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A PRODUCTION FUNCTION STUDY OF MANUFACTURING ESTABLISHMENTS OF FRANCE, INDIA, ISRAEL, JAPAN AND YUGOSLAVIA

G. S. MONGA

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THE DEGREE OF
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ABSTRACT

This thesis is an empirical inquiry into the nature of production functions of manufacturing establishments of France, India, Israel, Japan and Yugoslavia. It uses the difference between the nature of economic and technical variables to review several forms of production functions in the literature. Fifteen production relations are selected for a cross section analysis of the data of each country. Various criteria of grouping the establishment data are examined. It is found that meaningful results can be obtained from mixed establishment data which can represent the manufacturing sector of a country. It is found that in international comparisons based on production function analysis, nations are more relevant than industries or groups of manufacturing establishments. The intrinsic features of the data are best revealed when the production relation contains at least one suitable economic and one suitable technical variable on the explanatory side. By grouping the data according to various criteria and applying statistical tests, it is shown that there is homogeneity between groups of establishments within each country and that this homogeneity is revealed in almost all cases when the grouping of the data is based on a variable which is not a dependent variable in the production relation used in the analysis.

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CHAPTER ONE

INTRODUCTION

The Production Function

The neoclassical production function of the firm sums up in a single relation of continuously variable form, the technology of the firm and an exposition of a variety of results emanating from the actions of the firm. It is a general description of all outputs that can be obtained from all efficient combinations of inputs.

The efficiency of the production function involves technical efficiency in terms of an optimum relation between inputs and outputs and also economic efficiency in terms of the cost price relation between inputs and outputs. Thus the production function relates the maximum outputs available from a given set of inputs, resulting from a full consideration of production decisions made from the alternatives open to the individual decision making unit for the allocation of productive resources. This is done by removing what is technically infeasible, from the physical act of transformation of inputs into outputs. The production function is, thus, a mathematical artifice which provides a relation between inputs and outputs involving engineering laws and economic behaviour, but the associated assumptions and the type of firm envisaged by it may be more abstract than real.

The technical limitations on the entrepreneur as implied by the production function allow a certain degree of substitution between inputs. If the study is confined to single output production functions, it is assumed that it is the level of factor inputs rather than the level of output, which is controlled according to the formulation of the production function. The engineer maximises output for a given set of inputs through his choice of technology. The firm, with the strategy of maximum profits, under the market conditions in which it has to produce, subjects the production function to certain restrictions regarding the rewards to the inputs for their contribution to the outputs and combines the technological considerations suitably with economic requirements.

As an embodiment of technological constraints imposed on economic decisions, the production function is not supposed to include explicitly, certain economic variables like interest, prices and profits. This is because the form of the relationship between outputs and inputs is not based on economic decisions. But the behavioral and organisational aspects are not excluded.

In general, a production process may not be described as a complete process. Its characteristic features may vary depending upon the manner in which it may be related to other production processes. The concept of production function, which is a formalism, may, therefore, sometimes exhibit anomalies by producing different values of the same parameters under comparable though essentially different sets of circumstances.

The mathematical expression of a production function can take a wide variety of forms but the choice of an appropriate form should be made carefully as it may depend on a number of conditions.

* * * * *

The concept of production function is defined with respect to a given technology[1] and way of doing things. The production function corresponds to a given time period, it is the description of a continuous flow of inputs being transformed into output, within a feasible region of the production space.[2] It is not concerned with the detailed description of the processes that lead inputs into becoming output, a study of that aspect lies in the managerial or engineering field in which the definitions of the variables used may be entirely different from those used in economic production studies.

The description of a technical boundary of production by a production function is the concern of the economist for whom the production function is a tool which can be used to explain decisions that have already been made. The manager attempts to select the best production decision by elimination from a set of alternatives for actual implementation.

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1 A homogeneous technology with all the firms producing the same good. Firms producing even closely related items may have different production functions. This is because different items may involve the use of of different technologies.

Changes in technology alter the production function. The firm's technology includes all the technical information about the quantity of each input required to produce the output. The production function presumes technical efficiency and is interpreted to define the maximal output realisable from the input combination.

2 The transformation of inputs into output as a flow mechanism requires the specification of time dimension. During the time period for which the production function is defined, the management cannot alter the availability of inputs

The production function analysis can be used at different levels of aggregation; for individual processes, at the establishment level, at the level of a firm, an industry or an entire economy. But theoretically, the idea has been developed with reference to the firm. From the production function and the profit function, marginal productivity conditions of equilibrium may be derived under the assumption that the firm's aim is to maximise profits. Other assumptions about the firm adapting different advantageous strategies may be made. These should help the firm to make decisions about the optimal use of inputs to produce certain quantities of outputs.

All the points on the production surface are technically fully efficient and this implies the propriety of selecting any of them without any technical loss.[1] But different points may involve different costs of production and this leads the firm to seek concessions in the form of minimum costs by adjusting various input combinations suitably inaccordance with the input prices. This provides an association of technical efficiency and economic efficiency which are necessary and sufficient to arrive at minimum cost.[2] In practice, inputs have to be purchased in the market at the ruling prices with some knowledge of their productivity so that the requirements of economic efficiency may influence those of technical efficiency.

[1]The output points below the production surface are technically inefficient and those above the production surface are theoretically not possible.

2 Or, maximum profits.

Of the firm's two decision problems, the first is technical and the second is concerned with profit maximisation subject to the production function constraint. But the resulting production function from the solution of the technological problem may not possess the properties of continuity and nonvanishing partial derivatives."The effects of such decisions are .not adequately expressed by the theoretical operation of partial differentiation with respect to the quantities of separate inputs and outputs."[1]
A linear programming type technology, with a finite number of discontinuities in its derivatives, may be used to handle both decision problems simultaneously.

In spite of some doubts about the economic content of results obtained, several production models in common use have given remarkably good fits and consistent parameters for data obtained from a variety of sources, at different levels of aggregation and in different contexts Inconsistencies and meaningless results are obtained at times and it is not unusual to find the fault being associated with a bad choice of the model and the quality of data. Sometimes, the wide variety of theoretical explanations given in this connection can be disconcerting in the matter of testing the fundamental hypotheses of the theory of production.

But it cannot be denied that on purely empirical grounds the production function has done well even though it may sometimes

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1Dorfman R.(1951), Application of linear programming to the theory of the firm. Berkeley, Univ.of Calif.Press.

lack a theoretical basis A variety of production function models have been obtained by using technically and economically meaningful mathematical expressions which satisfy certain basic requirements. The variety of data covered by successful production function studies consists of time series as well as crosssectional data at different levels of aggregation.

Specification of the Production Function Model

The choice of any production function model depends not only on the properties of the model itself but also on other factors. It is not enough if the model is complex or involves a large number of variables or provides a good fit to some given data. Compared to a complicated model not easily applicable to real life results, a simpler, otherwise less satisfying model may be preferred provided its empirical attractiveness extracts meaningful structural relationships from the data. If the variables defined by the model are not found in a measurable form in practice , the model may not help unless express attempts to procure data on the required variables succeed. The theory and attraction of production functions are concerned with finding a form which is simple and provides a good fit to the data. If the fit is good, the ability of the model to provide a comprehensible picture of the underlying production process needs to be checked. Apart from these matters, the availability of statistical methods to derive the estimates of the parameters in the model is an essential element of any production function study.

It is usual to apply the production model to the available data without proper consideration for the level of aggregation. To be free from this problem and that of differences in technologies, a cross section of firms may be considered ideal for a production function study.[1]

If all the firms, working in a competitive industry, are identical in the matter of outputs, inputs, entrepreneurial abilities and face identical input prices, the production function would degenerate to a point and hence be meaningless. A scatter is essential for statistical results. This may happen if the firms face different input price ratios and that is possible if competition is not perfect. If perfect competition is not assumed, input and output prices may be treated as endogenous variables in which case, unfortunately the problem of identification arises.

One of the aims of this study is to make a search for appropriate forms of production functions which should be satisfactory theoretically as well as empirically. We shall go through a variety of forms available in the literature and consider the extensions in some cases. The problems associated with the estimation of some selected forms ill also be considered.

There are some surveys of production function studies available in the literature. The better known are those of Walter(1963), Hildebrand and Liu(1965) and Nerlove(1967). The next chapter will give a

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1A cross section sample is static in nature. The substitution, if any, actually taking place in firms, cannot be observed.

survey on different lines based on the factors entering into different production models and on the way different forms evolved along with their connecting links. Our survey is fairly ambitious and wades through a variety of popular as well as lesser known forms scattered throughout the literature of the last two decades. Perhaps most of the forms could be derived from a few generalised versions but that would conceal the essence of the development of the idea of the production function. We have also shown how further generalisations are possible in some cases. The intention is not to provide a mere catalogue but to present a systematic development of the production relations.

The Empirical Aspect of the Study

We have considered above one aspect of our study. It consists of a survey of a wide variety of production function forms in the literature. Out of these a few selected forms will be used for our empirical study. A rational choice out of these forms may be difficult to make; but, for empirical investigations, some forms may be unmanageable or undesirable For an empirical analysis, fifteen relations, most of them in common use, have been put into service.

We have a fairly good and reliable mass of data on individual manufacturing establishments of five countries. The data are crosssectional in nature and correspond to practically the same period for all the countries. Moreover they have been collected on a uniform basis in each of the countries under consideration, by U.N. experts. The five countries are France, India, Israel, Japan and Yugoslavia and the relatively larger establishments from each have been included. The empirical framework and the details of the data along with the accompanying inadequacies are given in chapters three and four.

The selected production relations have been fitted to the data on the manufacturing establishments of each of the five countries and the usual production function study of each carried out. The structural characteristics of the data and the variations resulting from the use of different forms of the production functions are carefully noted. At the same time the forms themselves come under a proper scrutiny from the point of view of their nature, content and results. A comparison of the characteristics of various forms has been made The development of some forms when gradually more variables are added, is analysed with an immediate comparison of the results for different countries. As the use of different techniques of estimation would make the results more involved, the ordinary least squares technique has been used throughout although the possibility of better results from the use of other techniques is not ruled out.

The Hypotheses

The twin aims of the econometric study of a variety of production function forms and manufacturing activity at the establishment level in five countries have been supplemented by a probe into the possibility of a certain degree of uniformity in the nature of manufacturing activity in the five countries belonging to different economic and political categories. This is done with the help of the production function study of the manufacturing establishment data. Here we regret the lack of adequate data which could have allowed us to carry out a more detailed comparative analysis. For instance, it would have been proper to carry out such an analysis had for data pertaining to individual industries. The availability of similar data in two different time periods would allow interesting comparison possibilities.[1]

For the purpose of analysis, we may resort to an artificial contrivance by assuming that each country under consideration made a beginning under different sets of circumstances in a certain initial period. This can be built on the basic framework that the economic experiences over years have not been identical for all countries. After the two world wars and particularly after the second, many significant changes can be noticed in the process of industrial development in different countries. Several consideration have played important roles in the common desire of most countries to have rapid industrialisation.

Immediately after the second world war, we find that practically all the nations, irrespective of their standing during the war period, were affected in some way or other by the war. This was at least one common factor for most countries at that time even though some of them were colonies, some newly born, some had still plenty of capital left with them and others were at different stages of development. The extent of manufacturing activity in these countries differed significantly. But each country was required to make some kind of fresh beginning during the period 1945-48. Concentrating on the manufacturing activity at the establishment level in the five selected countries which may be assumed to have made some kind of start, irrespective of their economic, social or political standards, we wish to examine the structure of the manufacturing activity of these countries, in the period of reference which is 1964-66.

[1]The empirical analyst almost religiously seeks data which should satisfy more and more of his theoretical requirements. It is not implied that our data are unsatisfactory.

Any artificial contrivance of a trend does not influence the purely crosssectional nature of the study.

We set out to test the hypothesis that in spite of a variety of experiences through which each country may have passed the material requirements of production can put severe limitations on the variety of forces of industrialisation that different societies may wish to adopt so that the structure of the production model tends to remain the same. In other words, we hypothesise that a production function analysis of the manufacturing establishments of different countries, each with a reasonably prominent industrial sector, should suggest a certain amount of stability in the structural characteristics of the production function and that this should happen irrespective of the industries or the prevalent economic systems. Empirical work based on a better variety of data could throw additional light on the conclusions. The analysis is given at the level of different size groups of establishments rather than the industry level in view of there being only a few establishments corresponding to most industries. But that does not seem to diminish the quality of our results. As we shall see it adds a new dimension to our study, which is in the form of an analysis of covariance and a scrutiny of the criteria of grouping the establishments.

While we make use of the production function analysis in this work, the use of alternative techniques for the same purpose is not ruled out. Similarly, the use of time series data can add to the meaning and utility of the results.

While comparing the results for different countries the use of several production relations enables us to compare their forms and the explanatory variables entering into them;

these are divided into technical and economic variables. The search for a good form leads us to test the hypothesis that the production function is a technical relationship but that, for overall satisfactory empirical performance, both technical as well as economic variables should enter a production relation.

We also test for the existence of constant returns to scale and nonunitary elasticity of substitution in all the countries under study. On the basis of the division of the manufacturing establishment data of each country into a suitable number of groups with the help of economic and technical criteria we test the hypothesis of the uniformity ofthe structural parameters of different groups in each country by means of analysis of covariance. It is also hypothesised that the technical criteria for grouping the data rather than economic criteria, should lead to the uniformity of the structural parameters.

The Plan of the Study

In this introductory chapter we have described the twin aims of this work, viz, a study of the extensions of the production function forms and an empirical crosssectional study of the production function of the manufacturing establishments of five countries. The hypotheses to be tested have been given.

Chapter two gives a survey of the production function literature on the basis of successive extensions obtained by the addition of certain variables to some basic relations like the Cobb Douglas form, the CES function or the productivity relation.

Chapter three gives the empirical framework of the study. This chapter provides a base for a production function analysis of mixed manufacturing establishment data. Technical and economic variables as used in this study are defined and various criteria for grouping the establishment data discussed. The hypotheses to be tested are also described in this chapter.

The details of the data and the variables are given in chapter four.

Chapter five presents the results of multiple regression analysis of the data. Only the pool regressions are considered in this chapter. An analysis of group regressions follows in chapter six.

The analysis of covariance of the results forms the subject matter of chapter seven. The stability and uniformity of the manufacturing sector of each country under study is tested in this chapter. This is followed by a summary and conclusions.

CHAPTER TWO

A SURVEY OF THE LITERATURE

Introduction

A large variety of different forms of production functions, evolved during the last two decades, has been systematically presented in this chapter. Although the literature on the subject has been highly scattered and seems to have developed at an uneven pace, concentrating on some aspect or the other of the subject at different periods of time depending on requirements, the vein of regular development within cannot fail to be noticed. What follows therefore, is not a mere catalogue of production function events but rather a uniform study of the development of the idea, without sacrificing the individual attempts which, though sometimes trivial, made some contribution to the subject. Indeed, most of the forms could have been derived as particular cases of some generalised forms. That would be one approach to the subject but the significance and the role of the individual forms would not come out. The bringing together of a variety of forms in one place has its own advantages. It has helped us to select suitable forms in the light of the data we have and the way we intend to carry out the analysis. While considering some extensions of any production function it has been possible to show in some cases that further extensions are not over; they continue to remain a possibility.

The need to formulate extensions and more generalised forms of production function arises also from the restrictive nature of the existing production function forms which lead to instability and inconsistencies of the estimates obtained from their empirical use. The amendments take the form of removal of some undesirable restrictions even at the cost of simplicity, inclusion of more inputs, making use of a corresponding cost function to estimate the production parameters or any other indirect estimation methods, and other techniques. The use of experience gained from empirical studies can be usefully incorporated into the production relation in a suitable manner For instance,[1] the assumption, usually based on experience, may be made about the nature or movement of some parameter of a production function by hypothesising a suitable relationship between it and some of the variables in the production function.

Whatever the procedure followed, it seems proper, in a study of various production function forms, to evaluate and build upon individual efforts rather than just derive several forms by taking special cases or by relaxing some restrictions The economic content of some forms may be questionable but even that needs to be considered in view of further improvement possibilities.

Though the existing number of forms of production functions surpasses anything similar in economics there is still no universally satisfactory form. Perhaps such a unique form does not exist and different situations and types of data demand the use of different forms on the basis of their merits Some forms have not been very popular empirically, some

1Sato(1965)makes the assumption that σ is a linear function of K/L and derives some forms of VES functions.Soskice(1968) assumes the joint returns to scale to be related to output and and derives variable returns to scale CES function.

others have been developed merely as theoretical exercises. unless one or the other of the existing forms, like the Cobb Douglas or the CES function which is simple enough or can be suitably simplified and is capable of satisfying various requirements, is found to be adequate, the new form has meaning only if it fares a little better. Complicated mathematical models are not necessarily the best answer as shown by the empirical results and the amount of computational work involved. Nonlinear procedures of estimation have their own drawbacks and problems in spite of the computer, some of the common complaints about them are the difficulty of obtaining a global optimum by iteration, difficulties with the statistical properties of the nonlinear estimates and multicollinearity problems.

Apart from a statistically convenient form, it may be desirable to have in a production function a number of properties that may make it less unrealistic. It may be desirable to have nonhomogeneity which implies variable returns to scale. The variability of elasticity of substitution along the isoquants as well as along the expansion path is equally desirable. It is usual to assume diminishing marginal products. This assumption should be relaxable to variable marginal products. Satisfying the economic rationale of the subject is another important requirement.

We look upon the production function as a relation made up of some technical and economic variables which enter it and bring about differences in its form, nature and subsequently, in the empirical results obtained with its help.

We begin with a simple form and go on developing its extensions with the help of variations brought about by the use of technical and economic factors. In the case of Cobb Douglas function with which w begin, we add one or more factors and arrive at more general forms with some remarkably different qualities. If we begin with th constant elasticity of substitution (CES) production function, we can consider a number of variations which amount to generalisations in one direction or the other. Other extensions and some independent forms are also given.

Notation Used

Q	Quantity of output
Y	Total output in money terms
V	Value added
X_i	ith input
K	Capital
L	Labour
M	Raw materials
$w=W/L$	Money wages per unit of labour
r	Rate of return on capital
p	Output price
w_i	Price of ith input
γ	Efficiency parameter
δ	Distribution parameter
ρ	Substitution parameter
ν	Returns to scale parameter
σ	Elasticity of substitution
ϵ_L	Output elasticity of labour
ϵ_K	Output elasticity of capital
s	Marginal rate of substitution
$y=Y/L$	Output labour ratio
$x=K/L$	Capital labour ratio
S_K	Capital share of total output
S_L	Labour share of total output

a, b, c, A, B, C, etc. Constants

The Definition of the Neoclassical Production Function

The neoclassical production function is a mathematical statement expressing the technological relationship between the output of a process and the inputs entering into the process with possibilities of substitution. Let $X = (X_1, \ldots, X_n)$ be the vector of n inputs $X_1, \ldots, X_n$ where each $X_i \geqslant 0$. Then, to each point in the input space there is a unique nonnegative output point. The general production function for a single output Q, produced from n variable inputs, may be written

$$Q = F(X_1, X_2, \ldots, X_n)$$

This function is assumed to be single valued and continuously differentiable.

There exists an economic region which is a subset of the input space in which output does not decrease as input increases. For any two vector points X^i and X^j $(\geqslant X^i)$ in the economic region we have $F(X^j) \geqslant F(X^i)$ which implies that the first partial derivatives or marginal products are nonnegative:

$$\partial F/\partial X_i \geqslant 0 \qquad i=1,2,\ldots,n.$$

The law of diminishing returns requires $\partial^2 F/\partial X_i^2 < 0$, $i=1,2,\ldots n$.

In a convex subset of the economic region, the Hessian matrix h is negative definite where

$$h = \frac{\partial^2 F}{\partial X^2} = \begin{bmatrix} \frac{\partial^2 F}{\partial X_1^2} & \frac{\partial^2 F}{\partial X_1 \partial X_2} & \cdots & \frac{\partial^2 F}{\partial X_1 \partial X_n} \\ \vdots & \vdots & & \vdots \\ \frac{\partial^2 F}{\partial X_n \partial X_1} & \frac{\partial^2 F}{\partial X_n \partial X_2} & & \frac{\partial^2 F}{\partial X_n^2} \end{bmatrix}$$

For the production function $Q = F(X)$, the returns to scale depicts the behaviour of output when all inputs are changed by the same proportion. If the inputs are multiplied by the scale factor ν (> 0) at a certain point in the input space, the function shows increasing, constant or decreasing returns to scale according as $F(\nu X) >, = \text{ or } < \nu F(X)$.

At any point $(X_1, X_2, \ldots, X_n)$ in the input space, the scale elasticity of production is the sum of the marginal elasticities of output with respect to various inputs

$$\epsilon(X) = \sum_{i=1}^{n} \epsilon_i(X)$$

where the elasticity of output with respect to the ith input is given by $\epsilon_i(X) = (X_i/F)\, \partial F/\partial X_i$, $i = 1, 2, \ldots, n$.

The production surface divides the points in the production space into two parts the attainable and the unattainable. The feasible region of the production space is the closed half space defined by $Q \leqslant F(X_1, X_2, \ldots, X_n)$

In the case of two inputs X_1, X_2, the marginal rate of technical substitution (MRS) is given by

$$\begin{aligned} MRS = s &= F_1/F_2 \\ &= -\, dX_2/dX_1 \end{aligned}$$

where the suffixes to F denote the appropriate partial derivatives.

Writing $x = X_2/X_1$, we have, for variations along an isoquant, the elasticity of substitution, in the case of two inputs X_1, X_2, corresponding to substitutions for a constant output level, defined by $\sigma = \frac{dx}{ds}\frac{s}{x}$.

It can be shown that

$$\sigma = -\frac{F_1 F_2 (X_1 F_1 + X_2 F_2)}{X_1 X_2 (F_{11} F_2^2 - 2F_{12} F_1 F_2 + F_{22} F_1^2)}$$

or $\sigma = \frac{s}{X_1 X_2} \frac{X_1 s + X_2}{d^2 X_2 / dX_1^2}$

which is nonnegative and lies between zero and infinity. Also σ is inversely proportional to changes in the isoquant slope.

For a homogeneous production function of degree one, it can be shown that $\sigma = F_1 F_2 / F F_{12}$

If we write $y = Q/X_1$, $x = X_2/X_1$, the formula for lasticity of substitution, instead of remaining a partial differential equation, can be written as a nonlinear differential equation

$$\sigma = - \frac{y' (y - x y')}{x y y''}$$

This is an extensively used relation in production function studies.

For the n-input production function $Q = F(X_1, X_2, \ldots X_n)$, homogeneous of degree one, the bordered Hessian may be written

$$H = \begin{vmatrix} 0 & F_1 & F_2 \ldots F_n \\ F_1 & F_{11} & F_{12} \ldots F_{1n} \\ F_2 & F_{21} & F_{22} \ldots F_{2n} \\ \ldots & & \ldots \\ F_n & F_{n1} & F_{n2} \ldots F_{nn} \end{vmatrix}$$

where the suffixes to F show appropriate partial derivatives.

Denoting the cofactor of F_{ij} by H_{ij} we have the Allen partial elasticity of substitution between X_i, X_j given by

$$\sigma_{ij} = \frac{\Sigma X_i F_i}{X_i X_j} \frac{H_{ij}}{H} = \sigma_{ji} \quad \text{(symmetry)} \quad i \neq j$$

Using Euler's theorem,

$$\sigma_{ij} = \frac{Q}{X_i X_j} \frac{H_{ij}}{H}$$

from which the result for the two input case follows easily.

The production function and related concepts like marginal productivity are meant to be applied to a single unit of production and not to an economy or even to a sector of an economy. When the concept came into prevalence, the difference between aggregate production function and the firm's production function was either not realised or ignored. As remarked by J. Schumpeter in the History of Economic Thought, "most of the leaders of that period, among them Bohm-Bawerk, J B.Clark, Wicksteed and Wicksell, took the existence of the aggregative social production function for granted, at least by implication, without realising that the logical right to use this concept must be aquired by proof. Many modern authors, especially, the Keynesians, are just as careless." Some modern authors do not believe in the production function at all.

While it is possible to examine the empirical appropriateness of a production function or the suitability of a given functional form for the data, it is not quite easy to examine the assumption of continuous substitutability.[1]

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1 "The so called general case - convex isoquants and smooth and continuous substitution - brushes too many allocation problems under the carpet, and the intermediate case - discontinuous factor substitution in the presence of bottlenecks - is surprisingly enough the more powerful tool for analysing the firm's maximisation problem."M.Blaug (1968), Economic Theory in Retrospect. Heinemann, London.

Technical and Economic Aspects of the Production Function

While defining a production function as a technical relationship between inputs and output, it is assumed that the producing unit is working in the most efficient manner possible, with the knowledge and materials at its disposal. In other words, at the existing state of technical knowledge, the production function is so defined as to express the maximum output available from the given input combination.

As defined, the production function involves technical variables and physical quantities. Moreover, the estimation of the production function is legitimate as long as it can be confidently assumed that the observations have been generated by the same production process which is sought to be estimated.[1] This requirement may not be fulfilled in practice. Also it is practically impossible to measure variables like output and capital and labour in uniform, physical units. Almost invariably, the difficulty arising from the measurement of physical quantities is overcome by using money values and the production function compares changes in output quantities which are price weighted with net investment or some other measure of capital and a physical measure of labour, usually in manhours or manyears and a cost measure of raw material input if any.

---

[1]Combining observations on different units is justified if it can be assumed a priori that the form of the individual production functions and all the corresponding parameters of different units are the same.

When a factor, intended to be a physical quantity by the definition of the production function is replaced by some kind of money equivalent, the production function idea may get somewhat distorted. However, although the replacement of technical variables by economic variaables[1] may influence the technically defined maximisation idea implied in the production function, this does not change the technical nature of the production function.[2]

The difficulties emanating from theoretical requirement and the need for being practical in empirical work also lead to the entry of economic factors into the field of the technical produnction function.

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[1]The use of the terms "technical" and "economic" in various different contexts is not uncommon in the literature. For instance, the concepts of "technical" and "economic" efficiency are well known. We will be making frequent use of the terms technical and economic in the sense described on p.125-28. These terms have at times been used in the literature practically in the same sense in which we have used them. For instance, see Hilton K. and H Dolphin(1970), Capital and capacity utilisation in the U K., Bul. Oxford Univ. Inst. of Econ. and Stat.Vol.32 where a reference will also be found to Klein L.R.(1960), Some theoretical issues on the measurement of capacity, Econometrica.

[2]In a production function, a factor of production is conceived as a technically defined input, constituting an important aspect of the neoclassical view of the economic process. Although capital and labour are physically expressed means of production and expected to be technical in nature, they evolve out of economic (and even social)relationships and are not independent of other economic relations.

It is difficult to avoid aggregating various different types of output, items of capital and various types of labour. With that follows the inevitable entry of economic factors like wages, rate of return on capital and costs in various forms into the production model either explicitly[1] or through some side relations which become essential to meet the practical requirements Also it is difficult for the firm to ignore economic factors because its expansion path for different output levels depends on the input price ratio. Moreover,

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[1]As for instance, in ACMS(1961) CES function and in Hanoch(1971) CDE function.

An interesting use of real money balances(not prices) as a factor of production contributing significantly to production can be found in

Sinai A. and H.Stokes(1972), Real money balances an omitted variable from the production function ? The Rev. of Econ. and Stat., p.290-96; A reply(1975).

Sinai and Stokes find many writers like Friedman, Johnson and Nadiri asserting that money belongs in the production function. They test the hypothesis that real money balances are a factor input. The rationale for including money balances in the production function relates in part, to the increased economic efficiency of a monetary economy compared with a barter economy. Although "the standard neoclassical production function is concerned with real output and real inputs...yet...labourers, owners of capital goods and entrepreneurs must go to the markets and exchange physical goods and services in return for services and goods." Also, "there are numerous implications of real money balances as a factor of production." The possibility of simultaneous bias is not ruled out but if a simultaneous model is attempted, "it should contain the production function and factor demand equations for capital, labour and money balances."

In this note, we have tried to point out an attempt to include in the production function an economic variable. However, in our study, no use has been made of money balances as a factor of production.

because of its dependence on economic processes, the state of technology cannot be categorically called a non-economic factor. The incorporation of some nontechnical factors into the production function may influence the nature of its empirical performance.[1]

Constant Technology Assumption

Changes in technology are not envisaged in our manufacturing establishment data which correspond to a given period of time. The production function forms considered in the following pages will not involve technology as a variable.[2]

The Survey

The plan of the rest of this chapter is as follows.

The first part gives a very brief description of a few basic models and some features of those that follow.

The second part begins with the Cobb Douglas function and proceeds to give several extensions obtained by the introduction into it of additional factors. Only technical factors are used in this part

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[1]This in no way is meant to question the technical nature of the production function. The production function remains a technical relation.

[2]The average production function may be an ambiguous concept without assumptions about the technical structure of establishments. A study of future structural changes can be made by comparing the best practice and average practice functions of manufacturing industry.

For the manufacturing establishment data, the production function resulting from the best practice technique is made up of the maximum-output-giving parts of the production functions of establishments.

The third part gives the CES function, its extensions and variations available in the literature.

In the fourth part, extensions have been made of the ACMS productivity wage relation by the addition of new factors.

The fifth part is a brief description of variable elasticity of substitution production functions.

Homothetic production functions are discussed in the sixth part. This is followed by a few miscellan-ous forms and a comment.

Some Basic Models

The simplest functional form for a production model is the linear form with two or more inputs The n input model $Q = a + \sum_{i=1}^{n} b_i X_i$ is a production function if Q and X_i are non-negative and $\partial Q/\partial X_i = b_i > 0$. The elasticity of substitution is infinity and the output elasticity is unity. This production model is not used in practice.

It is possible to think of a firm having only some discrete choices in the matter of inputs, their quantities and use. Based on this assumption of constancy of engineering and technological factors that determine the relationship between inputs we have the Leontief input-output function which is also linear. In the two input case, let c_1, c_2 stand for the amounts of inputs X_1, X_2 needed to produce one unit of output. The Leontieff function may be written

$$Q = \min(X_1/c_1 \ , \ X_2/c_2)$$

or $$X_1 \geqslant c_1 Q \quad , \quad X_2 \geqslant c_2 Q$$

Because the two inputs move together the marginal products remain undefined. The elasticity of substitution is 0. The output elasticity is unity provided $X_1/c_1 = X_2/c_2$

In the theory of production functions, the first major contribution was the Cobb Douglas production function which has had a long life span and is still being used on a large scale in spite of a number of other models now available. The Leontief fixed proportions case has important applications in a specialised input output framework. A commonly used form in recent years is the constant elasticity of substitution (CES) production function which is a generalisation of the Cobb Douglas function. The popularity and usefulness of these forms have given rise to a number of extensions and more complex forms which perform additional roles and are supposed to have certain desirable properties. Reference may be made to the surveys of Walters(1963), Hildebrand and Liu(1965) and Nerlove(1967).

Because of their relative simplicity and manageability, the Leontief, Cobb Douglas and the CES functions have yielded a number of useful and interesting production function studies. Recently some variable elasticity of substitution (VES) functions have also been used in empirical work. None of these functions has all the empirically desirable characterestics. Often additional qualities are brought into them at a price which consists of some simplifying and very likely, unrealistic assumptions. In any case, the mathematical representation of any form does not have much meaning unless supported by empirical results.

The Leontief function does not allow for substitution between inputs. The capital labour ratio is uniquely determined and has nothing to do with prices. It means that for any output there is only one production process. The Cobb Douglas function allows for factor substitution but the elasticity of substitution is restricted to unity. Along an isoquant, the proportional change in inputs for a given change in input price ratio is fixed. This is also the case with the CES function but the extent of this change is a parameter of the CES function and not fixed in advance as in the Cobb Douglas case. The VES function allows the variability along an isoquant, of the elasticity of substitution which is proportional to the input ratio. In the case of more than two inputs, we have the trans-log production function which is subject to a minimum number of prior restrictions and is amenable to tests of degree of returns to scale and separability.[1]

The Cobb Douglas Function and Some Extensions

We now carry out a brief analysis of the Cobb Douglas function and some other forms which may be considered as its extensions.

Harter-Carter-Hocking's(1960)Transcendental Production Function.

Vinod's(1972)Homogeneous Function of Variable Degree.

Chu-Aigner-Frankel's(1970)Log Quadratic Law of Production.

Sudit's(1973)Additive Nonhomogeneous Function.

Janvry's(1972)Generalised Power Production Function.

Kmenta's(1967)CES Approximation.

Christensen-Jorgensen-Lau's(1971)Translog Function.

The Cobb Douglas Production Function

The two input Cobb Douglas production function is usually written in the form

$$Q = A K^{\alpha} L^{\beta}$$

where K and L stand for capital and labour respectively. Q is the output produced. The function satisfies the neoclassical

1The restrictions of separability and aggregation can be imposed on the translog function as testable parametric restrictions.This is a very useful feature for empirical work.

requirements in that the marginal products are positive:

$$\partial Q/\partial K = \alpha Q/K > 0, \quad \partial Q/\partial L = \beta Q/L > 0.$$

Also $\partial^2 Q/\partial K^2 = \frac{\alpha(\alpha-1)}{K} Q/K < 0, \quad \partial^2 Q/\partial L^2 = \frac{\beta(\beta-1)}{L} Q/L < 0.$

α and β are output elasticities. The returns to scale are given by $\alpha + \beta$. A is the efficiency coefficient and α/β is the degree of input intensity. Writing x = K/L, the elasticity of substitution is given by

$$\sigma = d \ln x/d \ln s = 1 \quad \text{since } s = \frac{\partial Q/\partial L}{\partial Q/\partial K} = \frac{\beta K}{\alpha L}.$$

Written in the form $Q = A K^{\alpha} L^{\beta}$, the Cobb Douglas function involves only technical variables. But as it is difficult to measure the physical output Q in suitable units, Q is replaced by Y, total value of output.

It is difficult to express K, capital assets, in suitable physical terms and therefore, even K has to be used in value terms. As for the problem of lack of homogeneity, it is common to all inputs.

The use of money values in place of physical quantities introduces an economic element into the purely technical relationship of the production function. This is unavoidable in practice and may be the cause of some differences in conclusions drawn from empirical production function studies based on the assumption of a purely technical production relation.

If we write α_i for the output elasticity of X_i, the n-input Cobb Douglas function may be written

$$Q = A X_1^{\alpha_1} X_2^{\alpha_2} \dots X_n^{\alpha_n}$$

Also $\partial Q/\partial X_i = \alpha_i Q/X_i > 0$ and $\partial^2 Q/\partial X_i^2 = \frac{\alpha_i(\alpha_i - 1)}{X_i}\frac{Q}{X_i} < 0$, for $i = 1,2,\dots,n$. The returns to scale is given by $\sum\alpha_i$ and $\sigma_{ij} = 1, i \neq j$.

Cobb Douglas Function with Constant Returns to Scale

With the constraint $\alpha + \beta = 1$, the Cobb Douglas function with two inputs K and L may be written as a productivity relation between average productivity and capital intensity. The assumption of constant returns to scale made here may or may not be true. Dividing the unrestricted Cobb Douglas equation throughout by L, we have

$$Q/L = A\, K^{\alpha}/L^{1-\beta}$$

$$= A(K/L)^{\alpha} \quad \text{since } \alpha + \beta = 1$$

To test the hypothesis of constant returns to scale, that is, to test $\alpha + \beta = 1$, the relation may be written

$$Q/L = A(K/L)^{\alpha}\; L^{\alpha+\beta-1}$$

The significance of the coefficient of L can be used to verify the hypothesis that $\alpha + \beta$ adds up to unity.

The Extension Procedure

We now introduce additional explanatory factors into the Cobb Douglas relation. K/L is a good explanatory factor and may be introduced into the Cobb Douglas function. But since it leaves the latter unaltered in form, a term like $(\ln K/L)^2$ may be used in the log linear Cobb Douglas relation. This gives rise to a new productivity relation which happens to coincide with an approximation by Taylor's expansion of the CES as well as the VES functions which we shall consider later. The CES approximation is usually called Kmenta approximation.

From the procedure just mentioned it can be seen that the introduction of an additional factor into an existing form gives rise to a new form of production function. The resulting form may not necessarily continue to retain the original properties like those of homogeneity or constancy of returns to scale or unitary elasticity of substitution that may be present in the basic form with which we may start.

If to the right hand side of the Cobb Douglas function for n inputs,

$$Q = A \; X_1^{\alpha_1} \; X_2^{\alpha_2} \ldots X_n^{\alpha_n}$$

multiplicative exponents of inputs are introduced we get an earlier form of transcendental production function given by Harter, Carter and Hocking (1960). There are several nonhomogeneous variations of the Cobb Douglas function. Kmenta approximation obtained by adding $(\ln K/L)^2$ as an additional explanatory factor to the Cobb Douglas log linear relation results in a nonhomogeneous function. Another nonhomogeneous function is that of Vinod(1972) which is obtained by adding (ln K. ln L) in the two input Cobb Douglas loglinear relation. In other words, it is obtained by making each input exponent a linear function of the other input in the two input Cobb Douglas function. If instead, the exponents are made linear functions of input ratios, we get Sudit's(1973) homogeneous function which results in the addition of three terms to the Cobb Douglas linear relation, viz., $(\ln K)^2$, $(\ln L)^2$ and (ln K. ln L). If only two terms, viz., L ln K and K ln L are added the resulting production function is nonhomogeneous. Other nonhomogeneous functions which may be considered as some kinds of extensions of the Cobb Douglas function are those of Chu-Aigner-Frankel (1970), Janvry(1972) and the by now quite famous translog production function of Christensen, Jorgensen and Lau (1971).

Some Extensions of the Cobb Douglas Function

One of the earlier extensions of the Cobb Douglas function was obtained by using exponents of inputs as additional multiplicative factors in the Cobb Douglas form. Harter, Carter and Hocking(1960) called it the transcendental production function. It may be written

$$Q = A X_1^{\alpha_1} e^{\beta_1 X_1} \ldots X_n^{\alpha_n} e^{\beta_n X_n}.$$

In the two input case the marginal rate of technical substitution is given by

$$s = \frac{\alpha_2 + \beta_2 X_2}{\alpha_1 + \beta_1 X_1} \frac{X_1}{X_2} .$$

The function exhibits nonconstant elasticity of substitution. It has also the characteristic of allowing marginal products to rise before eventually falling.

The elasticities of production are given by

$$\epsilon_1 = \partial \ln Q / \partial \ln X_1 = \alpha_1 + \beta_1 X_1$$

$$\epsilon_2 = \partial \ln Q / \partial \ln X_2 = \alpha_2 + \beta_2 X_2$$

so that the scale of production elasticity is

$$\epsilon = \alpha_1 + \alpha_2 + \beta_1 X_1 + \beta_2 X_2$$

The Harter, Carter and Hocking relation adds a linear function of inputs to the Cobb Douglas function written in the log linear form

$$\ln Q = \ln A + \sum \alpha_i \ln X_i + \sum \beta_i X_i$$

The nonhomogeneous production function of Vinod(1972) provides an extension of the Cobb Douglas function by substituting α and β (of $Q = AK^{\alpha} L^{\beta}$) by linear functions of inputs. α is replaced by a linear function of L and β by a linear function of K. Thus

$$Q = A K^{a_1 + c_1 \ln L} L^{a_2 + c_2 \ln K}$$

This function adds an interactive term to the linear Cobb Douglas relation which with $a_3 = c_1 + c_2$ may be written

$$\ln Q = \ln A + a_1 \ln K + a_2 \ln L + a_3 \ln K . \ln L$$

If a_3 is not significantly different from zero, the Cobb Douglas function is implied.

The output elasticities are given by

$$\epsilon_L = a_2 + a_3 \ln K, \quad \epsilon_K = a_1 + a_3 \ln L$$

and the scale elasticity

$$\epsilon = \epsilon_L + \epsilon_K = a_1 + a_2 + a_3 \ln LK$$

which is variable and dependent on the input levels.

If in the elasticity of substitution expression given on p.19 or Allen(1938, p.342) we substitute

$$F_K = (a_1 + a_3 \ln L)Q/K, \quad F_L = (a_2 + a_3 \ln K)Q/L$$

$$F_{LL} = Q\epsilon_L (\epsilon_L - 1)/L^2, \quad F_{KK} = Q\epsilon_K(\epsilon_K - 1)/K^2$$

$$F_{KL} = (\epsilon_K \epsilon_L Q/L + a_3 Q/L)/K = (\epsilon_K \epsilon_L + a_3)Q/KL$$

we have

$$\sigma = \frac{-F_K F_L (KF_K + LF_L)/KL}{F_{KK}F_L^2 - 2F_{KL}F_K F_L + F_{LL}F_K^2}$$

$$= \frac{\epsilon_L + \epsilon_K}{\epsilon_L + \epsilon_K + 2a_3}$$

$$= \frac{a_1 + a_2 + a_3 \ln KL}{a_1 + a_2 + a_3(2 + \ln KL)}$$

which is less than unity if $\epsilon_L + \epsilon_K \gtrless 0$ and $a_3 \gtrless 0$.

The function is reasonably nonrestrictive and is a natural generalisation of the Cobb Douglas function.

If the α and β of the Cobb Douglas function are replaced by log linear functions of input ratios, we have Sudit's (1973) Homogeneous production function of variable degree with variable elasticity of substitution and returns to scale

$$Q = A\,K^{a_1 + c_1 \ln K/L}\,L^{a_2 + c_2 \ln L/K}$$

It is homogeneous of degree $a_1 + a_2 + c_1 \ln K/L + c_2 \ln L/K$, and implies that different production techniques as reflected by different input ratios generate different scale factors. It reduces to the Cobb Douglas form with $c_1 = 0 = c_2$. For estimation purposes it may be written

$$\ln Q = \ln A + a_1 \ln K + a_2 \ln L + a_{12} \ln K \ln L - c_1 (\ln K)^2 - c_2 (\ln L)^2 \quad \text{where } a_{12} = c_1 + c_2$$

or

$$\ln Q = \ln A + a_1 \ln K + a_2 \ln L + c_1 (\ln K \ln L - (\ln K)^2) + c_2 (\ln K \ln L - (\ln L)^2)$$

The output elasticities are given by

$$\epsilon_K = a_1 + (c_1 + c_2) \ln L - 2c_1 (\ln K)/K$$

$$\epsilon_L = a_2 + (c_1 + c_2) \ln K - 2c_2 (\ln L)/L$$

The elasticity of substitution

$$\sigma = \frac{\epsilon_L + \epsilon_K}{\epsilon_K + \epsilon_L + 2c_1(\ln L - 1)\frac{L\epsilon_L - \epsilon_K}{L\epsilon_L} + 2c_2(\ln K - 1)\frac{K\epsilon_K - \epsilon_L}{K\epsilon_K}}$$

= 1 if c_1 and c_2 are zero.

Although it is a more flexible form than the Cobb Douglas function and has variable elasticity of substitution and returns to scale, it may suffer from the effect of multicollinearity if K and L happen to be highly collinear. The scale elasticity varies only along the isoquants. Along the expansion path, this function retains the property of homogeneity.

Sudit's(1973) additive nonhomogeneous production function (ANH) has a number of desirable properties. The function written in the general form for two inputs

$$Q = a_1 X_1 + a_2 X_2 + a_{12} X_1 \ln X_2 + a_{21} X_2 \ln X_1$$

has marginal products which are functions of the input ratio and the remaining input

$$\partial Q/\partial X_1 = a_1 + a_{12} \ln X_2 + a_{21} X_2 / X_1$$

$$\partial Q/\partial X_2 = a_2 + a_{12} \ln X_1 + a_{21} X_1 / X_2$$

This implies that the abundance of a factor lowers its marginal product and the marginal cost of the other factor rises. The law of diminishing returns is thus satisfied. But the function is not necessarily restricted to diminishing returns since

$$\partial^2 Q/\partial X_1^2 = -a_{21} X_2/X_1^2 \quad \text{and} \quad \partial^2 Q/\partial X_2^2 = -a_{21} X_1/X_2^2$$

which means increasing returns from both inputs are possible for a_{12}, $a_{21} < 0$.

The shift in the marginal product of one input in response to a change in the other indicates the extent of their complementarity or competitiveness

$$\partial^2 Q/\partial X_1 \partial X_2 = \frac{a_{12}}{X_2} + \frac{a_{21}}{X_1}$$

The scale elasticity

$$\epsilon = \epsilon_1 + \epsilon_2 = \frac{X_1}{Q}\frac{\partial Q}{\partial X_1} + \frac{X_2}{Q}\frac{\partial Q}{\partial X_2} = 1 + \frac{a_{12}X_1 + a_{21}X_2}{Q}$$

which implies returns to scale are variable over the scale of production.

$\epsilon \gtrless 1$ if a_{12}, $a_{21} \gtrless 0$

$\epsilon = 1$ if a_{12}, $a_{21} = 0$ which means $Q = a_1 X_1 + a_2 X_2$

The ANH function is not constrained to be convex to the origin. The marginal rate of substitution is given by

$$\frac{dX_1}{dX_2} = -\frac{a_2 + a_{12}X_1/X_2 + a_{21}X_1}{a_1 + a_{21}X_2/X_1 + A_{12}X_2}$$

The elasticity of substitution is not constant and the function is a variable elasticity of substitution function.

We now consider the Chu, Aigner and Frankel's (1970) log quadratic law of production. Using $L(\geq 1)$ for labour, $K(\geq 1)$ for capital and $\bar{L}$, $\bar{K}$ for parameters which are, respectively, the maximising values of the labour and capital inputs that determine the highest total output, the Chu-Aigner-Frankel(CAF) function may be written

$$Q = A\left(\frac{L}{\bar{L}}\right)^{c_1(1-\ln L/\ln \bar{L})}\left(\frac{K}{\bar{K}}\right)^{c_2(1-\ln K/\ln \bar{K})}$$

or $\ln Q = a + a_1 \ln L + a_2 \ln K - b_1 (\ln L)^2 - b_2 (\ln K)^2$

where $a = \ln A - c_1 \ln \bar{L} - c_2 \ln \bar{K}$, $a_1 = 2c_1$, $a_2 = 2c_2$

$b_1 = c_1/\ln \bar{L}$, $b_2 = c_2/\ln \bar{K}$

The Chu-Aigner-Frankel(CAF) function is nonhomogeneous and has nonconstant factor shares. It is obtained by simply adding the squared terms, $(\ln K)^2$ and $(\ln L)^2$ to the loglinear Cobb Douglas function, and thus belongs to a family of log polynomials. If we equate to zero, the marginal products

$$\partial Q/\partial L = 2c_1(1-\ln L/\ln \bar{L})Q/L, \quad \partial Q/\partial K = 2c_2(1-\ln K/\ln\bar{K})Q/K$$

we get $L = \bar{L}$ and $K = \bar{K}$. Since total output is maximised at this point, $\bar{L}$ and $\bar{K}$ may be called the maximum total productivity parameters. Similarly, since the average productivities Q/L and Q/K are maximised when $L = \bar{L}^{\,1-1/2c_1}$ and $K = \bar{K}^{\,1-1/2c_2}$, c_1 and c_2 are the maximum average productivity parameters. They determine the maximum average productivities once $\bar{L}$ and $\bar{K}$ are fixed. This helps determine the economic region of the production function.

For the CAF function the marginal product of labour exceeds the average product before the latter is maximum and is less than the average product after that, so the function obeys the law of variable proportions. This enables us to categorise the behaviour of input productivities and hence to determine the most economic region without attaching any significance to the symmetry of the stages of production.

The returns to scale are variable according to the values taken by L and K. Replacing L and K by λL and λK in the CAF function, we have

$$A(\lambda L/\bar{L})^{c_1(1-\ln \lambda L/\ln \bar{L})} \; (K/\bar{K})^{c_2(1-\ln \lambda K/\ln \bar{K})}$$

$$= \left\{ (\bar{L}/L)^{c_1/\ln \bar{L}} \; (\bar{K}/K)^{c_2/\ln \bar{K}} \right\}^{\ln\lambda} Q$$

$$= z^{\ln\lambda} Q, \text{ say.}$$

If the inputs are increased by a multiple of λ, the ouput increased by a multiple of $z^{\ln\lambda}$ which itself is a function of inputs.

Janvry's(1972) generalised power production function (GPPF) allows for nonhomogeneity and also for variability of the returns to scale, marginal productivities, elasticities of production, marginal rates of substitution and elasticities of substitution. It includes as special cases the Cobb Douglas and transcendental production functions.

If $f^{j}(X)$ and $g(X)$ are polynomials of any degree in the arguments of the m dimensional input vector X, the GPF may be written

$$Q = A \prod_{j=1}^{m} X_j^{f^j(X)} e^{g(X)}$$

This reduces to the Cobb Douglas form if $f^{j}(X) = \alpha_j$ for all j and $g(X) = 0$. If $f^{j}(X) = \alpha_j$ for all j and $g(X) = \Sigma \gamma_m X_m$, the transcendental form results.

The marginal product of factor X_j is

$$\frac{\partial Q}{\partial X_j} = Q\left[\frac{f^j(X)}{X_j} + \frac{\partial g(X)}{\partial X_j} + \sum_{j=1}^{m} \frac{\partial f^j(X)}{\partial X_j} \ln X_j \right]$$

which can assume positive, zero or negative values depending on the specification of the polynomials and hence can describe all three stages of production for $g(X) \neq 0$.

The GPPF is homogeneous if and only if the polynomials $f^{j}(X)$, j=1,...,m, and $g(X)$ are homogeneous of degree zero.

The function exhibits variable returns to scale unless all $f^{m}(X)$ are independent of X which reduces the GPPF to the Cobb Douglas form.

The economic region of production is defined by the set of values of the X's such that $0 < \sum_{j}^{m} f^{j}(X) \leq 1$.

In the special two input case

$$Q = A\, X_1^{\alpha_1 + \beta_1 X_2} X_2^{\alpha_2} e^{\gamma_1 X_1}$$

the marginal products are

$$\partial Q/\partial X_1 = (\alpha_1 + \beta_1 X_2 + \gamma_1 X_1)\, Q/X_1$$

$$\partial Q/\partial X_2 = (\alpha_2 + \beta_1 X_2 \ln X_1)\, Q/X_2$$

For $X_1 = -(\alpha_1 + \beta_1 X_2)/\gamma_1$, $\partial Q/\partial X_1 = 0$. It is maximum for $\partial^2 Q/\partial X_1^2 = 0$, i.e. for $X_1 = -(\alpha_1 + \beta_1 X_2 + \sqrt{\alpha_1 + \beta_1 X_2})/\gamma_1$

Thus X_1 has a positive and decreasing marginal product in the interval

$$-\frac{1}{\gamma_1}(\alpha_1 + \beta_1 X_2 - \sqrt{\alpha_1 + \beta_1 X_2}) < X_1 < X_1 - \frac{1}{\gamma_1}(\alpha_1 + \beta_1 X_2)$$

which is a function of X_2. Also X_1 has a negative marginal product if X_2 exceeds the critical level $-\frac{1}{\beta_1}(\alpha_1 + \gamma_1 X_1)$

The elasticity of substitution of the GPPF is a variable parameter

$$\sigma = \frac{b(a + b)}{b^2 + a\alpha_2 + 2b\beta_1 X_2}$$

where $a = \alpha_1 + \beta_1 X_2$ and $b = \alpha_2 + \beta_1 X_2 \ln X_1$

If $\beta_1 = 0$, $\sigma = 1$ which is the Cobb Douglas case.

If we introduce a suitable multiplicative exponential into Vinod's(1972) two input nonhomogeneous production fun-ction, an extension of Janvry's form may be obtained. But the general form of Janvry's production function allows many more possibilities.

Kmenta Approximation

Kmenta approximation which was introduced as a Taylor series expansion up to the second order terms of the constant elasticity of substitution (CES) production function, is a commonly used relation in production function studies. We may look upon it as an obvious extension of Cobb Douglas function from which it may be obtained by the addition of some appropriate factors.

It is difficult to linearise and estimate the parameters of the CES function with nonconstant returns to scale

$$Q = \gamma\left[\delta L^{-\rho} + (1-\delta)K^{-\rho}\right]^{-\nu/\rho}$$

or $\ln Q/L = \ln\gamma + (\nu-1)\ln L - \frac{\nu}{\rho} f(\rho)$

$$f(\rho) = \ln\left[\delta + (1-\delta)(K/L)^{-\rho}\right]$$

$= f(0) + \rho f'(0) + \frac{1}{2}\rho^2 f''(0)$, when expanded around $\rho = 0$ with terms of order higher than the second omitted. Since

$$f(0) = 0, \quad f'(0) = -(1-\delta)\ln K/L$$

$$f''(0) = \delta(1-\delta)(\ln K/L)^2$$

we have

$$f(\rho) = -\rho(1-\delta)\ln K/L + \frac{1}{2}\rho^2\delta(1-\delta)(\ln K/L)^2$$

If we substitute $a_0 = \ln\gamma$, $a_1 = \nu - 1$, $a_2 = \nu(1-\delta)$, $a_3 = -\frac{1}{2}\nu\rho\delta(1-\delta)$, we have the Kmenta approximation

$$\ln Q/L = a_0 + a_1 \ln L + a_2 \ln K/L + a_3(\ln K/L)^2 \qquad (3a)$$

or equivalently,

$$\ln Q = a_{10} + \nu\delta\ln K + \nu(1-\delta)\ln L - \tfrac{1}{2}\rho\nu\delta(1-\delta)(\ln K - \ln L)^2$$

$$= a_{10} + a_{11}\ln K + a_{12}\ln L + a_{13}(\ln K - \ln L)^2.$$

The last term on the right disappears if $\rho = 0$ The approximation is better with ρ closer to zero. If a_{13} is not significantly different from zero, the Cobb Douglas form may not be rejected though the exact situation would be unpredictable as a more general production function could result if a_{13} is significantly different from zero. Moreover a_{13} also depends on δ and $1-\delta$ and that makes the test weak.

The estimates of the parameters a_1 and a_2 in (3a) and hence of δ and σ are not independent of the units of measurement.[1] So the elasticity of substitution may be evalu-

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1At K=L, the approximation is exact. For empirical work, the units of K and L may be so chosen in the sample as to equate their geometric averages.

ated at the mean level of a sample. The elasticity of substitution of this function depends on the input ratio and the function may be said to be homothetic. While we shall consider the concept of homotheticity in a later section, it may suffice to say that homotheticity implies that, if the expansion of the last term in the Kmenta approximation with fresh coefficients viz.,

$a_3 (\ln K/L)^2 = a_{31} (\ln K)^2 - 2a_{32} \ln K \ln L + a_{33} (\ln L)^2$

is tested in a linear hypothesis framework, it results in $a_{31} = a_{32} = a_{32} = a_3$. If it does not, a more general nonhomothetic polynomial function deserves to be considered Specifically, Kmenta approximation belongs to the special class of homothetic production functions in that its elasticity of substitution depends on the input ratio.

It is not necessary to expand f (ς) around $\varsigma = 0$. Any other appropriate value may be taken. But the error of approximation depends on the extent to which the actual value of ς deviates from the chosen value. It also depends on the input ratio as well as on the values of the other parameters in the function., The extent of the specification error resulting from the approximation depends on the closeness of the approximation.

Kmenta approximation of the CES function is linear in all parameters. Thus the best linear unbiased estimates can be obtained from it by using ordinary least squares method though the bias may have been caused by the dropping of the higher order terms.

The Translog Production Function

The production function underlying the cost theory is nonhomogeneous. The firm has decreasing returns to scale at low output levels, constant returns to scale at intermediate levels and increasing returns to scale at higher levels of output. Such a generalisation is not allowed by a homogeneous production function. A nonhomogeneous production function may allow these variations.

We have seen that nonhomogeneity can manifest itself when terms of second and higher order are added to the Cobb Douglas function.

A nonhomothetic, generalised formulation of the Cobb Douglas and Kmenta functions may be written

$$\ln Q = \ln A + \alpha \ln K + \beta \ln L + \gamma_{KK}(\ln K)^2 + \gamma_{LL}(\ln L)^2 + \gamma_{KL}\ln K.\ln L \quad (3b)$$

whose scale elasticity is given by

$$\epsilon = \alpha + \beta + (2\gamma_{KK}+\gamma_{KL})\ln K + (2\gamma_{LL}+\gamma_{KL})\ln L.$$

The bracketed terms of the scale elasticity vanish if $\gamma_{KK}=\gamma_{LL}= -\frac{\gamma_{KL}}{2}$ in which case the function has constant scale elasticity and becomes homogeneous. It leads to the Cobb Douglas function if $\gamma_{KL}= 0$. The function can be useful in testing the homotheticity of the Kmenta approximation.

The expansion (3b) is the two input case of the Christensen, Jorgensen and Lau's(1971) translog production function. The translog production function may be considered as a second order local approximation of some underlying function. It has both linear and quadratic terms and can admit an arbitrary number of inputs. It may be viewed as an improved generalisation of the Cobb Douglas and Kmenta's CES approximation in that, with more than two inputs, and under reasonably general conditions it enables us to estimate partial elasticities of substitution among all forms of inputs. In the case of Cobb Douglas and CES functions, the separability conditions have to be imposed i.e., specified a priori. In the translog case they can be tested.

Suppose there exists a technological relationship for output with three inputs : capital (K), labour (L) and raw material (M), viz.,

$$\ln Q = \ln A + F(\ln K, \ln L, \ln M).$$

For the n input case $\ln Q = \ln A + F(\ln X_1, \ln X_2, \ldots, \ln X_n)$, a second order Taylor series approximation in the neighbourhood about the point with inputs unity results in

$$\ln Q - \ln A = F(C) + \sum_{i=1}^{n} \ln X_i \frac{\partial F}{\partial \ln X_i} + \frac{1}{2} \sum_{i}^{n} \sum_{j}^{n} \ln X_i \ln X_j \frac{\partial^2 F}{\partial \ln X_i \partial \ln X_j}$$

We have, in the three input case, with suitable notational changes,

$$\begin{aligned} F = {} & \ln \alpha_0 + \alpha_K \ln K + \alpha_L \ln L + \alpha_M \ln M \\ & + \tfrac{1}{2} \gamma_{KK} (\ln K)^2 + \tfrac{1}{2} \gamma_{LL} (\ln L)^2 + \tfrac{1}{2} \gamma_{MM} (\ln M)^2 \\ & + \gamma_{KL} \ln K \ln L + \gamma_{LM} \ln L \ln M + \gamma_{MK} \ln M \ln K \end{aligned}$$

By substituting $\ln A + \ln \alpha_0 = \ln A\alpha_0$, we have

$$\ln Q = \ln A \alpha_0 + \alpha_K \ln K + \quad \text{etc.}$$

For positive marginal products we must have

$$\partial \ln Q / \partial \ln X_i = \partial F / \partial \ln X_i = \alpha_i + \gamma_{iK} \ln K + \gamma_{iL} \ln L + \gamma_{iM} \ln M$$
$$> 0 \quad \text{for } i = K, L, M$$

For the function to be quasi concave at every data point, the bordered Hessian matrix should be negative and semi-definite.

It is found that the translog function, being a second order approximation and a quadratic, is not globally well behaved. But it may be considered as a good representation of production possibilities for most data.

Cobb Douglas Splines

Foirier's(1974) piecewise splines permit U shaped cost curves and piecewise homotheticity although differentiability of the functions along lines parallel to the input axes is no longer possible. In the Cobb Douglas function, returns to scale are nonvarying so that the average cost curve is not U shaped. With the Cobb Douglas splines the structural change and behaviour of the function in each piece can be tested.

Let α_i and β_j be positive constants and θ so chosen as to make the Cobb Douglas spline $F(K,L) = \theta_{ij} K^{\alpha_i} L^{\beta_j}$ continuous over the positive quadrants formed by the IJ rectangles defined by the knots in the meshes

$$\Delta_L = \{L_1 < L_2 \ldots < L_{J-1}\}, \quad \Delta_K = \{K_1 < K_2 \ldots < K_{I-1}\}$$

Using the continuity conditions

$$\ln \theta_{i+1,j} = \ln \theta_{ij} + (\alpha_i - \alpha_{i+1}) \ln K_i, \ i = 1,2,\ldots,I-1 \text{ for all } j$$

$$\ln \theta_{i,j+1} = \ln \theta_{ij} + (\beta_j - \beta_{j+1}) \ln L_j, \ j = 1,2,\ldots,J-1 \text{ for all } i.$$

Defining $\bar{K} = \max(\ln K - \ln K_i, 0) \quad , i = 1,2,\ldots,I-1$

$\bar{L} = \max(\ln L - \ln L_j, 0) \quad , j = 1,2,\ldots,J-1$

we have for a given θ and for all K and L

$$\ln F(K,L) = \ln \theta_{11} + \alpha_1 \ln K + \beta_1 \ln L + \sum_{i=2}^{I-1} \alpha_i K_i + \sum_{j=2}^{J-1} \beta_j L_j$$

where α_i, β_j represent changes in the output elasticities of K & L. For a fixed output level Q_0, the isoquants over the rectangle(i,j); $K = (Q_0 L^{-\beta_j}/\theta_{ij})^{1/\alpha_i}$ are continuous though having corners along the grid lines. They are strictly convex if and only if each output elasticity is a decreasing step function of its respective output $\alpha_i \geq \alpha_{i+1}$, $\beta_j \geq \beta_{j+1}$, i=1,...,I-1; j=1,...,J-1.

It can be shown that F(K,L) exhibits increasing returns to scale over all rectangles below and to the left of rectangle (i,j) and decreasing returns above and to the right of the rectable (i,j).

The CES Production Function

The second stage in the development of the production function idea begins with the constant elasticity of substitution (CES) production function which was introduced by Arrow, Chenery, Minhas and Solow, briefly, ACMS(1961).and developed independently by Brown and de Cani(1961). Next to the Cobb Douglas function, the CES function has been the most used form in the literature on production functions. Uzawa(1962), Mukerji(1963), Mcfadden(1963), Harcourt(1966) and others have treated and developed it extensively at the theoretical level. Its empirical contents have been examined by several authors. As rightly pointed out by Heathfield(1971) "the CES function seems to have been born of empirical observations in much the same way as was the Cobb Douglas function." Minhas(1973) describes in detail the ACMS intercountry data and methods which were used to estimate the CES function. Minasian's (1961) study is also crosssectional in nature and estimates the elasticity of substitution parameters from the CES productivity-wage relation used on U.S. 1957 two digit manufacturing industries. Other interesting crosssectional studies are by Solow(1964) Dhrymes(1963) and Fuchs(1963). Ryan(1975) uses company data and estimates all the parameters of the CES function by nonlinear methods.

Compared to the Cobb Douglas function which is confined to unitary elasticity of substitution σ, the CES function allows σ to take any positive constant value. Yet, for a simple estimation procedure the restrictive assumptions of homogeneity and competitive conditions have to be made. Consistent estimates of the CES function are not easily obtained.

Several extensions of the CES function are concerned with generalisations to the multi-input cases. Unfortunately, they are not quite capable of removing some of the artificial restrictions on the elasticity of substitution parameter. Attempts made by Mukerji(1963),Uzawa(1962), Mcfadden(1963) and Sato(1964) in this connection are noted in the following pages. Soskice's(1968) CES function with variable returns to scale, offers a departure from the usual pattern. Hilhorst's (1971) variation does away with the input prices and enables us to estimate the CES function by means of relative factor shares.

Several extensions of the production function forms during the last fifteen years have been connected with or have emanated from the CES function which retains all the neoclassical properties of the Cobb Douglas function. It allows diminishing marginal products and variable returns to scale.

In their intial study, ACMS(1961) used a simplified version of the CES function by fitting a log linear relation between average productivity and wage per unit. Over a crosssection of seventeen countries, separate CES estimates were made for each of twenty four manufacturing industries. The slope of the relation which stands for the elasticity of substitution, was found to be consistently less than unity. This was supposed to challenge the Cobb Douglas assumption of unitary elasticity of substitution. Profit maximisation , perfect competition and optimising market behaviour are assumed and hypotheses on production efficiency and distributive shares are tested. Unfortunately,ACMS estimate the parameters of the CES function by ignoring possible international differences in production models. This implies that all production parameters are identical across countries But the countries thus brought together in a single production function range from highly developed countries like America and Japan to underdeveloped countries like India and Iraq so that an estimated production function may not be quite reliable. Owing to differences in efficiencies, the industries in the advanced countries may not be similar to those in the developing countries. The input intensities and technologies and the extent of the availability of capital may differ over countries.

The establishment data used by us does not suffer from this drawback. Although we have put together manufacturing establishments belonging to different industries they are not from different countries whose technologies are likely to be at uncomparably different levels. Moreover, the data need not be confined to the assumption of unitary elasticity of substitution. The CES function can be used to test if the latter is different from unity.

The CES function is given by

$$Q = \gamma [\delta X_1^{-\rho} + (1-\delta) X_2^{-\rho}]^{-1/\rho}$$

where X_1, X_2, are two inputs. It is linear and homogeneous with constant returns to scale. It belongs to a class of functions with mean value of order $-\rho$ Paroush(1964) proved by means of simple integration and without any assumption about market conditions that every linearly homogeneous production function , f in X_1 and X_2, with a constant elasticity of substitution σ is a mean value of order $-\rho$

$$f = \gamma [\delta X_1^{-\rho} + (1-\delta) X_2^{-\rho}]^{-1/\rho}$$

where γ and δ are constants.

Yasui(1965) has proved, also without any assumption about the market, the more general theorem that every production function f in X_1, X_2 and with a constant elasticity of substitution σ, is of the form

$$f = \phi[\delta X_1^{-\rho} + (1-\delta)X_2^{-\rho}]^{-1/\rho}$$

where ϕ is any differentiable function. It is arbitrary in so far as it does not contradict economic considerations.

As $\rho \to \infty$, the CES function reduces to the Leontief fixed proportions function. As $\rho \to 0$, it reduces to the Cobb Douglas function. The CES function has the marginal rate of technical substitution given by

$$MRS = w/r = \frac{1-\delta}{\delta}(K/L)^{1+\rho}$$

and the input share ratio

$$wL/rK = \frac{1-\delta}{\delta}(K/L)^{\rho}$$

The marginal products are positive and the second derivatives are negative, e.g. we have

$$\frac{\partial^2 Q}{\partial L^2} = \gamma^{-\rho}\frac{1-\delta}{\delta}Q^{1+\rho}L^{-\rho}\left[\frac{1-\delta}{\delta L^{\rho}K^{-\rho}+1-\delta} - 1\right]$$

which is negative since $1-\delta < 1$

Henceforth, we will use valueadded V, instead of physical quantity output Q, for the convenience of argument.

The Cobb Douglas to the CES Function

Under the assumptions of perfect competition and constant returns to scale the Cobb Douglas function is

$$V = A K^{\alpha} L^{1-\alpha}$$

where Q has been replaced by value added V, for convenience. The labour share

$$\alpha = wL/V \quad \text{or} \quad V/L = w/\alpha$$

which may be written as a regression model

$$\ln V/L = -\ln \alpha + b \ln w$$

where b can be shown to equal the elasticity of substitution between capital and labour under competitive conditions.

ACMS(1961) showed that the Cobb Douglas relation which confined b to unity and the Harrod Domar relation confining b to zero both came from a more general relation which is the CES production function $V = \gamma[\delta K^{-\rho} + (1-\delta)L^{-\rho}]^{-1/\rho}$ where γ is the efficiency parameter, δ the distribution parameter of capital, $1-\delta$ being that of labour, ρ the substitution parameter from which can be found the elasticity of substitution given by $\sigma = \frac{1}{1+\rho}$.

Discussing the crucial nature of elasticity of substitution, ACMS remarked that economic analysis based on zero or unitary elasticity of substitution often leads to restrictive conclusions. The instability of the Harrod Domar growth model results from the assumption of zero elasticity of substitution. It may not be quite reasonable to assume unit elasticity of substitution to agree with the supposed constancy of input share in some cases [1] All the sectors of an economy or different economies need not necessarily have either zero or unit elasticity of substitution.

