PARTON MODELS

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Abstract

We discuss the successes and failures of the naive quark and parton models and offer a possible remedy for the resonance symmetry versus vertex symmetry dilemma. Using the concept of current and constituent quarks related by a general Melosh-type transformation, we calculate matrix elements for excitation of the baryon octet ground state by weak and electromagnetic currents and find that the theory compares well with experiment except in the low ω region. Here some further SU(6) breaking mechanism is necessary for a good fit.

An alternative to the current and constituent quarks idea is to attempt a relativistic treatment of quarks, assuming current and constituent quarks to be fundamentally identical. Several models are discussed and a new model for mesons is presented. The solutions are similar to those obtained in other models apart from the inclusion of spin-orbit coupling terms, which have not previously appeared naturally in relativistic wavefunctions. We calculate some decay widths and find reasonable agreement with the data.

1. The development of the non-relativistic quark model

1.1 Introduction

The concept of an elementary particle, a building block out of which all matter is constructed, has haunted physics for centuries, and when finally the proton was identified, the concensus was that the elusive elementary particle had been found. The subsequent detection of many more similar particles did not suffice to change this view for some time.

When the SU(3) nature of the hadron spectrum began to emerge, the first scheme [1] was to assume the other particles to be bound states of some combination of protons, neutrons and Λ particles, the Λ being required to enable the construction of states of non-zero strangeness. This leads to predicted states and quantum numbers which are not observed, such as triply charged baryon states (a bound state of three protons).

Another way in which the situation may be viewed without losing the proton as an elementary particle, is to invoke democracy and say that all the particles are equally elementary. Then it is feasible to assume that the set of observed particles is unique, and is consistent with unitarity, analyticity, crossing symmetry and Lorentz invariance. Unfortunately there is no reason to suppose that there is a finite number of particles, and considering only a limited selection leads to inconsistency [2]. On the other hand, a solution of the problem taking into account an infinite number of particles is beyond the scope of present techniques.

One viable possibility is to drop one of the consistency requirements in the hope of being able to include it at a later stage. Neglect of unitarity, together with the assumption that all particles are stable, lying on straight Regge trajectories, is the course leading to dual models. The resulting spectrum is very encouraging, but in order to obtain it, a sacrifice must be made in the form of non-physical space dimensions. Also SU(3) symmetry is not included, though other quantum numbers are correct.

The problem then is two-fold; both the symmetry scheme and the dynamical behaviour of the particles must be incorporated in any successful model.

1.2 Symmetries of the strong Hamiltonian

When the atom was postulated to consist of a nucleus of neutrons and protons surrounded by a number of electrons, the nucleons were assumed to be elementary particles in the sense that they could not be broken down into smaller constituents, in the same way as previously the atom had been thought to be indivisible. In that case, one of the particles must be stable, otherwise mass could be converted into pure energy, and matter would not exist. Since β-decay of nuclei is observed, the stable particle must be the proton, while the neutron may decay into it. Hence we see empirically that both charge and the total number of baryons is conserved. With the increased sophistication of detection instruments, a wealth of new particles was discovered, including the strange particles. From the observed strong production of kaons in pairs, and their subsequent weak decays, we may add to charge and baryon number the conservation of strangeness.

Conservation of these quantities corresponds to the invariance properties of the Hamiltonian, which must satisfy the SU(1) symmetries related to the conserved quantum numbers. It should be noted that the strangeness symmetry is only exact under strong interactions, and is violated in weak processes, so that even this simple symmetry is broken.

To enlarge the scheme, we must first make the assumption that the apparent indistinguishability of the neutron and proton except by their charge is exact. We are then able to describe the neutron and proton as the two isotopic spin states of the nucleon. Clearly the value of the projection of the isotopic spin along some axis determines the charge on the particle, so that taking strange particles into account, we are led to the equation:

$$Q = I + \tfrac{1}{2}Y \tag{1.1}$$

where Y = baryon number + strangeness

Thus we have an invariance of the Hamiltonian under rotations in isospace about the z-axis, that is an SU(2) symmetry. This is good to about 1%, since it is broken by the electromagnetic interaction, but satisfied by the strong interaction.

Combining the SU(2) of isospin with the SU(1) of strangeness, the strong Hamiltonian must commute with the generators of an SU(3). Ordinary spin may also be included to extend the symmetry to SU(6). Finally,

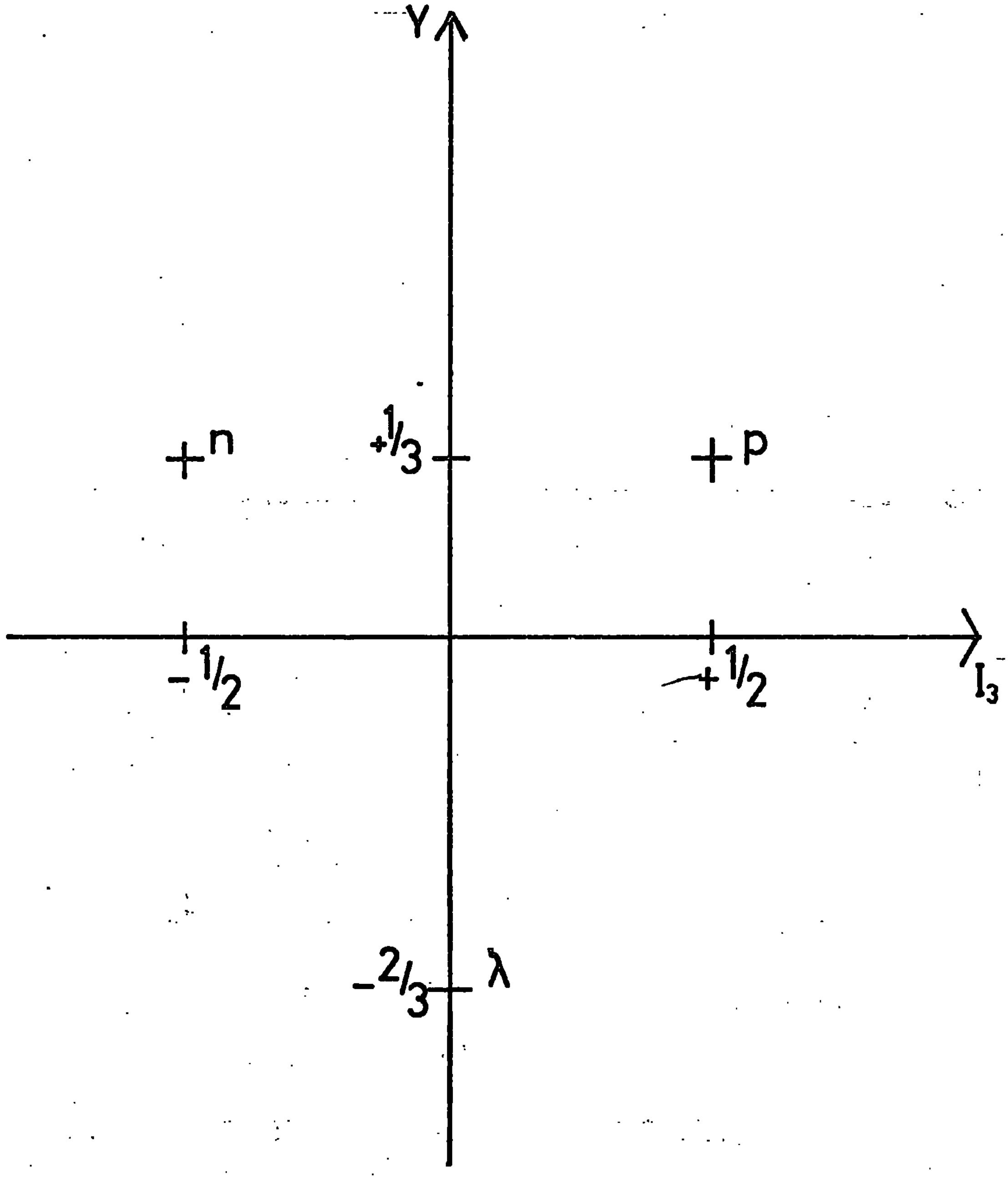


FIG. 1
The basic quark triplet.

invariance under space rotations lead to an SU(6) x O(3) symmetry.

This symmetry is approximately satisfied by the Hamiltonian, which describes the elementary particles of the system. The nucleon, however, has structure functions different from unity [3] and so cannot be a point (or elementary) particle. Another clue is that mesons were found to fall into octets with the same spin and parity, while baryons fitted into octets and decuplets. This suggests that the meson is formed from an elementary particle and an antiparticle, each obeying the SU(3) symmetry, giving meson multiplets of:

$$\underline{3} \otimes \underline{3}^* = \underline{8} \oplus \underline{1}$$

and that the baryon consists of three elementary particles giving baryon multiplets of:

$$\underline{3} \otimes \underline{3} \otimes \underline{3} = \underline{10} \oplus \underline{8} \oplus \underline{8} \oplus \underline{1}$$

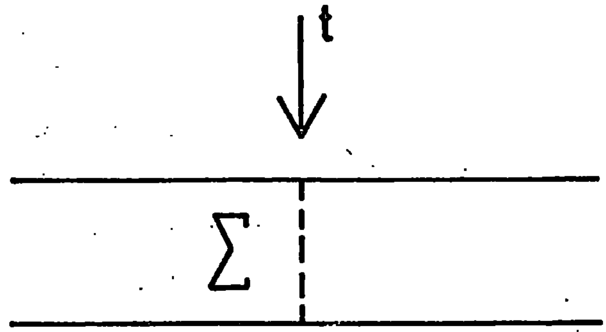
The symmetry may be extended to SU(6), though this is broken to about 25%. Then the mesons form multiplets of:

$$\underline{6} \otimes \underline{6}^* = \underline{35} \oplus \underline{1}$$

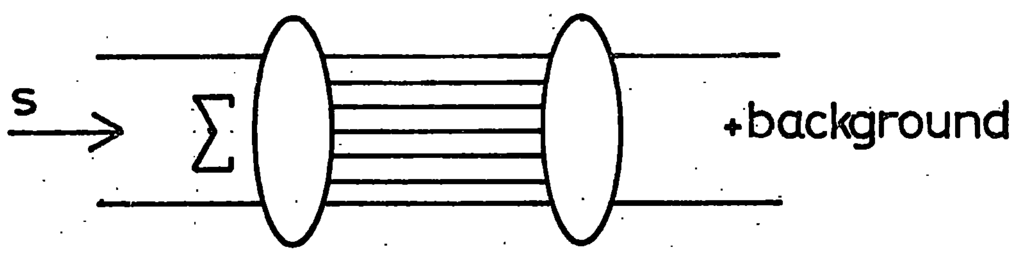
and the baryons multiplets of:

$$\underline{6} \otimes \underline{6} \otimes \underline{6} = \underline{20} \oplus \underline{56} \oplus \underline{70} \oplus \underline{70}$$

The elementary particles, or quarks, are assigned spin $\frac{1}{2}$ and quantum numbers as in fig 1, in order to reproduce the hadron quantum numbers correctly. Parity may also be predicted if the O(3) symmetry of orbital angular momentum is included. However, the rotation symmetry is included in such a way as to neglect any spin-orbit



(a)
High energy interaction



(b)

FIG 2
Low energy interaction

coupling, which is not a good assumption [4]. A further difficulty is that the symmetry is non-relativistic, applying only in the rest frame of the hadron. Nevertheless, for hadrons at rest, it works well [5].

1.3 Scattering processes

As well as a static scheme, some dynamic theory for the scattering of hadrons is required. High energy strong interactions can be well described by the exchange of Regge poles in the t-channel, whereas the low energy behaviour can be represented as a sum over direct s-channel resonances plus some non-resonating background (fig 2). In the intermediate energy region there are two possibilities. The first is the interference model [6], where the intermediate amplitude is assumed to be a superposition of the two types of description:

$$A = A^{Regge} + A^{Res} \qquad (1.2)$$

However the finite energy sum rules showed that this led to double counting of resonance contributions [7], and since the sum rules are based on analyticity, the interference model was rejected in favour of duality, which gives the amplitude as:

$$A = A^{Regge} + A^{Res} - \langle A \rangle^{Res} \qquad (1.3)$$

where $\langle A \rangle^{Res}$ denotes the locally averaged resonance amplitude.

This implies that at low energy, the Regge amplitude is cancelled by the averaged resonance contribution and this is semi-local duality. On the other hand, at high energy, the resonance contribution

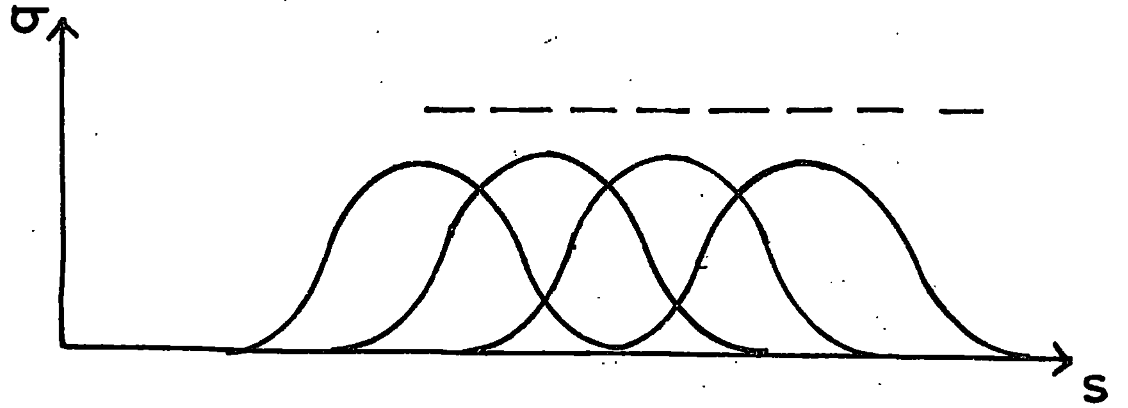
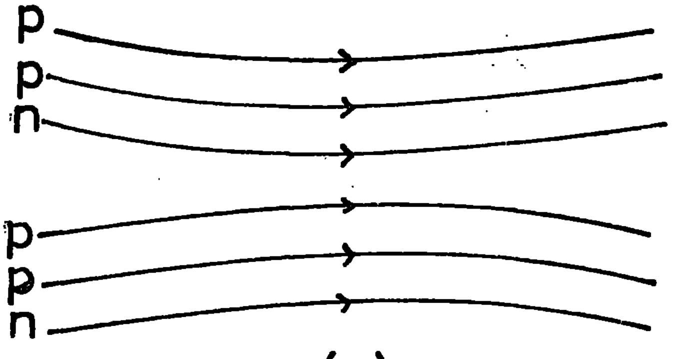
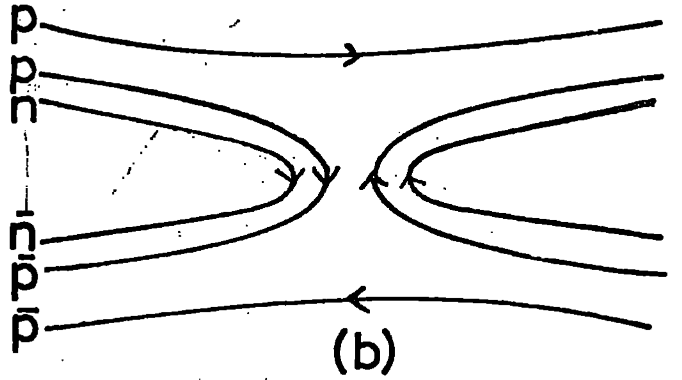


FIG. 3.
Sum over s-channel resonances to give smooth behaviour.



(a)
Proton-proton scattering.



(b)
Proton-antiproton scattering.
FIG. 4.

is equal to its average, leaving only the Regge term. This suggests that the s- and t-channel descriptions are equivalent and both separately complete.

Hence we are led to the idea of local duality, that the smooth behaviour at high energy can be described by Regge terms or by a sum over direct s-channel resonances, such that at any energy, the maxima of some resonances occur at the minima of others, summing to a smooth behaviour (fig 3). This may be expressed in terms of quark diagrams provided that some non-exoticity assumptions are made.

An exotic particle is one which cannot be made up as a combination of three quarks or a quark-antiquark pair (such as $qqqq\bar{q}$ or $q\bar{q}q\bar{q}$). The assumption that such particles cannot be produced and do not contribute in either the s- or the t-channel leads to some quite dramatic results. Consider as an example proton-proton scattering (fig 4a). At low energies, the process is dominated by the formation of direct s-channel resonances. Now the combination of the six quarks from the two incoming protons is exotic as defined above and therefore cannot contribute. Thus the process can take place only through the non-resonating background, that is by diffraction, and the cross-section should fall smoothly with energy. On the other hand, proton-antiproton scattering (fig 4b) has three incoming quarks and three antiquarks which may form a non-exotic combination. Hence there is a resonance contribution, so that the cross-section would be expected to show resonance

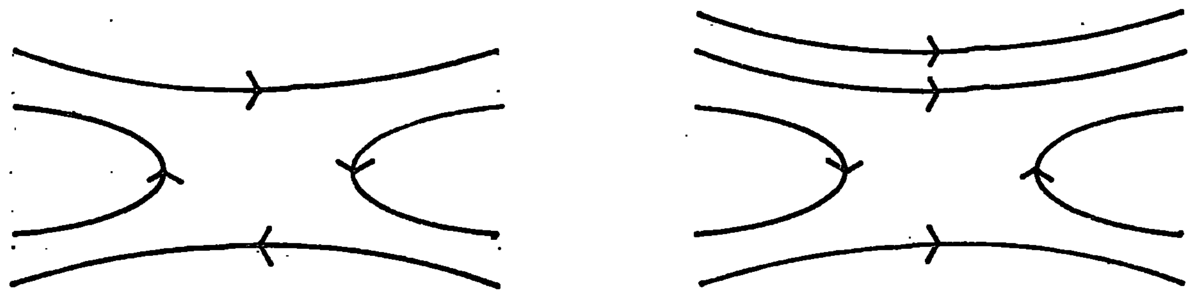


FIG. 5.
Allowed quark diagrams.

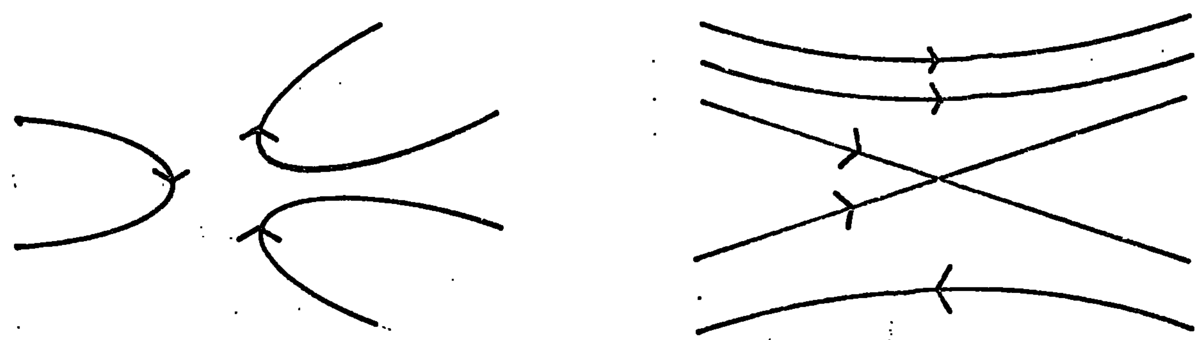


FIG. 6.
Forbidden quark diagrams.

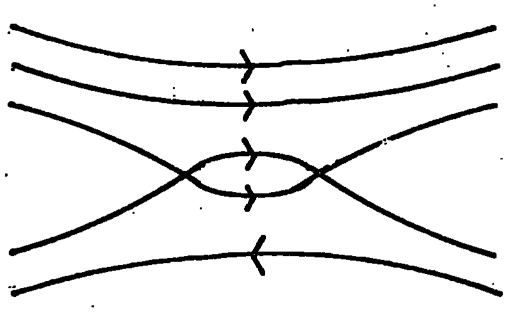


FIG. 7
Pomeron exchange.

structure. Both these predictions are verified by the data [8].

With the aid of this simple hypothesis, rules for the drawing of allowed diagrams can be postulated. These are [9]:

i) Each quark has its own line, which may not change its identity.

ii) An antiquark is represented by a line running in the opposite direction to the motion of the particle.

iii) The two ends of a single line may not belong to the same external particle.

iv) In any of the s-, t-, u-channels it is possible to cut the diagram in two by cutting only a non-exotic combination of lines.

These rules correspond simply to the assumption of SU(3) and to the equivalence of the descriptions in any channel.

Fig 5 shows some allowed processes and fig 6 some forbidden ones. Fig 7 shows a contribution to the s-channel background. There is no net exchange of quarks so the t-channel singularity has vacuum quantum numbers, and the two-component duality theory [10] identifies this singularity with the pomeron.

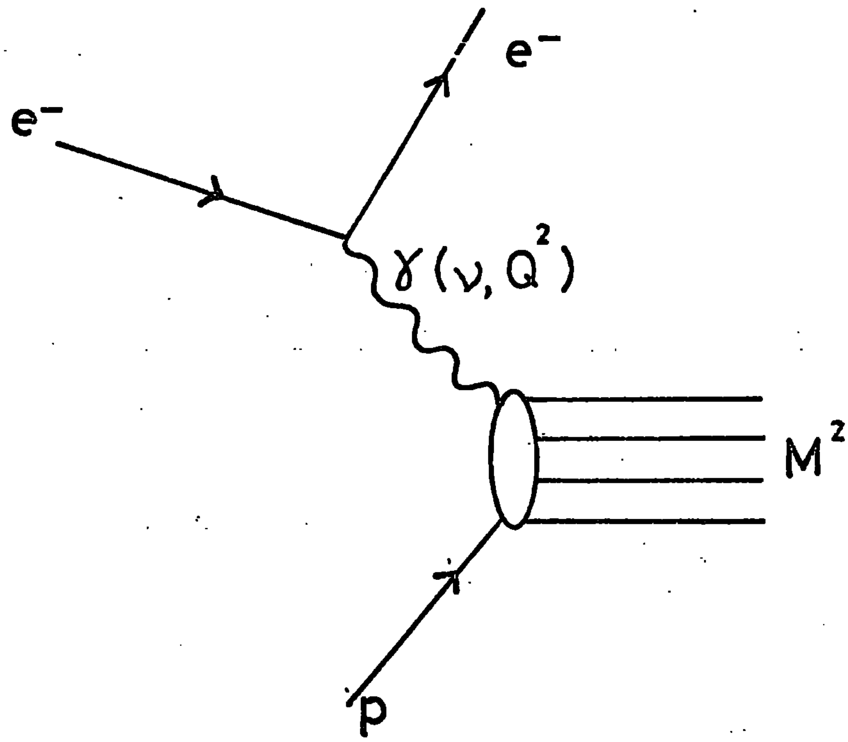


FIG. 8.
Deep inelastic scattering.

2. The naive parton model

2.1 Introduction

One question which arises immediately from the SU(3) nature of the hadron spectrum is that of whether the symmetry has a physical basis. That is, shall we find the same SU(3) internal structure by probing the hadron at distances short enough to reveal the existence of constituents? To resolve this entails studying processes with large momentum transfer, equivalent to short range probing, and a suitable candidate is the deep inelastic scattering of electrons on protons. This interaction has the advantage of involving only a single structured particle.(Fig 8)

We make the assumption that the process can be described in exactly the same way as electron-muon scattering, except that the structure of the proton is accounted for by two structure functions, W_1 and W_2, which are not required for the pointlike muon. There are two structure functions to allow for different behaviour of the proton depending on the polarisation of the virtual photon. A priori we might expect these functions to depend on both the momentum transfer, Q^2, and the photon energy, ν, so that there is no specific prediction that can be made for their energy dependence.

The data [11] shows that in fact W_1 and νW_2 are functions only of the ratio (ν/Q^2), the so-called phenomenon of scaling. The naive parton model is an attempt to explain this.

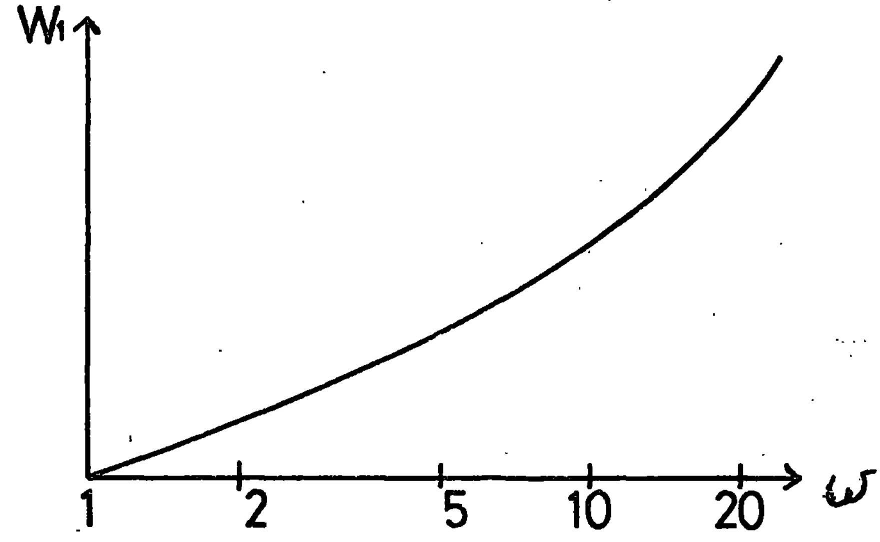


FIG. 9.

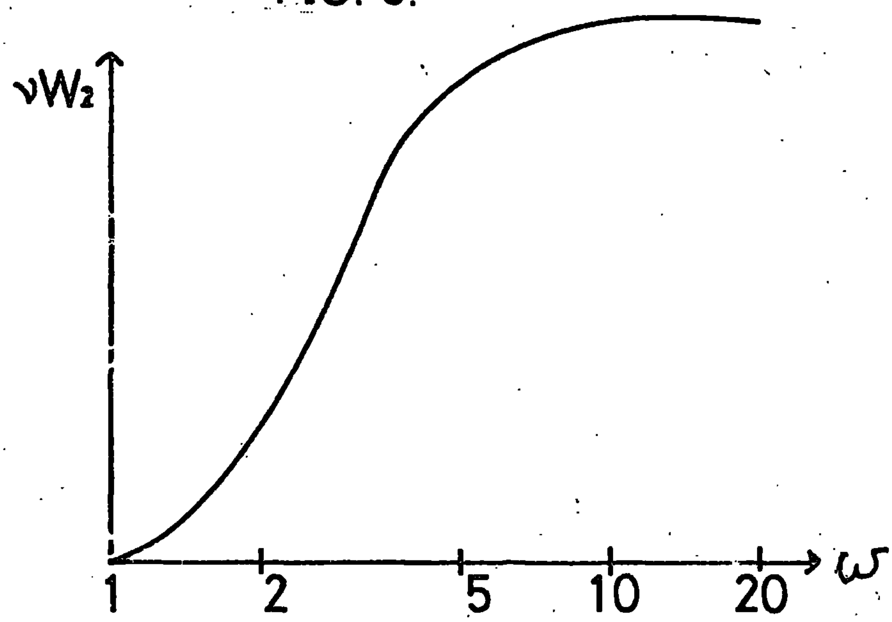


FIG. 10.

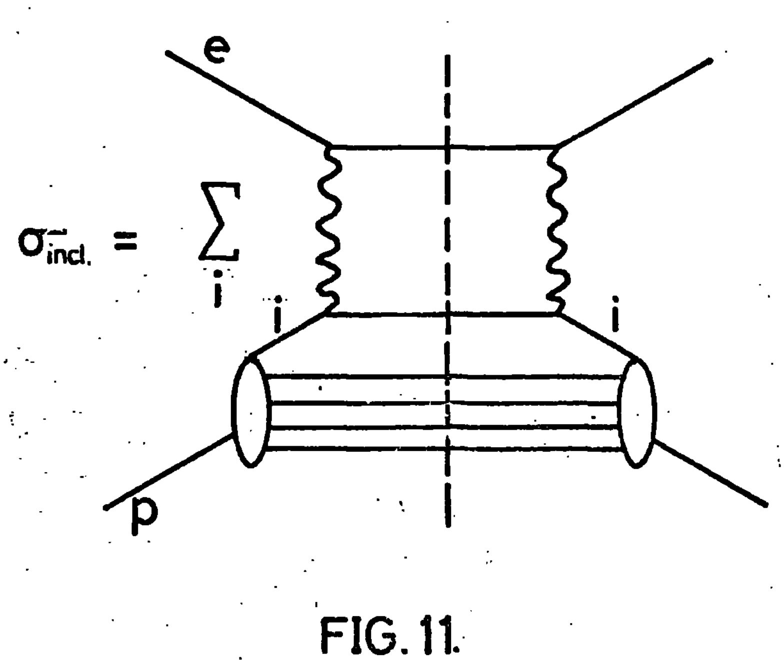


FIG. 11.

