Models and methods in hillslope profile morphometry

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MODELS AND METHODS IN HILLSLOPE PROFILE MORPHOMETRY

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Abstract

This thesis considers and evaluates mathematical models and methods of data analysis used in the quantitative study of hillslope profile form.

Models of hillslope profiles are brought together in a critical and comprehensive review. Modelling approaches are classified using five dichotomies (static/dynamic, deterministic/stochastic, phenomenological/representational, analytical/simulation, discrete/continuous).

Profile data to serve as examples were collected using a pantometer in a 100 km^2 square centred on Bilsdale in the North York Moors. Geomorphological interpretations put forward for this area include theses of profound lithological influence, polycyclic denudation history, proglacial lake overflow channels and profound cryonival influence.

Profile dimensions, profile shapes, angle and curvature frequency distributions and bedrock geology can be related via a fourfold grouping of profiles. The use of quantile-based summary measures and of a method of spatial averaging and differencing are advocated and illustrated.

Autocorrelation analysis of hillslope angle series appears to be of limited geomorphological interest, as autocorrelation functions tell a story of overall profile shape, which can be measured more directly in other ways. Problems of non-stationarity and estimator choice deserve greater emphasis.

Methods of profile analysis previously proposed by Ahnert, Ongley, Pitty and Young are all unsatisfactory. A method based on additive error partition and nonlinear smoothing is proposed as an interim alternative, and results related to bedrock geology.

An approach to model fitting is outlined which treats specification, estimation and checking in sequence. A power function due to Kirkby is used as an example and fitted to field data for components. The exercise works well if regarded as a minimum descriptive approach but much greater difficulties arise if process interpretation is attempted.

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Contents

Chapter 1

INTRODUCTION

Alain once said there are only two kinds of scholars: those who love ideas and those who loathe them. In the world of science these two attitudes continue to oppose each other; but both, by their confrontation, are necessary to scientific progress. One can only regret, on behalf of those who scorn ideas, that this progress, to which they contribute, invariably proves them wrong.

Jacques Monod, Chance and necessity, Ch. 8

1.1 Subject

1.2 Aims and structure

1.3 Presentation

1.1 Subject

This thesis lies in the field of hillslope profile morphometry. The term 'morphometry' is here used in a wide sense to include mathematical models and methods of sampling, measurement and data analysis used in the study of landsurface geometry. It is a convenient contraction of the more correct term 'geomorphometry' (cf. Tricart, 1947; Evans, 1972).

A hillslope profile is a line connecting a drainage divide at the crest of a hillslope with a drainage line at its base. It is a 'wiggly line' (Scheidegger, 1970, 7) in two dimensions (one vertical, one horizontal) corresponding to a maximum gradient path in three dimensions (one vertical, two horizontal).

The geometry of a hillslope profile is its form sensu stricto. Form sensu lato includes characteristics of soil, underlying bedrock, vegetation and climate. Hillslopes are modified by various processes operating on the hillslope and at its flanks and endpoints. Full understanding of hillslope form, process and development is to be sought in the interaction of form and process over time and space: this is the concern of hillslope geomorphology.

In attempting description and explanation of hillslope form, hillslope morphometry draws upon ideas and techniques of mathematics and statistics. It is convenient to distinguish various classes of problems arising in hillslope morphometry (illustrated here by examples from the field of modelling).

(i) Methodological problems (e.g. what degree of simplification is permissible?)

(ii) Theoretical problems (e.g. what are the geomorphological grounds for this model?)

(iii) Technical problems (e.g. what is the mathematical basis of this model?)

(iv) Empirical problems (e.g. is a particular model a realistic description of a given field profile?)

All these problems are important and need to be considered carefully.

Any choice of research problem is usually a reflection of an investigator's interests and competence and his perception of the importance of problems within a discipline. The choice made here reflects a strong interest in methodological, theoretical and technical questions and a belief that empirical investigation is worthless without the backing of sound ideas and methods. Contrary to the opinions of some fieldworkers in geomorphology, such backing is not available unless specialist investigation is undertaken into methodological, theoretical and technical questions.

Discussion of this issue often slides into a puerile debate over the relative merits of 'armchair' and 'muddy boots' approaches in geomorphology. The simple answer is that such approaches are complementary, not competing, and that it is not a matter of one approach being superior and the other being inferior. But this still leaves room for argument over details (exactly how do these approaches complement each other?), and the details are not a matter for abstract discussion: antagonists must start to consider specific circumstances.

1.2 Aims and structure

The aims of this thesis are as follows.

(i) To provide critical and comprehensive reviews of work in modelling and data analysis in hillslope morphometry, concentrating particularly on continuous models (Chs. 2, 3) and profile analysis (Ch. 8). It is felt that the absence of such reviews is a barrier to progress, and that as well as hindering communication within each field, it impedes understanding of models and methods by workers outside.

(ii) To place procedures on a firm mathematical basis and to evaluate their practical utility, concentrating particularly on autocorrelation analysis (Ch. 7), profile analysis (Ch. 8) and model curve fitting (Ch. 9). In each case it is shown that existing geomorphological practice has considerable technical shortcomings; alternative procedures are explained and evaluated empirically.

(iii) To consider the <u>empirical role</u> of hillslope morphometry in geomorphology. This entails consideration of the hypotheses put forward in the literature for the field area (Ch. 4) and analysis of the implications of morphometric results for these hypotheses (Chs. 6, 8, 9). The aim is not to solve field problems or to provide new interpretations of the geomorphology of the field area: that would require much more work on forms, deposits and processes than has been possible.

(iv) To set the field of hillslope morphometry within a <u>methodological and theoretical context</u>, particularly by relating ideas on continuous models to geomorphological theory, the philosophy of science, and the methodology of the natural and environmental sciences (Ch. 2). Here scholarship can provide a clear formulation of genuine problems and a dismissal of pseudoproblems.

The chapter structure is as follows.

Chapters 2 and 3 give a review of hillslope profile modelling. Chapter 2, on principles, is almost entirely non-mathematical: it examines methodological and theoretical issues at length. This chapter should serve as a nontechnical introduction for the general geomorphologist. Chapter 3 gives a detailed and comprehensive review of the models which have been proposed; it is presented in a unified notation, and includes some new results.

Chapter 4 supplies background information on the field area, a 100 km^2 square centred on Bilsdale in the North York Moors. It discusses the hypotheses which have been put forward and identifies the problems which may be attacked using morphometry. Chapter 5 briefly describes the sampling and measurement procedures used to collect hillslope profile data for this study. Chapter 6 describes the field profiles, outlining the variations which exist in profile form and in angle and curvature frequency distributions. Some technical innovations are employed.

Chapters 7, 8 and 9 place particular procedures on a firm mathematical basis and evaluate their practical utility. Autocorrelation analysis (Ch. 7) is a relatively new technique in hillslope geomorphology: its usefulness has not so far been examined systematically. Profile analysis (Ch. 8) is a field with several competing methods: an attempt is made to separate the wheat from the chaff. Model curve fitting (Ch. 9) has not received careful attention to date: even the relatively simple case discussed here, a one-parameter nonlinear model, presents some challenging problems.

Chapter 10 presents the conclusions of the thesis.

1.3 Presentation

Figures and Tables are not distinguished but are both regarded as Exhibits (cf. Tukey, 1977). This practice flouts convention, but saves separate labelling systems and encourages hybrid displays. Exhibits are labelled alphabetically within chapters (e.g. 2A, 2B), whereas chapters, sections and subsections are labelled numerically using a common hierarchical notation.

Algebraic notation is intended to be consistent within individual chapters and is collated at the end of each chapter. It has regrettably not been found possible to use a single system of notation throughout the thesis.

Appendix I lists profile data and Appendix II gives brief descriptions of computer programs.

Chapter 2

PRINCIPLES OF MODELLING HILLSLOPE PROFILES

About thirty years ago there was much talk that geologists ought only to observe and not theorise; and I well remember someone saying that at this rate a man might as well go into a gravel-pit and count the pebbles and describe the colours. How odd it is that anyone should not see that all observation must be for or against some view if it is to be of any service.

Charles Darwin in a letter to Henry Fawcett, 1861,quoted by P. B. Medawar, Induction and intuition in scientific thought, p. 11.

2.1 Geomorphology and hillslope morphometry

The science of geomorphology studies landforms and related processes, and aims to describe and explain the form of the land surface. Ideally such description and explanation should be rooted in a systematic theory which presents a coherent account of form, process and development.

Three systematic approaches are especially noteworthy:

(i) the land surface viewed as a continuous rough surface (Evans, 1972; Mark, 1975)

(ii) the land surface viewed as a hierarchy of drainage basins (Leopold et al,1964; Chorley, 1969; Gregory and Walling, 1973; Douglas, 1977)

(iii) the land surface viewed as a set of hillslopes corresponding to maximum gradient paths between drainage divides and drainage lines (Carson and Kirkby, 1972; Young, 1972).

These approaches all have great value. The idea of a continuous rough surface is the most general, whereas drainage basins and hillslopes are most readily identified where fluvial (slope and stream) processes are dominant. On the other hand, they are functioning systems as well as natural geometric entities. Hillslopes are conveniently simpler than drainage basins; however, reduction to one horizontal dimension loses the effects of plan curvature. Most of the landsurface is composed of valley slopes (Young, 1972, 1) and so geomorphology 'is by necessity mainly a study of slopes' (Ahnert, 1971,3).

The place of hillslope study within geomorphology, and the place of morphometry in hillslope geomorphology, deserve more detailed examination. Recent diversification of approaches within geomorphology has often seemed tantamount to disintegration of the discipline. Geomorphology is in motley disarray, a 'bandwaggon parade', to use Jennings' (1973) picturesque expression. There has been much concern that different 'schools' in geomorphology fail to understand one another. Indeed, Chorley (1967, 59) asked whether 'the study of landforms still exists as a discrete scholarly entity' and commented upon 'the inability of workers to identify broad common objectives of even the most general character, or even to communicate to one another their mutual objections'.

Hence it is important to outline a view of hillslope morphometry as part of a 'pluralist' geomorphology in which the existence of different approaches is recognised and resolved (Butzer, 1973). Butzer identified four major directions of primary research in the discipline:

(i) Quantitative study of geomorphological processes
(ii) Quantitative analysis of landforms
(iii) Quantitative and qualitative study of sediments
(iv) Systematic, regional studies of complex landform evolution through time and in the wake of environmental change.

Research may also be classified according to the geomorphological systems which are of primary interest (e.g. hillslopes, drainage basins, topographic surfaces). A cross-classification yields a simple matrix representation of contemporary geomorphology which allows hillslope morphometry to be precisely located within current research (2A). A glance at this diagram shows that hillslope morphometry can be linked to other approaches to hill-slopes (row linkages), and to other branches of morpho-metry (column linkages).

This matrix representation reflects a simplified yet structured view of the important directions of current research. It may seem entirely uncontroversial. Nevertheless, in stressing approaches based on replicated systems (such as hillslopes or drainage basins) this view to some extent stands opposed to a strong tradition in geomorphology, which concentrates on interesting 'features' and by comparison neglects supposedly 'featureless' areas. This bias has had unfortunate consequences: '. . . the geomorphologist at the present rate of knowledge can often say remarkably little by way of description or explanation about an ordinary "featureless" rolling landscape' (Lewin, 1969, 84). However, an allegedly featureless fluvial landscape can be discussed in terms of its constituent hillslopes and drainage basins. A concern for atypical and striking forms should be supplemented by an analysis of ordinary landscapes, which is of equal interest and importance.

The analyses considered in this thesis are <u>morphometric</u>, involving the use of mathematical models and

Topic of interest / System of interest	Processes	Morphometry	Sediments	Evolution	etc.
Hillslopes		*			
Streams					
Basins					
Surfaces					
etc.					

2A. <u>Hillslope morphometry within contemporary geomorphology</u>

of methods of sampling, measurement and data analysis (cf. the definition in Ch. 1.1 above). These analyses must not be unreflective, so this chapter continues with discussions of philosophical issues, approaches and major geomorphological problems in modelling. (Questions of sampling, measurement and data analysis receive attention in Chapters 5 to 9 below).

2.2 Philosophical issues in modelling

2.2.1 Critical rationalism and the inevitability of theory

Discussion of philosophical issues is rare in geomorphology, and the discipline is dominated by what Reynaud (1971, 95-9) called 'the spontaneous philosophy of geomorphologists', an antitheoretical empiricism underlain by methodological principles which are frequently implicit rather than explicit. In contrast this thesis adopts the viewpoint of critical rationalism set out by Popper (Popper, 1972, 1976; Medawar, 1969; Magee, 1973). According to this view, it is best to formulate ideas as clearly as possible; and to subject them to severe criticism. This applies, for example, at a methodological level (such as when a methodological principle is in question) and at an empirical level (such as when a hypothesis is in question). It is interesting to note the broad similarity of Gilbert's methodological ideas (cf. Gilbert, 1886; Gilluly, 1963; Kitts, 1973).

Such a view is inevitable once one realises that it is impossible to work without assumptions either at a methodological level or at an empirical level. There are often occasions when the assumptions are trivial or uncontroversial, but in general it is valuable to review the assumptions made in a piece of work, and to assess their validity.

Some theory about the world is inevitable. Even if we attempt merely to observe, we use implicit theory about what is noteworthy, and are influenced by preconceptions. '. . . The "facts" that enter our knowledge are already viewed in a certain way and are, therefore, essentially ideational . . . Experience arises <u>together</u> with theoretical assumptions <u>not</u> before them, and an experience without theory is just as incomprehensible as is (allegedly) a theory without experience' (Feyerabend, 1975, 19, 168).

2.2.2. <u>Key terms discussed</u>

Interest in philosophical matters is often associated with a preoccupation with questions of definition, meaning and terminology. It is, however, generally possible to avoid arguments over definitions. '. . . Since all definitions must use undefined terms, it does not, as a rule, matter whether we use a term as a primitive term or as a defined term' (Popper, 1972, 58; cf. Popper, 1966, 9-21; Geach, 1976, Ch. 9). Nevertheless, some key terms need to be discussed in detail, because they may be unfamiliar, or because they are in practice used in different senses.

One valuable distinction is between <u>epistemological</u> and <u>ontological</u> questions. Ontological questions are about the world, its character and its history. Epistemological questions are about knowledge of the world, or more precisely about the validity of claims to knowledge of the world. This simple distinction can help to clarify controversies (cf. Watson, 1969; Levins, 1970; Morales, 1975), as will be seen below.

The term _theory_ is here used in a general way to denote a set of ideas used in description and explanation. These ideas may be implicit or explicit; and if explicit, expressed verbally or mathematically.

The term _system_ denotes 'a collection of components which are either acting upon other components, being acted upon, or mutually interacting' (Birtwistle _et al_, 1973, 13): that is, a collection of components which are interrelated in some way (cf. Margalef, 1968, 2; Chapman, 1977, 79-82). In particular, 'system' is usually to be understood as 'real-world system of interest' (Wilson, 1972, 31): in this thesis, the systems of interest are hillslopes or fluvial systems at smaller or larger scales.

The term _model_ denotes ' a simplified picture of the system' (Birtwistle _et al_,1973, 14). In particular, 'model' is usually to be understood as 'formal representation of a theory' (Wilson, 1972, 32); informal schemes, whether verbal, graphical or implicit, are not here considered as models.

These definitions contrast with usages common in geomorphology. Many workers use the term 'model' in a catholic sense (especially Chorley, 1967; Thornes and Brunsden, 1977). Model and system concepts are often confused (tangled up) or conflated (taken as one) in the geographical and geomorphological literature (e.g. Harvey, 1969, Ch. 23; Chorley and Kennedy, 1971; Carson and Kirkby, 1972, Chs. 1 and 2; Andrews, 1975, 8-10; Sugden and John, 1976, 4). It is true that according to the definitions given above models themselves may be regarded as systems (Birtwistle et al, 1973, 15) but such conflation is not encouraged in this thesis; a sharp distinction is made between the two concepts. A system is essentially a part of the physical world (an ontological entity, in a sense) whereas a model is essentially a human creation (an epistemological entity, in a sense).

Both 'model' and 'system' are used in a great variety of senses both within and outside geomorphology, and there seems to be no very strong reason for regarding any particular senses as essentially correct. Any choice is admittedly arbitrary. That made here at least seems simple and straightforward, and it does allow various methodological and empirical issues to be formulated clearly. In several ways, one may ask how 'good' a model is as a representation of a given system. Even formulating such questions is greatly aided by a sharp distinction between 'model' and 'system'.

If geomorphology aims to explain the form of the land surface, then it is necessary to consider the criteria for an ideal explanation. This is a large issue, much debated both in philosophy and the sciences. Two criteria deserve special emphasis (Popper, 1972, 191-3). Firstly, an 'explanation' should be independently testable and not ad hoc or circular. Secondly, it should relate a set of conditions to one or more universal laws and initial conditions. These criteria remain as logical ideals even though many supposed explanations in geomorphology fail to satisfy them. (The character of the 'universal laws' invoked in geomorphology will be discussed below).

2.2.3. Why use mathematics?

It is reasonable to ask why the use of mathematics, an abstract and formal exercise, should be of value in understanding geomorphological systems. Why should a string of symbols on a sheet of paper have any relevance to the behaviour of masses of soil, rock and water? This apparently naive question raises some deep philosophical problems, yet there do not seem to be more than a few scattered comments upon the issue in the geomorphological literature. It is tacitly assumed either that the use of mathematics is futile, irrelevant or pretentious, or that as a standard scientific procedure it needs little or no justification.

It is difficult to find a satisfactory answer to this question, and some alleged solutions are unconvincing. Atiyah (1976, 292) suggested that mathematics may be viewed as the science of analogy, and that 'the widespread applicability of mathematics in the natural sciences, which has intrigued all mathematicians of a philosophical bent, arises from the fundamental role which comparisons play in the mental process we refer to as "understanding"'. This suggestion appears to be little more than a reformulation of the problem, for equally striking questions immediately arise: how and why are comparisons involved in understanding?, are some kinds 'better' than others, and if so, in what sense and why? And so on.

Hence a detailed consideration of the actual and possible roles of mathematics in geomorphology is in order. In what follows, mathematics is taken as 'given', a set of formal languages which may be of use in description or explanation. But it should be stated that the character of mathematics is itself a matter for considerable philosophical dispute (e.g. Körner, 1960, 1971; Lakatos, 1962, 1976a, 1976b; Kreisel, 1965; Steiner, 1975).

One fact frequently allowed to confuse the issue is that geomorphologists often find mathematical notation unfamiliar, puzzling or disturbing. Many would feel happy with the string of symbols 'the slope is steep' but not with the string '$\left|\frac{\partial z}{\partial x}\right| > \gamma$'. (The two could be construed as equivalent with conventional interpretations of ∂, $|\ |$ and $>$ and appropriate interpretations of z, x and γ). The unfamiliarity of algebraic notation is an important educational and psychological issue, and the mathematical weakness of most geomorphologists has far from trivial consequences , but neither has much bearing on the philosophical issue of the value of mathematics as a mode of expression.

Naturally it would be inappropriate to use mathematics in situations where ordinary language serves the purpose as well or better. The example just given might seem an excellent case in point. Notice, however, that as soon as we wish to say how steep a particular slope is, or to define 'steep', mathematics becomes the appropriate medium: numerical answers are required for these questions. Moreover, the customary arguments from abuse or misuse are overplayed: arguing generally against mathematical applications in geomorphology on the grounds that some past applications have been mistaken or misleading (which is certainly true) is logically on a par with blaming a weapon for a crime. No approach bears a guarantee of success, and it is absurd to ask for one. There are many cases, actual or potential, in which 'the mathematical formalism may be hiding as much as it reveals' (Schwartz, 1962, 360). Hence it is not a matter of asking for justification of a mathematical approach, but of seeking a clear view of how and why mathematics might be useful.

Another common misconception is the view that the usefulness of linguistic symbolism is obvious while the usefulness of mathematical symbolism requires explanation. Close examination shows that both kinds of symbolism raise deep problems (cf. Craik, 1967, Ch. 5; Skellam, 1972, 15). It is thus illegitimate to object to mathematical symbolism as symbolism unless we also object to linguistic symbolism.

It seems likely that mathematics might be useful to geomorphology in a variety of ways, nor is such multiplicity particularly surprising: it also applies to ordinary language.

(i) The idea of mathematics as a science of analogy or pattern, noted by Atiyah (1976), is certainly of importance. 'Any pattern we see in the universe will be one for which a mathematical treatment is possible; conversely, whenever a new mathematical insight occurs, we are able to recognise new kinds of patterns. If any of these occur in nature, we have a totally unexpected application of the theory. And this is how mathematics gets its power; for a pattern which is hard to recognise in one area may be obvious in another. By taking inspiration from the second we discover the existence of the first' (Anon, 1973, 658). Naturally the application of analogies must be careful: the relevance of an attractive analogy can easily be exaggerated and analogies require independent testing (cf. Wilson, 1969).

(ii) Ordinary language is essentially topological and it is possible to give a verbal account of the important variables in a system and indicate the connections and relationships between them. Conversely many of the topological 'box-and-arrow' diagrams in the geomorphological literature could be translated into words without appreciable loss of content. However, mathematical representations allow algebraic and arithmetic specification of relationships which indicate the form and size of connections (cf. Nelder, 1972, 368) and are thus more informative. It is more informative to specify that a hillslope obeys a power function than to specify that it is curved, and more informative still to specify parameter values.

(iii) An algebraic argument has the merit that in principle assumptions are made explicit, derivations are shown, and the character and form of solutions are made plain. The logical structure of the argument is exposed to public scrutiny, to criticism and to testing (cf. Ziman, 1968, 45). In contrast, a verbal argument may be astonishingly difficult to evaluate.

(iv) An entirely algebraic argument divorced from empirical evidence is usually regarded with suspicion. Nevertheless, it is important to note the possibility that mathematics may be used as 'a tool for gaining qualitative insight into real phenomena' (Smith and Bretherton, 1972, 1507). If assumptions made are demonstrably weak, or if results are qualitatively stable under perturbations of the axioms, then it may be possible to produce 'robust theorems' (cf. Schwartz, 1962, 357; Levins, 1970, 76: Levin, 1975, 785). Alternatively there are methods for investigating the qualitative properties of partially specified systems (May, 1973; Levins, 1974).

(v) One point of view goes beyond this and stresses the logical character of models, used as means of deriving the consequences of initial assumptions. Their empirical realism (truth or falsity) is regarded as secondary. Lewontin (1963, 224) argued that models are not contingent, but analytic; models should never be said 'to be true or false in an empirical sense'. 'A model is essentially a calculating engine designed to produce some output for a given input . . .

Models cannot produce really "new" knowledge, but can only demonstrate what is entailed by the theory from which the model is built' (225). Similarly Morales (1975, 335) deplored 'the mistake of using models as mirrors of reality rather than as heuristic constructs'. Models tend to be 'attacked, or defended, as ontologically true or false rather than epistemologically useful and insightful or obfuscating and ineffectual' (337). Neither Lewontin nor Morales advocated empirical irrelevance: they merely requested a clear view of models in which their logical character is appreciated.

(vi) It is sad but true that qualitative impressions obtained in the field may be seriously misleading as well as imprecise. '. . . A property which seems perfectly apparent, or an "obvious" relation of cause and effect, may upon careful measurement and analysis prove to be exactly the reverse of the "apparent" or the "obvious"' (Leopold et al, 1964, 8). Thus in 1712 it was generally believed that Northamptonshire was the highest county in England: reliable methods of height measurement were only developed later (Porter, 1977, 223). Recent psychological work on geological observation has undermined the widely-held belief that the impressions of experienced investigators are objective (Chadwick, 1975, 1976). Hence the value of quantitative measurement is great.

(vii) Where some geomorphological response is the result of several factors, it is important to determine their relative importance. This can only be done by quantitative methods (van Hise, 1904, 605; Jeffreys, 1918, 179).

(viii) A precise statement can be more easily refuted than a vague one, and can therefore be better tested (Popper, 1972, 356; cf. Ahnert, 1971, 11). In certain applications, a high degree of precision may be spurious or unnecessary, as has often been pointed out, but such superfluity needs to be demonstrated rather than asserted.

2.2.4. Modelling and simplification

While the actual or potential role of mathematics in geomorphology has received little detailed examination, issues of simplification have been discussed more frequently. In particular, it has often been alleged that models are unduly simplified. Consider, for example, the following remarks.

(i) 'Other scholars . . . have sought to contribute to the resolution of morphological problems by a mathematical treatment. Given the complexity of phenomena such a treatment can only be applied to very simple forms' (Hol, 1957, 198; translated from French).

(ii) 'The direct attack by mathematical methods would seem to offer very limited chances of success. Dealing as we inevitably are with infinitely variable mixtures of solids, liquids and gases, it is manifest that the parameters in our imagined equations will not be constant, but themselves unmanageably variable' (Wooldridge, 1958, 32).

(iii) '. . . Slope profiles are usually too irregular to be described in their entirety by formulae' (Pitty, 1970, 18).

(iv) 'Quantitative models almost invariably lead to simplification and to an increasing distance to reality beyond permissible limits' (Büdel, 1975, 2).

Clearly such charges need to be considered with care. While much will depend on the details of geomorphologist's purpose and geomorphological system, some general remarks on simplification are in order.

In the first place, geomorphological systems are widely recognised to be complex. Thornbury (1954, 21) regarded the principle that 'complexity of geomorphic evolution is more common than simplicity' as a 'fundamental concept' of geomorphology, while Schumm (1973, 1977; Schumm and Parker, 1973) has even proposed a principle of complex response. Yet it would be wrong to overemphasise the degree of complexity found in geomorphological systems. Milovidova (1970) showed how few of the logically possible landform types actually occur in a given region; Connelly (1972) put forward a similar view supported by results obtained with entropy measures from altitude data.

In any case, it is desirable that models should be simple. If a model is to be of any use, it must be a simplified representation of the system of interest. An exact copy would be useless in explanation (Hanson, 1971, 81). The desirability of simplicity is encapsulated in a celebrated logical maxim known as Ockham's razor; its correct version, is not, however, as sharp as is often supposed: 'frustra fit per plura quod potest equaliter fieri per pauciora' or '. . . it is vain to do by more what can equally be done by fewer' (Leff, 1975, 35; cf. Anderson, 1963, 176; Skellam, 1972, 27).

In hillslope geomorphology, simplification is justifiable in the many instances when the interest is 'in the _average_ properties of a hillside, not in all of its intricate details' (Scheidegger, 1970, 3; cf. Kirkby, 1974, 2; Mizutani, 1974, 4). Naturally, there are occasions on which the details are of interest, but the point in this context is that in modelling profiles the aim is merely to account for the overall form of the profile, and not for all the infinite (_sic_) variability (_pace_ Wooldridge), nor for the pròfile in its entirety (_pace_ Pitty).

Unfortunately, the tail can also wag the dog. Simplicity may be forced upon the modeller by the need for analytical tractability (Schwartz, 1962; Klemeš, 1974). 'For complicated phenomena like surface weathering and erosion it is impractical to include more than a few of the known physical effects. An attempt to do so would lead to a cumbersome model; indeed, a detailed study of the fluid motion alone would be possible only in highly idealised situations. It seems preferable instead to use as simple a model as possible, for if we describe only the overall macroscopic behaviour of the fluid we can rely largely on conservation laws, and these retain their validity even though the details of the small-scale motion are unknown' (Luke, 1974, 4035).

It is disturbing to find that opponents of modelling often regard it as unnecessary to substantiate charges of oversimplification. The possibility that models might be useful has been dismissed _ex cathedra_ by eminent historical and climatic geomorphologists like Wooldridge and Büdel.

Yet it is not clear that denudation chronologies and climatic-geomorphological regionalisations are self-evidently innocent of oversimplification: they are manifestly simplified schemes which deliberately omit a multitude of details. Furthermore, apparently simple models may have very complicated behaviour (May, 1976a): it may be difficult to assess the simplicity of a model itself.

Models should not be subjected to summary trial and execution on a charge of oversimplification. It is necessary to hear the case at length.

2.2.5 Models, domains and testing

The domain can be defined as the set of systems which satisfy the assumptions underlying the model (cf. Cohen, 1966, 66 on 'domain of applicability'; Harvey, 1969, 89; Scheidegger, 1970, 150 on 'domain of application'). Strictly speaking, a model should be tested empirically on a system which belongs to the domain. However, systems belonging to the domain may well be very rare (or nonexistent) if the assumptions behind a model are at all idealised. In practice, as the geographer Guelke (1971, 47) remarked ironically, the discrepancies between a model and reality have often been 'explained' by demonstrating that the test situation was not applicable to it. Guelke was rightly critical of such practice: 'Whatever the difficulties that a scientific investigator might encounter in attempting to test his models he may not regard the logical part of them as beyond criticism because the ideal conditions for which they were constructed do not exist'.

There are further difficulties. In the first place, it may be difficult to identify systems belonging to the domain if we have no independent evidence on the mode of hillslope development. Clearly it would be circular to infer past processes from present forms and then use those inferences to identify an appropriate model. Secondly, the idea of simplification is linked with that of approximation: we would rarely expect an exact fit to model predictions. This still leaves the important question of deciding how much discrepancy is acceptable. In practice this may come down to working with a poor model on the grounds that it is the best we have.

The possibility of equifinality is important in any discussion of testing. Equifinal systems are such that particular system states may be reached in different ways (von Bertalanffy, 1950). In geomorphological terms, a particular morphology may be produced by different processes or different combinations of processes. Equifinality is now widely accepted in geomorphology (cf. Chorley, 1962, 1964; Cooke and Warren, 1973; Cooke and Reeves, 1976), sometimes in the guise of 'convergence' (Wilhelmy, 1958; Twidale, 1971; Douglas, 1976; Gossmann, 1976) or 'homology' (King, 1953; Pitty, 1971), although it is still occasionally ignored (for example see Scheidegger, 1970, 120, 143).

The main methodological implication of equifinality is that its existence limits inference about formative processes from morphological evidence. This applies to mathematical explanations as well as to verbal explanations (Leopold et al, 1964, 500; among others). For example, a good fit obtained with a model based on a particular set of process assumptions does not necessarily imply that the assumptions are realistic. However, it does seem likely that equifinality is not an absolute condition but relative both to the degree of precision and to the set of descriptors which are used. Be that as it may, equifinality is certainly a major difficulty in model testing.

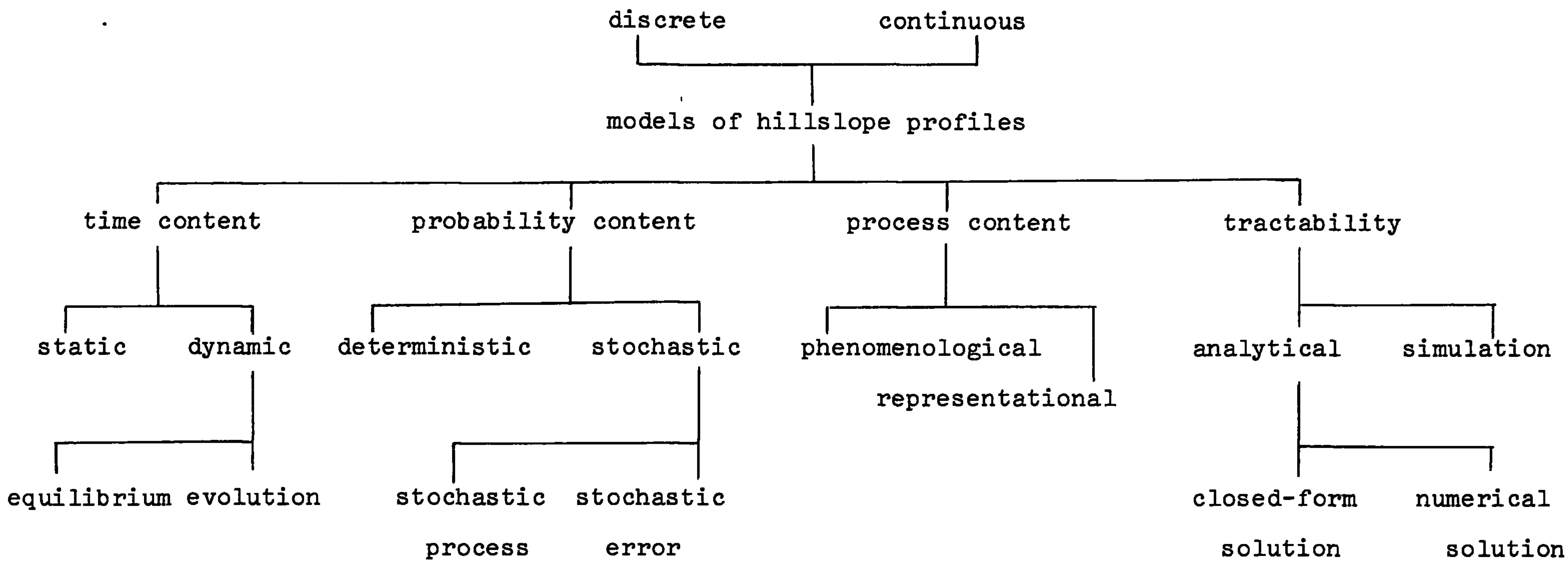
Identifying the domain, deciding on the discrepancy acceptable and equifinality of form are all important problems in testing, which must be combined with a realisation that highly complex models tend to be both unhelpful and intractable. As has been pointed out, however, a different view is possible, in which the empirical realism of a model is regarded as secondary, and its main role is that of a logical vehicle for deriving the necessary consequences of initial assumptions. There is much to be said for such a point of view. Ultimately, however, the characteristic that distinguishes a scientific theory from a mathematical argument is its applicability to empirical reality.

2.3 Approaches to modelling

A variety of approaches have been used in modelling hillslope profiles. These can be classified in a simple way using five dichotomies (2B).

(i) Static/dynamic: according to time content (2.3.1.)

(ii) Deterministic/stochastic: according to probability content (2.3.2.)



2B Classification of hillslope profile models

(iii) Phenomenological/representational: according to process content (2.3.3.)

(iv) Analytical/simulation: according to tractability (2.3.4.)

(v) Discrete/continuous: models treating hillslopes as combinations of discrete components and models treating hillslopes as continuous curves (Cox, 1977c). References to reviews of discrete models are given in Ch. 3.1 and a discussion of dividing profiles into discrete components is given in Ch. 8.

2.3.1. Static and dynamic models

The difference between static and dynamic models is simple: only dynamic models include elapsed time as a variable. Static models of hillslope profiles predict form at one time, often without any process implications. Dynamic models of hillslope profiles predict either some kind of invariant or equilibrium form; or a series of successive forms, that is, the evolution of the hillslope system. The idea of a static model is straightforward, but ideas of equilibrium and evolution require detailed examination.

The choice between equilibrium and evolutionary models depends ideally on whether the time it takes for a system of interest to reach equilibrium is short or long relative to the time span being considered (Kirkby, 1974, 3; cf. Schumm and Lichty, 1965). However, the decision must generally be taken in ignorance, since knowledge of reaction and relaxation times (Wolman and Gerson, 1978; Graf, 1977) and of rates of hillslope retreat (Young, 1974) is still fragmentary.

Ideas of equilibrium have a long history in geomorphology. Although most recent work follows the pioneer studies of Strahler (1950), Leopold and Maddock (1953), Hack (1960) and Chorley (1962), these authors acknowledge the influence of Gilbert (1880, 1909) (on whom cf. Pyne, 1976). Moreover, ideas of steady state formed one strand in classical 'uniformitarianism', especially that of Lyell (cf. Hooykaas, 1970; Rudwick, 1970, 1971; Gould, 1975, 1977; Porter, 1977).

With such a background, it is not surprising that 'equilibrium' has appeared in geomorphology in a variety of different guises (cf. Young, 1970a; 1972, 96-102 on concepts of equilibrium, grade and uniformity in hillslope geomorphology; Chorley and Kennedy, 1971, 201-3 on kinds of equilibrium in physical geography; Tricart and Cailleux, 1972 on morphoclimatic equilibrium; Statham, 1977, Ch. 1 on mechanical and chemical equilibrium in geomorphology). The issue is further complicated by a certain amount of confusion and disagreement over the meaning of some key terms: witness the treatment of 'dynamic equilibrium', 'equilibrium', 'quasi-equilibrium', 'steady-state' and 'time-independence' in the texts of Easterbrook (1969, 428), Small (1970, 189), Pitty (1971, 70), Gregory and Walling (1973, 18-19), Garner (1974, 29), Ruhe (1975a, 86), Butzer (1976, 81-2), Twidale (1976, 424), Douglas (1977, 227-8), Rice (1977, 222-4) and Schumm (1977, 4-5). No clear consensus emerges from these texts about whether these terms denote distinct conditions, nor about how they may be distinguished. Moreover, these texts indicate that the majority of geomorphologists understand equilibrium to be essentially a matter of 'adjustment' or 'balance', and that geomorphologists often use equilibrium terms in an inexact and metaphorical fashion.

One way to cut through such confusion, disagreement and inexactitude is to return to primary sources in a bid to isolate the fundamental ideas. The classic paper by Hack (1960) is the primary source on 'dynamic equilibrium' theory. Two strands of thought are intertwined in this paper. One is critical, both of the Davisian theory in particular, and more generally of historicism in geomorphology, the idea that explanations in geomorphology <u>must</u> be historical explanations (an idea very much alive: cf. Büdel, 1975). The other is constructive: the dynamic equilibrium theory itself. It is curious, and unfortunate, that while the first aspect has often been discussed, the second has largely escaped critical evaluation.

The central idea in the dynamic equilibrium theory is that of constant form. It is hypothesised (according to Hack 'It is assumed') that 'within a single erosional system all elements of the topography are mutually adjusted so that they are downwasting at the same rate' (Hack, 1960, 85).

The hypothesis of constant form could be tested in principle by measuring rates of downwasting. Even if the hypothesis were upheld, however, it is clear that constant form alone cannot explain the form of (say) a particular hillslope profile. As Ahnert (1967, 24) pointed out, 'a period of uniform downwearing must be preceded by one of differential downwearing' to establish the existing topography at a greater altitude . Or, to put it another way, it needs to be shown, firstly, that the system will converge towards a constant form condition, and, secondly, that this is a stable condition. Constant form must be first <u>attained</u> and then <u>maintained</u> (cf. Morse, 1949 and Lewontin, 1969 on stable and unstable equilibria). Hack (1960, 86) asserted that 'as long as diastrophic [sc. tectonic] forces operate gradually enough so that a balance can be maintained by erosive processes, then the topography will remain in a state of balance even though it may be evolving from one form to another'. This may well be true; but it takes more than asseveration to establish a case. Moreover, it is not at all clear that 'a state of balance' does not preclude evolution 'from one form to another'. The dynamic equilibrium theory becomes totally comprehensive, and hence totally vacuous, with the parenthetical admission (Hack, 1960, 94) that 'erosional energy changes through time and hence forms must change'. No definite hypotheses of any kind accompany this unexceptionable statement. A criticism made by the philosopher Gellner (1968, 165) in another context applies to Hack's theory with particular force: 'If a man says - "I have the idea X, which applies to things except in as far as it does not" - we pay scant attention to him: all ideas have the property of applying, except in as far as they do not. To postulate one of them with such a proviso is not much of an achievement'.

Hence Hack's (1960) dynamic equilibrium theory is a combination of two components: an imprecise hypothesis of 'adjustment' and 'balance', and a precise hypothesis of constant form, which is not supported independently. The first, which receives most emphasis in Hack's paper, is an alternative to hypotheses of 'relict' forms or 'polycyclic' development made by climatic or historical geomorphologists. The second, produced but not discussed in detail, has been neglected until quite recently. It is, however, straightforward to formulate mathematically, and is thus the component of dynamic equilibrium theory which finds explicit expression in models of hillslope development.

The idea of 'constant form' needs mathematical definition. Suppose that a hillslope profile is represented by a curve

$$z = z(x,t)$$

Here z denotes height above the base of the slope, x horizontal distance from the divide, and t elapsed time. The following possibilities must be distinguished.

(i) The slope profile itself remains constant

$$\frac{\partial z}{\partial t} = 0 \; ; \quad z(x,t) = z(x)$$

(ii) The rate of downwearing is constant over space and time; this contains (i) as a special case

$$\frac{\partial z}{\partial t} = -\alpha \, , \quad \alpha \geqslant 0$$

(iii) The rate of downwearing is constant over space; this contains (ii) as a special case

$$\frac{\partial z}{\partial t} = -\alpha(t), \quad \alpha \geqslant 0$$

It seems likely that (i) - (iii) would all be regarded as 'dynamic equilibrium', 'steady-state' or 'time-independent' conditions by many geomorphologists. But since all these terms are frequently used in other senses, a different term is preferable: 'constant form' (Smith and Bretherton, 1972) is perfectly adequate.

Note that these 'equilibrium' states are here defined in terms of properties which remain constant or invariant. This procedure would seem to have wider applicability. Hypotheses of constant soil depth or 'characteristic form' are further examples in which invariants can be specified unequivocally (cf. Carson and Kirkby, 1972), and such a clear specification might reduce the vagueness and confusion frequently characteristic of statements on equilibrium in geomorphological literature. However, the procedure cannot be applied to all kinds of equilibrium conditions; a class of counterexamples are states defined by variational principles (e.g. minimisation of work: Carson and Kirkby, 1972, 4; Kirkby, 1977b). Furthermore, invariant forms may not be equilibrium forms, although in large part this is a terminological issue. According to Kirkby (1974, 9-10) characteristic forms are not equilibrium forms.

The idea of constant form finds its strongest qualitative statement in Hack (1960), although it may be traced to Gilbert (1880, 1909). Constant form hypotheses have been incorporated in several models of hillslope development, although not always with reference to dynamic equilibrium theory. In reviewing this work, it is important to resolve a fundamental ambiguity by specifying whether (a) constant form prevails throughout geomorphological history, or (b) landforms converge on a constant form condition.

In either case, (i) above is an extremely implausible hypothesis. Either downwasting must be balanced exactly by uplift, or if uplift is not included in the model, downwasting must be identically zero: neither alternative seems at all likely (Schumm, 1963; Ahnert, 1970a; Smith and Bretherton, 1972, 1512).

In case (a), if (ii) above holds, then there is a straightforward solution:

$$z(x,t) = z(x,0) - \alpha t$$

The model was discussed briefly by Scheidegger (1961; 1970, 132-4) and Pollack (1968, 1969), although without reference to any equilibrium ideas. If (iii) holds, then we have the generalisation:

$$z(x,t) = z(x,0) - \bar{\alpha} t$$

where $\bar{\alpha}$ is the time average, $\frac{1}{t}\int_0^t \alpha(t')\,dt'$

t' is elapsed time.

Neither of these models is very interesting. It seems unlikely that either (i) or (ii) would hold for long periods of geological time. Moreover, it is necessary to specify an initial profile to obtain any particular solution, and to explain the initial profile in order to explain the profile at time t. Both of these requirements are difficult to meet.

In case (b) we seek a constant form solution to one or more equations specifying hillslope development. This was first done by Jeffreys (1918); in recent years the approach has been used by Smith and Bretherton (1972), Luke (1974), Hirano (1975, 1976) and Kirkby (1976a, 1976b; Wilson and Kirkby, 1975). It is no longer necessary to specify initial profiles and the stability properties of the constant form solution can be investigated analytically.

Equilibrium, in the guise of constant form, plays a dual role in such models. It is a mathematical convenience, a simplifying assumption which makes it easier to obtain closed-form solutions. It may serve as a first approximation to other solutions, as Kirkby has shown. It also represents a physical hypothesis, and naturally requires empirical testing. However, some theoretical support for constant form solutions is provided by the stability results of Jeffreys (1918) and Smith and Bretherton (1972), which indicate the conditions under which constant form will be maintained through feedback.

If there are no grounds for expecting any kind of equilibrium, then evolutionary models are necessary, which predict a series of successive hillslope profiles.

Evolutionary models are very difficult to test satisfactorily. Hillslope development is difficult to observe except in special circumstances, and, contrariwise, the special circumstances are so special that doubt must be cast on their typicality (e.g. badlands studied by Schumm, 1956a, 1956b). The major difficulty is clearly the discrepancy which may arise between human lifespans and time spans of geomorphological interest.

Given but one profile for each hillslope, it is possible to use the series of profiles predicted by the model as a set of templates, and choose the most realistic (cf. Pitty, 1972 on Davisian and Penckian predictions). This is a procedure almost forced upon us unless we know the appropriate elapsed time from other evidence (cf. Chappell, 1974). Such a procedure is unsatisfactory because it is more difficult to refute a model in these circumstances.

An attractive solution to the testing problem is to seek a spatial series of hillslope profiles which can be treated as if it were a temporal series. This kind of solution is often described as invoking ergodicity or the ergodic hypothesis, but such use of terminology is sometimes unjustified (for a fairly rigorous statement of ergodicity see Scheidegger, 1970, 267). A looser term such as 'space-time transformation' is preferable (cf. Chorley and Kennedy, 1971, 277-80 for applications in hillslope geomorphology; Thornes and Brunsden, 1977, 23-5 more generally). Despite widespread enthusiasm for the idea, it seems to be applicable only in very special situations (e.g. Savigear, 1952). In particular, the assumption that profiles within a drainage basin follow the same sequence but at different rates appears to be false (Carson and Kirkby, 1972, 9, 405).

Evolutionary models frequently require the specification of an initial profile z(x,0). This is difficult to supply in most cases, but there are some exceptions, notably the work of Mizutani (1974) on volcanoes and slag heaps.

2.3.2. Deterministic and stochastic models

If a model includes one or more random variables each specified by a probability distribution, it is stochastic. Otherwise it is deterministic.

There is a continuing controversy in geography (Harvey, 1969, 260-3), in geology (Watson, 1969, 491-2; Mann, 1970; Raup, 1977; Whitten, 1977), in geomorphology (Leopold and Langbein, 1963; Scheidegger and Langbein, 1966; Howard, 1972; Shreve, 1975; Thornes and Brunsden, 1977), and indeed in many other disciplines, over the extent to which explanations couched in probability terms are satisfactory. Debate on this issue can be traced to preSocratic philosophy.

Two questions have often been conflated. First, there is the ontological issue of whether the world is deterministic or stochastic, either as a whole or in part. This is a difficult issue which seems entirely open at present given the uncertain status of stochastic models in quantum mechanics. The question may even be undecidable in principle since the failure of a deterministic model need not be ascribed to the stochastic character of nature.

Second, there is the epistemological issue of whether stochastic explanations invoking random variables are satisfactory. Many scholars feel uneasy about hypothesising factors which by definition are unpredictable except in probability terms, and it has been suggested that chance is merely a label for our ignorance, used to cover a residue of fact as yet unexplained. Hence only deterministic explanations can be fully satisfactory. It does seem, however, that a particular stance on this epistemological issue need not entail a particular ontological view. It would be entirely possible to use stochastic explanations while holding that the world is essentially deterministic, and vice versa; and, *a fortiori*, to use one or other kind without committing oneself to any ontology. In Monod's (1974, 110-2) terminology, one can recognise 'operational uncertainty' while not necessarily admitting the existence of 'essential uncertainty'.

A further point which needs some clarification is the meaning of the term 'random'. Three senses need to be distinguished:

(i) Random in the sense of apparently haphazard or chaotic, a report of a subjective impression: this is in large part a psychological matter.

(ii) Random in the sense of equal and independent probabilities.

(iii) Random in the sense of a random variable characterised by a probability distribution: this is the standard mathematical sense and the sense adopted here. In this sense, random = stochastic.

Whatever the philosophical issues, the attitude of mathematicians to the inclusion of random variables in models is generally pragmatic. Whittle (1970, 19), in a text on probability theory, took as a premise 'that there is a certain amount of variability which we cannot explain but must accept', while Bard (1974, 18) similarly wrote that 'unpredictable disturbances are as much parts of physical reality as are the underlying exact quantities which appear in the model'.

Random variables may appear in stochastic models in many different ways; here we mention two (cf. Watson, 1972, 39-40)

(a) value at data point = value of deterministic function + random error

(b) value at data point = value at a point on a random function.

Mathematically these model families are not really distinct: the second can be regarded as a special case of the first in which the deterministic function is identically zero. However, in practice, models of type (a), which may be called stochastic error models, are usually quite distinct from models of type (b), which may be called stochastic process models. In case (a), variability is split into a systematic part, approximated by a deterministic function, and a residual or error part, treated as a random variable.

The need for a random variable arises from the fact of modelling life that no nontrivial data series would ever be fitted exactly by a deterministic model: sampling variation, measurement error, incorrectness of functional form, and uncontrolled variables intervene. In case (b), data are regarded as a realisation of a stochastic process, which is a mathematical process operating in time and/or space according to probability laws.

The usual idea is that deterministic functions capture 'smooth' behaviour while stochastic (random) variables mop up the remaining 'rough' behaviour. This idea is subject to two reservations. Firstly, 'rough' components need not be treated in a probabilistic or stochastic manner, especially in exploratory data analysis (cf. Tukey, 1977; McNeil, 1977). Secondly, there are some deterministic processes with extremely rough (apparently random) behaviour (May, 1976a; Lorenz, 1976), and, conversely, some stochastic processes with extremely smooth (apparently deterministic)behaviour (Cohen, 1976; May, 1976b). This is mentioned largely for completeness: these processes have not been applied as yet in geomorphology.

2.3.3. Phenomenological and representational models

The distinction between phenomenological and representational models is based on a distinction between phenomenological and representational theories made by the philosopher Bunge (1964). A phenomenological model attempts only to capture the phenomena (sc. surface appearances); a phenomenological model of a hillslope profile attempts only an approximation of hillslope form, and neither in assumptions nor in detailed structure does it try to reflect geomorphological processes or physical principles. A representational model is more ambitious, aiming to represent the underlying processes as well as surface appearances, ideally representing them in terms of physical principles.

Terminology here is an awkward matter. These terms are not entirely satisfactory, but more familiar and less cumbersome alternatives seem unsuitable in other ways. Parallel distinctions have been drawn between 'empirical' and 'rational' models in geomorphology (Mackin, 1963; Young, 1972, 18); between 'empirical' and 'conceptual' models (Clarke, 1973) or between 'operational' and 'physical' models (Klemeš, 1974) in hydrology; between 'empirical' and 'theoretical' models in ecology (Wiegert, 1975); and between 'homomorphic' and 'isomorphic' models in pedology (Huggett, 1975). None of these pairs is very satisfactory in capturing a contrast in process content.

The term 'process-response model' (Whitten, 1964; Carson and Kirkby, 1972; Young, 1972, Ch. 10), considered here equivalent to 'representational model', is not used, partly because it lacks an antonym, and partly to avoid confusion with the rather different term 'process-response system' (Chorley and Kennedy, 1971, Ch. 4).

It is widely accepted in geomorphology that, as far as possible, landforms and related processes should be explained in terms of mechanical and chemical principles (Strahler, 1952; Yatsu, 1966; Carson, 1971; Statham, 1977). This is merely an example of a more general attitude - that explanations should make reference to actual mechanisms. 'To explain a phenomenon, to explain some pattern of happenings, we must be able to describe the causal mechanism which is responsible for it' (Harré, 1972, 178). Hence there is a desire to relate geological knowledge to physical theory (Kitts, 1974), or to relate form to process in geography (Harvey, 1969).

Although a quest for causal explanation leads to the construction of representational models of hillslope profiles, phenomenological models may still be of considerable descriptive value (cf. Curry, 1967, 267). For example, if upslope convexities may be approximated by power functions, the parameter values provide a simple and efficient means of comparing different convexities (Hack and Goodlett, 1960). Furthermore, the importance of description should not be underplayed: detailed and systematic description has its place alongside explanatory theory.

If the aim is to build representational models, how is this to be done? Four levels may be distinguished in process study, not necessarily exclusive, sequential or exhaustive.

(i) Recognition of processes from incidental evidence (e.g. the supposed identification of creep from bending trees, bulging walls, etc.).

(ii) Measurement of rates of operation. This allows a decision on the processes to be modelled to be based on quantitative evidence.

(iii) Identification of controlling variables. This allows a choice of controlling factors to be included in any model. Usually hillslope profile models have been based upon the assumption that process rates are essentially functions of profile geometry (especially gradient, distance from divide, curvature). Non-geometric controls such as mantle strength, moisture, texture and vegetation have received less attention from modellers, but have often been included in field investigations.

(iv) Elucidation of physical mechanisms. Processes are analysed in terms of mechanical and chemical principles: many models fall short of such integration. At some stage, an empirical or phenomenological approach must be employed. Hillslope profile models cannot in practice be based on 'fundamental' physical theories such as quantum theory or relativity theory (cf. Schoener, 1972, 390 on ecological models). The 'universal laws' invoked in ideal geomorphological explanations are usually those of mechanics or chemistry (cf. Ch. 2.2.2).

A principle which is widely accepted in modelling hillslope development is that a mass balance rather than an energy balance provides an appropriate framework. Fluxes of energy performing geomorphological work are a negligible component of hillslope energy budgets (Carson and Kirkby, 1972, 28; Young, 1972, 21; Kirkby, 1974, 2-3; cf. Hare, 1973, 188). Hence fluxes of material are the central concern, and continuity equations allow these to be handled systematically (e.g. Carson and Kirkby, 1972, 107-9; Wilson and Kirkby, 1975, 205-6; Kirkby, 1976b, 9-10).

While 'phenomenological' and 'representational' are presented here as polar opposites, it must be admitted that in practice models exhibit continuous gradation in process content.

2.3.4. Analytical and simulation models

Analytical models ideally take the form of sets of equations possessing solutions in closed form, whereas simulation models include those expressed in the form of computer programs specifying sequences of operations. (No other category of simulation models will be considered here). In practice, these classes of model intergrade: solutions to many equations can only be obtained using methods of numerical analysis which must be implemented on a computer.

An ideal situation may be sketched as follows. The modeller writes down a set of equations (usually ordinary or partial differential equations, if a dynamic model is being constructed), which represent empirical knowledge, physical principles and any constraints which must be satisfied (e.g. conservation of mass or energy). These equations are then solved in general using 'standard' methods and in particular by inserting initial and boundary conditions.

If this ideal was always obtained, there would be no need for simulation models. The story is, however, something of a fable, not least because differential equations are strange beasts. Many apparently simple equations possess no simple closed form solution, and it is frequently necessary to compromise, by seeking an approximate solution or a particular kind of solution, such as an equilibrium solution. In the latter case, the mathematical fact that a particular solution exists does not support the physical hypothesis that such a solution will be attained. Such a hypothesis is a further statement requiring justification.

In contrast, a simulation model is much easier to build, requiring only an elementary knowledge of computer programming (FORTRAN, rather than a special simulation language such as SIMULA (Birtwistle et al, 1973), has generally been used in geomorphology). It is usually possible to build models more complex than (tractable) analytical models, while some difficulties facing analytical models (such as the existence of thresholds and the need to model magnitude and frequency distributions) may be of little account: such features can be handled easily. Correspondingly, the danger exists that a highly complex model will be impossible to investigate systematically, and it will never be clear which results are genuine and which artefactual: to use May's (1974, 682) delightful expression, the model may be 'a multi-parameter, computerised Goon show'. Howard (1972) has given a sober discussion of the problems of computer simulation in geomorphology, while Moon (1975) has reported a pioneer sensitivity analysis of Ahnert's (1973) model. Such sensitivity analyses are vitally necessary as complements to development sequences produced by simulation runs (for an excellent ecological example, cf. Steele, 1974).

2.4 Major geomorphological problems in modelling

Several kinds of complicating features are characteristic of hillslope systems, and hence should ideally be reflected in models of hillslope profiles. These are examined in turn below.

2.4.1. Polygenesis

'Polygenesis' is a term describing the common (if not universal) situation in which a landform has been produced by a combination of different processes. Polygenesis is a major kind of complexity frequently found for hillslope systems. In so far as hillslopes are polygenetic, models should reflect such an origin, although incorporation of polygenesis should preferably rest on quantitative evidence about the relative importance of different processes.

However, various kinds of polygenesis need to be distinguished. Firstly, there are situations in which different processes are acting more or less simultaneously (for example, creep and rainwash). Secondly, there are situations in which processes have very different return frequencies (for example, creep and large-scale failures). Thirdly, late Tertiary and Quaternary climatic change may have led to the succession of different suites of processes (Young, 1972, 240-6); for example, alternation between cryonival and warmer conditions in present day humid temperate areas (e.g. Rapp, 1967; Black, 1969 and cf. Ch. 4.4.3 below).

These situations may not be equally problematic. If hillslopes are especially subject to large-scale failure when steep and to slower processes when gentle, their form at any time may reflect one or the other rather than a combination: hence relatively simple models may still be applicable, at least to individual hillslope components. Climatic change is not problematic if slopes are essentially relict, or if the contrast between different regimes has been unduly exaggerated.

2.4.2. Feedback

Feedback loops are of great importance in geomorphological systems (Melton, 1958; King, 1970; Twidale et al, 1974, 1977; Crozier, 1977) and any realistic model must thus mimic the major loops in operation. The most general and most basic feedback relationship is between process and form. Not only do processes affect forms, but forms affect processes, both in general and on hillslopes (Chorley, 1964, 71; Ahnert, 1971, 3-4; Young, 1972, 104-5).

As Smith and Bretherton (1972, 1506) put it, the physical landscape may be idealised as a time-dependent, self-forming surface.

Even this basic relationship is not always mirrored in representational models. Some stochastic process models (cf. Ch. 2.3.2, Ch. 3), which are based on the idea that empirical data series = realisation of stochastic process, do not allow feedback, because the relation between generating process and generated series is asymmetric: the characteristics of the series do not affect those of the stochastic process. In this sense at least the stochastic process postulated is not analogous to the geomorphological processes in operation.

2.4.3. Thresholds

Many geomorphological systems contain thresholds or discontinuities (Chorley and Kennedy, 1971, 236-40; Reynaud, 1971, 47-50; Schumm, 1973, 1977). In the case of hillslopes, good examples are provided by the thresholds which must be crossed before slope failure occurs (e.g. Carson, 1976). The existence of discontinuities poses difficulties for the usual approach to modelling physical systems, centred around ordinary and partial differential equations, as Souchez (1966a, 212) and Aronsson (1973, 2) have remarked in a hillslope modelling context. The use of differential equations is generally based on the assumption that both functions and derivatives vary smoothly and continuously.

Hence there is good mathematical reason for a sharp division between the models of soil and rock mechanics, mainly concerned with the character of failure conditions, and the models of profile development considered here, mainly applicable to hillslopes subject to slower processes which may be 'averaged' over whatever discontinuities are present (cf. Carson and Kirkby, 1972, 110).

Naturally this division is unfortunate from a geomorphological point of view. If hillslope development must be attributed to a combination of threshold-dependent and threshold-independent processes, then a model of hillslope development should reflect such combination. This can be done, to a certain extent, in a simulation model (see Ch. 3.3 below for examples).

2.4.4. Magnitude and frequency

Most geomorphological processes are intermittent in their action: even apparently continuous processes such as soil creep may take place as a series of 'microcatastrophes'. Hence the magnitude and frequency of geomorphological events need to be considered both in general (Wolman and Miller, 1960; Leopold *et al*, 1964, 67-94; Wolman and Gerson, 1978) and for hillslopes (Carson and Kirkby, 1972, 102-4; Young, 1972, 85-7; 1974, 74-5; Starkel, 1976).

The 'Wolman-Miller thesis' is that events of *intermediate* magnitude and frequency have most geomorphological impact: major events have little effect in total because they are rare; frequent events have little effect in total because they are minor. This principle certainly holds if the frequency distribution of magnitudes above some threshold is lognormal, and the relationship between work achieved and event magnitude (again, expressed above a threshold) is a power function. However, although the most valuable and provocative generalisation available on magnitude and frequency, the Wolman-Miller thesis should not be considered as established truth. One important qualification (Wolman and Gerson, 1978) is that extreme events of particular magnitude and frequency must be seen in relation to the rates at which recovery of specific forms takes place between recurrences. For example, the speed of revegetation of bare ground is a major control of the effectiveness of extreme events.

Almost all continuous slope models handle the magnitude-frequency issue by (implicitly) averaging over frequency distributions and representing processes as continuous in time (but see Kirkby, 1976a, 1976b for an exception; and also Price, 1974, 1976 on alluvial fan deposition). This is almost certainly necessary if analytical solutions are sought, while conversely it is not a necessary assumption for simulation models. Averaging over distributions may be reasonable if processes such as creep and rainwash are in question, but probably not in the case of large-scale failure.

2.4.5. Laterality

A profile is only an idealisation, valid to the extent that horizontal or plan curvature (and thus lateral sediment flux) can be neglected. Ultimately the aim must be the modelling of landsurfaces, and some of the models reviewed below attempt to do this (although Ch. 3 does not cover all landsurface models). Complication comes not only in the form of another horizontal dimension (e.g. squaring computer storage requirements) but also in qualitatively new features which should be incorporated, especially the interaction of a drainage network and the intervening slopes (Sprunt, 1972; Armstrong, 1976).

The discussion in Carson and Kirkby (1972, 390-6) leads to an encouraging result: profile models are stable in the sense that slight curvatures lead to only slightly different predictions. Contrary evidence would imply that only surface models can be at all realistic.

2.4.6. Lithological variation and soil properties

The influence of lithological variation upon hillslope morphology has often been reported (cf. Young, 1972, Ch. 17 for a brief review) and the importance of rock characteristics in hillslope development needs little emphasis. It is, however, very difficult to incorporate rock properties in models of landform development in a satisfactory manner.

Although it seems clear that the relative importance of lithological variation should be assessed quantitatively, relationships between rocks and relief have often been discussed without any reference to morphometric variables (Cox, 1973, 3). In many cases 'resistant' beds stand out as steps in slope profiles: how important are they in the overall landscape? are they merely micro- or meso-features on the scale of the slope profile, or can lithological variation be held to dominate the landscape? These are properly quantitative questions.

If the lithology underlying a given profile is fairly homogeneous, then it may be permissible to omit rock properties from a model for that profile alone, although clearly any comparison of profiles on different lithologies should not follow this practice. Extreme heterogeneity might also be taken as near-homogeneity if much of the variability is on a micro-scale and can be averaged out. One possible example would be a rapidly alternating sandstone-shale succession.

In the more common intermediate situation, a naive tactic is to assign differing 'resistance' values to different strata. Several dynamic slope models allow such variations.

Three difficulties deserve note. Firstly, resistance values should be supplied independently in order to avoid the circular arguments unfortunately characteristic of rock-relief studies (Yatsu, 1966, 9-10; Sparks, 1971, 370; Tricart and Cailleux, 1972, 17). If resistance values are given in an arbitrary fashion, drawing upon intuitive ideas only, then nothing is done to break the circle. However, the situation cannot be improved substantially without overcoming some formidable problems of definition, sampling and measurement. Secondly, if a soil cover is present, and hillslope development is transport-limited, resistance is a matter of soil properties rather than rock properties: to use Chorley's (1959, 503) metaphor, 'bedrock is not to be considered a parent of the related topography, but rather a grandparent'. This simple point is not always observed in models. Thirdly, resistance is frequently defined in hillslope models as a single-valued property, whereas it is well known that several different properties of rocks and soils affect rates of mobilisation and transport (e.g. Bryan, 1968, 1977; Sparks, 1971, Ch. 2; Thornes, 1975; Statham, 1977, Ch. 2).

The lithology may allow extensive chemical removal of material from the hillslope, although most models focus on mechanical removal. Carson and Kirkby (1972, 257-71) reviewed appropriate models for chemical removal, while in later work Kirkby (1976 a.) suggested the simple approximation that chemical downwasting is spatially constant, which he regarded as invalid only when chemical removal is unimportant. However, the context of this remark is a model which assumes homogeneous lithology.

Soil depth has been included in several models, usually by postulating a relationship of some kind between soil depth and weathering rate. The main difficulty is that virtually nothing is known empirically about this relationship (Young, 1972, 46) although there is no shortage of hypotheses in the literature (e.g. Culling, 1965, 246; Souchez, 1966a, 190; Carson and Kirkby, 1972, 104-6; and others below).

2.4.7. Boundary conditions

What can be said about the behaviour of slope endpoints (Carson, 1969, 77) and about tectonic, isostatic and eustatic rates? Assumed answers to these questions appear as boundary conditions in some hillslope models.

In particular, (i) is removal of material at the base impeded or unimpeded? (Strahler, 1950; Savigear, 1952; Melton, 1960; Carson and Kirkby, 1972, 139-40).

(ii) is the stream downcutting and/or moving laterally? (Smith and Bretherton, 1972).

(iii) are divides migrating laterally? (Carson and Kirkby, 1972, 396-7).

All these possible forms of endpoint behaviour occur in nature and deserve investigation.

There are some grounds for supposing that tectonic and isostatic movements need not be modelled explicitly. Schumm (1963) argued that available quantitative evidence on rates of downwearing and uplift supports the classic Davisian hypothesis of relatively rapid uplift followed by relatively long stillstands (cf. also Carson and Kirkby, 1972, 21-5). If this were correct, uplift could be subsumed in an initial condition, much as Davis (1909) did in his cyclical scheme. But it seems dangerous to rely on this interpretation, mainly because in some regions epeirogenic movement seems to have occurred over a long period of time.

A more important reason for not modelling tectonic and isostatic movements explicitly is that if the area affected by uplift is large relative to the length of the profile, it may be possible to treat uplift as a regional rather than a local factor, or to treat it indirectly, through river downcutting. This is a less restrictive assumption than Schumm's quasi-Davisian hypothesis, and thus renders the latter unnecessary.

The case of eustatic fluctuations is more complex. Since sea level has varied over the latter part of geological time, it has been suggested that pulses of downcutting ('rejuvenation')have travelled up valleys leading to the formation of valley-in-valley forms, marked by breaks of slope and even terraces (Sparks, 1960, 220-4; Young, 1972, 239-40). However detailed process studies have revealed that such pulses may be quickly damped upstream; that the morphological response to 'rejuvenation' may be complex; and that forms attributable to 'rejuvenation' are equifinal (Leopold et al, 1964, 258-66, 442-5; Chorley, 1965a, 28; Schumm and Parker, 1973; Schumm, 1973, 1977). Hence it is difficult to know whether eustatic effects should be modelled at all.