1With unitary elasticity of substitution the effect of change in the relative input prices would be exactly compensated by the change in input ratio leaving the relative share unaltered. M.Bronfenbrenner(1960,J.of Pol.Economy) shows that constancy of input shares is possible for a wide range of values of the elasticity of substitution. If $S_L = wL/(wL+rK)$ is labour share, it can be shown that(writing z for L/K) $dS_L/dz = (w/r)(1-S_L)^2(\sigma-1)/\sigma$ which shows the change in the labour share as the labour capital ratio changes expressed as a function of elasticity of substitution. If $\sigma=1$, this equals zero implying a fixed labour share with changes in L/K. Bronfenbrenner points out that the right hand side of this expression does not differ much from zero when $\sigma \neq 1$ and concludes that σ is not necessarily exactly equal to unity in order for relative factor shares to be constant. Note that $\partial(dS_L/dz)/\partial\sigma = (w/r)(1-S_L)^2/\sigma^2$

If we write $y = V/L$ and $x = K/L$, the general form of the production function, $V = F(K,L)$, may be written

$$V/L = F(K/L, 1)$$

or

$$y = f(x).$$

Under competitive conditions

$$\partial V/\partial L = w = f(x) - xf'(x) = y - xy'$$

and

$$\partial V/\partial K = r = f'(x) = y'.$$

If in the relation

$$\ln V/L = -\ln \alpha + b \ln w$$

we substitute for w and write $\ln a = -\ln \alpha$, we have

$$\ln y = \ln a + b \ln (y - xdy/dx)$$

or

$$dy/dx = (a^{1/b} y - y^{1/b})/a^{1/b} x.$$

Set $\rho = -1 + 1/b$, $\delta_2 = a^{-1/b}$ and take $\frac{1}{\rho} \ln \delta_1$ as a constant of integration. We then have

$$y = (\delta_1 x^{-\rho} + \delta_2)^{-1/\rho} .$$

For marginal products to be positive, $\delta_1 > 0, \delta_2 > 0$. ρ ranges from -1 to 0 and σ from ∞ to 0 since $\sigma = \frac{1}{1+\rho}$.

This function comprises the family of production functions with constant elasticity of substitution for all values of K/L. If we write $\delta_1 + \delta_2 = \gamma^{-\rho}$ and $\delta_1 \gamma^{\rho} = \delta$, we have the usual form of the CES function with constant returns to scale,

$$V = \gamma[\delta K^{-\rho} + (1-\delta)L^{-\rho}]^{-1/\rho}, \quad \gamma > 0, \quad 0 < \delta \leq 1, \quad \rho > -1.$$

Brown and de Cani (1963) generalised this form by introducing ν, the returns to scale parameter

$$V = \gamma[\delta K^{-\rho} + (1-\delta)L^{-\rho}]^{-\nu/\rho}$$

$\nu \gtreqless 1$ implies increasing, constant or diminishing returns to scale, respectively.

The explicit form of the CES function can be obtained by a number of different approaches. The assumption of homogeneity and the constancy of elasticity of substitution uniquely determine the algebraic form of this function.

For a homogeneous function $V = F(K,L)$ of degree ν the elasticity of substitution is given by

$$\frac{F_K F_L}{(1-\nu)F_K F_L + \nu F F_{KL}}$$

where the suffixes to F denote appropriate partial derivatives. Since $\sigma = \frac{1}{1+\rho}$, we have $F_{KL}/F_L = (1+\rho/\nu)F_K/V$. Writing $C(L)$ for an arbitrary function of L and integrating

$$\ln F_L = \ln V^{1+\frac{\rho}{\nu}} + C(L)$$

or $$V^{1+\frac{\rho}{\nu}} = C(L)\,\partial V/\partial L.$$

Introducing $\theta_1(K)$ as an arbitrary function of K and $\theta_2(L)$ as a primitive of $C(L)$, we have

$$F = V = (-\rho/\nu)^{-\nu/\rho}[\theta_1(K) + \theta_2(L)]^{-\nu/\rho}$$

Homogeneity of degree ν implies

$$K F_K + L F_L = \nu V$$

Since $$F_K = (-\nu/\rho)^{\frac{\nu}{\rho}+1}[\theta_1(K) + \theta_2(L)]^{-\frac{\nu}{\rho}-1}\theta_1'(K)$$

$$F_L = (-\nu/\rho)^{\frac{\nu}{\rho}+1}[\theta_1(K) + \theta_2(L)]^{-\frac{\nu}{\rho}-1}\theta_2'(L)$$

we have, by substitution

$$K\,\theta_1'(K) + L\,\theta_2'(L) = -\rho[\theta_1(K) + \theta_2(L)]$$

Separating the two equations, we get

$$\frac{\theta_1'(K)}{\theta_1(K)} = -\frac{\rho}{K}, \quad \frac{\theta_2'(L)}{\theta_2(L)} = -\frac{\rho}{L}$$

or $$\theta_1(K) = \delta_1 K^{-\rho}, \quad \theta_2(L) = \delta_2 L^{-\rho}$$

Substituting

$$\gamma = (-\nu/\rho)^{\frac{\nu}{\rho}}(\delta_1+\delta_2)^{-\frac{\nu}{\rho}}, \quad \delta = \frac{\delta_1}{\delta_1+\delta_2}$$

we have

$$V = \gamma[\delta K^{-\rho} + (1-\delta)L^{-\rho}]^{-\nu/\rho}$$

The Cobb Douglas function can also be derived in this manner if during the integration process we write $\rho = 0$, Vazquez(1971). All the other related forms of production functions, like the variable elasticity of substitution function can be derived similarly

Alternatively, since, with nonconstant returns to scale, we can write

$$V = L^{\nu} F(1, K/L)$$

or $V/L^{\nu} = f(x)$ where $x = K/L$, we have

$$\frac{1}{1+\varsigma} = \sigma = \frac{f'(\nu f - xf')}{(\nu - 1)xf'^2 - \nu xff''}$$

The substitution $u = \nu\frac{f}{f'} - x$ results in

$$du/u = (\varsigma + 1)\, dx/x \quad \text{since } du/dx = \nu(1 - ff''/f'^2) - 1$$

Hence $\quad u = \nu f/f' - x = A x^{1+\varsigma}$

or $\quad f = B(A + x^{-\varsigma})^{-\nu/\varsigma}$

where A, B are constants of integration.

Setting $A = \frac{1-\delta}{\delta}$, $B = \gamma\,\delta^{-\nu/\varsigma}$, we get

$$V = \gamma\,[\delta K^{-\varsigma} + (1-\delta) L^{-\varsigma}]^{-\nu/\varsigma}$$

which is the required CES function.

Another alternative is to make use of the concepts of partial output elasticities with respect to labour(ϵ_L) and capital (ϵ_K).

$$\epsilon_L = LF_L/V = \frac{\nu f - xf'}{f} \quad \text{since } F_L = L^{\nu-1}(\nu f - xf')$$

$$\epsilon_K = KF_K/V = \frac{xf'}{f} \quad \text{since } F_K = L^{\nu-1} f'$$

With $\nu = \epsilon_L + \epsilon_K$, the MRS of labour for capital is given by

$$s = \frac{f'}{\nu f - x f'} = \frac{\nu - \epsilon_L}{x \epsilon_L}$$

Since

$$\frac{ds}{dx} = \frac{\epsilon_L(\epsilon_L - \nu) - \nu x \epsilon_L'}{x^2 \epsilon_L^2} \quad \text{where } \epsilon_L' = d\epsilon_L/dx$$

we have $\frac{1}{1+\varsigma} = \sigma = \frac{\epsilon_L(\nu - \epsilon_L)}{\epsilon_L(\nu - \epsilon_L) + \nu x \epsilon_L'}$ or $\frac{d\epsilon_L}{\epsilon_L} + \frac{d\epsilon_L}{\nu - \epsilon_L} = \varsigma\frac{dx}{x}$

$$\therefore A x^{\varsigma} = \frac{\epsilon_L}{\nu - \epsilon_L} = \frac{\nu f}{x f'} - 1$$

$$\therefore f = B(A + x^{-\varsigma})^{-\nu/\varsigma}$$

or $\quad V = \gamma\,[\delta K^{-\varsigma} + (1-\delta) L^{-\varsigma}]^{-\nu/\varsigma}$

It may be remarked that any homogeneous production function can be derived by using suitable assumptions about its parameters and the relationship between these parameters.

Several empirical studies show that the elasticity of substitution may vary with changes in input combinations. It may be possible to assume constant elasticity of substitution in the case of time series or data with a narrow range for the input ratio but in the case of establishment data, large variations in input ratios may be expected. For data at a higher level of aggregation, changes in input ratio may be noticed only marginally but in the case of establishment data, there may be a wide range of substitution possibilities. The Cobb Douglas or the CES function may be used for establishment data if it can be assumed that the input ratios are fairly constant. Such an assumption may hold if all the establishments are of nearly the same size and belong to the same industry But if they come from different industries or different countries as in ACMS(1961), this assumption may not be justified However, the CES function has produced reasonably satisfactory results in many cases.

Mayor(1969) pointed out six possible sources of bias in the elasticity of substitution estimates from cross section data with the CES function. There are variations in labour quality and output prices. The efficiency parameter and distribution parameters can vary. There is what he calls the problem of lags and dynamic adjustment.[1]

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1 The assumption of long run competitive equilibrium in a cross sectional model may require that wages equal the marginal value product of labour and productivity equal the value appropriate for that wage rate. The choice of technique may usually be based on permanent rather than the measured wages which are used in the cross sectional model. Also the firms do not remain on their optimal positions through fluctuations in market forces and the productivity has significant cyclical movements. These transitory elements in wages and productivity introduce a bias due to the problem of lags and dynamic adjustments which do not have an easily discernible effect on the estimates. The probable direction of bias, if any, is a matter of conjecture. See Mayor(1969).

Most of the ACMS(1961) assumptions with respect to the CES function were adopted by other theorists and empirical workers. These assumptions are constancy of returns to scale, exogeneity of wage rate, absence of correlation between output and labour prices and equality of factor reward and marginal productivity. Relaxation of one or more of these assumptions led to some new forms of production relations as in Hildebrand and Liu(1965) and Nerlove(1967).

The popularity of the CES function rests on its conveniently simplified version though empirically it may not always be superior to the Cobb Douglas function. The nonconfinement to unitary elasticity of substitution is its important feature though even that is not a very reliable characteristic. Nadiri(1970) and Katz(1969) find that the CES function gives elasticity of substitution estimates below unity in the case of time series data and above unity in the case of cross section data.[1] Nerlove(1965) remarked that

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1There is plenty of evidence to suggest that the crosssection estimates of the elasticity of substitution are greater than the time series estimates.which are likely to be of a short term nature as compared to the cross section estimates. According to Grilliches(1967), the downward bias in time series estimates arises from the omitted variables in the regression. Brown(1967) finds that "a pattern emerges of differences between cross section and time series estimates of elasticity of substitution, namely, the former are generally larger than the time series estimates." Ferguson(1965) attributes the downward bias in the time series estimates of the elasticity of substitution to changes in the quality of labour service.

The estimation technique and the specification of the model can influence the value of elasticity of substitution. Even the arrangement of data and the nature of capital series used can affect it See also Maddala(1965) and Lucas(1963).

even minor differences in specification or methods of estimation resulted in differing estimates of the elasticity of substitution parameter.

Gupta(1968) questioned the validity of the ACMS(1961) hypothesis of a common production function with neutral differences in the efficiency factor use for an industry across countries. These differences are neutral[1] only if the marginal rate of substitution is the same across countries for each combination of K and L.

For the CES function

$$V = \gamma[\delta_1 K^{-\rho} + \delta_2 L^{-\rho}]^{-1/\rho}$$

the marginal rate of substitution is given by

$$s = \frac{\partial V/\partial L}{\partial V/\partial K} = \frac{\delta_1}{\delta_2}(K/L)^{1/\sigma}$$

where $\sigma = 1/(1+\rho)$. If σ is assumed to be a constant, s is constant implying Hicks neutrality for each value of K/L provided $\xi = \delta_1/\delta_2$ is constant.

Since $\partial s/\partial \xi = (K/L)^{1/\sigma} = s/\xi$, we are able to test the neutrality hypothesis by examining the variance of ξ.

Assuming perfect competition which is a long term and not a short term condition we have $s = w/r$ and

$$\delta_1/\delta_2 = (L/K)^{1/\sigma} w/r$$

To test the neutrality hypothesis it would be inappropriate to use δ_1/δ_2 because the inequality of s and w/r anywhere will lead to various degrees of imperfection in product and factor markets, nonconstancy of returns and prevalence of disequilibrium in various markets, all of which together would amount to testing a highly composite hypothesis. Even if ACMS arrived at the neutrality hypothesis

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1Assuming unembodied neutral technical change applicable to crosssection observations.

their conclusion must be rejected.

Since $\delta_2 = 1-\delta_1$, $\delta_1/\delta_2 = \frac{\delta_1}{1-\delta_1}$

$$Var(\delta_1) \neq Var\left(\frac{\delta_1}{1-\delta_1}\right),$$

it would be wrong to test for neutrality by examining $Var(\delta_1)$ as ACMS have done, instead of $Var\left(\frac{\delta_1}{1-\delta_1}\right)$ which should be used for the purpose. Also, since $Var\left(\frac{\delta_1}{1-\delta_1}\right) > Var(\delta_1)$, the degree of non-neutrality will be underestimated as ACMS have used $Var(\delta_1)$ to test the same.

Again, ACMS found that the assumption of a common industry production function for all the countries in the sample was invalid because of neutral efficiency differences across countries. Consequently they revised their estimates of γ on the assumption that in each industry, the efficiency parameter varies positively among countries. This is because $\xi = \delta_1/\delta_2$ is highly sensitive to variations in σ . As noted above since we can write

$$\xi = \frac{w}{r}\left(\frac{L}{K}\right)^{1/\sigma}$$

it follows that

$$\frac{\partial \xi}{\partial \sigma} = -\frac{\delta_1}{1-\delta_1}\,\frac{1}{\sigma^2}\ln\frac{L}{K}$$

It is possible that in spite of differences in the input intensities of different countries, their elasticities of substitution may be comparable because of the relative flexibility of capital intensive technologies being similar to that of labour intensive technologies. ACMS allowed for parameter differences across countries by setting up the hypothesis of factor use efficiency differences, purely labour augmenting differences and capital augmenting differences. Assuming constant σ and writing

$$V = \gamma[\delta K^{-\rho}+(1-\delta)L^{-\rho}]^{-1/\rho} = [\delta_1 K^{\rho} + \delta_2 L^{-\rho}]^{-1/\rho}$$

the neutrality of input use implies constant δ_1/δ_2 for each country, allowing for proportional changes both in δ_1 and δ_2. The labour augmenting hypothesis implies a varying δ_2 and the capital augmenting hypothesis implies a varying δ_1. They found as a fair working hypothesis that international production function differences are contained in γ, the efficiency parameter.

According to Feldstein(1967), a weakness of the CES function lies in the assumption of the exogeneity of wage rate by ACMS(1961) in their estimation procedure, another weakness lies in the assumption of equality of wage rate and the marginal product of labour. The constant returns to scale aspect of the CES function is as questionable as it is in the case of the Cobb Douglas function.

For nonconstant returns to scale with wage rate equal to the marginal value product, the CES wage productivity relation gives a biased elasticity of substitution because, according to Feldstein, the appropriate relation is

$$\ln V/L = a' + b' \ln w + \frac{(1-\sigma)(\nu-1)}{\nu} \ln V \quad \text{where } a' = \sigma \ln \gamma^{\delta/\nu} / \nu(1-\delta)$$

The bias $= (1 - \sigma)\frac{\nu-1}{\nu}\beta$

where β is the expected value of b as obtained from the CES relation $\ln V/L = a + b \ln w$.

Thus the equality of the wage rate and the marginal value product of labour is in doubt as it depends on perfectly competitive product and labour markets with all cost minimising firms and absence of increasing returns to scale for each firm. This may lead to additional bias in the estimate of σ.

The CES Function under Imperfect Competition

If it is desired to give up the assumption of perfectly competitive markets, made in the case of most production function studies, including the CES function, then writing η_{pQ} for the elasticity of demand for output and η_{wL} for the elasticity of supply of labour, we may, with Dhrymes(1965), make a correction for the inequality between the marginal product of labour and the real wage rate, allowing for labour and output market conditions

$$\partial Q/\partial L = w \frac{1 + 1/\eta_{wL}}{1 + 1/\eta_{pQ}} = w\ \phi(I), \text{ say}$$

where the index $\phi(I)$ may depend on time or place, and Q stands for physical output Q.

We may then write

$$Q = \phi_0(I)\ [\ \phi_1(I)\ K^{-\rho} + \phi_2\ (I)\ L^{-\rho}\ ]^{-1/\rho}$$

where ϕ_0, ϕ_1, ϕ_2 are functions of ϕ. Also $\phi_1 + \phi_2 = 1$.

The CES productivity wage relation may be written

$$\ln Q/L = a + b \ln \phi + b \ln w$$

Unfortunately accurate estimates of η_{wL}, η_{pQ} cannot be easily obtained though Katz's(1969) attempt to apply it to cross sectional data may be noted.

With the net profit $\pi = pQ - wL - rK$, the profit maximising condition is given by

$$0 = \partial\pi/\partial L = p\,\partial Q/\partial L + Q(\partial p/\partial Q)(\partial Q/\partial L) - w - L\partial w/\partial L$$

and $\partial Q/\partial L = \dfrac{w + \partial w/\partial L}{p + \partial p/\partial Q}\ \dfrac{L}{Q} = \dfrac{1 + \eta_{wL}}{1 + \eta_{pQ}}\ w/p$

where $\eta_{wL} = (\partial w/\partial L)(L/w)$ and $\eta_{pQ} = (\partial p/\partial Q)(Q/p)$, respectively are the labour wage elasticity and output price elasticity in an imperfect market as mentioned above.

Under competitive conditions $\partial Q/\partial L = w/p$. We have, therefore,

$$\frac{w}{p}\;\frac{1+\eta_{wL}}{1+\eta_{pQ}} = \nu Q^{-\rho/\nu}(1-\delta)\,(Q/L)^{1+\rho}\,Q^{\frac{1-\nu}{\nu}\rho}$$

or, $$\ln Q/L = \frac{1}{1+\rho}\ln\frac{\nu(1-\delta)}{\gamma^{\rho/\nu}} + \frac{1}{1+\rho}\ln w - \frac{1}{1+\rho}\ln p + \frac{1}{1+\rho}\ln\frac{1+\eta_{wL}}{1+\eta_{pQ}} + \frac{\rho\nu-1}{1+\rho\nu}\ln Q$$

Writing $D = \frac{1+\eta_{wL}}{1+\eta_{pQ}}$ for the degree of imperfection in factor and commodity markets and $C = (1-\sigma)\frac{\nu-1}{\nu}$, we may write the above relation

$$\ln Q/L = a + \sigma\ln w + \sigma\ln D - \sigma\ln p + C\ln Q$$

Katz(1969) uses this relation in his cross regional study of the CES function. The hypothesis tested is whether the extent of market imperfection is different for different firms depending on their sizes (or time factor) if the sample consists of crosssection (or time series) data. In spite of the theoretically sound points, the empirical worth of this modification is not very high unles accurate estimates of the η's are available.

Some Multi-input Extensions

The CES function has been generalised not only to allow for any degree of homogeneity but also to include any number of inputs.

For the n inputs, $X_1, X_2, \ldots, X_n$,

Mukerji(1963) gives the generalisation

$$V = \gamma\left(\sum \delta_i X_i^{-\rho}\right)^{-1/\rho} \qquad \sum \delta_i = 1$$

where V is value added.

The production function is symmetrical with respect to all dimensions of factor intensity because it results in constant and identical partial elasticities of substitution.

A less unrealistic form was given later by Mukerji(1964) ·

$$V = \gamma \left(\sum_{i=1}^{n} \delta_i X_i^{-\rho_i} \right)^{-1/\rho}$$

which is shown to have constant ratios of partial elasticities of substitution (CRES) . This amounts to a partial generalisation of the C S function. It has the added quality of being nonhomogeneous and nonhomothetic. It is nonhomogeneous so long as ρ_i's are not equal among themselves and also not equal to ρ .

The marginal product of the ith input is given by

$$F_i = \frac{\partial V}{\partial X_i} = \delta_i \frac{\rho_i}{\rho} \frac{V^{1+\rho}}{X_i^{1+\rho_i}}$$ where $\gamma = 1$, without loss of generality

Since $F_{ii} = F_i \left(\frac{1+\rho}{V} F - \frac{1+\rho_i}{X_i} \right)$ and $F_{ij} = \frac{1+\rho}{V} F_i F_j$,

the partial elasticity of substitution is given by

$$\sigma_{ij} = \frac{1}{(1+\rho_i)(1+\rho_j)} \frac{\Sigma X_i F_i}{\Sigma X_i F_i/(1+\rho_i)} \quad i \neq j$$

which is not a constant. It depends on input levels and their combinations. The ratios of partial elasticities are constant though not necessarily equal.

$$\frac{\sigma_{ij}}{\sigma_{i'j'}} = \frac{(1+\rho_{i'})(1+\rho_{j'})}{(1+\rho_i)(1+\rho_j)} \qquad i \neq j,\ i' \neq j'$$

When all ρ_i's are equal to ρ , say, then $\sigma_{ij} = \frac{1}{1+\rho}$

When one of the subscripts coincides, the ratio is independent of the common subscript

$$\frac{\sigma_{ij}}{\sigma_{ij'}} = \frac{1+\rho_{j'}}{1+\rho_j} \qquad i \neq j \neq k$$

The Mukerji function may be estimated by nonlinear least squares but the difficulty will be greater, the larger the number of variables. If perfect competition is assumed, then, denoting by w_i the price of input X_i , we have

$$w_i = F_i = \frac{\delta_i \rho_i}{\rho} \frac{V^{1+\rho}}{X_i^{1+\rho_i}}$$

which with a log linear regression may be used to estimate δ_i, ρ_i, ρ.

The Mukerji function restricts the ratios of partial elasticities of substitution to be constant. This may be unjustifiable on economic grounds. But the function is more general than the Cobb Douglas and CES functions which may be derived as special cases.

Consider a three input Mukerji function to see how convenient it may be to make use of it in practice.

$$V = \gamma(\delta_1 X_1^{-\rho_1} + \delta_2 X_2^{-\rho_2} + \delta_3 X_3^{-\rho_3})^{-1/\rho}$$

Without loss of generality, we may assume $\gamma = 1$.

This function is homogeneous if and only if $\rho_1 = \rho_2 = \rho_3 = \rho$. Also $\nu \gtreqless 1$ according as $\rho_i \gtreqless \rho$

Since $\dfrac{\sigma_{12}}{\sigma_{13}} = \dfrac{1+\rho_3}{1+\rho_2}$

$$\sigma_{12} \gtreqless \sigma_{13} \quad \text{if} \quad \rho_3 \gtreqless \rho_2 \quad \text{or} \quad \rho_3 - \rho \gtreqless \rho_2 - \rho$$

Under conditions of perfect competition

$$w_i = \frac{\partial F}{\partial X_i} = \frac{\delta_i \rho_i}{\rho} \frac{V^{1+\rho}}{X_i^{1+\rho_i}}$$

$$\frac{F_2}{F_3} = \frac{w_2}{w_3} = \frac{\delta_2 \rho_2}{\delta_3 \rho_3} \frac{X_3^{1+\rho_3}}{X_2^{1+\rho_2}}$$

If we write $\delta_2\rho_2/\delta_3\rho_3 = A$, we have the regression equation

$$\ln w_2/w_3 = \ln A - b_2 \ln X_2 + b_3 \ln X_3$$

If the three variables are capital K, direct labour L_D and unskilled or indirect labour L_I, then, with suitable suffixes attached to the symbols, we have the regression equation

$$\ln w_D/w_I = \ln A - b_D \ln L_D + b_I \ln L_I$$

From this we may find the elasticity of substitution between capital and direct labour (σ_{KD}) and between capital and indirect labour (σ_{KI}). In the short run, the extent of complementarity between capital and direct labour and between capital and indirect labour may be found.

To verify if the partial elasticity between any two pairs of inputs in the multiple input production function with

$\rho_i = \rho$ = constant, may be equal in actual practice, Gujarati and Fabozzi(1972) proceed to test it empirically and come to the conclusion that in the case of the data they used, these partial elasticities are not the same. This means that the CES production function with more than two inputs and $\rho_i = \rho$ may not be considered a reliable form.

Taking at a time, two variables out of the three variables K, L_D and L_I Gujarati and Fabozzi make use of the first four of the following six relationships to estimate σ

$$V/K = a\, r^{\sigma} \text{ where r is the rate of return on capital}$$
$$V/L_D = b\, w_D^{\sigma}$$
$$V/L_I = c\, w_I^{\sigma}$$
$$L_D/L_I = \frac{c}{b}(w_I/w_D)^{\sigma}$$

and $$L_D/K = \frac{a}{b}(r/w_D)^{\sigma}, \quad L_I/K = \frac{a}{c}(r/w_I)^{\sigma}$$

For data on 18 industries they found that all the estimated partial elasticities differed from one another for almost all the industries. They concluded that for the three input production function

$$V = \gamma [\delta_K K^{-\rho} + \delta_D L_D^{-\rho} + \delta_I L_I^{-\rho}]^{-1/\rho}, \quad \delta_K + \delta_D + \delta_I = 1$$

the partial elasticities, though theoretically equal, may not be the same empirically.

To get over the restrictive difficulty of constant partial elasticities of substitution for all pairs of factors in a multi-input CES function Uzawa(1962) and Mcfadden(1963) arrived at a slightly different form which for the four input case may be written

$$V = \gamma [\delta_1 X_1^{-\rho_{12}} + \delta_2 X_2^{-\rho_{12}}]^{-\frac{\xi_1}{\rho_{12}}} [\delta_3 X_3^{-\rho_{34}} + \delta_4 X_4^{-\rho_{34}}]^{-\frac{\xi_2}{\rho_{34}}}$$

where $\xi_1 + \xi_2 = 1$, $\sigma_{12} = \frac{1}{1+\rho_{12}}$, $\sigma_{34} = \frac{1}{1+\rho_{34}}$

and $\sigma_{13} = \sigma_{14} = \sigma_{23} = \sigma_{24} = 1$

The set of four inputs has been divided into subgroups within each of which there is a common partial elasticity of substitution which need not be the same for all the subgroups. Unfortunately, the construction constrains the partial elasticity between variables of different subgroups to unity.

For three inputs K , L , M the mixed form may be written as a Cobb Douglas-CES hybrid.

$$V = (K^{-\rho_1} + L^{-\rho_1})^{-1/\rho_1} M^{\rho_2}$$

or $$V = \gamma K(M^{\rho_1 \rho_2} - (V/L)^{\rho_1})^{1/\rho}$$

The third input M may be expressed as a multiplicative exponential

$$V = \gamma (K^{-\rho_1} + L^{-\rho_1}) e^{-V/M}$$

or $$V = \gamma K(e^{\rho_1 V/M} - (V/L)^{\rho_1})^{1/\rho}$$

Although these forms are interesting, the difficulty of nonlinear estimation remains. Problems of estimation and artificial constraints are a common feature of Mukerji, Uzawa and McFadden's models. As is evident from the empirical results of Gujarati and Fabozzi(1972), the theory happens to be very different from practice.

We now consider Sato's multi-input extension of the CES function which is, relatively speaking, more realistic than some of the forms given above. It will be given first in the n input case and then to consider its practical aspect, in the three input case.

Sato's(1963) Two Level CES Function

Under certain conditions, Sato's two level CES function generates different partial elasticities of substitution between different pairs of inputs.

If the production function is

$$V = F(X)$$

$$= F(X_1, X_2, \ldots, X_n)$$

and X is partitioned into t subsets

$$X = (X^1, X^2, \ldots, X^t) \qquad X_i \in X^j,\ X_i \notin X^k,$$
$$i = 1,2,\ldots,n;\ j,k = 1,2,\ldots,t$$

then, if the production function is strongly separable with respect to such a partition, we may write

$$V = F[g_1(X^1) + g_2(X^2) + \ldots + g_t(X^t)] .$$

If σ_m is the partial elasticity of substitution within the mth subset in which it is constant, and if

$\delta_i^m > 0,\ -1 < \rho_m = \frac{1-\sigma_m}{\sigma_m}$

then a two level CES function may be constructed:

$$V_m = g_m(X^m) = [\sum_i (\delta_i^m X_i^m)^{-\rho_m}]^{-1/\rho_m}$$

$$V = F(V_m) = (\sum_m \alpha_m V_m^{-\rho})^{-1/\rho} \qquad \alpha_m > 0,\ -1 < \rho = \frac{1-\sigma}{\sigma}$$

where σ is the elasticity of substitution within the t input groups. We have the two level CES function given by

$$V = [\sum_m \alpha_m \{(\sum_i \delta_i^m X_i^m)^{-\rho_m}\}^{-\rho/\rho_m}]^{-1/\rho}$$

$\sigma_m = \frac{1}{1+\rho_m}$ is the intra subset and $\sigma = \frac{1}{1+\rho}$, the intersubset elasticity of substitution. The direct partial elasticity of substitution between any two inputs X_i, X_j is

$$\sigma_{ij} = \sigma_m \text{ if } X_i,\ X_j \in X^m$$

$$= \frac{a + b + c}{a/\sigma_p + b/\sigma_q + c/\sigma} \qquad \text{if } X_i \in X^p,\ X_j \in X^q,\ p \neq q$$

where $a = \frac{1}{S_i^p} - \frac{1}{S^p}$, $b = \frac{1}{S_j^q} - \frac{1}{S^q}$, $c = \frac{1}{S^p} + \frac{1}{S^q}$;

S_i^p is the relative share of the ith element of subset r

S^p is the relative share of the rth subset of inputs.

Thus the partial elasticities of any pair of inputs from different groups are harmonic means of $\sigma, \sigma_p, \sigma_q$ and depend on relative share sizes. In general they will be different.

Sato Three Input CES Function(Two Level)

The Sato two level CES function is homothetic because its marginal rate of thechnical substitution(MRS) depends only on the input ratio and is independent of the scale of production.

A strongly separable function with its MRS between any two inputs independent of other inputs does not allow interdependence among inputs. A weakly separable function may, therefore, be considered. If the three inputs are X_i, X_j, X_k and the output is Y, this function will be of the form

$$Y = F \{ (X_i, X_j), X_k \}$$

The partial elasticity of substitution between X_i, X_j does not depend on X_k.

The partial elasticity of substitution between X_j, X_k depends on X_i; that between X_i, X_k depends on X_j.

If the three inputs are capital(K), labour(L) and raw material(M), the weakly separable function is

$$Y = F \{ (K, L), M \}$$

This implies that in the first stage of production process, capital and labour are combined to produce the composite output V which, combined with M, leads to the final output

$$Y = F (V, M)$$

Let $\sigma_0 = \frac{1}{1+\rho_0}$ be the elasticity of substitution between K and L in the first stage CES function

$$V = (\delta_1 K^{-\rho_0} + \delta_2 K^{-\rho_0})^{-1/\rho_0}.$$

Let $\sigma = \frac{1}{1+\rho}$ be the elasticity of substitution between V and M at the second stage CES function

$$Y = [\alpha_1 (\delta_1 K^{-\rho_0} + \delta_2 L^{-\rho_0})^{\rho/\rho_0} + \alpha_2 M^{-\rho}]^{-1/\rho}$$

This is very different from the highly restrictive case of constant, equal partial elasticities of substitution

$$Y = (\beta_1 K^{-c} + \beta_2 L^{-c} + \beta_3 M^{-c})^{-1/c}$$

The Sato two level CES function provides an alternative method for purposes of comparison in empirical work though the amount of work involved is substantial. The constancy of elasticities between K and L and between V and M does not necessarily result in equal partial elasticites of substitution so that the rate of substitution between M and any other input can be found.

If $\rho = -1$ i.e. the elasticity of substitution between V and M is infinite, the two level CES function reduces to the usual two input CES function. As ρ tends to -1

$$Y = \alpha_1 [\delta_1 K^{-\rho_0} + \delta_2 L^{-\rho_0}]^{-1/\rho_0} + \alpha_2 M$$

or $$V = \frac{Y - \alpha_2 M}{\alpha_1} = (\delta_1 K^{-\rho} + \delta_2 L^{-\rho})^{-1/\rho_0}$$

If ρ is infinite, $\sigma = 0$ and again we arrive at the two input CES function which can describe the three input production process.

The CES Production Function with Variable Returns to Scale

Instead of assuming constant returns to scale it may be possible to assume that point returns to scale are related to output , as done by Soskice(1968). In that case, the common procedure for estimating the elasticity of substitution may be inconsistent.

For the production function V = F (K,L), assuming that the point returns to scale are a function of output, or value added, say h (V), we have, since dL/L =dK/K, the point returns to scale given by

$$(dV/dK)\ K/V = (\partial V/\partial K)\ K/V + (\partial V/\partial L)\ L/V = h\ (V)$$

If h(V) is suitably specified, say as a quadratic in V, we may write $h(V) = a_0 + a_1 V + a_2 V^2$, $a_1^2 > a_0 a_2$. If g numbers a CES isoquant in $g = \delta K^{1-1/\sigma} + (1-\delta)L^{1-1/\sigma}$ we have

$$\frac{dV}{dg}\left[\frac{\partial g}{\partial K}\frac{K}{y} + \frac{\partial g}{\partial L}\frac{L}{y}\right] = h(V)$$

or, $\frac{1}{V}\frac{dV}{dg}\,(1-1/\sigma)\,\delta K^{1-1/\sigma} + (1-1/\sigma)(1-\delta)L^{1-1/\sigma} = h(V)$

i.e. $(1 - 1/\sigma)\,\frac{dV}{dg}\,\frac{g}{V} = a_0 + a_1 V + a_2 V^2$

If α, β are the roots of the quadratuc $a_0 + a_1 V + a_2 V^2$ and $\ln \gamma$ is an arbitrary constant of integration, we can write the solution

$$V|V-\alpha|^{\frac{\beta}{\alpha-\beta}}|V-\beta|^{\frac{\alpha}{\alpha-\beta}} = \gamma\left[\delta K^{1-1/\sigma} + (1-\delta)L^{1-1/\sigma}\right]^{\frac{a_0}{1-1/\sigma}}$$

where a_0 is the returns to scale parameter in the usual CES function. The term beside V on the left, viz. $|V-\alpha|^{\frac{\beta}{\alpha-\beta}}|V-\beta|^{\frac{\alpha}{\alpha-\beta}}$, acts as a deflator or inflator of V depending on whether the actual returns to scale imply a higher or lower level of output than that implied by the unchanging returns to scale a_0.

Different specifications for h(V) would result in other forms of production functions with returns to scale varying with output. The CES case corresponds to $h(V)=a_0$.

Further, to obtain a consistent estimate of σ, it is usual to add ln V as an additional independent variable to the ACMS equation relating ln V/L and ln w. This is valid provided ln w is orthogonal to the error vector and this may not be the case usually. The use of an appropriate instrumental variable may be helpful. Moreover, it also requires the assumption of constant returns to scale to be made. If the returns to scale are not constant and $a_1 \neq o$, $a_2 \neq o$ in h (V), the regression of ln V/L on ln w and ln V yields an inconsistent estimate of σ. With variables measured from their means, this equation may be written without a constant term,

$$\ln V/L = \sigma \ln w + (1-\sigma) \frac{a_o - 1}{a_o} \ln V$$

Writing $\Psi = V \, |V - \alpha|^{\frac{\beta}{\alpha-\beta}} |V-\beta|^{\frac{\alpha}{\alpha-\beta}} = V \phi(V)$

we have, since $\frac{\partial \Psi}{\partial L} = \frac{\partial V}{\partial L} \frac{\partial \Psi}{\partial V} = w \frac{\partial \Psi}{\partial V}$

$$\ln \Psi/L = \sigma \ln w \frac{\partial \Psi}{\partial V} + (1-\sigma) \frac{a_o - 1}{a_o} \ln \Psi$$

$$\therefore \ln V/L = \sigma \ln w + (1-\sigma) \frac{a_o - 1}{a_o} \ln V + \sigma \ln \frac{\partial \Psi}{\partial V} + \left[(1-\sigma)\frac{a_o - 1}{a_o} - 1\right] \ln \phi(V)$$

which involves two additional variables, viz., $\ln \partial\Psi/\partial V$ and ln ϕ (V) which are missing from the original equation. Now ϕ (V) and $\partial\Psi/\partial V$ are generally highly correlated with V though the extent depends on the range of values of V. If ln w is orthogonal to ln V, the omitted variables do not matter significantly.

Hilhorst's CES Variation

Hilhorst's(1971) CES variation does not require data on factor prices. It is enough to have information about relative factor shares.

Hilhorst gives a more general formulation of the CES function

$$V = (\alpha L^a + \beta K^b)^{1/c}$$

which is homogeneous of degree h/c if a = b = h. It is linearly homogeneous when a = b = c but nonhomogeneous when a = b $\neq$ c.

The function makes use of the optimisation result that the entrepreneurs aim at minimising cost per unit of the product:

$$\frac{\partial V/\partial K}{\partial V/\partial L} = r/w = \frac{b\beta}{a\alpha} \frac{K^{b-1}}{L^{a-1}}$$

Since, with constant r turns to scale,

$$V = \alpha^{1/c} L^{a/c} \left(1 + \frac{\beta}{\alpha} \frac{K^b}{L^a} \right)^{1/c}$$

and $$V = \beta^{1/c} K^{b/c} \left(1 + \frac{\alpha}{\beta} \frac{L^a}{K^b} \right)^{1/c},$$

by taking logarithms and using, $\frac{\beta}{\alpha}\left(\frac{K^b}{L^a}\right) = \frac{r}{w} \frac{a}{b} \frac{K}{L}$,

we have two equations to estimate the parameters

These equations help us to t st the supposition of minimum costs and the effects of the size of the stablishm nt on the use of the inputs.

To carry out the empirical work, a suitable iteration method may be used. For instance, we may first assume a = b. From this a/c and b/c may be found approximately and the quantities $1 + \frac{a}{b} \frac{r}{w} \frac{K}{L}$ and $1 + \frac{b}{a} \frac{w}{r} \frac{L}{K}$ calculated. Hilhorst found the iteration process ending quickly.

Alternatively, writing the CES formulation in its usual notation

$$V = \gamma [\delta K^{-\rho} + (1 - \delta) L^{-\rho}]^{-\nu/\rho}$$

where ν is the returns to scale parameter, we may write

$$V = \gamma (1-\delta)^{-\nu/\rho} L^{\nu} [1 + \frac{\delta}{1-\delta} (K/L)^{-\rho}]^{-\nu/\rho}$$

Assuming the equality of supply elasticities of labour and capital, we have

$$\partial V/\partial L = \nu w, \quad \partial V/\partial K = \nu r$$

so that $$\frac{\partial V/\partial K}{\partial V/\partial L} = \frac{\delta}{1-\delta} \left(\frac{L}{K}\right)^{1+\rho} = \frac{r}{w}$$

We then have $V = \gamma (1-\delta) L^{\nu} (1 + rK/wL)^{-\nu/\rho}$

and $$\ln V = \ln \gamma - \frac{\nu}{\rho} \ln (1-\delta) + \nu \ln L - \frac{\nu}{\rho} \ln (1 + rK/wL)$$

This relationship gives the values of ν and ρ. δ may be found by iteration or by det rmining it exogen ously. with one of the several side relations of the CES function, like $\ln rK/wL = \ln \frac{\delta}{1-\delta} + \rho \ln L/K$.

Kadiyala's General Form

Kadiyala(1972) gives a form of production function which may be considered as a direct generalisation of Cobb Douglas and the CES functions. The variability of the resulting elasticity of substitution is not confined to a monotonic rise or fall with the input ratio as is the case with the usual VES functions; it is symmetrical with respect to the two endpoints of the input ratio and passes through maximum and minimum stages.

The Kadiyala n input case may be written

$$V = A \left[\sum_{i}^{n} \delta_i X_i^{2\rho} + 2 \sum_{i<j}\sum \delta_{ij} X_i^{\rho} X_j^{\rho} \right]^{\nu/2\rho}$$

where $\sum_i \delta_i + 2 \sum_{i<j}\sum \delta_{ij} = 1$. In the two input case with K and L,

$$V = A \left(\delta_1 K^{2\rho} + 2\delta_{12} K^{\rho_1} L^{\rho_1} + \delta_2 L^{2\rho} \right)^{\nu/2\rho} \text{ where } \delta_1 + 2\delta_{12} + \delta_2 = 1$$

For nonnegative marginal products and a homogeneous function of degree one we must have $\rho_1 + \rho_2 = 2\rho$

The function reduces to the CES form for $\delta_{12} = 0$, to the Cobb Douglas form for $\rho \to 0, \delta_{12} = 0$; to Lu Fletcher VES form for $\delta_2 = 0$ and to Sato Hoffman VES form for $\delta_1 = 0$.

There are maximum and minimum values of elasticity of substitution for $\rho > 0$ and for $\rho < 0$. If the ρ exponents are unequal, the function would be difficult to use.

Meyer and Kadiyala(1974) conducted empirical tests by fitting various constrained and unconstrained forms of production functions to agricultural experiment data. The Cobb Douglas, CES and Kadiyala functions were the three alternative forms used. Two sets of results were obtained one with the constraint of constant returns to scale and the other without the constraint. The nonconstant returns to scale version gave a significantly better fit which was improved by moving from the Cobb Douglas to the CES to the Kadiyala function. Nonlinear estimation methods were used.

Productivity Relations and Production Functions

We now consider some extensions of the production function forms beginning once again with the Cobb Douglas and CES functions. The approach followed is not different from that in the section on the case of Cobb Douglas function. Instead of adding only technical variables to the Cobb Douglas function, economic variables will also be added to arrive at some new forms which contain the essence of both the Cobb Douglas and the CES functions. The authors of the extensions presented in this section happen to have begun with the productivity instead of the production function, for the development of their forms.

There is a good scope for the addition of different types of variables in a productivity relation and using it for empirical purposes subject to the economic validity of the relationship. The procedure is useful in carrying out experiments regarding the consequences of stepwise addition or substitution of some factors beginning with any basic productivity relationship.

In view of the nature of our data and the availability of an adequate number of variables it should be possible to experiment with several productivity relationships and arrive at some convenient forms of production functions for our purpose. We have considered the theoretical development of some of the forms though later for empirical work we have selected only a few.

Beginning with the Cobb Douglas and CES productivity relations we shall consider the functions developed by Lu Fletcher(1968), Tsang and Yeung(1974), Vazquez (1971).

The Cobb Douglas function $V = A K^{\alpha} L^{\beta}$ may be written as a productivity relation

$$V/L = A (K/L)^{\alpha} L^{\alpha+\beta-1}$$

or $$\ln V/L = \ln A + \alpha \ln(K/L) + (\alpha+\beta-1) \ln L$$

If the assumptions of perfect competition and profit maximisation are made, the Cobb Douglas production function leads to a productivity relation between labour productivity and the wage rate ·

$$V/L = m w$$

where m is a constant and w is the wage rate which is equated to the marginal productivity of labour. If the steps are retraced by substituting $w = f(x) - x f'(x)$ where $x = K/L$, we arrive at the original Cobb Douglas function.

The CES productivity relation, as used empirically by ACMS(1961), is given by $V/L = m w^{b}$ where b is not confined to unity. This is usually written and used in the log-form

$$\ln y = a + b \ln w \quad \text{where } a = \ln m,\ y = V/L$$

The dependence of V/L on K/L as in the Cobb Douglas relation and on w as in the CES relation may be used to consider a joint dependence of V/L on both. Other possibilities may also be investigated.

Hildebrand and Liu(1965), in a study of two digit U.S. industries noted that a better fit for a production function could be obtained if some key variables determined the relation. They suggested the inclusion of K/L as an additional explanatory argument in the CES function. Lu and Fletcher's (1968) VES production function was derived from a linear relation between V/L and K/L and w.

$$\ln V/L = a + b \ln w + c \ln x \quad \text{where } x = K/L \qquad (4b)$$

This relation may be looked upon as a blend of the CES productivity relation $\ln V/L = a + b \ln w$ and the Cobb Douglas relation constrained by constant returns to scale, $\ln V/L = a' + b' \ln K/L$. We may also visualise it as obtained from one economic and one technical explanatory factor in the same relation instead of only one of the two explanatory variables. But this relation is neither the Cobb Douglas nor its generalisation, the CES function but includes both as special cases. Grilliches(1967), giving an alternative statistical interpretation to the relation and considering two equations(the Cobb Douglas and the CES relations given above) in the system points out an identification problem here.

We now consider the production function that may be derived from the relation (4b). It may be noted that the addition of any terms to the CES productivity relation is likely to make the elasticity of substitution parameter a variable quantity. In other words, the resulting production function becomes a variable elasticity of substitution(VES) production function. Of course, the variability of elasticity of substitution can be brought about in other ways so that describing a production function as a VES function does not give it some exclusive quality not to be found elsewhere.

The Lu Fletcher VES Production Function

In the Lu Fletcher relation (4b), where a, b, c are constants, if we assume homogeneity of degree one, we have, by substituting y for V/L and using the relations $w = f(x) - x f'(x)$ and $r = f'(x)$,

$$\ln y = \ln a + b \ln (y - x\, dy/dx) + c \ln x$$

which, when solved results in the production function

$$V = \left(\beta K^{1-1/b} + a^{-1/b} \tfrac{1-b}{1-b-c} (K/L)^{-c/b} L^{1-1/b} \right)^{b/(b-1)}$$

where β is a constant of integration. If we write

$\varrho = -1 + 1/b, \quad a = (1 - \delta)^{-b} \gamma^{1-b}, \quad \beta = \delta \gamma^{1-1/b}$

we have $V = \gamma [\delta K^{-\varrho} + (1 - \delta) \eta (K/L)^{-c(1+\varrho)} L^{-\varrho}]^{-1/\varrho}$

Except for the multiplicative factor associated with L this function has the same form as the CES function to which it reduces if c = 0. It has positive marginal products, downward sloping marginal product curves over the relevant ranges of the inputs and homogeneity of degree one. The variable elasticity of substitution varying with the input ratio is given by

$$\sigma = \frac{b}{1 - cf/xf'} = \frac{b}{1 - c [1 + (f-xf')/xf']}$$

$$= \frac{b}{1 - c(1 + wL/rK)}$$

under conditions of perfect competition. Lu and Fletcher give a manageable expression for σ as an explicit function of x.

$$\frac{b}{1 - c[1 + \frac{1-\delta}{\delta} \{ x^{1+c/b\; 1/b} - c/(b+c-1) \}^{-1}]}$$

By using side conditions for marginal productivity and calculating the average value of $\frac{\delta}{1-\delta}$, the value of σ at any point on the isoquant can be derived.

The Tsang-Yeung(1974) Productivity Relation

The Lu Fletcher relation (4b) has w and K/L as explanatory factors. We have looked upon this relation as a blend of the restricted Cobb Douglas function and the CES productivity relation. If we make a blend of the Cobb Douglas relation in the form

$$\ln V/L = a + \alpha \ln K/L + (\alpha + \beta - 1)\ln L$$

with the CES productivity relation, we find another relation with three explanatory factors, viz., w, K/L and L.

Alternatively, in the Lu Fletcher relation with one technical and one economic factor, one more technical factor L may be introduced on the ground that it does not find adequate representation in the Lu Fletcher relation. What we have described as a blend was given by Tsang and Yeung(1974) as a productivity relation between average productivity, wage rate, input ratio and total labour

$$\ln V/L = \ln a + b \ln w + c \ln w/L^{h-1} + d \ln L \quad .$$

Without loss of generality, we may write $d = (h-1)(1-b)$ and then we have

$$\ln y = \ln a + b \ln w/L^{h-1} + c \ln x$$

where $y = V/L^h$. If $h = 1$, then $y = V/L$.

If perfect competition is not assumed and η_V and η_L stand for the elasticities of product demand and labour supply, respectively, we have

$$p = \alpha' V^{\eta_V}, \quad w = \beta' L^{\eta_L}$$

where p and w are output and labour input prices respectively.

For maximum profit conditions

$$\frac{\partial V}{\partial L} = \frac{1 + 1/\eta_L}{1 + 1/\eta_V} \frac{w}{p}$$

$= m\ w$, say, assuming p to be constant.

If the function is homogeneous of degree h,

$$m\ w = \frac{\partial}{\partial L}(L^h y) = (h\ y - x\ dy/dx) L^{h-1}$$

Thus $\ln y = \ln a + b \ln(hy - xdy/dx)/m + c \ln x$

or $dy/dx = (h - a^{-\frac{1}{b}} m\ x^{-\frac{c}{b}} y^{\frac{1}{b}-1})\ y/x$

Using the substitution $z = y^{1-\frac{1}{b}}$, the solution is

$$z = y^{1-\frac{1}{b}} = x^{-h(\frac{1}{b}-1)} \left[\frac{a^{1-1/b}(1-b)}{h(1-b)-c} m x^{h(\frac{1}{b}-1)-\frac{c}{b}} + \beta \right]$$

where β is a constant of integration.

If we write $h = -1 + 1/b$

$$\alpha_0 = \frac{a^{-\frac{1}{b}}(1 - b)}{h(1 - b)-c}$$

$$\rho = h(-1 + 1/b)$$

we arrive at

$$z = x\ (\beta + m\ \alpha_0 x^{\rho-\frac{c}{b}})$$

The resulting production function is

$$V = \left[\beta K^{-\rho} + \alpha L^{-\rho} (K/L)^{-\rho_0} \right]^{-h/\rho} \quad \text{where } \rho_0 = c/b \quad \alpha = \alpha_0 m$$

The form resembles the CES function except for the multiplicative term in K/L associated with L. The resulting term $L_1 = L\ (K/L)^{\rho_0/\rho}$ may be regarded as a composite labour variable associated with capital intensity.

The Lu Fletcher production function has the same form as the Tsang Yeung function because the latter is derived from the Lu Fletcher productivity relation with the additional factor L which is absorbed in the existing L factor of the productivity relation, in the process of deriving the explicit production function

In this production function which is more of a firm model function, economic magnitudes like prices and elasticities have been explicitly included. It is homogeneous of degree h, has positive and diminishing marginal products for certain values of the parameters, variable elasticities of substitution and variable factor shares. It includes as special cases, the Cobb Douglas function

$$V = A \; K^{h\rho_0/\rho} \; L^{h - h\rho_0/\rho} \qquad \text{for} \quad \beta = 0, \; A = \alpha^{-h/\rho}$$

and the CES function

$$V = [\beta K^{-\rho} + \alpha L^{-\rho}]^{-h/\rho} \qquad \text{for } c = 0$$

Under conditions of perfect competition, α may be replaced by α_0 since, then m = 1.

It may not be quite useful to introduce more factors into the log linear productivity relation because the number of the available different factors is small and the effect of any additional factors may be contained in the existing factors. There is the risk of multicollinearity too. It was also noticed that the empirical results obtained from the use of successive productivity relations stopped showing any improvements when we reached the Tsang Yeung production function which added little to the results.

There does not seem to be much scope beyond the use of a few important technical and economic variables in the productivity relation.

Vazquez(1971) Linear Productivity Relation

We may now consider a variation in the Lu-Fletcher form by taking a linear productivity relation(without using logarithms) instead of a log linear productivity relation. If we assume a linear relationship between V/L, w and x under condition of perfect competition

$$V/L = a + b\,w + c\,x$$

we have $y = a + b(y - x\,dy/dx) + cx$

whose solution is given by $y = x(Ax^{-\frac{1}{b}} + c) + a/(1-b)$

where A is a constant of integration.

The explicit form of the production function is

$$V = A\,K^{1-\frac{1}{b}}L^{\frac{1}{b}} + \frac{a}{1-b}L + cK$$

The marginal productivities are given by

$$\frac{\partial V}{\partial L} = \frac{A}{b}(K/L)^{1-1/b} + \frac{a}{1-b}$$

$$\frac{\partial V}{\partial K} = A\,\frac{b-1}{b}(K/L)^{-\frac{1}{b}} + c$$

which are positive for any value of K/L with $a \leq 0$, $c \geqslant 0$.

If this production function is homogeneous of degree one, the elasticity of substitution depends on the input ratio

$$\sigma = \frac{y'(xy' - y)}{xyy''}$$

$$= \frac{(a + (b-1)y + cx)(y - a - cx)}{(b-1)y(y-cx) + ay}$$

The function reduces to the Cobb Douglas form if $a = 0 = c$. If only $c = 0$ we have Bruno's model

$$V = A\,K^{\frac{b-1}{b}}L^{\frac{1}{b}} - \frac{a}{b-1}L \quad \text{with} \quad \sigma = 1 - aL/V.$$

If only $a = 0$, we have

$$V = A\,K^{\frac{b-1}{b}}L^{\frac{1}{b}} + cK \quad \text{with} \quad \sigma = 1 - cK/(1-b)V.$$

If $b > 1$, $c > 0$, then σ is always greater than unity. As V/L increases, σ monotonically approaches unity from above.

If $c < 0$, σ is always less than unity and monotonically approaches unity from below as V/L increases.

The linear productivity relation given above is quite interesting because it can be handled without any transformation and it offers a straightforward generalisation of the Cobb Douglas form. In addition it offers a natural extension of the Bruno function which on its own may be considered rather asymmetrical between inputs. But it is not more general than the Lu-Fletcher productivity relation because it does not include the CES function as a particular case. So far as the fit is concerned we found there was nothing to choose between the two from the point of view of our data the linear as well as the log linear productivity relations both gave us similar fits. Between the two we prefer the Lu-Fletcher relation which therefore, we have used empirically in detail.

* * * *

This section on productivity relations may be looked upon as an extension of the sections on the Cobb Douglas as well as the CES functions. Because of its convenience, the use of productivity relations is a common feature of production function studies. We have considered above a few extensions of the CES productivity relations. Depending on the importance and the need for inclusion, more variables may be added to an existing relation. For instance, if it is known that $(\ln K/L)^2$ can play a significant explanatory explanatory role, the Lu Fletcher productivity relation may be extended to be in the form $\ln V/L = a + b \ln w + c \ln K/L + d(\ln K/L)^2$, though it is not likely that it may help much. However, a proper explicit form of production function cannot be derived from this relation

Extensions of other productivity relations may be obtained on similar lines but their value would lie in their economic validity.

VES Production Functions derived from Elasticity of Substitution Relation

Extensions of production function forms may be made by the use of the elasticity of substitution which is assumed to have a functional relationship with the input ratio or some other factor. This mode of derivation of production function forms is based on the argument that a constant or unitary elasticity of substitution is not a very realistic assumption.

The form of production function resulting from the relation between σ and the input ratio depends on the type of assumed relation. Moreover, there are two possibilities in the case of a two input function with K and L inputs. One may assume σ to depend on K/L or L/K. In either case, we do away with the tacit assumption of Cobb Douglas and CES functions which require some kind of fixed technical substitution between inputs.

While it is difficult to predict the behaviour of elasticity of substitution in practical cases it is certainly possible to derive some conclusions from observed results. Instead of assuming σ to be zero (Harrod Domar) or unity (Cobb Douglas) or infinity (straight line isoquant) or even a constant between zero and infinity (CES function), it may be useful to assume that σ varies with the input ratio. Several possibilities may be considered.

σ is small at low input ratios, rising as the ratio rises, reaches a maximum and decreases at a certain value of the input ratio.

Or, σ may be high at low input ratios, falling with a rise in the ratio.

Alternatively, σ is small at low input ratios and increases as the ratio rises. σ may be assumed to vary with any other factor or factors provided such a variation can be economically justified.

Unfortunately it is not easy to incorporate various qualities expected of the parameter σ in the same function. Attempts have been made in the literature to deal with simpler cases.

Empirically, if σ is found not to vary significantly with capital deepening the validity of the CES function follows. But the constancy of elasticity of substitution is not a characteristic of the real world. It is counterintuitive. This has led to a search for suitable functions with variable elasticity of substitution. As early as 1931, Hicks, in his Theory of Wages emphasised that elasticity of substitution increased with an increase in capital and that should result in the making and adopting of a labour-saving invention. A functional relation between elasticity of substitution and capital labour ratio was implied in Hicks', suggestion.

The concept of a variable elasticity of substitution (VES) production function if confined to a particular function is not quite viable inasmuch as any production function which is not a CES function is, by definition a VES function for which an infinity of possibilities can be discovered. Moreover a VES function, in the sense in which it is being understood, just allows for one's unwillingness to assume constancy along an isoquant. In that case, one should be equally unwilling to assume constancy along a ray, this is the approach of the homothetic production functions. But what is needed is an algebraic form which is sufficiently general, linear in parameters and convenient to estimate. The VES functions, derived with the help of the elasticity of substitution relation do not necessarily possess these qualities. However, depending on the assumptions made about the elasticity of substitution, some of these qualities may be introduced into the functions that are derived.

We will arrive at some known forms of production functions by giving different values to the elasticity of substitution. The VES functions will result by assuming that it is dependent on the capital labour ratio, K/L. ACMS(1961), in their study of the CES function, suggest such a dependence. Wise and Yeh(1965), in their inter-country study of wage and productivity differentials find that the elasticity of substitution increases to a certain point above unity as K/L increases and to less than unity as K/L decreases.

We will consider the forms developed by Sato(1965) and Revankar(1971). A more general form will also be given. The following formula will be found to be convenient to derive the forms referred to above.

$$\sigma = - \frac{y'(y - xy')}{xyy''} .$$

If it is assumed that $\sigma = 0$, we have $y/x = y'$. This leads to y=Ax, where A is a constant of integration. Since x=K/L, y=V/L, this relation may be written V = AK.

Another possible solution is $V = \beta L$ if the formula for elasticity of substitution uses $1/x = L/K$ instead of $x = K/L$. The complete solution is the linear production function:

$V = AK + \beta L$, with positive and constant marginal products. If the equations $V = AK$ and $V = \beta L$ are both true or only one has meaning, we have the Leontief fixed proportions case

$$V = \min (AK, \beta L).$$

If $\sigma = 1$, we have the differential equation

$$y'' + y'/x - y'^2/y = 0$$

whose solution is $y = A x^{\alpha}$, or $V/L = A (K/L)^{\alpha}$ which is the Cobb Douglas function.

The assumption σ = a constant and the resulting differential equation $y'' + y'/\sigma x - y'^2/\sigma y = 0$ lead to the CES function.

Sato(1965) derived the explicit forms of some VES production functions under the assumptions of perfect competition and constant technology. Let the elasticity of substitution be assumed to be a linear function of $x = K/L$,

i.e., $\sigma(x) = \dfrac{y'(y - xy')}{-xyy'} = a + bx$ $\sigma(x) > 0$, $x = K/L$, $y = V/L$

Substituting $u = y/y'$,

we have $\dfrac{1}{x(a+bx)} = \dfrac{1-u'}{x-u}$

or $\ln (u - x) = \dfrac{1}{a} \ln \dfrac{x}{a+bx} + c'$

$$u - x = c\left(\frac{x}{a+bx}\right)^{1/a}$$

$c > 0$ because $u - x$ = marginal rate of substitution > 0.

Writing A for an arbitrary constant

$$y = A \exp \int \frac{dx}{x + \left(\frac{c}{a+bx}\right)^{1/a}} \qquad A > 0$$

If an explicit solution is desired for the above expression then a simplification may be introduced by assuming a to be a rational number equal to n/m where n,m are positive integers. Giving suitable values to a, a variety of forms of production functions can be generated. Only some of these forms may be useful in practice.

We consider now some forms of VES functions. A generalisation is also suggested.

Revankar's VES production function, although a particular case of Sato(1965), was introduced independently and may be considered in its own right. Revankar(1969) makes the assumption that the elasticity of substitution is a function of the capital labour ratio

$$\sigma(x) = 1 + bx \qquad \text{where} \quad x = K/L.$$

This allows a test of the null hypothesis $b = 0$ to find if the function should be a Cobb Douglas function.

The explicit form of the production function is given by

$$V = \gamma K^{1-\delta\varsigma}\left[L + (\varsigma-1)K\right]^{\delta\varsigma}$$

where $b = \frac{\varsigma - 1}{1-\delta\varsigma}$. The parameters δ and ς are affected by the units of measurement so that it is always possible to secure the condition $0 < \delta < 1$ as a matter of convention.

To ensure that

$$\sigma = 1 + \frac{\varsigma - 1}{1-\delta\varsigma} K/L \geqslant 0$$

the restriction is $L/K \geqslant \frac{1-\varsigma}{1-\delta\varsigma}$

Revankar's function can be modified to be one with homogeneity of degree

$$V = \gamma K^{\nu(1-\delta\varsigma)}\left[L + (\varsigma-1)K\right]^{\nu\delta\varsigma}.$$

The CES function cannot be derived from Revankar's VES function. The latter is more general, however, in that, as against a constant elasticity of substitution independent of the level of output, at all points of an isoquant, it has the substitution parameter constant only along a ray while varying along an isoquant.

If we write $L_1 = L + (\varrho - 1) K$

the function becomes

$$V = \gamma K^{\nu(1-\delta\varrho)} L_1^{\nu\delta\varrho}$$

which means that the use of L instead of L_1 in the Cobb Douglas function involves specification error. With $\varrho \neq 1$, L_1 may be regarded as a composite labour input in the context of the Cobb Douglas function.

Alternatively, writing $V = \gamma A(K/L) K^{\nu(1-\delta\varrho)} L^{\nu\delta\varrho}$

where $A(K/L) = [1 + (\varrho - 1) K/L]^{\nu\delta\varrho}$,

we may compare it with

$$V = \gamma_1 K^{\nu(1-\delta\varrho)} L^{\nu\delta\varrho}.$$

Revankar's function satisfies the properties of a neoclassical production function

$$\partial V/\partial K = (1 - \delta\varrho)V/K + \nu\delta\varrho(\varrho - 1)V/[L+(\varrho-1)K] \geq 0$$

$$\partial V/\partial L = \nu\delta\varrho V/[L+(\varrho-1)K] \geq 0$$

$$1/s = 1/MRS = \frac{\varrho-1}{\delta\varrho} + \frac{1-\delta\varrho}{\delta\varrho}\frac{L}{K}, \quad \frac{\partial(1/s)}{\partial(K/L)} \geq 0$$

The factor shares are asymmetrical and nonconstant. They depend on the input ratio.

$$S_K = \frac{K}{V}\frac{\partial V}{\partial K} = (1-\delta\varrho) + \frac{\nu\delta\varrho}{1+\frac{1}{\varrho-1}\frac{L}{K}}$$

$$S_L = \frac{L}{V}\frac{\partial V}{\partial L} = \frac{\nu\delta\varrho}{1+(\varrho-1)\frac{K}{L}}$$

Revankar's VES function includes as special cases

Harrod Domar case: for $\varrho = 0$, $V = AK$

Cobb Douglas function: for $\varrho = 1$, $V = A K^{\nu(1-\delta)} L^{\nu\delta}$

The St.Line Isoquant fn.: for $\frac{1}{\varrho} > 1$, $V = \frac{A}{\delta\nu}[\delta L + (1-\delta)K]^{\nu}$

Sato and Hoffman(1968) using the definition of elasticity of substitution, derive a workable form of production function.

Since $\sigma = \dfrac{y'(y - xy')}{-x\,y\,y''}$

$$y = A \exp \int \frac{dx}{\beta e^{\frac{d \ln x}{x}} + x} = A \exp \int \psi(x)\,dx \text{ , say.}$$

To solve this it it necessary to get an explicit integration result for $\int \psi(x)\,dx$. If $S_L(x)$ stands for labour share, assume $\psi(x) = S_L(x)/x$.

Let $S_L(x)$ be a linear function of x :

$$S_L(x) = ax + x$$

then

$$\psi(x) = a + 1$$

$$y = A\, e^{ax}\, x^{b}$$

Thus , by making suitable assumptions about an expression involving K/L for σ , it is possible to generate a variety of forms of production functions. This form namely, $y = Ae^{ax}\, x^{b}$ is very easy to handle and is useful provided the assumptions associated with it are justified. The assumptions given above, viz., $\psi(x) = S_L(x)/x$ and $S_L(x) = ax + x$ have been made only to arrive at a simple form for the production function.

Sato and Hoffman fitted this VES function to U.S. and Japan time series data and found results which were more satisfactory than those obtained from the Cobb Douglas or CES function.

The dependence of σ on K/L has been justified on theoretical as well as empirical grounds. We have noted that Hicks suggested such a dependence as early as 1931 in his Theory of Wages. Although ACMS(1961) themselves didnot make use of such a connection they did suggest it.

As noted earlier(see p.78), no definite rule about the behaviour of σ with K/L has been noticed. On the same lines, we may expect the dependence of σ on L/K, with the behaviour of σ, with changes in L/K, not following any definite rule. Since a high K/L implies a low L/K, it is usual to think that a relation with one implies an inverse relation with the other. This may not be so in practice. It may be surmised that σ depends on both though not necessarily symmetrically and that the reaction of σ to changes in K/L need not obviate its reaction to changes in L/K. In other words, it is suggested that the capital intensity and labour intensity may not have any predictable inverse effects on elasticity of substitution. The substitution of capital for labour is a gradual process which has been going on for centuries. The substitution of labour for capital is relatively an uncommon phenomenon, it takes place sometimes under certain circumstances and in a manner, usually different from that of the substitution of capital for labour.

It may be useful to modify the assumption of σ depending on K/L alone. To avoid more complicated relations, let us assume a linear relation between σ and the two ratios K/L and L/K, in the form

$$\sigma = a + b\,K/L + c\,L/K$$

$$= a + b\,x + c/x \qquad \text{where } x = K/L$$

where a,b,c are unknown quantities and need to be determined.

From this relation it is possible to arrive at an explicit form of production function if we use the definition of elasticity of substitution

$$6 = -\frac{y'(y - xy')}{xyy'} = a + bx + c/x$$

which may be written

$$(c + ax + bx^2)(y''/y) - x(y'/y)^2 + y'/y = 0$$

or $$(c + ax + bx^2)(1 - \tfrac{d}{dx}\tfrac{1}{u}) - x + 1/u = 0 \quad \text{where } u = y'/y.$$

If we write $\phi = -\int \frac{dx}{c + ax + bx^2}$, we have

$$\frac{d}{dx}\frac{e^{\phi}}{u} = e^{\phi} + x\, e^{\phi}\, d\phi/dx$$

$$y'/y = [\, x + B\, e^{-\phi(x)}]^{1}$$

Hence $$y = A \exp \int \frac{dx}{x + Be^{-\phi(x)}}$$

where A, B are arbitrary constants. For an explicit solution we must have an explict solution for $\phi(x)$. We have

$$\phi(x) = -\frac{2}{\sqrt{4ab - c^2}} \tan \frac{2ax + c}{\sqrt{4ab - c^2}} \qquad \text{if } 4ab > c^2$$

$$= 2/(2ax + c) \qquad \text{if } 4ab = c^2$$

$$= -\frac{1}{\sqrt{c^2 - 4ab}} \ln \frac{2ax + c - \sqrt{c^2 - 4ab}}{2ax + c + \sqrt{c^2 - 4ab}} \qquad \text{if } 4ab < c^2$$

Concentrating on the last case with $4ab < c^2$ and substituting

$$a_0 = -\frac{c - \sqrt{c^2 - 4ab}}{2a}, \; b_0 = \frac{c + \sqrt{c^2 - 4ab}}{2a}, \quad c_0 = \frac{1}{\sqrt{c^2 - 4ab}}$$

we get $$-\phi(x) = \ln \left[\frac{x + a_0}{x + b_0} \right]^{c_0}$$

so that $$y = A \exp \int \left[x + B\left(\frac{x + a_0}{x + b_0}\right)^{c_0} \right]^{-1} dx$$

$$= A \exp \int \frac{(x + b_0)^{c_0}\, dx}{x(x + b_0)^{c_0} + B(x + a_0)^{c_0}}$$

$$= A \exp \int \frac{P(x)}{R(x)}\, dx, \quad \text{say,}$$

where $P(x)$, $R(x)$ are polynomials of a degree which will be assumed to be an integer as the non-integer case cannot be solved.

The solution may be written

$$y = A(x - \alpha_1)^{\beta_1}(x - \alpha_2)^{\beta_2} \ldots (x - \alpha_n)^{\beta_n}$$

where $\alpha_1, \alpha_2, \ldots, \alpha_n$ are the roots of R(x) and none of the roots is repeated so that $P(x)/R(x) = \sum_{i=1}^{n} \beta_i/(x-\alpha_i)$

If $\alpha_1, \alpha_2, \ldots, \alpha_n$ are repeated roots such that α_i is repeated m_i times, i = 1,2,...,n, then $R(x) = (x-\alpha_1)^{m_1} \ldots (x-\alpha_n)^{m_n}$ and $P(x)/R(x) = \sum_{i=1}^{n} \sum_{j=1}^{m_i} \beta_{ij}/(x-\alpha_i)^j$ so that

$$\int \frac{P(x)dx}{R} = \sum_{i=1}^{n} \sum_{j=2}^{m_i} \frac{\beta_{ij}}{(1-j)(x-\alpha_i)^{j-1}} + \sum_{i=1}^{n} \beta_{i1} \ln(x-\alpha_i) + \ln A$$

where ln A is a constant of integration. Writing $\beta_{i1} = \beta_i$, we have the explicit solution given by

$$y = A \left\{ \exp \sum_{i=1}^{n} \sum_{j=2}^{m_i} \frac{\beta_{ij}}{1-j} (x-\alpha_i)^{1-j} \right\} \prod_{i=1}^{n} (x-\alpha_i)^{\beta_i}.$$

If we write $\psi_i(x) = (x-\alpha_i)\, P(x)/R(x)$ and its t^{th} derivative is denoted by $\psi_i^{(t)}$, we can find

$$\beta_{im_i} = \frac{\psi_i^{(j-1)}(\alpha_i)}{(m_i - 1)!}$$

R(x) has repeated roots if R(x) and R'(x) have a common root. For instance, repeated roots are possible if $a_0 = b_0$, i.e. $(c - \sqrt{c^2 - 4ab})/2a = (c + \sqrt{c^2 - 4ab})/2a$ or $c^2 = 4ab$.

