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M. B. GIETZMANN

INCENTIVE ISSUES IN TRANSFER PRICING

Ph.D. THESIS

NOVEMBER 1984

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## ABSTRACT

This thesis is concerned with the transfer of resources within an organization. It is assumed that top level management wish to allocate scarce corporate resources as effectively as possible. However, top level management do not have enough information to fully assess the potential return from deployment of resources, in particular divisions of the corporation. Hence, top level management require divisional managers who hold such information to communicate it. Any resultant allocation of corporate resources will clearly affect the profit attained by a division. In addition, it is assumed divisional managers will be compensated partly on the basis of divisional profit, in order to promote higher levels of divisional management effort provision. The inter-relationship between the allocation of corporate resources and divisional management's resultant compensation, leads to an incentive problem. Divisional managers will perceive advantages from communicating information to top level management about the division's potential returns in a strategic (untruthful) fashion in order to improve their compensation.

In this thesis it is argued that opportunities for misrepresentation should be controlled. The major result of the thesis is to propose a method for allocating scarce corporate resources and compensating divisional managers, that induces them to tell the truth and provide appropriate levels of managerial effort.

## ACKNOWLEDGEMENTS

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## CHAPTER ONE

### INTRODUCTION TO THE PROBLEM OF TRANSFER PRICING

#### 1.1 General Concepts

As an economic organization grows in size and complexity, it becomes increasingly difficult for the top level decision makers (the central headquarters) to co-ordinate and control the lower levels of the organization. Therefore, there is a need to delegate responsibility and to establish a decentralized decision making process. The decentralized units that are formed will be called the divisions of the organization. Each division it is assumed will be run by a divisional manager. However, for this strategy of decentralization to be useful to the central headquarters, the central headquarters must be able to co-ordinate and control the divisional managers. This is partly done by actively appraising divisional managers' performance and linking the divisional managers' compensation to the outcome of the appraisal procedure.

The overall results of the interactions between the central headquarters and divisional managers is also appraised. For instance, shareholders external to the

decentralized decision making process of the organization will be interested in the performance of the organization. Profit is the most widely used general appraisal measure of organizations by those external to the organization. Thus, it may seem prudent for the central headquarters to appraise divisional managers' performance by measurement of divisional profitability and, hence,





encourage divisional managers to make profitable decisions. However, one of the problems the central headquarters faces in trying to measure divisional profitability is how to deal with the transfer of goods between divisions. The central headquarters has to determine some method by which to assign a "cost" for the transfer of the goods. This "cost" will be viewed as the revenue generated from the provision of the goods by the supplying division and, as the cost of ensuring provision of the goods by the receiving division. Hence, this "cost" will enter as part of the profit measure of both the supplying and receiving divisions. The "cost" of provision of a unit of the good is commonly known as the transfer price of the good.

There will be a number of procedures by which the central headquarters can assign a transfer price to interdivisional transfers. The importance of this is that different procedures may affect the performance of the various divisional managers in different ways. That is, different transfer pricing procedures may give rise to the divisional managers having different incentives to perform tasks. The central theme of this thesis is to consider the incentives for divisional managers under various transfer pricing procedures and how this affects the central headquarters.

## 1.2 Umapathy's Study of Transfer Pricing

Umapathy (1978) conducted a survey of companies in the USA which had decentralized operations. The results of the survey of 291 responding companies who claimed to have at least two profit centres (divisions) in their organization led Umapathy to conclude<sup>1</sup> that the survey

"shows that most large decentralized US manufacturing firms demonstrate some degree of interdependence between profit centres. About 85 per cent of the profit center's transfer goods and transfers of services and joint use of common facilities exist in 55 per cent and 71 per cent of the companies respectively. Thus, the notion of truly independent profit centres is rare if not absent in US manufacturing firms. The companies in our sample split up their operations into interdependent profit centers, even though this policy introduces problem areas such as transfer pricing administration and profit center performance evaluation."

It is worth noting at this point that, at the sample selection stage, Umapathy only needed to exclude the returned questionnaires of 17 firms who did not have two or more profit centres, in order to determine the sample of firms which had, at least, two profit centres. Thus, Umapathy's survey results suggest that transfer pricing procedures are widely used in large US companies.

Umapathy also requested those companies who transferred goods between profit centres to specify their transfer pricing policies that they used. Of the selected sample group, 96 per cent did so. He found<sup>2</sup> that 4 per cent of firms used a transfer pricing procedure, based on variable costing techniques, 26 per cent on full cost, 17 per cent on cost plus, 22 per cent on negotiation and 31 per cent on market price. In addition then Umapathy's study suggests that there is no one predominant transfer pricing procedure adopted by large US companies. An important question then is why do we observe such diversity in the pricing procedures adopted. One could hypothesise that the reason why more transfer pricing procedures were not based on a market price

measure is because, in many instances, a market price does not exist. This situation could easily arise as one division could be manufacturing a good which is used by another division to make the final product, and this good may be what makes the final product different from the competition's final product. Hence, it would not be in the interests of the company to have this special intermediate good sold externally to competitors and, thus, a market price for the intermediate good will not exist. However, given Umapathy's results, one must either assume that companies do not wish to use market price based transfer prices for some other reason, or that the amount of companies that can not use market price based transfer prices is a significant proportion of the overall sample.

To summarise then, Umapathy's survey suggest that transfer pricing is practised on quite a wide scale<sup>3</sup> in large US companies, but that there is a diversity of procedures adopted.

### 1.3 A Research Strategy for the Transfer Pricing Problem

Section 1.2 illustrates that transfer pricing is worthy of consideration by academics as many companies feel they have a need to establish transfer pricing procedures. From here, the academic could follow two pathways of thought to try and build a theory of transfer pricing.

One pathway could commence with the premise that, in the real world, the transfer pricing procedures adopted by companies work well because, otherwise, companies would not adopt them. Therefore, the role of developing a theory

of transfer pricing is to explain why different companies find it best to use different transfer pricing procedures. Expressed in another fashion, the role of such a theory would be to identify a company's environmental conditions that lead to a particular procedure being adopted.

However, one may wish to argue that, in the real world, firms encounter difficulties when trying to implement certain transfer pricing procedures.

Clearly, it would be desirable to demonstrate why this may be so, in order to follow this pathway. Having identified the major causes of problems, one can then attempt to identify transfer pricing procedures that overcome these problems in order to construct a theory of how transfer prices should be set.

This thesis follows the latter pathway. It is shown why existing transfer pricing procedures are problematical. One particularly important problem area which will be identified as "incentive compatibility" is discussed in detail. It is argued that, as a first step in constructing a theory of transfer pricing, this problem must be dealt with. Subsequently, a transfer pricing procedure which deals with the problem is constructed. The main evidence for adopting this view, that transfer pricing is likely to be problematical in many instances, is presented in Chapter 3. However, at this point, it would seem useful to this cause to quote some of the recent work of Kaplan (1984) which is sustentative to the view that companies find the issue of how to construct

transfer pricing procedures problematical. In Kaplan's (1984) discussion of "Developments since 1925 in Cost Accounting and Managerial Control", he comments<sup>4</sup> that

"The transfer price problem remained a thorny issue for vertically integrated or multi-divisional firms, though there are very few references to this subject until the most recent 30 years ..... the transfer price issue remains an open problem to this day, ..... it is probable that the distribution of transfer pricing practices among firms in 1983 would be indistinguishable from that of thirty years ago, when the transfer pricing problem first attracted the attention of academics."

#### 1.4 Outline of Thesis

Chapter 2 is a general discussion of "why, how and at what cost" organizations decentralize their operations and decision making. The discussion is intended to highlight the major environmental components which promote a move towards decentralization of operations and decision making. It is intended that Chapter 2 provides a conceptual background to why the issue of determining how to price interdivisional transfers arises in a modern organization. The views expressed in Chapter 2 are strongly influenced by the work of Vancil (1978).

The main role of Chapter 3 is to establish that a major problem with many proposed transfer pricing procedures is that they are not incentive compatible. Not being incentive compatible refers here to the situation where divisional managers will have a motive to lie about their division's operating conditions in order to bias the transfer pricing process in their favour. However, one may think to establish the generality of this problem, one would have to carry out the monumental feat of exhaustively considering all the proposed transfer pricing procedures and show-

ing why they do not guarantee incentive compatibility. However, in Chapter 3, instead of exhaustive consideration of all proposed procedures, attention is restricted to the most important seminal works on transfer pricing<sup>5</sup>. These are the works by Hirshleifer (1957) and the Baumol and Fabian (1964) interpretation of the Dantzig and Wolfe (1960) Decomposition Procedure. It is shown in detail that these procedures are not incentive compatible. In addition, it is suggested that, if the external environment under which divisional managers operate is characterized by uncertainty, this may further aggravate the situation by making it possible for divisional managers to lie more fully and easily about their operating conditions.

The chapter concludes with a detailed discussion of the work by Ronen and Mckinney (1970) on transfer pricing. It is argued that they were the first academic accountants who recognized the problem of non-incentive compatibility and who published a transfer pricing procedure that attempted to resolve the problem. However, it is shown in detail that they did not succeed in designing an incentive compatible transfer pricing procedure.

In Chapter 4, a Characterization Theorem will be presented which defines exactly, the rules that transfer pricing procedures must satisfy, in order to guarantee incentive compatibility. The number of transfer pricing procedures that satisfy these rules is very small and is discussed in Chapter 4. The Theorem thus provides the rationale for why exhaustive consideration was not given to the incentive-compatibility properties of all possible transfer pricing procedures, since only those defined by the Theorem guarantee incentive compatibility.

Whereas Chapter 3 was concerned with establishing and demonstrating that incentive compatibility was a problem for a range of transfer pricing procedures, Chapter 4 is the first chapter concerned with how the problem may be resolved. The chapter is based on the work of Groves and Loeb (1979). In order to appreciate the approach taken by Groves and Loeb, Chapter 4 commences with a brief introduction to the elements of game theory that are used by Groves and Loeb. Directly after this, the Groves and Loeb (1979) transfer pricing procedure is presented.

Groves and Loeb (1979) choose to describe those transfer pricing procedures that solve the central head-quarter's co-ordination problem<sup>6</sup>, while guaranteeing incentive compatibility as "optimal control mechanisms". The proof establishing that the Groves and Loeb (1979) procedure is an optimal control mechanism is therefore presented.

Next the work of Green and Laffont (1977) concerning the possibility of designing alternative optimal control mechanisms is presented. A Characterisation Theorem due to them is presented which establishes that the only optimal control mechanisms that can be designed must be of the general form of the Groves and Loeb (1979) procedure<sup>7</sup>.

Rather than recommend adoption of the Groves and Loeb (1979) transfer pricing procedure, it is argued that the procedure would encounter some difficulties if it were attempted to be implemented. Two major difficulties are highlighted and discussed. The first of the problems is

that, even though individual divisional managers may have no incentive to lie under the Groves and Loeb (1979) transfer pricing procedure, coalitions of divisional managers may gain from lying. This fact is proven and a discussion of how such coalitions may form is presented. It is, however, concluded that, although there is a potential for coalitions to form, this problem is not likely to be severe, as such coalitions would be highly unstable.

The second of the problems discussed relates to the way divisional managers would be compensated under the Groves and Loeb (1979) transfer pricing procedure. It is noted that, in Groves and Loeb (1979), they do not detail how exactly divisional managers should be compensated. They do, however, describe general properties that a compensation scheme should possess. This is sufficient for their purposes, as they assume that divisional managers will always attempt to maximize their division's evaluation measure, which is an adjusted profit figure. However, it is suggested that divisional managers will not solely be interested in maximizing their compensation. This is because the amount of effort required to carry out tasks also enters as an argument in the divisional manager's utility function. Since different tasks are likely to require differing amounts of effort, divisional managers will be primarily concerned with evaluating tasks (which achieve differing divisional profits) by considering the tradeoff between compensation and effort. It is proven that this influence destroys the guaranteed incentive compatibility properties of the Groves and Loeb (1979) procedure. The implications of this result are discussed



and it is argued that the problems of divisional managers lying and finding effort provision disutiliticious are closely related. It is argued that one can not solve one of these problems without simultaneously accounting for the other.

The chapter concludes with a discussion of a recommended modification to the original Groves and Loeb transfer pricing procedure, provided by Cohen and Loeb (1984). It is argued that this attempt to simultaneously deal with the two problems of lying and effort provision is unsatisfactory.

At this point, one may consider that the research has reached a "dead end". I say this because recall that Chapter 3 establishes that a problem with many transfer pricing procedures is that they do not ensure that divisional managers report truthfully to the central headquarters. Having established the desirability of ensuring truthfulness, Chapter 4 commences by showing that one can only guarantee truthfulness if one adopts a Groves and Loeb (1979) type transfer pricing procedure. However, the chapter concludes by discussing various undesirable properties of a Groves and Loeb (1979) type procedure.

It is, however, important to stress that this result does not end the search for transfer pricing procedures with desirable incentives for the divisional managers. The type of guarantee of truthfulness that Groves and Loeb (1979) require is very strong indeed. They require that, when one considers the incentives of one divisional manager to lie, one can only talk about guaranteeing the truthful-

ness of that divisional manager's communications if that manager would always tell the truth, no matter whether or not other divisional managers choose to lie.

This is an unnecessarily severe restriction. The concept of truthfulness required for chapters subsequent to Chapter 4 is somewhat different. It is assumed that one can talk about guaranteeing truthfulness (incentive compatibility), if a divisional manager always tells the truth, whenever all other divisional managers are telling the truth.

The above two paragraphs illustrate that, whereas Groves and Loeb (1979) could analyse a divisional manager's incentives in exclusion from those of other divisional managers' in this new approach one can not. In order to work within this new approach, one needs to consider explicitly the interactions between divisional managers. In order to facilitate modelling of these interactions, Chapter 5 is concerned with overviewing the "Theory of Games of Incomplete Information". This overview is presented to demonstrate that the theory is suitable for modelling essential aspects of the transfer pricing problem, when the interactions (subjective expectations) between divisional managers must be considered.

Chapter 6 commences with a presentation of how one can explicitly model the transfer pricing problem in a decentralized organization as a game played between players possessing incomplete information. In this model, incentive compatibility and effort provision issues are both considered. A procedure for solving transfer pricing problems

in organizations is presented and an example is provided. It is argued that the reason this procedure works is because, in a decentralized environment, divisional management must be explicitly compensated for the provision of effort and information services. It is the recommendation of payment for the latter service which is new to the literature on transfer pricing. A crucial result presented is that central management should value divisional managers' information services by determining how much divisional managers could gain by distorting the information they communicate to the central headquarters.

The model of the decentralized organization used in Chapter 6 is very simple as shown by the example in the chapter. This is considered acceptable, in order to make it possible to understand intuitively how the transfer pricing procedure works. Chapter 7 is concerned with applying the same general procedure to a far more complex environment.

NOTES

1. Umapathy (1978) p. 169. In addition, note that on p. 144, Umapathy suggests that he had not intended to just consider large firms. It just turned out that relatively few small firms responded to the survey questionnaire.
2. See Exhibit B-10, Umapathy (1978).
3. For additional references on surveys of transfer pricing procedures adopted see Thomas ( 1980 ), p. 128, for instance.
4. Kaplan (1984), p. 402.
5. Some procedures closely related to the Baumol and Fabian (1964) interpretation of the Dantzig and Wolfe (1960) Decomposition procedure are also discussed.
6. For a precise definition of optimal control mechanisms see section 4.1.
7. It may seem strange to the reader that the Green and Laffont (1977) work predates that of Groves and Loeb (1979). However, the transfer pricing procedure developed by Groves and Loeb (1979) is closely related to their work on public inputs presented in Groves and Loeb (1975). This 1975 work is the basis for the Green and Laffont (1977) analysis.
8. I talk of a tradeoff here as it is assumed that compensation provides utility whereas effort is disutilititious.

CHAPTER TWOCONTROLLED DECENTRALIZATION2.1 The Motivation for Decentralization

The prime forces motivating the adoption of managerial decentralization in an organization can be classified under two, non-mutually exclusive headings. The first source of motivation shall be described as "The Economics of Managerial Tasks", with the second source being described as "Decentralization as an Organizational Philosophy".

A simple example for each source will now be presented and constituent problems identified. The examples are intended to illustrate how the motivating forces may arise in a simple setting and are not intended to purport to show the only way the motivating forces may manifest themselves. It is recognised that there will be many other cases that motivate a move towards more decentralization.

2.1.1 The Economics of Managerial Tasks

Let us consider the case of a small, single product, owner-managed business. The owner wishes to become more prosperous and so is considering expanding the business. Expansion can be achieved in a number of ways. The owner

may wish the business to expand output of its single product. However, this option is eventually limited by overall market demand and competition from other producers. In addition, in order to internalize as far as possible, certain input and output market uncertainties, the business may attempt to take control (purchase) of the suppliers of raw materials and retailers of the final product. However, this may be limited by the large number of suppliers or retailers concerned. Also the size of the other operations of these suppliers and retailers may be such that such ventures would be extremely costly. A more likely line of expansion will be to introduce new product lines in order to reach new markets.

Given uncertainties in the product markets, the business may also wish to diversify its interests into some unrelated field in order to avoid the risk of being defenseless against damaging uncertain outcomes that may arise in one product markets●

The first part of this example illustrates that, as business grows in size, this puts increasing pressures on management time as there are now more decisions to make and monitor. Since management time is a scarce resource, one manager will not be able to manage effectively and so the need will arise to delegate decisions and monitoring to someone else. These other managers will be called local managers and, in order to avoid confusion, the overall controlling management will be

called central management. This need for central management to delegate certain day to day normal operating decisions to local management allows central management to spend more time considering crucial and difficult strategic decisions. This first constituent problem can, therefore, be summarised as "the need to conserve central management time".

As new product lines are introduced and the business diversifies into new fields, more information about new and unrelated markets will need to be gathered. As central management will have limited knowledge, information and time to observe all the relevant markets and assess all possibilities, these roles will also be partly delegated to local management. This second constituent problem can, therefore, be summarised as "the need to create information specialists".

Even though there may only be one person in a business collecting information on a product market, a local manager, there may be many other local managers from other businesses also collecting information. In a competitive product market, it is crucial that, if the full benefits from collecting information are to be gained, that the information be quickly acted on. It is important that some information can be acted upon by local managers as soon as they observe it, rather than it be required that they transmit the information to central management. This is because central management may take some time to assess the information and make decisions. For instance, a local manager may find a new superior supplier of some raw material used in the local manager's

sphere of operation. It is crucial that the manager be able to act upon such information in a timely fashion, rather than wait possibly a number of months for central management to process and agree with a change of supplier. Given that it must be established that local managers can freely act on certain decisions, it is also important that they be required not to take independent actions on certain decisions that may affect other parts of the business. For instance, if the new raw material purchased by the original local manager to produce some intermediate product, did not satisfy the needs of a local manager who is responsible for further processing of the intermediate product, then the unilateral decision may not be optimal. This third constituent problem is summarised as "the need to ensure certain decisions have timely local responses".

Given that local managers are required to communicate some information to central management and that, since central funds allocatable for use by local management are scarce, the central management will have to determine how to allocate funds between local managers. It is crucial at this stage that, in order to stop central management being overloaded with information it needs to process, that local managers can effectively select what information will be crucial, and communicate it in a clear summarised fashion. This fourth constituent problem is summarised as "the need to ease the burden of co-ordinational complexity".



### 2.1.2 Decentralization as an Organizational Philosophy

Whereas the previous sub-section considered the actual roles that local management needs to perform, this sub-section considers the attitudes that local managers will have to increased responsibility and independence and how this may affect their future performance.

Suppose that one of the production lines of our typical expanding business had been previously supervised by a foreman. In this role, the foreman had been required to communicate daily assessments on the functioning of the production line so that central management could assess and control the production line. In addition, suppose the inputs that were available for use on the production line, such as labour and raw materials, were determined centrally. Hence, the foreman's role is really of a passive nature.

Suppose central management decide to change the role of the foreman to a local manager by delegating the responsibility for production of the product. The previous sub-section indicates some of the new roles the local manager will need to perform. However, what is also worthy of consideration is whether the local manager still views the production process passively. Given new responsibility and additional authority, the newly created local manager

may respond by actively searching for improved methods of production and may be keen to implement such changes. The fifth problem then that provides an incentive for businesses to decentralise shall be summarised as "the need to motivate local management".

Another important problem is that, if central management wish to attract high potential applicants into the business to improve the quality of future decisions, it is important that new members be given challenges to stimulate them and are given experience. One of the best ways to achieve such training of new personnel is to make them local managers and give them some responsibility and authority so that they can learn to respond to these challenges. The sixth and final problem which arises in a business of some size that provides an incentive for decentralisation shall, therefore, be summarised as "the need to train local management".

## 2.2 Implementing Decentralization

The previous section indicated that decentralization may be adopted in order to overcome a number of organizational problems. This section considers how the strategy of managerial decentralization can be implemented.

Decentralization is synonymous with delegating authority and establishing responsibility. The central management of an organization can issue two types of authority to a local manager. The central management can place certain

physical resources, such as machinery, under the custody of a local manager, giving the local manager the power to decide how to utilize the physical resources. The central management can also indicate which decisions the local manager has authority to take without central management consultation.

At first, one may then assume that effective organizational decentralization will be achieved by making local managers' only responsible for those resources and decisions which they had authority over. This may be the case if organizational units did not interact. However, the very reason for a large organization coming into existence is so as to facilitate the co-ordination (interaction) of these units to achieve some prespecified goal. Thus, typically local managers' responsibility will exceed their authority to effect change. The intention of this practice is to motivate local managers to work together in unison, rather than in seclusion from others.

Before considering these important issues further, it is important that it is established how local managers are made responsible for actions taken by them.

### 2.2.1 Making local managers responsible

The central management of an organization monitor the performance of the organization using a financial measurement system. Thus, one way to make local managers feel responsible for a range of decisions is to measure the financial consequences of those decisions and link local managers' compensation to those financial consequences. For instance, a local manager may be rewarded on the basis of the profitability of the unit in the organization which the manager had authority over<sup>1</sup>. How exactly the central management should make financial measurements will be considered in later chapters. At this stage, what needs to be noted is that, in order that local managers should feel suitably responsible, the financial measurement system must function well to reflect these responsibilities.

### 2.2.2 Establishing an Organization Form

Decentralized organizations arise in many settings in response to some or all of the problems discussed in section 2.1. The form of decentralized organizations that arise in response to only a partial number of those problems is likely to be different from an organization

that decentralizes in response to all of these problems being present. Having said this, though, the six constituent problems of section 2.1 are likely to be present to a lesser or greater degree in any organization that resists decentralization. However, the second constituent problem, that of the need to create information specialists does not always arise, possibly because of the physical possibilities for production. For instance, on a production line making tin cans of only one size, the relationship between inputs and outputs is likely to be a stable one. Suppose in addition the suppliers of raw materials were stable and reasonably uniformly priced between alternate suppliers. Also assume the tin cans were only to be used by other units in the organization and not sold externally. In this type of case, the problem of requiring a local manager to be an information specialist in the market for the production of tin cans does not seem to be valid. What is needed is a competent local manager who meets the requirements of the five other problems. For instance, the local manager may respond well to the valuable training experience overseeing the production of tin cans.

This type of stable relationship between inputs and outputs gives rise to the formulation of the decentralized unit concerned as a Standard Cost Centre, responsible for the efficiency of production. However,

not only does the extent of the problems of section 2.1 vary between organizations, but also the extent of responsibility varies. For instance, some local managers may take responsibility for the level of financial assets employed in a decentralized unit. These units may then be appraised as Investment Centres. However, this study will restrict its analysis to the consideration of Profit Centres. A Profit Centre is defined here as a decentralized unit whose performance is measured by profit and is the responsibility of a local manager. Profit is required to be a meaningful concept in that the relationship between inputs and outputs is an uncertain one. For instance, if the supply price of the raw materials for tin can production had been uncertain, the actual price paid for the raw materials would reflect the ability of the local manager to find the lowest price supplier at the right time. In addition, assume that a number of advances are continually taking place in the technology of tin can production. The output of the production line, a number of tin cans costing a certain amount will also vary with the skill of the local manager. The reason for the above definition of a Profit Centre with its caveat will become apparent shortly when the methodology of this study is discussed. Basically, it results from the need to use one specific form of organizational form to concentrate study upon.

### 2.2.3 The Right Decentralized Environment (Controlled Autonomy)

The term autonomy refers to the willingness of local managers to take independent action to improve the performance of the organization. The term controlled autonomy is intended to imply that local managers recognise limits on their autonomy. The reason these limits are placed is because the actions of one local manager will often affect another local manager's operating conditions. In which case, both managers must be aware of their interaction and the need to take joint decisions. Thus, a crucial determinant of the effectiveness of decentralisation is whether or not local managers adopt an attitude of co-operation.

These sentiments are also expressed<sup>2</sup> by R.F. Vancil in the following passage:

"Autonomy is the word used by managers to permit them to talk about the ambiguity of a profit center manager's role. Decentralization is an organization philosophy that inevitably creates ambiguity in that role, by holding a profit center manager responsible for the financial performance of a business and, at the same time, withholding from him the functional authority to control shared resources .....

Decentralization works not only because it is a powerful concept but also because corporate managers work at making it work. Having designed an ambiguous role, they also design a management process and a set of management systems to help themselves and their profit center managers cope with the ambiguity. The result is, indeed, the best of both worlds: multiple centers of initiative and a spectrum of decision making processes that are used selectively to ensure that the benefits from interdependency are not lost".

This chapter is entitled Controlled Decentralization in order to convey an idea of the ambiguity that Vancil describes that decentralization gives rise to. Vancil clearly conveys the feeling that, when decentralization works, it does so to the great benefit of the interested parties. However, the next section considers the problems that may arise when trying to create the right decentralized environments for local managers to operate in.

### 2.3 Decentralization, at What Cost?

This section critically considers what may go wrong when organizations attempt to decentralize their operations. The problem of how to exactly create the right decentralized environment is soon encountered when



central management try to determine suitable (financial) performance measures for local management.

### 2.3.1 Performance Measure-Destruction of Co-operation

As has been mentioned before, local managers' responsibility will often exceed their authority and the local manager's performance measure must reflect this. Let us now consider a simple case that illustrates these issues. Suppose one local manager is dissatisfied with the product line being produced and retailed by his or her unit. Suppose the manager identifies a new product line which the manager believes will be very profitable. However, the new product line requires as an input a new intermediate product which would be required to be produced in another unit of the organization, as the local manager of this unit has the skill, knowledge and facilities to produce this new intermediate product. In order that production be realised, both local managers must co-operate to ensure that the right intermediate and final product are produced. The performance measure of the intermediate product producing local manager will impute some value to transfer of the intermediate product. Hence, one can see that the performance measure of this local manager is partly determined by an action instigated by another

local manager. In addition, the continued production of the intermediate product will depend on the success of the other local manager's choice and implementation of the new product line.

Hence, the performance measure of the two local managers is partly linked. However, if the performance measures do not reflect the relative efforts of the local managers to see that the intermediate and final products are successful, then this may cause disharmony between the local managers. If the local managers take the view that other local managers are free riding on their efforts and benefitting at cost to themselves, because the performance measures are not truly reflecting the relative shares of effort, this will foster an environment of non co-operation. Put another way, one local manager may view the improvement of another local manager's performance measure as purely a result of his or her own performance measure being set artificially low.

Clearly, if the performance measure fosters these types of views, it will not lead to effective decentralization where there is a co-operative spirit among local managers. This problem is also sometimes described as the problem of externalities in decentralized organizations.

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no

### 2.3.2 Performance Measure Generation of Strategic Information Specialists

In Section 2.1, it was indicated that one of the important roles of a local manager may be to collect specialised information on, for instance, new production possibilities, new suppliers and/or retailers. In addition, the local manager would be supposed to make this information freely available to central management in a clear concise form. However, because of the way the performance measure is specified, it may be in the interest of the local manager to conceal the information until a later stage.

For example, the performance measurement system may be budget orientated in that performance is measured relative to achievement of budget specified goals. One of the budget goals may be the expectation of being able to produce output at some cost consistent with an implied cost function. If the local manager concerned finds information about a new low cost production method for producing the required output, the manager may choose to conceal this information until after the budget has been set and then implement the new low cost production method if this improves his or her performance measure. At this stage, some observers may remark that this problem is easy to overcome because, at the end of the period the cost function will be audited and it will be found that the functional relationship is different from

that supposed in the budget. This type of comment will be discussed in more detail at a later stage within a more specific framework, but at this stage two points should be noted.

First, at the budget stage what is being asked for and formulated is expectations. It is not possible to audit an expectation in someone's mind. One of the main reasons why budgeting procedures are adopted is so that local managers hopefully communicate their expectations and possibly revise them in the light of information gleaned from other local managers' or central managers' expectations. If an expectation is not realised in an uncertain world, one could not identify what part of the variance was due to chance, poor performance, poor forecasting or misrepresentation of one's true expectations. This is particularly the case where the local manager concerned is, and is employed to be, the (possibly only) information specialist in a field and where others' knowledge and information is limited.

Secondly, the critical comment takes no account of the fact that, as an information specialist, a local manager must be rewarded for carrying out this role successfully. This is not to say that the reward should be achieved by the strategy of misrepresentation of available information. What is being argued is that it must be clearly recognised that local managers must be given an incentive (reward) to act as information specialists.

### 2.3.3 Goal Incongruent Performance Measures

Ideally, the performance measure by which a local manager is appraised should motivate the local manager to be able to identify and adopt the overall goals of the organization. However, the local manager's strong focus on their performance measure may lead them to substitute and realise their own personal goals for those of the organization. For instance, it may be that certain local managers may be able to improve their own performance measures by taking actions at the expense of organization goal fulfilment. This type of problem is commented on by Kaplan<sup>3</sup> in the following passage:

"The measure of performance tends to become an end to itself, more important than the economic performance that it attempts to represent..... Any single measure may be manipulated to benefit the decentralized unit at the expense of the corporation. This fundamental problem arises because, unlike the situation in the physical sciences, the act of measurement in social sciences and in management changes the event and the observer. Measurement is neither neutral nor objective. The measure chosen for evaluating performance acquires value and importance by the fact of being selected for attention. People within the system change their behaviour as a function of the measure chosen to summarise the economic performance of their organizational unit".

In addition, the performance measure may only be able to partly communicate the goals of the organization. This could easily happen when multiple organizational goals exist which are not totally internally consistent and need special consideration.

#### 2.3.4 Incompleteness of Performance Measures

Assume the financial performance measure that central management will be trying to optimize is the net financial performance of the organization after local managers have been compensated. Hence, the central management will not want to compensate local managers excessively. However, it is equally likely to be concerned with under-compensating good local managers. This is because, if good local managers are not easy to replace, low compensation may lead the good local managers to decide to leave the organization at some future cost to the organization and the central management.

The important issue here is whether or not local manager performance measures do reflect how good or bad a local manager is. This problem arises because often local manager performance measures can be affected by unexpected, unplanned market forces, which are not the result of anything local management has been able to do. For instance, the product sold by one local manager's

unit may suddenly increase in demand in an export market because of some unforeseen event occurring in that foreign market. In order to meet this excess demand, the local manager may not need to expend much extra effort. If this is the case, ideally the local manager's performance measure should take this into account. One way organizations often attempt to overcome this type of problem is by linking the performance measure to a flexible budget report.

To summarise then, care must be taken in constructing and interpreting performance measures, since they may only partially measure actual performance, although they often determine actual compensation to local management.

### 2.3.5 The Uncertainty in Performance Measures

Assuming that local managers operate in an uncertain environment, their performance measure will depend on the outcome of uncertain events. For instance, the sales of a local manager's unit may depend upon many uncertainties, such as fierceness of competition from other competitors or even the weather in the case of ice cream sales. Given that the central management of the organization have some attitude towards risk, another role of the performance measure should be to motivate local management to adopt projects consistent with the central management's attitude towards risk. This may

not always be realised, however, because the performance measure may often overexpose or overprotect local management from risk.

#### 2.4 Modelling Decentralization

As the previous section has indicated, the central tool by which central management can influence the benefits and costs that arise from decentralization is through the performance measurement system for local managers.

The aim of this thesis is to construct a theory of how a performance measurement system should function in order to gain the maximum achievable net benefits from decentralization. However, the motivation for decentralization gives rise to many different forms of organizational structure and performance measurement. This is in response to different external market conditions, different possibilities for innovation in production and different inter-relationships between organizational units for instance. The research methodology adopted in this thesis is to concentrate attention on one form of decentralized organizational structure and then to see how best a performance measurement system can be constructed for this particular decentralized organizational structure. It is then hoped that the findings made by detailed consideration of one particular structure will be able to be adapted to consider other structures.



#### 2.4.1 The Transfer of an Intermediate Product

This study will use as its illustrative organization, one in which there are two decentralized units called divisions, run by divisional managers appointed by a central management team called the central headquarters. The first division will be called the manufacturing division. It is assumed that the role of this division is to produce an intermediate product. However, the supply and price of at least some raw materials is uncertain. In addition, the cost function of the manufacturing division may change from one period to the next, since the production technology is assumed to be constantly changing. The second division will be denoted the distribution division. The role of this division is to further process the intermediate product and then sell the product on an uncertain external market, whose demand conditions may vary from period to period because of exogenous forces such as changing tastes. As a first step, let us assume the basis for performance measurement of the divisional managers is divisional profit. It is, therefore, crucial that a price must be determined at which the intermediate product is to be transferred. What is viewed as most important is the process by which the price is determined, rather than only the actual transfer price that results.

In order to see as clearly as possible how this process should, in general, take place, it will be assumed that there is no external market for the intermediate product. This is to ensure that the net benefits from decentralization can first be derived from internal organizational forces, rather than the result of a particular form of market existence. This no external market assumption is also made so as to hopefully give the most generaliseable results. That is to say, it is recognized that an external market for an intermediate product may take many forms between the poles of competitive and monopolistic operating conditions. Therefore, if one can construct a theory of transfer pricing that is not external market specific in its assumptions, then it is hoped that this type of theory will be generaliseable to most external market cases. In addition, it is assumed that the divisional managers are not held responsible for the level and type of investment in their division, but only for the short-term operating decisions that need to be made for the division. This means that transfer prices<sup>4</sup> based on the assumption that divisions are investment centres will not be considered. One reason why this may occur in practice is that divisional managers may not stay in a particular division long enough for the full benefits of the investment they instigated to be realised. The reason the divisional managers may be moved about is because the central headquarters want to ensure that a divisional manager does not start to think of a division's assets as "my assets" rather than "our assets".

The next chapter then considers a review of the transfer pricing literature for profit centres.

NOTES TO CHAPTER TWO

1. Typically, this profit measure will also depend on the performance of other units.
2. R.F. Vancil, "Decentralization: Managerial Ambiguity by Design", pp. 131 - 132.
3. R.S. Kaplan, "Advanced Management Accounting", page 441. A superscript referencing a note to the Heisenberg uncertainty principle has not been included in this presentation of the original passage.
4. For instance, return on investment and residual income. In addition, it will be assumed that working capital requirements do not vary with output. Hence, one does not need to consider the issue of whether or not a charge for working capital needs to be made to determine the marginal cost of production. See Ashton (1984) for more on this subject.

CHAPTER THREE

A REVIEW OF THE PROPERTIES OF SOME TRANSFER  
PRICING PROCEDURES WHEN THERE IS NO EXTERNAL  
MARKET FOR INTERMEDIATE PRODUCTS

3.1 Ideologies

There are a number<sup>1</sup> of established works reviewing transfer pricing procedures in detail. This chapter will, instead, try to focus attention mainly on the ideological background of specific procedures and then give an account of the procedures. The reason this line of approach is taken is related to the fact that, in the previous chapter, it was established intuitively that a transfer pricing procedure in a decentralized organization should play a number of roles. For instance, the procedure should ensure that central management time is conserved and <sup>that</sup> local management are motivated to perform well. However, research has tended to be of a piecemeal nature; instead of trying to design procedures satisfying all roles, researchers have concentrated on procedures that would satisfy a specific role. It is accepted that, given the difficult nature of the overall problem, a piecemeal research approach is a valid first step. However, this <sup>approach</sup> gives rise to problems when one tries to assess the relative superiority of different procedures which have implicitly assumed different (partial) roles for transfer pricing procedures.

It is recognized that all roles established on intuitive grounds in the previous chapter should be satisfied by a transfer pricing procedure wherever possible. However, if one is only able to design procedures that satisfy a partial

number of roles, one must try and establish a priority ranking over the roles. Thus, in this work, if two alternative procedures hold different priority ranking over roles, they are said to be based on different ideologies. A substantial part of this thesis is concerned with establishing<sup>a</sup> preferred partial priority ranking over the roles that a transfer pricing procedure must satisfy. Let us now consider two of the seminal works on transfer pricing in sections 3.2 and 3.3 respectively.

### 3.2 The Hirshleifer Analysis

Let us assume that the corporation under consideration has two divisions, a manufacturing and a distribution division. Each division produces one product and the divisions are independent except for the fact that the manufacturing division transfers all of its output to the distribution division for further processing<sup>2</sup>.

Hirshleifer addresses the problem of how to co-ordinate the local decisions of the divisional managers in order to ensure maximization of the corporate goal(s), which is assumed here to be<sup>single period</sup> profit maximization. Hirshleifer attempts to solve the problem by mimicing the operation of a competitive market. Specifically Hirshleifer demonstrates that, if the intermediate product were to be perfectly competitively priced, divisional managers would take local decisions that would ensure that corporate profits were maximised. This approach is also sometimes called the "economic approach", as it is based on the traditional lines of marginal economic analysis. Hirshleifer analysed the situation geometrically along the following lines<sup>3</sup>.

Let  $C(q_m)$  = manufacturing divisions cost of producing  $q_m$  units of the intermediate product.

$C_D(q_D)$  = distribution division's cost of selling  $q_D$  units of the final good and any additional costs incurred in transforming the intermediate product into the final product.

$R(q_D)$  = distribution division's revenue from selling  $q_D$  units of the final product.

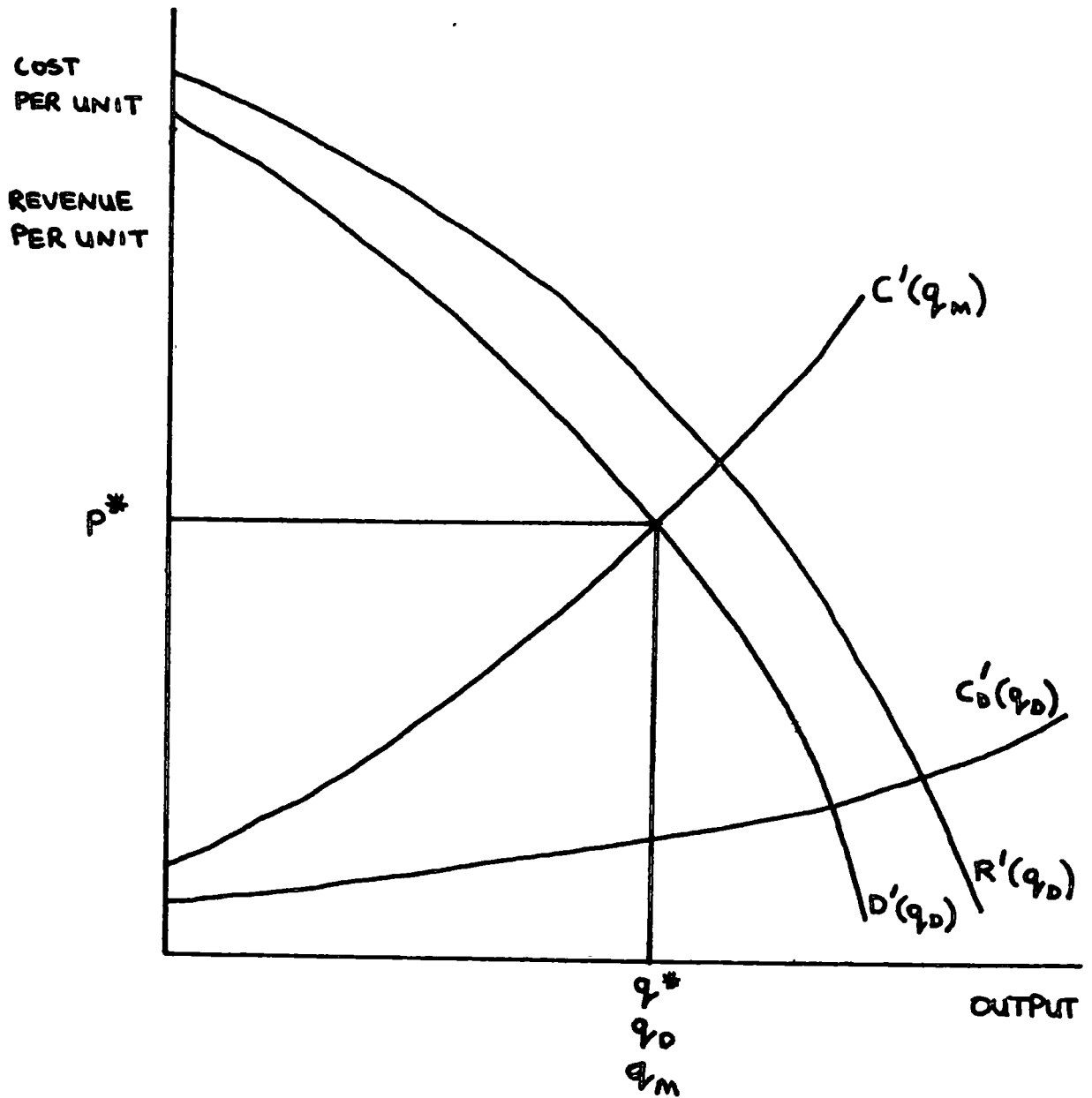
$D(q_D)$  = distribution division's net revenue function  
 $= R(q_D) - C_D(q_D)$ .

Let us assume that the distribution division sells the final product on an imperfectly competitive market<sup>4</sup> and the marginal cost and marginal revenue schedules are as presented in Figure 1.

To maximise corporate profits both divisions must agree on the quantity of the intermediate product that needs to be produced and transferred. The optimal output,  $q^*$ , and price,  $p^*$ , will be where the net marginal revenue of the distribution division equals the marginal cost of the manufacturing division for production of the intermediate product.

The important result of Hirshleifer's analysis is that once central management determines this price  $p^*$ , the manufacturing divisional manager would independently choose an output level  $q_m$  at which marginal manufacturing cost equalled  $p^*$ . Also the distribution divisional manager would independently wish to acquire  $q_D$  units of the intermediate product where net marginal revenue equalled  $p^*$ . Hence, since  $q_D = q_m = q^*$ , the local

FIGURE 1



THE SUPERScript (') DENOTES A FIRST DERIVATIVE  
FOR INSTANCE :

$$\frac{d R(q_D)}{d q_D} \equiv R'(q_D) \equiv \text{MARGINAL REVENUE}$$



decisions of the divisional managers are consistent with the decisions that central management would instruct divisional managers to take to maximise corporate profit.

Hirshleifer does not specifically establish how, in practice, his procedure should be made operational, but suggests<sup>5</sup> that "There are a variety of ways in which the optimum solution might be arrived at operationally. Some device like a neutral umpire might be employed to set an initial trial price  $p^*$ .... after which the divisions would respond by declaring tentative outputs  $q_m$  and  $q_D$ . If  $q_m$  exceeds  $q_D$ , the  $p^*$  should be adjusted downward by the umpire and the reverse if  $q_D$  exceeds  $q_m$ ..... until a  $p^*$  is found such that the planned outputs are co-ordinated".

However, Ronen and Mckinney (1970) have proposed an operational procedure for implementing the Hirshleifer transfer pricing procedure. They suggest that the marginal cost schedule of the manufacturing division be reported to the distribution division, which tells the distribution divisional manager how much would be produced by the manufacturing division at any transfer price  $p^*$ . The distribution divisional manager determines the division's average revenue curve, which is the difference between the market price of the final product  $p$  and the transfer price of the intermediate product  $p^*$ . The division will wish to produce the output for which

$$q_D^* \max pq_D - p^*q_D - C_D(q_D) \quad (1)$$

which satisfies the first order condition

$$p - p^* = C'_D(q_D) \quad (2)$$

That is, the manager determines the output at which average revenue equals marginal distribution costs. The manufacturing divisional manager will agree to produce  $q_m^* = q_D^*$  units at transfer price  $p^*$  as

$$p^* = C'_m(q_m) \quad (3)$$

which are the first order conditions for

$$q_m^* \max p^* q_m - C(q_m) \quad (4)$$

However, as shall be demonstrated below in section 3.4.1 there are a number of problems associated with trying to make the Hirshleifer procedure operational. Hirshleifer's procedure is best viewed as establishing that, in some instances, a unique transfer price can be established which will co-ordinate the local decision making activities of local managers to yield overall corporate optimization.

### 3.2.1 Regularity Requirements for the Existence of Hirshleifer Prices.

The analysis of Hirshleifer rests on the assumption that there exists a unique perfectly competitive price (satisfying the required marginal conditions) for the intermediate product, which will guide the divisional managers to choose unique optimal operating decisions. However, in some instances, such a price may not exist. This sub-

section investigates the type of operating conditions for divisions, which would ensure that a unique co-ordinating transfer price would always exist. These conditions will be called the regularity conditions for the problem.

Let us, therefore, consider the regularity conditions which are sufficient and necessary for:

- (i) the existence of such unique transfer prices, and
- (ii) the price to guide divisional management to make decisions which maximize the central headquarter's welfare.

The following mathematical discussion, based on Moeseke and Ghellink (1969) will make use of vector notation so there is no need to restrict attention to only two divisions with one intermediate product. Let us assume there are  $N$  divisions indexed  $i = 1, \dots, n$  and  $M$  common resource constraints indexed  $j = 1, \dots, m$ . The common resource constraints relate to limited corporate resources that need to be shared out among the divisions, or transfers between the divisions. Let the productive activities of division  $i$  be characterized by  $n_i$  variables, summarized in the vector  $x_i$  where

$$x_i = (x_{i1}, \dots, x_{in_i}) \quad i = 1, \dots, n$$

The profit function (preference function) of the central headquarters is assumed to be the sum of all divisional profit functions which are functions of the activity levels

in that division and can be written as the following sum of  $n$  functions:

$$\max \sum_{i=1}^n f_i(x_i) \quad (5)$$

The private constraints that only division  $i$  face can be expressed as:

$$x_i \in X_i \quad i = 1, \dots, n \quad (6)$$

where  $X_i$  = division  $i$ 's feasible (production) set.

The important assumption here is that division  $i$ 's private constraint set can be specified independently of the activities of other divisions. However, this is not possible for the  $m$  common resource constraints which are assumed to take the following general form:

$$\begin{aligned} g_{11}(x_1) + \dots + g_{1N}(x_N) &\leq b_1 & (7) \\ \vdots & & \\ g_{m1}(x_1) + \dots + g_{mN}(x_N) &\leq b_m \end{aligned}$$

In the situation where, for instance, an intermediate product is produced in one division and transferred to another division, one of the  $g_{ji}$  functions will be positive and another negative with the associated  $b_j$  value being zero, where  $j = 1, \dots, m$

Let the non-negative  $m$  vector  $u = (u_1, \dots, u_m)$  represent the transfer prices charged to divisions for each common resource. Each division trying to maximize divisional profit will, therefore, operate at the activity level  $x_i^*$  for which

$$x_i^* \max \{ f_i(x_i) - \sum_{j=1}^m u_j g_{ji}(x_i) \} \quad (8)$$

s.t.  $x_i \in X_i$

Proposition 3.1

Let  $x_1^*, \dots, x_n^*$  denote the activity vectors that solve equation (8) and satisfy (7) as well as the condition that

$$\sum_{i=1}^n g_{ji}(x_i^*) < b_j \quad \text{implies } u_j = 0 \quad (9)$$

for all  $j = 1, \dots, m$

then  $x_i^*$  solves the overall problem defined by (5) subject to (6) and (7).

PROOF: See Appendix 1.

Proposition 3.1 is a sufficiency theorem for establishing that, if the above conditions are fulfilled, then a solution to the overall corporate problem has been obtained. Note that no assumptions of differentiability, concavity or convexity of the functions  $f_i$ ,  $g_{ji}$  and sets  $X_i$ , is made.

However, the proposition does not say that the prices  $u_j$  required by the proposition, and solutions  $x_1^*, \dots, x_n^*$  also required by the proposition, will always exist. It only states that, if they do exist, then a solution to the overall corporate problem has been obtained by solving (co-ordinating) the  $n$  divisional subproblems.

Therefore, let us now consider the necessary and sufficient conditions for the existence of a unique set of transfer prices (one price for each common resource) that ensure that divisional subproblem optimization leads to overall corporate optimization.

Let  $D_0$  represent the following convex programming problem:

$$\begin{aligned}
 & \max f(x) \\
 & \text{s.t. } x \in X \\
 & h(x) \geq 0
 \end{aligned} \tag{16}$$

where  $h_j(x) = b_j - \sum_{i=1}^n g_{ji}(x_i)$  for  $j=1, \dots, m$  where  $X$  is convex, while  $f$  and  $h$  are concave. The Lagrangian of  $D_0$  is

$$L(x, u) \equiv f(x) + uh(x) \tag{17}$$

and is said to possess a saddlepoint if

$$L(x, u^*) \leq L(x^*, u^*) \leq L(x^*, u) \tag{18}$$

for all  $x \in X$  and all  $u \geq 0$

In addition, let  $D_0$  satisfy the Slater condition that:

$$\text{there exists an } x^0 \in X \text{ such that } h(x^0) > 0 \tag{18}'$$

**Kuhn-Tucher Saddlepoint Theorem:** If  $D_0$  is convex and satisfies condition (18)', then  $z^*$  solves  $D_0$  if, and only if, there is a non-negative  $u^*$  such that  $(x^*, u^*)$  is a saddlepoint of  $L$  defined by (17).

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This theorem is proven by Uzawa in/Arrow, Hurwicz and Uzawa (1958).

At this point, it should be noted that the usual form of the Kuhn-Tucher conditions assuming differentiability have not been used as it is intended that the following transfer price existence theorem be applicable to linear (non-differentiable) environments.

Let  $D_i$  represent the divisional subproblems:

$$\begin{aligned} \max f_i(x_i) \\ \text{s.t. } x_i \in X_i \\ h_i(x_i) \geq 0. \end{aligned}$$

Proposition 3.2

Let  $D_0$  be convex and satisfy (18).

(a) If the vectors  $x_i^*$  solve  $D_0$ , then there exists a  $u^* \geq 0$  such that, for  $u = u^*$ ,  $x_i^*$  solves  $D_i$  for all  $i=1, \dots, n$ .

(b) If  $D_0$  possesses a solution with corresponding saddlepoint member  $u^* \geq 0$  such that, for  $u = u^*$ , the  $x_i^*$  uniquely solve  $D_i$  for all  $i=1, \dots, n$ , then the vectors  $x_i^*$  solve  $D_0$ .

PROOF: See Appendix 2.

In addition, note that the saddlepoint  $(x_i^*, u^*)$  of proposition 3.2 satisfy (9) because by the second inequality of (18)

$$\sum_{i=1}^n f_i(x_i^*) + \sum_{j=1}^m u_j^* b_j - \sum_{i=1}^n \sum_{j=1}^m u_j^* g_{ji}(x_i^*) \leq \sum_{i=1}^n f_i(x_i^*) +$$

$$\sum_{j=1}^m u_j^* b_j - \sum_{i=1}^n \sum_{j=1}^m u_j^* g_{ji}(x_i^*)$$

for all  $u_j \geq 0$

Setting all  $u_j = 0$  reveals that

$$\sum_{j=1}^m u_j^* (b_j - \sum_{i=1}^n g_{ji}(x_i^*)) \leq 0$$

Since the reverse inequalities (7) hold, we have

$$\sum_{j=1}^m u_j^* (b_j - \sum_{i=1}^n g_{ji}(x_i^*)) = 0 \text{ which implies (9)}$$

To summarize then, proposition 3.2 ensures that for the set of vectors  $x_i^*$  that solve  $D_0$ , there exists a set  $u^* \geq 0$  of resource prices that sustain a once established equilibrium among divisions. The Kuhn-Tucher Saddlepoint

Theorem is interpreted as an existence theorem to establish what regularity conditions are sufficient and necessary to ensure the existence of suitable transfer prices.

However, if the role of a transfer price is not only to sustain a solution to  $D_0$  and  $D_i$ , but also to provide such a solution, one would require the transfer price that solved  $D_0$  to provide unique solutions to each  $D_i$ . Proposition 3.3 specifies the class of convex programming problems  $D_0$  for which such uniqueness is assured.

#### Proposition 3.3

If  $D_0$  has a solution, then either of the following conditions ensures the uniqueness of the solution  $x_i^*$  to  $D_i$  for  $u = u^*$

- (a)  $f_i$  is strictly concave on  $X_i$ ,
- (b) there is a  $j$  such that  $u_j^* > 0$  and  $g_{ji}$  is strictly convex on  $X_i$ .

PROOF: See Appendix 3.



Thus, only when the regularity conditions of Proposition 3.3 are satisfied are Hirshleifer's results ensured. In subsequent discussions of Hirshleifer's analysis, it will be assumed these conditions are met unless otherwise stated and, in addition, it will be assumed that the appropriate functions are (twice) differentiable, so that one can talk about marginal conditions unambiguously.

### 3.3 Decomposition Analysis

No matter how one wished to make the Hirshleifer procedure operational, somebody would have to collect detailed information on divisional cost and revenue structures, or/This may be extremely costly, time consuming and possibly infeasible, if a large number of divisional constraints and decision variables were involved. The decomposition procedures discussed below were designed originally as computational procedures for solving large scale constrained optimization problems, but they also have clear organizational interpretations<sup>6</sup>. The method by which these computational procedures work will be considered in some detail as this will have a bearing on organizational interpretation of specific procedures. One should, at this point, note that consideration will

first only be given to problems characterised by linear operating conditions. This assumption about operating conditions will be relaxed at a later stage.

Before discussing specific models, the general concept of organizational co-ordination via decomposition will be illustrated using a set theoretic representation of the problem as follows<sup>7</sup>. Let us assume that the overall corporate resource allocation problem can be expressed as a mathematical model which shall be denoted D. For example, three possible forms that D may take<sup>8</sup> are illustrated below:

Block angular structure (coupling constraint):

$$\begin{aligned}
 \max Z &= c_1 x_1 + c_2 x_2 + \dots + c_n x_n \\
 \text{s.t.} \quad &A_1 x_1 + A_2 x_2 + \dots + A_n x_n = b_0 \text{ coupling constraint (row)} \\
 &B_1 x_1 \leq b_1 \\
 &B_2 x_2 \leq b_2 \\
 &\quad \quad \quad \vdots \\
 &B_n x_n \leq b_n \\
 &x_1, x_2, \dots, x_n \geq 0
 \end{aligned}$$

Dual angular structure (coupling variable):

$$\begin{aligned}
 \max U &= b'_1 \Pi_1 + b'_2 \Pi_2 + \dots + b'_n \Pi_n + b'_0 \Pi_0 \\
 \text{s.t.} \quad &B'_1 \Pi_1 + A'_1 \Pi_0 \leq C'_1 \\
 &B'_2 \Pi_2 + A'_2 \Pi_0 \leq C'_2 \\
 &\quad \quad \quad \vdots \\
 &B'_n \Pi_n + A'_n \Pi_0 \leq C'_n
 \end{aligned}$$

where the superscript ( $\prime$ ) indicates the operation of transposition, so that, for instance,  $b_1'$  may represent the transposition of a column vector into a row vector.

Block angular structure with coupling constraints and variables:

$$\begin{aligned} \max Z_n &= c_1 x_1 + c_2 x_2 \dots + c_n x_n + c_0 y \\ \text{s.t.} \quad & A_1 x_1 + \dots + A_n x_n + E_0 y = b_0 \\ & B_1 x_1 \dots + E_1 y = b_1 \\ & \quad R_2 x_2 \dots + E_2 y = b_2 \\ & \quad \quad \quad \vdots \\ & \quad \quad \quad B_n x_n + E_n y = b_n \end{aligned}$$

with  $x_i \geq 0 \quad i=1, \dots, n \quad \Pi_i \geq 0 \quad i=0, 1, \dots, n, \quad y \geq 0$  throughout.

The overall problem D may be expressed as follows:

Out of all  $m \in M$  find one for which  $m \in \bar{M}$

where  $M$  = the feasible set

$\bar{M}$  = the acceptable set

Thus, any  $m \in (M \cap \bar{M})$  is a solution to D. This specification of D is general enough to include other objectives besides optimization, such as satisficing.

The central headquarters could attempt to collect enough information to fully characterize D and then attempt to solve D. However, there may be advantages gained by allowing only a subset of all the possible information that characterizes D to be transmitted. This could be achieved

if one partitioned the problem  $D$  into an equivalent multilevel problem with the following subproblems:

Let there be  $n$  subproblems on the lowest level called the infimal subproblems and denoted  $D_1(\cdot), \dots, D_n(\cdot)$

Let there be one subproblem denoted  $D_0$  on the top level, called the supremal subproblem.

What is hoped to be obtained by this transformation of the problem is that the two level representation  $D_0, D_1(\cdot), \dots, D_n(\cdot)$  will attain the same solution  $m \in (M \cap \bar{M})$  as the overall problem  $D$ .

The infimal subproblem  $D_j(\cdot)$  is specified as:  
 Out of all  $x_j \in X_j^\gamma$  find one for which  $x_j \in (X_j^\gamma \cap \bar{X}_j^\gamma)$  where  $X_j^\gamma$  is the feasible set.

$\bar{X}_j^\gamma$  is the acceptable set.

These sets are superscripted  $\gamma$  to indicate that both sets and, hence, the infimal subproblem depend on the parameter  $\gamma$  which is specified by the supremal subproblem, and is the means of co-ordinating the infimal subproblems. The parameter could, for instance, be a vector of real numbers representing allocation quantities of some resource. With regard to the feasible set, it shall be assumed that the underlying determinants of the specific form of the feasible set in an organizational context are technological and institutional factors independent of  $\gamma$ . However, the actual attainable level of feasibility is determined by  $\gamma$ . Thus, the role of the supremal subproblem  $D_0$  will be to determine a value of  $\gamma$  which leads to the infimal subproblems achieving the same solution as the central headquarters would have achieved had it been able to formulate and solve  $D$ .

Now let  $\bar{D}(\delta) = (D_1(\delta), \dots, D_n(\delta))$

and  $x = (x_1, \dots, x_n)$

To transform a solution  $x$  to  $\bar{D}(\delta)$  into a solution to  $D$ , one needs a mapping  $\Pi_m: x \rightarrow m$ . However, given some value of  $\delta$ , it is not to be expected that any collection of infimal subproblems  $\bar{D}(\delta)$ , will result in a solution to  $D$  by way of the mapping  $\Pi_m$ . For that purpose, the infimal subproblems must be co-ordinated by the supremal subproblem which can, hence, be stated as:

Find  $\delta$  such that

$$(1) (X_j^\delta \cap \bar{X}_j^\delta) \neq \emptyset \text{ for } j = 1, \dots, n.$$

$$(2) \Pi_m(x) \in (M \cap \bar{M})$$

$$\text{for any } x \in (X_1^\delta \cap \bar{X}_1^\delta) \times \dots \times (X_n^\delta \cap \bar{X}_n^\delta)$$

In other words then, the parameter  $\delta$  must first be such that each infimal subproblem has a solution and, secondly, that any candidate solution to the overall problem put together from infimal subproblem solutions, must solve the overall problem  $D$ .

Thus, decomposition is the act of partitioning the overall problem  $D$  into<sup>9</sup> a two-level hierarchy of subproblems  $(D_0, \bar{D}(\delta))$ . It can be successful if  $(D_0, \bar{D}(\delta))$  is co-ordinable relative to  $D$ . That is the parameter would have to satisfy conditions (1) and (2) for the supremal subproblem. The virtue of such a partitioning of the problem is seen by the fact that, when the central headquarters attempts to solve  $D_0$ , it need know nothing of the

form of  $x_j^\delta$  or  $\bar{x}_j^\delta$  for  $j=1, \dots, n$ . All the central headquarters needs to know is that the  $n$  infimal subproblems have solutions for some value of  $\delta$  and that these are the solutions that are communicated to the central headquarters by the divisional (subunit) managers.

Note the value of  $\delta$  which will co-ordinate the decomposed problem  $(D_o, \bar{D}(\delta))$  is unlikely to be immediately obvious, but if the value of  $\delta$  is redetermined after every iteration (attempted solution of  $(D_o, \bar{D}(\delta))$  according to some appropriate rule, then it may be the case that such a  $\delta$  is located eventually.

The above concept of co-ordinability can be viewed as an existence requirement, that is, a multilevel system is not co-ordinable unless a  $\delta$  satisfying conditions (1) and (2) exists. However, this is not to say that, given such existence of the required co-ordinating parameter, that the subunits (when solving their infimal subproblems) will know unequivocally at what level to set their solutions  $x$ , consistent with co-ordinability under the mapping  $\Pi_m(x)$ . An example of such a situation arising will be given in subsection 3.3.1.

### 3.3.1 The Dantzig and Wolfe Decomposition Procedure and Interpretation.

Let the overall problem  $D$  for an organization<sup>10</sup> be representable by the following mathematical model:

$$\begin{aligned}
 &\text{maximise} && c_1x_1 + c_2x_2 + \dots + c_nx_n \\
 &\text{s.t.} && A_1x_1 + A_2x_2 + \dots + A_nx_n \leq a \\
 &&& B_1x_1 \leq b_1 \\
 &&& \quad B_2x_2 \leq b_2 \\
 &&& \quad \quad \quad \vdots \\
 &&& \quad \quad \quad + B_nx_n \leq b_n \\
 &&& x_1, x_2, \dots, x_n \geq 0
 \end{aligned}$$

Here,  $x_j$  is an  $n_j$  vector,  $A_j$  is an  $m \times n_j$  matrix,  $B_j$  is an  $m_j \times n_j$  matrix,  $b_j$  is an  $m_j$  vector, for  $j=1, \dots, n$ , and  $a$  is an  $m$  vector. That is, it is assumed there are  $m$  coupling constraints and  $m_j$  private constraints faced by division  $j$  that produces  $n_j$  products, hence  $c_j$  is also an  $n_j$  vector.

This block-angular formulation has a special structure. If it were not for the coupling constraints  $A_1x_1 + A_2x_2 + \dots + A_nx_n \leq a$ , the problem would divide into  $n$  independent problems. Thus, this model has a clear organizational interpretation. Each division faces a set of private constraints not faced by any other division. The only source of interaction is through the common constraints. Hence, the central headquarters can influence

(and hopefully co-ordinate) the actions of the divisions through manipulating the common constraints without needing to know the private constraints.

Since the problem is a totally linear problem, the private feasible sets are defineable as:

$$X_j = \{x_j \mid B_j x_j \leq b_j, x_j \geq 0\} \text{ for each } j$$

Thus  $X_j$  is a closed convex set and any point in the set  $X_j$  can be expressed as some convex combination of the set of extreme points of  $X_j$  and its set of extreme rays.<sup>11</sup>

Thus, any  $x_j \in X_j$  can be expressed as:

$$(3) \quad x_j = \sum_{p=1}^{P(j)} \lambda_j^p x_j^p + \sum_{r=1}^{R(j)} \delta_j^r \tilde{x}_j^r$$

where for  $j=1, \dots, n$

$x_j^p$  with  $p=(1, \dots, P(j))$  are the extreme points of  $X_j$

$\tilde{x}_j^r$  with  $r=(1, \dots, R(j))$  identify the extreme rays of

$$\{x_j \mid B_j x_j \leq b_j, x_j \geq 0\}$$

$$\sum_{p=1}^{P(j)} \lambda_j^p = 1 \text{ and all } \lambda_j^p \geq 0, \delta_j^r \geq 0$$

Using (3) one can represent the block angular problem D now as:

$$\text{maximize } \sum_{j=1}^n c_j \left( \sum_{p=1}^{P(j)} \lambda_j^p x_j^p + \sum_{r=1}^{R(j)} \delta_j^r \tilde{x}_j^r \right)$$



$$\text{s.t.} \quad \sum_{j=1}^n A_j \left( \sum_{p=1}^{P(j)} \lambda_j^p x_j^p \right) + \sum_{r=1}^{R(j)} \delta_j^r \tilde{x}_j^r \leq a$$

$$\sum_{p=1}^{P(j)} \lambda_j^p = 1 \quad \text{for } j=1, \dots, n$$

$$\text{all } \lambda_j^p \geq 0, \quad \delta_j^r \geq 0$$

If one then uses the convention that

$$w_j^p = c_j x_j^p, \quad \tilde{w}_j^r = c_j \tilde{x}_j^r, \quad L_j^p = A_j^p x_j^p \quad \text{and} \quad \tilde{L}_j^r = A_j^r \tilde{x}_j^r$$

then the problem can be written as

$$(D_0) \quad \begin{aligned} &\text{maximize} \quad \sum_{j=1}^n \left( \sum_{p=1}^{P(j)} w_j^p \lambda_j^p + \sum_{r=1}^{R(j)} \tilde{w}_j^r \delta_j^r \right) \\ &\text{s.t.} \quad \sum_{j=1}^n \left( \sum_{p=1}^{P(j)} L_j^p \lambda_j^p + \sum_{r=1}^{R(j)} \tilde{L}_j^r \delta_j^r \right) \leq a \end{aligned}$$

$$\sum_{p=1}^{P(j)} \lambda_j^p = 1 \quad \text{for } j=1, \dots, n$$

$$\text{all } \lambda_j^p \geq 0, \quad \delta_j^r \geq 0$$

The original block angular problem with  $m + \sum_{j=1}^n m_j$  number of

rows of constraints has now been replaced with a problem

$D_0$  with  $m + n$  number of rows of constraints where  $\sum_{j=1}^n m_j$  is

likely to be much larger than  $n$ . However, the problem  $D_0$

has as many variables as there are extreme points and extreme rays.

The Dantzig and Wolfe decomposition procedure works by starting with a limited number of these points and rays and then progressively determines new points and rays as required. This process is called column generation. When the problem  $D'_0$  is formulated with only a number of the possible extreme points and rays, it shall be called the restricted master problem.

