Composite models for quarks and leptons

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COMPOSITE MODELS FOR QUARKS AND LEPTONS

by

Y A M I N A   B O U G U E N A Y A

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CONTENTS

INTRODUCTION

A long time ago it has been suggested that matter was made of basic constituents. This view has always influenced the study of matter.

The common rule is to seek simple and economic theories. In particular, economy and elegance have proved very helpful to a better understanding of the physical world. Thus any overabundance of fundamental object is disrupting; it seems that this is the way things are in nature.
Indeed, a spectral distribution of masses or energy levels often convey a manifestation of a compound physical system. The periodic table reflects this principle. Atoms which were thought to be fundamental, also appeared as a composite system of nuclei and electrons.

Moreover, in 1968 electron scattering experiments (SLAC) gave the first hint that point-like objects existed inside the protons, "the partons". Hence, confirming the proposition of Gell-Mann and Zweig (1964) that proton and other "elementary" particles were made from more basic entities, the quarks.

Later on, the quark model contributed to the success of the unification of the weak, strong and electromagnetic interactions in the same theory, the standard model. However, some problems required the extension of the group of quarks and leptons of the standard model which causes an increase in the number of fundamental entities.

Again a plethora is threatening, yielding the inevitable question: "is there a limit to this increase or is there some internal structure, a new spectroscopy?" Many attempts have been made to answer these questions and many views have been proposed, among them the compositeness of quarks and leptons which is rather a conventional answer.

The purpose of this dissertation is to actually describe this topic. The main ideas of composite models are more or less described.
First of all, the standard model is reviewed in the first chapter. In the

second chapter, the problems are brought out to see what motivated the search for grand unified models. The latter attempt to describe weak, electromagnetic and strong interactions but do not provide answers to all the existing problems. In particular, the spectrum of quarks and leptons remains unexplained.

It seems that this pattern (of observed fermionic masses) can be reproduced if the quarks and leptons are assumed to be made of more fundamental objects "the preons".

The number would be much smaller than the actual number of quarks and leptons. However there are many limitations and restrictions to this idea as it will be seen in the third chapter.

The fourth chapter deals with an overall and general study of different composite schemes.

Then, in the fifth chapter, problems common to all varieties of composite models are exposed. They consist mainly of 't Hooft conditions.

In the sixth and last chapter, the rishon model is exposed. It illustrates all the previous chapters and constitutes a concrete example of the ideas discussed by 't Hooft and reviewed in chapter five.

CHAPTER 1

THE STANDARD MODEL

1.1 Description

Although the basic phenomenology of low energy weak interactions has been known for a considerable time, a satisfactory theory was missing. The weak interactions were described by a current-current Fermi interaction (four-point interaction) lagrangian,

$$\mathcal{L}_{int} = -G_w/\sqrt{2}\ J^{\mu}(x) J_{\mu}^{\dagger}(x)$$

This was not, however, a complete theory since it was not renormalisable. It was successful only in lowest order where, due to the smallness of the weak interaction coupling constant Gw, the lagrangian could be used as an effective lagrangian involving a tree diagram and ignoring the divergent loops. Moreover, even considering lowest orders only (i.e. first order), the model yields a lack of unitarity. For instance, the evaluation of the cross-section for $e\nu$-scattering at high energies obtained by using Born approximation, is not sufficient; higher order terms must be added so that the scattering amplitude satisfies unitarity. In more detail the cross-section for $e\nu$-scattering is given by

$$d\sigma = \frac{1}{64\pi^2} |M_{fi}|^2 \frac{|p'|}{|p| \mathcal{E}^2} d\Omega$$

where the amplitude M_{fi} is

$$M_{fi} = -\frac{G}{\sqrt{2}}\left[\bar{u}(p')\gamma^{\alpha}(1+\gamma^5)u(p)\right]\left[\bar{u}(k')\gamma_{\alpha}(1+\gamma_5)u(k)\right]$$

where (p,k), (p',k') are the initial and final momenta respectively.
In an elastic scattering $|p| = |p'|$

and so

$$d\sigma = \frac{1}{64\pi^2} |M_{fi}|^2 \frac{d\Omega}{\mathcal{E}^2}$$

and finally, the total cross-section is

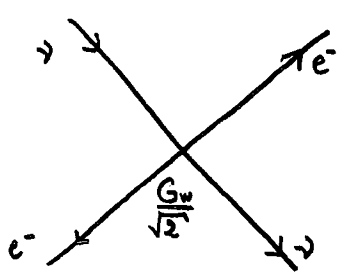
$$\sigma = \frac{4}{\pi} G^2 s$$

where s is the energy squared of the centre of mass, i.e. $s = (p_e + p_{\nu_e})^2$
But this is an s-wave process (due to the local character of the weak interaction). The cross-section σ only contributes to the $\ell = 0$ partial wave and hence by unitarity $\sigma \leq cste/s$, i.e. it cannot be greater than s^{-1}.
Therefore, the theory must fail at the "unitarity limit" $s \sim G^{-1}$ where σ violates unitarity.

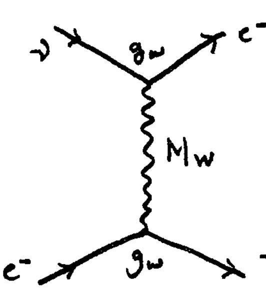
The situation has been improved by introducing massive vector mesons W_μ to mediate the weak interactions between currents.
The lagrangian became then

$$\mathcal{L}_{int} = - g_w \left(J^\mu W_\mu + J^\mu W_\mu^+ \right)$$

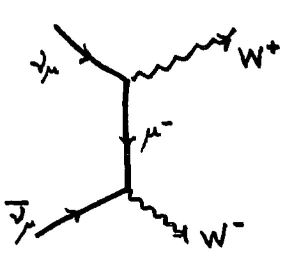
and



is replaced by



This modification does avoid the violation of unitarity in lowest order, at least for the $e\nu$-scattering process. But although it gives different results at higher energies it does not ensure that the unitarity bound is not violated for any Born process, for instance processes involving "external" W particles, e.g.,



$$\nu_\mu \bar{\nu}_\mu \to W^+ W^-$$

Hence the theory remained unrenormalisable.

Quite recently, theories possessing nice symmetry properties, the gauge theories[1] have known an active development.
In these theories conservation laws are not consequences of space-time symmetries.
For every quantum number, there corresponds a transformation on the fields which leaves the theory invariant. In the simple case of electromagnetism, the group of transformation on the fields is the abelian group U(1).
However, a theory may contain more than one conserved quantity and be invariant under a larger group of transformations than U(1). In 1954, Yang and Mills introduced SU(2), the group of isospin which is also the simplest non-abelian group. In a theory with isospin symmetry, there are no preferred directions in the fictitious isotopic space. Imagine that at each point of space time there is a set of axes in the isotopic space which define the isospin properties of a particle located at that space-time point. When all the axes are parallel, it is a global gauge invariance (or a transformation of the first kind in the case of EM).
The symmetry properties of the theory are improved if it is invariant when the axes are rotated independently. This is a local gauge transformation (or a transformation of the second kind in the case of EM).
This idea has been generalised to other internal symmetries. Such theories, which are locally invariant under internal symmetries are known as Yang-Mills theories or non-abelian gauge theories because the gauge groups are non-abelian. The gauge fields do not commute, they generate a Lie algebra.

As any gauge theory, Yang-Mills theories must contain interactions mediated by particles of spin 1, the gauge bosons. But, there was a sort of flaw in these theories; the Yang-Mills gauge particles have to be massless and apparently, except for the photon there were no other massless spin 1 particles. Yet, a subtle way out to this problem has been found. The non-vanishing masses of the gauge bosons have been attributed to the non-exact nature of the gauge invariance of the group concerned. The gauge invariance is said to be a broken symmetry.

To account for this breakdown, it is assumed that the vacuum is not invariant under the gauge group, for a simple addition of mass terms in the fundamental lagrangian would spoil the renormalisation of the theory leading to infinities in pertubative calculations.

That situation is called spontaneous symmetry breaking. The implementation of spontaneous symmetry breaking also requires a trick, the Higgs mechanism which is called after its inventor - P. Higgs as its name indicates.

The Higgs mechanism postulates the existence of additional new fields. It cures the gauge theories from the presence of threatening massless particles, the Goldstone bosons. In fact, the would-be Goldstone bosons combine with the would-be massless gauge bosons to produce massive gauge bosons.

The fruit of all these developments appeared in the form of models published by Weinberg and by Salam in 1968. However the flourishing of these models had to wait until 1971, when the quantisation and renormalisation of Yang-Mills theories have been established by G. 't Hooft[2]. Indeed, after many years of studies by many people, it has been realised that the spontaneous breakdown of the symmetry did not affect the divergences of the theory so that the same renormalisation procedure (counter-terms) remove the divergences from the theory whether the symmetry is spontaneously broken or not.

The Salam-Weinberg model[1] as well as many other models[3] proposed, unifies weak and electromagnetic interactions in the simplest gauge group. Furthermore, it constitutes the simplest form of the electro-weak theory according to experiments and thus it has been retained as the standard model. In this model, electro-weak interactions are described by the gauge group SU(2) x U(1). Four gauge particles are associated with SU(2) x U(1), the photon, the charged $W^{\pm}$ and a neutral vector meson.

A gauge theory is constructed out of SU(2) x U(1). It involves a triplet of gauge fields A_μ for SU(2) with charge g and a field B_μ for U(1) with a coupling constant g' .

The lagrangian reads

$$\mathcal{L}_0 = \mathcal{L}_{leptons} + \mathcal{L}_{gauge}$$

$$\mathcal{L}_{leptons} = \bar{\Psi}_L i\gamma^\mu (\partial_\mu + \frac{i}{2} g \underline{\tau} \cdot \underline{A}_\mu - \frac{i}{2} g' B_\mu) \Psi_L$$
$$+ \bar{e}_R i \gamma^\mu (\partial_\mu - i g' B_\mu) e_R$$

$$\mathcal{L}_{gauge} = - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} G_{\mu\nu} G^{\mu\nu}$$

with

$$F_{\mu\nu} = \partial_\mu \underline{A}_\nu - \partial_\nu \underline{A}_\mu - g \underline{A}_\mu \wedge \underline{A}_\nu$$
$$G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu .$$

There is no electron mass terms because it is forbidden by the chiral SU(2) invariance requirement.

The leptons have been grouped in doublets, eg. $\Psi = \begin{pmatrix} \nu_e \\ e_L \end{pmatrix}$ and singlets, e.g. $e_R = \frac{1}{2}(1+\gamma_5)e$.

Indeed, the standard model separates right and left handed particles into right handed singlets and left handed doublets. This is because neutrinos are massless and occur in nature only in left handed form and also because electromagnetism conserves parity whereas weak interactions do not.

To end up with three massives vector mesons and one massless boson (the photon), a doublet of complex Higgs (four degrees of freedom) is introduced, $h = \begin{vmatrix} h^+ \\ h_0 \end{vmatrix}$.

The addition of these scalar fields to the gauge theory with fermions originates three new terms in the lagrangian:

i. kinetic term of the type $|D_\mu h_i|^2$ with

$$D_\mu = \partial_\mu - i g\, G_i A^i_\mu$$

where g is the coupling constant and G_i the infinitesimal generators of the gauge group which contains the scalars.

ii. Yukawa interactions between the scalar fields and the fermions

$$f_{ijk} \bar{\Psi}_{iL} h_j \Psi_{kR} + h.c.$$

iii. Self-interactions of the scalar fields, denoted $V(h_i)$ and known as Higgs potential. A general renormalisable potential is

$$V(h^+h) = \mu^2 h^+ h + \lambda (h^+ h)^2$$

It determines the scale of the theory and the structure of the vacuum. Vacuum exceptation value at lowest order is given by,

$$\langle h_i \rangle = \lambda_i$$

with $\left.\frac{\partial V}{\partial h_i}\right|_{h_i=\lambda_j} = 0$ and $\left.\frac{\partial^2 V}{\partial h_i \partial h_j}\right|_{h_k=\lambda_k} < 0$

so $$\mathcal{L}_{Higgs} = (D_\mu h)^\dagger (D_\mu h) - V(h^\dagger h) - G_e \left[\bar{\Psi} e_R h + h^\dagger e_R \Psi_L\right]$$

where one electron only, is considered.

After performing a parametrisation and the SU(2) gauge transformation, the theory ends up with massive fermions. The electrons acquire masses and so do the charged vector fields $W^\pm$ and the neutral field Z. There remains one massless field A_μ, it corresponds to the physical photon. They are defined by

$$W_\mu^\pm = \frac{1}{\sqrt{2}} \left(A_\mu^1 \mp i A_\mu^2\right)$$

$$Z_\mu = \left(g^2 + g'^2\right)^{-1/2} \left(-g A_\mu^3 + g' B_\mu\right)$$

$$A_\mu = \left(g^2 + g'^2\right)^{-1/2} \left(g B_\mu + g' A_\mu^3\right)$$

The electromagnetic coupling to the photon A_μ is defined as

$$e = g' \cos\theta_W = g \sin\theta_W$$

where θ_W is the Weinberg angle.

The theory requires large masses for the W's, $M_W = (78 \pm 2)$ GeV, the Z^o is even heavier, $M_Z = (89 \pm 2)$ GeV.

These values result from the relations

$$M_W \simeq 37.3/\sin\theta_W \quad \text{GeV}$$

$$M_Z = M_W/\cos\theta_W$$

together with the experimental value for θ_W, $\sin^2\theta_W \simeq 0.2$.

In fact, weak interactions are suppressed with respect to electromagnetism up to energies sufficiently high to create the intermediate bosons as real particles. At energies of order M_W, the weak and electromagnetic interactions eventually become of comparable strength, thus manifesting unity.

But in general, exchange particles could have only an intermediary role and do not appear in initial and final states. They are "off the mass shall", i.e. virtual particles and contribute even in reactions where there is not enough energy to create a real particle, "on the shell mass".

The success of the unification of weak and electromagnetic interactions lead to a further unification with strong interactions at the level of quarks since they are at present the fundamental constituents of hadrons. As quarks come in three colours (statistics requirement) the gauge group is extended to $SU(3)_c \times SU(2) \times U(1)$.

The quarks are organised into left handed doublet $N_L = \begin{pmatrix} u \\ d_\theta \end{pmatrix}$

where $d_\theta = d \cos\theta_c + s \sin\theta_c$,

righthanded singlets U_R, d_R and s_R and a left handed singlet $s_{\theta L} = (s \cos\theta_c - d \sin\theta_c)_L$.

From the interaction lagrangian rewritten in terms of the fields $W^\pm$, Z and A, it can be seen that the neutral boson Z couples to the electromagnetic current and to a current $\bar{N}_L \tau_3 \gamma_\mu N_L$. The latter includes terms in $\sin\theta_c \cos\theta_c$.

Thus it causes problems because $\Delta S \neq 0$ together with $\Delta Q = 0$ currents are very suppressed.

To illustrate this, let us consider a type of strange mesons, the K-mesons. They frequently decay in processes like ,

$$K^+ \longrightarrow \mu^+ + \nu_\mu \,.$$
$$K^+ \longrightarrow \pi^0 + \mu^+ + \nu_\mu \,.$$

These decays involve charged currents since a charged μ combine with a neutral ν_μ.

A neutral current decay would correspond to

$$K^+ \longrightarrow \pi^+ + \nu_\mu + \bar{\nu}_\mu \,,$$

where the leptons are both neutral. However this decay does not occur in nature.

The problem was finally solved by Glashow-Ilipoulos and Maiani[4] (1970) by use of a trick called G-I-M mechanism. It consists in reordering the quarks in analogy with the leptonic pattern.

A fourth quark "charm", is introduced to make the left-handed doublets and righthanded singlets symmetrical in both leptonic and quark sectors.

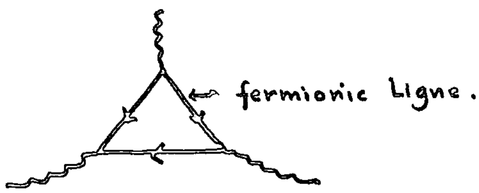
The left-handed singlet $s_{\theta L}$ becomes a doublet $\begin{pmatrix} c \\ s_{\theta L} \end{pmatrix}$.

This made the strangeness changing currents cancel and yielded an anomaly-free theory.

Indeed if the symmetry of the theory is violated, i.e. if it contains anomalies, the renormalisability of the gauge theory is spoilt.

