain procedure [39,72] we eventually obtain the expressions:

$$\frac{\partial F}{\partial w} - \frac{d}{dx}\left(\frac{\partial F}{\partial (dw/dx)}\right) + \frac{d^2}{dx^2}\left(\frac{\partial F}{\partial (d^2w/dx^2)}\right) = \phi . \tag{3.3}$$

The above is known as the Euler-Lagrange equation. Also, in addition to the above, the following may be obtained

$$\left[\frac{\partial F}{\partial (dw/dx)} - \frac{d}{dx}\left(\frac{\partial F}{\partial (dw/dx)}\right)\right]_{x_1}^{x_2} = \phi. \text{ and } \left[\frac{\partial F}{\partial (d^2w/dx^2)}\right]_{x_1}^{x_2} = \phi. \tag{3.4}$$

This is known as "the natural boundary conditions".

If they are satisfied they are called "free boundary conditions" or else if one of them is not satisfied then a corresponding set of equations must be satisfied instead. The latter are called "geometric boundary conditions" or "forced boundary conditions".

Instead of trying to solve the governing differential equation(3.3) we form a close approximation of the functional which is noted as the $\bar{I}$ (the dash above the symbol indicates the approximate one of the same nature). Hence if we find a solution $\bar{w}(x)$, for the functional $\bar{I}$ it can be assumed that this solution will be close enough to the exact solution $w(x)$ as well. Following the approximate solution approach the analysis proceeds as follows:

Firstly, by assuming a mathematical expression for the unknown function $\bar{w}(x)$, preferably a polynomial of x, so that the functional $\bar{I}$ becomes a function of the unknown coefficients of the polynomial.

Thus $\delta\bar{I} = \phi$. can be expressed and satisfied by the following set of equations

$$\frac{\partial \bar{I}}{\partial a_i} = \phi. \tag{3.5}$$

($a_i$ are the coefficients of the polynomial)

The use of a polynomial of x for expressing $\bar{w}$, possesses the advantage of an easy mathematical manipulation.

Secondly, by performing the integration, summing the subintegrals, of the function $\bar{I}$, of a finite number of subdomains which form the whole domain (finite elements).

Thus, combining the finite element method (outlined in Chapter 1 section 3) with the variational approach, the primary functional may be related to the individual element rather than the total domain. Hence the geometry of the overall body and the system of the boundary conditions are not unsolved obstacles, even for highly complex problems, as they were in the classical Rayleigh-Ritz approach, from which the finite element method is derived.

The polynomial for the unknown function mentioned above must be such that certain conditions are satisfied and consequently the convergence towards the exact solution can be achieved.

These conditions vary with the nature of the functional and the variational principle which is to be employed [72,80,81,82,83,84,87, 99,104,115]

These conditions can, however, be described in general as follows:

(a) The number of coefficients (terms) of the polynomal selected to represent the unknown function must be at least equal to the number of the degrees of freedom associated with the element.

(b) The chosen function should provide compatibility of certain quantities across element interfaces.

(c) A rigid body deformation and a constant curvature state should be included in the polynomial.

(d) The assumed function must be continuous and be differentiable to an order consistent with the variational principle expressing the problem.

## 4. VARIATIONAL PRINCIPLES (SMALL DEFLECTIONS)

The various approaches in the finite element analysis of solid continua are associated with several variational principles of solid mechanics, thus introducing different types of finite element methods.

These types have been classified as follows:

(a) The first derives from the principle of minimum potential energy and is based on the assumption of a continuous displacement field over the entire solid. The various models based in this approach are known as "displacement models".

39,41,43,72,74, 80,81,82,87,88, 97,104,115]

(b) The second derives from the principle of minimum complementary energy and is based on assumed equilibrium stress fields. The various models based in this approach are known as "the equilibrium models".

[5,39,41,43,45, 72,77,80,81,82, 86,88,104]

(c) The third derives from a modified complementary energy principle with assumed stress functions within the element and displacement functions at the element interfaces.

The various models based in this approach are known as the "hybrid models".

14,34,39,72,80, 81,82,88,104]

(d) The fourth derives from Reissners-variational principle with assumed continuous displacement field over the entire solid and assumed stress field for individual elements.

The various models based in this approach are known as "the mixed models".

38,39,72,80,81 82,88,95,104]

The models used in the present analysis are based either in the first or fourth approach. At the following sections the mathematical formulation of the first and fourth approach are presented in detail

## 4.1. DEFINITION OF SYMBOLS AND FUNDAMENTAL RELATIONSHIPS

$\{\delta_o^e\}$ overall vector of nodal degrees of freedom for an element

$\{\delta_o^w\}$ vector of nodal degrees of freedom as far as the transverse displacement models are concerned

$\{M_o^e\}$ vector of nodal degrees of freedom as far as the moment-models are concerned

$\{\delta_o^s\}$ vector of nodal degrees of freedom as far as the shear-models are concerned

$\{\delta_o^{uv}\}$ vector of nodal degrees of freedom as far as the in-plane displacement models are concerned

$\{\delta_o^\theta\}$ vector of nodal degrees of freedom as far as the total rotation models are concerned

$\{\delta_o\}$ vector of general displacements within an element corresponding to the nodal degrees of freedom

$[N]$ shape functions matrix relating the general displacement vector $\{\delta_o\}$ with the general vector of nodal degrees of freedom $\{\delta_o^e\}$

$$\{\delta_o\} = [N]\{\delta_o^e\} \tag{4.1}$$

$\{\varepsilon\}$ strains vector, as described in Chapter 2, section 2 by the equations (2.9), (2.10), (2.12), (2.13)

[B] strain-displacement matrix relating the strains vector $\{\varepsilon\}$ with the nodal degreees of freedom vector $\{\delta_o^e\}$

$$\{\varepsilon\} = [B]\ \{\delta_o^e\} \tag{4.2}$$

$\{\sigma\}$ stress-resultants vector as described in Chapter 2, section 2 by the equations (2.1), (2.9), (2.10), (2.12), (2.13), (Fig. 2.1)

[D] elasticity matrix relating the stress-resultants vector $\{\sigma\}$ with the strains vector $\{\varepsilon\}$ as described in Chapter 2, section 2 by the equations (2.9), (2.11), (2.12), (2.13)

$$\{\sigma\} = [D]\ \{\varepsilon\}$$

[C] elasticity matrix relating the strains vector $\{\varepsilon\}$ with the stress-resultants vector $\{\sigma\}$

$$\{\varepsilon\} = [C]\ \{\sigma\} \tag{4.3}$$

$$[C_b] = [D_b]^{-1}, \quad D_o = D_{xx}^{xx} \cdot D_{yy}^{yy} - D_{yy}^{xx} \cdot D_{xx}^{yy} \tag{4.4}$$

$[S_n^o]$ stress matrix relating the stress-resultants vector $\{\sigma\}$ with the vector of nodal degrees of freedom $\{\delta_o^e\}$.

$$\{\sigma\} = [S_n^o]\ \{\delta_o^e\} \tag{4.5}$$

w transverse displacement (corresponding to z axis) (Fig.2.2) known otherwise as deflection

u in plane displacement (corresponding to x axis) (Fig. 2.2)

v in plane displacement (corresponding to y axis) (Fig. 2.2)

| | |
|---|---|
| $\theta_x$ , $\theta_y$ | total rotation of the cross sections zx, zy respectively (Fig. 2.2) |
| $w'_x$ , $w'_y$ <br> $w'_{xx}$, $w'_{xy}$ , $w'_{yy}$ | first and second derivatives of the transverse displacement with respect to x or y axis (Physical meaning slopes and curvatures) |
| $\phi_x$, $\phi_y$ | transverse shear deformation of the zx, zy respectively identical with symbols used before as shear strains $\gamma_x$ , $\gamma_y$ for the cross section zx, zy respectively (Fig. 2.2) |
| $\{M_{ij}\}$ $(i,j = x,y)$ | moments vector |
| $\{Q_i\}$ $(i = x,y)$ | shear forces vector |
| $\{N_{ij}\}$ $(i,j = x,y)$ | in plane forces vector |
| $\{\bar{R}_o\}$ | prescribed nodal force vector (corresponding with the displacement vector $\{\delta_o^e\}$ as far as the work product is concerned) |
| $\bar{M}$, $\bar{Q}$, $\bar{\theta}$, $\bar{w}$ | prescribed quantities of the same nature as the ones noted above |
| $\bar{P} = \begin{Bmatrix} \bar{P}_x \\ \bar{P}_y \\ \bar{P}_z \end{Bmatrix}$ | distributed load vector (corresponding to the axes x, y, z respectively) |
| $A_n$ | Area of the $n^{th}$ element |

$S_{o,n}$ portion of the boundary where $(M_{nn}, M_{ns}, Q_n)$ are prescribed

$S_{n,n}$ portion of the boundary where $(w, \theta_n, \theta_s)$ are prescribed

$[K_n^o]$ the stiffness matrix of the nth element with respect to the local system

$[R_n^o]$ the load vector of the nth element with respect to the local system

$[K_n]$ the stiffness matrix of the nth element with respect to the global system

$\{R_n\}$ the load vector of the nth element with respect to the global system

$\{\delta^e\}$ the overall nodal degrees of freedom vector

$[K]$ the overall stiffness matrix

$[R]$ the overall load vector

## 4.2 Displacement-models

For the displacement models the functional which is employed is the potential energy of the continuum. The condition enforced through the variational principle, is such that it minimises the potential energy. The polynomials employed to approximate the unknown functions of the functional, are functions of certain nodal values (degrees of freedom) which from the structural analysis point of view are displacement or derivatives of the displacements. [8,17,21,29, 30,36,44,63,65,74,115] Continuity, compatability and completeness requirements will be discussed for each individual model.

The potential energy for a sandwich plate is

$$I = \sum_{1}^{n} \left\{ \frac{1}{2} \iint_{A_n} \{\varepsilon\}^T \{\sigma\} \, dA - \iint_{A_n} \{\delta_o\}^T \{\bar{P}\} \, dA - \{\delta_o^e\}^T \{\bar{R}_o\} \right\} \tag{4.6}$$

for the finite element approximation all the parameters must be expressed in the functional as functions of the unknown nodal values using the notation of Chapter 4, section 1).
For the diplsacements models the vector $\{\delta_o^e\}$ includes as nodal degrees of freedom displacements and derivatives of the displacements.
After the substitutions the functional has the form:-

$$\bar{I} = \sum_{1}^{n} \left\{ \frac{1}{2} \{\delta_o^e\}^T \iint_{A_n} [B]^T [D] [B] dA \, \{\delta_o^e\} - \{\delta_o^e\}^T \iint_{A_n} [N]^T \{\bar{P}\} \, dA - \{\delta_o^e\}^T \{\bar{R}_o\} \right\} \tag{4.7}$$

where $\Sigma$ is the summation symbol and n the number of the elements).

(Appendices II and III provide more details of the nature of the matrices involved in the above expression in the form they have been developed for the present applications.)

By assuming that the following equations are valid:-

$$[K_n^o] = \iint_{A_n} [B]^T [D][B] \, dA \qquad \{R_n^o\} = \iint_{A_n} [N]^T \{\bar{P}\} \, dA + \{\bar{R}_o\} \tag{4.8}$$

the expression (4.7) becomes:-

$$\bar{I} = \sum_1^n \left\{ \frac{1}{2} \{\delta_o^e\}^T [K_n^o] \{\delta_o^e\} - \{\delta_o^e\}^T \{R_n^o\} \right\} \tag{4.9}$$

Applying the variation of the functional $\bar{I}$ in the form

$$\frac{\partial \bar{I}}{\partial \{\delta_o^e\}} = \phi. \tag{4.10}$$

one obtains:-

$$\sum_1^n \left\{ [K_n^o] \{\delta_o^e\} - \{R_n^o\} \right\} = \phi. \tag{4.11}$$

or

$$[K] \{\delta^e\} - \{R\} = \phi. \tag{4.12}$$

### 4.3 Mixed models

For the mixed models the functional is of a different form than the one used for the displacement models.

The conditions enforced through the variational principle leads to a stationary value of the functional. The polynomials employed to approximate the unknown functions are functions of certain nodal values (degrees of freedom) which are from the structural analysis point of view displacement or derivatives of the displacement as well as stresses. [25,38,56,57,80,81,82,87,90,91,95,105,106,107]

The continuity, compatibility and completeness requirements can vary.

The functional has the form

$$I = \sum_{1}^{n} \left\{ \iint_{A_n} \left( \{\sigma_b\}\{\theta_o\}^T - \frac{1}{2}\{\sigma_b\}^T [C_b] \{\sigma_b\} + \frac{1}{2}\{\varepsilon_{uv}\}^T [D_{uv}]\{\sigma_{uv}\} \right) dA \right.$$

$$- \iint_{A_n} \begin{Bmatrix} u \\ v \\ w \end{Bmatrix} \{\bar{P}\}^T dA - \{\delta_o^e\}^T \{\bar{R}_o\}$$

$$- \int_{S_{o,n}} (\bar{M}_{nn}\, \theta_n + \bar{M}_{ns}\, \theta_s + \bar{Q}_n\, w)\, ds$$

$$\left. - \int_{S_{n,n}} [M_{nn}\, (\theta_n - \bar{\theta}_n) + M_{ns}\, (\theta_s - \bar{\theta}_s)\ [\ Q_n\, (w - \bar{w})\ ]\, ds \right\} \qquad (4.13)$$

(See equations 2.9, 2.10, 2.11, 2.12, 2.13)

where $\{\theta_o\}^T = \{-\theta_{x,x}, -\theta_{y,y}, -(\theta_{x,y} + \theta_{y,x}), -\theta_x + w_{,x}, -\theta_y + w_{,y}\}$

$$\{\sigma_b\}^T = \{ M_{xx}, \quad M_{yy}, \quad M_{xy}, \quad Q_x, \quad Q_y \}$$

$$\{\varepsilon_{uv}\}^T = \{ \varepsilon_{xx}, \quad \varepsilon_{yy}, \quad \varepsilon_{xy} \} \qquad (4.14)$$

$$\{\sigma_{uv}\}^T = \{ N_{xx}, \quad N_{yy}, \quad N_{xy} \}$$

$$[C_b] = [D_b]^{-1} = \begin{bmatrix} \frac{D^{yy}_{yy}}{D_o} & -\frac{D^{xx}_{yy}}{D_o} & \phi. & \phi. & \phi. \\ -\frac{D^{yy}_{xx}}{D_o} & \frac{D^{xx}_{xx}}{D_o} & \phi. & \phi. & \phi. \\ \phi. & \phi. & \frac{2}{D^{xy}_{xy}} & \phi & \phi. \\ \phi. & \phi. & \phi. & \frac{1}{S^{xz}_{xz}} & \phi. \\ \phi. & \phi. & \phi. & \phi. & \frac{1}{S^{yz}_{yz}} \end{bmatrix} \tag{4.15}$$

$$[D_{uv}] = \begin{bmatrix} E^{xx}_{xx} & \bar{E}^{xx}_{yy} & \phi. \\ E^{yy}_{xx} & E^{yy}_{yy} & \phi. \\ \phi. & \phi. & E^{xy}_{xy} \end{bmatrix} \tag{4.16}$$

The following expression is obtained as the functional using equations (4.13), assuming continuity for $M_{nn}$, $M_{ns}$, w, u, v across element interfaces, following the procedure of [25,38,56,57,80,81,82, 87,90,91,95,105,107].

$$I = \sum_{1}^{n} \left\{ \iint_{A_n} \left( \{\sigma_b\}^T \{\theta_1\} - \frac{1}{2} \{\sigma_b\}^T [C_b] \{\sigma_b\} + \frac{1}{2} \{\varepsilon_{uv}\}^T [D_{uv}] \{\sigma_{uv}\} \right) dA \right.$$

$$- \iint_{A_n} \begin{Bmatrix} u \\ v \\ w \end{Bmatrix} \{\bar{P}\}^T dA - \{\delta_o\} \{\bar{R}_o\}$$

$$\left. - \int_{S_{o,n}} \bar{Q}_n \, w \, ds + \int_{S_{n,n}} (M_{nn} \bar{\theta}_n + M_{ns} \bar{\theta}_s) \, ds \right\} \tag{4.17}$$

where

$$\{\theta_1\}^T = \{ \phi. , \phi. , \phi. , w_{,x} , w_{,y} \} \tag{4.18}$$

for the finite elements approximation all the parameters must be expressed in the functional as functions of the unknown nodal values using the notation of Chapter 4, section 1). (For more details see Appendices II and IV).

For the mixed models the nodal degrees of freedom are displacements together with moments.

After the substitutions the functional has the form

$$\bar{I} = \sum_{1}^{n} \left\{ \{M_o^e\}^T [K_n^{mw}] \{\delta_o^w\} + \frac{1}{2} \{M_o^e\}^T [K_n^{mq}] \{M_o^e\} + \frac{1}{2} \{\delta_o^{uv}\}^T [K_{uv}] \{\delta_o^{uv}\} \right.$$

$$\left. - \{M_o^e\} \{R_n^m\} - \{\delta_o^w\} \{R_n^w\} - \{\delta_o^{uv}\} \{R_n^{uv}\} \right\} \tag{4.19}$$

Applying the variation of the functional $\bar{I}$ in the form

$$\left.\begin{aligned} \frac{\partial \bar{I}}{\partial \{\delta_o^{uv}\}} &= \phi. \\ \frac{\partial \bar{I}}{\partial \{\delta_o^{w}\}} &= \phi. \end{aligned}\right\} \text{Force-displacements relationships} \tag{4.20}$$

$$\frac{\partial \bar{I}}{\partial \{M_o^e\}} = \phi. \quad \text{Equilibrium equations} \tag{4.21}$$

one obtains

$$\sum_{1}^{n} \left\{ [K_n^o] \{\delta_o^e\} - \{R_n^o\} \right\} = \phi. \tag{4.22}$$

$$[K_n^o] = \begin{bmatrix} [K_n^{uv}] & [\phi.] & [\phi.] \\ [\phi.] & [\phi.] & [K_n^{mw}]^T \\ [\phi.] & [K_n^{mw}] & [K_n^{mq}] \end{bmatrix} \tag{4.23}$$

$$\{\delta_o^e\} = \begin{Bmatrix} \{\delta_o^{uv}\} \\ \{\delta_o^{w}\} \\ \{M_o^e\} \end{Bmatrix} \tag{4.24}$$

$$\{R_n^o\} = \begin{Bmatrix} \{R_n^{uv}\} \\ \{R_n^{w}\} \\ \{R_n^{m}\} \end{Bmatrix} \tag{4.25}$$

or

$$[K] \{\delta_o^e\} - \{R\} = \phi. \tag{4.26}$$

Thus a system of linear simultaneous equations is obtained for the mixed models, which are of a similar form to those obtained for the displacement models (4.12).

The stiffness matrix [K] for both the displacements and the mixed models is symmetric and positively definite. When the boundary conditions are introduced the stiffness matrix becomes nonsingular.

Thus a solution can be obtained by using one of the techniques for solving a large system of equations taking advantage of the symmetry and the banded nature of the stiffness matrix [64,75,108]

The various characteristics and advantages of the different techniques are presented in references [39,72,115].

The technique employed in the present analysis is a modification of the frontal solution [20] as it has been developed by BETTESS, compatible with the M.T.S. system (N.U.M.A.C.). There is also a version of the same technique with the same alteration compatible with the OS system in Cambridge.

The frontal solution technique has been proved advantageous due to the nature of the analysed problems involving a very large number of unknowns and complex boundary conditions.

It has been combined with a data generation programme (to be outlined in Chapter 11 ) which reduces effectively the amount of work required for the solution of a specific problem.

## 5. MODELS EMPLOYED IN THE FORMATION OF THE SANDWICH PLATE AND SANDWICH DOME MODELS

As it has been presented in Chapter 4 starting from the functionals (4.6), (4.13) and following the variational approach outlined in Chapter 3 a system of linear simultaneous equations can be obtained (4.12), (4.26). According to this approach the different parameters of the functional are approximated with different finite element models which are to be presented in this chapter. The various models for the sandwich plate and sandwich dome problems are composed by suitable combinations of the various basic models which can be classified in the following five groups.

(a) Transverse displacement approximating models (w)

(b) Transverse shear deformation approximating models $(\phi_x, \phi_y)$

(c) In-plane displacements approximating models (u, v)

(d) Moments approximating models $(M_{xx}, M_{yy}, M_{xy})$

(e) Total rotations approximating models $(\theta_x, \theta_y)$

All the above five groups of finite elements in the present analysis are of triangular shape with corner nodes and may also have mid-side or centre nodes (Fig.5.1÷5.15).The triangular shaped models have the potentiality of being applicable to any shape of structure in the interest of the present work.

### 5.1 Transverse displacement approximating models

Numerous models have been developed for the analysis of classical plate bending (shear deformation neglected). The following have been chosen to be employed in the present analysis as they have been proved successful. [9,10,11,17,18,24,29,30,35, 36,44,48,49,63,92,97,111,112,113,114,115]

### 5.1.1 Non-conforming triangular finite element in plate bending

This element has been presented first in 1965 [12] Further investigation of its characeristics has been accomplished by several applications [26,115] Through the conclusions which have been obtained this element has been proved to be simple and successful. The discontinuity of normal slope across the interelement boundaries doesnot prevent the element from yielding accuracy and convergence occurs for regular element sub-divisions.

The transverse displacement w has been given by the relationship

Fig. 5.1

$$w = [N^w] \{\delta_o^w\} \tag{5.1}$$

$$\{\delta_o^w\}^T = [w_1, w_{,x1}, w_{,y1}, w_2, w_{,x2}, w_{,y2}, w_3, w_{,x3}, w_{,y3}] \tag{5.2}$$

subscript x, y indicates derivatives of x, y respectively and the number (1 ÷ 3) indicates the relevant node

$$[N^w] = [N_1^w, N_2^w, N_3^w, N_4^w, N_5^w, N_6^w, N_7^w, N_8^w, N_9^w] \tag{5.3}$$

$N_1^w = L_1 + L_1^2L_2 + L_1^2L_3 - L_1L_2^2 - L_1L_3^2 \Longrightarrow N_4^w, N_7^w$ from $N_1^w$ with circle-symmetrical substitution of subscripts 1, 2, 3

$N_2^w = C_3(L_1^2L_2 + \frac{1}{2} L_1L_2L_3) - C_2(L_1^2L_3 + \frac{1}{2} L_1L_2L_3) \Longrightarrow N_5^w, N_6^w$ from $N_2^w$ as above

$N_3^w = b_2(L_1^2L_3 + \frac{1}{2} L_1L_2L_3) - b_3(L_1^2L_2 + \frac{1}{2} L_1L_2L_3) \Longrightarrow N_6^w, N_9^w$ from $N_3^w$ as above

(5.4)

(see Appendix I for the geometric symbols in use).

### 5.1.2 Refined triangular plate bending (eighteen-degrees-of-freedom) finite element

The variation of the transverse displacement for this element is a quintic polynomial of $x, y$ (or $L_1, L_2, L_3$).
It derives from the full quintic polynomial of 21 terms assuming cubic variation of the normal slope $w_{n_i}$ along the interelement boundaries (see Appendix III). The main advantages of the refined element is that ensuring continuity of the normal slope the convergence is much faster, thus good accuracy can be obtained for coarse mesh idealizations. [18,35,36,37,115]

The discontinuity of the normal curvature and hence bending moment are much smaller than lower order elements. The disadvantage of this element is the difficulty in applying the boundary conditions due to the existence of higher order derivatives of the transverse displacements

Fig. 5.2

$$w = [N^w] \{\delta_o^w\} \tag{5.5}$$

$$\{\delta_o^w\}^T = [w_i, w'_{xi}, w'_{yi}, w'_{xxi}, w'_{xyi}, w'_{yyi}, \ldots\ldots]_{i = 1 \div 3} \tag{5.6}$$

$$[N^w] = [F] \quad [T]^{-1} \tag{5.7}$$

the formation of the matrices [F] and $[T]^{-1}$ is given in Appendix III.

### 5.1.3 Linear variation of the transverse displacement

This model is to be used in the mixed formulation [39,59,72,105,115] (See Appendix IV).

Fig. 5.3

$$w = [N^w] \quad \{\delta_o^w\} \tag{5.8}$$

$$\{\delta_o^w\}^T = [w_1, w_2, w_3] \tag{5.9}$$

$$[N^w] = [L_1, L_2, L_3] \tag{5.10}$$

### 5.1.4 Quadratic variation of the transverse displacement

This model is also to be used in the mixed formulation [39,59,72,105,115] (See Appendix IV).

Fig. 5.4

$$w = [N^w] \quad \{\delta_o^w\} \tag{5.11}$$

$$\{\delta_o^w\}^T = [w_1, w_2, w_3, w_4, w_5, w_6] \tag{5.12}$$

$$[N^w] = [(2L_1-1)L_1, \ (2L_2-1)L_2, \ (2L_3-1)L_3, \ 4L_2L_3, \ 4L_1L_3, \ 4L_1L_2] \tag{5.13}$$

### 5.1.5 Cubic variation of the transverse displacement

This element is to be used in the rotation element (see Chapter 6 ) [39,59,72,105,115]

Fig. 5.5

$$w = [N^w] \{\delta_o^w\} \tag{5.14}$$

$$\{\delta_o^w\}^T = \{w_i, \ldots\}_{i=1 \div 7} \tag{5.15}$$

$$[N^w] = [N_i^w, \ldots)_{i=1 \div 7} \tag{5.16}$$

$$\begin{aligned}
N_1^w &= (2L_1-1)L_1 + 3L_1L_2L_3 \\
N_2^w &= (2L_2-1)L_2 + 3L_1L_2L_3 \\
N_3^w &= (2L_3-1)L_3 + 3L_1L_2L_3 \\
N_4^w &= 4L_2L_3 - 12L_1L_2L_3 \\
N_5^w &= 4L_1L_3 - 12L_1L_2L_3 \\
N_6^w &= 4L_1L_2 - 12L_1L_2L_3 \\
N_7^w &= 27L_1L_2L_3
\end{aligned} \tag{5.17}$$

## 5.2 Transverse shear deformation approximating models. [39,59,72,105,115]

### 5.2.1 Linear variation of transverse shear deformation.

$$\phi_x = [N^s] \{\delta_1^s\} \tag{5.18}$$

$$\phi_y = [N^s] \{\delta_2^s\} \tag{5.19}$$

Fig. 5.6

$$\{\delta_1^s\}^T = [\phi_{x_1}, \quad \phi_{x_2}, \quad \phi_{x_3}] \tag{5.20}$$

$$\{\delta_2^s\}^T = [\phi_{y_1}, \quad \phi_{y_2}, \quad \phi_{y_3}] \tag{5.21}$$

$$[N^s] = [L_1, \quad L_2, \quad L_3] \tag{5.22}$$

### 5.2.2 Quadratic variation of transverse shear deformation

$$\phi_x = [N^s] \{\delta_1^s\} \tag{5.23}$$

$$\phi_y = [N^s] \{\delta_2^s\} \tag{5.24}$$

Fig. 5.7

$$\{\delta_1^s\}^T = [\phi_{x_i}, \ldots\ldots] \quad i = 1 \div 6 \tag{5.25}$$

$$\{\delta_2^s\}^T = [\phi_{yi}, \ldots\ldots] \quad i = 1 \div 6 \tag{5.26}$$

$$[N^s] = [(2L_1-1)L_1, \ (2L_2-1)L_2, \ (2L_3-1)L_3, \ 4L_2L_3, \ 4L_1L_3, \ 4L_1L_2] \tag{5.27}$$

5.2.3 Cubic variation of transverse shear deformation.

$$\phi_x = [N^s] \{\delta_1^s\} \tag{5.28}$$

$$\phi_y = [N^s] \{\delta_2^s\} \tag{5.29}$$

Fig. 5.8

$$\{\delta_1^s\}^T = \{\phi_{xi}, \ldots\ldots\} \quad i = 1 \div 7 \tag{5.30}$$

$$\{\delta_2^s\}^T = \{\phi_{yi}, \ldots\ldots\} \quad i = 1 \div 7 \tag{5.31}$$

$$[N^s] = [N_i^s, \ldots\ldots] \quad i = 1 \div 7 \tag{5.32}$$

$$\begin{aligned}
N_1^s &= (2L_1 - 1)L_1 + 3L_1L_2L_3 \\
N_2^s &= (2L_2 - 1)L_2 + 3L_1L_2L_3 \\
N_3^s &= (2L_3 - 1)L_3 + 3L_1L_2L_3 \\
N_4^s &= 4L_2L_3 - 12L_1L_2L_3 \\
N_5^s &= 4L_1L_3 - 12L_1L_2L_3 \\
N_6^s &= 4L_1L_2 - 12L_1L_2L_3 \\
N_7^s &= 27L_1L_2L_3
\end{aligned} \tag{5.33}$$

## 5.3 In plane displacement approximating models

The first successful examples of the application of the finite element method were the two dimensional elastic problems of plane stress. The majority of the various existing finite elements are based on the displacement approach [7,8,28,38,39,42,58,59,72,103,105,111,115] although there are several finite elements based on different approaches

In the present analysis four displacement triangular models have been chosen as the most suitable and simple.

### 5.3.1 Linear variation of the in plane displacements

Fig. 5.9

$$u = [N^{uv}] \{\delta_1^{uv}\} \tag{5.34}$$

$$v = [N^{uv}] \{\delta_2^{uv}\} \tag{5.35}$$

$$\{\delta_1^{uv}\}^T = [u_1, u_2, u_3] \tag{5.36}$$

$$\{\delta_2^{uv}\}^T = [v_1, v_2, v_3] \tag{5.37}$$

$$[N^{uv}] = [L_1, L_2, L_3] \tag{5.38}$$

(see Appendix II for the formation of $[B^{uv}]$ matrix.

### 5.3.2 Quadratic variation of the in plane displacements

Fig. 5.10

$$u = [N^{uv}] \{\delta_1^{uv}\} \tag{5.39}$$

$$v = [N^{uv}] \{\delta_2^{uv}\} \tag{5.40}$$

$$\{\delta_1^{uv}\}^T = [u_i, \ldots]_{i=1 \div 6} \tag{5.41}$$

$$\{\delta_2^{uv}\}^T = [v_i, \ldots]_{i=1 \div 6} \tag{5.42}$$

$$[N^{uv}] = [(2L_1-1)L_1, (2L_2-1)L_2, (2L_3-1)L_3, 4L_2L_3, 4L_1L_3, 4L_1L_2] \tag{5.43}$$

### 5.3.3 Cubic variation of the in plane displacements

The variation along the interelement boundaries is quadratic. This element has been developed to cope with transformation difficulties along plate interconnections for the dome structures.

Fig. 5.11

$$u = [N^{uv}] \{\delta_1^{uv}\} \tag{5.44}$$

$$v = [N^{uv}] \{\delta_2^{uv}\} \tag{5.45}$$

$$\{\delta_1^{uv}\}^T = [u_1, u_2, u_3, u_{,SS4}, u_{,SS5}, u_{,SS6}, u_7] \tag{5.46}$$

$$\{\delta_2^{uv}\}^T = [v_1, v_2, v_3, v_{,SS4}, v_{,SS5}, v_{,SS6}, v_7] \tag{5.47}$$

where

$$u_{,SS} = \frac{\partial^2 u}{\partial \bar{s}^2} \quad , \quad v_{,SS} = \frac{\partial^2 v}{\partial \bar{s}^2} \tag{5.48}$$

(where $\bar{s}$ is the vector along the sides $\overrightarrow{23}$, $\overrightarrow{31}$, $\overrightarrow{12}$ )

(see also Appendx I (8)).

$$[N^{uv}] = [N_i^{uv}, \ldots]_{i=1 \div 7} \tag{5.49}$$

$$N_1^{uv} = L_1 - 9L_1L_2L_3 \quad , \quad N_2^{uv} = L_2 - 9L_1L_2L_3 \quad , \quad N_3^{uv} = L_3 - 9L_1L_2L_3$$

$$N_4^{uv} = -0.5L_2L_3 + 1.5L_1L_2L_3 \quad , \quad N_5^{uv} = -0.5L_1L_3 + 1.5L_1L_2L_3 \quad , \quad N_6^{uv} = 0.5L_1L_2 + 1.5L_1L_2L_3$$

$$N_7^{uv} = 27L_1L_2L_3 \tag{5.50}$$

5.3.4 Cubic variation of the in plane-displacements.

$$u = [N^{uv}] \{\delta_1^{uv}\} \tag{5.51}$$

$$v = [N^{uv}] \{\delta_2^{uv}\} \tag{5.52}$$

Fig. 5.12

$$\{\delta_1^{uv}\}^T = [u_i \ , \ \ldots . \ ] \quad i = 1 \div 7 \tag{5.53}$$

$$\{\delta_2^{uv}\}^T = [v_i \ , \ \ldots . \ ] \quad i = 1 \div 7 \tag{5.54}$$

$$[N^{uv}] = [N_i^{uv} , \ \ldots . \ ] \quad i = 1 \div 7 \tag{5.55}$$

$$\begin{aligned}
N_1^{uv} &= (2L_1 - 1)L_1 + 3L_1L_2L_3 \\
N_2^{uv} &= (2L_2 - 1)L_2 + 3L_1L_2L_3 \\
N_3^{uv} &= (2L_3 - 1)L_3 + 3L_1L_2L_3 \\
N_4^{uv} &= 4L_2L_3 - 12L_1L_2L_3 \\
N_5^{uv} &= 4L_1L_3 - 12L_1L_2L_3 \\
N_6^{uv} &= 4L_1L_2 - 12L_1L_2L_3 \\
N_7^{uv} &= 27L_1L_2L_3
\end{aligned} \tag{5.56}$$

## 5.4 Moments approximating models

For the approximation of moments distribution in the mixed models approach several moments interpolation functions have been used. The nature and the order of these functions is defined by the continuity requirements associated with the functional and the variational principle used for the derivation of the model. [25,56,57,90,91,105,106,107]

The following have been chosen for the present analysis

### 5.4.1 Linear variation of moments

Fig. 5.13

$$M_{xx} = [N^m] \quad \{M_1^e\} \tag{5.57}$$

$$M_{yy} = [N^m] \quad \{M_2^e\} \tag{5.58}$$

$$M_{xy} = [N^m] \quad \{M_3^e\} \tag{5.59}$$

$$\{M_1^e\}^T = [M_{xx1}, \; M_{xx2}, \; M_{xx3}] \tag{5.60}$$

$$\{M_2^e\}^T = [M_{yy1}, \; M_{yy2}, \; M_{yy3}] \tag{5.61}$$

$$\{M_3^e\}^T = [M_{xy1}, \; M_{xy2}, \; M_{xy3}] \tag{5.62}$$

$$[N^m] = [L_1, \; L_2, \; L_3] \tag{5.63}$$

(See Appendix IV for more details)

### 5.4.2 Quadratic variation of moments

$$M_{xx} = [N^m] \quad \{M_1^e\} \tag{5.64}$$

$$M_{yy} = [N^m] \quad \{M_2^e\} \tag{5.65}$$

$$M_{xy} = [N^m] \quad \{M_3^e\} \tag{5.66}$$

Fig. 5.14

$$\{M_i^e\}^T = [M_{xxi} \ , \ \ldots.] \qquad i = 1 \div 6 \tag{5.67}$$

$$\{M_2^e\}^T = [M_{yyi} \ , \ \ldots.] \qquad i = 1 \div 6 \tag{5.68}$$

$$\{M_3^e\}^T = [M_{xyi} \ , \ \ldots.] \qquad i = 1 \div 6 \tag{5.69}$$

$$[N^m] = [(2L_1 - 1)L_1, \ (2L_2 - 1)L_2, \ (2L_3 - 1)L_3, \ 4L_2L_3, \ 4L_1L_3, \ 4L_1L_2] \tag{5.70}$$

## 5.5 Total rotation approximating models

For the total rotation finite element formulation (to be outlined in the Chapter 6) the following model has been employed. [39,59,72,105,115]

Fig. 5.15

$$\theta_x = [N^\theta] \{\delta_1^\theta\} \tag{5.71}$$

$$\theta_y = [N^\theta] \{\delta_2^\theta\} \tag{5.72}$$

$$\{\delta_1^\theta\}^T = [\theta_{xi}, \ldots]_{i=1 \div 6} \tag{5.73}$$

$$\{\delta_2^\theta\}^T = [\theta_{yi}, \ldots]_{i=1 \div 6} \tag{5.74}$$

$$[N^\theta] = [(2L_1-1)L_1,\ (2L_2-1)L_2,\ (2L_3-1)L_3,\ 4L_2L_3,\ 4L_1L_3,\ 4L_1L_2] \tag{5.75}$$

## 6. SANDWICH PLATE BENDING MODELS

A large number of various publications is available as far as the bending of sandwich plate is concerned

Various finite elements have also been developed to solve the problem [1,12,14,15,21,67,68,69,73,85,90,91]

Seven different models have been developed in the present analysis as a first step towards the solution of the polyhedral dome sandwich structures. These models deal with the sandwich plate bending problem and their classification coincides with the classification of the variational principles employed for the development of each individual model respectively, that is displacement models and mixed models (Chapter 4).

### 6.1 Displacement models

The variational principle outlined in Chapter 4 Section 2 has been employed for the development of the displacement models.

For the finite element approximation the strains and stresses in the functional are expressed as functions of nodal values of displacements (degrees of freedom).

Two different groups of sandwich plate displacement models have been developed depending on the form of strains-nodal displacements relationships.

The models classified in the first group are to be called Deflection-Shear Displacement models.

The strains as formulated in Chapter 2 have the form

$$
\begin{aligned}
a_{xx} &= w_{,xx} - \phi_{x,x} \\
a_{yy} &= w_{,yy} - \phi_{y,y} \\
a_{xy} &= 2w_{,xy} - \phi_{x,y} - \phi_{y,x} \\
\gamma_x &= \phi_x \\
\gamma_y &= \phi_y
\end{aligned}
\tag{6.1}
$$

The parameters at the right hand side of equations (6.1) are expressed as functions of nodal transverse displacements and their derivatives and nodal transverse shear deformations independently employing the models outlined in Chapter 5 (section 5.1, 5.2).

The models classified in the second group are to be called Total-Rotation displacement models.

The strains as formulated in Chapter 2 have the form

$$
\begin{aligned}
a_{xx} &= \theta_{x,x} \\
a_{yy} &= \theta_{y,y} \\
a_{xy} &= \theta_{x,y} + \theta_{y,x} \\
\gamma_x &= w_{,x} - \theta_x \\
\gamma_y &= w_{,y} - \theta_y
\end{aligned}
\tag{6.2}
$$

The parameters at the right hand side of equations (6.2) are expressed as functions of nodal total rotations and nodal transverse displacements independently, employing the models outlined in Chapter 5 (sections 5.1, 5.5).

The displacement models developed are the following:

6.1.1 <u>Deflection-shear model with 15 degrees of freedom</u>

For reference to this model the symbol PDS 15 has been employed (See Section 6.3 for more details).

This model has been developed employing the non-conforming triangular finite element in plate bending together with the linear variation of shear deformation model (5.1.1., 5.2.1.) for expressing the parameters in equations (6.2), (4.3). More details for the formation of the various matrices are presented in Appendix II

Fig. 6.1

The vector of the nodal degrees of freedom has the form

$$\{\delta_o^e\}^T = \{w_1, w'_{x1}, w'_{y1}, w_2, w'_{x2}, w'_{y2}, w_3, w'_{x3}, w'_{y3}, \phi_{x1}, \phi_{y1}, \phi_{x2}, \phi_{y2}, \phi_{x3}, \phi_{y3}\} \quad (6.3)$$

Employing the transformation relationships (to be outlined in Chapter 8) the transformed stiffness, stress and load matrices are obtained ÷

a) When the vector of the nodal degrees of freedom becomes ÷

$$\{\delta_o^e\}^T = \{w_i, w'_{xi}, w'_{yi}, \phi_{xi}, \phi_{yi}, \ldots\ldots\}_{i=1 \div 3} \quad (6.4)$$

b) For a node (i) which belongs to a boundary the set of degrees of freedom linked with this node becomes ÷

$$\{w_i, \theta_{ni}, w'_{si}, \phi_{ni}, \phi_{si}\} \quad (6.5)$$

(where $\bar{n}$ is the vector normal to the boundary $\bar{s}$)

### 6.1.2 Deflection-shear model with 21 degrees of freedom

Reference symbol PDS21

This model has been developed employing the non-conforming triangular finite element in plate bending together with the quadratic variation of shear deformation model (5.1.1., 5.2.2) for expressing the parameters in equations (6.1), (4.8). The same procedure for the formation of the various matrices is employed as for the PDS15 model (Appendix II).

Fig. 6.2

The vector of the nodal degrees of freedom has the form:

$$\{\delta_o^e\}^T = \{w_1, w_{,x1}, w_{,y1}, w_2, w_{,x2}, w_{,y2}, w_3, w_{,x3}, w_{,y3}, \phi_{x1}, \phi_{y1}, \phi_{x2}, \phi_{y2}, \phi_{x3}, \phi_{y3}, \phi_{x4}, \phi_{y4}, \phi_{x5}, \phi_{y5}, \phi_{x6}, \phi_{y6}\} \tag{6.6}$$

Employing the transformation relationships (Chapter 8) the transformed stiffness, stress and load matrices are obtained.

a) When the vector of the nodal degrees of freedom becomes ÷

$$\{\delta_o^e\}^T = \{w_i, w_{,xi}, w_{,yi}, \phi_{xi}, \phi_{yi}, \ldots, \phi_{xj}, \phi_{yj}, \ldots\} \quad \begin{matrix} i = 1 \div 3 \\ j = 4 \div 6 \end{matrix} \tag{6.7}$$

b) For a node (i) which belongs to a boundary ($\bar{s}$) the set of degrees of freedom linked with this node becomes ÷

$$\{w_i, \theta_{ni}, w_{,si}, \phi_{ni}, \phi_{si}\} \quad \text{for} \quad i \leq 3 \tag{6.8}$$

$$\{\phi_{ni}, \phi_{si}\} \quad \text{for } i > 3 \tag{6.9}$$

### 6.1.3 Deflection-shear model with 24 degrees of freedom

Reference symbol PDS24

This model has been developed employing the Refined triangular plate bending finite element (eighteen-degrees-of-freedom) together with the linear variation of shear deformation model (5.1.2., 5.2.1) for expressing the parameters in equations (6.1), (4.8).

More details for the formation of the various matrices are presented in Appendix III.

The vector of the nodal degrees of freedom has the form:

Fig. 6.3

$$\{\delta_o^e\}^T = \{w_i, w_{,xi}, w_{,yi}, w_{,xxi}, w_{,xyi}, w_{,yyi} \ldots\ldots, \phi_{xi}, \phi_{yi}, \ldots\ldots\}_{i=1 \div 3} \tag{6.10}$$

Employing the transformation relationships (Chapter 8) the transformed stiffness, stress and load matrices are obtained,

a) When the vector of the nodal degrees of freedom becomes

$$\{\delta_o^e\}^T = \{w_i, w_{,xi}, w_{,yi}, w_{,xyi}, w_{,yyi}, \phi_{xi}, \phi_{yi}, \ldots\ldots\}_{i=1 \div 3} \tag{6.11}$$

b) For a node (i) which belongs to a boundary ($\bar{s}$) the set of degrees of freedom linked with this node becomes ÷

$$\{w_i, \theta_{ni}, w_{,si}, w_{,nni}, w_{,sni}, w_{,ssi}, \phi_{ni}, \phi_{si}\} \tag{6.12}$$

### 6.1.4 Deflection-shear model with 30 degrees of freedom

Reference symbol PDS30

This model has been developed employing the refined triangular plate bending finite element (eighteen-degrees-of-freedom) together with the cubic variation of shear deformation model. (5.1.2, 5.2.3.) for expressing the parameters in equations (6.1), (4.8). The same procedure for the formation of the various matrices is employed as for the PDS24 model (Appendix III).

Fig. 6.4

The vector of nodal degrees of freedom after elimination of the degrees of freedom at the centre node (Chapter 8 Section 4) has the form:

$$\{\delta_o^e\}^T = \{w_i, w_{,xi}, w_{,yi}, w_{,xxi}, w_{,xyi}, w_{,yyi}, \ldots, \phi_{xj}, \phi_{yj}, \ldots\}^{i=1\div 3}_{j=1\div 7} \quad (6.13)$$

Employing the transformation relationships (Chapter 8) the transformed stiffness, stress and load matrices are obtained,

a) When the vector of the nodal degrees of freedom becomes ÷

$$\{\delta_o^e\}^T = \{w_i, w_{,xi}, w_{,yi}, w_{,xxi}, w_{,xyi}, w_{,yyi}, \phi_{xi}, \phi_{yi}, \ldots \phi_{xj}, \phi_{yj}, \ldots\}^{i=1\div 3}_{j=4\div 6} \quad (6.14)$$

b) For a node (i) which belongs to a boundary ($\bar{s}$) the set of degrees of freedom linked with this node becomes ÷

$$\{w_i, \theta_{ni}, w_{,si}, w_{,nni}, w_{,sni}, w_{,ssi}, \phi_{ni}, \phi_{si}\} \quad \text{for } i \leq 3 \quad (6.15)$$

$$\{\phi_{ni}, \phi_{si}\} \quad \text{for} \quad i > 3 \quad (6.16)$$

### 6.1.5 Total rotation model with 18 degrees of freedom

Reference symbol PRO18

This model has been developed employing the total rotation model together with the cubic variation of transverse displacement model (5.5, 5.1.5) for expressing the parameters in equations (6.2), (4.8)

More details for the formation of the various matrices are presented in appendix V

Fig. 6.5

The vector of nodal degrees of freedom after the elimination of the degree of freedom of the centre node (Chapter 8, Section 4) has the form:

$$\{\delta_o^e\}^T = \{\theta_{xi}, \theta_{yi}, \ldots\ldots, w_i, \ldots\} \quad i = 1 \div 6 \tag{6.17}$$

Employing the transformation relationships (Chapter 8) the transformed stiffness, stress and load matrices are obtained

a) when the vector of nodal degrees of freedom become ÷

$$\{\delta_o^e\}^T = \{w_i, \theta_{xi}, \theta_{yi}, \ldots\} \quad i = 1 \div 6 \tag{6.18}$$

(b) For a node (i) which belongs to a boundary ($\bar{s}$) the set of degrees of freedom linked with this node becomes ÷

$$\{w_i, \theta_{ni}, \theta_{si}\} \tag{6.19}$$

## 6.2 Mixed models

The variational principle outlined in Chapter 4, Section 3 has been employed for the development of the mixed models.

For the finite element approximation the various parameters of the functional are expressed as functions of nodal values of moments and displacements.

The mixed models developed are the following:

### 6.2.1 Mixed model with 12 degrees of freedom

Reference symbol PMX12

This model has been developed employing the linear variation of moments model together with the linear variation of the transverse displacement model (5.4.1 5.1.3) for expressing the various parameters in equations (4.19) (4.22).

The same procedure for the formation of the various matrices is employed as for the PMX24 model which is to be outlined in the next paragraph (see Appendix IV).

Fig. 6.6

The vector of the nodal degrees of freedom has the form

$$\{\delta_o^e\}^T = \{M_{xxi}, M_{yyi}, M_{xyi}, \ldots, w_i, \ldots\} \quad i = 1 \div 3 \tag{6.20}$$

Employing the transformation relationships (Chapter 8) the transformed stiffness, stress and load matrices are obtained.

a) when the vector of nodal degrees of freedom becomes

$$\{\delta_o^e\}^T = \{w_i, M_{xxi}, M_{yyi}, M_{xyi}, \ldots\} \quad i = 1 \div 3 \tag{6.21}$$

b) For a node (i) which belongs to a boundary ($\bar{s}$) the set of degrees of freedom linked with this node becomes ÷

$$\{w_i, M_{nni}, M_{ssi}, M_{sni}\} \tag{6.22}$$

### 6.2.2 Mixed model with 24 degrees of freedom

Reference symbol PMX24

This model has been developed employing the quadratic variation of moments model together with the quadratic variation of the transverse displacement model (5.4.2, 5.1.4) for expressing the various parameters in equations (4.19), (4.22).

More details for the formation of various matrices are presented in Appendix IV

Fig. 6.7

The vector of the nodal degrees of freedom has the form

$$\{\delta_o^e\}^T = \{M_{xxi}, M_{yyi}, M_{xyi}, \ldots\ldots w_i, \ldots\ldots\} \qquad i = 1 \div 6 \tag{6.23}$$

Employing the transformation relationships (Chapter 8 ) the transformed stiffness, stress and load matrices are obtained.

a) when the vector of nodal degrees of freedom becomes:

$$\{\delta_o^e\}^T = \{w_i, M_{xxi}, M_{yyi}, M_{xyi}, \ldots\ldots\} \qquad i = 1 \div 6 \tag{6.24}$$

b) For a node which belongs to a boundary ($\tilde{s}$) the set of degrees of freedom linked with the node becomes:

$$\{w_i, M_{nni}, M_{ssi}, M_{sni}\} \tag{6.25}$$

## 6.3 Symbols for the different elements

The first alphabetic character in the symbol indicates either a Plate element or a Dome element.

The next two alphabetic characters indicate:

a) Deflection-Shear model

b) total ROtation model

c) MiXed model.

The two last characters, the arithmetic ones, indicate the number of degrees of freedom per element.

Thus for the seven sandwich plate bending finite elements presented previously the following symbols have been employed here for each one respectively:

PDS15, PDS21, PDS24, PDS30, PRO18, PMX12, PMX24.

For the dome models which are to be presented in the following chapter (7). The following symbols have been employed in accordance with the above definition, for each one respectively:

DDS21, DDS33, DMX36, DRO30.

PLATE ELEMENTS (TRIANGULAR PLATE)
PDS15
PDS21
PMx12
PMx24
PDS24
PDS30
PRO18
m = 3
number of mesh subdivisions
m
for m = 12
365
460
630
930
990
1180
1300
total number of unknowns
10
20
30
40
50
60
70
80
90
100
200
300
400
500
600
700
800
900
1000
1500
2000

FIG. 6.8.

PDS15
PDS21
PMx12
PMx24
PDS24
PDS30
PRO18
n = 3
n
number of mesh subdivisions
f = 2
n = 12
680
850
1350
1450
1500
2250
2500
total number of unknowns
10
20
30
40
50
60
70
80
90
100
200
300
400
500
600
700
800
900
1000
1500
2000
3000
2
3
4
5
6
7
8
9
10
11
12
13

FIG. 6.9

## 7. DOME MODELS

The dome elements to be presented in this chapter are employed for the analysis of the behaviour of the polyhedral dome sandwich structures.

The four models which have been developed, derived from the sandwich plate bending finite elements presented in Chapter 6 in combination with the in-plane displacement models outlined in Chapter 5, section 3.

The choice of four sandwich plate bending finite elements from the total number of seven has been carried out taking into account the following important factors:

a) Accuracy obtained by each individual model for the sandwich plate bending problems in comparison with the number of degrees of freedom involved in it (see Figures 6.8, 6.9, 7.5, 7.6), (Chapter 12)

b) The set of displacements at nodes must be complete so that transformation along the plate interconnections of the polyhedral dome sandwich structure can be carried out (Chapter 8).

c) The ability to apply the necessary boundary condition without too much difficulty.

Further justification for the chosen models with regards to the above factors is to be presented in the conclusions. ( Chapter 12)

The variational principles outlined in Chapter 4 are once again employed and consequently three displacement models and one mixed model are derived in a similar way with the one presented for the sandwich plate bending problem (Chapter 6).

The way the reference symbols of the various dome models are generated is presented in Chapter 6, section 3.

For nodes belonging to a boundary, either external or internal (such as plates interconnection) certain transformation and condensation techniques are used (to be described in Chapter 8).

## 7.1 Displacement models

The two different approaches for expressing the strain-nodal displacement relationship are once again employed (see Chapter 6, section 1). As a result two of the models are referred to as Deflection-shear displacement models and one as a total rotation model.

### 7.1.1 Deflection shear model with 21 degrees of freedom

Reference symbol DDS21

This model derives from the sandwich plate bending finite element PDS15 (described in 6.1.1) in combination with the linear variation of the in-plane displacements model (described in 5.3.1).

Details relevant to the formation of the various matrices involved are presented in Appendix II.

Fig. 7.1

The vector of nodal degrees of freedom through certain transformation formulae becomes:

$$\{\delta_o^e\}^T = \{u_i, v_i, w_i, \theta_{xi}, \theta_{yi}, \phi_{xi}, \phi_{yi}, \ldots\ldots\} \quad i = 1 \div 3 \tag{7.1}$$

Consequently the transformed stiffness, stress and load matrices are obtained respectively (Chapter 8).

7.1.2 Deflection-shear model with 33 degrees of freedom

Reference symbol DDS33

This model derives from the sandwich plate bending finite element PDS21 (described in 6.1.2) in combination with the cubic variation of the in-plane displacement model (described in 5.3.3).

The same procedure for the formation of the various matrices is employed as for the DDS21 (Appendix II).

Fig. 7.2

The vector of nodal degrees of freedom through certain transformation formulae as well as elimination of the degrees of freedom at the centre node becomes:

$$\{\delta_o^e\} = \{u_i, v_i, w_i, \theta_{xi}, \theta_{yi}, \phi_{xi}, \phi_{yi}, \ldots, \phi_{xj}, \phi_{yj}, u_{ssj}, v_{ssj}, \ldots\}_{i=4\div6}^{i=1\div3} \qquad (7.2)$$

Consequently the transformed stiffness, stress and load matrices are obtained respectively (Chapter 8).

7.1.3 Total rotation model with 30 degrees of freedom

Reference symbol DRO30

This model derives from the sandwich plate bending finite element PRO18 (described in 6.1.5) in combination with the cubic variation of the in plane displacements model (described in 5.3.4).

Details relevant to the formation of the various matrices involved are presented in Appendix V

Fig. 7.3

The vector of nodal degrees of freedom, through certain transformation formulae as well as elimination of the degrees of freedom at the centre node, becomes:

$$\{\delta_o^e\} = \{u_i, v_i, w_i, \theta_{xi}, \theta_{yi}, \ldots\} \quad i = 1 \div 6 \tag{7.3}$$

Consequently the transformed stiffness, stress and load matrices are obtained respectively (Chapter 8)

7.2 Mixed models

Two models could have been derived, employing the variational principle outlined in Chapter 4, Section 2. as an extension of the sandwich plate bending mixed models, described in Chapter 6, Section 2. That is: First a model which derives from the sandwich plate bending finite element PMX12 (6.2.1) in combination with the linear variation of the in-plane displacements model (5.3.1) DMX18. Secondly, a model which derives from the sandwich plate bending finite element PMX24 (6.2.2) in combination with the quadratic variation of the in plane displacements model (5.3.2) DMX36.

Limitations of the present work with regards to time and space have allowed the derivation of only one.

Accuracy factors have led in the choice of the second model despite the advantage of the low number of degrees of freedom which the first possesses (see Figs. 7.1,7.2). The formation of the first model (DMX18) is similar to the one of the second DMX36 (to be presented next) and the results are believed to be of a similar nature with the results obtained from the application of the DMX36.

It will be seen from the discussion that the former model can easily be formed and applied.

7.2.1 Mixed model with 36 degrees of freedom

Reference symbol DMX36

Details relevant to the formation of the various matrices involved are presented in Appendix IV.

Fig. 7.4

The vector of nodal degrees of freedom through certain transformation formulae becomes:

$$\{\delta_o^e\}^T = \{u_i, v_i, w_i, M_{xxi}, M_{yyi}, M_{xyi}, \ldots\} \quad i = 1 \div 6 \qquad (7.4)$$

Consequently the transformed stiffness, stress and load matrices are obtained respectively (Chapter 8).

DOME ELEMENTS (TRIANGULAR PLATE)
n=3
DDS15
DDS33
DMX18
DMX36
DRO30
number of mesh subdivisions
n
for n=12
550
630
1570
1680
1940
total number of unknowns
10
20
30
40
50
60
70
80
90
100
200
300
400
500
600
700
800
900
1000
1500
2000
3000
1
2
3
4
5
6
7
8
9
10
11
12
13
14

FIG. 7.5.

DDS21
DDS33
DMX18
DMX36
DRO30
n = 3
number of mesh subdivisions
n
n = 12
1000
1180
3100
3300
3700
total number of unknowns
10
20
30
40
50
60
70
80
90
100
200
300
400
500
600
700
800
900
1000
2000
3000
4000
5000

FIG. 7.6.

## 8. TRANSFORMATION - CONDENSATION

To obtain the final set of equations (4.12), (4.26) by the combination of the variational approach and the finite elements approximation (Chapters 3 and 4) it is necessary to assemble the linear equations obtained in a matrix form for each individual element (4.11, 4.12, 4.22, 4.26). These equations are at first evaluated with respect to a local system of cartesian-coordinates (see Appendix I). Thus a common global coordinates system is needed and transformation from the local to the global system must be carried out for all the parameters involved.

For both the plate and the dome finite elements, described in Chapter 6 and 7 respectively, transformation must be introduced in order to assemble the elements and apply the boundary conditions.

### 8.1 Transformation - formulae

The form of the relationship to be transformed is as follows:

$$\sum_{i=1}^{n} \left\{ [K_n^o] \cdot \{\delta_o^e\} - [R_n^o] \right\} = \phi. \tag{8.1}$$

$\{\delta_o^e\}$ is the vector of the nodal degrees of freedom in the local system of cartesian coordinates.

The relationship between the nodal degrees of freedom with respect to the local system and the nodal degrees of freedom with respect to the global system has the form

$$\{\delta_o^e\} = [TR] \{\delta_g^e\} \tag{8.2}$$

where $[TR]$ is the transformation matrix

As the corresponding force component must perform the same amount of work in either system the following relationship is obtained

$$\{R_n\}^T \{\delta_g^e\} = \{R_n^o\}^T \{\delta_o^e\} \tag{8.3}$$

where $\{R_n\}$, $\{\delta_g^e\}$ are the load and nodal degrees of freedom vectors with respect to the global system.

Substituting equation (8.2) to equation (8.3) the following equation can be obtained:

$$\{R_n\} = [TR]^T \{R_n^o\} \tag{8.4}$$

Pre-multiplying equation (8.3) by $[TR]^T$ and substituting equation (8.4) the following relationship can be obtained:

$$\sum_1^n \left\{ [TR]^T [K_n^o] [TR] \{\delta_g^e\} - \{R_n\} \right\} = \phi. \tag{8.5}$$

or

$$\sum_1^n \left\{ [K_n] \{\delta_g^e\} - \{R_n\} \right\} = \phi. \tag{8.6}$$

So the stiffness, stress and load matrices $[K_n]$, $[S_n]$, $\{R_n\}$ with respect to a global system can be found from the corresponding matrices with respect to the local system $[K_n^o]$, $[S_n^o]$, $\{R_n^o\}$ through the expressions:

$$\begin{aligned} [K_n] &= [TR]^T [K_n^o] [TR] \\ [S_n] &= [TR] [S_n^o] \\ \{R_n\} &= [TR]^T \{R_n^o\} \end{aligned} \tag{8.7}$$

## 8.2 Transformation matrices for a plate

The transformation in sandwich plate bending problems is necessary for a node which belongs to boundary ($\bar{s}$) in order to apply the boundary conditions.

The parameters involved have the following form:

( FIG. 8.1 )

$$\begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{bmatrix} B_2 & G_2 \\ -G_2 & B_2 \end{bmatrix} \begin{Bmatrix} a_n \\ a_s \end{Bmatrix} \tag{8.8}$$

$a_n, a_s$ are the co-ordinates of the new system $\bar{n}$, $\bar{s}$) .

$$B_2 = \frac{y_2 - y_1}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

$$G_2 = \frac{x_2 - x_1}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}} \tag{8.9}$$

### 8.2.1 Displacement models

$$\begin{Bmatrix} w,_x \\ w,_y \end{Bmatrix} = \begin{bmatrix} B_2 & G_2 \\ -G_2 & B_2 \end{bmatrix} \begin{Bmatrix} w,_n \\ w,_s \end{Bmatrix}$$

$$\begin{Bmatrix} \phi_x \\ \phi_y \end{Bmatrix} = \begin{bmatrix} B_2 & G_2 \\ -G_2 & B_2 \end{bmatrix} \begin{Bmatrix} \phi_n \\ \phi_s \end{Bmatrix} \qquad \begin{Bmatrix} w,_{xx} \\ w,_{xy} \\ w,_{yy} \end{Bmatrix} = \begin{bmatrix} B_2^2 & 2B_2G_2 & G_2^2 \\ -B_2G_2 & (B_2^2 - G_2^2) & B_2G_2 \\ G_2^2 & -2B_2G_2 & B_2^2 \end{bmatrix} \begin{Bmatrix} w,_{nn} \\ w,_{sn} \\ w,_{ss} \end{Bmatrix} \tag{8.10}$$

$$\begin{Bmatrix} \theta_x \\ \theta_y \end{Bmatrix} = \begin{bmatrix} B_2 & G_2 \\ -G_2 & B_2 \end{bmatrix} \begin{Bmatrix} \theta_n \\ \theta_s \end{Bmatrix}$$

### 8.2.2 Mixed models

( FIG. 8.2 )

$$\begin{Bmatrix} M_{ss} \\ M_{nn} \\ 2M_{sn} \end{Bmatrix} = \begin{bmatrix} G_2^2 & B_2^2 & 2B_2G_2 \\ B_2^2 & G_2^2 & -2B_2G_2 \\ 2B_2G_2 & -2B_2G_2 & 2(B_2^2 - G_2^2) \end{bmatrix} \begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} \qquad (8.11)$$

$$\begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} B_2^2 & G_2^2 & 2B_2G_2 \\ G_2^2 & B_2^2 & -2B_2G_2 \\ -B_2G_2 & B_2G_2 & (B_2^2 - G_2^2) \end{bmatrix} \begin{Bmatrix} M_{nn} \\ M_{ss} \\ M_{sn} \end{Bmatrix} \qquad (8.12)$$

### 8.2.3 Rotation of an element

It is very common in many plate and dome cases that an element results from another one of the same dimensions by a single rotation of $180^{o}$

( FIG. 8.3 )

The various matrices of the second element can be evaluated by a simple transformation of the first element's corresponding matrices.

The following expressions relate the degrees of freedom for the 1st and 2nd element

$$
\begin{aligned}
u_2 &= -u_1 \\
v_2 &= -v_1 \\
w_2 &= w_1 \\
w'_{x2} &= -w_{x1} \\
w'_{y2} &= -w'_{y1} \\
w'_{xx2} &= w'_{xx1} \\
w'_{xy2} &= w'_{xy1} \\
w'_{yy2} &= w'_{yy2} \\
\phi'_{x2} &= -\phi'_{x1} \\
\phi'_{y2} &= -\phi'_{y1} \\
\theta'_{x2} &= -\theta'_{x1} \\
\theta'_{y2} &= -\theta'_{y1} \\
M_{xx2} &= M_{xx1} \\
M_{yy2} &= M_{yy1} \\
M_{xy2} &= M_{xy1}
\end{aligned}
\qquad (8.13)
$$

The subscripts 1.2 refer to the coordinate system $x_1oy_1$ for the first element and the coordinate system $x_2oy_2$ for the second element with regard to the relevant degree of freedom (Fig. 8.3 )

## 8.3 Transformation matrices for a dome model

As it has been mentioned previously in Chapter 7 a very important factor, which is decisive for the combination of the various bending and membrane models with regard to the derivation of the several dome models, is the completeness of the different sets of nodal degrees of freedom.

The completeness of a set, with respect to the three-dimensions space, is necessary regarding the assemblage of the finite elements with nodes belonging to plates interconnections, and leads to the accomplishment of the transformation.

The above requirement has been fulfilled for all the dome elements presented in Chapter 7 as far as the set of nodal displacements u, v, w is concerned (see Chapter 7).

The set of rotations or moments, however, is incomplete due to the absence of a rotation or a moment, as a degree of freedom, with respect to an axis normal to the plane of the element.

One approach to the problem has been suggested in reference [115] is that additional degrees must be introduced resulting in the completeness of the set.

Some work has been done to determine the form of the part of the stiffness matrix corresponding to these additional degrees of freedom..

A second approach is based on the selection of certain coordinate systems whereby a suitable orientation of the various axes enables the transformation to be performed with a considerable degree of accuracy. [21,31,115]

The second approach has been followed for the present analysis.

First of all the various matrices for a given element are evaluated with respect to a local coordinate system defined as follows:

The x axis of this local system is the intersection of the element's plane with the xoy of the global system of cartesian coordinate Oxyz (Fig.8.4)

The unit vectors of the local system are:

$$\vec{v}_{z'} = \begin{bmatrix} \lambda_{z'x} \\ \lambda_{z'y} \\ \lambda_{z'z} \end{bmatrix} = \frac{1}{DT} \begin{bmatrix} y_{ji}\, z_{mi} - z_{ji}\, y_{mi} \\ z_{ji}\, x_{mi} - x_{ji}\, z_{mi} \\ x_{ji}\, y_{mi} - y_{ji}\, x_{mi} \end{bmatrix} \tag{8.14}$$

$$\vec{v}_{x'} = \begin{bmatrix} \lambda_{x'x} \\ \lambda_{x'y} \\ \phi. \end{bmatrix} = \begin{bmatrix} \lambda_{z'y} / \sqrt{\lambda^2_{z'y} + \lambda^2_{z'x}} \\ -\lambda_{z'x} / \sqrt{\lambda^2_{z'y} + \lambda^2_{z'x}} \\ \phi. \end{bmatrix} \tag{8.15}$$

$$\vec{v}_{y'} = \begin{bmatrix} \lambda_{y'x} \\ \lambda_{y'y} \\ \lambda_{y'z} \end{bmatrix} = \begin{bmatrix} \lambda_{z'z}\ \lambda_{z'x} / \sqrt{\lambda^2_{z'y} + \lambda^2_{z'x}} \\ \lambda_{z'z}\ \lambda_{z'y} / \sqrt{\lambda^2_{z'y} + \lambda^2_{z'x}} \\ -\sqrt{\lambda^2_{z'y} + \lambda^2_{z'x}} \end{bmatrix} \tag{8.16}$$

$$\begin{Bmatrix} x' \\ y' \\ z' \end{Bmatrix} = \begin{bmatrix} \lambda_{x'x} & \lambda_{x'y} & \phi. \\ \lambda_{y'x} & \lambda_{y'y} & \lambda_{y'z} \\ \lambda_{z'x} & \lambda_{z'y} & \lambda_{z'z} \end{bmatrix} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} \tag{8.17}$$

Thus the transformation matrix from local to global has been obtained.

$$\begin{Bmatrix} x' \\ y' \\ z' \end{Bmatrix} = \begin{bmatrix} T_{eg} \end{bmatrix} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} \tag{8.18}$$

Another coordinates system $\bar{x}\ \bar{y}\ \bar{z}$ is defined for an element with a node (m) belonging to a plates-interconnection (1) (2) in the following way. (fig. 8.4)

The $\bar{x}$ axis of the system coincides with the line (1) (2). The $\bar{y}$ axis is normal to the vertical plate through the line (1) (2). And the $\bar{z}$ axis is defined as the cross product of the unit vectors of x and y respectively.

$$\vec{v}_{\bar{x}} = \begin{bmatrix} \mu_{\bar{x}x} \\ \mu_{\bar{x}y} \\ \mu_{\bar{x}z} \end{bmatrix} = \frac{1}{\ell_{12}} \begin{bmatrix} x_2 - x_1 \\ y_2 - y_1 \\ z_2 - z_1 \end{bmatrix} \tag{8.19}$$

$$\ell_{12} = \sqrt{(x_2 - x_1)^2 + (v_2 - v_1)^2 + (z_2 - z_1)^2} \tag{8.20}$$

$$\vec{v}_{\bar{y}} = \begin{bmatrix} \mu_{\bar{y}x} \\ \mu_{\bar{y}y} \\ \mu_{\bar{y}z} \end{bmatrix} = \begin{bmatrix} -\mu_{\bar{x}y} \Big/ \sqrt{\mu_{\bar{x}x}^2 + \mu_{\bar{x}y}^2} \\ \mu_{\bar{x}x} \Big/ \sqrt{\mu_{\bar{x}x}^2 + \mu_{\bar{x}y}^2} \\ \phi. \end{bmatrix} \tag{8.21}$$

$$\vec{v}_{\bar{z}} = \begin{bmatrix} \mu_{\bar{z}x} \\ \mu_{\bar{z}y} \\ \mu_{\bar{z}z} \end{bmatrix} = \begin{bmatrix} -\mu_{\bar{x}x} \cdot \mu_{\bar{x}z} \Big/ \sqrt{\mu_{\bar{x}x}^2 + \mu_{\bar{x}y}^2} \\ -\mu_{\bar{x}y} \cdot \mu_{\bar{x}z} \Big/ \sqrt{\mu_{\bar{x}x}^2 + \mu_{\bar{x}y}^2} \\ \sqrt{\mu_{\bar{x}x}^2 + \mu_{\bar{x}y}^2} \end{bmatrix} \tag{8.22}$$

$$\begin{Bmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{Bmatrix} = \begin{bmatrix} \mu_{\bar{x}x} & \mu_{\bar{x}y} & \mu_{\bar{x}z} \\ \mu_{\bar{y}x} & \mu_{\bar{y}y} & \mu_{\bar{y}z} \\ \mu_{\bar{z}x} & \mu_{\bar{z}y} & \mu_{\bar{z}z} \end{bmatrix} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} \tag{8.23}$$

$$\begin{Bmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{Bmatrix} = \left[T_{\bar{e}g}\right] \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} \tag{8.24}$$

Combining equations (8.18)(8.24) and considering the orthogonal nature of the matrices $[T_{eg}]$ , $[T_{\bar{e}g}]$ the following relationships can be obtained

$$\begin{Bmatrix} x' \\ y' \\ z' \end{Bmatrix} = [T_{e\bar{e}}] \begin{Bmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{Bmatrix} \tag{8.25}$$

$$[T_{e\bar{e}}] = \begin{bmatrix} v_{x'\bar{x}} & v_{x'\bar{y}} & v_{x'\bar{z}} \\ v_{y'\bar{x}} & v_{y'\bar{y}} & v_{y'\bar{z}} \\ v_{z'\bar{x}} & v_{z'\bar{y}} & v_{z'\bar{z}} \end{bmatrix} \tag{8.26}$$

$$[T_{e\bar{e}}] = [T_{eg}] \quad [T_{\bar{e}g}]^T \tag{8.27}$$

For co-planar nodes the assembling of the equations is peformed in the local co-ordinates system.

For nodes belonging to an interconnection of two plates or to an external boundary the displacements set u', v', w' is transformed employing the matrix $[T_{eg}]$ (8.18) to the global displacements u, v, w together with the relevant transformation procedure for the various matrices involved (8.7). The set of total rotation, transverse shear displacements and moments is transformed employing the matrix $[T_{e\bar{e}}]$ (8.25) to the coordinate system $\bar{x}\ \bar{y}\ \bar{z}$ eliminating at the same time the transverse shear displacement relevant to the cross section normal to the $\bar{x}$ axis .

For nodes belonging to an interconnection of more than two plates all the different parameters are transformed to the global system xyz employing the matrix $[T_{eg}]$ eliminating at the same time the transverse shear displacements.

( FIG. 8.4 )

## 8.4 Condensation

When the elimination of a number of nodal degrees of freedom $\{\delta_2^e\}$ is required from the total vector of $\{\delta_1^e\}$ with the remaining vector noted as $\{\delta_1^e\}$ the following relationships can be obtained.

The stiffness matrix can be partitioned with regards to the two sets of nodal degrees of freedom. $\{\delta_1^e\}$ , $\{\delta_2^e\}$

$$[K_{11}] \{\delta_1^e\} + [K_{12}] \{\delta_2^e\} = \{R_1\} \tag{8.28}$$

$$[K_{21}] \{\delta_1^e\} + [K_{22}] \{\delta_2^e\} = \{R_2\} \tag{8.29}$$

Solving equation (8.29) with respect to $\{\delta_2^e\}$ and substituting the value for equation (8.28) one obtains:

$$[K_{cd}] \{\delta_1^e\} - \{R_{cd}\} = \phi \tag{8.30}$$

$$[K_{cd}] = [K_{11}] - [K_{12}] [K_{22}]^{-1} [K_{21}] \tag{8.31}$$

$$[R_{cd}] = \{R_1\} - [K_{12}] [K_{22}]^{-1} \{R_2\} \tag{8.32}$$

In the same way the stress matrix after the condensation has the form

$$\{\sigma\} = ([S_1] - [S_2][K_{22}]^{-1}[K_{21}])\{\delta_1^e\} + [S_2][K_{22}]^{-1} \{R_2\} \tag{8.33}$$

# 9. EXPERIMENTAL WORK

## 9.1 Introduction

The experimental part of the present work consists of the construction and testing of two polyhedral domed sandwich structures.

(a) The 24 faced dome (see Photogr. a ). This dome is formed by six 4 faced pyramidal flat segments (see Chapter 13 ).

(b) The 36 faced dome (see Photog. b÷e). The second dome is an extension of the first formed by adding six "dormer" sections (see Chapter 13 ).

The dimensions of both the domes have been obtained through a computer programme (outlined in [85]). They are included in Chap. 13 and are presented in the relevant section. The experimental results obtained from the various loading cases are presented in the same section

## 9.2 Construction and Materials

The decisive factor for the determination of the dome's dimensions was the size of the basic orthogonal sandwich sheet, readily available from the manufacturers. The panels forming the two domes are all identical as far as their dimensions are concerned and have been formed as part of the basic orthogonal sandwich sheet, as shown in Fig. 9.1 , so that from each sheet two panels can be obtained with the least possible wastage of material.

AB = CD = 1.23 m

AD = BC = 2.25 m

AC = 2.42 m

FIG. 9.1

The sandwich panels used were composed from hardboard faces of 4.1 mm thickness and polyurethane core of 50.8mm thickness.

The elasticity moduli for the above mentioned materials, as well as for the composite sandwich structure used for the present analysis, were obtained by other researchers [21,46,85].

The properties of all the various sandwich panels used in the present work are presented in figs. 13.2,13.3.

The construction of the two domes was carried out in the following way:

(a) First the supports and the foundation of the structure were built. Detailed drawings of the foundation are presented in Fig.9.2÷9.7

The foundation must be functioning, at the final stage, in such a way that the displacements, with respect to all three global axes, are zero without constraining the corresponding rotations (pinned-joint function).

For the construction stage a limited amount of displacements with respect to x and y axes have been allowed so that dimensional inaccuracies could be overcome (see detailed drawings). 9.4÷9.6

(b) Second the six 4-faced pyramidal flat segments (Fig. 9.8.1) as well as the six "dormer" sections (Fig. 9.8.2) were built by joining the identical triangular sandwich panels together. The joining technique details are presented in Fig. 9.9

The joining technique has been tested and found satisfactorily strong and economical. Details with regard to the behaviour of the joints are presented in Chapter 10.

(c) At this stage the first dome was formed by erecting the six segments on site and by adjusting the previously mentioned mechanisms at the supports so that all six segments came together. Next, the joints of the adjoining segments, which were built using the same technique mentioned above, were secured by the fitting of the steel cover plates and bolts (See Fig. 9.10)

It is worth mentioning that the construction of the six segments was carried out in 10 days and the erection and formation of the first dome in one day, with the help of three departmental technicians.

(d) After the testing of the first dome was accomplished, the second dome was built by erecting and joining to the existing first dome the six "dormer" sections. The same joining technique was used as previously. (see fig. 9.11)

The second dome was then tested.

## 9.3 Testing of the Domes

The two domes were tested under concentrated loads acting at the centroids of certain groups of panels.

The first dome was tested under two loading cases:-

1st Concentrated load of 1216 N at all upper panel centroids.

2nd Concentrated load of 1216 N at all bottom panel centroids.

The second dome was tested under three loading cases:-

1st Concentrated load of 1216 Nt at all upper panel centroids.

2nd Concentrated load of 1216 Nt at all bottom panel centroids.

3rd Concentrated load of 1216 Nt at all "dormer" centroids.

The following assumptions made in the analysis of the structure were tested.

(a) The behaviour of the supports was found satisfactory.

The displacements measured at the supports were practically zero. (Approximately two orders of magnitude smaller than the maximum displacement).

(b) The behaviour of the structure under certain concentrated loads was found to be, with regard to the structure as a whole, such that the analytical assumption of an elastic behaviour can be considered as a fair approximation. Visco-elastic indentation of the faces was observed locally for the loaded panels in a limited area surrounding the loading point, which required a little time (1 hour) to recover.

(c) To minimize the effects of temperature and relative humidity changes as well as the wind effects, the testing of the domes under

various loading cases was accomplished under as similar weather conditions as possible. In addition, the surface of the panels was covered by two layers of "yacht varnished" for weather protection.

