Primary mathematical skills in Egypt and England

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PRIMARY MATHEMATICAL SKILLS IN EGYPT AND ENGLAND

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ABSTRACT

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As the intention of this research was to investigate the acquiring of skills in mathematics in primary schools, Egypt (which is considered to be a developing Country) and England (considered to be an advanced country) were chosen to provide different ends of the scale.

This piece of research is considered to be of high significance for a number of reasons: firstly, the acquiring of mathematical skills is an important aim in the school curriculum. Secondly, primary school level is an important stage as it is the basis for the other stages. Thirdly, Elementary school mathematical experience may serve in developing one's abilities to understand social institutions, and in equipping one to meet more effectively problems which occur in personal life. Fourthly, there is a deficiency in defining skills practically. Fifthly, there is a lack in evaluating skills objectively.

A practical definition and classification of skills have been adapted, developed, and modified. Objective tests for evaluating skills have been

designed for both Egypt and England. Children's performance in the test of skills has been analysed, and appropriate comparisons between Egyptian children and English children in acquiring skills have been made. General observations from the children's results have been made. It is hoped that this research will contribute in evaluating and improving the methods of teaching mathematics in primary school in general, and teaching mathematical concepts and skills in particular.

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CHAPTER ONE

INTRODUCTION AND STATEMENT OF THE PROBLEM

It has been said that the nineteenth century was the age of power and the twentieth century is the age of information. Since about 1950 the world has seen a tremendous growth in the science dealing with information. Within many countries there is a growing awareness that with the rapid development of technology there is a great need to plan ahead in order to produce the required number of different technologists and other kinds of highly trained manpower. In this context, the role of mathematics in various school systems is crucial. Recently, there has been a new publication from the Department of Education and Science (*), which stresses the need to relate the mathematics to everyday life, using a new classroom approach, and also the need for a new system of assessment for the acquisition of facts and

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(*) Department of Education and Science, MATHEMATICS FROM 5 TO 16, CURRICULUM MATTERS 3, AN H M I SERIES, London, 1985.

skills.

The skills required of modern, civilized man are different and more varied than those previously needed. Participation in a technological society requires highly developed technological skills as well as social skills. In particular men must learn to communicate with others and must acquire the complex social patterns of their community.

In order to discuss in more detail these skills and their implications for society, it is necessary to distinguish between two broad classes of learned skills:-

(a)Perceptual motor skills, which involve responses to real objects in the spatial world.

(b)Language skills, which can include the use of signs and symbols. Language skills, which include mathematics, begin by internal concepts producing comprehension and then externalising by expression of those thoughts and ideas through speech and later by symbols, reading, writing and mathematics.

Within the category of general language skills come the highly specialized and precise languages of mathematics, science and logic. Skill in these languages greatly facilitates the manipulation of symbols of certain types. Among the most recent of these special symbol systems are the computer languages, which scientists and engineers use to instruct computing machines. These languages are suitable for solving many problems involving the manipulation of symbols, and the uses to which these machines are put often resemble skills previously performed by man.

Learned skills vary according to mental and chronological age. Man is capable of learning an almost limitless number of skills. It is now time to consider how much improvement is possible in particular skills as a result of practice.

There are definite limits to the level of proficiency that an individual may reach in the performance of any particular skilled activity. The prediction of these limits is of major interest to human-performance theory. However, actual performance approaches these limits so slowly that it is seldom possible to say that a certain individual has reached the limits of his capacity in a particular activity. Moreover, people seldom attain the theoretical limits of their capacities, the evidence that skills can be improved almost invariably coming from learning curves, where performance is considered a function of days or months of practice under favourable conditions.

Snoddy 1926 (*) was probably one of the first psychologists to call attention to the continuous nature of improvement in motor skills. He used mirror drawing as the skill-learning task: the subject was required to trace with a pencil along a starshaped path, while viewing his hand in a mirror. This had the effect of reversing the normal eye-hand relations. One trial a day performance scores were based on time and errors. In order to emphasize the relatively slow changes taking place after extended practice, Snoddy plotted the logarithm of performance against the logarithm of trials.
The result indicated that improvement in performance continued over the entire sixty days of practice, but that the rate of improvement slowed down over a period of time. There have since been many studies in this field such as Grossman 1959 (**), Robert

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(*) G. S. Snoddy, 'Learning and stability', Appli.Psychol, 1926, 10, 1-36.
(**) E. R. F. W. Grossman, 'A theory of the acquisition of speed-skill', Ergonomics, 1959, 2, 153-166.

Seibel 1963 (*), Stevens and Savin 1962 (**).

What is also important is that there is a correspondence between behaviour which includes learned skills, and the breadth of educational objectives. An instructional objective may be viewed as having three dimensions:-

(I)- The objective should specify the behaviour that the learner is to achieve. This behaviour may be classified in various ways. One such classification separates behaviour into <u>cognitive</u>, <u>affective</u>, and <u>motor types</u>. Thus an objective should specify the kinds of thinking or feeling or action that the student is expected to achieve. An example of a cognitive objective is the student's ability to reason logically in mathematics. An example of an affective objective is the student's interest in mathematics; he wants to learn more mathematics. An example of an objective of motor (or psychomotor) type is the student's ability to make precise

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(*) R. Seibel, 'Discrimination reaction time for a 1,023 alternative task', <u>J.Exp.Psychology</u>, 1963, 66, 215-226.
(**)J. C. Stevens, and H. B. Savin, 'On the form of learning curves',<u>J.Exp.Anal.Behav</u>, 1962, 5, 15-18.

geometric drawings with rulers and compasses.

(II)Objectives have a content dimension. This specifies the subject matter about which the student is to think, in which he is to have an interest, or towards which he is to have a certain attitude. The subject matter may be specified much more precisely, for example, the student's ability to solve quadratic equations in one variable.

(III) Objectives have a dimension which has to do with the field of application of the knowledge or skill specified, for example, the student's ability to apply quadratic equations to the solution of appropriate problems of physical science.
In this example, the behaviour specified is "apply", the content is "quadratic equations", and the field of application is "physical science".

The programmes of evaluating basic mathematical skills should reflect several principal objectives such as :-

1-Defining the ways in which children acquire mathematical skills.

2-Defining the tools which affect the acquisition of the required mathematical skills.

3-To provide the educational researchers with more effective and objective criteria for evaluating mathematical skills.

4-To identify areas:-

(i) In which the child has particular strengths so that suitable and more advanced activities can be provided to build on these skills.

(ii) And where the child has particular weaknesses, to alleviate the child's difficulties and detect them early, thus preventing problems from arising.

Although initially it may appear that the assessments will be very time-consuming, it will soon become apparent that most of it can be carried out during normal classroom routine. Further, it will probably be found that many children will not require any special treatment, while some will be shown to have strengths in a particular skill.

This research and its materials are designed and conducted to help teachers to identify any particular child's strengths and weaknesses at an early stage before the difficulties become problems. It is fully appreciated that there is likely to be a hierarchy of mathematical skills. For example, if one is dealing with a child who has a difficulty in transference or procedural skills it is likely that he will display difficulties not only in performing operations but possibly also in applied skills. If he is not helped quickly he may also face problems with other skills. As a child grows he learns through his visual experiences the structures of objects and their relationships. As a result of children's personalities and home backgrounds, they arrive at school with differing attitudes towards school and its associated activities; they also adopt different learning styles and ways of tackling new problems. Some children approach a new task with enthusiasm and confidence, others adopt strategies which include delaying tactics, prevarication or even clowning to divert attention from the task in hand. It is essential that teachers understand that many children take up these varying approaches in order to cover up their weaknesses. If children are to learn efficiently they must be well motivated. Young children are naturally curious and eager to learn and teachers must make sure that the environment which they create is stimulating and alive. Learning can and should be fun for the young child. Even the most mundane activities can become interesting if they are linked with games or presented in a lively fashion by the teacher. A quick look at the class record sheet may indicate which areas need special attention for the whole class. For example, a series of poor marks for a particular topic may suggest that different teaching strategies need to be adopted.

It is argued that what differentiates mathematics from other subjects is the hierarchical and integrated nature of the concepts and operations involved. It is possible to learn about any one period of history without reference to any other. Not so with mathematics. In order to be able to perform at any given level in mathematics it is necessary to have learnt most, if not all, of the related subconcepts. This does not mean that because the children lack a given concept they are bound to fail at the next level, but it does mean that the more missing concepts there are, the less likely it is that they will succeed at subsequent levels. This raises immediately the problem of determining what concepts are involved in any given hierarchy and how these concepts are ordered and related. Finding answers to this question has been one of the principal objectives of other studies. This study tries to investigate what skills are involved in primary school mathematics and how these skills are ordered and related. There is interest in mathematical skills and their development in children of primary school age for particular reasons. It is quite common to find in Educational and Mathematical Journals references to the very important role that mathematical skills can play in modern society, especially in the development of technology. What is also important is the growing awareness that the rapid development of technology requires a range of different skills, such as communication, language, motor, social, scientific as well as mathematical skills. There is no doubt that, in this context, mathematical skills have a strategic role.

The question has been posed as to what mathematical concepts an individual would be able to assimilate and accommodate at any given point in time. It is argued that this depends in part on his existing cognitive structures or schemata relative to mathematics. Initially, therefore, it is important to:-

(i)Determine the state of the individual's existing, cognitive structures.

(ii)Determine the content and structure of what is to be learnt.

(iii)Match (i) with (ii) and make a programme of the individual's learning experiences such that he will develop good or adequate schemata.

All these considerations and constraints must be taken into account when planning the teaching of basic mathematical skills. There is no doubt that in many countries albeit not very technologically developed, there is a definite need for planning to produce trained personnel to cope with the developing needs of the country.

The primary school stage in Egypt is very important because while it is the initial step, it may also be the final one for some children who are not continuing their studies. This research is confined therefore to the primary school level and it was decided to do so considering England as a progressive country, so that sometimes comparisons could be made with Egypt.

There is one obvious importance of skills to a developing technological country, particularly in a mathematical curriculum to meet present and future challenges. It is the intention of this research to concentrate on the effectiveness of this development, to realize the objectives of teaching mathematics and in particular to study the development of basic mathematical skills in primary children. For this purpose, having defined and classified such skills, a questionnaire was designed based on the classifications of the skills that must be learnt in primary schools, and a series of tests were designed based on the skills which must be acquired. Content analysis of primary school books was also carried out in order to discover what skills were expected of the pupils.

In mathematics, more than other subjects, it is not just the degree of abstraction that is relevant but the relationship of one concept to another. Therefore the lack of any basic concept means that the solving of a particular problem will be more difficult and could become impossible. The child's efforts will become more frustrated, particularly as each concept relates hierarchically to the next. One could query why children are expected to work at a higher conceptual level before they have grasped the fundamental concepts. Possibly they have been "taught" the relevant concepts and are therefore assumed to have learnt them. Alternatively the assumption may be made that they have been taught them elsewhere, e.g. in their primary school and hence that they should know them. There may be a failure to appreciate their lack of knowledge. Rarely does a school examine a cross-section of pupils to establish what concepts are causing difficulty. Both the comprehension and acquisition of mathematical concepts and skills in primary schools is therefore highly important because the more missing concepts and skills there are, especially in the early stages, the more misunderstandings there are likely to be later on. The lack of mobility in schemata appears to be directly related to the way in which the pupil has been taught. If he has been taught the concepts in a particularly rigid format then his resulting cognitive structures are correspondingly likely to be immobile. For example, it has been shown that some pupils taught addition only in the usual vertical format have difficulty in applying it to problems set out horizontally. They have even greater difficulty when addition and subtraction are included in the calculation of one problem.

THE REASONS WHY RESEARCH IS NECESSARY

The development of the modern mathematical curricula causes great criticism from some mathematicians and parents. In spite of this most of them do not reject the development, but feel that the skills are perhaps neglected. This objection has been raised in all the countries where a modern curriculum has been developed. Max.A.Sobel in THE TEACHING OF SECONDARY SCHOOL MATHEMATICS (*) states that 'The development of skills has long been a recognized objective of the teaching of mathematics at all grade levels. In recent years, with the advent of the so-called modern curricula, there has been a tendency to emphasize concept building at the expense of building mathematical skills'.

In fact, the teachers of mathematics are seeking advice and guidance in their search for effective means to build skills while they also teach concepts.

(*) M. A. Sobel, THE TEACHING OF SECONDARY SCHOOL MATHEMATICS, National Council of Teachers of Mathematics, U. S. A, 1970, 33, 291.

Educationist groups who are concerned with the reform of the curriculum have emphasised the importance of skills. The report of the College Entrance Examination Board's Commission on Mathematics includes the following objective as the first of its nine-point programme: (*) 'strong preparation, both in concepts and skills, for college mathematics'. The commission on mathematics, after stating the need to teach appropriate manipulative skills, continues in its report (**) :'strong skills are surely needed, but they must be based on understanding and not merely on rote memorization. Once meaning has been achieved, then drill should be provided to establish skills - skills that can be performed, as Whitehead says, without thinking. In this way, the mind is liberated to grapple with new ideas'. Therefore, the commission realized the necessity for skill development but paid more attention to that based entirely on understanding. Educators have declared and announced this point of view through the years. Again it was included in the report of the Cambridge Conference on School Mathematics, which explained its plan for the teaching of mathematics : 'we proposed to gain three

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(*) College Entrance Examination Board. Commission on Mathematics, PROGRAMME FOR COLLEGE PREPARATORY MATHEMATICS, Princeton , N.J. The Board 1959, 21.
(**) Ibid, 33.

years through a new organization of the subject matter and the virtually total abandonment of drill for drill's sake, replacing the unmotivated drill of classical arithmetic with problems which illustrate new mathematical concepts' (*). The report continued to say: 'Lest there be any misunderstanding concerning our viewpoint, let it be stated that reasonable proficiency in arithmetic calculation and algebraic manipulations is essential to the study of mathematics. However, the means of imparting such skill need not rest on methodical drill' (**).

Other sources recognize the necessity for the development of skills. All emphasized the importance of drills which are based on meaning and understanding. They also explained that the vital need of the classroom teacher is for effective teaching which will provide meaningful experiences to build skills successfully. This point of view was introduced in the twenty-first year book of the National Council of Teachers of Mathematics

'More recently, drill, as a part of the learning process, is again respected. But it is not drill for

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(*)Cambridge Conference on School Mathematics,<u>GOALS FOR SCHOOL MATHEMATICS</u>,Houghton-Mifflin co, Boston, 1963, 7.
(**) Ibid, 8.

drill's sake that we respect, rather it is its contribution to meaningful learning that aims to become functional for the individual. In order to be most fruitful, drill must be employed with artistry. This is not easy mechanical, or formulated artistry, but it is one that requires a high level of discernment in knowing when, how much, where, and how to apply' (*). One of the learning processes is drill, but it is not drill for drill's sake, rather it is employed to produce a great contribution to meaningful learning that aims to be essential and crucial for the individual. So that, drill must be functional and fruitful. Sobel explained that in the last few years, with the appearance of what is called the modern curriculum, emphasis is placed on concepts and formation of concepts needed to develop mathematical skills. If mathematical skills are not developed the cause is the teaching and not the modern curriculum.

Acquiring mathematical skills is an important aim in teaching mathematics (**). These skills require understanding and knowing clearly what is to

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(*) B. A. Sueltz, 'Drill-Practice-Recurring Experience In The Learning of Mathematics: Its Theory And Practice', Teachers of Mathematics, Washington, 1953, 21, 192.

(**)F. E. Abd El Latif, CURRICULUM AND ITS PRINCIPAL, ITS ORGANIZATION, AN EVALUATION OF ITS EFFECTS, Cairo, 1962, 301.

be done and which methods should be used. Johnson explains in THE NEW MATHEMATICS IN OUR SCHOOL (*) that the great weakness of traditional mathematics results in many finishing their studies without a clear understanding of mathematical concepts, even though they may have acquired a certain level of skills. Since he states, 'Probably the greatest weakness of the traditional mathematics was the fact that thousands of students who were subjected to years of study of the subject came away from their courses with a certain degree of competence in skills, but with virtually no concept of what they were doing. Here, for example, is a list of questions relating to the "why" of certain simple arithmetic operations. If you can answer more than half of them, you had either extraordinary teachers or an unusual insight into the processes':

> When one divides a fraction by another (1/3 ÷ 1/7, for example) why invert the divisor(1/7) and multiply?
>
> When the subtraction is made such as

$$\begin{array}{r} 63 \\ -\ 5 \\ \hline \end{array}$$

What makes it possible for one to subtract the 5 from

(*) D. A. Johnson& R. Rahtz, THE NEW MATHEMATICS IN OUR SCHOOLS, New York, 1966, 4-5.

the 3?