Kirkby (1971, 28) has suggested that the theory of kinematic waves (Lighthill and Whitham, 1955) could be used to examine knickpoint propagation, but this idea has apparently not been followed up. Luke (1972, 1974) briefly remarked on the relationship of his models to kinematic wave theory.

2.5 Summary

(i) Hillslope geomorphology studies the forms, processes and development of hillslope systems. Hillslopes are usefully viewed as replicated systems, rather than as unique or restricted features, and their study is related to that of other replicated systems, notably drainage basins and topographic surfaces. Within hillslope geomorphology, morphometry - the quantitative analysis of land form - is a major approach alongside historical, process and sediment studies. (2.1)

(ii) Since theory is inevitable, it is necessary to examine ideas and assumptions critically. A few troublesome terms need to be defined carefully: in particular, a distinction is drawn here between 'system' (i.e. real-world system of interest) and 'model' (i.e. simplified formal representation). The role of mathematics in geomorphology, and the relationship between modelling and simplification, are frequently misunderstood: attempts are made to clarify the underlying issues. The testing of a model in its domain, including the problem of equifinality, needs close attention. (2.2)

(iii) Approaches to modelling hillslope profiles are considered using a simple classification based on five dichotomies: static/dynamic (including an extended examination of the important idea of equilibrium); deterministic/stochastic (including a discussion of the role of probability in models); phenomenological/representational (including a discussion of the representation of geomorphological processes); analytical/simulation (especially the particular advantages and limitations of each kind); and discrete/continuous (for further discussion see Chs. 3.1 and 8 below). (2.3)

(iv) Major geomorphological problems in modelling hillslope profiles are assembled and evaluated (polygenesis, feedback, thresholds, magnitude and frequency, laterality, lithological variation and soil properties, boundary conditions). (2.4)

2.6 Notation

d	in ordinary derivative or in integral
t, t'	elapsed time
x	horizontal coordinate
z	vertical coordinate
α	rate of downwearing
$\bar{\alpha}$	average rate of downwearing
Υ	constant
∂	in partial derivative
$\int$	in integral
$\vert\ \vert$	modulus

Chapter 3

A REVIEW OF CONTINUOUS MODELS OF HILLSLOPE PROFILES

... Prince Papadiamantopoulos turned out, in spite of his wonderfully promising title and name, to be a perfectly serious intellectual like the rest of us. More serious indeed; for I discovered, to my horror, that he was a first-class geologist and could understand the differential calculus.

Aldous Huxley, Those barren leaves, Pt. II, Ch. 1.

3.1 Introduction

This chapter gives a fairly complete review of models of hillslope profiles which treat a hillslope as a continuous curve. Work on modelling topographical profiles and land-surfaces is discussed when it is of direct interest, but not studies of stream and river profiles (e.g. Tanner, 1971).

Related work on hillslopes treated as combinations of discrete components (for example, free face and debris slope) is not reviewed here, partly because the applicability of such models will not be further considered in this thesis, and partly because there are good brief reviews by Carson and Kirkby (1972, 140-7), Scheidegger (1970, 120-32) and Young (1972, 105-9), which remain essentially up-to-date. By contrast, the field of continuous modelling lacks a comprehensive and up-to-date review. Models of special features such as talus slopes or mass failures are not covered.

There is little consensus in the field about notation, and there are even some workers who demonstrably use inconsistent or inappropriate notation. A unified notation has been adopted here which should aid comparison, and which may even encourage standardisation.

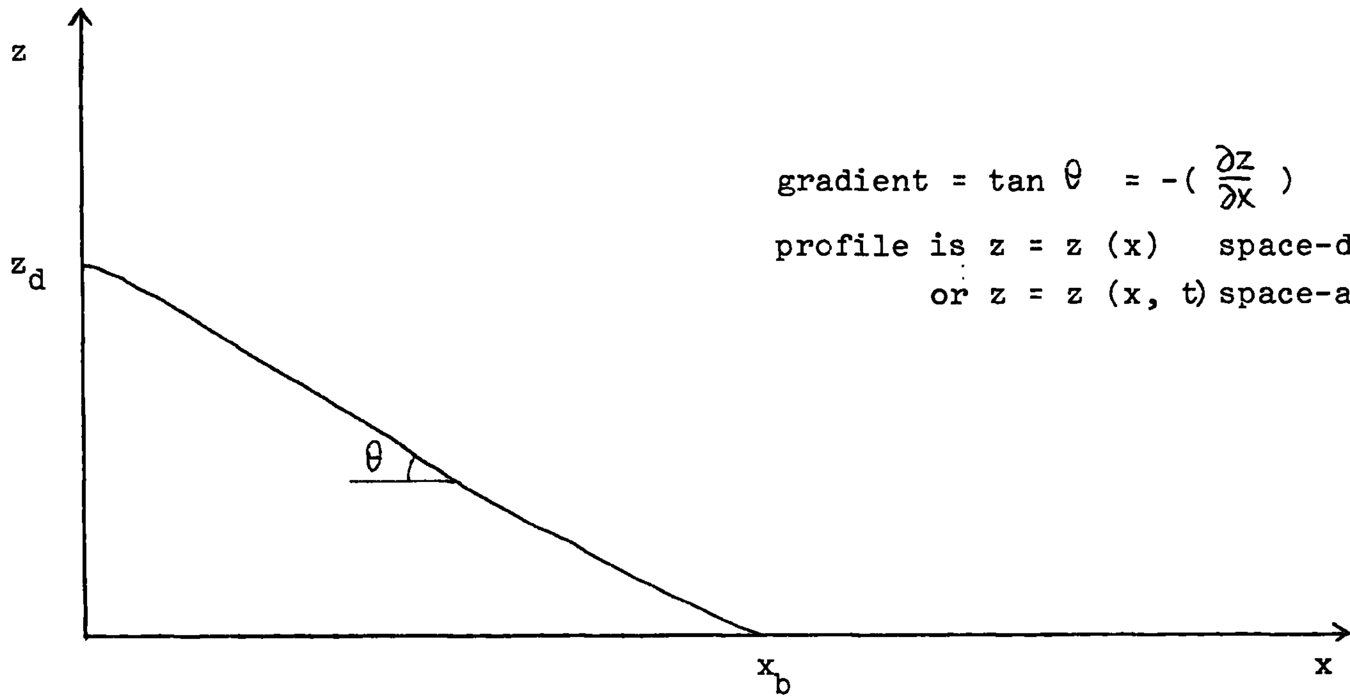
While an attempt has been made at completeness, there is clearly insufficient space to list every major equation in the literature, let alone to provide full proofs of every result, or to cast the discussion in the rigorous style of professional mathematicians. The surest guide to the character of particular models is the original literature: here only an overview is provided.

The sharpest dichotomy in practice between sets of models classified according to the scheme outlined above (Ch. 2.3) is between static and dynamic models, which are reviewed below in Chs. 3.2 and 3.3 respectively. A thematic review seems most natural for static models, while a historical review with connective summary appears best for dynamic models.

The profile notation adopted here as standard is given in 3A.

3.2 Static models

A list of static models proposed for hillslope profiles is given in 3B. This list includes some models used for hillslope components, glacially-moulded profiles and topographical profiles. Over the last century, and especially over the last twenty years, many functional forms have been discussed in the literature, usually as phenomenological rather than representational models. If a hillslope resembles an arc of a circle or a Gaussian curve, then that is that. If nothing more is claimed, then nothing more need be discussed. Here three of the more popular classes of model are singled out for attention, together with some stochastic models which are relatively novel.



gradient = $\tan\theta = -\left(\frac{\partial z}{\partial x}\right)$

profile is $z = z(x)$ space-dependent

or $z = z(x, t)$ space-and-time-dependent

z is vertical coordinate, x is horizontal coordinate, y is extra horizontal coordinate for surface $z = z(x, y, t)$

3A <u>Standard notation for hillslope profiles</u>

3B

Static models of hillslope profiles

Model	Reference	Comments
BINOMIAL or NORMAL (GAUSSIAN)	Tylor 1875	Cf. outline of Mt. Tabor (now Israel)
	Thompson 1942, 121	Hill outlines
	Ashton 1976	Mt. Piper, Victoria, Australia
POWER SERIES POLYNOMIAL	Tylor 1875	Parabola. Hirwaun, Wales.
	Lake 1928	Parabola. Components. Gwynedd, Wales.
	Savigear 1956, 1962	Devon and Cornwall.
	Troeh 1964, 1965	'Paraboloids of revolution' fitted to three-dimensional forms - quadratic in two dimensions
	Ruhe & Walker 1968	Iowa
	Lewin 1969	Incl. quintic. Yorkshire Wolds.
	Clark 1970	Parabola. Western U.S.A.
	Kleiss 1970	Quadratic. Components. Iowa.
	Ongley 1970	Linear. Components. N.S. Wales
	Young 1970b	Components. Mato Grosso, Brazil.
	Doornkamp & King 1971	Linear, quadratic, cubic discussed
	Woods 1972, 1974, 1975	Colorado and Kentucky. Cf. Cox 1975
	Parsons 1973, 1976b	Linear. Components. Italy, Morocco.
	Blong 1975	Linear. N. Island, New Zealand
	Cox 1975	Cf. Woods
	Toy 1977	Linear. U.S.A. Cubic for 'average' profile

3B (continued)

Model	Reference	Comments
CATENARY	Davis 1916	Glacial trough cross profiles, Montana. Idea mentioned by several later workers.
ARC OF CIRCLE	Lake 1928	Components. Gwynedd, Wales.
	Juvigné 1973	Components. Belgium.
POWER FUNCTION	Hack & Goodlett 1960	Components: Virginia. Cf. Drury & Nisbet
	White 1966	Components: Ohio
	Drury & Nisbet 1971	Components: Cf. Hack & Goodlett
	Bull 1975, 1977	Components: California, Arizona, Utah. Cf. Cox
	Cox 1977a	Components: Cf. Bull
	Toy 1977	Components: U.S.A.
	Svensson 1959	Glacial trough cross-profiles
	Graf 1970	Glacial trough cross-profiles
	Doornkamp & King 1971	Glacial trough cross-profiles
	Drewry 1972	Glacial trough cross-profiles
	Till 1973	Glacial trough cross-profiles
	King 1974	Glacial trough cross-profiles
	Aniya 1974	Glacial cirque long-profiles
	Graf 1976a	Glacial cirque long-profiles
EXPONENTIAL	Dury 1966, 1970	Tangent-distance regressions. Pediments. N.S. Wales
	Dury *et al* 1967	Tangent-distance regressions. Pediments. N.S. Wales
	Dury 1972	Semi-logarithmic regressions. Pediments. S. England

3B (continued)

Model	Reference	Comments
	Ruhe 1975b, 1977	Profiles. Iowa, N. Mexico, Hawaii, Puerto Rico. Cf. Cox
	Cox 1977b	Cf. Ruhe.
	Bridge & Beckman 1977	S. E. Queensland
ASYMPTOTIC GOMPERTZ CURVE	Lewin 1969	Yorkshire Wolds
COSINE CURVE	Clark 1970	Western U.S.A.
RECTANGULAR HYPERBOLA	Clark 1970	Western U.S.A.
	Doornkamp & King 1971	
LOGARITHMIC	Milne 1878, 1879	Volcano slopes
	Kleiss 1970	Components, Iowa
	Doornkamp & King 1971	
	Toy 1977	Attempted & dismissed for concavity. U.S.A.
ANGLE LINEAR FUNCTION OF INDEX	Pitty 1970	Components
	Abrahams & Parsons 1977	Components. N. S. Wales
ARMA	Thornes 1972, 1973	Iceland and unnamed area
	Haining 1977a	Cf. Thornes

3B (continued)

Model	Reference	Comments
FRACTIONAL BROWNIAN FUNCTIONS	Mandelbrot 1975a, 1975b, 1975c, 1977	Stochastic models
CYCLOID	Bridge & Beckman 1977	S. E. Queensland
OTHERS	Ruhe 1967	Deterministic (original not seen)

3.2.1 Power series polynomial

This class of model has the general form

$$z = \sum b_p x^p$$

Particular cases include p = O(1)1 (or linear)

$$z = b_o + b_1 x$$

p = O(1)2 (or quadratic; including parabolas)

$$z = b_o + b_1 x + b_2 x^2$$

p = O(1)3 (or cubic)

$$z = b_o + b_1 x + b_2 x^2 + b_3 x^3$$

A similar family of models has the general form

$$x = \sum b_p z^p$$

Most authors use low-order polynomials: the highest order employed in hillslope profile models appears to be 7 (Toy, 1977). A linear model will naturally only be a good approximation of a hillslope which has a nearly constant gradient. A quadratic model is appropriate for a smooth convex or concave slope; a cubic allows an inflexion; and higher orders allow increasingly complicated forms.

Polynomial models are phenomenological: the idea that they can 'explain' hillslope profiles (Woods, 1974, 416) is absurd, unless an extremely weak notion of explanation is adopted (cf. Ch. 2.2.2 above). Since the form of the function is not derived from geomorphological theory, the best that can be hoped for is a parsimonious summary of the data (Cox, 1975, 489; cf. Lewin, 1969, 72).

The linear model has been used (Doornkamp and King, 1971; Blong, 1975; Toy, 1977) to estimate average angle $\bar{\theta}$ from the relation

$$\left|\frac{dz}{dx}\right| = \left|b_1\right| = \tan\bar{\theta}$$

This seems pointless, if only because $\tilde{\theta}$ can be calculated directly from

$$\tan\tilde{\theta} = \frac{z_d}{x_b}$$

3.2.2 Power functions

In its simplest form, a power function is represented by

$$z_d - z = ax^b$$

although other versions are also found. Power functions have been fitted separately to individual components (convexities, straight slopes, concavities). They do not allow inflexions. The model is again essentially phenomenological, although one of Kirkby's models has a power function solution (see below). The reader is referred to a debate on the 'allometric' interpretation of power functions (Bull, 1975, 1976; Cox, 1977a; Bull, 1977) which will not be prolonged here. It is also of interest to note the use of power functions to model glacial trough cross profiles and glacial cirque long profiles.

3.2.3 Exponential function

In its simplest form, the exponential model has the form

$$z = ae^{-bx} \quad , b>0$$

where e is the transcendental number 2.71828..., the base of natural logarithms. The model is such that z tends to

0 asymptotically as x tends to infinity: it is an ever-more-gentle concavity. Since the model will not allow inflexions, it can hardly be appropriate for entire profiles, unless an upper convexity is absent or negligible. Nevertheless it has been applied to entire profiles (Ruhe, 1975b; cf. Cox, 1977b; Ruhe, 1977).

A more complicated relative is implicit in the tangent-distance regressions of Dury (1966, 1970; et al, 1967) on pediment profiles, and another relative is implicit in the 'semi-logarithmic' regression used, but not explained in detail, by Dury (1972).

3.2.4 Stochastic process models

In the last few years, a new kind of static model has been introduced, following the idea that hillslope profiles may be regarded as realisations of spatial stochastic processes.

Thornes (1972, 1973) has applied autoregressive models, moving average models and mixed autoregressive-moving average models borrowed from time series analysis to series of measured slope angles. These models are all special cases of the general ARMA model (Box & Jenkins, 1976; Chatfield, 1975, 41-51). The application of these models to hillslope profiles faces several problems, which deserve discussion.

Firstly, the models are essentially phenomenological. Secondly, in time series analysis it is usually natural to suppose that the present is influenced by the past, but not vice versa: the arrow of time limits plausible specifications.

On the other hand, a point on a slope is related not only to points upslope (downslope sediment flux) but also to points downslope (e.g. basal undercutting). Thus the unilateral ARMA models need to be replaced by bilateral models which allow influences to operate in both directions. This problem was recognised by Thornes (1976, 59), although Church (1972, 80) took a different view in a study of fluvioglacial stream profiles: he argued that it was unlikely that control from downstream would be effective for long distances. However, those bilateral models of spatial stochastic processes which have been discussed in the literature (Whittle, 1954; Haining, 1977a, 1977b) seem inappropriate for the geomorphological case in which upslope and downslope influences differ in kind. Thirdly, feedback between form and process is not captured by these models (cf. Ch. 2.4.3). Fourthly, it seems possible that in certain respects results may be artefacts of the measured lengths used in profile survey (Thornes, 1973).

Mandelbrot (1975a, 1975b, 1975c, 1977) considered a variety of stochastic process models for landsurfaces. The most interesting are based on the class of fractional Brownian functions. Given points P', P" which lie in a real Euclidean space, a fractional Brownian function B_H is defined by a property of local <u>differences</u>

$$B_H(P') - B_H(P'') = \Delta B_H, \text{ say}$$

ΔB_H is drawn from a normal (Gaussian) distribution with mean zero and variance $|P'P''|^{2H}$, or

$$\Delta B_H \sim N(0, |P'P''|^{2H})$$

H, the parameter of the process, lies in the interval $0<H<1$. Mandelbrot's model encompasses both profiles (space has dimension 1) and surfaces (dimension 2). $H\approx 0.7$ turns out to generate quite realistic landsurfaces. Although a Poisson approximation to a Brownian surface was motivated by an idea of random faults, this is geologically unrealistic and the model is best regarded as phenomenological. It is here classed as static: Mandelbrot, however, was not clear about whether the real Euclidean space in which points P lie could be interpreted as physical space-time, which would make his model a dynamic model.

Parsons (1973, 1976b) and Graf (1976b) have explored the use of Markov-type ideas and transition probability schemes for hillslope modelling. Such an approach treats hillslopes as combinations of discrete components, and will not be discussed here in further detail.

3.3 Dynamic models

The review of dynamic hillslope profile models which follows is historical, by dates of authors' first key publications in the field. This is the line of least resistance: no other sequence appears at all satisfactory, however. Some readers may prefer to read the connective summary (3.3.22) first, and then to refer to modellers of particular interest.

3.3.1 Jeffreys

Jeffreys (1918), best known for his distinguished work in geophysics, published the first continuous model of hillslope development, although it was not presented as such. This paper is of more than historical interest: its ideas and results remain of signal importance, and it has been sadly neglected by recent workers (cf. Cox, 1977c, for an appreciation).

Jeffreys gave a dynamical treatment of the flow of surface water during rain and then considered denudation by viscous flow. The rate of denudation with uniform soil was shown to be a function of d tan θ , where d is the depth of water and θ slope angle. If this product is constant, there is an interesting case: 'The surface sinks at a uniform rate all over retaining its size and shape, but progressively sinking. This represents one case of the "peneplain"' (Jeffreys, 1918, 184).

If a surface $z = z(x,y,t)$ is such that contours are parallel to the y-axis then the form of the 'peneplain' can be derived. This special case is clearly that of a hillslope profile $z = z(x,t)$. It is given parametrically by

$$z = a - b\ (2 \operatorname{cosec} \theta - \sin \theta)$$

$$x = c - b\ (\operatorname{cosec} \theta \cot \theta - \cos \theta)$$

where a is a function of time and b and c are constants. This constant form profile is concave upwards and almost parabolic except near the divide where it is nearly vertical. This last prediction is not very realistic, but was discussed at some length by Jeffreys.

The idea of constant form, although present in the 'dynamic equilibrium theory' of Gilbert (1880, 1909), appears to have been introduced independently by Jeffreys. He provided some motivation for this idea by considering the stability of the peneplain under small disturbances. He argued that such a peneplain was stable for corrugations running across the slope but not for those running down the slope. This kind of instability was thought to be counteracted much of the time by soil friability and vegetation.

This paper is remarkable for its elegant and rigorous approach and its concern to elucidate the mechanics of surface water flow and erosion. It originated two of the most valuable ideas of hillslope modelling: constant form and stability under perturbations. Constant form is introduced as a hypothesis, not as a theorem: Jeffreys made it clear that it was 'an interesting case', but did not claim wider validity for the idea. However, the results of stability analyses provide some motivation for such a hypothesis. Here there is a striking parallel with the later work of Smith (see below).

3.3.2 de Martonne and Birot

de Martonne and Birot (1944), in a paper on slope development in humid tropical climates, suggested that regolith depth w (measured normal to the surface in this case) followed

$$\frac{\partial w}{\partial t} = a \sin \theta$$

whence, using geometrical and trigonometrical identities, the hillslope follows

$$\frac{\partial x}{\partial t}\left[1+\left(\frac{\partial x}{\partial z}\right)^2\right] = a$$

(Here the partial differentiation symbol ∂ has been used, although de Martonne and Birot showed a cavalier disregard for the distinctions between δ (small difference), d (ordinary differential) and ∂). The movement of the stream at the slope base was given by a quadratic

$$z_b = b_1 t + b_2 t^2$$

The solution is cumbersome and given parametrically. Choosing physically admissible values for the basal slope, de Martonne and Birot considered the effects of different downcutting regimes.

This model is not very well presented. The central premise $\frac{\partial w}{\partial t} = a \sin\theta$ leads to the prediction that w increases fastest on vertical slopes, which is absurd if only because vertical slopes do not carry regoliths: hence the need to choose 'physically admissible' values. The main feature of interest is the treatment of different downcutting regimes permitted by the quadratic.

3.3.3 Culling

Culling (1960) suggested that the diffusion equation might serve as a phenomenological model for profiles and landsurfaces. In one horizontal dimension

$$\frac{\partial z}{\partial t} = a\,\frac{\partial^2 z}{\partial x^2}$$

i.e., rate of downcutting is proportional to horizontal rate of change of gradient. One salient advantage of this equation, which also applies to heat conduction, is that its properties are well understood (e.g. Carslaw and Jaeger, 1959).

It is easy to show that the diffusion equation follows from an assumption that sediment flux S is proportional to gradient, i.e.

$$S = a \tan \theta = a \left(- \frac{\partial z}{\partial x} \right)$$

From continuity

$$\frac{\partial z}{\partial t} = - \frac{\partial S}{\partial x} = - \frac{\partial}{\partial x} \left[a \left(- \frac{\partial z}{\partial x} \right) \right] = a \frac{\partial^2 z}{\partial x^2}$$

Culling (1963, 1965) argued that the diffusion equation was especially applicable to soil creep. The hypothesis that creep is the result of numerous randomly directed movements of minute extent made by individual particles was shown to lead to a diffusion equation. Various modifications and generalisations of the model were also discussed, which allow for non-random influences and mass transport. All three papers included several worked examples for particular initial and boundary conditions.

While the physical validity of Culling's stochastic hypothesis is doubtful, the diffusion model remains available for processes with sediment flux proportional to gradient. Indeed, if creep is regarded as proportional to sin θ ($\approx$ tan θ for $\theta < 20^{\circ}$) this is an appropriate model irrespective of the microscale mechanics which underlie the process.

3.3.4 Scheidegger

Scheidegger proposed several models of hillslope development in a series of papers published in the 1960s: a convenient summary was provided by Scheidegger (1970, Chs. 3.5, 3.6, 5.8).

A family of linear models have the general form

$$\frac{\partial z}{\partial t} = -af \quad , \quad a > 0$$

In particular cases (i) f = 1 (constant form) (ii) f = z (iii) $f = -\frac{\partial z}{\partial x}$. In each case straightforward analytical solutions exist, but the models are admitted to be simplistic at best.

A family of nonlinear models suggested as an improvement has the general form

$$\frac{\partial z}{\partial t} = -af \sqrt{\left[1 + \left(\frac{\partial z}{\partial x}\right)^2\right]}$$

The multiplying factor

$$\sqrt{\left[1 + \left(\frac{\partial z}{\partial x}\right)^2\right]} = \sqrt{\left[1 + \tan^2\theta\right]} = \sec\theta$$

reflects the assumption that 'weathering' acts normally to the slope. Since sec θ increases as θ increases between 0° and 90°, the effect of this factor in the nonlinear models is that steeper slopes downwaste relatively faster as compared with the linear models. Since analytical solutions are only available in special circumstances, numerical solutions are generally necessary. The special cases suggested are (i) to (iii) above, the last being regarded as most realistic.

Modifications and generalisations to these nonlinear models include generalisations to allow for lithological variation and for endogenetic effects (uplift or subsidence).

Although these models are motivated by various remarks about supposed processes, they are best regarded as phenomenological. The assumption that denudation acts normally to the slope seems more appropriate for weathering-limited development than for transport-limited development.

Writing in a more general context about large scale landscape development, Scheidegger (1970, Ch. 5.8) motivated a diffusion equation

$$\frac{\partial z}{\partial t} = a \left[\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} \right]$$

by an extended analogy between landscapes and thermodynamic systems. Illustrative solutions show the decay of an idealised range and of an idealised slope bank. Modifications and generalisations of this model were discussed by Scheidegger and Langbein (1966, 3-5). The analogy remains an analogy; these models are also phenomenological.

3.3.5 Takeshita

Takeshita (1963) discussed various models for different kinds of processes and presented results of simulations obtained with finite difference schemes. The models included erosion rates normal to the slope and derivatives with respect to arc length: they have been translated here into partial differential equations of functions and derivatives expressed vertically and horizontally.

Subduing recession follows

$$\frac{\partial z}{\partial t} = \frac{\partial}{\partial x}\left[a \tan \theta\right]$$

in the case of lift-and-drop processes or the similar relation

$$\frac{\partial z}{\partial t} = \frac{\partial}{\partial x}\left[a \sin \theta\right]$$

in the case of soil creep, unconcentrated wash and 'aqueous solifluction'. Parallel recession follows

$$\frac{\partial z}{\partial t} = b \tan \theta = b\left(-\frac{\partial z}{\partial x}\right)$$

in the case of rill wash and large-scale landslides while steepening recession follows

$$\frac{\partial z}{\partial t} = \left(-\frac{\partial z}{\partial x}\right)\left[b + c\sqrt{(z_d - z)}\right]$$

in the case of head-dependent processes such as fall, gullying and mudflow. In practice these modes of recession are frequently combined.

Takeshita appears to have been unaware that analytical solutions are available for some of these equations. Their major interest lies in the process motivation supplied by Takeshita.

3.3.6 Young

Young (1963) built a set of what would now be called simulation models, although his simulations were not assisted by any computer - evidently a Herculean labour. The models are centred around a continuity principle: eight equations presented in an elaborate argument (Young, 1963, 49-50) are in essence nothing more than a single continuity equation

$$\frac{\partial S}{\partial x} + \frac{\partial z}{\partial t} = 0$$

Much of the interest of these lies in their relation to Young's field experience, together with the variety of conditions investigated. Initial slopes included a straight 35° slope with level surface above, a 35° slope with level surfaces above and below, and a level surface.

Sediment fluxes were given by the following equations

$$S = C_1 \sin \theta \qquad \text{(i)}$$

$$S = C_2 \theta^2 \qquad \text{(ii)}$$

$$S = C_3 x \sin \theta \qquad \text{(iii)}$$

$$S = C_3 x \sin \theta, \text{ distances on concavity doubled} \qquad \text{(iv)}$$

$$S = C_4 w \sin \theta \qquad \text{(v)}$$

(i) and (v) may represent creep and (iii) wash. (iv) may yield an approximation to the effects of catenary variation in texture. (ii) is an example of conditions in which sediment transport is very much faster on steep slopes than on gentle slopes.

In addition, 'direct removal' was included in some models. This term covers processes such as solution which remove material from the slope profile in a time shorter than the time step used in simulation. Assumptions employed were

$$S = C_5 \quad , \quad \theta > 0$$

$$S = C_6 \sin \theta$$

The rate of weathering was included in two models to produce relative estimates of soil depth

$$W = C_7 \, w \sin \theta$$

$$W = C_8 / w$$

A variety of slope base conditions were considered: no vertical erosion, with basal removal impeded or unimpeded; varying rates of uniform vertical erosion; and alternating periods of vertical erosion and no erosion with unimpeded basal removal. One further feature of interest was that rapid mass movement was assumed at angles above 35°, reducing the angle to 35°.

These simulation models, which have influenced many subsequent workers, contain many interesting ideas, which in general are better investigated either analytically or by computer simulation. Parsons (1976a) has recently investigated similar models.

3.3.7 Trofimov

Trofimov and his coworkers at Kazan have published several papers since 1964 on continuous models of hill-slope development, mostly in Russian. It is not possible to give here an adequate summary and assessment of earlier work.

Trofimov and Moskovkin (1976a) derived an expression defining the stable equilibrium profile of a slope developing under sheet flood erosion, which turns out to be concave. They (1976b) used a continuity equation

$$\frac{\partial S}{\partial x} = -\frac{\partial z}{\partial t}$$

for

$$S = q\,\rho$$

where q is water discharge and ρ sediment concentration

$$\rho = (ax^2 + bx + c)\frac{\partial z}{\partial x}$$

The solution was obtained using Legendre polynomials: it is a sum of an infinite series. Trofimov and Moskovkin noted an important property of this kind of solution: initial details become increasingly irrelevant.

They also derived a solution of the diffusion equation

$$\frac{\partial z}{\partial t} = \alpha \frac{\partial^2 z}{\partial x^2}$$

for the boundary condition of constant basal recession, i.e.

$$z_b = z\,(bt,\ t) = 0$$

3.3.8 Souchez

Souchez (1966a; cf. 1961, 1963, 1964, 1966b) considered the development of a hillslope profile under 'viscous' and 'plastic' mass movements. It is, unfortunately, difficult to follow his argument: in addition to several minor errors, the presentation is marred by a deep-seated confusion over coordinate systems. The following is a reconstruction.

Both viscous and plastic flow follow

$$-\frac{dv}{du} = \frac{\tau - \tau_c}{\eta} \quad , \quad \tau > \tau_c$$

where v is speed of mass movement

u depth below surface, measured normally

$\tau = \sigma u \sin\theta$ tangential stress

σ specific weight

τ_c critical stress

η coefficient of viscosity

(cf. Strahler, 1952, 926).

For <u>viscous</u> flow $\tau_c = 0$. If speed v = 0 at $u = u_0$ then

$$v = \frac{\sigma \sin\theta}{2\eta} \left(u_0^2 - u^2\right) \quad , \quad u \leqslant u_0$$

by integration, and average speed

$$\bar{v} = u_0^{-1} \int_0^{u_0} v \; du = \frac{\sigma \sin\theta}{3\eta} u_0^2$$

The sediment flux

$$S = u_0 \bar{v} = \frac{\sigma u_0^3}{3\eta} \sin\theta$$

$$= \frac{\sigma u_0^3}{3\eta} \left(-\frac{\partial z}{\partial x}\right) \quad , \quad \theta < 20^\circ$$

From continuity

$$\frac{\partial z}{\partial t} = -\frac{\partial S}{\partial x} = \frac{\sigma u_0^3}{3\eta} \frac{\partial^2 z}{\partial x^2}$$

This is the diffusion equation yet again.

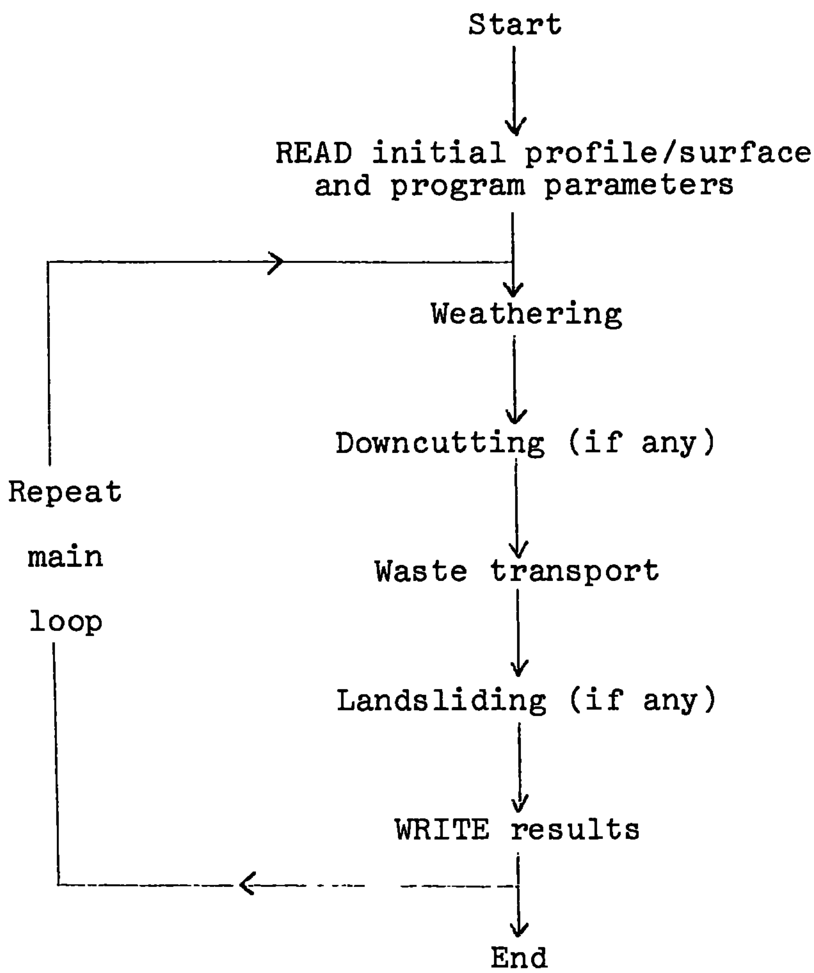
For plastic flow $\tau_c > 0$: the system exhibits a threshold. It is straightforward to obtain expressions for v, $\bar{v}$ and S but the continuity equation is valid only if the conditions for mass movement are met everywhere.

Apart from the difficulties surrounding his presentation, the problem with Souchez's model is that there seems to be little physical justification for positing either viscous or plastic flow (Scheidegger, 1970, 97-8; Young,1972, 112-3). It is interesting, however, to see another motivation for the diffusion equation, as a low-angle approximation to hillslope development under viscous flow.

3.3.9 Ahnert

Ahnert (1966, 1970b, 1971, 1972, 1972b, 1973, 1976a, 1976b, 1977; see also Mosley, 1973, Moon, 1975, 1977) has been developing simulation models of hillslope and landsurface development over the last decade. The emphasis here is on the latest versions (FORTRAN programs; COSLOP for hillslopes, SLOP3D for landsurfaces). The basic structure of both programs is similar, and each is written in a modular fashion.

Weathering rate depends on soil thickness. It may be mechanical (decreasing exponentially with thickness), chemical (increasing and then decreasing) or a combination of the two. More 'resistant' strata, either horizontal,



vertical or dipping, may be incorporated. Resistance results in reduced weathering rates. Infiltration properties may also vary with lithology.

Downcutting takesplace at one or two points. A variety of modes of baselevel lowering are possible.

Waste transport may be by splash, wash, plastic flow, viscous flow or sliding. Sediment flux for splash follows

$$S \propto \sin^{c_1} \theta$$

and for wash

$$S = r\, d^{c_2} \sin^{c_3} \theta$$

where r is a resistance variable (modelled as a power function of soil thickness) and d is depth of flow. Plastic flow depends on a threshold w'

$$S \propto (w \sin\theta - w'), \quad w \sin\theta \geq w'$$

while viscous flow occurs at all gradients

$$S \propto w^{c_4} \sin\theta$$

Debris sliding occurs so that no slope above 35° carries a regolith. (Note that rock faces above 45° are allowed).

These assumptions on waste transport (Ahnert, 1976b, 1977) are basically semi-empirical. They are a great improvement on previous sets of assumptions in earlier versions of Ahnert's models. The most obvious absentees are options for rotational failure as opposed to translational failure (cf. Hutchinson, 1968) and for gradient-dependent creep

$$S \propto \tan\theta$$

The option for viscous flow is clearly meant to be the alternative to such creep.

Assumptions about weathering and downcutting seem relatively plausible, but this is probably a reflection of present ignorance about these matters.

The great strength of these models is also their great weakness. Modular structure and versatility combine to make them attractive tools, although naturally they must be used with care. More sensitivity analyses, following Moon (1975), would be valuable.

3.3.10 Devdariani

Devdariani (1967a) drew an analogy between stream long-profile development and the conduction of heat in a thermodynamic system, thus motivating a diffusion equation

$$\frac{\partial z}{\partial t} = a \frac{\partial^2 z}{\partial x^2}$$

with general solution

$$z = \sum_{i=1}^{\infty} a_i' e^{-b_i t} \sin c_i x$$

where the a''s, the b's and the c's are constants, ordered so that $b_1 < b_2 < b_3 < \ldots < b_\infty$. As time t increases, the first term comes to dominate the series so that

$$z = a'e^{-bt} \sin cx$$

dropping the subscript $i = 1$. As $t \to \infty$, $z \to 0$.

The model was generalised to allow for spatially variable rock properties, stream discharge, etc., and for the existence of a limiting profile $z_{lim}(x)$ for which $\frac{\partial z_{lim}}{\partial t} = 0$. The solution now takes the form

$$z = z_{lim}(x) + \sum_{i=1}^{\infty} a_i' e^{-b_i t} c_i(x)$$

Devdariani suggested that models of this kind were applicable to hillslopes developing under mass movement or surface wash.

Assuming again that as t increases, terms indexed by $i \geqslant 2$ become negligible, the right-hand term becomes the product of $a'e^{-bt}$, a function of time alone, and $c(x)$, a function of distance alone. This is remarkably similar to characteristic form solutions sought by Kirkby (see below), although in the latter case, the term corresponding to $z_{lim}(x)$ is often identically zero.

The main difficulty with this model is the form of the solution which includes the sum of an infinite series. In general, these are no grounds for assuming that the b's and t are such that terms other than the first may be neglected. The form of the limiting profile $z_{lim}(x)$ is not specified theoretically, nor is it suggested how it might be determined from field data. This also applies to the functions $c_i(x)$. Moreover, the idea of a limiting profile stands in need of detailed geomorphological justification.

In a later paper, Devdariani (1967b) suggested an equation for sediment flux

$$S = -a \frac{\partial z_{rel}}{\partial x}$$

where z_{rel} is height above the limiting profile, so that

$$z(x, t) = z_{lim}(x) + z_{rel}(x, t)$$

Thus

$$\frac{\partial z}{\partial t} = \frac{\partial z_{rel}}{\partial t} \quad ; \quad \frac{\partial z}{\partial x} = \frac{\partial z_{lim}}{\partial x} + \frac{\partial z_{rel}}{\partial x}$$

The equation for S was generalised to $a = a(x)$. Using a continuity equation with an endogenetic term

$$\frac{\partial S}{\partial x} + \frac{\partial z}{\partial t} = f(x,t)$$

it may be seen that

$$\frac{\partial z}{\partial t} = -\frac{\partial S}{\partial x} + f(x,t) = -\frac{\partial}{\partial x}\left[-a(x)\frac{\partial z_{rel}}{\partial x}\right] + f(x,t)$$

with solution required for $z_{rel}(x, t)$. The model was again regarded as applicable to both hillslope and stream profiles. A series solution was obtained for general $f(x, t)$, and some special cases were investigated. This model is subject to the kind of criticisms made above, but remains of interest as a phenomenological model taking uplift into account.

3.3.11 Hirano

In addition to the papers discussed below (Hirano, 1968, 1972, 1975, 1976), Hirano has published ten other papers in Japanese.

Hirano (1968) proposed a composite linear model

$$\frac{\partial z}{\partial t} = a\frac{\partial^2 z}{\partial x^2} + b\frac{\partial z}{\partial x} + cz$$

Solutions were presented for the case of a 'finite mountain', symmetrical about a divide at $x = 0$, for the special case $c = 0$, and for a variety of boundary conditions. Lithological variations were treated by letting a, b and c become functions of x, z and t and endogenetic effects by adding a function f to the model

$$\frac{\partial z}{\partial t} = a\frac{\partial^2 z}{\partial x^2} + b\frac{\partial z}{\partial x} + cz + f(x,t)$$

In particular, the endogenetic function was assumed separable

$$f(x, t) = X(x)\,T(t)$$

Examples of T were instantaneous impulse, uniform effect and exponential decay. An example of X was a step function

$$X = 1 \quad x < x_b$$
$$= 0 \quad x > x_b$$

representing a fault scarp.

The basic equation, a second order linear partial differential equation, permits a wide variety of solutions depending on parameter values, initial conditions and boundary conditions.

Hirano suggested that the term $a\frac{\partial^2 z}{\partial x^2}$ represented creep (which is plausible); that the term $b\frac{\partial z}{\partial x}$ represented wash (thereby assumed distance-independent); and was at a loss to provide a process interpretation for the term cz. In fact, the model is essentially phenomenological, and Hirano's attempt to provide a process motivation is unconvincing. The rationale for the model is, in large part, its tractability which stems from its linearity in the derivatives.

Hirano (1972) reported some testing of this model on a fault scarp and a valley wall in the Hira Mountains, Japan.

Hirano (1975) motivated the linear model

$$\frac{\partial z}{\partial t} = a\frac{\partial^2 z}{\partial x^2} + b\frac{\partial z}{\partial x}$$

as an approximation to

$$\frac{\partial z}{\partial t} = \frac{\partial}{\partial x}\left[(a + bx)\frac{\partial z}{\partial x}\right]$$

itself regarded as a model of the combined effects of creep and wash. He presented several constant form solutions (although without reference to earlier work deriving such solutions) and some more general solutions, both analytical and numerical.

Most recently Hirano (1976) discussed a generalisation of his linear model to two horizontal dimensions

$$\frac{\partial z}{\partial t} = a\left(\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}\right) + b\left(\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y}\right) + f(x,y,t)$$

(In some applications the function f was taken to be identically zero). This equation can be transformed into a diffusion equation by substitution. Hirano paid especial attention to constant form solutions

$$a\left(\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}\right) + b\left(\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y}\right) + \alpha = 0$$

and to the relationship between channel and valley slopes.

3.3.12 Pollack

Pollack (1968) discussed various simple models briefly with special emphasis on conversion of partial differential equations to difference equations. The only original model is the uninteresting equation

$$\frac{\partial z}{\partial t} = \epsilon < 0$$

where ϵ is a random variable.

In a later and somewhat inaccessible paper Pollack (1969) proposed a modified diffusion model

$$\frac{\partial z}{\partial t} = \frac{\partial}{\partial x}\left[f_1(z)\frac{\partial z}{\partial x}\right] + f_2(x,z)$$

where f_1 and f_2 describe the characteristics of different strata influencing the rates and position of stream down-cutting and valley widening. Numerical experiments with the model attempted to simulate the development of the Grand Canyon, with some success. For an accessible summary of this model, see Harbaugh and Bonham-Carter (1970, 531-5).

3.3.13 Aronsson

Aronsson (1973) discussed a model in which denudation intensity f is a function of position f (x, z). Denudation was assumed to act normally to the slope, with transport of material so rapid that loose surficial debris does not affect the process (although in some applications a protective regolith was assumed to cover the hillslope surface). Hence the model is applicable to weathering-limited development, rather than transport-limited development.

If P (x, z) is a point initially inside the slope and t (x, z) the time elapsed before P appears at the surface, then t is given by

$$t\ (x,\ z) = \min \int \frac{ds}{f(s)},$$

where the minimum is taken over all curves which connect P and the initial profile z = z (x, 0), and where s is the arc length of the curve. This procedure resembles the use of Fermat's principle in optics (cf. Gelfand and Fomin, 1963, Appendix I). Knowledge of the function t (x, z) is in principle sufficient to describe the development of a hillslope profile. Curves of the form t (x, z) = constant describe profiles at particular times.

Unfortunately, no general method is known for calculating this function. However, a numerical procedure developed by Rydman is applicable to some special cases in which the initial profile is linear and denudational intensity (inversely proportional to resistance) is constant for each of a set of parallel horizontal layers which make up the hillside. Aronsson presented the results of a series of simulation experiments showing diagrammatically the sequences of hillslope development under various hypothetical conditions.

These results are implausible: many profiles include vertical or overhanging components, generally only possible in strong bedrock. Moreover, the procedure demands a specification of an initial profile z (x, 0) which is rarely practicable.

The central question is, however, whether the Fermat-type principle is a natural model for hillslope development. In an earlier, more difficult, paper Aronsson (1970) proved that the principle was a consequence of certain axioms. Hence the question becomes whether these axioms are appropriate. This is in doubt, particularly because of the assumption that hillslope development after some time t depends only on the profile at t (Aronsson, 1970; independence assumptions, 675; Condition D, 676).

3.3.14 Gossmann

Gossmann (1970, 1976) used a basic equation for sediment flux

$$S = a x^m \sin\theta \cos\theta + b \begin{cases} \tan\theta & \theta > \theta_c \\ 0 & \theta \leqslant \theta_c \end{cases}$$

Here the first term represents wash: the multiplying factor $\sin\theta \cos\theta$ takes a maximum at $\theta = 45^\circ$ and is intended to mimic the fact that wash erosion reaches a maximum at intermediate angles (although the peak appears to be much below 45°: Horton, 1945). The second term represents processes which are independent of distance x yet dependent on a threshold angle θ_c . This flux equation was combined with a continuity equation (unfortunately misquoted)

$$\frac{\partial S}{\partial x} + \frac{\partial z}{\partial t} = 0$$

Results from these equations for different parameter values and boundary conditions were presented graphically by Gossmann (1976). These results stem from a simulation version of the model. They were compared with firstly, semi-arid and arid conditions ('pedimentation' and surface wash assumed dominant); secondly, 'periglacial' conditions (solifluction and surface wash); and thirdly, tropical wet and dry conditions. Special equations were also employed; for example, in the periglacial case the equation

$$S = 3x^2 \sin\theta \cos\theta + \begin{cases} \sin 3\theta \cos 3\theta & 0 \leqslant \theta \leqslant 30^\circ \\ 0 & \theta > 30^\circ \end{cases}$$

is meant to mimic the decrease of solifluction caused by removal of fines by surface and subsurface wash. The term $\sin 3\theta \cos 3\theta$ reaches a maximum value at $\theta = 15^\circ$ (although this maximum is ½, not 1 as stated by Gossmann).

The most valuable feature of Gossmann's paper is the link forged between hillslope modelling and climatic geomorphology, especially as practised by Büdel and his disciples, although many of the underlying theses are accepted uncritically (cf. Stoddart, 1969). On the other hand, while hillslope development is modelled in a rational way, the form of sediment flux equations is not always motivated convincingly.

Subsequent simulation work in a similar style (Rohdenburg et al, 1976) used the basic sediment flux equations

$$S = a \tan \theta$$

$$S = a\, x^m \sin^n \theta$$

and explored the extent to which particular forms are process-specific, paying special attention to basal conditions.

3.3.15 Kirkby

Kirkby (1971, 1976a, 1976b, 1977a; Carson and Kirkby, 1972; Wilson and Kirkby, 1975) has produced some of the most valuable models of hillslope development at present available.

The first family of models (Kirkby, 1971; Carson and Kirkby, 1972, 107-9, 433-6) are best described through an example.

Sediment flux is given by

$$S = a x^{m} \left(-\frac{\partial z}{\partial x}\right)^{n}$$

and must satisfy a continuity equation

$$\frac{\partial S}{\partial x} + \frac{\partial z}{\partial t} = 0$$

whence

$$\frac{\partial}{\partial x}\left[a x^{m} \left(-\frac{\partial z}{\partial x}\right)^{n}\right] + \frac{\partial z}{\partial t} = 0$$

Kirkby sought 'characteristic form' solutions of the kind

$$z\ (x,\ t) = X\ (x).\ T\ (t)$$

i.e., the variables were assumed separable. This kind of solution is often associated with exponential decay

$$T\ (t) = c_1 e^{-c_2 t}$$

(Wilson & Kirkby, 1975, 215-7; but beware typographical errors in eqns. 6.214, 6.218, 6.224). With the boundary conditions of horizontally fixed divide and horizontally and vertically fixed base with unimpeded removal, an approximation to X(x) is given by

$$X(x) = z_d \left[1 - \left(\frac{x}{x_b}\right)^{\left[\frac{1-m}{n}+1\right]}\right]$$

or

$$\frac{z}{z_d} = 1 - \left(\frac{x}{x_b}\right)^{\left[\frac{1-m}{n}+1\right]}$$

Since both z/z_d and x/x_b are dimensionless this function gives the shape of the profile. The scale of the profiles varies as z_d since x_b is assumed constant. In this case z_d must decline exponentially for the solution to be valid: and, indeed, this is consistent with the empirical finding (but on a different scale, and over space rather than time) that denudation rate is proportional to relief (Ahnert, 1970a). Furthermore, some simulation results suggest that hillslopes may converge towards a state in which

$$\frac{dz}{dt} = -cz \quad , \quad c > 0$$

(Kirkby, 1976a, 259-60).

The special case m = 0, n = 1 yields the familiar diffusion equation, discussed by Culling and others. Conversely, it may be seen that Kirkby has generalised the diffusion equation, which now appears in its proper perspective as merely one possible transport law. Exact results available for this case encourage the contention that empirical convergence towards characteristic form solutions may be quite rapid.

Initial conditions do not need to be specified to obtain characteristic form solutions. A further point of interest is that the approximation to X (x) quoted above is a power function, discussed above as a static model.

Signal advantages of the family of models of which this is an example include, firstly, the empirical realism of the sediment transport laws for many <u>individual</u> processes such as creep and wash; secondly, the interesting and important idea of characteristic form; and thirdly, the way in which the model generalises the diffusion model. The main disadvantage of the solution obtained for X (x) is its dependence on a restrictive set of boundary conditions.

(Note that in the more general case

$$S = f(x)\left(-\frac{\partial z}{\partial x}\right)^n$$

$$\frac{\partial S}{\partial x} = f'(x)\left(-\frac{\partial z}{\partial x}\right)^n + n\, f(x)\left(-\frac{\partial z}{\partial x}\right)^{n-1}\left(-\frac{\partial^2 z}{\partial x^2}\right)$$

The n in the right-hand term is erroneously omitted from Carson and Kirkby, 1972, 433, eqn. B1)

(For applications of these models, see Kirkby and Kirkby, 1974, 1976; Richards, 1977).

The second family of models (Wilson & Kirkby, 1975, 186-8, 199-200, 216-7; Kirkby, 1976a) all centre on the sediment transport law

$$S = c_1\left(c_2 + x^2\right)\left(-\frac{\partial z}{\partial x}\right)$$

For example, with a continuity equation

$$\frac{\partial S}{\partial x} + \frac{\partial z}{\partial t} = 0$$

and constant form

$$\frac{\partial z}{\partial t} = -\alpha$$

the solution is

$$z = z_d - \frac{\alpha}{2c_1}\left[\ln\left(1 + \frac{x^2}{c_2}\right)\right]$$

(Wilson & Kirkby, 1975, 199-200).

The sediment transport law used for this family seems to be quite good at representing the combined effects of creep and wash, and leads fairly readily to a variety of solutions (other examples are given in the references cited).

The third family of models (Kirkby, 1976a, 1976b, 1977a) are exemplified by a simulation model which incorporates climatic and hydrological variables (Kirkby, 1976a). For other work including hydrological and pedological processes, see Kirkby (1976b, 1977a).

Daily rainfall falling on a hillslope profile is partitioned into overland flow, subsurface flow, evapotranspiration and change in soil water storage. Annual volumes are computed using an assumed frequency distribution of daily rainfall. Sediment fluxes for creep, wash and solution are computed as functions of hydrological and morphometric variables, and the rate of downwearing calculated from a continuity equation.

Creep (or splash and unconcentrated wash in arid areas) follows

$$S = 10 \tan \theta \qquad \text{cm}^2\ \text{yr}^{-1}$$

and soil wash

$$S = 170\, q^2 \tan \theta \qquad \text{cm}^2\ \text{yr}^{-1}$$

where q is annual overland flow flux in $m^2\ yr^{-1}$. Solution is calculated separately for each oxide using an approach developed from that of Carson and Kirkby (1972).

Simulation experiments reported were for a fixed slope base. Input was an initial slope profile, climatic parameters, and rock and soil parameters. Output included slope profiles, flow volumes, rates of lowering and soil thickness. The results included simulations of the variations in slope profile development and sediment yield with climatic parameters.

These simulation models, by explicitly incorporating climate, hydrology and pedology, represent a great leap forward in hillslope modelling.

3.3.16 Smith and Bretherton

Smith and Bretherton (1972; cf. also Smith, 1971, 1974) presented two very interesting models in an extremely important theoretical paper. Both were motivated by the example of sand-clay badlands, which have little vegetation and a fairly impermeable substrate, yet are asserted to embody the fundamental aspects of drainage basin evolution while retaining relative simplicity.

The 'smooth surface' model is aimed at the basic problem: how does an initial surface z = z (x, y, t) which slopes only in the x-direction, falling monotonically away from a divide at x = 0, evolve under rainfall-produced denudation, and how does it come to assume a form similar to that of known drainage basins?

The assumptions made in this model were set out very clearly. The substrate is homogeneous, rainfall is uniform and steady, and there are no losses of water through evaporation or infiltration. The surface z is smooth in the sense that it is (at least) twice continuously differentiable in the space domain, and (at least) once continuously differentiable in the time domain. Both water and sediment are assumed to flow downhill (rather than, for example, in the direction of the free water surface), and are constrained by continuity equations.

The key assumption, however, is that of a sediment transport law

$S = f(g, q)$

where $g = |\nabla z|$ = gradient

and S is constrained by the inequalities

$$f > 0, \quad \frac{\partial f}{\partial g} > 0, \quad \frac{\partial f}{\partial q} > 0 \quad \text{for} \quad g, q > 0$$

The operator ∇ is the directional differential operator which yields the vector field of maximum gradient when applied to landsurface altitude z. Taking the modulus gives the scalar field of maximum gradient. $|\nabla z| = g$ would equal $\frac{\partial z}{\partial x}$ if and only if $\frac{\partial z}{\partial y} = 0$, that is, there is no plan curvature.

This model was specialised first to one horizontal dimension to solve for topographic profiles $z = z(x, t)$ with the boundary conditions that no water or sediment crosses the divide. Solutions obtained were, firstly, 'constant form' solutions for which $\frac{\partial z}{\partial t} = -\alpha$; the conditions under which these are convex or concave were investigated; secondly, time-dependent solutions for the special case

$S = c\, q^m g^n \quad ; \; m, n > 0$

provided that there is no sediment sink

$$\int_0^\infty z \, dx = \text{constant}$$

and that $2n \neq m$, $n + 1 \neq m$.

The solutions have the form

$$z = c_1 t^{-c_2} \left[1 - \left(\frac{x}{x_b}\right)^{\frac{1-m+n}{n}}\right]^{\frac{n}{n-1}} \quad ; \quad 0 \leqslant x \leqslant x_b$$

$$= 0 \quad ; \quad x > x_b$$

where $c_2 = 1/(2m - n)$

$x_b = c_3 t^{c_2}$

and the constants c_1, c_3 reflect the no sediment sink constraint.

Roughly speaking, it seems that if $0 < m < 1$ there will be an inflexion of the slope profile, but if $m > 1$, the profile will be everywhere concave. Smith and Bretherton interpreted these solutions as models of stream channels and noted that they imply continuous declining development: steady state is not achieved because the time-dependent term never vanishes.

The problem of initiation of channel-like features was then treated using the method of stability analysis which is standard in hydrodynamics (e.g. Chandrasekhar, 1961; see also Allen, 1970, 61-5). The central question is whether constant form solutions are stable under perturbations of infinitesimal amplitude. In one horizontal dimension it turns out that for any transport law $S = f(g, q)$ the constant form solution is stable: for example, knickpoints (of infinitesimal amplitude) are always removed. In two horizontal dimensions, the result is that channel-like forms must grow on concavities, but will disappear on convexities. Smith and Bretherton suggested that these stability results may be extended from the constant form solutions to the more general time-dependent solutions.

According to Smith and Bretherton, a transport law, to be realistic, should have an associated constant form surface which is convex in the upper portion and concave in the lower portion: it is then termed a 'landscape-forming transport law'. Such a law should be able to account in broad terms for the existence of a stable channel network, which reflects an approximate balance between positive feedback processes which cause the necessary instability for channel growth and negative feedback processes which give the necessary stability to check unbridled channel growth.

A 'discrete channel model' is necessary, according to Smith and Bretherton, because neither analytical nor computer methods draw results on the later stages of channel development from the smooth surface model. The aim is to model a continuous surface consisting of discrete streams and valleys, but this involves a major difficulty: the need to specify a law of lateral channel migration.

Given that fluxes of water and sediment enter a channel from both side slopes, how does the stream move sideways? Or does it remain approximately fixed horizontally, downcutting in the same place? Smith and Bretherton hypothesised that a stream moves away from the side slope contributing the larger sediment flux, and so there is a tendency to equalise fluxes.

A model of a two-valley system, with side slopes at an angle of repose, and relatively straight streams with channel slopes much lower than side slopes, was subjected to a stability analysis. The constant form solution turned out to be unstable to perturbations of infinitesimal amplitude. On the other hand, a model with horizontally fixed streams produced embarrassing results:

small streams become perched high up on the valley side slopes of other streams. Since neither model seems very realistic empirically or very well developed, the problem of lateral channel migration remains unsolved.

Both the approaches and the results set out in this important paper merit long discussion. The use of a general model is linked with the aim of explaining typical or quintessential fluvial topography in qualitative terms. It follows that the Smith and Bretherton models cannot be employed to explain differences in hillslope form over space or time. However, it seems pointless to condemn the models for failing to fulfil an aim for which they were not designed.

The use of the transport law S = f (g, q), together with other features of the model, was motivated by badland situations in which surface wash is the dominant geomorphological process. Despite this, the results of the models seem to carry over to a large extent to situations in which other transport processes are important.

The use of constant form solutions is probably the greatest weakness of the model, and needs greater justification. Moreover, it is unclear how far stability under perturbations of infinitesimal amplitude implies stability under perturbations of finite amplitude. The only certainty is that the mathematics are far more difficult (Lewontin, 1969).

All in all, however, these models are among the best now available, not least in the links they forge between profile modelling and surface modelling.

3.3.17 Luke

Luke (1972, 1974, 1976) is a mathematician interested in geomorphology. In a recent series of papers he has shown some results of a qualitative, geometric approach.

Luke (1972) considered both the general case of a landsurface z = z (x, y, t) developing according to

$$\frac{\partial z}{\partial t} = -f\left(\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, z\right)$$

and the special case of a hillslope profile developing according to

$$\frac{\partial z}{\partial t} = -f\left(\frac{\partial z}{\partial x}, z\right) \quad \text{or} \quad \frac{\partial z}{\partial t} = -f\left(\frac{\partial z}{\partial x}\right)$$

Here f is an 'empirically determined function', although in Luke's examples it is always a phenomenological function of uncertain origin.

Luke showed that, given an initial profile or surface, these equations may be solved for any time t using the method of characteristics. This is something of a Pyrrhic victory, since the determination of f and the initial profile are generally nontrivial. Luke suggested a method for determining f from actual profiles but this depends on knowing the time t. Moreover, solving by characteristics may yield multivalued solutions (vertical or overhanging slopes): this difficulty is met by choosing the lowest z value, a procedure which requires more justification (but cf. Luke, 1976, for a note on multivalued profiles).

The landsurface model is interesting but extremely crude. Stream downcutting takes place on predetermined paths in the (x, y) plane and is combined with hillslope evolution.

In one example, Luke pointed out that after a certain time the profile depended only on the function f: a result which is relevant in particular to Aronsson's axioms (see above).

Luke (1974) considered a surface developing according to

$$\frac{\partial z}{\partial t} = -f\,(g, q, S)$$

where f, again, is an 'empirical function' and g is gradient $|\nabla z|$.

This model is a generalisation of one developed by Smith and Bretherton (1972) and applies mainly to surfaces subject to surface wash. Unfortunately, Luke's treatment of it fails between two stools. The general discussion is vague about geomorphological implications, while special solutions are given only for implausible situations which lack interest. Finally, not a word is said to motivate the constant form solutions which are derived.

3.3.18 Huggett

Huggett (1973a, 21-8; see also 1973b, 1975) put forward a model for landsurface development in the wider context of work on 'soil-landscape systems'. The key equation is

$$\frac{\partial z}{\partial t} = \frac{\partial}{\partial x}\left[D_x \frac{\partial z}{\partial x}\right] - \frac{\partial}{\partial x}\left[v_x z\right] + \frac{\partial}{\partial y}\left[D_y \frac{\partial z}{\partial y}\right] - \frac{\partial}{\partial y}\left[v_y z\right]$$

where D_x, D_y are diffusion coefficients, and v_x, v_y are speeds (Huggett wrote 'velocities') of bulk flow in x,y directions. Huggett suggested putting $v_x = -\frac{\partial z}{\partial x}$, $v_y = -\frac{\partial z}{\partial y}$ (which would be incorrect dimensionally) and he modified the model to allow D_x and D_y to be functions of x, y and z.

This model is a nonlinear diffusion equation with mass transport terms. It must be solved numerically, especially in its more complex form. It is best classed as phenomenological, since although Huggett interpreted various terms of the complex model in geomorphological terms not all the effects are very plausible. Apart from some simulation results (including the case of a heterogeneous bedrock) the model has been little developed or applied.

3.3.19 Mizutani

Using physical principles, Mizutani (1974) proposed two equations for the depth of erosion or deposition measured normal to the surface in conditions where surface wash and mass wasting are the dominant processes.

$$d_e = \frac{d}{dl}\left[a\, l^m \sin^n \theta\right] \qquad \text{(i)}$$

where l is slope length measured along the surface

$$d_e = a\, l^m \left(\sin\theta - \sin\theta_c\right)^n \qquad \text{(ii)}$$

where θ_c is a critical angle.

These equations were manipulated to yield partial differential equations for hillslope development. (i) becomes

$$\frac{\partial z}{\partial t} = b_1 x^m \frac{\partial^2 z}{\partial x^2} + b_2 x^{m-1} \frac{\partial z}{\partial x}$$

with the simplifying assumptions (a) $l = x$; (b) $\left(\frac{\partial z}{\partial x}\right)^2 = \tan^2 \theta$ may be neglected; (c) $n = 1$. Assumptions (a) and (b) both imply gentle slopes.

In the case $m = 2$ the equation may be solved analytically using some cunning substitutions. In fact, a different manipulation using only assumptions (a) and (b) would yield

$$\frac{\partial z}{\partial t} = -\frac{\partial}{\partial x}\left[a x^m \tan^n \theta\right]$$

which resembles a model proposed by Kirkby (1971).

Using (a) (b) (c) equation (ii) reduces to

$$\frac{\partial z}{\partial t} = a x^m \left(\frac{\partial z}{\partial x} + \sin \theta_c\right)$$

Mizutani suggested a generalisation

$$\frac{\partial z}{\partial t} = f_1(x,z)\, x^m \left[\frac{\partial z}{\partial x} + f_2(t)\right] + f_3(x,z)\frac{\partial z}{\partial x}$$

which may be solved using the method of characteristics.

These models were modified to produce upslope convexities, divide recession, plateau dissection and radial valley development, and to allow for lithological variation: in short, a variety of interesting applications.

A further point to commend is the representational character of the models, which are quite closely related to physical principles.

Earlier work by Mizutani is quoted and summarised in his 1974 paper. For later work in Japanese, see Mizutani (1976).

3.3.20 Grenander

Grenander (1975; 1976, 402-11) considered the height of a landsurface z as a function of location on a circle x of unit circumference, and of time t. The landsurface is considered to be the result of interaction between an erosion mechanism given by a diffusion equation

$$\frac{\partial z}{\partial t} = a \frac{\partial^2 z}{\partial x^2}$$

and a tectonic mechanism which operates instantaneously at discrete time points. The tectonic mechanism is a Poisson point process in time, and a set of independent stationary periodic processes in space. The total mass of the landform

$$\int_0^1 z \, dx = \text{constant}$$

which implies that 'average uplift'

$$\int_0^1 f(x) \, dx = 0$$

Grenander showed that the stochastic partial differential equation yielded by combining erosion and tectonic mechanisms defines a height field in statistical equilibrium. He derived expressions for time and space autocovariance functions, carried out some simulation experiments, and discussed the estimation of pattern parameters from spatial data and optimal retrospection of past topographical profiles.

Although elegantly and rigorously developed, Grenander's model is not particularly instructive. The assumptions are phenomenological and of dubious realism. The treatment of a circular profile is difficult to relate to other work. On a naive view, the finding of statistical equilibrium is not surprising given that total mass is conserved. Finally, all the important closed form results contain sums of doubly infinite series which would be difficult to use in practice.

Freiberger and Grenander (1977) considered the case of a surface developing according to a stochastic partial differential equation under the constraint of constant total mass. The basic process is a two-dimensional diffusion mechanism with two forcing terms, one additive random noise, the other a tangential force field representing drag forces supposed to act on the surface from the interior of the earth. According to the latter the occurrence, amplitude and direction of point disturbances are all random variables, and these disturbances are propagated over the surface according to a specified influence function. Analytical results were given for a square in the (x, y) plane, which was mapped on to a torus to avoid boundary effects. These results cover stochastic characteristics of various fields, optimal data compression and noise suppression, and the statistical geometry of random surfaces. The case of a height field on a sphere was also examined.

The planar case was simulated with a discrete space, discrete time FORTRAN program. The initial surface was flat.

The authors did not supply any detailed physical justification of their assumptions, but invited comments from geologists and geographers on the validity of their model. It does seem to bear very little relation to current ideas on geomorphological processes, whether exogenetic or endogenetic.

3.3.21 New results

In this section some new results are presented which extend Kirkby's models.

Constant form solutions of the form

$$\frac{\partial z}{\partial t} = -\alpha$$

can be derived for the equations

$$\frac{\partial S}{\partial x} + \frac{\partial z}{\partial t} = 0$$

$$S = a\, x^m \left(-\frac{\partial z}{\partial x}\right)^n$$

Combining equations

$$\frac{\partial}{\partial x}\left[a\, x^m \left(-\frac{\partial z}{\partial x}\right)^n\right] = \alpha$$

Integrating, and using a boundary condition S = 0, x = 0,

$$a\, x^m \left(-\frac{\partial z}{\partial x}\right)^n = \alpha x$$

Rearranging

$$\frac{\partial z}{\partial x} = -\left(\frac{\alpha}{a}\right)^{\frac{1}{n}} x^{\frac{1-m}{n}}$$

Integrating again, and using the definition $z = z_d$, $x = 0$

$$z = z_d - \left(\frac{\alpha}{a}\right)^{\frac{1}{n}} \left(\frac{1}{k}\right) x^k$$

where $k = \frac{1-m+n}{n}$.