If each $m_i = 1$, the simple case of nonrepeated roots follows. Assuming nonrepeated roots and substituting x = K/L, we get the production relation

$$V = AL(K/L-\alpha_1)^{\beta_1}(K/L-\alpha_2)^{\beta_2}\ldots(K/L-\alpha_n)^{\beta_n}$$

The expression $V/L = A \prod_{i=1}^{n} (K/L - \alpha_i)^{\beta_i}$ is a polynomial in K/L of degree $\sum_{i=1}^{n} \beta_i$.

If all the roots are equal or if there is a single root the expression reduces to the form $V/L = A(K/L - \alpha)^{\beta}$ which is a polynomial of degree β in K/L. This is the simplest expression that may be arrived at for practical work unless $\alpha = 0$ in which case it is reduced to the Cobb Douglas form with constant returns to scale. In this form, α may be considered as a correction factor for K/L.

The production relation

$$V/L = A\ (K/L - \alpha)^{\beta} \qquad (A)$$

satisfies neoclassical requirements if $0 < \frac{\beta}{x-\alpha} < 1$, so that $\partial V/\partial L = (1 - \frac{\beta x}{x-\alpha})V/L > 0$; $\partial V/\partial K = \frac{\beta}{x-\alpha} V/K > 0$.

The MRS is given by $s = \frac{1-\beta}{\beta} x - \alpha/\beta$ so that the elasticity of substitution is a function of $1/x$ in this simple case:

$\sigma = 1 - \frac{\alpha}{1-\beta}\frac{1}{x}$ which implies $a=1, b=0, c= -\frac{\alpha}{1-\beta}$, and which reduces to $\sigma = 1$ if $\alpha = 0$.

The estimation of the relation (A) requires the use of nonlinear regression technique unless the value of α is known. It should be possible to estimate the relation more easily if α (or $\alpha_1, \alpha_2, \ldots, \alpha_n$ in the n-root case) can be found. This may be done by estimating a,b and c first.

One way of estimating a,b,c is by the method of ordinary least squares. Assuming that the production relation $y = y(x)$ is a polynomial in x, it is possible to obtain approximate numerical values of the derivatives y' and y'' with the help of data on x and y. If the n sets of values of x,y,y',y'' are substituted in the formula for σ, the corresponding n values of σ_i are obtained. From the relation

$$\sigma_j = a + bx_i + c/x_j \qquad i = 1,2,\ldots,n$$

a,b,c may be estimated by ordinary least squares.

The production relation developed in this section is given as an interesting extension possibility. There may be a lot of computational work here.

* * * *

Inspit of the inadequacy of the VES functions the elasticity of substitution parameter received ample attention. It resulted in some interesting forms of production functions. The literature does not have many cases of estimation of the entire parameter set of any important functional forms. ACMS(1961) did it in their seminal paper but it was done in very few empirical studies thereafter. Perhaps this was because of the difficulties associated with their estimation and also because the production function phenomenon seems to have evolved in a manner that does not allow one to go far. Everything seems to revolve around a few chosen concepts and so much labour has gone into the study of the basis of these concepts that it has become difficult to give them up in spite of many failures. The parameters themselves happen to have been defined in such a way that many hypotheses can be tested even without estimating some of the parameters.

The CES vs VES Approximations

The Second order Approximations of the CES & VES Functions

Both the CES and VES functions are nonlinear in parameters. Nonlinear estimation methods used to estimate the parameters are uncertain and difficult to handle so that some indirect methods based on strong assumptions or approximations have to be used though the resulting estimates are not necessarily reliable. For instance, The CES function is sometimes replaced by Kmenta approximation.

As shown by Maddala and Kadane(1967), by the use of Monte Carlo methods, this procedure does not result in reliable estimates of the elasticity of substitution although it gives reliable estimates of the returns to scale parameter. Again, the use of the popular productivity-wage relationship, instead of the proper CES function, is based on some strong assumptions.

The estimation of the parameters of the VES function involves us in difficulties similar to those of the CES function. Although the VES function is expected to be superior to the CES in some respects, Corbo(1974) has shown that the second order approximations of both the CES and VES functions have the same form which, when considered as an approximation of the VES function is, in general, superior to the approximation of the CES function. According to Corbo, it is not proper to use the Kmenta approximation to "make inferences with respect to parameters of a CES function without strong independent evidence that the true production model is indeed a CES function." Kmenta(1967) also observes that if a function f_1 is an approximation to f_2, what is relevant is how well f_1 approximates f_2 within some range of practical importance.

Allowing for nonconstant returns to scale, the CES function is given by

$$V = \gamma[\delta K^{-\rho} + (1-\delta)L^{-\rho}]^{-\nu/\rho} \qquad 0 \leq \delta \leq 1;\ \rho > -1;\ \gamma, \nu > 0$$

The Bruno form of the VES function may be written

$$V = \gamma[\delta K^{-\rho} + (1-\delta)L^{-\rho(1-m)}K^{-m\rho}]^{-\nu/\rho} \qquad \nu, \gamma > 0,\ x = \frac{K}{L} > 0$$
$$0 < \delta < 1,\ \rho > -1$$
$$\delta + (1-\delta)x^{\rho(1-m)} > 0$$

The Taylor series expansion up to the second order term of each around $\rho = 0$ is given by

$$\text{CES} \quad \ln V = \ln \gamma + \nu\delta \ln K + \nu(1-\delta) \ln L - \frac{\nu\rho}{2}\delta(1-\delta)(\ln K - \ln L)^2$$

$$\text{VES} \quad \ln V = \ln \gamma + \nu[\delta + m(1-\delta)] \ln K - \nu(1-\delta)(m-1) \ln L - \frac{1}{2}\nu\rho\delta(1-\delta)(m-1)^2 (\ln K - \ln L)^2$$

As the two expressions have the same common form, the CES expression cannot be used to estimate the coefficients of a CES function without a priori information that the CES is indeed the true model. If this common form is

$$\ln y = \ln V/L = \ln \gamma + \gamma_1 \ln x + \gamma_2 (\ln x)^2 , \qquad x = K/L$$

there is the problem of multicollinearity if x, the capital labour ratio is limited in range. A high correlation between $\ln x$ and $(\ln x)^2$ may imply their varying inversely with each other and large standard errors of coefficients.

All VES functions do not necessarily have the same resultant approximation as that for the CES function. The Bruno VES form approximates to the Kmenta form, as noted above because the CES function is a special case of Bruno function and is nested within it. It may not be nested within some other VES functions. See Harvey(1977). A Taylor series expansion around $\rho = 1$ of Revankar VES function

$$V = \gamma K^{\nu(1-\delta\rho)} [L + (\rho - 1) K]^{\nu\delta\rho}$$

results in $\ln y = \ln \gamma + (\nu-1)\ln L + \nu(1-\delta\rho)\ln x + \nu\delta\rho(\rho-1)x$ which is different from the Kmenta approximation in that it has a term in x instead of $(\ln x)^2$ and enables an indirect estimation of all the parameters. As in the case of Kmenta approximation, the significance of the coefficient of x allows a test of the hypothesis that the true function is of the Cobb Douglas form.

Returns to Scale varying with Output. Homothetic Production Functions

The rate of returns to scale may or may not vary with the output level. When it varies inversly with the output, the optimum level of production can be determined. When it is constant which happens when the production function is homogeneous of degree one, the optimum production level does not exist.

If the rate of returns to scale varies directly with the output level the optimum production level may still be found for the whole data. In this case it would be some kind of average of the point returns to scale over different output levels and the production function is no more homogeneous. If, in such a case, a homogeneous production function is assumed, the nature of error will depend on the extent to which point returns to scale vary with output.

Homothetic production functions, introduced by Shephard (1953) provide a useful framework of production processes

As applied to the two input case, homotheticity implies the constancy and uniqueness of the slopes of the isoquants along a ray from the origin. The elasticity of substitution is constant along a ray but not necessarily so along an isoquant. No assumption about the elasticity of substitution needs to be made if the homotheticity property is to measure varying returns to scale of a production process.

Homotheticity is a geometric property. For the entire class of functions related to an underlying function, it is useful to study production behaviour based on the preservation of the homotheticity of the isoquant map.

Thus we can separate the variations in the output level from the elasticity of substitution characteristics of underlying production functions. Homogeneous functions are a subset of the homothetic functions.

It is logical to use homothetic functions which can offer both nonvariability in returns to scale as well as elasticity of substitution. Often it is difficult to justify the use of a function like the CES which allows nonvariable elasticity of substitution and constant returns to scale even though the U-shaped long-run average cost curves necessarily imply variable returns to scale.

A production function

$$V = F(X_1, X_2, \ldots, X_n) \quad V \geqslant 0$$

for which output increases steadily as one moves from the origin to the input space, is homothetic if every isoquant is a radial blow up of every other isoquant. All isoquants are related by transformations homothetic to the origin. Homotheticity is a property of the isoquants and is not affected by the output labels attached to them. No ray from the origin can intersect any isoquant more than once.

If F is homothetic so are g (F) and h (g)=h$\{g(F)\}$ where g and h are order preserving transformations. Here it may be noted that g (F) is homothetic if and only if F is homogeneous.

Again if F is homogeneous of degree ν, then

$$g\{F(\lambda x)\} = g\{\lambda^{v} F(x)\}$$

That is, in the homothetic production function, the marginal rate of substitution is independent of λ, the scale of production. This means that the elasticity of substitution is constant along a ray from the origin but may not be constant along an isoquant, or, that, along a ray from the origin, the slopes of the isoquants are unique and equal. The unique relation $s = \Psi(K/L)$ between the slopes of the isoquants and the slope of the ray intersecting it (which is the input ratio), is a differential equation of the first order which leads to the homothetic function

$$V = g(F(K, L))$$

Compared to the homogeneous functions, homothetic functions have the advantage that they reflect variable returns to scale varying with output in a production process. Even technical progress occuring in various forms can be tested with the help of homothetic functions. Homotheticity implies an even distribution of returns to scale among all the inputs.

It can be easily seen that any homogeneous function is homothetic . This follows if we prove s is a function of K/L only.

A homogeneous production function of degree h may be written

$$V = F(K, L)$$

$$= L^{h} f(K/L)$$

Since $F_K = L^{h-1} f'$ and $F_L = L^{h-1}(hf - (K/L)f')$,

we have the marginal rate of technical substitution

$$s = F_L / F_K = h f/f' - K/L$$

$$= \Psi(K/L)$$

i.e. the MRTS is a function of K/L only. For a homothetic function the MRTS is constant along a ray from the origin.

Clemhout(1968), Mayer(1970) and Zellner-Ravankar (1969) have developed some forms of homothetic production functions.

Zellner and Ravankar(1969) show that any neoclassical production function with an arbitrary degree of homogeneity and constant or variable returns to scale can be transformed into another neoclassical generalised production function (GPF) with the same elasticity of substitution and with the returns to scale variable and satisfying a preassigned relationship to the level of output. Clemhout's(1968) homothetic isoquant production function(HIPF) assumes homotheticity rather than the more restrictive homogeneity of the production function. The idea in each case and particularly in the case of Zellner and Ravankar functions is that certain features of the neoclassical production functions are retained and used to form a new production function with varying returns to scale. The new functions, GPF, exhibit a wide variety of the behaviour of returns to scale and can be used to reconcile the factor share controversy.

The GPF cannot be used at the micro level, because it does not obey the law of variable proportions, with the consequence that the inherent micro production functions remain unknown. It may still be used on data with a lower degree of aggregation. Purely micro data are difficult to obtain in practice.

Generalised Production Function

Zellner and Ravankar's (1968) generalised production function (G P F) results from the transformation of a neo-classical production function with constant or variable elasticity of substitution into a neoclassical G P F with the same elasticity of substitution and variable returns to scale satisfying a preassigned relationship to the level of output. Let the neoclassical production function in two inputs , F (K, L) be of an arbitrary degree of homogeneity Let

$$V = g(F) \quad , \; g(0) = 0 \; , \quad dg/df > 0 \quad \text{for } F \geq 0$$

so that $\partial V / \partial L = (dg/dF)(\partial F/\partial L) > 0$

$$\partial V / \partial K = (dg/dF)(\partial F/\partial K) > 0$$

g (F) is a continuous , positive , monotonic increasing function and has the same elasticity of substitution as F . It was called a homothetic production function by Shephard (1953) and its potential as a variable returns to scale function was revealed by Zellner and Ravankar.

V = g (F) satisfies the neoclassical conditions for a production function.

The MRS for this function is given by

$$s = \frac{\partial g/\partial L}{\partial g/\partial K} = \frac{\frac{dg}{dF}\frac{\partial F}{\partial L}}{\frac{dg}{dF}\frac{\partial F}{\partial K}} = \frac{\partial F/\partial L}{\partial F/\partial K}$$

so that its elasticity of substitution is

$$\sigma = \frac{s}{x}\frac{dx}{ds}$$

which is the same as that for F(K,L), the underlying function since s and x are the same for both.

For a preassigned returns to scale function $\epsilon(V)$, we have, by Euler's theorem

$$V\epsilon(V) = L\frac{\partial g}{\partial L} + K\frac{\partial g}{\partial K}$$
$$= \frac{dg}{dF}\left(L\frac{\partial F}{\partial L} + K\frac{\partial F}{\partial K}\right) = \frac{dV}{dF}F\epsilon_F$$

where F is assumed to be homogeneous of degree ϵ_F .

$$\therefore\ (dV/dF)(F/V) = \epsilon(V)$$

whose solution gives the production function V = g(F).

For the GPF, factor shares depend on the output, the input ratio and the elasticity of substitution.

If perfect competition prevails

$$w = \frac{\partial V}{\partial L} = \frac{\partial g}{\partial F}\frac{\partial F}{\partial L}$$

The labour share of V = g(F) is given by

$$S_L = \frac{L}{V}\frac{\partial V}{\partial L} = \frac{F}{V}\frac{dg}{dV}\quad\frac{L}{F}\frac{\partial F}{\partial L}$$
$$= \frac{\epsilon(V)}{V}S_l$$

where S_l is the labour share of F(K,L).

For example, if $F = \gamma\left[\delta K^{-\rho} + (1-\delta)L^{-\rho}\right]^{-\nu/\rho}$ $\qquad \sigma = \frac{1}{1+\rho}$

$$S_L = (1 - V/c)\ \frac{\nu(1-\delta)^{\sigma}\gamma^{\sigma-1}}{w^{\sigma-1}}$$

Thus S_L depends on V, K/L, σ and other factors.

For $\sigma > 1$, S_L falls as V and K/L inrease.

For $0 < \sigma < 1$, S_L rises to a maximum and then falls as V and K/L increase.

The GPF embodies variable returns to scale and allows for a varied behaviour of factor shares.

Zellner and Revankar(1969) apply to crosssection data the GPF resulting from

$$\epsilon(V) = \frac{\alpha}{1+\theta V} = \frac{\alpha'}{1+\theta'(V-b')}$$

where $\alpha = \alpha' h$, $h = (1-b'\theta')^{-1}$; $\theta = \theta' h$; $\alpha', b' > 0$, $b'\theta' < 1$.

If $\theta > 0$, $\epsilon(V)$ falls from α (at V=0) to zero(as $V \to \infty$)

If $\theta < 0$, $\epsilon(V)$ rises from α (at V=0) to α'(at V=b) to ∞ (as $V \to -\frac{1}{\theta}$)

If $\theta = 0$, $\epsilon(V) = \alpha$, a constant.

If we solve the equation

$$\frac{dV}{V} = \frac{dF}{F} \frac{h}{1 + \theta V}$$

we get $Ve^{\theta V} = c^h F^h$ where c^h is a constant of integration. If F is of the Cobb Douglas form: $F = A' K^{\alpha(1-\delta)} L^{\delta\alpha'}$, with returns to scale α' , we have

$$V e^{\theta V} = A K^{\alpha(1-\delta)} L^{\alpha\delta} \quad 0 < \delta < 1 ; \ A, \alpha > 0$$

or $\ln V + \theta V = \ln A + \alpha(1-\delta)\ln K + \alpha\delta \ln L = u_i$

where the error terms u_i are normally and independently distributed with zero mean and constant variance and ln K, ln L are independent of u .

The parameters may be estimated by the maximum likelihood method. Zellner and Revankar found returns to scale varying with output for 1957 state data for U.S. transportation equipment industry.

Nerlove(1963) suggested the form

$$\ln V + \theta V = \alpha \ln K/L + \beta \ln L$$

with the scale elasticity given by $\epsilon(V) = \frac{\alpha}{1+\theta V}$

Ringsted's(1971) preassigned returns to scale $\epsilon(V) = \epsilon \ln \theta/V, \ \theta > V$, along with an underlying Cobb Douglas function, $F = A K^{\alpha} L^{\beta}$, leads to

$$V = \theta \exp\{-A' K^{-\alpha'} L^{-\beta'}\} \quad \text{where } \alpha' = \alpha h, \ \beta' = \beta h$$

$$\alpha' + \beta' = h(\alpha + \beta) = h \epsilon_F$$

An infinite numb r of algebraic forms can be used to estimate a production function. Clemhout(1968) is not satisfied with the highly restrictive economic assumptions made in the available forms and suggests a homothetic isoquant production function(HIPF) which allows variable returns to scale. The underlying function is homogeneous but the transformation allows any form for the production function.

The function makes use of the property of homotheticity that the slope of an isoquant along a given ray from the origin is a function of the slope of that ray. Thus any economies or diseconomies that arise are of the neutral type.

Let F(K,L) be a neoclassical production function homogeneous of degree of one. The general form of the homothetic function may be written $V = g(F)$

The slope of the isoquant gives the definition of homotheticity : $-dK/dL = \psi(K/L)$

Writing $L\,f(K/L) = F(K,L)$, we have with $x = K/L$,

$$\psi(x) = -\frac{dK}{dL} = -\frac{\partial f/\partial L}{\partial f/\partial K}$$

$$= \frac{f(x) - x\,f'(x)}{f'(x)}$$

We have $f'/f = \phi(x) = \frac{1}{x - \psi(x)}$

$$\therefore f = e^{\int \phi(x)\,dx}$$

or $$F(K,L) \simeq L\,e^{\Sigma a_i x^i}$$

where the integral has been approximated by a polynomial to find a series for F.

$\phi(x)$ is not known but it may be found if some assumption about factor payments is made. Under constant returns to scale, marginal products equal factor prices. Then we have

$$\psi(x) = \frac{1}{x \pm w/r} = \frac{L}{K}\;\frac{1}{1 + wL/rK}$$

Meyer(1970) took an approximate value of $x = 2.9$ (nearly equal to the capital ratio of his series which was 3.07), approximated $\psi(x)$ by a Taylor's series and by adding successively higher powers of -2.9 by stepwise least squares arrived at a quadratic equation, the higher powers having been found to be insignificant. Integration of this quadratic gave $\int \phi(x)\,dx$ and hence F.

Out of the family of curves of various degrees for the polynomial $\sum a_i x^i$ that giving the best fit i.e. the one with the highest value of correlation between x and $\phi(x)$ series is chosen. We get the F series from which the HIPF is obtained.

The elasticity of substitution of the HIPF is free to vary for different K/L values.

$$\sigma = \frac{\psi(x)}{x\psi'(x)} = \frac{x\phi^2 - \phi}{x\phi^2 + x\phi'} \quad \text{where } \phi(x) = \frac{1}{x - \psi(x)}$$

Write the returns to scale function

$$\lambda = \frac{dV}{dF}\,\frac{F}{V} \qquad \text{and th transformation } V = F^{\lambda}.$$

Since V is observed and F can be calculated, an estimate of λ can be obtained. The final form of the production function is $V = A\, e^{\lambda t}\, F^{\lambda}$ which happens to be a homogeneous function. For regression purposes this form was used by Clemhout to determine a measure of returns to scale for data from private nonfarm domestic economy, U.S. 1929-1953, without any explicit formulation for returns to scale. Three different measures for capacity capital stock were used. The resulting coefficients were not very sensitive to variations in measurement.

Wolkowitz(1971) derives homothetic isoquant production functions from economic relationships drawn directly from the production process. The production process is not specified a priori and is therefore empirically a more useful concept.

If η_L and η_K are the supply elasticities of labour and capital respectively, we have, under equilibrium conditions, the marginal rate of substitution given by

$$s = w/r = \frac{MP_L(1+\eta_L)}{MP_K(1+\eta_K)} = \frac{MP_L}{MP_K} \quad \text{if } \eta_L = \eta_K$$

Since $s = \Psi(K/L) = -dK/dL$, we have, for $K/L = x$, the Clemhout result, $F(K,L) = L\, e^{\int \phi(x)dx}$ where $\phi(x) = \frac{1}{x+\Psi(x)}$ without an explicit solution for F. The numerical series is not easy to handle and depends on an approximation for small values of x.

Wolkowitz tries to satisfy both economic and mathematical criteria by selecting $\Psi(x)$ in such a way as to have $\phi(x)$ in an integrable form. Although it amounts to a constraint, an explicit solution for F can be guaranteed and some testable hypotheses may be established. Wolkowitz gives several explicit forms.

CRESH Production Function

Uzawa's n-input CES generalisation shows that for constant Allen elasticities further generalisation is not possible. McFadden, using alternative definitions of elasticities, proved that constancy of elasticities implies even more stringent restrictions on the production function. Sato's two level CES function, using a CES function among composite goods, each of which was a CES combination of several factors, gave a form which was difficult to estimate. Mukerji's constant ratios of elasticities of substitution (CRES) function is not homogeneous or homothetic so that individual elasticities of substitution vary with output as well as input combinations.

Gorman (1965) pointed out that in the Mukerji function, if δ_i and ρ_i are functions of output V, rather than constants, the CRES generalisation could still result. According to Hanoch (1971), this would alter the pattern of change of individual elasticities of substitution with output as well as the curvature of the expansion lines without removing the restrictions on combinations of elasticities while preserving the CRES property, but these elasticities would not be negative when the parameters vary with output. Hanoch has given a number of additional interesting properties of the Mukerji function. He points out that the Mukerji function does not necessarily lead to nonhomogeneous or nonhomothetic production function. His CRESH production functions are homothetic or homogeneous in addition to having the CRES property. For empirical work, he gives various cases including that for crosssectional studies of individual firms in competitive factor markets.

The CRESH function is useful in studying patterns of substitution or complementarity among three or more factors like different kinds of skills or forms of capital, where the CES model fails because it assumes away these differences.

Several well known forms are special cases of the CRESH function. An analysis from the empirical point of view, of the estimation of Hanoch's CRESH as well as HCDE(homothetic constant differences of elasticities of substitution) functions may be found in Hanoch(1971) and Weiss(1977).

Some other general forms

Vazquez(1971) assumes a functional relationship between the ratios of output elasticities (or relative shares) and K/L under perfect competition and homogeneity conditions :

$$S_L/S_K = a + b\,x^c \qquad \text{where } x = K/L$$

From this is derived a new and more general family of production functions with interesting properties.

Since $S_L = \dfrac{\nu f - x\,f'}{f}$, $S_k = \dfrac{x\,f'}{f}$

we have $\dfrac{dy}{y} = \dfrac{df}{f} = \dfrac{\nu\,d\,x}{x(1+a+b\,x^c)}$

$$y = A\left(\frac{x^c}{1+a+b\,x^c}\right)^{\frac{\nu}{c(1+a)}}$$

Where A is a constant of integration

Since $y = A\left[(1+a)\,x^{-c} + b\right]^{-\frac{\nu}{c(1+a)}}$

the production function may be written

$$V = A\left[\,b\,L^{-c(1+a)} + (1+a)\,(L/K)^{-ca}\,K^{-c(1+a)}\right]^{-\frac{\nu}{c(1+a)}}$$

The r turns to scale are not constant

a=0, c=-1 gives straight line isoquants,

a=0, b=0 gives right angled isoquants,

b=0 gives the Cobb Douglas function

a=0 gives the CES function

c=-1 gives Ravankar IVES function

$$V = \gamma L^{\alpha(1-\delta\rho)}\left[K + (\rho-1)\,L\right]^{\alpha\delta\rho}$$

for $\alpha=\nu$, $\delta = \dfrac{1}{1+a+b}$, $\rho = \dfrac{1+a+b}{a+1}$, $\gamma = A(1+a)^{\frac{\nu}{1+a}}$

If a, b are positive, the marginal productivities are positive:

$$\partial V/\partial K = (\nu V/K)(1+a+b\,x^c)$$

$$\partial V/\partial L = (\nu V/L)(a+b\,x^c)\,(1+a+b\,x^c)^{-1}$$

$$s = MRS = \frac{1}{x(a+b\ x^{c})}\ , \quad \frac{ds}{dx} = -\ \frac{a+b(1+c)x^{c}}{x^{2}(a+b\ x^{c})^{2}}$$

If $\frac{ds}{dx}$ is negative, the isoquants will have the correct convexity for all x if $c \geqslant -1$

For $c < -1$ and $x^{*} = \left| \frac{a}{b(1+c)} \right|^{1/c}$

the isoquants have a point of inflexion.

For $x < x^{*}$, the isoquants are well behaved.

For $x > x^{*}$, they are concave from below.

Since $$\frac{\partial^{2} V}{\partial K^{2}} = -\frac{\nu V}{K^{2}}\ \frac{1-\nu+a+b(1+c)x^{c}}{(1+a+b\ x^{c})^{2}}$$

$$\frac{\partial^{2} V}{\partial L^{2}} = -\ \frac{\nu V}{L}\ \frac{(1-\nu)\ (a+b\ x^{c})^{2}+a+b\ (1+c)x^{c}}{(1+a+b\ x^{c})^{2}}$$

This means that the marginal product curves are monotonically decreasing or increasing for suitable values of c, ν.

The elasticity of substitution is a variable quantity.

$$\sigma = \frac{a + b\ x^{c}}{a+b(1+c)x^{c}} \qquad \begin{array}{ll} 0 < \sigma \leqslant 1 & \text{for } c \geqslant 0 \\ \sigma > 1 & \text{for } -1 \leqslant c \leqslant 0 \\ \sigma > 1 & \text{for } c < -1 \text{ over } x \quad x^{*} \end{array}$$

When a,b have the same sign, the possibility of factor substitution diminishes with increasing amounts of capital per unit of labour.

<u>Sato and Beckman(1968)</u>

According to Sato and Beckman, it is possible to carry out multivariate analysis among subsets of variables if the two input restriction is dropped. Also instead of a prior specification of the production function form it is prefereable to test production relations empirically and then generate final forms. This is done by specifying a log linear relationship between all the commonly used variables in production function analysis and then proceeding to obtain alternative specifcations of the different forms.

The seven variables used by Beckman and Sato are

$y = V/L$	output labour ratio
$x = K/L$	capital labour ratio
$y/x = V/K$	output capital ratio
$w = y'$	wage rate
$r = y - xy'$	rate of return on capital
$s = (y-xy')/y'$	marginal rate of substitution
$S_L = xy'/y$	labour share
$\sigma = \frac{y'(y-xy')}{-xyy''}$	elasticity of substitution

The most general relationship may be written

$$\{(y-xy')/y\}^a (xy'/y)^b (y-xy')^q y'^d (y/x)^g y^h x^m = c \quad 0$$

where c is a constant. The authors fitted subsets of variables in stepwise regression analysis and came to the conclusion that only four forms deserved consideration

$\ln y = \text{const.} + \alpha \ln x$	Cobb Douglas
$\ln y = \text{const.} + b_1 \ln w$	CES
$\ln y = \text{const.} + b_2 \ln w + c_2 \ln x + d_2 \ln r$	Cobb Douglas-CES $b=0$ if $d \neq 0$ $d=0$ if $b \neq 0$
$\ln r = \text{const.} + b_3 \ln w + c_3 \ln x + d_3 \ln y$	Parametric form

The last one gave the best results free from multicollinearity which was suspected to be present in the Cobb Douglas-CES case though the latter gave good results also. The CES case yielded good results but suffered from multicollinearity effects. The Cobb Douglas form gave mostly good results.

* * *

An attempt has been made in this survey to bring together some of the production relations found scattered all over the literature on the subject. The survey is not complete and many more forms and ideas related to the subject have to be left out as we have already gone beyond the scope of our study. The empirical analysis in this study will be mainly concerned with the Cobb Douglas and the CES functions and also with some of their extensions. The inclusion of homothetic functions in this survey is only an attempt to make it rather less incomplete.

The procedure followed in developing the different forms makes use of the links of technical and economic variables which will be considered in the next chapter. The empirical work in this study depends very much on a distinction between these two types of variables. The method used to obtain extensions of production function forms seems to bring out the meaning of each form more clearly. It should be possible to arrive at a number of forms as special cases of some more general form or by making suitable assumptions about the parameters [1]

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1 For instance, (using the notation in the text), in the case of homogeneous production functions, the relationship between σ, s and ϵ , the output elasticity of labour, may be utilised to develop any suitable form of a homogeneous production function. Since, with returns to scale ν , we have $\epsilon = \nu - xf'/f$, it follows that $f = \exp \int \frac{\nu - \epsilon}{x} dx$ or, $V = L^{\nu} \exp \int \frac{\nu - LV_L/V}{x} dx$, where V_L is the marginal product of labour, given by $V_L = (\nu - xf'/f)V/L = \epsilon V/L$.

Similarly, from the relation MRS = s = $\nu f/f' - x$, we get $V = L^{\nu} \exp\int \frac{\nu}{s+x} dx$. From $\sigma = (dx/ds)(s/x)$ or $s = \exp\int \frac{1}{\sigma x} dx$ we also get $V = L^{\nu} \exp\int \frac{\nu \, dx}{x + e^{\int dx/x\sigma}}$ Here σ, s and ϵ are assumed to be functions of x.

Footnote Contd.

There are several other forms of production relations to be found in the literature. Sato's(1977) nonhomothetic functions provide a more general and more meaningful class of CTS functions. Hanoch's(1971) constant difference of elasticities of substitution(CDE) mentioned earlier is a multifactor production function which makes use of input variables and their prices. The duality between cost and production has now long been use in the literature, see for instance, Walters(1963). The theory of production function now allows a formal representation of almost any underlying production relationship even though it may not be easy to determine how these relationships determine the specific form of the production function.

If $\epsilon = \nu(1-\alpha)$, $V = L^{\nu} \exp \int \frac{\nu - \epsilon}{x} dx = AK^{\alpha\nu} L^{\nu - \alpha\nu}$, we have the Cobb Douglas function.

If $\epsilon = \nu(1-\xi)/(1+\theta x-x)$, we get the Revankar VLS function
$V = A K^{\nu\xi} L^{\nu-\nu\xi} (1+\theta x-x)^{\nu-\nu\xi}$

If we take, instead, $\epsilon = \frac{\nu(1-\delta)}{1-\delta+\delta x^{\rho}}$, we get the CES function
$V = A [\delta K^{-\rho} + (1-\delta) L^{-\rho}]^{-\nu/\rho}$

If $\epsilon = \nu\omega(1-\delta)/(1-\delta+\delta x^{-\omega\rho})$, we get the Hildebrand-Liu form of the VES function
$V = A[\delta K^{-\rho} + (1-\delta) L^{\rho} x^{(\omega-1)\rho}]^{-\nu/\rho}$.

CHAPTER THREE

THE EMPIRICAL FRAMEWORK AND THE HYPOTHESES

Introduction

On the basis of a few selected production models an econometric study of the manufacturing establishments of France, India, Israel, Japan and Yugoslavia is to be carried out. Perhaps a straightforward production function analysis of the observations from these five countries should fulfil the main aim of this study; this has been done in good detail with the help of fifteen selected forms of production functions and the results compared. But a reappraisal of the data on manufacturing establishments shows that there is ample scope for a study of the uniformity of the manufacturing sectors of the countries concerned. This is because the method of collecting the data, the definitions of the variables used and the period of reference are almost identical in all the countries.

With the help of different production models we intend to analyse the empirical evidence on the technical aspects of industrial production overshadowed by the economic aspects, on an internationally comparable basis. The adoption of the industrial system is at the root of modern economic development which is the common aspiration of all countries.

A proper comparison of the production performance of different countries would require , for a consideration of the changes taking place through industrialisation, the examination of the movements of gross domestic product of each country, the nature and quantum of its natural resources and its international trade, the relationship between its input cost and input use and the nature of its production function. If we compare any country with others on the basis of one or more of these considerations, we must, at the same time, ascertain that the remaining unconsidered factors do not vary significantly if a proper analysis is to be carried out. Unfortunately, it is not possible to do so because there are not many countries which show much resemblance in these matters.

The production function analysis in the following chapters cannot eliminate the differences which are too obvious to be concealed by any means. There are differences in the rates of capacity utilisation, levels of economic and technological development, availability and extent of resources and their utilisation.

Yet we must proceed to discover if the production functions have something comparable to offer, even though a proper comparison would require a consideration of all the factors mentioned above and perhaps many more.

In any production function analysis some assumptions are almost inevitable. It would be easy for instance, to make the assumption of smooth input substitutability, perfect competition in product and factor markets and a profit maximising firm behaviour. But it is not always necessary to make these assumptions.

The validity of the results depends on a proper statistical interpretation based on a correct approach in terms of economic theory. It is possible that the conclusions drawn from our results in the case of the five countries under study may not hold in the case of other countries.

But the contrary is more likely since our sample of countries is sufficiently representative and any conclusions commonly applicable to all the five countries may be expected to be more or less general. The validity of the results depends further on the assumption that the selected establishments constitute the main body of, or represent adequately, the industrial sector of, the country concerned. We make this assumption and the empirical results that will follow justify this.

The five countries are characterised by a number of individual features which are known to differ significantly from one another. In the midst of dissimilarities based on history, culture, technology, resource endowments, socio-economic constraints and environment it may seem improper to look for any similarity. Except for the characteristics of self interest and existence common to all nations, the one possibly common factor among these countries is gaining political independence or emergence or a rebirth following the second world war.

For France it was a fresh political start under stable conditions. The post liberation government in 1946 made a firm decision to guide the economy through proper planning.

India gained Independence in 1947 after a long period of colonisation and passed through some years of trial and error. In 1951, India resorted to planning with increasing emphasis on modern industry.

Israel was established as a state in 1948 and sucessfully scraped through major economic and political upheavals.

Japan's total defeat in 1945 left the economy in a state of disorder associated with large scale capital destruction. This was followed by rehabilitation and reconstruction at a high rate of recovery and growth.

The recreation of Yugoslavia began after the liberation in 1945 and the country proceeded towards the formation of a socialist society. The worker managed enterprise system passed through several phases leading to a continuous development of the economy.

We have thus five countries making some sort of beginning at about the same period of time. Had they all been identical in technology, resource endowments and other features in an intial period, the differences in these characteristics after a certain period of time would allow an interesting trend study. These five economies began not only with differences in environment, in technology and resource allocation but also with major differences in political, economic and social structures. There were differences in 1964-66, our period of reference, and even today, these countries are at various stages of industrialisation, following different political ideologies, possessing varying degrees of marginal competence and subject to a variety of socio-economic constraints. The quantity, quality and productivity of factors differ as also the strength of their currencies.

We can say that these economies have passed through significantly different patterns of industrialisation and could be considered as belonging to different categories.

To see if each of these happens to be typical of some category based on some accepted classification, we may consider Black's(1966) subdivision obtained by linking political factors to the development of industrialisation in different countries.

Black(1966) gives seven patterns which cover the world's 148 organised societies. The first pattern covers Britain and France. The second pattern consists of the United States, Canada and such other countries as have European settlers. The third is made of most West and East European countries which consolidated their leadership after the French revolution. The fourth pattern consists of Latin American countries. Countries like Russia, Japan and Turkey belong to the fifth pattern; these are such countries as were influenced by the patterns mentioned earlier but began to modernise internally and were not under colonial rule because of their military strength, inaccessibility or otherwise. The sixth pattern is formed by the countries which were once colonial societies like India, Egypt or Malaysia. Also belonging to this pattern are Israel and some countries which did not exist before. The seventh pattern comprises countries of the sub-Saharan Africa which were required to undergo the whole of the modernising sequence after their independence.

Of the countries whose data we are going to deal with, France belongs to the first pattern, Yugoslavia to the third, Japan to the fifth and India and Israel belong to the sixth pattern. Thus, we find that four patterns find a representation in our study.

A few representative characteristics of the selected countries may be noted.

France : A Western European industrially advanced economy; under the German occupation for some time during the second world war.

India : An Asian ex-colony with a developing economy; with the second largest population in the world , a deep rooted capitalist system experimenting with some socialism.

Israel : A Middle-East, fast developing economy; required to make a totally fresh start from scratch.

Japan : The only non-Western country to become a major industrialised capitalist economy. Almost the only country in Asia, Africa and Latin America that escaped being turned into a colony or dependency of Western Europe.

Yugoslavia: An Eastern European economy; relatively well advanced industrially; belonging to the socialist block but having an independent character of its own.

Some other countries need to be represented but we are handicapped by the nonavailability of adequate data. Several shades of ideologies and economies remain unrepresented . But we shall make the best use of what we have by bringing it under a common, comparable analysis and then see how far the differing factors interact with reference to the production structure of the countries.

Some issues

Do the boundaries set by the environment of a country play such an important role as to increase its individuality in the nature of its industrialisation, or, does some kind of internationalisation phenomenon tend to eliminate differences, if any, with the passage of time? The two sides of the question may be considered separately.

The individuality hypothesis would suggest that in spite of imitation of and borrowing from other established industrialised countries and drawing lessons from their successes and failures, each country tends to adopt an economic pattern which is most suited to its own resources, tradition and environment. Gradually, this emerging pattern is established and after a certain stage, it becomes difficult to make a shift away from it either because of sheer inertia or because of real or imaginary risks involved in doing so. This established pattern results in returns whichdiffer from one society to another because of the evolving differences and in spite of a discernible interdependence. Each such society develops its own innovative investment, technology and economic patterns. There is a certainty of returns in a well established society with advanced technology and fast growing economic patterns. There is what Easterbrook (1966) calls a persistence zone in a society with a traditional or colonial nature, suffering from a high risk situation leading to low and even negative returns, associated with simple economic and technological changes, with a dependence on external forces and internal and external pressures of uncertainty. There is a transformation zone in such countries as have a sufficient security of returns in which investment of resources is not automatically induced by the existing structures but is a result of autonomous decision making about investment. The risk is at a normal commercial level and institutional support is available from the society in all essential forms.

The other possibility is what may be called an an internationalisation phenomenon which explains away these differences. At a certain stage the industrial societies tend to look and even become similar and the differences are eliminated gradually. The less industrialised societies derive their industrialisation, partly or fully from the advanced societies. How this happens can be seen, variously, through the working of the factory system, specialisation, the manner of substitution among inputs and the nature of economies of scale for different countries. Perhaps, one could suggest that material requirements of production, consumption and subsistence can put severe limitations on the variety of forms of industrialisation that different societies may wish to adopt. As Turner(1975) says," technological forms are not infinitely variable and certain salient features of the material environment on which the technology has to operate will also exert their own limiting efforts,"..but. "man's ability to generate variety will hold out a possibility of a range of social forms to be explored." In addition the process of internationalisation goes on.

There are numerous forces behind the process of internationalisation. Technological development made some of the externalities of the production and consumption process international in character. As observed by Lindbeck(1977), "the accentuation of the returns to scale in some industries forced small and medium sized countries to specialise in a more and more narrow range of products, the increased role of technology in production management stimulated trade in technology which resulted not only in an expansion of trade in patents and machines but also, due to the complementarity between technology and management, in an internationalisation of entrepreneurship."

In addition to economic and technical factors, political decisions and institutional changes have also accelerated the internationalisation process. After a certain stage these changes are beyond control so that the political system practically gives in to the internationalised system. According to Lindbeck, "most important external effects of production and consumption are external to nations and not only to firms and households."

It is difficult to say categorically, as to which of the above contentions is correct. That of divergence and individuality or internationalisation. The difficulty is allthe greater because of the indeterminacy of the time of arrival and the nature of industrialisation in a given society. Ideally, we should like to begin with a number of different societies, resembling one another in some respects. We may then, after a certain period of time, say twenty five years, study the state of affairs in each of the societies under consideration and compare notes. Instead, we have five countries, with fundamentally different beginnings and circumstances. We impose an artifically common beginning for the countries and study the state of affairs after a period of about two decades. That does not make it a trend study because there may be any other intial period or none at all. This study remains a crosssectional study in any case and considers the happenings in the period of reference.

In view of the problems associated with the procedure to be followed in coming to a conclusion and in measuring and comparing the extent of internationalisation or divergence between the economies, we shall narrow our field of inquiry to the manufacturing establishments of the countries; we shall confine ourselves to considering the problem in the light of a production function analysis which need not necessarily answer all our queries but which could touch several major aspects of the problem.

Much of what we shall do depends on the quality and quantity of data we have. We have five countries each in an entirely different set of circumstances in the matter of resources, economic development, political and social structure. The only possible common factor is the period immediately following the last war. If in spite of differences we find similarities in the production function coefficients, hypothetically "after" a period of about two decades or any other period of time, there may be much to recommend the possibility of internationalisation or some kind of convergence to a common situation. If however, dissimilarities in the production parameters are noticed, the internationalisation phenomenon would stand refuted.

Some Related Studies

We have tried to reduce a comparative study of the structural characteristics of manufacturing sectors to that of a study of production functions of manufacturing establishments.

While trying to put the whole thing in a simple form, we are not unaware of the possibility that many complex processes may be at work within. But it may still be possible to find if the industries in different countries have been influenced similarly or differently by a variety of circumstances and whether this had led to any structural differences in their industrial sectors.

By making international comparisons, Hoffman(1958) analysed the general pattern of development of manufacturing industry and found striking similarities in the process of growth of various national economies. Dividing the whole industrial economy of each country into two broad sectors consisting of consumer goods and capital goods industries he postulated that the former develop first during the process of industrialisation and the latter soon develop faster than that. The ratio of value added of the consumer good industries continually declines as compared with that of capital goods industries. According to Hoffman, the date of industrialisation in any country has not affected these patterns of industry. On the basis of a time series analysis he arrived at the conclusion, applicable to any economy, that irrespective of relative amounts of factors of production, location factors and state of technology,"the structure of the manufacturing sector of the economy has always followed a uniform pattern."

Koropecky(1969) using a contemporary cross-section analysis of 2-digit standard industrial classification data on manufacturing found that uniformity could be caused by uniform factors that persisted. Relating th three hierarchies (i) wages and the proportions in which skills are distributed, (ii) capital and labour intensity and (iii) output per worker, each of which he found stable over time, he concluded that changes in the structure of manufacturing were markedly in the direction of the highest average output per worker but not in the direction of capital and labour intensive industries. Koropecky suggested that the pattern of change is likely to be "stable over time and these characteristics of structural change in the manufacturing sector extend beyond the contemporary crosssections."

The Average Production Function

The present study is a production function analysis at the disaggregated level of mixed establishment data for each country. The resulting production function and their characteristics are some kind of averages which do not represent any particular establishment but which can be quite useful for comparison purposes.

Owing to the nature of data at our disposal, the resulting production function is the average over industries since there are establishments belonging to different industries.

The concept of the average production function is useful in estimating the average output of a firm, given a certain input combination. But it is not useful in determining the efficiency, the degree of resource utilisation or the production capacity of an economy or industry. It is difficult to extend the idea to the manufacturing industry in general though, in that case, the output measured in value terms could still be used. Unfortunately, this value may not correspond to any good. The manufacturing industry production function constructed from establishment data must be distinguished from the industry's aggregate production function which relates aggregate outputs with aggregate inputs. The industry's aggregate production function may sometimes be approximated by the average production function in the absence of aggregate data. In practice, however, the reverse may be the case since firm data are more difficult to get. Hildebrand and Liu(1965) have suggested the use of representative establishments to estimate manufacturing production functions.

Firms facing difficult production constraints, may, when considered together, represent an average technology. Aigner and Chu(1968) suggested that"the estimated production function represents the average production surface for the industry". They consider the divergences from the production surface as random fluctuations due to chance.

In estimating the average output from a given set of inputs, if we note differences in expected average output, they should be due to differences in technical efficiency. The estimated production function may, then be used as a measure of relative efficiency. This suggests a useful method for handling our data which consist of individual establishments of different sizes and belong to different industries.

Bronfenbrenner(1944), Marschak and Andrews(1944) and Nerlove(1965) in their works on the estimation of microproduction functions assume the estimated function to be an average production function for the industry. This implies that some some firms could produce more than the average and the others less than the average. But the sense in which the term average is used may be questionable.

The average production function cannot be assumed to be a function for a firm of average size unless an assumption is also made that the parameters of the function are random variables with averages corresponding to those of the average sized firm. Similarly an average technology is not a feasible idea since it would require the consideration of an average corresponding to different inputs.

The productive capacity is meaningful only if the output level can be sustained. But the average production function cannot represent that because it does not embody the long run overtones associated with the argument.

If all the establishments belong to the same industry, the distinguishing features of the production function of establishments may be embodied in attainable values for certain technical parameters in the industry production function, any differences in them reflecting relative scales of operation, varying organisation structures etc. See Aigner and Chu(1968). If an envelope isoquant is constructed for the industry, as is done by Farrell(1957, 1962), the production function of any firm may conceptually be obtained from the industry function upto the extent of the firms ability to obtain the optimal parameter values of the industry. The idea may be extended from the industry production functions to the manufacturing industry production functions, since in practice, firms manufacturing identical, single products are impossible to obtain and the notion of value added from price weighted outputs is common. Thus we tend to look upon all establishments as giving rise to a manufacturing industry production function within the same country. The differences, if any, are expected to arise, less from differences in the nature of industry but much more from the differences in the quantity and quality of capital, labour and related factors.

It should be made clear at this point that production functions estimated by regression analysis are conditional median production functions, see Goldberger(1968). The probability is one half for any actual output point to lie outside the production surface. The neoclassical production function is different from the statistical production function because it expresses the maximum output that may be obtained from a given combination of inputs at the existing state of technology. The probability of a production point lying outside the defined production surface is zero in the case of a neoclassical production function. If production points tend to lie inside the surface a new technology may be implied. In an inter-firm crosssection study, an average production function can approximate an aggregate production function though it would be difficult to visualise a firm employing average capital, average labour or average technology.

Problems and Techniques of Estimation

There are several problems in the estimation of a production function These are described briefly in Appendix A. Some techniques of estimation, with special reference to the Cobb Douglas and the CES f nctions have also been given.

Use of Ordinary Least Squares Method

In view of the very large number of regressions attempted in this study and because of a large amount of computational work involved it has been decided to make use of ordinary least squares technique which seems to be adequate for our purpose. This should enable us to concentrate on the comparative analysis of a large number of results.

The least squares estimator has a number of desirable properties. By Gauss-Markov theorem it is the best among the class of unbiased linear estimates . If the random elements are independently and normally distributed, the least squares estimators maximise the likelihood function. But the least squares procedure is not efficient if the disturbances are heteroscedastic, the possibility of which is not ruled out in our data. Even when the method may give relatively lower standard errors of coefficients, the possibility of bias and inconsistency remains.

The use of simultaneous equation methods, two stage least squares or maximum likelihood , requires the construction of a complete production and input model. The ordinary least squares estimates are not consistent and suffer from simultaneous equation bias resulting from the correlation between the random error term and the dependent variable; the bias cannot be eliminated by increasing the sample size though the use of suitable instrumental variables may help with crosssectional data like ours. The possibility of a high correlation between the error term and the dependent variable is minimised if we assume a lack of any connecting factor among firms. Use of suitable additional variables may reduce the upward bias associated with ordinaryleast squares estimates.

A high degree of disaggregation in our data and noninclusion of very small establishments are likely to result in low biases which generally result from aggregation and specification errors. In addition, our data are reasonably reliable and uniformly collected. All this is not to deny the better role that could be played by a multi-equation model.

In the estimation of the production function by the ordinary least squares method, Zellner, Kmenta and Dreze (1966) restored faith in the single equation model under the assumption of identical production functions across firms with respect to form and parameters which are stochastic. This makes the firms' profit function random.

The optimality of the estimates of the production function by ordinary least squares depends on the assumption of maximisation of expected profits rather than profits with price given and known with certainty to the entrepreneur. This assumption may be fulfilled in a market economy where either perfect competition or monopolistic competition may prevail. In a socialist economy we can not be too sure of the correctness of such an assumption. One of the countries in our model is Yugoslavia which has a socialistic economy whose results may be expected to be different from those of other countries. The actual manpower or manhours employed by an establishment in a market economy may tend to equate marginal revenue product to wage rate though, owing to miscalculations, only a partial adjustment to equilibrium conditions may be attained. The situation may be entirely different in a socialist economy.

Following the Zellner-Kmenta-Dreze model, it is assumed in this study, that the entrepreneur, faced with the uncertainty of non-instantaneous production, seeks to maximise his expected profits while the production function is disturbed by the error term. The transmission of the disturbance from the production function to the marginal productivity relations is ignored.

Economic and Technical Variables

Throughout this study, use has been made of the distinction between economic and technical variables which occur in production function analysis. In the last chapter this distinction was utilised to assess the extensions of some production function forms. An attempt will be made now to clarify the meaning of the the concept of economic and technical variables.

The act of production being technical in nature, the factors entering it must have a technical character. Technical variables are specifications for inputs and outputs while economic variables tend to be specifications of input and output prices. In general, variables considered before the act of production are technical in nature. Variables subject to market forces, expressed in money units, may be called economic variables; usually they arise after the act of production has already taken place.

It is difficult to place certain variables like the number of shifts worked, the rate of capacity utilisation or the age of establishment in either of the two categories. They may influence, and be influenced by, both economic as well as technical considerations.

The money factor is always present, at least in the background, and influences all aspects of production. It influences technical as well as all the other variables with which a production relation may be concerned. If output prices are high, the rate of capacity utilisation or the number of shifts worked is likely to go up. The economic outlook of an older establishment may be quite different from that of a new establishment.

Because of the inconvenience in the use of physical units, when a technical variable has to be expressed in money terms, the process of change in the nature of units may not necessarily allow the variable to retain its original nature. Capital, though a technical variable, is almost invariably expressed in money terms but still retains its technical nature because items of capital are not purchased frequently and the variable may not be seriously influenced by price fluctuations. On the other hand, output, when expressed in value terms through price weighting, may lose its technical character and may become an economic variable in the form of total value of output or value added.

Wages may not behave as a technical variable because the requirement of labour in production does not necessarily depend on the wages. Even though, more capital intensive techniques can reduce the dependence on labour to some extent, yet a certain minimum requirement for labour still remains irrespective of high or low wages which may not necessarily be related to the quality or quantity of labour.

The same wages may not extract equal quantities of work from different units of labour. Moreover, the forces of demand and supply play an important role in the determination of wages.

Although w is equal to the marginal product in competitive equilibrium and marginal product is a technical matter, the equilibrium involves supply as well as demand which are economic forces. The manager of an establishment may try to move his production function to an appropriate position so that w may appear to be a technical variable but this may not be usually possible because of fluctuations in prices, trade union activities and the nature of work in the establishment which may not necessarily be related to labour productivity.

Practically the same arguments can be given in the case of rate of return on capital In competitive equilibrium it may tend to equal the marginal product of capital and since that involves forces of demand and supply of capital, the rate of return on capital is an economic variable.

The ratio w/r may tend to equal the marginal rate of technical substitution and may be expected to behave as a technical variable on the same lines as w or r. But under the influence of economic forces, in practice, it behaves as an economic variable.

As for the factor share ratio, "the intrusion of technical change between the simple facts of factor ratios and factor rewards"[1] gives it a character of its own and its effective nature may depend on the influence exerted by the economic and technical factor combination in it.

1Solow(1958)

The manner in which the production function is defined and used in actual practice does not seem to allow watertight compartments for economic and technical variables. Also, it may not be easy to claim equivalence of nature of different technical (or economic) variables. But the difference between technical and economic variables, in the manner we have considered it, will be used to examine various criteria used for grouping establishment data and also to consider the effect if any, of this difference, on the homogeneity of manufacturing establishment data of the countries under study.

It may be possible to look upon some technical variables as quantities and some economic variables as prices although it should be remembered that the relative importance of the price weights used in some quantities may vary from one variable to another and an exact categorisation may be difficult.

The effect of economic and technical[1] variables on the grouping of manufacturing establishment data and on pool and group regression analysis will be studied later. The results of analysis of covariance will also be considered with reference to this concept

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1The use of these terms is not uncommon in the literature. The concepts of "economic" and "technical" efficiency are well known. Recently we came across the use of these terms nearly made in the same sense as we have done This was in K.Hilton and H.Dolphin(1970),Capital and capacity utilisation in the U.K.,Bul.Ox.Univ.Inst.of Econ &Stat.,Vol.32 where a reference to "Klein L.R.(1960),Some theoretical issues on the measurement of capacity,Econometrica,Vol.28" is also to be found.

Establishment Size. Grouping of Establishment Data.

The size of establishment is a measure of crucial importance in any production function study of intercountry differentials. An important role is played by the size of establishment in group regression analysis which has been carried out in chapter six. The focus of study in that chapter and chapter seven will be the variations obtained by the use of economic and technical variables to group the establishment data.

For a suitable ordering and grouping of the establishments, we are not in a position to make use of standard industrial classification numbers because there are not enough establishments corresponding to each such number. We resort, therefore, to other criteria which can be in terms of economic performance and may be influenced by the prevalent economic system, the size of the market and the level of economic development, or, they can be related to the technology and may be useful in the study of certain problems with policy implications. Some causes of variations in factor elasticities, economies of scale and elasticity of substitution between inputs can be studied in this manner.

It is difficult to single out any criterion for a measure of establishment size or economic performance Even though technical performance is often expressed in economic terms and vice versa, it is difficult to make any general rule about the correlation between the two. There are no commonly agreed measures of establishment size or economic performance in spite of the two being bracketed together sometimes in practice.

Variables like capital assets and labour must exist before any economic performance is noticed. To a certain extent the nature of economic performance can be visualised from a knowledge of these variables which are basically technical in nature. Establishment size may also be determined by variables like gross or net output, output labour ratio, rate of return or sales which also happen to be measures of economic performance. These latter variables are ususally measured in value terms and as economic variables, most variables measuring economic performance, are, more or less, closely related to the economic rather than the technical aspect of production with which the establishment size variables are concerned. It may be expected, therefore, that a grouping of establishments on the basis of these two types of variables should give different results. We will set this up as an hypothesis and test it empirically.

We will now consider the relevance of some measures of establishment size and economic performance mainly from the point of view of grouping our data. Moreover, we take the opportunity to justify the use of a common production function for all manufacturing establishments or groups of establishments irrespective of industry considerations.

The economic performance of large companies is often compared by putting them together in the same analysis without reference to the industry to which they may belong. See, for example, Wood(1975) or the Fortune Directory of five hundred largest American firms which occurs periodically and has been a source of important information and can provide material for economic analysis. In comparisons like these the performance or "efficiency" of a firm is measured by and graded according to rate of return, sales turnover, value added, productivity or a suitable combination of these.

Measurements are usually carried out in money units as physical units are difficult to handle.

Sales turnover, on its own or in relation to capital employed, is a commonly used measure of efficiency by managers. Unfortunately, the money value of items going into sales may be so different from one firm to another and from time to time that this measure may not always be considered as a reliable measure.

Wood(1975) finds value added an attractive measure of economic performance. According to him, added value as a measure of performance can be considered to be particularly reliable because"it is not affected by depreciation policy, interest charges, development costs, government grants, wage levels,etc., .contained within the added value...(and) is only marginally affected by changes in stock valuation methods."Wood gives several reasons and examples in support of his contention.

For empirical analysis, one may choose to measure establishment size by the size of labour force, rather than by output or sales turnover. It would then imply a decrease in establishment size if the number of workers decreases even though output may have increased at the same time because of a high capital intensity. As Pryor(1973) says, "if the average employment size of establishments in a given branch of industry is determined by the level of technological knowledge available to all nations then any difference in the average size of industrial establishment can be explained by differences in the distribution of employment among industrial branches."

There are not many studies about international comparisons of establishment size or economic performance because of lack of data or because of difficulties of generalisation with the help of causal factors underlying establishment size.

Bain's(1970) study of international differences in industrial structure compares the plant sizes of eight countries as measured by the number of employees. In Lavigne's(1970) study of socialist economies, the comparisons made are qualitative in nature. Good comparative studies pertaining to the EEC countries are common

According to Pryor(1973), who carries out an analysis of size of production establishments in mining and manufacturing of communist and capitalist countries, the theoretical analysis of establishment size depends very much on the measure of size that is employed. On the grounds of available data he chooses to measure establishment size by labour force. In a crosssection analysis of manufacturing establishments of eight countries he finds that it is possible to rank countries according to establishment size and the results obtained by using different indicators of size are similar. There are several such indicators of size. One is the arithmetic mean(A.M.) of employees in all the establishments The second is the Entropy Index of Theil(1967), denoted by E in the expression $\ln E = \ln L - \sum t_i \ln \frac{1}{t_i}$ where L is total labour force and t_i is the proportion of labour employed in the ith productive unit. The Niehan's Index is $N = \sum t_i L_i$ where the labour force employed by the ith unit is denoted by L_i. On the basis of these indicators, Pryor finds the average employment size of manufacturing establishments in several countries. Given in the table below are the results for four of the five countries in our study as the details for India are not available. The table has been adapted from Pryor(1973) and in its original form, contains details about several other countries.

Average Employment Size of Manufacturing Establishments

	Year of reference	A.M. Index	Entropy Index	Niehan's Index	Percentage of Labour force in estbmnt.
France	1962	122	337	1620	20.3
Israel	1965	74	128	220	21.9
Japan	1963	87	224	667	16.6
Yugoslavia	1963	269	566	1064	33.1

The average establishment size in Yugoslavia in particular and in Eastern Europe in general is very large in comparison with that in Western Europe; on the average it is up to four times the size of the establishment in Western Europe. As noted by Pryor, the establishment size in Hungary and Poland is even larger than that. This is because the central direction of manufacturing in Yugoslavia has been very much less than in Hungary and Poland although many manufacturing establishments in Yugoslavia were built when the central government had considerable control over investment and the smaller establishments were discouraged. As compared to the other countries in our study, the political influence in the case of Yugoslavia is so significant as to single it out as a special case from the point of view of uniform development of manufacturing activity.

We now consider a few more measures of establishment size and economic performace of establishments. We will also consider the possibility of interchangeability of some of these measures.

According to Moyer(1968), "two important indices of performance in industry studies are the degree of utilisation of production capacity and some measure of rate of returns. Analysis usually focuses on various structural conditions that influence these and other measures of performance as well as on the effect of market conduct performance. This is because knowledge of the operating rate can help predict profit without one's being required to estimate costs, prices and other factors."

The study of the relationship between these factors is also useful for public policy purposes. The rate of return provides a basic level for project evaluation and its knowledge can be useful in studying the performance of different sectors of an economy or different groups of establishments. A malfunctioning of the capital market may be expected if significant differences in the rate of return in different sectors do not tend to narrow down over time and , in the case of establishments, a certain acceptable trend is not noticeable among groups of establishments formed on the basis of a certain size pattern.

Instead of grouping the establishments by their economic performance or by the size of one or the other variable we may group them on the basis of the rate of technical progress of the industries to which they belong. For instance, steel products, chemicals and machinery may belong to the high technical progress group and most consumer goods like food, tobacco and leather may belong to the low technical progress group. Other groups may also be formed. Unfortunately, these groups are likely to be subjective in nature and compact divisions are difficult to make between different industries. Morever, in international comparisons there is the possibility of variations in the rate of technical progress in different industries. The nature of a particular industry is not likely to be the same in different countries. Also some countries may be disproportionately represented by some industries.

Grouping of establishments may be based on the quality rather than the quantity of certain inputs used. For qualitative grouping the problem is that of defining and locating similar qualities of inputs in different kinds of data collected from a variety of sources. The quality of capital, may, for instance, be decided on the basis of age of establishment or the age of machinery and equipment used but there is no guarantee that any meaningful and detailed data on these or other suitable characteristics are usually available. Perhaps it may be easier to group the data on the basis of labour quality provided such classification is confined to the same country. Some ratios could prove to be useful as alternative modes of grouping. The capital labour ratio, the direct-indirect labour ratio or value added-labour ratio are some examples for the purpose.

As the distinction between economic and technical variables forms an important part of our study, it may be useful to place some of the variables above, which have not occurred earlier, in suitable categories.

Forming technical progress groups can be a useful technical criterion of grouping the manufacturing establishment data provided the divisions based on it are not subjective in nature and are comparable over countries. This criterion may, however, be handicapped by the variations in the quality and quantity of data in different countries.

As for the qualitative grouping based on the age of machinery or the age of establishment, it may be difficult to place the measure in a suitable category though it is nearer to being a technical grouping criterion based on the time factor and experience. The capital labour ratios of the older establishments are likely to be different from those of new establishments. But within each country, the technical and economic forces tend to reduce any differences resulting from this qualitative factor; the accummulated technical and economic experience of some establishments can be easily shared or neutralised by other establish-ments within the country or even from other countries. We can argue on the same lines about the accummulated experience in the form of quality of labour used as a grouping criterion. We may expect, then, the age of establishment as a grouping criterion, to have the characteristics of both a technical and an economic variable.

Utilisation of production capacity is another interesting factor which may be used as a grouping criterion.

It is subject to both economic and technical forces. In the technical sense it may be related to the maximum production in an establishment, the number of hours worked, the number of shifts worked, strain on machinery and equipment and so on. In the economic sense, it may be linked with the availability of resources and their costs, producing a given output at a minimum average cost, and with the attainment of economic efficiency in general. For mixed establishment data, this may be a better point of view but the technical sense is also important within each country. Utilisation of capacity may influence, and be influenced by, capital labour ratio, productivity of capital, rate of return and nature and size of an industry.

It may be interesting to compare groups of establishments of different sizes and having the same degree of capacity utilisation. But that would be difficult. However, it is possible to put together establishments belonging to certain class intervals of percentage capacity utilisation.

As a grouping criterion, the number of shifts worked may also be considered on practically the same lines as capacity utilisation. Both these factors, viz., percentage utilisation of capacity and the number of shifts worked, therefore, may be looked upon as having the features of technical as well as economic variables The age of establishment falls in that category.

A satisfactory expression for the size of an establishment may not necessarily be found in any one or more of the measures mentioned above. High wages may be associated with a relatively small establishment. Low capital assets in some establishment may be capable of yielding relatively high value added. Criteria other than the one under consideration, in determining the establishment size, may also play an important role. A measure combining the essential elements of various measures is desirable but difficult to construct.

A rough comparison of these measures may be made by means of Spearman's rank correlation. Table 7, appendix, gives the rank correlations, for all the countries under study, between every pair of the following variables: total value of production, value added, total labour, direct labour, net capital assets, machinery value, annual depreciation allowances, total wages, capital-labour ratio, value added-labour ratio, value added-capital ratio, rate of capacity utilisation, age of establishment, age of machinery, percentage of motors operated in shift one, electricity consumption in kwh and capacity of motors in kwh.

As can be seen from table 7, the value of rank correlation is different for different pairs of variables. This implies that the analyses based on different criteria of establishment size are likely to produce different results.

Great care in selecting the best measure is usually not taken in practice either because it is assumed that different measures are correlated and therefore interchangeable or because better measures are not available and use has to be made of whatever measure is found to be relatively more convenient. The rank correlation is not a very reliable method for the purpose of making a choice of the measure. Unless the practical consideration of the availability of the measures restricts the choice, economic and statistical factors should control the selection of the best criterion. The empirical results obtained by using different measures are not necessarily the same and rigorous conditions may need to be considered for interchangeability of different measures. Smith, Boyes and Peseau(1975) show one possible way of interchanging different measures of establishment size provided they are related linearly in some cases and log linearly in some other cases with unit elasticity.[1]

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1 Let Ψ_1 and Ψ_2 be two alternative measures of firm size. If there is perfect correlation between their logarithms so that $\Psi_1 = a_o \Psi_2^{b}$, $a_o, b > 0$, then b is the elasticity of Ψ_1 with respect to Ψ_2.

Writing $\xi_1 = \ln \Psi_1$ and $\xi_2 = \ln \Psi_2$, we have $\xi_1 = a + b\,\xi_2$ where $a = \ln a_o$.

If ξ_1 is used to study a characteristic $\mathbb{D}$, we may write

$\phi = c + d\,\xi_1$ where $\phi = \ln \mathbb{D}$

Hence, $\phi = c + ad + bd\,\xi_2$.

The alternative measure of firm size yields the same elasticity for $\mathbb{D}$ with respect to firm size if $bd = d$ or, $b = 1$.

Some comments on the data with reference to this production function study

In a study of inter-industry production functions or inter-establishment production functions it is necessary that the data satisfy certain requirements.

The labour and capital variables should be fairly homogeneous across industries or establishments. This is usually not possible. To a certain extent, we may get around the problem by assuming that different qualities of a variable stand for different variables.

For a proper production analysis, industries or establishments must be close to one another in the matter of technology and efficiency. A correction for differences in technology and efficiency may be made by introducing into the production model some possible measures of these factors. In practice this requirement can be only partially met. It may also be desirable to make a correction for capacity utilisation which may vary from one observation to another. This is because, strictly speaking, each establishment has its own real boundaries and each point on its production function stands for certain technical possibilities peculiar to itself and not to any generally applicable theory. In theory, as the production function represents all possible combinations of inputs resulting in an output, it amounts to the solution of an optimisation problem resulting from a consideration of all such combinations of inputs. This implies that no commodity can have different production functions wherever it may have been produced. Such an argument tends to ignore a number of happenings associated with each establishment in reality. Identical amounts of inputs in different regions can produce different amounts of output. The differences may be explained partly atleast by differences in input qualities, capacity utilisation and environmental factors.

In th present study it has not been possible to satisfy all the requirements . But this situation applies to all the studies on the subject. Moreover, different types and sizes of manufacturing establishments have been pooled together in this study. Some observations in this connection may be made.

Pooling Different Types of Establishments : a Justification

The nature and quantum of our data do not allow us to carry out our comparative study with reference to individual industrial sectors. We have, therefore, pooled together data on all the establishments and fitted production functions to a crosssection of individual establishments in each country without distinguishing between industries. Different types and sizes of establishments have been brought together though the relatively small ones are not included.

We begin by making the commonplace apology about the paucity of appropriate data which can force the analyst to make the best use of what is available, even at the cost of some exactness or reality. As for the establishment level data, it is hard to come by and many a production function study at a disaggregated level has been given up in favour of an aggregate study. We do not have fully satisfactory data but we are not far from it and would carry out the production function analysis on otherwise valuable data. There will be some amount of aggregation. For instance, the value added is found by weighting the net outputs of individual items by their prices. This is unavoidable as there are hardly any establishments in practice which produce one homogeneous product. Even if there were such firms, they could be producing items of varying quality.

Pooling of this type is not uncommon in the literature on production functions. Grilliches and Ringstad(9171) have done it in their famous production function study. Solomon and Forsyth(1977) and Pack(1976) do the same.

An advantage we have is in the form of a number of redeeming features associated with the data which are more reliable than most other similar data. They provide us with two measures of output, a few measures of labour and about a dozen measures of capital. We have in addition, good measures of capacity utilisation, inventories, intermediate inputs, fuel consumption, capacity of motors worked, age of machinery, age of establishment, shifts worked and the use of labour and percentage of motors operated in each shift. These are variables which production function studies have often to manage without.

The behaviour of a firm belonging to one industry may not conform to that of other industries. But it is true that the behaviour of individual firms does not necessarily conform to that of other units in the same industry or to what the whole industry does. At the same time, it may be difficult to prove the contrary. A firm belonging to any industry keeps a watchful eye on firms belonging to other industries. The workers already engaged in any industry and those yet to join some industry as well as the trade union leaders have a fairly good idea of the circumstances available in other industries. In a market economy with good mobility of labour, the element of competition among different jobs can not be easily avoided. The same can be said about capital which tends to move towards different industries according to the pattern of returns. It does not hesitate to desert certain industries according to the same pattern. Expected returns play an important role in determining the direction, quantity and quality of capital flowing into industries. While it is true that the use of additional capital in any establishment depends on the circumstances available in that establishment, it is also true that the introduction of substantial quantities of fresh capital or the use of capital in a new establishment depends on expected returns rather than on the nature of industry alone.

The implicit assumption about the constancy of production function for all establishments in the same industry may not necessarily be true. Such an assumption makes the selected productive techniques a function of relative factor prices. The same can be said about the assumption concerning the constancy of production function for all establishments irrespective of industry. If, therefore, we fail to discover such a constancy we will have open to us the alternative to test the extent and nature of variability of production functions in our data. This will be done by arranging the data according to certain criteria and fitting production functions to different parts of it and carrying out the analysis of covariance.

The existence of different production functions can enable us to examine the dual characteristics observed in the industrial sector of any country.