Ward-Takahashi identities must hold in order to have renormalisable theories. Anomalies[5] in these identities may occur in theories containing fermions, when the verification of these identities depends on the algebra of Dirac matrices $(\gamma^{\alpha}\gamma_5)$. This happens when a vertex with odd number of axial currents (with γ_5) cannot be regularised. The W-T identities are then broken. Such anomalies occurring in spontaneously broken gauge theories threaten the unitarity of the S-matrix. Therefore, they must be absent or cancel between each other.

The simplest anomalies are associated with the vertex of three currents,



called the anomaly triangle.

The absence of such triangles from the theory guarantee its renormalisability. This is a result of two lemmas :

i. Anomalies are not "renormalised", i.e. if there is no anomaly in lowest order there will not be any at all orders.

ii. All anomalies are related, i.e. if the simplest anomaly (triangle) is absent in a model, so are all the others.

1.2 General Features of the Standard Model

1.2.a. Cabibbo angles and fermion masses

Although the standard model[1] has had a lot of success in correlating electromagnetic and weak interaction data, it fails to give any explanation of the Cabibbo angles and the fermion masses.

In the framework of the standard model, the quarks and leptons come in left-handed doublets of the "weak isospin". Three such doublets are known at present.

The weak interactions of the quarks are described by the weak interaction mixing angles[6]. These angles arise since the weak interaction eigenstates are not eigenstates of the mass matrix. They are intimately related to the quark masses and could be regarded as elements of the mass matrix.

Let the three quark doublets be $\begin{pmatrix} u & c & t \\ d' & s' & b' \end{pmatrix}$,

the physical quark eigenstates d', s', b' are related to the mass eigenstates d, s, b.

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = A \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

A is a 3 x 3 transformation matrix such as

$$A = \begin{pmatrix} c_1 & s_1 c_3 & -s_1 c_3 \\ -s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 c_2 & -c_1 s_2 c_3 - c_2 s_3 e^{i\delta} & -c_1 s_2 s_3 + c_2 s_3 e^{i\delta} \end{pmatrix}$$

where $c_i(s_i) = \cos\theta_i \ (\sin\theta_i) \qquad i = 1, 2, 3.$

The three Euler angles $\theta_{1,2,3}$ generalise the conventional Cabibbo angle, giving all mixings between the (u, d) ; (c, s) and (t, b) doublets. The fourth parameters in A is the phase δ known as the Kobayashi-Maskawa phase. It induces CP-violation[7]. In general, there are three ways to account for CP-violation in Yang-Mills gauge theories with spontaneously symmetry breaking:

i. by introduction of complex phase parameter in the interactions of Higgs bosons;

ii. by introduction of right-handed charged current;

iii. by the increase of the number of quarks.

Taking into account the third possibility, the sixth flavor of quarks has been introduced. Indeed, the presence of the third generation permit to put a phase in the A-matrix leading to CP-violation.

An attractive aspect of the matrix A is the way different and apparently unrelated processes are determined by the four parameters θ_i and δ. These processes should measure the A-matrix elements but experimentally only θ_1 is known.

The weak mixing angles have no physical meaning in the leptonic sector. Indeed, if neutrinos are massless there are no mass-eigenstates and so no transformation matrix (analog of A). In the case of neutrinos with small masses, Cabbibo-like angles and KM-phase are involved. The effects due to the weak interaction mixing are confined to the phenomenon of the neutrino oscillations. The actual limits on neutrino masses are $m(\nu_e) < 6 \cdot 10^{-5}$ Mev; $m(\nu_\mu) < 0.5$ Mev; $m(\nu_\tau) < 200$ MeV . Massive neutrinos will make the theory even more complicated; increase in the number of free parameters, an additional mass scale to the mass hierarchy fermions, etc.

1.2.b. Higgs sector

Model independent analyses of neutral current data as well as the SLAC polarised electron-scattering experiment have verified a great deal of the predictions of the standard model. However, the data give information only about the symmetry nature of the neutral current and its relative strength to the charged current. It says nothing about Higgs bosons. The facts are in agreement with the model but do not prove the spontaneously broken symmetry nature of gauge theories.

In the standard model, it is necessary to introduce scalar fields "the Higgs scalars", to provide with masses the participating fermions and bosons. To some authors, this is a rather awkward explanation. They think that better methods should be found to account for the masses.

This is the case of Dimopoulos and Susskind[8] who built a model with dynamical symmetry breaking which induces fermion and boson masses.

In the standard model with left-handed doublets and right-handed singlets, the only scalar fields which induce fermion mass terms are SU(2) doublets. If non-doublet scalars are present in the theory, they do not lead Yukawa couplings and hence do not affect the fermion mass spectrum. However after symmetry breaking (VEV for neutral component of scalar multiples), their kinetic terms would affect the gauge boson spectrum. An SU(2) multiplet scalar does not contribute to the neutral boson mass and thus spoils the relation

$m_Z \cos\theta_W = m_W$.

This exclusion of all scalar multiplets except SU(2)-doublets follows from the actual available neutrino data. However, the number of scalar doublets is left arbitrary.

1.3 The Generation Puzzle

The observed spectrum of quarks and leptons shows a definite order. Quarks and leptons seem to come in generations. Except from masses, nothing else distinguishes among generations.

The fermions are classified into generations according to their masses[9],

1st	$\begin{pmatrix} \nu_e & u \\ e^- & d \end{pmatrix}$	lighter than 100 MeV.
2nd	$\begin{pmatrix} \nu_\mu & c \\ \mu^- & s \end{pmatrix}$	masses larger than 100 and smaller than 1500 MeV.
3rd	$\begin{pmatrix} \nu_\tau & t \\ \tau^- & b \end{pmatrix}$	heavier than 1500 MeV.

Several attempts have been made in order to give explanations to the generation puzzle, i.e. what distinguishes the fermionic generations.

Excluding Higgs couplings, the gauge lagrangian for the three generations possesses a global U(3) symmetry (among the generations). This symmetry breaks down when the fermions acquire their masses. The breaking can occur via several ways. Let us review some of them:

i. a "horizontal gauge symmetry"[10] U(1) is proposed to distinguish two fermionic generations.

The main assumptions are:

i.a) a hypercharge is assigned to the horizontal gauge groups Y', Y being the usual hypercharge of the standard model,

$$Y = \lambda_i y \ , \quad Y' = \lambda_i' y' \ ;$$

y and y' are quantum numbers in a given generation and λ_i and λ_i' are scale parameters called "seriality" of the i-th generation.

i.b) the model is anomaly free. The anomalies are cancelled within each generation in the Weinberg-Salam model. They might not vanish in the presence of the additional horizontal group but the overall anomaly vanishes.

i.c) a Higgs doublet is associated to each generation. There exists a Higgs singlet (zero W-S hypercharge) with vacuum expectation value such as the related mass M(Z') is made as heavy as wanted. Z' is the analog of Z the gauge boson associated with U(1) the horizontal group.

In this scheme three generations is not the end but suggests a fourth one and so on.

Horizontal gauge bosons (Z') must exist, but they induce undesirable features in the weak neutral current sector.

The major problem is that of anomalies. For more than two generations, complications arise. There is only one relation giving the "seriality" numbers

$$\sum_i \lambda_i' = \sum_i \lambda_i'^3 = 0$$

λ_i is not determined uniquely if more than two generations are involved. The model become than arbitrary. The anomaly constraints for Y and Y' cannot guarantee W-S universality any more.

ii. Another possibility[11] is the incorporation of some flavor symmetry into the unified gauge group along with SU(5) structure or equivalently incorporation of several generations in one "dynasty multiplet".

iii. A third alternative is "discrete symmetries". To each quark, lepton and Higgs field transforming under a discrete transformation, e.g. phase transformation, there corresponds an unbroken discrete symmetry. These symmetries attempt to relate fermion masses and generation mixing angles in order to understand the fermionic spectrum. However, this does not enlighten the description of the different generations.

Weinberg's model, for instance, induces that some masses of the first family can be obtained as a kind of radiative connection. Another interesting model[12] suggests that the different scales observed among generations come from the VEV of three distinct Higgs multiplets. The scale problem is then intrinsic, not accidental. This situation is absolutely plausible. One doublet scalar is indeed simpler but, on the other hand, with the proliferation of quarks and leptons, it can be argued that the scalars could also appear in several generations.

The conclusion is that no realistic model exists yet and that a lot more work needs to be done to be able to understand the spectrum of quarks and leptons. Therefore, the generation puzzle remains unsolved.

CHAPTER 2

COMPOSITE MODELS: motivations and alternatives

2.1 Parameters and Particles in the Standard Model

The standard model[1] $SU(3)_C \times SU(2) \times U(1)$ is compatible with all known facts and experimental results of particle physics. However it has been established only at low energies $\sim$ 100 GeV. It is probably an effective model at much higher energies. Whether this is right or not cannot be predicted since even some main ingredients of the standard model have not been verified yet because still out of reach of available energies. But soon, facilities will be provided with enough energy to produce them ($W^{\pm}$, ...).

Furthermore, the standard model is somewhat arbitrary; the three interactions are not truly unified in the sense that the model contains only one gauge coupling constant.
The family problem remains open as well as parity which is violated in weak interactions but not in strong interactions. The charge quantisation is unexplained, etc...

Due to this arbitreness, there are many free parameters in the theory; the fermion masses, the mixing angles, CP-violating phase etc... . They are summarised in table[6](1). Add to this, the proliferation of fundamental particles which reinforce the idea that the standard model is not the ultimate theory.

In the early 60's a fourth lepton ν_μ has been discovered and for the sake of esthetical analogy, a search for a fourth "fundamental baryon" has been proposed. Later on, in 1970 speculations on a fourth quark became necessity[4]. It has been pointed out that a simple gauge theory of weak interactions must have neutral current and that the absence of strangeness changing neutral currents can be reconciled with the presence of strangeness conserving netural currents (i.e. $|\Delta S| = 1$ neutral currents are not observable) only if a fourth quark is added. Thus providing a theoretical framework for

sector	particles	parameters
Gauge bosons	eight gluons photon (γ) $W^{\pm}, Z$	(α_{st} or Λ) (α, θ_W or g_1, g_2) M_W, M_Z
fermions	i. u, d, ν_e, e ii. c, s, ν_μ, μ iii. t, b, ν_τ, τ iv. ?	- left doublets, right singlets - number of generations - six quark masses - three charged lepton masses - three generalised Cabbibo angles - one Kobayashi-Maskawa phase - three leptonic Cabbibo angles - one leptonic K-M phase quark - lepton angles and phases
Higgs particles	ϕ more?	Number of Higgs particles Classification of Higgs particles Higgs masses (couplings)

table (1) Parameters in the Standard Model the search for the fourth quark.

Two years later[13], it became clear that the fourth quark or "charm" was needed to equalise the number of (left-handed) quark doublets and lepton doublets, to cancel the anomalies in the standard model.
Note that, the cancellation of anomalies require that for each new quark there has to be a new lepton and vice-versa. Moreover, CP-violation was cured by addition of quarks and consequently of leptons too. This, of course, means more parameters in the theory: generalised Cabibbo angles, K-M phase, new fermion masses.

2.2 Higgs Scalars

The Higgs mechanism has been well established; it gives a sort of explanation of how some particles are provided with masses. Nevertheless, many people object to the introduction of Higgs particles in any theory. Arbitrary free parameters are introduced into the theory through the Higgs potential and the Yukawa couplings. Hence, with the simplest Higgs structure (i.e. one doublet) and three fermion generations, the standard model is left with fourteen fundamental parameters outside the gauge sector:

m_H the physical mass of the scalar

nine fermion masses (neutrinos are assumed to be massless) and

three mixing angles plus one K-M phase.

The Higgs sector proves to be quite complicated and is not understood; the electron and τ-lepton for example have the same gauge couplings but their coupling to the Higgs scalar differ by $\sim$ 3000 factor!

In grand unified theories[14], the gauge sector is simplified and the different coupling constants are related to each other. However, this causes further complication of the Higgs sector. To guarantee the renormalisability of the theory, and according to Susskind and Wilson interpretation[15] a cut-off K of order of the Planck mass ($\sim 10^{19}$GeV) is required.

This huge mass put a strict constraint on the accuracy of the bare mass which must be of order 10^{-30}, i.e. 30 decimals. Such an adjustment is not "natural". The mass scale has to be put "by hand" and the theory ends up with very heavy particles ($\sim 10^{15}$ GeV), this is the hierarchy problem.

Let us illustrate this with an example. The self-energies of the scalars are quadratically divergent; the renormalised and unrenormalised scalar masses are related by

$$m^2 = m_0^2 + K^2 g_0^2 = m_0^2 + \Delta m^2$$
$$\mu_0 = m_0 / K$$

where μ_0 is the bare mass, m the physical mass and K the ultra-violet cut - off.

This implies

$$\mu_0^2 = \frac{m^2}{K^2} - g_0^2$$

In Grand Unified Theories, superheavy vector bosons (the leptoquarks) are needed to suppress unobserved interactions (see later section 2.3). They arise in the spontaneous breakdown to the observed interactions,

$$G \text{ (e.g., } SU(5)) \xrightarrow{10^{17}\,\text{GeV}} \text{observed interactions i.e. } SU(3)_c \times SU(2) \times U(1) \xrightarrow{10^{2}\,\text{GeV}} SU(3) \times U(1)$$

and have masses of about the Planck mass. Hence the cut-off K is taken to be approximately the Planck mass, i.e. $\sim 10^{19}$ GeV.

To obtain $m \sim 1$ GeV , μ_0^2 must be adjusted (five tunned) to about 10^{38} decimals otherwise it would come out to be $\sim 10^{19}$ GeV too. Indeed,

$$\left.\begin{array}{l} m^2 \sim 1 \text{ GeV}^2 \\ K^2 \sim 10^{38} \text{ GeV}^2 \end{array}\right\} \Rightarrow \mu_0^2 \sim 10^{-38} - g_0^2$$

These "unnatural" adjustments are caused by the quadratic divergences in the scalar particle masses.

Georgi[11] argued against this interpretation. He maintained that this is simply an artifact of the regularisation procedure. The fact remains that to avoid mass scales at $M_W \sim 10^2$ GeV and at 10^{14} GeV the large ratio of the hierarchy of masses must be obtained by use of the Higgs mechanism. This is apparently very difficult to obtain and involves problems; again incredible accuracy is needed.

Some people think that at this stage gravity should not be ignored any more and that an understanding of the mass spectrum, etc. will follow from a unified theory of all four interactions.

As such a theory does not exist yet, let us consider a less ambitious but interesting alternative solution, technicolor.

Technicolor schemes known as QTD[8], are defined by a technicolor gauge group $SU(N)_{TC}$. QTD is taken to be similar to QCD; they are parallel theories with different energy scales. In fact QTD is a sort of rescaling of QCD. For example technihadrons exist at a scale 10^3 higher than usual hadrons of QCD. The theory contains techni-quarks a family of massless fermions which carry a strong technicolor $\Lambda_{TD} \sim 1\,\mathrm{TeV}$ interaction.

The techniquarks are doublets under SU(2) x U(1), N-tuplets under $SU(N)_{TC}$ and singlets under color. They interact via unbroken interactions, $SU(N)_{TC}$. These technicolor binding forces generate a spontaneous breakdown of chiral flavor SU(2) x SU(2) exhibiting the existence of massless technipions. The technipions replace the usual scalar sector in yielding a mass matrix for the intermediate bosons but leaving the quarks and leptons massless. Therefore they leave the generation puzzle as it is since these theories do not have anything to say about generations.

However, these theories can be exploited. Indeed they have been combined to grand unified theories; the resulting synthesis possesses the unification of GUT together with the solution of the scalar problem offered by technicolor.

These extended technicolor theories give rise to massive $\sim 10^2\,\mathrm{TeV}$ gauge bosons connecting fermions to technifermions and allow for the dynamical technifermion masses to be of the order of ordinary fermions.

But this combination still involves the complications of a new technicolor gauge interaction and new technifermions.

This is an interesting and hopeful solution though it presents difficulties.

A simplification of the fundamental theory may be possible if the technifermions are composite of more fundamental objects (see later).

2.3 Alternative Models; GUT

The overabundance of parameters and fundamental fields in the SU(3) x SU(2) x U(1) theory in one hand and its many successes in the other hand lead us to think that the model was not incorrect but rather incomplete at higher energies and needed to be extended.

Many speculations have been made but without any concrete result, e.g.