Suppose  $\Pi$  is an optimal dual multiplier vector associated with the first  $m$  inequality restrictions of the restricted master problem at some iteration and let  $\alpha = (\alpha_1, \dots, \alpha_n)$  be the optimal dual multiplier vector associated with convex weighting constraints. For each division  $j=1, \dots, n$ , the central headquarters communicates the shadow price vector  $\Pi$  and asks the divisional managers to solve their divisional (infimal) subproblems.

$$\begin{aligned} \bar{D} \quad & \max (c_j - \Pi A_j) x_j \\ & \text{s.t. } B_j x_j \leq b_j \\ & x_j \geq 0 \end{aligned}$$

Clearly, the feasible solution to such a problem will occur at an extreme point or on an extreme ray of the divisional private feasible region. If the division determines the optimal solution to be at extreme point  $x_j^P \in X_j$  the division indicates this to the central headquarters and communicates the  $L_j^P$  characteristics of the point. The central headquarters then uses the simplex optimality

requirement<sup>12</sup> which, for extreme points is if  $w_j^p - L_j^p - \alpha > 0$  add the column  $(L_j^p, 0, \dots, 1, \dots, 0)$  to the restricted master problem. The first  $m$  components of this column vector are given by  $L_j^p$  and the last  $n$  components are zero, except for the  $j$ th, which is equal to unity. Similarly, if the solution to the  $j$ th divisional informal subproblem occurs along an extreme ray  $\tilde{x}_j^r$  of the set  $X_j$ , the central headquarters adds<sup>13</sup>  $(\tilde{L}_j^r, 0, \dots, 0)$  as a new column to the restricted master problem. One can depict the information flows as follows:

RESTRICTED MASTER PROBLEMSTART

Solve resource allocation problem with initial and generated columns to determine optimal weights  $\lambda_j^P, \delta_j^I$  and associated dual prices  $\Pi, \alpha$ .

determine  
Using reduced cost coefficients/whether for any  $j=1, \dots, n$ ,  
 $w_j^P - \Pi L_j^P - \alpha > 0$  or  $\hat{w}_j^I - \Pi L_j^I > 0$  which ever is appropriate, if so determine a new column  $(L_j^P, 0, \dots, 1, \dots, 0)$  with associated objective function value  $w_j^P$  or new column  $(L_j^I, 0, \dots, 0)$  with associated objective function value  $\hat{w}_j^I$  whichever is appropriate and go to START. If no such inequalities exist, STOP.

communicate dual prices  $\Pi$  to divisions



communicate reduced cost coefficients

Given the dual prices,  $\Pi$ , each division solves its allocation problem and communicates "reduced cost coefficients" for extreme point or extreme ray solutions, that solve the divisional allocation problem. The coefficients values are:

$w_j^P - \Pi L_j^P$  for an extreme point solution for division  $j$ .

$\hat{w}_j^I - \Pi L_j^I$  for an extreme ray solution for division  $j$ .

DIVISIONAL SUBPROBLEMS

To illustrate the decomposition procedure, suppose the overall corporate problem were

$$\max Z = x_1 + x_2 + 2y_1 + y_2$$

$$\text{s.t.} \quad x_1 + 2x_2 + 2y_1 + y_2 \leq 40$$

$$x_1 + 3x_2 \leq 30$$

$$2x_1 + x_2 \leq 20$$

$$y_1 \leq 10$$

$$y_2 \leq 10$$

$$y_1 + y_2 \leq 15$$

Thus, for two divisions  $i=1,2$

$$c_1 = (1, 1)$$

$$c_2 = (2, 1)$$

$$A_1 = (1, 2)$$

$$A_2 = (2, 1)$$

$$a = 40$$

$$B_1 = \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$b_1 = \begin{bmatrix} 30 \\ 20 \end{bmatrix}$$

$$b_2 = \begin{bmatrix} 1 & 1 \\ 10 \\ 10 \\ 15 \end{bmatrix}$$

$$X = \{x_1, x_2 \mid x_1 + 3x_2 < 30, 2x_1 + x_2 \leq 20, x_1, x_2 \geq 0\}$$

$$Y = \{y_1, y_2 \mid y_1 \leq 10, y_2 \leq 10, y_1 + y_2 \leq 15, y_1, y_2 \geq 0\}$$

ITERATION 0 = START

To start the iterative procedure, an initial basic feasible solution is required. By inspection one can choose:

$$x^0 = (0,0), y^0 = (0,0) \text{ as } (0,0) \in X \text{ and } (0,0) \in Y$$

$$\text{Thus, } A_1 x^0 = \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = 0$$

$$A_2 y^0 = \begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = 0$$

$$c_1 x^0 = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = 0$$

$$c_2 y^0 = \begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = 0$$

Thus, the initial restricted master problem is

$$\max \quad 0\lambda_x^0 + 0\lambda_y^0$$

$$\text{s.t. } 0\lambda_x^0 + 0\lambda_y^0 + s = 40$$

$$\lambda_x^0 = 1$$

$$\lambda_y^0 = 1$$

$$\lambda_x^0, \lambda_y^0, s \geq 0$$

Thus the (constraint) columns  $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ ,  $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$  have

been generated for the restricted master problem. The initial basic feasible solution to the initial restricted master problem is

$$(x^0, y^0, s, \lambda_x^0, \lambda_y^0) = (0, 0, 40, 1, 1)$$

with associated dual prices

$$(u^0, \alpha_x^0, \alpha_y^0) = (0, 0, 0)$$

#### ITERATION 1

The central headquarters informs the divisions that there is to be no charge ( $u^0=0$ ) for the corporate supplied common resource and asks the divisions to determine how much of the resource they therefore require. The divisional managers, therefore, solve the following problems:

for division x

$$\max z_x^1 = x_1 + x_2$$

$$\text{s.t. } x^1 \in X$$

for division y

$$\max z_y^1 = 2y_1 + y_2$$

$$\text{s.t. } y^1 \in Y$$

the solution  $x^1 = (6, 8)$

the solution  $y^1 = (10, 5)$

$$\text{Thus } c_1 x^1 = 14$$

$$c_2 y^1 = 25$$

$$A_1 x^1 = 22$$

$$A_2 y^1 = 25$$

$$\text{And } c_1 x^1 - u^0 A_1 x^1 - \alpha_x^0 = 14 > 0$$

$$c_2 y^1 - u^0 A_2 y^1 - \alpha_y^0 = 25 > 0$$

so the two new columns  $\begin{bmatrix} 22 \\ 1 \\ 0 \end{bmatrix}$  and  $\begin{bmatrix} 25 \\ 0 \\ 1 \end{bmatrix}$  should be added

to the restricted master problem which is now:

$$\max \quad 14\lambda_x^1 + 25\lambda_y^1$$

$$\text{s.t.} \quad 22\lambda_x^1 + 25\lambda_y^1 + s = 40$$

$$\lambda_x^0 + \lambda_x^1 = 1$$

$$\lambda_y^0 + \lambda_y^1 = 1$$

$$\lambda_x^0, \lambda_y^0, \lambda_x^1, \lambda_y^1, s \geq 0$$

The optimal solution to this problem is

$$\lambda_x^0 = \frac{7}{22}, \lambda_x^1 = \frac{15}{22}, \lambda_y^1 = 1, s = \lambda_y^0 = 0$$

with associated dual prices  $(u^1, \alpha_x^1, \alpha_y^1) = (\frac{14}{22}, 0, \frac{200}{22})$

## ITERATION 2

The central headquarters now informs the divisional managers that the unit price for the common resource is  $\frac{14}{22}$  and again requests them to solve their divisional problems, which are

$$\max z_x^2 = (c_1 - u^1 A_1) x^2 = \frac{4}{11} x_1^2 - \frac{3}{11} x_2^2$$

$$\text{s.t.} \quad x^2 \in X \qquad \text{s.t.} \quad x^2 \in X$$



$$\begin{aligned} \max z_v^2 &= (c_2 - u^1 A_2) y^2 = \frac{8y_1^2}{11} + \frac{4y_2^2}{11} \\ \text{s.t. } y^2 &\in Y & \text{s.t. } y^2 &\in Y \end{aligned}$$

The solutions are respectively  $x^2 = (10, 0)$   
 $y^2 = (10, 5)$

$$\begin{aligned} \text{with } c_1 x^2 &= 10 & c_2 y^2 &= 25 \\ A_1 x^2 &= 10 & A_2 y^2 &= 25 \end{aligned}$$

$$\text{and } c_1 x^2 = u^1 A_1 x^2 - \alpha_x^1 - \frac{40}{11} > 0$$

$$c_2 y^2 - u^1 A_2 - \alpha_y^1 = 0$$

so the two new columns  $\begin{bmatrix} 10 \\ 1 \\ 0 \end{bmatrix}$   $\begin{bmatrix} 25 \\ 0 \\ 1 \end{bmatrix}$  should be added

to the restricted master problem, which is now:

$$\max 14\lambda_x^1 + 25\lambda_y^1 + 10\lambda_x^2 + 25\lambda_y^2$$

$$\text{s.t. } 22\lambda_x^1 + 25\lambda_y^1 + 10\lambda_x^2 + 25\lambda_y^2 + s = 40$$

$$\lambda_x^0 + \lambda_x^1 + \lambda_x^2 = 1$$

$$\lambda_y^0 + \lambda_y^1 + \lambda_y^2 = 1$$

$$\lambda_x^0, \lambda_x^1, \lambda_x^2, \lambda_y^0, \lambda_y^1, \lambda_y^2, s \geq 0$$

The optimal solution to this problem is:

$$\lambda_x^1 = \frac{5}{12}, \lambda_x^2 = \frac{7}{12}, \lambda_y^1 = 1, s = \lambda_x^0 = \lambda_y^0 = \lambda_y^2 = 0$$

with associated dual prices  $(u^2, \alpha_x^2, \alpha_y^2) = (\frac{1}{3}, \frac{20}{3}, \frac{50}{3})$

### ITERATION 3

The central headquarters now informs the divisional managers that the common resources unit price will be  $\frac{1}{3}$  and again requests them to solve their divisional problems, which are:

$$\max z_x^3 = \frac{2}{3} x_1^3 + \frac{1}{3} x_2^3$$

$$\text{s.t.} \quad x^3 \in X$$

$$\max z_y^3 = \frac{4}{3} y_1^3 + \frac{2}{3} y_2^3$$

$$\text{s.t.} \quad y^3 \in Y$$

The solutions are respectively  $x^3 = (6, 8)$  OR  $(10, 0)$  OR some convex combination of these two extreme points  
 $y^3 = (10, 5)$

The overall optimal solution must, therefore have been obtained as the columns generated in this last iteration were added during previous iterations.

The iterative solution procedure's convergence to the optimal solution is guaranteed<sup>14</sup> in a finite number of iterations since there are only a finite number of extreme points and rays. However, the number of iterations may be

very large for some problems. For this reason, it may be desirable to terminate the iterations before optimality is obtained. A very attractive feature of the Dantzig and Wolfe decomposition method in this connection is that it provides a bound on the optimal solution value. Hence, one can develop stopping rules dependent on the size of the bounds.

To demonstrate this consider a restricted master problem at some iteration  $t$  having optimal solution value  $Z_t$ . For ease of notation, assume the problem only has extreme point and not extreme ray solutions. Let  $\lambda_p^*$ ,  $p=1, \dots, P$  give the optimal solution to the overall problem.

$$z^* = \sum_{p=1}^P w^p \lambda_p^*$$

s.t.

$$\sum_{p=1}^P L^p \lambda_p^* \leq a$$

$$\sum_{p=1}^P \lambda_p^* = 1$$

multiply the first constraint by its optimal dual price vector  $u$  and the second constraint by its optimal dual price  $\alpha$  and subtract these two expressions from the first equation. This gives

$$z^* - (ua + \alpha) = \sum_{p=1}^P \lambda_p^* (w^p - u l^p - \alpha)$$

The bracketed right-hand side quantities are the reduced cost coefficients and recall that the central headquarters tried to determine the most positive reduced cost coefficients to identify potential column entrants so that

$$z^* - (ua + \alpha) < \sum_{p=1}^P \lambda_p^* (\max_{p=1, \dots, P} (w^p - uL^p - \alpha))$$

Note the restricted master problem requires that

$\sum_{p=1}^P \lambda_p^* = 1$ . In addition, note that  $(ua + \alpha)$  is the optimal solution to the dual problem at iteration  $t$ .

Therefore, since both primal and dual are feasible, it must be that  $z_t = (ua + \alpha)$  by<sup>15</sup> the Fundamental Theorem of Duality for Linear Programming. Thus

$$z^* \leq z_t + \max_{p=1, \dots, P} (w^p - uL^p - \alpha)$$

Let us define

$$z_t + \max_{p=1, \dots, P} (w^p - uL^p - \alpha) \equiv \hat{z}_t$$

Thus  $z_t \leq z^* \leq \hat{z}_t$

where the left-hand side inequality follows from

$$0 \leq z^* - z_t \leq \max_{p=1, \dots, P} (w^p - uL^p - \alpha)$$

where the right-hand side inequality must always be positive except when  $z^t = z^*$ .

To illustrate, consider iteration 2 of the previous problem. The lower bound is clearly the solution to restricted master at iteration 2 which is

$$\begin{aligned} z_t &= 1 \times \left( \frac{5.6}{12} + \frac{7.10}{12} \right) + 1 \times \left( \frac{5.8}{12} \right) + 2 \times 10 + 1 \times 5 \\ &= \frac{110}{3} = 36.66 \end{aligned}$$

$$\text{The upper bound} = \frac{110}{3} + \frac{40}{11} = 40.30$$

The above phase in the Dantzig and Wolfe procedures shall be called the iterative phase. Once the optimal transfer prices have been determined and identified,

one must consider how to ensure that this solution is arrived at in practice. This phase is called the implementation phase.

The previous analysis discussing bounding of the optimal solution and convergence showed that it would be possible to stop the iterative process before optimality is obtained, when the central headquarters is satisfied with the approximate solution. The question then is how will the final operating decisions be implemented. The ideology assumed by the Hirshleifer procedure was that transfer prices would be sufficient to guide divisional managers to make unaided operating decisions that would be optimal for the corporation.

To illustrate this concept, let us use the previous example. The feasible regions and modified objective functions faced by the divisions at the beginning of iteration 2 are presented in Figure 2. Thus, division 1 will wish to produce 10 units of  $x_1$  given the prevailing unit transfer price of  $\frac{14}{22}$ , while division 2 will wish to produce 10 units of  $y_1$  and 5 units of  $y_2$ . Given these divisional plans, then two new columns are generated for the restricted master problem which is then solved generating a new solution (convex combination of extreme points) and a new unit transfer price of  $\frac{1}{3}$ . The feasible regions and modified objective functions faced by the division at the beginning of iteration 3 are presented in Figure 3. Division 1 will wish to produce  $(x_1, x_2) = \lambda(6, 8) + (1 - \lambda)(10, 0)$  for any value of  $\lambda$  satisfying  $1 \geq \lambda \geq 0$ . Division 2 will wish to produce 10 units of  $y_1$  and 5 units of  $y_2$  as before.

However, the optimal corporate solution is based on the assumption that division 1 produces  $x = \frac{5}{12} \times (6, 8) + \frac{7}{12} \times (10, 0) = (\frac{25}{3}, \frac{10}{3})$  and division 2 produces  $y = (10, 5)$ . Thus, in order to ensure overall optimality specifying the transfer price is not sufficient and the central headquarters must specify for division 1 exact production requirements<sup>16</sup>. This type of problem with implementing transfer pricing procedures using the Dantzig and Wolfe decomposition method has been known for some time. For instance, Baumol and Fabian (1964) comment that:

FIGURE 2

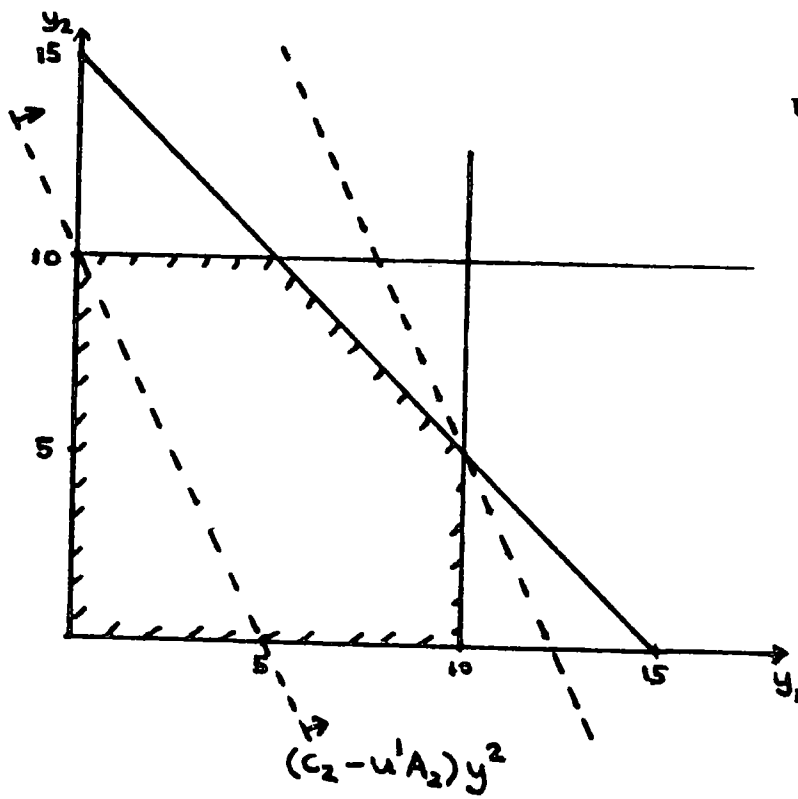
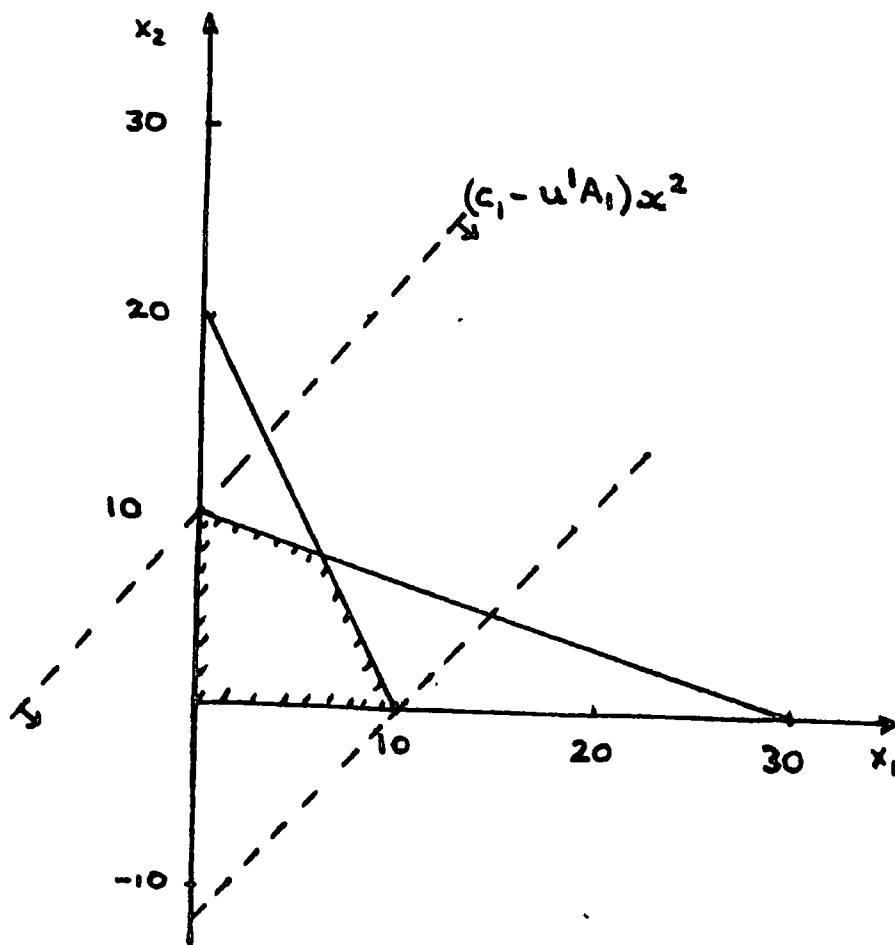
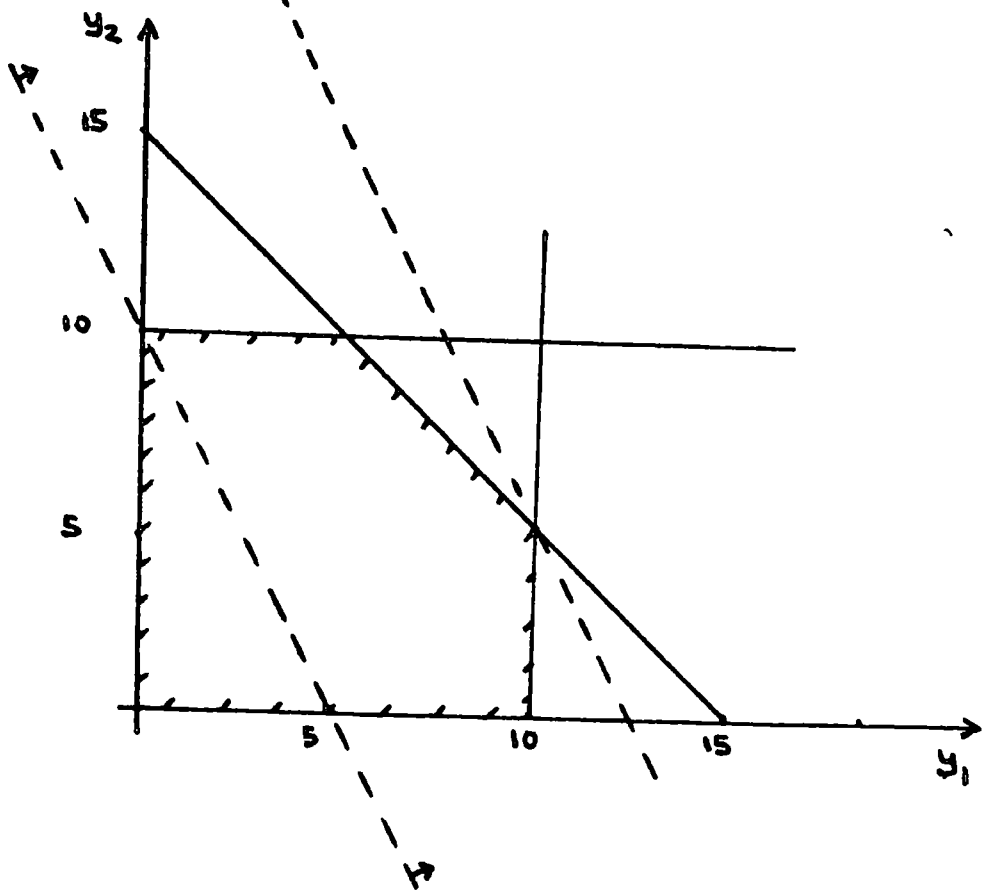
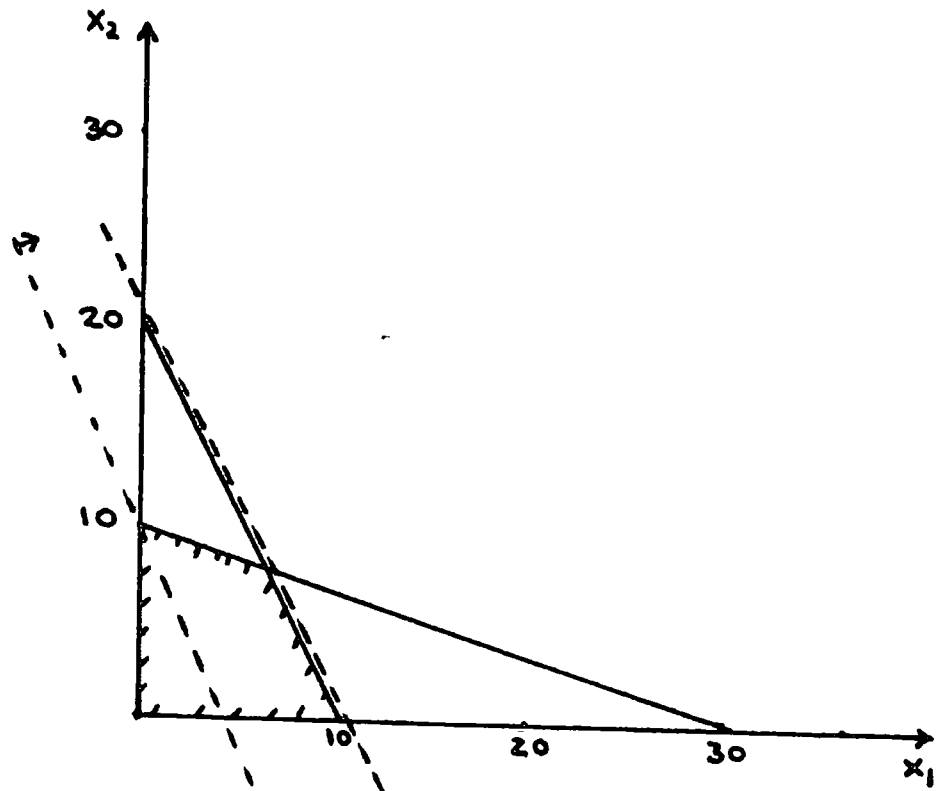


FIGURE 3





"It is to be emphasized that now division managers must be told by the company what weights they are to employ, i.e. what combination of their proposals the company desires them to produce. There is no automatic motivation mechanism which will lead division managers to arrive at such a combination of outputs of their own volition. In this way, the decentralization permitted by decomposition breaks down completely at this point".

In the example presented here, the problem has arisen because of the linearity of the model. One could get round this problem by having the central headquarters instruct the divisions to operate certain plans. However, this means that the only role for divisional management is to communicate information and carry out instructions. There seems little room for any decision to be made by divisional management.

### 3.3.2 Introducing Non-Linearities into the Dantzig and Wolfe Decomposition Method.

Given a completely linear environment, the Dantzig and Wolfe Decomposition method did not necessarily generate the transfer prices that would allow divisional managers to make their own operating decisions. This is because a linear programming model can only arrive at a solution (operating level) which is at an extreme point of the divisional constraint sets. However, the optimal solution to the original block angular corporate problem need not be such that any solution occurs at an extreme point of the divisional constraint sets. This is because the

extreme points of the original block angular corporate problem need not correspond to the extreme points of independent (divisional) diagonal blocks which only partly define the constraint set for the overall problem. The overall optimal solutions may well be interior to the independent (divisional) diagonal block constraints sets. Only by weighting certain extreme points of the independent divisional problems can these interior points be generated. Thus, divisional managers are being instructed to take specific operating decisions when such weights have to be communicated. However, if the divisional objective functions were non linear, then non-extreme points could be obtained without requiring specific weights to be specified. This approach of introducing non-linearities into the Dantzig-Wolfe Decomposition procedure has been considered by Jennergren (1973) and Hass (1968).

Jennergren's procedure involves using the Dantzig and Wolfe Decomposition procedure to locate the optimal corporate solution. Once this solution is obtained, the central headquarters then informs the divisional managers that common resources will be priced according to specified linear price schedules (supply functions) of the form

$$r_j^t + k a_{ji}$$

$$t = 1, 2, \dots \text{ indexes the iteration}$$

$$k > 0$$

Let  $r^t = (r_1^t, \dots, r_m^t)$  and  $a_i = (a_{1i}, \dots, a_{mi})$

for  $a_{ji} > 0$ , this represents the quantity of common resource  $j$  required by division  $i$  to operate at the optimal production level  $x_{ji}$ , which solves the revised divisional problem  $D_{JEN}$

for  $a_{ji} < 0$ , this represents the quantity of common resource  $j$  supplied by division  $i$  which operates at the optimal production level  $x_{ji}$  which solves  $D_{JEN}$

$$\text{Hence } a_i = x_i' A_i'$$

where the superscript ( $'$ ) indicates the operation of transposition.

The divisional problems with the Jennergren procedure then are:

$$\begin{aligned} D_{JEN}: \quad & \max (c_i - (r + kx_k' A_i') A_i) x_i \\ & \text{s.t. } x_i \in X_i \\ & \text{for each } i=1, \dots, n \end{aligned}$$

Thus the objective function of the divisional problems are quadratic. The procedure is implemented by the central headquarters choosing initial price schedules and asking the divisional managers what would be their optimal operating decisions (that solve  $D_{JEN}$ ) given these price schedules. Given this information at iteration  $t$ , the central headquarters then revises the price schedules so that at the next iteration, the divisional solutions will converge towards the optimal solution determined by the Dantzig and Wolfe method. Jennergren proved convergence of his procedure to the optimal solution, but only as the number of iterations becomes infinitely large.

Jennergren's procedure is best thought of as a possibility result which shows that optimal co-ordination via pricing is possible rather than as a practical approach. This is because Jennergren's procedure does not work unless the Dantzig and Wolfe decomposition method is first iteratively carried out until optimality is reached and, as was mentioned earlier, for practical applications, the iterations are likely to be stopped before optimality is attained.

Hass argues that often the corporation under consideration will be operating in imperfect markets, where the price it pays for inputs and the prices at which it sells outputs depends upon the quantity it purchases or sells of the product or related products also purchased or sold by the corporation. Thus, the divisions face downward sloping demand curves and upward sloping supply curves (price functions are of the form:

$$p_j = d + ea_j$$

where  $d$  and  $e$  are constants. The Hass procedure allows for any demand dependency which can be expressed in such a way that the divisional total revenue expression is a polynomial of degree two or less, that is the overall corporate problem and divisional problems are allowed to have quadratic objective functions. A crucial aspect of the Hass procedure, however, is that it does not assume separability of the objective functions of the divisions.

The procedure is not only applicable to quadratic programming problems, but also to problems with quadratic objective functions which have non-linear convex divisional constraints sets.

The Hass procedure works like the Dantzig and Wolfe procedure by column generation. That is, the divisions subproblems determine new feasible points<sup>17</sup>. The restricted master problem then determines the optimal weighting to give to all the feasible points known of by the central headquarters. Given a new optimal weighting, the central headquarters communicates revised price functions which incorporate the dual prices generated by solving the restricted master. That is, the revised price functions are adjusted for the current opportunity costs of the scarce corporate resources and for the current marginal values of the dependencies they impose or benefit by. The iterative procedure continues until the optimality criterion establishes that all the most recent divisional plans (feasible points) from the last iteration have been added already to the restricted master problem. One difficulty with the Hass procedure is that the divisional problems may be very difficult to solve, particularly if the divisional constraints are non-linear as well.

It should be noted, at this point, that the Dantzig and Wolfe Decomposition procedure can also be applied to corporate problems of the following form:

$$\begin{aligned}
 \max \quad & \sum_{i=1}^n f_i(x_i) \\
 \text{s.t.} \quad & \sum_{i=1}^n A_i x_i \leq a \\
 & x_i \in X_i \quad \text{for } i = 1, \dots, n
 \end{aligned}$$

where instead of being required to be linear functions as in the original Dantzig and Wolfe formulation, each function  $f_i$  is required to be continuous and concave and each set  $X_i$  is closed, bounded and convex<sup>18</sup>.

### 3.3.3 The Ten Kate Decomposition Method

Let us assume that the overall corporate problem has the following standard block angular structure as before.

$$\begin{aligned}
 \max \quad & c_1 x_1 + \dots + c_n x_n \\
 \text{s.t.} \quad & A_1 x_1 + \dots + A_n x_n \leq a \\
 & B_1 x_1 \leq b_1 \\
 & \quad B_2 x_2 \leq b_2 \\
 & \quad \quad \quad \vdots \\
 & \quad \quad \quad B_n x_n \leq b_n \\
 & x_1, \dots, x_n \geq 0
 \end{aligned}$$

where  $a$  is the  $m$ -vector  $a = (a_1, \dots, a_m)$

One can reformulate the problem by partitioning the problem so that one can explicitly consider the feasible production set for each division, including any common resource constraints as follows:

$$\begin{array}{rcll}
 \max & c_1 x_1 + \dots + c_n x_n & & \\
 \text{s.t.} & A_1 x_1 & -a_1 & \leq 0 \\
 & B_1 x_1 & & \leq b_1 \\
 & A_2 x_2 & -a_2 & \leq 0 \\
 & B_2 x_2 & & \leq b_2 \\
 & \vdots & & \vdots \\
 & A_n x_n & -a_n & \leq 0 \\
 & B_n x_n & & \leq b_n \\
 & a_1 + a_2 + \dots + a_n & & = a \\
 & x_1, x_2, \dots, x_n & & \geq 0
 \end{array}$$

These two formulations are clearly equivalent so that, if one problem has a solution, so will the other. The latter partition has clearly left the central headquarters with two groups of decision variables to determine,  $(x_1, \dots, x_n)$  and  $(a_1, \dots, a_n)$ . The Ten Kate decomposition method is a resource directive decomposition approach where the central headquarters will hope that, by selecting (co-ordinating) appropriate resource vectors  $(a_1, \dots, a_n)$ , it can determine

an optimal solution to the overall problem without needing to know all the information on divisional constraints. This is in contrast to the Dantzig Wolfe decomposition method that can be described as a price directive decomposition approach. The above discussions indicate that the two level hierarchy of subproblems takes the following specific form:

For each division  $i$  the divisional problem is:

$$\begin{aligned}
 z_i &= \max c_i x_i \\
 \text{s.t.} \quad & A_i x_i \leq a_i \\
 & B_i x_i \leq b_i \\
 & x_i \geq 0
 \end{aligned}$$

with the restricted master problem for the central headquarters being:

$$\begin{aligned}
 \max \quad & \{z_1 + z_2 + \dots + z_n\} \\
 \text{s.t.} \quad & \sum_{i=1}^n a_i = a
 \end{aligned}$$

The algorithm used for solving this problem, is based on using the dual problem to the overall corporate (primal) problem. Its role is to iteratively approximate the optimal  $z_i$  functions for  $i=1, \dots, n$ . The dual problem to the overall corporate problem (expressed in partitioned form) is as follows:



$$\min \sum_{i=1}^n \Pi_i b_i + \Pi a$$

$$\text{s.t. } u_i A_i + \Pi_i B_i \geq c_i$$

$$- u_i + \Pi = 0$$

$$u_i, \Pi_i \geq 0 \quad \text{for } i=1, \dots, n$$

where clearly  $u_i$  is the dual price on the  $i^{\text{th}}$  divisional common resource constraint set,  $\Pi_i$  is the dual price on the  $i^{\text{th}}$  divisional private constraint set and  $\Pi$  is the dual price on the total common resource share constraint.

Note that feasible solutions to the divisional sub-problems may not exist if inappropriate  $a_i$  vectors are chosen, for instance, if the vector contained only negative components. This would give rise to dual prices from the solution of the divisional problem being set equal to  $-\infty$ . The reason this occurs can be seen if one considers the dual of the divisional (primal) problems which is as follows:

$$z_i = \min \{u_i a_i + \Pi_i b_i\}$$

$$\text{s.t. } u_i A_i + \Pi_i B_i \geq c_i$$

$$u_i, \Pi_i \geq 0$$

for each  $i=1, \dots, n$ .

Since the objective function and constraint sets are linear, any solution must occur at an extreme point or along an extreme ray. Let  $(u_i^p, \pi_i^p)$  for  $p=1, \dots, P(i)$  represent the set of extreme points of  $\{(u_i, \pi_i) \mid u_i A_i + \pi_i B_i \geq c_i, u_i, \pi_i \geq 0\}$  and let  $(\hat{u}_i^r, \hat{\pi}_i^r)$  for  $r=1, \dots, R(i)$  represent the set of extreme rays for the cone  $\{(u_i, \pi_i) \mid u_i A_i + \pi_i B_i \geq 0, u_i, \pi_i \geq 0\}$

The divisional dual problem will have an unbounded solution  $(-\infty)$  if the solution occurs along an extreme ray of the defined cone. To prevent such inappropriate selections of  $a_i$  vectors by the central headquarters, the following constraints must be satisfied:

$$\hat{u}_i^r a_i + \hat{\pi}_i^r b_i \geq 0$$

Therefore, one can now define a set of possible divisional common resource allocations  $A_i$  as follows:

Any  $a_i \in A_i$  will ensure that the primal divisional problems have feasible solutions where:

$$A_i = \{a_i \mid \hat{u}_i^r a_i + \hat{\pi}_i^r b_i \geq 0 \text{ for } r=1, \dots, R(i)\}$$

The algorithm uses the dual divisional problems to approximate solutions to the primal divisional problems because of the existence by the Fundamental Theorem of Duality in Linear Programming of the following relationships:

$$\begin{aligned} & \max \{c_i x_i \mid A_i x_i \leq a_i, B_i x_i \leq b_i, x_i \geq 0\} \\ & = \min_{p=1, \dots, P(i)} \{u_i^p a_i + \pi_i^p b_i\} \\ & \leq u_i^p a_i + \pi_i^p b_i \quad \text{for any } p=1, \dots, P(i) \end{aligned}$$

for all  $i = 1, \dots, n$ .

Thus, the central headquarters can start the iterative procedure by offering certain allocations of common resources  $a = (a_1, \dots, a_n)$  to the divisions and asking them to solve their primal (or dual) divisional problems. Thus, the  $i^{\text{th}}$  division determines the dual prices at some iteration  $p$  to be  $u_i^p$  and  $\pi_i^p$ .

$$\text{Let} \quad y_i^p = \pi_i^p b_i$$

$$\text{and} \quad \hat{y}_i^r = \pi_i^r b_i$$

Thus, the central headquarters can ask the divisional managers to communicate the values for  $u_i^p$  and  $y_i^p$  and this generates the following constraint to the restricted master problem  $z_i \leq u_i^p a_i^p + y_i^p$  for  $i = 1, \dots, n$ .

Thus, what the central headquarters is attempting to do is iteratively determine divisional constraints so that the restricted master problem becomes

$$\max \{ z_1 + z_2 + \dots + z_n \}$$

$$\text{s.t. } z_i \leq u_i^P a_i^P + y_i^P \quad p = 1, \dots, P(i)$$

$$i = 1, \dots, n$$

$$a_i \in A_i \quad i = 1, \dots, n$$

$$\sum_{i=1}^n a_j = a$$

Suppose at some iteration  $t$  the solution to this restricted master problem is  $(z_1^t, \dots, z_n^t, a_1^t, \dots, a_n^t)$ . Each division is informed of its allocation of common resource  $a_i^t$ , and is asked to solve its primal (or dual) divisional subproblem.

1. IF the  $i^{\text{th}}$  divisional (primal) problem is infeasible at iteration  $t$ , the constraint

$$u_i^r a_i^t + v_i^r \geq 0 \text{ is added to the restricted master problem.}$$

2. IF the  $i^{\text{th}}$  divisional (primal) problem has a feasible solution at iteration  $t$ , the constraint

$$u_i^t a_i^t + y_i^t \geq z_i \text{ is added to the restricted master problem}$$

$$\text{if } u_i^t a_i^t + y_i^t < z_i$$

3. IF the  $i^{\text{th}}$  divisional (primal) problem has a feasible solution at iteration  $t$  and

$$u_i^t + y_i^t \geq z_i \text{ no constraint is added for division } i.$$

Once all the divisional problems have feasible solutions and no new constraints need to be added to the restricted master problem, the overall corporate problem has been solved.

The underlying economic rationale for Tenkate's decomposition method is that common resources should be reallocated at some iteration  $t$  by the central headquarters, unless each common resource yields equal marginal return in all uses<sup>20</sup>. To illustrate how the method generates such marginal return values, let  $(u_i^{t-1}, \pi_i^{t-1})$  be the optimal dual prices associated with the  $i^{\text{th}}$  division's (primal) problem when division  $i$ 's common resource allocation vector is  $a_i^{t-1} \in A_i$ . At the next iteration for  $a_i^t \in A_i$ .

$$z_i(a_i^t) = \min \{u_i^t a_i^t + y_i^t | u_i^t A_i + \pi_i^t B_i \geq c_i,$$

$$u_i, \pi_i \geq 0\}$$

$$\leq u_i^{t-1} a_i + y_i^{t-1}$$

$$\text{also } z_i(a_i^{t-1}) = u_i^{t-1} a_i + y_i^{t-1}$$

These two relations imply

$$z_i(a_i^t) = z_i(a_i^{t-1}) + u_i^{t-1} (a_i^t - a_i^{t-1})$$

Thus  $u_i^{t-1}$  indicates how the optimal solution value of the primal divisional problem would change in response to small changes in  $a_i^{t-1}$ .

To understand more clearly the relationship between the Dantzig and Wolfe Decomposition Method and the Tenkate Decomposition Method let us reconsider the original block angular overall corporate problem in partitioned form. The dual to this problem is:

$$\min \pi_1 b_1 + \pi_2 b_2 + \dots + \pi_n b_n + \Pi a$$

$$\text{s.t. } -u_i + \pi = 0 \quad \text{for } i = 1, \dots, n$$

$$u_i A_i + \pi_i B_i \geq c_i \quad \text{for } i = 1, \dots, n$$

$$u_i, \pi_i \geq 0 \quad \text{for } i = 1, \dots, n$$

One can now apply the Dantzig-Wolfe methodology of presenting this dual problem in terms of convex combinations of extreme points and extreme rays (defined by  $u_i A_i + \pi_i B_i \geq c_i$  divisional constraints) as follows:

$$\min \sum_{i=1}^n \left\{ \sum_{p=1}^{P(i)} \lambda_i^p (\pi_i)^p b_i + \sum_{r=1}^{R(i)} \delta_i^r (\hat{\pi}_i)^r b_i \right\} + \Pi a$$

$$\text{s.t. } - \sum_{p=1}^{P(i)} \lambda_i^p (u_i)^p - \sum_{r=1}^{R(i)} \delta_i^r (\hat{u}_i)^r + \pi = 0 \quad \text{for } i = 1, \dots, n$$

$$\sum_{p=1}^{P(i)} \lambda_i^p = 1 \quad \text{for } i = 1, \dots, n$$

$$\lambda_i^p, \lambda_i^r \geq 0$$

If one now takes the dual of this last formulation of the problem, with the first  $n$  dual variables being denoted  $a_i$  for  $i=1, \dots, n$  and the last  $n$  dual variables being denoted  $z_i$  for  $i = 1, \dots, n$  one has:

$$\max \quad z_1 + z_2 + \dots + z_n$$

$$\text{s.t.} \quad - u_i^p a_i + z_i \leq \pi_i^p b_i$$

$$- \hat{u}_i^r a_i \leq \hat{\pi}_i^r b_i$$

$$+ \sum_{i=1}^n a_i \leq a$$

which is just a slightly rearranged form of the restricted master problem in the Tenkate Decomposition Method. Thus, applying the Tenkate Decomposition Method is entirely equivalent to dualizing the overall corporate problem in partitioned form and then applying the Dantzig and Wolfe Decomposition Method to the dual, that is the Tenkate method can be viewed as the dual of the Dantzig and Wolfe method.

Given this close relationship, one can see that some characteristics of the Dantzig and Wolfe procedure, such as being able to compute upper and lower bounds will also be present with the Tenkate procedure. As was done for the Dantzig and Wolfe procedure, one can present a summary of the information flows with the Tenkate Decomposition procedure as follows:

RESTRICTED MASTER PROBLEM

START

determine allocation of common resources  
that solves restricted master at

$$\hat{u}_1^t + \hat{y}_1^t \gg 0$$

If divisional problem infeasible add constraint

iteration  $t$ , giving objective function

If divisional problem feasible add constraint

value  $z_1^t$

$$u_1^t a_1^t + y_1^t \geq z_1^t \text{ if } u_1^t a_1^t + y_1^t < z_1^t$$

However if  $u_1^t a_1^t + y_1^t \geq z_1^t$  do not add such a constraint

If no constraints to add STOP

communicate  
allocations  
to divisions

 $a_1^t$ 

communicate  
 $u_1^t, y_1^t$

Divisions solve divisional problems and determine dual prices  $u_1^t$  and constants  $y_1^t$ .

DIVISIONAL SUBPROBLEMS



Before comparing properties of the two decomposition procedures, it is important to consider whether it is appropriate to consider a resource directive allocation mechanism as a transfer pricing procedure. Clearly with a resource directive approach, the central headquarters no longer sets a common transfer price vector that all divisions face. However, the responses on dual prices by the divisional managers are clearly bid prices for certain allocations of common resources. These bid prices are clearly transfer prices, with the property that instead of the central headquarters instigating them, the divisional managers set them. So, in my opinion, both price and resource directive approaches can be viewed as transfer pricing procedures provided one recognizes that the transfer prices are generated by different groups and that the informational efficiency of prices in a price directive approach are not present to the same extent in a resource directive approach. Specifically in the price directive approach one price for each resource is communicated and summarizes a large amount of information. However, with the resource directive approach, a different resource allocation needs to be provided to every division and each division is likely to respond with a different transfer price for a given resource.

The principal reason for considering the Tenkate algorithm was because of the inability of the Dantzig and Wolfe price directive algorithm to co-ordinate optimally the operating decisions of divisional managers after the final

iteration when operating in a linear environment. This problem is clearly resolved with the resource directive approach at the final optimal iteration of the Tenkate algorithm each divisional manager can calculate what is the optimal divisional operating decision to take, given the allocation of common resources from the central headquarters and these decisions will be optimal for the central headquarters overall problem. However, as was mentioned earlier such resource allocation algorithms are unlikely to be iterated to optimality, hence one is concerned with properties of the algorithm at some arbitrary intermediate iteration. With the Tenkate algorithm co-ordinability is assured at any iteration provided the the common resource allocation vector is feasible. That is the central headquarters will be satisfied with the (decentralized) operating decisions that divisional managers take, given any feasible, but not yet optimal allocation of scarce common resources. However, during the iterative procedure, certain proposed allocations of common resources by the central headquarters will be unfeasible<sup>21</sup> as the central headquarters is not aware of all the constraints faced by divisions. If, though, the iterative procedure were to be halted at this point, the central headquarters could ensure co-ordinability by using a previous (stored) feasible allocation of common resources.

The above discussion is only a short introduction to the comparative properties of both algorithms. A much more detailed discussion can be found in Burton and Obel (1979). A more detailed discussion is not presented here as both

algorithms suffer from a common problem, which is discussed in Section 3.4.2. However, it is worth noting that the Tenkate algorithm can also be extended to non linear environments<sup>22</sup>. In addition, many non linear problems which can not be solved using pricing algorithms, because of non-convexities, may be solveable using resource directive algorithms<sup>23</sup>. Also a mixed algorithm which uses Dantzig and Wolfe price directive decomposition for some divisions and Tenkate resource directive decomposition for the remaining divisions has been formulated by Obel (1978).

#### 3.4 INCENTIVE COMPATIBILITY AND UNCERTAINTY

In Section 2.3 a general discussion was presented which considered the problems that may be encountered when organizations attempt to decentralize. This section will consider whether the Hirshleifer and Dantzig and Wolfe transfer pricing procedures overcomes these difficulties. To summarize, the problems that Section 2.3 identified were:

1. Performance measure destruction of co-operation
2. Strategic information specialists
3. Goal incongruent performance measures
4. Incompleteness of performance measures
5. Uncertainty in performance measures.

The first three problems are intimately related, as the following simple case study illustrates, and will be grouped under the heading Incentive Compatibility from here on.

Suppose the distribution divisional manager of the prototypical organization identifies a new potentially successful product line, provided it can be manufactured below a certain cost. The manufacturing divisional manager would then be asked to identify a suitable production technique (if one exists) and estimate the cost conditions for such an operation. The manufacturing divisional manager will not wish to co-operate unless the manager perceives some advantage from co-operation. In such a situation, a manager will respond with the information that is most advantageous to communicate from his own point of view which need not necessarily coincide with the interests of the corporation. The reason for this is the outcome of his performance measure (his reward) will be conditional on the information he chooses to communicate.

This simple case study illustrates that divisional performance measures that are incongruent can lead to un-co-operative behaviour which takes the form of restricted and or distorted information flows arising. A more rigorous example of this type of occurrence is presented in Section 3.4.1 when considering the incentive compatibility properties of the Hirshleifer transfer pricing procedure.

Problems four and five give rise to an important question in management accounting. Should divisional managers operating in an uncertain environment be required to share risk or should they be protected from risk.

The following discussion uses a model presented by<sup>24</sup> Kanodia (1979) to consider these issues:

Let  $Q_M$  denote the intermediate product

with  $q_M$  = quantity of  $Q_M$  produced by manufacturing division

$X_M$  denotes the sole input used by the manufacturing division

$x_M$  = quantity of  $X_M$  used in production of

$Q_D$  denotes the final product

$q_D$  = quantity of final product  $Q_D$  sold by distribution division

$X_D$  denotes the input used by the distribution division to convert the intermediate product into the final product

$x_D$  = quantity of  $X_D$  used in production

$q_m^D$  = quantity of intermediate product  $Q_M$  used by distribution division

$P_m$  = fixed price of  $X_M$  bought on a (certain) external market

$P_D$  = fixed price of  $X_D$  bought on a (certain) external market

$P^*$  = fixed price of  $Q_D$  sold on a (certain) external market

In addition assume the technology of the two divisions can be represented by the production functions:

$$q_m = F(x_m) \quad (1)$$

$$q_D = G(q_m^D, x_D) \quad (2)$$

where  $F$  and  $G$  are increasing, strictly concave, and continuously differentiable.

The overall corporate problem therefore is:

$$(D_0) \quad \max p^* q_D - p_m x_m - p_D x_D$$

$$\text{s.t. (1), (2)}$$

$$\text{and } q_m \geq q_m^D$$

$$q_m, q_m^D, x_m, x_D \geq 0$$

The Lagrangian for this problem  $D_0$  is:

$$\begin{aligned} \max L_{D_0} &= p^* q_D - p_m x_m - p_D x_D + \lambda_1 (F(x_m) - q_m) \\ &+ \lambda_2 (G(q_m^D, x_D) - q_D) \\ &+ \lambda_3 (q_m - q_m^D) \end{aligned}$$

First order conditions give

$$\frac{\delta L}{\delta q_D} = p^* - \lambda_2 = 0$$

$$\frac{\delta L}{\delta q_m} = -\lambda_1 + \lambda_2 \frac{\delta G(\dots)}{\delta q_m} = 0$$

$$\frac{\delta L}{\delta x_m} = -p_m + \lambda_1 \frac{\delta F(\dots)}{\delta x_m} = 0$$

$$\frac{\delta L}{\delta x_D} = -p_D + \lambda_2 \frac{\delta G(\dots)}{\delta x_D} = 0$$

from the first expression one can substitute  $\lambda_2 = p^*$  from the second expression and using the above substitution this gives

$$\lambda_1 = p^* \frac{\delta G(\dots)}{\delta q_m}$$

thus

$$p_m = p^* \frac{\delta G(\dots)}{\delta q_m} \frac{\delta F}{\delta x_m} \quad (3)$$

and 
$$p_D = p^* \frac{\delta G(\dots)}{\delta q_m} \quad (4)$$

Note that the final constraint of  $D_0$  will hold with equality and Equation (3) is the Hirshleifer rule, marginal cost equals net marginal revenue.

In Kanodia's model if one lets  $p'$  represent the transfer price for the intermediate product, then the divisional subproblems will be

$$\begin{aligned} (D_M) \quad & \max \quad p' q_M - p_m x_m \\ & \text{s.t.} \quad q_m = F(x_m) \\ & \quad \quad q_m, x_m \geq 0 \end{aligned}$$

and

$$\begin{aligned} (D_D) \quad & \max \quad p^* q_D - p' q_M^D - p_D x_D \\ & \text{s.t.} \quad q_D = G(q_M^D, d_D) \\ & \quad \quad q_D, q_M^D, x_D \geq 0 \end{aligned}$$

The Lagrangian for the manufacturing divisional subproblem is:

$$L_{DM} = p' q_m - p_m x_m + \lambda_m (F(x_m) - q_m)$$

First order conditions give:

$$\frac{\delta L}{\delta q_m} = p' - \lambda_m = 0$$

$$\frac{\delta L}{\delta x_m} = -p_m + \lambda_m \frac{\delta F(\cdot)}{\delta x_m} = 0$$

$$\text{which require } p_m = p' \frac{\delta F(\cdot)}{\delta x_m} \quad (5)$$

The Lagrangian for the distributional divisional subproblem is:

$$L_{D,D} = p^* q_D - p' q_m^D - p_D x_D + \lambda_D (G(q_m^D, x_D) - q_D)$$

First order conditions give:

$$\frac{\delta L}{\delta q_D} = p^* - \lambda_D = 0$$

$$\frac{\delta L}{\delta q_m^D} = -p' + \lambda_D \frac{\delta G(\cdot, \cdot, \cdot)}{\delta q_m^D} = 0$$

$$\frac{\delta L}{\delta x_D} = -p_D + \lambda_D \frac{\delta G(\cdot, \cdot, \cdot)}{\delta x_D} = 0$$

$$\text{which require } p' = p^* \frac{\delta G(\cdot, \cdot, \cdot)}{\delta q_m^D} \quad (6)$$

$$\text{and } p_D = p^* \frac{\delta G(\cdot, \cdot, \cdot)}{\delta x_D} \quad (7)$$

If  $p'$  is set correctly, it will induce the divisional managers to act such that  $q_m = q_D^M$

Thus equations (5) and (6) give (3) and (7) is identical to (4).



Let us now consider how the problem changes on the introduction of an element of uncertainty into the problem. Assume that the price of the final product  $p^*$  is not known with certainty<sup>25</sup>. Suppose it is commonly known that there are  $T$  possible prices for  $Q_D$ ,  $(p_1^*, \dots, p_T^*)$  but that the probability (of price occurrence) distribution is best known by the distributional divisional manager<sup>26</sup>. This assumption would seem to be consistent with the view that divisional managers were information specialists in their own markets. Assuming that  $Q_D$  is sold in a perfectly competitive market, the price  $p_t^*$  which is realized can not be influenced by any action of the divisional managers and is observed by both managers ex post. Let  $\Pi_{tm}$  and  $\Pi_{tD}$  denote the profits allocated to the respective divisions and assume the divisional managers are rewarded on the basis of linear compensation functions of the form:

$$s_{tM} = K_m + k_m \Pi_{tm} \quad K_m, k_m > 0 \quad (8)$$

$$s_{tD} = K_D + k_D \Pi_{tD} \quad K_D, k_D > 0 \quad (8)$$

Let  $U_m(s_m)$ ,  $U_D(s_D)$  represent the divisional managers' utility functions<sup>27</sup>. Suppose the central headquarters wishes to implement the "certainty equivalent" to the Firshleifer type procedure presented above. The central headquarters will wish to determine the transfer price  $\hat{p}'$  which equates demand<sup>and</sup> supply of the intermediate product.

Let  $\hat{q}_m$  denote the amount of  $Q_m$  produced and exchanged, and  $\hat{q}_D$  the amount of  $Q_D$  produced and sold, given  $\hat{p}'$ .

Thus the manufacturing division's manager will choose  $\hat{q}_m$  that solves:

$$\begin{aligned} \max \quad & \hat{p}' q_m - p_m x_m \\ \text{s.t.} \quad & q_m = F(x_m) \end{aligned} \tag{9}$$

Assume that  $(a_1, \dots, a_t, \dots, a_T)$  represents the probability distribution for the finished product price that the distributional divisional manager perceives. Given the argument of the distribution divisional manager's utility function, the manager will desire to maximize the expected utility from compensation that is:

$$\begin{aligned} \max \quad & \sum_{t=1}^T a_t U_D \{K_D + k_D (p_t^* q_D - \hat{p}' q_m^D - P_D x_D)\} \\ \text{s.t.} \quad & q_D = G(q_m^D, x_D) \end{aligned} \tag{10}$$

First order conditions for the Lagrangian for (10) require

$$\frac{\delta L}{\delta q_D} = \frac{\sum_{t=1}^T a_t \delta U_D(s_{tD})}{\delta q_D} - \lambda_D = 0 \tag{a}$$

where one can use the relationship

$$\frac{\delta U_D(s_{tD})}{\delta q_D} = \frac{\delta s_{tD}}{\delta q_D} \frac{\delta U(s_{tD})}{\delta s_{tD}}$$

and letting

$$U'_D(S_{tD}) = \frac{\delta U_D(S_{tD})}{\delta S_{tD}}$$

to write the first order condition (a) as:

$$\frac{\delta L}{\delta q_D} = k_D \sum_{t=1}^T a_t U'_D(S_{tD}) P_t^* - \lambda_D = 0 \quad (b)$$

the remaining first order conditions for the Lagrangian are:

$$\frac{\delta L}{\delta q_M} = \frac{\sum_{t=1}^T a_t \delta U_D(S_{tD})}{\delta q_M^D} + \frac{\lambda_D \delta G(\dots)}{\delta q_M^D} = 0$$

and since

$$\frac{\delta U_D(S_{tD})}{\delta q_M^D} = \frac{\delta S_{tD}}{\delta q_M^D} U'_D(S_{tD})$$

$$\frac{\delta L}{\delta q_M} = -k_D \sum_{t=1}^T a_t U'_D(S_{tD}) \hat{p}' + \lambda_D \frac{\delta G(\dots)}{\delta q_M^D} = 0 \quad (c)$$

and

$$\frac{\delta L}{\delta x_D} = \frac{\sum_{t=1}^T a_t \delta U_D(S_{tD})}{\delta x_D} + \lambda_D \frac{\delta G(\dots)}{\delta x_D} = 0$$

and since

$$\frac{\delta U_D(S_{tD})}{\delta x_D} = \frac{\delta S_{tD}}{\delta x_D} U'_D(S_{tD})$$

$$\frac{\delta L}{\delta x_D} = -k_D \sum_{t=1}^T a_t U'_D(S_{tD}) p_D + \lambda_D \frac{\delta G(\dots)}{\delta x_D} = 0 \quad (d)$$

The certainty equivalent price for  $Q_D$  shall be denoted  $\hat{p}^*$  is defined by the relationship:

$$\hat{p}^* \sum_{t=1}^T a_t U_D(S_{tD}) = \sum_{t=1}^T a_t U_D(S_{tD}) p_t^* \quad (e)$$

thus since equation (b) requires

$$k_D \sum_{t=1}^T a_t U'_D(S_{tD}) p_t^* = \lambda_D$$

one can express equation (c) as

$$\hat{p}' = \frac{k_D \sum_{t=1}^T a_t U'_D(S_{tD}) p_t^*}{k_D \sum_{t=1}^T a_t U'_D(S_{tD})} \cdot \frac{\delta G(\dots)}{\delta q_M^D}$$

using equation (e) this becomes

$$\hat{p}' = \hat{p}^* \frac{\delta G(\dots)}{\delta q_M^D} \quad (11)$$

Similarly one can express equation (d) as:

$$p_D = \frac{k_D \sum_{t=1}^T a_t U'_D(S_{tD}) p_t^*}{k_D \sum_{t=1}^T a_t U'_D(S_{tD})} = \frac{\delta G(\dots)}{\delta x_D}$$



using equation (c) this becomes

$$P_D = \hat{p}^* \frac{\delta G(\dots)}{\delta x_D} \quad (12)$$

Thus under uncertainty, the first order conditions are very similar to those under certainty, with the only difference being the certain price of the final product being replaced by its certainty equivalent when the distribution division operates under uncertain conditions.

In the case when the distribution division's manager is not risk neutral, the certainty equivalent price will not equal the expected price as the following result proves:

$$\sigma = \text{covariance} \{U_D'(\tilde{S}_D), \tilde{p}^*\}, \text{ that is}$$

$$\sigma = E \{U_D'(\tilde{S}_D), \tilde{p}^*\} - E \{U_D'(\tilde{S}_D)\} \cdot E \{\tilde{p}^*\}$$

using the above definition of the certainty equivalent price by equation (e) we have

$$\frac{\sigma}{E\{U_D'(\tilde{S}_D)\}} = \hat{p}^* - E \{\tilde{p}^*\}$$

rearranging gives

$$\hat{p}^* = E \{\tilde{p}^*\} + \frac{\sigma}{E\{U_D'(\tilde{S}_D)\}}$$

That is the certainty equivalent price is the expectation of the uncertain price plus a remainder term for a risk averse individual.

$$U_D'(\tilde{S}_D) > 0$$

and

$$U_D''(\hat{S}_D) = \frac{\delta U_D^2(S_D)}{\delta \tilde{S}_D^2} \leq 0$$

Thus the covariance is negative since as  $p^*$  increases,  $S_D$  increases and marginal utility drops making their covariance negative. Since marginal utility is positive, we have

$$\hat{p}^* < E\{\tilde{p}^*\}$$

Given this result, one would expect that the higher the risk aversion of distribution division's manager, the lower will be the certainty equivalent price  $\hat{p}^*$ , resulting in lower production of the final product  $Q_D$  and, hence, lower demand for the intermediate product  $Q_M$ .

One can express this intuitive result more formally as:

Kanodia Theorem: The more risk averse the distribution manager is, (i) the lower will be the division's production of  $Q_D$  and the division's demand  $Q_M^D$  for the intermediate product, at any given transfer price,  $p'$ ,

(ii) the lower will be the equilibrium transfer price,  $\hat{p}'$ , and

(iii) the lower will be the profits that accrue to the manufacturing division's manager.

This corresponds to Theorem 2 in Kanodia (1979) where the theorem is proven. The importance of the above analysis is that it shows that the distribution divisional manager will vary the production decisions taken, depending on his or her attitude towards risk. Given that the central headquarters may have a different attitude towards risk, this raises the question of whether the actions of the distributional divisional manager will optimize the central headquarters' preferences. If, given the different utility functions, both the distribution divisional manager and the central headquarters could always agree on which decision to take (output of  $Q_D$ ), then the problem would be just a question of how to share risk (uncertain compensation) between the divisional managers and the central headquarters. To express this in another fashion, if

there existed a group utility function which specified which decision to take given external uncertainty which all participants agreed with, then an optimal external decision (output of  $Q_D$ ) could always be identified, and the question of how different utility functional forms affect how risk needs to be shared could be considered separately from the external decision. The essential elements of the problem can be developed using the following simplified example due to Pratt<sup>28</sup>. The example shows that, in general, such a group utility function will not exist even in the simpler case of a corporate structure of a central headquarters and one division, never mind a structure with two divisions and a central headquarters, where all parties hold different attitudes towards risk.

Suppose the central headquarters has the utility function  $U_C$  and the distribution division's manager has utility function  $U_D$  as presented in Figure 2. Let the following lottery conceptualize the uncertain final product price environment.

## Lottery L

| state (price) | probability | payoff |
|---------------|-------------|--------|
| $P_1^*$       | .25         | -£300  |
| $P_2^*$       | .25         | -£100  |
| $P_3^*$       | .25         | +£100  |
| $P_4^*$       | .25         | +£300  |

The expected monetary value of L is zero and the expected utility of L is negative for each individual, since they are both risk averse. However, it is possible to partition L so that the expected utilities of the partitioned lotteries are both zero as below:

## Partition of L

| state (price) | Probability | payoff to C | payoff to D |
|---------------|-------------|-------------|-------------|
| $P_1^*$       | .25         | -£100       | -£200       |
| $P_2^*$       | .25         | -£100       | -£200       |
| $P_3^*$       | .25         | £ 0         | +£100       |
| $P_4^*$       | .25         | +£200       | +£100       |



$$E\{U_C(S_C)\} = -\frac{100}{4} - \frac{100}{4} + \frac{200}{4} = 0$$

$$E\{U_D(S_D)\} = -\frac{200}{4} + \frac{100}{4} + \frac{100}{4} = 0$$

Since the expected monetary value is zero, it can not be possible to repartition the lottery such that  $E\{U_C(S_C)\} > 0$  and  $E\{U_D(S_D)\} > 0$  given the risk aversity of the utility functions. Suppose the group members are free to partition the lotteries they select. Then for the pairs of utilities given in Figure 2, there is no group utility function  $U^*$  that the group can use to choose between lotteries. To see this, suppose to the contrary that such an appropriate group utility function  $U^*$  did exist. Then since a just acceptable partition can be determined for L, it must be that

$$\frac{1}{4} U^*(-£300) + \frac{1}{4} U^*(-£100) + \frac{1}{4} U^*(+£100) + \frac{1}{4} U^*(+£300) = 0$$

where we have arbitrarily let  $U^*(0) = 0$ .

Using Figure 2 again, one sees that a lottery L which gives a .5 chance of -£100 and a .5 chance of +£100 is just acceptable to either individual and there would be no partition of the lottery that the group members would strictly prefer to zero. Hence, it must be that

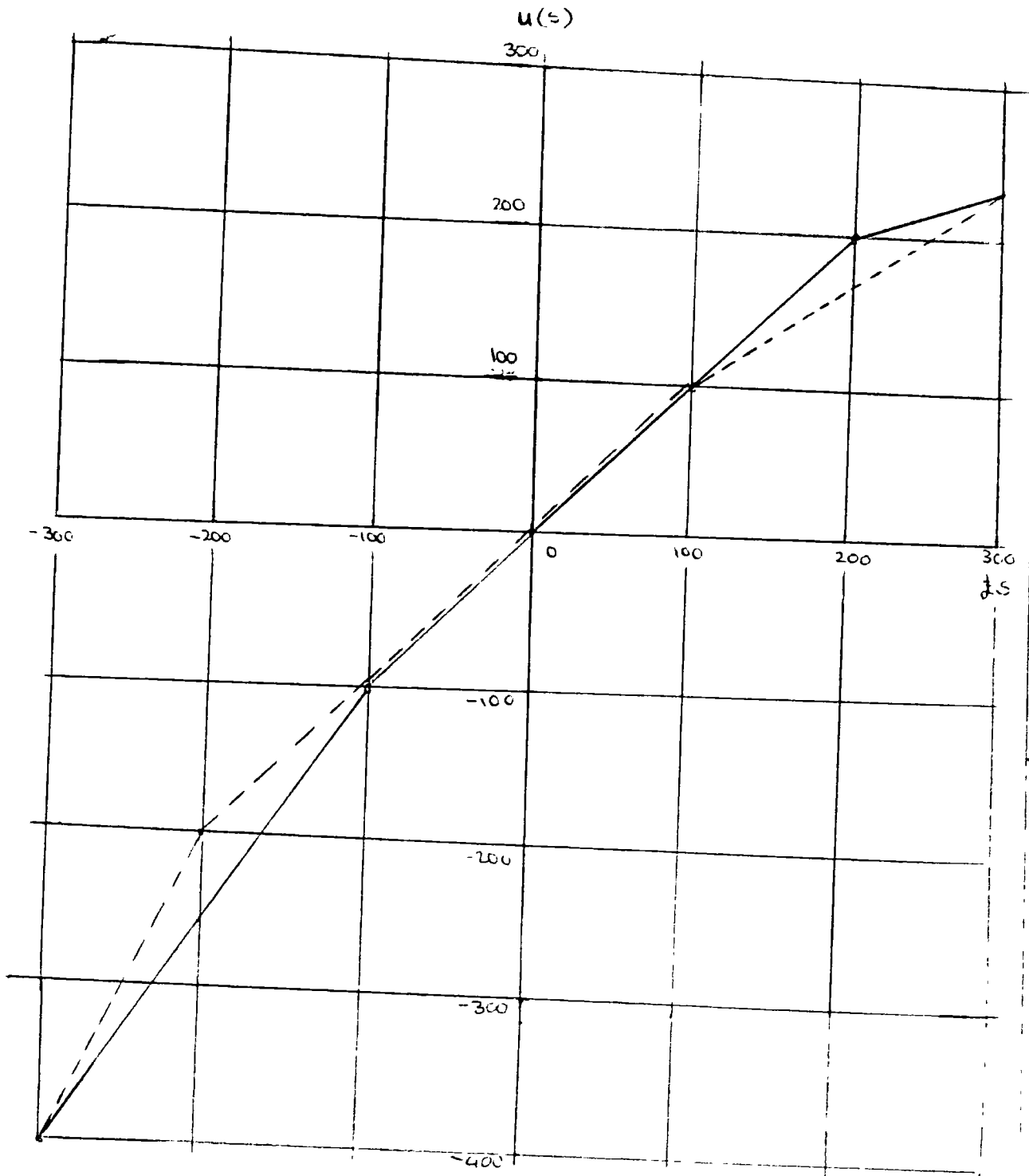
$$\frac{1}{2} U^*(-£100) + \frac{1}{2} U^*(+£100) = 0$$

One can then use the two equations above to see that  $U^*$  must satisfy the lottery  $L^+$

$$\frac{1}{2} U^*(-£300) + \frac{1}{2} U^*(+£300) = 0$$

FIGURE 2

————  $u_c$   
-----  $u_D$



However  $L^+$  can never be partitioned into (just barely) acceptable shares. To see this, we must have

$$(Z) S_{1C} + S_{1D} = -£300$$

$$(Y) S_{2C} + S_{2D} = +£300$$

$$(X) S_{1C} + S_{2C} \geq 0 \quad \text{for c's share to be acceptable}$$

$$(W) S_{1D} + S_{2D} \geq 0 \quad \text{for d's share to be acceptable}$$

adding equations (Z) and (Y) gives

$$(S_{1C} + S_{2C}) + (S_{1D} + S_{2D}) = 0$$

which implies equations (X) and (W) must hold with equality. This, in turn, implies that  $S_{1C}$  and  $S_{2C}$  must lie between -100 and +200 and  $S_{1D}$  and  $S_{2D}$  must lie between -200 and -100. The implications for c's acceptable partition mean that  $S_{1D}$  must be -£200 or less. This means that  $S_{2D}$  must be correspondingly +£200 or more. However, this means to remain acceptable for c, one must have a partition that is not acceptable to D. That is,  $L^+$  is not jointly acceptable, and no group  $U^*$  can exist. That is, the group can not act as a single decision maker would<sup>29</sup>.