In this multiplication

```
      52
*     56
-----------------
     312
    260
-----------------
    2912
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Why does one write the 0 of 260 under the 1 of 312 rather than under the 2?

In dividing 14 by 0.2, why is the answer 70 rather than 7 or 700?

If division is a process that breaks something into smaller parts, why, when one divides 1/3 by 1/6 is the answer, 2, larger than either 1/3 or 1/6?

When one adds 1/3 and 1/6, why must one first change 1/3 to 2/6?

One pays for a $0.65 tuna fish check with a dollar bill. If subtraction means "take away", why does the cashier check her subtraction.

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$1.00
-0.65
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by adding like this: $ 0.65 + $ 0.35 = $ 1.00?

Why is the product of two negative numbers a positive number, for example, -4 * -7 = +28?

In dividing 7.68 by 4.2 why "move" the decimal point in the computation?

How does one know that $3/5 = 60/100$?

Two of the main aims of the new curriculum are that, in the acquisition of these skills, both understanding and interest should be maintained. This is in fact the main distinction between modern and traditional mathematics curricula.

The Arab Organization of Education, Culture and Sciences Project (*) emphasized the necessity of supplying children with skills which served not only other subjects but also enabled them to understand scientific achievement. The curriculum should be suited to primary school children, with the necessary balance between concepts and skills, application and abstraction.

A publication (**) about the present state of modern mathematics programmes in general Education by Soad Anees in the National Committee for the International Mathematical Union and the African

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(*)The Arab Organization of Education, Culture and Sciences Project, A LEADER PROJECT TO DEVELOP TEACHING MATHEMATICS, FOR PRIMARY STAGE LEVEL, THE FIRST MEETING OF ARAB EXPERTS, ALEXANDRIA 8-14 JUL, 1972, 34.

(**) Arab Republic of Egypt, Academy of Scientific Research and Technology, National Committee for the International Mathematical Union and the African Mathematical Union, PROCEEDINGS OF THE CONFERENCE ON MATHEMATICS EDUCATION PRE-UNIVERSITY STAGE, CAIRO, 8-11 DECEMBER 1980, 2.

Mathematical Union states that the experiment of teaching modern mathematics in elementary, preparatory and secondary school in Egypt has influenced teachers and parents. It is not unusual to find difficulties related to content, textbook or the reactions of teachers and administrators. Efforts have been made to solve the resulting difficulties. Problems should not cause a return to "traditional" mathematics - a balanced programme is needed for all the students.

An evaluation of Educational elements has taken place in Egypt in an endeavour to combat the overall problems of educating primary school teachers and their inspectors as well as to study examinations and the problems found with textbooks.

Prof.William Ebeid in his article (*) presents some trends and prospects for the next ten years. Two ideas are recommended for Egyptian schools,

Emphasis on basic skills with a broader outlook which includes higher mental skills and is not restricted to the mechanical ones.

More attention to problem solving and applied mathematics in the content of any

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(*) Ibid, 21& 244-249.

future text books.

Curriculum planners and teachers are invited to work in the next decade on a simple principle
'Expose Less Information and Develop More Skills'

The future of teaching mathematics in Egypt is looked at in four dimensions:
Basic Education, Educational responsibility, Scientific research and methods of evaluation.
Basic Education aims at providing the learners with necessary values, good behaviour practice, knowledge and practical skills suitable for varying environments. Educational responsibilty aims at satisfying the desired objectives of Education and the development of the pupils. It is recommended that use is made of scientific research and that methods of evaluation are developed.

With both modern and traditional mathematics curricula, the question arises as to whether the aim to get pupils to learn mathematical skills is being achieved.

So the problem of this research is defined as 'PRIMARY MATHEMATICAL SKILLS IN EGYPT AND ENGLAND'.

In dealing with this problem the following questions were taken into account.

What are the practical definitions of the mathematical skills?.

What are the mathematical skills which are involved in primary school books in Egypt.?

What are the mathematical skills that should be achieved at primary school level?

Are these mathematical skills universal however the curriculum is studied?

How far do the Egyptian pupils acquire these skills?

How far do the British pupils acquire these skills?.

Are there any differences in the acquisition of skills according to:-

Sex, age, and countries (e.g. Egypt and England)

Does the acquisition of one skill influence the acquisition of another?

What are the differences in these skills amongst children as they move up the primary school?

What are the differences between Egyptian children and British children in primary schools?

THE AIM OF THE RESEARCH

The aim of this research is to recognize and identify mathematical skills in primary schools and thereby to design objective tests to evaluate them.

The most important part is to endeavour to evaluate the skills which are a part of the affective domain in the taxonomy of Bloom's objectives.
One aim of the research into mathematical teaching is to give some recommendations that support new methods, and to direct the teacher in establishing skills which are based on complete understanding, thus influencing positively the pupils future action.

SIGNIFICANCE OF THE STUDY

This research has its importance in that it attempts:-

To define clearly mathematical skills at primary school level.

To trace the development of mathematical skills in primary schools as the children move up through the school.

To design and apply for the first time in Egyptian studies of primary school children new objective tests which evaluate very important elements in teaching mathematics.

To introduce some recent views of contemporary educationists.

To identify areas, firstly where the child has particular strengths so that suitable, and more advanced, activities can be provided to build on these skills; secondly where the child has particular weaknesses to alleviate and to detect difficulties early, thus preventing problems from arising.

To consider objectively the current claims of educationalists regarding the necessity of paying more attention to the affective as well as the cognitive domain.

To contribute towards the objective measurement of the acquisition by children of basic mathematical skills.

ASSUMPTIONS

The research assumes that:-

The acquisition of mathematical skills is an essential aim in all stages of the curriculum.

The development of mathematical skills is an important aim of primary mathematics teaching.

There is no contradiction between the concern over mathematical concepts and the acquisition of such skills.

The acquisition of mathematical skills usually depends upon the concepts which are involved in these skills.

LIMITS OF THE STUDY

The research will be limited by the following:-

Samples were only taken from:-

Pupils in the state primary schools in the provincial administration of Elminya, Egypt, and

pupils from primary schools in the city and county of Durham, England.

The sample is selected from children aged seven to nine years, and nine to eleven years.

The tests designed are limited by the assessment of skills as defined by the researcher's classification and the primary school books.

SEQUENCE OF RESEARCH

To answer the questions the research proceeded as follows:-

Mathematical skills were defined.

A content analysis of Egyptian primary school books was made in the light of concepts, facts and the skills they are expected to acquire and the identified skills contained therein.

A questionnaire was designed to obtain aspects of these skills from educational experts in Egypt and England and the results compared.

Four tests of mathematical skills for both Egyptian and British children were designed to cover the primary school age range 7 to 9 years and 9 to 11 years.

Experts in mathematical education were provided with the operational definitions of the five main categories of mathematical skills as defined by the researcher and were asked to use their experience to assess the validity of the tests and the classification of items in the tests.

The tests were modified and this final classification of test items into a category of five main mathematical skills was made.

A pilot study of the above tests was carried out in an Egyptian primary school in London, two of the primary schools in Elminya and in the Blue Coat, Saint Margaret's, Saint Hild's College C.E.

Junior mixed and infant schools in Durham for British children. Tests were modified as necessary.

The main study of the tests was made in Egyptian primary schools in Elminya and in British primary schools in county Durham.

Results were studied, interpreted and suggestions and recommendations were made.

TOOLS OF THE STUDY

The following tools were used:-

Questionnaires

Content analysis of primary school mathematics books to identify and define the mathematical skills towards which they are aimed.

Tests which were designed to evaluate the mathematical skills at which the pupils and teaching methods are aimed.

STATISTICAL METHODS

To obtain the results statistically several statistical methods were used :-

Percentages, Means and Standard Deviations .

Frequency distribution tables, diagrams to display some results.

The T-Test to study the significance of differences in the acquisition of skills.

The coefficients of correlations to study the validity of the Egyptian test.

THE TERMINOLOGY OF THE RESEARCH SKILLS

The development of skills in children is one of the main aims in teaching mathematics. The majority of educationalists evaluate any mathematical curriculum by the success in acquiring the required skills and thus the strength or weakness of mathematical skills in children usually becomes the principal criterion of success or failure in curriculum development. Undoubtedly the importance of mathematical skills is accepted but this often conceals the problem of defining the actual skills in any practical and measurable way which isolates the individual skills.

Most research has concentrated on psychomotor skills. For example, Travers (*) in his book about learning devotes a complete chapter to studies and research of learning skills, but defines and discusses all in psychomotor terminology. Posner and Keele (**) state that the psychological terminology of skills is very broad and general and that most studies define skills as a product of performance characterised by experience, speed and accuracy. These skill definitions in terms of psychomotor and manual activities conceal mental activities involving symbols as used in language and thought. Bartlett and Fitts (***) agree that mental activities are important in defining skills. It should be noticed that most definitions are about the measurement of skills and not the skill itself. Invariably the discussion centres round the ability to perform the

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(*) C. W. Harris & M. R. Lila, <u>Encyclopaedia of Educational Research</u>, New York, 1960, 3, 1282-1287.
(**) M. I. Posner & S. W. Keele, "Skills learning", in R. M. W. Travers (ED.), <u>Second Handbook of Research on Teaching</u>, Chicago, 1973, 805-831.
(***) Ibid, 805.

work at a high level of accuracy using less effort and time if possible.

That is, skill is defined as being able to carry out a certain process with a level of speed and accuracy to save time. Such as Ahmed Abu El Abass 1974 (*), and R. Labib 1974 (**).

Definitions which combine the ideas of psychomotor and mental skills are :-

'Flexibility and accuracy in performing certain tasks' (***), 'Achievement of the task with accuracy and ease in the least possible time and effort maintaining safety and avoiding the dangers in human life' (****).

Discussing these ideas, Singer 1970 (*****) states that these skills refer to expertness in

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(*) A. Abu El Abass& M. A. Aly and others. THE EFFECT OF TEACHING MODERN MATHEMATICAL CURRICULUM AND TEACHING TRADITIONAL MATHEMATICAL CURRICULUM IN ACHIEVEMENT OF MATHEMATICS AT THE FIRST PRIMARY SCHOOL CHILDREN AND THE SECOND PRIMARY SCHOOL CHILDREN IN BAGDAD,, Bagdad, 1974, 18.
(**) R. Labib, SCIENCE TEACHER, HIS RESPONSIBILITY, THE METHODS OF HIS WORK HIS PREPARATION, HIS PROFIONAL AND SCIENTIFIC DEVELOPMENT, Cairo, 1974, 18.
(***) A. Z. Saleh, THE PSYCHOLOGY IN ADMINISTRATION AND INDUSTRY, Cairo, 1967, 120.
(****) D. Sarhan& A. A. M. Kamel, CURRICULA, Cairo, 1969, 2, 32.
(*****) R. M. Singer, MOTORLEARNING AND HUMAN PERFORMANCE, New York, 1970, 4.

carrying out certain tasks.

The Oxford Dictionary 1976 (*) defines skill as 'Expertness, Practised ability, Facility in an action or in doing or to do something, Dexterity, Tact'.

In Good's Dictionary 1973 (**) skill is defined as 'Any thing that the individual has learned to do with ease and precision, may be either a physical or a mental performance'.

This study will concentrate on mental skills in connection with mathematics, so that the idea of skill here is closer to the Good definition which is as follows:-

'Mathematical skill is the ability to use the operational techniques of mathematics for example computation, induction, deduction and abstraction'. (***).

Fundamental skill is a skill that is basic to

(*) H. W. Fowler & F. G. Fowler, THE CONCISE OXFORD DICTIONARY OF CURRENT ENGLISH, Oxford, 1976, 1071.
(**) C. V. Good, DICTIONARY OF EDUCATION, New York, 1973, 537.
(***) Ibid, 537.

the mastery of a school subject, such as addition or subtraction in arithmetic. William Ebeid 1978 (*) defines mathematical skill as performing tasks at a higher level than mechanical performance ranging between mere application of any rule and that which needs connection between mental operations.
He gives two examples of mathematical skills at a lower level as performing arithmetical or algebraic operations etc, and at higher level as identifying common characteristics, abstraction from a given situation, reforming statements and expressing them differently and induction and generalization from special cases.

These ideas were reflected in the international conference to develop mathematics in the third world which was held in Khartoum in 1978. Here it was emphasised that mathematical skills play an important role in solving the urgent and palpable problems of Third World countries (**).

For the purpose of this research a general definition of skill as 'The ability to do work with

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(*) W. T. Ebeid, 'ANALYSIS OF MATHEMATICAL CONTENT IN PREPARATORY STAGE', THE ARAB ORGANIZATION OF EDUCATION, CULTURE AND SCIENCES, Cairo, 1978, 2.
(**) The Arab Organization of Education, Culture and Sciences, the international conference to develop the mathematics in the third world, PUBLICATION OF TEACHING SCIENCE AND MATHEMATICS, October 1978, 1, 5.

accuracy, speed, and understanding' is taken.

From the mathematical point of view, skill is taken as the mental operation concept connected with mathematical work i.e mathematical skill is 'A certain mental process connected with the scheduled mathematical units in the primary school curriculum'. This definition is expanded in a later chapter (chapter seven).

AN EVALUATION OF SKILL

There are many definitions of evaluation. The DICTIONARY OF EDUCATION 1981 (*) under evaluation, gives 'In the U K this term tends to mean identifying the effects and judging the effectiveness of some learning experience, e.g.a lesson or of a course, or of a complete curriculum. In the U S the term is also used for what is in the U K called assessment (of student attainment). Assessing student attainment before and after the learning experience might be an important part of evaluation but so too are interviews with teachers, administrators,

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(*) R. Derek, A DICTIONARY OF EDUCATION, London, 1981, 84-85.

students, parents and other people in the community, critical analysis of teaching materials, observation of teaching and learning activity, and so on. Some evaluators incline more towards a quantitative approach, while others emphasise the qualitative aspects. Some will combine the two approaches'.

AGRO-BOTANY EVALUATION. (*)

'A term describing evaluation based on judging the extent to which several pre-determined criteria of educational success have been achieved. This approach is based largely on that of the physical sciences - agricultural research in particular, emphasises aspects of learning that are quantifiable; and makes considerable use of statistical techniques. Contrasts with illuminative evaluation'.

ILLUMINATIVE EVALUATION (**)

'An approach to evaluation, pioneered in the U K by Parlett and Hamilton, and by Stake in the U S avoids the quantitative emphasis of the so-called Agro-Botany approach and concentrates more on ascertaining how participants in the curriculum feel about the experience. The chief means of collecting data are observation, conversation, discussions, content analysis of student writings, and so on, and the overall strategy is meant to be flexible enough

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(*) Ibid, 8.
(**) Ibid, 123.

to develop along with the situation being investigated'.

The National Centre of Educational Research in Cairo 1979 (*), defines evaluation as
'A process accompanying an educational operation which measures divergence or convergence from planned objectives in a scientific way characterised by continuance, comprehension, accuracy and purpose'.

Other 1964 (**), definitions define evaluation as 'An instrument to improve and develop the material and the various methods of teaching situations and learning, and for increasing its effectiveness in realising educational objectives'.

Fooad Abu Hatab 1976 (***), defines evaluation as 'The judgement of value of things or individuals or subjects'.

(*) The National Centre of Educational Research sharing with the development centre of teaching sciences, AN EVALUATION AS INTRODUCTION TO IMPROVEMENT, Cairo, 1979, 87.
(**) A. K. Kazim, & G. Abd El Hameed, EDUCATIONAL AIDS AND CURRICULUM, Cairo, 1964, 75.
(***) F. Abu Hatab& S. A. Authman, THE PSYCHOLOGICAL EVALUATION, Cairo, 1976, 2, 9.

Zeinab Mohamed Fareed 1974 (*), defines it as 'Being aware of the value and judgement of a situation and being able to clarify the advantages and disadvantages of it'.

Ezat Abd El Mawgood 1977 (**), sees evaluation as 'Using collecting, classifying, analysing and explaining quantitative or qualitative data about phenomena or situations and using this to make a judgement or decision'.

From the above it can be seen that there are many diverse definitions of evaluation. This research takes evaluation to be the measure or instrument by which the effectiveness of the school schedule in achieving the required educational objective is measured. It involves the clarification of the advantages or disadvantages of the objectives, along with suggestions and development of material or methods to improve the teaching situation in a continuous, comprehensive, accurate and purposeful manner.

In summary, this chapter states clearly the

(*) Z. M. Fareed, 'An evaluation of educational modern', J. Education, Cairo, March, 1974, 2, 43.
(**) M. E. Abd El Mawgood& others, THE FUNDAMENTAL OF CURRICULUM AND ITS APPLICATION, Cairo, 1977, 154.

problem and the necessity of the research. It has also discussed the aim, assumptions, limits and the sequence of the research. Furthermore it has explained the tools employed and statistical methods. Above all it has clarified the terminology and the practical terms that have been adapted and modified in this research. In this chapter no attempt has been made to deal with all the complexities of the entire field. Instead, the emphasis was on the topic which has been treated and is important in understanding children's performance in primary school mathematics. This includes the importance, the practical definitions and hierarchical organization of mathematical skills in the primary stage, the general ideas of the thesis's content as a whole, and the simple explanation of the sequential processes. Skills vary in complexity. The study of skilled tasks must cover the complete spectrum from very simple to very complex processes, including the use of mathematical symbols. It is hoped that this analysis of the thesis will give some insight into children's mathematical skills in the primary stage which children bring to and need for complex tasks and higher stages of study.

CHAPTER TWO

THE MATHEMATICAL BACKGROUND IN EGYPT AND ENGLAND

Chapters two and three in this thesis respectively discuss the mathematical background in Egypt and England. A general review of the changes in the teaching of mathematics in recent years will be covered in those chapters, which summarises the main trends, some of the difficulties to be overcome, suggests possible future development and relates these to what has happened in Egypt and England.

THE MATHEMATICAL BACKGROUND IN EGYPT

Most countries have shown a trend in teaching towards an increased use of concrete representation and practical exercises- "LEARNING BY DOING". This is consistent with the recognition that education is an active process and indeed, the present feature is the growth in the importance of "ACTIVITY-METHODS".

In all countries there has been an effort to make all mathematical work more realistic and related to the pupil's experience.

Closely related to this trend is the change in function of the teacher. There has been a marked decrease in dogmatism and a corresponding growth of a more subtle role, one in which the teacher guides rather than directs, encourages rather than drives, and altogether takes up a far less dominant position. Also related are the rising claims of democracy and the growth of a more liberal conception of education. These have led to a tremendous increase in the secondary school population and also to a more widely based mathematical curriculum.

Mathematics is a cumulative science, with its past forever assimilated in its present and future, but perhaps cloaked in different forms which contemporary fashion may dictate.

Historically, Egypt is a leading country in providing for the physical, mental and emotional well-being of its people. According to Egypt's permanent constitution, free education is a right guaranteed by the state to every citizen. Egyptian education aims at developing the individual's spiritual, mental and physical attributes. It also considers moral aspects and self-determination.

Egyptian education has, until recently, been divided into four stages. First, primary education, to which parents send their children at the age of six and where they stay for six years. The children learn reading, writing and arithmetic. The examinations system can be divided into two methods. One is organized by the teachers and senior teachers and covers the first five years. The other is organized by the local administration and constitutes the final examination of this stage. Secondly, after finishing the primary stage, pupils enter the preparatory school, where they stay for three years until the age of fifteen. The examination system, as in the primary stage, is by two methods. One is organized by the teachers and senior teachers for the first two years, the other is organized by the local administration for the third year. Thirdly, the secondary stage, which is of two different types; general schools which may lead on to university; and vocational schools (technical secondary schools) which include agricultural secondary schools, commercial secondary schools, schools of administration, secretarial secondary schools, and industrial secondary schools. This stage covers the three years after preparatory school from the age of fifteen to the age of eighteen. The examinations system is the same as before but there is one difference in that the final examination of the general secondary schools is given by the Ministry of Education and all students who wish to enter university must first pass this examination and achieve a higher mark in their general secondary certificate. In the case of general secondary schools, the students study academic subjects, while the students in vocational or technical secondary schools study a few academic subjects with a concentration on technical subjects. The Egyptian state educational system is administered and financed by the government. Fourthly, there is higher education, those students who have finished the general secondary and obtained high marks can enter a faculty in higher education. The minimum period of study is four years and the maximum six years, according to the type of faculty.

In Egypt, education statistics revealed that in 1983 (*) :- The number of public school students at the elementary level is 4,884,000, the number of secondary school students, all divisions included is

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(*) State Information Service, <u>EGYPT</u>, <u>FACTS AND FIGURES</u>, <u>PRESS AND INFORMATION BUREAU</u>, <u>EMBASSY OF EGYPT</u>, London, 1983.

1,350,000, and the number of students in universities and higher institutions is 530,000. Total number of faculties is 36. (A faculty in Egypt is the equivalent of a department or school of the English University and Polytechnic system).

Education is the issue of the time - the issue of all times. This is because education is the system by which man is developed to be social, and is trained to utilize his potentialities to their optimum levels. In the final analysis, man is the very cultural history of the nation.

Contemporary civilization, having a universal function and structure, leans heavily on the accomplishments of natural and mathematical sciences and its technology for the material side, and on the vast participation of the masses in public life for the social side. Contemporary knowledge is no longer removed from reality since it is a functional knowledge in its very basis. Because of this, there is a world-wide interest in promoting science and mathematics education in both developed and developing countries. Through such interest and with the purpose of achieving more prosperity and progress in the different aspects of life, developing countries have had to move at two speeds, the first to overcome backwardness that was imposed on them for different reasons, and the second to bridge the expanding gap between them and the more developed countries.

ALECSO (*) had to undertake its responsibilities towards science and mathematics education within the framework of intellectual and educational Arab unity. In this field, ALECSO planned and put into practice several projects that gained great encouragement from all Arab countries. The aim of the project is to develop human resources to reflect the life and future aspirations of the Arab region so that the style and level of modern society is developed to meet what responsible Arab thought sees as both desirable and important for the future.

Educators in the Arab states often inquire - with good reason - about the situation of science and mathematics education in the Arab region as compared with what goes on elsewhere in the world. Many are fascinated by projects which take place in some developed countries that are undoubtedly valuable and well funded, and to which facilities are given to reach all parts of the world. The fast technological

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(*) The Arab Organization of Education, Culture and Sciences, Teaching Mathematics and Sciences, june 1976.

progress in some countries and its impact on the preparation of scientists and technicians may suggest that there is a large gap between those countries and the Arab region in the field of educational technology with regard to science and mathematics.

Professor Saber states in the Journal of the Arab Organization (*) that 'Nevertheless, international meetings where the most modern ideas and projects are presented, meetings of preparatory committees with local, regional and international bodies and the Arab expert, meetings where foreign experts participate, all give the feeling that science and mathematics education in the Arab region is not much behind'

Arab countries have all accelerated the rate of advancement in the field of science and mathematics education, all are convinced that science education, reinforced with Arab heritage and spiritual values is essential for all forms of development, the most important of which is the development of human resources capable of leading society towards progress and prosperity.

(*) Arab Organization of Education, culture and sciences, Teaching Mathematics and Sciences, , june 1977.

The State has adopted a method of scientific planning as a starting point, with the goal of providing its people with a high degree of efficiency to meet the needs of society. Education is provided free of cost for all students and at all levels. The State linked education to its policy of striving to raise the standard of living and increase national income. New types of courses have been created in order to keep abreast of academic and technological development.

In addition, the State has sent students abroad to be trained, reading for higher academic degrees which will benefit their country when they return, and has employed foreign experts, benefiting from their knowledge and expertise in advanced fields of science and technology.

Arithmetic is now augmented by algebra, geometry and numerical trigonometry, and these studies are part of the instruction for all normal children up to fifteen years of age. Dealing with the main trends in the separate branches, arithmetic has been reduced in content by curtailing elementary artificial problems and obsolete commercial practices. Complicated fractions and involved questions have been removed. The space so gained has usually been filled by the preliminary stages of geometry and practical drawing. Arithmetic has been made the stepping stone to algebra replacing the former traditional introduction, which was by means of symbols, expressions and the use of the four rules.

Mohamed Ahmed El-Hawary, the superintendent of mathematics in the Ministry of Education (*), states that 'many developments have occurred in the so-called traditional syllabuses of mathematics since 1956 when the educational system came to be 6-3-3. Although the literary-section in the secondary stage is deprived from having any mathematics, the scientific section, which attracts the majority of our students, includes a reasonable course in mathematics. Both the science and mathematics branches study geometry, algebra, trigonometry, calculus and mechanics with more time and advanced content for the mathematics branch students. Many suggestions are proposed to raise the level of teaching mathematics in preparatory and secondary stages'.

Samy Samaan, general supervisor of mathematics

(*) M. A. El-Hawary, 'The present state of traditional mathematics programmes in preparatory and secondary stages', <u>National Committee for the International Mathematical Union and the African Mathematical Union</u>, Cairo 8-11 December, 1980.

at the Ministry of Education writing on the present state of mathematics curricula in technical schools, (*) states that 'in most of the cases, mathematics is taught only in industrial schools. Very recently, some courses in mathematics have been suggested to special types of commercial and agricultural schools. The five-year-industrial schools have a high level course in mathematics. However, the three-year-industrial schools have a relatively poor and traditional syllabus in mathematics'.

Professor William Ebeid, who holds the chair of mathematics education at the Faculty of Education in Ain Shams university, (**) states in his article Mathematics In The Seventies (An Evaluative Outlook) that 'modern mathematics programmes have been introduced on an experimental basis in secondary, then preparatory schools and in 1970-1971 the first application of the new mathematical curriculum took place in primary schools in Egypt. UNESCO, then ALECSO, participated in the implementation of these programmes. There have been both positive and negative effects of this modernization. A major problem resulted from the duality of inputs in

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(*) S. A. Samaan, Ibid. p. 3.
(**) W. T. Ebeid, 'The action and reaction of the new mathematical experiment', <u>Educational Journal</u>, Cairo, April 1975, p.61-64.

mathematics teaching as related to admission to further education, particularly to university. Development is a continuous process. A developmental programme which reflects modern trends, satisfies local needs and is based on local realities is still needed'.

The experiment of teaching modern mathematics in elementary, preparatory and secondary schools in Egypt has influenced teachers and parents. It is not unusual to face difficulties related to the content, textbook, reaction of teachers, parents and administrators. Efforts have been made to undo the arising difficulties. Problems do not mean a return back to "traditional" mathematics. A balanced programme is needed for all the students. This research gives a brief historical note showing the needs to develop primary and secondary school mathematics. It also mentioned the role of UNESCO and Arab Organization in developing mathematics education in the Arab countries , particularly Egypt.

The arithmetic curriculum has been rearranged to take into account the modern trend of topics' being in psychological rather than subject logical order and the needs and stages of development of the child. The difficulties facing the teaching of arithmetic in primary school may be due to the general problems of teaching such as preparing primary school teachers, professional inspection, examinations and the problem of evaluation, audio-visual aids, overcrowded class rooms, and the problem of textbooks. Methods of teaching mathematics are being revised, in view of the increasing movements to modernize school mathematics during the last ten years. Problems have arisen in training teachers and in practising modern mathematics in schools.

In December-1980 the European Journal of Science Education, in the article entitled 'The Reform Movement In Mathematical Curricula In Egypt And The UNESCO Mathematics Project For The Arab' (*) states that 'during the last ten years, schools in many different countries have used new mathematical curricula to update mathematics education. These new curricula had previously been introduced by various projects, some of which had started as early as the 1950's. The experience during these years has resulted in a general desire to revert to the traditional curricula. This tendency which has been expressed in the phrase "back to basics", has become

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(*) G. N. Malaty, 'The reform movement in mathematical curricula in Egypt and the UNESCO Mathematics Project For The Arab States', European Journal Of Science Education, vol2. n.4 Oct-Dec 1980. p.449-455.

international after certain specialists pronounced the new curricula as failures'. It also stated the problems in Egypt (*) saying that 'the main problems in the Egyptian case are (a) the training of teachers to teach the new curriculum and (b) the follow-up research to evaluate it. The use of electronic calculators by students and teachers in some countries may be a reason for weakness in arithmetical skills. Other factors accompanying the use of the new curricula can be the reasons which have led specialists describing the previous movement of updating mathematical curricula (known as the new mathematics) to be a failure. But the two problems, mentioned above in the case of Egypt, can be met in any attempt to introduce a new curriculum and can prevent us from establishing any new programme in the future. The teacher is the main factor in any reform, and scientific evaluation is essential to obtain necessary improvements in the curriculum'.

The learning of mathematics in elementary schools is a process of development from the concrete or real world to the semi-abstract, and then to the abstract world of numbers, operations, measurements, shapes, spaces and relationships. It takes into consideration as its starting point the background

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(*) Ibid, p. 449.

experience of each child and aims to put the pupil in a position to develop his powers of logical thinking, to apply such powers to the solution of everyday problems, and lastly to prepare himself for higher levels of mathematical learning. In relation to these problems it would be useful to study in more detail the experience of Egypt in reforming the mathematical curricula.

Primary education in Egypt is governed by article 3 of the constitution, which states that it is to be free and compulsory for all Egyptian children, irrespective of race, sex and religious beliefs. As a result, the vast majority of Egyptian children attend primary school between the ages of six and twelve years, following a course of six grades of primary education, at the end of which they should obtain the certificate of primary education which is an indispensable requirement for getting paid employment in the country.

For about 50 years prior to 1970 the form and content of the school mathematics curriculum, especially at the first two stages, had remained relatively stable in content and methodology. Within the traditional curricula, the mathematicians and mathematics educators in Egypt made some positive improvements, for instance, differential and integral calculus were included in the syllabus of the secondary school. Also, an attempt was made to unify mathematics in what can be called "general mathematics" in grade seven at the beginning of the preparatory school. In general mathematics, pupils studied mathematics using one textbook dealing with all subjects instead of studying it by means of different textbooks for different topics.

A review of primary school mathematical textbook content over the last seventy years has been done in this research to see the changes through these years. A complete translation of the primary mathematics curriculum since 1916 until now in Egypt (*) has been reviewed in detail in appendices. For example the framework for 1916 and 1925 are shown in detail in appendices I and II.

There are slight differences in content of arithmetic text books in Egyptian primary schools in 1916 and 1925. The main difference is an increased mathematical content, and more concern to relate the arithmetical studies to practical life. Also the 1930 content of primary arithmetic school textbooks

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(*) The Arab States Education Conference, The Primary Education Stage in Egypt, 1954.

is given in appendix III. It is noticed that pupils started to learn simple geometry in the fourth and fifth years. The scheme of the framework in 1937 is given in detail in appendix IV.

There were two different types of primary schools, one called Obligatory Schools, and the other Elementary Schools. These became one type of primary school after revolution in 1952. There were two slightly different types of arithmetic curricula in the two types of school before 1952. The last scheme in appendix IV was from Elementary Schools in 1937. The differences in Obligatory schools for that year is given in appendix V. The scheme of the curriculum arithmetical framework for schools in 1947 is given in detail in appendix VI. The scheme of mathematics in primary school in 1949 is given in appendix VII. These were all before the Egyptian revolution in 1952. After the revolution there were some changes in curriculum. The arithmetic and geometry curriculum scheme for 1953, 1959, 1960, 1965, 1967, 1971, and the contemporary framework for 1983 are described in detail in appendices VIII to XIV respectively.

Rapid revolutions are not common in Egyptian education and, before describing the change and saying what the researcher thinks about it, it may be worth indicating briefly how it has been brought about. Changes of this nature and magnitude probably occur only when there exists a fairly widespread dissatisfaction with the current state of affairs and a predisposition to look in new directions. The dissatisfaction had certainly been there for many years and it was not confined to Egypt. It was associated with the growing need of society for mathematics at an advanced level. Those who supported the accepted ways argued that a sound mechanical foundation was essential before anything more adventurous could be attempted and that children must learn to walk before they tried to run. There was, however, a growing conviction that the accepted approach laid too exclusive an emphasis on mechanical operations, was too little concerned with the practical uses of mathematics, and that the traditional syllabuses included much useless number work.

For thirty years or more attempts had been made both by Egyptian educational researchers and by the writers of textbooks to make arithmetic more practical and more interesting, but it was not until a mathematical rather than a purely arithmetical, approach began to be made, that the whole subject began to take on a new look. The various kinds of thought for the use of primary schools, none of which was perhaps essential to the change that has taken place, have helped teachers to think in a fresh way about number and broken down some of the misgivings that many primary teachers had about mathematics as distinct from "infant number". Many of these researchers came to realise the contribution of experience to the formation of concepts and the limited value of processes learnt by rote. Books, too, had their influence on primary mathematics, in particular Piaget's research and books about the nature of mathematics and the use of educational aids to enable children to have a deeper understanding of mathematics. <u>EDUCATIONAL AIDS IN MATHEMATICAL SCIENCES</u> (*) was written by Prof.Ahmed Abu El Abass, who held a chair at the Faculty of Education, Ain Shams University. The first edition of that book in 1958 provided a tremendous encouragement to help teachers to change methods of teaching mathematics in the primary school. The book illustrated the use of visual aids in teaching mathematics and pointed out the advantages to be gained when these aids were used in the right conditions to achieve a high level of understanding by children. At the same time the

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(*) A. A. El Abass, <u>EDUCATIONAL AIDS IN MATHEMATICAL SCIENCES</u>, Cairo, 1958.

children would become less anxious and more interested in learning mathematics. It explained how educational aids helped pupils to have better comprehension in mathematics if they are being used in the right way. In addition to that it simply explained how to make these educational aids and how they could be made cheaply and create more interest by using objects in the environment which were familiar to the small child.

All schools reflect the views of society, or of some section of society, about the way children should be brought up, whether or not these views are consciously held or defined. The Egyptian elementary school derived, in part at least, from the principles of education for all children which were established by the revolution in 1952. The main aim was to provide for what were then thought to be the educational needs of the revolution society irrespective of race, sex, class and religion. Egyptian education is strictly geared to particular political and social beliefs.

The aims of primary education in Egypt after the revolution derived from what was then called the United Arab Republic Ministry of Education, which issued the primary stage curriculum in 1959 and an outline of primary school aims, one obvious purpose of which is to fit children for the society into which they will grow up. In that book the primary arithmetic curriculum scheme in 1959 is given as well (*).

The most notable development in the field of arithmetic during the thirty years since the publication of the 1952 mathematics curriculum has been a series of changes clustering around the phrase MEANINGFUL ARITHMETIC, and PRACTICAL ARITHMETIC, APPLICATION OF ARITHMETIC TO REAL LIFE. One of these changes has been an emphasis on an objective of a better understanding of the number system and an increased ability in quantitative thinking. This could be seen through the arithmetic scheme and the objectives during different years in Egypt which are entirely set out and explained in detail, for the period 1916 until the present, in the appendices I to XIV respectively. Another change has resulted in an attempt to identify those arithmetical meanings around which the content of arithmetic should be organized. Still another change has been the broadening of the base for evaluating outcomes of the teaching of arithmetic, which go much beyond the

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(*) Ministry of Education, United Arab Republic, <u>THE PRIMARY STAGE CURRICULA</u>, Cairo, 1959, 97-116.

earlier objectives of computational efficiency. While the ideology of an improved programme of arithmetic has been more clearly defined, the realization of its benefits is hampered by the lack of research to match the scheme. For example, teachers who work for the development of meaningful outcomes are frustrated by the application of achievements tests which measure only the computational outcomes of a drill programme. Yet the improvement of testing must wait for research on the techniques of evaluation to provide more effective ways of measuring the meaningful outcomes that are desired. The purpose of instructional research is a functional one, namely, to provide the knowledge, both specific and general, that is needed to improve the instructional programme. Research must supply the answers to take the place of the guessing which so frequently guides the scheme. Research must supply the evidence from which valid principles of teaching may be derived. These principles, in turn, must be the sources for deducing the proper applications to particular teaching situations. When research findings are interpreted and translated into applications, it is not action research, but research in action, producing actual functional outcomes. Mathematics in the past has very much affected the present. To take in mathematics it is necessary to take in all, old and new, past and present.

It is not surprising then to find that knowledge of mathematics in the past will help to grasp better mathematics in the present, and even enable the educationalists to look ahead with a better perspective.

In the early 1960's it became obvious to the mathematicians and mathematics educators that there was a need for constructive innovation in the school mathematics curriculum. Three main reasons for reform existed. First, there were the difficulties of emphasizing understanding within the framework of the traditional programme. All attempts to prepare good teachers and to improve and modify school mathematics curricula could not alter the fact that mathematics teaching lacked desirable goals. Despite the increase in the number of class hours of mathematics instruction in the 50 s, mathematics was in serious trouble. Traditionally both the examiners and students prepared for examinations by studying specific types of problems, whose solution required only a mechanical response. Therefore school mathematics became a set of empirical rules, tricks and mechanical drills. This undesirable trend increased with the introduction of free education in the 50's for six million students. For instance, the examination at the end of the twelfth grade suddenly became a hard race for more than 100 000 students. To enter a profession, a student's overall mark had to be high, which is not possible without a high mark in mathematics. The work of the teachers focused on training these students to solve various kinds of problems. The students tried to solve a large number of similar problems with the help of special books and tutors. Mathematicians and mathematics educators in the early 1960's were of the opinion that there was no hope of solving this dilemma without replacing the traditional programme by a contemporary one. The other reason is the wide gap between the mathematics programme in Egyptian schools and the mathematics taught at the universities.

In the early 60's, while school mathematics was still traditional, Egyptian university courses had already begun to include some new topics, in an attempt to have an integrated modern course. In addition, there was a growth of the movement of reform in school mathematics in advanced and developed countries. This movement, which had been started in the 50's, was increased in the early 60's. The aims of the international movement of reform in school mathematics as a means of advancing mathematics as a science and thus advancing applied science and technology at the same time, were an incentive to think about the reform of school mathematics in Egypt. For these three main reasons various new programmes were begun in the 1970's (*).

This research is intended to display the changes of the primary arithmetical textbook's content through previous years. This chapter is an attempt to describe and understand the process of the planned arithmetic curriculum change that Egypt has experienced over the past seventy years. It has been tried in this chapter to chart the history of the curriculum development movement, to explain its aims and to describe its processes and structures in action. The last decade has seen a change of mood. The optimism of the pioneer innovators is now muted, in part because planned change has turned out to be a more formidable problem than was initially assumed. There are advantages to be gained from developing an inclusive arithmetic framework, not least the opportunity to identify how Egypt's own cultural idiosyncracies have shaped and differentiated Egyptian responses to a commonly perceived problem. It also seems useful to treat the systems

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(*) The Ministry Of Central Education, THE UNITED STUDY CURRICULUM OF PRIMARY STAGE, Cairo, 1960, 152-179.

chronologically, which will mean beginning with the Egyptian system before the revolution in 1952 and moving on to the system after the revolution until now. It is believed that this procedure will at least help to maintain the links between past and present.

The declared aims of the primary mathematics curriculum suggested by the Ministry of Education in 1960 (*), were: to extend the student's knowledge and background of the principles and content of curriculum for children of ages 6 to 12. In addition the children were to be given an opportunity of undertaking mathematical activity in a manner closely approximating to environmental conditions. They were also to be given a type of arithmetic which will bring children into closer contact with the physical world around them, which will give them something more significant to do than repetitive calculation, and which will lead them to the formation and expression of personal judgements. The introduction of practical arithmetic has taken many forms in different types of schools, but it generally follows the pattern of problem assignments, using familiar materials and situations from the child's own environment and employing measuring apparatus of all

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(*) Ibid, 176-179.

kinds. Depending on circumstances, this kind of activity is based on the mathematics corner, the mathematics table, or the mathematics trolley. In one particularly enthusiastic school associated with Ain Shams University, mathematics has become the central subject in the imaginative life of the school, having relationships with all other subjects and mathematics appears in every corner. The effects of this on language, art, poetry and the general attitude to work have been far-reaching. This is clearly a contrived and concentrated environment, designed to stimulate mathematical interest. But even if the environment can be called artificial, there is no doubt about the genuineness of its appeal to the exploring fingers and immediate interests of the children around whom it has developed. In primary schools, particularly in infant schools, the normal environment possesses many mathematical possibilities. If these are to be used for the discovery of relationships in number, quantity and space, materials such as cups and saucers, knives and forks, spoons, plates, chairs, pencils, books, balls, marbles, rice, beans, were suggested as being valuable at the school so that they can be brought into use when required. Collections of attractive objects for sorting and matching (one to one correspondence) were also advocated.

This brief review of the arithmetic curriculum development leads to some important implications in the development of Egyptian children's mathematical thinking within the school environment. One of these is the need to provide throughout the first school continuing opportunities to manipulate materials. A wide range of materials and situations is also necessary in view of the personal nature of learning. Some children may develop concepts of reversibility primarily, for example, through extensive use of the water tray, others through the use of fabrics for sewing, while others may achieve it as a result of experiences in a variety of different situations. Real learning results from interaction with the environment, and mathematical understanding like all understanding is achieved by personal routes. Special apparatus and structural materials may help the learner to organize and codify this understanding, but do not provide fundamental meanings. For example, in learning about shape, the use of wooden templates alone would lead to inadequate concepts, but could help children to identify, codify and name shapes already observed and observable in flowers, furniture, buildings. The realization that in total experience the mathematical element is one among many only comes to a child as he discovers this to be so in different situations. It is through such individual and co-operative activity over a range of experiences that the place of mathematics in human experience is understood.

In 1963, the Ministry of Education issued the united study curricula for the primary stage, which contained the same aims and instruction for primary school subjects as in 1960 (*). The primary school framework scheme in that year is given in appendix X.

As early as the end of the nineteenth century, research began to supplement general observation of children's methods of learning. In Egypt, in March 1963 (**), Educational research into the ways in which children learn arithmetic was produced and discussed. The main ideas in that book were translated from Morton's book (***), (****). It was also influenced by other books such as those by Buckingham (*****), Rosenquist (******), and Spencer.

(*) Ministry Of Education, United Arab Republic, THE UNITED STUDY CURRICULUM FOR PRIMARY STAGE, Cairo, 1963, 143-158.
(**) L. Gorgy, A. Z. Mohamed, and M. Aly, EDUCATIONAL RESEARCHES TO HELP TEACHERS IN TEACHING ARITHMETIC, Cairo, 1963, 17-69.
(***) R. L. Morton, 'Estimating quotient figures when dividing by two place numbers', Elementary School Journal, November 1947, 48, 141-148.
(****) R. L. Morton, TEACHING CHILDREN ARITHMETIC, New York, 1953, 566.
(*****) B. R. Buckingham, ELEMENTARY ARITHMETIC, ITS MEANING AND PRACTICE, Boston, 1947, 744.
(******) L. L. Rosenquist, YOUNG CHILDREN LEARN TO USE ARITHMETIC, Boston, 1949, 174.

These are concerned with the importance of understanding in learning arithmetic. Prolonged periods of routine practice in, for example, computation reduce rather than improve accuracy. Children can only learn efficiently from concrete situations as lived or described, and from these situations, children acquire concepts in every area of the curriculum. A child who has no immediate incentive for learning arithmetic is unlikely to succeed because of warnings about the disadvantages of innumeracy in adult life (*). Children vary in the degree of interest and readiness for learning arithmetic, but can be led to want to learn arithmetic, provided that they are sufficiently mature. At every stage of learning children need rich and varied materials and situations, though the pace at which they should be introduced may vary according to the individual. Although children think and reason in different ways they all pass through certain stages depending on their chronological and mental ages and their experience. Practice is necessary to fix a concept once it has been understood, therefore practice should follow, and not precede, discovery. There is the evident enjoyment that many teachers themselves display with enthusiasm

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(*) P. L. Spencer, M. Brydegaard, BUILDING MATHEMATICAL CONCEPTS IN THE ELEMENTARY SCHOOL, New York, 1952, 372.

on encountering mathematics in its new methods of teaching. Educationists also emphasised the importance of using aids to learning and to teaching mathematics. Children and teacher must be taught to use aids profitably and to associate them with learning as well as with entertainment. There is a particular need for the school to provide direct experiences in which all the senses come into play. Teachers must, therefore, consider how they can use them best to enrich the ways in which children can learn. Many changes have taken place in the past thirty years in the mathematical thinking about methods of teaching arithmetic in primary school. The causes of the main changes include the increased development of education in Egypt, the greater earning power of skilled workers abroad, and the greater appreciation of children and their needs that has spread through the population.

A teacher has a dual responsibility in the development of mathematical concepts, first in creating the basic environment within which the learning can take place, and second by intervening in children's learning when appropriate, to help them focus on the relevant mathematical elements. Throughout this thesis the role of mathematical skill in clarifying, refining, and extending children's ideas is emphasized, and it is crucial in helping children to focus their thoughts on mathematical relationships. New objective tests to evaluate mathematical children's skill in primary school aged 7 to 9 and 9 to 11 were constructed, devised, modified, and adjusted. These tests, it is hoped, will help teachers to recognize, identify, clarify, and determine children's strengths and weaknesses in mathematics so that a diagnostic programme can be built for those particular children's needs and individuality.

A. E. Esmael and others 1965 (*), in THE INSTRUCTOR IN ARITHMETIC FOR THE FIRST PRIMARY CLASS, stressed knowledge and skills that would be useful to adult life, and teacher responsibilty for the direction which things take. Also in the same year (**) The Egyptian Educational Ministry's report made clear to the teacher what he was supposed to do, arranged a concerted programme of mutually supportive teaching, and provided institutional backing for such measures as the teacher had to take. The elementary school was authoritarian in its ethos, with most of

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(*) A. F. Esmael, and others, THE INSTRUCTOR IN ARITHMETIC FOR THE FIRST PRIMARY CLASS, Cairo, 1965, 7-129.
(**) The Ministry Of Education, FOUNDATION OF EDUCATION, AND ITS APPLICATIONS TO PRIMARY SCHOOL FOR INSTITUTIONS OF TEACHERS, Cairo, 1965, 5-112.

the pressure to work coming from the teacher, aided by various systems of artificial incentives.

The Ministry of Education 1966, issued <u>THE DEVELOPED PRIMARY STAGE CURRICULUM</u>, (*) which contained a study planned for application in 1965-1966. This gave the aims of primary school subjects, aids and ways to achieve these goals, general instruction and the content of the primary school framework in every subject. In this context, the school content in mathematics is given in appendix XI.

In 1967 (**), the Ministry of Education issued <u>THE INSTRUCTOR IN ARITHMETIC FOR THE PRIMARY SCHOOL STAGE</u>, which contained the same direction and content framework scheme and curriculum for the primary school as in 1966.

In March (***) and May (****) 1967, Prof. W. T. Ebeid, in his two articles attacked the copy

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(*) The Ministry of Education, <u>THE DEVELOPED PRIMARY STAGE CURRICULUM</u>, Cairo, 1966, 3-263.
(**) The Ministry of Education, <u>THE INSTRUCTOR IN ARITHMETIC FOR PRIMARY SCHOOL STAGE</u>, Cairo, 1967, 3-91.
(***) W. T. Ebeid, 'How new mathematics curriculum meet modern life', <u>Educational Journal</u>, Cairo, 1967, 3, 59-63.
(****) W. T. Ebeid, 'Preparing primary school teacher for mathematics', <u>Educational Journal</u>, Cairo, 1967, 4, 26-35.

methods which were still in vogue in many schools, and said that a good education should aim at giving a pupil power, capacity, seriousness, method, intellectual insight, and not specific preparation for this or that department of the world's busines. He stressed the basic skills and the narrow way in which these were conceived, and that in contemporary life, in which the pattern and character of occupations is rapidly changing. The equipment of narrow skills drilled into elementary school children would no longer be adequate, or even sometimes necessary, as more and more low-grade work is mechanized. Increasingly, firms prefer to give their own particular kinds of training. Increasingly, also, jobs require a theoretical ingredient, not just a practical know-how to be gained simply by long apprenticeship. Employers look to the schools for a general education which will produce people who are adaptable and flexible (*) and adaptability is as much a function of having general concepts and principles at one's command as it is of an attitude of readiness to try something different. Not surprisingly, therefore, mathematics and science are at the forefront of the minds of those who now call for a general education. W. T. Ebeid outlined in his report; special characteristics of mathematics

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(*) Ibid, 59-63.

and its educational implications, assumptions about curriculum, his conception of the aims of teaching mathematics, the beginning and development of the contemporary movement for reforming mathematics education in general and in Egypt in particular, and some evaluative remarks on the experience of developing mathematics education in Egypt. Also in that context, Mohamed Kamel 1969 (*), the director's assistant in Educational government, emphasized the above mentioned idea and gave thorough explanations for preparing a good arithmetic lesson, methods of teaching, general properties of children's learning of arithmetic, how to use educational aids to help teachers to achieve good results, and what factors could be taken into consideration when teachers teach arithmetic to primary school children.

It would be worthwhile to mention those educationists who did their best to make improvements in the field of teaching mathematics such as Yehia Hendam and Gaber Abd El Hameed (**). In the three hundred pages of their book, <u>TEACHING ARITHMETIC AND ITS EDUCATIONAL AND PSYCHOLOGICAL BASIS</u>, they

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(*) M. M. Kamel, <u>INSTRUCTIONS AND DIRECTIONS ON TEACHING MATHEMATICS FOR PRIMARY STAGE</u>, El Minia, 1969, 25-32.
(**) Y. Hendam, G. Abd El Hameed, <u>TEACHING ARITHMETIC AND ITS EDUCATIONAL AND PSYCHOLOGICAL BASIS</u>, Cairo, 1966, 5-300.

reviewed thoroughly the psychological and educational basis for learning arithmetic, addition, subtraction, multiplication, division, fractions, decimal fractions, applications of percentages, measurements, solving problems, educational aids, and evaluations in arithmetic. Also M. El Atrony 1968 (*), stressed in his article 'In teaching mathematics', the great importance of teaching new mathematics, and he gave in detail the need to apply new mathematics in Egyptian schools. In that year 1968 (**), Professor William Ebeid, in his article, 'Adjusting, developing mathematics curricula', gave his view about developing mathematics curricula for primary school children. He presented a model of the structure of an integrated syllabus making allowance for the general culture of the teacher as well as the educational subjects with one principal specialization in mathematics and another subsidiary one.

In 1967-1968 (***) 1968-1969 (****) , the

(*) M. El Atrony, 'In teaching mathematics', Educational Journal, Cairo, January 1968, 2, 64-69.
(**) W. T. Ebeid, 'Adjusting, developing mathematics curricula', Educational Journal, Cairo, March 1968, 20 th year, 3, 76-83.
(***) Ministry of Education, THE EDUCATIONAL PSYCHOLOGY FOR PREPARING PRIMARY SCHOOL TEACHERS, Cairo, 1967-1968, 5-251.
(****) Ministry of Education, THE DEVELOPED PRIMARY STAGE CURRICULA, Cairo, 1968-1969, 124-147.

Ministry of Education in THE EDUCATION AND PSYCHOLOGY FOR PREPARING PRIMARY SCHOOL TEACHERS, and THE DEVELOPED PRIMARY STAGE CURRICULA, similarly gave the aims of teaching mathematics at the primary stage and general directions for the primary school teacher in every class as mentioned before in 1965 and 1966. It also gave the content of framework scheme curricula in primary school as given in appendix XII.

Professor William Ebeid in March 1969 (*), and Mohamed Shok (**), published two articles. One article 'The international movement to develop teaching mathematics', presented some trends and prospects for the future. Some ideas were recommended for Egyptian schools, such as emphasis on basic skills with a broader view, to include higher mental skills and not be restricted to the manipulative ones. Also more attention should be given to problem solving and mathematical applications as a basis for the future content. He also suggested that curriculum planners and teachers should be invited to work, in the next decade, on a simple principle; Expose Less Information and Develop

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(*) W. T. Ebeid, 'The international movement to develop teaching mathematics', Educational Journal, Cairo, March 1969, 21 th year, 3, 38-45.

(**) M. Shok, 'The contemporary thought in mathematics', Educational Journal, Cairo, March 1969, 21st year, 3, 46-49.

More Skills.

The other article, 'The contemporary thought in mathematics', focused on developing new teaching methods to make mathematics learning meaningful. He mentioned that researchers and curriculum developers orientated toward conceptual approaches seem to agree on the importance of fostering in children a strong intuitive understanding of the underlying structures of mathematics. He looked at several converging lines of thought suggesting that an understanding of mathematical structures is fundamental to meaningful learning. He also described some structure-orientated teaching methods and materials designed to promote meaningful conceptual development. Throughout his article it was assumed that each new mathematical idea must develop in a child's mind through his own activities, handling objects, comparing measurements or shapes, reflecting on the results of his experiments and communicating them to others. Above all he characterized the aims of the required contemporary mathematical teaching.

For all these reasons various new programmes were begun in the 1970s. These efforts, among other things, led to the following projects:-

(I) The UNESCO Mathematics Project for Arab

States (UMPAS) designed for secondary schools (grades 9-12). This was started in 1970-1971 (*).

(II) The ALECSO (**) (Arab UNESCO) Project for the preparatory schools (grades 7-9). This was begun in 1974-1975.

(III) The Modern Mathematics Curriculum (***) for the elementary schools (grades 1-6). It was begun in 1970-1971, but it was basically an injection of modern topics into the traditional syllabus.

(IV) The Normal Schools Modern Mathematics Project (****). This was begun in 1972-1973.

The international co-operation in the realization of the UNESCO Project and its place as the earliest among the education projects in Egypt, are good reasons for discussing it here.

(*) Academy of Scientific Research and Technology, Proceedings of The Conference On Mathematics Education, Pre-University Stage, NATIONAL COMMITTEE FOR THE INTERNATIONAL MATHEMATICAL UNION AND THE AFRICAN MATHEMATICAL UNION, 8-11 December 1980, 7-107.

(**) The Arab Organization of Education, Culture and Sciences Journal Of Teaching Mathematics And Sciences, Cairo, 1976, 1st year, 1.

(***) The Arab Organization of Education, Culture and Sciences Journal Of Teaching Mathematics And Sciences, Cairo, June 1977, 1.

(****) The Arab Organization of Education, Culture and Sciences, Journal Of Teaching Mathematics And Sciences, Cairo, December 1977, 2nd year, 2.

UNESCO was interested in co-operating with its member states to improve science and mathematics instruction. This interest can be shown in the resolutions of the general conferences of UNESCO in 1962, 1964 and 1966. The interest of UNESCO was translated into UNESCO's Programme for science education .

In this programme, projects in physics, chemistry, and biology were targeted and were implemented in 1965-1966 in many countries in South America, Asia and Africa. All these projects for curricular reform in science education were designed for secondary schools (age 15-18) as the quickest way to update the courses in higher education and consequently reap their benefits in the economy. While Egypt was a participant in the biology project, it expressed interest to UNESCO in modernizing mathematics teaching as fundamental to reform in other curricula.

One of the significant factors in UNESCO's (*) establishing UMPAS was the recommendations of the

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(*) UNESCO, UNESCO MATHEMATICS PROJECT FOR THE ARAB STATES, 8-17 March 1969, Regional seminar held in Cairo.

conference of ministers of education and ministers responsible for economic planning in the Arab States held in April 1966. These recommendations expressed the need for qualitative improvements in education, especially in teaching mathematics and physical sciences.

UNESCO began UMPAS in 1967 with the co-operation of a number of Arab Universities and Ministries of Education. The activities of the project can be summarized as follows:-

(I) National study-groups were formed towards the end of 1967 to study and prepare for the project. One of their tasks was to study the materials published by similar groups in other countries.

Another aspect of the activities of these study groups was the evaluation of teaching mathematics in Arab countries, each group in its own country. Some of the findings of the study were published in a UNESCO (*) Report School Mathematics In Arab Countries 1969, which served as a guide for the study groups,

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(*) UNESCO, UNESCO MATHEMATICS PROJECT FOR THE ARAB STATES SCHOOL MATHEMATICS IN THE ARAB COUNTRIES, PARIS, 1969.

and were investigated by Malaty's thesis (*).

(II) A planning seminar was held in Cairo in 1969 under UNESCO auspices, with participants from the study-groups and six UNESCO consultants from various advanced countries. A new syllabus for secondary schools (grades 10-12) was proposed by the Egyptian study-group and was subsequently adopted. Other outcomes of the seminar included the preparation of different recommendations for the work of realizing the project.

(III) Four writing sessions were held during 1969-1970, each writing session lasting for two weeks. As a result, three textbooks, one for each of the three secondary grades, were published in English by UNESCO and subsequently translated. These textbooks were written by 22 Arab and UNESCO experts.

(IV) Three inter-Arab training seminars were held in July 1970, 1971 and 1972 to train teachers, selected from all those who were working in experimental schools, with the textbooks of the corresponding grades 10, 11, and 12.

(*) G. N. Malaty, 'The characteristics of achievement concepts of new mathematical curricula at the Egyptian secondary school', Ph.D Thesis, Moscow, unpublished, Academy of Pedagogical Sciences of the USSR, 1977.

(V) The Nice (France) evaluation and planning seminar was held in September 1970. At this seminar representatives from Arab countries discussed with consultants for UNESCO the steps of the project and prepared a different set of recommendations.

In the school year 1970-1971, a few secondary (*) schools in Egypt began using the UMPAS curriculum. Table 1 shows the way the UMPAS curriculum was introduced into Egyptian secondary schools. The special schools are language schools and others, in which instruction is not free. In state schools, about 80% of all students are at the secondary stage. In general, a state school is more than twice as large as a special school.

Table 1: number of schools using the UMPAS curriculum

School year	State schools	Special schools
1970 - 1971	3	3
1971 - 1972	12	12
1976 - 1977	13	26

(*) Ibid .

The UMPAS syllabus is as follows:-

Grade 10 (five periods per week , each lasting 45 minutes); sets, mappings, relations, operational systems, introduction to logic, affine geometry, rational numbers, real numbers, transformation geometry, solution of sentences, co-ordinate geometry, statistics and probability and trigonometry.

Grade 11 (*) (six periods per week each lasting 45 minutes); matrices, groups, vector geometry, complex numbers, combinatorics and induction, sequences, polynomials and rational functions, exponents and logarithms, circular functions.

Grade 12 (six periods per week, each lasting 45 minutes); limits and continuity, differential calculus, differential calculus applied, integral calculus, integral calculus applied, probability, differential equations.

The details of this syllabus, which can be seen in the UNESCO textbooks (**), show that the curriculum of UMPAS is a modern one. Its main aim is to provide unified material.

To evaluate the success of the UMPAS curriculum

(*) Ibid .
(**) Ibid .
(***) Ibid .

(***), some of the main findings that have resulted from testing, questionnaires, interviews and the study of the documents of UMPAS and from the critical reports obtained from teachers will be presented here. On the positive side, it was found that, the teaching of mathematics gave more attention to understanding; and most of the teachers and students showed favourable attitudes towards the modern curriculum. On the negative side, a number of findings emerged which are as follows (*):-

(i)It was not recommended that a modern curriculum be taught after the study of traditional curriculum for 9 grades (age 6-15).

(ii)Testing of students in grade 7 (ALESCO Project) and students in grade 10 (UMPAS) showed that the students in grade 7 had a better understanding of the concepts of sets. The experimental details are these:- A multiple-choice test was used to measure the understanding of the concepts of sets which are included in the UMPAS' grade 10 programme (**). The main sample in that research consisted of 11 groups from 11 different secondary schools (grade 10) . Each group comprised a class of about 40 students. These groups were selected overall to represent

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(*) UNESCO, <u>SCIENCE EDUCATION, 29 NOVEMBER-10 DECEMBER 1971</u>, Regional seminar held in Cairo.
(**) UNESCO, <u>MATHEMATICS PROJECT FOR THE ARAB STATES, GRADE 10, 1970</u>.

different levels. They gave the test to a group of 39 grade 7 students just as they gave it to the main sample. The test contained 57 items concerned with 15 different concepts which are studied at the same level by students in grades 7 and 10. It should also be mentioned that the teacher of the grade 7 class had not received any higher education and that he had followed a training course given by inspectors who had recently trained at Ain Shams University. It also mentioned that the term used in the test for the concept of "set" was the same as that in grade 10 but different from the term used in grade 7. Despite these factors, unexpected results showed that the students of the grade 7 group were better than those in the main sample, i.e the grade 10 students;

Test results revealed unexpected weaknesses in the understanding of basic concepts, such as relations. The tests showed that the crucial factors in understanding the concepts were the teacher and the textbooks.

Retraining of the teachers had clearly not been sufficient. The brief nature of the training course is sufficient only to provide a course in mathematics equivalent to the school course without imparting a deep understanding of the modern treatment of mathematics. The new content impressed on the teachers the need to consider understanding more than had been the case with the traditional one. On the other hand, training courses did not touch on teaching techniques - discovery and other new methods were still unfamiliar to teachers in general. Unfortunately, the new teachers who have recently finished their university course and studied modern mathematics cannot get posts in the experimental schools teaching mathematics. The majority of these new teachers teach traditional curricula, while the majority of teachers who work in experimental schools are those whose education involved the study of traditional mathematics.

The time devoted to writing the textbooks in English and translating them into Arabic was not sufficient. As a result of the involvement of 22 Arab and foreign experts on each textbook for only two weeks there was only a loose connection between chapters, each of which had been written by a different author. The following example is a significant one. In the English textbook for grade 10 the sentence "we have seen that for a rational number the representation was a repeating decimal number" (*) was found in chapter 8 on real numbers. Yet, students would not have met this representation

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(*) UNESCO, MATHEMATICS PROJECT FOR THE ARAB STATES, GRADE 10, 1970.

before. Many mistakes appeared in the translation of the textbooks, the most crucial mistakes being in the translation of some of the ideas, and these gave rise not only to methodological errors but also to mathematical ones.

The traditional examination system in Egypt is an impediment to understanding UMPAS mathematics. This problem of time is becoming more serious with the appearance of special "non-governmental textbooks" and special teachers. The experience of teachers in relation to specific problems which occur more frequently in examinations, has increased with time and consequently encouraged them to write their own books. These books have become more necessary to students than the official textbooks in that they lay stress on preparation for examinations.

The preparation of the first group of teachers (*) to start working in 1970-1971 was better than the preparation of subsequent groups. For instance, for the first group only, weekly meetings throughout the school year were organized to enable teachers to discuss their problems with mathematics specialists. For this first group of teachers for whose training

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(*) UNESCO, MATHEMATICS PROJECT FOR THE ARAB STATES, GRADE 11, 1970.

UNESCO was partly responsible, the Egyptian specialists worked enthusiastically to provide training. Since then, the specialists have, for different reasons, become less interested in UMPAS. The difficulties in expanding UMPAS, the limitations of training budgets, and new interests in working on newer projects, particularly the ALECSO Project, were important factors in leaving subsequent groups less well prepared. Another reason was that more than half the specialists who trained the first group (who at the same time formed the national study-group) left Egypt subsequently to work in other Arab countries (*).

The fatal mistake in the realization of UMPAS is connected with the follow-up of the recommendations, which were made in the report of UNESCO school mathematics in Arab countries, 1969 and the more important recommendations, which were made at the UNESCO seminars; the Cairo regional seminar held in 1969; the Nice evaluation and planning seminar held in 1970; and the Cairo science education regional seminar held in 1970.

In these seminars many recommendations were made

(*) UNESCO, <u>MATHEMATICS PROJECT FOR THE ARAB STATES, GRADE 12</u>, 1971.

in different fields, such as writing text books, retraining teachers, writing the teacher's commentary, publicizing the project, evaluating the programme and improving the textbooks, the formation and activities of the inter-Arab committee on mathematics, co-ordination with the teaching of other subects and the holding of a revision seminar in 1972. Most of these recommendations were not realized. For instance, in relation to the writing of textbooks, recommendations were made to include diagnostic tests, to study function as a particular type of relation and to study logic along with the study of sets, relations and functions.

In practice, the UNESCO textbooks did not contain diagnostic tests; the study of functions came before the study of relations and the study of logic became a separate chapter after the study of sets, functions, and relations. Despite the positive attitudes embodied in the recommendations, the actual possibility of Arab countries implementing them, even with the help of UNESCO, would need much more time than was allowed for the preparation and introduction of the new curriculum. A significant example of this is the students who started teaching the new curriculum in Egypt in 1970. When they went to their schools they found that they were without textbooks as these had not yet been translated; they were later to receive the first chapters in separate parts. UNESCO did not convene the recommended revision seminar in 1972; also, whereas during 1969-1970 UNESCO was entirely involved in the UMPAS, after the training of teachers in July 1972 it became disengaged from it.

UMPAS expanded in Egypt slowly, and from 1971-1972 to 1976-1977 the number of state schools which used UMPAS increased by only one school out of 310. Without doubt, there are some indviduals in Egypt who object to UMPAS and who can prevent UMPAS from expanding. It may also be said that without a serious follow-up to all the recommendations of different seminars of UNESCO and ALECSO, it would be a mistake to expand UMPAS. When it was discussing the future of UMPAS, it is possible to close one's eyes to the present international trends. Many countries have decided that it is better to go back to basics, some have even moved forward to basics. The first slogan is more relevant to Egypt than the second because of two factors. First, there is the pressure of certain mathematicians, particularly those working in applied mathematics (mechanics), who were set against the new mathematics from the beginning. The second factor is the greater feeling of familiarity of many teachers with the traditional mathematics rather than with the new, and this applies especially to those who have not taught the new curriculum before.

On the other hand, it may be argued that for UMPAS or any other project, evaluation should be the criterion by which the decisions have been taken. Pedagogically nothing can be refused, excluded or included without the use of objective evaluation. It is now appropriate to devote time and effort to an evaluation of these projects. The continuous evaluation of any curriculum can determine the proper improvements that should be made and, step-by -step, a more fully satisfactory curriculum can be achieved. It is true that evaluation needs time, but it has to be remembered that whereas schools had worked with traditional curricula for hundreds of years, they have used the new curricula for only about 10 years, while evaluation of them has proceeded for even less time. In Egypt it also has to be remembered, that the back-to-basics movements is not connected solely with the new mathematics or even with the conditions inside the schools. It is also related to other external social factors. It should be mentioned here that this does not mean supporting one trend as opposed to another, but it is supporting the principle of evaluation. In other words, judgement must come before decision.

It is attempted in the pages that follow to condense a rather diffuse and sometimes inaccessible area of educational curriculum. The Ministry of Education in 1971 (*) published, THE CURRICULA OF THE CULTURAL UNIT FOR THE PRIMARY SCHOOL, and THE PRINCIPLES OF THE NEW MATHEMATICS FOR THE PRIMARY SCHOOL (**). These two books concerned the aims of teaching of geometry and arithmetic in primary school, general instruction for arithmetic primary school teaching, schemas of work and curricula for arithmetic, and the concepts of new mathematics. Since one of the aims of this chapter is to chart the development of the arithmetic framework scheme in primary schools during recent years, the arithmetic framework scheme curricula in primary school for 1971 is given in appendix XIII.

Yehia Hendam 1973 (***), published TEACHING NEW ARITHMETIC FOR THE BEGINNERS BY USING EDUCATIONAL

(*) Ministry of Education, THE CURRICULA OF THE CULTURAL UNIT FOR THE PRIMARY SCHOOL, Cairo, 1971, 127-145.
(**) Ministry of Education, THE PRINCIPLES OF THE NEW MATHEMATICS FOR THE PRIMARY SCHOOLS, Cairo, 1971, 5-56.
(***) Y. Hendam, TEACHING NEW ARITHMETIC FOR THE BEGINNERS BY USING EDUCATIONAL AIDS, Cairo, 1973.

AIDS, which contained advice on how to use aids usefully and successefuly. It also included researches about how children learn arithmetic. Dr.Yehia Hendam held a chair in the Girls' Faculty at Ain Shams University, department of curriculum and teaching methods. These researches will be explained later in the chapter which gives a review of the literature.

Prof. W. Ebeid 1974 (*), issued THE REQUIRED MATHEMATICAL SKILLS FOR STUDYING SCIENCES FOR PREPARATORY STAGE, which contained principally:- determining the plan of the research, mathematics for use in first, second, and third grades of preparatory schools and recommendation and a questionnaire for science teachers in preparatory stage and its results: it also gave a definition of a mathematical skill and produced a content analysis of two subjects of the preparatory stage in both mathematics and the sciences.

Yehia Hendam 1975 (**), published his book which contained a new attitude to studying new mathematics, basic concepts on which logical and mathematical

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(*) W. T. Ebeid, THE REQUIRED MATHEMATICAL SKILLS FOR STUDYING SCIENCES FOR PREPARATORY STAGE, Cairo, 1974.
(**) Y. Hendam& A. Ebraheem, INSTRUCTING A CHILD THE NEW MATHEMATICS BY ACTIVITIES, Cairo, 1975, 6-74.

thinking are based and activities to help to improve and develop mathematical thinking in kindergarden stage. Other articles were issued on that subject by The Arab Organization of Education, Culture and Sciences. One of these was in June 1976, which explained how important education is and, in particular, the teaching of mathematics (*). Another was in June 1977 (**), which explained the science and education of the Arab world among modern life in the world. A third, in December 1977 (***), contained 'Computing in school science and mathematics', it also included the ALECSO Pilot Project in mathematics. Above all the Arab Educational, Cultural and Scientific Organization continued to issue articles in the field of education in Arab societies as a guide to improve and develop education, teaching mathematics and science in particular and pointing out their weaknesses in Arab societies. One issued in June 1981 (****) discussed,

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(*) The Arab Organization of Education, Culture and Science, _Journal of Teaching Mathematics and Sciences_, Cairo, June 1976, 1st year, 1.

(**) The Arab Organization of Education, Culture and Science, 'Among modern life in the world', _Journal of Teaching Mathematics and Sciences_, Cairo, June 1977, 1, 1-7.

(***) ALECSO, 'Computing in school science and mathematics', _Journal of Teaching Mathematics and Sciences_, Cairo, December 1977, 2nd year, 2, 7& 20-21.

(****) The Arab Organization of Education, Culture and Science, 'An evaluation of educational programme in Arab societies', _Journal of Teaching Mathematics and Sciences_, Cairo, June 1981.

'An evaluation of educational programme in Arab societies'.

Lilah Riad, who is the Head of the Nursery and Kindergarten department at Ramasis College for girls (ex American College) published a non-government Kindergarten book PRE-PRIMARY MATHEMATICS in 1980 (*). This book for the nursery stage had the aim of introducing the concept of numbers as an answer to the child's growing sense of logic and for the need to quantify it, i.e.the logic of number. The manipulation of numbers was introduced. All numbers were essentially multiples of "one", hence the manipulation of "one" as the basis of increase and decrease in numbers from one to ten had been stressed throughout.

At first the child was made to count sets of successively increasing numbers of similar objects but of a different nature each time. Writing the number in letters was simultaneously introduced. The associative nature of numbers was introduced by one to one correspondence of objects naturally linked together, e.g.cup and saucer etc. Finger counting as resorted to by children intuitively, was actually a

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(*) L. T. Riad, PRE-PRIMARY MATHEMATICS, Cairo, 1980.

form of one to one correspondence. As the child learns more of number manipulation skills, finger counting will be abandoned spontaneously. Synthesis and decomposition of numbers was introduced, where the shift of one was emphasized by drawings proceeding in a step-like manner for the different components of each number.

In a pre-primer course the book's emphasis was not on solving sums of addition and subtraction, but simple patterns of addition and subtraction were introduced as a way to emphasize manipulation skills. Ordination e.g.first, second, etc...was introduced. Ordination and cardination (simple mentioning of numbers serially) should be linked together in the child's mind. Halving was used to emphasize the nature of "one" as a whole number. As an example of the practical usage of numbers, spatial relationship was resorted to e.g objects diversely were placed at top, bottom, left and right in relation to each other. More instruction was introduced in the nature of geometric figures and their inter-relationship. The main aim of that course was to present to the child mathematics without tears.

In 1981, another edition of <u>PRE-PRIMARY MATHEMATICS</u>, (*), was produced for the same stage by the same writer. This book, over twenty-two pages, pointed out that the child, by use of the sense of sight, learns to identify different objects. Soon the child classifies similar objects as belonging together to form one set. The next step in the child's growing sense of logic is to attempt to quantify what he was able to classify. Hence there arises the need to answer the question: How many?. This is at first answered by quantifying words such as more, less or equal. A more refined way of quantifying is further felt by the child. Hence the need of numbers or the logic of number presents itself. The main aim of the course for the nursery stage was to introduce the concept of number to the child. The child first resorts to collective quantifying i.e.viewing the aggregates as a whole. As the rings in the drawings were added one by one in the rack, they got higher. After this initial approach, spacing of similar objects was done. The child was made to count similar objects forming a set. The next step was the counting of random objects and matching them with the appropriate number on the page. So far a counting procedure was used and the concept of the quantifying role of numbers had been grasped. As a further step, the sequential

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(*) L. T. Riad, <u>PRE-PRIMARY MATHEMATICS</u>, Cairo, 1981.

nature of numbers in the abstract was introduced. This was cardination. Cardination was the mere sequential mentioning of numbers in ascending and descending order. For this, familiar nursery rhymes were a help e.g. One, Two, Buckle My Shoe. Actually these were number mnemonics. Vividly coloured charts, not in the book, were further used in the class. Various class activities were also done, some by the children as group games and some by the teacher alone. Examples of the latter were counting how many times a ball was bounced or a note was played on the piano. A correct approach following the logical sequence of events in a child's mind simplifies the mathematical procedure once and for all. Geometric figures had been introduced because they answer the child's questioning about spatial perception.

It is intended to chart the history of the arithmetic curriculum development movement in primary school. The last decade has seen changes in the framework by which these schemes have been reviewed since 1916. The content analysis of the recent primary mathematical school books used in Egypt in 1983 has been reviewed in appendix VIX. These books have been thoroughly analysed by this thesis into objective content of concepts involved, facts and the expected skills for both traditional and modern mathematics in every grade. This deliberate analysis for twelve primary school mathematics books is to build on research's measurements for evaluating basic mathematical skills. These mathematical skill tests, of which evaluation is made, will be well explained in the following chapters.

This chapter mainly consists of two parts. First, there is a brief discussion of some problems of research and school framework scheme as they relate to arithmetic; and, second, there is a group of specific proposals for research submitted by some educational members of the profession who have an active interest in the teaching of arithmetic. The educationists hope that teachers interested in some of the particular problems that are suggested will correspond with the persons who proposed them, so that all may benefit from sharing their interest in the problems. In that sense, this chapter may function as a clearinghouse for those interested in doing research in this field.

This has been an attempt to describe and understand the process of planned mathematics curriculum change that Egypt has experienced over the past seventy years. The process has reached a hiatus, perhaps a permanent halt, and it may be that the day of the national curriculum development project is over. On the other hand, centralised curriculum innovation preceded a current feature of educational systems throughout the Arab hemisphere.

It has been tried in this chapter to chart the history of the mathematics curriculum development movement in Egypt, to explain its aims and to describe its processes and structures in examples. The last decade has seen a change in the content of the arithmetic framework scheme. The optimism of the pioneer innovators is now muted, in part because planned change has turned out to be a more formidable problem than was initially assumed.

It started from a view of arithmetic curriculum development as the systematic planning, production and use of new practice and has seen attempts to institutionalise new practice as a central feature of the change process. The focus was on arithmetic framework scheme curriculum changes as an example of the more general phenomenon of planned curriculum change. The curriculum reform movement has been approached from two different perspectives. First it looked at schemes which have been developed to account for changes in society, and for improved communications which allow closer contact with other cultures. Second, it was influenced by an evaluation of studies and projects in other countries. In this way they were able to benefit from the experience of leaders in curriculum reform in Egypt and abroad, learning from the difficulties faced during application, as well as from the approaches used to overcome these obstacles. Thus the research continues from work already started elsewhere, rather than going over the same ground again.

CHAPTER THREE

AN HISTORICAL BACKGROUND TO THE TEACHING OF MATHEMATICS IN ENGLAND

Recent and present interest in arithmetic can be best understood in the light of the historical development of the subject. Without such an historical background much of the research relating to the curriculum would appear to be meaningless.

Arithmetic was the first branch of mathematics to take a place in primary and early education (*). There were two purposes, if rightly taught, as Sir Joshua Fitch, who was Her Majesty's Inspector of Training Colleges and Assistant Commissioner to the Endowed Schools Commission, (**) states in his book 'Its rules become of real service in helping us to solve the problems of daily life; and its laws and principles, if rightly investigated serve to set

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(*) D. E. Smith, THE TEACHING OF ARITHMETIC, London, 1909.
(**) J. Fitch, LECTURES ON TEACHING, London, 1898, 286.

particular mental faculties in operation, and so to further the improvement and development of the learner'. He defined the aims of arithmetic both as an art and as a science, 'You cannot measure its intellectual usefulness by looking only at its immediate aims. It is, in fact, both an art and a science:- an art because it contemplates the doing of actual work, the attainment of definite and useful results; a science because it investigates principles, because he who unearths the truths which underlie the rules of arithmetic, is being exercised, not merely in the attainment of a particular kind of truth about numbers, but in the processes by which truth of many other kinds is to be investigated and attained' (*). He follows this by stating that 'Such a paedagogue, who could do sums of surprising length and intricacy, and set them down in beautiful figures in a book duly garnished with flourishes, passed then for the good arithmetician. The scholar who could work out the largest and most dexterous methods was the winner of all prizes, and so long as he produced right answers, the extent to which he had understood the processes he employed was a matter of small concern' (**). Adamson (***), who used to hold a

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(*) Ibid, 287.
(**) Ibid, 287.
(***) J. W. Adamson, THE PRACTICE OF INSTRUCTION, London, 1907, 288.

chair in King's College at London, in his book THE PRACTICE OF INSTRUCTION , puts forward similar views.

It is assumed that Robert Recorde was the writer of the first series of mathematics texts in English although Cuthbert Tunstall completed an earlier one in 1522, a few days before his nomination as Bishop of London. However the books written by Recorde were much better known and used more extensively for another reason as Howson (*) states in his book A HISTORY OF MATHEMATICS EDUCATION IN ENGLAND, 'Recorde was the first mathematics educator. Not only did Recorde teach mathematics, but his writings show clearly - both implicitly and explicity - that he had also given serious consideration to the problems of learning and teaching mathematics'.

Recorde's first arithmetic text book and his most famous and important one was THE GROUND OF ARTES which was first published in 1543, and appeared in numerous other issues and editions - at least forty five - and under various editors . This was twenty one years after Tunstall's book DE ARTE SUPPUTANDI (**) which was published in seven editions in Paris and Strasbourg and was based on Italian models.

(*) A. G. Howson, A HISTORY OF MATHEMATICS EDUCATION IN ENGLAND, Cambridge, 1982, 6.
(**) Ibid, 13.

Recorde's first book was used for nearly a century and a half, the last edition coming out in 1699. He also wrote THE PATHWAY TO KNOWLEDGE , it was first published in 1551 (also in 1574 and 1602) which contained 'the first principles of geometrie, as they may moste aptly be applied unto practise, bothe for the use of instrumentes geometricall, and astronomicall and also for projection of plattes (plans) in everye kinde, and therefore much necessary for all sortes of men' (*). Then followed, THE GATE OF KNOWLEDGE (1556), dealing with measuring and the use of the quadrant, THE CASTLE OF KNOWLEDGE (1556), dealing with astronomy, and THE WHETSTONE OF WITTE (1557) which was the last of Recorde's works, much concerned with arithmetic and algebra, dealing with more advanced work in arithmetic.

The importance of Recorde's work lay in the fact that (**) he was one of the earliest writers to show how the rules of arithmetic were used in the ordinary life of the people. These ideas have continued to the present century, and have been of considerable importance in the formation of present methods of teaching mathematics in primary schools. He was also the first British mathematics educator to emphasise

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(*) Ibid, 15.
(**) Ibid, 20.

the limitations of learning by rote.

The arithmetical terms he introduced were in common usage and it is clear that he identified some key problems of mathematical education and sought his own solutions to them. 'Since Recordè's time the majority of authors - from Cocker, Wingate, Vyse, Dilworth, to Walkinghame and Colenso - have treated arithmetic from the utilitarian point of view exclusively. Their books give few or no demonstrations of the theory of numbers, but are filled with what are called commercial rules' (*).

Recorde was followed by Edward Locker - his "arithmetic" came out posthumously in 1677, having been published by J. Hawkins. This book was popular and used for two centuries. By 1750, just over seventy years after its publication, a fifty-third edition had been brought out. Cocker copied largely from Recorde. Both produced text-books of arithmetic consisting of long and tedious repetitions of one or two operations, performed by rule of thumb, leading to an often uncomprehended result known as the "answer". Neither recognised that once dexterity with accuracy is achieved the process is valueless

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(*) J. Fitch, <u>LECTURES ON TEACHING</u>, Cambridge, 1898, 290.

unless it is then used as an instrument to attack a further difficulty.

Cocker's book was much criticised by the nineteenth century reformers, and particularly by De Morgan. De Morgan was one of the earliest to insist on an explanation of the principles underlying the operations. He therefore, contrary to the usual practice, advocated the use of small numbers because 'the powers of the mind cannot be the directed to two things at once; if the complexity of the numbers used requires all the student's attention, he cannot observe the principle of the rule which he is following' (*). He used diagrams to represent multiplication, and was far in advance of his time in this recognition of the value of a concrete approach to new ideas. In addition he was one of the first to recognize the relationship between the several branches of mathematics. De Morgan published his own text-book of arithmetic in 1830 but unfortunately it met with no great success - it was said to be too difficult.

In spite of De Morgan's efforts there was no appeal to originality, no scientific investigation of underlying principles in the many other arithmetic

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(*) A. De Morgan THE STUDY OF MATHEMATICS, 1902, 21.

text books which appeared during the remaining years of the century. Colenso's <u>ARITHMETIC</u>, which was published in 1850, was widely used and this book more than any other single influence helped to overthrow the merely lengthy, repetitive processes which had served as arithmetical study. By means of Colenso's systematic arrangement, clearly stated "rules", and well devised examples, it was possible to do quite a large amount of work of varied interest. As to why anything was done was another matter; arithmetic still consisted of the solution by rule-of-thumb methods of problems classified according to type. There was no change of approach until 1900.

The usual practice during the eighteenth century was for the teacher to dictate rules and solutions of problems which the pupil slavishly reproduced in his copy book; <u>ORAL</u> examples were a novelty; text books were on the whole written for the teachers and used by them as a source of the requisite rules. Thus John Mair, in his <u>ARITHMETIC, RATIONAL AND PRACTICAL</u>, published in 1766, described the method for multiplying 73 by 29 as shown here:-