Using the definition $z = 0$, $x = x_b$

$$0 = z_d - \left(\frac{\alpha}{a}\right)^{\frac{1}{n}} \left(\frac{1}{k}\right) x_b^{\,k}$$

$$z_d = \left(\frac{\alpha}{a}\right)^{\frac{1}{n}} \left(\frac{1}{k}\right) x_b^{\,k}$$

Dividing through

$$\frac{z}{z_d} = 1 - \left(\frac{x}{x_b}\right)^k$$

This constant form solution, which is exact, is equal to the approximation derived by Kirkby (1971) to the space-dependent function which appears in the variables-separable solution

$$z\ (x,\ t) = X\ (x)\ T\ (t)$$

3.3.22 Connective summary

A historical review of ideas and results in dynamic hillslope modelling over the last sixty years must clearly be supplemented by a connective summary. This is attempted here in a cross-classification of dynamic models (3C) and

3C Dynamic models: a cross-classification

Name	Date of first key publication discussed	Country	Deterministic or stochastic	Phenomenological or representational	Analytical or simulation
Jeffreys [1]	1918	U.K.	D	R	A
de Martonne & Birot [2]	1944	France	D	P	A
Culling [3]	1960	U.K.	Both	P	A
Scheidegger [4]	1961	now Austria	D	P	A
Takeshita [5]	1963	Japan	D	R	S
Young [6]	1963	U.K.	D	Mixed	S
Trofimov [7]	1964	U.S.S.R.	D	R	A
Souchez [8]	1966	Belg.	D	R	A
Ahnert [9]	1966	now B.R.D.	D	R	S
Devdariani [10]	1967	U.S.S.R.	D	P	A
Hirano [11]	1968	Japan	D	P	A
Pollack [12]	1968	U.S.A.	D	P	S

Aronsson [13]	1970	Sweden	D	P	A
Gossmann [14]	1970	B.R.D.	D	P	S
Kirkby [15]	1971	U.K.	D	R	Both
Smith & Bretherton [16]	1972	U.S.A.	D	R	A
Luke [17]	1972	U.S.A.	D	P	A
Huggett [18]	1973	U.K.	D	P	S
Mizutani [19]	1974	Japan	D	R	A
Grenander [20]	1975	U.S.A.	S	P	Both
Mercier	1976	France	Insufficient information available		
Cox [21]	this thesis	U.K.	D	R	A

Numbers in brackets refer to sub sections in this chapter in an index of particular topics (3D). Note that it is not always easy to decide which category should receive each model in 3C. In particular, the categories 'phenomenological' and 'representational' intergrade continuously. The dictionary in 3D is not exhaustive of the topics covered by particular workers, but it does serve as a systematic guide to ideas and results of value.

Some general remarks on dominant themes are in order by way of conclusion.

One recurrent assumption is that hillslopes may be treated as self-modifying geometric systems. While such an assumption is a mathematical version of the basic idea that process and form are related by feedback (cf. Ch. 2.4.3 above), it has the important consequence that distance, height and especially gradient are regarded in most models as the major controls of processes. By contrast, independent variations in climate, hydrology and soil properties have received little attention. Lithological properties have been incorporated in several models, but generally in an extremely crude manner (cf. Ch. 2.4.7 above).

Failure and solution are the most frequently neglected processes. In the case of failure, neglect can be attributed to intractability: threshold-dependent processes cannot be handled easily in models based on differential equations (cf. Ch. 2.4.4 above). In the case of solution, ignorance is probably the major cause of neglect.

3D Dynamic models: a topical index

Topic	Models
angle or gradient as variable	1-12, 14-21
boundary conditions and endogenetic effects	2-4, 6-7, 9-11 15-16, 19-20 Rohdenburg
circular profile	20
climate and hydrology	14-15
constant form	1, 4, 11, 15-17, 21
continuity principle	6-8, 10, 14-16, 21
creep and solifluction	3, 5-6, 10-11, 14-15, 19, Rohdenburg
diffusion	3-4, 7-8, 10-12, 15, 18, 20
distance as variable	6, 14-15, 19, 21, Rohdenburg
failure	5-6, 9
height as variable	4-5, 11-13, 17-19
historical interest	1-2
limiting profile	10

Topic	Models
lithology	4, 9-13, 15, 18-19
method of characteristics	17, 19
profile shape	15
qualitative results	16-17
soil properties, weathering rate	2, 6, 8-9, 15
solution	6, 15
splash	9, 15
stability	1, 7, 16
surface development	1, 3-4, 9, 11, 16-18, 20
variables-separable solutions	10, 15
variational approach	13
viscous and plastic flow	8-9
wash	1, 5-7, 9-11, 14-17, 19, Rohdenburg
weathering-limited development	4, 13

Continuous hillslope development models have usually been devised for transport-limited situations: weathering-limited situations have received little attention.

Continuity equations are a firm and indispensable basis for hillslope modelling, even though the value and impoбtance of continuity principles has only become fully apparent quite recently.

Two key ideas, constant form and stability properties, although introduced in the earliest work in this field, have been rediscovered since 1972 (cf. Ch. 2.3.1 above). The strategy of seeking robust qualitative results is a striking innovation in a field which has until recently been characterised by a marked lack of mathematical sophistication.

While many modellers of hillslope development have also addressed the more challenging and more general problem of surface development, relatively little progress has been made on this front, largely because plan curvature effects, slope endpoint behaviour and network development remain poorly understood (cf. Ch. 2.4.6, 2.4.8 above).

There is little relationship between static and dynamic models of hillslope profiles: the major exception is that power functions arise as equilibrium (invariant) solutions to some dynamic models.

Most dynamic models have been deterministic. Ideas of spatial stochastic processes introduced quite recently have failed to yield important new insights, although the use of stochastic errors in model fitting remains an interesting possibility (cf. Ch. 2.3.2 above, Ch. 9 below).

Both analytical and simulation approaches are of value. Simulation is necessary to handle thresholds and frequency distributions, although simulation should be accompanied by sensitivity analyses (cf. Ch. 2.3.4 above).

Despite increasing knowledge about process rates, controls and mechanisms, phenomenological models remain popular. If models are to be of explanatory value, their process content must be exposed to searching critical examination (cf. Ch. 2.3.3 above).

3.4 Notation

a, b, c	constants or functions with local meaning
B_H	fractional Brownian function
d, d_e	depth of water, depth of erosion
d	in ordinary derivative or in integral
D	diffusion coefficient
e	base of natural logarithms, 2.71828...
f, f'	function, its derivative
g	gradient
H	parameter of B_H
i	subscript
k	Kirkby parameter
l	length
m, n	exponents
N (,)	Normal (Gaussian) distribution
p	integer power in polynomial
P, P', P"	points in space
q	discharge
r	resistance
s	arc length
S	sediment flux
t	elapsed time
T	function of t only
u, u_o	depths normal to surface
v, $\bar{v}$	speed, average speed
w, w'	regolith depth, threshold
W	rate of weathering
x, x_b	horizontal coordinate, of slope base

X	function of x only
y	horizontal coordinate
z, z_b, z_d	vertical coordinate, of slope base, of divide
z_{lim}, z_{rel}	height of limiting profile, height above it.
α	rate of downwearing
δ, Δ	difference operator
ϵ	random variable
η	viscosity
θ, $\bar{\theta}$, θ_c	angle, average angle, critical angle
ρ	sediment concentration
σ	specific weight
Σ	summation operator
τ, τ_c	tangential stress, critical value
$\propto$	is proportional to
∂	in partial derivative
∇	directional differential operator
∞	infinity
$\sim$, $\approx$	is drawn from, approximately equals
$\int$	in integral
$\vert\ \vert$	modulus
$\rightarrow$	tends to

Chapter 4

THE FIELD AREA

The nobly silent hills loom up on high
In peace that stills my question whence or why.

Goethe, *Faust*, Pt. II, Act IV.

4.1. Introduction

The field area in which slope profiles were measured for this thesis is the 10km x 10km grid square SE 59, which lies in the western part of the North York Moors. It is approximately bisected by the valley of Bilsdale. The area was chosen mainly for its relatively simple geological and geomorphological history.

The general character of the Jurassic upland was well described by Fox-Strangways (1892, 408). 'The Jurassic rocks of Yorkshire form an isolated range of hills cut off from the rest of the county, and from the elevated ground composed of other geological formations, by a series of large valleys, which form the great lines of drainage of this part of England. From their peculiar geological construction these hills present a bold front to the north and west, overlooking the great plains of the Tees and the Ouse, while to the south and east they gradually fall away to low ground beneath the escarpment of the Wolds, or are cut off by the sea'.

In this chapter, a review of knowledge of geological and geomorphological history is presented for the field area, drawing where appropriate on studies made in other parts of the Moors and on field observations. The criterion of relevance is that geological and geomorphological findings should have direct or indirect implications for hillslope profile morphometry.

4.2 Geological background

4.2.1 Jurassic strata

All the solid rocks of the field area date from the Jurassic Period (190-140 million yr). They were mapped for the Geological Survey by Fox-Strangways *et al* (1885, 1886, 1892): the resulting Memoirs remain the most valuable accounts for geomorphologists, despite a large volume of subsequent work (cf. Hemingway, 1974 for a comprehensive review).

Salient characteristics of the various Jurassic formations are given in 4A, which is based on Geological Survey sources, except that the Estuarine Beds have been termed Deltaic (Hemingway, 1949). The Middle Jurassic nomenclature proposed by Hemingway and Knox (1973) is not employed because it cannot be correlated exactly with Geological Survey mapping units. It is, however, clearly preferable to the older terminology.

One important characteristic of the geological succession is the marked vertical and lateral variability found within formations. The lithological descriptions given in 4A are generalised: the examples of vertical variability given in 4B provide a counterweight to such generalisation.

4A

<u>Jurassic formations found in the study area</u>

Formation		G.S. symbol	Lithology (generalised)	Thickness (m) West (G.S. Sheet 42)	Thickness (m) East (G.S.Sheet 4[illegible]
Middle Calcareous Grit	MIDDLE OOLITE	g^{11^4}	sandstones	0-7	10
Lower Limestone	MIDDLE OOLITE	g^{11^3}	limestone	0-24	16
Lower Calcareous Grit	MIDDLE OOLITE	g^{11^1}	sandstones, shales	30-59	30-46
Oxford Clay	MIDDLE OOLITE	g^{10}	sandy shales	23-25	14-29
Kellaways Rock	MIDDLE OOLITE	$g^{\underline{10}}$	sandstones, shales	19-22	14-29
Cornbrash	LOWER OOLITE	g^{9}	limestone and sandstone	0	2
Deltaic Beds	LOWER OOLITE	$g^{5^{1111}}$	shales with sandstones; at base massive sandstones	41	70
Grey Limestone Series	LOWER OOLITE	$g^{5^{111}}$	shales with siliceous and calcareous bands and gritstones	14	6
Eller Beck Bed*	LOWER OOLITE	$g^{5^{\underline{11}}}$	flaggy sandstones, shales, iron-		
Deltaic Beds	LOWER OOLITE	$g^{5^{11}}$	stone bands; sandstones and shales, some fireclays and thin coals	70	79
Dogger	LOWER OOLITE	$g^{5^{1}}$	sandstone or ironstone or limestone	0-7	0-10

4A (continued)

Formation		G.S. symbol	Lithology (generalised)	Thickness (m) West (G.S. Sheet 42)	Thickness (m) East (G.S. Sheet 43)
U. Lias Alum Shale	LIAS	g^{3111}	shales	62-70	59-68
U. Lias Jet Rock	LIAS	g^{311}			
U. Lias Grey Shale	LIAS	g^{31}			
M. Lias Ironstone Series	LIAS	g^{211}	shales with ironstone bands	16	22
M. Lias Sandy Series	LIAS	g^{21}	thin flaggy sandstones and sandy shales	23	23
Lower Lias	LIAS	g^{1}	sandy shales with hard bands	185	60

* Occurs within g^{511}

Sources: Geological Survey Memoirs and maps

4B Examples of sections in Bilsdale showing lithological variability

1. Novey House, Ladhill Beck, in Grey Limestone Series

	in.	cm. (rounded)
Shales with fossils		
Grit		
Hard, sandy siliceous beds	36	90
Sandy shales	60	150
Siliceous and calcareous beds	48	120
Shales		

2. Blow Gill Farm, in Eller Beck Bed

Sandstone		
Shale	60	150
Thin ironstone	4	10
Shale	36	90
Ironstone	6	15
Shale		

3. Tarn Hole Beck, in Ironstone Series

Ferruginous shale	60	150
Ironstone	5	13
Shale	24	60
Ironstone	15	38
Shale	42	105
Sandy band	4	10
Shale	48	120
Ironstone	4	10
Micaceous sandstone	24	60
Shale	144	365
Ironstone	21	53
Shale	60	150

Sources: Geological Survey Memoirs

4.2.2. Structural history

The structural history of the North York Moors has been relatively simple (cf. Kent, 1974).

The dominant post-Cretaceous movement has been easterly tilting (cf. Smalley, 1967). As the top of the Chalk subsided beneath the Tertiary in the North Sea basin, the Pennine area correspondingly rose by 500 to 700 m. This movement has been regarded as contemporary with Alpine orogenesis, and thus essentially Miocene, but the evidence from the North Sea supports a picture of continuing subsidence and it thus seems likely that the rise of the Pennine area was a long-continued epeirogenic movement initiatedin the early Tertiary.

In addition, the area of the Moors has suffered anticlinal warping with an amplitude on the Dogger of at least 500 m. relative to the Derwent Valley syncline to the south. The crest of the fold is divided into separate culminations by crossing meridional trends. A series of domes may be distinguished from west to east along the Cleveland anticline: the Chop Gate dome, the Danby Head dome, the Egton dome and the Robin Hood's Bay dome. The first of these takes its name from Chop Gate (559996) in SE 59.

Taking account of uncertainty about magnitude and spatial variation, it can be seen that post-Cretaceous uplift in the field area has been of the order of 100 to 1000 m, an average of some 1.5 to 15 mm per 1000 yr.

Known faults are few in number and relatively minor.

4.3 Geomorphological interpretations

4.3.1 The influence of lithology

Archibald Geikie was in little doubt about the influence of structure and lithology when he wrote his prefatory Notice to the Eskdale and Rosedale Memoir (Fox-Strangways *et al*, 1885).

'. . . This region may be regarded as a vast model exemplifying, in a striking manner, the relations of topographical feature to the nature and disposition of the rocks underneath. The strata being nearly horizontal and little disturbed by dislocations, the valleys radiating from the tableland can be traced out as the results of erosion, with a precision and completeness unattainable in other districts of the country where the geological structure is less simple'.

Fox-Strangways (1892, Ch. 17; 1894) argued the case in greater detail. He regarded the forms of hills and valleys in the Jurassic upland as 'entirely due to sub-aerial agents' (1892, 411) and related large-scale forms, such as the drainage pattern, to structure, and small-scale forms to lithological variation.

'. . . The main features of the district . . . occur where there is the greatest geological difference between succeeding strata, for instance where a thick bed of porous sandstone succeeds to a considerable thickness of shale; by the weathering away of which the rock stands out in a bold feature overlooking the beds below. The thickest beds of shale are the Lias and Oxford Clay, and therefore it is just above their outcrop that we get the main features of the district' (1892, 417).

Broadly similar views have been put forward by Elgee (1912, Ch. 12), Palmer (1973, Ch. 4) and de Boer (1974, 281). Palmer (1956) examined the relationship between differential weathering and tor formation at the Bridestones (SE 8791) in the eastern Tabular Hills. The only quantitative study of the relationship between lithology and relief, however, is the work of Gregory and Brown (1966) in Eskdale. They discussed areal frequency distributions of slope angle for different geological formations, derived from a comparison of a morphological map with Geological Survey sheets. Weighted mean angles and resistance values from their paper are reproduced in 4C.

Although these results are extremely interesting, the methods and interpretations of Gregory and Brown require criticism on several grounds.

(i) Morphological mapping is an unsatisfactory method of data collection. It is predicated on the assumption of geomorphological atomism (see Ch. 8.2 below), it lacks replicability and it ignores internal variability.

(ii) As Doornkamp and King (1971, 126) pointed out more generally, comparison of a morphological map and a geological map 'is only a helpful exercise if it is known that the geology was not mapped from surface form in the first instance. "Feature mapping" of geology is bound to

4C

Weighted mean angle and resistance for various geological formations in parts of Eskdale according to Gregory and Brown (1966)

Formation	Weighted mean angle (deg)	Resistance
Kellaways	2.26	High
Cornbrash	4.92	Very high
Upper Deltaic	3.36	Moderate
Moor Grit	3.00	Very high
Grey Limestone	3.92	Very high
Middle Deltaic	5.03	High
Eller Beck	12.95	Very high
Lower Deltaic	7.11	High
Dogger	22.24	Very high
Alum Shale		Low
Jet Rock	9.32	Moderate
Grey Shale		Low
Ironstone Series	4.59	High
Sandy Series	6.49	High
Lower Lias	8.72	Moderate

lead to the conclusion that there is a clear relationship between geology and slope form. Where there has been sufficient confidence in this relationship for feature mapping to have taken place, the correlation between form and geology may indeed be close. Feature mapping of the geology tends, however, to overstate a relationship which may not always be so precise in reality'.

(iii) The resistance values given by Gregory and Brown, without any explanation of derivation, are open to objection. Presumably they are based on impressions obtained in the field of physical characteristics of the various rocks. Some shales, for example, may literally fall to pieces in the hand, while even well weathered grits can be broken only with difficulty. While such impressions are unlikely to be misleading, they may be irrelevant. Where a soil cover is present, 'resistance' properties are pedological, not lithological, and the very idea of a single measure of resistance is dubious since several different processes are operating (cf. Ch. 2.4.6 above).

(iv) Neither the strength of the relationship between lithology and slope form, nor the possible variation in the relationship with scale, is explored in any detail.

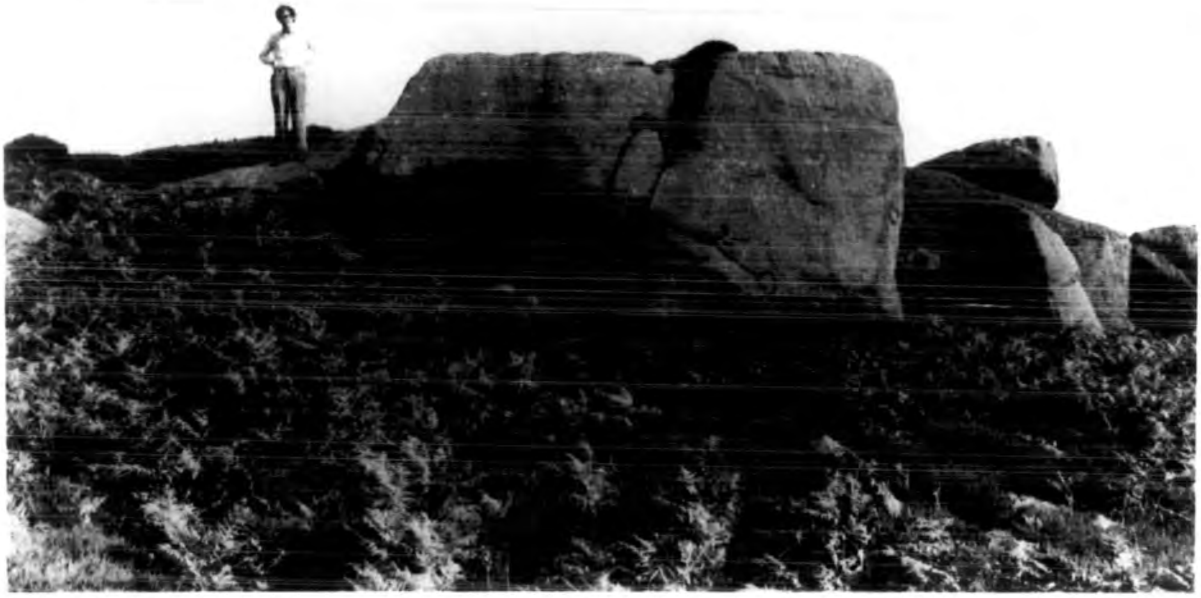
(v) The existence of particular modal or characteristic angles is not demonstrated convincingly (cf. Speight, 1971, for a fuller discussion of this issue).

(vi) Gregory and Brown tend to play down lithological variation within formations (for example, their Fig. 7

misleadingly describes each mapping unit as either shale, or sandstone, or limestone, or grit: qualifying symbols denoting variable succession or lateral variation do not sufficiently offset the impression of uniformity).

(vii) Palmer (1973, 45) criticised Gregory and Brown for neglecting the position of the rock in relation to the streams, a major control in 'an area that is still in a relatively youthful stage of landform evolution'. For example, moderately steep slopes occurring in the relatively weak Lower Lias can be explained by their proximity to recent river incisions in some valley floors. While Palmer's criticism may be couched in questionable terminology, this kind of relationship must be remembered. However, the position of strata in itself explains nothing: what are important are the processes associated with that position.

Some field observations in SE 59 throw light on the question of lithological influence and can best be considered in terms of scale. Much of the lithological variation is on a microscale, and it finds morphological expression at that scale. This is most clearly seen in stream long profiles. Hard sandstone or gritstone bands frequently outcrop as small waterfalls or rapids, commonly 1 m - 4 m in height (e.g. Blow Gill, 528932; Tarn Hole, around 593981; Arns Gill, 533965; Proddale confluence, 518970; Ladhill Beck, 548925). Valleyside crags and tors



4D Valleyside tor above Tarn Hole

4E Escarpment of the Hambleton and Tabular Hills

4F Streamside bluff in Tripsdale

(e.g. above Tarn Hole, 4D) are frequently larger features, but the essential point remains valid: 'features' attributable to lithological variation are often microforms when compared with complete profiles.

On a larger scale, the relation between relief and mapping units is clear in the broad contrast between escarpments emphasised especially by Fox-Strangways (1892, 414). In particular, the Middle Oolite corresponds to the escarpment of the Hambleton and Tabular Hills (illustrated in 4E). What remains least clear, ironically enough, is the importance of lithological variation at the scale of the hillslope profile: a major task of a morphometric approach to hillslope profiles is to throw some light on this issue.

Finally, two bluffs cut in the Upper Liassic shales in Tripsdale (584995) provide small illustrations of Palmer's criticisms (one is shown in 4F). Although apparently very unresistant, these bluffs are steep solely because of their position, being undercut by the stream.

4.3.2. Supposed planation surfaces (see 4G and 4H)

Davis (1895), writing on 'The development of certain English rivers', regarded the topography as the result of subaerial rather than marine denudation, and suggested that it was in the mature stage of a second cycle of denudation. However, the North York Moors, as only a

4G Bilsdale from the east, showing the subhorizontal interfluve

4H Incised valley in Tripsdale, of the kind attributed to rejuvenation small part of the area discussed by Davis, received but passing reference. A more detailed interpretation of 'The geological history of the rivers of east Yorkshire' was given by Reed (1901), who postulated six cycles of denudation, of which not all were cycles in the Davisian sense. Superimposition of the drainage from a Cretaceous cover was followed by early Tertiary planation. Versey (1939) regarded much of the upland surface of the Moors as part of a 'Wolds Peneplane [sic] ', the very highest parts standing above the Peneplane as monadnocks. He also placed planation in the early Tertiary, followed by uplift, warping and faulting. Hemingway (1958, 24; 1966, 16), on the other hand, considered planation to have been marine, although he did not argue this case in any detail.

Peel and Palmer (1955) introduced a dissenting note. Since the 'peneplain' truncates the Cleveland Dyke, dated by Dubey and Holmes (1929) at 26 million years (but see below), they inferred a late Tertiary age, thus producing an interpretation similar to that suggested for the 'peneplain' of southern England. They also suggested that uplift of the peneplain was discontinuous.

Palmer (1967) later showed willingness to entertain a hypothesis of marine planation. 'There is no strong argument against most of the surfaces having an initially marine origin . . . the absence of contemporary marine or, for that matter, residual land deposits is no embarrassment, except to those who hold the conservative view that the upland surfaces coincide exactly with reconstructed Tertiary surfaces' (1967, 17).

Palmer (1973) has recently extended this view, producing an imperfect echo of Davis nearly eighty years earlier: '. . . The main features of the relief may be said to result from the incision of streams during the early part of the present uncompleted cycle of erosion into a peneplain produced towards the end of a previous cycle' (1973, 22). The peneplain on the Moors is 'certainly of late-Tertiary age' (32) since it truncates mid-Tertiary folds and fault-produced irregularities.

However, '. . . what has been taken for a peneplain is really a stepped surface, each step represented by a series of hilltops and bevelled spurs that lie within a restricted height range . . . Consequently a more appropriate model than a simple tilted peneplain is one where the peneplain was covered by the sea and then rose intermittently around the Cleveland axis to allow the sea to trim benches in it' (33).

A 'tentative model' for denudation chronology in north-east Yorkshire was given by Palmer (1973, Ch. 5) and some key figures are reproduced in 4I, together with metric equivalents, revised Quaternary stage names (Mitchell _et al_, 1973) and Pliocene dates (Berggren, 1973).

In the Pliocene (according to Palmer) there was discontinuous uplift of the peneplain from the sea, while

41

Heights of marine benches and valley widening stages in north-east Yorkshire according to Palmer (1973) Figures in feet (m)

	Age	Height
TERNARY	Flandrian	
	Devensian	
	Ipswichian	0
	Wolstonian	
	Hoxnian	?80 (24)
	Anglian	
	Cromerian	225-175 (69-53)
	Beestonian	
	Pastonian	280-250 (85-76)
	Baventian	
	Antian	350-300 (107-91)
	Thurnian	
	Ludhamian	550-475 (167-145)
	Waltonian	
	1.8 million years	
		750-625 (229-191)
OCENE		1100-850 (335-259)
		1500-1200 (457-366)
	5 million years	

during the Quaternary glacially controlled sea-level fluctuations were superimposed upon the pattern. The assignment of benches higher than 550 ft. to the Pliocene, and those lower to the Quaternary, is based on an analogy with southern England. The absence of diagnostic deposits once again is 'no embarrassment':

'There are no known beach or sea-floor deposits on the benches,which is hardly surprising since there are hardly any glacial deposits either that are older than the Weichselian [sc. Devensian] , or last glacial age. The wasting of the interfluves by weathering and soil movement took care of such deposits, as it has taken care of any Pliocene soils' (Palmer, 1973, 51).

The interpretation given by Palmer to hillslope development is related to this 'tentative model' of denudation chronology. The hillslopes around the Bridestones were described as rejuvenated (Palmer, 1956), and the hillslopes of the Moors as a whole were described in essentially Davisian terms, emphasising stage of development and attributing valley-in-valley forms to rejuvenation (Palmer, 1973, Chs. 4 and 5).

de Boer (1974), reviewing studies of the physiographic evolution of Yorkshire, struck a more sceptical note, writing of a growing appreciation of 'the uncertainties of explanation' (271). He pointed out that the most recent dating of the Cleveland Dyke (at least 58 million years, Evans <u>et al</u>, 1973) implies that early Tertiary planation is 'once more a tenable hypothesis' (272). de Boer favoured the idea of a mainly subaerial, polycyclic denudation history, postulated for the Moors by Davis, Reed and Versey, including superimposition of drainage from a Cretaceous cover.

Such ideas on planation surfaces are open to objection on a variety of methodological grounds. An extended discussion of these grounds is unnecessary because there are excellent accounts elsewhere: it is sufficient to rehearse the major points briefly, and to cite appropriate references.

The various interpretations put forward over a period of eighty years differ in several respects, particularly on the number, date and origin of planation surfaces, but they all share the major assumption that accordance of flats implies former base-levelling (cf. Linton, 1964, 118). However, the existence of a planation surface cannot be inferred simply from accordance; it must be demonstrated independently, and the alternative hypothesis must be considered that the present landforms can be explained without resort to several cycles (cf. Leopold et al, 1964, 499-500). Such hypotheses were put forward by contemporaries of Davis, such as Tarr (1898), Shaler (1899) and Smith (1899); by workers on Gipfelfluren, or summit surfaces in mountain ranges (cf. Daly, 1905; Baulig, 1952, 173-7; Hewitt, 1972, 27-30); and by proponents of the 'dynamic equilibrium' approach to landscape evolution (Hack, 1960; Chorley, 1962, 1965a, 1965b).

Hypotheses of planation surfaces are very difficult to test satisfactorily, for two basic reasons. Firstly, landscapes including planation surfaces can assume a very large variety of forms, given that a peneplain may be undulating, uplifted, tilted, warped, faulted, dissected, and even destroyed almost completely, yet still be detectable by convinced enthusiasts. (The grossest anomalies can also be explained as former monadnocks.) Secondly, there is a lack of independent evidence on residual deposits and tectonic history.

A final point is that any analogies with the supposedly better understood geomorphological evolution of southern England, and especially with the celebrated scheme of Wooldridge and Linton (1955), should take account of the fact that detailed geological and soil mapping in that area has overturned accepted hypotheses (Worssam, 1973; Hodgson et al, 1974; Shepard-Thorn, 1975; Catt and Hodgson, 1976).

These objections go far beyond the sceptical remarks of de Boer (1974). Any alternative scheme of denudation history would have to meet these objections, which would be unlikely in the present state of knowledge. A more realistic task for a morphometric approach is to provide quantitative evidence on the hillslope forms which exist.

4.4 Quaternary events

4.4.1. Glaciations

The Quaternary, the last period of geological time, included six cold phases in Britain. Glaciations are known to have occurred in the last three, Anglian, Wolstonian and Devensian (Sparks and West, 1972, 4-5). In Yorkshire, very little is known about glaciations before the Devensian (Penny, 1974). The Devensian glacial limit, however, is clearly defined in the field area. During the last glaciation much of the North York Moors stood above the ice. In grid square SE 59 only about 0.1 km^2 is covered by glacial deposits, in Scugdale in the extreme northwest. The escarpment of the Moors here acted as a rampart against the ice. The date of the Late Devensian glacial maximum in England was about 18000 B.P. (Penny, 1974, 254; Jones, 1977).

The 'Older Drift' (i.e. pre-Devensian) on the Moors is neither abundant nor revealing. None has been reported from SE 59. It may represent Anglian or Wolstonian glaciations or both (Bisat, 1940; Penny, 1974; cf. Elgee, 1912 and Versey, 1939 for the older idea of Tertiary marine deposition). SE 59 may have been glaciated during one or both of these stages, although no features have apparently been attributed to glacial erosion, and nothing is known for certain about this possibility. As Palmer (1973, 51) remarked, the Moors are unlikely to have supported their own glaciers, but would rather have been overridden by an icesheet from outside.

4.4.2 Supposed overflow channels

In his paper hypothesising 'A system of glacier-lakes in the Cleveland Hills' Kendall (1902) briefly discussed some features to the north and west of the field area, which he identified as former overflow channels that drained proglacial lakes impounded between the icesheet and the northern escarpment. Two 'channels' in particular might have allowed water to escape southwards into Raisdale and Bilsdale, and hence into SE 59.

(i) Between Carlton Bank and Cringle Moor at 528032, regarded by Kendall (1902, 514) as of minor importance, and not mentioned by Elgee (1908), Best (1956) or Arnett (1971a) in subsequent geomorphological accounts. In the field this feature is not very convincing as an overflow channel: a broad flattish col, it could have allowed overflow if a lake existed, but there is no sign of strong linear erosion (see 4J).

(ii) In the Ingleby Greenhow Corner: 'The re-entrant angle of the Cleveland escarpment at the eastern end is breached by a splendid overflow-channel' (Kendall, 1902, 514). This feature is difficult to place from its description, and reference to 'the eastern end' of the re-entrant angle is puzzling. All later workers have construed the description as a reference to the col at Hasty Bank (573032), or have independently regarded this feature as an overflow channel. But if Kendall meant Hasty Bank, why did he not say so directly? If he meant somewhere else, where could it be?

Best (1956) included Hasty Bank in a group of high-level channels which he attributed to an earlier glaciation,

4J Supposed overflow channel, Carlton Bank

4K Supposed overflow channel, Hasty Bank on the grounds of their weathered appearance and their lack of association with low-level channels in the Vales of York and Stokesley. Arnett (1971a, 14-15) suggested that meltwater pouring through the Hasty Bank channel led to considerable enlargement of Bilsdale, although no other geomorphologist appears to have suggested meltwater erosion beyond the channels. The Hasty Bank col is more convincing in the field as a candidate overflow channel: it falls away rapidly in a steep-sided valley towards Bilsdale (see 4K).

The presence of cols at these sites can credibly be attributed to the beheading of Raisdale and Bilsdale by scarp retreat (Fox-Strangways *et al*, 1885, 56; Elgee, 1912, 236; Palmer, 1973, 41), but there appears to be no strong independent evidence for the hypothesis of overflow channels. Moreover, proglacial drainage of the kind postulated is now believed to be rare (cf. Bonney, 1915, who made this objection; Kendall, 1916, who ignored it; Sissons, 1960, 1961 and Gregory, 1962a, 1962b, 1965, who reinterpreted Kendall's work in Eskdale).

4.4.3 Cryonival conditions

During the Quaternary the North York Moors repeatedly experienced cold climates, including cryonival conditions in which frost and snow activity was significant. (The term 'cryonival' is used as a more precise alternative to the debased term 'periglacial': cf. Linton, 1969).

Several geomorphologists have described cryonival features and deposits from the North York Moors. Palmer (1973, 58) voiced a general interpretation: 'Although information is not as good as it should be, one gains the impression that the hillslopes and their associated deposits owe much of their present appearance to the effects wrought by a rigorous periglacial climate'.

Dimbleby (1952) identified a system of fossil ice wedges in the eastern Tabular Hills, but failed to find any elsewhere in the Moors. These wedges were associated with till, erratic pebbles and solifluction material and were assigned to a pre-Devensian period.

Palmer (1956) regarded valley infill in Dovedale as a solifluction deposit, although he was inclined to play down the role of cryonival conditions in tor formation and hillslope development. Solifluction deposits were also identified by Gregory (1965) in Eskdale; by Tufnell (1969) on Murk Mire Moor, Levisham Moor, and Lockton Low Moor, and around Robin Hood's Bay; by Imeson (1970, 1974) in the upper part of Bransdale; by Bendelow and Carroll (1976) around Pickering Moor and Troutsdale; and by Jones (1977) in Kildale. According to Arnett (1971a, 18) in cryonival conditions 'colluvial activity on the slopes was far greater than at present': he cited 'angular rubble' found beneath soil profiles in Caydale.

Gregory (1966) described small 'nivation benches' in Eskdale, attributing them to snow patch erosion, and assigned inactive mass movement phenomena (rotational slumps, earth flow tongues) to cryonival conditions, although without arguing the case in detail. Tufnell (1969) alluded to frost shattered sandstone on the North York Moors and to erected stones on Murk Mire Moor. Palmer (1973, 57) cited angular boulders exposed by peat fires as evidence for frost riving. Catt et al (1974) reported loess in the Hambleton Hills.

Simmons and Cundill (1974) quoted evidence that peat growth in two landslip bogs (NZ 683028, SE 674996) started in the early Flandrian; this gives minimum ages for the landslips, which may have been associated with cryonival conditions.

Many sections in SE 59 show material which is clearly colluvial rather than in situ (e.g. stream side sections at 592979 and 508977; trackside sections at 538954 and 549907). The colluvial origin is clearly demonstrated when sandstone cobbles and boulders are incorporated in a mantle resting on shales, as at 592979 and 508977. The thickness of colluvium is not great, being generally of the order of 1 m. Many other sections, however, are small and not obviously colluvial. Whether the colluvium is a solifluction deposit is difficult to say, although it is plausible when the colluvium incorporates very coarse material. Surface boulders are also common (for example, a conspicuous spread around 549918), but it is not clear that these must all be attributed to cryonival frost riving. An apparently inactive and well vegetated landslip scar such as Kay Nest (583985; see 4L) may also be plausibly attributed to cryonival

4L Landslip scar, Kay Nest

4M Gullying in Black Intake

conditions of the early Flandrian or earlier.

Assessment of cryonival influence on hillslope development is relatively easy in principle. It is a matter of establishing how much development must be attributed to (i) pre Quaternary time; (ii) relatively warm conditions during the Quaternary; (iii) cryonival conditions during the Quaternary; (iv) postglacial time; and also to glacial erosion if this operated. This is basically a question of the rates of slope retreat during these periods and their durations (Young, 1974, 44). Unfortunately, their durations are still very much a matter for controversy, while rates of retreat must naturally be inferred from contemporary observations of forms, processes and deposits.

Two theses of cryonival influence may be distinguished (Carson and Kirkby, 1972, 322).

(i) (Weak thesis) Many hillslopes display cryonival features and deposits apparently little modified. Therefore, postglacialslope retreat has been negligible and hillslopes may be regarded as relict.

(ii) (Strong thesis) In cryonival periods a marked acceleration of slope processes took place, leading to profound alterations of profile geometry and basal accumulations of cryonival deposits.

Such theses, both in general and as far as the North York Moors are concerned, must be considered in the light of the following points.

Firstly, there is a lack of quantitative evidence on past slope retreat under cryonival processes. From the available evidence, Young (1972, 244) has suggested that total retreat was of the order of 1 - 10 m. If this is correct, cryonival influence may well have been exaggerated.

Secondly, there are problems of reporting bias and sampling basis. Reports of cryonival features and deposits are frequent in the literature, but workers failing to find any in an area will write about something else instead (Young, 1972, 241). Large and striking features and sections are most likely to be reported, leading to a biased estimate of the importance of cryonival activity.

Thirdly, as Young (1972, 244; 1974, 74) argued, the unmodified condition of cryonival features and deposits does not imply that postglacial activity is less intense than cryonival. It may be accounted for by the shortness of postglacial time.

Fourthly, it is difficult to say how many traces of cryonival action have been destroyed or truncated, or to assess the overall importance of preglacial or interglacial retreat (Williams, 1968, 311; Sparks and West, 1972, 116).

Fifthly, many of these features reported (e.g. wedges, erected stones, angular boulders and even nivation benches) appear to be only micro- or meso- forms compared with the scale of the hillslope profiles.

Sixthly, the cryonival origin of many features is not always clear cut. The criteria used to identify cryonival deposits are 'often very vague' (Carson and Kirkby, 1972, 322) while there is growing appreciation of the difficulties of distinguishing solifluction deposits from other colluvial deposits (Benedict, 1976).

Seventhly, it is difficult to understand how any appreciable modification of profile geometry could take place without a great deal of rock weathering to supplement downslope movement of the mantle (Carson and Kirkby, 1972, 323).

Eighthly, it is not clear that cryonival processes would even tend to produce a radically different profile geometry, although existing theory on cryonival slope development is very weak (cf. Carson and Kirkby, 1972, Ch. 12; Jahn, 1975, Ch. 17; French, 1976, Ch. 7). For example, while lobes and terraces are distinctive products of gelifluction (Washburn, 1973, 189; Embleton and King, 1975, 112-9; Benedict, 1976; French, 1976, 139-41) these are commonly micro- or meso- forms: formation of such features is not inevitably associated with fundamentally different profile shapes.

Hence the many reports of apparently cryonival features and deposits must be interpreted cautiously. There are several grounds for doubting a thesis of profound cryonival influence.

4.4.4 <u>Contemporary erosion</u>

Imeson (1970, 1971a, 1971b, 1973, 1974) has studied contemporary erosion in upper Bransdale. He clearly documented the relationship between heather burning and soil erosion (Imeson, 1971a) . The broader picture is one in which unvegetated areas and channel sides contribute much more sediment than vegetated areas, although in a 19 km^2 basin most is deposited as colluvium or alluvium and does not pass the outlet (Imeson, 1974).

Incidental observations by Palmer (1973, 59) and in SE 59 underscore the importance of vegetation as established systematically by Imeson. Rilling, sheet wash and dissection of peat can be observed, e.g. above Crookleth Crags, around 555968. These are often associated with overgrazing and burning. There is occasional slight gullying, notably along footpaths. The footpath up from Beacon Guest is deeply inset around 564965, suggesting former gullying. It is quite well vegetated now, which may reflect the great decrease in its use. Similarly the old footpath through Black Intake (around 575992) seems to have been gullied and then stabilised, although there are further signs of erosion at the present (see 4M). The most striking example, at 586916, is a case in which a gully has exposed bedrock over a reach of 7m, removing 90 cm of soil (see 4N). Occasional landslips, as at 543905 (all dimensions ~ 20 m; see 4O), have been active recently. A further interesting feature is the 'bunker' or 'sheep scar' (McVean and Lockie, 1969, 29; Evans, 1974), a crescent-shaped scar enlarged by sheep, as at 543907, 532927 or 532935. Apart from areas of peat erosion, however, all these bare areas suffering erosion are minor and localised.

4N Gullying near Roppa Wood

4O Landslip at Hawnby Hill

4.5 Summary

(i) Geological Survey accounts of the Jurassic formations in the field area remain the most valuable for the geomorphologist, even though the usefulness of their stratigraphical classification is limited by vertical and lateral variations in lithology (4.2.1). Post-Cretaceous structural history can be summarised very simply as a combination of epeirogenic tilting and anticlinal warping (4.2.2).

(ii) The following theses have been advanced about the geomorphological development of the field area, although none has received very much critical examination.

(a) The thesis of profound lithological influence (4.3.1)

(b) The thesis of polycyclic denudation history (4.3.2)

(c) The thesis of proglacial lake overflow channels (4.4.2)

(d) The thesis of profound cryonival influence (4.4.3)

(iii) There is complete ignorance about the effects of pre-Devensian glaciations on the field area (4.4.1)

(iv) The question of scale of feature is crucial, particularly as far as features attributable to lithological or cryonival influence are concerned (4.3.1, 4.4.3).

Chapter 5

SAMPLING AND MEASUREMENT PROCEDURES

> 'Hill. Yes, that was it. But it is a hasty word for a thing that has stood here ever since this part of the world was shaped.'

J.R.R. Tolkien, The Lord of the Rings, Bk. III, Ch. 4

5.1 Introduction

In this chapter sampling and measurement procedures used to collect hillslope profile data in the field area are discussed in the light of general principles. Although this study does not offer innovations in procedures, the need to give a brief report allows the introduction of some methodological ideas previously neglected in hillslope profile studies.

5.2 Sampling

The sampling problem is simple in essence. Given a set of objects (a population), choose a smaller subset (a sample) for detailed study. There are two fundamentally different approaches:

(i) Choose objects quite arbitrarily, for example, those which are interesting or accessible.

(ii) Choose objects according to some definite rule designed to ensure that the sample is representative of the population.

This distinction corresponds fairly closely to that often made between purposive and probability sampling (e.g. Harvey, 1969, 356-69), but such terms are not very appropriate: procedures covered by (i) may lack a definite purpose, in the sense that choice may be essentially haphazard; while procedures covered by (ii) may not be probabilistic, in the sense that the 'definite rule' is deterministic.

It is clear that unless some definite rule is followed to ensure a representative sample, there can be no strong grounds for generalising from the sample to the population, and any statements made about the sample should not be attributed wider applicability. This should always be recognised explicitly: whether it is important depends on the purpose of the exercise.

Sampling theory is, in large part, a study of the definite rules devised to generate representative samples in various circumstances. It will be worthwhile considering different kinds of sampling problems to see how far they arise in hillslope geomorphology, not least because there is some controversy and confusion in this field over sampling principles and procedures. (The classification used here is not exhaustive. The terminology employed is original).

The classical sampling problem arises when the population consists of discrete individuals which are a set in the strict sense; that is, they possess no natural ordering and are not indexed by time or space coordinates. Random selection is the fundamental solution to the classical sampling problem: objects are labelled numerically and a sample chosen using a list of pseudorandom numbers or some equivalent procedure. Although a variety of other procedures exists, in essence they are modifications of simple random selection (e.g. stratification, multistage sampling). See Stuart (1976) for an introduction to classical sampling.

Classical sampling procedures might be appropriate in hillslope geomorphology if the landsurface could be regarded as a combination of discrete units, a view here termed geomorphological atomism (for fuller discussion, see Ch. 8.2 below). This view is open to objection as a partial theory, but here it is sufficient to note two limitations to an atomistic approach to hillslope profile sampling. Firstly, an initial survey stage is necessary to define the units of a landscape before such units can be used as a framework for choosing a sample of profiles. Secondly, attributes of neighbouring units would tend to be autocorrelated, implying that random selection is not appropriate.

The serial sampling problem arises when the population is a single-valued continuous series indexed by time or space coordinates. Systematic selection is the fundamental solution to the serial sampling problem: thus time series are generally recorded at regular intervals, while various grids (with square or triangular meshes) are increasingly being recognised as the preferred class of sampling schemes for spatial series (cf. Holmes, 1970; Evans, 1969, 1972; McCammon, 1975; but see Hammersley, 1975, for a dissenting note on abstruse technical grounds). Many continuous series arise in hillslope geomorphology, and point sampling schemes have been used to measure slope over a length centred at a point (e.g. Strahler, 1956; Juvigné, 1973) or to measure soil properties or process rates in plots centred on a point (e.g. Reynolds, 1975a, 1975b; Anderson, 1977). Systematic schemes have not, however, been universally used. In process studies particularly, some kind of stratification is common, where sampling is linked to an experimental design which aims to assess the effects of controlling variables.

The path sampling problem arises when the population is a set of paths on a surface. It does not seem to arise outside geomorphology, nor is there a theory of paths on surfaces which would lead to recommendations about sampling procedures (cf. Longuet-Higgins, 1962; Switzer, 1976 for theory on random surfaces). Since hillslope profiles are maximum-gradient paths, this means that there is no firm theoretical basis for choosing hillslope profile samples. By default, hillslope profile sampling has been treated as a serial sampling problem, a problem of selecting points, whether these are to be endpoints or intermediate points on profiles.

It is first necessary to decide whether the target population of hillslope profiles includes all profiles or merely some subset. The most important consideration here is plan curvature, and, as a first approximation, profiles may be divided into three classes.

(a) Profiles with negligible plan curvature (straight contours), e.g. on valley sides. 'Negligible' to be defined operationally (see, for example, Abrahams and Parsons, 1977).

(b) Profiles convex in plan, e.g. on spurs

(c) Profiles concave in plan, e.g. in valley heads

Usually either (a) alone is regarded as the target population, or (a), (b) and (c) combined.

Given the target population, it is possible to choose a set of points on a map and identify profiles which extend upslope and downslope from those points. (Any profiles not belonging to the target population will naturally have to be rejected). Whatever the details of point selection, this approach is at best only an approximate solution to the hillslope profile sampling problem.

From a geomorphological point of view, a more direct and natural approach to profile sampling is to use the stream network as a basis for selection. Broadly similar schemes of this kind have been used or suggested by Arnett (1971b), Chorley and Kennedy (1971, 50-55), Young et al (1974, 17-19), Summerfield (1976) and Abrahams and Parsons (1977). In the simplest situation, slopes with straight contours have bases at or near the midpoints of links in the stream network, while slopes with curved contours have bases at or near source or junction nodes. This approach presupposes well-integrated fluvial topography, in which hillsides are in clear and unambiguous relationships with streams.

While ideas from sampling theory, such as random and systematic selection and stratification, appear in such schemes, the choice of the stream network as sampling framework is difficult to evaluate theoretically, however attractive it may be for the geomorphologist. Moreover, the identification of the stream network itself may not be straightforward. Stream sources may be difficult to locate and some permanent watercourses may appear to lack geomorphological significance. There is much to be said for the view that the valley network rather than the stream network is appropriate for a profile sampling framework. Indeed, one ideal would be for a sample of profiles to be chosen by an iterative selection procedure which identified a representative set from a sufficiently detailed altitude matrix.

However a sample of profiles is chosen, it is quite likely that some profiles will prove unsurveyable (e.g. land use may not permit survey; access may be forbidden or dangerous). Hence in practice it may be necessary to modify an initially chosen sample. There is, naturally, no guarantee that such forced omissions will not induce biases in representation.

The target population for this study was the set of hillslope profiles with straight contours. Profiles with negligible plan curvature are simplest to interpret, and they are particularly appropriate for testing hillslope models which neglect laterality.

As a first step the stream network in the field area was delimited from Ordnance Survey 1:63,360 and 1:25,000 sheets, and a list drawn up of stream 'systems' and 'major subsystems', the terms being used in one-off senses (5A). A list was then prepared of 'possible profiles' using specific criteria (5B).

This list includes 19 profiles. In fact only 11 of these were surveyed, for two reasons. As research progressed, it became clear that an adequate treatment of methodological, theoretical and technical questions - the major foci of the thesis, in short - would imply that the treatment of empirical questions in the project would be less extensive than was originally envisaged. In addition, this number of profiles seemed sufficient to allow illustration and examination of the methods developed in this study.

5A

Stream systems and major subsystems in SE 59

1. Stream systems i.e. discrete networks within field area

(i) Scugdale Beck system	-	flows into Leven, and thence into Tees
(ii) Rye system	-	flows into Derwent
(iii) Ladhill Beck system	-	flows into Rye
(iv) Seph system	-	flows into Rye
(v) Riccal system	-	flows into Rye

2. Major subsystems

(ii) Rye system

Stonymoor Sike, Proddale Beck, Arns Gill, Wheat Beck, Locker Beck, Blowgill, Eskerdale Beck, Thorodale Beck*

(iv) Seph system

Raisdale Beck, Hollow Bottom Beck, Ledge Beck (Tripsdale Beck, Tarnhole Beck), Fangdale Beck, Todhill Beck

(v) Riccal system

Bogmire Gill, Bonfield Gill, Potter House Beck*

Notes

* ad hoc names devised by author

(a) Some short streams in upper Bilsdale are not marked beyond the B1257 road. Carlton Watercourse (marked on 1:25,000) is clearly artificial. The upper course of Kyloe Cow Beck is also problematic.

(b) The 'major subsystems' are the larger tributaries, larger in terms of mainstream length and/or valley depth: however, the decisions on these were fairly subjective, and no precise criteria used.

5B

Pбssible profiles and actual profiles

Possible profile	Crest GR	Base GR	
Scugdale	522995	520989	
Stonymoor Sike	508978	515980	X
Proddale Sike	522970	524965	X
Arns Gill	528962	524965	X
Wheat Beck	506947	505960	
Locker Beck	510937	510927	
Blow Gill	534946	531953	X
Eskerdale Beck	519921	510927	
Thorodale Beck*	516907	515915	
Ladhill Beck	549924	555926	X
Raisdale Beck	546996	540987	
Hollow Bottom Beck	554982	551987	X
Tripsdale Beck	581984	575987	X
Tarn Hole Beck	590976	598974	X
Fangdale Beck	560948	553948	X
Todhill Beck	571912	575905	X
Bogmire Gill	595929	585930	
Bonfield Gill	600962	594964	X
Potter House Beck*	594912	590605	

* ad hoc names devised by author   X actually surveyed (cf. 6L)

Selection procedure

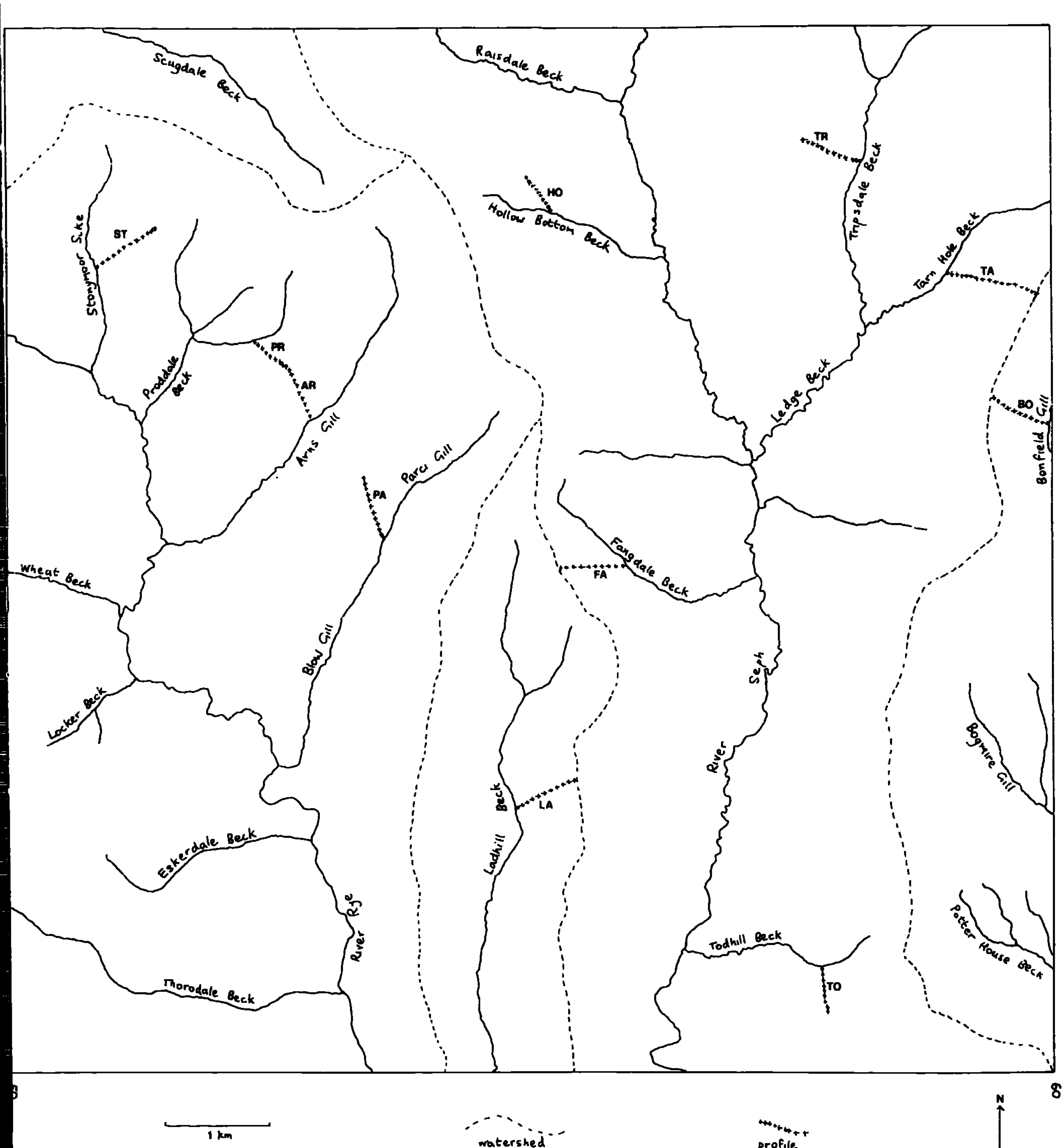
Take 'systems' and 'major subsystems' as given. Choose one profile for each subsystem and one for each system without sub-systems. Take a base approximately midway along relevant stream link, or along part of relevant link within field area. Valleyside should have approximately straight contours and be relatively free of obstructions such as roads. It would be convenient to have profiles on opposite sides of a ridge.

Arbitrary within these broad specifications.

It is necessary, therefore, to stress the limitations of this sample of profiles. Firstly, the number of profiles is rather small to allow clearcut generalisations about the field area. Secondly, a haphazard element entered into the choice of profiles, given that research plans narrowed in focus while data collection proceeded intermittently, and that the original sample was not completed. Thirdly, it is not demonstrated convincingly that the profile selection procedure used produces a representative sample, although it is not clear precisely what other procedure would achieve this aim.

When slope endpoints were located in the field, it was sometimes found necessary to move slightly, to avoid local obstructions or local plan curvature (cf. 6L below for actual endpoint positions).

These profiles are shown on a map in 5C, together with major streams and major watersheds.



5C Measured profiles, major streams and major watersheds in SE 59

5.3 Measurement

A profile may be recorded in two ways, which are numerically equivalent:

(i) As a series of measured lengths, l_i; $i = 1,\ldots,n$, together with a corresponding series of measured angles θ_i; $i = 1,\ldots,n$.

(ii) As a series of coordinates, x_i, z_i; $i = 1,\ldots,n$. Naturally if (i) is chosen, and constant l is used, then the horizontal interval, Δx , say, will not be constant in general; and conversely, if (ii) is chosen, and constant Δx is used, then l will not be constant in general.

The method of measured lengths and angles was used in the present study and attention here is focused on this case.

It seems clear that measured lengths should be constant and relatively short (Pitty, 1967; cf. Young et al, 1974, 31-3). Many hillslope profiles have been surveyed with variable measured lengths. In extreme cases, breaks and changes of slope have been identified visually and used to bound measured lengths. Naturally such a survey procedure prejudices any interpretation of profile data, and (what is worse) prejudices it in an unknown and complicated manner. The case for constant measured lengths is thus very strong.

Gerrard and Robinson (1971) discussed the choice of measured length in some detail. They warned that the results of slope profile survey may vary considerably with the measured length adopted. This warning was illustrated by angle frequency distributions obtained from repeated surveys using different measured lengths. They also warned that microrelief would influence readings made at very short measured lengths. It does seem, however, that such warnings may be slightly exaggerated. The effect of repeated survey is to mix scale variation with measurement error (Gilg, 1973), while their homily against microrelief rests on a dichotomy between microrelief and 'the true nature of the slope' (Gerrard and Robinson, 1971, 50) which seems dubious. There do not appear to be any firm physical grounds for distinguishing sharply between two components of topography, while results from spectral analysis (e.g. Sayles and Thomas, 1978) support the contrary idea that variations can be observed at all spatial frequencies of geomorphological interest.

A short measured length (5 m. or less) seems preferable to allow detailed recording of hillslope profiles. Naturally superfluous detail can always be ignored or removed, while detail finer than that recorded cannot be added without resurvey.

A pantometer (Pitty, 1968) was used to survey hillslope profiles in this study. It was made by Dr. E. W. Anderson and produces angle measurements for a constant measured length of 1.52 m. (5 ft.) (Anderson, 1977, 100-102). Angles were measured to the nearest 0.5° (cf. Young et al, 1974, 13). The procedure near the crest of each profile was to continue measurement along an orthogonal until the crest was clearly passed: the exact position of the crest was identified later from the data.

Measured angle series, together with notes on vegetation, natural and artificial features, and basal stream characteristics, are given in Appendix I.

5.4 Notation

i subscript

l measured length

n number of observations

x horizontal coordinate

z vertical coordinate

Δ difference operator

θ angle

Chapter 6

DESCRIPTION OF MEASURED PROFILES

... Without words, there is no possibility of reckoning of Numbers; much lesse of Magnitudes, of Swiftnesse, of Force, and other things, the reckonings whereof are necessary to the being, or well-being of man-kind.

Thomas Hobbes, Leviathan, Ch. 4.

6.1 Profile form

If l_i; $i = 1, \ldots, n$ and θ_i; $i = 1, \ldots, n$ denote measured lengths and measured angles in crest to base sequence, then profile coordinates may be calculated from

$$\left.\begin{aligned} x_j &= \sum_{i=1}^{j} l_i \cos\theta_i \\ z_j &= \sum_{i=j+1}^{n} l_i \sin\theta_i \end{aligned}\right\} \quad j = 1, \ldots, n$$

Profile height z_d and profile length x_b may be derived from

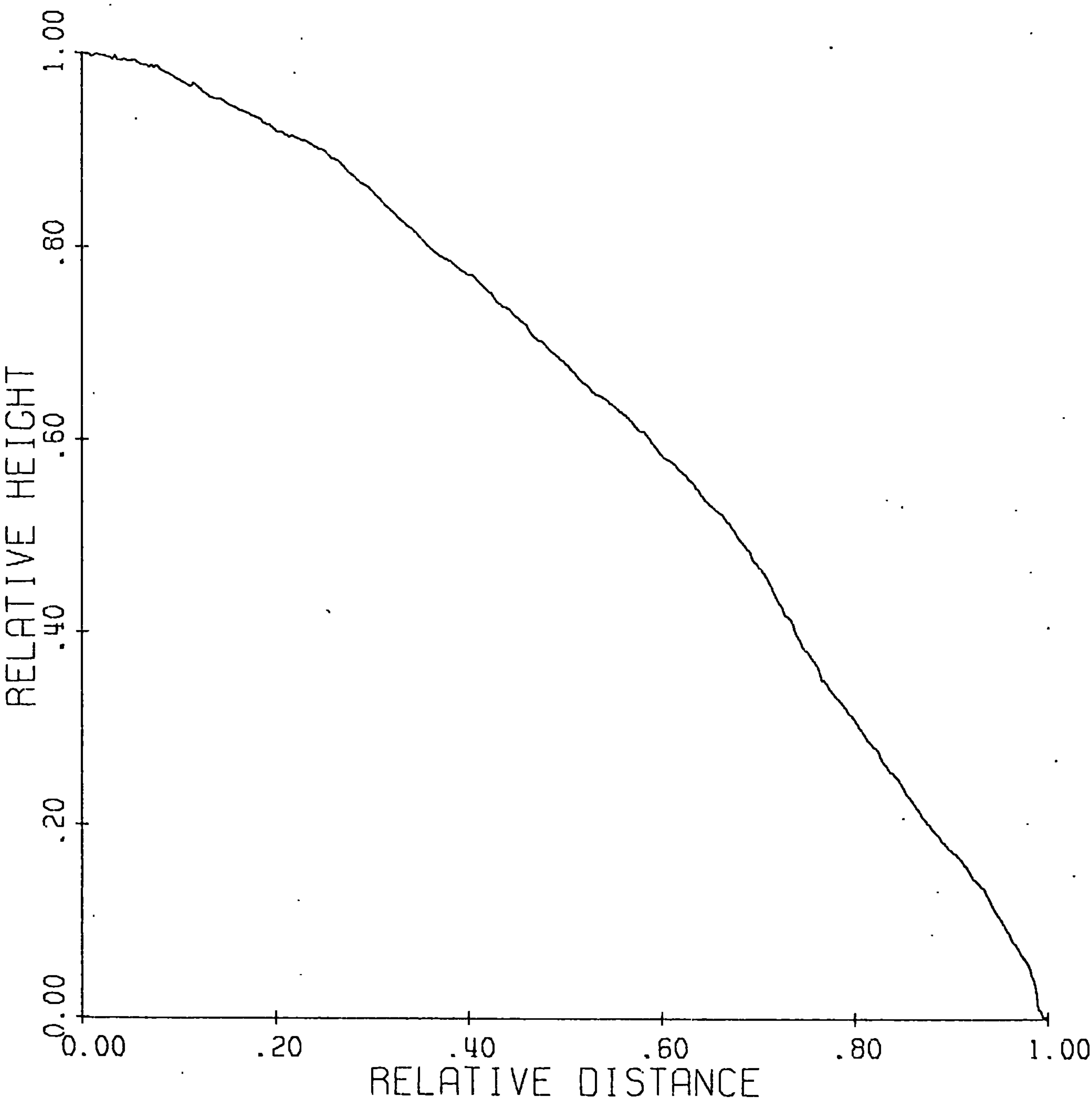
$$z_d = \sum_{i=1}^{n} l_i \sin\theta_i$$

$$x_b = \sum_{i=1}^{n} l_i \cos\theta_i$$

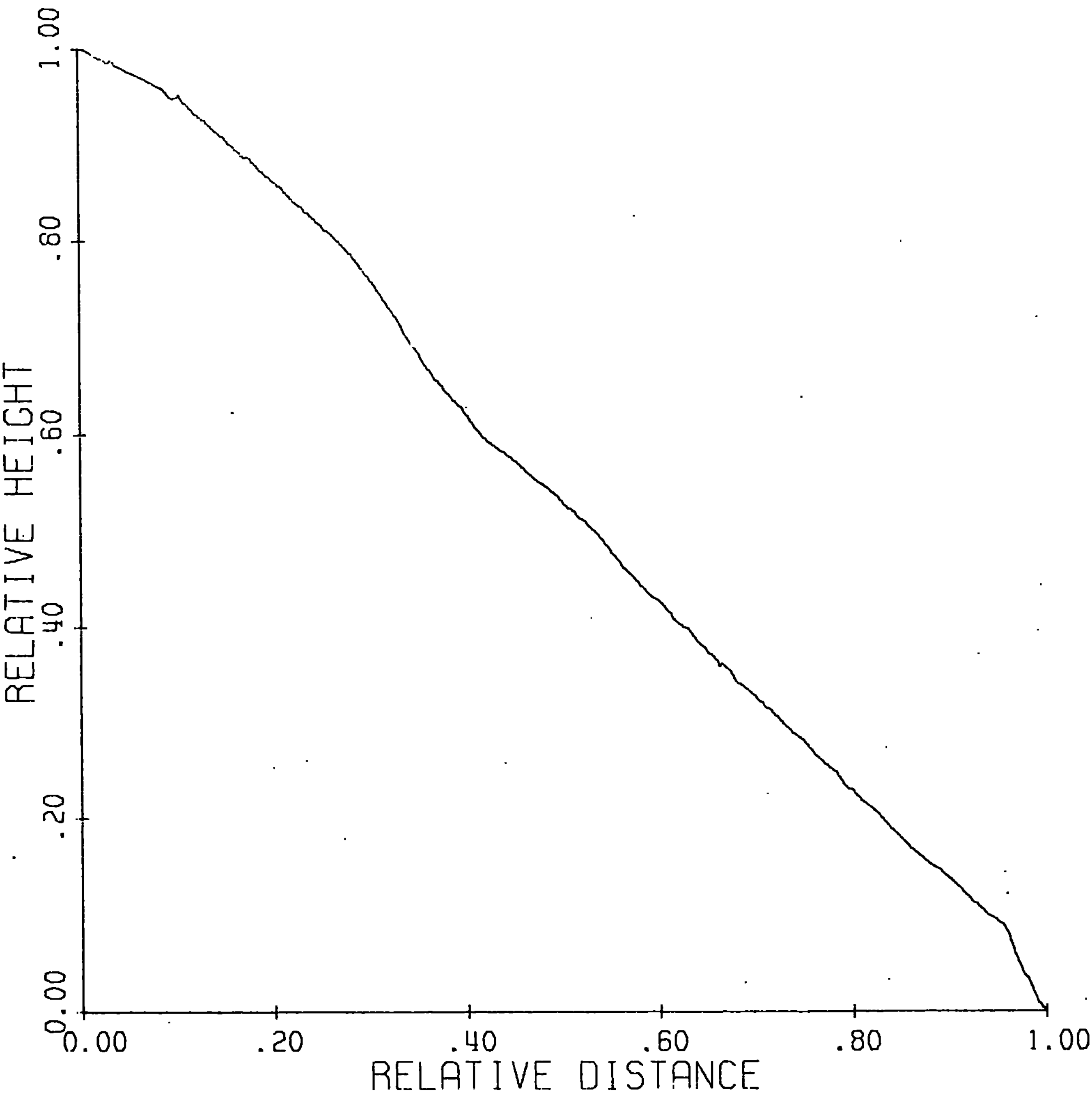
and average angle $\bar{\theta}$ is given by arctan (z_d/x_b) (cf. 3A). These relations were used to produce profile coordinates for the eleven profiles measured in the field area. Dimensionless plots, in which z/z_d is shown against x/x_b, are given in 6A to 6K. Number of observations n, estimated profile dimensions z_d and x_b, average angle $\bar{\theta}$, endpoint locations and bedrock geology are shown for each profile in 6L. Identifiers are here introduced for each profile and for each geological formation (see 6M for key).