A comparison of production functions of different countries is usually done on an aggregate level. A study of establishments belonging to different industries, pooled together may throw additional light on the results.

The technical standards of all the establishments included in such a study would generally be not the same, more so when they belong to different industries. Whether or not this may be the case , the outcome needs to be carefully analysed. There may, for instance, be some internal peculiarities in some establishments and not in others.

If there are major differences they may be revealed to a certain extent, in a production function analysis. It is not unlikely that the conclusion drawn from a partial set of establishments from each country may arouse suspicion because several aspects of these countries may not have been taken into account. For instance, there may be tremendous differences in establishment sizes in different countries. There may be difficulty in measuring the establishment size according to an accepted pattern. There may be dissimilarities in the patterns of industries and their percentage representation.

But some or more of these difficulties will always be present in any production function study.and inspite of that we are looking for some uniformity of pattern. The intention is not to ignore the differences but to carry out a comparative analysis of different countries under different circumstances and influences. The differences arising from the pattern and proportion of industries cannot be eliminated. But the mobility of factors and their adjustment eliminate a number of differences, whatever the industry. The factors continue to compete with one another even when they belong to different industries.

The pooling of manufacturing establishments for a production function study can be a useful exercise. At times pooled data may prove to be superior to the data of individual industries. According to Steindl(1964) who has carried out a stochastic study of firms of different countries, "if the mass of firms is divided according to individual lines of manufacturing or trade, the distribution often becomes irregular. A neat division of firms, if it goes beyond the broad division of manufacturing, trade, etc., is artificial because of the arbitrary allocation of many firms, and because, firms in growing spread from one business to the other, the stochastic process which accounts for the regularity is more applicable to the broad field of all firms than to narrow industrial divisions."

The regression analysis in chapter five is based on pooled manufacturing establishment data The group regression analysis in chapter six is also carried out without reference to the industries to which the establishments may belong. As we will see, the use of various other criteria, instead of industry-wise grouping can also produce interesting results.

The Hypotheses

On the basis of a crosssectional production function analysis of the manufacturing establishments of France, India, Israel, Japan and Yugoslavia during the period of reference 1964-66, we set up the hypothesis that, irrespective of the industries to which the manufacturing establishments of a country may belong, the pool regressions for all the establishments taken together should provide statistically meaningful results provided the size of the establishments is not unduly small and provided appropriate production models are used for the analysis.

It is usual to group manufacturing establishment data according to standard industrial classification numbers. It is suggested that economically meaningful groups can also be formed by means of criteria other than those of standard industrial classification. We set up the hypothesis that from the point of view of crosssectional production function analysis, such groups are statistically meaningful provided the number of establishments in any group is reasonable from a statistical point of view and suitable criteria are used to form the groups.

It is hypothesised that the pool regressions, if based on a more or less complete set of manufacturing establishment data of a country should provide statistically and economically superior results to the group regressions based on any subset of the complete data. The pool data and the production functions fitted to them should, therefore, represent the manufacturing sector of any economy more fully than any groups and the production functions fitted to them.

In the absence of time series data, it is not possible to compare results at two different points of time. Instead, therefore, with the help of our manufacturing establishment data, we set up the hypothesis that, for countries which are industrialised or which are on their way to industrialisation, the process of internationalisation may be reflected in the almost identical reaction of each country to any appropriate production model, provided a fair sample of all types of manufacturing activity within the country is taken. In other words, the technical aspect of the production function should be noticeable in the comparable values of the technical parameters of different countries. This may be verified by pool regressions.

The hypotheses of constancy of returns to scale and non unitary elasticity of substitution will be verified by means of production function analysis of the manufacturing establishment data.

The production function being a technical relationship, technical factors should be able to explain this relationship fully. In practice, it is difficult to avoid the influence of economic forces which play a prominent role in production.[1] If technical factors are found to be inadequate as explanatory factors, the use of economic magnitudes should help. Depending on the method of ordinary least squares, we set up the hypothesis that the explanatory power of a production model may be improved if technical as well as economic variables

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1For instance, the SMAC(1961) CES function and Hanoch (1971)CDE or constant difference of elasticities of substitution function have, included in them, economic magnitudes like prices and elasticities wrt prices are used together as explanatory factors. In other words, even though, theoretically, the production function is a purely technical concept, in practical applications, the use of technical as well as economic magnitudes as explanatory factors should improve the statistical results. This hypothesis will be tested with the manufacturing establishment data of each country under study with the help of pool regressions as well as group regressions, in chapters five and six.

Chapter seven presents the analysis of covariance of group regressions for each country

Several technical and economic criteria of size will be used to divide the manufacturing establishment data of each country into three nearly equal groups, each containg about one third the total number of establishments, arranged in an increasing order of magnitude of the size criterion used.

We will test the hypothesis that within each country, the corresponding technical parameters of any production relation fitted to each group within a country, are statistically not different. In other words, with the help of analysis of covariance, we will verify the homogeneity of different groups within each country. Since, heterogeneity of groups resulting from grouping by the dependent variable in a production relation is a statistical artifact arising from sample selection bias, our concern will be mainly with independent variables in the production relation or other suitable variables available in the data. It is suggested that, in the revelation of homogeneity by the use of different criteria, some discrepancies can be introduced because of differences in the economic and technical nature of the grouping criteria.

The conclusions drawn from the analysis of covariance will test the stability and uniformity of the manufacturing sector within each country.

CHAPTER FOUR

THE DATA AND THE VARIABLES

Introduction

There are numerous data problems in any econometric study. Apart from the problem of availability of satisfactory data, there is the serious question of the implicitly assumed homogeneity of inputs like capital and labour. No operationally satisfactory solution that could take care of the non-homogeneity of inputs has been found in production function analysis. Other problems are concerned with input utilisation and the role of some inputs whose measures are either not available or not reliable. It is not clear whether allowing for these defects or developing techniques to take care of the shortcomings is worth the additional complications that may be introduced. Our data may be considered reasonably satisfactory particularly because they have been uniformly and systemically collected by experts.

We begin by describing the nature of the data used. This is followed by a consideration of the variety of measures available in the data for each variable; a suitable choice of a few variables out of these has been made.

Nature of the Data Used

Any effort to evaluate and measure the performance of an economy, particularly in the industrial sector, must concentrate on the individual establishment or the firm which is the decision making unit and which along with other similar or dissimilar units in the economy is the source of useful information. A study of the relatively larger manufacturing establishments using large quantities of capital and labour can be highly rewarding .

We have at our disposal detailed information about the manufacturing establishments of France, India, Israel Japan and Yugoslavia. For each of five countries we wish to carry out a production function analysis of the cross-section over individual establishments. The year of reference is 1964-66 and the data have been obtained from the U.N. Profiles of Industrial Establishments published from 1969 onwards. The data were collected on the same uniform pattern to elicit information from selected manufacturing establishments in each of the countries mentioned here. The U.N. representatives from the section on the development and organisation of industries visited and carried out an extensive survey of their establishments. The participating personnel of selected establishments were briefed and guided by a special expert group organised in each country to ascertain the uniformity of the collected data which were later checked for consistency.

The number of observations selected from each country is as follows:

France	64
India	117
Israel	69
Japan	63
Yugoslavia	145

The pattern of data collection remains the same throughout. In the case of France, India, Israel and Japan the observations are on what may be described as individual establishments. In the case of Yugoslavia the term used is enterprises which are described as financially and managerially self contained units. The pattern of the establishment size differs from country to country and does not necessarily reflect the size of the underlying technological units. The concept of size itself needs some clarification and has been considered separately.

Establishments with less than ten workers or negative value added are not included in our data. Wages for direct and indirect labour were usually available separately but on a few occassions had to be estimated on the basis of comparable, related establishments . In the later stages, the analysis did not require the use of separated wages .

The Establishment, the Local Unit, the Enterprise

The difference between the establishment, the local unit and the enterprise may be noted in the light of the definition given in The Growth of World Industry 1938-1961 : National Tables, New York, 1963.

The establishment is an economic unit which engages under a single ownership or control, in one or predominantly one kind of industrial activity at a single location : e.g. the individual workshop, factory or generating station.

The local unit comprises all the industrial activities carried on at a single location under a single ownership or control.

The enterprise, a legal entity, is an individual proprietorship or any association of persons or organisation owning and carrying on a business undertaking, engaged in one or more industrial activities at one or more locations. It may be divisible into establishments or local units. In this study the Yugoslav enterprises are treated as establishments.

THE VARIABLES

The manufacturing establishment data which we are going to use, offers us ample opportunity in the matter of choice of variables. There are two measures of output· total value of production and value added, the latter obtained from the former by the subtraction of the value of intermediate inputs. Neither of the two measures can be used in physical terms because with a single output production function and with mixed industry data, we are forced to use money measures. In any case, it would be hard to find many establishments of the same type, producing one and the same commodity.

We have several measures of capital or its proxies. There are various difficulties associated with its measurement. But we have been able to get over the difficulty to a certain extent by putting into service as many as a dozen different measures of capital. Such diverse items as net value of capital assets, consumption of electricity in kwh, value of electricity consumed in money terms and capacity of motors were fitted into the same production model, with the same set of data. Alternative sets of data and production models were also tried. The results obtained were found to be remarkably close to one another. A partial description of this exercise is given in the next section on Selection of Variables. Highly significant values of rank correlations between pairs of these variables were also found, see appendix table 7.

A sorting of other measures was also carried out on similar lines.

It may be remarked that the intrinsic heterogeneity of physical measures of industrial production renders them unfit for purposes of comparison and aggregation which are important requirements in many economic studies. Weighting of physical quantities by prices results in homogeneous common units which may enable one to evaluate the contribution of various stages of production within an establishment. But the use of the concept of market prices may not be quite appropriate because, in spite of the assumption usually made about them, the markets are not perfect. In the absence of perfect specialisation and distinct activities there can be a wide range of costs and prices in the same market, as for example, in the empirical measurement of the value of total or net output.

Only a limited number of variables will be used subsequently although the description of the variables given below includes that of some which were tried but which did not justify their inclusion in the detailed study. Even though some variables did not find a place in the analysis they were found useful as criteria for grouping the establishments.

Total Value of Production

The dependent variable in our study is almost invariably some measure of output or output per unit of labour or output per unit of capital. The nonavailability of a physical measure of output forces us to use the value of total output or the value of net output. The latter is described under the name of value added. We will consider here the ingredients of total value of output.

This refers to the gross value of goods and services produced during the year under study. This does not include internally consumed items but any products sold for revenue or meant to be sold regardless of their having been sold or not are included in it. This variable thus includes shipments for sale, transfers to sister establishments, fees received for any contract or commission work performed in materials supplied by others as well as other revenues arising from use of productive facilities like repairs, installation., transportation, storage etc., marketable by-products and processing wastes. Total value of production is calculated by finding the values of individual products at unit values which are factory delivery prices excluding sales tax where different products are lumped together for convenience; their unit price represents a weighted average of the prices of the specific components.

It is difficult to imagine a truly homogeneous unit of output from all establishments in an empirical study. The total or average of all the goods and series produced can be done only in money terms in most cases and avoids the problem of variation in the intermediate goods mix used. Unfortunately, by this method, the resources would appear to be used with equal efficiency. The problem does not vanish if we are more strict about the choice of establishments working on comparable lines · there will not be many such cases.

Value Added

According to Wood(1976) ,"All productive economic activity is designed to add value to materials by using the skills and efforts of the people coupled with capital resources in the form of machinery and buildings."

The variable "value added" in our data is equal to the difference between the total value of production and the cost of all "intermediate inputs", defined separately.

Alternatively, value added as used here is the sum of a number of ingredients

i) wages and salaries inclusive of income taxes and social security contribution as also of bonus or payments in hand, if any

ii) administrative or welfare expenses on employees

iii) annual depreciation and royalties paid out, if any

iv) certain indirect taxes (other than sales taxes) charged to corporations and not transferred overtly to customers

v) rentals

vi) subsidies as a negative component of value added

vii) other gross business income obtained as a residual : total gross value of production minus total consumption of materials, energy and non factor services etc. this equals the sum of corporate income taxes , dividends, retained profits and interest paid on financial liabilities.

It is common to use value added as a proxy for physical output. It is a good proxy provided the price implicitly used in it does not change systematically with one or more of the explanatory variables. In a competitive industry excess profits resulting from technological progress may have been eliminated so that the price will be the lowest possible. The regression of value added on the other variables does not reflect the exact situation implicit in a production function based on physical output idea. In particular , the technological progress is concealed. Thus the assumption made about price loses meaning in a competitive, profit maximising situation. Also according to Euler's theorem, since payments satisfying marginal productivity conditions exhaust all output, value added must maintain the same relation to inputs irrespective of their productivities under conditions of constant returns to scale.

In spite of some problems associated with value added as a measure of output, it is one of the most convenient and commonly used measures. Value added as well as value of physicalcapital may usually be employed directly, without being deflated in crosssection analysis.

Capital

We have the following measures that may represent capital.

1. Gross capital assets
2. Net capital assets
3. Depreciation allowance for the year, being the difference between gross and net assets
4. Fixed capital excluding machinery and equipment
5. Machinery and equipment
6. Quantity of fuel consumed : electricity, gas, coal, water
7. Value of fuel consumption
8. Electricity consumed in KWH
9. Value of electricity consumed
10. Capacity of electricity motors
11. Capital stock including preference and equity
12. Net worth of capital stock

The value of fixed assets in our data is the book value after depreciation. It is obtained by subtracting total depreciation charged upto a given date from the value before depreciation which is the accummulation of the historical purchase values. In the case of Yugoslavia we have data on revalued capital assets which are appreciated values. These have been taken under the same heading as capital assets after depreciation.

Machinery and equipment are lumped together. This variable can be easily used as an alternative measure of capital. Another measure of capital, which may represent the total of liquid and fixed capital is the capital stock i.e. the total of preferred and common stock. The net worth of total assets, is the sum of capital stock plus reserves and retained earnings.

Annual depreciation is in terms of a percentage of book-value after depreciation at the end of the previous year. The measure does not necessarily reflect the rate of capitaluse. The method of calculation may differ from one establishment to another.

Replacement value is estimated as the cost of replacing an existing unit by a net functional equivalent which may be zero in the case of an antiquated unit.

Average age of machinery

The average age of machinery in years is calculated beginning with the year of make which is assumed to be the same as the first year of its use. Details about this variable are not properly available over all establishments.

Labour

The two major categories into which total labour is divided are direct labour and indirect labour. Direct production labour(denoted by L_D) consists of production workers engaged in manufacturing proper as well as in important ancillary operations such as preparation of raw materials, inspection and packaging. Indirect labour(L_I) consists of management and other auxiliary activities like production planning, research and development, accounting, sales and purchases, clerical work and those unrelated to the primary manufacturing process. We also have a cross classification between educated labour including those with certain skills(L_E) and other labour(L_O) with no significant training period. The total of L_D and L_I is total labour(L) which is also the total of L_E and L_O. Thus the direct/indirect and educated/other classifications of labour are not mutually exclusive.

We also have the option of measuring labour by the number of working hours. This is because the actual number of hours worked in each establishment are available. If H_D relates to direct labour working hours and H_I to indirect working hours then total working hours of all labour may be written $H = H_D + H_I$.

Yet another measure of labour or working hours may be constructed. Thus, we may define net working hours, which we may denote by H_N, as the sum of the direct working hours and suitably weighted indirect working hours. The weights could be obtained, for instance, from the ratio of indirect to direct wages for each establishment Such weighting could have the effect of arriving at a more appropriate measure of working hours.

Capital as well as labour are both likely to work at different degrees of intensity which can change rapidly and are difficult to measure.

Wages

There are three complementary measures of wage earnings :

W_D wage earnings of direct labour

W_I wage earnings of indirect labour

W_x other expenditure on employees

The total wages are given by $W = W_D + W_I + W_x$

Most of the time we have made use of wages per unit of total labour which is given by $w = W/L$

The other corresponding measures are direct and indirect wages per unit :

$$w_D = W_D / L_D \quad \text{and} \quad w_I = W_I / L_I$$

Intermediate Inputs

Purchases from outside, intra-firm transfers and net withdrawls from the stock of materials together make the intermediate inputs which include the following items :

i) work performed by sub-contractors

ii) repairs and maintainance

iii) materials(not including materials purchased on capital account

iv) material input for auxiliary activities

v) non-factor service inputs

Capacity Utilisation

In our data, the figure for capacity utilisation is only a rough estimate supplied by the management personnel of each establishment. It cannot be considered to be an objective estimate. In the survey data, it was found to have been given in the form of "a potential percentage increase in the following year over the actual production this year provided certain additional production facilities are made available to the management." Taking this potential percentage figure given by the management in express quantitative terms, as representing full capacity utilisation, we have derived the figures for actual capacity utilisation for the year of reference. The figures arrived at are bound to be of a subjective nature.

At the individual firm level, profit maximisation or cost minimisation are supposed to guide the entrepreneur to arrive at an equilibrium level of output which is most efficient from the profit or cost points of view respectively. This output level is the capacity output of the firm. The problems associated with the use of the concept of capacity output are the usual problems associated with an aggregate production function. Even if we use establishment data, problems like dissimilarites of production functions, technology and entrepreneurial abilities for different firms, nonhomogeneous inputs and outputs and dissimilarity of supply constraints still remain.

Inventories

Inventories are the sum of direct production materials, other input materials like energy, packing, repairs and maintenance and auxiliary activity materials, work in process and finished products valued at selling price or production costs. Differences in valuation procedures introduce a possible element of error which is not expected to be very significant In the case of Yugoslavia , the data on inventories are not available. Therefore, in the case of Yugoslavia only, another variable, which has nothing to do with inventories, has been included; this is the interest paid by the enterprise for the loan taken. This is a feature peculiar to Yugoslavia where each enterprise pays this interest every year; the interest covers only the contributions to the central investment fund which are different from interest on loans and are levied on business operational funds, fixed assets and working capital. The rate varies from one to six per cent for different industries.

Number of Shifts

The number of shifts is either one, two or three. We do have an approximate measure of capital utilisation data. But in the absence of actual capital utilisation data, according to Nadiri and Rosen(1973), a direct relation between capital utilisation and the number of shifts may be assumed. Roughly speaking, full utilisation may be assumed to correspond to three shifts where each normal shift is of eight hours.

Two shifts may be linked to two-third utilisation of capacity and one shift to one-third utilisation, only for convenience.

SELECTION OF VARIABLES

The number of variables in our data being large, it may be possible to make a choice between several different claimants to represent some of the variables for the purpose of production function analysis. Since we are going to use several production relations for the data of five countries, the computation work may prove to be rather heavy. We will select therefore, only one representative for each variable required, so far as possible. To this end, the Cobb Douglas function was fitted to the data of each of the five countries using different claimants for each variable; the one giving the best overall results was selected for final analysis.

Selection of the Capital and Output Variables

We have two measures of output in value terms· gross value of output, Y and net value added, V. There are several measures of capital, K. The results obtained by the use of some of these measures of K along with total labour, L, in the Cobb Douglas functions

$$Y = A K^{\alpha} L^{\beta} \quad \text{and} \quad V = A K^{\alpha} L^{\beta}$$

will now be given. The equations given below have been selected from a larger set of results in which not only the measures of capital described earlier have been tried but also several other measures with corrections applied for capacity utilisation and the effects of intermediate inputs and inventories. The corrections and manipulations in the measures of capital did not seem to help much. For a proper comparison to be made of the results emanating from different measures of capital and output, we shall use the following suffixed notation:

Capital Variable		Exponent of K	Exponent of L,
K	Net capital assets	α	β
K_{MC}	Machinery and equipment	α_{MC}	β_{MC}
K_{DP}	Depreciation	α_{DP}	β_{DP}
K_{FV}	Value of fuel consumed	α_{FV}	β_{FV}
K_{EL}	Electricity consumed	α_{EL}	β_{EL}
K_{EV}	Value of electricity consumed	α_{EV}	β_{EV}

The rank correlation between several pairs of these different capital measures was found to be highly significant for each pair taken. This should, to a good extent, rule out the need to fit production functions with different claimants.

The following production functions were obtained in the case of India, one of the five countries in our data.

	R^2
$V = A\ K^{0.465}\ L^{0.605}$	0.91
$V = A\ K_{MC}^{0.390}\ L^{0.663}$	0.89
$V = A\ K_{DP}^{0.441}\ L^{0.605}$	0.89
$V = A\ K_{FV}^{0.321}\ L^{0.739}$	0.86
$V = A\ K_{EV}^{0.277}\ L^{0.773}$	0.86
$V = A\ K_{EL}^{0.265}\ L^{0.751}$	0.87

	R^2
$Y = A K^{.466} L^{.519}$	.85
$Y = A K^{.378} L^{.569}$	.84
$Y = A K^{.430} L^{.533}$	.83
$Y = A K^{.384} L^{.582}$	.82
$Y = A K^{.285} L^{.618}$	.82
$Y = A K^{.303} L^{.656}$	.81

It is difficult to choose between V and Y from the above results but perhaps the results with V may be considered preferable. In any case we have chosen to use V in our empirical work. Many empirical production function studies have oreferred to use V instead of Y, for different reasons See for instance, Grilliches and Ringstad(1971).

As for the capital variable, we decided to make use of net capital assets.[1] It would be difficult to handle several representatives of capital in an analysis which is comparative in nature and makes use of a number of production relations. This is not to deny the important role which variables like intermediate inputs, capacity utilisation, percentage of motors operated[2] and capacity of motors used can play in this connection, in improving the quality of the capital variable.

1Some empirical results given on this page and the last were not the final determining factors in the selection of the capital input. Data of other countries were also used Moreover, several, though not all, permutations of variables were tried and the differences in various sets of results were not very large. This is not the best criterion of selecting the variable. An attempt has been made to select a fairly representative variable out of several available

2See for instance, Bautista R.(1975), Industrial capital utilisation in the Phillipphines, Mimeo, IBRD, Heathfield D.(1972) The measurement of capital usage using electricity consumption data for the UK, J of the Royal St.Soc.II,135, Hilton K.(1970), Capital and capacity utilisation in the U.K., Disc.Paper 7003, Econometric Model Progress Paper A6, Univ.of Southampton; Kim Y. & G.Kwon(1973) Capital utilisation in Korean mfg., Morawetz D. (1975), The electricity measure of capital utilisation, The Maurice Falk Inst.for Econ.Research in Israel, Discussion Paper 755.

Choice of the Labour Variable

There are several measures of labour input in its different aspects available to us

L total labour

L_D direct labour

L_I indirect labour

L_E educated labour

L_O other labour

L_N net labour

H total working hours

H_D direct working hours

H_I indirect working hours

H_N net working hours.

It may be noted that

$$L = L_D + L_I = L_E + L_O$$

$$H = H_D + H_I$$

$$L_N = L_D + L_I . w_I / w_D$$

$$H_N = H_D + H_I . w_I / w_D$$

where w_I and w_D are the wage earnings of direct and indirect workers respectively. The rank correlation[1] between pairs of some of these variables, like L, L_D, L_N was found to be significant. Also total working man hours instead of total manyears did not improve upon the latter in that they differ more or less by a constant multiple.

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[1] Derived independently

Using a procedure similar to that for capital, it was felt that total labour could be the appropriate choice for our study. The use of net labour or net working hours did not seem to improve upon the results obtained with the help of total labour. The reason possibly may lie in that, from a technical point of view, which is more relevant for production function analysis, weighting by wages may be inappropriate as it is likely to introduce an element of bias against indirect labour.[1]

1It is not implied that indirect labour contributes to production in the same way as direct labour. Direct labour without support from indirect labour is not all that helpless but the "indirect" still remain to be done. The use of direct labour to do these jobs in the absence of indirect labour implies loss of resources in the same way as the use of one type of equipment to do some job which could be more appropriately handled by a different type of equipment.

As for the administrative functions, the role and range of activities performed by the administrative personnel have increased considerably in an age of rapid technological progress which is accompanied by a growing complexity of production and financial planning. High managerial skills are essential not only in advanced economies but also in developing economies which may import their technology from advanced economies and are thus obliged to use their methods too.

* * * *

An examination of capital and labour variables together in the different Cobb Douglas fits suggested an obvious choice of net capital assets and total labour for purposes of production analysis.

To sum up, only four basic variables will be used subsequently although in some cases, a few other variables will be needed. The four main variables are K,L,w and V. The other variables used in some models will be L_D (direct labour), L_I (indirect labour), M (raw material), Y (total value of output) and r (rate of return). The four main variables will also be used in certain combinations in different functional forms and with some transformations. Although we have arrived at this small number of variables after a careful choice made out of numerous possibilities and also after a very large number of regression fits, several regressions will still have to be estimated. We will have fifteen production models in eleven of which V, value added or V/L, value added per unit of labour or V/K, value added per unit capital will be the dependent variable; in one case Y, total value of output will be the dependent variable. Other dependent variables have been used in the last three cases.

Nature of Aggregation in our Establishment Data

The estimates of micro production functions should directly reflect the micro technologies because at low levels of aggregation extraneous influences should be less important. As observed by Bosworth(1976), the "problems of aggregationcan result in failure of aggregate production functions to reflect the underlying technology of production. Such fears give micro studies much of their appeal."

Our establishment data, collected systematically by experts, may be assumed to be relatively free from extraneous influences. But they are not free from a certain amount of aggregation which is in the form of addition of outputs as well as inputs corresponding to various processes within each establishment. In any case, strictly homogeneous outputs and inputs are difficult to obtain. An establishment is generally required to produce a variety of outputs which are usually added by weighting by prices. There are differences in the quality and intensity of input use and intermediate products. Different items of capital are not all homogeneous, alternative items may produce the same output per unit of time. There may be variations in the durability, quality and performance of different items of capital. The same can be said about labour.

The problem of homogeneous output is not solved even if we are strict about the choice of the establishments by selecting only those that should be working on comparable lines and producing similar if not identical outputs. The variations in output per unit of input will not be easily observable and the efficiency in the use of resources may be different. Also, market imperfections may affect the establishments differently.

As we would like to be confined to the single output production functions, the output of each establishment will be measured in value terms. This will not involve the problem of variations in the mix of intermediate goods used.

At the level of individual establishment data, it is possible to construct appropriate and more accurate series with an improved degree of homogeneity in outputs and inputs though the extent of improvement depends on the number and complexity of the underlying production processes. Unfortunately we cannot take full advantage of the facility of having a more homogeneous series because, the number of observations for each industry being small, we have to form a pooled series of establishments belonging to different industries.

Our aggregation is in value terms so that the marginal products will also be in value terms. This is a convenient compromise because we have different industries pooled together. That would be difficult unless we brought in the medium of value.

Such aggregation is not quite valid unless there are constant returns to scale. This condition would apply even when the outputs are all identical and that is a condition which is hard to obtain. Our results may suffer, therefore, from some aggregation bias which cannot be avoided in any case because the basic conditions of identity of production functions and constant returns to scale may not necessarily be satisfied in practice.

POOL REGRESSIONS

GROUP REGRESSIONS

ANALYSIS OF COVARIANCE

CHAPTER FIVE

EMPIRICAL RESULTS

POOL REGRESSIONS

Introduction

This chapter gives the main statistical results and a summary of findings. The description of our data has already been given. There are 64 manufacturing establishments from France, 117 from India, 69 from Israel, 63 from Japan and 145 from Yugoslavia. The observations from different countries have not been pooled together : the data for each countryhave been analysed separately and comparable results and conclusions presented. In view of the nonavailability of a sufficiently large number of establishments belonging to each of the several standard industrial classification (SIC) numbers or even related SIC numbers all establishments in each country have been looked upon as productive units which make use of different quantities of certain inputs and show differences in economic performance. Different criteria with their various interpretations have been exploited to rearrange the establishments in a suitable order which helped in forming possibly more homogeneous groups whose characteristics could be subjected to a proper analysis of covariance.

The Three Stages of Analysis

The results are analysed in three stages. In the first stage, all the establishments in each country have been taken as observation units and various production models have been fitted to them on identical lines. These have been called pool regressions.

The results for each of these regressions, corresponding to different production models are given in alphabetical order for all the five countries in the first row of each block in tables 1 to 6, Appendix

The second stage consists of the corresponding results for the groups of establishments formed on the basis of K, the capital assets of establishments in each country. Each group includes nearly one third of the total number of establishments belonging to the small K, medium K and large K categories. The results obtained from grouping based on alternative criteria are also given and compared.

In the third stage, the analysis of covariance is given. Various sets of results are obtained but the main results are for groups formed on the basis of K.

The tables showing the results for some alternative criteria for grouping are given and analysed.

The Production Relations Used in the Empirical Study

For the empirical analysis, fifteen forms of production relations have been selected. All these relations, derived from the original forms, have been in use in empirical studies in the literature though some of them like the Cobb Douglas and the CES functions have been used more commonly.

The explanatory side of these relations will be looked upon as having been composed of technical and /or economic variables. The effect of the inclusion of different variables in the relations will be studied with the help of regression analysis. The ordinary least squares method will be used throughout.

The regressions fitted to the total number of manufacturing establishments in each country will be called pool I regressions. In the next chapter where grouping of the establishments is done, the regression equations fitted to the groups will be called group regressions for which the same production relations and ordinary least squares method as for the pool I regressions will be used. Table 0 gives the mathematical expressions for the fifteen relations used and the order in which they will occur in the analysis.

Table 0 is in six parts.

Part one shows the Cobb Douglas production function (1a) with two inputs K,L and (1b) with three inputs K,L,M.

Part two shows the Cobb Douglas function (2a) with three inputs K, L_D, L_I and also(2b) with three inputs K, L_E, L_C.

Part three has (3a) the Kmenta approximation and (3b) the translog production function.

Part four consists of (4a) the CESL relation between output per unit of labour and wages per unit; (4b) the VESL1 relation between labour productivity and wages per unit and capital labour ratio as explanatory variables, it has been denoted by VESL1 to distinguish it from (4c) the VESL2 relation between labour productivity and wages per unit, capital labour ratio and labour as explanatory variables.

Part five has three relations corresponding to those of part four. (5a) The CESK is the relation between output per unit of capital and rate of return, (5b) the VESK1, the relation between output per unit of capital and rate of return, r and labour capital ratio L/K, and (5c) the VESK2, the relation between output per unit of capital and r,L/K and K.

Part six has (6a), (6b), two side relations of the CES function and (6c) derived from the CES function.

Table O

The Fifteen Production Relations used in the Empirical Analysis

PART ONE

1a) Cobb Douglas Two inputs K,L $V = A K^{\alpha} L^{\beta}$	1b) Cobb Douglas Three inputs K,L,M $Y = A_M K^{\alpha_M} L^{\beta_M} M^{\gamma_M}$

PART TWO

2a) Cobb Douglas Three inputs K, L_D, L_I $V = A_{DI} K^{\alpha_{DI}} L_D^{\beta_D} L_I^{\beta_I}$	2b) Cobb Douglas Three inputs K, L_E, L_O $V = A_{EO} K^{\alpha_{EO}} L_E^{\beta_E} L_O^{\beta_O}$

PART THREE

3a) Kmenta Approximation $\ln V = A_K + \alpha_K \ln K + \beta_K \ln L + \delta_K (\ln K/L)^2$	3b) Translog Prod. Fn. $\ln V = A_T + \alpha_{T1} \ln K + \beta_{T1} \ln L + \alpha_{T2} (\ln K)^2 + \beta_{T2} (\ln L)^2 + \gamma_T \ln K . \ln L$

PART FOUR

4a) CESL $\ln V/L = a_L + b_L \ln w$	4b) VESL1 $\ln V/L = a_{L1} + b_{L1} \ln w + c_{L1} \ln K/L$	4c) VESL2 $\ln V/L = a_{L2} + b_{L2} \ln w + c_{L2} \ln K/L + d_{L2} \ln L$

PART FIVE

5a) CESK $\ln V/K = a_K + b_K \ln r$	5b) VESK1 $\ln V/K = a_{K1} + b_{K1} \ln r + c_{K1} \ln L/K$	5c) VESK2 $\ln V/K = a_{K2} + b_{K2} \ln r + c_{K2} \ln L/K + d_{K2} \ln K$

PART SIX

6a) $\ln K/L = a_D + b_D \ln w/r$	6b) $\ln wL/rK = a_S + \frac{1-b_S}{b_S} \ln K/L$	6c) $\ln L = a_V - b_V \ln w + c_V \ln V$

The Multiple Regression Results. Tables 1-6.

The empirical results for the fifteen production relations have been summarised in six tables which correspond to the six parts of table 0.

Tables 1, 2 and 3 give two forms each and tables 4, 5 and 6 cover three forms each. The tables give for each form, the ordinary least squares regression coefficients which in some cases have a straightforward economic meaning while in others the meaning can be worked out.

The vertical lines in each of the tables 1-6 separate the results for different production relations. The value of R^2 is given in each case. The first row in each block gives the pooled regression coefficients along with their t-values. The other rows in each block correspond to groups and will be considered in the following chapter.

The result of the pool regression of each model will be studied on its own as well as in comparison with other models. Changes in results obtained as we move from one production relation to the other will be noted. In particular, the effects of alterations made in a production form and of the addition of technical and economic variables in a form will be considered.

The numbering of the equations is in strict correspondence with the numbering of the tables. The two sides of table 1 correspond to two production equations which are numbered (1a) and (1b) throughout. Table four has three parts which correspond to production equations numbered (4a), (4b) and (4c), and so on. Th correspondence between the table numbers and equation numbers is maintained in all the following chapters.

POOL REGRESSIONS

The results and analysis of each of the fifteen production relations follows. There will be references to the tables in the appendix but it will not be always necessary to refer to them because the essential parts of each table are given along with the text.

The Cobb Douglas Function

The left side of table 1, appendix, gives the results for the two input Cobb Douglas function

$$V = A\ K^{\alpha} L^{\beta} \tag{1a}$$

On the right are the results of the three input Cobb Douglas function

$$Y = A_M\ K^{\alpha_M} L^{\beta_M} M^{\gamma_M} \tag{1b}$$

where K,L,M,V,Y stand for capital, labour,raw materials, value added and value of total output respectively. The first line of eachblock in the table shows that the coefficients of all the factors in both (1a) and (1b) are statistically significant. The fits are good and the Cobb Douglas function seems to represent the data very well.

Table 1a(1) gives the summary results for all the countries in the case of the two input Cobb Douglas function (1a). The returns to scale parameter $\nu = \alpha + \beta$ is near unity for all the countries. To check for the constancy of returns to scale, the two input Cobb Douglas function in the form

$$V/L = A\ (K/L)^{\alpha}\ L^{\alpha+\beta-1}$$

was fitted to the data and the significance of $h = \alpha + \beta - 1$ was tested against the alternative h = 0. All the h values were found to be insignificant and we conclude that constant returns to scale prevail for the manufacturing establishments of all the countries under consideration.

Table 1a (1)

The Two Input Cobb Douglas Production Function $V = A\,K^{\alpha}L^{\beta}$

		R^2	$\nu = \alpha + \beta$
France	64 firms		
$\ln V = 2.88 + 0.197 \ln K + 0.752 \ln L$		0.89	0.95
(4.6) (13.2)			
India:	117 firms		
$\ln V = 0.51 + 0.465 \ln K + 0.605 \ln L$		0.91	1.07
(11.0) (10.5)			
Israel	69 firms		
$\ln V = 2.97 + 0.157 \ln K + 0.656 \ln L$		0.84	0.81
(3.8) (9.53)			
Japan:	63 firms		
$\ln V = 1.46 + 0.456 \ln K + 0.529 \ln L$		0.96	0.99
(7.0) (6.1)			
Yugoslavia:	145 firms		
$\ln V = 0.17 + 0.335 \ln K + 0.641 \ln L$		0.85	0.98
(8.3) (13.7)			

Use of Raw Material Input

The omission of material input from production function studies is implicitly assumed with value added as a measure of output. It may be useful to check if raw material input(M) has a role to play with the help of Cobb Douglas function.

According to Klein(1962), M as an input should be treated the same way as K and L because it may not have any fixed relation with output; at different levels of operation there may be economies or diseconomies of scale in the use of M. Minhas(1963)takes raw materials as a constant proportion of output, and thus assumes a zero elasticity of substitution between material input and value added since $M = aY$, $V=Y-M=(1-a)F(K,L)$ so that V may be used instead of Y if a is a constant or is uncorrelated with K or L. Alternatively,the assumption may be made that the elasticity of substitution between V and M in $g[F(K,L),M] = g(V,M)$ is infinite.According to Griliches and Ringstad(1971),"the role of materials may be intermediate or more complicated than either one of these two extreme models."

Different countries may face different sets of circumstances in procuring their raw materials. Costs may be different depending on time and place and the production function may not remain uninfluenced by the introduction of this input. In particular, it may be interesting to know if the production function results for different countries show some uniformity of reaction with respect to raw material input. We will make use of the three input Cobb Douglas function for the purpose.

With all factors in physical terms in the production relation $Q=F(K,L,M)$,the situation could be easily analysed if all the establishments produced identical goods with uniform, identical units of all inputs. In practice, every establishment manufacturs a number of items and uses a large variety of inputs Both the sides, therefore need to be suitably weighted. Unfortunately the measure of capital weighted by price is not necessarily related to that for the output. L is usually measured in man-days or man-hours. M is measured in terms of costs incurred which, again, may have no proper connection with output or finished goods prices. The anomalies are difficult to reconcile as the quantities on the two sides of the production function correspond to different units or to different systems of pricing. There may be a significant time and place lag between the sets of prices of raw material and output. The capital component may be valued, revalued, depreciated or deflated in different ways. The valuation prices may be highly unreliable or inconsistent in most cases. The depreciation rates may differ from one establishment to another.

In spite of these difficulties the money values of the concepts can help and we have replaced Q by Y, the gross value of output (not by V, the value added) and assumed that K and M, instead of being physical quantities, have been valued suitably in money terms. This makes it easier for us to put together establishments belonging to different industries.

On the right side of table 1 appendix, we have the detailed results of the three-input Cobb Douglas production function

$$Y = A K^{\alpha_M} L^{\beta_M} M^{\gamma_M} \quad (1b)$$

where Y is the value of total output

Table 1a(2)

Regression Coefficients of the Three Input(K,L,M) Cobb Douglas Function

Country	Constant	α_M	β_M	γ_M	$\alpha_M+\beta_M+\gamma_M$	R^2
France	2.45	.14	.67	.14	.96	.90
		(3.1)	(10.4)	(2.4)		
India	-0.08	.33	.48	.28	1.09	.93
		(7.2)	(8.2)	(5.1)		
Israel	2.21	.08	.64	.18	.89	.86
		(1.7)	(9.7)	(2.9)		
Japan	0.90	.31	.53	.19	1.04	.93
		(4.2)	(6.6)	(3.2)		
Yugoslavia	0.37	.24	.48	.31	1.03	.88
		(6.9)	(10.3)	(6.8)		

The value of the constant term has gone down for all the countries except for Yugoslavia. The returns to scale are still constant though more emphatically now than in the two input case.

The coefficients γ_M are significant throughout. The entry of raw material has reduced to a certain extent the statistical significance of the coefficients α_M and β_M. The raw material input does seem to have a decisive influence on other inputs and hence on the production function. Relatively speaking, the coefficient of capital is affected more than that of labour by the entry of raw materials.

Grilliches and Ringstad(1971) justify the exclusion of raw materials from the list of inputs because this facilitates the comparison of results for different industries with different material use intensities and improves the comparability of data for individual establishments even within the same industry. It facilitates aggregation by reducing double counting as the product crosses industry lines on its way to final consumption. The estimation and interpretation problems become simpler because of the elimination of raw material from both sides of the production relation. Raw material is an "asymmetric" input, related to gross production output level so that its inclusion as an independent variable may obscure the production relation and lead to a simultaneous equation bias inasmuch as this factor is more endogenous than labour or capital.

We find that the raw material input does play an important role as an explanatory factor in production function analysis. In an establishment, it is an input factor like capital or labour. It results in a more complete and better model nearer to reality. We note that its inclusion does influence the results of other inputs and comparable results for different countries are obtained, the technologies in different countries all seem to be influenced by the raw material factor.

Quality of Labour

We now consider the three input Cobb Douglas function with a difference. The quality of labour has been known to make a considerable difference to the production function, as should be evident from the study of Layard et al(1971) who treat several education groups in an interplant study in the electrical engineering industry. It may be assumed that the productivity of direct labour is different from that of indirect labour and that the costs and benefits pertaining to the two are different.

If therefore, we divide labour into two parts, say direct (L_D) indirect (L_I), we have a three-input (K, L_D, L_I) Cobb Douglas function to compare the results with those of the two-input Cobb Douglas form(1a). By comparing the roles of different qualities of labour we shall also consider whether such a three-input case brings different countries closer together in their production pattern or tends to pull them apart.

Two different sets of Cobb Douglas results are shown in table 2, appendix. (or table 2a(i) below) The left sid shows them when direct (L_D) and indirect (L_I) labour are used along with net capital assets (K) :

$$V = A_{DI} K^{\alpha_{DI}} L_D^{\beta_D} L_I^{\beta_I} \qquad (2a)$$

and the right side shows the results with educated (L_E) and other (L_O) labour along with K

$$V = A_{EO} K^{\alpha_{EO}} L_E^{\beta_E} L_O^{\beta_O} \qquad (2b)$$

The suffixes used are self-explanatory and total labour

$$L = L_D + L_I = L_E + L_O$$

We note that all the coefficients are significant and the fits are good throughout. The returns to scale are constant.

The relation (2a) can be useful in examining the contribution of marginal product of indirect labour as compared to that of direct labour. Also, if the estimate of β_I, the output elasticity with respect to indirect labour happens to be insignificantly different from zero it would mean that indirect labour contributes little to output. A significant β_I would imply a justification of its use and a separate consideration. Any results obtained would be qualified to a certain extent in that the establishments do not face similar technological possibilities nor do they belong to the same industry.

Table 2a(1)

Regression Coefficients of the Three Input Cobb Douglas Production Function

$V = A_{DI} K^{\alpha_{DI}} L_D^{\beta_D} L_I^{\beta_I}$					$V = A_{EO} K^{\alpha_{EO}} L_E^{\beta_E} L_O^{\beta_O}$			
α_{DI}	β_D	β_I	$\alpha_{DI}+\beta_D+\beta_I$		α_{EO}	β_E	β_O	$\alpha_{EO}+\beta_E+\beta_O$
.14 (3.5)	.33 (5.3)	.45 (7.2)	.92	France	.19 (4.5)	.19 (3.6)	.60 (9.7)	.97
.46 (10.0)	.48 (6.9)	.13 (1.9)	1.08	India	.44 (9.1)	.16 (2.8)	.48 (8.8)	1.07
.16 (3.7)	.44 (5.5)	.21 (2.9)	.81	Israel	.16 (3.7)	.26 (4.5)	.40 (5.6)	.82
.42 (7.0)	.13 (1.6)	.36 (6.6)	.92	Japan	.46 (7.6)	.26 (5.7)	.17 (3.3)	.89
.35 (10.2)	.28 (4.1)	.34 (6.5)	.96	Yugo-slavia	.29 (8.6)	.30 (6.2)	.36 (6.0)	.95

Tables 1a(1), 1a(2) and 2a(1) corresponding to the two-input and three-input Cobb Douglas cases show that returns to scale for the establishment data tend to be nearly constant. It is also noticed that raw material as well as different varieties of labour are all essential factors in the study of the Cobb Douglas production function as applied to our data. The introduction of raw material influences the significance ofthe capital variable in the two-input case but the quality-wise break-up of labour does not influence capital. While it is true that each type of labour has a different role to play, it is also true that different types of labour may be pooled together. This may be due to our inability, in the absence of adequate data, to make a more elaborate classification of different types of labour or due to an inadequate number of classes. Also ag regation and valuation in money terms, of net output, may conceal the proper role played by different types of labour. It is also likely that the disaggregation of only one of the variables may not be adequate . This implies that production is a joint effort on the part of all the factors. The significant values of the regression coefficients show that both direct and indirect labour are essential for production in any country. The same may be said about educated and other labour.

The alternative classifications of labour do not seem to influence the role of capital whose regression coefficients remain practically unaltered.

In France, Japan and Yugoslavia, where the difference between the wages of direct and indirect labour is not very large, the regression coefficient of indirect labour has a higher value than that of direct labour. In the case of the division based on educated and other labour, the higher share of other labour, as compared to that of educated labour is discernible in all the countries.

The capital share, α , is proportionately higher inthe relatively less developed countries and lower in the relatively advanced countries in our data. The labour share, β , is relatively lower in developing countries. In this respect, Japan, though economically an advanced country is an exception and shows a high capital share in manufacturing with relatively less being offered to labour. With the transfer of technology from the developed to the developing countries, perhaps, a gradual elimination of differences between the β's of different countries may be expected.

Coming back to table 2a(1), we notice that the three input(one capital and two labour inputs) case bears a resemblance with the two input case of equation (1a). The coefficients of capital from (1a) are similar to those obtained from (2a) and (2b). At the same time the coefficients of labour from the two input case seem to have just split into two parts for each country This is evident from table 2a(2).

Table 2a(2)

The Coefficients of Capital and Labour in the Two Input($V=AK^{\alpha}L^{\beta}$) and the Three Input($V=AK_{DI}^{\alpha_{DI}}L_D^{\beta_D}L_I^{\beta_I}$) cases

Equation	France	India	Israel	Japan	Yugoslavia
(1a) α	.20	.47	.16	.46	.34
(2a) α_{DI}	.14	.46	.16	.42	.35
(1a) β	.75	.61	.66	.53	.64
(2a) $\beta_D+\beta_I$	.78	.61	.65	.49	.62

The closeness of results between countries is remarkable and suggests a uniformity of pattern in all the countries.[1]

[1] For some statistical tests see p 208a-e

Kmenta Approximation and the Translog Production Function.

Table 3, Appendix

The Taylor series expansion up to the second order terms of the CES function

$$V = \gamma [\delta K^{-\rho} + (1-\delta) L^{-\rho}]^{-\nu/\rho}$$

may be written

$$\ln V = \ln \gamma + \nu\delta \ln K + \nu(1-\delta) \ln L - \tfrac{1}{2}(1-\delta)\nu\rho\delta(\ln K/L)^2 \qquad \text{or,}$$

$$\ln V = \ln \gamma_K + \alpha_K \ln K + \beta_K \ln L + \delta_K \ln (K/L)^2 \qquad (3a)$$

where the suffix K is for the first letter of Kmenta and

$$\gamma_K = \gamma \;, \quad \alpha_K = \nu\delta \;, \quad \beta_K = \nu(1-\delta), \quad \delta_K = -\tfrac{1}{2}\nu\rho\delta(1-\delta)$$

In the Kmenta approximation δ depends on the units of measurements. The parameters are constant over the whole range of output, as in the Cobb Douglas case. Kmenta (1967) has shown empirically that if the second order term is included the error resulting from neglecting higher order terms is not serious unless both K/L and ρ are either very high or very low though according to Nerlove(1967), the error can be substantial. Since the coefficient of the omitted term is likely to be negative the Cobb Douglas function will tend to yield a low output elasticity of capital and a high output elasticity of labour. Moreover capital and labour are likely to be related to the residual of the production function.

In the case of manufacturing establishments it is difficult to imagine capital and labour to be uncorrelated with the residual in the regression. If the residual were a stochastic element, expected profit maximisation by the entrepreneurs would make the independence of capital and labour from the residual valid. According to Nerlove(1967), in a crosssection of firms, it is generally more reasonable

Table 3a(1)

A Comparison of the Values of Regression Coefficients of Cobb Douglas function and Kmenta Approximation

Cobb Douglas Function				Kmenta Approximation		
Coefficient of	α (ln K)	β (ln L)		α_K (ln K)	β_K (ln L)	δ_K $(\ln K/L)^2$
	.20 (4.6)	.75 (13.2)	France	-.09 (.57)	1.05 (6.4)	.05 (1.9)
	.47 (11.0)	.61 (10.5)	India	.34 (2.3)	.73 (5.1)	.04 (.93)
	.16 (3.8)	.66 (9.5)	Israel	-.11 (.80)	.93 (6.2)	.05 (2.1)
	.46 (7.0)	.53 (6.1)	Japan	.43 (1.7)	.56 (2.1)	.01 (.12)
	.34 (9.3)	.64 (13.7)	Yugo-slavia	.12 (1.8)	.88 (11.5)	.09 (3.8)

to assume that the residuals reflect differences among firms such as "the possession of nonmeasured amounts of other factors and so are known to the decision makers who then allow for such differences in optimising input levels, thus producing a correlation between them and the residuals."

Equation (3a) may be looked upon as an extension of Cobb Douglas function with an additional explanatory factor involving the capital labour ratio. The equation also results from a Taylor series expansion of certain variable elasticity of substitution functions. For instance, see p.89-91.

Kmenta approximation is homothetic and nonhomogeneous and is an interesting form on its own.[1]

As can be seen from table 3, appendix, or table 3a(1) hereinabove, the pool regression fits for the Kmenta approximation are satisfactory but the regression coefficients are not all statistically significant δ_K, the coefficient of $(\ln K/L)^2$, is not significant except in the case of Israel and Yugoslavia though the overall results are unsatisfactory in all the cases. The introduction of the additional term $(\ln K/L)^2$ alters the Cobb Douglas results in a major way.

1An advantage of using the Kmenta approximation is that it does not require any assumptions to be made about factor rewards. Although α_K and β_K are not invariant with regard to units in which K and L are measured, there is no such problem with $\alpha_K + \beta_K$ and δ_K in equation (3a).

It may be noted, however, that the Kmenta approximation can be used to test the null hypothesis that the elasticity of substitution is unity provided the basic production function is of the CTS form.

Both the labour and the capital coefficients are affected by the introduction of $(\ln K/L)^2$ as an explanatory factor in the Cobb Douglas function. All the t values are now much diminished. This still leaves the coefficients of labour significant but the coefficients of capital are affected much more and are rendered insignificant in practically all the cases. The ratio K/L contains the effect of both K and L but it could be easily used as a substitute for K rather than for L. Being a measure of intensity of capital per unit of labour, K/L can prove to be a suitable explanatory factor in some cases. In the case of some countries, capital intensity may be a more decisive factor in production than the absolute value of capital.

There may be other reasons for the insignificant or negative values of the capital coefficients. There may be a high correlation between the independent variables and this may make it difficult to separate their effects on the dependent variable. The addition of the explanatory factor $(\ln K/L)^2$ may also be looked upon as a misspecification. The assumptions associated with the production function may, at times, lead to anomalous results.

The collinearity between K and L need not have the same effect on all the samples It is not a statistical problem, it is a problem arising out of the insufficiency of information that could separate the contribution of capital from that of labour.

Douglas and Bronfenbrenner(1939) arrive at these "nonsense parameters" in their cross section study of American industry in which they refer to the reality of the instability problem

There are several production function studies which show that multicollinearity may lead to problems of structural estimation and specification error.

The Translog Production Function

The translog function may be looked upon as an extension of the Cobb Douglas function as well as the Kmenta approximation. It is a nonhomogeneous form with all technical explanatory factors and may be written

$$\ln V = \ln A + \alpha_{T1} \ln K + \beta_{T1} \ln L + \alpha_{T2} (\ln K)^2 + \beta_{T2} (\ln L)^2 + \gamma_T \ln K . \ln L \quad (3b)$$

As can be seen from table 3, appendix, where the pool regression results are given, the statistical fit for all the countries are very good but the presence of multicollinearity may be suspected. Some estimators have relatively large variances and may be imprecise. The coefficients of ln L and lnK.lnL in almost all the cases seem to suggest that these variables are redundant for the data. On the whole, it would be difficult to place much confidence in the individual parameter estimates.

This function will not be used for analysis of group regressions in the next chapter although it will be subjected to analysis of covariance with reference to our data.

The CES and VES Functions   Table 4 and 5 Appendix.

The CES function in the form

$$V = \gamma [\delta K^{-\rho} + (1-\delta) L^{-\rho}]^{-\nu/\rho} \qquad (3)$$

requires the use of nonlinear methods for the estimation of its parameters. One way out is the use of Kmenta approximation which has given good results in some production function studies.

A simplified version of (3) under the assumptions of constant returns to scale, perfect competition in the factor and production markets and profit maximising conditions is the productivity relation connecting labour productivity V/L with the wage rate w.

$$\ln V/L = a + b \ln w \qquad (4a')$$

where $a = b \ln \gamma/(1-\delta)$ is a constant and b is found to be the elasticity of substitution parameter so that $b = \sigma = 1/(1+\rho)$.

Since capital assets K and rate of return on capital r are not used in this equation and since there is a parallel relation connecting these two we may rewrite (4a') with suitably suffixed parameters and call it

$$CESL \cdot \ln V/L = a_L + b_L \ln w \qquad (4a)$$

the parallel relation using capital is

$$CESK \quad \ln V/K = a_K + b_K \ln r \qquad (5a)$$

The relation (5a) is not in common use because of lack of data on K and r and the bias resulting from the way V,K and r are measured. Of course, the relation (4a) is also not free from bias. Pederson(1972) has investigated the direction of bias when capital data are used in the estimation. He has found empirical support for the hypothesis that the bias in the value of b is towards unity when labour data are used. Pederson tested the hypothesis by varifying the results for two digit manufacturing industries in the U.S.

Tables 4 and 5, Appendix, show that all the regression coefficients b_L and b_K which stand for elasticities of substitution using labour and capital data respectively, are significant for all the countries. Pederson's hypotheses that b_k should be less than b_L is consistently satisfied. This seems to suggest that CESK tends to give lower estimates of elasticity of substitution than CESL does. Moreover, with only one exception, all the b values are less than unity as Pederson's hypotheses suggest. See table 4a(1)

It would be convenient to study the CES function results along with those of the VES functions as the latter may be considered as extensions ofthe former. The introduction of the V S functions, at this stage can simplify the comparison.

Table 4a(1)

The Elasticity of Substitution obtained from

CESL b_L		CESK b_K
.81	France	.78
.79	India	.75
.87	Israel	.89
.85	Japan	.62
1.36	Yugoslavia	.93

Note. All the t values are significant at 5/ level

For the VES function we shall use the Lu-Fletcher(1968) generalisation

$$V = \gamma \left[\delta K^{-\rho} + (1-\delta)\eta (K/L)^{-\lambda(1+\rho)} L^{-\rho}\right]^{-1/\rho}$$

This results in the productivity relation

$$\ln V/L = a + b \ln w + c \ln K/L$$

which being an extension of the CES productivity relation, is obtained from it by the addition of the factor K/L. It will be written with the suggestive suffixes involving L ·

$$\ln V/L = a_{L1} + b_{L1} \ln w + c_{L1} \ln K/L \qquad (4b)$$

to distinguish it from the parallel relation

$$\ln V/K = a_{K1} + b_{K1} \ln r + c_{K1} \ln L/K \qquad (5b)$$

with suffixes corresponding to the capital variable K.

These two relations will be called, respectively VESL1 and VESK1 to distinguish them from yet another set of relations.

The addition of the explanatory factor L to the VESL1 results in

$$\ln V/L = a_{L2} + b_{L2} \ln w + c_{L2} \ln K/L + d_{L2} \ln L \; (4c)$$

This will be called VESL2. It is due to Tsang and Yeung (1974)

A parallel relation based on K may be called VESK2 and written

$$\ln V/K = a_{K2} + b_{K2} \ln w + c_{K2} \ln L/K + d_{K2} \ln K \; (5c)$$

The results for the equations (4a), (4b), (4c) are given in the form of the consolidated table 4a(2)

Table 4a(2)

POOL REGRESSION COEFFICIENTS

CESL	VESL1			VESL2		
b_L Coef. of w	b_{L1} Coef. of w	c_{L1} Coef. of K/L		b_{L2} Coef. of w	c_{L2} Coef. of K/L	d_{L2} Coef. of L
.81	.75	.18	France	.74	.18	-.<u>01</u>
.79	.50	.37	India	.49	.37	.<u>02</u>
.87	.84	.09	Israel	.78	.12	-.12
.85	.68	.39	Japan	.86	.41	-.12
1.36	1.12	.25	Yugoslavia	1.12	.25	-.<u>02</u>

All t values except those in the underlined cases are significant at 5% level.

Table 4a(2) presents the gradual change in the values of the parameters as the CFSL relation is generalised by first inserting K/L as an additional explanatory factor into the CFSL and then introducing L as a further explanatory factor into the resulting VFSL1 relation As we move from the CESL to the VESL1, a point easily noted is that while the coefficient of w continues to remain significant, that of K/L also emerges significant in all cases. However, a larger change in the coefficient of w is noticed in the case of India and Japan where the role of capital is quite prominent as we found with the help of Cobb Douglas function.

The results do not continue to improve when we move from the VFSL1 to the VESL2 While the coefficients of w and K/L remain practically unchanged, the significance of L is shown in the case of two out of five countries. The possibility of obtaining significant values of the coefficient of L for some other data is not completely ruled out.

It can be said that output per unit depends both on the wage rate as well as the capital intensity and other factors and not on the wage rate alone. The addition of a technical factor to the CESL relation with only an economic explanatory factor or the addition of an economic explanatory factor like w to the Cobb Douglas

Table 5a(1)

POOL REGRESSION COEFFICIENTS

CESK	VESK1			VESK2		
b_K	b_{K1}	c_{K1}		b_{K2}	c_{K2}	d_{K2}
Coef. of r	Coef. of r	Coef. of L/K		Coef. of r	Coef. of L/K	Coef. of K
.78	.36	.62	France	.36	.56	-.07
.75	.57	.32	India	.57	.41	08
.89	.43	.50	Israel	.42	.58	-.02
.62	.51	.47	Japan	.52	.50	.02
.86	.67	.33	Yugoslavia	.67	.32	-.01

All t values except those in the underlined cases are significant at 5% level.

relation in the form $V/L = A\ (K/L)^{\alpha}$ involving only one technical explanatory factor improves the results remarkably. Thus both K/L and w are essential to the VESL1 productivity relation The use of this relation also leads to a reduction in the value of the constant term as obtained from the CES relation for all the countries. At the same time, the statistical significance of regression coefficients shows an overall improvement All this goes to prove the superiority of the VESL1 over the CESL relation.

The movement from the VESL1 to the VESL2 relation does not suggest any noteworthy improvement in the regression fits for all the countries, as obtained from the addition of another explanatory factor L in the VESL1 relation. But while the regression coefficients of L are insignificant in the case of France, India and Yugoslavia they are statistically significant in the case of Israel and Japan showing the greater importance of the role of labour in the manufacturing establishments of Israel and Japan as evidenced by this production function analysis.

It may be noted that, algebraically speaking, the mathematical expression for the VESL2 relation has the same form as that for the VESL1 relation and the consideration of L as a separate factor may be used to test the significance of L in the VESL2 relation.

On the same lines, the inclusion of K as an additional explanatory factor may help in testing the significance of L in the VESK1 relation. From the empirical results given in table 5, appendix, it can be seen that in a change from the VESK1 to the VESK2 function, the coefficients of r and L/K remain practically unaltered in almost all the cases. The pool regression results are given in table 5a(1).

Just as we found that the CESK relation produced better results than the CESL relation on an overall basis so does the VESK1 relation seem to give better overall results than the VESL1 relation. We also note that the change from the CESL to VESL1 (or from the CESK to VESK1) is accompanied by significant coefficients of K/L(or L/K) while the coefficients of w (or r) remain significant and have practically the same absolute values. This implies the incompleteness of the CES productivity relation which on the whole, gives rather unsatisfactory results. The addition of a technical explanatory factor produces immediately improved results and collinearity does not seem to be a serious problem as shown by the results.

The pattern of results obtained with the help of the VES relation is common to all the countries under study and shows the importance of the inclusion of both K/L and w (or L/K and r) as explanatory factors in the production relation.

* * *

We have now analysed the results obtained by a transition from the Cobb Douglas and the CES functions to the VES function . In the matter of its explanatory power as well as in the significance of its coefficients, the VES function seems to do better than the Cobb Douglas as well as the CES function in the case of our manufacturing establishment data. There may certainly be scope for further improvement but this improvement may be difficult to obtain from the use of additional explanatory variables in the production relation, the effect of most variables in common use may be found in w or K/L. Even the effect of w may be contained in K/L.. We have also noticed the not very encouraging results obtained by the use of labour(or capital) as additional explanatory factors in the VESL1 (or VESK1) relations and it seems difficult to go beyond that.

An improvement in results may be possible if, instead of using mixed industry establishment data, use is made of data belonging to individual industries. We are not in a position to do that but we have divided the data into groups of establishments based on various criteria and carried out the analysis in the next chapter

* * * * *

Three Variations from the CES Function

From the CES function

$$V = \gamma[\delta K^{-\rho} + (1-\delta)L^{-\rho}]^{-1/\rho}$$

we can derive the two side relations for the estimation of some of the parameters of the function. If we assume the CES function to be valid and make the assumptions of perfect competition along with cost minimisation with output fixed exogenously, we arrive at

$$K/L = \left(\frac{\delta}{1-\delta}\right)^{\sigma}(w/r)^{\sigma}$$

where $\sigma = \frac{1}{1+\rho}$. This can be written

$$\ln K/L = \sigma \ln \frac{\delta}{1-\delta} + \sigma \ln w/r \qquad (6a)$$

Since neither the returns to scale nor the output price enter this relation, it may be found to be quite useful for the purpose of estimation of the CES parameters. However, the data on capital labour ratio and input price ratio should be available

The relation (6a) between capital labour ratio and the input price ratio connects technical forces and their economic rewards without reference to output. It suggests an interference in the technical process of production by economic factors in that any particular input combination would have created the same amount of output anywhere had it not been for variations in rewards that might bring about major differences in results.

If we keep K/L constant then $\delta/(1-\delta)$ becomes a function of w/r. That means the distribution parameters tend to acquire new values on the basis of factor rewards which differ between countries and hence change $\delta/(1-\delta)$ as well as K/L.

The equation (6a) may be considered to be the expansion path of the firm and implies that the firm decides changes in capital labour ratio in response to changes in input price ratio without any allowance for time lag in the adjustment process. The equation suggests the long run pattern of attaining optimum production under the available technological conditions. If relative prices are independent of the disturbance term or, in other words, given to the firm, the equation (6a) allows the estimation of the distribution parameter δ and the elasticity of substitution σ . There will be simultaneous equation bias if input prices are influenced by decisions about the input ratio. This is less likely to happen in the case of firm level data. Once $\hat{\delta}$ and $\hat{\rho}$ are found the remaining parameters of the CES function can be estimated in the second stage from the additional relation $\ln V = \ln \gamma - \frac{\nu}{\rho}[\hat{\delta} K^{-\hat{\rho}} + (1-\hat{\delta}) L^{-\hat{\rho}}]$ provided the last term on the right is independent of the disturbance term in the production function; or with Kmenta(1964) and Nerlove(1967) it does not give consistent estimates of ν and γ unless K and L are independent of the residual in the production function. They suggest that direct methods of estimating the production function seem more useful. In the CES function, γ is assumed to be a neutral efficiency parameter with a uniform impact on capital and labour efficiency. If there are variations in γ across observations, the capital labour ratio K/L and factor price ratio w/r remain unaltered because γ cancels out in the numerator and denominator, in equation (6a). As no measurement errors are involved an unbiased estimate of σ may be obtained. Moreover, if there is a crosssectional measurement error in L and a corresponding proportional error in K, the matter is statistically indistinguishable from crosssectional variation in γ. If the error in K is not proportional, the estimate of σ is unbiased.

In view of the debate about the appropriate measure of capital as a stock or as a flow of services, an error is likely to arise if one is used instead of the other. Moroney(1970) has shown that if the specification (6a) is used, any measurement error resulting from the wrong use merges with the disturbance term and in that case σ is not likely to be biased under suitable assumptions.

Suppose capital measured as services is λK where λ is some fraction of the observed capital stock. Then, using u for the disturbance term, (6a) becomes[1]

$$\lambda K/L = \left(\frac{\delta}{1-\delta}\right)^{\sigma}(w/r)^{\sigma}\, e^{u}$$

or
$$K/L = \left(\frac{\delta}{1-\delta}\right)^{\sigma}(w/r)^{\sigma}\, e^{u'}$$

where $u' = u - \ln\lambda$ is a new disturbance term, the form (6a) remaining the same. If λ is a constant or is stochastically independent of w/r then σ remains unbiased. λ may be considered as a correction term for crosssection variation in capital quality and hence may be assumed to be independent of w/r. If λ is interpreted as a departure from full capacity utilisation, it is likely to very among industries. As we have combined observations on industries, the σ obtained from (6a) may be biased to a certain extent. At the same time, although industrial production cycles differ, resulting in different values of λ for different industries, λ may be assumed to be fairly constant for all industries <u>within</u>

1 See page 202a

Footnote continued from p.202

The relation

$$\lambda K/L = \left(\frac{\delta}{1-\delta} \right)^{\sigma} (w/r)^{\sigma} \qquad (6aa)$$

was fitted to the data in the form

$$\ln K/L = \sigma \ln \left(\frac{\delta}{1-\delta} \right) + \sigma \ln (w/r) \qquad (6ab)$$

The empirical results were not found to be satisfactory. While the estimates of elasticity of substitution, for France, India, Israel, Japan and Yugoslavia respectively, were

	France	India	Israel	Japan	Yugoslavia
	.85	.89	1.3	.94	.88
t-values	(2.5)	(6.0)	(10.0)	(3.7)	(6.8)

all of them statistically significant, the values of R^2 were extremely low suggesting very poor fits. The introduction of λ did seem to make a difference inasmuch as it pulled up the elasticity of substitution estimates of all the countries except that of Yugoslavia inwhich case the effect was the opposite. This certainly brought out the effect on the manufacturing establishment of different countries of the factor capacity utilisation which seems to even out, to a certain extent, the values of elasticity of substitution leading them closer to unity.

Unfortunately, the group regressions produced much poorer results, with practically all regression coefficients statistically insignificant and much lower values of R^2 than even in the case of pool regressions. Also the F values from analysis of covariance were extremely large.

The fitting of the relation

$$\ln K/L = a_0 + b_0 \ln w/r + c_0 \ln \lambda \qquad (6ac)$$

produced consistently low values of the coefficient of λ for pool as well as group regressions associated with statistically poor fits. All c_0's were insignificant.

Both the relations (6ab) and (6ac) have not been considered in the following chapters. This does not necessarily reflect the inadequacy of the relations.

Table 6a(1)

The Values of Elasticity of Substitution obtained from

	(4a)	(5a)	(6a)	(6b)	(6c)
France	.81	.78	.66	1.87	.80
India	.79	.75	.64	1.79	.54
Israel	.87	.89	.80	1.43	.88
Japan	.85	.62	.32	4.00	.63
Yugoslavia	1.36	.93	.97	1.72	1.26

Note All t values are signficant at 5% level

$$\ln V/L = a_L + b_L \ln w \qquad (4a)$$

$$\ln V/K = a_K + b_K \ln r \qquad (5a)$$

$$\ln K/L = a_D + b_D \ln w/r \qquad (6a)$$

$$\ln wL/rK = a_S + \frac{1-b_s}{b_s} \ln K/L \qquad (6b)$$

$$\ln L = a_V - b_V \ln w + c_V \ln V \qquad (6c)$$

each country but not between countries. The extent to which this assumption is violated, may be shown by the values of δ.

The relation (6a) allows the bringing together of different qualities of labour and different vintages of capital whose rates and manner of utilisation may differ from one establishment to another. The left side of appendix table 6 gives the results for (6a). The fits are all poor although the regression coefficients representing the elasticity of substitution are all significant. Practically the same was the case with the CESL relation (4a) which, however, yielded more uniform values of elasticity of substitution. These values along with some other comparable results are given in table 6a(1).

The middle of appendix table 6 gives the results for

$$\ln wL/rK = \ln \frac{1-\delta}{\delta} + \varsigma \ln K/L \qquad (6b)$$

which is the regression of input share ratio on capital labour ratio. This follows if (6a) is written as

$$w/r = \frac{1-\delta}{\delta} (K/L)^{\frac{1}{\sigma}} \quad \text{or } wL/rK = \frac{1-\delta}{\delta} (K/L)^{\varsigma}.$$

The equation (6b) also fits the data rather poorly though the regression coefficients have significant values as can be seen from table 6a(1).

ACMS(1961), using the relation (6b) found contradictory results and blamed the discrepancy on the difference in the assumptions about the error terms implied in the different modes of estimation.

* * *

A Labour Demand Relation

If we allow for the possibility of specification error in the CES productivity-wage relation, we arrive at a statistically more accurate version of the CES function. The specification error can arise out of errors in the measurement of inputs or variability of γ , the index of efficiency, or p, the output price. Moroney(1970) has shown that the consequences of these omissions may be conveniently analysed within a common specification error framework.