```
73
 4
-------------
292        ( 4*73 )
  7
--------------
2044       ( 4*7*73=28*73 )
  73
--------------
2117       ( 28*73 +1*73 )
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He gives the following rule : 'If your multiplier consists of two or more figures, multiply continually by its component parts, or by the component parts of the composite number that comes nearest it; and then multiply the given multiplicand by the difference of the multiplier and the nearest composite number; the sum or difference of these two products is the answer'.

Again, at the beginning of the nineteenth century the following lines were in vogue as aids to the beginner in division:

" Mick quot ? Multiplica, subduc, transferque sequentem.

First ask how oft ? In quot the answer make; then multiply, subtract and down a figure take".

Again, during the eighteenth and the greater part of the nineteenth centuries, questions involving proportion were dealt by the "Rule of three"; the pupil learned a set of rules covering every possible case of direct and inverse proportion. Thus in James Gray's <u>INTRODUCTION TO ARITHMETIC</u> published in 1797, the necessary rules are stated thus: 'simple proportion teaches from three given numbers to find a fourth. Of the three given numbers two are always of the same kind, the other is of the same kind as the fourth, or number required in the question.

Rule (I) Write down that number, which is of the same kind or species with the number required, in the middle, with two points before it : and four after it :: .

(II) Consider from the sense of the question, whether the answer ought to be greater or less than this number ; if greater, place the lesser of the other two numbers on the left hand, for the first, and the other on the right; but if less place the greatest for the first.

(III) If the first and third numbers are of different denominations reduce them into the same, and the second number into the lowest denomination mentioned.

(IV) Multiply the second and third numbers together, and divide their product by the first, the quotient is the answer if there be no remainder and is always of the same denomination with the second number.

(V) If there be a remainder, reduce it to the next lower denomination, and divide by the same divisor: proceed thus with all the remainders till you have reduced them to the lowest denomination which the second number admits of, and the several quotients will be the answer required' (*).

This was the "rule of three" as stated in a very popular text-book which by 1825 had run to twenty-one editions.

The first writer to break away from this method of "teaching" proportion was De Morgan. In his ELEMENTS OF ARITHMETIC which was first published in 1830, his explanation of the rule of three comes very near to the "unitary" method. The majority of authors wrote books which provided little demonstration of the theory of numbers but were filled with what were called "commercial" rules. The aims of teaching arithmetic were outlined by Sir Joshua Fitch as being 'very clearly defined, and all the progress towards it is regulated accordingly. The successful arithmetician is to be a good computer, a skilful tradesman, a land surveyor, or an exciseman; and the whole object of the art is to fit him to perform one or other of these important functions' (**). The late eighteenth and early nineteenth centuries had thus seen the publication of many text-books suitable for pupils, suggesting that

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(*) W. Gray, 'The teaching of elementary mathematics in Scotland in the nineteenth century', M.A Thesis, Unpub., Edinburgh, 1952, 151- 158.
(**) S. J. Fitch, LECTURES ON TEACHING, Cambridge, 1898, 290.

exercises or good sums should be given in words, not in figures, since actual life presented them in words not in the shape of sums. Children were often puzzled by this approach, first asking what rule is it, and how to set it down. Nevertheless many teachers continued to follow the old system of rule-of-thumb methods and dictation of solutions. Much time was spent in teaching clever tricks of computation of the most limited application and there was no appeal to general principles. This mechanical method of teaching a rule and its application was roundly attacked by the best teachers during the nineteenth century, but many who professed a belief in the value of an understanding of the principles involved continued to write text-books in which the emphasis was on the clear statement of rules.

Generally speaking, the arithmetic course during the nineteenth century was overloaded with much heavy work that had little or no educational value, was of no practical utility and was often of an artificial character.

In the mid-nineteenth century those concerned with the practice of arithmetic were by no means ready for the scientific and logical ideas of De Morgan, but it was beginning to be felt that arithmetic should not be taught only for utilitarian reasons. One of the first to advocate a more liberal approach in the teaching of arithmetic was Sir Joshua Fitch who realized that 'a good education should aim at giving a pupil power, capacity, seriousness, method, intellectual insight; not specific preparation for this or that department of the world's business. All plans of education are degraded and vulgarized directly this principle is forgotten. They are ennobled in just the proportion in which it is steadfastly and faithfully borne in mind' (*).

Eight years later, in a series of lectures on teaching given at Cambridge during the Lent term, 1880, he attacked the "copy" methods which were still in vogue in many schools: 'Making out a fair copy of a sum in a book, garnished with ruled red ink lines and flourishes, is a favourite employment in some schools, and consumes a good deal of time. It has its utility, of course, as an exercise in neatness and arrangement, and in the mere writing of figures. Moreover, it is liked by some teachers because it pleases parents, and is the only visible evidence of arithmetical progress which can be appreciated at

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(*) J. Fitch, <u>METHODS OF TEACHING ARITHMETIC</u>, London, 1872, VOL.XIX, 36.

home. Yet as a device for increasing or strengthening a child's arithmetical knowledge, it is very useless' (*). He deplored the use, in manuals of mental arithmetic, of rules founded upon accidental facilities afforded by particular numbers, applicable only in isolated cases: 'the student who has his memory filled with these rules is not helped, but rather hindered, by them' (**). He called for discrimination in the list of tables children were expected to learn by heart; he agreed that they must know those tables of weights and measures that were in constant use, but pointed out that it was not worth while to learn apothecaries' weight, cloth measure, or ale and beer measure, because these measures were no longer in actual or legal use, and the sums on them which the books contained were only survivals from an earlier age. 'Here, as elsewhere, (we must) abstain from giving to the verbal memory that which has no real value, and is not likely to come into use' (***). He mentioned the value of estimating a rough answer before doing a sum and advocated the broadening of the range of topics usually dealt with in arithmetic to include, for example, the computation of the time of falling

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(*) J. Fitch, LECTURES ON TEACHING DELIVERED IN THE UNIVERSITY OF CAMBRIDGE DURING THE LENT TERM, 1880, Cambridge, 1884, 298- 299.
(**) Ibid, 300.
(***) Ibid, 305.

bodies; actual measurement of the playground and elementary land surveying; the use of logarithmic tables and the solution of triangles by means of them; their application to the determination of the heights of mountains and spires, or the breadths of rivers; the difference of time between various places whose longitude is given; the measurement of distances on a map which has a scale of miles attached to it; the reading of the thermometer and the conversion of Fahrenheit to centigrade; the statistics of attendance in the school itself and the method of computing its average attendance. He pointed out that the value of these applications is not in utility but in the cultivation of general power, fertility of resource and quickness in dealing with numbers, all of which are of importance in the intellectual and in the practical life of all boys and girls. He was in favour of the use of unitary method instead of the "Rule of three" in dealing with problems on proportion 'because "the rule of three" is a great stumbling block to learners. It comes much too early in the course, and learned empirically as it too often is, is not readily capable of application to problems' (*). Thus he was one of the first to recognize the importance of the time-placement of a subject. He suggested that decimals

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(*) Ibid, 335- 336.

should be used in money questions and gave the "farthings rule" for the decimalisation of money.

Professor Perry in a suggested syllabus of 1901, advocated the early use of decimals and went on to make the revolutionary proposal of the use of logarithms and slide rules in the arithmetic course and of an experimental approach to the teaching of mensuration. In 1902, the committee of the British Association, set up to report upon improvements that might be affected in the teaching of elementary mathematics, urged the adoption of a decimal system of weights and measures in this country (a simplification still not fully achieved eighty years later) and reinforced Professor Perry's plea that, in the early stages, constant appeal should be made to concrete illustrations. One of those who emphasized concrete illustrations in the early stages was John William Adamson who was Professor of Education at King's College, London, who states in his book THE PRACTICE OF INSTRUCTION that (*) 'In a discussion arising out of the report on the improvements which may be effected in teaching mathematics, presented to the British Association in 1902, Professor Perry remarks that the Anglo-Saxon boy has never been

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(*) J. W. Adamson, THE PRACTICE OF INSTRUCTION, London, 1907, 286.

educated except through the senses'. The Association committee regretted that in arithmetic examinations, too many questions were tests rather of mechanical facility than of clear thinking or of knowledge.

In the same year the Mathematical Association issued its report on <u>THE TEACHING OF ELEMENTARY MATHEMATICS</u>. It stated that 'the Committee consider that there is considerable danger of the true educational value of arithmetic and algebra being seriously impaired by reason of a tendency to sacrifice clear understanding to mere mechanical skill', and went on 'In view of this the Committee recommend:

(i) That easy viva voce examples should be frequently used in both arithmetic and algebra;

(ii) That great stress should be laid on fundamental principles;

(iii) That, as far as possible, the rules which a pupil uses should be generalisations from his own experience;

(iv) That, whenever practicable, geometry should be employed to illustrate arithmetic and algebra, and, in particular, that graphs should be used extensively;

(v) That many of the harder rules and heavier types of examples, which examinations alone compel us to retain in a school curriculum, should be postponed' (*).

The report then made detailed recommendations as to the treatment of the individual branches of mathematics. Dealing with arithmetic, it stated that, in examinations in arithmetic, the use of algebraic symbols should not be prohibited; and further that 'the committee consider that the mental training afforded by arithmetic is largely impaired by the existing tendency of text-books to classify problems and to establish each type as a separate rule' (**).

M. E. Boole took up the last point in her book LECTURES ON THE LOGIC OF ARITHMETIC. She stated that 'teachers of such subjects as Electricity complain of the difficulty of getting pupils to apply what they know of mathematics (at whatever level) to the analysis and manipulation of real forces. It is not that the pupil does not know enough (of arithmetic or algebra, or the calculus as the case may be) but he too often does not see, and cannot be got to see, how to apply what he knows. Some faculty has been paralysed during his school life; he lacks

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(*) Mathematical Association, THE TEACHING OF ELEMENTARY MATHEMATICS, 1902, 6.
(**) Ibid, 7.

something of what should constitute a living mathematical intelligence' (*). She went on to say that 'some of the worst mental habits are induced by the practice of teachers making a statement as if _ex cathedra_, and then proceeding to bring forward proofs of its truth. Anything (the teacher) intends to prove should never be stated; children should be led up to find it out for themselves by successive questions' (**). Although this is an improvement on the earlier system which consisted largely of the dictation of rules, many today will feel that it does not go far enough and that the child, by experimental investigation, should have to the full the joy of the discoverer.

W. P. Turnbull dealt with the same point in his book, _THE TEACHING OF ARITHMETIC_, writing of the proper treatment of rules, he suggested that 'It is a vital point of good method that no rule should be given. This does not mean that no rule should ever be known and used; far from it. But the rule should be taught in a proper way, and dictation is not the proper way. A rule should be discovered by the children under the teacher's guidance and it should be carefully formulated with the teacher's help. The

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(*) M. E. Boole, _LECTURES ON THE LOGIC OF ARITHMETIC_, London, 1903, 7- 8.
(**) Ibid, 16.

formulated rule should be learned by heart verbatim and should be applied until the children have a mechanical mastery over the operation. It should not, however, be discovered once for all; but, as often as necessary, the teacher should in some fresh instance lead the children to discover the rule anew, even though they know it by heart already. What spoils arithmetical teaching is taking steps in a wrong order, putting practice before understanding instead of understanding before practice' (*). Turnbull's remarks on the rediscovery of known rules are reminiscent of A. N. Whitehead's conception of inert ideas, expounded in his AIMS OF EDUCATION (**), first published in England in 1932. Whitehead considered an idea inert if it was not rethought genuinely in experience every time it was repeated. He based his fundamental criticism of orthodox education on the fact that dogmas were being handed on without being re-examined or revitalized by being made true to experience.

Turnbull also suggested that children should be allowed to invent problems, thereby encouraging their inventive powers and also training them to

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(*) W. P. Turnbull, THE TEACHING OF ARITHMETIC, London, 1903, 95.
(**) A. N. Whitehead, THE AIMS OF EDUCATION, London, 1932.

distinguish between essential and inessential data; that arithmetic should be made as realistic as possible; and that any division of arithmetic and algebra into distinct subjects was to be deprecated.

All the points mentioned in the last paragragh were restated in the Board of Education's SUGGESTIONS FOR THE CONSIDERATION OF TEACHERS AND OTHERS CONCERNED IN THE WORK OF PUBLIC ELEMENTARY SCHOOLS, in the chapter dealing with the teaching of arithmetic. It was further stated that 'If from the very first the aim of the teaching were to make the children understand the reasons of the processes that they employ, the time given to arithmetic in many schools might be curtailed without the effect of the instruction being impaired. It is important that arithmetic should be treated not merely as the art of performing certain numerical operations. It should be taught with the view of making the scholars think clearly and systematically about number. It is thus clear that written arithmetic should be an appendage to mental work rather than the reverse' (*). This is the first official mention of the value of mental arithmetic, and of the possibility of a reduction in

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(*) Board of Education, SUGGESTIONS FOR THE CONSIDERATION OF TEACHERS AND OTHERS CONCERNED IN THE WORK OF PUBLIC ELEMENTARY SCHOOLS, H. M. S. O, 1905, 40.

the time spent on the study of the subject; it was this saving of time that was to make possible the introduction of trigonometry and calculus into the main school syllabus of secondary schools.

By 1906 some of the suggestions made in the Mathematical Association Report of 1902 had been adopted. T. J. Garstang, writing on the teaching of mathematics in public schools, stated that 'fortunately for teaching, the use of algebraical symbols is now permitted in examinations on arithmetic, and the best method available may be adopted in finding the solution of a given problem. Large as such innovations may appear, it remains doubtful whether they have gone far enough. The commercial arithmetic still exacted through examinations is largely either a survival of past commercial method or a collection of artificial fictions' (*). He went on to describe the value of a practical approach to the subject: 'Many of the public schools have recently equipped laboratories for practical mathematics; youth will thus make real experience of tangible objects and instruments of measurement. If practical problems must be given to boys, the most fundamental principle of education

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(*) T. J. Garstang, <u>THE PUBLIC SCHOOLS FROM WITHIN</u>, London, 1906, 16.

demands that there should be firsthand experience of the concrete subject matter referred to in the problems. History amply justifies this latest innovation. Not once but many times have great mathematical truths been discovered through the difficulties presented in dealing with things' (*).

Four years later P. B. Ballard gave a more detailed explanation of the value of the experimental approach in a book entitled HANDWORK AS AN EDUCATIONAL MEDIUM . In the preface he stated that 'there has been in the minds of many of us who are intimately concerned with education a steadily growing conviction that the side of school work that specially needs strengthening is the expressional as distinct from the receptive. The child should do more for himself; he should have fuller opportunity of exercising his natural tendency to plan, to manipulate and to construct' (**).

In a chapter entitled Pitfalls In The Teaching Of Arithmetic, he discussed this in detail: 'the newer method of regarding educational problems from the standpoint of the child and the various stages of his development, as distinguished from the standpoint

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(*) Ibid, 18.
(**) P. B. Ballard, HANDWORK AS AN EDUCATIONAL MEDIUM, London, 1910, VII.

of the adult, has not yet had time to leaven completely the scheme of instruction in this subject. We see in all branches of instruction a tendency to bring the mind of the pupil, in the early stages of his school career, into closer contact with the concrete facts of life, and to postpone to a later period the systematic and formal study of the data there presented. We are coming to consider it desirable that the early stages of instruction in arithmetic should be mainly concerned with the application of commonsense to the numerical relations of concrete objects with which the child is brought into daily contact. The reasonableness of this order is obvious. Not merely are the stages of development in the child's brain followed, but the knowledge thus acquired is firmly rooted in reality, and is in vital touch with that system of knowledge which is brought into use in everyday life' (*). So he emphasised that there must be continuity of interest, and that the operations in question should serve as a means to an end and not as ends in themselves. The lack of variety in the exercises was another source of ineffective training. He advocated the use of short problems. These were of more value than long problems in sustaining interest, and went on to say that 'the advantages of the short, concrete problem

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(*) Ibid, 159- 160.

are numerous, not the least being the large number of ideas brought into use during the lesson' (*). He mentioned the importance of oral arithmetic, and recommended that 'the barrier between oral and written arithmetic should be broken down, and the point of transition from one to the other be recognised as varying with the individual pupil' (**).

He attacked the ideas of this division between the several branches of mathematics as well as the isolation of arithmetic from practical life, 'while the isolation of arithmetic from the practical side of life often leads to an unintelligent jugglery with figures, its isolation from the other branches of mathematics is equally undesirable, resulting in a loss of effectiveness for each. The incursion of geometry into the arithmetical realm in the way of graphs and mensuration is heartily welcomed by most teachers, but there still remains a reluctance to use algebraic symbols and processes in the arithmetic lesson' (***). He suggested that the time spent in applying the principle of proportion to questions in interest, profit and loss, partnership, stocks, discount and other commercial transactions could be

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(*) Ibid, 165.
(**) Ibid, 166.
(***) Ibid, 166.

put to better use in a more rigid and scientific investigation of the principles of pure number.

In general, Ballard stated that 'the arithmetic course should consist of a gradual progress from familiar concrete cases to abstract principles, leading up to the still more abstract science of algebra. This does not mean that there should be no abstract arithmetic at the beginning of the course and no concrete at the close, but that the teacher's main aim should at first be to graft the arithmetic firmly upon the actual facts of life, and afterwards to proceed to the investigation and application of the laws of number' (*). In developing systems of knowledge in the child's mind isolation always means weakness. The divorce of school life from home life and the dissolution of the natural partnership between arithmetic, algebra and geometry, are equally regrettable. They lead to looseness of mental grip. Too much time spent in example-grinding to promote facility secures mechanical results at the expense of intelligence......'To promote the complete understanding of the principles of numbers is the main concern of the teacher of arithmetic, and to this must be subordinated all other aims, even that of securing accuracy and speed in mechanical

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(*) Ibid, 170.

processes; for the system that enables a pupil to compute with ease and think with difficulty stands self-condemned' (*).

G. W. Palmer in 1912, who was a mathematical master at Christ's Hospital suggested that in the earlier stages of learning arithmetic (that is, the part dealing with numeration, the four rules, compound, quantities, metric system, decimal fractions and vulgar fractions), the methods of five years ago were still very largely followed, but with differences such as:-

(I) The principles were explained more thoroughly, since diagrams were being used for this purpose.

(II) There was a more serious attempt to explain the necessary rules.

(III) Fractions was confined to the simpler kinds.

Palmer stated that it was assumed that graphical work would be used throughout as a means of illustration and occasionally, as a means of calculation, as in proportion and logarithms.

He emphasised that the problems should be miscellaneous; they should not be separated into types.

Summarising the conclusions arrived at after

(*) Ibid, 171.

twenty-five years of experiment, Palmer stated that 'the reformers would clear away entirely parts of the subject which do not seem essential and also clear away elaborate and artificial developments of many other parts. Rules should not be used until a considerable effort has been made to explain their meaning. Sets of examples should be so varied that they cannot be worked by a purely imitative process but must necessarily require thought. New ideas should be made as real as possible by concrete illustration. This should include geometrical illustrations such as diagrams, graphical work on squared paper and also laboratory work in close connection with mathematical teaching. Questions should, as far as possible, be taken from the arithmetic of everyday life, and at any rate they should not offend one's common sense'.

(*) 'In all these changes examining bodies can do a great deal to help or hinder. There are many reforms that could be introduced tomorrow if it were not for the demands of examining bodies. The majority of the examination papers set at the present time are an obstacle to many changes which nearly all teachers regard as beneficial and are anxious to

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(*) G. W. Palmer, THE TEACHING OF ARITHMETIC IN SECONDARY SCHOOLS; BOARD OF EDUCATION SPECIAL REPORTS ON THE TEACHING OF MATHEMATICS, 1912, 256.

introduce. So long as these papers remain unaltered, the hands of teachers are tied and progress is necessarily slow'.

Despite this Palmer felt that during the previous ten years a great improvement in the teaching of arithmetic had begun to enter the English schools; and he felt that this would be even greater if the teachers of arithmetic had been mathematicians. He mentioned that only in a small percentage of schools was the teaching of mathematics entirely in the hands of properly qualified mathematicians. So from his view point many of the teaching weaknesses in mathematics were due to the shortage of mathematics specialists. In the following year, the Board of Education issued a revised edition of its pamphlet of 1905 on the teaching of arithmetic. It was suggested that only simple examples of each rule should be given, so that there should be no delay in passing on to new ideas. By this means, the child's interest would be maintained. It pointed out that 'by concrete methods in this report is meant practical work done by the children. This should not belong just to the early stages but in suitably varied form should accompany much of the more advanced teaching. The practical work, if properly directed may not only introduce the rules but also lead the children to establish rules for themselves. Children who have been so trained will develop habits of mental activity and self-reliance when faced with new problems, habits which can be but little fostered by the mere dogmatic teaching of the rules' (*).

Ballard in TEACHING THE ESSENTIALS OF ARITHMETIC states that 'Let children learn the multiplication table so that it can be reproduced, item by item, with mechanical precision and promptitude; fix the routine of the simple rules so that they absorb the minimum of creative thought; foster the formation of useful habits so that intelligence may be kept at work in its proper sphere. Habit is a servant; see that it is a good servant. Intelligence is a master; see that it is not allowed to concern itself too much with life below-stairs' (**).

The Mathematical Association Report on THE TEACHING OF ARITHMETIC IN SCHOOLS advocated rule-of-thumb methods, but with an approach very different from that prevailing during the eighteenth

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(*) Board of Education Circular 807, SUGGESTIONS FOR THE CONSIDERATION OF TEACHERS AND OTHERS CONCERNED WITH THE WORK OF PUPLIC ELEMENTARY SCHOOLS; INSTALMENT NO.3- SUGGESTIONS FOR THE TEACHING OF ARITHMETIC, 1912, 7.
(**) P. B. Ballard, TEACHING THE ESSENTIALS OF ARITHMETIC, 1928, 14- 15.

and nineteenth centuries - the rules were to be first suggested by the pupils themselves. 'All teaching of general principles should begin from simple concrete examples, and the preliminary work leading up to a broad generalisation should be mainly oral. The final generalisation into rule-of-thumb should be delayed until it is almost spontaneously suggested by the pupils themselves. It is vitally important to give a clear justification for any rule-of-thumb; some pupils will understand it, a few will remember it, but all should realise that there is a justification and that it is logic and not magic. On the other hand, it is most important that this final crystallisation of a fluid method into a hard-and-fast rule should be well defined. The pupil must know and use such rules-of-thumb, and the habit of using them must be developed. Mathematics essentially aims at an economy of thought, and the more operations a boy can perform without having to stop and think, the more he will learn about the subject. "Civilisation", as Whitehead says, 'advances by extending the number of important operations we can perform without thinking about them' (*).