The profiles fall readily into four classes:

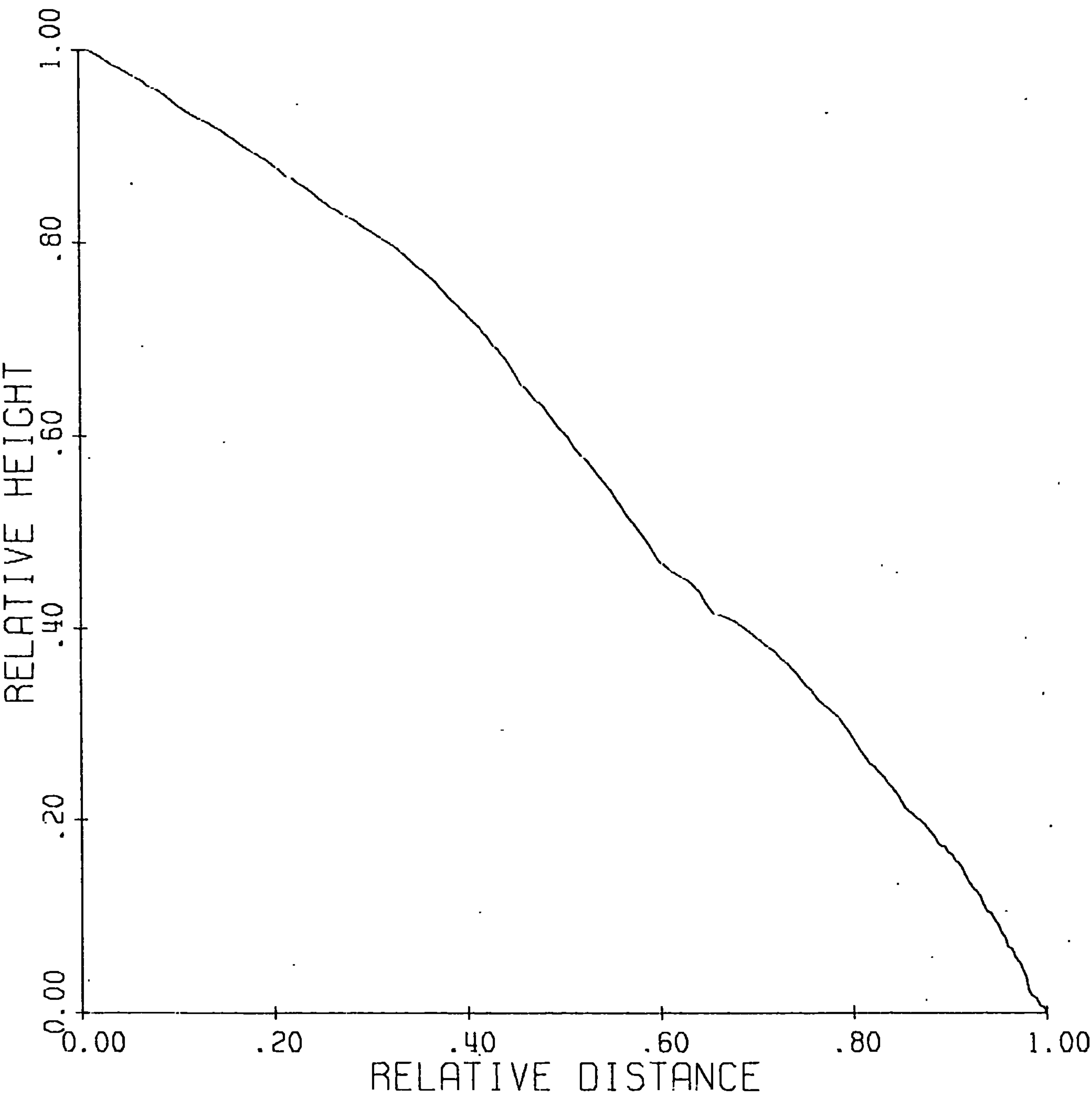
(i) BO, ST, PR, PA: the four gentlest profiles, all on the formation del.



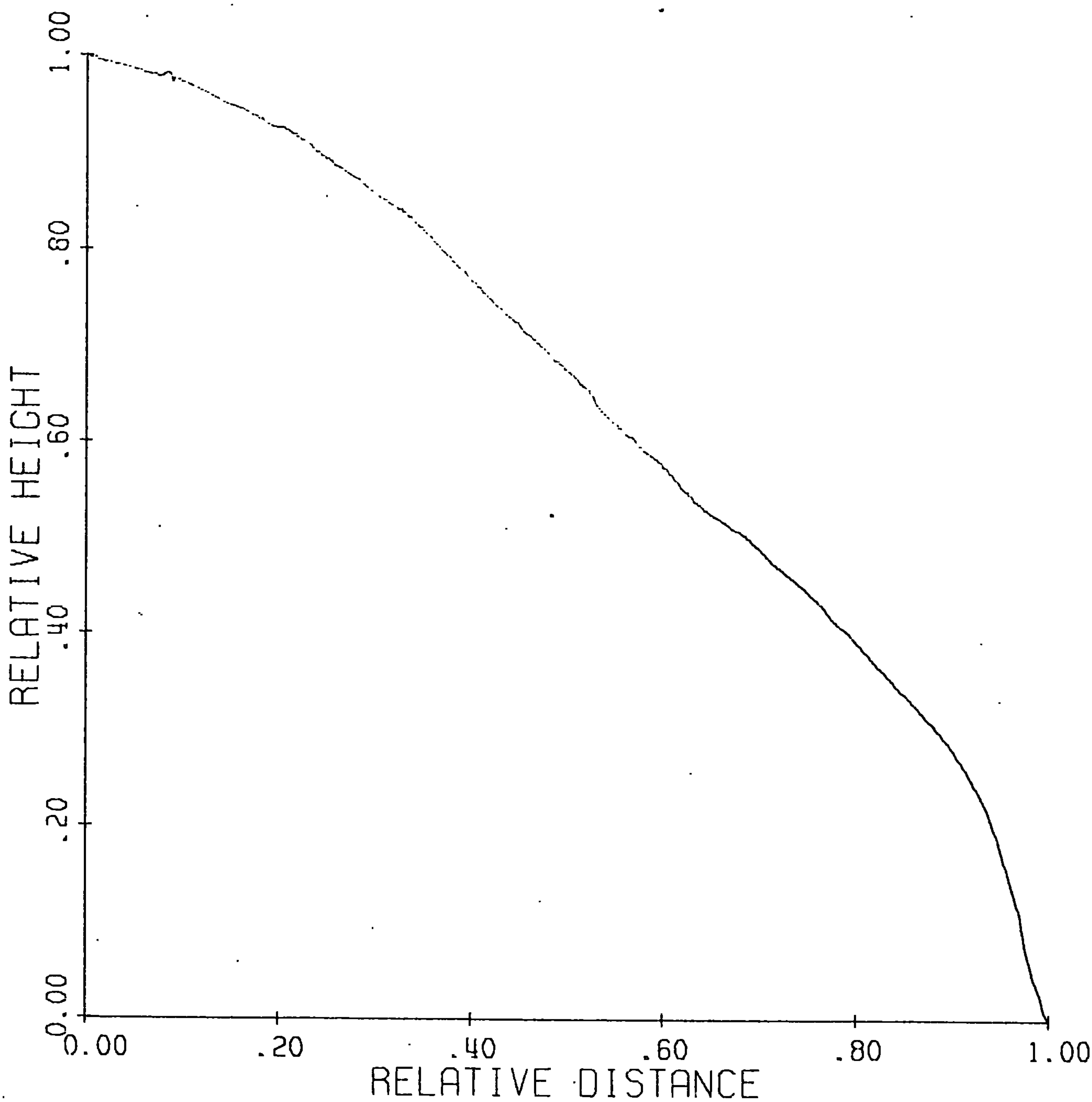
6A Profile BO



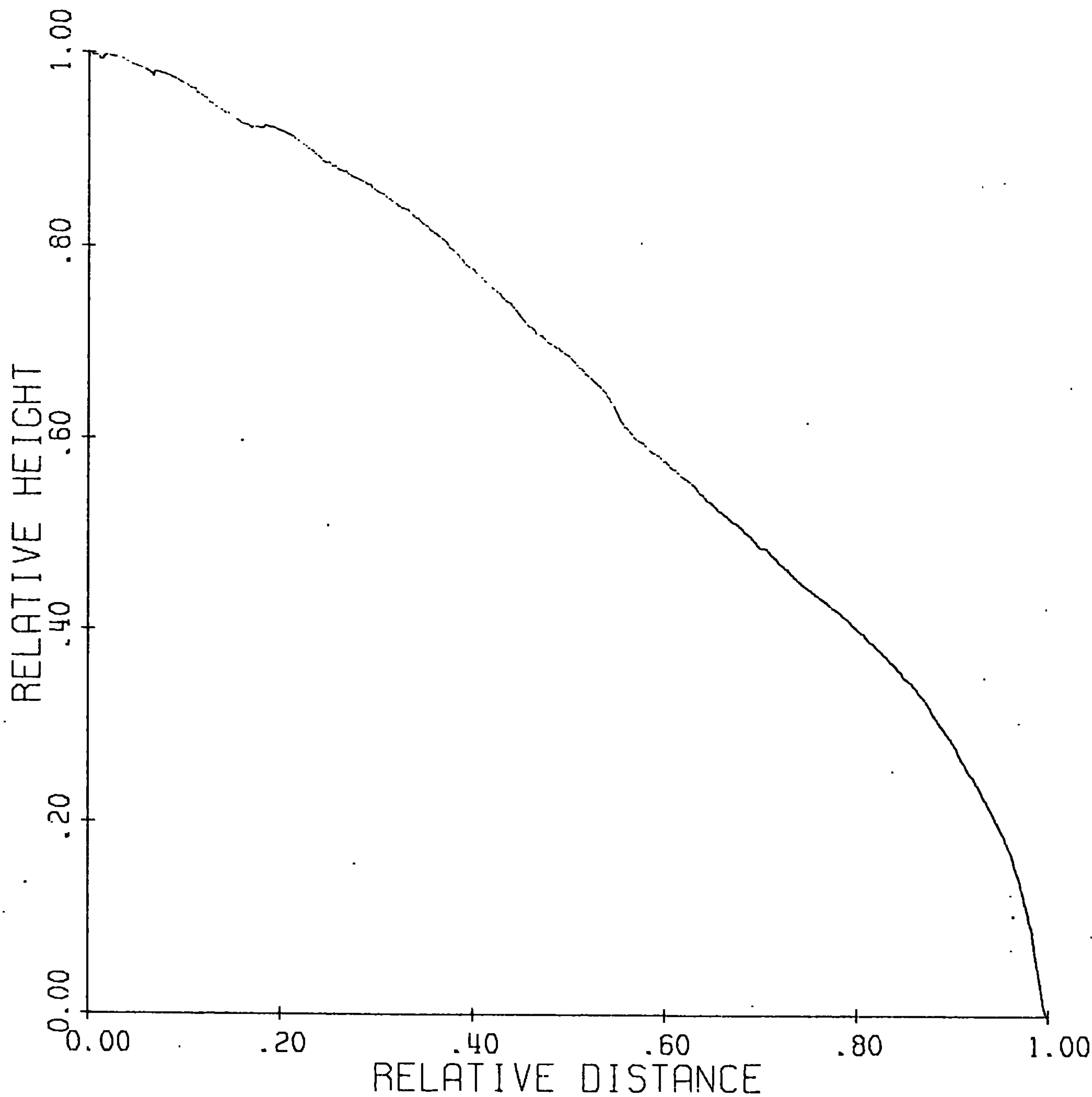
6B Profile ST



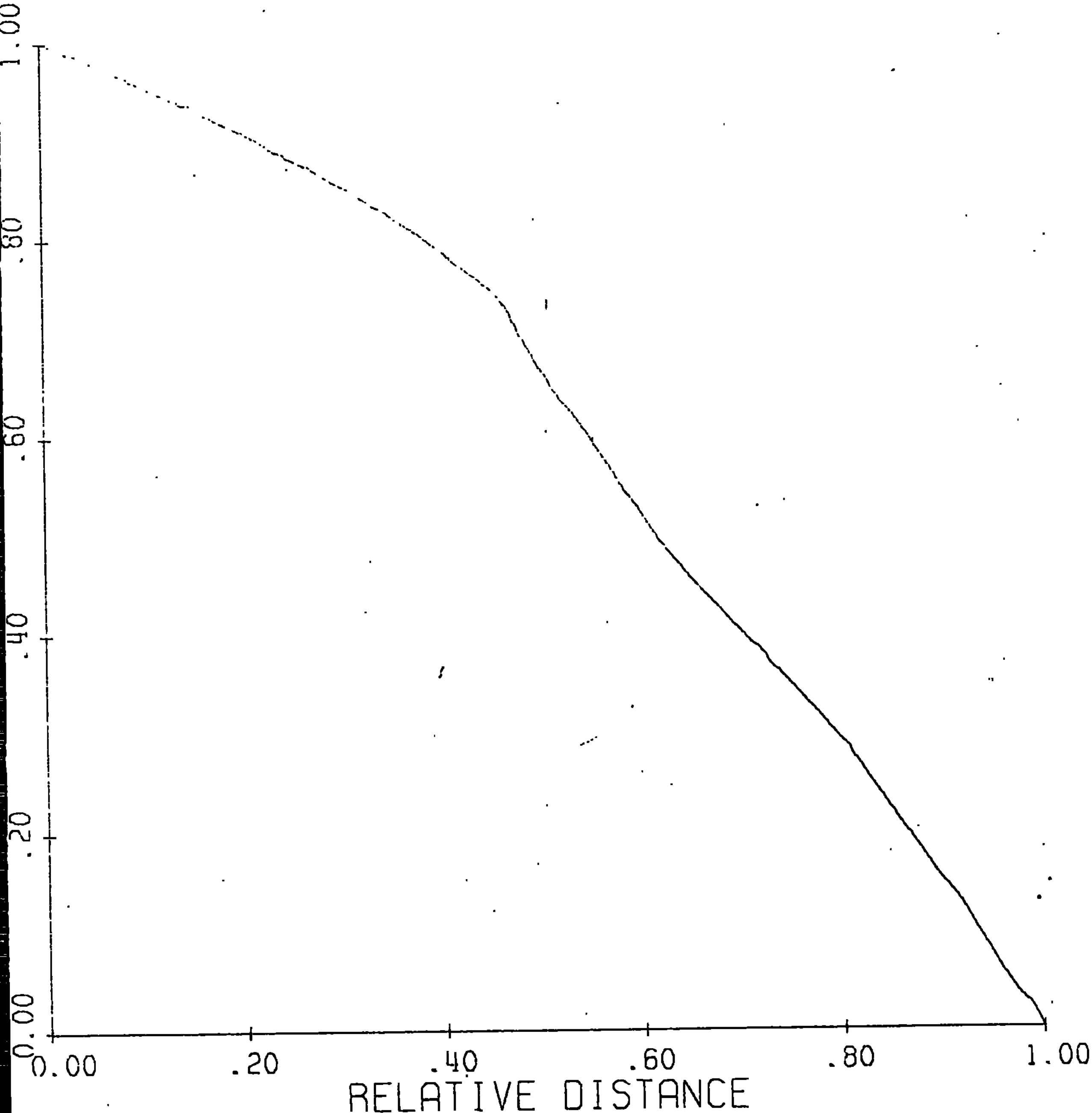
6C Profile PR



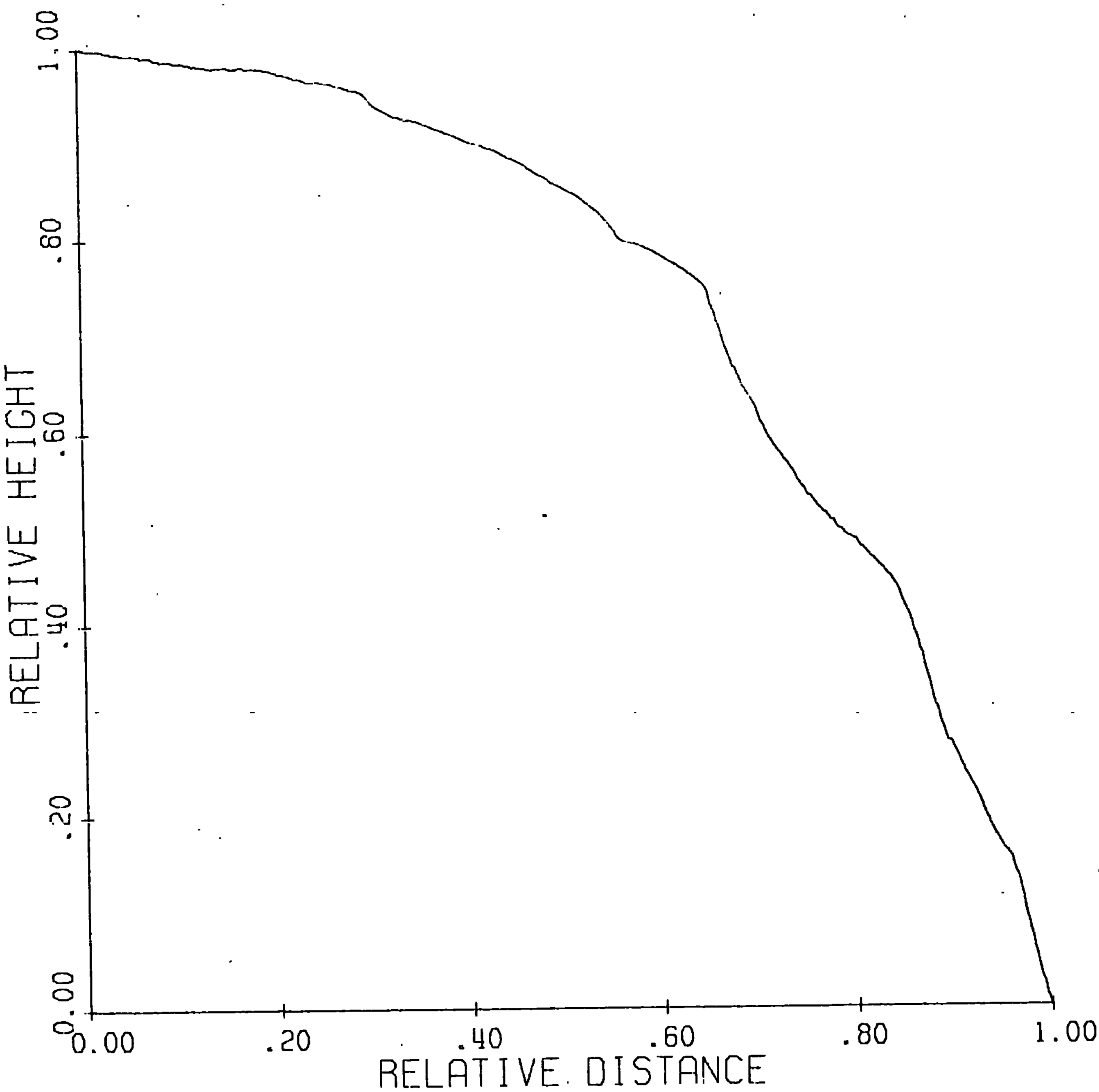
6D Profile PA



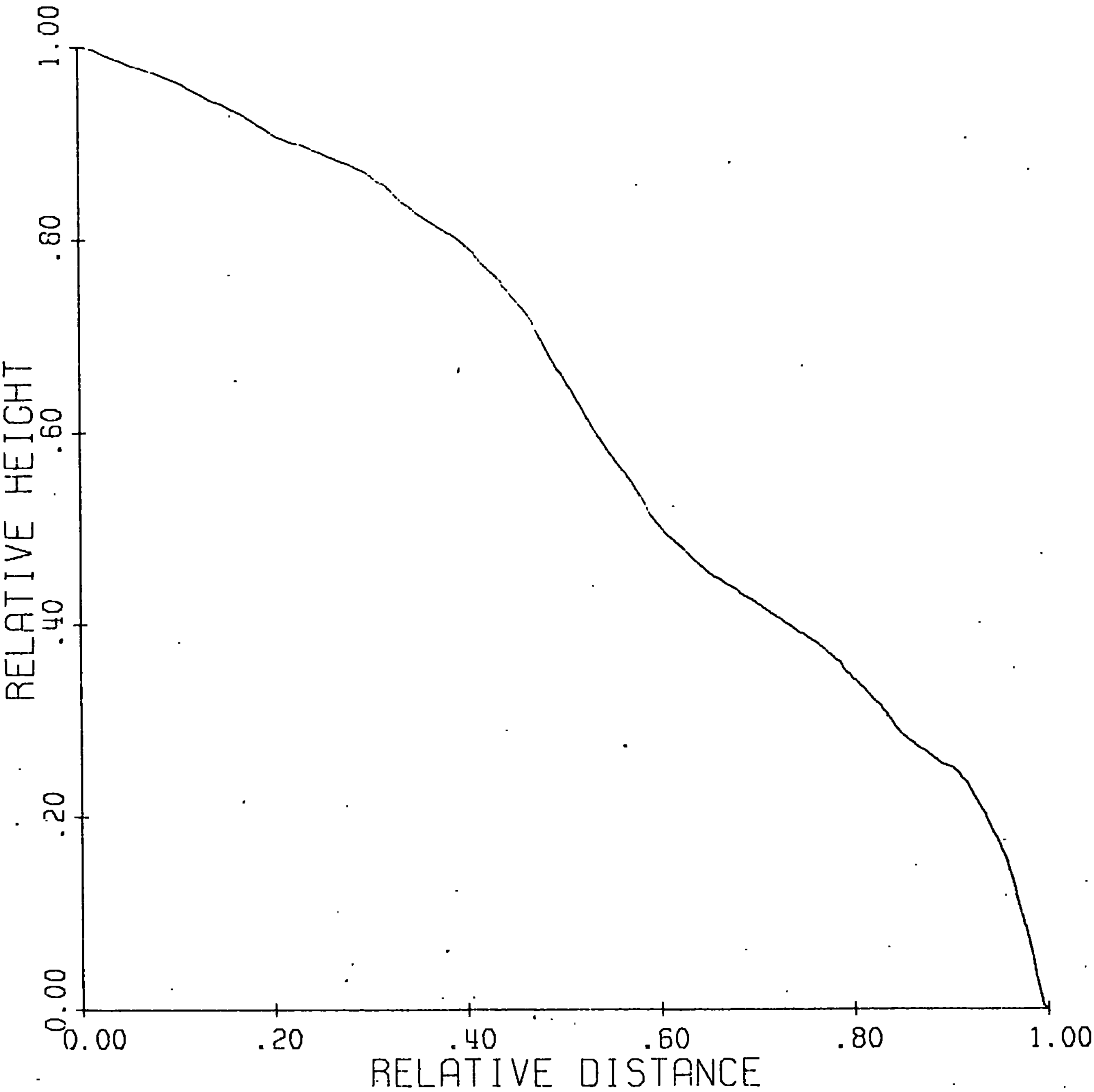
6E Profile FA



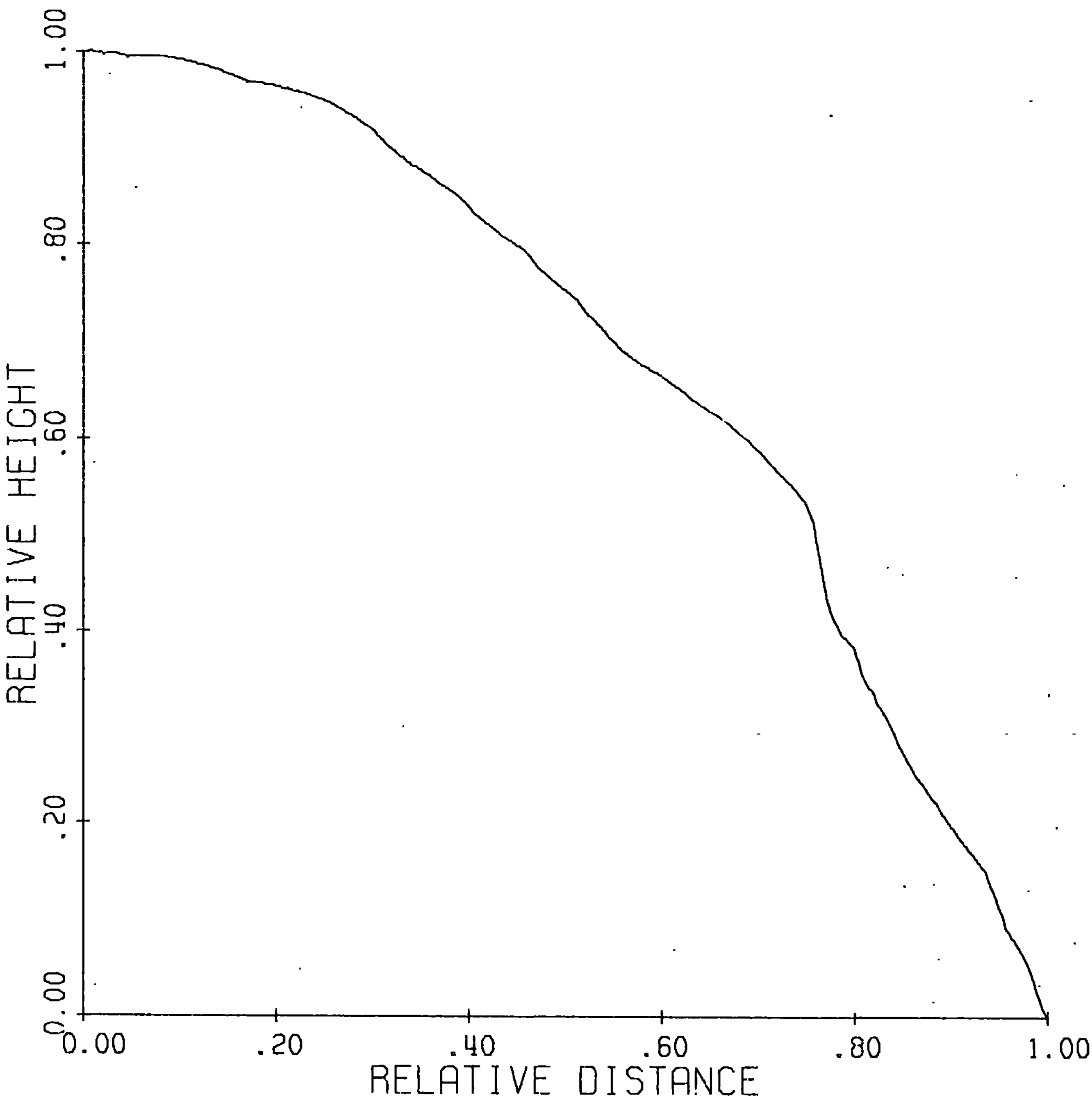
6F Profile LA



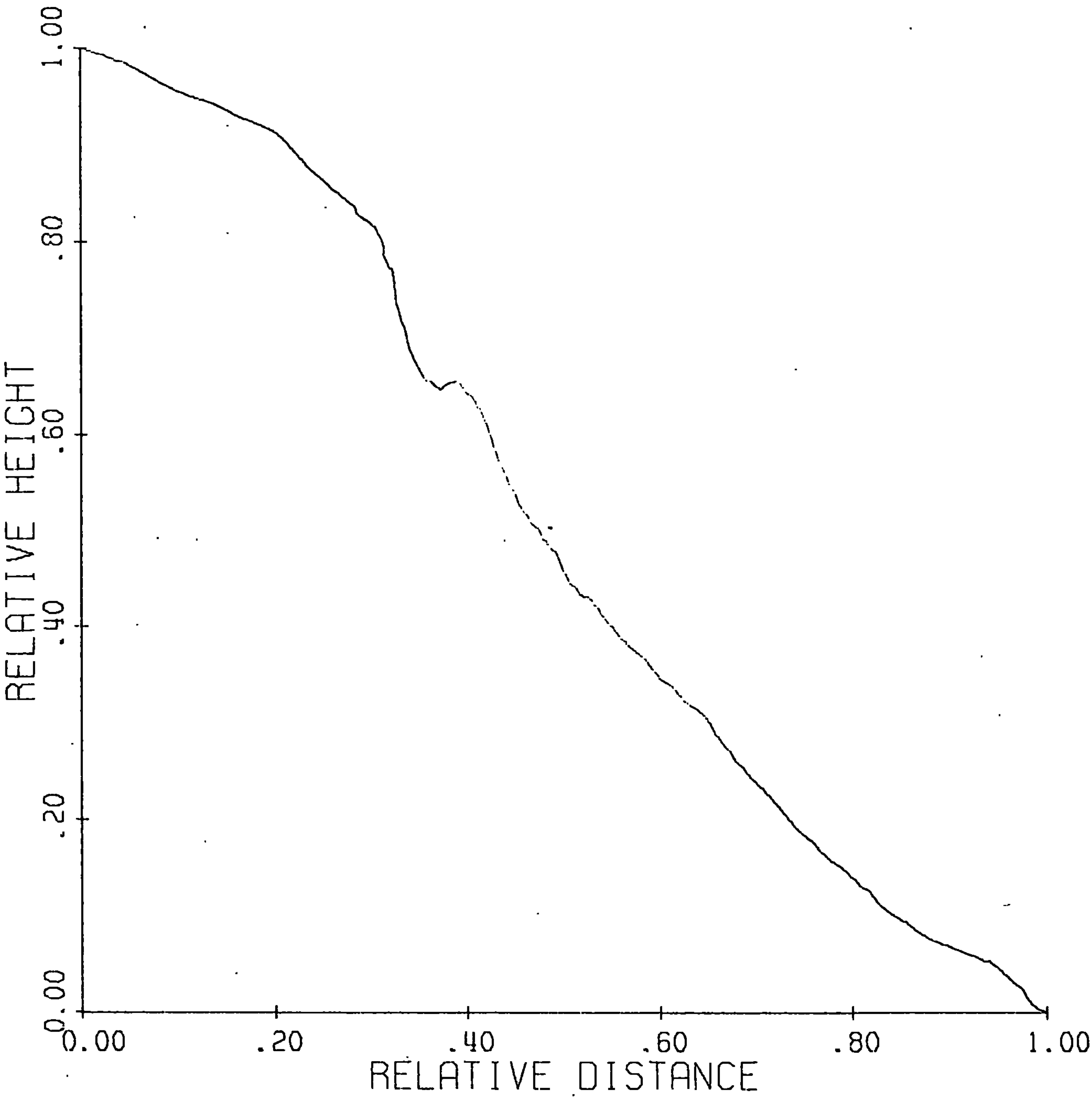
6G Profile TR



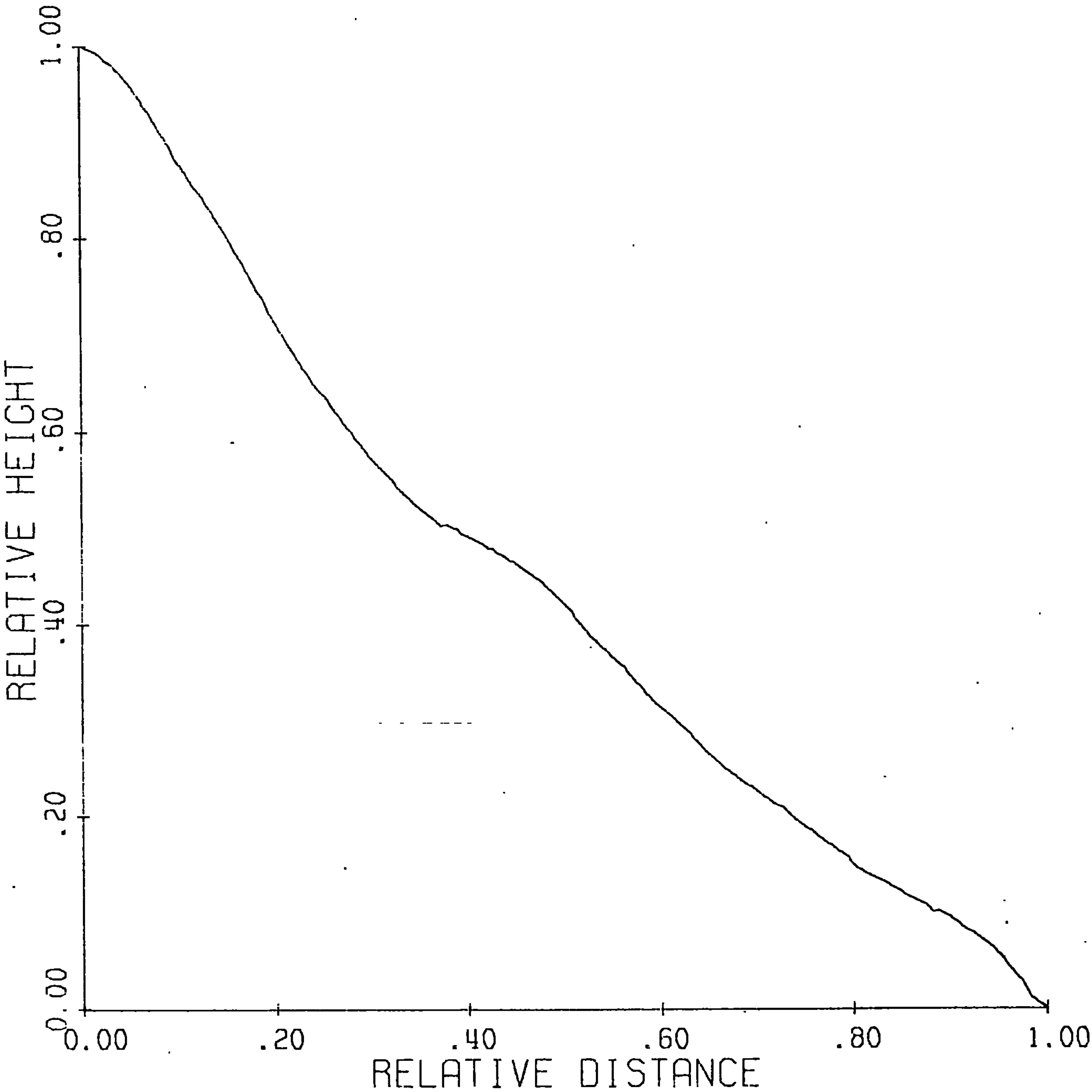
6H Profile AR



6I Profile TA



6J Profile HO



6K Profile TO

(ii) FA, LA: the next gentlest profiles, both gls/del.

(iii) TR, AR, TA: the next gentlest, all with bases on uli.

(iv) HO, TO: the steepest, crossing four and five formations respectively.

The dimensionless plots facilitate comparison of profile shape. They give an immediate impression of overall form. The vertical exaggeration of each plot is cot $\tilde{\theta}$ (= x_b/z_d).

(i) The four gentlest profiles, all on del, are slightly convex in overall form, with steepening towards the base (6A to 6D).

(ii) The next two, both on gls/del, are also slightly convex. Indeed PA and FA are almost identical in shape (i.e. dimensionless form) (6E and 6F).

(iii) TR, AR and TA, all with bases on uli, are more strongly convex, with suggestions of distinct breaks of slope bounding separate components. The most striking feature on these profiles is Tarn Hole Crag (estimated at 50^o) on TA (6G to 6I).

(iv) HO and TO are the steepest profiles. The most dramatic feature on HO is the failure scar below the crag. TO is a remarkably steep profile on the scarp of the Tabular Hills, being steepest near the crest (6J and 6K).

Dimensionless plots are used here partly as one method of standardised comparison, and partly because dimensionless curves originally obtained by Kirkby (1971)

6L. Measured hillslope profiles

Name	Identifier	Stream	Crest GR	Base GR	Geology	n	z_d (metres)	x_b (metres)	$\bar{\theta}$ (degrees)
Bonfield	BO	B. Gill	595965	600962	del	394	60	594	5.8
Stonymoor	ST	S. Sike	515981	508977	del	453	83	682	6.9
Proddale	PR	P. Sike	527967	524970	del	272	50	410	6.9
Parci Gill	PA	P.G.	534957	536952	del	421	83	630	7.5
Fangdale	FA	F. Beck	552948	559948	gls/del	506	116	755	8.7
Ladhill	LA	L. Beck	555928	548925	gls/del	469	113	701	9.1
Tripsdale	TR	T. Beck	577989	582987	del/uli	403	102	594	9.7
Arnsgill	AR	A.G.	527967	529962	del/uli	363	100	537	10.6
Tarn Hole	TA	T.H. Beck	599974	590977	gls/del/uli	567	174	830	11.8
Hollow Bottom	HO	H.B. Beck	550986	553982	del/dog/uli/mli	414	156	597	14.6
Todhill	TO	T. Beck	579905	578910	lcg/oxf/kel/cor/del	309	153	439	19.3

6M. Key to geological formation identifiers

Formation	Identifier
Lower Calcareous Grit	lcg
Oxford Clay	oxf
Kellaways Rock	kel
Cornbrash	cor
Deltaic Beds	del
Grey Limestone Series	gls
Dogger	dog
Upper Lias	uli
Middle Lias	mli

will be fitted to these profiles. Against this must be set an important reservation, that shape (dimensionless form) must not be considered independently of size: hillslope profiles must not be assumed isometric.

6.2 Angle and curvature frequency distributions

The study of angle frequency distributions is an accepted part of hillslope geomorphology (cf. Strahler, 1950, 1956; Young, 1961, 1972; Gregory and Brown, 1966; Pitty, 1969, 1970; Speight, 1971; Gerrard and Robinson, 1971; Juvigné, 1973; Statham, 1975; Carson, 1976, 1977; Evans, 1977 among several others). Angle frequency distributions have been compiled variously for all kinds of slopes and for particular kinds, such as straight components, while there has been interest both in the general form of frequency distributions, and in the identification and interpretation of modal or characteristic angles.

Curvature is a property which has received less attention. It is properly defined as the rate of change of angle θ with arc length s, that is $\frac{d\theta}{ds}$ (cf. Ferguson, 1973 on river meanders). In hillslope geomorphology, a variety of related curvature measures have been proposed (e.g. Ahnert, 1970c, 78-9; Young, 1972, 137-43; Demirmen, 1975, 258-60). In this study, constant measured lengths $l\ (=\Delta s)$ allow the use of the simplest measure, the first forward difference

$$\Delta\theta_i = \theta_{i+1} - \theta_i$$

(cf. Wilkes, 1966, 18; Ferguson, 1975 on river meanders).

Frequency distributions are most commonly summarised by moment-based measures, such as mean, standard deviation, skewness and kurtosis (see Evans 1972, 1977 for major examples of such an approach). The main disadvantage of moment-based measures is their lack of resistance (robustness) to wild observations or outliers (see Mosteller and Tukey, 1977, Ch. 10 for a contemporary introduction to resistance and robustness). The alternative approach taken here is to use resistant measures which are quantiles (order statistics) or functions of quantiles. As Pitty (1970) pointed out, this approach is more appropriate for the kinds of frequency distributions characteristic of hillslope profile data, which often include outliers.

Given a sample of data, the order statistics are the values ranked in numerical order, say

smallest has rank 1 ,..., largest has rank n

for the sake of argument. The minimum (min) and the maximum (max) are easily defined as the extreme order statistics. The median (med) is the middle ranked value, defined by the following rule:

(i) if n is odd

med = value with rank (n+1)/2

(ii) if n is even

med = average of value with rank (n+2)/2 and value with rank n/2

For defining other quantiles, it is helpful to introduce the average function, ave ( ), and the floor function (Iverson, 1962, 12), which gives the largest integer less than or equal to its argument: for example, floor (4.7) = 4 and floor (4.0) = 4.

The quartiles (loq, upq) may be estimated quickly by the following rules:

(i) if n is not divisible by 4

loq = value with rank of floor $(\frac{n+3}{4})$

upq = value with rank of n + 1 - floor $(\frac{n+3}{4})$

(ii) if n is divisible by 4

loq = ave (value with rank of floor $(\frac{n+3}{4})$, next larger value)

upq = ave (value with rank of n + 1 - floor $(\frac{n+3}{4})$, next smaller value)

(For these estimators of quartiles, see Andrews et al, 1972; for the labels min, max, med, loq, upq, see McNeil, 1977; for the label ave, see Blackman and Tukey, 1959).

Other quantiles may be estimated by analogous rules: the 5% and 95% points (p5, p95) are used here.

The spread (dispersion or scale) of a distribution may be measured by differences of the form

an upper quantile - its corresponding lower quantile

Range (max - min), 90% spread (p95 - p5) and midspread or interquartile range (upq-loq) show increasing resistance to outliers.

Finally quantile-based measures of 'asymmetry' and 'tailedness' may be devised as alternatives to moment-based skewness and kurtosis. Various such measures have often been used in sedimentology (Griffiths, 1967, 107-8), but those used here appear to be new.

The ratio

$$\frac{\text{upq} - \text{med}}{\text{med} - \text{loq}}$$

measures asymmetry. It has a lower limit of 0, a value of 1 for any symmetric distribution (e.g. Gaussian or normal), and indefinitely large values for increasingly right-skewed distributions.

An 'outer' quantile may be defined as one nearer an extreme than an 'inner quantile'. Ratios of the form

$$\frac{\text{spread between outer quantiles}}{\text{spread between inner quantiles}}$$

can be used in

$$\frac{\text{value of ratio for a distribution} - 1}{\text{value of ratio for Gaussian} - 1}$$

which is a measure of tailedness. It has a lower limit of 0, a value of 1 for a Gaussian distribution, and indefinitely large values for increasingly long-tailed distributions. If outer quantiles are p5 and p95, and inner quantiles are loq and upq, then the Gaussian has a spread ratio of 2.44. This particular tailedness measure is used here.

Summary measures for angle frequency distributions obtained from the measured profiles are given in 6N, which

6N. Absolute angle frequency distributions: order-based summary measures

Profile	min	p5	loq	med	upq	p95	max	range	90%spr	midspr	asymmetry	tailedness
BO	0.0	1.0	3.0	5.5	8.5	13.0	41.0	41.0	12.0	5.5	1.20	0.82
ST	0.0	2.0	5.0	7.0	8.5	14.0	23.5	23.5	12.0	3.5	0.75	1.69
PR	1.0	3.0	4.5	6.5	8.7	13.0	32.5	31.5	10.0	4.2	1.10	0.96
PA	0.0	2.0	4.5	6.5	9.0	17.5	45.0	45.0	15.5	4.5	1.25	1.70
FA	0.0	1.5	5.0	7.7	11.0	24.0	50.0	50.0	22.5	6.0	1.22	1.91
LA	0.0	2.5	5.0	9.0	12.5	17.0	25.5	25.5	14.5	7.5	0.88	0.65
TR	0.0	0.5	2.5	6.0	15.0	31.0	46.0	46.0	30.5	12.5	2.57	1.00
AR	0.0	3.0	5.0	8.5	13.0	25.5	47.5	47.5	22.5	8.0	1.29	1.26
TA	0.0	1.0	6.0	10.0	16.0	32.5	50.0	50.0	31.5	10.0	1.50	1.49
HO	0.0	4.5	8.0	12.5	19.0	36.0	90.0	90.0	31.5	11.0	1.44	1.29
TO	1.0	8.0	13.5	18.5	25.0	31.0	39.0	38.0	23.0	11.5	1.30	0.69

Note:Asymmetry and tailedness are dimensionless: other measures in degrees

Asymmetry of 1.0 indicates symmetry: tailedness of 1.0 indicates similarity to Gaussian in this respect.

lists min, p5, loq, med, upq, p95, max, range, 90% spread, midspread, asymmetry and tailedness. These measures are based on absolute measured angles (that is, ignoring the negative sign of angles on reversed slopes). Supporting diagrams are given in 6O and 6P.

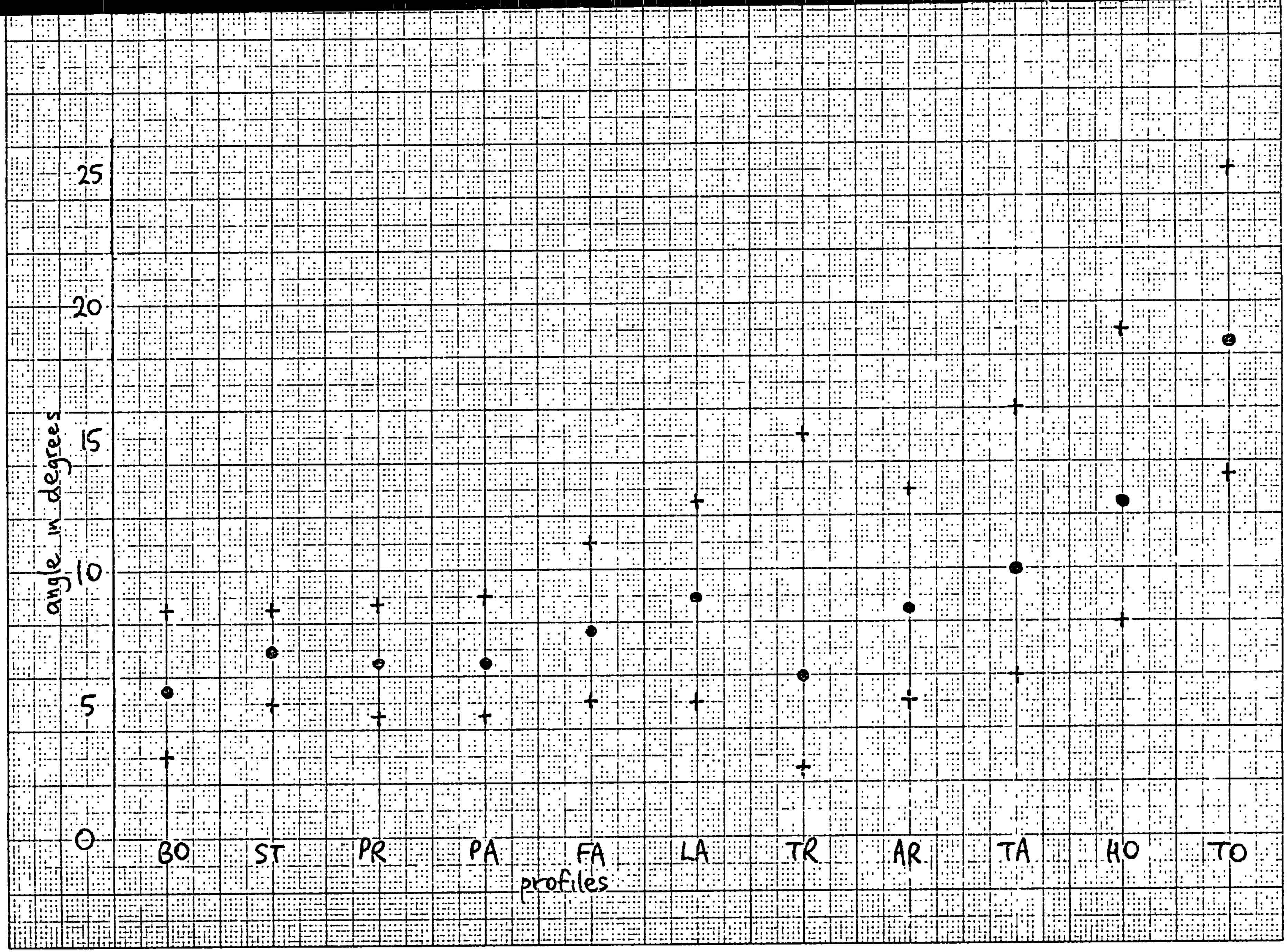
Medians and quartiles (6O) are easily related to the four classes identified above (6.1). BO, ST, PR, and PA are clearly similar with low medians and midspreads; FA and LA have higher values of both medians and midspreads. The marked curvature of TR is reflected by large spread and high asymmetry. The remaining profiles, AR, TA, HO and TO have increasingly large medians and midspreads.

Medians and midspreads (6P) supplement this picture. There is a general tendency for midspreads to increase with median, although TO (relatively low midspread) and particularly TR (very low median) are discrepant.

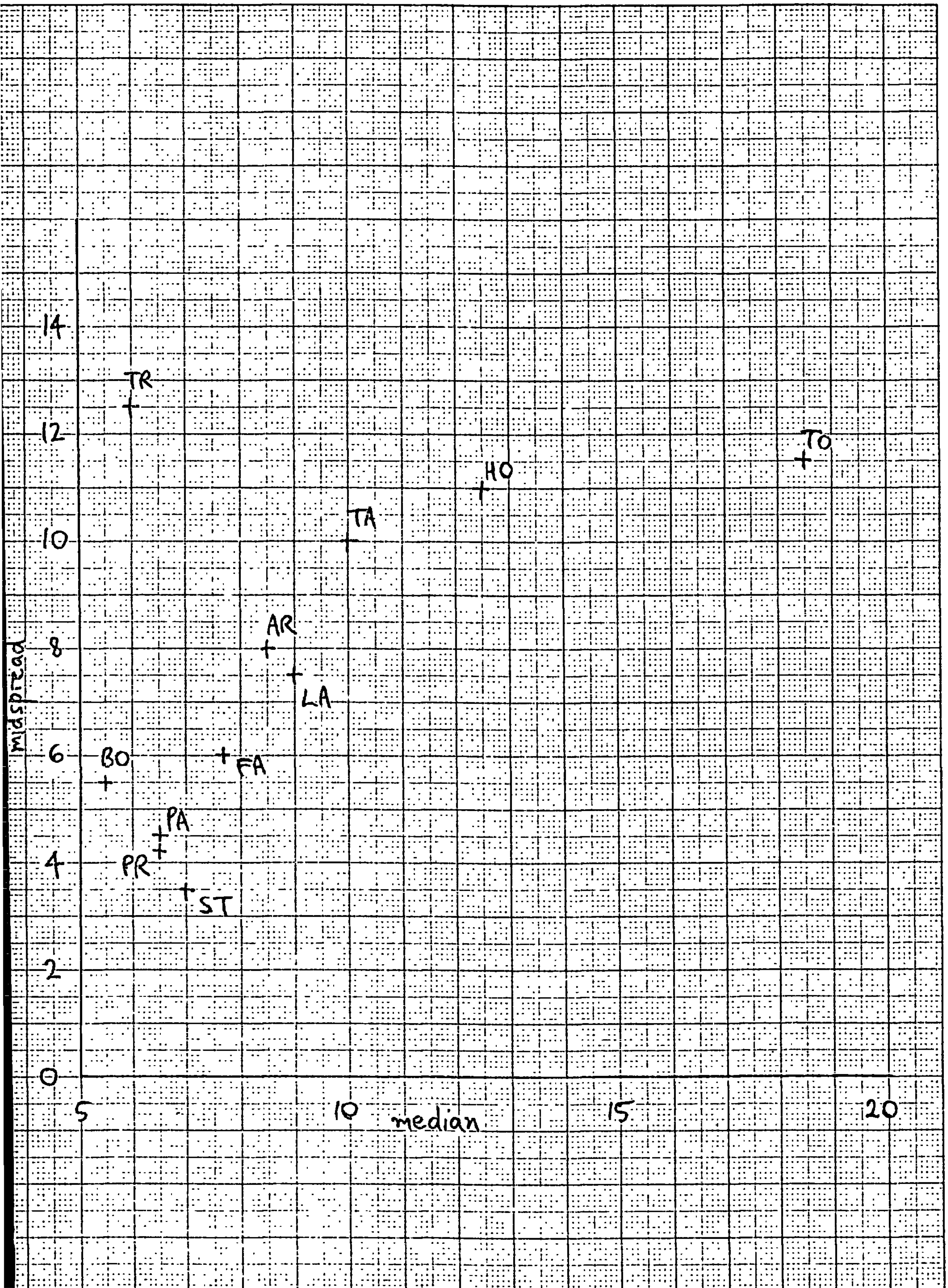
Angle distributions mostly vary from approximate symmetry to moderate right-skewedness; TR is notably most extreme. They range from being moderately short-tailed to being moderately long-tailed.

These results on asymmetry and tailedness raise the possibility of transforming the data to more nearly Gaussian values (cf. Evans, 1977 for a recent review of this issue). A transformation is monotonic if, for all values, it satisfies the identity

$$\text{rank of transformed value} \equiv \text{rank of original value}$$



60 Medians and quartiles of angle frequency distributions



6P Medians and midspreads of angle frequency distributions

The transformations employed for angle data are monotonic over the usual interval $0° \leq \theta \leq 90°$, and so the commutative property

quantile from transformed data = transform of original quantile

can be used to reduce the work in choosing appropriate transformations. As an example, the transform ln tangent has been used (6Q to 6S).

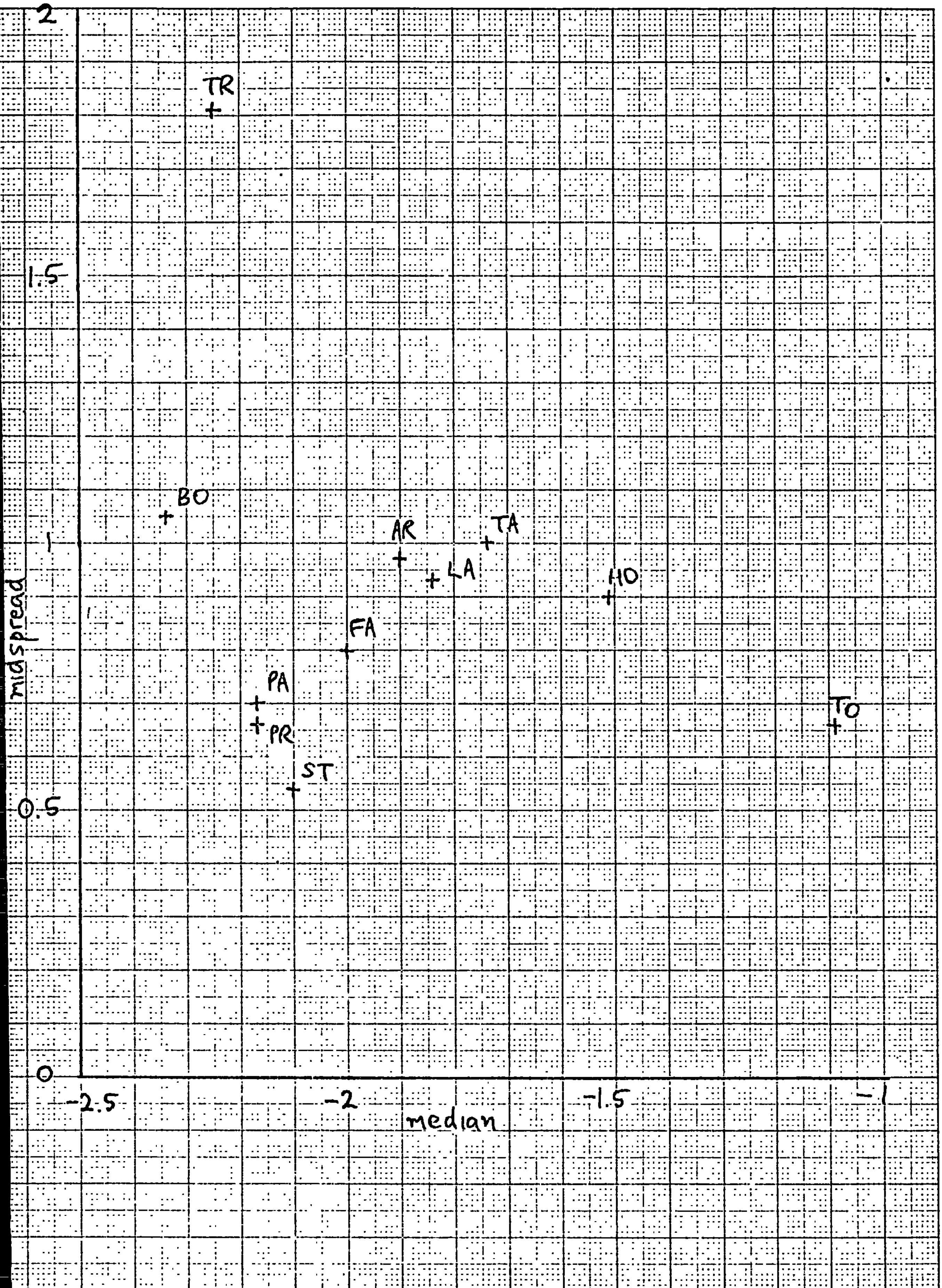
In this particular case the ln tangent transformation is only a mixed success. It removes most of the relationship between midspread and median, except that TR remains discrepant (6R). Asymmetry and tailedness are sometimes improved and sometimes worsened (6S), the pattern being as follows:

asymmetry & tailedness better: PA, FA, AR, HO, TO
asymmetry better, tailedness worse: TR, TA
asymmetry worse, tailedness better: BO, LA
asymmetry & tailedness worse: ST, PR.

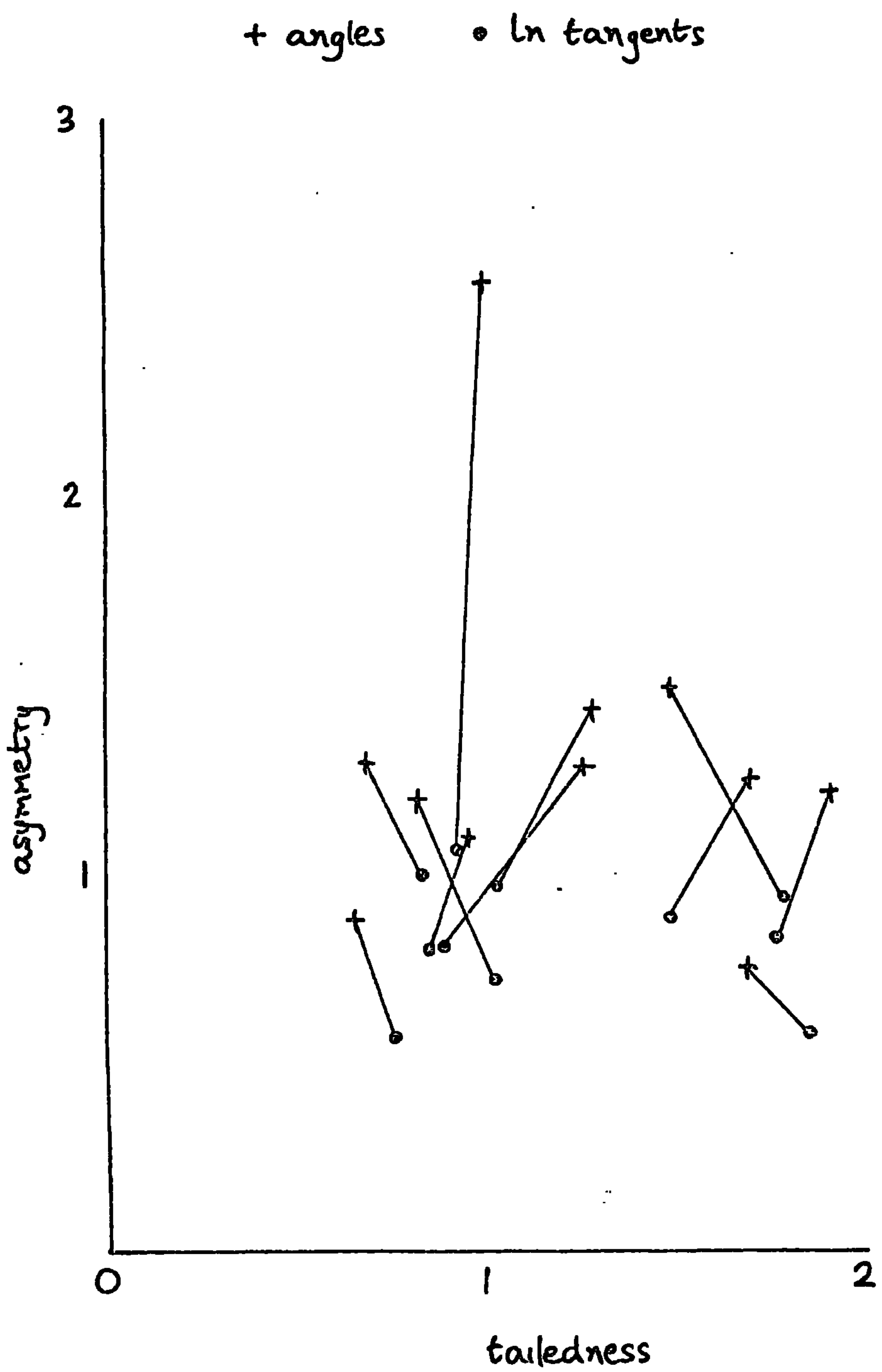
Curvature frequency distributions (6T) are all centred near zero, approximately symmetrical, and rather longer-tailed than Gaussian. They are distinguished mainly by their spreads.

6Q. Some measures for ln tangent transformed data

Profile	med	midspread	asymmetry	tailedness
BO	-2.34	1.05	0.72	1.02
ST	-2.10	0.54	0.58	1.85
PR	-2.17	0.66	0.80	0.85
PA	-2.17	0.70	0.89	1.49
FA	-2.00	0.80	0.83	1.77
LA	-1.84	0.93	0.57	0.76
TR	-2.25	1.81	1.07	0.93
AR	-1.90	0.97	0.81	0.88
TA	-1.74	1.00	0.94	1.79
HO	-1.51	0.90	0.97	1.03
TO	-1.09	0.66	1.00	-0.83



6R Medians and midspreads of transformed data



6S Asymmetry and tailedness of transformed data

6T. <u>Curvature frequency distributions: order-based summary measures</u>

Profile	min	p5	loq	med	upq	p95	max	range	90%spr	midspr	asymmetry	tailedness
BO	-39.5	-8.0	-2.5	0.0	3.0	7.5	20.0	59.5	15.5	5.5	1.20	1.26
ST	-37.0	-6.5	-2.0	0.0	2.5	7.0	28.5	65.5	13.5	4.5	1.25	1.39
PR	-25.0	-5.0	-2.0	0.0	1.5	4.5	19.0	44.0	9.5	3.5	0.75	1.19
PA	-31.5	-5.5	-2.0	0.0	2.0	6.5	19.5	51.0	12.0	4.0	1.00	1.39
FA	-45.5	-7.0	-2.5	0.0	2.5	6.5	22.0	67.5	13.5	5.0	1.00	1.18
LA	-23.0	-5.0	-1.5	0.0	1.5	5.0	20.0	43.0	10.0	3.0	1.00	1.62
TR	-24.0	-9.5	-2.5	0.0	2.5	10.0	25.5	49.5	19.5	5.0	1.00	2.01
AR	-32.0	-6.0	-2.5	0.0	2.0	6.5	16.5	48.5	12.5	4.5	0.80	1.23
TA	-30.5	-9.0	-3.0	0.0	3.0	9.5	20.5	51.0	18.5	6.0	1.00	1.45
HO	-60.0	-12.0	-3.5	-0.5	3.5	11.5	71.0	131.0	23.5	7.0	0.75	1.64
TO	-27.5	-11.0	-4.0	0.5	3.7	11.0	27.5	55.0	22.0	7.7	0.71	1.29

Note: Curvatures are based on <u>signed</u> angles. Asymmetry and tailedness are dimensionless: other measures in degrees.

6.3 Frequency distributions at different scales

It is important to consider whether results depend unduly on measured length, as argued by Gerrard and Robinson (1971) in particular. One way of investigating this question is to average adjacent angle values, compute curvatures as differences between adjacent averages, and recompile angle and curvature frequency distributions. (On averaging and differencing methods, see Blackman and Tukey, 1959; Curry, 1972; Cox, 1973).

In general, given a series u_i; $i = 1,\ldots,n$, an appropriate method would be to calculate averages for subseries of length n'

$$\bar{u}_j = \text{ave (values in subseries } j) \quad j = 1,\ldots,p$$

$$\bar{u}_j = \text{ave (values in subseries } j) \quad j = p + 1;\ n \text{ modulo } n' \neq 0$$

where p is given by

$$n - pn' = n \text{ modulo } n'$$

(i.e., the remainder on dividing n by n')

The corresponding differences are given by

$$\Delta\bar{u}_j = \bar{u}_{j+1} - \bar{u}_j$$

Varying subseries length n' then amounts to averaging and differencing at different scales.

In the case of hillslopes, subseries of angles of length n' do not in general correspond to measured lengths of n' times the original constant measured length, because slopes may be locally convex or concave. In addition, the appropriate averaging function appears to be

$$\bar{\theta} = \arctan\left(\sum \sin\theta \Big/ \sum \cos\theta\right)$$

Averaging and differencing were carried out for subseries lengths n' of 1, 2, 5 and 10 (corresponding to total measured lengths of 1.52m, 3.04, 7.60, 15.20m). In interpreting the results two points should be borne in mind. Firstly, as n' increases, the number of values available to compile each frequency distribution decreases (it is approximately n/n'), and results inevitably reflect a combination of scale variation and sampling variation, the latter increasing with subseries length. Secondly, since original data were measured to 0.5°, differences of this order are in no sense surprising.