2.2 The model

The differential cross-section for the deep inelastic scattering of electrons on protons, with an unpolarised beam and target and only the outgoing electron observed, may be written:

$$\frac{d^2\sigma}{d\Omega\, dE'} = \frac{\alpha^2}{4E^2 \sin^4\theta/2} \left(W_2 \cos^2\theta/2 + 2W_1 \sin^2\theta/2\right) \qquad (2.1)$$

where E,E' are the ingoing and outgoing electron energies

θ is the electron scattering angle.

$W_1(\nu,Q^2)$ and $W_2(\nu,Q^2)$ are structure functions which depend on the internal structure of the proton target. For the case of elastic scattering from a point target, these functions may be specified as [12]:

$$\begin{aligned} W_1^{pt}(\nu,Q^2) &= Q^2/4M^2\, \delta(Q^2/2M - \nu) \\ W_2^{pt}(\nu,Q^2) &= \delta(Q^2/2M - \nu) \end{aligned} \qquad (2.2)$$

where M is the mass of the target.

We may now examine the data to see how it compares with the pointlike elastic structure functions, and sketches of W_1 and νW_2 as functions of $\omega = 2M\nu/Q^2$ are shown in figs 9 and 10. Both W_1 and νW_2 appear to be independent of Q^2, and this suggests that the proton may consist of 'partons' which interact in a pointlike manner, so that the inelastic cross-section is the incoherent sum of the elastic cross-sections from the individual partons, as in fig 11. Thus: [13]

$$W_{1,2} = \sum_i W_{1,2}^{pt(i)}\, e_i^2 \qquad (2.3)$$

where $$W_1^{pt(i)} = Q^2/4m_i^2 \ (Q^2/2m_i - \nu)$$
$$W_2^{pt(i)} = (Q^2/2m_i - \nu)$$

and m_i is the parton mass

e_i is the ratio of the charge of the parton to that of the electron.

To proceed further we must adopt some assumption for the dividing of the various dynamical variables between the partons. The most general is to assume a mass distribution for the partons such that the parton mass is a fraction x_i of the proton mass and $f(x_i)dx_i$ is the probability of finding a parton of mass between x_iM and $(x_i + dx_i)M$ in the proton, where M is the proton mass. Then:

$$\begin{aligned} W_1(\nu,Q^2) &= \sum_i Q^2/4m_i^2 \ \delta(Q^2/2m_i - \nu) \ e_i^2 \\ &= \sum_i e_i^2 \int_0^1 dx_i \ f(x_i) \ Q^2/4x_iM^2 \ \delta(Q^2/2x_iM - \nu) \end{aligned} \quad (2.4)$$

Similarly:

$$W_2(\nu,Q^2) = \sum_i e_i^2 \int_0^1 dx_i \ f(x_i) \ \delta(Q^2/2x_iM - \nu) \quad (2.5)$$

Then, since the δ-function may be written as $x_i/\nu \ \delta(1/\omega - x_i)$ we have:

$$\begin{aligned} W_1(\nu,Q^2) &= \sum_i e_i^2 \int_0^1 dx_i \ f(x_i) \ Q^2/4x_iM^2\nu \ \delta(1/\omega - x_i) \\ &= \sum_i e_i^2 f(1/\omega) \ Q^2\omega/4M^2\nu \\ &= \sum_i e_i^2 f(1/\omega) \ 1/2M \\ &= W_1(\omega) \end{aligned} \quad (2.6)$$

and $$\nu W_2(\nu,Q^2) = \nu W_2(\omega) \quad (2.7)$$

Thus the structure functions have the desired scaling property.

The process may alternatively be described in terms of the cross-sections for the absorption of transverse and scalar photons. There are two of these for unpolarised targets, since the photon helicity +1 and -1 cross-sections are equal by parity invariance. Hence naturally W_1 and W_2 may be related to σ_S and σ_T [14], and:

$$W_1 = K\sigma_T$$

$$W_2 = K(\sigma_T + \sigma_S)\; Q^2/(Q^2+\nu^2)$$

where K is a constant.

The ratio $R \equiv \sigma_S/\sigma_T$ is an interesting quantity, since if the partons were to have spin 0, σ_T must be zero and $R \to \infty$. On the other hand, the prediction obtained by assuming muon-like point behaviour as above is:

$$R = W_2/W_1\,(1+\nu^2/Q^2) - 1 \qquad (2.9)$$

and assuming the mass distribution f(x),

$$\nu W_2(\omega) = 2MxW_1(\omega)$$

$$= Q^2/\nu\; W_1(\omega)$$

so that:

$$R = Q^2/\nu^2(1+\nu^2/Q^2) - 1$$

$$= Q^2/\nu^2$$

$$= 4M^2/Q^2\omega^2 \qquad (2.10)$$

Thus R is predicted to be small in the regions where $Q^2 \to \infty$ with ω fixed, and $\nu \to \infty$ with Q^2 fixed. The data [15] shows that R is indeed small in the appropriate regions.

At this point, having ascertained that if partons exist they have spin $\frac{1}{2}$, the suggestion is that these partons are in fact the quarks of Gell-Mann and Zweig [16], in which case the quarks would have the usual quantum numbers. A difficulty now arises in that if partons are quarks, they must all have some fixed fraction x of the total proton mass and the structure functions should be δ-functions. However this may be resolved by allowing the partons to have some Fermi energy so that the δ-function appears smeared out about the value $\omega = 1/x$, and hence knowledge of the structure functions should tell us how many partons there are in a nucleon. Since we do not know how to treat the diffractive parts of processes in this model, we must use data with the diffractive part removed, and the simplest way to do that is to consider the difference $(\nu W_2^p - \nu W_2^n)$ so that the diffractive parts in each cancel out.

From the model we expect:

$$\int d\omega/\omega(\nu W_2^p - \nu W_2^n) = \sum_i (e_i^{2p} - e_i^{2n}) = 1/3 \qquad (2.11)$$

The data [17] shows a peak around $x = 1/3$ and the area under the curve to be 0.28 ± 0.06, if the cross-section is assumed to be dominated by Regge exchange at high ω.

The diffractive parts may be included and will behave suitably to give the above cancellation in the duality scheme [18], where on the basis of duality diagrams the following equivalences are proposed:

$$\begin{array}{ccc} \text{3 valence quarks} & + & \text{sea of } q\bar{q} \text{ pairs} \\ \updownarrow & & \updownarrow \\ \text{resonances} & + & \text{background} \\ \updownarrow & & \updownarrow \\ \text{non-diffractive} & + & \text{diffractive} \end{array}$$

The sea of $q\bar{q}$ pairs is assumed to be a singlet with respect to SU(3) so that there is no contribution to nucleon excitation, i.e. to non-diffractive processes. Such a sea would indeed contribute equally to νW_2^p and νW_2^n so that the difference $(\nu W_2^p - \nu W_2^n)$ would contain no diffractive part.

2.3 Sum rules

We may now proceed a step further in evaluating the structure functions. Writing:

$$\begin{aligned} F_2(\omega) &\equiv \nu W_2(\omega) \\ &= \sum_i e_i^2 \; dx\, f(x)\, x\, \delta(x - 1/\omega) \end{aligned}$$

Then: $$F_2(x) = \sum_i e_i^2\, x f(x)$$

where now $x = 1/\omega = Q^2/2M\nu$

Assuming that the partons may be ascribed quark quantum numbers, this becomes explicitly:

$$\begin{aligned} F_2^{\gamma P}(x) = {} & 4/9 \left[u_p^P(x) + u_{\bar{p}}^P(x)\right] \\ & + 1/9 \left[u_n^P(x) + u_{\bar{n}}^P(x) + u_\Lambda^P(x) + u_{\bar{\Lambda}}^P(x)\right] \end{aligned} \tag{2.12}$$

where $u(x) = x\, f(x)$

and $u_i^P(x)$ refers to parton i in the proton P.

By isospin reflection, we may relate amplitudes for finding a p quark in the proton and an n quark in the neutron, and the set of relations is:

$$u_p^P = u_n^N$$

$$u_n^P = u_p^N$$

$$u_\lambda^P = u_\lambda^N$$

and similarly for antiquarks.

Writing all the amplitudes in terms of proton target amplitudes and dropping the superscript, we have:

$$F_2^{\gamma p}(x) = 4/9 \left[u_p(x) + u_{\bar{p}}(x)\right] + 1/9 \left[u_n(x) + u_{\bar{n}}(x) + u_\lambda(x) + u_{\bar{\lambda}}(x)\right] \quad (2.13)$$

$$F_2^{\gamma n}(x) = 4/9 \left[u_n(x) + u_{\bar{n}}(x)\right] + 1/9 \left[u_p(x) + u_{\bar{p}}(x) + u_\lambda(x) + u_{\bar{\lambda}}(x)\right] \quad (2.14)$$

Hence immediately follows the constraint [19]:

$$\tfrac{1}{4} \leqslant F_2^{\gamma n}(x)/F_2^{\gamma p}(x) \leqslant 4 \quad (2.15)$$

which appears to be satisfied by the data [20].

This constraint includes both valence quarks and the sea, i.e. both diffractive and non-diffractive parts, and so may be dissected further by evaluating the two parts separately. For the non-diffractive part,

$$u_p = 2u_n$$

$$u_\lambda = u_{\bar{\lambda}} = u_{\bar{p}} = u_{\bar{n}} = 0$$

so that; $\quad (F_2^{\gamma n}/F_2^{\gamma p})_{ND} = 2/3$

For the diffractive part, the sea alone contributes and since this is by hypothesis an isoscalar,

$$(F_2^{\gamma n}/F_2^{\gamma p})_D = 1$$

To obtain the lower limit, the current must couple only to the active quark, i.e. that with the same isospin quantum numbers as the target - p in the proton and n in the neutron. Thus as $x \to 0$, i.e. $\omega \to \infty$ diffractive processes seem more important, which is reasonable since this corresponds to high energies when q^2 is large. For smaller ω, non-diffractive processes become more important, while for very small ω, some mechanism causing coupling to the active quark only becomes dominant.

In a similar way, we can examine the weak interaction structure functions. Since the neutrino couples to the isospin of the target, we have the relations:

$$F_2^{\nu p} = F_2^{\bar{\nu} n}$$
$$F_2^{\nu n} = F_2^{\bar{\nu} p}$$

while for the quark amplitudes:

$$\nu n \to p$$
$$\nu p = \nu\lambda = \nu\bar{n} = \nu\bar{\lambda} = 0$$
$$\nu\bar{p} \to \bar{n}$$

Then neglecting strangeness-changing currents, i.e. taking the Cabibbo angle as zero:

$$F_2^{\nu p} = u_n(x) + u_{\bar{p}}(x) \quad (2.16)$$
$$F_2^{\nu n} = u_p(x) + u_{\bar{n}}(x) \quad (2.17)$$

Using the same approximations for regions of ω as in the electromagnetic case:

$$(F_2^{\nu p}/F_2^{\nu n})_{ND} = \tfrac{1}{2} \qquad x \sim 1/3$$

$$(F_2^{\nu p}/F_2^{\nu n})_D = 1 \qquad x \to 0$$

and $$(F_2^{\nu p}/F_2^{\nu n})_{active} \to 0 \qquad x \to 1$$

As yet, there is no data good enough to test these predictions, but the neutrino and photon predictions combined may be tested as follows.

From equations 2.13,14,16,17:

$$F_2^{\gamma p} - F_2^{\gamma n} = 1/3\ (F_2^{\nu n} - F_2^{\nu p}) \qquad (2.18)$$

since $u_{\bar{p}} = u_{\bar{n}}$ if the sea is isoscalar.

And $$F^{\gamma p} + F^{\gamma n} \geqslant 5/9\ (F_2^{\nu n} + F_2^{\nu p}) \qquad (2.19)$$

This would be an equality if the electromagnetic current did not couple to strange quarks, so that taking the small ω region, where strange quarks appear not to contribute the equality should hold. This seems to be fairly well supported by the data [14].

Other sum rules may be deduced by consideration of the conserved quantum numbers of the proton:

$$\text{Strangeness} = 0 = \int \left[u_\lambda(x) - u_{\bar{\lambda}}(x)\right] dx \qquad (2.20)$$

Using this result:

$$\text{Charge} = 1 = \int \left\{ 2/3 \left[u_p(x) - u_{\bar{p}}(x)\right] - 1/3 \left[u_n(x) - u_{\bar{n}}(x)\right] \right\} dx \qquad (2.21)$$

$$\text{Baryon number} = 1 = \int 1/3 \left[u_p(x) + u_n(x) - u_{\bar{p}}(x) - u_{\bar{n}}(x)\right] dx \qquad (2.22)$$

Writing these in terms of the structure functions:

$$\int_1^\infty d\omega/\omega \left[F_2^{\nu n}(\omega) - F_2^{\nu p}(\omega)\right] = 1 \qquad (2.23)$$

(Adler sum rule [21])

$$\int_1^\infty d\omega/\omega \left[F_2^{\gamma p}(\omega) - F_2^{\gamma n}(\omega)\right] = 1/3 \qquad (2.24)$$

(c.f. equation 2.11)

The fact that the second relation is almost satisfied suggests that the Adler sum rule may also be similarly good.

2.4 Quark partons

The identification of partons with quarks in section 2.2 enabled specific predictions to be made for various ratios which were obtained from the incoherent impulse approximation. It is interesting to view the same process entirely from the quark model standpoint, where the non-diffractive part of the Compton amplitude may be deduced by considering the couplings of various baryon states that can be excited from the target nucleon, and summing over all possible excited states. The structure functions are essentially the imaginary part of the forward Compton amplitude. (Fig 12)

In exciting any member of a <u>56</u>- or <u>70</u>-plet, the matrix element will contain a Clebsch-Gordan coefficient which is known explicitly for each particular member of a multiplet and some other quantity describing all the unknown dynamics, which is different for <u>56</u>- and <u>70</u>- plets. These unknown quantities remain the same for proton or neutron targets, since this only alters isospin assignment, and for weak and electromagnetic currents, provided the quarks have an SU(6) x O(3) symmetry, this is irrelevant.

Expressing the contributions to the structure functions in this form, we obtain the results in Table I. Regarding the quantities A,B as sums over all possible 56-, 70-plets, we then find:

$$
\begin{aligned}
F_2^{\gamma n} &= 12A + 6B \\
F_2^{\gamma p} &= 17A + 10B \\
F_2^{\nu n} &= 33A + 21B \\
F_2^{\nu p} &= 24A + 3B
\end{aligned}
$$

A and B may be related by demanding no exotic t-channel exchanges and this gives simply $A = B$.

With this condition,

$$
\begin{aligned}
F_2^{\gamma n} &= 18A \\
F_2^{\gamma p} &= 27A \\
F_2^{\nu n} &= 54A \\
F_2^{\nu p} &= 27A
\end{aligned}
$$

i.e. exactly the same ratios hold as found before. Thus it is clear that the quark parton model results depend on two implicit assumptions:

i) that SU(6) is exact

ii) that there are no t-channel exotics.

From the mass spectrum alone, we know that SU(6) is broken to at least 20 to 25%, so that these results are also good only to a similar degree. The question remains as to how the symmetry is broken.

One possibility is to treat the nucleon as a quark and a core. If we consider the region $\omega \leqslant 3$, strange quarks and antiquarks should be unimportant and we can divide the nucleon into an interacting quark and a non-interacting core consisting of the remaining quarks. This we regard as a quasi-particle. [22] It must be either isovector or isoscalar in order to give a combined isospin of core and interacting quark of $\frac{1}{2}$. Under SU(6) the isovector and isoscalar cores should be equally likely, but suppose now that this is not true. Similarly we may remove the degeneracy in the spin case also, so that the probabilities of spin 0 and 1 cores are not equal. Nucleon wavefunctions may then be deduced [14] and the structure functions calculated as before. The results depend crucially on the relative probability of the isoscalar to isovector core and reduce to the old SU(6) results when the probabilities are made equal. If we write

$$\phi_1(x)/\phi_0(x) = 1 - x$$

where $\phi_1(x)$ is the probability for an isovector core

$\phi_0(x)$ is the probability for an isoscalar core

then we find that the ratio of the structure functions for neutron and proton targets is:

$$\frac{F_2^{\gamma n}(x)}{F_2^{\gamma p}(x)} = \frac{4 - 3x}{6 - 2x}$$

$$\rightarrow \quad 1/4 \qquad \text{as } x \rightarrow 1$$

as is indeed observed in the data [20].

It is also interesting to note that the relation equation 2.19 may still be derived in this SU(6) broken model, since it holds for $\phi_1(x)$ and $\phi_0(x)$ separately. Further tests of this model are given in ref 22, but these are as yet not verifiable against data.

2.5 Further symmetry

In the parton model, the calculation of e^+e^- annihilation cross-sections is simple. In particular, the ratio

$$R \equiv \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$

is given by:

$$R = \sum_i Q_i^2$$

where $Q_i e$ is the charge of the i^{th} parton.

Muons are assumed to be pointlike, as are the partons, so that the ratio removes all dynamical factors, leaving simply the ratio of the charges squared. Hence for the quark parton model, the ratio is:

$$\begin{aligned} R &= 4/9 + 1/9 + 1/9 \\ &= 2/3 \end{aligned}$$

This is much too small to agree with the data [23] which shows the ratio R to be about 2 to 3 in the region $q^2 \leqslant 9$ Gev^2. Clearly the only way to achieve this ratio and yet maintain the quark parton model in its standard form is to increase the number of quarks to nine, where three have the charge of the p-quark, three that of the n-quark and three that of the λ-quark.

This provides us with the second hint that more than three quarks are required, the first having arisen with the introduction of the SU(6) model. The baryon ground state made up of three quarks is the $\underline{56}$-plet containing the $\frac{1}{2}^+$ nucleon and the $3/2^+$ Δ decuplet. This is a completely symmetric state under the interchange of any two of the constituent quarks. The space part of the wavefunction is also symmetric, since there is zero orbital angular momentum, which means that the whole three quark wavefunction is symmetric. Yet the baryon is a fermion and so should have an overall antisymmetric wavefunction. This difficulty may be resolved by the introduction of an extra degree of freedom, colour. The ground state baryon wavefunction may then be assumed [24] to be antisymmetric in colour, i.e. a colour singlet, so that the complete wavefunction is indeed antisymmetric. To prevent coloured states being observable, it must be assumed that the singlet is the lowest energy state.

The symmetry problem together with the observed value of R suggests that there are nine quarks, the extra number occuring because of the required extra degree of freedom, the SU(3) of colour.

The situation was recently further upset by new data for the value of R [25], which shows that in the region $9 \leqslant q^2 \leqslant 25$ Gev^2, the ratio rises steeply to between 4 and 6. It is not yet clear whether the data shows a bump in this region or the beginning of a considerable rise. If the former is the case, it is conceivable that it represents a threshold of some new quantum number which at lower energies does not appear. This could be because the quark carrying this new quantum number might be heavier than the others, i.e. the new larger symmetry is badly broken.

A similar extra quark is required in order to unify the weak and electromagnetic interactions [26], and in this guise the extra quantum number has been called charm, giving quarks with quantum numbers as shown in fig 13. It is hoped that the rise in R may also be caused by the charm threshold.

More recently still, new resonances, the ψ(3100) and ψ'(3700) have been found [27] in e^+e^- annihilation and it has been suggested [28] that these are indeed charmed states. The quark model then implies that several other states should be close by in energy to form suitable multiplets, but as yet these have not been seen.

On the other hand, if the ratio R continues to climb, it is difficult to find any simple explanation. All single photon exchange processes to describe e^+e^- annihilation to hadrons lead to the prediction of R as a constant [29]. By invoking various symmetries the value of the constant can be manipulated to whatever is desired, but a rise in R cannot be reproduced. The hope is that eventually the ratio will settle at some well defined asymptotic value, and the model can then be adjusted to include this value, by altering the number of quarks, or their charges, or both.

3. SU(6) breaking

3.1 Introduction

The first two chapters have shown that the SU(6) scheme describes satisfactorily the observed hadron spectrum, and the interactions of a single structured particle with a point particle, while the predictions of the parton model for the non-diffractive parts of hadron cross-sections are not quite so successful. We now turn to the problem of interactions between two structured particles, in particular hadron decay processes.

Decays which take place via the strong interaction are characterised by a very short decay time ($\sim 10^{-24}$ secs) and such processes might therefore be expected to conserve those quantities which are invariant under the strong interaction, that is isospin and its third component, strangeness, spin and its third component, and orbital angular momentum. Applying these simple constraints enables decays to be categorised into those which are allowed, forbidden, doubly forbidden, etc. Processes which are forbidden via the strong interaction may of course be allowed via the weak or electromagnetic interaction, but for the purposes of this discussion we shall consider only the strong interaction.

As well as this simple selection of allowed processes, we may relate the decay rates of particular particles in the same multiplet, since the coupling constant for any particle in a given multiplet to two other particles is universal. This is well verified for SU(3) coupling constants [30], but is not satisfied when the symmetry is extended to SU(6) [31], i.e. SU(6) appears to be broken for coupling constants. In this chapter, we attempt to find a basis for the breaking.

3.2 The symmetry $SU(6)_W$

Several dynamical quark models have been proposed and although all naturally reproduce the SU(3) results, it is difficult to estimate their validity outside these results. It is therefore interesting to investigate the purely algebraic structure of the quark model with a view to differentiating between those results which depend on the quark dynamics and those which may be deduced simply from the algebra.

First we consider the spectrum of resonances as observed experimentally. This may be built up as a representation of the group SU(6) x O(3) where the SU(6) symmetry expresses the invariance of the strong interaction under isospin, hypercharge and spin transformations, and O(3) allows for orbital excitations. This scheme leads to prediction of mass degeneracy between the $L=0$, $J=0$ and $J=1$ mesons, and between the $L=0$, $J=\frac{1}{2}$ and $J=3/2$ baryons, which is clearly very badly broken, but nevertheless we are enabled to view all particles as fitting into a unified picture.

We may now attempt to predict which decays are allowed within the confines of the model, simply by demanding conservation of spin and strangeness, as well as the usual dynamical variables (energy, etc.). Consider the decay $\rho \to \pi\pi$. According to SU(6) ideas, this process is forbidden, since spin is not conserved, and similarly in the corresponding baryon case, $\Delta \to N\pi$. However, experiment shows that these decays certainly do take place [32], and this leads to the conclusion that any scheme which conserves intrinsic spin is not suitable for the description of particle decays.

To gain some insight into what sort of symmetry might be suitable, consider a two-body decay in the centre of mass frame. Then the product particles move collinearly. It may thus be helpful to consider transformations which leave the momentum in a given direction (say the z-direction) invariant. These are:

i) rotations about the z-axis

ii) rotations through π about the x-axis, followed by reflection in the x-y plane

iii) rotations through π about the y-axis, followed by reflection in the x-y plane.

The generators of these transformations may then be used to form a new symmetry scheme, $SU(6)_W$. Note that only the first transformation is through an arbitrary angle θ, while the other two are through the specific angle π, so that the transformations are single members of the group of rotations. We then have to guess that the full group is the one appropriate for vertices.

In this scheme the quark operators are unchanged, so that the baryon spectrum is undisturbed, while the antiquark operators are affected according to [33]:

$$\begin{array}{lllll} W_x = \sigma_x & & \overline{W_x} = -\sigma_x \\ W_y = \sigma_y & & \overline{W_y} = -\sigma_y \\ W_z = \sigma_z & & \overline{W_z} = \sigma_z \end{array}$$

This reorganises the meson spectrum, though it still decomposes into the same multiplets except with W-S flip, so that in the new scheme, the $|S=0, S_z=0\rangle$ state becomes $-|W=1, W_z=0\rangle$ and the $|S=1, S_z=0\rangle$ becomes $|W=0, W_z=0\rangle$. Hence the π forms a spin triplet with ρ^1 and ρ^{-1}, and ρ^0 is a spin singlet (where the superscripts refer to helicity). This then removes the problem of ρ-decay since ρ^0 has $W=0$, $W_z=0$ and may decay to two π's with $W=1$, $W_z=0$. An analogous argument also allows $\Delta \rightarrow N\pi$ in the SU(6) scheme.

We now have a symmetry which is invariant under Lorentz transformations in the z-direction, but it is still not suitable to describe even collinear strong interaction vertices. As an example consider the process $B \rightarrow \omega\pi$. Since the B is a state of $|W = 1, W_z = 0\rangle$ under $SU(6)_W$, the decay to the helicity +1 state of the ω does not conserve W_z. Only the decay to the helicity 0 state should occur, whereas experimentally the decay is observed to be predominantly to the helicity 1 state. [34]. There are several similar examples of violation of W-spin conservation both in meson and baryon decays.

Another problem with $SU(6)_W$ is that of the anomalous magnetic moment of the quarks. The proton is made up of three quarks in an s-state, so that its magnetic moment μ_p is given by the sum $\sum_i \mu_i$ of the moments of each of the quarks. Thus the individual quarks also have anomalous moments, and so do not couple simply (as $e\gamma_\mu A^\mu$) to the electromagnetic field.

Both of these difficulties may be resolved by the following simple idea. We know that systems at rest are well represented by the $SU(6)_W$ scheme, but that the scheme fails for vertices. Suppose then that there is a different $SU(6)_W$ for vertices. This may be alternatively expressed by postulating current quarks, different from constituent quarks, such that a current proton, made up of three current quarks in an s-state, is a superposition of constituent states. Then current quarks couple simply to the electromagnetic current and have no anomalous magnetic moment, while constituent quarks, being a superposition of current states do not couple simply. At vertices, the simple conservation rules no longer apply, and depending on the relation between the two different $SU(6)_W$ schemes, new rules may be substituted.

3.3 Current algebra

The first problem is to see whether it is possible to construct an $SU(6)_W$ of coupling constants. That the vector (Q_α) and axial vector ($Q_{\alpha 5}$) charges should form an algebra was first suggested by Gell-Mann [35], where:

$$Q^\alpha(t) = \int d^3x \, V^\alpha(\underline{x},t)$$
$$Q^{\alpha 5}(t) = \int d^3x \, A^\alpha(\underline{x},t)$$

where $\alpha = 1,2\ldots8$

and V^α, A^α are the vector and axial vector current octets.

Their equal time commutation relations are (dropping the explicit time dependence):

$$\begin{aligned} [Q^\alpha, Q^\beta] &= i f^{\alpha\beta\gamma} Q^\gamma \\ [Q^\alpha, Q^{\beta 5}] &= i f^{\alpha\beta\gamma} Q^{\gamma 5} \\ [Q^{\alpha 5}, Q^{\beta 5}] &= i f^{\alpha\beta\gamma} Q^\gamma \end{aligned} \qquad (3.2)$$

where $f^{\alpha\beta\gamma}$ are the SU(3) structure constants. Rewriting these in terms of right- and left-handed charges $Q^\alpha \pm Q^{\alpha 5}$:

$$[Q^\alpha \pm Q^{\alpha 5}, Q^\beta \pm Q^{\beta 5}] = 2i f^{\alpha\beta\gamma} [Q^\gamma \pm Q^{\gamma 5}]$$
$$[Q^\alpha + Q^{\alpha 5}, Q^\beta - Q^{\beta 5}] = 0$$

These relations demonstrate the SU(3) x SU(3) nature of the algebra.

From the third of equations 3.2, taking matrix elements and inserting a complete set of states on the left hand side:

$$\sum_n \langle f | Q^5 | n \rangle \langle n | Q^5 | i \rangle \sim \langle f | Q | i \rangle$$

This sum includes multiparticle diagrams (fig 14) as well as the simple first order diagram, and this makes it useful to work in the infinite momentum frame, where the former are suppressed like 1/P [36]. Considering the relation in this frame, and invoking PCAC to identify the divergence of the axial current with the pion, the difference ($\sigma^{\pi^+ p} - \sigma^{\pi^- p}$) of the πp total cross-sections may be evaluated in terms of of G_A/G_V, the ratio of axial to vector coupling constants constants. [37]. This is the Adler-Weisberger sum rule and has been verified experimentally to 20%. As yet there is no evidence to suggest that the algebra of currents is not exact.