(*) The Mathematical Association, <u>THE TEACHING OF ARITHMETIC IN SCHOOLS</u>, 1932, 9 & 10.

The report suggested that everything in the teaching of arithmetic inside class-rooms should be closely related with the life of the pupil outside it.

What could be concluded concerning the teaching of arithmetic was that there was no such insistent demand for reform as in the case of geometry. The changes that were made were in the same direction, but of a less revolutionary kind than in geometry. The teacher tries to establish his pupil's arithmetical knowledge on a basis of experience of the concrete, and the nature of the requisite experience is selected so as to be in accordance with the interests suitable to the pupil's age and environment. In addition, the new arithmetic course combined a happy use of practical work with the discipline of more precise learning. The child was to enjoy multiplication tables rather than consider them a drudgery - even in the early stages of the secondary school course.

In his book <u>THE AIMS OF EDUCATION</u>, Whitehead remarked that 'any serious change in the intellectual outlook of human society must necessarily be followed by an educational revolution. The law is inexorable that education to be living and effective must be directed to informing pupils with those ideas and to creating for them those capacities which will enable them to appreciate the current thought of their epoch' (*).

There has been such a serious change in the intellectual outlook of human society and a very cursory study reveals many of the reasons. The last hundred years have seen the emergence of a world of mass-production, the marvels first of steam and then of electricity, the steady rise of urbanisation and of industrial and commercial life as it is known today. New relationships among people were inevitable.

There were new conditions of work and of leisure, new possibilities, new fears, new revelations of the power of man's hand and still more of the power and creations of his brain. The world changed. Human relationships and human feelings assumed a new importance: men working with men, men working for men, men housed in thousands in new townships, the problems of men seeking promotion, of others who needed training for the new tasks; the new problems of work and recreation - and through all

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(*) A. N. Whitehead, <u>THE AIMS OF EDUCATION</u>, <u>London</u>, <u>1947</u>, <u>116</u>.

these separate movements there was a growing theme - the rise of man's assessment of his real position and intrinsic worth in the welter of the new creation.

In the year 1900 the English educational system was a heterogeneous mixture of types of schools. It was indeed a land of two nations; elementary education, which was only compulsory from 1891, (described as "a regrettable necessity" (*)), was perhaps administered in a grudging spirit and on the cheapest lines to the masses of poor children; while the public school and grammar school educated the sons of the more fortunate, and had little or no connection with the former type. Education in the elementary school was minimal. From the three R'S of 1870 there was only slight amelioration in curricula up to the abolition of payment by results in 1897. Teachers were badly trained and a large proportion of the staff were either unqualified or youthful pupil-teachers. Training facilities were totally inadequate, the pay and general conditions of work were such that all save those with a vocation for the task were frightened away into other occupations. The twentieth century has seen a steady growth in appreciation of the importance of education in

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(*) M. L. Jacks, MODERN TRENDS IN EDUCATION, 1950, 21.

national life. The term education is no longer synonymous with instruction or the mere acquisition of knowledge. These newer ideas are well illustrated by 'The chief end of education is to foster the full and harmonious development of the individual and the function of the school is :-
To provide a rich social environment where adolescence grows in character and understanding through the interplay of personalities rather than by the imparting of knowledge' (*).

Again, education became commonly regarded not as a privilege, not as a necessity, but as the right of all free men.

H. J. Spencer stated in his report in 1912 'great changes amounting almost to a revolution have taken place in the aims and methods of our schools during the last decade and particularly in mathematical teaching' (**).

Because of the importance of this report, it is perhaps wise to summarise the features it proposed in the teaching of mathematics which will help in

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(*) Advisory Council On Education In Scotland, SECONDARY EDUCATION, 1947, 9- 10.
(**) H. J. Spencer, THE TEACHING OF MATHEMATICS IN ENGLISH ELEMENTARY SCHOOLS, 1912, VOL 26, 32.

tracing the embodiment of those ideas in the modern teaching.

The main features were:-

(i) Mathematics has an essential unity and must be taught as a subject - elementary mathematics - rather than as an agglomeration of separate branches, arithmetic, algebra, geometry, trigonometry.

(ii) It merits its place in the curriculum because it is one of the main lines which the creative spirit of man has followed in its development.

(iii) It has an inward and an outward aspect- previous teaching was far too greatly concerned with the former, hence the accumulation of unrealistic practice and the growth of manipulative work. Future mathematical teaching must be concerned with a few broad principles and plentiful illustration. The outward aspect will thus be developed if the examples are chosen from the natural sciences and everyday phenomena within the child's experience.

(iv) From (iii) above, there must be greater effort directed towards making mathematics a set of mental "tools" able to be applied to all manner of problems met in actual life.

(v) With the above aims, mathematics will not be taught merely for mental disciplinary reasons, for utilitarian purposes, for civic uses or for cultural reasons, rather will they all be incorporated in the wider aim.

(vi) Regard for the historical development of the subject would help to promote an appreciation of how the subject grew in response to human needs, while topics would be seen in the right perspective and in correct relationship to one another.

The teaching of mathematics (mainly arithmetic) in primary school must be viewed in its general context - the curriculum which, the Consultative Committee Report (Sir W. H. Hadow, C. B. E. Chairman) stated, 'is to be thought of in terms of activity and experience rather than of knowledge to be acquired and facts to be stored' (*).

The Board of Education's pamphlet no. 101- SENIOR SCHOOL MATHEMATICS, (**) pointed out that while arithmetic has utilitarian values, these are 'substantially less than the content of the ordinary school course up to 14 years' (***). Cultural and civic requirements, on the other hand, are much less restricted.

(*) Consultative Committee Report (W. H. Hadow, C. B. E. Chairman), On THE PRIMARY SCHOOL, 1931, 93.
(**) The Board Of Education, SENIOR SCHOOL MATHEMATICS, 1935.
(***) Ibid, 5.

Dealing with the link with the junior school the pamphlet stated that 'the best fruit of earlier arithmetical teaching is in the power of dealing intelligently with numbers and numerical data, and this power is best developed by a balanced syllabus which gives due weight, not only to the acquirement of accuracy in computation but also to adequate practical work, to facility in mental arithmetic, and to the solution of simple and varied problems' (*) a statement quite in line with the developments traced so far.

There (**) have been so many challenges placed before primary school teachers in the last few decades that it would not be surprising if they had given up thinking. These challenges may be viewed in the context of general educational trends in which three broad stages may be identified. Before the 1960s, although some reorganisation of school structures is apparent and there is evidence of the introduction of some progressive ideas into primary school, there was really little change in the technology of teaching.

During the 1960s there were years of

(*) Ibid, 9.
(**) W. Taylor, <u>RESEARCH AND REFORM IN TEACHER EDUCATION</u>, N.F.E.R, 1978.

unparalleled expansion at all levels. Interest was expressed in innovation both in content and in method and numerical growth provided by expanding resources, which resulted in new projects, programmes, journals and numerous conferences and meetings. Developing sociology and psychology provided background theory. However, many of these influences came from outside the school. The 1970s showed little change after the progress of the 1960s but gradually emphasis began to centre on the teacher as the organiser of teaching resources and as a school-based curriculum designer.

In addition, teaching had become a much less private activity both within a school, namely team teaching and more open classrooms, and outside the school, namely cooperation with social workers and the demands for accountability and standards.

However, two dimensions of change and their inter-relationship must be taken into consideration in this discussion. They are both mentioned in the Mathematical Association Report 1955 (*) and are :- curriculum content and the learning processes. The report considered both aspects, stating that 'The aim of primary teaching is the laying of a foundation of

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(*) Mathematical Association, <u>THE TEACHING OF MATHEMATICS IN PRIMARY SCHOOLS</u>, London, 1955.

mathematical thinking about the numerical and spatial aspects of the objects and activities which children of this age encounter' (*). But the emphasis of the report was on children and their learning: 'children developing at their own individual rates learn through their active response to the experiences which came to them; through constructive play, experiment and discussion children become aware of relationships and develop mental structures which are mathematical in form and are the only sound basis for mathematical techniques' (**). Two aspects of understanding were considered important: the teacher's understanding of how children learn and the child's understanding of concepts. In 1956 the Mathematical Association published a report (***) intended "for consideration by all concerned with the development of young children". The background to the report is of some interest, for the work on it began in 1938 at which time the committee concentrated on what should be the content of the syllabus in junior schools. The war brought that committee's deliberations to an end, and when it was reconstituted eight years later the emphasis had changed. The new committee did not share the belief

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(*) Ibid, V.
(**) Ibid, V.
(***) Mathematical Association, THE TEACHING OF MATHEMATICS IN PRIMARY SCHOOLS, 1956.

that a curriculum should be drawn up prescribing the mathematics to be taught at each stage of primary years. Instead, the members discussed children's approach to number through play and the use of apparatus. The committee was led to make deeper enquiries into the nature of children's mathematical development and found that the assumptions that a syllabus could be prescribed and that good teaching could be produced by careful analysis of the logical steps involved, had become inconsistent with the newer principles taking shape. A new doctrine had emerged finally to be elaborated by a third committee which adopted it as its guiding principle.

Improvements in primary education were to come about, then, not through the introduction of new content, or the prescription of syllabuses, but by treating children as individuals, by guiding their responses to everyday experiences and constructive play, and by the development of appropriate mental structures. New comments about children learning appeared, such as chronological age is not of itself a test for readiness for a new process: a child's experience is total, mathematics is part of it and can only be isolated in the mind of the teacher: in group work it is assumed that the teacher can ascertain the child's rate of progress: there is a quantity of apparatus on the market and teachers need to examine it critically before adopting it....the criterion is whether children in fact learn from it and this can only be decided by the teacher after observation. All the above were noted in PRIMARY EDUCATION 1959 (*). But the summary of research on mathematics teaching was mainly related to arithmetic and showed that teachers too lacked experience and activity of their own on which to reflect. However, MATHEMATICS IN PRIMARY SCHOOLS, 1966 (**) showed that thinking and activity were taking place. It provided an illustrated account of both children and teachers exploring mathematics, indicating in considerable detail the type of activity which was being undertaken in classrooms, schools and teachers' centres at this time. This experience was the background to the second Mathematical Association Report in which there was much more conscious awareness of the changes teachers were having to face:

(i) Mathematical education rather than arithmetic techniques.

(ii) Understanding and developing a child's mathematical learning.

(*) Board of Education, PRIMARY EDUCATION, H. M. S. O, 1959.
(**) Board of Education, MATHEMATICS IN PRIMARY SCHOOLS, H. M. S. O, 1966, no.1.

(iii) New ideas in mathematical content requiring both personal understanding and the development of appropriate teaching methods. It is clear that planning is vital and cannot be achieved by teachers in isolation. Teachers are encouraged to make continuing attempts to develop their professional judgement and MATHEMATICS 5-11, (*) indicates areas in which this judgement may be required and some means by which this development may be achieved. In a survey made of primary education in England by H M Inspectors of Schools, Board of Education 1978 (**) in chapter 5, sections 6.28- 6.31 ; chapter 8, sections 8.21- 8.22, the findings do not support the view which is sometimes expressed that primary schools neglect the practice of the basic skills in arithmetic. In the classes inspected considerable attention was paid to computation, measurement and calculations involving sums of money, though the results of these efforts were disappointing in some respects. In describing what their schools set out to achieve in mathematics, Heads' comments indicated clearly that they attached considerable importance to children achieving competence in the basic skills of arithmetic and understanding mathematical processes.

(*) Board of Education, MATHEMATICS 5-11, H. M. S. O, London, 1979.
(**) Board of Education, PRIMARY EDUCATION IN ENGLAND, H. M. S. O, London, 1978.

There was almost universal reference to the rules of addition, subtraction, multiplication and division, to computation and to concepts such as weight and number. This common view was reflected in the statement of one Head who said his intentions were: 'to teach the children their tables. To teach the four rules of numbers in relation to money, decimals, fractions, time and measurement, along with some basic geometry. To show why and how these processes work so that children can understand them and use them accurately' (*). Many Heads also referred to the importance of children gaining confidence, enjoyment and satisfaction from their work in mathemtics.

The whole of the chapter shows how the teaching of mathematics has developed over the years, particularly since the turn of the century. Arithmetic has been regarded from the earliest times as a vital area of study in primary school. However, it has often been taught by people with no mathematical training; and this problem persists up to the present day. As a result, the subject has been presented initially in too great abstraction, not gradually building concepts from the concrete to the formal stage; and teaching methods have failed to

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(*) Ibid.

emphasise learning by means of children's own discovery, concentrating instead on rote methods. Even though the importance of mathematics in solving everyday problems was realised by early educationists, this had little effect on teaching. Also, mathematics has tended to be taught as a series of disjointed subjects, arithmetic, geometry and so on, rather than as an integrated whole. Nevertheless, efforts were always made to use educational aids, and to make learning a pleasurable activity; and many reports, such as those of Spencer in 1912 and of the Mathematical Association, have brought out the criticisms listed above, and suggested innovations in the content and methods of teaching. As a result, there have been many improvements in recent years; children are now expected to learn through exploration and understanding of situations where mathematics is used, and gone are the days of learning by rote the mathematical rules which for the vast majority had no meaning.

It is hoped that the study of the history of mathematics teaching will help to improve teachers' understanding of children's conceptual development, leading to more effective acquisition of the mathematical skills needed in today's society.

CHAPTER FOUR

VIEWS ON MATHEMATICAL LEARNING IN PRIMARY SCHOOLS

This chapter seeks to outline some of the historical factors which have determined the mathematical curriculum of the primary schools today. These factors supply a background for the subsequent discussion of more recent experimental studies carried out by Piaget and others in the field of concept development.

Fleming 1952 (*) has suggested that the the modern investigations into teaching methods began with experimental work in learning theory made by the psychologists of the mid-nineteenth century. Fleming cites G. M. Whipple (**) as being a pioneer in the use of a test of computation which led to the introduction of survey tests and the subsequent realisation that the rate of conceptual development differed from one individual to another. Fleming

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(*) C. M. Fleming, RESEARCH AND THE BASIC CURRICULUM, London, 1952, second edition, 39.
(**) G. M. Whipple, MANUAL OF MENTAL AND PHYSICAL TESTS, Baltimore, 1910.

explains that the nature of arithmetical instruction at this time was that of verbal instruction given to the entire class of children, regardless of ability or conceptual maturity. A large proportion of the school day was set aside for arithmetic which comprised verbal explanation by the teacher and mechanical exercises performed by the pupils.

English research workers such as Ballard and Schonell were influenced by American research, outlined by Dutton 1964 (*). Although this work refers to elementary schools in the U.S.A it provides a useful guideline to be considered in the practice of teaching arithmetic in England. Dutton reports four major periods in the methods of the teaching of arithmetic. The first period which he refers to as the "DRILL THEORY OF TEACHING", was in evidence before 1900. Dutton describes it as having reached its peak in America by 1920. This method had a profound influence on English arithmetic teaching.

Other psychologists advanced various theories which also influenced the teaching of arithmetic in English elementary school. Thorndike (**) propounded

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(*) W. M. Dutton, EVALUATING PUPILS, UNDERSTANDING OF ARITHMETIC, New Jersey, 1964. 4.
(**) E. L. Thorndike, THE PSYCHOLOGY OF ARITHMETIC, New York, 1922.

the theory of stimulus-response, i.e that for every stimulus there was a response which was tied to it by a specific "bond". This view of learning led to a consideration of elementary mathematics as a vast range of isolated, unrelated skills in specific areas, these skills being most easily learnt through the establishment of various "bonds".

The customary analysis of the operations required in performing the four rules of number with regard to the teaching of mathematics in the junior school has been derived from the work of Thorndike. An emphasis was placed on the skill of computation when attempts were made to limit the arithmetic curriculum to the social "usefulness" of various functions. Fleming (*) refers to this social utility approach and draws attention to the "social usefulness" survey conducted by G. S. Wilson (**) as early as 1917. Dutton assumes that this approach was replaced by what he specifies (***) as "the meaningful approach" in the 1930's, but it was unfortunate that this emphasis on computation, produced by the contemporary practice of drill and

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(*) Fleming, Ibid.
(**) G. M. Wilson, 'A survey of the social and business uses of arithmetic' in the <u>SIXTEENTH YEAR BOOK OF THE NATIONAL SOCIETY FOR THE STUDY OF EDUCATION</u>, Public School Publishing Company, Illinois, 1917, Part I, Chapter VIII.
(***) Dutton, Ibid, 8.

stimulus-response learning, became associated with attempts to limit the curriculum.

Ballard (*) firmly established a pattern of analysis of the content of elementary mathematics. This analysis sets out the relative difficulty of various combinations, the distribution of practice, and the treatment of zero combinations and remained largely unchallenged in this country until the publication of the Mathematical Associations Report in 1956 (**).
Certainly in the nine schools visited for the particular purpose of that report it was quite clear that the approach suggested by Ballard, with the subsequent modifications produced by Schonell, was still followed by the majority of teachers. It therefore seems appropriate to comment rather more fully on his work.

In all the schools visited, an early start was made in the Junior first year classes on the formal learning of multiplication tables. For example all the classes were given some time in each week for the chanting of tables. Four of the classes had

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(*) p. B. Ballard, TEACHING THE ESSENTIALS OF ARITHMETIC, London, 1928
(**) Mathematics Association Report, THE TEACHING OF MATHEMATICS IN THE PRIMARY SCHOOL, G. Bell and Son, London, 1956.

multiplication table charts displayed while none of the classes provided apparatus of any kind for building-up the tables. All the class teachers agreed that the following statement by Ballard (*) represented a reasonable description of their aim 'Let the children learn the multiplication table so that it is reproduced, item by item, with mechanical precision and promptitude; fix the routine of simple rules so that they absorb the minimum of thought'. Ballard's further comment provides an apt summary of the classroom practice observed by the experimenter in the visits: 'There is, I fear, no help for it. We must face the cold fact that arithmetic, however much it is doctored or dressed-up, is not an interesting subject to the ordinary young child'. Obviously once a teacher has this view there are very real limits to the changes in the nature of the learning situation that they can be expected to initiate. It would seem that this view of the nature of the activity held by the teachers visited during the experimental study for that research, partially explains their continued support of a pattern of work which is based on careful analysis of examples and a limitation of possible responses. This was done in order to ensure speed and accuracy on the part of the child. Indeed the pursuit of speed and accuracy assumed an

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(*) P. B. Ballard, Ibid.

importance which quite unbalanced the curriculum. Ballard, again, illustrates the point the researcher is making:-

(I) 2
6
3

(II) 2+6+3

(I) is better than (II) because (I) is the king's highway while (II) is not. It can be proved experimentally that children find it easier to deal with figures in the (I) form than in the (II) form. They take less time over it and they are more accurate.

The children finally selected for the experimental study came from those whose teachers and head teachers were prepared to adopt this criterion for the work they initiated in their classes. Generally it was seen during the initial visits to schools, the visits to the three schools selected for pilot study and the much more frequent visits to the school finally selected for the actual experimental study, that much of the present classroom practice could be understood by reference to the period of "drill" associated with Thorndike and Ballard and the particular variation of this basic pattern devised by Schonell. Before discussing the work of Schonell and subsequent workers in the field it is proposed to discuss the historical summary developed by Hyde (*). Hyde's summary covers much the same ground discussed by Dutton, but classifies (perhaps too neatly) the psychological explanations of the approaches under five main headings: (i) Historical approach. (ii) Associationist approach. (iii) Biographical, clinical approach. (iv) Statistical approach and (v) Diagnostic/ Remedial approach.

Hyde's "Historical approach" is the psychological theory of recapitulation or the "Cultural epoch theory". Thorndike 1922 (**) indicated that very few supporters of this theory ever applied it to the teaching of arithmetic, and this remains true to this day. Indeed, the original psychological theory is generally regarded to be in error. However, it is also true that many text-book writers trace the evolution of number ideas in primitive tribes and provide "equivalent" experiences for young children. C. Stern 1949 (***) discusses

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(*) D. M. Hyde, 'An investigation of Piaget's theories of the development of the concept of number', Ph.D Thesis, Unpub., London, 1959.
(**) E. L. Thorndike, <u>THE PSYCHOLOGY OF ARITHMETIC</u>, New York, 1922.
(***) C. Stern, <u>CHILDREN DISCOVER ARITHMETIC</u>, New York, 1949.

the historical evolution of "number" in order to introduce her own theories of learning mathematics, though the central theme of her work is based on the theories of Gestalt psychology. This particular aspect is discussed in other studies which are more related to psychology in connection with Dutton's and Hyde's classification of the various approaches to learning. Many text-books (for example Flavell's PRIMARY MATHEMATICS, published by Methuen and Thyra Smith's THE STORY OF NUMBERS, published by Basil Blackwell) show a similar historical influence and often the order of introduction of various topics parallels the historical developmental sequence. Of far greater importance in relation to its impact on schools is Hyde's second category, "the Associationist Approach". This has already been discussed in some detail in relation to Dutton's category, the "drill theory" of teaching and the work of Ballard in this country. This theory depends upon an organised hierarchy of habits, careful training and appropriate drills.

As early as 1937 Wheat (*) was critical of this approach: 'A century ago Pestalozzi was engaged in the task of organising instruction on the principle

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(*) H. G. Wheat, THE PSYCHOLOGY AND TEACHING OF ARITHMETIC, Boston, 1937, 157.

of proceeding from the simple to the complex. His plan was to divide and sub-divide a subject into its minutest parts and to teach these "simple" parts to his pupils one by one'.

Wheat indicates that other fields of curriculum experience have moved far away from this piecemeal view of learning but that primary teachers remain, however, in the Pestallozzian period of meticulous dissection in the teaching of arithmetic. This subject has been analysed for the pupil into a multitude of combinations, processes, formulas, rules, types of problem, etc., and the pupil is taught each in turn as a separate item of experience. Often, when he has completed the course, he knows only those parts he can still remember and they all seem to him as separate and unrelated combinations, processes, formulas, rules and types of problem to be solved.

Finally, when his memory for these separate items fails him, he has nothing left to carry into his adult world but the remembrances of a series of meaningless, uninteresting and unpleasant experiences that his classes in arithmetic seem to have provided for him. Wheat's comment on drill is particularly appropriate: 'drill has come to be the type of classroom procedure that is peculiar to, and characteristic of, arithmetic. Drill has come to assume an importance far in excess of its merits'. Some allowance must be made for the date of publication, 1937, but there are many residual elements in the present day practice.

Though Hyde does not link the Associationist Approach with the later work of Schonell and the Diagnostic/ Remedial Approach it would seem that there is a carry-over of the ideas of drill and practice into this approach. A discussion of the common elements is attempted in the section of this chapter devoted to the work of Schonell. Before discussing Schonell's work it would seem appropriate to comment on Hyde's other two groupings: "Biographical, Clinical Approach" and the "Statistical Approach". They both seem less significant from a curriculum point of view though the experimental techniques developed with these approaches have been widely used. The "Biographical, Clinical Approach" involved studies of development by controlled observations. Gessell's studies (*) of development are a major source of this type of material. Gessell provides "norms" for different age groups, lists of accomplishments and attitudes

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(*) A. Gessell, <u>THE CHILD FROM FIVE TO TEN</u>, London, 1946.

towards formal teaching. The differences between American norms and British conditions have prevented any widespread use of this material in the field of arithmetic studies. Hyde's classification of researches under the term, the "Statistical Approach" includes such studies as those undertaken by the Scottish Council for Educational Research (*) (**) (***). The psychometrist has had a far-reaching influence on the curriculum. Some schools have attempted to group children for mathematical work on the results of standard scores in attainment tests. Whether this form of testing has been worthwhile or not is open to question. Vernon 1960 (****) suggests that it is widely known that 'the correlation between G and V tests and general attainment is very high'. The need, therefore, to add a battery of attainment tests to the intelligence testing programme seems problematical. However, these questions seem to be more directly related to the type of research undertaken rather than the classroom practice. It is Hyde's final category, "Diagnostic / Remedial", that seems to be a vital factor in curriculum provision prior to the late 1950s. Schonell is most closely

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(*) The Scottish Council For Educational Research, STUDIES IN ARITHMETIC, London, 1939, VOL I, XIII.
(**) Ibid, 1941, VOL II, XVIII.
(***) Ibid, ADDITION AND SUBTRACTION FACTS AND PROCESSES, 1948, XXVIII.
(****) P. E. Vernon, INTELLIGENCE AND ATTAINMENT TESTS, London, 1960, 183.

associated with this approach in this country (*).

Schonell analysed the difficulties experienced by children when manipulating numbers and solving arithmetical problems. He then produced a graded order of difficulty, together with suggestions as to the means of overcoming and minimising them. Schonell's main concern - despite his attention to such topics as "Reading Age" and "Number Age" that involved maturational factors - was with the sequence and the degree of difficulty of the tasks themselves. His work in this series; RIGHT FROM THE START ARITHMETIC, published by Oliver and Boyd, 1937, has been a significant influence on the curriculum of perhaps the majority of junior schools. Attempts have been made from time to time in some areas to establish a common syllabus which generally reflects the work of Schonell and his concern with the logical analysis of the arithmetical material.

R. H. Adams (**) drew heavily upon the work of Schonell and in his actual programme of remedial teaching he used Schonell's series; RIGHT FROM THE START . For the actual diagnosis of difficulties he

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(*) F. J. Schonell, DIAGNOSIS OF INDIVIDUAL DIFFICULTIES IN ARITHMETIC, Edinburgh, 1937.
(**) R. H. Adams, 'An investigation into backwardness in arithmetic in the junior school', M. A Thesis, Unpublished, London, 1940.

used Schonell's DIAGNOSIS OF INDIVIDUAL DIFFICULTIES IN ARITHMETIC (*). Adams suggested that 'the process, when analysed, needs to be introduced to the children by detailed teaching in the initial stages, followed by a carefully planned, sufficiently graded practice in the later stages. A multiplicity of examples is desirable, class arithmetic books do not, as a rule, give a sufficient number or variety of examples'. This aspect of Schonell's approach has certainly been assimilated into the curriculum of many junior schools, but his pleas for building a wide base of experience - and thus ensuring understanding - has not been acted upon to the same extent. Indeed Schonell gave comparatively little weighting to this aspect in his own writings.

Adams analysed the frequency of errors and found that 9+5 was not of the same order of difficulty as 5+9. Adams made the comment that the teaching steps were not sufficiently related to each other when discussing the "four rules of number". It is true that many teachers do not, even today, look upon "four rules" as a series of interrelated groupings.

The approach adopted by Schonell and Fleming

(*) Schonell, Ibid.

sought a solution to the problem of learning mathematics through the provision of carefully graded individual work and the use of diagnostic tests, practice games, supplementary aids such as flash cards and the separation of "problem" and "mechanical" and spaced practice were typical of the activities associated with this approach (*).

One particular difficulty associated with the approach of such writers as Schonell and Fleming concerned the treatment of zero facts. Zero was treated as another number fact and the intensive practice of "zero facts" was a special feature of the schemes they devised for children. Wheat 1937 (**) discussed the appearance of zero items in test material and the fact that children were confused by them resulted in the introduction of the "teaching" of these items. As Wheat indicated, 'teachers have tried to teach them, but since they mean nothing explanations have added to the confusion'. It was just this sort of difficulty that escaped detection in the large-scale survey-type tests. Wheat suggested that the solution to this particular problem is to omit them 'both from the teaching and

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(*) C. M. Fleming, MANUAL TO THE BEACON ARITHMETIC, London, 1939 and 1948.
(**) H. G. Wheat, THE PSYCHOLOGY AND TEACHING OF ARITHMETIC, Boston, 1937.

testing, since they are both meaningless and useless' (*). Wheat also discusses this problem in his more recent publication (**). The meaningful treatment of zero as a place holder provides, perhaps, the clearest single example of the change from the curriculum approach that was established by Ballard's work to a curriculum seeking to develop understanding and concept formation. For some schools the change has yet to take place, though it is probable that the publication of the Mathematical Association's Report (***) marked the beginning of the process of change for many. E. Biggs (****) supports the view that this publication of The Mathematical Association 1956 served as a catalyst: 'The publication of the Mathematical Association's THE TEACHING OF MATHEMATICS IN PRIMARY SCHOOLS evoked an unprecedented interest in the teaching of mathematics from teachers in schools of all types'. The period since the publication of THE TEACHING OF MATHEMATICS IN PRIMARY SCHOOLS will be discussed subsequently. Before doing so, it seems appropriate to consider the contribution of Gestalt psychology to the changes in the mathematical curriculum. In an experimental

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(*) Ibid.
(**) H. G. Wheat, HOW TO TEACH ARITHMETIC, New York, 1956, 415-416.
(***) The Mathematical Association, Ibid, 1965.
(****) E. E. Biggs, in School Council For The Curriculum and Examinations Curriculum Bulletin no.1, MATHEMATICS IN THE PRIMARY SCHOOL, London, 1956, XV.

context the "field theories" of learning that stemmed from Gestalt psychology made their impact in America in the late 1920s. Dutton (*) suggests that 'Gestalt psychology challenged Thorndike's laws of exercise and effect and brought to a close the heavy emphasis upon drill'. This would seem to be an unduly optimistic verdict and it is certain that English curriculum provision did not undergo a change of this nature at the time suggested by Dutton for American schools. Indeed Dutton (**) later writes: 'unfortunately there are numerous traditional teachers who still cling to the outmoded drill theory of teaching'. This would certainly be true of some English experiences.

The first example of a publication that attempts to present a mathematical curriculum (for the 7+age group) in terms of Gestalt psychology is Catherine Stern's <u>CHILDREN DISCOVER ARITHMETIC</u> published in Britain in 1953. Wertheimer's (***) essays provide examples of the Gestalt approach applied to problem solving. He provided open-ended problem-solving situations and discussed children's responses in terms of Gestalt theory. R. A. Wheeler 1935 (****)

(*) Dutton, Ibid, 5.
(**) Ibid, 5.
(***) M. Wertheimer, <u>PRODUCTIVE THINKING</u>, New York, 1945; London, 1959.
(****) R. A. Wheeler, 'The new psychology of

made a strong plea for the teaching of mathematics in Gestalt terms 'Therefore forget drill. Prepare work logically and concentrate on relations'. In practice the influence of Gestalt psychology can also be seen in the "meaning theory" or "meaningful arithmetic" approach, that evolved prior to the impact of the experimental work of Piaget on the curriculum. W. A. Brownell is probably the most widely known advocate of this approach. He outlined the psychological considerations that needed to be taken into account in <u>THE TENTH YEARBOOK OF THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS</u> (*****). One might describe the theory in the following terms: drill is recognised as a means of securing retention when an idea or process is already understood; arithmetic is to be viewed as a closely-knit system of understandable ideas, principles and processes. Brownell 1941 (******) describes "meaningful arithmetic" in these terms: 'Social situations supplemented by learning activities deliberately designed to (i) Make number operations sensible.

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learning', in New York Teachers College, Columbia University (ED.). <u>THE TENTH YEARBOOK OF THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS</u>, New York, 1935.

(*****) W. A. Brownell, 'Psychological considerations in the learning and teaching of arithmetic', <u>THE TENTH YEARBOOK OF THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS</u> . New York, 1935.

(******) W. A. Brownell, <u>ARITHMETIC IN GRADES I AND II - A CRITICAL SUMMARY OF NEW AND PREVIOUSLY REPORTED RESEARCH</u>, Durham, 1941.

(ii) Encourage children as rapidly as they safely may to adopt procedures that make for arithmetical efficiency'. Dutton's comments (*) on the work of Brownell in 'The place of meaning in the teaching of arithmetic', Elementary School Journal, January 1947, 47, 257-258, aimed at investigating the efficiency of teaching a process with "meaning method" and "mechanical method", the study was concerned with the teaching of subtraction (using "decomposition" and "equal additions") and taking "meaningful" and "mechanical" as the approaches. In the meaningful approach, place value was used and the use of "crutches" was taught. With the "mechanical method" the children were not given explanations or introduced to "crutches". The study invoved 1400 children in matched groups for "C A" and "M A". The responses were evaluated for speed, accuracy, degree of understanding, length of relation and transfer rules involved. The conclusion was that the "meaning" method led to much better relations and a much greater ability to transfer to new situations.

Much of the "meaning method" is very influenced by Gestalt theories of learning. The stress is on the development of insight rather than rote learning. With the Catherine Stern material and programme, one

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(*) W. H. Dutton, Ibid.

has examples of material planned so that each learning experience follows "naturally" from the previous experience and each new factor is presented as an orderly extension of ideas already known and understood. The teacher's role is to help the child structure his ideas and symbolise them so as to develop an understanding of the notational system and number relations. Referring to the work of Stern, Wertheimer (*) writes: 'In many years of study with children Dr. Catherine Stern has developed tools and methods for the teaching of arithmetic in which genuine discovery in tasks of a structural nature plays an essential role. The results in learning - and happiness - seem extraordinarily good as compared with the usual teaching by drill which focuses on forming associative bonds, etc.'. The term "structive" refers to the perceptual organisation of the material and reveals the Gestalt psychologists' concern with the "laws" of "proximity", "closure", etc. Stern's own comments reveal this concern with "structures" and non-counting approach to arithmetic. She describes her material in the following terms: 'Instead of developing the first number concepts by counting the elements of unstructured groups of objects, the child works with structures that show from the start the relations between the numbers of

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(*) M. Wertheimer, Ibid.

our system. Our devices differ from other materials used to make numbers concrete; the child's experiments with our materials are based on measurement which introduces the basic mathematical concepts at the very start of the child's number work' (*).

The Stern material is one of many different forms of what have become known as "structured Apparatus" that are now available for use in schools. These materials are discussed rather more fully in the concluding section of this chapter following the discussion of the particular contribution of Piaget's work to any reconsideration of the curriculum. The fundamental nature of the changes that have taken place in the study of conceptual development (and in particular the development of mathematical concept) is seen by comparing the comments of Judd 1926 (**), Adams 1940 (***) and Buswell 1951 (****). There is a similarity of comment that is surprising in view of the span of years covered.

(*) Ibid, XXIII.
(**) C. H. Judd, 'Research in elementary education', Journal Of Educational Psychology, 1926, VOL. 17, 217-225
(***) R. H. Adams, Ibid.
(****) G. T. Buswell, THE TEACHING OF ARITHMETIC, FIFTIETH YEARBOOK, N.S.S.E, Chicago, 1951, part II, 282- 297.

C. H. Judd's comment on teaching provides an appropriate summary for much of the work that preceded the pioneering studies of Piaget in the field of conceptual development: 'We have had tests and more tests since Rice made his first results in 1901. The tests have told us where arithmetic fails and where more effective teaching should attack the subject but the methods of the new attack and the methods of further study have until recently been held back by the universal devotion to tests'.

Despite Judd's optimism Adams in 1940 (*) could still state that progress in the teaching of arithmetic had been less than in reading because of the static methods used - 'Testing but little or no new methods of attack'. Buswell (**) 'Needed research in arithmetic reviews the previous twenty years of research and indicates that too much time had been spent in dealing with incidental problems such as the analysis of textbooks, curricula, comparisons of specific techniques of teaching given topics, and far too little on the thinking done by children as they learn. Buswell's plea was for the systematic study of the learning process, rather than the concentration on the incidental problems

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(*) R. H. Adams, Ibid.
(**) G. T. Buswell, Ibid.

associated with learning.

J. B. Biggs (*) also comments on the emphasis of previous research: 'although arithmetic itself has been the subject of research for many years, much of the reported material in the extensive literature tends to be fragmentary and disjointed'. Biggs is anxious that the psychologist should build up a basic psychological scheme and that this should be used as the foundation for a body of knowledge that would be more directly useful to teachers. In this sense Biggs looks to the work of Piaget who aims to create a basic interpretative scale for all mental activity. Piaget has, in fact, been the major influence on the recent attempts to reconsider the nature of the mathematical experiences provided for children in primary schools.

Piaget's work has been central to the recent investigations (**) undertaken in this country into the development of various concepts and it therefore seems appropriate to include a brief outline of his work. It is well known that the delay in the general acceptance of Piaget's work was partly due to certain

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(*) J. B. Biggs, <u>EDUCATIONAL RESEARCH</u>, February 1959, VOL 1, No.2.
(**) E. A. Lunzer, in N. F. E. R, <u>RECENT STUDIES IN BRITAIN BASED ON THE WORK OF J. PIAGET</u>, London, 1961, No.4.

defects in the early experimental work. S. Isaacs (*) voiced some of these doubts: 'the all important question is still how far the kind of "egocentric" behaviour described by Piaget is specific to his experimental situation and how far it extends to the everyday learning of the child in the classroom and outside'. In LANGUAGE AND THOUGHT OF CHILDREN 1923 and JUDGEMENT AND REASONING 1924 Piaget attempted by direct questioning and answer methods to discover the nature of a child's reasoning. Susan Isaacs (**), commenting on Piaget's work, concluded 'the difference between adults and children is not that the former do not reason or that they only reason in the form of a perceptual judgement or practical manipulation, but that the children's reasoning which is essentially based on their personal concrete problems has less need for clear verbal formulation'.

Despite a critical reception to his early studies Piaget continued with his work and by 1936 he had formulated a general theory of child development. THE BIRTH OF INTELLIGENCE 1936 gives an outline of Piaget's theories for the growth of reasoning. Piaget's work has a biological bias and the use of such terms as "assimilation", "adaptation",

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(*) S. Isaacs, INTELLECTUAL GROWTH IN YOUNG CHILDREN, London, 1930, 49- 110.
(**) S. Isaacs, Ibid.

"accommodation" and "integration" are indications of his belief that formal reasoning derives, ultimately, from the sensory-motor development of the infant. Piaget's aim is to relate this development to the nervous structures themselves and he looks to neurological sources for the final support of his analysis.

Piaget's <u>THE CHILD'S CONCEPTION OF NUMBER</u> 1952 (*) marks an important stage in the development of his experimental method. He attempted to take account of some of the criticisms of his earlier work and his approach now included behavioural situations rather than purely verbal ones. The approach involved the setting-up of "concrete problem situations" and the child was then questioned in relation to his own process of arriving at a solution. With this method Piaget was able to externalise the processes of concept formation to an extent that had never before been achieved. It is without doubt that Piaget's work has been the major significant influence in educational psychology and educational practice associated with young children since the early 1950s.

(*) J. Piaget, <u>THE CHILD'S CONCEPTION OF NUMBER</u>, London, 1952.

Though the work of Piaget occupies an obviously central position in the field of conceptual development, it has been criticised on two counts. The particular logical framework (*) 'Every psychological explanation comes sooner or later to lean either on biology or logic' used by Piaget to structure his experimental work has been questioned, while his use of the concept "stages of development" needs careful consideration. Piaget's studies are not longitudinal (**) and the data for each stage does not come from a consideration of a sequence of development of any one individual child, but rather represents a selection of experimental results that fit the developmental scheme. There is also some criticism of the detail of actual problem-solving situations.

Despite these criticisms follow-up work in 1957; 1959 (***), (****) broadly confirms the work of Piaget. Lovell (*****) notes that 'experiments

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(*) J. Piaget, THE PSYCHOLOGY OF INTELLIGENCE, London, 1950, 3.
(**) E. A. Lunzer, 'Some points of Piagetian theory in the light of experimental criticism', J. CH. Psychol. and Psychiatry, 1960, I No. 3.
(***) R. Beard, 'An investigation of Piaget's theories of the development of the concept of number', Ph.D Thesis, Unpub., London, 1957.
(****) D. M. Hyde, 'An investigation of Piaget's theories of the development of the concept of number', Ph.D Thesis, Unpub., London, 1959.
(*****) K. Lovell, THE GROWTH OF BASIC MATHEMATICAL AND SCIENTIFIC CONCEPTS IN CHILDREN, London, 1961,

carried out under the writers' direction among primary school and E. S. N. Special school pupils, into children's ability to consider the whole-part relationship, have confirmed Piaget's findings in the main'.

The impetus to research work provided by Piaget is seen in the studies quoted above. E. A. Lunzer 1961 (******) provides a further indication of the extent and importance of Piaget's influence. The influence has been of particular importance in the field of mathematics. The contemporary curriculum shows a far greater concern for conceptual development and opportunities to explore mathematical ideas in terms of "concrete" experience, involving the manipulation of materials than would have been so twenty or even ten years ago. This work has been undertaken against the background of Piaget's outline of the development of thought and his particular studies of mathematical development.