> The extension of the electroweak gauge group to a left-right symmetry $SU(2)_L \times SU(2)_R \times U(1)$ gives basically the same results as the usual standard model.

However, one quite successful extension is the grand unified theory. These theories constitute a fertile field of particle physics. The weak and electromagnetic interactions are unified in a grand unified theory with SU(5) gauge symmetry; SU(5) is one among many possibilities proposed.

Many reasons motivated this search. The reduction of the number of gauge coupling constants from three to one, the explanation of the quantisation of change (i.e. as the condition of renormalisality is $\sum_i Q_i = \sum_{quarks} Q_i + \sum_{leptons} Q_i = 0$, $3 \sum_i Q_{quark} = \sum_i Q_{lepton}$ in doublet-sector); etc.

Unification proved to be the answer to some of the questions such as

- why is electric charge quantised?
- why do leptons carry integral charge while quarks carry fractional charges? etc.

Furthermore, it has been shown by Georgi and Quinn and Weinberg[14] that the three gauge coupling constants are equal within numerical coefficients in the symmetry limit.

However, unification does not go without problems, the most serious being the hierarchy problem.

The aim of grand unification is to remedy some of the standard problems by considering one large gauge group incorporating both electroweak and strong interactions and one large representation including all gauge bosons, in other words it is based on a group G such that

$$G \supset SU(3)_c \times SU(2) \times U(1)$$

The rank of G determines the number of conserved additive quantum numbers. The smallest group is SU(5), i.e. the smallest which does not lead to arbitrary parameters. It allows for two quantum numbers of $SU(3)_c$, the electric charge and a "weak charge" (usually associated with Z^0) and possibly additional quantum numbers increasing with the rank of G.

The first three are exactly conserved and are coupled to massless particles (gluons and photons), the rest correspond to broken symmetries and massive bosons since no other massless bosons are known.

It follows that it is not possible to have an exactly conserved baryon number and lepton number operators as generator of G. The many grand unified models belong to two classes:

a. "minimal schemes", a typical example being the SU(5) group. The total number of colorless weak bosons is three ($W^{\pm}, Z^0$). All the fermions in one generation are related to each other but there is no connection between two different generations. In fact, the number of generations is undetermined and could take any value.

b. "maximal schemes" in which all quarks and leptons belong to one irreducible representation of G and thus fundamental fermions are connected. On the contrary of the minimal schemes, the $SU(N)_f$ flavor group acting on the N-quark flavors is a subgroup of G, i.e.

$$G \supset SU(N)_f \times SU(3)_c$$

This means, many colorless weak bosons exist, including bosons which relate the different quarks to each other.

Examples of maximal scheme groups are

$$E(7), \qquad SU(4)_f \times SU(4)_c$$

Both classes of grand unified models present common features. They contain gluons, at least three colorless weak bosons and the photon, all of which are included in the adjoint representation of G and are coupled to the generators of

a. $SU(3)_c \times SU(2) \times U(1)$

b. $SU(3)_c \times SU(N)_f$

In addition to these, there are bosons carrying the quantum numbers of $SU(3)_c$ and $SU(2)$, and leptoquarks.

Leptoquarks are bosons called so because they can convert a quark into a lepton. Thus they carry color, baryon number and lepton number. They respond to both strong and weak interactions and they are presumably confined, because colored. As already mentioned, the baryon and/or lepton numbers are not exactly conserved and practically this means that the proton is unstable! However, the proton is known to live for at least 10^{30} years, it follows that the gauge bosons (leptoquarks) responsible for proton's decay are very heavy about some 10^{14} GeV, e.g.

$$G \xrightarrow{10^{14}\,\mathrm{GeV}} SU(3)_c \times SU(2) \times U(1) \xrightarrow{10^{2}\,\mathrm{GeV}} SU(3) \times U(1)$$

As the reduction of gauge coupling constants from three to one was a motivation for the search of grand unified models, it naturally characterizes them all. This is possible only if the magnitudes of the weak electromagnetic and strong interaction couplings become the same at some high energy. This energy is taken to be the mass scale at which G is broken down, i.e. 10^{14} GeV. The weak and the EM interactions are presumably comparable at energies above W-masses ($\sim 10^2$ GeV). The gauge theory of colored quarks and gluons, QCD, is asymptotically free at sufficiently high energies and its "running" coupling constant may decrease to the level of weak and EM interactions. According to Georgi and Quinn and Weinberg[14], below 10^{14} GeV the three couplings evolve as follows:

(renormalisation Group)

$$SU(3)_c : \alpha_s^{-1}(\mu) = \alpha_G^{-1} + (6\pi)^{-1} \ln\left(\frac{M}{\mu}\right)(4N-33)$$

$$SU(2) : \sin^2\theta\, \alpha^{-1}(\mu) = \alpha_G^{-1} + (6\pi)^{-1} \ln\left(\frac{M}{\mu}\right)(4N-22)$$

$$U(1) : \frac{3}{5}\cos^2\theta\, \alpha^{-1}(\mu) = \alpha_G^{-1} + (6\pi)^{-1} \ln\left(\frac{M}{\mu}\right)(4N) ,$$

Where N is the number of families of fermions.

At $\mu = M$ all the three couplings are proportional to each other. To deduce these formula, it is assumed that G breakdown to SU(3) x SU(2) x U(1) and not to another group. The three formula may be combined and reduced to one:

$$\cos 2\theta/\alpha - (2/3\ \alpha_s) = (44/6\pi)\ \ln(M/\mu)$$

which is independent of N and α_G.

Unfortunately, these relations are untestable by known experiments. So the theory ends up with predictions such as fundamental fermions, determination of Weinberg angle, etc. the validity of which cannot always be checked. Hence, there remains many unanswered problems. In particular the theory gives no explanation for the experimental fact that more than one family exists; the incorporation of more than one family is possible but there is no theoretical reason for it. It seems then, that not only unification does not answer all the questions it was built for, but also it leads to the serious hierarchy problem and does not reduce the number of fundamental particles.

2.4 Quarks and Leptons

In the first generation at the level of the leptons ν_e, e and the u, d quarks it can already be noticed that both quarks and leptons are (V-A) weak currents and must come together in order to have an anomaly free (V-A) theory. They are similar; leptons are $J=1/2$ point-like objects in the sense of minimal coupling in weak and EM processes similarly to quarks which are $J=1/2$ point-like constituents of hadrons. Their charges are quantised, e.g. 3 Q(d) = Q (e).

The second generation is similar to the first one but the "s" and "d" quarks are mixed (Cabbibo angles) together and in general the two generations are split by a large mass difference. If (ν_e, e^-) and (ν_μ, μ^-) doublets undergo an identical way all interactions, what causes the difference mass scale of the two lepton doublets? and similarly for quarks. This became even more pronounced when the third generation is included.

Therefore, in all generations there exists similarity between quarks and leptons. It may be wondered what is the cause of this connection. Two possible answers are being sorted out. The grand unification and the composite models. The latter are based on the ambitious and rather exciting idea that quarks and leptons are composites of the same fundamental objects. This was suggested by the increasing number of fundamental particles especially as the spectroscopy of quarks and leptons is rather regular and could eventually be compared to nucleous spectroscopy.

At present, the origin of quark and lepton masses have been explained by different mechanisms. Namely Yukawa interactions of quarks and leptons with elementary scalars that develop vacuum expectation values by spontaneous symmetry breaking and new gauge interactions that connect the quarks and leptons with very heavy fermions whose masses are provided by a dynamical spontaneous breakdown. None of these explanations is completely satisfying.

Another mechanism has been suggested[16]. It says that the strong and electroweak interactions could be responsible for the fermion masses. This is not obvious, at first sight. The gauge couplings of the strong and electroweak interactions that connect the quarks or the leptons with each other preserve enough chiral symmetry so that if the quarks and leptons are massless when these interactions are switched off they do not seem to get any mass when the interactions are taken into account.

Weinberg argued that this difficulty is over if the quarks and leptons are composites of more elementary fermions.

As the quarks and leptons are known to be point-like objects, the gauge coupling that binds them would be a very large energy Λ_{HC}. This extra strong force "hypercolor"[17, 18, 19] would provide the means of breaking the chiral symmetries which could not be broken by color and electroweak interactions.

In fact no serious model has been proposed except may be the rishon model which of course has been influenced by the many incomplete models available but which is still at an "embryonic" stage too. This "braking" is due to the restrictions and limitations to which composite models are subjected. This is just the topic of the next chapter.

CHAPTER 3

RESTRICTIONS AND LIMITATIONS ON COMPOSITE MODELS

3.1 The Scale Problem

In the last decade, some discoveries have deeply influenced the understanding of particle physics; the successful Weinberg-Salam model, the approximate scaling of deep inelastic structure functions, etc.
The results of these various experiments indicate that the quarks and leptons do not show any sign of inner structure down to $\Lambda^{-1} \sim \tau$ (τ is the effective radius of a quark or a lepton) where Λ equals 10 to 10^2 GeV.
The quarks and leptons behave like Dirac point particles and any substructure can, to a good approximation, be ignored at least down to 10^{-16} cm. The size of a lepton or a quark may be as small as the Planck length, i.e. 10^{-33} cm.

Compositeness should then be expected at energy scale well above Λ or equivalently at distances below 10^{-16} cm. Furthermore, if the quarks and leptons are made of the same constituents (preons), baryon and lepton number violation are likely to occur, leading to proton decay. The present limit on proton's lifetime indicates that this can probably happen only at distances below 10^{-29} cm or momenta somewhere above 10^{15} GeV.

At such scales (~ 1 TeV) the masses of the observed quarks and leptons are very small, i.e. $M \ll \Lambda \sim 1/r$ or $Mr \ll 1$. This is rather unusual. All known composite systems have $Mr \gg 1$: atoms have $Mr \simeq 10^6$, nuclei $\simeq 10^2$ and nucleons ≈ 5.

The evidence of point-like behaviour of leptons and quarks resulting from experimental and theoretical facts put severe constraints and limitations on composite models.
For instance, the contribution to the anomalous magnetic moment from the quark and lepton substructure, is at first sight difficult to evaluate. Indeed, the mass scale of the moment does not coincide with the mass scale of the composite system. Lipkin[20] argued that it would be a peculiar accident for the

magnetic moment to combine with the Dirac moment into a minimal coupling.

Other authors[21], however, claimed that assuming $M\tau \ll 1$ everything else follows quite automatically. Thus the minimal coupling is consistent with a renormalisable effective theory.
The renormalisabity of the theory at the composite level would emerge naturally if the gauge bosons are composites (see later).

Furthermore, all low energy particles are relatively massless if the scale Λ is very large.
This presumably reveals the presence of some symmetry[22] which insures the masslenesss of these particles; chiral symmetry for fermions, gauge symmetry for vector bosons, etc.
A composite model should then possess a chiral symmetry. A common belief is that knowledge of the macroscopic world follows from the microscopic $\not{s}$ one.
Accordingly, the effective lagrangian should be derived from the fundamental lagrangian which describes the fundamental fields and do not include any composite particle.
Therefore, it would be nice if the symmetry appears at the fundamental level, i.e. if the fundamental fermions are massless.
Nevertheless, this is not sufficient since the symmetry may be broken spontaneously allowing the composite fermions to be massive. Thus the requirement to help this situation is that the chiral symmetry of the fundamental lagrangian remains, at least, partially unbroken at the composite level.
Now, the massleness of the composite quarks and leptons is guaranteed. However, this does not tell anything about the scale Λ or about the fundamental fermions; why are they not seen? what binds them together?, etc.

To answer this, let us follow a traditional way.
Assume a new color-like degree of freedom, "hypercolor". The fundamental fermions carry hypercolor [17, 18, 19]. They are confined into hypercolor singlet composite fermions with radius $r \sim \Lambda_{HC}^{-1}$ by hypercolor forces

characterized by a scale parameter Λ_{HC}. The singlet quarks and leptons are observed at momenta below Λ_{HC}.

Even though the construction of hypercolor is similar to QCD-color it is somewhat different. Indeed, in QCD $SU(2)_L \times SU(2)_R$ is broken spontaneously into a vector SU(2); in other words the theory ends up with massless gauge vector particle but no massless composite fermions. The chiral symmetry is completely broken in two-flavor massless QCD and probably in higher flavor QCD as well (see later in Chapter 5).

The way the hypercolor model has been defined, it is isomorphic to the corresponding number of flavor QCD, i.e. although these "colors" are generated in apparently different ways, they have in fact identical structure and therefore they lead to exactly similar symmetry patterns.

Here comes out a serious problem. How can some chiral symmetry of the hypercolor model remains unbroken if the model behaves like QCD?

This looks inconsistent. Nevertheless, it did not discourage theorists from looking for a way out. They made many hypotheses which could eventually be formulated into a model with the required pattern of chiral symmetry breaking (see later).

In some of these models it is assumed that left handed and right handed fermions have different transformation properties under the gauge group. Such models are not isomorphic to QCD since a fermion-antifermion condensate (a scalar) cannot break the chiral symmetry without breaking the gauge symmetry.

Here, people opted for two alternatives. Either no condensation exists[18] at all or the gauge symmetry breaks itself[19] into a smaller subgroup. But, the fact remains that both considerations have not succeeded. It might be that the pattern of chiral symmetry breaking is flavor-number dependent or that color and electroweak interactions have different influences on QCD and hypercolor wonders Harari[2].

His arguments are that if these possibilities have not been proved wrong, they could be right.

All things considered, a proper dynamics of composite models is needed in order to enlighten the chiral symmetry breaking pattern and thus the generation puzzle. This latter remains unsolved since all the plausible explanations suggested are not supported by convincing proof.

It has been assumed that, as the generations do not differ by a conserved quantity and apparently cannot be labelled by a quantum number, higher generations could be thought of as an analog of the first one[17]. The splitting between generations will be due to an excitation (radial or orbital) and hence masses of this excitation of composite quarks and leptons are expected to be of the same order as the scale $\Lambda \sim (1\,\mathrm{TeV})$ which is the inverse of the radius of the system, i.e. $\Lambda^{-1} \sim r$.

But this is not in agreement with the values of mass splitting among generations (100 MeV – GeV).

Could it be that excitations are different in each case?

In the following chapters we will see how different models handle this problem.

3.2 Anomalous Magnetic Moment

It is traditionally expected that internal structure contributes to the anomal magnetic moment.

The usual electromagnetic vertex  is described by

$$\Gamma^{\mu} = \gamma^{\mu} f(k) - \frac{1}{2m} \sigma^{\mu\nu} q_{\nu}\, g(k),$$

where f and g are the form factors.

The value $g(0)$ has an important physical meaning. It gives the radiative correction to the magnetic moment.

For instance, let us assume the scattering amplitude is

$$M_{fi} = -e\, \gamma^{\mu}_{fi} A_{\mu}(k)$$

the term corresponding to the form factor g is

$$\delta M_{fi} = \frac{e}{2m}\, g(k^2)\, (\bar{u} \sigma^{\mu\nu} u)\, k_{\nu}\, A_{\mu}(k)$$

where e is the electric charge and m the mass of the spin-½ fermion considered (e^- or μ).

For a pur magnetic field $A^{\mu} = (0, A)$,

$$\delta M_{fi} = \frac{e}{2m}\, g(-k^2)\, (\bar{u} \Sigma u)\, i\, k \wedge A(k) \qquad \text{(3 dimensions)}$$

A is constant and implies $k^\mu = (0, k)$.

Replace $i k_\lambda A_k$ by H_k the magnetic field then take the limit $k \to 0$,

$$\delta M_{fi} = e\, g(0)\, H_k\, (\omega_2^* \sigma\, \omega_1)$$

W_1 and W_2 are defined by

$$\bar{u}_2 = \sqrt{2m}\, (\omega_2^*, 0) \quad \text{and} \quad u_1 = \sqrt{2m} \begin{pmatrix} \omega_1 \\ 0 \end{pmatrix}$$

The scattering amplitude in a scalar potential ϕ_k (i.e. by an electric field) is

$$M_{fi} = -e\, (\bar{u}_2 \gamma\, u_1)\, \phi_k$$

$$\simeq -e\, \phi_k\, 2m\, (\omega_2^* \omega_1)$$

By analogy, we could attribute to the electron (muon) in a magnetic field a potential energy

$$-\frac{e}{2m}\, g(0)\, \sigma\, H_k$$

This means that the electron (muon) has an anomalous magnetic moment

$$\mu' = \frac{e\hbar}{2mc}\, g(0)$$

which is an extra moment to the Dirac normal magnetic moment

$$e\hbar / 2mc$$

If quarks and leptons are composites there should be new corresponding corrections to the magnetic moment.