It seems the implications of the above analysis are that, in general, since the external decisions have to be linked to how risk is shared internally, then there will need to be T contingent transfer prices. Kanodia (1979) deals with this problem by establishing conditions under which divisional managers will view the external decisions and utility from risky compensation as separate problems. He achieves this result by specifying a complete internal contingent market for the uncertain optimal  $Q_D$  decisions,

$Q_{1D}, \dots, Q_{tD}, \dots, Q_{TD}$ . The divisional manager's purchase contingent quantities at contingent prices using what Kanodia calls their ex-ante income to satisfy their risk preferences. Hence, they clearly wish to maximize their ex-ante income. Kanodia then defines ex-ante income in such a manner that the divisional managers maximizing ex-ante income will maximize the central headquarters preference (local risk sharing). This is achieved by setting the transfer price that defines ex-ante income equal to the central headquarters certainty equivalent price.

Kanodia also presents a global risk sharing transfer pricing procedure which allows for risk sharing among all three parties. This is based on the local sharing procedure, but with the contingent prices specified to ensure that all contingent markets clear.

The above discussion has implied the problem that arises under uncertainty is that optimal risk sharing agreements between divisional managers and the central headquarters have to be determined. However as Kaplan (page 612, 1982) puts it:

"in practice capital markets provide a myriad of economic agents to offer risk sharing arrangements for the owners of firms. Managers will typically be hired, not because of their desirable risk sharing abilities, but because they have specialized information, are good at making and implementing decisions and have the capability for advancement to higher-level managerial positions and responsibility . . . . . , Our problem will probably be the opposite one: how to shield the managers from the negative consequences of random outcomes from their decisions".

In other words, instead of trying to determine optimal risk sharing contracts with managers, uncertainty gives rise to the problem of how to shield managers' compensation from random outcomes. Thus, typically some form of budgetting

procedure which incorporates divisional management forecasts needs to be included. However, one must recognize that this leads to another problem. Since we have recognized operations are occurring in an uncertain environment, one can not expect forecasts and actualities to always correspond. Once divisional managers realize that information they are providing to construct budgets will also be used to evaluate their performance, an incentive is created for them to distort or bias information to improve their performance evaluation measure. Divisional managers can achieve such misrepresentations undetected because, when they act in their role as information specialists and are asked to forecast some variable, since they have the best information on the variable, when the central headquarters observes a variance between the forecast and the actual value of the variable after the event, the central headquarters can not be sure whether the variance is due to random chance, poor forecasting caused by poor skill or insufficient information, misrepresentation, or some combination of these factors.

One can sum up the discussion of this section by concluding that this section was intended to highlight two problems,

External uncertainty and asymmetric access to information (to assess the likelihood of uncertain events in the world external to the corporation) can give rise to internal uncertainty (incentive compatibility issues) with regard to the perceived truthfulness of information communicated by divisional managers.

The following two sections will illustrate these issues by considering whether divisional managers have incentives to report truthfully in the Hirshleifer and Dantzig and Wolfe transfer pricing procedures.

### 3.4.1 Incentives to Misrepresent in the Hirshleifer Procedure.

Let us recall the two divisional Hirshleifer transfer pricing procedure and notation presented in Section 3.2. Since we have assumed no external market for the intermediate product, the distribution division's manager will clearly perceive that he or she is a monopsonistic buyer of the manufacturing division's production. If the distribution division's manager chooses to exercise monopsonistic power, the manager will realise that the price that must be paid for any given quantity of the intermediate product, is given by the manufacturing divisions supply curve, that is the  $MC_M$  schedule. This means the monopsonistic profits attributable to the distribution division can be optimized by choosing the output  $q$  for which:

$$\begin{aligned} q \max D(q) - p^* q \\ \text{s.t. } p^* &= C'(q) \end{aligned}$$

that is

$$q \max D(q) - C'(q).q$$

First order conditions require:

$$D'(q) = C''(q).q + C'(q)$$

Assuming that the distribution division's net marginal revenue schedule is decreasing and the manufacturing division's marginal cost schedule is increasing, we have:

$$C''(q) \cdot q > 0$$

which means that

$$D'(q_{(\text{MONOPS})}) > D'(q_{(\text{COMPET})})$$

where  $q_{(\text{MONOPS})}$  is the output of the distribution division when it acts monopsonistically and  $q_{(\text{COMPET})}$  is the output of the distribution division if it acted competitively (as a price taker) as required by the Hirshleifer procedure. Thus, when the distribution divisional manager perceives the division's monopsonistic power, the manager can increase the profits of the division and, hence, his or her compensation<sup>30</sup> by purchasing less of the intermediate product than if the manager acted as a price taker. This case is illustrated in Figure 3. Note that since  $C'(q)$  is increasing, this implies that the transfer price in the monopsonistic case will also be lower than in the competitive case. The diagram also illustrates the rationale for the conditions derived from the first order requirements. That is, the additional divisional profit attributable to the distribution division when its manager acts monopsonistically is  $\text{AREA } (P^*_{(\text{COMPET})} P^*_{(\text{MONOPS})} DC) - \text{AREA } (ABC)$ . If this monopsonistic action merely leads to a redistribution of profits between the two divisions, the central headquarters may not be unduly concerned. However, the effect of the distribution division's manager's monopsonistic actions is to reduce total company profits by  $ABD$ .

The question is how would the distribution division's manager be able to realise such monopsonistic power undetected if, at the budgeting stage, the manager is required to forecast the divisions expected <sup>marginal</sup> net/revenue function. Figure 4 shows clearly that, if the distribution division's manager understates the division's <sup>marginal</sup> expected net/revenue function as below, he will be able to achieve the monopsonistic position without the central headquarters or manufacturing division's manager realising.

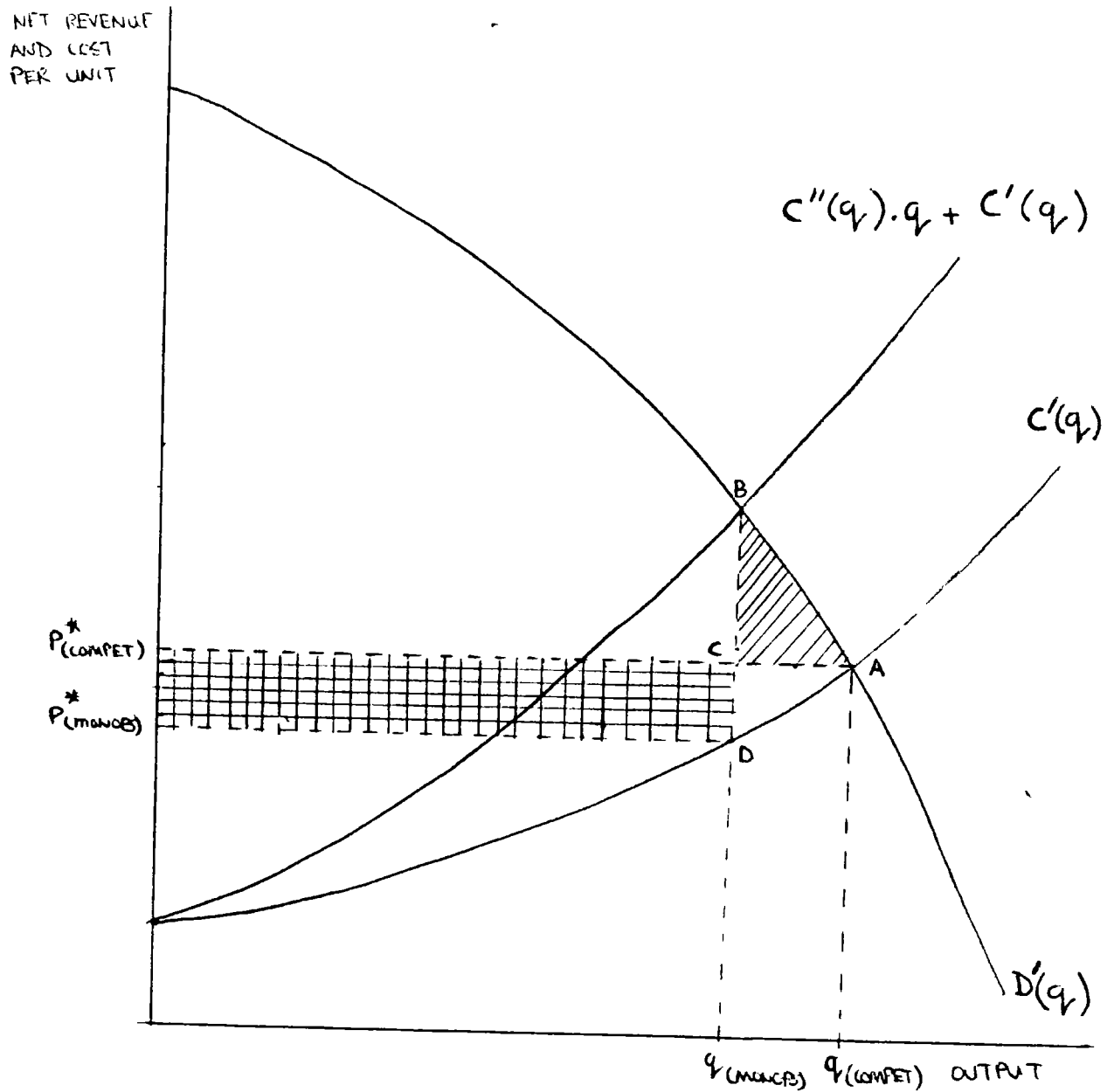
Note the incentive to misrepresent arises because one can not expect a limited number of transactors on an internal market not to recognise they have some market power. Thus, one would expect that, in this model, the manufacturing division's manager perceives that he or she has some market power as their division is the monopolistic seller of the intermediate product.

Whereas in the monopsonistic case, the distribution division's manager did not have an input demand schedule, but instead selected the output on the manufacturing division's supply schedule that maximized the distribution division's profits, here the manufacturing division will not have an output supply schedule relating price and quantity. The manufacturing division's manager will select the output on the distribution division's demand schedule that maximises the manufacturing division's profits, that is:

$$\begin{aligned} q \max p^* q - C(q) \\ \text{s.t. } p^* &= D'(q) \end{aligned}$$

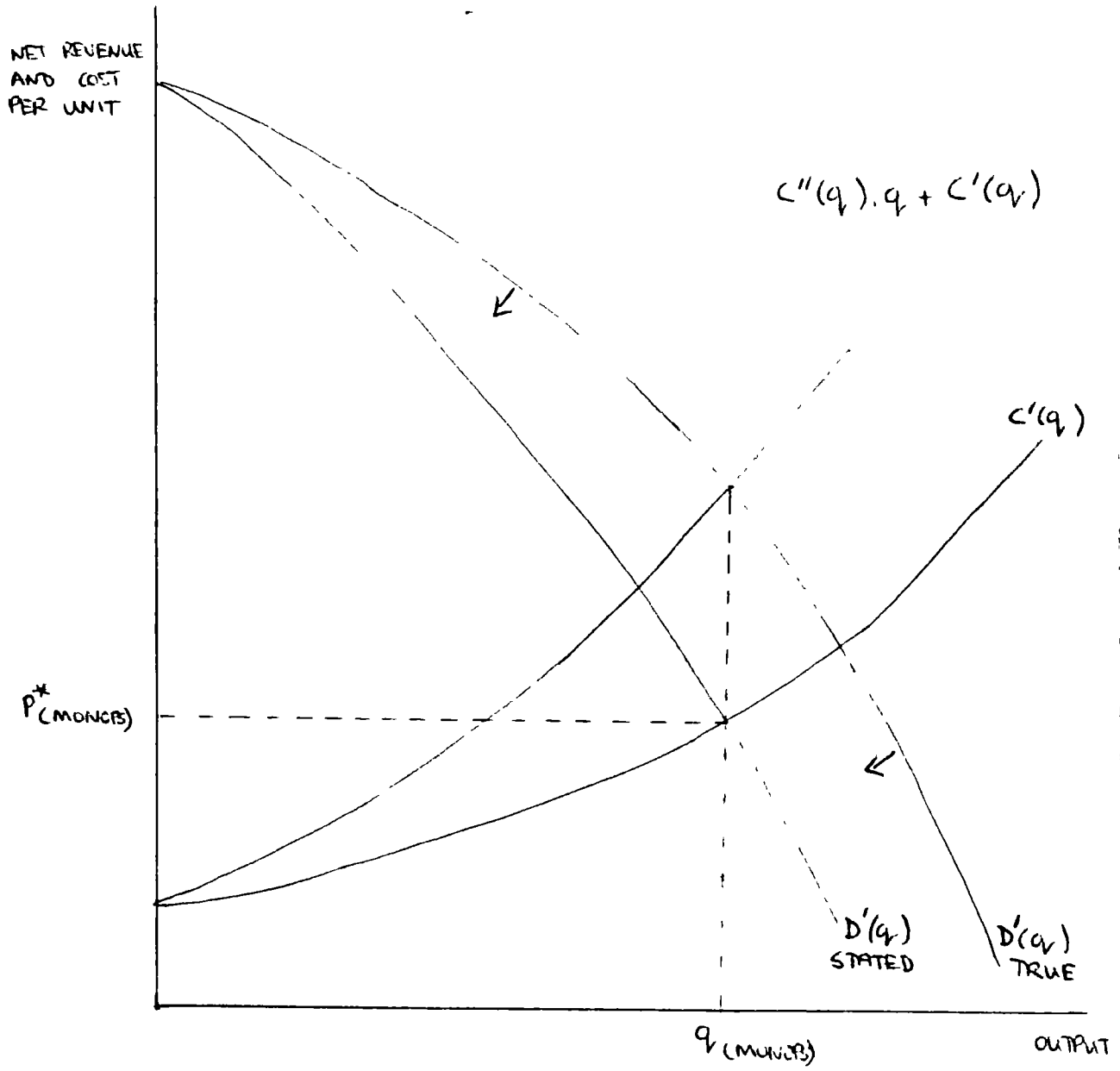


FIGURE 3



$$\text{AREA } (P^*_{(COMPET)}, P^*_{(MONOP)}, D, C) > \text{AREA } (A, B, C)$$

FIGURE 4



that is

$$q \max D'(q) \cdot q - C(q)$$

First order conditions require

$$D'(q) + D''(q) \cdot q = C'(q)$$

Assuming that the distribution division's net marginal revenue schedule is decreasing and the manufacturing division's marginal cost schedule is increasing, we have:

$$D''(q) \cdot q < 0$$

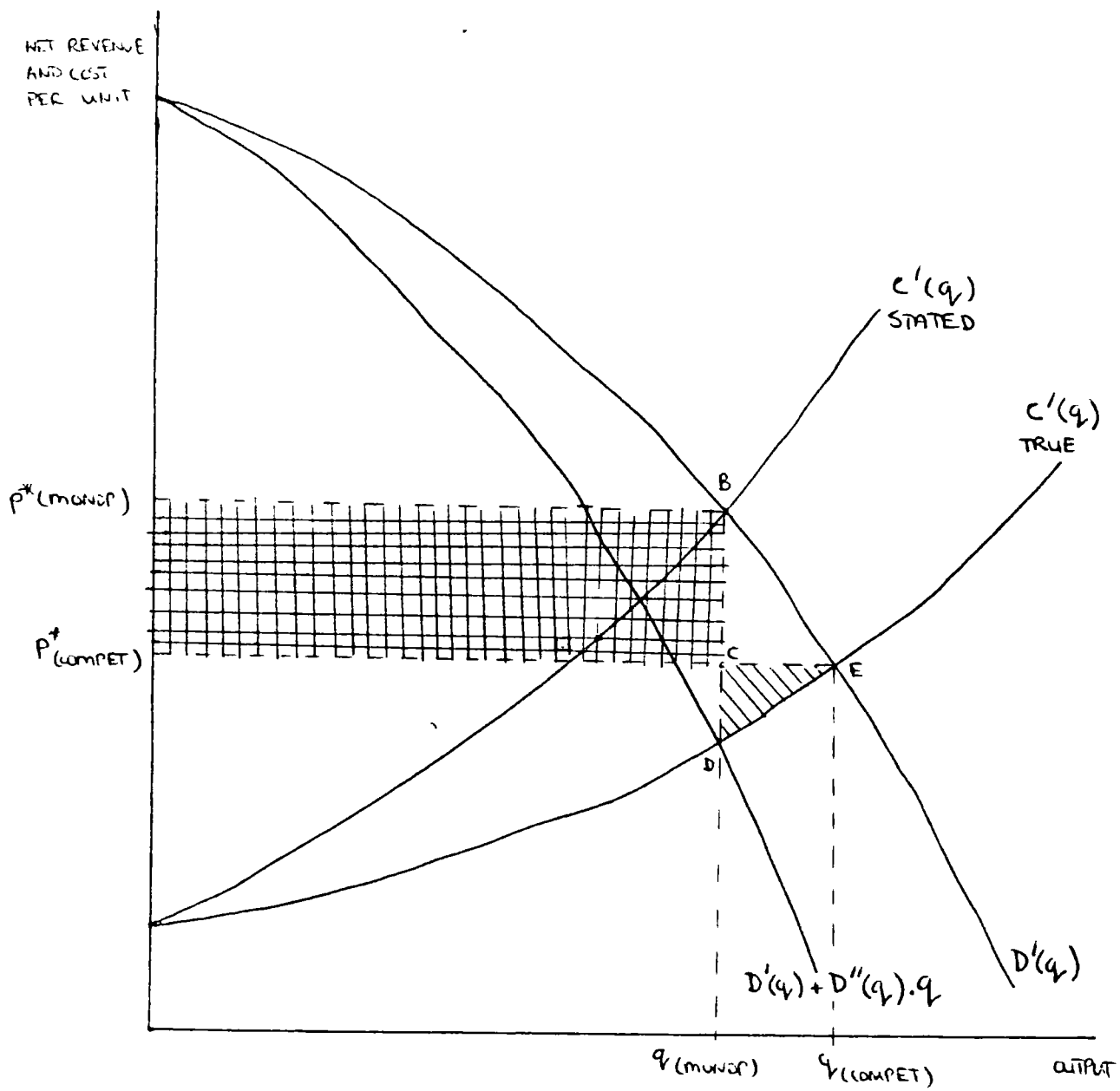
which means that

$$C'(q_{(MONOP)}) < C'(q_{(COMPET)})$$

where  $q_{(MONOP)}$  is the output of the manufacturing division when it acts monopolistically with  $q_{(COMPET)}$  being the output of the manufacturing division, if it acted competitively as required by the Hirshleifer procedure. This situation is depicted in Figure 5 which also shows how the manufacturing division's manager is able to realise such monopolistic power without the central headquarters or distribution division's manager realising. Also the diagram illustrates that the differential monopolistic profit is large and given by  $AREA(P^*_{(COMPET)} P^*_{(MONOP)} BC) - AREA(CDE)$  and the reduction in the overall corporate profits is  $AREA(BDE)$ . Thus, in this environment, the manufacturing divisional manager has a clear incentive to overstate the division's marginal cost conditions.

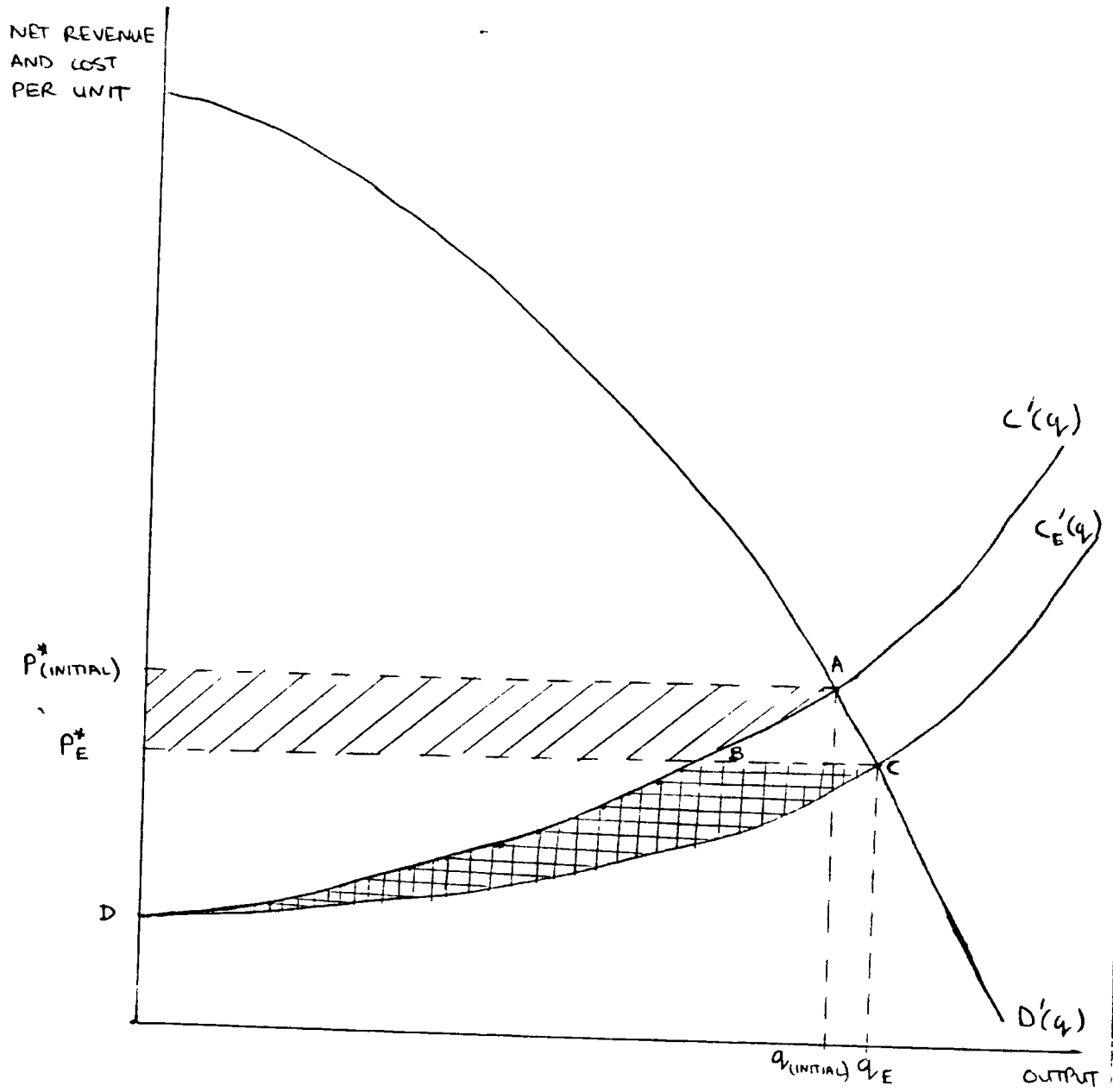
Let us consider a third factor that may give rise to a divisional manager having an incentive to mis-state a schedule or hide it. Suppose Figure 6 depicts the situation

where the distribution and manufacturing divisional managers report truthfully their net marginal revenue and marginal cost schedules initially. Thus, the agreed transfer will be  $q_{(INITIAL)}$  units of the intermediate product at the transfer price  $P_{(INITIAL)}^*$ . Assume immediately after these conditions are established by the budgeting procedure that the manufacturing division's manager learns of a more efficient process by which to manufacture the intermediate good, such that the marginal cost schedule now becomes  $c'_E(q)$  as depicted in Figure 6. If the manufacturing division's manager immediately reported this change in operating conditions, a new transfer price  $P_E^*$  could be determined which would give rise to a resultant increase in the output of the intermediate product. However, the change in the manufacturing division's profit would be  $AREA(DBC) - AREA(P_E^* P_{(INITIAL)}^* AB)$  and, since the latter area is greater than the former area, the division's profit and hence manager's compensation would fall. This result clearly is related to the monopolist solution which requires the manufacturing division's manager to overstate the division's marginal cost schedule to increase the division's profit. The result is clearly of some significant importance as it raises the question of why should a manufacturing division's manager ever want to introduce new technology of this type, since it will not benefit the manager concerned, since the benefits are passed on to the distribution division and its manager.



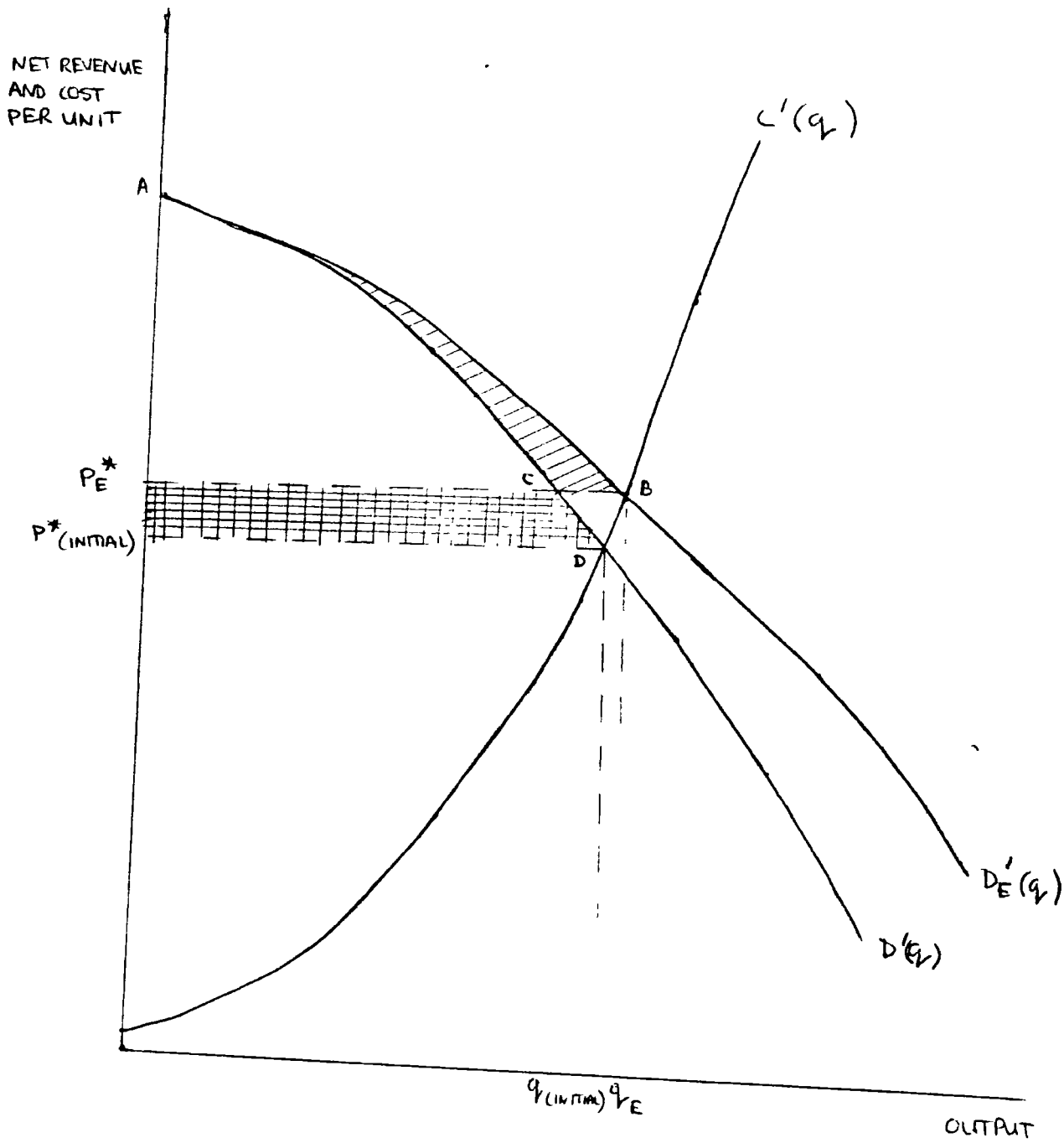
$$\text{AREA} ( P^*_{(\text{COMPET})} P^*_{(\text{MONOP})} B C ) > \text{AREA} ( C D E )$$

FIGURE 6.



I.C.

FIGURE 7



An analogous result can hold when one considers whether the distribution division's manager has no incentive to decrease distribution costs or increase distribution revenue so net marginal revenue now becomes  $D_E(q)$  as depicted in Figure 7. If the distribution division's manager reported this change in operating conditions, a new transfer price  $P_E^*$  could be determined which would give rise to a resultant increase in the output of the intermediate product. However, the change in the distribution division's profit would be  $\text{AREA (ABC)} - \text{AREA (CDP}^*_{\text{INITIAL}} P_E^*)$  and, since the latter is greater than the former area, the division's profit and, hence, the manager's compensation, would fall.

Note that, whereas the monopsonistic and monopolistic problems can arise for any cost and revenue conditions satisfying the regularity conditions, this is not the case for the "cost efficiency" and "revenue efficiency" problems. For instance, if in the "cost efficiency" problem, if the net marginal revenue schedule of the distribution division is relatively shallow and the marginal cost schedule of the manufacturing division relatively steep, the manufacturing division's manager will have an incentive to introduce new more efficient processes. For the "revenue efficiency" problem, if the net marginal revenue schedule of the distribution division is relatively steep and the marginal cost schedule of the manufacturing division relatively shallow, the distribution division's manager will have an incentive to introduce new more efficient distribution and sales techniques.



### 3.4.2 Incentives to Misrepresent in the Dantzig and Wolfe Decomposition Procedure for Transfer Pricing

Jennergren (1971) considers in detail the incentives for divisional managers to misrepresent in the Dantzig and Wolfe procedure. He assumes<sup>32</sup> a two division and central headquarters structure, so that the restricted master problem,  $D_0$ , takes the form:

$$\begin{aligned} \lambda_1, \lambda_2 \max \quad & \sum_{p=1}^{P(1)} W_1^p \lambda_1^p + \sum_{p=1}^{P(2)} W_2^p \lambda_2^p \\ \text{s.t.} \quad & \sum_{p=1}^{P(1)} L_1^p \lambda_1^p + \sum_{p=1}^{P(2)} L_2^p \lambda_2^p \leq a \\ & \sum_{p=1}^{P(1)} \lambda_1^p = 1, \quad \sum_{p=1}^{P(2)} \lambda_2^p = 1 \\ & \text{all } \lambda_1^p, \lambda_2^p \geq 0 \end{aligned}$$

Let

$$L_1 = (L_1^1, \dots, L_1^{P(1)})$$

$$L_2 = (L_2^1, \dots, L_2^{P(2)})$$

$$W_1 = (W_1^1, \dots, W_1^{P(1)})$$

$$W_2 = (W_2^1, \dots, W_2^{P(2)})$$

$$\lambda_1^T = (\lambda_1^1, \dots, \lambda_1^{P(1)})$$

where T denotes the operation of transposition

$$\lambda_2^T = (\lambda_2^1, \dots, \lambda_2^{P(2)})$$

and let  $e_1$  be defined as a  $P(1)$  row vector with the number one in every position and  $e_2$  be defined as a  $P(2)$  row vector with the number one in every position. Thus, one expresses the restricted master problem more succinctly as:

$$\lambda_1, \lambda_2 \quad \max \quad W_1 \lambda_1 + W_2 \lambda_2$$

$$\text{s.t. } L_1 \lambda_1 + L_2 \lambda_2 \leq a$$

$$e_1 \lambda_1 = 1$$

$$e_2 \lambda_2 = 1$$

$$\lambda_1 \lambda_2 \geq 0$$

In order to distinguish between the set of all conceivable reduced cost coefficients and those which are communicated to the central headquarters let:

$$\hat{L}_1 = (\hat{L}_1^1, \hat{L}_1^2, \hat{L}_1^3, \dots)$$

$$\hat{W}_1 = (\hat{W}_1^1, \hat{W}_1^2, \hat{W}_1^3, \dots)$$

represent the reduced cost coefficients communicated by division one's manager at iterations 1, 2, 3, .....

In order to analyse deterministically how division one's manager could optimally cheat, it is assumed that the manager knows the overall problem  $D$  and that the central headquarters is using Dantzig Wolfe decomposition to try and solve the problem. For the moment, it is assumed that the iterative process of resolution will be continued until an optimal solution to  $C_0$  has been determined. In addition, it is assumed that division two's manager does not wish to cheat. Jennergren (1971) also assumes that division one's manager will not be able to cheat undetected by appealing to the argument that uncertainty in forecasting has caused the discrepancy between forecasted and actual values. That is, he assumes no external environmental uncertainty and, in so doing, strengthens the case for arguing that misrepresentation can take place successfully, if he can show that, under these more restrictive conditions, divisions can still cheat. Thus, the cheating (mis representation) will be in the spirit of the examples in which divisional managers had no incentive to introduce new more efficient techniques in the Hirshleifer procedure, that is the cheating involves "hiding" some feasible production schemes.

The divisional manager is assumed to be able to benefit from misrepresentation by being able to exploit information about constraints faced by the other division and the central headquarters. Thus, given the assumption that division one's manager knows the form of  $D_0$ , which the central headquarters faces, in order to implement any transfer price and associated allocation (and, hence, level of compensation), the manager must ensure that the information he/she communicates satisfies the constraints that specify  $D_0$ , that is:

$$\hat{L}_1 \hat{\lambda}_1 + L_2 \lambda_2 \leq a \quad (i)$$

$$\hat{e}_1 \hat{\lambda}_2 = 1 \quad (ii)$$

$$e_2 \lambda_2 = 1 \quad (iii)$$

With regard to the dimension of the variables communicated by division one's manager and associated variables, this will depend on the number of iterations it takes before the central headquarters thinks they have determined the optimal solution. Thus, if  $Z$  iterations are required  $\hat{L}_1, \hat{e}_1, \hat{\lambda}_1^T$  would be  $Z$  row vectors.

In order for the central headquarters to think they do not need to carry out any further iterations, the optimality conditions for the restricted master problem must be satisfied, that is:

$$\pi \hat{L}_1 + \alpha_1 \hat{e}_1 \geq \hat{W}_1 \quad (iv)$$

$$\pi L_2 + \alpha_2 e_2 \geq W_2 \quad (v)$$

The first three constraints may be called the primal feasibility constraints for  $D_0$  and the next two are the dual feasibility constraints. In addition to these relations, one can determine which constraints are binding in the primal problem. The principle of Complementary Slackness shows that, if a constraint is non-binding at the optimal solution, then the corresponding dual variable must

be zero. Expressed in another fashion, the principle means that, if the  $i^{\text{th}}$  slack variable for constraint  $i$  in the primal problem is not zero, then the  $i^{\text{th}}$  dual variable must be zero, that is the dual price of the  $i^{\text{th}}$  input (resource) is zero at that level of operations. Thus, the complementary slackness conditions for the above inequality constraints are:

$$\pi(\hat{L}_1 \hat{\lambda}_1 + L_2 \lambda_2 - a) = 0 \quad (\text{vi})$$

$$\hat{\lambda}_1(\hat{W}_1 - \pi \hat{L}_1 - \alpha_1 \hat{e}_1) = 0 \quad (\text{vii})$$

$$\lambda_2(W_2 - \pi L_2 - \alpha_2 e_2) = 0 \quad (\text{viii})$$

The sign restrictions for optimal weightings and transfer price are that they must be positive however the dual prices associated with the constraints (ii) and (iii) are unrestricted in sign since the constraints are equalities<sup>34</sup>, that is:

$$\hat{\lambda}_1, \lambda_2, \pi > 0, \alpha_1, \alpha_2: \text{ no sign restriction}$$

where  $\alpha_1$  and  $\alpha_2$  are scalars.

Finally, since the model assumes no external uncertainty and since the central headquarters can determine how much of the corporate resource each division should use and the associated divisional profit, each division must carry out the operations indicated by the optimal solution. Otherwise, the central headquarters would realise any discrepancy could only be due to cheating. Thus the final optimal cheating

constraint requires that a plan<sup>of</sup> operations chosen by the (central headquarters) optimal solution can actually be implemented exactly by a division one's manager, that is:

$$\hat{L}_1 \hat{\lambda}_1 = L_1 \lambda_1, \quad \hat{\lambda}_1 \hat{W}_1 - \Pi \hat{L}_1 \hat{\lambda}_1 = \lambda_1 W_1 - \Pi L_1 \lambda_1$$

(ix)

for some  $\lambda_1$  such that  $e_1 \lambda_1 = 1$  and  $\lambda_1 \geq 0$

One would expect the objective function for division one's optimal cheating problem to be:

$$\hat{L}_1, \hat{W}_1, \hat{\lambda}_1, \lambda_2, \Pi, \alpha_1, \alpha_2 \quad \max \quad \hat{W}_1 \hat{\lambda}_1 - \Pi \hat{L}_1 \hat{\lambda}_1$$

However, Jennergren (1971) notes that degenerate solutions (where there may be many optimal solutions to the restricted master problem) can often occur. It is likely that different optimal solutions will give rise to different possibilities and benefits from cheating. Given our earlier assumptions, division one's manager will also be aware of this and so may wish to restrict communications so that certain degenerate solutions with low or no possibilities for cheating are not located. That is, division one's manager will wish to consider all the values

$$\hat{L}_1, \hat{W}_1, \hat{\lambda}_1, \lambda_2, \Pi, \alpha_1, \alpha_2 \quad \min \quad \hat{W}_1 \hat{\lambda}_1 - \Pi \hat{L}_1 \hat{\lambda}_1$$

and then choose the greatest of these values to determine what to communicate. However, the maximum with respect to  $\hat{L}_1$  and  $\hat{W}_1$  may not be attained because of the degeneracy problem so instead one would look for the least upper bound called the supremum. Thus, division one's optimal cheating problem is:

$$\hat{L}_1, \hat{w}_1, \hat{\lambda}_1, \lambda_2, \Pi, \alpha_1, \alpha_2 \quad \text{SUPREMUM } (\min\{\hat{w}_1 \lambda_1 - \Pi \hat{L}_1 \hat{\lambda}_1\})$$

In order to understand how division one's manager will go about cheating suppose that  $\hat{L}_1^0, \hat{w}_1^0, \hat{\lambda}_1^0, \lambda_2^0, \Pi^0, \alpha_1^0, \alpha_2^0$  were to constitute an optimal solution to the optimal cheating problem. It must be that  $\hat{\lambda}_1^0, \lambda_2^0$  are optimal in the restricted master problem

$$\hat{\lambda}_1^0, \lambda_2^0 \quad \max \quad \hat{w}_1^0 \hat{\lambda}_1 + w_2 \lambda_2$$

$$\text{s.t. } \hat{L}_1^0 \hat{\lambda}_1 + L_2^1 \lambda_2 \leq a$$

$$\hat{e}_1 \hat{\lambda}_1 = 1, e_2 \lambda_2 = 1, \hat{\lambda}_1 \geq 0, \lambda_2 \geq 0$$

That is the strategy of the cheating division is to determine the restricted master problem which gives the highest divisional profit. To illustrate, suppose the overall corporate problem D is:

$$\max \quad 3X_{11} + 2X_{21} + 3X_{12} + X_{22}$$

$$\text{s.t. } 2X_{11} + X_{21} + X_{12} + X_{22} \leq 5$$

$$(X_{11}, X_{21}) \in X_1 = \{(X_{11}, X_{21}) \mid X_{11} + X_{21} \leq 2,$$

$$X_{11} \geq 0, X_{21} \geq 0\}$$

$$(X_{12}, X_{22}) \in X_2 = \{(X_{12}, X_{22}) \mid X_{12} + 2X_{22} \leq 2,$$

$$X_{12} \geq 0, X_{22} \geq 0\}$$

This problem has unique optimal solution

$$X_{11} = 1, X_{21} = 1, X_{12} = 2, X_{22} = 0$$

with corporate profit of 11 units and transfer price of 1 unit, imputing profit of 2 units to division one.

In this problem, the extreme points of division one's private constraint set are  $X_1^1 = (0,0)$ ,  $X_1^2 = (2,0)$ ,  $X_1^3 = (0,2)$ . Suppose division one's manager chooses to keep extreme point  $X_1^2$  secret. The overall problem which will then be solved by Dantzig Wolfe decomposition would be:

$$\max 3X_{11} + 2X_{21} + 3X_{12} + X_{22}$$

$$\text{s.t. } 2X_{11} + X_{21} + X_{12} + X_{22} \leq 5$$

$$(X_{11}, X_{21}) \in \hat{X}_1 = \{(X_{11}, X_{21}) \mid (X_{11}, X_{21}) \text{ is a convex combination of } (0,0) \text{ and } (0,2)\}$$

$$(X_{12}, X_{22}) \in X_2 = \{(X_{12}, X_{22}) \mid X_{12} + 2X_{22} \leq 2,$$

$$X_{12} \geq 0, X_{22} \geq 0\}$$

This problem has the unique optimal solution

$$X_{11} = 0, X_{21} = 2, X_{12} = 2, X_{22} = 0$$

with corporate profit of 10 units, zero transfer price imputing profit of 4 units to division one.



The optimal cheating problem defined above is a very difficult one mathematically. However, the above example illustrates how a division can use an "approximate" solution procedure for the optimal cheating problem. This "approximate" solution procedure involves determining a subset of all the divisional extreme points which are to be hidden and not communicated.

There are two particularly questionable assumptions made in the above discussion on optimal cheating. First of all, it is unrealistic to assume that a divisional manager will have full information about the overall corporate problem, especially when one is assuming decomposition analysis is being used, because the central headquarters only has limited information. Secondly, in practice, the iterative solution procedure will be terminated after only a few iterations - most likely before the optimal solution is determined. In this case, an optimal cheating problem can not be specified. However, this is not to say that divisional managers will not be able to still cheat profitably. Jemmergren and Muller (1973) conducted a simulation study of cheating with the Dantzig Wolfe Decomposition procedure. They terminated iterations before optimality and assumed that division  $i$ 's manager adopted the "approximate" cheating strategy of communicating

$$\hat{L}_i = B_i L_i$$

$$0 \leq B_i < 1$$

where  $B_i$  is called the cheating parameter. This implies that division  $i$ 's manager is hiding part of the division's feasible production area and will, hence, be in effect communicating a lower demand for the corporate resource than if  $B_i = 1$ . In their simulation study, they assumed a central headquarters and two divisional structures and allowed both divisions to cheat simultaneously defining light cheating as a situation where  $B_1 + B_2 > 1.6$ . Jennergren and Muller (1973) found that:

"Light cheating under the Dantzig-Wolfe rule in two cases results in rather small improvements in local profits for one or both divisions.....In some cases, light cheating under the Dantzig-Wolfe rule even results in small decreases in local profits for one or both divisions.....However, fairly large improvements in local profits can be attained under the Dantzig-Wolfe rule through heavier divisional cheating".

Thus, even in these more realistic cases, it seems that divisional managers will be able to form an "approximate" cheating strategy quite easily and will probably gain from such cheating, dependent on the particular situation.

### 3.4.3 The Generality of the existence of incentives to misrepresent.

Sections 3.4.1 and 3.4.2 have demonstrated that, in the two transfer pricing procedures presented, there exists a clear incentive for divisional managers to misrepresent the facts, that they communicate to the central headquarters. Also, the discussions have shown that such cheating will be relatively easy to carry out and likely to be quite successful. Clearly this type of result is of fundamental importance for Transfer Pricing Theory as it negates many of the advantages that transfer pricing is purported to convey in a decentralized organization.

However, the above analysis only establishes the potential prevalence of the misrepresentation problem for two transfer pricing procedures. There have been many other transfer pricing procedures proposed, however, and one may wish to argue therefore that, before one can establish that misrepresentation is a relatively pervasive problem, one must determine whether all the other proposed transfer pricing procedures suffer from this problem. However, this lengthy process is unnecessary. In Chapter 4, a Characterization Theorem will be presented which establishes a set of rules which define the class of transfer pricing procedures for which divisional managers will not have an incentive to misrepresent the facts. Given this theorem then, one can study whether any arbitrarily proposed transfer pricing procedure satisfies the rules of the theorem. If

not then, misrepresentation is possible. It turns out that the Characterization Theorem defines a relatively narrow class of potential procedures that would be incentive compatible (managers having no incentive to misrepresent).

Before presenting the Characterization Theorem, however, it will be instructive to end this chapter by considering the transfer pricing procedure proposed by Ronen and Mckinney (1970) which was the first serious, but unsuccessful, attempt to deal with the problem of the incentive compatibility of transfer pricing procedures.

#### 3.4.4 The Ronen and Mckinney tax subsidy transfer pricing procedure.

Ronen and Mckinney (1970) base their approach implicitly on the results of analysis of the type presented in Section 3.4.1. That is, they identify the major reason why divisional managers have incentives to cheat as arising because of the existence of their respective monopsonistic or monopolistic positions, with regard to the intermediate product. They argue that, in order to achieve incentive compatible transfer pricing, one should allow the divisional managers to freely and openly exert and realize their respective monopsonistic and monopolistic powers and, in so doing, achieve incentive compatibility. However, this is at the cost of dissenting the goal of having a "perfectly competitive" transfer price co-ordinate the operating plans of the divisions. Note, though, their

procedure does implement an output of the intermediate product identical to the output that would be achieved under "perfectly competitive" transfer pricing, if incentive compatibility were not a problem, as assumed in the Hirshleifer analysis.

In the Ronen and McKinney procedure, the central headquarters communicates a set of transfer prices to the manufacturing division's manager and asks the manager to determine the output of the intermediate product that the manager would be prepared to produce at each constituent price. Hence, assuming<sup>35</sup> no external market for the intermediate product, the central headquarters is effectively requiring the manufacturing division's manager to communicate the division's marginal cost schedule  $C'(q)$ .

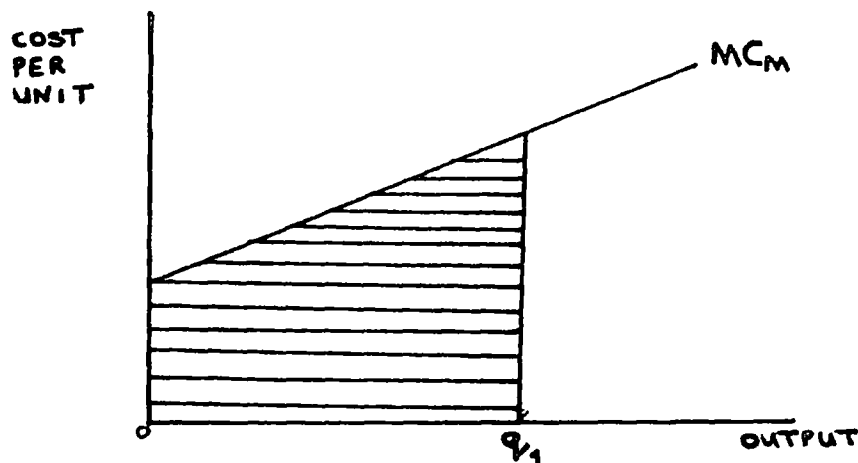
The central headquarters then uses the above generated data to derive the average cost curve for the manufacturing division, which will be denoted  ${}_A C(q)$ . Ronen and McKinney argue that this is possible because assuming the manufacturing division's total cost function, denoted  $C(q)$  has variable and fixed components respectively so that it can be represented as:

$$C(q) = \varnothing(q) + b$$

then  $C'(q) = \varnothing'(q)$

also  $\int \frac{\varnothing'(q)}{q} dq = \frac{\varnothing(q) + b}{q} = {}_A C(q)$

If it is difficult to conceive the central headquarters integrating the manufacturing division's marginal cost schedule, there is an analogous but equivalent interpretation. From the standard definition of the operation of integration, we know that it gives us the area under the curve. So all the central headquarters



need do is determine the area under the curve between 0 and  $q_1$  and divide this by  $q_1$  to give the average cost at output  $q_1$  for the manufacturing division.

However, note there is a problem with Ronen and McKinney's argument. Loeb (1975) points out that, when calculating, for example, an average cost schedule from a marginal cost schedule, the only schedule that can be derived by integration is the average variable cost curve since

$$\int \phi'(q) dq = \phi(q) + b$$

where  $b$  is a constant not specified by the marginal cost curve. Thus, for the Ronen and McKinney procedure to be operational, the central headquarters must possess information on the fixed costs for a division beforehand.

Possibly concurrently with its communications with the manufacturing division's manager, the central headquarters communicates a set of prospective transfer prices to the distribution division's manager and asks the manager to indicate at how much of the intermediate product would be demanded by the manager at each constituent transfer price. Thus, with no external market for the intermediate product, the central headquarters will be collecting information on the distribution division's net marginal revenue schedule  $D'(q)$ . As above, but with some caveat assuming that fixed net revenue is known, the central headquarters derives the net average revenue schedule  ${}_A D(q)$  for the distribution division, that is let

$$R(q) = \delta(q), R'(q) = \delta'(q)$$

$$C_D(q) = \epsilon(q) + d, C_D'(q) = \epsilon'(q)$$

so  $D'(q) = \delta'(q) - \epsilon'(q)$

$$\begin{aligned} {}_A D(q) &= \frac{\int [\delta'(q) - \epsilon'(q)] dq}{q} \\ &= \frac{\delta(q) - \epsilon(q) - d}{q} \end{aligned}$$

Next the central headquarters communicates the  ${}_A C(q)$  schedule to the distribution division's manager, where this schedule represents the inverse <sup>36</sup> supply schedule for the intermediate product that the distribution division faces.

That is, for any given demand for the intermediate product, where that output cuts the  ${}_A C(q)$  schedule determines the transfer price for that level of transfer of the intermediate product. Thus, the schedule can be denoted  $\bar{P}(q)$  to show explicitly this relationship. Concurrently, the central headquarters communicates the  ${}_A D(q)$  schedule to the manufacturing division's manager. This schedule will represent the inverse demand schedule for the intermediate product that the manufacturing division faces. That is, for any given supply of the intermediate product, the schedule determines the transfer price that the manufacturing division will be paid for the transfer. Again to show explicitly this relationship, the schedule shall be denoted  $P^*(q)$ . At this stage, one may wonder if the central headquarters might encounter difficulties in ensuring that the quantities supplied and demanded are identical. However, this is not a problem and it is easy to show that both divisions will, in fact, agree on a common output  $q$ . This is because the manufacturing division's manager will choose the output  $q_m$  which satisfies the first order conditions.

$$C'(q'_m) = D'(q'_m)$$

while the distribution division will similarly choose the output  $q_D$  that satisfies

$$D'(q'_D) = C'(q'_D)$$

that is

$$q'_m = q'_D = q'$$



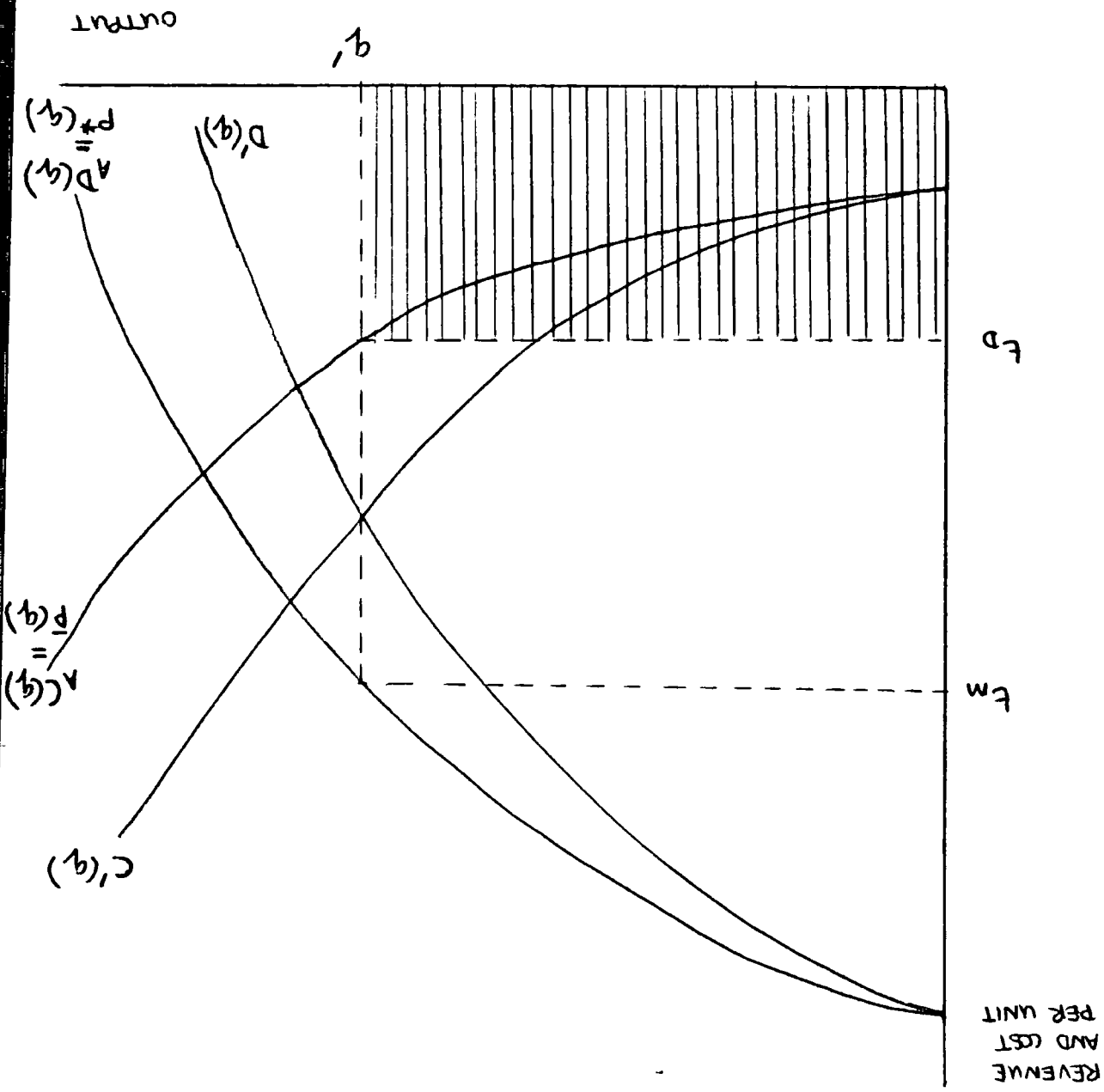
Once the level of output is determined as  $q$  by both divisions, the central headquarters charges the distribution division  $\bar{P}(q)$  per unit for the transferred intermediate product and credits the manufacturing division with  $p^*(q)$  per unit for the transfers. This credit is made up of the payment from the distribution division and a subsidy from the central headquarters of

$$\{P^*(q) - \bar{P}(q)\} q \quad \text{when } P^*(q) > \bar{P}(q)$$

if  $P^*(q) < \bar{P}(q)$  the subsidy becomes negative and the manufacturing division is being taxed.

To demonstrate the procedure, consider the following example illustrated by Figure 8 where, for instance, one is assuming the final product is sold on an imperfect market. After the above specified communications have taken place, the manufacturing division's manager will choose to produce the output of the intermediate product at which  $D'(q) = C'(q)$ , after deriving the  $D'(q)$  schedule from the  ${}_A D(q)$  schedule communicated by the central headquarters. Thus for transferring  $q$  of the intermediate product, the manufacturing division knows it will be credited with  $t_m$  per unit of the intermediate product. Similarly, using the communicated  ${}_A C(q)$  schedule, the distribution division's manager can derive the  $C'(q)$  schedule and determine the output for which  $D'(q) = C'(q)$ . For this supply  $q'$  of the intermediate product, the distribution division realises it will have to pay  $t_D$  per unit.

FIGURE 8



The shaded area of Figure 8 thus represents the amount the distribution division will pay to the manufacturing division for the supply of  $q$  of the intermediate product. The subsidy paid by the central headquarters to the manufacturing division is thus represented by the area  $(t_m - t_D)q'$ .

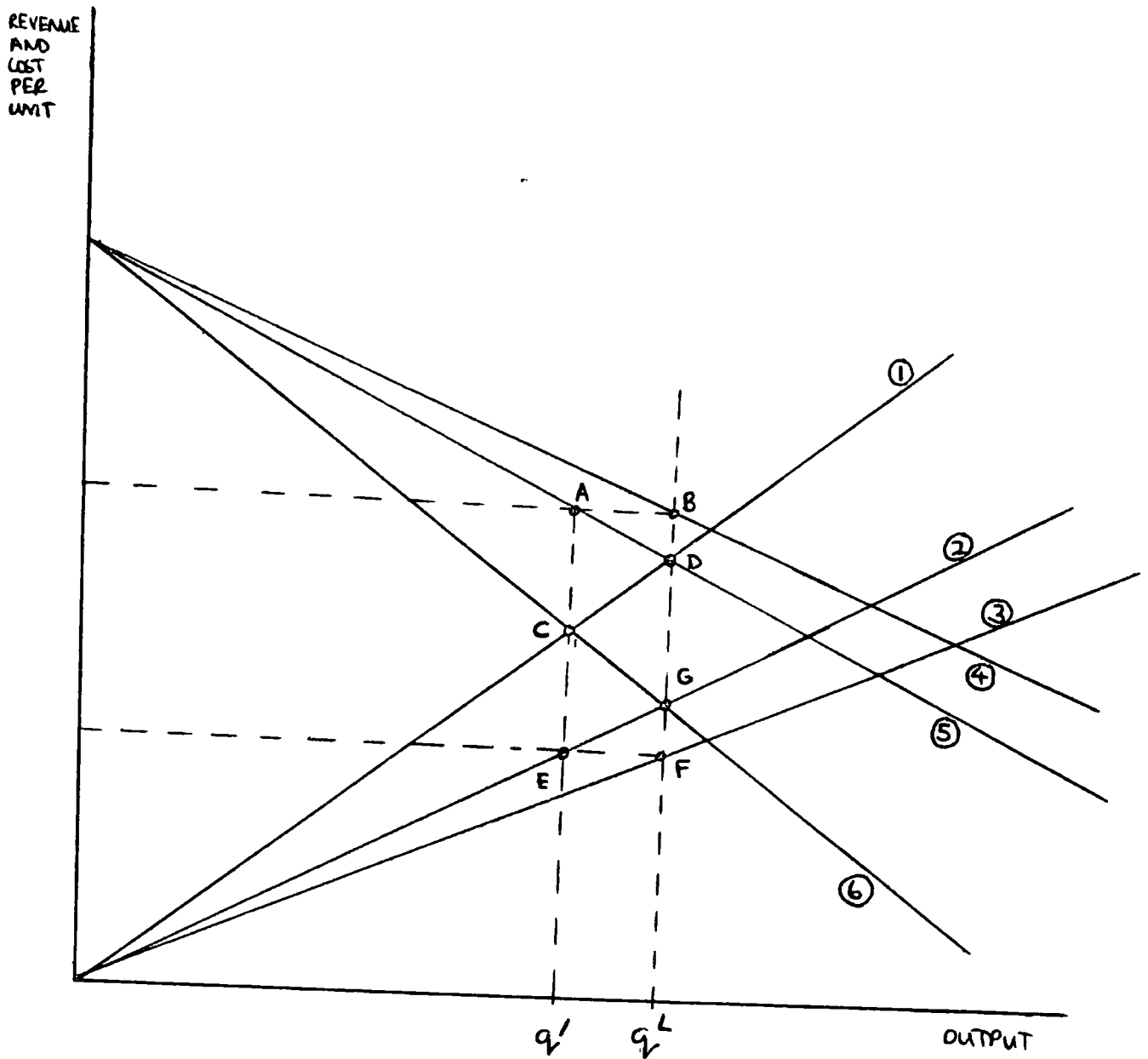
The advantages of Ronen and McKinney procedure then seems to be that the level of output  $q'$  chosen by the divisions is once again the level of output of the intermediate product consistent with overall corporate profit maximisation. In addition, it is claimed that the system encourages truthful communication by the divisional managers (incentive compatibility).

The Ronen and Mckinney paper was an important development in transfer pricing theory as it was the first paper to recognise why misrepresentation occurred<sup>and</sup> that attempted to deal with this problem systematically. However, it has been proven by Groves and Loeb (1976) that the Ronen and Mckinney procedure is not incentive compatible. Groves and Loeb argue that, since the divisions are free to choose  $q_m$  and  $q_D$  and, since the central headquarters is committed to subsidising the manufacturing division, there will be possibilities for the divisions to game against the central headquarters. Expressing this more formally<sup>37</sup>, Groves and Loeb (1976) note that

"While the joint strategy of the two divisions to report truthfully is a Nash, non co-operative equilibrium, it is not a dominant strategy in the sense that other equilibriums exist where divisions report untruthfully but are both better off".

To illustrate this possibility, Figure 9 depicts a situation where both divisions game against the central headquarters. In this particular situation, it is assumed the cheating strategy both divisions have chosen to adopt is: instead of communicating their true marginal schedules  $(C'(q)_{\text{TRUE}}, D'(q)_{\text{TRUE}})$ , they choose to communicate to the central headquarters their true average schedules, claiming that these are, in fact, their marginal schedules. Thus, the communicated schedules can be denoted  $(C'(q)_{\text{GIVEN}}, D'(q)_{\text{GIVEN}})$ . The central headquarters then derives the fictitious average schedules denoted  $(A^C(q)_{\text{CALC}}, A^D(q)_{\text{CALC}})$  and communicates these to the appropriate division. As Figure 9 clearly illustrates, this results in an equilibrium situation, because the amount the distribution division wants to buy is  $Q^L$  (where  $D'(q)_{\text{TRUE}} = C'(q)_{\text{GIVEN}}$ ), which is identical to the amount the manufacturing division wishes to supply (where  $C'(q)_{\text{TRUE}} = D'(q)_{\text{GIVEN}}$ ). In this situation, both divisions realise greater profits and this leads to corporate suboptimization, because the extra subsidy the centre pays AB EF (when the divisions lie) is greater than the extra profit generated by the two divisions achieving the greater output  $(Q^L - Q)$  by lying.

142.  
FIGURE 9



① =  $C'(q)_{\text{TRUE}}$

② =  $C'(q)_{\text{GIVEN}} = AC(q)_{\text{TRUE}}$

③ =  $AC(q)_{\text{CALC}}$

⑥ =  $D'(q)_{\text{TRUE}}$

⑤ =  $D'(q)_{\text{GIVEN}} = AD(q)_{\text{TRUE}}$

④ =  $AD(q)_{\text{CALC}}$

$\pi_M^{\text{EXTRA}} = \text{ABDG}$

$\pi_D^{\text{EXTRA}} = \text{CGFE}$

EXTRA SUBSIDY PAID BY CENTRAL HEADQUARTERS = ABFE

WHERE CLEARLY  $\text{ABFE} > \pi_M^{\text{EXTRA}} + \pi_D^{\text{EXTRA}}$  (BY CDG)

### 3.5 CONCLUSION

This chapter has demonstrated that, with two transfer pricing procedures, divisional managers can report untruthfully to the central headquarters. One method of cheating involves "hiding" production possibilities. This can occur when a divisional manager learns of a new technology for producing or selling a product. The examples presented in this chapter show that divisional managers may have an incentive not to report (hide) these findings.

The problem also arises when divisional managers are asked to forecast the value of uncertain variables, which they have expert knowledge of. When the forecasts affect how transfer prices will be set and, hence, how divisional managers are compensated, divisional managers can clearly gain by reporting untruthful forecasts, in order to bias transfer prices in their favour. When the actual values of the ex ante uncertain variables are realised, the central headquarters will not be able to detect whether divisional managers have lied or not. This is because any difference between forecasted and realised values could have been due to a number of factors. For instance, the variance could arise from poor forecasting, chance, lying or a combination of all these factors.

If such divisional cheating occurs, all of the problems listed at the beginning of Section 3.4 arise, when the central headquarters attempts to implement controlled decentralization by establishing a transfer pricing procedure.

The role of this thesis from here on will, therefore, be to attempt to consider or design transfer pricing procedures which ensure that divisional managers have no incentive to lie.

## APPENDIX 1

PROOF;

For any set of vectors  $x_i \in X_i$

$$f_i(x_i^*) - \sum_{j=1}^m u_j g_{ji}(x_i^*) \geq f_i(x_i) - \sum_{j=1}^m u_j g_{ji}(x_i) \quad (10)$$

for all  $i = 1, \dots, n$  by condition (8).