Specialising now to a current quark model, the current densities may be written:

$$\begin{aligned} V_\mu^\alpha &= \bar{q}\,\gamma_\mu \lambda^\alpha/2\; q \\ A_\mu^\alpha &= \bar{q}\,\gamma_\mu \gamma_5 \lambda^\alpha/2\; q \end{aligned} \qquad (3.3)$$

where λ^α are the SU(3) generators.

The algebra may also be generalised by replacing γ_μ by Γ, the Dirac covariants, to give a U(12). However, transforming to the infinite momentum frame so that multiparticle diagrams may be neglected, also causes most of the generators to vanish. Those that remain, the so-called good operators, are those components that are infinite in the infinite momentum frame. The z- and t-components become infinite and equal, while the x- and y-components are finite and so negligible. Hence those that remain are:

$$\begin{aligned}
F^{\alpha}(t) &= \int V_0^{\alpha}(\underline{x},t)\, d\underline{x} &&= \int q^{\dagger} \lambda^{\alpha}/2\; q\, d\underline{x} \\
F_z^{\alpha}(t) &= \int A_z^{\alpha}(\underline{x},t)\, d\underline{x} &&= \int q^{\dagger} \sigma_z \lambda^{\alpha}/2\; q\, d\underline{x} \\
F_x^{\alpha}(t) &= \int V_{yz}^{\alpha}(\underline{x},t)\, d\underline{x} &&= \int q^{\dagger} \beta \sigma_x \lambda^{\alpha}/2\; q\, d\underline{x} \\
F_y^{\alpha}(t) &= \int V_{xz}^{\alpha}(\underline{x},t)\, d\underline{x} &&= \int q^{\dagger} \beta \sigma_y \lambda^{\alpha}/2\; q\, d\underline{x}
\end{aligned} \qquad (3.4)$$

$$\text{where } V_{\mu\nu}^{\alpha} = \bar{q}\, \sigma_{\mu\nu} \lambda^{\alpha}/2\; q$$

Although these tensor currents do not appear to couple directly to anything in nature, they do arise naturally from commuting the observable currents with their divergences. Thus we have 36 good operators, forming an $SU(6)_W$ of currents.

3.4 <u>The Melosh transformation</u>

The real transformation from current to constituent states must superimpose many current states and also must create $q\bar{q}$ pairs. As yet, such a transformation has not been found.

Melosh's transformation [38] treats only the free quark problem, where the difficulties are much reduced, since only single quark states can be related. The sea of $q\bar{q}$ pairs is also neglected. Since the $SU(6)_W$'s may not be identified, Melosh chose the next most simple possibility, i.e. that the transformation is unitary so that the group generators are related by:

$$W^i = V F^i V^{-1} \qquad (3.5)$$

where V is a unitary matrix.

Several desirable properties of V may be injected [33] so as to preserve the good parts of $SU(6)_W$ (constituent):

i) Invariance under rotations about the z-axis is required, so that:

$$[J_z, V] = 0 \qquad (3.6)$$

ii) CVC is good for decay processes, so it should not be destroyed by the transformation.

Thus:
$$F^i_o = W^i_o$$
and
$$[F^i_o, V] = 0 \qquad (3.7)$$

iii) Conservation of W-spin is not good for decays, so that:

$$F^i_z \neq W^i_z$$
and
$$[F^i_z, V] \neq 0 \qquad (3.8)$$

Using the free quark approximation, the Hamiltonian H may be written:

$$H = \int d^3x \, \bar{q} \, (-i\alpha . \partial + \beta m) \, q$$
and
$$[W^i, H] = 0 \qquad (3.9)$$

Clearly, when the Hamiltonian is transformed into current space, it should commute with the transformed generators:

$$[F^i, V^{-1} H V] = 0 \qquad (3.10)$$

Writing the transformed generators explicitly:

$$
\begin{aligned}
F_o^i &= \int d^3x \;\; q^\dagger \lambda^i/2\, q \\
F_z^i &= \int d^3x \;\; q^\dagger \sigma_z \lambda^i/2\, q \qquad (3.11) \\
F_{x,y}^i &= \int d^3x \;\; q^\dagger \beta \sigma_{x,y} \lambda^i/2\, q
\end{aligned}
$$

Now these generators contain only 1, $\beta\sigma_{x,y}$, σ_z and so commute with any operator containing only 1, β, α_z and $\beta\alpha_z$, so that the transformed Hamiltonian must contain only these. Hence the transformation V must transform away the $\underline{\alpha}_\perp$ part of the constituent space Hamiltonian H. Such a transformation is realised in the form:

$$V = \exp(iY) \qquad (3.12)$$

where $$Y = \int d^3x\, \bar{q} \arctan \left[\underline{\gamma}_\perp . \underline{\partial}_\perp / m\right]$$

giving a current space Hamiltonian:

$$V^{-1} H V = \int d^3x\, \bar{q} \left[-i \alpha_z \partial_z + \beta \sqrt{m^2 + (\underline{\gamma}_\perp . \underline{\partial}_\perp)^2}\right] q$$

This is identical in form to the Foldy-Wouthuysen transformation [39], except that the z-direction is picked out as a special direction as a result of choosing $SU(6)_W$ in preference to SU(6).

The transformed operators must now be boosted to the infinite momentum frame to remove the bad components.

3.5 Models with direct SU(6) breaking

i) The ^3P_o model

This is an attempt to incorporate the

breaking of W-spin conservation without invoking the idea of current and constituent quarks. Returning to the example of the decay $B \rightarrow \omega\pi$, data [34] shows that the decay from the helicity 1 state of the B is dominant over that from the helicity 0 state, contrary to the predictions of $SU(6)_W \times O(2)_{L_z}$, where only the 0 state should be allowed to decay. Similar contradictions occur in baryon decays.

A general decay of this type to a ground state hadron and pion (or kaon) is:

$$A(J^A, L, S^A; \lambda) \rightarrow B(J^B, L=0, S^B = J^B; \lambda) + C(J^C = 0) \tag{3.13}$$

where L,S are the quark orbital and spin angular momenta,

the z-axis is the direction of the momentum of B,

is the projection of J^A along this axis,

A,B,C are members of multiplets of static $SU(6) \times O(3)$.

The decay is now assumed to take place by the creation of a quark-antiquark pair (fig 16), one of whose members appears in each final hadron. The pair is assumed to have the quantum numbers of the vacuum, i.e. it is an SU(3) singlet, $J^{PC} = 0^{++}$. Using spectrographic notation, the state may be described as $^3P_0$. The model is motivated by duality diagrams (cf. e.g. fig 5).

Initially a further restriction was made on the singlet pair, namely that the transverse momenta of the quarks may be neglected compared to the longitudinal momenta in collinear processes such as decays. Then:

$$S_z = L_z = 0$$

and again the old $SU(6)_W \times O(2)_{L_z}$ results are obtained. However Rosner and Colglazier [40] suggested that this restriction be relaxed so as to allow also $L_z = \pm 1$, on the grounds that although the restriction is plausible for high energy processes such as deep inelastic scattering, it is less so for low energy decays. Thus the ratio of the different helicity amplitudes becomes arbitrary, i.e. a new parameter is introduced. Then although no relationship between the helicity amplitudes of the same decay may be predicted, ratios between amplitudes for different decays may be related. For example:

$$2\,(g_1/g_0)_{A_1\to\rho\pi} = (g_0/g_1)_{B\to\omega\pi} + 1 \qquad (3.14)$$

where g_0, g_1 are the couplings to helicity 0,1. This relation seems consistent with the data [41].

The theory was generalised by Rosner and Petersen [42] so that the helicity amplitudes for the general process in equation 3.13 are:

$$M_\lambda(A\to BC) = \sum_i \begin{pmatrix} A & \Big| & B & 35 \\ \alpha, a & \Big| & \beta, b & 8, 3 \end{pmatrix}_i \begin{pmatrix} \alpha & \Big| & \beta & 8 \\ A & \Big| & B & C \end{pmatrix}_i \times X_\lambda(A\to BC) \qquad (3.15)$$

where $X_\lambda(A\rightarrow BC) = \sum_{L_z} (J^B, \lambda, 1 - L_z \mid S^A, \lambda - L_z)$
$$\times (S^A, \lambda - L_z, L, L_z \mid J, \lambda) a_L^{L_z} \quad (3.16)$$

Here the first term is an SU(3) scalar factor with $\underline{A}$,$\underline{B}$,$\underline{35}$ labelling SU(6) representations and (α,a), (β,b) and (8,3) labelling SU(3) x SU(2) representations. The second term is an isoscalar factor with A,B,C labelling specific isomultiplets. The sum over i corresponds to the various possible couplings, for example d and f when α,β are octets.

Physically, the $^3P_0$ singlet has been combined with the pseudoscalar meson to form an effective (8,3) member of the $\underline{35}$-plet of SU(6). Thus the formalism is equivalent to $SU(6)_W \times O(2)_{L_z}$ if $a_L^{\pm 1}$ are put equal to zero.

In $X_\lambda(A\rightarrow BC)$, the first coefficient describes quark spin conservation, while the second describes the L-S coupling to form a total spin J. The reduced matrix elements $a_L^{L_z}$ are assumed to depend only on L_z and the specific multiplets involved in the decay. Equation 3.14 may be derived directly from equation 3.15.

The helicity amplitudes may therefore be calculated, and if symmetry breaking due to centrifugal barrier terms is taken into account, may be compared reasonably successfully with data.

ii) l-broken $SU(6)_W$

Another way of introducing symmetry breaking is to calculate matrix elements of Melosh transformed currents between mixed states of $SU(6)_W$, mixing taking place amongst states of the same isospin and hypercharge, both at the SU(3) and the SU(6) levels. [31] The effect of the Melosh transformation is to introduce extra terms into the currents, breaking the $SU(6)_W$ symmetry. In calculating decay widths, some angular momentum barrier factor must be included, although the results do not seem to depend strongly on the form chosen.

Testing at the SU(3) level only, shows that resonance decay couplings do fit in well with predictions of SU(3) [30]. However, the SU(6) nature of the hadron spectrum encourages a search for a similar larger symmetry for coupling constants. Using the scheme suggested above, the bad predictions of $SU(3)_W$ are avoided.

The mixing of the $SU(6)_W$ states may be parametrised in terms of mixing angles, according to, for strangeness 0 components:

$$\begin{pmatrix} N_1 \\ N_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} {}^48 \\ {}^28 \end{pmatrix} \qquad (3.17)$$

for states in any given $\underline{70}$-plet. Thus, for example, the physical N^* state contains a small admixture

ii) l-broken $SU(6)_W$

An alternative way of introducing symmetry breaking, is to incorporate a mixing scheme into $SU(6)_W$ whereby states of the same isospin and hypercharge are expected to mix both at the SU(3) and the SU(6) level. [31] Thus, for example, the physical N^* state contains a small admixture of the $\underline{70}\ {}^4\underline{8}$, instead of being pure $\underline{56}\ {}^2\underline{8}$ (or vice versa), according to:

$$\begin{pmatrix} N_1 \\ N_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} {}^4 8 \\ {}^2 8 \end{pmatrix} \qquad (3.17)$$

Testing at the SU(3) level only, shows that resonance decay couplings do fit in well with predictions of the symmetry [30]. However, the SU(6) nature of the hadron spectrum encourages a search for a similar larger symmetry for coupling constants. To avoid the problems of $SU(6)_W$ the mixing scheme above may be adopted, giving in effect the most general possible SU(6) structure. In calculating decay widths, some angular momentum barrier factor must be included, although the results do not seem to depend strongly on the form chosen [31].

The mixing may be parametrised in terms of mixing angles, as in equation 3.17, and these may be determined from a fit to the data. In this way, Cashmore, Hey and Litchfield show

[31] that the scheme can describe quite well decays of the types:

$$\underline{56},\ 2^{+} \rightarrow \underline{56},\ 0^{+} \quad \text{pseudoscalar meson}$$

$$\underline{70},\ 1^{-} \rightarrow \underline{56},\ 0^{+} \quad \text{pseudoscalar meson}$$

and photoproduction of the $\underline{70},\ 1^{-}$ and $\underline{56},\ 2^{+}$.

Having established such a model, it is then possible to use it to test the plausibility of the multiplet assignments of various resonances. In particular, they suggest that some of the supposedly positive parity $\underline{70}$-plets are in fact radial excitations of the $\underline{56}$, and they can find no resonance which is indisputably in a positive parity $\underline{70}$-plet.

This scheme allows a more comprehensive breaking of $SU(6)_W$ than the $^3P_0$ model, which does not contain all the terms generated by the Melosh transformation. The fact that the SU(6) structure of the decay couplings is shown to hold approximately, encourages a measure of confidence in the validity of the ideas on which the Melosh transformation is based.

4. Implications of the current and constituent quark hypothesis

4.1 Introduction

The Melosh transformation is a unitary transformation between two different $SU(6)_W$ algebras, those of free constituent and free current quarks. To apply it to the real world, we must assume that the structure is unchanged by the interaction between quarks. To see whether this assumption is viable, it is necessary to test the algebraic structure of the model against that which is actually observed. Unless this confrontation is successful, there is little to be gained from attempting to include the dynamic properties of quarks.

In order to test the algebra of the quark model, we must first write the Melosh transformation in a way independent of the quark dynamics, that is, we must extract the $SU(6)_W \times O(2)_{L_z}$ properties of the transformation [43]. From the way in which it was initially constructed, using the commutation properties of the operators, it is clear that the appropriate algebraic properties of the operator generating the transformation are:

$$\left\{\underline{35};\ W = 1,\ W_z = \pm 1,\ L_z = \mp 1\right\}$$
$$\left\{SU(3),\ \underline{1}\right\} \qquad Y = C = +$$

The transformation in its algebraic form may now be applied to the simple currents, as written in the current basis, in order to construct the current in the constituent basis.

To simplify the problem still further, we make the assumption that a current only interacts with a single quark. This is also made in the explicit free quark model, but there is no physical justification for it. In the algebraic formalism, the assumption is equivalent to restricting the currents to an $\{SU(3), \underline{8}\}$ part of a $\underline{35}$-plet.

4.2 The transformation of the currents

In the current basis, a transverse current with $\Delta J_z = 1$ may be written as:

$$J_T^{+} \sim \{\underline{35};\ W = 1,\ W_z = 1,\ L_z = 0\}$$

where $J_T^{\pm} = 1/\sqrt{2}\ (\mp J_x - iJ_y)$

and $YJ_T^{\pm} Y^{-1} = -J_T^{\mp}$, where Y is the x-y plane reflection operator.

Applying the Melosh transformation, the current may be written in the constituent basis as:

$$\begin{aligned} J_T^{+} \sim\ & \{\underline{35};\ W = 0,\ W_z = 0,\ L_z = 1\} \\ & + \{\underline{35};\ W = 1,\ W_z = 1,\ L_z = 0\} \\ & + \{\underline{35};\ W = 1,\ W_z = 0,\ L_z = 1\} \\ & + \{\underline{35};\ W = 1,\ W_z = -1,\ L_z = 2\} \end{aligned} \tag{4.1}$$

where the individual terms appear in arbitrary combinations. If the explicit quark model is used, the combinations are fixed and depend on parameters in the dynamics, such as quark mass.

It is interesting to note that the form of the current obtained by application of the Melosh transformation is in fact the most general which may be written, consistent with the restriction to single quark interactions.

In order to calculate matrix elements of the transverse current between free quark states, the current must be expressed in terms of single quark operators:

$$J_T^{+} = (aL^{+} + ibS^{+} + icL^{+}S^{o} + idL^{+}L^{+}S^{-})\, J \qquad (4.2)$$

where L^{+} is the $+$ part of a vector operator

$S^{\pm,o}$ are the quark spin operators

and $L^{\pm} = 1/\sqrt{2}\,(\mp L_x - iL_y)$

$S^{\pm} = 1/\sqrt{2}\,(S_x \pm iS_y)$, $\quad S^{o} = S_z$

and J is an SU(3) octet operator.

Thus a,b,c,d are $SU(6)_W$ singlet operators, which commute with L_z. Applying the reflection operator Y:

$$J_T^{-} = (aL^{-} + ibS^{-} - icL^{-}S^{o} + idL^{-}L^{-}S^{+})\, J$$

since $YL^{\pm}Y^{-1} = -L^{\mp}$

$YS^{\pm}Y^{-1} = -S^{\mp}$, $YS^{o}Y^{-1} = -S^{o}$

Taking the Hermitian conjugate:

$$(J_T)^{\dagger} = -\bar{\bar{J}}_T \qquad \text{for } J^{\dagger} = \bar{J}$$

and $(L^{\dagger})^{\dagger} = -L^{-}$

Hence a,b,c,d are Hermitian.

The validity of this derivation of the form of the transverse current may be questioned, since the matrix elements of $J_T^{\pm}$ taken between $SU(6)_W \times O(2)_{L_z}$ states are not invariant under boosts in the z-direction, whereas the Melosh transformation is expected to be meaningful only when applied to good operators. However, the transformation has the same form in an interacting theory as in the free case for which it was constructed, so it may be argued that the operators may be abstracted from the free case. Since no particular dynamical behaviour is specified, it may be hoped that the difficulty may be avoided and the current operator $J_T^{\pm}$ used with impunity. At $Q^2 = 0$, with real photons, $J_T^{\pm}$ can be related to good operators via current conservation [44], but not in general for $Q^2 \neq 0$.

This problem does not arise for the good longitudinal currents. For the longitudinal part of a vector current, the transformed current may be written:

$$\begin{aligned} J_L \sim \; & \{\underline{35};\ W = 0,\ W_z = 0,\ L_z = 0\} \\ & + \{\underline{35};\ W = 1,\ W_z = -1,\ L_z = +1\} \\ & + \{\underline{35};\ W = 1,\ W_z = 1,\ L_z = -1\} \end{aligned}$$

where the first term alone is the untransformed current, and again each term has an arbitrary coefficient.

Rewriting in terms of single quark operators:

$$J_L = (\alpha + i\beta L^- S^+ + i\gamma L^+ S^-) J$$

Applying the reflection conditions as before, except $YJ_LY^{-1} = J_L$, the coefficients are reduced to two, since $\beta = \gamma$.

Thus: $$J_L = (\alpha + i\beta [L^- S^+ + L^+ S^-]) J \qquad (4.3)$$

Taking the Hermitian conjugate, α and β are found to be Hermitian.

The axial current may be treated similarly, the transformed current being:

$$\begin{aligned} \tilde{J}_L \sim & \{\underline{35};\ W = 1,\ W_z = 0,\ L_z = 0\} \\ & + \{\underline{35};\ W = 1,\ W_z = -1,\ L_z = 1\} \\ & + \{\underline{35};\ W = 1,\ W_z = 1,\ L_z = -1\} \end{aligned}$$

where the first term is the untransformed current.

Rewriting:

$$\tilde{J}_L = (\tilde{\alpha} S^0 + \tilde{\beta} L^+ S^- + \tilde{\gamma} L^- S^+) J$$

Applying the reflection conditions, with $Y\tilde{J}_LY^{-1} = -\tilde{J}_L$ for an axial current, the coefficients are again reduced to two, with $\beta = -\gamma$, so that:

$$\tilde{J}_L = (\tilde{\alpha} S^0 + \tilde{\beta}[L^+ S^- - L^- S^+]) J \qquad (4.4)$$

Again $\tilde{\alpha}$ and $\tilde{\beta}$ are found to be Hermitian.

It is now possible to calculate the helicity amplitudes for the excitation of any resonance. For collinear initial baryon state B and final resonance state R, these are defined by:

$$
\begin{aligned}
A_{3/2} &= \langle R\ 3/2 \,|\, J_T^{+} \,|\, B\ \tfrac{1}{2} \rangle \\
&= \eta_R\,(-)^{J_R-\frac{1}{2}} \langle R\ \tfrac{1}{2} \,|\, J_T^{-} \,|\, B\ 3/2 \rangle \\
A_{\frac{1}{2}} &= \langle R\ \tfrac{1}{2} \,|\, J_T^{+} \,|\, B\ -\tfrac{1}{2} \rangle \\
&= \eta_R\,(-)^{J_R-\frac{1}{2}} \langle R\ -\tfrac{1}{2} \,|\, J_T^{-} \,|\, B\ \tfrac{1}{2} \rangle \\
A_L &= \langle R\ \tfrac{1}{2} \,|\, J_L \,|\, B\ \tfrac{1}{2} \rangle \\
&= \eta_R\,(-)^{J_R-\frac{1}{2}} \langle R\ -\tfrac{1}{2} \,|\, J_L \,|\, B\ \tfrac{1}{2} \rangle \\
\tilde{A}_L &= \langle R\ \tfrac{1}{2} \,|\, \tilde{J}_L \,|\, B\ \tfrac{1}{2} \rangle \\
&= -\eta_R\,(-)^{J_R-\frac{1}{2}} \langle R\ -\tfrac{1}{2} \,|\, \tilde{J}_L \,|\, B\ \tfrac{1}{2} \rangle
\end{aligned}
\tag{4.5}
$$

where η_R is the parity of resonance R

J_R is the spin of resonance R.

The wavefunctions used for the initial and final states are the standard SU(6) x O(3) wavefunctions with given angular momentum L, and definite parity, since baryons are unaffected by W-S flip.

4.3 Matrix elements and the non-diffractive contribution to inelastic structure functions

Transition matrix elements for transverse octet currents to excite members of 56- and 70-plets representations for arbitrary L are given in Table I, including the specialised cases of electromagnetic and weak $\Delta Q = 1$ currents for proton or neutron targets. This table also covers transverse axial currents, although the detailed coefficients are different. The corresponding matrix elements for longitudinal vector and axial currents are given in

Tables II and III. For the special case of L = 1 70-plets and L = 2 56-plets, of most practical interest, the results for transverse photons and pions reduce to those of refs. 44 and 45, save for matters of sign convention which are discussed in Appendix III. Predictions for strangeness-changing or arbitrary neutral octet currents may also easily be deduced from the tables.

The non-diffractive contribution to the inelastic structure functions for weak or electromagnetic currents may now be obtained by summing over the individual resonance contributions using the orthogonality relations for Clebsch-Gordan coefficients. The results for proton and neutron targets are given in Tables IV and V.

We assume that SU(6) x O(3) is an exact symmetry and that each supermultiplet contributes at a definite mass. Then since each 56- and 70-plet has the same form of contribution, independent of L, the total contributions from all 56's and 70's, weighted appropriately by their masses, is simply obtained by summing over the individual multiplets and replacing the coefficients a, b etc. by A, B which are the weighted sums of the a, b etc. for each multiplet. These total contributions are given in Tables VI and VII.

The most striking result arising from

the above treatment is that independent of relative abundances and couplings of 56's and 70's

$$A^{\gamma n} \equiv \frac{\sigma_{\frac{1}{2}}^{\gamma n} - \sigma_{3/2}^{\gamma n}}{\sigma_T^{\gamma n}}$$

$$= \quad 0$$

and $\quad \sigma_{TL}^{\gamma n} = 0$

where $\quad \sigma_{\frac{1}{2},3/2} = \sum A^2_{\frac{1}{2},3/2}$

$$\sigma_T = \sigma_{\frac{1}{2}} + \sigma_{3/2}$$

$$\sigma_{TL} = \sum A_L A_{\frac{1}{2}}$$

These relations are true for each 56 and 70 separately. Hence within this framework a non-zero value for $A^{\gamma n}$ or $\sigma_{TL}^{\gamma n}$ will be direct evidence for violation of the SU(6) symmetry assumption.

Apart from these two relations, the expression for the cross-sections arising from 56- and 70-plet final states differ, so that with no information on the relative importance of 56- and 70-plets, it is only possible to set bounds for various ratios rather than obtain absolute predictions.

Consider as an example the ratio $\sigma^{\gamma n}/\sigma^{\gamma p}$. If only 56's contribute, then for either transverse or electromagnetic currents, the ratio may be constrained by:

$$0 \leqslant \sigma^{\gamma n}/\sigma^{\gamma p} \leqslant 12/17 \qquad (4.6)$$

The lower bound results for A or α dominant (the spin zero terms) while the upper bound arises if B,C,D or β dominate (the spin one terms).

Similarly for only 70-plets present:

$$3/5 \leqslant \sigma^{\gamma n}/\sigma^{\gamma p} \leqslant 1$$

Thus from SU(6) assumptions only, the ratio is restricted to values between 0 and 1.

The lower bound of 0 is not that usually obtained in the quark parton model, where the value is 1/4, but the difference arises because the quark parton model usually includes an assumption of the incoherent impulse approximation. The lower bound obtained above is for a contribution from 56-plets only, which necessitates exotic t-channel exchanges, corresponding to non-incoherent impulse contributions (Fig 16).

If we now impose the restraint of only non-exotic exchanges in the t-channel, we find that this is exactly what is required to enforce exchange degeneracy of the 56- and 70-plet contributions. From Appendix III:

$$\sum_{B'J'M'} \left| \left\langle B^{56,L}_{J'M'} \right| J^+_T \left| B^{56,0}_{JM} \right\rangle \right|^2 = 3U_+ + 6E_+$$

$$\sum_{B'J'M'} \left| \left\langle B^{70,L}_{J'M'} \right| J^+_T \left| B^{56,0}_{JM} \right\rangle \right|^2 = 6U'_+ - 6E'_+ \qquad (4.7)$$

where $$U_+ = (\bar{B}^{56,0}_{JM})_{\alpha a,\beta b,\gamma c} (B^{56,0}_{JM})^{\alpha' a',\beta b,\gamma c} \times (\bar{J}J)^{\alpha}_{\alpha'} \times \left[(A^2 + C^2 + \tfrac{1}{2}B^2 + \tfrac{1}{2}D^2) + (2AC - \tfrac{1}{2}B^2 + \tfrac{1}{2}D^2)\sigma^0 \right]^a_{a'}$$

$$E_+ = (\bar{B}^{56,0}_{JM})_{\alpha a,\beta b,\gamma c} (B^{56,0}_{JM})^{\alpha' a',\beta' b',\gamma c} \times \bar{J}^{\alpha}_{\alpha'} J^{\beta}_{\beta'} \times \left[(A + C\sigma^0)^a_{a'} (A + C\sigma^0)^b_{b'} + B^2 \sigma^{-a}_{a'} \sigma^{+b}_{b'} + D^2 \sigma^{+a}_{a'} \sigma^{-b}_{b'} \right]$$

and U', E' are obtained by replacing A, B etc. by A', B' etc.

The terms E_+, E'_+ have a tensor structure and thus correspond to the currents landing on different quarks, which includes exotic exchanges. To exclude these contributions, there must be cancellation between E_+ and E'_+. This requires:

$$E_+ = E'_+$$

which is only true if:

$$(A \pm C)^2 = (A' \pm C')^2, \quad B^2 = B'^2, \quad D^2 = D'^2 \qquad (4.8)$$

Thus the coefficients for 56- and 70-plet contributions must be identical, representing exchange degeneracy between the couplings to transverse vector currents.

Similarly, by considering σ_L, $\tilde{\sigma}_L$ and σ_{TL}, the constraints may be extended to all the couplings:

$$\alpha^2 = \alpha'^2, \quad \beta^2 = \beta'^2, \quad \tilde{\alpha}^2 = \tilde{\alpha}'^2, \quad \tilde{\beta}^2 = \tilde{\beta}'^2$$

$$\beta(A \pm C) = \beta'(A' \pm C'), \qquad \alpha B = \alpha' B'$$

Substituting these values into the cross-sections, we obtain:

$$\sigma^{\gamma p} : \sigma^{\gamma n} : \sigma^{W^+ p} : \sigma^{W^+ n} = 1 : 2/3 : 1 : 2 \qquad (4.9)$$

for either transverse or longitudinal vector currents, in agreement with the results from the naive parton model [18], neglecting the transformation between current and constituent quarks.

On the other hand, the polarisation symmetries are affected by the inclusion of the transformation:

$$\begin{array}{ll} A^{\gamma p} = 5/9\ X & A^{\gamma n} = 0 \\ A^{W^+ p} = -1/3\ X & A^{W^+ n} = 2/3\ X \end{array} \qquad (4.10)$$

where $X = \dfrac{-4AC + B^2 - D^2}{2(A^2 + C^2) + B^2 + D^2} \qquad (-1 \leqslant X \leqslant 1)$

Excluding the transformation corresponds to $A = C = D = 0$ so $X = 1$, and all the asymmetries have a definite sign. However, taking the transformation into account, even the signs cannot be predicted though the magnitudes are bounded and the ratios are independent of X:

$$A^{\gamma n} : A^{\gamma p} : A^{W^+ p} : A^{W^+ n} = 0 : 5 : -3 : 6 \qquad (4.11)$$

Experimentally, the ratio of $\sigma^{\gamma n}/\sigma^{\gamma p}$ does not satisfy 4.9 near $\omega \sim 1$ (where $\omega = Q^2/2M\nu$), but the nature of the failure of the assumptions made cannot be further investigated without knowledge of the behaviour of $A^{\gamma n}$ and $A^{\gamma p}$ as well. One possibility is that SU(6) is violated, for instance by the nucleon containing a 70-plet part in its wavefunction.