(d) The structure was assumed to be symmetrical. The dimension inaccuracies observed during the construction stage were negligible.

The symmetrical behaviour of the domes was tested next and found to be satisfactory. First, the displacements at certain symmetrical points under a symmetrical loading case were measured and found to be almost symmetrical (with a maximum deviation of 10 per cent). Second, the displacements (of a point on an axis of symmetry), normal to the axis of symmetry, for a symmetrical loading case were measured and found to be negligible.

Due to the above reasons, the assumption of the symmetrical behaviour of the structure under symmetrical load was considered to be valid and consequently only 1/12 of the two domes was numerically modelled for the loading cases (symmetrical) mentioned previously.

(e) Time-dependent behaviour was also observed. To minimize the effects of this behaviour the measurements of the points by the theodolites have been taken in the minimum possible time starting always from the loaded panel where the time-dependent behaviour is expected to occur.

(f) The application of the high density concentrated load of 1216 Nt per loading point, as already mentioned, was dictated by

the high rigidity of the structure and the accuracy of the method used for measuring the displacements, to be outlined in the next section. Special steel cylindrical devices were used at the loading points to secure the vertical application and spreading of the load.

## 9.4 Displacement Measurement

Under a certain loading case the displacements of certain points were measured (see Chapter 13)

For this a combination of two theodolites was used. The theodolites were secured so that they were at the same horizontal level separated by a constant distance.

For each modelling point on the structure eight readings were obtained, four with the structure unloaded and four with the structure loaded. The four readings in each case consist of one horizontal and one vertical angle from both the theodolites.

The global displacements have been obtained for each point from the above measurements by the following mathematical formulation, through a computer program based on it.

The first and second theodolites are considered to be at points $0_1$, $0_2$ respectively (see Fig. 9.12)

A modelling point, at $A_1$ before the application of the load, is to be considered.

The coordinates of $A_1$ with respect to the two cartesian coordinate systems $x_1 0_1 y_1$, $x_2 0_2 y_2$ are $(a_1', a_2', a_3')$, $(a_1', a_2' + \ell, a_3)$ where $\ell$ is the constant distance between $0_1$ and $0_2$ which has already been measured.

The following angles can be measured by the two theodolites:

| | For Point $A_1$ | | For Point $A_1'$ | | |
|---|---|---|---|---|---|
| | angle $\phi$ | $+ g\ \phi$ | angle $\phi'$ | $+ g\ \phi'$ | |
| Horizontal angle | $Y_1 0_1 B_1$ | $h_1$ | $Y_1 0_1 B_1'$ | $h_1'$ | From Point $0_1$ |
| Vertical angle | $B_1 0_1 A_1$ | $v_1$ | $B_1' 0_1 A_1'$ | $v_1'$ | |
| Horizontal angle | $Y_2 0_2 B_1$ | $h_2$ | $Y_2 0_2 B_1'$ | $h_2'$ | From Point $0_2$ |
| Vertical angle | $BB_1 0_2 A_1$ | $v_2$ | $B_1' 0_2 A_1'$ | $v_2'$ | |

By the following expressions the coordinates of the points can be related to the measured angles:

$$
\begin{aligned}
a_1 &= a_2 h_1 \\
a_1 &= (a_2 + \ell)\ h_2 \\
a_1' &= a_2'\ h_1' \\
a_1' &= (a_2' + \ell)\ h_2'
\end{aligned}
\qquad (9.1)
$$

From (1) the following can be obtained:

$$
\begin{aligned}
a_1 &= \frac{\ell\ h_1\ h_2}{h_1 - h_2} \\
a_2 &= \frac{\ell\ h_2}{h_1 - h_2} \\
a_1' &= \frac{\ell\ h_1'\ h_2'}{h_1' - h_2'} \\
a_2' &= \frac{\ell\ h_2'}{h_1' - h_2'}
\end{aligned}
\qquad (9.2)
$$

The following expressions related the horizontal coordinates $a_1$, $a_2$, $a_1'$, $a_2$ obtained by equations (2) with the third coordinate $a_3$, $a_3'$ and the vertical angles. (Mid value is being taken)

$$a_3 = \frac{v_1\sqrt{a_1^2 + a_2^2} + v_2\sqrt{a_1^2 + (a_2 + \ell)^2}}{2}$$

$$a_3' = \frac{v_1'\sqrt{a_1'^2 + a_2'^2} + v_2'\sqrt{a_1'^2 + (a_2' + \ell)^2}}{2} \qquad (9.3)$$

the displacements for the point $A_1$ can be obtained with respect to the cartesian system $x_1 0_1 y_1$ (or $x_2 0_2 y_2$)

$$u_{A_1} = a_1' - a_1$$

$$v_{A_1} = a_2' - a_2 \qquad (9.4)$$

$$w_{A_1} = a_3' - a_3$$

The displacements with respect to any cartesian coordinate system $x_n 0_n y_n$ can be obtained from the $u_{A_1}$, $v_{A_1}$, $w_{A_1}$ employing the appropriate transformation expressions.

$A_1$ a modelling point before loading

$A_1'$ the same modelling point after loading

$O_1O_2$ = constant = $\ell$

$B_1$, $B_1'$ the projections of $A_1$, $A_1'$ respectively on the horizontal plane $x_2O_2y_2 \equiv x_1O_1y_1$

FIG. 9.12

PHOTOGRAPH a.

PHOTOGRAPH b

PHOTOGRAPH C

PHOTOGRAPH d.

PHOTOGRAPH e.

9.8.1. 4. FACED PYRAMIDAL SEGMENT.

9.8.2. DORMER

FIG. 9.8.

## VALLEY

SCALE : 1cm = 1cm

## RIDGE

SCALE: 1cm = 1cm

FIG. 9.9.

ERECTION OF
24-FACED DOME.

FIG. 9.10.

ERECTION
OF 36-FACED DOME

FIG. 9.11.

# 10. BEHAVIOUR OF THE JOINT

## 10.1. INTRODUCTION

The behaviour of the joint, as it has been formed for the construction of the two domes, the 24-faced dome and the 36-faced dome, is to be outlined in the present chapter.

The behaviour of the joints belonging to the rest of the domes analysed in the present work is assumed to be as outlined in ref. [85] .

## 10.2. TESTS

The strength of the joint under axial tension has been evaluated experimentally by a series of tests for various forms of arrangement at the position of the bolts.

It has been established by the results of these tests that the presence of a wooden insert (as shown in figs. 10.1 - 10.4 ) significantly increases the tensile strength of the joint.

The ultimate axial load of the joint with a wooden cylinder, inserted at the position of the bolts, as shown in fig. 10.2 , is 4.33 times greater than the ultimum axial load of the joint without any insert at all.

The ultimum axial load at the joint in the form used in the construction of the two domes, which has a wooden cube as an insert, as shown in fig. 10.3 , is 10 times greater than the ultimum axial load at the joint without any insert at all.

There are five wooden inserts per metre of joint length for each of the joints in the two domes. This number has been assumed empirically to be sufficient for the function of the joints with regard to the tests that were to be performed with the two domes.

The function of the joint is to be analysed in the following section.

## 10.3. ANALYSIS OF THE FUNCTION OF THE JOINT

### 10.3.1. AXIAL TENSILE STRENGTH

The strength of a unit length sandwich panel in axial tension, assuming constant distribution of the stresses across the thickness at the faces and that the contribution of the core is negligible (see chapter 2 ). is given by the relationship:

$$P^s = 2 \cdot f \cdot \sigma_f = 2 \times 4.1\ 10^{-3} \times 2.5\ 10^7 = 2.05\ 10^5\ \text{Nt} \qquad (10.1)$$

(see figs. 10.1 , 10.4)

where $P^s$ is the axial tensile load for a sandwich panel of one metre in length, and $\sigma_f$ the value of the normal stress at the faces.

For a joint line with 5 inserts per metre the axial tensile load is given by the relationship.

$$P^j = n_{in} \cdot P^i = 5 \times 15\ 10^3 = 0.75\ 10^5\ \text{Nt} \qquad (10.2)$$

(see figs.10.3 , 10.4)

Where $n_{in}$ is the number of inserts per metre and $P^i$ the experimental value of the axial load per insert (see fig. 10.4 )

### 10.3.2 STRENGTH UNDER BENDING

The active width $\ell_{ac.}$ of the steel plate at a joint line functioning under bending is assumed to be the one corresponding to the total length of the wooden inserts per metre.

For a bending moment acting at the joint line the stresses which are developed, assuming that the distribution of the stresses across the thickness of the hardboard faces or the steel plate is constant and that the contribution of the core is negligible, are given by the following relationships:

a) For the faces when the sandwich panel only is to be considered

$$M = 1 . f . \sigma_f \ (c + f) \qquad (10.3)$$

b) For the steel covering plates only

$$M = \ell_{ac}.\ t_s.\sigma_s\ (c + 2f + t_s) = \ell_{in}\ n_{in}.\ t_s.\sigma_s.\ (c + 2f + t_s) \qquad (10.4)$$

c) At the position of the bolt

$$M = P^c\ n_{in}\ .\ (c + 2f + t_s) \qquad (10.5)$$

where $P^c = 0.9P^i$ (see previous section) (10.6)

The maximum values of the bending moments which can be applied in each case can be evaluated by substituting at the above relationships the relevant maximum values of stresses or forces as follows

a) For the faces

$$\sigma_f\ max = 2.5\ 10^7\ Nt/m^2 \qquad Mmax = 5.63\ 10^3\ Ntm/m \qquad (10.7)$$

b) For the steel plates

$$\sigma_s\ max = 1.025\ 10^9\ Nt/m^2 \qquad Mmax = 15.38\ 10^3\ Ntm/m \qquad (10.8)$$

c) For the bolt

$$P^i = 15\ 10^3\ Nt \qquad Mmax = 4.12\ 10^3\ Ntm/m \qquad (10.9)$$

## 10.4. ANGULAR FUNCTION OF THE JOINT UNDER BENDING

For valleys under the action of a bending moment as shown in fig. 10.5.3 the top steel plate can be deformed as shown in fig. 10.5.4.

For a bending moment acting with an opposite sign the presence of the wooden insert restricts such deformation.

The same applies for ridges with a bending moment acting with an opposite sign as shown in figs. 10.5.1 , 10.5.2.

The following analysis has been attempted for evaluating this angular deformation of the joint under bending.

The static system and the applied load are as shown in fig. 10.6.

The resulting stressing condition of the plate is as shown in fig.10.6.

Through this the angular deformation can be evaluated as follows:

$$\omega = 2\Delta\phi = \frac{2\ell^2}{12E_s I_s} \cos\phi \sin\phi \frac{M}{d} \qquad (10.10)$$

$$I_s = \ell ac.t_s^3 /12 \qquad (10.11)$$

$$\omega = \frac{2\ell^2}{E_s \ell ac.t_s^3} \cos\phi \sin\phi \frac{M}{d} \qquad (10.12)$$

$$\frac{\omega}{2\phi} = \varepsilon_{\Delta\phi} = \frac{l^2}{E_s \ell ac.t_s^3 d} \frac{\cos\phi \sin\phi}{\phi} \cdot M \qquad (10.13)$$

$d = 60$ mm $\qquad \ell ac = 0.250$ m $\qquad \ell = 0.025$ m

$$\boxed{\delta_1 = \frac{\cos\phi \sin\phi}{\phi}} \qquad (10.14)$$

For $\varepsilon_{\Delta\phi} = 0.4$ % $\qquad \boxed{M = 20/\delta_1 \text{ Ntm/m}}$ (10.15)

For $\varepsilon_{\Delta\phi} = 0.8$ % $\qquad \boxed{M = 40/\delta_1 \text{ Ntm/m}}$ (10.16)

The values of $\delta_1$ and M are given by the graphs where they are plotted against the value of the corresponding angle

## 10.5. CONCLUSIONS FROM THE BEHAVIOUR OF THE JOINT

The type of the joint used for the construction of the two domes, the 24-faced dome and the 36-faced dome, is sufficiently strong in comparison with the strength of the sandwich panels joined by it, when the angular function of the joint is negligable due to the sign of the acting bending moment as explained in section 4.

It becomes obvious from the analysis of sections 3. and 4. that when angular deformation of the joint is expected, the joint does not act as an absoloutely fixed joint.

With the joint angle decreasing and/or the distance between the bolt and the joint line increasing the joint tends to act as a hinge.

The values of joint angles which are formed by the various adjacent sandwich panels forming the two domes, the 24-faced dome and the 36-faced dome, are presented in fig. 10.7.

$P^i_{failure} = 1.5\ 10^3\ Nt$

FIG. 10.1.

$P^i_{failure} = 6.5\ 10^3$ Nt

SCALE: 1cm = 1cm

FIG. 10.2.

$P^i_{failure} = 15 \cdot 10^3$ Nt

SCALE: 1 cm = 1 cm

FIG. 10.3.

a
b
M
P
steel-plate $t_s$=1mm thick
hardboard face $t_f$=4.1mm
polyurethane core
C=50.8 mm
45mm
45mm
wooden inserts 50mm long
5 inserts/m
steel-plate
$t_s$=1mm thick
SECTION a-a
$t_s$=1mm
$t_f$=4.1mm
C= 52mm
steel-plate
hardboard face
polyurethane core
hardboard face
steel-plate $t_s$=1mm thick
wooden insert
(5 inserts/m)
150mm
$l_{in}$=50mm
SECTION b-b
polyurethane core
C = 50.8mm thick
hardboard face $t_f$ = 4.1mm thick
hardboard face $t_f$ = 4.1mm thick

FIG. 10.4.

FIG. 10.5.

l=25mm
l=25mm
φ
φ
a
a
a
a
l sinφ
l sinφ
P= M/d
steel-plate
a - a
ts=1mm
lac=250mm
l/2 cosφ sinφ M/d
MOMENT'S DIAGRAM
l/2 cosφ sinφ M/d
P= M/d
l/2 cosφ sinφ M/d
1"/l
1"
1"/l
MOMENT'S DIAGRAM:
1"/l
1"/l

FIG. 10.6.

| DESCRIPTION | SYMBOL | degrees |
|---|---|---|
| PYRAMID LONG RIDGE | a–a | ↑ 61.67 |
| PYRAMID SHORT RIDGE | b–b | ↑ 76.41 |
| TOP VALLEY | c–c | ↓ 72.93 |
| DORMER RIDGE | d–d | ↑ 56.50 |
| DORMER VALLEY | e–e | ↓ 56.00 |

FIG. 10.7.

# 11. COMPUTER PROGRAMS

## 11.1 Introduction

For the formation of the stiffness, stress and load matrices (see Chapter 4 ) eleven separate subroutines have been developed. The first seven deal with the sandwich plate bending models and the remaining four with the dome models each of them corresponding to the relevant model as they were presented in Chapters 6 and 7.

The listing of each individual subroutine is presented in the Appendix.

All eleven subroutines are consistent with the main routine, which is a modified version, by Bettess,of the frontal solution technique for solving a large system of simultaneous equations as it has been developed by Irons.

The main routine assembles the various matrices for the total number of elements (Chapter 4. ) and for a given set of boundary conditions proceeds with the solution of the system of simultaneous equations. [20,21,64]

The results for all the various models include for every element: (a) the relevant nodal displacements, (b) the stresses. (See Chapters 6 and 7 and Tables 11.1 ÷ 11.11 )

For the displacement and rotation, plate and dome models the stresses are evaluated at the centroid of the element in the following order.

For the sandwich plate bending models:

$$\{M_{xx_c}, M_{yy_c}, M_{xy_c}, Q_{x_c}, Q_{y_c}\}$$

For the dome models

$$\{M_{xx_c}, M_{yy_c}, M_{xy_c}, Q_{x_c}, Q_{y_c}, N_{xx_c}, N_{yy_c}, N_{xy_c}\}$$

For the mixed model PMX12 the shear forces are evaluated at the centroid of the element

$$\{ Q_{x_c} , Q_{y_c} \}$$

For the mixed model PMX24 the shear forces are evaluated at the first and second nodes of the element

$$\{ Q_{x1} , Q_{y1} , Q_{x2} , Q_{y2} \}$$

For the mixed model DMX36 the shear forces are evaluated at the first and second nodes and the in-plane forces at the centroid of the element.

$$\{ Q_{x1} , Q_{y1} , Q_{x2} , Q_{y2} , N_{xx_c} , N_{yy_c} , N_{xy_c} \}$$

Due to the considerable amount of work involved for the preparation of the data necessary for the solution of a problem, an additional routine has been developed which can generate the data for a problem suitable for all eleven models.

This routine, for most of the cases, considerably reduces the required amount of work, particularly if a series of solutions with all of the different models involved are to be obtained.

The above mentioned routine has been employed successfully for most of the cases which have been solved as part of the present analysis. It is believed that it could, therefore, be similarly usefully employed by any future users.

The different parameters involved in the data generation routine are outlined in the next section.

## 11.2 DATA GENERATION ROUTINE INPUT

11.2.1 First the structure has to be divided into triangular elements.

It is very advantageous as far as the computer time and consequently the cost per run are concerned to employ as few types of elements as possible. The term "types of elements" indicates elements of the same nature (one model can be employed for the solution of a problem at one time) but with different dimensions and elasticity moduli.

For an element "similar" to a previous one with regards to the above mentioned parameters (dimensions and elasticity moduli of an element) the various matrices which have already been calculated and stored for the first of the similar elements can be used again for all the remaining similar elements.

For this purpose storage space for two different sets of matrices is allocated.

The polyhedral sandwich dome structures (see Chapter 1) take full advantage of the above mentioned principle.

The numbering of the nodes is then carried out employing single numbers for each node as if the node had only one degree of freedom.

The numbering order is totally insignificant.

If a particular problem is to be solved by more than one model (including mid-side nodes models) it is advisable to number the mid-side nodes together with the corner nodes.

11.2.2 The second step is the numbering of the elements.

It is of great importance that a node which belongs to more than one element appears in such a way that the difference

between the smallest number of the element, where the node first appears, and the largest number of the element, where the node last appears, is the minimum possible.

Another important factor which must be taken into account in the numbering of the elements in combination with the above mentioned rule is that one must use the storage facility for similar element matrices in the most efficient way as far as the calculation of the various matrices for an element is concerned (see paragraph 9 INFO(3)).

This can be achieved by numbering the similar elements in consecutive order. Note that a similar element can be obtained by using the option of $180^{\circ}$ rotation of an element (see Chapter 8 Section 2 and paragraph 3 ).

11.2.3 Six numbers are punched via FORMAT (6I5).

The first number (NEIDOS) indicates the code number of the model to be employed (see tables 11.1 ÷ 11.11 ).

The second number (NELEM) indicates the total number of elements involved in the problem.

The third number (NKIND) indicates the number of different elements as far as the coordinates of the elements are concerned, (maximum 30).

Note that a set of coordinates resulting by pure translation from a previous one is considered identical to the latter.

The fourth number (NSTIF) indicates the number of different stiffness sets as regards the elasticity moduli which are to be considered in the problem, (maximum 10).

The fifth number (NBOUL) indicates the total number of different boundary lines (internal or external) as regards the

boundary conditions and the transformation which are to be considered in the problem, (maximum 16).

The sixth number (NBOUP) indicates the number of different individual points with a singularity as far as the boundary conditions and the transformation are concerned so that they have to be considered separately and not as part of one of the previously mentioned boundary lines.

(NBOUP + NBOUL < 26)

Note: All the above set limits can be increased by changing the size of the relevant arrays in the data generation routine.

For changes which particularly affect the limit of the numbers NSTIF and NBOUL changes in the arrays of the main solving routine have to be introduced.

11.2.4. Nine numbers are to be punched via FORMAT (6D 10.3,/,3D 10.3) for each of the different coordinate sets. (see NKIND paragraph 3).

They represent the coordinates of the three corner nodes of the relevant element with respect to a global cartesian coordinate system in the following order:

$$(x_i \ , \ y_i \ , \ z_i)_i \ = \ 1,3$$

The numerical subscript indicates the 1st, 2nd and 3rd node respectively.

11.2.5. Twelve numbers are to be punched via FORMAT (6D 10.3,/,6D 10.3) for each of the different stiffness sets (see paragraph 3 NSTIF).

They represent the elasticity moduli of a set in the following order:

$$D^{xx}_{xx}, \quad D^{yy}_{xx}, \quad D^{yy}_{yy}, \quad D^{xx}_{xy}, \quad S^{xz}_{xz}, \quad S^{yz}_{yz},$$

$$E^{xx}_{xx}, \quad E^{yy}_{xx}, \quad E^{xx}_{yy}, \quad E^{yy}_{yy}, \quad E^{xy}_{xy}, \quad D^{xx}_{yy}$$

(see Chapter 2, Section 2 )

Note:

(a) That $D^{xx}_{yy}$ has to be punched only if it is different from $D^{yy}_{xx}$

(b) For sandwich plate bending problems the moduli relevant to plane stress ($E^{ij}_{rs}$) can be substituted by zeros.

11.2.6. A code number (see paragraph 3 NBOUL, NBOUP) must be punched for each individual boundary line and boundary point via FORMAT (26 I 3).

The first one represents the code number for the first boundary line and the (NBOUL)th one represents the code number for the last boundary line.

The (NBOUL+1)th one represents the code number for the first boundary point and the (NBOUL + NBOUD)th one represents the code number for the last boundary point (see tables

| 1st | 2nd | 3rd | 4th | ..... | NBOULth | (NBOUL+1)th | ..... | (NBOUL+NBOUP)th |
|---|---|---|---|---|---|---|---|---|
| 1st B. Line | 2nd B. Line | 3rd B. Line | 4th B. Line | ..... | Last B. Line | 1st B. Point | ..... | Last B. Point |

NBOUL + NBOUP < 26

11.2.7. A set of six numbers must be punched via FORMAT (6D 10.3) for each of the boundary lines (see paragraph 3 and 6).

They represent the coordinates of two points $A_1$, $A_2$ on the relevant boundary line with respect to the global cartesian co-ordinate system in the following order:

$$x_2 , \quad x_1 , \quad y_2 , \quad y_1 , \quad z_2 , \quad z_1$$

The numerical subscripts 1, 2 indicate point $A_1$ and $A_2$ respectively.

11.2.8. Generally for all eleven models six numbers must be punched next via FORMAT (6IS) for each individual element in the structure from the first to the last element in a consecutive order.

They represent the numbers of the nodes from the 1st to the 6th as they have been set out. (See paragraph 1).

Note: (a) The order of the nodes must be consistent with the setting of their coordinates (see paragraph 4 and Appendix I ).

(b) For models with only corner nodes the first three numbers (relevant to the corner nodes) are necessary although all six can be punched as well. (See paragraph 1).

11.2.9. Sixteen numbers must be punched next via FORMAT (16IS) for every element.

The first number (INFO(1)) indicates the relevant stiffness set to be used for the calculation of the various matrices (from the NSTIF sets. See paragraph 3).

Note: Even if the various matrices are to be taken ready from the storage space, in the case of a similar element, the above number must be set greater than zero.

The second number (INFO(2)) indicates the relevant co-ordinate set to be used for the calculations of the various matrices (from the NKIND sets. See paragraph 3).

Note: If set to zero the coordinates of the nodes are to be set as zero. This option can only be used in a case of a similar element.

The third number (INFO(3)) indicates the storage technique to be used and has the following significance with regard to the setting of the value.

If 11: it evaluates the various matrices, stores them in position I of the storage, and uses them.

If 12: it evaluates the various matrices, stores them in position II of the storage and uses them.

If 1: it takes and uses the various matrices stored in position I of the storage.

If 2: it takes and uses the various matrices stored in position II of the storage.

If 0: it evaluates and uses the various matrices without storing them.

If -1: it takes the various matrices from position I of the storage and uses them after transforming them by a rotation of 180° (see Chapter 8, Section 2, paragraph 3).

If -2: it takes the various matrices from position II of the storage and uses them after transforming them by a rotation of 180° (see Chapter 8, Section 2, paragraph 3).

The 4th, 6th, 8th, 10th, 12th, 14th numbers are set as follows

| Number | 4th | 6th | 8th | 10th | 12th | 14th | The operations listed below are to be carried out depending on the setting of the corresponding number (as shown) |
|---|---|---|---|---|---|---|---|
| Refers to | 1st node | 2nd node | 3rd node | 4th node | 5th node | 6th node | |
| if it is set to | 1 | 2 | 3 | 4 | 5 | 6 | Transformation and boundary conditions |
| >> | -1 | -2 | -3 | -4 | -5 | -6 | Boundary conditions |
| >> | 0 | 0 | 0 | 0 | 0 | 0 | Unchanged |

The 5th, 7th, 9th, 11th, 13th, 15th numbers are set as follows (to be set, if the previously defined six numbers which the present numbers are combined with are different from zero).

| Number Refers to | 5th 1st node | 7th 2nd node | 9th 3rd node | 11th 4th node | 13th 5th node | 15th 6th node | Indicates that the relevant node is linked with the Nth. boundary line or point (paragraph 6) as follows |
|---|---|---|---|---|---|---|---|
| combined with | 4th number | 6th number | 8th number | 10th number | 12th number | 14th number | As previously defined |
| is set to | N | N | N | N | N | N | WHERE N indicates the nth boundary line or point in the order they have been defined in paragraph 6. The transformation is to be performed with respect to the system defined by the line $A_1$ $A_2$ |
| is set to | -N | -N | -N | -N | -N | -N | (only if the corresponding number of the previously defined set is positive) N as above. The transformation is to be performed with respect to the global coordinate system. |

See paragraph 7 and Chapter 8.

The 16th number indicates the loading condition for the element and is set in a value as follows

0 If no load is to be applied

n Where n is the number of the non zero terms of the load matrix which is of the following form

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $P_x$ | $P_y$ | $P_z$ | $R_x^1$ | $R_y^1$ | $R_z^1$ | $R_x^2$ | $R_y^2$ | $R_z^2$ | $R_x^3$ | $R_y^3$ | $R_x^3$ | |

$P_i$ the component of the uniformly distributed load with respect to i global axis

$R_i^j$ the component of the concentrated load at node j with respect to i global axis

If the 16th number is 0 the next card is the card with the 16 numbers for the next element.

If the 16th number is different from zero the next card (cards) must be inserted in the following way via FORMAT (I5,D 10.3). The first number to be punched indicates the column of the non-zero term in the load matrix and the second number indicates the magnitude of the non-zero term.

# PLATE MODEL

REFERENCE SYMBOL
PDS 15

CODE NUMBER
1

STRESSES EVALUATED AT CENTROID
$M_{xx_c}, M_{yy_c}, M_{xy_c}, Q_{x_c}, Q_{y_c}$

TOTAL NUMBER OF D.O.F.
15

| | NBOUL : NBOUD | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | DEGREES OF FREEDOM BEFORE THE TRANSFORMATION | DEGREES OF FREEDOM AFTER THE TRANSFORMATION | FREE EDGE | FREE EDGE WITH STIFFENER | SIMPLY SUPPORTED EDGE | SIMPLY SUPPORTED EDGE WITH STIF. | CLAMPED EDGE | CLAMPED EDGE WITH STIF. | AXIS OF SYMMETRY | AXIS OF SYMMETRY WITH STIF. | POINT SUPPORT | CORNER POINT | CORNER POINT | CORNER POINT | CORNER POINT | CORNER POINT | CORNER POINT | CORNER POINT |
| CORNER NODES | $W$ | $W$ | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 |
| | $W_x$ | $\theta_n$ | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 0 |
| | $W_y$ | $W_s$ | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 0 |
| | $\phi_x$ | $\phi_n$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 1 |
| | $\phi_y$ | $\phi_s$ | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 |
| MIDSIDE NODES (IF DIFFERENT FROM CORNER NODES) | | | | | | | | | | | | | | | | | | |

1 INDICATES CONSTRAINED D.O.F.

0 INDICATES FREE D.O.F.

TABLE 11.1.

| NODE | DEGREES OF FREEDOM BEFORE THE TRANSFORMATION | $w$ | $w_x$ | $w_y$ | $\theta_x$ | $\theta_y$ | DEGREES OF FREEDOM AFTER THE TRANSFORMATION | $w$ | $\theta_n$ | $w_s$ | $\theta_n$ | $\theta_s$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H | FREE EDGE | 0 | 0 | 0 | 0 | 0 | | | | | | |
| P | FREE EDGE WITH STIFFENER | 0 | 0 | 0 | 0 | 1 | | | | | | |
| U | SIMPLY SUPPORTED EDGE | 1 | 0 | 1 | 0 | 0 | | | | | | |
| - | SIMPLY SUPPORTED EDGE WITH STIF. | 1 | 0 | 1 | 0 | 1 | | | | | | |
| U | CLAMPED EDGE | 1 | 1 | 1 | 1 | 0 | | | | | | |
| O | CLAMPED EDGE WITH STIF. | 1 | 1 | 1 | 1 | 1 | | | | | | |
| - | AXIS OF SYMMETRY | 0 | 1 | 0 | 1 | 0 | | | | | | |
| O | AXIS OF SYMMETRY WITH STIF. | 0 | 1 | 0 | 1 | 1 | | | | | | |
| U | POINT SUPPORT | 1 | 0 | 0 | 0 | 0 | | | | | | |
| V | CORNER POINT | 1 | 1 | 1 | 0 | 0 | | | | | | |
| - | CORNER POINT | 1 | 1 | 1 | 1 | 1 | | | | | | |
| - | CORNER POINT | 1 | 0 | 1 | 1 | 2 | | | | | | |
| - | CORNER POINT | 1 | 1 | 1 | 1 | 1 | | | | | | |
| - | CORNER POINT | 1 | 1 | 0 | 0 | 0 | | | | | | |
| - | CORNER POINT | 1 | 1 | 1 | 1 | 0 | | | | | | |
| - | CORNER POINT | 1 | 1 | 0 | 0 | 1 | | | | | | |

CORNER NODES

MIDSIDE NODES (IF DIFFERENT FROM CORNER NODES)

| NODE | $\theta_x$ | $\theta_y$ |
|---|---|---|
| 1 | 0 | 0 |
| 2 | 0 | 1 |
| 3 | 0 | 0 |
| 4 | 0 | 1 |
| 5 | 1 | 0 |
| 6 | 1 | 1 |
| 7 | 1 | 0 |
| 8 | 1 | 1 |

TABLE 11.2.

1 INDICATES CONSTRAINED D.O.F.

0 INDICATES FREE D.O.F.

| | |
|---|---|
| REFERENCE SYMBOL | PDS21 |
| CODE NUMBER | 2 |
| STRESSES EVALUATED AT CENTROID | $M_{xc}, M_{yc}, M_{xyc}, Q_{xc}, Q_{yc}$ |
| TOTAL NUMBER OF D.O.F. | 21 |

| MIDSIDE NODES (IF DIFFERENT FROM CORNER NODES) | | | | CORNER NODES | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | $M_{xy}$ | $M_{yy}$ | $M_{xx}$ | $W$ | DEGREES OF FREEDOM BEFORE THE TRANSFORMATION | | | NBOUD |
| | | | | $M_{ns}$ | $M_{ss}$ | $M_{nn}$ | $W$ | DEGREES OF FREEDOM AFTER THE TRANSFORMATION | | | NBOUD |
| | | | | 1 | 0 | 1 | 0 | FREE EDGE | | | 1 |
| | | | | 0 | 0 | 1 | 0 | FREE EDGE WITH STIFFENER | | | 2 |
| | | | | 1 | 1 | 1 | 1 | SIMPLY SUPPORTED EDGE | | | 3 |
| | | | | 0 | 1 | 1 | 1 | SIMPLY SUPPORTED EDGE WITH STIF. | | | 4 |
| | | | | 1 | 0 | 0 | 1 | CLAMPED EDGE | | | 5 |
| | | | | 0 | 0 | 0 | 1 | CLAMPED EDGE WITH STIF. | | | 6 |
| | | | | 1 | 0 | 0 | 0 | AXIS OF SYMMETRY | | | 7 |
| | | | | 0 | 0 | 0 | 0 | AXIS OF SYMMETRY WITH STIF. | | | 8 |
| | | | | 0 | 1 | 1 | 1 | POINT SUPPORT | | | 9 |
| | | | | 1 | 0 | 0 | 1 | CORNER POINT | | | 10 |
| | | | | 0 | 0 | 0 | 1 | CORNER POINT | | | 11 |
| | | | | 1 | 0 | 0 | 0 | CORNER POINT | | | 12 |
| | | | | 0 | 0 | 0 | 0 | CORNER POINT | | | 13 |
| | | | | 0 | 1 | 1 | 0 | CORNER POINT | | | 14 |
| | | | | 1 | 0 | 0 | 1 | CORNER POINT | | | 15 |
| | | | | 0 | 0 | 0 | 1 | CORNER POINT | | | 16 |

TABLE 11.3.

1 INDICATES CONSTRAINED D.O.F.

0 INDICATES FREE D.O.F.

PLATE MODEL

REFERENCE SYMBOL PMX12

CODE NUMBER 3

STRESSES EVALUATED AT CENTROID $Q_{x_2}, Q_{y_2}$

TOTAL NUMBER OF D.O.F. 12

| | NBOUL, NBOUD | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| DEGREES OF FREEDOM BEFORE THE TRANSFORMATION | DEGREES OF FREEDOM AFTER THE TRANSFORMATION | FREE EDGE | FREE EDGE WITH STIFFENER | SIMPLY SUPPORTED EDGE | SIMPLY SUPPORTED EDGE WITH STIF. | CLAMPED EDGE | CLAMPED EDGE WITH STIF. | AXIS OF SYMMETRY | AXIS OF SYMMETRY WITH STIF. | POINT SUPPORT | CORNER POINT | CORNER POINT | CORNER POINT | CORNER POINT | CORNER POINT | CORNER POINT | CORNER POINT |
| CORNER NODES: W | W | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 |
| $M_{xy}$ | $M_{nn}$ | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| $M_{yy}$ | $M_{ss}$ | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| $M_{xy}$ | $M_{ns}$ | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 |
| MIDSIDE NODES (IF DIFFERENT FROM CORNER NODES) | | | | | | | | | | | | | | | | | |

PLATE MODEL

REFERENCE SYMBOL

PMX24

CODE NUMBER

4

STRESSES EVALUATED AT 1st and 2nd NODE

$Q_{x_1}, Q_{y_1}, Q_{x_2}, Q_{y_2}$

TOTAL NUMBER OF D.O.F.

24

1 INDICATES CONSTRAINED D.O.F.

0 INDICATES FREE D.O.F.

TABLE 11.4.

# PLATE MODEL

REFERENCE SYMBOL: PDS24

CODE NUMBER: 5

STRESSES EVALUATED AT CENTROID

$M_{xx_c}, M_{yy_c}, M_{xy_c}, Q_{x_c}, Q_{y_c}$

TOTAL NUMBER OF D.O.F.: 24

| | NBOUL | NBOUP | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | DEGREES OF FREEDOM BEFORE THE TRANSFORMATION | DEGREES OF FREEDOM AFTER THE TRANSFORMATION | FREE EDGE | FREE EDGE WITH STIFFENER | SIMPLY SUPPORTED EDGE | SIMPLY SUPPORTED EDGE WITH STIF. | CLAMPED EDGE | CLAMPED EDGE WITH STIF. | AXIS OF SYMMETRY | AXIS OF SYMMETRY WITH STIF. | POINT SUPPORT | CORNER POINT | CORNER POINT | CORNER POINT | CORNER POINT | CORNER POINT | CORNER POINT | CORNER POINT |
| CORNER NODES | $W$ | $W$ | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 |
| | $W_x$ | $\theta_m$ | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 0 |
| | $W_y$ | $W_s$ | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 0 |
| | $W_{xx}$ | $W_{mm}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 |
| | $W_{xy}$ | $W_{sm}$ | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 |
| | $W_{yy}$ | $W_{ss}$ | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| | $\phi_x$ | $\phi_m$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 1 |
| | $\phi_y$ | $\phi_s$ | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 |
| MIDSIDE NODES (IF DIFFERENT FROM CORNER NODES) | | | | | | | | | | | | | | | | | | |

1 INDICATES CONSTRAINED D.O.F.

0 INDICATES FREE D.O.F.

TABLE 11.5.

# PLATE MODEL

REFERENCE SYMBOL
PDS30

CODE NUMBER
6

STRESSES EVALUATED AT CENTROID

$M_{xx_c}, M_{yy_c}, M_{xy_c}, Q_{x_c}, Q_{y_c}$

TOTAL NUMBER OF D.O.F.
30

1 INDICATES CONSTRAINED D.O.F.

0 INDICATES FREE D.O.F.

| | NBOUL, NBOUD | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | DEGREES OF FREEDOM BEFORE THE TRANSFORMATION / DEGREES OF FREEDOM AFTER THE TRANSFORMATION | FREE EDGE | FREE EDGE WITH STIFFENER | SIMPLY SUPPORTED EDGE | SIMPLY SUPPORTED EDGE WITH STIF. | CLAMPED EDGE | CLAMPED EDGE WITH STIF. | AXIS OF SYMMETRY | AXIS OF SYMMETRY WITH STIF. | POINT SUPPORT | CORNER POINT | CORNER POINT | CORNER POINT | CORNER POINT | CORNER POINT | CORNER POINT | CORNER POINT |
| CORNER NODES | $W$ / $W$ | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 |
| | $W_x$ / $\theta_M$ | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 0 |
| | $W_Y$ / $W_S$ | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 0 |
| | $W_{xx}$ / $W_{MM}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 |
| | $W_{XY}$ / $W_{SM}$ | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 |
| | $W_{YY}$ / $W_{SS}$ | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| | $\phi_x$ / $\phi_M$ | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 1 |
| | $\phi_Y$ / $\phi_S$ | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 |
| MIDSIDE NODES (IF DIFFERENT FROM CORNER NODES) | $\phi_x$ / $\phi_M$ | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | | | | | | | | |
| | $\phi_Y$ / $\phi_S$ | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | | | | | | | | |
| | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | |

TABLE 11.6.

| | NBOUL, NBOUD | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | DEGREES OF FREEDOM BEFORE THE TRANSFORMATION | FREE EDGE | FREE EDGE WITH STIFFENER | SIMPLY SUPPORTED EDGE | SIMPLY SUPPORTED EDGE WITH STIF. | CLAMPED EDGE | CLAMPED EDGE WITH STIF. | AXIS OF SYMMETRY | AXIS OF SYMMETRY WITH STIF. | POINT SUPPORT | CORNER POINT | CORNER POINT | CORNER POINT | CORNER POINT | CORNER POINT | CORNER POINT | CORNER POINT |
| | DEGREES OF FREEDOM AFTER THE TRANSFORMATION | | | | | | | | | | | | | | | | |
| CORNER NODES | $W$ / $W$ | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 |
| | $\theta_x$ / $\theta_M$ | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| | $\theta_y$ / $\theta_S$ | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 1 |
| | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | |
| MIDSIDE NODES (IF DIFFERENT FROM CORNER NODES) | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | |

PLATE MODEL

REFERENCE SYMBOL
PR018

CODE NUMBER
7

STRESSES EVALUATED AT CENTROID

$M_{xx_c}, M_{yy_c}, M_{xy_c}, Q_{x_c}, Q_{y_c}$

TOTAL NUMBER OF D.O.F.
18

1 INDICATES CONSTRAINED D.O.F.

0 INDICATES FREE D.O.F.

TABLE 11.7.

DOME MODEL

REFERENCE SYMBOL

DDS21

CODE NUMBER

8

STRESSES EVALUATED AT CENTROID

$M_{xx_c}, M_{yy_c}, M_{xy_c}, Q_{x_c}, Q_{y_c}, N_{xx_c}, N_{yy_c}, N_{xy_c}$

TOTAL NUMBER OF D.O.F.

21

| | NBOUL, NBOUP | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | DEGREES OF FREEDOM BEFORE THE TRANSFORMATION / DEGREES OF FREEDOM AFTER THE TRANSFORMATION | FREE EDGE | FREE EDGE WITH STIF. | AXIS OF SYMMETRY | AXIS OF SYMMETRY WITH ST. | AXIS OF SYMMETRY | AXIS OF SYMMETRY WITH ST. | SIMPLY SUPPORTED EDGE | SIMPLY SUPPORTED WITH STIF. | SIMPLY SUPPORTED EDGE | SIMPLY SUPPORTED WITH STIF. | SIMPLY SUPPORTED EDGE | SIMPLY SUPPORTED WITH STIF. | SIMPLY SUPPORTED EDGE | SIMPLY SUPPORTED WITH STIF. | POINT SUPPORT | | | | | | PLATE INTERCONNECTION | | | | | | | | | |
| CORNER NODES | $u^{local}$ / $u^{global}$ | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 |
| | $v^e$ / $v^g$ | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| | $w^e$ / $w^g$ | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | $\theta_x^e$ / $\theta_m$ | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | $\theta_y^e$ / $\theta_s$ | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | $\phi_x^e$ / $\phi_m$ | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | $\phi_y^e$ / $\phi_s$ | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

MIDSIDE NODES (IF DIFFERENT FROM CORNER NODES)

1 INDICATES CONSTRAINED D.O.F.

0 INDICATES FREE D.O.F.

TABLE 11.8.

DOME MODEL

REFERENCE SYMBOL

DDS33

CODE NUMBER

9

STRESSES EVALUATED AT CENTROID

$M_{xx_c}, M_{yy_c}, M_{xy_c}, Q_{x_c}, Q_{y_c}, N_{xx_c}, N_{yy_c}, N_{xy_c}$

TOTAL NUMBER OF D.O.F.

33

1 INDICATES CONSTRAINED D.O.F.

0 INDICATES FREE D.O.F.

| | NBOUL, NBOUP | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | DEGREES OF FREEDOM BEFORE THE TRANSFORMATION | DEGREES OF FREEDOM AFTER THE TRANSFORMATION | FREE EDGE | FREE EDGE WITH STIF. | AXIS OF SYMMETRY | AXIS OF SYMMETRY WITH ST. | AXIS OF SYMMETRY | AXIS OF SYMMETRY, WITH ST. | SIMPLY SUPPORTED EDGE | SIMPLY SUPPORTED WITH STIF. | SIMPLY SUPPORTED EDGE | SIMPLY SUPPORTED WITH STIF. | SIMPLY SUPPORTED EDGE | SIMPLY SUPPORTED WITH STIF. | SIMPLY SUPPORTED EDGE | SIMPLY SUPPORTED WITH STIF. | POINT SUPPORT | >> | ; | ; | ; | PLATE INTERCONNECTION | ; | ; | ; | ; | ; | ; | ; | ; | ; |
| CORNER NODES | $u^{local}$ | $u^{global}$ | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 |
| | $v^e$ | $v^g$ | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| | $w^e$ | $w^g$ | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | $\theta_x^e$ | $\theta_n$ | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | $\theta_y^e$ | $\theta_s$ | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | $\phi_x^e$ | $\phi_n$ | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | $\phi_y^e$ | $\phi_s$ | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| MIDSIDE NODES (IF DIFFERENT FROM CORNER NODES) | $\phi_x^e$ | $\phi_n$ | | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | | | | | | | | | | | | | | | | |
| | $\phi_y^e$ | $\phi_s$ | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | | | | | | | | | | | | | | | | |
| | $u_{ss}$ | $u_{ss}$ | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | | | | | | | | | | | | | | | | |
| | $v_{ss}$ | $v_{ss}$ | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | | | | | | | | | | | | | | | | |

TABLE 11.9.

| | NBOUL, NBOUP | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | DEGREES OF FREEDOM BEFORE THE TRANSFORMATION | FREE EDGE | FREE EDGE WITH STIF. | AXIS OF SYMMETRY | AXIS OF SYMMETRY WITH ST. | AXIS OF SYMMETRY | AXIS OF SYMMETRY, WITH ST. | SIMPLY SUPPORTED EDGE | SIMPLY SUPPORTED WITH STIF. | SIMPLY SUPPORTED EDGE | SIMPLY SUPPORTED WITH STIF. | SIMPLY SUPPORTED EDGE | SIMPLY SUPPORTED WITH STIF. | SIMPLY SUPPORTED EDGE | SIMPLY SUPPORTED WITH STIF. | POINT SUPPORT | | | | | | PLATE INTERCONNECTION | | | | | | | | |
| CORNER NODES | DEGREES OF FREEDOM AFTER THE TRANSFORMATION | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | $u^{local}$ / $u^{global}$ | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 |
| | $v^{l}$ / $v^{g}$ | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| | $w^{l}$ / $w^{g}$ | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | $M_{xx}^{l}$ / $M_{nn}$ | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| | $M_{yy}^{l}$ / $M_{ss}$ | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| | $M_{xy}^{l}$ / $M_{sn}$ | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 |
| MIDSIDE NODES (IF DIFFERENT FROM CORNER NODES) | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |

DOME MODEL

REFERENCE SYMBOL

DMX36

CODE NUMBER

11

STRESSES EVALUATED

AT 1st and 2nd NODE and at CENTROID

$Q_{x_1}, Q_{y_1}, Q_{x_2}, Q_{y_2}$

$N_{xx_c}, N_{yy_c}, N_{xy_c}$

TOTAL NUMBER OF D.O.F.

36

1 INDICATES CONSTRAINED D.O.F.

0 INDICATES FREE D.O.F.

TABLE 11.10.

DOME MODEL

REFERENCE SYMBOL: D R O 3 0

CODE NUMBER: 12

STRESSES EVALUATED AT CENTROID: $N_{xxc}, M_{yyc}, M_{xyc}, Q_{xc}, Q_{yc}, N_{xxc}, N_{yyc}, N_{xyc}$

TOTAL NUMBER OF D.O.F.: 30

1 INDICATES CONSTRAINED D.O.F.

0 INDICATES FREE D.O.F.

| | NBOUP, NBOUP | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | DEGREES OF FREEDOM BEFORE THE TRANSFORMATION | FREE EDGE | FREE EDGE WITH STIF. | AXIS OF SYMMETRY | AXIS OF SYMMETRY WITH ST. | AXIS OF SYMMETRY | AXIS OF SYMMETRY WITH ST. | SIMPLY SUPPORTED EDGE | SIMPLY SUPPORTED WITH STIF. | SIMPLY SUPPORTED EDGE | SIMPLY SUPPORTED WITH STIF. | SIMPLY SUPPORTED EDGE | SIMPLY SUPPORTED WITH STIF. | SIMPLY SUPPORTED EDGE | SIMPLY SUPPORTED WITH STIF. | POINT SUPPORT | | | | | | PLATE INTERCONNECTION | | | | | | | | | |
| | DEGREES OF FREEDOM AFTER THE TRANSFORMATION | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| CORNER NODES | $u^{local}$ / $u^{global}$ | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 |
| | $v^{l}$ / $v^{g}$ | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| | $w^{l}$ / $w^{g}$ | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | $\theta_x^{l}$ / $\theta_n$ | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | $\theta_y^{l}$ / $\theta_s$ | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| MIDSIDE NODES (IF DIFFERENT FROM CORNER NODES) | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |

TABLE 11.11.

## 12. SANDWICH PLATE BENDING RESULTS

The various sandwich plate bending models, (see fig. 12.1), outlined in chapter 6, were tested by solving a series of problems, for which other solutions have already been found by other researchers.

The characteristics of each individual model had to be established by the analysis of the obtained results.

As outlined in chapter 7 the sandwich dome models were formed by extending certain sandwich plate bending models.

The selectionof these models was based on the conclusions drawn from the comparative analysis of the results from the plate models presented in this chapter.

The analysed problems are classified as follows.

### 12.1 Theoretical results.

Most of the existing theoretical results are obtained from numerical solutions of certain sandwich plate bending problems (small deflections).[21 ]

The following were chosen for the analysis.

#### 12.1.1. Comparison with Dynamic Relaxation Method [16]

Square isotropic sandwich plate under uniformly distributed load.

The plate is simply supported with the twisting moment fixed to zero (Mns = 0) all along the boundaries. (see figs. 12.5, 12.6, CASE 1)

A second solution was obtained with the twisting moment acting (Mns $\neq$ 0) all along the boundaries. (see figs. 12.5, 12.7, CASE 2)

The deflection curves obtained for the various models compare very well with those of reference [16]. The model PDS21 is more flexible than the rest. The same applies for the models PDS30 to a lesser extent.

For the mixed model PMX24 the agreement is very good; the mixed model PMX12 being less accurate.

The various models are very accurate with regard to the distribution of moment. The agreement with regard to the distribution of shear stresses varies from very good for the models PDS21 and PMX24 to fair for the models PDS15 and PMX12 [16].

### 12.1.2. Comparison with Finite Difference Method [110]

Square orthotropic sandwich plate under uniformly distributed load.

The plate is simply supported with the twisting moment acting ($Mns \neq 0$) all along the boundaries.

Six sets of results were obtained by varying the properties of the plate. (see fig. 12.5, CASES 3 ÷ 8)

The results for each case are presented in figs. 12.8 ÷ 12.11

The various models behave in the same way as presented in the previous section 12.1.1.

### 12.1.3. Comparison with Finite Difference Method [110]

Square orthotropic sandwich plate under uniformly distributed load.

The plate is clamped with the twisting moment acting ($Mns \neq 0$) all along the boundaries.

Three sets of results were again obtained by varying the properties of the plate. (see fig. 12.5, CASES 9 ÷ 11)

The results obtained for each individual case are presented in figs. 12.12 ÷ 12.20.

The maximum values for the deflection, moment and shear stress obtained through the various models are plotted against the values of shear stiffness for the different cases. (see figs. 12.19, 12.20 CASE 9 ÷ 12)

The deviation of the deflection obtained through the model PDS21 increases for small values of shear stiffness as the deflection due to shear becomes predominant in the increase of the total deflection.

The effect of the shear stiffness on the moments and shear stresses is negligible.

### 12.1.4. Comparison with Finite Element Models [21, 97]

Skew isotropic sandwich plate under uniformly distributed load.

The plate is simply supported with the twisting moment acting ($Mns \neq 0$) all along the boundaries. (see figs. 12.21 ÷ 12.24, CASE 1)

A second solution was obtained with the twisting moment fixed to zero ($Mns = 0$) all along the boundaries. (see figs. 12.21 ÷ 12.27, CASE 2)

The models PMX12, PMX24 appear to be stiff and the models PDS21, PDS30 flexible for CASE 1 with regard to the deflection although the

agreement for CASE 2 is very good.

For the distribution of the moment My for CASE 1 the agreement is reasonable for most of the displacement models, although the mixed models differ because the My was fixed to zero at the corner A. For the distribution of the Mx for CASE 1 the mixed models are more accurate.

For CASE 2, all the models are quite accurate with regard to the distribution of the moments Mx, My, the mixed models being superior in this respect.

### 12.1.5. Comparison with Fourier Series Method [66]

Skew isotropic sandwich plate under uniformly distributed load [66]

The plate is clamped with the twisting moment acting ($Mns \neq 0$) all along the boundaries.

Four sets of results were obtained by varying the skew angle of the plate. (see figs. 12.28 ÷ 12.37, CASES 1 ÷ 4)

For all the cases the mixed models as well as most of the displacement models are seen to be accurate, the mixed models being superior in this respect.

The higher order models PDS24, PDS30 appear to be less accurate.

## 12.2 Experimental Results

### 12.2.1. Comparison with Sandwich Plates tested by Bettess

Square sandwich plate under concentrated load at the centre of the plate. [21]

The plate was supported at all four corners and the edges were unstiffened (Mns = 0 all along the boundaries.)

Seven cases were analysed corresponding to seven different sandwich panels tested by Bettess. [21] (see figs. 12.38 ÷ 12.49, CASES 1 ÷ 7)

The above cases proved to be the most difficult modelling tests for the various elements.

The models PDS21, PDS30, PRO18 show an increased flexibility (up to 45%) for most cases excepting CASE 6. (see fig. 12.39)

Due to the above behaviour additional boundary conditions were introduced for the above models, by restraining the rotations at the four supports.

For the quadratic mixed model PMX24 the modelling was performed with the twisting moment acting all along the free edge. Despite this, the

model behaves in an increasingly flexible manner for cases 1, 2 and 5, but becomes stiffer for case 6.

The displacement models PDS15 and PDS24 proved more succesful than the rest with the exception of case 5, generally agreeing very well with the experimental results.

### 12.2.2. Comparison with triangular sandwich plates under concentrated load tested by COLLINS [21, 33, 85]

Six cases were analysed varying the shape, the loading point and the boundary conditions of the plate.

The properties of the plate were the same as Bettes's Plate 3.

For all the models the agreement with the experimental results varies from good to very good. The displacement models PDS15 and PDS24 appear to behave in a less flexible manner in comparison with the experimental results, the latter being closer to the behaviour of the models PDS21 and PDS30 which behave as in the previously analysed cases with an increased flexibility. The behaviour of the mixed models PMX12, PMX24 and the rotation model PRO18 is very satisfactory.

## 12.3 Conclusions

Seven different models were used in the present analysis to solve sandwich plate bending problems as presented in section 12.1 and 12.2 serving the purpose of comparison between the mixed variational approach and the displacement variational approach. (chapters 4 and 6)

An attempt was made by simulating the theoretical and experimental results obtained by other researchers, to understand the behaviour of the various models and from the comparative study to select the most efficient and appropriate for the sandwich dome problem as explained in chapters 7 and 12.

The following can be concluded from the problems analysed in sections 12.1, 12.2.

From the convergence study presented in figs. 12.2 ÷ 12.5 it is evident that all the models converge, the higher order in a better way as expected.

The improvement in the accuracy obtained from the higher order displacement models PDS24 and PDS30 in comparison with the models PDS15 and PDS21 does not provide enough justification for the considerable increase in computational effort, due to the increase in the total number of

unknowns for a problem. (see fig. 6.1, 6.2,12.2 ÷ 12.5)

Another important factor is the boundary conditions for the higher order models PDS24 and PDS30. They include restraintsof second derivations of deflections ($W_{nn}$, $W_{sn}$, $W_{ss}$) which adds one more difficulty to the already delicate problem of simulating the physical boundary restraints.

This proved to have a very great influence in the case of the square plates supported at the four corners under concentrated load, as mentioned in Section 12.2.

The displacement models tend to show better accuracy for problems with few displacement constraints(free edges) than the mixed models. On the other hand for problems with a large number of displacement constraints (clamped edges) but few constraints for the mixed models, the latter are more accurate in their behaviour.

The same is valid from the numerical and computational effort point of view when the boundary constraints are introduced as computer data.

The mixed models proved advantageous with regard to the distribution of the stresses and in particular to the distribution of moments, which is obtained from nodal values, as for the mixed models the moments represent degrees of freedom (see chapters 4, 5 and 6)

As expected the increase in the order of the shear variation for the models PDS21 and PDS30 produce more flexible models because of the increase, as mentioned in section 12.1.1, in the part of the total deflection due to shear.

This was proved to cope in a better way with problems where the ratio shear stiffness over bending stiffness $S_{xz}^{xz}/D_{xx}^{xx}$ (see chapter 2) had small values of the range between 1 ÷ 15, as some of the cases tested experimentally. (12.2)

## 12.4 The extension from Plate to Dome Models

As already mentioned in chapter 7, four plate models were extended to form the four dome models (figs. 7.1, 7.2 and 13.1). These are PDS15, PDS21, PMX24 and PRO18.

The two higher order elements PDS24 and PDS30 were rejected because, as outlined in section 12.3, the accuracy obtained does not justify the increase in the total number of unknowns. Another decisive factor was the difficulty which the second derivatives present in transformation and boundary conditions.

The mixed models PMX12 was rejected for reasons outlined in chapter 7.

The first models PDS15 (DDS21) has the advantage of possessing the least degrees of freedom as a sandwich dome model, this being of great significance from the computational effort point of view. The behaviour of the model when tested by the different plate bending problems was found, on average to be very satisfactory.

The second model PDS21 (DDS33) although it appeared to behave flexibly in most cases, was included in the set of dome models. The reason for this inclusion was that most of the dome cases to be analysed, presented a ratio of shear stiffness over bending stiffness ($S_{xz}/D_{xx}$) of a fairly low value. As outlined in the previous section, this particular high shear model proved quite successful in this respect.

The third model PMX24 (DMX36) was very successful in a large number of cases but not so successful in others, as explained in the previous section. Its application to the dome problem will provide comparative results between the mixed and displacement approach for the dome problem.

Special advantages of the mixed dome model include the higher variation with regard to plane stress (second order) and the presence of moments as degrees of freedom, the latter being very significant for the analysis of dome structures (see chapters 10 and 13).

The fourth model PRO18 (DRO30) was successful throughout the various sandwich plate bending problems. It was included in order to obtain comparative results from the total rotation model category which it represents.

FIG. 12.1.

# SANDWICH PLATE MODELS

| REFERENCE SYMBOL<br>DEGREES OF FREEDOM | SANDWICH - PLATE MODELS | STRESSES |
|---|---|---|
| **PDS15**<br>$w, w_x, w_y, \phi_x, \phi_y$<br>ISPLACEMENT MODEL | | $M_{xx_c}$, $M_{yy_c}$, $M_{xy_c}$<br>$Q_{x_c}$, $Q_{y_c}$<br>at the centroid |
| **PDS21**<br>$w, w_x, w_y, \phi_x, \phi_y$<br>$\phi_x, \phi_y$<br>ISPLACEMENT MODEL | | $M_{xx_c}$, $M_{yy_c}$, $M_{xy_c}$<br>$Q_{x_c}$, $Q_{y_c}$<br>at the centroid |
| **PMX12**<br>$w, M_{xx}, M_{yy}, M_{xy}$<br>IXED MODEL | | $Q_{x_c}$, $Q_{y_c}$ at the centroid |
| **PMX24**<br>$w, M_{xx}, M_{yy}, M_{xy}$<br>IXED MODEL | | $Q_{x_1}, Q_{y_1}$ at 1st node<br>$Q_{x_2}, Q_{y_2}$ at 2nd node |
| **PDS24**<br>$w, w_x, w_y, w_{xx}, w_{xy}, w_{yy}$<br>$\phi_x, \phi_y$<br>ISPLACEMENT MODEL | | $M_{xx_c}$, $M_{yy_c}$, $M_{xy_c}$<br>$Q_{x_c}$, $Q_{y_c}$<br>at the centroid |
| **PDS30**<br>$w, w_x, w_y, w_{xx}, w_{xy}, w_{yy}$<br>$\phi_x, \phi_y$<br>$\phi_x, \phi_y$<br>ISPLACEMENT MODEL | | $M_{xx_c}$, $M_{yy_c}$, $M_{xy_c}$<br>$Q_{x_c}$, $Q_{y_c}$<br>at the centroid |
| **PRO18**<br>, $w, \theta_x, \theta_y$<br>DISPLACEMENT MODEL | | $M_{xx_c}$, $M_{yy_c}$, $M_{xy_c}$<br>$Q_{x_c}$, $Q_{y_c}$<br>at the centroid |

n = 3
PDS15
PDS21
PMX12
PMX24
PDS24
PDS30
PRO18
n number of mesh subdivisions
n = 10
n = 9
n = 8
n = 7
n = 6
n = 5
n = 4
n = 3
196
245
399
392
413
537
560
n = 6
n = 3
10
20
30
40
50
60
70
80
90
100
200
300
400
500
600
700
800
900
1000
total number of degrees of freedom

FIG. 12.2.

# CONVERGENCE TOWARDS SERIES SOLUTION

FIG. 12.3.

# CONVERGENCE TOWARDS SERIES SOLUTION

FIG. 12.4.

# SQUARE PLATE

UNDER

UNIFORMLY DISTRIBUTED LOAD

$1\ Nt/m^2$

| KEY | |
|---|---|
| PDS15 | ∘ |
| PDS21 | △ |
| PMX12 | × |
| PMX24 | -·-·- |
| PDS24 | □ |
| PDS30 | + |
| PRO18 | ▽ |

| CASE | BOUNDARY CONDITIONS | $D^{xx}_{xx}$ (Nt-m) | $D^{yy}_{xx}=D^{xx}_{yy}$ (Nt-m) | $D^{yy}_{yy}$ (Nt-m) | $D^{xy}_{xy}$ (Nt-m) | $S^{xz}_{xz}$ (Nt/m) | $S^{yz}_{yz}$ (Nt/m) | COMPARISON with —·— |
|---|---|---|---|---|---|---|---|---|
| 1. | SIMPLY SUPPORTED $M_{ns}=0.$ along the supports | 1.0 | 0.3 | 1.0 | 0.7 | 16. | 16. | FINITE DIFFERENCES [16] (BASU and DAWSON) |
| 2. | SIMPLY SUPPORTED $M_{ns}\neq 0.$ along the supports | 1.0 | 0.3 | 1.0 | 0.7 | 16. | 16. | FINITE DIFFERENCES (BASU and DAWSON) [16] |
| 3. | >> | 1.1 | 0.33 | 1.1 | 0.769 | 50. | 50. | FINITE DIFFERENCES [110] (CHAPMAN and WILLIAMS) |
| 4. | >> | 1.08 | 0.27 | 0.9 | 0.631 | 50. | 60. | >> |
| 5. | >> | 1.07 | 0.229 | 0.764 | 0.535 | 50. | 70. | >> |
| 6. | >> | 1.06 | 0.198 | 0.663 | 0.464 | 50. | 80. | >> |
| 7. | >> | 1.055 | 0.176 | 0.586 | 0.409 | 50. | 90. | >> |
| 8. | >> | 1.045 | 0.157 | 0.522 | 0.367 | 50. | 100. | >> |
| 9. | CLAMPED $M_{ns}\neq 0.$ along the supports | 1.1 | 0.33 | 1.1 | 0.769 | 50. | 50. | >> |
| 10. | >> | 1.1 | 0.33 | 1.1 | 0.769 | 250. | 250. | >> |
| 11. | >> | 1.1 | 0.33 | 1.1 | 0.769 | 500. | 500. | >> |

FIG. 12.5.

# CASE 1.

FIG. 12.6.

0.5m
A
C
0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
10.
11. 10^-3 m
W
0.5m
A
C
0.
1.
2.
3.
4.
5. 10^-2 Nt-m/m
Mxx
0.5m
D
B
0.
1.
2.
3.
4. 10^-1 Nt/m
Qy

FIG. 12.7.

CASE 3.

FIG. 12.8.

FIG. 12.9.

$w_c$ (central deflection)
$10^{-3}$ m
9.
8.
7.
6.
5.
4.
3.
2.
1.
0.
50.
60.
70.
80.
90.
100.
$S_{yz}^{yz}$ (Nt/m)
CASE 3.
CASE 4.
CASE 5.
CASE 6.
CASE 7.
CASE 8.

FIG. 12.10.

Mxx, Myy (central moments)
10^-2 Nt·m/m
9.
8.
7.
6.
5.
4.
3.
2.
1.
Mxx
Myy
50.
60.
70.
80.
90.
100.
S^yz_yz (Nt/m)
CASE 3.
CASE 4.
CASE 5.
CASE 6.
CASE 7.
CASE 8.

FIG. 12.11.

CASE 9

FIG. 12.12

CASE 9.

FIG. 12.13.

# CASE 10.

FIG. 12.14.

# CASE 10.

FIG. 12.15.

# CASE 11

FIG. 12.16.

# CASE 11.

FIG. 12.17.

| | CASE 9. | | | | | CASE 10. | | | | | CASE 11. | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| MESH 3 x 3 | $W_c$ | $M_{xx(max)}$ | $M_{xx(min)}$ | $M_{xy(max)}$ | $Q_{y(max)}$ | $W_c$ | $M_{xx(max)}$ | $M_{xx(min)}$ | $M_{xy(max)}$ | $Q_{y(max)}$ | $W_c$ | $M_{xx(max)}$ | $M_{xx(min)}$ | $M_{xy(max)}$ | $Q_{y(max)}$ |
| PDS15 | $2.71\ 10^{-3}$ | $2.15\ 10^{-2}$ | $-2.29\ 10^{-2}$ | $6.32\ 10^{-3}$ | $-2.74\ 10^{-1}$ | $1.53\ 10^{-3}$ | $2.17\ 10^{-2}$ | $-2.28\ 10^{-2}$ | $4.60\ 10^{-3}$ | $-3.00\ 10^{-1}$ | $1.38\ 10^{-3}$ | $2.17\ 10^{-2}$ | $-2.28\ 10^{-2}$ | $4.28\ 10^{-3}$ | $-3.15\ 10^{-1}$ |
| PDS21 | $3.41\ 10^{-3}$ | $2.23\ 10^{-2}$ | $-2.14\ 10^{-2}$ | $6.74\ 10^{-3}$ | $-3.65\ 10^{-1}$ | $1.77\ 10^{-3}$ | $2.19\ 10^{-2}$ | $-2.56\ 10^{-2}$ | $4.42\ 10^{-3}$ | $-3.82\ 10^{-1}$ | $1.52\ 10^{-3}$ | $2.18\ 10^{-2}$ | $-2.61\ 10^{-2}$ | $4.38\ 10^{-3}$ | $-3.93\ 10^{-1}$ |
| PMX12 | $2.63\ 10^{-3}$ | $2.56\ 10^{-2}$ | $-4.24\ 10^{-2}$ | $7.24\ 10^{-3}$ | $-2.83\ 10^{-1}$ | $1.44\ 10^{-3}$ | $2.53\ 10^{-2}$ | $-4.60\ 10^{-2}$ | $5.94\ 10^{-3}$ | $-3.01\ 10^{-1}$ | $1.28\ 10^{-3}$ | $2.53\ 10^{-2}$ | $-4.70\ 10^{-2}$ | $5.37\ 10^{-3}$ | $-3.08\ 10^{-1}$ |
| PMX24 | $2.72\ 10^{-3}$ | $2.35\ 10^{-2}$ | $-4.32\ 10^{-2}$ | $6.67\ 10^{-3}$ | $-3.68\ 10^{-1}$ | $1.49\ 10^{-3}$ | $2.26\ 10^{-2}$ | $-4.54\ 10^{-2}$ | $4.42\ 10^{-3}$ | $-3.94\ 10^{-1}$ | $1.33\ 10^{-3}$ | $2.24\ 10^{-2}$ | $-4.56\ 10^{-2}$ | $4.13\ 10^{-3}$ | $-3.90\ 10^{-1}$ |
| PDS24 | $2.74\ 10^{-3}$ | $2.25\ 10^{-2}$ | $-2.12\ 10^{-2}$ | $6.57\ 10^{-3}$ | $-3.08\ 10^{-1}$ | $1.49\ 10^{-3}$ | $2.23\ 10^{-2}$ | $-2.50\ 10^{-2}$ | $5.15\ 10^{-3}$ | $-3.43\ 10^{-1}$ | $1.33\ 10^{-3}$ | $2.22\ 10^{-2}$ | $-2.58\ 10^{-2}$ | $4.85\ 10^{-3}$ | $-3.53\ 10^{-1}$ |
| PDS30 | $2.55\ 10^{-3}$ | $2.30\ 10^{-2}$ | $-2.34\ 10^{-2}$ | $5.49\ 10^{-3}$ | $-3.08\ 10^{-1}$ | $1.49\ 10^{-3}$ | $2.28\ 10^{-2}$ | $-2.41\ 10^{-2}$ | $2.89\ 10^{-3}$ | $-3.43\ 10^{-1}$ | $1.34\ 10^{-3}$ | $2.27\ 10^{-2}$ | $-2.46\ 10^{-2}$ | $2.80\ 10^{-3}$ | $-3.72\ 10^{-1}$ |
| PRO18 | $2.66\ 10^{-3}$ | $2.22\ 10^{-2}$ | $-2.65\ 10^{-2}$ | $5.91\ 10^{-3}$ | $-3.00\ 10^{-1}$ | $1.46\ 10^{-3}$ | $2.16\ 10^{-2}$ | $-2.72\ 10^{-2}$ | $4.54\ 10^{-3}$ | $-2.99\ 10^{-1}$ | $1.28\ 10^{-3}$ | $2.13\ 10^{-2}$ | $-2.71\ 10^{-2}$ | $4.14\ 10^{-3}$ | $-2.74\ 10^{-1}$ |
| CHAPMAN AND WILLIAMS | $2.70\ 10^{-3}$ | $2.30\ 10^{-2}$ | $-4.35\ 10^{-2}$ | $7.0\ 10^{-3}$ | $-3.70\ 10^{-1}$ | $1.50\ 10^{-3}$ | $2.25\ 10^{-2}$ | $-4.70\ 10^{-2}$ | $5.00\ 10^{-3}$ | $-4.00\ 10^{-1}$ | $1.35\ 10^{-3}$ | $2.25\ 10^{-2}$ | $-5.00\ 10^{-2}$ | $4.30\ 10^{-3}$ | $-4.10\ 10^{-1}$ |
| (UNITS) | (m) | (Nt m/m) | (Nt m/m) | (Nt m/m) | (Nt/m) | (m) | (Nt m/m) | (Nt m/m) | (Nt m/m) | (Nt/m) | (m) | (Nt m/m) | (Nt m/m) | (Nt m/m) | (Nt/m) |

FIG. 12.18.

4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
CASE 9.
CASE 10.
CASE 11.
0.
100
200
300
400
500
600
700
800
900
$S_{YZ}^{YZ}$ (Nt/m)

FIG. 12.19.

$M_{xx}$
$Q_Y$
$5.\ 10^{-1}$ (Nt/m)
4.
3.
2.
1.
0.
(Nt m/m) 3.0 $10^{-2}$
2.5
2.0
$Q_Y$
$M_{xx}$
CASE 9.
CASE 10.
CASE 11.
0.
100.
200.
300.
400.
500
(Nt/m)

FIG. 12.20.

# SKEW SIMPLY SUPPORTED PLATE UNDER UNIFORMLY DISTRIBUTED LOAD

$1\ Nt/m^2$

| KEY | |
|---|---|
| PDS15 | ○ |
| PDS21 | △ |
| PMX12 | × |
| PMX24 | —·—·— |
| PDS24 | □ |
| PDS30 | + |
| PRO18 | ▽ |

| CASE | BOUNDARY CONDITIONS | $D^{xx}_{xx}$ (Nt-m) | $D^{yy}_{xx} = D^{xx}_{yy}$ (Nt-m) | $D^{yy}_{yy}$ (Nt-m) | $D^{xy}_{xy}$ (Nt-m) | $S^{xz}_{xz}$ (Nt/m) | $S^{yz}_{yz}$ (Nt/m) | COMPARISON with | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | SIMPLY SUPPORTED $M_{ns} \neq 0$. along the boundaries | 1.0 | 0.3 | 1.0 | 0.7 | 218.4 | 218.4 | BETTESS —+— [21] | SANDERS ——— [97] |
| 2. | SIMPLY SUPPORTED $M_{ns} = 0$. along the boundaries | 1.0 | 0.3 | 1.0 | 0.7 | 218.4 | 218.4 | BETTESS —+— [21] | SANDERS ——— [97] |

FIG. 12.21.

0.966 m
A
C
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7 $10^{-3}$ m
W
0.259 m
B
C
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7 $10^{-3}$ m
W

FIG. 12.22.

FIG. 12.23.

Q
2.6 $10^{-1}$ Nt/m
2.4
2.2
2.04
2.08
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
$Q_y$
$Q_x$
$Q_\eta$
C
B
A
0.259 m
0.966 m

FIG. 12.24.

CASE 2.

FIG. 12.25.

# CASE 2.

FIG. 12.26.

2.2 10⁻¹ Nt/m
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
2.15
2.08
$Q_n$
$Q_Y$
$Q_x$
C
B
A
0.259m
0.966m

FIG. 12.27.

UNDER

UNIFORMLY DISTRIBUTED LOAD 1Nt/m²

| KEY | |
|---|---|
| PDS15 | ○ |
| PDS21 | △ |
| PMX12 | × |
| PMX24 | -·-·- |
| PDS24 | □ |
| PDS30 | ------- or + |
| PRO18 | ▽ |

| CASE | BOUNDARY CONDITIONS | $\Theta$ (dg.) | $D^{xx}_{xx}$ (Nt-m/m) | $D^{xx}_{yy}=D^{yy}_{xx}$ (Nt-m/m) | $D^{yy}_{yy}$ (Nt-m/m) | $D^{xy}_{xy}$ (Nt-m/m) | $S^{xz}_{xz}$ (Nt/m) | $S^{yz}_{yz}$ (Nt/m) | COMPARISON with —+— |
|---|---|---|---|---|---|---|---|---|---|
| 1. | CLAMPED $M_{ns}=0$. along the boundaries | 15° | $1.6527\ 10^4$ | $0.5289\ 10^4$ | $1.6527\ 10^4$ | $1.1238\ 10^4$ | $0.8979\ 10^5$ | $0.8979\ 10^5$ | KENNEDY [66] |
| 2. | >> | 30° | >> | >> | >> | >> | >> | >> | >> |
| 3. | >> | 45° | >> | >> | >> | >> | >> | >> | >> |
| 4. | >> | 60° | >> | >> | >> | >> | >> | >> | >> |

FIG. 12.28.

# CASE 1.

FIG. 12.29.

# CASE 1.

FIG. 12.30.

0.880 m
A
C
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9 $10^{-6}$ m
W
0.508 m
B
C
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8 $10^{-6}$ m
W

FIG. 12.31.

0.880m
-3.0
-2.0
-1.0
0.0
1.0
2.0
3.0 $10^{-2}$ Nt-m/m
C
A
$M_{xx}$
0.508m
-3.0
-2.0
-1.0
0.0
1.0
2.0
3.0 $10^{-2}$ Nt-m/m
C
B
$M_{YY}$
$M_m$
-4.0 $10^{-2}$ Nt-m/m
-3.0
-2.0
-1.0
0.0
A
B
1.016m

FIG. 12.32.

0.939 m
A
C
0.0
0.1
0.2
0.3
0.4
0.5
0.6 $10^{-6}$ m
W
0.389 m
B
C
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7 $10^{-6}$ m
W

FIG. 12.33.

0.939 m
A
C
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
2.0 $10^{-2}$ Nt m/m
$M_{xx}$
0.389 m
B
C
-2.0
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
2.0
2.5 $10^{-2}$ Nt m/m
$M_{yy}$
$M_n$
-2.5 $10^{-2}$ Nt m/m
-2.0
-1.5
-1.0
-0.5
0.0
A
B
1.016 m

FIG. 12.34.

0.981 m
A
C
0.0
0.05
0.10
0.15
0.20
0.25
0.30
0.35 $10^{-6}$ m
W

0.263 m
B
C
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35 $10^{-6}$ m
W

FIG. 12.35.

0.981 m
-0.75
-0.50
-0.25
0.00
0.25
0.50
0.75
1.00
1.25 $10^{-2}$ Nt m/m
$M_{xx}$
A
C
0.263 m
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
$M_{yy}$
B
C
$M_n$
-2.0 $10^{-2}$ Nt m/m
-1.5
-1.0
-0.5
0.0
A
B
1.016 m

FIG. 12.36.

| | CASE 1. ($\theta = 15°$) | | | CASE 2. ($\theta = 30°$) | | | CASE 3. ($\theta = 45°$) | | | CASE 4. ($\theta = 60°$) | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $W_c$ | $M_{max}$ | $M_{edge}$ | $W_c$ | $M_{max}$ | $M_{edge}$ | $W_c$ | $M_{max}$ | $M_{edge}$ | $W_c$ | $M_{max}$ | $M_{edge}$ |
| PDS15 | 0.841 | 2.22 | -2.35 | 0.674 | 1.74 | -2.30 | 0.448 | 1.25 | -1.21 | 0.218 | 0.703 | -0.721 |
| PDS21 | 0.899 | 2.33 | -1.85 | 0.707 | 2.06 | -1.90 | 0.442 | 1.37 | -1.41 | 0.188 | 0.643 | -1.25 |
| PMX12 | 0.851 | 2.86 | -3.99 | 0.673 | 2.43 | -3.10 | 0.445 | 1.76 | -1.99 | 0.221 | 0.964 | -0.940 |
| PMX24 | 0.888 | 2.65 | -4.30 | 0.732 | 2.24 | -3.54 | 0.509 | 1.64 | -2.53 | 0.270 | 0.934 | -1.35 |
| PDS24 | 0.848 | 2.56 | -1.78 | 0.657 | 2.44 | -1.91 | 0.411 | 2.15 | -1.71 | 0.173 | 1.120 | -1.66 |
| PDS30 | 0.729 | 2.66 | -2.40 | 0.588 | 2.22 | -2.37 | 0.398 | 1.52 | -2.35 | 0.204 | 0.881 | -1.95 |
| PRO18 | 0.885 | 2.65 | -2.35 | 0.729 | 2.18 | -1.90 | 0.507 | 1.54 | -1.47 | 0.269 | 0.828 | -0.950 |
| KENNEDY | 0.912 | 2.53 | -4.36 | 0.748 | 2.19 | -3.66 | 0.524 | 1.63 | -3.59 | 0.282 | 0.956 | -2.15 |
| | $10^{-6}$ m | $10^{-2}$ Nm/m | $10^{-2}$ Nm/m | $10^{-6}$ m | $10^{-2}$ Nm/m | $10^{-2}$ Nm/m | $10^{-6}$ m | $10^{-2}$ Nm/m | $10^{-2}$ Nm/m | $10^{-6}$ m | $10^{-2}$ Nm/m | $10^{-2}$ Nm/m |

FIG. 12.37.