Results for angle are given in 6U for selected measures. The patterns shown are broadly as would be expected. Minima and maxima approach each other as subseries length increases and bumps and ruts are averaged out. (The maxima for TA are an apparent exception, but the stability in this case reflects a string of 12 measured lengths on Tarn Hole Crag which could not be traversed with a pantometer: an Abney level was used to estimate the overall angle, and the distance over the string was measured by tape). These figures show quite clearly that maximum angle, favoured by Strahler (1950) and others, depends strongly on measured length. On all profiles except TA, maximum slopes for $n' = 10$ are 7 to 25 degrees gentler than for $n' = 1$. More thought must be given to this problem of scale variation by those seeking special process interpretations for maximum angles. It is also

Angle frequency distributions: variation of summary measures with scale

Profile	n′	min	loq	med	upq	max	midspread
BO	1	-10.0	3.0	5.0	8.0	41.0	5.0
	2	-3.0	3.5	5.5	7.7	36.0	4.2
	5	0.3	3.8	5.4	7.9	23.2	4.1
	10	0.8	3.7	5.7	8.1	16.0	4.4
ST	1	-14.5	5.0	7.0	8.5	23.5	3.5
	2	-3.7	5.2	6.7	8.2	20.7	3.0
	5	1.3	5.6	6.6	8.1	18.5	2.5
	10	2.9	5.9	6.7	7.8	13.5	1.9
PR	1	-1.5	4.5	6.5	8.7	32.5	4.2
	2	1.0	4.7	6.2	8.9	20.0	4.2
	5	2.5	4.8	7.0	8.7	14.4	3.9
	10	3.4	4.7	7.1	8.5	13.4	3.8
PA	1	-17.5	4.5	6.5	9.0	45.0	4.5
	2	-8.2	4.7	6.5	8.5	44.8	3.8
	5	-1.7	4.8	6.5	7.8	36.1	3.0
	10	0.2	4.9	6.5	8.0	35.7	3.1
FA	1	-21.0	5.0	7.5	10.5	50.0	5.5
	2	-9.2	5.5	7.5	10.0	44.2	4.5
	5	-1.0	5.6	8.0	9.5	36.6	3.9
	10	1.7	5.5	8.1	9.4	30.9	3.9
LA	1	-11.0	5.0	9.0	12.5	25.5	7.5
	2	-2.7	5.2	9.2	12.5	22.0	7.3
	5	2.2	5.2	9.7	12.9	18.0	7.7
	10	3.2	4.8	9.8	12.9	16.2	8.1
TR	1	-5.0	2.5	6.0	15.0	46.0	12.5
	2	-2.0	2.5	6.0	14.5	43.2	12.0
	5	-0.3	2.5	6.1	15.3	36.0	12.8
	10	0.1	2.7	6.2	18.0	33.9	15.3
AR	1	-2.0	5.0	8.5	13.0	47.5	8.0
	2	1.2	5.5	8.9	13.5	43.2	8.0
	5	2.3	5.7	7.9	13.7	41.6	8.0
	10	3.4	6.4	8.1	14.6	37.7	8.2
TA	1	-11.5	5.5	10.0	16.0	50.0	10.5
	2	-3.5	5.4	9.7	16.0	50.0	10.6
	5	-2.2	5.8	9.3	16.2	50.0	10.4
	10	0.1	6.9	10.0	16.8	50.0	9.9
HO	1	-21.0	8.0	12.5	19.0	90.0	11.0
	2	-15.2	8.0	12.7	18.8	65.0	10.8
	5	-7.1	8.0	13.7	18.2	52.7	10.2
	10	1.2	7.8	13.8	17.1	46.5	9.3
TO	1	-10.5	13.5	18.5	25.0	39.0	11.5
	2	-3.0	13.7	18.7	24.7	35.0	11.0
	5	3.2	13.8	18.6	25.0	34.0	11.2
	10	7.9	13.7	19.1	24.5	31.7	10.8

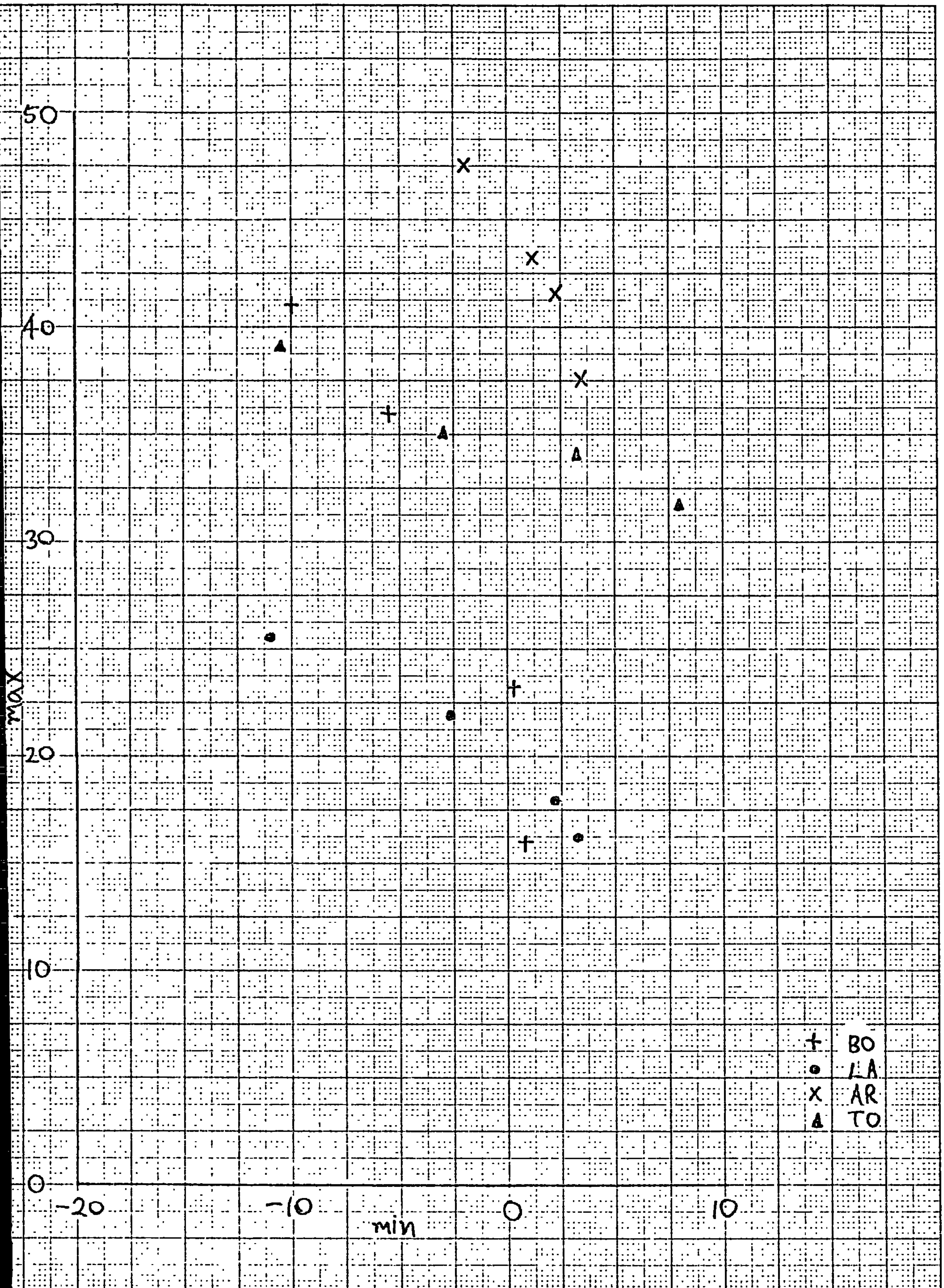
important to note from these results that only on HO and TA are angles of 40^{o} or more other than localised. The minima are of less interest: they generally arise from local slope reversals, often of human origin (cf. Appendix I).

By contrast median and midspread are relatively stable measures, as would be expected from their resistant characteristics.

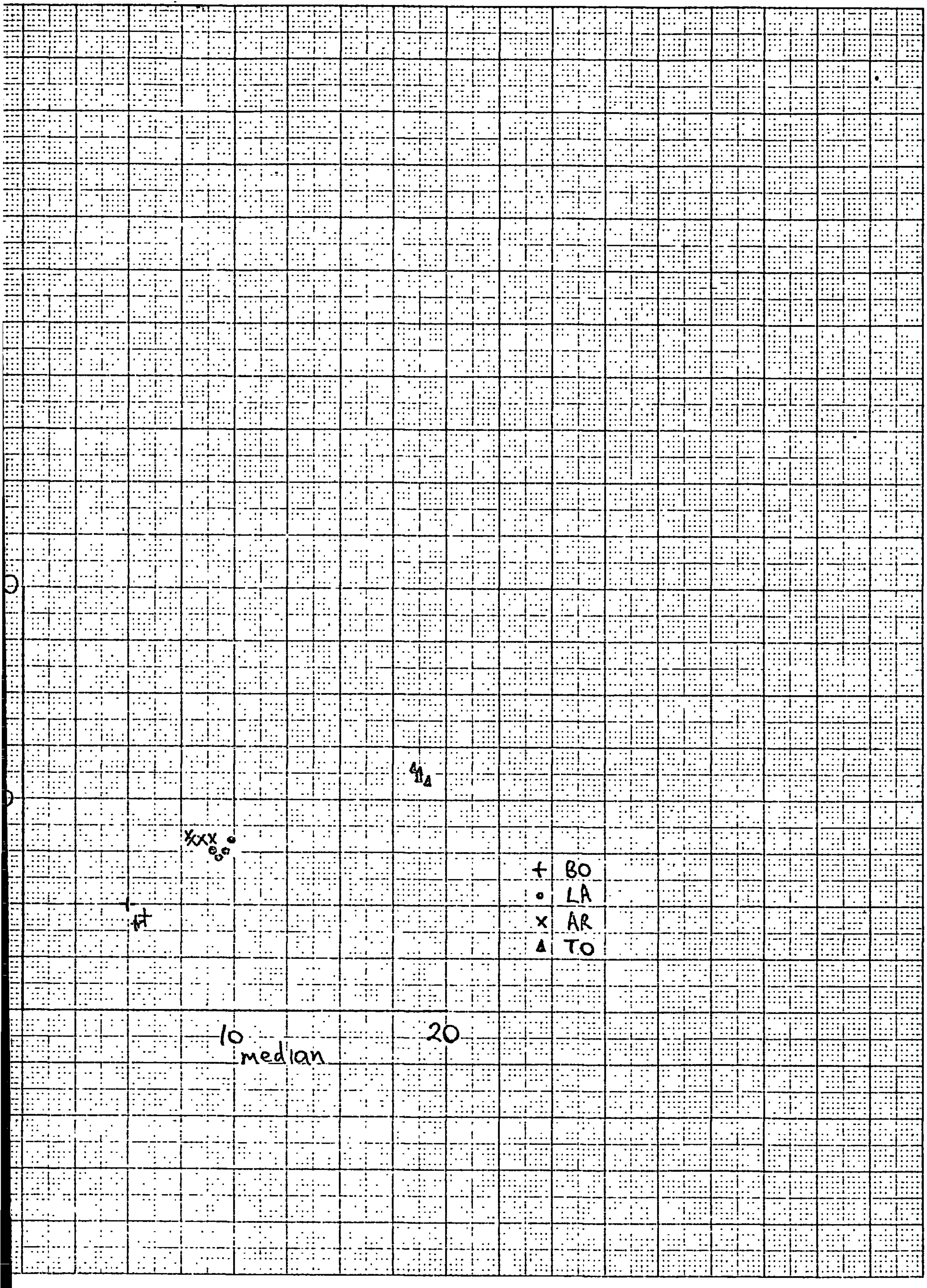
Results are displayed graphically for four selected profiles, one from each of four groups distinguished earlier (BO, LA, AR, TO) in 6V and 6W. The marked contrast between unstable min and max (6V) and stable med and midspread (6W) is very clear.

Results for curvature (6X) show again that extremes are unstable and the median stable. There is also a systematic tendency for quartiles to approach the median, which is expectable on geometric grounds (note again that the curvature measure used here is $\Delta\theta$; no account has been taken of varying Δs).

Averaging and differencing thus appears to be a useful method of exploring scale variation, and of identifying scale-variable and scale-constant morphometric measures. The question of scale variation will arise again in other ways in the following two chapters.



6V Scale variation in min and max angles for selected profiles



Scale variation in median and midspread of angles for selected profiles urvature frequency distributions: variation of summary measures with scale

Profile	n′	min	loq	med	upq	max	midspread
BO	1	-39.5	-2.5	0.0	3.0	20.0	5.5
	2	-26.2	-2.0	0.2	2.1	20.5	4.1
	5	-17.2	-0.8	0.1	1.1	14.3	1.9
	10	-10.0	-0.7	0.2	1.2	6.5	1.9
ST	1	-37.0	-2.0	0.0	2.5	28.5	4.5
	2	-18.0	-1.5	0.0	1.7	9.0	3.2
	5	-6.4	-1.2	0.1	1.1	10.0	3.3
	10	-4.9	-0.8	0.2	1.0	7.5	1.8
PR	1	-25.0	-2.0	0.0	1.5	19.0	3.5
	2	-10.7	-1.0	0.5	1.2	7.5	2.2
	5	-6.4	-0.7	0.3	1.1	3.7	1.8
	10	-5.4	-0.5	0.5	1.2	2.8	1.7
PA	1	-31.5	-2.0	0.0	2.0	19.5	4.0
	2	-24.5	-1.5	0.0	1.5	17.5	3.0
	5	-27.3	-0.8	0.0	1.5	10.0	2.3
	10	-27.7	-0.7	0.4	1.3	12.5	2.0
FA	1	-45.5	-2.5	0.0	2.5	22.0	5.0
	2	-14.7	-1.7	0.0	1.7	21.7	3.4
	5	-11.4	-0.8	0.2	1.1	5.9	1.9
	10	-7.5	-1.0	0.7	1.5	8.3	2.5
LA	1	-23.0	-1.5	0.0	1.5	20.0	3.0
	2	-11.2	-1.0	0.0	1.2	13.2	2.2
	5	-3.6	-0.9	0.2	0.9	6.6	1.8
	10	-3.3	-0.6	0.3	0.9	7.3	1.5
TR	1	-24.0	-2.5	0.0	2.5	25.5	5.0
	2	-16.7	-1.7	0.0	2.0	19.2	3.7
	5	-11.0	-1.2	0.2	1.5	19.4	2.7
	10	-7.9	-1.3	0.5	1.9	13.7	3.2
AR	1	-32.0	-2.5	0.0	2.0	16.5	4.5
	2	-19.5	-1.5	0.3	1.5	10.7	3.0
	5	-21.2	-1.2	0.3	2.0	12.0	3.2
	10	-17.3	-0.9	0.3	2.1	10.1	3.0
TA	1	-30.5	-3.0	0.0	3.0	20.5	6.0
	2	-15.0	-2.0	0.0	2.3	20.0	4.3
	5	-14.6	-1.6	0.0	1.7	18.1	3.3
	10	-20.0	-1.4	0.2	1.5	26.4	2.9
HO	1	-60.0	-3.5	-0.5	3.5	71.0	7.0
	2	-29.0	-3.2	-0.3	3.0	35.5	6.2
	5	-16.4	-2.8	-0.6	3.4	20.4	6.2
	10	-29.9	-2.6	-0.5	2.2	21.2	4.8
TO	1	-27.5	-4.0	0.5	3.7	27.5	7.7
	2	-20.5	-2.5	0.0	3.2	10.7	5.7
	5	-13.6	-1.8	0.2	1.8	9.4	3.6
	10	-9.0	-2.5	0.1	2.9	12.8	5.4

6.4. Summary

(i) Profile form is discussed for the profiles measured in the field in terms of profile dimensions and profile shape. A four-fold grouping is outlined, which is closely related to variations in bedrock geology (6.1).

(ii) Angle and curvature frequency distributions are summarised using quantile-based measures which are considered more appropriate for data containing wild observations. Median and midspread of angle can be readily interpreted in terms of the four-fold grouping proposed earlier (6.2).

(iii) Spatial averaging and differencing of angle series throws light on the scale variation of distribution summary measures. Minimum and maximum angles are extremely unstable, but median and midspread of angles are satisfactorily stable (6.3).

6.5. Notation

d	in ordinary derivative
i	subscript
j	subscript
l	measured length
n	number of observations
n'	number in subseries
p	related to number of subseries
s	arc length
u	value of series
$\bar{u}$	average value
x	horizontal coordinate
x_b	slope length
z	vertical coordinate
z_d	slope height
Δ	difference operator
θ	angle
$\bar{\theta}$	average angle
Σ	summation operator

Mnemonics

ave	average
loq	lower quartile
max	maximum
med	median
min	minimum
p5	5% point
p95	95% point
upq	upper quartile

Chapter 7

AUTOCORRELATION ANALYSIS OF HILLSLOPE SERIES

Schoolmaster: 'Suppose x is the number of sheep in the problem'. Pupil: 'But, Sir, suppose x is not the number of sheep'. [I asked Prof. Wittgenstein was this not a profound philosophical joke, and he said it was.]

J. E. Littlewood, A mathematician's miscellany, p. 41.

7.1 The idea of autocorrelation

The idea of autocorrelation is probably best approached from the more widely-known idea of correlation. The correlation r between two variables, u and v say, is a measure with the following general properties:

(i) $-1 \leq r \leq 1$.

(ii) r is positive if u and v are associated directly, and negative if they are associated inversely.

(iii) The absolute value of r is 0 if u and v are not associated (uncorrelated), 1 if they are perfectly associated, and between 0 and 1 for intermediate cases.

Particular measures of correlation may be distinguished according to the precise criterion of 'association' which is employed. If linear association is being considered, then it is natural to take the Pearson product-moment correlation coefficient

$$r = \frac{\mathrm{cov}(u,v)}{\sqrt{[\mathrm{var}(u)\,\mathrm{var}(v)]}} = \frac{\mathrm{cov}(u,v)}{\mathrm{std}(u)\,\mathrm{std}(v)}$$

where cov, var and std denote covariance, variance and standard deviation.

For a set of observations u_i, v_i; $i=1,\dots,n$ we have, cancelling a divisor common to numerator and denominator,

$$r = \frac{\sum_{i=1}^{n}(u_i - \bar{u})(v_i - \bar{v})}{\sqrt{\left[\left(\sum_{i=1}^{n}(u_i - \bar{u})^2\right)\left(\sum_{i=1}^{n}(v_i - \bar{v})^2\right)\right]}}$$

where $\bar{u}$, $\bar{v}$ are means

$$\bar{u} = \frac{1}{n}\sum_{i=1}^{n} u_i \qquad \bar{v} = \frac{1}{n}\sum_{i=1}^{n} v_i$$

Putting $u_i - \bar{u} = a_i$, $v_i - \bar{v} = b_i$ then

$$r = \frac{\sum_{i=1}^{n} a_i b_i}{\sqrt{\left[\sum_{i=1}^{n} a_i^2 \sum_{i=1}^{n} b_i^2\right]}}$$

For any real numbers a_i, b_i; $i=1, \ldots, n$ the Cauchy-Schwarz inequality (e.g. Stephenson, 1971, 14) gives

$$\left(\sum_{i=1}^{n} a_i b_i\right)^2 \leqslant \sum_{i=1}^{n} a_i^2 \sum_{i=1}^{n} b_i^2$$

whence

$$\left|\sum_{i=1}^{n} a_i b_i\right| \leqslant \sqrt{\left[\sum_{i=1}^{n} a_i^2 \sum_{i=1}^{n} b_i^2\right]}$$

and

$$|r| = \left|\frac{\sum_{i=1}^{n} a_i b_i}{\sqrt{\left[\sum_{i=1}^{n} a_i^2 \sum_{i=1}^{n} b_i^2\right]}}\right| \leqslant 1$$

The Pearson correlation coefficient is not the only possible measure of correlation; other criteria of association lead to other measures. If <u>monotonic</u> association is being considered, then it is natural to compute measures based on ranks only, such as Spearman's rank correlation coefficient.

Autocorrelation involves correlating one variable (say u) with itself, rather than two variables (say u and v) with each other. In particular, take the case of a one-dimensional series u_i; i=1,... ,n where the subscripts i refer to position in a sequence, whether temporal or spatial. (The case of observations in two or more dimensions will not be considered here. In the two-dimensional case, autocorrelation can be defined by a straightforward analogy with the one-dimensional case if observations refer to a regular lattice: a different approach is necessary for an irregular lattice, for which see Cliff and Ord, 1973.)

Autocorrelation is the correlation between a series and itself displaced. Thus by analogy with the Pearson coefficient

$$r_k = \frac{\operatorname{cov}(u_i, u_{i+k})}{\sqrt{[\operatorname{var}(u_i)\ \operatorname{var}(u_{i+k})]}}$$

Here k is the lag or spacing between observations u_i; i=1, ... , n-k, which are being correlated with observations u_{i+k}; i+k = 1+k,...,n. The covariance is known in this case as the autocovariance. For k = 0, $r_k = r_o$, the correlation between a series and itself, which is exactly 1. For k = 0,1,2,3,... , we have the autocorrelation function,

r_0, r_1, r_2, r_3,.... A plot of the autocorrelation function is a correlogram.

Some basic texts on autocorrelation and related subjects are as follows:

(i) Agterberg (1974) and Schwarzacher (1975), reliable texts intended for earth scientists;

(ii) Kendall (1973) and Chatfield (1975), general texts on time series analysis.

7.2 Autocorrelation and hillslope profiles

Autocorrelation analysis of spatial series is now an accepted technique in geomorphology, either in isolation, or more usually in conjunction with spectral analysis or model fitting. It has been used, for example, in the study of stream and gully profiles (Melton, 1962; Bennett, 1976; Richards, 1976; Alexander, 1977), of stream plans (Speight, 1965, 1967; Ferguson, 1975, 1976), and of topographic profiles (Evans, 1972, 33-6; Drewry, 1975; Pike and Rozema, 1975; Webster, 1977). Empirical autocorrelation functions were computed from hillslope profile data first by Nieuwenhuis and van den Berg (1971), and subsequently by Thornes (1972, 1973).

Why study autocorrelation properties of hillslope profile data? There seem to be four main reasons.

Firstly, many standard statistical methods assume independence of observations. If a variable v is independent of u, then the conditional probability distribution pr(v|u) is identical to the marginal distribution pr(v). Loosely, knowing u provides no extra information which would help in predicting v. If v is independent of u, then u and v are uncorrelated, i.e. r (u,v) = 0. (The converse is not necessarily true, as may be seen from a counterexample (Plackett, 1971, 65): if u is symmetrically distributed about 0, then v = |u| and u are uncorrelated, even though v may be predicted exactly from u.)

However, spatial data tend to be strongly autocorrelated, which implies that tabulated sampling distributions of various standard test statistics such as Student's t and Fisher's F are definitely invalid (cf. Moran, 1973; Hepple, 1974; Box, 1976; Haggett et al., 1977, Chs. 10-11). Nieuwenhuis and van den Berg (1971) were the first to discuss the problem of invalidity of standard methods in a hillslope context. Their results suggested that a 10% sample of tangents measured over adjacent 10 m measured lengths would consist of approximately independent observations.

Hence the strength of autocorrelation present in hillslope data affects the validity of some standard statistical tests.

Secondly, it is possible that autocorrelation functions and/or autocovariance functions might serve as descriptors of surface roughness at different scales. It is a matter of common observation that slopes are rough, displaying 'bumps', 'ruts', 'microrelief' and even 'nanorelief' (Young, 1972, 201). Autocorrelation would seem a possible inverse measure of roughness, for the extent to which values of surface geometry can be predicted from neighbouring values is inversely related to surface roughness.

Thirdly, some stochastic models (reviewed in Ch. 3 above) yield predictions of autocorrelation functions which can be tested empirically.

Fourthly, models which are combinations of deterministic and stochastic components can be fitted by assuming independence of stochastic errors. The residuals, or differences between actual and predicted values, should then be examined for autocorrelation (Pitty, 1970; cf. Ch. 9 below).

The autocorrelation properties of hillslope profile data are thus of interest for several reasons, and clearly merit further exploration.

7.3. Stationarity, nonstationarity and autocorrelation

Calculated autocorrelations can be taken simply as descriptive measures. For example, any measure of the form

$$r = \frac{\sum_{i=1}^{n} a_i b_i}{\sqrt{\left[\sum_{i=1}^{n} a_i^2 \sum_{i=1}^{n} b_i^2\right]}}$$

which satisfies $|r| \leq 1$ has a natural geometric interpretation as the cosine of the angle between the vectors $(a_1, \ldots, a_n)$ and $(b_1, \ldots, b_n)$. It could be argued that actual values of autocorrelations are the only subject of interest: what values might have been is a hypothetical issue.

Nevertheless, one standard approach to autocorrelation is centred on such inferential questions. The assumption which is made is that the empirical series u_i; $i=1,\ldots,n$ is one possible realisation of a chance or stochastic process U. The inferential problem is to estimate the parameters of this generating process.

Almost all the theory behind such an approach considers the special case of stationary stochastic processes, which are defined by the characteristic that expectations of statistics are everywhere the same: that is to say, the averages over all possible realisations are constant in time and/or space. In particular, a process is second-order stationary if expectations of first-order and second-order statistics are invariant under translation of the origin. (Higher order statistics might vary.) Such a process is defined by constant mean

$$E[U] = \mu$$

and autocovariance function

$$E[(U_i - \mu)(U_{i+k} - \mu)] = \gamma_k$$

Here E is the expectation operator (see, for example, Whittle, 1970, Ch. 2).

Putting k = 0 we deduce constant variance

$$\gamma_0 = \sigma^2$$

and scaling we deduce constant autocorrelation function

$$\frac{\gamma_k}{\gamma_0} = \rho_k$$

Synonyms for second-order stationarity are covariance, mean square, weak, wide-sense stationarity.

It is usually argued that it only makes sense to compute the autocorrelation function if the generating process is second-order stationary (in a geological context see Agterberg, 1974, 314-5; Schwarzacher, 1975, 166). Otherwise the autocorrelation function is effectively a set of variables, not a set of parameters, and it is indeed extremely doubtful whether there is an estimation problem in the classical sense. If this is true, it is very restrictive: for if stationarity is a requirement, only straight slopes can be studied by autocorrelation analysis of angle data, and only slopes of constant curvature (single convexities or single concavities) by autocorrelation analysis of curvature data. An approach so restricted would be geomorphologically useless.

However, attempts to step outside the framework provided by stationarity face a general problem explained clearly by Whittle (1963, 83): 'In dropping the assumption of stationarity, one is left with scarcely any restriction upon one's model. For this reason, it is all the more difficult to specify a model, or even to specify some of the statistical properties of the variates (such as first and second moments)'.

This view is perhaps a little conservative. Independence or dependence of observations is an important issue, whether or not there is any question of a stationary generating process. Empirical autocorrelation functions might be of some use as averaged descriptors even in nonstationary situations. And increasing interest is being shown by statisticians in nonstationary processes, notably in the class of ARIMA processes discussed by Box and Jenkins (1976).

Nevertheless the standard response to nonstationarity is to operate on the empirical series to produce a new, approximately stationary, series, usually by dividing it into nonstationary and stationary components. Detrending by polynomial approximation, variate differencing, the application of moving averages, demodulation and remodulation, and variance-stabilising transformations have been the main methods used. None of these are universally effective or free from secondary complications but in processing geomorphological series the most important single issue must be the physical rationale of any method for producing approximate stationarity. For this reason autocorrelation analysis finds its major application in analysing residuals from process-based or process relevant models: any arbitrary specification of trend is likely to produce geomorphologically irrelevant results. (However, first differencing is of interest, because curvature deserves attention in its own right.)

The sampling distribution theory of autocorrelation functions of stationary processes has been extensively investigated. One approximate but fairly robust result for the null case $\rho_k = 0$ (k fairly small) is

$$r_k \sim N\left(-\frac{1}{n}, \frac{1}{n}\right)$$

Both bias and variance are $O\left(\frac{1}{n}\right)$. Bias may thus be neglected in practice for $n \gg 200$. This analytical result is broadly supported by Monte Carlo sampling experiments (Cox, 1966; Wallis and O'Connell, 1972).

(The term stationarity is used here to denote invariance under translation of the origin, referring uniformly to temporal, spatial and temporal-spatial stochastic processes. Note, however, that homogeneity is used by some authors for the spatial case. A further point of importance is that processes in two or more dimensions may also be characterised by the different property of isotropy, invariance under rotation of coordinate axes.)

7.4 Choice of estimator

Four different estimators of autocorrelation are widely used. Different means are distinguished below by

$$\bar{u} = \frac{1}{n}\sum_{i=1}^{n} u_i$$, the mean of the whole series

$$\bar{u}' = \frac{1}{n-k}\sum_{i=1}^{n-k} u_i$$, the mean of leading values

$$\bar{u}'' = \frac{1}{n-k}\sum_{i=1}^{n-k} u_{i+k}$$, the mean of lagging values

(1) (e.g. Kendall, 1973, 40; Schwarzacher, 1975, 164)

$$r_k = \frac{\sum_{i=1}^{n-k}(u_i - \bar{u}')(u_{i+k} - \bar{u}'')}{\sqrt{\left[\sum_{i=1}^{n-k}(u_i - \bar{u}')^2 \sum_{i=1}^{n-k}(u_{i+k} - \bar{u}'')^2\right]}}$$

This is the exact analogue of the Pearson coefficient, and, from the Cauchy-Schwarz inequality, satisfies $-1 \leqslant r_k \leqslant 1$.

(2) (e.g. Granger and Hatanaka, 1964, 8; Quimpo, 1968, 367)

$$r_k = \frac{\frac{1}{n-k}\sum_{i=1}^{n-k}(u_i - \bar{u}')(u_{i+k} - \bar{u}'')}{\frac{1}{n}\sum_{i=1}^{n}(u_i - \bar{u})^2}$$

In this case the denominator only contains one variance term, the variance of the whole series.

(3) (e.g. Kendall, 1973, 40; Agterberg, 1974, 315; Båth, 1974, 179; Drewry, 1975, 194; Richards, 1976, 77; Webster, 1977, 199)

$$r_k = \frac{\frac{1}{n-k}\sum_{i=1}^{n-k}(u_i - \bar{u})(u_{i+k} - \bar{u})}{\frac{1}{n}\sum_{i=1}^{n}(u_i - \bar{u})^2}$$

In this case there is a further simplification: both terms in the covariance are referred to the mean of the whole series, rather than to separate means of leading and lagging values.

(4) (e.g. Jenkins and Watts, 1968, 180-2; Fishman, 1969, 82-3; Chatfield, 1975,24; Box and Jenkins, 1976, 32)

$$r_k = \frac{\sum_{i=1}^{n-k}(u_i - \bar{u})(u_{i+k} - \bar{u})}{\sum_{i=1}^{n}(u_i - \bar{u})^2}$$

The final simplification here is that the factor $\frac{n}{n-k}$ has been dropped.

What justification could there be for using (2), (3) or (4) in place of (1)?

Firstly, use of the grossly biased (4) in particular ensures positive estimates of variance spectrum ordinates; otherwise meaningless negative estimates might be produced. This is relevant only if spectral analysis is being undertaken.

Secondly, the gross bias of (4) may be practically neglected for $k \ll n$.

Thirdly, (2), (3) and (4) may be calculated more quickly than (1). This is relevant only if computing facilities are not available.

Fourthly, it is often asserted that approximate estimators are more efficient (i.e. have smaller variance) than the Pearson analogue estimator (1), basically because the denominator is a constant, not a function of lag, and because the use of one mean for the whole series rather than separate means for leading and lagging values would reduce variability. However, sampling experiments for stationary processes suggest that the gain may not be appreciable (cf. Cox, 1966; Wallis and O'Connell, 1972; Lenton and Schaake, 1973 in estimating ρ_1 in null and Markov cases).

Fifthly, expectations of (1), (2) and (3) coincide under second-order stationarity. This is the argument of computational simplicity in another guise, and may be rejected on the groundsthat estimation should not assume second-order stationarity as a matter of course.

Note again that only (1) satisfies $|r_k| \leq 1$: the others may lead to 'improper' estimates ($|r_k| > 1$), although in practice this is likely only at long lags.

Examples of autocorrelation analysis in geomorphology (7A) include cases in which estimators have not been specified, cases in which estimators have not been made completely clear, and cases of incorrect formulae. There does seem to be a general preference for estimators (3) and (4), partly because several authors used spectral analysis.

7A Examples of autocorrelation analysis in geomorphology

Reference	Original data	Estimator used	Comments
Melton 1962	stream gradient	? (1)	
Bennett 1976	stream height	not stated	
Richards 1976	" "	(3)	
Alexander 1977	gully gradient, width etc	? (3)	confuses autocorrelation and autocovariance
Speight 1965, 1967	stream direction	? (4)	confuses autocorrelation and autocovariance
Ferguson 1975, 1976	" "	not stated	
Drewry 1975	height	(3)	
Pike & Rozema 1975	"	? (4)	omits reference to subtraction of mean
Webster 1977	"	(3)	
Nieuwenhuis & van den Berg 1971	slope gradient	circular estimator	
Thornes 1972, 1973	" angle	not stated	

It is concluded that the Pearson analogue estimator should be used, and a fortiori that the chosen estimator should be stated and justified. A safe procedure is to use both (1) and (3): the difference between estimates is a measure of second-order nonstationarity, which is of interest in its own right. Such a procedure is also preferable to the elaborate and rather impracticable tests for nonstationarity which have been proposed (e.g. Priestley and Subba Rao, 1969).

7.5 Empirical results

In this section empirical results are presented for autocorrelation functions calculated for each profile, for angle and curvature series, using two separate estimators for lags k = 0 (1) 50. Various rules of thumb exist for the minimum number of lags to be computed, partly because a large number of autocorrelation coefficients are usually redundant, and partly because coefficients become increasingly unstable as k increases, since they are based on fewer pairs of values. Davis (1973, 236), Chatfield (1975, 25) and Box and Jenkins (1976, 33) all recommended not going beyond n/4. The rule of thumb suggested here after some experimentation (which included computing all possible lags!) is to compute up to the smaller of 50 and floor (n/4). (For the floor function, see Ch. 6.2 above, or Iverson, 1962, 12).

Autocorrelations for lags k = 1 (1) 5 (5) 50, for angle and curvature, for all profiles and for 'Pearson' and 'abbreviated' estimators, i.e. (1) and (3) of Ch. 7.4, are displayed in 7B, 7C, 7E and 7F. Differences between estimates are displayed in 7D and 7G. These tables summarise 44 correlograms which cannot all be shown here. However, angle and curvature series and the resulting correlograms for the Pearson estimator are shown in 7H to 7O for two contrasting profiles, ST and AR.

In interpreting these autocorrelations, two sets of questions, geomorphological and statistical, must be considered. The most basic feature of 7B (Pearson for angle) is the fact, expected on geomorphological grounds, that angle autocorrelations do not in general dampen to zero. 143 out of 154 autocorrelations in the table lie outside the 0.99 confidence interval for $\rho_k=0$, which was calculated on the assumption that estimator bias could be neglected and hence $r_k \sim N(0,\frac{1}{n})$. This disposes of any notion that these angle observations can be taken as statistically independent. Nor would a sampling procedure akin to that suggested by Nieuwenhuis and van den Berg (1971) work for these data.

The next idea to go is the fairly naive notion that autocorrelation functions might capture the local property of surface roughness. The correlograms in fact tell a story of overall profile shape, as is clear from the ranking on r_1, putting the profile with smallest value first: ST, BO, PR, TO, HO, LA, FA, PA, TR, TA, AR. This ranking can be compared with the groups distinguished in Ch. 6.1, repeated here for convenience:

7B Selected autocorrelations, angle series, Pearson estimator (1 lag = 1.52 m)

	Lag														Confidence limits	
Profile	1	2	3	4	5	10	15	20	25	30	35	40	45	50	0.95	0.99
BO	286	356	348	292	271	299	386	330	229	210	321	284	254	336	±099	±130
ST	227	327	220	240	197	185	141	$\underline{112}$	$\underline{117}$	140	$\underline{013}$	$\underline{058}$	$\underline{061}$	$-\underline{010}$	±092	±121
PR	437	436	415	385	373	345	356	257	281	280	270	331	294	259	±119	±156
PA	780	751	762	740	731	665	593	502	464	443	475	472	415	366	±096	±126
FA	760	743	731	724	731	697	639	604	559	504	448	417	380	384	±087	±115
LA	679	665	691	642	646	600	598	585	574	615	582	564	526	554	±091	±119
TR	824	817	829	781	789	675	606	550	527	452	481	523	546	555	±098	±128
AR	853	833	828	802	775	726	588	503	357	278	179	217	232	261	±103	±135
TA	846	830	819	790	763	644	519	528	539	539	479	445	430	431	±082	±108
HO	670	622	551	497	533	373	160	$\underline{026}$	$-\underline{031}$	$\underline{034}$	144	267	259	228	±096	±127
TO	639	646	602	595	576	493	368	249	$\underline{072}$	$-\underline{066}$	-222	-404	-416	-490	±112	±147

Note: Zero and decimal point elided. Values underlined fall inside 0.99 confidence interval for $\rho_k = 0$.

7C <u>Selected autocorrelations, angle series, abbreviated estimator</u>

	Lag														Confidence limits	
Profile	1	2	3	4	5	10	15	20	25	30	35	40	45	50	0.95	0.99
BO	286	356	344	290	247	254	319	268	181	159	247	218	188	253	±099	±130
ST	227	326	220	240	194	180	133	<u>097</u>	<u>102</u>	122	<u>001</u>	<u>043</u>	<u>043</u>	-<u>028</u>	±092	±121
PR	436	431	409	379	369	305	310	215	231	232	218	278	235	192	±119	±156
PA	780	750	732	696	666	481	374	294	262	236	233	229	193	167	±096	±126
FA	760	743	728	708	712	668	552	461	397	328	268	237	199	172	±087	±115
LA	677	659	684	632	634	595	580	569	551	586	552	530	495	523	±091	±119
TR	821	814	816	768	775	634	538	478	457	386	406	439	451	441	±098	±128
AR	854	835	813	776	734	613	449	347	233	170	<u>098</u>	<u>123</u>	<u>133</u>	154	±103	±135
TA	847	831	819	786	756	629	501	508	521	507	439	400	384	384	±082	±108
HO	670	622	552	497	535	377	164	<u>027</u>	-<u>033</u>	<u>035</u>	154	291	285	254	±096	±127
TO	638	645	602	599	583	514	398	279	<u>085</u>	-<u>067</u>	-236	-444	-446	-513	±112	±147

Note: Zero and decimal point elided. Values underlined fall <u>inside</u> 0.99 confidence interval for $\rho_k = 0$.

7D Differences between estimates, Pearson - abbreviated, for angle autocorrelations

Profile	Lag 1	2	3	4	5	10	15	20	25	30	35	40	45	50	med (absolute value)
BO	000	000	004	002	024	045	067	062	048	051	074	066	066	083	050
ST	000	001	000	000	003	005	008	015	015	018	012	015	018	018	010
PR	001	005	006	006	004	040	046	042	050	048	052	053	059	067	044
PA	000	001	030	044	065	164	219	208	202	207	242	243	222	199	201
FA	000	000	003	016	019	031	087	163	162	176	180	180	181	212	125
LA	002	006	007	010	012	005	018	016	023	029	030	034	031	031	017
TR	003	003	013	013	014	041	068	072	070	066	075	084	095	114	067
AR	-001	-002	015	026	041	113	139	156	124	108	081	094	099	107	097
TA	-001	-001	000	004	007	015	018	020	018	032	040	045	046	047	018
HO	000	000	-001	000	-002	-004	-004	-001	-002	001	-010	-024	-026	-026	002
TO	001	001	000	-004	-007	-021	-030	-030	-013	-001	-014	-040	-030	-023	014
med (absolute value)	001	001	004	006	012	031	046	042	048	048	052	053	059	067	

Note: Zero and decimal point elided.

7E Selected autocorrelations, curvature series, Pearson estimator

Profile	Lag 1	2	3	4	5	10	15	20	25	30	35	40	45	50	Confidence limits 0.95	0.99
BO	-528	026	063	-042	-020	-143	119	-029	-156	-070	126	004	-087	111	±099	±130
ST	-584	162	-080	033	-062	-073	050	-119	-039	108	-079	045	108	-060	±092	±121
PR	-494	006	021	-028	-024	-151	027	-064	-022	-033	036	074	002	055	±119	±156
PA	-349	-121	123	-068	011	041	089	-048	007	023	070	045	-004	-020	±096	±126
FA	-527	076	040	-082	018	018	-115	-067	028	-023	003	074	-066	-005	±087	±115
LA	-424	-142	183	-135	016	-056	-057	-044	-044	088	009	-021	-080	014	±091	±119
TR	-471	-064	168	-147	099	099	018	-018	028	-182	-002	-138	-194	-022	±098	±128
AR	-432	048	026	030	-012	040	-070	-076	-160	057	-013	055	-076	038	±103	±135
TA	-456	007	072	-016	-002	133	-020	055	-017	-015	-011	002	-018	009	±082	±108
HO	-429	036	-025	-135	097	058	038	-054	-057	-092	011	067	-021	063	±096	±127
TO	-470	056	-071	039	-071	023	086	000	004	002	086	-158	061	-141	±112	±147

Note: Zero and decimal point elided. Values underlined fall outside 0.99 confidence interval for $\rho_k = 0$.

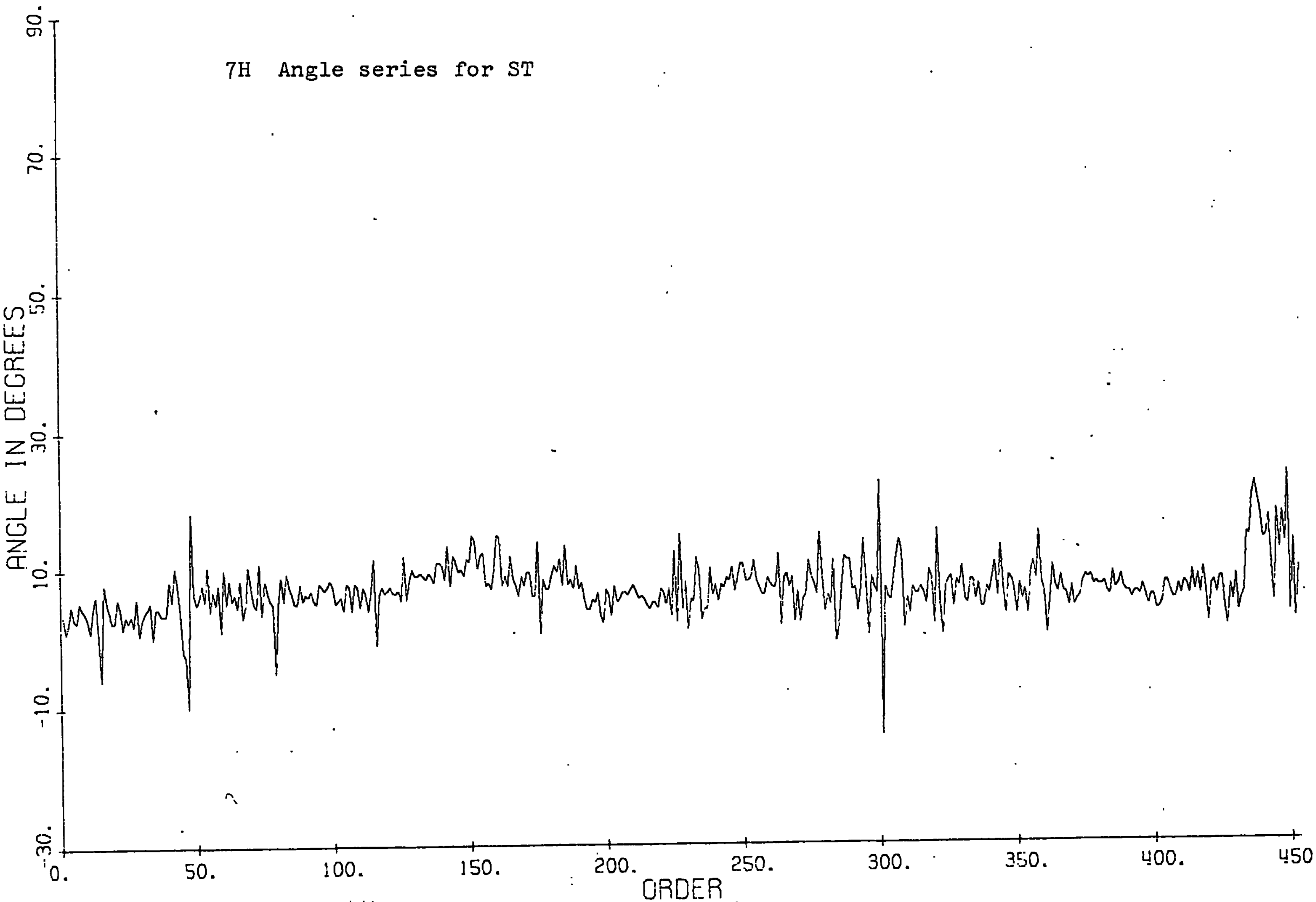
7F Selected autocorrelations, curvature series, abbreviated estimator

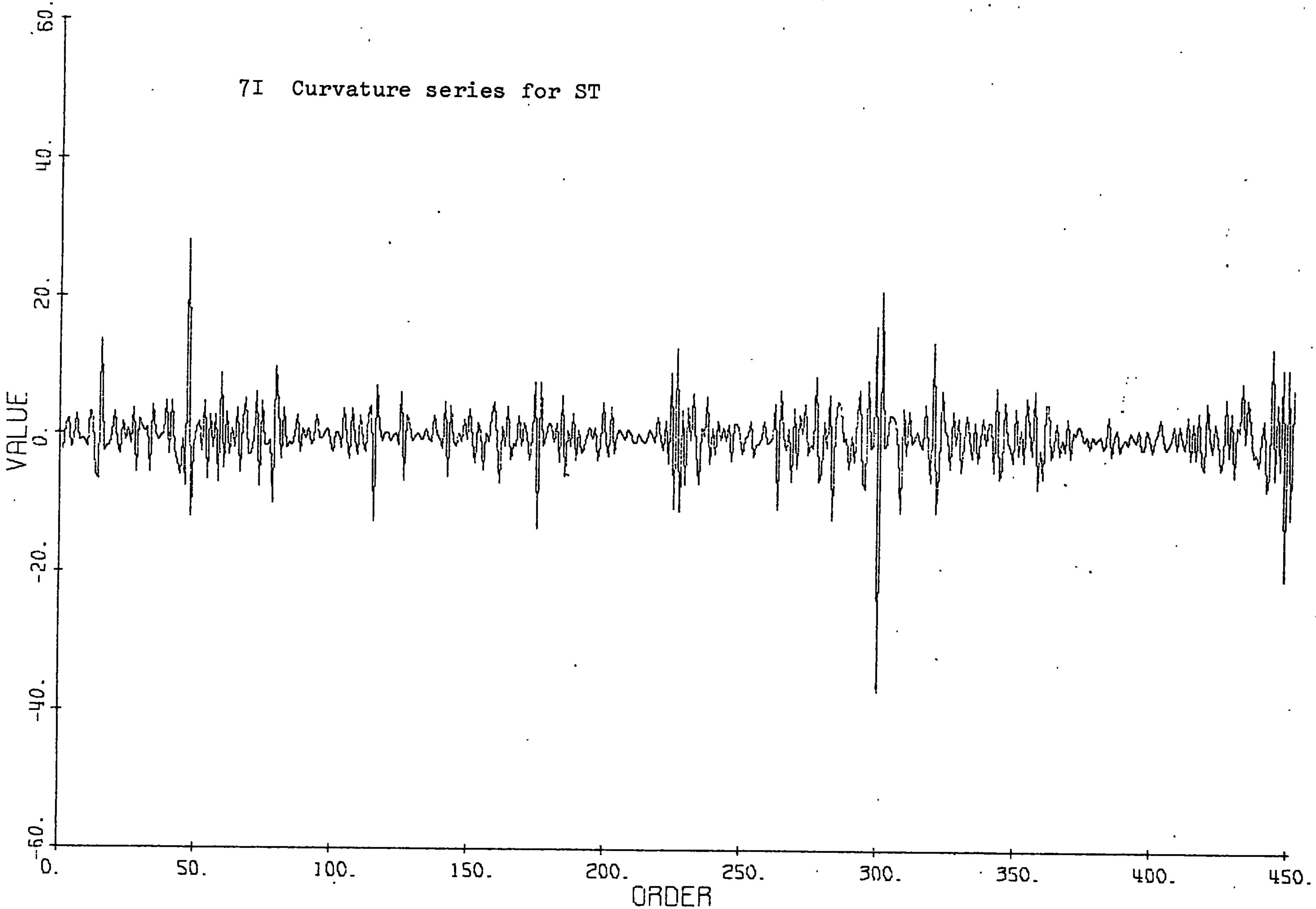
Profile	Lag 1	2	3	4	5	10	15	20	25	30	35	40	45	50
BO	-$\underline{529}$	025	062	-038	-018	-124	104	-026	-$\underline{139}$	-062	112	004	-078	100
ST	-$\underline{583}$	$\underline{161}$	-079	032	-060	-069	048	-115	-038	106	-078	045	109	-061
PR	-$\underline{488}$	006	020	-027	-021	-122	020	-045	-016	-022	024	050	001	038
PA	-$\underline{345}$	-112	113	-063	010	036	079	-043	006	020	063	040	-003	-018
FA	-$\underline{528}$	076	039	-080	017	017	-111	-064	026	-021	003	070	-063	-005
LA	-$\underline{425}$	-$\underline{143}$	$\underline{184}$	-$\underline{136}$	016	-057	-059	-046	-046	092	010	-023	-087	015
TR	-$\underline{470}$	-063	$\underline{163}$	-$\underline{142}$	095	094	017	-017	027	-$\underline{174}$	-002	-$\underline{134}$	-$\underline{185}$	-020
AR	-$\underline{433}$	044	024	028	-011	037	-065	-070	-$\underline{149}$	051	-012	050	-070	035
TA	-$\underline{456}$	007	072	-016	-002	$\underline{133}$	-020	056	-017	-015	-011	002	-017	009
HO	-$\underline{430}$	036	-025	-$\underline{136}$	098	059	039	-056	-059	-097	012	073	-023	070
TO	-$\underline{470}$	056	-071	040	-071	023	085	000	004	002	087	-$\underline{159}$	062	-140

Note: Zero and decimal point elided. Values underlined fall outside 0.99 confidence interval for $\rho_k = 0$.

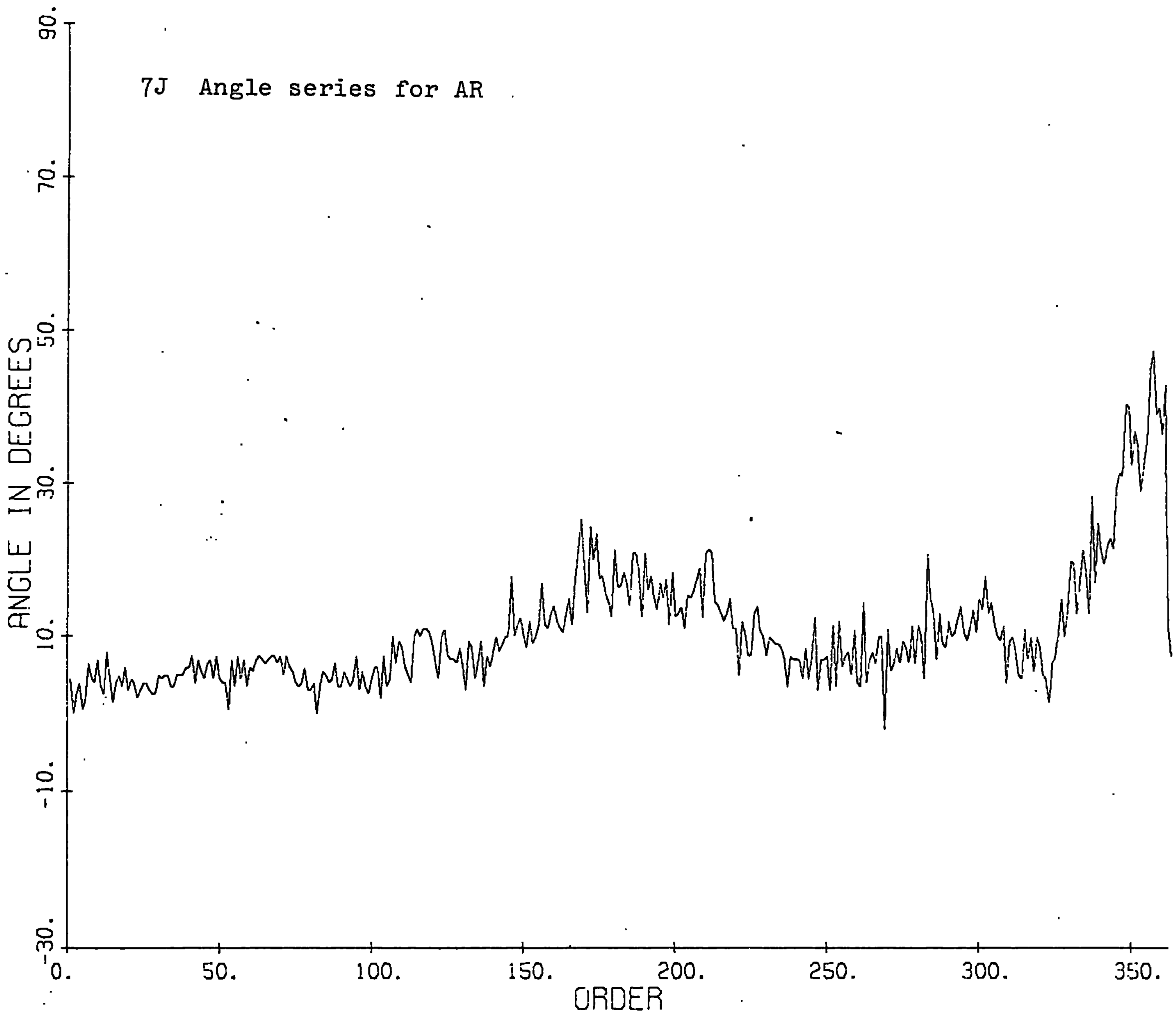
7G <u>Differences between estimates, Pearson - abbreviated, for curvature autocorrelations</u>

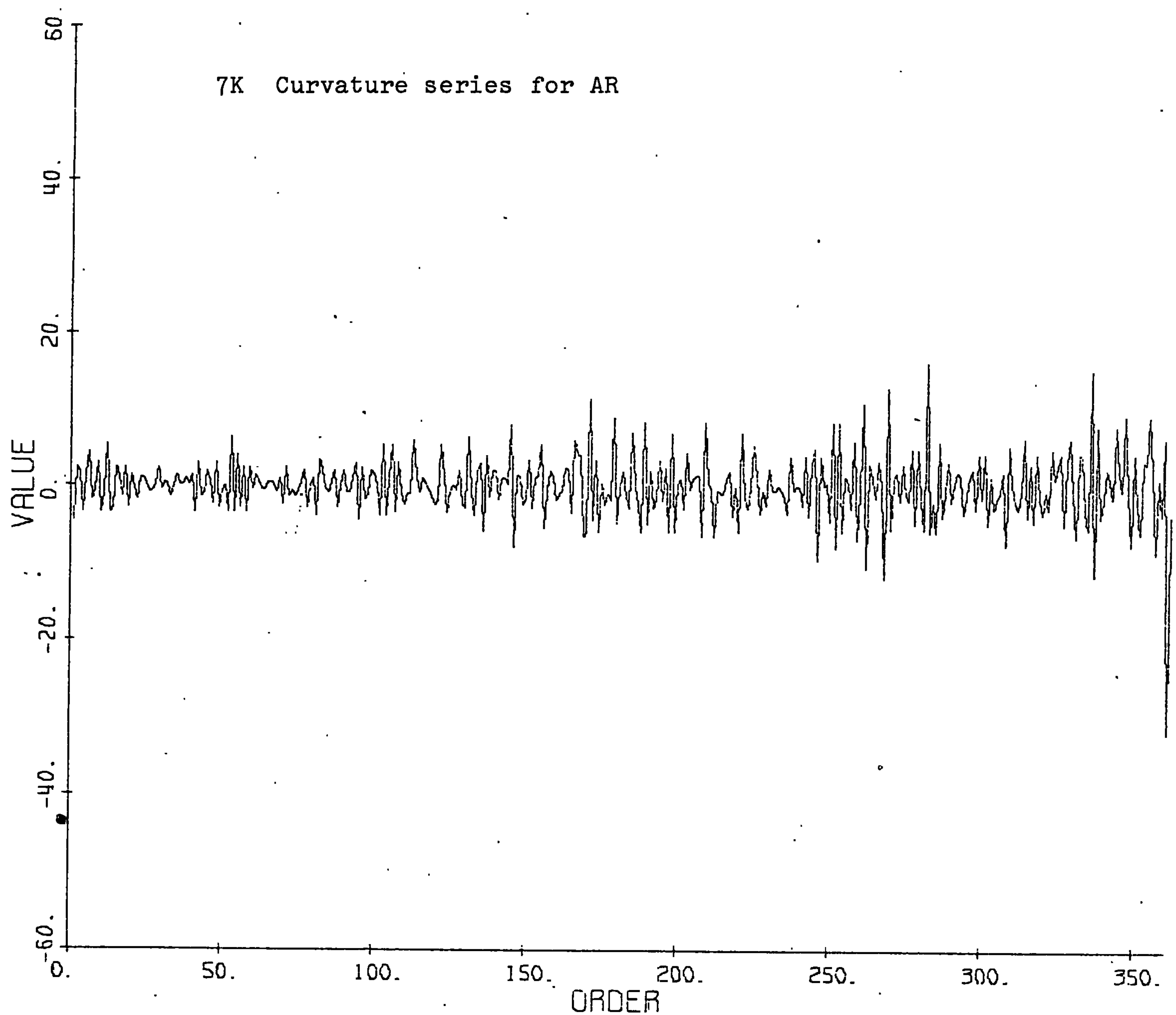
	Lag														
Profile	1	2	3	4	5	10	15	20	25	30	35	40	45	50	med (absolute value)
BO	001	001	001	-004	-002	-019	015	-003	-017	-008	014	000	-009	011	006
ST	-001	001	-001	001	-002	-004	002	-004	-001	002	-001	000	-001	-001	001
PR	-006	000	001	-001	-003	-029	007	-019	006	-011	012	024	001	017	007
PA	-004	-009	010	-005	001	005	010	-005	001	003	007	005	-001	-002	005
FA	001	000	001	-002	001	001	-004	-003	002	-002	000	004	-003	000	002
LA	001	001	-001	001	000	001	002	002	002	-004	-001	002	007	-001	001
TR	-001	-001	005	-005	004	005	001	-001	001	-008	000	-004	-009	-002	003
AR	001	004	002	002	-001	003	-005	-006	-011	006	-001	005	-006	003	004
TA	000	000	000	-001	000	000	001	000	000	000	-001	001	-001	-001	000
HO	001	000	000	001	-001	-001	-001	002	002	005	-001	-006	002	-007	001
TO	000	000	000	-001	000	000	001	000	000	000	-001	001	-001	-001	000
med (absolute value)	001	001	001	001	001	003	002	003	002	004	001	004	002	002	

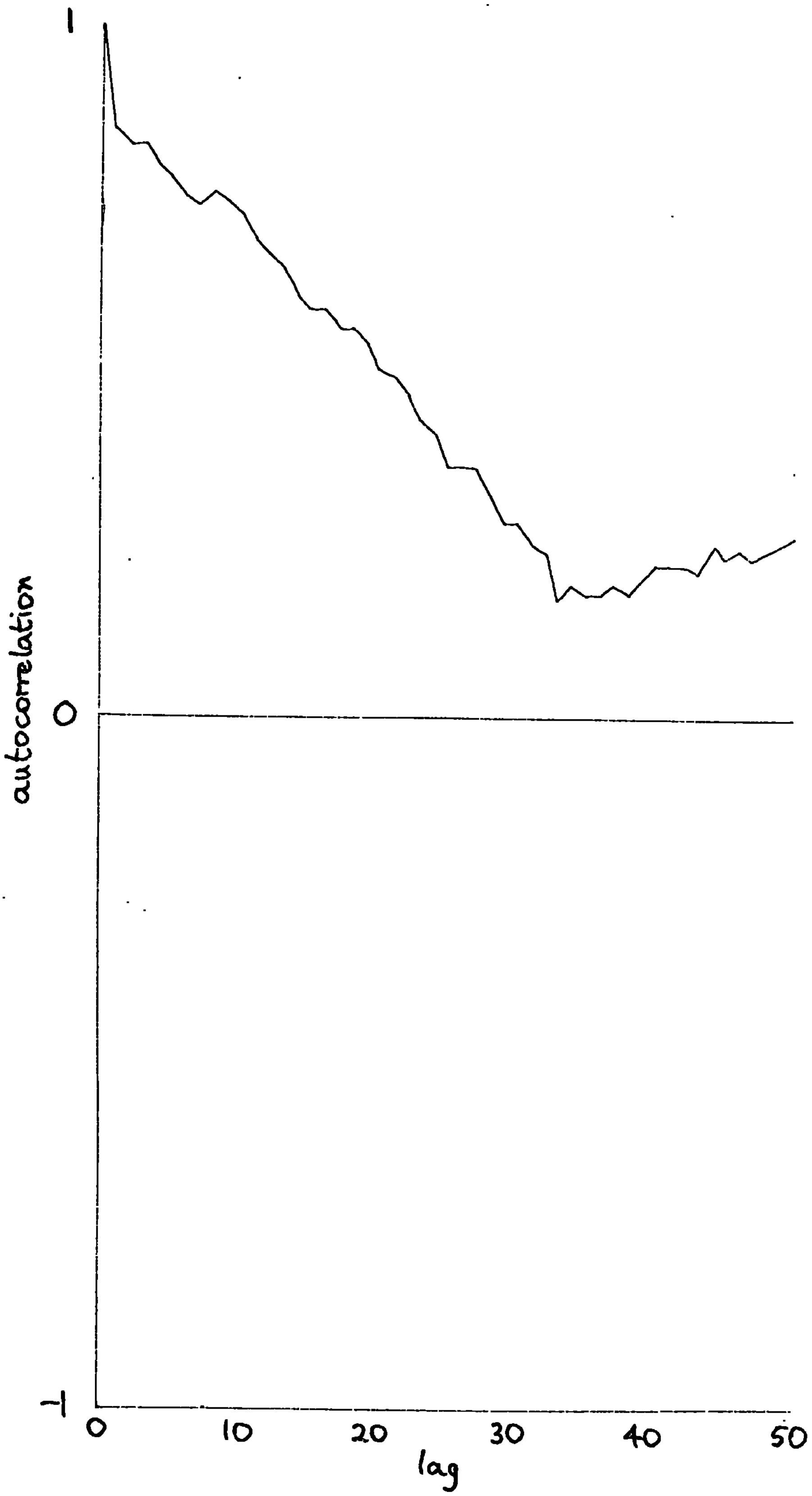

7H Angle series for ST
ANGLE IN DEGREES
90.
70.
50.
30.
10.
-10.
-30.
0.
50.
100.
150.
200.
250.
300.
350.
400.
450.
ORDER



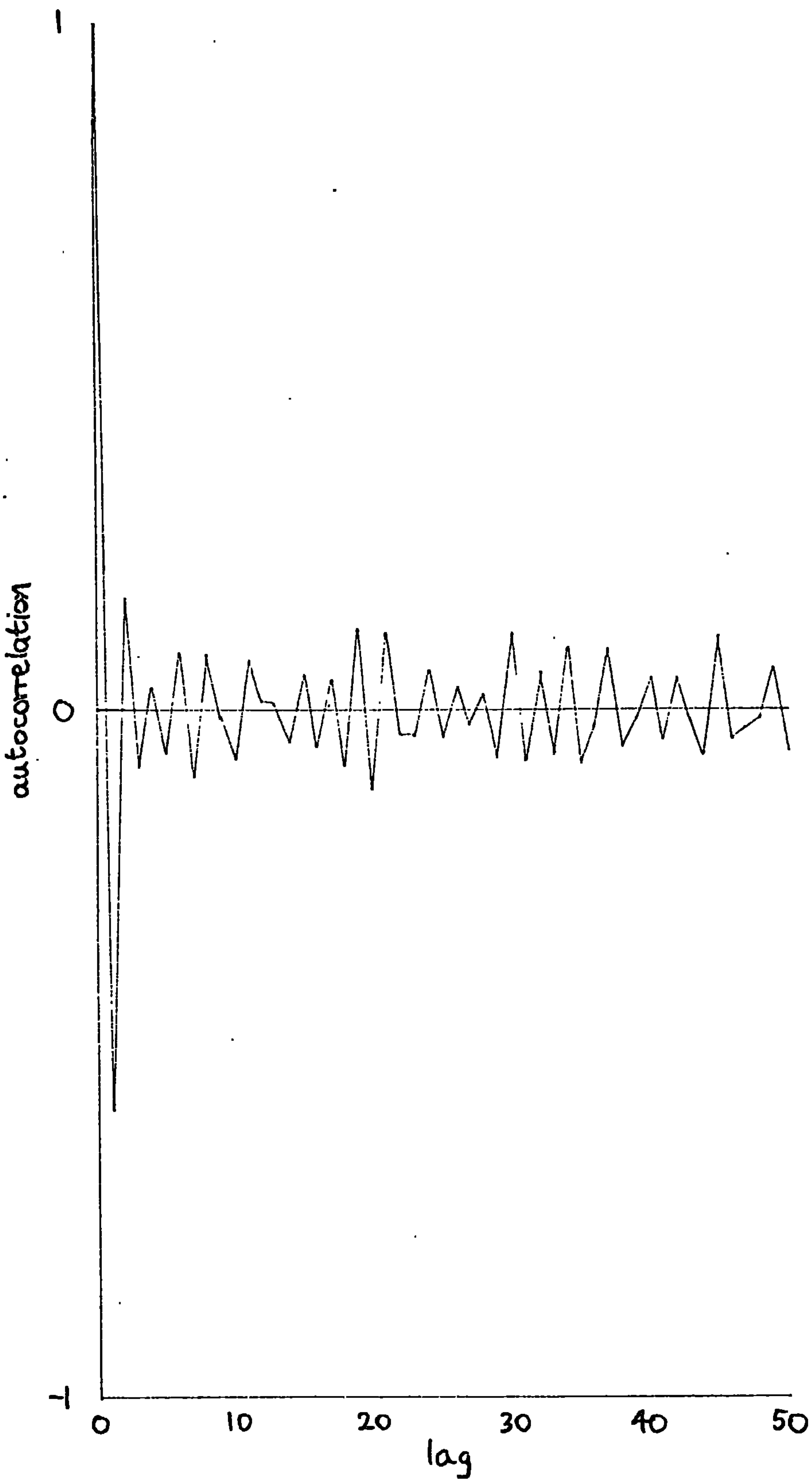
7I Curvature series for ST


7J Angle series for AR
ANGLE IN DEGREES
90.
70.
50.
30.
10.
-10.
-30.
0.
50.
100.
150.
200.
250.
300.
350.
ORDER