If, in the CES function $V = \gamma[\delta K^{-\rho} + (1-\delta) L^{-\rho}]^{-\nu/\rho}$ $\nu \neq 1$, we use $\partial V/\partial L = w$, we have the modified relation

$$\ln V/L = \sigma \ln \frac{\gamma^{\rho/\nu}}{\nu(1-\delta)} + \sigma \ln w + (1-\sigma)\frac{\nu-1}{\nu} \ln V$$

instead of $\ln V/L = \sigma_1 \ln \frac{\gamma^{\rho}}{1-\delta} + \sigma_1 \ln w$. Also

$$E(\sigma_1) = \sigma + (1-\sigma)\frac{\nu-1}{\nu}\frac{Cov(\ln V, \ln w)}{Var(\ln w)} .$$

σ is biased unless $\sigma = 1$ or there is no correlation between ln w and ln V. Other cases of specification error may be considered on the same lines.

Since V enters the modified CES relation exogeneously, although it is specified as endogeneous, it is not quite proper to use V as an additional explanatory variable. If it is assumed that w and V are exogeneous and L is endogeneous we have another model for our empirical study

$$\ln L = \sigma \ln \frac{\nu(1-\delta)}{\gamma^{\rho/\nu}} - \sigma \ln w + \left(\frac{1}{\nu} + \frac{\sigma(\nu-1)}{\nu}\right) \ln V \qquad (6c)$$

The right side of appendix table 6 (table 6a(1) above) gives the results for (6c). The fits are good for all the countries. The values of elasticity of substitution are comparable with those of the CESL relation. They are slightly lower than those of the CESL though the differences in the case of India and Japan are significant as can be seen from table 6a(2).

Table 6a(2)

The Elasticity of Substitution obtained from (4a) and (6c)

	France	India	Israel	Japan	Yugoslavia
Eqn.(6c)	.80	.54	.88	.63	1.26
	(6.9)	(5.5)	(10.5)	(3.0)	(11.2)
Eqn.(4a)	.81	.79	.87	.85	1.36
	(6.9)	(8.4)	(10.7)	(5.2)	(12.4)

In the case of Yugoslavia the difference is small. These differences are likely to be the result of the returns to scale factor which has been introduced into the equation (6c). In the case of India, Japan and Yugoslavia the returns to scale are slightly increasing, and the case of France and Israel slightly decreasing though in either case they are not significantly different from unity. The equation (6c) emphasises these differences by pulling down the values of elasticity of substitution in the relevant cases.

In the case of India, Japan and Yugoslavia, the demand for labour is likely to increase with increasing returns to scale because the total quantity of labour is not as limited as in the case of France and Israel. This is reflected in the results in spite of a very nominal difference in returns to scale

From the results obtained so far, it may not be possible to arrive at any definite conclusions about the uniformity of manufacturing patterns on the basis of mixed manufacturing establishment data. But we find that the parameter values obtained for different countries from the same model tend to suggest such uniformity. The uniformity comes out more clearly in the case of the Cobb Douglas function and its extensions obtained with the addition of technical variables only. The differences seem to come in with the use of economic variables. But, then, economic factors cannot be ignored in any production model. From a practical point of view, the expression for the production function must make use of both technical as well as economic factors in spite of the production function being a technical relationship. This does not change our stand about the basically technical nature of the production function.

The differences that are noticed in the results are also suggestive of differences in the nature and pace of the economies. We cannot assume a similarity of situation in the countries under consideration. While it is true that some kind of postwar beginning was made by each of the countries under consideration, they were different economically, politically and historically. Their rate of development was different and so were the quality and quantity of labour and capital and experience available to them. Yet, on the whole, there is reason to believe that each of the production models used, provides ample evidence of the similarity of the nature of manufacturing establishments of the five countries. This happens through the nature of fits which are either uniformly good or uniformly bad for practically all the countries with reference to any of the various models used. The returns to scale are constant in all the cases. The elasticity of substitution is found to be not differing much from 0.8 for all the countries except Yugoslavia if we draw our conclusions from the CES function. Perhaps a further improvement in the similarity of results is a matter of time which may differ for each country. Unfortunately the argument cannot be substantiated unless data for more countries and individual industries are available.

We now analyse some results based on the Cobb Douglas and CES functions.

The percentage shares of capital and labour as obtained from the two input Cobb Douglas function allow, roughly, the following groups to be formed [1]

	France, Israel	Yugoslavia	India, Japan
Capital Labour share	20·80	35:65	45·55

Although the Japanese economy is advanced the wage pattern has not changed much there. In Yugoslavia worker participation seems to help the labour share go up. The maximum labour share is to be found in the case of France and Israel, 20:80, perhaps an ideal for other countries.

These differences in input shares are int-rnal, and exist in spite of nearly constant returns to scale available in all the cases. The range of capital share is rather wide though that for labour is not so wide. Some details of the internal differences will be revealed when we consider groups of manufacturing establishments in the next chapter where a similar production function analysis is carried out for them

We now consider the CESL and CESF relations (4a) and (5a). Assuming the elasticity of substitution to be constant we note that the σ values for all the 'capitalist' countries are below unity and that for the 'socialist' Yugoslavia above unity.

1 For some statistical tests see page 208a-e

The hypothesis $\sigma = 0$ is rejected for all the countries. The hypothesis $\sigma = 0.8$ is supported by all the countries except Yugoslavia.

The results obtained from the VES function justify the hypothesis that the explanatory power of a production model in practice can be improved only if technical as well as economic variables are included as explanatory factors. Even though, theoretically, only the technical aspect is emphasised by the production function, the economic factors should not be ignored in an economic world.

The good fits obtained in the case of several production models and the satisfactory values of parameters show that if we have data on individual manufacturing establishments, the nonavailability of industry-wise data may not necessarily be a handicap.

A support for these hypotheses and others mentioned in chapter three may be available from an analysis of group regressions and analysis of covariance which are the subject matter of the next two chapters.

THE FIFTEEN PRODUCTION RELATIONS USED IN THE EMPIRICAL ANALYSIS

The equations for the fifteen production relations used for empirical analysis in this study, their numbers used throughout this study and the order in which they occur in all the tables 1-6 anywhere in the text are given below.

Table 1

1a) $V = A\,K^{\alpha}\,L^{\beta}$

1b) $Y = A_M K^{\alpha_M}\,L^{\beta_M}\,M^{\gamma_M}$

Table 2

2a) $V = A_{DI}\,K^{\alpha_{DI}}\,L^{\beta_D}\,L^{\beta_I}$

2b) $V = A_{EO}\,K^{\alpha_{EO}}\,L^{\beta_E}\,L^{\beta_O}$

Table 3

3a) $\ln V = A_K + \alpha_K \ln K + \beta_K \ln L + \delta_K (\ln K/L)^2$

3b) $\ln V = A_T + \alpha_{T1} \ln K + \beta_{T1} \ln L + \alpha_{T2} (\ln K)^2 + \beta_{T2} (\ln L)^2 + \gamma_T \ln K . \ln L$

Table 4

4a) $\ln V/L = a_L + b_L \ln w$

4b) $\ln V/L = a_{L1} + b_{L1} \ln w + c_{L1} \ln K/L$

4c) $\ln V/L = a_{L2} + b_{L2} \ln w + c_{L2} \ln K/L + d_{L2} \ln L$

Table 5

5a) $\ln V/K = a_K + b_K \ln r$

5b) $\ln V/K = a_{K1} + b_{K1} \ln r + c_{K1} \ln L/K$

5c) $\ln V/K = a_{K2} + b_{K2} \ln r + c_{K2} \ln L/K + d_{K2} \ln K$

Table 6

6a) $\ln K/L = a_D + b_D \ln w/r$

6b) $\ln wL/rK = a_S + \frac{1-b_S}{b_S} \ln K/L$

6c) $\ln L = a_V - b_V \ln w + c_V \ln V$

Some Statistical Tests

The purpose of this note is to present some statistical tests which were used to make certain statements occurring in a few places in the text.

In the multiple regression model

$$Y = b_1 + b_2X_2 + b_3X_3 + \dots + b_mX_m$$

the test of the hypothesis that b_1 equals a specified number β_1 i.e. $H_0 \cdot b_1 = \beta_1$, can be carried out[1] by the use of the statistic $t_{n-m} = (b_1 - \beta_1)/s_1$ where s_1 is the standard error of b_1.

To test if, with the Cobb Douglas function(p.186), the capital share for France and Israel is 20%, for Yugoslavia 35% and for India and Japan 45%, so far as our manufacturing establishment data are concerned, we make use of the abovementioned t statistic and find that for France, India, Israel, Japan, Yugoslavia, respectively t= 0.304, .827, .799, .062, .139 all of which are statistically insignificant at 5% or 1% level of significance The null hypothesis was not rejected.

The test of the hypothesis $\beta_D + \beta_I = \beta$ was carried out by using[2] $t = (\beta_D + \beta_I - \beta)/s_{\beta_D + \beta_I}$. All the t values were found to be nearly zero and statistically insignificant. The null hypothesis was not rejected.

To test if the elasticity of substitution as obtained with the help of the CESL relation is around 0.8 for all the countries except Yugoslavia, we make use of the t test once again.

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[1] Kmenta J.(1971), Elements of Econometrics, Macmillan.p.366.

[2] Ibid.p.372.

The t values for France, India, Israel, Japan, Yugoslavia respectively are t = 0.086, 0.106, 0.861, 0.306, 5.105. If the null hypothesis is σ = 0.80, it is not rejected for France, India, Israel and Japan. In the case of Yugoslavia it is rejected.

The hypothesis that the corresponding regression coefficients of all the countries under study are equal was carried out by using the F statistic. The hypothesis was rejected in the case of almost all the production relations we have used in this chapter. It was decided therefore to compare the results for pairs of countries in our study by using the F statistic. As can be seen from table C0 on p.208d, ten pairs of countries were considered and the F statistic was calculated for all these pairs in respect of the fifteen production relations we have been using. Col.1 in the table shows the number of the production relation used and the names of the countries in other columns are abbreviated to the first three letters in each case. Cols.2-11 give the F values for the pairs mentioned above the columns. Only a brief analysis will be given.

Cols. 2,6 and 10 indicate that there is no compatibility between the pairs of countries. France and India, India and Israel; Israel and Yugoslavia, with reference to almost all the production relations. The only relations which bring almost all the countries together are 1. the relation (6a) which does not involve value added and into which neither the returns to scale parameter nor the output price enter, and 2. the relation (6c) which is the labour demand relation. However, TABLE **00**

COMPARISON OF REGRESSION PARAMETERS OF PAIRS OF COUNTRIES

F VALUES ONLY

1 FRC FEL	2 FRA IND	3 FRA ISR	4 FRA JAP	5 FRA YUG	6 IND ISR	7 IND JAP	8 IND YUG	9 ISR JAP	10 ISR YUG	11 JAP YUG
1a)	6.5**	2.0	4.0**	2.0	15.**	1.5	2.8	9.6**	6.5**	1.1
1b)	6.6**	2.2	4.1**	2.4	14.**	2.1	3.0	10.**	6.8**	1.9
2a)	6.7**	2.6	3.8**	2.5	11.**	2.6	3.0	7.4**	6.5**	1.5
2b)	4.6**	2.5	8.2**	2.3	10.**	3.9**	3.3	6.0**	3.5**	2.3
3a)	5.7**	1.7	3.8**	5.9**	12.**	2.1	1.4	7.7**	9.7**	1.5
3b)	5.1**	3.8**	5.0**	3.0	12.**	1.9	1.2	10.**	9.8**	2.2
4a)	25.**	3.1	13.**	5.8**	4.4	11.8**	5.4**	5.2**	6.1**	3.0
4b)	4.2**	1.6	4.1**	4.7**	9.4**	2.9	7.4**	9.1**	6.3**	4.0**
4c)	[illegible]	2.6	5.2**	3.6	8.3**	1.7	5.8**	8.7**	8.0**	4.2**
5a)	15.**	1.7	1.1	2.1	1.9	1.1	5.6**	3.2	6.0**	9.6**
5b)	15.**	1.4	3.8	31.**	11.**	3.2	4.0**	1.9	20.**	9.9**
5c)	16.**	2.5	4.6**	25.**	9.9**	4.0 *	7.6**	1.5	15.**	8.2**
6a)	44.**	1.1	1.7	3.1	1.2	1.6	4.5	4.2	1.4	10.**
6b)	30.**	2.3	1.8	7.5**	1.4	2.2	8.3**	5.3**	11.**	14.**
6c)	2.5	2.2	4.5**	1.1	2.6	2.4	1.2	3.5	1.9	3.7

** INDICATES ONE PER CENT LEVEL OF SIGNIFICANCE there are some pairs of countries which seem to go together with respect to almost all the production relations and some other pairs of countries which show similarity for either the production relations involving only the technical explanatory factors or for the production relations involving only the economic explanatory factors. The pairs France,Israel and India,Japan exhibit common features with respect to almost all the relations. This was noticed also when the pool regressions were being analysed in this chapter. Moreover, a part of this result has already been considered with the help of the t test when the capital shares from the Cobb Douglas function and the elasticities of substitution from the CES function were compared.

An interesting feature of the results is that out of three countries(France,Israel,Yugoslavia in the table) if two pairs (France, Israel and France, Yugoslavia) exhibit some common characteristics with respect to several production relations(Col.3 and Col.5), this is no guarantee that the third pair(Israel,Yugoslavia,Col.10) should necessarily exhibit common characteristics with respect to those production relation.

From the results given in table OO for our manufacturing establishment data it cannot be maintained that all the countries in our study have common production parameters with reference to any of the production relations except perhaps (6a) and (6c) but it can be said that there are groups or at least some pairs of countries which do have common production characteristics with reference to almost any production relation in this study.

CHAPTER SIX

THE GROUP REGRESSIONS

There are several economic and technical criteria which can be used to divide the manufacturing establishment data into a given number of parts. In this chapter, for a detailed study of group regressions, use will be made of the money value of net capital assets, K, to divide the total number of establishments into three nearly equal sized groups. A production function analysis will be carried out on each group so formed. The assumptions, the procedure and the forms of production relations used will be the same as those in the case of pooled data. The nature of the technical parameters in the groups and the problem of returns to scale will be considered.

The group regressions allow a comparative study of the parameters corresponding to the groups. We may be able to study how the parameters change when we move from the group of small size establishments to the group of large size establishments. We can find if the pool regression parameters are supported by identical corresponding values of the group regression parameters or are merely the averages of the values of the group regression parameters.

The three groups into which the establishment data of each country have been divided according to the value of K, will be called Small K, Medium K and Large K groups, each containing about one third of the total number of establishments in the case of each country. The details are given in the table below.

Table Showing the Number of Establishments in the Groups

	France	India	Israel	Japan	Yugoslavia
Total Number	64	117	69	63	145
Small K Group	21	39	23	21	49
Medium K Group	22	39	23	21	48
Large K Group	21	39	23	21	48

The small K group consists of establishments which correspond to the lowest third of the total number of establishments when they are arranged in increasing order of magnitude of K. There is no lower limit in terms of capital but there are no establishments with less than ten workers.

The medium K group consists of the middle third of the establishments and has clearly defined limits for each country. The large K group has the upper third of the establishments. The lower limit is defined but the upper limit is open.[1]

[1] The limits in each case are the limits of the data themselves. This means the limits for different countries are not comparable. This is not uncommon in such studies. Had we insisted on the equality of limits for all the countries there would be problems about the adequacy of the number of establishments within each group Some groups would not contain a single establishments and the purpose of the study would be lost

In any case, in our study, we have not compared the corresponding groups of different countries. Any comparisons that will be made will be for overall results, tendencies over groups or for pooled data.

The frequency distributions of several variables pertaining to the establishment data of France, India, Israel, Japan and Yugoslavia are given in table 8, appendix. As can be seen from the cumulative percentage frequency figures, the distributions for different countries are dissimilar in the case of most variables.

Because of the inadequacy of data, only a fraction of the total frequency is represented in the case of some frequency distributions. Fortunately, complete data are available for the more important variables, but the number of observations corresponding to each class interval and the total number of establishments in each country are not all comparable in size. Some of these variables will be used as grouping criteria in the analysis of covariance which will be carried out later.

* * * *

Tables 1 to 6, appendix, give the empirical results for pool regressions and group regressions. The first line in each block corresponds to pool regressions which have been analysed in the last chapter. The next three lines in each block correspond to the small K, medium K and large K group regressions. As in the case of pool regressions, fifteen production relations have been used for the analysis. The main emphasis will be on the Cobb Douglas, CES and VES relations.

It may not be necessary to refer to the appendix tables continuously to study group regression results as the summarised results are given all along the body of the text.

* * * * * * *

In this chapter, all the group regression results will be studied in detail and if an overall uniformity will be noticed for all the countries in the matter of the production function results, it will be taken as a possibility, though not as a conclusive evidence, of uniformity within the manufacturing sector of each country. The final decision will rest with the results of analysis of covariance which has been carried out in the next chapter.

The group regression results will be compared with the results of pool regressions, in the matter of statistical fit, significance of regression coefficients and overall performance. We will test the hypothesis that, for a proper representation of the manufacturing sector of an economy, the production function for the pool data of manufacturing establishments, rather than that for any group of those establishments, is the appropriate mode of expression. This will be done with the help of several production function forms.

GROUP REGRESSION RESULTS

Cobb Douglas Production Function

Table 1, appendix gives the group regression results for the Cobb Douglas production function with two and three inputs

$$V = A K^{\alpha} L^{\beta} \qquad (1a)$$

$$Y = A K^{\alpha_M} L^{\beta_M} M^{\gamma_M} \qquad (1b)$$

Considering first the results for (1a) we note that the fits are satisfactory and all the labour coefficients are significant as they were in the case of pool regressions. But all the capital coefficients are not significant except in the case of Japan. The large K groups, however, yield significant capital coefficients for all the countries. This implies that there is the possibility, particularly in the case of small and medium groups, of the existence of multicollinearity which vanishes in the case of pool regressions.

We also make the observation that the pooled data fit is decisively superior to the group fits for all the countries. This should imply that the pooled data are reasonably representative of total manufacturing establishment data of each country since they seem to take care of all the constituent parts of manufacturing activity.

If the regression coefficients and their significance in statistical terms can be considered as indicators, we may say that invariably, the labour coefficient remains the more important variable for all the groups and for all the countries.

Except for some differences in the values of regression coefficients, we note that the regression pattern remains practically the same for all countries under consideration. To arrive at some more definite conclusions, some additional features of the results should be considered.

Table Ib(1)

The Cobb Douglas Function $V = A K^{\alpha} L^{\beta}$

	CONST.	α	β	$\alpha+\beta$	R2
FRANCE					
ALL 64 FIRMS	2.88	.197	.752	0.95	.89
		(4.6)	(13.2)		
POOLED K GROUPS					
SMALL K 21 FIRMS	2.78	.140	.847	0.99	.88
		(1.5)	(8.83)		
MEDIUM K 22 FIRMS	1.57	.295	.864	1.15	.89
		(1.3)	(12.0)		
LARGE K 21 FIRMS	3.25	.292	.549	0.84	.56
		(2.[illegible])	(3.95)		
INDIA					
ALL 117 FIRMS	0.51	.465	.605	1.07	.91
		(11.0)	(10.5)		
POOLED K GROUPS					
SMALL K 39 FIRMS	1.62	.320	.538	0.86	.73
		(1.30)	(7.38)		
MEDIUM K 39 FIRMS	1.32	.262	.735	1.00	.63
		(1.20)	(7.37)		
LARGE K 39 FIRMS	0.98	.515	.472	0.99	.79
		(6.00)	(4.96)		
ISRAEL					
ALL 69 FIRMS	2.97	.157	.656	0.81	.[illegible]4
		([illegible].8)	(9.53)		
POOLED K GROUPS					
SMALL K 23 FIRMS	3.89	.061	.586	0.65	.54
		(.68)	(4.63)		
MEDIUM K 23 FIRMS	3.24	.143	.601	0.74	.74
		(.66)	(6.44)		
LARGE K 23 FIRMS	0.55	.295	.850	1.15	.69
JAPAN					
ALL 63 FIRMS	1.46	.456	.529	0.99	.96
		(7.0)	(6.12)		
POOLED K GROUPS					
SMALL K 21 FIRMS	2.56	.394	.328	0.72	.63
		(2.9)	(2.57)		
MEDIUM K 21 FIRMS	0.75	.629	.159	0.78	.75
		(3.8)	(2.87)		
LARGE K 21 FIRMS	0.14	.416	.785	1.20	.87
		(3.9)	(5.61)		
YUGOSLAVIA					
ALL 145 FIRMS	0.17	.335	.641	0.98	.85
		(9.3)	(13.7)		
POOLED K GROUPS					
SMALL K 49 FIRMS	0.06	.155	.852	1.01	.82
		(1.6)	(11.1)		
MEDIUM K 49 FIRMS	2.66	.068	.572	0.64	.51
		(0.3)	(6.8)		
LARGE K 48 FIRMS	0.21	.429	.524	0.95	.65
		(4.1)	(6.4)		

Returns to Scale

Table 1b(1) shows the group regression coefficients along with the coefficients for pool regressions, for the Cobb Douglas function (1a). The returns to scale are given by $\nu = \alpha + \beta$. Whereas the null hypothesis $\alpha + \beta = 1$ was not rejected by the pool regression results of all the countries and constant returns to scale seemed to be the characteristic feature of manufacturing establishment data, we now find that the groups do not exhibit constant returns to scale everywhere.

In the case of France, the small establishments exhibit constant returns, the medium size establishments show increasing returns while the large size establishments show diminishing returns. Beyond the medium size, diseconomies of scale seem to prevail. The conclusion is supported by Carré, Dubois and Malinvaud(1976)

In the case of India, the medium and large size establishments seem to be at the stage of constant returns to scale. It is difficult to say if still larger establishments would realise increasing returns but the possibility is not ruled out. For India, 1964, our period of reference may be said to be a crucial year in the third five year plan which, as a result of top priority given to rapid industrialisation and diversification of the economy in the second plan, was associated with large capital investments. Our results find a support in Cheema's(1975)analysis of large and small factories during 1963-64. Cheema's observations are based on a detailed study in a more general context. It is likely that the high productivity of small establishments may be due to cheaper inputs obtained from the large establishments.

Between 1963 & 1964, the average size of fixed assets of factories rose by more than eleven per cent The possibility of low productivity of large establishments because of new investments driving up the capital assets without an immediate proportionate increase in output is not ruled out.

In the case of Israel, increasing returns are discernible as the size of the establishments increases. Compared to the rise in returns to scale from the small to medium establishments, that from the medium to large establishments is substantially large. This may be because, in the case of Israeli data, the small and medium size establishments are much smaller than the large size establishments on the basis of different grouping criteria and particularly on the basis of net capital assets used.

In the case of Japan, although the coefficients of capital and labour are very different from those of Israel, the returns to scale factor follows the same pattern as that of Israel. There is a small rise from the small to medium and a large rise from the medium to large size establishments so far as the returns to scale are concerned. Not surprisingly, the size distribution pattern of manufacturing establishments of Japan happens to be practically the same as that for Israel, as can be seen from the appendix table 8. Both in the case of Israel as well as Japan, imported technology played an important role in economic modernisation. In the case of Japan, this finds mention in Okhawa and Rosovosky(1973).

In the case of the socialist Yugoslav economy, even the small enterprises are sufficiently large and show constant returns to scale. The large enterprises do the same but the medium size enterprises show diminishing returns. These results for the returns to scale figures for Yugoslavia materialise because of the unique structure of her enterprises most of which are not autonomous and are often grouped with other enterprises under a variety of arrangements and for a number of reasons According to a World Bank Report(1975) on Yugoslavia, "the units function rather like divisions with separate accounts within a decentralised concern."

If, for the time being, we leave Yugoslavia out of consideration, we note that for all the countries, the change from the small to medium size establishments is similar in the matter of returns to scale as obtained from the Cobb Douglas function. The behaviour of large establishments seems to be rather erratic, although, it can be said that roughly speaking, a tendency for increasing returns is discernible in the case of India, Israel and Japan. This may be because, while other groups have some kind of limits imposed by the mode of grouping the data, the large establishments do not have any well defined upper limit so that the extent of largeness would differ from one country to another and the results may be neither uniform nor comparable for different countries. In the case of pool regressions, the discrepancies arising from such a situation can be concealed.

Table 1b(2)

The Cobb Douglas Function $Y=A_M K^{\alpha_M} L^{\beta_M} M^{\gamma_M}$

	CONST.	α_M	β_M	γ_M	$\alpha_M+\beta_M+\gamma_M$	R2
FRANCE						
ALL 64 FIRMS	2.45	.144	.673	.141	0.96	.90
		(3.1)	(10.4)	(2.35)		
POOLED K						
SMALL K 21 FIRMS	2.22	.124	.795	.112	1.03	.89
		(1.3)	(7.79)	(1.33)		
MEDIUM K 22 FIRMS	1.41	.183	.789	.155	1.13	.90
		(.82)	(10.4)	(1.85)		
LARGE K 21 FIRMS	0.86	.298	.268	.395	0.96	.68
		(2.5)	(1.64)	(2.57)		
INDIA						
ALL 117 FIRMS	-.08	.332	.476	.281	0.99	.93
		(7.2)	(8.20)	(5.08)		
POOLED K						
SMALL K 39 FIRMS	0.71	.363	.380	.206	0.95	.77
		(2.7)	(2.78)	(2.23)		
MEDIUM K 39 FIRMS	0.40	.245	.631	.197	1.07	.66
		(1.1)	(5.62)	(1.81)		
LARGE K 39 FIRMS	-1.1	.352	.349	.449	1.15	.86
		(4.5)	(4.24)	(4.39)		
ISRAEL						
ALL 69 FIRMS	2.21	.081	.637	.176	0.89	.86
		(1.7)	(9.69)	(2.85)		
POOLED K						
SMALL K 23 FIRMS	2.71	-.108	.662	.253	0.81	.74
		(1.1)	(5.78)	(2.66)		
MEDIUM K 23 FIRMS	2.81	.053	.582	.145	0.78	.77
		(.25)	(6.43)	(1.62)		
LARGE K 23 FIRMS	-.25	.224	.748	.237	1.21	.73
		(1.9)	(4.40)	(1.72)		
JAPAN						
ALL 63 FIRMS	0.90	.314	.534	.190	1.04	.93
		(4.2)	(6.63)	(3.23)		
POOLED K						
SMALL K 21 FIRMS	2.44	.346	.351	.043	0.74	.64
		(1.9)	(2.47)	(0.42)		
MEDIUM K 21 FIRMS	0.52	.452	.353	.245	1.05	.83
		(2.9)	(2.51)	(2.80)		
LARGE K 21 FIRMS	-.81	.260	.782	.252	1.29	.90
		(2.1)	(6.13)	(2.16)		
YUGOSLAVI						
ALL 145 FIRMS	.37	.238	.482	.309	1.03	.88
		(6.9)	(10.3)	(6.79)		
POOLED K						
SMALL K 49 FIRMS	-.19	.059	.766	.201	1.03	.83
		(.58)	(9.20)	(2.21)		
MEDIUM K 49 FIRMS	1.43	.085	.407	.296	0.79	.69
		(.49)	(5.50)	(5.18)		
LARGE K 48 FIRMS	-.28	.233	.249	.509	0.99	.79
		(2.6)	(3.10)	(5.45)		

Perhaps, comparable limits imposed on size may bring the results for large establishments in line with those of small and medium size establishments for all the countries.

It is likely that a certain size group of one country may exhibit returns to scale similar to those of a different size group of another country. This may be because of a correspondence between such groups whose sizes may be comparable or whose constituent parts may match with each other. The small K establishments of France allow nearly the same returns to scale as the large K establishments of India. The medium K group of France seems to correspond to the large K group of Israel.

The results, given in table 1b(2) for the three input Cobb Douglas function (1b) show that the pattern of group regressions, after the introduction of the third input M, raw material, compares well with the two input case. The pool regression fit is superior to that of group regressions. The labour coefficient is highly significant but the capital coefficient is not while the coefficient of M emerges significant at the cost of both labour and capital for all the groups. The raw material coefficient shows a distinct tendency of becoming more important as the establishment size increases. It rises from the small to the large K groups for all the countries. The ranking of the capital coefficients is similar to that of M while the labour coefficient behaves as in the two input case.

Constant returns to scale are evident for all the groups though the large K groups of India, Israel and Japan show returns to scale on the increasing side. The pattern resembles that of the two input case.

Table 2b(1)

The Cobb Douglas Production Function

$V=A_{DI}K^{\alpha_{DI}}L^{\beta_D}L^{\beta_I}$ $V=A_{EC}K^{\alpha_{EC}}L^{\beta_L}L^{\beta_C}$

	CONST	α_{DI}	β_D	β_I	ν	R^2	CONST	α_{EC}	β_C	β	ν	ω
FRANCE												
ALL 64 FIRMS	3 89	.144	.28	459	0 9[illegible]	91	3 33	.189 (4 5)	188 (3.6)	596 (9 7)	0 97	.90
POOLED K GROUPS												
SMALL K 21 FIRMS	3 42	116 (1 21)	5 0 (5 0)	3 70 (3 2)	1 02	89	3 05	141 (1 4)	119 (89)	754 (7 2)	1.01	87
MEDIUM K 22 FIRMS	3 54	168 (1 03)	3 01 (4 4)	2[illegible] (8 6)	0 99	95	0 02	.558 (2 8)	22 4 (3 1)	65 8 (7 3)	1 42	.92
LARGE K 21 FIRMS	3 82	2 45 (1 79)	23[illegible] (1 6)	3 86 (2 2)	0 86	62	4 95	167 (1 2)	23 8 (2 4)	.3 45 (2 4)	0 75	.64
INDIA												
ALL 117 FIRMS	0 82	464 (10 0)	481 (6 9)	13 0 (1 9)	1 08	91	0.96	.441 (9 1)	1 57 (2.8)	.475 (8.8)	1.07	.91
POOLED K GROUPS												
SMALL K 39 FIRMS	1.90	3 01 (2 0)	478 (3 0)	075 (76)	0.85	74	1 88	3 89 (2 7)	12 6 (1.8)	3 49 (3 4)	0 86	71
MEDIUM K 39 FIRMS	0 90	3 3 (1 7)	770 (6 3)	- 072 (59)	1 05	68	1 43	297 (1 [illegible])	033 (26)	671 (6 4)	1 00	63
LARGE K 39 FIRMS	1 82	42 7 (4 6)	22 0 (2 1)	3[illegible]8 (2 9)	1 01	81	1 98	.3 84 (4 1)	3 2 6 (3 4)	2 81 (3 1)	0 99	85
ISRAEL												
ALL 69 FIRMS	3 45	155 (3 72)	440 (5 5)	2 13 (2 9)	0 81	84	3 54	.256 (3 7)	2 61 (4 5)	409 (5 6)	0.82	83
POOLED K GROUPS												
SMALL K 23 FIRMS	4 3 4	0 43 (0 45)	42 6 (3 3)	167 (1 6)	0 64	65	4 49	018 (21)	2 48 (3 1)	3 8[illegible] (3 8)	0 65	69
MEDIUM K 23 FIRMS	3 65	13 9 (0 64)	441 (3 3)	158 (1 3)	0 74	76	3 2	12 0 (55)	- 02 1 (1 5)	619 (4 0)	0 72	76
LARGE K 23 FIRMS	1 2 5	287 (2 29)	3 1 (3 1)	33 5 (2 [illegible])	1 15	69	1 47	2 70 (2 4)	42 0 (4 4)	.483 (3 4)	1 17	73
JAPAN												
ALL 63 FIRMS	2 47	42 0 (7 0)	13 1 (1 6)	3 64 (6 6)	0 92	94	2.40	.456 (7 5)	268 (5 7)	168 (3 3)	0 89	92
POOLED K GROUPS												
SMALL K 21 FIRMS	2 76	411 (3 6)	099 (0 9)	298 (3 6)	0 81	75	2 99	3 70 (3 1)	1 59 (2.9)	188 (1 9)	0 72	71
MEDIUM K 21 FIRMS	2 10	612 (3 9)	- 048 (24)	3 44 (3 1)	0 91	79	1 96	560 (3 1)	277 (2 8)	114 (1 3)	0 95	76
LARGE K 21 FIRMS	0 96	[illegible]94 (3 3)	42 (2 8)	32 9 (2 7)	1 18	87	1 36	42 7 (3 2)	411 (3.7)	2 56 (3 3)	1 09	85
YUGOSLAVIA												
ALL 145 FIRMS	0 92	3 46 (10 2)	279 (4 1)	33 8 (6 5)	0 96	86	1 23	294 (8 6)	295 (6 [illegible])	3 62 (6 0)	0.95	.87
POOLED K GROUPS												
SMALL K 49 FIRMS	1 2 5	11[illegible] (1 25)	410 (3 4)	42 4 (4 3)	0 95	84	0 50	142 (1 4)	101 (1 0)	749 (6 1)	0.99	82
MEDIUM K 48 FIRMS	2 94	090 (0 41)	404 (3 1)	158 (1 6)	0 65	51	3.27	068 (36)	3 11 (4 3)	3 08 (3 4)	0.69	.64
LARGE K 48 FIRMS	1 6[illegible]	3 93 (4 47)	062 (0 6)	417 (5 8)	0 87	76	2 01	3[illegible]1 (3 6)	403 (5 0)	140 (1 4)	0.87	.76

The Three Input Cobb Douglas Function with Two Labour Inputs

We will now consider the group regression results with the help of a three input Cobb Douglas function, using net capital assets K and two labour inputs obtained by dividing total labour into two parts corresponding to two qualities of labour. Two cross classifications of total labour are used. Direct labour, L_D and indirect labour, L_I add up to total labour, L which is also the total of educated labour, L_I and other labour L_O .

Table 2, appendix or table 2b(1) herewith gives the group regression results alongside those of pool regressions in the case of the Cobb Douglas functions

$$V = A_{DI}\ K^{\alpha_{DI}}\ L_D^{\beta_D}\ L_I^{\beta_I} \qquad (2a)$$

$$V = A_{EO}\ K^{\alpha_{EO}}\ L_E^{\beta_E}\ L_O^{\beta_O} \qquad (2b)$$

The statistical results for the expressions (2a) and (2b) resemble each other in some respects. The coefficients of capital are practically equal in all the cases and the role of direct labour is played by other labour while the role of indirect labour seems to correspond to that of educated labour, in most cases This implies that in the actual running of a manufacturing establishment, educated labour mostly plays the part of indirect labour.

Comparing the results with the two input Cobb douglas function results, we find that while α is nearly equal to α_{DI} or α_{EO} ,the sum $\beta_D+\beta_I$ or the sum $\beta_E+\beta_O$ of the labour coefficients nearly equal to β .

This situation, being common to all the countries, may be generalised and considered to be their usual feature and supports the hypothesis of a uniform pattern in the matter of distribution of total labour share between direct and indirect labour. But, although the pattern is the same, the values of the shares are not.

The inequalities of income distribution between direct and indirect labour are obvious in the case of Israel, and more particularly in the case of India where the wages for indirect labour are generally extremely low. In the case of France, Japan and Yugoslavia, indirect labour is well paid. In the case of more advanced countries, as the establishment size increases, the proportion of direct to indirect labour share decreases as can be seen from the results for Yugoslavia and France. It may be surmised that with the passage of time and further industrialisation this situation becomes the common feature of all the countries. A proper verification of this possibility requires an analysis of similar data in a later period of time.

The constancy of returns is proved, once again, as in the two input case, for all the groups except small K group, Israel, and medium K group, Yugoslavia. Even in these cases, the pool regressions continue to show constancy of returns.

Despite quantitative differences, the pattern of various characteristics in different countries shows uniformity when analysed with the help of two input and three input Cobb Douglas functions.

Kmenta Approximation and Translog Production Function

The left side of table 3, appendix, shows the group regression results alongside those of the pool regression for the Kmenta approximation

$$\ln V = \ln \gamma + \nu\delta \ln K + \nu(1-\delta)\ln L - \tfrac{1}{2}(1-\delta)\nu\delta\varrho(\ln K/L)^2 \qquad (3a)$$

$$= \ln \gamma_k + \alpha_k \ln K + \beta_k \ln L + \delta_k(\ln K/L)^2$$

The fits are satisfactory and the coefficients of K and L follow the same pattern as that of the pool regression, that is , the coefficients of capital are not significant but those of labour are. The same is true of the additional term $(\ln K/L)^2$ whose coefficients are not significant along with those of K, in most cases, except in the case of large K groups of France, Japan and Yugoslavia. This implies that capital-labour ratio does play an important role in large establishments. India and Israel do not come into the picture because, relatively speaking, their establishments cannot be said to be large enough.

The hypothesis of constant returns to scale is justified in the case of pooled data and also in the case of small and medium K groups but not in the case of large K groups of France and Japan For France, it is diminishing returns to scale and for Japan, it is increasing returns to scale. Except for these cases, there is support for the hypothesis of constant returns to scale for all the groups of the countries being studied.

As can be seen from the results there is a possibility of high correlation between some of the explanatory variables in group regressions. The presence of multicollinearity cannot be ruled out.

As mentioned in the last chapter the group regression results for the translog function are given in the appendix table 3. However, the analysis of covariance results based on the translog function will be given in the next chapter.

<u>CFSL and CFSK</u>

The left side of table 4, appendix gives the results for the CFSL relation which will be considered along with the CFSK relation the results for which are on the left side of table 5, appendix. The two relations are

CFSL $\quad \ln V/L = a_L + b_L \ln w \quad$ (4a)

CFSK $\quad \ln V/K = a_K + b_K \ln r \quad$ (5a)

Each of these two relations is either a description of

Table 4b(1)

Elasticities of Substitution b_L and b_K Obtained from CESL and CESK Functions

	France		India		Israel		Japan		Yugoslavia	
	b_L	b_K	b_L	b_K	b_L	b_K	b_L	b_K	b_L	b_K
All	.81	.78	.79	.75	.87	.89	.85	.62	1.36	.86
Small K	.78	.50	.94	.73	.80	.59	.58	.36	1.36	.87
Medium K	.63	.80	.94	.85	1.08	.69	1.06	.49	1.46	.77
Large K	.96	.85	.22	.57	.78	1.01	.94	.87	1.21	.98

Note. The t-values are all statistically significant. at 5% level of significance.

the wage rate explaining labour productivity in money terms or that of the rate of return on capital explaining capital productivity in money terms.

The fits are not very good though they are better in the case of CESK. The regression coefficients, b_L and b_K, which stand for the elasticity of substitution in each case are significant for all the groups of all the countries. The values of b_L and b_K are given in table 4b(1). The CESK gives lower estimates of the elasticity of substitution than does the CESL for almost all the groups. This supports Pederson's(1972) hypothesis that b_K should be less than b_L. Another contention of Pederson is also supported in that almost all b_K values are less than unity.

In the case of pool regressions, the elasticities of substitution for all the countries, except Yugoslavia, were found to be close to each other as derived from CESL and CESK.

For most of the groups, the difference between the values of b_L and b_K is fairly large and does not seem to follow any pattern. This is because the size of establishment and the level of economic development of each countrycontributes decisively to the difference. This also implies that, as noticed in chapter three, the nature of wage rate and rate of return depends on the country to which they belong. The values of elasticity of substitution obtained from (4a) and (5a), therefore, vary according to the extent to which the economic forces ascertain themselves in the face of technical factors.

The average effect of the pool regression values of b_L and b_K suggests that the pooled data rather than the group data, can well represent the important aspects of the manufacturing industry within each country.

A generalisation about the values of the elasticity of substitution may be difficult but it can be said that except for the CFSL case of Yugoslavia, the values are, indeed, on the lower side of unity for the pool as well as group regressions. As the group size increases, the elasticity of substitution values tend to go up though it is difficult to give a hard and fast rule for this. The movement of the b_K values is relatively more smooth than that of the b_L values. With the exception of the large K group of India, the b_K values rise, as the group size rises, in a more decisive manner than the b_L values. It may be remembered that the grouping of the establishments is based on K values. This means that the elasticity of substiution depends on K, and possibly on K/L. But the extent and manner of dependence do not seem to be predictable or identical for different countries. Perhaps, it may be possible to make similar remarks about the dependence of b_L on L or L/K if the data are regrouped according to the L values. It may be contended that the elasticity of substitution depends on both K/L and L/K. It is on the basis of this contention that we developed the production relation given on page 86 in chapter two.

As for some different results obtained in the case of Yugoslav establishments it should be remembered that they seem to work under conditions different from those in capitalist establishments. They are labour managed and are constrained to make the best utilisation of resources, even if it has to be done by withholding present consumption in favour of capital formation. Moreover, they have no capital of their own which is all borrowed and adds substantially to their responsibility.

In spite of different values of regression coefficients obtained for different groups within each country with the help of (4a) and (5a), it cannot be asserted that the groups are structurally different. The homogeneity of the groups can be decided by means of analysis of covariance in the next chapter. But we have been able to notice, with the help of the CESL and CESK relations, that there are some production characteristics which may be found in all the countries uniformly.

On the basis of pool as well as group regressions, if we were to make a choice between the CESL and CESK functions, we may say that the over-rall performance of CESK is better than that of the CESL in that it gives a better fit and more consistent and significant values of the regression coefficients.

This implies that international comparisons as well as those between groups, when based on a production relation involving capital productivity, reveal the subject matter more clearly than when based on a relation involving labour productivity. The international market for machinery and equipment is quite competitive ;the prices of items of capital and capital productivity tend to be equal in different countries.

The CES results are based on the assumption of constant returns to scale.[1] The Cobb Douglas function indicated constancy of returns to scale emphatically in the case of pooled data but not so emphatically in the case of group data. On that basis, the elasticity of substitution as obtained from the pooled data of each country may be considered more reliable than those obtained from groups. Improved values of the elasticity may be expected if allowance is made for nonconstancy of returns to scale where necessary.

Although both the CESL and CESK functions reveal a variety of interesting results and point towards some kind of uniformity of the pattern of production relation for different countries, it has not been possible to come to any definite conclusions. Both suffer from inadequate explanatory power and the addition of a technical explanatory variable may be expected to improve the results. This takes us to a consideration of the VESL and VESK relations.

1If there are nonconstant returns to scale, we can use the modified relation discussed earlier

$$\ln V/L = \text{const.} + \sigma \ln w + (1-\sigma)\,\frac{\nu-1}{\nu}\,\ln V$$

The assumption of constant returns to scale sets the last coefficient in this equation zero.

VESL and VESK

The middle parts of tables 4 and 5 in the appendix, give the results obtained for the VESL1 and VESK1 relations

VESL1 $\ln V/L = a_{L1} + b_{L1} \ln w + c_{L1} \ln K/L$ (4b)

VESK1 $\ln V/K = a_{K1} + b_{K1} \ln r + c_{K1} \ln L/K$ (5b)

The fits for the groups are good and the regression coefficients are all statistically significant though the overall performance of the VESK1 is better than that of VESL1.

The input ratio comes out as an important explanatory factor both in the case of VESL1 and VESK1. Its inclusion as an additional explanatory factor seems to lead to an improvement of results.[1] The addition of the technical factor K/L(or L/K) in the CES relation does not suggest a fall in the importance of w(or r). In most cases the coefficients of w remain practically the same as in the case of CESL and the coefficients of r register a slight fall in several cases.

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[1]There is a prominent improvement in fit in all the groups and the coefficients of w(or r) and K/L(or L/K) are all significant.

According to Grilliches(1967)(in M Brown(Ed.),The Theory and Empirical Analysis of Production), there may be an identification problem here. Also, according to him since all the effects of w are contained in K/L, its coefficient should not only be significant but"should actually swamp the effect of w" Since K/L is rarely "measured without error and w is related to the systematic component of K/L, the latter variable may perform as a proxy for the correct K/L measure and not be forced out."

According to Nerlove(1967), "what remains striking is the diversity of results and their sensitivity to small changes in the specification of the equation fitted or of the data used."Nerlove suspects the possibility of collinearity which we notice is not the case with our results Moreover, we find that the correlation between w and K/L is very poor for all the countries.

Table 4b(2)

A Comparison of the Coefficients of

CESL	VESL1			CESK	VESK1	
b_L Coef. of w	b_{L1} Coef. of w	c_{L1} Coef. of K/L		b_K Coef. of r	b_{K1} Coef. of r	c_{K1} Coef. of L/K
			France			
.81	.75	.18	All	.78	.36	.62
.78	.72	.08	Small K	.50	.21	.72
.63	.61	.15	Medium K	.80	.34	.65
.96	.79	.21	Large K	.85	.58	.42
			India			
.79	.50	.37	All	.75	.57	.32
.94	.85	.21	Small K	.73	.63	.34
.94	.93	.19	Medium K	.85	.63	.41
.22	.03	.52	Large K	.57	.45	.35
			Israel			
.87	.84	.09	All	.89	.43	.50
.80	.79	.05	Small K	.59	.31	.74
1.08	.86	.19	Medium K	.64	.44	.47
.78	.78	.26	Large K	1.01	.63	.55
			Japan			
.85	.62	.39	All	.62	.51	.47
.58	.56	.54	Small K	.36	.43	.51
1.06	.85	.59	Medium K	.49	.54	.47
.94	.84	.33	Large K	.87	.63	.37
			Yugoslavia			
1.36	1.12	.25	All	.86	.67	.33
1.36	1.32	.07	Small K	.87	.62	.39
1.46	1.16	26	Medium K	.77	.60	.34
1.21	.99	.36	Large K	.98	.80	.23

All t-values except those in the underlined cases are significant. at 5% level of significance.

The empirical results for the VESL1 and the VESK1 relations, are given in a summary form in table 4b(2). It shows the contribution of the input ratio in the VESL1 VESK1 relations in the case of all the groups and for all the countries in our study It is quite likely that the VESL1 and the VESK1 relations may provide good results in the case of the data of other countries as well. Between the two movements from the CESL to the VESL1 and from the CESK to the VESK1, the latter provides better results on the whole.

If we compare the VESL1 coefficients of w and K/L with the VESK1 coefficients of r and L/K, we find that in almost all the groups, the coefficients of w are larger than those of K/L but the coefficients of r are smaller than the coefficients of L/K. If these two factors in the two production relations could be likened to the two inputs in the case of the Cobb Douglas function, a similarity in the values as well as the pattern of the values of these coefficients can be easily noticed. However, this is suggested only as a rough analogy but it helps to see that whatever the form in which a production relation is expressed, the presence of the two factors, capital and labour, in some form or other in the production relation is difficult to avoid. This is not meant to imply that the change from the Cobb Douglas relation to the VES relation has nothing different to offer.

The VES relation is a blend of the CES productivity relation and the Cobb Douglas relation with constant returns to scale. But it is neither the Cobb Douglas nor the CES relation although both are its particular cases. There may be an identification problem here, as Griliches(1967)[1] points out if the VES relation is split into the two relations (viz., the Cobb Douglas and the CES relations referred to above).

A useful characteristic of the VES function is that it has the property of variable elasticity of substitution which can be useful in some cases. We should not be too concerned with what the specific value of elasticity of substitution at some mean point is because it will change as one moves along the isoquant. What is important is that the resulting parameters emanate from a variable elasticity of substitution production function whose special characteristics may be expected to be reflected to a certain extent in these parameters.

We now consider the results obtained from the extension of the VESL1 and VESK1 functions to the VESL2 and VESK2 functions in the case of group regressions on the same lines as was done in the case of pool regressions.

The addition of L as a further explanatory factor in the VESL1 and the addition of K to the VESK1 lead to

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[1] Griliches(1967), in M.Brown(Ed), The Theory and Empirical Analysis of Production.

VTSL2: $\ln V/L = a_{L2}+b_{L2}\ln w+c_{L2}\ln K/L+d_{L2}\ln L$ (4c)

VTSK2: $\ln V/K = a_{K2}+b_{K2}\ln r+c_{K2}\ln L/K+d_{K2}\ln K$ (5c)

The empirical results for these two relations are given on the right side of tables 4 and 5, appendix. There is no noteworthy improvement in the regression results and the coefficients of L and K are statistically insignificant for almost all the groups. At the same time, the the regression coefficients of w,K/L and of r,L/K remain practically unaltered in almost all the cases. These two explanatory factors, w,K/L or r,L/K, between themselves, seem to manage well the explanatory requirement of the relation for all the groups in all the countries. The pattern is almost consistent for all the countries and is suggestive of the uniform reaction of manufacturing establishment data of all the countries to the VES relation involving these two factors.

So far as the analysis of group results is concerned we can say that it is not necessary to move from the VTSL1 to the VTSL2 or from the VTSK1 to the VTSK2 relation as no major improvement seems to be made in the empirical analysis by this.

CES Variations

The group regression results for the relation between the input ratio and the input price ratio

$$\ln K/L = b_D \ln \frac{\delta}{1-\delta} + b_D \ \ln w/r \qquad (6a)$$

are on the left side of table 6. The coefficient b_D stands for the elasticity of substitution as derived from this relation and δ is the distribution parameter of the CES function.

The regression fits are very poor for the groups. This was also the case with pool regressions. But except in the cases of small and medium groups of Japan, the values of b_D are significant in all the cases. The negative values of b_D in the case of Japan are associated with extremely poor regression fits. This implies that in the small and medium size establishments of Japan, the increase in the use of capital is associated with a simultaneous increase in the quantity of labour. It also implies that the rate of return on capital has risen more than proportionately with the rise in wage rate. This is in support of a similar conclusion derived with the help of the Cobb Douglas function.

The elasticity of substitution values b_D, as obtained from the relation (6a) are comparable with the values b_L as obtained from the CESL relation (4a). If we leave out Yugoslavia, the b_L values seem to cluster around 0.8 and the b_D values around 0.6. The small and medium groups of Japan are an exception. The values of b_D and b_L are relatively higher in the case of Yugoslavia.

The relation (4a) is constrained by the assumption of constant returns to scale while (6a) is free from such a restriction and does not contain the value added variable. While this should be considered a merit of (6a), some difficulty arises when an economic factor like w/r is used to explain a technical factor like K/L. Even if (6a) may be theoretically a better relation, the CESI relation (4a) seems to fare better because it incorporates the constancy of returns to scale which may be considered to be a feature of our data or because the explanation of labour productivity by wage rate is a better method of describing the state of affairs in the data

The relationship between the factor share ratio wL/rK and the input ratio K/L given by

$$\ln wL/rK = \ln \frac{1-\delta}{\delta} + \frac{1-b_s}{b_s} \ln K/L \qquad (6b)$$

where b_S is the input elasticity of substitution in this relation, is a straight derivation from the relation (6a) and may be expected to give empirical results comparable with those of (6a). The middle part of table 6, appendix, gives the detailed results for (6b). Table 6b(1) gives the values of b_S along with the values of b_D, b_L and b_V, the last of which will be obtained from the relation (6c) whose details will soon follow.

The relation (6b) produces poor regression fits although the values of regression coefficients are all significant with only one exception. The range of values of the regression coefficients for all the groups remain within reasonable limits, the small and medium group of Japan are the exceptions which remind us of a similar situation obtained with (6a). The high values of the regression coefficients in these two cases suggest that in the small and medium size establishments of Japan, the share of labour has not gone up with the rise in capital intensity. This was also the conclusion drawn from (6a) and the Cobb Douglas function

In the appraisal of various technical and economic variables in chapter three, the factor share ratio,wL/rK

Table 6b(1)

The Values of Elasticities of Substitution Obtained from

		(4a)	(5a)	(6a)	(6b)	(6c)
France	Pooled	.81	.78	.66	1.87	.80
	Small K	.78	.50	.39	1.42	.78
	Medium K	.63	.80	65	1.72	.70
	Large K	.96	.85	.74	1.93	.94
India	Pooled	.79	.75	.64	1.32	.54
	Small K	.94	.73	.46	1.67	.97
	Medium K	.94	.85	.69	1.66	1.03
	Large K	.<u>22</u>	.57	.29	2.27	.<u>12</u>
Israel	Pooled	.87	.89	.79	1.41	.88
	Small K	.80	.59	.37	1.85	.85
	Medium K	1.08	.64	.61	1.32	1.01
	Large K	.78	1.01	.65	2.22	.66
Japan	Pooled	.85	.62	.32	4.00	.63
	Small K	.<u>58</u>	.36	.<u>10</u>	-7.69	.58
	Medium K	1.06	.49	.<u>12</u>	-9.09	.48
	Large K	.94	.87	.76	2.17	.67
Yugoslavia	Pooled	1.36	.86	.97	1.72	1.26
	Small K	1.36	.87	.79	1.25	1.34
	Medium K	1.46	.77	.62	1.73	1.45
	Large K	1.21	.98	1.07	1.93	1.36

All t values except those in the underlined cases are significant at 5% level

Note.

$$\ln V/L = a_L + b_L \ln w \qquad (4a)$$

$$\ln V/K = a_K + b_K \ln r \qquad (5a)$$

$$\ln K/L = a_D + b_D \ln w/r \qquad (6a)$$

$$\ln wL/rK = a_S + \frac{1-b_s}{b_s} \ln K/L \qquad (6b)$$

$$\ln L = a_V - b_V \ln w + c_V \ln V \qquad (6c)$$

was found to lie somewhere near the middle of our technical-economic spectrum. Its value seems to fluctuate under the influence of economic forces. But there are no major fluctuations between the groups of the same country, the Japanese groups being an exception. It should be possible to establish this uniformity more firmly with the help of analysis of covariance

The relation (6b), which amounts to explaining an economic variable with the help of a technical variable, although statistically not very successful for our data, unfolds more clearly what was obtained earlier with other relations. It is useful because it brings in explicitly, the problem of explaining factor shares in a production relation

In the context of a discussion of various simultaneous equations difficulties which may arise in the estimation of CES production functions, Nerlove(1967) suggests that it is simpler to use (6a) if data are available. The equation "does not involve γ and so must hold exactly unless there are imperfections in profit maximisation. These in turn might make it impossible to estimate (the equation) by ordinary least squares." What applies to (6a) applies to (6b) also. It is possible to say that the regression results for (6a) and (6b) are not quite satisfactory and this might be caused, among other factors, by imperfections in profit maximisation in different groups and different countries. It is likely that the use of simultaneous equations method may improve the results but the results already obtained show the prominent influence of economic factors in bringing about differences in the technical act of production.

A Labour Demand Relation

We now consider the labour demand relation derived from the CTS function after allowing for the specification bias. This should possibly yield more accurate estimates of the elasticity of substitution. The relation

$$\ln L = b_v \ln \nu(1-\delta)/ \quad - b_v \ln w + \left(\frac{1}{\nu} + b_v \frac{\nu-1}{\nu}\right) \ln V \qquad (6c')$$

or

$$\ln L = a_V - b_V \ln w + c_V \ln V \qquad (6c)$$

is essentially an economic relation connecting demand for labour with the wage rate and value added, even though it is based on a technical relation

The empirical results are given on the left side of table 6, appendix. The summary table 6b(1) gives the results along with those of some other relations. Practically all the group regression fits obtained for the relation (6c) are good and all the regression coefficients are statistically significant with the exception of the small and medium establishment cases of Japan, this is in line with the results obtained from (6a) and (6b). The values of elasticity of substitution are, with the exception of Japan, practically the same as those obtained with the CES relation (4a). This implies that the correction for specification bias intended to be made by (6c) is not quite necessary. The CLSL relation based on the assumption of constant returns to scale seems to be adequate and does not need to be replaced by (6c).

An interesting finding from the empirical results of (6c) is that c_V, the coefficient of ln V, has a value which is around unity in the case of pool as well as group regressions. Once again, this implies constancy of returns to scale in practically all the cases. A uniformity in the nature of the parameters of different groups within each country is discernible but it is difficult to say if this uniformity pervades over all the countries. However, the reaction of the manufacturing establishment data of each country to the relation (6c) is similar. The reaction is also similar to the other CES variations (4a),(5a),(6a-c).

In overall performance, the relation (6c) which is a labour demand relation, is superior to any of the CES variations including the CESL and CESK. It also gives the elasticity of substitution estimates free from bias. But since the main results obtained from it and from the CESL are similar for our data, the CESL may as well be depended upon for certain conclusions Between the CESL and the CESK, the performance of the CESK is definitely better. These observations are from the point of view of the quality of empirical results as obtained from the use of different production relations. But so far as the reation of the manufacturing establishment data of different countries is concerned we find that it is practically the same irrespective of the production relation used.

We will now compare the empirical results as obtained from the pool regressions and group regressions.

* * * *

The Pool Regressions and The Group Regressions

In the case of all the countries under study, the pool regressions have provided better statistical fits than the group regressions. They have also produced regression coefficients which are always significant except in the case of the translog production function while there are several cases of group regression coefficients being statistically insignificant. It implies that the pooled data provide a better representative of and constitute a more complete set of observations from the manufacturing sectors of the countries under study than do the group data. We say this from a comparison point of view. It is not meant to be said that the quality of group results is poor or that they exhibit heterogeneity of structure within the countries under the study; this aspect will be dealt with under the heading of analysis of covariance in chapter seven. Here we have been able to show that it is possible to carry out production function analysis without reference to industry and that meaningful results can be obtained even when the constituent units are looked upon as arranged according to the quantity of capital assets used by them. Other criteria to arrange the establishments may also be used We did carry out the production function analysis on the basis of other criteria and noted certain interesting results which were found to depend significantly on the nature of the criteria used. As the nature and extent of differences in results become evident more easily from an analysis of covariance of the group regressions, the main results obtained from the use of other criteria are in the next chapter.

From the results obtained so far we conclude that the nonavailability of data according to standard industrial classification numbers is not necessarily a handicap and that a production function study can still be carried out with useful results.

We also note that the explanatory power of pool regressions as well as group regressions is prominently increased when the explanatory side of the production relation consists of both technical and economic factors. Good results may not be expected always in pracice if any one of these two types of explanatory factors is missing from the production relation.

The parameters of the production relations are not identical for all the countries. The impact of the prevalent political system, the historical factor and the level of economic development of the country on the nature of the empirical results cannot be completely ruled out. For instance, the nature of results in the case of Yugoslavia can be easily attributed to the political forces though even then, the pattern of results does not differ much from that of other countries.

We note that in some cases the variations between the values of group regression coefficients are large but, almost invariably, their average values are very close to the values of the pool regression coefficients. Unfortunately, the group establishment sizes of different countries are not the same so that the groups may not be expected to give comparable values of the coefficients. The pool regressions even out a number of differences in the group regression results.

In spite of some differences, the values of the parameters of the pool regressions as well as group regressions remain within reasonable limits. Except in the case of Yugoslavia, the value of elasticity of substitution is found to be around 0.8. The returns to scale parameter in most cases is around unity. Constant returns to scale are emphasised more by the pool regressions than by the group regressions.

So far as the ability to represent the manufacturing sector of a country is concerned, the pooled data are quite superior to the group data. While the pool regression results between different countries can be easily compared, the group regression results, based on different sizes of groups of different countries are not comparable. But a comparison of the pool regressions of different countries makes meaning only if there is stability of group regressions within each country. This leads us to the analysis of covariance of the data in the next chapter.

CHAPTER SEVEN

ANALYSIS OF COVARIANCE

A satisfactory estimation of a production function for the manufacturing establishments of a country, pooled together, is meaningful if the underlying structure is stable. Such a stability may imply the structural uniformity of the manufacturing sector of the country.

If the establishments are divided into groups and if a production function fitted to each of the individual groups shows signs of similarity among the groups as well as with the pooled data production function, the argument of stability and uniformity is emphasised. If the group results fail to establish uniformity in spite of a satisfactory pool regression there is still the possibility of uniformity of data in some groups and not in others unless it is categorically proved that uniformity and stability for the groups are completely ruled out.

We have carried out the regression analysis for our manufacturing establishment data in two parts. In the first part, pool regressions have been obtained for each country and studied in detail on their own as well as on a comparative basis. Several production function forms have been used. In the second part, the establishments in each country, have been divided into three groups and a regression analysis similar to the first one, along with a comparative study, has been carried out. The same production functions as in the first part have been used here.

<u>The Chow Test of Equality of Regression Coefficients</u>

We have divided the manufacturing establishment data of each country under study into three groups. We are interested in finding if the group regression parameters, which represent the structural characteristics of the respective groups, differ significantly from one group to another. In other words, we would like to know if there are any structural differences between different groups of each country.

Let b_1, b_2, b_3 stand for the vectors of the regression coefficients of groups one, two and three respectively. We wish to test the hypothesis $b_1 = b_2 = b_3$ against the alternative hypothesis that the regression parameters of different groups are different, or what is the same thing, their structural characteristics are not the same.

If there are n establishments in a country and the production function fitted to the data has m explanatory variables in the regression equation, we have the Chow test to verify our null hypothesis $b_1 = b_2 = b_3$, which makes use of the F statistic given by

$$F = \frac{(T - \sum_{i=1}^{3} T_i)/2(m+1)}{(\sum_{i=1}^{3} T_i)/(n-4m-4)}$$

where T is the sum of the squared residuals for the pool regression and T_i is the sum of the squared residuals of the regression for group i. The degrees of freedom are given by 2(m+1) and n-4m-4. We will make use of this test in the analysis of covariance.

Analysis of Covariance Tables

There are several sets of tables in the appendix which give the anlysis of covariance results based on different grouping criteria. Corresponding to each grouping criterion and the fifteen production relations listed in table 0, the analysis of covariance results are spread over six tables as in the case of pool and group regression. The results for a few criteria are given briefly in some consolidated tables.

Tables 1A to 6A are based on K groups

Tables 1B to 6B are based on L groups

Tables 1C to 6C are based on V groups

Tables 1D to 6D are based on K/L groups

Tables 1E onwards are based on miscellaneous grouping criteria. They are in consolidated form.

The pool regression analysis in chapter five and the group regression analysis in chapter six were given in detail for K groups only. On the same lines and in the same order, the analysis of covariance results will be discussed in detail in the case of K groups. A brief account of the analysis of covariance results will also be given for other grouping criteria along with some tables.

* * * * *

The Cobb Douglas Function

We begin by considering the results of the analysis of covariance of the grouped manufacturing establishment data subject to the two and three input Cobb Douglas functions

$$V = A\,K^{\alpha} L^{\beta} \qquad (1a)$$

$$Y = A\,K^{\alpha_M} L^{\beta_M} M^{\gamma_M} \qquad (1b)$$

In both cases the explanatory factors are technical variables and the grouping of the data is based on K which is a technical variable but which is not a dependent variable in the equation Under these circumstances we wish to know if there is homogeneity between groups of establishments.

Table 1A in the appendix shows that practically all the F values, obtained by analysis of covariance, corresponding to (1a) and (1b) are insignificant. This implies that the corresponding production function parameters of different groups of each country when compared on the basis of Cobb Douglas function compare well with one another. We may say that the estimates of regression coefficients which correspond to one another in different groups, are randomly distributed. The grouping does not seem to lead to different production relations for different groups and the pool regressions for the total manufacturing establishment data of each country may be considered as valid.

It is likely that the groups may be in the form of continuing segments with the same slope and intercept. It is also likely that in spite of variation within the groups the levelling[1] effect equates slopes and intercepts. In either case

[1] I.e. even if some groups show more variation than others their scatters may remain on the same slope

the conclusions remain unaffected. In particular, the variations within the groups represent a wide variety of experience and the F values are suggestive of the stability of the parameters of the production relations

We thus find that homogeneity between groups of each country is revealed and the pooled data of each country suggest an underlying stable structure when the grouping of establishments is done by means of a dependent variable like net capital assets. It is likely that the technical nature of this variable may also be contributing to a better revelation of the homogeneity that is there between groups. Whether this is so needs to be tested by means of other dependent variables or other factors with different characteristics This will be done later.

We conclude from the empirical results that the use of the Cobb Douglas function with two and three inputs along with a grouping of the data by K, fully confirms the presence of homogeneity between groups and justifies the pooling of all manufacturing establishment data for the production function analysis.

<u>The Cobb Douglas Function</u> with two labour inputs.

Table 1B, appendix, gives the analysis of covariance results corresponding to the three input Cobb Douglas relations

$$V = A_{DI} K^{\alpha_{DI}} L_D^{\beta_D} L_I^{\beta_I} \qquad (2a)$$

$$V = A_{EO} K^{\alpha_{EO}} L_E^{\beta_E} L_O^{\beta_O} \qquad (2b)$$

where the two types of labour, L_D(direct) and L_I(indirect) add up to total labour L which is also the sum of L_E(educated) and L_O(other labour). The analysis of covariance results support the hypotheses of the last section very well.
All the explanatory variables in (2a) and (2b) are technical in nature and the grouping criterion K is technical too.
The stability of the production relation for the pool data of each country is more firmly established. If the analysis of covariance results obtained so far are meaningful, we may expect similar results with other production relations under comparable circumstances.

<u>Kmenta Approximation and the Translog Function</u>

The explanatory factors in the case of Kmenta approximation and the Translog Production function are all technical in nature

$$\ln V = \gamma_K + \alpha_K \ln K + \beta_K \ln L + \delta_K (\ln K/L)^2 \qquad (3a)$$

$$\ln V = A_T + \alpha_{T1} \ln K + \beta_{T1} \ln L + \alpha_{T2} (\ln K)^2 + \beta_{T2} (\ln L)^2 + \gamma_T \ln K . \ln L \qquad (3b)$$

With the grouping criterion K, the situation resembles, in either case, that in the case of the Cobb Douglas function above. The analysis of covariance table 3A, appendix, shows that every F value is insignificant. This happens in spite of unsatisfactory regression results. But here we are concerned with the stability of pool regressions and the homogeneity between groups which are much in evidence and fully support the hypotheses referred to earlier.

CESL and CESK

We now consider the analysis of covariance results given in tables 4A and 5A, appendix for the relations

CESL $\quad \ln V/L = a_L + b_L \ln w \quad$ (4a)

CESK $\quad \ln V/K = a_K + b_K \ln r \quad$ (5a)

Since a separate reasoning may be given for the CESK, we will consider the CESL results. The grouping is by K which is not a dependent variable in the CESL relation which has an economic explanatory variable. With such a combination of circumstances, homogeneity is revealed in the case of France, Israel and Japan; the heterogeneity in the case of India and Yugoslavia may be introduced for a variety of reasons. In the case of both of these countries, the distribution of establishments, according to grouping by K, is rather uneven as compared to that of any of the other countries. This can be seen from the appendix table 8. Both the countries make much more use of direct labour as well as indirect labour in production than other countries in our study. The pattern of wages is uneven in either case; the differences in wages of two similar establishments in these countries can be large. Moreover, in the case of Yugoslavia, the enterprise is worker managed and the situation is different from a capitalist type of enterprise. There is no specialised economic administration directing enterprises nor any imperative planning of their activity Referring to the worker managed enterprise in Yugoslavia, Lavigne(1970) remarks that "income distribution is the chosen ground of self management...and the enterprise must fulfil its social obligations ...The purpose of obliging the enterprise to establish funds is to prevent all the revenue being distributed to workers "

Coming to the analysis of covariance results for the CESK given in the appendix table 5A, we find that the F values of all the countries are significant. This may suggest that if the CESK is the right relation for the data, group homogeneity does not exist in any of the countries. But the reason for this heterogeneity may lie elsewhere. Although the grouping variable K is not a dependent variable yet it seems to affect the dependent variable in a systematic manner. As the quantity K increases from the small K to large K groups, the quantity V/K in the dependent variable goes on diminishing thus forming three distinct groups based more or less on a dependent variable. This should lead to the suggestion of heterogeneity as would also be the case with the use of a dependent grouping variable in any production relation. However, if the grouping variable, in addition to being a part of the dependent variable, also occurs as an explanatory variable or as a part of an explanatory variable, the chances of heterogeneity being revealed may be diminished, depending upon how far any explanatory variable contains the effect of other explanatory variables. For instance, according to Grilliches(1967)[1],all the effects of w may be contained in K/L in the VESL relation.

1Ibid.

With the help of the CTS production relations (4a) and (5a) as also from the earlier results in this chapter, we have been able to show that even when homogeneity is known to be present in the data, the method of grouping can introduce heterogeneity A proper choice of a grouping variable is essential to reveal the presence of homogeneity.

From the results we also note that it is not quite necessary to group manufacturing establishment data according to standard industrial classification numbers only. Use may be made of other grouping criteria which can help in the analysis of homogeneity of groups and stability of pool regressions. Of course, the nature of the variables used in the production relation does play a role in this connection.

We will consider now what changes, if any, are brought about in the analysis of covariance results when additional explanatory factors are introduced into the CES relation.

VFSL and VFSK

The production relations

VFSL1 $\ln V/L = a_{L1}+b_{L1}\ln w + c_{L1}\ln K/L$ (4b)

VFSK1 $\ln V/K = a_{K1}+b_{K1}\ln r + c_{K1}\ln L/K$ (5b)

are obtained from the corresponding CES relation by the introduction of an input ratio, K/L or L/K which is a technical factor.

So far as the grouping criterion K is concerned, it does not occur either as a dependent variable or as an effective part of the dependent variable. In addition, it occurs as part of an independent variable. The situation, therefore, may be said to be a little better than the corresponding case of the CESL relation. Further, as we have noted earlier with reference to Griliches(1967) the effect of w may be contained in V/L.

From the analysis of covariance table 4A in the appendix, we find that the results are an improved version of the CESL results inasmuch as India and Yugoslavia do show significant F values as in the CESL case but with a diminished F. This is for the same reasons as given in the case of the CESL relation.