Piaget (*******) outlined three stages of thought:

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(******) E. A. Lunzer, RECENT STUDIES IN BRITAIN BASED ON THE WORK OF PIAGET, N. F. E. R, London, 1961, No.4.
(*******) J. Piaget, THE PSYCHOLOGY OF INTELLIGENCE, London, 1950.

(I) Non-operational (Birth to 2 years), also called the Sensory-motor stage.

(II) Pre-operational stage, subdivided into (i) Transductive period (2-4 years) and (ii) Intuitive period (4-7 years).

(III) Operational (i) Concrete (7-11 years) (ii) Formal (11 years and over).

For Piaget it is a process of progressive differentiation of operations. In the Pre-operational period the child has no means of representing an object. When a ball goes from sight it ceases to exist for the very young child. In the Intuitive period the child begins to represent absent objects but his scheme of reality is patchy and is limited by his actual perceptual configurations or represented spatial configurations in imagination. The Intuitive stage (4 to 7/8 years) is the stage when perceptions dominate the child's thinking. Around seven to eight years of age Piaget sees the development of formal concrete operations. A child is able to form concepts of classes, relations and numbers, together with ideas of space and time, but there are still limits to the extent to which the environment can be understood. The operations are not yet logical operations 'they are only mastered in concrete situations where the child is actively manipulating data, which he can see, touch and fixate..'.

In the period of Formal operations possibilities as distinct from hard facts can be dealt with and Logical operations concerned with the relations between statements expressing operations are now possible. It is Piaget's view that these Logical operations "grow out" of simple overt activities such as classifying, serializing or enumerating beads and other objects.

Thomson 1959 (*) quotes Reichard, Schneider and Rapport's discussion of the three stages '....Three levels of development; a concretistic level where classification tends to be made on the basis of non-essential incidental features of the objects; a functional level where classification is a basis of use, value, etc..., and a conceptual level where the child classifies more nearly on the abstract properties or relations'.

Piaget does not suggest that his classification provides a set of "age norms" but only that they provide, for particular tests, a framework for the discussion of conceptual development. 'There is in

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(*) R. Thomson, Quates Reichard, Schneider and Rapport, 'The development of concept formation in children', <u>Amer.J.Orthopsychiat.</u>, 1944 in <u>THE PSYCHOLOGY OF THINKING</u>, London, 1959.

fact a wide age range represented in the children whose responses Piaget quotes as illustrative of the general course of development. In some instances a single stage is illustrated with responses from children of five years and seven and a half years (*). Indeed Beard (**) Hyde (***) and Lovell (****) all provide evidence for this point of view. Piaget's work seems to suggest broad developmental stages that are associated with certain types of performance. The behavioural responses are regarded by Piaget as indications of inner mental structures. The build-up of mental structures is the process of concept formation.

The development of concepts is particularly concerned with the period of "concrete operations" and further discussion of this period seems appropriate. J. H. Flavell (*****) describes the period in these terms: 'acquisition of a well structured and coherent framework within which to represent and operate upon the concrete perceivable world of things and events'. It is seen that the

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(*) E. M. Churchill, 'Piaget's findings and the teacher', National Froebel Foundation Bulletin, London, Oct. 1960, No.126, 2.
(**) R. Beard, loc cit.
(***) D. M. Hyde, loc cit.
(****) K. Lovell, loc cit.
(*****) J. H. Flavell, THE DEVELOPMENTAL PSYCHOLOGY OF PIAGET, New York, 1963, 165.

child at this stage 'behaves, in a wide variety of tasks, as though a rich and assimilatory organization were functioning in equilibrium or balance with a finely tuned discriminative accommodatory mechanism' (*). The child during this period is building a solid cognitive bedrock out of which - if at all - the next stage will develop, i.e the stage of Formal operations.

The stage of "concrete operations" is receiving increasing attention from educational psychologists and others in relation to the need to provide guidance to teachers in the use of "concrete material". This is at a stage of education when the child with the traditional curriculum is involved in the learning of the "four rules of number". Piaget suggests a group structure for these arithmetical operations:

(i) The additive group of whole numbers with the corresponding group properties;

composition $1+1=2$, $2+1=3$,

associativity $(1+1) + 1= 1+ (1+1)$.

Inverses -1 -2 etc...and identity,0.

In all groups there is iteration

$1+1=2$, $1+2=3$ etc...,

(*) J. H. Flavell, Ibid, 165.

(ii) Multiplicative group of positive numbers with its group properties;

composition $1*1=1$, $1*2=2$

associativity $(1*2) * 3= 1* (2*3)$

inverses / 1 / 2 and identify 1.

Lovell (*) writes that 'for Piaget the concept of number is not based on images or mere ability to use symbols verbally as the formation and systemisation in the mind of two operations; classification' which involves the simultaneous awareness of equivalence and distinguish-ability 'and seriation, for the concept to form in the mind these two operations must blend'. The number system is 'the union of classification and ordering, for the idea of the number 8, say, depends upon the child grasping in his mind eight objects to form a class and upon placing 8 between 7 and 9 ; that is, in relation'.

Some brief references have been made to the work of Piaget in relation to their place in the pattern of curriculum development from the "drill theory" through to the "meaningful approach". The influence of Gestalt psychology in relation to the learning of elementary mathematics has already been outlined and

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(*) K. Lovell, Ibid, 51.

before attempting to comment on the contemporary scene, it is necessary to indicate in what ways the Piagetian viewpoint differs from Gestalt psychology thus emphasising the influence of Piaget's work in moving forward to the present view of the nature of the appropriate mathematical activity for a young child.

It has been indicated already that Piaget sees the dual nature of intelligence as involving both biological and logical studies. He admits (*) that Gestalt psychology indicates that behaviour involves the total field, embracing subject and object but that 'from the Gestalt point of view there exists only the field linking the objects and subject with neither activity on his part or the isolated experience of the object' (**). Piaget then continues with a discussion of "operations" and then mentions their dynamic nature, in contrast to the static perceptual organisation associated with Gestalt psychology: 'The problem is, therefore, to understand how operations arise out of material action, and what laws of equilibrium govern their evolution; operations are thus conceived as grouping themselves of necessity into complex systems

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(*) J. Piaget, THE PSYCHOLOGY OF INTELLIGENCE, London, 1950, 5.
(**) Ibid, 15.

comparable to the "configurations" of the Gestalt theory, but these, far from being static and given from the start are mobile and reversible, and round themselves off only when the limit of the individual and the social genetic process that characterizes them is reached' (*). It is this operational view of intelligence that has made such a difference to the contemporary discussions of the methods of teaching and learning. Piaget's concern with this question was shown in the account of one of his lectures reported in the National Froebel Foundation Bulletin (**) 'There were in fact three main methods of teaching or learning: (i) Verbal or formal; (ii) Intuitive; (iii) Active. Thus if one took the case of mathematics there was the traditional way of making the children learn the rules first and hoping that they would come to understand them later. There were the more modern methods based on Gestalt psychology, getting the children to attend to and study different configurations and their relations to one another. And finally there was the active approach, by which children were stimulated to handle and experiment and thus discover for themselves, or reinvent, mathematical relationships and laws. The

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(*) Ibid, 17.
(**) J. Piaget, 'Children's thinking- the figural aspect and the operational aspect', NATIONAL FROEBEL FOUNDATION BULLETIN, London, Dec. 1960, No.127, 2.

intuitive methods were certainly well ahead of the verbal or formal ones, but they were not enough. They took the child only as far as configurations not transformations. And the former, in fact, set up a sort of verbalism or literalism of their own, a verbalism of the image'. Turning from the purely curriculum considerations to the psychological, Piaget puts the difference between his approach and Gestalt views in the following terms 'the question that contemporary thought is really concerned with is nothing less than : what does knowledge consist of ?. Is it contemplation in one's mind of copies of things? Or is it a construction resulting from the subject's action on the object, a group of transformations derived from interaction between the structure of the object and that of the subject' (*). Piaget then suggests 'that only the transformations which he effects in them by his own actions turn them into knowledge. To know an object is to reconstruct or reconstitute it. Knowledge is action on objects, not the kind of action which is merely intended to serve utilitarian ends, but action that is in effect a transformation of the object into a construction in the subject's mind' (**).

(*) Ibid, 2- 3.
(**) Ibid, 3.

This view of the nature of intelligence for the period of concrete operations which forms the focus of this study requires further comment on the term "operations". Berlyne's article 1957 (*) 'Recent developments in Piaget's work' comments on the period of concrete operations 'Concepts which figure in operational thought are called operations because they are internalized responses. They grow out of certain overt actions in exactly the same way as images grow out of imitation'. The systems of operations are called groupings, their stability depends on their having five properties: (i) Closure, 2+3=5, any two operations can be combined to form a third operation.

(ii) Reversibility, for any operation there is an opposite operation which cancels it, 2+3=5 but 5-3=2.

(iii) Associativity (previously discussed).

(iv) Inverses and (v) Iteration (previously discussed).

The active adaptation implied in the operational view of intelligence involves, for Piaget, the use of the terms "accommodation" and "assimilation". 'Assimilation (**) is thus the incorporation of objects into patterns of behaviour, these patterns

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(*) D. E. Berlyne, 'Recent developments in Piaget's work', British Journal of Educational Psychology, Feb. 1957, VOL XXVII, 8.
(**) J. Piaget, THE PSYCHOLOGY OF INTELLIGENCE, London, 1950, 8.

being none other than the whole gamut of actions capable of active repetition'. "Accommodation" is defined in the following terms: 'Conversely, the environment acts on the organism and, following the practice of biologists, we can describe this converse action by the term "Accommodation", it being understood that the individual never suffers the impact of surrounding stimuli as such, but they simply modify the assimilatory cycle by accommodating him to themselves. Psychologically, we again find the same process in the sense that the pressure of circumstances always leads, not to passive submission to them, but to a simple modification of the action affecting them' (*). The term "adaptation" is regarded by Piaget 'as an equilibrium between assimilation and accommodation, which amounts to the same as an equilibrium between subject and object' (**).

In summarising the act of intelligence the researcher may use Piaget's own description 'thus the act of intelligence consists of grouping or co-ordinating operations: however operations are actions which are internalised and have become reversible, like addition which is derived from the action of

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(*) Ibid, 8.
(**) Ibid, 8.

bringing together and can be reversed in the form of subtraction' (*).

It is now proposed to discuss the psychological background to the changing views on the nature of the learning activity in relation to the recent attempts in many primary schools to base their curriculum on the sequences of "concept development", and the use of "performance materials".

Much of the work undertaken in junior schools is linked to the achievement of skill in computation. The application of these carefully fostered skills and the development of mathematical insights has long been a problem to many teachers and is, perhaps, epitomised in the often repeated comment "he can't do problems". In any curriculum provision that emphasises the acquisition of skills in computation, the problem involved in the use of these skills (and the generation of "insights") has generally been solved by the teaching of limited problem-solving techniques, barely distinguishable from the purely mechanical practice.

(*) B. Inhelder& J. M. Tanner (Editors) DISCUSSIONS ON CHILD DEVELOPMENT, London, 1960, VOL.4, 10.

H. B. Beech's (*) widely distributed book is a typical example of this approach. Lovell (**) comments generally on this approach: 'the great weaknesses of the method were that it did nothing to stimulate enquiry or discussion, it gave no place to imagination and it provided no scope for the child to build up concepts by his own activity or experience'.

In more recent years (see Mathematical Association Publication THE TEACHING OF MATHEMATICS IN THE PRIMARY SCHOOL (***)) there has been a growing interest in the creation of a favourable attitude to mathematics and emphasis on "understanding". With the growth in the demand for "numeracy" one notes that teachers are placing less reliance on rule learning and memorisation. As Williams 1964 (****) suggests: 'in many situations, an exploitation of the derivability of arithmetic offers us opportunity to side-step much painstaking memory work'. "The exploitation of the derivability of arithmetic" must mean, for many teachers, a change in their teaching approach. In many classrooms there is a high level of "cognitive passivity" and an almost total neglect

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(*) H. B. Beech, PROBLEMS IN ARITHMETIC, TEACHERS BOOK, A and C Black, London, (n.d.).
(**) K. Lovell, loc cit, 31.
(***) 1956.
(****) J. D. William, EDUCATIONAL RESEARCH, July 1964, VOL. VI, No. 3, 194.

of the possibilities of discussion. This means that there is no encouragement to a child to interpret his actions. If a child loses his place in a calculating routine he generally lacks any means of constructing or "deriving" his own moves and the only strategy he can adopt, if any, is to start again from the beginning. Throughout, the course of calculation is prescribed by the teacher and the basic operations are often concealed by the "short-cut" methods dictated by the teacher. Flavell (*) reminds us that intellectual development is an organisational process and that it is the active intellectual operations that are organised. It is now our purpose to examine some of the problems associated with viewing various forms of apparatus in relation to this "organisational process" of intellectual development. In particular, various forms of "structural" apparatus will be considered. The term "structure" is interpretable in the sense that children are actually handling mathematical "analogues" when using the materials rather than as purely perceptual "Gestalt" structures.

The majority of the terms of "structural" apparatus provide for the immediate recognition of the numerical values by the child. The basic group

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(*) J. H. Flavell, loc cit, 168.

structures are embodied in discrete physical objects whose sizes (length) are proportional to the groups themselves. Beard (*) ('Does Piaget count in our number syllabus?') Found that many schools assumed that a sufficient basis of practical work had been provided in the infant school and that it was, therefore, reasonable to proceed directly to "sums" in the "four rules". Commenting further on the purpose of the apparatus Beard writes: 'Piaget finds in primary school children that mathematical thinking depends on the use of concrete aids and in general it is not until adolescence that facility with language and symbols makes it possible to reason from a hypothesis, to imagine various possibilties and to deduce the consequences which should follow'. This research fully supports Beard's thesis that 'a child who lacks experience with concrete aids will not perform adequate mental operations and so cannot develop the abstractions of more advanced mathematics'(page 5).

The problem facing many teachers of first year junior children is their traditional "hasty retreat" from the use of the "concrete materials" of the infant school. In this context the "two stages" of

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(*) R. M. Beard, TEACHING ARITHMETIC, London, Autumn 1963, VOL.1, No.3, 5.

apparatus should be noted. Many infant schools provide a rich variety of counting situations (milk bottle tops, stones, peas, etc.) But use little apparatus that could be regarded as "a bridge" from number in the concrete situation to the use of number as a complete abstraction. As a result of the general climate created by "Piaget type" researches there is a much greater interest in the role of apparatus with young children. Teachers are now more aware of the problems associated with conceptual development. In particular, Piaget's view that operations arise from physical actions and that operations extend the scope of the action by internalizing it, has had a significant influence on the provision of apparatus in the junior school.

It is true that any provision of material in a classroom is likely to have some effect on the curriculum and the social organisation of the class. Indeed Wheeler (*) suggests that the absorption of a new method into a preconceived attitude to the teaching of the subject is most unlikely to bring any substantial improvement - if it brings any at all. It is certainly assumed by the majority of writers that the provision of "structural" materials will

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(*) D. Wheeler, 'Structural materials in the primary school', MATHEMATICS TEACHING, Spring 1963, No.22, 42.

have some effect on the curriculum and the organisation of the learning situation. Biggs (*) suggests that structural apparatus is rewarding in itself, hence praise and blame from teachers is reduced, marked anxiety does not develop and learning is facilitated. Dienes (**) supports the use of apparatus (as well as for other reasons) because it allows children to discover concepts for themselves with the minimum of direction from the teacher. Dienes seeks the creation of a non-anthoritarian directed learning situation and it would seem that much of the apparatus available today (e.g. Cuisenaire, Dienes, Stern) can only be fully and appropriately used in such circumstances. It seems clear that the mere act of electing to provide some form of apparatus in a classroom forces the teacher to reconsider the nature of the learning pattern in the class - at least within the field of mathematics. Teachers who provide one of the forms of structural apparatus are more likely to be influenced by the actual programme devised for the material rather than the traditional textbook or the "scheme" produced by the head teacher. It would seem, from discussions with many teachers, that teachers using apparatus

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(*) J. B. Biggs, <u>THE PSYCHOLOGY OF ARITHMETIC</u>, April 16th 1962.
(**) Z. P. Dienes, <u>BUILDING-UP MATHEMATICS</u>, London, 1960.

encourage the development of mathematical ideas (e.g ordinal number with the use of number tracks or ladders) of which other teachers are not actually aware. Structural apparatus leads to a greater awareness of interrelated processes and mathematical relationships. The role of learning by rule becomes less significant. The teacher becomes more aware of the need to develop operational thinking. A criterion for a true operation could be whether it arises as a result of a child understanding two relationships in interaction, rather than one. A child who knows that $A<B$ but cannot reason $A<C$ is not thinking operationally. A child who can reason $A<B$, $B<C$, therefore $A<C$ is thinking operationally. Dienes (*) describes the learning of anything new and, in particular, mathematics, in the following terms: '(i) Sorting events into classes or categories so that any event is immediately recognised as either belonging or not belonging to a class or category, or of course as being irrelevent to it. (ii) Becoming aware of the relationship to each other of the classes or categories constructed'.

Piaget's account in DISCUSSIONS ON CHILD DEVELOPMENT (**) is a helpful explanation of the

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(*) Z. P. Dienes, THE POWER OF MATHEMATICS, London, 1964, 21.
(**) B. Inhelder and J. M. Tanner (Editors)

nature of this activity: 'Thus the act of intelligence consists in grouping or co-ordinating operations: however operations are actions which are interiorized and have become reversible, like addition which is derived from the action of bringing together and can be reversed in the form of subtraction'.

Within the field of concrete operations various forms of apparatus can be used to structure the "groupments" of Piaget. One can undertake propositional operations by postulating a hypothesis which one then attempts to verify by manipulating apparatus. For example, the Stern blocks can be structured to show:

(i) Combinativity 1+2 = 3, 3+4 = 7

(ii) Reversibility 3-1 = 2, 3-2 = 1

(iii) Associativity (1+2) + 4 = 1+ (2+4) = 7

(iv) Identity 1-1 = 0 , 3-3 = 0.

As Lovell (***) points out, Piaget does not suggest that mathematical concepts are derived from the materials themselves, but rather from an

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<u>DISCUSSIONS ON CHILD DEVELOPMENT</u>, London, 1960, VOL.4, 10.
(***) K. Lovell, <u>THE GROWTH OF BASIC MATHEMATICAL AND SCIENTIFIC CONCEPTS</u>, London, 1961, 44.

appreciation of the significance of the operations performed with the materials: 'Mere perception and imagery in relation to the apparatus do not necessarily permit these operations, e.g true thinking'.

When one examines the range of apparatus available it is seen that much of the more recent material is based upon measurement rather than counting activities. Earlier types of apparatus were more frequently based on counting activities and it is felt that this approach still has a significant contribution to make. Its role is discussed in this chapter. Counting is an extremely versatile activity that has many "real life" applications but it does not always provide the most appropriate tool for solving a problem. The piecemeal element in many counting situations can be, as Stern (*) suggests, a disadvantage. A further difficulty is the fact that many teachers have not been sufficiently aware in their classroom work of the vital significance of one-to-one correspondence situations and the ordinal aspect of counting. In the past many schools based their work on two factors; concrete "real life" experience at the infant school stage and cardinal

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(*) C. Stern, CHILDREN DISCOVER ARITHMETIC, New York, 1949, and London, 1953, 18.

counting activities. In such circumstances (and with the complete break from concrete counting situations of the infant school to the pencil and paper tasks of the junior school) the counting procedures developed were often laborious and unsystematic; failing to match the intrinsic structure of mathematics.

Piaget (*) writes: 'If a child has not reached a certain level of understanding, which characterises the beginning of the third stage, counting aloud has no effect on the mechanisms of numerical thought'. There is a need on the part of the teacher to appreciate the appropriate moment for the introduction of counting activities. He continues 'It is therefore no exaggeration to say that the verbal factor plays little part in the development of correspondence or equivalence... At the point at which correspondence becomes quantifying, thereby giving rise to the beginning of equivalence, counting aloud may, no doubt, hasten the process of evaluation. Our only contention is that the process is not begun by numerals as such'.

J. B. Biggs (**) echoes these words: 'mere

(*) J. Piaget, THE CHILD'S CONCEPTION OF NUMBER, London, 1952, 64.
(**) J. B. Biggs, EDUCATIONAL RESEARCH, 1959, VOL.1, No.2, 21.

verbal counting does not adequately account for all the ordinal properties although it is vital for both cardinal and ordinal properties'.

Churchill (*) provides a more detailed consideration of the role of counting. Her experimental programme aimed to teach something about number through the provision of informal play activities. Materials and activities were provided that gave experience of ordinal and cardinal numbering. Churchill discusses the fact that though the Piaget test items penetrate beyond the child's verbal response she feels justified in taking a more cautious view of counting than that outlined by Piaget. Commenting on the responses of children she says (**) 'They are words you say in certain situations that carry no quantitative meaning at all. This state of affairs is a reflection of the diffuse, synthetic and rigid nature of the little child's thought processes; until the structures become more mobile, reversible operations are not possible, hence number cannot be constructed... Little children use words in the numbers language so commonly, and work

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(*) E. M. Churchill 'Early number concepts, an experimental study of the growth of numerical ideas in 5 year old children', M. A. Thesis, Unpub., Leeds, 1956.
(**) E. M. Churchill, 'The number concepts of the young child', Researches and Studies, Institute of Education, Leeds, 1958, No.18 (see also No.17).

out little sums so accurately that we have assumed they carry the same meaning as we should intend were we using them'.

However, Churchill has reservations concerning Piaget's comment that 'counting aloud has no effect on the mechanism of thought'. She supports his view that the process "correspondence and equivalence" is not begun by numbers as such, but goes on to postulate 'that through repeated counting acts, particularly if accompanied by finger pointing as they so often are in little children, the child becomes aware of the units comprising the group, one of the basic factors necessary to the number concept'. In other words counting might contribute to the structuring which changes global, synthetic judgements into analytical ones.

Other investigations (*), (**) have shown that any "experience programme" involving materials needs to give particular attention to the development of an

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(*) A. Phemister, 'An investigation into children's understanding of number on school entery and the effectiveness of infant classroom teaching based on Piaget's theory', Dissertation, Unpub., Manchester, 1960.

(**) J. A. Tough, 'Study of the effect of relevant experience on the building-up of the concepts of seriation and ordinal correspondence in a group of five year-old children', Froebel Trainers Diploma, National Froebel Foundation, Oct. 1960.

appropriate use of counting techniques and that an established and accurate counting process cannot be assumed. They also revealed the need for a very careful evaluation of the mathematical materials available in the classroom. Tough's study commented on the need to provide materials that led to discussion while Phemister found that children were not always aware of the fact that they could use their counting experience to help them solve, for example, conservation problems. In this present study the writer found that many First Year Junior children did not possess an adequate counting technique.

One of the problems associated with counting activities has been the failure to produce appropriate forms of apparatus. To some extent the influence of Gestalt psychology, with its emphasis on the perceptual process, improved the design of much of the material and certainly much of the Stern material can be discussed in relation to this point, even though "measurement" was stressed rather than counting.

The Stern "Pattern Boards" provide a more mathematically worthwhile pattern for cardinal numbers than do, for example, playing cards or dominoes. The "Number Track" leads to "counting-on" rather than through and helps considerably towards the development of an efficient counting technique. The stress on "structure" from the Gestalt psychologist made it very clear that there could be no justification for "fishing games", "marble games" and similar items in any mathematical programme. It is indeed unfortunate that the "counting approach" is so often associated with this type of material rather than, for example, a Number Ladder. The failure to produce appropriate forms of counting apparatus also meant an almost total neglect of ordinal numbers which is now generally recognised to be essential to any development of number ideas.

There are now available various forms of what is generally termed "structural apparatus". With many of these forms of apparatus the numerical values are immediately recognised by the child. The basic group structures are embodied in discrete physical objects whose sizes (length) are proportional to the groups themselves. The introduction of this type of apparatus into the junior school reflects the recognition (previously mentioned) that "concrete operations" are part of the junior school as well as the infant school together with a desire to stress relationships. Descriptions of many of the various forms of structural apparatus can be found in Educational Research for 1962 (*). One advantage derived from the introduction of "structural apparatus" has been the incentive for many teachers to study in detail the appropriate rules for counting, and its significance. Certainly in the testing of skills for this study it was seen that many children could not count efficiently and it was subsequently seen that the use of structural materials (generally the Number Track) actually helped children to improve their counting techniques. The use of apparatus enabled them to correlate the perceptual and action situation. This study is particularly concerned with the period of concrete operations and the discussion of structural materials is in relation to this stage as observed in the 7+ age group. Vernon 1956 (**) considers that Piaget has shown that 'the young child gradually builds up or acquires his perceptions and habits and thinking through contacts with his physical and social environment; and that the level that he reaches at any age is affected considerably by the kind and amount of intellectual stimulation that environment has provided up to that age'. This statement reminds

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(*) J. D. Williams, 'Teaching arithmetic by concrete analogy. II. structural apparatus', Educational Research, June 1962, VOL.IV No.3.
(**) P. E. Vernon, THE MEASUREMENT OF ABILITIES, London, 1956.

educators of the need to consider with a great deal of caution the nature of the experience and the type of apparatus that are provided at the stage of "concrete operations". Works already cited (*), (**), (***) provide illustrations of the problem with infant school children. The writings of Dienes (****), (*****), Gattegno (******), Goutard (*******) and others provide examples of discussions of appropriate activities (viewed from the point of view of using particular forms of apparatus) for junior children. All these writers reveal the influence of Piaget on the current scene though this is not to say that Piaget would be in agreement with all the developments (see National Froebel Foundation Bulletin No. 127, December, 1960).

It has already been noted that Piaget was dissatisfied with the Gestalt emphasis on perceptual images and that he found the discussion of images to be conceived in terms that were too static to reflect the nature of concept development. It is because the

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(*) E. M. Churchull, unpub. thesis, loc cit.
(**) A. Phemister, loc cit.
(***) J. Tough, loc cit.
(****) Z. P. Dienes, BUILDING-UP MATHEMATICS, London, 1960.
(*****) Z. P. Dienes, MATHEMATICS IN THE PRIMARY SCHOOL, London, 1964.
(******) C. Gattegno, NUMBERS IN COLOUR, London, 1954.
(*******) M. Goutard, MATHEMATICS AND CHILDREN, Reading, 1964.

term "images" is too restrictive that in other studies use is made of the term "action-images" in the evaluation of the use made by children of various forms of apparatus. The idea is seen not to reside in the materials themselves but in the actions "operations" carried out on the materials. With a Number Ladder it is the action of making a step that leads to a concept of addition, not the contemplation of one particular rung on the Number Ladder. The same comment can be made about the Number Track and the use of Stern blocks.

In recent years one particular form of apparatus has attracted a great deal of attention - Cuisenaire rods. Gattegno has increasingly adopted a Piagetian viewpoint (unlike Stern who has not added to her original statement (*) concerning the use of her apparatus) though Lovell (**) reports that neither Cuisenaire nor Gattegno has provided a convincing theory linking the interplay of perception and actions, or of the relation between them and the mental structures that result therefrom....But the apparatus does present a structured situation to the child in which it is comparatively easy for him to discover many mathematical relationships. Lovell

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(*) C. Stern, loc cit.
(**) K. Lovell, <u>THE GROWTH OF BASIC MATHEMATICAL AND SCIENTIFIC CONCEPTS</u>, London, 1961, 46.

considers that the Cuisenaire apparatus does meet some of the objections to the type of apparatus based on the fairly rigid perceptual structures of the Gestalt psychologist: it clearly enables the child to appreciate the significance of his own actions through the rearrangement of the materials, it yields concepts which are mathematically valuable, and it relies only in part upon visual perception and imagery. Structural materials present the child with a clear and accurate representation of number system in a concrete form so that the child, in manipulating the material, is able to discover and systematize his ideas of number relationships. Piaget's (*) comments on one of his own experimental situations (using cards rather similar to Cuisenaire or Stern type material) are of interest in relation to the question of apparatus. The child is required, in the experimental situation, to reconstruct a series of cards: 'with cards arranged as a staircase the material represents with the maximum of intuitive clarity the law governing the formation of the first ten finite numbers, each ordinal corresponding to each cardinal and vice-versa'.

Gattegno (**) writes of the contribution of

(*) J. Piaget, <u>CHILD'S CONCEPTION OF NUMBER</u>, London, 1952, 137.
(**) C. Gattegno, <u>NUMBER IN COLOUR</u>, London, 1954.

Cuisenaire in the following terms: 'Psychologically, the value of Cuisenaire's contribution lies in the fact that by providing a semi-abstract material' (the need now recognised by many junior teachers for a halfway "bridge" between number in the concrete situation and number as an abstraction) 'he has overcome the gap between active and intellectual thought. Our minds are swift when dealing with images and representations but they move slowly in the actual performance of an action. We can move mountains in our imagination whereas the removal of a heavy piece of furniture may be a strenuous and exhausting activity involving a number of persons. By the elimination of actual steps which are replaced by virtual ones the mind gains in power and that is the significance of mathematics'.

The use of structural materials follows the "Dynamic Principle" suggested by Dienes (*).

A child is first able to play with the components, which is then followed by structured play and this is followed by formation of concepts and the introduction of practice. Dienes further suggests a "constructivity principle" and indicates that

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(*) Z. P. Dienes, BUILDING-UP MATHEMATICS, London, 1960, 44.

construction should always precede analysis, which is almost altogether absent from children's learning until the age of 12. The Dienes type material has become more widely used in schools. The emphasis in the use of structural apparatus is to provide a more reasonable learning situation. Dienes (*) explains this point of view: 'Those who have begun to think along such lines have realized that mathematical insights are very seldom generated on blackboards. The need for mathematical apparatus is beginning to be appreciated'.

In this chapter, the nature of mathematical learning for young children has been reviewed. A concise discussion of the development of ideas on effective techniques of teaching has been covered from the drill theory through to the "Gestalt"-inspired apparatus of Stern and finally to the "action image" experiences presumed by Dienes, Gattegno and others.

Nevertheless it would seem, in the light of this review, that structural materials have a rightful place in the early years of the junior school. The effective teacher of arithmetic seeks to use

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(*) Z. P. Dienes, BUILDING-UP MATHEMATICS, , Ibid, 27.

instructional procedures which aid the learner in gaining an understanding of basic ideas, concepts, facts, or principles of elementary mathematics; in developing proficiency in various skills and applications; in developing intellectual curiosity, a discovery attitude, and flexibility in thinking; in gaining ability to analyze, to make judgements, and to generalize; and in developing an inclination for the subject and an appreciation of its role in society. Structural materials allow children to discover concepts and facts for themselves with the minimum of direction from the teachers. Arithmetic has been a fertile field for the application of some of the developed theories of learning.

Generally, there are some agreements among educators on some principles of learning that have been derived from these theories (*). Some of the accepted principles of learning which apply to the teaching of arithmetic are viewed.

Readiness and motivation are important factors in effective learning. Readiness is a function of mental maturity, experience, interest, and attitude.

(*) F. G. Lankford, 'Implications of the psychology of learning for the teaching of mathematics', <u>THE GROWTH OF MATHEMATICAL IDEAS</u>: <u>GRADES K-12</u>, <u>THE TWENTY-FOURTH YEARBOOK</u>, The National Council of Teachers of Mathematics, Washington, 1959.

It is better when the pupil has the desire to learn and has gained those background skills and understandings which are related to the new concept to be learned. Teachers who understand arithmetic well and present it meaningfully can do a great deal toward motivating pupils to learn arithmetic.

Interesting applications by which the pupil sees purpose in learning activities has motivational value. It is also desirable that students should be enthusiastic about learning mathematics because they find the content interesting. When the learners get insight into the systematic character of the number system, satisfaction in learning results.

Learning is understanding rather than mechanical memorization and includes seeing relationships and making appropriate generalizations. Learning arithmetic meaningfully includes understanding the structure, or system of relationships, on which it is based.

Meaningful teaching helps to reveal the relationships and understandings inherent in arithmetic to the learner by methods of his own discovery. The clever teacher advises and guides the children into making meaningful generalizations.

Elementary school children do not learn best just by being told or being shown how to do something. Children learn more when given the opportunity to question, think through, explore, and engage in active participation through varied learning activities and with varied learning materials making for more effective learning. Children who are actively involved in exploring different ways of handling a quantitative situation are more likely to be active thinkers in the learning process.

The learner gradually reaches more mature levels of insight because learning is a developmental process. Generally learning proceeds from the simple to the complex and is a continuous process of integrating previously learned concepts with new concepts. The development of understanding in arithmetic in each stage is an outgrowth of a previous stage. Simple ideas are used as a basis for more generalized ideas. Ideas and concepts continue to expand into more mature and abstract concepts.

The most effective learning is that appropriate for individuals. Learners differ in their rate of learning, interest, attitude, experience, and ability to learn. Such consideration may result in differentiation in content, teaching methods, learning activities, and instructional materials.

Understanding of the basic principles of what is being learned followed by practice is necessary for proficiency and effective learning. The development of understanding does not exclude the need for practice. Practice is essential to the automatic mastery of facts and processes of arithmetic. However, the amount of practice and review needed is decreased when arithmetic is mathematically meaningful to the learner. Practice should be provided in accordance with the needs of the learners.

The emphasis on meaningful generalizations and on the application of generalizations in a variety of situations increased retention, transfer, and application of learning. It would be better for the learner if he discovered relationships and principles then applied them in a variety of situations. In that case he would be able to transfer his learning to new situations.

Learners should be aware of their own progress as it contributes to achieving effective learning. It is important to evaluate the progress in learning for both the teacher and the learner. The indications of progress in learning motivate and stimulate the learner to compete with himself for further and continued growth in learning. Knowledge of mistakes provides the learner with evidence as to topics on which restudy is needed. The child will assume more responsibilty for his own progress when kept alert to his progress. It is not the purpose of this chapter to evaluate the various types of apparatus but merely to summon examples in order to explain the changes in the nature of the mathematical curriculum and the purpose of apparatus. However, the teacher's view of the apparatus' place and purpose is the vital factor in the use of it. The function of apparatus would be useless unless its employ is in relation to a basic consideration of the aims of mathematical learning.

However, it would seem that the new trends or concerns sees in the use of structural apparatus the encouragement to the child to make his own moves, adopt his own strategies, develop his own descriptive language and determine his own level of abstraction. Learners would be able to know what to do and make the right decision in an appropriate time.

One further development is helpful and hoped for in learning situations. The co-operation between primary and secondary schools would be of benefit, both to the children and to the teachers concerned. The consultation of the content of syllabus and the methods used in different operations will contribute to decrease the gaps of learning experiences during the individual's life. Such a relation between stages would introduce an integrated course of arithmetic. It also gives teachers in different types of schools indications where child's difficulties are and raises teacher's understandings of the child's problems in other stages. Above all, it might lead to effective teaching and smooth out some of the children's difficulties.

One of the problems included in the arrangement between schools lie in the numerous and varied numbers and types of the schools; some central co-ordinating body is essential. It could be suggested that the institutes and schools of education might step into the breach and lead the way to greater unity in the arithmetic course.

CHAPTER 5

REVIEW OF THE PERTINENT LITERATURE

In this chapter an attempt is made to review the most important research which has been done hitherto concerning the learning of mathematical concepts by young children and to decide what influence their findings ought to have on the teaching of early stages of mathematics.

The first point that needs to be made is that in respect of the precise subject of this research, PRIMARY MATHEMATICAL SKILLS IN EGYPT AND ENGLAND, no literature exists to be reviewed. No previous cross-cultural study of mathematical skills in primary schools between Egyptian and British children has been made. The present study may therefore claim to be a pioneering one. Other cross-cultural studies, the results of which relate generally to the present study, have been made and the most important of these are reviewed.

The science of child psychology has done much during the past two decades to increase the teachers' knowledge of children. The teacher-child relationship, and the appearance and experiences of the primary school, especially in the case of the infants and younger junior school children, have been transformed by the knowledge arising from "child-centred" educational practice.

A major contribution to the knowledge of the various national and cultural differences in the mathematical attainment of school children is provided by The International Project for the Evaluation of Education Achievement (I E A). An outcome of the project was Hussein's work in 1967 (*), The International Study of Achievement in Mathematics which concentrated on the effects of differences among the school systems, and on differences in the achievements, interests, and attitudes of school pupils, in twelve countries (Australia, Belgium, England, Federal Republic of Germany, Finland, France, Israel, Japan, the Netherlands, Scotland, Sweden and the U.S.A). Another purpose was to test the degree of

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(*) T. Hussein, INTERNATIONAL STUDY OF ACHIEVEMENT IN MATHEMATICS: A COMPARISON OF TWELVE COUNTRIES, INTERNATIONAL PROJECT FOR THE EVALUATION OF EDUCATIONAL ACHIEVEMENT, 1967, VOL I, II.

universality of certain relationships which had been established in some countries, e.g the effects of sex and home background on achievement. The children tested were all in the secondary school age range. The range of test mean scores for the twelve countries covered more than one standard deviation of the combined distribution for the thirteen-year-old sample, and was even larger for older children. Differences in the variability of scores within countries were also marked. The test scores were correlated with forty-five other variables related to the school, the teacher or the student-between-country. Correlations were generally low, though within-country correlations were, for the most part, higher. Sex differences in performances were found to vary from country to country, being obviously affected by environmental factors no less than by genetic considerations. Generally girls were better on verbal problems, and boys on computational ones. An aspect of this study which is of particular interest is classification of objectives arrived at from an integration of the lists submitted by the participating countries. The objectives in the teaching and examining of mathematics were formulated as follows:-

Knowledge and information: recall of definitions, notations, concepts; Technique and skills:

manipulation and computation; Translation of data into symbols and schema and vice versa; Comprehension, capacity to analyse problems, to follow reasoning; Inventiveness, reasoning creatively in mathematics.

It is interesting to note the extent to which this structuring of objectives agrees with the more general taxonomy proposed by Bloom 1956 (*). Bloom's attempt to partition the cognitive domain generally - not just in the field of mathematics - resulted in the following:-

Knowledge of:-

specifics; the ways and means of dealing with specifics, universals and abstractions in the field:

Comprehension, including translation, interpolation and extrapolation:

Application, applying to new situations or problems:

Analysis of:- elements, relationships, organizational principles:

Synthesis, the producing of a communication, or plan, and the derivation of abstract relationships:

(*) B. S. Bloom et al, TAXONOMY OF EDUCATIONAL OBJECTIVES: THE CLASSIFICATION OF EDUCATIONAL GOALS, HANDBOOK 1, COGNITIVE DOMAIN, 1956.

Evaluation, judgement in terms of internal evidence, and also of external evidence.

This organization of educational objectives is hierarchical, in that each category builds on those that precede it, thus one cannot comprehend without first achieving the relevant knowledge and one cannot apply the knowledge without both knowing and comprehending. As compared with the I E A list of objectives, the Bloom taxonomy extends in more detail to the higher-order objectives (though inventiveness could embrace all three of analysis, synthesis and evaluation). Both the I E A "knowledge and information" and "technique and skills" can be subsumed in the wider "knowledge" category of Bloom, while the I E A "translation" appears as part of comprehension. A very similar structuring too was made by the School Mathematics Study Group in the U. S. A, Romberg and Wilson 1966 (*), and also by the Indian National Council of Educational Research and Training 1966 (**), while in Britain Wood in 1967

(*) T. A. Romberg and J. W. Wilson, THE DEVELOPMENT OF MATHEMATICS ACHIEVEMENT TESTS FOR THE NATIONAL LONGITUDINAL STUDY OF MATHEMATICS STUDIES, School Mathematics Study Group, Stanford, California, 1966.

(**) Indian National Council of Educational Research and Training, REPORT OF THE TRAINING COURSE ON EDUCATIONAL EVALUATION, DEPARTMENT OF CURRICULUM AND EVALUATION, New Delhi, 1966.

(***) R. Woods, THE ITEM BANK PROJECT: A PILOT STUDY OF AN ALTERNATIVE METHOD OF CALIBRATING

(***) advocated much the same scheme. Thus while it may be reasonable to restrict the structuring of mathematical objectives for primary school children to the first half of the Bloom taxonomy, a decision in principle to assess for separate objectives along these lines would seem to be in general accord with accepted practice in many countries.