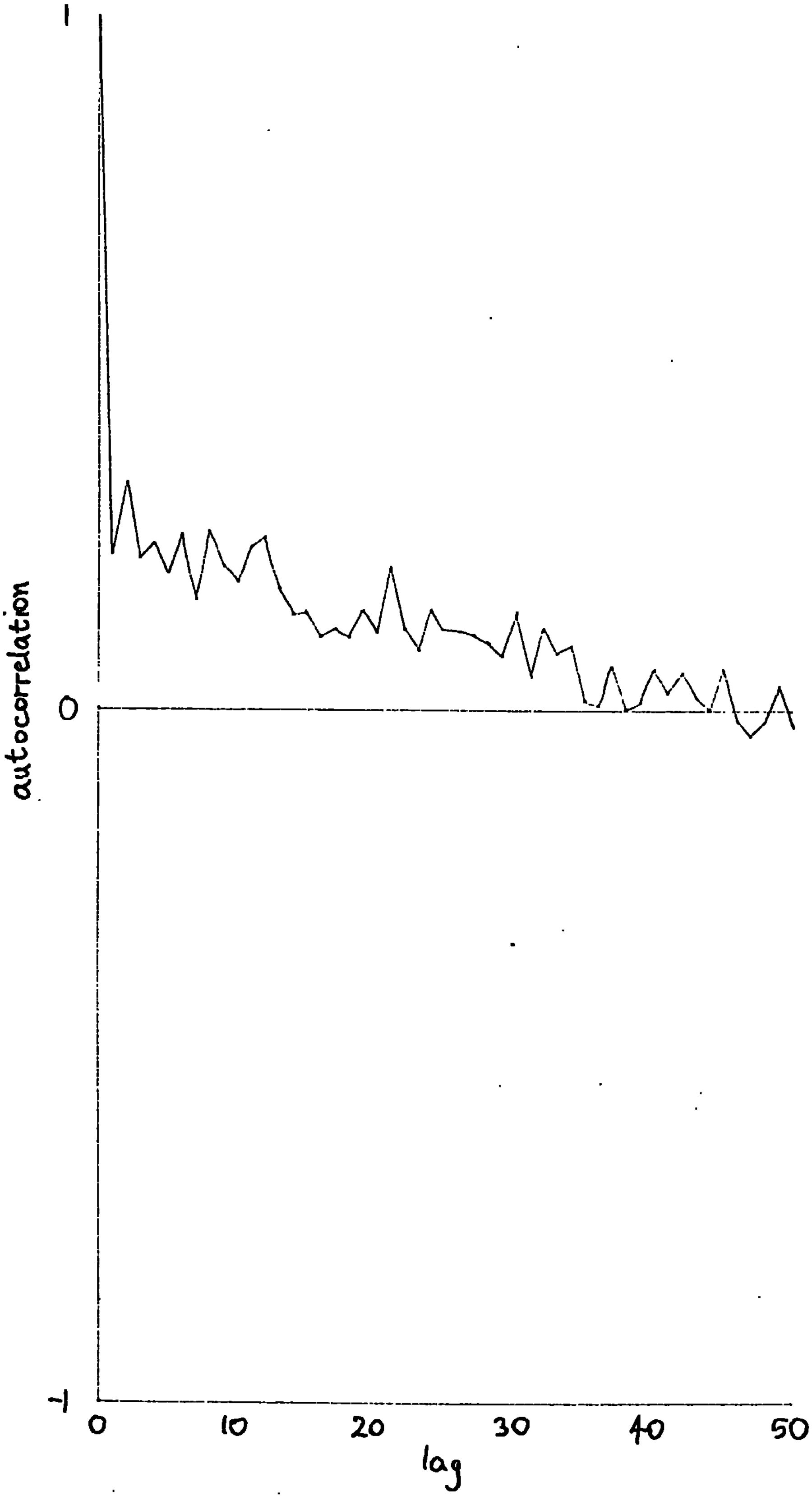

7K Curvature series for AR
VALUE
60.
40.
20.
0.
-20.
-40.
-60.
0.
50.
100.
150.
200.
250.
300.
350.
ORDER



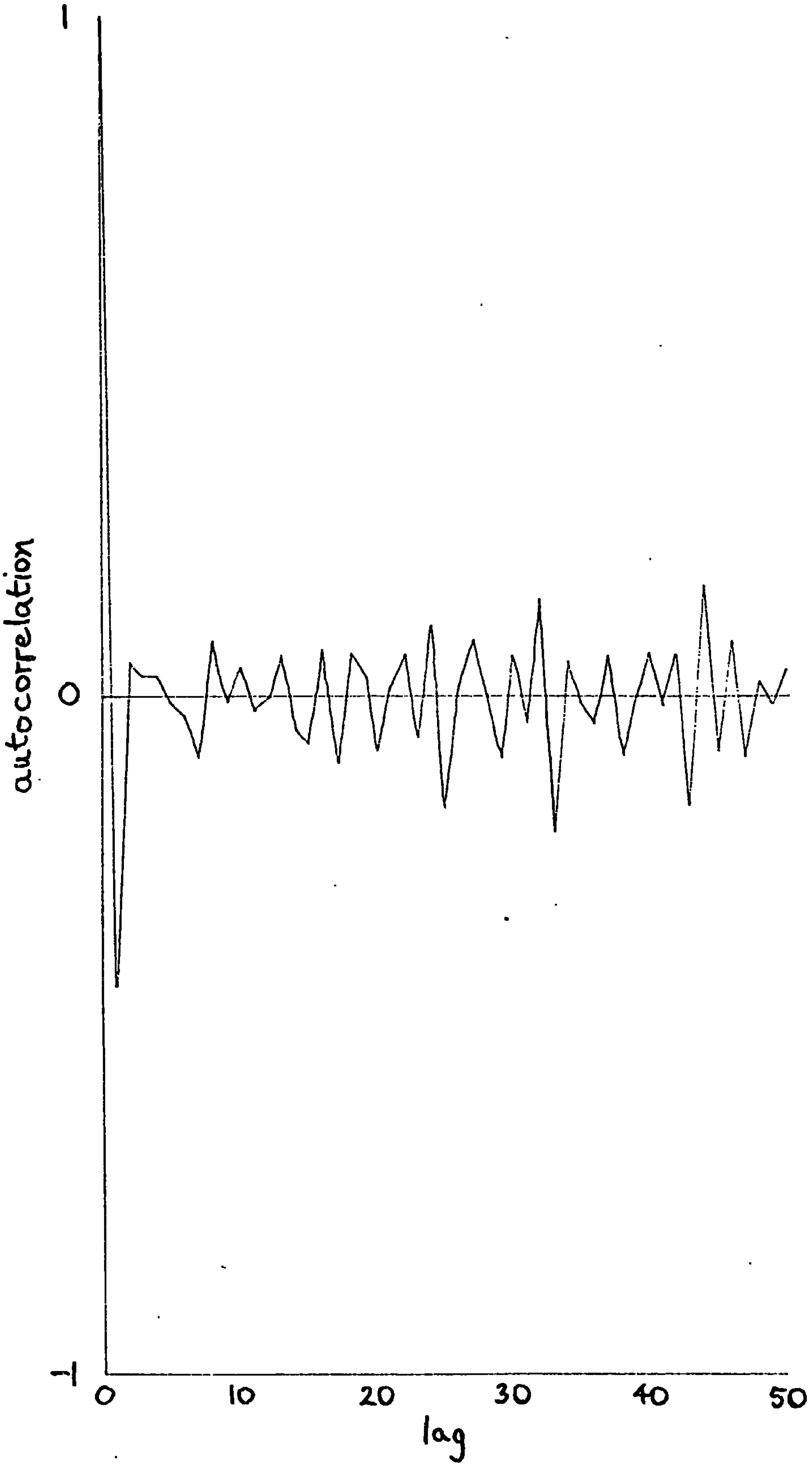
7L Correlogram for ST angles



7M Correlogram for ST curvatures



7N Correlogram for AR angles



70 Correlogram for AR curvatures

(i) BO, ST, PR, PA: the four gentlest, slightly convex

(ii) FA, LA: the next gentlest, more convex

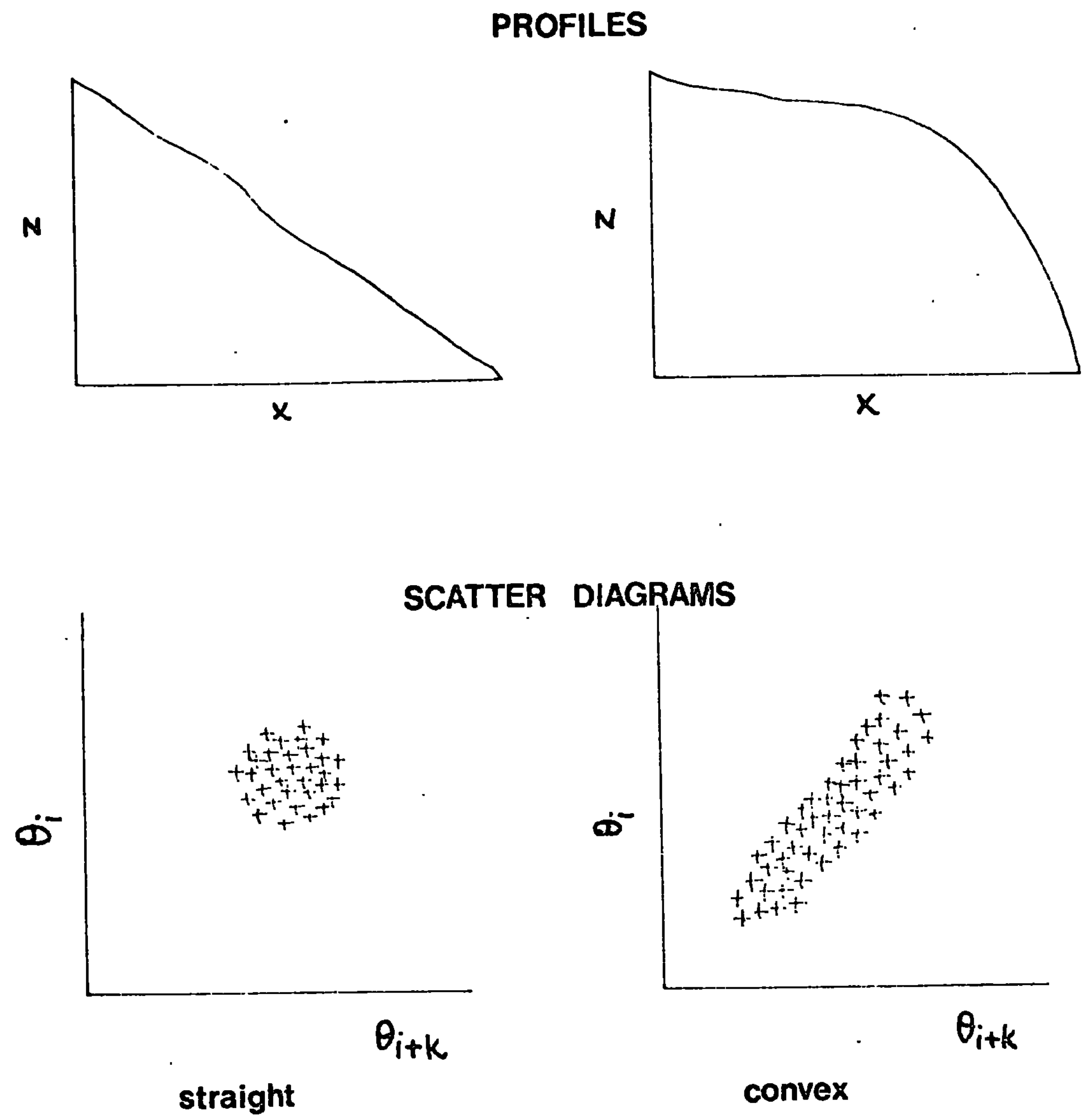
(iii) TR, AR, TA: the next gentlest, yet more convex

(iv) HO, TO: the steepest, more complex in form

Group (iii) with the strongest convexities (the strongest trends in angle series) produces the three highest values of r_1. Group (ii) produces rather lower values, while the multicomponent form of group (iv) results in still lower values. With one puzzling exception (PA), the nearly straight slopes of group (i) have the lowest values: they almost lack trends in angle series.

The reason for this result, that positive autocorrelations at short lags reflect profile convexity (or concavity), can be seen by considering 7P. A straight slope would correspond to a small region of the scatter diagram of leading against lagging values, within which values would be fairly evenly spread. By contrast a convex slope would correspond to a larger region of such a scatter diagram, oriented at an angle approaching 45^{o}. Hence the autocorrelation (correlation between leading and lagging values) would tend to be higher for the convex slope than for the straight slope. The global property of convexity has an influence on the local property of autocorrelation, because autocorrelation is based on a combination of paired values from all parts of the slope.

Marked convexity (or indeed concavity) of slope is, in statistical terms, marked nonstationarity of angle series.



7P Schematic relationships between leading and lagging values

It has been considered whether autocorrelation functions calculated from nonstationary series might be of use as sets of averaged descriptors, contrary to the orthodox view (Ch. 7.3 above). Moreover, the Pearson estimator of autocorrelation does not depend on the stationarity of the series to satisfy $-1 \leqslant r_k \leqslant 1$, and it allows leading and lagging values to have different means and variances. In these senses the Pearson estimator adjusts for nonstationary behaviour. Nevertheless, the results in 7A show that low-lag autocorrelations tell a story of profile shape, or of nonstationarity. The naive idea that low-lag autocorrelations would reflect local roughness collapses and it can be seen that combination of results from different parts of the slope allows autocorrelations to reflect overall profile shape.

Hence the interest of 7B appears to be statistical rather than geomorphological, because profile shape can be studied more directly and more efficiently by other means.

7C (abbreviated for angle) is best viewed through 7D (Pearson - abbreviated for angle). Differences tend to increase with lag. Apart from that, the differences serve as measures of nonstationarity. The results presented for each profile are summarised by the median absolute value; the median being taken over the lags listed, k = 1(1)5(5)50. The ranking (smallest first) is HO, ST, TO, LA, TA, PR, BO, TR, AR, FA and PA. There is no simple story here, but it is clear that differences between estimators can be appreciable even for low lags and series which visually appear to be almost trend-free.

Pearson estimates tend to exceed abbreviated estimates. The 154 values in 7D range from 0.243 to -0.040. 129 are positive and 25 negative. This implies that the abbreviated estimator imparts a downward bias, and reduces the number of autocorrelations which would be correctly recognised as significantly different from the null case.

Results for curvature (7E to 7G) differ strikingly from those for angle. All r_1 values in 7E are strongly negative and fall outside the 0.99 confidence interval for $\rho_k = 0$. However, curvature autocorrelation functions dampen much more readily than angle autocorrelation functions. Only 27 out of 154 values in 7E fall outside the 0.99 confidence interval for $\rho_k = 0$, and only 16 out of 143 at lags of 2 or more.

7F (abbreviated for curvature) is best viewed through 7G (Pearson - abbreviated for curvature) which shows a straightforward picture. All tabulated differences are very small, showing that the first differencing process which produces curvature is sufficient to yield second-order stationary series. It may be that r_1 of curvature is a fair measure of roughness. Whether this is the case or not, it is fairly conservative, ranging from -0.349 (PA) to -0.584 (ST).

7.6 Summary

(i) Autocorrelation properties of hillslope series are of interest if only because it is important to know whether values are statistically dependent. Angle series are strongly autocorrelated, whereas curvature series dampen much more readily after strong negative values at lag one. Otherwise, autocorrelation of angle is of fairly limited geomorphological interest. It does not measure surface roughness, but rather tends to reflect overall profile shape, which can be defined and measured more directly in other ways.(7.2 and 7.5)

(ii) The related problems of nonstationarity and estimator choice deserve more attention than is customary in geomorphology. It is important to state and justify the estimator used. The Pearson analogue estimator is here recommended strongly. (7.3 and 7.4)

7.7 Notation

a,b	real numbers
E	expectation operator
F	Fisher's statistic
i	subscript
k	lag
n	number of observations
N(,)	Normal (Gaussian) distribution
O	of the order of
r	autocorrelation, correlation
t	Student's statistic
u	real variable
$\bar{u}$, $\bar{u}'$, $\bar{u}''$	means of whole series, leading values, lagging values
U	generating process
v	real variable
$\bar{v}$	mean of v
γ	autocovariance of process
μ	mean of process
ρ	autocorrelation of process
σ	standard deviation of process
Σ	summation operator
$\sim$	is drawn from
$\mid$	given
$\lvert\ \rvert$	modulus

Mnemonics

cov	covariance
med	median
pr	probability
std	standard deviation
var	variance

Chapter 8

PROFILE ANALYSIS

... If a man's wit be wandering, let him study the mathematics; for in demonstrations, if his wit be called away never so little, he must begin again ...

Francis Bacon, Essays L: Of studies

8.1 The problem of profile analysis

Profile analysis may be defined as the partition of a hillslope profile into components which are relatively homogeneous in some explicit sense. In other words, within a component the value of some variable (say u) is approximately constant. (The term 'partition' is used here both as a verb, to denote the process, and as a noun, to denote the result).

If u is angle θ , then the components have approximately constant angle, and may be termed segments. If u is curvature $\frac{d\theta}{ds}$, then the components have approximately constant curvature, and may be termed elements. Since constant angle implies constant (zero) curvature, segments are a subset of elements (but cf. Young, 1971, 1972 and Parsons, 1977, who regarded segments and elements as disjoint sets). This terminology is compared with that of other workers in 8A.

For various reasons no need may be perceived for any special method of profile analysis. Firstly, the existence and bounds of distinct components such as 'free face' or 'debris slope' may be judged entirely obvious. Secondly, the number of observations may not be sufficiently large to require (or to justify) sophisticated analysis. Thirdly, basic features of the data may emerge quite clearly from graphical analysis (e.g. Pitty, 1969, 43-53), or from inspection of averages and differences (cf. Kulinkovich et al,1966; Hawkins and

8A

Profile analysis terminology

Reference(s)	Straight	Curved	Either
Leopold *et al.* 1964	facet	element	segment
Savigear 1965	facet	segment	
Savigear 1967			unit, component
Pitty 1969, 1970			component
Ahnert 1970c			segment
Caine 1971			segment, component
Young 1971, 1972	segment	element	unit
Demirmen 1975			segment
Jahn 1975			sector
Graf 1976b			component
Toy 1977			segment
This thesis	segment	element	component

Note: The term 'component slope' is used by Young (1972,4) for a different purpose.

Merriam, 1975). In contrast, attention is directed here towards situations in which the existence of components is considered problematic, the number of data is reasonably large (number of values >100, say) and the need is felt for a numerical method of profile analysis.

The partition of a series of values u_i; $i=1,\ldots,n$ into k subseries may be regarded as the placing of k-1 markers to indicate the bounds of the chosen subseries. k may take any value between 1 and n.

Profile analysis may be considered as a combinatorial problem (cf. Cliff et al, 1975, Ch. 2; their maximally constrained case is equivalent to profile analysis, but is denoted by their Fig. 2.5b, not by Fig. 2.5a). The series u could be represented by a 'chain' with n-1 'links'. Neighbouring values of u could be placed either in the same component or in neighbouring components: each link could be either 'intact' or 'broken'. There are thus 2^{n-1} possible partitions of a profile of n observations. This number increases explosively with n, with the simple but important implication that as n increases it soon becomes impracticable to inspect all possible partitions, and it is advisable, if at all possible, to use a method other than inspection to find the best partition, given some optimality criterion.

The total number of possible partitions 2^{n-1} is for all the possibilities for $k=1,\ldots,n$; for example, the 1 possibility for $k=1$, the $n-1$ possibilities for $k=2$, and so on. In general

$$2^{n-1} = \sum_{k=1}^{n} \binom{n-1}{k-1} = \sum_{k=1}^{n} \frac{(n-1)!}{(k-1)!(n-k)!}$$

$\binom{n-1}{k-1}$ may be much less than 2^{n-1}, depending on k, but it grows rapidly with n, and with $k < \frac{n}{2}$. For example when n=100, 200 and k=2(1)5

k	$\binom{100-1}{k-1}$	$\binom{200-1}{k-1}$
2	99	199
3	4 851	19 701
4	156 849	1 293 699
5	3 764 376	63 391 251

The problem of profile analysis is formally equivalent to some other problems in data analysis, such as the problem of dividing a time series into epochs or periods (Fisher, 1958; Guthery, 1974; Hartigan, 1975) and the problem of stratigraphical zonation met in geology and palaeoecology (Gordon, 1973; Gordon and Birks, 1972; Hawkins and Merriam, 1973, 1975). In each case, the fundamental issue is the partition of a one-dimensional series into subseries which are homogeneous in some sense: this is a special problem in numerical classification. Such partition problems, which are sometimes described as piecewise approximation or segmentation problems, arise in many disciplines: see, for example, the papers which frequently appear upon the subject in the <u>IEEE Transactions on Computers</u>, such as that by Blumenthal <u>et al</u>. (1977).

The two-dimensional case is also of great interest: consider the role of regionalisation in, for example, geography (Grigg, 1967; Spence and Taylor, 1970; Cliff *et al.*, 1975; Haggett *et al.*, 1977), geomorphology (Gellert, 1972; Mather, 1972), geology (Henley, 1976) and terrain evaluation (Mitchell, 1973; Ollier, 1977).

The basic similarity of partition problems emerges most clearly from a formal statement. This allows ideas and techniques to be borrowed where appropriate, and leads to the embedding of profile analysis within numerical classification. It is both remarkable and unfortunate that profile analysis has not been recognised widely within geomorphology as a numerical classification problem (although note some passing discussion in Parsons, 1973).

The data u could in general be vector-valued, leading to a multivariate classification problem (cf. Webster, 1973; Hawkins and Merriam, 1974; Hawkins, 1976), but the multivariate case of profile analysis will not be further examined here.

8.2 Geomorphological considerations

The primary purpose of profile analysis, as with any classification, must be parsimony in description. If n data values may be summarised efficiently by the attributes of k components, where $k \ll n$, then profile analysis yields a convenient simplification. Profile analysis as discussed here is a morphometric technique, although the method could be applied to any one-dimensional data series. While it may be hoped that a partition of a profile might be of interest to students of process or development, or to practitioners of applied disciplines, such purposes are here regarded as secondary to the aim of morphometric description.

Nevertheless, it is important to discuss the geomorphological grounds for profile analysis. Clearly profile analysis is a valid approach if a hillslope may be legitimately regarded as a combination of discrete components (or, more generally, if a landscape may be considered as a mosaic of discrete units). Two views may be identified on this issue: one emphasising the <u>atomistic</u> character of the landscape, the other emphasising its <u>continuous</u> character. (It is commonplace in the physical sciences to contrast atomistic and continuum views, or particle and field theories: see, for example, the remarks of Holton (1973, 1978) on these views as 'themata' in the history of scientific thought.)

The atomistic view finds its strongest expression in the morphological mapping procedures of the 'Sheffield school', which are centred on 'the concept that there is a small basic indivisible unit of terrain' (Mitchell, 1973, 49). Waters (1958) and Savigear (1965) were the main proponents of morphological mapping, while Gregory and Brown (1966) and Doornkamp and King (1971, Ch. 6) provided further discussion and worked examples. Morphological mapping in particular and geomorphological atomism in general seem to have three theoretical bases, as follows:

(i) The idea that a polycyclic denudation history associated with a fluctuating base level would produce a landsurface which was a mosaic of slopes and flats (Wooldridge, 1932; Linton, 1951).

(ii) The idea that process discontinuities would be associated with morphological discontinuities, found for example in the work of King (1967) and in the so-called 'Nine Unit Landsurface Model' (Dalrymple et al., 1968; Conacher and Dalrymple, 1977).

(iii) The idea that morphological discontinuities may be associated with lithological discontinuities.

While in individual cases these ideas may be very plausible, it still remains necessary to test such a view as part of profile analysis, and thus to consider the alternative possibility that a hillslope profile is essentially a smoothly-changing continuous curve. An emphasis on continuity does not seem to possess any theoretical justification. It is rather that ideas of continuous curves and surfaces are both natural and convenient for any modelling approach centred on differential equations, or for any morphometric approach centred on the statistical analysis of spatial series.

The atomism-continuity issue is perhaps best approached by comparing variability between and within components. One possible pitfall here is that variability measures may be unduly sensitive to the measured length(s) used in profile survey (cf. Gerrard and Robinson, 1971; Gerrard, 1978). This could be investigated to some extent by the aggregation approach used in Ch. 6.3 above for analysing angle and curvature frequency distributions.

8.3 Basic principles

The following principles are suggested to underlie profile analysis.

(i) Objectivity. A call for objectivity implies that methods should be explicit and replicable. It does not amount to a denial of the need or value of individual interpretation or experience. In practice a full specification of the algorithm used is required; for example, in the form of a computer program. Making methods explicit and replicable means that they can be discussed and evaluated, and, if need be, modified or rejected.

(ii) Against adhockery. *Ad hoc* procedures, such as arbitrary cut-off or allocation rules, should be avoided as far as possible. Since profile analysis is a numerical classification problem it seems eminently sensible to embed the problem within the fields of cluster analysis and numerical taxonomy (cf. Jardine and Sibson, 1971; Sneath and Sokal, 1973; Everitt, 1974; Sokal, 1974; Hartigan, 1975). If profile analysis has special features which need to be handled by special procedures, then this case must be argued explicitly. As a distinguished statistician once wrote, 'we make no mockery of honest adhockery' (Good, 1965, 56): but if a systematic procedure is available, adhockery deserves all the mockery it may receive.

(iii) Data analysis. A certain open-mindedness should be entertained about the basic features of the data. In particular

(a) While many geomorphologists use the term 'microrelief' it is by no means clear whether microrelief is really a distinctive source of variation in principle, let alone how it may be distinguished in practice (cf. Ch. 5.3 above).

(b) It will rarely be clear *prima facie* how many components exist, or indeed whether they really exist *qua* components.

(c) It should be easy to vary the number of components k. While for a variety of geomorphological and psychological reasons the value of k chosen will often be between 1 and 9, it is nevertheless vital that it should be straightforward to consider different values of k. A satisfactory method for profile analysis will not prejudge the amount of detail required by the user, while k should be changed if only to determine the sensitivity of the resulting partitions.

(d) It is important that there should be some check of the validity of supposed components in the form of a comparison of variability between and within components.

(iv) Decency assumptions made explicit. If a method of data analysis is regarded as a transformation of one set of numbers into another set, it can often be shown that there are conditions in which the transformations work best according to given optimality criteria. The corresponding 'decency assumptions' about the ideal character of data should be made explicit, and the consequences of suboptimal conditions should be known as far as possible. (The excellent term 'decency assumption' is taken from Levins, 1970, 74).

(v) Direction-invariance. Results from profile analysis should not vary with direction of data processing, base to crest or crest to base (cf. Gerrard, 1974). Formally, results from the series $u_1,\ldots,u_n$ and the reversed series $u_n,\ldots,u_1$ should be equivalent. A component is a component whether one is climbing up or sliding down.

(vi) Principles and practice. A further requirement is that any method of profile analysis must also be useful geomorphologically, which is in large part an issue for the fieldworker. However, it seems vital that this principle should not be allowed to override other principles, so that a statistically dubious method is regarded as acceptable geomorphologically, merely because it produces apparently sensible results.

8.4 Existing methods

8.4.1 Ahnert's method

Ahnert (1970c) proposed that segments (i.e. components in the terminology proposed here) should be regarded as straight (sc. segments) if angle does not change twice in the same direction in two successive measured lengths and total variation in angle does not exceed 3°. Concave and convex segments (sc. components) are then distinguished by the direction of angle increase.

The choice of 3° for angle range is clearly arbitrary and implies that the number of segments (sc. components) is strongly influenced by measured length.

Juvigné (1973) independently proposed a broadly similar method, which is open to similar objections.

8.4.2 Ongley's method

The method devised by Ongley (1970) tackles the problem of identifying (rectilinear) segments rather than that of profile analysis sensu stricto. Subseries of profile coordinates x and z are entered in local regressions with the implicit model

$$z = \alpha + \beta x + \epsilon$$

where α, β are parameters and ϵ stochastic error. Choice of subseries for regression is determined by a complicated algorithm: the program works its way up a profile. A subseries is accepted as a segment if no residual exceeds a prespecified tolerance in absolute size. Note that in general segments may overlap.

Apart from the fact that Ongley's method is of no use for identifying elements, it is unsatisfactory for several reasons (cf. also Gerrard, 1974, 1978). In practice it is difficult to choose an appropriate tolerance value. At high levels considerable overlaps occur; at low levels there may be a multitude of very short segments. In either case, it is not easy to decide on a value for tolerance which would lead to much better results. Consequently it may be necessary to rerun the program several times before obtaining output satisfactory for the purpose in mind.

The algorithm for 'walking' the program upslope is extremely clumsy. It is not surprising that it may lead to direction-variant results: if data are reversed, and the program instructed to walk downslope, completely different results have been found to occur (contrary to the suggestion by Gerrard, 1974). This seems to be a fatal defect.

Ongley's method was applied to survey data for profiles TO and TR to illustrate these remarks. Tolerance was set at 5m somewhat arbitrarily. (For his slopes on the Cobar pediplain, New South Wales, Ongley used 0.3 ft. (90mm), but these slopes were evidently both gentle and smooth.) The data for TO and TR were read in both base-crest and crest-base directions. Results for TO are given in 8B, which lists terminal index numbers for each segment identified. The indexes run in crest-base sequence in both cases for comparability.

The most striking feature of the TO results is that 54 segments were identified in one case, and 107 in the other. More detailed inspection shows the lack of correspondence between the two sets of results: in fact, no segment occurs in both lists. An alternative method of comparing results is through frequency distributions of segment lengths (8C): the differences are quite clear.

8B Results for Ongley's method on TO

Tolerance = 5m.

Base-crest

1,44	6,45	9,46	11,47	13,48	14,49	16,50	18,51
20,52	21,54	22,55	25,56	26,57	28,58	30,59	32,60
33,61	34,62	36,63	39,64	40,65	42,66	41,67	43,68
43,69	45,71	46,72	48,73	49,74	50,77	52,78	54,79
55,80	56,82	57,83	58,85	59,87	60,89	61,90	62,91
63,94	64,97	65,100	66,102	67,103	68,106	69,107	70,112
71,113	72,116	73,177	74,204	75,287	76,309		

segments = 54

Crest-base

1,118	2,119	3,120	11,121	79,122	86,123	90,124	95,126
96,127	97,128	100,129	102,130	104,131	105,132	106,133	107,134
108,135	110,137	111,138	112,139	113,140	114,140	115,141	116,142
117,142	118,142	119,143	120,142	121,147	122,150	123,153	124,155
125,158	126,157	127,158	128,162	129,168	130,174	131,180	132,189
133,195	134,200	135,206	136,220	137,219	138,224	139,225	142,226
152,227	154,228	155,231	156,232	157,233	161,234	164,235	170,236
171,237	172,238	173,239	174,240	176,241	182,242	183,243	184,244
186,245	188,246	190,253	192,254	196,255	199,256	201,257	203,258
206,259	207,260	209,261	211,262	214,265	216,266	217,267	219,268
222,269	224,270	225,271	226,274	233,275	235,276	236,277	237,278
238,279	239,280	240,284	241,285	242,289	243,291	244,293	245,294
246,294	248,292	249,291	250,290	251,292	252,293	253,293	254,296
255,300	256,308	257,309					

segments = 107

(# measured angles = 309)

8C <u>Segment length distributions for Ongley on TO at 5m</u>

segment length (#measured angles)	#segments Base-crest	Crest-base
0-9	0	0
10-19	0	0
20-29	22	19
30-39	23	13
40-49	5	25
50-59	0	18
60-69	0	15
70-79	0	8
80-89	0	5
90-99	0	0
100-199	2 (105,131)	4 (111,118,118,118)
200-299	2 (213,234)	0

	Base-crest	Crest-base
loq	27	32
med	31.5	48
upq	37	63

A user faced with the results in 8B and 8C and wishing to continue would presumably rerun with lower tolerance, but what value would he choose? Intelligent guesswork would be the only guide.

The results for TR (8D and 8E) tell a similar story. 237 segments are identified by processing in one direction, while only 93 are identified by processing in the other direction. Once again no segment occurs in both lists. There is a strong tendency to identify a multiplicity of segments of approximately similar length as the program works its way round a concavity or convexity. This is hardly surprising since Ongley's method is, after all, attempting to identify segments.

Parsons (1973, 1976b) independently produced a superficially similar method also based on local linear regressions. Various ad hoc devices reduce the influence of outliers and lead to an analysis of the profile into disjoint segments. This method is, however, also direction-variant (Parsons, personal communication, 1977).

8.4.3 Pitty's method

Pitty (1970, 30-44) proposed regressions of angle against index number with the implicit model

$$\theta = \alpha + \beta \iota + \epsilon$$

where α, β are parameters and ϵ stochastic error. A cusum test was suggested for changes in slope and a test using

8D Results for Ongley's method on TR

Tolerance = 5m

Base-crest

1,42	2,43	3,44	4,45	5,46	6,47	7,48	8,49
9,50	10,51	11,52	12,53	13,54	15,55	16,56	17,57
19,58	20,59	22,60	23,61	24,62	25,63	27,64	28,65
29,66	31,67	33,68	34,69	35,70	37,71	37,72	39,73
40,74	41,75	42,76	43,77	44,78	45,79	46,80	46,81
47,82	49,83	50,84	51,85	52,86	53,87	54,88	55,89
56,90	57,91	58,93	59,94	60,95	61,96	62,97	63,98
64,100	65,101	66,102	67,103	68,104	69,105	70,106	71,107
72,108	73,109	74,110	75,111	76,112	77,113	78,114	79,115
80,116	81,118	82,119	83,120	84,121	85,122	86,123	87,124
88,125	89,126	90,127	91,129	92,130	93,131	94,132	95,133
96,135	97,137	98,138	99,139	100,140	101,141	102,142	104,143
105,144	106,145	107,146	108,147	110,148	111,149	112,149	113,151
115,152	117,153	118,154	119,155	120,156	121,157	122,158	123,159
124,160	126,161	127,162	129,163	130,164	131,165	132,166	134,167
135,168	136,170	138,171	139,172	140,173	141,174	142,176	144,177
145,178	146,179	147,180	148,181	149,182	150,183	151,184	152,185
153,186	154,187	155,188	156,189	157,190	158,191	159,192	160,193
161,195	162,196	163,197	164,198	165,199	166,200	167,201	168,202
169,204	170,205	171,206	172,207	173,208	174,209	175,210	176,211
177,213	178,214	179,215	180,216	181,217	182,218	183,219	185,221
186,222	187,223	188,224	189,225	190,226	191,227	192,228	193,229
195,230	198,231	198,232	200,233	202,234	204,235	205,236	207,237
209,238	210,239	211,240	212,241	214,242	216,243	217,244	218,245
219,246	222,247	223,248	224,249	225,251	226,252	227,253	228,255
229,256	230,258	231,260	232,261	233,268	234,284	235,296	236,302
237,309	238,311	239,312	249,313	267,314	270,315	276,316	281,317
284,318	289,319	288,320	290,321	293,322	294,323	295,324	297,325
300,326	300,327	301,328	303,329	305,330	305,331	306,332	308,333
309,335	310,336	312,337	313,402	314,403			

segments = 237

Crest-base

1,256	170,257	178,258	186,259	198,260	205,261	216,262	231,263
238,264	243,265	245,266	247,267	248,268	249,269	250,270	251,271
252,272	253,273	254,272	255,272	256,274	257,277	258,278	259,279
260,281	261,282	262,285	263,287	264,290	265,293	266,295	267,297
268,298	269,300	270,303	271,306	272,307	273,309	274,309	275,312
276,314	277,316	278,323	279,331	280,333	281,339	283,240	288,341
296,342	301,343	310,344	316,345	318,346	326,347	330,348	331,349
334,350	335,351	337,352	338,353	339,354	340,355	341,356	342,358
343,359	344,360	345,361	346,362	347,365	348,368	349,370	350,374
351,376	352,380	353,381	354,384	355,387	357,388	361,389	365,390
369,391	370,392	374,393	376,394	379,398	381,396	382,397	384,398
385,399	386,400	387,401	388,402	389,403			

segments = 93

(# measured angles = 4Ø3)

8E Segment length distributions for Ongley on TR at 5m

segment length (#measured angles)	# segments Base-crest	Crest-base
0-9	0	0
10-19	0	27
20-29	26	32
30-39	169	18
40-49	33	4
50-59	1	7
60-69	3	1
70-79	3	1
80-89	0	2
90-99	2	0
100-199	0	0
200-299	0	(256) 1

	Base-crest	Crest-base
loq	34	19
med	36	23
upq	38	34

regression output for breaks of slope.

Pitty did not discuss the choice of angle subseries for regression. There is considerable confusion in his account between three logically distinct issues: the validity of the null hypothesis $H_0 : \beta = 0$, the adequacy of the model (assessed perhaps by a global lack-of-fit statistic) and the independence of residuals. It is unfortunate that two tests rather than one are proposed for this problem: there seems to be no justification for this complication. The cusum test has recently been shown to be totally unsuitable for autocorrelated data (Johnson and Bagshaw, 1974; Bagshaw and Johnson, 1975), while the test for breaks of slope implicitly assumes the adequacy of the regression model which is one of the issues at stake. Finally, Pitty's use of algebraic notation is inconsistent, which makes understanding of his proposals difficult.

8.4.4 Young's methods

Young (1971) proposed three techniques of profile analysis: best segments analysis, best elements analysis and best units analysis (hereafter BSA, BEA, BUA). They are a development of previous subjective methods developed by Savigear and Young.

The variability of a subseries in either angle or curvature (either case denoted here by u) is measured by its coefficient of variation i.e. $\frac{\text{standard deviation}}{\text{mean}} \cdot 100$

$$= \frac{\sqrt{\left(\frac{\sum l_i u_i^2}{\sum l_i} - \bar{u}\right)}}{\bar{u}} \cdot 100$$

where $\bar{u} = \frac{\sum l_i u_i}{\sum l_i}$ and l_i denotes measured length.

Curvature is in fact defined rather unsatisfactorily by Young as

$$c_1 = \frac{200\,(\theta_2 - \theta_1)}{l_1 + l_2}$$

$$c_i = \frac{200\,(\theta_{i+1} - \theta_{i-1})}{l_{i-1} + 2l_i + l_{i+1}} \qquad i = 2, \ldots, n-1$$

$$c_n = \frac{200\,(\theta_n - \theta_{n-1})}{l_{n-1} + l_n}$$

In BSA a subseries is accepted as a segment if the coefficient of variation of angle does not exceed a prespecified maximum. In BEA a subseries is accepted as an element if the coefficient of variation of curvature does not exceed a second prespecified maximum. BUA allows for both segments and elements, depending on two prespecified maxima.

There is an immediate difficulty here. Since segments are a subset of elements, BSA and BEA are not independent. Furthermore, the existence of BUA is both unnecessary and confusing.

In these techniques, a measured length may fall into two or more components. This difficulty has to be resolved if components are not to overlap. For example, in BUA, 'a measured length which falls into two or more slope units, each within the specified maximum variability, is allocated to the longest unit; if two units are of equal length, it is allocated to that with the lowest coefficient of variation; if the coefficients are also equal, allocation is to a segment in preference to an element' (Young, 1971, 5). Hence since similarity and contiguity criteria do not always produce a satisfactory partition into components, Young's method assigns measured lengths whose status is in doubt to 'the longest acceptable unit', defined in an ad hoc manner.

Another fundamental difficulty is that like all ratios, the coefficient of variation is not a stable measure (cf. Kendall and Stuart, 1969, 47-8). As denominators become smaller, values of the coefficient tend to become extremely large. This produces a bias towards short components for low mean angles or curvatures ('flats' become short segments, 'segments' become short elements). Young tackled this difficulty by replacing means below 2 by a value of 2, but this is clearly not a very satisfactory solution.

The coefficient of variation has the further disadvantage (Lewontin, 1966; Gilbert, 1973, 54), that it rests on the implicit assumption that standard deviation is proportional to the mean.

Young (1971) recommended coefficients of variation of 10.0 (angle) and 25.0 (curvature), but remarked (personal communication, 1975) that higher values might be necessary. 8F gives the numbers of segments and elements obtained with maxima of 10.0 (10.0) 90.0 for profile Tripsdale (TR). These results show that for this kind of data, obtained with short measured lengths, very high coefficients are needed to produce few-component partitions (but, once again, the user would have to fall back on intelligent guesswork in choosing new values for rerunning the program). It is also clear that results are direction-variant: different numbers of components may be produced by different directions of processing, although the differences in numbers are relatively small.

8G and 8H focus on a specific case: segments for 50.0. The instability of the coefficient of variation leads to many short gentle segments, particularly on the upper part of the convexity. It is also clear how results are basically an artefact of direction of processing: only 56 out of 403 measured lengths are allocated to exactly the same segment in both cases. (The direction-variant character of Young's method was first pointed out by Gerrard, 1974). Results for 90.0 which are given in 8I, are even worse: only 5 out of 403 lengths are allocated to exactly the same segment.

Results for profile TO are given in 8J, which tells a similar story: while the numbers of components are approximately equal for different directions of processing, the actual components may once again differ markedly. The example of segments at 50.0 is particularly striking.

8F <u>Results of best segments analysis and best elements analysis on TR, by direction of processing</u>

	# segments		# elements	
Maximum coefficient of variation	Base-crest	Crest-base	Base-crest	Crest-base
10.0	284	284	379	379
20.0	208	210	357	356
30.0	139	130	339	340
40.0	95	100	313	315
50.0	57	64	296	294
60.0	35	35	274	274
70.0	25	26	262	260
80.0	21	15	236	230
90.0	11	8	224	216

(#measured angles = 403)

8G Results of best segments analysis on TR: maximum coefficient of variation = 50.0

Base-crest

1,4	5	6	7	8,9	10	11,16	17
18	19	20	21	22	23	24,25	26
27	28,29	30	31	32,35	36,40	41	42,45
46,47	48	49,50	51,53	54,65	66	67,68	69,72
73,89	90,91	92	93,98	99	100,102	103,115	116,120
121,161	162	163	164	165,167	168	169,217	218,222
223,224	225	226,228	229	230	231,242	243,311	312
313,403							

segments = 57

Crest-base

1,4	5	6	7	8,9	10	11	12,16
17	18	19	20	21	22	23	24,25
26	27	28,29	30	31	32,35	36,40	41
42,45	46,47	48	49,50	51,53	54	55	56,65
66,67	68	69	70,74	75,90	91	92	93,98
99	100,103	104	105,114	115,129	130,131	132,134	135
136,169	170	171,175	176	177,182	183	184	185,226
227	228,229	230	231	232,255	256,257	258,347	348,403

segments = 64

8H. <u>Segment length distributions for Young on TR at 50.0</u>

segment length (#measured angles)	# segments Base-crest	# segments Crest-base
0-4	44	49
5-9	5	6
10-14	3	2
15-19	1	2
20-24	0	1
25-29	0	0
30-34	0	1
35-39	0	0
40-44	1	1
45-49	1	0
50-59	0	1 (56)
60-69	1 (69)	0
70-79	0	0
80-89	0	0
90-99	1 (91)	1 (90)

	Base-crest	Crest-base
loq	1	1
med	2	1
upq	4	4

81

Results of best segments analysis on TR:
maximum coefficient of variation = 90.0

Base-crest

1,18 19 20 21,22 23 24,25 26 27
28,30 31,70 71,404

#segments = 11

Crest-base

1,25 26 27 28,30 31 32,116 117,164 165,403

#segments = 8

Ordered segment lengths

Base-crest

1,1,1,1,1,2,2,3,18,40,334

loq = 1 med = 2 upq = 18

Crest-base

1,1,1,3,25,48,85,239

loq = 1 med = 14 upq = 66.5

8J

Results of best segments analysis and best elements analysis on TO

Maximum coefficient of variation	#segments Base-crest	#segments Crest-base	#elements Base-crest	#elements Crest-base
10.0	157	155	291	291
20.0	55	58	267	266
30.0	22	22	253	253
40.0	9	10	238	238
50.0	5	1	223	225
60.0	1	1	200	200
70.0	1	1	182	187
80.0	1	1	169	170

(#measured angles = 309)

Case of coefficient maximum = 50.0

Base-crest Segment	Actual coefficient
1,272	41.41
273	0.0
274	0.0
275	0.0
276,309	41.50

Crest-base Segment	Actual coefficient
1,309	42.79

While the whole profile can be accepted as a single segment at 42.79, this is not revealed by base-crest processing which divides the profile into five segments, and thus fails to find the optimum.

Recently Parsons (1977) discussed applications of Young's methods (and elaborations of them) to a large amount of profile data. For a critique of this paper and a reply, see Cox (1978) and Parsons (1978). A further paper (Abrahams and Parsons, 1977) is open to similar objections.

8.4.5 Assessment of existing methods

This survey has concentrated on Ongley's and Young's methods as the most popular among geomorphologists. These methods may now be considered in the light of the principles proposed in Ch. 8.3 above.

The most striking feature is the ad hoc and arbitrary character of existing methods, which have basically been developed in ignorance of the large body of work on numerical classification. Both Ongley's and Young's methods may be highly direction-variant, which seems impossible to justify.

In neither case is it easy to vary the number of components, since the parameters controlled are related to the number of components in a complex and unknown manner. Nor does either method allow comparison of between-component and within-component variation. The problem of sensitivity to measured length has been largely ignored, although it would be straightforward to apply these methods to aggregated data.

Hence no existing method can be regarded as satisfactory. Attention now turns to a method drawn from numerical classification, which was originated by Fisher (1958), generalised by Hartigan (1975), and which produces 'additive error partitions'. This method is discussed in some detail before being applied to profile data from the field area.

8.5 Additive error partitions in principle

Fisher (1958) proposed a weighted least-squares criterion for partitioning a series u of length n into k contiguous subseries

$$\text{minimise} \quad \sum_{i=1}^{n} w_i \, (u_i - \bar{u}_i)^2$$

Here w_i is a weight, $\bar{u}_i$ is the average of the subseries which includes u_i, and minimisation is over the $\binom{n-1}{k-1}$ possible partitions. This criterion is one of a family of least-squares criteriawhich also includes criteria for vector-valued u and piecewise functional approximation. Various members of this family have been used in geology and palaeoecology, mainly for stratigraphical zonation (Gordon and Birks, 1972; Gordon, 1973; Hawkins and Merriam, 1973, 1975).

Hartigan (1975, Ch.6) considered additive error criteria of the general form

$$\text{minimise} \quad \sum_{j=1}^{k} d_j$$

Here d_j is a diameter for subseries j, and minimisation is over the possible partitions. Hartigan mentioned, as special cases of d_j

$$d_j = \sum |\text{value} - \text{median}|$$
$$d_j = \sum |\text{value} - \text{mean}|^2$$

where summation is over values of u which belong to subseries j. Clearly both these can be seen as members of the family

$$d_j = \sum |\text{value} - \text{typical value}|^p$$

In principle the choice of diameter might be justified by considering conditional probability distributions for each subseries (Hartigan, 1975, 135), each of the form

$$\text{pr (value of u} \mid \text{local parameter)}$$

Classical results carry over so that p = 1 is the maximum likelihood procedure for double exponential (Laplace) distributions, and p = 2 that for normal (Gaussian) distributions, assuming independent observations. However, such conditional probability distributions are in practice unknown (the task of profile analysis being in essence to estimate local parameters); they may not follow any classical distributions (results for marginal frequency distributions discussed in Ch. 6.2 above show a broad general tendency to long-tailedness); the form of distributions may vary from subseries to subseries; and mutual independence of observations from each distribution may well be a very strong assumption (even though weaker than mutual independence of all observations, discussed in Ch. 7 above).

Such important reservations aside, additive error partition methods produce optimal components for k=1,2,3.... The number of components is controlled directly, and both generalised and detailed partitions may be produced in a single program run. Component attributes (typical value, variation, boundaries) are of course of great interest, while partitions as a whole can also be compared for different values of k, and for different profiles, perhaps by considering the measure

$$\frac{1}{n}\left[\text{minimum}\left(\sum_{j=1}^{k} d_j\right)\right] = V \quad \text{, say.}$$

Finding the optimal partition is nontrivial computationally given the combinatorial explosion of $\binom{n-1}{k-1}$. However, it can be tacked by a dynamic programming algorithm (Bellman and Dreyfus, 1962; Hartigan, 1975). Such an algorithm will always find an optimum, although it may not be unique. This raises the question whether different directions of processing would find different optima with equal values of the objective function. It is conjectured here that for real-valued objective functions of the kind considered below, this is in principle an event of probability zero (in the exact sense of the expression) and direction-invariance can thus be assumed in practice. This conjecture is supported by Hartigan (personal communication, 1977).

8.6 Additive error partitions in practice

Additive error partitions were produced for curvature series u = $\Delta\theta$ using the least-squares criterion

$$\text{minimise} \sum_{i=1}^{n} (u_i - \bar{u}_i)^2$$

for all eleven hillslope profiles, for k = 1(1)10.

The results showed a strong tendency for the method to yield a large number of very short components. For each partition of n into k, the k components can be ordered numerically by their lengths (i.e. numbers of observations included). This can be repeated for each profile, and order statistics and functions of them calculated over all profiles. Medians and midspreads over profiles of ordered component lengths are given in 8K.

There is a clear pattern in these results. Least-squares partitions pick up a large amount of local roughness, 'bumps' and 'ruts', and declare many such features to be components. This seems undesirable. Accordingly it was decided to smooth the data before partition. The original data are angles θ which are differenced to produce curvatures $\Delta\theta$. If smoother series are desired, then there are various possibilities:

(i) smooth angles, then compute curvatures

(ii) do not change angles, but smooth curvatures

(iii) smooth angles, then compute and smooth curvatures.

The first possibility seems the most straightforward and was therefore adopted.

8K <u>Additive error partitions of curvature series by least squares criterion:</u>

<u>medians and midspreads over profiles of ordered component lengths</u>

Rank of length (shortest = 1, longest = k)

median (midspread)

# components = k	1	2	3	4	5	6	7	8	9	10
1	413(106)									
2	2(3)	389(150)								
3	1(0)	44(127)	360(134)							
4	1(0)	1(1)	44(118)	299(125)						
5	1(0)	1(0)	3(11)	88(98)	282(145)					
6	1(0)	1(0)	1(1)	3(23)	92(117)	282(137)				
7	1(0)	1(0)	1(0)	1(2)	6(22)	92(101)	281(131)			
8	1(0)	1(0)	1(0)	1(1)	3(8)	4(21)	95(99)	260(89)		
9	1(0)	1(0)	1(0)	1(0)	1(1)	3(2)	11(40)	88(82)	281(131)	
10	1(0)	1(0)	1(0)	1(0)	1(1)	1(2)	3(6)	12(34)	88(59)	260(123)

Naturally many smoothing methods are possible to produce smoothed series v from original series u. Linear smoothers of the general form

$$v_i = \sum_I w_I u_I ,$$

where $\sum_I w_I = 1$ and values I are those in the neighbourhood of i, have been frequently used in geography and other disciplines (e.g. Holloway, 1958; Rayner, 1971, 65-74; Chatfield, 1975, 17-20; Tobler, 1975; see Pitty, 1969, 45-9 for a hillslope example). Nonlinear smoothers have attracted much less attention but are both simple and advantageous (Beaton and Tukey, 1974; Velleman, 1977; McNeil, 1977, Ch. 6; Tukey, 1977, Chs. 7 and 16).

The simplest nonlinear smoothers are running medians: the gentlest is of length 3 whereby

$$v_i = \text{med }(u_{i-1}, u_i, u_{i+1}),$$

with appropriate end-value rules, here

$$v_1 = \text{med }(u_1, u_2)$$

$$v_n = \text{med }(u_{n-1}, u_n)$$

Running medians are attractive as resistant or robust smoothers which are less sensitive to bumps and ruts than comparable running means (moving averages). (For a contemporary introduction to resistance and robustness, see Mosteller and Tukey, 1977, Ch. 10).

Nonlinear smoothing can be followed by linear smoothing, for example by Hanning (Blackman and Tukey, 1959, 171 and references on nonlinear smoothing just cited):

$$v_i = \frac{1}{4} u_{i-1} + \frac{1}{2} u_i + \frac{1}{4} u_{i+1}$$

Again end-values must be tackled in some way: here

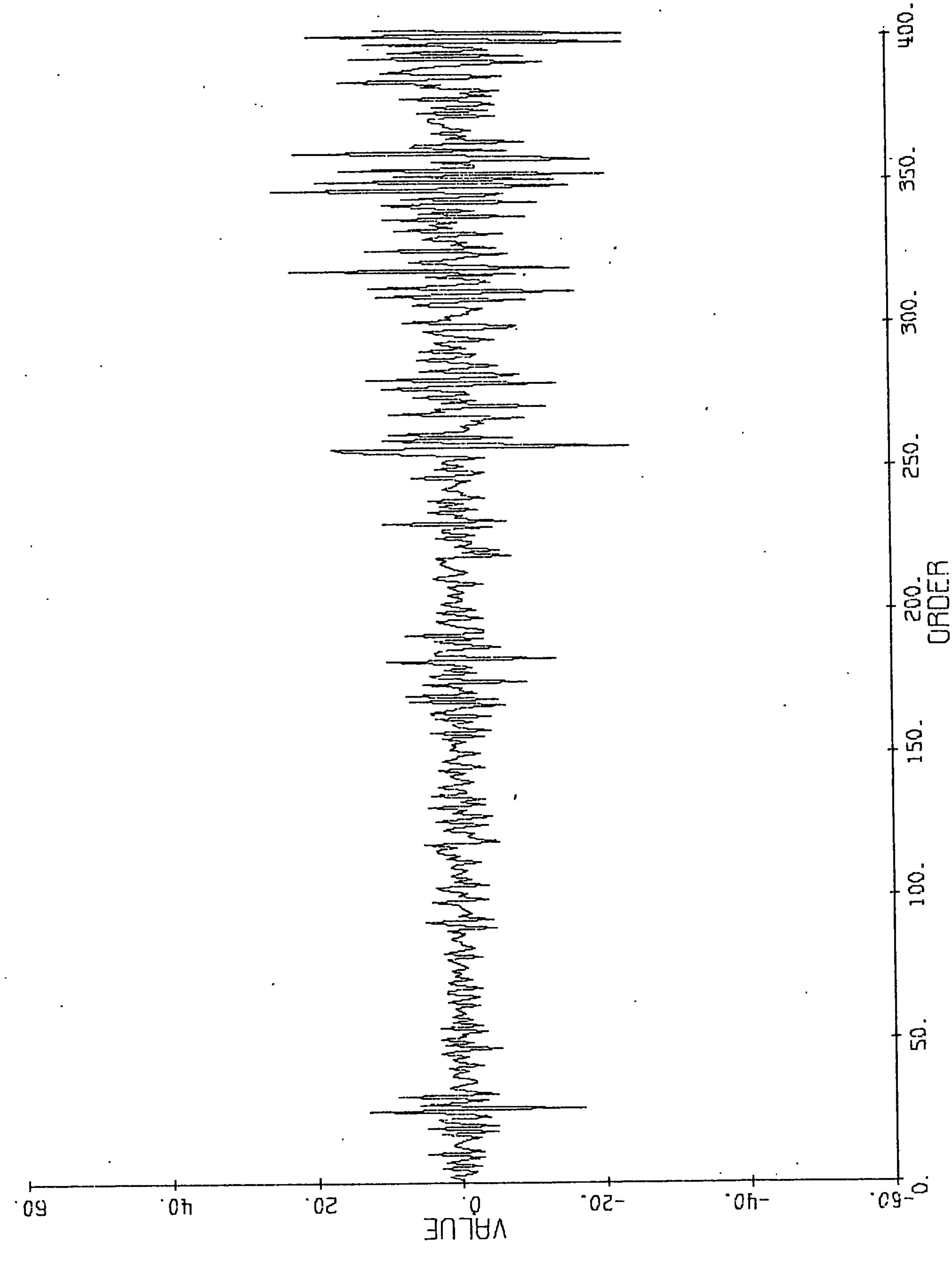
$$v_1 = \frac{2}{3} u_1 + \frac{1}{3} u_2$$

$$v_n = \frac{2}{3} u_n + \frac{1}{3} u_{n-1}$$

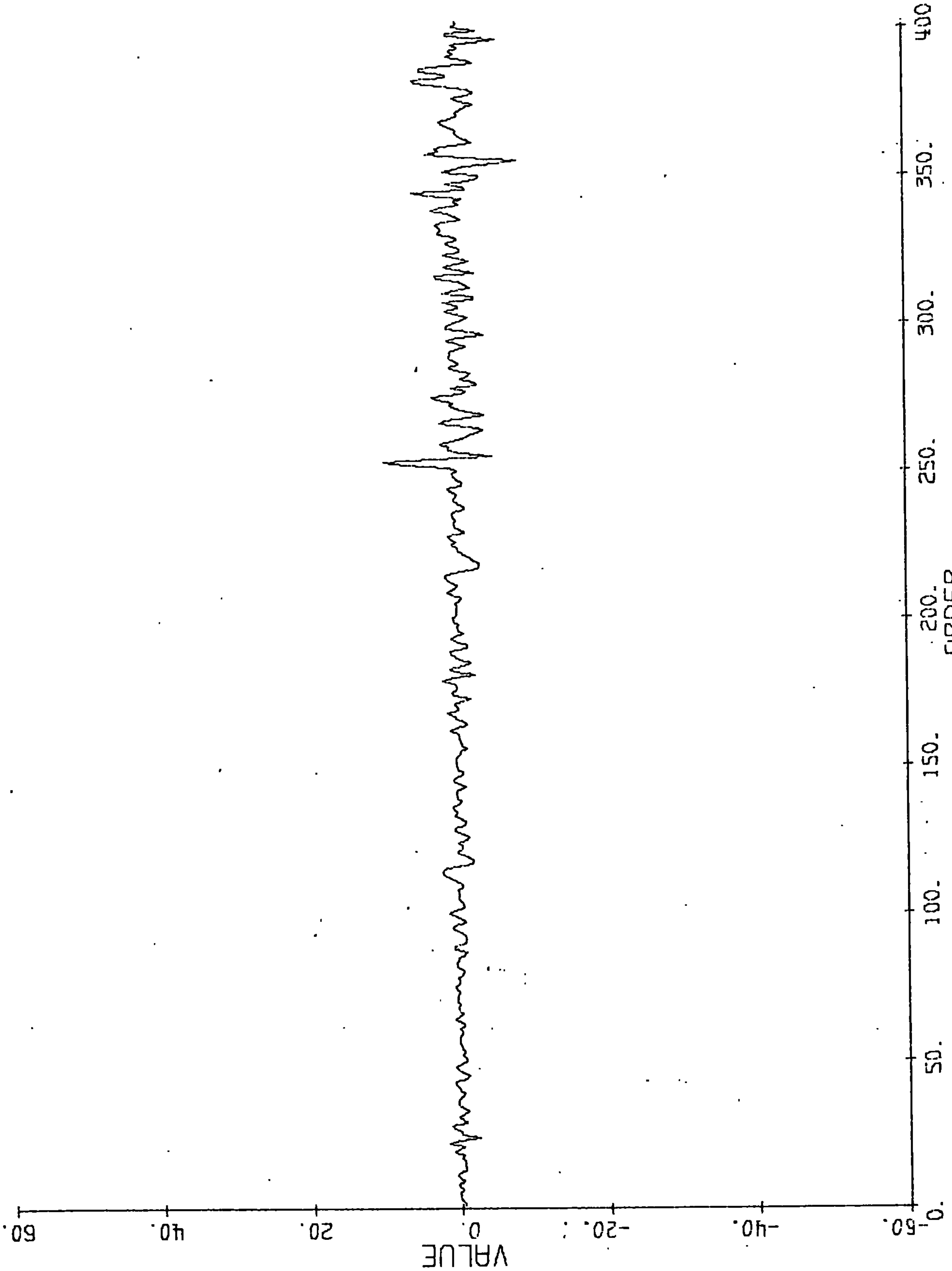
Some preliminary experiments with running median of length 3, followed by Hanning (3H for short), indicated that it was too gentle. The procedure was thus repeated, hence the smoother 3H3H. Smoothed angles, rough angles (rough = data - smooth), and curvatures derived from smooth angles were computed and plotted in each case. This particular smoother is fairly conservative: it still leaves much local variability, as can be seen from a comparison of original and smooth curvatures for profile TR in 8L and 8M, and in other plots for the remaining profiles. The smoother removes some 69 to 78% of variation on a linear scale (8N).

Additive error partitions were produced for these relatively smooth curvature series. The distribution of component lengths (80) shows that a strong tendency to produce a multitude of very short components persists despite smoothing. There is, however, a broad tendency for the shortest components to grow slightly longer, while some (but by no means all) of the longest components contract in length.

With this problem in mind, the component breaks can be examined (8P). Partitions are given for differing



8L Original curvatures for TR



8M Smooth curvatures for TR

8N

Variability of curvature series before and after smoothing

Profile	std		% reduction	midspread		% reduction
BO	5.6	1.30	77	5.5	1.4	75
ST	4.8	1.14	76	4.5	1.1	76
PR	3.7	0.95	74	3.5	0.9	74
PA	4.4	1.26	71	4.0	1.0	75
FA	4.8	1.29	73	5.0	1.1	78
LA	3.8	0.96	75	3.0	0.8	73
TR	6.1	1.60	74	5.0	1.2	76
AR	4.3	1.34	69	4.5	1.1	76
TA	5.7	1.65	71	6.0	1.5	75
HO	9.4	2.88	69	7.0	2.2	69
TO	7.0	1.82	74	7.7	2.2	71

80 Additive error partitions of smooth curvature series by least squares criterion: medians and midspreads over profiles of ordered component lengths

Rank of length (shortest = 1, longest = k)

median (midspread)

	1	2	3	4	5	6	7	8	9	10
1	413(106)									
2	7(10)	394(167)								
3	2(2)	38(122)	382(141)							
4	2(1)	3(5)	38(96)	373(137)						
5	2(1)	3(1)	6(12)	43(89)	381(138)					
6	2(1)	3(1)	4(3)	16(21)	83(69)	307(154)				
7	2(1)	2(1)	3(3)	6(3)	14(19)	84(78)	298(146)			
8	2(1)	2(1)	3(1)	4(3)	6(10)	16(24)	84(78)	295(144)		
9	2(1)	2(1)	3(1)	4(2)	5(2)	10(14)	13(22)	89(84)	251(223)	
10	1(1)	2(1)	2(1)	3(2)	4(2)	5(2)	10(19)	20(23)	86(62)	251(223)

8P <u>Component bounds</u>

BO #curvatures = 393

k												
2											386	
3									382		386	
4									382		386	390
5								381		385	386	390
6						298	299		382		386	390
7						298	299	381		385	386	390
8				295	297		299	381		385	386	390
9	285	287	292			298	299		382		386	390
10	285	287	292			298	299	381		385	386	390

ST #curvatures = 452

k													
2													445
3		43	45										
4	39	43	45										
5		43	45						429	433			
6	39	43	45						429	433			
7	39	43	45						429		434	440	
8	39	43	45	296	298				429	433			
9	39	43	45	296	298	303			429	433			
10	39	43	45	296	298		304	306	429	433			

PR #curvatures = 271

k									
2							264		
3							264	265	
4					261		264	265	
5			174	177			264	265	
6			174	177	261		264	265	
7			174	177	261	263	264	265	
8	159	161	174	177	261		264	265	
9	159	161	174	177	261	263	264	265	
10	159	161	174	177	261	263	264	265	266

PA #curvatures = 420

k										
2								415		
3								415		418
4					392			415		418
5						408	410		416	418
6						408	410	415	416	418
7			24	26		408	410		416	418
8			24	26		408	410	415	416	418
9	2	4	24	26		408	410		416	418
10	2	4	24	26		408	410	415	416	418

8P (continued)

FA #curvatures = 505

k									
2						489			
3		38	41						
4		38	41	43					
5		38	41	43		489			
6		38	41	43	483	489			
7		38	41	43	483	489	491		
8		38	41	43	483	489		497	500
9	34	38	41	43	483	489		497	500
10	34	38	41	43	483	489	491	497	500

LA #curvatures = 468

k									
2								459	
3	65	66							
4					373	376	378		
5	65	66						459	462
6	65	66			373	376	378		
7	65	66	219	221				459	462
8	65	66			373	376	378	459	462
9	65	66	219	221	373	376		459	462
10	65	66	219	221	373	376	378	459	462

TR #curvatures = 402

k											
2											394
3	251	253									
4	251	253							379		
5	251	253							379	387	
6	251	253				353	355			387	
7	251	253				353	355		379	387	
8	251	253				353	355	359	379	387	
9	251	253		342	344	353	355		379	387	
10	251	253	314	342	344	353	355		379	387	

AR #curvatures = 362

k												
2											357	
3											357	360
4			320								357	360
5			320							354	357	360
6			320					351	353		357	360
7					340	345	350		353		357	360
8			320		340	345	350		353		357	360
9			320	327	340	345	350		353		357	360
10	163	165	320		340	345	350		353		357	360

8P (continued)

TA #curvatures = 566

k												
2												560
3	411	418										
4			423				443	447				
5	411	418					443	447				
6	411	418					443		448	455		
7	411	418					443		448	455		560
8	411	418		427			443		448	455		560
9	411	418			428	431	443		448	455		560
10	411	418			428	431	443		448	455	458	560

HO #curvatures = 413

k											
2							131				
3				128		130					
4		124	127			130					
5	118	124	127			130					
6	118	124	127			130				155	
7	118	124	127			130				155	175
8	118	124	127			130		152	154		175
9	118	124	127	128		130		152	154		175
10	118	124	127	128	129	130		152	154		175

TO #curvatures = 318

k										
2	16									
3							270	271		
4							270	271	300	
5							270	271	300	303
6	16						270	271	300	303
7				116	118		270	271	300	303
8				116	118	123	270	271	300	303
9	16			116	118	123	270	271	300	303
10		28	30	116	118	123	270	271	300	303

numbers of components in a format which allows comparison of the effects of varying the level of generalisation. It seems likely that only genuine breaks will be maintained as the number of components varies: breaks which appear and disappear relatively quickly are likely to be spurious. Moreover, very short components will be disregarded as lacking in geomorphological interest.