However, the magnetic moments of the electrons are known with such a good accuracy that experimental results are in perfect agreement with theoretical predictions (up to 10^{-19} decimals) including QED + weak and QCD corrections.

$$g(0) = \frac{\alpha}{4\pi} \int_1^\infty \frac{dx}{x^{3/2}\sqrt{x-1}} = \frac{\alpha}{2\pi}$$

it results that

$$\mu_e = \frac{e\hbar}{2m_e c} \left(1 + \frac{\alpha}{2\pi}\right) \qquad \text{at first order;}$$

it was first established by Schwinger in 1949[23].

At second order,

$$g^{(2)}(0) = \left(\frac{\alpha}{\pi}\right)^2 \left(\frac{197}{144} + \frac{\pi^2}{12} - \frac{\pi^2}{2} \ln 2 + \frac{3}{4} O(3)\right)$$

$$= -0{,}328\, \frac{\alpha^2}{\pi^2}$$

therefore

$$\mu_\mu = \frac{e\hbar}{2m_\mu c} \left(1 + \frac{\alpha}{2\pi} + 0{,}76\, \frac{\alpha^2}{\pi^2}\right).$$

For the muon, the magnetic moment is

$$\mu_\mu = \frac{e\hbar}{2m_\mu c}\left(1 + \frac{\alpha}{2\pi} + 0{,}76\,\frac{\alpha^2}{\pi^2}\right)$$

And using units of Bohr magnetons

$$g = 2\left(1 + \frac{\alpha}{2\pi} + O(\alpha^2)\right)$$

where $\alpha = e^2/4\pi$ and $\mu_B = \hbar|e|/2m_e c = 0{,}927 \cdot 10^{-23}$ MKSA.

Finally,

$$a = (g-2) = \frac{\alpha}{2\pi} + O(\alpha^2)$$

where g is the gyromagnetic ratio also called factor of Landé.

$$a_\mu^{exp} - a_\mu^{theor} = (0.1 \pm 1.9) \cdot 10^{-8}$$

$$a_e^{exp} - a_e^{theor} = -(251 \pm 154) \cdot 10^{-2} \text{ [24]}.$$

Thus it is hard to find any room for any extra corrections which would eventually indicate the compositeness nature of quarks and leptons.

Even more serious difficulties are posed by Lipkin[20] and other authors[25]. They claimed that the obtention of the Dirac moments of the electrons in composite models is difficult because the mass scale of the moment must coincide with the mass scale of the composite system and because the spin and magnetic moment are a linear function of the spins and moments of the constituents and involve Clebshes.

For instance, for an elementary spin-$\frac{1}{2}$ particle, the mass appearing in the formula giving the magnetic moment is the mass of the Dirac equation. But this is not true for a composite system because there is no simple relation between a mass of a composite system and its constituents. And apparently there is no reason why the different constituent couplings should combine into a minimal coupling depending only on the four-momentum of the system and not of the masses of the constituents.

Moreover, as the spins of the constituents are parallel or anti-parallel to the spin of the bound state, the spin and magnetic moment of the composite system does not result from a simple algebraic sum as it is the case for electric charge.

Hence, the ratio of the magnetic moment of the composite systems to its electric charge is given by,

$$\mu/Q = \sum_i c_i \mu_i / \sum_i q_i$$

where μ_i, q_i are the constitutent magnetic moments and electric charges respectively, c_i are the functions of Clebsh-Gordon coefficients in the coupling of the spins of the constituents to the total spin.
In general μ/Q is completely different from μ_i/q_i.

These additional difficulties due to magnetic moments are actually avoided by authors[21] who argue that the above suggestions[20] are non-relativistic. They first note that the high accuracy of ($g-2$) is not really an obstacle. The weak and QCD contributions to the magnetic moment predicted by the standard model are very small and could have been neglected. If this did not put off the standard model, it should be possible to add new small corrections due to the compositeness of quarks and leptons.

The basic assumption is that quarks and leptons are light bound states involving $Mr \ll 1$. This problem has been discussed in the previous section. Shaw-Silverman and Slansky[21] assumed $Mr \ll 1$. Then they deduced a sidewise dispersion relation for the anomalous magnetic moment which is given by the form factor in the expression of the amplitude at a minimal coupling.
The dispersion relation is expanded in terms of mass parameters, taking into account $Mr \ll 1$. It results that the new contributions of the form factor (from the composite nature of the spin-$\frac{1}{2}$ fermion considered) obtained, gives the new binding corrections $O(m/m_c)$.

The magnetic moment of a composite system with mass m, charge e and spin-$\frac{1}{2}$ is

$$\frac{e}{2m}\left[1 + \text{usual (QED + Weak + QCD) corrections} + O(m/m_c)\right]$$

m_c are the constitutent masses. If the constituents are a boson with mass m_B and a fermion with mass m_F, for $m_B \gg m_F$, the additive binding corrections to ($g-2$) are $O(m m_F/m_B^2)$.

3.3 The Gauge Bosons (of QCD)

The gluons are the non-abelian analog to the photon. Except that they do not exist as free particles, they couple to themselves and come in eight colors, gluons and photons are similar.

All calculations and experiments are consistent with massless gluons and photons.

In general, massless particles including composites occur together with a special symmetry which guarantee their massleness. In this case, this means exact local gauge symmetry and if these exactly massless gauge bosons appear in the fundamental lagrangian, this latter is exactly gauge invariant under the corresponding gauge group .

The effective lagrangian (i.e. the lagrangian involving only light particles) and the fundamental lagrangian (i.e. the lagrangian considered at higher energies when the structure of the particles is revealed; it is usually taken to be free of any mass term) possess the same local gauge symmetry if the gauge invariance of the former is not broken by small corrections and because the symmetry of the latter cannot be broken by higher dimension terms proportional to Λ_{HC}^{-n} $(n>0)$.

It follows that the two lagrangians contain the corresponding gauge bosons as fundamental fields.

Therefore gluons and photons would not be composites.

If they were composites, the fundamental constituents should also possess an exact gauge symmetry which remains unbroken at the composite level to ensure that the composite gluons and photons do not gain any mass. This proved difficult to realise and involves many serious problems.

Actually, the gluons and the photons are assumed to be fundamental which is very likely.

The gauge bosons left are the massive $W^{\pm}$ and Z. As seen before, difficulties in the construction of composite gauge bosons are mainly due to their massleness.

This makes the consideration of composite W and Z easier and more plausible.

In particular, if the Higgs scalars which are responsible for the longitudinal components of W and Z are composites, it follows that the W and Z are, at least, partly composites and may not appear in the fundamental lagrangian.

M_W and M_Z are very small before Λ_{HC}, hence the gauge bosons W and Z are almost massless at this scale and a symmetry principle is needed. But unfortunately, it seems that there is neither a symmetry argument nor a dynamical argument for light composite W and Z in a hypercolor theory with fundamental fermions.
The symmetries of the fundamental lagrangian must be such as the electroweak symmetry emerges at the composite level.
If the W and Z bosons are composites, the corresponding forces, the weak interactions are not fundamental any more. They became probably residual hypercolor forces with short range action.

All the gauge symmetries in the fundamental lagrangian are exact symmetries of nature.
The fundamental gauge symmetries are exactly conserved while other symmetries are spontaneously broken.
Hopefully, after the breaking, the usual gauge group of the standard model will emerge at the overlying level.

If however, the W and Z bosons are elementary, the electroweak group exists already in the underlying theory.

Also, Higgs scalars must be present at the fundamental level to give masses to the fundamental W and Z. As the presence of scalars [15, 28] in the fundamental lagrangian is a flaw to the "naturalness"[28] of the theory, it is reasonable to assume that the W and Z bosons are composites as well as the scalars.

CHAPTER 4

COMPOSITE MODELS

4.1 General Features of a Composite Model

Before considering some specific examples, it might be helpful to know roughly what is required from a composite model. Taking into account, in one hand the predictions which would confirm the validity of the model and in the other hand the problems to avoid, a theoretical framework can be constructed. This will be the criteria for a typical composite model and therefore actual available composite models are not expected to fulfil these requirements.

As any model, a composite model of quarks and leptons

i. should be expressed in simple and elegant mathematical terms which will hopefully yield neat, economic and simple answers;

ii. should be realistic for it is expected to reproduce at least roughly, the observed spectrum of quarks and leptons and in general to describe as much as possible the real world. Hence a realistic model should survive its predictions. The experiments predicted should be observed otherwise the model is simply ruled out.

Furthermore, it should respond some basic requirements we have come across in the preceding chapters.

i. The anomaly constraints should be satisfied (see Chapter 5).

ii. The generation puzzle should be solved.

iii. The pattern of chiral symmetry breaking (if present in the model) should be clarified and described by an appropriate dynamics.

Model building and dynamics

A model can be built differently depending on the hypotheses imposed at the start. It follows that the specific requirements differ from a variety of models to another. Thus putting different constraints in each case. Here are some examples.

i. The fundamental fermions could presumably be massive or massless; the main thing is that they are bound into approximately massless composite fermions (compared with Λ_H). In the former case, it is hard to explain the situation but in the latter case two solutions are being studied. The first one is the chiral symmetry which has been discussed before and which shall be seen again in the context of particular models. The second is supersymmetry; the massless composite fermions could be identified to the Goldstone fermions of a broken supersymmetry[27].

ii. Equally crucial is the problem of the dynamics involved. Indeed, some force should bind the preons together to make quarks and leptons. Many speculations have been made as to the nature of this force.

ii.a. The problem may be analog to QCD involving non-abelian confining forces known as hypercolor[17], QDD[18], QSD[10] and so on. 't Hooft[28] call these binding gauge forces "metacolor". The metacolor forces become strong at energies of order Λ_H (the compositeness scale). The preons possess some chiral symmetry which, if it is not broken at the composite level, would yield massless spin-$\frac{1}{2}$ bound fermions. SU(N) metacolor group did not lead to consistent results for $N > 2$; it gives rise to anomalies and hence cannot be gauged, (see Chapter 5). Thus, Barbieri et al[29] studied the case where the metacolor group is $O(n)$ with n odd. Preons transform as the n-dimensional, vector representation.

They claim that binding can be obtained without spontaneous chiral breaking. The realisation of chiral symmetry breaking is in fact very difficult and probably requires something like technicolor. Some of the scalar composites which do not acquire a vacuum expectation value in the metacolor dynamics could do so at an energy scale where some of the metaflavor (i.e. symmetries external to metacolor) interactions become strong. However, the addition of technipreons may lead to the breaking of the SU(N) subgroup considered, i.e. the local symmetry of the preonic langrangian which at low energies generate (hopefully) the standard model. Further, it may yield normal fermion masses if their global chirality is explicitly broken by their residual metacolor interactions.

ii.b. The dynamics might also occur via abelian forces with magnetic and electric component [30, 31]. These magnetic models are characterised by a strong coupling ($\frac{e\gamma}{2\pi} = 1$), a possibility of generation of spin by Saha's mechanism[7] and an economy of degrees of freedom.

Moreover, grand unified theories can be deduced from such models, e.g. SU(10)[31]
SU(5)[32].

The flavor symmetry is explicitly, but slightly broken by electric charges and hence except for gluons there are no massless particles. The color symmetry is exact at the preonic level. The vector mediant of this magnetic theory is called luxon. The photon is a massless composite of vector bosons. Photonic and luxonic charges are proportional so that the exact quantisation of observable electric charge is kept.

Criteria for composite models of leptons only

Greenberg and Nelson[32] consider composite models of leptons only; the quarks being fundamental. They describe ten criteria which a composite model of leptons should satisfy.

It should predict:

i. the chiral algebra of lepton charges;

ii. muon-electron universality;

iii. separate conservation of muon and electron numbers;

iv. that odd-half-integral (integral) spin particles have odd (even) value of lepton and baryon numbers;

v. lepton magnetic moments;

vi. G_V and G_A for weak lepton currents;

vii. mass relations for leptons and the low mass of the known leptons;

viii. massless two-component neutrinos;

ix. lepton-hadron universality;

x. absence of strong lepton interactions at low energy.

They actually present two models which will be considered later on and which satisfy some of these requirements.

4.2 Models Based on One Fermion and One or More Bosons

Greenberg[33] proposed a model in which a quark is a composite of a Fermi, spin-$\frac{1}{2}$ colorless flavored object and a Bose, spin-0, flavorless colored object. The fermion Q is an elt of $SU(3)_F$-triplet representation and the boson C is an element of $SU(3)_C$-triplet representation.

Pati et al[34] suggested a similar scheme with the additional feature that the fermion carries charm as an additional quantum number and the boson carries lepton number as a fourth "color".

In Pati et al model, the description is very general. The interpreon forces are hardly discussed. They note that massless bound states are difficult to obtain, for instance the zero-mass leptonic composites, i.e. the neutrinos. They suspect that the neutrinos, like the photon are rather special composites whose existence should be guaranteed by a symmetry principle. The relevant symmetry in the case of the photon would be gauge invariance. As for the neutrinos, supersymmetry may possibly be incorporated.

Greenberg has some discussion of the dynamics involved. Mesons are four-body $Q\bar{C}\bar{Q}C$ systems and baryons are six-body $Q\bar{C}Q\bar{C}Q\bar{C}$ systems.

Ignoring the centre of mass motion the mesons have three degrees of freedom,

Authors	General Features	Dynamics	Predictions	Problems
Greenberg[33]	fermion Q and boson C $Q = (3, 1)$, $C = (1, 3)$ under $SU_F(3) \times SU_c(3)$	harmonic oscillator	i) mesons = $Q\bar{C}\bar{Q}C$ baryons = $Q\bar{C}Q\bar{C}Q\bar{C}$ ii) particles with narrow width namely the ψ's resonances	not discussed
Pati et al[34]	similar to Greenberg's but Q carries charm and C carries Lepton number			i) massless bound states (e.g. neutrinos) are difficult to obtain ii) no dynamics
Kalman[35]	heavy quarks q' are q-quark compounds	harmonic oscillator	i) the number of generations is limited ii) and the heavy quarks decay $q' \to q + W$; W is the weak boson	
Krolbrowski[37]	the preons Q and C transform as $Q = (3, -1/3)$, $C = (3, -1/3)$ under $SU(3)_c \times U(1)_{EM}$	the preons interact with a color-octet of gluons	i) a second Z-boson if the photon is elementary	i) unexplained origin of weak and strong interactions ii) problems in expressing the lepton-magnetic moment (massless neutrinos)

one fermion and one boson.

..... cont.

Authors	General Features	Dynamics	Predictions	Problems
Ne'eman [38]	one fermion doublet/singlet colorless set: $(\alpha^0,\alpha^-)_L, \alpha^0_R, \alpha^-_R$ two bosons $SU(2)_L$ - singlets $\beta_B^{2/3}, \beta_S^0$ + QCD. $\beta_B = SU(3)_c$-triplet B = baryon number S = seriality		i) proton decay $p \to \alpha + \beta_B$ and $M_{\beta_B} \simeq$ Planck mass. ii) $\mu \to e + \gamma$ induces $M_{\beta_S} \gtrsim 300$ GeV.	i) no dynamics ii) origin of β_S and β_B unknown.
Shaw[39]	heavy quarks q_α are compounds of q = (u, d, s, c) and a colored neutral boson α.	q is strongly binded to α.	i) explanation of Υ-system	i) origin of binding unexplained, i.e. missing dynamics ii) predictions are not in accord with known experiments
Greenberg and Sucher[19]	3 preons = (one fermion + 2 bosons) quarks, leptons, weak bosons and Higgs are composite states. It incorporates Cabbibo structure, GIM-mechanism and spontaneously broken left-right symmetry for weak interactions.	SU(N) local gauge "QSD" interaction; a QCD-like confining theory.	i) strong and weak interactions are residual effects of flavor-independent QSD- and QCD-interactions ii) mass degeneracies are due to orbital and radial excitations iii) generations of narrow states up to the W-masses	i) no reason for neutrinos to be massless ii) unlimited number of generations

one fermion and two bosons.

<u>Models based on one fermion and one or more bosons</u>

three modes of space excitation. The usual quark model corresponds to the excitations of the centre of mass of $Q\bar{C}$ and $C\bar{Q}$ systems, called the r-mode. The two other possibilities are the ρ and R-modes. For the R-mode, Q and $\bar{Q}$ and also $\bar{C}$ and C oscillate in opposite directions as in the r-mode, while for the ρ-mode Q and $\bar{Q}$, $\bar{C}$ and C oscillate in the same direction as in the anti-symmetric stretching mode of the molecules.