Condition (9) ensures that

$$u_j (b_j - \sum_{i=1}^n g_{ji}(x_i^*)) = 0 \quad (11)$$

for all  $j = 1, \dots, m$

since either  $u_j = 0$  or  $b_j - \sum_{i=1}^n g_{ji}(x_i^*) = 0$

Thus for all  $x_i$  satisfying (7)

$$u_j (b_j - \sum_{i=1}^n g_{ji}(x_i)) \geq u_j (b_j - \sum_{i=1}^n g_{ji}(x_i^*)) \quad (12)$$

Summing (10) over all  $i$  yields

$$\sum_{i=1}^n f_i(x_i^*) - \sum_{i=1}^n \sum_{j=1}^m u_j g_{ji}(x_i^*) \geq \sum_{i=1}^n f_i(x_i) - \sum_{i=1}^n \sum_{j=1}^m$$

$$u_j g_{ji}(x_i) \quad (13)$$

Summing (12) over all  $j$  yields

$$\sum_{j=1}^m u_j b_j - \sum_{i=1}^n \sum_{j=1}^m u_j g_{ji}(x_i) \geq \sum_{j=1}^m u_j b_j - \sum_{i=1}^n \sum_{j=1}^m u_j g_{ji}(x_i^*) \quad (14)$$



which is equivalent to

$$\sum_{j=1}^m u_j b_j + \sum_{i=1}^n \sum_{j=1}^m u_j g_{ji}(x_i^*) \geq \sum_{j=1}^m u_j b_j + \sum_{i=1}^n \sum_{j=1}^m u_j g_{ji}(x_i) \quad (14)$$

Adding 13 to (14) yields

$$\sum_{i=1}^n f_i(x_i^*) + \sum_{j=1}^m u_j b_j \geq \sum_{i=1}^n f_i(x_i) + \sum_{j=1}^m u_j b_j \quad (15)$$

implying the desired result Q.E.D.

## APPENDIX 2

PROOF:

(a) By the Kuhn Tucher Saddlepoint Theorem, there is a  $u^* \geq 0$  such that  $x_i^*$  and  $u^*$  constitute a saddlepoint of the Lagrangian of  $D_0$ . Hence, by the first inequality of (18) for all  $x_i \in X_i$

$$\begin{aligned} \sum_{i=1}^n f_i(x_i) + \sum_{j=1}^m u_j^* b_j - \sum_{i=1}^n \sum_{j=1}^m u_j^* g_{ji}(x_i) &\leq \sum_{i=1}^n f_i(x_i^*) + \\ \sum_{j=1}^m u_j^* b_j - \sum_{i=1}^n \sum_{j=1}^m u_j^* g_{ji}(x_i^*) & \end{aligned} \quad (19)$$

subtraction of  $\sum_{j=1}^m u_j^* b_j$  from (19) yields

$$\sum_{i=1}^n f_i(x_i) - \sum_{i=1}^n \sum_{j=1}^m u_j^* g_{ji}(x_i) \leq \sum_{i=1}^n f_i(x_i^*) - \sum_{i=1}^n \sum_{j=1}^m u_j^* g_{ji}(x_i^*)$$

which by (8) holds for all  $x_i \in X_i$ , if and only if,  $x_i^*$  solves all  $D_i$  for  $i=1, \dots, n$  for  $u = u_j^*$ .

(b) Any set of vectors  $\bar{x}_i$  that, for  $u = u^*$ , solve  $D_0$  by (a) also solve all the  $D_i$  which possess unique solutions  $x_i^*$ . Hence, for all  $i=1, \dots, n$ ,  $\bar{x}_i = x_i^*$  Q.E.D.

## APPENDIX 3

PROOF:

By proposition 3.2(a) a solution  $x_i^*$  to  $D_i$  exists.

Let there be another solution  $x_i \in X_i$  and denote  $\bar{x}_i \equiv tx_i^*$

+ (1-t) $x_i$  where  $0 < t < 1$ . By strict convexity

$\bar{x}_i \in X_i$ . In either of the cases (a) or (b), the maximand

$Z = f_i(x_i) - \sum_{j=1}^m u_j g_{ij}(x_i)$  in (8) is strictly concave as

a non-negative linear combination of concave functions of which at least one with a positive weight is strict.

Hence,  $Z(\bar{x}_i) > tZ(x_i^*) + (1-t)Z(x_i) = Z(x_i^*)$  which is a

contradiction.

1. For instance, Abdel-Khalik and Lusk (1974), Thomas (1980) and Tomkins (1973).
2. This environment will be assumed when discussing all other procedures, unless otherwise stated.
3. Hirshleifer's (1957) exact presentation is slightly different from the presentation here.
4. Thus, the marginal revenue schedule is downward sloping. In the case of a perfectly competitive market for the final product, the schedule would be horizontal. However, for the following procedure, it is not crucial which one of the two assumptions one makes. In addition, Hirshleifer does not make explicit any assumptions about the convexity or concavity of the schedules. However, this issue will be considered in more detail shortly.
5. Hirshleifer (1957).
6. Computational procedures, such as the compact inverse method which have no such organizational interpretation are not discussed. For a discussion of the compact inverse procedure see Lasdon (1970).
7. As developed by Jennergren (1976).
8. For Section 3.3 the assumption of only two divisions is relaxed. The dimensions of the vectors and matrices here will be fully specified later when each structure is discussed in turn. They are presented here only to demonstrate generally that a specific problem  $D$  may take a number of forms.
9. More generally, decomposition is the act of partitioning the overall problem  $D$  into a multilevel hierarchy.

10. The Dantzig and Wolfe procedure can also be used to solve many other problems of different structure. The reason attention is being limited to this structure is because it has a specific organizational interpretation.

11. This theorem has not been proven as it is a commonly used theorem in management accounting when linear programming is studied. However, a detailed discussion can be found in Rockafellar (1970), Sections 18 and 19, for instance.

12. This condition is easily derivable from the Lagrangian of  $D_0$

$$L(\Pi, \alpha) = -\Pi a + \sum_{j=1}^n \left\{ \max_{p=1}^{p(j)} (w_j^p - \Pi L_j^p - \alpha) \lambda_j^p \right\} +$$

$$\sum_{j=1}^n \left\{ \max_{r=1}^{R(j)} (\hat{w}_j^r - \Pi \hat{L}_j^r) \delta_j^r \right\}$$

If  $(w_j^s + \Pi L_j^s - \alpha) > 0$  is an, as yet, unconsidered extreme point  $s$ ,  $\lambda_j^s = 0$ ; the value of the Lagrangian could be increased by giving some "weight" to this extreme point in the restricted master problem.

13. Since no constraint is set of the extreme ray weights  $\delta_j^r$ .

14. Assuming there are no pathological degeneracy cases.

15. Again, as this is a commonly used result, it is not proven here. However, a detailed discussion can be found in Kolman and Beck (1980), Section 3.1. Also note here that the dual price  $\Pi$  in Note 12 is being denoted by  $u$ .

16. In addition, transfer prices may fail to direct divisional managers correctly, if the corporate optimal solution lies interior to the divisional feasible region as discussed by Baumol and Fabian (1964), page 16.

17. The optimality criterion to decide whether or not to add new feasible points to the restricted master problem must be applied. See Hass (1968), pp. B - 318.

18. See Sekine (1963) for instance.

19. Feasibility can be assured by adding constraints of the form  $\hat{u}_j^r a_{ji} + \hat{\pi}_i^r b_i > 0$  when appropriate.

20. The general economic principle of requiring equal marginal return of a resource in alternative uses is a sufficient condition for an optimal allocation, but not a necessary condition. Freeland and Moore (1977) demonstrated that, before equal marginal returns are achieved in an iterative algorithm, some of the divisional problems are likely to become degenerate, so that optimality conditions for these algorithms are more involved than the usual marginal returns condition.

21. That is, extreme ray solutions of the dual divisional problem.

22. In fact, the Benders (1962) resource directive algorithm, which was designed to function in special types of non linear environments, preceded the Tenkate algorithm.

23. See, for instance, Heal (1973).

24. Note, however, that I do not draw the same conclusion as Kanodia.

25. Kanodia also assumed that markets external to the firm were incomplete in the Arrow-Debreu sense, that is, the firm can not diversify away all its risk by making contingent contracts in external markets.

26. Note Kanodia does not make this assumption. He assumes that an objective commonly known probability distribution exists.

27. It is assumed that a divisional manager's only significant source of income is compensation received from the corporation and that they can't sell risky securities in capital markets which represent claims to this income. Hence, one can ignore portfolio opportunities in external markets and assume <sup>that</sup> divisional managers wish to maximise the expected utility derived from divisional compensation.

28. As presented in Raiffa (1968), page 211.

29. Note there are a few special cases in which members' preferences and judgements can be combined into group utility and probability functions. See Wilson (1968) for instance.

30. Assume divisional managerial compensation is a positive linear function of divisional income.

31. Note that by more efficient I mean that, at every level of output, the cost of production of the intermediate product is less.

32. The results of the following analysis are also readily generaliseable to the case of more than two divisions. In addition, it is assumed that the problem is bounded so that extreme ray solutions need not be considered.

33. See, for instance, page 158, Kilman and Beck (1980).

34. For an explanation of this result see page 148, Kilman and Beck (1980).

35. Ronen and Mckinney also discuss situations under which this assumption is relaxed.

36. Where the price paid depends on the quantity supplied.

37. The concept of a Nash equilibrium will be discussed in some detail in the following chapter.

## CHAPTER FOUR

### THE GROVES AND LOEB MECHANISM AND A CHARACTERIZATION THEOREM

#### 4.1 Information Strategies

The transfer pricing mechanisms discussed in the review chapter were unsatisfactory because they were open to manipulation by the divisional managers. By responding untruthfully to the questions of the centre, divisional managers could bias a transfer price in their favour.

Given the above argument, it would seem useful, if one could embed into a transfer pricing mechanism (procedure) the possibilities for the strategic use of information. Subsequently, one could then attempt to design transfer pricing mechanisms which promoted divisional managers to choose truth telling strategies.

In the decentralised corporate environment considered here divisional managers will, in general, not know the private constraints of other divisions and will not be forming binding agreements with other divisions to share information; that is, it is assumed there is no preplay communication. These aspects mean that it is appropriate to formulate the problem as non co-operative game.

However, it is not usually assumed<sup>in game theory</sup> that the strategies of players is the information they choose to communicate.

In fact, the implicit assumption of non co-operative game theory is that each player has complete information about



the game. This assumption implies that each player knows the structure of the game, other players' utility functions and the probabilities that other players attach to playing certain strategies. Consequently, a game describing a decentralised transfer pricing mechanism is one where each player has incomplete information. In certain circumstances, though, the incompleteness of information can be ignored. For instance, in the situation where a player can choose a strategy irrespective of the strategies chosen by other players, information about other players' utility functions is not of value. For the present, assuming this special case holds, the problem of incompleteness of information can be ignored. The task now is to formulate a non co-operative game which conceptualises the decentralised transfer pricing process.

It seems reasonable to assume that the centre can specify which variables it requires divisions to communicate. For instance, it may require divisions to communicate price signals, demands or conditional profit functions, reflecting a division's willingness to pay for a common corporate resource<sup>1</sup>. Thus, the centre can determine the language  $M$  in which divisions communicate with the centre. Each element  $M^i$  defines the  $i^{\text{th}}$  divisional message (strategy) space, so that for the  $n$  divisions, the joint message (strategy) space is:

$$M = M^1 \times \dots \times M^n$$

Specification of the game in normal form requires definition of the set of players, the strategy space and the payoff functions, that is:

N = set of players = number of divisions

M = joint strategy space = joint message space =  
language

E = payoff functions = divisional evaluation measures

It is assumed here that the centre is a non playing<sup>2</sup> participant in the game, following some rule of behaviour such as trying to allocate corporate resources so as to maximise overall corporate profit given the messages (strategies) of the divisions.

In some cases, in order to be able to describe which strategy a given player will choose, it is necessary to assign a utility function to a player. This is because when the player has some expectation about the strategies to be played by other agents, the player's attitude towards risk will determine which strategy is optimal, given particular expectations. If, however, there exists a strategy such that no superior outcome could be attained by any other strategy in any event, then the player's expectations and attitude towards risk would not determine the optimal strategy.

Given the specification of the game, there is also a need to consider what is the appropriate solution (equilibrium) concept for the game. In the following analysis, it will be appropriate to consider three solution concepts which will now be presented in order<sup>3</sup> of "strength".

Nash Equilibrium:

In a situation where some player  $i$  expects the other players to choose strategies  $(m_1, \dots, m_{i-1}, m_{i+1}, \dots, m_n)$  the player will select that strategy  $m_i^*$  that maximises his payoff (evaluation measure), given his anticipation of the other players' behaviour. Thus, if

$$(m/m_i^*) = (m_1, \dots, m_i^*, \dots, m_n)$$

then it is possible to define the  $i^{\text{th}}$  player's best response  $m_i^*$  as

$$E_i(m/m_i^*) \geq E_i(m) \quad \text{for all } m_i \in M_i$$

$$\text{where } m = (m_1, \dots, m_n)$$

However, if the same degree of rationality is attributed to the other players, player  $i$  will expect them to follow the same decision rule. This may lead to an expectational circle, with no stable strategies. By stable it is meant that, if each player  $i$  tentatively announced a commitment to a strategy  $m_i^*$ , no player would be induced to reconsider his commitment on finding out the commitments of other players. Therefore, only if a strategy  $n$ -tuple  $m^*$  exists where

$$E_i(m^*) \geq E_i(m^*/m_i) \quad \text{for all } m_i \in M_i \text{ and all } i \in N$$

will the expectational circle be broken. This defines a Nash equilibrium. That is, the messages chosen by players constitute a Nash equilibrium if no agent can unilaterally improve his payoff as long as others do not change their messages. Unfortunately, in many instances, though, the

Nash equilibrium may be non-unique. The importance of this fact is that, if one were to consider whether players will tell the truth showing that truth telling is a Nash equilibrium strategy, does not ensure that players will tell the truth. For example, if a player suspects that another player will choose a strategy of lying, it may be in his interest to lie as well. That is, there may also be a Nash equilibrium where all agents choose to lie.

In order to ensure truth telling by a player acting individually, requires the stronger notion of a dominant strategy equilibrium.

Dominant Strategy Equilibrium:

A dominant strategy equilibrium occurs when there is a strategy  $m_i \in M^i$  for every  $i$  such that

$$E_i(m_{>i}, m_i) \geq E_i(m_{>i}, m'_i)$$

for all  $m_{>i} \in M$  and all  $m'_i \in M^i$

where  $m_{>i} = (m_1, \dots, m_{i-1}, m_{i+1}, \dots, m_n)$

That is, each player  $i$  has the same (best) response  $m_i$  no matter what the response  $m_{>i} = (m_1, \dots, m_{i-1}, m_{i+1}, \dots, m_n)$  is of the other players. This means that, if it can be established that truth is a dominant strategy for all players, it follows no player acting alone can make himself better off by reporting untruthfully. There may be possibilities, though, for a group of players acting together to make themselves better off by reporting untruthfully, even though truth is a dominant strategy when

they act individually. There is, however, an equilibrium concept that can ensure that any group of players will also report truthfully.

**Strong Equilibrium:**

A strong equilibrium point occurs when there is some message  $m^* \in M$  for which

$$\{E_i(m^*)\}_{i \in C} \geq \{E_i(m_C, m_{n \setminus C}^*)\}_{i \in C}$$

for all  $m_C \in M^C$

$$\text{where } m_{n \setminus C}^* = \{m_j^*\}_{j \neq i \in C} \in M^{n \setminus C}$$

Thus, to qualify as a strong equilibrium, it must be:

- (i) impossible for one player to change his message (all others remaining the same) and increase his evaluation measure,
- (ii) it must be impossible for a subset  $C$  of players to jointly change their strategies (all other  $n \setminus C$  remaining the same) and increase their payoffs,
- (iii) it must also be impossible for all players jointly to change their strategies and increase their payoffs.

Although the strong equilibrium involves the actions of groups of players, it does not involve the use of binding agreements at the equilibrium point and, thus, may be considered a non co-operative concept (even though

the actual act of improving by a group at a non-equilibrium point may be co-operative). Another attractive feature of the strong equilibrium concept is that it does not require that all logically possible coalitions may actually form<sup>4</sup>.

Existence:

After defining appropriate equilibrium notions it is necessary to consider under what conditions the existence<sup>5</sup> of a (Nash) non co-operative equilibrium is ensured.

If, as above, one lets  $m_{>ic} = (m_1, \dots, m_{i-1}, m_{i+1}, \dots, m_n)$ , the best response for a player  $i$  can be defined for any  $m_{>ic} \in M^{>ic}$  which rival players may choose. This may be denoted  $E_i(m_{>ic})$ , so that for any  $m \in M$ , there is a best response vector valued function

$$E(m) = \{E_1(m_{>1c}), \dots, E_n(m_{>nc})\}$$

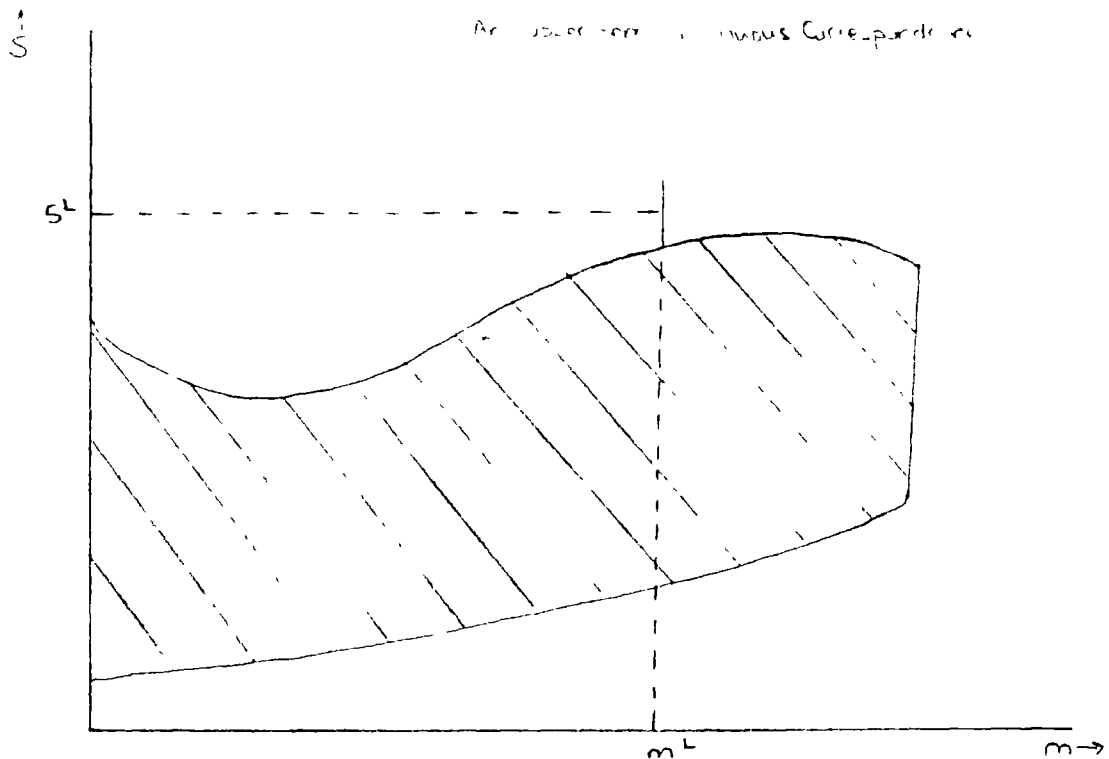
As the discussion of a Nash equilibrium given earlier specifies, a non co-operative equilibrium is a fixed point of the function  $E$ , so that if  $m^*$  is a non co-operative equilibrium message  $n$ -tuple, it satisfies  $m^* = E(m^*)$ , or more generally, if  $R$  is a correspondence<sup>6</sup>  $m^* \in R(m^*)$ . Proof of existence then requires the definition of the best response mapping and showing that it must have a fixed point. This, therefore, requires proof that the best response mapping satisfies the conditions of a 'fixed point theorem.

There are several fixed point theorems, but only one shall be discussed here, since (i) it is the one most appropriate for the Groves and Loeb transfer pricing model, and (ii) it is quite general. In the following discussion, the concept of truthfulness will be defined with respect to some decision rule  $k(m)$ , where truthful messages lead to optimal decisions being made. For any given truthful divisional message  $m_i$ , the addition of a constant to the message, so that  $\hat{m}_i$  is communicated (where  $\hat{m}_i = m_i + \text{constant}$ ) still leads to optimal decisions being made. Hence, the best response mapping is a correspondence. Kakutani's fixed point theorem considers correspondences and will now be considered<sup>7</sup>.

#### KAKUTANI'S FIXED POINT THEOREM:

If  $R(m)$  is an upper semicontinuous correspondence which maps a compact convex subset  $M$  of  $R^N$  into a closed convex subset of  $M$ , then there exists a  $m^* \in M$  such that  $m^* \in R(m^*)$  ●

$R(m)$  is an upper semicontinuous correspondence at the point  $m^*$  if, when the sequence  $m^L$ ,  $L = 1, 2, \dots$  converges to  $m^*$ ,  $S^L \in R(m^L)$  and  $S^L$ ,  $L = 1, 2, \dots$  converges to  $S^* \in R(m^*)$ .  $R$  is upper semicontinuous if it is upper semicontinuous at each point of its domain  $M$ .



Thus, in order to ensure that the information game satisfies the conditions of a fixed point requires that the set  $k$  of external decision be a compact space and that the language set  $M$  be the collection of all upper semi-continuous real valued functions  $m_i(\cdot)$  of  $k$ , where  $m^* \in M$ .

Assuming also that the evaluation measure of the  $i^{\text{th}}$  player is a scalar valued function defined for all  $m \in M$  if it is continuous and bounded everywhere for  $i=1, \dots, N$ , then the conditions of Kakutani's fixed point theorem are met.



#### 4.2 The Groves and Loeb transfer pricing mechanism

Groves and Loeb construct a mechanism where the centre asks divisions to report conditional profit functions and then, using this information, determines the allocation of some common or intermediate product. The centre charges the divisions a cost share that has the property that reporting the truthful profit function to the centre is a dominant strategy for the divisions in this game defined by the mechanism. In the mechanism the centre does not set a transfer price as such, but the resultant forecasted net profit function can be viewed as implicitly defining a transfer price.

The model considers a firm consisting of  $n$  divisions and a corporate centre. The players, therefore, can be indexed by  $i = 0, 1, 2, \dots, N$ , where the index  $i = 0$  refers to the centre. The language (or message space) for the game is the true or false profit functions that the divisions claim to have. The centre will, in general, not generate revenues and so its profits will probably be negative, reflecting central costs which have not been allocated to the divisions. After receiving messages from the divisions the centre then determines the optimal allocation of resources conditional on the divisional messages. In keeping with normal accounting terminology the divisional payoffs will be referred to as the divisional evaluation measures, and we shall assume divisional managers are rewarded partly on the basis of their divisional evaluation measure<sup>8</sup>.

As noted before, the divisions have to make private decisions which only affect their own profitability, and also common decisions which will affect other divisions' profitability as well. A vector of private decisions will be denoted by  $L = (L_0, L_1, L_2, \dots, L_N)$ . Private decisions may take the form of the specification of quantities of inputs, outputs or choice of technique, when they have no direct effect on the other divisions. A vector of common decisions or what can also be called common inputs is denoted by  $k$ . This classification is quite general and, for instance, allows for the presence of public goods, externalities and inter-divisional transfers.

The global profits of the firm can be denoted

$$\bar{\pi}(L, k) = \sum_{i=0}^n \bar{\pi}_i(L_i, k)$$

In order to understand how divisional managers will formulate their strategies, it is important to specify what information they have about others. It is assumed here that the organisational form is such that the  $i^{\text{th}}$  division knows only its own profit function  $\bar{\pi}_i(L_i, k)$  and that the centre knows only  $\bar{\pi}_0(L_0, k)$  and not the profit functions of the divisions at the commencement of the game. In other words, this is a game characterised by incomplete information, because in a non co-operative game of complete information, it is assumed that every player knows all the sets of pure strategies  $M_1, \dots, M_N$ . However, if all players have a dominant strategy to play, there is no need to explicitly formulate the game as one with in-

complete information. The reason for this is because the existence of a dominant strategy allows individual players to ignore what strategic options other players may have.

In order for the centre to allocate the common input  $k$ , the divisions are required to communicate their profitability conditional on various allocations of  $k$ . The centre then determines what allocation of  $k$  is consistent with global profit maximisation, given the divisional messages. By considering only those evaluation measures which are strictly increasing in a division's own profit, Groves and Loeb claim that divisional managers will be motivated to take those local decisions which maximise their own division's profits, given the predetermined level of common inputs. Provided the centre only has the objective of global profit maximisation, it will apportion  $k$  consistent with this objective. Given the above discussion, this means that attention need only be focused on the apportionment of common input levels. Therefore let  $\Pi_i(k) = \max_{L_i} \bar{\Pi}_i(L_i, k)$  for  $i = 0, 1, 2, \dots, N$ , so that

global profits can be denoted  $\Pi(k) = \sum_{i=0}^N \Pi_i(k)$ .

Let  $m_i(k)$  denote a division's reported profit function remembering that this may not be the division's actual profit function  $\Pi_i(k)$  if it decides to lie. If the argument of the division's communicated profit function is

suppressed, the vector of messages that the centre receives can be denoted  $m = (m_1, m_2, \dots, m_n)$ .

Expost after the appropriate allocation of  $k$  and after the divisions have realised their profits, the centre will want to evaluate divisions and divisional managers. This evaluation should be on the basis of controllable performance, but there is a difficulty in trying to invoke such separability when there are such interdependencies as common inputs. Groves and Loeb take the view that a division is responsible for the externality it enforces on other divisions when it is allocated some of  $k$ . The exact method of assessment of the externality will be given below. At this point all that need be said is that it will depend on other divisions' messages, for example, alternative bids for  $k_i$  allocated to division  $i$ . Therefore, division  $i$ 's evaluation measure will depend on the division's own realised profits and the messages of all the divisions, this can be denoted as  $E_i(\Pi_i(k), m)$ .

To summarise the above, we can therefore describe the corporate problem as one of determining an optimal control mechanism defined as a pair  $C = \{k(m), \langle E_i(\Pi_i, m) \rangle_{i=1}^N\}$  where  $k(m)$  is the centre's decision rule for allocation of the common resource. To be optimal, the mechanism must satisfy two requirements. First, it must be the case that there exists a message  $\hat{m}_i$  that maximises  $\hat{E}_i(\Pi_i(k), m)$  for every  $m^{>i} \in M^{>i}$  for each  $i$ , that is each division must have a dominant strategy so that the  $i^{\text{th}}$  division's best message is independent of the messages of other divisions.

Secondly, the mechanism must be efficient so that global profits are maximised, that is, for all n-tuples  $\hat{m}$  satisfying the first requirement, the centre's common resource decisions  $\hat{k}(\hat{m})$  maximise  $\Pi(k)$ .

Before considering whether the Groves and Loeb mechanism is optimal, there is a need to specify regularity conditions which describe the environment in which divisions operate. Specification of the size of the message space is also required in order to determine what messages are allowable. The message space  $M$  must be sufficiently large so that, for every collection of profit functions admitting a solution to the joint profit maximisation, there are messages  $\hat{m}_i$  in  $M$  allowing an efficient solution. In general, the set of possible common resource allocations  $k$  is assumed to be a compact space and the message space  $M$  the collection of all upper semi-continuous real-valued functions  $m_i(k)$ . The division's true profit functions  $\Pi_i(k)$  are also assumed to be members of the space  $M$ . These regularity conditions ensure that the centre's decision problem has a solution, though this may not be unique. That is, the centre decision rule  $k(m)$  is such that for every

$$m \in M \quad \hat{k}(m) \text{ maximises } \sum_{i=1}^N m_i(k)$$

If the concept of truthfulness is defined such that truthful divisional messages  $m_i^*$  give rise to  $k(m)$  maximising

$$\sum_{i=1}^N \Pi_i(k), \text{ then } m_i(k) = \Pi_i(k) \text{ is not the only truthful}$$

divisional message, since the addition of a constant to  $m_i^*$  will not change the decision  $\hat{k}(m)$ <sup>9</sup>.

Groves and Loeb define a control mechanism  $C^*$  which is optimal. To understand this, consider first the centre's decision rule  $k^*(m)$ .

$$k^*(m) = k^* \text{ which maximises } \Pi_0(k) + \sum_{i=1}^N M_i(k) + \text{constant}$$

with respect to  $k$  for any constant.

With the regularity conditions stated above, a maximum will exist, though this may not be unique, but  $k^*(m)$  will select a particular maximiser, that is  $k^*(m)$  is the rule maximising reported profits of the firm.

If it were the case that divisions reported their true profit functions, the centre's rule would lead to global profits being maximised. The evaluation measure, therefore, should be designed so as to encourage divisional managers to respond truthfully. If the  $i^{\text{th}}$  divisional evaluation measure were<sup>10</sup>

$$E_i^O(\Pi_i, m) = \Pi_0(k^*(m)) + \Pi_i + \sum_{j \neq i}^N m_j(k^*(m)) \text{ for } i=1, \dots, N.$$

it is clear that this should induce truthfulness, since the centre has driven the divisional manager to have exactly the same objective as the centre, that is, global profit maximisation. Expressed in another manner, the  $i^{\text{th}}$  division's message to the centre only affects  $E_i^O$  through

its influence on the centre's choice of common resource level  $k$ . If a division lies, this will give rise to the division being allocated a less favourable amount of  $k$  than if it were truthful.

In its present form, the evaluation measure seems somewhat trivial. The important factor is that it can be modified in a certain manner and still preserve its incentive properties. In general, let  $\alpha_i(m_{>iC})$  be any strictly positive function of all divisions' messages except the  $i^{\text{th}}$ , and let  $B_i(m_{>iC})$  be an arbitrary function of all other divisions' messages. Then any evaluation measure of the form

$$E_i^*(\Pi_i, m) = \alpha_i(m_{>iC}) \cdot E_i^O(\Pi_i, m) + B_i(m_{>iC})$$

will also motivate the  $i^{\text{th}}$  division to communicate truthfully. Thus, there is a whole class of optimal control mechanisms of the form

$$C^* = \{k^*(m), \langle E_i^*(\Pi_i, m) \rangle_{i=1}^N \}$$

It should be noted that, in general, the optimal evaluation measures, plus the centre's profits, will not sum to equal overall firm profits for all possible messages  $m$ .

Up until this point, the issue of controllability of divisional evaluation measures has not been considered. Groves and Loeb argue that/for the evaluation measures to provide motivational benefits for the divisional managers,

it is crucial that the evaluation measures be controllable by the divisional managers. As noted earlier, this is not possible in a strict sense because of the interrelationships introduced by the use of common resources. Given the information transmitted in the process, it is possible, though, to calculate the opportunity cost of a division consuming common resources.

$$\text{Let } \alpha_i(m_{>iC}) = 1$$

$$\text{and } B_i(m_{>iC}) = -\{\Pi_0(k^i) + \sum_{j \neq i} m_j(k^i)\}$$

$$\text{where } k^i = k^i(m_{>iC}) \text{ maximises } \Pi_0(k) + \sum_{j \neq i} m_j(k)$$

with respect to  $k$ .

This evaluation measure may be denoted by  $\bar{E}_i(\cdot)$  and is given by:

$$\bar{E}_i(\Pi_i, m) = \Pi_0(k^*(m)) - \Pi_0(k^i) + \Pi_i + \sum_{j \neq i} \{m_j(k^*(m)) - m_j(k^i)\}$$

This evaluation measure accounts for the impact one division's consumption of common resources has on other divisions and is, hence, a measure of the opportunity cost of having the  $i^{\text{th}}$  division. It is controllable in the limited sense that a division can choose to communicate messages, such that it does not affect<sup>11</sup> the centre's allocation of  $k$ . This, of course, would not be optimal



but is a matter the divisional manager has control over. However, it seems unlikely that a division would ever exercise this right. The relative non-controllability aspect is an inevitable result of divisional interactions through the use of common resources and is not to be considered as a fault of the Groves and Loeb evaluation measure. Another related factor is that, since the Groves and Loeb method relies mainly on forecasted data, efficiency or forecasting variances in other divisions are allocated solely to the responsible division through the realised profit term. An important aspect of the divisional evaluation measure in such an environment is how the centre distinguishes between the performance of a division and the performance of that division's manager, for example, in the case where a good manager is placed in an unprofitable division. The centre may, therefore, wish to adjust the evaluation measure  $\bar{E}_i$  but still retain the incentive properties of an optimal control mechanism. A target level  $T_i$  could be set so that a divisional manager's evaluation measure became

$$E_i(\Pi_i, m) = \bar{E}_i(\Pi_i, m) - T_i$$

In order to maintain the incentive properties though, this target level must be set prior to transmittal of messages and the determination of common input levels.

#### 4.3 Characterisation theorem on the possibility of finding other optimal control revelation mechanisms

In the previous section, it is shown that the Groves and Loeb mechanism is optimal, in the sense that it leads to efficient allocation of common resources, while at the same time, eliminating strategic interaction among individual agents, because of the existence of dominant strategies. In order to assess the relative importance of the Groves and Loeb mechanism, one must consider whether there are other revelation mechanisms that are optimal. Green and Laffont (1977) considered this question and so their analysis will be presented now.

Given that the centre lacks information about the specific environments that divisions operate within, it will not be until after divisions report their profitability that the centre can determine charges or payments for common resources. From hereafter these charges or payments will be referred to generally as transfers and denoted  $t_i$ . That is divisions after transfer profits can be expressed as:

$$e_i(k,t) = \Pi_i(k) + t_i$$

and the centre's profit can be denoted  $\Pi_0(k)$ .

The messages that divisions choose to communicate will depend on the transfer pricing mechanism that the centre has chosen to use. To be more specific, the outcome of a

transfer pricing mechanism will depend on the message (strategy) space of the divisions, the decision function of the centre  $k(m)$  and the transfer rule  $t(m)$  that the centre employs. In general, there are many different messages that the centre may want divisions to communicate. If, though, divisional messages are restricted to be only valuation functions for common resources, then it is possible to define<sup>12</sup> a revelation mechanism R.M. as

$$RM = \{M, f\}$$

where

$$M_i = \{m_i \mid m_i : \kappa \rightarrow R\}$$

$\kappa$  = set of all possible allocations of

$$M = M_1 \times \dots \times M_N$$

and the outcome function  $\{f(m) = \{k(m), t_1(m), \dots, t_N(m)\}$

from  $X_i M_i = M$  into  $\kappa \times R^N$  such that

for each joint message space  $m \in M$ .

- (1) the allocation of common resources is determined by the decision function  $k(m)$ ,
- (2) the transfer to division  $i$  is  $t_i(m)$  for  $i=1, \dots, N$ .

In the revelation mechanism, the divisions will be asked to give their valuations of the common resources, that is  $\Pi_i(k)$ . It is possible that divisions may not report their true valuation functions  $\Pi_i(k)$ , so let  $m(k) = \{m_1(k), \dots, m_N(k)\}$  represent the reported valuation function, or if the argument is suppressed, it can be denoted  $m = \{m_1, \dots, m_N\}$

In order to ensure that the centre uses the decision rule which maximises reported profit (i.e. allocated the common resources optimally, given the information it has), attention is restricted to direct revelation mechanisms (D.R.M.) defined as follows:

DRM =  $\{m, f\}$  is a revelation mechanism for which

$$k^*(m_1, \dots, m_N) \text{ MAXIMISES } \Pi_0(k) + \sum_{i=1}^N m_i(k) \text{ for } k \in K$$

Since the maximum may not be unique, and  $\kappa$  is a compact set, a sufficient condition is that the valuation functions be restricted to be upper semi-continuous on  $\kappa$ . The allowance of set valued mechanisms means that reference can be made to extended direct revelation mechanisms EDRM defined as

$$\text{E.D.R.M.} = \{M, F\}$$

where  $M = M_1 \times \dots \times M_N$

and  $F: M \rightarrow \kappa \times \mathbb{R}^N$   $F(m) = (K^*(m), T_1(m), \dots, T_N(m))$

is a correspondence such that for every  $m \in M$ ,  $K^*(m)$  is a subset of the maximisers of  $\Pi_0(k) + \sum_{i=1}^N m_i(k)$

It is now possible to define a Groves<sup>13</sup> mechanism G.M. as follows:

GM =  $\{m, f\}$  is a direct revelation mechanism with the specific transfer rule:

$$t_i(m) = \sum_{\substack{j=1 \\ j \neq i}}^N m_j(k^*(m)) + h_i(m_{-i})$$

where  $h_i$  is an arbitrary deterministic function of the messages of the other divisions.

In order to allow the Groves' mechanism to be operated when transfers and decisions are not uniquely defined, there needs to be a definition of an extended Groves' mechanism  $EGM = (M, F)$  where  $K(m)$  is a decision correspondence from  $M$  into  $\kappa$  such that each selection  $k(m) \in K(m)$  maximises  $\Pi_0(k) + \sum_{i=1}^N m_i(k)$  and

$$t_i(m) = \sum_{\substack{j=1 \\ j \neq i}}^N m_j(k^*(m)) + h_i(m_{-i})$$

where  $t_i(m) \in T_i(m)$  and  $T_i(m)$  is an arbitrary deterministic correspondence from  $X_{\substack{j=1 \\ j \neq i}}^N M_j$  into  $R$ . For notational ease let  $\Sigma_{j \neq i}$  denote  $\sum_{\substack{j=1 \\ j \neq i}}^N$  from now.

Green and Laffont consider public goods in their work and so their terminology is somewhat different. They show the Groves' mechanism is strongly individually incentive compatible (s.i.i.c.) and successful. This corresponds to showing that a Groves' mechanism has truth telling as the dominant strategy for each individual and that the centre's decision rule allocates common resources so as to maximise corporate profits, that is, they show that a Groves' mechanism is optimal.

PROOF: Suppose that there exists some dominant strategy  $\Pi'_i$  such that  $\Pi'_i - \Pi_i$  is not constant. Then there exist  $\epsilon > 0, K^* \in K, K^{**} \in K$  such that:

$$\Pi'_i(k^*) = \Pi_i(k^*) + \alpha$$

$$\Pi'_i(k^{**}) = \Pi_i(k^{**}) + \alpha + \epsilon$$

Now choose  $m_{>i\epsilon}$  to be the upper semicontinuous function defined by:

$$\Pi_0(k^*) + \sum_{j \neq i} m_j(k^*) = -\Pi_i(k^*) - \alpha$$

$$\Pi_0(k^{**}) + \sum_{j \neq i} m_j(k^{**}) = -\Pi_i(k^{**}) - \alpha - \frac{\epsilon}{2}$$

$$\Pi_0(k) + \sum_{j \neq i} m_j(k) = -\sup_{k \in K} \{ \sup_{i} \Pi_i(k), \sup_{i} \Pi_i(k) \} - \alpha - \epsilon$$

for  $k \in K, k \neq k^*, k \neq k^{**}$

Since the divisional manager wishes to maximise

$$\Pi_0(k) + \Pi_i(k) + \sum_{j \neq i} m_j(k)$$

then answer  $\Pi_i$  leads to allocation  $k^*$  being chosen, while the answer  $\Pi'_i$  leads to allocation  $k^{**}$  being chosen, and since

$$\pi_1(k^*) + \pi_0(k^*) + \sum_{j \neq i} m_j(k^*) > \pi_1(k^{**}) + \pi_0(k^{**}) + \sum_{j \neq i} m_j(k^{**})$$

$\pi_1'$  cannot be a dominant strategy by contradiction. That is all the dominant strategies must differ from  $\pi_1$  by a constant.

Given that the normalised strategy for a divisional manager is such that  $m_i(0) = 0$ , by displacing  $m_i$  by non zero constant would lead to the destruction of the normalisation. The normalised dominant strategy ( $\alpha_i=0$ ) is therefore unique. Q.E.D.

Since each individual has a unique normalised dominant strategy of telling the truth, it must be that the centre's decision maximises the sum of the true profitability functions, that is, the Groves' mechanism is successful.

Having proven that the Groves' mechanism is optimal, Green and Laffont then consider the converse question of whether there are any other optimal direct revelation mechanisms.

A D.R.M. =  $(m, f)$  is said to satisfy the property of transfer independence and compensation if

(i)  $t_i(m)$  is independent of  $m_i$  at  $k^*$ , that is for  $m_{>ic}, m_i, m_i'$  such that

$$k^*(m_{>ic}, m_i) = k^*(m_{>ic}, m_i')$$

then  $t_i(m_{>ic}, m_i) = t_i(m_{>ic}, m_i')$

$$(ii) \quad t_i(m_{-i}, m_i) - t_i(m_{-i}, m_i') = \sum_{j \neq i} m_j(k^*) -$$

$$\sum_{j \neq i} m_j(k^{*'}) \text{ where}$$

$$k^* \text{ maximises } \sum_{j \neq i} m_j(k) + m_i(k) \text{ over } \kappa$$

$$\text{and } k^{*'} \text{ maximises } \sum_{j \neq i} m_j(k) + m_i'(k) \text{ over } \kappa$$

Part (i) represents the idea that a division's transfer depends on the division's own message only through its influence on the centre choice of allocation  $k$ . Part (ii) reflects the underlying ideology that, if a division by changing its message, alters the allocation  $k$ , then the cost of making the switch in terms of the altered  $t_i$  payment is exactly the cost it imposes on the other divisions.

This means we can now characterise a direct revelation mechanism as being a Groves' mechanism if, and only if,

$$(i) \quad k(m) \text{ maximises } \sum_i m_i \text{ over } \kappa \text{ and}$$

(ii)  $t_i$  satisfies the property of transfer independence and compensation for every  $i$ .

Characterisation Theorem: All optimal mechanisms are Groves' mechanisms.

PROOF: Consider, in turn, the negation of the two parts of the transfer independence and compensation property.



If (i) fails, there exists  $m_{>i}, m_i, m'_i$  which lead to the same  $k^*$  such that:  $t_i(m_{>i}, m_i) > t_i(m_{>i}, m'_i)$

Let  $m'_i = \Pi_i$

Then  $t_i(m_{>i}, m_i) + \Pi_i(k^*) > t_i(m_{>i}, \Pi_i) + \Pi_i(k^*)$

and  $\Pi_i$  is not a dominant strategy.

If (ii) fails, there exists  $m_{>i}, m_i, m'_i$  such that:

$k^*$  maximises  $\sum_{j \neq i} m_j + m_i$  over  $\kappa$

$k^{*/}$  maximises  $\sum_{j \neq i} m_j + m'_i$  over  $\kappa$

and  $t_i(m_{>i}, m_i) - t_i(m_{>i}, m'_i) = \sum_{j \neq i} m_j(k^*) -$

$\sum_{j \neq i} m_j(k^{*/}) + \epsilon$

for some  $\epsilon > 0$

Let  $\hat{m}'_i$  be defined as

$$\hat{m}'_i(k^*) = - \sum_{j \neq i} m_j(k^*)$$

$$\hat{m}'_i(k^{*/}) = - \sum_{j \neq i} m_j(k^{*/}) + \frac{\epsilon}{2}$$

$$\hat{m}'_i(k) = -c \text{ for } k \neq k^* \text{ or } k \neq k^{*/} \text{ with } c > \max_{k \in \kappa} \sum_{j \neq i} m_j(k)$$

$\hat{m}'_i$  is upper semi-continuous.

Note that  $\max_k \hat{m}_i'(k) + \sum_{j \neq i} m_j(k)$  is solved at  $k=k^{*'}$ , and

therefore by the first part of the proof

$$t_i(m_{>ic}, \hat{m}_i') = t_i(m_{>ic}, \hat{m}_i')$$

so that we also have that

$$\begin{aligned} t_i(m_{>ic}, m_i) - t_i(m_i, \hat{m}_i') &= \sum_{j \neq i} m_j(k^*) - \sum_{j \neq i} m_j(k^{*'}) + \varepsilon \\ &= -\hat{m}_i'(k^*) + \hat{m}_i'(k^{*'}) + \frac{\varepsilon}{2} \end{aligned}$$

Therefore  $t_i(m_{>ic}, m_i) + \hat{m}_i'(k^*) > t_i(m_{>ic}, \hat{m}_i') + \hat{m}_i'(k^{*'})$

Hence, when  $\Pi_i \equiv \hat{m}_i'$  the announcement of  $m_i$  would be superior which contradicts the fact that the mechanism has truth as the dominant strategy for each player  $\theta$

The importance of this result is that the only mechanisms that can be optimal are Groves' mechanisms. Green and Laffont also show that optimal extended mechanisms (where  $k$  is a correspondence) are isomorphic to extended Groves' mechanisms.

The class of Groves' mechanisms is quite large, given the form of transfer function allowed. However, the particular transfer function proposed by Groves and Loeb seems to be the most appropriate for the transfer pricing problem. However, the above results have been established

with the  $\alpha_i(m_{pic})$  of the evaluation measure  $E_i^*(\Pi_i, m)$  of the previous section set equal to unity. It is, however, possible to keep the incentive properties established and define a profit sharing type mechanism where  $\alpha_i > 0$ ,  $B_i = 0$  and  $\sum_{i=1}^N \alpha_i = 1$ .

However, with such a mechanism a division would be rewarded partly on the basis of the operational efficiency of other divisions even though they may be responding truthfully at the planning stage. Given this argument, the modified profit sharing type mechanism will not be considered further.

Some of the literature on public economics has been concerned with the fact that, in general, the Groves' mechanism does not guarantee a full allocation of profits, regardless of the divisions' messages. Though, this may raise efficiency problems when considering how to share the full cost of a public good, it does not necessarily cause any difficulty in setting transfer prices, since it is not necessary that the sum of divisional evaluation measures equals corporate profit.

Another important question with Green and Laffont's characterisation theorem is that, in deriving their uniqueness theorem, they assume a large domain of possible conditional profit functions. It would be interesting to see that if their universal domain assumption was dispensed with and some more restrictive domain were assumed to hold, whether their uniqueness characterisation still held.

Laffont and Maskin (1980) and Holmstrom (1979) consider this issue and it will be commented on here because their results allow us to derive the generic Groves mechanism in a more constructive manner and also because the approach provides a bridge between the dominant strategy approach and the expected utility maximisation approach (used in games of incomplete information).

Laffont and Maskin assume that the conditional profit functions for each division are parametrised by a variable  $\theta_i$  which the centre is ignorant of. In addition, it is assumed the centre knows the form of the functions  $\Pi_i(k, \theta_i)$ . That is, one can express the divisional after transfer profits as

$$E_i(k, t) = \Pi_i(k, \theta_i) + t_i \quad i = 1, \dots, n$$

The problem the centre now faces then is to choose the mechanism which will yield the optimal level of allocation of  $k$ , given that it has imperfect information about divisions states. That is, a mechanism in this setting is a mapping  $f(\cdot)$  from the strategy space  $\Theta = \prod_{i=1}^n \Theta_i$  into  $\overset{\circ}{R}_+ \times R^n$  when the quantity of the common resource has the range  $\overset{\circ}{R}_+ = ]0, \infty[$ . That is  $k(\cdot)$  associates to any  $n$ -tuple  $\theta$  of announced parameters an allocation  $k(\theta)$ , while  $t_i(\theta)$  represents the transfer payment to or from division  $i$ . Therefore, a division's after transfer profits will depend on the reported value of parameter  $\theta_i$  through the decision function  $k(\cdot)$  and

the resulting transfer function determined. Also, since the conditional profit function will depend on the true value of the parameter  $\hat{\theta}_i$  which may be different from the reported  $\theta_i$ , one can express the division's after transfer profits as

$$\pi_i(k(\theta), \hat{\theta}_i) + t_i(\theta) \quad i = 1, \dots, n$$

An allocation mechanism  $f(\cdot)$  will have truth telling for each division as a dominant strategy if for each  $i$ , and any  $\theta \in \Theta$

$$\begin{aligned} & \pi_i(k(\hat{\theta}_i, \hat{\theta}_{-i}), \hat{\theta}_i) + t_i(\hat{\theta}_i, \hat{\theta}_{-i}) \\ & \geq \pi_i(k(\theta_i, \hat{\theta}_{-i}), \hat{\theta}_i) + t_i(\theta_i, \hat{\theta}_{-i}) \end{aligned}$$

The restriction Laffont and Maskin place on the domain of the problem is that they require:

for  $i = 1, \dots, n$ ,  $\Theta_i$  be an open interval in  $\mathbb{R}$  and

$\pi_i: \mathbb{R}_+ \times \Theta_i \rightarrow \mathbb{R}$  be a continuously differentiable function such that for any  $\theta \in \Theta = \prod_{i=1}^n \Theta_i$  there exists  $k^*(\hat{\theta}) \in \mathbb{R}_+$  for which

$$(i) \quad \sum_{i=1}^n \pi_i(k^*(\hat{\theta}), \hat{\theta}_i) = \max_{k > 0} \sum_{i=1}^n \pi_i(k, \hat{\theta}_i)$$

(ii)  $k^*(\theta)$  is continuously differentiable.

It should be noted that Holmstrom has derived a similar uniqueness characterisation but assuming a more general restriction requiring only that the conditional profit

functions be smoothly connected in particular convex domains. Also, Laffont and Maskin note that the results are generalisable to multidimensional  $k$  allocation space.

If one were to assume that the divisional parameter spaces were one dimensional, it is possible to derive the Groves' mechanism in a simple fashion when a suitable differentiability condition is assumed. Assuming then that  $\theta_i \in \mathbb{R}^1$  for  $i=1, \dots, n$  and that  $\Pi_i(k^*(\theta_i, \hat{\theta}_{-i}), \hat{\theta}_i)$  and  $t_i(\theta_i, \hat{\theta}_{-i})$  are differentiable with respect to  $\theta_i$ , from the above restriction (i) and the definition of a dominant strategy it follows that:

$$\frac{\delta}{\delta \theta_i} \left\{ \sum_{i=1}^N \Pi_i(k^*(\hat{\theta}), \hat{\theta}_i) \right\} = 0$$

$$\text{and } \frac{\delta}{\delta \theta_i} \{ \Pi_i(k^*(\hat{\theta}), \hat{\theta}_i) + t_i(\hat{\theta}) \} = 0$$

for every  $\hat{\theta}$

On substituting the second equation into the first yields

$$\frac{\delta}{\delta \theta_i} t_i(\hat{\theta}) = \frac{\delta}{\delta \theta_i} \sum_{j \neq i} \Pi_j(k^*(\hat{\theta}), \hat{\theta}_j)$$

which integrates to the family of solutions

$$t_i(\hat{\theta}) = \sum_{j \neq i} \Pi_j(k^*(\hat{\theta}), \hat{\theta}_j) + h_i(\hat{\theta}_{-i})$$

That is, the solution is unique up to the arbitrary function  $h_i(\hat{\theta}_{-i})$  which is independent of  $\theta_i$ . The transfer rule that has been derived here is, of course, the transfer rule that characterises Groves' mechanisms. It

also clearly indicates that the mechanism works by driving the manager of division  $i$  and the centre to having the objective of wishing to maximise

$$\sum_{j=1}^n \pi_j(k^*(\theta_i, \hat{\theta}_{>ic}), \hat{\theta}_i) \text{ for every } \hat{\theta}$$

(when viewed as functions of  $\theta_i$  above).

Hence divisional managers would wish the same allocation of  $k$  as the centre, even if they knew  $\hat{\theta}_{>ic}$

#### 4.4 Limitations of the Groves' and Loeb transfer pricing mechanism

##### 4.4.1 Incentives for coalition formation.

The preceding discussion established that it is possible to design transfer pricing mechanisms that have the desirable property that truth telling is the dominant strategy for divisional managers acting in their own best interests. It did not establish whether truth telling would still be a dominant strategy if one considered it possible that divisional managers may co-operate among themselves to jointly determine their messages. In fact, it has been proven by Bennet and Conn (1977) that Groves' mechanisms are susceptible to manipulation by coalitions.

The Bennet and Conn proof is quite general, but in their manuscript they choose to consider a restricted outcome space which is not appropriate to the transfer pricing problem. A proof will, therefore, be provided with the appropriate outcome space and then the implications of this result will be discussed in some detail.

In the following discussion, let  $C_s$  denote a coalition of divisional managers, where the number of divisional managers in  $C_s$  is  $\#C_s$ . A generic strategy for the group  $C_s$  is  $m^s = (m_i)_{i \in C_s}$ .



For each coalition  $C_S \subset N$  the coalitional evaluation measure is

$$e^S(m) = \sum_{i \in C_S} \{ \Pi_i(k(m)) - t_i(m) \}$$

The undominated strategy set for a coalition will be the set of all joint strategies for the coalition, such that no alternative joint strategy exists which results in a higher coalitional payoff. It is then possible to denote a mechanism as being group incentive compatible if truthful revelation of conditional profits is in the undominated strategy set for every coalition. That is, the undominated strategy set for  $C_S$  is defined as

$$T^S \equiv \{ \hat{m}_S \in R^{\#C_S} : e^S(\hat{m}_S, m_{\supset S C}) \geq e^S(m_S, m_{\supset S C}) \}$$

for all  $m_S \in R^{\#C_S}$ , and for all  $m_{\supset S C} \in R^{N-\#C_S}$

so that the transfer pricing mechanism will be group incentive compatible for the coalition  $C_S$  if  $\Pi_S(k) \in T^S$ .

A Groves' mechanism is characterised by the following outcome function and transfer rules:

$$k^*(m) \max \Pi_0(k) + \sum_{i=1}^N m_i(k) \text{ for } k \in K$$

$$\text{and } \sum_{i=1}^n t_i(m) = \sum_{\substack{j=1 \\ j \neq i}}^n m_j(k^*(m)) + h_i(m_{\supset i C})$$

On combining this characterisation with the definition of a group incentive compatible mechanism, the following proof demonstrates that the class of mechanisms for which both conditions hold is, in fact, empty.

PROOF:

Consider two coalitions  $S_1$  and  $S_2$  such that  $\#S_1 \geq 1$  and  $S_2 = S_1 \cup \{s\}$  where  $s \notin S_1$ .

Given the definition of group incentive compatibility, it is necessary that both

$$\sum_{i \in S_1} t_i(m) = \sum_{j \notin S_1} m_j(k^*(m)) + h^1(m_{>S_1 \subset})$$

$$\text{and } \sum_{i \in S_2} t_i(m) = \sum_{j \notin S_2} m_j(k^*(m)) + h^2(m_{>S_2 \subset})$$

From which it follows that it must be the case that

$$\begin{aligned} t_s(m) &= \sum_{i \in S_2} t_i(m) - \sum_{i \in S_1} t_i(m) \\ &= - \sum_{j \in S} m_j(k^*(m)) + h^2(m_{>S_2 \subset}) - h^1(m_{>S_1 \subset}) \end{aligned}$$

Since  $\{s\}$  is a coalition in  $N$ , it is further necessary that

$$t_s(m) = \sum_{j \notin S} m_j(k^*(m)) + h^S(m_{>S \subset})$$

In order that the two expressions may be compatible requires that it must be that

$$\sum_{j \in N} m_j(k^*(m)) + h^S(m_{>S_2^c}) + h^1(m_{>S_1^c}) - h^2(m_{>S_2^c}) = 0$$

for all  $m \in \mathbb{R}^N$ .

$$\text{Let } h^0(m_{>0^c}) = h^2(m_{>S_2^c}) - h^1(m_{>S_1^c})$$

then it must be that

$$\sum_{j \in N} m_j(k^*(m)) = h^0(m_{>0^c}) - h^S(m_{>S^c})$$

if it were possible to prove that this condition does not in fact hold then the result is established.

Now suppose that for  $\bar{m}$ ,  $\bar{m}_s = \sum_{i \notin s} \bar{m}_i + \epsilon$ ,  $\epsilon > 0$  and

$\hat{m}$  is such that

$$\hat{m}_j = \begin{cases} \bar{m}_j, & j \neq s \\ \sum_{j \neq s} \bar{m}_j - \epsilon & j = s \end{cases}$$

For  $\bar{m}$  and  $\hat{m}$  so defined  $h^S(\bar{m}_{>S^c}) = h^S(\hat{m}_{>S^c})$

$$\text{and } \sum_{j \in N} \bar{m}_j(k^*(\cdot)) - \sum_{j \in N} \hat{m}_j(k^*(\cdot))$$

$$= h^0(\bar{m}_{>0^c}) - h^S(\bar{m}_{>S^c}) - h^0(\hat{m}_{>0^c}) + h^S(\bar{m}_{>S^c})$$

$$= h^0(\bar{m}_{>0^c}) - h^0(\hat{m}_{>0^c})$$

$$= 2 \sum_{i \notin s} \bar{m}_i + \epsilon - 2 \sum_{i \notin s} \bar{m}_i = \epsilon > 0$$

Now consider two alternative messages  $\tilde{m}$  and  $m^*$  such that

$$\tilde{m}_i = \begin{cases} \bar{m}_i, & i \notin S_1 \\ \bar{m}_i + \frac{\delta}{\#S_1}, & 0 < \delta < \epsilon, i \in S_1 \end{cases}$$

and

$$m_i^* = \begin{cases} \bar{m}_i, & i \notin S_2 \\ \hat{m}_i, & i = s \\ \bar{m}_i + \frac{\delta}{\#S_1}, & i \in S_1 \end{cases}$$

Then  $h^S(\tilde{m}_{>sc}) = h^S(m_{>sc}^*)$  and

$$\begin{aligned} & \sum_{i \in N} \tilde{m}_i(k^*(\cdot)) - \sum_{i \in N} m_i^*(k^*(\cdot)) \\ &= h^0(\tilde{m}_{>oc}) - h^0(m_{>oc}^*) \\ &= \sum_{i \in k} \bar{m}_i - \sum_{i \in S_1} \left[ \bar{m}_i + \frac{\delta}{\#S_1} \right] - \sum_{i \notin S_2} \bar{m}_i + \epsilon \\ &= \delta + \epsilon \end{aligned}$$

Now since  $(\tilde{m}_{>oc}) = (\bar{m}_{>oc})$

$$\text{(or } \tilde{m}_s = \bar{m}_s)$$

and  $(m_{>oc}^*) = (\hat{m}_{>oc})$

it follows that  $h(\hat{m}_{>0c}) = h(\bar{m}_{>0c})$

and  $h(m^*_{>0c}) = h(\hat{m}_{>0c})$

so that  $\delta + \epsilon = h(\hat{m}_{>0c}) - h(m^*_{>0c}) = h(\bar{m}_{>0c}) - h(\hat{m}_{>0c}) = \epsilon$ .  $\textcircled{4}$

This contradiction completes the impossibility proof  $\textcircled{4}$ . It should be noticed that one could have  $\#S_1 = 1$  and  $s$  representing a single division, in which case the above results hold for any coalition of size two or greater.

The above result is of great importance because it seems to imply that the desirable properties of any Groves' and Loeb transfer pricing mechanism will be lost, because divisional managers will find it advantageous to collude and misrepresent their conditional profit functions.

Before this line of argument is accepted, it seems appropriate to recall what assumptions have been made about the organisational form of the problem. It has been assumed that divisional managers only have knowledge about their own divisional profit functions and that the centre only has knowledge about its own profit function. Given this argument, one must consider how a coalition could compute its optimal (untruthful) message. In order to do this, the coalition would need to have detailed knowledge of all other non-coitional conditional profit functions. The possibility of divisional managers having data on non-coitional divisions is ruled out by the organisation form which has been specified

above, but even if this specification had not been made, it seems unlikely that a coalition would have access to such information since it is clearly not in the interests of a non-coitional division to make such information freely available.

This argument would not strictly hold, though, if the grand coalition of all divisions were considered. However, it would be an heroic assumption to assume that all the divisions would join together and truthfully share each others' information and then would be in a position to calculate their optimal response to the centre.

However, the above problem of collusion could still be a very serious problem as the example discussed below shows. Here the divisions forming a coalition do not need detailed information about other non-coitional divisions' operating conditions to construct "approximate" cheating strategies. I would now like to present a diagrammatic example which shows just how easy it might be for a coalition of two divisions to cheat. This type of diagrammatic representation was first suggested by Vickrey (1961) and later discussed in more detail by Loeb (1977), however he did not consider cheating.

Suppose that there are four divisions, two distribution divisions,  $i=1,2$  and two manufacturing divisions  $i = 3,4$ . Assume the intermediate product that is transferred is denoted  $k$ . In such a situation, the net marginal revenue schedules of the distribution divisions

will define inverse demand functions here denoted  $d_i(k_i)$  where the area under the respective functions from 0 to  $K_i$  are defined as

$$D_i(K_i) = \int_0^{K_i} d_i(k) dk$$

Similarly, the marginal cost schedules of the manufacturing divisions define inverse supply functions here denoted  $s_i(k_i)$  where the area under the respective functions from 0 to  $K_i$  are defined as

$$S_i(K_i) = \int_0^{K_i} s_i(k) dk$$

Note that with respect to our general model, in terms of conditional profit functions, the manufacturing divisions' increased levels of provision of the intermediate good will decrease the conditional profit of the division since the total cost of producing the intermediate good increases as output increases.

It is possible to characterise the Groves' and Loeb mechanism by the outcome functions which determines  $\hat{k} = (\hat{k}_1, \dots, \hat{k}_4)$  such that:

$$\hat{k} \text{ maximises } \sum_{i=1}^2 D_i(K_i) - \sum_{i=3}^4 S_i(K_i)$$

with respect to  $k \in F \equiv \{(k_1, \dots, k_4) \mid k_i \geq 0 \text{ } i=1, \dots, 4 \text{ and}$

$$k_1 + k_2 \leq k_3 + k_4\}$$

and where a distribution division,  $i = 1$  for instance, has the evaluation measure

$$D_1(\hat{k}_1) - t_1$$

$$t_1 = -D_2(\hat{k}_2) + \sum_{j=3}^4 S_j(\hat{k}_j) + \{ \max_{k_{>1c} \in F_{>1c}} D_2(k_2) - \sum_{j=3}^4 S_j(k_j) \}$$

where  $F_{>1c} = \{(k_2, k_3, k_4) \mid (0, k_2, k_3, k_4) \in F\}$

Similarly, a manufacturing division,  $i=4$ , for instance, has an evaluation measure

$$- (S_4(\hat{k}_4) - t_4)$$

$$t_4 = \sum_{j=1}^2 D_j(\hat{k}_j) - S_3(\hat{k}_3) - \{ \max_{k_{>4c} \in F_{>4c}} \sum_{j=1}^2 D_j(k_j) - S_3(k_3) \}$$

where  $F_{>4c} = \{(k_1, k_2, k_3) \mid (k_1, k_2, k_3, 0) \in F\}$

The example of coalitional cheating will consider two distribution divisions forming a coalition. First, therefore, one must calculate the divisional evaluation measures using the Groves' and Loeb mechanism when the two distribution divisions do not form a coalition.



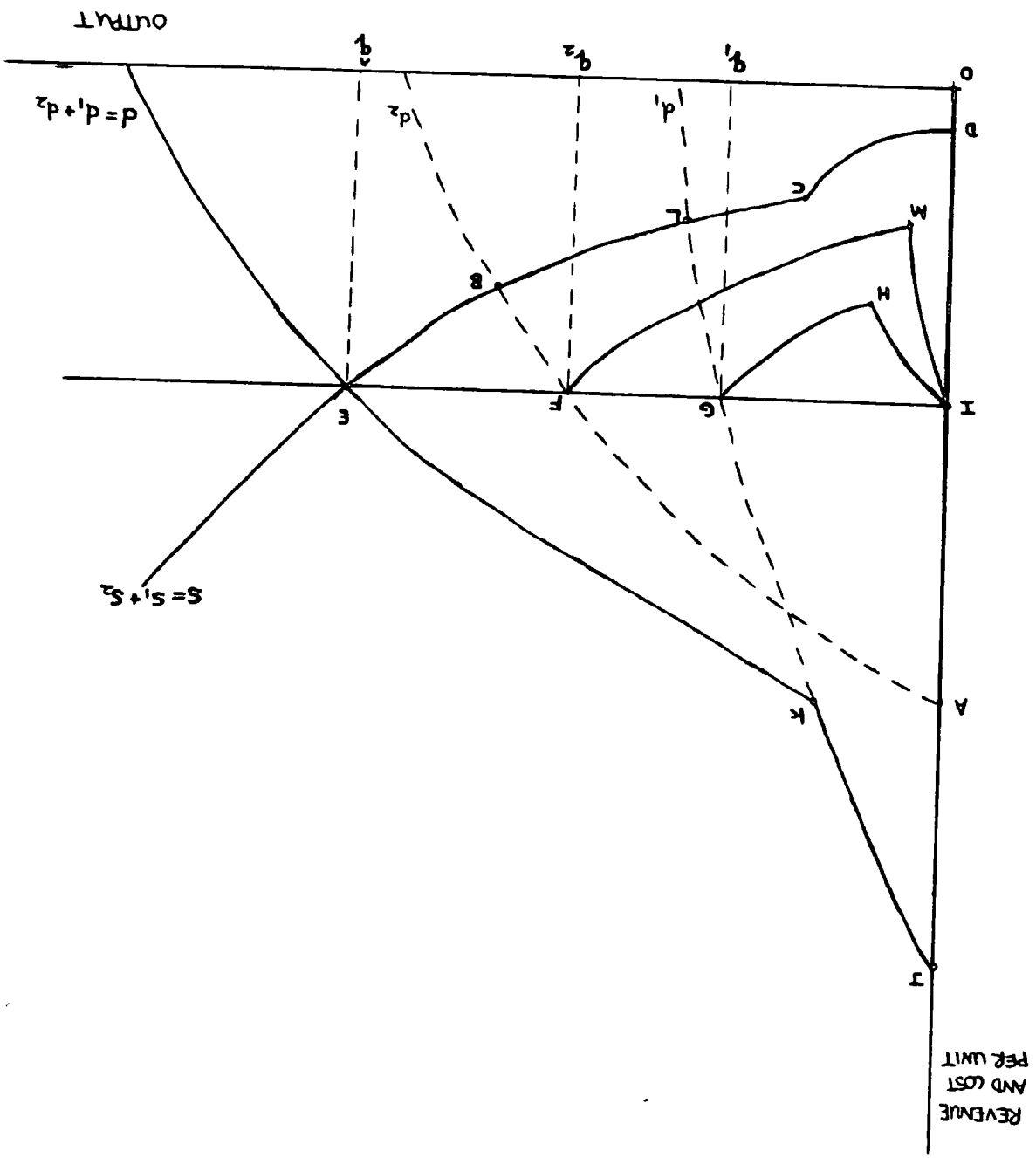
If the schedules are as represented in Figure 1, then the divisional evaluation measure for division one can be calculated as follows:

$$\begin{aligned}
 t_1 &= -D_2(\hat{k}_2) = -OAFq^2 \\
 &+ \sum_{j=3}^4 S_j(\hat{k}_j) = ODCBFq^{\hat{}} \\
 &+ \{ \cdot \} = + ABCD \\
 &\quad \underline{\underline{q^2 qEBF}}
 \end{aligned}$$

This is equal to the area  $q^1GHI$  since  $GHI$  is congruent to  $EBF$  by construction, implying that the divisional evaluation measure is  $JkGHI$  which is clearly greater than the divisional evaluation measure using a Hirshleifer type mechanism.