Turning now from mathematics to cultural differences in general ability, one inevitably encounters the problem of how to make valid assessments, and of distinguishing if possible between the genetic and environmental components of such ability. Anastasi 1961 (****) discusses the difficulties of test constructors in this field. The distinction between Intelligence A, "genetic potentiality" and Intelligence B, "the interaction of intelligence A with environment" made by Hebb 1949 (*****), and the somewhat similar distinction between "fluid" and "crystallized" intelligence made by Cattell 1963 (******) both illustrate this. Any assessment for different cultures - such as that of

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ATTAINMENT IN THE C. S. E. EXAMINATION WITH SPECIAL REFRENCE TO MATHEMATICS, N. F. E. R, 1967.
(****) A. Anastasi, PSYCHOLOGICAL TESTING, 1961.
(*****) D. O. Hebb, THE ORGANIZATION OF BEHAVIOUR, 1949.
(******) R. B. Cattell, 'Theory of fluid and crystallized intelligence: a critical experiment', J. Educ. Pschol., 1963, 54, 1- 22.

Egypt and England - must of necessity be concerned with intelligence B or the "crystallized" intelligence of Cattell. The "culture fair" test of Cattell may be noted as an attempt to construct an instrument suitable for cross-cutural comparisons. In a comprehensive study of cultural differences in intelligence by Eells et al. 1951 (*) it was shown that in order to minimize the effects of social status the test used should be composed primarily of pictures, geometrical designs and stylized drawings. Such items are the least "class-loaded", and are also suitable for groups of children heterogeneous in respect of ethnicity and race. Attention to the cumulative disadvantages of an impoverished environment has also resulted from the researches in England of Wiseman 1964 (**) and Douglas 1964 (***). A comprehensive review of these, and related problems is provided by Vernon 1969 (****).

In summary, a research into the differences of the skills in mathematics in primary school, as the children grow up, of Egyptian and English children

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(*) K. Eells, INTELLIGENCE AND CULTURAL DIFFERENCES, Chicago, 1951.
(**) S. Wiseman, EDUCATION AND ENVIRONMENT, Manchester, 1964.
(***) J. W. B. Douglas, THE HOME AND THE SCHOOL, 1964.
(****) P. E. Vernon, INTELLIGENCE AND CULTURAL ENVIRONMENT, 1969.

will be the first of its kind; studies of the performance in mathematics of older children in other countries suggest a structuring of assessment in categories following the Bloom taxonomy; and any concurrent assessment of ability should be attempted by a non-verbal intelligence test with pictures and patterned items.

Specific suggestions to develop skills have been presented by Max A. Sobel in THE TEACHING OF SECONDARY SCHOOL MATHEMATICS (*). He explained in detail, giving examples from mathematics and his suggestions to develop skills are reviewed here. DEVELOP UNDERSTANDING BEFORE SKILLS. This would be by developing new skills in as meaningful a manner as possible, the students performing better in skills if they understand what they are doing and why they are doing it. Let children discover the formula for the area of a parallelogram by a dynamic approach instead of telling them the formula in which to subtitute figures. Laboratory techniques are effective in fixing skills, more attention should be given to discovery methods and laboratory techniques, to enable children to discover on their own. AVOID ROUTINE DRILL THAT TENDS TO BE MECHANICAL. Clearly

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(*) M. A. Sobel, THE TEACHING OF SECONDARY SCHOOL MATHEMATICS, National Council of Teachers of Mathematics, U. S. A, 1970, 291- 308.

young children need to be given different kinds of educational situations and varied experiences which will lead them to master the concepts and grasp the skills which depend entirely on understanding and not mere mechanical drill. A similar idea was one of the conclusions in experiments conducted many years ago which showed that students who are given routine tasks to perform tend to acquire rigid mental sets (Luchins 1942 (*); Luchins and Luchins 1950 (**)). They are inclined to follow procedures blindly, often overlooking more direct methods of solution because of their adherence to rules. Inflexible patterns of thought blind a student to the meaning behind an operation and retard his mathematical growth. Many textbooks promote rigidity of thought by presenting drill exercises in sets grouped to be all alike (for a single skill). The student learning the skill throws an automatic mental switch to perform the required exercises. True, in performing the exercises he may learn the particular skill, but his performance of the operations contributes nothing to his mathematical understanding. Furthermore, skills learned in such a manner are likely to be soon

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(*) A. S. Luchins, 'Mechanization in problem solving: the effect of einstellung', Psychological Monographs, 1942, 54, No.6, whole no. 248.
(**) A. S. Luchins, H. Luchins (ED.), 'New experimental attempts at preventing mechanization in problem solving', Journal of General Psychology, April 1950, 42, 279- 297.

forgotten. It should not be standard practice for sets of drill problems to pave the way for the introduction and development of new skills. ENCOURAGE AND REWARD ORIGINALITY. The skilful teacher often looks for problems that permit original solutions and incorporates them in sets of practical exercises. Displays of originality should be appreciated, developed and encouraged; the student should not be forced to do routine work and follow particular patterns. Children should be provided with success in a variety of experiences. This kind of experience eventually produces mathematical mastery. REVIEW SKILLS AT THE TIME THEY ARE NEEDED. The time to introduce new facts to a child is when his current knowledge has been insufficient to solve a problem he is facing. This is the time when his interest for learning will be keenest. Children should only be provided with exercises when they understand the need for them, and these exercises should be made stimulating and enjoyable. EMPLOY NOVEL IDEAS TO FIX SKILLS. Many children grow up disliking mathematics because they have spent many hours on routine, tedious, dull and meaningless exercises - just drill for drill's sake. The intelligent teacher knows that the art of teaching demands developing a variety of procedures for fixing skills and presents interesting opportunities for drill. This is possible by associating skills with unusual events, offering interesting puzzles to introduce new topics and perhaps using computers to develop and maintain skills. RELATE NEW SKILLS TO PREVIOUSLY LEARNED SKILLS. Assignments can be organised to include several exercises from preceding units of work which contain previously learned skills. The maintenance of skills is an important teaching objective. PROVIDE FOR INDIVIDUALIZATION OF INSTRUCTION. A teacher might provide a collection of exercises, adequately graded and differentiated, from which the student, with teacher guidance, may make selections according to his need. Sets of drill problems and materials geared to individual needs can be offered in a similar way. CAPITALIZE ON ERRORS. When the teacher notices the children repeating basic errors, then he should stop immediately and investigate the problem and the skill in a more meaningful way. Perhaps a better way is to teach the student to search for different ways to approach a problem to increase flexibility. ANALYZE ALL POSSIBLE ASPECTS OF A SKILL. Before a teacher introduces a skill, he should list all possible sources of student error so that every effort can be made to alleviate these difficulties from the beginning. It is essential to distinguish the different components of mathematical skills, and to programme the teaching of each (indentifiably different) new skill. This is an important way by which a high level of skill will be attained, while the pupil remains confident and interested. A particular problem may be approached by a sequence of exercises that increases in level of difficulty while leading the student along in relatively small increments. GENERATE ENTHUSIASM. Topics should be approached with enthusiasm and interest to improve learning, so that students willingly attempt to learn basic skills and develop a positive attitude. As Bulter and Wren wrote: 'Enthusiasm is contagious, and sane enthusiasm backed by sympathetic and enlightened competence is the only real guarantee of the effective maintenance of student interest' (*). If this is done, such learning will be both pleasurable and memorable. A similar point of view was expressed in the Cambridge Conference report 1963 (**), 'Both the practitioner of applied mathematics and the creator of pure mathematics spend much of their time and effort on HERE'S A SITUATION, EXPLORE IT, not only on HERE'S A PROBLEM, SOLVE IT or HERE'S A THEOREM, PROVE IT. It is good to admit this to the students, and to let them work on mathematics in this

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(*) C. H. Bulter and F. L. Wren, THE TEACHING OF SECONDARY MATHEMATICS, New York, 1960, 127.
(**) Cambridge Conference On School Mathematics, GOALS FOR SCHOOL MATHEMATICS, Boston, 1963, 80.

manner themselves'. The role of interesting problems in motivating the learner of a skill is stressed throughout the previous studies. Teachers should strive to use many open-ended problems so that students have opportunities to make creative discoveries on their own. In all aspects of teaching mathematics the attitude of the teacher is a prime factor in setting the stage, creating interest, generating enthusiasm. In both teaching and maintaining skills a variety of approaches is essential.

In practice, teaching mathematical skills may not seem to be an especially rewarding task. It is hard work, often frustrating, and it can be far from exciting. Yet for these very reasons it can be regarded as a great challenge. The art of teaching includes work with students toward the mastery of basic skills in ways that make the task palatable to both student and teacher.

Considerable efforts have been made in every major curriculum group to stress the need for skill development. Even programmes that place heavy emphasis on concept building and discovery techniques are firm in their insistence that skills must accompany ideas. Presenting exercises and drill effectively is the task of the teacher.

In an Egyptian study carried out by Professor William Ebeid, MATHEMATICAL SKILLS ARE REQUIRED FOR STUDYING SCIENCES IN A PREPARATORY STAGE, (*), a model was presented of the structure of an integrated syllabus between mathematics and sciences in preparatory school, making suggestions for the mathematical skills that are required to study sciences at that stage. The required skills were classified as:- qualitative, performable, quantitative, related to shapes and practical skills. The necessity and importance of integration between mathematics and science, both of which have the same general educational objectives, was emphasised and stressed in Prof. Ebeid's study. This classification was in the light of the needs of studying sciences, and not all the mathematical skills are mentioned in that study.

Eissa Michaeel Hadad 1977 (**) in 'The mathematical achievement level at the end of a primary stage', carried out a study to recognize the level of mathematical achievement of primary stage children, comparing it with the expected level of the

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(*) W. T. Ebeid, MATHEMATICAL SKILLS ARE REQUIRED FOR STUDYING SCIENCES IN A PREPARATORY STAGE, Cairo, 1974.
(**) E. M. Hadad, 'The mathematical achievement level of the end of a primary stage', M.A Thesis, Faculty of Education, Unpub., Oman, 1977.

required mathematical objectives in that stage. He tried to investigate whether there were significant differences in achievement between boys and girls and any common mistakes among children by the end of a primary stage. The results of this study revealed a decrease in the standard of children's achievement in mathematics generally. They also demonstrated the weaknesses of children in operating the required, basic concepts which had been supposedly established and learned earlier in school life in order to form a basis for all learning in the future.

Abd El Fatah El Sharkawy's study 1978 (*) is considered as one of the evaluation studies, the purpose of which was to recognize the contemporary mathematical concepts in the first year in secondary schools in Kuwait and to establish how far the children had acquired these concepts. He pointed out the basic concepts: twenty concepts concerning groups, fifteen concepts on relations, fifteen concepts on applications. He designed an achievement test on these concepts. As a result of applying his test he found that some concepts were unclear in children's minds, such as the concept of group,

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(*) Abd El Fatah El Sharkawy, 'The contemporary mathematical concepts of first class secondary school in Kuwait and how far children understand these concepts', M. A. Thesis, Faculty of Education and Art, Unpub., Kuwaite, 1978.

element, relationships, empty group and two equivalent groups among others. He also observed that boys were better than girls in understanding groups but he could not find differences between them in the understanding of relations and applications. The level of understanding changed as the children moved up to higher classes.

Many reports have been presented to explain the learning of mathematics at the different general education stages in Arabic countries. One of these reports, prepared by William Ebeid (*), studied 'Teaching of new mathematics in Egyptian schools'. He explained that there are some mathematics curricula in Egyptian schools which are called "new mathematics" and listed them as follows:

> Unesco Curriculum for Secondary Stage.
>
> Arabic Organization of Curriculum for Education, Culture and Sciences in the Preparatory Stage.
>
> New Mathematical Curriculum for the Institute of Teachers.
>
> New Mathematical Curriculum for the Primary Stage.

He found that it is not right to claim that the

(*) Academy of Scientific Research and Technology of Trade and Scientific Unions, A REPORT OF TEACHING NEW MATHEMATICS IN EGYPTIAN SCHOOLS, Cairo, 1979.

current teaching in mathematics is "New Mathematics", because the New Mathematics has three dimensions:-

Content which contains new topics in the pre-university stage which was not studied before.

Reconstruction of concepts and topics of traditional mathematics to be in harmony with the "new" topics in a united mathematical form.

More effective and active methods of teaching, concentrating on discovery learning and not just learning by mere rote, the aim being to let children acquire creative skills of understanding and not mere mechanical operations without thinking.

This report reviewed the problems which were faced in introducing the "New Mathematics" explaining that these problems were to be expected and usual in any development. They could be overcome by more application, experience, experimenting, modifying and improving the syllabus as a result. Efforts should be made to solve the arising difficulties. Modern mathematics programmes have been introduced on an experimental basis in secondary and then preparatory schools in Egypt. Unesco and then Alecso participated in the implementation of these programmes. There have been both positive and negative effects of this modernization. A major problem resulted from the duality of inputs in mathematics teaching as related to admission to further education, particularly to university. Development is a continuous process, so a development programme which reflects modern trends and satisfies local needs and is based on local realities is still needed.

Developing mathematics curricula in general education in Egypt, was studied by F. M. Mina 1980 (*) who is in the Faculty of Education, Ain Shams University. This study concluded that mathematics curricula should be developed for general education in Egypt. He presented some ideas to achieve his aim which concentrated on the following areas:-

Special characteristics of mathematics and their educational implications.

Assumptions about curriculum.

The writer's understanding of the aims of teaching mathematics.

The beginning and development of the contemporary movement to reform mathematics education in general and in Egypt in particular.

Some evaluative remarks on the experience of developing mathematics education in Egypt.

'The present state of modern mathematics

(*) F. M. Mina, 'Developing mathematics curricula in general education in Egypt', <u>Educational Journal</u>, Cairo, October 1980, 32.nd year, 1, 61- 79.

programmes in general education', a study presented by Soad Anees 1977 (*), who was a general director of preparatory education, Ministry of Education in Egypt, explained how the experiment of teaching modern mathematics in elementary, preparatory and secondary schools in Egypt has influenced teachers and parents. It was not unusual to face difficulties related to the content, textbooks, reaction of teachers and administrators. Efforts have been made to overcome the arising difficulties. Problems do not mean a return to "traditional" mathematics. A balanced programme was needed for all the students.

'Difficulties of teaching arithmetic in primary schools' was a study presented by Mourice Aiad Hanua (**), Ministry of Education, which discussed the difficulties facing the teaching of arithmetic in the light of the general problems of teaching in primary schools. It considers the general problems of preparing primary school teachers, professional inspection, examinations and the problem of evaluation, audio-visual aids, overcrowding in class

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(*) S. A. Anees, THE CONTEMPORARY WORLD ATTITUDES IN TEACHING MATHEMATICS, A STUDY WAS IN THE CENTRE OF DEVELOPING SCIENCES, Ain Shams University, Cairo, December 1977.
(**) National Committee For The International Mathematical Union And The African Mathematical Union, PROCEEDINGS OF THE CONFERENCE ON MATHEMATICS EDUCATION PRE-UNIVERSITY STAGE, 8-11 DECEMBER 1980, Cairo, 64- 71.

rooms, and the problems of textbooks.

Research in arithmetic is in danger of becoming crystallized into patterns which have proved useful in dealing with problems of learning through practice, in analyzing the content of textbooks and courses of study, in making surveys of social usage of arithmetic, and in comparing the effects of method A with method B as they apply to the teaching of various aspects of arithmetic. Some research studies of this type were valuable, and much improvement in the teaching of arithmetic has resulted from them. However, if teachers are to become sensitive to the meanings that arithmetic has for the children who are learning and if desirable habits of quantitative thinking are to be realized as outcomes, then techniques of research must be devised which reveal the thinking being done by children as they learn. There are few models to follow for this kind of research. The need is for ingenuity and inventiveness. Egyptian educationalists may need to learn to be patient with failures in some pilot studies, but they must also persevere in research related to the new and significant concepts in the teaching of arithmetic which have arisen to prominence during the last two decades.

It was the hope of the Egyptian educationalists that a sampling of members of the profession who have already published studies on arithmetic might yield some proposals for research which would reflect the developing interest in meaningful outcomes. Accordingly, letters requesting suggestions regarding problems on which there is a recognized need for research were sent to a sample list including public school teachers, supervisors, and members of college and university faculties who had published articles or studies on the teaching of arithmetic. Suggestions or information regarding certain factors considered essential to an appropriate plan for research into a specific problem included the significance of the problem relating present knowledge concerning arithmetic to needs in the teaching of arithmetic; possible sources of data concerning the problem; suggested methods and techniques for carrying out a study and consideration of the question as to whether the outcome would be a principal contribution to new knowledge about teaching and learning of arithmetic, new techniques of research, or some other aspect.

It is hoped that this thesis will provide useful and applicable insights and guidance to aid the endeavours of teachers and students who may wish to participate in the search for much needed solutions to problems in the field of arithmetic such as those proposals indicated below:-

A study of how children think about number, how concepts and skills are developed, and how understanding is established. Present knowledge does not provide the reader with information as to what goes on in the mind of a child when confronted with situations involving number concepts and number sense. The concept of number is a dynamic concept. How can the child's concept of number be extended most effectively to include fractions, decimals, approximation, graphical scale etc. Actual classroom situations are sources of data under the direction of teachers interested in organizing their programme in such a manner as to learn and record the thought processes of their students as their number concepts become enriched through guided experience. Teachers concerned with this type of study should be provided with competent help essential to the securing and recording of data which reflects accurately the thinking processes of their students. This method of study would require much patience and many individual conferences with each student. If a child could be encouraged to talk as he works, to "think out loud", and if a recording could be made, the results would be helpful. The principal contribution would be "new knowledge" which should prove to be exceedingly helpful in improving the teaching of arithmetic.

A second proposal is the determination of the effectiveness of the use of manipulative materials in the teaching of arithmetic. The significance of this problem is directly related to the question of teaching arithmetic meaningfully. Manipulative materials should enable the pupil to deal with things so that he can discover relationships among quantities more effectively than by use of symbolic and pictorial materials. The classroom should be used as sources of data for an experiment to show the value of manipulative materials with, for example, two equated groups at different grade levels, such as grades 3 and 4, as a method of study. One group could use a well illustrated textbook, preferably a book from a series published since 1980, while the other group could use the same text, but this group should supplement the text with the use of manipulative materials. The first group would not use these materials. A certain area of subject matter, such as carrying in addition, compound subtraction, understanding of fractions, and the like, could be selected for experimental purposes. If the results from the group using manipulative materials are significantly superior to those attained by the other group, these results may be regarded as proof that beginning a pupil at a low level of abstraction is an effective means of instruction in arithmetic.

Another proposal is to establish methods of evaluating a pupil's ability to use arithmetic in functional situations. A functional situation is one that is encountered in experience and may present itself in writing, in speech, in a visual or kinesthetic impression, or in a combination of these. The problem is important because arithmetic aims to be meaningful to pupils and to be functionally useful to them. The need is for a type of evaluation that deals with real materials in real situations. Sources of data depend upon setting up new methods of testing and evaluation which must be used experimentally. To carry on this study, the validity of tools of measurement must be established and developed, and used in a comprehensive study. This study would yield new procedures and techniques of measurement and evaluation.

It appears that there are two very important factors, besides the child, that are likely to affect the development of mathematical concepts. First there is the mathematical understanding of the teacher and secondly the environment in which the child is reared. Any discussion on educational achievement or cross-cultural research is based on the premise that individuals differ in their abilities and achievements. Within any particular nation there are subgroups determined by sex, colour, skills, education, values and many other characteristics. Even more so, therefore, nations differ from one another, sometimes in very pronounced ways. No one has denied that each different culture, no matter how culture is defined, has its own peculiar emphases, customs and values.

In this chapter a concise review of the most relevant research was investigated. Interest was focussed on learning mathematical skills and on discovering how missing concepts and skills make gaps in the individual's knowledge which affect his subsequent learning of mathematical skills. Although faults and weaknesses in the diagnostic tests used soon became apparent the results did provide encouragement to the view that in principle it is possible to measure and describe the nature of the individual's cognitive structures including skills relative to mathematics. This conclusion soon leads to the issue as to how the various skills might be well defined, identified and classified. In other words what sort of scale or measure can be used to describe and classify skills ? Such a measure should emphasise the constructive nature of learning relating the skills to the content and structure of the material to be learnt. Hitherto, in Egypt there has been no such measure available, and where attempts have been made to measure mathematical achievement, then such measures have been based on mechanical learning and operations. An attempt to overcome this problem by producing an objective test to evaluate the skills will be more fully described in the chapters that follow.

The findings of the preliminary study also highlight the need to establish, in a much more vigorous fashion, the structure involved in a particular hierarchy of skills as related to a given "complex" mathematical task. This is an extremely difficult problem, as noted in the earlier discussion, of the difficulties which arise in attempting to measure the degree of skill in a particular learning situation, and has frustrated the efforts of a number of other researchers.

If the view is accepted that learning skill is an important aim in teaching mathematics, then it is logical to argue that defining the structure and concepts of the hierarchy of skill involved in "what is to be learnt" should be a first priority.

Consequently in the following chapters the emphasis will be principally directed to that task.

CHAPTER SIX

THE QUESTIONNAIRE RESULTS FOR EGYPT AND ENGLAND

This chapter is divided into two sections which give the questionnaire results for Egypt and England respectively. A general review of the questionnaire design, its procedure and results will be covered in this chapter, which summarises the main structure, some of the processes to be undertaken, as well as suggesting possible future developments.

PART ONE

EGYPTIAN RESULTS OF THE QUESTIONNAIRE

The questionnaire as a means for research (*) is a form of planned collection of data which sets out

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(*) H. H. Hyman, SURVEY DESIGN AN ANALYSIS, PRINCIPLES CASES, AND PROCEDURES, Glencose, III, 1955.

to analyze the relationships between certain variables. Questionnaire and interviews are necessary to gather data for social surveys. The questionnaire (*) is essentially a scientific instrument for measuring and collecting particular kinds of data. Like all such instruments, it has to be specially designed according to certain specifications and with specific aims in mind. This one attempts to answer such questions as:

Is this classification of skills satisfactory?.

Are there any more skills that should be added?.

Is the level of primary school mathematics sufficiently high?.

It was also designed to collect data about the reasons why the level of primary mathematics is not felt to be satisfactory to study the present way of teaching mathematics in general, to investigate the teaching methods of mathematics and to consider the opinions of various educationalists as regards its importance in everyday life.

<u>The procedure of questionnaire design</u> (**)

There are several steps which were taken into consideration in designing this questionnaire:

(*) C. A. Moser, <u>SURVEY METHODS IN SOCIAL INVESTIGATION</u>, London, 1958.
(**) A. N. Oppenhein, <u>QUESTIONNAIRE DESIGN AND ATTITUDE MEASUREMENT</u>, London, 1966.

FIRST :- the subjects of the questionnaire fall into two categories: the classification of mathematical skills which were suggested by the researcher and the methods of teaching in primary school mathematics with reference to the reasons for poor level of performance of the children.

SECONDLY :- all aspects of these two areas were covered by the questions drawn up. The questions were formulated in as simple a way as possible with care taken to avoid ambiguity so that the questionnaire could be answered by the majority of respondents (teachers, advisers, etc..).

Most questions were of the closed kind so that long answers could be avoided and analysis of replies would be simpler. Nevertheless a few open ended questions were included, enabling those answering to elaborate on some points.

THIRDLY :- a pilot questionnaire was circulated and the answers were used to rephrase and rearrange the questions in order that there be a logical connection between them and that the final questionnaire be an integral unit.

The early questions were simple and interesting in the hope that no one would be deterred from answering.

On the advice of acknowledged experts in this field some items were deleted or reformulated.

FOURTHLY :- the final questionnaire in its ultimate form was, once again, looked over by experts to ensure that it would fulfil the requirements of the survey.

FIFTHLY :- validity and reliability of the questionnaire. The reliability was tested by using the questionnaire on the same group of people on two occasions approximately a month apart, and validity was checked by the judgement of experts in this field (*).

Some rules were taken into consideration when the items were designed; words were chosen with as concise a meaning as possible; to avoid making the question complex; a simple word order was chosen ; unnecessary words were omitted; questions not directly related to the area of study were avoided; and the elements of the questionnaire were arranged in the most logical way both for the study and the respondents.

Two types of questions featured in the survey: those concerning issues of a general nature which would not be affected by the respondent's individual

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(*) J. Jaber, and A. K. Kazem, THE METHODS OF THE RESEARCH IN EDUCATION AND PSYCHOLOGY, Cairo, 1978.

position, such as the necessity of problem solving skills in adult life. Those of a more controversial nature requiring deep thought and full detail were put at the end of the questionnaire.

Generally it could be said that those questions requiring little thought were placed at the beginning while those requiring more thought and detail were put at the end of the questionnaire. Care was taken to ensure that the questionnaire was not too long. In Egyptian schools interviews ensured that those who had replied to the questionnaire had responded faithfully to their own views and did not feel constrained to answer according to the theory.

The results of the Egyptian questionnaire

The questionnaire and all tables referred to in this chapter are to be found in Appendices (XV) to (LVII). (Tables are arranged so as to correspond with those of the questions).

The SPSS program was used to analyse the answers and likewise, the questions were coded. In analyzing the data one must remember that some respondents did not answer all the questions. Nevertheless information required but not given is termed as "missing data", which denotes the percentage of people who did not reply to any specific question. For each question two figuers are given: the first one indicates the percentage based upon the total number of respondents irrespective of whether they answered; the second figure, which appears in brackets, stands for the percentage of responses for that specific question whatever the category, excluding those respondents who missed out the question completely.

Question 1A Jobs

The key to the codes for the first table is as follows: code 1- Primary school inspector; code 2- Primary school Head teacher; code 3- Principal teacher in primary school; code 4- Primary school Deputy-Head teacher; code 5- Ordinary teachers with responsibilty for mathematics; code 6- Primary school teacher; code 7- Psychologist at University, Department of Psychology; code 8- Lecturer in methods of teaching mathematics at Faculty of Education; code 9- Lecturer in Foundation of Education; code 10- Assistant Lecturer at the Faculty of Education; code 11- Administrator at Faculty of Education; code 12- Mathematics Inspector; code 13- Other status; and code 99- Missing data.

Table (XV) shows that most of those answering were from code 4, i.e Deputy-Head teachers at primary school. They account for 20.7 percent of the respondents (21.4 percent).

The next largest category consists of ordinary teachers with responsibility for mathematics, who account for 17.4 percent (17.9 percent).

Head teachers, account for 10.7 percent (11.1 percent).

The principal teachers in primary school (a category roughly the equivalent to that of senior teacher in a Secondary School in England) account for exactly the same proportion of the whole, i.e 10.7 percent (11.1 percent).

Furthermore, inspectors in primary school account for 9.9 percent (10.3 percent) are precisely the same percentage as that of the primary school teachers.

Moreover, 8.3 percent (8.5 percent) of the total number of respondents had other jobs. They include lecturers in mathematics Department at the Faculty of Science, managers in Education Department, directors in educational administration.

In addition, the mathematical inspectors at primary school account for 5 percent (5.1 percent).

Finally one must consider the faculty administrators, assistant lecturers, lecturers in foundation of education and psychologists in psychology depts. These form (1.7 percent), (0.8 percent), (0.8 percent), and (0.8 percent) of the whole respectively.

<u>Question 1B The qualifications of respondents</u>

Table (XVI) gives the various qualifications of those replying.

Code 1- Holders of a certificate in general secondary; code 2- Holders of a three years teaching Diploma in education; code 3- Holders of a Five years teaching Diploma; code 4- Holders of a certificate in general secondary followed by a two years special teaching Diploma in mathematics; code 5- Teachers with B.Sc in mathematics; code 6- Teachers with a B.Sc in mathematics and education; code 7- Graduates with M.Sc in mathematics or mathematics and education; code 8- Those who are Doctors of education; code 9- Holders of Old Certificate in primary education; code 10; Holders of an Industrial Diploma; code 11- Two years studies in education preceded by Industrial Diploma; code 12- Those who are Doctors in mathematics; and code 13- Those who have other qualifications in education e.g Azhar certificate in primary stage, primary teachers institution certificate, etc....

One must now consider according to decreasing numerical order of importance, the different categories of qualifications. It was noteworthy that the largest number of responses came from people who had a five year teaching Diploma. They constitute 37.2 (39.1 percent) of the total. The next most significant group of respondents in numerical terms were those who had a certificate in general secondary education and in addition a two year special teaching Diploma in mathematics. They make up 21.5 percent (22.6 percent) of the respondents. They are followed by those who had a three year teaching Diploma, who add up to 10.7 percent (11.3 percent). The rest had a B.Sc in mathematics and education, B.Sc in mathematics or industrial Diplomas with two years studies. Other qualifications included industrial Diploma, old certificate in primary education, Doctorates in mathematics and certificate in general secondary education. They represented 5.8, 5.8, 3.3, 2.5, 1.7, 1.7,and 0.8 percent (6.1, 6.1, 3.5, 2.6, 1.7, 1.7, and 0.9 percent) respectively.

Question 2: Teaching experience in terms of years

Code 1- 1 to 5 years; code 2- 6 to 10 years; code 3- 11 to 15 years; code 4- 16 to 20 years; code 5- 21 to 25 years; code 6- 26 to 30 years; code 7- 31 to 35 years; code 8-36 to 40 years; and code 9- missing data. Table (XVII) shows that most of those answering had worked 11- 15 years, since they form 19.8 percent (22.9 percent) of the total. They are followed by those who had worked 16 to 20 years, who form 19 percent (21.9 percent).

Finally came those who have worked for 21 to 25 years, 6 to 10 years, 1 to 5 years, 26 to 30 years, 31 to 35 years or more than 35 years make up 12.4 percent, 9.9 percent, 8.3 percent, 6.6 percent and 2.5 percent (14.3%, 11.4%, 9.5%, 7.6%, and 2.9%) respectively.

It would seem that most people who answered this questionnaire had had experience in working in this area for a quite long time.

Question 3: Sex

Code 1- male; code 2- female; and code 9- missing data.

As can be seen from table (XVIII), most of the respondents were males, who constitute 68.6 percent

(69.7 percent). Female responses form 29.8% (30.3%). The code number in all the following questions in this section are:

Code 1- strongly agree; code 2- agree; code 3- undecided; code 4- disagree; and code 5- strongly disagree.

Question 4

Table (XIX) presents the breakdown of respondents' opinions about whether ACQUIRING BASIC MATHEMATICAL SKILLS IS AN IMPORTANT AIM IN TEACHING MATHEMATICS IN PRIMARY SCHOOL. There were no responses indicating disagreement. It seems, therefore, that the majority acquiesces, accepts and knows that mathematical skill is a fundamental element in the teaching of mathematics.

Question 5

Table (XX) gives the data that results from the question concerning the level of mathematical skills reached by the normal primary school child. As can be seen in that table, a large percentage of respondents believed the level of mathematical skills reached by the average primary school child to be unsatisfactory. These represented 68.4 percent, as against the 28.2 percent, who believed the level reached to be satisfactory, and 3.4 percent who remained undecided. Consequently one can assert that the majority feel that the level is not satisfactory. The question which immediately posed itself was what were the reasons for this unsatisfactory level?

From the responses to the open ended questions, various causes could be identified. These were:-

(i) The methods of teaching are not sufficiently varied or creative.

(ii) The evaluation at the end of every mathematical lesson includes inadequate questions covering elementary basic mathematical skills.

(iii) There are no specifically qualified teachers to initiate the children in the concepts and skills of mathematics in primary schools. Teachers are often entirely dependent on the textbook's examples which they tend to follow blindly.

(iv) The primary school curriculum does not put enough emphasis on the concepts and skills of mathematics.

(v) Parents and pupils are inclined to be apathetic about ensuring an understanding of the skills which should have been acquired at primary school level. One can ascribe this in part to the systematic transition of the child, at the end of each school year, into the class above. This is based on no other objective criteria than that of age.

(vi) Overcrowded classes along with insufficient number of mathematics lessons do not provide any opportunity for the pupils to concentrate on the concepts and skills of mathematics.

(vii) The lack of structural apparatus and audio-visual aids etc.. Hinders children's progress and understanding.

(viii) Some of the textbooks used were other than those recommended by the educational board.

(ix) Parents and pupils were more interested in the test results than in the assimilation of mathematical concepts and skills in general with a consequent narrowing of the mathematical curriculum.

(x) The mathematics teaching is incoherent and the development of mathematical skills was impaired due to the lessons in the recommended books not being sufficiently integrated.

The aforesaid views came from those who responded to the questionnaire, all of whom are regarded as experienced people. Their opinions reflect their concern and a feeling of dissatisfaction with the current state of mathematical attainments of primary children.

Question 6

Table (xxi) sets out the information that ensued from the question about whether all Egyptian primary school pupils should be taught basic mathematical skills. It reveals that 57 percent (60 percent) fervently believed that they should. Whilst, in addition, 33.9 percent (35.7 percent) concurred. So with the adjusted frequency 95.7 percent assented that Egyptian primary school pupils should be taught basic mathematical skills

The reasons for this affirmative reply were that:-

(i) Mathematical skills are a fundamental basis for mathematics.

(ii) Mathematical skills help the pupils to cope with and understand their environment.

(iii) Most practical work involves mathematical skills to some extent.

(iv) Commerce - buying and selling - depends on having mastered certain mathematical skills.

(v) Mathematical skills are essential in developing accuracy and speed in arithmetic operations.

(vi) Mathematical skills are necessary to secure the children's ability to use geometrical tools correctly.

(vii) Mathematical skills are required, both to verbalize arithmetical problems and to solve them logically.

(viii) Mathematical skills are needed to be able to extract the necessary information from a school textbook.

(ix) The capacity to read or to know mathematical language, its terminology and various symbols depends on mathematical skills.

(x) Further study of mathematics hinges on mathematical skills acquired earlier.

(xi) Since mathematics is relevant to many fields of knowledge, the pupils should gain mathematical skills, in order to facilitate their progress in other subjects.

(xii) It is particularly necessary to acquire basic mathematical skills at the primary level for those pupils who will only be given primary education.

(xiii) The mathematical skills acquired by the pupils help them to solve their practical problems in daily life with accuracy and speed.

(xiv) Induction and deduction all depend on the individual's mastery of mathematical skills.

(xv) The instruction of mathematical skills gives the pupils self-confidence and a better grasp of mathematics for their further school work.

(xvi) In order to perform various arithmetical operations easily and with accuracy, the pupils need to have a basic understanding of mathematical skills.

The above is the summary of the respondents' views on the importance of mathematical skills in primary school.

Question 7

This question raised the problem of whether or not basic mathematical skills should be taught to primary school pupils throughout the world. The results were as follows:

As is evinced by table (xxii), 51.2 percent (53 percent) strongly believe it should, while 29.8 percent (30.8 percent) acquiesced. In general, 83.8 percent assented that basic mathematical skills should be taught in primary schools all over the world.

Many reasons for the agreement are the same as for the previous question and consequently have already been elaborated. However, it is necessary to put forward some other reasons peculiar to this issue:

(I) This is a modern technological age, and mathematical skills have become increasingly important as they contribute to and improve technology.

(II) They are fundamental for all sciences.

(III) They are the basis for various mathematical operations.

(IV) Mathematical skills are essential for developing mental agility, logical thought processes and better thinking.

(V) Because of the cultural exchanges between countries everyone should possess basic mathematical skills so as to increase the understanding between races.

(VI) In order to master the ability to work imaginatively, basic mathematical skills are necessary.

(VII) Instruments, projects etc, which come from other countries require extensive knowledge of the basic mathematical skills pertaining to that country if they are to be of use.

(VIII) It is essential to train a new generation to be able to handle scientific thinking correctly.

Question 8

This question dealt with the issue of whether basic mathematical skills are of the same level everywhere. The results are given in table (xxiii).

As table (xxiii) corroborates, 41.3 percent (44.2 percent) definitely believe that they are, added to which 35.5 percent (38.1 percent) think that they are. Thus 82.3 percent concurred that the basic mathematical skills are the same everywhere.

Question 9

Here the respondents are asked whether there are any differences between mathematical skills and basic mathematical skills.

Here 63.6 percent assented that there are differences between basic mathematical skills and mathematical skills (see table (xxiv)).

The differences given were the following:

The basic mathematical skills are a bedrock for mathematical skills. Hence it is necessary to acquire basic mathematical skills before developing general mathematical skills.

Basic mathematical skills are imperative from the start to build up all mathematical skills and to establish mathematical concepts.

The four basic arithmetical rules constitute a minimum level of arithmetic which help the individual to perform successfully in life, and to acquire basic mathematical skills which are a subset of mathematical skills in general.

The straightforward application of the arithmetical operations and the ability to extract results by addition, subtraction, multiplication, and division requires basic mathematical skills, whereas thinking, abstraction, logic, and the ability to translate relationships between various situations into mathematical language, depend on mathematical skills in general.

Addition, subtraction, multiplication, division, simple numerical manipulation, common denominator, common divisor, squares, cubes and roots require basic mathematical skills. However basic mathematical skills along with transference skills, mathematical modelling, applied skills, and problem solving skills all constitute general mathematical skills. Mathematical skills are built upon basic ones since mathematical skills are wider than just the basic skills.

All fields of life require basic mathematical skills. However mathematical skills in general may only be applicable in special cases.

<u>Question 10</u>

This question asked whether transferable skills are an important aim in primary mathematics teaching.

Table (xxv) establishes that the majority of those answering, acquiesced that transferable skills are a major objective. They represented 46.3 percent (54.9 percent) who strongly concurred, in addition to 28.9 percent (34.3 percent) who agreed. So for the adjusted frequency 89.2 percent acquiesced that this is an important goal.

Question 11:

This question required people to decide whether various stated skills were transferable.

(11 A) Practical skills, as in practical geometry.

(11 B) Reading skills of mathematical language as in reading mathematical symbols and terms and being aware of their meaning.

(11 C) Transferring a table of data to a graph.

(11 D) Expressing a mathematical law, equation or relation in words.

(11 E) Creating a mathematical model of a physical situation in terms of symbols, equations or formulae.

As is substantiated by the results of the tables (xxvi), (xxvii), (xxviii), (xxix), and (xxx) although in all cases approximately 45% of the sample did not respond to these particular questions, the percentage of agreement of those who did was extremely high, at approximately 90%. Looking at the adjusted frequency in table (xxvi), 94.9% assented, 97.1% acquiesced in table (xxvii), 97.1% concurred in table (xxviii), 93.9% agreed in table (xxix), and 90.3% approved in table (xxx). Therefore a high percentage of the responses accept all the items in question eleven.

Question 12:

This question asked whether acquiring procedural skills is an important aim in primary mathematics teaching.

According to table (xxxi), a total of 95.1 percent agreed that this skill is an important aim in primary mathematics teaching.

Question 13:

This question enquired whether various stated skills are procedural skills:

(13 A) Being able to do mechanical addition, subtraction, multiplication.

(13 B) Being able to calculate the square or cube of a number.

(13 C) Finding a common divisor, denominator or common multiple of two numbers.

(13 D) Finding the union or intersection of two sets.

(13 E) Being able to change a number from one base to another (e.g being able to change 12 base ten to a binary number).

(13 F) Being able to do simple four rule examples in multibase arithmetic.

One can gauge from tables (xxxii), (xxxiii), (xxxiv), (xxxv), (xxxvi), and (xxxvii) by looking at the adjusted frequencies that the case for procedural skills and its subcategories has a high percentage of agreement. The percentages are 98.9%, 96.2%, 97.5%, 94.9%, 85.3%, and 96.2 percent respectively.

Procedural skills which were added by respondents

In addition to the list of procedural skills provided in the questionnaire, the respondents mentioned the following:

The aptitude of utilizing various units of measurements and the relationships among them such as lengths, weights, currency and reading time.

The ability to perform some arithmetical operations orally or mentally and reaching their answers or solutions faster when not writing out the calculations.

The faculty of differentiating the dissimilarities in terms of all numbers (less than and greater than).

Finding of percentages and interest rates.

Calculation or finding of volumes and areas of certain shapes in the environment.

Question 14:

This question examined whether acquiring applied skills is one of the main targets in primary mathematics teaching.

According to table (xxxviii), 92 percent think it is.

Question 15:

This question requested people to decide whether various stated skills were applied skills:

(15 A) Direct application of mathematical laws or relations (e.g using the formula for area of a rectangle).

(15 B) Expressing a variable as a combination of other variables (e.g the perimeter of a rectangle is the sum of the lengths of the four sides).

As can be discovered from tables (xxxix) and (xxxx), the percentage of agreements about the subcategory of applied skills were (97.4%) and (87.3 perecent) respectively.

Applied skills which were added by respondents

In addition to the list of applied skills referred to in the questionnaire, the respondents mentioned the following:

Practical application of arithmetical and geometrical data in life relies on applied skills.