# SQUARE PLATE WITH FOUR CORNER SUPPORTS

UNDER

1 Nt. CENTRAL LOAD

| KEY | |
|---|---|
| PDS15 | ○ |
| PDS21 with stif-corners | △ |
| PMX12 | × |
| PMX24 | ---- |
| PDS24 | □ |
| PDS30 with stif-corners | + |
| PRO18 with stif-corners | ▽ |
| PDS21 | ▲ |
| PDS30 | ◆ |
| PRO18 | ▼ |

| CASE | DESCRIPTION OF THE PLATE | $a$ (m) | $D^{xx}_{xx}$ (Nt-m) | $D^{xx}_{YY} = D^{YY}_{xx}$ (Nt-m) | $D^{YY}_{YY}$ (Nt-m) | $D^{xY}_{xY}$ (Nt-m) | $S^{xz}_{xz}$ (Nt/m) | $S^{Yz}_{Yz}$ (Nt/m) | COMPARISON WITH — + — |
|---|---|---|---|---|---|---|---|---|---|
| 1. | PLYWOOD PLATE (BETTES PLATE 1.) $\frac{c}{t} = 15.7$ | 0.5 | 9700 | 2600 | 6800 | 7100 | 59000 | 59000 | BETTESS EXPERIMENT [21] |
| 2. | PLYWOOD PLATE (BETTES PLATE 3.) $\frac{c}{t} = 7.5$ | 0.5 | 2540 | 690 | 1800 | 1900 | 32000 | 32000 | » |
| 3. | FIBREGLASS PLATE (BETTES PLATE 4.) | 0.5 | $0.29\ 10^4$ | $0.86\ 10^3$ | $0.29\ 10^4$ | $0.20\ 10^4$ | $2.5\ 10^5$ | $2.5\ 10^5$ | » |
| 4. | FIBREGLASS PLETE (BETTES PLATE 5.) | 0.5 | $0.15\ 10^4$ | $0.46\ 10^3$ | $0.15\ 10^4$ | $0.11\ 10^4$ | $0.18\ 10^6$ | $0.18\ 10^6$ | » |
| 5. | FIBREGLASS PLATE (BETTES PLATE 6.) | 0.5 | $0.59\ 10^3$ | $0.18\ 10^3$ | $0.59\ 10^3$ | $0.41\ 10^3$ | $1.0\ 10^5$ | $1.0\ 10^5$ | » |
| 6. | ALUMINIUM PLATE (BETTES PLATE 7.) $\frac{c}{t} = 22.6$ | 1.0 | $0.29\ 10^4$ | $0.72\ 10^3$ | $0.29\ 10^4$ | $0.17\ 10^4$ | $0.37\ 10^6$ | $0.37\ 10^6$ | » |
| 7. | HARDBOARD PLATE (BETTES PLATE 8.) $\frac{c}{t} = 15.3$ | 1.0 | $0.26\ 10^5$ | $0.79\ 10^4$ | $0.26\ 10^5$ | $0.92\ 10^4$ | $1.4\ 10^5$ | $1.4\ 10^5$ | » |

FIG. 12.38.

0.25m
A
C
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6 10^-5 m
W
0.25m
F
E
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6 10^-5 m
W
J
G
0.0
0.2
0.4
0.6
0.8
1.0
1.2 10^-5 m
W
D
B
0.0
0.2
0.4
0.6
0.8
1.0
1.2 10^-5 m
W

FIG. 12.39.

# CASE 2.

FIG. 12.40.

0.25m
A
C
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8 10^-5 m
W
0.25m
F
E
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7 10^-5 m
W
J
G
0.0
0.1
0.2
0.3
0.4
0.5
0.6 10^-5 m
W
D
B
0.0
0.1
0.2
0.3
0.4
0.5
0.6 10^-5 m
W

FIG. 12.41.

CASE 4.

FIG. 12.42.

# CASE 5.

FIG. 12.43.

0.5m
A
C
0.0
0.5
1.0
1.5
2.0 10^-5 m
W
0.5m
E
0.0
0.5
1.0
1.5 10^-5 m
W
J
G
0.0
0.5
1.0
1.5 10^-5 m
W
D
B
0.0
0.5
1.0
1.5 10^-
W

FIG. 12.44.

FIG. 12.45.

3.0 Nt/m
2.0
1.0
0.0
$Q_Y$
2.0 Nt/m
1.0
A
D
C
B
0.5m
$Q_Y$
0.0
1.0
2.0 Nt/m
1.0
2.0 Nt/m
0.5m
$Q_Y$

FIG. 12.46.

CASE F.

FIG. 12.47.

D
A
B
C
0.5m
$M_{xx}$
$M_{xx}$
0.3 Nt-m/m
0.2
0.1
0.0
0.0
0.1
0.2
0.3 Nt-m/m
0.1
0.2
0.3 Nt-m/m
0.5m
$M_{xx}$

FIG. 12.48.

$Q_Y$
Nt/m 2.0
1.0
$Q_Y$
2.0 Nt/m
1.0
0.0
A
D
0.5m
C
$Q_Y$
B
0.0
1.0
2.0 Nt/m
1.0
2.0 Nt/m
0.5m
$Q_Y$

FIG. 12.49.

$D^{xx}_{xx} = 0.254\ 10^4\ Nt_m$ , $D^{YY}_{xx} = D^{xx}_{YY} = 0.690\ 10^3\ Nt_m$

$D^{YY}_{YY} = 0.180\ 10^4\ Nt_m$ , $D^{xY}_{xY} = 0.190\ 10^4\ Nt_m$

$D^{xz}_{xz} = D^{Yz}_{Yz} = 0.320\ 10^5\ Nt/_m$

| KEY | |
|---|---|
| PDS15 | ∘ |
| PDS21 | ▲ |
| PMX12 | × |
| PMX24 | -·-·- |
| PDS24 | □ |
| PDS30 | + |
| PRO18 | ▼ |

| CASE | $L_1$ (m) | $L_2$ (m) | $\phi$ (dg.) | BOUNDARY CONDITION OF THE SIDE AB | BOUNDARY CONDITION OF THE SIDE BC | BOUNDARY CONDITION OF THE SIDE CA | 1. Nt. CON. LOAT AT THE POINT | COMPARISON WITH + |
|---|---|---|---|---|---|---|---|---|
| 1. | 1. | 1. | 90° | simply supported | simply supported | simply supported | G | COLLINS [33] EXPERIMENT |
| 2. | 1. | 1. | 90° | >> | simply supported | free | E | >> |
| 3. | 1. | 1. | 90° | >> | free | simply supported | G | >> |
| 4. | 0.75 | 0.75 | 60° | >> | simply supported | simply supported | G | >> |
| 5. | 0.75 | 0.75 | 60° | >> | free | simply supported | D | >> |
| 6. | 0.75 | 0.75 | 60° | >> | free | simply supported | G | >> |

FIG. 12.50.

M4
M1
0.0
0.4
0.8 10^-5 m
W
M5
G
0.0
0.4
0.8
1.2
1.6
2.0 10^-5 m
W
F
M2
0.0
0.4
0.8 10^-5 m
W
M6
M3
0.0
0.4
0.8 10^-5 m
W
C
E
D
1m
M1
G
M2
M3
90°
A
M6
F
M5
M4
B
1m

FIG. 12.51.

A
C
0.
2.
4.
6.
8. 10^-5 m
W
M2
M1
0.
2.
4.
6. 10^-5 m
W
M4
M3
0.
2.
4. 10^-5 m
W
ΔY
C
M1
M3
D
E
1m
90°
A
M2
M4
F
B
x
1m

FIG. 12.52.

B
D
0.
1.
2.
3.
4. 10^-5 m
W
M2
M1
0.
1.
2.
3.
4. 10^-5 m
W
M3
G
0.
1.
2.
3.
4.10^-5 m
W
Y
C
1m
E
D
M1
G
M4
90°
A
F
M3
M2
B
X
1m
F
M4
0.
1.
2. 10^-5 m
W

FIG. 12.53.

# CASE 4.

FIG. 12.54.

B
D
0.
1.
2.
3.
4.
5.
6. 10^-5 m
W
M2
M1
0.
1.
2.
3.
4. 10^-5 m
W
M3
G
0.
1.
2.
3.
4. 10^-5 m
W
F
M4
0.
1. 10^-5 m
W
Y
C
0.75 m
E
D
M1
G
M4
60°
F
M3
M2
B
X
0.75 m

FIG. 12.55.

B
D
0.0
0.4
0.8
1.2
1.6
2.0
2.4 10⁻⁵ m
W
M₂
M₁
0.0
0.4
0.8
1.2
1.6
2.0 10⁻⁵ m
W
M₃
G
0.0
0.4
0.8
1.2
1.6
2.0
2.4 10⁻⁵ m
W
F
M₄
0.0
0.4
0.8
1.2
1.6 10⁻⁵ m
W
Y
C
0.75 m
E
D
M₁
G
M₄
60°
A
F
M₃
M₂
B
X
0.75 m

FIG. 12.56.

## 13. RESULTS FROM THE SANDWICH DOME MODELS

As presented in the previous chapter (see section 12.4), four sandwich dome models were developed.

They were tested with a series of sandwich dome problems and the obtained results were compared with the experimental results obtained for the same problems by previous researchers as well as by the author himself.

These analysed problems are as follows:

| | | | |
|---|---|---|---|
| (a) | Tetrahedral dome | (fig. 13.6 ÷ 13.17) | [85] |
| (b) | Square Pyramid | (fig. 13.18 ÷ 13.32) | [21] |
| (c) | 16-faced dome | (fig. 13.33 ÷ 13.33) | [85] |
| (d) | 24-faced dome | (fig. 13.59 ÷ 13.87) | (see chapter 9) |
| (e) | 36-faced dome | (fig. 13.88 ÷ 13.132 ) | (see chapter 9) |

The elastic properties of the sandwich panels from which the above structures were constructed are listed in fig. 13.3.

The elastic properties of the materials, from which the sandwich panels themselves are composed, are outlined in fig. 13.2 (see also references [21, 85]).

## 13.1 Tetrahedral Dome (figs. 13.6 ÷ 13.17) [85]

The shape, support conditions and dimensions of the structure are shown in figures 13.6 and 13.7. (For a more detailed presentation see references [21, 85] ).

This dome was analysed for one loading case. The structure was loaded with 1Nt vertical concentrated load applied at the centroid of face ABC as shown in figures 13.6 and 13.7. Due to symmetry one half of the structure was modelled.

The displacements normal to the face are plotted against experimental results [85], for the loaded panel. (figs. 13.13, 13.13).

It is evident that the presence of a free edge at the loaded panel increases the deformations due to shear, especially as the sandwich panels forming the tetrahedral dome have low shear rigidity. (see fig. 13.3)

It is for this reason that the results obtained from the higher shear variation model, DDS33, appear more accurate when compared with the experimental results.

The horizontal displacements obtained from the various models are plotted in figures 13.8 ÷ 13.11, for half of the whole structure.

The moments and shear forces distribution of the loaded panel ABC, are shown in figures 13.14 ÷ 13.17. The results obtained from the various models when compared with each other appear to agree fairly well in their respective distribution form.

For the mixed model DMX36, the moment $M_{nn}$, where n is the vector normal to the relevant joint line (see fig. 8.2), was fixed to zero all along the various joint lines of the structure, with the result that the joints acted like hinges.

This constraint of the moment $M_{nn}$ was justified by the way these joints were constructed. (see chapter 10 and reference [85]).

## 13.2 Square Pyramid (figs. 13.18 ÷ 13.32) [21]

The shape, support conditions and dimensions of the structure are shown in figures 13.18 and 13.19. (For a more detailed presentation see reference [21]).

This structure was analysed when loaded with 1 Nt vertical concentrated load at all four face centroids, as shown in figures 13.18 and 13.19. Due to symmetry only one half of the structure was modelled.

The horizontal displacements for half of the whole structure and the in-plane stresses at specific points of the loaded face EBC are shown in figures 13.20 ÷ 13.23.

The agreement between the results obtained from the various models and the experiment with regard to the in-plane stresses is very good. The models

with higher order variation of the in-plane displacements (DDS33, DMX36, DRO30) show increased values of horizontal displacements, as expected, when compared with the linear variation model with regard to the in-plane displacements. (DDS21).

Displacements normal to the face are plotted against experimental results [21], for the loaded panel EBC. (see figs. 13.24, 13.27). The agreement between the experimental results and the result obtained from the various models is good, especially for the models DDS21 and DMX36.

The free edge does not influence the behaviour of the loaded panel in the same way as outlined in the previous section.

This is due to the high shear rigidity of the aluminium sandwich panels from which the square pyramid is composed. (see fig. 13.3).

The distribution of moments for the loaded panel EBC is shown in figures 13.28 ÷ 13.32.

The results obtained from the various models, when compared with each other, appear to agree very well in their respective distribution form. Moreover, the agreement with the experimental results is also very good.

For the mixed model DMX36, the moment $M_{nn}$, where n is the vector normal to the relevant joint line (see fig. 8.2), was fixed to zero all along the various joint lines so that the joints acted like hinges. This constraint of the moment $M_{nn}$ was justified by the way these joints were constructed. (see chapter 10 and reference [85]).

## 13.3 16-faced domes (figs. 13.33 ÷ 13.58) [85]

The shape, support conditions and dimensions of the structure are shown in figures 13.33, 13.34. (For a more detailed presentation see reference [85]).

This structure was analysed for two loading cases. Due to symmetry only $^1/_8$ of the structure was modelled.

The first loading case consisted of 1Nt vertical concentrated load acting at all eight upper panel centroids. (see figs. 13.35 ÷ 13.46).

For the second case the load was 1 Nt vertical concentrated load applied at all eight bottom panel centroids. (see figs. 13.47 ÷ 13.58).

The experimental results are presented in reference [85]. For each loading case, the horizontal and vertical displacements for $^1/_8$ of the whole structure as well as the normal displacements of the loaded face, obtained by employing the four individual dome models, are plotted against the experimental results. [85]. (see figs. 13.39 ÷ 13.42, 13.51 ÷ 13.54)

The agreement between the results obtained from the various models and the experimental results varies from quite good for the first model (DDS21)

to very good for the remaining models (DDS33, DMX36, DRO30). (see figs. 13.39 ÷ 13.42, 13.51 ÷ 13.54)

The reason for this is the linear variation of the in-plane displacements for the first model (DDS21) in comparison with the higher order variation for the same parameters for the other models:

For the second loading case the presence of the free edge and the low shear rigidity of the sandwich panels forming the structure, resulted in the same behaviour, with regard to the higher order shear model DDS33, as the one observed in the tetrahedral dome (see section 13.1), in that the latter model showed increased flexibility.

For the mixed model, DMX36, the moment $M_{nn}$, where n is the vector normal to the relevant joint line (see fig. 8.2), was fixed to zero all along the various joint lines of the structure, with the result that the joints acted like hinges.

This constraint of the moment $M_{nn}$ was justified by the way these joints were constucted. (see chapter 10 and reference [85]).

## 13.4 24-faced dome (figs. 13.59 ÷ 13.87)

The shape, support conditions and dimensions of the structure are shown in figs. 13.59, 13.60.

Details relevant to the construction and testing of this dome are presented in chapter 9.

This structure was analysed for two loading cases. Due to symmetry, only $^1/_{12}$ of the structure was modelled.

The first loading case consisted of 1 Nt vertical concentrated load acting at all twelve upper panel centroids. (see figs. 13.62 ÷ 13.74)

In the second loading case the vertical concentrated load of 1 Nt was applied at all twelve bottom panel centroids. (see figs. 13.75 ÷ 13.87).

The experimental results are presented in figures 13.62 and 13.75 for each loading case respectively.

The horizontal and vertical displacements for $^1/_{12}$ of the whole structure as well as the normal displacements of the loaded face were obtained for both loading cases by employing all four individual dome models.

These results are plotted against experimental results obtained by testing the structure as presented in chapter 9.

For both loading cases, the higher order variation of the in plane displacement models DDS33, DMX36 and DRO30, yield better accuracy than the linear variation of the in plane displacement model DDS21.

The accuracy obtained is satisfactory. All the models agree in their deformation pattern with the experimental results. The higher order shear variation model DDS33 agrees well with the experimental results especially for the second loading case, where the presence of the free edge at the loaded panel influences the results in the same way as explained previously. (see sections 13.1, 13.3).

For the mixed model DMX36, the moment $M_{nn}$, where n is the vector normal to the joint-line AB (see fig. 8.2) was constrained to zero all along the line for both loading cases. This was enforced in order to simulate the joint action as outlined in chapter 10.

Some further conclusions drawn from the analysis of the 24-faced dome are as follows:

First, with regard to the joint-lines, and, in particular those unaffected by the joint action described in chapter 10, the covering steel plates (see chapters 9. 10) have a stiffening effect on the structure in the longitudinal direction of the joint. As a result, a more detailed analysis must be carried out by modelling the area adjacent to the joint lines with elements of a higher stiffness.

The second conclusion refers to the support conditions of the overall structure. In the analysis it was assumed that the structure is supported at points with all three displacements u, v, w as well as the moments $M_z$, and $M_x$ (where x, and z the gloval axes, see figs. 13.59, 13.60) fixed to zero.

The supports at the actual structure were constructed in such a way that the above assumption is fulfilled (see chapter 9).

However, due to the considerable stiffness of those parts of the supports which are joined to the sandwich panels the assumption of points supports is not valid.

Thus for a more correct representation of the supports' behaviour a line support instead of a point support is recommended.

In addition, the elements adjacent to this line support must be considered with an increased stiffness.

## 13.5 36-faced dome (see figs. 13.88 ÷13.132 )

The shape, support conditions and dimensions of the structure are shown in figures 13.88 ÷ 13.90

Details relevant to the construction and testing of this dome are presented in chapter 9. This structure was analysed for three loading cases. Due to symmetry only $^1/_{12}$ of the whole structure was modelled.

The first loading case consisted of 1 Nt vertical concentrated load acting at all twelve upper panel centroids. (see figs. 13.91 ÷ 13.104 )

In the second loading case a vertical concentrated load of 1 Nt was applied at all twelve bottom panel centroids. (see figs. 13.105 ÷ 13.118 )

For the third loading case the same load was applied at all twelve centroids of the panels forming the dormer sections. (see figs. 13.119 ÷ 13.132 )

The horizontal and vertical displacements for $^1/_{12}$ of the whole structure as well as the normal displacements of the loaded face, were obtained for each individual loading case, employing the four dome models.

These results are plotted against the experimental results obtained by testing the structure as presented in chapter 9. The same conclusions as stated in the previous section are again valid for this dome.

The models DDS33, DMX36 and DRO30 are more accurate than the model DDS21. All the models agree in their deformation pattern with the experimental results.

The higher shear order element, DDS33, agrees well with the experimental results especially for the third loading case where the presence of the free edges on the dormer section panels influences the results in the same way as discussed in sections 13.1 and 13.3.

For the mixed model, DMX36, the moment $M_{nn}$, where n is a vector normal to a line (see fig. 8.2), was constrained to zero all along the joint-lines, AB and BD, for every loading case.

This was enforced in order to simulate the joint action as explained in chapter 10.

An investigation of the distribution of the normal moment, $M_{nn}$, all along the joint lines for the different models, was carried out from the obtained results.

It is evident from the cases, that when the load is applied on a panel with a joint in the form of a valley adjacent to it then this joint will experience normal moments $M_{nn}$, of a considerable magnitude exceeding the limits found in the analysis of chapter 10 with regard to the joint action.

Further investigation and adaptation of the various computer programs, developed in the present work, which will incorporate the function of the joints, is therefore recommended. The mixed model, DMX36, has the most potential in that respect due to the presence of moments as nodal degrees of freedom.

A solution to the problem would be to introduce elastic constraints so that the normal moments could be transferred at a joint line between adjacent panels only when its magnitude does not exceed a certain limit. This limit can be decided on by a semi-analytical method, (chapter 10) or by purely experimental investigations.

## 14. CONCLUSIONS

Four finite element models were developed for the analysis of the polyhedral dome sandwich structures.

After a comparative study between the results obtained from these four dome models and the experimental results of the five specific analysed domes (see previous sections) it is evident that the accuracy obtained from each model varies from fair to very good depending on the nature of the problem.

A convergency study carried out for the square pyramid (fig. 13.5) indicates that the various models converge in a monotonic manner.

All the examined structures were analysed, using a subdivision of 100 triangular elements for each panel of the polyhedron.

Each individual model carries particular features which makes it more accurate for certain cases. These are as follows:

The first model,DDS21, has the advantage of possessing the lowest number of degrees of freedom.

For problems where the bending action is predominant, when compared with the membrane action, this model yields accurate results.

For structures where the membrane action becomes predominant, as in the case of the last three polyhedra, the finite element models, with a variation of the in plane displacements of an order higher than linear, yield better results. This is due to the slow convergency of the linear variation of the in-plane displacement finite element which is part of the first dome model DDS21. As a result the model becomes relatively stiffer and produces less accurate results.

As stated for the examined sandwich plate bending, as well as the sandwich dome problems, structures with free edges experience high deformations of the free edge when loaded at the relevant panel.

This is particularly valid for structures composed of sandwich panels with core of low shear rigidity. In this case a representation of the shear deformation with a variation higher than linear yields better results.

As discussed previously, the joint action could be of considerable significance for the dome structures. Further research must be carried out either analytically or experimentally in this direction as suggested.

This can be achieved numerically by modifying the models developed in the present work so that they can accommodate the desired joint action. The mixed formulation is more succesful in this respect.

The boundary conditions is another important factor and has to be treated with great care as it has a very decisive effect on the results.

Point supports, as mentioned for the sandwich plate bending and sandwich dome problems, present difficulties which can lead to incorrect physical representation of the structure. They can also lead to numerical inadequacies.

Depending on the degree of accuracy desired and the funds available a more detailed study can be performed including all these factors. The developed programs are able to accommodate such options.

For a complete understanding of the behaviour of the structures analysed in the present work the effects of geometric nonlinearities (large deflections) and the time dependent phenomena (creep) must be considered.

These structures when loaded for a long period become subject to considerable deformation which is largely due to the creep effect. This is evident from the experiments performed by Parton as well as from those carried out by the author. [85]

The first model DDS21 has the lowest number of degrees of freedom and consequently is the most efficient for use in any future analysis which includes the creep effects.

This is due to the iteration procedure which must be employed in order to solve the time dependent equations.

The author investigated these aspects but more research in this direction is necessary.

| REFERENCE SYMBOL<br>DEGREES OF FREEDOM | SANDWICH – DOME MODELS | STRESSES |
|---|---|---|
| **DDS21**<br>⊙ $u, v, w, w_x, w_y, \phi_x, \phi_y$<br>DISPLACEMENT MODEL | | $M_{xx_c}, M_{yy_c}, M_{xy_c}$<br>$Q_{x_c}, Q_{y_c}$<br>$N_{xx_c}, N_{yy_c}, N_{xy_c}$<br>} at the centroid |
| **DDS33**<br>⊙ $u, v, w, w_x, w_y, \phi_x, \phi_y$<br>× $\phi_x, \phi_y, u_{SH}, v_{BS}$<br>DISPLACEMENT MODEL | | $M_{xx_c}, M_{yy_c}, M_{xy_c}$<br>$Q_{x_c}, Q_{y_c}$<br>$N_{xx_c}, N_{yy_c}, N_{xy_c}$<br>} at the centroid |
| **DMX36**<br>⊙ $u, v, w, M_{xx}, M_{yy}, M_{xy}$<br>MIXED MODEL | | $Q_{x_1}, Q_{y_1}$ at 1st node<br>$Q_{x_2}, Q_{y_2}$ at 2nd node<br>$N_{xx_c}, N_{yy_c}, N_{xy_c}$ at the centroid |
| **DRO30**<br>⊙ $u, v, w, \theta_x, \theta_y$<br>DISPLACEMENT MODEL | | $M_{xx_c}, M_{yy_c}, M_{xy_c}$<br>$Q_{x_c}, Q_{y_c}$<br>$N_{xx_c}, N_{yy_c}, N_{xy_c}$<br>} at the centroid |

FIG. 13.1. SANDWICH DOME MODELS

| MATERIALS (REFERENCE [21]) | | | | thickness measured (m) | ELASTIC PROPERTIES (Nt/m²) | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | $C_{xx}^{xx}$ / $10^{10}$ | $C_{yy}^{yy}$ / $10^{10}$ | $C_{yy}^{xx}$ / $10^{10}$ | $C_{xy}^{xy}$ / $10^{10}$ | $C_{yz}^{yz}$ / $10^{7}$ | $C_{xz}^{xz}$ / $10^{7}$ |
| 1 | PLYWOOD | FACE | ORTHOTROPIC | 0.00166 | 1.2 ÷ 1.4 | 0.78 ÷ 0.91 | 0.32 ÷ 0.38 | 0.55 | | |
| 2 | ALUMINIUM | FACE | ISOTROPIC | 0.000486 | 6.8 | 6.8 | 2.0 | 4.8 | | |
| 3 | FIBREGLASS | FACE | >> | VARIED | | | | | | |
| 4 | HARDBOARD (ICI) | FACE | >> | 0.00408 | 0.56 | 0.56 | 0.168 | 0.392 | | |
| 5 | HARDBOARD | FACE | >> | 0.00340 | 0.42 | 0.42 | 0.126 | 0.294 | | |
| 6 | EXPANDED POLYURETHANE | CORE | >> | 0.0260 | | | | | 0.2 | 0.2 |
| 7 | EXPANDED POLYURETHANE | CORE | >> | 0.0190 | | | | | 0.2 | 0.2 |
| 8 | EXPANDED POLYURETHANE | CORE | >> | 0.0125 | | | | | 0.2 | 0.2 |
| 9 | EXP. POLYURETHANE BONDED TO THE HARDBOARD | CORE | >> | 0.0260 | | | | | 0.2 | 0.2 |
| 10 | EXPANDED POLYVINYLCHLORIDE | CORE | >> | 0.0110 | | | | | 1.0 | 1.0 |
| 11 | EXPANDED POLYURETHANE | CORE | >> | 0.0190 | | | | | 0.14 | 0.14 |

FIG. 13.2. ELASTIC PROPERTIES

| | POLYHEDRAL DOME CONSTRUCTED BY SANDWICH PANELS | COMPOSED FROM MATERIALS | ELASTIC PROPERTIES OF SANDWICH PANELS (REFERENCE [21]) | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | $D^{xx}_{xx}$ | $D^{xx}_{yy}=D^{yy}_{xx}$ | $D^{yy}_{yy}$ | $D^{xy}_{xy}$ | $S^{xz}_{xz}$ | $S^{yz}_{yz}$ | $E^{xx}_{xx}$ | $E^{xx}_{yy}$ | $E^{yy}_{xx}$ | $E^{yy}_{yy}$ | $E^{xy}_{xy}$ |
| | | | $10^4$ Nt/m | $10^3$ Nt/m | $10^4$ Nt/m | $10^4$ Nt/m | $10^5$ Nt/m | $10^5$ Nt/m | $10^8$ Nt/m | $10^7$ Nt/m | $10^7$ Nt/m | $10^8$ Nt/m | $10^8$ Nt/m |
| 1 | TETRAHEDRAL DOME PLATES (1) $h=0.031$ m $c/\ell=15.66$ | 1 and 6 | 0.97 | 2.6 | 0.68 | 0.71 | 0.59 | 0.59 | 0.46 | 1.3 | 1.3 | 0.30 | 0.37 |
| 2 | TETRAHEDRAL DOME PLATES (2) $h=0.023$ m $c/\ell=11.44$ | 1 and 7 | 0.54 | 1.5 | 0.38 | 0.39 | 0.45 | 0.45 | 0.46 | 1.3 | 1.3 | 0.30 | 0.37 |
| 3 | TETRAHEDRAL DOME PLATES (3) $h=0.016$ m $c/\ell=7.5$ | 1 and 8 | 0.254 | 0.69 | 0.18 | 0.19 | 0.32 | 0.32 | 0.46 | 1.3 | 1.3 | 0.30 | 0.37 |
| 4 | SQUARE PYRAMID $h=0.013$ m $c/\ell=22.63$ | 2 and 10 | 0.29 | 0.72 | 0.29 | 0.17 | 3.7 | 3.7 | 0.66 | 1.9 | 1.9 | 0.66 | 0.47 |
| 5 | 16 FACED DOME $h=0.016$ m $c/\ell=7.5$ | 1 and 8 | 0.19 | 0.52 | 0.135 | 0.14 | 0.32 | 0.32 | 0.35 | 0.98 | 0.98 | 0.225 | 0.27 |
| 6 | 24 FACED DOME $h=0.058$ m $c/\ell=12.75$ | 4 and 9 | 2.6 | 7.9 | 2.6 | 0.92 | 1.4 | 1.4 | 0.41 | 1.2 | 1.2 | 0.41 | 0.29 |
| 7 | 36 FACED DOME $h=0.058$ $c/\ell=12.75$ | 4 and 9 | 2.6 | 7.9 | 2.6 | 0.92 | 1.4 | 1.4 | 0.41 | 1.2 | 1.2 | 0.41 | 0.29 |
| 8 | HEXAGONAL DOME $h=0.032$ $c/\ell=7.03$ | 4 and 9 | 0.57 | 1.7 | 0.57 | 0.4 | 0.99 | 0.99 | 0.41 | 1.2 | 1.2 | 0.41 | 0.29 |

FIG. 13.3. ELASTIC PROPERTIES OF SANDWICH PANELS

# DOME ELEMENTS (POLYHEDRAL STUCTURE OF TWO PLATES

FIG. 13.4.

FIG. 13.5. CONVERGENCE TOWARDS SERIES SOLUTION

TETRAHEDRAL DOME (PLATES 3)

SCALE : $1cm = 10^{-1} m$

FIG. 13.6. GENERAL ARRANGEMENT

TETRAHEDRAL DOME (PLATES 3)

1Nt VERTICAL LOAD AT CENTROID ($M_6$) OF FACE ABC

FIG. 13.7. GENERAL ARRANGEMENT

1Nt VERTICAL LOAD AT CENTROID ($M_6$) OF FACE ABC

HORIZONTAL DISPLACEMENTS

SCALES:

$1cm = 10^{-1}m$ LENGTH

$1cm = 10^{-5}m$ DISPLACEMENTS

TETRAHEDRAL DOME (PLATES 3)

FIG. 13.8.

1Nt VERTICAL LOAD AT CENTROID ($M_6$) OF FACE ABC

HORIZONTAL DISPLACEMENTS

SCALES :

$1cm = 10^{-1} m$ LENGTH

$1cm = 10^{-5} m$ DISPLAC.

TETRAHEDRAL DOME (PLATES 3)

FIG. 13.9.

1 Nt VERTICAL LOAD AT CENTROID ($M_6$) OF FACE ABC

HORIZONTAL DISPLACEMENTS

SCALES:

$1cm = 10^{-1} m$ LENGTH

$1cm = 10^{-5} m$ DISPLAC.

TETRAHEDRAL DOME (PLATES 3)

FIG. 13.10.

1Nt VERTICAL LOAD AT CENTROID ($M_6$) OF FACE ABC

HORIZONTAL DISPLACEMENTS

SCALES:

$1cm = 10^{-1} m$ LENGTH

$1cm = 10^{-5} m$ DISPLAC.

TETRAHEDRAL DOME (PLATES 3)

FIG. 13.11.

1Nt VERTICAL LOAD AT CENTROID ($M_6$) OF FACE ABC

NORMAL DISPLACEMENTS OF LOADED FACE

SCALES :

1cm = 4cm LENGTH

1cm = $0.5\,10^{-5}$ m DISPL.

FIG. 13.12. TETRAHEDRAL DOME (PLATES 3)

1Nt VERTICAL LOAD AT CENTROID ($M_G$) OF FACE ABC

NORMAL DISPLACEMENTS OF LOADED FACE

SCALES:

1 cm = 4 cm LENGTH

1 cm = 0.5 $10^{-5}$ m DISPLAC.

FIG. 13.13. TETRAHEDRAL DOME (PLATES 3)

MOMENTS $M_{xx}$ OF LOADED FACE

TETRAHEDRAL DOME (PLATES 3)

FIG. 13.14.

1Nt VERTICAL LOAD AT CENTROID ($M_6$) OF FACE ABC

MOMENTS $M_{yy}$ OF LOADED FACE

SCALES:

1cm = 4cm LENGTH

1cm = $2 \cdot 10^{-2}$ Ntm/m MOMENTS

TETRAHEDRAL DOME (PLATES 3)

FIG. 13.15.

1 Nt VERTICAL LOAD AT CENTROID ($M_6$) OF FACE ABC

SHEAR FORCES $Q_Y$ OF LOADED FACE

TETRAHEDRAL DOME (PLATES 3)

FIG. 13.16.

1 Nt VERTICAL LOAD AT CENTROID ($M_6$) OF FACE ABC

SHEAR FORCES $Q_x$ OF LOADED FACE

TETRAHEDRAL DOME (PLATES 3)

FIG. 13.17

# SQUARE PYRAMID

GENERAL ARRANGEMENT

SCALE: $1cm = 10^{-1} m$

FIG. 13.18.

SQUARE PYRAMID
GENERAL ARRANGEMENT
SCALE:
1cm = 5 · 10⁻² m
KEY
DDS21
DDS33
DMX36
DRO30
EXPERIMENT (REF. [21])
Y
E
M13
M12
M11
M10
M9
M8
M7
M6
B
M5
M4
M3
M2
M1
C
X
0.5m
0.42m
0.42m

FIG. 13.19.

# SQUARE PYRAMID

1Nt VERTICAL LOAD AT ALL 4 FACE CENTROIDS

FIG. 13.20.

# SQUARE PYRAMID

1Nt VERTICAL LOAD AT ALL 4 FACE CENTROIDS

FIG. 13.21.

# SQUARE PYRAMID

1Nt VERTICAL LOAD AT ALL 4 FACE CENTROIDS

IN-PLANE STRESSES

FIG 13.22.

# SQUARE PYRAMID

1Nt VERTICAL LOAD AT ALL 4 FACE CENTROIDS

FIG. 13.23.

FIG. 13.24. SQUARE PYRAMID

1Nt VERTICAL LOAD AT ALL 4 FACE CENTROIDS

NORMAL DISPLACEMENTS OF LOADED FACE

SCALES: $1cm = 2.5\ 10^{-2}$ m LENGTH

$1cm = 0.5\ 10^{-6}$ m DISPLAC.

FIG. 13.25. SQUARE PYRAMID

1 Nt VERTICAL LOAD AT ALL 4 FACE CENTROIDS

NORMAL DISPLACEMENTS OF LOADED FACE

SCALES : $1cm = 2.5\ 10^{-2}\ m$ LENGTH

$1cm = 0.5\ 10^{-6}\ m$ DISPLAC.

FIG. 13.26. SQUARE PYRAMID

1Nt VERTICAL LOAD AT ALL 4 FACE CENTROIDS

NORMAL DISPLACEMENTS OF LOADED FACE

SCALES: 1cm = $5 \cdot 10^{-2}$ m LENGTH

1cm = $0.5\ 10^{-6}$ m DISPLAC.

FIG. 13.27. SQUARE PYRAMID

1Nt VERTICAL LOAD AT ALL 4 FACE CENTROIDS

NORMAL DISPLACEMENTS OF LOADED FACE

SCALES: $1cm = 5 \cdot 10^{-2} m$ LENGTH

$1cm = 0.5\ 10^{-6} m$ DISPLAC.

FIG. 13.28 SQUARE PYRAMID

1Nt VERTICAL LOAD AT ALL 4 FACE CENTROIDS

MOMENTS OF LOADED FACE

SCALES: 1cm = 5 $10^{-2}$m LENGTH

1cm = 2 $10^{-2}$ Ntm/m MOMENTS

FIG. 13.29 SQUARE PYRAMID

1Nt VERTICAL LOAD AT ALL 4 FACE CENTROIDS

MOMENTS OF LOADED FACE

SCALES : 1cm = 5 $10^{-2}$ m LENGTH

1cm = 2 $10^{-2}$ Ntm/m MOMENTS

FIG. 13.30. SQUARE PYRAMID

1Nt VERTICAL LOAD AT ALL 4 FACE CENTROIDS

MOMENTS OF LOADED FACE

SCALES $1cm = 5 \cdot 10^{-2} m$ LENGTH

$1cm = 2 \cdot 10^{-2} Ntm/m$ MOMENTS

FIG. 13.31. SQUARE PYRAMID

1 Nt VERTICAL LOAD AT ALL 4 FACE CENTROIDS

MOMENTS OF LOADED FACE

SCALES : $1\,cm = 5 \cdot 10^{-2}\,m$ LENGTH

$1\,cm = 2 \cdot 10^{-2}\,Nt\,m/m$ MOMENTS

FIG. 13.32. SQUARE PYRAMID

1Nt VERTICAL LOAD AT ALL 4 FACE CENTROIDS

MOMENTS OF LOADED FACE

SCALES: $1cm = 5 \cdot 10^{-2} m$ LENGTH

$1 cm = 2 \; 10^{-2} Ntm/m$ MOMENTS

# 16 FACED DOME

## GENERAL ARRANGEMENT

SCALE: 1m = 25m

FIG 13.33.

# 16 FACED DOME

FIG. 13.34. GENERAL ARRANGEMENT

FIG. 13.35. 16 FACED DOME

1 Nt VERTICAL LOAD AT ALL 8 UPPER PANEL CENTROIDS

HORIZONTAL DISPLACEMENTS

SCALES:

LENGTH $1cm = 10^{-1} m$

DISPLAC. $1cm = 10^{-6} m$

FIG. 13.36. 16 FACED DOME

1 Nt VERTICAL LOAD AT ALL 8 UPPER PANEL CENTROIDS

HORIZONTAL DISPLACEMENTS

SCALES:

LENGTH $1cm = 10^{-1} m$

DISPLAC. $1cm = 10^{-6} m$

FIG. 13.37. 16 FACED DOME

1 Nt VERTICAL LOAD AT ALL 8 UPPER PANEL CENTROIDS

HORIZONTAL DISPLACEMENTS

SCALES :

LENGTH $1cm = 10^{-1} m$

DISPLAC. $1cm = 10^{-6} m$

FIG. 13.38. 16 FACED DOME

1Nt VERTICAL LOAD AT ALL 8 UPPER PANEL CENTROIDS

HORIZONTAL DISPLACEMENTS

SCALES:

LENGTH $1cm = 10^{-1} m$

DISPLAC. $1cm = 10^{-6} m$

GLOBAL DISPLACEMENTS W, U

SCALES:

LENGTH $1cm = 10^{-1} m$

DISPL. $1cm = 2.5\ 10^{-6} m$

FIN. ELEMENT VERTICAL DEFLECTION $W_{G_1} = -9.20\ 10^{-6} m$

EXPERIMENTAL VERTICAL DEFLECTION $W_{G_1} = -13.66\ 10^{-6} m$

1Nt VERTICAL LOAD AT ALL B UPPER FACE CENTROIDS ($G_1$)

FIG. 13.39. 16 FACED DOME

GLOBAL DISPLACEMENTS W, U

SCALES:

LENGTH $1cm = 10^{-1} m$

DISPL. $1cm = 2.5\,10^{-6} m$

FIN. ELEMENT VERTICAL DEFLECTION $W_{G_1} = -12.56\,10^{-6} m$

EXPERIMENTAL VERTICAL DEFLECTION $W_{G_1} = -13.66\,10^{-6} m$

1Nt VERTICAL LOAD AT ALL 8 UPPER PANEL CENTROIDS ($G_1$)

FIG. 13.40. 16 FACED DOME

GLOBAL DISPLACEMENTS W, U

SCALES

LENGTH $1cm = 10^{-1} m$

DISPL. $1cm = 2.5\ 10^{-6} m$

FIN. ELEMENT VERTICAL DEFLECTION $W_{G_1} = -12.40\ 10^{-6} m$

EXPERIMENTAL VERTICAL DEFLECTION $W_{G_1} = -13.66\ 10^{-6} m$

1Nt VERTICAL LOAD AT ALL 8 UPPER PANEL CENTROIDS ($G_1$)

FIG. 13.41. 16 FACED DOME

GLOBAL DISPLACEMENTS W, U

SCALES:

LENGTH $1cm = 10^{-1} m$

DISPL. $1cm = 2.5\ 10^{-6} m$

FINITE ELEMENT VERTICAL DEFLECTION $W_{G_1} = -12.69\ 10^{-6} m$

EXPERIMENTAL VERTICAL DEFLECTION $W_{G_1} = -13.66\ 10^{-6} m$

1Nt VERTICAL LOAD AT ALL 8 UPPER PANEL CENTROIDS ($G_1$)

FIG. 13.42. 16 FACED DOME

1Nt VERTICAL LOAD AT ALL 8 UPPER PANEL CENTROIDS

NORMAL DISPLACEMENTS OF LOADED FACE

FIG. 13.43. 16 FACED DOME

1Nt VERTICAL LOAD AT ALL 8 UPPER PANEL CENTROIDS

NORMAL DISPLACEMENTS OF LOADED FACE

FIG. 13.44. 16 FACED DOME

1Nt VERTICAL LOAD AT ALL 8 UPPER PANEL CENTROIDS

NORMAL DISPLACEMENTS OF LOADED FACE

FIG. 13.45. 16 FACED DOME

1Nt VERTICAL LOAD AT ALL 8 UPPER PANEL CENTROIDS
NORMAL DISPLACEMENTS OF LOADED FACE

FIG. 13.46. 16 FACED DOME

FIG. 13.47. 16 FACED DOME

1Nt VERTICAL LOAD AT ALL 8 BOTTOM PANEL CENTROIDS

HORIZONTAL DISPLACEMENTS

SCALES :

LENGTH $1cm = 10^{-1} m$

DISPLAC. $1cm = 10^{-5} m$

FIG. 13.48. 16 FACED DOME

1Nt VERTICAL LOAD AT ALL 8 BOTTOM PANEL CENTROIDS

HORIZONTAL DISPLACEMENTS

SCALES:

LENGTH $1cm = 10^{-1} m$

DISPLAC. $1cm = 10^{-5} m$

FIG. 13.49. 16 FACED DOME

1Nt VERTICAL LOAD AT ALL 8 BOTTOM PANEL CENTROIDS

HORIZONTAL DISPLACEMENTS

SCALES:

LENGTH $1cm = 10^{-1} m$

DISPLAC. $1cm = 10^{-5} m$

FIG. 13.50. 16 FACED DOME

1Nt VERTICAL LOAD AT ALL 8 BOTTOM PANEL CENTROIDS

HORIZONTAL DISPLACEMENTS

SCALES:

LENGTH $1cm = 10^{-1} m$

DISPLAC. $1cm = 10^{-5} m$

GLOBAL DISPLACEMENTS W, U

SCALES:

LENGTH $1cm = 10^{-1} m$

DISPLAC. $1cm = 10^{-5} m$

FIN. ELEMENT VERTICAL DEFLECTION $W_B = -0.11\ 10^{-6} m$

EXPERIMENTAL VERTICAL DEFLECTION $W_B = -0.16\ 10^{-6} m$

FIN. ELEMENT VERTICAL DEFLECTION $W_{G_2} = -10.63\ 10^{-6} m$

EXPERIMENTAL VERTICAL DEFLECTION $W_{G_2} = -12.71\ 10^{-6} m$

1Nt VERTICAL LOAD AT ALL 8 BOTTOM PANEL CENTROIDS ($G_2$)

FIG. 13.51. 16 FACED DOME

GLOBAL DISPLACEMENTS W, U

SCALES:

LENGTH $1cm = 10^{-1} m$

DISPLAC. $1cm = 10^{-5} m$

FIN. ELEMENT VERTICAL DEFLECTION $W_B = -0.11\ 10^{-6} m$

EXPERIMENTAL VERTICAL DEFLECTION $W_B = -0.16\ 10^{-6} m$

FIN. ELEMENT VERTICAL DEFLECTION $W_{G_2} = -14.18\ 10^{-6} m$

EXPERIMENTAL VERTICAL DEFLECTION $W_{G_2} = -12.71\ 10^{-6} m$

1Nt VERTICAL LOAD AT ALL 8 BOTTOM PANEL CENTROIDS ($G_2$)

FIG. 1352. 16 FACED DOME

GLOBAL DISPLACEMENTS W, V

SCALES:

LENGTH $1cm = 10^{-1} m$

DISPLAC. $1cm = 10^{-5} m$

FIN. ELEMENT VERTICAL DEFLECTION $W_B = 0.0$ m

EXPERIMENTAL VERTICAL DEFLECTION $W_B = -0.16\ 10^{-6}$ m

FIN. ELEMENT VERTICAL DEFLECTION $W_{G_2} = -13.70\ 10^{-6}$ m

EXPERIMENTAL VERTICAL DEFLECTION $W_{G_2} = -12.71\ 10^{-6}$ m

1Nt VERTICAL LOAD AT ALL 8 BOTTOM PANEL CENTROIDS ($G_2$)

FIG. 13.53. 16 FACED DOME

GLOBAL DISPLACEMENTS W, U

SCALES:

LENGTH $1cm = 10^{-1} m$

DISPLAC. $1cm = 10^{-6} m$

FIN. ELEMENT VERTICAL DEFLECTION $W_B = 0.02\ 10^{-6} m$

EXPERIMENTAL VERTICAL DEFLECTION $W_B = -0.16\ 10^{-6} m$

FIN. ELEMEN VERTICAL DEFLECTION $W_{G2} = -13.24\ 10^{-6} m$

EXPERIMENTAL VERTICAL DEFLECTION $W_{G2} = -12.71\ 10^{-6} m$

1Nt VERTICAL LOAD AT ALL 8 BOTTOM PANEL CENTROIDS ($G_2$)

FIG. 13.54. 16 FACED DOME

1Nt VERTICAL LOAD AT ALL 8 BOTTOM PANEL CENTROID

NORMAL DISPLACEMENTS OF LOADED FACE

FIG. 13.55. 16 FACED DOME

1Nt VERTICAL LOAD AT ALL 8 BOTTOM PANEL CENTROIDS

NORMAL DISPLACEMENTS OF LOADED FACE

FIG. 13.56. 16 FACED DOME

1Nt VERTICAL LOAD AT ALL 8 BOTTOM PANEL CENTROIDS

NORMAL DISPLACEMENTS OF LOADED FACE

FIG. 13.57. 16 FACED DOME

1Nt VERTICAL LOAD AT ALL 8 BOTTOM PANEL CENTROIDS

NORMAL DISPLACEMENTS OF LOADED FACE

FIG. 13.58. 16 FACED DOME

# FIG. 13.59. 24. FACED DOME

GENERAL ARRANGEMENT

SCALE:
1cm = 0.5m LENGTH

FIG. 13.60. 24. FACED DOME

GENERAL ARRANGEMENT

SCALE
1cm = 0.5 m LENGTH

PLAN

ELEVATION

FIG. 13.61 24 FACED DOME

GENERAL ARRANGEMENT

SCALE :

1cm = $2 \cdot 10^{-1}$ m LENGTH

| KEY | |
|---|---|
| DDS21 | ⊙ |
| DDS33 | △ |
| DMX36 | × |
| DRO30 | ⊡ |
| EXPERIMENT | —+— |

1Nt VERTICAL LOAD AT ALL 12 UPPER PANEL CENTROIDS

EXPERIMENTAL VALUES OF GLOBAL DISPL. $u, v, w$

| POINT | $u$ $10^{-6}m$ | $v$ $10^{-6}m$ | $w$ $10^{-6}m$ | POINT | $u$ $10^{-6}m$ | $v$ $10^{-6}m$ | $w$ $10^{-6}m$ | POINT | $u$ $10^{-6}m$ | $v$ $10^{-6}m$ | $w$ $10^{-6}m$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| A | -0.05 | -0.07 | -0.66 | $M_{13}$ | -1.15 | -1.60 | -3.50 | $N_{11}$ | 0.00 | -1.32 | -2.13 |
| $M_1$ | -1.30 | -1.89 | -2.14 | $M_{14}$ | 0.25 | -0.66 | -3.19 | $N_2$ | 0.05 | -1.06 | -3.28 |
| $M_2$ | -0.32 | -1.07 | -1.89 | $M_8$ | -0.34 | -2.30 | -2.46 | $N_3$ | -0.98 | -0.41 | -1.31 |
| $M_3$ | -0.74 | -1.31 | -3.45 | B | -0.82 | -1.56 | -3.28 | $N_{12}$ | -1.15 | -0.82 | -2.21 |
| $M_9$ | 0.00 | 0.00 | -3.03 | $M_{15}$ | -0.47 | -1.39 | -2.87 | $N_{13}$ | -0.51 | -0.65 | -1.56 |
| $M_4$ | -0.25 | -1.64 | -2.29 | $M_{16}$ | 0.00 | -1.56 | -2.63 | $N_4$ | -0.08 | -0.82 | -1.39 |
| $M_5$ | -0.71 | -1.43 | -4.26 | $M_{17}$ | 0.08 | -1.64 | -1.97 | $N_5$ | -0.33 | -0.57 | -1.06 |
| $M_{10}$ | -1.97 | -1.39 | -4.01 | $M_{18}$ | 0.30 | -1.31 | -1.80 | $N_{14}$ | -0.65 | -0.41 | -1.23 |
| $M_{11}$ | -0.98 | -2.71 | -3.93 | C | -0.25 | -1.64 | -2.38 | $N_6$ | -0.08 | -1.01 | -0.98 |
| $M_6$ | 0.08 | -1.31 | -3.12 | $N_1$ | -0.82 | -1.48 | -2.54 | $N_7$ | -0.33 | -0.41 | -0.33 |
| $M_7$ | -0.62 | -1.31 | -3.61 | $N_9$ | -1.48 | -1.48 | -2.13 | $N_8$ | 0.00 | -0.51 | 0.00 |
| $M_{12}$ | -1.15 | -1.47 | -3.93 | $N_{10}$ | -1.23 | -1.72 | -1.88 | | | | |

FIG. 13.62. 24 FACED DOME

FIG. 13.63. 24 FACED DOME

1Nt VERTICAL LOAD AT ALL 12 UPPER PANEL CENTROIDS

HORIZONTAL DISPLACEMENTS

SCALES :

$1cm = 2 \cdot 10^{-1} m$ LENGTH

$1cm = 10^{-6} m$ DISPLAC.

FIG. 13.64. 24 FACED DOME

1Nt VERTICAL LOAD AT ALL 12 UPPER PANEL CENTROIDS

HORIZONTAL DISPLACEMENTS

SCALES :

$1cm = 2 \cdot 10^{-1} m$ LENGTH

$1cm = 10^{-6} m$ DISPLAC.

FIG. 13.65. 24 FACED DOME

1Nt VERTICAL LOAD AT ALL 12 UPPER PANEL CENTROIDS

HORIZONTAL DISPLACEMENTS

SCALES:

$1cm = 2 \cdot 10^{-1} m$ LENGTH

$1cm = 2 \cdot 10^{-6} m$ DISPLAC.

FIG. 13.66. 24 FACED DOME

1Nt VERTICAL LOAD AT ALL 12 UPPER PANEL CENTROIDS

HORIZONTAL DISPLACEMENTS

SCALES:

$1cm = 2 \cdot 10^{-1} m$ LENGTH

$1cm = 2 \cdot 10^{-6} m$ DISPLAC.

GLOBAL DISPLACEMENTS W, U

SCALES:

$1cm = 2 \cdot 10^{-1} m$ LENGTH

$1cm = 10^{-6} m$ DISPLAC.

1Nt VERTICAL LOAD AT ALL 12 UPPER-PANEL CENTROIDS

FIG. 13.67. 24 FACED DOME

GLOBAL DISPLACEMENTS U, W

SCALES

$1cm = 2 \cdot 10^{-1} m$ LENGTH

$1cm = 10^{-6} m$ DISPLAC.

1Nt VERTICAL LOAD AT ALL 12 UPPER PANEL CENTROIDS

1Nt FIG. 13.68. 24 FACED DOME

GLOBAL DISPLACEMENTS W, U

SCALES:

$1cm = 2 \cdot 10^{-1} m$ LENGTH

$1cm = 2 \cdot 10^{-6} m$ DISPLAC.

1Nt VERTICAL LOAD AT ALL 12 UPPER PANEL CENTROIDS

FIG. 13.69. 24 FACED DOME

GLOBAL DISPLACEMENTS W, U

SCALES:

1cm = $2 \cdot 10^{-1}$ m LENGTH

1cm = $2 \cdot 10^{-6}$ m DISPLAC.

1Nt VERTICAL LOAD AT ALL 12 UPPER PANEL CENTROIDS

FIG. 13.70. 24 FACED DOME

1Nt VERTICAL LOAD AT ALL 12 UPPER PANEL CENTROIDS

NORMAL DISPLACEMENTS OF LOADED FACE

SCALES:

1cm = $10^{-1}$ m LENGTH

1cm = $2 \cdot 10^{-7}$ m DISPLAC.

| | FINITE ELEMENT (PDS21) VALUE OF NORMAL DISPLAC. W ($10^{-6}$m) | EXPERIMENTAL VALUE OF NORMAL DISPLAC. W ($10^{-6}$m) |
|---|---|---|
| $M_1$ | -1.10 | -2.26 |
| $M_2$ | -1.12 | -1.85 |
| $M_3$ | -1.28 | -3.23 |
| $M_9$ | -1.40 | -2.57 |
| $M_4$ | -1.28 | -2.23 |
| $M_5$ | -1.32 | -3.95 |
| $M_{10}$ | -2.39 | -3.73 |
| $M_{11}$ | -2.36 | -3.97 |
| $M_6$ | -1.60 | -2.95 |
| $M_7$ | -1.20 | -3.37 |
| $M_{12}$ | -1.65 | -3.68 |
| $M_{13}$ | -2.18 | -3.35 |
| $M_{14}$ | -1.65 | -2.86 |
| $M_8$ | -1.19 | -2.63 |

FIG. 13.11. 24 FACED DOME

1nt VERTICAL LOAD AT ALL 12 UPPER PANEL CENTROIDS

NORMAL DISPLACEMENTS OF LOADED FACE

SCALES:

1cm = $10^{-1}$ m. LENGTH

1cm = $2 \cdot 10^{-7}$ m DISPLAC.

| | FINITE ELEMENT (DDS33) VALUE OF NORMAL DISPLAC. W ($10^{-6}$ m) | EXPERIMENTAL VALUE OF NORMAL DISPLAC. W ($10^{-6}$ m) |
|---|---|---|
| $M_1$ | -1.53 | -2.26 |
| $M_2$ | -1.58 | -1.85 |
| $M_3$ | -1.91 | -3.23 |
| $M_9$ | -2.06 | -2.57 |
| $M_4$ | -1.89 | -2.23 |
| $M_5$ | -1.97 | -3.95 |
| $M_{10}$ | -3.36 | -3.73 |
| $M_{11}$ | -3.31 | -3.97 |
| $M_6$ | -2.15 | -2.95 |
| $M_7$ | -1.85 | -3.37 |
| $M_{12}$ | -2.43 | -3.68 |
| $M_{13}$ | -3.00 | -3.35 |
| $M_{14}$ | -2.56 | -2.86 |
| $M_8$ | -1.84 | -2.63 |

FIG. 13.72. 24 FACED DOME

FIG. 13.73. 24 FACED DOME

1 Nt VERTICAL LOAD AT ALL 12 UPPER PANEL CENTROIDS

| | FINITE ELEMENT (DM×36) VALUE OF NORMAL DISPLAC. W ($10^{-6}$ m) | EXPERIMENTAL VALUE OF NORMAL DISPLAC. W ($10^{-6}$ m) |
|---|---|---|
| $M_1$ | -3.25 | -2.26 |
| $M_2$ | -3.32 | -1.85 |
| $M_3$ | -4.00 | -3.23 |
| $M_9$ | -4.07 | -2.57 |
| $M_4$ | -4.01 | -2.23 |
| $M_5$ | -4.12 | -3.95 |
| $M_{10}$ | -5.10 | -3.73 |
| $M_{11}$ | -5.05 | -3.97 |
| $M_6$ | -4.12 | -2.95 |
| $M_7$ | -3.95 | -3.37 |
| $M_{12}$ | -4.30 | -3.68 |
| $M_{13}$ | -4.62 | -3.35 |
| $M_{14}$ | -4.40 | -2.86 |
| $M_8$ | -3.77 | -2.63 |

NORMAL DISPLACEMENTS OF LOADED FACE

1Nt VERTICAL LOAD AT ALL 12 UPPER PANEL CENTROIDS

| | FINITE ELEMENT (DR030) VALUE OF NORMAL DISPLAC. W ($10^{-6}$ m) | EXPERIMENTAL VALUE OF NORMAL DISPLAC. W ($10^{-6}$ m) |
|---|---|---|
| $M_1$ | -3.25 | -2.26 |
| $M_2$ | -3.32 | -1.85 |
| $M_3$ | -3.95 | -3.23 |
| $M_9$ | -4.01 | -2.57 |
| $M_4$ | -3.95 | -2.23 |
| $M_5$ | -4.04 | -3.95 |
| $M_{10}$ | -5.06 | -3.73 |
| $M_{11}$ | -5.03 | -3.97 |
| $M_6$ | -4.01 | -2.95 |
| $M_7$ | -3.85 | -3.37 |
| $M_{12}$ | -4.21 | -3.68 |
| $M_{13}$ | -4.57 | -3.35 |
| $M_{14}$ | -4.30 | -2.86 |
| $M_8$ | -3.82 | -2.63 |

NORMAL DISPLACEMENTS OF LOADED FACE

FIG. 13.74. 24 FACED DOME

1Nt VERTICAL LOAD AT ALL 12 BOTTOM PANEL CENTROIDS

EXPERIMENTAL VALUES OF GLOBAL DISPL. $u$, $v$, $w$

| POINT | $u$ $10^{-6}$ m | $v$ $10^{-6}$ m | $w$ $10^{-6}$ m | POINT | $u$ $10^{-6}$ m | $v$ $10^{-6}$ m | $w$ $10^{-6}$ m | POINT | $u$ $10^{-6}$ m | $v$ $10^{-6}$ m | $w$ $10^{-6}$ m |
|---|---|---|---|---|---|---|---|---|---|---|---|
| A | -0.08 | -0.21 | 3.45 | $M_{13}$ | -0.74 | -1.89 | -1.15 | $N_{11}$ | -0.33 | -2.13 | -2.62 |
| $M_1$ | -0.33 | -1.39 | 2.13 | $M_{14}$ | -0.33 | -1.15 | -1.56 | $N_2$ | 0.09 | -0.82 | -3.11 |
| $M_2$ | -0.35 | -1.07 | 2.54 | $M_8$ | -0.16 | -1.80 | -0.98 | $N_3$ | -2.75 | -3.44 | -5.14 |
| $M_3$ | -0.57 | -0.90 | 0.33 | B | -1.89 | -3.75 | -4.02 | $N_{12}$ | -1.80 | -3.85 | -4.47 |
| $M_9$ | -0.16 | -1.31 | 1.23 | $M_{15}$ | -1.23 | -3.52 | -3.85 | $N_{13}$ | -0. 5 | -2. 5 | - |
| $M_4$ | -0.33 | -1.64 | 1.07 | $M_{16}$ | -0.25 | -3.03 | -3.11 | $N_4$ | 0.08 | -1.00 | -1.80 |
| $M_5$ | -0.74 | -3.11 | -1.39 | $M_{17}$ | -1.05 | -2.98 | -2.62 | $N_5$ | -0.65 | -2.66 | -3.66 |
| $M_{10}$ | -0.36 | -1.15 | -0.98 | $M_{18}$ | -0.18 | -2.38 | -2.54 | $N_{14}$ | -0.16 | -1.89 | -2.29 |
| $M_{11}$ | 0.00 | -1.89 | -0.41 | C | -0.75 | -3.03 | -2.29 | $N_6$ | 0.16 | -0.74 | -1.25 |
| $M_6$ | -0.33 | -2.87 | 0.25 | $N_1$ | -1.72 | -3.46 | -4.65 | $N_7$ | 0.00 | -0.75 | -1.90 |
| $M_7$ | -1.80 | -3.52 | -2.62 | $N_9$ | -1.39 | -3.85 | -3.85 | $N_8$ | 0.21 | 0.00 | -0.45 |
| $M_{12}$ | -1.07 | -3.11 | -2.46 | $N_{10}$ | -0.66 | -2.38 | -3.52 | | | | |

FIG. 13.75 24 FACED DOME

FIG. 13.76. 24 FACED DOME

1Nt VERTICAL LOAD AT ALL 12 BOTTOM PANEL CENTROIDS

HORIZONTAL DISPLACEMENTS

SCALES :

$1cm = 2 \cdot 10^{-1} m$ LENGTH

$1cm = 4 \cdot 10^{-6} m$ DISPLAC.

FIG. 13.77.

# 24 FACED DOME

1Nt VERTICAL LOAD AT ALL 12 BOTTOM PANEL CENTROIDS

HORIZONTAL DISPLACEMENTS

SCALES:

$1cm = 2 \cdot 10^{-1} m$ LENGTH

$1cm = 4 \cdot 10^{-6} m$ DISPLAC.

FIG. 13.78.

# 24 FACED DOME

1Nt VERTICAL LOAD AT ALL 12 BOTTOM PANEL CENTROIDS

HORIZONTAL DISPLACEMENTS

SCALES :

$1cm = 2 \cdot 10^{-1} m$ LENGTH

$1cm = 4 \cdot 10^{-6} m$ DISPLAC.

FIG. 13.79.

# 24 FACED DOME

1Nt VERTICAL LOAD AT ALL 12 BOTTOM PANEL CENTROIDS

HORIZONTAL DISPLACEMENTS

SCALES:

$1cm = 2 \cdot 10^{-1} m$ LENGTH

$1cm = 4 \cdot 10^{-6} m$ DISPLAC.

GLOBAL DISPLACEMENTS W, U

SCALES :

$1cm = 2 \cdot 10^{-1} m$ LENGTH

$1cm = 2 \cdot 10^{-6} m$ DISPLAC.

1Nt VERTICAL LOAD AT ALL 12 BOTTOM PANEL CENTROIDS

FIG. 13.80. 24 FACED DOME

GLOBAL DISPLACEMENTS W, U

SCALES

$1cm = 2 \cdot 10^{-1} m$ LENGTH

$1cm = 2 \cdot 10^{-6} m$ DISPLAC.

1Nt VERTICAL LOAD AT ALL 12 BOTTOM PANEL CENTROIDS

FIG. 13.81. 24 FACED DOME

GLOBAL DISPLACEMENTS W, U

SCALES :

$1cm = 2 \cdot 10^{-1} m$ LENGTH

$1cm = 2 \cdot 10^{-6} m$ DISPLAC.

1Nt VERTICAL LOAD AT ALL 12 BOTTOM PANEL CENTROIDS

FIG. 13.82.

24 FACED DOME

GLOBAL DISPLACEMENTS W, U

SCALES:

$1cm = 2 \cdot 10^{-1} m$ LENGTH

$1cm = 2 \cdot 10^{-6} m$ DISPLAC.

1Nt VERTICAL LOAD AT ALL 12 BOTTOM PANEL CENTROIDS

FIG. 13.83. 24 FACED DOME

FIG. 13.84. 24 FACED DOME

NORMAL DISPLACEMENTS OF LOADED FACE

SCALES:

$1cm = 10^{-1} m$ LENGTH

$1cm = 4 \cdot 10^{-7} m$ DISPLAC.

| | FINITE ELEMENT (DDS21) VALUE OF NORMAL DISPLAC. W ($10^{-6}$ m) | EXPERIMENTAL VALUE OF NORMAD DISPLAC. W ($10^{-6}$ m) |
|---|---|---|
| $N_1$ | -3.64 | -5.84 |
| $N_9$ | -3.43 | -5.45 |
| $N_{10}$ | -3.13 | -3.97 |
| $N_{11}$ | -2.14 | -3.10 |
| $N_2$ | -1.12 | -2.37 |
| $N_3$ | -5.51 | -6.62 |
| $N_{12}$ | -4.64 | -6.03 |
| $N_{13}$ | -3.20 | -4.23 |
| $N_4$ | -1.18 | -1.70 |
| $N_5$ | -4.56 | -4.23 |
| $N_{14}$ | -2.85 | -2.67 |
| $N_6$ | -1.11 | -1.16 |
| $N_7$ | -2.56 | -1.64 |
| $N_8$ | -0.70 | -0.18 |

1Nt VERTICAL LOAD AT ALL 12 BOTTOM PANEL CENTROIDS

FIG. 13.85.

# 24 FACED DOME

| | FINITE ELEMENT (DDS33) VALUE OF NORMAL DISPLAC. W ($10^{-6}$ m) | EXPERIMENTAL VALUE OF NORMAL DISPLAC. W ($10^{-6}$ m) |
|---|---|---|
| $N_1$ | -5.30 | -5.84 |
| $N_9$ | -5.07 | -5.45 |
| $N_{10}$ | -4.60 | -3.97 |
| $N_{11}$ | -3.41 | -3.10 |
| $N_2$ | -2.09 | -2.37 |
| $N_3$ | -7.94 | -6.62 |
| $N_{12}$ | -6.76 | -6.03 |
| $N_{13}$ | -4.87 | -4.23 |
| $N_4$ | -1.91 | -1.70 |
| $N_5$ | -6.83 | -4.23 |
| $N_{14}$ | -4.56 | -2.67 |
| $N_6$ | -2.20 | -1.16 |
| $N_7$ | -4.86 | -1.64 |
| $N_8$ | -1.90 | -0.18 |

1Nt VERTICAL LOAD AT ALL 12 BOTTOM PANEL CENTROIDS

FIG. 13.86. 24 FACED DOME

| | FINITE ELEMENT (DMX36) VALUE OF NORMAL DISPLAC. W ($10^{-6}$ m) | EXPERIMENTAL VALUE OF NORMAL DISPLAC. W ($10^{-6}$ m) |
|---|---|---|
| $N_1$ | -6.07 | -5.84 |
| $N_9$ | -6.13 | -5.45 |
| $N_{10}$ | -6.05 | -3.97 |
| $N_{11}$ | -5.58 | -3.10 |
| $N_2$ | -5.04 | -2.37 |
| $N_3$ | -7.36 | -6.62 |
| $N_{12}$ | -7.16 | -6.03 |
| $N_{13}$ | -6.45 | -4.23 |
| $N_4$ | -5.42 | -1.70 |
| $N_5$ | -6.36 | 4.23 |
| $N_{14}$ | -5.89 | -2.67 |
| $N_6$ | -5.49 | -1.16 |
| $N_7$ | -4.66 | -1.64 |
| $N_8$ | -4.89 | -0.18 |

1Nt VERTICAL LOAD AT ALL 12 BOTTOM PANEL CENTROIDS

FIG. 13.87 24 FACED DOME

| | FINITE ELEMENT (DR030) VALUE OF NORMAL DISPL. W ($10^{-6}$ m) | EXPERIMENTAL VALUE OF NORMAL DISPLAC. W ($10^{-6}$ m) |
|---|---|---|
| $N_1$ | -6.76 | -5.84 |
| $N_9$ | -6.55 | -5.45 |
| $N_{10}$ | -6.21 | -3.97 |
| $N_{11}$ | -5.42 | -3.10 |
| $N_2$ | -4.56 | -2.37 |
| $N_3$ | -8.41 | -6.62 |
| $N_{12}$ | -7.63 | -6.03 |
| $N_{13}$ | -6.56 | -4.23 |
| $N_4$ | -4.88 | -1.70 |
| $N_5$ | -7.32 | 4.23 |
| $N_{14}$ | -6.11 | -2.67 |
| $N_6$ | -5.06 | -1.16 |
| $N_7$ | -5.48 | -1.64 |
| $N_8$ | -4.46 | -0.18 |

1Nt VERTICAL LOAD AT ALL 12 BOTTOM PANEL CENTROIDS

FIG. 13.88. 36 FACED DOME

FIG. 13.89. 36 FACED DOME

SCALE:

1cm = 0.5m LENGTH

FIG. 13.90. 36 FACED DOME

GENERAL ARRANGEMENT

SCALE :

$1cm = 2 \cdot 10^{-1} m$ LENGTH

| KEY | |
|---|---|
| DDS21 | ⊙ |
| DDS33 | △ |
| DMX36 | × |
| DRO30 | ⊡ |
| EXPERIMENT | —+— |

1 Nt VERTICAL LOAD AT ALL 12 UPPER PANEL CENTROIDS

EXPERIMENTAL VALUES OF GLOBAL DISPLAC. $u$, $v$, $w$

| POINT | $u$ $10^{-6}$ m | $v$ $10^{-6}$ m | $w$ $10^{-6}$ m | POINT | $u$ $10^{-6}$ m | $v$ $10^{-6}$ m | $w$ $10^{-6}$ m |
|---|---|---|---|---|---|---|---|
| A | -0.04 | -0.06 | -0.53 | $M_{13}$ | 0.16 | 0.57 | -0.94 |
| $M_2$ | -0.08 | 0.52 | -1.64 | $M_{14}$ | 0.04 | 1.15 | -1.43 |
| $M_3$ | +0.28 | 0.72 | -2.28 | $M_{15}$ | -0.08 | 1.05 | -1.07 |
| $M_4$ | -0.13 | 0.48 | -1.73 | $M_{16}$ | 0.04 | 1.15 | -1.07 |
| $M_5$ | +0.78 | 0.98 | -4.34 | $M_{17}$ | 0.45 | 0.41 | -0.78 |
| $M_6$ | 0.49 | 1.80 | -2.42 | $M_{18}$ | 0.04 | 0.16 | -0.78 |
| $M_7$ | 0.61 | 1.48 | -2.05 | $M_{19}$ | 0.16 | 0.57 | -0.82 |
| T | 0.33 | 0.86 | -0.98 | $M_{20}$ | 0.29 | 0.90 | -0.91 |
| $M_9$ | 0.37 | 1.15 | -1.54 | $M_{21}$ | 0.25 | 0.80 | -1.25 |
| B | 0.66 | 2.21 | -2.25 | $M_{22}$ | 0.45 | 0.16 | -0.66 |
| $M_{11}$ | 0.21 | 1.97 | -1.64 | $M_{23}$ | 0.15 | 0.51 | -0.66 |
| C | -0.08 | 1.11 | -1.19 | | | | |

FIG. 13.91. 36 FACED DOME

FIG. 13.92 36 FACED DOME

1N VERTICAL LOAD AT ALL 12 UPPER PANEL CENTROIDS

HORIZONTAL DISPLACEMENTS

SCALES:

$1cm = 2.10^{-1} m$ LENGTH

$1cm = 1.10^{-6} m$ DISPLAC.

FIG. 13.93 36 FACED DOME

1Nt VERTICAL LOAD AT ALL 12 UPPER - PANEL CENTROIDS

HORIZONTAL DISPLACEMENTS

SCALES:

$1cm = 2.10^{-1} m$ LENGTH

$1cm = 1.10^{-6} m$ DISPLAC.

FIG. 13.94 36 FACED DOME

1Nt VERTICAL LOAD AT ALL 12 UPPER-PANEL CENTROIDS

HORIZONTAL DISPLACEMENTS

SCALES:

$1cm = 2.10^{-1} m$ LENGTH

$1cm = 2.10^{-6} m$ DISPLAC.

FIG. 13.95 36 FACED DOME

1Nt VERTICAL LOAD AT ALL 12 UPPER-PANEL CENTROIDS

HORIZONTAL DISPLACEMENTS

SCALES:

$1cm = 2.10^{-1} m$ LENGTH

$1cm = 2.10^{-6} m$ DISPLAC.

SCALES:

$1cm = 2.10^{-1} m$ LENGTH

$1cm = 1.10^{-6} m$ DISPLAC.

1 Nt VERTICAL LOAD AT ALL 12 UPPER-PANEL CENTROIDS

FIG. 13.96 36 FACED DOME

SCALES:

$1cm = 2.10^{-1} m$ LENGTH

$1cm = 2.10^{-6} m$ DISPLAC.

1Nt VERTICAL LOAD AT ALL 12 UPPER PANEL CENTROIDS

FIG. 13.97 **36 FACED DOME**

SCALES:

$1cm = 2.10^{-1} m$ LENGTH

$1cm = 2.10^{-6} m$ DISPLAC.

1 Nt VERTICAL LOAD AT ALL 12 UPPER-PANEL CENTROIDS

FIG. 13.98 36 FACED DOME

SCALES:

$1cm = 2.10^{-1} m$ LENGTH

$1cm = 2.10^{-6} m$ DISPLAC.

1Nt VERTICAL LOAD AT ALL 12 UPPER-PANEL CENTROIDS

FIG. 13.99 36 FACED DOME

FIG. 13.100 — 36 FACED DOME

1 KNt VERTICAL LOAD AT ALL 12 UPPER PANEL CENTROIDS

SIGN CONVENTION

LINE VECTORS:
$\overrightarrow{CA}$, $\overrightarrow{BA}$, $\overrightarrow{CB}$
$\overrightarrow{DC}$, $\overrightarrow{DB}$, $\overrightarrow{DT}$, $\overrightarrow{TB}$

MOMENTS DISTRIBUTION

SCALES:

1cm = $2 \cdot 10^{-1}$ m LENGTH

1 cm = 50 Nt m/m MOMENTS

FIG. 13. 101 36 FACED DOME

1 KNt VERTICAL LOAD AT ALL UPPER PANEL CENTROIDS

SIGN CONVENTION

LINE VECTORS:

$\overrightarrow{CA}$, $\overrightarrow{BA}$, $\overrightarrow{CB}$

$\overrightarrow{DC}$, $\overrightarrow{DB}$, $\overrightarrow{DT}$, $\overrightarrow{TB}$

MOMENTS DISTRIBUTION

SCALES:

$1cm = 2 \cdot 10^{-1} m$ LENGTH

$1 cm = 50 Ntm/m$ MOMENTS

FIG. 13.102 **36 FACED DOME**

1 KNt VERTICAL LOAD AT ALL 12 UPPER PANEL CENTROIDS

SIGN CONVENTION

LIVE VECTORS:
$\overrightarrow{CA}$, $\overrightarrow{BA}$, $\overrightarrow{CB}$
$\overrightarrow{DC}$, $\overrightarrow{DB}$, $\overrightarrow{DT}$, $\overrightarrow{TB}$

MOMENTS DISTRIBUTION

SCALES:

$1cm = 2 \cdot 10^{-1} m$ LENGTH

$1 cm = 50 Ntm/m$ MOMENTS

FIG. 13.103 · 36 FACED DOME

1 KNt VERTICAL LOAD AT ALL UPPER PANEL CENTROIDS

SIGN CONVENTION

LINE VECTORS:

$\overrightarrow{CA}$, $\overrightarrow{BA}$, $\overrightarrow{CB}$

$\overrightarrow{DC}$, $\overrightarrow{DB}$, $\overrightarrow{DT}$, $\overrightarrow{TB}$

MOMENTS DISTRIBUTION

SCALES:

$1cm = 2 \cdot 10^{-1} m$ LENGTH

$1cm = 50 Nt\,m/m$ MOMENTS

# 36 FACED DOME

SCALES:
1cm = $10^{-1}$ m LENGTH
1cm = $5 \cdot 10^{-7}$ m DISPLAC.

| | EXPERIMENTAL VALUE — OF NORMAL DISPLACEMENT W | FINITE ELEMENT VALUE (DDS21) ⊙ OF NORMAL DISPLACEMENT W | FINITE ELEMENT VALUE (DDS33) △ OF NORMAL DISPLACEMENT W | FINITE ELEMENT VALUE (DMX36) X OF NORMAL DISPLACEMENT W $M_{nn}=0$ along AB, BD | FINITE ELEMENT VALUE (DRO30) ⊡ OF NORMAL DISPLACEMENT W |
|---|---|---|---|---|---|
| $M_2$ | -1.55 | -1.11 | -1.62 | -3.51 | -3.55 |
| $M_3$ | -2.05 | -1.70 | -2.30 | -3.92 | -3.97 |
| $M_4$ | -1.44 | -1.11 | -1.61 | -3.50 | -3.54 |
| $M_6$ | -2.71 | -1.99 | -2.80 | -4.24 | -4.27 |
| $M_7$ | -2.38 | -2.02 | -2.78 | -4.20 | -4.21 |
| | $10^{-6}$ m | $10^{-6}$ m | $10^{-6}$ m | $10^{-6}$ m | $10^{-6}$ m |

1Nt VERTICAL LOAD AT ALL 12 UPPER PANEL CENTROIDS

1 Nt VERTICAL LOAD AT ALL 12 BOTTOM PANEL CENTROIDS

EXPERIMENTAL VALUES OF GLOBAL DISPLAC. $u, v, w$

| POINT | $u$ $10^{-6}$ m | $v$ $10^{-6}$ m | $w$ $10^{-6}$ m | POINT | $u$ $10^{-6}$ m | $v$ $10^{-6}$ m | $w$ $10^{-6}$ m |
|---|---|---|---|---|---|---|---|
| A | 0.00 | 0.02 | 0.37 | $M_{13}$ | 0.62 | 2.38 | -1.48 |
| $M_2$ | 0.61 | 1.15 | -0.12 | $M_{14}$ | 0.62 | 2.13 | -1.72 |
| $M_3$ | 0.33 | 1.15 | -0.53 | $M_{15}$ | 0.86 | 2.54 | -2.62 |
| $M_4$ | 0.21 | 1.39 | -0.53 | $M_{16}$ | 0.45 | 1.89 | -1.84 |
| $M_5$ | 0.25 | 1.31 | -0.98 | $M_{17}$ | 0.74 | 2.13 | -1.35 |
| $M_6$ | 0.37 | 1.80 | -1.52 | $M_{18}$ | 1.56 | 3.07 | -2.79 |
| $M_7$ | -0.04 | 0.74 | -0.70 | $M_{19}$ | 0.29 | 1.19 | -0.70 |
| T | 0.90 | 2.38 | -0.62 | $M_{20}$ | 0.49 | 1.80 | -1.56 |
| $M_9$ | 0.61 | 2.66 | -1.35 | $M_{21}$ | 0.66 | 0.86 | -1.27 |
| B | 0.49 | 2.21 | -2.54 | $M_{22}$ | 0.49 | 1.48 | -1.56 |
| $M_{11}$ | 0.03 | 2.21 | -1.89 | $M_{23}$ | 0.04 | 1.15 | -0.94 |
| C | -0.12 | 2.13 | -1.56 | | | | |

FIG. 13.105 36 FACED DOME

FIG. 13.106 **36 FACED DOME**

1Nt VERTICAL LOAD AT ALL 12 BOTTOM-PANEL CENTROIDS

HORIZONTAL DISPLACEMENTS

SCALES:

$1cm = 2 \cdot 10^{-1} m$ LENGTH

$1cm = 2 \cdot 10^{-6} m$ DISPLAC.