4.4 Current algebra and sum rules

Several sum rules may be derived from the commutation relations of current operators, and from these may be derived various constraints when saturated with resonances belonging to 56- and 70-plets, for arbitrary L. In the model of $SU(6)_W$, only those sum rules involving forward scattering amplitudes may be discussed, because of the collinear nature of the symmetry.

The Adler-Weisberger sum rule is obtained from the commutator of two axial current operators:

$$\left[F_i^5 \; , \; F_j^5\right] \; = \; i \; f_{ijk} \; F_k \qquad (4.12)$$

At $Q^2 = 0$, the matrix elements of this equation between any states of the L = 0 56-plet, summed over any 56- or 70-plet with arbitrary L, giving for a 56:

$$\sum_{J'M'B'} \left\langle B_{J,M}^{56,0} \middle| \tilde{J}_{L,i} \middle| B_{J'M'}^{\prime 56,L} \right\rangle \left\langle B_{J'M'}^{\prime 56,L} \middle| \tilde{J}_{L,j} \middle| B_{JM}^{56,0} \right\rangle - i \leftrightarrow j \; = \; (\tilde{\alpha}^2 + \tilde{\beta}^2) i f_{ijk} V_k \qquad (4.13)$$

where $V_k \; = \; 3 \; (B_{JM}^{56,0})_{\alpha a, \beta b, \gamma c} \; (B_{JM}^{56,0})^{\alpha' a', \beta b, \gamma c} \times \; (\tfrac{1}{2}\lambda_k)_{\alpha'}^{\alpha} \; {}_{a'}^{a}$

and similarly a 70, except $\frac{1}{\sqrt{2}}\tilde{\alpha}, \frac{1}{\sqrt{2}}\tilde{\beta}$ are replaced by $\tilde{\alpha}', \tilde{\beta}'$.

V_k is the matrix element of the vector charge F_k between any members of the $L = 0$ 56-plet, so the sum rule reads:

$$\sum_{56} (\tilde{\alpha}^2 + \tilde{\beta}^2) + 2\sum_{70} (\tilde{\alpha}'^2 + \tilde{\beta}'^2) = 1 \qquad (4.14)$$

From Table III, $\tilde{A}_L^{W^+ n} = 5/3\,\tilde{\alpha}$, so that the old quark parton model result of $g_A/g_V = 5/3$ corresponds to assuming the Adler-Weisberger relation to be saturated just by the lowest 56-plet, with $L = 0$. The Melosh transformation requires additional terms, so that $g_A/g_V < 5/3$.

It should be noted that taking matrix elements of equation 4.12 between octet and decuplet states, or taking the difference of 4.12 between $M = 3/2$ and $M = \frac{1}{2}$ decuplet states, leads to superconvergence relations which are automatically satisfied within each 56- or 70-plet.

The commutator of the vector current operators is trivially satisfied for each SU(3) multiplet because of the assumption of exact SU(3) symmetry.

We may also consider the Cabibbo-Radicati [46] and Drell-Hearn-Gerasimov [47] sum rules decomposed according to isospin, with isovector and isoscalar currents between nucleon states. These read:

$$2F_1'^V(0) = \mu_A^{V\,2} - 16/9\,\mu^2 + \sum_R \frac{2}{(M_R^2 - M^2)^2}\left[\sigma_T^{W^+ n}(R) - \sigma_T^{W^+ p}(R)\right] \qquad (4.15)$$

where $F_1'^V(0) > 0$

and $F_1^V(0) = 1$ is the usual isovector form factor.

$$\mu_A^V = \mu_p - \mu_n - 1/2M$$
$$\mu_p = \mu, \quad \mu_p = -3/2\mu_n$$

and:

$$(\mu_A^V)^2 = 16/9\mu^2 + \sum_R \frac{2}{(M_R^2-M^2)^2}\left[\sigma_{3/2}^{W^+n}(R) + \sigma_{3/2}^{W^+p}(R) - \sigma_{\frac{1}{2}}^{W^+n}(R) - \sigma_{\frac{1}{2}}^{W^+p}(R)\right] \quad (4.16)$$

For photon-neutron scattering, the Drell-Hearn-Gerasimov sum rule is very simple, since $(\sigma_{3/2}^{\gamma n} - \sigma_{\frac{1}{2}}^{\gamma n})$ is zero for each <u>56</u>- and <u>70</u>-plet by itself, so the rule just reads:

$$\mu_n^2 = 4/9\mu^2$$

where only the Δ makes a contribution. This is in agreement with the usual SU(6) result for μ_n.

Similarly, for isoscalar photons:

$$(\mu_A^S)^2 = \sum_R \frac{4}{(M_R^2 - M^2)^2}\left[\sigma_{3/2}^{\gamma_s p}(R) - \sigma_{\frac{1}{2}}^{\gamma_s p}(R)\right] \quad (4.17)$$

where $\mu_A^S = \mu_p + \mu_n - 1/2M$

and $\sigma^{\gamma_s p} = \sigma^{\gamma_s n} = \sigma^{\gamma p} + \sigma^{\gamma n} - \frac{1}{2}(\sigma^{W^+p} + \sigma^{W^+n})$

may be easily deduced from our results.

Then substituting our results for the cross-sections in these sum rules, we obtain:

$$2F_1'^V(0) = (\mu_A^V)^2 - 16/9\mu^2 + \sum_{56} \frac{2}{(M_{56}^2 - M^2)^2}\left[2(A^2 + C^2) + B^2 + D^2\right] + \sum_{70} \frac{2}{(M_{70}^2 - M^2)^2}\left[2(A'^2 + C'^2) + B'^2 + D'^2\right] \quad (4.18)$$

$$(\mu_A^V)^2 = 16/9\mu^2 + \sum_{56} \frac{2}{(M_{56}^2 - M^2)^2}\left[20/3\ AC - B^2 + D^2\right] + \sum_{70} \frac{4}{(M_{70}^2 - M^2)^2}\left[8/3\ A'C' - B'^2 + D'^2\right] \tag{4.19}$$

$$(\mu_A^S)^2 = \sum_{56} \frac{2}{(M_{56}^2 - M^2)^2}\left[4/3\ AC - 1/9(B^2 - D^2)\right] + \sum_{70} \frac{4}{(M_{70}^2 - M^2)^2}\left[1/9(B' \quad - D' \quad)\right] \tag{4.20}$$

Taking the simplest possible approximation, neglecting all higher resonant states, we obtain for (4.19) and (4.20):

$$(\mu_A^V)^2 = 16/9\mu^2 \quad \text{and} \quad (\mu_A^S)^2 = 0$$

$$\text{or} \quad \mu_p = 3/2M\ , \quad \mu_n = -1/M \tag{4.21}$$

Even this crude simplification is already quite close to experiment. Including an admixture of <u>70</u> L = 1 (where D' = 0), it is clear that:

$$(\mu_A^S)^2 = B'^2 = 0 \tag{4.22}$$

is the only consistent solution, indicating that μ_A^S and B' for <u>70</u> L = 1 are likely to remain small. In general, the main conclusions to be drawn from equations 4.18,19,20 are that a consistent saturation requires the presence of C or D (C' or D') for some excited <u>56</u>- (<u>70</u>-) plet. Further, each set of excited supermultiplets contributes positively to $F_1'^V(0)$ in accord with its known positive result.

4.5 Conclusions

The results for the asymmetries $A^{\gamma n}$ etc. are interesting for several reasons. In the calculation of matrix elements, there is no restriction in the value of Q^2, so that the results should be valid over the whole range of the momentum transfer. We may therefore consider the sum rules which apply to both photo- and electro-production. In photoproduction, the Drell-Hearn-Gerasimov sum rule requires that the asymmetry on protons be negative over a substantial region, which is supported by some data [48], while for large Q^2 the Bjorken sum rule [49] suggests the asymmetry should be positive. Thus it seems probable that the asymmetry will show an interesting Q^2 variation.

On the basis of naive parton models, or light cone analysis [50], $X(Q^2)$ is expected to scale for large Q^2. Now, in the resonance region it seems that if $X(Q^2)$ is changing sign at all as Q^2 goes from zero to space-like, then it is doing so very slowly with increasing Q^2. On the other hand, the unpolarised structure functions scale down to very low Q^2. Thus it seems possible that either the asymmetry remains negative in the deep inelastic region and Bjorken's rule fails, or that scaling obtains only for very large Q^2 for the polarised structure functions.

It is also interesting to consider

role of the approach in this chapter. Since we have made no dynamical assumptions at all, the coefficients A, B etc. are completely arbitrary, so that scaling is not in any way either predicted or necessary to the theory. On the other hand, the theory is not troubled by the main defect of the naive parton model, that free quarks appear in the final state. Our results show that many of the good deductions from the parton model are in fact independent of the incoherent impulse approximation. If we include the assumption of no exotic t-channel exchanges we may also derive some other sum rules familiar in the quark-parton model. Starting from the Adler sum rule for vector currents:

$$\int_1^\infty d\omega/\omega \left[F_2^{\nu n}(\omega) - F_2^{\nu p}(\omega)\right] = 1 \quad (4.23)$$

and including the no exotic constraints:

$$\int_1^\infty d\omega/\omega \left[F_2^{\gamma p}(\omega) - F_2^{\gamma n}(\omega)\right] = 1/3 \quad (4.24)$$

and if the vector-axial interference is maximal and negative (as predicted in the naive model):

$$\int_1^\infty d\omega/\omega \left[F_3^{\nu p}(\omega) + F_3^{\nu n}(\omega)\right] = -6 \quad (4.25)$$

Thus we can see how our general results from SU(6) symmetry alone are incorporated into more specific models.

4.6 Postscript - the asymmetries paradox

In Section 4.3, we obtained the relation:

$$A^{\gamma p} = 5/9\, X\,, \quad -1 \leqslant X \leqslant 1$$

This seems to conflict directly with Bjorken's sum rule:

$$\int dx/x \left[A^{\gamma p}\, F_2^{\gamma p}(x) - A^{\gamma n}\, F_2^{\gamma n}(x)\right] = 1/3\, g_A/g_V$$

Since $A^{\gamma n}$ is predicted to be zero, this requires that $A^{\gamma p}$ be positive over a large range in x, since SU(6) gives $g_A/g_V = 5/3$.

In connection with this apparent paradox, it is interesting to see what happens when calculations are done in the current basis, i.e. transforming the constituent states into current states. It can be easily shown [51] that the transformed states may be written:

$$|\phi\rangle_c = \cos\theta\,|\psi\rangle + \sin\theta\,|\delta\psi\rangle$$

where $\quad V|\psi\rangle = |\psi\rangle + \lambda|\delta\psi\rangle$

and V is the Melosh transformation.

The result $(\sigma^{\gamma n}/\sigma^{\gamma p})_{ND} = 2/3$ is obtained as before, and the asymmetries are:

$$A^{\gamma n} = 0,$$

$$A^{\gamma p} = 5/9(\cos^2\theta - \sin^2\theta)$$

$$\equiv 5/9 \cos 2\theta$$

Thus it is clear that the effect of the transformation V is simply a rotation through the angle θ of the quark spins. This does not change the relative probabilities of spin 1 to spin 0 core, so that the SU(6) remains unbroken, hence the bad value for the ratio $(\sigma^{\gamma n}/\sigma^{\gamma p})_{ND}$.

Thus we see from this section that the Melosh transformation does not break SU(6), so that it can never lead to the empirical result $F_2^{\gamma n}/F_2^{\gamma p} \rightarrow \frac{1}{4}$ as $x \rightarrow 1$.

If we now calculate g_A/g_V in this formalism, we find that:

$$g_A/g_V = 5/3 \cos 2\theta \qquad (4.26)$$

so that the Bjorken sum rule is again realised.

Empirically, $g_A/g_V \approx 5/4$, so that we can find a value for $\theta = \tan^{-1}(1/\sqrt{7})$. This quantity is also interesting since it measures the amplitudes for finding $L_z = 0$ and $L_z \neq 0$. in the nucleon ground state in current space. This leads us to deduce that 40% of the amplitude has $L_z \neq 0$. This figure coincides with that obtained by Sehgal [52] in the parton model through a completely different approach.

Such a large proportion of $L_z \neq 0$ in the ground state shows why the $SU(6)_W$ of constituent quarks is inadequate to describe the behaviour of current quarks. But this section also shows that the Melosh transformation in its turn is not sufficient to explain the low results. Clearly some other mechanism which splits the spin 0 core from the spin 1 core results is necessary.

5. Relativistic quark models

5.1 Introduction

In the previous chapters, we have treated quarks strictly as a convenient description of the algebraic structure of hadrons, with no assumptions of their dynamics beyond that of their pointlike behaviour. Combined with the use of the Melosh transformation, this has led to a scheme for hadron interactions which seems to agree quite well with experimental observations, except for the region of low ω, where it is clear that SU(6) breaking, probably in the form of of unequal probabilities for a spin 0 and spin 1 core, must occur. The Melosh transformation does not introduce such breaking, merely rotating the quark spins, and as such is not sufficient to allow a full description of hadron interactions.

Having examined the algebraic structure of the quark model plus Melosh transformation with some success, it is interesting to consider the position of the transformation in a dynamic model. There are two distinct types of model possible, because of the problem of individual quarks not being seen outside the hadron. Either quarks are light and weakly bound but somehow contained within the hadron, or they are heavy and very strongly bound. In either case they must behave as quasi-free particles in order to produce the observed symmetries, so that it is reasonable to treat quarks as free Dirac particles, ignoring mass and binding problems for the purposes of investigating the basis for the Melosh transformation.

5.2 Relativistic quarks

The Hamiltonian for a free Dirac quark is:

$$H_o = \int d^4x \ \delta(t) \ \psi^\dagger (\underline{\alpha}.\underline{p} + \beta m) \psi \qquad (5.1)$$

where ψ is a superposition of solutions u(p,s) of the free Dirac equation.

None of the SU(6) symmetry operators (apart from unity) commute with this Hamiltonian. In fact O'Raffertaigh [53] showed that it is not possible to combine relativistic invariance with SU(6) invariance except by a direct product of the symmetry groups.

Thus to find any symmetry at all, we must look in a particular frame, the most obviously useful being the rest frame. The solutions u(p,s) of the Dirac equation are obtained by Lorentz boost of the rest frame solutions w(O):

$$u(p) = L(p) \ w(O) \qquad (5.2)$$

However the Lorentz boost is not a unitary transformation, which we require in order to leave the Hamiltonian unchanged:

$$H_o = \int d^4x \; \delta(t)\, \psi^{\dagger} e^{-iS}\, e^{iS}\, (\underline{\alpha}.\underline{p} + \beta m) \times e^{-iS}\, e^{iS}\, \psi$$

$$= \int d^4x \; \delta(t)\, \phi^{\dagger} e^{iS}\, (\underline{\alpha}.\underline{p} + \beta m)\, e^{-iS} \phi \qquad (5.3)$$

where $u(p,s) = e^{iS}\, w(0)$

$$= L(p)\, w(0)$$

and ϕ is a superposition of rest frame wave-functions w.

This simply entails renormalising w(0) such that $w^{\dagger} w = u^{\dagger} u$ (instead of $\bar{w}w = \bar{u}u$), and this defines S.

Hence we can find:

$$e^{iS}\, (\underline{\alpha}.\underline{p} + \beta m)\, e^{-iS} = \sqrt{m^2 + \underline{p}^2}\; \beta$$

and the free Hamiltonian is:

$$H_o = \int d^4x \; \delta(t)\, \phi^{\dagger} \beta \sqrt{m^2 + \underline{p}^2}\; \phi \qquad (5.4)$$

This Hamiltonian is invariant under transformations of the form:

$$\phi \rightarrow \phi - i\Lambda\phi$$

where
$$[\Lambda , \beta] = 0$$

Thus Λ is precisely the set of SU(6) x SU(6) matrices, and the transformation manifestly demonstrates that relativistic free quarks have this symmetry in the rest frame.

Another useful frame is that in which one component of momentum is boosted to infinity. Choosing the z-direction, states of large momentum may be obtained by boosting states of fixed momentum. Boosting with rapidity ω:

$$\begin{aligned} x' &= x \\ y' &= y \\ z' &= z\cosh\omega + t\sinh\omega \\ t' &= t\cosh\omega + z\sinh\omega \end{aligned} \qquad (5.5)$$

The quantities which are most interesting from the point of view of calculations are the charges $F(\Lambda)$:

$$F(\Lambda) = \int d^4x\ \delta(t)\,\psi^\dagger\Lambda\psi \qquad (5.6)$$

These charges do not commute with the free Hamiltonian, but the SU(6) generators $W(\Lambda)$ do, where $W(\Lambda)$ are related to $F(\Lambda)$ by the Foldy-Wouthuysen transformation [40]. It would have simplified the problem greatly had the charges $F(\Lambda)$ commuted with the Hamiltonian instead. They do form amongst themselves an SU(6) algebra however, so it may be that they can be used as symmetry operators after all. Transforming the operators to the infinite momentum frame:

$$\begin{aligned} \hat{F}(\Lambda) &= e^{\frac{1}{2}\omega\alpha_3}\ F(\Lambda) \\ &= 2\int d^4x\ \delta(t + z\tanh\omega)\psi^\dagger\Lambda'\psi \end{aligned} \qquad (5.7)$$

where

$$\begin{aligned} \Lambda' &= e^{\frac{1}{2}\omega\alpha_3}\ \Lambda\ e^{-\frac{1}{2}\omega\alpha_3} \\ &= \frac{1+\alpha_3\tanh\omega/2}{2}\ \frac{\Lambda\, 2\cosh^2\omega/2}{\cosh\omega/2}\ \frac{1+\alpha_3\tanh\omega/2}{2} \end{aligned}$$

and allowing $\omega \to \infty$:

$$\hat{F}(\Lambda) = 2\int d^4x\ \delta(t+z)\psi^\dagger\tfrac{1}{2}(1+\alpha_3)\Lambda\tfrac{1}{2}(1+\alpha_3)\psi \qquad (5.8)$$

These new operators should still have the same symmetry properties as before, since they have simply been Lorentz boosted. However, many of the SU(6) x SU(6) operators anticommute with α_3, and hence the operators corresponding to these values of Λ vanish identically. Those which do not vanish commute with α_3, forming a subalgebra, $SU(6)_W$:

$$\Lambda = (1, \lambda_i)\left[1, \beta\sigma_1, \beta\sigma_2, \sigma_3\right] \qquad (5.9)$$

where λ_i are the SU(3) matrices.

The corresponding good operators also satisfy this algebra, provided limits of commutators are identified with commutators of limits for these operators [54]. They also commute with the free Hamiltonian.

Thus relativistic quarks satisfy an SU(6) x SU(6) algebra in the infinite momentum limit.

A further point to be noticed concerning the new charges $\hat{F}(\Lambda)$ is that $\delta(t)$ has been replaced by $\delta(t+z)$ so that the combination $p_3 + p_0$ is now conserved. Nevertheless, if null-plane anticommutation relations are assumed for the wavefunctions ψ, only those for the good components coincide with the infinite momentum limit method, indicating that it is not always reliable because of the assumption which had to be made regarding limits of commutators.

The first hope was that $\hat{F}(\Lambda)$ would be suitable particle operators, but this was dismissed since these operators lead to zero anomalous magnetic moment for nucleons, in contradiction with the large moment observed. This being the case, Melosh suggested that perhaps the $W(\Lambda)$ when boosted to infinite momentum might prove more suitable. Since the $W(\Lambda)$ contain p_3 which is not invariant under boosts in the z-direction, Melosh modified the generators by using a Foldy-Wouthuysen transformation which removed only α_1 and α_2, but not α_3 from the Hamiltonian. This then removes the term $\alpha_3 p_3$ from $W(\Lambda)$ and the Hamiltonian is:

$$H_0 = \int d^4x \; \delta(t) \; \phi^\dagger \left[\beta\sqrt{m^2 + p_\perp^2} + \alpha_3 p_3 \right] \phi \qquad (5.10)$$

This then means that the symmetry of the $W(\Lambda)$ is reduced to invariance under transformations such that:

$$\begin{aligned} [\Lambda, \beta] &= 0 \quad \text{as before, but also} \\ [\Lambda, \alpha_3] &= 0 \end{aligned} \qquad (5.11)$$

This is not a disadvantage however, since the set of Λ is now the set of $SU(6)_W$ operators and these are the ones most relevant to the problem. Boosting the generators $W'(\Lambda)$ now produces the operators $\hat{W}(\Lambda)$, which also have an $SU(6)_W$ in a similar way to the $\hat{F}(\Lambda)$. Then the question arises as to whether the $\hat{W}(\Lambda)$

are any more suitable than the $\hat{F}(\Lambda)$, since the original problem was simply to find a symmetry, which the $W(\Lambda)$ and $\hat{F}(\Lambda)$ already possess. Performing two transformations each of which leads to a symmetry is just solving the problem twice.

One way of choosing a preferable set of operators is to consider the rotation properties, since we know what properties we require for particles. Because the rotation matrices anticommute with α_3, complicated relations ensue for the $F(\Lambda)$ [54] such that it is not possible for a particle to have a definite spin under the $F(\Lambda)$ and at the same time belong to a definite representation of $SU(6)_W$ under the $\hat{F}(\Lambda)$. On the other hand, the relations are simply satisfied for the $W'(\Lambda)$ and $\hat{W}(\Lambda)$.

With this in mind we may now start again from the Foldy-Wouthuysen transformation and simply renormalise it slightly differently in order to achieve the required rotation properties, instead of arbitrarily removing the p_3 and α_3 parts as for $W'(\Lambda)$. This leads to the second Melosh transformation:

$$\phi = \frac{m + |p_0 + p_3| + i\,\gamma_\perp . p_\perp}{\sqrt{(m + |p_0 + p_3|)^2 + p_\perp^2}}\,\psi \tag{5.12}$$

Unfortunately, this procedure has

resulted in the loss of longitudinal boost invariance, which the original $W(\Lambda)$ generators had, so that it seems that one difficulty is removed only at the expense of creating another. The new generators then have lost all boost invariance, so that they should be symmetries only for systems at rest, while for moving systems, one must deboost to rest and then reboost.

In the free quark case, the modified generators $\hat{W}(\Lambda)$ are constants of the motion, as are all the null-plane charges, and in fact are the same as the Foldy-Wouthuysen transformed generators $\overline{W}(\Lambda)$. Whether the transformation remains in any way effective for a system with interaction must depend solely on the strength and type of interaction.

5.3 Quark model philosophies

At this stage, we have pointed out the problems arising from an attempt to describe vertex symmetries by the same $SU(6)_W$ as the resonance spectrum and considered the resolution of this difficulty by means of the Melosh transformation. Both of the suggestions put forward by Melosh have their disadvantages, although as can be seen from Chapter 4 the algebraic scheme is extremely useful. From it can be deduced that the free transformation is moderately appropriate at least, although clearly $SU(6)_W$ symmetry is broken to a minimum of 20 to 25%.

In view of this it is not unreasonable to look for alternative methods of resolving the constituent quark-current quark dilemma. Such a possibility is to reject the idea that there is any fundamental difference between the two types of quark, save that one is at rest while the other is moving. The difference is then expressed in the Lorentz boost. It is interesting therefore to investigate relativistic formulations of the quark model in order to discover whether the effects of the Melosh transformation may be described in this way, purely by invoking dynamics.

In section 5.1, we saw that free relativistic quarks with spin-$\frac{1}{2}$ satisfy an SU(6) x SU(6) x O(3) algebra in the rest frame. In the discussion of hadrons, we have a choice in deciding how to view the quarks. Either they are light with weak binding, so that the symmetries still hold, or they are heavy with strong binding and small effective mass, but somehow the symmetry is carried over to the effective quarks. From the failure of the search for quarks experimentally, we are forced to favour the second case, though then the success of SU(6) for classification becomes almost incredible, since there is no compelling reason why the bound quarks should behave as if they were free. A further alternative is to suppose that free quarks do not exist, and that the quark model is simply a phenomenological description of the internal symmetries of hadrons.

In formulating models appropriate to any of these viewpoints, there are two important criteria to be incorporated. The first is that to have any physical significance, the model must reduce to the non-relativistic quark model in the rest frame, since it is so successful there. Secondly, the interaction between quarks is severely restricted because of the observed linearly rising hadron mass spectrum. The physical origin of the behaviour is not clear, so that the best that can be done is to accept it as a fundamental fact, and adjust the interaction accordingly. If we assume that the rise of the trajectories is exactly linear, we must inject a harmonic oscillator form of interaction.

With these points in mind, we are forced to reject any model in which the interaction potential is singular at the origin. Thus a simple field theory, treating quarks as fundamental particles bound by a neutral vector gluon field, is not suitable, in spite of its success in the formulation of current commutation relations and light cone algebra.

Many relativistic models which do satisfy the above criteria have been put forward, encompassing all the standpoints mentioned previously as to the nature of quarks. It is the object of the following sections to discuss some of these models.

5.4 Dual models

In these models an attempt is made to include the known crossing and analyticity constraints in the description of hadron interactions. Veneziano [55] succeeded in parametrising the invariant amplitude for the process $\pi\pi \rightarrow \omega\pi$, which was then extended to a formula for an n-point function [56]. The amplitude contained both s- and t-channel descriptions without allowing double poles or

any unwanted singularities. However, since linearly rising trajectories were an input of the model, unitarity has been neglected, and it was hoped that this would turn out to be an acceptable approximation.

These results may also be reached by treating the hadron as made up of an infinite number of partons, which may be real constituents or simply degrees of freedom. Then since each particle consists of an infinite number of these, interactions take place by an infinite number of interactions between the individual partons. Low order diagrams with small numbers of interactions become meaningless and the high order diagrams dominate (Fig.17). Because of the infinite number of partons, there exists a symmetry between the channels, and if the partons are treated as scalars, a statistical model [57] gives the Veneziano amplitudes.

The model may also be approached from a different angle, by considering the free Hamiltonian of the constituents of the hadron. Treating them as scalars, this may be written as an infinite sum of covariant oscillators:

$$H = \frac{1}{2} \sum_{n=0}^{\infty} \left[p_n^2 + \omega_n^2 q_n^2 \right] \tag{5.13}$$

where $\omega_{n+1} - \omega_n = \omega$, $n = 0,1,2,\ldots$
It is assumed that the physical limit is that when $\omega_0 \to 0$, when the zeroth mode becomes translational. Thus p_0 is the centre of mass momentum, while p_n, q_n are the internal momenta and positions.

In this limit, all the physical observables of the system may be written as averages over the period of internal cycles. The internal time, τ, is defined by means of the Heisenberg equations:

$$[H , f] = i\, df/d\tau \qquad (5.14)$$

where f is any operator.
The fundamental cycle is taken to be:

$$-\pi/\omega \leqslant \tau \leqslant \pi/\omega \qquad (5.15)$$

In particular, if $a^{\dagger}_{n\mu}(\tau)$ and $a_{n\mu}(\tau)$ are the harmonic oscillator creation and annihilation operators corresponding to the internal motion, then:

$$a_{n\mu}(\tau) = e^{i\omega_n \tau} a_{n\mu} \qquad (5.16)$$

The total momentum of the particle may be written:

$$\begin{aligned} P_\mu &= \sum_{n=0}^{\infty} p_{n\mu} \\ &= p_{o\mu} + \sum_{n=1}^{\infty} \sqrt{\omega_n/2}\,\big(a^{\dagger}_{n\mu}\, e^{i\omega_n \tau} \\ &\qquad + a_{n\mu}\, e^{-i\omega_n \tau}\big) \end{aligned} \qquad (5.17)$$

and the observed momentum is the time-averaged

P_μ:

$$p_\mu = \langle P_\mu \rangle = p_{o\mu} \tag{5.18}$$

We are now in a position to generalise the Klein-Gordan equation as follows:

$$p^2 - m_o^2 = \langle P \rangle . \langle P \rangle - m_o^2 \rightarrow \langle :P^2: \rangle - m_o^2$$

Then:

$$(p^2 - m_o^2)\phi \rightarrow (p_o^2 + \sum_{n=1}^{\infty} \omega_n a^\dagger_{n\mu} a_n^\mu - m_o^2)\phi \tag{5.19}$$

where the normal ordering is to eliminate the zero-point energy.