The ability to distinguish between and compare various shapes found in the environment calls for applied skills.

The faculty to employ geometrical lessons in other subjects such as agricultural or arts lessons etc, requires the acquisition of applied skills.

The ability to investigate the environment's problems and its needs, and to keep trying to overcome these problems makes demands of applied skills.

Deduction and induction of geometrical relations, such as finding the unknown angle by knowing two angles of a triangle, along with understanding the relation between the triangle's sides and its angles necessitates applied skills.

Question 16:

This question studied whether acquiring logical skills is an important aim in primary mathematics teaching.

As can be seen from table (xxxxi), there was a large percentage of agreements - 92.6 percent - if the missing data was ignored.

Question 17:

This question investigates whether the following skills are logical skills:

(17 A) Proof of simple geometrical or algebraic relation (e.g if a=b and b=c then a=c).

(17 B) Being able to solve a problem by various methods (e.g being able to decide the longest side of a triangle by measurement of length or realizing it is opposite the largest angle).

(17 C) The faculty to use an inversion process (e.g solving 5- ?=3 by calculating 5-3).

As can be seen from tables (xxxxii), (xxxxiii) and (xxxxiv), although the percentage of non-replies exceeded any other category, still quite high percentages of those who replied agree, 93.6%, 98.4%, and 100 percent if the missing data is omitted.

<u>Logical skills which were added by respondents</u>

In addition to the given list of logical skills provided in the questionnaire, the respondents mentioned the following:

The capacity to define the problem and to try and solve this problem by using what is known, involves logical skills.

The technique of answering why and how a particular answer and process is chosen to solve a problem depends upon logical skills.

The aptitude of consciously investigating the relationships between the results and what is given deliberately to order the solution of a problem to achieve the aim, requires logical skills.

The ability to deduce the necessary results of a particular issue in the light of what is given, implies logical skills.

Comparison between various areas at school or at home presupposes logical skills.

To solve the following relations would require logical skills:-

(i) If $A > B$ and $B > C$ Then $A > C$.

(ii) If $X = 3$ Then $X*X = 9$.

(iii) If $3x + 5 = 29$ Then $x = 8$.

(iv) All sides of a triangle are equal if and only if its angles are equal.

<u>Question 18</u>:

This question explored whether acquiring problem solving skills is a fundamental objective in primary mathematics teaching.

As can be shown from table (xxxxv), a total of 86 percent assented that acquiring problem solving skills is an important aim in primary mathematics teaching (if the missing data is removed from consideration).

Other reasons given by the respondents why the acquisition of problem solving skills is of crucial importance to primary mathematics teaching were:

Pupils are given the confidence and know-how to behave properly in different situations.

The Mastery of problem solving skills may help to reform or adjust beneficially the personality of pupils.

Problem solving skills basically exercise the higher mental operations of pupils such as thinking criticizing, creating, reforming, and concentrating, investigating, analyzing, accurate observation and aid correct organization of scientific thinking.

Acquiring the acceptable moral and social attitudes.

By becoming acquainted with problem solving skills, an individual becomes able to cope with the present era. It may well lead to mathematics becoming a favourite subject.

Being able to solve practical problems creates a greater self confidence and sense of responsibility.

Having mastered some problem solving skills, children are better motivated towards mathematics as a subject, since they are able to overcome a problem and obtain the satisfaction of having done so.

The assimilation of problem solving skills enables the individual to increase his capacity for thinking and increases the ability to use numbers in expressing himself.

As a result of acquiring problem solving skills along with increased mathematical ability, pupils are better able to understand their environment.

Children's natural inquisitiveness is more readily satisfied if problem solving skills have been learnt.

Question 19:

This question inquired whether the following skills are problem solving skills:

(19 A) Being able to establish the relationship between given data and the required result.

(19 B) Being able to arrive at a general result from specific examples.

(19 C) Solving a problem which puts to the test several skills.

(19 D) Solving non-standard problems.

(19 E) Being able to formulate a simple scientific problem in mathematical terms (e.g establishing the relation between the height a ball from which is dropped and the rebound height).

(19 F) Being able to use mathematical theories in solving mathematical or non-mathematical issues.

It can be seen from tables (xxxxvi), (xxxxvii), (xxxxviii), (xxxxix), (l), and (li) that, although a large number of the sample did not answer the previous question, among those who did, the percentage of agreement was extremely high. The percentages of agreements about problem solving skills and its subcategory were (97.4%, 95.2%, 90%, 91.9%, 88.9%, and 94.9%) respectively.

<u>Problem solving skills which were added by respondents</u>

In addition to the list of problem solving skills enumerated in the questionnaire, the respondents mentioned the following:

(i) Practical experience through the use of sensory aids (educational aids).

(ii) Skill in guessing.

(iii) The understanding of scientific models.

(iv) Solving some puzzles such as, if a number and its half and its quarter equal 14, what is the number ?.

(v) The skill in solving mental problems.

(vi) Making sure of results by using different methods.

<u>Question 20 A</u>:

This puts forward the statement that the basic mathematical skills are well covered in the scheme of mathematics used in your school.

As can be seen from table (lii), there is an indication here that many teachers do not think that their present mathematics schemes cover all of the mathematical skills. In fact, the percentage of disagreement accounted for as many as (46.4%) forty six point four percent of the sample. The percentage of agreement, on the other hand, accounted for only about (21.4) twenty one point four percent if the missing data is omitted.

<u>Question 20 B</u>:

This question asked whether these skills are covered by:

(i) Discovery learning. (iii) Practical approaches.

(ii) Problem solving. (iv) Programmed learning approaches.

(v) Other learning methods (please state).

Code 1- Discovery learning. Code 2- problem solving.

Code 3- Practical approaches. Code 4- Programmed learning approaches.

Code 5- First two. Code 6- First three.

Code 7- All. Code 8- 1+ 3.

Code 9- 1+ 4. Code 10- 2+3.

Code 11- 2+4. Code 12- 3+4.

Code 13- 1+3+4. Code 14- 2+3+4.

Code 15- Other learning methods. Code 99- Missing data.

As can be ascertained from table (liii), looking at the adjusted frequency, some respondents tend to prefer discovery learning in teaching concepts and skills. They made up 16.3 percent. A number of respondents show a preference for the use of all previous methods to cover all skills. They formed 15.2 percent if the missing data is neglected. Several respondents favour practical approaches, representing 10.9 percent. A majority of people are inclined to opt for problem solving or the three latter methods or the former three methods. They represent 8.7%, 8.7%, or 7.6 percent respectively. A few people are inclined to a programmed learning approach. They represent 2.2 percent.

Some points of view which were added by respondents and which may help in acquiring these skills are:-

(i) Collective work such as in the school canteen, going on trips, etc.

(ii) Using educational aids.

(iii) The encouragement of the pupils' extra curricular and class activities.

(iv) Discussion groups.

(v) Visits to the surrounding area.

(vi) Self learning.

(vii) Trial and error.

(viii) Individual teaching.

Question 21:

This question considered whether the basic mathematical skills are best covered in a :-

(i) Modern syllabus.

(ii) Traditional syllabus.

(iii) Mixed syllabus.

Code 1- Modern syllabus. Code 3- Mixed syllabus.

Code 2- Traditional syllabus. Code 9- Missing data.

As can be established from table (liv), a larger percentage of respondents approved of a mixed syllabus rather than any other kind of syllabus. In fact, mixed syllabi were favoured by as many as seventy two percent of respondents. Modern syllabi and traditional syllabi, on the other hand, accounted for only 21.3 percent and 6.5 percent respectively.

Question 22:

This propounded that to cover the basic mathematical skills it is best to use

(I) Blackboard, chalk, talk.

(II) Structural apparatus.

(III) The environment.

(IV) Other equipment (please list).

Code 1- Blackboard, chalk, talk. Code 5- 1+ 3.

Code 2- Structural apparatus. Code 6- 2+ 3.

Code 3- The environment. Code 7- 1+ 2+ 3.

Code 4- 1+ 2. Code 8- Other equipment.
Code 9- Missing data.

According to table (lv), most respondents tend to prefer using all these elements in teaching; they represented 41.2 percent. Then come those who thought that these skills should be covered by using blackboard only; they reresented 15.7 percent if the missing data is deleted. Finally come those who thought that the basic skills were best covered by the blackboard in addition to the environment. They represented only two percent if the missing data is left out.

Question 23:
The basic mathematical skills are well learnt and understood by primary school children using current teaching methods.

It can be seen from table (lvi) that a larger percentage of respondents differed and strongly demurred than agreed. In fact, disagreements accounted for as many as sixty four percent of the respondents. Agreements, on the other hand, accounted for only about nine percent. There is an indication here that many respondents do not think that their current teaching methods promote a better understanding of all of the mathematical concepts and skills.

Question 24: Basic mathematical skills are well covered in initial teacher training courses.
As indicated in table (lvii), 95 percent concurred that basic mathematical skills are well covered in initial teacher training courses.

A SUMMARY OF THE EGYPTIAN QUESTIONNAIRE'S RESULTS

On the positive side, it can be established from the tables that there were some similarities in the responses given and differences in others. For example table (xix), which evinced that acquiring basic mathematical skills is an important aim in teaching mathematics in primary school, and table (xxi), which illustrated the opinion that all Egyptian primary school pupils should be taught basic mathematical skills, are similar in that they both have a high percentage of agreement from those answering, since the adjusted frequencies were 100% and 95.7 percent respectively. Also table (xxii) which points to the necessity of teaching basic mathematical skills for primary school pupils throughout the world, and table (xxiii) for the basic skills being the same everywhere, are very similar in that they both have almost the same percentage of respondents' agreement where the cumulative frequencies were 83.8% and 82.3 percent respectively. Furthermore, with regard to the tables (xxv), (xxxi), (xxxviii), (xxxxi) and (xxxxv) which prove, the importance of acquiring transferable, procedural, applied, logical, and problem solving skills in primary mathematics teaching, there is a correlation in that they all have approximately the same percentage of respondents, the cumulative frequencies being 89.2%, 95.1%, 92%, 92.6%, and 86 percent respectively. Although the first and last percentages are slightly lower than the others, there is still a high degree of correspondence. An additional factor shown by the tables was that the subcategory of every main classification of skill, had about the same percentage of agreement. In the case of transferable skills and its subcategories, practical skills, reading skills in mathematical language, transferring a table of data into a graph, expressing a mathematical law (equation or relation) in words and creating a mathematical model of a physical situation, were found to be related to transferable skills since they have a similarly high percentage of respondents of 94.9%, 97.1%, 97.1%, 93.9% and 90.3 percent respectively.

This table also reflects the fact that the percentage of agreement between respondents for the subcategories of transferable skills are quite high, more than 90%. Similarly, there is a high percentage of respondents in the categories of other skills such as those procedural skills, of being able to do the four mechanical arithmetical rules, calculate the square or cube of a number, find a common divisor, denominator or common multiple of two numbers, find the union or intersection of two sets, change a number from one base to another, and do simple four rule examples in multibase arithmetic; these have a very similar high percentage of respondents of 98.9%, 96.3%, 97.5%, 94.9%, 85.3%, and 96.2 percent respectively. Also, of course, the supposed subcategory of applied skills, such as direct application of mathematical laws, expressing a variable as a combination of other variables, as well as the subdivision of logical skills, such as proof of a simple geometrical or algebraic relation, solving a problem by various methods and using an inversion process, obtained about the same high percentage of agreement among respondents of 97.4%, 87.3%, 93.6%, 98.4% and 100 percent respectively. Similarly comes the suggested subcategory of problem solving skills such as establishing the relationship between given data and the required result, arriving at a general result from particular examples, solving a problem which requires several skills, solving a non-standard problem, formulating a simple scientific problem in mathematical terms and using mathematical methods in solving mathematical or non-mathematical issues, which got a high percentage of respondents' agreement in every case, too, of 97.4%, 95.2%, 90%, 91.9%, 88.9% and 94.9 percent respectively. Thus it is quite easy to investigate the similarities in these tables.

On the other hand, with regard to the percentage of agreement between respondents, the above tables reached high percentages which are quite different in this respect from the following tables which showed a lower percentage of agreement with regard to questions on the satisfaction with the level of mathematical skills reached by the normal primary school child. There are differences between mathematical skills and basic mathematical skills. Basic mathematical skills are well discussed in the scheme of mathematics used in most schools, and basic mathematical skills are well learnt and understood by the majority of primary children using current teaching methods. These scored lower percentages of agreement rather than the former questions since they got only 28.2%, 63.6%, 21.4% and 9.8% respectively.

This brief discussion of the correspondence of views between respondents indicates that while there is a low percentage of agreement concerning satisfaction with the level of mathematical skills reached by the normal school child and the methods used in teaching in primary schools now, the large measure of agreement concerning the necessity of acquiring basic mathematical skills suggests recognition of the fact that such skills are vital to success in virtually all subjects and also to life in general. There was an agreement of about 90% with classification of skills which had been suggested by the researcher. It should also be mentioned here that there were some skills in every category which could be added and they were taken into consideration in reformulating the final list of mathematical classification and definitions of skills which are dealt with in the next chapter (chapter seven). They were also considered in designing the tests of evaluating these skills. The general feeling of those questioned was that the present level attained by children and the methods used in teaching mathematics in primary school were not satisfactory. The respondents mentioned the reasons for dissatisfaction, and through analyzing the responses it appeared that the present methods of teaching mathematics were traditional (blackboard and chalk).

There was no attempt at using concrete objects at the initial stages to build-up concepts and skills correctly along with clear understanding. All efforts starting with abstract concepts ended up by not being understood.

It should be mentioned here that KENDALL Correlation Coefficients was used to see the relationships among variables.

<u>A summary of KENDALL Correlation Coefficients</u>

There are positive correlation coefficients between the level of qualification and each of the following; the percentage of agreements about differences between mathematical skills and basic mathematical skills; the percentage of agreements about transferrable skills and its category are an important aim in primary mathematics teaching; the percentage of agreements about the necessity of acquiring procedural skills and its category; the percentage of agreements about acquiring logical skills is an important aim in primary mathematics teaching.

Generally speaking, the higher the

qualification, the more opportunity to have a good job; the higher the status, the more agreements there are about differences between mathematical skills and basic mathematical skills, and these classifications of skills and the importance of this category in primary mathematics teaching. On the other hand there are negative correlation coefficients between qualification and work experience in primary school.

There are negative correlation coefficients between qualification and satisfaction with the level of mathematical skills reached by the normal primary school children, in other words, the higher the qualification which the individual has the more he felt that the level was unsatifactory. The majority of responses about differences between mathematical skills in general and basic mathematical skills came from people who have high qualifications. People who are highly qualified accept the classification of skills and its categories along with their views for some modifications and suggestions. They consider these skills to be an important feature in teaching mathematics in primary school.

With regard to work experience, there are positive coefficient correlations between work of experience and satisfaction with present schemes and methods of teaching. People who have worked for a short time are dissatisfied with the present systems and present methods of teaching. The significance was 0.05.

More illustrations and simple explanations for the results of this questionnaire will be reviewed in the following charts.

Chart 1: shows the percentage of agreement among respondents about the importance of mathematical skills in teaching primary school mathematics. The vertical axis shows the percentage of agreement among respondents in the different types of classification. The horizontal axis shows the classification of skills (compares five types of skills).

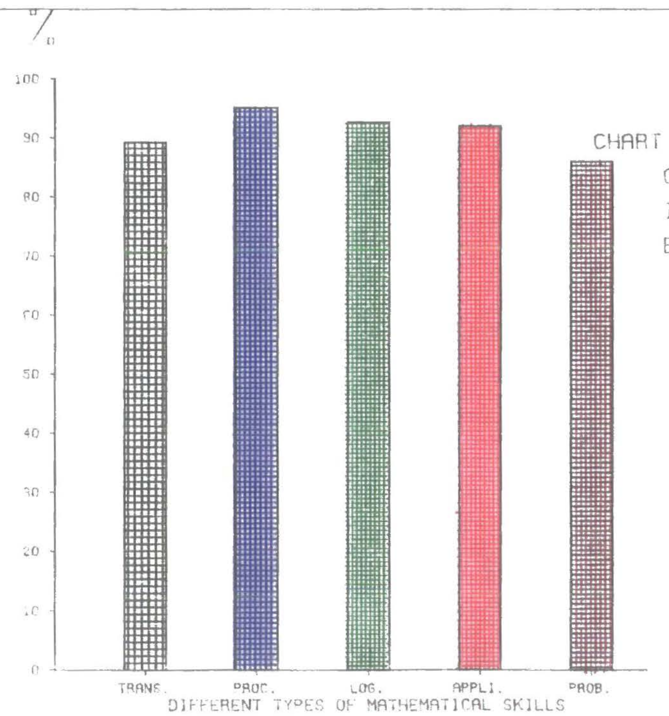


CHART 1: THE CORRESPONDENCE IN RESPONSES, CONCERNING THE IMPORTANCE OF SKILLS IN TEACHING PRIMARY MATHEMATICS, IN EGYPT EXPRESSED AS A PERCENTAGE

Chart 2: shows the percentage of correspondence in the different categories of the first kind of classification of skills (transferable skills). The vertical axis shows the percentage of agreement among respondents in the different types of transferable skills. The horizontal axis compares five types of transferable skills.

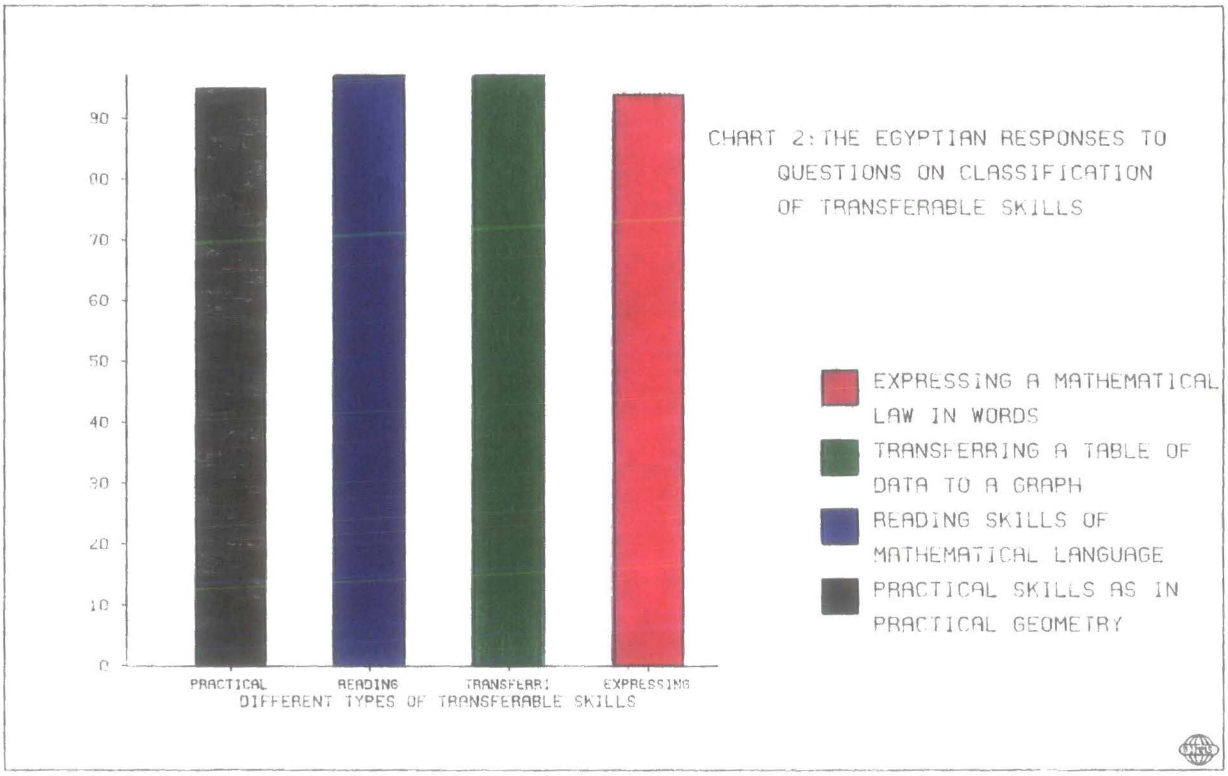


CHART 2: THE EGYPTIAN RESPONSES TO QUESTIONS ON CLASSIFICATION OF TRANSFERABLE SKILLS

Chart 3: shows the percentage of correspondence in the different category of the second type of classification of skills (procedural skills). The vertical axis shows the percentage of agreement among respondents in the different types of procedural skills. The horizontal axis compares six types of procedural skills.

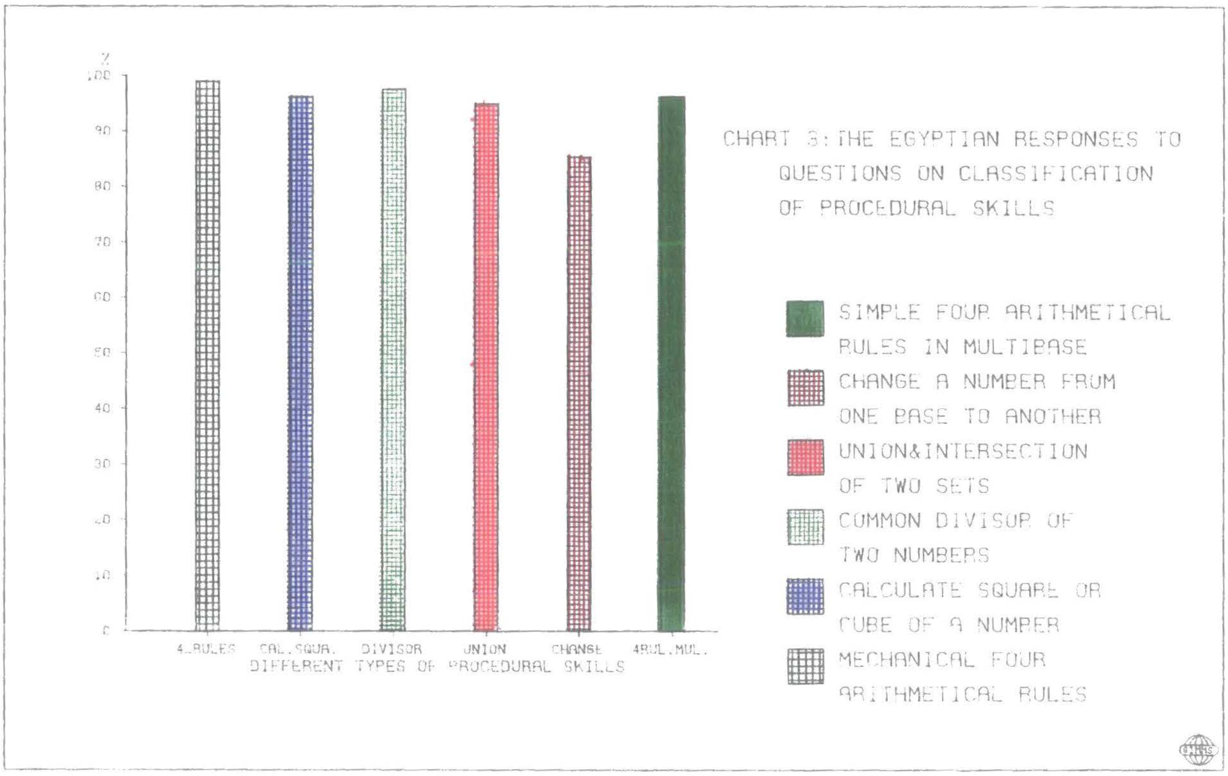


CHART 3: THE EGYPTIAN RESPONSES TO QUESTIONS ON CLASSIFICATION OF PROCEDURAL SKILLS

Chart 4: shows the percentage of correspondence in the different types of the third category of classification of skills (logical skills). The vertical axis shows the percentage of agreement among respondents in the different types of logical skills. The horizontal axis compares three types of logical skills.

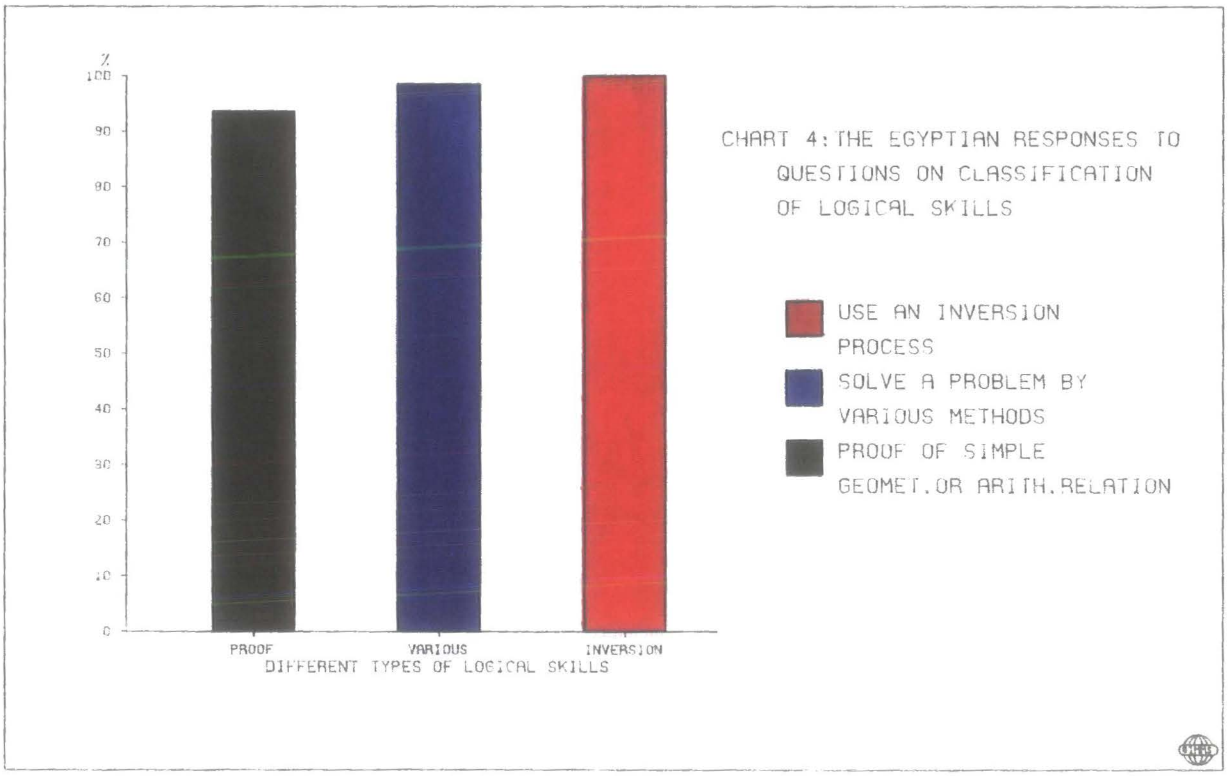

%
100
90
80
70
60
50
40
30
20
10
0
PROOF
VARIOUS
INVERSION
DIFFERENT TYPES OF LOGICAL SKILLS
CHART 4: THE EGYPTIAN RESPONSES TO QUESTIONS ON CLASSIFICATION OF LOGICAL SKILLS
USE AN INVERSION PROCESS
SOLVE A PROBLEM BY VARIOUS METHODS
PROOF OF SIMPLE GEOMET. OR ARITH. RELATION

Chart 5: shows the percentage of correspondence in the different types of the fourth category of classification of skills (applied skills). The vertical axis shows the percentage of agreement among respondents in the different types of logical skills. The horizontal axis compares two types of applied skills.

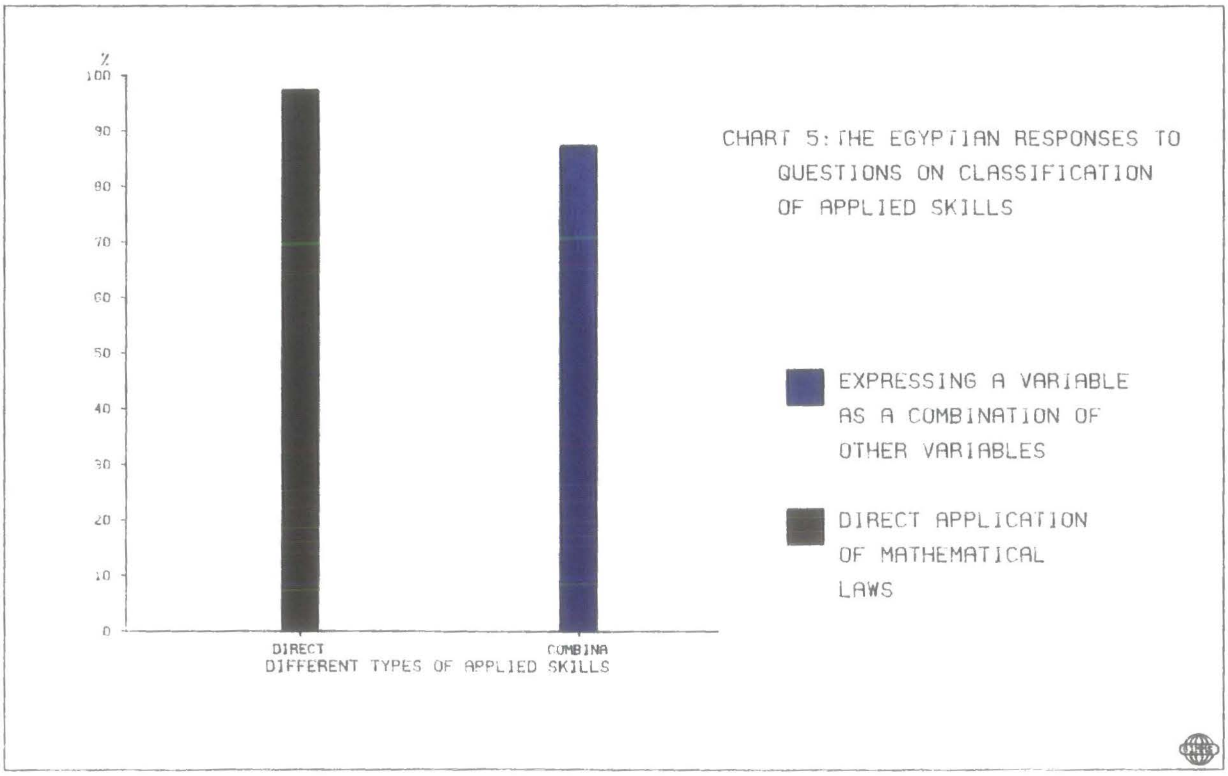

%
100
90
80
70
60
50
40
30
20
10
0
DIRECT
COMBINA
DIFFERENT TYPES OF APPLIED SKILLS
CHART 5: THE EGYPTIAN RESPONSES TO QUESTIONS ON CLASSIFICATION OF APPLIED SKILLS
EXPRESSING A VARIABLE AS A COMBINATION OF OTHER VARIABLES
DIRECT APPLICATION OF MATHEMATICAL LAWS

Chart 6: shows the percentage of correspondence in the different types of the fifth category of classifications of skills (problem solving skills). The vertical axis shows the percentage of agreement among respondents in the different types of problem solving skills. The horizontal axis compares six types of problem solving skills.

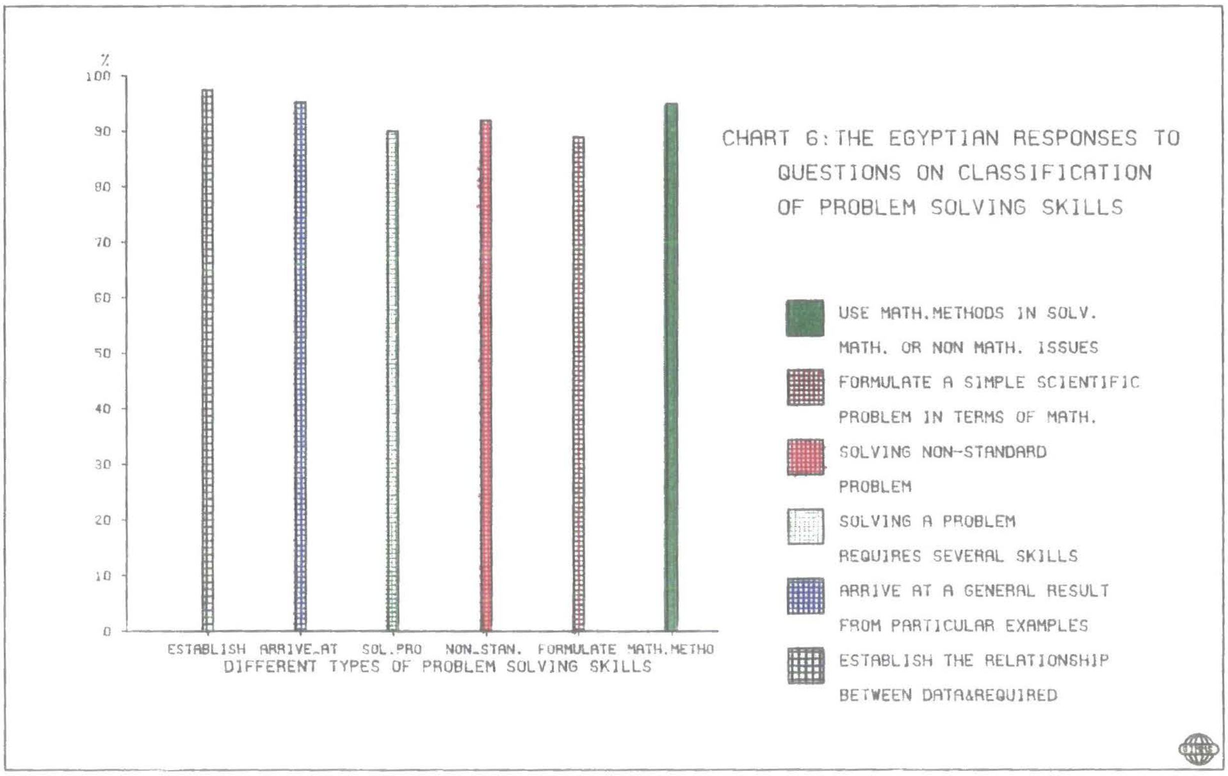


CHART 6: THE EGYPTIAN RESPONSES TO QUESTIONS ON CLASSIFICATION OF PROBLEM SOLVING SKILLS
%
100
90
80
70
60
50
40
30
20
10
0
ESTABLISH
ARRIVE_AT
SOL.PRO
NON_STAN.
FORMULATE
MATH.METHO
DIFFERENT TYPES OF PROBLEM SOLVING SKILLS
USE MATH.METHODS IN SOLV. MATH. OR NON MATH. ISSUES
FORMULATE A SIMPLE SCIENTIFIC PROBLEM IN TERMS OF MATH.
SOLVING NON-STANDARD PROBLEM
SOLVING A PROBLEM REQUIRES SEVERAL SKILLS
ARRIVE AT A GENERAL RESULT FROM PARTICULAR EXAMPLES
ESTABLISH THE RELATIONSHIP BETWEEN DATA&REQUIRED

CHAPTER SIX

PART TWO

ENGLISH RESULTS OF THE QUESTIONNAIRE

The terms opinion, attitude, belief have no specific meanings in literature: generally they refer to a person's adherence to one side of a question rather than to another. Opinions, attitudes and beliefs are rational and/or emotional judgments on such questions. They differ from each other in their generality or in the intensity with which they are held:- opinions commonly refer to topical short-term judgments, attitudes are somewhat more enduring and all-embracing, whilst beliefs are more basic and have to do with central values. In other words, opinions are concerned with impressions, attitudes with views or convictions, and beliefs are sometimes called values or sentiments. There are however, no hard and fast rules about the definitions of the terms, so that one man's opinion may be another's attitude and yet another's belief.

OPINION-ATTITUDE QUESTIONNAIRE

The English questionnaire included twenty four questions, four of which involved personal questions, about the respondents' jobs, qualification, years of work, and sex. Fourteen of the questions involved selecting an answer which corresponded more or less to their own view or opinion: on the importance of acquiring mathematical skills; the level of mathematical skills reached by the normal primary school child, and if this is not satisfactory, to state why; on whether all English primary school children should be taught basic mathematical skills; on whether they should be acquired by all primary school children in the world, and whether these skills are the same everywhere; and on the differences between basic mathematical skills and mathematical skills. These were then followed by questions about the importance of every classification of skills; Transferable; Procedural; Logical; Applied; and Problem solving skills in the aims of primary school teaching; about whether the various stated skills in the questionnaire were under the correct classification, with the opportunity given to add others, delete, or comment; on whether the basic mathematical skills are well covered in the scheme of mathematics used in the schools; concerning the methods used in teaching mathematics; whether mathematical skills would be well covered by modern syllabuses, traditional syllabuses, or a mixture of both; asking if the best materials could be used to obtain better achievement in these skills at that young age, when children need more activity with concrete objects; on whether the current methods of teaching in primary schools are powerful enough to acquire them beneficially; and if these skills are well covered in the initial teacher training courses.

The SPSS programme in MTS was used in order to analyze the English questionnaire's results as well as in the Egyptian one. The computer programme's data was prepared by coding for jobs, qualifications, years of work, sex, and for the various questions in the questionnaire.

The statistical analysis of each question including absolute frequency, relative frequency, adjusted frequency, cumulative frequency, histograms, mean, mode, minimum, maximum, standard deviation, median, range, variance was calculated for the most part in the program. There is, however, no attempt to display these results in detail as they appeared in the computer output, so that concise and precise conclusions would be produced.

GENERAL RESULTS OF THE PERSONAL QUESTIONS

The results of the four personal questions were as follows: the respondents were mostly primary teachers, who represent 72.1 percent; head teachers in primary schools who account for 19.0 percent; primary teachers with responsibility for mathematics, totaling 6.1 percent; then comes the smallest percentage of all the sample of 180 persons; lecturers in the university Education Department who form 1.1 percent; lecturers in methods of teaching mathematics in a school of Education university who also make up 1.1 percent; and Psychologists in a University at Education Department who add up to 0.6 percent.

The qualification was the second personal question; thirty seven point nine of the respondents had O-level, followed by those having O-level along with teaching certificate, representing thirty three point three percent; next those with O-level and A-level qualifications and a teaching certificate, constituting eleven point three percent; and lastly come those who are qualified with a university degree with a P.G.C.E, or teaching certificate, representing ten point two percent, and one point one respectively.

The work experience of the respondents was as follows: in descending order; the majority of answers came from those who had worked for a period of 16-20 years in primary schools, with twenty eight point one percent; then those who had worked for a period of 6-10 years who added up to twenty six point four percent; and those who had worked for 11-15 years, of whom there were twenty five point eight percent; the smallest percentages of respondents were included under the following year ranges; 21-25 years, nine percent; 26-30 years, six point two percent; 31-35 years, two point eight percent; and over 35 years, point six percent .

Forty six point nine percent of the sample were male and fifty three point one percent were female.

General Attitude to Importance of acquiring Mathematical Skills

Question 4, in the second part of the questionnaire, considered to what extent they thought that: mathematical skills are a major element in primary school curriculum. They then were asked to select one of the following answers: strongly agree(SA), agree(A), undecided(U), disagree(D), or strongly disagree(SD).

Most of the respondents strongly agreed, accounting for eighty six point seven percent, with eleven point seven percent agreeing.
It is worth pointing out that nobody expressed the view that they are not important and one point seven percent did not answer.

Level of Mathematical Skills Reached by the Normal School Child is satisfactory.

The results showed quite clearly that only a very small minority of the respondents were "not satisfactory", since the percentage of those who demurred was six point seven, with one point one percent strongly differed. The majority of the respondents were "satisfactory": 11.2% strongly assenting, and 62.0% concurring, they consider that the attainment of a primary school child in mathematical skills is satisfactory, whilst 19.0% remained undecided in this matter.

<u>Fundamental Mathematical Skills are the Same Everywhere and Should Be Taught to Primary School Pupils throughout the World</u>.

The answers of the respondents have been analysed as follows. It will be seen that the majority of them tend to acquiesce with the statement, with those who strongly agree with the first part representing 11.1%, 48.8% assenting, as against 13.3% who demurred and 1.7% who strongly differed. On the second part, 33.0% strongly acquiesce, 54.2% agree, as against 3.4% differing and 1.7% strongly disagreeing. Surprisingly there was a proportion of disagreement among those respondents. It was to be expected that as a result of the importance of fundamental mathematical skills in primary school aims, certain ideas would be developed and generated, and nobody would disagree with these skills being taught all over the world. Generally speaking, it is evident that those respondents were found the matter under investigation interesting as they had reflected on what they themselves had learnt and experienced in primary school mathematics. However, the need for revising the standard definition of skills was clearly explained to those who were going to answer the questionnaire.