FIG. 13.10x 36 FACED DOME

1Nt VERTICAL LOAD AT ALL 12 BOTTOM PANEL CENTROIDS

HORIZONTAL DISPLACEMENTS

SCALES:

$1cm = 2 \cdot 10^{-1} m$ LENGTH

$1cm = 2 \cdot 10^{-6} m$ DISPLAC.

FIG. 13.108 36 FACED DOME

1Nt VERTICAL LOAD AT ALL 12 BOTTOM PANEL CENTROIDS

HORIZONTAL DISPLACEMENTS

SCALES:

$1cm = 2 \cdot 10^{-1} m$ LENGTH

$1cm = 2 \cdot 10^{-6} m$ DISPLAC.

FIG. 13.109 **36 FACED DOME**

1Nt VERTICAL LOAD AT ALL 12 BOTTOM PANEL CENTROIDS

HORIZONTAL DISPLACEMENTS

SCALES:

$1cm = 2 \cdot 10^{-1} m$ LENGTH

$1cm = 2 \cdot 10^{-6} m$ DISPLAC.

SCALES:

$1cm = 2 \cdot 10^{-1} m$ LENGTH

$1cm = 2 \cdot 10^{-6} m$ DISPLAC.

1Nt VERTICAL LOAD AT ALL 12 BOTTOM-PANEL CENTROIDS

FIG. 13.110 36 FACED DOME

SCALES:

$1cm = 2 \cdot 10^{-1} m$ LENGTH

$1cm = 2 \cdot 10^{-6} m$ DISPLAC.

1 Nt. VERTICAL LOAD AT ALL 12 BOTTOM-PANEL CENTROIDS

FIG. 13.111 36 FACED DOME

SCALES:

$1cm = 2 \cdot 10^{-1} m$ LENGTH

$1cm = 2 \cdot 10^{-6} m$ DISPLAC.

1Nt VERTICAL LOAD AT ALL 12 BOTTOM PANEL CENTROIDS

FIG. 13.112 36 FACED DOME

SCALES:

1cm = $2 \cdot 10^{-1}$ m  LENGTH

1cm = $2 \cdot 10^{-6}$ m  DISPLAC.

1Nt VERTICAL LOAD AT ALL 12 BOTTOM PANEL CENTROIDS

FIG. 13.113  36 FACED DOME

FIG. 13.114 36 FACED DOME

1 KNt VERTICAL LOAD AT ALL 12 BOTTOM-PANEL CENTROIDS

SIGN CONVENTION

LINE VECTORS:
$\overrightarrow{CA}$, $\overrightarrow{BA}$, $\overrightarrow{CB}$
$\overrightarrow{DC}$, $\overrightarrow{DB}$, $\overrightarrow{DT}$, $\overrightarrow{TB}$

MOMENTS DISTRIBUTION

SCALES:

$1cm = 2 \cdot 10^{-1} m$ LENGTH

$1cm = 100 Nt m/m$ MOMENTS

FIG. 13.115

# 36 FACED DOME

1 KNt VERTICAL LOAD AT ALL 12 BOTTOM PANEL CENTROIDS

SIGN CONVENTION

LINE VECTORS:

$\overrightarrow{CA}$, $\overrightarrow{BA}$, $\overrightarrow{CB}$

$\overrightarrow{DC}$, $\overrightarrow{DB}$, $\overrightarrow{DT}$, $\overrightarrow{TB}$

MOMENTS DISTRIBUTION

SCALES:

$1cm = 2 \cdot 10^{-1} m$ LENGTH

$1 cm = 100 Nm/m$ MOMENTS

FIG. 13.116

# 36 FACED DOME

1 KNt VERTICAL LOAD AT ALL 12 BOTTOM - PANEL CENTROIDS

SIGN CONVENTION

LINE VECTORS:

$\overrightarrow{CA}$, $\overrightarrow{BA}$, $\overrightarrow{CB}$

$\overrightarrow{DC}$, $\overrightarrow{DB}$, $\overrightarrow{DT}$, $\overrightarrow{TB}$

MOMENTS DISTRIBUTION

SCALES:

$1cm = 2 \cdot 10^{-1} m$ LENGTH

$1 cm = 100 \, Nm/m$ MOMENTS

FIG. 13.117

# 36 FACED DOME

1 KNt VERTICAL LOAD AT ALL 12 BOTTOM PANEL CENTROIDS

SIGN CONVENTION

LINE VECTORS:

$\overrightarrow{CA}$, $\overrightarrow{BA}$, $\overrightarrow{CB}$

$\overrightarrow{DC}$, $\overrightarrow{DB}$, $\overrightarrow{DT}$, $\overrightarrow{TB}$

MOMENTS DISTRIBUTION

SCALES:

$1cm = 2 \cdot 10^{-1}$ m LENGTH

$1cm = 100$ Nt-m/m MOMENTS

FIG. 13.11B 36 FACED DOME

NORMAL DISPLACEMENTS OF LOADED FACE

SCALES:

1cm = $10^{-1}$ m LENGTH

1cm = $5 \cdot 10^{-7}$ m DISPLAC.

NORMAL DISPLACEMENTS OF LOADED FACE

| | EXPERIMENTAL VALUE OF NORMAL DISPLACEMENT W (+) | FINITE ELEMENT VALUE (DDS21) OF NORMAL DISPLACEMENT W (⊙) | FINITE ELEMENT VALUE (DDS33) OF NORMAL DISPLACEMENT W (△) | FINITE ELEMENT VALUE (DMx36) OF NORMAL DISPLACEMENT W ($M_{nn}=0$ at 2-4, AB, BD) (x) | FINITE ELEMENT VALUE (DRO30) OF NORMAL DISPLACEMENT W (□) |
|---|---|---|---|---|---|
| $M_{15}$ | -3.62 | -1.58 | -2.46 | -3.72 | -3.75 |
| $M_{16}$ | -2.54 | -1.65 | -2.49 | -3.88 | -3.92 |
| $M_{21}$ | -1.63 | -0.80 | -1.57 | -2.93 | -2.88 |
| $M_{22}$ | -2.13 | -1.29 | -2.05 | -3.42 | -3.44 |
| $M_{23}$ | -1.33 | -0.96 | -1.69 | -3.30 | -3.34 |
| | $10^{-6}$ m | $10^{-6}$ m | $10^{-6}$ m | $10^{-6}$ m | $10^{-6}$ m |

1Nt VERTICAL LOAD AT ALL 12 BOTTOM PANEL CENTROIDS

1 Nt VERTICAL LOAD AT ALL 12 DORMER CENTROIDS

EXPERIMENTAL VALUES OF GLOBAL DISPL. $u, v, w$

| POINT | $u$ $10^{-6}$ m | $v$ $10^{-6}$ m | $w$ $10^{-6}$ m | POINT | $u$ $10^{-6}$ m | $v$ $10^{-6}$ m | $w$ $10^{-6}$ m |
|---|---|---|---|---|---|---|---|
| A | -0.12 | -0.10 | 0.16 | $M_{13}$ | -1.37 | 1.73 | -3.79 |
| $M_2$ | -0.33 | 1.23 | -1.39 | $M_{14}$ | -0.49 | 2.54 | -3.21 |
| $M_3$ | 0.08 | 0.95 | -0.41 | $M_{15}$ | 0.25 | 1.64 | -2.71 |
| $M_4$ | 0.00 | 1.15 | -0.61 | $M_{16}$ | -0.08 | 1.60 | -2.62 |
| $M_5$ | 0.08 | 1.37 | -1.85 | $M_{17}$ | -1.89 | 3.28 | -5.90 |
| $M_6$ | 0.10 | 1.52 | -1.95 | $M_{18}$ | 0.00 | 1.47 | -1.45 |
| $M_7$ | 0.00 | 1.47 | -1.52 | $M_{19}$ | -1.56 | 2.30 | -3.85 |
| T | 0.90 | 2.35 | -1.45 | $M_{20}$ | -0.57 | 2.87 | -3.28 |
| $M_9$ | 0.82 | 2.04 | -1.85 | $M_{21}$ | 0.49 | 1.31 | -1.64 |
| B | 0.57 | 1.96 | -1.95 | $M_{22}$ | 0.41 | 0.33 | -1.47 |
| $M_{11}$ | 0.00 | 1.50 | -1.46 | $M_{23}$ | 0.14 | 0.64 | -0.96 |
| C | -0.16 | 0.76 | -1.31 | | | | |

FIG. 13.119 36 FACED DOME

FIG. 13.120 36 FACED DOME

1Nt VERTICAL LOAD AT ALL 12 DORMER CENTROIDS

HORIZONTAL DISPLACEMENTS

SCALES:

$1cm = 2 \cdot 10^{-1} m$ LENGTH

$1cm = 10^{-6} m$ DISPLAC.

FIG. 13.121 36 FACED DOME

1Nt VERTICAL LOAD AT ALL 12 DORMER CENTROIDS

HORIZONTAL DISPLACEMENTS

SCALES:

$1cm = 2 \cdot 10^{-1} m$ LENGTH

$1cm = 10^{-6} m$ DISPLAC.

FIG. 13.122 36 FACED DOME

1Nt VERTICAL LOAD AT ALL 12 DORMER CENTROIDS

HORIZONTAL DISPLACEMENTS

SCALES:

$1cm = 2 \cdot 10^{-1} m$ LENGTH

$1cm = 2 \cdot 10^{-6} m$ DISPLAC.

FIG. 13.123 36 FACED DOME

1Nt VERTICAL LOAD AT ALL 12 DORMER CENTROIDS

HORIZONTAL DISPLACEMENTS

SCALES:

$1cm = 2 \cdot 10^{-1} m$ LENGTH

$1cm = 2 \cdot 10^{-6} m$ DISPLAC.

SCALES:

$1cm = 2 \cdot 10^{-1} m$ LENGTH

$1cm = 2 \cdot 10^{-6} m$ DISPLAC.

1Nt VERTICAL LOAD AT ALL 12 DORMER CENTROIDS

FIG. 13.124 36 FACED DOME

SCALES:

$1cm = 2 \cdot 10^{-1} m$ LENGTH

$1cm = 2 \cdot 10^{-6} m$ DISPLAC.

1 Nt VERTICAL LOAD AT ALL 12 DORMER CENTROIDS

FIG. 13.125 36 FACED DOME

SCALES:

$1cm = 2 \cdot 10^{-1} m$ LENGTH

$1cm = 2 \cdot 10^{-6} m$ DISPLAC.

1Nt VERTICAL LOAD AT ALL 12 DORMER CENTROIDS

FIG. 13.126 36 FACED DOME

SCALES:

$1cm = 2 \cdot 10^{-1} m$ LENGTH

$1cm = 2 \cdot 10^{-6} m$ DISPLAC.

1Nt VERTICAL LOAD AT ALL 12 DORMER CENTROIDS

FIG. 13.127 36 FACED DOME

FIG. 13.128

# 36 FACED DOME

1 KNt VERTICAL LOAD AT ALL 12 DORMER CENTROIDS

SIGN CONVENTION

LINE VECTORS:

$\overrightarrow{CA}$, $\overrightarrow{BA}$, $\overrightarrow{CB}$

$\overrightarrow{DC}$, $\overrightarrow{DB}$, $\overrightarrow{DT}$, $\overrightarrow{TB}$

MOMENTS DISTRIBUTION

SCALES:

$1cm = 2 \cdot 10^{-1} m$ LENGTH

$1 cm = 100 \, Ntm/m$ MOMENTS

FIG. 13.129

# 36 FACED DOME

1 KNt VERTICAL LOAD AT ALL 12 DORMER CENTROIDS

SIGN CONVENTION

LINE VECTORS:

$\overrightarrow{CA}$, $\overrightarrow{BA}$, $\overrightarrow{CB}$

$\overrightarrow{DC}$, $\overrightarrow{DB}$, $\overrightarrow{DT}$, $\overrightarrow{TB}$

MOMENTS DISTRIBUTION

SCALES:

$1cm = 2 \cdot 10^{-1}$ m LENGTH

$1cm = 100 Nt m/m$ MOMENTS

FIG. 13.130

# 36 FACED DOME

1 KNt VERTICAL LOAD AT ALL 12 DORMER CENTROIDS

SIGN CONVENTION

LINE VECTORS:
$\overrightarrow{CA}$, $\overrightarrow{BA}$, $\overrightarrow{CB}$
$\overrightarrow{DC}$, $\overrightarrow{DB}$, $\overrightarrow{DT}$, $\overrightarrow{TB}$

MOMENTS DISTRIBUTION

SCALES:

$1cm = 2 \cdot 10^{-1} m$ LENGTH

$1 cm = 100 Nt m/m$ MOMENTS

FIG. 13.131

# 36 FACED DOME

1 KNt VERTICAL LOAD AT ALL 12 DORMER CENTROIDS

SIGN CONVENTION

LINE VECTORS:
$\overrightarrow{CA}$, $\overrightarrow{BA}$, $\overrightarrow{CB}$
$\overrightarrow{DC}$, $\overrightarrow{DB}$, $\overrightarrow{DT}$, $\overrightarrow{TB}$

MOMENTS DISTRIBUTION

SCALES:

1cm = $2 \cdot 10^{-1}$ m LENGTH

1 cm = 100 Ntm/m MOMENTS

FIG. 13.132 36 FACED DOME

SCALES:

1cm = $10^{-1}$ m LENGTH

1cm = $5 \cdot 10^{-7}$ m DISPLAC.

| | + EXPERIMENTAL VALUE OF NORMAL DISPLACEMENT W | ⊙ FINITE ELEMENT VALUE (DDS21) OF NORMAL DISPLACEMENT W | △ FINITE ELEMENT VALUE (DDS33) OF NORMAL DISPLACEMENT W | × FINITE ELEMENT VALUE (DMX36) OF NORMAL DISPLACEMENT W $M_{mn}=0$ along AB, BD | ⊡ FINITE ELEMENT VALUE (DRO30) OF NORMAL DISPLACEMENT W |
|---|---|---|---|---|---|
| $M_{13}$ | -4.19 | -3.23 | -4.57 | -4.67 | -4.97 |
| $M_{14}$ | -3.40 | -2.20 | -3.20 | -3.82 | -4.02 |
| $M_{19}$ | -4.45 | -3.15 | -4.90 | -4.38 | -4.76 |
| $M_{20}$ | -3.57 | -2.35 | -3.45 | -3.82 | -4.02 |
| $M_{21}$ | -1.35 | -0.85 | -1.85 | -2.65 | -2.60 |
| | $10^{-6}$ m | $10^{-6}$ m | $10^{-6}$ m | $10^{-6}$ m | $10^{-6}$ m |

1Nt VERTICAL LOAD AT ALL 12 DORMER CENTROIDS

## NOTATION

| | | |
|---|---|---|
| $a_i, b_i, c_i$ / $a'_i, b'_i, c'_i$ $(i = 1,3)$ | Parameters for natural coordinates system. | APPENDIX I-V |
| $a_i$ $(i = 1 \div n)$ | Polynomial coefficients | CHAPTER 3<br>APPENDIX III |
| $a_n, a_s$ | Skew coordinates with respect to system $\bar{n}, \bar{s}$ . | CHAPTER 8 |
| $A_n$ | Area of the $n^{th}$ element | |
| $a_{ij}$ $(i,j = x,y)$ | Bending strains | |
| $a_i, a'_i$ $(i = 1 \div 3)$ | Parameters related to the angles measured by the theodolite | CHAPTER 9 |
| $\{B\}$ | Vector of polynomial coefficients | APPENDIX III |
| $B_i$ $(i = 1 \div 18)$ | Polynomial coefficients | APPENDIX III |
| $[B]$ | Strains-displacements matrix | |
| $B_{ij}$ | Elements of the [B] matrix | APPENDICES II-V |
| $[B^w]$ | Bending strains-transverse displacements matrix | APPENDICES II,III |
| $[B^{s1}]$ | Bending strains - shear displacements matrix | APPENDICES II,III |
| $[B^{s2}]$ | Shear strains - shear displacements matrix | APPENDICES II,III |

| | | |
|---|---|---|
| $[B_1^\theta]$ | Bending strains - total rotations matrix | APPENDIX V |
| $[B_2^\theta]$ | Shear strains - total rotations matrix | APPENDIX V |
| $[B^w]$ | Shear strains - transverse displacements matrix | APPENDIX V |
| $[B^{uv}]$ | In-plane strains - in-plane displacements matrix | APPENDICES II - V |
| $B_2$, $G_2$ | Parameters for the transformation with respect to the skew coordinates system $\bar{n}$, $\bar{s}$ . | CHAPTER 8 |
| C | Core thickness | |
| $C_{rs}^{ij}$ $(i,j,r,s = x,y)$ | Cartesian elasticity tensor. When with a prefix f or c refers to the faces or the core respectively. | |
| [C] | Elasticity matrix relating the strains vector to the stress resultants vector | CHAPTER 4 |
| $[C_b]$ | Elasticity matrix relating the bending and shear stress resultants vector | CHAPTER 4 and APPENDIX 4 |
| $d_i$ $(i = 1 \div 3)$ | Perpendiculars of a triangle | APPENDIX I |
| DT | Twice the area of a triangle | |
| [D] | Elasticity matrix relating the stress resultants vector to the strains vector | |
| $D_{rs}^{ij}$ $(i,k,r,s = x,y)$ | Elements of the elasticity matrix [D] with respect to the bending strains. | |

| | | |
|---|---|---|
| $D_o$ | Elastic constant | CHAPTER 4 |
| $[D_b]$ | Part of the elasticity matrix [D] with regard to the bending strains and shear strains | |
| $[D_{uv}]$ | Part of the elasticity matrix [D] with regard to the in-plane strains | |
| $E^{ij}_{rs}$ $(i,j,r,s = x,y)$ | Elements of the elasticity matrix [D] with respect to the in plane strains | |
| $e_{rs}$ $(r,s = x,y,z)$ | Cartesian strain tensor | |
| f | Face thickness | |
| F( ) | Function | |
| [F], $[F_{gen}]$, [GC] | Matrices for the formation of the shape functions matrix | CHAPTER 5 and APPENDIX III |
| h | Plate thickness | |
| $h_1, v_1, h_1', v_1'$ | Parameters related to the angles measured by the theodolite | CHAPTER 9 |
| I | Functional | |
| $\bar{I}$ | Approximate expression of the functional | CHAPTER 3 |
| $[K^o_n]$ | The stiffness matrix of the $n^{th}$ element with repect to a local coordinates system | |

| | | |
|---|---|---|
| $[K_n]$ | The stiffness matrix of the $n^{th}$ element with respect to a global coordinate system | |
| $[K]$ | The overall stiffness matrix | |
| $[K_n^{uv}]$ | Part of the stiffness matrix referring to the in-plane displacements of the $n^{th}$ element | CHAPTER 4 and APPENDIX IV |
| $[K_n^{mw}]$ | Part of the stiffness matrix (for the mixed formulation) referring to the transverse displacement of the $n^{th}$ element | CHAPTER 4 and APPENDIX IV |
| $[K_n^{mq}]$ | Part of the stiffness matrix (for the mixed formulation) referring to the moments as generalised displacements of the $n^{th}$ element | CHAPTER 4 and APPENDIX IV |
| $K_i$ $(i = 1 \div 3)$ | Parameters for the formation of various matrices | APPENDIX III |
| $L_i$ $(i = 1 \div 3)$ | Natural coordinates for a triangle | |
| $\ell_{in}$ | Length of the side in | CHAPTER 8 |
| $\ell_{in}$ | Length of an insert at a joint | CHAPTER 10 |
| $\ell_{ac}$ | Active length of a joint | CHAPTER 10 |
| $m_i$ $(i = 1 \div 5)$ | Parameters for the formation of the matrix $[F_{gen}]^{-1}$ | APPENDIX III |
| $M_{ij}$ $(i,j = x,y$ or $n,s)$ | Bending srress resultants (Moments) | |

| | | |
|---|---|---|
| $\{M_o^e\}$ | Vector of nodal degrees of freedom with regard to the moments models | |
| $\{M_i^e\}$ $(i = 1 \div 3)$ | Vector of nodal degrees of freedom with regard to the moments $M_{xx}$, $M_{yy}$, $M_{xy}$ respectively | |
| $\bar{M}_{ij}$ $(i,j = x,y$ or $n,s)$ | Moments as prescribed quantities | |
| $N_{ij}$ $(i,j = x,y)$ | In-plane stress resultants | |
| $[N]$ | Shape functions matrix relating the general displacements vector to the general vector of degrees of freedom for an element | |
| $N_i$ | Element of the shape functions matrix referring to the $i^{th}$ node | |
| $[N^w]$ | Shape functions matrix with respect to the transverse displacements models | |
| $[N^s]$ | Shape functions matrix with respect to the shear models | |
| $[N^\theta]$ | Shape functions matrix with respect to the total rotations models | |
| $[N^m]$ | Shape functions matrix with respect to the moments models | |
| $[N_m^o]$ | Shape functions matrix (mixed model) | APPENDIX IV |
| $[N_m^1]$ | Shape functions matrix (mixed model) | APPENDIX IV |
| $[N^{uv}]$ | Shape functions matrix with respect to the in-plane models | |

| | | |
|---|---|---|
| $\bar{n},\bar{s}$ | Coordinates of skewed coordinates system | |
| $n_{in}$ | Number of inserts per metre of joint-line | CHAPTER 10 |
| $P^i$ | Load carried through an insert at a joint | CHAPTER 10 |
| $P^j$ | Load carried by the inserts per metre of joint-line | CHAPTER 10 |
| $P^s$ | Load carried by the hardboard faces per metre of joint-line | CHAPTER 10 |
| $P^c$ | Reduced by 10% value of load carried by an insert | CHAPTER 10 |
| $\bar{P}$ | Distributed load vector | |
| $Q_i$ $(i,j = x,y$ or $n,s)$ | Shear stress resultants (Shear Forces) | |
| $\bar{Q}_i$ $(i,j = x,y$ or $n,s)$ | Shear forces prescribed quantities | |
| $\{\bar{R}_o\}$ | Prescribed load vector, corresponding to the general displacements vector $\{\delta_o^e\}$ | |
| $\{R_n^o\}$ | Generalised load vector for the $n^{th}$ element with respect to a local coordinates system corresponding to the vector of nodal degrees of freedom $\{\delta_o^e\}$ | |

| | | |
|---|---|---|
| $\{R_n\}$ | Generalised load vector for the $n^{th}$ element with respect to a global coordinates system corresponding to the vector of nodal d.o.f. $\{\delta_g^e\}$ | |
| $\{R\}$ | Overall load vector | |
| $\{R_n^w\}$ | Load vector corresponding to the nodal values of transverse displacements as d.o.f. for the $n^{th}$ element | CHAPTER 4 and APPENDIX IV |
| $\{R_n^m\}$ | Load vector corresponding to the nodal values of moments as d.o.f. for the $n^{th}$ element | CHAPTER 4 and APPENDIX IV |
| $\{R_n^{uv}\}$ | Load vector corresponding to the nodal values of in-plane displacements as d.o.f. for the $n^{th}$ element | CHAPTER 4 and APPENDIX IV |
| $S_{jz}^{iz}$ $(i,j=x,y)$ | Elements of the elasticity matrix [D] with respect to the shear strains | |
| $[S_n^o]$ | Stress matrix with respect to a local coordinates system for the $n^{th}$ element | |
| $[S_n]$ | Stress matrix with respect to a global coordinates system for the $n^{th}$ element | |
| $S_{o,n}$ | Part of the boundary where $M_{nn}$, $M_{ns}$, $Q_n$ are prescribed | |

| | | |
|---|---|---|
| $S_{n,n}$ | Part of the boundary where $w, \theta_n, \theta_s$ are prescribed | |
| $t_s$ | Steel plate thickness | CHAPTER 10 |
| $t_{ij}$ $(i,j = x,y,z)$ | Cartesian stress tensor | |
| $[T]^{-1}$ | Matrix for the formation of the shape functions matrix | CHAPTER 5 and APPENDIX III |
| $[TR]$ | Transformation matrix from a local to a global coordinates system | CHAPTER 8 |
| $[T_{\bar{e}g}]$ | Transofmration matrix relating the x', y', z' and x,y,z sets of coordinates | CHAPTER 8 |
| $[T_{\bar{e}e}]$ | Transformation matrix relating the x',y',z' and $\bar{x},\bar{y},\bar{z}$ sets of coordinates | CHAPTER 8 |
| $u$ | In-plane displacement corresponding to x axis | |
| $u_i$ | Nodal value of the in-plane displacement u at the $i^{th}$ node | |
| $u_{,i}$ $(i = x,y$ or $n,s)$ | First derivatives of the in-plane displacement u with respect to the i axis | |
| $u_{,ij}$ $(i,j = x,y$ or $n,s)$ | Second derivatives of the in-plane displacement u with respect to the i,j axes | |

| | | |
|---|---|---|
| $v$ | In-plane displacement corresponding to the y axis | |
| $v_i$ | Nodal value of the in-plane displacement v at the $i^{th}$ node | |
| $v,_i$ $(i = x,y$ or $n,s)$ | First derivatives of the in-plane displacement v with respect to the $i^{th}$ axis | |
| $v,_{ij}$ $(i,j = x,y$ or $n,s)$ | Second derivatives of the in-plane displacement with respect to the i,j axes | |
| $v_{i,\bar{j}}$ $(i,j = x,y,z)$ | Elements of the transformation matrix $[T_{e\bar{e}}]$ | |
| $w_{(x)}$ | Function of x | |
| $\bar{w}_{(x)}$ | Approximate expression of the above function | CHAPTER 3 |
| $w$ | Transverse displacement corresponding to the z axis (deflection) | |
| $\bar{w}$ | Prescribed quantity of the transverse displacement | CHAPTER 4 |
| $w_i$ | Nodal value of the transverse displacement at the $i^{th}$ node | |
| $w,_i$ $(i = x,y$ or $n,s)$ | First derivatives of the transverse displacement with respect to the $i^{th}$ axis | |
| $w,_{ir}$ $(i = x,y$ $r = 1 \div 3)$ | The above quantities at the $r^{th}$ node | |

| | | |
|---|---|---|
| $w_{,ij}$ $(i,j = x,y$ or $n,s)$ | Second derivatives of the transverse displacement with respect to i,j axes | |
| $w_{,ijr}$ $(i,j = x,y$ or $n,s,$ $r = 1 \div 3)$ | The above quantities at the $r^{th}$ node | |
| x,y,z | Cartesian coordinates | |
| x,y,z | Global coordinates system | CHAPTER 8 |
| x',y',z' | Local coordinates system | CHAPTER 8 |
| $\bar{x},\bar{y},\bar{z}$ | Plates-interconnection coordinates system | CHAPTER 8 |
| $\gamma_i$ $(i = x,y)$ | Shear strains. With a prefix c refers to the core | |
| $\delta$ | Variational operator | CHAPTER 3 |
| $\delta_1$ | Trigonometric function | CHAPTER 10 |
| $\{\delta o\}$ | General displacements vector | |
| $\{\delta^e\}$ | Overall nodal generalised displacements (degrees of freedom) vector | |
| $\{\delta_o^e\}$ | Nodal generalised displacements (degrees of freedom) vector for an element with respect to a local coordinates system | |
| $\{\delta_g^e\}$ | Nodal generalised displacements (d.o.f.) vector for an element with respect to a global system | CHAPTER 8 |

| | |
|---|---|
| $\{\delta^w\}$ | Vector of nodal degrees of freedom with regard to the transverse displacement models |
| $\{\delta^s\}$ | Vector of nodal d.o.f. with regard to the shear models |
| $\{\delta_i^s\}$ $(i=1,2)$ | The above quantities referring to the shear angles $\phi_x$, $\phi_y$ respectively |
| $\{\delta^\theta\}$ | Vector of nodal d.o.f. with regard to the total rotation models |
| $\{\delta_i^\theta\}$ $(i=1,2)$ | The above quantities referring to the total rotations $\theta_x$, $\theta_y$ respectively |
| $\{\delta^{uv}\}$ | Vector of nodal d.o.f. with regard to the in-plane displacements models |
| $\{\delta_i^{uv}\}$ $(i=1,2)$ | The above quantities referring to the in-plane displacements u,v, respectively |
| $\varepsilon_{\Delta\phi}$ | Strain referring to the change of angle for a joint |
| $\{\varepsilon\}$ | Strains vector |
| $\{\varepsilon_{uv}\}$ | In-plane strains vector |
| $\{\varepsilon_b\}$ | Bending and shear strains vector |
| $\theta_i$ $(i=x,y \text{ or } n,s)$ | Total rotations |
| $\theta_{i,r}$ $(i=x,y \text{ or } r=1\div 6)$ | Total rotations at the $r^{th}$ node |
| $\bar{\theta}_i$ $(i=x,y \text{ or } n,s)$ | Prescribed quantities of the total rotations |

| | | |
|---|---|---|
| $\lambda_{i,j}$ $(i,j = x,y,z)$ | Elements of the transformation matric $[T_{eg}]$ | CHAPTER 8 |
| $\mu_{\bar{i},j}$ $(i,j = x,y,z)$ | Elements of the transformation matrix $[T_{\bar{e}g}]$ | CHAPTER 8 |
| $\{\sigma\}$ | Stress resultants vector | |
| $\{\sigma_b\}$ | Vector of bending and shear stress resultants | |
| $\{\sigma_{uv}\}$ | Vector of in-plane stress resultants | |
| $\sigma_s$ | Stress of the steel plate | CHAPTER 10 |
| $\sigma_f$ | Stress of the hardboard face | CHAPTER 10 |
| $\tau_i$ $(i = 1 \div 3)$ | Distance of α point P within a triangle from the sides of the triangle | APPENDIX I |
| $O, \phi_.$ | Zero | |
| $\Phi$ | Function | |
| $\phi$ | Angle | |
| $\phi_i$ $(i = x,y$ or $n,s)$ | Shear angle | |
| $\phi_{i,r}$ $(i = x,y$ or $n,s$ $r = 1 \div 6)$ | Shear angle at the $r^{th}$ node | |
| $\omega$ | Change of angle at a joint | |

| | |
|---|---|
| $\Sigma$ | Indicates summation |
| $\nabla$ | Hamilton's operator |
| $\{\ \}$ | Vector |
| $[\ ]$ | Matrix of two dimensions |
| $[\ ]^{-1}$ | Inverse of a matrix |
| $[\ ]^T$, $\{\ \}^T$ | Transpose of a matrix or a vector |
| $[n_1, n_2, n_3 \ldots]$ | Where $n_i$ are numbers indicating the references |
| Nt | Newton weight units |

## REFERENCES


1. J.F. ABEL and E.P. POPOV, "Static and dynamic finite element analysis of sandwich structures," 2nd. Conf. Matrix Methods in Struct. Mech. Air Force Inst. of Tech. Wright Patterson Base, Ohio, 1968.
2. S. AHMAD, B.M. IRONS and O.C. ZIENKIEWICZ, "Curved thick shell and membrane elements with particular reference to axi-symmetric problems," 2nd. Conf. Matrix methods in Struct. Mech. Air Force Inst. of Technology, Wright Patterson Base, Ohio, 1968.
3. H.G. ALLEN, "Analysis and design of structural sandwich panels," Pergamon 1969.
4. A.M. ALWAN, "Large deflection of Sandwich plates with orthotropic cores," AIAA Journal, Vol.2, No.10, Oct. 1964.
5. R.S. ALWOOD and G.M.M. CORNES, "A polygonal finite element for plate bending problems using the assumed stress approach," Int. J. Num. Meth. Eng. 1, 1969.
6. Anon. "Foams, a 26 million dollar market," Canadian Plastics, Vol.2B, 3, 1970.
7. J.H. ARGYRIS, "Triangular elements with linearly varying strain for the matrix displacement method," Journal of the Royal Aeronautical Society, Vol.69, Oct.1965.
8. J.H. ARGYRIS, "Continua and Discontinua," Pros. Conf. on Matrix Methods in Struct. Mech., Wright Patterson Air Force Base, Ohio, 1965.
9. J.H. ARGYRIS, "On the Analysis of Complex Elastic Structures" Appl. Mech. Rev., 11, 1958.

10. J.H. ARGYRIS, D.W. SCHARPF, "The SHEBA family of shell elements for the displacement method," Journal of Royal Aeronautical Society, Vol.72, No.694, 1968.

11. J.H. ARGYRIS, I. FRIED and D.W. SCHARPF, "The TUBA family of plate elements for the matrix displacement method," Journal of the Royal Aeronautical Society, Vol.72, Aug. 1968

12. J.H. ARGYRIS and D.W. SCHARPF, "Finite element theory of plates and shells including transverse shear strain effects," Proc. Symposium of IUTAM, Aug. 1970, University of Liege, Belgium, 1971.

13. R. BANHAM, "Guide to modern Architecture," Van Nostrand, 1963.

14. A.J. BARNARD, "The finite element method applied to the analysis of sandwich plate and shell structures," M.Sc. Thesis, Loughborough University of Technology, January 1972.

15. G. BARTELDS and H.H. OTTENS, "Finite element analysis of sandwich panels," Proc. Symposium of IUTAM, Aug. 1970, University of Liege, Belgium, 1971.

16. A.K. BASU and J.M. DAWSON, "Orthotropic sandwich plates," Proc. of Inst. Civil Eng. 1970 Supplement (IV).

17. G.P. BAZELEY, Y.K. CHEUNG, B.M. IRONS and O.C. ZIENKIEWICZ, "Triangular elements in bending-comforming and non-conforming solutions," Proc. Conf. Matrix Methods in Struct. Mech. Air Force Inst. Tech., Wright Patterson A.F. Base, Ohio, 1965.

18. K. BELL, "A refined triangular plate bending element," Int. J. of Num. Meth. in Eng., 1, 1969.

19. B.S. BENJAMIN, "Structural design with plastics," Van Nostrand, 1969.

20. P. BETTESS, "The Irons Programm," University of Durham Dept. of Eng. Science, Inter. Report, 1970.

21. P. BETTESS, "Analysis of polyhedral domed structures composed of flat plates of sandwich material," Ph.D. Thesis, University of Durham, 1971.

22. R. BOYD, "New directions in Japanese Architecture," Studio Vista, 1968.

23. C. BREBBIA and J.J. CONNOR, "Geometrically non-linear finite element analysis," Proc. of ASCE, Journal of Engineering Mechanics Division, April 1969.

24. G.A. BUTLIN and R. FORD, "A compatible plate bending element," University of Leicester, Eng. Dept. Report 1968.

25. A. CHATTERJEE and A.V. SETLUR, "A mixed finite element formulation for plate problems," Int. J. Num. Meth. Eng., Vol.4, 1972.

26. V.K. CHEUNG, I.P. KING and U.C. ZIENKIEWICZ, "Slab bridges with arbitrary shape and support conditions - a general method of analysis based on finite elements," Proc. Inst. of Civil Engineers, 40, 1968.

27. K.H. CHU and S.G. PINJARKAR, "Analysis of horizontally curved box girder bridges," Proc. ASCE, Str. DM, St.10, October 1971.

28. R.W. CLOUGH, "The finite element in plane stress analysis," Proc. 2nd. A.S.C.E. Conf. on Electronic Computation, Pittsburgh Pa., Sept. 1960.

29. R.W. CLOUGH and C.A. FELIPPA, "A refined quadrilateral element for analysis of plate bending," Proc. of 2nd Conf. in Struct. Mech., Air Force Inst. of Tech., Wright Patterson Air Force Base, Ohio, 1968.

30. R.W. CLOUGH and G.L. TOCHER, "Finite element stiffness matrices for analysis of plate bending," Proc. Conf. on matrix methods in Struct. Mech. Air Force Inst. of Technology, Wright Patteron Air Force Base, Ohio, Oct.1968

31. R.W. CLOUGH and C.P. JOHNSON, "A finite element approximation for the analysis of thin shells," J. Solids and Struct. 4, 1968.

32. J.J. CONNOR and C. BREBBIA, "Stiffness matrix for a shallow rectangular shell element," Journal of the Engineering Mechanics Division, ASCE, Oct.1967.

33. D.L. COLLINS, "The flexibility of triangular sandwich plates," Third year project report, Dept. Eng. Science, University of Durham, 1972.

34. R.D. COOK, "Two hybrid elements for analysis of thick, thin and sandwich plates," Int. I. Num. Meth. Eng., Vol.5, 1972.

35. G.R. COWPER, E. KOSKO, G.M. LINDBERG and M.D. OLSON, "Formulation of new triangular plate bending element," Trans. Canada Aero-Space Inst. 1, 1968.

36. G.R. COWPER, E. KOSKO, G.M. LINDBERG and M.D. OLSON, "A high precision triangular plate bending element," National Research Council of Canada Aero-Report LR-514 1968.

37. G.R. COWPER, G.M. LINDBERG and M.D. OLSON, "A shallow shell finite element of triangular shape," Int. J. of Solids and Struct. 6, 1970.

38. R. DURHAM and K. PISTER, "A finite element application of the Helinger-Reissner Variational Theorem," 2nd. Conf. in Struct. Mech., Wright Patterson Air Force Base, Oct. 1968.

39. C.S. DESAI and J.F. ABEL, "Introduction to the finite element method," Van Nostrand, New York, 1972.

40. R.M. DAVIES, "Plastics in building construction," Proc. Conf. on plastics in building construction, 25th Sept. 1964.

41. B. FRAIJS DE VEUBEKE, "Displacement and Equilibrium models in the finite element method," In "Stress Analysis," edited by O.C. Zienkiewicz and G. Holister, John Wiley, 1965.

42. B. FRAIJS DE VEUBEKE, "Bending and stretching of plates," Proc. of Conf. on Matrix methods in Struct. Mech., Air Force Inst. of Tech., Wright Patterson Air Force Base, Ohio, 1965.

43. B. FRAIJS DE VEUBEKE and D.C. ZIENKIEWICZ, "Strain Energy bounds in finite element analysis by slab analogy," J. of Strain Analysis, 2, 1967.

44. B. FRAIJS DE VEUBEKE, "A conforming finite element for plate bending," Int. J. Solids Struct. 4, 1968.

45. B. FRAIJS DE VEUBEKE, "An equilibrium model for plate bending," Int. J. Solids Struct. 4, 1968.

46. D.J. ELLIOT, "Structural properties of flat sandwich panels," M.Sc. Thesis, University of Durham, 1970.

47. J. ERGATOUDIS, B.M. IRONS and O.C. ZIENKIEWICZ, "Three-Dimensional analysis of arch dams and their foundations," Symposium on arch dams at the Inst. of Civil Engineers, March, 1968.

48. J. ERGATOUDIS, B.M. IRONS, O.C. ZIENKIEWICZ, "Curved, Iso-parametric, quadrilateral elements for finite element analysis," Int. J. Solids and Structures, Vol.7, 1968.

49. C.A. FELIPPA, "Refined finite element analysis of linear and non-linear two-dimensional structures," Civil Engr. Dept., University of California, Berkeley, Rept. No.66-22, 1966.

50. G. FEUERSTEIN, "New directions in German Architecture," Studio Vista, 1968.

51. I. FRIED, "Shear in $C^o$ and $C^1$ bending finite elements," Int. J. Solids Structures, Vol.9, 1973.

52. I. FRIED, "Some aspects of the natural coordinate system in the finite element method," A.I.A.A. Journal, Vol.7, No.7, July 1969.

53. W. FLUGGE, "Stresses in Shells," Springer-Verlag, Berlin 1960.

54. A.E. GREEN and W. ZERNA, "Theoretical Elasticity," Oxford, the Clarendon Press, 1954.

55. R.L. GRIEVE et al., "New generation of structural foam polymers," J. Cell Plast. Vol.6, 4, 1970.

56. L.R. HERRMANN, "Finite element bending analysis for plates," Proc. A.S.C.E., St. Dn, EMS, Oct. 1967.

57. L.R. HERRMANN and D.M. CAMPBELL, "A finite element analysis for thin shells," A.I.A.A. Journal, Vol.6, No.10, Oct. 1968.

58. I. HOLAND and P.G. BERGAN, "Higher-order finite element for plane stress," Discussion, Proceedings A.S.C.E., EM2, April 1968.

59. I. HOLAND and K. BELL, "Finite element methods in stress analysis," T.A.P.I.R., The Technical University of Norway, 1970.

60. B.M. IRONS, "Lagrange multiplier techniques in structural analysis," J.A.I.A.A., 3, 6, 1172, 1965.

61. B.M. IRONS, "Engineering application of numerical integration in stiffness method," Journal of A.I.A.A., Vol.4, 1966.

62. B.M. IRONS, "Roundoff criteria in direct stiffness solutions, A.I.A.A. Journal, Vol.6, No.7, July 1969.

63. B.M. IRONS, "A conforming quartic triangular element for plate bending," Int. Journal of Numerical Methods in Engineering, 1, 1969.

64. B.M. IRONS, "A frontal solution program for finite element analysis," Int. Journal of Num. Methd. in Eng., Vol.2, No.6, 1970.

65. M.W. JOHNSON and R.W. McLAY, "Convergence of the finite element method in the theory of Elasticity," Trans. A.S.M.E. June 1968.

66. J.B. KENNEDY, "On the deformation of parallelogrammic sandwich panels," The Aero J. of the Royal Aeronautical Society, Vol. 74, June 1970.

67. S.W. KEY and Z.E. BEISINGER, "The analysis of thin shells with transverse shear strains by the finite element method," Proc. 2nd Conf. Matrix Meth. Str. Mech., Air Force Inst. Tech., Wright Patterson Base, Ohio, 1969.

68. T.P. KHATUA and V.K. CHEUNG, "Bending and vibration of multilayer sandwich beams and plates," Int. Journal Num. Met. Eng., Vol.6, 1973.

69. V. KOLAR and I. NEMEC, "The efficient finite element analysis of rectangular and skew laminated plates," Int. J. Num. Met. Eng. Vol.7, 1973.

70. R. LANDAU, "New directions in British Architecture," Studio Vista, 1968.

71. C. LIBOVE and S.B. BATDORF, "A general small deflection theory for flat sandwich plates," N.A.C.A. Report, No. B99, 1948.

72. H.C. MARTIN and G.F. CAREY, "Introduction to finite element analysis," McGraw-Hill, New York, 1973.

73. A.S. MAWENYA and J.D. DAVIES, "Finite element bending analysis of multilayer plates," Int. Journal Num. Meth. Engin. Vol.8, 1974.

74. R.J. MELOSH, "Basis for derivation of matrices for the direct stiffness method," J.A.I.A.A., 1, 1631-7, 1963.

75. R.J.MELOSH and R.M. BAMFORD, "Efficient Solution of load deflection Equations," Proc. A.S.C.E., J. Str. Dn., Vol.95, ST4, April 1969.

76. C. MEYER, A.C. SCORDELIS, "Analysis of curved folded plate structure," Proc. A.S.C.E., St. Dn, St.10, Oct. 1971.

77. L.S.D. MORLEY, "A triangular Equilibrium element with linearly varying bending moments for plate bending problems," Journal of the Royal Aeronautical Society, Vol.72, Aug. 1968.

78. G. NITSIOTAS, " Theory of Plates and Shells", (in Greek), Thessaloniki, 1968.

79. H.G. NODEAN, Progress in thermosetting foams," Plast. Technology, Vol.14, 10, 1966.

80. E.R. ARRANTES OLIVEIRA, "Theoretical foundations of the finite element methods," Int. J. Solid Struct., Vol.4, No.10, Oct. 1968.

81. E.R. ARRANTES OLIVEIRA, "Completeness and convergence in the finite element method," Proc. 2nd Conf. on Matrix Meth. in Str. Mech., Air Force Inst. Tech., Wright Patterson Air Force Base, Ohio, 1968.

82. E.R. ARRANTES OLIVEIRA, "The Convergence theorems and their role in the theory of structures," Proc. Symposium of IUTAM, Aug. 1970, University of Liege, Belgium, 1971.

83. J.T. ODEN, "A general theory of finite elements," :
I. "Topological considerations;" II. "Applications"
Int. Journal of Numer. Methods in Engineering, 1, 1969.

84. J.T. ODEN, "Finite elements of nonlinear continua,"
McGraw-Hill, New York, 1972.

85. G.M. PARTON, "The structural behaviour of polyhedral sandwich shells," Ph.D. Thesis, University of Durham, 1972.

86. T.H.H. PIAN, "Derivation of element stiffness matrices by assumed stress distribution," A.I.A.A. J. 2, 1964.

87. T.H.H. PIAN and P. TONG, "Basis of finite element methods for Solid Continua," Int. Journal for Numerical Methods in Engineering, Vol.1, No.1, Jan-March 1969.

88. T.H.H. PIAN and P. TONG, "Variational formulation of finite-displacement analysis," Proc. of Sym. I.U.T.A.M. University Liege, Belgium , 1971.

89. F.J. PLANTEMA, "Sandwich Construction," Wiley, 1967.

90. C.A. PRATO, "Shell finite element method via Reissner's principle," Int. J. Solids and Structures, Vol.5, 1969.

91. C.W. PRYOR, R.M. BARKER and D. FREDERICK, "Finite element bending analysis of Reissner plates," Proc. A.S.C.E., St. Dn. EM6, Dec. 1970.

92. H. RAMSTAD and I. HOLAND, "The finite element method for analysis of skew plates in bending by use of parallelogram elements," International Symposium on the use of Electronical Digital Computers in Structural Engineering, Newcastle-upon-Tyne, 1966.

93. E. REISSNER, "Finite deflections of sandwich plates," J. Inst. Aeronautical Sciences, Vol.15, July 1948.

94. E. REISSNER, "Small bending and stretching of sandwich type shells," N.A.C.A., 975, Washington D.C. 1950.

95. E. REISSNER, "On a variational theorem in elasticity," J. Math. Phys. Vol.29, 1950.

96. M.G. SAMUCHIN and J. DUNDURS, "Transmission of concentrated forces into prismatic shells - II," Int. J. Solids Struct. Vol.9, 1973.

97. G. SANDER, "Applications de la methode des elements finis a la flexion des plagues," Collection des publications, University de Liege, Faculte des Sciences Appliquees, 1969.

98. A.C. SCORDELIS and K.S. LO, "Computer analysis of cylindrical shells," J. Am. Concr. Inst. 61, May, 1964.

99. J.M. SMITH and W. DUNCAN, "The effectiveness of nodal continuities in finite element analysis of thin rectangular and skew plates in bending," Int. J. of Num. Meth. in Eng. 2, 1970.

100. R.A.M. STERN, "New directions in American Architecture, Studio Vista, 1968.

101. S.P. TIMOSHENKO and J.N. GOODIER, "Theory o f Elasticity," 2nd Edition, McGraw-Hill, 1951.

102. S.P. TIMOSHENKO and S. WOINOWSKY-KRIEGER, "Theory of plates and shells," Second edition, McGraw-Hill, New York, 1959.

103. J.L. TOCHER and B.J. HARTZ, "Higher order finite element for plane stress," Proceedings A.S.C.E., EM4, Aug. 1967.

104. P. TONG, T.H.H. PIAN, "The Convergence of the finite element method in solving linear elastic problems," Int. J. Solids Structures, Vol.3, 1967.

105. H. TOTTENHAM and C.A. BREBBIA, "Finite element techniques in structural mechanics," Southampton University Press, 1970.

106. W. VISSER, "A refined mixed-type plate bending element," A.I.A.A. Journal, Vol.7, No.9, Sept. 1969.

107. W. VISSER, "The application of a curved mixed type shell element," Proc. Sym. of I.U.T.A.M., University of Liege, Belgium, 1971.

108. R.N. WHITE, "Optimum solution techniques for finite difference equations," Proc. A.S.C.E. Dn, Vol. B9, STR. Aug. 1963.

109. C.S. WHITNEY, B.G. ANDERSON and H. BIRNBAUM, "Reinforced concrete folded plate construction," Proc. A.S.C.E., St. Dn. St. B, Oct. 1959.

110. D.G. WILLIAMS and S.C. CHAPMAN, "Effect of shear deformation on uniformly loaded rectangular orthotropic plates," Proc. o f Inst. Civil Engin. Supplem. Voltume, 1969.

111. E.L. WILSON, "Structural analysis of axisymmetric solids," A.I.A.A. Journal, Vol.3, No.12, Dec. 1965.

112. O.C. ZIENKIEWICZ and V.K. CHEUNG, "The finite element method for analysis of elastic isotropic and orthotropic slabs," Proc. Inst. Civ. Engrs. 2B, Aug. 1974.

113. O.C. ZIENKIEWICZ, V.K. CHEUNG, "Finite element procedures in the solution of plate and shell problems," in "Stress Analysis," ed. O.C. Zienkiewicz and G.S. Holister, S. Wiley, 1965.

114. O.C. ZIENKIEWICZ, C.J. PAREKH and I.P. KING, "Arch dams analysed by a linear finite element shell solution programm," Proc . Symp. Arch Dams, Inst. Civ. Eng. London 1968.

115. O.C. ZIENKIEWICZ, "The finite element method in engineering science," McGraw-Hill, London, 1971.

## APPENDIX I

## GEOMETRICAL RELATIONSHIPS FOR A TRIANGLE

The area coordinates $L_1$, $L_2$, $L_3$ can be defined as follows:- [39,52,59,63 105,115]

$$L_1 = \frac{\tau_1}{d_1}, \qquad L_2 = \frac{\tau_2}{d_2}, \qquad L_3 = \frac{\tau_3}{d_3} \tag{AI.1}$$

If $(x_1,y_1)$, $(x_2,y_2)$, $(x_3,y_3)$ are the cartesian sets of coordinates for the vertices $P_1$, $P_2$, $P_3$ respectively, and $(x,y)$, $(L_1, L_2, L_3)$ the cartesian and area coordinates respectively for the point P the following relationships are valid:

$$\begin{aligned} x_1L_1 + x_2L_2 + x_3L_3 &= x \\ y_1L_1 + y_2L_2 + y_3L_3 &= y \\ L_1 + L_2 + L_3 &= 1 \end{aligned} \tag{AI.2}$$

The above give the cartesian set $(x,y)$ for any set of area coordinates $L_1$, $L_2$, $L_3$.

In reverse we can evaluate the area coordinates $L_1$, $L_2$, $L_3$ for any set of cartesian coordinates $(x,y)$ through the relationships

$$L_i = \frac{1}{DT}\,(a_i + b_i\,x + c_i y_i) \quad i = 1 \div 3 \tag{AI.3}$$

$$\left.\begin{aligned} a_1 &= x_2y_3 - x_3y_2 \\ b_1 &= y_2 - y_3 \\ c_1 &= x_3 - x_2 \end{aligned}\right\} \tag{AI.4}$$

$a_2$, $b_2$, $c_2$

$a_3$, $b_3$, $c_3$

from $a_1$, $b_1$, $c_1$ with circle symmetrical substitution of the subscripts 1, 2, 3

DT = twice the area of the triangle = $b_1c_2 - b_2c_1 = b_2c_3 - b_3c_2 = b_3c_1 - b_1c_3$

(AI.5)

From (AI.3) $\Longrightarrow$

$$\frac{\partial L_i}{\partial_x} = \frac{b_i}{DT}, \qquad \frac{\partial L_i}{\partial_y} = \frac{c_i}{DT} \qquad i = 1 \div 3 \qquad \text{(AI.6)}$$

We have a function of the form $\Phi(L_1, L_2, L_3)$ and we want to express the derivatives of $\phi$ at the direction $\bar{n}_1$, $\bar{n}_2$, $\bar{n}_3$ which are vectors normal to the sides $\overline{23}$ ($L_1 = \phi$), $\overline{31}$ ($L_2 = \phi$), $\overline{12}$ ($L_3 = \phi$) respectively.

For easier formulation we have

$$b'_i = \frac{b_i}{DT}, \qquad c'_i = \frac{c_i}{DT} \qquad i = 1 \div 3 \qquad \text{(AI.7)}$$

$$\frac{\partial\Phi}{\partial_{n_i}} = \bar{n}_i \nabla\Phi = \bar{n}_i \frac{\partial\Phi}{\partial L_1}\nabla L_1 + \frac{\partial\Phi}{\partial L_2}\nabla L_2 + \frac{\partial\Phi}{\partial L_3}\nabla L_3) \qquad \text{(AI.8)}$$

where $\nabla$ is Hamilton's operator. For two dimensions has the form

$$\nabla F = \frac{\partial F}{\partial x}\bar{i} + \frac{\partial F}{\partial y}\bar{j}$$

(i,j the unit vectors at x,y)

so

$$\nabla L_1 = \begin{bmatrix} b_1' \\ c_1' \end{bmatrix}, \quad \nabla L_2 = \begin{bmatrix} b_2' \\ c_2' \end{bmatrix}, \quad \nabla L_3 = \begin{bmatrix} b_3' \\ c_3' \end{bmatrix} \tag{AI.9}$$

Substituting the (A.9) to (A.8) and naming as

$$\Phi, i = \frac{\partial \Phi}{\partial L_i} \tag{AI.10}$$

we obtain -

$$\frac{\partial \Phi}{\partial n_i} = \bar{n}_i \left\{ \Phi,_1 \begin{bmatrix} b_1' \\ c_1' \end{bmatrix} + \Phi,_2 \begin{bmatrix} b_2' \\ c_2' \end{bmatrix} + \Phi,_3 \begin{bmatrix} b_3' \\ c_3' \end{bmatrix} \right\} \tag{AI.11}$$

so taking the right values for $\bar{n}_1$, $\bar{n}_2$, $\bar{n}_3$ we can write

$$\frac{\partial \Phi}{\partial \bar{n}_1} = \left[ \Phi,_1 (b_1^2 + c_1^2) + \Phi,_2 (b_1 b_2 + c_1 c_2) + \Phi,_3 (b_1 b_3 + c_1 c_3) \right] / DT \times (P_2 P_3)$$

$$\frac{\partial \Phi}{\partial \bar{n}_2} = \left[ \Phi,_1 (b_1 b_2 + c_1 c_2) + \Phi,_2 (b_2^2 + c_2^2) + \Phi,_3 (b_2 b_3 + c_2 c_3) \right] / DT \times (P_1 P_3) \tag{AI.12}$$

$$\frac{\partial \Phi}{\partial \bar{n}_3} = \left[ \Phi,_1 (b_1 b_3 + c_1 c_3) + \Phi,_2 (b_2 b_3 + c_2 c_3) + \Phi,_3 (b_3^2 + c_3^2) \right] / DT \times (P_1 P_2)$$

## APPENDIX II

FORMATION OF MATRICES (Reference to DDS15, DDS21)

### Formation of [B]

$$\{\varepsilon\} = \begin{Bmatrix} a_{xx} \\ a_{yy} \\ a_{xy} \\ \gamma_x \\ \gamma_y \\ \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \end{Bmatrix} = [B] \quad \{\delta_o^e\} \qquad \text{(AII.1)}$$

$$\left.\begin{array}{llll} a_{xx} = w,_{xx} - \phi_{x,x} & & & \\ a_{yy} = w,_{yy} - \phi_{y,y} & & & \\ a_{xy} = 2W,_{xy} - \phi_{x,y} & - \phi_{y,x} & & \\ \gamma_x = \phi_x , & \gamma_y = \phi_y & & \\ \varepsilon_{xx} = u,_x & \varepsilon_{yy} = v,_y & & \\ \varepsilon_{xy} = u,_y + u,_x & & & \end{array}\right\} \qquad \text{(AII.2)}$$

$$\{\delta_o^e\} = \begin{Bmatrix} \delta_o^w \\ \delta_o^s \\ \delta_o^{uv} \end{Bmatrix} \qquad \text{(AII.3)}$$

$\{\delta_o^w\}$ deflections and derivatives of the deflections as degrees of freedom

$\{\delta_o^s\}$ shear angles as degrees of freedom

$\{\delta_o^{uv}\}$ membrane moves as degrees of freedom.

$$\{\delta_o^w\}^T = \{w_1, w_{,x1}, w_{,y1}, w_2, w_{,x2}, w_{,y2}, w_3, w_{,x3}, w_{,y3}\} \quad \text{(AII.4)}$$

$$\{\delta_o^s\}^T = \{\phi_{x1}, \phi_{y1}, \phi_{x2}, \phi_{y2}, \phi_{x3}, \phi_{y3}\} \quad \text{(AII.5)}$$

$$\{\delta_o^{uv}\}^T = \{u_1, v_1, u_2, v_2, u_3, v_3\} \quad \text{(AII.6)}$$

the subscript , x, y indicates derivatives of x, y respectively and the number (1 ÷ 6) indicates the relevant node.

$$\{\varepsilon\} = \begin{bmatrix} [B^w] & [B^{s1}] & [\phi.] \\ [\phi.] & [B^{s2}] & [\phi.] \\ [\phi.] & [\phi.] & [B^{uv}] \end{bmatrix} \begin{Bmatrix} \{\delta_o^w\} \\ \{\delta_o^s\} \\ \{\delta_o^{uv}\} \end{Bmatrix} \quad \text{(AII.7)}$$

<u>Formation of $[B^w]$ matrix</u>

$[B^w]$ is a matrix of 3 rows and 9 columns. We will indicate as $B_{ij}^w$ on of its elements at the i row and j column.

$$B_{1,1}^w = \frac{1}{(DT)^2}\left[-2b_1(L_1b_1+L_2b_2+L_3b_3)+2(b_1-b_2)(L_2b_1+L_1b_2)+2(b_1-b_3)(L_1b_3+L_3b_1)\right]$$

$B_{1,4}^w$, $B_{1,7}^w$ from the above will circle symmetrical substitution of the subscript 1,2,3.

$$B_{1,2}^w = \frac{1}{(DT)^2}\left[2b_1^2(c_3L_2-c_2L_3)+4L_1b_1(b_2c_3-b_3c_2)+(c_3-c_2)(b_1b_2L_3+p_1p_3L_2+b_2b_3L_1)\right]$$

$B_{1,5}^w$, $B_{1,8}^w$ from the above with circle-symmetrical substitution of the subscripts 1,2,3.

$$B^W_{1,3}=\frac{1}{(DT)^2}\left[2b_1^2(b_2L_3-b_3L_2)+(b_2-b_3)(b_1b_2L_3+b_1b_3L_2+b_2b_3L_1)\right]$$

$B^W_{1,6}$, $B^W_{1,9}$ from the above will circle symmetrical substitution of the subscripts 1,2,3.

$$B^W_{2,1}=\frac{1}{(DT)^2}\left[-2c_1(L_1c_1+L_2c_2+L_3c_3)+2(c_1-c_2)(L_2c_1+L_1c_2)+2(c_1-c_3)(L_1c_3+L_3c_1)\right]$$

$B^W_{2,4}$, $B^W_{2,7}$ from the above as previously.

$$B^W_{2,2}=\frac{1}{(DT)^2}\left[2c_1^2(c_3L_2-c_2L_3)+(c_3-c_2)(c_1c_2L_3+c_1c_3L_2+c_2c_3L_1)\right]$$

$B^W_{2,5}$, $B^W_{2,8}$ from the above as previously.

$$B^W_{2,3}=\frac{1}{(DT)^2}\left[2c_1^2(b_2L_3-b_3L_2)+4L_1c_1(c_3b_2-c_2b_3)+(b_2-b_3)(c_1c_2L_3+c_1c_3L_2+c_2c_3L_1)\right]$$

$B^W_{2,6}$, $B^W_{2,9}$ from the above as previously.

$$B^W_{3,1}=\frac{1}{(DT)^2}\left[-2b_1(c_1L_1+c_2L_2+c_3L_3)+2(b_1-b_2)(L_2c_1+L_1c_2)+2(b_1-b_3)(L_1c_3+L_3c_1)\right]$$

$B^W_{3,4}$, $B^W_{3,7}$ from the above as previously.

$$B^W_{3,2}=\frac{1}{(DT)^2}\left\{2b_1c_1(c_3L_2-c_2L_3)+2c_1L_1(b_2c_3-b_3c_2)+\frac{c_3-c_2}{2}\left[L_1(c_3b_2+c_2b_3)+L_2(c_1b_3+c_3b_1)+L_3(c_1b_2+c_2b_1)\right]\right\}$$

$B^W_{3,5}$, $B^W_{3,8}$ from the above as previously.

$$B^W_{3,3}=\frac{1}{(DT)^2}\left\{2b_1c_1(b_2L_3-b_3L_2)+2b_1L_1(c_3b_2-b_3c_2)+\frac{b_2-b_3}{2}\left[L_1(c_3b_2+c_2b_3)+L_2(c_1b_3+c_3b_1)+L_3(c_1b_2+c_2b_1)\right]\right\}$$

$B^W_{3,6}$, $B^W_{3,9}$ from the above as previously.

Formation of $[B^{s1}]$ Matrix

$[B^{s1}]$ is a matrix of 3 rows and 6 columns.

$$[B^{s1}] = \frac{1}{DT} \begin{bmatrix} -b_1 & \phi. & -b_2 & \phi. & -b_3 & \phi. \\ \phi. & -c_1 & \phi. & -c_2 & \phi. & -c_3 \\ -c_1 & -b_1 & -c_2 & -b_2 & -c_3 & -b_3 \end{bmatrix}$$

Formation of $[B^{s2}]$ Matrix

$[B^{s2}]$ is a matrix of 2 rows and 6 columns

$$[B^{s2}] = \begin{bmatrix} L_1 & \phi. & L_2 & \phi. & L_3 & \phi. \\ \phi. & L_1 & \phi. & L_2 & \phi. & L_3 \end{bmatrix}$$

Formation of $[B^{uv}]$ Matrix

$[B^{uv}]$ is a matrix of 3 rows and 6 columns

$$[B^{uv}] = \frac{1}{DT} \begin{bmatrix} b_1 & \phi. & b_2 & \phi. & b_3 & \phi. \\ \phi. & c_1 & \phi. & c_2 & \phi. & c_3 \\ c_1 & b_1 & c_2 & b_2 & c_3 & b_3 \end{bmatrix}$$

The stiffness, stress and load matrices $[K_n]$, $[S]$, $\{R_n\}$ are obtained through equations (4.8), (4.5) employing the numerical integration formulae of Appendix VI.

## APPENDIX III

FORMATION OF MATRICES (Reference to PDS24)

The deflection is a complete 5th order polynomial with 21 terms

$$w \equiv \phi(L_1,L_2) = a_1 + a_2L_1 + a_3L_2 + a_4L_1^2 + a_6L_2^2 + a_7L_1^3 + a_8L_1^2L_2 + a_9L_1L_2^2 +$$

$$+ a_{10}L_2^3 + a_{11}L_1^4 + a_{12}L_1^3L_2 + a_{13}L_1^2L_2^2 + a_{14}L_1L_2^3 + a_{15}L_2^4 + a_{16}L_1^3 + a_{17}L_1^4L_2 + a_{18}L_1^3L_2^2 \pm$$

$$+ a_{19}L_1^2L_2^3 + a_{20}L_1L_2^4 + a_{21}L_2^5 ,$$

with $\phi,_3 = \phi$. the relationships (AI.1) Appendix I .

$$\frac{\partial\phi}{\partial\bar{n}_1} = \left[(b_1^2+c_1^2)\phi,_1 + (b_1b_2+c_1c_2)\phi,_2\right]/(P_2P_3)DT$$

$$\frac{\partial\phi}{\partial\bar{n}_2} = \left[(b_1b_2+c_1c_2)\phi,_1 + (b_2^2+c_2^2)\phi,_2\right]/(P_1P_3)DT$$

$$\frac{\partial\phi}{\partial\bar{n}_3} = \left[(b_1b_3+c_1c_3)\phi,_1 + (b_2b_3+c_2c_3)\phi,_2\right]/(P_1P_2)DT$$

$$\begin{aligned} k_1 &= b_1b_2+c_1c_2 \\ k_2 &= b_2b_3+c_2c_3 \\ k_3k_3 &= b_1b_3+c_1c_3 \\ k_1+k_2 &= -(b_2^2+c_2^2) \\ k_2+k_3 &= -(b_3^2+c_3^2) \\ k_1+k_3 &= -(b_1^2+c_1^2) \end{aligned} \qquad \text{(AIII.1)}$$

4th Order

$$\phi,_1 = 5a_{16}L_1^4 + 4a_{17}L_1^3L_2 + 3a_{18}L_1^2L_2^2 + 2a_{19}L_1L_2^3 + a_{20}L_2^4$$

4th Order

$$\phi,_2 = a_{17}L_1^4 + 2a_{18}L_1^3L_2 + 3a_{19}L_1^2L_2^2 + 4a_{20}L_1L_2^3 + a_{21}L_2^4$$

$$\text{side } \overline{23}\ L_1 = \phi.\ \frac{\partial w}{\partial \bar{n}_1} \text{ cubic} \Rightarrow 5k_1 a_{21} - (k_1+k_3) a_{20} = \phi.$$

$$\text{side } \overline{31}\ L_2 = \phi.\ \frac{\partial w}{\partial \bar{n}_2} \text{ cubic} \Rightarrow 5k_1 a_{16} - (k_1+k_2) a_{17} = \phi. \tag{AIII.2}$$

$$\text{side } \overline{12}\ L_1+L_2 = 1.\ \frac{\partial w}{\partial \bar{n}_3} \text{ cubic} \Rightarrow$$

$$5k_3 a_{16} + (k_2-4k_3) a_{17} + (3k_3-2k_2) a_{18} + (3k_2-2k_3) a_{19} + (k_3-4k_2) a_{20} + 5k_2 a_{21} = \phi.$$

from the above three equations we can express the coefficients $a_{20}$, $a_{17}$, $a_{19}$ as functions of the rest $a_{16}$, $a_{18}$, $a_{21}$ if we substitute these expressions in the initial polynomial form we will have a new polynomial with 18 terms which will fulfil the conditions $\frac{\partial w}{\partial n_i} = \phi.$ $i = 1 \div 3$ so the normal slope will be continuous across interelement boundaries. if $\{w_i, w_{xi}, w_{yi}, w_{xxi}, w_{xyi}, w_{yyi}\}$ $i = 1,3$ at three corner modes are chosen as degrees of freedom the polynomial is of the form

$$\begin{aligned} w = F(L_1,L_2) = {} & B_1+B_2L_1+B_3L_2+B_4L_1^2+B_5L_1L_2+B_6L_2^2+B_7L_1^3+B_8L_1^2L_2+B_9L_1L_2^2+ \\ & +B_{10}L_2^3+B_{11}L_1^4+B_{12}L_1^3L_2+B_{13}L_1^2L_2^2+B_{14}L_1L_2^3+B_{15}L_2^4+B_{16}[L_1^5+m_1L_1^4L_2+m_2L_1^2L_2^3]+ \\ & +B_{17}[L_1^3L_2^2+m_3L_1^2L_2^3]+B_{18}[L_2^5+m_4L_1L_2^4+m_5L_1^2L_2^3] \end{aligned} \tag{AIII.3}$$

$$m_1 = \frac{5k_1}{k_1+k_2} \qquad m_2 = \frac{5(k_3k_2+k_1k_2-3k_1k_3)}{(k_1+k_2)(2k_3-3k_2)} \qquad m_3 = \frac{3k_3-2k_3}{2k_3-3k_2} \tag{AIII.4}$$

$$m_4 = \frac{5k_1}{k_1+k_3} \qquad m_5 = \frac{5(k_3k_2+k_1k_3-3k_1k_2)}{(k_1+k_3)\ (2k_3-3k_2)}$$

are in a matrix form $w = [F]\{B\}$, $[F] = [F_i]$ $i = 1 \div 18$ a row matrix with 18 elements and $\{B\}$ a vector with 18 elements.

The elements of matrix [F] are as follows

$$F_1 = 1,\ F_2 = L_1,\ F_3 = L_2,\ F_4 = L_1^2,\ F_5 = L_1L_2,\ F_6 = L_2^2,\ F_7 = L_1^3,\ F_8 = L_1^2L_2$$

$$F_9 = L_1L_2^2,\ F_{10} = L_2^3,\ F_{11} = L_1^4,\ F_{12} = L_1^3L_2,\ F_{13} = L_1^2L_2^2,\ F_{14} = L_1L_2^3,\ F_{15} = L_2^4 \qquad \text{(AIII.5)}$$

$$F_{16} = L_1^5 + m_1L_1^4L_2 + m_2L_1^2L_3^2,\quad F_{17} = L_1^3L_2^2 + m_3L_1^2L_3^2,\quad F_{18} = L_2^5 + m_4L_1L_2^4 + m_sL_1^2L_2^3$$

The displacement within the element can be expressed in the form

$$\begin{Bmatrix} w \\ w,_x \\ w,_y \\ w,_{xx} \\ w,_{xy} \\ w,_{yy} \end{Bmatrix} = \begin{array}{|c|c|c|c|c|c|c|} \hline 1 & \phi. & \phi. & \phi. & \phi. & \phi. & [F] \\ \hline \phi. & b_1/DT & b_2/DT & \phi. & \phi. & \phi. & [F],_1 \\ \hline \phi & c_1/DT & c_2/DT & \phi. & \phi. & \phi. & [F],_2 \\ \hline \phi. & \phi. & \phi. & b_1^2/(DT)^2 & 2b_1b_2/(DT)^2 & b_2^2/(DT)^2 & [F],_{11} \\ \hline \phi. & \phi. & \phi. & b_1c_1/(DT)^2 & (b_1c_2+b_2c_1)/(DT)^2 & b_2c_2/(DT)^2 & [F],_{21} \\ \hline \phi. & \phi. & \phi. & c_1^2/(DT)^2 & 2c_1c_2/(DT)^2 & c_2^2/(DT)^2 & [F],_{22} \\ \hline \end{array} \{B\}$$

(AIII.6)

the subscripts at [F] indicates derivatives of $L_1$, $L_2$ respectively so

$$[F],_1 = \partial[F]/\partial L_1 \qquad [F],_{12} = \partial^2[F]/\partial L_1\partial L_2 \quad \text{etc.}$$

thus

$$\begin{Bmatrix} w \\ w,_x \\ w,_y \\ w,_{xx} \\ w,_{xy} \\ w,_{yy} \end{Bmatrix} = [GC]\ [Fgen]\ \{B\} \qquad \text{(AIII.7)}$$

$$\{\delta_o^W\} = \begin{bmatrix} [GC] & [\phi.] & [\phi.] \\ [\phi.] & [GC] & [\phi.] \\ [\phi.] & [\phi.] & [GC] \end{bmatrix} \begin{bmatrix} [Fgen]^{L_1=1}_{L_2=0} \\ [Fgen]^{L_1=0}_{L_2=1} \\ [Fgen]^{L_1=0}_{L_2=0} \end{bmatrix} \{B\} \qquad \text{(AIII.8)}$$

$$\{\delta_o^W\}^T = \left\{ W_i,\ W,_{xi},\ W,_{yi},\ W,_{xxi},\ W,_{xyi},\ W,_{yyi},\ \ldots\ldots \right\}_{i=1 \div 3} \qquad \text{(AIII.9)}$$

$$\{\delta_o^W\}^T = [GCdiag]\ [Fgen]_{all}\ \{B\} \qquad \text{(AIII.10)}$$

$$\{B\} = [Fgen]^{-1}_{all}\ [GCdiag]^{-1}\ \{\delta_o^W\} = [T]^{-1}\ \{\delta_o^W\} \qquad \text{(AIII.11)}$$

$$[GCdiag]^{-1} = \begin{bmatrix} [GC]^{-1} & [\phi.] & [\phi.] \\ [\phi.] & [GC]^{-1} & [\phi.] \\ [\phi.] & [\phi.] & [GC]^{-1} \end{bmatrix} \qquad \text{(AIII.12)}$$

$[GC]^{-1} =$

| | | | | | |
|---|---|---|---|---|---|
| 1 | $\phi.$ | $\phi.$ | $\phi.$ | $\phi.$ | $\phi.$ |
| $\phi.$ | $c_2$ | $-b_2$ | $\phi.$ | $\phi.$ | $\phi.$ |
| $\phi.$ | $-c_1$ | $b_1$ | $\phi.$ | $\phi.$ | $\phi.$ |
| $\phi.$ | $\phi.$ | $\phi.$ | $c_2^2$ | $-2c_2b_2$ | $b_2^2$ |
| $\phi.$ | $\phi.$ | $\phi.$ | $-c_1c_2$ | $c_1b_2 + c_2b_1$ | $-b_1b_2$ |
| $\phi.$ | $\phi.$ | $\phi.$ | $c_1^2$ | $-2c_1b_1$ | $b_1^2$ |

(AIII.13)

$[F_{gen}]^{-1}_{all}$ is a matrix of 18 x 18 size. We will indicate as (i,j) the element of the ith row and jth column. The non-mentioned elements are equal zero.