BO is probably best regarded as an upper component (1,380) with an irregular middle section (286,299); and a relatively short basal component (381,393). On ST, the breaks around 43 are caused by a track, while the very short components around 300 are also attributable to purely local irregularity. There is a good case for identifying a basal component (430,452) and a long upper component (1,429). Similarly on PR breaks in midslope may be discounted and a break made near the base: (1,264) and (265,271). Yet again on PA possible breaks near the crest do not seem to have substantial significance and the clearest break is between a long upper (1,408) and a short lower component (408,420).

In the case of FA the breaks near the crest are caused by a ditch and once again the basal part is best distinguished from an upper part, say (1,489) and (490,505). On LA, there are basically four possible breaks: around 65, around 220, around 376, and around 459. The first three correspond to local irregularity, whereas the last can be distinguished as the start of a short basal component (460,468).

On TR three components may be distinguished: a long convexity (1,252), a shorter convexity (253,353) and an irregular basal component (354,402). AR is best taken as an upper component (1,320) together with an irregular basal component (321,362). On TA an upper component (1,411) is evident. To this may be added an intermediate (and irregular) crags component (412,455), a long lower component (456,560) and a short basal component (561,566).

HO is a complicated case. The most appropriate division would seem to be into an upper component (1,118), an irregular crags component (119,175), and a third component reaching to the base (176, 413). Finally on TO the most important break is at 270. A basal component may be distinguished (271,308) including two minor irregularities. Breaks around 118 are attributable to a track while that at 16 is not persistent: this leaves a long upper component (1,270).

The component bounds accepted as of probable geomorphological significance are listed in 8Q, together with underlying geological formations repeated from 6L. The most obvious common feature, observed for every profile except HO, is a distinct, relatively short and often irregular basal component. Apart from that, there is a broad but by no means perfect relationship between the number of components accepted and the complexity of the geology. BO, ST, PR, PA, FA and LA, all within the Lower Oolite, are all accepted as one-component slopes above their basal components. TR, TA and HO (although not AR),

8Q Component bounds and geology

Profile	Geology	Component bounds
BO	del	1,380 381,393
ST	del	1,429 430,452
PR	del	1,264 265,271
PA	del	1,408 409,420
FA	gls/del	1,489 490,505
LA	gls/del	1,459 460,468
TR	del/uli	1,252 253,353 354,402
AR	del/uli	1,320 321,362
TA	gls/del/uli	1,411 412,455 456,560 561,566
HO	del/dog/uli/mli	1,118 119,175 176,413
TO	lcg/oxf/kel/cor/del	1,270 271,318

extending from Lower Oolite to Lias, all have appreciably more complex forms. On the other hand TO, which extends across five mapped formations in the Lower and Middle Oolites, is accepted as one component above its basal component.

These results suggest that diversity of underlying strata is not inevitably associated with complexity of hillslope forms in the sense of a multiplicity of distinct components. Examination of the variation of within-component mean of square V with k helps to complete the picture (8R). The ranking of profiles by V is related to geology, but much of the variability is on a microscale and does not find expression at component scale, that is, between components. (This conclusion is strengthened when it is remembered that original data have been smoothed before computing curvatures).

8.7 Summary and discussion

This chapter includes three distinct contributions to the theory and practice of profile analysis.

(i) An extended formulation embeds the problem within data analysis (8.1) and geomorphology (8.2); guiding principles are suggested (8.3).

(ii) A critique of existing methods shows that none are really acceptable (8.4).

(iii) Methods for producing additive error partitions are attractive alternatives (8.5). In practice, however, 8R

Within component mean square V for various partitions

k	BO	ST	PR	PA	FA	LA	TR	AR	TA	HO	TO
1	1.70	1.31	0.91	1.59	1.68	0.92	2.55	1.79	2.73	8.28	3.30
2	1.50	1.28	0.87	1.27	1.59	0.90	2.52	1.33	2.63	8.19	3.21
3	1.32	1.20	0.75	1.16	1.42	0.86	2.20	1.15	2.45	7.07	3.00
4	1.22	1.13	0.70	1.10	1.20	0.84	2.17	1.07	2.31	6.34	2.89
5	1.18	1.09	0.67	1.01	1.11	0.80	1.95	1.01	2.12	5.70	2.66
6	1.13	1.03	0.63	0.98	1.03	0.78	1.90	0.96	2.03	5.26	2.57
7	1.10	1.01	0.60	0.92	0.96	0.75	1.72	0.91	1.93	4.95	2.38
8	1.08	0.95	0.58	0.89	0.90	0.72	1.67	0.86	1.87	4.76	2.29
9	1.04	0.91	0.56	0.83	0.86	0.69	1.58	0.83	1.81	4.60	2.19
10	1.01	0.87	0.54	0.79	0.82	0.67	1.53	0.80	1.77	4.46	2.12

least squares partitions of curvature series need to be preceded by smoothing of original data. Components identified on field profiles allow some inferences about the relationship between profile morphometry and underlying geology (8.6).

The method of profile analysis adopted here, a combination of nonlinear smoothing and additive error partition, is rather arbitrary and perhaps unduly complex. It can only be regarded as an interim solution. Future work would be best directed at these two families of methods - nonlinear smoothing and additive error partition - probably with the aim of eliminating one for the sake of simplicity.

It has been argued above that optimality criteria for partitions need to be discussed in relation to the appropriate decency assumptions about the ideal character of data. The case of Hartigan's additive error partitions is instructive in this respect. It can be shown theoretically that optimal subseries diameter depends on the conditional distribution of subseries values, but it is also clear empirically that this may well vary, both within and between profiles. Hence no single diameter can be optimal.

In such a situation it is important to consider adaptive methods of profile analysis in which the components are distinguished by procedures which depend on the properties of the data. Ironically enough, apparently suitable adaptive methods have only received prominence since the work reported here was undertaken, notably adaptive methods of resistant/robust estimation (Mosteller and Tukey, 1977), and of nonlinear smoothing (McNeil, 1977; Tukey, 1977). Future work will examine these as possible bases for profile analysis methods.

8.8 Notation

c	curvature
d	in ordinary derivative
d	subseries diameter
H_o	null hypothesis
i	subscript
I	subscript
j	subscript
k	number of components
l	measured length
n	number of observations
p	power
s	arc length
u, $\bar{u}$	value of series, average value
v	value of series
V	within component variation
w	weight
x	horizontal coordinate
z	vertical coordinate
α	intercept parameter
β	slope parameter
Δ	difference operator
ϵ	stochastic error
θ	angle
Σ	summation operator
#	number of
$\mid$	given
$\mid \; \mid$	modulus

Mnemonics

loq	lower quartile
med	median
pr	probability
std	standard deviation
upq	upper quartile

Chapter 9

FITTING CONTINUOUS CURVES TO HILLSLOPES

The Saturnian and the Sirian exhausted themselves in conjectures upon this subject, and after abundance of argumentation equally ingenious and uncertain, were obliged to return to matters of fact.

Voltaire, Micromegas, Ch. II.

9.1 The general situation

It is clear that visual evaluation of model predictions ('the model profiles seem fairly realistic') cannot be accepted as a satisfactory means of model testing: mere 'eyeballing' leaves too much scope for subjective and arbitrary judgements. Model profiles should be fitted to actual profiles statistically, and goodness-of-fit assessed quantitatively. This much seems almost indisputable, yet the many models which have been proposed in the literature have received greatly varying amounts of statistical testing. Some, indeed, have received none at all. Moreover, model fitting has often been characterised by a cavalier disregard for the statistical difficulties which arise in the process (see, for example, Cox, 1975, on Woods, 1974 and Cox, 1977a on Bull, 1975). This chapter provides a systematic discussion of fitting time-invariant continuous deterministic models to hillslope profiles, illustrated by the derivation and application of fitting procedures for a model function originally obtained by Kirkby (1971).

A framework for discussion is given in 9A. Model fitting is here regarded as a three-stage process, as is common in modelling literature (cf. Matalas and Maddock, 1976).

(i) Specification: the form of the model is decided

(ii) Estimation: the parameters are estimated

(iii) Checking: the residuals are analysed.

9A Hillslope model fitting as a three-stage process

SPECIFICATION

Deterministic model function

↓

Decide on variables, known constants, unknown parameters

↓

Choose estimators of parameters

Decide on stochastic error structure

Choose measure of discrepancy

Use linearising transformation?

Errors in variables?

Autocorrelated errors?

↓

ESTIMATION

Estimate parameters by minimising discrepancy

(analytical solution or search algorithm)

↓

CHECKING

Comparison of estimates

Lack of fit measures

Spatial distribution of residuals

9.2 Specification and estimation: first approximation

The function

$$\frac{z}{z_d} = 1 - \left(\frac{x}{x_b}\right)^k$$

was first obtained by Kirkby (1971) as an approximate characteristic form solution to a particular continuity equation. It is also an exact constant form solution to the same continuity equation (cf. Ch. 3.3.15, 3.3.21 above). Moreover, it is related to the power functions used as empirical static models by Hack and Goodlett (1960) and others (cf. Ch. 3.2.2 and 3B above). k is here termed the Kirkby parameter.

The central question is the following: How can such a function be fitted to an actual hillslope profile or a component of such a profile? (The generalisation to cover components is straightforward. Henceforth 'base' and 'crest' imply base and crest of profile or component).

A first task in specification is to distinguish between variables, known constants and unknown parameters. In this case z and x are variables, z_d and x_b are known constants and k is an unknown parameter. Further algebraic simplification is possible by scaling coordinates to dimensionless variables. If we write

$$\text{relative fall} = v = \frac{z_d - z}{z_d}$$

$$\text{relative distance} = u = \frac{x}{x_b}$$

Then

$v = u^k$

It is now necessary to make three related decisions:

(a) To choose estimators of the parameters

(b) To specify the structure of stochastic errors

(c) To choose a measure of discrepancy between model and data.

(a) An estimator is a procedure for estimating a parameter; with any particular set of data it produces an estimate, an actual number. The properties of estimators are well discussed by Bard (1974), Plackett (1971) and Silvey (1970), among many others. In practice four properties are particularly important (Bard, 1974, 44):

(i) Small bias On average the estimator should produce a value as near as possible to the actual value.

(ii) Small variance (efficiency) The spread of estimates around the average estimate should be as small as possible.

(iii) Robustness The estimator should be stable under slight changes in the probability distribution of stochastic errors: in particular, it should not be thrown out by the occurrence of outlying or wild observations.

(iv) Computability It should be easy to calculate.

(b) A model is often written in standard reduced form (Bard, 1974, 26):

dependent variable(s)	=	deterministic function of independent variable(s) and parameter(s)	+ stochastic error(s)

Why are stochastic errors included in a model? The stochastic terms capture the <u>remainder</u> of the variation not captured by the deterministic function (Gilbert, 1973). This remainder will reflect one or more of the following:

(i) the influence of variables not included in the analysis

(ii) inappropriate choice of model function

(iii) real but random effects

(iv) measurement error

(v) interaction among the other four

(Mark and Church, 1977, 71)

In the example of this chapter a first approximation is to write

$$v = u^k + \epsilon$$

where ϵ is stochastic error. The following assumptions about ϵ are standard :

zero mean	$E(\epsilon_i) = 0$	
constant variance or homoscedasticity:	$E(\epsilon_i^2) = $ a constant	
uncorrelated:	$E(\epsilon_i \epsilon_j) = 0$	$i \neq j$

(c) Fitting the model may usually be viewed as minimising some measure of the <u>discrepancy</u> between the model and the data (Nelder, 1975,7). The residuals may be defined as a function of the parameter k

$$e_i(k) = v_i - u_i^k \qquad ; i=1,\ldots,n$$

The sum of squared residuals is then

$$\sum_{i=1}^{n} e_i^2 \quad , \text{ a function of } k.$$

Minimising the sum of squared residuals to estimate an unknown parameter is known as ordinary least squares estimation (OLS).

Why should this particular measure of discrepancy be used for estimation? Clearly it is necessary to consider OLS in the light of the criteria emphasised by Bard (1974,44): small bias, small variance, robustness and computability.

There is an important result for linear models, the Gauss-Markov theorem, which is roughly that if the errors ϵ satisfy the three standard assumptions in (b) above, then the OLS estimators are unbiased, and have the smallest variance of any linear unbiased estimator. (For fuller statements, see Wonnacott and Wonnacott, 1970, 48-51 or Silvey, 1970, 51-4). Hence there are some theoretical grounds for expecting small bias and small variance.

OLS is not robust under all circumstances, and is well-known to be unstable in the presence of wild observations. Consequently many alternative procedures have been proposed (e.g. Mosteller and Tukey, 1977; McNeil, 1977). However, the hillslopes considered here are well-behaved in the sense that bumps and ruts are microscale features. Hence only least-squares criteria will be considered for the discrepancy function. This has two major advantages: they are relatively easy to compute and they are fairly well understood in principle.

Returning to the Kirkby model the sum of squared residuals may be written

$$\sum_{i=1}^{n} (v_i - u_i^k)^2$$

This quantity is nonlinear in k and there is thus no closed-form expression for the OLS estimator of k. This problem could be tackled by using a search algorithm to find the minimum value of the discrepancy and hence an estimate of k(cf. Bard, 1974, Chs. 5 and 6, and Chambers, 1977, Ch. 6, on search algorithms). It can also be broached by a fairly straightforward trick at the expense of some complications. In the next section this trick (a linearising transformation) will be discussed together with ways of overcoming two further problems (errors in variables and autocorrelated errors).

9.3 Specification and estimation: further approximations

9.3.1 Linearising transformations

The deterministic function

$$v = u^k$$

can be reexpressed in logarithmic terms

$$\ln v = k \ln u$$

(Here and below natural logarithms are used). If it is supposed that additive stochastic errors perturb this deterministic function then

$$\ln v = k \ln u + \epsilon'$$

whence residuals may be defined

$$e_i(k) = \ln v_i - k \ln u_i$$

and the sum of squared residuals formed

$$\sum e_i^2(k) = \sum(\ln v_i - k \ln u_i)^2$$

The limits of summation have not been specified in this expression because the use of a logarithmic transformation brings a minor problem in its wake. Near the crest of a slope relative fall may be identically zero, and hence the logarithm may be indeterminate. At the base relative fall and relative distance are identically one. If measured lengths of zero gradient occur at the crest they should be omitted from the calculation, and an equal number of measured lengths omitted symmetrically at the base. Otherwise all summations will be $\sum_{i=1}^{n}$. Indices will be omitted from the expressions below, and it should be understood that the limits of summation are determined by this procedure.

The substitution $v' = \ln v$ and $u' = \ln u$ in the model above yield the obvious linear model

$$v' = k u' + \epsilon'$$

with OLS discrepancy (or sum of squared residuals)

$$\sum(v' - ku')^2$$

The analytical derivation of the OLS estimator will now be given. Expanding the term in parentheses

$$\begin{aligned}\sum(v' - ku')^2 &= \sum(v'^2 - 2ku'v' + k^2u'^2) \\ &= \sum v'^2 - 2k\sum u'v' + k^2\sum u'^2\end{aligned}$$

For a minimum it is sufficient that

$$\frac{\partial}{\partial k}\left[\sum(v' - ku')^2\right] = 0$$

whence

$$-2\sum u'v' + 2k\sum u'^2 = 0$$

$$k = \sum u'v' / \sum u'^2$$

$$= \frac{\sum \ln u . \ln v}{\sum (\ln u)^2}$$

The use of a linearising transformation is frequently recommended: the problems are less frequently emphasised. The account given here draws on Wonnacott and Wonnacott (1970, 91-8), Johnston (1972, 27-53), Goldf eld and Quandt (1972, Ch. 5) and Bard (1974, 78-80).

The linear model

$$\ln v = k \ln u + \epsilon'$$

corresponds to the model

$$v = u^k \exp(\epsilon')$$

with multiplicative errors. While it leads to a closed-form estimator, the assumption of stochastic errors which are multiplicative in the original metric needs substantive justification. It implies heteroscedasticity (unequal variances) together with some other problems (Goldf eld and Quandt, 1972, 136). As these authors wrote: 'In spite of the possibility that there might be a priori reasons for specifying the error term to be of a particular type, in most cases the multiplicative form seems to be chosen for its computational convenience. The two basic or pure alternatives . . . are rarely contrasted.' Moreover, the data ought to be allowed to reveal which of the two hypotheses about errors is acceptable (Goldf eld and Quandt, 1972, 137).

Goldf eld and Quandt proposed a generalised model incorporating both additive and multiplicative error terms. However, a simpler method of overcoming this difficulty seems more appropriate for the hillslope model case. Following a suggestion by Bard (1974, 79) a linearising transformation is used together with weighted least squares as a way of tackling the heteroscedasticity which is assumed to be associated with the transformation. The connection is given by a theorem which gives the relationship between the variances of two random variables (s and t, say)

$$\mathrm{var}(t) \approx \mathrm{var}(s)\left(\frac{\partial t}{\partial s}\right)^2$$

(see, e.g., Plackett, 1971, 59-60).

If an additive homoscedastic model is appropriate, then

$$\mathrm{var}(v|u) = \mathrm{var}(\epsilon)$$

but if we transform by logarithms

$$\mathrm{var}(\ln v|\ln u) = \mathrm{var}(\epsilon') \approx \mathrm{var}(\epsilon)\left(\frac{1}{v}\right)^2$$

because

$$\frac{\partial}{\partial v}(\ln v) = \frac{1}{v}$$

Since in this situation the variance is not constant, it is best to allow for this and weight each residual. The WLS discrepancy (sum of weighted squared residuals) for a linear model

$$v' = ku' + \epsilon'$$

with weights w is

$$\sum (v' - ku')^2 w = \sum w v'^2 - 2k \sum w u' v' + k^2 \sum w u'^2$$

For a minimum it is sufficient that

$$\frac{\partial}{\partial k}\left[\sum (v' - ku')^2 w\right] = 0$$

whence

$$-2\sum wu'v' + 2k\sum wu'^2 = 0$$

$$k = \sum wu'v' / \sum wu'^2$$

In the hillslope case, conditional variance of the stochastic errors is proportional to $(\frac{1}{v})^2$. It is logical to weight squared residuals inversely by variance, to discount inherently more variable fluctuations. Hence $w = v^2$ and the WLS estimator is

$$k = \frac{\sum v^2 \ln v \ln u}{\sum v^2 (\ln u)^2}$$

9.3.2 Errors in variables

It has been tacitly assumed so far that v is a dependent variable, which is error-prone, and that u is an independent variable, which is error-free. In geomorphological terms, however, there is no justification at all for any such asymmetric distinction. v is not a 'response' to 'factor' u any more than u is a 'response' to 'factor' v (to use the excellent terminology of Tukey, 1977, 125-6). Nor is u held at fixed values to see the resulting change in v. Both v and u must therefore be regarded as subject to stochastic fluctuation, a situation known as 'errors in variables'.

A variety of methods has been devised for this situation. A thorough review is given by Moran (1971). In the geological and geographical literature the problem has been discussed by McCammon (1973), Till (1973), Mark and Church (1977), Kuhry and Marcus (1977), and Mark and Peucker (1978), among others. McCammon (1973) suggested minimising the discrepancy

$$\sum \left(w_1 e_1^2 + w_2 e_2^2 \right)$$

where e_1 and e_2 are residual distances measured perpendicular to horizontal and vertical axes, and w_1 and w_2 are corresponding weights. McCammon gave details of programs implementing a minimisation algorithm. The main problem with this method is the need for specifying weights beforehand, which really requires detailed knowledge of error structure.

Till (1973) criticised the use of standard linear regression in geomorphological situations where both variables are subject to error. He recommended instead the use of the reduced major axis, the line which bisects the angle between the two standard regression lines. This is the correct solution if the stochastic errors perturbing the two variables have equal variances. Till reworked some examples of fitting power functions to glacial valleyside profiles given by Doornkamp and King (1971). He fitted a reduced major axis to the logarithmically transformed data, but using the assumption of multiplicative errors and without commenting on obviously autocorrelated residuals. The use of reduced major axes is clearly not a general solution to this problem given the strong assumption of equal variances.

Mark and Church (1977), discussing the problem of errors in variables, criticised the misuse of regression in earth science, especially in geomorphometry. (See also Mark and Peucker, 1978, on geographical applications). They reviewed solutions appropriate in different circumstances. Most emphasis is placed on estimation procedures which can be used when the ratio of error variances is known. The main disadvantage of such methods is that they require considerable knowledge about each variable. Kuhry and Marcus (1977) recommended a covariance ratio method which requires observations on a third variable, which is not possible in this case.

None of these solutions seems acceptable for the Kirkby model, and so the only possibility is to estimate parameters for polar situations ((i)v a response, u a factor (ii) u a response, v a factor) and consider the variation in results (cf. Moran, 1971, 251).

By symmetry OLS and WLS estimators can be derived for the case in which u is regarded as dependent and v as independent.

The OLS estimator of 1/k in the multiplicative model

$$\ln u = (1/k) \ln v + \epsilon'$$

is

$$\frac{\sum \ln v \ln u}{\sum (\ln v)^2}$$

The WLS estimator of 1/k in the additive model is

$$\frac{\sum u^2 \ln v \ln u}{\sum u^2 (\ln v)^2}$$

If it is assumed that

estimate of k = 1/(estimate of 1/k)

then these estimators may be compared with others proposed.

9.3.3 Autocorrelated errors

One of the standard assumptions about stochastic errors ϵ is that pairs of terms are uncorrelated

$$E(\epsilon_i \epsilon_j) = 0 \qquad i \neq j$$

Hence one possible problem is autocorrelation among error terms, especially if data are time or space series. The problems which arise when error terms are autocorrelated, and methods for overcoming these problems, have received much attention, especially in econometrics where linear models are often fitted to time series (e.g. Wonnacott and Wonnacott, 1970, 136-45; Johnston, 1972, Ch. 8; Stewart, 1976, 137-46). This work has recently been extended to spatial series by statistical geographers (Cliff and Ord, 1973, Ch. 5; Martin, 1974).

Johnston (1972, 246-9) outlined the consequences of using OLS estimators on a linear model when error terms are autocorrelated. Firstly, estimators of the parameters have large variances. Secondly, standard significance tests are invalid. Thirdly, prediction using the model is inefficient.

Such problems may be sufficiently serious to warrant the use of other methods as alternatives to OLS if there are grounds for suspecting autocorrelated errors. If the model is being applied to time series, then a first order autoregressive (Markov) scheme may be appropriate for the errors

$$\epsilon_i = \rho\epsilon_{i-1} + \eta_i$$

where

$$E(\eta_i) = 0$$

$$E(\eta_i^2) = \text{a constant}$$

$$E(\eta_i \eta_j) = 0 \qquad i \neq j$$

This Markov model assumes unilateral influences: the past is assumed to affect the present, but not vice versa. If the model is being applied to space series then bilateral or multilateral influences must be allowed. With a further generalisation to allow the possibility of unequal weighting, the formulation of Cliff and Ord (1973, 90) is obtained:

$$\epsilon_i = \rho \sum_{j \neq i} w_{ij} \epsilon_j + \eta_i \quad ; \sum_{j \neq i} w_{ij} = 1$$

The bilateral case is more appropriate for hillslope profiles.

If ρ is unknown, then there are two possibilities: it can be estimated iteratively from the data, or a value can be assumed <u>a priori</u>. OLS implicitly assumes $\rho = 0$; the polar possibility is to assume $\rho = 1$ (Martin, 1974). However, the assumption $\rho = 1$ brings some theoretical problems in its wake, even for the linear model and time series case (cf. Wonnacott and Wonnacott, 1970, 141; Johnston, 1972, 245; Stewart, 1976, 145-6). Moreover, the estimators used by Cliff and Ord (1973) and Martin (1974) would need further adjustment for nonlinearity and errors in variables. Finally, such estimators are rather unstable numerically because they are based on products and squares of local differences which tend to be very small in the case of hillslope series.

Since estimator formulation is difficult in this case, and since no attempt will be made at either prediction or significance testing, attention to autocorrelation will here be confined to inspecting autocorrelation functions of residuals.

9.4 Deterministic estimation of the Kirkby parameter

The case of the model function originally derived by Kirkby (1971) is unusual because it is also possible to estimate the Kirkby parameter k without any reference to stochastic errors. This section is an intermezzo developing this point.

Kennedy (1967, 22) introduced a 'height-length integral' for hillslope profileswhich was defined by a graphical example. This example was repeated by Chorley and Kennedy (1971, 54) accompanied by an algebraic definition which is in fact both meaningless and incorrect. For relative fall v and relative distance u, a proper definition of the integral (here called the Kennedy integral) is

$$\int_0^1 (1 - v)\, du = K \text{ , say}$$

The integral is a measure of the shape of a hillslope profile.

$0 < K < 1$; $K = 0.5$ for straight slopes, $K > 0.5$ for convex, $K < 0.5$ for concave.

Kennedy (personal communication, 1977) computed values of K graphically. It is also possible to compute K directly from relative coordinates u and v. Putting $y = 1 - v$, the integral is the sum of n trapezia:

$$K = \sum_{i=1}^{n} \left(\frac{y_{i-1} + y_i}{2} \right)(u_i - u_{i-1})$$

where

$$y_0 = 1 , \quad u_0 = 0$$

As Chorley and Kennedy (1971, 290) hinted, there is a simple relationship between Kennedy integral K and Kirkby parameter k. Given that

$$K = \int_0^1 (1 - v) \, du$$

and $v = u^k$

then

$$K = \int_0^1 (1 - u^k) \, du$$

$$= \left[u - \frac{1}{k+1} u^{k+1} \right]_0^1$$

$$= 1 - \frac{1}{k+1} = \frac{k}{k+1}$$

and inversely

$$k = \frac{K}{1 - K}$$

Since the appropriate error structure for Kirkby's model is not obvious, such a simple deterministic estimator is highly attractive.

9.5 Checking

9.5.1 General remarks

Each method of estimation yields an estimate of the parameter, a set of fitted values for the 'dependent variable' and a set of estimated residuals.

residual = observed - fitted

These results must be analysed carefully to check the adequacy of the model. The basic approach is simple: 'a good fit does not prove that the model is correct . . . a lack of fit constitutes strong grounds for rejecting, or at least amending the model' (Bard, 1974, 198).

9.5.2 Comparison of estimators

In earlier sections several estimators have been derived for the Kirkby parameter k, including one without any reference to stochastic errors. Since it is never certain in practice which assumptions are most appropriate, it seems best to employ all the estimators, and to compare the results. The underlying principle is that great variability between estimators indicates an inadequate model, while conversely if remainder terms are all small, it should matter relatively little which assumptions are invoked.

9.5.3 Analysis of residuals

The set of residuals deserves to be studied in detail (McNeil, 1977; Tukey, 1977). In analysing residuals, it is best to supplement numerical investigation with graphical display.

Measures of overall lack of fit may readily be defined. Three are used here: root mean square residual, midspread of residuals and range of residuals.

The spatial distribution of residuals is worth consideration, for as Cliff and Ord (1973, 71) reported, good aspatial fit and good spatial fit are not necessarily associated. Residuals should be plotted in serial order and their autocorrelation properties investigated.

9.5.4 Against significance testing

Kirkby (1976b) and Moon (1977) have regarded the assignment of significance levels to fitted models as an important aspect of model checking. However, this approach seems to be both unnecessary and problematic, for three reasons.

(i) Individual hillslope profiles can be treated on their own merits, without any reference to hypothetical sets of approximately identical profiles of which they are supposedly representative.

(ii) Those significance tests which at first glance appear appropriate turn out on closer inspection to be inappropriate for the hillslope case. For example, the Durbin-Watson test for autocorrelated residuals is appropriate only for a model with an intercept term and with stochastic errors following a unilateral Markov property (cf. Wonnacott and Wonnacott, 1970, 142-3; Johnston, 1972, 250-2; Stewart, 1976, 147-50).

(iii) In hypothesis testing of the standard kind, we decide between null and alternative hypotheses with specified probabilities of error. The value of such a procedure is very much in doubt, and is a matter of considerable controversy among statisticians (Edwards, 1969; Barnett, 1973; Cox and Hinkley, 1974), although it is rarely questioned in geography (but cf. Cox and Anderson, 1978). Naive significance testing reduces model checking to a binary decision based on one number (which may be wildly inaccurate) and a null hypothesis (which may be totally irrelevant) (cf. Cox, 1977a on Bull, 1975). It is better to base any decisions on the indications provided by all the model results.

9.6 Results for field profiles

The Kirkby parameter k was estimated by the five estimators outlined above (see 9B) on the fourteen components identified in Ch. 8.6 above which were based on 100 or more observations (see 8Q and 9C). For each estimate, residuals were calculated of the form

$$v_i - u_i^k \qquad i = 1,\ldots,n$$

The frequency distribution of residuals is here summarised by min, loq, upq, max, range, midspread and root mean square (rms). Autocorrelation functions were calculated for each residual series up to a maximum lag determined by the rule of thumb given in Ch. 7.5 above (the smaller of 50 and floor (n/4)). 9D shows the number of angles in each component; the Kennedy integral; the set of estimates; summary measures for residual distributions (zero and decimal point elided);

9B Estimators of the Kirkby parameter

Identifier	Assumptions and definition
KMU	Errors multiplicative; u controlling factor (9.3.1) $$\frac{\sum \ln u \,.\, \ln v}{\sum (\ln u)^2}$$
KAU	Errors additive; u controlling factor (9.3.1) $$\frac{\sum v^2 \ln v \ln u}{\sum v^2 (\ln u)^2}$$
KMV	Errors multiplicative; v controlling factor (9.3.2) $$\frac{\sum (\ln v)^2}{\sum \ln v \ln u}$$
KAV	Errors additive; v controlling factor (9.3.2.) $$\frac{\sum u^2 (\ln v)^2}{\sum u^2 \ln v \ln u}$$
KKEN	Uses theoretical relation between Kirkby parameter and Kennedy integral (9.4) discrete version of $\int_0^1 (1-v)\,du$

9C Components fitted by Kirkby curves

Profile	Indices	Dimensions (m) x_b	z_d	(deg) $\bar{\theta}$
BO	1,380	573	56	5.5
ST	1,429	647	75	6.6
PR	1,264	398	47	6.8
PA	1,408	614	73	6.8
FA	1,489	732	103	8.0
LA	1,459	687	109	9.0
TR	3,250	375	24	3.7
TR	253,353	144	45	17.5
AR	1,320	479	75	8.9
TA	6,406	602	78	7.3
TA	456,560	147	56	20.8
HO	2,117	173	27	9.0
HO	176,413	345	96	15.5
TO	1,270	383	136	19.6

9D Results of fitting Kirkby curves

BO (1,380) # 380 K = 0.618

	Estimate	Residuals min	loq	upq	max	range	midspr	rms	autocorrelation
KMU	1.579	-035	-008	002	012	047	010	012	lag 44 0.124
KAU	1.619	-027	-003	007	018	045	010	010	lag 38 0.132
KMV	1.588	-033	-006	003	013	047	010	012	lag 43 0.115
KAV	1.612	-029	-003	006	017	046	010	010	lag 39 0.131
KKEN	1.615	-028	-003	007	017	045	010	010	lag 39 0.123

ST (1,429) # 429 K = 0.516

	Estimate	Residuals min	loq	upq	max	range	midspr	rms	autocorrelation
KMU	1.196	-018	000	053	070	087	053	039	[lag 50 0.641]
KAU	1.054	-044	-028	016	024	068	044	024	[lag 50 0.653]
KMV	1.201	-017	001	054	071	088	053	040	[lag 50 0.641]
KAV	1.092	-036	-020	025	036	073	045	024	[lag 50 0.648]
KKEN	1.066	-042	-026	019	027	069	044	023	[lag 50 0.651]

PR (1,264) # 264 K = 0.572

	Estimate	Residuals min	loq	upq	max	range	midspr	rms	autocorrelation
KMU	1.255	-040	-027	-001	016	056	027	021	lag 30 0.153
KAU	1.324	-024	-016	008	034	058	024	014	lag 31 0.131
KMV	1.259	-039	-027	-000	017	056	026	020	lag 31 0.108
KAV	1.329	-024	-015	009	035	059	024	014	lag 31 0.137
KKEN	1.335	-023	-014	010	036	060	024	014	lag 31 0.144

9D (continued)

PA (1,408)   # 408   K = 0.618

	Estimate	Residuals min	loq	upq	max	range	midspr	rms	autocorrelation
KMU	1.445	-085	-041	-010	009	095	031	037	[lag 50 0.626]
KAU	1.572	-073	-020	010	025	098	029	029	[lag 50 0.653]
KMV	1.455	-084	-039	-008	011	095	031	036	[lag 50 0.629]
KAV	1.556	-075	-022	007	023	097	029	029	[lag 50 0.652]
KKEN	1.618	-069	-012	019	036	105	031	029	[lag 50 0.653]

FA (1,489)   # 489   K = 0.625

	Estimate	Residuals min	loq	upq	max	range	midspr	rms	autocorrelation
KMU	1.446	-093	-053	-016	003	096	037	043	[lag 50 0.759]
KAU	1.604	-082	-025	012	025	107	037	032	[lag 50 0.796]
KMV	1.458	-092	-051	-014	004	096	037	042	[lag 50 0.764]
KAV	1.593	-082	-027	010	022	105	037	033	[lag 50 0.795]
KKEN	1.664	-078	-017	025	038	116	041	033	[lag 50 0.796]

LA (1,459)   # 459   K = 0.606

	Estimate	Residuals min	loq	upq	max	range	midspr	rms	autocorrelation
KMU	1.389	-082	-039	-007	006	088	032	034	[lag 50 0.483]
KAU	1.538	-045	-006	012	030	075	018	018	[lag 50 0.329]
KMV	1.401	-079	-036	-005	006	085	032	032	[lag 50 0.472]
KAV	1.563	-039	-003	015	036	075	018	018	[lag 50 0.309]
KKEN	1.540	-045	-006	012	030	075	018	018	[lag 50 0.328]

9D (continued)

TR (3,250) # 248 K = 0.652

	Estimate	min	loq	upq	max	range	midspr	rms	autocorrelation
		Residuals							
KMU	1.491	-115	-089	-008	012	127	081	068	lag 41 0.134
KAU	1.885	-033	-016	017	051	084	033	020	lag 25 0.158
KMV	1.529	-106	-080	-005	016	122	075	062	lag 40 0.151
KAV	1.876	-034	-018	016	050	084	034	020	lag 26 0.162
KKEN	1.877	-034	-018	016	050	085	034	020	lag 26 0.160

TR (253,353) # 101 K = 0.510

	Estimate	min	loq	upq	max	range	midspr	rms	autocorrelation
		Residuals							
KMU	0.982	-123	-072	041	062	185	112	062	[lag 25 0.324]
KAU	0.948	-128	-076	028	049	177	104	062	[lag 25 0.313]
KMV	1.003	-120	-070	047	069	189	117	062	[lag 25 0.328]
KAV	1.047	-113	-059	062	085	197	121	064	[lag 25 0.330]
KKEN	1.043	-113	-060	061	083	197	121	064	[lag 25 0.330]

AR (1,320) # 320 K = 0.572

	Estimate	min	loq	upq	max	range	midspr	rms	autocorrelation
		Residuals							
KMU	1.351	-064	-031	027	083	147	059	042	[lag 50 0.272]
KAU	1.322	-072	-039	023	077	148	062	043	[lag 50 0.285]
KMV	1.360	-061	-029	029	085	147	058	042	[lag 50 0.267]
KAV	1.434	-042	-013	044	102	144	057	044	[lag 50 0.238]
KKEN	1.338	-067	-034	025	080	148	060	042	[lag 50 0.278]

9D (continued)

TA (6,406) # 401 K = 0.622

	Estimate	Residuals min	loq	upq	max	range	midspr	rms	autocorrelation
KMU	1.640	-051	-018	018	058	109	036	028	[lag 50 0.563]
KAU	1.610	-057	-022	014	052	110	035	028	[lag 50 0.565]
KMV	1.688	-043	-013	024	066	109	038	029	[lag 50 0.562]
KAV	1.762	-031	-008	038	079	109	045	034	[lag 50 0.563]
KKEN	1.648	-050	-017	019	059	109	036	028	[lag 50 0.563]

TA (456,560) # 105 K = 0.522

	Estimate	Residuals min	loq	upq	max	range	midspr	rms	autocorrelation
KMU	0.999	-084	-039	002	016	100	040	036	lag 22 0.245
KAU	1.062	-068	-026	016	036	105	042	030	lag 25 0.219
KMV	1.001	-084	-038	002	016	100	041	035	lag 22 0.249
KAV	1.086	-063	-024	021	044	106	045	030	lag 25 0.246
KKEN	1.093	-061	-024	023	046	107	046	030	lag 26 0.215

HO (2,117) # 116 K = 0.603

	Estimate	Residuals min	loq	upq	max	range	midspr	rms	autocorrelation
KMU	1.330	-113	-059	002	016	129	060	050	lag 23 0.239
KAU	1.482	-079	-025	024	049	128	049	036	lag 24 0.234
KMV	1.335	-112	-057	002	018	129	060	050	lag 24 0.196
KAV	1.487	-078	-024	025	050	128	049	036	lag 24 0.234
KKEN	1.518	-072	-020	030	056	127	051	036	lag 24 0.235

9D (continued)

HO (176,413) # 238 K = 0.407

	Estimate	min	loq	upq	max	range	midspr	rms	autocorrelation
		Residuals							
KMU	0.730	-014	003	030	043	057	027	021	lag 32 0.126
KAU	0.691	-025	-012	019	032	057	031	016	lag 40 0.151
KMV	0.733	-014	004	031	043	057	027	022	lag 31 0.157
KAV	0.687	-027	-013	018	031	058	031	016	lag 41 0.158
KKEN	0.686	-027	-013	018	031	058	031	016	lag 41 0.161

TO (1,270) # 270 K = 0.431

	Estimate	min	loq	upq	max	range	midspr	rms	autocorrelation
		Residuals							
KMU	0.995	-017	036	096	135	153	059	080	lag 42 0.127
KAU	0.744	-077	-014	016	030	107	031	031	lag 45 0.140
KMV	1.029	-013	042	108	148	160	066	089	lag 42 0.150
KAV	0.760	-071	-008	020	038	109	028	030	lag 43 0.154
KKEN	0.757	-073	-009	019	036	109	028	031	lag 44 0.128

and autocorrelation characteristics, summarised by the first value falling on or below the upper bound of the 0.01 confidence interval for null autocorrelation, or failing that, by the value for the maximum lag calculated.

In interpreting these results, it is necessary to establish which estimates are best for each component, and then to consider the geomorphological implications of the best estimates. In doing this the figures of 9D are usefully supplemented by plots of residual series in spatial order.

The estimators were ordered from best to worst for each component, using the criteria (i) low rms (ii) low midspread (iii) low range (iv) low autocorrelation (v) symmetry of residual distributions about zero. Ties on any criterion were resolved by invoking criteria lower in this list. The criteria used here, and their ordering in this list, are arbitrary to some extent, but it will be seen that the results are sufficiently clearcut to remove serious doubts about such arbitrariness. The choice of rms residual as the most important criterion stems from the general approach adopted here of using least squares estimators. Naturally any monotonic transformation of root mean square residual (e.g. to mean square residual) would give the same ordering.

Estimators are thus ordered in 9E which gives estimators and rms for each estimator and component. 9F

9E Estimators ordered by performance with estimate and rms

Rank	1	2	3	4	5
BO (1,380)	KAU	KKEN	KAV	KMV	KMU
	1.619	1.615	1.612	1.588	1.579
	0.010	0.010	0.010	0.012	0.012
ST (1,429)	KKEN	KAU	KAV	KMU	KMV
	1.066	1.054	1.092	1.196	1.201
	0.023	0.024	0.024	0.039	0.040
PR (1,264)	KAU	KAV	KKEN	KMV	KMU
	1.324	1.329	1.335	1.259	1.255
	0.014	0.014	0.014	0.020	0.021
PA (1,408)	KAV	KAU	KKEN	KMV	KMU
	1.556	1.572	1.618	1.455	1.445
	0.029	0.029	0.029	0.036	0.037
FA (1,489)	KAU	KAV	KKEN	KMV	KMU
	1.604	1.593	1.664	1.458	1.446
	0.032	0.033	0.033	0.042	0.043
LA (1,459)	KAV	KKEN	KAU	KMV	KMU
	1.563	1.540	1.538	1.401	1.389
	0.018	0.018	0.018	0.032	0.034
TR (3,250)	KAU	KAV	KKEN	KMV	KMU
	1.885	1.876	1.877	1.529	1.491
	0.020	0.020	0.020	0.062	0.068
TR (253,353)	KAU	KMU	KMV	KKEN	KAV
	0.948	0.982	1.003	1.043	1.047
	0.062	0.062	0.062	0.064	0.064

9E (continued)

Rank	1	2	3	4	5
AR (1,320)	KMV	KMU	KKEN	KAU	KAV
	1.360	1.351	1.338	1.322	1.434
	0.042	0.042	0.042	0.043	0.044
TA (6,406)	KAU	KMU	KKEN	KMV	KAV
	1.610	1.640	1.648	1.688	1.762
	0.028	0.028	0.028	0.029	0.034
TA (456,560)	KAU	KAV	KKEN	KMV	KMU
	1.062	1.086	1.093	1.001	0.999
	0.030	0.030	0.030	0.035	0.036
HO (2,117)	KAV	KAU	KKEN	KMU	KMV
	1.487	1.482	1.518	1.330	1.335
	0.036	0.036	0.036	0.050	0.050
HO (176,413)	KAU	KAV	KKEN	KMU	KMV
	0.691	0.687	0.686	0.730	0.733
	0.016	0.016	0.016	0.021	0.022
TO (1,270)	KAV	KKEN	KAU	KMU	KMV
	0.760	0.757	0.744	0.995	1.029
	0.030	0.031	0.031	0.080	0.089

and 9G give further views of estimator performance. 9F shows the absolute performance

(rms for this estimator - rms for best estimator) $\geqslant$ 0

and 9G the relative performance

$$\frac{\text{rms for this estimator}}{\text{rms for best estimator}} \geqslant 1$$

Good estimators score low on each measure.

It is clear that it can make a great difference which estimator is employed. KMU and KMV are sometimes very poor performers: the assumption of heteroscedastic errors isusually unwarranted. KAV generally does well, while KKEN and KAU are the two best estimators. The good performance of KKEN is pleasing, since it does not require any recourse to ideas of stochastic errors. However, KAU appears to perform marginally better overall.

This analysis of estimator performance leans heavily on rms residual as a numerical summary. It would clearly be possible to base the analysis on other criteria. A second approach tried was the use of

this midspread - min midspread

and of

this midspread/min midspread

as analogues to the measures of performance given in 9F and 9G which are based on rms. The results, not reported here but readily obtainable from 9D, support those already given, but show a clear edge for KAU over KKEN as the best overall estimator.

9F <u>Absolute performance for each estimator on each component</u>

Quantity tabulated is
(rms residual for this estimator on this component
 - rms residual for best estimator on this component) x 1000

	KMU	KAU	KMV	KAV	KKEN
BO (1,380)	2	0	2	0	0
ST (1,429)	16	1	17	1	0
PR (1,264)	7	0	6	0	0
PA (1,408)	8	0	7	0	0
FA (1,489)	11	0	10	1	1
LA (1,459)	16	0	14	0	0
TR (3,250)	48	0	42	0	0
TR (253,353)	0	0	0	2	2
AR (1,320)	0	1	0	2	0
TA (6,406)	0	0	1	6	0
TA (456,560)	6	0	5	0	0
HO (2,117)	14	0	14	0	0
HO (176,413)	5	0	6	0	0
TO (1,270)	50	1	59	0	1
max	50	1	59	6	2
upq	16	0	14	1	0
med	8	0	7	0	0
loq	2	0	2	0	0
min	0	0	0	0	0

9G Relative performance for each estimator on each component

Quantity tabulated is

$$\frac{\text{rms residual for this estimator on this component}}{\text{rms residual for best estimator on this component}}$$

	KMU	KAU	KMV	KAV	KKEN
BO (1,380)	1.20	1.00	1.20	1.00	1.00
ST (1,429)	1.70	1.04	1.74	1.04	1.00
PR (1,264)	1.50	1.00	1.43	1.00	1.00
PA (1,408)	1.28	1.00	1.24	1.00	1.00
FA (1,489)	1.34	1.00	1.31	1.03	1.03
LA (1,459)	1.89	1.00	1.78	1.00	1.00
TR (3,250)	3.40	1.00	3.10	1.00	1.00
TR (253,353)	1.00	1.00	1.00	1.03	1.03
AR (1,320)	1.00	1.02	1.00	1.05	1.00
TA (6,406)	1.00	1.00	1.04	1.21	1.00
TA (456,560)	1.20	1.00	1.17	1.00	1.00
HO (2,117)	1.39	1.00	1.39	1.00	1.00
HO (176,413)	1.31	1.00	1.38	1.00	1.00
TO (1,270)	2.67	1.03	2.97	1.00	1.03
max	3.40	1.04	3.10	1.21	1.03
upq	1.70	1.00	1.74	1.03	1.00
med	1.33	1.00	1.35	1.00	1.00
loq	1.20	1.00	1.17	1.00	1.00
min	1.00	1.00	1.00	1.00	1.00

9H gives a further picture of variation among estimators. The quantity displayed is the difference between the parameter estimate for a given estimator and that for the best estimator for a given component, as given in 9E. KMU and KMV are again indicted by their poor performance and KAU once more emerges as better than its nearer competitors KKEN and KAV.

It is now appropriate to draw together the best estimates (as defined using the list of criteria above), rms residuals and Kennedy integrals (9I). At this stage some concreteness is also introduced:multiplying rms residual by component height gives a dimensioned measure of lack of fit. It is now helpful also to list data on slopes above and below fitted components, omitting trivially short sections (9J). In 9I and 9J components have been ordered by value of Kirkby parameter k. The fourteen components may be considered in three groups. The largest group contains upper convexities for TR, BO, TA, FA, LA, PA, HO, AR, PR, and ST. The remaining groups are approximately straight midslopes for TA and TR and concavities for TO and HO.

How are these fitted components to be interpreted? At a minimum, the family of power functions yields descriptive summaries of the hillslope profiles. This minimum approach resembles that of Hack and Goodlett (1960), although they employed graphical estimation, and paid no attention to either specification or checking. The value of the power function approach is simplicity, or more precisely, parsimony

9H Variability in parameter estimates for each component

Quantity tabulated is

(parameter estimate for this estimator - parameter estimate for best estimator) x 1000

	KMU	KAU	KMV	KAV	KKEN
BO (1,380)	-40	0	-31	-7	-4
ST (1,429)	130	-12	135	26	0
PR (1,264)	-69	0	-65	5	11
PA (1,408)	-111	16	-101	0	62
FA (1,489)	-158	0	-146	-11	60
LA (1,459)	-174	-25	-162	0	-23
TR (3,250)	-394	0	-356	-9	-8
TR (253,353)	34	0	55	99	95
AR (1,320)	-9	-38	0	74	-22
TA (6,406)	30	0	78	162	38
TA(456,560)	-63	0	-61	24	31
HO (2,117)	-157	-5	-152	0	31
HO(176,413)	39	0	42	-4	-5
TO (1,270)	235	-16	269	0	-3
max	235	16	269	162	95
upq	34	0	55	26	38
med	-52	0	-46	0	6
loq	-157	-12	-146	-4	-5
min	-394	-38	-356	-11	-23

9I Summary of best results for fitted components

	k	K	rms	rms (in metres)
TR (3,250)	1.885	0.652	0.020	0.5
BO (1,380)	1.619	0.618	0.010	0.6
TA (6,406)	1.610	0.622	0.028	2.2
FA (1,489)	1.604	0.625	0.032	3.3
LA (1,459)	1.563	0.606	0.018	2.0
PA (1,408)	1.556	0.618	0.029	2.1
HO (2,117)	1.487	0.603	0.036	1.0
AR (1,320)	1.360	0.572	0.042	3.1
PR (1,264)	1.324	0.572	0.014	0.7
ST (1,429)	1.066	0.516	0.023	1.7
TA (456,560)	1.062	0.522	0.030	1.7
TR (253,353)	0.948	0.510	0.062	2.8
TO (1,270)	0.760	0.431	0.030	4.1
HO (176,413)	0.691	0.407	0.016	1.5

9J Slopes above and below fitted components

Fitted component		Slope above x_b	z_d	$\bar{\theta}$	Fitted component x_b	z_d	$\bar{\theta}$		Slope below x_b	z_d	$\bar{\theta}$
TR (3,250)					375	24	3.7	(251,403)	216	77	19.7
BO (1,380)					573	56	5.5	(381,394)	20	5	13.1
TA (6,406)					602	78	7.3	(407,567)	220	96	23.6
FA (1,489)					732	103	8.0	(490,506)	22	12	29.0
LA (1,459)					687	109	9.0	(460,469)	15	4	13.8
PA (1,408)					614	73	6.8	(409,421)	16	10	32.3
HO (2,117)					173	27	9.0	(118,414)	421	129	17.0
AR (1,320)					479	75	8.9	(321,363)	58	26	23.7
PR (1,264)					398	47	6.8	(265,272)	12	3	12.6
ST (1,429)					647	75	6.6	(430,453)	35	8	12.5
TA (456,560)	(1,455)	674	73	6.2	147	56	20.8	(561,567)	9	5	26.2
TR (253,353)	(1,252)	382	25	3.7	144	45	17.5	(354,403)	68	31	24.8
TO (1,270)					383	136	19.6	(271,309)	56	17	16.7
HO (176,413)	(1,175)	250	60	13.5	345	96	15.5				

Note: Units of x_b and z_d are metres. Units of $\bar{\theta}$ are degrees and intelligibility. Each component is specified by two known constants (total height and total length) and one parameter estimate (readily interpreted as a measure of convexity or concavity). The theoretical relationship between the Kirkby parameter and the Kennedy integral, supported in practice by the generally good performance of KKEN, also aids interpretation.

One obvious pitfall here is the 'magic number syndrome' (Cox and Anderson, 1978) whereby a single-valued characterisation is sought which in some mysterious way captures all the information in a set of data. Such a tendency for the lure of simplicity to triumph over the many-sidedness of phenomena has been identified by discerning investigators in several fields (cf. Evans, 1972, 21-2 on general geomorphometry; MacArthur, 1972, 197-8 on diversity measures in ecology; Philip, 1974, 268 on soil physics; Medawar, 1977, 13 on demography, economics and psychology; Holton, 1978, 207-9 on science assessment). 'Magic number syndrome' is suggested as an ironic term for this pitfall: its applicability to cases in the social or environmental sciences need not be limited by the fact that 'magic numbers' are genuine entities in atomic physics.

This pitfall is easily avoided. Residuals and their summaries provide a readily intelligible picture of the inadequacy of a single-valued characterisation, since they show how far the one-parameter model yields an accurate reconstruction of the dimensionless profile. Hence we have a measure of the <u>cost</u> of a simple characterisation.

This minimum descriptive approach yields composite model-based summaries of field profiles (9K). This list is based on results presented earlier, together with interpretations of the plots of residuals in spatial order. Each power function is constrained by definition to pass exactly through the crest and base of each component, and so residuals tend to zero as endpoints are approached. However, their behaviour over the length of the component is a guide to the adequacy of the model which supplements the summary lack-of-fit measures. Several changes of residual sign (from positive to negative, or vice versa) within a component indicate small-scale residual variation which may happily be averaged out. On the other hand, simple structure, especially in the form of a changeover from residuals of one sign near the crest to residuals of the opposite sign near the base, throws some doubt on the acceptability of the model: in such cases, concavo-convex (or convexo-concave) deviations from the model suggest that two distinct components might have been combined. Such simple 'two regime' residual structures occur on 6 out of 14 fitted components, on ST (1,429), FA (1,489), TR (253 ,353), AR (1,320), TA (456, 560) and HO (2,117). They are noted in 9K. However, despite the fact that these 6 components tend to have higher lack-of-fit (rms) than the other eight components, the rather low values obtained seem to justify keeping the models as they stand, and thus regarding the residual variation as secondary.

9K Composite model-based summary of field profiles

BO Upper part (1,380) a long gentle convexity (z_d = 56m, x_b = 573m, $\bar{\theta}$ = 5.5°, k = 1.619) with gentle fluctuations (rms = 0.6m); lower part (381,394) a short steeper basal slope (z_d = 5m, x_b = 20m, $\bar{\theta}$ = 13.1°)

ST Upper part (1,429) a long gentle convexity (z_d = 75m, x_b = 647m, $\bar{\theta}$ = 6.6°, k = 1.066) with moderate fluctuations (rms = 1.7m, residuals show changeover from negative near crest to positive near base of component); lower part (430,453) a short steeper basal slope (z_d = 8m, x_b = 35m, $\bar{\theta}$ = 12.5°)

PR Upper part (1,264) a long gentle convexity (z_d = 47m, x_b = 398m, $\bar{\theta}$ = 6.8°, k = 1.324) with gentle fluctuations (rms = 0.7m); lower part (265,272) a short steeper basal slope (z_d = 3m, x_b = 12m, $\bar{\theta}$ = 12.6°)

PA Upper part (1,408) a long gentle convexity (z_d = 73m, x_b = 614m, $\bar{\theta}$ = 6.6°, k = 1.556) with moderate fluctuations (rms = 2.1m); lower part (409,421) a short steeper basal slope (z_d = 10m, x_b = 16m, $\bar{\theta}$ = 32.3°)

FA Upper part (1,489) a long gentle convexity (z_d = 103m, x_b = 732m, $\bar{\theta}$ = 8.0°, k = 1.604) with moderate fluctuations (rms = 3.3m, residuals show changeover from positive near crest to negative near base of component); lower part (490,506) a short steeper basal slope (z_d = 12m, x_b = 22m, $\bar{\theta}$ = 29.0°)

9K (continued)

LA Upper part (1,459) a long gentle convexity (z_d = 109m, x_b = 687m, $\bar{\theta}$ = 9.0°, k = 1.563) with moderate fluctuations (rms = 2.0m); lower part (460,469) a short steeper basal slope (z_d = 4m, x_b = 15m, $\bar{\theta}$ = 13.8°)

TR Upper part (3,250) a long gentle convexity (z_d = 24m, x_b = 375m, $\bar{\theta}$ = 3.7°, k = 1.885) with gentle fluctuations (rms = 0.5m); middle part (253,353) an approximately straight steep slope (z_d = 45m, x_b = 145m, $\bar{\theta}$ = 17.5°, k = 0.948) with moderate fluctuations (rms = 2.8m, residuals mostly positive near crest, negative towards base); lower part (354,403) a shorter steeper basal slope (z_d = 31m, x_b = 68m, $\bar{\theta}$ = 24.8°)

AR Upper part (1,320) a long gentle convexity (z_d = 75m, x_b = 479m, $\bar{\theta}$ = 8.9°, k = 1.360) with moderate fluctuations (rms = 3.1m, residuals mostly negative near crest, positive towards base); lower part (321,363) a shorter steeper basal slope (z_d = 26m, x_b = 58m, $\bar{\theta}$ = 23.7°)

TA Upper part (6,406) a long gentle convexity (z_d = 78m, x_b = 602m, $\bar{\theta}$ = 7.3°, k = 1.610) with moderate fluctuations (rms = 2.2m); a shorter steeper slope (407,455) (z_d = 36m, x_b = 64m, $\bar{\theta}$ = 29.2°); an approximately straight slope (456,560) (z_d = 56m, x_b = 147m, $\bar{\theta}$ = 20.8°, k = 1.062) with moderate fluctuations (rms = 1.7m, residuals show changeover from positive near crest to negative near base); lower part (561,567) short steeper basal slope (z_d = 5m, x_b = 9m, $\bar{\theta}$ = 26.2°)

9K (continued)

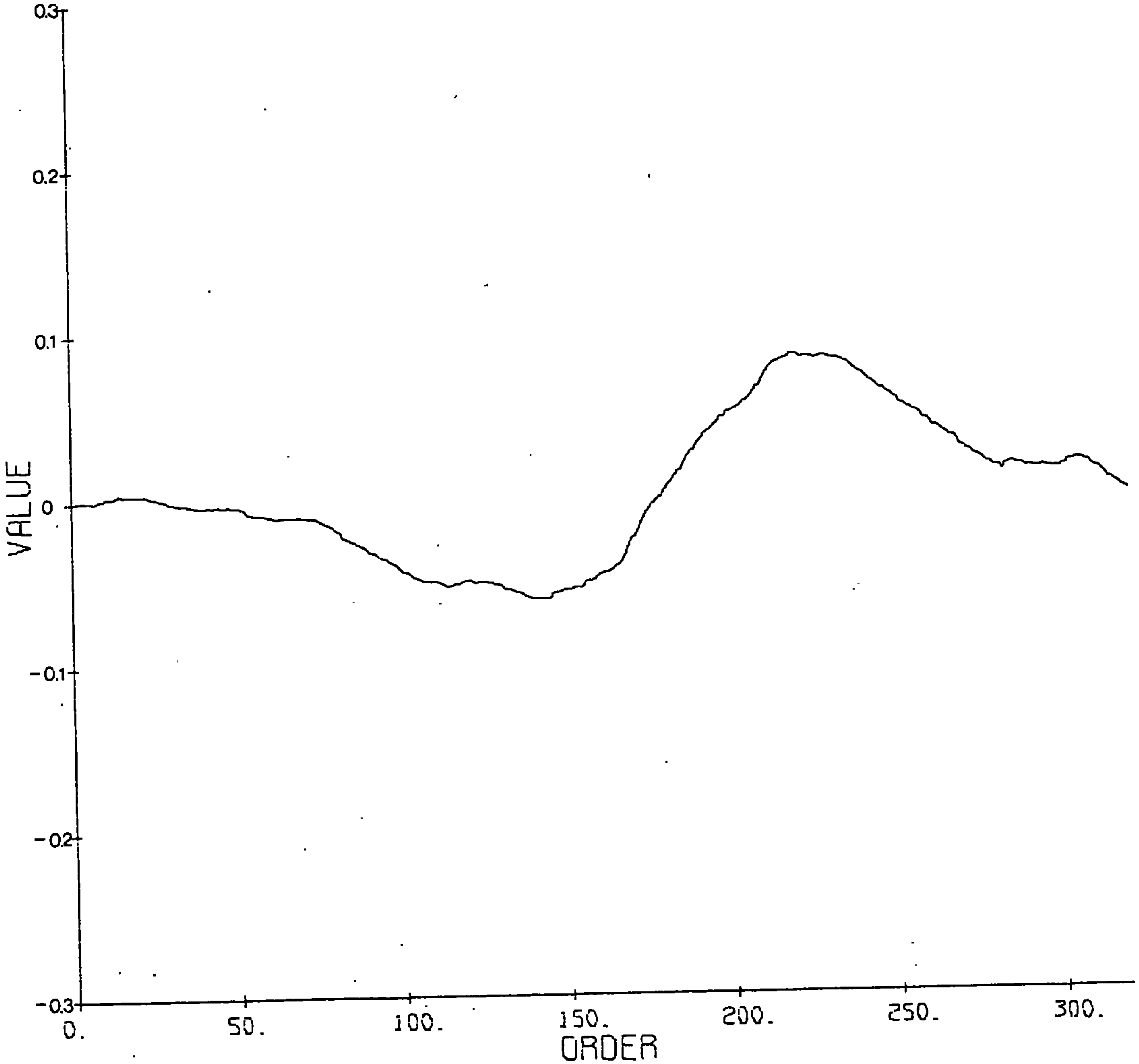
HO Upper part (2,117) a long gentle convexity (z_d = 27m, x_b = 173m, $\bar{\theta}$ = 9.0°, k = 1.487) with gentle fluctuations (rms = 1.0m, residuals positive near crest, mostly negative near base); middle part (118,175) a steeper slope (x_b = 75m, z_d = 33m, $\bar{\theta}$ = 23.6°); lower part a steep concavity (x_b = 345m, z_d = 96m, $\bar{\theta}$ = 19.6°, k = 0.691) with moderate fluctuations (rms = 1.5m)

TO Upper part (1,270) a long steep concavity (z_d = 136m, x_b = 383m, $\bar{\theta}$ = 19.6°, k = 0.760) with large fluctuations (rms = 4.1m); lower part a shorter gentler basal slope (z_d = 17m, x_b = 56m, $\bar{\theta}$ = 16.7°)

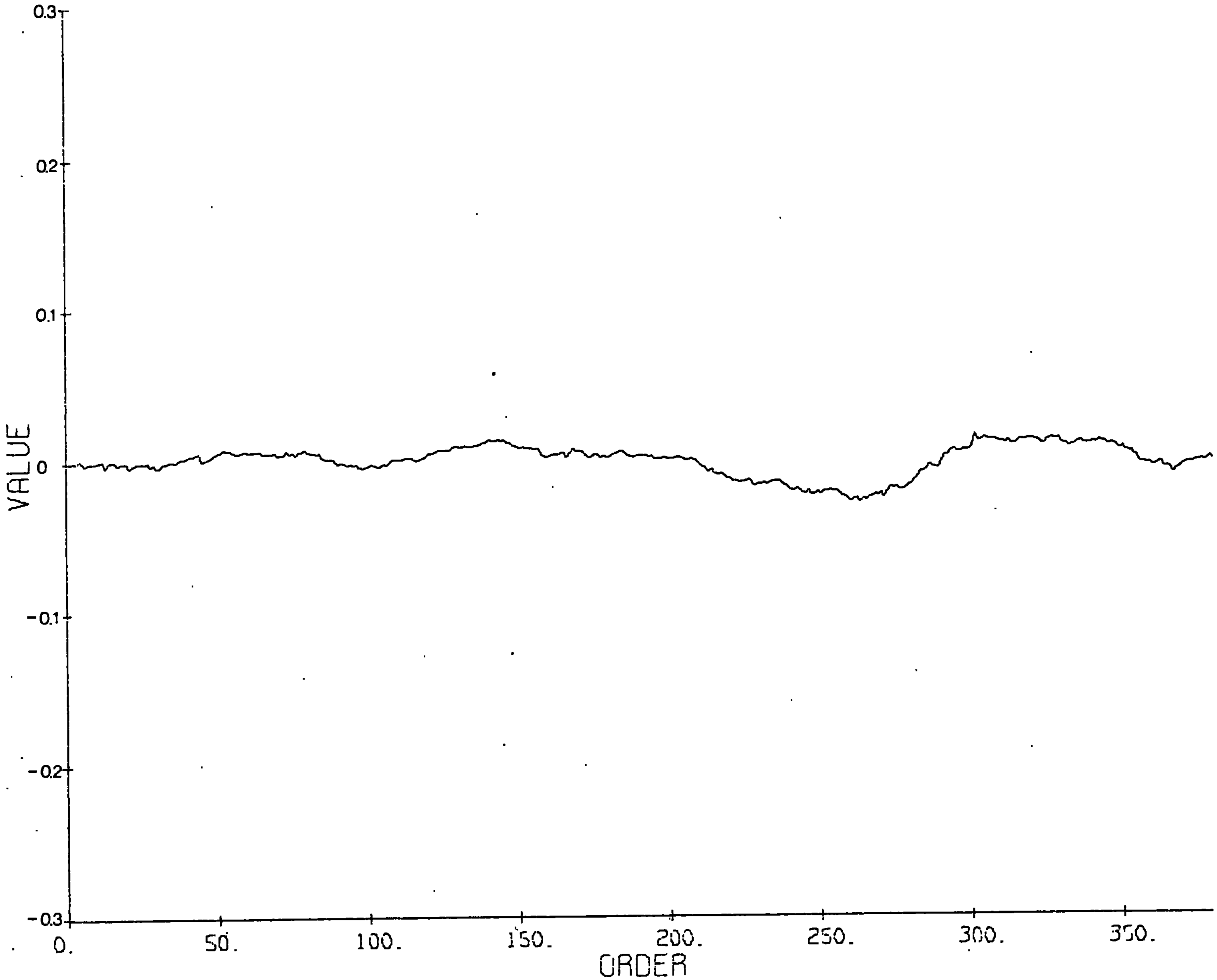
9L and 9M are examples of residual plots for AR (1,320) and KMV, and for BO (1,380) and KAU.

Difficulties arise when we go beyond a minimum descriptive approach, and consider interpreting the power functions as solutions to a dynamic model in which sediment flux is a power function of distance and gradient (cf. Ch. 3.3.15, 3.3.21 above). 9N shows the values of the Kirkby parameter which correspond to process exponents given by Kirkby (1971, 21). All the convexities are less convex than the k = 2 predicted for the single process of soil creep. However, this particular prediction is not very well founded, and many process studies of soil creep have found gradient to be a weak control of creep rates (e.g. Anderson, 1977). Moreover, values less than 2 seem compatible with the combination of soil creep <u>and other processes</u> which almost certainly moulded these convexities. This suggestion is merely intuitive, and no analytical results have been produced for a polygenetic generalisation of the model in question. The ten <u>upper convexities</u> vary considerably in Kirkby parameter from 1.885 to 1.066 (9I) which cannot be explained easily. In fact this indication of varying convexity is not obviously related either to geology (no clear difference between FA, LA and TA with crests on gls, or Grey Limestone Series, and the other seven in this group, with crests on del, or Deltaic Beds), or to profile dimensions (no clear relationship with z_d, x_b or $\overline{\theta}$).