An analogous discussion of modes can hold for the baryons as a $Q\bar{C}$ $Q\bar{C}$ $Q\bar{C}$ system. By an appropriate choice of the force constants and masses, the r-mode will be lowest, the ρ-mode occurring next. Then, Greenberg[33] presents a table giving the quantum numbers for various modes in the harmonic oscillator model for states with I = Y = 0 in ordinary SU(3). The ground state corresponds to an identity mode with $J^{PC} = 0^{-+}$ or 1^{--}. The first excited state contains an r-mode with $J^{PC} = 1^{+-}$ or $(0, 1, 2)^{++}$ and a ρ mode with $J^{PC} = 1^{++}(I^G = 0^+)$ or $(0, 1, 2)^{+-}$ $(I^G = 0^-)$.

Because its dipole moment vanishes, the ρ-mode can make dipole transitions to other ρ-modes but not to ordinary hadrons. Thus, the model predicts a set of narrow neutral states or in other words particles with narrow width.

The main idea of this model is that the ψ-resonances are narrow for hadronic decay because of the conservation of color-SU(3). Moreover, the ψ's are narrow for radiative decays to the usual hadrons.

Kalman[40] showed that the spectrum of the quarks is in reasonable agreement with the rough approximation of the harmonic oscillator model. He refers to Hongoh's work[36] in which a compound-quark model based on a dynamical group of the oscillator is used to describe the interactions. He notes that the number of generations is limited if the very heavy quarks decay via the reaction,

$$q \longrightarrow q' + W$$

where W is the weak intermediate boson.

Lichtenberg has suggested that even without a calculation the fact that particle width for the decay,

$$W \longrightarrow \pi + \gamma$$

is approximately 1 MeV, might mean that very heavy quarks have usually short lifetimes.

In Krolkowski's model[37], the fermion and the boson are both color-triplets with charge-$\frac{1}{3}$. They interact with a color-octet of gluons. The leptons and quarks of the lowest generation are then bound states;

$\nu_e = [Q\bar{C}]_s$ color-singlet

$e = [Q\,(CC)_P]_s$ color-singlet

$u = [\bar{Q}\,\bar{C}]_s$ color-triplet

$d = [\bar{Q}\,(CC)_P]_s$ color-triplet,

where the labels P and S refer to the relative angular momentum 1 and 0 states respectively. This is almost the unique interpretation of preon bound states. Some of the other combinations are probably unstable. The intermediate weak bosons $W^{\pm}$ and Z are given by the combinations

$W^- = [C\,(CC)_P]_s$ color-singlet

Z = a linear combination of color-singlets (Q)

and $(CC)_p$.

The photon may be described by an orthogonal combination of the latter states. However, if the photon is elementary like the gluons, a second neutral intermediate weak boson may describe it.

Krolkowski[37] raises a crucial question for the model, which is why leptons do not undertake any strong interactions while hadrons, being also color singlets, do. Moreover, large forces are needed to provide zero-mass for the neutrino ν_e, so its internal dynamics is very relativistic. Therefore, non-relativistic approximations cannot be applied to the internal motion of preons inside leptons. But as such approximations are needed to introduce preon magnetic moments into the lepton internal dynamics, it follows that lepton magnetic moment, i.e. magnetic interactions cannot be expressed in terms of preon magnetic moments.

To solve the generation problem, Ne'eman[38] introduced a second Bose field

β_S which carries a new quantum number called seriality and hence takes account of the generations explicitly.

The model consist in a fermion doublet/singlets colorless set,

$\begin{pmatrix} \alpha^0 \\ \alpha^- \end{pmatrix}_L$, α^0_R , α^-_R the alphon and two Bose fields singlets under $SU(2)_L$, $\beta_B^{2/3}$ and β_S^0 the beitons, in addition to the $SU(2)_L$ x U(1) and QCD gauge fields.

The β_B boson carries "hadronicity", i.e. baryonic quantum number as well as SU(3)-color triplet features. Thus it changes a lepton set into a quark set. Indeed, the alphons combined with β_S^0 give leptons with different seriality. For each fixed seriality S a combination with $\beta_B^{2/3}$ give the analog quark. The proton persumably decay via the process $p \longrightarrow \alpha + \beta_B$. Hence β_B should be as heavy as the Planck mass. Similarly, from the present bound on

$$\mu \longrightarrow \alpha + \gamma ,$$

it can be deduced that if β_S is a gauge vector particle in a unified theory, its mass is larger than 300 GeV. However, there is no direct indication of the nature of either β_B or β_S .

Shaw[39] proposed a model in which the u, d, s and c quarks are elementary. However, heavier quarks q_α are "flavor excitation" of the four flavors q due to strong binding to a colored neutral boson α with a new quantum number. He uses this model to explain the Υ-system but the predictions he ends up with are not quite confirmed by experiments. Furthermore, it is not certain that the model can be modified to make it viable.

Nevertheless, Shaw claims that it can have some value in the sense that it could be reconsidered if new heavier quark flavors are found.

Kalman[40] notes that even with the naive harmonic oscillator potential, the present quarkonium structure can be understood by the assumption that the heavy quarks are compound structures.

In addition to his single boson plus fermion model Greenberg noted with Sucher[19] that another alternative is to introduce a second boson. They propose a composite model of quarks, leptons, weak vector bosons and Higgs mesons. Most of these particles are two-body composites and most of the two-body composites correspond to desired particles. They are confined by an $SU(N)_S$ local gauge "QSD" interaction. The photon and the $SU(3)_c$ octet of gluons can either be kept elementary, consistent with the idea that exact local symmetries are associated with fundamental fields, or can be constructed out of preons being then composites.

The strong and the weak interactions of quarks and leptons are both residual effects of flavor-independent local gauge interactions, QSD and QCD respectively. The generations are accounted for, in a schematic way. There is no reason for the neutrinos to be massless and different generations are due to orbital and radial excitations. Their number is not limited by the theory. Moreover, this model predicts generations of narrow states up to the W-mass and definitely not above.

The model has Cabbibo structure, GIM mechanism and spontaneously broken left-right symmetry for the weak interactions. Whether this scheme is realistic in the sense that it leads to the observed mass spectrum of quarks and leptons is an open question, since such models require energies large enough (at least in the TeV-range) to probe the constituent structure of quarks and leptons, which is in fact the case of all composite models.

Terazawa and Akama[41] presented a model in which a lepton or a quark is made of a spinor preon and a scalar preon. Yang-Mills theory for interactions of a lepton or a quark and Einstein's general relativity for gravity appear as effective theories in which gauge bosons and gravitons are composites of a preon-antipreon pair. There exists a universal short distance cut-off at about the Planck length. As a result the gauge coupling and newtonian gravitational constants are related with each other. Also, the anomalous magnetic moment of a lepton (quark) due to the preon structure is proportional to the ratio of a lepton (quark) mass to a preon mass. They suggest that the photon could possibly be extremely light but not exactly massless. This is because in the strong coupling limit the photon (or gauge boson) mass squared can become small but never vanishes precisely because of the lagrangian involved.

4.3 Models Based on a Three-Fermion System

4.3.a. two types of fermions

Harari[44] and Shupe[42] considered a scheme in which all leptons and quarks are made of two spin-$\frac{1}{2}$ fields with charge 0 and $1/3$.
The constituents are assumed to be neutral and fractionally charged ($1/3$) in order to reproduce the electron without dealing with fractional couplings to the photon.
Notions of color and flavor have meaning only at the composite level. The leptons and quarks are formed by superpositions and involve states such as,

11V, 11V1V, ... and permutations.

The $J = 1/2$, $Q = 1/3$ preon is labelled T
$J = 1/2$, $Q = 0$ preon is labelled V;

it is assumed that $\bar{V}$, the antiparticle corresponding to the neutral consituent exists.
The fermions of the first generation can be built with three constituents combined in eight different ways.

$$\begin{aligned} TTT &\longrightarrow e^{+} \\ TTV,\ TVT,\ VTT &\longrightarrow u \\ TVV,\ VTV,\ VVT &\longrightarrow \bar{d} \\ VVV &\longrightarrow \nu_e\,. \end{aligned}$$

Leptons have only one allowed arrangement and therefore are not colored.
But quark states are degenerated and correspond to exact color symmetry.
However, a mechanism other than ordering is used by Squires[18] to "explain" color. In this scheme T and V transform under $SU(3)_c \times SU(3)_H$ ($SU(3)_H$ is a hypercolor gauge group) as

T: $(\bar{3}, 3)$ and V: $(1, 3)$;

and thus color is just a label on the T-fermion.

Authors	General Features	Dynamics	Predictions	Problems
Harari[44] / Shupe[42]	three spin-$\frac{1}{2}$ preons with electric charges $0, \pm 1/3$ color is a consequence of preon ordering and have a meaning only at the composite level as well as flavor		i) proton decay; $u + u \rightarrow \bar{d} + e^{+}$ ii) B–L is conserved while B and L are both not conserved	i) why leptons are free when quarks are confined (answer: $SU(3)_H$) ii) generation puzzle unsolved
Squires[18]	color $SU(3)_c$ is accounted for by allowing the charged preon to be a color $\bar{3}$. The rest is similar to Harari's and Shupe's Models	a strong confining force "hypercolor" SU(3) binds the preons together		i) unexplained fermionic spectrum
Adler[43]	similar to Harari's and Shupe's identification of preons	quaternionic QCD, i.e. n = 2 of SU(n) QCD using U(2)	i) the usual photon ii) gluons coupled to quarks in a pattern resembling to SU(3) QCD iii) gluons mediating color-singlet states, i.e. replacing the conventional intermediate bosons	i) presence of a second photon coupling to neutrinos and quarks

..... cont.

Authors	General Features	Dynamics	Predictions	Problems
Terazawa and Chikashige and Akama[45]	quarks, leptons, gauge bosons and Higgs scalars are spin-$\frac{1}{2}$ preon composites, one preon carry flavor; the second carry color and it is a singlet/triplet when a lepton/quark is involved. The third preon is color and flavor-singlet		i) $M_{W\pm} \simeq \sqrt{3} M_{preon}$ ii) very heavy leptons and quarks	
Pati and Salam and Strathdee[34]	Higgs scalars are fundamental flavor and color-singlet states and are not composites	$U(1)_A \times U(1)_B$, two abelian symmetries generating gauge forces		
Taylor[46]	3 spin-$\frac{1}{2}$ preons - one carry flavor, the second carry color, the third carry generation number. The photon is presumably coupled to the flavored preon only	"super-glue" gauge group SU(m)	i) B and L conserved, i.e. stable proton	i) spin-$\frac{3}{2}$ composite states; this is a common problem to all 3-spin composite models ii) composite states do not have automatically the correct doublet-singlet structure

Models based on a three-fermion system

Proton decay occur via

$$u + u \longrightarrow \bar{d} + e^{+}$$

i.e. (TTV) + (TTV) (TVV) + (TTT)

In such processes B and L are not conserved, however their difference is conserved and the neutrality of matter is preserved.

The second and third generations contain presumably the same set of states at different energy levels. But the calculation of the different masses and the transitions among generations require a better acquaintance of the dynamics involved.

In his model Squires[48] notes that the ground states do not have the same statistics under permutation of the $SU(3)_H \times SU(3)_C$ indices. Therefore there should be another sort of label on the preons. This might be a possible "reason" for the existence of generations. However, the situation is quite complicated and the procedure not clear at all.

Moreover, assuming that the eventual dynamics of these models lead to the Salam-Weinberg model at the overlying level, the structure of the neutral current Z_0 is put by hand and the neutral current coupling follows with $\sin^2\theta \simeq 0.25$.

A serious problem in these models is to explain why TTT and VVV states, i.e. the leptons are free when TTV, etc. , i.e. the quarks are confined. Add to this, the unexplained spectrum of quarks and leptons (see later in Chapter 6, the final version of the rishon model).

Adler[43] proposed quaternionic QCD (i.e. n = 2 of SU(n) QCD using U(2)) as a theory of quarks and leptons. He made a preliminary study of the dynamics of the residual interactions of three spinor composites, using the quark and lepton identification of Harari[44] and Shupe[42].

He concluded that three types of interactions appear:

i. a color-singlet, flavor-diagonal photon coupling to the electron, quarks and neutrinos with correct charge assignment and a second photon coupling to the neutron and quarks but not to the electron;

ii. color-changing, flavor-diagonal gluons coupling to the quarks in a pattern resembling but not identical to $SU(3)_{QCD}$.

iii. color-changing, flavor-changing gluons, three exchanges of which can produce a weak flavor-changing transition between color-singlet states without requiring the existence of conventional intermediate bosons.

This is not a realistic scheme; the symmetric limit of the theory is rather different from the standard phenomenology. For instance, the existence of a second photon is certainly undesirable. Nevertheless, this model may be on the right track. In particular, if U(2) gauge symmetry could be broken down to a U(1) gauge symmetry in such a way that only the a = 0 + 3 components of the gauge fields survive as massless excitations, a single photon could be obtained (a is the label on the usual Hermitian bases for U(2) τ^a, a = 0, 1, 2, 3).
It would probably have the correct couplings and a single set of flavor-conserving color gluons.
Adler[43] suggests the use of the Higgs mechanism to implement such a symmetry breaking scheme. Although, he notes that the introduction of scalar fields may not be necessary.

4.3.b. <u>three types of fermions</u>

Terazawa, Chikashige and Akama[45] proposed a model in which the gauge bosons and Higgs scalars as well as leptons and quarks are all composites of spin $\frac{1}{2}$-subquarks.
One carry flavor quantum number, the second carry color and the third one is a singlet in color and flavor; it is composed of a doublet ω_L and two singlets ω_{1R} and ω_{2R} under the Weinberg-Salam-SU(2) group.
According to whether a quark or a lepton is involved, the second preon is a triplet or a singlet under $SU(3)_c$. This scheme predicts the mass of the charged weak bosons to be approximately $\sqrt{3}$ times the preon mass. As a result, the authors suggest that there exist much heavier leptons and/or quarks whose masses reach or go beyond the weak-vector boson masses or that the masses of the

Higgs scalars and weak vector bosons are close to the threshold of preon-pair production, if any.

Later on, Pati, Salam and Shathdee[34] noted that the present quark spectrum can be accounted for in a similar model based on three fermionic preons. However, the singlet in flavor and color is not composed of other objects. The Higgs scalars are also fundamental. The preons are bind together to make quarks and leptons by means of forces arising through two vectorial abelian symmetries $U(1)_a \times U(1)_B$ generating two spin-1 gauge particles. The corresponding electric and magnetic charges are operative only at the preonic level, they are hidden at the composite level. They note that the preons may be scalar particles possessing intrinsic spin-0, the half-integer spins of the composites being contributed by the force field. It is due to half-integer angular momentum associated with the field created by the two reciprocal charges Q_A and Q_B. This is analogous to the case of angular momentum possessed by the EM-field of an electric charge in the presence of a magnetic monopole, though this treatment is non-relativistic and needs further elaboration.

The quark-lepton gauge symmetry is interpreted as an effective low-energy symmetry arising at the composite level.

Taylor[25] considered a model similar to Ne'eman's[16] but with three spin-$\frac{1}{2}$ fermions instead of one fermion and two bosons. One carry flavor, the second carry color and the third one carry generation number.

This model presents some difficulties. He notes that Ne'eman's model leads to quark composites which automatically have the correct doublet-singlet structure, i.e. the correct $SU(2)_L \times U(1)_{L+R}$ transformation properties and that this is not so in his model. To ensure this, he assumes that the correct flavor group is the gradded group $SU(2/1)$. The total symmetry is then $SU(2/1) \times SU(3)_c \times SU(m)$, where $SU(m)$ is a "superglue" gauge group that binds the preons together.

In this scheme the proton is absolutely stable and baryon and lepton numbers are separately conserved.

To give the correct assignments of charge to the preons, Taylor[46] takes into account the difficulty raised by Lipkin[20] and Glück[25] (see Chapter 3). To preserve these predictions he suggests that the photon is coupled only to the flavored preon.

The most crucial problem in this scheme as in any theory containing three fermions is the existence of spin-$\frac{3}{2}$ composites[47]. Fundamentally massless bound-states cannot develop with spin-$\frac{3}{2}$ or higher. 't Hooft[28] argues that the occurrence of massless bound states with spin-$\frac{3}{2}$ are forbidden by potential problems of unitarity violations and non-renormalisability.