( 1,13)=1

( 2,14)=1

( 3,15)=1

( 4.16)=0.5

( 5,17)=1

( 6,18)=0.5

( 7, 1)=10 ,( 7, 2)=-4 ,( 7, 4)=0.5 ,( 7,13)=-10 ,( 7,14)=-6 ,( 7,16)=-1.5

( 8, 1)=$6m_1$ ,( 8, 2)=$-3m_1$ ,( 8, 3)=3 ,( 8, 4)=$0.5m_1$ ,( 8, 5)=-1 ,( 8,13)=$-6m_1$

( 8,14)=$-3m_1$,( 8,15)=-3 ,( 8,16)=$0.5m_1$,( 8.17)=-2

( 9, 7)=$6m_4$ ,( 9, 8)=3 ,( 9, 9)=$-3m_4$ ,( 9,11)=-1 ,( 9,12)=$0.5m_4$,( 9,13)=$-6m_4$

( 9,14)=-3 ,( 9,15)=$-3m_4$ ,( 9,17)=-2 ,( 9,18)=$-0.5m_4$

(10, 7)=10 ,(10, 9)=-4 ,(10,12)=0.5 ,(10,13)=-10 ,(10,15)=-6 ,(10,18)=-1.5

(11, 1)=-15 ,(11, 2)=7 ,(11, 4)=-1 ,(11,13)=15 ,(11,14)=8 ,(11,16)=1.5

(12, 1)=$-12m_1$,(12, 2)=$6m_1$ ,(12, 3)=-2 ,(12, 4)=$-m_1$ ,(12, 5)=1 ,(12,13)=$12m_1$

(12,14)=$6m_1$ ,(12,15)=2 ,(12,16)=$m_1$ ,(12,17)=1

(13, 1)=$-6(m_1+m_2)/(1-m_3)$ ,(13, 2)=$3(m_1+m_2)/(1-m_3)$ ,(13, 3)=$-3/(1-m_3)$

(13, 4)=$-(m_1+m_2)/2(1-m_3)$,(13,5)=$1/(1-m_3)$,(13,6)=$-m_3/2(1-m_3)$,(13,7)=$6(m_3m_4-m_5)/(1-m_3)$

(13, 8)=$3m_3/(1-m_3)$, (13,9)=$3(m_3-m_3m_4)/(1-m_3)$,(13,10)=$1/2(1-m_3)$,(13,11)=$-m_3/(1-m_3)$

(13,12)=$(m_3m_4-m_5)/2(1-m_3)$,(13,13)=$6(m_1+m_2+m_3m_4)/(1-m_3)$

(13,14)=$3(m_1+m_2-m_3)/(1-m_3)$,(13,15)=$3(1+m_5-m_3m_4)/(1-m_3)$

(13,16)= $(m_1+m_2-1)/2(1-m_3)$,(13,17)=2, (13,18)=$(m_3+m_3-m_3m_4)/2(1-m_3)$

(14, 7)=$-12m_4$,(14, 8)=-2 ,(14, 9)=$6m_4$ ,(14,11)=1 ,(14,12)=$-m_4$

(14,13)=$12m_4$,(14,14)=2 ,(14,15)=$6m_4$ ,(14,17)=1 ,(14,18)=$m_4$

(15, 7)=-15 ,(15, 9)=7 ,(15,12)=-1 ,(15,13)=15 ,(15,15)=8 ,(15,18)=1.5

(16, 1)=6 ,(16, 2)=-3 ,(16, 4)=0.5 ,(16,13)=-6 ,(16,14)=-3 ,(16,16)=-0.5

(17, 1)=$6(m_1+m_2)/(1-m_3)$ ,(17, 2)=$-3(m_1+m_2)/(1-m_3)$, (17, 3)=$3/(1-m_3)$

(17, 4)=$(m_1+m_2)/2(1-m_3)$ ,(17, 5)=$-1/(1-m_3)$ , (17, 6)=$1/2(1-m_3)$

(17, 7)=$6(m_3-m_4)/(1-m_3)$ ,(17, 8)=$-3/(1-m_3)$ , (17, 9)=$3(m_4-m_5)/(1-m_3)$

(17,10)=$-1/2(1-m_3)$ ,(17,11)=$1/(1-m_3)$ , (17,12)=$(m_3-m_4)/2(3-m_3)$

(17,13)=$6(m_4-m_5-m_1-m_2)/(1-m_3)$, (17,14)=$3(1-m_1-m_2)/(1-m_3)$

(17,15)=$3(m_4-m_5-1)/(1-m_3)$,(17,16)=$(1-m_1-m_2)/2(1+m_3)$

(17,18)=$(m_4-m_5-1)/1(1-m_3)$

(18, 7)=6 , (18, 9)=-3 ,(18,12)=0.5 , (18,13)=-6, (18,15)=-3, (18,18)=-0.5

$$1 - m_3 = \frac{k_2 + K_3}{3k_2 - 2k_3} = - \frac{b_3^2 + c_3^2}{b_3(3b_2 - 2b_1) + c_3(3c_2 - 2c_1)}$$

has always a real value different from zero for a triangle.

So the inverse matrix $[Fgen]^{-1}_{all}$ can be always evaluated

$$\{\varepsilon\} = \begin{bmatrix} [B^w] & [B^{s1}] \\ \phi. & [B^{s2}] \end{bmatrix} \begin{bmatrix} \{\delta^w_o\} \\ \{\delta^s_o\} \end{bmatrix} \qquad \text{(AIII.14)}$$

$[B^{s1}]$, $[B^{s2}]$ identical with the ones defined for PDS15 - DDS21.

$[B^w]$ is a matrix of 18 x 18 size defined by the following relationships.

As it has been written at (AIII.6) the second derivatives (part of the strains ) are given by the form

$$\begin{Bmatrix} w,_{xx} \\ w,_{yy} \\ 2w,_{xy} \end{Bmatrix} = \begin{bmatrix} b_1^2/(DT)^2 & 2b_1b_2/(DT)^2 & b_2^2/(DT)^2 \\ c_1^2/(DT)^2 & 2c_1c_2/(DT)^2 & c_2^2/(DT)^2 \\ 2b_1c_1/(DT)^2 & 2(b_1c_2 + b_2c_1)/(DT)^2 & 2b_2c_2/(DT)^2 \end{bmatrix} \begin{bmatrix} [F],_{11} \\ [F],_{21} \\ [F],_{22} \end{bmatrix} \{B\} \qquad \text{(AIII.15)}$$

$[F],_{11} = \partial^2[F]/\partial L_1^2$ etc. as defined previously.

Thus:

$$\begin{Bmatrix} w,_{xx} \\ w,_{yy} \\ 2w,_{xy} \end{Bmatrix} = [B_1^w] \{B\} = [B_1^w] [T]^{-1} \{\delta^w_o\} \qquad \text{(AIII.16)}$$

with $[B^w] = [B_1^w] [T]^{-1}$ (AIII.17)

The stiffness, stress and load matrices $[k_n]$, $[S]$, $\{R_n\}$ are obtained through equations (4.8), (4.5) employing the numerical integration formulae of Appendix VI.

## APPENDIX IV

Formation of Stiffness Matrix for the model PMX24

$$\begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} = [N_m^o]\ \{M_o^e\},\quad \begin{Bmatrix} Q_x \\ Q_y \end{Bmatrix} = \begin{Bmatrix} M_{xx,x} + M_{xy,y} \\ M_{yy,y} + M_{xy,z} \end{Bmatrix} = [N_m^1]\{M_o^e\},\quad \begin{Bmatrix} W_{,x} \\ W_{,y} \end{Bmatrix} = [N_w^2]\ \{\delta_o^w\}$$

$$\{M_o^e\}^T = \{M_{xxi},\ M_{yyi},\ M_{xyi}\}_{i=1\div 6} \qquad \{\delta_o^w\}^T = \{w_i\}_{i=1\div 6}$$

$$[N_m^o] = \begin{bmatrix} N_1 & \phi. & \phi. & N_2 & \phi. & \phi. & N_3 & \phi. & \phi. & N_4 & \phi. & \phi. & N_5 & \phi. & \phi. & N_6 & \phi. & \phi. \\ \phi. & N_1 & \phi. & \phi. & N_2 & \phi. & \phi. & N_3 & \phi. & \phi. & N_4 & \phi. & \phi. & N_5 & \phi. & \phi. & N_6 & \phi. \\ \phi. & \phi. & N_1 & \phi. & \phi. & N_2 & \phi. & \phi. & N_3 & \phi. & \phi & N_4 & \phi. & \phi. & N_5 & \phi. & \phi. & N_6 \end{bmatrix}$$

$$N_1 = (2L_1 - 1)L_1\ ,\qquad N_2 = (2L_2 - 1)L_2\ ,\qquad N_3 = (2L_3 - 1)L_3$$

$$N_4 = 4L_2L_3\ ,\qquad N_5 = 4L_1L_3\ ,\qquad N_6 = 4L_1L_2\ .$$

$$[N_m^1] = \begin{bmatrix} N_{,x1} & \phi. & N_{,y1} & N_{,x2} & \phi. & N_{,y2} & N_{,x3} & \phi. & N_{,y3} & N_{,x4} & \phi. & N_{,y4} & N_{,x5} & \phi. & N_{,y5} & N_{,x6} & \phi. & N_{,y6} \\ \phi. & N_{,y1} & N_{,xi} & \phi. & N_{,y2} & N_{,x2} & \phi. & N_{,y3} & N_{,x3} & \phi. & N_{,y4} & N_{,x4} & \phi. & N_{,y5} & N_{,x5} & \phi. & N_{,x6} & N_{,x6} \end{bmatrix}$$

$$[N_w^2] = \begin{bmatrix} N_{,x1} & N_{,x2} & N_{,x3} & N_{,x4} & N_{,x5} & N_{,x6} \\ N_{,y1} & N_{,y2} & N_{,y3} & N_{,y4} & N_{,y5} & N_{,y6} \end{bmatrix} \qquad \{\sigma b\} = \begin{bmatrix} [N_m^o] \\ [N_m^1] \end{bmatrix} \{M_o^e\} = [N_{mq}]\ \{M_o^e\}$$

$$N'_{x1} = b_1(4L_1 - 1)/DT \qquad N'_{y1} = c_1(4L_1 - 1)/DT$$

$$N'_{x2} = b_2(4L_2 - 1)/DT \qquad N'_{y2} = c_2(4L_2 - 1)/DT$$

$$N'_{x3} = b_3(4L_3 - 1)/DT \qquad N'_{y3} = c_3(4L_3 - 1)/DT$$

$$N'_{x4} = 4(b_2L_3 + b_3L_2)/DT \qquad N'_{y4} = 4(c_2L_3 + c_3L_2)/DT$$

$$N'_{x5} = 4(b_1L_3 + b_3L_1)/DT \qquad N'_{y5} = 4(c_1L_3 + c_3L_1)/DT$$

$$N'_{x6} = 4(b_2L_1 + b_1L_2)/DT \qquad N'_{y6} = 4(c_2L_1 + c_2L_2)/DT$$

$$[K_n^{mw}] = \iint_{A_n} [N_m^1]^T \; [N_w^2] \; dA$$

$$[K_n^{mq}] = -\iint_{A_n} [N_{mq}]^T \; [C_b] \; [N_{mq}] \; dA$$

The integrals are evaluated employing the numerical integration formulae of Appendix VI.

## APPENDIX V

FORMATION OF MATRICES (Reference to PRO18, DRO30)

Formation of [B]

$$\{\varepsilon\} = \begin{Bmatrix} a_{xx} \\ a_{yy} \\ a_{xy} \\ \gamma_x \\ \gamma_y \\ \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \end{Bmatrix} = [B] \quad \{\delta_o^e\} \qquad \text{(AV.1)}$$

$$\left.\begin{aligned}
a_{xx} &= \theta_{x,x} \\
a_{yy} &= \theta_{y,y} \\
a_{xy} &= \theta_{x,y} + \theta_{y,x} \\
\gamma_x &= w_{,x} - \theta_x \\
\gamma_y &= w_{,y} - \theta_y \\
\varepsilon_{xx} &= u_{,x} \\
\varepsilon_{yy} &= v_{,y} \\
\varepsilon_{xy} &= u_{,y} + V_{,x}
\end{aligned}\right\} \qquad \text{(AV.2)}$$

$$\{\delta_o^e\} = \begin{Bmatrix} \delta_o^\theta \\ \delta_o^w \\ \delta_o^{uv} \end{Bmatrix} \tag{AV.3}$$

$\{\delta_o^\theta\}$ vector of nodal degrees of freedom with nodal total rotation s.

$\{\delta_o^w\}$ vector of nodal degrees of freedom with nodal transverse displacements.

$\{\delta_o^{uv}\}$ vector of nodal degrees of freedom with nodal in plane displacements.

$$\{\delta_o^\theta\}^T = [\theta_{xi}, \theta_{yi}, \ldots\ldots] \quad i = 1 \div 6 \tag{AV.4}$$

$$\{\delta_o^w\}^T = [w_i, \ldots\ldots\ldots\ldots] \quad i = 1 \div 7 \tag{AV.5}$$

$$\{\delta_o^{uv}\}^T = [u_i, v_i, \ldots\ldots] \quad i = 1 \div 7 \tag{AV.6}$$

$$\{\varepsilon\} = \begin{bmatrix} [B_1^\theta] & [\phi.] & [\phi.] \\ [B_2^\theta] & [B^w] & [\phi.] \\ [\phi.] & [\phi.] & [B^{uv}] \end{bmatrix} \begin{bmatrix} \{\delta_o^\theta\} \\ \{\delta_o^w\} \\ \{\delta_o^{uv}\} \end{bmatrix} \tag{AV.7}$$

## Formation of $[B_1^\theta]$ matrix

$[B_1^\theta]$ is a matrix of 3 rows and ]2 columns.

| | | |
|---|---|---|
| $B_{1,1}^\theta$ | $= b_1(4L_1-1)/DT$ | $B_{1,3}^\theta$, $B_{1,5}^\theta$ form $B_{1,1}^\theta$ with circle symmetrical substitution of the subscripts 1, 2, 3. |
| $B_{3,2}^\theta$ | $= B_{1,1}^\theta$ | $B_{3,4}^\theta$, $B_{3,6}^\theta$ from $B_{3,2}^\theta$ as previously |
| $B_{1,7}^\theta$ | $= (4b_2L_3 + 4b_3L_2)/DT$ | $B_{1,9}^\theta$, $B_{1,11}^\theta$ from $B_{1,7}^\theta$ as previously |
| $B_{3,8}^\theta$ | $= B_{17}^\theta$ | $B_{3,10}^\theta$, $B_{3,12}^\theta$ from $B_{3,8}^\theta$ as previously |
| $B_{2,2}^\theta$ | $= c_1(4L_1 - 1)/DT$ | $B_{2,4}^\theta$, $B_{2,6}^\theta$ from $B_{2,2}^\theta$ as previously |
| $B_{3,1}^\theta$ | $= B_{2,2}^\theta$ | $B_{3,3}^\theta$, $B_{3,5}^\theta$ from $B_{3,1}^\theta$ as previously |
| $B_{2,8}^\theta$ | $= (4C_2L_3 + 4C_3L_2)/DT$ | $B_{2\ 10}^\theta$, $B_{2,12}^\theta$ from $B_{2,8}^\theta$ as previously |
| $B_{3,7}^\theta$ | $= B_{2,8}^\theta$ | $B_{3,9}^\theta$, $B_{3,11}^\theta$ from $B_{3,7}^\theta$ as previously. |

The rest of the elements in the matrix are zero.

$$[B_2^{\theta}] = -\begin{bmatrix} (2L_1-1)L_1 & \phi. & (2L_2-1)L_2 & \phi. & (2L_3-1)L_3 & \phi. & 4L_2L_3 & \phi. & 4L_1L_3 & \phi. & 4L_1L_2 & \phi. \\ \phi. & (2L_1-1)L_1 & \phi. & (2L_2-1)L_2 & \phi. & (2L_3-1)L_3 & \phi. & 4L_2L_3 & \phi. & 4L_1L_3 & \phi. & 4L_1L_2 \end{bmatrix}$$

$$DIM2 = (b_1L_2L_3 + b_2L_1L_3 + b_3L_1L_2)/DT \qquad DIM3 = (c_1L_2L_3 + c_2L_1L_3 + c_3L_1L_2)/DT$$

$$N_{x1} = b_1(4L_1 - L)/DT + 3DIM2 \quad , \quad N_{x2} = b_2(4L_2-1)/DT + 3DIM2 \quad , \quad N_{x3} = b_3(4L_3 - 1)/DT + 3DIM2$$

$$N_{x4} = 4(b_2L_3 + b_3L_2)/DT - 12DIM2 \quad , \quad N_{x5} = 4(b_1L_3 + b_3L_1)/DT - 12DIM2 \quad , \quad N_{x6} = 4(b_1L_2 + b_2L_1)/DT - 12DIM2$$

$$N_{x7} = 27DIM2$$

$$N_{y1} = c_1(4L_1 - 1)/DT + 3DIM3 \quad , \quad N_{y2} = c_2(4L_2 - 1)/DT + 3DIM3 \quad , \quad N_{y3} = c_3(4L_3 - 1)/DT + 3DIM3$$

$$N_{y4} = 4(c_2L_3 + c_3L_2)/DT - 12DIM3 \quad , \quad N_{y5} = 4(c_1L_3 + c_3L_1)/DT - 12DIM3 \quad , \quad N_{y6} = 4(c_1L_2 + c_2L_1)/DT - 12DIM3$$

$$N_{y7} = 27DIM3$$

$$[B^{w}] = \begin{bmatrix} N_{x1} & N_{x2} & N_{x3} & N_{x4} & N_{x5} & N_{x6} & N_{x7} \\ N_{y1} & N_{y2} & N_{y3} & N_{y4} & N_{y5} & N_{y6} & N_{y7} \end{bmatrix}$$

$$[B^{uv}] = \begin{bmatrix} N_{x1} & \phi. & N_{x2} & \phi. & N_{x3} & \phi. & N_{x4} & \phi. & N_{x5} & \phi. & N_{x6} & \phi. & N_{x7} & \phi. \\ \phi. & N_{y1} & \phi. & N_{y2} & \phi. & N_{y3} & \phi. & N_{y4} & \phi. & N_{y5} & \phi. & N_{y6} & \phi. & N_{y7} \\ N_{y1} & N_{x1} & N_{y2} & N_{x2} & N_{y3} & N_{x3} & N_{y4} & N_{x4} & N_{y5} & N_{x5} & N_{y6} & N_{x6} & N_{y7} & N_{y7} \end{bmatrix}$$

The stiffness, stress and load matrices $[K_n]$, $[S]$, $\{R_n\}$ are obtained through equations (4.8), (4,5) employing the numberical integration formulae of Appendix VI.

## APPENDIX VI

## NUMERICAL INTEGRATION

The quintic numerical integration formulae have been employed as follows

[28,39,61,63,72,115]

| | $L_1$ | $L_2$ | $L_3$ |
|---|---|---|---|
| a | 1/3 | 1/3 | 1/3 |
| b | $\alpha_1$ | $\beta_1$ | $\beta_1$ |
| c | $\alpha_1$ | $\alpha_1$ | $\beta_1$ |
| d | $\beta_1$ | $\beta_1$ | $\alpha_1$ |
| e | $\alpha_2$ | $\beta_2$ | $\beta_2$ |
| f | $\beta_2$ | $\alpha_2$ | $\beta_2$ |
| g | $\beta_2$ | $\beta_2$ | $\alpha_2$ |

$\alpha_1 = 0.05971587$

$\beta_1 = 0.47014206$

$\alpha_2 = 0.79742699$

$\beta_2 = 0.10128651$

where it was possible the integration has been applied straightforward by the use of the formulae

$$\iint_{P_1P_2P_3} L_1^a \; L_2^b \; L_3^c \; dx\,dy = \frac{a!\,b!\,c!}{(a+b+c+2)!}\,DT$$

DT = twice the area of the triangle $P_1P_2P_3$

SANDWICH PLATE BENDING MODELS

COMPUTER LISTING

| | Reference symbol |
|---|---|
| 1. | PDS15 |
| 2. | PDS21 |
| 3. | PMX12 |
| 4. | PMX24 |
| 5. | PDS24 |
| 6. | PDS30 |
| 7. | PRO18 |

1. Reference symbol PDS15

```
 P=1 . PRCLTE=PURH CCPIES=4
= UNIVERSITY, BATCH
WAS: 10:26:39
SIGNED ON AT  11:36:1  ON MON SEP 22/75
PRINT
 *** PLATE DISPLAC.T  MODEL  15 DEGR. OF FREEDOM W, WX, WY, FX, FY  3 NODES**
***** STRESSES  AT THE CENTROID  MX, MY, MXY, QX, QY   ***************
*****   TRANSFORMATION  TO  W, DN, WS, FN, FS   ***********************
****  CUBIC  VARIATION  OF W   LINEAR OF FX,FY   ************************
      SUBROUTINE STIFF
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/STI/X(3,2 ),YCUNC(12),STUCK(36,36),STICK(8,36),FORCE(36),
     1INFO(20)
      COMMON/MAN/BOL(2,36,26),COL(2,8,36),DOL(2,36),CRAN(16,16),NGRAM
      COMMON/PAT/CO2(3,3),XC(3),YC(3)
      DIMENSION TRAN(15,15),SMK(15,15),SMK1(15,15),STR(5,15),TTK(5,15),
     1DADG(15),XM(16,2),YM(16,2),CO3(4,4)
      DO 100 I=1,36
      DO 100 J=1,36
  00  STUCK(J,I)= .
      DO 101 I=1,36
      DO 101 J=1,8
   1  STICK(J,I)= .
      DO 102 I=1,36
  02  FORCE(I)=0.
      DO 103 I=1,3
      XC(I)=X(1,I)
  3   YC(I)=X(2,I)
      IF(NGRAM.EQ.0) GO TO 517
      DO 516 M=1,NGRAM
      DO 516 I=1,2
      XM(M,I)=GRAM(M,I)
  16  YM(M,I)=GRAM(M,I+2)
  17  CONTINUE
      NLAK=INFO(1)
      IF(NLAK.EQ.1.OR.NLAK.EQ.2) GO TO 1 4
      CALL SUBTI
      IF(NLAK.EQ.0) GO TO 105
      NLAK=NLAK-1
      DO 106 I=1,15
      DO 106 J=1,15
   6  BOL(NLAK,J,I)=STUCK(J,I)
      DO 107 I=1,15
      DO 107 J=1,5
   7  COL(NLAK,J,I)=STICK(J,I)
      DO 108 I=1,15
  08  DOL(NLAK,I)=FORCE(I)
      GO TO 105
  04  DO 109 I=1,15
      DO 1 9 J=1,15
  09  STUCK(J,I)=BOL(NLAK,J,I)
      DO 110 I=1,15
      DO 11  J=1,5
  10  STICK(J,I)=COL(NLAK,J,I)
      DO 111 I=1,15
  11  FORCE(I)=DOL(NLAK,I)
  05  CONTINUE
      NLIK=INFO(2)
      IF(NLIK.EQ.0.OR.NLIK.EQ.99) GO TO 112
      DO 113 I=1,15
```

```
      DO 113 J=1,15
  113 SMK(J,I)=STICK(J,I)
      DO 114 I=1,15
      DO 114 J=1,5
  114 STR(J,I)=STICK(J,I)
      DO 115 I=1,NTIK
      K=(I-1)*2+3
      L=INFC(K)
      K1=INFC(K+1)
      DO 583 II=1,15
      DO 584 IJ=1,15
  584 TRAN(IJ,II)= .
  583 TRAN(II,II)=1.
      CALL TRANL(XM,YM,K1,CC3)
      KK=(I-1)*5
      DO 587 IN=1,4
      DO 587 IL=1,4
  587 TRAN(KK+IN+1,KK+IL+1)=CC3(IN,IL)
      CALL TIMES(SMK,TRAN,SMK1,15,15,15,1)
      CALL TIMES(TRAN,SMK1,SMK,15,15,15,2)
      CALL TIMES(STR,TRAN,TIK,5,15,15,1)
      DO 116 NI=1,15
      DO 116 NJ=1,5
  116 STR(NJ,NI)=TIK(NJ,NI)
  115 CONTINUE
      DO 117 I=1,15
      DO 117 J=1,15
  117 STICK(J,I)=SMK(J,I)
      DO 118 I=1,15
      DO 118 J=1,5
  118 STICK(J,I)=STR(J,I)
  112 CONTINUE
      BBRD=0.
      DO 604 J=6,8
      DO 604 I=1,3
  604 IF(X(I,J).NE.0)BBRD=1.
      IF(BBRD.NE.1.)GO TO 560
      DO 273 I=1,15
  273 DADG(I)=0.
      DADG(1)=X(3,6)
      DADG(6)=X(3,7)
      DADG(11)=X(3,8)
      DO 274 I=1,15
  274 FORCE(I)=FORCE(I)+DADG(I)
  560 CONTINUE
      RETURN
      END
      SUBROUTINE TRANL(X,Y,K,TRL)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION X(16,2),Y(16,2),TRL(4,4),X2(2),Y2(2)
      DO 515 J=1,2
      X2(J)=X(K,J)
  515 Y2(J)=Y(K,J)
      G2=X2(2)-X2(1)
      B2=Y2(2)-Y2(1)
      GLEN=DSQRT(G2**2+B2**2)
      DO 100 I=1,4
      DO 100 J=1,4
  100 TRL(I,J)=0.
      IF(GLEN.EQ.0) WRITE(6,700)
```

```
      TRL(1,1)=B2/GLEN
      TRL(1,2)=G2/GLEN
      TRL(1,3)=B2/GLEN
      TRL(2,1)=-G2/GLEN
      TRL(2,2)=B2/GLEN
      TRL(2,3)=-G2/GLEN
      TRL(3,3)=B2/GLEN
      TRL(3,4)=G2/GLEN
      TRL(4,3)=-G2/GLEN
      TRL(4,4)=B2/GLEN
60    FORMAT(/,'      ****ERROR****  SUBROUTINE  TRANI    GLEN----',/)
      RETURN
      END
       **********          VE      THA     VORTHTA        TOY        THEOY    **********
       **********          VE      THA     VORTHTA        TOY        THEOY    **********
         SUBROUTINES        **********************
         SUBROUTINES        **********************
      THE   STIFFNESS   MATRIX    ST
      THE   STRESS      MATRIX    S
      THE   LOAD        MATRIX    FND
      INPUT   FOR   EACH POINT A THE NODA COORDINATES,B THE VALUES OF LOAD
      INPUT   FOR   EACH ELEMENT THE COEFFICIENTS   B(3) ,G(3)
      INPUT   FOR   EACH ELEMENT THE ELASTICITY   MATRIX   E(6)
      SUBROUTINE   SHAPEF(A,B,C,DT,NEK)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/JON/ST(15,15),S(5,15),D(5,5)
      COMMON/JAC/EMP(15,1),PIM(15),P(5),PI(15)
      DIMENSION A(3),B(3),G(3),BE(5,15),EU(5,15)
      DIMENSION SW(3),SX(3),SY(3),SWX(3),SXX(3),SYX(3),SWY(3),SXY(3),SYY
     1(3),SWXX(3),SXXX(3),SYXX(3),SWYY(3),SXYY(3),SYYY(3),SWXY(3),SXXY(3
     2),SYXY(3),SU(3),SV(3),SEY(3),SEY(3),SUX(3),SUY(3),SVX(3),SVY(3)
      DO 2 1 I=1,3
      K=I+1
      IF(K.GT.3)K=K-3
      J=I+2
      IF(J.GT.3)J=J-3
      SW(I)=A(I)+A(I)**2*A(K)+A(I)**2*A(J)-A(I)*A(K)**2-A(I)*A(J)**2
      SY(I)=B(K)*(A(I)**2*A(J)+0.5*A(I)*A(K)*A(J))-B(J)*(A(K)*A(I)**2+
     1C.5*A(I)*A(K)*A(J))
      SX(I)=G(J)*(A(I)**2*A(K)+ .5*A(I)*A(K)*A(J))-G(K)*(A(J)*A(I)**2+
     10.5*A(I)*A(K)*A(J))
      SWX(I)=(1./DT)*(2.*B(I)*(A(I)*A(K)+A(I)*A(J)+A(K)*A(J))+2.*A(I)*
     1A(K)*(B(I)-B(K))+2.*A(I)*A(J)*(B(I)-B(J)))
      SYX(I)=(1./DT)*(2.*B(I)*A(I)*(B(J)*A(K)-B(K)*A(J))+0.5*(B(J)-B(K))
     1*(B(I)*A(K)*A(J)+B(K)*A(I)*A(J)+B(J)*A(I)*A(K)))*(-1. )
      SXX(I)=(1./DT)*(2.*B(I)*A(I)*(G(J)*A(K)-G(K)*A(J))+A(I)**2*(B(K)*
     1G(J)-B(J)*G(K))+0.5*(G(J)-G(K))*(B(I)*A(K)*A(J)+B(K)*A(I)*A(J)+
     2B(J)*A(I)*A(K)))
      SWY(I)=(+1./DT)*(2.*G(I)*(A(I)*A(K)+A(I)*A(J)+A(K)*A(J))+2.*A(I)*
     1A(K)*(G(I)-G(K))+2.*A(I)*A(J)*(G(I)-G(J)))
      SYY(I)=(-1./DT)*(2.*G(I)*A(I)*(B(J)*A(K)-B(K)*A(J))+A(I)**2*(G(K)*
     1B(J)-G(J)*B(K))+0.5*(B(J)-B(K))*(G(I)*A(K)*A(J)+G(K)*A(I)*A(J)+
     2G(J)*A(I)*A(K)))
      SXY(I)=(+1./DT)*(2.*G(I)*A(I)*(G(J)*A(K)-G(K)*A(J))+0.5*(G(J)-G(K)
     1)*(G(I)*A(K)*A(J)+G(K)*A(I)*A(J)+G(J)*A(I)*A(K)))
      SU(I)=A(I)
      SV(I)=A(I)
      SEY(I)=A(I)
      SEX(I)=A(I)
      SWXX(I)=(+1./DT**2)*(-2.*B(I)*(A(I)*B(I)+A(K)*B(K)+A(J)*B(J))+
```

```
 12.*(B(I)-B(K))*(A(K)*B(I)+A(I)*B(K))+2.*(B(I)-B(J))*(A(I)*B(J)+
 2A(J)*B(I)))
  SYXX(I)=(-1./DT**2)*(2.*B(I)**2*(B(J)*A(K)-B(K)*A(J))+(B(J)-B(K)
 1)*(B(I)*B(K)*A(J)+B(I)*B(J)*A(K)+B(K)*B(J)*A(I)))
  SXXX(I)=(+1./DT**2)*(2.*B(I)**2*(G(J)*A(K)-G(K)*A(J))+4.*A(I)*B(I)
 1*(B(K)*G(J)-B(J)*G(K))+(G(J)-G(K))*(B(I)*B(K)*A(J)+B(I)*B(J)*A(K)+
 2B(K)*B(J)*A(I)))
  SYYY(I)=(+1./ DT**2)*(-2.*G(I)*(A(I)*G(I)+A(K)*G(K)+A(J)*G(J))+2.*
 1(G(I)-G(K))*(A(K)*G(I)+A(I)*G(K))+2.*(G(I)-G(J))*(A(I)*G(J)+A(J)*
 2G(I)))
  SYYY(I)=(-1./DT**2)*(2.*G(I)**2*(B(J)*A(K)-B(K)*A(J))+4.*A(I)*G(I)
 1*(G(K)*B(J)-G(J)*B(K))+(B(J)-B(K))*(G(I)*G(K)*A(J)+G(I)*G(J)*A(K)+
 2G(K)*G(J)*A(I)))
  SXYY(I)=(+1./DT**2)*(2.*G(I)**2*(G(J)*A(K)-G(K)*A(J))+(G(J)-G(K)
 1)*(G(I)*G(K)*A(J)+G(I)*G(J)*A(K)+G(K)*G(J)*A(I)))
  SWXY(I)=(2./DT**2)*(-2.*B(I)*(G(I)*A(I)+G(K)*A(K)+G(J)*A(J))+2.*
 1(B(I)-B(K))*(A(K)*G(I)+A(I)*G(K))+2.*(B(I)-B(J))*(A(I)*G(J)+A(J)*
 2G(I)))
  SYXY(I)=(2./DT**2)*(2.*B(I)*G(I)*(B(J)*A(K)-B(K)*A(J))+2.*B(I)*
 1A(I)*(G(K)*B(J)-G(J)*B(K))+0.5*(B(J)-B(K))*(A(I)*(G(J)*B(K)+G(K)*
 2B(J))+A(K)*(G(I)*B(J)+G(J)*B(I))+A(J)*(G(I)*B(K)+G(K)*B(I))))
 3*(-1.0)
  SXXY(I)=(2./DT**2)*(2.*B(I)*G(I)*(G(J)*A(K)-G(K)*A(J))+2.*G(I)*
 1A(I)*(B(K)*G(J)-B(J)*G(K))+0.5*(G(J)-G(K))*(A(I)*(G(J)*B(K)+G(K)*
 2B(J))+A(K)*(G(I)*B(J)+G(J)*B(I))+A(J)*(G(I)*B(K)+G(K)*B(I))))
  SUX(I)=(1./DT)*B(I)
  SVX(I)=(1./DT)*B(I)
  SUY(I)=(1./DT)*G(I)
  SVY(I)=(1./DT)*G(I)
201 CONTINUE
  DO 401 I=1,15
  DO 401 J=1,5
401 BE(J,I)=0.
  DO 202  J=1,7,3
  I=J
  IF(I.LE.3)K=I
  IF(I.GT.3.AND.I.LE.6)K=I-2
  IF(I.GT.6)K=I-4
  BE(1,J)=SWXX(K)
  BE(2,J)=SWYY(K)
  BE(3,J)=SWXY(K)
  BE(1,J+1)=SYXX(K)
  BE(2,J+1)=SYYY(K)
  BE(3,J+1)=SYXY(K)
  BE(1,J+2)=SXXX(K)
  BE(2,J+2)=SXYY(K)
  BE(3,J+2)=SXXY(K)
202 CONTINUE
  DO 203 J=10,14,2
  I=J-9
  IF(I.LE.2)K=I
  IF(I.GT.2.AND.I.LE.4)K=I-1
  IF(I.GT.4)K=I-2
  BE(1,J)=-SUX(K)
  BE(3,J)=-SUY(K)
  BE(4,J)=SEX(K)
  BE(2,J+1)=-SVY(K)
  BE(3,J+1)=-SVX(K)
  BE(5,J+1)=SEY(K)
203 CONTINUE
```

```
     STIFNESS MATRIX        BY MULTIPLICATION  B *D*B
     STIFNESS MATRIX    SM
     STRESS   MATRIX    S  BY  MULTIPLICATION    D*B
    CALL TIMES(D,BE,S,5,5,15,1)
    CALL TIMES(BT,S,SM,15,5,15,2)
    DO 402 I=1,15
    DO 4 2 J=1,5
02  EN(J,I)=0.
    PROGRAMM    FOR  THE  SHAPE  FUNCTION  MATRIX   N
    DO  213  J=1,7,3
    I=J
    IF(I.LE.3)K=I
    IF(I.GT.3.AND.I.LE.6)K=I-2
    IF(I.GT.6)K=I-4
    EN(1,J)=SN(K)
    EN(2,J)=SNY(K)
    EN(3,J)=SNX(K)
    EN(1,J+1)=SY(K)
    EN(2,J+1)=SYY(K)
    EN(3,J+1)=SYX(K)
    EN(1,J+2)=SX(K)
    EN(2,J+2)=SXY(K)
    EN(3,J+2)=SXX(K)
13  CONTINUE
    DO 214 J=10,14,2
    I=J-9
    IF(I.LE.2)K=I
    IF(I.GT.2.AND.I.LE.4)K=I-1
    IF(I.GT.4)K=I-2
    EN(4,J)=SEX(K)
    EN(5,J+1)=SEY(K)
14  CONTINUE
    CALL TIMES(EN,P,ENP,15,5,1,2)
    RETURN
    END
    SUBROUTINE  SURTI
    IMPLICIT REAL*8 (A-H,O-Z)
    COMMON/STI/X(3,20),ELMOD(12),STUCK(36,36),STICK(8,36),FORCE(36)
   1,INFO(20)
    COMMON/RAT/CO2(3,3),XI(3),Y(3)
    COMMON/JON/SM(15,15),S(5,15),D(5,5)
    COMMON/JAC/ENP(15,1),PIN(15),P(5),PI(15)
    DO 100 I=1,5
    DO 100 J=1,5
    D(J,I)=1.
    D(1,1)=ELMOD(1)
    D(1,2)=ELMOD(2)
    D(2,2)=ELMOD(3)
    D(3,3)=ELMOD(4)/2.
    D(4,4)=ELMOD(5)
    D(5,5)=ELMOD(6)
    FORM   THE  WHOLE  SYMMETRIC   MATRIX  D
    DO 21  J=1,5
    DO 210  I=1,5
    D(I,J)=D(J,I)
1   CONTINUE
    DDDD=ELMOD(12)
    IF(DDDD.NE.0.) D(2,1)=DDDD
    DO 224 I=1,5
24  P(I)=0.
```

```
      P(1)=X(3,5)
      DIMENSION A(3),B(3),C(3),LENG(3),W(7),A1(7),A2(7),A3(7),ENK(15),
     1SMK(15,15),TIK(5,15),M1(15),SMK1(15,15)
      DO 200  I=1,3
      K=I+1
      IF(K.GT.3)K=K-3
      J=I+2
      IF(J.GT.3)J=J-3
      B(I)=Y(K)-Y(J)
      C(I)=XI(J)-XI(K)
      LENG(I)=DSQRT(B(I)**2+C(I)**2)
200   CONTINUE
      DT=B(1)*C(2)-B(2)*C(1)
QUINTIC  INTEGRATION  FOR STIFFNESS MATRIX  STUCK
      AREA   COORDINATES   L1  L2  L3    ARCO        A(I)
 WEIGHTS  OF  INTEGRATION          W(I)
      DATA A1/ .33333333D 00,0.5971587D 00,2*0.47014206D 00,0.79742698D
     1 00,2*0.10128651D 00/,A3/0.33333333D 00,2*0.47014206D 00,0.05971587
     2D 00,2*0.10128651D 00,0.79742698D 00/,W/ .225D 00,3* .132394
     315D 00,3*0.12593918D 00/
      DO 211 I=1,15
      DO 211 J=1,15
211   SMK(J,I)=0.
      DO 222 I=1,15
222   ENK(I)= .
      DO 212 K=1,7
      A(1)=A1(K)
      A(3)=A3(K)
      A(2)=1.-A(1)-A(3)
      CALL SHAPEF(A,B,C,DT, )
      IF(K.NE.1) GO TO 801
      DO 802 I=1,15
      DO 802 J=1,5
802   TIK(J,I)=S(J,I)
801   CONTINUE
      DO 221 I=1,15
221   ENK(I)=ENK(I)+DT/2.*W(K)*ENP(I,1)
      DO 215 I=1,15
      DO 215 J=1,15
215   SMK(J,I)=SMK(J,I)+DT/2.*W(K)*SM(J,I)
212   CONTINUE
      DATA M1/1,3,2,10,11,4,6,5,12,13,7,9,8,14,15/
      DIMENSION TOT(15,15)
      DO 247 I=1,15
      DO 247 J=1,15
247   TOT(J,I)=0.
      DO 248 I=1,15
248   TOT(M1(I),I)=1.
      CALL TIMES(SMK,TOT,SMK1,15,15,15,1)
      CALL TIMES(TOT,SMK1,SMK,15,15,15,2)
      CALL TIMES(TIK,TOT,S,5,15,15,1)
      CALL TIMES(TOT,ENK,ENP,15,15,1,2)
      DO 216 I=1,15
      DO 216 J=1,15
216   STUCK(J,I)=SMK(J,I)
      DO 217 I=1,15
      DO 217 J=1,5
217   STICK(J,I)=S(J,I)
      DO 218 I=1,15
218   FORCE(I)=ENP(I,1)
```

```
      RETURN
      END
      SUBROUTINE TIMES(A,B,P,N,M,L,KOK)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION A(1),B(1),P(1)
      KOK=1   A(M,N) , B(N,L) , P(M,L)    REGULAR    A*B
      KOK=2   A(M,N) , P(M,L) , B(N,L)    TRANSPOSE  A*B
      IR=1
      DO 100 K=1,L
      DO 100 J=1,N
      B(IR)=0.
      GO TO(101,102),KOK
  101 CONTINUE
      DO 103 I=1,M
      IA=M*(I-1)+J
      IB=M*(K-1)+I
  103 B(IR)=B(IR)+A(IA)*P(IB)
      GO TO 100
  102 CONTINUE
      DO 104 I=1,M
      IA=M*(J-1)+I
      IB=M*(K-1)+I
  104 B(IR)=B(IR)+A(IA)*P(IB)
  100 IR=IR+1
      RETURN
      END
```

2. Reference symbol PDS21

```
) P=100 OUTPUT=DUPE COPIES=4
= UNIVERSITY, PATCH
WAS: 11:36:10
SIGNED ON AT  11:36:26 ON MON SEP 22/75
PRINT*
  *** PLATE DISPLAC.MODEL 21 DEG. OF FREED.  W,WX,WY,FX,FY  3 CORNER NODES ***
  ****  +   FX,FY  3 MIDSIDE NODES                          ******************
  ****CUBIC VARIATION FOR W ,  QUADRATIC FOR FX,FY          ******************
  *** STRESSES  AT CENTROID  MXX,MYY,MXY,QX,QY    ***********************
  *** TRANSFORMATION  W, PN, WS, FN, FS            ***********************
    SUBROUTINE STIFF
    IMPLICIT REAL*8 (A-H,O-Z)
    COMMON/STI/X(3,20),YCONC(12),STUCK(36,36),STICK(8,36),FORCE(36),
   1INFO(20)
    COMMON/MAN/BOL(2,36,36),COL(2,8,36),DOL(2,36),GRAM(16,16),NGRAM
    COMMON/PAT/CO2(3,3),XO(3),YO(3)
    DIMENSION TRAN(21,21),SMK(21,21),SMK1(21,21),STR(5,21),TIK(5,21),
   1DAMG(21),XM(16,2),YM(16,2),CO3(4,4)
    DO 100 I=1,36
    DO 100 J=1,36
100 STUCK(J,I)=0.
    DO 101 I=1,36
    DO 101 J=1,8
101 STICK(J,I)=0.
    DO 102 I=1,36
102 FORCE(I)= .
    DO 103 I=1,3
    XO(I)=X(1,I)
103 YO(I)=X(2,I)
    IF(NGRAM.EQ.0) GO TO 517
    DO 516 M=1,NGRAM
    DO 516 I=1,2
    XM(M,I)=GRAM(M,I)
516 YM(M,I)=GRAM(M,I+2)
517 CONTINUE
    NLAK=INFO(1)
    IF(NLAK.EQ.1.OR.NLAK.EQ.2) GO TO 104
    CALL SUBTI
    IF(NLAK.EQ.0) GO TO 105
    NLAK=NLAK-10
    DO 106 I=1,21
    DO 106 J=1,21
106 BOL(NLAK,J,I)=STUCK(J,I)
    DO 107 I=1,21
    DO 107 J=1,5
107 COL(NLAK,J,I)=STICK(J,I)
    DO 108 I=1,21
108 DOL(NLAK,I)=FORCE(I)
    GO TO 105
104 DO 109 I=1,21
    DO 109 J=1,21
109 STUCK(J,I)=BOL(NLAK,J,I)
    DO 110 I=1,21
    DO 110 J=1,5
110 STICK(J,I)=COL(NLAK,J,I)
    DO 111 I=1,21
111 FORCE(I)=DOL(NLAK,I)
105 CONTINUE
    NLIK=INFO(2)
    IF(NLIK.EQ.0.OR.NLIK.EQ.99) GO TO 112
```

```
      DO 113 I=1,21
      DO 113 J=1,21
113   SMK(J,I)=STUCK(J,I)
      DO 114 I=1,21
      DO 114 J=1,5
114   STR(J,I)=STICK(J,I)
      DO 115 I=1,NLTK
      K=(I-1)*2+2
      L=INPU(K)
      K1=INPU(K+1)
      DO 584 II=1,21
      DO 584 JJ=1,21
584   TRAN(JJ,II)=0.
583   TRAN(II,II)=1.
      CALL TRANL(XY,YY,K1,CCB)
      IF(L.GT.3) GO TO 119
      KK=(L-1)*5
      DO 587 IN=1,4
      DO 587 IL=1,4
587   TRAN(KK+IN+1,KK+IL+1)=CCB(IN,IL)
      GO TO 120
119   KK=2*L+7
      DO 121 IN=1,2
      DO 121 IL=1,2
121   TRAN(KK+IN,KK+IL)=CCB(IN,IL)
122   CONTINUE
      CALL TIMES(SMK,TRAN,SMK1,21,21,21,1)
      CALL TIMES(TRAN,SMK1,SMK,21,21,21,2)
      CALL TIMES(STR,TRAN,TIK,5,21,21,1)
      DO 116 NI=1,21
      DO 116 NJ=1,5
116   STR(NJ,NI)=TIK(NJ,NI)
115   CONTINUE
      DO 117 I=1,21
      DO 117 J=1,21
117   STUCK(J,I)=SMK(J,I)
      DO 118 I=1,21
      DO 118 J=1,5
118   STICK(J,I)=STR(J,I)
112   CONTINUE
      BERR0=0.
      DO 604 J=6,8
      DO 604 I=1,3
604   IF(X(I,J).NE.0)BRRR0=1.
      IF(BRRR.NE.1.)GO TO 56
      DO 273 I=1,21
273   DACG(I)=0.
      DACG(1)=X(3,6)
      DACG(6)=X(3,7)
      DACG(11)=X(3,8)
      DO 274 I=1,21
274   FORCE(I)=FORCE(I)+DACG(I)
56    CONTINUE
      RETURN
      END
      SUBROUTINE TRANL(X,Y,K,TRL)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION X(16,2),Y(16,2),TRL(4,4),X2(2),Y2(2)
      DO 515 J=1,2
      X2(J)=X(K,J)
```

```
15 Y2(J)=Y(K,J)
   G2=X2(2)-X2(1)
   B2=Y2(2)-Y2(1)
   GLEN=DSQRT(G2**2+B2**2)
   DO 100 I=1,4
   DO 1   J=1,4
00 TRL(I,J)=0.
   IF(GLEN.EQ.0) WRITE(6,700)
   TRL(1,1)=B2/GLEN
   TRL(1,2)=G2/GLEN
   TRL(1,3)=B2/GLEN
   TRL(2,1)=-G2/GLEN
   TRL(2,2)=B2/GLEN
   TRL(2,3)=-G2/GLEN
   TRL(3,3)=B2/GLEN
   TRL(3,4)=G2/GLEN
   TRL(4,3)=-G2/GLEN
   TRL(4,4)=B2/GLEN
00 FORMAT(/,'      ****ERROR****  SUBROUTINE  TRANL    GLEN----',/)
   RETURN
   END
    ***** ****        MC     THN     VORTHIA       TOY        THEOY    **********
    ***** ****        MC     THN     VORTHIA       TOY        THEOY    **********
      SUBROGRAMMS        ********************
      SUBROGRAMMS        ********************
   THE   STIFFNESS   MATRIX    SM
   THE   STRESS      MATRIX    S
   THE   LOAD        MATRIX    FNP
   INPUT   FOR   EACH POINT A THE AREA COORDINATES,P THE VALUES OF LOAD
   INPUT   FOR   EACH ELEMENT THE COEFFICIENTS   B(3) ,G(3)
   INPUT   FOR   EACH ELEMENT THE ELASTICITY  MATRIX  D(8)
   SUBROUTINE  SHAPEF(A,B,G,DT,NOK)
   IMPLICIT REAL*8 (A-H,O-Z)
   COMMON/JOB/SM(21,21),S(5,21),D(5,5)
   COMMON/JAC/FNP(21,1),PIN(15),P(5),PI(15)
   DIMENSION A(3),B(3),G(3),BF(5,21),FN(5,21)
   DIMENSION SW(3),SX(3),SY(3),SWX(3),SXX(3),SYX(3),SWY(3),SXY(3),SYY
  1(3),SWXX(3),SXYX(3),SYXX(3),SWYY(3),SXYY(3),SYYY(3),SWXY(3),SXXY(3
  2),SYXY(3),SF(6),SFX(6),SFY(6)
   DO 201 I=1,3
   K=I+1
   IF(K.GT.3)K=K-3
   J=I+2
   IF(J.GT.3)J=J-3
   SW(I)=A(I)+A(I)**2*A(K)+A(I)**2*A(J)-A(I)*A(K)**2-A(I)*A(J)**2
   SY(I)=B(K)*(A(I)**2*A(J)+0.5*A(I)*A(K)*A(J))-B(J)*(A(K)*A(I)**2+
  10.5*A(I)*A(K)*A(J))
   SX(I)=G(J)*(A(I)**2*A(K)+0.5*A(I)*A(K)*A(J))-G(K)*(A(J)*A(I)**2+
  1 .5*A(I)*A(K)*A(J))
   SWX(I)=(1./DT)*(2.*B(I)*(A(I)*A(K)+A(I)*A(J)+A(K)*A(J))+2.*A(I)*
  1A(K)*(B(I)-B(K))+2.*A(I)*A(J)*(B(I)-B(J)))
   SYX(I)=(1./DT)*(2.*B(I)*A(I)*(B(J)*A(K)-B(K)*A(J))+0.5*(B(J)-B(K))
  1*(B(I)*A(K)*A(J)+B(K)*A(I)*A(J)+B(J)*A(I)*A(K)))*(-1.0)
   SXX(I)=(1./DT)*(2.*B(I)*A(I)*(G(J)*A(K)-G(K)*A(J))+A(I)**2*(B(K)*
  1G(J)-B(J)*G(K))+0.5*(G(J)-G(K))*(B(I)*A(K)*A(J)+B(K)*A(I)*A(J)+
  2B(J)*A(I)*A(K)))
   SWY(I)=(+1./DT)*(2.*G(I)*(A(I)*A(K)+A(I)*A(J)+A(K)*A(J))+2.*A(I)*
  1A(K)*(G(I)-G(K))+2.*A(I)*A(J)*(G(I)-G(J)))
   SYY(I)=(-1./DT)*(2.*G(I)*A(I)*(B(J)*A(K)-B(K)*A(J))+A(I)**2*(G(K)*
  1B(J)-G(J)*B(K))+ .5*(B(J)-B(K))*(G(I)*A(K)*A(J)+G(K)*A(I)*A(J)+
```

```
 2G(J)*A(I)*A(K)))
  SXY(I)=(+1./DT)*(2.*G(I)*A(I)*(C(J)*A(K)-G(K)*A(J))+ .5*(G(J)-G(K)
 1)*(G(I)*A(K)*A(J)+G(K)*A(I)*A(J)+G(J)*A(I)*A(K)))
  SWXX(I)=(+1./DT**2)*(-2.*B(I)*(A(I)*B(I)+A(K)*B(K)+A(J)*B(J))+
 12.*(B(I)-B(K))*(A(K)*B(I)+A(I)*B(K))+2.*(B(I)-B(J))*(A(I)*B(J)+
 2A(J)*B(I)))
  SYXX(I)=(-1./DT**2)*(2.*B(I)**2*(B(J)*A(K)-B(K)*A(J))+(B(J)-B(K)
 1)*(B(I)*B(K)*A(J)+B(I)*B(J)*A(K)+B(K)*B(J)*A(I)))
  SXXX(I)=(+1./DT**2)*(2.*B(I)**2*(G(J)*A(K)-G(K)*A(J))+4.*A(I)*B(I)
 1*(B(K)*G(J)-B(J)*G(K))+(G(J)-G(K))*(B(I)*B(K)*A(J)+B(I)*B(J)*A(K)+
 2B(K)*B(J)*A(I)))
  SWYY(I)=(+1./ DT**2)*(-2.*G(I)*(A(I)*G(I)+A(K)*G(K)+A(J)*G(J))+2.*
 1(G(I)-G(K))*(A(K)*G(I)+A(I)*G(K))+2.*(G(I)-G(J))*(A(I)*G(J)+A(J)*
 2G(I)))
  SYYY(I)=(-1./DT**2)*(2.*G(I)**2*(B(J)*A(K)-B(K)*A(J))+4.*A(I)*G(I)
 1*(G(K)*B(J)-G(J)*B(K))+(B(J)-B(K))*(G(I)*G(K)*A(J)+G(I)*G(J)*A(K)+
 2G(K)*G(J)*A(I)))
  SXYY(I)=(+1./DT**2)*(2.*G(I)**2*(G(J)*A(K)-G(K)*A(J))+(G(J)-G(K)
 1)*(G(I)*G(K)*A(J)+G(I)*G(J)*A(K)+G(K)*G(J)*A(I)))
  SWXY(I)=(2./DT**2)*(-2.*B(I)*(G(I)*A(I)+G(K)*A(K)+G(J)*A(J))+2.*
 1(B(I)-B(K))*(A(K)*G(I)+A(I)*G(K))+2.*(B(I)-B(J))*(A(I)*G(J)+A(J)*
 2G(I)))
  SYXY(I)=(2./DT**2)*(2.*G(I)*G(I)*(B(J)*A(K)-B(K)*A(J))+2.*B(I)*
 1A(I)*(G(K)*B(J)-G(J)*B(K))+0.5*(B(J)-B(K))*(A(I)*(G(J)*B(K)+G(K)*
 2B(J))+A(K)*(G(I)*B(J)+G(J)*B(I))+A(J)*(G(I)*B(K)+G(K)*B(I))))
 3*(-1.0)
  SXXY(I)=(2./DT**2)*(2.*B(I)*G(I)*(G(J)*A(K)-G(K)*A(J))+2.*G(I)*
 1A(I)*(B(K)*G(J)-B(J)*G(K))+ .5*(G(J)-G(K))*(A(I)*(G(J)*B(K)+G(K)*
 2B(J))+A(K)*(G(I)*B(J)+G(J)*B(I))+A(J)*(G(I)*B(K)+G(K)*B(I))))
  SF(I)=(2.*A(I)-1.)*A(I)
  SF(I+3)=4.*A(K)*A(J)
  SFX(I)=B(I)*(4.*A(I)-1.)/DT
  SFX(I+3)=(4.*B(K)*A(J)+4.*B(J)*A(K))/DT
  SFY(I)=G(I)*(4.*A(I)-1.)/DT
2.1 SFY(I+3)=(4.*G(K)*A(J)+4.*G(J)*A(K))/DT
  DO 401 I=1,21
  DO 401 J=1,5
 1 BE(J,I)=0.
  DO 202  J=1,7,3
  I=J
  IF(I.LE.3)K=1
  IF(I.GT.3.AND.I.LE.6)K=I-2
  IF(I.GT.6)K=I-4
  BE(1,J)=SWXX(K)
  BE(2,J)=SWYY(K)
  BE(3,J)=SWXY(K)
  BE(1,J+1)=SYXX(K)
  BE(2,J+1)=SYYY(K)
  BE(3,J+1)=SYXY(K)
  BE(1,J+2)=SXXX(K)
  BE(2,J+2)=SXYY(K)
  BE(3,J+2)=SXXY(K)
02 CONTINUE
  DO 105 I=1,6
  J=8+2*I
  BE(1,J)=-SFX(I)
  BE(3,J)=-SFY(I)
  BE(4,J)=SF(I)
  BE(2,J+1)=-SFY(I)
  BE(3,J+1)=-SFX(I)
```

```
 5 BE(5,J+1)=SF(I)
    STIFNESS MATRIX        BY MULTIPLICATION  B*D*B
    STIFNESS MATRIX    SM
    STRESS    MATRIX    S   BY   MULTIPLICATION     D*B
   CALL TIMES(D,BE,S,5,5,21,1)
   CALL TIMES(BE,S,SM,21,5,21,2)
   DO 402 I=1,21
   DO 402 J=1,5
 2 EN(J,I)= .
   PROGRAMM     FOR   THE   SHAPE   FUNCTION   MATRIX    N
   DO  213   J=1,7,3
   I=J
   IF(I.LE.3)K=I
   IF(I.GT.3.AND.I.LE.6)K=I-2
   IF(I.GT.6)K=I-4
   EN(1,J)=SM(K)
   EN(2,J)=SMY(K)
   EN(3,J)=SMX(K)
   EN(1,J+1)=SY(K)
   EN(2,J+1)=SYY(K)
   EN(3,J+1)=SYX(K)
   EN(1,J+2)=SX(K)
   EN(2,J+2)=SXY(K)
   EN(3,J+2)=SXX(K)
13 CONTINUE
   DO 110 I=1,6
   J=8+2*I
   EN(4,J)=SF(I)
10 EN(5,J+1)=SF(I)
   CALL TIMES(EN,D,ENP,21,5,1,2)
   RETURN
   END
   SUBROUTINE  SUBTI
   IMPLICIT REAL*8 (A-H,O-Z)
   COMMON/STI/X(3,20),ELMOD(12),STUCK(36,36),STICK(8,36),FORCE(36)
  1,INFO(20)
   COMMON/PAT/CO2(3,3),XI(3),Y(3)
   COMMON/JCN/SM(21,21),S(5,21),D(5,5)
   COMMON/JAC/ENP(21,1),PIN(15),P(5),PI(15)
   DO 100 I=1,5
   DO 100 J=1,5
 1 D(J,I)=0.
   D(1,1)=ELMOD(1)
   D(1,2)=ELMOD(2)
   D(2,2)=ELMOD(3)
   D(3,3)=ELMOD(4)/2.
   D(4,4)=ELMOD(5)
   D(5,5)=ELMOD(6)
   FORM   THE  WHOLE  SYMMETRIC   MATRIX  D
   DO 210 J=1,5
   DO 210 I=1,5
   D(I,J)=D(J,I)
1 CONTINUE
   IF(ELMOD(12).NE.0.) D(2,1)=ELMOD(12)
   DO 224 I=1,5
24 P(I)=0.
   P(1)=X(3,5)
   DIMENSION A(3),B(3),C(3),LENG(3),H(7),A1(7),A2(7),A3(7),ENK(21),
  1SMK(21,21),TIK(5,21),M1(21),SMK1(21,21)
   DO 200  I=1,3
```

```
      K=I+1
      IF(K.GT.3)K=K-3
      J=I+2
      IF(J.GT.3)J=J-3
      B(I)=Y(K)-Y(J)
      C(I)=XI(J)-XI(K)
      LENG(I)=DSQRT(B(I)**2+G(I)**2)
00    CONTINUE
      DT=B(1)*G(2)-B(2)*G(1)
QUINTIC  INTEGRATION  FOR STIFFNESS MATRIX  STUCK
      AREA   COORDINATES   L1  L2  L3     ARCC        A(I)
 WEIGHTS  OF  INTEGRATION          W(I)
      DATA A1/0.33333333D 00,0.05971587D 00,2*0.47014206D 00,0.79742699D
     1    ,2*0.1 1236651D    /,A3/ .33333333D 00,2*0.47014206D 00,0.0597158
     27D 00,2*0.10128651D 00,0.79742699D 00/,W/0.22500000D 00,3*0.132394
     315D 00,3*0.12593918D 00/
      DO 211 I=1,21
      DO 211 J=1,21
11    SMK(J,I)=0.
      DO 222 I=1,21
22    ENK(I)=0.
      DO 212 K=1,7
      A(1)=A1(K)
      A(3)=A3(K)
      A(2)=1.-A(1)-A(3)
      CALL SHAPEF(A,B,G,DT,0)
      IF(K.NE.1) GO TO 801
      DO 802 I=1,21
      DO 802 J=1,5
802   TIK(J,I)=S(J,I)
801   CONTINUE
      DO 221 I=1,21
221   ENK(I)=ENK(I)+DT/2.*W(K)*ENP(I,1)
      DO 215 I=1,21
      DO 215 J=1,21
215   SMK(J,I)=SMK(J,I)+DT/2.*W(K)*SM(J,I)
212   CONTINUE
      DATA M1/1,3,2,1 ,11,4,6,5,12,13,7,9,8,14,15,16,17,19,18,20,21/
      DIMENSION TOT(21,21)
      DO 247 I=1,21
      DO 247 J=1,21
247   TOT(J,I)=0.
      DO 248 I=1,21
248   TOT(M1(I),I)=1.
      CALL TIMES(SMK,TOT,SMK1,21,21,21,1)
      CALL TIMES(TOT,SMK1,SMK,21,21,21,2)
      CALL TIMES(TIK,TOT,S,5,21,21,1)
      CALL TIMES(TOT,ENK,ENP,21,21,1,2)
      DO 216 I=1,21
      DO 216 J=1,21
216   STUCK(J,I)=SMK(J,I)
      DO 217 I=1,21
      DO 217 J=1,5
217   STICK(J,I)=S(J,I)
      DO 218 I=1,21
218   FORCE(I)=ENP(I,1)
      RETURN
      END
      SUBROUTINE TIMES(A,B,R,N,M,L,KCK)
      IMPLICIT REAL*8 (A-H,O-Z)
```

```
      DIMENSION A(1),B(1),C(1)
      KOK=1   A(N,M) , B(M,L) , C(N,L)    REGULAR    A*B
      KOK=2   A(M,N) , B(M,L) , C(N,L)    TRANSPOSE  A'*B
      IC=1
      DO 100 K=1,L
      DO 100 J=1,N
      C(IC)=0.
      GO TO(101,102),KOK
101   CONTINUE
      DO 103 I=1,M
      IA=M*(J-1)+J
      IB=M*(K-1)+I
103   C(IC)=C(IC)+A(IA)*B(IB)
      GO TO 100
102   CONTINUE
      DO 104 I=1,M
      IA=M*(J-1)+I
      IB=M*(K-1)+I
104   C(IC)=C(IC)+A(IA)*B(IB)
      IC=IC+1
      RETURN
      END
```

3. Reference symbol PMX12

```
 P=100 PRCUTE=OUBH COPIES=4
= UNIVERSITY, BATCH
WAS: 11:36:26
IGNED ON AT  11:37:14 ON MON SEP 22/75
RINTR
 ***** MIXED  MODEL  12 DEGREES OF FREEDOM P, MXX, MYY, MXY  3 MODES *****
    ***** STRESSES   CONSTANT   WITHIN  AN ELEMENT  QX ,QY ***********
   ****** TRANSFORMATION   M , MNN , MSS , MNS        ******************
   ****** LINEAR   VARIATION    OF THE  FUNCTIONS     ******************
   SUBROUTINE  STIFF
   IMPLICIT REAL*8 (A-H,O-Z)
   COMMON/STI/X(3,20),YOUNG(12),STUCK(36,36),STICK(8,36),FORCE(36),
  1INFO(20)
   COMMON/MAN/BOL(2,36,36),COL(2,8,36),DOL(2,36),GRAM(16,16),NGRAM
   COMMON/COOR/X1(3),Y1(3)
   DIMENSION TRAN(12,12),SMK(12,12),SMK1(12,12),STR(2,12),
  1TIK(2,12),DADG(12),XM(16,2),YM(16,2),CC3(3,3)
   DO 100 I=1,3
   XL(I)=X(1,I)
   Y1(I)=X(2,I)
   DO 101 I=1,36
   DO 101 J=1,36
 1 STUCK(J,I)=0.
   DO 102 I=1,36
   DO 102 J=1,8
102 STICK(J,I)=0.
   DO 103 I=1,36
 3 FORCE(I)= .
   IF(NGRAM.EQ.0) GO TO 513
   DO 516 M=1,NGRAM
   DO 516 I=1,2
   XM(M,I)=GRAM(M,I)
516 YM(M,I)=GRAM(M,I+2)
513 CONTINUE
   NLAK=INFO(1)
   IF(NLAK.EQ.1.OR.NLAK.EQ.2)GO TO 104
   CALL  SUBTI
   IF(NLAK.EQ.0)GO TO 105
   NLAK=NLAK-1
   DO 106 I=1,12
   DO 106 J=1,12
 6 BOL(NLAK,J,I)=STUCK(J,I)
   DO 113 I=1,12
   DO 113 J=1,2
113 COL(NLAK,J,I)=STICK(J,I)
   DO 108 I=1,12
108 DOL(NLAK,I)=FORCE(I)
   GO TO 105
104 DO 109 I=1,12
   DO 109 J=1,12
109 STUCK(J,I)=BOL(NLAK,J,I)
   DO 114 I=1,12
   DO 114 J=1,2
114 STICK(J,I)=COL(NLAK,J,I)
   DO 111 I=1,12
111 FORCE(I)=DOL(NLAK,I)
105 CONTINUE
   NLIK=INFO(2)
   IF(NLIK.EQ.0.OR.NLIK.EQ.99) GO TO 112
   DO 213 I=1,12
```

```
      DO 213 J=1,12
213   SMK(J,I)=STUCK(J,I)
      DO 214 I=1,12
      DO 214 J=1,2
214   STR(J,I)=STICK(J,I)
      DO 115 I=1,NITK
      K=(I-1)*2+3
      L=INFO(K)
      K1=INFO(K+1)
      DO 583 II=1,12
      DO 584 IJ=1,12
584   TRAN(IJ,II)= .
583   TRAN(II,II)=1.
      CALL TRANL(XM,YM,K1,CC3)
      KK=(L-1)*4
      DO 587 IN=1,3
      DO 587 IL=1,3
587   TRAN(KK+IN+1,KK+IL+1)=CC3(IN,IL)
      CALL TIMES(SMK,TRAN,SMK1,12,12,12,1)
      CALL TIMES(TRAN,SMK1,SMK,12,12,12,2)
      CALL TIMES(STR,TRAN,TIK,2,12,12,1)
      DO 116 NI=1,12
      DO 116 NJ=1,2
116   STR(NJ,NI)=TIK(NJ,NI)
115   CONTINUE
      DO 117 I=1,12
      DO 117 J=1,12
117   STUCK(J,I)=SMK(J,I)
      DO 118 I=1,12
      DO 118 J=1,2
118   STICK(J,I)=STR(J,I)
112   CONTINUE
      BRPO=0.
      DO 604 J=6,8
      DO 604 I=1,3
604   IF(X(I,J).NE.0.)BRPO=1.
      IF(BRPO.NE.1.) GO TO 560
      DO 273 I=1,12
273   DADG(I)=0.
      DADG(1)=X(3,6)
      DADG(5)=X(3,7)
      DADG(9)=X(3,8)
      DO 274 I=1,12
274   FORCE(I)=FORCE(I)+DADG(I)
560   CONTINUE
      RETURN
      END
      SUBROUTINE SUBTI
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/STI/X(3,20),YOUNG(12),STUCK(36,36),STICK(8,36),FORCE(36),
     1INFO(20)
      COMMON/COOR/XL(3),YL(3)
      COMMON/TREA/B(3),G(3),EL(3),DT
      DIMENSION ENM1(2,9),ENM2(2,3),EFB(3,3),EFS(2,2),STMM(9,9),ENM(2,9
     1),STMO(9,9),STN(9,9),STMM(9,3),TT(12,12),SIM(12,12),M1(12),STR(2,
     212),STRI(2,12),GE(12,12)
      DO 100 I=1,3
      K=I+1
      J=I+2
      IF(K.GT.3)K=K-3
```

```
      IF(J.GT.3)J=J-3
      B(I)=YL(K)-YL(J)
      C(I)=XL(J)-XL(K)
100   EL(I)=B(I)**2+C(I)**2
      DT=B(1)*C(2)-B(2)*C(1)
      D11=YOUNG(1)
      D12=YOUNG(2)
      D22=YOUNG(3)
      D33=YOUNG(4)/2.
      D44=YOUNG(5)
      D55=YOUNG(6)
      D21=D12
      DDDD=YOUNG(12)
      IF(DDDD.NE.0.) D21=DDDD
      DO 101 I=1,3
      DO 101 J=1,3
101   EFB(J,I)=0.
      DO 102 I=1,2
      DO 102 J=1,2
102   EFS(J,I)=0.
      DDT=D11*D22-D12*D21
      IF(DDT.EQ.0.)WRITE(6,700)
700   FORMAT(/,'      *****  ERROR    ****  ELASTICITY  MATRIX ',/)
      EFB(1,1)=D22/DDT
      EFB(1,2)=-D12/DDT
      EFB(2,1)=-D21/DDT
      EFB(2,2)=D11/DDT
      EFB(3,3)=1./D33
      EFS(1,1)=1./D44
      EFS(2,2)=1./D55
      DO 103 I=1,9
      DO 103 J=1,2
103   ENM1(J,I)=0.
      DO 104 I=1,3
      J=3*I-2
      ENM1(1,J)=B(I)/DT
      ENM1(2,J+1)=C(I)/DT
      ENM1(1,J+2)=G(I)/DT
104   ENM1(2,J+2)=B(I)/DT
      DO 105 I=1,3
      ENM2(1,I)=B(I)/DT
105   ENM2(2,I)=G(I)/DT
      DO 106 I=1,9,3
      DO 106 J=1,9,3
      AS=DT/24.
      K=I-J
      IF(K.LT.3.AND.K.GT.-3) AS=DT/12.
      STMM(I,J)=EFB(1,1)*AS
      STMM(I+1,J)=EFB(2,1)*AS
      STMM(I+2,J)=EFB(3,1)*AS
      STMM(I,J+1)=EFB(1,2)*AS
      STMM(I+1,J+1)=EFB(2,2)*AS
      STMM(I+2,J+1)=EFB(3,2)*AS
      STMM(I,J+2)=EFB(1,3)*AS
      STMM(I+1,J+2)=EFB(2,3)*AS
106   STMM(I+2,J+2)=EFB(3,3)*AS
      CALL TIMES(EFS,ENM1,ENM,2,2,9,1)
      CALL TIMES(ENM1,ENM,STMG,9,2,9,2)
      CALL TIMES(ENM1,ENM2,STMH,9,2,3,2)
      DO 107 I=1,9
```

```
      DO 107 J=1,9
107   STM(J,I)=STMM(J,I)+DT/2.*STMD(J,I)
      DO 110 I=1,12
      DO 110 J=1,12
      TT(J,I)=0.
110   SIM(J,I)=0.
      DO 111 I=1,3
      DO 111 J=1,9
111   SIM(J+3,I)=STMW(J,I)*DT/2.
      DO 112 I=1,9
      DO 112 J=1,3
112   SIM(J,I+3)=STMW(I,J)*DT/2.
      DO 113 I=1,9
      DO 113 J=1,9
113   SIM(J+3,I+3)=-STM(J,I)
      DO 114 I=1,12
      DO 114 J=1,2
114   STR(J,I)=0.
      DO 115 I=1,9
      DO 115 J=1,2
115   STR(J,I+3)=DMM1(J,I)
      DATA M1/1,4,5,6,2,7,8,9,3,10,11,12/
      DO 116 I=1,12
116   TT(M1(I),I)=1.
      CALL TIMES(SIM,TT,GE,12,12,12,1)
      CALL TIMES(TT,GE,SIM,12,12,12,2)
      CALL TIMES(STR,TT,STR1,2,12,12,1)
      DO 117 I=1,12
      DO 117 J=1,12
117   STUCK(J,I)=SIM(J,I)
      DO 118 I=1,12
      DO 118 J=1,2
118   STICK(J,I)=STR1(J,I)
      PG=X(3,5)
      FORCE(1)=DT/6.*PG
      FORCE(5)=DT/6.*PG
      FORCE(9)=DT/6.*PG
      RETURN
      END
      SUBROUTINE TIMES(A,B,R,N,M,L,KOK)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION A(1),B(1),R(1)
      KOK=1  A(N,M) , B(M,L) , R(N,L)    REGULAR    A*B
      KOK=2  A(M,N) , B(M,L) , R(N,L)    TRANSPOSE  A'*B
      IR=1
      DO 100 K=1,L
      DO 100 J=1,N
      R(IR)=0.
      GO TO(101,102),KOK
101   CONTINUE
      DO 103 I=1,M
      IA=N*(I-1)+J
      IB=M*(K-1)+I
103   R(IR)=R(IR)+A(IA)*B(IB)
      GO TO 100
102   CONTINUE
      DO 104 I=1,M
      IA=M*(J-1)+I
      IB=M*(K-1)+I
104   R(IR)=R(IR)+A(IA)*B(IB)
```

```
CO  IR=IR+1
    RETURN
    END
    SUBROUTINE TRANL(X,Y,K1,TPL)
    IMPLICIT REAL*8 (A-H,O-Z)
    DIMENSION X(16,2),Y(16,2),X2(2),Y2(2),TPL(3,3)
    DO 515 J=1,2
    X2(J)=X(K1,J)
15  Y2(J)=Y(K1,J)
    G2=X2(2)-X2(1)
    B2=Y2(2)-Y2(1)
    GLEN=DSQRT(B2**2+G2**2)
    IF(GLEN.EQ.0.) WRITE(6,700)
    COB=B2/GLEN
    SIB=-G2/GLEN
    TRL(1,1)=COB**2
    TRL(1,3)=-2.*COB*SIB
    TRL(1,2)=SIB**2
    TRL(2,1)=SIB**2
    TRL(2,3)=2.*COB*SIB
    TRL(2,2)=COB**2
    TRL(2,1)=COB*SIB
    TRL(3,3)=COB**2-SIB**2
    TRL(3,2)=-COB*SIB
7   FORMAT(/,'        ***** ERROR  ******  GLEN    SUB. TRANL',/)
    RETURN
    END
```