Thus the new mass operator is:

$$\mathcal{M}^2 = m_o^2 + \sum_{n=1}^{\infty} \omega_n a^\dagger_{n\mu} a_n^\mu \tag{5.20}$$

A difficulty now arises since the time-like states have negative normalisation and are thus unphysical. They are most simply removed by gauge conditions:

$$p_\mu a_n^\mu \,|\text{physical state}\rangle = 0 \tag{5.21}$$

i.e. $\langle P_\mu(\tau) \rangle \langle e^{i\omega_n \tau} P^\mu(\tau) \rangle \,|\text{physical state}\rangle$

$$= 0 \tag{5.22}$$

Replacing $\langle P_\mu(\tau) \rangle \langle e^{i\omega_n \tau} P^\mu(\tau) \rangle$ by $\langle e^{i\omega_n \tau} :P_\mu P^\mu: \rangle$ we obtain:

$$\langle e^{i\omega_n \tau} :P_\mu P^\mu: \rangle \,|\text{physical state}\rangle = 0 \tag{5.23}$$

These are exactly the Virasoro gauge conditions [58]. They can be satisfied provided the dimension of space is allowed to vary, and for 25 space dimensions and 1 time, there are no unphysical states. Also it is found that in this dimension of space, the pomeron may be represented simply by a pole, instead of the more complicated singularity necessary in any other dimension in this model.

To this point, the hadron constituents, or partons, have been treated as scalars. To incorporate spin in a dynamical way, Ramond made an analogous generalisation of the Dirac equation [59]. In order to do this it is necessary to find some generalisation of the Dirac matrices, such that the average over a cycle should reproduce the usual matrices:

$$\langle \Gamma_\mu(\tau) \rangle = \gamma_\mu \tag{5.24}$$

The simplest anticommutation relations consistent with those of the usual matrices are:

$$\left\{ \Gamma_\mu(\tau), \Gamma_\mu(\tau') \right\} = 2g_{\mu\nu}\delta\left[\omega/2\pi\ (\tau - \tau')\right] \tag{5.25}$$

Again appealing to simplicity, we have for the adjoint matrices:

$$\Gamma^\dagger_\mu(\tau) = \gamma_0 \Gamma_\mu(\tau) \gamma_0 \tag{5.26}$$

Assuming that $\Gamma_\mu(\tau)$ may be expressed as a Fourier series over the internal cycle, the matrices may be written:

$$\Gamma_\mu(\tau) = \gamma_\mu + i\sqrt{2}\,\gamma_5 \sum_{n=1}^{\infty} (b^\dagger_{n\mu} e^{i\omega_n\tau} + b_{n\mu} e^{-i\omega_n\tau}) + i\omega_0\tau\,\delta_\mu \qquad (5.26)$$

provided the coefficients obey the anti-commutation relations:

$$\begin{aligned} \{b_{n\mu}, b_{m\nu}\} &= \{b^\dagger_{n\mu}, b^\dagger_{m\nu}\} = 0 \\ \{b^\dagger_{n\mu}, b_{m\nu}\} &= g_{\mu\nu}\,\delta_{mn} \end{aligned} \qquad (5.27)$$

The last term is neglected in the limit $\omega_0 \to 0$, under the assumption that δ_μ is not singular in this limit.

The Dirac equation may now be generalised in an analagous way to the Klein-Gordan equation:

$$\begin{aligned} (p_\mu \gamma^\mu - m_0)\,\phi &= 0 \\ \to \quad [\langle P_\mu\rangle\langle\Gamma^\mu\rangle - m]\,\phi &= 0 \end{aligned} \qquad (5.28)$$

Writing this in terms of creation and annihilation operators:

$$\left\{ \gamma_\mu p_0^\mu - m_0 - \gamma_5 \sum_{n=1}^{\infty} \omega_n (a^\dagger_{n\mu} b_n^\mu + b^\dagger_{n\mu} a_n^\mu) \right\} \phi = 0 \qquad (5.29)$$

Squaring the equation, we obtain:

$$\begin{aligned} &\left\{ \langle \Gamma_\mu P^\mu\rangle\langle\Gamma_\mu P^\mu\rangle - m_0^2 \right\}\phi \\ &= \left[\langle P^2\rangle - \tfrac{1}{4} i \langle \Gamma_\mu \dot{\Gamma}^\mu\rangle - m_0^2\right]\phi = 0 \end{aligned} \qquad (5.30)$$

where $\dot{\Gamma}_\mu = d/d\tau\,(\Gamma_\mu)$

from the anticommutation relations for Γ_μ

and since $\{P_\mu(\tau), P_\nu(\tau)\} = -\frac{1}{2} i g_{\mu\nu}\, d/d\tau\,\delta[\omega/2\pi(\tau - \tau')]$

In terms of creation and annihilation operators:

$$\left\{ p_0^2 - m_0^2 + \sum_{n=1}^{\infty} \omega_n (a^\dagger_{n\mu} a_n^\mu + b^\dagger_{n\mu} b_n^\mu) \right\}\phi = 0 \qquad (5.31)$$

This leads to linear trajectories in the physical limit $\omega_n \to n\omega$, with masses of excited states given by:

$$m_l^2 = m_0^2 + l\omega \qquad (5.32)$$

where $l = 0,1,2,\ldots$

As with the Klein-Gordan equation, gauge conditions are necessary to remove unphysical states, and these may be written:

$$\left\langle e^{\pm i\omega_n \tau} \Gamma_\mu P^\mu \right\rangle \mid \text{physical state} \rangle = 0 \qquad (5.33)$$

corresponding to:

$$p_\mu b_n{}^\mu \mid \text{physical state} \rangle = 0 \qquad (5.34)$$

Again the non-physical states can only be decoupled completely in an unphysical number of dimensions, though less than for the scalar parton case. It is hoped that if further symmetries can be included in the model, the critical number of space dimensions will drop to three, though at present it is not clear how symmetries such as isospin may be incorporated non-trivially.

5.5 The Bethe-Salpeter equation

The difficulties of the dual model are such as to leave it unphysical at its present stage, so that perhaps unitarity should not be approximated in this way. A manifestly relativistic approach can be made through the Bethe-Salpeter equation [60] for the bound state of a fermion and antifermion.

In momentum space this may be written:

$$(\not{p}_1 - m_1)_{\alpha\gamma} \psi_{\alpha\beta}(p_1 - p_2)(\not{p}_2 + m_2)_{\beta\delta} = \lambda \int d^4k/(2\pi)^4 \, V_{\gamma\delta\rho\sigma}(p_1, p_2, k)\psi_{\rho\sigma}(k) \tag{5.35}$$

Rewriting in terms of centre of mass variables:

$$(\tfrac{1}{2}\not{P} + \not{q} - m)\psi(q)(\tfrac{1}{2}\not{P} - \not{q} + m) = \lambda \int d^4k/(2\pi)^4 \, V(P,q,k)\psi(k) \tag{5.36}$$

where we have taken:

$$m_1 = m_2$$

and:

$$P = p_1 + p_2$$

$$q = \tfrac{1}{2}(p_1 - p_2)$$

ψ is a covariant wavefunction and may be expressed in terms of the Fermi bilinears Γ. The binding is due to the repeated action of a potential V. Hence the equation may be represented diagramatically as in Fig. 18.

Provided the potential depends only on the exchange momentum, i.e. if:

$$V(p_1, p_2, k) = V(k - \overline{p_1 - p_2}) \tag{5.37}$$

the equation may be re-expressed in position space by the ladder approximation:

$$(\tfrac{1}{2}\not{P} + i\not{\partial} - m)\psi(x)(\tfrac{1}{2}\not{P} - i\not{\partial} + m) = \lambda V(x)\psi(x) \tag{5.38}$$

The wavefunctions are normalised to the charge on the constituents as in Fig. 18:

$$\langle\psi | j^A(q=0)/e^A|\psi\rangle = \int d^4k/(2\pi)^4 \, \mathrm{Tr}\left[\bar{\psi}(k)\gamma_\mu \psi(k) \times (\tfrac{1}{2}\not{P} + \not{k} + m)\right] \equiv 2P_\mu \tag{5.39}$$

where e^A is the charge on particle A,
and $\bar{\psi} = \gamma_0 \psi^\dagger \gamma_0$

The equation is extremely complicated to solve, but can be rendered more tractable by considering massless bound states as suggested by Bohm, Joos and Krammer [61], since the equation then exhibits O(3.1) symmetry. Continuing analytically the q_0 variable to pure imaginary values [62] the equation has O(4) symmetry leading to a hyperradial eigenvalue equation for λ. Having solved this, Bohm et al. (B.J.K.) argue that quarks are very heavy so that:

$$m_{hadron} \ll m_{quark}$$

so that putting the bound state mass equal to zero is satisfactory for a first approximation. Then to improve the results, a perturbation expansion in m_{hadron}/m_{quark} may be made.

The potential V(k - q) must be further restricted to the harmonic oscillator form in order to produce linear trajectories, and to give the required charge conjugation and parity spectrum. This is done by the choice of bilinears, and the suitable ones are:

$$\begin{aligned} V(x) &= (\gamma_5 + \gamma_\mu + 1)(\alpha + \beta x^2) \\ V(x) &= (\gamma_5 + \sigma_{\mu\nu} - \gamma_5\gamma_\mu)(\alpha + \beta x^2) \end{aligned} \qquad (5.40)$$

Both interactions lead to some negative normalisation states, and no spin-orbit coupling is included. When applied to meson decay amplitudes, the predicted widths are in reasonable agreement with experiment, correct to about 20 to 25% [63].

5.6 The quark model of Feynman, Kislinger and Ravndal (FKR)

Rather than using the Bethe-Salpeter equation as a basis, FKR [64] start from the non-relativistic oscillator quark model [65] and generalise it in an idiosynchratic way to include relativity.

The Hamiltonian for two particles bound by a harmonic oscillator potential is:

$$H = (1/2m)\underline{p}_1^2 + (1/2m)\underline{p}_2^2 + m\omega_0^2(\underline{x}_1 - \underline{x}_2)^2 \qquad (5.41)$$

or, writing:

$$\underline{q} = \tfrac{1}{2}(\underline{p}_1 - \underline{p}_2)$$

$$\underline{x} = (\underline{x}_1 - \underline{x}_2)$$

in the rest frame:

$$H = (1/m)q^2 + m\omega_0^2 x^2 \qquad (5.42)$$

$$\text{i.e.} \quad 4mH = 4(q^2 + \omega^2 x^2) + \text{constant} \qquad (5.43)$$

where $\omega = m\omega_0$

Now the relativistic energy squared is:

$$m_{meson}^2 = (2m + H)^2$$

and since $H \ll m$,

$$m_{meson}^2 \approx 4m^2 + 4mH$$

Hence, adding $4m^2$ to 5.43, we obtain:

$$m^2_{meson} = 4(\underline{q}^2 + \omega \underline{x}^2) + \text{constant} \tag{5.44}$$

Equation 5.44 may easily be generalised to a covariant form:

$$K = -2\left[p_1^2 + p_2^2 + 2\omega^2(x_1 - x_2)^2\right] \tag{5.45}$$

where now $p_{1\mu}, p_{2\mu}, x_{1\mu}, x_{2\mu}$ are four-vectors.

In the centre of mass frame:

$$K = P^2 - 4(q^2 + \omega^2 x^2) \tag{5.46}$$

where $P_\mu = p_{1\mu} + p_{2\mu}$

Thus comparing with equation 5.44, $K - P^2$ may be identified with the mass squared operator. Defining creation and annihilation operators:

$$\begin{aligned} \sqrt{2\omega}\, a_\mu^\dagger &= q_\mu + i\omega x_\mu \\ \sqrt{2\omega}\, a_\mu &= q_\mu - i\omega x_\mu \end{aligned} \tag{5.47}$$

we obtain:

$$K - P^2 = \mathcal{M}^2 = -\Omega\, a_\mu^\dagger a^\mu + \text{constant} \tag{5.48}$$

where $\Omega = 8\omega$.

This leads to straight Regge trajectories of slope Ω^{-1}. The ground state meson corresponds to the oscillator vacuum state (apart from SU(3) labels) and excited states are generated by the action of the creation operators. As in dual models, the problem of negative normalisation timelike states arises, but here FKR eliminate them by fiat. They adopt the condition:

$$P.a|\psi\rangle = 0 = m.a_0|\psi\rangle \tag{5.49}$$

in the rest frame. This means that the set of physical states is no longer complete, so unitarity is violated. Consequently the decay matrix elements will be increasingly too big with rising energy, and a correction factor must be introduced to compensate for this.

As yet no mention has been made of spin, and this is artificially introduced by the interpretation of p_i in equation 5.45 as $\not{p}_i\not{p}_i$, where $\not{p}_i = p_{i\mu}\gamma_i^{\mu}$. It is then assumed that Dirac relations are obeyed individually for each quark. These restrictions are equivalent to assuming quasi-free quarks within the hadron. Note that both quark and antiquark are treated as quarks, one with the opposite charge, presumably to maintain a close correspondence to the non-relativistic model.

Under these conditions, the electromagnetic current is easy to deduce. In the presence of an electromagnetic potential A_μ, the operator $\not{p}$ is replaced by $(\not{p} - e\not{A})$, where e is the charge on the interacting particle, so that the mass squared operator is perturbed by an amount:

$$\delta K = 3\sum_{a} e_a(\not{p}_a \not{A}_a + \not{A}_a \not{p}_a) \qquad (5.50)$$

summing over all three quarks, and neglecting the second order term $(e\not{A})^2$. Taking the case of a plane wave carrying momentum q_μ and with polarisation vector ϵ_μ, we obtain:

$$\delta K = 3\sum_{a} e_a(\not{p}_a \gamma_\mu e^{iq.ua} + \gamma_\mu e^{iq.ua} \not{p}_a)\epsilon^\mu \qquad (5.51)$$

so that the vector current is:

$$j_\mu^V = 3\sum_{a} e_a(\not{p}_a \gamma_\mu e^{iq.ua} + \gamma_\mu e^{iq.ua} \not{p}_a) \qquad (5.52)$$

where u_a is the position of quark a.

Similarly the axial current is:

$$j_\mu^A = 3i\sum_{a} e_a(\not{p}_a \gamma_\mu \gamma_5 e^{iq.ua} + \gamma_5 \gamma_\mu e^{iq.ua} \not{p}_a) \qquad (5.53)$$

and its divergence is:

$$\partial_\mu j^{A\mu} = 3\sum_{a} e_a(\not{p}_a \gamma_5 \not{q} e^{iq.ua} + \gamma_5 \not{q} e^{iq.ua} \not{p}_a) \qquad (5.54)$$

These currents may now be expressed explicitly in terms of centre of mass variables and creation and annihilation operators, and specific matrix elements calculated. The results obtained are mostly within 20% of the experimental data, although there are several bad disagreements. Some of these arise because a decay to two pseudoscalar mesons, for example, may be calculated by treating either as the current, Then since the calculations depend on the mass of this particle, different answers are obtained in each case.

The scheme may be simply extended to include baryons also and the same difficulties appear again.

With regard to the Melosh transformation, it should be noted that the electromagnetic current contains no spin-orbit coupling term. For example, in the decay of the $D_{13}(1520)$, the amplitudes $A^p_{3/2}$ and $A^n_{3/2}$ are of equal magnitude but opposite sign, and whereas the data [66] is just consistent with this, it is thought that [67]

$$A^n_{3/2} \approx -2/3\, A^p_{3/2} \tag{5.55}$$

which would be the case if there existed a spin-orbit term of the same strength as the orbital term i.e. A = C in the Melosh transformed vector current.

However, there is some spin-orbit coupling contained in the axial current, so that the bad $SU(6)_W$ results for the decay $B \to \omega\pi$ are improved. Partly therefore, this model succeeds in duplicating the effects of the Melosh transformation, though by no means entirely. Possibly this partial failure is due to the way in which spin is so artificially introduced, and a more natural treatment might lead to greater success.

6. A relativistic quark model for mesons

6.1 Introduction

In the previous chapter, some dynamical models attempting to construct relativistic descriptions of hadrons were discussed, and none of them were able to reproduce fully the effects of the Melosh transformation. Nor has there arisen any fundamental resolution of the difficulty that quarks are apparently both very strongly bound, since they must be heavy to escape observation, and quasi-free within the hadron. Taking the infinite mass limit for quarks is however theoretically useful, since in this case it has been shown [68] that there exists a relation between bound state quark dynamics and the dual model. This may be demonstrated by allowing the quark mass to become infinite in the Bethe-Salpeter equation for two bound scalar particles, when the equation reduces to a linear form. When the equation is solved for a harmonic oscillator interaction, it is possible to find an expression for the four-point function in the narrow resonance approximation which is closely related to the Veneziano formula. This is essentially also the treatment in the FKR model, which we may therefore regard as a lowest order approximation to the Veneziano model. The fact that the FKR model only has one creation and annihilation operator, rather than a sum over an infinite number, is interpreted as being a neglect of the sea of $q\bar{q}$ pairs.

In a similar way, it is possible [69] to solve the Bethe-Salpeter equation for two spin-$\frac{1}{2}$ paticles, and thus obtain a lowest order approximation to the Ramond model.

6.2 The FKR model from the Bethe-Salpeter equation

We consider first the alternative method of deriving the FKR model suggested above, starting from the Bethe-Salpeter equation for scalar particles:

$$(p_1^2 - m^2)\psi(p_2^2 - m^2) = V\psi' \qquad (6.1)$$

assuming $m_1 = m_2 = m$

Taking the limit $m \rightarrow$ large:

$$(p_1^2 + p_2^2 - m^2 + U)\psi = 0 \qquad (6.2)$$

where $U = V/m^2$

In order to obtain linearly rising trajectories, U must be of harmonic oscillator form, giving:

$$(p_1^2 + p_2^2 + 2\omega^2 x^2 + C)\psi = 0 \qquad (6.3)$$

Separating into centre of mass variables:

$$\left[\tfrac{1}{2}P^2 + 2(q^2 + \omega^2 x^2) + C\right]\psi = 0 \qquad (6.4)$$

where
$$P = p_1 + p_2$$
$$q = \tfrac{1}{2}(p_1 - p_2)$$
$$C = \text{constant}$$

The potential U includes a constant part in order to cancel out the large mass and allow the bound states to have only a small mass.

Introducing the standard harmonic oscillator creation and annihilation operators:

$$\begin{aligned} \sqrt{2\omega}\, a_\mu^\dagger &= q_\mu + i\omega x_\mu \\ \sqrt{2\omega}\, a_\mu &= q_\mu - i\omega x_\mu \end{aligned} \qquad (6.5)$$

and substituting into equation 6.4 we obtain the FKR model:

$$(P^2 + \Omega\, a_\mu^\dagger a^\mu + 2C)\psi = 0 \qquad (6.6)$$

where
$$\Omega = 8\omega$$
$$P^2 = \mathcal{M}^2 = (\text{meson mass})^2$$

6.3 The model

We now attempt to repeat this treatment for spin-$\frac{1}{2}$ quarks, again starting from the Bethe-Salpeter equation:

$$(\not{p}_1 - m)\psi(\not{p}_2 - m) = V\psi \qquad (6.7)$$

where $\not{p}_i = p_{i\mu}\gamma_i^\mu$

In fact, from the form of the Bethe-

Salpeter equation in equation 6.7:

$$\gamma_{2\mu} = \gamma_{1\mu}{}^{T} \qquad (6.8)$$

and $\gamma_{1\mu}$ operates from the right and $\gamma_{2\mu}$ from the left.

We have here written down the quark-quark equation, for simplicity in working out the charge conjugation properties (see ref 76, A4). The $q\bar{q}$ results are obtained by the use of the charge conjugation matrix.

Taking the large mass limit, the equation becomes:

$$(\not{p}_1 + \not{p}_2)\psi = (m - U)\psi \qquad (6.9)$$

where $U = V/m$

Rewriting in terms of centre of mass variables as before:

$$\left[\tfrac{1}{2}P_\mu(\gamma_1{}^\mu + \gamma_2{}^\mu) + q_\mu(\gamma_1{}^\mu - \gamma_2{}^\mu) - m + U\right]\psi = 0 \qquad (6.10)$$

We now wish to chose a suitable interaction U, such that the squared equation represents a harmonic oscillator. This requirement leads to two restraints, since the squared equation has two forms. In order to obtain equation 6.6 from squaring, we linearise in spin $\frac{1}{2} \otimes \frac{1}{2}$ space to get:

$$\tfrac{1}{2}P_\mu(\gamma_1{}^\mu + \gamma_2{}^\mu) \sim \mathcal{M} \sim \sqrt{2\omega}\,(\alpha_{+\mu}a^\mu + \alpha_{-\mu}a^{\dagger\mu}) \qquad (6.11)$$

Secondly we require that equation 6.10 when squared should be of the form of equation 6.4, and this introduces various conditions on the potential.

From the first restraint, to ensure harmonic oscillator form, we must eliminate terms quadratic in a_μ and $a^\dagger_\mu$, i.e.:

$$\alpha_+^\mu \alpha_+^\nu = \alpha_-^\mu \alpha_-^\nu \equiv 0 \qquad (6.12)$$

Comparison with equation 6.10 suggests the form:

$$\alpha_\pm^\mu = (1 + A_\pm)(\gamma_1^\mu - \gamma_2^\mu) \qquad (6.13)$$

and with equation 6.12 that:

$$A_\pm = \pm\gamma_5^1\gamma_5^2 \qquad (6.14)$$

Thus:

$$\sqrt{2\omega}\,(\alpha_+ a + \alpha_- a^\dagger) = q_\mu(\gamma_1^\mu - \gamma_2^\mu) - i\omega\gamma_5^1\gamma_5^2(\gamma_1^\mu - \gamma_2^\mu)x_\mu \qquad (6.15)$$

To determine the exact form of the potential, we now explicitly square equation 6.10:

$$\left[\tfrac{1}{2}P^2 + q^2 - m^2 + U^2 + \left\{\tfrac{1}{2}P_\mu(\gamma_1^\mu + \gamma_2^\mu), U\right\} + \left\{q_\mu(\gamma_1^\mu - \gamma_2^\mu), U\right\} = 0 \qquad (6.16)$$

For a harmonic oscillator form:

$$U^2 \sim -\omega^2 x^2 + \text{constant} \qquad (6.17)$$

and $\left\{\tfrac{1}{2}\not{P}, U\right\} = \left\{\not{q}, U\right\} = 0 \qquad (6.18)$

It is not possible to satisfy all these requirements at once, so we neglect condition 6.18, and then we obtain:

$$U = -i\omega\,\gamma_5^1\gamma_5^2(\gamma_1^\mu - \gamma_2^\mu)x_\mu + \gamma_5^1\gamma_5^2 B \qquad (6.19)$$

where B is an arbitrary constant.

The form of the interaction, linear in position variables, which, like the momenta, are contracted with Dirac matrices, arises from the position-momentum symmetry characteristic of the harmonic oscillator. Equation 6.19 is clearly invariant under the interchange:

$$q_\mu \leftrightarrow - i\omega \gamma_5^1 \gamma_5^2 x_\mu$$

The second term $\gamma_5^1 \gamma_5^2 B$ was not included in the original model [70], and as a result, some confusion arose as to the magnitude of the quark mass, since without this term, the ground state mass of the meson is the same as the quark mass, which is clearly not desirable. However the new term removes this difficulty, leaving the ground state mass free as an extra parameter (see Appendix V). The large quark mass enables us to ignore quark propagation effects, establishing a close relation between our relativistic results and the non-relativistic ones.

A further advantage of the large quark mass scheme is that the internal motion of the quarks within the hadron may be treated as non-relativistic (though this entails neglecting the relative energy in the centre of mass). In this approximation, the four dimensional creation and annihilation operators become three dimensional in the rest frame, so we may replace a_μ, $a_\mu^\dagger$ by:

$$\eta_\mu = (0,\underline{a}) \quad \text{in the rest frame}$$

Generalising to any frame:

$$\eta_\mu = a_\mu - P_\mu \, P.a/M^2 \tag{6.20}$$

where $P_\mu P^\mu = M^2$

This also has the effect of removing the negative norm timelike states, which had to be artificially decoupled from physical states in the FKR model, leading to a violation of unitarity and the necessity of introducing an adjustment factor to compensate for this in the calculation of matrix elements.

6.4 Properties of the wave functions

The solutions to equation 6.10 when the form of the interaction has been substituted in:

$$\left[\tfrac{1}{2}P_\mu(\gamma_1^\mu + \gamma_2^\mu) - m_0 + \tfrac{1}{2}\sqrt{\Omega}(\alpha_+ a + \alpha_- a^\dagger)\right]\psi = 0 \tag{6.21}$$

can be built up in a Fock space with a vacuum $|0_p\rangle$ defined by:

$$\eta_\mu |0_p\rangle = 0 \tag{6.22}$$

Since η_μ is simply the non-relativistic annihilation operator in the rest frame, this state is the non-relativistic ground state in the rest frame. A general solution may be written:

$$|\Psi_p\rangle = \Psi_p(\eta_\mu, \eta_\mu^\dagger)\,|0_p\rangle \qquad (6.23)$$

where Ψ_p is a 4 x 4 matrix, and in calculations, quark operators act on Ψ_p from the right, while antiquark operators act from the left.

The invariance properties of the solutions under charge conjugation, parity and Lorentz transformations are defined in the same way as for the Bethe-Salpeter wave functions (see ref 76,A4).

In the evaluation of matrix elements, we are not able to use the usual invariant measure d^4q, since the wave function does not depend on the relative energy as a result of the approximation used in the replacement of the creation and annihilation operators a_μ and $a_\mu^\dagger$ by η_μ and $\eta_\mu^\dagger$ (i.e. as a result of the large quark mass approach). Hence we must redefine the scalar product in a more appropriate way. This we do in a way prompted by the FKR treatment. That is, we replace $|0_p\rangle$ by $|0\rangle$, the four-dimensional vacuum state, which also satisfies equation 6.23. Defining the spin part of the scalar product in the usual way, we have:

$$\langle \psi_{p_2} | \psi_{p_1} \rangle = \langle 0 | \mathrm{Tr}\ \bar{\psi}_{p_2}(\eta_2^\dagger, \eta_2)\, \psi_{p_1}(\eta_1^\dagger, \eta_1) | 0 \rangle \tag{6.24}$$

We normalise the wave functions in the same way as the Bethe-Salpeter wave functions, by using the matrix element of the quark current at zero momentum transfer to define the quark charge, e_q (fig 19). Thus we obtain:

$$\begin{aligned} &\langle \psi_p | j_\mu^q(0)/e_q | \psi_p \rangle \\ &= -A \langle 0 | \mathrm{Tr}\ \bar{\psi}_p(\eta^\dagger, \eta)\, \gamma_\mu \psi_p(\eta, \eta^\dagger) | 0 \rangle \\ &\equiv 2P_\mu \end{aligned} \tag{6.25}$$

where A is a normalisation factor.

We use the minimal vector current for the quarks, which may be written:

$$\begin{aligned} V_\mu^q &= -e_q \gamma_\mu e^{iq.x} \\ &= -e_q \gamma_\mu F e^{q.a^\dagger/\sqrt{\Omega}} e^{-q.a/\sqrt{\Omega}} \end{aligned} \tag{6.26}$$

where $F = e^{-q^2/2\Omega}$

and the centre of mass phase factor is neglected.

Then the axial current is:

$$A_\mu^q = -\lambda_q \gamma_5 \gamma_\mu F e^{q.a^\dagger/\sqrt{\Omega}} e^{-q.a/\sqrt{\Omega}}$$

The antiquark operators, neglecting unitary spin factors, are given by:

$$O_\mu^{\bar{q}} = F C \Gamma_\mu^T C^{-1} e^{-q.a^\dagger/\sqrt{\Omega}} e^{q.a/\sqrt{\Omega}} \tag{6.27}$$

where Γ_μ is either γ_μ or $\gamma_5\gamma_\mu$.

The corresponding antiquark matrix elements are:

$$\begin{aligned} \langle \psi_{p_2} | O^{\bar{q}} | \psi_{p_1} \rangle = F \langle 0 | \mathrm{Tr}\ \bar{\psi}(\eta_2^\dagger, \eta_2)\ & e^{-q.a^\dagger/\sqrt{\Omega}} \\ \times\, e^{q.a/\sqrt{\Omega}}\, \psi(\eta_1, \eta_1^\dagger)\, & C \Gamma^T C^{-1} | 0 \rangle \end{aligned} \tag{6.28}$$

The unitary spin parts are calculated under the assumption of no $\eta\eta'$ mixing in the 0^{-+} nonet, and ideal mixing of ω and ϕ in the 1^{--} nonet.