4.4 Other Varieties of Models

Greenberg and Nelson[32] gave ten criteria for composite models of leptons (see 4.1) and presented two models which satisfy some of these criteria. Three triplets of leptonic objects analog of quarks and called "leptoquarks" are used in both models.

The first model uses fractionally charged leptoquarks. It satisfies the first six requirements listed in 4.1. However it does not deal with the dynamical problems.

In the second scheme, the leptoquarks are integrally charged Fermi particles. Greenberg and Nelson speculated about the possibility that leptoquarks and quarks are identical. In this case it seems impossible to isolate leptons from strong interactions. Nevertheless, ignoring this difficulty the identification of quarks with leptoquarks may possibly lead to a unified description of leptonic and hadronic phenomena. It should be possible to relate the masses of leptons and baryons.

Finally, in both models the leptons are assigned to non-singlet representations; this presumably yields leptons whose masses are larger than hadron masses. Here, the authors emphasised that a plausible mass formula allows the known leptons to be less massive than baryons in the case of identical quarks and leptoquarks.

In addition to their unified three-spinor-preon model, Terazawa Chikashige and Akama[45] proposed a model in which the photon, weak vector bosons and Higgs scalars are composites of lepton-antilepton pair or quark-antiquark pair. It follows that the Weinberg angle is given by $\sin^2\theta = 3/8$ for fractionally charged quarks.

All the gauge coupling constants are related to a single coupling constant, the fine structure. The gluon coupling constant is determined to be 8/3 times the fine structure. This suggests that the model could be extended to a larger symmetry than SU(3) x SU(2) x U(1).

The weak vector bosons acquire mass after the spontaneous breaking of the SU(2) x U(1) symmetry through the Higgs mechanism.

However, the relations between the masses of the weak bosons and those of leptons and quarks are specific to the model.

Nowack, Sucher and Woo[48] studied the possibility that leptons are bound to a non-leptonic core to form hadrons. They note that such a model has to deal with three main problems:

i. the reconciliation of the weak interactions of leptons with the strong interactions necessary to bind the leptons into a hadron;

ii. the magnetic moment of the electron which is large on the scale of hadronic magnetic moment;

iii. and the massleness of some leptons which may be difficult to trap into bound states.

They suggest that the binding occurs via a massive neutral vector meson. They show roughly that the magnetic moments of the composite system are in accordance with present experimental values. A major effect would be a change in the value of

$$R = \sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$$

for non-charmed mesons.

Also, as the lagrangian reads

$$\mathcal{L} = \mathcal{L}_{strong}(U_{NLM}) - f\,\gamma_\mu V^\mu$$

where $\gamma_\mu = \gamma_\mu^{\ell} + \gamma_\mu^{NLM}$, NLM = Non Leptonic Matter,

sharply rising cross-sections in eP scattering should be seen at some point. Furthermore, the model requires the existence of observable cores whose masses are presumably as low as four to six GeV.

Zee[49] presented a semi-empirical study of the quark-mass distribution. In particular he noted from empirical fact that,

$$\frac{m_{\psi/J}}{m_\phi} \simeq \frac{m_\Upsilon}{m_{\psi/J}} \simeq 3$$

This induced a mass formula for the n-th quark,

$$m_n \simeq 3^n m_0 .$$

Actually, he reminds the reader that this relation has never been explained satisfactorily.

Nevertheless, a class of models known as "Cluster models" are based on this mass formula.

Taking this result into account, Carrigan[50] suggests that all quarks could be regarded as made up of u and d quarks and two "shadow" quarks $d'_{2/3}$ (Q = $2/3$) and $u'_{1/3}$ (Q = $-1/3$); the prime indicates shadow quarks. Shadow quarks are quarks with abnormal quark charge. They are necessary to get the right charge balance while maintaining the mass formula, Further quarks are given by the following combinations:

$$\begin{array}{llll}
3. & s_{-1/3} = d'_{2/3}\,\bar{d}'_{-2/3}\,d_{-1/3} , & s'_{2/3} = u'_{-1/3}\,\bar{u}'_{1/3}\,u_{2/3} ; \\
4. & c_{2/3} = s_{-1/3}\,\bar{s}_{1/3}\,s'_{2/3} , & c'_{-1/3} = s'_{2/3}\,\bar{s}'_{-2/3}\,s_{-1/3} ; \\
5. & b_{-1/3} = c_{2/3}\,\bar{c}_{-2/3}\,c'_{-1/3} , & b'_{2/3} = c'_{-1/3}\,\bar{c}'_{1/3}\,c_{2/3} ; \\
 & etc \ldots &
\end{array}$$

The normal quarks have negative binding energy, but shadow quarks have positive binding energy and therefore are unstable when not bound to normal quarks. It follows that a compound-quark cannot decay strongly.

Phillips[51] proposed a scheme containing ten leptons and ten subquarks, each comes in nine colors. The proton is predicted to be stable, baryon and lepton numbers are separately conserved. New heavy leptons and five generations of quarks are also predicted.

In this model the simplicity of subquark structure is lost.

Derman[52] has considered a model for composite quarks and leptons in which the Higgs bosons H in the standard SU(3) x SU(2) x U(1) gauge models is responsible for strong binding. All quarks and leptons are constructed of one fundamental massive lepton and quark generation and a heavy neutral Higgs boson. The mass of the latter, M_H can be estimated if it causes light bound fermion states. He shows how the wave functions of such composites could produce point-like behaviour in the limit of $M_H \rightarrow 0$; he suggests $M_H \simeq 15$ MeV. The major problems are the obtention of the Dirac magnetic moment for a composite lepton (quark) whose mass is that of the bound state rather than that of its massive constituents and the suppressed radiative decays, e.g. $\mu \longrightarrow e\gamma$ and of course a proper dynamics which is crucial for any composite model to be viable.

CHAPTER 5

PROBLEMS IN COMPOSITE MODELS

5.1 't Hooft Conditions

In physics, symmetry principles are closely connected with conservation laws. Conservation of momentum and energy follows from invariances of spatial and temporal translations respectively. Symmetry principles, e.g. translation invariance, imply conservation laws and vice-versa. For instance, continuous symmetry lead to additive conservation laws, discrete symmetry correspond to multiplicative conservation laws, etc.

Among the symmetrical forms of quantum field theory, there are the famous gauge theories. 't Hooft[28] argued that in such theories if some parameters are very small it cannot be an accident; it must be the consequence of some symmetry. The smaller the parameter, the higher the symmetry or else the theory is "unnatural".

In unified gauge theories, the effective interactions at low energy scale μ_1 should follow from the properties at higher energy scale μ_2 (or smaller length, i.e. microscopic induces macroscopic).

In natural theories this should not require that the different parameters at the energy scale μ_2 match with an accuracy of order μ_1/μ_2. However, if at the energy μ_2 some parameters are very small, say

$$\alpha(\mu_2) = \Theta(\mu_1/\mu_2),$$

this may still be considered as natural, if this property is not spoilt by higher order effects.

At any energy scale μ, a physical parameter $\alpha_i(\mu)$ can take small value provided that the replacement $\alpha_i(\mu) = 0$ increases the symmetry of the theory.

For instance, at a mass scale $\mu = 50$ GeV,

the electron mass is $m_e = 10^{-5}$

This is a small parameter; $m_e \doteq 0$ imply an additional chiral symmetry corresponding to conservations of left-handed and right.

handed electrons. This guarantees that the renormalisations of m_e are proportional to m_e itself.

In the same way, gauge coupling constants and other interaction constants may be small because replacing them with zero would free the gauge bosons or other particles so that they are separately conserved.

Imposing "naturalness" on gauge theories comes out with restrictions and conditions. This is the aim of 't Hooft[28] who attempts to construct models with improved naturalness, the ideal case being of course a complete natural theory. To achieve this, more QCD-like theories are needed besides QCD itself. In all known theories naturalness seems to be lost beyond a certain mass scale μ_0 which is about 10^3 GeV ; it is referred to as NBMS, Naturalness Breakdown Mass Scale. All elementary particle interactions do not involve unnatural parameters in the range of available energies. This excludes quantum gravity which does not obey the formation of naturalness.

As for the standard model it is natural up to energy scale $\mu_0 \simeq 10^3$ GeV . Above μ_0 , difficulties with unnatural mass parameters occur. These unwanted parameters are present only in theories with scalar fields. The electrons, for example have the same gauge couplings but their couplings to the Higgs scalar differ by huge factors. Even in GUT where the various coupling constants are related to each other, two mass scales are needed; they are apparently unrelated and sort of fixed "by hand". This is the hierarchy problem.

The Higgs mass squared m_H^2 is a fundamental parameter in the lagrangian (up to a coefficient); it is small at energy scale $\mu \gg m_H$. Taking the limit $m_H \to 0$ does not seem to indicate a symmetry. To cure the situation 't Hooft[28] emphasised with Dimopoulos and Susskind[2] that the "physical" Higgs field must be composite.

Note here that "natural" refers to the original lagrangian of the theory and does not include the Higgs mechanism itself. Therefore, if the Higgs field is a composite field it does not appear in the fundamental lagrangian and hence do not disturb its naturalness.

Dimopoulos and Susskind[8] employed confining gauge forces (technicolor) to obtain scalar bound states which could substitute the Higgs field (see Chapter 2). The fermions are still considered as elementary. The Higgs field is a fermion-antifermion composite field. The quarks and leptons must couple to this composite field in order to produce their masses. This coupling requires again new scalar fields that cause naturalness to break down at some 30 TeV which is more satisfying than 10^3 GeV; naturalness has therefore been improved.

At this stage, 't Hooft[28, 26] speculated that it can be even more improved if the observed quarks and leptons are composites (see later).

Compared with the energy scale on which the binding forces take place, the composite fermions must be almost massless. On the other hand the underlying theory should lead to the standard model at the overlying level. These facts suggest that the quarks and leptons are bound states of some unbroken "strong color" SU(N) group.

As the massleness of the quarks and leptons cannot be an accident, a chiral symmetry must be present in the fundamental theory. However, this chiral symmetry can lead to inconsistency in the theory once it is spontaneously broken as in QCD. Hence, a likely solution is that the chiral symmetry is not completely broken. The partial breaking would leave some states massless which hopefully will coincide with the observed quarks and leptons.

The problem now is to find these QCD-like models where the chiral symmetry is not spontaneously broken.

't Hooft considered a set of fundamental fermions bound by some hypercolor gauge group G_c with a global flavor symmetry, G_F. In general, the confinement at Λ_{HC} will be accompanied by some spontaneous breaking of the global symmetry $G_F \longrightarrow H_F$, with fermions protected from acquiring mass by the preserved chiral symmetry in H_F. This will be illustrated in a concrete example, the rishon model in the next chapter.

To determine which and how many H_F-representations of massless composite fermions appear, 't Hooft[28] postulated several consistency conditions. The proposed conditions on the spectrum of massless bound states provide severe constraints on any composite model.

In 't Hooft example there are two types of anomalies associated with flavor transformations:

i. those associated with $H_F \times G_C$. Only U(1) invariant subgroups of H_F contribute here. They correspond to small violations of H_F symmetry. Only the anomaly free part of the flavor groups is considered. Thus in QCD with N flavors H_F is not

U(N) x U(N)

but SU(N) x SU(N) x U(1);

ii. those associated with H_F and which are removed by adding new "spectator" fermions coupled to H_F alone.

At small distance (i.e. high energies $\sim \Lambda_{HC}$) the group seen is the gauge group $G_C \times H_F$ with chiral fermions in several representations of this group. Fermions with trivial representation under G_C (i.e. singlets) are put in the original theory if necessary to make the theory anomaly free. All the anomalies are therefore cancelled by construction.

At low energies, lower than Λ_C, only H_F and its gauge fields are seen. Light bound states are coupled to these gauge fields and form new representations r of H_F with left or right-handed chirality. The number of left-handed and right-handed fermion fields in the representations are given by unknown indices $\ell(r)$. The spectator fermions are unchanged.

As the original theory is anamoly free and thus renormalisable, the derived effective theory must also be renormalisable. Hence the anomalies must vanish at the overlying level as well. The spectator fermions are the same in both the original lagrangian and the effective lagrangian, they are therefore discarded without any effect on the anomaly cancellation requirement. The latter states that the values of the H_F- anomalies should not depend on whether they are calculated in terms of fundamental fields or in terms of composite fields.

't Hooft studied the constraints on flavor group for SU(3) and SU(5) gauge theories. He did not succeed in finding a model which conserves chiral symmetry. The set of $\{\ell(r)\}$ he was looking for was not physically acceptable (non- integer values).

The net contribution of representation r to an anomaly is $\ell(r)\,A(r)$, where $A(r)$ is the anomaly of a single member of representation. The anomalies are required to be the same whether evaluated in terms of preons $(1, \square)$ and $(\square, 1)$ (under $SU(N)_L \times SU(N)_R$) or composite states. This means that summing over all composite representations r ,

$$\sum_r \ell(r)\,A(r) = A(\square) \qquad (1)$$

where $\square$ is Young-tableau representation for the fundamental representation of G_F ie r_0 .

$$\sum_r \ell(r)\,A(r) \equiv \left(\sum_L - \sum_R\right) d^{abc}(r)$$
$$A(\square) \equiv n_c \left(d^{abc}(r_0)_L - d^{abc}(r_0)_R\right).$$

This put a constraint on $\ell(r)$. Since $A(r)$ depends on H_F , equation (1) is N-dependent. However 't Hooft argued that hypercolor which is responsible for the binding of preons and thus for determining the $\{\ell(r)\}$ should not depend on the number of flavors, i.e. the number of fundamental fermions. Equations (1) do not fix the values of $\ell(r)$ completely and hence additional equations for the $\{\ell(r)\}$ are needed. They are called the Appelquist-Carrazone-Symanzik (A-C-S) decoupling equations. They follow from requiring the composites to be such that, when one of the fundamental fermions acquire a large mass, the remaining unbroken chiral symmetries allow all composite particles containing this fermion to get a mass also. This decoupling theorem applies to any renormalisable theory with different mass scales (Λ_{HC} and Λ_C).

In more details G_F being $SU_L(N) \times SU_R(N) \times U(1)$, when one of the N fermions becomes massive with mass m, G_F is reduced into $G_F' \subset G_F$

$$G_F' = SU(N-1)_L \otimes SU(N-1)_R \otimes U(1) \otimes U(1)_H$$

where $U(1)_H$ corresponds to the heavy fermion.

When $m \rightarrow \infty$. the effects due to this fermion should also disappear and they can do this only by becoming heavy . To become heavy, they should contain an equal number of left-handed and right-handed fermions, i.e. they form representation r' of G'_F with total index $\ell'(r') = 0$.

$\ell'(r')$ is the number of massless multiplets of G'_F in r' .

Therefore $$\ell'(r') = \sum_{r \text{ with } r' \subset r} \ell(r)$$ must vanish .

The main result is that when the A.C.S. theorem consequences for the fermion spectrum are taken into account along with the anomaly constraints, there are no solutions for any number of color.

't Hooft concluded that the maximal flavor group is SU(2). For instance he shows that in a model where $G_c = SU(3)$

$$G_F = SU(N)_L \times SU(N)_R \times U(1)$$

cannot completely preserve its global flavor symmetry for $n > 2$. In other words, the number of zero mass quarks must be smaller than three.

Frishman et al[53] analysed the nature of zero-mass singularities implied by 't Hooft axial anomaly equations. They emphasised with 't Hoof that massive states do not contribute to the anomaly equation in absence of chiral symmetry breaking. They claimed that the axial anomaly of three-point functions of color-singlet currents (e.g. $J^\mu_R \, J^\mu_{B-L} \, J^\mu_Y$, $U(1)_Y$ being the axial anomaly in the rishon model) in quark confining theories implies the presence of massless bound states in the physical spectrum. These can be either fermions or Goldstone bosons. The latter possibility is realised in QCD where the anomaly equations with fermions do not hold since the number of zero-mass quarks is larger than two. Therefore a breakdown of chirality is inevitable.

There remains, however the possibility that chiral symmetry is only partially broken as in the rishon model, leaving a few massless chiral bound states. The success of such models resides in the reproduction of the observed quark and lepton spectrum.