4. Reference symbol PMX24

```
C P=100 PRCUTP=DURH CCPIES=4
= UNIVERSITY, BATCH
WAS: 11:37:14
SIGNED ON AT  11:37:19 ON MCN SEP 22/75
PRINT:
      ************     MIXED  MODEL  24   DEGREES OF  FREEDOM  W, MX, MY, MXY,  A ND
      ************   STRESSES  QX, QY AT FIRST AND SECOND NODES  ****************
      ************     TRANSFORMATION   TO U,  MMN,  MSS,  MSN   ******************
      ************      QUADRATIC  VARIATION   OF THE FUNCTIONS        ************
      SUBROUTINE  STIFF
      IMPLICIT REAL*8 (A-H,C-Z)
      COMMON/STI/X(3,2C),YOUNG(12),STUCK(36,36),STICK(8,36),FORCE(36),
     1INFO(20)
      COMMON/MAN/BCL(2,36,36),CCL(2,8,36),DCL(2,36),GRAM(16,16),NGRAM
      COMMON/COOR/X1(3),Y1(3)
      DIMENSION TRAN(24,24),SMK(24,24),SMK1(24,24),STP(4,24),TIK(4,24),
     1DADG(24),XM(16,2),YM(16,2),CO3(3,3)
      DO 100 I=1,3
      X1(I)=X(1,I)
100   Y1(I)=X(2,I)
      DO 101 I=1,36
      DO 101 J=1,36
101   STUCK(J,I)=0.
      DO 102 I=1,36
      DO 102 J=1,8
102   STICK(J,I)=0.
      DO 103 I=1,36
103   FORCE(I)=0.
      IF(NGRAM.EQ.0) GO TO 517
      DO 516 M=1,NGRAM
      DO 516 I=1,2
      XM(M,I)=GRAM(M,I)
516   YM(M,I)=GRAM(M,I+2)
517   CONTINUE
      NLAK=INFO(1)
      IF(NLAK.EQ.1.OR.NLAK.EQ.2)GO TO 104
      CALL SUBTI
      IF(NLAK.EQ.0)GO TO 105
      NLAK=NLAK-10
      DO 106 I=1,24
      DO 106 J=1,24
106   BCL(NLAK,J,I)=STUCK(J,I)
      DO 113 I=1,24
      DO 113 J=1,4
113   CCL(NLAK,J,I)=STICK(J,I)
      DO 108 I=1,24
108   DCL(NLAK,I)=FORCE(I)
      GO TO 105
104   DO 109 I=1,24
      DO 109 J=1,24
109   STUCK(J,I)=BCL(NLAK,J,I)
      DO 214 I=1,24
      DO 214 J=1,4
214   STICK(J,I)=CCL(NLAK,J,I)
      DO 111 I=1,24
111   FORCE(I)=DCL(NLAK,I)
105   CONTINUE
      NLIK=INFO(2)
      IF(NLIK.EQ.0.OR.NLIK.EQ.99) GO TO 112
      DO 213 I=1,24
```

```
      DO 213 J=1,24
13    SMK(J,I)=STUCK(J,I)
      DO 114 I=1,24
      DO 114 J=1,4
14    STR(J,I)=STICK(J,I)
      DO 115 I=1,NLIK
      K=(I-1)*2+3
      L=INFO(K)
      K1=INFO(K+1)
      DO 583 II=1,24
      DO 584 IJ=1,24
84    TRAN(IJ,II)=0.
83    TRAN(II,II)=1.
      CALL TRANL(XM,YM,K1,CO3)
      KK=(L-1)*4
      DO 587 IN=1,3
      DO 587 IL=1,3
87    TRAN(KK+IN+1,KK+IL+1)=CO3(IN,IL)
      CALL TIMES(SMK,TRAN,SMK1,24,24,24,1)
      CALL TIMES(TRAN,SMK1,SMK,24,24,24,2)
      CALL TIMES(STR,TRAN,TIK,4,24,24,1)
      DO 116 NI=1,24
      DO 116 NJ=1,4
16    STR(NJ,NI)=TIK(NJ,NI)
15    CONTINUE
      DO 117 I=1,24
      DO 117 J=1,24
17    STUCK(J,I)=SMK(J,I)
      DO 118 I=1,24
      DO 118 J=1,4
18    STICK(J,I)=STR(J,I)
12    CONTINUE
      BRPO=0.
      DO 604 J=6,11
      DO 604 I=1,3
04    IF(X(I,J).NE.0.)BRPO=1.
      IF(BRPO.NE.1.) GO TO 560
      DO 273 I=1,24
73    DAOG(I)=0.
      DAOG(1)=X(3,6)
      DAOG(5)=X(3,7)
      DAOG(9)=X(3,8)
      DAOG(13)=X(3,9)
      DAOG(17)=X(3,10)
      DAOG(21)=X(3,11)
      DO 274 I=1,24
74    FORCE(I)=FORCE(I)+DAOG(I)
60    CONTINUE
      RETURN
      END
      SUBROUTINE SUBTI
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/STI/X(3,20),YO(12),STUCK(36,36),STICK(8,36),FORCE(36),
     1INFO(20)
      COMMON/TRDA/B(3),G(3),EL(3),DT
      COMMON/OLCEP/EMM(18,18),EMO(18,18),STMW(18,6),STR(4,24)
      COMMON/COOR/XL(3),YL(3)
      DIMENSION STM(18,18),TT(24,24),M1(24),SIM(24,24),CE(24,24)
     1,STR1(4,24)
      DO 10 I=1,3
```

```
      K=I+1
      J=I+2
      IF(K.GT.3)K=K-3
      IF(J.GT.3)J=J-3
      B(I)=YL(K)-YL(J)
      G(I)=XL(J)-XL(K)
00    EL(I)=B(I)**2+G(I)**2
      DT=B(1)*G(2)-B(2)*G(1)
      DO 111 I=1,24
      DO 111 J=1,4
11    STR(J,I)=0.
      CALL INTSU
      DO 1 1 I=1,18
      DO 101 J=1,18
01    STM(J,I)=EMM(J,I)+EMG(J,I)
      DO 102 I=1,24
      DO 102 J=1,24
      TT(J,I)=0.
02    SIM(J,I)=0.
      DO 103 I=1,6
      DO 1 3 J=1,18
03    SIM(J+6,I)=STMW(J,I)
      DO 104 I=1,18
      DO 104 J=1,6
04    SIM(J,I+6)=STMW(I,J)
      DO 1 5 I=1,18
      DO 105 J=1,18
05    SIM(J+6,I+6)=-STM(J,I)
      DATA M1/1,7,8,9,2,1 ,11,12,3,13,14,15,4,16,17,18,5,19,20,21,
     16,22,23,24/
      DO 107 I=1,24
07    TT(M1(I),I)=1.
      CALL TIMES(SIM,TT,GE,24,24,24,1)
      CALL TIMES(TT,GE,SIM,24,24,24,2)
      CALL TIMES(STR,TT,STR1,4,24,24,1)
      DO 106 I=1,24
      DO 1 6 J=1,24
06    STUCK(J,I)=SIM(J,I)
      DO 112 I=1,24
      DO 112 J=1,4
12    STICK(J,I)=STR1(J,I)
      PG=X(3,5)
      FORCE(13)=DT/6.*PG
      FORCE(17)=DT/6.*PG
      FORCE(21)=DT/6.*PG
      RETURN
      END
      SUBROUTINE TRANL(X,Y,K1,TRL)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION X(16,2),Y(16,2),X2(2),Y2(2),TRL(3,3)
      DO 515 J=1,2
      X2(J)=X(K1,J)
15    Y2(J)=Y(K1,J)
      G2=X2(2)-X2(1)
      B2=Y2(2)-Y2(1)
      GLEN=DSQRT(B2**2+G2**2)
      IF(GLEN.EQ.0.) WRITE(6,700)
      COB=B2/GLEN
      SIB=-G2/GLEN
      TRL(1,1)=COB**2
```

```
      TRL(1,3)=-2.*COB*SIB
      TRL(1,2)=SIB**2
      TRL(2,1)=SIB**2
      TRL(2,3)=2.*COB*SIB
      TRL(2,2)=COB**2
      TRL(3,1)=COB*SIB
      TRL(3,3)=COB**2-SIB**2
      TRL(3,2)=-COB*SIB
      FORMAT(/,'        ***** ERROR  ******  GLEN   SUB. TRANL',/)
      RETURN
      END
      SUBROUTINE INTSU
      IMPLICIT REAL*8 (A-H,O-Z)
      D21=YOUNG(12)
      COMMON/TRCA/B(3),C(3),EL(3),DT
      COMMON/OLDCP/EMM(18,18),EMQ(18,18),EMW(18,6),STR(4,24)
      COMMON/STI/X(3,2 ),YOUNG(12),ST(36,36),SF(8,36),FCR(36),INFO(20)
      DIMENSION A1(7),A2(7),A3(7),W(7),EFB(3,3),EFS(2,2),A(3),
     1FN(6),FE(6),FF(6),ENM0(3,18),ENW2(2,6),ENM1(2,18)
     2,EMM1(18,6),EMM1(3,18),EMM2(18,18),EMQ1(2,18),EMQ2(18,18)
      DATA A1/0.333333330 00,0.05971587D 00,2*0.47014206D 00,0.797426990
     1    ,2* .10128651D   /,A3/ .33333333D 0 ,2* .47 142 6D   ,0. 597158
     27D 00,2*0.10128651D 00,0.797426990 00/,W/0.22500000D 00,3*0.132394
     315D 00,3*0.125939180 00/
      D11=YOUNG(1)
      D12=YOUNG(2)
      D22=YOUNG(3)
      D33=YOUNG(4)/2.
      D44=YOUNG(5)
      D55=YOUNG(6)
      D21=D12
      DDDD=YOUNG(12)
      IF(DDDD.NE.0.)D21=DDDD
      DO 100 I=1,3
      DO 100 J=1,3
      EFB(J,I)=0.
      DO 101 I=1,2
      DO 101 J=1,2
101   EFS(J,I)=0.
      DOT=D11*D22-D12*D21
      IF(DOT.EQ. .)WRITE(6,700 )
700   FORMAT(/,'        *****  ERROR   ****  ELASTICITY  MATRIX ',/)
      EFB(1,1)=D22/DOT
      EFB(1,2)=-D12/DOT
      EFB(2,1)=-D21/DOT
      EFB(2,2)=D11/DOT
      EFB(3,3)=1./D33
      EFS(1,1)=1./D44
      EFS(2,2)=1./D55
      DO 107 I=1,18
      DO 107 J=1,18
      EMM(J,I)=0.
107   EMQ(J,I)=0.
      DO 108 I=1,6
      DO 108 J=1,18
108   EMW(J,I)=0.
         INTEGRATION
      DO 102 K=1,9
      IF(K.EQ.8) A(1)=1.
      IF(K.EQ.8) A(2)=0.
```

```
      IF(K.EQ.8) A(3)=0.
      IF(K.EQ.3) A(1)=0.
      IF(K.EQ.9) A(2)=1.
      IF(K.EQ.9) A(3)=0.
      IF(K.GT.7)GO TO 112
      A2(K)=1.-A1(K)-A3(K)
      A(1)=A1(K)
      A(2)=A2(K)
      A(3)=A3(K)
12    CONTINUE
      EN(1)=(2.*A(1)-1.)*A(1)
      EN(2)=(2.*A(2)-1.)*A(2)
      EN(3)=(2.*A(3)-1.)*A(3)
      EN(4)=4.*A(2)*A(3)
      EN(5)=4.*A(1)*A(3)
      EN(6)=4.*A(1)*A(2)
      EE(1)=B(1)*(4.*A(1)-1.)/DT
      EE(2)=B(2)*(4.*A(2)-1.)/DT
      EE(3)=B(3)*(4.*A(3)-1.)/DT
      EE(4)=4.*(B(2)*A(3)+B(3)*A(2))/DT
      EE(5)=4.*(B(1)*A(3)+B(3)*A(1))/DT
      EE(6)=4.*(B(1)*A(2)+B(2)*A(1))/DT
      EE(1)=C(1)*(4.*A(1)-1.)/DT
      EE(2)=C(2)*(4.*A(2)-1.)/DT
      EE(3)=C(3)*(4.*A(3)-1.)/DT
      EE(4)=4.*(C(2)*A(3)+C(3)*A(2))/DT
      EE(5)=4.*(C(1)*A(3)+C(3)*A(1))/DT
      EE(6)=4.*(C(1)*A(2)+C(2)*A(1))/DT
      DO 103 I=1,18
      DO 103 J=1,3
103   EMM (J,I)= .
      DO 203 I=1,18
      DO 203 J=1,2
203   EMM1(J,I)= .
      DO 104 J=1,6
      DO 104 I=1,3
      M=3*J+I-3
104   EMMO(I,M)=EN(J)
      DO 105 J=1,18,3
      M=(J+2)/3
      EMM1(1,J)=EE(M)
      EMM1(1,J+2)=EE(M)
      EMM1(2,J+1)=EE(M)
105   EMM1(2,J+2)=EE(M)
      IF(K.GT.7)GO TO 113
      DO 106 J=1,6
      EMW2(1,J)=EE(J)
106   EMW2(2,J)=EE(J)
      CALL TIMES(EMM1,EMW2,EMW1,18,2,6,2)
      CALL TIMES(EEB,EMM ,EMM1,3,3,18,1)
      CALL TIMES(EMMO,EMM1,EMM2,18,3,18,2)
      CALL TIMES(EES,EMM1,EMO1,2,2,18,1)
      CALL TIMES(EMM1,EMO1,EMO2,18,2,18,2)
      DO 108 I=1,18
      DO 109 J=1,18
      EMM(J,I)=EMM(J,I)+W(K)*EMM2(J,I)*DT/2.
109   EMO(J,I)=EMO(J,I)+W(K)*EMO2(J,I)*DT/2.
      DO 110 I=1,6
      DO 110 J=1,18
110   EMW(J,I)=EMW(J,I)+W(K)*EMW1(J,I)*DT/2.
```

```
13 CONTINUE
   IF(K.LT.8) GO TO 102
   L=0
   IF(K.EQ.9) L=2
   DO 114 I=1,18
   DO 114 J=1,2
14 STR(L+J,I+6)=FNM1(J,I)
02 CONTINUE
   RETURN
   END
   SUBROUTINE TIMES(A,B,R,N,M,L,KOK)
   IMPLICIT REAL*8 (A-H,O-Z)
   DIMENSION A(1),B(1),R(1)
   KOK=1  A(N,M) , B(M,L) , R(N,L)    REGULAR    A*B
   KOK=2  A(M,N) , B(M,L) , R(N,L)    TRANSPOSE  A'*B
   IR=1
   DO 100 K=1,L
   DO 100 J=1,N
   R(IR)=0.
   GO TO(101,102),KOK
01 CONTINUE
   DO 103 I=1,M
   IA=N*(I-1)+J
   IB=M*(K-1)+I
03 R(IR)=R(IR)+A(IA)*B(IB)
   GO TO 100
02 CONTINUE
   DO 104 I=1,M
   IA=M*(J-1)+I
   IB=M*(K-1)+I
04 R(IR)=R(IR)+A(IA)*B(IB)
00 IR=IR+1
   RETURN
   END
```

5. Reference symbol PDS24

```
P=100 PROUTE=PURE PORTES=4
= UNIVERSITY, BATCH
WAS: 11:37:10
IGNED ON AT  11:48:40 ON MON SEP 22/75
RINTR
  **** PLATE  DISPL. ELEMENT 24 DG. OF FREEDOM AT 3 NODES *****
  ****  CUBIC  VARIATION OF W ( CUBIC VAR. OF WL ALONG THE SIDES )*****
  ****  LINEAR   VARIATION OF SHEAR ANGLE  BX,BY          ******
  ****  STRESSES SXX,SYY,SXY,QX,QY   AT CENTROID         ********
   **** MODULI OF ELASTISITY THROUGH  YOUNG(12)     **********
  ****TRANSFORMATION TO  W, BN, BS, WBN, WSN, WSS, BN, BS  ****************
   SUBROUTINE TRANL(X,Y,Z,K,TRL)
   IMPLICIT REAL*8 (A-H,O-Z)
   DIMENSION X(16,2),Y(16,2),Z(16,2),TRL(7,7),X2(2),Y2(2)
   DO 515 J=1,2
   X2(J)=X(K,J)
15 Y2(J)=Y(K,J)
   C2=X2(2)-X2(1)
   B2=Y2(2)-Y2(1)
   GLEN2=DSQRT(C2**2+B2**2)
   IF(GLEN2.LT. .1D-12.AND.GLEN2.GT.- .1D-12) WRITE(6,200)
   B2=B2/GLEN2
   C2=C2/GLEN2
   DO 516 I=1,7
   DO 516 J=1,7
16 TRL(J,I)=0.
   TRL(1,1)=B2
   TRL(1,2)=C2
   TRL(1,6)=B2
   TRL(2,1)=-C2
   TRL(2,2)=B2
   TRL(2,6)=-C2
   TRL(3,3)=B2**2
   TRL(3,4)=2.*B2*C2
   TRL(3,5)=C2**2
   TRL(4,3)=-B2*C2
   TRL(4,4)=B2**2-C2**2
   TRL(4,5)=B2*C2
   TRL(5,3)=C2**2
   TRL(5,4)=-2.*B2*C2
   TRL(5,5)=B2**2
   TRL(6,6)=B2
   TRL(6,7)=C2
   TRL(7,6)=-C2
   TRL(7,7)=B2
00 FORMAT('      *****     ERROR     TRANL    ******')
   RETURN
   END
   SUBROUTINE  STIFF
   IMPLICIT REAL*8 (A-H,O-Z)
   COMMON/STI/X(3,2 ),YOUNG(12),STUCK(36,36),STICK(8,36),FORCE(36),
  1INFO(20)
   COMMON/MAN/BCD(2,36,36),CDL(2,8,36),DCD(2,36),GRAM(16,16),NGRAM
   COMMON/CLOK/CO2(3,3),XO(3),YO(3),CGM(3,3)
   DIMENSION TRAN(24,24),SMK(24,24),STR(5,24),DAOG(24),
  1XM(16,2),YM(16,2),ZM(16,2),XL(3),YL(3),ZL(3),CC3(7,7)
  2,C1(3),B1(3),G1(3),STU(24,24),TIK(5,24),FOR(24)
   DO 100 I=1,3
   XO(I)=X(1,I)
   DO 101 I=1,36
```

```
      DO 1 1 J=1,36
01    STUCK(J,I)=0.
      DO 102 I=1,36
      DO 1 2 J=1,5
02    STICK(J,I)=0.
      DO 103 I=1,36
03    FORCE(I)=0.
      DO 607 I=1,24
07    DADG(I)=0.
      IF(NGRAM.EQ.0) GO TO 517
      DO 516 M=1,NGRAM
      DO 516 I=1,2
      XM(M,I)=GRAM(M,I)
      ZM(M,I)=GRAM(M,I+4)
16    YM(M,I)=GRAM(M,I+2)
17    CONTINUE
      CNLD=0.
      DO 432 I=1,3
      DO 432 J=6,9
32    IF(X(I,J).NE. .) CNLD=1.
      IF(CNLD.NE.1.)GO TO 437
      DO 433 I=6,9
      J=BB(I-6)+1
33    DADG(J)=X(3,I)
37    CONTINUE
      NLAK=INFO(1)
      IF(NLAK.EQ.1.OR.NLAK.EQ.2)GO TO 104
      CALL SUBTT
      IF(NLAK.EQ.0)GO TO 105
      NLAK=NLAK-10
      DO 1 6 I=1,24
      DO 106 J=1,24
06    BCL(NLAK,J,I)=STUCK(J,I)
      DO 113 I=1,24
      DO 113 J=1,5
13    CCL(NLAK,J,I)=STICK(J,I)
      DO 1 9 I=1,24
08    DCL(NLAK,I)=FORCE(I)
      GO TO 105
04    DO 1 9 I=1,24
      DO 109 J=1,24
09    STUCK(J,I)=BCL(NLAK,J,I)
      DO 214 I=1,24
      DO 214 J=1,5
14    STICK(J,I)=CCL(NLAK,J,I)
      DO 111 I=1,24
11    FORCE(I)=DCL(NLAK,I)
15    CONTINUE
      DO 438 I=1,24
38    FORCE(I)=FORCE(I)+DADG(I)
      NLIK=INFO(2)
      IF(NLIK.EQ.0.OR.NLIK.EQ.99) GO TO 115
      DO 601 I=1,24
      DO 601 J=1,24
01    STU(J,I)=STUCK(J,I)
      DO 6 2 I=1,24
      DO 602 J=1,5
02    TIK(J,I)=STICK(J,I)
      DO 6 3 I=1,24
03    FOR(I)=FORCE(I)
```

```
      DO 116 I=1,NLTK
      K=(I-1)*2+3
      L=INFO(K)
      K1=INFO(K+1)
      DO 583 II=1,24
      DO 584 IJ=1,24
   84 TRAN(IJ,II)=0.
   83 TRAN(II,II)=1.
      KK=(L-1)*8
      CALL TRANM(XM,YM,ZM,K1,CO3)
      DO 222 IL=1,7
      DO 222 JL=1,7
   22 TRAN(KK+IL+1,KK+JL+1)=CO3(IL,JL)
      CALL TIMES(STU,TRAN,SMK,24,24,24,1)
      CALL TIMES(TRAN,SMK,STU,24,24,24,2)
      CALL TIMES(TIK,TRAN,STR,5,24,24,1)
      CALL TIMES(TRAN,FOR,FACC,24,24,1,2)
      DO 232 IL=1,24
      DO 232 JL=1,5
   32 TIK(JL,IL)=STR(JL,IL)
      DO 233 IL=1,24
   33 FOR(IL)=FACC(IL)
   16 CONTINUE
      DO 604 I=1,24
      DO 604 J=1,24
   04 STUCK(J,I)=STU(J,I)
      DO 605 I=1,24
      DO 605 J=1,5
   05 STICK(J,I)=TIK(J,I)
      DO 606 I=1,24
   06 FORCE(I)=FOR(I)
   15 CONTINUE
      RETURN
      END
      SUBROUTINE  SHAPEF(A,B,G,DT)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/JCN/SM(24,24),S(5,24),D(5,5),P(8),END(24,1)
      DIMENSION  PF(5,24),BET(3,18),CET(3,3),A(3),B(3),C(3)
     1,BED(3,18),CBI(6,6),TEM(6,18),CEN(6,18),EN(8,24)
      COMMON/DOPA/CM(5),OK(3)
      CET(1,1)=(B(1)/DT)**2
      CET(1,2)=2.*B(1)*B(2)/DT**2
      CET(1,3)=(B(2)/DT)**2
      CET(2,1)=(G(1)/DT)**2
      CET(2,2)=2.*G(1)*G(2)/DT**2
      CET(2,3)=(G(2)/DT)**2
      CET(3,1)=2.*G(1)*B(1)/DT**2
      CET(3,2)=2.*(G(1)*B(2)+G(2)*B(1))/DT**2
      CET(3,3)=2.*G(2)*B(2)/DT**2
      DO 100 I=1,3
      DO 100 J=1,18
      BET(I,J)=0.
  00 CONTINUE
      DO 1 1 I=1,5
      DO 101 J=1,24
      PF(I,J)=0.
  01 CONTINUE
      BET(1,4)=2.
      BET(1,7)=6.*A(1)
      BET(1,8)=2.*A(2)
```

```
      BET(1,11)=12.*A(1)**2
      BET(1,12)=6.*A(1)*A(2)
      BET(1,13)=2.*A(2)**2
      BET(1,16)=20.*A(1)**3+12.*DM(1)*A(1)**2*A(2)+2.*DM(2)*A(2)**3
      BET(1,17)=6.*A(1)*A(2)**2+2.*DM(3)*A(2)**3
      BET(1,18)=2.*A(2)**3*DM(5)
      BET(2,5)=1.
      BET(2,8)=2.*A(1)
      BET(2,9)=2.*A(2)
      BET(2,12)=3.*A(1)**2
      BET(2,13)=4.*A(1)*A(2)
      BET(2,14)=3.*A(2)**2
      BET(2,16)=4.*DM(1)*A(1)**3+6.*DM(2)*A(1)*A(2)**2
      BET(2,17)=6.*A(1)**2*A(2)+6.*DM(3)*A(1)*A(2)**2
      BET(2,18)=6.*A(1)*A(2)**2*DM(5)+4.*DM(4)*A(2)**3
      BET(3,6)=2.
      BET(3,9)=2.*A(1)
      BET(3,10)=6.*A(2)
      BET(3,13)=2.*A(1)**2
      BET(3,14)=6.*A(1)*A(2)
      BET(3,15)=12.*A(2)**2
      BET(3,16)=6.*DM(2)*A(1)**2*A(2)
      BET(3,17)=2.*A(1)**3+6.*DM(3)*A(1)**2*A(2)
      BET(3,18)=20.*A(2)**3+12.*DM(4)*A(1)*A(2)**2+6.*DM(5)*A(1)**2*A(2)
      CALL TIMES(CET,BET,BED,3,3,18,1)
      DO 102 I=1,3
      DO 102 J=1,18
      BE(I,J)=-BED(I,J)
  102 CONTINUE
      DO 103 J=1,3
      K=(J-1)*2+1
      BE(1,18+K)=B(J)/DT
      BE(2,19+K)=C(J)/DT
      BE(3,18+K)=C(J)/DT
      BE(3,19+K)=B(J)/DT
  103 CONTINUE
      BE(4,19)=A(1)
      BE(4,21)=A(2)
      BE(4,23)=A(3)
      BE(5,20)=A(1)
      BE(5,22)=A(2)
      BE(5,24)=A(3)
      CALL TIMES(D,BE,S,5,5,24,1)
      CALL TIMES(BE,S,SM,24,5,24,2)
      DO 104 I=1,6
      DO 104 J=1,6
      CBI(I,J)=0.
  104 CONTINUE
      DO 105 I=1,6
      DO 105 J=1,18
      TEN(I,J)=0.
  105 CONTINUE
      DO 106 I=1,8
      DO 106 J=1,24
      EN(I,J)=0.
  106 CONTINUE
      CBI(1,1)=1.
      CBI(2,2)=B(1)/DT
      CBI(2,3)=B(2)/DT
      CBI(3,2)=C(1)/DT
```

```
      CBT(3,3)=G(2)/DT
      CBT(4,4)=(B(1)/DT)**2
      CBT(4,5)=2.*B(1)*B(2)/DT**2
      CBT(4,6)=(B(2)/DT)**2
      CBT(5,4)=G(1)*B(1)/DT**2
      CBT(5,5)=(G(2)*B(1)+G(1)*B(2))/DT**2
      CBT(5,6)=G(2)*B(2)/DT**2
      CBT(6,4)=(G(1)/DT)**2
      CBT(6,5)=2.*G(1)*G(2)/DT**2
      CBT(6,6)=(G(2)/DT)**2
      TEN(1,1)=1.
      TEN(1,2)=A(1)
      TEN(1,3)=A(2)
      TEN(1,4)=A(1)**2
      TEN(1,5)=A(1)*A(2)
      TEN(1,6)=A(2)**2
      TEN(1,7)=A(1)**3
      TEN(1,8)=A(1)**2*A(2)
      TEN(1,9)=A(1)*A(2)**2
      TEN(1,10)=A(2)**3
      TEN(1,11)=A(1)**4
      TEN(1,12)=A(1)**3*A(2)
      TEN(1,13)=A(1)**2*A(2)**2
      TEN(1,14)=A(1)*A(2)**3
      TEN(1,15)=A(2)**4
      TEN(1,16)=A(1)**5+CM(1)*A(1)**4*A(2)+CM(2)*A(1)**2*A(2)**3
      TEN(1,17)=A(1)**3*A(2)**2+CM(3)*A(1)**2*A(2)**3
      TEN(1,18)=A(2)**5+CM(4)*A(1)*A(2)**4+CM(5)*A(1)**2*A(2)**3
      TEN(2,2)=1.
      TEN(2,4)=2.*A(1)
      TEN(2,5)=A(2)
      TEN(2,7)=3.*A(1)**2
      TEN(2,8)=2.*A(1)*A(2)
      TEN(2,9)=A(2)**2
      TEN(2,11)=4.*A(1)**3
      TEN(2,12)=3.*A(1)**2*A(2)
      TEN(2,13)=2.*A(1)*A(2)**2
      TEN(2,14)=A(2)**3
      TEN(2,16)=5.*A(1)**4+4.*CM(1)*A(1)**3*A(2)+2.*CM(2)*A(1)*A(2)**3
      TEN(2,17)=3.*A(1)**2*A(2)**2+2.*CM(3)*A(1)*A(2)**3
      TEN(2,18)=CM(4)*A(2)**4+2.*CM(5)*A(1)*A(2)**3
      TEN(3,3)=1.
      TEN(3,5)=A(1)
      TEN(3,6)=2.*A(2)
      TEN(3,8)=A(1)**2
      TEN(3,9)=2.*A(1)*A(2)
      TEN(3,10)=3.*A(2)**2
      TEN(3,12)=A(1)**3
      TEN(3,13)=2.*A(1)**2*A(2)
      TEN(3,14)=3.*A(1)*A(2)**2
      TEN(3,15)=4.*A(2)**3
      TEN(3,16)=CM(1)*A(1)**4+3.*CM(2)*A(1)**2*A(2)**2
      TEN(3,17)=2.*A(1)**3*A(2)+3.*CM(3)*A(1)**2*A(2)**2
      TEN(3,18)=5.*A(2)**4+4.*CM(4)*A(1)*A(2)**3+3.*CM(5)*A(1)**2*A(2)**
     12
      TEN(4,4)=2.
      TEN(4,7)=6.*A(1)
      TEN(4,8)=2.*A(2)
      TEN(4,11)=12.*A(1)**2
      TEN(4,12)=6.*A(1)*A(2)
```

```
      TEN(4,13)=2.*A(2)**2
      TEN(4,16)=20.*A(1)**3+12.*OM(1)*A(1)**2*A(2)+2.*OM(2)*A(2)**3
      TEN(4,17)=6.*A(1)*A(2)**2+2.*OM(3)*A(2)**2
      TEN(4,19)=2.*OM(5)*A(2)**3
      TEN(5,5)=1.
      TEN(5,8)=2.*A(1)
      TEN(5,9)=2.*A(2)
      TEN(5,12)=3.*A(1)**2
      TEN(5,13)=4.*A(1)*A(2)
      TEN(5,14)=3.*A(2)**2
      TEN(5,16)=4.*OM(1)*A(1)**3+6.*OM(2)*A(1)*A(2)**2
      TEN(5,17)=6.*A(1)**2*A(2)+6.*OM(3)*A(1)*A(2)**2
      TEN(5,19)=4.*OM(4)*A(2)**3+6.*OM(5)*A(1)*A(2)**2
      TEN(6,6)=2.
      TEN(6,9)=2.*A(1)
      TEN(6,10)=6.*A(2)
      TEN(6,13)=2.*A(1)**2
      TEN(6,14)=6.*A(1)*A(2)
      TEN(6,15)=12.*A(2)**2
      TEN(6,16)=6.*OM(2)*A(1)**2*A(2)
      TEN(6,17)=2.*A(1)**3+6.*OM(3)*A(1)**2*A(2)
      TEN(6,19)=20.*A(2)**3+12.*OM(4)*A(1)*A(2)**2+6.*OM(5)*A(1)**2*A(2)
      CALL TIMES(CBT,TEN,DEN,6,6,18,1)
      DO 107  I=1,6
      DO 107  J=1,18
      EN(I,J)=DEN(I,J)
107   CONTINUE
      EN(7,19)=A(1)
      EN(7,21)=A(2)
      EN(7,23)=A(3)
      EN(8,20)=A(1)
      EN(8,22)=A(2)
      EN(8,24)=A(3)
      CALL TIMES(EN,P,ENP,24,8,1,2)
      RETURN
      END
      SUBROUTINE  SUBTI
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/STI/X(3,2 ),ELMOD(12),STUCK(36,36),SLICK(8,36),FORCE(36)
     1,INFO(20)
      COMMON/JCN/SM(24,24),S(5,24),D(5,5),P(8),ENP(24,1)
      COMMON/DOPA/OM(5),CK(3)
      COMMON/COCP/CO2(3,3),XO(3),YO(3),COM(3,3)
      DO 208 I=1,5
      DO 208 J=1,5
208   D(J,I)=0.
      D(1,1)=ELMOD(1)
      D(1,2)=ELMOD(2)
      D(2,2)=ELMOD(3)
      D(3,3)=ELMOD(4)/2.
      D(4,4)=ELMOD(5)
      D(5,5)=ELMOD(6)
      D(2,1)=D(1,2)
      IF(ELMOD(12).NE.0.)D(2,1)=ELMOD(12)
      DATA ALF/0.33333333D00/,ALF1/0.05971587D00/,BET1/0.47014206D00/
      DATA ALF2/0.797426985D00/,BET2/0.101286510D00/
      DATA WE1/0.225000D00/,WE2/0.13239415D00/,WE3/0.12593918D00/
      DIMENSION  B(3),G(3),ARCO(2,7),W(7),A(3),DAOM(24,1),LENG(3),
     1T(24,24),SOM(24,24),FOR(24),TTK(5,24),V1(24)
      ARCO(1,1)=ALF
```

```
      ARCO(2,1)=ALF
      ARCO(1,2)=BET1
      ARCO(2,2)=BET1
      ARCO(1,3)=ALF1
      ARCO(2,3)=BET1
      ARCO(1,4)=BET1
      ARCO(2,4)=ALF1
      ARCO(1,5)=BET2
      ARCO(2,5)=BET2
      ARCO(1,6)=ALF2
      ARCO(2,6)=BET2
      ARCO(1,7)=BET2
      ARCO(2,7)=ALF2
      W(1)=WF1
      W(2)=WF2
      W(3)=WF2
      W(4)=WF2
      W(5)=WF3
      W(6)=WF3
      W(7)=WF3
      DO 223 I=1,8
223   P(I)=0.
      P(1)=X(3,5)
      DO 200 I=1,3
      K=I+1
      IF(K.GT.3)K=K-3
      J=I+2
      IF(J.GT.3)J=J-3
      B(I)=YC(K)-YC(J)
      G(I)=XC(J)-XC(K)
      LENG(I)=DSQRT(B(I)**2+G(I)**2)
200   CONTINUE
      CK(1)=B(1)*B(2)+G(1)*G(2)
      CK(2)=B(2)*B(3)+G(2)*G(3)
      CK(3)=B(3)*B(1)+G(3)*G(1)
      DT=B(1)*G(2)-B(2)*G(1)
      CM(1)=5.*CK(1)/(CK(1)+CK(2))
      CM(2)=5.*(CK(3)*CK(2)+CK(1)*CK(2)-3.*CK(1)*CK(3))/
     1((CK(1)+CK(2))*(2.*CK(3)-3.*CK(2)))
      CM(3)=(3.*CK(3)-2.*CK(2))/(2.*CK(3)-3.*CK(2))
      CM(4)=5.*CK(1)/(CK(1)+CK(3))
      CM(5)=5.*(CK(3)*CK(2)+CK(1)*CK(3)-3.*CK(1)*CK(2))/
     1((CK(1)+CK(3))*(2.*CK(3)-3.*CK(2)))
      CALL ANTI(B,G,DT,T)
      DO 214 I=1,24
      DO 214 J=1,24
      SOM(I,J)=0.
214   CONTINUE
      DO 314 I=1,24
      DACL(I,1)=0.
314   CONTINUE
      DO 213  K=1,7
      A(1)=ARCO(1,K)
      A(2)=ARCO(2,K)
      A(3)=1.-A(1)-A(2)
      CALL  SHAPEE(A,B,G,DT)
      IF(K.NE.1)GO TO 510
      DO 511 I=1,24
      DO 511 J=1,5
511   TTK(J,I)=S(J,I)
```

```
1   CONTINUE
    DO 313 I=1,24
    DACL(I,1)=DACL(I,1)+DT/2.*PM(K)*ENP(I,1)
13 CONTINUE
    DO 213 J=1,24
    DO 213 I=1,24
    SDM(I,J)=SDM(I,J)+DT/2.*PM(K)*SM(I,J)
13 CONTINUE
    CALL TIMES(SDM,T,SM,24,24,24,1)
    CALL TIMES(T,SM,SDM,24,24,24,2)
    CALL TIMES(TIK,T,S,5,24,24,1)
    CALL TIMES(T,DACL,FRE,24,24,1,2)
    DATA M1/1,2,3,4,5,6,19,20,7,8,9,10,11,12,21,22,
   113,14,15,16,17,18,23,24/
    DO 224 I=1,24
    DO 224 J=1,24
24  T(J,I)= .
    DO 222 I=1,24
22  T(M1(I),I)=1.
    CALL TIMES(SDM,T,SM,24,24,24,1)
    CALL TIMES(T,SM,SDM,24,24,24,2)
    CALL TIMES(S,T,TIK,5,24,24,1)
    CALL TIMES(T,FRE,DACL,24,24,1,2)
    DO 550 I=1,24
    DO 55  J=1,24
50  STUCK(J,I)=SDM(J,I)
    DO 551 I=1,24
    DO 551 J=1,5
51  STICK(J,I)=TIK(J,I)
    DO 552 I=1,24
52  FORCE(I)=DACL(I,1)
    RETURN
    END
    SUBROUTINE  ANTI(B,C,DT,T)
    IMPLICIT REAL*8 (A-H,O-Z)
    DIMENSION TI(18,18),BCI(18,18),TO(18,18),T(24,24),B(3),C(3)
    COMMON/DORA/OM(5),OK(3)
    DO 100 I=1,18
    DO 1   J=1,18
    TI(I,J)=0.
00  CONTINUE
    DO 11  I=1,24
    DO 110 J=1,24
    IF(I.EQ.J.AND.I.GT.18) GO TO 111
    T(I,J)=0.
    GO TO 110
11  T(I,J)=1.
10  CONTINUE
    OMI=1.-OM(3)
    IF(OMI.EQ. )GO TO 1 7
    TI(1,13)=1.
    TI(2,14)=1.
    TI(3,15)=1.
    TI(4,16)=0.5
    TI(5,17)=1.
    TI(6,18)=0.5
    TI(7,1)=10.
    TI(7,2)=-4.
    TI(7,4)=0.5
    TI(7,13)=-10.
```

```
TI(7,14)=-6.
TI(7,16)=-1.5
TI(8,1)=6.*CM(1)
TI(8,2)=-3.*OM(1)
TI(8,3)=3.
TI(8,4)=0.5*CM(1)
TI(8,5)=-1.
TI(8,13)=-6. *CM(1)
TI(8,14)=-3.*OM(1)
TI(8,15)=-3.
TI(8,16)=- .5*CM(1)
TI(8,17)=-2.
TI(9,7)=6.*CM(4)
TI(9,8)=3.
TI(9,9)=-3.*CM(4)
TI(9,11)=-1.
TI(9,12)=0.5*OM(4)
TI(9,13)=-6.*OM(4)
TI(9,14)=-3.
TI(9,15)=-3.*OM(4)
TI(9,17)=-2.
TI(9,18)=-0.5*CM(4)
TI(10,7)=10.
TI(1 ,9)=-4.
TI(10,12)=0.5
TI(10,13)=-10.
TI(1 ,15)=-4.
TI(10,18)=-1.5
TI(11,1)=-15.
TI(11,2)=7.
TI(11,4)=-1.
TI(11,13)=15.
TI(11,14)=8.
TI(11,16)=1.5
TI(12,1)=-12.*CM(1)
TI(12,2)=6.*OM(1)
TI(12,3)=-2.
TI(12,4)=-CM(1)
TI(12,5)=1.
TI(12,13)=12.*CM(1)
TI(12,14)=6.*OM(1)
TI(12,15)=2.
TI(12,16)=CM(1)
TI(12,17)=1.
TI(13,1)=-6.*(CM(1)+CM(2))/CMI
TI(13,2)=3.*(OM(1)+OM(2))/CMI
TI(13,3)=-3./OMI
TI(13,4)=-(CM(1)+CM(2))/(2.*OMI)
TI(13,5)=1./OMI
TI(13,6)=-CM(3)/(2.*CMI)
TI(13,7)=6.*(OM(3)*CM(4)-OM(5))/CMI
TI(13,8)=3.*OM(3)/CMI
TI(13,9)=3.*(OM(5)-CM(3)*OM(4))/OMI
TI(13,10)=1./(2.*OMI)
TI(13,11)=-CM(3)/CMI
TI(13,12)=(CM(3)*CM(4)-CM(5))/(2.*CMI)
TI(13,13)=6.*(OM(1)+CM(2)+OM(5)-OM(3)*OM(4))/CMI
TI(13,14)=3.*(CM(1)+CM(2)-CM(3))/OMI
TI(13,15)=3.*(1.+CM(5)-CM(3)*CM(4))/CMI
TI(13,16)=(OM(1)+OM(2)-1.)/(2.*CMI)
```

```
      TI(13,17)=2.
      TI(13,18)=(DM(5)+DM(3)-DM(3)*DM(4))/(2.*DMI)
      TI(14,7)=-12.*DM(4)
      TI(14,8)=-2.
      TI(14,9)=6.*DM(4)
      TI(14,11)=1.
      TI(14,12)=-DM(4)
      TI(14,13)=12.*DM(4)
      TI(14,14)=2.
      TI(14,15)=6.*DM(4)
      TI(14,17)=1.
      TI(14,18)=DM(4)
      TI(15,7)=-15.
      TI(15,8)=7.
      TI(15,12)=-1.
      TI(15,13)=15.
      TI(15,15)=8.
      TI(15,18)=1.5
      TI(16,1)=6.
      TI(16,2)=-3.
      TI(16,4)=0.5
      TI(16,13)=-6.
      TI(16,14)=-3.
      TI(16,16)=-0.5
      TI(17,1)=6.*(DM(1)+DM(2))/DMI
      TI(17,2)=-3.*(DM(1)+DM(2))/DMI
      TI(17,3)=3./DMI
      TI(17,4)=(DM(1)+DM(2))/(2.*DMI)
      TI(17,5)=-1./DMI
      TI(17,6)=1./(2.*DMI)
      TI(17,7)=6.*(DM(5)-DM(4))/DMI
      TI(17,8)=-3./DMI
      TI(17,9)=3.*(DM(4)-DM(5))/DMI
      TI(17,10)=-1./(2.*DMI)
      TI(17,11)-1./DMI
      TI(17,12)=(DM(5)-DM(4))/(2.*DMI)
      TI(17,13)=6.*(DM(4)-DM(5)-DM(1)-DM(2))/DMI
      TI(17,14)=3.*(1.-DM(1)-DM(2))/DMI
      TI(17,15)=3.*(DM(4)-DM(5)-1.)/DMI
      TI(17,16)=(1.-DM(1)-DM(2))/(2.*DMI)
      TI(17,18)=(DM(4)-DM(5)-1.)/(2.*DMI)
      TI(18,7)=6.
      TI(18,9)=-3.
      TI(18,12)= .5
      TI(18,13)=-6.
      TI(18,15)=-3.
      TI(18,18)=-0.5
      DO 105 J=1,18
      DO 1 5 I=1,18
      BCI(I,J)=0.
105   CONTINUE
      BCI(1,1)=1.
      BCI(2,2)=G(2)
      BCI(2,3)=-B(2)
      BCI(3,2)=-G(1)
      BCI(3,3)=B(1)
      BCI(4,4)=G(2)**2
      BCI(4,5)=-2.*G(2)*B(2)
      BCI(4,6)=B(2)**2
      BCI(5,4)=-G(1)*G(2)
```

```
      BCI(5,5)=G(2)*B(1)+G(1)*B(2)
      BCI(5,6)=-B(1)*B(2)
      BCI(6,4)=G(1)**2
      BCI(6,5)=-2.*G(1)*B(1)
      BCI(6,6)=B(1)**2
      DO 106 K=6,12,6
      DO 106 J=1,6
      DO 106 I=1,6
      BCI(K+J,K+I)=BCI(J,I)
106   CONTINUE
      CALL TIMES(TI,BCI,TC,18,18,18,1)
      DO 109 I=1,18
      DO 109 J=1,18
      T(I,J)=TO(I,J)
109   CONTINUE
      GO TO 108
107   WRITE(6,600)
108   CONTINUE
      FORMAT(1*********      ERROR      ANTISTRESS      *********1)
      RETURN
      END
      SUBROUTINE TIMES(A,B,P,N,M,L,KOK)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION A(1),B(1),P(1)
      KOK=1   A(N,M) , B(M,L) , P(N,L)    REGULAR    A*B
      KOK=2   A(M,N) , B(M,L) , P(N,L)    TRANSPOSE  A'*B
      IP=1
      DO 100 K=1,L
      DO 100 J=1,N
      P(IP)=0.
      GO TO(101,102),KOK
  1   CONTINUE
      DO 103 I=1,M
      IA=N*(I-1)+J
      IB=M*(K-1)+I
103   P(IP)=P(IP)+A(IA)*B(IB)
      GO TO 100
102   CONTINUE
      DO 104 I=1,M
      IA=M*(J-1)+I
      IB=M*(K-1)+I
104   P(IP)=P(IP)+A(IA)*B(IB)
100   IP=IP+1
      RETURN
      END
```

6. Reference symbol PDS30

```
P=100 PRCUTE=PUPH CCPILS=4
= UNIVERSITY, PATCH
WAS: 11:48:4
IGNED ON AT  11:48:46 ON MON SEP 22/75
RINTS
  **** PLATE  DISPL. ELEMENT 3  DG. OF FREEDOM AT 4 NODES *****
  ****  QUINTIC  VARIATION OF W ( CUBIC VAR. OF WN ALONG THE SIDES )****
  ****  CUBIC   VARIATION OF SHEAR ANGLE  FX,FY          *****
  ****  STRESSES MXX,MYY,MXY,QX,QY   AT CENTROID         ********
    **** MODULI OF ELASTISITY THROUGH  YOUNG(12)     *********
  ****TRANSFORMATION TO  W, PN, MS, MNN, MSN, MSS, FN, FS  ***************
   SUBROUTINE TRANL(X,Y,Z,K,TRL)
   IMPLICIT REAL*8 (A-H,O-Z)
   DIMENSION X(16,2),Y(16,2),Z(16,2),TRL(7,7),X2(2),Y2(2)
   DO 515 J=1,2
   X2(J)=X(K,J)
15 Y2(J)=Y(K,J)
   G2=X2(2)-X2(1)
   B2=Y2(2)-Y2(1)
   GLEN2=DSQRT(G2**2+B2**2)
   IF(GLEN2.LT.0.1D-12.AND.GLEN2.GT.-0.1D-12) WRITE(6,700)
   E2=B2/GLEN2
   G2=G2/GLEN2
   DO 516 I=1,7
   DO 516 J=1,7
16 TRL(J,I)=0.
   TRL(1,1)=B2
   TRL(1,2)=G2
   TRL(1,6)=B2
   TRL(2,1)=-G2
   TRL(2,2)=B2
   TRL(2,6)=-G2
   TRL(3,3)=B2**2
   TRL(3,4)=2.*B2*G2
   TRL(3,5)=G2**2
   TRL(4,3)=-B2*G2
   TRL(4,4)=B2**2-G2**2
   TRL(4,5)=B2*G2
   TRL(5,3)=G2**2
   TRL(5,4)=-2.*B2*G2
   TRL(5,5)=B2**2
   TRL(6,6)=B2
   TRL(6,7)=G2
   TRL(7,6)=-G2
   TRL(7,7)=B2
00 FORMAT(1      *****     ERROR     TRANL     ******)
   RETURN
   END
   SUBROUTINE CNDNS(K)
   IMPLICIT REAL*8 (A-H,O-Z)
   COMMON/CON/ST(32,32),S(5,32),FCR(32)
   DIMENSION B(32)
   A=1./ST(K,K)
   DO 40 I=1,32
40 B(I)=ST(K,I)
   DO 41 J=1,32
   DO 41 I=1,32
41 ST(J,I)=ST(J,I)-B(I)*B(J)*A
   DO 42 I=1,5
   D=S(I,K)
```

```
      DO 42 J=1,32
   42 S(I,J)=S(I,J)-P(J)*A*D
      D=FOR(K)
      DO 43 I=1,32
   43 FOR(I)=FOR(I)-P(I)*D*A
      RETURN
      END
      SUBROUTINE  STIFF
      IMPLICIT DOUBLE (A-H,O-Z)
      COMMON/STI/X(3,2 ),YOUNG(12),STUCK(36,36),STICK(8,36),FORCE(36),
     1INFO(20)
      COMMON/MAN/BOL(2,36,36),COL(2,8,36),DOL(2,36),GRAM(16,16),NGRAM
      COMMON/COOR/CO2(3,3),XC(3),YC(3),CCM(3,3)
      DIMENSION TRAN(30,30),SMK(30,30),STR(5,30),DAOG(30),
     1XM(16,2),YM(16,2),ZM(16,2),X1(3),Y1(3),Z1(3),CO3(7,7)
     2,C1(3),B1(3),C1(3),STU(30,30),TIX(5,30),FOR(30)
      DO 100 I=1,3
      XO(I)=X(1,I)
  100 YO(I)=X(2,I)
      DO 101 I=1,36
      DO 101 J=1,36
  101 STUCK(J,I)=0.
      DO 102 I=1,36
      DO 102 J=1,8
  102 STICK(J,I)=0.
      DO 103 I=1,36
  103 FORCE(I)= .
      DO 607 I=1,30
  607 DAOG(I)=0.
      IF(NGRAM.EQ.0) GO TO 517
      DO 516 M=1,NGRAM
      DO 516 I=1,2
      XM(M,I)=GRAM(M,I)
      ZM(M,I)=GRAM(M,I+4)
  516 YM(M,I)=GRAM(M,I+2)
  517 CONTINUE
      CNLD=0.
      DO 432 I=1,3
      DO 432 J=6,8
  432 IF(X(I,J).NE.0.) CNLD=1.
      IF(CNLD.NE.1.)GO TO 437
      DO 433 I=6,8
      J=8*(I-6)+1
  433 LAOG(J)=X(3,I)
  437 CONTINUE
      NLAK=INFO(1)
      IF(NLAK.EQ.1.OR.NLAK.EQ.2)GO TO 104
      CALL SUBTI
      IF(NLAK.EQ.0)GO TO 105
      NLAK=NLAK-10
      DO 106 I=1,30
      DO 106 J=1,20
  106 BOL(NLAK,J,I)=STUCK(J,I)
      DO 113 I=1,3
      DO 113 J=1,5
  113 COL(NLAK,J,I)=STICK(J,I)
      DO 108 I=1,30
  108 DOL(NLAK,I)=FORCE(I)
      GO TO 105
  104 DO 109 I=1,30
```

```
      DO 109 J=1,30
    9 STUCK(J,I)=BOL(NLAK,J,I)
      DO 214 I=1,30
      DO 214 J=1,5
   14 STICK(J,I)=COL(NLAK,J,I)
      DO 111 I=1,30
   11 FORCE(I)=DCL(NLAK,I)
   05 CONTINUE
      DO 438 I=1,30
   38 FORCE(I)=FORCE(I)+DADG(I)
      NLIK=INFO(2)
      IF(NLIK.EQ. .OR.NLIK.EQ.0) GO TO 115
      DO 601 I=1,30
      DO 601 J=1,30
   01 STU(J,I)=STUCK(J,I)
      DO 602 I=1,30
      DO 602 J=1,5
   02 TIK(J,I)=STICK(J,I)
      DO 603 I=1,30
    3 FOR(I)=FORCE(I)
      DO 116 I=1,NLIK
      K=(I-1)*2+3
      L=INFO(K)
      K1=INFO(K+L)
      DO 583 II=1,3
      DO 584 IJ=1,30
   84 TRAN(IJ,II)=0.
   83 TRAN(II,II)=1.
      CALL TRAN1(XM,YM,ZM,K1,CO3)
      IF(L.GT.3) GO TO 608
      KK=(I-1)*8
      DO 222 IL=1,7
      DO 222 JL=1,7
   22 TRAN(KK+IL+1,KK+JL+1)=CO3(IL,JL)
      GO TO 609
    8 KK=24+(I-4)*2
      DO 610 IL=1,2
      DO 610 JL=1,2
   10 TRAN(KK+IL,KK+JL)=CO3(IL,JL)
   09 CONTINUE
      CALL TIMES(STU,TRAN,SMK,30,30,30,1)
      CALL TIMES(TRAN,SMK,STU,30,30,30,2)
      CALL TIMES(TIK,TRAN,STR,5,30,30,1)
      CALL TIMES(TRAN,FOR,DADG,30,30,1,2)
      DO 232 IL=1,3
      DO 232 JL=1,5
   32 TIK(JL,IL)=STR(JL,IL)
      DO 233 IL=1,3
   33 FOR(IL)=DADG(IL)
  116 CONTINUE
      DO 604 I=1,3
      DO 604 J=1,30
   04 STUCK(J,I)=STU(J,I)
      DO 605 I=1,30
      DO 605 J=1,5
   05 STICK(J,I)=TIK(J,I)
      DO 606 I=1,30
   06 FORCE(I)=FOR(I)
  115 CONTINUE
      RETURN
```

```
      END
      SUBROUTINE  SHAPEF(A,B,G,DT)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/JON/SM(32,32),S(5,32),D(5,5),P(3),ENP(32,1)
      DIMENSION  BE(5,32),BET(3,18),CET(3,3),A(3),B(3),G(3)
     1,BED(3,18),CBI(6,6),TEN(6,18),DEN(6,18),EM(8,32)
     2,SE(7),SEX(7),SEY(7)
      COMMON/SOBA/OM(5),OK(3)
      CET(1,1)=(B(1)/DT)**2
      CET(1,2)=2.*B(1)*B(2)/DT**2
      CET(1,3)=(B(2)/DT)**2
      CET(2,1)=(G(1)/DT)**2
      CET(2,2)=2.*G(1)*G(2)/DT**2
      CET(2,3)=(G(2)/DT)**2
      CET(3,1)=2.*G(1)*B(1)/DT**2
      CET(3,2)=2.*(G(1)*B(2)+G(2)*B(1))/DT**2
      CET(3,3)=2.*G(2)*B(2)/DT**2
      DO 100 I=1,3
      DO 100 J=1,18
      BET(I,J)= .
100   CONTINUE
      DO 101 I=1,8
      DO 1 1 J=1,32
      BE(I,J)=0.
101   CONTINUE
      BET(1,4)=2.
      BET(1,7)=6.*A(1)
      BET(1,8)=2.*A(2)
      BET(1,11)=12.*A(1)**2
      BET(1,12)=6.*A(1)*A(2)
      BET(1,13)=2.*A(2)**2
      BET(1,16)=20.*A(1)**3+12.*OM(1)*A(1)**2*A(2)+2.*OM(2)*A(2)**3
      BET(1,17)=6.*A(1)*A(2)**2+2.*OM(3)*A(2)**3
      BET(1,18)=2.*A(2)**3*OM(5)
      BET(2,5)=1.
      BET(2,8)=2.*A(1)
      BET(2,9)=2.*A(2)
      BET(2,12)=3.*A(1)**2
      BET(2,13)=4.*A(1)*A(2)
      BET(2,14)=3.*A(2)**2
      BET(2,16)=4.*OM(1)*A(1)**3+6.*OM(2)*A(1)*A(2)**2
      BET(2,17)=6.*A(1)**2*A(2)+6.*OM(3)*A(1)*A(2)**2
      BET(2,18)=6.*A(1)*A(2)**2*OM(5)+4.*OM(4)*A(2)**3
      BET(3,6)=2.
      BET(3,9)=2.*A(1)
      BET(3,10)=6.*A(2)
      BET(3,13)=2.*A(1)**2
      BET(3,14)=6.*A(1)*A(2)
      BET(3,15)=12.*A(2)**2
      BET(3,16)=6.*OM(2)*A(1)**2*A(2)
      BET(3,17)=2.*A(1)**3+6.*OM(3)*A(1)**2*A(2)
      BET(3,18)=20.*A(2)**3+12.*OM(4)*A(1)*A(2)**2+6.*OM(5)*A(1)**2*A(2)
      CALL TIMES(CET,BET,BED,3,3,18,1)
      DO 102 I=1,3
      DO 1 2 J=1,18
      BE(I,J)=-BED(I,J)
102   CONTINUE
      DIM1=A(1)*A(2)*A(3)
      DIM2=(B(1)*A(2)*A(3)+B(2)*A(1)*A(3)+B(3)*A(1)*A(2))/DT
      DIM3=(G(1)*A(2)*A(3)+G(2)*A(1)*A(3)+G(3)*A(1)*A(2))/DT
```

```
      DO 108 I=1,3
      K=I+1
      J=I+2
      IF(K.GT.3)K=K-3
      IF(J.GT.3)J=J-3
      SF(I)=(2.*A(I)-1.)*A(I)+3.*DIM1
      SF(I+3)=4.*A(K)*A(J)-12.*DIM1
      SFX(I)=B(I)*(4.*A(I)-1.)/DT+3.*DIM2
      SFX(I+3)=(B(K)*4.*A(J)+B(J)*4.*A(K))/DT-12.*DIM2
      SFY(I)=G(I)*(4.*A(I)-1.)/DT+3.*DIM3
    8 SFY(I+3)=(G(K)*4.*A(J)+G(J)*4.*A(K))/DT-12.*DIM3
      SF(7)=27.*DIM1
      SFX(7)=27.*DIM2
      SFY(7)=27.*DIM3
      DO 103 J=19,31,2
      K=(J-19)/2+1
      BF(1,J)=SFX(K)
      BF(2,J+1)=SFY(K)
      BF(3,J)=SFY(K)
      BF(3,J+1)=SFX(K)
      BF(4,J)=SF(K)
  03  BF(5,J+1)=SF(K)
      CALL TIMES(F,BF,S,5,5,32,1)
      CALL TIMES(CF,S,SM,32,5,32,2)
      DO 104 I=1,6
      DO 104 J=1,6
      CBI(I,J)=0.
  04  CONTINUE
      DO 105 I=1,6
      DO 105 J=1,18
      TEN(I,J)=0.
  05  CONTINUE
      DO 106 I=1,8
      DO 106 J=1,32
      SM(I,J)=0.
  06  CONTINUE
      CBI(1,1)=1.
      CBI(2,2)=B(1)/DT
      CBI(2,3)=B(2)/DT
      CBI(3,2)=G(1)/DT
      CBI(3,3)=G(2)/DT
      CBI(4,4)=(B(1)/DT)**2
      CBI(4,5)=2.*B(1)*B(2)/DT**2
      CBI(4,6)=(B(2)/DT)**2
      CBI(5,4)=G(1)*B(1)/DT**2
      CBI(5,5)=(G(2)*B(1)+G(1)*B(2))/DT**2
      CBI(5,6)=G(2)*B(2)/DT**2
      CBI(6,4)=(G(1)/DT)**2
      CBI(6,5)=2.*G(1)*G(2)/DT**2
      CBI(6,6)=(G(2)/DT)**2
      TEN(1,1)=1.
      TEN(1,2)=A(1)
      TEN(1,3)=A(2)
      TEN(1,4)=A(1)**2
      TEN(1,5)=A(1)*A(2)
      TEN(1,6)=A(2)**2
      TEN(1,7)=A(1)**3
      TEN(1,8)=A(1)**2*A(2)
      TEN(1,9)=A(1)*A(2)**2
      TEN(1,10)=A(2)**3
```

```
TEN(1,11)=A(1)**4
TEN(1,12)=A(1)**3*A(2)
TEN(1,13)=A(1)**2*A(2)**2
TEN(1,14)=A(1)*A(2)**3
TEN(1,15)=A(2)**4
TEN(1,16)=A(1)**5+OM(1)*A(1)**4*A(2)+OM(2)*A(1)**2*A(2)**3
TEN(1,17)=A(1)**3*A(2)**2+OM(3)*A(1)**2*A(2)**3
TEN(1,18)=A(2)**5+OM(4)*A(1)*A(2)**4+OM(5)*A(1)**2*A(2)**3
TEN(2,2)=1.
TEN(2,4)=2.*A(1)
TEN(2,5)=A(2)
TEN(2,7)=3.*A(1)**2
TEN(2,8)=2.*A(1)*A(2)
TEN(2,9)=A(2)**2
TEN(2,11)=4.*A(1)**3
TEN(2,12)=3.*A(1)**2*A(2)
TEN(2,13)=2.*A(1)*A(2)**2
TEN(2,14)=A(2)**3
TEN(2,16)=5.*A(1)**4+4.*OM(1)*A(1)**3*A(2)+2.*OM(2)*A(1)*A(2)**3
TEN(2,17)=3.*A(1)**2*A(2)**2+2.*OM(3)*A(1)*A(2)**3
TEN(2,18)=OM(4)*A(2)**4+2.*OM(5)*A(1)*A(2)**3
TEN(3,3)=1.
TEN(3,5)=A(1)
TEN(3,6)=2.*A(2)
TEN(3,8)=A(1)**2
TEN(3,9)=2.*A(1)*A(2)
TEN(3,10)=3.*A(2)**2
TEN(3,12)=A(1)**3
TEN(3,13)=2.*A(1)**2*A(2)
TEN(3,14)=3.*A(1)*A(2)**2
TEN(3,15)=4.*A(2)**3
TEN(3,16)=OM(1)*A(1)**4+3.*OM(2)*A(1)**2*A(2)**2
TEN(3,17)=2.*A(1)**3*A(2)+3.*OM(3)*A(1)**2*A(2)**2
TEN(3,18)=5.*A(2)**4+4.*OM(4)*A(1)*A(2)**3+3.*OM(5)*A(1)**2*A(2)**
12
TEN(4,4)=2.
TEN(4,7)=6.*A(1)
TEN(4,8)=2.*A(2)
TEN(4,11)=12.*A(1)**2
TEN(4,12)=6.*A(1)*A(2)
TEN(4,13)=2.*A(2)**2
TEN(4,16)=2 .*A(1)**3+12.*OM(1)*A(1)**2*A(2)+2.*OM(2)*A(2)**3
TEN(4,17)=6.*A(1)*A(2)**2+2.*OM(3)*A(2)**3
TEN(4,18)=2.*OM(5)*A(2)**3
TEN(5,5)=1.
TEN(5,8)=2.*A(1)
TEN(5,9)=2.*A(2)
TEN(5,12)=3.*A(1)**2
TEN(5,13)=4.*A(1)*A(2)
TEN(5,14)=3.*A(2)**2
TEN(5,16)=4.*OM(1)*A(1)**3+6.*OM(2)*A(1)*A(2)**2
TEN(5,17)=6.*A(1)**2*A(2)+6.*OM(3)*A(1)*A(2)**2
TEN(5,18)=4.*OM(4)*A(2)**3+6.*OM(5)*A(1)*A(2)**2
TEN(6,6)=2.
TEN(6,9)=2.*A(1)
TEN(6,10)=6.*A(2)
TEN(6,13)=2.*A(1)**2
TEN(6,14)=6.*A(1)*A(2)
TEN(6,15)=12.*A(2)**2
TEN(6,16)=6.*OM(2)*A(1)**2*A(2)
```

```
      TEN(6,17)=2.*A(1)**3+6.*DM(3)*A(1)**2*A(2)
      TEN(6,18)=2.*A(2)**3+12.*DM(4)*A(1)*A(2)**2+6.*DM(5)*A(1)**2*A(2)
      CALL TIMES(CRT,TEN,DEN,6,6,18,1)
      DO 107  I=1,6
      DO 107  J=1,18
      EN(I,J)=DEN(I,J)
107   CONTINUE
      DO 1 9 J=19,31,2
      K=(J-19)/2+1
      EN(7,J)=SF(K)
 9    EN(8,J+1)=SF(K)
      CALL TIMES(EN,P,EMP,32,8,1,2)
      RETURN
      END
      SUBROUTINE  SUBTT
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/STI/X(3,20),ELMOD(12),STUCK(36,36),STTCK(8,36),SDECE(32)
     1,INED(20)
      COMMON/JCN/SM(32,32),S(5,42),D(5,5),B(3),EMP(32,1)
      COMMON/CORA/CM(5),DK(3)
      COMMON/CORP/CD2(3,3),XC(3),YC(3),CCM(3,3)
      COMMON/CON/SDM(32,32),TIK(5,32),DACL(32)
      DO 208 I=1,5
      DO 208 J=1,5
208   D(J,I)=0.
      D(1,1)=ELMOD(1)
      D(1,2)=ELMOD(2)
      D(2,2)=ELMOD(3)
      D(3,3)=ELMOD(4)/2.
      D(4,4)=ELMOD(5)
      D(5,5)=ELMOD(6)
      D(2,1)=D(1,2)
      IF(ELMOD(12).NE.0.)D(2,1)=ELMOD(12)
      DATA ALF/0.333333330D00/,ALF1/0.059715870D00/,BET1/0.470142064D00/
      DATA ALF2/ .797426985D  /,BET2/ .1 128651D  /
      DATA WE1/0.225000000/,WE2/0.132394152D00/,WE3/0.125939180D00/
      DIMENSION  B(3),G(3),ARCO(2,7),W(7),A(3),LENG(3),
     1T(32,32),ESF(32),M1(32)
      ARCO(1,1)=ALF
      ARCO(2,1)=ALF
      ARCO(1,2)=BET1
      ARCO(2,2)=BET1
      ARCO(1,3)=ALF1
      ARCO(2,3)=BET1
      ARCO(1,4)=BET1
      ARCO(2,4)=ALF1
      ARCO(1,5)=BET2
      ARCO(2,5)=BET2
      ARCO(1,6)=ALF2
      ARCO(2,6)=BET2
      ARCO(1,7)=BET2
      ARCO(2,7)=ALF2
      W(1)=WE1
      W(2)=WE2
      W(3)=WE2
      W(4)=WE2
      W(5)=WE3
      W(6)=WE3
      W(7)=WE3
      DO 223 I=1,8
```

```
23 B(I)=0.
   B(1)=X(3,5)
   DO 200 I=1,3
   K=I+1
   IF(K.GT.3)K=K-3
   J=I+2
   IF(J.GT.3)J=J-3
   B(I)=YC(K)-YC(J)
   C(I)=XC(J)-XC(K)
   LENG(I)=DSQRT(B(I)**2+C(I)**2)
00 CONTINUE
   CK(1)=B(1)*B(2)+C(1)*C(2)
   CK(2)=B(2)*B(3)+C(2)*C(3)
   CK(3)=B(3)*B(1)+C(3)*C(1)
   DT=B(1)*C(2)-B(2)*C(1)
   CM(1)=5.*CK(1)/(CK(1)+CK(2))
   CM(2)=5.*(CK(3)*CK(2)+CK(1)*CK(2)-3.*CK(1)*CK(3))/
  1((CK(1)+CK(2))*(2.*CK(3)-3.*CK(2)))
   CM(3)=(3.*CK(3)-2.*CK(2))/(2.*CK(3)-3.*CK(2))
   CM(4)=5.*CK(1)/(CK(1)+CK(3))
   CM(5)=5.*(CK(3)*CK(2)+CK(1)*CK(3)-3.*CK(1)*CK(2))/
  1((CK(1)+CK(3))*(2.*CK(3)-3.*CK(2)))
   CALL ANTI(3,C,DT,T)
   DO 214 I=1,32
   DO 214 J=1,32
   SCM(I,J)=0.
14 CONTINUE
   DO 314 I=1,32
   DACL(I)= .
14 CONTINUE
   DO 213  K=1,7
   A(1)=ARCD(1,K)
   A(2)=ARCD(2,K)
   A(3)=1.-A(1)-A(2)
   CALL  SHAPEF(A,B,C,DT)
   IF(K.NE.1)GO TO 510
   DO 511 I=1,32
   DO 511 J=1,5
11 TIK(J,I)=S(J,I)
1  CONTINUE
   DO 313 I=1,32
   DACL(I)=DACL(I)+DT/2.*W(K)*ENP(I,1)
13 CONTINUE
   DO 213 J=1,32
   DO 213 I=1,32
   SCM(I,J)=SCM(I,J)+DT/2.*W(K)*SM(I,J)
13 CONTINUE
   CALL TIMES(SCM,T,SM,32,32,32,1)
   CALL TIMES(T,SM,SCM,32,32,32,2)
   CALL TIMES(TIK,T,S,5,32,32,1)
   CALL TIMES(T,DACL,FRE,32,32,1,2)
   DATA M1/1,2,3,4,5,6,10,2 ,7,8,9,10,11,12,21,22,
  113,14,15,16,17,18,23,24,25,26,27,28,29,30,31,32/
   DO 224 I=1,32
   DO 224 J=1,32
24 T(J,I)=0.
   DO 222 I=1,32
22 T(M1(I),I)=1.
   CALL TIMES(SCM,T,SM,32,32,32,1)
   CALL TIMES(T,SM,SCM,32,32,32,2)
```

```
      CALL TIMES(S,T,TIK,5,32,32,1)
      CALL TIMES(T,F35,FACL,32,32,1,2)
      CALL CADNS(32)
      CALL CADNS(31)
      DO 550 I=1,3
      DO 550 J=1,30
550   STICK(J,I)=SCM(J,I)
      DO 551 I=1,3
      DO 551 J=1,5
551   STICK(J,I)=TIK(J,I)
      DO 552 I=1,3
552   FORCE(I)=FACL(I)
      RETURN
      END
      SUBROUTINE  AMTI(B,G,DT,T)
      IMPLICIT REAL*8 (A-F,O-Z)
      DIMENSION TI(18,18),BCI(18,18),TO(18,13),T(32,32),B(3),G(3)
      COMMON/DORA/OM(5),DK(3)
      DO 100 I=1,18
      DO 100 J=1,18
      TI(I,J)=0.
100   CONTINUE
      DO 110 I=1,32
      DO 110 J=1,32
      IF(I.EQ.J.AND.I.GT.18) GO TO 111
      T(I,J)=0.
      GO TO 110
111   T(I,J)=1.
110   CONTINUE
      OMI=1.-OM(3)
      IF(OMI.EQ.0)GO TO 107
      TI(1,13)=1.
      TI(2,14)=1.
      TI(3,15)=1.
      TI(4,16)=0.5
      TI(5,17)=1.
      TI(6,18)=0.5
      TI(7,1)=10.
      TI(7,2)=-4.
      TI(7,4)=0.5
      TI(7,13)=-10.
      TI(7,14)=-6.
      TI(7,16)=-1.5
      TI(8,1)=6.*OM(1)
      TI(8,2)=-3.*OM(1)
      TI(8,3)=3.
      TI(8,4)=0.5*OM(1)
      TI(8,5)=-1.
      TI(8,13)=-6. *OM(1)
      TI(8,14)=-3.*OM(1)
      TI(8,15)=-3.
      TI(8,16)=- .5*OM(1)
      TI(8,17)=-2.
      TI(9,7)=6.*OM(4)
      TI(9,8)=3.
      TI(9,9)=-3.*OM(4)
      TI(9,11)=-1.
      TI(9,12)=0.5*OM(4)
      TI(9,13)=-6.*OM(4)
      TI(9,14)=-3.
```

```
TI(9,15)=-3.*CM(4)
TI(9,17)=-2.
TI(9,18)=-0.5*CM(4)
TI(10,7)=10.
TI(10,9)=-4.
TI(10,12)=0.5
TI(10,13)=-10.
TI(10,15)=-6.
TI(10,18)=-1.5
TI(11,1)=-15.
TI(11,2)=7.
TI(11,4)=-1.
TI(11,13)=15.
TI(11,14)=3.
TI(11,16)=1.5
TI(12,1)=-12.*CM(1)
TI(12,2)=6.*CM(1)
TI(12,3)=-2.
TI(12,4)=-CM(1)
TI(12,5)=1.
TI(12,13)=12.*CM(1)
TI(12,14)=6.*CM(1)
TI(12,15)=2.
TI(12,16)=CM(1)
TI(12,17)=1.
TI(13,1)=-6.*(CM(1)+CM(2))/CMI
TI(13,2)=3.*(CM(1)+CM(2))/CMI
TI(13,3)=-3./CMI
TI(13,4)=-(CM(1)+CM(2))/(2.*CMI)
TI(13,5)=1./CMI
TI(13,6)=-CM(3)/(2.*CMI)
TI(13,7)=6.*(CM(3)*CM(4)-CM(5))/CMI
TI(13,8)=3.*CM(3)/CMI
TI(13,9)=3.*(CM(5)-CM(3)*CM(4))/CMI
TI(13,10)=1./(2.*CMI)
TI(13,11)=-CM(3)/CMI
TI(13,12)=(CM(3)*CM(4)-CM(5))/(2.*CMI)
TI(13,13)=6.*(CM(1)+CM(2)+CM(5)-CM(3)*CM(4))/CMI
TI(13,14)=3.*(CM(1)+CM(2)-CM(3))/CMI
TI(13,15)=3.*(1.+CM(5)-CM(3)*CM(4))/CMI
TI(13,16)=(CM(1)+CM(2)-1.)/(2.*CMI)
TI(13,17)=2.
TI(13,18)=(CM(5)+CM(3)-CM(3)*CM(4))/(2.*CMI)
TI(14,7)=-12.*CM(4)
TI(14,8)=-2.
TI(14,9)=6.*CM(4)
TI(14,11)=1.
TI(14,12)=-CM(4)
TI(14,13)=12.*CM(4)
TI(14,14)=2.
TI(14,15)=6.*CM(4)
TI(14,17)=1.
TI(14,18)=CM(4)
TI(15,7)=-15.
TI(15,9)=7.
TI(15,12)=-1.
TI(15,13)=15.
TI(15,15)=8.
TI(15,18)=1.5
TI(16,1)=6.
```

```
      TI(16,2)=-3.
      TI(16,7)=0.5
      TI(16,13)=-6.
      TI(16,14)=-3.
      TI(16,16)=-0.5
      TI(17,1)=6.*(CM(1)+CM(2))/CMI
      TI(17,2)=-3.*(CM(1)+CM(2))/CMI
      TI(17,3)=3./CMI
      TI(17,4)=(CM(1)+CM(2))/(2.*CMI)
      TI(17,5)=-1./CMI
      TI(17,6)=1./(2.*CMI)
      TI(17,7)=6.*(CM(5)-CM(4))/CMI
      TI(17,8)=-3./CMI
      TI(17,9)=3.*(CM(4)-CM(5))/CMI
      TI(17,10)=-1./(2.*CMI)
      TI(17,11)=1./CMI
      TI(17,12)=(CM(5)-CM(4))/(2.*CMI)
      TI(17,13)=6.*(CM(4)-CM(5)-CM(1)-CM(2))/CMI
      TI(17,14)=3.*(1.-CM(1)-CM(2))/CMI
      TI(17,15)=3.*(CM(4)-CM(5)-1.)/CMI
      TI(17,16)=(1.-CM(1)-CM(2))/(2.*CMI)
      TI(17,18)=(CM(4)-CM(5)-1.)/(2.*CMI)
      TI(18,7)=6.
      TI(18,9)=-3.
      TI(18,12)=0.5
      TI(18,13)=-6.
      TI(18,15)=-3.
      TI(18,18)=-0.5
      DO 105 J=1,18
      DO 105 I=1,18
      BCI(I,J)=0.
    5 CONTINUE
      BCI(1,1)=1.
      BCI(2,2)=G(2)
      BCI(2,3)=-B(2)
      BCI(3,2)=-G(1)
      BCI(3,3)=B(1)
      BCI(4,4)=G(2)**2
      BCI(4,5)=-2.*G(2)*B(2)
      BCI(4,6)=B(2)**2
      BCI(5,4)=-G(1)*G(2)
      BCI(5,5)=G(2)*B(1)+G(1)*B(2)
      BCI(5,6)=-B(1)*B(2)
      BCI(6,4)=G(1)**2
      BCI(6,5)=-2.*G(1)*B(1)
      BCI(6,6)=B(1)**2
      DO 106 K=6,12,6
      DO 106 J=1,6
      DO 106 I=1,6
      BCI(K+J,K+I)=BCI(J,I)
   06 CONTINUE
      CALL TIMES(TI,BCI,TO,18,18,18,1)
      DO 109 I=1,18
      DO 109 J=1,18
      T(I,J)=TO(I,J)
    9 CONTINUE
      GO TO 108
   07 WRITE(6,600)
   08 CONTINUE
  00 FORMAT(1**********      ERROR      ANTISTROFIS      **********)
```

```
      RETURN
      END
      SUBROUTINE TIMES(A,B,P,N,M,L,KOK)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION A(1),B(1),P(1)
      KOK=1   A(N,M)  ,  B(M,L)  ,  P(N,L)    REGULAR    A*B
      KOK=2   A(M,N)  ,  B(M,L)  ,  P(N,L)    TRANSPOSE  A'*B
      IP=1
      DO 100 K=1,L
      DO 100 J=1,N
      P(IP)=0.
      GO TO(101,102),KOK
101   CONTINUE
      DO 103 I=1,M
      IA=N*(I-1)+J
      IB=M*(K-1)+I
103   P(IP)=P(IP)+A(IA)*B(IB)
      GO TO 100
102   CONTINUE
      DO 104 I=1,M
      IA=M*(J-1)+I
      IB=M*(K-1)+I
104   P(IP)=P(IP)+A(IA)*B(IB)
100   IP=IP+1
      RETURN
      END
```