Similarly division two has the evaluation measure  $AFMI$ . In order to determine in what fashion the divisions optimally cheat, it would be necessary to determine what expectations or facts a coalition has about non-coalitional inverse demand and inverse supply functions. However, in the following examples, it is assumed that coalitions use "approximate" cheating strategies, which can readily be formulated. For example, one such readily formulateable strategy would be for two distribution divisions to agree to each understate their respective inverse demand schedules by ten percent below their true values.

FIGURE 1



To demonstrate this approach, let us consider a number of possible prospective "approximate" cheating strategies.

I. Suppose distribution division one (d.d.1) understates its inverse demand schedule as in Figure 2. The effect of such cheating will only benefit distribution division two (d.d.2) directly by increasing d.d.2's evaluation by  $IMFF'M'I$ . In fact, this cheating will reduce d.d.1's evaluation measure by  $WXVYZ$ , as illustrated by Figure 3. To see this, note that D.D.1 loses  $FBB'F' - IVXI' + YGZ$ . In this case, note that  $IMFF'M'I > WXVZYG$  so there does seem to be some gains from cheating. Hence, d.d.2 may offer to pay d.d.1 some amount to make it worthwhile to d.d.1 to understate the inverse demand schedule  $d_1$ . The important question is how will a suitable payment be determined. Given the assumption that the distribution divisions do not know, in detail, the inverse supply schedules, they will wait until after the operating plans have been specified by the central headquarters. At this point in time, they will know the value of the inverse supply schedule at output  $q_1 + q_2$  and may determine what it would approximately have been at output  $q_1 + q_2$ . The divisions can then determine what the net resultant gains (if there are any) from having d.d.1 cheat and then use some prespecified rule such as "split the difference" in the net gain to determine the suitable payment.

FIGURE 2

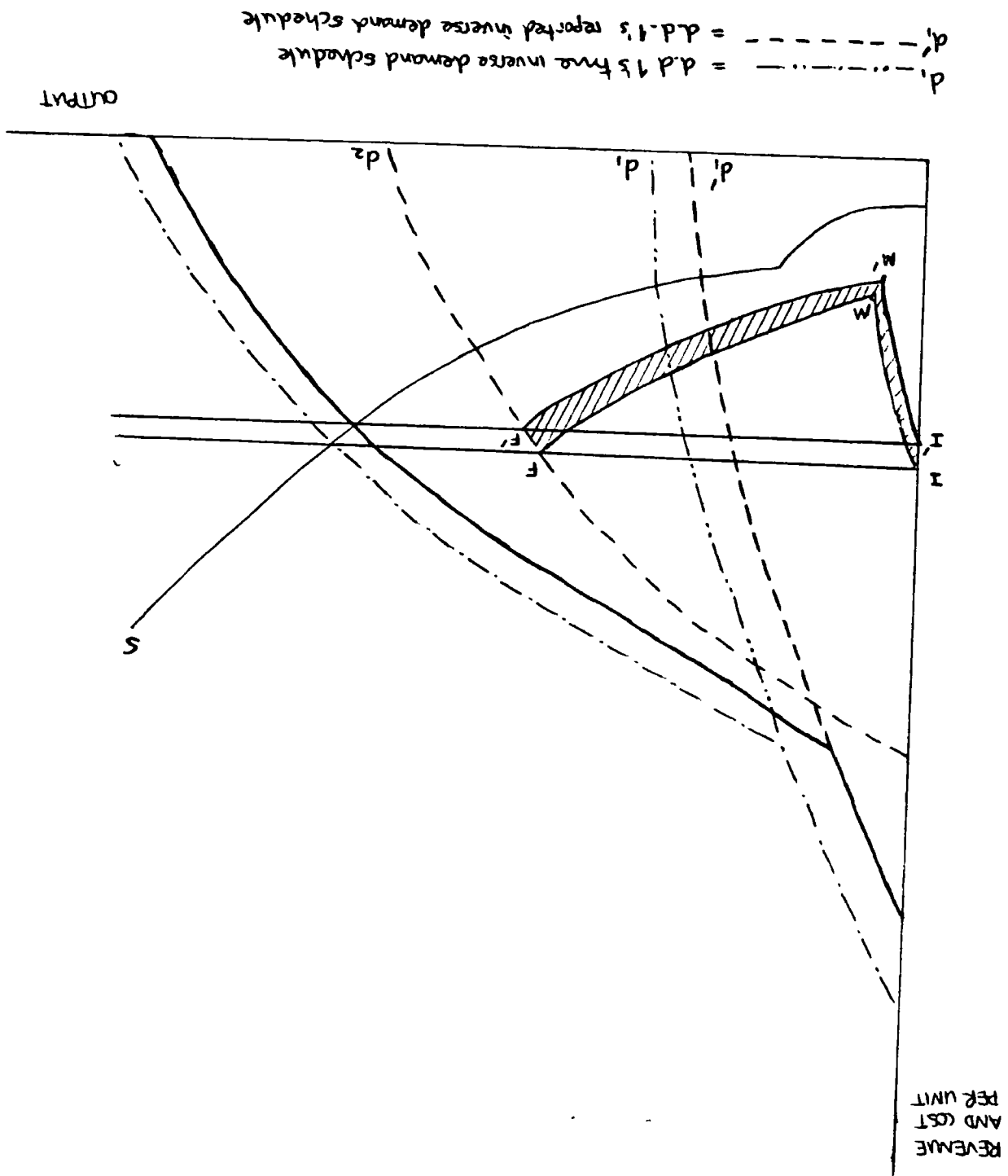
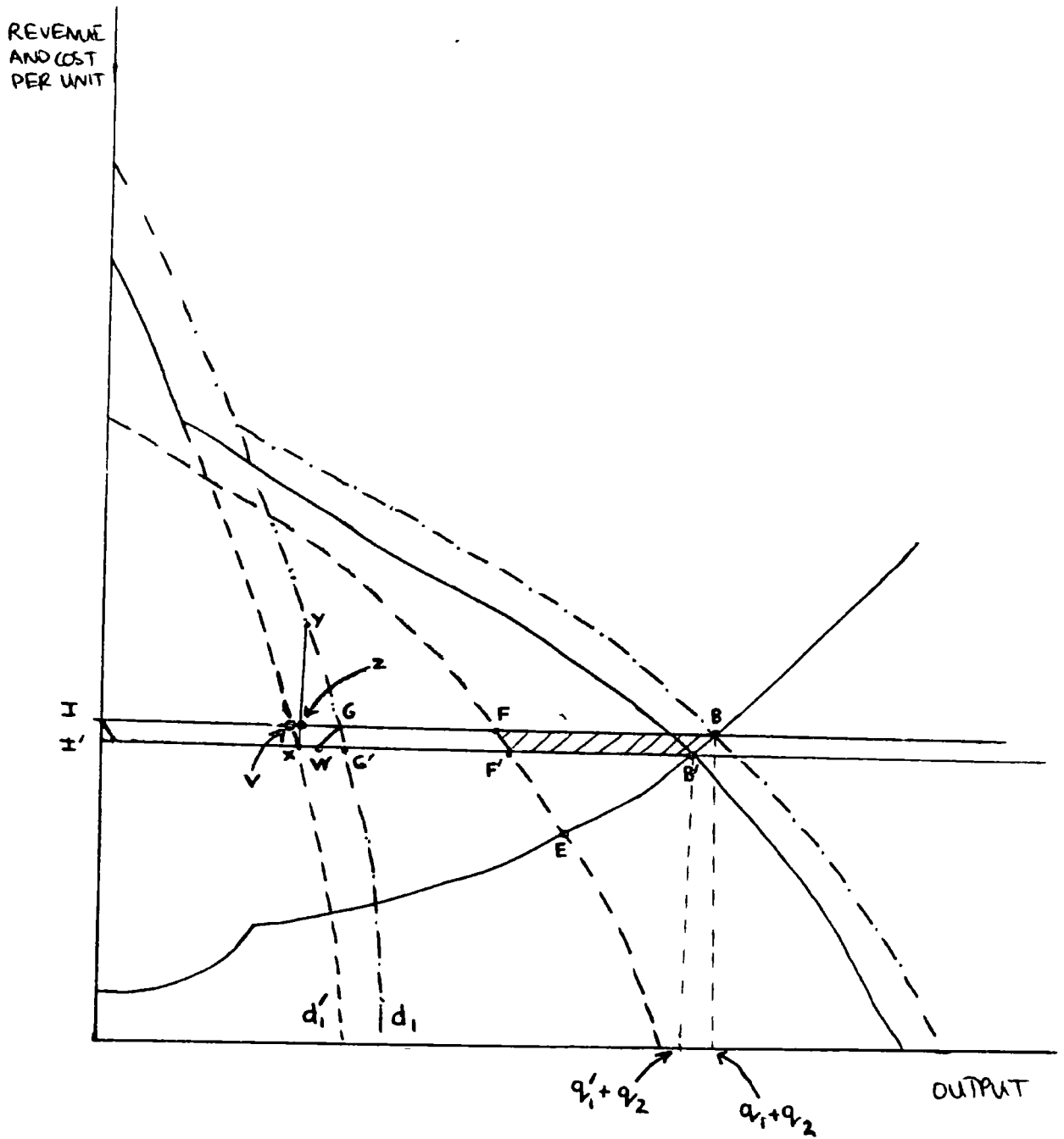


FIGURE 3



$d_1$  ..... = d.d.1's true inverse demand schedule  
 $d_1'$  - - - - = d.d.1's reported inverse demand schedule

This strategy, however, seems questionable as it is hard to see how d.d.2's manager can "pay" d.d.1's manager without the central headquarters becoming aware. A far more likely type of procedure is a reciprocal type agreement as detailed in II.'

II. Suppose d.d.1's manager and d.d.2's manager agree to a strategy in which d.d.1's manager understates the division's inverse demand schedule by some set amount and, in return d.d.2's manager understates that division's inverse demand schedule by some set amount. This type of agreement is illustrated in Figure 4 and Figure 5.

Figure 4 demonstrates that d.d.1's gain from d.d.2's lying is  $IHWG'H'$  while losing  $TGW$  by lying for d.d.2, with net gain in this case being positive<sup>14</sup>. Figure 5 demonstrates that d.d.2's gains from d.d.1's lying are  $IQQ'I'$  while losing  $RSQ$  by lying for d.d.1, with the net gain clearly being positive<sup>15</sup>. To recap, note that since under the Groves' and Loeb procedure, it is a division's actual inverse demand schedule ex post that is used to evaluate the division, plus the extra opportunity cost term and the central headquarters profits. Thus, the evaluation measure of division 2 in this instance will be  $A'RQ'I' + Q'HI'$  plus the central headquarter's profits. In the Groves' and Loeb procedure, the central headquarter's profit level is not the sum of divisional profits, but is the cost of centrally providing services and resources to the divisions. Since the lying results in a lower output

FIGURE 4

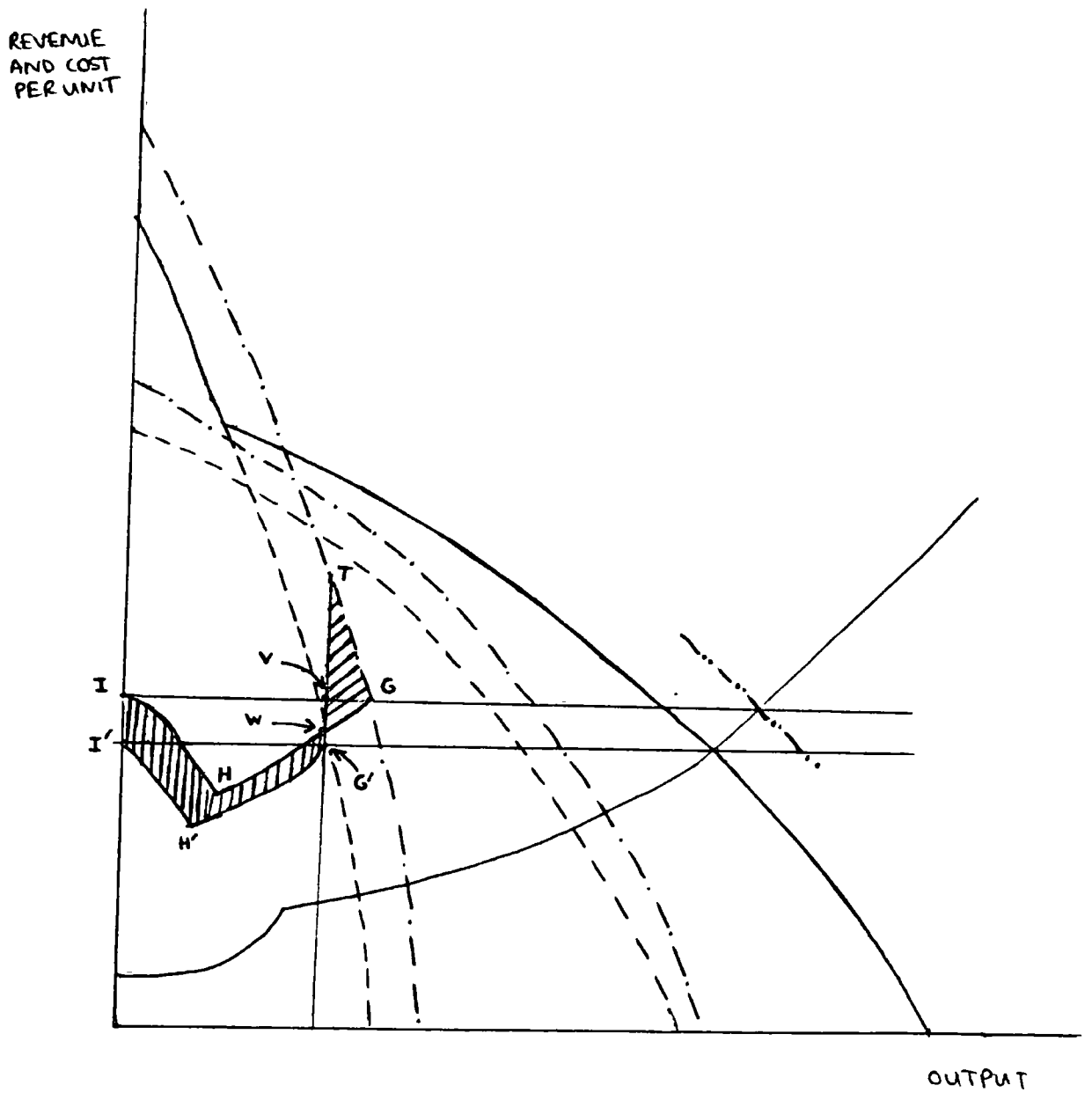
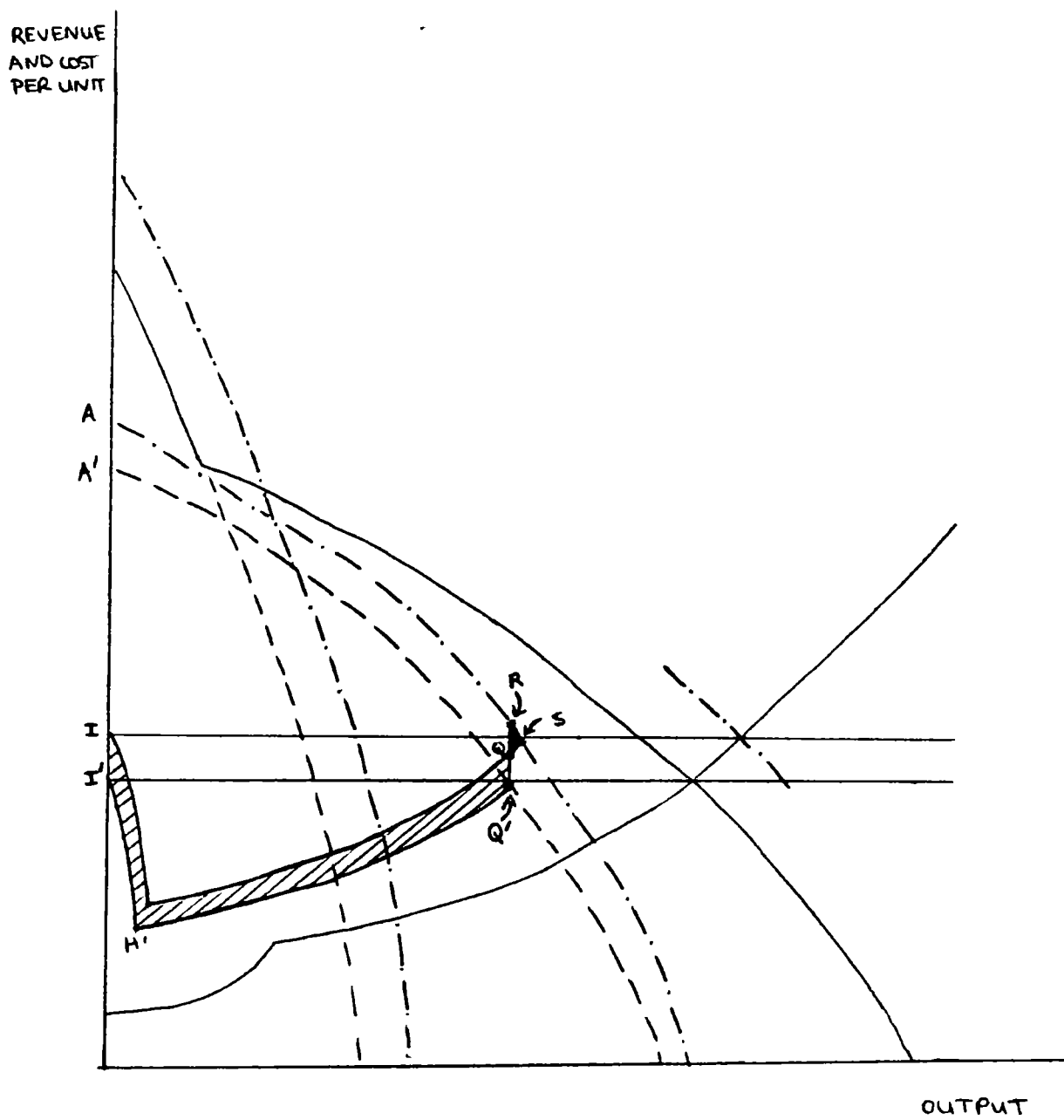


FIGURE 5



- = d.d.1's or d.d.2's true inverse demand schedule
- = d.d.1's or d.d.2's reported inverse demand schedule
- = true aggregate inverse demand schedule



any total variable components of the central headquarter's costs will be less than if the divisions had reported truthfully. Hence, by lying, the divisions can clearly improve their evaluation measures.

However, as the above discussion of net gains from lying illustrates, coalitions between divisions seemingly prepared to form reciprocal lying agreements are likely to be highly unstable. This is because it would be better for the division to have

another division lie on its behalf, thinking that the other division was reciprocating, but that division would not, in fact reciprocate, and may even choose to overstate its inverse demand schedule. Figure 6 illustrates such a situation. The gain to d.d.2 by overstating its inverse demand schedule and having d.d.1 understate its inverse demand schedule is CAEDB - EDF, which equals CAEGB - DFG. Note in this example, it has been assumed that the amount d.d.2 overstates its inverse demand schedule is directly proportionate to the amount d.d.1 understates its inverse demand schedule, such that the aggregate inverse supply schedule intersects the aggregate inverse demand schedule at the same output that would result if all schedules were communicated truthfully. Figure 7 provides a composite comparison of the approximate cheating strategies assumed for Figure 5 and Figure 6. The diagram illustrates that the approximate cheating strategy of Figure 6 is clearly preferable to d.d.2 because

FIGURE 6

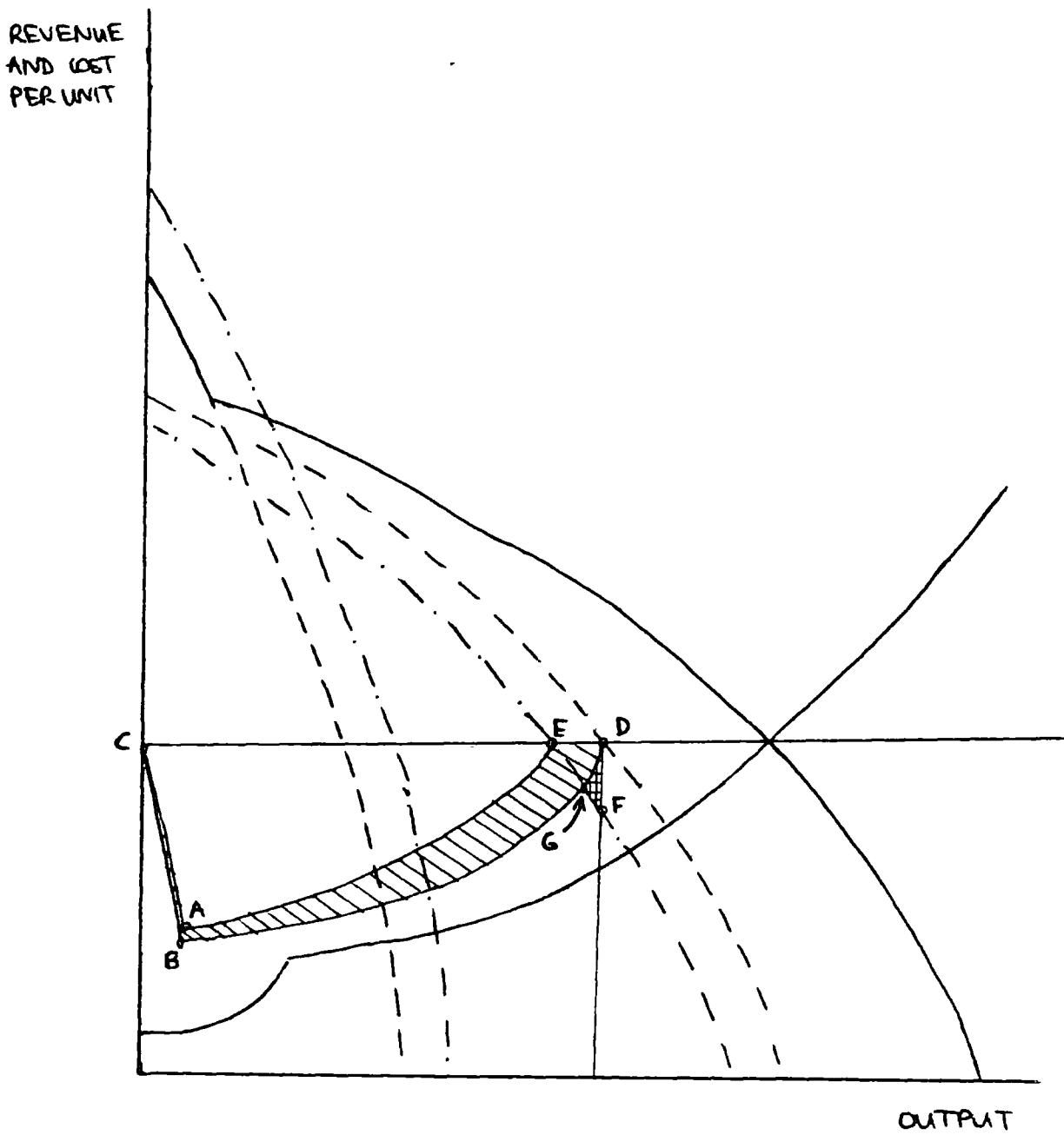
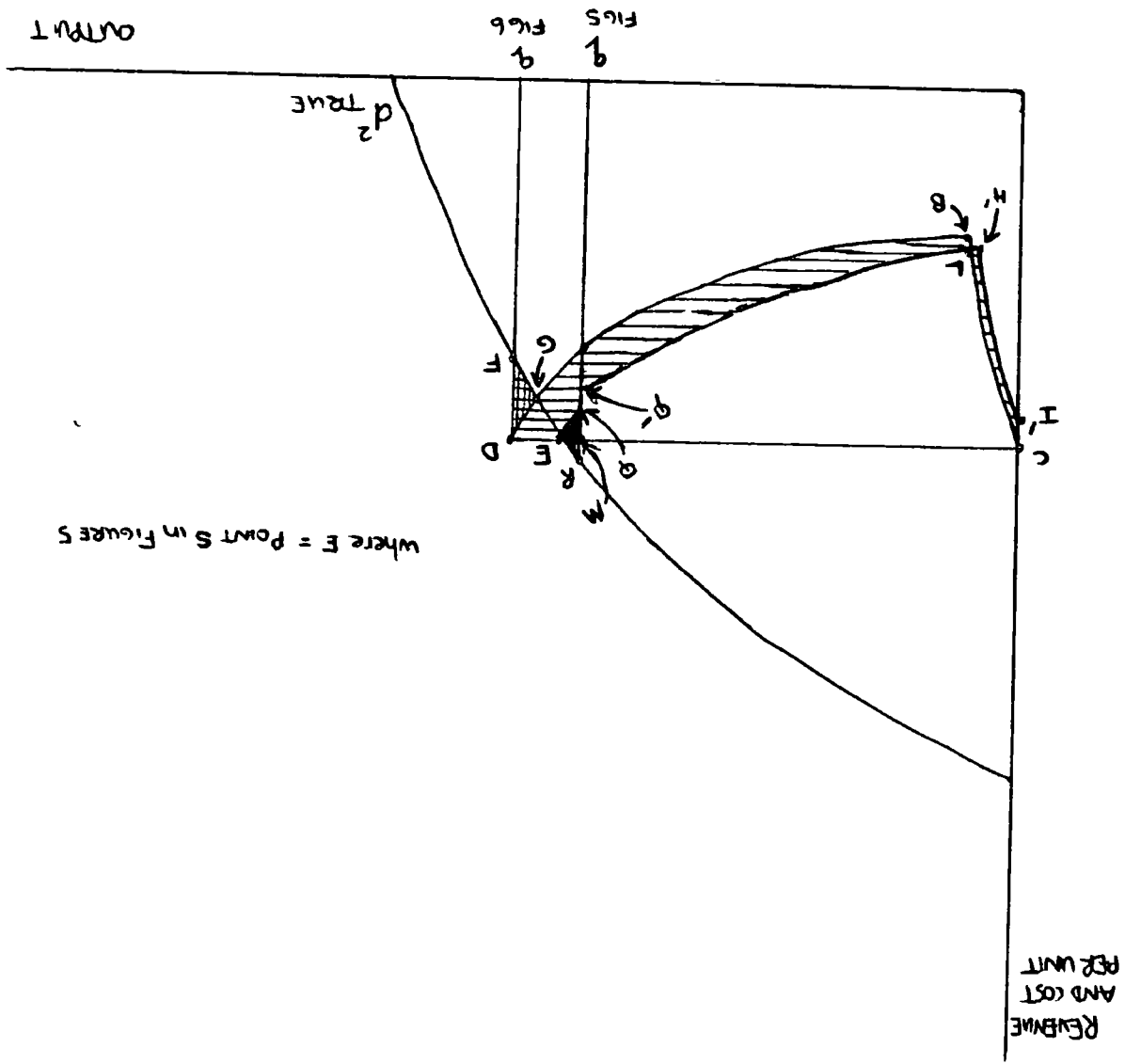


FIGURE 7



where E = point S in figures 5

FIGS 1  
q  
q

OUTPUT

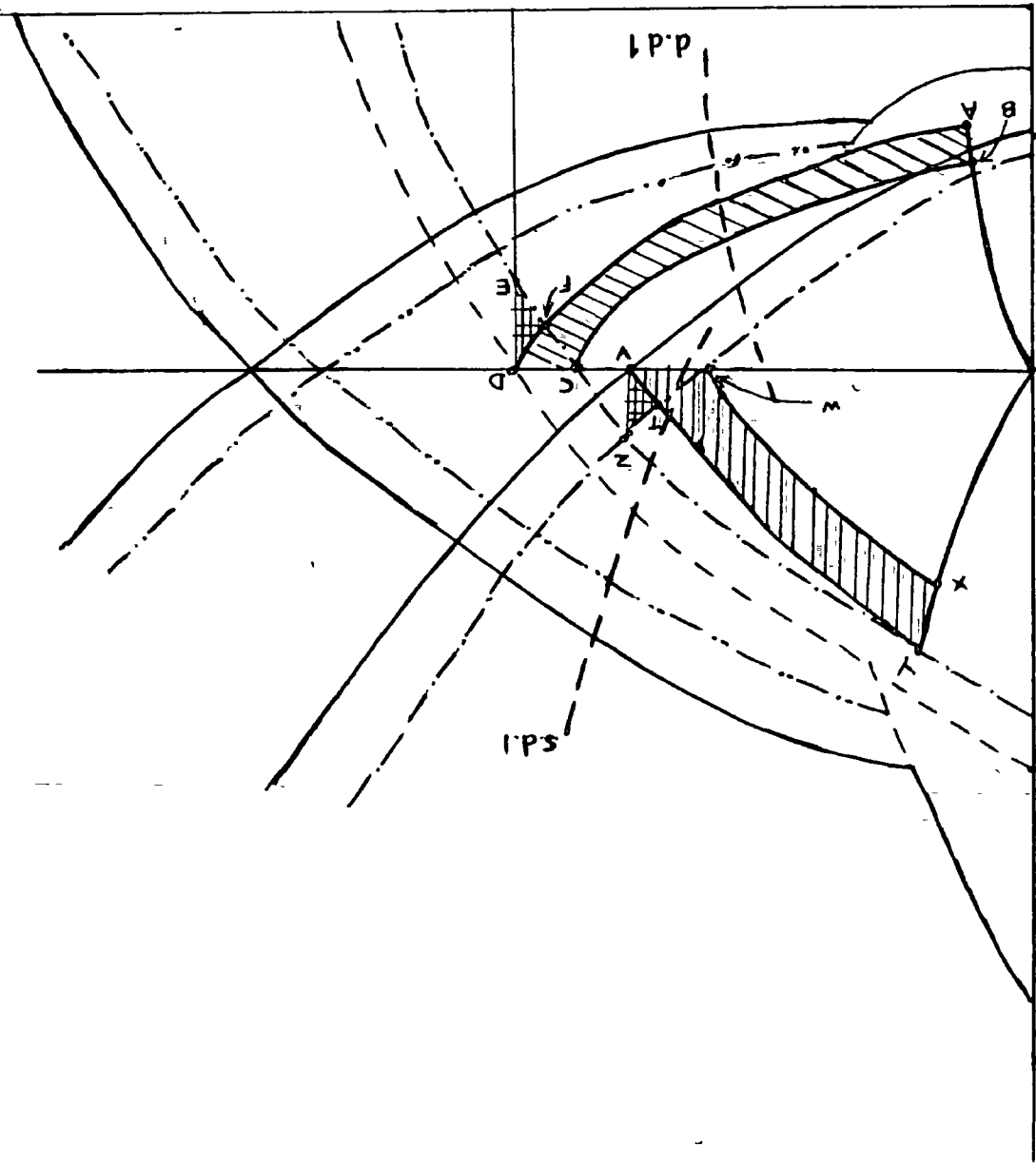
its evaluation measure is greater by  $BLQ'_{MD} + QRE - DFG - CI'_{HL}$ . In this particular example,  $DFG$  is approximately equal to  $REQ$  and  $CI'_{HL}$  is very small. Hence, d.d.2 is approximately  $BLQ'_{MD}$  better off. To understand the intuition behind this result, one should note that, under the Groves' and Loeb procedure, the distribution division that consumes the most amount of the intermediate product has the largest opportunity cost<sup>16</sup> added to its evaluation measure. Thus, to recap then, although two distribution divisions have an incentive to form a coalition and cheat by understating their inverse demand schedules, each division's manager will also have an incentive to renege the agreement, hoping that the other keeps to its part of the agreement, thus such potential coalitions are inherently unstable and this will be recognisable by the distribution divisions' managers.

A detailed discussion of possible manufacturing division coalitions will not be presented here, as it was for the distribution divisions, as the two cases are analogous, except that the manufacturing divisions' counterpart to strategy I would be to overstate inverse supply schedules.

Another possibility for coalition formation to cheat would be between a distribution division and a manufacturing division. Figure 8 demonstrates the possible advantages of such a coalition. However, the approximate cheating strategies are somewhat unconventional, in that the distribution division d.d.2 is required to overstate its

= d.d.2's (or s.d.2's) true inverse demand (inverse supply) schedule  
 = d.d.2's (or s.d.2's), reported inverse demand (inverse supply) schedule  
 = true aggregate inverse demand or inverse supply schedule

OUTPUT



REVENUE AND COST PER UNIT

FIGURE 8

inverse demand schedule and the manufacturing division, labeled s.d.2 is required to understate its inverse supply schedule. This diagram has been specifically drawn so that the cheating strategies result in the aggregate supply and demand schedules intersecting at the same Hirshleifer price. This is to simplify the already considerably complex diagram and it should be noted that the general results do not require this to be the case.

The evaluation measure of d.d.2 increased by  $ABCD - DFC$  which equals  $ABCF - DFE$ , while the evaluation measure of s.d.2 increases by  $XYVW - ZVW$  which equals  $XYTW - ZVT$ . Once again, this type of collusion may be unstable, since each division has an incentive to renege on any reciprocal agreement. Alternatively, what divisions may do is overstate their true schedule to potential coalitional partners and then, when they are, say, required to understate their inverse demand schedule, they do so, and end up communicating to the central headquarters, an inverse demand schedule close to their true schedule's value.

The above discussion has been applied to a vertically organised organisation with manufacturing and distributing divisions. However, the possibility of coalitions forming to cheat in horizontally organised corporations, as in the Dantzig-Wolfe type model, is equally likely. This is because the only difference would be that the inverse supply schedule (of some scarce resource) would be fixed.

The above discussion has demonstrated how divisions may form coalitions to cheat under the Groves' and Loeb procedure, but that such coalitions are likely to be inherently unstable.

#### 4.4.2 Divisional managers' disutility from effort provision.

The discussion of the Groves' and Loeb mechanism has previously always assumed that the manager of a division  $i$  holds as sole objective maximisation of the performance measure  $\bar{E}_i$ . However, while carrying out divisional management tasks, a divisional manager will need to expend effort. It is important to recognize that divisional managers will experience disutility from the provision of their effort services. If the objective of the central headquarters is the arrangement of interdivisional transfers to facilitate maximisation of the sum of divisional performance measures (and, hence, corporate profits), the following problem arises. Since different divisional tasks will require different divisional managerial effort and attract differing amounts of divisional managerial compensation, each divisional manager will be concerned with the tradeoff between his effort and compensation associated with a given task (plan of operations). In addition, divisional managers will recognise that the central headquarters will be choosing tasks on the basis of information

communicated by the divisional managers and the task selection procedure employed by the central headquarters will not take account of divisional managers experiencing disutility from effort provision. Hence, given asymmetric access to divisional information, the divisional managers will perceive advantages from communicating strategically (untruthfully) to maximise their utility. That is, divisional managers may choose to communicate strategically in such a fashion, such that, unwittingly the central headquarters will attempt to implement solutions, which not only take account of divisional preference for compensation, but also the disutility that arises from any associate divisional managerial effort provision. Thus, the incentive compatibility properties of the Groves' and Loeb mechanism do not hold when it is recognised that divisional managers will consider the tradeoff between utility gained from compensation and the associated disutility of effort provision. This result is proven<sup>17</sup> by Miller and Murrel (1981). An alternative proof and a discussion of the implications of this result will now be presented.

Suppose we are considering a firm consisting of  $n$  divisions, indexed  $i = 1, \dots, n$  and a central headquarters. It must now be explicitly recognised that the  $i^{\text{th}}$  divisional profits do not just functionally depend on the



level of intermediate product transfers or the level of provision of common corporate resources  $k_i$ , but also the level of effort provided by the  $i^{\text{th}}$  divisional manager, denoted  $e_i$ . Thus, one can express division  $i$ 's profit as:

$$\bar{\pi}_i = \bar{\pi}_i(k_i, e_i)$$

where  $\bar{\pi}_{ik} = \frac{\partial \bar{\pi}_i}{\partial k_i} > 0$  for  $i=1, \dots, n$

$$\begin{aligned} \bar{\pi}_{ie} &= \frac{\partial \bar{\pi}_i}{\partial e_i} > 0 \text{ for } e_i < \bar{e}_i, \\ &= 0 \text{ for } e_i \geq \bar{e}_i \end{aligned}$$

Also  $\bar{\pi}_i(k_i, 0) \geq d$  where  $d$  is a positive constant for all  $k_i > 0$

The penultimate requirement is based on the assumption that there is a finite level of effort beyond<sup>18</sup> which a divisional manager can not increase divisional profitability by increased effort, given other resources are fixed. The usual declining marginal productivity assumption is also made, that is

$$\begin{aligned} \frac{\partial^2 \bar{\pi}_i}{\partial e_i^2} &< 0 \text{ for } e_i < \bar{e}_i, \\ &= 0 \text{ for } e_i \geq \bar{e}_i \end{aligned}$$

Each divisional manager is assumed to maximise their utility function which is separable in reward  $R_i$  and effort  $e_i$ , where the function is increasing in reward and decreasing in effort, so that one can represent the utility function as:

$$U_i(R_i, e_i) = R_i - g_i(e_i)$$

where clearly  $\frac{\partial g_i}{\partial e_i} > 0$  for all levels of  $e_i$  since divisional managers gain disutility from provision of effort. Also assume  $\frac{\partial^2 g_i}{\partial e_i^2} > 0$  and  $g_i(0) = 0$ .

An important question is whether the central headquarters wishes to maximise corporate profits before or after managerial compensation is charged. One can present the objective function of the central headquarters as requiring

$$k(m) \max \sum_{i=1}^n m(k_i, e_i) - C \left( \sum_{i=1}^n k_i \right) - B \sum_{i=1}^n R_i$$

where  $m(\cdot, \cdot)$  is the reported profit function for division  $i$ . In this model,  $B$  is a constant which takes on the values 0 or 1 depending on whether the centre explicitly recognises the costs of managerial compensation in determining the optimal solution.

Let us first consider the case when  $B$  takes the value zero, so that the centre's objective is maximisation of corporate profit before managerial compensation is charged.

The central headquarters will be aware that for any allocation  $k_i$  the  $i^{\text{th}}$  divisional manager will choose the effort level  $e_i^*(k_i)$  defined as:

$$e_i^*(k) = \operatorname{argmax} \{ \bar{\pi}_i(k_i, e_i) - g_i(e_i) \}$$

$$\text{since } R_i = \bar{\pi}_i(\cdot, \cdot) + \sum_{j \neq i} m_j(k_j(m)) - C \left( \sum_{j=1}^n k_j(m) \right) - A_{>ic}$$

Thus, in order to determine the corporate optimal allocation vector  $k$ , the central headquarters will require information, either explicitly or implicitly on the function  $g_i$ . Thus, in addition to sending information about the divisional profit function Miller and Murrell claim that the central headquarters will require divisional managers to communicate a message,  $q_i$  reporting to the centre the nature of the effort disutility function  $g_i$ . Note that instead one could consider the case where the message sent by a divisional manager concerning the nature of  $g_i$  could be sent implicitly if the divisional manager reports instead  $m_i^*(k_i)$  where

$$\begin{aligned} m_i^*(k_i) &= \max_{e_i} \{ \bar{\pi}_i(k_i, e_i) - g_i(e_i) \} \\ &= \bar{\pi}_i(k_i, e_i^*(k_i)) - g_i(e_i^*(k_i)) \end{aligned}$$

Theorem 1. There exists no reward system that simultaneously enables the centre to maximise corporate profit (before charging managerial compensation) and which induces honesty as the optimal strategy for divisional managers when the reward systems must be solely functions of messages from the divisional managers and the centre's observations of realised divisional profitability.

PROOF: The proof proceeds by contradiction. Assume that such a reward system exists. One can express such a reward system as  $R_i(m_i^*(k_i), \bar{\pi}_i(k_i, e_i))$  where all other arguments of the reward system, such as other divisional messages are omitted, as they are not directly relevant to this proof. Any optimal divisional management reward given the central headquarters objective function should encourage managers of division  $i$  to provide  $\bar{e}_i$  effort level. Thus

$$R_i(\bar{m}_i^*(\bar{k}_i), \bar{\pi}_i(\bar{k}_i, \bar{e}_i))$$

must maximise the utility function of the manager of division  $i$  where  $\bar{m}_i^*(\cdot)$  is the optimal message:

$$\bar{m}_i^*(\bar{k}_i) = \bar{e}_i \operatorname{argmax} \{ \bar{\pi}_i(\bar{k}_i, e_i) - g_i(e_i) \}$$

with  $\bar{k}_i$  being the corporate profit maximising allocation.

Given the above assumed regularity conditions, this central headquarter's optimum must be unique and, hence, the divisional manager's reward system at  $\bar{e}_i$  must satisfy the following inequality:

$$R_j(\bar{m}_i^*(\bar{k}_i), \bar{\pi}_i(\bar{k}_i, \bar{e}_i)) > R_i(m_i^0(\bar{k}_i), \bar{\pi}(\bar{k}_i, e_i^0))$$

for any  $e_i^0 < \bar{e}_i$

$$\text{and where } m_i^0(\bar{k}_i) = \bar{\pi}_i(\bar{k}_i, e_i^0) - g_i(e_i^0)$$

Thus there must be some  $e_i^0$  for which

$$R_i(\bar{m}_i^*(\bar{k}_i), \bar{\pi}_i(\bar{k}_i, \bar{e}_i)) = \theta\{q_i(\bar{e}_i) - g_i(e_i^0)\} + R_i(m_i^0(\bar{k}_i),$$

$$\bar{\pi}(\bar{k}_i, e_i^0))$$

where assume that  $0 < \theta < 1$

Suppose the divisional manager determines the scalar value,  $\phi > 0$  for which

$$e_i^0 = \operatorname{argmax} \bar{\pi}_i(\bar{k}_i, e_i) - (1 + \phi)g_i(e_i)$$

Existence of an appropriate scalar is guaranteed by the regularity conditions. Thus, if the division communicates

$$m_i^0(\bar{k}_i) = e_i^0 \operatorname{argmax} \bar{\pi}_i(\bar{k}_i, e_i) - (1 + \phi)g_i(e_i)$$

it can expect to receive reward

$$R_i(m_i^0(\bar{k}_i), \bar{\pi}(\bar{k}_i, e_i^0))$$

Thus, the divisional manager's utility when reporting respectively  $\bar{m}_i^*(\bar{k}_i)$  and  $\bar{m}_i^0$ , when using the scalar  $\phi$  in the latter instance, must satisfy the inequality

$$R_i(\bar{m}_i^0(\bar{k}_i), \bar{\pi}(\bar{k}_i, e_i^0)) + \theta\{g_i(\bar{e}_i) - g_i(e_i^0)\} - g_i(\bar{e}_i)$$

$$> R_i(\bar{m}_i^0(\bar{k}_i), \bar{\pi}(\bar{k}_i, e_i^0)) - g_i(e_i^0)$$

which requires

$$\theta\{q_i(\bar{e}_i) - q_i(e_i^0)\} > g_i(\bar{e}_i) - g_i(e_i^0)$$

but this inequality does not hold since  $0 < \theta < 1$ .  $\square$

This result contradicts the assumption that the reward system will induce managers to transmit information that allows the central headquarters to determine the corporate profit maximising plan of operations.

Note that the proof centres on the observation that divisional managers can always increase the size of their bonus by exaggerating the disutility of effort in this case by a proportion  $\phi$ .

Let us now consider the case when B takes the value of one so that the central headquarters objective is maximisation of corporate profits net of managerial compensation. In order to focus on the effect of changing the central headquarter's objective to 'net' instead of 'gross', it will be assumed that effort does not affect output. The Groves' and Loeb mechanism has been

formulated for situations where the centre has no preferences on the level of managerial compensation. Thus, the level of managerial compensation relative to  $\bar{\pi}_1 = \bar{\pi}_1(k_1)$  is not uniquely specified in the mechanism.

**Theorem 2.** There exists no reward system that simultaneously enables the centre to maximise net corporate profits and which induces honesty as the optimal strategy for divisional managers when the reward system must be solely functions of messages from divisional managers and the centre's observations of realised divisional profitability.

Before proceeding with the proof, it shall be assumed that there is a minimum level of divisional managerial reward,  $\bar{R}_i$ ,  $i=1, \dots, n$ , below which divisional managers will not be prepared to work. That is, reward for working in the corporation must be greater than or equal to some prespecified level  $\bar{R}_i$ .

PROOF: if  $k^* = \operatorname{argmax}_k \sum_{i=1}^n \bar{\pi}_i(k_i) - C(\sum_{i=1}^n k_i)$

one can denote  $\bar{\pi}(k^*)$  as the maximum gross corporate profit. Thus, given the previous assumption maximum net corporate profits can be expressed as:

$$\bar{\pi}(k^*) - \sum_{i=1}^n \bar{R}_i$$

The Groves' and Loeb mechanism requires<sup>19</sup> that

$$R_i = \alpha_i \bar{E}_i(\pi_i, m) - T_i, \quad \alpha_i \text{ is constant with } \alpha_i \geq 0$$

Thus, the only Groves' and Loeb mechanism that could achieve maximisation of net corporate profits is a Groves' and Loeb mechanism with  $R_i = \bar{R}_i$  at the optimal level of provision of  $k$ . Since the optimal level of provision of  $k$  is not known before divisions choose to (strategically or truthfully) communicate,  $R_i = \bar{R}_i$  can only be guaranteed if  $\alpha_i = 0$  and  $T_i = -\bar{R}_i$ . However  $\alpha_i = 0$  destroys the dominance of truthful communication for the Groves' and Loeb mechanism since a divisional manager will receive  $\bar{R}_i$  regardless of what is communicated. ©

The work of Miller and Murrell (1981) clearly shows that, if divisional managers are effort averse, the original Groves' and Loeb mechanism will not maintain its incentive properties. Miller and Murrell (1981) conclude in the paper that

"the results of this paper thus significantly generalise those of Pryor by showing that, not only do existing schemes fail, but also it is impossible to construct any bonus scheme that is entirely satisfactory from the centre's viewpoint. The search for the elusive optimal bonus scheme in command economies (Berliner, 1976) is thus bound to be a failure."

However, in my view, Miller and Murrell's impossibility claim is based on a fundamental mistaken premise, as the discussion below will attempt to highlight.

In the preceding chapters of this thesis, it has been argued that the central headquarters will wish to employ divisional managers, to collect and process specialised information and to carry out divisional tasks which the central headquarters does not have time to carry out.



That is, one of the fundamental reasons why divisional managers are employed is for provision of their effort services. Clearly, the central headquarters recognises this and will also recognise that divisional managers are effort averse and require some inducement to provide their effort services. Thus Theorem 1 due to Miller and Murrell is based on the premise that the central headquarters is attempting to maximise corporate profits by requiring truthful information provision from divisional managers, but that the centre does not recognise that divisional managers need be rewarded for their effort services. However, I would argue that central headquarters do recognise that divisional managers need to be rewarded for their effort services.

The above argument in this setting can be expressed in an alternative fashion if one uses the following taxonomy for incentive problems. "Moral hazard" refers to the problem of inducing divisional managers to supply proper amounts of effort when their true divisional profit function is not known by the central headquarters. "Adverse selection" refers to a situation where the realised profit (a scalar) of a division can be observed, but it cannot be verified whether the realised profit was the correctly chosen one from the range of possibilities specified by the profit "function", which only the divisional manager knows completely.

Thus it is argued that in a decentralised corporation, one can not deal with adverse selection problems without also simultaneously considering moral hazard issues. This is because, as the Miller and Murrell results show, moral hazard issues may often be the cause of adverse selection problems. Thus,

what is wrong with the Miller and Murrell impossibility claim is that they assume that the central headquarters is not aware of, or is not prepared to deal with moral hazard issues.

Recently, Cohen and Loeb (1984) have presented a modified version of the Groves' and Loeb mechanism, which not only takes account of adverse selection problems, but also considers simultaneously moral hazard issues. Their model will now be presented.

To recap, the profits of the firm gross of the costs of rewarding the divisional managers and the central headquarter's management can be expressed as:

$$\Pi(k, e) = \sum_{i=1}^n \bar{\pi}_i(k_i, e_i) - C\left(\sum_{i=1}^n k_i\right) \quad (1)$$

where  $k = (k_1, \dots, k_n)$  and  $e = (e_1, \dots, e_n)$ .

The divisional manager's utility functions are separable in reward and effort, being increasing in the former argument and decreasing later.

$$U_i(R_i, e_i) = R_i - g_i(e_i) \quad (2)$$

In a Groves' and Loeb mechanism, the manager of division  $i$  sends a message function  $m_i(k_i)$  to the central headquarters. The headquarters then determines the allocation

$$k(m) = \operatorname{argmax} \sum_{i=1}^n m_i(k_i) - C\left(\sum_{i=1}^n k_i\right) \quad (3)$$

The Groves' and Loeb divisional management reward system can be expressed as:

$$R_i = \bar{\pi}_i + \sum_{j \neq i} m_j(k_j(m)) - C\left(\sum_{j=1}^n k_j(m)\right) - A_{>iC} \quad (4)$$

By combining equations (2) and (4), one sees that the  $i^{\text{th}}$  divisional manager selects  $(m_i^*, e_i^*)$  to maximise the function  $G_i(m_i, e_i)$ , defined as:

$$G_i(m_i, e_i) = \bar{\pi}_i(k_i(m), e_i) + \sum_{j \neq i} m_j(k_j(m)) - C\left(\sum_{j=1}^n k_j(m)\right) - A_{>iC} - g_i(e_i) \quad (5)$$

Note that for any conditional allocation  $k_i$ , the  $i^{\text{th}}$  divisional manager will choose the effort level  $e_i^*(k_i)$  defined by:

$$e_i^*(k_i) = \operatorname{argmax} \{\bar{\pi}_i(k_i, e_i) - g_i(e_i)\} \quad (6)$$

given the form of equation (5). This equation represents the idea that divisional managers can choose message strategies with the knowledge that they can optimally adjust for effort after the central headquarters choose a  $k_i$  value.

LEMMA 1. Under the Groves' and Loeb mechanism in an environment characterised by effort averse divisional managers, there exists a best message for the  $i^{\text{th}}$  divisional manager, irrespective of the messages other divisional managers choose to communicate and the effort levels the others choose to provide. The specific form of the dominant strategy for the  $i^{\text{th}}$  divisional manager is to report:

$$\begin{aligned} m_i^*(k_i) &= \max_{e_i} \bar{\pi}_i(k_i, e_i) - g_i(e_i) \\ &= \bar{\pi}_i(k_i, e_i^*(k_i)) - g_i(e_i^*(k_i)) \end{aligned} \quad (7)$$

for any  $(n-1)$  tuple of strategies of other divisional managers.

PROOF: the  $i^{\text{th}}$  divisional manager's problem is to solve

$$\begin{aligned} &\max_{m_i, e_i} \bar{\pi}_i(k_i(m), e_i) + \sum_{j \neq i} m_j(k_j(m)) - C \left( \sum_{j=1}^n k_j(m) \right) - \\ &A_{>i} - g_i(e_i) \\ &= \max_{m_i} \bar{\pi}_i(k_i(m), e_i^*(k_i(m))) + \sum_{j \neq i} m_j(k_j(m)) \\ &- C \left( \sum_{j \neq i} k_j(m) \right) - A_{>i} - g_i(e_i^*(k_i(m))) \end{aligned} \quad (8)$$

By equation (7)

$$\begin{aligned}
 & \bar{\pi}_i(k_i(m_{>ic}, m_i^*), e_i^*(k_i(m_{>ic}, m_i^*))) + \sum_{j \neq i} m_j(k_j(m_{>ic}, m_i^*)) \\
 & - C\left(\sum_{j=1}^n k_j(m_{>ic}, m_i^*)\right) - A_{>ic} - g_i(e_i^*(k_i(m_{>ic}, m_i^*))) \\
 & = m_i^*(k_i(m_{>ic}, m_i^*)) + \sum_{j \neq i} m_j(k_j(m_{>ic}, m_i^*)) \\
 & - C\left(\sum_{j=1}^n k_j(m_{>ic}, m_i^*)\right) - A_{>ic} \\
 & \geq m_i^*(k_i) + \sum_{j \neq i} m_j(k_j) - C\left(\sum_{j=1}^n k_j\right) - A_{>ic} \tag{9}
 \end{aligned}$$

for all  $k$ .

Thus we have:

$$\begin{aligned}
 & m_i^*(k_i(m_{>ic}, m_i^*)) + \sum_{j \neq i} m_j(k_j(m_{>ic}, m_i^*)) \\
 & - C\left(\sum_{j=1}^n k_j(m_{>ic}, m_i^*)\right) - A_{>ic} \\
 & \geq m_i^*(k_i(m)) + \sum_{j \neq i} m_j(k_j(m)) \\
 & - C\left(\sum_{j=1}^n k_j(m)\right) - A_{>ic}
 \end{aligned}$$

for all  $m_i \in M_i$

Hence,  $m_i^*$  maximises the maximand of (8).

Thus, when the central headquarters is trying to solve the allocation problem, given the form of the messages from divisional managers, the central headquarters will actually be trying to determine the solution  $t_{\alpha}^-$ :

$$\max_{e, k} \sum_{i=1}^n \bar{\pi}_i(k_i, e_i) - C\left(\sum_{j=1}^n k_j\right) - \sum_{i=1}^n g_i(e_i) \quad (10)$$

Thus, although the central headquarters selects allocations to maximise a measure of total corporate profits gross of rewards  $R_i$ , it can be seen that in equilibrium the headquarters can be sure that the divisional managers have reported truthfully as the central headquarters has allowed for divisional managers experiencing disutility from the provision of effort.

The above suggested amendment to the original Groves' and Loeb mechanism by Cohen and Loeb does seem, at first, to deal with the problems that arise when it is recognised that divisional managers are effort averse. However, closer consideration should be given to the implied objective function which the central headquarters is adopting. The objective function is given by equation (10):

$$\max_{e, k} \sum_{i=1}^n \bar{\pi}_i(k_i, e_i) - C\left(\sum_{j=1}^n k_j\right) - \sum_{i=1}^n g_i(e_i)$$

Using equation (2) this can be rewritten as:

$$\max_{(e,k)} \sum_{i=1}^n \bar{\pi}_i(k_i, e_i) - C\left(\sum_{j=1}^n k_j\right) + \sum_{i=1}^n U_i(R_i, e_i) - \sum_{i=1}^n R_i$$

However, the objective function of central headquarters is much more likely to be:

$$\max_{(e,k)} \sum_{i=1}^n \bar{\pi}_i(k_i, e_i) - C\left(\sum_{j=1}^n k_j\right) - \sum_{i=1}^n R_i$$

That is maximisation of the central headquarters residual claim to corporate profits, after compensating divisional management. Thus, the Cohen and Loeb amendment does not lead to a resolution of the moral hazard-adverse selection problem that arises in decentralised organisations, as it does not take correct account of the central headquarter's preferences. It appears that Cohen and Loeb (1984, page 21) are aware of this, as they indicate that they only "present the model .....as a partial solution to the problem of moral hazard and resource allocation".

#### 4.5 Evaluation of Research Position

In the above analysis, attention is restricted to the possibility of designing transfer pricing procedures in which divisional managers have dominant strategies which require truthful communication. It is shown above that the only procedures guaranteeing this dominance are of the Groves and Loeb type, provided there are no moral hazard

problems. When moral hazard issues are present, it is shown that the Groves and Loeb procedure and suggested modification by Cohen and Loeb is unsatisfactory.

In Chapter 6, an alternative transfer pricing procedure is therefore designed. There it is argued that the transfer pricing problem is best viewed as a multi-player game with incomplete information. Instead of trying to design a procedure which has dominant truth telling strategies as the equilibrium of the communication game, a procedure which has Bayesian truth telling strategies as the equilibrium will be designed. In order to introduce these new concepts Chapter 5, therefore, presents an overview of games with incomplete information and Bayesian equilibria.

However, care must be taken at this point. In Chapters 3 and 4 it is argued that one need not analyse all transfer pricing procedures to see if untruthful communication can arise. This is because the Green and Laffont Characterisation Theorem establishes that only Groves and Loeb type procedures guarantee truthfulness when moral hazard issues are not considered. However, it is now argued above that the Groves and Loeb type procedures are undesirable and that research will be conducted with a new concept of guaranteed truthfulness, that of a Bayesian "truthful" equilibrium. Thus, it will now be assumed that, unless a transfer pricing procedure has a Bayesian equilibrium in which all divisional managers choose to report truthfully (even when moral hazard issues arise), that untruthful communication can arise.



NOTES

1. Note that while the division is required to communicate information about some variable, this information need not be correct, if it is not in the interest of the division to communicate truthfully.
2. Non playing in the sense that the centre does not choose a strategy to play.
3. Here "strength" refers to the immunity of the mechanism to manipulation. There are other possible equilibrium concepts, such as the Bayesian equilibrium, but these other solution concepts will not be considered until later when, for instance, incompleteness of information is formally modelled.
4. Additional desirable aspects of the Strong equilibrium will be discussed later in this chapter.
5. The idea here is to establish existence conditions for the "weakest" equilibrium concept first and then consider existence issues for other concepts when they are discussed in a transfer pricing setting.
6. Point to set mapping.
7. For proof of the theorem see Friedman (1977).
8. The exact details of a divisional manager's evaluation measure will be established later.
9. That is, the best response of a division is not unique and there are a set of equally preferable best responses.
10. Here  $\Pi_i$  represents realised profit.

11. That is, communicate a message such as a constant so that the division is not allocated any common resources.
12. This notation is somewhat different from that which Green and Laffont use.
13. The Groves' and Loeb transfer pricing mechanism is a special case of the general Groves' mechanism. Green and Laffont also assume  $\alpha_i(m_{>ic}) = 1$ , which need not always be the case.
14. Note the net gain may not always be positive depending on d.d.2's lying and the form of the inverse supply schedules.
15. Corollary of note 14.
16. That is  $IMF > IHG$  in Figure 1.
17. Theorems 1 and 3 in Miller and Murrel (1981).
18. In addition, it is assumed that effort cannot take on non-positive values.
19. See page 168 for instance.
20. Where other players' strategic options are irrelevant.

GAMES WITH INCOMPLETE INFORMATION5.1 Introduction

When discussing the Groves and Loeb mechanism, it was noted that in a decentralised allocation game, the divisions have incomplete information about the other divisions environmental conditions. Also the centre has incomplete information about the environmental conditions of the divisions. However, an important feature of the dominant strategy mechanisms of Groves and Loeb was that the dominance property was such that divisions could ignore the strategies of other divisions and, hence, did not need to take account of the incompleteness of their information. In fact, this property has led some authors<sup>1</sup> to describe the divisional decision making process in the Groves and Loeb mechanism as decision making under complete ignorance. Here the ignorance, of course, refers to the knowledge a division has of another division and not to a division's knowledge of its own private decision position.

It has been argued in the previous chapter that the Groves and Loeb transfer pricing mechanism<sup>2</sup> has some inadequacies. It will be shown in the next chapter that some of these inadequacies can be dealt with by defining new allocation rules (transfer pricing procedures) that take account of the incomplete informational knowledge of divisions and the centre. This chapter will present a discussion of the theory of games with incomplete information proposed by Harsanyi.<sup>3</sup>

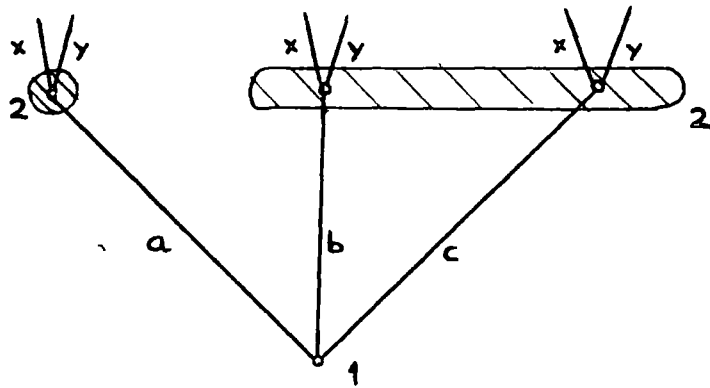
## 5.2 Games with Incomplete Information Played by Bayesian Players

Games of incomplete information arise when some, or all, of the players lack full information about the 'rules' of the allocation game. This could occur when a player has incomplete information about other players payoff functions and available strategies and/or the information other players have about aspects of the game. The incompleteness may also refer to the lack of knowledge players may have of their own payoff functions and/or available strategies.

An analytical difficulty arises when one wants to consider how players will form their expectations about other players and formulate strategies when they have incomplete information. The difficulty is that the incompleteness of information gives rise to an infinite regress in reciprocal expectations on the part of players.<sup>4</sup> However, it is possible to overcome this difficulty if it is possible to show that the game of incomplete information is game theoretically equivalent to a game of complete information, since one can then analyse the equivalent game of complete information in which, under suitable regularity conditions, a stable set of expectations will exist.

Harsanyi follows this approach and shows that with a game of incomplete information, it is possible to construct an equivalent game of complete but imperfect information. A game of (complete and) perfect information occurs when all players know all previous moves, including chance moves, whereas a game of (complete and) imperfect information occurs when a player is ignorant of some past move, or set of moves. For example, poker is a game of (complete and) imperfect

information because the first round of cards is dealt face down and a player does not know the very first move of the game (the cards dealt to his opponents). However, the player can calculate the chance of an opponent being dealt a particular card, hence it is a game of complete information. An alternative presentation of the concept of imperfect information is to use a game tree and consider players information sets. Suppose player 1 moves first and has three possible strategies available. After player 1 moves, player 2 must choose to adopt one of two alternative strategies. Suppose the game tree is as illustrated below.



The set of nodes indexed by player 2's index is sub-partitioned into two sets containing one and two nodes, respectively. The elements of this sub-partition are called the "information sets" for player 2 and indicate the information available to him at his move. If player 1 chose alternative a, then by circling the node at the end of arc a above, one is indicating that player 2 knows unambiguously what player 1 did on his previous move. However, if player 1 chooses alternative b or c, then by jointly circling the nodes at the ends of these arcs, we indicate that player 2 cannot distinguish

whether  $b$  or  $c$  was chosen. He cannot differentiate which node in that information set he is at and, therefore, has imperfect information when it is his turn to move.

It is possible to always represent the problem of incomplete knowledge about the normal form of the game as incomplete knowledge by players, of the payoff functions.

For instance, the assumption that some strategy  $m_i$  is not available to player  $i$  is equivalent game theoretically to the assumption that player  $i$  will never actually use  $m_i$  because it has an extremely low payoff associated with it. Therefore, provided one defines player  $i$ 's strategy space  $M_i$  to include all those strategies other players think  $i$  has available, as well as all those strategies  $i$  knows are available, one can assume that this set  $M_i$  is known to all players, since any lack of information on the part of some player  $j$  about  $M_i$  can be represented as a lack of information about the numerical values of player  $i$ 's payoff function. Similarly, incomplete knowledge about the possible physical outcomes of the game is game theoretically equivalent to incomplete knowledge about strategies of opponents if strategies are suitably defined. It is now possible to consider the appropriate form of a game of incomplete information (about payoff functions).

#### 5.2.1 The Appropriate Game Form for Games with Incomplete Information

A game of complete information in extensive form consists of:

(i) a game tree, each move in which is assigned to one, and only one, of the players  $i \in \{\xi, 1, \dots, n\}$ , where  $\xi$  represents the possibility of chance moves (by nature).

(ii) an assignment of each of player  $i$ 's moves to some information set  $C_{ij}$  of  $i$ , for each  $i \in \{\xi, 1, \dots, n\}$ .

(iii) a description of the consequence of each possible full history of the game and an assignment of that consequence to the corresponding end point.

(iv) an assessment by each player  $i \in \{1, \dots, n\}$  of his utility function  $U_i: E \rightarrow R^1$  on the set  $E$  of all potential consequences.

(v) an assessment by each player  $i \in \{1, \dots, n\}$  of his probability function on each of  $\xi$ 's information sets  $C_{\xi j}$ .

(vi) full knowledge of (I) - (V) on the part of each player  $\{1, \dots, n\}$ .

Games in extensive form are somewhat cumbersome for the purposes of strategic analysis. However, they can be reformulated more compactly as games in normal form<sup>5</sup>. Essentially, what the normal form does is to define a complete pure strategy set  $\{m_{i1}, m_{i2}, \dots, m_{iJ(i)}\}$ , consisting of one choice  $m_{ij}$  from each of  $i$ 's information sets  $C_{ij}$  when the number of information sets  $i$  faces totals to  $J(i)$ . However, the implicit assumption with normal form games is that players simultaneously choose their (complete) strategies before the game is commenced. To express this in another way, we are assuming that players will choose their strategies which determine the outcome before chance<sup>6</sup>, determines at which information set they are located. Harsanyi gives some examples which show that the use of normal form games, in games of incomplete information give rise to counter intuitive solutions to the games. One of these examples will now be presented.

Let us consider a two person, zero sum game where each player may belong to one of two possible attribute classes. It is the uncertainty which a player has about the other players type of attribute class that conceptualizes incompleteness of information in the game. Let the possible attribute classes for player 1 be denoted  $a_1$  and  $a_2$  and those of player 2 be denoted  $b_1$  and  $b_2$ . Assume that the joint probability distribution of attribute classes for the game is:

|       |       |       |
|-------|-------|-------|
|       | $b_1$ | $b_2$ |
| $a_1$ | .01   | .00   |
| $a_2$ | .09   | .90   |

Player 1 will now be able to use the following conditional probabilities (subjective probabilities) once he or she finds out the attribute class they are.

|              |       |       |
|--------------|-------|-------|
|              | $b_1$ | $b_2$ |
| $p(b_j a_1)$ | 1.00  | .00   |
| $p(b_j a_2)$ | .09   | .91   |

where  $j = 1, 2$ .