6.5 The solutions

The complexities of equation 6.21 may be considerably reduced by writing the equation in the rest frame. When the effects of parity transformations and charge conjugation are taken into account, the wavefunction ψ_p may be separated into four components, each 2 x 2 matrices. The equation may then be expressed as a set of coupled equations at the Pauli spinor level, and reduced algebraically to a single equation (see ref 76, A4).

The application of charge conjugation and parity invariance allows the solutions to be divided into two classes characterised by the value of PC. In the non-relativistic model, the set with PC = -1 corresponds to spin zero states, while that with PC = +1 corresponds to spin one states. In the relativistic wavefunctions, this relation is maintained in the large components, but not in the small. With some manipulation the large components may be shown to be eigenstates of the number operator, and hence the full solution may be constructed. This solution, rewritten in covariant form, may then be demonstrated to be a solution of equation 6.21 by direct substitution.

i) PC = -1

As a consequence of choosing a three dimensional interaction in the rest frame as in the non-relativistic model, we obtain the naive quark model result that mesons with $P = (-1)^J$ and $C = (-1)^{J+1}$ are forbidden. Had we not wished for the close correspondence to the non-relativistic model, the interaction would in general have been four dimensional in the rest frame, and in this case the forbiddenness depends crucially on the spin-space structure of the interaction.

The solution for a PC = -1 state of mass m and spin quantum numbers (J, J_z) is:

$$|\psi\rangle = 1/2\sqrt{m_0}\ \gamma_5 \left[(m - m_0 \not{P}/m) + \sqrt{\Omega}\ \not{P}/m\ \not{q}\right] |N, J, J_z\rangle \qquad (6.29)$$

where m_0 is the effective quark mass and only states with $P = (-1)^{J+1}$, $C = (-1)^J$ exist.

$|N,J,J_z\rangle$ is an eigenstate of the number operator $\eta_\mu^\dagger \eta^\mu$, with eigenvalue N.

In the limit $\Omega \to 0$, the solution reduces to that for a free (mass m) quark and antiquark with relative angular momentum zero, that is, there is no admixture of negative energy states. The ground state solution:

$$|\psi_o\rangle = 1/2\sqrt{m_o}\ \gamma_5(m - m_o \not{P}/m)|0\rangle$$

is simply a boosted non-relativistic wave-function. It has the same spin structure as postulated by Gudehus [71]. The BJK solution has the same form, but with $\not{P}/m$ replaced by $\not{P}/m_{quark}$.

The particles lie on straight Regge trajectories:

$$m^2 = m_o^2 + \Omega N \qquad (6.30)$$

where $N = 0,1,2...$

We choose the slope of the trajectory to be unity, i.e. $\Omega^{-1} = 1$, and take for m_o^2 the average of the square of the pseudoscalar meson masses, $m_o^2 = 0.25$ Gev^2. With these choices, we reproduce reasonably this section of the mass spectrum (fig 20).

In fixing the parameter Ω as above, we have also automatically fixed the size of the hadron [72]. The space part of the ground state solution is:

$$\langle \underline{x} = \underline{x}_1 - \underline{x}_2 | 0 \rangle \sim \exp(-\omega^2 x^2/2)$$

$$\sim \exp(-x^2/2R^2) \qquad (6.31)$$

where $R^2 = 1/\omega = 8/\Omega = 8$ natural units.

R is essentially the average separation of the quarks and determines the size of the meson. Taking a nearest order of magnitude approximation, the cross-section of the meson is $\sim \pi R^2 \sim 24$ natural units $\sim$ 10 mb, which is not unreasonable.

ii) PC = + 1

Using the same method as for PC = -1, we find that the solutions are not always unique, because the eigenvalue equation at the Pauli spinor level demands only an eigenstate of the number operator. Thus the quark spin and the orbital angular momentum are decoupled, so that for a given meson spin, there are two allowed values of orbital angular momentum. This same mechanism however, ensures the desirable feature that there is no spin-orbit splitting of the trajectories. In fact the ambiguity of the solutions does not manifest itself in every state, and the states on the leading trajectory and those with the spin equal to the orbital angular momentum are unique.

Hence the first ambiguous case occurs for a spin one meson with N = 2, so that the effect is irrelevant for practical considerations. To define a general solution, we require that the rest frame wavefunction should have no spin zero components, in accord with the unambiguous solutions. Then the unnormalised PC = +1 states are:

$$|\psi\rangle = 1/2\sqrt{m_0}\left[(m + m_0 \not{P}/m)(\not{\epsilon} + \Omega\, \eta^{\dagger}\epsilon.\eta/(m^2 - m_0^2)) - i\sqrt{\Omega}\,\gamma_5/m\, \mathcal{E}(P\eta^{\dagger}\epsilon\gamma)\right]|N,\ell,\ell_z\rangle \quad (6.32)$$

where $P = C = (-1)^{\ell}$,

ϵ_μ is a spin one wavefunction

$\mathcal{E}(P\eta^{\dagger}\epsilon\gamma)$ is the Levi-Civita tensor dotted into four four-vectors.

The solution has not been separated into its possible spin states of $\ell+1$, ℓ and $\ell-1$.

As in the PC = -1 solutions, we note that in the limit $\Omega \to 0$, the solution reverts to that for a free quark and anti-quark with relative momentum zero, showing that there is no negative energy admixture.

The term $\eta^{\dagger}\epsilon.\eta/(m^2 - m_0^2)$ which vanishes in the case $J = \ell$, is responsible for adding the extra orbital angular momentum in the large components for the cases $J = \ell \pm 1$, where the solution is not unique.

The significance of the other terms is most easily understood through consideration of the ground state solution:

$$|\psi_0\rangle = 1/2\sqrt{m_0}\left[(m + m_0 \not{P}/m)\not{\epsilon} - i\sqrt{\Omega}\,\xi/m\,\mathcal{E}(P\eta^{\dagger}\epsilon\gamma)\right]|0\rangle \tag{6.33}$$

The first term is closely related to a boosted non-relativistic wavefunction, and is identical if $m_0 = m$, when the solution becomes that of Gudehus [71]. It is also similar to the solution of BJK:

$$|\psi\rangle = (1 + \not{P}/m_q)\not{\epsilon}\,|0\rangle \tag{6.34}$$

with $\langle q|0\rangle = e^{-q^2/2\Omega}$

In the second term, the quark spin is coupled to a P-wave orbital state (in the small components) and hence corresponds to a spin-orbit interaction. This feature is absent from the solutions of BJK and Gudehus. Experiments, on the other hand, indicate the necessity of this type of coupling (see section 3.2) and we regard this property as most desirable. This spin-orbit coupling in the wave-functions is independent of the presence or absence of spin-orbit splitting of the trajectories.

The mass spectrum is again linear and is not affected by the ambiguity in the solutions:

$$m^2 = m_0^2 + \Omega(N + 2) \qquad (6.35)$$

With the parameters fixed as previously, the mass of the vector meson is predicted to be 1.58 Gev, which is about twice the correct value. This devolves from too strong a spin-spin interaction, though clearly some such interaction is required to make the vector mesons more massive than the pseudoscalar nonet. In calculations, we reset the value of m_0 (since β is free) to be:

$$m_0'^2 = m_0^2 - 1.5\Omega \qquad (6.36)$$

and use the trajectory:

$$\begin{aligned} m^2 &= m_0'^2 + \Omega(N + 2) \\ &= (m_0^2 + 0.5\Omega) + \Omega N \end{aligned} \qquad (6.37)$$

The ground state mass squared $(m_0^2 + 0.5\Omega)$ is again chosen to be the average value of the vector meson mass squared. The mass spectrum is then reasonable, with the quantum numbers identical to those predicted in the non-relativistic quark model (fig 21).

Having obtained the specific wave-functions for relativistic mesons, it is clear that they do indeed incorporate a spin-orbit coupling term, and this is suggestive of the Melosh transformation. In the FKR treatment, $SU(6)_W$ wavefunctions were chosen for the meson states, while the vector and axial currents were more complicated than the usual simple minimal currents. In this model, the reverse is true, with the usual currents being used to calculate matrix elements between complicated states. This may be compared with the calculations of chapter 4 in constituent space and current space, except that in FKR spin-orbit coupling appears only in the axial current and not in the vector current, where it is also required. The natural occurrence of this coupling in our model is its most attractive feature.

6.6 Applications

Following FKR, we calculate experimental observables from current matrix elements, neglecting quark propagation effects because of the large mass approximation. We have the additional problem that our vector currents are not in general conserved, while those of FKR are, so a momentum term must be added to the coupling to ensure conservation. Then the current is of the form:

$$V_\mu = -e_q F(g_1 \gamma_\mu + g_2 P_\mu) e^{q.a^\dagger/\sqrt{\Omega}} \times e^{-q.a/\sqrt{\Omega}} \qquad (6.38)$$

where g_1, g_2 are constrained to ensure current conservation

and $g_1^2 + g_2^2 = 1$

and $\partial_\mu A^\mu = -i \lambda_q F \gamma_5 (g_1 \not{q} + g_2 q.P)$
$\qquad\qquad x\ e^{q.a^\dagger/\sqrt{\Omega}}\ e^{-q.a/\sqrt{\Omega}}$

Since this introduces an extra parameter, we take $g_2 = 0$ for our calculations as a first approximation. In the cases where the decay is an equal mass transition or a PC $= +1$ meson to pseudoscalar meson transition, the current is exactly conserved with $g_2 = 0$ and $g_1 = 1$, so that for practical purposes, vector current matrix elements are all exact, and only the axial current matrix elements for the decays $1^+ \rightarrow 1^-$ and $2^+ \rightarrow 1^-$ are not exact.

i) Lepton decays

The decay matrix for a pseudoscalar meson to annihilate into a lepton pair is:

$$\langle \text{vacuum} \,|\, A_\mu(0) \,|\, 0^{-+}\ \text{meson} \rangle = \mathrm{Tr}\, \gamma_\mu \gamma_5 \psi_p (x_\mu = 0)$$
$$\equiv f_p P_\mu \qquad (6.39)$$

where $\psi_p(x_\mu = 0)$ is the pseudoscalar wavefunction at the origin

f_p is the pseudoscalar meson form factor.

Evaluating the trace leads to:

$$f_p = \sqrt{m_o}/m_p\ (\Omega/4\pi) \qquad (6.40)$$

The fact that this contains a factor of $1/m_p$ means that we cannot use the physical masses to calculate f_p, since this would lead to a very large symmetry breaking which is not observed, i.e. we are faced with the Van Royen-Weisskopf paradox [73]. Instead we are forced to use the predicted unbroken masses to avoid the difficulty. Then:

$$m_p = m_o$$

and $f_\pi = f_k = 0.112$ Gev

as compared with the experimental values [74]:

$$f_k = 0.105 \text{ Gev} \qquad f_\pi = 0.095 \text{ Gev}$$

Similarly the calculation may be repeated for a vector meson decay:

$$\langle \text{vacuum} \,|\, V_\mu(0) \,|\, 1^{-+} \text{meson} \rangle = \mathrm{Tr}\, \gamma_\mu \psi_V(x_\mu = 0)$$
$$= g_V m_V \epsilon_{V\mu} \qquad (6.41)$$

which leads to:

$$3/\sqrt{2}\, g_\phi = 3 g_\omega = g_p = 1/\sqrt{2m_o}(\Omega/4\pi)$$
$$= 0.08 \text{ Gev}$$

as compared with the experimental values:

$$3/\sqrt{2}\, g_\phi = 0.158 \text{ Gev} \qquad 3g_\omega = 0.156 \text{ Gev}$$
$$g_p = 0.160 \text{ Gev}$$

This result is comparable with those of other quark models such as BJK, who predict g_V to be about twice the experimental value and FKR, who have:

$$g_p = f_\pi = f_k \text{ etc}$$

ii) Matrix elements of the vector current

The form factor for $K\pi$ coupling in $K\ell_3$ decay is calculated from the matrix element of the vector current:

$$\langle \pi | V_\mu(0) | K \rangle = F/2\sqrt{2}\ (m_\pi/m_k - m_k/m_\pi) \times \left[(p_k + p_\pi)_\mu + (p_k - p_\pi)_\mu \right]$$
$$\equiv f_+(q^2)(p_k + p_\pi)_\mu \qquad (6.42)$$
$$f_-(q^2)(p_k - p_\pi)_\mu$$

Evaluating this using unbroken masses, we obtain:

$$f_\pm(q^2) = 0$$

in disagreement with the experimental result [74] of:

$$f_-(0) \approx f_+(0) \approx 1.0$$

This disagreement is clearly attributable to the use of unbroken masses, and a symmetry breaking scheme must be added to our model if these reults are to be well described. Had we used physical masses in equation 6.42, we would have achieved good results but violated our calculational prescription.

Symmetry breaking is not so important in the electromagnetic decays of the vector mesons, where we obtain much better results. Neglecting the unitary spin factor, we obtain for the matrix element of the quark current:

$$T^q = -i \langle \pi | \epsilon^{\mu *} J_\mu^q(0) | 1^{+-} \text{ meson} \rangle$$
$$= \frac{-em_2}{m_1 m_0} F \left[1 + (m_0/m_2)^2\right] \mathcal{E}(q \epsilon_2^* p_1 \epsilon_1) \qquad (6.43)$$

where the kinematics are indicated in fig 22.

The first term in equation 6.43 corresponds to an orbital magnetic moment, while the second is analogous to an intrinsic moment. The antiquark amplitude differs only in the signs of the various terms, so combining with the unitary spin factor, we obtain the results in Table V1.

The coupling constant, g is defined by:

$$T = eg_\gamma \mathcal{E}(q \epsilon_2^* p_1 \epsilon_1) \qquad (6.44)$$

and we have used unbroken masses in its evaluation. The quantities dotted into the Levi-Civita tensor are taken at their physical values, primarily for convenience, as any other procedure has a minor effect on the results.

iii) Decays by emission of a pseudoscalar meson

Following FKR, we calculate the amplitude for pseudoscalar emission by replacing the pseudoscalar meson interaction by the divergence of the axial vector current, which is given by:

$$\partial_\mu A^\mu = -i \lambda_q F \gamma_5 \not{q} \, e^{q \cdot a^\dagger/\sqrt{\Omega}} \, e^{-q \cdot a/\sqrt{\Omega}} \qquad (6.45)$$

The overall strength of the

interaction is determined by the adjustable parameter, f, and the decay amplitude from an initial state, i, to a final state, j, by emission of a pseudoscalar, is:

$$T = -if \sum_{\alpha} \langle j | \partial_\mu A^\mu_\alpha | i \rangle \qquad (6.46)$$

where the summation is over quarks and antiquarks.

Expressions for the matrix elements with the unitary factor λ_q removed are given in Table VII.

Decay widths are calculated using the formula:

$$\Gamma = R/(2J_i + 1) \sum |T|^2 \; (\underline{q} / 2m_i^2) \qquad (6.47)$$

where $\underline{q}$ is the three-momentum of the decay products in the rest frame of the initial particle

and the summation is over the initial and final spins

The factor R is to account for the different charge modes allowed in the decay [64].

In calculating the decay widths, the dynamical quantities are input using unbroken masses, while for the kinematic phase space factors we use physical masses.

For example, the decay width:

$$\Gamma\,(1^{--} \rightarrow 0^{-+}\, 0^{-+}) = R\; 2/3(g/m_i)^2\, |\underline{q}|^3 \qquad (6.48)$$

is calculated by putting physical masses into $\underline{q}$ and unbroken masses into (g/m_i).

As can be seen from Table VIII, this prescription yields good agreement with data, as might be expected since this is simply reflecting the SU(3) symmetry of the coupling constants. In the FKR model, the relative values of decay widths for a given decay type are in less good agreement with data, but this is due to the symmetry breaking introduced by using physical masses throughout. This problem is particularly acute in the case of decays into two pseudoscalar mesons, where different results are obtained depending on which meson is replaced by the axial current. The use of unbroken masses avoids this asymmetry.

The value of the coupling constant, f, is expected to be close to that given by PCAC theory, that is:

$$f_T = f^{\frac{1}{2}}_{\pi NN}/m_\pi g_A = 1.65$$

Using this value, the decay widths are found to be too large, and reduction to a value $f = 1.46 \text{ Gev}^{-1}$ is necessary to give a good fit to the data. Our best fit to the decay width data, together with that of FKR, is shown in Table VIII.

The results for the vector and tensor decay widths are all within 20% of the experimental values, which is encouraging when the fact that we have not introduced symmetry breaking is taken into account, and is an improvement on FKR. For the $2^{+} \rightarrow 0^{-}1^{-}$ decays, our results are less good than those of FKR, though we have not used a conserved current and can improve our results by rectifying this (Table IX).

In the other two types of decay considered, $1^{+-} \rightarrow 1^{-}0^{-}$ and $1^{++} \rightarrow 1^{-}0^{-}$, we again have the problem that the underlying vector current is not conserved so that our results can be improved. The approximate results are however correct to an order of magnitude, and are comparable to those of FKR. On the other hand, the helicity proerties, which depend crucially on the type of coupling, are much worse in our model unless a conserved current is used (Table X).

The unfavourable comparison with experiment for the decays $K^{**}(1240) \rightarrow K^{*}\pi$ and $A_1 \rightarrow \rho\pi$ is perhaps mitigated by possible contamination of the resonances by the Deck effect [74].

6.7 Conclusions

The harmonic oscillator character of our wavefunctions was an input from the beginning in order to ensure linear trajectories, and is not crucial to the lowest two states, which are the only ones for which calculations were made. On the other hand, it does generate the spin structure on which our results are entirely dependent. Hence the harmonic oscillator input is indirectly responsible for any relevance of the model.

The large components of the wavefunctions are analogous to boosted non-relativistic wavefunctions and occur in all relativistic models. The main difference arises from whether m_{quark} or m_{hadron} is regarded as the fundamental mass. The novel feature of our model is the inclusion of a spin-orbit term in the small components, which nevertheless does not cause splitting of the trajectories. This extra orbital term is needed to produce some of our good results, the least ambiguous case being the electromagnetic decays, where the orbital term contributes half the result. We regard this success in the electromagnetic case as our most inportant result, as it does not depend on any arbitrary factors.

The significance of the results for the pseudoscalar decay widths is harder to evaluate because the prescription which replaces the pseudoscalar interaction by the divergence of the axial vector current is not quite as well established as the corresponding formula for the vector current. Further, we do not adhere strictly to PCAC theory as we use a coupling constant slightly different from that given by the theory.

Given that the relative success for any one decay type just depends on SU(3) symmetry and the prescription to use unbroken masses, the fact that the coupling constant is an independent parameter means Table VIII contains only four independent results. Also, because the data for the $1^{++} \rightarrow 1^{-}0^{-}$ decay is ambiguous, there are really only three numbers to compare with experiment. Of these, two compare well, while the third, $B \rightarrow \omega\pi$, is not so good.

To summarise, we have proposed an equation in which the internal quark motion is closely related to that in a non-relativistic model, but is treated in a covariant manner. Our model is most closely related to that of FKR, but has the advantage of incorporating spin in a more dynamical way. However, their simple treatment of spin enables them to include baryons in their scheme. The extension of our model to include baryons is clearly the next step.

7. Conclusions

In the preceding chapters, we have attempted to discuss the dilemma caused by the non-identity of current and constituent quarks from as many different approaches as possible. In general, they all achieve some degree of success, though some are less specific than others.

In the first chapter, we introduced the naive quark model, demonstrating its remarkable success in predicting particle states in the mass spectrum. Unfortunately, it is easily shown that the symmetry SU(6) of resonances is not suitable for predicting final states in decay processes. Chapter 2 relates the same story from the opposite direction, showing that the partons of the naive parton model cannot be identified with quarks, without some modification of the symmetry SU(6).

We discuss some possible methods of adapting the symmetry in Chapter 3. The first obvious step is to introduce some form of relativistic invariance and though this cannot be attained in full, invariance in a chosen direction leads to the symmetry $SU(6)_W$. However, this is still not sufficient to produce a satisfactory description of both the resonance spectrum and decay processes, and some further modification is necessary. This can be achieved in several ways, though all are based on the idea of SU(6) breaking and one of these is the Melosh transformation, where the concept of two different types of quarks is invoked, these two types being related by the transformation from one space to the other. Hence to calculate the matrix element of a current between two constituent states, the current must first be transformed into constituent space (or the constituent states into current space).

This idea can then be extended and used to calculate observables as in Chapter 4, where a considerable degree of success is achieved. The spin-orbit coupling observed experimentally is reproduced but the strength of the coupling is a free parameter of the theory. It is also shown that even the use of a general Melosh-type transformation is not sufficient to duplicate the experimental results in the low ω region, and it is clear that the transformation does not in fact break the symmetry $SU(6)_W$ but merely rotates the quark spins. Thus some further mechanism is required to give different probabilities to spin 1 and spin 0 cores, isospin 1 and 0 cores.

In Chapter 5, we examine the theoretical basis for the Melosh transformation and find that although the transformation is plausible, it is difficult to justify it theoretically without severely limiting the sphere of its usefulness. We then turn to an alternative method of viewing the underlying dilemma, that is, we regard constituent and current quarks as fundamentally the same, except that the first are at rest while the second are in motion. Then, treating quarks relativistically, we can investigate whether the same effects as those of the Melosh transformation are produced.

We introduce a relativistic quark model for mesons in Chapter 6. The meson ground state mass is a free parameter of the theory, but once that is fixed, the mass spectrum obtained is the same as that produced by the non-relativistic model. Following FKR, we use a minimal vector current to calculate matrix elements and replace the pion by the divergence of the axial vector current. However, in our model, the minimal current is not always conserved so that in some cases we are forced to correct it to a conserved form. In this way we achieve some reasonable results, within 20% of the data. Most notable is the natural appearance of spin-orbit coupling in our model in a comparable manner to that arising from the Melosh transformation, without causing at the same time any splitting of the trajectories.

One effect of including spin in a dynamical way is to make the equation for the three quark case rather complicated, so that at present the model only encompasses mesons. It is hoped that the model may be extended to the baryon case in the future.

References

1. S.Sakata Prog.Th.Phys. 16 (1956) 686
2. P.D.B.Collins, R.C.Johnson and E.J.Squires Phys.Lett. 26B (1968) 223
3. J.Bernstein Elementary Particles and Their Currents W.H.Freeman and Co., London 1968.
4. G.Ascoli, H.B.Crawley, D.W.Mortara and A.Shapiro Phys.Rev.Lett. 21 (1968) 1411
5. R.H.Dalitz and M.Horgan Nucl.Phys. B66 (1973) 135
6. V.Barger and D.Cline Phys.Rev.Lett. 16 (1966) 913
   V.Barger and M.Olsson Phys.Rev. 151 (1966) 1123
7. Dolen, Horn and Schmidt Phys.Rev. 166 (1968) 1768
8. V.Barger and R.J.N.Phillips Phys.Rev.Lett. 24 (1970) 291
9. J.Rosner Phys.Rev.Lett. 22 (1969) 689
   H.Harari Phys.Rev.Lett. 22 (1969) 562
10. P.G.O.Freund Phys.Rev.Lett. 20 (1968) 235
    H.Harari Phys.Rev.Lett. 20 (1968) 1385
11. F.J.Gilman Phys.Reports 4C (1972)
12. J.D.Bjorken and S.D.Drell Relativistic Quantum Mechanics McGraw-Hill New York 1964
13. J.D.Bjorken Phys.Rev. 179 (1969) 1547
    R.P.Feynman Phys.Rev.Lett. 23 (1969) 1415
    J.D.Bjorken and E.A.Paschos Phys.Rev. 185 (1969) 1975
14. F.E.Close Partons and Quarks Daresbury Lecture Note Series No. 12 DNPL/R31
15. C.H.Llewellyn-Smith Phys.Reports 3C (1972) 264
16. M.Gell-Mann Phys.Lett. 8 (1964) 214
    G.Zweig CERN preprint TH 401 (1964) 412

17. H.Kendal Proc. of electron photon symposium (1971) Cornell
18. J.Kuti and V.Weisskopf Phys.Rev. D4 (1971) 3418
P.V.Landshoff and J.C.Polkinghorne Nucl.Phys. B28 (1971) 240
19. O.Nachtmann Nucl.Phys. B38 (1972) 397
20. A.Bodek et al. Phys.Rev.Lett. 30 (1973) 1087
21. S.Adler Phys.Rev. 143 (1966) 1144
22. F.E.Close Phys.Lett. 43B (1973) 422
M.Chaichan and S.Kitikado CERN TH 1640 (1973)
23. A.Litke et al. Phys.Rev.Lett. 30 (1973) 189
24. H.Fritsch and M.Gell-Mann Proc. of the 16th International Conference on High Energy Physics NAL Batavia (1972)
25. G.Tarnopolosky et al. Phys.Rev.Lett. 32 (1974) 432
26. S.L.Glashow, J.Iliopoulos and L.Maiani Phys.Rev. D2 (1970) 1285
27. S.Ting et al. Phys.Rev.Lett. 33 (1974) 1404
B.Richter et al. Phys.Rev.Lett. 33 (1974) 1406
28. see e.g. S.Pakvasa, G.Rajasekaran and S.F.Tuan Phys.Rev. 11 (1975) 1345
29. B.J.Bjorken Proc. of the 6th International Symposium on Electron and Photon Interactions at High Energies Bonn (1973)
30. D.E.Plane et al. Nucl.Phys. B22 (1970) 93
31. A.J.G.Hey, P.J.Litchfield and R.J.Cashmore CERN preprint TH1886 (1974)
32. H.A.Gordon Phys.Rev. D8 (1973) 779
33. J.Weyers Constituent Quarks and Current Quarks Lectures given at the International Summer School on Particle Interactions at Very High Energies Louvain (1973)
34. M.Afzal Nuovo Cimento 15A (1973) 61
35. R.Dashen and M.Gell-Mann Phys.Rev.Lett. 17 (1966) 340

36. S.Drell Phys.Rev. D1 (1970) 1035
37. S.L.Adler Phys.Rev. 140 (1965) 736
R.Weisberger Phys.Rev. 143 (1965) 1302
38. H.J.Melosh Ph.D. Thesis Caltech (1973) unpublished
39. L.L.Foldy and S.A.Wouthuysen Phys.Rev. 78 (1950) 29
S.Tani Proc.Th.Phys. Kyoto 6 (1951) 267
M.H.L.Price Proc.Roy.Soc. A195 (1948) 62
40. J.Rosner and W.Colglazier Nucl.Phys. B27 (1971) 349
41. G.Ascoli Phys.Rev.Lett. 26 (1971) 929
42. J.Rosner and W.Petersen Phys.Rev. D6 (1972) 820
43. F.E.Close, H.Osborn and A.M.Thomson Nucl.Phys. B77 (1974) 281
44. A.J.G.Hey and J.Weyers Phys.Lett. 48B (1974) 69
F.J.Gilman and I.Karliner Phys.Lett. 46B (1973) 426
45. F.J.Gilman and M.Kugler Phys.Rev.Lett. 30 (1973) 518
A.J.G.Hey and J.Weyers Phys.Lett. 44B (1973) 263
F.J.Gilman, M.Kugler and S.Meshkov SLAC-PUB-1286 (1973)
46. N.Cabibbo and L.A.Radicati Phys.Lett. 19 (1966) 697
47. S.D.Drell and A.C.Hearn Phys.Rev.Lett. 16 (1966) 908
S.B.Gerasimov Soviet J.Nucl.Phys. 2 (1966) 430
48. I.Karliner Phys.Rev. D7 (1973) 2717
49. J.D.Bjorken Phys.Rev. 148 (1966) 1467
50. A.J.G.Hey and J.Mandula Phys.Rev. D5 (1972) 2610
C.Carlson and Wu-Ki Tung Phys.Rev. D5 (1972) 721
D.Wray Weizman Institute Report (1972)
51. F.E.Close CERN preprint TH.1875 (1974)
52. L.M.Sehgal Aachen preprint (1974)
53. L.O'Raifeartaigh Phys.Rev. 139 (1965) 1052

54. J.S.Bell Schladming Lecture Notes TH.1851-CERN

55. G.Veneziano Nuovo Cimento 57A (1968) 190

56. Z.Koba and H.B.Nielsen Nucl.Phys. B10 (1969) 633

57. H.B.Nielsen and P.Olesen Phys.Lett. 32B (1970) 203

58. H.A.Virasoro Phys.Rev. D1 (1970) 2933

59. P.Ramond Phys.Rev. D3 (1971) 2415

60. E.E.Salpeter and H.A.Bethe Phys.Rev. 82 (1951) 1232

61. M.Bohm, H.Joos and M.Krammer Nucl.Phys. B51 (1973) 397

62. G.C.Wick Phys.Rev. 96 (1954) 1124

63. M.Bohm, H.Joos and M.Krammer CERN preprint TH.1715 (1973)

64. R.P.Feynman, M.Kislinger and F.Ravndal Phys.Rev. D3 (1971) 2706

65. O.W.Greenberg Phys.Rev.Lett. 13 (1964) 598
R.H.Dalitz Lectures at the 2nd Hawaii Conference (1967)
D.Faiman and A.W.Hendry Phys.Rev. 173 (1968) 1720
L.A.Copley, G.Karl and E.Obryk Phys.Lett. 29B (1969)117

66. R.L.Walker Phus.Rev. 182 (1969) 1729

67. R.G.Moorhouse and H.Oberlack Phys.Lett. 43B (1973) 44

68. L.Susskind Phys.Rev.Lett. 24 (1968) 944

69. L.Susskind Phys.Rev. D1 (1970) 1182

70. P.J.Walters, A.M.Thomson and F.D.Gault J.Phys.A 7 (1974) 1681

71. T.Gudehus Phys.Rev. 184 (1969) 1788

72. A.Le Yaouanc, L.Oliver, P.Pene and J.C.Raynal
Orsay preprint (1972) LPTHE 72/6

73. R.van Royen and V.Weisskopf Nuovo Cimento 50 (1967) 617
R.van Royen and V.Weisskopf Nuovo Cimento 51 (1967) 583

74. Particle Data Group Rev.Mod.Phys. 45 (1973)

75. R.Carlitz and M.Kislinger Phys.Rev. D2 (1970) 336

76. P.J.Walters Ph.D. Thesis Durham (1974) unpublished

Appendix 1

Basic Conventions

a) Units

Natural units are used throughout with

$\hbar = c = 1$ and 1 Gev = 1 nat. unit.