7. Reference symbol PRO18

```
 P=100 PRCUTE=DURF CCPIES=4
= UNIVERSITY, FATCH
WAS: 11:48:46
IGNED ON AT  11:48:53 ON MON SEP 22/75
RINT*
  *** PLATE ROTATION ELEMENT WITH 18 DEGREES OF FR.  AT 6 NODES W,RX,RY*****
  **** CUBIC VARIATICN FCR THE DIS. W QUADRATIC FOR THE ROTATIONS     *******
  **** 5  STRESSES AT THE CENTRCID   MXX,MYY,MXY,QX,QY                *******
  ****  TRANSFCRMATICN  TO  RM  RS                                    *******
  ****  ELASTICITY  MCDULI  VIA  YCUNG(12)                            *******
   SUBRCUTINE STIFF
   IMPLICIT REAL*8 (A-H,O-Z)
   CCMMCN/STI/X(3,2 ),YCUNG(12),STUCK(36,36),STICK(8,36),FORCE(36)
  1,INFC(20)
   CCMMCN/MAN/RCL(2,36,36),CCL(2,8,36),DCL(2,36),GRAM(16,16),NGRAM
   CCMMCN/CCDR/X1(3),Y1(3)
   DIMENSICN TRAN(18,18),SMK(18,18),SMK1(18,18),STR(5,18),TIK(5,18),
  1PAIG(18),XM(16,2),YM(16,2),CO3(2,2)
   DO 100 I=1,3
   X1(I)=X(1,I)
00 Y1(I)=X(2,I)
   DO 101 I=1,36
   DO 101 J=1,36
 1 STUCK(J,I)= .
   DO 102 I=1,36
   DO 102 J=1,8
 2 STICK(J,I)=0.
   DO 103 I=1,36
03 FORCE(I)=0.
   IF(NGRAM.EQ.0) GO TO 517
   DO 516 M=1,NGRAM
   DO 516 I=1,2
   XM(M,I)=GRAM(M,I)
16 YM(M,I)=GRAM(M,I+2)
17 CONTINUE
   NLAK=INFO(1)
   IF(NLAK.LE.1.OR.NLAK.EQ.2) GO TO 104
   CALL SUBTI
   IF(NLAK.EQ. ) GO TO 1 5
   NLAK=NLAK-10
   DO 106 I=1,18
   DO 106 J=1,18
06 RCL(NLAK,J,I)=STUCK(J,I)
   DO 113 I=1,18
   DO 113 J=1,5
13 CCL(NLAK,J,I)=STICK(J,I)
   DO 1 8 I=1,18
08 DCL(NLAK,I)=FORCE(I)
   GC TO 105
04 DO 109 I=1,18
   DO 109 J=1,18
 9 STUCK(J,I)=RCL(NLAK,J,I)
   DO 214 I=1,18
   DC 214 J=1,5
14 STICK(J,I)=CCL(NLAK,J,I)
   DO 111 I=1,18
11 FORCE(I)=DCL(NLAK,I)
05 CCNTINUE
   NLIK=INFO(2)
   IF(NLIK.EQ.0.OR.NLIK.EQ.99) GO TO 112
```

```
      DO 213 I=1,18
      DO 213 J=1,18
13    SMK(J,I)=STUCK(J,I)
      DO 114 I=1,18
      DO 114 J=1,5
14    STR(J,I)=STICK(J,I)
      DO 115 I=1,NLTK
      K=(I-1)*2+3
      L=INFC(K)
      K1=INFC(K+1)
      DO 583 II=1,18
      DO 584 IJ=1,18
84    TRAN(IJ,II)=0.
83    TRAN(II,II)=1.
      CALL TRANL(XM,YM,K1,CC3)
      KK=(L-1)*3
      DO 587 IN=1,2
      DO 587 IL=1,2
87    TRAN(KK+IL+1,KK+IN+1)=CO3(IL,IN)
      CALL TIMES(SMK,TRAN,SMK1,18,18,18,1)
      CALL TIMES(TRAN,SMK1,SMK,18,18,18,2)
      CALL TIMES(STR,TRAN,TIK,5,18,18,1)
      DO 116 NI=1,18
      DO 116 NJ=1,5
16    STR(NJ,NI)=TIK(NJ,NI)
15    CONTINUE
      DO 117 I=1,18
      DO 117 J=1,18
17    STUCK(J,I)=SMK(J,I)
      DO 118 I=1,18
      DO 118 J=1,5
18    STICK(J,I)=STR(J,I)
12    CONTINUE
      BPPC=0.
      DO 604 J=6,11
      DO 604 I=1,3
04    IF(X(I,J).NE. .)BPPC=1.
      IF(BPPC.NE.1) GO TO 560
      DO 273 I=1,18
73    DAOG(I)= .
      DAOG(1)=X(3,6)
      DAOG(4)=X(3,7)
      DAOG(7)=X(3,8)
      DAOG(10)=X(3,9)
      DAOG(13)=X(3,1 )
      DAOG(16)=X(3,11)
      DO 274 I=1,18
74    FORCE(I)=FORCE(I)+DAOG(I)
60    CONTINUE
      RETURN
      END
      SUBROUTINE TRANL(X,Y,K1,TRL)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION X(16,2),Y(16,2),X2(2),Y2(2),TRL(2,2)
      DO 515 J=1,2
      X2(J)=X(K1,J)
15    Y2(J)=Y(K1,J)
      G2=X2(2)-X2(1)
      B2=Y2(2)-Y2(1)
      GLEN2=DSQRT(G2**2+B2**2)
```

```
      IF(GLEN2.LT. .1D-12.AND.GLEN2.GT.- .1D-12) WRITE(4,70 )
      B2=B2/GLEN2
      G2=G2/GLEN2
      TRL(1,1)=B2
      TRL(1,2)=G2
      TRL(2,1)=-G2
      TRL(2,2)=B2
00    FORMAT('        ******    ERROR   TRANL       *********')
      RETURN
      END
      SUBROUTINE CONDNS(K)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/CON/ST(19,19),T(5,19)
      DIMENSION B(19)
      A=1./ST(K,K)
      DO 40 I=1,19
40    B(I)=ST(K,I)
      DO 41 J=1,19
      DO 41 I=1,19
41    ST(J,I)=ST(J,I)-B(I)*B(J)*A
      DO 42 I=1,5
      D=T(I,K)
      DO 42 J=1,19
42    T(I,J)=T(I,J)-B(J)*A*D
      RETURN
      END
      SUBROUTINE SHAP(A,B,G,DT)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION A(3),B(3),G(3),BF(5,19),SR(6),SRX(6),SRY(6),
     1SWX(7),SWY(7)
      COMMON/JCN/SN(19,19),S(5,19),D(5,5)
      DIM1=A(1)*A(2)*A(3)
      DIM2=(B(1)*A(2)*A(3)+B(2)*A(1)*A(3)+B(3)*A(1)*A(2))/DT
      DIM3=(G(1)*A(2)*A(3)+G(2)*A(1)*A(3)+G(3)*A(1)*A(2))/DT
      DO 100 I=1,3
      K=I+1
      J=I+2
      IF(K.GT.3)K=K-3
      IF(J.GT.3)J=J-3
      SR(I)=(2.*A(I)-1.)*A(I)
      SR(I+3)=4.*A(K)*A(J)
      SRX(I)=B(I)*(4.*A(I)-1.)/DT
      SRX(I+3)=(B(K)*4.*A(J)+B(J)*4.*A(K))/DT
      SRY(I)=G(I)*(4.*A(I)-1.)/DT
      SRY(I+3)=(G(K)*4.*A(J)+G(J)*4.*A(K))/DT
      SWX(I)=SRX(I)+3.*DIM2
      SWX(I+3)=SRX(I+3)-12.*DIM2
      SWY(I)=SRY(I)+3.*DIM3
      SWY(I+3)=SRY(I+3)-12.*DIM3
100   CONTINUE
      SWX(7)=27.*DIM2
      SWY(7)=27.*DIM3
      DO 101 I=1,19
      DO 101 J=1,5
101   BF(J,I)=0.
      DO 102 I=1,11,2
      K=(I+1)/2
      BF(1,I)=SRX(K)
      BF(2,I+1)=SRY(K)
      BF(3,I)=SRY(K)
```

```
      BE(3,I+1)=SRX(K)
      BE(4,I)=-SR(K)
      BE(5,I+1)=-SR(K)
102   CONTINUE
      DO 103 I=13,19
      K=I-12
      BE(4,I)=SWX(K)
103   BE(5,I)=SWY(K)
      CALL TIMES(D,BE,S,5,5,19,1)
      CALL TIMES(BE,S,SW,19,5,19,2)
      RETURN
      END
      SUBROUTINE SLBTT
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/STT/X(3,20),YCUNC(12),STICK(36,36),STICK(8,36),FORCE(36)
     1,INFO(20)
      COMMON/JCM/SR(19,19),S(5,19),D(5,5)
      COMMON/CCCR/XC(3),YC(3)
      COMMON/CDM/ST(19,19),TIK(5,19)
      DIMENSION A1(7),A2(7),A3(7),U(7),B(3),G(3),A(3),M1(19),T(19,19)
      DATA A1/ .33333333D    , .59715377D    ,2* .47 142 4D    , .79742699D
     1 00,2*0.10128651D 00/,A3/0.33333333D 00,2*0.47014206D 00,0.05971572
     2D 00,2*0.10128651D 00,0.79742699D 00/,U/0.22500000D 00,3*0.12593918
     3D    ,3* .12593918D    /
      DO 200 I=1,5
      DO 200 J=1,5
      D(J,I)= .
      D(1,1)=YCUNC(1)
      D(1,2)=YCUNC(2)
      D(2,1)=D(1,2)
      D(2,2)=YCUNC(3)
      D(3,3)=YCUNC(4)/2.
      D(4,4)=YCUNC(5)
      D(5,5)=YCUNC(6)
      IF(YCUNC(12).NE. .) D(2,1)=YCUNC(12)
      DO 210 I=1,19
      DO 210 J=1,19
210   ST(J,I)= .
      DO 211 I=1,3
      K=I+1
      J=I+2
      IF(K.GT.3)K=K-3
      IF(J.GT.3)J=J-3
      B(I)=YC(K)-YC(J)
211   G(I)=XC(J)-XC(K)
      DT=B(1)*G(2)-B(2)*G(1)
      DO 212 K=1,7
      A2(K)=1.-A1(K)-A3(K)
      A(1)=A1(K)
      A(2)=A2(K)
      A(3)=A3(K)
      CALL SHAP(A,B,G,DT)
      IF(K.NE.1) GO TO 213
      DO 214 I=1,19
      DO 214 J=1,5
214   TIK(J,I)=S(J,I)
213   CONTINUE
      DO 215 I=1,19
      DO 215 J=1,19
215   ST(J,I)=ST(J,I)+U(K)*SW(J,I)*DT/2.
```

```
12 CONTINUE
   DO 217 I=1,19
   DO 217 J=1,19
17 T(J,I)=0.
   DATA M1/13,1,2,14,3,4,15,5,6,16,7,8,17,9,10,18,11,12,19/
   DO 218 I=1,19
18 T(M1(I),I)=1.
   CALL TIMES(ST,T,SM,19,19,19,1)
   CALL TIMES(T,SM,ST,19,19,19,2)
   CALL TIMES(TIK,T,S,5,19,19,1)
   DO 219 I=1,19
   DO 219 J=1,5
19 TIK(J,I)=S(J,I)
   CALL CNDNS(19)
   DO 220 I=1,18
   DO 22  J=1,19
20 STICK(J,I)=ST(J,I)
   DO 221 I=1,18
   DO 221 J=1,5
21 STICK(J,I)=TIK(J,I)
   PG=X(3,5)
   FORCE(1 )=PG*DT/6.
   FORCE(13)=PG*DT/6.
   FORCE(16)=PG*DT/6.
   RETURN
   END
   SUBROUTINE TIMES(A,B,R,N,M,L,KOK)
   IMPLICIT REAL*8 (A-H,O-Z)
   DIMENSION A(1),B(1),R(1)
   KOK=1 A(N,M) , B(M,L) , R(N,L)  REGULAR A*B=R
   KOK=2 A(M,N) , B(M,L) , R(N,L) TRANSPOSE  A*B=R
   IR=1
   DO 100 K=1,L
   DO 100 J=1,N
   R(IR)=0.
   GO TO(101,102),KOK
01 CONTINUE
   DO 103 I=1,M
   IA=M*(J-1)+J
   IB=M*(K-1)+I
03 R(IR)=R(IR)+A(IA)*B(IB)
   GO TO 100
02 CONTINUE
   DO 1 4 I=1,M
   IA=M*(J-1)+I
   IB=M*(K-1)+I
04 R(IR)=R(IR)+A(IA)*B(IB)
00 IR=IR+1
   RETURN
   END
```

SANDWICH DOME MODELS

COMPUTER LISTING

| | Reference symbol |
|---|---|
| 1. | DDS21 |
| 2. | DDS33 |
| 3. | DMX36 |
| 4. | DRO30 |

1. Reference symbol DDS21

```
0 P=100 PRCLTE=PURH CPFIES=4
 = UNIVERSITY, RATCH
 WAS: 15:52:34
SIGNED ON AT  15:52:40 ON MON SEP 22/75
PRINT*
   **** DOME ELEMENT  21 DG. OF FREEDOM AT 3 CORNER NODES ********
      ****   CUBIC   VARIATION OF W (W,WX,WY)   AND LINEAR OF U,V,EX,EY ********
   ****  TRANSFORMATION AT GLOBAL SET OF U,V,W AND TOTAL ROTATIONS  ********
   **** STRESSES AT CENTROID  MXX,MYY,MXY QX,QY,NXX,NYY,NXY   ***********
    SUBROUTINE  STIFF
    IMPLICIT REAL*8 (A-H,O-Z)
    COMMON/STI/X(3,20),YOUNG(12),STUCK(36,36),STICK(8,36),FORCE(36),
   1INFO(2 )
    COMMON/MAN/BOL(2,36,36),COL(2,8,36),DOL(2,36),GRAM(16,16),NGRAM
    COMMON/PAT/CO2(3,3),XO(3),YO(3)
    DIMENSION X1(3),Y1(3),Z1(3),G1(3),B1(3),C1(3),CO1(3,3),CO3(3,3)
    DIMENSION XM(16,2),YM(16,2),ZM(16,2),TRAN(21,21),MCL(3)
    DIMENSION CO4(3,3),SOK(21,21),SIK(8,21),DADL(21),FADG(21)
    COMMON/COND/SOK1(21,21),SIK1(8,21),SEK1(21)
    DATA MCL/3,10,17/
    DO 599 I=1,21
599 FORCE(I)=0.
    X1 , Y1 , Z1   GLOBAL  COORDINATES  OF  THE  VERTICES  (1,2,3)
    DO 415 I=1,3
    X1(I)=X(1,I)
    Y1(I)=X(2,I)
415 Z1(I)=X(3,I)
  XM , YM , ZM (M,2)  GLOBAL  COORDIN.  OF THE JOINT LINE (DIRECTION 1,2)
  M= THE CODE NUMBER OF THE JOINT LINE (MAX=16)
    IF(NGRAM.EQ.0)GO TO 517
    DO 516 M=1,NGRAM
    DO 516 I=1,2
    XM(M,I)=GRAM(M,I)
    YM(M,I)=GRAM(M,I+2)
516 ZM(M,I)=GRAM(M,I+4)
517 CONTINUE
    DO 400 I=1,3
    K=I+1
    J=I+2
    IF(K.GT.3)K=K-3
    IF(J.GT.3)J=J-3
    G1(I)=X1(J)-X1(K)
    B1(I)=Y1(J)-Y1(K)
400 C1(I)=Z1(J)-Z1(K)
  CO2(3,3)  TRANSF.  MATRIX FROM GLOBAL TO LOCAL  D' = < CO2 > D
  D'  =  LOCAL     D  =  GLOBAL
    CALL  TRANS(G1,B1,C1)
  XO(3) , YO(3) THE LOCAL COORDIN.  OF THE VERTICES (1,2,3)
    DO 592 I=1,3
    DO 592 J=1,3
592 CO4(I,J)=X(I,J)
    CALL TIMES(CO2,CO4,CO1,3,3,3,1)
    DO 414 J=1,3
    XO(J)=CO1(1,J)
414 YO(J)=CO1(2,J)
    NLAK=INFO(1)
    METRO=0
    IF(NLAK.EQ.-1.OR.NLAK.EQ.-2) GO TO 840
    IF(NLAK.EQ.1.OR.NLAK.EQ.2) GO TO 575
    CALL SUBTI
```

```
      IF(NLAK.EQ.0) GO TO 58
      NLAK=NLAK-10
      DO 900 I=1,21
9     DOL(NLAK,I)=FORCE(I)
      DO 576 J=1,21
      DO 576 I=1,21
576   BOL(NLAK,I,J)=STUCK(I,J)
      DO 577 J=1,21
      DO 577 I=1,8
577   COL(NLAK,I,J)=STICK(I,J)
      GO TO 580
840   NLAK=-NLAK
      METRO=-1
575   DO 578 J=1,21
      DO 578 I=1,21
578   STUCK(I,J)=BOL(NLAK,I,J)
      DO 579 J=1,21
      DO 579 I=1,8
579   STICK(I,J)=COL(NLAK,I,J)
      DO 901 I=1,21
901   FORCE(I)=DOL(NLAK,I)
      IF(METRO.NE.-1) GO TO 580
      DO 861 I=1,21
      DO 861 J=1,21
      E1=0.
      IF(I.NE.J) GO TO 862
      E1=-1.
      DO 863 II=1,3
      IF(I.EQ.MOL(II)) E1=1.
863   CONTINUE
862   TRAN(J,I)=E1
861   CONTINUE
      DO 1 I=1,21
      DO 1 J=1,21
    1 SOK1(J,I)=STUCK(J,I)
      DO 2 I=1,21
      DO 2 J=1,8
    2 SIK(J,I)=STICK(J,I)
      DO 3 I=1,21
    3 DAOL(I)=FORCE(I)
      CALL TIMES(SOK1,TRAN,SOK,21,21,21,1)
      CALL TIMES(TRAN,SOK,SOK1,21,21,21,2)
      CALL TIMES(SIK,TRAN,SIK1,8,21,21,1)
      CALL TIMES(TRAN,DAOL,SOK1,21,21,1,2)
      DO 4 I=1,21
      DO 4 J=1,21
    4 STUCK(J,I)=SOK1(J,I)
      DO 5 I=1,21
      DO 5 J=1,8
    5 STICK(J,I)=SIK1(J,I)
      DO 6 I=1,21
    6 FORCE(I)=SOK1(I)
580   CONTINUE
  INFO(1)=0  FINDS LOCAL STIFF. AND STRESS MATR. WITOUT STORING
  INFO(1)=1  TAKES LOCAL STIFF. AND STRESS MATR. OF THE 1ST. STORAGE
  INFO(1)=2  TAKES LOCAL STIFF AND STRESS MATR.  OF THE 2ND. STORAGE
  INFO(1)=11 FINDS LOCAL STIFF. AND STRESS MATR. AND STORES IN THE 1ST.
  INFO(1)=12 FINDS LOCAL STIFF. AND STRESS MATR. AND STORES IN THE 2ND.
  INFO(2)=0  KEEPS LOCAL WITHOUT  TRANSFORMATION
  INFO(2)=X  X  NO NODES IN THE ELEMENT NEED TRANSFORMATION (MAX=3)
```

```
      IF(INFO(2).EQ. .OR.INFO(2).EQ.99) GO TO 581
      DO 801 I=1,21
      DO 801 J=1,21
  801 SOK1(J,I)=STICK(J,I)
      DO 802 I=1,21
      DO 802 J=1,8
  802 SIK1(J,I)=STICK(J,I)
      DO 806 I=1,21
  806 SEK1(I)=FORCE(I)
      NLIK=INFO(2)
      DO 582 I=1,NLIK
    INFO(3) AYXCN ARITHMES TIS MODE
    INFO(4) CODE NUMBER OF NODE--JOINT LINE(MAX=16) (IF.EQ.0 ALL GLOBAL,
    IF.EQ.-1 ONLY U,V,W TRANSF)
    INFO(5),INFO(7) THE SAME AS INFO(3)
    INFO(6),INFO(8) THE SAME AS INFO(4)
      K=(I-1)*2+3
      L=INFO(K)
      K1=INFO(K+1)
      DO 583 II=1,21
      DO 584 IS=1,21
  584 TRAN(IS,II)=0.
  583 TRAN(II,II)=1.
      KK=(L-1)*7
      DO 585 IN=1,3
      DO 585 IL=1,3
  585 TRAN(KK+IN,KK+IL)=CO2(IN,IL)
      IF(K1.EQ.-1) GO TO 582
      IF(K1.EQ.0) GO TO 586
    INTERCONNECTION OF TWO PLATES
      CALL TRANL(XM,YM,ZM,K1,CO3)
    X AXE OF THE NEW SYSTEM THE JOINT LINE
    FY  SHEAR ANGLE CONDENSED OUT
      DO 587 IN=1,2
      DO 587 IL=1,2
      TRAN(KK+IN+3,KK+IL+3)=CO3(IN,IL)
  587 TRAN(KK+IN+5,KK+IL+5)=CO3(IN,IL)
      CALL TIMES(SOK1,TRAN,SOK,21,21,21,1)
      CALL TIMES(TRAN,SOK,SOK1,21,21,21,2)
      DO 902 IM=1,21
  902 DACL(IM)=SEK1(IM)
      DO 593 JM=1,21
      DO 593 IM=1,8
  593 SIK(IM,JM)=SIK1(IM,JM)
      CALL TIMES(SIK,TRAN,SIK1,8,21,21,1)
      CALL TIMES(TRAN,DACL,SEK1,21,21,1,2)
      KUK=KK+7
      CALL CNDNS(KUK)
      SOK1(KUK,KUK)=1.
      GO TO 582
    INTERCONNECTION OF MORE THAN TWO PLATES
  586 CALL CNDNS(KK+6)
      CALL CNDNS(KK+7)
      SOK1(KK+7,KK+7)=1.
      DO 588 IN=1,3
      DO 588 IL=1,3
  588 TRAN(KK+IN+3,KK+IL+3)=CO2(IN,IL)
      CALL TIMES(SOK1,TRAN,SOK,21,21,21,1)
      CALL TIMES(TRAN,SOK,SOK1,21,21,21,2)
      DO 903 IM=1,21
```

```
903 DACL(IM)=SEK1(IM)
    DO 594 JM=1,21
    DO 594 IM=1,8
594 SIK(IM,JM)=SIK1(IM,JM)
    CALL TIMES(SIK,TRAN,SIK1,8,21,21,1)
    CALL TIMES(TRAN,DACL,SEK1,21,21,1,2)
582 CONTINUE
    DO 803 I=1,21
    DO 803 J=1,21
803 STUCK(J,I)=SOK1(J,I)
    DO 804 I=1,21
    DO 804 J=1,8
804 STICK(J,I)=SIK1(J,I)
    DO 805 I=1,21
805 FORCE(I)=SEK1(I)
581 CONTINUE
    DO 610 I=1,21
610 DAOL(I)=0.
    BRPO=0.
    DO 604 J=6,8
    DO 604 I=1,3
604 IF(X(I,J).NE.0.)BRPO=1.
    IF(BRPO.NE.1.) GO TO 56
    DO 273 I=1,21
273 DAOG(I)=0.
    KN=0
    DO 274 I=1,15,7
    KN=KN+1
    DAOG(I)=X(1,5+KN)
    DAOG(I+1)=X(2,5+KN)
274 DAOG(I+2)=X(3,5+KN)
    DO 595 I=1,21
    DO 596 J=1,21
596 TRAN(J,I)=0.
595 TRAN(I,I)=1.
    DO 812 KM=1,15,7
    L=KM-1
    KN=L/7+1
    GL=0.
    IF(NLIK.EQ.0)GO TO 810
    DO 811 IN=1,NLIK
    K=(IN-1)*2+3
811 IF(KN.EQ.INFO(K))GL=1.
    IF(GL.EQ.1.)GO TO 812
810 CONTINUE
    DO 597 I=1,3
    DO 597 J=1,3
597 TRAN(I+L,J+L)=CO2(I,J)
812 CONTINUE
    CALL TIMES(TRAN,DAOG,DACL,21,21,1,1)
560 CONTINUE
    DO 602 I=1,21
602 FORCE(I)=FORCE(I)+DAOL(I)
    RETURN
    END
    SUBROUTINE TRANL(X,Y,Z,K,TRL)
    IMPLICIT REAL*8 (A-H,O-Z)
    COMMON/PAT/TRG(3,3),XG(3),YG(3)
    DIMENSION X(16,2),Y(16,2),Z(16,2),TRL(3,3),TR(3,3)
    DIMENSION X2(3),Y2(3),Z2(3)
```

```
      DO 515 J=1,2
      X2(J)=X(K,J)
      Y2(J)=Y(K,J)
515   Z2(J)=Z(K,J)
      G2=X2(2)-X2(1)
      B2=Y2(2)-Y2(1)
      C2=Z2(2)-Z2(1)
      GLEN2=DSQRT(G2**2+B2**2+C2**2)
      IF(GLEN2.EQ.0.)WRITE(6,700)
      TR(1,1)=G2/GLEN2
      TR(2,1)=B2/GLEN2
      TR(3,1)=C2/GLEN2
      TR(1,2)=TRG(3,2)*TR(3,1)-TRG(3,3)*TR(2,1)
      TR(2,2)=TRG(3,3)*TR(1,1)-TRG(3,1)*TR(3,1)
      TR(3,2)=TRG(3,1)*TR(2,1)-TRG(3,2)*TR(1,1)
      TR(1,3)=TRG(3,1)
      TR(2,3)=TRG(3,2)
      TR(3,3)=TRG(3,3)
      CALL TIMES(TRG,TR,TRI,3,3,3,1)
700   FORMAT(10X,'      **** ERROR ****  GLEN2',/)
      RETURN
      END
      SUBROUTINE CNDNS(K)
      IMPLICIT REAL*8 (A-H,O-Z)
 CONDENSES OUT THE  K TH.  DEGREE OF FREEDOM
      COMMON/COND/SIM(21,21),TIM(8,21),VIM(21)
      DIMENSION B(21)
      A=1./SIM(K,K)
      DO 40 I=1,21
4     B(I)=SIM(K,I)
      DO 41 J=1,21
      DO 41 I=1,21
41    SIM(I,J)=SIM(I,J)-B(I)*B(J)*A
      DO 42 I=1,8
      D=TIM(I,K)
      DO 42 J=1,21
42    TIM(I,J)=TIM(I,J)-B(J)*A*D
      D=VIM(K)
      DO 43 I=1,21
43    VIM(I)=VIM(I)-B(I)*D*A
      RETURN
      END
       *********       ME     THN     VCHTHIA       TCY       THEDY    ********
      THE  STIFFNESS  MATRIX   SM
      THE  STRESS     MATRIX   S
      THE  LOAD       MATRIX   FNP
      INPUT  FOR  EACH POINT A THE AREA COORDINATES,P THE VALUES OF LOAD
      INPUT  FOR  EACH ELEMENT THE COEFFICIENTS  B(3) ,C(3)
      INPUT  FOR  EACH ELEMENT THE ELASTICITY  MATRIX  D(8)
      SUBROUTINE  SHAPEF(A,B,G,DT,NCK)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/JON/SM(21,21),S(8,21),D(8,8),FNP(21,1),P(7)
      DIMENSION A(3),B(3),C(3),BE(8,21),FN(7,21)
      DIMENSION SW(3),SX(3),SY(3),SWX(3),SXX(3),SYX(3),SWY(3),SXY(3),SYY
     1(3),SWXX(3),SXXX(3),SYXX(3),SWYY(3),SXYY(3),SYYY(3),SWXY(3),SXXY(3
     2),SYXY(3),SU(3),SV(3),SFX(3),SFY(3),SUX(3),SUY(3),SVX(3),SVY(3)
      DO 201 I=1,3
      K=I+1
      IF(K.GT.3)K=K-3
      J=I+2
```

```
      IF(J.GT.3)J=J-3
      SW(I)=A(I)+A(I)**2*A(K)+A(I)**2*A(J)-A(I)*A(K)**2-A(I)*A(J)**2
      SY(I)=B(K)*(A(I)**2*A(J)+0.5*A(I)*A(K)*A(J))-B(J)*(A(K)*A(I)**2+
     1 .5*A(I)*A(K)*A(J))
      SX(I)=C(J)*(A(I)**2*A(K)+0.5*A(I)*A(K)*A(J))-G(K)*(A(J)*A(I)**2+
     1C.5*A(I)*A(K)*A(J))
      SWX(I)=(1./DT)*(2.*B(I)*(A(I)*A(K)+A(I)*A(J)+A(K)*A(J))+2.*A(I)*
     1A(K)*(B(I)-B(K))+2.*A(I)*A(J)*(B(I)-B(J)))
      SYX(I)=(1./DT)*(2.*B(I)*A(I)*(B(J)*A(K)-B(K)*A(J))+ .5*(B(J)-B(K))
     1*(B(I)*A(K)*A(J)+B(K)*A(I)*A(J)+B(J)*A(I)*A(K)))*(-1.0)
      SXX(I)=(1./DT)*(2.*B(I)*A(I)*(C(J)*A(K)-G(K)*A(J))+A(I)**2*(B(K)*
     1G(J)-B(J)*G(K))+ .5*(G(J)-G(K))*(B(I)*A(K)*A(J)+B(K)*A(I)*A(J)+
     2B(J)*A(I)*A(K)))
      SWY(I)=(+1./DT)*(2.*G(I)*(A(I)*A(K)+A(I)*A(J)+A(K)*A(J))+2.*A(I)*
     1A(K)*(G(I)-G(K))+2.*A(I)*A(J)*(G(I)-G(J)))
      SYY(I)=(-1./DT)*(2.*G(I)*A(I)*(B(J)*A(K)-B(K)*A(J))+A(I)**2*(G(K)*
     1B(J)-G(J)*B(K))+ .5*(B(J)-B(K))*(G(I)*A(K)*A(J)+G(K)*A(I)*A(J)+
     2G(J)*A(I)*A(K)))
      SXY(I)=(+1./DT)*(2.*G(I)*A(I)*(C(J)*A(K)-G(K)*A(J))+0.5*(G(J)-G(K)
     1)*(G(I)*A(K)*A(J)+G(K)*A(I)*A(J)+G(J)*A(I)*A(K)))
      SU(I)=A(I)
      SV(I)=A(I)
      SFY(I)=A(I)
      SFX(I)=A(I)
      SWXX(I)=(+1./DT**2)*(-2.*B(I)*(A(I)*B(I)+A(K)*B(K)+A(J)*B(J))+
     12.*(B(I)-B(K))*(A(K)*B(I)+A(I)*B(K))+2.*(B(I)-B(J))*(A(I)*B(J)+
     2A(J)*B(I)))
      SYXX(I)=(-1./DT**2)*(2.*B(I)**2*(B(J)*A(K)-B(K)*A(J))+(B(J)-B(K)
     1)*(B(I)*B(K)*A(I)+B(I)*B(J)*A(K)+B(K)*B(J)*A(I)))
      SXXX(I)=(+1./DT**2)*(2.*B(I)**2*(C(J)*A(K)-G(K)*A(J))+4.*A(I)*B(I)
     1*(B(K)*G(J)-B(J)*G(K))+(G(J)-G(K))*(B(I)*B(K)*A(J)+B(I)*B(J)*A(K)+
     2B(K)*B(J)*A(I)))
      SWYY(I)=(+1./ DT**2)*(-2.*G(I)*(A(I)*G(I)+A(K)*G(K)+A(J)*G(J))+2.*
     1(G(I)-G(K))*(A(K)*G(I)+A(I)*G(K))+2.*(G(I)-G(J))*(A(I)*G(J)+A(J)*
     2G(I)))
      SYYY(I)=(-1./DT**2)*(2.*G(I)**2*(B(J)*A(K)-B(K)*A(J))+4.*A(I)*G(I)
     1*(G(K)*B(J)-G(J)*B(K))+(B(J)-B(K))*(G(I)*G(K)*A(J)+G(I)*G(J)*A(K)+
     2G(K)*G(J)*A(I)))
      SXYY(I)=(+1./DT**2)*(2.*G(I)**2*(G(J)*A(K)-G(K)*A(J))+(G(J)-G(K)
     1)*(G(I)*G(K)*A(J)+G(I)*G(J)*A(K)+G(K)*G(J)*A(I)))
      SWXY(I)=(2./DT**2)*(-2.*B(I)*(G(I)*A(I)+G(K)*A(K)+G(J)*A(J))+2.*
     1(B(I)-B(K))*(A(K)*G(I)+A(I)*G(K))+2.*(B(I)-B(J))*(A(I)*G(J)+A(J)*
     2G(I)))
      SYXY(I)=(2./DT**2)*(2.*B(I)*G(I)*(B(J)*A(K)-B(K)*A(J))+2.*B(I)*
     1A(I)*(G(K)*B(J)-G(J)*B(K))+0.5*(B(J)-B(K))*(A(I)*(G(J)*B(K)+G(K)*
     2B(J))+A(K)*(G(I)*B(J)+G(J)*B(I))+A(J)*(G(I)*B(K)+G(K)*B(I))))
     3*(-1.0)
      SXXY(I)=(2./DT**2)*(2.*B(I)*G(I)*(C(J)*A(K)-G(K)*A(J))+2.*G(I)*
     1A(I)*(B(K)*G(J)-B(J)*G(K))+0.5*(G(J)-G(K))*(A(I)*(G(J)*B(K)+G(K)*
     2B(J))+A(K)*(G(I)*B(J)+G(J)*B(I))+A(J)*(G(I)*B(K)+G(K)*B(I))))
      SUX(I)=(1./DT)*B(I)
      SVX(I)=(1./DT)*B(I)
      SUY(I)=(1./DT)*G(I)
      SVY(I)=(1./DT)*G(I)
201   CONTINUE
 FORM     MATRIX    B
      DO 301 J=1,21
      DO 301 I=1,8
      BE(I,J)=0.
301   CONTINUE
```

```
      DO 2 2 K=1,3
      J=3*K-2
      BE(1,J)=SWXX(K)
      BE(2,J)=SWYY(K)
      BE(3,J)=SWXY(K)
      BE(1,J+1)=SYXX(K)
      BE(2,J+1)=SYYY(K)
      BE(3,J+1)=SYXY(K)
      BE(1,J+2)=SXXX(K)
      BE(2,J+2)=SXYY(K)
      BE(3,J+2)=SXXY(K)
2 2   CONTINUE
      DO 203 K=1,3
      J=2*K+8
      BE(1,J)=-SUX(K)
      BE(3,J)=-SUY(K)
      BE(4,J)=SEX(K)
      BE(2,J+1)=-SVY(K)
      BE(3,J+1)=-SVX(K)
      BE(5,J+1)=SEY(K)
203   CONTINUE
      DO 204 K=1,3
      J=2*K+14
      BE(6,J)=SUX(K)
      BE(8,J)=SUY(K)
      BE(7,J+1)=SVY(K)
      BE(8,J+1)=SVX(K)
204   CONTINUE
       STIFNESS  MATRIX        BY MULTIPLICATION  B *D*B
       STIFNESS  MATRIX    SM
       STRESS    MATRIX    S    BY  MULTIPLICATION    D*B
      CALL TIMES(D,BE,S,8,8,21,1)
      CALL TIMES(BE,S,SM,21,3,21,2)
      PROGRAM    FOR  THE  SHAPE  FUNCTION  MATRIX   N
      DO 300 J=1,21
      DO 300 I=1,7
      EN(I,J)=0.
300   CONTINUE
      DO 213 K=1,3
      J=3*K-2
      EN(1,J)=SW(K)
      EN(2,J)=SWY(K)
      EN(3,J)=SWX(K)
      EN(1,J+1)=SY(K)
      EN(2,J+1)=SYY(K)
      EN(3,J+1)=SYX(K)
      EN(1,J+2)=SX(K)
      EN(2,J+2)=SXY(K)
      EN(3,J+2)=SXX(K)
213   CONTINUE
      DO 214 K=1,3
      J=2*K+8
      EN(4,J)=SEX(K)
      EN(5,J+1)=SEY(K)
214   CONTINUE
      DO 215 K=1,3
      J=2*K+14
      EN(6,J)=SU(K)
      EN(7,J+1)=SV(K)
215   CONTINUE
```

```
      FORM THE  MATRIX  ENP   BY MULTIPLICATION    EN*P
      EN    SHAPE  FUNCTION  MATRIX
      P     LOAD   MATRIX  WITH SEVEN COMPONENTS  AT EACH POINT
      CALL TIMES(EN,P,ENP,21,7,1,2)
      RETURN
      END
      FORM  MATRIX  D     PROPERTIES  OF MATERIAL
         SANDWICH         PLATES
      ISOTROPIC   MATERIAL      FACES
      ORTHOTROPIK     MATERIAL    CORE
      EF  ERASTICITY   MODULUS  OF THE FACES
      ENF   POISSON   RATIO    OF  THE FACES
     GF    MODULUS   OF   RIGITY  OF  THE   FACES
     GCY   MODULUS  OF  RIGITY  OF  THE  CORE   IN  THE    YZ    LEVEL
     GCX   MODULUS  OF  RIGITY  OF   CORE    IN  THE     XZ   LEVEL
      THF    THICKNESS    OF  THE  FACE
      THC    THICKNESS    OF  THE  CORE
      A2M    MOMENT   OF AREA   OF  THE SANDWICH   SECTION  OF  THE   FACES
      SUBROUTINE  SUBTT
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/STI/X(3,20),ELMOD(12),SMK(36,36),TIK(8,36),PIIK(36)
     1,INED(20)
      COMMON/JON/SM(21,21),S(8,21),D(8,8),ENP(21,1),P(7)
      COMMON/PAT/VAV(3,3),XI(3),Y(3)
      DIMENSION A(3),B(3),G(3),GLE(3),W(7),W1(4),W2(4)
      DIMENSION PG(7),PI(7),VAV1(7,7),TOT(21,21),SMK1(21,21)
      DO 800 I=1,8
      DO 800 J=1,8
800   D(J,I)=0.
      D(1,1)=ELMOD(1)
      D(1,2)=ELMOD(2)
      D(2,2)=ELMOD(3)
      D(3,3)=ELMOD(4)/2.
      D(4,4)=ELMOD(5)
      D(5,5)=ELMOD(6)
      D(6,6)=ELMOD(7)
      D(6,7)=ELMOD(8)
      D(7,7)=ELMOD(10)
      D(8,8)=ELMOD(11)
      FORM   THE  WHOLE  SYMMETRIC   MATRIX  D
      DO 210 J=1,8
      DO 210 I=1,8
      D(I,J)=D(J,I)
210   CONTINUE
      D(7,6)=ELMOD(9)
      IF(ELMOD(12).NE. 0.) D(2,1)=ELMOD(12)
         PG(7)    THE    GLOBAL   DISTRIBUTED   LOAD
      DO 224 I=1,7
224   PG(I)=0.
      PG(1)=X(1,5)
      PG(2)=X(2,5)
      PG(3)=X(3,5)
      DO 430 I=1,7
      DO 430 J=1,7
430   VAV1(I,J)=0.
      DO 431 I=1,3
      DO 431 J=1,3
431   VAV1(I,J)=VAV(I,J)
      DO 432 K=3,5,2
      DO 432 I=1,2
```

```
      DO 432 J=1,2
432   VAV1(I+K,J+K)=VAV(I,J)
      CALL TIMES(VAV1,PG,PL,7,7,1,1)
      DO 435 I=1,5
435   P(I)=PL(I+2)
      P(6)=PL(1)
      P(7)=PL(2)
 COORDINATES OF VERTECES OF TRIANGLE  X(I,7)  I=1,6
      DIMENSION ENK(21),ARCO(2,7),ARCO2(4),SL(8,21)
      DO 200  I=1,3
      K=I+1
      IF(K.GT.3)K=K-3
      J=I+2
      IF(J.GT.3)J=J-3
      B(I)=Y(K)-Y(J)
      C(I)=XI(J)-XI(K)
      CLE(I)=DSQRT(B(I)**2+C(I)**2)
200   CONTINUE
      DT=P(1)*G(2)-P(2)*G(1)
 QUINTIC  INTEGRATION  FOR STIFFNESS MATRIX  STUCK
      AREA   COORDINATES   L1  L2  L3    ARCO        A(I)
  WEIGHTS  OF  INTEGRATION          W(I)
      DATA ALF/ .33333333D  /,ALF1/ .5971587D00/,BET1/0.47014206D00/
      DATA ALF2/0.797426985D00/,BET2/0.10128651D00/
      DATA WF1/0.225000D0/,WF2/0.132394152D  /,WF3/ .125939180  /
      DATA ALFA1/ .5D0  /,ALFA2/ . D  /,WF/0.33333333D00/
      ARCO(1,1)=ALF
      ARCO(2,1)=ALF
      ARCO(1,2)=ALF1
      ARCO(2,2)=BET1
      ARCO(1,3)=BET1
      ARCO(2,3)=ALF1
      ARCO(1,4)=BET1
      ARCO(2,4)=BET1
      ARCO(1,5)=ALF2
      ARCO(2,5)=BET2
      ARCO(1,6)=BET2
      ARCO(2,6)=ALF2
      ARCO(1,7)=BET2
      ARCO(2,7)=BET2
      W(1)=WF1
      W(2)=WF2
      W(3)=WF2
      W(4)=WF2
      W(5)=WF3
      W(6)=WF3
      W(7)=WF3
      DO 222 I=1,21
222   ENK(I)=0.
      DO 211 J=1,21
      DO 211 I=1,21
      SMK1(I,J)=0.
211   CONTINUE
      DO 212  K=1,7
      A(1)=ARCO(1,K)
      A(2)=ARCO(2,K)
      A(3)=1.-A(1)-A(2)
      CALL SHAPEF(A,B,G,CT,1)
      IF(K.NE.1) GO TO 801
      DO 802 I=1,21
```

```
      DO 8 2 J=1,8
302   S1(J,I)=S(J,I)
301   CONTINUE
      DO 221 I=1,21
221   ENK(I)=ENK(I)+DT/2.*W(K)*ENP(I,1)
      DO 212 J=1,21
      DO 212 I=1,21
      SMK1(I,J)=SMK1(I,J)+DT/2.*W(K)*SM(I,J)
212   CONTINUE
 AREA  COORDINATES   L1  L2  L3   ARCC1       A(I)
 WEIGHTS   OF  INTEGRATION        W1(I)
       P(1,7)   REPRESENTS  DISTRIBUTED  LOADS  CONSTAND IN  ELEM  AREA DT/M2
      ENK(21)  REPRESENTS THE SAME LOADS IN  THE THREE NODES  1,2,3  AS  STUCK
       RECLASSIFICATION    OF    THE  DEGREES  OF  FREEDOM
      DO 425 J=1,21
      DO 425 I=1,21
425   TOT(I,J)=0.
      DO 45  K1=1,7,3
      K2=7*(K1+2)/3-4
      TOT(K1,K2)=1.
      DO 450 K3=1,3,2
      TOT(K1+1,K2+K3+1)=1.
450   TOT(K1+2,K2+K3)=1.
      DO 451 K1=6,20,7
      K2=2*(K1+1)/7+8
      DO 451 K3=1,6,5
      K4=6*(K3-1)/5
      TOT(K2+K4,K1-K3+1)=1.
451   TOT(K2+K4+1,K1-K3+2)=1.
      CALL TIMES(SMK1,TOT,SM,21,21,21,1)
      CALL TIMES(TOT,SM,SMK1,21,21,21,2)
      CALL TIMES(S1,TOT,S,8,21,21,1)
      CALL TIMES(TOT,ENK,ENP,21,21,1,2)
      DO 5 1 I=1,21
      DO 501 J=1,21
501   SMK(J,I)=SMK1(J,I)
      DO 5 2 I=1,21
      DO 502 J=1,8
502   TIK(J,I)=S(J,I)
      DO 803 I=1,21
803   PINK(I)=ENP(I,1)
      RETURN
      END
      SUBROUTINE TRANS(G1,B1,C1)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/PAT/TRMA(3,3),XO(3),YO(3)
      DIMENSION B1(3),G1(3),C1(3)
    FORM  THE TRANSFORMATION MATRIX    LOCAL=(TRMA)*GLOBAL
      DO 10 I=1,3
      DO 1  J=1,3
 10   TRMA(I,J)=0.
      AA=B1(2)*C1(3)-B1(3)*C1(2)
      AB=G1(3)*C1(2)-G1(2)*C1(3)
      AC=B1(3)*G1(2)-G1(3)*B1(2)
      IF((AA.GT.-.1D-12.AND.AA.LT.0.1D-12).AND.(AB.GT.-.1D-12.AND.AB.
     1  LT. .1D-12)) GO TO 20
      AD=DSQRT(AA**2+AB**2+AC**2)
      AE=DSQRT(AA**2+AB**2)
      TRMA(3,1)=AA/AD
      TRMA(3,2)=AB/AD
```

```
      TEMA(3,3)=AC/AD
      TEMA(1,1)=-AB/AE
      TEMA(1,2)=AC/AE
      TEMA(2,1)=-AA*AC/(AE*AD)
      TEMA(2,2)=-AB*AC/(AE*AD)
      TEMA(2,3)=(AA**2+AB**2)/(AE*AD)
      RETURN
   20 DO 30 I=1,3
   30 TEMA(I,J)=1.
      RETURN
      END
      SUBROUTINE    TIMES(A,B,C,N,M,L,KOK)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION  A(1),B(1),C(1)
      KOK=1   A(N,M),B(M,L),C(N,L)    REGULAR     A*B
      KOK=2   A(M,N),B(M,L),C(N,L)    TRANSPOSE   A'*B
      IC=1
      DO  100   K=1,L
      DO  100   J=1,N
      C(IC)=0.
      GO TO(101,102),KOK
  101 CONTINUE
      DO 103 I=1,M
      IA=N*(I-1)+J
      IB=M*(K-1)+I
  103 C(IC)=C(IC)+A(IA)*B(IB)
      GO TO 100
  102 CONTINUE
      DO 104  I=1,M
      IA=M*(J-1)+I
      IB=M*(K-1)+I
  104 C(IC)=C(IC)+A(IA)*B(IB)
  100 IC=IC+1
      RETURN
      END
```

2. Reference symbol DDS33

```
0 P=100 PRCUTE=DUPH CCPIES=4
 = UNIVERSITY, BATCH
 WAS: 15:52:4
SIGNED ON AT  15:52:46 ON MON SEP 22/75
PRINT
   **** PLANE ELEMENT  23 DG. OF FREEDOM  AT 6 NODES  *****
   **** CUBIC VARIATION OF W(U,WX,WY) AND QUADRATIC OF U,V,EX,EY ********
   ****** TRANSFORMATION AT GLOBAL SET OF U,V,W AND TOTAL ROTATIONS  *******
   **** STRESSES AT CENTROID  MXX,MYY,MXY QX,QY,NXX,NYY,NXY   **********
    SUBROUTINE STIFF
    IMPLICIT REAL*8 (A-H,O-Z)
    COMMON/STI/X(3,20),YOUNG(12),STUCK(36,36),STICK(8,36),FORCE(36),
   1INFO(20)
    COMMON/MAN/BOL(2,36,36),COL(2,8,36),DCL(2,36),GRAM(16,16),NGRAM
    COMMON/PAT/CO2(3,3),XO(3),YO(3)
    DIMENSION X1(3),Y1(3),Z1(3),G1(3),B1(3),C1(3),CO1(3,3),CO3(3,3),
   1XM(16,2),YM(16,2),ZM(16,2),TRAN(36,36),CO4(3,3),SCK(36,36),STK(8,3
   26),STI(36),DADL(36),MOL(3)
    DATA MCL/3,1 ,17/
    X1,Y1,Z1  GLOBAL COORDINATES OF THE VERTICES  (1,2,3)
    DO 100 I=1,3
    X1(I)=X(1,I)
    Y1(I)=X(2,I)
100 Z1(I)=X(3,I)
    XM,YM,ZM (M,2) GLOBAL COORD  OF THE JOINT LINE  (DIRECTION  1,2)
    M=  THE  CODE  NUMBER OF  THE   JOINT   (MAX=16)
    IF(NGRAM.EQ. )GO TO 1 1
    DO 102 I=1,2
    DO 102 M=1,NGRAM
    XM(M,I)=GRAM(M,I)
    YM(M,I)=GRAM(M,I+2)
102 ZM(M,I)=GRAM(M,I+4)
101 CONTINUE
    DO 103 I=1,3
    K=I+1
    J=I+2
    IF(K.GT.3)K=K-3
    IF(J.GT.3)J=J-3
    C1(I)=X1(J)-X1(K)
    B1(I)=Y1(J)-Y1(K)
103 C1(I)=Z1(J)-Z1(K)
    CO2(3,3) TRANSF. MATRIX FROM GLOBAL TO LOCAL  D' = < CO2 > D
    D' = LOCAL   D  = GLOBAL
    XO(3),YO(3)  THE  LOCAL  COORDINATES OF THE VERTICES (1,2,3)
    CALL TRANS(G1,B1,C1)
    DO 104 J=1,3
    DO 104 I=1,3
104 CO4(I,J)=X(I,J)
    CALL TIMES(CO2,CO4,CO1,3,3,3,1)
    DO 105 J=1,3
    XO(J)=CO1(1,J)
105 YO(J)=CO1(2,J)
    DO 106  I=1,36
106 FORCE(I)=0.
    INFO(1)=-1 TAKES LOCAL STIF. STR, FROM 1ST STORE AND REVERSES
    THE APPROPRIATE D.O.F
    INFO(1)=-2 THE SAME AS BEFORE FROM THE 2ND STORA
    INFO(1)=0  FINDS LOCAL STIFF. STR. AND D.LOAD  WITHOUT STORING
    INFO(1)=1  TAKES LOCAL STIFF. STR. AND D.LOAD  FROM THE  1ST  STORAGE
    INFO(1)=2  TAKES LOCAL STIFF. STR. AND D.LOAD  FROM THE  2ND  STORAGE
```

```
      INFO(1)=11 FINDS LOCAL STIFF. STR. AND D.LOAD  AND  STORES AT 1ST STORAGE
      INFO(1)=12 FINDS LOCAL STIFF. STR. AND D.LOAD  AND  STORES AT 2ND STORAGE
      NLAK=INFO(1)
      METRC=0
      IF(NLAK.EQ.-1.OR.NLAK.EQ.-2) GO TO 340
      IF(NLAK.EQ.1.OR.NLAK.EQ.2)GO TO 107
      CALL  SUBTT
      IF(NLAK.EQ.0)GO TO 108
      NLAK=NLAK-10
      DO 1 0 I=1,33
  109 COL(NLAK,I)=FORCE(I)
      DO 140 I=1,33
      DO 140 J=1,8
  140 COL(NLAK,J,I)=STICK(J,I)
      DO 141 I=1,33
      DO 141 J=1,33
  141 BOL(NLAK,J,I)=STUCK(J,I)
      GO TO 1 6
  340 NLAK=-NLAK
      METRC=-1
  107 DO 111 I=1,33
  111 FORCE(I)=COL(NLAK,I)
      DO 142 I=1,33
      DO 142 J=1,8
  142 STICK(J,I)=COL(NLAK,J,I)
      DO 143 I=1,33
      DO 143 J=1,33
  143 STUCK(J,I)=BOL(NLAK,J,I)
      IF(METRC.NE.-1) GO TO 108
      DO 861 I=1,36
      DO 861 J=1,36
      E1=0.
      IF(I.NE.J) GO TO 862
      E1=-1.
      DO 863 II=1,3
      IF(I.EQ.MOL(II)) E1=1.
  863 CONTINUE
  862 TRAN(J,I)=E1
  861 CONTINUE
      CALL  TIMES(STUCK,TRAN,SCK,36,36,36,1)
      CALL  TIMES(TRAN,SCK,STUCK,36,36,36,2)
      CALL  TIMES(STICK,TRAN,STK,3,36,36,1)
      CALL  TIMES(TRAN,FORCE,DACL,36,36,1,2)
      DO 5 I=1,33
      DO 5 J=1,8
    5 STICK(J,I)=STK(J,I)
      DO 6 I=1,33
    6 FORCE(I)=DACL(I)
  108 CONTINUE
      INFO(2)=0 KEEPS LOCAL  WITHOUT  TRANSFORMATION
      INFO(2)=X X NO  NODES  OF THE   ELEMENT  NEED   TRANSFORMATION  (MAX=5)
      NLIK=INFO(2)
      IF(NLIK.EQ.0.OR.NLIK.EQ.99) GO TO 120
      DO 120 I=1,NLIK
      INFO(3)   NUMBER  OF THE NODE  (1 6)
      INFO(4)   CODE NO OF NODE--JOINT LINE (MAX=16) (IF.EQ.0 ALL GLOBAL,
      IF.EQ.-1  ONLY   U , V , W   TRANSF)
      INFO(5), INFO(7), INFO(9), INFO(11)  THE SAME AS INFO(3)
      INFO(6), INFO(8), INFO(1 ),INFO(12)  THE SAME AS INFO(4)
      K=(I-1)*2+3
```

```
      L=INI(K)
      K1=INFC(K+1)
      DO 117 IS=1,36
      DO 116 IT=1,36
116   TRAN(IT,IS)=C.
117   TRAN(IS,IS)=1.
      IF(L.GT.3)GO TO 118
      KK=(L-1)*7
      DO 119 IL=1,3
      DO 119 IN=1,3
119   TRAN(KK+IN,KK+IL)=CO2(IN,IL)
      IF(K1.EQ.-1)GO TO 12
      IF(K1.EQ.0) GO TO 121
      CALL TRANL(XM,YM,ZM,K1,CO3)
      X  AXE OF THE NEW  SYSTEM  THE  JOINT LINE
      EX   SHEAR   ANGLE   IS  CONDENSED  OUT INTERCONECTION OF TWO PLATES
      DO 122 IL=1,2
      DO 122 IN=1,2
      TRAN(KK+IN+3,KK+IL+3)=CO3(IN,IL)
122   TRAN(KK+IN+5,KK+IL+5)=CO3(IN,IL)
      CALL TIMES(STUCK,TRAN,SOK,36,36,36,1)
      CALL TIMES(TRAN,SOK,STUCK,36,36,36,2)
      DO 123 JM=1,36
123   SIL(JM)=FORCE(JM)
      DO 144 JM=1,36
      DO 144 IM=1,8
144   SIK(IM,JM)=STICK(IM,JM)
      CALL TIMES(STK,TRAN,STICK,8,36,36,1)
      CALL TIMES(TRAN,SIL,FORCE,36,36,1,2)
      KUK=KK+7
      CALL CNDNS(KUK)
      STUCK(KUK,KUK)=1.
      GO TO 120
      INTERCONECTION  OF  MORE  THAN  TWO PLATES
121   CALL CNDNS(KK+6)
      CALL CNDNS(KK+7)
      STUCK(KK+7,KK+7)=1.
      DO 124 IL=1,3
      DO 124 IN=1,3
.24   TRAN(KK+IN+3,KK+IL+3)=CO2(IN,IL)
      CALL TIMES(STUCK,TRAN,SOK,36,36,36,1)
      CALL TIMES(TRAN,SOK,STUCK,36,36,36,2)
      DO 125 JM=1,36
.25   SIL(JM)=FORCE(JM)
      DO 145 JM=1,36
      DO 145 IM=1,8
.45   SIK(IM,JM)=STICK(IM,JM)
      CALL TIMES(STK,TRAN,STICK,8,36,36,1)
      CALL TIMES(TRAN,SIL,FORCE,36,36,1,2)
      GO TO 12
      MIDSIDE  NODES  TRANSFORMATION  AT INTRCONECTION  OF TWO PLATES
      X AXE OF THE NEW SYSTEM  THE JOINT-LINE  SHEAR ANGE EY  IS CONDENSED OUT
18    KK=(L-4)*4+21
      CALL TRANL(XM,YM,ZM,K1,CO3)
      DO 126 IL=1,2
      DO 126 IN=1,2
126   TRAN(KK+IN,KK+IL)=CO3(IN,IL)
      CALL TIMES(STUCK,TRAN,SOK,36,36,36,1)
      CALL TIMES(TRAN,SOK,STUCK,36,36,36,2)
      DO 127 JM=1,36
```

```
127 SIL(JM)=FORCE(JM)
    DO 146 JM=1,36
    DO 146 IM=1,8
146 SIK(IM,JM)=STICK(IM,JM)
    CALL TIMES(SIK,TRAN,STICK,8,36,36,1)
    CALL TIMES(TRAN,SIL,FORCE,36,36,1,2)
    KUK=KK+2
    CALL CONDNS(KUK)
    STICK(KUK,KUK)=1.
120 CONTINUE
    CNSL= .
    DO 131 I=1,36
131 DAOL(I)=0.
    DO 129 J=6,8
    DO 129 I=1,3
129 IF(X(I,J).NE.0.)CNSL=1.
    IF(CNSL.NE.1.)GO TO 128
    DO 130 I=1,36
130 SIL(I)=0.
    KO=0
    DO 132 I=1,15,7
    KO=KO+1
    SIL(I)=X(1,5+KO)
    SIL(I+1)=X(2,5+KO)
132 SIL(I+2)=X(3,5+KO)
    DO 134 I=1,36
    DO 133 J=1,36
133 TRAN(J,I)= .
134 TRAN(I,I)=1.
    DO 135 KM=1,15,7
    L=KM-1
    KN=L/7+1
    GL=0.
    IF(NLIK.EQ.0)GO TO 137
    DO 136 IN=1,NLIK
    K=(IN-1)*2+3
136 IF(KN.EQ.INFO(K))GL=1.
    IF(GL.EQ.1.)GO TO 135
137 CONTINUE
    DO 138 J=1,3
    DO 138 I=1,3
138 TRAN(I+L,J+L)=CC2(I,J)
135 CONTINUE
    CALL TIMES(TRAN,SIL,DAOL,36,36,1,1)
128 DO 139 I=1,36
139 FORCE(I)=FORCE(I)+DAOL(I)
    RETURN
    END
    SUBROUTINE  SHAP(A,B,C,DT,M)
    IMPLICIT REAL*8 (A-H,O-Z)
    DIMENSION  SW(3),SWX(3),SWY(3),SWXX(3),SWYY(3),SWXY(3),SX(3),SXX(3
   1),SXY(3),SXXX(3),SXYY(3),SXXY(3),SY(3),SYX(3),SYY(3),SYXX(3),SYYY(
   23),SYXY(3),SU(7),SUX(7),SUY(7),SF(6),SFX(6),SFY(6),A(3),B(3),G(3),
   3BE(8,35),EN(7,35)
    COMMON/JCN/SM(35,35),S(8,35),D(8,8),P(7),ENP(35,1)
    D1=A(1)*A(2)*A(3)
    D2=B(1)*A(2)*A(3)+B(2)*A(1)*A(3)+B(3)*A(1)*A(2)
    D3=G(1)*A(2)*A(3)+G(2)*A(1)*A(3)+G(3)*A(1)*A(3)
    DO  100 I=1,3
    K=I+1
```

```
 J=I+2
 IF(K.GT.3)K=K-3
 IF(J.GT.3)J=J-3
 SW(I)=A(I)+A(I)**2*A(K)+A(I)**2*A(J)-A(I)*A(K)**2-A(I)*A(J)**2
 SY(I)=B(K)*(A(I)**2*A(J)+0.5*A(I)*A(K)*A(J))-B(J)*(A(K)*A(I)**2+ .
15*A(I)*A(K)*A(J))
 SX(I)=C(J)*(A(I)**2*A(K)+0.5*A(I)*A(K)*A(J))-C(K)*(A(J)*A(I)**2+0.
15*A(I)*A(K)*A(J))
 SWX(I)=(1./DT)*(2.*B(I)*(A(I)*A(K)+A(I)*A(J)+A(K)*A(J))+2.*A(I)*A(
1K)*(B(I)-B(K))+2.*A(I)*A(J)*(B(I)-B(J)))
 SYX(I)=(1./DT)*(2.*B(I)*A(I)*(B(J)*A(K)-B(K)*A(J))+0.5*(B(J)-B(K))
1*(B(I)*A(K)*A(J)+B(K)*A(I)*A(J)+B(J)*A(I)*A(K)))*(-1. )
 SXX(I)=(1./DT)*(2.*B(I)*A(I)*(C(J)*A(K)-C(K)*A(J))+A(I)**2*(B(K)*C
1(J)-B(J)*C(K))+0.5*(C(J)-C(K))*(B(I)*A(K)*A(J)+B(K)*A(I)*A(J)+B(J)
2*A(I)*A(K)))
 SWY(I)=(1./DT)*(2.*C(I)*(A(I)*A(K)+A(I)*A(J)+A(K)*A(J))+2.*A(I)*A(
1K)*(C(I)-C(K))+2.*A(I)*A(J)*(C(I)-C(J)))
 SYY(I)=(-1./DT)*(2.*C(I)*A(I)*(B(J)*A(K)-B(K)*A(J))+A(I)**2*(C(K)*
1B(J)-C(J)*B(K))+0.5*(B(J)-B(K))*(C(I)*A(K)*A(J)+C(K)*A(I)*A(J)+C(J
2)*A(I)*A(K)))
 SXY(I)=(1./DT)*(2.*C(I)*A(I)*(C(J)*A(K)-C(K)*A(J))+0.5*(C(J)-C(K))
1*(C(I)*A(K)*A(J)+C(K)*A(I)*A(J)+C(J)*A(I)*A(K)))
 SWXX(I)=(1./DT**2)*(-2.*B(I)*(A(I)*B(I)+A(K)*B(K)+A(J)*B(J))+2.*(B
1(I)-B(K))*(A(K)*B(I)+A(I)*B(K))+2.*(B(I)-B(J))*(A(I)*B(J)+A(J)*B(I
2)))
 SYXX(I)=(-1./DT**2)*(2.*B(I)**2*(B(J)*A(K)-B(K)*A(J))+(B(J)-B(K))*
1(B(I)*B(K)*A(J)+B(I)*B(J)*A(K)+B(K)*B(J)*A(I)))
 SXXX(I)=(1./DT**2)*(2.*B(I)**2*(C(J)*A(K)-C(K)*A(J))+4.*A(I)*B(I)*
1(B(K)*C(J)-B(J)*C(K))+(C(J)-C(K))*(B(I)*B(K)*A(J)+B(I)*B(J)*A(K)+B
2(K)*B(J)*A(I)))
 SWYY(I)=(1./DT**2)*(-2.*C(I)*(A(I)*C(I)+A(K)*C(K)+A(J)*C(J))+2.*(C
1(I)-C(K))*(A(K)*C(I)+A(I)*C(K))+2.*(C(I)-C(J))*(A(I)*C(J)+A(J)*C(I
2)))
 SYYY(I)=(-1./DT**2)*(2.*C(I)**2*(B(J)*A(K)-B(K)*A(J))+4.*A(I)*C(I)
1A(C(K)*B(J)-C(J)*B(K))+(B(J)-B(K))*(C(I)*C(K)*A(J)+C(I)*C(J)*A(K)+
2C(K)*C(J)*A(I)))
 SXYY(I)=(1./DT**2)*(2.*C(I)**2*(C(J)*A(K)-C(K)*A(J))+(C(J)-C(K))*(
1C(I)*C(K)*A(J)+C(I)*C(J)*A(K)+C(K)*C(J)*A(I)))
 SWXY(I)=(2./DT**2)*(-2.*B(I)*(C(I)*A(I)+C(K)*A(K)+C(J)*A(J))+2.*(B
1(I)-B(K))*(A(K)*C(I)+A(I)*C(K))+2.*(B(I)-B(J))*(A(I)*C(J)+A(J)*C(I
2)))
 SYXY(I)=(-2./DT**2)*(2.*B(I)*C(I)*(B(J)*A(K)-B(K)*A(J))+2.*B(I)*A(
1I)*(C(K)*B(J)-C(J)*B(K))+ .5*(B(J)-B(K))*(A(I)*(C(J)*B(K)+C(K)*B(J
2))+A(K)*(C(I)*B(J)+C(J)*B(I))+A(J)*(C(I)*B(K)+C(K)*B(I))))
 SXXY(I)=(2./DT**2)*(2.*B(I)*C(I)*(C(J)*A(K)-C(K)*A(J))+2.*C(I)*A(I
1)*(B(K)*C(J)-B(J)*C(K))+ .5*(C(J)-C(K))*(A(I)*(C(J)*B(K)+C(K)*B(J)
2)+A(K)*(C(I)*B(J)+C(J)*B(I))+A(J)*(C(I)*B(K)+C(K)*B(I))))
 SU(I)=A(I)-9.*D1
 SU(I+3)=-0.5*A(K)*A(J)+1.5*D1
 SUX(I)=(1./DT)*(B(I)-9.*D2)
 SUX(I+3)=(1./DT)*(- .5*(B(K)*A(J)+B(J)*A(K))+1.5*D2)
 SUY(I)=(1./DT)*(C(I)-9.*D3)
 SUY(I+3)=(1./DT)*(-0.5*(C(K)*A(J)+C(J)*A(K))+1.5*D3)
 SF(I)=(2.*A(I)-1.)*A(I)
 SF(I+3)=4.*A(K)*A(J)
 SFX(I)=(1./DT)*(B(I)*(4.*A(I)-1.))
 SFX(I+3)=(1./DT)*(4.*B(K)*A(J)+4.*B(J)*A(K))
 SFY(I)=(1./DT)*(C(I)*(4.*A(I)-1.))
 SFY(I+3)=(1./DT)*(4.*C(K)*A(J)+4.*C(J)*A(K))
 SU(7)=27.*D1
```

```
      SUX(7)=27.*D2
      SUY(7)=27.*D3
      FORM   MATRIX   BE(8,35)
      DO 103 I=1,35
      DO 103 J=1,8
103   BE(J,I)=0.
      DO 104  I=1,3
      J=3*I-2
      BE(1,J)=SXXX(I)
      BE(2,J)=SXYY(I)
      BE(3,J)=SXXY(I)
      BE(1,J+1)=SXXX(I)
      BE(2,J+1)=SXYY(I)
      BE(3,J+1)=SXXY(I)
      BE(1,J+2)=SYXX(I)
      BE(2,J+2)=SYYY(I)
104   BE(3,J+2)=SYXY(I)
      DO 105 I=1,6
      K1=8
      IF(I.GT.3)K1=14
      J=K1+2*I
      BE(1,J)=-SEX(I)
      BE(3,J)=-SEY(I)
      BE(4,J)=SE(I)
      BE(2,J+1)=-SEY(I)
      BE(3,J+1)=-SEX(I)
105   BE(5,J+1)=SE(I)
      DO 106 I=1,7
      K1=14
      IF(I.GT.3)K1=20
      J=K1+2*I
      BE(6,J)=SUX(I)
      BE(8,J)=SUY(I)
      BE(7,J+1)=SUY(I)
106   BE(8,J+1)=SUX(I)
      CALL TIMES(D,BE,S,8,8,35,1)
      CALL TIMES(BE,S,SM,35,8,35,2)
      FORM   MATRIX   EN   SHAPE   FUNCTIONS
      DO 108 J=1,35
      DO 108 I=1,7
108   EN(I,J)=0.
      DO 109 I=1,3
      J=3*I-2
      EN(1,J)=SW(I)
      EN(2,J)=SWX(I)
      EN(3,J)=SWY(I)
      EN(1,J+1)=SX(I)
      EN(2,J+1)=SXX(I)
      EN(3,J+1)=SXY(I)
      EN(1,J+2)=SY(I)
      EN(2,J+2)=SYX(I)
109   EN(3,J+2)=SYY(I)
      DO 110 I=1,6
      K1=8
      IF(I.GT.3)K1=14
      J=K1+2*I
      EN(4,J)=SE(I)
110   EN(5,J+1)=SE(I)
      DO 111 I=1,7
      K1=14
```

```
      IF(I.GT.3)K1=20
      J=K1+2*I
      EN(6,J)=SU(I)
111   EN(7,J+1)=SU(I)
      CALL TIMES(EN,P,ENP,35,7,1,2)
      RETURN
      END
      SUBROUTINE SUBTI
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/STI/X(3,20),ELMOD(12),SMK(36,36),TIK(3,36),DINK(36),
     1INFO(20)
      COMMON/JCN/SM(35,35),S(8,35),D(8,8),P(7),ENP(35,1)
      COMMON/PAT/VAV(3,3),XL(3),YL(3)
      DIMENSION A(3),B(3),C(3),J(7),ARCO(2,7),SMK1(35,35),ENK(35)
      DO 100 J=1,8
      DO 100 I=1,8
100   D(I,J)=0.
      D(1,1)=ELMOD(1)
      D(1,2)=ELMOD(2)
      D(2,2)=ELMOD(3)
      D(3,3)=ELMOD(4)/2.
      D(4,4)=ELMOD(5)
      D(5,5)=ELMOD(6)
      D(6,6)=ELMOD(7)
      D(6,7)=ELMOD(8)
      D(7,7)=ELMOD(10)
      D(8,8)=ELMOD(11)
      DO 101 J=1,8
      DO 101 I=1,8
101   D(I,J)=D(J,I)
      D(7,6)=ELMOD(9)
      IF(ELMOD(12).NE.0.) D(2,1)=ELMOD(12)
      DIMENSION PG(7),PL(7),VAV1(7,7),TOT(35,35),S1(8,35)
      PG(7)   THE  GLOBAL DISTRIBUTED  LOAD
      DO 111 I=1,7
111   PG(I)=0.
      PG(1)=X(1,5)
      PG(2)=X(2,5)
      PG(3)=X(3,5)
      DO 112 J=1,7
      DO 112 I=1,7
112   VAV1(I,J)=0.
      DO 113 J=1,3
      DO 113 I=1,3
113   VAV1(I,J)=VAV(I,J)
      DO 114  K=3,5,2
      DO 114  I=1,2
      DO 114  J=1,2
114   VAV1(I+K,J+K)=VAV(I,J)
      CALL TIMES(VAV1,PG,PL,7,7,1,1)
      DO 115  I=1,5
115   P(I)=PL(I+2)
      P(6)=PL(1)
      P(7)=PL(2)
      DATA ALF/0.33333333D00/,ALF1/0.05971587D00/,BET1/0.47014206D00/,
     1ALF2/ .79742698D 00/,BET2/ .1 1286581D 00/,WF1/ .225 0D 00/,WF2/ .13239
     2415D00/,WF3/0.12593918D00/,ALFA1/0.5D00/
      ARCO(1,1)=ALF
      ARCO(2,1)=ALF
      ARCO(1,2)=ALF1
```

```
      ARCO(2,2)=BET1
      ARCO(1,3)=BET1
      ARCO(2,3)=ALF1
      ARCO(1,4)=BET1
      ARCO(2,4)=BET1
      ARCO(1,5)=ALF2
      ARCO(2,5)=BET2
      ARCO(1,6)=BET2
      ARCO(2,6)=ALF2
      ARCO(1,7)=BET2
      ARCO(2,7)=BET2
      W(1)=WF1
      W(2)=WF2
      W(3)=WF2
      W(4)=WF2
      W(5)=WF3
      W(6)=WF3
      W(7)=WF3
      DO 102 I=1,3
      K=I+1
      J=I+2
      IF(K.GT.3)K=K-3
      IF(J.GT.3)J=J-3
      B(I)=YL(K)-YL(J)
102   C(I)=XL(J)-XL(K)
      DT=B(1)*C(2)-B(2)*C(1)
      DO 103 I=1,35
103   ENK(I)=0.
      DO 121 I=1,35
      DO 121 J=1,35
121   SMK1(J,I)=0.
      DO 122 K=1,7
      A(1)=ARCO(1,K)
      A(2)=ARCO(2,K)
      A(3)=1.-A(1)-A(2)
      CALL SHAP(A,B,C,DT,2)
      IF(K.NE.1) GO TO 502
      DO 501 I=1,35
      DO 501 J=1,8
501   S1(J,I)=S(J,I)
502   CONTINUE
      DO 104 I=1,35
104   ENK(I)=ENK(I)+DT/2.*W(K)*ENP(I,1)
      DO 122 I=1,35
      DO 122 J=1,35
122   SMK1(J,I)=SMK1(J,I)+DT/2.*W(K)*SM(J,I)
      LOCAL  RECLASSIFICATION  OF THE  DEGREES OF FREEDOM
      DO 105 J=1,35
      DO 105 I=1,35
105   TOT(I,J)=0.
      DO 106 K1=1,7,3
      K2=7*(K1+2)/3-4
      TOT(K1,K2)=1.
      DO 106 K3=1,3,2
      TOT(K1+1,K2+K3)=1.
106   TOT(K1+2,K2+K3+1)=1.
      DO 107 K1=6,20,7
      K2=2*(K1+1)/7+8
      DO 107 K3=1,6,5
      K4=6*(K3-1)/5
```

```
      TOT(K2+K4,K1-K3+1)=1.
107   TOT(K2+K4+1,K1-K3+2)=1.
      DO 108 K1=22,30,4
      K2=K1/2+11
      DO 108 K3=1,3,2
      K4=6*(K3-1)/2
      TOT(K2+K4,K1+K3-1)=1.
108   TOT(K2+K4+1,K1+K3)=1.
      TOT(34,34)=1.
      TOT(35,35)=1.
      CALL TIMES(SMK1,TOT,SM,35,35,35,1)
      CALL TIMES(TOT,SM,SMK1,35,35,35,2)
      CALL TIMES(SL,TOT,S,9,35,35,1)
      CALL TIMES(TOT,ENK,ENP,35,35,1,2)
      DO 109 I=1,35
109   PINK(I)=ENP(I,1)
      DO 110 I=1,35
      DO 110 J=1,9
110   TIK(J,I)=S(J,I)
      DO 124 I=1,35
      DO 124 J=1,35
124   SMK(J,I)=SMK1(J,I)
      CALL CNDNS(35)
      CALL CNDNS(34)
      RETURN
      END
      SUBROUTINE TRANS(G1,B1,C1)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/PAT/TRMA(3,3),XO(3),YO(3)
      DIMENSION B1(3),G1(3),C1(3)
      FORM  THE TRANSFORMATION MATRIX LOCAL=(TRMA)*GLOBAL
      DO 10 J=1,3
      DO 10 I=1,3
10    TRMA(I,J)= .
      AA=B1(2)*C1(3)-B1(3)*C1(2)
      AB=G1(3)*C1(2)-G1(2)*C1(3)
      AC=B1(3)*G1(2)-G1(3)*B1(2)
      IF((AA.GT.-.1D-12.AND.AA.LT.0.1D-12).AND.(AB.GT.-.1D-12.AND.AB.LT.
     1 .1D-12))GO TO 2
      AD=DSQRT(AA**2+AB**2+AC**2)
      AE=DSQRT(AA**2+AB**2)
      TRMA(3,1)=AA/AD
      TRMA(3,2)=AB/AD
      TRMA(3,3)=AC/AD
      TRMA(1,1)=-AB/AE
      TRMA(1,2)=AA/AE
      TRMA(2,1)=-AA*AC/(AE*AD)
      TRMA(2,2)=-AB*AC/(AE*AD)
      TRMA(2,3)=(AA**2+AB**2)/(AE*AD)
      RETURN
20    DO 30 I=1,3
30    TRMA(I,I)=1.
      RETURN
      END
      SUBROUTINE  TRANL(X,Y,Z,K,TRL)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/PAT/TRG(3,3),XG(3),YG(3)
      DIMENSION X(16,2),Y(16,2),Z(16,2),TRL(3,3),TR(3,3),X2(3),Y2(3),Z2(
     13)
      DO 100 J=1,2
```

```
      X2(J)=X(K,J)
      Y2(J)=Y(K,J)
      Z2(J)=Z(K,J)
      C2=X2(2)-X2(1)
      B2=Y2(2)-Y2(1)
      C2=Z2(2)-Z2(1)
      GLEN2=DSQRT(C2**2+B2**2+C2**2)
      IF(GLEN2.LT. .10-12)WRITE(6,7   )
      TR(1,1)=C2/GLEN2
      TR(2,1)=B2/GLEN2
      TR(3,1)=C2/GLEN2
      TR(1,2)=TRG(3,2)*TR(3,1)-TRG(3,3)*TR(2,1)
      TR(2,2)=TRG(3,3)*TR(1,1)-TRG(3,1)*TR(3,1)
      TR(3,2)=TRG(3,1)*TR(2,1)-TRG(3,2)*TR(1,1)
      TR(1,3)=TRG(3,1)
      TR(2,3)=TRG(3,2)
      TR(3,3)=TRG(3,3)
      CALL TIMES(TRG,TR,TRL,3,3,3,1)
    7 FORMAT(1 X,'     **** ERROR  GLEN2 ****',/)
      RETURN
      END
      SUBROUTINE CNDNS(K)
      IMPLICIT REAL*8 (A-H,O-Z)
      CONDENSES OUT  THE  K TH. DEGREE OF FREEDOM
      COMMON/STI/X(3,20),YCONC(12),SIM(36,36),TIM(8,36),VIM(36),INFO(20)
      DIMENSION B(36)
      A=1./SIM(K,K)
      DO 40  I=1,35
   40 B(I)=SIM(K,I)
      DO 41  J=1,35
      DO 41  I=1,35
   41 SIM(I,J)=SIM(I,J)-B(I)*B(J)*A
      DO 42  I=1,8
      D=TIM(I,K)
      DO 42  J=1,35
   42 TIM(I,J)=TIM(I,J)-B(J)*A*D
      D=VIM(K)
      DO 43 I=1,35
   43 VIM(I)=VIM(I)-B(I)*D*A
      RETURN
      END
      SUBROUTINE   TIMES(A,B,R,N,M,L,KOK)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION  A(1),B(1),R(1)
      KOK=1  A(N,M),B(M,L),R(N,L)   REGULAR     A*B
      KOK=2  A(M,N),B(M,L),R(N,L)   TRANSPOSE   A'*B
      IR=1
      DO  100   K=1,L
      DO  100   J=1,N
      R(IR)=0.
      GO TO(101,102),KOK
  101 CONTINUE
      DO 103 I=1,M
      IA=N*(I-1)+J
      IB=M*(K-1)+I
  103 R(IR)=R(IR)+A(IA)*B(IB)
      GO TO 100
  102 CONTINUE
      DO 104 I=1,M
      IA=M*(J-1)+I
```

```
      IB=M*(K-1)+I
104   R(IP)=R(IB)+A(IA)*B(IP)
1     IP=IP+1
      RETURN
      END
```