The results for the VFSK1 relation given in table 5A in the appendix, follow the pattern of the CFSK results. Arguing on the same lines as in the case of the VFSL1 relation, we may say that L/K is now the effective explanatory factor and the results should be influenced by its presence. All the F values are diminished now and two of the five values have even become insignificant.

It may be noted that while K is present in the dependent variable it is also effectively present in an explanatory factor.

To find if the explicit presence of the grouping variable on the explanatory side of the production relation does make a notable difference in the results, it is necessary to consider the VTSK2 relation which has K as an additional explanatory factor. However, we will consider the results for both the VTSL2 and VTSK2.

VFSL2 $\ln V/L = a_{L2}+b_{L2}\ln w+c_{L2}\ln K/L+d_{L2}\ln L$ (4c)

VTSK2 $\ln V/K = a_{K2}+b_{K2}\ln r+c_{K2}\ln L/K+d_{K2}\ln K$ (5c)

From the appendix tables 4A and 5A we note that the introduction of the grouping variable explicitly or of another variable with similar characteristics further improves the prospect of revealing the homogeneity in the data. The drop in the value of F is noticed both in the case of VTSL2 and VTSK2, though more so in the case of the latter because of the presence of K as an explanatory factor. Whereas all the F values were significant with the CTSK, all the F values except that of Yugoslavia, are statistically insignificant now with VTSK2.

We note that in the transition from the CFSL to VFSL1 to VTSL2 or from the CTSK to VFSK1 to VTSK2 the homogeneity of the manufacturing establishment data of each country comes out gradually if, by the introduction of additional explanatory factors in the production relation, any affinity between the grouping criterion and the dependent variable becomes less. For this conclusion, the net effect of the explanatory factors in the production relation should be taken into account.

<u>The CES Variations</u>

In the relation

$$\ln K/L = a_D + b_D \ln w/r \qquad (6a)$$

the dependent variable K/L has a close affinity with the grouping criterion K. Table 6A in the appendix gives the analysis of covariance results for this relation with K grouping and as may be expected all the F values are statistically highly significant, thus showing that grouping by a dependent variable in a production relation must reveal the presence of heterogeneity in the data.

The same table gives the analysis of covariance results for

$$\ln wL/rK = a_S + b_S \ln K/L \qquad (6b)$$

which explains the factor share ratio by means of the capital labour ratio. The grouping variable K has a strong affinity with the explanatory factor K/L and homogeneity in the data should be expected to be revealed. A more important reason for this expectation may be the absence of an affinity between K and the dependent variable. Table 6A shows that with the relation (6b), the F values for almost all the countries are statistically insignificant. The conclusions drawn in the case of earlier production relations are supported by (6a) and (6b).

We now consider the demand for labour relation given by

$$\ln L = a_V - b_V \ln w + c_V \ln V \qquad (6c)$$

The demand for labour relation

$$\ln L = \text{const.} - b_V \ln w + c_V \ln V \qquad (6c)$$

is not a production relation although it can be looked upon as an alternative method of expressing the CLSL relation (4a), with a correction for specification bias. As can be seen from the analysis of covariance table 6A, appendix, the F values are significant only in the case of India and Yug slavia, for reasons similar to those given in the case of (4a).

* * * *

All the analysis of covariance results obtained with the help of the technical grouping criterion K, seem to confirm almost categorically, the hypothesis, that when the explanatory variables in a production relation are technical in nature, a grouping of manufacturing establishment data based on a technical criterion, results in homogeneous groups within each country and indicates the stability of pool regressions. With economic explanatory variables, group homogeneity is absent; it is possible to reveal group homogeneity by the addition of technical explanatory variables to the production relation, provided the technical effect is strong enough when compared with the economic effect of the existing variables.

We now consider the results with some other grouping criteria

L Groups

Tables 1B to 6B in the appendix give the analysis of covariance results for groups of establishments formed on the basis of total labour L.

The F values corresponding to the two and three input Cobb Douglas function and the Kmenta approximation given in tables 1B,2B,3B are all statistically not significant and support the contention that when the grouping is based on a variable not related to the dependent variable, homogeneity between groups should be revealed and the stability of pool regressions established. The consistency of the results obtained in the case of equations (1a), (1b), (2a), (2b), (3a), (3b), ... matches well with that obtained with K grouping results.

The results for the CRS and VRS relations as also those for the CRS side relations (4-6) follow practically the same pattern as that of the K groups, though they are not an exact replica of the K group results. As shown by table 7, appendix, the rank correlation between K and L is not perfect. This implies that in spite of a somewhat different constitution of the groups now formed, the results point towards the same conclusions.

The technical variables K and L are usually to be found on the explanatory side of a production relation. We now consider another grouping criterion, V which is usually a dependent variable in a production relation.

Grouping Based on Value Added, V

Tables 1C to 6C in the appendix give the analysis of covariance results based on the grouping criterion V, which, in most of our production relations occcurs on the left hand side.

As we have considered earlier, the grouping based on dependent variable is most likely to suggest heterogeneity between groups for obvious statistical reasons. Thus even when there is homogeneity between groups, a grouping by a dependent variable will tend to conceal it and heterogeneity will appear. We will now see if this is what actually happens with the dependent variable V.

The analysis of covariance results for the Cobb Douglas and other relations (1 to 3) yield F values which are all statistically insignificant. This follows at once because V is a dependent variable in these relations.

In the case of the CTS and VTS relations, we have seen that the net effect of the explanatory variables influences the results. The presence of another variable in the dependent variable also makes a difference in results. The argument can be given on the same lines as that given in the case of K groups. As for the demand for input relation, the results are more straightforward. In this relation(6a), V is one of the explanatory factors so that all the F values are statistically insignificant.

We can say that in almost all cases, significant heterogeneity arises when grouping is done by the dependent variable.

V/L Groups

It is common to have V/L as a dependent variable in production function studies. For this reason and to obtain a support for the conclusions arrived at with the help of V groups , V/L was put to use as a grouping criterion. As expected, almost all the F values, where V/L was a dependent variable or had a strong affinity to the dependent variable in the production relation, were found to be statistically significant

* * * *

K/L Groups

The grouping by K/L was carried out to gain support for the earlier results In almost all the cases homogeneity was revealed when K/L was not a dependent variable or had no affinity with the dependent variable of the production relation under study.

* * * *

We have found homogeneity in our manufacturing establishment data with the help of several grouping criteria that occur on the explanatory side of a production relation. We will now consider some other grouping criteria which cannot be categorised as dependent or independent variables But they are certainly not dependent variables in the production relation. They have their own peculiarities some of which we will consider now.

Analysis of Covariance with some Other Grouping Criteria

Several other grouping criteria, in addition to those considered so far, were utilised in this study for group regression analysis and analysis of covariance. Some of the results obtained from the use of certain criteria will be considered now for analysis of covariance. The consolidated tables showing only the F values corresponding to each production relation are given in the appendix. The numbering of the expressions for the production relations remains the same as before.

Age of Establishment. The age of establishments may be looked upon as a technical factor, but, as noticed in chapter three, it is also a qualitative factor which interacts with technical as well as economic forces. As shown by table 7, appendix, the rank correlation between the age of establishment(denoted by NRYRS in the table) and capital labour ratio(CAPLT) is negative, though statistically not significant. To a certain extent, the older establishments may be expected to have a low capital labour ratio and vice versa. It can be easily said that for our data, the subsets of establishments formed by the grouping criterion of age of establishment is likely to be quite different from those formed by capital labour ratio or by net capital assets(CAPAS) which has a low rank correlation with the age of establishment(NRYRS).

Table 1E in the appendix gives only the F values obtained from the analysis of covariance based on the regression analysis of the three groups of establishments formed with the help of the age of establishment criterion. The results for all the countries have been put together in the table in which the number of the production relation is shown on the left.

Practically all the F values are statistically insignificant and reveal the homogeneity of groups formed on the basis of the age of establishment. This happens irrespective of the nature of the production relation used. The age of establishment is thus an interesting criterion for grouping establishment data and firmly supports the hypothesis of uniformity of the manufacturing sector within each country.

Capacity Utilisation and Shifts Worked. Both, the percentage utilisation of capacity and the number of shifts worked may be looked upon as technical factors but, being related to the rate of return and the value of output, they are both subject to economic forces Table 7, appendix, shows insignificant rank correlations between either of the two and K, L or K/L. The subsets of establishments formed by the grouping criteria, capacity utilisation and shifts worked, are quite different from each other, particularly because the sizes of groups formed by capacity utilisation can be made equal but the sizes formed by shifts worked may not necessarily be equal. The groups formed on the basis of number of shifts worked need not be a third of the total number of establishments.

Tables 2F and 3E show, for all the fifteen production relations, only the F values obtained from the analysis of covariance based on the grouping criteria of percentage utilisation of capacity and the number of shifts worked. As can be seen from the tables, practically all the F values are statistically insignificant and reveal the homogeneity of groups within each country. We have thus several criteria which support the conclusion of homogeneity between groups of manufacturing establishments irrespective of the nature of the production relation used

Summary and Conclusions

This study has been carried out in two fairly distinct parts. The first part reviews the extensions of some of the simple production function forms in the literature. For this purpose, use is made of the concept of technical and economic variables.

Fifteen relations have been selected for an empirical production function study which constitutes the second part of this work. Use is made of the manufacturing establishment data of France, India, Israel, Japan and Yugoslavia. This is the main part of the work.

Instead of making use of industry-wise establishment data, this work relies on pool data of mixed industry establishments for an empirical production function analysis. Instead of a stop after a straightforward production function analysis of pool regressions, we proceed further, divide the data of each country into three nearly equal sized groups and carry out group regression analysis as well as analysis of covariance based on different grouping criteria. The procedure followed in the case of group regressions is the same as that for pool regressions. The detailed analysis of covariance results are based on the K group criterion which also forms the basis of group regressions.

Several conclusions can be drawn from the variety of results obtained by means of pool regressions, group regressions and analysis of covariance.

In a production relation it is possible to describe some factors as technical and some others as economic variables. There can be some variables which are difficult to put in either of these two categories. The entry of an economic factor into a production relation may take place in a variety of ways. For instance, it can take place by a direct use of an economic factor as in the case of Sinai and Stokes' use[1] of real money balances as a factor of production; it may be through the inevitable weighting by money units of some of the technical variables which cannot be otherwise expressed in suitable homogeneous physical units, or, as is often the case, the entry of an economic factor into a production relation may take place through some side relations of the production function. Usually the interpretation and analysis will be different in different cases. But any of these happenings does not change the basic concept that the production function is, and remains a technical relation in the neoclassical view of the economic process.

The choice of suitable technical and economic variables entering into a production relation must be based on logical economic considerations. Use can be made of the concept of technical and economic variables in the development of several production function forms in the literature. Several extensions of some basic production function forms can be obtained by the use of additional technical and/or economic explanatory factors in some basic production relation. Production relations can be also derived or their extensions obtained, on the

[1] Ibid.

basis of information obtained from empirical results and suitable assumptions about the nature of the parameters of the production relation For instance, it is usual to assume the dependence of elasticity of substitution on capital labour ratio. The development of a variety of production function forms has been considered in this study.

Some production relations and hence the technical and economic variables in those relations, capture the prominent features of a production process more vividly than some other production relations In empirical work on production function studies, the use of more than one production relation is useful for the purpose of analysis.

In this study, several production relations have been used in the cross section analysis of manufacturing establishments of five countries It is found that the presence of one suitable economic variable and one suitable technical variable on the explanatory side of the production relation leads to statistically more satisfactory results provided an appropriate choice of these explanatory variables is made. In most cases, the use of further explanatory variables in the production relation does not necessarily help. We find that the essence of the technical and economic features of the data is well captured by the combined use of technical and economic explanatory factors in the Lu Fletcher VES relation. Other production relations are also helpful in various contexts.

All the production relations used in the empirical analysis suggest that in international comparisons made on the basis of production function analysis, nations are more relevant than industries or groups of manufacturing establishments. It is legitimate to pool together establishments belonging to different industries provided the size of establishments included in the analysis is not very small. In the case of each country, the pool regressions yield statistically and economically meaningful results. The pool data and the associated production relations are better representatives of the manufacturing sector of an economy and can describe it more fully than any group or groups of establishments and the associated production relations.

So far as the question of forming groups of manufacturing establishments is concerned, it is not necessary to adhere to the classification scheme based on standard industrial classification numbers unless the analysis specifically requires the use of such a classification scheme. Other criteria can also be used to form economically meaningful groups of establishments. There are several technical and economic criteria which can be used for the purpose. The quantitive nature of these criteria makes it possible to arrange the establishments in a certain order and to form a certain number and type of groups depending on the scope and analysis of the project.

Within each country, the group regressions provide many useful results and form a good basis for internal comparisons. Group results between countries can be compared provided the groups are of comparable sizes and have more or less the same range for the grouping criterion.

The process of internationalisation affects all the industrialised countries and also those countries which are on their way to industrialisation. However, in spite of the working of the internationalisation process, the parameters of the production functions of all such countries may not be identical, at least in the short run.

While we find that all the countries in our study do not exhibit common characteristics with reference to certain production relations, there are some pairs of countries which do yield almost identical production parameters by the F test as the epitomised results in table OO have shown. We conclude that it is possible to form subsets of countries with similar characteristics so far as their production relations are concerned. All the same, it is easily noticed that there is considerable uniformity across nations in the ability of almost all the production relations used in this study to capture the main features of the production process. The parameters vary, of course, but the statistical properties reveal a remarkable uniformity.

Constancy of returns to scale is noticed in all the countries under study. There is evidence of a nonunitary elasticity of substitution which is found to be around 0.8 for all the countries under study except Yugoslavia where it is significantly above unity.

Within each country, if groups of establishments are formed on the basis of certain criteria, homogeneity of the technical parameters of the production functions between groups is revealed. In almost all cases, as it should be expected statistically, significant heterogeneity arises when grouping is done by the dependent variable in the production relation. However, the homogeneity of manufacturing establishment data within each country is decisively revealed in almost all cases when the grouping of data is done by an independent variable in the production relation or by a variable not related to the dependent variable.

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APPENDIX A

Some Estimation Problems and Techniques

APPENDIX TABLES

APPENDIX A

The General Linear Model, Estimation Problems associated with the Production Function, Some Estimation Techniques.

The General Linear Model

The general linear model in the matrix form may be written $\underset{n\times 1}{y} = \underset{n\times m}{X}\underset{m\times 1}{\beta} + \underset{n\times 1}{u}$

which is a linear relationship between the dependent variable y, a column vector of n observations and the m explanatory variables $X_1, X_2, \ldots, X_m$. X is the n×m matrix of observations on x's,

β is the vector of true regression coefficients and u is the vector of unobserved stochastic terms such that $E\,u = 0$, $E\,(u\,u') = \sigma_0^2 I_n$. This assumes that u_i's are uncorrelated random variables with zero expectation and variance σ_0^2.

Another assumption of the general linear model is that the independent variables are free from errors of measurement. Thus X is nonstochastic. The stochastic term arises because of sampling variations in y for given values of X.

If it is assumed that the rank of X is m<n, it implies a unique least squares solution resulting in an unbiased and efficient estimate i.e. the best linear unbiased estimate of β.

If the u's are normally distributed the least squares estimates are the maximum likelihood estimates. If we write $\underset{n\times 1}{\hat{y}} = \underset{n\times m}{X}\underset{m\times 1}{\hat{\beta}}$ we have the vector of observed errors term given by

$$\underset{n\times 1}{\varepsilon} = \underset{n\times 1}{y} - \underset{n\times 1}{\hat{y}}$$

Assuming the inverse of X'X exists, we have, by minimising $\sum \epsilon_i^2$

$$\hat{\beta} = (X'X)^{-1} X'y \quad .$$

All the assumptions may not necessarily be satisfied in the case of all data. It is almost impossible to test if the independent variables are measured without error. Moreover, in the case of economic variables, the entry of some measurement errors is unavoidable since several imperfect adjustments may have been made in the data.

It may be difficult to assume that all sets of X's are linearly independent If some sets are not independent or if there is a high degree of correlation between one or more pairs of X's, the problem of multicollinearity may arise. Also, the assumption of homoscedasticity or the constancy of error variance may not be satisfied.

Additional assumptions regarding the distribution of u terms may be necessary, in the case of the general linear model. Either it may be assumed that the u's are independently, normally distributed or, no explicit assumption about the form of the distribution of u may be made and instead, resort may be made to the Cental Limit theorem.[1]

If it is found that the regressors are random variables, not necessarily normally distributed but having an arbitrary distribution it may still be possible to use regression analysis with some modifications. An outline of such models and related discussion is given by Johnston(1963) and Goldberger(1964) and others

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1The Central Limit theorem may be invoked to justify the normality of the distribution of u as a fair approximation. For instance see W Fellner(1957), An introduction to probability theory. John Wiley.

The Production Function and the Estimation Problem

The regression analysis is the most commonly used method to estimate the parameters of the production function. In spite of its repeated and fruitful use, the method suffers from certain drawbacks. The important issues involved in a production function study are those of identification, bias and inconsistency of estimates. The Cobb Douglas and the CES functions have been the most studied models in this respect. A brief account of some estimation methods with reference to these is given here. The results can be extended to other models.

The simplest economic production process could be represented by a single unilateral causal relation and hence by the single equation approach which allows computational simplicity. The researcher implicitly hopes, as Heady and Dillon(1960) put it, "that the single equation estimates are not greatly biased (while) a system of equations might be more appropriate - at least theoretically, although perhaps not computationally. The researcher may not have any idea of the extent of the bias in not using a multiequation model."

The Problem of Identification and Estimation of the Production Function

Nerlove's(1965) study of the identification problem in production function analysis has been done with special reference to the Cobb Douglas function. According to Nerlove, ordinary least squares estimates do not result in unbiased estimates of the parameters if capital and labour inputs influence the prices of these inputs in the economy.

He considers the problem on the basis of different assumptions about the input and output markets and their alternative measures of inputs and outputs.

It would be difficult to estimate a function if the nature of the system in which it exists is not known and the identifiablity conditions are not satisfied. The function is only a part of the system and has some variables determined somewhere else in the system. The economist has to take all the values of the variables as they come , produced by the mechanism outside the control of the economist. According to Marshack and Andrews(1944), "this mechanism is expressed by a system of simultaneous equations, as many of them as there are variables. The experimenter can isolate one such equation, substituting his own action for all the other equations. The economist cannot". While the whole system may be working smoothly the part under consideration may not.

The use of direct least squares to estimate the production function would therefore result in simultaneous equation bias. The resulting estimates may be biased and inconsistent.

Simultaneity and interrelationships are common features of production function analysis. For each firm or industry, output is a cause as well as effect so that it can occur as an independent or as a dependent variable. In most production function studies ordinary least squares technique is used in preference to the simultaneous equation or maximum likelihood approach because of lack of necessary data and computational problems though the fulfilment of certain assumptions may make the ordinary least squares technique adequate for most purposes.

The Cobb Douglas Production Function

Consider the production function of a firm with two inputs K and L and working under conditions of perfect competition. This implies that prices of inputs and outputs are given exogenously. As shown by Marschak and Andrews(1944) such a production relation, remains underidentified and cannot be estimated meaningfully. In the technical relationship in the form of a production function, the actual input quantities used are a result of economic and behavioural decisions. Assuming that the firm maximises profits subject to the constraint of the production function which we take to be the Cobb Douglas production function,

$Q = A K^{\alpha} L^{\beta}$, the profit is given by

$\pi = p Q - wL - rK.$

If we maximise $\pi = pQ - wL - rK - \lambda(Q - F)$

where $Q = F(K,L) = A K^{\alpha} L^{\beta}$, we have

$$\frac{\partial \pi}{\partial Q} = \frac{\partial \pi}{\partial K} = \frac{\partial \pi}{\partial L} = \frac{\partial \pi}{\partial \lambda} = 0$$

so that $p = \lambda$, $w = \lambda F_L$, $r = \lambda F_K$, $Q = F$.

The marginal productivities are given by

$$F_K = \alpha Q/K = w/p$$

$$F_L = \beta Q/L = r/p$$

The complete production model is described by the three relations :

Output supply	$Q = A K^{\alpha} L^{\beta}$
Demand for capital	$\alpha Q/K = r/p$
Demand for labour	$\beta Q/L = w/p$

These may be written

$$\ln Q - \alpha \ln K - \beta \ln L = \ln A = \lambda_0, \text{ say}$$

$$\ln Q - \ln L = \ln w/p\beta = \lambda_2, \text{ say}$$

$$\ln Q - \ln K = \ln r/p\alpha = \lambda_1, \text{ say}$$

It is assumed that input and output prices and hence λ_1 and λ_2 are the same for all firms.

Writing $x_0 = \ln Q$, $x_1 = \ln K$, $x_2 = \ln L$, the equations may be written

$$x_0 - \alpha x_1 - \beta x_2 = \lambda_0$$
$$x_0 - x_1 = \lambda_1$$
$$x_0 - x_2 = \lambda_2$$

or,

$$\begin{bmatrix} 1 & -\alpha & -\beta \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \lambda_0 \\ \lambda_1 \\ \lambda_2 \end{bmatrix}$$

which for a given set of prices, will not be identified because they will generate only a single point on the production function. There is no estimation problem if prices vary over time. For a crosssection of firms, each with the same production function and working under conditions of perfect competition the same problem would arise when estimation is carried out at any time with the assumption of fixed prices. If however, enterpreneurial talents are assumed to be unequal and distributed randomly, we may introduce into the production function a random variable corresponding to the technical ability of the enterpreneur of the ith firm. The production function $Q = A\ K^{\alpha}\ L^{\beta}$ will, for the ith firm, now take the form $Q_i = A_i\ K_i^{\alpha_i}\ L_i^{\beta_i}$ where $A_i = Ae^{u_{0i}}$ which is the coefficient of technical efficiency and is different for each firm. Using u_{1i} and u_{2i} for the random variables included in the marginal productivity relations, we have the set of simultaneous equations

$$x_0 - \alpha x_{1i} - \beta x_{2i} = \lambda_0 + u_{0i}$$
$$x_0 - x_{1i} = \lambda_1 - u_{1i}$$
$$x_0 - x_{2i} = \lambda_2 - u_{2i}$$

To allow for market imperfections resulting in the nonsatisfaction of marginal productivity conditions two constraints R_1 and R_2 may be introduced such that $rR_1 = \lambda F_K$ and $wR_2 = \lambda F_L$. In the above system of equations, λ_1 and λ_L will then be defined by $\lambda_1 = \ln rR_1/\alpha p$ and $\lambda_L = \ln wR_L/\beta p$.

If the data are crosssectional, λo, $\lambda 1$, $\lambda 2$, are constants if prices are fixed, so that, there being no exogeneous variables, the production function will not be identified. Moreover the equations in the simultaneous system given above are not independent because the random variables may be correlated with entrepreneurial abilities. Even if we assume that in the short run capital and labour are exogeneous, the assumption may not hold in the long run in which case the simultaneous equation bias cannot be avoided.

Since this may lead to biased and inconsistent parameters alternative estimation methods based on different assumptions about the profit maximisation conditions have been developed.

Zellner, Kmenta and Dreze(1966) assume that the firm is working under conditions of uncertainty and the production process is not instantaneous. The random variable is made up of unpredictable variations in input performance, weather conditions and other factors beyond the entrepreneur's control. The prices are known with certainty. If they are not known they are independent of the production function and their expected values are known. The firms try to maximise expected profits under these conditions. If u_{o1} is normally distributed with zero mean and variance S_{oo}, the expected value of Q in the Cobb Douglas function is $E(Q) = A K^{\alpha} L^{\beta} e^{\frac{1}{2}S_{oo}}$ Since prices may not be known exactly and may have to be anticipated, random factors do affect the entrepreneur's economic activity.

Writing $\lambda_1' = (\ln rR / p\alpha) - S_0^2/2$, $\lambda_2' = (\ln wR / p\beta) - S_{00}^2/2$ and assuming that u_0 and u_1 as well as u_c and u_2 are uncorrelated we have a consistent system of simultaneous equations.

$$x_{ci} - \alpha x_{1i} - \beta x_{2i} = \lambda_0 + u_{ci}$$
$$x_{ci} - x_{1i} = \lambda_1' + u_{0i} + u_{1i}$$
$$x_{ci} - x_{2i} = \lambda_2' + u_{0i} + u_{2i}$$

In the reduced form x_{1i} and x_{2i} do not depend on u_{ci}. This model can be used for crosssection data. Simple least squares estimation of the model

$$\ln Q = \ln A + \alpha \ln K + \beta \ln L + u_c$$

gives consistent estimates of the parameters provided only unexpected factors bring about variations in the production function from one firm to another.

The Cobb Douglas Function with Constant Returns to Scale

If the assumption is made that marginal products are equal to factor prices so that total output is exhausted in the constant returns to scale Cobb Douglas function

$$Q/L = A\,(K/L)^{\alpha}\, u$$

we have

$$\partial Q/\partial L = \alpha Q/L = w/p$$

This gives us a complete system of linear equations

$$\ln Q/L = \ln A + \alpha \ln K/L + \ln u$$
$$\ln Q/L = \ln 1/\alpha + \ln w/p$$

from which the parameters can be easily derived.

Indirect Least Squares

Constant estimates of the parameters of the production function may also be obtained by the use of indirect least squares. In the Cobb Douglas relation

$\ln Q = \ln A + \alpha \ln K + \beta \ln L + \ln u$, if we write $z_1 = \ln K - \ln Q$, $z_2 = \ln L - \ln Q$, $A_1 = \frac{\ln A}{1-\alpha-\beta}$, $\alpha_1 = \frac{\alpha}{1-\alpha-\beta}$, $\alpha_2 = \frac{\beta}{1-\alpha-\beta}$, we obtain $\ln Q = A_1 + \alpha_1 z_1 + \beta_1 z_2 + u_1$ from which can be found

$$\alpha = \frac{\alpha_1}{1+\alpha_1+\beta_1} \quad , \quad = \frac{\beta_1}{1+\alpha_1+\beta_1}$$

The least squares estimates based on this equation are consistent if $F(uv_1) = F(uv_2) = 0$, where v_1 and v_2 are the economic disturbances in the relations

$$\ln K - \ln Q = C_1 + v_1 \quad , \quad \ln L - \ln Q = C_2 + v_2$$

which are derived from profit maximising conditions The parameters will be efficient if in this two step procedure, the error terms in the technical and behavioural equations are independent or at least uncorrelated.

If there is only one input or if one of the two inputs is fixed, say L is fixed, then only one equation is enough for consistent least squares estimation of the parameters

$$\ln Q = A_1 + \beta_1 \ln L + \alpha_1 \ln z_1 + u_1$$

where $z_1 = \ln K - \ln Q$, $\alpha_1 = \frac{\alpha}{1-\alpha}$, $\beta_1 = \frac{\beta}{1-\beta}$, $u_1 = \frac{u}{1-\alpha}$

Grilliches has observed that reasonable estimates are not available from indirect least squares method which is a full information method but "this information while full is apparently not very good as it leads to unreasonable coefficients and very high standard errors."

Note. Only a brief description of various simultaneous equation difficulties that may arise in the estimation of the Cobb Douglas and the CES functions has been given here. We do not have any serious identification problem in our analysis. In any cross section study like ours, an exogenous explanation for differing prices is a desirable situation. It may be noted however, that dispersion through price differences is an induced dispersion, the firms scattering through production space in pursuit of their targets.

Klein's Factor Share Method

In the case of Cobb Douglas function, Klein(1953) method of factor share does not make use of the least squares procedure and α and β are estimated as the shares of capital and labour in total output

$$\alpha = rK/pQ, \qquad \beta = wL/pQ$$

The estimates are found by using the geometric mean over n observations of input shares

$$\hat{\alpha} = \left(\prod_{i=1}^{n} r_i K_i / p_i Q_i\right)^{1/n}, \qquad \hat{\beta} = \left(\prod_{i=1}^{n} w_i L_i / p_i Q_i\right)^{1/n}$$

which can be written

$$\ln\hat{\alpha} = \sum (\ln r_i K_i - \ln p_i Q_i)/n, \quad \ln\hat{\beta} = \sum (\ln w_i L_i - \ln p_i Q_i).$$

As shown by Dhrymes(1962), the estimates are asymptotically unbiased and have the minimum variance. However $e^{\ln\hat{\alpha}}$ is not an unbiased estimate of $e^{\ln\alpha}$.

The procedure does not estimate the production function directly. It uses the information that the input demand equations are jointly derived along with the production function during the course of profit maximisation

$$\ln Q = \ln A + \alpha \ln K + \beta \ln L + u_0$$
$$\ln rK = \ln \alpha + \ln pQ + v_1$$
$$\ln wL = \ln \beta + \ln pQ + v_2$$

There are two nonlinear restrictions here in that the last two equations involve $\ln\alpha$ and $\ln\beta$ whereas the first one has α and β. It is assumed that input and output prices are not only exogenous constant but are also observable.

The method cannot be used to test hypotheses about economies of scale because the parameters will tend to sum to unity if the accounting identity connecting Q,K and L has been used.

Hoch(1958) proposed an estimation procedure in the case of the Cobb Douglas function which removed the single equation least squares bias from the estimates.

If u_0 , v_1 , v_2 are the error variables in the production function and the two input demand relations with variances S_{oo}, S_{11} and S_{22} respectively, Hoch's estimates of α and β , using α' and β' , the ordinary least squares estimates, are given by,

$$\alpha = \alpha'(1+ Soo/S_{11} + Soo/S_{22}) - Soo/S_{\alpha\alpha}$$

$$\beta = \beta'(1+ Soo/S_{11} + Soo/S_{22}) - Soo/S_{\beta\beta}$$

By calculating m_{oo} the sample variance of Q, $m_{\alpha\alpha}$, $m_{\beta\beta}$ the sample variances of K and L and $m_{o\alpha}$, $m_{o\beta}$ the sample covariances of (K,Q) and (L,Q) and using the relation

$$Soo = m_{oo} - \hat{\alpha} m_{o\alpha} - \hat{\beta} m_{o\beta}$$

Hoch estimates

$$\hat{S}oo = Soo\ (1-Soo/S_{11} - Soo/S_{22})^{-1}$$

$$\hat{S}_{\alpha\alpha} = m_{oo} + m_{\alpha\alpha} - 2\ m_{o\alpha}$$

$$\hat{S}_{\beta\beta} = m_{oo} + m_{\beta\beta} - 2\ m_{o\beta}$$

If L is exogenously determined and the remaining disturbances are uncorrelated, $E(u_0 v_1) = 0$ so that

$$\hat{S}oo = Soo\ (1- Soo/S_{11})$$

$$\alpha = \alpha'(1 + Soo/S_{11}) - Soo/S_{11}$$

$$\beta = \beta'(1 + Soo/S_{11})$$

If $E(v_1 v_2) = 0$, then $E(u_0 v_1) = E(u_0 v_2) = 0$, that is, economic disturbances are correlated with each other but not with technical disturbances. Hoch gives additional results also without any consideration for prices.

The CES Production Function

The parameters of the CES function may be estimated directly by nonlinear methods or, as is usually the case, with the help of marginal productivity relations.

Nonlinear Estimation of the CES Function

There are several nonlinear approaches used to estimate the CES parameters. Basically they amount to iterative least squares regression by specifying an intial value θ for each unknown parameter θ in $Y = F(X, \theta) = F(X_1, \ldots X_n, \theta_1, \ldots \theta_n)$ which is written as a Taylor's series expension around θ. The difference between $F(X, \theta)$ and $F(X, \theta_0)$ which is approximated to the first order derivatives, is minimised to arrive at the new set of parameters θ'. The best solution is obtained iteratively. There is no guarantee that the solution may be obtained. There is the possibility that the unknown a priori values of θ may be wrongly chosen. The problem of multicollinearity may not be ruled out. See Goldfeld and Quandt(1971).

The CES Function, Simultaneous Equations

The usual method of determining the CES function coefficients makes use of the marginal productivity relations given below :

$$\partial V/\partial K = \nu \delta V^{1+\rho/\nu} / \gamma^{\rho/\nu} K^{1+\rho}$$

$$\partial V/\partial L = \nu(1-\delta) V^{1+\rho/\nu} / \gamma^{\rho/\nu} L^{1+\rho}$$

Here , a desirable property of neoclassical production function is satisfied in that the marginal productivities are positive. The second order condition is not satisfied unless $\nu = 1$. The elasticity of substitution is given by $\sigma = 1/1+\rho$. Since ρ may take any constant value, σ may do so too. ρ, δ and γ are features of the ruling technology in a crosssectional context.

Assuming perfectly competitive factor and product markets, the behavioural equations result by equating the marginal productivities to their respective factor prices. A consistent and efficient estimation will require the use of three simultaneous equations: the production function and the two marginal productivity relations, all of which lead to reduced forms, nonlinear in parameters. A single equation least squares method by using one of the two marginal productivity relations can help. Since under our assumptions

$$w = (1-\delta) V^{1+\rho/\nu} / \gamma^{\rho/\nu} L^{1+\rho}$$

we may have L as the explained variable

$$\ln L = \frac{1}{1+\rho} \ln \nu(1-\delta)\gamma^{-\rho/\nu} - \frac{1}{1+\rho} \ln w + \frac{\nu+\rho}{\nu(1+\rho)} \ln V + u_1 \qquad (4a_c)$$

$$\text{Similarly, } \ln K = \frac{1}{1+\rho} \ln \nu\delta\gamma^{-\rho/\nu} - \frac{1}{1+\rho} \ln r + \frac{\nu+\rho}{\nu(1+\rho)} \ln V + u_2 \qquad (5a_c)$$

If only these two equations are used it may be possible to estimate all the four parameters ρ, ν, γ and δ, provided both give initially similar estimates of ρ and ν. This may not necessarily be the case particularly in the case of crosssection data. But it is possible to put these equations to use for testing the validity of the model.

The difference in the values of the parameters from the two equations may be due to errors of observation in the capital data, misspecification in one of the derived demands ignoring the dynamic element in the relationship or the institutional factors. They may also be due to the breach of the assumption that the explanatory variables should not be correlated with the disturbance term in the ordinary least squares method used. While these possibilities are likely the more likely possibility is that the value added may be related to the disturbance term in the behavioural relations.

If however we assume that no scale economies exist so that the production function becomes homogeneous with $\nu = 1$. we have the relation

$$\ln L = \frac{1}{1+\rho} \ln (1-\delta)\gamma^{-\rho} - \frac{1}{1+\rho} \ln w + \ln V$$

so that $\ln V/L = -\frac{1}{1+\rho} \ln (1-\delta) \gamma^{-\rho} + \frac{1}{1+\rho} \ln w + u_1$

similarly $\ln V/K = -\frac{1}{1+\rho} \ln \delta \gamma^{-\rho} + \frac{1}{1+\rho} \ln r + u_2$

Both of these can be estimated by ordinary least squares which will give biased and inconsistent estimates if $\nu \neq 1$. If the parameter estimates happen to be similar from the two equation they can be used in the production function to linearise it in a two step procedure.

A division of $(4a_0)$ by $(5a_0)$ results in

$$\ln K/L = -\frac{1}{1+\rho} \ln \frac{1-\delta}{\delta} + \frac{1}{1+\rho} \ln \frac{w}{r} + u_3 \qquad (6a)$$

$\hat{\delta}$ and $\hat{\rho}$ from this used in $\xi = [\delta K^{-\rho} + (1-\delta) L^{-\rho}]$, an instrumental variable, can lead to the estimation of ν and γ

δ and ρ are optimally estimated as (6a) does not make use of endogeneous explanatory variables. But the estimates of ν and γ may not be efficient unless K and L used as explanatory variables in the second step are uncorrelated with the error term.

The Two Step Procedure for the CES Function

For the CES function $Q = \gamma[\delta K^{-\rho} + (1-\delta) L^{-\rho}]^{-\nu/\rho}$, equating factor rewards to the marginal productivities under the assumption of perfect competition, we get

$$\frac{\partial Q/\partial K}{\partial Q/\partial L} = r/w = \frac{\delta}{1-\delta} (K/L)^{-1-\rho}$$

which when written in the form

$$\ln r/w = \ln (\delta/1-\delta) - (1+\rho) \ln K/L + \ln u$$

gives the estimates of δ and ρ. The latter when substituted in th relation

$$\ln Q = \ln \gamma - \frac{\nu}{\hat{\rho}} \ln [\hat{\delta} K \quad + (1-\hat{\delta}) \quad L^{-\hat{\rho}}] + \ln u$$

can enable us to estimate γ and ν . We assume that u is normally distributed with zero mean and constant variance.

Under the assumption of the general linear model the equation $\ln r/w = \ln (\delta/1-\delta) - (1+\rho) \ln K/L$ yields unbiased estimates of $\ln (\delta/1-\delta)$ and ρ . It does not follow that $\hat{\delta}$ obtained from the estimate of $\ln (\delta/1-\delta)$ is unbiased. If this is taken as the first stage regression it should be remembered that in the second stage regression of the CES production function, $\hat{\rho}$ and $\hat{\delta}$ thus estimated, are subject to errors of estimation. Since only point estimates of $\hat{\rho}$ and $\hat{\delta}$ can be used, the standard errors of ρ and δ have to be ignored so that the second stage regression does not fulfill the assumption of the general linear model. This is because the independent variables are not measured without error. The resulting fit in the second stage gives R^2 applicable to the second stage only, subject to the errors of the first stage.

Kmenta Approximation

It is possible to estimate the parameters of the CES function by nonlinear least squares procedures. But the convergence to well defined values of the parameters is not always guaranteed. Kmenta approximation of the CES function has often been used to do away with the nonlinearity problem associated with the CES function:

$$\ln Q = \ln \gamma + \nu\delta \ln K + \nu (1-\delta) \ln L - \tfrac{1}{2} \rho\nu\delta (1-\delta)(\ln K - \ln L)^2.$$

Except for the last term on the right hand side , this equation represents the Cobb Douglas function. The last term $\frac{1}{2}\rho\nu\delta (1-\delta)(\ln K - \ln L)^2$ may be looked upon as a correction term reflecting the departure of ρ from zero It disappears if $\rho = 0$. If ρ is not significantly different from zero, the CES function in the form of Kmenta approximation, may be rejected in favour of the Cobb Douglas function. $\rho = 0$ means $\sigma = 1$. A significant value of ρ means an elasticity of substitution different from unity. If the elasticity of substitution is significantly different from unity it does not lead to an automatic acceptance of the CES function. This is because several other production models are compatiable with the idea of nonunitary or variable elasticity of substituion. Moreover the Kmenta approximation ignores the third and higher order terms whose effects are unknown and unpredictable. The coefficient of $(\ln K/L)^2$, namely, $\frac{1}{2}\rho\nu\delta(1-\delta)$ on which we base our conclusion is likely to be biased on the lower side since its constitutent parts are mostly less than unity each. The assumption of $\rho = 0$ ignores the possibility of any other ofthese constitutents contributing to the results. The estimates are not independent of the units of measurement. Recently Corbo(1974) has found that the expression in the Kmenta approximation does not follow from the CES function alone. It can equally well result from a similar approximation of some variable elasticity of substitution production functions.

Grilliches and Ringstad(1971) have shown that since the estimate of the coefficient of $(\ln K/L)^2$ is not independent of units of measurement of K and L there are advantages in evaluating the parameters at the geometric mean level of inputs. The accuracy is improved when the units of measurement are so chosen as to equate to zero the means of ln K and ln L. If K = L the approximation isexact so that the sample observations of K and L are distributed about the line K = L. For improved results, $\overline{\ln K}$, $\overline{\ln L}$ and $\overline{\ln K/L}$ may be subtracted from ln K, ln L and ln K/L in the approximation .

The Kmenta approximation may be used in the estimation of the CES function by indirect least squares. The CES production function and the marginal productivity relations

$$Q = \gamma[\delta K^{-\rho} + (1-\delta) L^{-\rho}]^{-\nu/\rho} e^{u_0}$$

$$r/p = \nu\delta\gamma^{-\rho/\nu} Q^{1+\rho/\nu} K^{-1-\rho} e^{-u_1}$$

$$w/p = \nu(1-\delta)\gamma^{-\rho/\nu} Q^{1+\rho/\nu} L^{-1-\rho} e^{-u_2}$$

may be written

$$\ln Q = \ln \gamma - \nu/\rho \ln [\delta K^{-\rho} + (1-\delta) L^{-\rho}] + u_0$$

$$(1+\rho/\nu) \ln Q - (1+\rho) \ln K = \ln \frac{\gamma^{\rho/\nu}}{\rho\nu\delta} r + u_1$$

$$(1+\rho/\nu) \ln Q - (1+\rho) \ln L = \ln \frac{\gamma^{\rho/\nu}}{\rho\nu(1-\delta)} w + u_2$$

For constant or nonconstant returns to scale and for an elasticity of substitution of unity or otherwise we have to deal with a simultaneous system of equations linear in parameters whose number is large. There is a possibility of identification if prior restrictions are imposed on the parameters. Kmenta assumed a diagonal variance covariance matrix of the disturbances and replaced the CES expression by using Taylor's series approximation so that the system becomes

$$\ln Q - \nu\delta \ln K - \nu(1-\delta) \ln L + \tfrac{1}{2}\nu\rho\delta(1-\delta) (\ln K - \ln L)^2 = \lambda_0 + u_0$$

$$(1+\rho/\nu) \ln Q - (1+\rho) \ln K = \lambda_1 + u_1$$

$$(1+\rho/\nu) \ln Q - (1+\rho) \ln L = \lambda_2 + u_2$$

where $\lambda_1 = \ln \frac{\gamma^{\rho/\nu}}{\rho\nu\delta} r$, $\lambda_2 = \ln \frac{\gamma^{\rho/\nu}}{\rho\nu(1-\delta)} w$

The last two equations, which are input demand relations are first identified and can yield consistent estimates of $\frac{1+\rho/\nu}{1+\rho}$ provided the two relations give the same results.

Let $\xi = \frac{1+\rho/\nu}{1+\rho}$

and define $z_1 = \xi \ln Q - \ln K$, $z_2 = \xi \ln Q - \ln L$ and

$$z_3 = (\ln K - \ln L)^2$$

we then have to estimate the regression equation

$$\ln Q = a_0 + a_1 z_1 + a_2 z_2 + a_3 z_3 + u'$$

Here u' is proportional to u_0 and the coefficients are identified.

Hodges(1969) extended to the CES function the result of Zellner-Kmenta-Dreze applied to the Cobb Douglas function.

Janvry(1972) shows that under the behavioural assumption of maximisation of expected profits, direct estimation of the production function from crosssection data on firms is always free from simultaneous equation bias, whatever the functional form specified. For the general class of stochastic functions

$$Q = f(X)\, e^{v_0}, \quad F(e^{v_0}) = \mu$$

let the firm objective be to maximise conditional expectation of profits under conditions of perfect competition

$$E(\pi) = p\, E(Q) - \sum_{j=1}^{m} p_j X_j$$

where p is the output price and p_j, j= 1,...,m, the m input prices. Since $E(Q) = f(X)\mu$, we have the m additional

equations $\quad p\, f_j(X)\mu - p_j = 0$

or $\quad p\, f(X)\, g_j(X) = p_j \qquad j = 1, 2, \ldots, m$

If we write $c_j = \ln p_j / p\mu$, we get m + 1 simultaneous equations similar to those of Zellner-Kmenta-Dreze.

$$\ln Q = \ln f(X) + v_0,$$

$$\ln Q + \ln g_j(X) = c_j + v_0 + v_j, \quad j = 1,2,\ldots,m.$$

In the reduced form equations, v_0 is independent of v_j so that consistent ordinary least squares estimates of the parameters are obtained for any form of production function.

A similar result can be obtained in the estimation of the general class of stochastic production functions

$$Q = f(X) + v_0, \quad F(v_0) = 0.$$

A similar result has been given independently by Kalejian(1971).

<u>Specification Bias resulting from inclusion of irrelevant variables or exclusion of relevant variables.</u>

Consider a general linear model which may be assumed to be the true model, with three independent variables

$$y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i$$

where y is a column of n observations on output and X_1, X_2, X_3 are columns of n observations each on the three independent variables. In the matrix notation,

$$\underset{n\times 1}{y} = \underset{n\times 4}{X}\ \underset{4\times 1}{\beta} + \underset{n\times 1}{u}$$

where a column of unity has been associated with β_0.

If the model wrongly specified is

$$y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + u_i'$$

or

$$\underset{n\times 1}{y} = \underset{n\times 3}{\bar{X}}\ \underset{3\times 1}{\bar{\beta}} + \underset{n\times 1}{\bar{u}}$$

the rank of X is likely to be higher than the rank of $\bar{X}$.

$$\hat{\bar{\beta}} = (\bar{X}'\bar{X})^{-1}\bar{X}'y$$

$$= (\bar{X}'\bar{X})^{-1}\bar{X}'X\beta = G\beta$$

where $G = (\bar{X}'\bar{X})^{-1}\bar{X}'X$. No bias in the parameter estimates of the regression equation is involved if X_3 is not correlated with X_1 and X_2. But the returns to scale parameter will be biased.

In the case of the Cobb Douglas function if the specification is $Y = A K^{\alpha} L^{\beta}$ instead of $Y = A K^{\alpha} L^{\beta} M^{\gamma}$, the returns to scale will be noted as $\alpha + \beta$ rather than $\alpha + \beta + \gamma$. The bias will be downward in this case. It will be upward if an irrelevant variable is included.

If the omitted variable X_3 is positively correlated with X_1 or X_2, either β_1 or β_2 or both will be biased upward.

If proportional changes in X_1 or X_2 correspond to less than proportional changes in X_3, the returns to scale will be underestimated. They will be overestimated if the correspondence of proportional changes in X_1 or X_2 is with more than proportional changes in X_3.

If X_3 changes in the same proportion as X_1 and X_2, there is no bias involved.

The Aggregation Problem

The aggregation problem arises because of a jump from the microeconomics of one unit to the macroeconomics of many units, usually assuming a pattern for the latter similar to the one for the former.

In the case of a firm, the technological relationship of the production function is managed by the entrepreneur who decides the output levels and the required input proportions. But who does the job in the case of an industry ? As Walters(1963) says in his survey, "one difficulty is immediately apparent those factors which we regard as fixed for the individual firm are not necessarily fixed for the industry, e.g.,entrepreneurial ability." This means that full fledged aggregation or even part aggregation results in the misspecification of the variables and hence of the associated parameters.

There is no profit maximising macro-decision maker. There is no reasonable analogy in equating marginal productivity of inputs to their rewards nor the equilibrium available under perfect competition. In spite of the success associated with a number of macro models, the usual basis of such aggregation is intuition which may introduce an illogical element in the subject.

According to Green(1964), aggregation is a process "whereby a part of th information available for the solution of a problem is sacrificed for the purpose of making the problem more easily manageable." Aggregation may be deemed to be satisfactory if, in spite of not using detailed data, the reliability of results is not lowered significantly while the costs are lowered at the same time. In other words, the transition from the micro to macro should not significantly affect the viability of aggregation of variables and relations to be used in the process of aggregation.

If the macroeconomic variables are set up to correspond to the microeconomic variables, one may question whether the identical looking macro variables define the same phenomenon as the micro variables. Alternatively, one may ask if the macro variables can be so defined as to be consistent with the microeconomic theory. Some aspects of this problem may be considered with reference to production functions.

Specification Bias due to Aggregation

The aggregate production function, when used to compare interindustry efficiency or intertemporal technical progress, tends to ignore quality and structural differences in inputs. It is essential to take care of the structural aspect of labour for a proper specification of the labour input in the production function which may suffer from bias as a result of almost stable differences in the qualitities of different types of labour. The assumption of homogeneity of labour and capital without any corrective for heterogeneity involves serious policy implications apart from bias. Unfortunately, it is not possible to do much about it in empirical work for reasons of non-availability of data and difficulties of estimation.

The aggregation problem may be considered as a special case of specification bias. Reasoning in terms of aggregates is a long standing tradition of economic theory though, often, its use may not be justified.

Each additional level of disaggregation may be expected to improve the homogeneity and accuracy of the data. But there are limits. For instance, many different items are used as capital to produce goods which pass through several processes;and prices of capital items used are also not identical. Also, the alternative items of capital may not necessarily produce the same output per unit of time. The durability of different units of output may not be the same. Yet different units of output or different units of capital may be aggregated without proper justification.

Thus, strict disaggregation is difficult because even a single establishment or plant employs a variety of capital and labour and must, in general, produce several kinds of outputs. It is practically impossible to obtain strictly homogeneous outputs, capital and labour in all the observations and therefore a certain degree of aggregation or a limited disaggregation has to be allowed.

Any establishment produces more than one output though most production function studies are based on a single output. The aggregation problem obviously arises because the single output used in the empirical analysis is the result of a price weighted combination of several items which may, in some cases, be very different from one another. Any errors arising out of aggregation will be larger, the higher the degree of aggregation. Similarly, labour and capital units, even though nonhomogeneous and differing in numerous ways, have to be combined. Usually, detailed data are not available. But even if they were, it would be difficult to make use of them in view of the variations in quality, age, experience and a variety of other circumstances over different establishments.

Aggregation in Production Function Studies

It has been observed by Bosworth(1976) that 'problems of aggregation can result in the failure of aggregate production functions to reflect the underlying technology of production. Such fears give micro studies much of their appeal. At a very low level of aggregation extraneous influences should be less important and estimates should directly reflect the micro technologies."

Fisher(1969,1971) has pointed out how rigorous aggregation, almost invariably resorted to in production function analysis, is possible only under very stringent conditions. All these restrictions are in addition to the many other unrealistic assumptions associated with the subject.

The concept of the production function, as originally envisaged, was concerned with quantifying the relationship between the output and inputs of an individual firm. According to Nelson(1964), the extension of this relationship to a "single and stable relationship between a measure of aggregate outputs is uncertain at best." The uncertainties and doubts associated with the problem of aggregation may be many. For a firm, the production function is a technological relationship between its output and inputs to be used. Is it the same for an industry or an economy ? It is difficult to visualise a whole industry or an economy from the same technological angle as the firm. Inputs have a different meaning for a firm compared to that for an industry.

The main difficulty due to aggregation arises when use is made of the same form of the production function in the macro case as in the micro case. The use of a particular form of the aggregate production function cannot be questioned. What is objectionable, however, is the implicit assumption that aggregation does not make any difference to the situation.

For our manufacturing establishment data, in so far as we carry out some kind of addition of all outputs and inputs corresponding to various production processes within each establishment, without distinguishing between different types and qualities of inputs or outputs, our production functions do become roughly aggregative in nature. Yet the intrinsic features of the technologies within the establishments do not remain unreflected by this. Our aggregation is in value terms so that the marginal products will also be in value terms. Such an aggregation is not valid unless there areconstant returns to scale. Our results may suffer, therefore, from some aggregation bias but we realise at the same time that it is neither possible nor feasible to separate data in each product which is likely to differ from one establishment to another. Nor are the basic conditions of identity of production functions and constant returns to scale likely to be satisfied.

* * *

According to Klein (1946), if there exist production functions relating outputs to inputs from the individual firm, functions connecting aggregate outputs and inputs for the economy as a whole should also exist. Further, if individual firms are assumed to be maximising their profits according to certain marginal productivity relations then the aggregate variables should satisfy analogous equations. Alternatvely , it should be possible to derive from the individual production functions an economy production function of aggregate variables which may be looked upon as technological variables of an economy-wide process. This requirement is independent of the satisfaction of any equilibrim conditions for maximum profits since technology alone is involved In equilibrim however, the aggregate variables for the economy should satisfy the classical marginal productivity conditions Pu (1946) objected by saying that Klein's assumption that aggregate output is independent of input distributions is not proper and to assume that a unique macro production function exists, is unnecessary and arbitrary because some other function can as well take care of the profit maximising conditions Pu observed that it was possible to imagine a transfer of factors between firms resulting in a rise or fall of the aggregate production function although the total quantity of factors remained unchanged. It is necessary to include in an aggregate production function variables showing the distribution of factors. Aggregation should be based on fixed patterns of distribution of the values of the micro-variables in each aggregate. In other words, the existence of such fixed patterns is essential for aggregation. According to Shephard(1971), the production relation to be aggregated reflects some optimisation or equilibrium decisions and describes a limited arrangement of inputs to outputs relative to those available in the technology. If in the aggregates, the microvariables have a fixed pattern of distribution, then the aggregate production function does not describe the alternatives in the technology and becomes a statement of the net effect of individual optimising decisions. For some given set of prices and circumstances, Klein's individual production functions reflect the full range of the alternatives available whether realised or not by the firms at a given time. The aggregate production function thus constructed from aggregate variables can be successfully used for the purpose of prediction and explanation. In the very short run, Fu's assumption of fixed distribution of microvariables may be fulfilled but not in the long run.

Using Theil's(1954) procedure of aggregation as applied to the Cobb Douglas function, Grilliches(1957) shows that when the micro parameters are not identical, the estimates of the macro parameters resulting from aggregation are averages of the corresponding micro parameters along with a bias depending on all the elements of each of the micro parameters. The macro coefficient of labour in a two input model is influenced by the micro coefficients of labour as well as capital.

Zellner(1969) considers another aspect of the aggregation problem in terms of regression models with random coefficients. He shows that under certain conditions, the usual macro two stage least square estimator is a consistent estimator for the mean of individual coefficient vectors. Theil's results relate to models with fixed coefficients. Zellner's"consistent" results with no aggregation bias relate to models with random coefficients which have been found to be useful in some situations.

APPENDIX TABLES

THE FIFTEEN PRODUCTION RELATIONS USED IN THE EMPIRICAL ANALYSIS

The equations for the fifteen production relations used for empirical analysis in this study, their numbers used throughout this study and the order in which they occur in all the tables 1-6 anywhere in the text are given below.

Table 1

1a) $V = A K^{\alpha} L^{\beta}$

1b) $Y = A_M K^{\alpha_m} L^{\beta_m} M^{\gamma_m}$

Table 2

2a) $V = A_{DI} K^{\alpha_{DI}} L^{\beta_D} I^{\beta_I}$

2b) $V = A_{EO} K^{\alpha_{EO}} L^{\beta_E} I^{\beta_O}$

Table 3

3a) $\ln V = A_1 + \alpha_1 \ln K + \beta_1 \ln L + \delta_K (\ln K/L)^2$

3b) $\ln V = A_T + \alpha_{T1} \ln K + \beta_{T1} \ln L + \alpha_{T2} (\ln K)^2 + \beta_{T2} (\ln L)^2 + \gamma_1 \ln K . \ln L$

Table 4

4a) $\ln V/L = a_L + b_L \ln w$

4b) $\ln V/L = a_{L1} + b_{L1} \ln w + c_{L1} \ln K/L$

4c) $\ln V/L = a_{L2} + b_{L2} \ln w + c_{L2} \ln K/L + d_{L2} \ln L$

Table 5

5a) $\ln V/K = a_K + b_K \ln r$

5b) $\ln V/K = a_{K1} + b_{K1} \ln r + c_{K1} \ln L/K$

5c) $\ln V/K = a_{K2} + b_{K2} \ln r + c_{K2} \ln L/K + d_{K2} \ln K$

Table 6

6a) $\ln K/L = a_s - b_s \ln w/r$

6b) $\ln wL/rK = a_s + \frac{1-b_s}{b_s} \ln w/r$

6c) $\ln L = a_v - b_v \ln w + c_v \ln V$

POOL REGRESSIONS

AND

GROUP REGRESSIONS

TABLE 1

PRODUCTION FUNCTIONS FOR ALL FIRMS
AND FOR GROUPS OF FIRMS

THREE K GROUPS
FIRMS ARRANGED IN INCREASING ORDER OF K(CAPITAL ASSETS)

COBB DOUGLAS PRODUCTION FUNCTION

(1a) $V = A K^{\alpha} L^{\beta}$ (1b) $Y = A K^{\alpha_M} L^{\beta_M} M^{\gamma_M}$

	CONST.	α	β	$\alpha+\beta$	$\bar{R}^2$	CONST.	α_M	β_M	γ_M	$\alpha_M+\beta_M+\gamma_M$	$\bar{R}^2$
FRANCE											
ALL 64 FIRMS	2.88	.197	.752	0.95	.89	2.45	.144	.673	.141	0.96	.90
		(4.6)	(13.2)				(3.1)	(10.4)	(2.35)		
POOLED K GROUPS											
SMALL K 21 FIRMS	2.78	.140	.847	0.99	.88	2.22	.124	.795	.112	1.03	.89
		(1.5)	(8.83)				(1.3)	(7.79)	(1.33)		
MEDIUM K 22 FIRMS	1.57	.295	.854	1.15	.89	1.41	.183	.789	.155	1.13	.90
		(1.3)	(12.0)				(.82)	(10.4)	(1.85)		
LARGE K 21 FIRMS	3.25	.292	.549	0.84	.56	0.86	.298	.268	.395	0.96	.69
		(2.2)	(3.95)				(2.5)	(1.64)	(2.57)		
INDIA											
ALL 117 FIRMS	0.51	.405	.605	1.07	.91	-.08	.332	.476	.201	0.99	.93
		(11.0)	(10.5)				(7.2)	(8.20)	(5.08)		
POOLED K GROUPS											
SMALL K 39 FIRMS	1.62	.320	.538	0.86	.73	0.71	.365	.360	.206	0.95	.77
		(1.33)	(7.38)				(2.7)	(2.78)	(2.23)		
MEDIUM K 39 FIRMS	1.32	.262	.735	1.00	.63	0.40	.245	.631	.197	1.07	.66
		(1.20)	(7.37)				(1.1)	(5.62)	(1.81)		
LARGE K 39 FIRMS	0.98	.515	.472	0.99	.79	-1.1	.352	.349	.449	1.15	.86
		(6.00)	(4.96)				(4.5)	(4.24)	(4.39)		
ISRAEL											
ALL 69 FIRMS	2.97	.157	.656	0.81	.84	2.21	.081	.637	.176	0.89	.86
		(3.8)	(9.53)				(1.7)	(9.69)	(2.85)		
POOLED K GROUPS											
SMALL K 23 FIRMS	3.89	.061	.596	0.65	.64	2.71	-.108	.662	.253	0.81	.74
		(.68)	(4.63)				(1.1)	(5.78)	(2.66)		
MEDIUM K 23 FIRMS	3.24	.143	.601	0.74	.74	2.81	.053	.582	.145	0.78	.77
		(.66)	(6.44)				(.25)	(6.43)	(1.62)		
LARGE K 23 FIRMS	0.55	.295	.860	1.16	.69	-.25	.224	.748	.237	1.21	.73
		(2.55)	(5.23)				(1.5)	(4.40)	(1.72)		
JAPAN											
ALL 63 FIRMS	1.46	.456	.529	0.99	.96	0.90	.314	.534	.190	1.04	.93
		(7.0)	(6.12)				(4.2)	(6.63)	(3.23)		
POOLED K GROUPS											
SMALL K 21 FIRMS	2.56	.394	.328	0.72	.61	2.44	.346	.351	.043	0.74	.64
		(2.9)	(2.57)				(1.9)	(2.47)	(0.42)		
MEDIUM K 21 FIRMS	0.75	.629	.159	0.78	.75	0.52	.452	.352	.245	1.05	.83
		(3.8)	(2.87)				(2.9)	(2.51)	(2.80)		
LARGE K 21 FIRMS	0.14	.416	.785	1.20	.87	-.81	.260	.782	.252	1.29	.90
		(2.9)	(5.61)				(2.1)	(6.13)	(2.16)		
YUGOSLAVIA											
ALL 145 FIRMS	0.17	.335	.641	0.98	.85	.37	.238	.482	.309	1.03	.89
		(9.3)	(13.7)				(6.9)	(10.3)	(6.79)		
POOLED K GROUPS											
SMALL K 49 FIRMS	0.06	.155	.852	1.01	.82	-.19	.059	.766	.201	1.03	.83
		(1.6)	(11.1)				(.58)	(9.20)	(2.21)		
MEDIUM K 49 FIRMS	2.66	.068	.572	0.64	.51	1.43	.085	.407	.296	0.79	.69
		(0.3)	(6.8)				(.49)	(5.50)	(5.18)		
LARGE K 48 FIRMS	0.21	.429	.524	0.95	.65	-.28	.223	.249	.509	0.99	.79
		(4.1)	(6.4)				(2.6)	(3.10)	(5.45)		

THE t-VALUES FOR THE REGRESSION COEFFICIENTS ARE SHOWN IN BRACKETS

TABLE 2

PRODUCTION FUNCTIONS FOR ALL FIRMS
AND FOR GROUPS OF FIRMS

THREE K GROUPS
FIRMS ARRANGED IN INCREASING ORDER OF K(CAPITAL ASSETS)

COBB DOUGLAS PRODUCTION FUNCTION

(2a) $V = AK^{\alpha_{DI}} L_D^{\beta_D} L_I^{\beta_I}$ (2b) $V = AK^{\alpha_{EO}} L_E^{\beta_E} L_O^{\beta_O}$

	(2a) CONST	α_{DI}	β_D	β_I	$\alpha+\beta+\gamma$	R^2	(2b) CONST	α_{EO}	β_E	β_O	$\alpha+\beta+\gamma$	R^2
FRANCE												
ALL 64 FIRMS	3.89	.144	.228	.450	0.92	.91	0.33	.189 (4.5)	.188 (3.6)	.596 (9.7)	0.97	.90
POOLED K GROUPS												
SMALL K 21 FIRMS	3.42	.116 (1.21)	.530 (5.0)	.370 (3.2)	1.02	.89	3.05	.141 (1.4)	.119 (.89)	.754 (7.2)	1.01	.87
MEDIUM K 22 FIRMS	3.54	.168 (1.03)	.301 (4.4)	.522 (8.6)	0.99	.95	0.02	.558 (2.8)	.224 (3.1)	.638 (7.3)	1.42	.92
LARGE K 21 FIRMS	3.82	.243 (1.79)	.233 (1.6)	.336 (2.2)	0.86	.62	4.55	.167 (1.2)	.238 (2.4)	.345 (2.4)	0.75	.64
INDIA												
ALL 117 FIRMS	0.82	.464 (10.0)	.481 (6.9)	.130 (1.9)	1.08	.91	0.56	.441 (9.1)	.157 (2.8)	.475 (8.8)	1.07	.91
POOLED K GROUPS												
SMALL K 39 FIRMS	1.90	.201 (2.0)	.478 (3.0)	.075 (.76)	0.85	.74	1.88	.389 (2.7)	.126 (1.8)	.349 (3.4)	0.86	.71
MEDIUM K 39 FIRMS	0.90	.353 (1.7)	.770 (6.3)	-.072 (.59)	1.05	.68	1.43	.297 (1.3)	.033 (.26)	.671 (6.4)	1.00	.63
LARGE K 39 FIRMS	1.82	.427 (4.6)	.220 (2.1)	.338 (2.[illegible])	1.01	.81	1.98	.284 (4.1)	.326 (3.4)	.281 (3.1)	0.99	.83
ISRAEL												
ALL 69 FIRMS	3.45	.155 (3.72)	.440 (5.5)	.213 (2.7)	0.81	.64	3.34	.156 (3.7)	.261 (4.6)	.400 (5.6)	0.82	.83
POOLED K GROUPS												
SMALL K 23 FIRMS	4.34	.043 (0.45)	.436 (3.3)	.167 (1.6)	0.64	.65	4.49	.018 (.21)	.248 (3.1)	.386 (3.8)	0.65	.69
MEDIUM K 23 FIRMS	3.65	.139 (0.64)	.441 (3.3)	.158 (1.5)	0.74	.76	3.52	.120 (.55)	-.021 (.15)	.619 (4.0)	0.72	.76
LARGE K 23 FIRMS	1.25	.287 (2.29)	.531 (3.1)	.335 (2.2)	1.15	.69	1.47	.270 (2.4)	.420 (4.4)	.483 (3.4)	1.17	.73
JAPAN												
ALL 63 FIRMS	2.47	.420 (7.0)	.131 (1.6)	.364 (6.6)	0.92	.94	2.40	.456 (7.5)	.268 (5.7)	.168 (3.3)	0.89	.92
POOLED K GROUPS												
SMALL K 21 FIRMS	2.76	.411 (3.6)	.050 (0.9)	.293 (3.6)	0.81	.75	2.99	.370 (3.1)	.159 (2.9)	.188 (1.9)	0.72	.71
MEDIUM K 21 FIRMS	2.10	.612 (3.9)	-.043 (.24)	.344 (3.1)	0.91	.79	.96	.560 (3.1)	.277 (2.8)	.114 (1.3)	0.95	.76
LARGE K 21 FIRMS	0.56	.394 (3.3)	.452 (2.8)	.369 (2.7)	1.18	.87	1.36	.427 (3.2)	.411 (3.7)	.256 (3.3)	1.09	.85
YUGOSLAVIA												
ALL 145 FIRMS	0.92	.346 (10.2)	.279 (4.1)	.336 (6.5)	0.96	.86	1.23	.294 (8.6)	.255 (6.2)	.362 (6.0)	0.95	.87
POOLED K GROUPS												
SMALL K 49 FIRMS	1.35	.118 ([illegible])	.410 (3.4)	.427 (4.3)	0.95	.84	0.60	.142 (1.4)	.111 (1.0)	.749 (6.1)	0.99	.8
MEDIUM K 4[illegible] FIRMS	2.[illegible]4	.090 (0.41)	.404 (3.1)	.158 (1.6)	0.65	.51	3.27	.058 (.36)	.311 (4.3)	.338 (3.4)	0.69	.64
LARGE K 48 FIRMS	1.[illegible]3	.3[illegible] (4.47)	.062 (0.6)	.417 (5.8)	0.87	.76	2.01	.331 (3.6)	.403 (5.0)	.140 (1.4)	0.67	.76

THE t-VALUES FOR THE REGRESSION COEFFICIENTS ARE SHOWN IN BRACKETS.

$$\ln V = const + \alpha_K \ln K + \beta_K \ln L + \delta_K (\ln K/L)^2 \qquad (3a)$$

$$\ln V = const + \alpha_{T1} \ln K + \beta_{T1} \ln L + \alpha_{T2} (\ln K)^2 + \beta_{T2} (\ln L)^2 + \gamma_T \ln K \ln L \qquad (3b)$$

TABLE 3

PRODUCTION FUNCTIONS FOR ALL FIRMS
AND FOR GROUPS OF FIRMS

THREE K GROUPS
FIRMS ARRANGED IN INCREASING ORDER OF K (CAPITAL ASSETS)

	(3a)					3(b)						
	CONST	α_K	β_K	δ_K	R2	CONST	α_{T1}	β_{T1}	α_{T2}	β_{T2}	γ_T	R2
FRANCE												
ALL 64 FIRMS	3.14	−.087 (.57)	1.050 (6.35)	.050 (1.9)	.90	3.0	−.03 (.11)	.15 (4.1)	.01 (.84)	−.07 (2.1)		.90
POOLED K GROUPS												
SMALL K 21 FIRMS	2.01	.074 (.23)	.917 (2.76)	.020 (.22)	.88							
MEDIUM K 22 FIRMS	0.80	1.173 (3.1)	−.073 (.207)	−.159 (2.7)	.92							
LARGE K 21 FIRMS	5.23	−1.06 (1.9)	1.943 (3.38)	.187 (2.5)	.68							
INDIA												
ALL 117 FIRMS	0.64	.335 (2.3)	.725 (5.1)	.035 (.53)	.91	0.86	.81 (4.4)	..	−.02 (1.8)	.05 (9.9)	.	.91
POOLED K GROUPS												
SMALL K 39 FIRMS	1.62	−.087 (.25)	.967 (2.8)	.229 (1.3)	.75							
MEDIUM K 39 FIRMS	2.13	−.071 (.17)	.575 (3.5)	.086 (.91)	.64							
LARGE K 39 FIRMS	−.01	1.146 (7.6)	−.112 (.79)	−.127 (.16)	.80							
ISRAEL												
ALL 69 FIRMS	3.21	−.108 (.80)	.928 (6.24)	.052 (2.1)	.85	2.7	−.32 (1 [illegible])		.03 ([illegible] 5)	.07 (10.2)		.86
POOLED K GROUPS												
SMALL K 23 FIRMS	4.01	−.143 (.66)	.779 (3.44)	.083 (1.0)	.66							
MEDIUM K 23 FIRMS	3.34	.054 (.11)	.693 (1.43)	.016 (.19)	.74							
LARGE K 23 FIRMS	4.31	−1.44 (1.5)	2.465 (2.72)	.241 (1.8)	.73							
JAPAN												
ALL 63 FIRMS	1.49	.426 (1.7)	.559 (2.1)	.006 (.12)	.92	1.4	.67 (2.8)		−.03 (1.0)	.055 (6.5)		.92
POOLED K GROUPS												
SMALL K 21 FIRMS	2.64	.726 (1.3)	−.075 (.11)	−.111 (.61)	.64							
MEDIUM K 21 FIRMS	1.58	−.198 (.24)	1.289 (1.5)	.181 (1.0)	.76							
LARGE K 21 FIRMS	0.83	REF	1.184 (11.1)	.108 (4.1)	.88							
YUGOSLAVIA												
ALL 145 FIRMS	0.09	.120 (1.8)	.877 (11.5)	.088 (3.8)	.86	0.1		--	.02 (8.9)	.05 (13.9)	..	.85
POOLED K GROUPS												
SMALL K 49 FIRMS	0.26	.221 (2.0)	.761 (7.2)	−.124 (1.2)	.82							
MEDIUM K 48 FIRMS	2.73	−.200 (.77)	.842 (4.9)	.114 (1.8)	.54							
LARGE K 48 FIRMS	0.15	−.051 (.21)	1.062 (4.2)	.125 (2.2)	.69							