Hence one natural unit of length $= \frac{\hbar c}{1\ \mathrm{Gev}} = 1.973 \times 10^{-14}$ cm

and 1 mb $= 10^{-27}\mathrm{cm}^2 = 0.3893$ nat. units.

b) 4-vectors

Contravariant 4-vector $A^\mu = (A^0, A^1, A^2, A^3)$ $(A^0, \underline{A})$

Metric $g_{\mu\nu} = (1,-1,-1,-1)$ with $A_\mu = g_{\mu\nu} A^\nu$

where all repeated indices are summed, Greek indices from 0 to 3 and Latin indices from 1 to 3.

4-position $x^\mu = (x^0, \underline{x})$ where x^0 is the time and $\underline{x}$ the spatial position.

4-momentum operator $p^\mu = i\partial^\mu = i\frac{\partial}{\partial x_\mu} = (i\frac{\partial}{\partial x^0}, -i\underline{\nabla})$

c) Dirac matrices

Anti-commutation relations $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$

$\gamma_0 = \beta$ and $\underline{\gamma} = \beta\underline{\alpha}$

In the Pauli Dirac representation

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad \underline{\gamma} = \begin{pmatrix} 0 & \underline{\sigma} \\ -\underline{\sigma} & 0 \end{pmatrix}$$

Pauli matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

where $\sigma^{\mu\nu} = i/2\,[\gamma^\mu, \gamma^\nu]$, $\sigma^{ij} = \begin{pmatrix} \sigma^k & 0 \\ 0 & \sigma^k \end{pmatrix} \varepsilon^{ijk}$

$$\sigma^{0i} = i\alpha^i, \quad \gamma_5 = \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Feynman slash $A\!\!\!/ = A_\mu \gamma^\mu$

Free particle Dirac spinor $u_\lambda(p)$ is defined by the equation:

$$(p\!\!\!/ - m)u_\lambda(p) = 0$$

The anti-spinor v (p) satisfies the equation:

$$(p\!\!\!/ - m)v_\lambda(p) = 0$$

In both cases $p_0 > 0$

Adjoint spinor $\bar{u}_\lambda(p) = u^\dagger_\lambda(p)\,\gamma_0$

and if u transforms under a Lorentz transformation Λ to $u' = S(\Lambda)u$ then $\bar{u} \xrightarrow{\Lambda} \bar{u}S^{-1}(\Lambda)$

Normalisation conditions

$$\bar{u}_\lambda(p)\ u_\lambda(p) = 2m \quad \text{and} \quad \bar{v}_\lambda(p)\ v_\lambda(p) = -2m$$

where the index λ defines the spin direction.

Appendix II

The good representation of SU(6):

$$(\underline{\alpha}, \beta) \rightarrow V(\underline{\alpha}, \beta)V^{-1}$$
$$C \rightarrow VCV^{-1}$$

$$\underline{\alpha}_\perp = \begin{pmatrix} 0 & -\underline{\sigma}_T \\ -\underline{\sigma}_T & 0 \end{pmatrix} \qquad \alpha_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\beta = \begin{pmatrix} 0 & \sigma_3 \\ \sigma_3 & 0 \end{pmatrix} \qquad \gamma_5 = \begin{pmatrix} -\sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix}$$

$$\underline{\gamma} = \begin{pmatrix} i\sigma_3\underline{\sigma}_T & 0 \\ 0 & i\sigma_3\underline{\sigma}_T \end{pmatrix} \qquad \gamma_3 = \begin{pmatrix} 0 & i\sigma_3 \\ -i\sigma_3 & 0 \end{pmatrix}$$

$$\sigma_1 = \begin{pmatrix} 0 & \sigma_1\sigma_3 \\ \sigma_1\sigma_3 & 0 \end{pmatrix} \qquad \sigma_2 = \begin{pmatrix} 0 & \sigma_2\sigma_3 \\ \sigma_2\sigma_3 & 0 \end{pmatrix}$$

$$\sigma_3 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix}$$

Appendix III

This appendix outlines the $SU(6) \times O(3)_L$ wavefunctions [75] for the $\underline{56}$ and $\underline{70}$ representations of baryon states with arbitrary orbital angular momentum L. We use $\alpha, \beta, \gamma \ldots = 1,2,3\ldots$ as SU(3) indices, $a,b,c\ldots = 1,2$ for SU(2) spin indices, which are combined $A = (\alpha a)$, $B = (\beta b)\ldots$ to display the properties under SU(6). Octet, $\underline{8}$, baryon states are described by a matrix:

$$B^{\alpha}_{\beta} = \begin{array}{c} \\ \alpha\downarrow \end{array}\!\!\begin{array}{l} \beta\rightarrow \\ \begin{pmatrix} \frac{1}{\sqrt{6}}\Lambda + \frac{1}{\sqrt{2}}\Sigma^0 & -\Sigma^+ & p \\ \Sigma^- & \frac{1}{\sqrt{6}}\Lambda - \frac{1}{\sqrt{2}}\Sigma^0 & n \\ -\Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix} \end{array}$$

while decuplets, $\underline{10}$, are represented by a completely symmetric tensor:

$$D^{111} = \Delta^{++} \qquad D^{112} = \tfrac{1}{\sqrt{3}}\Delta^{+} \qquad D^{122} = \tfrac{1}{\sqrt{3}}\Delta^{0}$$
$$D^{222} = \Delta^{-} \qquad D^{113} = \tfrac{1}{\sqrt{3}}\Sigma^{*+} \qquad D^{123} = \tfrac{1}{\sqrt{6}}\Sigma^{*0}$$
$$D^{133} = \tfrac{1}{\sqrt{3}}\Xi^{*0} \qquad D^{233} = \tfrac{1}{\sqrt{3}}\Xi^{*-} \qquad D^{333} = \Omega^{-}$$

Spin $\frac{1}{2}$, $2S + 1 = 2$, states are given in terms of the usual spinors:

$$\chi^{a}_{\frac{1}{2}} = \begin{array}{c} a\uparrow \end{array}\!\begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad\qquad \chi^{a}_{-\frac{1}{2}} = \begin{array}{c} a\downarrow \end{array}\!\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

while for spin 3/2, $2S + 1 = 4$:

$$\xi^{a}_{m} = \sum_{r,s} \langle 3/2\, m | \tfrac{1}{2} r | s \rangle \, \chi^{a}_{r} \, \underline{\varepsilon}^{(s)}$$

where $\underline{\varepsilon}^{(\pm 1)} = 1/\sqrt{2}(\mp\hat{\underline{x}} - i\hat{\underline{y}}) \qquad \underline{\varepsilon}^{(0)} = \hat{\underline{z}}$

$$(\underline{\sigma})^{a}_{b} \cdot \xi^{b}_{m} = 0$$

with $\underline{\sigma}$ the conventional Pauli matrices.
We utilise $\epsilon^{\alpha\beta\gamma} = \epsilon_{\alpha\beta\gamma}$ as the totally antisymmetric SU(3) singlet tensor and $C^{ab} = C_{ab}$ for the corresponding tensor in SU(2), having the properties:

$$C^{12} = -C^{21} = 1, \qquad (\underline{\sigma}C)^{ab} = (\underline{\sigma}C)^{ba}$$
$$(\underline{\sigma}C)^{ab*} = -(\underline{\sigma}C)_{ab}, \qquad C^{ab}C_{bc} = -\delta^a_c$$

With this notation, the completely symmetric $\underline{56}$-plet, for total angular momentum J and z-component M, is decomposed into ${}^2\underline{8}$ and ${}^4\underline{10}$ representations under SU(3) x SU(2)$_s$ according to:

$$\begin{aligned}|B^{56,L}_{JM}\rangle & (\alpha a)(\beta b)(\gamma c) \\ &= 1/3\sqrt{2} \sum_{\ell_3,m} \left\{ \epsilon^{\alpha\beta\sigma} B^{\gamma}_{\sigma} C^{ab}\chi^c_m + \epsilon^{\beta\gamma\sigma} B^{\alpha}_{\sigma} C^{bc}\chi^a_m \right. \\ &\qquad \left. + \epsilon^{\gamma\alpha\sigma} B^{\beta}_{\sigma} C^{ca}\chi^b_m \right\} |L\ell_3\rangle\langle JM|L\ell_3,\tfrac{1}{2}m\rangle \\ &\quad + 1/\sqrt{2} \sum_{\ell_3,m} D^{\alpha\beta\gamma} (\underline{\sigma}C)^{ab}\cdot\xi^c_m |L\ell_3\rangle\langle JM\ L\ell_3,3/2m\rangle \end{aligned} \qquad \text{III.1}$$

Here $|L\ell_3\rangle$ are the orbital angular momentum L, z-component ℓ_3, states of the three quark system of the appropriate symmetry. For the mixed symmetry $\underline{70}$-plet, the decomposition into ${}^2\underline{8}$, ${}^4\underline{8}$, ${}^2\underline{10}$ and ${}^2\underline{1}$ representations is, respectively:

$$\begin{aligned}|B^{70,L}_{JM}\rangle & [(\alpha a)(\beta b)](\gamma c) \\ &= 1/\sqrt{6} \sum_{\ell_3,m} \left\{ \epsilon^{\beta\gamma\sigma} B^{\alpha}_{\frac{1}{2}\sigma} C^{bc}\chi^a_m - \epsilon^{\alpha\gamma\sigma} B^{\beta}_{\frac{1}{2}\sigma} C^{ac}\chi^b_m \right\} |L\ell_3\rangle\langle JM|L\ell_3,\tfrac{1}{2}m\rangle \\ &+ \tfrac{1}{2} \sum_{\ell_3,m} \epsilon^{\alpha\beta\sigma} B^{\gamma}_{3/2\,\sigma} (\underline{\sigma}C)^{ab}, \xi^c_m |L\ell_3\rangle\langle JM|L\ell_3,3/2m\rangle \\ &+ 1/\sqrt{2} \sum_{\ell_3,m} D^{\alpha\beta\gamma} C^{ab}\chi^c_m |L\ell_3\rangle\langle JM|L\ell_3,\tfrac{1}{2}m\rangle \\ &+ 1/6 \sum_{\ell_3,m} \epsilon^{\alpha\beta\gamma} Y (C^{ac}\chi^b_m + C^{bc}\chi^a_m) |L\ell_3\rangle\langle JM|L\ell_3,\tfrac{1}{2}m\rangle \end{aligned} \qquad \text{III.2}$$

where Y represents the SU(3) singlet baryon.

In principle, there are ambiguities of sign in relating quark spin S = 3/2 to S = ½ states, decuplets to octets and singlets, and also in the choice of LS or SL coupling; we consistently use the signs in equations 111.1 and 111.2. For observable cross-sections the sign conventions are of course immaterial, but they are crucial at intermediary stages of calculation. The wavefunctions are normalised so that:

$$\left\langle B^{56,L}_{J'M'}\right|_{ABC}\left|B^{56,L}_{JM}\right\rangle^{ABC} = \delta_{J'J}\,\delta_{M'M}\left\{\mathrm{Tr}(\bar{B}B) + \bar{D}_{\alpha\beta\gamma}D^{\alpha\beta\gamma}\right\}$$

$$\left\langle B^{70,L}_{J'M'}\right|_{[AB]C}\left|B^{70,L}_{JM}\right\rangle^{[AB]C} = \delta_{J'J}\,\delta_{M'M}\left\{\mathrm{Tr}(\bar{B}_{\frac{1}{2}}B_{\frac{1}{2}}) + \mathrm{Tr}(\bar{B}_{\frac{3}{2}}B_{\frac{3}{2}}) + \bar{D}_{\alpha\beta\gamma}D^{\alpha\beta\gamma} + \bar{\gamma}\gamma\right\} \qquad \text{III.3}$$

The completeness properties are:

$$\sum_{JMBD}\left|B^{56,L}_{JM}\right\rangle^{ABC}\left\langle B^{56,L}_{J'M'}\right|_{A'B'C'} = \sum_{\ell_3}|L\ell_3\rangle\langle\ell_3 L| \times 1/6\left\{\delta^A_{A'}\delta^B_{B'}\delta^C_{C'} + \delta^B_{A'}\delta^C_{B'}\delta^A_{C'} + \delta^C_{A'}\delta^A_{B'}\delta^B_{C'} + \delta^B_{A'}\delta^A_{B'}\delta^C_{C'} + \delta^C_{A'}\delta^B_{B'}\delta^A_{C'} + \delta^A_{A'}\delta^C_{B'}\delta^B_{C'}\right\}$$

$$\sum_{JMB_{\frac{1}{2}}B\,DY}\left|B^{70,L}_{JM}\right\rangle^{[AB]C}\left\langle B^{70,L}_{J'M'}\right|_{[A'B']C'} = \sum_{\ell_3}|L\ell_3\rangle\langle\ell_3 L| \times 1/6\left\{2\delta^A_{A'}\delta^B_{B'}\delta^C_{C'} - \delta^B_{A'}\delta^C_{B'}\delta^A_{C'} - \delta^C_{A'}\delta^A_{B'}\delta^B_{C'} - 2\delta^B_{A'}\delta^A_{B'}\delta^C_{C'} + \delta^C_{A'}\delta^B_{B'}\delta^A_{C'} + \delta^A_{A'}\delta^C_{B'}\delta^B_{C'}\right\} \qquad \text{III.4}$$

The usefulness of 111.4 for us is that it may be employed to obtain:

$$\sum_{J'M'B'D'}\left|\left\langle B'^{56,L}_{J'M'}\right|_{A'BC} J^A_{A'}\left|B^{56,0}_{JM}\right\rangle^{ABC}\right|^2 = 1/3U + 2/3E$$

$$\sum_{J'M'B'B'D'Y'}\left|\left\langle B'^{70,L}_{J'M'}\right|_{A'BC} J^A_{A'}\left|B^{56,0}_{JM}\right\rangle^{ABC}\right|^2 = \tfrac{1}{2}U - \tfrac{1}{2}E \qquad \text{III.5}$$

$$\text{where } U = \sum_{\ell_3} (\bar{B}^{56,0}_{JM})_{ABC} (B^{56,0}_{JM})^{A'BC} (J^{*}_{\ell_3} J_{\ell_3})^{A}_{A'} \qquad \text{III.6}$$

$$E = \sum_{\ell_3} (\bar{B}^{56,0}_{JM})_{ABC} (B^{56,0}_{JM})^{A'B'C} (J^{*}_{\ell_3})^{A}_{A'} (J_{\ell_3})^{B}_{B'}$$

where $(B^{56,0}_{JM})^{ABC}$ represents just the ground state SU(6) 56 wavefunctions and:

$$(J_{\ell_3})^{B}_{B'} = \langle L\ell_3 | J^{B}_{B'} | 0 \rangle$$

$|0\rangle$ being the L = 0 ground state orbital wavefunction. The term E in III.5 contains the contributions of the only exotic t-channel 405 representation (and in a quark picture corresponds to the currents coupling to different quarks as in fig. 15(b)).

To calculate the couplings of the L = 0 octet baryons to excited 56 and 70 states under the action of a current:

$$J^{(\alpha a)}_{(\beta b)} = J^{\alpha}_{\beta} j^{a}_{b}$$

belonging to a 35, the main step is to evaluate:

$$\left\langle B^{56,L}_{JM} \right|_{ABC} J^{A}_{A'} \left| B^{56,0}_{J'M'} \right\rangle^{A'BC}$$

$$= \sum_{\ell_3 m} \left\langle JM | L\ell_3, \tfrac{1}{2}m \right\rangle 1/9 \left\langle L\ell_3 \right| \bar{\chi}_m j \chi_{m'} \left[\mathrm{Tr}(\bar{B}BJ) + 5\mathrm{Tr}(B\bar{B}J)\right]$$
$$- j^{a}_{a} \bar{\chi}_m \chi_{m'} \left[2\mathrm{Tr}(\bar{B}BJ) + \mathrm{Tr}(B\bar{B}J)\right] |0\rangle$$
$$+ \sum_{\ell_3 m} \left\langle JM | L\ell_3, 3/2m \right\rangle 1/3 \left\langle L\ell_3 \right| \bar{\xi}_m \cdot j\sigma \chi_{m'} \bar{D}_{\alpha\beta\gamma} J^{\alpha}_{\alpha'} \epsilon^{\alpha'\beta\sigma} B^{\gamma}_{\sigma} | 0 \rangle \qquad \text{III.7}$$

and:

$$\langle B^{70,L}_{JM} | {}_{[AB]C} J^{A}_{A'} | B^{56,0}_{J'M'} \rangle^{A'BC} \qquad \text{III.8}$$

$$= \sum_{\ell_3 m} \langle JM | L\ell_3, \tfrac{1}{2}m \rangle \, 1/6\sqrt{3} \, \langle L\ell_3 | [\bar{\chi}_m j \chi_{m'} \{ J^{\alpha}_{\alpha} Tr(\bar{B}_{\frac{1}{2}} B) - Tr(\bar{B}_{\frac{1}{2}} BJ) + 4Tr(B\bar{B}_{\frac{1}{2}} J) \} - j^{a}_{a} \bar{\chi}_m \chi_{m'} \{ 2J^{\alpha}_{\alpha} Tr(\bar{B}_{\frac{1}{2}} B) - 2Tr(\bar{B}_{\frac{1}{2}} BJ) - Tr(B\bar{B}_{\frac{1}{2}} J) \}] | 0 \rangle$$

$$+ \sum_{\ell_3 m} \langle JM | L\ell_3, 3/2m \rangle \, 1/6\sqrt{2} \, \langle L\ell_3 | \bar{\chi}_m \cdot j\underline{\sigma} \chi_{m'} [2J^{\alpha}_{\alpha} Tr(\bar{B}_{3/2} B) - 2Tr(\bar{B}_{3/2} BJ) - Tr(B\bar{B}_{3/2} J)] | 0 \rangle$$

$$+ \sum_{\ell_3 m} \langle JM | L\ell_3, \tfrac{1}{2}m \rangle \, 1/6 \, \langle L\ell_3 | (2j^{a}_{a} \bar{\chi}_m \chi_{m'} - \bar{\chi}_m j \chi_{m'}) \times \bar{D}_{\alpha\beta\gamma} J^{\alpha}_{\alpha'} \epsilon^{\alpha'\beta\sigma} B^{\gamma}_{\sigma} | 0 \rangle$$

$$+ \sum_{\ell_3 m} \langle JM | L\ell_3, \tfrac{1}{2}m \rangle \, 1/2\sqrt{2} \, \langle L\ell_3 | \bar{\chi}_m j \chi_{m'} \bar{\gamma} Tr(JB) | 0 \rangle \quad \ldots.$$

where the discarded terms are irrelevant for initial octet states.

For the transverse current, from 4.2, we have:

$$(j_T^{+})^{a}_{b} = aL^{+}\delta^{a}_{b} + ib(S^{+})^{a}_{b} + icL^{+}(S^{0})^{a}_{b} + idL^{++}(S^{-})^{a}_{b}$$

with S^{+}, S^{0} represented by the usual spin matrices and it is now easy to obtain the results of Table I where:

$$\begin{aligned} A &= 1/3 \langle L1 | aL^{+} | 0 \rangle = n_L/3 \langle L\,-1 | aL^{-} | 0 \rangle \\ B &= i/3 \langle L0 | b | 0 \rangle = in_L/3 \langle L0 | b | 0 \rangle \\ C &= i/3 \langle L1 | cL^{+} | 0 \rangle = in_L/3 \langle L\,-1 | cL^{-} | 0 \rangle \\ D &= i/3 \langle L2 | dL^{+}L^{+} | 0 \rangle = n_L/3 \langle L\,-2 | dL^{-}L^{-} | 0 \rangle \end{aligned} \qquad \text{III.9}$$

We have used in III.9:

$$Y | L\ell_3 \rangle = n_L (-)^{\ell_3} | L, -\ell_3 \rangle$$

n_L is the orbital normality, the parity being $(-)^L n_L$. For A',B',C',D' the results are identical save that the constant of proportionality in

III.9 is 1/2 3 instead of 1/3. States with n_L = -, when B = 0, have not as yet been identified among low-lying resonances, but there is no general reason for their non-existence in highly excited SU(6) multiplets.

For the longitudinal vector and axial currents, from 4.4 and 4.5:

$$(j_L)^a_b = \alpha \delta^a_b + i\beta(S^+L^- + S^-L^+)^a_b$$
$$(\tilde{j}_L)^a_b = \tilde{\alpha}(S^o)^a_b + \tilde{\beta}(-S^+L^- + S^-L^+)^a_b$$

so that:

$$\alpha = 1/3 \langle L0|\alpha|0\rangle, \quad \tilde{\alpha} = 1/3 \langle L0|\tilde{\alpha}|0\rangle$$
$$\beta = i/3 \langle L1|\beta L^+|0\rangle = in_L/3 \langle L\ -1|\beta L^-|0\rangle \qquad \text{III.10}$$
$$\tilde{\beta} = 1/3 \langle L1|\tilde{\beta} L^+|0\rangle = n_L/3 \langle L\ -1|\tilde{\beta} L^-|0\rangle$$

where for n_L = -, $\alpha = \tilde{\alpha} = 0$ and in the same way $\alpha, \tilde{\alpha}, \beta, \tilde{\beta}$ can be shown to be real. For $\alpha', \tilde{\alpha}', \beta', \tilde{\beta}'$ the results are obtained as above, except with $1/2\sqrt{3}$ instead of 1/3.

Appendix IV

To clarify the possibly surprising result that the polarisation asymmetry for virtual photon-proton scattering could be negative, we present the following simple argument. From the proton's SU(6) wavefunction, the probabilities to find p or n quarks with spin along ($\uparrow$) or opposite ($\downarrow$) to the proton spin, taken along its direction of motion are:

$$P_{p\uparrow} = 5/9 \quad , \quad P_{p\downarrow} = 1/9$$
$$P_{n\uparrow} = 1/9 \quad , \quad P_{n\downarrow} = 2/9$$

The cross-sections for virtual photon scattering on quarks with spins parallel and antiparallel to the photon spin are Q^2y and Q^2z, where Q is the quark charge, and from Section 4.2:

$$y = 3(A + C)^2 + 3D^2$$
$$z = 3(A - C)^2 + 3B^2$$

Hence the cross-section for virtual photon spin parallel to the proton spin is:

$$\sigma^{\gamma p}_{3/2} = 3(5/9.4/9 + 1/9.1/9)y + 3(1/9.4/9 + 2/9.1/9)z$$
$$= 7/9y + 2/9z$$

the overall factor 3 measuring the three quarks.

Similarly:

$$\sigma^{\gamma p}_{\frac{1}{2}} = 7/9z + 2/9y$$

so that: $\sigma^{\gamma p}_{T} = y + z$

and: $A^{\gamma p} = 5/9(z - y)/(z + y)$

This calculation ignores interference effects between different quarks, but these are conventionally supposed absent in the deep inelastic scaling region.

In naive quark models, without the Melosh transformation, $y = 0$, leading to $A^{\gamma p} = 5/9$.

A similar discussion for $\sigma^{\gamma n}$ leads directly to $\sigma^{\gamma n}/\sigma^{\gamma p} = 2/3$ and $A^{\gamma n} = 0$.

Appendix V

i) The form of the interaction for the linearised oscillator model.

The squared interaction must be of the form $\omega^2 x^2$ + constant, see ref 76, Chapter 6, and this is satisfied by the original form chosen:

$$U = -i\omega\gamma_5^1\gamma_5^2(\gamma_{1\mu} - \gamma_{2\mu})x^\mu$$

However it is also satisfied by:

$$U = -i\omega\gamma_5^1\gamma_5^2(\gamma_{1\mu} - \gamma_{2\mu})x^\mu + \gamma_5^1\gamma_5^2 B$$

where B is an arbitrary constant.

Using this form, the equations to be solved at the Pauli spinor level are:

$$(\mu-\epsilon)|1\rangle + \beta|4\rangle = \tfrac{1}{2}\underline{a}\left[\underline{\sigma}|3-2\rangle + |3-2\rangle\underline{\sigma}^T\right] + \tfrac{1}{2}\underline{a}^\dagger\left[\underline{\sigma}|3+2\rangle - |3+2\rangle\underline{\sigma}^T\right]$$

$$(\mu+\epsilon)|4\rangle + \beta|1\rangle = \tfrac{1}{2}\underline{a}\left[\underline{\sigma}|3-2\rangle + |3-2\rangle\underline{\sigma}^T\right] - \tfrac{1}{2}\underline{a}^\dagger\left[\underline{\sigma}|3+2\rangle - |3+2\rangle\underline{\sigma}^T\right]$$

$$\mu|2\rangle + \beta|3\rangle = \tfrac{1}{2}\underline{a}\left[\underline{\sigma}|4-1\rangle - |4-1\rangle\underline{\sigma}^T\right] + \tfrac{1}{2}\underline{a}^\dagger\left[\underline{\sigma}|4+1\rangle + |4+1\rangle\underline{\sigma}^T\right]$$

$$\mu|3\rangle + \beta|2\rangle = \tfrac{1}{2}\underline{a}\left[\underline{\sigma}|4-1\rangle - |4-1\rangle\underline{\sigma}^T\right] - \tfrac{1}{2}\underline{a}^\dagger\left[\underline{\sigma}|4+1\rangle + |4+1\rangle\underline{\sigma}^T\right]$$

(V.1,2,3,4)

ii) Solutions for the PC = − case

Using the same method as in ref 76, A4 we find that $|1\rangle$ and $|4\rangle$ are pure spin 0 so that:

$$\underline{\sigma}|4\pm1\rangle + |4\pm1\rangle\underline{\sigma}^T = 0 .$$

Hence V.3,4 reduce to:

$$\mu|2\rangle + \beta|3\rangle = \mu|3\rangle + \beta|2\rangle = \tfrac{1}{2}\underline{a}\left[\underline{\sigma}|4-1\rangle - |4-1\rangle\underline{\sigma}^T\right]$$

i.e. $|3\rangle = |2\rangle$

Substituting into V.1,2:

$$(\mu-\epsilon)|1\rangle + \beta|4\rangle = -(\mu+\epsilon)|4\rangle - \beta|1\rangle$$

$$= \tfrac{1}{2}\underline{a}^\dagger\left[\underline{\sigma}|3+2\rangle - |3+2\rangle\underline{\sigma}^T\right]$$

so that: $|4\rangle = (\epsilon - \overline{\mu+\beta})/(\epsilon + \overline{\mu+\beta})|1\rangle$

Thus:

$$\left\{(\mu-\epsilon)+\beta\left[\epsilon-\sqrt{\mu+\beta}\right]/\left[\epsilon+\sqrt{\mu+\beta}\right]\right\}|1\rangle = \underline{a}^{+}\left[\underline{\sigma}|3\rangle - |3\rangle\underline{\sigma}^{T}\right]$$
$$= 2\underline{a}^{\dagger}.\underline{a}/(\mu+\beta)\left\{(\epsilon-\sqrt{\mu+\beta})/(\epsilon+\sqrt{\mu+\beta}) - 1\right\}|1\rangle$$

Hence the eigenvalue equation is:

$$\epsilon^2 = \mu^2 - \beta^2 + 4N$$

ii) Solutions for the PC = + case

Adding and subtracting equations V.1,2 and V.3,4, we may rewrite them:

$$(\mu-\epsilon+\beta)|1\rangle + (\mu+\epsilon+\beta)|4\rangle = \underline{a}\left[\underline{\sigma}|3-2\rangle + |3-2\rangle\underline{\sigma}^{T}\right] \qquad \text{V.5}$$
$$(\mu-\epsilon-\beta)|1\rangle + (\beta-\mu-\epsilon)|4\rangle = \underline{a}^{\dagger}\left[\underline{\sigma}|3+2\rangle + |3+2\rangle\underline{\sigma}^{T}\right] \qquad \text{V.6}$$
$$(\mu+\beta)|2+3\rangle = \underline{a}\left[\underline{\sigma}|4-1\rangle - |4-1\rangle\underline{\sigma}^{T}\right] \qquad \text{V.7}$$
$$(\mu-\beta)|2-3\rangle = \underline{a}^{\dagger}\left[\underline{\sigma}|4+1\rangle + |4+1\rangle\underline{\sigma}^{T}\right] \qquad \text{V.8}$$

From charge conjugation, $|1\rangle, |4\rangle$ are pure spin 1, so that from V.7, $|2+3\rangle$ is pure spin 0 or $|2\rangle = -|3\rangle$. Similarly from V.8, $|2-3\rangle$ is pure spin 1 or $|2\rangle = |3\rangle$. Consider first the possibility that $|2+3\rangle$ is spin 0 and $|2\rangle = |3\rangle$. Then from V.6 :

$$(\mu-\epsilon-\beta)|1\rangle + (\beta-\mu-\epsilon)|4\rangle = 0$$

But from V.5:

$$(\mu-\epsilon+\beta)|1\rangle + (\mu+\epsilon+\beta)|4\rangle = 0$$

which is the trivial solution $|1\rangle = |4\rangle = 0$.