Similarly for player 2

|       |              |              |
|-------|--------------|--------------|
|       | $p(a_j b_1)$ | $p(a_j b_2)$ |
| $a_1$ | .10          | .00          |
| $a_2$ | .90          | 1.00         |



Also assume that the **four** possible combinations of attribute classes and payoffs to player 1 will be:

|       | $z_1$ | $z_2$ |
|-------|-------|-------|
| $y_1$ | (2)   | 5     |
| $y_2$ | -1    | 20    |

In case  $(a_1, b_1)$  where  $z_1, z_2$  are player 2's two possible pure strategies and  $y_1, y_2$  are player 1's two possible pure strategies.

|       | $z_1$ | $z_2$ |
|-------|-------|-------|
| $y_1$ | -24   | -36   |
| $y_2$ | (0)   | 24    |

In case  $(a_1, b_2)$

|       | $z_1$ | $z_2$ |
|-------|-------|-------|
| $y_1$ | 28    | (15)  |
| $y_2$ | 40    | 4     |

In case  $(a_2, b_1)$

|       | $z_1$ | $z_2$ |
|-------|-------|-------|
| $y_1$ | (12)  | 20    |
| $y_2$ | 2     | 13    |

In case  $(a_2, b_2)$

where the circled element in each payoff box represents the equilibrium in each game where each player knows their own and their opponent's attribute class with certainty<sup>8</sup>.

In the game of incomplete information where player 1 is unsure of player 2's attribute class (and where player 1 formulates his or her optimal strategy before finding out what attribute class he or she belongs to during any one play of the game), the normal form of the game is:

|          | $z^{11}$ | $z^{12}$ | $z^{21}$ | $z^{22}$ |
|----------|----------|----------|----------|----------|
| $y^{11}$ | 13.34    | 20.54    | 11.20    | 19.4     |
| $y^{12}$ | 5.42     | 15.32    | 2.21     | 12.11    |
| $y^{21}$ | 13.31    | 20.51    | 12.35    | 19.55    |
| $y^{22}$ | 5.39     | 15.29    | 2.36     | 12.25    |

where the normalized strategy for player 1 are of the form  $s_1^* = y_{n,t} = (y_n, y_t)$  where  $s_1 = y_n, (n = 1, 2)$  is the ordinary pure strategy player 1 will choose if the players' attribute class is  $a_1$ , whereas  $s_1 = y_t (t = 1, 2)$  is the ordinary pure strategy the player would choose if his or her attribute class were  $a_2$ . A similar argument can be presented for player 2's strategies.

Hence, the strategy pair  $(y^{12}, z^{11})$  implies that:

player 1 adopts strategy  $y_1$  when the players' attribute class is  $a_1$

player 1 adopts strategy  $y_2$  when the players' attribute class is  $a_2$

player 2 adopts strategy  $z_1$  regardless of what attribute class the player belongs to.

Thus, the total payoff expectations corresponding to this strategy pair is:

$$(.01 \times 2) + (0.00 \times -24) + (.09 \times 40) + (.90 \times 2) = 5.42$$

The only equilibrium point of this game is  $(y^{21}, z^{21})$ .

Thus player 1's optimal (normalized) strategy is to:

play  $y_2$  when class  $a_1$

play  $y_1$  when class  $a_2$

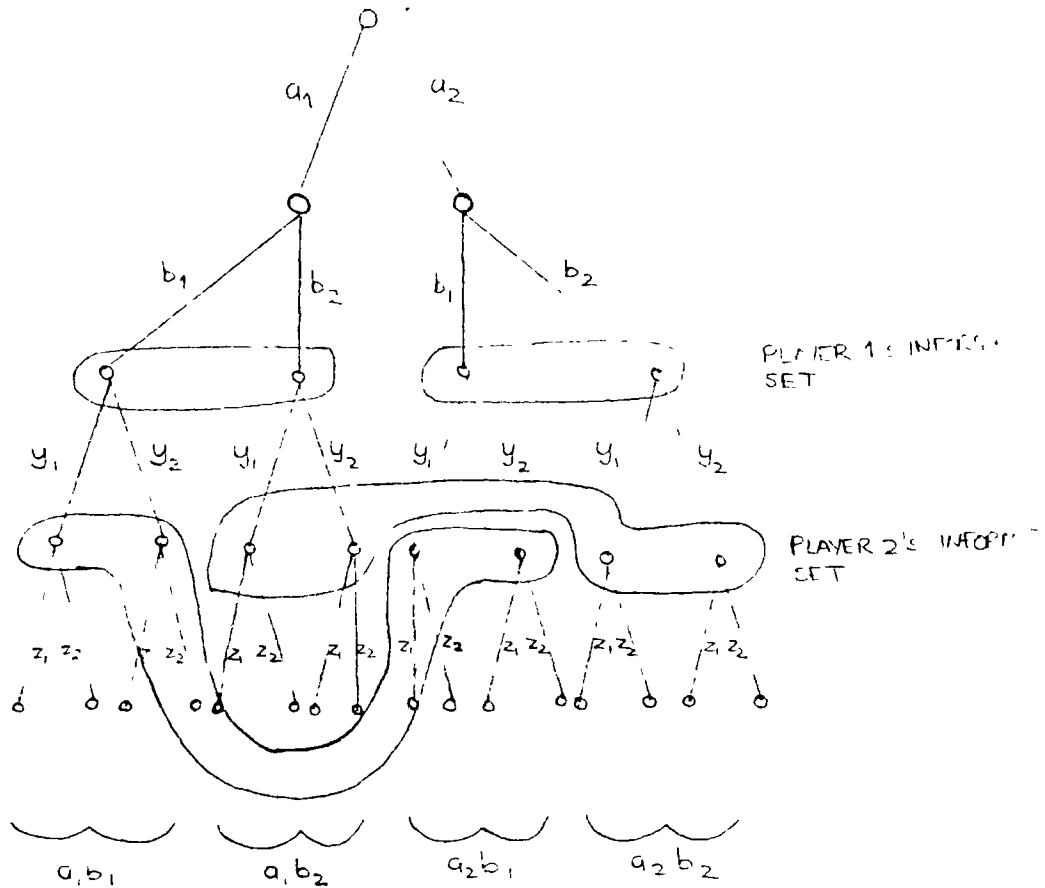
and player 2's optimal (normalized) strategy is to:

play  $z_2$  when class  $b_1$

play  $z_1$  when class  $b_2$

Suppose, however, player 1 actually belongs to attribute class  $a_1$ . Given this information, player 1 will be able to infer (by conditional probabilities) that player 2 must belong to attribute class  $b_1$ . In addition, since player 1 knows the conditional probabilities which player 2 believes, player 1 will be able to conclude that player 2 is assigning a near unity probability (.90) to the mistaken hypothesis that player 1 belongs to class  $a_2$ . Thus, player 1 realizes that player 2 is expecting the constituent game  $(a_2, b_1)$  to apply. Thus, player 1 will expect player 2 to adopt strategy  $z_2$ . If case  $(a_2, b_1)$  did represent the actual situation then player 1's best reply to strategy  $z_2$  would be strategy  $y_1$ . However, here we are assuming that player 1 knows that the actual constituent game being played is  $(a_1, b_1)$ . Hence, player 1's best reply to strategy  $z_2$  is  $y_2$  not  $y_1$ . Thus, by choosing  $y_2$ , player 1 will be able to exploit player 2's mistaken

belief that player 1 probably belongs to attribute class  $a_2$ . The extensive form of the game where player 1's attribute class is known only by player 1 and player 2's attribute class is known only by player 2 can be represented as:



The above example illustrated how the normal form of game obscures the difficulties involved in the updating of subjective (conditional) probabilities when players learn of some information, such as which attribute class they will belong to, before a game commences. It would, therefore seem more appropriate to explicitly recognise that games of incomplete information are games characterized by delayed commitment, where a

player will (want to) wait until he finds out at which information set he has located before choosing a strategy. Therefore, Harsanyi chose to formulate games of incomplete information in the standard form. The standard form is an intermediate form between the extensive form and the normal form. The standard form shows, therefore, how strategies are made up of choices at information sets. The standard form can be derived from the extensive form by assuming that each information set of player  $i$  is administered by a separate agent, indexed  $ij$ . Therefore, the standard form does not only have a player set  $N$ , but also an agent set, or type set  $C_i$  for every player  $i \in N$ . Player  $i$ 's agent set  $C_i$  is assumed to be a non-empty finite set of pairs of integers of the form  $ij$ . The union of all  $C_i$  with  $i \in N$  is denoted  $C$ .

Each agent  $ij$  (at information set  $C_{ij}$ ) has a non-empty finite strategy set,  $M_{ij}$  of available strategies  $m_{ij}$ . A pure strategy  $m_i$  of player  $i$  may, therefore, be thought of as a collection of strategies for his agents.

$$(1) \quad m_i = (m_{ij})_{C_i}$$

The lower index  $C_i$  indicates that  $m_i$  contains one element  $m_{ij}$  for every  $ij \in C_i$ . That is, player  $i$ 's pure strategy set  $M_i$  is the set of all these collections.

$$(2) \quad M_i = \prod_{ij \in C_i} M_{ij}$$

A pure strategy combination is a collection of pure strategies  $m = (m_i)_N$  containing one pure strategy for each player  $i \in N$ .

Alternatively, one could view a pure strategy combination as a collection of strategies  $m = (m_{ij})_C$  containing one strategy  $m_{ij} \in M_{ij}$  for each agent  $ij \in C$ . There is no need to make a distinction between  $(m_i)_N$  and  $(m_{ij})_C$  if both prescribe the same strategies to the agents in  $C$ . The pure strategy combination  $M$  is the set of all the collections  $m$ :

$$(3) \quad M = \prod_{i \in N} M_i = \prod_{ij \in C} M_{ij}$$

Payoffs are defined only for players and not agents. A payoff function  $E$  on  $M$  assigns a payoff vector

$$E(m) = \{E_i(m)\}_N \quad \text{to each } m \in M$$

Once again, the lower index  $N$  indicates that  $E(m)$  contains one component  $E_i(m)$  for every  $i \in N$ .

One can view  $M$  as a structure endowed with all the information on the sets  $N$ ,  $C_i$  and  $M_{ij}$ .  $M$  is said to be admissible if all these sets are finite and non-empty. We are now in a position to give a formal definition of a game in standard form.

Standard form: A game in standard form  $G = (M, E)$  consists of an admissible set of pure strategy combinations  $M$  with the structure indicated by equation (3), together with a payoff function  $E$  on  $M$ .

Normal form: The normal form of a standard form game  $G = (M, E)$  has the same structure as  $G$ , except that the information on the internal structure of the pure strategy sets given by equation (2) is suppressed. Notationally then, one need not make any distinction between a standard form and its normal form.

## 5.2.2

The standard form of a game with incomplete information

Let the set of vectors  $a_{0i}, a_{1i}, \dots, a_{ii}, \dots, a_{ni}$  take the following interpretation. The first component is the vector of parameters of the payoff function  $E_i$  of player  $i$ , which player  $j$  views as unknown to all players, whereas the other vectors represent the parameters of  $E_i$  which, in player  $j$ 's opinion, are unknown to some of the players but known to the players indexed by the first subscript to  $a$ .

The vector that, in player  $j$ 's opinion, represents the knowledge player  $k$  has of the other players' payoff functions  $E_1, \dots, E_n$ , excluding that information that player  $j$  thinks all  $n$  players share, can therefore be denoted as  $a_k = (a_{k1}, \dots, a_{kn})$  for  $k = 0, 1, \dots, i, \dots, n$ .

Here the vector  $a_0$  summarises the information that player  $j$  believes none of the players have about the payoff functions  $E_1, \dots, E_n$ .

It is, therefore, possible to express what, in player  $j$ 's opinion, is incompletely known by some or all players about the payoff functions  $E_1, \dots, E_n$  as being represented by the vector  $a = (a_1, \dots, a_n)$ .

Now define a function  $V_i^*$  whose mathematical form is known to all players such that

$$(1) \quad E_i(m_1, \dots, m_n) = V_i^*(m_1, \dots, m_n, a_0, a)$$

$$\text{also let } a_{>i} = (a_1, \dots, a_{i-1}, a_{i+1}, \dots, a_n)$$

This means it is possible to express incomplete knowledge about the form of the original payoff function in a game theoretically equivalent form, where the form of the new payoff function is known to all, and incomplete knowledge is represented by the vectors  $a_0, a$ .

For any given player  $i$ , the  $n$  component vectors of  $a_0, a_{>ic}$  will represent unknown variables, therefore assuming the players are Bayesian, they will assign a subjective joint probability distribution to the unknown vectors. Player  $i$ , however, will have incomplete knowledge about the subjective probability distributions that the other players adopt. It is possible to deal with this problem in the same manner as incomplete knowledge of the form of payoff functions was dealt with. That is one can define a set of vectors  $b_{>ic} = (b_1, \dots, b_{i-1}, b_{i+1}, \dots, b_n)$  to denote the parameter vectors of the subjective probability distributions that the other players use. The subjective probability distributions that the other players use can be denoted

$$P_{>ic} = P_1, \dots, P_{i-1}, P_{i+1}, \dots, P_N$$

Therefore, it is possible to redefine the subjective probability distribution that  $i$  has over its unknown variables  $a_0, a_{>ic}$  and  $b_{>ic}$  as follows:

$$(2) P_i(a_0, a_{>ic}; b_{>ic}) = R_i(a_0, a_{>ic}, b_{>ic} | b_i)$$

where  $R_i$ , unlike  $P_i$  is a function whose mathematical form is known to all  $n$  players. Here  $b_i$  is the vector consisting of those parameters of the function  $P_i$ , which in player  $j$ 's opinion, are unknown to some or all of the other players besides  $i$ . That is, the function  $R_i$  is a function yielding for each specific value of the vector  $b_i$  (which is known to  $i$ ) a probability distribution over the vector range space  $A_0 \times A_{>ic} \times B_{>ic}$ .



It is possible to eliminate the vector  $a_0$ , unknown to all players from equations one and two. For equation one expected values are taken with respect to  $a_0$  in terms of player  $i$ 's own subjective probability distribution. That is one can define

$$(3) \quad V_i(m_1, \dots, m_n, a, b_i) \\ = \int_{A_0} V_i^*(m_1, \dots, m_n, a_0, a) d(a_0) R_i(a_0, a_{>ic}, b_{>ic} | b_i)$$

so that we can express the expected value of player  $i$ 's payoff in terms of his own subjective probability distribution as:

$$(4) \quad x_i = V_i(m_1, \dots, m_n, a, b_i)$$

For equation (2), it is possible to eliminate  $a_0$  by taking the appropriate marginal probability distribution.

So define

$$(5) \quad P_i(a_{>ic}, b_{>ic}) = \int_{A_0} d(a_0) P_i(a_0, a_{>ic}, b_{>ic})$$

and define

$$(6) \quad R_i(a_{>ic}, b_{>ic} | b_i) = \int_{A_0} d(a_0) R_i(a_0, a_{>ic}, b_{>ic} | b_i)$$

thus

$$(7) \quad P_i(a_{>ic}, b_{>ic}) = R_i(a_{>ic}, b_{>ic} | b_i)$$

For reasons that will become apparent shortly when we define a vector  $c$ , it will be helpful to re-express equation (4) as

$$(8) \quad x_i = V_i(m_1, \dots, m_n, a, b_i, b_{>ic}) = V_i(m_1, \dots, m_n, a, b)$$

(where the vector  $b_{>ic}$  occurs only vacuously), and re-express equation (7) as

$$(9) \quad P_i(a_{>ic}, b_{>ic}) = R_i(a_{>ic}, b_{>ic} | a_i, b_i)$$

(where on the right hand side vector  $a_i$  occurs only vacuously).

It is possible to simplify the overall notation now that the concepts have been developed. Let us define

$$c_i = (a_i, b_i)$$

$$c = (a, b)$$

$$c_{>ic} = (a_{>ic}, b_{>ic})$$

where  $c_i$  represents the total information available to player  $i$  in the game, if one disregards public information, which is information available to all  $n$  players. The vector  $c_i$  is sometimes referred to as player  $i$ 's information vector or, alternatively, player  $i$ 's attribute vector or player  $i$ 's type. Clearly  $c_i$  summarises some crucial parameters of player  $i$ 's own payoff function  $E_i$  as well as the main parameters of his beliefs. So in the model players' incomplete information about the true nature of the game situation can be represented by the assumption that in general the actual value of a player's attribute vector will be known only to the player himself and will not be known by any of the other players.

This means it is possible to write equations (8) and (9) respectively in more concise form as

$$(10) \quad x_i = V_i(m_1, \dots, m_n, c) = V_i(m_1, \dots, m_n, c_1, \dots, c_n)$$

$$(11) \quad P_i(c_{>ic}) = R_i(c_{>ic} | c_i)$$

OR

$$(12) \quad P_i(c_1, \dots, c_{i-1}, c_{i+1}, \dots, c_n) = R_i(c_1, \dots, c_{i-1}, c_{i+1}, \dots, c_n | c_i)$$

Equations (10) and (12) can be regarded as the standard form equations defining the particular game of incomplete information considered from the point of view of some player  $j$ .

It is possible to define the standard form of the game of incomplete information faced by player  $j$ , therefore, by the ordered set  $G$ , such that

$$(13) G = (M_1, \dots, M_N; C_1, \dots, C_N; V_1, \dots, V_N, R_1, \dots, R_N)$$

where for  $i = 1, \dots, n$  we<sup>9</sup> write  $M_i = \{m_i\}$

$$C_i = \{c_i\}$$

where  $V_i$  is a function from the set  $M_1 \times \dots \times M_N \times C_1 \times \dots \times C_N$  to player  $i$ 's utility line  $\mathbb{H}_i$  (which is a copy of the real line) and where for any value of  $c_i$  the function  $R_i = R_i(c_{-i} | c_i)$  is a probability distribution over the set.

$$c_{-i} = C_1 \times \dots \times C_{i-1} \times C_{i+1} \times \dots \times C_N$$

### 5.2.3 The Bayesian Equivalent Game

A game of complete information  $G^*$  game theoretically equivalent to the game of incomplete information  $G$  would be a game  $G^*$  with the same payoff functions  $V_i$  and the same strategy spaces  $M_i$ . The vectors  $c_i$  would now be interpreted as random vectors (chance moves) with the objective joint probability distribution  $R^* = R^*(c_1, \dots, c_n) = R^*(c)$  known to all  $n$  players.

Therefore, it is possible to define the Bayesian equivalent game of complete information hereafter only denoted as  $G^*$  as

$$G^* = \{M_1, \dots, M_N, C_1, \dots, C_N, V_1, \dots, V_N, R^*\}$$

Thus the ordered set  $G^*$  differs from the ordered set  $G$  only in the fact that the  $n$ -type  $R_1, \dots, R_N$  occurring in  $G$  is replaced in  $G^*$  by the singleton  $R^*$ . Therefore, in order to obtain equivalence between the two games, it is necessary that each player  $i$  should assign the same numerical probability to any given specific event. This, in turn, requires equivalence between the subjective probability distribution  $R_i(C_{>ic} | C_i)$  and the objective conditional probability distribution  $R^*(C_{>ic} | C_i)$  generated by  $R^*(C)$ . That is, in order that the two games be Bayes-equivalent for player  $j$

1. The two games must have the same strategy range spaces  $M_1, \dots, M_N$  and the same types range spaces  $C_1, \dots, C_N$ .
2. They must have the same payoff functions  $V_1, \dots, V_N$ .
3. Also, the subjective probability distribution  $R_i$  of each player in  $G$  must satisfy the following relationships.

$$(14) \quad R_i(C_{>ic} | C_i) = R^*(C_{>ic} | C_i)$$

$$\text{where } R^*(C) = R^*(C_{>ic}, C_i)$$

then

$$(15) \quad R^*(C_{>ic} | C_i) = \frac{R^*(C_{>ic}, C_i)}{\int_{C_{>ic}} d_{(C_{>ic})} R^*(C_{>ic}, C_i)}$$

given (14) and (15) we also have

$$(16) R^*(C) = R^*(C_{>i|c}, C_i) = R_i(C_{>i|c} | C_1) \cdot \int_{C_{>i|c}} d_{(C_{>i|c})} R^*(C_{>i|c}, C_1)$$

Therefore, the question of whether there exists the Bayesian equivalent game of complete information  $G^*$  depends on whether the subjective probability distributions  $R_1(C_{>1|c} | C_1), \dots, R_n(C_{>n|c} | C_n)$  are, in effect, derived from an underlying probability distribution  $R^*(C_1, \dots, C_n)$  which satisfies equation (16).

The essential argument for defending such an existence is that one may wish to argue that any differences in the subjective probabilities that players hold are due exclusively to differences in information. That is, any difference between the subjective probabilities that players entertain must be due to the knowledge of the outcome of the chance event which determines their type  $C_i$ , in the model presented here. More generally, it is assumed that people who have always received precisely the same information would have no rational reason for maintaining different subjective probabilities. However, Aumann<sup>11</sup> argues that, because of psychological factors, it may seem that, "reconciling subjective probabilities makes sense if it is a question of implicitly exchanging information, but not if we are talking about 'innate' differences in priors". He is arguing that the information difference argument may not be universally valid because of inborn psychological differences between how people calculate and perceive things or how they wish to calculate and perceive.

However, if instead of considering people in general as the players, we restrict consideration to the set of divisional managers, these inborn psychological differences may be presumed to be very small differences. The reason this may be the case is because, in order to attain the position of manager, potential candidates will have had to have been successful in selection trials. For example, those candidates chosen to be managers may be those candidates whose psychological attributes are consistent with those of company policy. For instance, those candidates who are "dynamic and assertive" will have the best chance of being chosen to be managers by a company who want dynamic and assertive managers. In other words, what I am arguing is that managers are partly chosen for their psychological attributes, so one would expect that the "innate differences" that Aumann talks of would be small. To illustrate this by a general example, if it were the case that the weightlessness sickness that some potential astronauts could encounter, were totally due to psychological reasons, then it is likely that the vulnerable potential astronauts would have been detected in tests.

Harsanyi describes the beliefs of the players as mutually consistent if there does exist a basic probability distribution  $R^*(C_1 \dots C_n)$  from which the players' subjective probability distributions can be derived as conditional probability distributions,  $R_1(C_{2c} | C_1), \dots, R_n(C_{nc} | C_n)$ . Therefore, in trying to estimate the probability distribution  $R^*(C_1 \dots C_n)$ , player  $j$

should try and use the information common to all  $n$  players, and then make use of any additional information he has about the game situation. Harsanyi argues that, if player  $j$  uses this estimation procedure, even though it will guarantee the internal consistency between  $n$  probability distributions  $R_1, \dots, R_n$ , that player  $j$  uses to analyse the game, it may not guarantee external consistency among the probability distributions used by different players, because of the "innate differences" that Aumann talks of. Harsanyi argues, however, that the procedure whereby each player tries to estimate  $R^*$  as if they were an outside observer using that information that was common knowledge goes as far as an estimation procedure can go in promoting external consistency. In addition, if one assumes there has been some pre-selection carried out to determine the players as discussed above, then one might be reasonably confident of external consistency.

In the case where player  $j$  has information that player  $i$  is using a subjective probability distribution inconsistent with his, then this difference may be explained in terms of differences of information that different players have about the process determining the game situation, in which case one could redefine the players' attribute vectors  $C$  to ensure that the  $n$  players are using mutually consistent subjective probability distributions. In other words, player  $j$  must use not only the information he has obtained independently about the game, but also information he can infer from what he knows about the other players' assessment of probabilities.

If he were to assign probability  $P_j(e)$  to event  $e$  and discover information that player  $i$  assigns a very different probability  $P_i(e) \neq P_j(e)$  to the event, then player  $j$  cannot just rest the matter with the conclusion that he and player  $i$  are assessing the probabilities on the basis of two rather different sets of information. Instead he must consider which assessment is likely to be based on more correct and more complete information.

Alternatively, if one wished to conclude that the players' subjective probability distributions were, in fact, inconsistent then, in order to analyse the problem, one must use a model of the underlying Bayesian game suggested by Selten as an alternative to the prior lottery model in which the game of incomplete information is replaced by a Bayesian game of complete information, in which nature first conducts a lottery in accordance with  $R^*(C_1, \dots, C_n)$  to determine which subgame will be played.

Selten's model is called the posterior-lottery model. Suppose that the attribute vector  $C_i$  of player  $i$  can take  $J_i$  different values in the game. Assume that the role of player  $i$  will be played at the same time by  $J_i$  different players, each representing a different value of the attribute vector  $C_i$ . The set  $J_i$  can be called the role class  $i$ . Different individuals in the same role class  $i$  will be distinguished by subscripts as players  $i_1, i_2, \dots, i_{J_i}$ . Therefore, the total number of players in the game will be

$$J = \sum_{i=1}^N J_i$$



Each player  $ij$  from a given role class  $i$  will choose some strategy  $m_i$  from player  $i$ 's strategy space  $M_i$ . After all  $J$  players have chosen their strategies, one player  $ij$  from each role class  $i$  will be randomly selected as an active player. The probability of selecting individuals with any specific combination of attribute vectors  $C_1 = C'_1, \dots, C_n = C'_n$  will be governed by the probability distribution  $R^*(C_1, \dots, C_n)$ .

To model a situation of mutually inconsistent subjective probability distributions, we assume that when all  $J$  players have chosen their strategies, there will be a separate lottery  $L(i_j)$  for every player  $ij$ , instead of there being one grand lottery  $L^*$  for all  $J$  players. For each player  $ij$  in the role class  $i$ , his lottery will choose  $(n-1)$  players as his partners to define the subgame he participates in. If player  $ij$  himself has the attribute vector  $C_i = C'_i$ , then the probability of his  $(n-1)$  partners having any specific combination of attribute vectors  $C_1 = C'_1, \dots, C_{i-1} = C'_{i-1}, C_{i+1} = C'_{i+1}, \dots, C_n = C'_n$  will be governed by the subjective probability distribution  $R_i = R_i(C_{\neq i} | C_i = C'_i)$  which player  $i$  himself entertains.

Under this model, partnership is not necessarily a symmetric relationship, since if lottery  $L(ij)$  of player  $ij$  choose a player  $rj$  as a partner, then it does not follow that lottery  $L(rj)$  of player  $rj$  will likewise choose player  $ij$  as a partner for player  $rj$ .

Another difficulty that may arise is that, if there is an admissible probability distribution  $R^*(C)$  from which the subjective probability distributions can be said to be deriveable from, will this admissible probability distribution be unique. If uniqueness did not hold, then it may be seen as problematic, since this would give rise to the possibility of multiplicity of Bayesian games  $G^*$  Bayes equivalent to a given game of incomplete information. However, Harsanyi argues that one must then determine the decomposed component games of the incomplete game and proves that, for each indecomposable component game  $G^M = G(D^M)$ , every admissible distribution  $R^*(C)$  will generate the same uniquely determined conditional probability distribution.

$R^M = R^M(C) = R^*(C|CED^M)$  whenever this conditional probability distribution  $R^M$  generated by this admissible distribution  $R^*$  is well defined. Here  $D^M$  is the defining cylinder of the game  $G^M$ .

Myerson (1983), however, has analysed the issue in another fashion and has argued that the requirement of consistency of beliefs is not an issue of basic importance when studying general Bayesian games, provided one can rule out the requirement of interpersonal comparability of utility.

The case of interpersonal comparisons of utility can be ruled out, because for games of incomplete information each player will already know their own type before making any decision. Thus, when the game is played, intertype comparisons of utility are decision-theoretically meaningless, since once a

player knows his type, he can not be asked to choose it. For example, it is decision-theoretically meaningless to try and determine whether a player would prefer to be a dynamic (type) manager or a non-dynamic (type) manager, when in the game under consideration the player's type has already been determined and the player knows whether he is a dynamic type or not.

This result means that utility scales of different types can be specified separately. From decision theory, it is well known that von Neumann Morgenstern utility scales can only be defined up to increasing linear transformations. Thus, two Bayesian games with the same decision sets<sup>12</sup> and type sets

$$G = (D_1, \dots, D_N, T_1, \dots, T_N, P_1, \dots, P_n, U_1, \dots, U_n)$$

and

$$\hat{G} = (D_1, \dots, D_N, T_1, \dots, T_N, \hat{P}_1, \dots, \hat{P}_n, \hat{U}_1, \dots, \hat{U}_n)$$

are what Myerson (1983) calls utility equivalent if and only if they have the same conditional probability distributions ( $\hat{P}_i = P_i$  for all  $i$ ) and there exists numbers  $a_i(t_i)$  and  $b_i(t_i)$  for each  $i$  and each  $t_i$  in  $T_i$ , such that

$$a_i(t_i) > 0 \text{ and } \hat{U}_i(d, t) = a_i(t_i)U_i(d, t) + b_i(t_i)$$

for all  $d \in D$  and all  $t \in T$ .

Thus utility equivalent Bayesian games differ only in that the utility functions of some types of some players may be linearly rescaled, and the Bayesian equilibria of the two utility equivalent games will be the same.

In Bayesian games, whenever a player chooses an action or decision, his criterion for the best decision is that it should give him the highest conditionally expected utility, given his actual type. Expected utility is determined by multiplying utilities times probabilities and then summing over all possible values of the unknowns. For example, if some function  $\delta: T \rightarrow D$  determined how the players' decisions depend on their types, then the conditionally expected utility for type  $t_i$  of player  $i$  would be

$$\sum_{t_{-i} \in T_{-i}} P_i(t_{-i} | t_i) U_i(\delta(t), t)$$

Thus, one can define  $z_i: D \times T \rightarrow R$  by

$$z_i(d, t) = P_i(t_{-i} | t_i) U_i(d, t)$$

for all  $d \in D$  and all  $t \in T$

and call  $z_i$  the evaluation function for player  $i$ . Since only the product of probability times utility matters in computing expected utilities, one can say that two Bayesian games are probability-equivalent if and only if they have the same decision sets  $D_i$  and type sets  $T_i$  and evaluation functions  $z_i$  for all players, that is

$$\hat{P}_i(t_{-i} | t_i) \hat{U}_i(d, t) = P_i(t_{-i} | t_i) U_i(d, t)$$

for all  $d \in D$  and all  $t \in T$

Given this result, consistency of beliefs is not an issue of basic importance, because by suitable specification of utility scales, one can ensure that Bayesian games like  $G$  and  $\hat{G}$  will have the same Bayesian equilibria.

Thus, a more general equivalence relation can be defined among Bayesian games. Two Bayesian games  $G$  and  $\hat{G}$  with the same decision sets and type sets are evaluation-equivalent if, and only if, for every player  $i$ , there exist functions

$$a_i: T_i \rightarrow R$$

and  $b_i: T \rightarrow R$  such that

$$a_i(t_i) > 0 \quad \text{every } t_i \text{ and}$$

$$\hat{P}_i(t_{-i} | t_i) \hat{U}_i(d, t) = a_i(t_i) P_i(t_{-i} | t_i) U_i(d, t) + b_i(t)$$

for all  $d \in D$  and all  $t \in T$ .

Note here that the additive constant can depend on all players' types, while the multiplicative constant can only depend on  $i$ 's type. Thus, Bayesian equilibria will be invariant under any evaluation-equivalent transformation of the game.

#### 5.2.4 Bayesian equilibrium strategies

The next issue to consider is when one can derive some Bayesian game  $G^*$  equivalent to the game of incomplete information, will it have an equilibrium point, and if so, what form will the strategies take. The following definitions will always refer to a fixed game in standard form.

A mixed strategy  $q_i$  of player  $i$  is a probability distribution over player  $i$ 's set of pure strategies, where  $q_i(m_i)$  denotes the probability assigned to  $m_i$ . A mixed strategy is said to be completely mixed if  $q_i(m_i)$  is positive for every pure strategy  $m_i \in M_i$ . The set of all mixed strategies of player  $i$  is denoted  $Q_i$ .

A combination  $q = (q_i)_N$  of mixed strategies is called a mixed combination and contains a mixed strategy  $q_i$  for every  $i \in N$ . The set of all combinations of this kind is denoted by  $Q$ . For  $q = (q_i)_N$  and  $m = (m_i)_N$  it is convenient to use the notation

$$q(m) = \prod_{i \in N} q_i(m_i)$$

That is,  $q(m)$  is the product of all  $q_i(m_i)$  with  $i \in N$ . The product  $q(m)$  is called the realisation probability of  $m$  under  $q$ . The definition of the payoff function  $E$  is extended from  $M$  to  $Q$ .

$$E(q) = \sum_{m \in M} q(m)E(m)$$

The equations for  $E$  are vector equations which hold for every component  $E_i$  of  $E$ .

An  $i$ -incomplete combination of mixed strategies is a combination which contains one mixed strategy for every player with the exception of  $i$ . An  $i$ -incomplete combination of mixed strategies is denoted  $q_{>i}$ . Similarly  $M_{>i}$  denotes the set of all  $i$ -incomplete combinations of pure strategies and  $Q_{>i}$  denotes the set of all  $i$ -incomplete mixed combinations. Also let  $q_i q_{>i}$  denote that  $q \in Q$ , containing the components  $q_i$  and  $q_{>i}$ . If for all players, with the exception of player  $i$ , the strategies in  $q_{>i}$  agree with those in  $q$ , we call  $q_{>i}$  prescribed by  $q$ .

Another (related) type of strategy warranting consideration is behavioural strategies. For these we need first to define local strategies. A local strategy  $b_{ij}$  of an agent  $ij$  is a probability distribution over his choice set  $m_{ij}$ . The probability assigned to  $m_{ij} \in M_{ij}$  by  $b_{ij}$  is denoted  $b_{ij}(m_{ij})$ .

The set of all possible local strategies of agent  $i_j$  is denoted  $B_{ij}$ .

Therefore a behavioural strategy for a player  $i$

$$b_i = (b_{ij})_{C_i}$$

is a collection of local strategies containing one for each of player  $i$ 's agents. The set of all possible behavioural strategies of player  $i$  is denoted  $B_i$ . It will be convenient to use the following notation sometimes -

$$b_i(m_i) = \prod_{ij \in C_i} b_{ij}(m_{ij})$$

$$\text{for } b_i = (b_{ij})_{C_i} \text{ and } m_i = (m_{ij})_{C_i}$$

Let  $b_i(m_i)$  be called the realisation probability of  $m_i$  under  $b_i$ . Therefore,  $b_i$  can be looked upon as a special type of mixed strategy and from the realisation probabilities  $b_i(m_i)$ , one can reconstruct the local strategies which make it up. Thus, a behavioural strategy is uniquely determined by its realisation probabilities. Thus, behavioural strategies can be looked upon as a special type of mixed strategy where  $B_i$  is identified with some subset of  $Q_i$ .

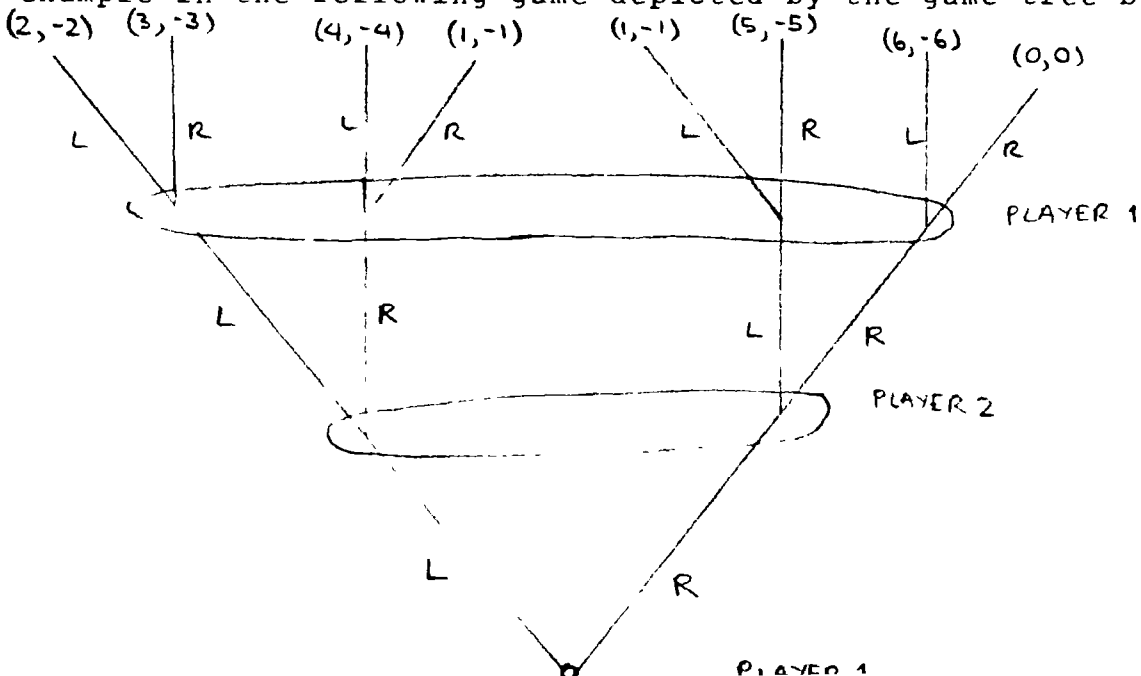
With a mixed strategy, a player must construct all possible pure strategies or plans, assign a probability to each one, and then perform a random experiment with the above determined probability weights to determine his choice of pure strategy to play. The construction of such a strategy, therefore, may involve a great amount of computation and

information storage. Behavioural strategies are much more "informationally efficient" because, unlike a mixed strategy which assigns a probability distribution to each pure strategy, a behavioural strategy simply assigns a probability distribution to each local information set which defines the probability weight that the player will use in deciding upon any one of his alternative actions at that set.

To illustrate the efficiency argument, consider the following example. A player is given a card from a deck of fifty-two cards and is then required to pass or bet a fixed amount. The total number of conceivable pure strategies is  $2^{52}$  since there are fifty-two cards, each giving rise to two alternatives. Since mixed strategies are all points in a unit simplex, having a dimension of one less than the set they are defined over, the set of mixed strategies is of dimension  $2^{52} - 1$ . A behavioural strategy, however, gives the probability of betting with each hand, hence there are only 52 such strategies.

In some games, however, optimal mixed strategies of interest are not realisable by use of behavioural strategies, for

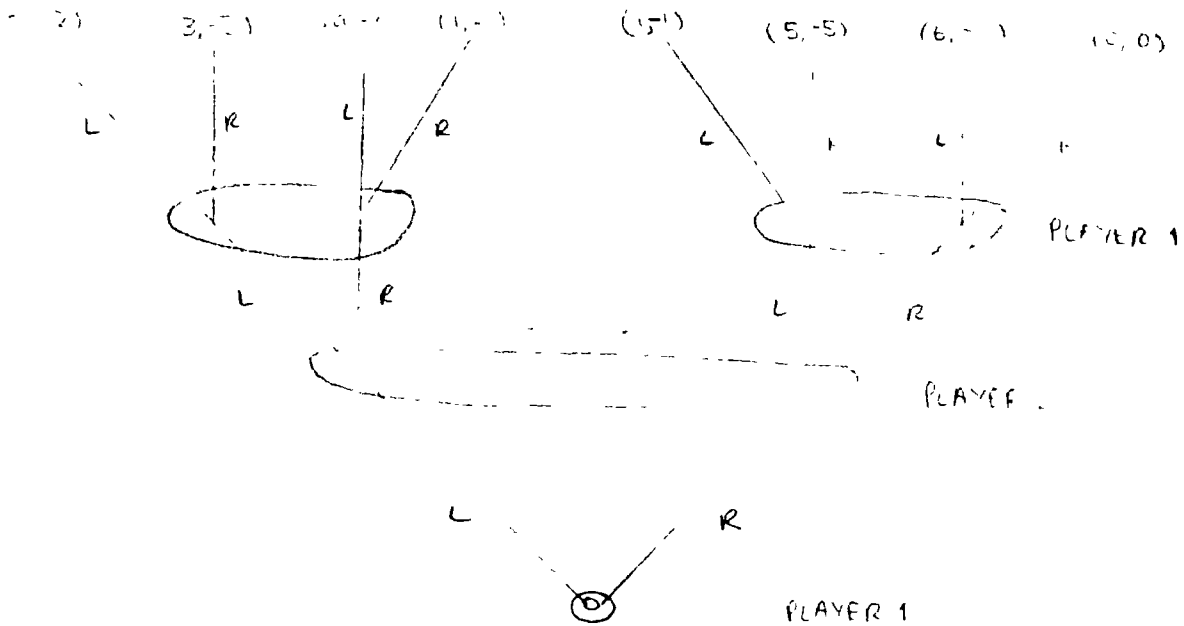
example in the following game depicted by the game tree below.





In this game, player one's pure strategies are LL, LR, RL and RR. A mixed strategy is a four dimensional probability vector  $q = (q_1, q_2, q_3, q_4)$  specifying the probability with which player one will use any of his four pure strategies. A behavioural strategy in this game is a pair of two dimensional probability vectors  $(b_1, 1-b_1), (b_2, 1-b_2)$  where  $b_1$  is the probability of moving left at move one and  $1-b_1$  is the probability of moving right; similarly for  $b_2, 1-b_2$  at move two. Note that for any choice of  $b_1$  and  $b_2$ , a mixed strategy is determined in which the probabilities attached to the pure strategies LL, LR, RL and RR are given as  $(b_1)(b_2), (b_1)(1-b_2), (1-b_1)(b_2),$  and  $((1-b_1)(1-b_2))$ . Since the game is a zero sum game, mini-max strategies will be optimal. In this game, the optimal mixed strategy<sup>13</sup> is  $(0, \frac{5}{7}, \frac{2}{7}, 0)$ . Note that no matter what values are chosen for  $b_1$  and  $b_2$ , a player will not be able to replicate the mixed strategy by use of a behavioural strategy. However, the above game is one of imperfect recall, which arises when a player makes a move when he is not able to remember one or more of his previous moves. An important theorem due to Kuhn (1953) is that in every game with perfect recall, where a player remembers all his previous moves (even though he may be uncertain about previous moves of other players), a realisation equivalent behavioural strategy can be found for every mixed strategy of a real player  $i$ .

To illustrate this, reconsider the previous game with redefined information set as below -



The analysis of mixed strategies is as before. However, the behavioural strategies are now defined by three, two dimensional probability vectors,  $(b_1, 1-b_1)$ ,  $(b_2, 1-b_2)$  and  $(b_3, 1-b_3)$ , since there are now three information sets.

Here,  $b_1$ ,  $b_2$  and  $b_3$  are the probability weights attached to moving left at any one of player one's information sets, labelled A, B and C respectively. The optimal mixed strategy can now be achieved by setting  $b_1 = \frac{5}{7}$ ,  $1-b_1 = \frac{2}{7}$ ,  $b_2 = 0$ ,  $1-b_2 = 1$ ,  $b_3 = 1$  and  $1-b_3 = 0$ .

Games of imperfect recall will be excluded from this analysis, as they are only of importance when one wants to examine games where some players are teams of agents in which each agent moves at a different information set of the player of which he is a member. It must also be assumed that the team agents cannot communicate freely among each other.

Nash's theorem, on the existence of equilibrium points for finite games, guarantees that every game in standard form has, at least, one equilibrium point in mixed strategies (Nash 1951). Since attention will be restricted to games with

perfect recall where Kuhn's theorem holds, we shall be able to rely on the existence of equilibrium points in behavioural strategies.

An equilibrium of a Bayesian game is a set of conjectures about how each player would choose his strategy<sup>14</sup> as a function of their type so as to maximise conditionally expected utility. Formally  $(m_1, \dots, m_n)$  is a Bayesian equilibrium of the Bayesian game  $G^*$  if, and only if, for every player  $i$ ,  $m_i$  is a function from  $i$ 's type vector  $T_i$  to their strategy space  $M_i$ , such that for every  $t_i$  in  $T_i$

$$\begin{aligned} & \sum_{t_{-i} \in T_{-i}} p_i(t_{-i} | t_i) U_i(m(t), t) \\ = & \max_{m_i^* \in M_i} \sum_{t_{-i} \in T_{-i}} p_i(t_{-i} | t_i) U_i((m_{-i}(t_{-i}), m_i^*), t) \end{aligned}$$

where  $m(t) = (m_1(t_1), \dots, m_n(t_n))$

and  $(m_{-i}(t_{-i}), m_i^*) = (m_1(t_1), \dots, m_{i-1}(t_{i-1}), m_i^*, m_{i+1}(t_{i+1}), \dots, m_n(t_n))$

The above equation requires that if player  $i$  is of type  $t_i$  and player  $i$  expects the other players to employ strategies  $m_{-i}(t_{-i})$ , then the strategy  $m_i(t_i) = m_i^*$  would be optimal for him, in that it maximises his conditional expected utility.

The function of a Bayesian equilibrium seems a desirable construct as, unless the predictions of the players constitute a Bayesian equilibrium, at least when one player is one type, they would expect to gain by using some unpredicted decision. That is, a player's behaviour can be rationally self-fulfilling if, and only if, it is a Bayesian equilibrium.

The following simple example illustrates the notion of a Bayesian equilibrium. Suppose there are two players with

$$M_1 = (a_1, b_1), M_2 = (a_2, b_2), T_1 = (1), T_2 = (2y, 2z)$$

$P_1(2y|1) = 0.9, P_1(2z|1) = 0.1$  and the two type dependent payoff matrices are

$$t_1 = 1, t_2 = 2y$$

|       | $a_2$ | $b_2$ |
|-------|-------|-------|
| $a_1$ | 3, 3  | 9, 4  |
| $b_1$ | 8, 5  | 6, 6  |

$$t_1 = 1, t_2 = 2z$$

|       | $a_2$ | $b_2$ |
|-------|-------|-------|
| $a_1$ | 4, 10 | 2, 9  |
| $b_1$ | 8, 8  | 5, 7  |

In this game,  $b_2$  is a dominant strategy for type  $2y$  and  $a_2$  is a dominant strategy for type  $2z$ . Therefore, player one wants to play either  $a_1$  or  $b_1$ , depending on what type of player one expects player two to be. Given the probabilistic beliefs of player one about what type of player two is, the expected value of playing

$$\text{strategy } a_1 \text{ is } = 0.9 \times 9 + 0.1 \times 4 = 8.5$$

$$\text{strategy } b_1 \text{ is } = 0.9 \times 6 + 0.1 \times 8 = 6.2$$

that is, player will choose to adopt strategy  $a_1$  and the unique Bayesian equilibrium of the game will be

$$m_1(1) = a_1, m_2(2y) = b_2, m_2(2z) = a_2$$

The example shows that the Bayesian equilibrium is different in nature from the Nash equilibrium of a complete information game, where player two's type is common knowledge. In the complete information case, the Nash equilibrium strategies would be determined by analysing each bimatrix separately, where the equilibrium strategies of the game would be  $(a_1, b_2)$  if player two's type is 2y and  $(b_1, a_2)$  if player two's type were 2z. Further, it is worth noting that, in this case, player two has no incentive to communicate his type to player one. This is because player two prefers  $(b_1, a_2)$  to  $(a_1, b_2)$  when he is of type 2y and he prefers  $(a_1, b_2)$  to  $(b_1, a_2)$  if he is of type 2z. Thus, if communication were allowed, player two would attempt to mislead player one by claiming he was type 2z when his true type was 2y, and claim he was type 2y when his true type was 2z, in order to attain the preferred outcome.

### 5.3 Conclusion

The above discussion has shown how players can determine rational strategies when they have incomplete information about the payoff functions in the game they are playing. Let us now recall our discussion of the Groves and Loeb transfer pricing mechanism, including the Cohen and Loeb suggested changes.

The initial discussion of the Groves and Loeb mechanism clearly established that one must recognize that divisional managers' strategies are the information they choose to communicate. In addition, Cohen and Loeb demonstrated that, when divisional managers are effort averse, it must be recognized that divisional managers' strategies are made up of two components; the information they choose to communicate and the effort level they wish to provide. In addition, the problems with the Cohen and Loeb suggested changes indicate that the central headquarter's preferences should be modelled and that the central headquarters should be considered as a "player" in the game rather than just a passive observer. That is, the central headquarters is allowed to act strategically. Thus, if we wish to model the transfer pricing problem as a game of incomplete information, it must allow for communication between the divisional (managers) players and the central headquarters, where divisional players' strategies involve communicating information and providing certain levels of effort. The type of strategy the central headquarters should adopt should also be considered and the preferences of the central headquarters should be modelled. In addition, the information held by various players should also be modelled.

The next chapter presents such a model of a Bayesian game, equivalent to a game with incomplete information, which allows for communication in a decentralized organizational environment. The original generalized model was due to Myerson (1982).

NOTES TO CHAPTER 5

1. See d'Aspremont and Gerard-Varet (1979), for instance.
2. And that the Cohen and Loeb amendment is inadequate.
3. Harsanyi (1967 and 1968).
4. Consider a two player game of incomplete information. Player 1's strategy choice depends on what player 1 expects to be player 2's payoff function  $U_2$ , as the form of this function will influence the way player 2 will play the game. This expectation about  $U_2$  can be called player 1's first order expectation. However, player 1 will realize that  $U_2$  is not the only information player 2 uses to choose a strategy. Player 2's first order expectation about player 1's payoff function  $U_1$ , will influence player 2's view of what strategy he or she thinks player 1 will play and, hence, which strategy he or she should choose. The expectation of player 1 about what first order expectation player 2 adopts can be called player 1's second order expectation. However, player 1's choice of strategy will also depend on what player 1 expects to be player 2's second order expectation. This may be called player 1's third order expectation - and so on ad infinitum. Harsanyi shows that this kind of sequential expectations model, where a game of incomplete information has to be analysed in terms of infinite sequences of subjective probability distributions over subjective probability distributions can be replaced by a model which analyses the game of incomplete information in terms of one unique probability distribution.

5. A game of complete information in normal form consists of, (i) a non empty set  $M_i$  consisting of all pure strategies  $m_i$  for player  $i$ , for each player  $i \in N$ , (ii) a utility function  $U_i: M_1 \times \dots \times M_N \rightarrow R^1$  for each player  $i \in N$  and (iii) full knowledge of (i) and (ii) on the part of each player  $i \in N$ .

6. This can be achieved because condition (vi) implies that every player knows (i) the sets  $\xi, M_1, \dots, M_N$  of pure strategies, (ii) each player  $i$ 's utility  $U_i(e(\xi, m_1, \dots, m_n))$  for every  $(\xi, m_1, \dots, m_n) \in \xi \times M_1 \times \dots \times M_N$ , (iii) each player  $i$ 's probability function  $P_{\xi_i}(\cdot)$  on  $\xi$ . Hence, each player  $i$ 's utility evaluation

$$U_i(m_1, \dots, m_n) = \sum_{j=1}^k u_i(e\{\xi^j, m_1, \dots, m_n\}) P_{\xi_i}(\xi^j)$$

of every  $(m_1, \dots, m_n)$  in  $M_1 \times \dots \times M_N$  is common knowledge in the set  $N = (1, \dots, n)$  of players when nature can adopt any of  $k$  "strategies".

7. A much more detailed discussion of attribute classes and related issues is presented in the next section.

8. In a two person zero sum game, the point  $(y_i^*, z_i^*)$  is an equilibrium point if, and only if,  $y_i^*$  is a maximin strategy for player 1 and  $z_i^*$  is a minimax for player 2.

9. This notation is intended to indicate that  $M_i$  and  $C_i$  are the range spaces of possible  $m_i$  and  $c_i$  respectively.

10. Harsanyi assumes  $x_i$  is player  $i$ 's payoff expressed in utility units so that one can write  $x_i = U_i(m_1, \dots, m_n) = V_i(m_1, \dots, m_n, c_1, \dots, c_n)$ .



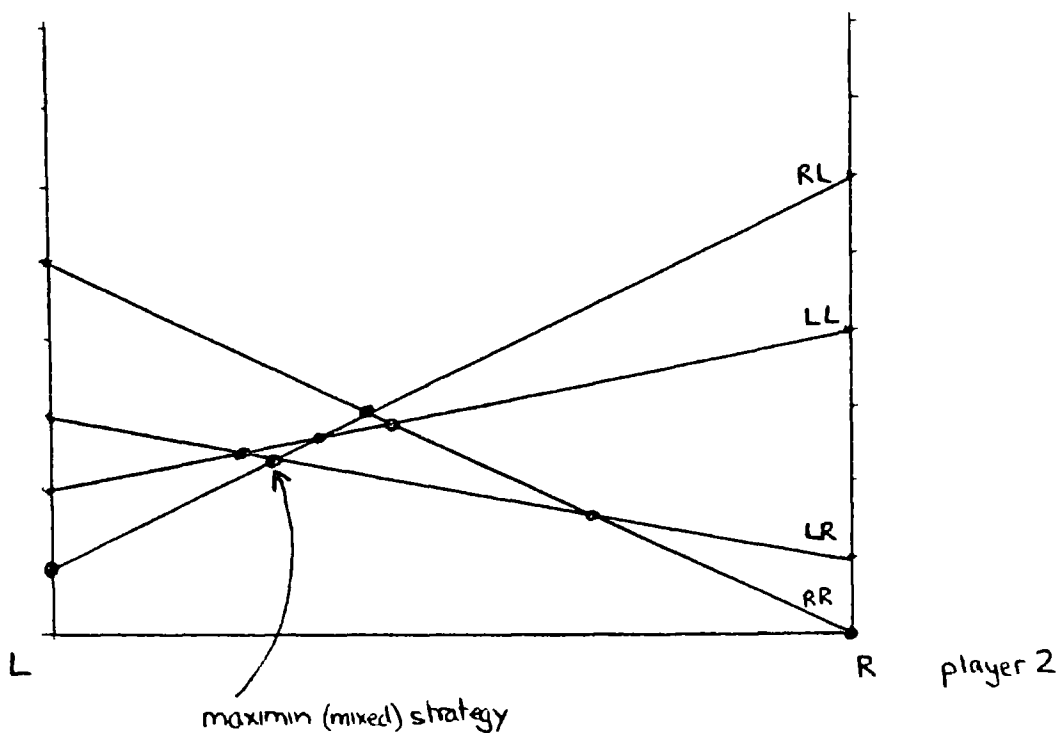
11. Aumann (1976).

12. Given the form of earlier notation, the decision sets  $D_1, \dots, D_N$  correspond to the strategy sets  $M_1, \dots, M_N$ , the type sets  $T_1, \dots, T_N$  correspond to the attribute sets  $C_1, \dots, C_N$ . The probability distributions  $P_1, \dots, P_N$  correspond to the probability distributions  $R_1, \dots, R_N$  and the utility sets  $U_1, \dots, U_N$  correspond to the payoff sets  $V_1, \dots, V_n$ .

13. To see this note that the payoff matrix for player 1 is:

|    | L | R |
|----|---|---|
| LL | 2 | 4 |
| LR | 3 | 1 |
| RL | 1 | 6 |
| RR | 5 | 0 |

This can be represented diagrammatically as:



So the maximin strategy is some mixed strategy with RL and LR having positive weights. If  $p$  is the probability that player 2 adopts strategy L, then we can determine the optimal strategy since

$$1p + (1-p).6 = 3p + (1-p).1$$

$$\text{gives } p = \frac{5}{7} \text{ and } 1-p = \frac{2}{7}$$

That is, the optimal mixed strategy is  $(0, \frac{5}{7}, \frac{2}{7}, 0)$ .

14. Note here that the types set  $T_1 X \dots X T_n$  corresponds to the attribute set  $C_1 X \dots X C_N$ .

6.1 A Generalised Principal-Agent Game of Incomplete Information

In the Bayesian games considered in the previous chapter, each player's strategy depended solely on their type. In order to be able to apply the above principles of decision making with incomplete information to resolve allocation problems such as transfer pricing, it is necessary to allow for communication between the central headquarters and divisional management. For instance, the central headquarters may require a division to report its type and, on the basis of this information, the central headquarters may take decisions which directly affect a division. The central headquarters may also wish to recommend that divisional managers take certain actions that are unobservable by the central headquarters. For example, divisional managers may be required to report their true inverse demand function (type) as in the Groves and Loeb mechanism. The central headquarters can then utilise this information to determine transfer prices. The central headquarters may also recommend that divisional managers provide certain levels of effort in order to implement the central headquarters plans. It is now possible to formulate the above general problem as a Bayesian incentive game of incomplete information with communication.

1

let  $D_0$  = the set of enforceable actions that the central headquarters takes, such as the setting of a transfer price, or the allocation of centrally provided input.

$D_i$  = the set of all private actions controlled by divisional managers, such as effort levels.

The term private in this context means that the action is unobservable by the central headquarters and other divisions.

$D = D_0 \times D_1 \times \dots \times D_N$  = the set of all possible combinations of actions taken by the central headquarters and the divisional managers.

where  $d = (d_0, d_1, \dots, d_n)$  = an outcome in  $D$ .

$T = (T_1 \times \dots \times T_N)$  = the set of all possible combinations of individual types.

where  $t = (t_1, \dots, t_n)$  = a specific state in  $T$ .

It is assumed that  $T$  and  $D$  are non-empty finite sets.

The above formulation means that the strategy of a divisional manager now involves communication about some variable through the message (strategy)  $m_i$ , and the provision of some private action  $d_i$ . The role of the central headquarters is to influence divisional managers' behaviour by choice of  $d_0$ . The domain of the utility functions  $u_i$ ,  $i = 0, 1, \dots, n$  are thus

$$DXT = (D_0 \times D_1 \times \dots \times D_N) \times (T_1 \times \dots \times T_N)$$

Thus  $u_0: DXT \rightarrow R$  = the central headquarters utility function for  $i = 0$ .

$u_i: DXT \rightarrow R$  = divisional manager (player)  $i$ 's utility function for  $i = 1, \dots, n$ .

So for any  $(d,t) \in (DXT)$ ,  $u_i(d,t)$  represents the expected utility payoff for divisional manager  $i$ , or the central headquarters when  $i=0$ . The payoff is measured by some von Neumann-Morgenstern utility scale, where management acts according to the decisions  $d$ , and where divisional managers' information is represented by the vector of types  $t$ . Any random variable not observed by anyone should be intergrated out, using their conditional distributions given  $t$ , to compute the expected utility  $u_i(d,t)$ .

$P^*$  = the probability distribution on  $T$  such that  $P^*(t)$  is the probability that  $t = (t_1, \dots, t_n)$ , as assessed by the central headquarters or by any divisional manager ex ante, before any divisional manager has their type revealed to them.

However, once a game commences, each divisional manager will be aware of their type and may wish to revise his conjectures about other divisional managers' types, given this information. That is, the conditional distributions or beliefs  $(p_1, \dots, p_n)$  that the players will hold should be computed from  $P^*(t)$  by Bayes theorem, that is

$$(1) \quad p_i(t_{-i} | t_i) = \frac{P^*(t)}{p_i^*(t_i)}$$

for all  $t_i \in T_i$  and all  $t_{-i} \in T_{-i}$ .

where  $p_i^*(t_i) = \sum_{t_{-i} \in T_{-i}} P^*(t)$

for all  $t_i \in T_i$ .

Here a convention is being used that, whenever  $t, t_{j_1} \subset$  and  $t_1$  appear in the same formula, then  $t$  is the vector of types with the  $1^{th}$  component  $t_1$  and all other components as in  $t_{j_1} \subset$ .

Since the central headquarters is taking decisions on the basis of information communicated to it by the divisions let  $\Pi(d_0, d_1, \dots, d_n | t_1, \dots, t_n)$  denote the conditional probability that the central headquarters will choose  $d_0$  in  $D_0$  and recommend that each divisional manager use  $d_1$  in  $D_1$  if each (player) divisional manager reported his or her type to be  $t_i$ . The numbers  $\Pi(d|t)$  must, therefore, satisfy the following probability distribution constraints.

$$(2) \quad \sum_{d \in D} \Pi(d|t) = 1 \quad \text{and} \quad \Pi(d|t) \geq 0$$

for all  $d \in D$  and all  $t \in T$ .

A function  $\Pi: D \times T \rightarrow R$  that satisfies this condition is denoted a mechanism for the Bayesian game.

If every player reports their type honestly and obeys the recommendations of the central headquarters, then the conditional expected utility for type  $t_1$  of divisional manager 1 from mechanism  $\Pi$  would be:

$$(3) \quad U_1(\Pi | t_1) = \sum_{t_{j_1} \subset \in T_{j_1} \subset} \sum_{d \in D} P_i(t_{j_1} \subset | t_1) \Pi(d|t) u_1(d, t)$$

Note that care must be taken not to confuse the conditional expected utility  $U_i$  and the expected utility  $u_i$  from  $(d, t)$ .

However, it may be that each divisional manager may choose to lie or disobey the central headquarter's instructions, since the players' types can not be verified by the central headquarters, and each selection of a private action  $d_i \in D_i$  for  $i = 1, \dots, n$  is controlled by divisional manager  $i$ . Thus the co-ordination mechanism induces a communication game  $G_{\Pi}$  in which each divisional manager must select his or her type report and plan for choosing a private action in  $D_i$  as a function of the central headquarter's recommendation. The central headquarters must also take some public action  $D_0$ . Thus, formally one can define  $G_{\Pi}$  as

$$G_{\Pi} = (D_0, D_1, \dots, D_n, T_1, \dots, T_n, P_1, \dots, P_n, u_1, \dots, u_n)$$

where for  $i = 1, \dots, n$

$$D_i = \{ (m_i, \delta_i) \mid m_i \in T_i \text{ and } \delta_i : D_i \rightarrow D_i \}$$

Here  $(m_i, \delta_i)$  represents divisional manager  $i$ 's participation strategy or plan, where  $m_i$  is a function which determines what type a divisional manager will report he or she is, given his or her true type and  $\delta_i$  is a function which determines what private action  $\hat{d}_i \in D_i$  a divisional manager should employ given the central headquarter's recommendations that player  $i$  should adopt  $d_i$ .

Therefore, in general, it is possible to express divisional manager  $i$ 's conditional expected utility, given the participation strategies of all divisional managers and their types and the central headquarters public decision as

$$\begin{aligned} & \hat{U}_1((d_0, (m_1, \delta_1), \dots, (m_n, \delta_n)), t) \\ = & \sum_{t_{>1c} \in T_{>1c}} \sum_{d \in D} P_i(t_{>1c} | t_1) \Pi(d | m_1, \dots, m_n) \\ & \cdot u_i((d_0, (\delta_1(d_1), \dots, \delta_n(d_n))), t) \end{aligned}$$

We shall assume that each divisional manager communicates with the central headquarters separately and confidentially, so that it seems plausible to assume that divisional manager 1's private action does not depend on the recommendations to the other players.

Suppose divisional manager  $i$  was of type  $t_i$  and chose to use the participation strategy  $(m_i, \delta_i)$  in the communication game  $G_{\Pi}$  where  $m_i \neq t_i$ . If all other divisional managers were expected to report their types honestly and choose their private actions obediently, then divisional manager  $i$ 's conditional expected utility would be:

$$\begin{aligned} (4) \quad U_i^*(\Pi, \delta_i, m_i | t_i) = & \sum_{t_{>1c} \in T_{>1c}} \sum_{d \in D} P_i(t_{>1c} | t_i) \Pi(d | t_{>1c}, m_i) \\ & \cdot u_i((d_{>1c}, \delta_i(d_i)), t) \end{aligned}$$

where  $(d_{>1c}, \delta_i(d_i)) = (d_0, d_1, \dots, d_{i-1}, \delta_i(d_i), d_{i+1}, \dots, d_n)$

and  $(t_{>1c}, m_i) = (t_1, \dots, t_{i-1}, m_i, t_{i+1}, \dots, t_n)$



A mechanism is said to be Bayesian incentive compatible if, and only if, it is a Bayesian equilibrium for all players to report their types honestly and to obey the central head-quarter's recommendations when mechanism  $\Pi$  is used. Thus  $\Pi$  is Bayesian incentive compatible if, and only if

$$(5) \quad U_1(\Pi | t_1) \geq U_1^*(\Pi, \delta_1, m_1 | t_1)$$

for all  $i = 1, \dots, n$

for all  $t_1 \in T_1$

for all  $m_1 \in M_1$

for all  $\delta_1 : D_1 \rightarrow D_1$

Besides lying about their type and/or being disobedient, divisional managers may refuse to take part in the proceedings by refusing to communicate, or they may simply resign from their post. Therefore, a mechanism must also be constrained to be individually rational for the divisional managers, in order for it to be implementable. How we specify the individual rationality constraint depends on the model. For instance, we might require that the mechanism provides a divisional manager with a conditionally expected utility greater than he or she could gain in some other form of employment. Alternatively, if the private action  $d_1$  of a divisional manager represented his or her effort level and the divisional manager preferred less effort to more effort and he or she was rewarded in monetary terms, then the conditionally expected utility the manager gained from monetary reward must be greater than, or equal to, the conditionally expected disutility he or she

incurs from providing effort. The latter case will be used here, so we can say a mechanism  $\Pi$  is individually rational if, and only if

$$(6) \quad U_i(\Pi|t_i) \geq 0 \quad \text{for } \overset{\text{all}}{t_i} \in T_i \text{ and all } i = 1, \dots, n.$$

It should be noted at this point that there may be many different Bayesian equilibria of a communication game  $G_\Pi$ , even if  $\Pi$  is incentive compatible. Also attention has been restricted to direct mechanisms where

$$m_i(t_i) \in T_i$$

and the central headquarters private action messages have been direct, that is drawn from the central headquarter's message space  $D_{i,oc}$ .

However, one may want to consider indirect mechanisms which do not require direct revelation. Therefore, the direct revelation requirement will now be dropped so that consideration could be given to indirect revelation mechanisms. Thus, we now allow for divisional indirect messages for which  $m_i(t_i) \notin T_i$ . Also, the central headquarter's recommendations for provision of private actions may be indirect. Thus, we denote the central headquarter's messages as  $c_i$ , where it may be that  $c_i \notin D_i$ .

In order to allow for this extension, we will now redefine the possible message and directive spaces as follows:

$M_i$  = the  $i^{\text{th}}$  divisional message space.

$C_i$  = the  $i^{\text{th}}$  private action directive space.

Thus, we can also define

$$M = M_1 \times \dots \times M_N$$

$$C = C_1 \times \dots \times C_N$$

Also we now have

$$U_1(\Pi, \delta_i, m_i | t_i) = \sum_{t_{>ic} \in T_{>ic}} \sum_{d_o \in D_o} \sum_{c \in C} P_i(t_{>ic} | t_i) \Pi(d_o, c | m(t)) \cdot u_i(d_o, \delta(c, t), t)$$

where  $m(t) = (m_1(t_1), \dots, m_n(t_n))$

$$(\delta(c, t)) = (\delta_1(c_1, t_1), \dots, \delta_n(c_n, t_n))$$

The participation strategies  $(m_i, \delta_i)$  for each  $i$  will form an equilibrium if, and only if, for every  $i$  and every alternative participation strategy  $(\hat{m}_i, \hat{\delta}_i)$

$$(7) \quad U_i(\Pi, \delta_i, m_i | t_i)$$

$$\geq \hat{U}_i(\Pi, \hat{\delta}_i, \hat{m}_i | t_i) = \sum_{t_{>ic} \in T_{>ic}} \sum_{d_o \in D_o} \sum_{c \in C} P_i(t_{>ic} | t_i) \Pi(d_o, c | m_{>ic}(t_{>ic}), \hat{m}_1(t_1)) \cdot u_i(d_o, \delta_{>ic}(c_{>ic}, t_{>ic}), \hat{\delta}_i(c_i, t_i), t)$$

for all  $t_i \in T_i$  for all  $i = 1, \dots, n$ .

where

$$(m_{>ic}(t_{>ic}), \hat{m}_i(t_i)) = (m_1(t_1), \dots, m_{i-1}(t_{i-1}), \hat{m}_i(t_i), m_{i+1}(t_{i+1}), \dots, m_n(t_n))$$

$$(\delta_{>ic}(c_{>ic}, t_{>ic}), \hat{\delta}_i(c_i, t_i)) = (\delta_1(c_1, t_1), \dots, \delta_{i-1}(c_{i-1}, t_{i-1}), \hat{\delta}_i(c_i, t_i), \delta_{i+1}(c_{i+1}, t_{i+1}), \dots, \delta_n(c_n, t_n))$$

Since there are many conceivable indirect revelation mechanisms, it may seem that analysis to determine whether there are any indirect revelation mechanisms that perform better than direct revelation mechanisms would be important, but difficult. However, it can be shown that for any indirect revelation mechanism, there exists a direct revelation mechanism which performs at least as well as the indirect revelation mechanism. This result, due to Myerson (1979) is known as the Revelation Principle and this proposition will now be formally stated and proven.