5.2 Alternative Models Satisfying 't Hooft Conditions

G_c-groups other than $SU(N)$ have been considered. For instance the $O(n)$ gauge group which is responsible for the binding of quarks and leptons into composite states of preons in the model of Barbieri, Maiani and Petronzio[29]. The metaflavor symmetry is an SU(N) global ungauged symmetry. U(1) is eliminated because it leads to Adler-Bell-Jackiw (A-B-J) anomalies. The preons transform as the n-dimentional (n-odd), vector, real representation of $O(n)$ and metacolor currents are anomaly free. n is restricted to be odd, to allow for composite $O(n)$-singlet states with half-integer spin. Moreover n satisfies

$$n > 2 + 2N/11 ,$$

in order to keep metacolor IR.(infra-red) divergent, i.e. asymptotically free. Then, the observed interactions are obtained by gauging the metaflavor group. The gauging of the full SU(N) is however not allowed by anomalies with the assumed representation of preons.
They found solutions to the consistency conditions proposed by 't Hooft. The simplest solution is precisely n-families of quarks and leptons. It can be interpreted as composite states of one preon and $O(n)$-glue.
In this scheme[29], the binding has been obtained without spontaneous breaking of the chiral symmetry. The latter is not easily "made to break". It requires technicolor besides $O(n)$ forces. The global chirality of normal fermions could be explicitly broken by their residual metacolor interactions. Hence, the techni-preons could be responsible for the breaking of the gauged metaflavor subgroup and also for the normal fermion masses.

Bars and Yankielowicz[54] proposed a model for composite fermions which satisfies the anomaly constraints. The decoupling theorem is also obeyed when the preons acquire mass and accordingly when a preon mass becomes very large the composite becomes very massive too by breaking the chiral symmetry spontaneously. However, preons of small mass can bind into massless composites. This weaker form of the decoupling theorem is adopted to allow for solutions in left-right symmetric QCD-like metacolor theories. Indeed, if the A-C-S decoupling theorem is not taken into account, there are solutions of the anomaly constraint equations for $SU(n)_c \times SU(N)_L \times SU(N)_R \times U(1)$ theories. It has even been argued[56] that the A-C-S theorem is not absolutely reliable and may be dropped.

The model[54] is realistic enough to account for $SU(3)_c \times SU(2) \times U(1)$ at the overlying level. It predicts new, presumably heavy fermions in addition to the observed quarks and leptons, but the number of composites is still larger than the number of preons.

The electroweak interactions are not residual forces of broken meta-flavor symmetries in the composite model of Barbieri et al[55]. They are directly related to Λ_H , the scale of metacolor (or hypercolor).

The model consists of fundamental preons, two of which are fermions, two others are scalars. It is left-right symmetric and it satisfies the anomaly constraints. The standard model is not exactly recovered at the overlying level. Moreover this model predicts pseudo-Goldstone bosons, one Goldstone boson - a Mojoron - associated with spontaneous breaking of lepton number and required by the existence of massive neutrino .

5.3 Proton Decay

The stability of proton is usually attributed to the conservation of baryon number.

Baryon number was believed to be an exactly conserved quantity and any violation of this conservation law occurring in a model meant that the model had to be reconsidered to make sure the proton was stable.

In composite models, quarks and leptons are bound states of the same objects, the preons. Hence, baryon and lepton numbers are not well defined at the fundamental level. Then, violations of baryon and lepton numbers are expected at the composite level, leading to proton decay among other things.

Let us contrast the conservation of baryon number with another conservation law which is not questioned, that of electric charge.

The electric charge, on the contrary of baryon number is directly associated with a gauge symmetry. Any violation of electric charge would lead to a massive photon. However as m_γ is estimated to be smaller than 6.10^{-22} MeV, it is reasonable to assume that Q, the electric charge, is exactly conserved. For baryon number, the long range forces associated with it couple to mass (not to baryon number). It is not associated with a gauge force symmetry.

Therefore, if baryon number is exactly conserved, it must be because of an unbroken global symmetry as in the standard model.

It is however possible to have interactions which violate the quantum number by a small amount without causing further difficulties.

Actually, the validity of baryon number conservation is considered as an experimental question, although all known interactions are not likely to violate baryon conservation at experimentally observable rate.

Indeed, the time scale of weak interactions is about 10^{-10} second and that of strong and electromagnetic interactions is even less, while the present lower limit on the proton lifetime is $\sim 10^{30}$ years. Therefore, these interactions probably conserve baryon number, although a small violation cannot be ruled out. 't Hooft[59] suggested that baryon and lepton numbers are violated by vacuum tunneling effects of weak interactions, e.g. reactions like

$$p + n \longrightarrow e^+ + \bar{\nu}_\mu$$

and

$$p + n \longrightarrow \mu^+ + \bar{\nu}_e$$

will occur in SU(2) x U(1) model. However, this source of baryon violation is negligible in practice; the decay and cross-sections are proportional to

$$\exp(-4\pi \sin\theta_W/\alpha) = \exp(-400) \quad !$$

Therefore, it is likely that if proton does decay at observable rate, it is because of a new interaction.

This is the case of GUT models where baryon number is explicitly violated by gauge and Yukawa couplings of fermions to the new bosons in the theory. The quarks and leptons belong to the same representation of a group G; they are connected by means of superheavy gauge bosons. Thus baryon number violation is attributed to the fact that leptoquarks and Higgs bosons are very heavy; the GUT mass scale is about 10^{15} GeV (see section 2.3).

This is not the case of composite models where the compositeness scale of quarks and leptons, Λ_H could be as low as $500 - 10^3$ GeV. Theoretically, it is hoped that Λ_H is not too large, especially if the weak bosons (W and Z) are composites.

In fact, in composite models there are "light" particles with very small masses in comparison with Λ_H (e.g. quarks, leptons, gluons, W, Z ...) and presumably "heavy" particles with masses of order Λ_H .

Processes involving light particles can exchange heavy particles through non-renormalisable high-dimension terms in the effective langrangian. Thus, a high dimension term in the effective Lagrangian involving only light particles will not induce proton decay. In general, the simplest baryon number violating (6-dimensional) term is an effective four-fermion interaction of the form $uude^-$. It contains a coefficient Λ_H^{2} , possibly indicating the exchange of heavy composite vector particles. Such a term yields a proton lifetime:

$$\tau_p \sim \Lambda_H^4 / m_p^5 \sim 10^{30} \quad \text{years} ,$$

leading a limit of $\Lambda_H \gtrsim 10^{15}$ GeV .

Therefore, proton decay must be forbidden in lowest order, in composite models allowing such a term in order to allow for smaller values of Λ_H .

In the Rishon model for instance, the leading proton decay violate (B - L), e.g.

$$p \longrightarrow \nu \pi^+$$

and allows $\Lambda_H \sim 3 . 10^7$ GeV .

CHAPTER 6

THE RISHON MODEL

6.1 The Model

6.1.a. An economic scheme

As seen briefly in the third section of the fourth chapter, the Rishon model [42, 44] consists of two spin-$\frac{1}{2}$ objects, the T-rishon charge $\frac{1}{3}$ and the neutral V-rishon. Quarks and leptons are composites of three rishons or three anti-rishons.

In this scheme the standard model appears only at the composite level. It is described by an effective lagrangian at low energies. The fundamental lagrangian includes massless fermions and gauge bosons: the photon, the gluons and (their $SU(N)_H$ analog) the hypergluons.

The weak bosons and the scalars are composites and hence not fundamental (see later).

The theory is locally gauge invariant under the direct product of the gauge groups present, i.e. color, electromagnetism and hypercolor. Hypercolor is introduced to ensure the construction of (almost) massless composites from massless rishons. It is a color-type symmetry which keeps the rishons confined (or else they would be observed).

The minimal gauge group is $SU(N)_H \times SU(3)_c \times U(1)_{EM}$ (the photon is assumed to be fundamental). The massless rishons belong to the N-representation of $SU(N)_H$, hence a composite hypercolor fermion can only be made of N rishons (Nodd). The most economic N, is therefore N = 3. Moreover, as the smallest values taken by Q and B-L are $\frac{1}{3}$ and as $|Q| \leqslant 1$, $|B-L| \leqslant 1$, three rishons with $|Q| = 1/3, 0$ and $|B-L| = 1/3$ (table 2.) are sufficient to construct composites with the right values of Q and B-L.

In this argument two values of Q are needed, thus at least two rishons are necessary. It is crucial to keep the smallest number of different fundamental constituents otherwise there will be too many of them.

The composite fermions, quarks and leptons are hypercolor singlets (see later); furthermore they come in three colors. Therefore the Tand V rishons cannot have the same color.

Consider the quantisation of charge; it implies that 3Q equal the color triality. If T and V are assigned to any pair of different color trialities, the color-charge relation is guaranteed since a quark (or a lepton) is made of three rishons.

The simplest representations of $SU(3)_c$ are 1, 3 and $\bar{3}$.

The color assignments of T and V can then be,

$$(3, \bar{3}), \qquad (1, 3) \qquad \text{and} \qquad (\bar{3}, 1).$$

$(3, \bar{3})$ assignment is retained because it satisfies Fermi statistics (at the composite level). It follows that T and V transform like $(3, 3)_{\frac{1}{3}}$, $(3, \bar{3})_0$ under $SU(3)_H \times SU(3)_c \times U(1)_{EM}$, the local gauge group of the theory.

6.1.b. lagrangian and symmetries

For the theory to be realistic, some conditions are required. It must have at least three generations of massless quarks and leptons as well as very light $W^{\pm}$ and Z bosons (in comparison with Λ_H, the hypercolor scale) at least for $SU(2) \times U(1)$ but preferably for $SU(2)_L \times SU(2)_R \times U(1)$ since the model is left-right symmetric (see section 2.b.).

The massless particles appearing in the underlying lagrangian are the rishons and seventeen gauge bosons. The latter correspond to the color octet (gluons), the hypercolor octet (hypergluons) and the photon. Their masslessness is certain since hypercolor, color and electromagnetism are exact gauge symmetries guaranteed by the construction of the model; they are not broken at any stage.

The fundamental lagrangian[22] of the rishon model is:

$$\begin{aligned} \mathcal{L} = \; & \bar{T}_{jj'} \left(i\, \delta^j_k \delta^{j'}_{k'} \cdot \not{\partial} + g_H\, \delta^j_k (\lambda^a)^{j'}_{k'} \cdot A^a_H + g_c\, \delta^{j'}_{k'} (\lambda^a)^j_k A^a_c + \right. \\ & \left. + \tfrac{1}{3} e\, \delta^j_k \delta^{j'}_k \not{A} \right) T^{kk'} + \bar{V}^j_{j'} \left(i\, \delta^k_j \delta^{j'}_{k'} \not{\partial} + g_H\, \delta^k_j (\lambda^a)^{j'}_{k'} A^a_H \right. \\ & \left. + g_c\, \delta^{j'}_{k'} (\lambda^a)^k_j A^a_c \right) V^{k'}_k - \tfrac{1}{4} (F_{EM})_{\mu\nu} (F_{EM})^{\mu\nu} + \\ & - \tfrac{1}{4} (F^a_c)_{\mu\nu} (F^a_c)^{\mu\nu} - \tfrac{1}{4} (F^a_H)_{\mu\nu} (F^a_H)^{\mu\nu} \; ; \end{aligned}$$

$$(F_H^a)_{\mu\nu} = \partial_\mu A^a_{H\nu} - \partial_\nu A^a_{H\mu} + g_H f_{abd} A^b_{H\mu} A^d_{H\nu};$$

$$(F_{EM})_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu , \ldots$$

Upper and lower color indices correspond to the $\underline{3}$ and $\underline{\bar{3}}$ representations where T and V are Dirac four-spinors representing a right-handed and left-handed massless fermion.

$a = 1, \ldots 8$ index for SU(3) generators; λ^a are the corresponding 3 x 3 matrices.

$j, k = 1,2,3$ are the color indices

and $j', k' = 1,2,3$ are the hypercolor indices.

The lagrangian includes the couplings of rishons to gluons and hypergluons, as well as gluon and hypergluon self-couplings, but no mass terms.

$A^a_{H\mu}$, $A^a_{c\mu}$, A_μ are the hypergluon, gluon and photon fields.

The free parameters of the lagrangian are g_H, g_c and e. The two coupling constants g_H and g_c are presumably inequal, say $g_c < g_H$.

If Λ_H is the scale at which $g_H \simeq 1$ then $g_c(\Lambda_H) < 1$. The two scale parameters then obey $\Lambda_c \ll \Lambda_H$.

This lagrangian conserves n_T and n_V, the net numbers of T and V rishons. It therefore possesses a global U(1) x U(1) symmetry. The two U(1) quantum numbers are chosen to be

$R = \frac{1}{3}(n_T + n_V)$ the total rishon number

and $Y = \frac{1}{3}(n_T - n_V)$ such as $Y = B-L$ (baryon number minus lepton number).

Since,

$$Q = \frac{1}{3} n_T = \frac{1}{2}(R + Y)$$

it follows that,

$$\frac{1}{2} R = (I_{3L} + I_{3R})$$

Moreover, as there are no masses in the fundamental lagrangian, there is a chiral $[U(1)]^4$ symmetry besides two axial U(1) symmetries. One of them, $U(1)_Y$ corresponds to a divergent current $Y_\mu = \bar{T}\gamma_\mu\gamma_5 T - \bar{V}\gamma_\mu\gamma_5 V$ which however yields only an electromagnetic anomaly.

Thus the axial charge Y is conserved and could eventually be identified to $(I_{3L} - I_{3R})$ so that,

$$\frac{1}{2} Y = I_{3L} - I_{3R}$$

note that $$Y(T_L) = -Y(T_R) = -Y(V_L) = Y(V_R) = 1$$

and $$R = \begin{cases} 1/3 & \text{for } T \text{ and } V \\ -1/3 & \text{for } \bar{T} \text{ and } \bar{V} \end{cases}$$

note also that the two vector quantum numbers[4] can be chosen to be $T = Q = \frac{1}{3} n_T$ and $\Upsilon = \frac{1}{3} n_V = V$.

One of the two vector symmetries is then $U(1)_{EM}$. Its corresponding current is $J^T_\mu = \bar{T}\gamma_\mu T \equiv J_\mu^{EM}$. The second vector current is $J^V_\mu = \bar{V}\gamma_\mu V$.

The second $U(1)_X$ symmetry is not conserved. The divergence $\partial_\mu X_\mu$ of the current $X_\mu = \bar{T}\gamma_\mu\gamma_5 + \bar{V}\gamma_\mu\gamma_5 V$ depends on non-abelian anomalies. $U(1)_X$ must therefore be broken.

The full symmetry of the lagrangian is then $SU(3)_H \times SU(3)_C \times U(1)_R \times U(1)_{B-L} \times U(1)_\Upsilon$. The model contains an exact flavor symmetry

$$U(1)_R \times U(1)_{B-L} \times U(1)_\Upsilon .$$

The three U(1) charges R, B-L and Y correspond to the three charges of V the global flavor symmetry $SU(6)_L \times SU(6)_R \times U(1) = G_F$,

(There are six left and right-handed rishons in the theory each is in the 3-dimensional representation of $SU(3)_H$.).

The fourth axial charge X is defined to be X = 1 for left-handed fermions and antifermions while X = -1 for the right-handed ones. The non-conserved axial $U(1)_X$ symmetry is broken (by hypercolor instantons effects)[59] to a discrete conserved axial subgroup Z_{12}. The global axial charge X is a suitable candidate for a generation label. Higher generations are presumably excitations of the first one. However, they can neither be radial nor orbital, since the mass-splittings between the generations are smaller than the inverse radius of the composite system (see Chapter 3).

They may be formed by addition of fermion pairs $T_L \bar{T}_L V_L \bar{V}_L$ or $T_R \bar{T}_R V_R \bar{V}_R$ which carry $X = \pm 4$. These added fermions do not carry quantum numbers under the Lorentz group, hypercolor, color and the electro-weak group. The unique allowed quantum number is X. It distinguishes between analog fermions from different generations which except for this are the same.

At this stage quarks and leptons are massless. Hence, the chiral symmetry is dynamically broken by composite scalar fields in order to generate masses. The scalar fields (with $X \neq 0$) may develop VEV leading to a dynamical breaking of Z_{12} symmetry (see 6.3.a.).

Different scalar fields would then lead to different matrix elements in the fermion mass matrix. It follows relations between Cabbibo angles and fermion masses[9]. The physical quarks and leptons are eigenstates of the diagonalised mass matrix.

6.2 Composite Fermions - Weak Interactions

6.2.a. quarks and leptons

As the energy decreases below Λ_H , for instance at Λ_C , all hypercolor non-singlets become confined and only $SU(3)_H$ -singlets survive as physical particles. Such particles are three-rishons or three-antirishons states (TTT, TTV, TVV and VVV and their antiparticles).