9L Residual series for AR (1, 320) and KMV



9M Residual
series for BO
(1, 380) and
KAU



9N Theoretical predictions for Kirkby parameter

Process	Distance exponent	Gradient exponent	k
soil creep	0	1	2
rainsplash	0	1-2	2-15
soil wash	1.3-1.7	1.3-2	0.46-0.85
rivers	2-3	3	0.33-0.67

Source: After Kirkby (1971,21)

The other four components are the approximately straight mid slopes of TA and TR and the concavities of TO and HO. Here the dynamic model approach breaks down, for these are steeper slopes ($\bar{\theta}$ = 20.8, 17.5, 19.6, 15.5 degrees) and one or more kinds of threshold-dependent failure processes has probably been operative in each case. Without any investigation of regolith properties it would be rash indeed to hazard a process interpretation of three components. Variations in lithology may also be more important in these cases.

Finally, some remarks are in order on the field profiles interpreted as combinations of components (9K). All but two include basal slopes shorter and steeper than the components above; these components were not fitted to models because they were so short. 9O gives an overview of basal slopes. It shows how they are mostly secondary features on the scale of the hillslope profile, and how they exhibit considerable variation in form. Whether such basal slopes may be attributed to rejuvenation remains an open question.

9.7 Summary and discussion

(i) Model predictions should be fitted to actual hillslope profiles statistically, and goodness-of-fit assessed quantitatively. Model fitting may be regarded as a three - stage process, involving specification, estimation and checking (9.1).

90 Basal slopes

	x_b	z_d	$\bar{\theta}$	
BO	20	5	13.1	
ST	35	8	12.5	
PR	12	3	12.6	
PA	16	10	32.3	
FA	22	12	29.0	
LA	15	4	13.8	steeper, shorter than above
TR	68	31	24.8	
AR	58	26	23.7	
TA	9	5	26.2	
HO	345	96	19.6	gentler, longer than above
TO	56	17	16.7	gentler, shorter than above
max	345	96	32.3	
upq	58	26	26.2	
med	29	11	21.7	
loq	15	5	13.1	
min	9	3	12.5	

(ii) In the case of a model function derived by Kirkby (1971), a first approximation in specification and estimation leads to a sum of squared residuals discrepancy which is nonlinear in the parameter to be estimated. A linearising transformation leads to a closed-form estimator, but may induce heteroscedasticity, which can be tackled by weighting residuals. Since both variables in the model are error-prone, estimators should be developed for the polar situations in which each variable is regarded as a response. Although autocorrelated errors may well be present, no estimation procedure is known for this model which takes account of autocorrelation in a satisfactory manner. The Kirkby parameter can also be estimated deterministically as a function of the Kennedy integral (9.2, 9.3, 9.4).

(iii) Checking the model should be based on a variety of estimators since it is never certain in practice which assumptions are appropriate. Lack of fit measures, the spatial pattern of residuals, and their autocorrelation properties all deserve attention. An approach to model checking based on significance testing seems both unnecessary and problematic (9.5).

(iv) Five estimators were employed on fourteen components based on 100 or more observations. Residuals were calculated in each case, and extremes, quartiles, range, root mean square, midspread and autocorrelation function of residuals were derived, supplemented by plots of residual series in spatial order. The best estimates were identified for each component mainly on the basis of root mean square residual, although other criteria were considered as well. At the same time performance of the individual estimators was reviewed, indicating that KMU and KMV were the poorest, while KAU is better than its nearer competitors KAV and KKEN.

The Kirkby power function model can be used in a minimum descriptive approach, attractive because of its parsimony and intelligibility. The 'magic number syndrome' is readily avoided, since the cost of a single-valued characterisation is considered. A more ambitious approach in which power functions are regarded as dynamic model solutions runs into greater difficulties, largely because of polygenetic development and the presumed operation of failure processes on steeper slopes.

The viewpoint in this chapter is that an appropriate methodology for model fitting must be developed before the important empirical issue of identifying adequate models can be tackled in a satisfactory manner. The one-parameter model used here in examplifying an approach to model fitting is attractive as a candidate descriptive and explanatory model, yet is sufficiently simple to serve as a starting point in developing such a methodology. The results in this chapter support this point of view: even in such a relatively simple case technical issues are not trivial in any sense. It does matter which estimator is used; statistical theory does provide guidance over procedures; and the attitude that residuals should be investigated pays dividends in practice. Some important technical questions remain for consideration. The relative value of robust-resistant fitting procedures has not been evaluated. More complex models would need to be fitted by optimisation algorithms. Simulation models would require sensitivity analyses in addition to some degree of parameter estimation.

The rough adequacy of the power function model for the components fitted here is almost guaranteed by the fact that component bounds have been identified by profile analysis. This procedure may appear to possess an element of circularity, although it seems to make little sense to fit a model for a component to anything but a component. There might be some value in fitting power functions to entire profiles to consider the adequacy of such crude models, but this has not been attempted here. A final point is that since the profile analysis methods used here are regarded as only interim, new analyses would require new model fits.

9.8 Notation

d	in ordinary derivative or in integral
e	residual
E	expectation operator
i	index
j	index
k	Kirkby parameter
K	Kennedy integral
n	number of observations
s	random variable
t	random variable
u	relative distance
u'	ln u
v	relative fall
v'	ln v
w	weight
x	horizontal coordinate
x_b	slope length
y	1 - v
z	vertical coordinate
z_d	slope height
ϵ	stochastic error
ϵ'	" "
η	" "
$\bar{\theta}$	average angle
ρ	autoregressive parameter
Σ	summation operator
∂	in partial derivative
$\int$	integral
#	number

Mnemonics

loq	lower quartile
max	maximum
med	median
min	minimum
OLS	ordinary least squares
rms	root mean square
upq	upper quartile
var	variance

Chapter 10

CONCLUSIONS

"When you say 'hill' ", the Queen interrupted, "I could show you hills, in comparison with which you'd call that a valley."

Lewis Carroll, Through the Looking-Glass and what Alice found there, Ch. II.

10.1 Retrospect

'Hillslope profile morphometry' is taken in this thesis in a wide sense to include mathematical models and methods of sampling, measurement and data analysis used in the study of hillslope profiles. The aims of the work reported here were set out in Ch. 1.2, but may be repeated here for convenience.

(i) To provide critical and comprehensive reviews of work in modelling and data analysis in hillslope morphometry, concentrating particularly on continuous models (Chs. 2, 3) and profile analysis (Ch. 8).

(ii) To place procedures on a firm mathematical basis and to evaluate their practical utility, concentrating particularly on autocorrelation analysis (Ch. 7), profile analysis (Ch. 8) and model curve fitting (Ch. 9).

(iii) To consider the empirical role of hillslope morphometry in geomorphology, which entails consideration of the hypotheses put forward in the literature for the field area (Ch. 4) and analysis of the implications of morphometric results for these hypotheses (Chs. 6, 8, 9).

(iv) To set the field of hillslope morphometry within a methodological and theoretical context, particularly by relating ideas on continuous models to geomorphological theory, the philosophy of science, and the methodology of the natural and environmental sciences (Ch. 2).

The emphases of this thesis are therefore on forms, rather than processes or development, and on methodological, theoretical and technical problems rather than empirical problems.

Major achievements and conclusions may now be summarised.

Models (simplified formal representations) of hillslope profile systems are brought together in a critical and comprehensive review, presented in a unified notation. Fundamental philosophical issues and major geomorphological problems arising in modelling are considered and modelling approaches expounded through a classification based on five dichotomies (static/dynamic, deterministic/stochastic, phenomenological/ representational, analytical/simulation, discrete/continuous). Many functional forms have been suggested for static continuous models, particularly power series polynomial, power function and exponential function. Dynamic continuous models have been based on the assumption that hillslopes may be treated as self-modifying geometric systems: independent variations in climate, hydrology and soil properties have received little attention. Processes of failure and solution have been relatively neglected. The important ideas of constant form and stability properties, used by Jeffreys in 1918, have been rediscovered since 1972. Extensions from modelling profiles to modelling surfaces face difficulties in tackling plan curvature effects, endpoint behaviour and stream network development. Ideas of spatial stochastic processes have failed to yield important new insights. Simulation, preferably accompanied by sensitivity analyses, is valuable for handling thresholds and frequency distributions. As knowledge about processes improves, the onus is increasingly on modellers to produce representational models (Chs. 2 and 3).

Profile data to serve as examples were collected in a 100 km^2 square centred on Bilsdale in the North York Moors using a pantometer. Geomorphological interpretations previously enunciated for this field area include theses of profound lithological influence, polycyclic denudation history, proglacial lake overflow channels and profound cryonival influence. Identifying scale variations emerges as a fundamental task for a morphometric approach (Chs. 4 and 5).

Profile dimensions and profile shapes allow a fourfold grouping of measured profiles which is closely related to variations of bedrock geology. Angle and curvature frequency distributions are summarised using quantile-based measures which are considered appropriate for data containing outliers. Median and midspread of angle can be related to the fourfold grouping. A novel method of spatial averaging and differencing shows clearly the scale variation of extreme values (Ch. 6).

Autocorrelation analysis of hillslope series is a relatively new technique: its usefulness has not so far been examined systematically. Angle series are strongly autocorrelated, whereas curvature series autocorrelations dampen much more readily. However, angle autocorrelations reflect not surface roughness but overall profile shape which can be measured more directly in other ways. The Pearson analogue estimator is recommended and it is emphasised that nonstationarity is an important problem in practice (Ch. 7).

Profile analysis (the division of a profile into discrete components) is a long established approach in hillslope geomorphology, but it has not been widely recognised as a numerical classification problem and it has usually been based on an uncritically held atomistic view of the landscape. The problem is formulated more carefully and embedded more deeply in data analysis and geomorphology. Methods previously proposed by Ahnert (1970c), Ongley (1970), Pitty (1970) and Young (1971) are shown to be unacceptable. A method based on additive error partition and nonlinear smoothing is presented as an interim alternative. Results are interpreted with reference to bedrock geology. Satisfactory methods of profile analysis must necessarily be adaptive (Ch. 8).

Fitting continuous models to hillslope profiles (or components) may be regarded as a sequence of specification, estimation and checking. A one-parameter power function due to Kirkby (1971) is used as an example. Problems of linearising transformations, errors in variables and autocorrelated errors are considered carefully when developing stochastic estimators. A deterministic estimator may be employed which is a transform of an integral proposed by Kennedy (1967). Model checking should be based not on significance testing but on lack of fit measures, spatial pattern of residuals and autocorrelation properties of residuals, all compared for different estimators. Results with field data allow identification not only of best estimates of the Kirkby parameter but also of good estimators. The power function model can be used in a minimum descriptive approach but attempts at process interpretation raise greater difficulties. Important technical problems remain, however, before the empirical issue of identifying adequate models can be tackled in a satisfactory manner (Ch. 9).

10. 2 Prospect

It has often been pointed out that research rarely produces final solutions to problems: new problems and new formulations of problems arise from conceptual and empirical inquiries. The work reported here includes contributions to various geomorphological problems, methodological, theoretical, technical and empirical. It is appropriate to close by indicating some of the major problems which remain outstanding and which will be attacked in future research.

The development of landscapes is now recognised to be a much more complex affair than was implied by the cyclical schemes which dominated English-language geomorphology until the 1960s. The key ideas which are currently being found useful include those of equilibrium states, feedback loops, threshold conditions, system stability and response, and intermittence of events. There is, however, great scope for further work on these concepts. Ideas of equilibrium, for example, are at present the subject of considerable confusion, disagreement and inexactitude, and it is important that rigorous views are more widely adopted. Notions of feedback and threshold - to give a second example - have earned many a passing aside but relatively little detailed analysis.

Scale variations of morphometric properties continue to deserve attention if only because geomorphology is concerned with forms at a range of spatial scales. (This can hardly be a banality when many contemporary interpretations fail to specify relevant scales: witness various hypotheses of lithological and cryonival influence). In the case of hillslope profile studies, it remains unclear how far morphometric results are artefacts of the measured length used in profile survey. It would be naive indeed to seek the best, still less the correct, measured length. A better strategy is to use a relatively short measured length and different levels of generalisation (e.g. through averaging and differencing, smoothing or profile analysis), and thus to identify properties which are sensitive or insensitive to scale over the range of interest.

The division of a hillslope profile into discrete components remains problematic: no method proposed is really satisfactory. There can be little point in empirical applications of published methods until better alternatives have been developed. It does seem that any method will have to be adaptive to be acceptable, whereas those published are tied up with restrictive decency assumptions about the ideal character of profile data. Additive error partition and nonlinear smoothing are attractive families of methods which merit further examination.

Future work in hillslope profile morphometry should draw upon resistant methods of data analysis. Slavish adherence to moment-based measures and least-squares estimation would prove very limiting. Fitting complicated models will require recourse to search algorithms for discrepancy minimisation, while simulation modelling will need to be supplemented by sensitivity analyses. In these instances, as in general, good work in hillslope profile morphometry will flow from applications of the best techniques available, and not from an introverted muddling through with ad hoc procedures.

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APPENDICES

Appendix I

Hillslope profile data

This appendix gives data for each measured profile. Exhibits IA to IK show measured angles in crest to base order. Units are degrees. The following supplementary data are given below for each profile:

(i) Notes on vegetation, natural and artificial features. Numbers given index measured angles.

(ii) Notes on the basal stream: estimated 'bankfull width' (average of about 5 measurements) and remarks on stream detritus.

Bonfield (BO)

(i)	1 - 8	Heather
	9 - 21	Eroding peat, some heather
	22 - 387	Heather
	388 - 394	Mixed vegetation, stream bank

(ii) bfw 1.0m

some boulders about 500mm, many cobbles 50 - 100mm.

Stonymoor (ST)

(i)	1 - 42	Heather
	43 - 48	Track
	49 - 72	Heather
	73 - 79	Track
	80 - 202	Heather
	203 - 281	Recovering burnt heather
	282 - 299	Heather

0.5	1.0	0.0	4.5	-2.5	-3.0	2.5	1.0	0.5	3.0	0.5	5.0	-10.0	10.0	1.0	1.0
-3.5	4.5	0.0	2.5	-5.5	5.5	3.5	2.5	1.5	2.0	4.0	-5.0	6.0	-5.5	3.0	7.5
3.0	3.5	2.0	0.5	5.0	6.0	1.0	5.5	3.0	3.5	4.0	4.0	-10.0	4.0	5.0	4.5
7.5	4.0	1.5	7.0	1.0	3.0	0.5	0.0	2.5	5.0	5.0	2.0	2.5	2.0	3.0	6.0
-1.0	3.0	2.5	3.5	4.5	0.0	3.0	2.5	9.5	0.5	4.5	0.0	6.5	4.5	5.5	0.0
1.0	5.0	1.5	7.0	-6.0	3.0	2.5	4.5	1.0	0.5	4.0	3.0	2.0	5.0	2.0	6.0
-1.0	2.5	5.0	6.5	7.0	2.0	2.0	2.5	8.0	3.5	7.0	9.0	3.0	4.5	4.5	6.5
5.0	3.5	2.5	3.0	6.5	4.5	6.5	9.0	5.5	5.0	7.0	4.0	5.5	5.0	4.5	8.5
6.0	4.0	3.0	7.5	2.0	4.5	7.0	4.5	7.0	7.0	5.5	8.5	3.5	5.0	6.5	2.0
7.0	2.0	4.0	2.0	2.5	2.5	7.0	4.5	5.0	5.0	3.0	7.0	2.0	-2.0	3.5	7.5
6.5	5.0	6.0	6.0	5.5	1.0	11.0	11.0	2.5	4.5	5.0	1.5	2.0	6.0	8.0	6.5
1.5	8.5	3.5	4.0	10.0	7.5	8.5	6.0	4.5	0.0	5.0	6.0	6.5	6.5	6.5	4.5
4.5	7.5	1.5	7.0	5.0	9.0	4.0	5.5	7.0	8.0	5.0	3.5	5.5	5.5	8.5	5.0
2.5	2.0	3.5	2.5	3.0	9.5	1.0	5.0	7.0	5.0	2.0	6.0	3.0	6.5	5.0	7.0
8.0	7.5	5.0	0.0	7.5	7.0	9.0	5.5	7.5	9.5	6.5	7.5	3.0	3.0	4.5	2.5
5.0	10.0	5.0	3.5	5.0	12.5	1.5	6.0	10.5	4.0	11.0	7.0	8.5	5.0	7.0	4.0
4.5	5.0	5.0	2.5	10.0	10.5	0.5	12.0	5.0	11.0	9.0	8.5	6.5	10.0	2.0	15.5
13.0	5.0	9.5	3.0	8.5	9.5	11.0	9.0	15.0	11.0	13.0	7.5	11.5	14.5	3.0	5.0
9.0	20.0	13.0	8.0	14.5	9.0	3.0	9.0	9.0	7.5	7.5	12.0	27.5	-3.0	7.0	12.0
5.5	8.0	8.0	6.0	5.5	8.0	7.0	11.0	4.5	8.5	9.0	12.0	6.5	10.0	8.0	8.5
5.0	7.5	5.0	5.0	17.0	10.0	6.0	8.5	11.0	0.0	7.0	5.5	9.0	9.5	12.0	9.5
5.0	6.5	6.0	9.5	6.0	13.0	7.0	9.0	4.0	10.0	6.5	5.0	5.0	13.0	3.0	6.5
9.0	2.5	6.0	2.0	7.5	7.0	11.5	3.5	9.0	11.5	10.0	2.5	7.5	8.5	2.5	15.0
9.5	13.0	11.5	9.0	8.5	10.0	9.0	11.0	7.5	11.5	13.0	4.0	9.5	11.5	8.5	6.5
8.5	13.0	20.0	11.0	31.0	41.0	1.5	18.0	-1.0	5.5						

IA BO measured angles

3.5	1.0	2.5	5.0	3.0	2.5	5.5	4.5	4.0	3.0	1.0	4.5	6.5	0.5	-6.0	8.0
5.5	4.0	2.5	2.5	6.0	4.5	1.5	3.5	2.5	3.5	2.0	6.0	0.5	3.0	4.0	4.5
5.5	6.0	4.5	4.5	3.5	3.5	3.5	8.5	5.5	10.5	8.0	4.0	-2.0	-2.5	-10.0	18.5
6.5	5.0	6.0	8.0	5.5	10.5	4.0	7.0	5.0	8.0	1.0	10.0	5.0	8.5	5.5	6.5
4.5	6.5	3.0	5.0	10.5	7.5	5.0	4.5	11.0	3.5	8.5	7.0	5.5	5.0	-5.0	5.0
9.0	5.5	9.5	7.5	6.5	5.0	5.0	8.0	5.5	6.5	6.0	7.0	5.5	5.0	8.0	7.5
7.0	7.5	8.5	7.5	5.0	5.5	6.0	4.0	8.0	7.5	4.0	8.0	7.5	4.5	7.5	6.5
4.0	7.0	11.5	-1.0	6.5	7.5	6.5	7.0	7.5	6.5	6.5	7.0	5.5	12.0	5.5	8.5
10.0	9.0	9.0	9.5	9.0	8.5	9.5	9.0	8.0	11.0	11.0	10.5	8.5	13.5	7.5	12.0
11.0	9.5	10.0	9.0	11.5	11.0	15.0	14.0	10.0	12.0	12.5	7.5	8.0	7.0	10.0	15.0
14.5	7.5	9.0	7.5	12.0	8.5	7.5	6.0	9.0	7.5	9.5	9.5	6.0	6.0	14.0	0.5
6.5	7.0	7.0	9.0	10.5	9.5	11.5	7.5	13.5	7.5	8.5	7.0	10.5	7.0	8.0	5.5
4.0	4.0	5.5	5.0	6.5	3.0	2.0	7.0	6.0	3.0	7.5	5.0	5.5	6.5	6.5	6.0
7.0	7.5	6.5	5.5	6.0	5.5	4.5	4.0	5.0	5.0	4.0	7.0	6.5	4.5	7.0	3.0
12.5	2.0	15.0	4.0	8.0	1.0	5.5	5.0	11.5	9.5	2.5	4.0	4.0	10.0	6.0	7.5
5.0	7.5	7.0	8.5	8.0	10.0	6.5	8.5	10.5	10.5	8.0	8.0	8.5	11.0	8.0	7.0
8.0	6.0	6.5	7.5	7.0	7.0	12.0	1.5	8.5	9.0	7.0	8.5	2.0	6.5	2.0	5.5
6.0	11.0	8.5	7.5	6.0	15.0	8.5	3.5	6.0	4.5	11.0	-1.0	1.5	7.0	11.5	11.0
11.0	6.5	7.0	3.5	7.0	14.0	7.5	0.0	8.5	7.0	6.0	22.5	-14.5	7.0	5.5	5.0
2.5	11.5	14.0	12.0	1.0	5.5	3.0	7.0	6.0	6.0	7.0	6.0	4.5	9.5	8.0	1.5
15.5	4.5	0.0	7.0	8.0	8.5	4.0	8.0	7.0	10.0	5.0	4.5	8.0	8.0	5.0	7.5
4.0	4.0	7.0	6.0	8.5	10.5	5.5	13.0	7.0	3.0	8.5	8.0	6.5	3.0	7.5	5.0
6.5	3.0	9.0	10.5	8.0	15.0	7.5	6.0	0.0	5.0	10.0	7.0	6.0	8.5	6.0	6.0
4.0	7.0	4.0	5.0	5.5	7.5	8.5	8.0	8.5	7.0	7.5	7.0	7.0	7.5	6.0	5.5
9.0	6.5	7.0	8.5	6.5	6.0	6.0	5.0	6.0	6.0	5.5	7.0	5.5	4.0	5.5	5.5
3.5	3.5	4.5	7.5	7.5	6.0	5.0	5.0	7.0	5.5	7.5	7.0	5.5	9.0	6.0	8.5
5.5	9.5	6.0	1.5	7.0	7.5	5.5	8.0	8.0	3.5	1.0	7.0	4.5	8.5	3.0	5.0
6.0	14.5	14.0	20.0	22.0	19.5	17.5	13.5	14.0	17.0	9.5	4.5	18.0	12.0	17.5	13.0
23.5	3.0	13.5	2.0	9.5											

IB ST measured angles

300 - 301 Gully

302 - 328 Heather

329 - 339 Recovering burnt heather

340 - 431 Heather

432 - 453 Mixed vegetation

(ii) bfw 1.4m

cobbles, small boulders up to 900mm.

Proddale (PR)

(i) 1 - 122 Mixed low heath

123 - 130 Eroding peat

131 - 163 Mixed low heath

164 - 167 Eroding peat

168 - 172 Mixed low heath

173 - 176 Eroding peat

177 - 184 Mixed low heath

185 - 193 Eroding peat

194 - 241 Vegetation, some boulders

242 - 272 Juncus, Sphagnum, heath

(ii) bfw 0.8m

a few stones about 200mm.

Parci Gill (PA)

(i) 1 - 4 Heath

5 - 7 Ditch

8 - 27 Heath

28 - 29 Old gully

30 - 288 Heath

289 - 296 Track and track banks

297 - 374 Heath, many boulders

375 - 421 Bracken, banks

5.0	-1.5	2.0	3.5	3.5	3.5	5.5	4.0	5.5	3.0	4.0	5.0	3.0	4.0	5.5	2.5
3.0	3.5	6.5	4.0	1.5	4.0	4.5	4.5	6.5	3.5	6.5	6.0	6.0	4.0	5.5	2.5
4.0	3.5	3.0	4.0	5.0	4.0	3.5	4.0	5.0	4.5	5.0	4.5	6.0	5.0	5.0	4.5
4.5	2.5	5.5	3.5	5.0	6.0	5.5	4.5	7.0	5.5	3.5	2.5	7.0	3.5	5.0	5.0
6.0	5.0	6.0	5.0	4.5	6.5	3.5	4.0	4.5	5.5	1.5	3.0	6.0	4.0	7.0	4.5
2.5	4.5	4.5	4.5	5.0	3.5	4.5	5.0	3.5	7.5	5.5	7.5	5.5	6.0	7.5	4.5
4.5	7.0	5.0	8.5	7.0	8.0	8.0	6.5	7.5	5.5	9.0	5.0	7.5	8.0	6.0	6.5
11.0	6.5	9.0	11.0	3.5	10.5	7.5	9.0	11.0	11.5	12.5	10.0	5.0	7.0	10.0	8.0
3.0	6.5	10.0	10.5	7.5	10.0	8.5	6.5	9.0	10.5	11.0	8.0	8.5	3.5	10.0	8.5
10.0	6.0	10.0	8.0	8.0	11.5	10.0	11.5	10.5	9.5	8.5	9.0	10.0	8.5	9.5	10.0
12.0	11.5	11.0	3.0	6.0	5.5	5.0	4.0	3.5	4.5	4.0	7.0	7.0	6.5	14.5	10.0
9.5	10.0	3.0	-1.0	3.0	2.5	3.0	5.0	4.5	5.0	5.0	5.5	5.0	6.0	5.5	6.5
5.5	3.5	6.5	5.5	7.5	7.0	5.5	9.0	6.5	8.0	10.5	7.0	7.0	8.0	10.0	6.5
7.5	5.0	6.0	6.5	6.0	8.5	12.0	8.0	12.5	10.5	11.5	10.5	7.0	9.5	6.0	7.5
7.0	7.5	11.5	6.5	7.5	12.0	11.0	11.0	7.5	5.0	7.0	4.5	7.0	8.5	7.0	9.0
13.5	7.0	1.0	10.0	6.0	11.0	6.5	8.5	16.0	14.0	7.5	7.5	8.5	20.0	10.0	6.0
10.5	13.0	14.5	8.5	18.5	5.5	15.5	9.0	13.5	13.5	32.5	7.5	5.0	13.5	3.0	13.0

IC PR measured angles

0.5	4.0	2.5	-1.0	14.5	-2.0	-10.0	-1.5	-0.5	5.5	-4.5	4.5	-1.0	3.0	4.5	4.5
3.5	4.5	4.0	1.5	3.0	2.5	2.5	4.5	4.0	5.0	2.0	14.0	-17.5	1.0	0.0	4.5
1.0	4.5	1.0	4.5	4.0	2.5	5.5	4.5	3.5	4.0	5.5	5.5	3.5	2.5	8.0	6.5
5.0	4.5	6.5	9.0	6.5	5.0	4.5	5.5	5.0	3.5	7.0	2.0	4.0	7.0	5.0	4.5
4.5	7.0	4.0	3.0	-0.5	6.0	3.0	-6.0	3.0	2.0	4.0	-3.0	-7.5	3.0	3.0	0.0
2.5	3.5	3.5	2.0	4.5	4.0	4.5	5.0	4.0	3.5	4.5	5.0	7.0	6.0	8.0	5.5
4.0	8.5	5.5	6.5	5.5	8.0	4.5	-2.0	8.5	4.5	2.0	8.0	3.0	1.5	2.0	6.0
4.5	4.5	3.5	3.5	2.0	5.5	2.5	5.5	4.0	0.0	12.0	5.0	6.0	4.5	1.5	5.0
6.5	4.5	5.5	4.0	6.0	6.0	4.5	3.0	3.0	2.0	8.5	6.5	7.0	4.0	7.0	7.0
8.0	5.0	6.5	5.5	6.0	6.0	6.0	3.5	7.0	8.0	7.0	6.5	10.5	5.0	8.5	9.0
9.5	8.0	7.5	8.0	6.0	5.5	5.0	10.5	7.5	6.0	5.5	8.0	3.5	7.0	9.0	6.5
5.5	7.5	5.5	8.0	8.0	5.5	6.5	11.0	10.5	6.5	10.5	7.0	9.0	10.0	7.5	4.0
7.5	8.0	6.0	4.0	5.5	7.5	8.5	5.5	3.0	3.5	4.5	4.0	7.5	3.5	7.5	6.0
8.0	5.0	7.0	9.5	8.0	7.0	6.5	9.5	4.0	6.0	7.5	8.0	6.0	6.5	7.5	10.5
9.5	11.0	14.0	12.5	15.5	15.0	14.0	12.5	11.0	7.5	10.0	8.0	9.0	6.5	3.5	5.0
3.5	10.0	8.0	7.5	4.5	5.0	5.0	6.0	6.0	8.5	6.5	6.5	9.0	5.0	1.5	7.5
7.5	9.5	4.0	4.5	4.0	4.0	8.0	9.5	13.0	7.0	4.5	11.0	7.5	4.5	3.5	10.0
7.5	9.0	3.5	6.0	5.5	6.0	6.0	9.0	4.5	5.5	6.0	6.5	7.0	8.5	6.0	5.5
9.5	7.5	11.5	6.5	4.0	0.0	0.0	10.0	10.0	8.0	6.0	8.5	8.0	3.5	10.0	3.5
10.0	9.0	7.5	5.0	6.5	5.5	8.0	7.0	5.5	4.5	7.5	3.5	6.5	7.5	3.5	7.0
5.0	7.5	7.5	4.0	5.0	5.5	6.0	8.5	6.0	6.5	7.0	8.0	6.5	5.0	9.0	7.0
4.5	10.0	7.0	7.0	5.0	7.0	9.5	6.0	8.0	9.0	1.0	11.5	10.5	5.0	8.0	14.5
1.0	10.5	15.0	5.5	5.0	4.5	10.5	9.5	10.5	9.0	7.5	8.0	12.5	11.0	17.5	11.0
11.0	13.5	11.0	9.0	10.5	9.0	9.5	14.0	12.5	16.0	14.0	15.5	12.0	14.0	14.0	10.0
6.5	13.5	12.5	12.0	18.0	13.0	9.5	19.0	13.0	14.0	13.0	12.5	17.5	16.0	16.5	17.5
15.5	18.0	19.5	26.5	22.0	21.5	27.5	27.0	27.5	27.0	29.0	25.5	45.0	44.5	36.5	38.0
39.5	34.0	45.0	20.0	8.0											

ID PA measured angles

(ii) bfw 1.7m

boulders up to 1m, much smaller material.

Fangdale (FA)

(i)	1 - 41	Heath, some rock at surface
	42 - 46	Ditch
	47 - 98	Heath, some rock at surface
	99 - 124	Bracken
	125 - 127	Heath
	128 - 132	Track
	134 - 287	Heath
	288 - 469	Moss, bracken, rush
	470 - 506	Bracken

(ii) bfw 1.0m

some boulders 1-2m, mostly smaller material.

Ladhill (LA)

(i)	1 - 12	Heath
	13	Path
	14 - 67	Heath
	68 - 71	Tumulus
	72 - 110	Heath
	111 - 114	Tumulus
	115 - 206	Heath
	207	Path
	208 - 224	Bracken, stones at surface
	225 - 338	Pasture
	339 - 342	Up to and away from wall
	343 - 378	Pasture
	379 - 381	Track and line of wall

2.5	2.0	1.5	3.0	1.0	6.5	2.0	0.5	6.0	2.5	-8.0	7.5	0.0	4.0	5.5	1.0
1.0	1.0	2.5	4.0	1.5	3.0	0.0	3.5	3.0	0.0	4.5	4.0	4.0	2.5	2.0	0.5
2.5	2.0	2.0	3.0	6.5	0.5	1.0	-3.5	-9.5	-9.0	6.5	18.5	24.5	-21.0	1.0	6.5
4.5	3.5	3.5	3.0	5.0	4.5	3.0	0.5	6.0	5.0	8.5	2.0	4.0	3.0	6.0	7.5
3.0	4.5	6.0	4.0	7.0	4.0	4.0	2.5	4.0	2.5	5.5	2.0	1.0	6.5	-0.5	5.0
3.5	5.5	4.0	4.0	7.0	2.5	4.0	5.0	7.0	4.0	2.5	8.0	7.0	6.0	3.0	4.5
5.0	1.0	2.0	3.0	-0.5	0.0	5.0	3.5	6.0	6.5	5.0	1.0	11.0	7.0	7.5	2.5
3.0	7.5	7.5	5.5	10.0	8.5	9.0	3.0	8.0	8.0	6.0	5.5	4.5	8.0	3.5	11.5
9.0	0.5	2.5	5.0	6.0	7.0	6.5	4.5	7.5	6.0	4.5	5.5	4.5	8.0	11.5	5.5
8.0	8.5	6.5	6.0	6.5	6.0	4.5	6.0	5.0	7.0	5.0	6.0	6.0	3.5	7.0	6.0
5.0	6.0	3.5	6.0	7.0	7.0	8.5	3.5	10.0	11.0	6.0	10.0	10.5	1.0	12.5	8.5
9.0	8.5	8.0	10.0	11.0	9.0	12.0	8.0	4.0	10.0	8.0	8.0	13.0	8.0	7.5	12.0
6.5	7.0	6.0	8.0	11.0	10.5	12.0	9.5	2.5	8.5	11.0	7.5	11.5	11.0	6.0	9.0
10.5	6.5	7.0	11.0	7.5	8.5	7.0	6.0	11.5	7.0	6.5	3.5	9.0	11.5	2.5	8.5
8.5	11.5	10.5	7.5	7.0	6.0	8.0	12.5	0.5	12.5	6.5	8.5	8.5	8.0	8.0	11.5
9.0	8.5	8.5	4.5	6.0	7.5	5.5	8.5	8.5	8.0	8.5	5.5	9.0	7.0	9.0	12.0
9.5	8.5	4.5	9.5	7.5	12.0	13.5	15.0	17.5	15.0	8.0	14.0	10.0	7.5	9.5	6.5
5.0	11.5	6.0	6.5	10.0	5.5	10.5	7.0	6.0	8.0	3.0	2.0	15.0	7.0	13.0	8.5
8.0	11.0	5.0	5.0	4.5	9.0	6.0	6.0	4.5	9.0	10.0	12.0	6.0	7.0	9.5	11.0
6.5	13.5	11.5	10.0	11.5	13.0	9.0	4.0	10.5	8.5	8.0	10.0	11.0	7.0	7.5	8.0
7.0	11.0	5.5	5.5	7.0	4.5	6.0	6.0	4.0	7.0	5.0	5.5	6.0	4.5	10.0	6.0
5.0	6.0	0.5	5.0	5.5	5.0	6.5	11.5	4.5	9.0	8.0	10.5	8.0	7.0	7.5	11.0
3.5	14.5	8.5	10.5	9.5	8.0	6.0	8.5	6.0	6.0	8.0	6.0	7.5	6.0	8.0	5.0
8.0	8.5	8.0	4.5	9.0	5.5	12.0	9.0	11.0	6.0	9.0	4.5	15.0	4.0	7.0	12.5
10.5	16.5	9.5	12.5	8.0	6.0	12.0	5.0	9.5	3.5	5.5	9.0	8.0	8.5	11.0	11.0
11.5	9.5	9.0	10.0	10.0	7.0	11.0	11.5	8.0	11.5	10.5	10.5	7.5	8.0	8.5	8.0
12.0	3.0	11.5	6.5	20.5	5.5	7.5	11.0	8.0	2.5	8.0	14.5	5.0	13.0	7.5	8.5
10.5	13.5	6.5	10.5	13.5	7.5	9.5	6.5	10.0	9.5	12.0	10.0	9.0	11.0	12.0	9.5
11.5	12.0	13.0	12.0	15.5	12.5	13.0	10.5	14.5	14.5	13.5	15.0	16.5	16.0	13.0	14.0
17.0	15.5	18.0	18.5	15.5	21.5	19.0	24.0	19.0	27.0	20.5	27.5	25.5	28.5	30.0	27.5
23.0	34.5	24.0	28.0	26.5	21.0	33.5	28.0	33.0	44.5	38.5	50.0	39.5	30.0	25.0	32.0
25.0	22.0	21.0	22.0	20.0	24.0	38.5	24.5	17.0	16.5						

IE FA measured angles

3.0	-1.0	5.0	2.5	1.5	5.0	9.0	3.0	4.0	2.5	7.5	6.5	-11.0	9.0	1.0	5.0
0.5	4.5	5.0	4.0	6.0	2.0	15.0	-3.0	2.0	5.0	4.0	3.5	5.0	2.5	3.5	3.0
3.5	6.5	8.0	3.0	7.0	-1.0	5.0	6.0	8.0	3.5	2.0	3.0	6.0	2.0	4.5	2.5
4.5	6.0	5.0	3.0	5.0	3.0	6.0	4.0	3.5	2.5	3.0	6.0	3.5	5.0	5.0	4.5
8.0	2.0	4.0	-9.5	6.0	15.0	3.5	4.0	5.5	5.0	3.0	6.0	5.0	3.0	5.5	4.5
3.5	4.0	3.0	7.0	4.0	4.5	3.0	4.0	5.0	5.0	4.0	5.0	5.5	2.0	6.0	5.0
5.5	4.0	5.0	5.0	6.5	9.0	5.0	7.5	3.0	8.0	5.5	5.0	6.5	2.5	2.5	1.0
4.0	15.0	5.5	3.0	3.5	2.5	2.5	6.0	5.0	4.5	5.0	1.0	6.0	5.0	6.5	4.0
9.0	2.0	9.0	6.5	5.5	4.0	6.5	6.0	4.5	4.0	5.0	5.0	4.5	4.0	5.0	6.5
6.0	5.5	6.0	8.5	6.5	6.5	5.5	5.0	5.0	6.5	7.0	4.5	5.0	3.5	3.5	7.0
8.0	5.0	6.0	7.0	8.0	4.5	6.5	7.0	5.0	6.5	6.0	5.0	6.5	5.5	4.5	8.5
7.0	7.5	6.5	7.0	7.5	7.5	5.0	5.0	8.0	5.0	10.0	8.5	10.0	9.0	9.5	6.5
7.0	7.0	5.0	7.0	7.0	7.0	6.5	7.0	6.5	6.5	6.0	6.0	7.0	13.5	0.0	8.5
9.0	3.5	12.5	10.0	10.0	14.0	12.0	11.0	14.0	12.5	21.0	23.0	12.0	22.5	25.5	2.5
17.0	17.5	14.0	14.5	17.0	19.0	21.0	13.0	10.0	10.0	14.5	16.0	17.5	16.5	17.5	10.0
14.0	12.0	10.5	12.0	11.0	11.0	10.5	7.0	13.5	11.5	12.0	13.0	13.5	11.5	16.5	12.5
16.0	13.5	11.0	12.5	12.0	15.5	14.0	12.5	14.5	14.5	14.0	15.0	16.0	13.0	14.5	15.5
14.0	10.5	10.5	13.0	13.0	10.5	12.0	16.0	12.5	14.0	13.5	14.0	14.5	13.5	9.5	15.0
17.5	11.5	12.0	11.0	13.0	10.5	11.5	10.0	10.0	12.5	11.0	9.5	15.0	10.0	13.5	10.0
8.0	12.5	6.5	10.0	10.5	11.5	9.5	9.0	10.5	10.5	8.5	8.5	10.0	9.5	12.5	10.0
11.5	11.5	7.5	11.0	8.5	9.0	10.5	8.0	8.5	11.0	8.5	10.5	9.5	5.0	5.5	11.0
9.5	6.0	15.5	20.0	14.0	6.5	10.0	6.0	9.5	8.5	10.0	9.0	10.0	10.5	10.0	9.5
11.0	7.5	11.0	12.0	8.5	12.0	7.5	9.5	10.5	11.0	9.0	10.0	11.0	12.0	9.5	12.5
10.5	9.0	11.0	10.0	10.0	10.0	8.0	6.5	11.5	8.5	24.0	18.0	6.5	13.0	13.0	15.0
14.0	14.5	13.0	11.0	14.0	13.0	12.5	12.5	13.0	13.5	14.5	14.0	13.5	12.0	15.0	13.0
13.0	13.5	11.5	11.5	11.0	9.0	12.5	13.0	14.5	13.0	12.5	14.0	15.0	13.5	13.5	13.5
13.0	12.5	13.0	10.0	12.0	10.5	11.0	10.0	10.5	9.0	11.5	12.5	13.0	12.5	11.5	16.0
14.5	16.0	15.5	16.5	15.0	15.5	17.0	14.0	15.5	13.5	12.5	19.0	13.0	17.0	17.0	15.5
15.5	14.5	14.0	14.5	13.0	13.0	12.5	12.5	12.5	10.5	8.0	9.0	7.5	7.0	9.0	15.0
18.5	19.5	18.0	17.0	17.5											

IF LA measured angles

382 - 465	Pasture
466 - 469	Trees

(ii) bfw 3.3m

boulders up to 1.5m, mostly smaller cobbles, some fines.

Tripsdale (TR)

(i)	1 - 22	Calluna
	23 - 27	Track
	28 - 253	Calluna
	254 - 256	Bracken, Calluna
	257	Scar 1.8m
	258 - 263	Bracken, Calluna
	264 - 283	Some boulders 1-2m
	284 - 345	Bracken, Calluna
	346 - 357	Some boulders 1-2m
	358 - 384	Bracken, Calluna
	385 - 403	Sparse wood, mixed ground vegetation, some boulders

(ii) bfw 7m

many boulders up to 2m, much smaller material.

Arnsgill (AR)

(i)	1 - 96	Mixed low heath, some rock at surface
	97 - 98	Track
	99 - 117	Mosses, some bare ground
	118 - 160	Mosses, much bare ground, small boulders
	161 - 262	Bracken, medium boulders
	263 - 310	Mosses, medium boulders
	311 - 363	Bracken, bank

(ii) bfw 1.5m

boulders up to 1.5m, mostly small material

0.0	0.0	2.0	1.0	-1.0	2.0	-0.5	2.0	2.0	-1.0	4.0	1.0	1.0	2.5	3.0	2.5
0.0	5.0	-2.0	3.0	-2.0	0.5	4.0	0.0	-1.0	12.0	-5.0	1.0	1.5	-2.0	7.0	2.0
1.0	3.0	3.0	1.0	-0.5	1.0	2.0	1.0	3.0	0.0	0.5	2.0	1.0	4.0	4.5	-1.0
1.5	0.0	3.0	2.5	2.5	-1.0	2.0	-1.0	0.5	-1.0	0.5	0.0	0.5	1.5	-1.0	1.0
0.5	-2.0	0.0	2.0	-1.0	-0.5	1.5	0.0	1.0	1.0	2.5	1.5	1.5	2.5	4.5	1.5
4.0	4.0	2.0	2.0	2.0	3.5	3.5	3.0	5.0	0.0	1.0	6.0	1.5	2.0	0.5	0.0
0.0	0.5	4.5	0.5	1.0	0.0	2.5	6.0	2.0	3.0	1.5	3.0	3.0	2.0	3.5	4.0
1.0	3.0	3.0	6.0	9.5	10.0	15.0	9.5	7.0	6.0	4.0	6.5	7.0	3.0	6.5	6.0
1.5	2.5	1.5	6.0	2.5	2.5	-1.0	3.0	2.0	1.5	4.5	4.5	2.5	3.0	4.5	3.5
6.5	3.0	2.0	4.5	5.0	5.0	2.0	3.5	4.0	5.0	4.0	6.5	3.0	7.0	2.5	2.5
3.5	1.5	5.0	0.5	4.5	5.5	5.0	-1.5	5.5	0.0	7.5	5.0	5.0	4.5	9.5	0.0
2.5	6.5	4.0	6.0	4.0	5.5	15.5	2.0	5.0	8.5	9.5	3.5	3.0	6.5	3.0	10.5
7.0	3.5	3.5	5.0	8.0	4.5	4.0	7.0	4.5	5.0	7.5	7.0	6.5	8.0	7.0	7.5
9.0	5.5	6.5	10.0	10.5	9.0	9.5	11.0	13.5	15.0	18.0	10.5	10.0	4.0	4.5	2.5
0.5	3.5	1.0	3.0	5.0	0.0	10.5	3.5	4.0	3.0	7.0	4.0	6.5	5.5	9.5	5.5
5.0	4.0	6.0	6.0	6.0	4.0	10.5	8.5	4.5	7.5	6.5	8.5	10.0	6.0	11.0	26.5
44.0	20.0	19.0	29.5	21.5	31.0	30.0	28.0	27.5	23.5	21.5	12.0	21.5	23.0	26.0	13.5
15.5	12.5	18.5	16.5	15.5	26.0	27.5	13.5	26.0	21.0	12.0	17.0	16.5	10.5	10.0	15.5
13.5	10.5	15.5	14.0	15.5	18.5	13.0	12.5	15.0	19.5	12.5	4.0	11.5	12.0	13.5	11.5
10.0	6.0	12.0	14.5	4.5	15.5	16.0	-0.5	11.5	12.5	7.5	4.5	8.5	0.0	23.0	7.0
6.0	12.5	12.5	10.5	3.0	15.5	9.5	10.0	8.0	12.5	14.0	7.0	15.5	15.5	19.0	18.5
28.5	18.5	23.5	20.5	25.0	35.0	23.5	31.0	27.5	20.5	46.0	40.5	24.5	44.0	30.0	38.5
17.5	33.5	30.5	27.5	30.5	11.5	1.5	24.0	16.5	22.5	27.5	17.5	18.5	16.5	19.5	17.0
16.5	19.5	23.0	26.5	20.5	25.5	20.5	23.5	17.5	14.0	21.5	16.0	15.0	8.5	13.0	14.5
30.5	29.5	22.5	32.5	38.5	42.5	39.5	27.0	41.5	31.5	40.5	35.5	33.0	45.5	22.0	23.0
43.5	20.0	31.0													

IG TR measured angles

4.5	0.0	2.5	4.0	0.5	2.0	6.5	4.5	4.0	7.0	3.5	2.5	8.0	4.5	1.5	4.0
5.0	3.5	6.0	3.0	4.5	4.0	2.0	3.0	4.0	4.0	3.0	2.5	2.5	5.0	4.5	5.0
5.0	3.5	3.5	5.0	5.0	5.0	6.0	6.0	7.5	4.0	7.0	5.5	4.5	6.5	7.0	4.5
7.5	4.5	4.0	4.0	0.5	7.0	3.5	7.5	4.5	7.0	3.5	6.0	5.5	7.0	7.5	7.0
6.5	7.0	7.5	7.5	6.5	7.5	5.0	7.5	6.0	5.5	4.0	3.5	4.0	6.0	3.0	3.0
4.0	0.0	3.5	5.5	5.0	4.0	4.5	6.5	3.5	3.5	5.5	4.5	3.5	4.5	7.5	3.0
5.5	3.5	2.5	4.5	6.0	6.0	2.0	7.5	3.5	4.5	10.0	6.5	9.5	8.5	6.0	5.0
4.0	10.0	11.0	10.0	11.0	11.0	10.5	9.0	6.5	4.5	10.0	11.0	7.5	7.0	7.0	6.5
6.5	6.0	3.0	9.5	8.5	4.5	6.5	9.5	3.5	7.5	6.0	8.0	10.0	8.0	9.0	10.0
10.0	18.0	10.0	11.5	12.5	10.0	8.5	12.0	9.0	10.0	11.5	17.0	11.5	11.0	13.0	14.0
12.0	11.0	10.5	13.0	15.0	11.5	17.5	21.5	25.5	19.0	13.0	24.5	20.0	23.5	17.5	18.0
15.5	14.5	12.5	21.5	16.5	16.5	18.5	17.0	14.0	21.0	21.0	18.5	12.5	21.0	16.0	18.0
15.0	13.5	17.0	15.0	17.5	11.5	18.5	12.5	13.0	14.0	11.0	15.5	15.0	16.0	17.5	19.0
12.5	21.0	21.5	21.0	14.5	14.0	13.0	12.0	13.0	15.0	11.0	11.0	5.0	12.0	10.5	7.5
7.5	13.0	14.0	10.5	10.0	7.5	10.0	9.5	9.0	9.0	8.5	7.0	3.5	7.5	7.0	7.0
7.0	4.5	6.5	4.5	7.5	12.5	3.0	7.0	7.0	7.5	3.0	11.5	3.5	12.0	6.0	7.5
8.0	5.0	11.0	4.0	3.5	14.5	4.0	7.0	8.0	6.5	10.0	10.0	-2.0	11.0	5.5	6.5
8.5	6.5	9.5	8.5	6.5	11.5	6.5	11.5	10.0	4.5	21.0	15.0	13.0	7.0	13.0	9.0
8.5	12.0	10.0	10.5	12.5	14.0	10.5	9.5	11.0	13.5	10.5	15.0	13.5	18.0	13.0	14.5
11.5	10.0	9.5	11.5	4.0	9.5	10.0	8.0	5.0	4.5	11.0	7.0	10.0	5.5	10.0	8.5
5.0	4.5	1.5	6.5	7.5	10.5	15.0	10.0	13.5	20.0	19.5	13.0	17.5	21.5	18.5	13.0
28.5	17.0	25.0	21.0	19.5	22.0	23.0	21.5	29.5	31.5	31.0	40.5	40.0	32.5	37.0	35.0
29.0	32.5	35.5	45.0	47.5	39.0	40.0	36.5	43.0	11.0	7.5					

IH AR measured angles

Tarn Hole (TA)

(i)	1 - 3	Eroding peat
	4 - 20	Heather
	21 - 37	Eroding peat
	38 - 76	Heather
	77 - 93	Eroding peat
	94 - 150	Heather, bare ground, rock at surface
	151 - 152	Boulder
	153 - 169	Heather, bare ground, rock at surface
	170 - 242	Heather, some boulders
	243 - 281	Mosses
	282 - 419	Bracken, boulders
	420 - 431	Crag (Abney level estimate for angle)
	432 - 526	Bracken, boulders in upper part
	527 - 567	Open woodland, mixed ground vegetation

(ii) bfw 3m

many sandstone boulders up to 3m in bed and banks, clasts of other sizes down to shale fragments.

Hollow Bottom (HO)

(i)	1 - 13	Heather moor
	14 - 17	Path
	18 - 91	Heather moor
	92 - 122	Pasture grasses, Juncus
	123 - 131	On crags
	132 - 135	Crag (Abney level estimate for angle)
	136 - 173	Mixed moor vegetation, bedrock at 145
	174	Shale exposure, boulders around
	175 - 269	Shale spoil heaps, Juncus, bracken

4.5	0.0	-3.5	0.5	-1.5	5.0	2.5	-1.0	1.5	-4.0	12.5	3.0	-6.5	1.0	2.0	2.0
-3.0	-4.0	-2.0	9.0	1.0	3.0	5.0	5.5	9.0	-9.0	-7.0	4.5	3.5	-3.0	-1.0	-4.0
3.5	-1.5	0.0	-0.5	2.5	1.0	2.0	-1.0	-1.0	0.0	-2.0	2.0	2.5	-1.0	2.0	1.0
1.5	3.0	-0.5	2.5	6.0	-1.0	2.0	0.0	5.0	4.0	4.5	0.0	1.0	6.5	3.0	4.5
3.0	5.0	0.5	1.0	4.0	6.5	1.5	5.5	3.5	3.5	3.0	5.0	0.0	7.0	9.0	8.0
-0.5	8.0	3.5	0.5	5.0	8.0	0.5	7.5	5.0	6.0	2.0	5.0	19.0	-11.5	9.0	-1.0
-4.5	1.5	4.0	0.0	3.0	2.0	7.0	1.0	1.0	1.0	4.5	1.0	3.5	1.5	4.5	2.5
6.5	2.0	4.5	-2.0	6.5	3.5	1.5	6.5	2.0	1.0	3.0	3.0	2.5	3.0	5.0	5.5
3.0	4.0	3.5	6.0	4.0	2.5	1.0	6.0	7.5	2.5	8.5	3.0	6.5	8.0	6.0	9.5
4.0	6.0	9.5	8.0	9.5	5.0	7.0	6.5	10.0	5.5	0.0	16.0	8.0	8.5	8.0	6.0
9.0	6.0	9.5	9.0	7.5	16.0	15.0	13.5	11.5	7.0	16.5	11.5	12.5	12.5	5.0	17.0
4.5	11.0	11.0	9.5	9.0	4.5	13.0	12.5	6.0	5.0	15.0	6.0	1.5	7.5	6.5	7.0
11.0	12.0	3.0	5.5	10.0	7.5	10.0	9.5	12.0	9.0	9.5	6.0	6.5	8.0	5.0	8.5
9.0	9.5	10.0	7.5	10.0	10.0	8.5	13.0	8.5	13.5	10.0	18.5	7.0	17.5	17.0	12.0
7.5	11.0	7.5	7.5	15.0	6.0	10.5	9.0	9.0	17.0	1.0	7.5	13.0	15.0	4.5	5.5
6.5	13.0	6.0	5.0	7.0	10.0	6.5	7.0	9.0	7.5	8.0	11.0	7.5	10.5	18.0	18.0
17.5	15.5	17.5	12.5	10.0	8.5	10.5	10.5	11.5	9.0	10.0	10.5	10.0	10.0	12.0	7.0
10.0	8.5	8.0	9.0	9.0	11.0	9.0	9.0	10.5	10.5	15.0	14.0	17.5	11.0	19.0	13.5
17.0	9.0	7.5	12.0	10.5	14.0	9.5	14.0	16.5	10.0	17.0	16.5	14.0	10.0	11.0	12.5
12.0	13.0	12.0	11.5	10.0	9.0	6.5	10.0	10.0	11.0	6.5	10.0	7.0	9.0	6.5	6.5
7.5	4.0	7.5	7.5	8.5	7.5	6.5	2.0	10.0	8.0	5.0	7.0	11.0	10.0	12.0	2.5
10.5	8.0	13.0	3.0	6.5	9.0	8.5	8.0	10.5	10.5	12.0	10.5	8.5	8.0	7.0	8.5
9.0	8.0	5.0	7.5	13.5	5.0	7.0	6.5	10.0	6.5	7.0	6.0	7.0	12.5	9.5	6.0
7.5	6.5	10.5	14.0	8.0	13.5	11.5	9.0	10.0	10.0	12.0	5.0	11.0	8.0	16.0	10.5
7.0	15.0	12.5	5.0	18.0	10.5	15.0	12.5	9.5	17.0	12.5	11.5	18.5	8.5	11.5	15.5
9.0	12.0	13.0	12.0	10.0	6.0	17.0	16.0	12.5	15.0	16.5	15.0	13.0	12.0	21.0	22.0
28.5	28.0	31.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	42.0
38.0	29.0	22.0	34.0	13.0	26.0	22.0	24.0	26.0	15.5	3.0	18.0	6.0	12.5	10.0	17.5
32.5	35.0	40.0	37.0	33.0	30.0	24.0	24.5	22.0	14.0	9.0	29.0	22.0	32.5	20.0	20.0
14.5	25.0	15.0	21.0	22.5	26.5	22.0	21.0	22.0	25.5	38.0	19.0	19.0	22.0	23.5	23.0
19.5	19.0	18.5	19.0	21.0	22.0	18.0	15.0	18.0	15.0	16.0	16.5	19.0	15.0	18.5	11.0
14.0	17.5	19.0	22.0	21.0	17.5	14.5	13.0	20.5	15.0	19.0	13.5	15.0	19.0	20.0	12.0
17.0	19.5	16.5	5.5	18.0	16.0	13.0	13.0	19.5	13.0	16.0	12.0	21.5	14.5	11.0	30.0
32.0	22.0	30.0	29.5	21.5	30.0	45.0	30.0	20.0	29.5	30.5	29.0	40.0	15.5	18.0	21.5
13.0	22.0	19.5	12.5	23.5	19.5	21.0	17.5	25.0	26.0	19.5	30.5	35.5	30.0	33.0	33.0
30.5	25.0	35.0	37.0	24.5	17.5	14.0									

II TA measured angles

6.0	2.0	2.0	4.5	5.5	5.0	6.0	-2.5	4.5	6.5	9.0	8.0	5.0	14.0	5.0	-5.0
5.0	5.0	7.0	9.5	8.0	10.0	6.0	9.5	6.0	9.0	8.0	8.5	8.5	8.5	10.0	8.0
10.0	6.0	6.0	10.0	5.0	10.0	7.0	6.0	3.5	4.5	7.0	8.0	5.5	6.0	3.0	5.0
4.5	5.5	4.5	5.0	7.0	5.5	6.0	6.0	5.0	11.0	3.5	9.5	8.0	8.5	8.0	7.5
8.0	6.0	6.0	5.5	6.0	4.0	7.0	8.0	5.0	6.0	9.0	5.0	7.5	6.0	10.0	9.5
12.5	12.5	12.5	14.5	18.5	17.0	13.5	17.0	16.0	20.5	10.0	19.0	20.0	14.0	15.5	12.0
11.0	14.0	12.0	12.5	12.5	12.0	18.0	14.0	4.0	12.0	13.5	14.0	8.0	19.0	10.0	8.5
11.0	45.0	13.5	8.5	13.5	9.5	9.0	11.0	12.5	12.0	26.5	23.0	33.5	32.5	90.0	30.0
36.0	26.0	-6.0	65.0	65.0	65.0	65.0	44.0	51.0	36.0	34.5	40.0	46.0	50.0	34.0	32.5
32.0	22.0	21.0	33.0	27.0	13.0	-7.0	10.0	17.5	15.0	11.0	13.5	-21.0	-9.5	-10.0	-8.0
-1.5	-4.0	-9.5	17.5	27.0	14.5	18.0	7.5	16.0	25.0	24.5	17.0	21.0	42.0	26.5	31.5
38.0	42.5	42.0	37.5	31.0	42.0	30.0	26.0	33.0	31.0	29.5	22.5	30.5	26.0	32.5	29.0
21.5	21.0	18.0	21.5	23.0	13.0	16.5	8.0	36.0	24.0	10.0	29.5	11.0	24.0	8.0	31.0
28.0	37.0	33.5	23.0	22.5	31.0	11.0	9.5	12.5	36.0	5.0	16.5	-10.0	12.0	14.0	25.5
16.5	14.0	27.5	15.0	23.0	20.5	14.5	7.0	25.0	16.5	18.0	16.0	13.0	14.0	12.0	18.0
10.5	13.5	9.5	14.5	11.0	13.0	21.0	20.0	17.5	23.5	12.0	21.5	10.5	11.0	9.0	7.5
12.0	22.0	10.0	17.5	16.0	17.5	15.0	8.0	14.5	7.5	6.0	12.0	13.0	10.5	15.5	15.5
25.0	23.0	27.0	23.0	17.0	21.0	19.0	18.5	17.0	14.0	19.0	23.0	19.5	16.0	13.0	14.5
22.0	19.0	17.0	16.0	9.0	18.5	14.5	6.5	16.0	17.5	13.0	20.5	16.5	19.0	14.0	19.0
19.0	17.5	22.5	12.5	17.5	13.0	19.0	12.0	15.5	11.0	13.0	9.5	12.0	16.5	16.0	21.5
14.0	9.0	15.0	19.0	14.5	5.0	11.0	6.0	11.0	13.5	13.0	15.0	15.5	11.0	11.5	19.5
16.0	10.5	7.0	7.0	13.0	20.0	17.5	24.5	16.0	10.5	14.0	12.0	10.5	8.0	13.5	9.5
2.0	16.0	10.0	2.0	15.5	8.5	10.0	12.5	11.0	10.0	9.0	7.5	11.0	6.0	10.0	6.5
5.5	6.0	7.0	3.5	3.5	5.0	8.0	7.5	6.0	6.0	7.0	7.0	6.0	4.5	4.0	6.0
7.0	6.0	8.5	13.0	-1.0	-6.0	17.0	12.5	8.0	12.5	10.5	19.0	11.0	14.5	15.0	18.0
8.5	9.5	8.0	18.0	24.5	23.5	23.0	17.5	10.5	17.0	6.0	7.0	6.0	8.5		

IJ HO measured angles

270 - 275	Grasses
276 - 315	Bracken, grasses
316 - 397	Pasture grasses, Juncus
398 - 414	Pasture grasses, some trees

(ii) bfw 1.0m

boulders in bank up to 2m, shale pebbles, intermediate boulders.

Todhill (TO)

(i)

1 - 36	Vaccinium, heath
37 - 120	Open conifers, Vaccinium
121 - 129	Grass, track
130 - 141	Bracken
142	Path
143 - 175	Bracken
176 - 178	Path
179 - 240	Bracken
241 - 264	Bracken, heath
265 - 266	Path
267 - 273	Bracken, heath
274 - 275	Path
276 - 309	Bracken, heath

(ii) bfw 2.4m

one boulder 2.7m, mostly smaller boulders below 1m, cobbles and pebbles.

6.5	8.0	7.5	10.0	12.5	12.0	16.5	19.5	12.0	15.0	23.0	12.0	25.5	21.0	28.0	25.0
23.5	28.5	31.0	30.0	26.0	27.0	33.5	27.5	32.0	30.5	25.0	32.5	27.5	35.0	39.0	31.0
20.0	28.0	29.5	30.0	28.5	23.5	22.0	24.5	31.5	29.5	30.5	30.0	31.5	28.0	30.0	29.0
35.0	30.5	37.0	30.0	22.5	29.0	39.0	28.5	36.0	33.5	34.0	27.5	28.0	38.0	38.0	31.5
34.5	24.0	35.5	23.0	35.5	25.0	30.0	27.0	36.0	26.5	28.5	21.0	30.5	28.0	25.0	24.0
20.0	22.0	23.0	30.5	24.0	27.5	22.0	28.0	23.5	24.5	18.5	29.5	22.0	25.5	21.5	26.5
22.5	22.5	21.0	16.0	22.5	19.5	19.5	20.5	21.0	27.5	19.5	18.5	17.0	18.0	20.0	18.0
15.0	16.0	14.0	15.5	14.5	19.0	16.0	19.0	-8.5	2.5	5.5	10.0	6.5	24.0	12.5	8.5
9.0	9.0	10.0	10.0	13.5	14.0	14.0	-1.0	24.0	6.0	9.5	14.0	17.5	1.5	13.0	14.0
14.0	14.5	16.0	13.0	15.0	14.0	16.0	19.0	22.5	15.5	20.5	19.0	19.0	21.0	15.0	23.0
29.5	25.0	24.0	23.0	23.5	20.5	20.0	18.0	18.0	23.0	13.5	19.5	21.0	18.0	16.0	23.5
5.0	32.5	24.0	20.0	24.5	18.5	24.0	27.0	12.0	31.0	17.5	17.5	15.5	17.0	14.0	17.0
19.0	19.0	17.5	17.0	17.5	18.0	23.5	17.0	22.0	26.5	17.0	22.0	12.5	19.5	17.5	16.0
19.0	11.5	18.0	12.5	16.0	17.5	12.5	14.5	8.0	12.5	17.5	14.5	18.5	10.0	15.0	10.5
11.5	11.0	5.5	16.0	20.0	19.0	20.0	14.0	16.0	13.5	14.5	8.5	19.5	17.5	12.0	17.0
14.0	8.5	17.5	13.0	12.5	12.5	13.5	31.5	19.0	19.5	11.0	12.5	14.0	10.0	7.0	9.0
12.0	10.5	9.5	12.5	9.0	10.0	8.0	18.5	17.0	4.5	12.5	10.5	7.0	10.0	11.0	16.0
27.0	5.0	-10.5	10.5	11.0	10.5	16.0	12.5	12.0	18.5	19.0	8.5	11.0	13.5	14.0	14.0
16.5	11.5	14.5	14.5	25.0	17.5	21.5	28.5	22.5	18.5	30.0	20.0	14.5	38.0	30.5	35.5
12.0	15.5	10.0	14.0	22.0											

IK TO measured angles

Appendix II

Brief program descriptions

This appendix includes notes on various original computer programs used in this study. All were written in FORTRAN and run on the NUMAC system at the University of Durham. Listings are available from the author.

PROFILE. Reads characterstring header, number of angles and measured angle series (crest first). Measured length given as constant in DATA statement.

Calculates profile coordinates and average angle and produces dimensionless profile plot and angle series plot.

AVEDIFF. Reads characterstring header, number of data values, data series, number of subseries lengths, output device numbers and subseries lengths.

For each subseries length, subseries averages and differences produced in serial order. Then each series sorted to numerical order, median and quartiles produced, frequency distribution compiled and histogram plotted. Repeated for absolute values.

Subroutines for median and quartiles based on subroutines in Andrews et al. (1972, Appendix 11). Subroutine for sorting based on code in Day (1972, 72-3). Subroutine for histogram plotting based loosely on subroutine in Davis (1973, 229).

AUTO. Reads characterstring header, number of data values and data series. Calculates autocorrelation function for lags up to smaller of 50, floor (number of values /4), together with 0.05 and 0.01 confidence limits. Plots function against lag. Repeats for first differences of data.

Subroutine for calculation in two versions, for Pearson and abbreviated estimators. Subroutine for plotting based loosely on subroutine in Davis (1973, 229).

ONGLEY. Reads characterstring header, number of angles, measured angle series and tolerance. Writes details of acceptable segments, including regression equation, residuals, length of regression line, residual sum and average residual.

Based on program by Ongley (1970), revised to allow angle input; several minor modifications.

YOUNG. Reads characterstring header, number of angles, measured angle series and maximum coefficients of variation. Writes details of acceptable segments, elements and units.

Based on program SLOPEUNITS by A. Young, University of East Anglia, kindly supplied by the author. Rewritten to remove ICL FORTRAN features; several minor modifications.

FISHER. Reads characterstring header, number of angles and measured angle series. Calculates first differences and computes least squares partitions of difference series into 1 (1) 10 components. Writes indices, length, mean, standard deviation of each component, and within-component error for each partition.

Based in part on subroutines given by Hartigan (1975, 141-2).

SLOTH. Reads characterstring header, number of angles and measured angle series. Computes and plots curvatures, 3H3H smooth and rough angles, and curvatures from 3H3H smooth angles. Writes smooth curvatures.

KIRFIT. Reads characterstring header, number of measured angles, measured angle series and terminal indices of subseries to be fitted. Fits Kirkby curve to subseries using different estimators. For each estimator, writes estimate; mean square and root mean square residual; ordered residuals; quartiles, midspread, extremes and range of residuals; and autocorrelation function of residuals. Plots residuals in spatial order.