Revelation Principle: Given any equilibrium of participation strategies  $(m_i, \delta_i)_{i=1}^n$  in any co-ordination mechanism (direct or indirect) defined by  $(M_i, C_i)_{i=1}^n, \Pi$ , there exists an incentive compatible direct mechanism  $\Pi^*$  in which the central headquarters gets the same expected utility as in the given equilibrium of the given mechanism. Thus, the optimal incentive compatible direct co-ordination mechanism is also optimal in the class of all co-ordination mechanisms.

PROOF: Given some equilibrium of participation strategies  $(m_i, \delta_i)_{i=1}^n$  let  $\delta^*(d, t)$  be the set of all messages to the divisional managers such that each divisional manager  $i$  would respond by choosing private action  $d_i$  is his type were  $t_i$ . That is

$$\delta^*(d, t) = \{c \mid \delta_i(c_i, t_i) = d_i \text{ for all } i\}$$

Then define  $\Pi^* : D \times T \rightarrow R$  so that

$$\Pi^*(d|t) = \sum_{c \in \delta^*(d,t)} \Pi(d_0, c | m_1(t_1), \dots, m_n(t_n))$$

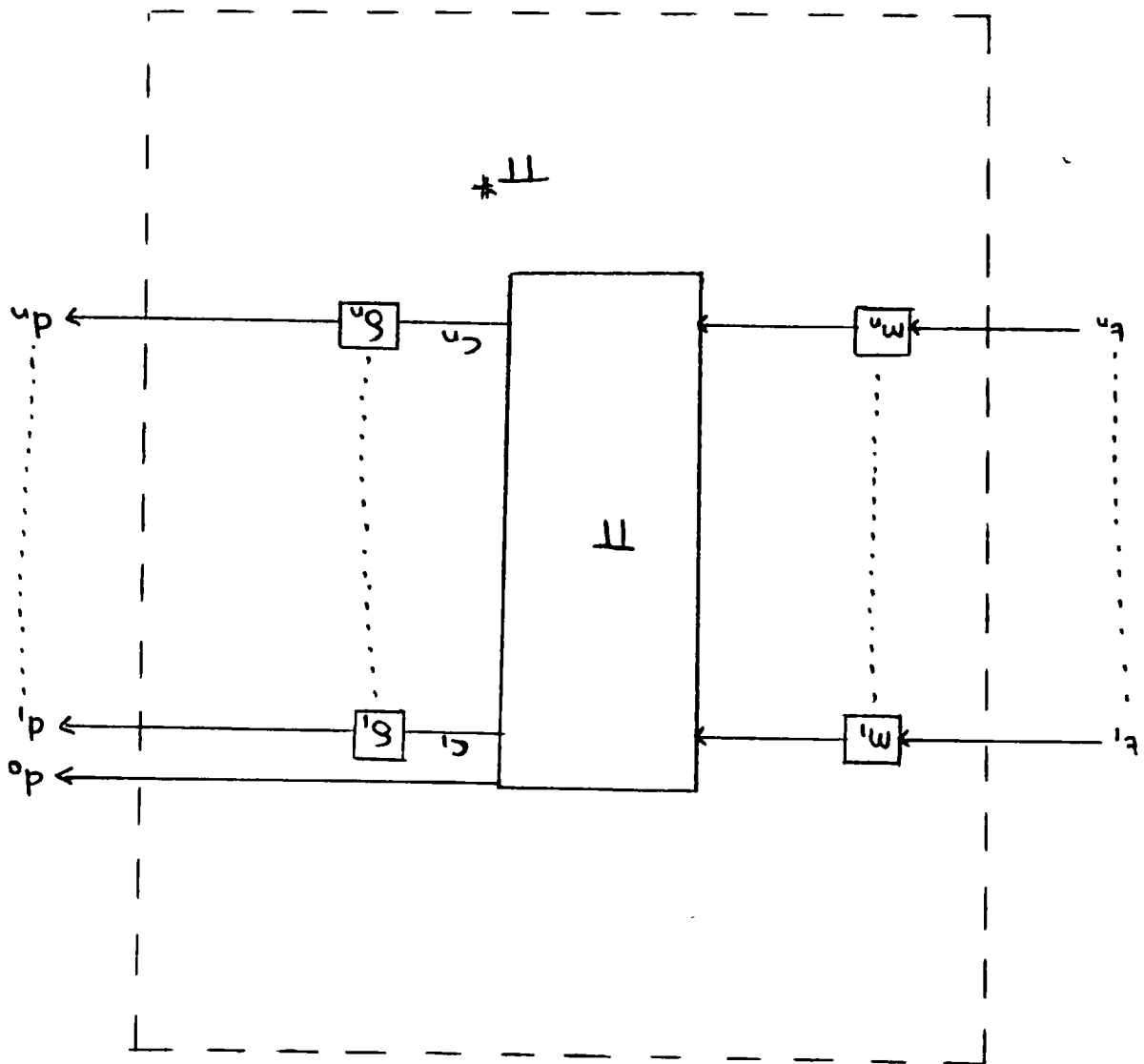
$\Pi^*$  is the direct co-ordination mechanism which simulates the overall effect of the original mechanism. That is  $\Pi^*(d,t)$  is the probability that the central headquarters will choose  $d_0$  and each divisional manager will choose  $d_1$  if  $t$  is the vector of types, when each divisional manager  $i$  chooses his reporting strategy according to  $m_i$ . The central headquarters decisions and the messages to the agents are determined from these reports by  $\Pi$  and each divisional manager  $i$  uses  $\delta_i$  to translate his received message into a decision in  $D_i$ .  $\Pi^*$  gives the same expected utility to the central headquarters and the same conditionally expected utility to the divisional managers as the originally given mechanism, since the probability distribution over decision vectors for any type vector is the same. To check that  $\Pi^*$  is incentive compatible suppose (to the contrary) that divisional manager  $i$  could gain when  $\tau_i$  is his type by reporting  $\hat{\tau}_i$  and then determining his private action by  $\delta_i : D_i \rightarrow D_i$  in violation of equation (5). Then if the participation strategies  $(\hat{m}_i, \hat{\delta}_i)$  for the original indirect mechanism, where

$$\hat{m}_i(\tau_i) = m_i(\tau_i), \quad \hat{m}_i(t_i) = m_i(t_i) \text{ if } t_i \neq \tau_i$$

$$\hat{\delta}_i(c_i, \tau_i) = \bar{\delta}_i(\delta_i(c_i, \tau_i)), \quad \hat{\delta}_i(c_i, t_i) = \delta_i(c_i, t_i) \text{ if } t_i \neq \tau_i$$

That is  $(\hat{m}_i, \hat{\delta}_i)$  differs from  $(m_i, \delta_i)$  in that, when  $\uparrow_i$  is  $i$ 's true type,  $i$  sends reports as if  $\hat{\uparrow}_i$  were  $i$ 's type and then determines planned responses by  $\bar{\delta}$  from what he would have done if  $\hat{\uparrow}_i$  were  $i$ 's type. Thus, if player  $i$  was type  $\uparrow_i$  and used the direct participation strategy  $(\hat{\uparrow}_i, \bar{\delta})$  this would violate equation (5) for  $\Pi^*$ , and similarly the indirect participation strategy  $(\hat{m}_i, \hat{\delta}_i)$  would violate (7) for the originally given indirect mechanism, which would contradict the fact that  $(m_i, \delta_i)$  is an equilibrium strategy for  $i$ . Thus  $\Pi^*$  must be incentive compatible. Q.E.D.

Another possible interpretation of the Revelation Principle is to note that an equilibrium in a finite-move extensive form game with explicit description of sequencing of information exchange and decision making will always correspond to some equilibrium in a one-shot game with suitably augmented message spaces. The one-shot game can then be reduced to a game where truthful information is passed. Also, randomisation in the possibly very complex extensive form game will still be captured by simple draws from a uniform distribution (honestly reported in equilibrium). In the previous discussion a one-shot game corresponds to direct revelation and finite move extensive form game corresponds to indirect revelation. That is a direct co-ordination mechanism simulates the overall effect of the (finite move) indirect co-ordination mechanism as the diagram below illustrates.



Expressed in yet another fashion, the Revelation Principle implies that any vector of expected utility points for the central headquarters and divisional managers which can be achieved in a mechanism in which divisional managers lie and/or disobeyingly provide their effort services, can be achieved by a mechanism which induces honesty and obedience. Thus, the central headquarters can restrict its attention to outcomes that are possible under direct mechanisms when trying to find the best achievable outcome using any possible mechanism.

## 6.2 Application of the Generalised Principal-Agent Game of Incomplete Information to the Transfer-Pricing Problem

In Section 6.1, a very general model of a Principal-Agency problem was presented. In order to show that this analysis is applicable to the transfer pricing problem, one must specify in much more detail the communication process between the central headquarters and divisional managers for this specific problem. For instance, the public action space  $D_0$  must be specified in much more detail.

Included in the specification of  $D_0$  must be the method by which divisional managers are to be compensated. It is important that the compensation functions be determined within the model and not imposed exogenously. That is to say, it is not admissible to, for instance, assume that divisional managers are paid a fixed share of their



divisions' profit. This is because the central headquarters has the ability to determine how it wishes to compensate divisional managers and so one must consider how the central headquarters could determine the best method by which to compensate divisional managers. Different compensation functions will affect how divisional managers perform and, hence, the net corporate profits that will accrue to the central headquarters.

Before undertaking this task, though, let us recall the role of the central headquarters in the Generalised Principal-Agency model. The central headquarters will wish to determine the direct revelation mechanism, which induces divisional managers to adopt honest and obedient participation strategies and which maximise the central headquarter's utility, while still being individually rational for divisional managers.

In the formulation presented here, the central headquarters possesses no private information, which means that the central headquarter's problem can be characterised as:

$$\underset{\Pi}{\text{maximise}} \quad U_0(\Pi) = \sum_{t \in T} \sum_{d \in D} P(t) \Pi(d|t) u_0(d, t)$$

subject to (2), (5) and (6).

In the following presentation, in order to simplify the presentation, without destroying the essential aspects of the problem, it will be assumed that all divisional managers' beliefs are consistent with a common prior (as in equation (1)). Furthermore, it will be assumed that

divisional managers' types are independent random variables in the common prior, that is:

$$P^*(t) = \prod_{i=1}^N p_i^*(t_i)$$

for all  $t \in T$

Here  $p_i^*(t_i)$  is the marginal probability that player  $i$  is type  $t_i$ .

Let us assume there are two divisions, a manufacturing and a distribution division, respectively. The manufacturing division produces an intermediate product  $y$  using two inputs  $x_1$  and  $x_2$ , which have associated factor input prices of  $e_m$  and  $t$  respectively. The distribution division utilises  $y$  and another input  $z$  to produce a final product  $q$ . The input  $z$  has a unit price of  $e_D$  and the transfer price associated with  $y$  is denoted  $p$ . In addition, it is assumed that the final product is sold on a competitive external market at price  $\theta$ . It will be assumed that the specific production technologies for the model are as follows:

$$y = f(x_1, x_2) = x_1^{.3} x_2^{.5}$$

$$q = g(y, z) = y^{.5} z^{.1}$$

The corporate objective function can be expressed as:

$$x_1, x_2, z \max \theta q - e_D z - e_m x_1 - t x_2$$

$$\text{s.t. } q = y^{.5} z^{.1}$$

$$y = x_1^{.3} x_2^{.5}$$

Let us introduce incomplete knowledge of information by assuming that there is uncertainty regarding what the factor price  $t$ , for the factor input  $x_2$  will be, for any forthcoming period. It is common knowledge that the factor price marginal probability distribution is

$$p^*(t) = \begin{cases} 0.1 & \text{for } t = 5 \\ 0.3 & \text{for } t = 4 \\ 0.4 & \text{for } t = 3 \\ 0.2 & \text{for } t = 2 \end{cases}$$

It is assumed that the manufacturing divisional manager observes the actual value of  $t$  at the beginning of the period under consideration and, hence, knows the true value of  $t$  privately. One could similarly assume that the final product price  $\theta$  is uncertain and that the distribution divisional manager privately observes its value, whereas others only have common knowledge of the marginal probability distribution of  $\theta$ . However, in order to simplify the example, it will be assumed that  $\theta$  can only take on one value which is  $\theta = 7$ . To introduce managerial effort as an argument in the model, it could be assumed that  $x_1$  is the manufacturing division's manager's level of effort provided and  $z$  is the distribution division's manager's level of effort provided. It will be assumed that the associated factor cost  $e_D$  of effort  $z$  is 1 and for  $x_1$  the associated factor cost  $e_m$  is 1.

$$\text{Let } d = (d_0, d_1, d_2)$$

$$\text{where in this model } d_1 = x_1$$

$$d_2 = z$$

$$\text{Let } d_0 = (d_{op}^m, d_{op}^D, d_{oy}^m, d_{oq}^D, d_{oc}^m, d_{oc}^D, d_{o\pi}^m, d_{o\pi}^D)$$

where  $d_{op}^m$  specifies the transfer price that the manufacturing division faces

$d_{op}^D$  specifies the transfer price<sup>3</sup> that the distribution division faces

$d_{oy}^m$  specifies the output of the intermediate product required from the manufacturing division

$d_{oq}^D$  specifies the output of the final product required from the distribution division

$d_{oc}^m, d_{oc}^D$  specifies the compensation accruing to the respective divisional managers

$d_{o\pi}^m, d_{o\pi}^D$  specifies the profit a division should earn

The divisional problems<sup>4</sup> can be expressed as:

$$\text{manufacturing division } x_1, x_2 \max py - e_m x_1 - tx_2$$

$$\text{s.t. } y = x_1^3 x_2^5$$

$$\text{distribution division } y, z \max \theta q - py - e_D z$$

$$\text{s.t. } q = y^5 z^1$$

In addition, assume that the sole argument of the central headquarters utility function is corporate profit and that it is risk neutral, so that  $U_0(f_0(d,t)) = f_0(d,t)$ .

Given the above notation, one can clearly write the central headquarter's problem as:

$$\begin{aligned} d \quad & \max \quad f_0(d,t) \\ & \text{s.t. (5) and (6)} \end{aligned}$$

if the central headquarters wished to choose a single pure strategy  $d$ .

However, if it is recognised that the types' variable  $t$  can take on a number of values with associated marginal probability  $p(t)$ , then the central headquarters may wish to specify its choice of strategy  $d$  as conditional on the types variable of the manufacturing division. Note that, since this is privately known only by that division's manager, the central headquarters will be basing its decision on a communicated value of  $t$ , which may not be the true value. Also the central headquarters may wish to choose a mixed strategy<sup>5</sup>, rather than a pure strategy if it benefits from this. Therefore, the appropriate specification for the central headquarter's objective function becomes:

$$\begin{aligned} \Pi(d|t) \quad & \max \quad P(t)\Pi(d|t)f_0(d,t) \\ & \text{s.t. (2), (5) and (6)} \end{aligned}$$

Suppose that divisional managers' utility functions are of the form:

$$U_m(c_m, e_m, x_1) = c_m - e_m x_1$$

$$\text{and } U_D(c_D, e_D, z) = c_D - e_D z$$

where  $c_m$  and  $c_D$  denotes the compensation received by manufacturing and distribution division's managers respectively. In addition, assume that individual rationality constraints required that:

$$U_m(\cdot, \cdot, \cdot) \geq 0$$

$$\text{and } U_D(\cdot, \cdot, \cdot) \geq 0$$

Suppose, for the moment, that there were no incentive compatibility problems as the central headquarters could perfectly observe  $t$ ,  $\theta$ ,  $x$ , and  $z$ . Then the central headquarter's overall problem<sup>6</sup> reduces to:

$$(d|t) \max \theta q - e_D z - e_m x_1 - t x_2$$

$$\text{s.t. } q = y^{.5} z^{.1}$$

$$y = x_1^{.3} x_2^{.5}$$

$$c_m = e_m x_1$$

$$c_D = e_D z$$

where the final constraints ensure individual rationality. It is proven in the Appendix to this chapter that, in this environment, the optimal transfer price given perfect observability and, hence, the guarantee of truthful communication of types variables and obedient provision of divisional managerial effort is:

$$p = 2.7263t^{.4}$$

Thus the optimal transfer price for :

$$t = 5 \quad \text{is} \quad P = 5.1899$$

$$t = 4 \quad \text{is} \quad P = 4.7468$$

$$t = 3 \quad \text{is} \quad P = 4.2308$$

$$t = 2 \quad \text{is} \quad P = 3.5974$$

In this model, where  $d_{op}^m = d_{op}^D = p$ , once the transfer price is determined the optimal output of each division can be determined in each case as:

$$\text{for } t = 5, \quad y = .3770$$

$$q = .5590$$

$$t = 4 \quad y = .4608$$

$$q = .6247$$

$$t = 3 \quad y = .5970$$

$$q = .7217$$

$$t = 2 \quad y = .8599$$

$$q = .8838$$

(See the end of the Appendix for derivation of these values.) Under each transfer pricing regime, the central headquarters would compensate the divisional managers as follows:

$$p = 5.1899 \quad x_1^* = .5869 = r_m$$

$$z^* = .3913 = r_D$$

provided their divisions earn the profits

$$\Pi_m = .3914$$

$$\Pi_D = 1.5651$$

$$p = 4.7468 \quad x_1^* = .6563 = r_m$$

$$z^* = .4375 = r_D$$

provided the divisions earn the profits

$$\Pi_m = .4373$$

$$\Pi_D = 1.7479$$

$$p = 4.2308 \quad x_1^* = .7578 = r_m$$

$$z^* = .5052 = r_D$$

provided the divisions earn the profits

$$\Pi_m = .5051$$

$$\Pi_D = 2.0210$$

$$p = 3.5974 \quad x_1^* = .7734 = r_m$$

$$z^* = .6187 = r_D$$

provided the divisions earn the profits

$$\Pi_m = .6185$$

$$\Pi_D = 2.4745$$

The above discussion demonstrates that, once the central headquarters knows the types variable for the manufacturing division, it can specify a suitable transfer price and associated controls such as output targets and budgeted profits. It is thus possible to specify a payoff table as below where the other  $d_0$  arguments besides  $p = d_{op}^m$  have been suppressed.



PAYOFF TABLE

|       | P = 5.1899 | P = 4.748 | P = 4.2308 | P = 3.5974 |
|-------|------------|-----------|------------|------------|
| t = 5 | X          |           |            |            |
| t = 4 |            | X         |            |            |
| t = 3 |            |           | X          |            |
| t = 2 |            |           |            | X          |

The cross that is found in any one row indicates the optimal transfer price for the central headquarters, given any known value of the types variable.

However, let us now relax the assumption that the manufacturing division's manager is honest and obedient. When such incentive compatibility issues need to be considered, one must consider the payoffs that would accrue to the manufacturing division's manager in the non-crossed boxes of the payoff table. This is because these are the outcomes that would arise if the manufacturing divisional manager chose to lie about the division's types variable and the central headquarters uses the originally derived controls  $d_0$ . Thus, the full payoff table will now be specified. The entry in each payoff box is the utility the manufacturing divisional manager achieves. If the central headquarters can ensure that divisional managers are honest and obedient, it can ensure that they

PAYOFF TABLE

|         | P = 5.1899 | P = 4.7468 | P = 4.2308 | P = 3.5974 |
|---------|------------|------------|------------|------------|
| t = 5   | 0          | -.2953     | - 1.0174   | - 3.4995   |
| t = 4   | .1822      | 0          | - .4661    | - 2.1165   |
| t = 3   | .3363      | .2501      | 0          | - 1.0504   |
| t = 2   | .4594      | .4497      | .3724      | 0          |
| $\Pi_c$ | 1.9565     | 2.1852     | 2.5261     | 3.093      |

are "just" paid enough to make it individually rational for them to carry out central headquarter's directives. Hence, the entries on the diagonal of the payoff table are all zero. In some cases, the manufacturing divisional manager would not benefit from lying and so those parts of the payoff table contain negative entries in the boxes. The final row indicates the profits that accrue to the central headquarters and is determined by adding the two divisions' profits under each transfer pricing regime respectively. To illustrate how the utility of the manufacturing divisional manager is determined, let us consider the case where the manufacturing division's manager communicates that the division's types variable is  $t = 3$ .

In this situation, the central headquarters would have set<sup>8</sup> the transfer price at 4.2308 and required the manufacturing division to produce .597 units of the intermediate product, while the distribution division is required to produce .7217 units of the final product. In addition, the manufacturing divisional manager will be compensated with .7578 units of compensation<sup>9</sup>, provided the division earns .5051 units of profit. The distribution divisional manager will be compensated with .5052 units of compensation, provided the division earns 2.0210 units of profit. It will now be demonstrated that, if the manufacturing division's true types variable were  $t = 4$ , that the divisional manager would find it disutilitous to communicate the division's types variable as being  $t = 3$ , if the central headquarters uses  $P = 4.2308$  and associated controls. When the division's true types variable is  $t = 4$  and the manager communicates the value  $t = 3$  to the central headquarters then specifies the controls

$$d_{op}^m = 4.2308$$

$$d_{oy}^m = .597$$

$$d_{oc}^m = .7578$$

$$d_{o\Pi}^m = .5051$$

and requests that:

$$d_1 = .7578, \text{ remembering that the central headquarters will not be able to observe the value of } d_1.$$

As far as the central headquarters is concerned

$$d_{011}^m = py - c_m - tx_2 = .5051$$

since  $c_m$  is set equal to  $e_m x_1$ . Since  $py$  and  $c_m$  are set (controlled) by the central headquarters, once the divisional manager communicates the types variable, it must ensure that, if the division's true types variable  $\hat{t}$  is different from the communicated value  $t^*$ , that  $\hat{t}x_2 = t^*x_2^*$  in order to achieve the budgeted profit. In this case:

$$t^*x_2^* = 3 \times (.4210) = 1.263$$

$$\text{so if } \hat{t} = 4$$

$$\hat{x}_2 = \frac{1.263}{4} = .31575$$

In order to meet the production requirement of the intermediate product, it must be that the divisional manager provides enough effort ( $\hat{x}_1$ ) so that

$$\hat{x}_1^{\hat{\alpha}} \hat{x}_2^B = .597 \quad (\text{with } B = .5)$$

$$\hat{x}_1^{\hat{\alpha}} = \frac{.597}{(.31575)^{\frac{1}{2}}} \quad (\text{with } \alpha = .3)$$

$$\hat{x}_1 = 1.2235$$

$$\text{and } u_m(c_m, e_m, \hat{x}_1) = .7578 - 1.2239 = -.4661$$

A similar argument can be presented for  $\hat{t} = 5$ ,  $t^* = 3$ , showing that  $u_m(\cdot, \cdot, \cdot)$  would be negative.

However, suppose that  $\hat{t} = 2$  and  $t^* = 3$ , when the central headquarters sets the same controls and requests the same effort level from the divisional manager.

$$\hat{x}_2 = \frac{1.263}{2} = .6315$$

$$\hat{x}_x^\alpha \hat{x}_2^B = .597$$

$$\text{gives } \hat{x}_1^\alpha = \frac{.597}{(.6315)^{\frac{1}{2}}} \quad (\text{with } B = .5)$$

$$\text{gives } \hat{x}_1 = .3854 \quad (\text{with } \alpha = .3)$$

$$\text{and } u_m(c_m, e_m, \hat{x}_1) = .7578 - .3854 = .3724$$

Clearly, the divisional manager can benefit from overstating the value of the division's types variable, because the manager can achieve the budgeted level of profit by using more of input  $x_2$  which is, in fact, cheaper than the central headquarters expects and less effort, while still receiving the same compensation.

Let us use the convention that:

$$d_o^1 = (d_{op}^m = d_{op}^D = 5.18, d_{oy}^m = .377, d_{oq}^D = .559, d_{oc}^m = .5869,$$

$$d_{oc}^D = .3913, d_{o\pi}^m = .3914, d_{o\pi}^D = 1.5651)$$

$$\text{ie } d_o^1 = (5.1899, .377, .559, .5869, .3913, .3914, 1.5651)$$

$$d_o^2 = (4.7468, .4608, .6247, .6563, .4375, .4373, 1.7479)$$

$$d_0^3 = (4.2308, .597, .7217, .7578, .5052, .5051, 2.0210)$$

$$d_0^4 = (3.5974, .8599, .8838, .7734, .6187, .6185, 2.4745)$$

$$\text{and that } d_1^1 = .5869$$

$$d_1^2 = .6563$$

$$d_1^3 = .7578$$

$$d_1^4 = .7734$$

Let

$$\Pi_5^1 = \Pi(d_0^1, d_1^1, d_2^1 | t^* = 5, \theta = 7) \quad \Pi_4^1 = \Pi(d_0^1, d_1^1, d_2^1 | t^* = 4, \theta = 7)$$

$$\Pi_5^2 = \Pi(d_0^2, d_1^2, d_2^2 | t^* = 5, \theta = 7) \quad \Pi_4^2 = \Pi(d_0^2, d_1^2, d_2^2 | t^* = 4, \theta = 7)$$

$$\Pi_5^3 = \Pi(d_0^3, d_1^3, d_2^3 | t^* = 5, \theta = 7) \quad \Pi_4^3 = \Pi(d_0^3, d_1^3, d_2^3 | t^* = 4, \theta = 7)$$

$$\Pi_5^4 = \Pi(d_0^4, d_1^4, d_2^4 | t^* = 5, \theta = 7) \quad \Pi_4^4 = \Pi(d_0^4, d_1^4, d_2^4 | t^* = 4, \theta = 7)$$

$$\Pi_3^1 = \Pi(d_0^1, d_1^1, d_2^1 | t^* = 3, \theta = 7) \quad \Pi_2^1 = \Pi(d_0^1, d_1^1, d_2^1 | t^* = 2, \theta = 7)$$

$$\Pi_3^2 = \Pi(d_0^2, d_1^2, d_2^2 | t^* = 3, \theta = 7) \quad \Pi_2^2 = \Pi(d_0^2, d_1^2, d_2^2 | t^* = 2, \theta = 7)$$

$$\Pi_3^3 = \Pi(d_0^3, d_1^3, d_2^3 | t^* = 3, \theta = 7) \quad \Pi_2^3 = \Pi(d_0^3, d_1^3, d_2^3 | t^* = 2, \theta = 7)$$

$$\Pi_3^4 = \Pi(d_0^4, d_1^4, d_2^4 | t^* = 3, \theta = 7) \quad \Pi_2^4 = \Pi(d_0^4, d_1^4, d_2^4 | t^* = 2, \theta = 7)$$

The expected utility to the central headquarters from the direct mechanism is:

$$\sum_{t \in T} \sum_{d \in D} P(t) \Pi(d|t, \theta) f_0(d, t, \theta)$$

where  $T = (t=5, t=4, t=3, t=2)$

$$D = (d_0^1 d_1^1 d_2^1, d_0^2 d_1^2 d_2^2, d_0^3 d_1^3 d_2^3, d_0^4 d_1^4 d_2^4)$$

It is required that  $\Pi(d|t, \theta) \geq 0$

$$\text{and } \sum_{d \in D} \Pi(d|t, \theta) = 1$$

for all  $d \in D$  and all  $t \in T$

in order to ensure that  $\Pi(d|t)$  is a valid conditional probability distribution. In addition, incentive compatibility constraints for the manufacturing division require that:

$$\sum_{t \in T} \sum_{d \in D} P(t) \Pi(d|t, \theta) U_m(d, t, \theta)$$

$$\geq \sum_{t \in T} \sum_{d \in D} P(t) \Pi(d|\hat{t}, \theta) U_m(d_{>1c}, \hat{\delta}_1(d_1), t)$$

for all  $t \in T$  and all  $\hat{t} \in T$  and all  $\hat{\delta}_1: d_1 \rightarrow d_1$

where  $d = (d_0 d_1 d_2)$

and  $(d_{>1c}, \hat{\delta}_1(d_1)) = (d_0, \hat{\delta}_1(d_1), d_2)$

Also the individual rationality constraints for the manufacturing division require

$$U_m(\pi|t) \geq 0 \quad \text{for all } t \in T$$

Thus, we can write the overall problem for the central headquarters as:

max

$$\begin{aligned} & \{ \cdot 1957 \pi_5^1 + \cdot 2185 \pi_5^2 + \cdot 2526 \pi_5^3 + \cdot 3093 \pi_5^4 \\ & + \cdot 5870 \pi_4^1 + \cdot 6556 \pi_4^2 + \cdot 7578 \pi_4^3 + \cdot 9279 \pi_4^4 \\ & + \cdot 7826 \pi_3^1 + \cdot 8741 \pi_3^2 + 1 \cdot 0104 \pi_3^3 + 1 \cdot 2372 \pi_3^4 \\ & + \cdot 3913 \pi_2^1 + \cdot 4370 \pi_2^2 + \cdot 5052 \pi_2^3 + \cdot 6186 \pi_2^4 \} \end{aligned}$$

$$\text{s.t.} \quad \pi_5^1 + \pi_5^2 + \pi_5^3 + \pi_5^4 = 1 \quad (1)$$

$$\pi_4^1 + \pi_4^2 + \pi_4^3 + \pi_4^4 = 1 \quad (2)$$

$$\pi_3^1 + \pi_3^2 + \pi_3^3 + \pi_3^4 = 1 \quad (3)$$

$$\pi_2^1 + \pi_2^2 + \pi_2^3 + \pi_2^4 = 1 \quad (4)$$

$$\cdot 4594 \pi_2^1 + \cdot 4497 \pi_2^2 + \cdot 3724 \pi_2^3 \quad (5)$$

$$\geq \cdot 4594 \pi_3^1 + \cdot 4497 \pi_3^2 + \cdot 3724 \pi_3^3$$



$$\cdot 4594 \pi_2^1 + \cdot 4497 \pi_2^2 + \cdot 3724 \pi_2^3 \quad (6)$$

$$\geq \cdot 4594 \pi_4^1 + \cdot 4497 \pi_4^2 + \cdot 3724 \pi_4^3$$

$$\cdot 4594 \pi_2^1 + \cdot 4497 \pi_2^2 + \cdot 3724 \pi_2^3 \quad (7)$$

$$\geq \cdot 4594 \pi_5^1 + \cdot 4497 \pi_5^2 + \cdot 3724 \pi_5^3$$

$$\cdot 3363 \pi_3^1 + \cdot 2501 \pi_3^2 - 1 \cdot 0504 \pi_3^4 \quad (8)$$

$$\geq \cdot 3363 \pi_2^1 + \cdot 2501 \pi_2^2 - 1 \cdot 0504 \pi_2^4$$

$$\cdot 3363 \pi_3^1 + \cdot 2501 \pi_3^2 - 1 \cdot 0504 \pi_3^4 \quad (9)$$

$$\geq \cdot 3363 \pi_4^1 + \cdot 2501 \pi_4^2 - 1 \cdot 0504 \pi_4^4$$

$$\cdot 3363 \pi_3^1 + \cdot 2501 \pi_3^2 - 1 \cdot 0504 \pi_3^4 \quad (10)$$

$$\geq \cdot 3363 \pi_5^1 + \cdot 2501 \pi_5^2 - 1 \cdot 0504 \pi_5^4$$

$$\cdot 1822 \pi_4^1 - \cdot 4661 \pi_4^3 - 2 \cdot 1165 \pi_4^4 \quad (11)$$

$$\geq \cdot 1822 \pi_5^1 - \cdot 4661 \pi_5^3 - 2 \cdot 1165 \pi_5^4$$

$$\cdot 1822 \pi_4^1 - \cdot 4661 \pi_4^3 - 2 \cdot 1165 \pi_4^4 \quad (12)$$

$$\geq \cdot 1822 \pi_3^1 - \cdot 4661 \pi_3^3 - 2 \cdot 1165 \pi_3^4$$

$$\cdot 1822 \pi_4^1 - \cdot 4661 \pi_4^3 - 2 \cdot 1165 \pi_4^4 \quad (13)$$

$$\geq \cdot 1822 \pi_2^1 - \cdot 4661 \pi_2^3 - 2 \cdot 1165 \pi_2^4$$

$$\begin{aligned} & -\cdot 2953\pi_5^2 - 1\cdot 0174\pi_5^3 - 3\cdot 4995\pi_5^4 & (14) \\ \geq & -\cdot 2953\pi_4^2 - 1\cdot 0174\pi_4^3 - 3\cdot 4995\pi_4^4 \end{aligned}$$

$$\begin{aligned} & -\cdot 2953\pi_5^2 - 1\cdot 0174\pi_5^3 - 3\cdot 4995\pi_5^4 & (15) \\ \geq & -\cdot 2953\pi_3^2 - 1\cdot 0174\pi_3^3 - 3\cdot 4995\pi_3^4 \end{aligned}$$

$$\begin{aligned} & -\cdot 2953\pi_5^2 - 1\cdot 0174\pi_5^3 - 3\cdot 4995\pi_5^4 & (16) \\ \geq & -\cdot 2953\pi_2^2 - 1\cdot 0174\pi_2^3 - 3\cdot 4995\pi_2^4 \end{aligned}$$

$$-\cdot 2953\pi_5^2 - 1\cdot 0174\pi_5^3 - 3\cdot 4995\pi_5^4 \geq 0 \quad (17)$$

$$\cdot 1822\pi_4^1 - \cdot 4661\pi_4^3 - 2\cdot 1165\pi_4^4 \geq 0 \quad (18)$$

$$\cdot 3363\pi_3^1 + \cdot 2501\pi_3^2 - 1\cdot 0504\pi_4^4 \geq 0 \quad (19)$$

$$\cdot 4594\pi_2^1 + \cdot 4497\pi_2^2 + \cdot 3724\pi_2^3 \geq 0 \quad (20)$$

$$\text{all } \pi_i^i \geq 0$$

The final basis for this linear programming problem has  
objective function value = 1.9566

$$\pi_5^1 = 1 \qquad S_{18} = \cdot 1822$$

$$\pi_4^1 = 1 \qquad S_{19} = \cdot 3363$$

$$\pi_3^1 = 1 \qquad S_{20} = \cdot 4594$$

$$\pi_2^1 = 1$$

Where the S-variables are surplus variables and where the subscript indicates the constraint they pertain to.

Thus, the optimal mechanism would be implementable by the following decision rule. No matter what type the manufacturing divisional manager claims to be, the central headquarters will always set the ordered  $d_0$ -tuple to be:

$$d_0^1 = (5.1899, .377, .559, .5869, .3913, .3914, 1.5651)$$

$$d_1^1 = .5869$$

$$d_2^1 = .3913$$

It is quite easy to see that the reason why this allocation mechanism ensures incentive compatibility is because  $d_0$  is constant no matter what type is reported and the manufacturing divisional manager will always prefer  $d_0^1, d_1^1, d_2^1$ , to be implemented (whatever type the division is) rather than another set of controls and recommendations.

It is interesting to compare this incentive compatible solution with the solution that would arise if the central headquarters had perfect information, being able to observe all decision variables and outcomes. If the manufacturing divisional manager claimed his or her division's types variable was  $t=4$ , in the perfectly observable world, the central headquarters would set  $\pi_4^2 = 1$ , that is use controls and recommendations  $d_0^2 d_1^2 d_2^2$ . This would mean that the transfer price would be set at 4.7468 and the manufacturing

division will be requested to produce .4608 units of the intermediate product being paid .6563 units provided the division earns the target level of profit of .4373 units.

In the example above, where the central headquarters does not have perfect observability, the central headquarters would need to set  $\Pi_4^1 = 1$  (plus other variables as specified in the solution) in order to ensure incentive compatibility. Clearly, the manufacturing divisional manager expects to be .1822 units better off when there is not perfect observability and the central headquarters requires incentive compatibility. Also the central headquarters can expect to be .2287 units (2.1852 - 1.9565) worse off. However, note that the central headquarters does not have a conscious choice between perfect and imperfect observability. It must do the best it can with imperfect observability. Thus, one can interpret the .1822 units extra as an expected premium which the divisional manager is paid for disseminating private information. I find this interpretation appealing, because throughout the analysis we have been assuming that divisional managers provide two categories of services to the corporation. These are the effort services they provide and the specialist information that they collect and can convey. Given that both services are valuable the divisional managers should be paid for providing both. That is, they should not only be compensated for providing effort, but also for providing information.

On accepting these principles, this gives rise to the difficult problem of how then could a central headquarters value the information transmission services that divisional managers provide. This thesis suggests that divisional managers should be rewarded just enough for their information provision services, so as to ensure they can never gain from transmitting untruthful information at cost to the central headquarters.

In addition, note that the direct transfer pricing procedure proposed here whereby the central headquarters chooses a pure or mixed strategy in terms of transfer prices that would have been set under complete observability is not the only procedure possible. For instance, given the Revelation principle, there may be an indirect transfer pricing procedure that would implement the same outcome.

### 6.3 Virtual Utility as an Explanation of Bayesian Incentive Compatible Mechanisms.

So far, it has been implicitly assumed that the central headquarters is free to choose whatever allocation mechanism it wishes and that it will always choose an individually rational, incentive compatible mechanism. However, let us consider what conditions need to be satisfied in order that such mechanisms exist.

Recall the overall problem for the central headquarters is:

$$\Pi(d|t) \max_{t \in T} \sum_{d \in D} P(t) \Pi(d|t) u_0(d, t)$$

s.t. (2), (5) and (6)

The Lagrangian for the problem where one only considers<sup>10</sup> the incentive compatibility constraints can be written as

$$\begin{aligned} & \sum_{t \in T} \sum_{d \in D} P(t) \Pi(d|t) u_0(d, t) \\ + & \sum_{i=1}^n \sum_{t_i \in T_i} \sum_{\hat{t}_i \in T_i} \sum_{\delta_i: D_i \rightarrow D_i} \alpha_i^0(\hat{\delta}_i, \hat{t}_i | t_i) \{ U_i(\Pi | t_i) - \\ & U_i^*(\Pi, \hat{\delta}_i, \hat{t}_i | t_i) \} \end{aligned} \quad (9)$$

The saddlepoint conditions of Lagrangian analysis require that:

$$\sum_{t \in T} \sum_{d \in D} P(t) \Pi^*(d|t) u_0(d, t) = \max_{\pi(d|t) \in \Pi} \left( \sum_{t \in T} \sum_{d \in D} P(t) \pi(d|t) u_0(d, t) \right)$$

$$+ \sum_{i=1}^n \sum_{t_i \in T_i} \sum_{\hat{t}_i \in T_i} \sum_{\delta_i: D_i \rightarrow D_i} \alpha_i^0(\hat{\delta}_i, \hat{t}_i | t_i) \{ u_i(\pi | t_i) -$$

$$u_i^*(\pi, \hat{\delta}_i, \hat{t}_i | t_i) \}$$

$\Pi =$  set of allocation mechanisms that satisfy constraint (2).

$$\text{and } \alpha_i^0(\hat{\delta}_i, \hat{t}_i | t_i) \{U_i(\Pi | t_i) - U_i^*(\Pi, \hat{\delta}_i, \hat{t}_i | t_i)\} = 0 \quad (9')$$

for all  $i$ , all  $t_i \in T_i$ , all  $\hat{t}_i \in T_i$  and all  $\hat{\delta}_i: D_i \rightarrow D_i$

$$\text{with } U_i(\Pi | t_i) - U_i^*(\Pi, \hat{\delta}_i, \hat{t}_i | t_i) \geq 0$$

for all  $i$ , all  $t_i \in T_i$ , all  $\hat{t}_i \in T_i$  and all  $\hat{\delta}_i: D_i \rightarrow D_i$

Note that  $\alpha_i^0(\hat{\delta}_i, \hat{t}_i | t_i)$  refers to the Lagrangian multiplier for lying and being disobedient, where it is claimed that the types variable is  $\hat{t}_i$  when really it is  $t_i$ , and then reinterpreting the central headquarters recommendations by utilising function  $\hat{\delta}_i: D_i \rightarrow D_i$ . However, it is possible to separate the Lagrange multiplier into constituent parts as follows. One can determine the Lagrangian multiplier for lying by claiming the types variable to be  $\hat{t}_i$  when it really is  $t_i$  and reinterpreting the central headquarters recommendations in any feasible fashion as:

$$\alpha_i(\hat{t}_i | t_i) = \sum_{\hat{\delta}_i: D_i \rightarrow D_i} \alpha_i^0(\hat{\delta}_i, \hat{t}_i | t_i) \quad (10)$$

with  $\alpha_i(\hat{t}_i | t_i) \geq 0$  for all  $i$ , all  $t_i \in T_i$ , all  $\hat{t}_i \in T_i$

all  $d_i \in D_i$  and all  $\hat{d}_i \in D_i$

However, we are interested in the case where the player lies and chooses the best method by which to reply to the recommendation  $d_i$ , given that the player claims the types variable to be  $\hat{t}_i$  when it really is  $t_i$ .

Let  $\gamma_i(\hat{d}_i | d_i, \hat{t}_i, t_i)$  represent a choice of reaction to the recommendation  $d_i$  when it is claimed that the types variable is  $\hat{t}_i$  when it really is  $t_i$ . Note it may be optimal for the player to choose a mixed strategy over reactions in reply to the recommendation when lying.

Hence we require:

$$\gamma_i(\hat{d}_i | d_i, \hat{t}_i, t_i) \geq 0 \quad (11)$$

for all  $i$ , all  $t_i \in T_i$ , all  $\hat{t}_i \in T_i$ , all  $d_i \in D_i$  and all  $\hat{d}_i \in D_i$

$$\text{and } \sum_{\hat{d}_i \in D_i} \gamma_i(\hat{d}_i | d_i, \hat{t}_i, t_i) = 1 \quad (12)$$

for all  $i$ , all  $t_i \in T_i$ , all  $\hat{t}_i \in T_i$  and all  $d_i \in D_i$

where  $\gamma_i(\hat{d}_i | d_i, \hat{t}_i, t_i)$  is the mixed strategy over reactions which is optimal for the player, that is

$$\begin{aligned} & \sum_{t_{>ic} \in T_{>ic}} \sum_{d \in D} \sum_{\hat{d}_i \in D_i} P_i(t_{>ic} | t_i) \Pi(d | t_{>ic}, \hat{t}_i) \gamma_i(\hat{d}_i | d_i, \hat{t}_i, t_i) \\ & \quad \cdot u_i(d_{>ic}, \hat{d}_i, t) \\ = & \max_{\hat{\delta}_i: D_i \rightarrow D_i} U_i^*(\Pi, \hat{\delta}_i, \hat{t}_i | t_i) \end{aligned} \quad (13)$$

for all  $i$ , all  $\hat{t}_i \in T_i$  and all  $t_i \in T_i$



Thus, when  $\gamma_i(\hat{d}_i | d_i, \hat{t}_i, t_i)$  satisfies (11), (12) and (13) we can determine the Lagrangian multiplier for lying and claiming the types variable to be  $\hat{t}_i$  when it really is  $t_i$  and reacting optimally to the central headquarters recommendations to be

$$\alpha_i(\hat{t}_i | t_i) = \frac{\sum_{\{\hat{\delta}_i | \hat{\delta}_i(d_i) = \hat{d}_i\}} \alpha_i^0(\hat{\delta}_i, \hat{t}_i | t_i)}{\gamma_i(\hat{d}_i | d_i, \hat{t}_i, t_i)}$$

for all  $i$ , all  $d_i \in D_i$ , all  $\hat{d}_i \in D_i$ , all  $t_i \in T_i$  and all  $\hat{t}_i \in T_i$ .

In the case where  $\alpha_i(\hat{t}_i | t_i) = 0$ , then  $\delta_i(\hat{d}_i | d_i, \hat{t}_i, t_i)$  still must be chosen to satisfy (13).

Thus we can now, for instance, express constraint (9)' as requiring:

$$0 = \alpha_i(\hat{t}_i | t_i) \{U_i(\pi | t_i) - \max_{\hat{\delta}_i} U_i^*(\pi, \hat{\delta}_i, \hat{t}_i | t_i)\} = 0 \quad (14)$$

for all  $i$ , all  $\hat{t}_i \in T$  and all  $t_i \in t_i$ .

Note that the saddlepoint conditions require:

$$\alpha_i^0(\hat{t}_i | t_i) \{U_i(\Pi | t_i) - U_i^*(\Pi, \hat{\delta}_i, \hat{t}_i)\} = 0 \quad (15)$$

for all  $i$ , all  $t_i \in T_i$  all  $\hat{t}_i \in T_i$  and all  $\hat{\delta}_i: D_i \rightarrow D_i$

$$\text{and } \alpha_i^0(t_i | \hat{t}_i) \{ U_i(\Pi | \hat{t}_i) - U_i^*(\Pi, \hat{\delta}_i, t_i | \hat{t}_i) \} = 0 \quad (16)$$

for all  $i$ , all  $t_i \in T_i$ , all  $\hat{t}_i \in \hat{T}_i$  and all  $\hat{\delta}_i: D_i \rightarrow D_i$

$$\begin{aligned} \text{then also } & \sum_{i=1}^n \sum_{t_i \in T_i} \sum_{\hat{t}_i \in \hat{T}_i} \sum_{\hat{\delta}_i: D_i \rightarrow D_i} \alpha_i^0(\hat{t}_i | t_i) \{ U_i(\Pi | t_i) - \\ & U_i^*(\Pi, \hat{\delta}_i, \hat{t}_i | t_i) \} = 0 \end{aligned}$$

$$\begin{aligned} \text{and } & \sum_{i=1}^n \sum_{t_i \in T_i} \sum_{\hat{t}_i \in \hat{T}_i} \sum_{\hat{\delta}_i: D_i \rightarrow D_i} \alpha_i^0(t_i | \hat{t}_i) \{ U_i(\Pi | \hat{t}_i) - \\ & U_i^*(\Pi, \hat{\delta}_i, t_i | \hat{t}_i) \} = 0 \end{aligned}$$

$$\begin{aligned} \text{now for each } i & \sum_{t_i \in T_i} \sum_{\hat{t}_i \in \hat{T}_i} \alpha_i(\hat{t}_i | t_i) U_i(\Pi | t_i) \\ & = \sum_{t_i \in T_i} \sum_{\hat{t}_i \in \hat{T}_i} \alpha_i(t_i | \hat{t}_i) U_i(\Pi | \hat{t}_i) \end{aligned}$$

$$\begin{aligned} \text{so that for each } i & \sum_{t_i \in T_i} \sum_{\hat{t}_i \in \hat{T}_i} \alpha_i(\hat{t}_i | t_i) U_i^*(\Pi, \hat{\delta}_i, \hat{t}_i | t_i) \\ & = \sum_{t_i \in T_i} \sum_{\hat{t}_i \in \hat{T}_i} \alpha_i(t_i | \hat{t}_i) U_i^*(\Pi, \hat{\delta}_i, t_i | \hat{t}_i) \end{aligned}$$

Now let us define the following function:

$$\begin{aligned}
v_0(d, t, \alpha, \delta) = & u_0(d, t) + \sum_{i=1}^n \left[ \left( \sum_{\hat{t}_i \in T_i} \alpha_i(t_i | \hat{t}_i) u_i(d, t) \right. \right. \\
& - \sum_{\hat{t}_i \in T_i} \alpha_i(\hat{t}_i | t_i) \sum_{\hat{d}_i \in D_i} \gamma_i(\hat{d}_i | d_i, t_i, \hat{t}_i) \\
& \left. \left. \cdot u_i((d_{sic}, \hat{d}_i), (t_{sic}, \hat{t}_i)) \right) / p_i^*(t_i) \right] \quad (17)
\end{aligned}$$

Thus, equation (9) can be expressed as

$$\sum_{t \in T} P^*(t) \sum_{d \in D} \Pi(d|t) V_0(d, t, \alpha, \delta) \quad (18)$$

where we require:

$$\sum_{d \in D} \Pi(d|t) V_0(d, t, \alpha, \delta) = \max_{d \in D} V_0(d, t, \alpha, \delta) \quad (19)$$

for all  $t \in T$ .

Thus, given the relationship described by equation (17), we can state the following theorem:

**Theorem:** If  $\Pi$  is an incentive-compatible allocation mechanism then  $\Pi$  will maximise the central headquarters utility if and only if there exists vectors  $\alpha$  and  $\delta$  such that constraints (10), (11), (12), (13), (14), (19) are satisfied. As the above analysis shows, the conditions of the Theorem follow from the relationship

$$\sum_{t \in T} \sum_{d \in D} P(t) \Pi(d|t) u_0(d, t)$$

$$+ \sum_{i=1}^n \sum_{t_i \in T_i} \sum_{\hat{t}_i \in T_i} \sum_{\hat{\delta}_i: D_i \rightarrow D_i} \alpha_i^0(\hat{\delta}_i, \hat{t}_i | t_i) \{U_i(\Pi | t_i) - U_i^*(\Pi, \hat{\delta}_i, \hat{t}_i | t_i)\}$$

$$= \sum_{t \in T} p^*(t) \sum_{d \in D} \Pi(d|t) v_0(d, t, \alpha, \gamma)$$

and saddlepoint conditions of Lagrangian analysis.

The function  $v_0(d, t, \alpha, \gamma)$  will be referred to as the central headquarter's virtual utility function. To understand how the central headquarters determines an incentive compatible allocation mechanism, one must therefore consider how the central headquarters virtual utility differs from the utility the central headquarters could achieve if there were no incentive compatibility problems.

Since one of the requirements of Lagrangian analysis is that

$$U_i(\Pi | t_i) - U_i^*(\Pi, \hat{\delta}_i, \hat{t}_i | t_i) \geq 0$$

for all  $i$ , all  $t_i \in T_i$ , all  $\hat{t}_i \in T_i$  and all  $\hat{\delta}_i: D_i \rightarrow D_i$

and since it is also required that

$$\alpha_i(t_i | \hat{t}_i) \geq 0 \text{ and } p_i^*(t_i) \geq 0$$

for all  $t_i \in T_i$  and all  $\hat{t}_i \in T_i$

This means that the bracketed term for equation (17) is positive. That is, it is costly to the central headquarters to ensure incentive compatibility. The level of the cost to the central headquarters is determined by how much extra the central headquarters must pay to the  $i=1, \dots, n$  divisional managers in order to ensure they have no incentive to lie or be disobedient. Thus, when incentive-compatibility problems arise, the strategy of the central headquarters should be to choose the allocation mechanism that maximises the central headquarters' virtual utility.

#### 6.4 Conclusion

In this chapter, I have demonstrated how one can adapt Myerson's Generalized Principal-Agent Model to analyse the problem of transfer pricing. The result of such adaptation is that I have been able to formulate a new method of transfer pricing. One of the major features of the new method is that it requires explicit recognition to be given to the role of divisional managers as information specialists. It is argued that, unless divisional managers are rewarded for carrying out this role, they will use their asymmetric access to information to further their own goals, in a way which is non-optimal for the organization as a whole. In addition, it is shown that the setting of appropriate rewards is difficult. It is demonstrated that the reward structure has to be derived from a linear program which models the incentive problems within the organization.

It is demonstrated in Section 6.3 that the above arguments for divisional managers being rewarded for being information specialists (in order to stop them exploiting their asymmetric access to information) can be derived explicitly from the central headquarter's general utility maximization problem. In the section, the central headquarter's virtual utility function is defined. It is shown that, when incentive compatibility problems arise, the central headquarters should adopt an objective of virtual utility maximization. The central headquarter's virtual utility differs from conventional utility (when there is no incentive compatibility problems) because it includes the costs of ensuring incentive compatibility when such problems could arise.

CHAPTER 6NOTES

1. The original Generalised Principal-Agent Model is due to Myerson (1982).
2. As will be clearly shown later, it will be assumed that  $d_0$  also specifies divisional manager's compensation functions.
3. This allows us to model a situation where a supplying and buying division face different transfer prices for the same intermediate product, if required.
4. Before introducing uncertainty regarding the value of  $t$ .
5. The rationale for choice of a mixed strategy will be discussed in some detail below.
6. In this case, the central headquarters need only consider pure strategies.
7. This is assuming the divisions achieve their budgeted profit level which they will do if they are truthful and obedient or they locate at one of the boxes below the diagonal. At points above the diagonal, they could not achieve their budgeted level of profit using individually rational methods.
8. The "would have" proviso refers to the fact that this is the strategy the central headquarters would follow if it was convinced that the manufacturing divisional manager was honest and obedient. If it is not so convinced, it may adopt an alternative strategy.
9. For instance, assume the units of compensation and profits are thousands of pounds sterling.
10. That is, where all payoffs are defined so that they are individually rational and all mechanisms are required to be feasible.

CHAPTER 6  
APPENDIX

LET US CONSIDER A SUPPLYING DIVISION WITH THE FOLLOWING PRODUCTION FUNCTION FOR AN INTERMEDIATE PRODUCT :

$$y = f(x_1, x_2) = \alpha x_1^\alpha x_2^B$$

$$\text{with } 0 < \alpha + B < 1$$

$$\text{Thus } \pi_s = p f(x_1, x_2) - e_s x_1 - t x_2$$

1<sup>ST</sup> ORDER CONDITIONS FOR A MAXIMUM REQUIRE

$$\frac{\partial \pi}{\partial x_1} = p \frac{\partial f(\cdot, \cdot)}{\partial x_1} - e_s = 0 \quad (1)$$

$$\frac{\partial \pi}{\partial x_2} = p \frac{\partial f(\cdot, \cdot)}{\partial x_2} - t = 0 \quad (2)$$

2<sup>ND</sup> ORDER CONDITIONS ARE SATISFIED SINCE IT SHALL BE ASSUMED THAT  $0 < \alpha + B < 1$

$$\left. \begin{array}{l} \textcircled{1} \text{ gives } \alpha x_1^{\alpha-1} x_2^B = \frac{e_s}{p} \\ \textcircled{2} \text{ gives } B x_1^\alpha x_2^{B-1} = \frac{t}{p} \end{array} \right\} \begin{array}{l} \text{taking} \\ \text{logs} \\ \text{gives} \end{array} \left. \begin{array}{l} \log \alpha + (\alpha-1) \log x_1 + B \log x_2 = \log \frac{e_s}{p} \quad \textcircled{1}' \\ \log B + \alpha \log x_1 + (B-1) \log x_2 = \log \frac{t}{p} \quad \textcircled{2}' \end{array} \right.$$

$$\textcircled{1}' \times \alpha = \textcircled{3} \quad \alpha \log \alpha + \alpha(\alpha-1) \log x_1 + \alpha B \log x_2 = \alpha \log \frac{e_s}{p}$$

$$\textcircled{2}' \times (\alpha-1) = \textcircled{4} \quad (\alpha-1) \log B + \alpha(\alpha-1) \log x_1 + (\alpha-1)(B-1) \log x_2 = (\alpha-1) \log \frac{t}{p}$$

$$\textcircled{3} - \textcircled{4} = \textcircled{5} \quad \alpha \log \alpha - (\alpha-1) \log B + (\alpha+B-1) \log x_2 = \alpha \log \frac{e_s}{p} - (\alpha-1) \log \frac{t}{p}$$

rearranging terms gives

$$(\alpha+B-1) \log x_2 = \alpha \log \frac{e_s}{p} - (\alpha-1) \log \frac{t}{p} - \alpha \log \alpha + (\alpha-1) \log B$$



$$\text{thus } x_2^* = \left[ \left( \frac{e_s}{P} \right)^\alpha \left( \frac{t}{P} \right)^{-(\alpha-1)} \alpha^{-\alpha} B^{(\alpha-1)} \right]^{\frac{1}{\alpha+B-1}}$$

and

$$x_2^* = \left[ e_s^{-\alpha} t^{(\alpha-1)} p \alpha^\alpha B^{(1-\alpha)} \right]^{\frac{1}{1-\alpha-B}}$$

Now CONSIDER THE FIRST ORDER CONDITIONS AGAIN

$$\textcircled{1}' \times (B-1) = \textcircled{3}' \quad (B-1) \log \alpha + (B-1)(\alpha-1) \log x_1 + (B-1)B \log x_2 = (B-1) \log \frac{e_s}{P}$$

$$\textcircled{2}' \times B = \textcircled{4}' \quad B \log B + B\alpha \log x_1 + (B-1)B \log x_2 = B \log \frac{t}{P}$$

$$\textcircled{3}' - \textcircled{4}' = \textcircled{5}' \quad (B-1) \log \alpha - B \log B + (1-\alpha-B) \log x_1 = (B-1) \log \frac{e_s}{P} - B \log \frac{t}{P}$$

rearranging terms gives

$$(1-\alpha-B) \log x_1 = B \log B + (1-B) \log \alpha + (B-1) \log \frac{e_s}{P} - B \log \frac{t}{P}$$

$$\text{thus } x_1^* = \left( B^B \alpha^{(1-B)} \left( \frac{e_s}{P} \right)^{(B-1)} \left( \frac{t}{P} \right)^{-B} \right)^{\frac{1}{1-\alpha-B}}$$

$$x_1^* = \left( B^B \alpha^{(1-B)} e_s^{(B-1)} t^{-B} P \right)^{\frac{1}{1-\alpha-B}}$$

Now

$$\pi^*(e_s, t, P) = p f(x_1^*, x_2^*) - e_s x_1^* - t x_2^*$$

$$= p \left( B^B \alpha^{(1-B)} e_s^{(B-1)} t^{-B} P \right)^{\frac{\alpha}{1-\alpha-B}} \cdot \left( e_s^{-\alpha} t^{(\alpha-1)} p \alpha^\alpha B^{(1-\alpha)} \right)^{\frac{B}{1-\alpha-B}}$$

$$- e_s \left( B^B \alpha^{(1-B)} e_s^{(B-1)} t^{-B} P \right)^{\frac{1}{1-\alpha-B}}$$

$$- t \left( e_s^{-\alpha} t^{(\alpha-1)} p \alpha^\alpha B^{(1-\alpha)} \right)^{\frac{1}{1-\alpha-B}}$$

$$= \left[ e_s^{-\alpha} t^{-\beta} \alpha^\alpha \beta^\beta p \right]^{\frac{1}{1-\alpha-\beta}} \cdot (1 - \alpha - \beta) \quad (6)$$

NOW CONSIDER A DISTRIBUTION DIVISION WITH THE FOLLOWING PRODUCTION FUNCTION FOR THE FINAL PRODUCT :

$$q = g(y, z) = y^\delta z^\delta$$

with  $0 < \delta + \delta < 1$

$$\text{Thus } \pi_D = \theta g(y, z) - p y - e_D z$$

1<sup>ST</sup> ORDER CONDITIONS FOR A MAXIMUM REQUIRE :

$$\frac{\partial \pi}{\partial y} = \theta \frac{\partial g(\cdot, \cdot)}{\partial y} - p = 0 \quad (7)$$

$$\frac{\partial \pi}{\partial z} = \theta \frac{\partial g(\cdot, \cdot)}{\partial z} - e_D = 0 \quad (8)$$

2<sup>ND</sup> ORDER CONDITIONS ARE SATISFIED SINCE  $0 < \delta + \delta < 1$

$$\left. \begin{array}{l} (7) \text{ gives } \delta y^{\delta-1} z^\delta = \frac{p}{\theta} \\ (8) \text{ gives } \delta y^\delta z^{\delta-1} = \frac{e_D}{\theta} \end{array} \right\} \begin{array}{l} \text{taking} \\ \text{logs} \\ \text{gives} \end{array} \left. \begin{array}{l} \log \delta + (\delta-1) \log y + \delta \log z = \log \frac{p}{\theta} \quad (7)' \\ \log \delta + \delta \log y + (\delta-1) \log z = \log \frac{e_D}{\theta} \quad (8)' \end{array} \right\}$$

$$(7)' \times (\delta-1) = (9) = (\delta-1) \log \delta + (\delta-1)(\delta-1) \log y + (\delta-1)\delta \log z = (\delta-1) \log \frac{p}{\theta}$$

$$(8)' \times \delta = (10) = \delta \log \delta + \delta \log y + (\delta-1)\delta \log z = \delta \log \frac{e_D}{\theta}$$

$$(9) - (10) = (11) \quad (\delta-1) \log \delta - \delta \log \delta + (1-\delta-\delta) \log y = (\delta-1) \log \frac{p}{\theta} - \delta \log \frac{e_D}{\theta}$$

$$= \frac{1}{1-\alpha-\beta} \left( \beta^\alpha \alpha^\alpha \Gamma_{-\beta} \rho \right) \left( \beta^\alpha \alpha^\alpha \Gamma_{-\beta} \rho \right) = \frac{1}{\beta} \left( \beta^\alpha \alpha^\alpha \Gamma_{(1-\alpha)} \rho \right) \left( \beta^\alpha \alpha^\alpha \Gamma_{(1-\alpha)} \rho \right)$$

$$\textcircled{12} \quad f(x_1^*, x_2^*) = \left( \beta^\alpha \alpha^\alpha \Gamma_{(1-\beta)} \rho \right) \left( \beta^\alpha \alpha^\alpha \Gamma_{(1-\beta)} \rho \right) = \frac{1}{\alpha} \left( \beta^\alpha \alpha^\alpha \Gamma_{(1-\beta)} \rho \right)$$

from supplying divisions problem we have

$$z^* = \left( \beta^\alpha \alpha^\alpha \Gamma_{(1-\beta)} \rho \right) \left( \beta^\alpha \alpha^\alpha \Gamma_{(1-\beta)} \rho \right) = z^* \log \beta + (1-\beta) \log \alpha + \beta \log \rho$$

rearranging terms gives:

$$\textcircled{9} - \textcircled{10} = \textcircled{11}, \quad \beta \log \alpha - (1-\beta) \log \alpha + (1-\beta) \log \beta = z^* \log \beta + \beta \log \rho - \beta \log \alpha$$

$$\textcircled{8} \times (1-\beta) = \textcircled{10} = (1-\beta) \log \beta + \beta \log \alpha + (1-\beta) \log \alpha + (1-\beta) \log \beta = z^* \log \beta + \beta \log \rho$$

$$\textcircled{7} \times \beta = \textcircled{9} = \beta \log \alpha + (1-\beta) \log \alpha + \beta \log \rho = z^* \log \beta + \beta \log \rho$$

now consider the first order conditions again

$$\text{thus } y^* = \left( \beta^\alpha \alpha^\alpha \Gamma_{(1-\beta)} \rho \right) \left( \beta^\alpha \alpha^\alpha \Gamma_{(1-\beta)} \rho \right) = \frac{1}{1-\beta-\alpha}$$

$$(1-\beta-\alpha) \log y = -(\beta-\alpha) \log \beta + \beta \log \alpha + (1-\beta) \log \beta - \beta \log \alpha - \beta \log \rho$$

rearranging terms gives:

FROM DISTRIBUTION DIVISIONS PROBLEM WE HAVE

$$y^* = \left( \gamma^{(1-\delta)} \delta^\delta \rho^{(\delta-1)} e_s^{-\delta} \theta \right)^{\frac{1}{1-\delta-\delta}}$$

We require  $f(x_1^*, x_2^*) = y^*$

$$\left( B^{-B} \alpha^\alpha e_s^{-\alpha} t^{-B} \right)^{\frac{1}{1-\alpha-B}} P^{\frac{\alpha+B}{1-\alpha-B}} = \left( \gamma^{(1-\delta)} \delta^\delta e_s^{-\delta} \theta \right)^{\frac{1}{1-\delta-\delta}} P^{\frac{\delta-1}{1-\delta-\delta}}$$

rearranging terms gives :

$$P^{\frac{1-\delta-\alpha\delta-B\delta}{(1-\alpha-B)(1-\delta-\delta)}} = \left( \gamma^{(1-\delta)} \delta^\delta e_s^{-\delta} \theta \right)^{\frac{1}{1-\delta-\delta}} \cdot \left( B^{-B} \alpha^\alpha e_s^\alpha t^B \right)^{\frac{1}{1-\alpha-B}}$$

$$P^{\alpha\delta+B\delta+\delta-1} = \left( \gamma^{(\delta-1)} \delta^{-\delta} e_s^\delta \theta^{-1} \right)^{1-\alpha-B} \cdot \left( B^{-B} \alpha^\alpha e_s^\alpha t^B \right)^{(\delta+\delta-1)} \quad (12)$$

NOW CONSIDER THE CENTRAL HEADQUARTER OVERALL PROBLEM:

$$z, x_1, x_2 \max \theta q - e_D z - e_S x_1 - t x_2$$

$$\text{s.t. } q = y^\delta z^\delta$$

$$y = x_1^\alpha x_2^B$$

that is

$$z, x_1, x_2 \max \theta (x_1^\alpha x_2^B z^\delta) - e_D z - e_S x_1 - t x_2$$

1<sup>ST</sup> ORDER CONDITIONS FOR A MAXIMUM REQUIRE :

$$\frac{\partial \Pi}{\partial x_1} = 0 \Rightarrow \alpha \delta x_1^{(\alpha\delta-1)} x_2^{B\delta} z^\delta = \frac{e_S}{\theta} \quad (13)$$

substituting derived equations for  $x_1$  and  $z$  into (13) gives (13)

$$z = B^{-1} \delta R^{-1} x_2 e^{-\frac{\theta}{\rho} t}$$

$$\log z = \log B + \log \delta - \log R + \log x_2 + \log t - \log e^{\frac{\theta}{\rho} t}$$

rearranging terms gives

$$\textcircled{14}' \quad \log z = \log B + \log \delta - \log R + \log x_2 + \log t - \log e^{\frac{\theta}{\rho} t} = \textcircled{17} = \textcircled{15}' - \textcircled{14}$$

$$\textcircled{15}' \quad \log \delta + \alpha \delta \log x_1 + B \delta \log x_2 + (\delta - 1) \log z = \log e^{\frac{\theta}{\rho} t}$$

$$\textcircled{14}' \quad \log B + \log \delta + \alpha \delta \log x_1 + (B \delta - 1) \log x_2 + \delta \log z = \log t e^{\frac{\theta}{\rho} t}$$

$$x_1 = \alpha B^{-1} x_2 e^{\frac{\theta}{\rho} t}$$

$$\log x_1 = \log \alpha - \log B + \log x_2 - \log e^{\frac{\theta}{\rho} t} + \log t$$

rearranging terms gives:

$$\textcircled{13}' - \textcircled{14}' = \textcircled{16} = \log \alpha - \log B - \log x_1 - \log x_2 = \log e^{\frac{\theta}{\rho} t} - \log t$$

$$\textcircled{14}' \quad \log B + \log \delta + \alpha \delta \log x_1 + (B \delta - 1) \log x_2 + \delta \log z = \log t e^{\frac{\theta}{\rho} t}$$

$$\textcircled{13}' \quad \log \alpha + \log \delta + (\alpha \delta - 1) \log x_1 + B \delta \log x_2 + \delta \log z = \log e^{\frac{\theta}{\rho} t}$$

taking logs for (13) and (14) gives

$$0 < \alpha \delta + B \delta + \delta < 1$$

2<sup>ND</sup> ORDER CONDITIONS ARE SATISFIED SINCE WE ARE ASSUMING

$$\frac{\partial \pi}{\partial z} = 0 \Rightarrow B \alpha^{\alpha \delta} x_1^{\alpha \delta} x_2^{B \delta - 1} z^{\delta - 1} = \frac{\theta}{\rho} e^{\frac{\theta}{\rho} t} \quad \textcircled{15}$$

$$\frac{\partial \pi}{\partial x_2} = 0 \Rightarrow B \delta \alpha^{\alpha \delta} x_1^{\alpha \delta} x_2^{B \delta - 1} z^{\delta} = \frac{\theta}{\rho} e^{\frac{\theta}{\rho} t} \quad \textcircled{14}$$

$$\textcircled{13}'' \quad \alpha \delta (\alpha B^{-1} x_2 e_s^{-1} t)^{\alpha \delta - 1} x_2^{B \delta} (B^{-1} \delta \delta^{-1} x_2 t e_D^{-1})^\delta = \frac{e_s}{\theta}$$

$$* x_2 = \left( \alpha^{-\alpha \delta} \gamma^{(\delta-1)} B^{(\delta+\alpha \delta-1)} e_s^{\alpha \delta} t^{(1-\alpha \delta-\delta)} \delta^{-\delta} e_D^\delta \theta^{-1} \right)^{\frac{1}{\alpha \delta + B \delta - 1 + \delta}}$$

from the SUPPLYING DIMENSIONS PROBLEM

$$x_2^* = \left[ e_s^{-\alpha} t^{(\alpha-1)} \alpha^\alpha B^{(1-\alpha)} \right]^{\frac{1}{1-\alpha-B}} p^{\frac{1}{1-\alpha-B}}$$

and we require  $p$  to be such that  $x_2^* = * x_2$  for the central headquarters. That is we require

$$p^{(\alpha \delta + B \delta - 1 + \delta)} = \left[ e_s^\alpha t^{(1-\alpha)} \alpha^{-\alpha} B^{(\alpha-1)} \right]^{(\alpha \delta + B \delta - 1 + \delta)} \cdot \left( \alpha^{-\alpha \delta} \gamma^{(\delta-1)} B^{(\delta+\alpha \delta-1)} e_s^{\alpha \delta} t^{(1-\alpha \delta-\delta)} \delta^{-\delta} e_D^\delta \theta^{-1} \right)^{1-\alpha-B}$$

$$p^{(\alpha \delta + B \delta - 1 + \delta)} = \left( e_s^\alpha t^B \alpha^{-\alpha} B^{-B} \right)^{(\delta+\delta-1)} \cdot \left( \gamma^{(\delta-1)} \delta^{-\delta} e_D^\delta \theta^{-1} \right)^{(1-\alpha-B)} \quad \textcircled{18}$$

Note that  $\textcircled{18}$  is identical to  $\textcircled{12}$ , that is the central headquarters optimal transfer price ensures that  $y^* = f(x_1^*, x_2^*)$

Thus given values for  $\alpha, B, \delta, \delta, e_s, t, e_D, \theta$  the central headquarters can always determine the optimal transfer price via the relationship specified by equation  $\textcircled{18}$

## AN EXAMPLE

$$\text{SUPPOSE } \alpha = 0.3, \beta = 0.5, \gamma = 0.5, \delta = 0.1$$

$$\theta = 7, e_s = 1, e_D = 1$$

UTILISING EQUATION (16) WE HAVE

$$p = (1 \times t^{0.4} \times 1.335 \times 1.3195) \times (0.7792 \times 0.912 \times 1 \times 2.1779)$$

$$\approx 2.7263 t^{0.4}$$

thus if  $t$  takes the value  $t = 5$

$$p = 5.1899$$

given that we have derived the equation

$$y^* = \left( \gamma^{(1-\delta)} \delta^\delta p^{(\delta-1)} e_D^{-\delta} \theta \right)^{\frac{1}{1-\delta-\delta}}$$

for the above assumed values

$$y^* = (0.21 \times 0.5623 \times 0.0246 \times 1 \times 129.6418)$$

$$\approx 0.377$$

and similarly we use the derived equation for  $z^*$  to find that

$$z^* = 0.3913$$

hence

$$q^* = y^{*\delta} z^{*\delta} = 0.559$$

Also  $x_1^*$  and  $x_2^*$  can be determined in a similar fashion by substituting in values into the derived equations.