THE t-VALUES FOR THE REGRESSION COEFFICIENTS ARE SHOWN IN BRACKETS.

$$\ln V/L = a_L + b_L \ln w \qquad (4a)\ \text{CESL}$$

$$\ln V/L = a_{L1} + b_{L1} \ln w + c_{L1} \ln K/L \qquad (4b)\ \text{VESL1}$$

$$\ln V/L = a_{L2} + b_{L2} \ln w + c_{L2} \ln K/L + d_{L2} \ln L \qquad (4c)\ \text{VESL2}$$

TABLE 4

PRODUCTION FUNCTIONS FOR ALL FIRMS

THREE K GROUPS

FIRMS ARRANGED IN INCREASING ORDER OF K(CAPITAL ASSETS)

CES AND VES PRODUCTION FUNCTIONS USING LABOUR

	(4a) CESL			(4b) VESL1				(4c) VESL2				
	CONST	b_L	R2	CONST	b_{L1}	c_{L1}	R2	CONST	b_{L2}	c_{L2}	d_{L2}	R^2
FRANCE												
ALL 64 FIRMS	0.94	0.81	.44	0.67	.75	.18	.63	0.72	.74	.18	-.01	.63
		(6.93)			(7.82)	(5.7)			(7.58)	(5.6)	(.22)	
POOLED K GROUPS												
SMALL K 21 FIRMS	0.89	0.78	.48	0.92	.72	.08	.51	0.74	.73	.09	.03	.52
		(4.16)			(3.69)	(1.2)			(3.62)	(1.2)	(.39)	
MEDIUM K 22 FIRMS	1.43	0.63	.37	1.15	.61	.15	.55	1.45	.62	.11	-.04	.55
		(3.39)			(3.75)	(2.8)			(3.57)	(.58)	(.22)	
LARGE K 21 FIRMS	0.66	0.56	.52	0.21	.79	.21	.74	0.76	.78	.25	-.06	.74
		(4.[illegible]7)			(4.74)	(3.8)			(4.49)	([illegible].6)	(.50)	
INDIA												
ALL 117 FIRMS	0.97	0.79	.38	0.64	0.50	.37	.65	0.53	0.49	.37	.02	.6[illegible]
		(9.4)			(6.4)	(5.3)			(6.0)	(9.3)	([illegible])	
POOLED K GROUPS												
SMALL K 37 FIRMS	0.64	0.94	.69	0.50	0.85	.21	.74	1.11	0.83	.13	-.10	.77
		(9.1)			(6.3)	(2.7)			(8.5)	(1.5)	(2.0)	
MEDIUM K 39 FIRMS	0.72	0.99	.63	0.53	0.93	.19	.72	1.38	0.94	.09	-.11	.72
		(7.9)			(8.3)	(3.3)			(8.3)	(.51)	(.41)	
LARGE K 39 FIRMS	1.[illegible]	0.22	.04	0.85	0.03	.52	.55	1.00	0.04	.51	-.02	.55
		(1.2)			(.24)	(6.3)			(.27)	(5.6)	(.21)	
ISRAEL												
ALL 69 FIRMS	0.69	0.67	.63	0.51	.84	.09	.69	1.16	.78	.12	-.12	.75
		(10.7)			(11.1)	([illegible].5)			(11.1)	(4.7)	(3.9)	
POOLED K GROUPS												
SMALL K 23 FIRMS	0.82	0.80	.60	0.78	.79	.05	.61	2.15	.67	.02	-.25	.77
		(5.61)			(5.33)	(.70)			(5.[illegible])	(.36)	(3.6)	
MEDIUM K 23 FIRMS	0.26	1.08	.71	0.19	.86	.19	.78	1.34	.83	.05	-.15	.80
		(7.21)			(5.43)	(2.5)			(5.21)	(.32)	(1.1)	
LARGE K 23 FIRMS	0.52	0.78	.60	0.04	.78	.26	.82	0.07	.78	.26	-.01	.82
		(5.63)			(8.34)	(5.0)			(7.92)	(4.4)	(.05)	
JAPAN												
ALL 6[illegible] FIRMS	0.65	0.85	.31	0.40	0.68	.39	.63	0.66	0.86	.41	-.12	.68
		(5.2)			(5.2)	(7.2)			(6.4)	(8.1)	(3.1)	
POOLED K GROUPS												
SMALL K 21 FIRMS	1.21	0.58	.11	0.36	0.56	.54	.61	1.76	0.58	.38	-.29	.72
		(1.5)			(2.2)	(4.8)			(2.5)	(3.2)	(2.6)	
MEDIUM K 21 FIRMS	1.06	1.06	.20	-.18	0.85	.59	.69	0.11	0.89	.56	-.06	.69
		(2.2)			(3.6)	(5.3)			(3.4)	(4.3)	(.45)	
LARGE K 21 FIRMS	0.80	0.94	.49	0.12	0.84	.33	.76	0.24	0.87	.32	-.02	.76
		(4.3)							(4.6)	(4.2)	(.26)	
YUGOSLAVIA												
ALL 145 FIRMS	0.99	1.36	.52	0.64	1.12	.25	.71	0.7[illegible]	1.12	.25	-.02	.71
		([illegible].4)			(12.6)	(9.8)			(12.6)	(9.6)	(.67)	
POOLED K GROUPS												
SMALL K 49 FIRMS	0.88	1.36	.70	0.83	1.32	.07	.72	0.78	1.32	.08	.01	.72
		(10.6)			(10.3)	(1.8)			(10.2)	(1.5)	(.12)	
MEDIUM K 48 FIRMS	0.58	1.46	.44	0.61	1.16	.26	.58	1.07	1.13	.21	-.07	.58
		(6.0)			(5.1)	([illegible].8)			(4.7)	(1.1)	(.31)	
LARGE K 48 FIRMS	1.11	1.21	.53	0.42	.99	.36	.79	1.00	1.02	.26	-.14	.81
		(7.2)			(8.5)	(7.5)			(9.0)	(3.9)	(-2.1)	

THE t-VALUES FOR THE REGRESSION COEFFICIENTS ARE SHOWN IN BRACKETS.

$$\ln V/K = a_K + b_K \ln r \qquad \text{(5a) CESK}$$

$$\ln V/K = a_{K1} + b_{K1} \ln r + c_{K1} \ln L/K \qquad \text{(5b) VESK1}$$

$$\ln V/K = a_{K2} + b_{K2} \ln r + c_{K2} \ln L/K + d_{K2} \ln K \qquad \text{(5c) VESK2}$$

TABLE 5

PRODUCTION FUNCTIONS FOR ALL FIRMS
AND FOR GROUPS OF FIRMS

THREE K GROUPS
FIRMS ARRANGED IN INCREASING ORDER OF K(CAPITAL ASSETS)

CES AND VES PRODUCTION FUNCTIONS USING CAPITAL

	(5a) CESK			(5b) VESK1				(5c) VESK2				
	CONST	b_K	R2	CONST	b_{K1}	c_{K1}	R2	CONST	b_{K2}	c_{K2}	d_{K2}	R^2
FRANCE												
ALL 64 FIRMS	1.16	0.78 (33.3)	.59	2.39	0.36 (8.21)	.62 (16.8)	.93	2.79	0.36 (8.66)	.56 (12.5)	-.07 (2.4)	.93
POOLED K GROUPS												
SMALL K 21 FIRMS	1.4[illegible]	0.50 (4.13)	.47	2.56	0.21 (3.47)	.72 (9.42)	.91	2.99	0.22 (3.65)	.67 (7.47)	-.08 (1.1)	.92
MEDIUM K 22 FIRMS	1.13	0.80 (5.84)	.63	2.48	0.34 (5.28)	.65 (11.6)	.95	1.23	0.34 (5.30)	.67 (11.6)	.16 (1.0)	.96
LARGE K 21 FIRMS	0.88	0.95 (6.55)	.59	1.92	0.58 (7.42)	.42 (7.03)	.92	2.45	0.57 (7.14)	.39 (5.17)	-.07 (.69)	.92
INDIA												
ALL 117 FIRMS	0.62	0.75 (17.4)	.72	1.00	0.57 (18.7)	.32 (13.6)	.89	0.53	0.57 (20.5)	.41 (14.5)	.08 (4.8)	.91
POOLED K GROUPS												
SMALL K 39 FIRMS	0.72	0.73 (9.43)	.71	0.97	0.63 (9.79)	.34 (4.95)	.82	0.79	0.64 (9.29)	.34 (4.92)	.03 (.61)	.83
MEDIUM K 39 FIRMS	0.74	0.85 (12.0)	.79	1.17	0.63 (13.2)	.41 (8.79)	.96	0.37	0.[illegible] (1[illegible].3)	.42 (8.89)	.11 (1.1)	.94
LARGE K 39 FIRMS	0.23	0.57 (6.17)	.64	1.00	0.45 (10.7)	.35 (8.50)	.88	0.11	0.48 (11.8)	.40 (9.29)	.11 ([illegible].7)	.90
ISRAEL												
ALL 69 FIRMS	1.06	0.89 (14.2)	.7[illegible]	1.96	0.43 (9.50)	.59 (14.2)	.94	2.06	0.42 (7.78)	.58 (11.3)	-.02 (.53)	.94
POOLED K GROUPS												
SMALL K 23 FIRMS	1.12	0.5[illegible] (3.94)	.43	2.16	0.[illegible]1 (4.07)	.74 (8.45)	.87	2.64	0.24 (2.27)	.66 (5.46)	-.14 (1.0)	.88
MEDIUM K 23 FIRMS	0.67	0.64 (4.83)	.5[illegible]	1.61	0.44 (5.71)	.47 (7.21)	.87	2.37	0.42 (5.33)	.47 (7.19)	-.10 (.77)	.87
LARGE K 23 FIRMS	1.15	1.01 (5.43)	.59	2.21	0.63 (5.39)	.55 (6.98)	.88	1.31	0.62 (5.39)	.64 (5.68)	.13 (1.2)	.89
JAPAN												
ALL 63 FIRMS	0.64	0.62 (7.71)	.49	1.55	0.51 (12.8)	.47 (13.9)	.88	1.45	0.52 (12.8)	.50 (11.0)	.02 (1.0)	.88
POOLED K GROUPS												
SMALL K 21 FIRMS	0.68	0.[illegible]6 (2.76)	.29	1.[illegible]6	0.43 (6.00)	.51 (6.97)	.80	1.65	0.42 (4.88)	.50 (5.44)	-.02 (.20)	.90
MEDIUM K 21 FIRMS	0.63	0.49 (3.61)	.41	1.58	0.54 (6.58)	.47 (5.92)	.80	1.57	0.5[illegible] (6.31)	.47 (5.23)	.002 (.03)	.80
LARGE K 21 FIRMS	0.65	0.[illegible]7 (7.[illegible]5)	.76	1.40	0.63 (8.03)	.[illegible]7 ([illegible])	.94	0.63	0.60 (8.25)	.48 (6.39)	.12 (2.2)	.94
YUGOSLAVIA												
ALL 145 FIRMS	0.56	0.93 (30.97)	.87	0.54	0.67 (39.1)	.[illegible]3 (24.5)	.98	0.62	0.67 (39.1)	.32 (20.0)	-.01 ([illegible].1)	.98
POOLED K GROUPS												
SMALL K 4[illegible] FIRMS	0.61	0.87 ([illegible]0.53)	.9[illegible]	0.52	0.6[illegible] (24.[illegible])	.39 (14.4)	.98	0.49	0.62 (24.3)	.39 (14.0)	.01 (.27)	.98
MEDIUM K 48 FIRMS	0.41	0.77 (15.60)	.84	0.48	0.60 (26.2)	.34 (1[illegible].2)	.97	0.91	0.60 (25.5)	.33 (14.4)	-.06 (.94)	.97
LARGE K 48 FIRMS	0.[illegible]	0.[illegible]8 (19.38)	.90	0.[illegible]7	0.[illegible] (24.4)	.23 (9.[illegible]5)	.97	0.25	0.81 (24.6)	.24 (9.[illegible]5)	.04 (1.3)	.97

THE t-VALUES FOR THE REGRESSION COEFFICIENTS ARE SHOWN IN BRACKETS.

$$\ln K/L = \text{const} + b_D \ln w/r \quad (6a)$$

$$\ln wL/rK = \text{const} + \frac{1-b_s}{b_s} \ln K/L \quad (6b)$$

$$\ln L = \text{const} - b_v \ln w + c_v \ln V \quad (6c)$$

TABLE 6

PRODUCTION FUNCTIONS FOR ALL FIRMS
AND FOR GROUPS OF FIRMS

THREE K GROUPS
FIRMS ARRANGED IN INCREASING ORDER OF K(CAPITAL ASSETS)

	(6a) CONST.	b_D	R^2	(6b) CONST	$(1-b_s)/b_s$	R^2	(6c) CONST	c_v	$-b_v$	R^2
FRANCE										
ALL 64 FIRMS	0.25	0.664 (5.84)	.26	2.03	-.466 (5.1)	.29	-.42	0.937 (26.0)	-0.758 (6.54)	.92
POOLED K GROUPS										
SMALL K 21 FIRMS	0.56	0.385 (2.93)	.30	1.77	-.240 (.89)	.04	-.39	0.936 (15.3)	-0.783 (4.17)	.93
MEDIUM K 22 FIRMS	0.34	0.65 (3.48)	.38	1.84	-.420 (2.5)	.24	-2.0	1.089 (15.5)	-0.657 (3.66)	.93
LARGE K 21 FIRMS	0.46	0.74 (3.26	.36	2.14	-.516 (3.5)	.39	-.16	0.942 (6.31)	-0.935 (4.14)	.72
INDIA										
ALL 117 FIRMS	0.58	.635 (8.02)	.36	0.69	-0.44 (6.2)	.25	0.21	0.822 (24.3)	-0.540 (5.51)	.85
POOLED K GROUPS										
SMALL K 39 FIRMS	0.37	.459 (3.82)	.28	0.62	-0.39 (2.4)	.14	-.94	1.051 (18.1)	-0.966 (9.02)	.91
MEDIUM K 39 FIRMS	0.32	.693 (5.11)	.41	0.61	-0.40 (3.4)	.24	-1.07	1.047 (14.1)	-1.029 (7.40)	.85
LARGE K 39 FIRMS	1.94	.286 (2.32)	.13	1.05	-0.56 (2.9)	.19	0.32	0.751 (7.02)	-0.124 (.709)	.58
ISRAEL										
ALL 69 FIRMS	-.08	0.793 (9.09)	.55	1.54	-.304 (4.0)	.19	-.74	1.008 (29.1)	-0.676 (10.5)	.93
POOLED K GROUPS										
SMALL K 23 FIRMS	0.65	0.376 (2.05)	.20	1.52	-.459 (1.9)	.15	-1.7	1.150 (10.9)	-0.851 (5.91)	.87
MEDIUM K 23 FIRMS	0.54	0.613 (3.20)	.34	2.01	-.443 (2.6)	.25	-.94	1.088 (12.8)	-1.055 (7.00)	.92
LARGE K 23 FIRMS	0.75	0.654 (2.34)	.21	2.97	-.685 (5.1)	.55	-.20	0.885 (7.45)	-0.657 (3.55)	.75
JAPAN										
ALL 63 FIRMS	1.34	0.323 (2.34)	.08	1.76	-0.75 (6.8)	.43	-0.52	0.858 (17.3)	-0.633 (2.99)	.87
POOLED K GROUPS										
SMALL K 21 FIRMS	1.85	-.102 (0.51)	.01	2.35	-1.13 (4.4)	.50	-1.21	0.598 (4.23)	-0.578 (1.39)	.51
MEDIUM K 21 FIRMS	2.23	-.119 (0.50)	.01	2.27	-1.11 (5.1)	.58	0.06	0.780 (4.47)	-0.475 (1.05)	.57
LARGE K 21 FIRMS	0.58	.757 (3.20)	.35	1.48	-0.54 (3.7)	.42	-0.08	0.861 (7.19)	-0.665 (2.08)	.80
YUGOSLAVIA										
ALL 145 FIRMS	0.19	0.968 (13.5)	.56	.24	-.421 (9.8)	.40	-.32	0.912 (30.5)	-1.257 (11.2)	.87
POOLED K GROUPS										
SMALL K 49 FIRMS	0.05	0.787 (8.95)	.63	.13	-.199 (2.2)	.10	-.70	0.974 (25.3)	-1.337 (9.96)	.94
MEDIUM K 48 FIRMS	0.35	0.621 (5.05)	.36	.33	-.425 (3.7)	.23	-.37	0.914 (9.37)	-1.454 (5.96)	.72
LARGE K 48 FIRMS	0.44	1.068 (7.03)	.52	.33	-.515 (7.5)	.55	-2.4	1.153 (12.5)	-1.364 (7.20)	.78

THE t-VALUES FOR THE REGRESSION COEFFICIENTS ARE SHOWN IN BRACKETS.

ANALYSIS OF COVARIANCE TABLES

A ** indicates significance at 1% level

TABLE 1A

A N A L Y S I S O F C O V A R I A N C E

THREE K GROUPS

FIRMS ARRANGED IN INCREASING ORDER OF
CAPITAL ASSETS, K

COBB DOUGLAS PRODUCTION FUNCTION

(1a) $V = A K^{\alpha} L^{\beta}$ (1b) $V = A K^{\alpha_M} L^{\beta_M} M^{\gamma_M}$

	(1a) D F	(1a) MEAN SQ	(1a) F RATIO	(1b) D F	(1b) MEAN SQ	(1b) F RATIO
FRANCE						
REGRESSION OVER GRPS	6	0 165	1 205	8	0 250	2 0[illegible]
RESIDUAL WITHIN GRPS	55	0 135		52	0 115	
INDIA						
REGRESSION OVER GRPS	6	0 444	1 831	8	0 268	1 [illegible]
RESIDUAL WITHIN GRPS	108	0 24[illegible]		105	0 [illegible]	
ISRAEL						
REGRESSION OVER GRPS	6	0 218	1 834	8	0 192	1 84
RESIDUAL WITHIN GRPS	60	0 119		57	0 105	
JAPAN						
REGRESSION OVER GRPS	6	0 314	2 69	8	0 238	[illegible] 40
RESIDUAL WITHIN GRPS	54	0 117		51	0 099	
YUGOSLAVIA						
REGRESSION OVER GRPS	6	0 264	1 93	8	0 236	3 0[illegible]**
RESIDUAL WITHIN GRPS	136	0 137		135	0 097	

TABLE 2A

A N A L Y S I S O F C O V A R I A N C E

THREE K GROUPS

FIRMS ARRANGED IN INCREASING ORDER OF
CAPITAL ASSETS, K

COBB DOUGLAS PRODUCTION FUNCTION

(2a) $V = A K^{\alpha_{DI}} L_D^{\beta_D} L_I^{\beta_I}$ (2b) $V = A K^{\alpha_{EO}} L_E^{\beta_E} L_O^{\beta_O}$

	(2a) D F	(2a) MEAN SQ	(2a) F RATIO	(2b) D F	(2b) MEAN SQ	(2b) F RATIO
FRANCE						
REGRESN OVER GROUPS	8	0 110	1 0[illegible]7	8	0 161	1 310
RESID WITHIN GROUPS	52	0 115		52	0 1[illegible]	
INDIA						
REGRESN OVER GROUPS	8	0 559	2 615	8	0 509	2 389
RESID WITHIN GROUPS	105	0 2[illegible]		105	0 2[illegible]3	
ISRAEL						
REGRESN OVER GROUPS	[illegible]	0 [illegible]	1 516	[illegible]	0 281	2 510
RESID WITHIN GROUPS	[illegible]	0 1[illegible]		57	0 11[illegible]	
JAPAN						
REGRESN OVER GROUPS	8	0 156	1 320	8	0 196	1 661
RESID WITHIN GROUPS	51	0 105		51	0 118	
YUGOSLAVIA						
REGRESN OVER GROUPS	8	0 279	2 380	8	0 271	2 445
RESID WITHIN GROUPS	1[illegible]	0 117		1[illegible]	0 111	

TABLE 3A

A N A L Y S I S   O F   C O V A R I A N C E

THREE K GROUPS
FIRMS ARRANGED IN INCREASING ORDER OF
CAPITAL ASSETS, K

(3a) $\ln V = cons + \alpha_K \ln K + \beta_L \ln L + \delta_K (\ln K/L)^2$

(3b) $\ln V = cons + \beta_T \ln \frac{K}{L} + \delta_T (\ln \frac{K}{L})^2 + \gamma_T \ln K \ln L$

	D F (3a)	MEAN SQ (3a)	F RATIO (3a)	D F (3b)	MEAN SQ (3b)	F RATIO (3b)
FRANCE						
REGRESN OVER GROUPS	8	0 260	2 304	4	0 187	1 856
RESID WITHIN GROUPS	52	0 115		58	0 101	
INDIA						
REGRESN OVER GROUPS	8	0 455	1 817	8	0 440	1 83[illegible]
RESID WITHIN GROUPS	105	0 240		105	0 240	
ISRAEL						
REGRESN OVER GROUPS	8	0 168	1 452	8	0 199	1 709
RESID WITHIN GROUPS	57	0 111		57	0 116	
JAPAN						
REGRESN OVER GROUPS	7	0 315	2 708	8	0 294	2 570
RESID WITHIN GROUPS	52	0 115		51	0 115	
YUGOSLAVIA						
REGRESN OVER GROUPS	8	0 1[illegible]6	1 0[illegible]2	8	0 1[illegible]5	1 020
RESID WITHIN GROUPS	1[illegible]	0 1[illegible]0		1[illegible]	0 130	

TABLE 4A

A N A L Y S I S   O F   C O V A R I A N C E

THREE K GROUPS

FIRMS ARRANGED IN INCREASING ORDER OF
CAPITAL ASSETS, K

CES AND VES PRODUCTION FUNCTIONS USING
LABOUR

(4a) CESL   (4b) VESL1   (4c) VESL2

	DF (4a)	MEANSQ (4a)	F RATIO (4a)	DF (4b)	MEANSQ (4b)	F RATIO (4b)	DF (4c)	MEANSQ (4c)	F RATIO (4c)
FRANCE									
REGRESN OVER GRPS	4	0 187	1 856	6	0 077	1 100	5	0 06[illegible]	1 11[illegible]
RESID WITHIN GRPS	58	0 101			0 070		9	0 07[illegible]	
INDIA									
REGRESN OVER GRPS	4	3 019	12 727**	6	1 167	8 467**	8	0 90[illegible]	6 504**
RESID WITHIN GRPS	111	0 237		108	0 158		[illegible]05	0 159	
ISRAEL									
REGRESN OVER GRPS	4	0 056	1 016	6	0 1[illegible]	2 824	8	0 075	1 867
RESID WITHIN GRPS	65	0 065		60	0 047		57	0 041	
JAPAN									
REGRESN OVER GRPS	4	0 152	1 326	6	0 2[illegible]6	5 016	8	0 14[illegible]	[illegible] 000
RESID WITHIN GRPS	57	0 175		54	0 079		51	0 07[illegible]	
YUGOSLAVIA									
REGRESN OVER GRPS	4	0 077	6 657**		0 186	2 990**	8	0 168	2 757**
RESID WITHIN GRPS	1[illegible]9	0 097		1[illegible]6	0 062		1[illegible]	0 062	

TABLE 5A

ANALYSIS OF COVARIANCE

THREE K GROUPS

FIRMS ARRANGED IN INCREASING ORDER OF
CAPITAL ASSETS, K

CES AND VES PRODUCTION FUNCTIONS USING
CAPITAL

(5a) CESK (5b) VESK1 (5c) VESK2

	DF	MEANSQ	F RATIO	DF	MEANSQ	F RATIO	DF	MEANSQ	F RATIO
FRANCE									
REGRESN OVER GRPS	4	1 371	6 949**	6	0 225	4 494**	8	0 141	2 804
RESID WITHIN GRPS	53	0 263		55	0 050		2	0 050	
INDIA									
REGRESN OVER GRPS	4	0 945	6 814**	6	0 2 5	4 70[illegible]**	8	0 03[illegible]	1 807
RESID WITHIN GRPS	111	0 139		108	0 054		105	0 02[illegible]	
ISRAEL									
REGRESN OVER GRPS	4	0 789	3 397	6	0 111	1 736	8	0 105	1 705
RESID WITHIN GRPS	6[illegible]	0 [illegible]2		60	0 062		57	0 062	
JAPAN									
REGRESN OVER GRPS	4	0 719	6 315**	6	0 0[illegible]1	1 2[illegible]6	8	0 0[illegible]	1 187
RESID WITHIN GRPS	57	0 114		54	0 0[illegible]8		51	0 0[illegible]8	
YUGOSLAVIA									
REGRESN OVER GRPS	4	0 721	16 47**	[illegible]	0 07[illegible]	7 7[illegible]2*	8	0 050	5 064**
RESID WITHIN GRPS	1[illegible]9	0 044		1[illegible]6	0 009		15[illegible]	0 009	

TABLE 6A

ANALYSIS OF COVARIANCE

THREE K GROUPS

FIRMS ARRANGED IN INCREASING ORDER OF
CAPITAL ASSETS, K

(6a) ln K/L $= const + b_D \ln \frac{w}{r}$ (6b) ln wL/rK $= const + \frac{1-b_s}{b_s} \ln K/L$ (6c) ln L $= c$ [illegible] $- b_V \ln w + c_V \ln V$

	DF	MEANSQ	F RATIO	DF	MEANSQ	F RATIO	DF	MEANSQ	F RATIO
FRANCE									
REGRESN OVER GRPS	4	[illegible] [illegible]8	5 488**	4	0 [illegible]3[illegible]	1 955	6	0 196	1 0[illegible]1
RESID WITHIN GRPS	55	0 [illegible]05		55	[illegible]49		55	0 10[illegible]	
INDIA									
REGRESN OVER GRPS	4	7 4[illegible]	1[illegible] 34*		[illegible] 201	[illegible] 578**	6	1 19[illegible]	5 [illegible]0**
RESID WITHIN GRPS	111	0 559		1[illegible]1	[illegible] 7[illegible]0		10[illegible]	0 2[illegible]0	
ISRAEL									
REGRESN OVER GRPS	4	1 675	5 66**	6	0 077	1 20[illegible]	4	1 1[illegible]3	2 748
RESID WITHIN GRPS	6[illegible]	0 458		[illegible]0	0 064		65	0 422	
JAPAN									
REGRESN OVER GRPS	4	[illegible] 676	7 39**	4	1 220	[illegible] 433	6	0 057	[illegible] 088
RESID WITHIN GRPS	57	0 36[illegible]		57	[illegible] [illegible]43		54	0 176	
YUGOSLAVIA									
REGRESN OVER GRPS	4	[illegible] 184	1[illegible] 18**	[illegible]	0 455	[illegible] 297	6	0 355	5 730**
RESID WITHIN GRPS	1[illegible]9	0 26[illegible]		133	0 198		156	0 095	

TABLE 1B

ANALYSIS OF COVARIANCE

THREE L GROUPS

FIRMS ARRANGED IN INCREASING ORDER OF
L, TOTAL LABOUR

COBB DOUGLAS PRODUCTION FUNCTION

(1a) $V = A K^{\alpha} L^{\beta}$ (1b) $Y = A K^{\alpha_M} L^{\beta_M} M^{\gamma_M}$

	(1a) D F	MEANSQ	F RATIO	(1b) D F	MEANSQ	F RATIO
FRANCE						
REGRESN OVER GRPS	6	0 197	1 497	8	0 162	1 313
RESID WITHIN GRPS	55	0 13[illegible]		52	0 123	
INDIA						
REGRESN OVER GRPS	6	0 459	1 898	8	0 231	1 119
RESID WITHIN GRPS	108	0 242		105	0 205	
ISRAEL						
REGRESN OVER GRPS	6	0 197	1 6[illegible]4	8	0 203	1 957
RESID WITHIN GRPS	60	0 121		57	0 103	
JAPAN						
REGRESN OVER GRPS	6	0 322	2 771	8	0 187	1 [illegible]8
RESID WITHIN GRPS	54	0 316		51	0 107	
YUGOSLAVIA						
REGRESN OVER GRPS	6	0 229	1 654	8	0 233	2 317
RESID WITHIN GRPS	136	0 1[illegible]8		133	0 10[illegible]	

TABLE 2B

ANALYSIS OF COVARIANCE

THREE L GROUPS

FIRMS ARRANGED IN INCREASING ORDER OF
L, TOTAL LABOUR

COBB DOUGLAS PRODUCTION FUNCTION

(2a) $V = A K^{\alpha_{DI}} L_D^{\beta_D} L_I^{\beta_I}$ (2b) $V = A K^{\alpha_{EO}} L_E^{\beta_E} L_O^{\beta_O}$

	(2a) D F	MEANSQ	F RATIO	(2b) D F	MEANSQ	F RATIO
FRANCE						
REGRESN OVER GRPS	8	0 140	1 286	8	0 30[illegible]	[illegible] 969**
RESID WITHIN GRPS	52	0 109		52	0 102	
INDIA						
REGRESN OVER GRPS	8	0 408	1 711	8	0 474	1 962
RESID WITHIN GRPS	10[illegible]	0 239		10[illegible]	0 24[illegible]	
ISRAEL						
REGRESN OVER GRPS	8	0 173	1 403	8	0 214	1 759
RESID WITHIN GRPS	57	0 12[illegible]4		57	0 121	
JAPAN						
REGRESN OVER GRPS	8	0 208	2 258	8	0 240	2 1[illegible]
RESID WITHIN GRPS	51	0 09[illegible]		51	0 111	
YUGOSLAVIA						
REGRESN OVER GRPS	8	0 221	1 82[illegible]	8	0 145	1 221
RESID WITHIN GRPS	133	0 121		133	0 118	

TABLE 3B

A N A L Y S I S   O F   C O V A R I A N C E

THREE L GROUPS

FIRMS ARRANGED IN INCREASING ORDER OF
L, TOTAL LABOUR

(3a) $\ln V = cons + \alpha_K \ln K + \beta_K \ln L + \delta_K (\ln K/L)^2$

(3b) $\ln V = cons + \beta_T \ln K/L + \delta_T (\ln \frac{K}{L})^2 + \gamma_T \ln K \ln L$

	D F	MEANSQ	F RATIO	D F	MEANSQ	F RATIO
FRANCE						
REGRESN OVER GRPS	8	0 156	1 211	8	0 1[illegible]2	1 197
RESID WITHIN GRPS	52	0 129		52	0 128	
INDIA						
REGRESN OVER GRPS	8	0 372	1 521	[illegible]	0 397	1 630
RESID WITHIN GRPS	105	0 214		105	0 243	
ISRAEL						
REGRESN OVER GRPS	8	0 106	1 170	8	0 137	1 093
RESID WITHIN GRPS	57	0 124		57	0 125	
JAPAN						
REGRESN OVER GRPS	7	0 311	2 682	8	0 324	2 992 **
RESID WITHIN GRPS	[illegible]	0 116		51	0 110	
YUGOSLAVIA						
REGRESN OVER GRPS	8	0 171	1 340	8	0 167	1 311
RESID WITHIN GRPS	133	0 127		133	0 127	

TABLE 4B

A N A L Y S I S   O F   C O V A R I A N C E

THREE L GROUPS

FIRMS ARRANGED IN INCREASING ORDER OF
L, TOTAL LABOUR

CES AND VES PRODUCTION FUNCTIONS USING LABOUR

	(4a) CESL			(4b) VESL1			(4c) VESL2		
	DF	MEANSQ	F RATIO	DF	MEANSQ	F RATIO	DF	MEANSQ	F RATIO
FRANCE									
REGRESN OVER GRPS	4	0 095	1 271	6	0 369	1 029	8	0 108	1 616
RESID WITHIN GRPS	58	0 101		5[illegible]	0 371		[illegible]	0 366	
INDIA									
REGRESN OVER GRPS	1	1 312	4 391**	6	0 395	2 185	8	0 465	2 689**
RESID WITHIN GRPS	111	0 299		[illegible]0	0 181		105	0 172	
ISRAEL									
REGRESN OVER GRPS	1	0 069	1 073	5	[illegible]	2 767	8	0 055	1 258
RESID WITHIN GRPS	6[illegible]	0 064		[illegible]	0 047		57	0 044	
JAPAN									
REGRESN OVER GRPS	1	0 946	[illegible] 299**	6	[illegible]	4 557**	8	0 159	2 095
RESID WITHIN GRPS	57	0 191		51	[illegible]		51	0 071	
YUGOSLAVIA									
REGRESN OVER GRPS	1	0 211	2 224	6	0 034	2 0[illegible]3	8	0 229	2 414
RESID WITHIN GRPS	139			1[illegible]	0 069		13[illegible]	0 079	

TABLE 5B

ANALYSIS OF COVARIANCE

THREE L GROUPS

FIRMS ARRANGED IN INCREASING ORDER OF L, TOTAL LABOUR

CES AND VES PRODUCTION FUNCTIONS USING CAPITAL

	(5a) CESK DF	MEANSQ	F RATIO	(£b) VESK1 DF	MEANSQ	F RATIO	(5c) VESK2 DF	MEANSQ	F RATIO
FRANCE									
REGRESN OVER GRPS	4	0 070	5 6[illegible]9**	6	0 083	1 270	8	0 016	1 413
RESID WITHIN GRPS	58	0 394		55	0 065		57	0 055	
INDIA									
REGRESN OVER GRPS	4	0 272	1 671	6	0 275	5 29**	8	0 1[illegible]	2 505
RESID WITHIN GRPS	111	0 16[illegible]		108	0 052		105	0 049	
ISRAEL									
REGRESN OVER GRPS	4	0 202	1 335	6	0 1[illegible]5	2 256	8	0 105	1 69[illegible]
RESID WITHIN GRPS	63	0 269		60	0 060		57	0 062	
JAPAN									
REGRESN OVER GRPS	4	0 472	3 508	6	0 063	1 863	8	0 076	2 481
RESID WITHIN GRPS	57	0 1[illegible]1		54	0 0[illegible]4		51	0 0[illegible]1	
YUGOSLAVIA									
REGRESN OVER GRPS	4	0 016	4 00[illegible]**	6	0 0[illegible]4	3 049**	8	0 030	2 694**
RESID WITHIN GRPS	1[illegible]9	0 064		136	0 011		1[illegible]	0 011	

TABLE 6B

ANALYSIS OF COVARIANCE

THREE L GROUPS

FIRMS ARRANGED IN INCREASING ORDER OF L TOTAL LABOUR

(6a) $\ln K/L = \text{Cons} + b_D \ln \frac{w}{r}$

(6b) $\ln WL/rK = \text{const} + \frac{1-b_S}{b_S} \ln \frac{K}{L}$

(6c) $\ln L = \text{cons} - b_V \ln w + c_V \ln V$

	(6a) DF	MEANSQ	F RATIO	(6b) DF	MEANSQ	F RATIO	(6c) DF	MEANSQ	F RATIO
FRANCE									
REGRESN OVER GRPS	4	0 1[illegible]	6 787**	4	1 [illegible]14	1 710	6	0 267	3 1[illegible]5
RESID WITHIN GRPS	58	0 [illegible]		56	0 600		55	0 005	
INDIA									
REGRESN OVER GRPS	1	0 865	1 096	6	0 [illegible]5	2 051	6	2 893	23 01**
RESID WITHIN GRPS	111	0 797		108	0 72[illegible]		108	0 126	
ISRAEL									
REGRESN OVER GRPS	4	0 352	1 494	4	1 5[illegible]	4 164**	6	0 159	3 077
RESID WITHIN GRPS	6[illegible]	0 541		6[illegible]	0 [illegible]92		60	0 05[illegible]	
JAPAN									
REGRESN OVER GRPS	1	1 500	3 [illegible]75	4	1 211	3 458	6	0 592	5 059**
RESID WITHIN GRPS	57	0 444		57	0 [illegible]50		5[illegible]	0 117	
YUGOSLAVIA									
REGRESN OVER GRPS	1	0 112	3 135	1	0 29[illegible]	3[illegible] [illegible]7**	6	0 95	1[illegible] 84**
RESID WITHIN GRPS	139	0 [illegible]50		1[illegible]9	0 [illegible]11		[illegible]5	0 [illegible]	

TABLE 1C

ANALYSIS OF COVARIANCE

THREE V GROUPS

FIRMS ARRANGED IN INCREASING ORDER OF
V, VALUE ADDED

COBB DOUGLAS PRODUCTION FUNCTION

(1a) $V = A K^{\alpha} L^{\beta}$ (1b) $V = A K^{\alpha_M} L^{\beta_M} M^{\gamma_M}$

	D F	MEANSQ	F RATIO	D F	MEANSQ	F RATIO
FRANCE						
REGRESN OVER GRPS	6	0 640	7 668*	8	0 471	6 209**
RESID WITHIN GRPS	[illegible]5	0 08[illegible]		52	0 076	
INDIA						
REGRESN OVER GRPS	6	2 098	1[illegible] 94**	8	1 126	8 169**
RESID WITHIN GRPS	108	0 151		1[illegible]5	0 1[illegible]8	
ISRAEL						
REGRESN OVER GRPS	6	0 467	4 [illegible]58**	8	0 [illegible]	3 7[illegible]**
RESID WITHIN GRPS	60			[illegible]7	0 086	
JAPAN						
REGRESN OVER GRPS	6	0 6[illegible]8	7 660**	8	0 [illegible]	4 7[illegible]**
RESID WITHIN GRPS	54	0 082		51	0 078	
YUGOSLAVIA						
REGRESN OVER GRPS	6	1 [illegible]91	18 [illegible]0**	8	0 79[illegible]	12 02**
RESID WITHIN GRPS	1[illegible]6	0 08[illegible]		1[illegible]	0 066	

TABLE 2C

ANALYSIS OF COVARIANCE

THREE V GROUPS

FIRMS ARRANGED IN INCREASING ORDER OF
V, VALUE ADDED

COBB DOUGLAS PRODUCTION FUNCTION

(2a) $V = A K^{\alpha_{DI}} L_D^{\beta_D} L_I^{\beta_I}$ (2b) $V = A K^{\alpha_{EO}} L_E^{\beta_E} L_O^{\beta_C}$

	D F	MEANSQ	F RATIO	D F	MEANSQ	F RATIO
FRANCE						
REGRESN OVER GRPS	8	0 [illegible]61	4 [illegible]49**	8	0 [illegible]03	7 114**
RESID WITHIN GRPS	52	0 074		52	0 071	
INDIA						
REGRESN OVER GRPS	8	[illegible]36	1[illegible] [illegible]5*	8	1 [illegible]	11 58**
RESID WITHIN GRPS	10[illegible]	0 [illegible]9		10[illegible]	0 149	
ISRAEL						
REGRESN OVER GRPS	8	[illegible]9[illegible]	4 254*	8	0 [illegible]7	4 [illegible]10**
RESID WITHIN GRPS	57	0 09[illegible]		57	0 09[illegible]	
JAPAN						
REGRESN OVER GRPS	8	0 2[illegible]	3 [illegible]45*	8	0 [illegible]80	4 577**
RESID WITHIN GRPS	51	0 079		[illegible]1	0 08[illegible]	
YUGOSLAVIA						
REGRESN OVER GRPS	8	1 0[illegible]	14 [illegible]9**	8	0 [illegible]21	12 8[illegible]**
RESID WITHIN GRPS	1[illegible]	0 071		[illegible]	0 072	

TABLE 3C

ANALYSIS OF COVARIANCE

THREE V GROUPS

FIRMS ARRANGED IN INCREASING ORDER OF
V, VALUE ADDED

(3a) $\ln V = const + \alpha_K \ln K + \beta_K \ln L + \delta_K (\ln K/L)^2$

(3b) $\ln V = const + \beta_T \ln \frac{K}{L} + \bar{\sigma}_T (\ln \frac{K}{L})^2 + \gamma_T \ln K \ln L$

	D F	MEANSQ	F RATIO	D F	MEANSQ	F RATIO
FRANCE						
REGRESN OVER GRPS	8	0 458	5 569**	8	0 47[illegible]	5 987**
RESID WITHIN GRPS	52	0 082		52	0 079	
INDIA						
REGRESN OVER GRPS	8	1 6[illegible]1	10 89**	8	1 645	11 10**
RESID WITHIN GRPS	105	0 149		105	0 148	
ISRAEL						
REGRESN OVER GRPS	8	0 [illegible]02	3 [illegible]19**	8	0 [illegible]7	[illegible]8**
RESID WITHIN GRPS	57	0 097		57	0 098	
JAPAN						
REGRESN OVER GRPS	8	0 [illegible]10	7 115**	8	0 550	7 392**
RESID WITHIN GRPS	51	0 076		51	0 074	
YUGOSLAVIA						
REGRESN OVER GRPS	8	0 975	12 [illegible]6**	8	0 981	1[illegible] [illegible]0**
RESID WITHIN GRPS	1[illegible]5	0 079		1[illegible]	0 079	

TABLE 4C

ANALYSIS OF COVARIANCE

THREE V GROUPS

FIRMS ARRANGED IN INCREASING ORDER OF
V, VALUE ADDED

CES AND VES PRODUCTION FUNCTIONS USING
LABOUR

	(4a) CESL			(4b) VESL1			(4c) VESL2		
	DF	MEANSQ	F RATIO	DF	MEANSQ	F RATIO	DF	MEANSQ	F RATIO
FRANCE									
REGRESN OVER GRPS	4	0 077	1 403	6	0 031	2 419	8	0 17[illegible]	3 079**
RESID WITHIN GRPS	58	0 108		55	0 075		52	0 05[illegible]	
INDIA									
REGRESN OVER GRPS	4	1 9[illegible]	6 950*	6	1 1[illegible]	8 0[illegible]3**	[illegible]	1 2[illegible]	11 75**
RESID WITHIN GRPS	111	0 277		108	0 140		10[illegible]	0 111	
ISRAEL									
REGRESN OVER GRPS	4	0 045	1 467	6	0 10[illegible]	[illegible] 060	8	0 130	3 91[illegible]**
RESID WITHIN GRPS	6[illegible]	0 066		60	0 050		57	0 8[illegible]	
JAPAN									
REGRESN OVER GRPS	[illegible]	0 149	1 101	[illegible]	0 1[illegible]0	1 [illegible]21	[illegible]	0 204	3 256**
RESID WITHIN GRPS	57	0 17[illegible]		54	0 091		[illegible]	0 06[illegible]	
YUGOSLAVIA									
REGRESN OVER GRPS	4	0 244	2 254	6	0 2[illegible]	4 111**	8	0 477	11 11**
RESID WITHIN GRPS	1[illegible]9	0 1[illegible]		[illegible]	0 060		1[illegible]	0 04[illegible]	

TABLE 5C

ANALYSIS OF COVARIANCE

THREE V GROUPS

FIRMS ARRANGED IN INCREASING ORDER OF
V, VALUE ADDED

CES AND VES PRODUCTIN FUNCTIONS USING
CAPITAL

	(5a) CESK			(5b) VESK1			(5c) VESK2		
	DF	MEANSQ	F RATIO	DF	MEANSQ	F RATIO	DF	MEANSQ	F RATIO
FRANCE									
REGRESN OVER GRPS	4	0.495	1.360	6	0.839	1.373	8	0.154	3.170**
RESID WITHIN GRPS	58	0.[illegible]64		55	0.06[illegible]		[illegible]2	0.048	
INDIA									
REGRESN OVER GRPS	4	0.150	1.446	6	0.447	10.40**	8	0.191	4.398**
RESID WITHIN GRPS	111	0.138		108	0.047		10[illegible]	0.043	
ISRAEL									
REGRESN OVER GRPS	4	0.349	1.341	6	0.143	2.427	8	0.169	3.187**
RESID WITHIN GRPS	63	0.260		60	0,059		57	0,053	
JAPAN									
REGRESN OVER GRPS	4	0.573	4.621**	6	0.043	1.134	8	0.071	2.[illegible]50
RESID WITHIN GRPS	57	0.124		51	0.0[illegible]6		51	0.031	
YUGOSLAVIA									
REGRESN OVER GRPS	4	0.277	4.907**	6	0.010	1.200	8	0.047	4.709**
RESID WITHIN GRPS	139	0.056		136	0.012		13[illegible]	0.010	

TABLE 6C

ANALYSIS OF COVARIANCE

THREE V GROUPS

FIRMS ARRANGED IN INCREASING ORDER OF
V, VALUE ADDED

(6a) $\ln K/L = const + b_D \ln \frac{w}{r}$

(6b) $\ln \frac{wL}{rK} = const + \frac{1-b}{b_s} \ln \frac{K}{L}$

(6c) $\ln L = const - b_v \ln w + c_v \ln V$

	(6a)			(6b)			(6c)		
	DF	MEANSQ	F RATIO	DF	MEANSQ	F RATIO	DF	MEANSQ	F RATIO
FRANCE									
REGRESN OVER GRPS	4	0.865	1.114	4	0.9[illegible]2	1.534	6	0.066	1.621
RESID WITHIN GRPS	58	0.776		58	0.608		55	0.107	
INDIA									
REGRESN OVER GRPS	4	1.591	2.064	4	[illegible]	3.124	6	0.791	3.265
RESID WITHIN GRPS	111	0.771		11[illegible]	[illegible]		108	0.242	
ISRAEL									
REGRESN OVER GRPS	4	0.539	1.138	4	1.664	4.269*	6	0.084	1.[illegible]24
RESID WITHIN GRPS	63	0.[illegible]		63	0.[illegible]90		60	0.064	
JAPAN									
REGRESN OVER GRPS	4	2.395	6.267**	4	0.911	2.454	6	0.[illegible]19	1.386
RESID WITHIN GRPS	57	0.38[illegible]		57	0.371		54	0.158	
YUGOSLAVIA									
REGRESN OVER GRPS	4	1.286	4.068**	4	0.947	5.148**	6	0.092	1.164
RESID WITHIN GRPS	139	0.316		139	0.184		136	0.107	

TABLE 1D

ANALYSIS OF COVARIANCE

THREE K/L GROUPS

FIRMS ARRANGED IN INCREASING ORDER OF
K/L, CAPITAL LABOUR RATIO

COBB DOUGLAS PRODUCTION FUNCTION

(1a) $V = A K^{\alpha} L^{\beta}$ (1b) $V = A K^{\alpha_M} L^{\beta_M} M^{\gamma_1}$

	D F	MEAN SQ	F RATIO	D F	MEAN SQ	F RATIO
FRANCE						
REGRESSION OVER GROUPS	6	[illegible]	1 768	8	0 2[illegible]5	2 217
RESIDUAL WITHIN GROUPS	5[illegible]	0 1[illegible]8		2	0 111	
INDIA						
REGRESSION OVER GROUPS	6	0 250	1 001	8	0 2[illegible]3	1 131
RESIDUAL WITHIN GROUPS	108	0 2[illegible]5		10[illegible]	0 206	
ISRAEL						
REGRESSION OVER GROUPS	[illegible]	0 [illegible]91	1 5[illegible]	[illegible]	0 248	2 56[illegible]
RESIDUAL WITHIN GROUPS	[illegible]	0 1[illegible]		57	0 0[illegible]7	
JAPAN						
REGRESSION OVER GROUPS	6	0 241	1 923	[illegible]	0 1[illegible]2	1 [illegible]2
RESIDUAL WITHIN GROUPS	[illegible]4	0 1[illegible]5		51	0 113	
YUGOSLAVIA						
REGRESSION OVER GROUPS	6	[illegible]	[illegible] 36	8	0 304	5 16[illegible]**
RESIDUAL WITHIN GROUPS	156	133		133	0 096	

TABLE 2D

ANALYSIS OF COVARIANCE

THREE K/L GROUPS

FIRMS ARRANGED IN INCREASING ORDER OF
K/L CAPITAL LABOUR RATIO

COBB DOUGLAS PRODUCTION FUNCTION

(2a) $V = A K^{\alpha_{DI}} L_D^{\beta_D} L_I^{\beta_I}$ (2b) $V = A K^{\alpha_{EO}} L_E^{\beta_E} L_O^{\beta_O}$

	DF	MEANSQ	F RATIO	DF	MEANSQ	F RATIO
FRANCE						
REGRESN OVER GRPS	8	0 1[illegible]5	1 2[illegible]4	[illegible]	0 176	1 450
RESID WITHIN GRPS	[illegible]	0 199		2	0 1[illegible]1	
INDIA						
REGRESN OVER GRPS	8	0 217	1 1[illegible]	8	0 [illegible]	1 10[illegible]
RESID WITHIN GRPS	1[illegible]	0 [illegible]5		10[illegible]	0 2[illegible]0	
ISRAEL						
REGRESN OVER GRPS	[illegible]	[illegible] 18[illegible]	1 26	8	0 204	1 6[illegible]
RESID WITHIN GRPS	[illegible]7	0 1[illegible]		57	0 1[illegible]	
JAPAN						
REGRESN OVER GRPS	6	0 1[illegible]0	1 1[illegible]9	8	0 1[illegible]1	1 [illegible]17
RESID WITHIN GRPS	[illegible]	0 10[illegible]		[illegible]1	0 1[illegible]	
YUGOSLAVIA						
REGRESN OVER GRPS	[illegible]	0 2[illegible]	2 [illegible]	8	0 25[illegible]	2 277
RESID WITHIN GRPS	1[illegible]	0 11[illegible]		1[illegible]	0 11[illegible]	

TABLE 3D

A N A L Y S I S   O F   C O V A R I A N C E

THREE K/L GROUPS

FIRMS ARRANGED IN INCREASING ORDER OF
K/L, CAPITAL LABOUR RATIO

	(3a) DF	(3a) MEANSQ	(3a) F RATIO	(3b) DF	(3b) MEANSQ	(3b) F RATIO
FRANCE						
REGRESN OVER GRPS	5	0 240	1 958	7	0 211	1 749
RESID WITHIN GRPS	55	0 12[illegible]		55	0 1[illegible]1	
INDIA						
REGRESN OVER GRPS	6	0 2[illegible]3	1 209	7	0 199	1 [illegible]96
RESID WITHIN GRPS	107	0 2[illegible]		106	0 2[illegible]8	
ISRAEL						
REGRESN OVER GRPS	6	0 1[illegible]7	1 041	6	0 1[illegible]6	1 261
RESID WITHIN GRPS	59	0 1[illegible]		59	0 1[illegible]	
JAPAN						
REGRESN OVER GRPS	5	0 2[illegible]	2 [illegible]41	6	0 2[illegible]5	2 006
RESID WITHIN GRPS	[illegible]	[illegible] 1[illegible]		55	0 1[illegible]	
YUGOSLAVIA						
REGRESN OVER GRPS	7	[illegible] 1[illegible]1	1 042	7	0 1[illegible]7	[illegible] 0[illegible]
RESID WITHIN GRPS	134	[illegible] 1[illegible]		1[illegible]1	[illegible]	

TABLE 4D

A N A L Y S I S   O F   C O V A R I A N C E

THREE K/L GROUPS

FIRMS ARRANGED IN INCREASING ORDER OF
K/L, CAPITAL LABOUR RATIO

CES AND VES PRODUCTION FUNCTIONS USING
LABOUR

	(4a) CESL DF	(4a) CESL MEANSQ	(4a) CESL F RATIO	(4b) VESL1 DF	(4b) VESL1 MEANSQ	(4b) VESL1 F RATIO	(4c) VESL2 DF	(4c) VESL2 MEANSQ	(4c) VESL2 F RATIO
FRANCE									
REGRESN OVER GRPS	4	0 135	5 201**	6	0 20[illegible]	[illegible] 560**	[illegible]	0 171	3 11[illegible]**
RESID WITHIN GRPS	5[illegible]	0 084		5[illegible]	0 0[illegible]		[illegible]	0 0[illegible]6	
INDIA									
REGRESN OVER GRPS	4	1 6[illegible]3	[illegible] 5 84**	6	0 891	[illegible] 819**	9	0 699	4 5[illegible]**
RESID WITHIN GRPS	111	0 1[illegible]		10[illegible]	0 13		10[illegible]	0 15[illegible]	
ISRAEL									
REGRESN OVER GRPS	4	0 29[illegible]	[illegible] 66[illegible]**	6	0 09[illegible]	794	8	0 07[illegible]	1 7[illegible]7
RESID WITHIN GRPS	6[illegible]	0 0[illegible]		[illegible]	[illegible]		[illegible]7	0 0[illegible]1	
JAPAN									
REGRESN OVER GRPS	4	[illegible] 0[illegible]	9 989*	6	0 07[illegible]	1 [illegible]3	8	0 1[illegible]7	[illegible] 24[illegible]
RESID WITHIN GRPS	[illegible]7	0 108		51	0 096		51	0 [illegible]70	
YUGOSLAVIA									
REGRESN OVER GRPS	4	0 711	7 590**	6	0 317	5 6[illegible]8**	[illegible]	0 [illegible]39	[illegible] 161**
RESID WITHIN GRPS	139	0 095		36	0 056		13[illegible]	0 [illegible]57	

TABLE 5D

A N A L Y S I S O F C O V A R I A N C E

THREE K/L GROUPS

FIRMS ARRANGED IN INCREASING ORDER OF
K/L, CAPITAL LABOUR RATIO

CES AND VES PRODUCTION FUNCTIONS USING
CAPITAL

	(5a) CESK			(5b) VESK1			(5c) VESK2		
	DF	MEANSQ	F RATIO	DF	MEANSQ	F RATIO	DF	MEANSQ	F RATIO
FRANCE									
REGRESN OVER GRPS	1	97	1 9 **	6	0 20	878**	8	0 11	85
RESID WITHIN GRPS	58	0 12 4		5	0 09		2	0 0 9	
INDIA									
REGRESN OVER GRPS	1	509	30 51**	6	3 1	131	8	0 36	1 18
RESID WITHIN GRPS	111	0 03		0	0 030		105	0 05	
ISRAEL									
REGRESN OVER GRPS	4	1 51	11 15**	6	0 11	37	[illegible]	0 59	047**
RESID WITHIN GRPS	6	0 26		69	0 05		57	0 05	
JAPAN									
REGRESN OVER GRPS	4	1 535	50 41**	6	0 070	11	8	0 07	49
RESID WITHIN GRPS	57	0 02		54	0 03		51	0 031	
YUGOSLAVIA									
REGRESN OVER GRPS	4	1 205	40 4 **	6	0 3 6	3 2 0**	8	0 07	3 89
RESID WITHIN GRPS	1 9	0 05 0		156	0 011		1	0 011	

TABLE 6D

A N A L Y S I S O F C O V A R I A N C E

THREE K/L GROUPS

FIRMS ARRANGED IN INCREASING ORDER OF
K/L, CAPITAL LABOUR RATIO

	(6a)			(6b)			(6c)		
	DF	MEANSQ	F RATIO	DF	MEAN SQ	F RATIO	DF	MEANSQ	F RATIO
FRANCE									
REGRESN OVER GRPS	4	7 0	7 5*	1	0 55	1 1	6	0 291	3 51**
RESID WITHIN GRPS	58	0 29		5	0 6 4		55	0 05	
INDIA									
REGRESN OVER GRPS	4	17 5	9 1 **	4	0 0	2 1 **	6	6	1 7 **
RESID WITHIN GRPS	111	0 15		111	0 7		1	0 1 7	
ISRAEL									
REGRESN OVER GRPS	1	0 5	0 81**	1	5 5	1 91	[illegible]	1 7	16
RESID WITHIN GRPS	6	2		5	0 461		[illegible]	0 05	
JAPAN									
REGRESN OVER GRP	1	5 658	7 05**	4	5	1 2 5		6	6 07**
RESID WITHIN GRPS	7	0 15		57	0 40		54	0 13	
YUGOSLAVIA									
REGRESN OVER GRPS	1	6 0	4 2 **	4	0 651	3 8	6	0 451	4 96 **
RESID WITHIN GRPS	1 9	0 178		1 9	0 19		1 6	0 0 1	

Table 1E

ANALYSIS OF COVARIANCE

GROUPING CRITERION : AGE OF ESTABLISHMENT

Production Relation	France	India	F VALUES ONLY Israel	Japan	Yugoslavia
(1a)	1.8	1.9	2.2	2.4	1.1
(1b)	1.1	1.6	1.9	2.2	2.6
(2a)	2.1	1.8	1.6	1.3	1.2
(2b)	2.5	2.1	1.8	1.2	1.4
(3a)	1.2	1.7	2.3	2.4	1.0
(3b)	1.2	1.8	2.5	2.2	1.0
(4a)	1.4	2.4	3.0	1.3	1.5
(4b)	2.1	2.3	1.6	1.9	1.1
(4c)	1.8	2.2	1.2	1.8	1.0
(5a)	1.6	1.7	1.8	1.8	1.1
(5b)	2.7	5.4**	3.7**	1.6	2.6
(5c)	2.5	3.0**	3.4**	1.6	2.5
(6a)	2.5	2.8	2.9	1.6	1.1
(6b)	2.2	2.2	2.5	1.7	1.9
(6c)	1.3	8.9**	2.3	1.4	1.3

Table 2E

ANALYSIS OF COVARIANCE

F values only

GROUPING CRITERION : PERCENTAGE CAPACITY UTILISATION

Production Relation	France	India	Israel	Japan	Yugoslavia
(1a)	1.2	1.3	2.0	1.5	1.0
(1b)	1.1	2.1	2.3	1.3	1.2
(2a)	1.2	1.1	1.9	2.0	1.2
(2b)	1.1	2.1	1.8	1.5	1.3
(3a)	1.5	2.1	1.8	1.3	1.4
(3b)	1.6	2.1	1.9	1.7	1.3
(4a)	2.1	3.9**	1.2	1.9	1.1
(4b)	1.1	2.6	1.6	1.7	1.2
(4c)	1.2	2.6	1.4	1.2	1.1
(5a)	1.1	1.2	6.0**	2.2	1.1
(5b)	1.0	3.9**	1.3	2.2	1.2
(5c)	2.0	2.4	1.2	2.2	1.7
(6a)	1.8	1.0	2.0	2.1	1.0
(6b)	1.5	1.3	3.6**	1.9	1.2
(6c)	1.1	3.1**	1.0	2.0	1.3

Table 3E

ANALYSIS OF COVARIANCE

F values only

GROUPING CRITERION : NUMBER OF SHIFTS WORKED

Production Relation	France	India	Israel	Japan	Yugoslavia
(1a)	1.5	1.3	1.4	2.1	2.9
(1b)	1.0	1.2	1.0	1.7	2.2
(2a)	1.1	1.1	1.3	1.4	2.6
(2b)	1.4	1.8	1.1	1.8	1.7
(3a)	1.9	1.7	1.0	3.7	1.8
(3b)	2.3	1.6	1.0	2.3	1.8
(4a)	1.8	1.5	1.7	1.6	1.7
(4b)	1.2	1.8	2.6	1.0	2.0
(4c)	1.1	1.7	1.6	1.4	3.0**
(5a)	1.6	2.3	1.3	2.9	2.1
(5b)	1.8	3.1**	2.4	1.7	1.5
(5c)	1.3	2.3	1.2	1.3	1.5
(6a)	1.3	1.5	1.3	4.4**	2.6
(6b)	1.9	2.3	5.2**	1.4	5.5**
(6c)	1.0	2.1	1.7	2.0	1.1

Table 7

RANK CORRELATION BETWEEN CERTAIN PAIRS OF VARIABLES

The notation used in this table is given below. The corresponding notation, if any, used in the text is given in brackets.