Hence $|2\rangle = -|3\rangle$ and $|2-3\rangle$ is spin 1.

Substituting into V.5,6:

$$(\mu-\epsilon+\beta)|1\rangle + (\mu+\epsilon+\beta)|4\rangle = 2\underline{a}\left[\underline{\sigma}|3\rangle + |3\rangle\underline{\sigma}^{T}\right]$$
$$(\mu-\epsilon-\beta)|1\rangle + (\beta-\mu-\epsilon)|4\rangle = 0$$

Hence solving as before:

$$(\mu^2-\beta^2-\epsilon^2)|1\rangle = -4\underline{a}^{\dagger}.\underline{a}|1\rangle$$

and the eigenvalue equation is:

$$\epsilon^2 = \mu^2 - \beta^2 + 4N$$

TABLE I

Transition matrix elements for transverse currents from initial baryon octet states to arbitrary states in 56- or 70-plet representations of SU(6).

SU(6); $^{2S+1}$ SU(3)		
56 ; ^{2}8	$A_{3/2}$	$A\langle J\tfrac{3}{2}\|L1,\tfrac{1}{2}\tfrac{1}{2}\rangle f + (C\langle J\tfrac{3}{2}\|L1,\tfrac{1}{2}\tfrac{1}{2}\rangle + D\langle J\tfrac{3}{2}\|L2,\tfrac{1}{2}-\tfrac{1}{2}\rangle) \times 1/3(2f+3d)$
	$A_{\frac{1}{2}}$	$A\langle J\tfrac{1}{2}\|L1,\tfrac{1}{2}-\tfrac{1}{2}\rangle f + (B\langle J\tfrac{1}{2}\|L0,\tfrac{1}{2}\tfrac{1}{2}\rangle - C\langle J\tfrac{1}{2}\|L1,\tfrac{1}{2}-\tfrac{1}{2}\rangle) \times 1/3(2f+3d)$
56 ; ^{4}10	$A_{3/2}$	$\sqrt{2/3}(\sqrt{3}B\langle J\tfrac{3}{2}\|L0,\tfrac{3}{2}\tfrac{3}{2}\rangle - 2C\langle J\tfrac{3}{2}\|L1,\tfrac{3}{2}\tfrac{1}{2}\rangle - D\langle J\tfrac{3}{2}\|L2,\tfrac{3}{2}-\tfrac{1}{2}\rangle)\Delta$
	$A_{\frac{1}{2}}$	$\sqrt{2/3}(B\langle J\tfrac{1}{2}\|L0,\tfrac{3}{2}\tfrac{1}{2}\rangle - 2C\langle J\tfrac{1}{2}\|L1,\tfrac{3}{2}-\tfrac{1}{2}\rangle - 3D\langle J\tfrac{1}{2}\|L2,\tfrac{3}{2}-\tfrac{3}{2}\rangle)\Delta$
70 ; ^{2}8	$A_{3/2}$	$A'\langle J\tfrac{3}{2}\|L1,\tfrac{1}{2}\tfrac{1}{2}\rangle\tfrac{1}{2}(f+3d) + (C'\langle J\tfrac{3}{2}\|L1,\tfrac{1}{2}\tfrac{1}{2}\rangle + D'\langle J\tfrac{3}{2}\|L2,\tfrac{1}{2}-\tfrac{1}{2}\rangle) \times 1/6(5f+3d)$
	$A_{\frac{1}{2}}$	$A'\langle J\tfrac{1}{2}/L1,\tfrac{1}{2}-\tfrac{1}{2}\rangle\tfrac{1}{2}(f+3d) + (B'\langle J\tfrac{1}{2}\|L0,\tfrac{1}{2}\tfrac{1}{2}\rangle - C'\langle J\tfrac{1}{2}\|L1,\tfrac{1}{2}-\tfrac{1}{2}\ ) \times 1/6(5f+3d)$
70 ; ^{4}8	$A_{3/2}$	$(\sqrt{3}B'\langle J\tfrac{3}{2}/L0,\tfrac{3}{2}\tfrac{3}{2}\rangle - 2C'\langle J\tfrac{3}{2}\|L1,\tfrac{3}{2}\tfrac{1}{2}\rangle - D'\langle J\tfrac{3}{2}\|L2,\tfrac{3}{2}-\tfrac{1}{2}\rangle) \times 1/6(-f+3d)$
	$A_{\frac{1}{2}}$	$(B'\langle J\tfrac{1}{2}\|L0,\tfrac{3}{2}\tfrac{1}{2}\rangle - 2C'\langle J\tfrac{1}{2}\|L1,\tfrac{3}{2}-\tfrac{1}{2}\rangle - 3D'\langle J\tfrac{1}{2}\ L2,\tfrac{3}{2}-\tfrac{3}{2}\rangle) \times 1/6(-f+3d)$
70 ; 10	$A_{3/2}$	$-3((A'-1/3C')\langle J\tfrac{3}{2}\|L1,\tfrac{1}{2}\tfrac{1}{2}\rangle -1/3D'\langle J\tfrac{3}{2}\|L2,\tfrac{1}{2}-\tfrac{1}{2}\rangle)\Delta$
	$A_{\frac{1}{2}}$	$-3((A'+1/3C')\langle J\tfrac{1}{2}\|L1,\tfrac{1}{2}-\tfrac{1}{2}\rangle -1/3B'\langle J\tfrac{1}{2}\|L0,\tfrac{1}{2}\tfrac{1}{2}\rangle)\Delta$
70 ; 1	A	$\sqrt{3/2}((A'+C')\langle J\tfrac{3}{2}/L1,\tfrac{1}{2}\tfrac{1}{2}\rangle + D'\langle J\tfrac{3}{2}\|L2,\tfrac{1}{2}-\tfrac{1}{2}\rangle)x$
	$A_{\frac{1}{2}}$	$\sqrt{3/2}((A'-C')\langle J\tfrac{1}{2}/L1,\tfrac{1}{2}-\tfrac{1}{2}\rangle + B'\langle J\tfrac{1}{2}\|L0,\tfrac{1}{2}\tfrac{1}{2}\rangle)x$

A ,$A_{\frac{1}{2}}$ are defined in 4.5 and $f = \mathrm{Tr}([B,\bar{B}]J)$, $d = \mathrm{Tr}(\{B,\bar{B}\}J)$, $\Delta = -\bar{D}_{\alpha\beta\gamma} J^{\alpha}{}_{\alpha'} \epsilon^{\alpha'\beta\sigma} B^{\gamma}_{\sigma}$, $x = \mathrm{Tr}(JB)$ are the SU(3) invariant couplings between two octets, between an octet and a decuplet and between an octet and a singlet under the action of an octet current J. If $n_L = (-)^L \eta_R = -$ then $B = 0$. For $J_{e.m.} = \frac{1}{2}(\lambda_1 + 1/\sqrt{3}\,\lambda_8)$ on a proton, $f = 1$, $d = 1/3$, $\Delta = -1/\sqrt{3}$,

$x = 0$ and on a neutron, $f = 0$, $d = -2/3$, $\Delta = -1/\sqrt{3}$, $x = 0$. For $J_{wk} = \frac{1}{2}(\lambda_1 + i\lambda_2)$ on a proton, $f = d = 0$, $\Delta = 1$, $x = 0$ and on a neutron $f = d = 1$, $\Delta = 1/\sqrt{3}$, $x = 0$.

TABLE II

Transition matrix elements for longitudinal vector currents from initial baryon octet states to arbitrary states in 56- or 70-plet representations of SU(6).

SU(6); $^{2S+1}$[SU(3)]	A_L
56 ; ^{2}8	$\alpha\langle J\frac{1}{2}\vert L0,\frac{1}{2}\frac{1}{2}\rangle f + \beta\langle J\frac{1}{2}\vert L1,\frac{1}{2}-\frac{1}{2}\rangle 1/3(2f+3d)$
56 ; ^{4}10	$-\sqrt{2/3}\,\beta\left(\langle J\frac{1}{2}\vert L1,\frac{3}{2}-\frac{1}{2}\rangle - n_L\sqrt{3}\langle J\frac{1}{2}\vert L-1,\frac{3}{2}\frac{3}{2}\rangle\right)\Delta$
70 ; ^{2}8	$\alpha'\langle J\frac{1}{2}\vert L0,\frac{1}{2}\frac{1}{2}\rangle\frac{1}{2}(f+3d) + \beta'\langle J\frac{1}{2}\vert L1,\frac{1}{2}-\frac{1}{2}\rangle 1/6(5f+3d)$
70 ; ^{4}8	$-\beta'\left(\langle J\frac{1}{2}\vert L1,\frac{3}{2}-\frac{1}{2}\rangle - n_L\sqrt{3}\langle J\frac{1}{2}\vert L-1,\frac{3}{2}\frac{3}{2}\rangle\right)1/6(-f+3d)$
70 ; ^{2}10	$-\sqrt{3}\left(\alpha'\langle J\frac{1}{2}\vert L0,\frac{1}{2}\frac{1}{2}\rangle - 1/3\beta'\langle J\frac{1}{2}\vert L1,\frac{1}{2}-\frac{1}{2}\rangle\right)\Delta$
70 ; ^{2}1	$\sqrt{3/2}\left(\alpha'\langle J\frac{1}{2}\vert L0,\frac{1}{2}\frac{1}{2}\rangle + \beta'\langle J\frac{1}{2}\vert L1,\frac{1}{2}-\frac{1}{2}\rangle\right)x$

A_L is defined in 4.5, and the other quantities are defined as in Table I. $n_L = (-)^L\eta_R$ and if $n_L = -$, $\alpha = 0$.

TABLE III

Transition matrix elements for longitudinal axial currents from initial baryon octet states to arbitrary states in 56- or 70-plet representations of SU(6).

SU(6); $^{2S+1}$[SU(3)]	$\tilde{A}_L$
$\underline{56}$; $^2\underline{8}$	$(\tilde{\alpha}\langle J\tfrac{1}{2}\vert L0,\tfrac{1}{2}\tfrac{1}{2}\rangle + \tilde{\beta}\langle J\tfrac{1}{2}\vert L1,\tfrac{1}{2}-\tfrac{1}{2}\rangle)1/3(2f+3d)$
$\underline{56}$; $^4\underline{10}$	$-\sqrt{2/3}(2\tilde{\alpha}\langle J\tfrac{1}{2}\vert L0,\tfrac{3}{2}\tfrac{1}{2}\rangle + \tilde{\beta}(\langle J\tfrac{1}{2}\vert L1,\tfrac{3}{2}-\tfrac{1}{2}\rangle + n_L\sqrt{3}\langle J\tfrac{1}{2}\vert L-1,\tfrac{3}{2}\tfrac{3}{2}\rangle))\Delta$
$\underline{70}$; $^2\underline{8}$	$(\alpha'\langle J\tfrac{1}{2}\vert L0,\tfrac{1}{2}\tfrac{1}{2}\rangle + \beta'\langle J\tfrac{1}{2}\vert L1,\tfrac{1}{2}-\tfrac{1}{2}\rangle)1/6(5f+3d)$
$\underline{70}$; $^4\underline{8}$	$-(2\alpha'\langle J\tfrac{1}{2}\vert L0,\tfrac{3}{2}\tfrac{1}{2}\rangle + \beta'(\langle J\tfrac{1}{2}\vert L1,\tfrac{3}{2}-\tfrac{1}{2}\rangle + n_L\sqrt{3}\langle J\tfrac{1}{2}\vert L-1,\tfrac{3}{2}\tfrac{3}{2}\rangle))1/6(-f+3d)$
$\underline{70}$; $^2\underline{10}$	$1/\sqrt{3}(\alpha'\langle J\tfrac{1}{2}\vert L0,\tfrac{1}{2}\tfrac{1}{2}\rangle + \beta'\langle J\tfrac{1}{2}\vert L1,\tfrac{1}{2}-\tfrac{1}{2}\rangle)\Delta$
$\underline{70}$; $^2\underline{1}$	$\sqrt{3/2}(\alpha'\langle J\tfrac{1}{2}\vert L0,\tfrac{1}{2}\tfrac{1}{2}\rangle + \beta'\langle J\tfrac{1}{2}\vert L1,\tfrac{1}{2}-\tfrac{1}{2}\rangle)x$

A_L is defined in 4.5 and the other quantities are defined as in Table I. $n_L = (-)^L\eta_R$ and if $n_L = -$, $\tilde{\alpha} = 0$.

TABLE IV

Contributions to the cross-sections $\sigma_{\frac{1}{2}}, \sigma_{\frac{3}{2}}, \sigma_L, \sigma_{TL}$, defined in section 4.3, for $J = J_{e.m.} = \frac{1}{2}(\lambda_3 + 1/\sqrt{3}\lambda_8)$ and for proton and neutron targets, $\sigma^{\gamma p}, \sigma^{\gamma n}$.

SU(6) ; $^{2S+1}$[SU(3)]	$\sigma^{\gamma p}_{\frac{1}{2}}$	$\sigma^{\gamma p}_{\frac{3}{2}}$	$\sigma^{\gamma n}_{\frac{1}{2}}$	$\sigma^{\gamma n}_{\frac{3}{2}}$
$\underline{56}$; $^2\underline{8}$	$(A-C)^2 + B^2$	$(A+C)^2 + D^2$	$4/9(B^2 + C^2)$	$4/9(C^2 + D^2)$
$\underline{56}$; $^4\underline{10}$	$2/9(B^2+4C^2+3D^2)$	$2/9(3B^2+4C^2+D^2)$	$2/9(B^2+4C^2+3D^2)$	$2/9(3B^2+4C^2+D^2)$
$\underline{70}$; $^2\underline{8}$	$(A'-C')^2 + B'^2$	$(A'+C')^2 + D'^2$	$(A'-\frac{1}{3}C')^2 + \frac{1}{9}B'^2$	$(A'+\frac{1}{3}C')^2 + \frac{1}{9}D'^2$
$\underline{70}$; $^4\underline{8}$	0	0	$\frac{1}{9}(B'^2+4C'^2+3D'^2)$	$\frac{1}{9}(3B'^2+4C'^2+D'^2)$
$\underline{70}$; $^2\underline{10}$	$(A'+\frac{1}{3}C')^2 + \frac{1}{9}B'^2$	$(A'-\frac{1}{3}C')^2 + \frac{1}{9}D'$	$(A'+\frac{1}{3}C')^2 + \frac{1}{9}B'^2$	$(A'-\frac{1}{3}C')^2 + \frac{1}{9}D'^2$

SU(6) ; $^{2S+1}$[SU(3)]	$\sigma^{\gamma p}_{L}$	$\sigma^{\gamma p}_{TL}$	$\sigma^{\gamma n}_{L}$	$\sigma^{\gamma n}_{TL}$
$\underline{56}$; $^2\underline{8}$	$\alpha^2+\beta^2$	$\alpha B + \beta(A-C)$	$4/9\beta^2$	$-4/9\beta C$
$\underline{56}$; $^4\underline{10}$	$8/9\beta^2$	$4/9\beta C$	$8/9\beta^2$	$4/9\beta C$
$\underline{70}$; $^2\underline{8}$	$\alpha'^2 + \beta'^2$	$\alpha' B' + \beta'(A'-C')$	$\alpha'^2 + \frac{1}{9}\beta'^2$	$\frac{1}{3}\alpha' B' + \frac{1}{3}\beta'(A-\frac{1}{3}C)$
$\underline{70}$; $^4\underline{8}$	0	0	$4/9\beta'^2$	$2/9\beta' C'$
$\underline{70}$; $^2\underline{10}$	$\alpha'^2+\frac{1}{9}\beta'^2$	$-\frac{1}{3}\alpha' B' - \frac{1}{3}\beta'(A'\ \frac{1}{3}C')$	$\alpha'^2 + \frac{1}{9}\beta'^2$	$-\frac{1}{3}\alpha' B' - \frac{1}{3}\beta'(A'+\frac{1}{3}C')$

TABLE V

Contributions to the cross-sections $\sigma_{\frac{1}{2}}$, $\sigma_{\frac{3}{2}}$, σ_L, $\tilde{\sigma}_L$, defined in section 4.3, for $J = J_{wk} = \frac{1}{2}(\lambda_1 + i\lambda_2)$ and for proton and neutron targets σ^{W^+p}, σ^{W^+n}, $\sigma^{W^-p} = \sigma^{W^+p}$.

SU(6); $^{2S+1}$[SU(3)]	$\sigma_{\frac{1}{2}}^{W^+p}$	$\sigma_{\frac{3}{2}}^{W^+p}$	$\sigma_{\frac{1}{2}}^{W^+n}$	$\sigma_{\frac{3}{2}}^{W^+n}$
<u>56</u> ; ^{2}<u>8</u>	0	0	$(A-\frac{5}{3}C)^2+\frac{25}{9}B^2$	$(A+\frac{5}{3}C)^2+\frac{25}{9}D^2$
<u>56</u> ; ^{4}<u>10</u>	$\frac{2}{3}(B^2+4C^2+3D^2)$	$\frac{2}{3}(3B^2+4C^2+D^2)$	$\frac{2}{9}(B^2+4C^2+3D^2)$	$\frac{2}{9}(3B^2+4C^2+D^2)$
<u>70</u> ; ^{2}<u>8</u>	0	0	$4(A-\frac{2}{3}C)^2+\frac{16}{9}B^2$	$4(A+\frac{2}{3}C)^2+\frac{16}{9}D^2$
<u>70</u> ; ^{4}<u>8</u>	0	0	$\frac{1}{9}(B^2+4C^2+3D^2)$	$\frac{1}{9}(3B^2+4C^2+D^2)$
<u>70</u> ; ^{2}<u>10</u>	$3(A+\frac{1}{3}C)^2+\frac{1}{3}B^2$	$3(A-\frac{1}{3}C)^2+\frac{1}{3}D^2$	$(A+\frac{1}{3}C)^2+\frac{1}{9}B^2$	$(A-\frac{1}{3}C)^2+\frac{1}{9}D^2$

SU(6); $^{2S+1}$[SU(3)]	$\sigma_L^{W^+p}$	$\tilde{\sigma}_L^{W^+p}$	$\sigma_L^{W^+n}$	$\tilde{\sigma}_L^{W^+n}$
<u>56</u> ; ^{2}<u>8</u>	0	0	$\alpha^2+\frac{25}{9}\beta^2$	$\frac{25}{9}(\tilde{\alpha}^2+\tilde{\beta}^2)$
<u>56</u> ; ^{4}<u>10</u>	$\frac{8}{3}\beta^2$	$\frac{8}{3}(\tilde{\alpha}^2+\tilde{\beta}^2)$	$\frac{8}{9}\beta^2$	$\frac{8}{9}(\tilde{\alpha}^2+\tilde{\beta}^2)$
<u>70</u> ; ^{2}<u>8</u>	0	0	$4(\alpha'^2+\frac{4}{9}\beta'^2)$	$\frac{16}{9}(\tilde{\alpha}'^2+\tilde{\beta}'^2)$
<u>70</u> ; ^{4}<u>8</u>	0	0	$\frac{4}{9}\beta'^2$	$\frac{4}{9}(\tilde{\alpha}'^2+\tilde{\beta}'^2)$
<u>70</u> ; ^{2}<u>10</u>	$3\alpha'^2+\frac{1}{3}\beta'^2$	$\frac{1}{3}(\tilde{\alpha}'^2+\tilde{\beta}'^2)$	$\alpha'^2+\frac{1}{9}\beta'^2$	$\frac{1}{9}(\tilde{\alpha}'^2+\tilde{\beta}'^2)$

TABLE VI

Coupling constants for electromagnetic decays.

Decay	g_γ (Gev^{-1})	g_γ^{exp}(Gev^{-1})
$\omega^0 \rightarrow \pi^0\gamma$	-2.2	-2.89 ± 0.26
$\phi^0 \rightarrow \pi^0\gamma$	0	-0.16 ± 0.02
$\phi^0 \rightarrow \eta^0\gamma$	-1.2	-0.82 ± 0.12

TABLE VII

Amplitudes for decays by emission of a pseudoscalar meson.

Decay type	Quark amplitude $-i\langle f\vert \partial_\mu A^{\mu q}\vert i\rangle$	Antiquark amplitude $-i\langle f\vert \partial_\mu A^{\mu q}\vert i\rangle$
$1^{--}\to 0^{-+}0^{-+}$	$-\frac{m_1 m_2}{m_0} FX_2 q\cdot\epsilon_1$	$\frac{m_1 m_2}{m_0} FX_2 q\cdot\epsilon_1$
$2^{+}\to 0^{-+}0^{-+}$	$\frac{m_1 m_2}{m_0} FX_2 q_\mu q_\nu T^{(2)\mu\nu}$	$\frac{m_1 m_2}{m_0} FX_2 q_\mu q_\nu T^{(2)\mu\nu}$
$2^{+}\to 1^{--}0^{-+}$	$\frac{-iF}{m_1 m_2\sqrt{\Omega}}(\Sigma+\Omega)T^{(2)\mu\nu}\epsilon_{\mu\rho\sigma\lambda}(P_1^\rho P_2^\sigma \epsilon_2^\lambda) \times q_\nu$	$\frac{iF}{m_1 m_2\sqrt{\Omega}}(\Sigma+\Omega)T^{(2)\mu\nu}\epsilon_{\mu\rho\sigma\lambda}(P_1^\rho P_2^\sigma \epsilon_2^\lambda) \times q_\nu$
$1^{+}\to 1^{--}0^{-+}$	$\frac{-F\sqrt{\Omega}}{m_0}\left\{\epsilon_1\cdot\epsilon_2\left(\frac{P_1\cdot P_2 Z_2}{m_1 m_2}\right) + q\cdot\epsilon_1\, q\cdot\epsilon_2\left(\frac{m_1 m_2}{\Omega}X_2 + \frac{Z_2}{m_1 m_2}\right)\right\}$	$\frac{-F\sqrt{\Omega}}{m_0}\left\{-\epsilon_1\cdot\epsilon_2\left(\frac{P_1\cdot P_2 Z_2}{m_1 m_2}\right) + q\cdot\epsilon_1\, q\cdot\epsilon_2\left(\frac{m_1 m_2}{\Omega}X_2 + \frac{Z_2}{m_1 m_2}\right)\right\}$
$1^{++}\to 1^{--}0^{-+}$	$\frac{F}{\sqrt{2\Omega}\, m_1 m_2}\left\{\epsilon_1\cdot\epsilon_2\left[(\Omega-\Sigma)Y - 2ZP_1\cdot P_2\Omega\right] + q\cdot\epsilon_1\, q\cdot\epsilon_2\left[(P_1\cdot P_2 - 2m_1^2)\Omega + P_1\cdot P_2\Sigma\right]\right\}$	$\frac{F}{\sqrt{2\Omega}\, m_1 m_2}\left\{\epsilon_1\cdot\epsilon_2\left[-(\Omega+2m_2^2)Y + 2ZP_1\cdot P_2\Omega\right] + q\cdot\epsilon_1\, q\cdot\epsilon_2\left[-\Omega(P_1\cdot P_2 - 2m_1^2) + 2m_1^2 m_2^2\right]\right\}$

Abbreviations:

$$X_i = \left[1 + (m_0/m_i)^2\right]$$
$$Z_i = m_i^2 - P_1\cdot P_2$$
$$Y = \left[m_1^2 m_2^2 - (P_1\cdot P_2)^2\right]$$
$$\Sigma = m_1^2 + m_2^2 + m_3^2$$
$$P_1\cdot P_2 = \tfrac{1}{2}(m_1^2 + m_2^2 - m_3^2)$$

$T^{(2)\mu\nu}$ is a spin-two wave-function.

TABLE VIII

Widths for decays by emission of a pseudoscalar meson.

Decay type	State	Mode	Γ (Mev)	Γ^{exp} (Mev)	Γ^{FKR} (Mev)
$1^{--} \to 0^{-+}0^{-+}$	ϕ (1099)	$K\bar{K}$	3.14	2.5 ± 0.3	9
		$\rho\pi$	0	< 0.6	0
	ω (784)	$\pi\pi$	0	0.13 ± 0.03	0
	K^*(892)	$K\pi$	46.2	50.1 ± 1.1	59.5
		πK	46.2	50.1 ± 1.1	144
	ρ (765)	$\pi\pi$	117	146 ± 10	142
$1^{+-} \to 1^{--}0^{-+}$	B (1235)	$\omega\pi$	66	120 ± 20	76.5
$1^{++} \to 1^{--}0^{-+}$	K^*(1240)	$K^*\pi$	47	~ 100	54
	A_1 (1070)	$\rho\pi$	100	200 - 400	145
$2^{++} \to 0^{-+}0^{-+}$	f'(1514)	$K\bar{K}$	62	40 ± 10	93
		$\pi\pi$	0	~ 0	0
	f (1260)	$K\bar{K}$	5.1	8 ± 5	12
		$\pi\pi$	153	130 ± 12	220
	K^*(1420)	$K\pi$	54.7	55 ± 6	78
		πK	54.7	55 ± 6	126
		$K\eta$	2	~ 2	4.5
		ηK	2	~ 2	3.6
	A_2 (1300)	$\eta\pi$	13.8	15 ± 1.5	20
		$\pi\eta$	13.8	15 ± 1.5	40
		$K\bar{K}$	7.8	4.7 ± 1	15
$2^{++} \to 0^{-+}1^{--}$	f'(1514)	$\bar{K}K^* + K\bar{K}^*$	9.3	< 14	13.5
	K (1420)	$K^*\pi$	18.2	29.5 ± 6	20
		ρK	5.7	9.2 ± 3	7
		ωK	1.4	4.4 ± 2	1.8
	A_2 (1300)	$\rho\pi$	53	72 ± 7	60

TABLE IX

Ratio of helicity amplitudes for $1^{+} \rightarrow 1^{-}0^{-}$ decays.

Decay	(T_{00}/T_{11})	$(T_{00}/T_{11})^{exp}$	$(T_{00}/T_{11})^{FKR}$
$B \rightarrow \omega\pi$	1.0	$0.2 \rightarrow 0.7$	0.19
$A_1 \rightarrow \rho\pi$	1.0	$2.0 \rightarrow 1.1$	1.3

TABLE X

An example of corrected results, with a conserved vector current

Decay	(T_{00}/T_{11})	$(T_{00}/T_{11})^{exp}$	Γ	Γ^{exp}
$B \rightarrow \omega\pi$	0.21	$0.2 \rightarrow 0.7$	78 Mev	120 ± 20 Mev