3. Reference symbol DMX36

```
=10 P=100 EXECUTE=SUBR COPIES=4
TE = UNIVERSITY, BATCH
ON WAS: 13:45:12
 SIGNED ON AT  15:51:57 ON MON SEP 22/75
*PRINT*
      ****** DONE  MIXED  ELEMENT  36 DG. OF FREEDOM AT 6 NODES ******
      ****** QUADRATIC VARIATION OF U,V,W,MXX,MYY,MXY          ********
      ***** TRANSFORMATION  TO GLOBAL  SET OF  U,V,W,    AND MNN,MSS,MNS***
      ***** STRESSES  AT 1ST VERTIX QX1,QY1, 2ND VERTIX QX2,QY2 ******
      ****** STRESSES AT CENTROID   MXX,MYY,MXY                 ************
      ****** MODULI OF ELASTISITY   THROUGH  YOUNG(12)    ********
      SUBROUTINE TRANS(G1,B1,C1)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/COCR/TRDF(3,3),XG(3),YG(3),TRM(3,3)
      DIMENSION B1(3),G1(3),C1(3),CN(3,3)
C     TRDF  TRANSFORMATION MATRIX FOR THE DEFLACTION  'LOCAL'=TRDF*'GLOBAL'
C     TRM   TRANSFORMATION MATRIX FOR THE MOMENTS     'LOCAL'=TRM *'GLOBAL'
      DO 10 I=1,3
      DO 10 J=1,3
      TRM(J,I)=0.
   10 TRDF(J,I)=0.
      AA=B1(2)*C1(3)-B1(3)*C1(2)
      AB=C1(3)*G1(2)-G1(2)*C1(3)
      AC=B1(3)*G1(2)-G1(3)*B1(2)
      IF((AA.GT.-.10-12.AND.AA.LT.0.10-12).AND.(AB.GT.-.10-12.AND.AB.LT.
     10.10-12)) GO TO 20
      AD=DSQRT(AA**2+AB**2+AC**2)
      AE=DSQRT(AA**2+AB**2)
      TRDF(3,1)=AA/AD
      TRDF(3,2)=AB/AD
      TRDF(3,3)=AC/AD
      TRDF(1,1)=-AB/AE
      TRDF(1,2)=AA/AE
      TRDF(2,1)=-AA*AC/(AE*AD)
      TRDF(2,2)=-AB*AC/(AE*AD)
      TRDF(2,3)=(AA**2+AB**2)/(AE*AD)
      GO TO 40
   20 DO 30 I=1,3
   30 TRDF(I,I)=1.
   40 A2=TRDF(1,1)
      A1=DSIN(DARCOS(A2))
      A3=TRDF(3,3)
      TRM(1,1)=A1**2
      TRM(1,2)=A2**2*A3
      TRM(1,3)=2.*A1*A2*A3
      TRM(2,1)=A2**2
      TRM(2,2)=A1**2*A3
      TRM(2,3)=-2.*A1*A2*A3
      TRM(3,1)=-A1*A2
      TRM(3,2)=A1*A2*A3
      TRM(3,3)=(A1**2-A2**2)*A3
      RETURN
      END
      SUBROUTINE TRANI(X,Y,Z,K,TRM)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/COCR/TRG(3,3),XG(3),YG(3),CCM(3,3)
      DIMENSION X(16,2),Y(16,2),Z(16,2),TRL(3,3),TR(3,3),TRM(3,3)
     1,X2(3),Y2(3),Z2(3)
      DO 100 J=1,2
      X2(J)=X(K,J)
```

```
      Y2(J)=Y(K,J)
  100 Z2(J)=Z(K,J)
      G2=X2(2)-X2(1)
      B2=Y2(2)-Y2(1)
      C2=Z2(2)-Z2(1)
      GLEN2=DSQRT(G2**2+B2**2+C2**2)
      IF(GLEN2.LT.0.1E-12) WRITE(6,700)
      TR(1,1)=G2/GLEN2
      TR(2,1)=B2/GLEN2
      TR(3,1)=C2/GLEN2
      TR(1,2)=TRG(3,2)*TR(3,1)-TRG(3,3)*TR(2,1)
      TR(2,2)=TRG(3,3)*TR(1,1)-TRG(3,1)*TR(3,1)
      TR(3,2)=TRG(3,1)*TR(2,1)-TRG(3,2)*TR(1,1)
      TR(1,3)=TRG(3,1)
      TR(2,3)=TRG(3,2)
      TR(3,3)=TRG(3,3)
      CALL TIMES(TRG,TR,TRL,3,3,3,1)
  700 FORMAT(10X,'    ****  ERROR   TRANI   ****     ',/)
      A2=TRL(1,1)
      A1=DSIN(DARCOS(A2))
      A3=TRL(3,3)
      TRM(1,1)=A1**2
      TRM(1,2)=A2**2*A3
      TRM(1,3)=2.*A1*A2*A3
      TRM(2,1)=A2**2
      TRM(2,2)=A1**2*A3
      TRM(2,3)=-2.*A1*A2*A3
      TRM(3,1)=-A1*A2
      TRM(3,2)=A1*A2*A3
      TRM(3,3)=(A1**2-A2**2)*A3
      RETURN
      END
    ****    MIXED  MODEL  24   DEGREES OF  FREEDOM  W, MX, MY, MXY,  6 NODES**
    *********   STRESSES  QX, QY AT FIRST AND SECOND NODES*****************
    *********      TRANSFORMATION   TO W,  MNN,  MSS,  MSN*****************
    *** QUADRATIC  VARIATION  OF ALL   THE FUNCTIONS  W, MXX, MYY, MXY *****
      SUBROUTINE  STIFF
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/STI/X(3,36),YOUNG(12),STUCK(36,36),STICK(8,36),FORCE(36),
     1INFO(20)
      COMMON/MAN/BOL(2,36,36),COL(2,8,36),DCL(2,36),GRAM(16,16),NGRAM
      COMMON/COOR/CO2(3,3),XO(3),YO(3),COM(3,3)
      DIMENSION  TRAN(36,36),SMK(36,36),STR(8,36),DAOL(36),MOL(12),
     1DAOC(36),XM(16,2),YM(16,2),ZM(16,2),X1(3),Y1(3),Z1(3),CO3(3,3)
     2,G1(3),B1(3),C1(3),CO1(3,3),CO4(3,3)
      DATA MOL/1,2,7,8,13,14,19,20,25,26,31,32/
      DO 100 I=1,3
      X1(I)=X(1,I)
      Z1(I)=X(3,I)
  100 Y1(I)=X(2,I)
      DO 101 I=1,36
      DO 101 J=1,36
  101 STUCK(J,I)=0.
      DO 102 I=1,36
      DO 102 J=1,8
  102 STICK(J,I)=0.
      DO 103 I=1,36
      DACC(I)=0.
  103 FORCE(I)=0.
      IF(NGRAM.EQ.0) GO TO 517
```

```
      DO 516 M=1,NGRAM
      DO 516 I=1,2
      XM(M,I)=GRAM(M,I)
      ZM(M,I)=GRAM(M,I+4)
  516 YM(M,I)=GRAM(M,I+2)
  517 CONTINUE
      DO 203 I=1,3
      K=I+1
      J=I+2
      IF(K.GT.3)K=K-3
      IF(J.GT.3)J=J-3
      G1(I)=X1(J)-X1(K)
      B1(I)=Y1(J)-Y1(K)
  203 C1(I)=Z1(J)-Z1(K)
      CALL TRANS(G1,B1,C1)
      DO 204 I=1,3
      DO 204 J=1,3
  204 CO4(J,I)=X(J,I)
      CALL TIMES(CO2,CO4,CO1,3,3,3,1)
      DO 205 J=1,3
      XO(J)=CO1(1,J)
  205 YO(J)=CO1(2,J)
      CNLD=0.
      DO 432 I=1,3
      DO 432 J=6,11
  432 IF(X(I,J).NE.0.) CNLD=1.
      IF(CNLD.NE.1.)GO TO 437
      DO 433 I=6,11
      J=6*(I-6)+1
      DACG(J)=X(1,I)
      DACG(J+1)=X(2,I)
  433 DACG(J+2)=X(3,I)
      DO 434 I=1,36
      DO 435 J=1,36
  435 TRAN(J,I)=0.
  434 TRAN(I,I)=1.
      DO 436 K=1,36,6
      DO 436 I=1,3
      DO 436 J=1,3
  436 TRAN(K+J-1,K+I-1)=CO2(J,I)
      CALL TIMES(TRAN,DACG,DACL,36,36,1,1)
  437 CONTINUE
      NLAK=INFO(1)
      METRO=1
      IF(NLAK.EQ.-1.OR.NLAK.EQ.-2) GO TO 840
      IF(NLAK.EQ.1.OR.NLAK.EQ.2)GO TO 104
      CALL SUBTI
      IF(NLAK.EQ.0)GO TO 105
      NLAK=NLAK-1
      DO 106 I=1,36
      DO 106 J=1,36
  106 BOL(NLAK,J,I)=STICK(J,I)
      DO 113 I=1,36
      DO 113 J=1,8
  113 COL(NLAK,J,I)=STICK(J,I)
      DO 108 I=1,36
  108 DCI(NLAK,I)=FORCE(I)
      GO TO 105
  840 NLAK=-NLAK
      METRO=-1
```

```
104 DO 105 I=1,36
    DO 109 J=1,36
109 STUCK(J,I)=BCL(NLAK,J,I)
    DO 214 I=1,36
    DO 214 J=1,8
214 STICK(J,I)=CCL(NLAK,J,I)
    DO 111 I=1,36
111 FORCE(I)=DOL(NLAK,I)
    IF(METEO.NE.-1) GO TO 105
    DO 841 I=1,36
    DO 841 J=1,8
    E1=1.
    DO 842 II=1,12
842 IF(I.EQ.MOL(II)) E1=-1.
841 STICK(J,I)=STICK(J,I)*E1
105 CONTINUE
    DO 438 I=1,36
438 FORCE(I)=FORCE(I)+DADL(I)
    NLIK=INFO(2)
    IF(NLIK.EQ.0.OR.NLIK.EQ.99) GO TO 115
    DO 115 I=1,NLIK
    K=(I-1)*2+3
    I=INFO(K)
    K1=INFO(K+1)
    DO 583 II=1,36
    DO 584 IJ=1,36
584 TRAN(IJ,II)=0.
583 TRAN(II,II)=1.
    KK=(I-1)*6
    DO 219 IL=1,3
    DO 219 JL=1,3
219 TRAN(KK+IL,KK+JL)=CC2(IL,JL)
    IF(K1.EQ.-1) GO TO 212
    IF(K1.EQ.0) GO TO 215
    CALL TRANL(XM,YM,ZM,K1,CO3)
    DO 222 IL=1,3
    DO 222 JL=1,3
222 TRAN(KK+IL+3,KK+JL+3)=CO3(IL,JL)
    GO TO 212
215 DO 216 IL=1,3
    DO 216 JL=1,3
216 TRAN(KK+IL+3,KK+JL+3)=CCM(IL,JL)
212 CONTINUE
    CALL TIMES(STUCK,TRAN,SMK,36,36,36,1)
    CALL TIMES(TRAN,SMK,STUCK,36,36,36,2)
    CALL TIMES(STICK,TRAN,STR,8,36,36,1)
    CALL TIMES(TRAN,FORCE,DADG,36,36,1,2)
    DO 232 IL=1,36
    DO 232 JL=1,8
232 STICK(JL,IL)=STR(JL,IL)
    DO 233 IL=1,36
233 FORCE(IL)=DADG(IL)
115 CONTINUE
    RETURN
    END
    SUBROUTINE SUBTI
    IMPLICIT REAL*8 (A-H,O-Z)
    COMMON/STI/X(3,20),YO(12),STUCK(36,36),STICK(8,36),FORCE(36),
   1INFO(2 )
    COMMON/TRCA/B(3),G(3),EL(3),DT
```

```
      COMMON/GLOLP/EMM(18,18),EMO(18,18),STMM(18,6),STD(6,36)
     1,STUV(12,12)
      COMMON/CCCR/CO2(3,3),XL(3),YL(3),COM(3,3)
      DIMENSION STM(18,18),TT(36,36),M1(36),SIM(36,36),CE(36,36),PG(3),
     1PL(3)
      DO 100 I=1,3
      K=I+1
      J=I+2
      IF(K.GT.3)K=K-3
      IF(J.GT.3)J=J-3
      F(I)=YL(K)-YL(J)
      G(I)=XL(J)-XL(K)
100   EL(I)=F(I)**2+G(I)**2
      DT=F(1)*G(2)-F(2)*G(1)
      DO 111 I=1,36
      DO 111 J=1,8
111   STR(J,I)=0.
      CALL INTSU
      DO 101 I=1,18
      DO 101 J=1,18
101   STM(J,I)=EMM(J,I)+EMO(J,I)
      DO 102 I=1,36
      DO 102 J=1,36
      TT(J,I)=0.
102   SIM(J,I)=0.
      DO 103 I=1,6
      DO 103 J=1,18
103   SIM(J+6,I)=STMM(J,I)
      DO 104 I=1,18
      DO 104 J=1,6
104   SIM(J,I+6)=STMM(I,J)
      DO 105 I=1,18
      DO 105 J=1,18
105   SIM(J+6,I+6)=-STM(J,I)
      DO 200 I=1,12
      DO 200 J=1,12
200   SIM(J+24,I+24)=STUV(J,I)
      DATA M1/25,26,1,7,8,9,27,28,2,10,11,12,29,30,3,13,14,15,31,32,4
     1,16,17,18,33,34,5,19,20,21,35,36,6,22,23,24/
      DO 107 I=1,36
107   TT(M1(I),I)=1.
      CALL TIMES(SIM,TT,CE,36,36,36,1)
      CALL TIMES(TT,CE,STICK,36,36,36,2)
      CALL TIMES(STR,TT,STICK,8,36,36,1)
      PG(1)=X(1,5)
      PG(2)=X(2,5)
      PG(3)=X(3,5)
      CALL TIMES(CO2,PG,PL,3,3,1,1)
      FORCE(19)=PL(1)*DT/6.
      FORCE(20)=PL(2)*DT/6.
      FORCE(21)=PL(3)*DT/6.
      FORCE(25)=PL(1)*DT/6.
      FORCE(26)=PL(2)*DT/6.
      FORCE(27)=PL(3)*DT/6.
      FORCE(31)=PL(1)*DT/6.
      FORCE(32)=PL(2)*DT/6.
      FORCE(33)=PL(3)*DT/6.
      RETURN
      END
      SUBROUTINE INTSU
```

```
      IMPLICIT REAL*8 (A-H,O-Z)
      D21=YOUNG(12)
      COMMON/TRDA/B(3),G(3),EL(3),DT
      COMMON/CLCEP/EMM(18,18),EMO(18,18),EMW(18,6),STR(8,36),STUV(12,12)
      COMMON/STI/X(3,2 ),YOUNG(12),ST(36,36),SE(8,36),ECR(36),INFO(20)
      DIMENSION A1(7),A2(7),A3(7),W(7),EEB(3,3),EES(2,2),A(3),
     1EN(6),EE(6),EF(6),EMM3(3,18),EMM2(2,6),EMM1(2,18)
     2,EMW1(18,6),EMM1(3,18),EMM2(18,18),EMO1(2,18),EMO2(18,18)
     3,SUV(3,12),BUV(3,12),D(3,3),EUV(12,12)
      DATA A1/0.33333333D 00,0.05971587D 00,2*0.47014206D 00,3.79742698D
     1    ,2* .1 128651D   /,A3/ .33333333D    ,2* .47 142 6D    , . 597158
     27D 00,2*0.10128651D 00,0.79742698D 00/,W/0.22500000D 00,3*0.132394
     315D 00,3*).12593918D 00/
      D11=YOUNG(1)
      D12=YOUNG(2)
      D22=YOUNG(3)
      D33=YOUNG(4)/2.
      D44=YOUNG(5)
      D55=YOUNG(6)
      D21=D12
      DDDD=YOUNG(12)
      IF(DDDD.NE. .)D21=DDDD
      D66=YOUNG(7)
      D67=YOUNG(8)
      D76=YOUNG(9)
      D77=YOUNG(10)
      D88=YOUNG(11)
      DO 100 I=1,3
      DO 100 J=1,3
100   EEB(J,I)=0.
      DO 101 I=1,2
      DO 101 J=1,2
101   EES(J,I)=0.
      DO 200 I=1,3
      DO 2   J=1,3
200   D(J,I)=0.
      DET=D11*D22-D12*D21
      IF(DET.EQ. .)WRITE(6,7  )
700   FORMAT(/,'       *****  ERROR    ****  ELASTICITY  MATRIX ',/)
      EEB(1,1)=D22/DET
      EEB(1,2)=-D12/DET
      EEB(2,1)=-D21/DET
      EEB(2,2)=D11/DET
      EEB(3,3)=1./D33
      EES(1,1)=1./D44
      EES(2,2)=1./D55
      D(1,1)=D66
      D(1,2)=D67
      D(2,1)=D76
      D(2,2)=D77
      D(3,3)=D88
      DO 107 I=1,18
      DO 107 J=1,18
      EMM(J,I)=0.
107   EMO(J,I)=0.
      DO 108 I=1,6
      DO 108 J=1,18
108   EMW(J,I)=0.
      DO 208 I=1,12
      DO 208 J=1,12
```

```
208 STUV(J,I)=0.
C          INTEGRATION
      DO 102 K=1,9
      IF(K.EQ.2) A(1)=1.
      IF(K.EQ.8) A(2)=0.
      IF(K.EQ.8) A(3)=0.
      IF(K.EQ.9) A(1)= .
      IF(K.EQ.9) A(2)=1.
      IF(K.EQ.9) A(3)=0.
      IF(K.GT.7)GO TO 112
      A2(K)=1.-A1(K)-A3(K)
      A(1)=A1(K)
      A(2)=A2(K)
      A(3)=A3(K)
112   CONTINUE
      EN(1)=(2.*A(1)-1.)*A(1)
      EN(2)=(2.*A(2)-1.)*A(2)
      EN(3)=(2.*A(3)-1.)*A(3)
      EN(4)=4.*A(2)*A(3)
      EN(5)=4.*A(1)*A(3)
      EN(6)=4.*A(1)*A(2)
      EE(1)=B(1)*(4.*A(1)-1.)/DT
      EE(2)=B(2)*(4.*A(2)-1.)/DT
      EE(3)=B(3)*(4.*A(3)-1.)/DT
      EE(4)=4.*(B(2)*A(3)+B(3)*A(2))/DT
      EE(5)=4.*(B(1)*A(3)+B(3)*A(1))/DT
      EE(6)=4.*(B(1)*A(2)+B(2)*A(1))/DT
      EE(1)=G(1)*(4.*A(1)-1.)/DT
      EE(2)=G(2)*(4.*A(2)-1.)/DT
      EE(3)=G(3)*(4.*A(3)-1.)/DT
      EE(4)=4.*(G(2)*A(3)+G(3)*A(2))/DT
      EE(5)=4.*(G(1)*A(3)+G(3)*A(1))/DT
      EE(6)=4.*(G(1)*A(2)+G(2)*A(1))/DT
      DO 103 I=1,18
      DO 103 J=1,3
103   ENMO(J,I)=0.
      DO 303 I=1,18
      DO 303 J=1,2
303   ENM1(J,I)=0.
      DO 203 I=1,12
      DO 203 J=1,3
203   BUV(J,I)=0.
      DO 205 J=1,12,2
      M=(J+1)/2
      BUV(1,J)=EE(M)
      BUV(2,J+1)=EE(M)
      BUV(3,J)=EE(M)
205   BUV(3,J+1)=EE(M)
      CALL TIMES(D,BUV,SUV,3,3,12,1)
      CALL TIMES(BUV,SUV,EUV,12,3,12,2)
      DO 104 J=1,6
      DO 104 I=1,3
      M=3*J+I-3
104   ENMO(I,M)=EN(J)
      DO 105 J=1,13,3
      M=(J+2)/3
      ENM1(1,J)=EE(M)
      ENM1(1,J+2)=EE(M)
      ENM1(2,J+1)=EE(M)
105   ENM1(2,J+2)=EE(M)
```

```
      IF(K.GT.7)GO TO 113
      DO 106 J=1,6
      ENW2(1,J)=EE(J)
106   ENW2(2,J)=EH(J)
      CALL TIMES(ENM1,ENW2,ENW1,18,2,6,2)
      CALL TIMES(AFB,ENM0,ENM1,3,3,18,1)
      CALL TIMES(EAM ,ENM1,ENM2,18,3,18,2)
      CALL TIMES(EES,ENM1,EMO1,2,2,18,1)
      CALL TIMES(ENM1,EMO1,EMO2,18,2,18,2)
      DO 109 I=1,18
      DO 109 J=1,18
      ENM(J,I)=ENM(J,I)+W(K)*ENM2(J,I)*DT/2.
109   EMO(J,I)=EMO(J,I)+W(K)*EMO2(J,I)*DT/2.
      DO 110 I=1,6
      DO 110 J=1,18
110   EMW(J,I)=EMW(J,I)+W(K)*EMW1(J,I)*DT/2.
      DO 210 I=1,12
      DO 210 J=1,12
210   STUV(J,I)=STUV(J,I)+W(K)*EUV(J,I)*DT/2.
113   CONTINUE
      IF(K.NE.1)GO TO 224
      DO 225 I=1,12
      DO 225 J=1,3
225   STR(J+4,I+24)=SUV(J,I)
224   CONTINUE
      IF(K.LT.9) GO TO 102
      L=0
      IF(K.EQ.9) L=2
      DO 114 I=1,18
      DO 114 J=1,2
114   STR(L+J,I+6)=ENM1(J,I)
102   CONTINUE
      RETURN
      END
      SUBROUTINE TIMES(A,B,R,N,M,L,KOK)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION A(1),B(1),R(1)
C     KOK=1  A(N,M) , B(M,L) , R(N,L)   REGULAR    A*B
C     KOK=2  A(M,N) , B(M,L) , R(N,L)   TRANSPOSE  A'*B
      IR=1
      DO 100 K=1,L
      DO 100 J=1,N
      R(IR)=0.
      GO TO(101,102),KOK
101   CONTINUE
      DO 103 I=1,M
      IA=N*(I-1)+J
      IB=M*(K-1)+I
103   R(IR)=R(IR)+A(IA)*B(IB)
      GO TO 1
102   CONTINUE
      DO 104 I=1,M
      IA=N*(J-1)+I
      IB=M*(K-1)+I
104   R(IR)=R(IR)+A(IA)*B(IB)
100   IR=IR+1
      RETURN
      END
```

4. Reference symbol DRO3O

```
10 P=100 PROUTE=DUPE COPIES=4
E = UNIVERSITY, BATCH
N WAS: 15:51:57
 SIGNED ON AT  15:52:26 ON MON SEP 22/75
*PRINT*
    *** DOME  ROTATION ELEMENT WITH 30 DEG. OF FR. AT 6 NODES U,V,W,RX,RY*****
    ***** CUBIC VARIATION FOR THE DIS. & QUADRATIC FOR THE ROTATIONS      ******
    **** 5  STRESSES AT THE CENTROID   MXX,MYY,MXY,QX,QY                  ******
    ***  TRANSFORMATION  TO  RM  CS                                       ******
    ****  ELASTICITY  MODULI  VIA  YOUNG(12)                              ******
     SUBROUTINE STIFF
     IMPLICIT REAL*8 (A-H,O-Z)
     COMMON/STI/X(3,20),YOUNG(12),STUCK(36,36),STICK(8,36),FORCE(36)
    1,INFC(20)
     COMMON/MAN/BCL(2,36,36),CCL(2,8,36),DCL(2,36),GRAM(16,16),NGRAM
     COMMON/COOR/CO2(3,3),XO(3),YO(3)
     DIMENSION TRAN(30,30),SMK(30,30),SMK1(30,30),STR(8,30),TIK(8,30),
    1DADG(30),XM(16,2),YM(16,2),CO3(2,2),DACL(30),MDL(6),FOR(30)
    2,ZM(16,2),G1(3),B1(3),C1(3),X1(3),Y1(3),Z1(3),CO4(3,3),CO1(3,3)
     DATA MDL/3,8,13,18,23,28/
     DO 100 I=1,3
     X1(I)=X(1,I)
     Z1(I)=X(3,I)
 100 Y1(I)=X(2,I)
     DO 101 I=1,36
     DO 1 1 J=1,36
 101 STUCK(J,I)=0.
     DO 102 I=1,36
     DO 102 J=1,8
 102 STICK(J,I)=0.
     DO 1 3 I=1,36
 103 FORCE(I)=0.
     DO 460 I=1,30
     DADL(I)=0.
 460 DADG(I)=0.
     IF(NGRAM.EQ.0) GO TO 517
     DO 516 M=1,NGRAM
     DO 516 I=1,2
     XM(M,I)=GRAM(M,I)
     ZM(M,I)=GRAM(M,I+4)
 516 YM(M,I)=GRAM(M,I+2)
 517 CONTINUE
     DO 400 I=1,3
     K=I+1
     J=I+2
     IF(K.GT.3)K=K-3
     IF(J.GT.3)J=J-3
     G1(I)=X1(J)-X1(K)
     B1(I)=Y1(J)-Y1(K)
4    C1(I)=Z1(J)-Z1(K)
     CALL TRANS(G1,B1,C1)
     DO 401 I=1,3
     DO 4 1 J=1,3
 401 CO4(I,J)=X(I,J)
     CALL TIMES(CO2,CO4,CO1,3,3,3,1)
     DO 4 2 J=1,3
     XO(J)=CO1(1,J)
 402 YO(J)=CO1(2,J)
     CNLD=0.
     DO 432 I=1,3
```

```
      DO 432 J=6,8
  432 IF(X(1,J).NE.0.)CNLD=1.
      IF(CNLD.NE.1.) GO TO 437
      DO 683 II=1,30
      DO 684 IJ=1,30
  684 TRAN(IJ,II)=0.
  683 TRAN(II,II)=1.
      DO 433 I=6,8
      J=5*(I-6)+1
      DAOG(J)=X(1,I)
      DAOG(J+1)=X(2,I)
      DAOG(J+2)=X(3,I)
  433 CONTINUE
      DO 436 K=1,30,5
      DO 436 I=1,3
      DO 436 J=1,3
  436 TRAN(K+J-1,K+I-1)=CC2(J,I)
      CALL TIMES(TRAN,DAOG,DAOL,30,30,1,1)
  437 CONTINUE
      NLAK=INFO(1)
      METRO=0
      IF(NLAK.EQ.-1.OR.NLAK.EQ.-2) GO TO 840
      IF(NLAK.EQ.1.OR.NLAK.EQ.2) GO TO 104
      CALL SUBTI
      IF(NLAK.EQ.0) GO TO 105
      NLAK=NLAK-10
      DO 106 I=1,30
      DO 106 J=1,30
  106 BOL(NLAK,J,I)=STUCK(J,I)
      DO 113 I=1,30
      DO 113 J=1,8
  113 COL(NLAK,J,I)=STICK(J,I)
      DO 108 I=1,30
  108 DOL(NLAK,I)=FORCE(I)
      GO TO 105
  840 NLAK=-NLAK
      METRO=-1
  104 DO 109 I=1,30
      DO 109 J=1,30
  109 STUCK(J,I)=BOL(NLAK,J,I)
      DO 214 I=1,30
      DO 214 J=1,8
  214 STICK(J,I)=COL(NLAK,J,I)
      DO 111 I=1,30
  111 FORCE(I)=DOL(NLAK,I)
      IF(METRO.NE.-1) GO TO 105
      DO 861 I=1,30
      DO 861 J=1,30
      F1=0.
      IF(I.NE.J) GO TO 862
      F1=-1.
      DO 863 II=1,6
      IF(I.EQ.MOL(II)) F1=1.
  863 CONTINUE
  862 TRAN(J,I)=F1
  861 CONTINUE
      DO 1 I=1,30
      DO 1 J=1,30
    1 SAK1(J,I)=STUCK(J,I)
      DO 2 I=1,30
```

```
      DO 2 J=1,8
    2 TIK(J,I)=STICK(J,I)
      DO 3 I=1,30
    3 DACG(I)=FORCE(I)
      CALL TIMES(SMK1,TRAN,SMK,30,30,30,1)
      CALL TIMES(TRAN,SMK,SMK1,3 ,3 ,3 ,2)
      CALL TIMES(TIK,TRAN,STR,8,30,30,1)
      CALL TIMES(TRAN,DACG,FOR,30,30,1,2)
      DO 4 I=1,3
      DO 4 J=1,30
    4 STICK(J,I)=SMK1(J,I)
      DO 5 I=1,3
      DO 5 J=1,8
    5 STICK(J,I)=STR(J,I)
      DO 6 I=1,30
    6 FORCE(I)=FOR(I)
  105 CONTINUE
      DO 438 I=1,30
  438 FORCE(I)=FORCE(I)+DACL(I)
      NLIK=INFO(2)
      IF(NLIK.EQ.0.OR.NLIK.EQ.99) GO TO 112
      DO 213 I=1,30
      DO 213 J=1,3
  213 SMK(J,I)=STICK(J,I)
      DO 114 I=1,30
      DO 114 J=1,8
  114 STR(J,I)=STICK(J,I)
      DO 439 I=1,3
  439 DACL(I)=FORCE(I)
      DO 115 I=1,NLIK
      K=(I-1)*2+3
      L=INFO(K)
      K1=INFO(K+1)
      DO 583 II=1,30
      DO 584 IJ=1,30
  584 TRAN(IJ,II)=0.
  583 TRAN(II,II)=1.
      KK=(L-1)*5
      DO 588 IN=1,3
      DO 588 JN=1,3
  588 TRAN(KK+IN,KK+JN)=CO2(IN,JN)
      IF(K1.LT.0) GO TO 589
      IF(K1.EQ. ) GO TO 59
      CALL TRANM(XM,YM,K1,CO3)
      DO 587 IN=1,2
      DO 587 IL=1,2
  587 TRAN(KK+IL+3,KK+IN+3)=CO3(IL,IN)
      GO TO 589
   59 DO 591 IN=1,2
      DO 591 JN=1,2
  591 TRAN(KK+JN+3,KK+IN+3)=CO2(JN,IN)
  589 CONTINUE
      CALL TIMES(SMK,TRAN,SMK1,30,30,30,1)
      CALL TIMES(TRAN,SMK1,SMK,3 ,3 ,3 ,2)
      CALL TIMES(STR,TRAN,TIK,8,30,30,1)
      CALL TIMES(TRAN,DACL,DACG,30,30,1,2)
      DO 116 NI=1,30
      DO 116 NJ=1,8
  116 STR(NJ,NI)=TIK(NJ,NI)
      DO 440 II=1,30
```

```
440 FACL(II)=FACC(II)
115 CONTINUE
    DO 117 I=1,30
    DO 117 J=1,3)
117 STUCK(J,I)=SMK(J,I)
    DO 118 I=1,30
    DO 118 J=1,8
118 STICK(J,I)=STP(J,I)
    DO 441 I=1,30
441 FORCE(I)=FACL(I)
112 CONTINUE
    RETURN
    END
    SUBROUTINE TRANL(X,Y,K1,TR1)
    IMPLICIT REAL*8 (A-H,O-Z)
    DIMENSION X(16,2),Y(16,2),Z(16,2),Z2(2),X2(2),Y2(2),TRL(2,2)
   1,TR(3,3),TR1(3,3)
    COMMON/COORD/TRG(3,3),X0(3),Y0(3)
    DO 515 J=1,2
    X2(J)=X(K1,J)
    Z2(J)=Z(K1,J)
515 Y2(J)=Y(K1,J)
    G2=X2(2)-X2(1)
    B2=Y2(2)-Y2(1)
    C2=Z2(2)-Z2(1)
    GLEN2=DSQRT(G2**2+B2**2+C2**2)
    IF(GLEN2.LT. .1D-12.AND.GLEN2.GT.- .1D-12) WRITE(6,700)
    B2=B2/GLEN2
    G2=G2/GLEN2
    C2=C2/GLEN2
    TR(1,1)=G2
    TR(2,1)=B2
    TR(3,1)=C2
    TR(1,2)=TRG(3,2)*TR(3,1)-TRG(3,3)*TR(2,1)
    TR(2,2)=TRG(3,3)*TR(1,1)-TRG(3,1)*TR(3,1)
    TR(3,2)=TRG(3,1)*TR(2,1)-TRG(3,2)*TR(1,1)
    TR(1,3)=TRG(3,1)
    TR(2,3)=TRG(3,2)
    TR(3,3)=TRG(3,3)
    CALL TIMES(TRG,TR,TR1,3,3,3,1)
    DO 10 I=1,2
    DO 10 J=1,2
 1  TRL(J,I)=TR1(J,I)
700 FORMAT(' ******   ERROR  TRANL      ********')
    RETURN
    END
    SUBROUTINE CNDNS(K)
    IMPLICIT REAL*8 (A-H,O-Z)
    COMMON/CON/ST(33,33),T(8,33)
    DIMENSION B(33)
    A=1./ST(K,K)
    DO 40 I=1,33
 40 B(I)=ST(K,I)
    DO 41 J=1,33
    DO 41 I=1,33
 41 ST(J,I)=ST(J,I)-B(I)*B(J)*A
    DO 42 I=1,8
    D=T(I,K)
    DO 42 J=1,33
 42 T(I,J)=T(I,J)-B(J)*A*D
```

```
      RETURN
      END
      SUBROUTINE SHAP(A,B,C,DT)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION A(3),B(3),G(3),BE(8,33),SR(6),SRX(6),SRY(6),
     1SWX(7),SWY(7)
      COMMON/JCN/SM(33,33),S(8,33),D(8,8)
      DIM1=A(1)*A(2)*A(3)
      DIM2=(B(1)*A(2)*A(3)+B(2)*A(1)*A(3)+B(3)*A(1)*A(2))/DT
      DIM3=(G(1)*A(2)*A(3)+G(2)*A(1)*A(3)+G(3)*A(1)*A(2))/DT
      DO 100 I=1,3
      K=I+1
      J=I+2
      IF(K.GT.3)K=K-3
      IF(J.GT.3)J=J-3
      SR(I)=(2.*A(I)-1.)*A(I)
      SR(I+3)=4.*A(K)*A(J)
      SRX(I)=B(I)*(4.*A(I)-1.)/DT
      SRX(I+3)=(B(K)*4.*A(J)+B(J)*4.*A(K))/DT
      SRY(I)=G(I)*(4.*A(I)-1.)/DT
      SRY(I+3)=(G(K)*4.*A(J)+G(J)*4.*A(K))/DT
      SWX(I)=SRX(I)+3.*DIM2
      SWX(I+3)=SRX(I+3)-12.*DIM2
      SWY(I)=SRY(I)+3.*DIM3
      SWY(I+3)=SRY(I+3)-12.*DIM3
100   CONTINUE
      SWX(7)=27.*DIM2
      SWY(7)=27.*DIM3
      DO 101 I=1,33
      DO 101 J=1,8
101   BE(J,I)=0.
      DO 102 I=1,11,2
      K=(I+1)/2
      BE(1,I)=SRX(K)
      BE(2,I+1)=SRY(K)
      BE(2,I)=SRY(K)
      BE(3,I+1)=SRX(K)
      BE(4,I)=-SR(K)
      BE(5,I+1)=-SR(K)
102   CONTINUE
      DO 103 I=13,19
      K=I-12
      BE(4,I)=SWX(K)
103   BE(5,I)=SWY(K)
      DO 104 I=20,33,2
      K=(I-18)/2
      BE(6,I)=SWX(K)
      BE(7,I+1)=SWY(K)
      BE(8,I)=SWY(K)
104   BE(8,I+1)=SWX(K)
      CALL TIMES(D,BE,S,8,8,33,1)
      CALL TIMES(BE,S,SM,33,8,33,2)
      RETURN
      END
      SUBROUTINE SUBTI
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/STI/X(3,2 ),YOUNG(12),STUCK(36,36),STICK(8,36),FORCE(36)
     1,INFO(20)
      COMMON/JCN/SM(33,33),S(8,33),D(8,8)
      COMMON/COOR/CO2(3,3),XO(3),YO(3)
```

```
      COMMON/CON/ST(33,33),TIK(8,33)
      DIMENSION A1(7),A2(7),A3(7),W(7),B(3),G(3),A(3),M1(33),T(33,33)
     1,PC(3),PL(3)
      DATA A1/0.33333333D 00,0.059715870 00,2*0.470142060 00,0.797426990
     1 ,2* .1 1286510   /,A3/ .333333330  0,2*0.470142060 00,0.0597158
     270 00,2*0.101286510 00,0.797426990 00/,W/0.225000000 00,3*0.132394
     315D 00,3*0.125939180 00/
      DO 2.  I=1,8
      DO 200 J=1,8
  200 D(J,I)=0.
      D(1,1)=YOUNG(1)
      D(1,2)=YOUNG(2)
      D(2,1)=D(1,2)
      D(2,2)=YOUNG(3)
      D(3,3)=YOUNG(4)/2.
      D(4,4)=YOUNG(5)
      D(5,5)=YOUNG(6)
      D(6,6)=YOUNG(7)
      D(6,7)=YOUNG(8)
      D(7,6)=YOUNG(9)
      D(7,7)=YOUNG(10)
      D(8,8)=YOUNG(11)
      IF(YOUNG(12).NE.0.) D(2,1)=YOUNG(12)
      DO 21  I=1,33
      DO 210 J=1,33
  210 ST(J,I)=0.
      DO 211 I=1,3
      K=I+1
      J=I+2
      IF(K.GT.3)K=K-3
      IF(J.GT.3)J=J-3
      P(I)=YC(K)-YC(J)
  211 G(I)=XC(J)-XC(K)
      DT=P(1)*G(2)-P(2)*G(1)
      DO 212 K=1,7
      A2(K)=1.-A1(K)-A3(K)
      A(1)=A1(K)
      A(2)=A2(K)
      A(3)=A3(K)
      CALL SHAP(A,B,G,DT)
      IF(K.NE.1) GO TO 213
      DO 214 I=1,33
      DO 214 J=1,8
  214 TIK(J,I)=S(J,I)
  213 CONTINUE
      DO 215 I=1,33
      DO 215 J=1,33
  215 ST(J,I)=ST(J,I)+W(K)*SM(J,I)*DT/2.
  212 CONTINUE
      DO 217 I=1,33
      DO 217 J=1,33
  217 T(J,I)=0.
      DATA M1/20,21,13,1,2,22,23,14,3,4,24,25,15,5,6,26,27,16,7,8,
     128,29,17,9,1 ,3 ,31,10,11,12,32,33,19/
      DO 218 I=1,33
  218 T(M1(I),I)=1.
      CALL TIMES(ST,T,SM,33,33,33,1)
      CALL TIMES(T,SM,ST,33,33,33,2)
      CALL TIMES(TIK,T,S,8,33,33,1)
      DO 219 I=1,33
```

```
      DO 219 J=1,8
219   TIK(J,I)=S(J,I)
      CALL CNDNS(33)
      CALL CNDNS(32)
      CALL CNDNS(31)
      DO 220 I=1,30
      DO 220 J=1,30
220   STUCK(J,I)=ST(J,I)
      DO 221 I=1,30
      DO 221 J=1,8
221   STICK(J,I)=TIK(J,I)
      PG(1)=X(1,5)
      PG(2)=X(2,5)
      PG(3)=X(3,5)
      CALL TIMES(CC2,PG,PL,3,3,1,1)
      FORCE(16)=PL(1)*DT/6.
      FORCE(17)=PL(2)*DT/6.
      FORCE(18)=PL(3)*DT/6.
      FORCE(21)=PL(1)*DT/6.
      FORCE(22)=PL(2)*DT/6.
      FORCE(23)=PL(3)*DT/6.
      FORCE(26)=PL(1)*DT/6.
      FORCE(27)=PL(2)*DT/6.
      FORCE(28)=PL(3)*DT/6.
      RETURN
      END
      SUBROUTINE TIMES(A,B,R,N,M,L,KOK)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION A(1),B(1),R(1)
      KOK=1 A(N,M) , B(M,L) , R(N,L)  REGULAR A*B=R
      KOK=2 A(M,N) , B(M,L) , R(N,L) TRANSPOSE  A'*B=R
      IR=1
      DO 100 K=1,L
      DO 100 J=1,N
      R(IR)=0.
      GO TO(101,102),KOK
101   CONTINUE
      DO 103 I=1,M
      IA=N*(I-1)+J
      IB=M*(K-1)+I
103   R(IR)=R(IR)+A(IA)*B(IB)
      GO TO 100
102   CONTINUE
      DO 104 I=1,M
      IA=M*(J-1)+I
      IB=M*(K-1)+I
104   R(IR)=R(IR)+A(IA)*B(IB)
100   IR=IR+1
      RETURN
      END
      SUBROUTINE TRANS(G1,B1,C1)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/COOR/TR(3,3),XO(3),YO(3)
      DIMENSION B1(3),G1(3),C1(3)
      DO 10 I=1,3
      DO 10 J=1,3
10    TR(J,I)=0.
      AA=B1(2)*C1(3)-B1(3)*C1(2)
      AB=G1(3)*C1(2)-G1(2)*C1(3)
      AC=B1(3)*G1(2)-G1(3)*B1(2)
```

```
      IF((AA.GT.-.1D-12.AND.AA.LT.0.1D-12).AND.(AB.GT.-.1D-12.AND.AB.
     1LT.0.1D-12)) GO TO 20
      AD=DSQRT(AA**2+AB**2+AC**2)
      AE=DSQRT(AA**2+AB**2)
      TR(3,1)=AA/AD
      TR(3,2)=AB/AD
      TR(3,3)=AC/AD
      TR(1,1)=-AB/AE
      TR(1,2)=AA/AE
      TR(2,1)=-AA*AC/(AE*AD)
      TR(2,2)=-AB*AC/(AE*AD)
      TR(2,3)=(AA**2+AB**2)/(AE*AD)
      RETURN
   20 DO 30 I=1,3
   30 TR(I,I)=1.
      RETURN
      END
```

# DATA GENERATION ROUTINE

```
$SIG ESN6 T=10 P=100 PROUTE=CURF COPIES=4
CHARGING RATE = UNIVERSITY, BATCH
**LAST SIGNON WAS: 10:24:00
 USER "ESN6" SIGNED ON AT  11:17:08 ON TUE OCT 14/75
$C GENE(3001) TO *PRINT*
C        ******    DATA GENERATION RUTINE  **********
C       ******  FOR 7 PLATE- ELEMENTS  AND  5 DOME-ELEMENTS  ****
C       ******     NEIDOS  FROM  1-12  RESPECTIVELY   ***********
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/ONE/NEIDOS,NSTIF,YOUNG(10,12),NBOU,NBOUL,XB(16,6),INFO(16),
     1INF(26),KBOU(26),COOR(30,3,3),X(3,3),N(6),NIN,INF1(26),NDF1,NDF2,
     2M1(8,30),M2(4,30),NF(26),KK
      READ(5,601) NEIDOS,NELEM,NKIND,NSTIF,NBOUL,NBOUP
C      ALLOWED 30  DIFFERENT  COORDINATE  SETS
      DO 100  I=1,NKIND
      READ(5,607)((X(L,M),L=1,3),M=1,3)
      DO 100 J=1,3
      DO 100 K=1,3
 100  COOR(I,J,K)=X(J,K)
C      ALLOWED 10  DIFFERENT  STIFFNESS   SETS
      READ(5,606)((YOUNG(J,I),I=1,12),J=1,NSTIF)
C      ALLOWED 26  DIFFERENT  BOUNDARY    SETS  IN  A PROBLEM
      NBOU=NBOUL+NBOUP
      READ(5,609)(KBOU(I),I=1,NBOU)
      READ(5,606)((XB(I,J),J=1,6),I=1,NBOUL)
      CALL SFORM
      DO 101 KK=1,NELEM
      READ(5,602)(N(I),I=1,6)
      DO 511 MOM=1,5
      MOL=MOM+1
      NANA=N(MOM)
      DO 510 MON=MOL,6
      IF(NANA.EQ.N(MON)) GO TO 512
 510  CONTINUE
 511  CONTINUE
      DO 102 J=1,6
 102  N(J)=N(J)*10+1
      WRITE(6,602)(N(J),J=1,6)
      WRITE(6,608)
 101  CONTINUE
      WRITE(6,608)
      WRITE(6,608)
      DO 103 KK=1,NELEM
      READ(5,601)(INFO(I),I=1,16)
      IF(KK.GT.1) GO TO 209
      CALL BCON
 209  CONTINUE
      DO 109 I=1,26
 109  INF(I)=0
      L=3
      DO 108 I=4,NIN,2
 108  IF(INFO(I).GT.0)L=L+2
      WRITE(6,603) INFO(1),L
      KIND=INFO(2)
      IF(KIND.EQ.0)GO TO 205
      WRITE(6,606)((COOR(KIND,J,M3),J=1,3),M3=1,3)
      GO TO 206
 205  WRITE(6,608)
      WRITE(6,608)
 206  CONTINUE
```

```
      IF(INFO(16).EQ.0)GO TO 105
      CALL LFORM
      GO TO 106
  105 WRITE(6,608)
      WRITE(6,608)
  106 CONTINUE
      INF(1)=INFO(3)
      INF(2)=L/2-1
      L1=1
      DO 110 I=4,NIN,2
      IF(INFO(I).LT.0.AND.INFO(I+1).LT.0) GO TO 517
      IF(INFO(I).LE.0)GO TO 110
      L1=L1+2
      INF(L1)=INFO(I)
      IF(INFO(I+1).LT.0)GO TO 210
      INF(L1+1)=INFO(I+1)
      GO TO 110
  210 INF(L1+1)=0
      INFO(I+1)=-INFO(I+1)
  110 CONTINUE
      L2=L1+2
      DO 112 I=1,26
  112 INF1(I)=0
      L1=0
      DO 114 I=4,NIN,2
      IF(INFO(I).EQ.0)GO TO 114
      IF(INFO(I).LT.0)INFO(I)=-INFO(I)
      L=INFO(I)
      K1=INFO(I+1)
      IF(L.NE.0.AND.K1.EQ.0) GO TO 515
      K=KBOU(K1)
      IF(L.GT.3.AND.NDF2.NE.0) GO TO 116
      DO 115  J=1,NDF1
      NF(J)=M1(J,K)*(J+(L-1)*NDF1)
      IF(NF(J).EQ.0) GO TO 115
      L1=L1+1
      INF1(L1)=NF(J)
  115 CONTINUE
      GO TO 114
  116 DO 117 J=1,NDF2
      NF(J)=M2(J,K)*(3*NDF1+J+(L-4)*NDF2)
      IF(NF(J).EQ.0) GO TO 117
      L1=L1+1
      INF1(L1)=NF(J)
  117 CONTINUE
  114 CONTINUE
      IF(L1.EQ.0) INF(L2)=0
      IF(L1.NE.0) INF(L2)=99
      WRITE(6,609)(INF(I),I=1,L2)
      IF(L1.EQ.0) GO TO 103
      WRITE(6,611) L1,(INF1(I),I=1,L1)
  103 CONTINUE
      WRITE(6,608)
      GO TO 520
  512 WRITE(6,612) KK
      GO TO 521
  515 WRITE(6,614) KK
      GO TO 521
  517 WRITE(6,618) KK
  521 A=1.
```

```
      B=0.
      C=A/B
      GO TO 520
601 FORMAT(16I5)
602 FORMAT(6I5)
603 FORMAT('  999',I5,'      ',I5)
606 FORMAT(6D10.3)
607 FORMAT(6D10.3,/,3D10.3)
608 FORMAT('      ')
609 FORMAT(26I3)
611 FORMAT(I3,/,26I3)
612 FORMAT(10X,' ****  ERROR  1 ****  NICKNAMES OF ELEMENT ',I10,//)
614 FORMAT(10X,' ****  ERROR  2 **** INFORMATION OF ELEMENT ',I10,/,
     1'    INFO(3) NOT 0  INFO(4) = 0 FOR  NODE 1  ',/,
     2'     (5)-(6) FOR NODE 2  (7)-(8) FOR NODE 3 ETC.   ')
618 FORMAT(10X,' ****  ERROR  3 **** INFORMATION OF ELEMENT ',I10,/,
     1'          INFO(3) INFO(4) BOTH NEGATIVE   FOR NODE 1',/,
     2'     (5)-(6) FOR NODE 2  (7)-(8) FOR NODE 3 ETC.   ')
520 STOP
      END
      SUBROUTINE BCON
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/ONE/NEIDOS,NSTIF,YOUNG(10,12),NBOU,NBOUL,XB(16,6),INFO(16),
     1INF(26),KBOU(26),COOR(30,3,3),X(3,3),N(6),NIN,INF1(26),NDF1,NDF2,
     2M1(8,30),M2(4,30),NF(26),KK
      IF(KK.GT.1) RETURN
      DO 16 I=1,8
      DO 16 J=1,30
 16   M1(I,J)=0
      DO 17 I=1,4
      DO 17 J=1,30
 17   M2(I,J)=0
      GO TO (1,2,3,4,5,6,7,8,9,10,11,12),NEIDOS
  1   NIN=8
      NDF1=5
      NDF2=0
      CALL DA1
      GO TO 15
  2   NIN=14
      NDF1=5
      NDF2=2
      CALL DA2
      GO TO 15
  3   NIN=8
      NDF1=4
      NDF2=0
      CALL DA34
      GO TO 15
  4   NIN=14
      NDF1=4
      NDF2=0
      CALL DA34
      GO TO 15
  5   NIN=8
      NDF1=8
      NDF2=0
      CALL DA5
      GO TO 15
  6   NIN=14
      NDF1=8
```

```
      NDF2=2
      CALL CA6
      GO TO 15
    7 NIN=14
      NDF1=3
      NDF2=0
      CALL CA7
      GO TO 15
    8 NIN=8
      NDF1=7
      NDF2=0
      CALL CA8
      GO TO 15
    9 NIN=14
      NDF1=7
      NDF2=4
      CALL CA8
      GO TO 15
   10 NIN=8
      NDF1=6
      NDF2=0
      CALL CA10
      GO TO 15
   11 NIN=14
      NDF1=6
      NDF2=0
      CALL CA10
      GO TO 15
   12 NIN=14
      NDF1=5
      NDF2=0
      CALL CA8
   15 CONTINUE
      RETURN
      END
      SUBROUTINE SFORM
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/ONE/NEIDOS,NSTIF,YOUNG(10,12),NBOU,NBOUL,XB(16,6),INFO(16),
     1INF(26),KBOU(26),COOR(30,3,3),X(3,3),N(6),NIN,INF1(26),NDF1,NDF2,
     2M1(8,30),M2(4,30),NF(26),KK
      WRITE(6,701)
      WRITE(6,702)
      WRITE(6,703)
      GO TO(101,102,103,104,105,106,107,108,109,110,111,112),NEIDOS
  101 WRITE(6,704)
      WRITE(6,706)
      WRITE(6,707)
      WRITE(6,708)
      WRITE(6,709)
      WRITE(6,710)
      WRITE(6,711) NSTIF
      GO TO 114
  102 WRITE(6,705)
      WRITE(6,706)
      WRITE(6,707)
      WRITE(6,708)
      WRITE(6,712)
      WRITE(6,710)
      WRITE(6,713) NSTIF
      GO TO 114
```

```
103 WRITE(6,704)
    WRITE(6,706)
    WRITE(6,707)
    WRITE(6,708)
    WRITE(6,717)
    WRITE(6,715)
    WRITE(6,718) NSTIF
    GO TO 114
104 WRITE(6,705)
    WRITE(6,706)
    WRITE(6,707)
    WRITE(6,708)
    WRITE(6,714)
    WRITE(6,715)
    WRITE(6,716) NSTIF
    GO TO 114
108 WRITE(6,704)
    WRITE(6,706)
    WRITE(6,707)
    WRITE(6,708)
    WRITE(6,723)
    WRITE(6,724)
    WRITE(6,725) NSTIF
    GO TO 114
109 WRITE(6,705)
    WRITE(6,706)
    WRITE(6,707)
    WRITE(6,708)
    WRITE(6,726)
    WRITE(6,724)
    WRITE(6,727) NSTIF
    GO TO 114
111 WRITE(6,705)
    WRITE(6,706)
    WRITE(6,707)
    WRITE(6,708)
    WRITE(6,728)
    WRITE(6,724)
    WRITE(6,729) NSTIF
    GO TO 114
105 WRITE(6,704)
    WRITE(6,706)
    WRITE(6,707)
    WRITE(6,708)
    WRITE(6,731)
    WRITE(6,710)
    WRITE(6,730) NSTIF
    GO TO 114
106 WRITE(6,705)
    WRITE(6,706)
    WRITE(6,707)
    WRITE(6,708)
    WRITE(6,733)
    WRITE(6,710)
    WRITE(6,732) NSTIF
    GO TO 114
107 WRITE(6,705)
    WRITE(6,706)
    WRITE(6,707)
    WRITE(6,708)
```

```
      WRITE(6,735)
      WRITE(6,710)
      WRITE(6,734) NSTIF
      GO TO 114
  110 WRITE(6,704)
      WRITE(6,706)
      WRITE(6,707)
      WRITE(6,708)
      WRITE(6,737)
      WRITE(6,724)
      WRITE(6,736) NSTIF
      GO TO 114
  112 WRITE(6,705)
      WRITE(6,706)
      WRITE(6,707)
      WRITE(6,708)
      WRITE(6,739)
      WRITE(6,724)
      WRITE(6,738) NSTIF
  114 WRITE(6,720)((YOUNG(J,I),I=1,12),J=1,NSTIF)
      WRITE(6,721) NBCUL
      WRITE(6,720)((XB(I,J),J=1,6),I=1,NBCUL)
      WRITE(6,722)
  701 FORMAT(1X,'(13I3)')
  702 FORMAT(1X,'(6D10.3,/,6D10.3)')
  703 FORMAT(1X,'(6I5)')
  704 FORMAT(1X,'(3I5,/,12I5)')
  705 FORMAT(1X,'(6I5,/,12I5)')
  706 FORMAT(1X,'(I5,5(I5,D10.3))')
  707 FORMAT(1X,'(4I5,4(/,6D10.3))')
  708 FORMAT(1X,'(25I3)')
  709 FORMAT(1X,'(3(1X,5D10.3,/))')
  710 FORMAT(1X,'(/,1X,5D10.3)')
  711 FORMAT('  1  3  8  3',I3,' 12  5  5  5  5')
  712 FORMAT(1X,'(3(1X,5D10.3,/),1X,6D10.3,/)')
  713 FORMAT('  2  6  8  3',I3,' 12  5  5  5  5  2  2  2')
  714 FORMAT(1X,'(6(1X,4D10.3,/))')
  715 FORMAT(1X,'(/,1X,2D10.3)')
  716 FORMAT('  4  6  8  3',I3,' 12  4  4  4  4  4  4  4')
  717 FORMAT(1X,'(3(1X,4D10.3,/))')
  718 FORMAT('  3  3  8  3',I3,' 12  2  4  4  4')
  719 FORMAT('      ')
  720 FORMAT(6D10.3)
  721 FORMAT(25I3)
  722 FORMAT('    1    1    0    1 1000 1000')
  723 FORMAT(1X,'(3(1X,7D10.3,/))')
  724 FORMAT(1X,'(/,1X,8D10.3)')
  725 FORMAT('  8  3  8  3',I3,' 12  8  7  7  7')
  726 FORMAT(1X,'(3(1X,7D10.3,/),3(1X,4D10.3,/))')
  727 FORMAT('  9  6  8  3',I3,' 12  8  7  7  7  4  4  4')
  728 FORMAT(1X,'(6(1X,6D10.3,/))')
  729 FORMAT(' 11  6  8  3',I3,' 12  8  6  6  6  6  6  6')
  730 FORMAT('  5  3  8  3',I3,' 12  5  8  8  8')
  731 FORMAT(1X,'(3(1X,8D10.3,/))')
  732 FORMAT('  6  6  8  3',I3,' 12  5  8  8  8  2  2  2')
  733 FORMAT(1X,'(3(1X,8D10.3,/),1X,6D10.3,/)')
  734 FORMAT('  7  6  8  3',I3,' 12  5  3  3  3  3  3  3')
  735 FORMAT(1X,'(6(1X,3D10.3,/))')
  736 FORMAT(' 10  3  8  3',I3,' 12  8  6  6  6')
  737 FORMAT(1X,'(3(1X,6D10.3,/))')
```

```
738 FORMAT(' 12  6  8  3',I3,' 12  8  5  5  5  5  5  5')
739 FORMAT(1X,'(6(1X,5D10.3,/))')
    RETURN
    END
    SUBROUTINE LFORM
    IMPLICIT REAL*8 (A-H,O-Z)
    COMMON/ONE/NEIDOS,NSTIF,YOUNG(10,12),NBOU,NBOUL,XB(16,6),INFC(16),
   1INF(26),KBOU(26),CCOR(30,3,3),X(3,3),N(6),NIN,INF1(26),NDF1,NDF2,
   2M1(8,30),M2(4,30),NF(26),KK
    DIMENSION P(12)
    K=INFC(16)
    DO 100 I=1,12
100 P(I)=0.
    DO 101 J=1,K
    READ(5,780) I,A
101 P(I)=A
    WRITE(6,781)(P(I),I=1,12)
780 FORMAT(I5,1D10.3)
781 FORMAT(6D10.3)
    RETURN
    END
    SUBROUTINE DA1
    IMPLICIT REAL*8 (A-H,O-Z)
    COMMON/ONE/NEIDOS,NSTIF,YOUNG(10,12),NBOU,NBOUL,XB(16,6),INFC(16),
   1INF(26),KBOU(26),CCOR(30,3,3),X(3,3),N(6),NIN,INF1(26),NDF1,NDF2,
   2M1(8,30),M2(4,30),NF(26),KK
    DIMENSION N1(30),N2(30),N3(30),N4(30),N5(30),N6(30),
   1N7(30),N8(30),N9(30),N10(30),N11(30),N12(30)
   2,MODE(12)
    DATA MODE/9,9,9,5,8,0,0,0,0,0,0,0/
    DATA N1/3,4,5,6,9,10,11,15,16,21*0/
    DATA N2/5,6,7,8,10,11,12,13,15,21*0/
    DATA N3/3,4,5,6,10,11,12,13,15,21*0/
    DATA N4/11,13,14,15,16,25*0/
    DATA N5/2,4,6,8,11,13,14,16,22*0/
    DATA N6/30*0/
    DATA N7/30*0/
    DATA N8/30*0/
    DATA N9/30*0/
    DATA N10/30*0/
    DATA N11/30*0/
    DATA N12/30*0/
    DO 210 LL=1,12
    KCK=MODE(LL)
    IF(KCK.LT.1) GO TO 211
    GO TO (301,302,303,304,305,306,307,308,309,310,311,312),LL
301 DO 401 I=1,KCK
401 M1(1,N1(I))=1
    GO TO 211
302 DO 402 I=1,KCK
402 M1(2,N2(I))=1
    GO TO 211
303 DO 403 I=1,KCK
403 M1(3,N3(I))=1
    GO TO 211
304 DO 404 I=1,KCK
404 M1(4,N4(I))=1
    GO TO 211
305 DO 405 I=1,KCK
405 M1(5,N5(I))=1
```

```
      GO TO 211
306 DO 406 I=1,KCK
406 M1(6,N6(I))=1
      GO TO 211
307 DO 407 I=1,KCK
407 M1(7,N7(I))=1
      GO TO 211
308 DO 408 I=1,KCK
408 M1(8,N8(I))=1
      GO TO 211
309 DO 409 I=1,KCK
409 M2(1,N9(I))=1
      GO TO 211
310 DO 410 I=1,KCK
410 M2(2,N10(I))=1
      GO TO 211
311 DO 411 I=1,KCK
411 M2(3,N11(I))=1
      GO TO 211
312 DO 412 I=1,KCK
412 M2(4,N12(I))=1
211 CONTINUE
210 CONTINUE
      RETURN
      END
      SUBROUTINE DA2
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/ONE/NEIDOS,NSTIF,YOUNG(10,12),NBOU,NBOUL,XB(16,6),INFC(16),
     1INF(26),KBOU(26),CCCR(30,3,3),X(3,3),N(6),NIN,INF1(26),NDF1,NDF2,
     2M1(8,30),M2(4,30),NF(26),KK
      DIMENSION N1(30),N2(30),N3(30),N4(30),N5(30),N6(30),
     1N7(30),N8(30),N9(30),N10(30),N11(30),N12(30)
     2,MODE(12)
      DATA MODE/9,9,9,9,8,0,0,0,4,4,0,0/
      DATA N1/3,4,5,6,9,10,11,15,16,21*0/
      DATA N2/5,6,7,8,10,11,12,13,15,21*0/
      DATA N3/3,4,5,6,10,11,12,13,15,21*0/
      DATA N4/5,6,7,8,11,13,14,15,16,21*0/
      DATA N5/2,4,6,8,11,13,14,16,22*0/
      DATA N6/30*0/
      DATA N7/30*0/
      DATA N8/30*0/
      DATA N9/5,6,7,8,26*0/
      DATA N10/2,4,6,8,26*0/
      DATA N11/30*0/
      DATA N12/30*0/
      DO 210 LL=1,12
      KCK=MODE(LL)
      IF(KCK.LT.1) GO TO 211
      GO TO (301,302,303,304,305,306,307,308,309,310,311,312),LL
301 DO 401 I=1,KCK
401 M1(1,N1(I))=1
      GO TO 211
302 DO 402 I=1,KCK
402 M1(2,N2(I))=1
      GO TO 211
303 DO 403 I=1,KCK
403 M1(3,N3(I))=1
      GO TO 211
304 DO 404 I=1,KCK
```

```
404 M1(4,N4(I))=1
    GO TO 211
305 DO 405 I=1,KCK
405 M1(5,N5(I))=1
    GO TO 211
306 DO 406 I=1,KCK
406 M1(6,N6(I))=1
    GO TO 211
307 DO 407 I=1,KCK
407 M1(7,N7(I))=1
    GO TO 211
308 DO 408 I=1,KCK
408 M1(8,N8(I))=1
    GO TO 211
309 DO 409 I=1,KCK
409 M2(1,N9(I))=1
    GO TO 211
310 DO 410 I=1,KCK
410 M2(2,N10(I))=1
    GO TO 211
311 DO 411 I=1,KCK
411 M2(3,N11(I))=1
    GO TO 211
312 DO 412 I=1,KCK
412 M2(4,N12(I))=1
211 CONTINUE
210 CONTINUE
    RETURN
    END
    SUBROUTINE DA34
    IMPLICIT REAL*8 (A-H,O-Z)
    COMMON/ONE/NEIDCS,NSTIF,YOUNG(10,12),NBCU,NBOUL,XB(16,6),INFO(16),
   1INF(26),KBOU(26),COOR(30,3,3),X(3,3),N(6),NIN,INF1(26),NDF1,NDF2,
   2M1(8,30),M2(4,30),NF(26),KK
    DIMENSION N1(30),N2(30),N3(30),N4(30),N5(30),N6(30),
   1N7(30),N8(30),N9(30),N10(30),N11(30),N12(30)
   2,MODE(12)
    DATA MODE/9,6,4,7,0,0,0,0,0,0,0,0/
    DATA N1/3,4,5,6,9,10,11,15,16,21*0/
    DATA N2/1,2,3,4,9,14,24*0/
    DATA N3/3,4,9,14,26*0/
    DATA N4/1,3,5,7,10,12,15,23*0/
    DATA N5/30*0/
    DATA N6/30*0/
    DATA N7/30*0/
    DATA N8/30*0/
    DATA N9/30*0/
    DATA N10/30*0/
    DATA N11/30*0/
    DATA N12/30*0/
    DO 210 LL=1,12
    KCK=MODE(LL)
    IF(KCK.LT.1) GO TO 211
    GO TO (301,302,303,304,305,306,307,308,309,310,311,312),LL
301 DO 401 I=1,KCK
401 M1(1,N1(I))=1
    GO TO 211
302 DO 402 I=1,KCK
402 M1(2,N2(I))=1
    GO TO 211
```

```
303 DO 403 I=1,KCK
403 M1(3,N3(I))=1
    GO TO 211
304 DO 404 I=1,KCK
404 M1(4,N4(I))=1
    GO TO 211
305 DO 405 I=1,KCK
405 M1(5,N5(I))=1
    GO TO 211
306 DO 406 I=1,KCK
406 M1(6,N6(I))=1
    GO TO 211
307 DO 407 I=1,KCK
407 M1(7,N7(I))=1
    GO TO 211
308 DO 408 I=1,KCK
408 M1(8,N8(I))=1
    GO TO 211
309 DO 409 I=1,KCK
409 M2(1,N9(I))=1
    GO TO 211
310 DO 410 I=1,KCK
410 M2(2,N10(I))=1
    GO TO 211
311 DO 411 I=1,KCK
411 M2(3,N11(I))=1
    GO TO 211
312 DO 412 I=1,KCK
412 M2(4,N12(I))=1
211 CONTINUE
210 CONTINUE
    RETURN
    END
    SUBROUTINE DAS
    IMPLICIT REAL*8 (A-H,O-Z)
    COMMON/ONE/NEIDOS,NSTIF,YOUNG(10,12),NBCU,NBOUL,XB(16,6),INFO(16),
   1INF(26),KBOU(26),COOR(30,3,3),X(3,3),N(6),NIN,INF1(26),NDF1,NDF2,
   2M1(8,30),M2(4,30),NF(26),KK
    DIMENSION N1(30),N2(30),N3(30),N4(30),N5(30),N6(30),
   1N7(30),N8(30),N9(30),N10(30),N11(30),N12(30)
   2,MODE(12)
    DATA MODE/9,9,9,3,5,6,5,8,0,0,0,0/
    DATA N1/3,4,5,6,9,10,11,15,16,21*0/
    DATA N2/5,6,7,8,10,11,12,13,15,21*0/
    DATA N3/3,4,5,6,10,11,12,13,15,21*0/
    DATA N4/10,11,15,27*0/
    DATA N5/7,8,12,13,16,25*0/
    DATA N6/3,4,5,6,10,11,24*0/
    DATA N7/11,13,14,15,16,25*0/
    DATA N8/2,4,6,8,11,13,14,16,22*0/
    DATA N9/30*0/
    DATA N10/30*0/
    DATA N11/30*0/
    DATA N12/30*0/
    DO 210 LL=1,12
    KOK=MODE(LL)
    IF(KCK.LT.1) GO TO 211
    GO TO (301,302,303,304,305,306,307,308,309,310,311,312),LL
301 DO 401 I=1,KCK
401 M1(1,N1(I))=1
```

```
      GO TO 211
  302 DO 402 I=1,KCK
  402 M1(2,N2(I))=1
      GO TO 211
  303 DO 403 I=1,KCK
  403 M1(3,N3(I))=1
      GO TO 211
  304 DO 404 I=1,KCK
  404 M1(4,N4(I))=1
      GO TO 211
  305 DO 405 I=1,KCK
  405 M1(5,N5(I))=1
      GO TO 211
  306 DO 406 I=1,KCK
  406 M1(6,N6(I))=1
      GO TO 211
  307 DO 407 I=1,KCK
  407 M1(7,N7(I))=1
      GO TO 211
  308 DO 408 I=1,KCK
  408 M1(8,N8(I))=1
      GO TO 211
  309 DO 409 I=1,KCK
  409 M2(1,N9(I))=1
      GO TO 211
  310 DO 410 I=1,KCK
  410 M2(2,N10(I))=1
      GO TO 211
  311 DO 411 I=1,KCK
  411 M2(3,N11(I))=1
      GO TO 211
  312 DO 412 I=1,KCK
  412 M2(4,N12(I))=1
  211 CONTINUE
  210 CONTINUE
      RETURN
      END
      SUBROUTINE DA6
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/ONE/NEIDOS,NSTIF,YOUNG(10,12),NBOU,NBOUL,XB(16,6),INFC(16),
     1INF(26),KBOU(26),COOR(30,3,3),X(3,3),N(6),NIN,INF1(26),NDF1,NDF2,
     2M1(8,30),M2(4,30),NF(26),KK
      DIMENSION N1(30),N2(30),N3(30),N4(30),N5(30),N6(30),
     1N7(30),N8(30),N9(30),N10(30),N11(30),N12(30)
     2,MODE(12)
      DATA MODE/9,9,9,3,5,6,9,8,4,4,0,0/
      DATA N1/3,4,5,6,9,10,11,15,16,21*0/
      DATA N2/5,6,7,8,10,11,12,13,15,21*0/
      DATA N3/3,4,5,6,10,11,12,13,15,21*0/
      DATA N4/10,11,15,27*0/
      DATA N5/7,8,12,13,16,25*0/
      DATA N6/3,4,5,6,10,11,24*0/
      DATA N7/5,6,7,8,11,13,14,15,16,21*0/
      DATA N8/2,4,6,8,11,13,14,16,22*0/
      DATA N9/5,6,7,8,26*0/
      DATA N10/2,4,6,8,26*0/
      DATA N11/30*0/
      DATA N12/30*0/
      DO 210 LL=1,12
      KCK=MODE(LL)
```

```
      IF(KOK.LT.1) GO TO 211
      GO TO (301,302,303,304,305,306,307,308,309,310,311,312),LL
  301 DO 401 I=1,KCK
  401 M1(1,N1(I))=1
      GO TO 211
  302 DO 402 I=1,KCK
  402 M1(2,N2(I))=1
      GO TO 211
  303 DO 403 I=1,KCK
  403 M1(3,N3(I))=1
      GO TO 211
  304 DO 404 I=1,KCK
  404 M1(4,N4(I))=1
      GO TO 211
  305 DO 405 I=1,KCK
  405 M1(5,N5(I))=1
      GO TO 211
  306 DO 406 I=1,KCK
  406 M1(6,N6(I))=1
      GO TO 211
  307 DO 407 I=1,KCK
  407 M1(7,N7(I))=1
      GO TO 211
  308 DO 408 I=1,KCK
  408 M1(8,N8(I))=1
      GO TO 211
  309 DO 409 I=1,KCK
  409 M2(1,N9(I))=1
      GO TO 211
  310 DO 410 I=1,KCK
  410 M2(2,N10(I))=1
      GO TO 211
  311 DO 411 I=1,KCK
  411 M2(3,N11(I))=1
      GO TO 211
  312 DO 412 I=1,KCK
  412 M2(4,N12(I))=1
  211 CONTINUE
  210 CONTINUE
      RETURN
      END
      SUBROUTINE DA7
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/ONE/NEIDCS,NSTIF,YOUNG(10,12),NBCU,NBOUL,XB(16,6),INFO(16),
     1INF(26),KBOU(26),COOR(30,3,3),X(3,3),N(6),NIN,INF1(26),NCF1,NCF2,
     2M1(8,30),M2(4,30),NF(26),KK
      DIMENSION N1(30),N2(30),N3(30),N4(30),N5(30),N6(30),
     1N7(30),N8(30),N9(30),N10(30),N11(30),N12(30)
     2,MCDE(12)
      DATA MCDE/9,11,10,0,0,0,0,0,0,0,0,0/
      DATA N1/3,4,5,6,9,10,11,15,16,21*0/
      DATA N2/5,6,7,8,10,11,12,13,14,15,16,19*0/
      DATA N3/2,4,6,8,10,11,12,13,14,16,20*0/
      DATA N4/30*0/
      DATA N5/30*0/
      DATA N6/30*0/
      DATA N7/30*0/
      DATA N8/30*0/
      DATA N9/30*0/
      DATA N10/30*0/
```

```
      DATA N11/30*0/
      DATA N12/30*0/
      DO 210 LL=1,12
      KOK=MODE(LL)
      IF(KOK.LT.1) GO TO 211
      GO TO (301,302,303,304,305,306,307,308,309,310,311,312),LL
  301 DO 401 I=1,KOK
  401 M1(1,N1(I))=1
      GO TO 211
  302 DO 402 I=1,KOK
  402 M1(2,N2(I))=1
      GO TO 211
  303 DO 403 I=1,KOK
  403 M1(3,N3(I))=1
      GO TO 211
  304 DO 404 I=1,KOK
  404 M1(4,N4(I))=1
      GO TO 211
  305 DO 405 I=1,KOK
  405 M1(5,N5(I))=1
      GO TO 211
  306 DO 406 I=1,KOK
  406 M1(6,N6(I))=1
      GO TO 211
  307 DO 407 I=1,KOK
  407 M1(7,N7(I))=1
      GO TO 211
  308 DO 408 I=1,KOK
  408 M1(8,N8(I))=1
      GO TO 211
  309 DO 409 I=1,KOK
  409 M2(1,N9(I))=1
      GO TO 211
  310 DO 410 I=1,KOK
  410 M2(2,N10(I))=1
      GO TO 211
  311 DO 411 I=1,KOK
  411 M2(3,N11(I))=1
      GO TO 211
  312 DO 412 I=1,KOK
  412 M2(4,N12(I))=1
  211 CONTINUE
  210 CONTINUE
      RETURN
      END
      SUBROUTINE DA8
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/ONE/NEIDOS,NSTIF,YOUNG(10,12),NBOU,NBOUL,XB(16,6),INFO(16),
     1INF(26),KBOU(26),COOR(30,3,3),X(3,3),N(6),NIN,INF1(26),NOF1,NOF2,
     2M1(8,30),M2(4,30),NF(26),KK
      DIMENSION N1(30),N2(30),N3(30),N4(30),N5(30),N6(30),
     1N7(30),N8(30),N9(30),N10(30),N11(30),N12(30)
     2,MODE(12)
      DATA MODE/13,13,12,5,7,10,13,0,4,7,6,6/
      DATA N1/3,4,9,10,13,14,15,16,17,20,25,26,27,17*0/
      DATA N2/5,6,11,12,13,14,15,17,18,20,28,29,30,17*0/
      DATA N3/7,8,9,10,11,12,13,14,15,16,18,19,18*0/
      DATA N4/3,4,5,6,17,25*0/
      DATA N5/4,6,8,10,12,14,17,23*0/
      DATA N6/3,4,5,6,15,16,17,18,19,20,20*0/
```

```
      DATA N7/2,4,6,8,10,12,14,15,16,17,18,19,20,17*0/
      DATA N8/30*0/
      DATA N9/3,4,5,6,26*0/
      DATA N10/2,4,6,8,10,12,14,23*0/
      DATA N11/3,4,9,10,13,14,24*0/
      DATA N12/5,6,11,12,13,14,24*0/
      DO 210 LL=1,12
      KCK=MCDE(LL)
      IF(KCK.LT.1) GO TO 211
      GO TO (301,302,303,304,305,306,307,308,309,310,311,312),LL
  301 DO 401 I=1,KCK
  401 M1(1,N1(I))=1
      GO TO 211
  302 DO 402 I=1,KCK
  402 M1(2,N2(I))=1
      GO TO 211
  303 DO 403 I=1,KCK
  403 M1(3,N3(I))=1
      GO TO 211
  304 DO 404 I=1,KCK
  404 M1(4,N4(I))=1
      GO TO 211
  305 DO 405 I=1,KCK
  405 M1(5,N5(I))=1
      GO TO 211
  306 DO 406 I=1,KCK
  406 M1(6,N6(I))=1
      GO TO 211
  307 DO 407 I=1,KCK
  407 M1(7,N7(I))=1
      GO TO 211
  308 DO 408 I=1,KCK
  408 M1(8,N8(I))=1
      GO TO 211
  309 DO 409 I=1,KCK
  409 M2(1,N9(I))=1
      GO TO 211
  310 DO 410 I=1,KCK
  410 M2(2,N10(I))=1
      GO TO 211
  311 DO 411 I=1,KCK
  411 M2(3,N11(I))=1
      GO TO 211
  312 DO 412 I=1,KCK
  412 M2(4,N12(I))=1
  211 CONTINUE
  210 CONTINUE
      RETURN
      END
      SUBROUTINE DA10
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/ONE/NEIDCS,NSTIF,YOUNG(10,12),NBCU,NBOUL,XB(16,6),INFO(16),
     1INF(26),KBOU(26),COOR(30,3,3),X(3,3),N(6),NIN,INF1(26),NDF1,NDF2,
     2M1(8,30),M2(4,30),NF(26),KK
      DIMENSION N1(30),N2(30),N3(30),N4(30),N5(30),N6(30),
     1N7(30),N8(30),N9(30),N10(30),N11(30),N12(30)
     2,MCDE(12)
      DATA MCDE/13,13,12,24,16,10,0,0,0,0,0,0/
      DATA N1/3,4,9,10,13,14,15,16,17,20,25,26,27,17*0/
      DATA N2/5,6,11,12,13,14,15,17,18,20,28,29,30,17*0/
```

```
      DATA N3/7,8,9,10,11,12,13,14,15,16,18,19,18*0/
      DATA N4/1,2,7,8,9,10,11,12,13,14,15,16,18,19,20,22,23,24,25,26,27,
     128,29,30,6*0/
      DATA N5/7,8,9,10,11,12,13,14,15,16,18,19,20,23,26,29,14*0/
      DATA N6/1,3,5,7,9,11,13,24,27,30,20*0/
      DATA N7/30*0/
      DATA N8/30*0/
      DATA N9/30*0/
      DATA N10/30*0/
      DATA N11/30*0/
      DATA N12/30*0/
      DO 210 LL=1,12
      KCK=MCDE(LL)
      IF(KCK.LT.1) GO TO 211
      GO TO (301,302,303,304,305,306,307,308,309,310,311,312),LL
  301 DO 401 I=1,KCK
  401 M1(1,N1(I))=1
      GO TO 211
  302 DO 402 I=1,KCK
  402 M1(2,N2(I))=1
      GO TO 211
  303 DO 403 I=1,KCK
  403 M1(3,N3(I))=1
      GO TO 211
  304 DO 404 I=1,KCK
  404 M1(4,N4(I))=1
      GO TO 211
  305 DO 405 I=1,KCK
  405 M1(5,N5(I))=1
      GO TO 211
  306 DO 406 I=1,KCK
  406 M1(6,N6(I))=1
      GO TO 211
  307 DO 407 I=1,KCK
  407 M1(7,N7(I))=1
      GO TO 211
  308 DO 408 I=1,KCK
  408 M1(8,N8(I))=1
      GO TO 211
  309 DO 409 I=1,KCK
  409 M2(1,N9(I))=1
      GO TO 211
  310 DO 410 I=1,KCK
  410 M2(2,N10(I))=1
      GO TO 211
  311 DO 411 I=1,KCK
  411 M2(3,N11(I))=1
      GO TO 211
  312 DO 412 I=1,KCK
  412 M2(4,N12(I))=1
  211 CONTINUE
  210 CONTINUE
      RETURN
      END
$SIG OFF
```

LIBRARY

DETAILS OF SUPPORTS
FOR
24 and 36 FACED DOMES

SCALE : 1CM = 5CM

FIG. 9.3.

SECTION D-D.

SECTION B-B

SECTION A - A

SECTION C - C

PLAN SCALE: 1cm = 5mm

DETAILS OF SUPPORTS FOR 24 and 36 FACED DOMES | FIG. 9.4. | G. MANOS

# SECTION A-A

SCALE: 1cm = 5mm

DETAILS OF SUPPORTS
FOR 24 and 36 FACED DOMES

G. MANOS

FIG. 9.5.

SCALE: 1cm = 5mm

G. MANOS

VIEW C – C

SCALE 1cm = 5mm

DETAILS OF SUPPORTS FOR 24 and 36 FACED DOMES

G. MANOS

FIG. 9.7.

PLAN

DETAILS OF SUPPORTS FOR 24 and 36 FACED DOMES

SECTION B-B

SCALE: 1cm = 5mm

DETAILS OF SUPPORTS FOR 24 and 36 FACED DOMES

G. MANOS

FIG. 9.6.

DETAILS OF PROPOSED SITE FOR

24 – 36 FACED DOME ROOF SECTIONS

SCALE : 1cm = 1M.

DATE : 14th OCT 71.

DRAWN : M.

FIG. 9.2.

PLAN OF PART OF UPPER GROUND FLOOR.

WEST ELEVATION.