TVPRD(Y)	Total value of production
VADTD(V)	Value added
LTOTL(L)	Total labour
LDRCT(L_D)	Direct labour
CAPAS(K)	Net capital assets
MCHNY(K_{MC})	Machinery and equipment
DALAN(K_{DA})	Depreciation allowances per annum
WAGES(W)	Total wages
CAPLT(K/L)	Capital labour ratio
VADLT(V/L)	Value added labour ratio
VACAS(V/K)	Value added capital ratio
CAPUT	Percentage capacity utilisation
NRYRS	Age of establishment
MCAGE	Age of machinery
PMOS1	Percentage of motors operated in shift one
ELCWH	Electricity consumed in kwh
MOTKW	Capacity of motors in kwh
N	Number of observations
SIG	Level of significance

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VARIABLE PAIR		VARIABLE PAIR		VARIABLE PAIR		VARIABLE PAIR		VARIABLE PAIR		VARIABLE PAIR	
TVPRD WITH VADED	0.8692 N(64) SIG .001	TVPRD WITH LTOTL	0.8272 N(64) SIG .001	TVPRD WITH LDRCT	0.7305 N(64) SIG .001	TVPRD WITH CAPAS	0.7901 N(64) SIG .001	TVPRD WITH MCHNY	0.7227 N(64) SIG .001	TVPRD WITH DALAN	0.7816 N(64) SIG .001
TVPRD WITH WAGES	0.8074 N(64) SIG .001	TVPRD WITH CAPLT	0.2756 N(64) SIG .014	TVPRD WITH VADLT	0.2701 N(64) SIG .015	TVPRD WITH VACAS	-0.1849 N(64) SIG .072	TVPRD WITH CAPUT	0.0383 N(63) SIG .383	TVPRD WITH NRYRS	0.0756 N(63) SIG .278
TVPRD WITH MCAGE	0.0756 N(31) SIG .343	TVPRD WITH PMOS1	0.4217 N(28) SIG .013	TVPRD WITH ELCWH	0.6256 N(61) SIG .001	TVPRD WITH MOTKW	0.7733 N(53) SIG .001	VADED WITH LTOTL	0.9160 N(64) SIG .001	VADED WITH LDRCT	0.8326 N(64) SIG .001
VADED WITH CAPAS	0.7650 N(64) SIG .001	VADED WITH MCHNY	0.6887 N(64) SIG .001	VADED WITH DALAN	0.8334 N(64) SIG .001	VADED WITH WAGES	0.9443 N(64) SIG .001	VADED WITH CAPLT	0.0929 N(64) SIG .233	VADED WITH VADLT	0.2347 N(64) SIG .031
VADED WITH VACAS	-0.0066 N(64) SIG .479	VADED WITH CAPUT	0.0114 N(63) SIG .465	VADED WITH NRYRS	0.1908 N(63) SIG .067	VADED WITH MCAGE	0.1530 N(31) SIG .206	VADED WITH PMOS1	0.4357 N(28) SIG .010	VADED WITH ELCWH	0.6195 N(61) SIG .001
VADED WITH MOTKW	0.7832 N(53) SIG .001	LTOTL WITH LDRCT	0.9549 N(64) SIG .001	LTOTL WITH CAPAS	0.7072 N(64) SIG .001	LTOTL WITH MCHNY	0.6126 N(64) SIG .001	LTOTL WITH DALAN	0.7063 N(64) SIG .001	LTOTL WITH WAGES	0.9465 N(64) SIG .001
LTOTL WITH CAPLT	-0.0322 N(64) SIG .400	LTOTL WITH VADLT	-0.0990 N(64) SIG .218	LTOTL WITH VACAS	0.0230 N(64) SIG .428	LTOTL WITH CAPUT	0.0768 N(63) SIG .275	LTOTL WITH NRYRS	0.2270 N(63) SIG .037	LTOTL WITH MCAGE	0.1087 N(31) SIG .280
LTOTL WITH PMOS1	0.4060 N(28) SIG .016	LTOTL WITH ELCWH	0.5106 N(61) SIG .001	LTOTL WITH MOTKW	0.7472 N(53) SIG .001	LDRCT WITH CAPAS	0.5923 N(64) SIG .001	LDRCT WITH MCHNY	0.4949 N(64) SIG .001	LDRCT WITH DALAN	0.5830 N(64) SIG .001
LDRCT WITH WAGES	0.8879 N(64) SIG .001	LDRCT WITH CAPLT	-0.1463 N(64) SIG .124	LDRCT WITH VADLT	-0.2069 N(64) SIG .050	LDRCT WITH VACAS	0.0934 N(64) SIG .231	LDRCT WITH CAPUT	0.1096 N(63) SIG .195	LDRCT WITH NRYRS	0.2399 N(63) SIG .029
LDRCT WITH MCAGE	0.1204 N(31) SIG .259	LDRCT WITH PMOS1	0.3936 N(28) SIG .019	LDRCT WITH ELCWH	0.4682 N(61) SIG .001	LDRCT WITH MOTKW	0.6760 N(53) SIG .001	CAPAS WITH MCHNY	0.9554 N(64) SIG .001	CAPAS WITH DALAN	0.8608 N(64) SIG .001
CAPAS WITH WAGES	0.6886 N(64) SIG .001	CAPAS WITH CAPLT	0.6286 N(64) SIG .001	CAPAS WITH VADLT	0.1950 N(64) SIG .061	CAPAS WITH VACAS	-0.5920 N(64) SIG .001	CAPAS WITH CAPUT	0.0424 N(63) SIG .371	CAPAS WITH NRYRS	0.0171 N(63) SIG .447

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CAPAS WITH MCAGE	0.0914 N(31) SIG .313	CAPAS WITH PMOS1	0.3934 N(28) SIG .019	CAPAS WITH ELCWH	0.6793 N(61) SIG .001	CAPAS WITH MOTKW	0.8736 N(53) SIG .001	MCHNY WITH DALAN	0.8212 N(64) SIG .001	MCHNY WITH WAGES	0.5836 N(64) SIG .001
MCHNY WITH CAPLT	0.6713 N(64) SIG .001	MCHNY WITH VADLT	0.2257 N(64) SIG .036	MCHNY WITH VACAS	-0.6264 N(64) SIG .001	MCHNY WITH CAPUT	0.0399 N(63) SIG .378	MCHNY WITH NRYRS	0.0024 N(63) SIG .493	MCHNY WITH MCAGE	0.1283 N(31) SIG .246
MCHNY WITH PMOS1	0.3148 N(28) SIG .051	MCHNY WITH ELCWH	0.6663 N(61) SIG .001	MCHNY WITH MOTKW	0.8183 N(53) SIG .001	DALAN WITH WAGES	0.7404 N(64) SIG .001	DALAN WITH CAPLT	0.4185 N(64) SIG .001	DALAN WITH VADLT	0.3251 N(54) SIG .004
DALAN WITH VACAS	-0.3260 N(64) SIG .004	DALAN WITH CAPUT	-0.0389 N(63) SIG .381	DALAN WITH NRYRS	0.0044 N(63) SIG .486	DALAN WITH MCAGE	-0.0414 N(31) SIG .412	DALAN WITH PMOS1	0.3181 N(28) SIG .050	DALAN WITH ELCWH	0.6470 N(61) SIG .001
DALAN WITH MOTKW	0.7847 N(53) SIG .001	WAGES WITH CAPLT	-0.0249 N(64) SIG .423	WAGES WITH VADLT	0.0913 N(64) SIG .236	WAGES WITH VACAS	0.0790 N(64) SIG .268	WAGES WITH CAPUT	0.0256 N(63) SIG .421	WAGES WITH NRYRS	0.2229 N(63) SIG .040
WAGES WITH MCAGE	0.1633 N(31) SIG .190	WAGES WITH PMOS1	0.3462 N(28) SIG .036	WAGES WITH ELCWH	0.5015 N(61) SIG .001	WAGES WITH MOTKW	0.7110 N(53) SIG .001	CAPLT WITH VADLT	0.3465 N(64) SIG .003	CAPLT WITH VACAS	-0.9327 N(64) SIG .001
CAPLT WITH CAPUT	-0.0213 N(63) SIG .434	CAPLT WITH NRYRS	-0.2143 N(63) SIG .046	CAPLT WITH MCAGE	-0.0439 N(31) SIG .407	CAPLT WITH PMOS1	0.0093 N(28) SIG .481	CAPLT WITH ELCWH	0.3890 N(61) SIG .001	CAPLT WITH MOTKW	0.4331 N(53) SIG .001
VADLT WITH VACAS	-0.0429 N(64) SIG .368	VADLT WITH CAPUT	-0.1690 N(63) SIG .093	VADLT WITH NRYRS	-0.1464 N(63) SIG .126	VADLT WITH MCAGE	-0.0388 N(31) SIG .418	VADLT WITH PMOS1	0.1426 N(28) SIG .235	VADLT WITH ELCWH	0.2372 N(61) SIG .033
VADLT WITH MOTKW	0.0952 N(53) SIG .249	VACAS WITH CAPUT	-0.0446 N(63) SIG .364	VACAS WITH NRYRS	0.1889 N(63) SIG .069	VACAS WITH MCAGE	0.0572 N(31) SIG .380	VACAS WITH PMOS1	0.0489 N(28) SIG .402	VACAS WITH ELCWH	-0.3324 N(61) SIG .004
VACAS WITH MOTKW	-0.4291 N(53) SIG .001	CAPUT WITH NRYRS	0.1406 N(62) SIG .138	CAPUT WITH MCAGE	-0.0694 N(31) SIG .355	CAPUT WITH PMOS1	0.1635 N(27) SIG .208	CAPUT WITH ELCWH	-0.0231 N(60) SIG .430	CAPUT WITH MOTKW	0.0409 N(52) SIG .387
NRYRS WITH MCAGE	0.3681 N(30) SIG .023	NRYRS WITH PMOS1	0.1703 N(27) SIG .198	NRYRS WITH ELCWH	0.1789 N(60) SIG .086	NRYRS WITH MOTKW	0.1575 N(52) SIG .132	MCAGE WITH PMOS1	-0.1685 N(14) SIG .282	MCAGE WITH ELCWH	0.5366 N(29) SIG .001
MCAGE WITH MOTKW	0.2037 N(28) SIG .149	PMOS1 WITH ELCWH	0.1398 N(25) SIG .253	PMOS1 WITH MOTKW	0.3149 N(26) SIG .059	ELCWH WITH MOTKW	0.7351 N(50) SIG .001				

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VARIABLE PAIR		VARIABLE PAIR		VARIABLE PAIR		VARIABLE PAIR		VARIABLE PAIR		VARIABLE PAIR	
TVPRD WITH VADED	0.9480 N(117) SIG .001	TVPRD WITH LTOTL	0.8661 N(117) SIG .001	TVPRD WITH LDRCT	0.8472 N(117) SIG .001	TVPRD WITH CAPAS	0.8809 N(117) SIG .001	TVPRD WITH MCHNY	0.8754 N(117) SIG .001	TVPRD WITH DALAN	0.8758 N(117) SIG .001
TVPRD WITH WAGES	0.8691 N(117) SIG .001	TVPRD WITH CAPLT	0.3454 N(117) SIG .001	TVPRD WITH VADLT	0.5480 N(117) SIG .001	TVPRD WITH VACAS	-0.0535 N(117) SIG .283	TVPRD WITH CAPUT	0.4125 N(117) SIG .001	TVPRD WITH NRYRS	0.2125 N(117) SIG .011
TVPRD WITH MCAGE	0.4433 N(51) SIG .001	TVPRD WITH PMOS1	0.0625 N(91) SIG .278	TVPRD WITH ELCWH	0.8693 N(116) SIG .001	TVPRD WITH MOTKW	0.8138 N(114) SIG .001	VADED WITH LTOTL	0.9037 N(117) SIG .001	VADED WITH LDRCT	0.8867 N(117) SIG .001
VADED WITH CAPAS	0.9004 N(117) SIG .001	VADED WITH MCHNY	0.8888 N(117) SIG .001	VADED WITH DALAN	0.9072 N(117) SIG .001	VADED WITH WAGES	0.9263 N(117) SIG .001	VADED WITH CAPLT	0.3411 N(117) SIG .001	VADED WITH VADLT	0.5894 N(117) SIG .001
VADED WITH VACAS	-0.0057 N(117) SIG .476	VADED WITH CAPUT	0.5100 N(117) SIG .001	VADED WITH NRYRS	0.2188 N(117) SIG .009	VADED WITH MCAGE	0.4309 N(61) SIG .001	VADED WITH PMOS1	0.1492 N(91) SIG .079	VADED WITH ELCWH	0.8657 N(116) SIG .001
VADED WITH MOTKW	0.8529 N(114) SIG .001	LTOTL WITH LDRCT	0.9905 N(117) SIG .001	LTOTL WITH CAPAS	0.7828 N(117) SIG .001	LTOTL WITH MCHNY	0.7829 N(117) SIG .001	LTOTL WITH DALAN	0.8125 N(117) SIG .001	LTOTL WITH WAGES	0.9375 N(117) SIG .001
LTOTL WITH CAPLT	0.0567 N(117) SIG .272	LTOTL WITH VADLT	0.2232 N(117) SIG .008	LTOTL WITH VACAS	0.0409 N(117) SIG .331	LTOTL WITH CAPUT	0.4748 N(117) SIG .001	LTOTL WITH NRYRS	0.3757 N(117) SIG .001	LTOTL WITH MCAGE	0.5114 N(61) SIG .001
LTOTL WITH PMOS1	0.1988 N(91) SIG .029	LTOTL WITH ELCWH	0.7931 N(116) SIG .001	LTOTL WITH MOTKW	0.8175 N(114) SIG .001	LDRCT WITH CAPAS	0.7544 N(117) SIG .001	LDRCT WITH MCHNY	0.7548 N(117) SIG .001	LDRCT WITH DALAN	0.7934 N(117) SIG .001
LDRCT WITH WAGES	0.9292 N(117) SIG .001	LDRCT WITH CAPLT	0.0232 N(117) SIG .402	LDRCT WITH VADLT	0.2082 N(117) SIG .012	LDRCT WITH VACAS	0.0721 N(117) SIG .220	LDRCT WITH CAPUT	0.4880 N(117) SIG .001	LDRCT WITH NRYRS	0.3787 N(117) SIG .001
LDRCT WITH MCAGE	0.5282 N(61) SIG .001	LDRCT WITH PMOS1	0.2215 N(91) SIG .017	LDRCT WITH ELCWH	0.7931 N(116) SIG .001	LDRCT WITH MOTKW	0.8115 N(114) SIG .001	CAPAS WITH MCHNY	0.9806 N(117) SIG .001	CAPAS WITH DALAN	0.9348 N(117) SIG .001
CAPAS WITH WAGES	0.8126 N(117) SIG .001	CAPAS WITH CAPLT	0.6255 N(117) SIG .001	CAPAS WITH VADLT	0.5793 N(117) SIG 001	CAPAS WITH VACAS	-0.3933 N(117) SIG .001	CAPAS WITH CAPUT	0.3994 N(117) SIG .001	CAPAS WITH NRYRS	0.0383 N(117) SIG .341

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CAPAS WITH MCAGE	0.2729 N(61) SIG .017	CAPAS WITH PMOS1	0.1349 N(91) SIG .101	CAPAS WITH ELCWH	0.8426 N(116) SIG .001	CAPAS WITH MOTKW	0.8058 N(114) SIG .001	MCHNY WITH DALAN	0.9386 N(117) SIG .001	MCHNY WITH WAGES	0.8056 N(117) SIG .001
MCHNY WITH CAPLT	0.6013 N(117) SIG .001	MCHNY WITH VADLT	0.5546 N(117) SIG .001	MCHNY WITH VACAS	-0.3813 N(117) SIG .001	MCHNY WITH CAPUT	0.3914 N(117) SIG .001	MCHNY WITH NRYRS	0.0292 N(117) SIG .377	MCHNY WITH MCAGF	0.2742 N(61) SIG .016
MCHNY WITH PMOS1	0.1276 N(91) SIG .114	MCHNY WITH ELCWH	0.8317 N(116) SIG .001	MCHNY WITH MOTKW	0.8028 N(114) SIG .001	DALAN WITH WAGES	0.8532 N(117) SIG .001	DALAN WITH CAPLT	0.5137 N(117) SIG .001	DALAN WITH VADLT	0.5616 N(117) SIG .001
DALAN WITH VACAS	-0.2726 N(117) SIG .001	DALAN WITH CAPUT	0.4245 N(117) SIG .001	DALAN WITH NRYRS	0.0917 N(117) SIG .163	DALAN WITH MCAGF	0.3280 N(61) SIG .005	DALAN WITH PMOS1	0.1653 N(91) SIG .059	DALAN WITH ELCWH	0.8406 N(116) SIG .001
DALAN WITH MOTKW	0.8054 N(114) SIG .001	WAGES WITH CAPLT	0.1718 N(117) SIG .032	WAGES WITH VADLT	0.3941 N(117) SIG .001	WAGES WITH VACAS	0.0505 N(117) SIG .294	WAGES WITH CAPUT	0.5017 N(117) SIG .001	WAGES WITH NRYRS	0.2800 N(117) SIG .001
WAGES WITH MCAGE	0.4328 N(61) SIG .001	WAGES WITH PMOS1	0.1975 N(91) SIG .030	WAGES WITH ELCWH	0.8001 N(116) SIG .001	WAGES WITH MOTKW	0.8310 N(114) SIG .001	CAPLT WITH VADLT	0.6862 N(117) SIG .001	CAPLT WITH VACAS	-0.7180 N(117) SIG .001
CAPLT WITH CAPUT	0.0695 N(117) SIG .228	CAPLT WITH NRYRS	-0.4315 N(117) SIG .001	CAPLT WITH MCAGE	-0.3663 N(61) SIG .002	CAPLT WITH PMOS1	-0.0182 N(91) SIG .432	CAPLT WITH ELCWH	0.3638 N(116) SIG .001	CAPLT WITH MOTKW	0.3026 N(114) SIG .001
VADLT WITH VACAS	-0.0654 N(117) SIG .242	VADLT WITH CAPUT	0.2364 N(117) SIG .005	VADLT WITH NRYRS	-0.1957 N(117) SIG .017	VADLT WITH MCAGE	0.0349 N(61) SIG .395	VADLT WITH PMOS1	-0.0869 N(91) SIG .206	VADLT WITH FLCWH	0.5014 N(116) SIG .001
VADLT WITH MOTKW	0.4198 N(114) SIG .001	VACAS WITH CAPUT	0.0604 N(117) SIG .259	VACAS WITH NRYPS	0.3988 N(117) SIG .001	VACAS WITH MCAGE	0.2701 N(61) SIG .018	VACAS WITH PMOS1	-0.0520 N(91) SIG .312	VACAS WITH FLCWH	-0.1371 N(116) SIG .071
VACAS WITH MOTKW	-0.0793 N(114) SIG .201	CAPUT WITH NRYRS	0.1471 N(117) SIG .057	CAPUT WITH MCAGE	0.2696 N(61) SIG .018	CAPUT WITH PMOS1	0.2986 N(91) SIG .002	CAPUT WITH ELCWH	0.4104 N(116) SIG .001	CAPUT WITH MOTKW	0.4353 N(114) SIG .001
NRYRS WITH MCAGE	0.5835 N(61) SIG .001	NRYRS WITH PMOS1	0.1125 N(91) SIG .144	NRYRS WITH ELCWH	0.1878 N(116) SIG .022	NRYRS WITH MOTKW	0.2097 N(114) SIG .013	MCAGF WITH PMOS1	0.1860 N(46) SIG .108	MCAGF WITH FLCWH	0.4261 N(61) SIG .001
MCAGE WITH MOTKW	0.3946 N(61) SIG .001	PMOS1 WITH ELCWH	0.0674 N(91) SIG .263	PMOS1 WITH MOTKW	0.1839 N(90) SIG .041	ELCWH WITH MOTKW	0.8219 N(114) SIG .001				

ISRAEL CONTD

CAPAS WITH MCAGE	-0.4464 N(29) SIG .008	CAPAS WITH PMOS1	0.0354 N(59) SIG .395	CAPAS WITH ELCWH	0.8015 N(69) SIG .001	CAPAS WITH MOTKW	0.7166 N(64) SIG .001	MCHNY WITH DALAN	0.9308 N(68) SIG .001	MCHNY WITH WAGES	0.6954 N(69) SIG .001
MCHNY WITH CAPLT	0.7480 N(69) SIG .001	MCHNY WITH VADLT	-0.0490 N(69) SIG .345	MCHNY WITH VACAS	-0.7901 N(59) SIG .001	MCHNY WITH CAPUT	0.2923 N(59) SIG .007	MCHNY WITH NRYRS	-0.2838 N(69) SIG .009	MCHNY WITH MCAGE	-0.5347 N(29) SIG .001
MCHNY WITH PMOS1	0.0201 N(59) SIG .440	MCHNY WITH ELCWH	0.7789 N(69) SIG .001	MCHNY WITH MOTKW	0.6839 N(64) SIG .001	DALAN WITH WAGES	0.7068 N(68) SIG .001	DALAN WITH CAPLT	0.7300 N(68) SIG .001	DALAN WITH VADLT	0.0354 N(68) SIG .387
DALAN WITH VACAS	-0.7353 N(68) SIG .001	DALAN WITH CAPUT	0.2515 N(68) SIG .019	DALAN WITH NRYRS	-0.2437 N(68) SIG .023	DALAN WITH MCAGE	-0.3931 N(28) SIG .019	DALAN WITH PMOS1	0.0764 N(58) SIG .284	DALAN WITH ELCWH	0.7904 N(68) SIG .001
DALAN WITH MOTKW	0.7268 N(63) SIG .001	WAGES WITH CAPLT	0.2531 N(69) SIG .018	WAGES WITH VADLT	-0.0548 N(69) SIG .328	WAGES WITH VACAS	-0.2600 N(69) SIG .015	WAGES WITH CAPUT	0.1962 N(69) SIG .053	WAGES WITH NRYRS	0.1136 N(69) SIG .176
WAGES WITH MCAGE	-0.3838 N(29) SIG .020	WAGES WITH PMOS1	-0.0198 N(59) SIG .441	WAGES WITH ELCWH	0.5837 N(69) SIG .001	WAGES WITH MOTKW	0.6379 N(64) SIG .001	CAPLT WITH VADLT	0.2615 N(69) SIG .015	CAPLT WITH VACAS	-0.9137 N(69) SIG .001
CAPLT WITH CAPUT	0.2958 N(69) SIG .007	CAPLT WITH NRYRS	-0.2714 N(69) SIG .012	CAPLT WITH MCAGE	-0.2641 N(29) SIG .083	CAPLT WITH PMOS1	0.1010 N(59) SIG .223	CAPLT WITH ELCWH	0.6215 N(69) SIG .001	CAPLT WITH MOTKW	0.5592 N(64) SIG .001
VADLT WITH VACAS	0.0995 N(69) SIG .208	VADLT WITH CAPUT	0.0337 N(69) SIG .392	VADLT WITH NRYRS	0.1508 N(69) SIG .108	VADLT WITH MCAGE	0.0448 N(29) SIG .409	VADLT WITH PMOS1	0.0370 N(59) SIG .390	VADLT WITH ELCWH	-0.0167 N(69) SIG .445
VADLT WITH MOTKW	0.0993 N(64) SIG .217	VACAS WITH CAPUT	-0.3083 N(69) SIG .005	VACAS WITH NRYRS	0.3754 N(69) SIG .001	VACAS WITH MCAGE	0.3042 N(29) SIG .054	VACAS WITH PMOS1	-0.0881 N(59) SIG .254	VACAS WITH ELCWH	-0.6525 N(69) SIG .001
VACAS WITH MOTKW	-0.5293 N(64) SIG .001	CAPUT WITH NRYRS	0.0241 N(69) SIG .422	CAPUT WITH MCAGE	-0.1761 N(29) SIG .180	CAPUT WITH PMOS1	0.2487 N(59) SIG .029	CAPUT WITH ELCWH	0.2586 N(69) SIG .016	CAPUT WITH MOTKW	0.1438 N(64) SIG .129
NRYRS WITH MCAGE	0.4823 N(29) SIG .004	NRYRS WITH PMOS1	-0.0537 N(59) SIG .343	NRYRS WITH ELCWH	-0.0614 N(69) SIG .308	NRYRS WITH MOTKW	-0.0257 N(64) SIG .420	MCAGE WITH PMOS1	-0.1810 N(24) SIG .199	MCAGE WITH ELCWH	-0.3668 N(29) SIG .025
MCAGE WITH MOTKW	-0.5711 N(27) SIG .001	PMOS1 WITH ELCWH	0.1809 N(59) SIG .085	PMOS1 WITH MOTKW	0.1003 N(56) SIG .231	ELCWH WITH MOTKW	0.7252 N(64) SIG .001				

ISRAEL - - - - - - - - - - S P E A R M A N   C O R R E L A T I O N   C O E F F I C I E N T S - - - - - - - - - - - - -

VARIABLE PAIR		VARIABLE PAIR		VARIABLE PAIR		VARIABLE PAIR		VARIABLE PAIR		VARIABLE PAIR	
TVPRD WITH VADED	0.8363 N(69) SIG .001	TVPRD WITH LTOTL	0.7246 N(69) SIG .001	TVPRD WITH LDRCT	0.6872 N(69) SIG .001	TVPRD WITH CAPAS	0.8029 N(69) SIG .001	TVPRD WITH MCHNY	0.7851 N(69) SIG .001	TVPRD WITH DALAN	0.7896 N(68) SIG .001
TVPRD WITH WAGES	0.7749 N(69) SIG .001	TVPRD WITH CAPLT	0.5533 N(69) SIG .001	TVPRD WITH VADLT	0.1967 N(69) SIG .053	TVPRD WITH VACAS	-0.4859 N(69) SIG .001	TVPRD WITH CAPUT	0.1808 N(69) SIG .069	TVPRD WITH NRYRS	-0.0130 N(69) SIG .458
TVPRD WITH MCAGE	-0.4617 N(29) SIG .006	TVPRD WITH PMOS1	0.1167 N(59) SIG .189	TVPRD WITH ELCWH	0.7103 N(69) SIG .001	TVPRD WITH MOTKW	0.6595 N(64) SIG .001	VADED WITH LTOTL	0.9213 N(69) SIG .001	VADED WITH LDRCT	0.8961 N(69) SIG .001
VADED WITH CAPAS	0.8054 N(69) SIG .001	VADED WITH MCHNY	0.7924 N(69) SIG .001	VADED WITH DALAN	0.8068 N(68) SIG .001	VADED WITH WAGES	0.9542 N(69) SIG .001	VADED WITH CAPLT	0.3756 N(69) SIG .001	VADED WITH VADLT	0.0465 N(69) SIG .352
VADED WITH VACAS	-0.3526 N(69) SIG .001	VADED WITH CAPUT	0.2051 N(69) SIG .045	VADED WITH NRYRS	0.0118 N(69) SIG .462	VADED WITH MCAGE	-0.4892 N(29) SIG .004	VADED WITH PMOS1	-0.0257 N(59) SIG .423	VADED WITH ELCWH	0.6633 N(69) SIG .001
VADED WITH MOTKW	0.6934 N(64) SIG .001	LTOTL WITH LDRCT	0.9801 N(69) SIG .001	LTOTL WITH CAPAS	0.7619 N(69) SIG .001	LTOTL WITH MCHNY	0.7576 N(69) SIG .001	LTOTL WITH DALAN	0.7407 N(68) SIG .001	LTOTL WITH WAGES	0.9322 N(69) SIG .001
LTOTL WITH CAPLT	0.2474 N(69) SIG .020	LTOTL WITH VADLT	-0.3082 N(69) SIG .005	LTOTL WITH VACAS	-0.3571 N(69) SIG .001	LTOTL WITH CAPUT	0.1547 N(69) SIG .102	LTOTL WITH NRYRS	-0.0208 N(69) SIG .433	LTOTL WITH MCAGE	-0.4519 N(29) SIG .007
LTOTL WITH PMOS1	-0.0316 N(59) SIG .406	LTOTL WITH ELCWH	0.6445 N(69) SIG .001	LTOTL WITH MOTKW	0.6017 N(64) SIG .001	LDRCT WITH CAPAS	0.7363 N(69) SIG .001	LDRCT WITH MCHNY	0.7380 N(69) SIG .001	LDRCT WITH DALAN	0.7302 N(68) SIG .001
LDRCT WITH WAGES	0.9055 N(69) SIG .001	LDRCT WITH CAPLT	0.2247 N(69) SIG .032	LDRCT WITH VADLT	-0.3275 N(69) SIG .003	LDRCT WITH VACAS	-0.3386 N(69) SIG .002	LDRCT WITH CAPUT	0.1780 N(69) SIG .072	LDRCT WITH NRYRS	-0.0775 N(69) SIG .263
LDRCT WITH MCAGE	-0.4426 N(29) SIG .008	LDRCT WITH PMOS1	-0.0052 N(59) SIG .484	LDRCT WITH ELCWH	0.6308 N(69) SIG .001	LDRCT WITH MOTKW	0.5852 N(64) SIG .001	CAPAS WITH MCHNY	0.9688 N(69) SIG .001	CAPAS WITH DALAN	0.9494 N(68) SIG .001
CAPAS WITH WAGES	0.7162 N(69) SIG .001	CAPAS WITH CAPLT	0.7930 N(69) SIG .001	CAPAS WITH VADLT	-0.0154 N(69) SIG .450	CAPAS WITH VACAS	-0.8202 N(69) SIG .001	CAPAS WITH CAPUT	0.2829 N(69) SIG .009	CAPAS WITH NRYRS	-0.2211 N(69) SIG .034

JAPAN - - - - - - - - - - - S P E A R M A N   C O R R E L A T I O N   C O E F F I C I E N T S - - - - - - - - - - - - -

VARIABLE PAIR		VARIABLE PAIR		VARIABLE PAIR		VARIABLE PAIR		VARIABLE PAIR		VARIABLE PAIR	
TVPRD WITH VADED	0.9063 N(63) SIG .001	TVPRD WITH LTOTL	0.8038 N(63) SIG .001	TVPRD WITH LDRCT	0.7526 N(63) SIG .001	TVPRD WITH CAPAS	0.8698 N(63) SIG .001	TVPRD WITH MCHNY	0.7843 N(63) SIG .001	TVPRD WITH DALAN	0.8021 N(63) SIG .001
TVPRD WITH WAGES	0.8305 N(53) SIG .001	TVPRD WITH CAPLT	0.4925 N(63) SIG .001	TVPRD WITH VADLT	0.5577 N(63) SIG .001	TVPRD WITH VACAS	-0.2191 N(63) SIG .042	TVPRD WITH CAPUT	-0.0744 N(63) SIG .281	TVPRD WITH NRYRS	0.2096 N(63) SIG .050
TVPRD WITH MCAGE	-0.1941 N(63) SIG .064	TVPRD WITH PMOS1	-0.1418 N(63) SIG .134	TVPRD WITH ELCWH	0.8058 N(63) SIG .001	TVPRD WITH MOTKW	0.7908 N(63) SIG .001	VADED WITH LTOTL	0.9140 N(63) SIG .001	VADED WITH LDRCT	0.8610 N(63) SIG .001
VADED WITH CAPAS	0.9145 N(63) SIG .001	VADED WITH MCHNY	0.8629 N(63) SIG .001	VADED WITH DALAN	0.8581 N(63) SIG .001	VADED WITH WAGES	0.9420 N(63) SIG .001	VADED WITH CAPLT	0.3849 N(63) SIG .001	VADED WITH VADLT	0.5030 N(63) SIG .001
VADED WITH VACAS	-0.1328 N(63) SIG .150	VADED WITH CAPUT	-0.0404 N(63) SIG .377	VADED WITH NPYRS	0.1945 N(63) SIG .063	VADED WITH MCAGE	-0.2313 N(63) SIG .034	VADED WITH PMOS1	-0.0701 N(63) SIG .293	VADED WITH ELCWH	0.8218 N(63) SIG .001
VADED WITH MOTKW	0.8189 N(63) SIG .001	LTOTL WITH LDRCT	0.9624 N(63) SIG .001	LTOTL WITH CAPAS	0.8842 N(63) SIG .001	LTOTL WITH MCHNY	0.8036 N(63) SIG .001	LTOTL WITH DALAN	0.7962 N(63) SIG .001	LTOTL WITH WAGES	0.9643 N(63) SIG .001
LTOTL WITH CAPLT	0.1979 N(63) SIG .060	LTOTL WITH VADLT	0.1873 N(63) SIG .071	LTOTL WITH VACAS	-0.1925 N(63) SIG .065	LTOTL WITH CAPUT	-0.0039 N(63) SIG .488	LTOTL WITH NRYRS	0.2602 N(63) SIG .020	LTOTL WITH MCAGE	-0.2787 N(63) SIG .013
LTOTL WITH PMOS1	-0.1492 N(63) SIG .122	LTOTL WITH ELCWH	0.7650 N(63) SIG .001	LTOTL WITH MOTKW	0 7385 N(63) SIG .001	LDRCT WITH CAPAS	0.8816 N(63) SIG .001	LDRCT WITH MCHNY	0.7994 N(63) SIG .001	LDRCT WITH DALAN	0.7720 N(63) SIG .001
LDRCT WITH WAGES	0.9043 N(63) SIG .001	LDRCT WITH CAPLT	0.2277 N(63) SIG .036	LDRCT WITH VADLT	0.0955 N(63) SIG .228	LDRCT WITH VACAS	-0.2903 N(63) SIG .010	LDRCT WITH CAPUT	0.0125 N(63) SIG .461	LDRCT WITH NRYRS	0.2553 N(53) SIG .022
LDRCT WITH MCAGE	-0.2560 N(53) SIG .021	LDRCT WITH PMOS1	-0.0977 N(63) SIG .223	LDRCT WITH ELCWH	0.7453 N(63) SIG .001	LDRCT WITH MOTKW	0.6855 N(63) SIG .001	CAPAS WITH MCHNY	0.9173 N(63) SIG .001	CAPAS WITH DALAN	0.8682 N(63) SIG .001
CAPAS WITH WAGES	0.8738 N(63) SIG .001	CAPAS WITH CAPLT	0.5647 N(63) SIG .001	CAPAS WITH VADLT	0.3697 N(63) SIG .001	CAPAS WITH VACAS	-0.4758 N(53) SIG .001	CAPAS WITH CAPUT	-0.0289 N(63) SIG .411	CAPAS WITH NRYRS	0.1215 N(63) SIG .171

JAPAN CONTD

CAPAS WITH MCAGE	-0.2314 N(63) SIG .034	CAPAS WITH PMOS1	-0.1564 N(63) SIG .110	CAPAS WITH ELCWH	0.8595 N(63) SIG .001	CAPAS WITH MOTKW	0.8155 N(63) SIG .001	MCHNY WITH DALAN	0.8169 N(63) SIG .001	MCHNY WITH WAGES	0.8164 N(63) SIG .001
MCHNY WITH CAPLT	0.5620 N(63) SIG .001	MCHNY WITH VADLT	0.3940 N(63) SIG .001	MCHNY WITH VACAS	-0.4242 N(63) SIG .001	MCHNY WITH CAPUT	0.0642 N(63) SIG .309	MCHNY WITH NRYRS	0.1207 N(63) SIG .173	MCHNY WITH MCAGE	-0.3049 N(63) SIG .008
MCHNY WITH PMOS1	-0.1550 N(63) SIG .113	MCHNY WITH ELCWH	0.8680 N(63) SIG .001	MCHNY WITH MOTKW	0.8469 N(63) SIG .001	DALAN WITH WAGES	0.8055 N(63) SIG .001	DALAN WITH CAPLT	0.5148 N(63) SIG .001	DALAN WITH VADLT	0.4853 N(63) SIG .001
DALAN WITH VACAS	-0.3084 N(63) SIG .007	DALAN WITH CAPUT	-0.0087 N(63) SIG .473	DALAN WITH NRYRS	0.0758 N(63) SIG .277	DALAN WITH MCAGE	-0.2328 N(63) SIG .033	DALAN WITH PMOS1	-0.0452 N(63) SIG .362	DALAN WITH ELCWH	0.7470 N(63) SIG .001
DALAN WITH MOTKW	0.7577 N(63) SIG .001	WAGES WITH CAPLT	0.2272 N(63) SIG .037	WAGES WITH VADLT	0.2980 N(63) SIG .009	WAGES WITH VACAS	-0.1277 N(63) SIG .159	WAGES WITH CAPUT	-0.0154 N(63) SIG .452	WAGES WITH NRYRS	0.2400 N(63) SIG .029
WAGES WITH MCAGE	-0.2416 N(63) SIG .028	WAGES WITH PMOS1	-0.1519 N(63) SIG .117	WAGES WITH ELCWH	0.7837 N(63) SIG .001	WAGES WITH MOTKW	0.7651 N(63) SIG .001	CAPLT WITH VADLT	0.6337 N(63) SIG .001	CAPLT WITH VACAS	-0.7055 N(63) SIG .001
CAPLT WITH CAPUT	-0.1485 N(63) SIG .123	CAPLT WITH NRYRS	-0.0645 N(63) SIG .308	CAPLT WITH MCAGE	-0.0564 N(63) SIG .330	CAPLT WITH PMOS1	-0.0451 N(63) SIG .363	CAPLT WITH ELCWH	0.5454 N(63) SIG .001	CAPLT WITH MOTKW	0.5076 N(63) SIG .001
VADLT WITH VACAS	0.0288 N(63) SIG .411	VADLT WITH CAPUT	-0.1865 N(63) SIG .072	VADLT WITH NRYRS	0.0310 N(63) SIG .405	VADLT WITH MCAGE	0.0226 N(63) SIG .430	VADLT WITH PMOS1	0.1063 N(63) SIG .203	VADLT WITH ELCWH	0.4276 N(63) SIG .001
VADLT WITH MOTKW	0.5191 N(63) SIG .001	VACAS WITH CAPUT	0.0228 N(63) SIG .429	VACAS WITH NRYRS	0.1246 N(63) SIG .165	VACAS WITH MCAGE	0.1264 N(63) SIG .162	VACAS WITH PMOS1	0.1806 N(63) SIG .078	VACAS WITH ELCWH	-0.3802 N(63) SIG .001
VACAS WITH MOTKW	-0.2714 N(63) SIG .016	CAPUT WITH NRYRS	-0.0812 N(63) SIG .263	CAPUT WITH MCAGE	-0.1962 N(63) SIG .062	CAPUT WITH PMOS1	-0.0569 N(63) SIG .329	CAPUT WITH ELCWH	-0.0458 N(63) SIG .361	CAPUT WITH MOTKW	-0.0196 N(63) SIG .440
NRYRS WITH MCAGE	-0.0983 N(63) SIG .222	NRYRS WITH PMOS1	-0.0582 N(63) SIG .325	NRYRS WITH ELCWH	0.2354 N(63) SIG .032	NRYRS WITH MOTKW	0.1606 N(63) SIG .104	MCAGE WITH PMOS1	0.1156 N(63) SIG .184	MCAGE WITH ELCWH	-0.2096 N(63) SIG .050
MCAGE WITH MOTKW	-0.1956 N(63) SIG .062	PMOS1 WITH ELCWH	-0.0802 N(63) SIG .266	PMOS1 WITH MOTKW	-0.1453 N(63) SIG .128	ELCWH WITH MOTKW	0.8643 N(63) SIG .001				

YUGOSLAVIA CONTD.

CAPAS WITH MCAGE	0.0794 N(145) SIG .171	CAPAS WITH PMOS1	0.0208 N(145) SIG .402	CAPAS WITH ELCWH	0.7986 N(145) SIG .001	CAPAS WITH MOTKW	0.8728 N(145) SIG .001	MCHNY WITH DALAN	0.9594 N(145) SIG .001	MCHNY WITH WAGES	0.6356 N(145) SIG .001
MCHNY WITH CAPLT	0.6368 N(145) SIG .001	MCHNY WITH VADLT	0.3833 N(145) SIG .001	MCHNY WITH VACAS	-0.5310 N(145) SIG .001	MCHNY WITH CAPUT	0.2203 N(145) SIG .004	MCHNY WITH NRYRS	0.1197 N(145) SIG .076	MCHNY WITH MCAGE	0.0847 N(145) SIG .155
MCHNY WITH PMOS1	-0.0242 N(145) SIG .386	MCHNY WITH ELCWH	0.7996 N(145) SIG .001	MCHNY WITH MOTKW	0.8644 N(145) SIG .001	DALAN WITH WAGES	0.6734 N(145) SIG .001	DALAN WITH CAPLT	0.5965 N(145) SIG .001	DALAN WITH VADLT	0.4413 N(145) SIG .001
DALAN WITH VACAS	-0.4441 N(145) SIG .001	DALAN WITH CAPUT	0.2373 N(145) SIG .002	DALAN WITH NRYRS	0.1089 N(145) SIG .096	DALAN WITH MCAGE	0.0218 N(145) SIG .397	DALAN WITH PMOS1	0.0001 N(145) SIG .499	DALAN WITH ELCWH	0.8194 N(145) SIG .001
DALAN WITH MOTKW	0.8730 N(145) SIG .001	WAGES WITH CAPLT	-0.0314 N(145) SIG .354	WAGES WITH VADLT	0.1806 N(145) SIG .015	WAGES WITH VACAS	0.1553 N(145) SIG .031	WAGES WITH CAPUT	0.1526 N(145) SIG .033	WAGES WITH NRYRS	0.1657 N(145) SIG .023
WAGES WITH MCAGE	0.1191 N(145) SIG .077	WAGES WITH PMOS1	-0.0864 N(145) SIG .151	WAGES WITH ELCWH	0.6145 N(145) SIG .001	WAGES WITH MOTKW	0.6391 N(145) SIG .001	CAPLT WITH VADLT	0.5141 N(145) SIG .001	CAPLT WITH VACAS	-0.8441 N(145) SIG .001
CAPLT WITH CAPUT	0.2200 N(145) SIG .004	CAPLT WITH NRYRS	-0.0453 N(145) SIG .294	CAPLT WITH MCAGE	0.0131 N(145) SIG .438	CAPLT WITH PMOS1	0.1112 N(145) SIG .092	CAPLT WITH ELCWH	0.5050 N(145) SIG .001	CAPLT WITH MOTKW	0.5459 N(145) SIG .001
VADLT WITH VACAS	-0.0401 N(145) SIG .316	VADLT WITH CAPUT	0.1332 N(145) SIG .055	VADLT WITH NRYRS	-0.0417 N(145) SIG .309	VADLT WITH MCAGE	-0.1042 N(145) SIG .105	VADLT WITH PMOS1	-0.1337 N(145) SIG .054	VADLT WITH ELCWH	0.4692 N(145) SIG .001
VADLT WITH MOTKW	0.3942 N(145) SIG .001	VACAS WITH CAPUT	-0.1883 N(145) SIG .012	VACAS WITH NRYRS	0.0475 N(145) SIG .285	VACAS WITH MCAGE	-0.0541 N(145) SIG .259	VACAS WITH PMOS1	-0.1909 N(145) SIG .011	VACAS WITH ELCWH	-0.3356 N(145) SIG .001
VACAS WITH MOTKW	-0.4163 N(145) SIG .001	CAPUT WITH NRYRS	0.1346 N(145) SIG .053	CAPUT WITH MCAGE	0.1176 N(145) SIG .080	CAPUT WITH PMOS1	0.1703 N(145) SIG .020	CAPUT WITH ELCWH	0.2832 N(145) SIG .001	CAPUT WITH MOTKW	0.2717 N(145) SIG .001
NRYRS WITH MCAGE	0.2029 N(145) SIG .007	NRYRS WITH PMOS1	-0.0584 N(145) SIG .243	NRYRS WITH ELCWH	0.1420 N(145) SIG .044	NRYRS WITH MOTKW	0.1189 N(145) SIG .077	MCAGE WITH PMOS1	-0.0437 N(145) SIG .301	MCAGE WITH ELCWH	0.0683 N(145) SIG .207
MCAGE WITH MOTKW	0.1297 N(145) SIG .060	PMOS1 WITH ELCWH	-0.0764 N(145) SIG .181	PMOS1 WITH MOTKW	-0.0652 N(145) SIG .218	ELCWH WITH MOTKW	0.8238 N(145) SIG .001				

YUGOSLAVIA- - - - - - - - S P E A R M A N   C O R R E L A T I O N   C O E F F I C I E N T S - - - - - - - - - - - - - -

VARIABLE PAIR		VARIABLE PAIR		VARIABLE PAIR		VARIABLE PAIR		VARIABLE PAIR		VARIABLE PAIR	
TVPRD WITH VADED	0.9172 N(145) SIG .001	TVPRD WITH LTOTL	0.8000 N(145) SIG .001	TVPRD WITH LDRCT	0.7823 N(145) SIG .001	TVPRD WITH CAPAS	0.7323 N(145) SIG .001	TVPRD WITH MCHNY	0.7149 N(145) SIG .001	TVPRD WITH DALAN	0.7240 N(145) SIG .001
TVPRD WITH WAGES	0.8503 N(145) SIG .001	TVPRD WITH CAPLT	0.2051 N(145) SIG .007	TVPRD WITH VADLT	0.4075 N(145) SIG .001	TVPRD WITH VACAS	0.0199 N(145) SIG .406	TVPRD WITH CAPUT	0.1469 N(145) SIG .039	TVPRD WITH NRYRS	0.1494 N(145) SIG .036
TVPRD WITH MCAGE	0.1063 N(145) SIG .102	TVPRD WITH PMOS1	-0.0647 N(145) SIG .220	TVPRD WITH ELCWH	0.7092 N(145) SIG .001	TVPRD WITH MOTKW	0.7021 N(145) SIG .001	VADED WITH LTOTL	0.8505 N(145) SIG .001	VADED WITH LDRCT	0.8279 N(145) SIG .001
VADED WITH CAPAS	0.7707 N(145) SIG .001	VADED WITH MCHNY	0.7575 N(145) SIG .001	VADED WITH DALAN	0.8006 N(145) SIG 001	VADED WITH WAGES	0.9235 N(145) SIG .001	VADED WITH CAPLT	0.1765 N(145) SIG .017	VADED WITH VADLT	0.4444 N(145) SIG .001
VADED WITH VACAS	0.0585 N(145) SIG .242	VADED WITH CAPUT	0.1887 N(145) SIG .012	VADED WITH NRYRS	0.1102 N(145) SIG .093	VADED WITH MCAGE	0.0623 N(145) SIG .228	VADED WITH PMOS1	-0.0815 N(145) SIG .165	VADED WITH ELCWH	0.7195 N(145) SIG .001
VADED WITH MOTKW	0.7368 N(145) SIG .001	LTOTL WITH LDRCT	0.9916 N(145) SIG .001	LTOTL WITH CAPAS	0.6600 N(145) SIG .001	LTOTL WITH MCHNY	0.6376 N(145) SIG .001	LTOTL WITH DALAN	0.6408 N(145) SIG .001	LTOTL WITH WAGES	0.9438 N(145) SIG .001
LTOTL WITH CAPLT	-0.0680 N(145) SIG .208	LTOTL WITH VADLT	-0.0387 N(145) SIG .322	LTOTL WITH VACAS	0.0780 N(145) SIG .176	LTOTL WITH CAPUT	0.1220 N(145) SIG .072	LTOTL WITH NRYRS	0.1571 N(145) SIG .030	LTOTL WITH MCAGE	0.1538 N(145) SIG .032
LTOTL WITH PMOS1	-0.0444 N(145) SIG .298	LTOTL WITH ELCWH	0.5417 N(145) SIG .001	LTOTL WITH MOTKW	0.6137 N(145) SIG .001	LDRCT WITH CAPAS	0.6628 N(145) SIG .001	LDRCT WITH MCHNY	0.6401 N(145) SIG .001	LDRCT WITH DALAN	0.6366 N(145) SIG .001
LDRCT WITH WAGES	0.9200 N(145) SIG .001	LDRCT WITH CAPLT	-0.0578 N(145) SIG .245	LDRCT WITH VADLT	-0.0721 N(145) SIG .195	LDRCT WITH VACAS	0.0492 N(145) SIG .278	LDRCT WITH CAPUT	0.1309 N(145) SIG .058	LDRCT WITH NRYRS	0.1714 N(145) SIG .020
LDRCT WITH MCAGE	0.1733 N(145) SIG .019	LDRCT WITH PMOS1	-0.0194 N(145) SIG .408	LDRCT WITH ELCWH	0.5379 N(145) SIG .001	LDRCT WITH MOTKW	0.6127 N(145) SIG .001	CAPAS WITH MCHNY	0.9845 N(145) SIG .001	CAPAS WITH DALAN	0.9546 N(145) SIG .001
CAPAS WITH WAGES	0.6545 N(145) SIG .001	CAPAS WITH CAPLT	0.6421 N(145) SIG .001	CAPAS WITH VADLT	0.3661 N(145) SIG .001	CAPAS WITH VACAS	-0.5404 N(145) SIG .001	CAPAS WITH CAPUT	0.2190 N(145) SIG .004	CAPAS WITH NRYRS	0.0960 N(145) SIG .125

Table 8

FREQUENCY DISTRIBUTIONS OF MANUFACTURING ESTABLISHMENTS OF FRANCE, INDIA, ISRAEL, JAPAN AND YUGOSLAVIA

This table gives the frequencies, relative frequencies and and percentage cumulative frequencies for the following

Net capital assets
Machinery and equipment
Value added
Total value of production
Depreciation allowances
Intermediate inputs
Value of fuel consumed
Electricity consumed in kwh
Value of electricity consumed
Motors operated in kwh
Inventories
Direct labour
Indirect labour
Educated labour
Other labour
Direct labour in shift one
Direct labour in shift two
Direct labour in shift three
Percentage of motors operated, shift one
Percentage of motors operated, shift two
Percentage of motors operated, shift three
Breakeven point percentage
Age of establishment in number of years
Age of machinery
Percentage capacity utilisation
Number of shifts worked.

Notation
/ Percentage of Establishments
Cum/ Cumulative Percentage

NET CAPITAL ASSETS	FRANCE NO.OF FIRMS	%	CUM. %	INDIA NO.OF FIRMS	%	CUM. %	ISRAEL NO.OF FIRMS	%	CUM. %	JAPAN NO.OF FIRMS	%	CUM. %	YUGOSLAVIA NO.OF FIRMS	%	CUM %
0- 100	1	2	2	3	3	3	3	4	4	2	3	3			
100- 500	3	5	6	24	21	23	10	14	19	17	27	30	15	10	10
500- 1000	9	14	20	7	9	29	11	16	35	10	16	45	23	16	26
1000- 2000	8	13	33	19	16	45	10	14	49	4	6	52	28	19	46
2000- 3000	9	14	47	15	13	58	9	13	62	9	14	66	28	19	65
3000- 4000	5	8	55	5	4	62	2	3	65	7	11	77	9	6	71
4000- 5000	6	9	64	6	5	68	5	7	72	2	3	80	5	3	74
5000- 10000	8	13	77	13	11	79	8	12	84	8	13	92	23	16	90
10000- 25000	7	11	88	12	10	89	8	12	96	2	3	95	13	9	99
25000- 50000	5	8	96	6	5	94	2	3	99	2	3	98			
50000-100000	3	5	100	2	2	96	1	1	100	1	2	100	1	1	100
100000-500000				5	4	100									

MACHINERY	FRANCE NO.OF FIRMS	%	CUM. %	INDIA NO.OF FIRMS	%	CUM. %	ISRAEL NO.OF FIRMS	%	CUM. %	JAPAN NO.OF FIRMS	%	CUM. %	YUGOSLAVIA NO.OF FIRMS	%	CUM %
0- 50	1	2	2	4	3	3	2	3	3	10	16	16			
50- 250	5	8	9	19	16	20	8	12	14	13	20	36	14	10	10
250- 500	9	14	23	18	15	35	9	13	28	8	13	48	21	14	24
500- 1000	13	20	44	13	11	46	14	20	48	5	8	56	26	18	42
1000- 2000	4	6	50	12	10	56	5	7	55	8	13	69	20	14	56
2000- 3000	5	8	58	7	6	62	7	10	65	9	14	83	14	10	66
3000- 4000	1	2	59	2	2	64	3	4	70				8	6	71
4000- 5000	12	19	78	11	9	74	7	10	80	6	9	92	25	17	88
5000- 10000	5	8	86	17	15	88	10	14	94	3	5	97	14	10	98
10000- 25000	5	8	94	6	5	93	2	3	97	1	2	98	2	1	99
25000- 50000	3	5	98	3	3	96	1	1	99				1	1	100
50000-250000	1	2	100	5	4	100	1	1	100	1	2	100			

VALUE ADDED	NO.OF FIRMS	%	CUM %	NO.OF FIRMS	%	CUM %	NO.OF FIRMS	%	CUM %	NO.OF FIRMS	%	CUM %	NO.OF FIRMS	%	CUM %
0. 100				1	1	1									
100- 500				18	15	16	5	7	7	12	19	19	28	19	19
500- 1000	3	5	5	19	16	32	15	22	29	12	19	38	34	23	43
1000- 2000	7	11	16	13	11	44	21	30	59	10	16	53	43	30	72
2000- 3000	6	9	25	11	9	53	10	14	74	8	13	66	18	12	85
3000- 4000	5	8	33	5	4	57	6	9	83	3	5	70	8	6	90
4000- 5000	3	5	38	3	3	60	4	6	88	5	8	78	7	5	95
5000- 10000	11	17	55	18	15	75	6	9	97	10	16	94	5	3	99
10000- 25000	22	34	89	19	16	91	2	3	100	3	5	98	2	1	100
25000- 50000	7	11	100	4	3	95									
50000-100000				5	4	99				1	2	100			
100000-500000				1	1	100									

	FRANCE			INDIA			ISRAELI			JAPAN			YUGOSLAVIA		
TOTAL VALUE OF PRODUCN.	NO.OF FIRMS	%	CUM %	NO.OF FIRMS	%	CUM %	NO.OF FIRMS	%	CUM %	NO.OF FIRMS	%	CUM %	NO.OF FIRMS	%	CUM %
100- 500				1	1	1				1	2	2	2	1	1
500- 1000	1	2	2	7	6	7	2	3	3	4	6	8	10	7	8
1000- 2000	2	3	5	15	13	20	7	10	13	7	11	19	22	15	23
2000- 3000				11	9	29	11	16	29	9	14	33	33	23	46
3000- 4000	2	3	8	6	5	34	4	6	35	4	6	39	16	11	57
4000- 5000	1	2	9	2	2	36	6	9	43	3	5	44	12	8	66
5000- 10000	13	20	30	17	15	50	22	32	75	16	25	69	31	21	87
10000- 25000	15	23	53	22	19	69	14	20	96	12	19	88	17	12	99
25000- 50000	18	28	81	19	16	85	3	4	100	7	11	98	2	1	100
~~50000-10000~~	~~7~~	~~11~~	~~92~~	~~9~~	~~8~~	~~93~~									
100000-500000	5	8	100	8	7	100				1	2	100			

DEPRECIATN. ALLOWANCES	FRANCE NO.OF FIRMS	%	CUM %	INDIA NO.OF FIRMS	%	CUM %	ISRAEL NO.OF FIRMS	%	CUM %	JAPAN NO.OF FIRMS	%	CUM %	YUGOSLAVIA NO.OF FIRMS	%	CUM %
0- 10				4	3	3	3	4	4				6	4	4
10- 25	2	3	3	5	4	8	4	6	10	3	5	5	18	12	17
25- 50	2	3	6	11	9	17	6	9	19	12	19	24	26	18	34
50- 100	1	2	8	18	15	32	11	16	35	12	19	43	31	21	56
100- 250	10	16	23	20	17	50	11	16	51	8	13	56	33	23	79
250- 500	11	17	41	16	14	63	15	22	72	11	17	73	19	13	92
500- 1000	13	20	61	12	10	74	11	16	88	11	17	90	10	7	99
1000- 2000	10	16	77	11	9	83	5	7	96	5	8	98	1	1	99
2000- 3000	1	2	78	8	7	90	3	4	100				1	1	100
3000- 4000	6	9	88	2	2	91									
4000- 5000	1	2	89	4	3	95									
5000- 10000	5	8	97	3	3	97									
10000- 25000	2	3	100	3	3	100				1	2	100			
25000- 50000															

INTERMED. INPUTS	FRANCE NO.OF FIRMS	%	CUM %	INDIA NO.OF FIRMS	%	CUM %	ISRAEL NO.OF FIRMS	%	CUM %	JAPAN NO.OF FIRMS	%	CUM %	YUGOSLAVIA NO.OF FIRMS	%	CUM %
100- 500				7	6	6	2	3	3	6	9	9	8	6	6
500- 1000	2	3	3	10	9	15	5	7	10	3	5	14	23	16	21
1000- 2000	3	5	8	19	16	31	15	22	32	10	16	30	43	30	51
2000- 3000	5	8	16	6	5	36	7	10	42	8	13	42	22	15	66
3000- 4000	3	5	20	7	6	42	8	12	54	6	9	52	13	9	75
4000- 5000	2	3	23	3	3	44	6	9	62	2	3	55	11	8	83
5000- 10000	15	23	47	15	13	57	16	23	86	11	17	72	18	12	95
10000- 25000	14	22	69	28	24	81	8	12	97	16	25	97	7	5	100
25000- 50000	13	20	89	11	9	91	2	3	100	1	2	98			
50000-100000	5	8	97	7	6	97				1	2	100			
100000-500000	2	3	100	4	3	100									

VALUE OF FUEL CONS.		FRANCE NO.OF FIRMS	%	CUM. %	INDIA NO.OF FIRMS	%	CUM. %	ISRAEL NO.OF FIRMS	%	CUM. %	JAPAN NO.OF FIRMS	%	CUM. %	YUGOSLAVIA NO.OF FIRMS	%	CUM %
0-	10	2	3	3	3	3	3	8	12	12	11	17	17	9	6	6
10-	25	2	3	6	9	8	10	10	14	26	13	20	38	20	14	20
25-	50	6	10	16	13	11	21	13	19	45	10	16	53	23	16	36
50-	100	4	6	22	9	8	29	9	13	58	5	8	61	22	15	51
100-	250	16	25	48	31	26	56	16	23	81	12	19	80	38	26	77
250-	500	8	13	60	11	9	65	5	7	88	3	5	84	18	12	90
500-	1000	8	13	73	18	15	80	5	7	96	5	8	92	11	8	97
1000-	2000	7	11	84	6	5	85	1	1	97	4	6	98	3	2	99
2000-	3000	4	6	90	4	3	89	1	1	99						
3000-	4000															
4000-	5000	1	2	92	1	1	90							1	1	100
5000-	10000	4	6	98	7	6	96	1	1	100						
10000-	25000	1	2	100	4	3	99				1	2	100			
25000-	50000				1	1	100									

ELEC.CONS. IN KWH.		FRANCE NO.OF FIRMS	%	CUM. %	INDIA NO.OF FIRMS	%	CUM. %	ISRAEL NO.OF FIRMS	%	CUM. %	JAPAN NO.OF FIRMS	%	CUM. %	YUGOSLAVIA NO.OF FIRMS	%	CUM %
0-	100	3	5	5	18	15	15	6	9	9	19	30	30	3	2	2
100-	500	15	23	28	32	27	43	20	29	38	14	22	52	23	16	18
500-	1000	7	11	39	11	9	52	12	17	55	7	11	63	19	13	31
1000-	2000	6	9	48	17	15	67	8	12	67	9	14	77	19	13	44
2000-	3000	8	13	61	3	3	69	6	9	75				14	10	54
3000-	4000	3	5	66	3	3	72	3	4	80	6	9	86	8	6	59
4000-	5000	2	3	69	3	3	74	3	4	84				9	6	66
5000-	10000	6	9	78	7	6	80	6	9	93	3	5	91	28	19	85
10000-	25000	3	5	83	11	9	90	2	3	96	4	6	97	10	7	92
25000-	50000	2	3	86	4	3	93	2	3	99	1	2	98	9	6	98
50000-	100000	7	11	97	4	3	97	1	1	100				2	1	99
100000-	500000	2	3	100	4	4	100				1	2	100	1	1	100

VALUE OF ELEC.CONS.	FRANCE NO.OF FIRMS	%	CUM %	INDIA NO.OF FIRMS	%	CUM %	ISRAEL NO.OF FIRMS	%	CUM %	JAPAN NO.OF FIRMS	%	CUM %	YUGOSLAVIA NO.OF FIRMS	%	CUM %
0- 10	3	5	5	11	9	9	16	23	23	19	30	30	26	18	18
10- 25	10	16	21	20	17	27	15	22	45	11	17	47	26	18	36
25- 50	7	11	33	14	12	39	11	16	61	10	16	63	30	21	57
50- 100	10	16	49	16	14	53	9	13	74	7	11	73	28	19	76
100- 250	9	15	64	19	16	69	12	17	91	9	14	88	20	14	90
250- 500	7	11	75	9	8	77	2	3	94	2	3	91	11	8	97
500- 1000	5	8	84	9	8	89	2	3	97	9	6	97	3	2	99
1000- 2000	5	8	92	7	6	91	1	1	98	1	2	98	1	1	100
2000- 3000				4	3	94	1	1	100						
3000- 4000	1	2	93	1	1	95									
4000- 5000	2	3	97	2	2	97									
5000- 10000	2	3	100	2	2	98				1	2	100			
10000- 25000															
25000- 50000				2	2	100									

MOTORS OPER. IN KWH	FRANCE NO.OF FIRMS	%	CUM %	INDIA NO.OF FIRMS	%	CUM %	ISRAEL NO.OF FIRMS	%	CUM %	JAPAN NO.OF FIRMS	%	CUM %	YUGOSLAVIA NO.OF FIRMS	%	CUM %
0- 100	4	6	6	22	19	19	15	22	22	23	36	36	4	3	3
100- 500	15	23	30	37	32	50	21	30	52	17	27	63	28	19	22
500- 1000	10	16	45	13	11	62	9	13	65	10	16	78	30	21	43
1000- 2000	7	11	56	13	11	73	8	12	77	2	3	81	32	22	65
2000- 3000	2	3	59	7	6	79	6	9	86	3	5	86	13	9	79
3000- 4000	3	5	64	1	1	79				1	2	88	10	7	81
4000- 5000	3	5	69	3	3	82	2	3	88	1	2	89	6	4	85
5000- 10000	6	9	78	9	8	90	2	3	91	3	5	94	13	9	94
10000- 25000	3	5	83	7	6	96	1	1	93				8	6	99
25000- 50000				1	1	97				1	2	95	1	1	100
50000-100000	11	17	100	3	3	99	5	7	100	3	5	100			
100000-500000				1	1	100									

	FRANCE			INDIA			ISRAEL			JAPAN			YUGOSLAVIA*		
INVEN-TORIES	NO.OF FIRMS	%	CUM. %	NO.OF FIRMS	%	CUM. %	NO.OF FIRMS	%	CUM. %	NO.OF FIRMS	%	CUM. %	NO.OF FIRMS	%	CUM %
0- 50	2	3	3							2	3	3	29	20	20
50- 250	3	5	8				10	14	14	14	22	25	74	51	71
250- 500	4	6	14				10	14	29	11	17	42	27	19	90
500- 1000	3	5	19				13	19	48	12	19	61	10	7	97
1000- 1500	8	13	31				10	14	62	4	6	67	1	1	97
1500- 2000	3	5	36				6	9	71	1	2	69	3	2	99
2000- 2500	6	9	45				4	6	77	5	8	77			
2500- 5000	8	13	58				9	13	90	10	16	92	1	1	100
5000- 12500	10	16	73				6	9	99	5	8	100			
12500- 25000	8	13	86				1	1	100						
25000- 50000	5	8	94												
50000-100000	4	6	100												

*INTEREST ON CAPITAL

	FRANCE			INDIA			ISRAEL			JAPAN			YUGOSLAVIA		
DIRECT LABOUR	NO.OF FIRMS	%	CUM. %	NO.OF FIRMS	%	CUM. %	NO.OF FIRMS	%	CUM. %	NO.OF FIRMS	%	CUM. %	NO.OF FIRMS	%	CUM %
10- 25	2	5	5	2	2	2	9	10	10	5	8	8			
25- 50	5	8	13	2	2	3	7	10	20	6	9	17			
50- 100	8	13	25	14	12	15	19	28	48	17	27	44	4	3	3
100- 250	22	34	59	26	22	38	21	30	78	21	33	77	17	12	14
250- 500	14	22	81	21	18	56	13	19	97	10	16	92	23	16	30
500- 1000	7	11	92	21	18	74	2	3	100	5	8	100	50	34	65
1000- 2000	5	8	100	16	14	87							38	26	91
2000- 3000				5	4	91							8	6	97
3000- 4000				4	3	95							3	2	99
4000- 5000				2	2	97							1	1	99
5000- 10000				4	3	100							1	1	100

INDIRECT LABOUR		NO.OF FIRMS	%	CUM %	NO.OF FIRMS	%	CUM %	NO.OF FIRMS	%	CUM %	NO.OF FIRMS	%	CUM %	NO.OF FIRMS	%	CUM %
0-	10	3	5	5	9	8	8	7	10	10	14	22	22	3	2	2
10-	25	5	8	13	18	15	23	17	25	35	7	11	33	7	5	7
25-	50	11	17	30	19	12	35	20	29	64	15	23	56	13	9	16
50-	100	7	11	41	23	20	55	13	19	83	11	17	73	30	21	37
100-	250	15	23	64	20	17	72	10	14	97	8	13	86	61	42	79
250-	500	7	11	75	19	16	88	2	3	100	7	11	97	18	12	91
500-	1000	7	11	86	6	5	93				1	2	98	10	7	98
1000-	2000	5	8	94	6	5	98					2	100	2	1	99
2000-	3000				1	1	99									
3000-	4000	3	5	98	1	1	100							1	1	100
4000-	5000	1	2	100												

FRANCE INDIA ISRAEL JAPAN YUGOSLAVIA

EDUCATED LABOUR		NO.OF FIRMS	%	CUM %	NO.OF FIRMS	%	CUM %	NO.OF FIRMS	%	CUM %	NO.OF FIRMS	%	CUM %	NO.OF FIRMS	%	CUM %
0-	10	22	35	35	13	11	11	14	20	20	13	20	20	12	8	8
10-	25	22	35	70	13	11	22	24	35	55	6	9	30	22	15	23
25-	50	5	8	78	27	23	45	14	20	75	11	17	47	34	23	47
50-	100	7	11	89	20	17	62	13	19	94	9	14	61	43	30	77
100-	250	7	11	100	26	22	85	3	4	99	13	20	81	23	16	92
250-	500				9	8	92	1	1	100	8	13	94	8	6	98
500-	1000				3	3	95				3	5	98	2	1	99
1000-	2000				5	4	99				1	2	100			
2000-	3000				1	1	100							1	1	100

OTHER LABOUR		FRANCE NO.OF FIRMS	%	CUM. %	INDIA NO.OF FIRMS	%	CUM. %	ISRAEL NO.OF FIRMS	%	CUM. %	JAPAN NO.OF FIRMS	%	CUM %	YUGOSLAVIA NO.OF FIRMS	%	CUM %
0-	10				1	1	1	1	1	1	2	3	3			
10-	25	3	5-	5	2	2	3	2	3	4	7	11	14			
25-	50	3	5-	10	2	2	4	10	14	19	8	13	27			
50-	100	7	11-	21	14	12	16	12	17	36	18	28	55	2	1	1
100-	250	14	22-	43	24	21	37	27	39	75	19	30	84	15	10	12
250-	500	17	27-	70	22	19	50	12	17	93	7	11	95	22	15	27
500-	1000	11	17-	87	19	16	72	5	7	100	3	5	100	46	32	59
1000-	2000	6	10-	97	16	14	85							44	30	89
2000-	3000	2	3-	100	7	6	91							9	6	95
3000-	4000				3	3	94							3	2	97
4000-	5000				2	2	96							2	1	99
5000-	10000				5	4	100							2	1	100

DIR.LAB.IN SHIFT ONE		FRANCE NO.OF FIRMS	%	CUM. %	INDIA NO.OF FIRMS	%	CUM. %	ISRAEL NO.OF FIRMS	%	CUM. %	JAPAN NO.OF FIRMS	%	CUM. %	YUGOSLAVIA NO.OF FIRMS	%	CUM %
0-	10							1	1	1						
10-	25	6	10	10	2	2	2	8	12	13	5	8	8			
25-	50	7	11	21	5	4	6	11	16	29	8	13	20	3	2	2
50-	100	6	10	30	20	17	23	23	33	62	18	28	48	5	3	6
100-	250	23	37	67	31	26	50	20	29	91	20	31	80	29	20	26
250-	500	12	19	86	29	25	74	6	9	100	10	16	95	44	31	55
500-	1000	5	8	94	15	13	87				3	5	100	44	31	87
1000-	2000	4	6	100	9	8	95							15	10	97
2000-	3000				4	3	98							1	1	98
3000-	4000				1	1	99							3	2	100
4000-	5000				1	1	100									

DIR.LAB.IN SHIFT TWO	FRANCE NO.OF FIRMS	%	CUM %	INDIA NO.OF FIRMS	%	CUM %	ISRAEL NO.OF FIRMS	%	CUM %	JAPAN NO.OF FIRMS	%	CUM %	YUGOSLAVIA NO.OF FIRMS	%	CUM %
0- 10	1	4	4	1	1	1	18	17	17	6	27	27	1	1	1
10- 25	3	13	17	6	7	8	14	29	46	4	18	45	6	4	5
25- 50	3	13	30	8	9	17	10	21	67	6	27	73	8	6	11
50- 100	4	17	48	15	17	34	9	19	85	2	9	82	16	12	22
100- 250	8	35	83	22	25	59	7	15	100	4	18	100	53	38	61
250- 500	3	13	96	16	18	77							38	28	88
500- 1000	1	4	100	12	14	91							13	9	98
1000- 2000				4	5	95							3	2	100
2000- 3000				3	3	99									
3000- 4000				1	1	100									

DIR.LAB.IN SHIFT THREE	FRANCE NO.OF FIRMS	%	CUM %	INDIA NO.OF FIRMS	%	CUM %	ISRAEL NO.OF FIRMS	%	CUM %	JAPAN NO.OF FIRMS	%	CUM %	YUGOSLAVIA NO.OF FIRMS	%	CUM %
0- 10	1	6	6	1	2	2	11	27	27	2	22	22	7	6	6
10- 25	3	19	25	5	8	10	13	32	60	3	33	56	17	14	20
25- 50	3	19	44	7	12	22	8	20	80	1	11	67	22	19	39
50- 100	5	31	75	10	17	38	7	17	97				21	18	57
100- 250	4	25	100	12	20	58	1	2	100	3	33	100	38	32	89
250- 500				14	23	82							12	10	99
500- 1000				6	10	92							1	1	100
1000- 2000				4	7	98									
2000- 3000				1	2	100									

% MOTOR OPER.SH.1		FRANCE NO.OF FIRMS	%	CUM %	INDIA NO.OF FIRMS	%	CUM %	ISRAEL NO.OF FIRMS	%	CUM %	JAPAN NO.OF FIRMS	%	CUM %	YUGOSLAVIA NO.OF FIRMS	%	CUM %
40-	50	1	4	4	3	3	3	3	5	5				1	1	1
50-	60	1	4	7	5	5	9	3	5	10						
50-	70	2	7	14	6	7	15	5	8	19	5	19	19	4	3	3
70-	80	5	18	32	13	14	30	10	17	36	4	15	35	9	6	10
80-	90	8	29	61	23	25	55	15	25	61	13	50	85	4	3	12
90-	100	11	39	100	41	45	100	23	39	100	4	15	100	127	88	100

%MOTOR OP.SH.2		FRANCE NO.OF FIRMS	%	CUM %	INDIA NO.OF FIRMS	%	CUM %	ISRAEL NO.OF FIRMS	%	CUM %	JAPAN NO.OF FIRMS	%	CUM %	YUGOSLAVIA NO.OF FIRMS	%	CUM %
0-	10							1	2	2	1	7	7	6	4	4
10-	20				2	3	3	1	2	4	2	14	21	4	3	7
20-	30				4	5	8				2	14	36	11	8	15
30-	40				7	9	18	4	9	13	1	7	43	16	12	27
40-	50	1	9	9	6	8	26	7	15	28				10	7	34
50-	60	1	9	18	8	11	36	7	15	43				12	9	42
60-	70				10	14	50	5	11	54	3	21	64	7	5	47
70-	80				14	19	69	3	7	61	1	7	71	10	7	55
80-	90	6	55	73	12	16	85	7	15	76	1	7	79	24	17	72
90-	100	3	27	100	11	15	100	11	24	100	3	21	100	39	28	100

		FRANCE			INDIA			ISRAEL			JAPAN			YUGOSLAVIA		
% MOTOR OP.SH.3		NO.OF FIRMS	%	CUM. %	NO.OF FIRMS	%	CUM. %	NO.OF FIRMS	%	CUM. %	NO.OF FIRMS	%	CUM. %	NO.OF FIRMS	%	CUM %
0-	10				1	2	2	3	8	8				27	23	23
10-	20				7	13	15	1	3	11	1	25	25	18	15	38
20-	30	1	20	20	4	8	23							7	6	44
30-	40				6	11	34	5	13	24				4	3	47
40-	50				5	9	43	4	11	34				9	8	55
50-	60				5	9	53	5	13	47	1	25	50	4	3	58
60-	70				7	13	66	3	8	55	1	25	75	6	5	63
70-	80	1	20	40	10	19	85	3	8	63	1	25	100	8	7	70
80-	90	1	20	60	1	2	87	7	18	82				16	13	83
90-	100	2	40	100	7	13	100	7	18	100				20	17	100

		FRANCE			INDIA			ISRAEL			JAPAN			YUGOSLAVIA		
BREAKEV. POINT.		NO.OF FIRMS	%	CUM. %	NO.OF FIRMS	%	CUM. %	NO.OF FIRMS	%	CUM. %	NO.OF FIRMS	%	CUM. %	NO.OF FIRMS	%	CUM %
0-	10															
10-	20				1	1	1				1	2	2	1	1	1
20-	30				5	4	5				1	2	3	6	4	5
30-	40	1	2	2	11	9	15							32	22	27
40-	50	2	3	5	14	12	26				4	6	9	29	20	47
50-	60	2	3	8	26	22	49	4	6	6	7	11	20	33	23	70
60-	70	9	15	23	20	17	66	10	14	20	10	16	36	22	15	85
70-	80	9	15	38	25	21	87	12	17	38	20	31	67	17	12	97
80-	90	15	25	62	6	5	92	23	33	71	16	25	92	4	3	99
90-	100	23	38	100	7	6	98	20	29	100	3	5	97	1	1	100
100-	125										2	3	100			
125-	150				2	2	100									

NUMBER OF YEARS	FRANCE NO.OF FIRMS	%	CUM %	INDIA NO.OF FIRMS	%	CUM %	ISRAEL NO.OF FIRMS	%	CUM %	JAPAN NO.OF FIRMS	%	CUM %	YUGOSLAVIA NO.OF FIRMS	%	CUM %
0- 5	6	10	10	23	20	20	19	28	28	7	11	11			
5- 10	6	10	19	15	13	32	13	19	46	11	17	29	15	10	10
10- 20	3	5	24	31	26	59	18	26	72	24	38	67	23	16	26
20- 30	6	10	33	19	16	75	14	20	93	13	21	87	28	19	46
30- 40	11	17	51	10	9	84	4	6	99	3	5	92	28	19	65
40- 50	11	17	68	9	8	91	1	1	100	5	8	100	9	6	71
50- 75	13	21	89	7	3	95							5	3	74
75- 100	2	3	92	8	5	100							23	16	90
100- 150	5	8	100										13	9	99
150- 200													1	1	100

AGE OF MACHINERY	FRANCE NO.OF FIRMS	%	CUM %	INDIA NO.OF FIRMS	%	CUM %	ISRAEL NO.OF FIRMS	%	CUM %	JAPAN NO.OF FIRMS	%	CUM %	YUGOSLAVIA NO.OF FIRMS	%	CUM %
0- 2	1	3	3	1	2	2	2	7	7	3	6	6	5	3	3
2- 5	11	35	39	13	21	23	8	28	34	26	51	57	16	11	14
5- 10	13	42	81	23	38	61	15	52	86	21	41	98	121	83	98
10- 15	3	10	90	11	18	79	3	10	97	1	2	100	3	2	100
15- 20	3	10	100	5	8	87									
20- 25				5	8	95	1	3	100						
25- 30				2	3	98									
30- 40				1	2	100									

CAPACITY UTILISATN.		FRANCE NO.OF FIRMS	%	CUM. %	INDIA NO.OF FIRMS	%	CUM. %	ISRAEL NO.OF FIRMS	%	CUM. %	JAPAN NO.OF FIRMS	%	CUM. %	YUGOSLAVIA NO.OF FIRMS	%	CUM %
10-	20				1	1	1	1	1	1						
20-	30							1	1	1				1	1	1
30-	40				4	3	4	9	13	17				1	1	1
40-	50	2	3	3	6	5	9	7	10	28				2	1	3
50-	60	1	2	5	4	3	13	6	9	36				8	6	8
60-	70	2	3	8	9	8	21	9	13	49				20	14	22
70-	80	13	21	29	24	21	41	16	23	72	14	22	22	34	23	46
80-	90	18	29	57	26	22	63	4	6	78	23	36	58	41	28	74
90-	100	27	43	100	43	37	100	15	22	100	27	42	100	38	26	100

NUMBER OF SHIFTS.	FRANCE NO.OF FIRMS	%	CUM. %	INDIA NO.OF FIRMS	%	CUM. %	ISRAEL NO.OF FIRMS	%	CUM. %	JAPAN NO.OF FIRMS	%	CUM. %	YUGOSLAVIA NO.OF FIRMS	%	CUM %
1-	41	65	65	31	26	26	21	30	30	43	67	67	6	4	4
2-	6	10	75	26	22	49	8	12	42	12	19	86	20	14	18
3-	16	25	100	60	51	100	40	58	100	9	14	100	119	82	100