Assuming that for each of these combinations, the only light state is the lowest color state (see table 1), the observed spectrum of quarks and leptons in one generation is recovered. Combinations involving rishons and antirishons at the same time, e.g. $TT\bar{V}$, $T\bar{V}\bar{V}$, etc. correspond to confined "hyperfermions" (non-singlets). Their effective masses are probably of order Λ_H .

A priori, the masses of the hypercolor-singlets cannot be predicted since the corresponding confinement mechanism is unknown. However, it is suspected that if some composites have small masses their overlying theory must be "natural" and in particular the effective lagrangian of the small-mass composites should be renormalisable. Although, this does not guarantee that the fundamental lagrangian induces the effective lagrangian, it is a necessary condition.

The minimal color-multiplet of table 1, i.e. TTT-singlet, TTV-singlet, TVV-antitriplet, etc., are assumed to be approximately massless on the scale of Λ_H . Hopefully, they obey the requirements of a natural theory. They reproduce precisely the quantum numbers of one generation of fermions (i.e. electric charges, (B-L) values and colors). Also, for each value of color and (B-L) there are two different values of R (see table 1) and hence two different composite fermions. They can be identified with the doublets of $SU(2)_L \times SU(2)_R$.

As to the rishon wave-function, it must be antisymmetric in order to satisfy Fermi-statistics,

e.g. $\quad e^+_L = (T_R T_R) T_L \quad , \qquad \nu_{eL} = (V_R V_R) V_L \quad$ etc..

Let us consider TTT-singlet for instance; it is a $(1, 1)$ lepton of $SU(3)_C \times SU(3)_H$ totally antisymmetric in color and hypercolor. Left and right-handed rishons transform like $(1/2, 0) + (0, 1/2)$ under the Lorentz group. The TTT-state must therefore transform like $(0, 1/2) + (1/2, 0)$ under $SU(2)_L \times SU(2)_R$, thus giving rise to a composite e^+ with $J = 1/2$; $J = 3/2$ cannot occur. However, in the case of quarks, although $J = 1/2$ states are consistent with Fermi-statistics, $J = 3/2$ states are not eliminated. Eventually, it could be argued that massless particles cannot develop with $J = 3/2$ or higher [28, 47] the problem is left open.

In a word, provided the lowest color state only are allowed, the model reproduces the correct spectrum of the first generation of quarks and letpons with all quantum numbers.

There remains some questions concerning the $SU(2)_L \times SU(2)_R \times U(1)$ approximate gauge symmetry, the anomaly constraints and the massleness of these states.

hypercolor		1	1
Minimal color	R/2 / B-L	1/2	-1/2
1	1	TTT (e^+)	$\bar{V}\bar{V}\bar{V}$ ($\bar{\nu}_e$)
3	1/3	TTV (u)	$\bar{V}\bar{V}\bar{T}$ (d)
$\bar{3}$	-1/3	TVV ($\bar{d}$)	$\bar{V}\bar{T}\bar{T}$ ($\bar{u}$)
1	-1	VVV (ν_e)	$\bar{T}\bar{T}\bar{T}$ (e^-)

table 1: hypercolor - singlet 3-rishon or 3-antirishon composite fermions assuming the minimal color.

6.2.b. the weak interactions

Hypercolorless leptons contain hypercolored rishons inside a radius of order Λ_H^{-1}. Two colorless hadrons containing colored quarks interact via hadronic forces. Similarly, two leptons interact with each other via short range residual hypercolor forces, which hopefully are identified with the weak interactions. To see how much this is possible, let us consider the "observed" properties of the short range hypercolor forces.

At the overlying level, hypercolor is confined and only $SU(3)_H$-singlets appear in the effective langrangian. Since the original symmetry is $SU(3)_C \times SU(3)_H \times U(1)_R \times U(1)_{B-L} \times U(1)_Y$, the symmetry of the effective lagrangian would appear to be $SU(3)_C \times U(1)_R \times U(1)_{B-L} \times U(1)_Y$.

However, at the composite level there are pairs of hypercolor fermions (VVV and $\bar{V}\bar{V}\bar{V}$) with the same $SU(3)_C \times U(1)_{B-L}$ properties. This corresponds to an SU(2) global symmetry of the low energy lagrangian.

This weak interaction group SU(2) is not of course a gauge symmetry of the fundamental lagrangian; that is to say, there are no fundamental massless gauge bosons corresponding to $W^{\pm}$ and Z. It is not even a global symmetry since the T and V rishons transform differently under $SU(3)_H \times SU(3)_C$. However at the overlying level, all composite fermions are hypercolor singlets and a global symmetry $SU(2)_L \times SU(2)_R$ emerges due to the interchange of

$$T_L \longrightarrow \bar{V}_L$$
$$T_R \longrightarrow \bar{V}_R$$

(see table 2).

Hopefully, massive $W^{\pm}$ and Z will appear in the effective lagrangian. The latter is renormalisable. Moreover the theory posses composite Higgs fields which provide the masses. Therefore, the effective theory must be like the standard model since there are no known theories possessing these features.

To make sure the effective theory is renormalisable, it is required that the composite bosons are very light compared with Λ_H. It follows an effective lagrangian locally invariant under the gauge theory of the vector boson couplings and therefore renormalisable. Indeed, if the effective lagrangian

Fermion	Color	$SU(2)_L \times SU(2)_R$	I_{3L}	I_{3R}	B-L	Q
$e^+_L = (TTT)$	1	$(0, 1/2)$	0	1/2	1	1
$\bar{\nu}_{eL} = (\bar{V}\bar{V}\bar{V})$	1	$(0, 1/2)$	0	- 1/2	1	0
$u_L = (TTV)$	3	$(1/2, 0)$	1/2	0	1/3	2/3
$d_L = (\bar{V}\bar{V}\bar{T})$	3	$(1/2, 0)$	- 1/2	0	1/3	- 1/3
$\bar{d}_L = (TVV)$	$\bar{3}$	$(0, 1/2)$	0	1/2	- 1/3	1/3
$\bar{u}_L = (\bar{V}\bar{T}\bar{T})$	$\bar{3}$	$(0, 1/2)$	0	- 1/2	- 1/3	- 2/3
$\nu_{eL} = (VVV)$	1	$(1/2, 0)$	1/2	0	- 1	0
$e^-_L = (\bar{T}\bar{T}\bar{T})$	1	$(\frac{1}{2}, 0)$	- 1/2	0	- 1	- 1

<u>table 2</u>: left_handed fermions of the first generation

is renormalisable (except for Λ_H^{-n} terms) it must be approximately gauge invariant under $SU(2)_L \times SU(2)_R \times U(1)$, with higher dimension terms breaking the symmetry.

However, the small masses of the weak bosons requires a new symmetry principle. All it can be suggested is that the composite weak bosons as well as the fermions acquire mass through a Higgs mechanism governed by scalar condensates similarly to technicolor.

Technicolor has been first introduced to replace the usual Higgs mechanism. It involves new complication due to the technicolor gauge interactions and the new technifermions (see Chapter 2).

However, King[61] argues that, if the technifermions are composite of preons they will appear only at the composite level and the underlying theory may be simple.

He introduces technipreons T' and V' in addition to the T and V rishons. They transform as

$$V = (3, 1, 3, 0) \qquad T = (3, 1, \bar{3}, -1/3)$$
$$V' = (3, M, 1, 0) \qquad T' = (3, \bar{M}, 1, -1/3)$$

under the local gauge group $SU(3)_H \times SU(M)_{TC} \times SU(3) \times U(1)_{EM}$, where $SU(M)_{TC}$ is the technicolor gauge group.

The hypercolor singlets appearing at the composite level are the quarks, the leptons and the technifermions. The latter condensate at $\Lambda_{TC} \sim 1$ TeV ($\Lambda_{HC} \gg \Lambda_{TC}$) forming the usual Higgs scalars. $SU(3)_c$ and $SU(M)_{TC}$ can be unfi ed into a gauge group $SU(N)_S$, $N = M+3$. At some energy Λ_S the symmetry breaks down, $SU(N)_S \longrightarrow SU(M)_{TC} \times SU(3)_c$.

Thus the model contains again two preons only which transform under $SU(3)_H \times SU(N)_S \times U(1)_{EM}$ as $V = (3, N, 0)$ and $T = (3, \bar{N}, -1/3)$.

However, the origin of this symmetry breaking is unknown along with the spectrum of the technifermions.

6.3 Difficulties and Problems

6.3.a. chiral symmetry breaking

't Hooft[28] analysed the restriction imposed by axial anomaly equations on the fermionic spectrum in confining theories with massless quarks (see Chapter 5).

In the rishon model, one of the two axial currents X_μ, presents a non-vanishing anomaly at the underlying level. It follows axial anomaly equations corresponding to this current which should remain true at every level (in energy) of the theory. A problem arises at zero-momentum where massive composite fermions do not obey the consistency conditions. Massless particles only contribute to the cancellation of the vanishing of the amplitude. Whereas, massive states do not contribute to the anomaly equations in the absence of chiral symmetry breaking. The continuous chiral symmetry $U(1)_Y$ is then broken to a chiral (discrete) sub-symmetry Z_6 which must remain intact to prevent quarks and leptons from acquiring masses of order Λ_H. A further breaking of $U(1)_Y$ will provide the composite fermions with masses at the effective level.

Two ways of breaking $U(1)_Y$ are suggested. A Goldstone boson is needed in both cases. It exists within the theory and decouples from some composite fermions kept massless by the discrete Z_6 symmetry. In either ways, the symmetry breaking occurs via complicated procedure. Whether this is justified or not can be answered by a proper dynamics.

King[58] noted that it is quite sensible to take the limit $g_c \to 0$, $\alpha \to 0$ at energies $\sim \Lambda_{HC}$; g_c and α are the QCD and QED couplings respectively. This is equivalent to switch off $SU(3)_c \times U(1)_{EM}$ and yields the global flavor symmetry,

$$G_F = SU(6)_L \times SU(6)_R \times U(1)$$

which exactly coincides with 't Hooft's example with $n=6$ (see Chapter 5). It is therefore expected to be spontaneously broken to $SU(6)_{L+R}$ with all fermions acquiring mass $\sim \Lambda_{HC}$. That is to say that whereas quarks and leptons are almost massless, masses tend to Λ_H when g_α, α tend to zero, (Hierarchy problem).

Thus, if the rishon model and 't Hooft conditions are correct there is only one way out: a partial breaking of G_F; $G_F \to H_F$ where H_F is in principle unknown.

Squires argued that

$$SU(6)_L \times SU(6)_R \longrightarrow SU(3)_L \times SU(3)_R$$

was the minimal breaking allowed consistent with the A-P-C decoupling and therefore it was assumed that

$$H_F = SU(3)_L \times SU(3)_R \times U(1)_R \times U(1)_{B-L}$$

with $\quad 6 \to 3 + \bar{3}$

and at energies $\ll \Lambda_{HC}$, QCD is given by

$$SU(3)_L \times SU(3)_R \longrightarrow SU(3)_{L+R} \equiv SU(3)_c$$

and with

$$T_L = (\underline{3}, \underline{1}, 1/3, 1/3) \qquad T_R = (\underline{1}, \underline{3}, 1/3, 1/3)$$
$$V_L = (\underline{\bar{3}}, \underline{1}, 1/3, -1/3) \qquad V_R = (\underline{1}, \underline{\bar{3}}, 1/3, -1/3)$$

H_F can be regarded as the $g_c = 0$, $\alpha \neq 0$ limit of the rishon model with $U(1)_Y$ broken as in Harari and Seiberg scheme.

The first generation of quarks and leptons is probably recovered. However all the particles are still massless. H_F must be broken, to allow for masses. The color forces may be responsible for this breaking[16] and hence,

$$H_F \longrightarrow H'_F = SU(3)_c \times U(1)_{B-L} \times U(1) \times Z_6$$

where Z_6 is a subgroup of H_F. More specifically, it is a $U(1)_Y$ subsymmetry as in Harari and Seiberg scheme.

6.3.b. <u>neutrino masses</u>

The neutral V-rishon has no trivial color and hypercolor; these acquire a meaning only at the composite level. V and $\bar{V}$ differ by some conserved quantity and cannot mix. Therefore, the massless V-rishon cannot acquire a Majorana mass.

At the composite level, the fermions are (VVV) = ν_e and $(\bar{V}\bar{V}\bar{V})$ combinations.

They are still neutral but they are now color and hypercolor singlets. They are not distinguished by a quantum number and can obtain a Majorana mass through a $6V$-condensate, for example.

Six V-rishons are the simplest Lorentz scalar combinations possessing a net R-number and therefore conserving hypercolor, color and electric charge. They can be identified with the Higgs scalars of $SU(2)_L \times SU(2)_R \times U(1)$. They can be used to break R-number B-L, P and C at the same time. Parity (P) and charge conjugarson (C) are spontaneously broken by the residual weak interactions through the $6V$-condensates although hypercolor, color and EM interactions conserve C and P.

Moreover, at energies above Λ_H, the number of T, $\bar{T}$, V and $\bar{V}$ is equal; matter-antimatter symmetry is conserved, e.g. Hydrogen atom has $4(T + \bar{T}) + 2(V + \bar{V})$ and it is neutral.

At the scale in which $6V$- condensates form the number of V and $\bar{V}$ is not equal any more, (B-L) is not conserved. The neutrinos and W_R-boson acquire masses which are determined by the VEV $\langle 6 \rangle$; the left handed neutrino mass depends on its corresponding lepton ($\sim m_e^2/m(\nu_R)$ very small).

These $6V$-condensates put everything in order. Unfortunately, the dynamics to manipulate it is missing, for instance $\langle 6 \rangle$ calculation.

6.3.c. <u>proton decay</u>

In a (B-L)-conserving composite model, the proton may decay through the process

$$u + u \longrightarrow \bar{d} + e^+ \qquad (1)$$

or equivalently in terms of rishons

$$(TTV) + (TTV) \longrightarrow (TTT) + (TVV)$$

If such a process proceeds in lowest order of the basic interaction of the fermionic constituents, the relevant scale is $\Lambda \sim 10^{15}$ GeV (the GUT scale) which is not consistent with the scale of Higgs in composite models. Squires[40] suggested that this may be inconsistent at first sight only. This suggestion is based on the following naive argument: as the reaction (1)

is suppressed by a factor proportional to M_q/Λ_H where M_q is the quark mass, the relevant scale may be adjusted by the suppression factor, thus leading approximately the radius of grand unification mass, i.e. 10^{15} GeV.
In the rishon model however, a lowest order proton decay is not allowed when considering the quarks and leptons of table (1). A second order decay may be allowed but the dominant processes are (B-L)-violating reactions[60], e.g.

$$p \longrightarrow e^- \pi^+ \pi^+ .$$

CONCLUSION

A variety of theoretical ideas associated with the notion that quarks and leptons are composites have been reviewed. There remains the problem of assembling them and unifying them in the same theory for at present, there is no completely satisfactory composite model of quarks and leptons.

From an experimental point of view, it is hoped that future high energy experiments will indicate some new physics beyond the standard model, perhaps revealing evidence of quark and lepton substructure.

The possible effects are expected from,

i. the existence of more flavors and of fermions in new (exotic) color representations;

ii. the improvement of the bounds on quarks and lepton form factors, muon ($g-2$) and rare weak decays (i.e. deviations of the weak charged currents of leptons and quarks from the V-A form); at some stage there may be evidence for non-zero radii.

From the theoretical point of view, the open problems are:

i. the implementation of chiral symmetry breaking;

ii. the fulfilment of various constraints (see Chapter 3) and 't Hooft consistency requirements (see Chapter 5);

iii. the resolution of the mass spectrum of quarks and leptons.

The present experimental and theoretical restrictions are so severe that the construction of a consistent model has not yet been possible.

However, it may be that the world is just not as simple and that a radical new idea is missing. It is interesting to compare the present situation with that which occurred some twenty years ago when the search was on for the structure of hadrons. The quarks had been originally proposed as a mathematical tool to explain the SU(3) flavor symmetry in the hadron spectrum. Because of Pauli statistics it had also been hypothesised that quarks possess a radically new degree of freedom, "color". This concept appeared very artificial at the time but it has since been confirmed and has of course formed the basis of the theory of strong interactions (QCD).

Are quarks and leptons composites? This question is far from being answered. But the situation is not hopeless; it seems rather promising.

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