CHAPTER SEVENTOWARD A GENERAL THEORY OF TRANSFER PRICING WITH  
INCOMPLETE INFORMATION7.1 INTRODUCTION

In Chapter Six, the details of how the central headquarters could implement a transfer pricing procedure were presented for a situation where the incompletely known types variables were discrete random variables, but where a divisional manager observed the actual realised value of the types variable just before the commencement of the communication process. However, it is possible to extend the model to allow analysis of the problem when the types variables are continuous random variables. This extension is most useful as it allows general statements about how transfer pricing mechanisms should be set in an environment of incomplete information, to be presented and proven.

This chapter will, therefore, be concerned with extending the analysis of the previous chapter to the case where the types variables are distributed continuously and where a divisional manager observes the actual value of the types variable.

7.2 Constructing individually rational, incentive compatible  
transfer pricing mechanisms under incomplete information

Let us assume there are  $n$  manufacturing divisions indexed  $i = 1, \dots, n$  and that there are  $m$  distribution divisions indexed  $i = 1, \dots, m$ .

It is assumed that it is common knowledge that the manager of the  $j^{\text{th}}$  manufacturing division attempts to maximise the division's profits by choosing appropriate inputs such that:



$$\begin{aligned}
 x_1^j, x_2^j \max \quad & p y^j - e_m^j x_1^j - t^j x_2^j \\
 \text{s.t.} \quad & y^j = (x_1^j)^{\alpha_j} (x_2^j)^{\beta_j}
 \end{aligned} \tag{1}$$

Likewise, it is assumed that it is common knowledge that the manager of the  $i^{\text{th}}$  distribution division attempts to maximise the division's profits by choosing appropriate inputs such that:

$$\begin{aligned}
 y^i, z^i \max \quad & \theta^i q^i - p y^i - e_D^i z^i \\
 \text{s.t.} \quad & q^i = (y^i)^{\gamma_i} (z^i)^{\delta_i}
 \end{aligned} \tag{2}$$

Assume it is common knowledge that all  $t^j$ ,  $j=1, \dots, n$  are bounded within known intervals  $\underline{t}^j$  to  $\bar{t}^j$  ( $\underline{t}^j < \bar{t}^j$ ) and that it is common knowledge that all  $\theta^i$ ,  $i=1, \dots, m$  are bounded within known interval  $\underline{\theta}^i$  to  $\bar{\theta}^i$  ( $\underline{\theta}^i < \bar{\theta}^i$ )

Following the Bayesian approach, we shall assume that the central headquarters and all divisional managers, ex ante, hold some subjective prior probability distribution for the unknown parameters,  $t^j, \theta^i$  for  $j = 1, \dots, n$ ,  $i = 1, \dots, m$ . Let  $f_j(t^j)$  and  $h_i(\theta^i)$  be the density functions for the probability distributions for  $i=1, \dots, n$ ,  $j=1, \dots, m$ . It is assumed all density functions are continuous in their arguments with  $f_j(t^j) > 0$  over the interval  $\{\underline{t}^j, \bar{t}^j\}$  and  $h_i(\theta^i) > 0$  over the interval  $\{\underline{\theta}^i, \bar{\theta}^i\}$  with  $F_j(t^j)$  and  $H_i(\theta^i)$  denoting the respective cumulative distribution functions. Let the joint density function for all the unknown input prices and uncertain output prices (types variables) be denoted:

$$g(t, \theta) = \prod_{j=1}^n f_j(t_j) \cdot \prod_{i=1}^m h_i(\theta^i) \quad (3)$$

It shall be assumed that the central headquarters wishes to maximise expected profits, that is

$$\begin{aligned} & q^i, z^i, x_1^j, x_2^j \quad \max \quad \sum_{i=1}^m \sum_{j=1}^n \int_{\underline{t}^1}^{\bar{t}^1} \dots \int_{\underline{t}^n}^{\bar{t}^n} \int_{\underline{\theta}^m}^{\bar{\theta}^m} g(t, \theta) \\ & \quad \cdot \{ \theta^i q^i - e_D^i z^i - e_m^j x_1^j - t^j x_2^j \} dt^1 \dots dt^n, \\ & \quad d\theta^1 \dots d\theta^m) \\ & \text{s.t. } q^i = (y^i)^{\gamma_i} (z^i)^{\delta_i} \\ & \quad y^j = (x_1^j)^{\alpha_j} (x_2^j)^{\beta_j} \end{aligned} \quad (4)$$

for  $i=1, \dots, n, j=1, \dots, m$ .

The method by which the central headquarters actively attempts to achieve this goal is by using a transfer pricing mechanism. Thus, the central headquarters first determines the set of enforceable and recommended controls ( $d = d_0 d_i d_j$ ) that it would adopt if it could perfectly observe the types variables and would be sure that divisional managers followed its recommendations and satisfied its controls (budgets). It then chooses the probability distribution (co-ordination mechanism) over control schemes

that it will adopt, conditional on information of the realised value of types variables communicated to it by the divisional managers. Thus, we can now specify the central headquarters problem as:

$$\Pi(d|t, \theta) \max \int \dots \int g(t, \theta) \Pi(d|t, \theta) f_0(d, t, \theta) dt d \theta dd \quad (5)$$

$$\text{where } f_0(d, t, \theta) = \sum_{i=1}^m \sum_{j=1}^n \theta^i q^i - e_D^1 z^1 - e_m^j x_1^j \quad (5)$$

$$- t^j x_2^j$$

$$\text{s.t. } q^i = (y^i)^{\delta_i} (z_i)^{\delta_i}$$

$$y^j = (x_1^j)^{\alpha_j} (x_2^j)^{\beta_j}$$

$$d = (d_{op}, d_{oy}, d_{oq}, d_{oc}, d_{o\pi})$$

$$\text{with } d_{op} = p$$

$$d_{oy}^j = y^j$$

$$d_{oq}^i = q^i$$

$$d_{oc}^j = e_m^j x_1^j$$

$$d_{oc}^i = e_D^i z^i$$

$$d_{o\pi}^j = p y^j - e_m^j x_m^j - t^j x_2^j$$

$$d_{o\pi}^i = \theta^i q^i - p y^1 - e_D^i z^i$$

for  $j=1, \dots, n$ ,  $i=1, \dots, m$ .

Note that in this instance

$$\int \dots \int dt d\theta dd$$

$$= \int_{\underline{t}^1}^{\bar{t}^1} \int_{\underline{t}^n}^{\bar{t}^n} \int_{\underline{\theta}^1}^{\bar{\theta}^1} \int_{\underline{\theta}^m}^{\bar{\theta}^m} \int_{\underline{d}_{o1}}^{\bar{d}_{o1}} \int_{\underline{d}_{on+m}}^{\bar{d}_{on+m}} \int_{\underline{d}_1}^{\bar{d}_1} \int_{\underline{d}_{n+m}}^{\bar{d}_{n+m}}$$

$$dt^1 \dots dt^n, d\theta^1, \dots, d\theta^m, dd_{o1}, \dots, dd_{on+m}, dd_1, \dots, dd_{n+m}$$

Given the central headquarters chooses some co-ordination mechanism  $\Pi(d|t, \theta)$  the expected utility of honest and obedient the/manager of the  $j^{\text{th}}$  manufacturing division (who observes the division's types variable to be  $t^j$ ) from the co-ordination mechanism, if all other divisional managers report truthfully and act obediently is

$$u_j(\Pi|t^j) = \int \dots \int g(t^{>j}, \theta|t^j) \Pi(d|t, \theta) \cdot u_j(d, t, \theta) dt^{>j} d\theta dd \quad (6)$$

where  $\int \dots \int dt^{>j} d\theta dd$  is/as above except that a term for the expectation over  $t^j$ , ( $\int_{\underline{t}^j}^{\bar{t}^j} dt^j$ ) is not included as the manager has observed the actual value of the types variable for the division in the forthcoming period.  
honest and obedient

Similarly for the/manager of distribution division  $i$ , who observes privately the output price  $\theta^1$ , we have

$$U_i(\Pi|\theta^1) = \int \dots \int g(t, \theta^{1c} | \theta^1) \Pi(d|t, \theta) \cdot u_i(d, t, \theta) dt d \theta^{1c} dd \quad (7)$$

where this time no  $\int_{\underline{\theta}^i}^{\bar{\theta}^i} d\theta^i$  term appears.

Given that it is being assumed that the division's types variables are independent random variables in the common prior

$$g(t^{>jc}, \theta|t^j) = \frac{g(t, \theta)}{f_j(t^j)}$$

Thus, the expected utility to divisional manager  $j$ , ex ante, before his or her division's type is revealed (from being honest and obedient)

$$\int_{\underline{t}^j}^{\bar{t}^j} f_j(t^j) U_j(\Pi|t^j) dt^j = \int_{\underline{t}^1}^{\bar{t}^1} \int_{\underline{d}_{n+m}}^{\bar{d}_{n+m}} g(t, \theta) \Pi(d|t, \theta) u_j(d, t, \theta) dt d \theta dd \quad (8)$$

Similarly for divisional manager  $i$ , ex ante, before his or her division's type is revealed his or her expected utility is: (from being honest and obedient)

$$\begin{aligned}
 & \int_{\underline{\theta}^1}^{\bar{\theta}^1} h_i(\theta^1) u_i(\Pi|\theta_i) d\theta^1 \\
 & = \int_{\underline{t}^1}^{\bar{t}^1} \dots \int_{\underline{d}_{n+m}}^{\bar{d}_{n+m}} g(t, \theta) \Pi(d|t, \theta) u_i(d|t, \theta) dt d\theta dd
 \end{aligned} \tag{9}$$

In order to ensure that  $\Pi(d|t, \theta)$  is a valid conditional probability function we require that:

$$\begin{aligned}
 & \Pi(d|t, \theta) \geq 0 \\
 & \text{and } \int_{\underline{d}_{01}}^{\bar{d}_{01}} \dots \int_{\underline{d}_{n+m}}^{\bar{d}_{n+m}} \Pi(d|t, \theta) dd = 1
 \end{aligned} \tag{10}$$

for all  $(t, \theta) \in T \times \Theta$

Let us now consider the incentive compatibility constraints. We require, that if division  $j$ 's types variable is  $t^j$ , that the divisional manager does at least as well by being honest and obedient as by reporting  $\hat{t}^j$  and then using  $\delta_j(d_j)$  when requested to use  $d_j$ , given that other managers are planning to be honest and obedient. Thus it is required that:

$$\begin{aligned}
 & U_j(\Pi|t^j) \geq U_j^*(\Pi, \delta_j, \hat{t}^j|t^j) \\
 & \text{for any } \hat{t}^j \neq t^j \text{ where } \hat{t}^j \in T^j
 \end{aligned} \tag{11}$$

for all  $j=1, \dots, n,$

Here  $U_j^*(\Pi, \delta_j, \hat{t}^j | t^j)$  denotes the expected utility of divisional manager  $j$  if he or she untruthfully communicated the  $j^{\text{th}}$  division's types variable to be  $\hat{t}^j$ , when it really was  $t^j$ , and where effort level  $\delta_j(d_j)$  was used rather than the recommended  $d_j$ . Thus we have:

$$U_j^*(\Pi, \delta_j, \hat{t}^j | t^j) = \int_{\underline{t}^1}^{\bar{t}^1} \dots \int_{\underline{d}_{n+m}}^{\bar{d}_{n+m}} g(t^{>jc}, \theta | t^j) \Pi(d | t^{>jc}, \hat{t}^j, \theta) \cdot u_j((d_{>jc}, \delta_j(d_j)), t, \theta) dt^{>jc} d\theta dd \quad (12)$$

where  $(t^{>jc}, \hat{t}^j, \theta) = (t^1, \dots, t^{j-1}, \hat{t}^j, t^{j+1}, \dots, t^n, \theta^1, \dots, \theta^m)$

and  $(d_{>jc}, \delta_j(d_j)) = (d_0, d_1, \dots, d_{j-1}, \delta_j(d_j), d_{j+1}, \dots, d_n, d_{n+1}, \dots, d_{n+n})$

Similarly then we require:

$$U_i(\Pi | \theta^i) \geq U_i^*(\Pi, \delta_i, \hat{\theta}^i | \theta^i) \quad (13)$$

for any  $\hat{\theta}^i \neq \theta^i$  where  $\hat{\theta}^i \in \theta^i$

for all  $i=1, \dots, m$

$$\text{where } U_i(\Pi, \delta_i, \hat{\theta}^i | \theta^i) = \int_{\underline{t}^1}^{\bar{t}^1} \dots \int_{\underline{d}_{n+m}}^{\bar{d}_{n+m}} g(t, \theta^{>i<} | \theta^i)$$

$$\Pi(d | t, \theta^{>i<}, \hat{\theta}^i) \tag{14}$$

$$\cdot u_i(d_{>n+i<}, \delta_i(d_{n+i}) | t, \theta) dt d\theta^{>i<} dd$$

$$\text{Where } (t, \theta^{>i<}, \hat{\theta}^i) = (t^1, \dots, t^n, \theta^1, \dots, \theta^{i-1}, \hat{\theta}^i, \theta^{i+1}, \dots, \theta^m)$$

$$\text{and } (d_{>n+i<}, \delta_i(d_{n+i})) = (d_0, d_1, \dots, d_n, d_{n+1}, \dots, d_{n+i-1},$$

$$\delta_i(d_{n+i}), d_{n+i+1}, \dots, d_{n+m})$$

$$\text{Let } \bar{\Pi}(d | \theta^i, \cdot, \cdot) = \int_{\underline{t}^1}^{\bar{t}^1} \dots \int_{\underline{d}_{n+m}}^{\bar{d}_{n+m}} g(\cdot, \cdot | \theta^i) \Pi(d | \theta^i, \cdot, \cdot)$$

$$\cdot dt^j d\theta^{>i<} dd$$

$$\text{And } \bar{\Pi}(d | t^j, \cdot, \cdot) = \int_{\underline{t}^1}^{\bar{t}^1} \dots \int_{\underline{d}_{n+m}}^{\bar{d}_{n+m}} g(\cdot, \cdot | t^j) \Pi(d | t^j, \cdot, \cdot)$$

$$\cdot dt^{>j<} d\theta dd$$

Individual rationality constraints require

$$U_j(\Pi | t^j) \geq 0 \text{ for all } j=1, \dots, n \tag{15}$$

$$U_i(\Pi | \theta^i) \geq 0 \text{ for all } i=1, \dots, m \tag{16}$$



Thus, the central headquarter's objective is to determine the feasible allocation mechanism  $\Pi(d|t, \theta)$  which defines a probability distribution over transfer prices, production, compensation and divisional profits, which maximises the expected profits that accrues to the central headquarters. That is

$$\Pi(d|t, \theta) \max \int_{\underline{t}^1}^{\bar{t}^1} \cdots \int_{\underline{d}_{n+m}}^{\bar{d}_{n+m}} g(t, \theta) \Pi(d|t, \theta) f_0(d, t, \theta) \cdot dt d\theta dd \quad (17)$$

subject to (5)', (10), (11), (13), (15), (16)

**THEOREM 1.** There exists controls and recommendations  $d$  such that the co-ordination mechanism  $\Pi(d|t, \theta)$  is an incentive compatible and individually rational mechanism if, and only if,  $\bar{\Pi}(d|t^j, \cdot, \cdot)$  is a non-increasing function for all manufacturing divisional managers  $j=1, \dots, n$  and  $\bar{\Pi}(d|\theta^i, \cdot, \cdot)$  is a non-decreasing function for all distribution divisional managers  $i=1, \dots, m$ . Furthermore, it is required that

$$\sum_{j=1}^n \int_{\underline{t}^1}^{\bar{t}^1} \cdots \int_{\underline{d}_{n+m}}^{\bar{d}_{n+m}} \left( u_j(d, t, \theta) + \frac{F_j(t^j) - 1}{f_j(t^j)} \right) g(t, \theta) \cdot \Pi(d|t, \theta) dt d\theta dd$$

$$\begin{aligned}
& + \sum_{i=1}^m \int_{\underline{t}^1}^{\bar{t}^1} \dots \int_{\underline{d}_{n+m}}^{\bar{d}_{n+m}} \left( u_i(d, t, \theta) - \frac{H_i(\theta^i)}{h_i(\theta^i)} \right) g(t, \theta) \Pi(d|t, \theta) \\
& \quad \cdot dt d\theta dd
\end{aligned}$$

$\geq 0$

In addition, given any individually rational incentive compatible mechanism, for all  $i$  and  $j$ ,  $U_j(\Pi|t^j)$  is non-increasing,  $U_i(\Pi|\theta^i)$  is non-decreasing and

$$\begin{aligned}
& \sum_{j=1}^n U_j(\Pi|\bar{t}^j) + \sum_{i=1}^m U_i(\Pi|\underline{\theta}^i) \\
= & \sum_{j=1}^n \min_{t^j \in \{\underline{t}^j, \bar{t}^j\}} U_j(t^j) + \sum_{i=1}^m \min_{\theta^i \in \{\underline{\theta}^i, \bar{\theta}^i\}} U_i(\theta^i) \\
= & \sum_{j=1}^n \int_{\underline{t}^1}^{\bar{t}^1} \dots \int_{\underline{d}_{n+m}}^{\bar{d}_{n+m}} \left( u_j(d, t, \theta) + \frac{F_j(t^j) - 1}{f_j(t^j)} \right) g(t, \theta) \Pi(d|t, \theta) \\
& \quad \cdot dt d\theta dd \\
& + \sum_{i=1}^m \int_{\underline{t}^1}^{\bar{t}^1} \dots \int_{\underline{d}_{n+m}}^{\bar{d}_{n+m}} \left( u_i(d, t, \theta) - \frac{H_i(\theta^i)}{h_i(\theta^i)} \right) g(t, \theta) \Pi(d|t, \theta) \\
& \quad \cdot dt d\theta dd
\end{aligned}$$

Proof of the theorem is given in the Appendix to this chapter.

Theorem 1 establishes the conditions that transfer pricing mechanisms must satisfy if they are to be individually rational and incentive compatible for divisional managers. The central headquarters should, therefore, attempt to determine the mechanism  $\Pi^*(\bar{d}|t, \theta)$  for which

$$\Pi^*(d|t, \theta) \max \int_{\underline{t}^1}^{\bar{t}^1} \int_{\underline{d}_{n+m}}^{\bar{d}_{n+m}} g(t, \theta) \Pi(d|t, \theta) U_0(d, t, \theta) dt d \theta_{00}$$

subject to the conditions of Theorem 1 being satisfied.

### 7.3 Conclusion

Even though the central headquarters's problem developed in section 7.2 may be a very difficult one to solve, its specification alone gives us some valuable intuition as to qualitatively how transfer prices should be set. One of the requirements of Theorem 1 is that  $U_j(\Pi|t, \theta^j, \cdot)$  be non-increasing and  $U_i(\Pi|\theta, \theta^i, \cdot)$  be non-decreasing. Recall  $t^j$  is the cost manufacturing division  $j$ 's manager incurs in buying in raw material  $x_2$ . Thus, manufacturing divisional managers can only expect to increase their expected utility by buying in raw materials at a lower cost. Similarly, distributional divisional managers can only expect to increase their expected utility by being able to sell the final product at higher prices, given the monotonicity requirements.

## APPENDIX

Proof of the theorem consists of the three lemmas that follow

LEMMA 1. If  $\pi(d|t, \theta)$  is an incentive compatible mechanism then for all  $i$  and  $j$ ,  $\bar{\pi}(d|t^j, \cdot, \cdot)$  and  $U_j(\pi|t^j)$  are non increasing while  $\bar{\pi}(d|\theta^i, \cdot, \cdot)$  and  $U_i(\pi|\theta^i)$  are non decreasing. Furthermore we have

$$U_j(\pi|t^j) = U_j(\pi|\bar{t}^j) + \int_{\underline{t}^j}^{\bar{t}^j} \bar{\pi}(d|t^j, t^{>jc}, \theta) dt^j$$

$$\text{and } U_i(\pi|\theta^i) = U_i(\pi|\underline{\theta}^i) + \int_{\underline{\theta}^i}^{\theta^i} \bar{\pi}(d|\theta^i, t, \theta^{>i c}) d\theta^i$$

## PROOF OF LEMMA 1

Let us consider a typical manufacturing division  $j$ . Recall equation (6).

$$(6) U_j(\pi|t^j) = \int_{\underline{t}^j}^{\bar{t}^j} \dots \int_{\underline{d}_{n+m}}^{\bar{d}_{n+m}} g(t^{>jc}, \theta|t^j) \pi(d|t, \theta)$$

$$\cdot u_j(d, t, \theta) dt^{>jc} d\theta dd.$$

so similarly we have

$$(38) U_j(\pi|\hat{t}^j) = \int_{\underline{t}^j}^{\bar{t}^j} \dots \int_{\underline{d}_{n+m}}^{\bar{d}_{n+m}} g(t^{>jc}, \theta|\hat{t}^j) \pi(d|t^{>jc}, \hat{t}^j, \theta)$$

$$\cdot u_j(d, t^{>jc}, \hat{t}^j, \theta) dt^{>jc} d\theta dd$$

From (11) and (12) for incentive compatibility

$$U_j(\pi|t^j) \geq U_j^*(\pi, \delta_j, \hat{t}^j|t^j)$$

$$= \int_{\underline{t}^j}^{\bar{t}^j} \dots \int_{\underline{d}_{n+m}}^{\bar{d}_{n+m}} g(t^{>jc}, \theta|t^j) \pi(d|t^{>jc}, \hat{t}^j, \theta)$$

$$\cdot u_j((d_{>jc}, \delta_j(d_j)), t, \theta) dt^{>jc} d\theta dd$$

so similarly once again, we require

$$\begin{aligned}
 (39) \quad U_j(\pi | \hat{t}^j) &\geq U_j^*(\pi, \delta_j, t^j | \hat{t}^j) \\
 &= \int_{\underline{t}^j}^{\bar{t}^j} \cdots \int_{\underline{d}_{n+m}}^{\bar{d}_{n+m}} g(t^{2jc}, \theta | \hat{t}^j) \pi(d | t, \theta) \\
 &\quad \cdot u_j((d_{2jc}, \delta_j(d_j)), t^{2jc}, \hat{t}^j, \theta) dt^{2jc} d\theta dd.
 \end{aligned}$$

Now use the convention that

$$g(\cdot, \cdot) \equiv g(t^{2jc}, \theta) = g(t^{2jc}, \theta | \hat{t}^j)$$

where the equality holds since we are assuming that the divisions types are independent random variables. Thus we have

$$\begin{aligned}
 (40) \quad U_j(\pi | t^j) - U_j(\pi | \hat{t}^j) &\geq \int_{\underline{t}^j}^{\bar{t}^j} \cdots \int_{\underline{d}_{n+m}}^{\bar{d}_{n+m}} g(\cdot, \cdot) \pi(d | t^{2jc}, \hat{t}^j, \theta) \\
 &\quad \cdot u_j((d_{2jc}, \delta_j(d_j)), t, \theta) dt^{2jc} d\theta dd. \\
 &- \int_{\underline{t}^j}^{\bar{t}^j} \cdots \int_{\underline{d}_{n+m}}^{\bar{d}_{n+m}} g(\cdot, \cdot) \pi(d | t^{2jc}, \hat{t}^j, \theta) \\
 &\quad \cdot u_j(d, t^{2jc}, \hat{t}^j, \theta) dt^{2jc} d\theta dd
 \end{aligned}$$

Also we have

$$\begin{aligned}
 (41) \quad u_j(\pi | t^j) - u_j(\pi | \hat{t}^j) &\leq \int_{\underline{t}^j}^{\bar{t}^j} \cdots \int_{\underline{d}_{n+m}}^{\bar{d}_{n+m}} g(\cdot, \cdot) \pi(d | t, \theta) \\
 &\quad \cdot u_j(d, t, \theta) dt^{2j^c} d\theta dd. \\
 &\quad - \int_{\underline{t}^j}^{\bar{t}^j} \cdots \int_{\underline{d}_{n+m}}^{\bar{d}_{n+m}} g(\cdot, \cdot) \pi(d | t, \theta) \\
 &\quad \cdot u_j((d_{2j^c}, \delta_j(d_j)), t^{2j^c}, \hat{t}^j, \theta) dt^{2j^c} d\theta dd
 \end{aligned}$$

thus from (40) we have

$$\begin{aligned}
 (42) \quad u_j(\pi | t^j) - u_j(\pi | \hat{t}^j) &\geq \bar{\pi}(d | t^{2j^c}, \hat{t}^j, \theta) \\
 &\quad \cdot [u_j((d_{2j^c}, \delta_j(d_j)), t, \theta) - u_j(d, t^{2j^c}, \hat{t}^j, \theta)] \\
 \text{with } \bar{\pi}(d | t^{2j^c}, \hat{t}^j, \theta) &= \int_{\underline{t}^j}^{\bar{t}^j} \cdots \int_{\underline{d}_{n+m}}^{\bar{d}_{n+m}} g(\cdot, \cdot) \pi(d | t^{2j^c}, \hat{t}^j, \theta) \\
 &\quad \cdot dt^{2j^c} d\theta dd.
 \end{aligned}$$

From (41) we have

$$\begin{aligned}
 (43) \quad u_j(\pi | t^j) - u_j(\pi | \hat{t}^j) &\leq \bar{\pi}(d | t, \theta) \\
 &\quad \cdot [u_j(d, t, \theta) - u_j((d_{2j^c}, \delta_j(d_j)), t^{2j^c}, \hat{t}^j, \theta)] \\
 \text{with } \bar{\pi}(d | t, \theta) &= \int_{\underline{t}^j}^{\bar{t}^j} \cdots \int_{\underline{d}_{n+m}}^{\bar{d}_{n+m}} g(\cdot, \cdot) \pi(d | t, \theta) dt^{2j^c} d\theta dd.
 \end{aligned}$$

It is now possible to formally express the above utilities in terms of the decision variables of a manufacturing divisions problem

Suppose the division's true types variable is  $\hat{t}^j$  and the divisional manager reports this to the central headquarters. For some vector  $d$  let it be such that

$$d = (\hat{d}_0, \hat{d}_j, \hat{d}_i)$$

$$= (\hat{p}, \hat{y}^j, \hat{q}^i, \hat{c}_j, \hat{c}_i, \hat{\pi}_j, \hat{\pi}_i, \hat{d}_j, \hat{d}_i)$$

for  $j = 1, \dots, n$   
 $i = 1, \dots, m$

For divisional manager  $j$  we have

$$(44) \quad u_j(d, t^{j,c}, \hat{t}^j, \theta) = \hat{c}_j - \hat{x}_1^j$$

with 
$$\hat{y}^j = (\hat{x}_1^j)^{\alpha_j} (\hat{x}_2^j)^{\beta_j}$$

thus it is also possible to express (44) as follows

$$(45) \quad u_j(d, t^{j,c}, \hat{t}^j, \theta) = \hat{c}_j - \left( \frac{\hat{y}^j}{(\hat{x}_2^j)^{\beta_j}} \right)^{\frac{1}{\alpha_j}}$$

Suppose however that division  $j$ 's true types variable was  $t^j$  and that the divisional manager reported it to be  $\hat{t}^j$  to the central headquarters. Thus given the same  $d$ , in order to achieve the target level of profit  $\hat{\pi}_j$  the divisional manager can choose

$$x_2^j = \frac{\hat{t}^j \hat{x}_2^j}{t^j}$$

Given the production function constraint we therefore have

$$(46) u_j((d_{2jc}, \delta_j(d_j)), t, \theta) = \hat{c}_j - \left( \frac{\hat{y}_j}{\left( \frac{(\hat{t}_j \hat{x}_2^j)^{\beta_j}}{\hat{t}_j} \right)^{\alpha_j}} \right)^{\frac{1}{\alpha_j}}$$

$$\begin{aligned} \text{thus } u_j((d_{2jc}, \delta_j(d_j)), t, \theta) &- u_j(d, t^{2jc}, \hat{t}_j, \theta) \\ &= \left( \frac{\hat{y}_j}{(\hat{x}_2^j)^{\beta_j}} \right)^{\frac{1}{\alpha_j}} - \left( \frac{\hat{y}_j}{\left( \frac{(\hat{t}_j \hat{x}_2^j)^{\beta_j}}{\hat{t}_j} \right)^{\alpha_j}} \right)^{\frac{1}{\alpha_j}} \\ &= \frac{(\hat{y}_j)^{\frac{1}{\alpha_j}} \left[ (\hat{t}_j)^{\frac{\beta_j}{\alpha_j}} - (t_j)^{\frac{\beta_j}{\alpha_j}} \right]}{(\hat{t}_j)^{\frac{\beta_j}{\alpha_j}} (\hat{x}_2^j)^{\frac{\beta_j}{\alpha_j}}} \\ &= \left( \frac{\hat{y}_j \left[ (\hat{t}_j)^{\beta_j} - (t_j)^{\beta_j} \right]}{(\hat{t}_j)^{\beta_j} (\hat{x}_2^j)^{\beta_j}} \right)^{\frac{1}{\alpha_j}} \end{aligned} \quad (47)$$

Let us now consider the utility expressions for equation (43). Suppose here that division  $j$ 's true types variable was  $t_j$  and that the divisional manager reported it to be  $\hat{t}_j$  to the central headquarters. In order to achieve the targeted level of profit  $\hat{\pi}_j$ , the divisional manager chooses

$$\hat{x}_2^j = \frac{t_j x_2^j}{\hat{t}_j}$$

thus we have

$$(48) u_j((d_{2jc}, \delta_j(d_j)), t^{2jc}, \hat{t}_j, \theta) = \hat{c}_j - \left( \frac{\hat{y}_j}{\left( \frac{(t_j x_2^j)^{\beta_j}}{\hat{t}_j} \right)^{\alpha_j}} \right)^{\frac{1}{\alpha_j}}$$



and similarly

$$(49) \quad u_j(d, t, \theta) = \hat{c}_j - \left( \frac{\hat{y}_j}{(x_2^j)^{\beta_j}} \right)^{\frac{1}{\alpha_j}}$$

thus  $u_j(d, t, \theta) - u_j((d_{j^c}, \delta_j(d_j)), t^{j^c}, \hat{t}^j, \theta)$

$$= \left( \frac{\hat{y}_j}{\left( \frac{t^j x_2^j}{\hat{t}^j} \right)^{\beta_j}} \right)^{\frac{1}{\alpha_j}} - \left( \frac{\hat{y}_j}{(x_2^j)^{\beta_j}} \right)^{\frac{1}{\alpha_j}}$$

$$= \left( \frac{\hat{y}_j \left[ (\hat{t}^j)^{\beta_j} - (t^j)^{\beta_j} \right]}{(t^j)^{\beta_j} (x_2^j)^{\beta_j}} \right)^{\frac{1}{\alpha_j}} \quad (50)$$

thus we can now express (42) and (43) as

$$\bar{\pi}(d | t, \theta) \left( \frac{\hat{y}_j \left[ (\hat{t}^j)^{\beta_j} - (t^j)^{\beta_j} \right]}{(\hat{t}^j)^{\beta_j} (x_2^j)^{\beta_j}} \right)^{\frac{1}{\alpha_j}}$$

$$\geq u_j(\pi | t^j) - u_j(\pi | \hat{t}^j)$$

$$\geq \bar{\pi}(d | t^{j^c}, \hat{t}^j, \theta) \left( \frac{\hat{y}_j \left[ (\hat{t}^j)^{\beta_j} - (t^j)^{\beta_j} \right]}{(\hat{t}^j)^{\beta_j} (x_2^j)^{\beta_j}} \right)^{\frac{1}{\alpha_j}} \quad (51)$$

thus if  $\hat{t}^j > t^j$  we must have

$$\bar{\pi}(d | t^{j^c}, \hat{t}^j, \theta) \leq \bar{\pi}(d | t, \theta)$$

so  $\bar{\pi}(d | t^{2j^c}, \cdot, \theta)$  is a non increasing function. Since  $\bar{\pi}(d | t^{2j^c}, \cdot, \theta)$  is monotonic, dividing (51) by

$$\left( \frac{\hat{y}^j \left[ (\hat{t}^j)^{\beta_j} - (t^j)^{\beta_j} \right]}{(\hat{t}^j)^{\beta_j} (\hat{x}_2^j)^{\beta_j}} \right)^{\frac{1}{\alpha_j}}$$

and taking the limit

as  $\hat{t}^j \rightarrow t^j$  gives the result that

$$u_j'(\pi | t^j) = -\bar{\pi}(d | t^{2j^c}, t^j, \theta) \text{ almost everywhere}$$

Also because of its monotonicity  $\bar{\pi}(d | t^{2j^c}, \cdot, \theta)$  is Riemann integrable so that

$$u_j(\pi | t^j) = u_j(\pi | \bar{t}^j) + \int_{\bar{t}^j}^{t^j} \bar{\pi}(d | t^{2j^c}, t^j, \theta) dt^j \quad (52)$$

Consider now a typical distribution dimension  $i$  :

Recall equation (6)

$$(6) \quad U_i(\pi | \theta^i) = \int_{\underline{t}^i}^{\bar{t}^i} \dots \int_{\underline{d}_{n+m}}^{\bar{d}_{n+m}} g(t, \theta^{2ic} | \theta^i) \pi(d | t, \theta) \cdot u_i(d, t, \theta) dt d\theta^{2ic} dd.$$

so similarly

$$(53) \quad U_i(\pi | \hat{\theta}^i) = \int_{\underline{t}^i}^{\bar{t}^i} \dots \int_{\underline{d}_{n+m}}^{\bar{d}_{n+m}} g(t, \theta^{2ic} | \hat{\theta}^i) \pi(d | t, \theta^{2ic}, \hat{\theta}^i) \cdot u_i(d, t, \theta^{2ic}, \hat{\theta}^i) dt d\theta^{2ic} dd$$

From (13) and (14)

$$\begin{aligned}
 (54) \quad U_i(\pi | \theta^i) &\geq U_i^*(\pi, \delta_i, \hat{\theta}^i | \theta^i) \\
 &= \int_{\underline{t}^i}^{\bar{t}^i} \cdots \int_{\underline{d}_{n+m}}^{\bar{d}_{n+m}} g(t, \theta^{2ic} | \theta^i) \pi(d | t, \theta^{2ic}, \hat{\theta}^i) \\
 &\quad \cdot u_i((d_{2n+1c}, \delta_i(d_{n+i})), t, \theta) dt d\theta^{2ic} dd.
 \end{aligned}$$

so similarly we also require

$$\begin{aligned}
 (55) \quad U_i(\pi | \hat{\theta}^i) &\geq U_i^*(\pi, \delta_i, \theta^i | \hat{\theta}^i) \\
 &= \int_{\underline{t}^i}^{\bar{t}^i} \cdots \int_{\underline{d}_{n+m}}^{\bar{d}_{n+m}} g(t, \theta^{2ic} | \hat{\theta}^i) \pi(d | t, \theta^{2ic}, \theta^i) \\
 &\quad \cdot u_i((d_{2n+1c}, \delta_i(d_{n+i})), t, \theta^{2ic}, \hat{\theta}^i)
 \end{aligned}$$

as before assume

$$g(\cdot, \cdot) \equiv g(t, \theta^{2ic}) = g(t, \theta^{2ic} | \theta^i)$$

Hence we have

$$\begin{aligned}
 (56) \quad U_i(\pi | \theta^i) - U_i(\pi | \hat{\theta}^i) &\geq \int_{\underline{t}^i}^{\bar{t}^i} \cdots \int_{\underline{d}_{n+m}}^{\bar{d}_{n+m}} g(\cdot, \cdot) \pi(d | t, \theta^{2ic}, \hat{\theta}^i) \\
 &\quad \cdot u_i((d_{2n+1c}, \delta_i(d_{n+i})), t, \theta) dt d\theta^{2ic} dd \\
 &\quad - \int_{\underline{t}^i}^{\bar{t}^i} \cdots \int_{\underline{d}_{n+m}}^{\bar{d}_{n+m}} g(\cdot, \cdot) \pi(d | t, \theta^{2ic}, \hat{\theta}^i) \\
 &\quad \cdot u_i(d, t, \theta^{2ic}, \hat{\theta}^i) dt d\theta^{2ic} dd
 \end{aligned}$$

In addition we require

$$\begin{aligned}
 (57) \quad u_i(\pi|\theta^i) - u_i(\pi|\hat{\theta}^i) &\leq \int_{\underline{t}^i}^{\bar{t}^i} \dots \int_{\underline{d}_{n+m}}^{\bar{d}_{n+m}} g(\cdot, \cdot) \pi(d|t, \theta) \\
 &\quad \cdot u_i(d, t, \theta) dt d\theta^{n+c} dd \\
 &\quad - \int_{\underline{t}^i}^{\bar{t}^i} \dots \int_{\underline{d}_{n+m}}^{\bar{d}_{n+m}} g(\cdot, \cdot) \pi(d|t, \theta^{n+c}, \hat{\theta}^i) \\
 &\quad \cdot u_i((d_{n+i+c}, \delta_i(d_{n+i})), t, \theta^{n+c}, \hat{\theta}^i) \\
 &\quad dt d\theta^{n+c} dd.
 \end{aligned}$$

Equation (56) gives

$$\begin{aligned}
 (58) \quad u_i(\pi|\theta^i) - u_i(\pi|\hat{\theta}^i) &\geq \bar{\pi}(d|t, \theta^{n+c}, \hat{\theta}^i) \\
 &\quad \cdot \left[ u_i((d_{n+i+c}, \delta_i(d_{n+i})), t, \theta) \right. \\
 &\quad \left. - u_i(d, t, \theta^{n+c}, \hat{\theta}^i) \right]
 \end{aligned}$$

where

$$\bar{\pi}(d|t, \theta^{n+c}, \hat{\theta}^i) = \int_{\underline{t}^i}^{\bar{t}^i} \dots \int_{\underline{d}_{n+m}}^{\bar{d}_{n+m}} g(\cdot, \cdot) \pi(d|t, \theta^{n+c}, \hat{\theta}^i) dt d\theta^i dd$$

Equation (57) gives

$$(59) \quad u_i(\pi | \theta^i) - u_i(\pi | \hat{\theta}^i) \leq \bar{\pi}(d | t, \theta) \cdot \left[ u_i(d, t, \theta) - u_i((d_{n+i}, \delta_i(d_{n+i})), t, \theta^{n+i}, \hat{\theta}^i) \right]$$

where

$$\bar{\pi}(d | t, \theta) = \int_{\underline{t}^i}^{\bar{t}^i} \cdots \int_{\underline{d}_{n+m}}^{\bar{d}_{n+m}} g(\cdot, \cdot) \pi(d | t, \theta) dt d\theta^{n+i} dd.$$

Let us now express the above utilities in terms of the decision variables of a distribution divisions problems. In the first instance suppose the divisions true types variable is  $\hat{\theta}^i$  and the divisional manager reports this to the central headquarters. For some  $d$  let it be such that:

$$d = (\hat{p}, \hat{y}_j, \hat{q}_i, \hat{c}_j, \hat{c}_i, \hat{\pi}_j, \hat{\pi}_i, \hat{d}_j, \hat{d}_i)$$

$$\text{for } j = 1, \dots, n \\ i = 1, \dots, m.$$

Let us consider in detail the  $i^{\text{th}}$  divisional managers utility.

$$(60) \quad u_i(d, t, \theta^{n+i}, \hat{\theta}^i) = \hat{c}_i - \hat{z}_i$$

when the divisional manager truthfully reports the divisions types variable to be  $\hat{\theta}^i$  and is obedient. However if the divisional manager chose to report the divisions types variable to be  $\hat{\theta}^i$  when it really was  $\theta^i$  then we would have

$$(61) \quad u_i((d_{n+1}^i, \delta_i(d_{n+1}^i)), t, \theta) = \hat{c}_i + (\hat{\theta}^i - \theta^i) \hat{q}_i \\ + \hat{p}(y_i - \hat{y}_i) - \hat{z}_i$$

since the budgeted level of profit will only be achieved if

$$\theta^i y_i \hat{x}_i \delta_i - \hat{p} y_i - z_i = \hat{\theta}^i \hat{y}_i \hat{x}_i \delta_i - \hat{p} \hat{y}_i - \hat{z}_i$$

thus 
$$z_i = (\theta^i - \hat{\theta}^i) \hat{q}_i + \hat{p}(\hat{y}_i - y_i) + \hat{z}_i$$

and we require that 
$$\hat{q}_i = \hat{y}_i \hat{x}_i \delta_i = y_i \hat{x}_i \delta_i$$

Thus it is possible to express equation (58) as

$$(62) \quad u_i(\pi | \theta^i) - u_i(\pi | \hat{\theta}^i) \geq \bar{\pi}(d | t, \theta^{sic}, \hat{\theta}^i) \\ \cdot (\hat{\theta}^i - \theta^i) \hat{q}_i + \hat{p}(y_i - \hat{y}_i)$$

Alternatively if a divisional manager truthfully reports the divisions types variable to be  $\theta^i$  and is obedient

$$(63) \quad u_i(d, t, \theta) = c_i - z_i$$

and if the divisional manager reports the divisions types variable to be  $\theta^i$  when it really was  $\hat{\theta}^i$

$$(64) \quad u_i((d_{n+1}^i, \delta_i(d_{n+1}^i)), t, \theta^{sic}, \hat{\theta}^i) = c_i - (\hat{\theta}^i - \theta^i) \hat{q}_i \\ - \hat{p}(y_i - \hat{y}_i) - z_i$$

as the budgeted profit can only be achieved if

$$(65) \quad \hat{\theta}^c \hat{y}_i^{\delta_i} \hat{z}_i^{\delta_i} - \hat{p} \hat{y}_i - \hat{z}_i = \theta^i y_i^{\delta_i} z_i^{\delta_i} - \hat{p} y_i - z_i$$

$$\hat{z}_i = (\hat{\theta}^c - \theta^i) \hat{q}_i + \hat{p}(y_i - \hat{y}_i) + z_i$$

$$\text{with } \hat{q}_i = \hat{y}_i^{\delta_i} \hat{z}_i^{\delta_i} = y_i^{\delta_i} z_i^{\delta_i}$$

Thus it is now possible to express equation (54) as

$$(66) \quad U_i(\pi | \theta^c) - U_i(\pi | \hat{\theta}^c) \leq \bar{\pi}(d | t, \theta) \cdot (\hat{\theta}^c - \theta^i) \hat{q}_i + \hat{p}(y_i - \hat{y}_i)$$

From equation (65) we know that if  $\hat{\theta}^c > \theta^i$  that

$$(67) \quad \hat{p} \hat{y}_i + \hat{z}_i > \hat{p} y_i + z_i$$

that is we require that

$$(68) \quad \hat{p}(y_i - \hat{y}_i) < \hat{z}_i - z_i$$

From the divisional problem stated by equation (2), it is apparent that as  $\theta$  rises, so would the optimal purchase of the intermediate product, that is

$$\text{if } \begin{matrix} \hat{\theta}^c > \theta^i \\ \hat{y}_i > y_i \end{matrix}$$

Now we can express  $z_i$  as

$$(69) \quad z_i = \left( \frac{\hat{q}_i}{(\hat{y}_i)^{\delta_i}} \right)^{\frac{1}{\delta_i}}$$

If  $\hat{y}_i > y_i$  we can sign the expression

$$(70) \quad \hat{z}_i - z_i = (\hat{q}_i)^{\frac{1}{\delta_i}} \left[ \frac{1}{(\hat{y}_i)^{\frac{\delta_i}{\delta_i}}} - \frac{1}{(y_i)^{\frac{\delta_i}{\delta_i}}} \right] < 0$$

$$\text{since } \frac{\delta_i}{\delta_i} > 0$$

Also from equation (65), if we rearrange terms we have

$$(\hat{\theta}^i - \theta^i) \hat{q}_i = -\hat{p}(y_i - \hat{y}_i) + \hat{z}_i - z_i$$

and if  $\hat{\theta}^i > \theta^i$ ,  $\hat{y}_i > y_i$  and by (68) and (70) we have

$$-\hat{p}(y_i - \hat{y}_i) > (\hat{\theta}^i - \theta^i) \hat{q}_i$$

so that

$$(71) \quad (\hat{\theta}^i - \theta^i) \hat{q}_i + \hat{p}(y_i - \hat{y}_i) < 0$$

Utilising this inequality in equations (62) and (66) mean that we have

$$(72) \quad \bar{\pi}(d|t, \theta^{sic}, \hat{\theta}^i) \geq \bar{\pi}(d|t, \theta)$$

so that  $\bar{\pi}(d|t, \theta^{sic}, \cdot)$  is non decreasing. Also we have

$$u_i'(\pi|\theta^i) = \bar{\pi}(d|t, \theta^{sic}, \theta^i) \text{ almost everywhere and}$$



$$(73) \quad u_i(\pi | \theta^i) = u_i(\pi | \underline{\theta}^i) + \int_{\underline{\theta}^i}^{\theta^i} \bar{\pi}(d | t, \theta^{i-1}, \theta^i) d\theta^i$$

Inspection of equations (52) and (73) shows that  $u_j(\pi | t^j)$  is non increasing and  $u_i(\pi | \theta^i)$  is non decreasing •

LEMMA 2 If  $\pi(d | t, \theta)$  is an incentive compatible mechanism then

$$\begin{aligned} & \sum_{j=1}^n u_j(\pi | \bar{t}^j) + \sum_{i=1}^m u_i(\pi | \underline{\theta}^i) \\ &= \sum_{j=1}^n \min_{t^j \in [\underline{t}^j, \bar{t}^j]} u_j(t^j) + \sum_{i=1}^m \min_{\theta^i \in [\underline{\theta}^i, \bar{\theta}^i]} u_i(\theta^i) \\ &= \sum_{j=1}^n \int_{\underline{t}^j}^{\bar{t}^j} \dots \int_{\underline{d}_{n+m}}^{\bar{d}_{n+m}} \left( u_j(d, t, \theta) + \frac{F_j(t^j) - 1}{f_j(t^j)} \right) g(t, \theta) \pi(d | t, \theta) dt d \theta \\ &+ \sum_{i=1}^m \int_{\underline{t}^i}^{\bar{t}^i} \dots \int_{\underline{d}_{n+m}}^{\bar{d}_{n+m}} \left( u_i(d, t, \theta) - \frac{H_i(\theta^i)}{h_i(\theta^i)} \right) g(t, \theta) \pi(d | t, \theta) dt d \theta \\ &\geq 0 \end{aligned}$$

PROOF OF LEMMA 2 .

The sum of expected gains to the divisional managers from using the transfer pricing procedure is

$$\sum_{j=1}^n \left[ \int_{\underline{t}^j}^{\bar{t}^j} u_j(\pi | t^j) f_j(t^j) dt^j \right] + \sum_{i=1}^m \left[ \int_{\underline{\theta}^i}^{\bar{\theta}^i} u_i(\pi | \theta^i) h_i(\theta^i) d\theta^i \right] \quad (74)$$

$$\begin{aligned}
&= \sum_{j=1}^3 \left[ \int_{\underline{t}^j}^{\bar{t}^j} \int_{\underline{t}^j}^{\bar{t}^j} \int_{\underline{d}_{n+m}}^{\bar{d}_{n+m}} g(t, \theta) \pi(d | t, \theta) u_j(d, t, \theta) dt d \theta dd \right] \\
&+ \sum_{i=1}^3 \left[ \int_{\underline{t}^i}^{\bar{t}^i} \dots \int_{\underline{t}^i}^{\bar{t}^i} \int_{\underline{d}_{n+m}}^{\bar{d}_{n+m}} g(t, \theta) \pi(d | t, \theta) u_i(d, t, \theta) dt d \theta dd \right] \quad (75)
\end{aligned}$$

However if for equation (74) we consider the case where  $t^j = \hat{t}^j$  and  $\theta^i = \hat{\theta}^i$  we can use equations (52) and (73) to reexpress equation (74) as

$$\begin{aligned}
&\sum_{j=1}^3 \left[ u_j(\pi | \bar{t}^j) + \int_{\underline{t}^j}^{\bar{t}^j} \int_{\hat{t}^j}^{\bar{t}^j} \bar{\pi}(d | \cdot, \hat{t}^j, \cdot) f_j(t^j) dt^j d\hat{t}^j \right] \\
&+ \sum_{i=1}^3 \left[ u_i(\pi | \underline{\theta}^i) + \int_{\underline{\theta}^i}^{\hat{\theta}^i} \int_{\underline{\theta}^i}^{\hat{\theta}^i} \bar{\pi}(d | \cdot, \cdot, \hat{\theta}^i) h_i(\theta^i) d\theta^i d\hat{\theta}^i \right] \quad (76) \\
&= \sum_{j=1}^3 \left[ u_j(\pi | \bar{t}^j) + \int_{\underline{t}^j}^{\bar{t}^j} \bar{\pi}(d | \cdot, \hat{t}^j, \cdot) (1 - F_j(\hat{t}^j)) dt^j \right] \\
&+ \sum_{i=1}^3 \left[ u_i(\pi | \underline{\theta}^i) + \int_{\underline{\theta}^i}^{\hat{\theta}^i} \bar{\pi}(d | \cdot, \cdot, \hat{\theta}^i) H_i(\hat{\theta}^i) d\theta^i \right] \\
&= \sum_{j=1}^3 \left[ u_j(\pi | \bar{t}^j) + \int_{\underline{t}^j}^{\bar{t}^j} \dots \int_{\underline{t}^j}^{\bar{t}^j} \int_{\underline{d}_{n+m}}^{\bar{d}_{n+m}} (1 - F_j(t^j)) \pi(d | \cdot, t^j, \cdot) g(t^j, \theta) \right. \\
&\quad \left. dt d \theta dd \right] \\
&+ \sum_{i=1}^3 \left[ u_i(\pi | \underline{\theta}^i) + \int_{\underline{t}^i}^{\bar{t}^i} \int_{\underline{\theta}^i}^{\bar{\theta}^i} \int_{\underline{d}_{n+m}}^{\bar{d}_{n+m}} H_i(\theta^i) \pi(d | \cdot, \cdot, \theta^i) g(t, \theta^i) \right]
\end{aligned}$$

$$\begin{aligned}
& dt d\theta dd ] \\
= & \sum_{j=1}^n \left[ u_j(\bar{t}^j) + \int_{\underline{t}^j}^{\bar{t}^j} \int_{\underline{t}}^{\bar{t}^j} \int_{\underline{d}_{n+m}}^{\bar{d}_{n+m}} \pi(d|t, \theta) \left( \frac{1 - F_j(t^j)}{f_j(t^j)} \right) \right. \\
& \left. g(t, \theta) dt d\theta dd \right] \\
+ & \sum_{i=1}^m \left[ u_i(\underline{\theta}^i) + \int_{\underline{t}^i}^{\bar{t}^i} \int_{\underline{\theta}^i}^{\bar{\theta}^i} \int_{\underline{d}_{n+m}}^{\bar{d}_{n+m}} \pi(d|t, \theta) \frac{H_i(\theta^i)}{h_i(\theta^i)} \right. \\
& \left. g(t, \theta) dt d\theta dd \right] \tag{77}
\end{aligned}$$

equating equations (77) and (75) gives

$$\begin{aligned}
& \sum_{j=1}^n u_j(\pi | \bar{t}^j) + \sum_{i=1}^m u_i(\pi | \underline{\theta}^i) \\
= & \sum_{j=1}^n \left[ \int_{\underline{t}^j}^{\bar{t}^j} \int_{\underline{t}^j}^{\bar{t}^j} \int_{\underline{d}_{n+m}}^{\bar{d}_{n+m}} g(t, \theta) \pi(d|t, \theta) u_j(d, t, \theta) dt d\theta dd \right] \\
& + \sum_{i=1}^m \left[ \int_{\underline{t}^i}^{\bar{t}^i} \int_{\underline{\theta}^i}^{\bar{\theta}^i} \int_{\underline{d}_{n+m}}^{\bar{d}_{n+m}} g(t, \theta) \pi(d|t, \theta) u_i(d, t, \theta) dt d\theta dd \right] \\
& + \sum_{j=1}^n \left[ \int_{\underline{t}^j}^{\bar{t}^j} \int_{\underline{t}^j}^{\bar{t}^j} \int_{\underline{d}_{n+m}}^{\bar{d}_{n+m}} g(t, \theta) \pi(d|t, \theta) \left( \frac{F_j(t^j) - 1}{f_j(t^j)} \right) dt d\theta dd \right] \\
& - \sum_{i=1}^m \left[ \int_{\underline{t}^i}^{\bar{t}^i} \int_{\underline{\theta}^i}^{\bar{\theta}^i} \int_{\underline{d}_{n+m}}^{\bar{d}_{n+m}} g(t, \theta) \pi(d|t, \theta) \frac{H_i(\theta^i)}{h_i(\theta^i)} dt d\theta dd \right] \tag{78}
\end{aligned}$$

LEMMA 1 showed that for all  $j$  and  $i$   $u_j(\pi | \cdot)$  are non increasing and  $u_i(\pi | \cdot)$  are non decreasing, so that  $u_j(\pi | \cdot)$  and  $u_i(\pi | \cdot)$  respectively attain their minimum

values at  $\bar{t}^j$  and  $\underline{\theta}^i$  respectively, therefore

$$\begin{aligned} & \sum_{j=1}^n u_j(\pi | \bar{t}^j) + \sum_{i=1}^m (\pi | \underline{\theta}^i) \\ &= \sum_{j=1}^n \min_{t^j \in [t^j, \bar{t}^j]} u_j(\pi | t^j) + \sum_{i=1}^m \min_{\theta^i \in [\underline{\theta}^i, \bar{\theta}^i]} u_i(\pi | \theta^i) \\ &\geq 0 \end{aligned} \quad (79)$$

where the final inequality is generated by the individual rationality constraints (15) and (16)  $\circ$

Thus Lemma 1 and Lemma 2 have proven the "only if" and characterization parts of Theorem 1. To complete the proof one need now only prove the "if" part of the Theorem

LEMMA 3 If  $\bar{\pi}(d | t^j, \cdot, \cdot)$  is non increasing and  $\bar{\pi}(d | \theta^i, \cdot, \cdot)$  is non decreasing for all  $i$  and  $j$  and satisfy the requirements of LEMMA 2, then  $\pi(d | t, \theta)$  is an incentive compatible individually rational allocation mechanism

PROOF OF LEMMA 3

Note that

$$u_j(\pi | t^j) = \bar{\pi}(d | t^j, \cdot, \cdot) \cdot [c_j(t^j, \cdot, \cdot) - x_i^j]$$

$$\text{and } u_j(\pi | \hat{t}^j) = \bar{\pi}(d | \hat{t}^j, \cdot, \cdot) \cdot [c_j(\hat{t}^j, \cdot, \cdot) - \hat{x}_i^j]$$

therefore

$$\bar{\pi}(d | t^j, \cdot, \cdot) c_j(t^j, \cdot, \cdot) - \bar{\pi}(d | \hat{t}^j, \cdot, \cdot) c_j(\hat{t}^j, \cdot, \cdot)$$

$$= U_j(\pi | t^j) - U_j(\pi | \hat{t}^j) + \bar{\pi}(d | t^j, \cdot) x_i^j - \bar{\pi}(d | \hat{t}^j, \cdot) \hat{x}_i^j$$

which given equation (52)

$$= \int_{\hat{t}^j}^{t^j} \bar{\pi}(d | t^j, \cdot) dt^j + \bar{\pi}(d | t^j, \cdot) x_i^j - \bar{\pi}(d | \hat{t}^j, \cdot) \hat{x}_i^j \quad (80)$$

Our normal notion of incentive compatibility has required both honesty and obedience however a special limiting case just requires honesty. This would be the case if a divisional manager's true type is  $t^j$  it should not be beneficial to untruthfully report  $\hat{t}^j$  and then be obedient. This notion of incentive compatibility requires

$$\begin{aligned} & \bar{\pi}(d | t^j, \cdot) [c_j(t^j, \cdot) - x_i^j] \\ & \geq \bar{\pi}(d | \hat{t}^j, \cdot) [c_j(\hat{t}^j, \cdot) - \hat{x}_i^j] \end{aligned}$$

that is

$$\begin{aligned} & \bar{\pi}(d | t^j, \cdot) c_j(t^j, \cdot) - \bar{\pi}(d | \hat{t}^j, \cdot) c_j(\hat{t}^j, \cdot) \\ & \geq \bar{\pi}(d | t^j, \cdot) x_i^j - \bar{\pi}(d | \hat{t}^j, \cdot) \hat{x}_i^j \quad (81) \end{aligned}$$

thus for equation (80) we must have

$$\int_{\hat{t}^j}^{t^j} \bar{\pi}(d | t^j, \cdot) dt^j \geq 0$$

that is with  $\hat{t}^j > t^j$ ,  $\bar{\pi}(d | t^j, \cdot)$  must be nonincreasing.

Individual rationality follows from assumption that conditions of Lemma 2 are satisfied.

For a typical distribution division  $i$  note that

$$u_i(\pi | \theta^i) = \bar{\pi}(d | \theta^i, \cdot) [c_i(\theta^i, \cdot) - z_i]$$

$$u_i(\pi | \hat{\theta}^i) = \bar{\pi}(d | \hat{\theta}^i, \cdot) [c_i(\hat{\theta}^i, \cdot) - \hat{z}_i]$$

therefore

$$\bar{\pi}(d | \theta^i, \cdot) c_i(\theta^i, \cdot) - \bar{\pi}(d | \hat{\theta}^i, \cdot) c_i(\hat{\theta}^i, \cdot)$$

$$= u_i(\pi | \theta^i) - u_i(\pi | \hat{\theta}^i) + \bar{\pi}(d | \theta^i, \cdot) z_i$$

$$- \bar{\pi}(d | \hat{\theta}^i, \cdot) \hat{z}_i$$

which given equation (73) gives

$$= \int_{\theta^i}^{\hat{\theta}^i} \bar{\pi}(d | \theta^i, \cdot) d\theta^i + \bar{\pi}(d | \theta^i, \cdot) z_i$$

$$- \bar{\pi}(d | \hat{\theta}^i, \cdot) \hat{z}_i \quad (82)$$

As for the manufacturing division  $j$ , incentive compatibility requires

$$\bar{\pi}(d | \theta^i, \cdot) [c_i(\theta^i, \cdot) - z_i] \geq \bar{\pi}(d | \hat{\theta}^i, \cdot) [c_i(\hat{\theta}^i, \cdot) - \hat{z}_i]$$

that is

$$\bar{\pi}(d | \theta^i, \cdot) c_i(\theta^i, \cdot) - \bar{\pi}(d | \hat{\theta}^i, \cdot) c_i(\hat{\theta}^i, \cdot)$$

$$\geq \bar{\pi}(d | \theta^i, \cdot) z_i - \bar{\pi}(d | \hat{\theta}^i, \cdot) \hat{z}_i \quad (83)$$

thus for equation (82) we require that

$$\int_{\theta^i}^{\hat{\theta}^i} \bar{\pi}(d | \theta^i, \cdot) d\theta^i \geq 0$$

that is with  $\hat{\theta}^i > \theta^i$ ,  $\bar{\pi}(d | \theta^i, \cdot)$  must be non decreasing

Individual rationality follows from the assumption that conditions of Lemma 2 are satisfied. ●

Lemmas 1, 2 and 3 together imply Theorem 1.

CHAPTER 8CONCLUSION

It has been argued by some industrial commentators that manufacturing industry is starting to go through a new industrial revolution. It is an industrial revolution where production techniques are becoming increasingly automated. However, instead of the thrust of the revolution being towards mass production of like items, success in the new industrial revolution depends on the ability of manufacturers to produce efficiently a wide range of differentiated products, in a timely fashion, not necessarily on a vast scale and with product lives being possibly quite short. In such an environment introducing innovative production techniques and products is crucial. Under these operating conditions, divisional managers are increasingly required to be specialists in their fields who can communicate accurately and concisely to the central headquarters about the need to, and possibility to, change production techniques and products. However, there may be a problem if divisional managers are assessed according to management accounting practices that were developed to aid assessment when production techniques and products were relatively stable. Rather than using their expert information to further the goals of the organization, divisional managers may use their superior access to information in a strategic fashion to further their own goals. This may occur particularly if the management accounting practices of the organization do not convince divisional managers



that it is in their interests to freely make known their specialist information. This thesis considers one important management accounting practice that can aid assessment of production and divisional management performance.

The practice of transfer pricing.

In my thesis, I argue that divisional managers who hold specialist information may find it in their own best interests to withhold information and report strategically to the central headquarters, if care is not taken when devising transfer pricing procedures. My major contribution is to adapt the Generalized Principal Agent Model of Myerson (1982) to model specifically the transfer pricing problem. This model has allowed me to make new proposals for how transfer prices should be set. In addition, I have shown that the model can be adapted (along the lines recommended by Myerson and Satterthwaite (1983)) to be applicable to quite general transfer pricing settings.

It is hoped that the theoretical transfer pricing procedure developed here will go some way to help in designing transfer pricing procedures that can be adopted by organizations. This transition from theory to practice in my view will only be achieved if it is clearly recognized that divisional managers should be rewarded for their information specialism as well as their effort services.

A central issue then is to determine how to evaluate the value to the organization of a divisional manager's specialist information. In this thesis, it has been argued that the least value one can assign to such information is determined by how much a divisional manager could gain by not reporting information truthfully.

It would be useful to extend this rationale for transfer pricing to other organizational environments. For instance, consideration of the case where divisions are assessed as investment centres would be an important extension. Another useful extension would be to where the division's type's variable is a probability distribution over uncertain values for some parameter. Here, incentive compatibility would refer to whether or not divisional managers reported the true probability distribution for the uncertain event or not.

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