<u>Are Transferable, Procedural, Logical, Applied, and Problem Solving skills an important aim in Primary Mathematics Teaching</u>?

<u>Is This List of Attached Examples of These Skills Correct</u>?

When they were questioned about the importance of acquiring these mentioned skills in the aims of teaching mathematics, 46.7% strongly agreed and 52.8% agreed, against 0.6% disagreeing on transferable skills; 54.7% strongly agreed and (44.7) Concurred with (0.6) Undecided on procedural skills; 52.0% strongly acquiesced and 44.6% agreed against 2.3% demurred and 1.1% undecided on logical skills; 38.8% strongly concurred and 53.9% assented against 2.8% disagreeing, 1.7% strongly disagreeing and 2.8% undecided on Applied skills; and 47.2% strongly acquiesced and 52.2% concurred against 0.6% disagreeing on problem solving skills.

An analysis of their responses to, and written comments on, the attached examples of each classification of the suggested practical definition of skills showed that 31.3% strongly concurred, 67.6% acquiesced against 1.1% disagreeing to practical skills, as in practical geometry; 28.4% strongly concurred, 69.3% assented against 2.3% demurring for reading skills of mathematical language as in reading mathematical symbols and terms and being aware of their meaning; 34.1% strongly agreed, 65.9% agreed on changing a table of data to a graph; 33.5% strongly acquiesced, 63.6% agreed against 0.6% disagreeing and 2.3% undecided to expressing a mathematical law, equation or relation in words; and 33.0% strongly agreed, 63.1% agreed against 1.1% differing and 2.8% undecided on thinking about a mathematical model of a physical situation in terms of symbols, equations or formulae.

The results of the responses about the list of examples of procedural skills were 44.6% strongly assented, 55.4% agreed with no objection to being able to do four arithmetical rules as one of them; 43.4% strongly agreed, 52.0% agreed and 4.6% were undecided for being able to calculate the square or cube (power) of a number; 42.3% strongly acquiesced, 55.4% agreed and 2.3% undecided about finding a common divisor, denominator or common multiple of two numbers; 38.9% strongly assented, 54.9% acquiesced and 6.3% undecided where finding the union or intersection of two sets is concerned; 40.6% strongly concurred, 53.7% agreed, 1.1% differed, with 4.6% undecided to being able to change a number from one base to another; and as a final example, 42.9% strongly agreed, 52.6% agreed, 1.1% demurred and 3.4% were undecided as regards being able to do simple four rule examples in multibase arithmetic.

Despite the percentages of agreement being found to be very high, it was decided to examine their comments closely. Revisions and alterations were continuously undertaken in the process of reaching a reasonable and satisfactory list of mathematical skills definitions, as well as the classifications and their examples.

Question 17 asked the respondents whether the attached examples of logical skills are correct, and to give their comments. The following results ensued: 45.7% strongly agreed, 54.3% assented, and there was no demur to the simple proof of a simple geometrical or algebraic relation; 45.7% strongly concurred, 52.0% agreed, 2.3% undecided and no objection on being able to think by various methods when facing a problem; 50.9% strongly agreed, 47.4% agreed, 1.2% who were undecided, and 0.6% disagreed to being able to rationalize and understand an inversion process.

Not all respondents, of course, answered the question concerned with their opinions about the enclosed examples of applied skills, but the results showed quite clearly that only a very small proportion of the respondents thought the second example had no place in applied skills. The majority of them, 64.2%, agreed, 35.8% strongly agreed, and no objection occurred on direct application of mathematical laws or relations as an example of applied skills; 61.3% agreed, 38.2% strongly agreed, with 0.6% disagreement about expressing a variable as a combination of other variables as another example. Respondents who worked through the questionnaire conscientiously gave their opinions on various responses.

The question concerned with responses to examples of the final classification of Mathematical Skills, which is Problem Solving Skills, found that 48.3% strongly concurred, 50.0% agreed, and 1.7% were undecided on being able to establish the relationship between given data and the required result being a problem solving skill; 50.6% agreed, 46.0% strongly agreed, 1.1% strongly disagreed, and 2.3% were undecided in the case of being able to arrive at a general result from particular examples; 49.4% strongly agreed, 43.1% agreed, 1.1% strongly disagreed, and 6.3% were undecided where solving a non-standard problem was concerned; 47.7% strongly agreed, 44.8% agreed, 1.1% strongly disagreed, and 6.3% were undecided when asked about being able to formulate a simple scientific problem in terms of mathematics; 42.3% strongly agreed, 56.6% agreed, with 1.1% undecided, and there was no objection about being able to use mathematical methods in solving mathematical, or apparently non-mathematical problems as examples of that category.

Of the 173 responses which were made to the question concerned with whether the basic mathematical skills are well covered in the scheme of mathematics pursued in their schools, 35.3% strongly agreed, 59.5% agreed, while 1.2% disagreed and 4.0% were undecided.

The majority of respondents, 63.3%, preferred a variety of teaching methods in order to acquire mathematical skills. The 93.3% were inclined to favour a mixture of traditional and modern syllabi, whereas 0.6% favour a modern syllabus only and 5.1% favoured a traditional syllabus.

It was significant that 48.0% concurred, and 3.5% strongly assented that basic mathematical skills are well learnt and understood by primary children when using current teaching methods, while 11.7%

disagreed, 0.6% strongly disagreed, and 36.3% were undecided.

The answers to the last question on the questionnaire, which was concerned with the initial teacher training courses, can be divided up as follows; only 1.1% strongly agreed, 10.9% agreed, while a high percentage of 35.4% disagreed, and 13.7% strongly disagreed, being dissatisfied about the course, with a high percentage of those undecided, namely 38.9%. They also commented that, in an overloaded teacher training programme, there is seldom enough time for a thorough mathematics course, or indeed any opportunity to demonstrate that teaching methods in mathematics is a worthwhile intellectual study.

Generally speaking, the questionnaire was well received by the majority of respondents taking part in the sample. Indeed a high percentage of respondents displayed their appreciation by expressing their personal opinions in their answers.

From this analysis of the responses made in connection with acquiring skills, one can safely conclude that the present methods of teaching mathematics do not build up a sufficient awareness of the problem of helping pupils to acquire mathematical skills and prevent them from making many absurd errors.

It may be helpful to draw a general picture of the respondents' point of view, expressed in their answers to the open ended questions. There are many reasons for their dissatisfaction at the level of mathematical skills reached by primary school children. One of the principle causes is the confusion resulting from metric and imperial measures. Another is attempting work outside the capacity of young children. In addition, there is a lack of practice in basic mathematical work. Furthermore, the new trends in education did not affect school work.

British primary school children should be taught mathematical skills for many reasons; children need to be provided with a sound basis from which to develop more advanced mathematical techniques; mathematical skills are essential for life and are applicable to other science subjects; numeracy is as important as literacy; as a basis for further study in education; it is essential for carrying out future complicated tasks; even those children who do not develop into "mathematicians" require basic number skills in order to cope with the arithmetic of daily life (e.g working out change in shops, calculating room measurement when re-decorating, shopping, determining wages, paying for bills, household accounts etc); moreover, a child will have extremely limited employment prospects without these skills. However, if he has mastered them, it will give him confidence for further work, especially problems; mathematics is increasingly important in the technology so that basic numeracy is even more crucial to the next generation than to those before it.

Children, it was held, should be taught mathematical skills throughout the world because the "language" of mathematics is universal; numeracy is important in all societies throughout the world; it is a basic requirement of any education; everyone needs to be able to manage money and practise economy at some level; a basic knowledge of mathematics amongst politicians might help to solve the world's monetary problems; all children should have equal educational opportunity; to help develop a process of logical thought; to stimulate individual confidence, feelings of competence and self esteem in everyday social interaction with peers; and to provide greater freedom and opportunity of movement, e.g from a rural to an industrial environment, and employment (manual to management).

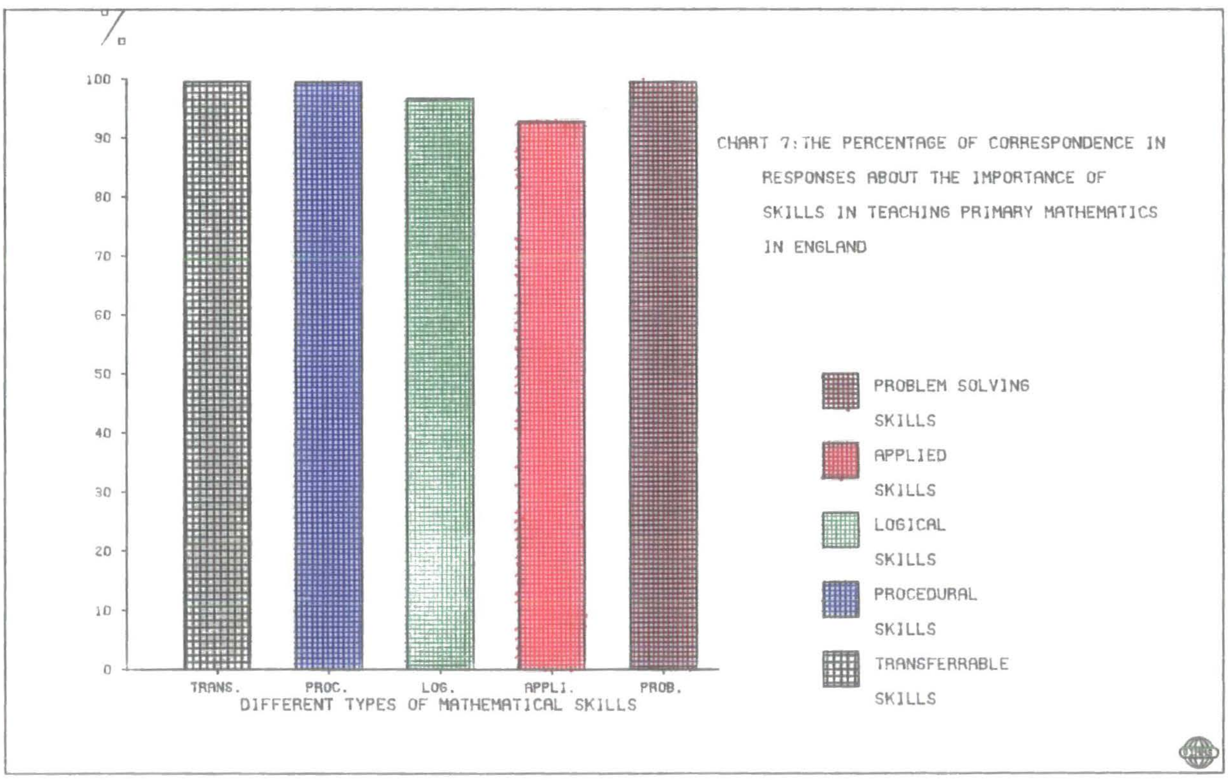
On the other hand, basic skills are fundamental to an understanding of mathematics, whereas mathematical skills involve further development in the learning of equations, formulae, etc; basic mathematical skills are essential to all other skills (e.g simple addition is necessary for addition of fractions and multiplication); basic skills can be taught to children who have little innate ability; mathematical skills depend largely on basic skills; degree of mathematical skill is based upon the specific needs to which it is to be applied; basic skills are fundamental if one is to progress steadily in a subject. Mathematical skills are of a higher standard once the basic skills have been mastered; these will be obtained through practice over a long period; basic skills are only the beginning of mathematical experience; and mathematical skills are an extension of the basic skills, but not every one will be able to achieve this extension.

More skills were added as a transferable skill, such as the ability to use mathematical aids, e.g calculators and computers; and map reading; procedural skills, such as working of percentages, averages, time, the calendar, complement of sets, and identification of geometric shapes; logical skills, such as understanding the meaning of the symbol "if and only if", sequences and series of numbers, being able to understand similar groups, reasoning from available data to formulate a simple theory which can then be tested, and to be able to extrapolate trends from a limited amount of information e.g to continue to plot a line graph beyond the data available; applied skills, such as area of irregular shapes, comprehending the concept "average" and applying it, and constructing figures to a given specification e.g triangles with given sides etc; problem solving skills, such as ability to deduce and propound; knowing how to display open-mindedness, imagination, hypothesis-testing and evaluation; unorthodox methods of using known data; and the degree to which these skills are taught and need to be taught in primary schools.

A more detailed analysis of the answers, however, shows that better qualified respondents in England favoured these definitions of skills and the classifications of mathematical skills. They appreciated this investigation, and they gave more ideas on these definitions, which were helpful in modifying, revising, adjusting, and developing the final list of definitions. Meanwhile, the less qualified tended to be undecided in their answers, which may somehow reflect inability to give a clear opinion.

Chart 7: shows the percentage of agreement among respondents about the importance of mathematical skills in teaching primary school mathematics for England. The vertical axis shows the percentage of agreement among English respondents in the different types of classification.

The horizontal axis shows the classification of skills (compares five categories of skills).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DIFFERENT TYPES OF MATHEMATICAL SKILLS
CHART 7: THE PERCENTAGE OF CORRESPONDENCE IN RESPONSES ABOUT THE IMPORTANCE OF SKILLS IN TEACHING PRIMARY MATHEMATICS IN ENGLAND
PROBLEM SOLVING SKILLS
APPLIED SKILLS
LOGICAL SKILLS
PROCEDURAL SKILLS
TRANSFERRABLE SKILLS

Chart 8: shows the percentage of English correspondence in the different categories of the first kind of classification of skills (transferable skills). The vertical axis shows the percentage of English agreement among respondents in the different types of transferable skills. The horizontal axis compares four types of transferable skills.

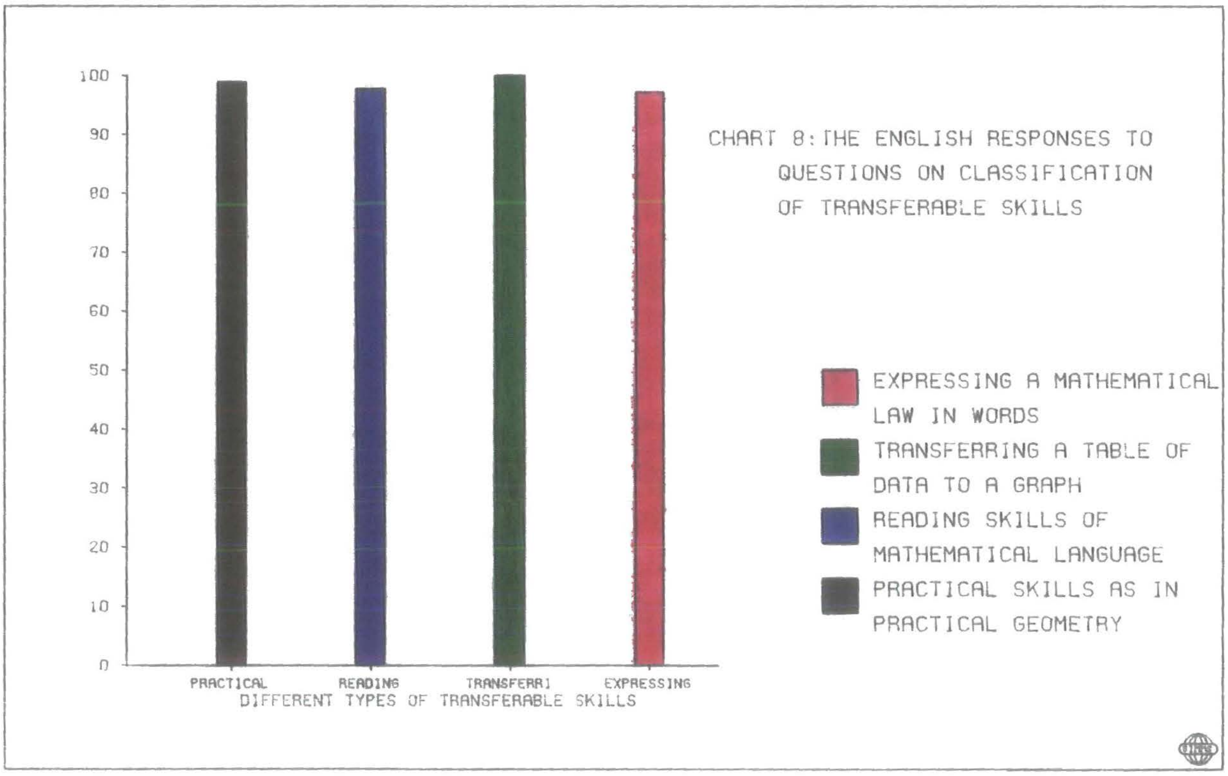
CHART 8: THE ENGLISH RESPONSES TO QUESTIONS ON CLASSIFICATION OF TRANSFERABLE SKILLS
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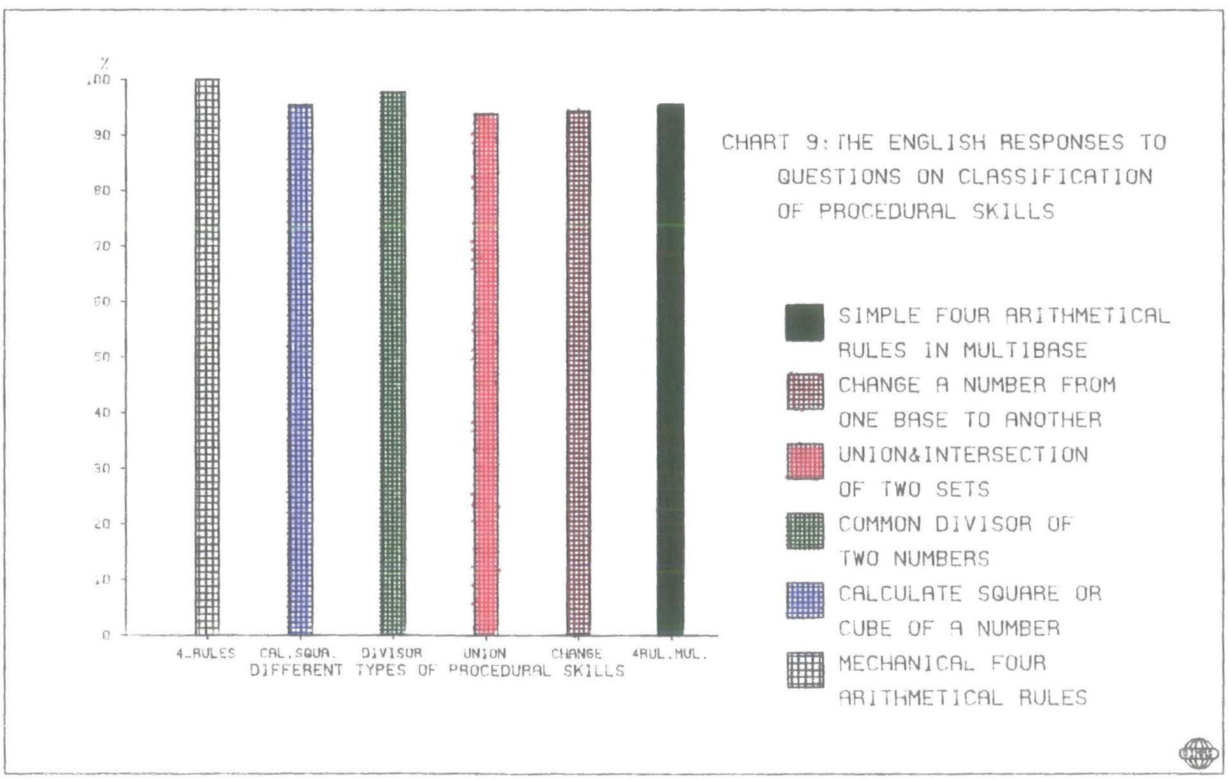
DIFFERENT TYPES OF TRANSFERABLE SKILLS
EXPRESSING A MATHEMATICAL LAW IN WORDS
TRANSFERRING A TABLE OF DATA TO A GRAPH
READING SKILLS OF MATHEMATICAL LANGUAGE
PRACTICAL SKILLS AS IN PRACTICAL GEOMETRY

Chart 9: shows the percentage of correspondence in the different category of the second type of classification of skills (procedural skills). The vertical axis shows the percentage of English agreement among respondents in the different types of procedural skills. The horizontal axis compares six types of procedural skills.



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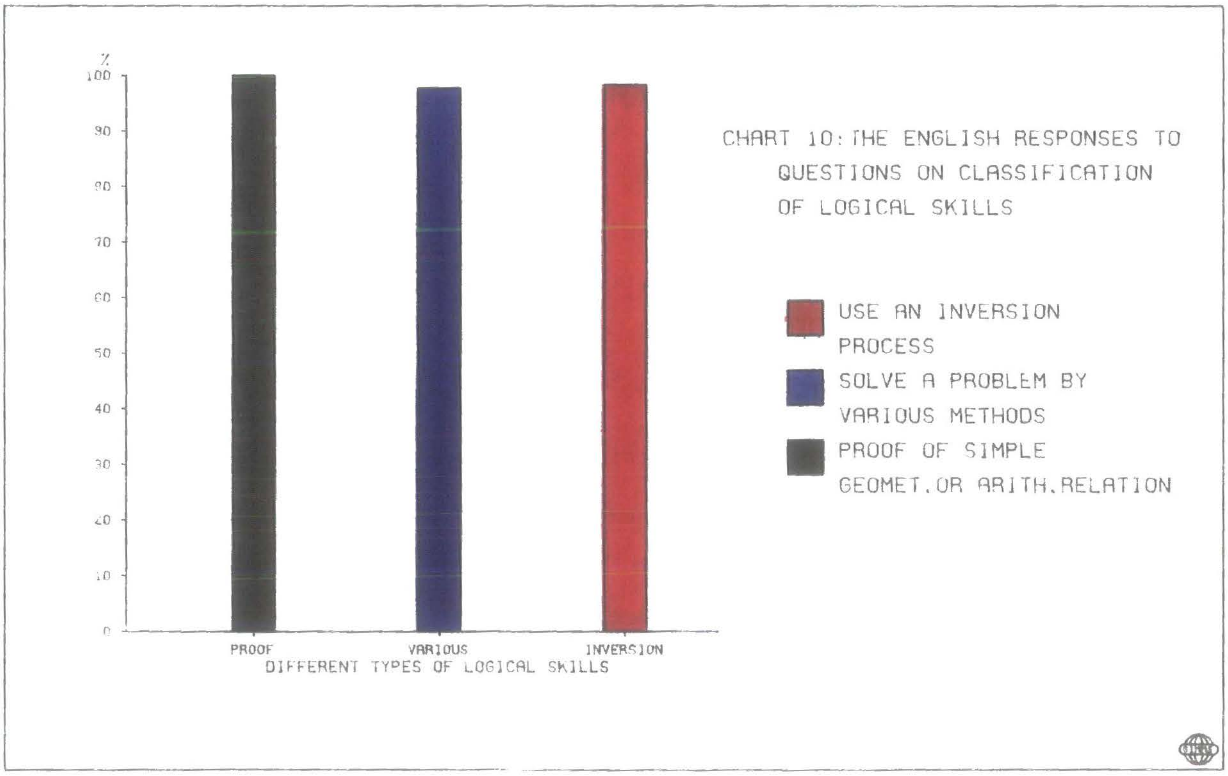
DIFFERENT TYPES OF PROCEDURAL SKILLS
CHART 9: THE ENGLISH RESPONSES TO QUESTIONS ON CLASSIFICATION OF PROCEDURAL SKILLS
SIMPLE FOUR ARITHMETICAL RULES IN MULTIBASE
CHANGE A NUMBER FROM ONE BASE TO ANOTHER
UNION&INTERSECTION OF TWO SETS
COMMON DIVISOR OF TWO NUMBERS
CALCULATE SQUARE OR CUBE OF A NUMBER
MECHANICAL FOUR ARITHMETICAL RULES

Chart 10: shows the percentage of correspondence in the different category of the third type of classification of skills (logical skills). The vertical axis shows the percentage of English agreement among respondents in the different types of logical skills. The horizontal axis compares three types of logical skills.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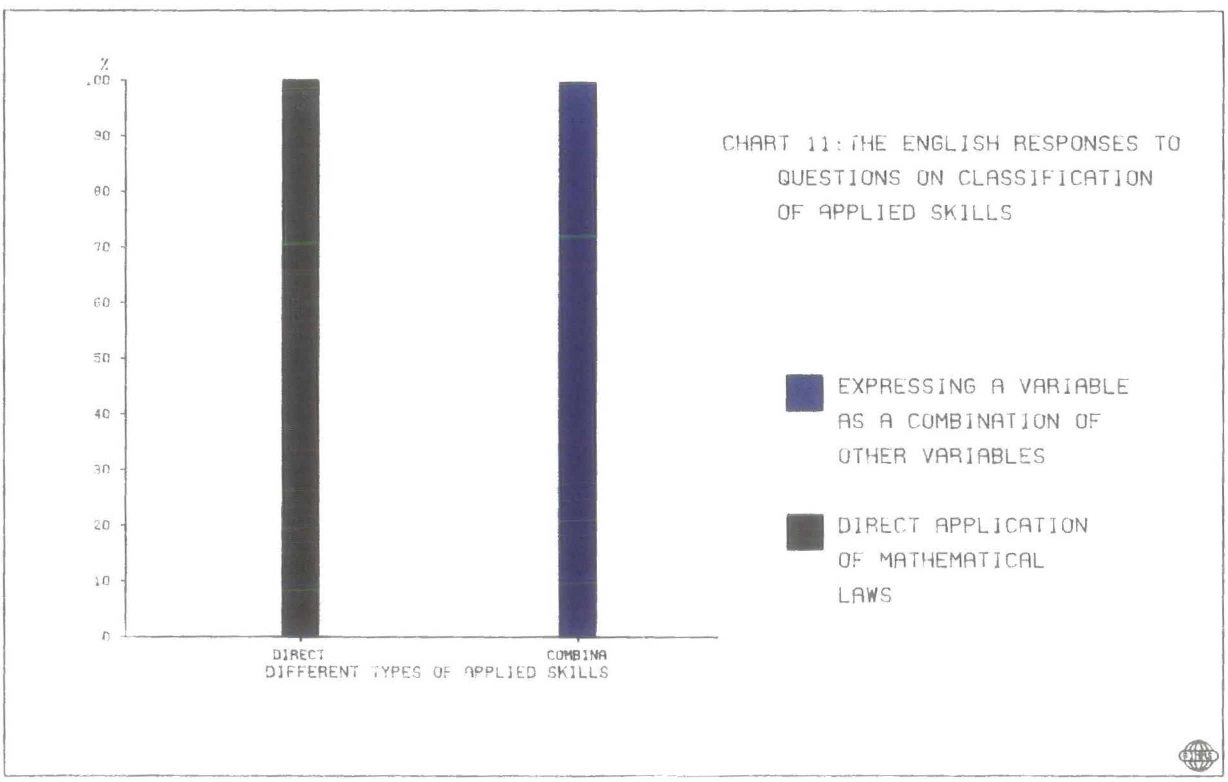
INVERSION
DIFFERENT TYPES OF LOGICAL SKILLS
CHART 10: THE ENGLISH RESPONSES TO QUESTIONS ON CLASSIFICATION OF LOGICAL SKILLS
USE AN INVERSION PROCESS
SOLVE A PROBLEM BY VARIOUS METHODS
PROOF OF SIMPLE GEOMET. OR ARITH. RELATION

Chart 11: shows the percentage of correspondence in the different category of the fourth category of classification of skills (applied skills). The vertical axis shows the percentage of English agreement among respondents in the different types of applied skills. The horizontal axis compares two types of applied skills.



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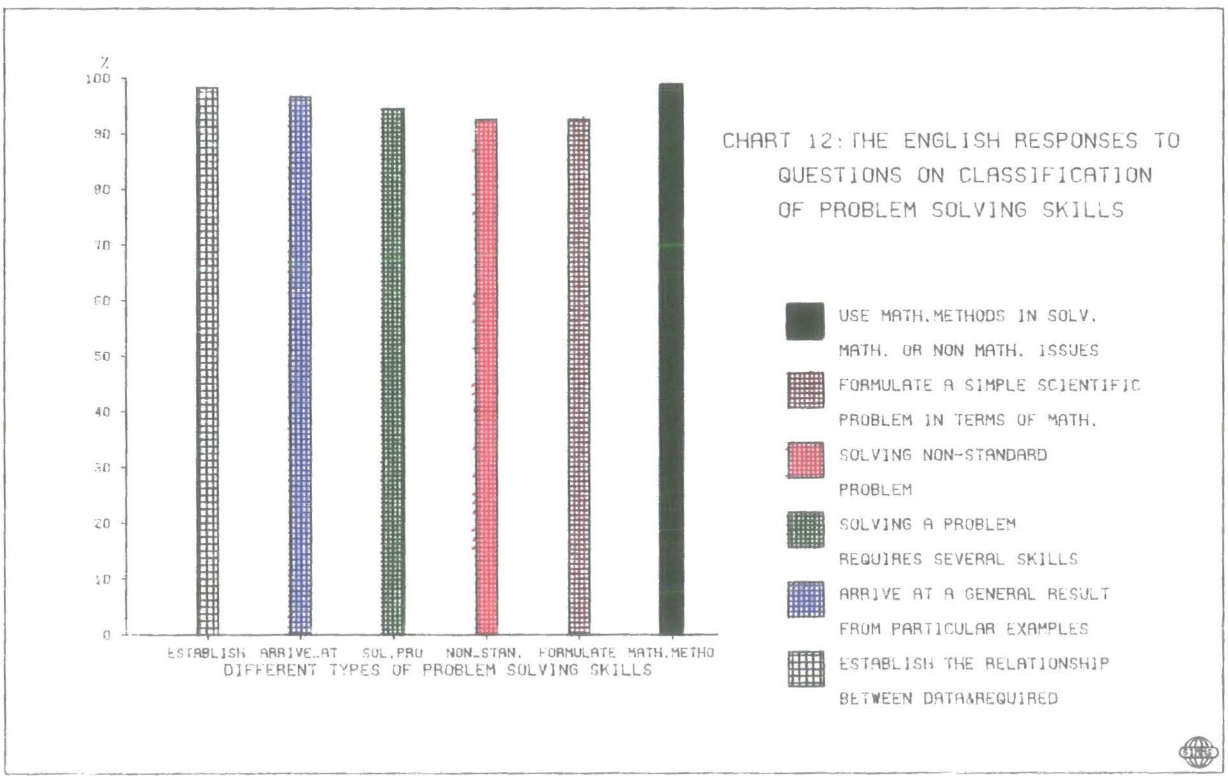
DIFFERENT TYPES OF APPLIED SKILLS
CHART 11: THE ENGLISH RESPONSES TO QUESTIONS ON CLASSIFICATION OF APPLIED SKILLS
EXPRESSING A VARIABLE AS A COMBINATION OF OTHER VARIABLES
DIRECT APPLICATION OF MATHEMATICAL LAWS

Chart 12: shows the percentage of correspondence in the different category of the fifth category of classification of skills (problem solving skills). The vertical axis shows the percentage of English agreement among respondents in the different types of problem solving skills. The horizontal axis compares six types of problem solving skills.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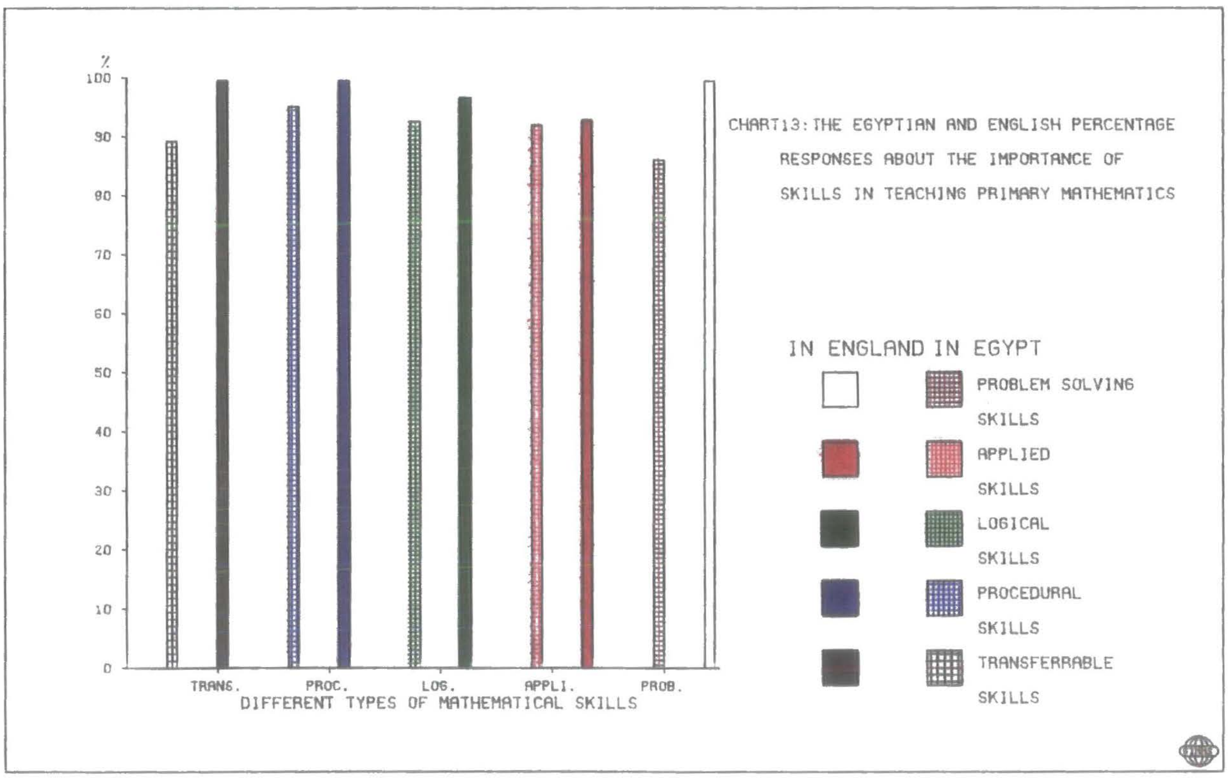
DIFFERENT TYPES OF PROBLEM SOLVING SKILLS
CHART 12: THE ENGLISH RESPONSES TO QUESTIONS ON CLASSIFICATION OF PROBLEM SOLVING SKILLS
USE MATH.METHODS IN SOLV. MATH. OR NON MATH. ISSUES
FORMULATE A SIMPLE SCIENTIFIC PROBLEM IN TERMS OF MATH.
SOLVING NON-STANDARD PROBLEM
SOLVING A PROBLEM REQUIRES SEVERAL SKILLS
ARRIVE AT A GENERAL RESULT FROM PARTICULAR EXAMPLES
ESTABLISH THE RELATIONSHIP BETWEEN DATA&REQUIRED

Chart 13: shows the percentage of Egyptian and English correspondence with regard to the importance of mathematical skills in teaching primary school mathematics. The vertical axis shows the percentage of Egyptian and English agreement among respondents in the different types of classification. The horizontal axis shows the classification of skills (comparison between Egypt and England in these skills).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DIFFERENT TYPES OF MATHEMATICAL SKILLS
CHART13: THE EGYPTIAN AND ENGLISH PERCENTAGE RESPONSES ABOUT THE IMPORTANCE OF SKILLS IN TEACHING PRIMARY MATHEMATICS
IN ENGLAND IN EGYPT
PROBLEM SOLVING SKILLS
APPLIED SKILLS
LOGICAL SKILLS
PROCEDURAL SKILLS
TRANSFERRABLE SKILLS

This chapter has attempted to outline the results of the questionnaires for Egypt and England respectively. The aim of these was to investigate the definitions of mathematical skills, and to collect suggestions of other skills to be included. As a result, the final practical definition of skills was drawn up, and is given in greater detail in the next chapter.

The questionnaires also tried to gather experts' points of view on matters such as present teaching methods and use of educational aids in primary schools, the importance of mathematical skills on the objectives of the primary curriculum and in daily life, and on the current teacher-training courses in mathematics at primary level; and these findings are also summarised in this chapter. The chapter highlights the classification of skills which led to the design of the appropriate test objectives for the age ranges considered.

CHAPTER SEVEN

A PRACTICAL DEFINITION AND CLASSIFICATION OF MATHEMATICAL SKILLS.

MATHEMATICAL SKILLS INVOLVED IN PRIMARY SCHOOL BOOKS.

The main problems dealt with in this chapter are the task of defining and classifying combined with that of devising measures of mathematical skills and item difficulty. The concept of a mathematical skill is first defined and discussed, followed by a content analysis of primary mathematical school books in Egypt. As a result, mathematical skill tests were designed, devised and developed through a pilot study for both Egyptian and English children. The reliabilty and validity of the designed tests of skills were also calculated. A brief outline of this technique is given here, but is more fully described in a later chapter.

The development of mathematical skills in young children is best acquired through an inductive process. Experiences with real objects and events in the child's environment should be the starting point for the growth of mathematical ideas and skills. Moreover, the abstracting of mathematical concepts from situations where children are at play or at work requires perceptive teaching.

Mathematics may then be thought of as a special language which enables man to communicate ideas about quantity, shape and relationships. Its grammar is logic and its vocabulary includes a wide variety of symbols. The basic teaching method is necessarily inductive, with emphasis upon the discovery of principles by each child himself.

If teaching is to be fully effective, regular evaluation of progress is essential. Teaching should be adapted to the needs of the child, in the knowledge of the level of achievement reached. The teacher with this knowledge is in a better position to assess the situation and decide what adaptations of teaching, if any, are required. Hence the need for an objective test of mathematical skills.

Primary school education in Egypt starts at the age of six years, and the mathematics syllabus for primary school stage is shown in appendices. The

main components of the syllabus, on which different test items of the various classifications of mathematical skills were designed, devised and developed, were studied so that the final form of mathematical skills tests would be according to the children's needs and age.
These two mathematical skills tests were modified, adjusted and adapted for two age ranges covering the primary school, one test for age 7-9, and the other for age 9-11. Each of these tests contained the main five classifications of mathematical skill definitions which were suggested, modified, developed and adapted through the research technique. The items of these tests were derived partly from school book syllabuses.

The first suggested classification of the practical definitions of mathematical skills is transference skills, such as comparing groups of different sizes; practical skills as in a practical geometry, reading skills of mathematical language as in reading mathematical symbols and terms and being aware of their meaning; transferring a table of data to a graph or understanding the significance of a data on a graph or histogram and expressing mathematical laws, equations or relations in words. These skills of the first classification of the practical definitions were covered by the first items of the designed tests of mathematical skills.

The second suggested classification is procedural skills, such as being able to use the four arithmetical rules; being able to calculate the square or cube of a number; finding a common divisor, denominator or common multiple of two numbers; finding the union or intersection of two sets; being able to change a number from one base to another and being able to do simple four rule examples in multibase arithmetic. These skills were covered by the second set of items in the tests.

The third supposed category is logical skills, such as proof of simple geometrical or algebraic relation; being able to solve a problem by various methods (e.g being able to decide the longest side of a triangle by measurement of length or realizing it is opposite the largest angle, or being able to choose the shortest way to go a particular place from other ways), and being able to use an inversion process (e.g solving 5- ?=3 by calculating 5-3). These were dealt with by the third set of items in the tests.

The fourth assumed classification is applied skills, such as direct application of mathematical laws or relations (e.g using the formula for area of a rectangle), and expressing a variable as a combination of other variables (e.g the perimeter of a rectangle is the sum of the lengths etc). Applied skills are tested by the fourth set of items.

The fifth presumed category of mathematical skills is problem solving skill. Examples of these are being able to establish the relationship between given data and the required result; being able to arrive at a general result from particular examples; solving a problem which requires several skills; solving non-standard problems; being able to formulate a simple scientific problem in terms of mathematics (e.g establishing the relation between the height a ball is dropped and rebound height); being able to use mathematical methods in solving mathematical or non-mathematical issues; and creating a mathematical model of a physical situation in terms of symbols, equations or formulae. The fifth group of items in the tests covered problem solving skill.

In this thesis the problem of defining the mathematical skills in primary school and the structure in a given learning hierarchy of mathematical skills has been considered. It is assumed that the skills and operations constituting the elements of a hierarchy form an ascending scale. Such a scale may be defined as an orderly arrangement in which the child, in acquiring the concepts and performing the operations at a given level, is related to his performance of the operations and acquisition of the concepts and skills at the previous level, continuing back to the most basic level involved. This implies that every item pair in the hierarchy should be positively correlated, with the degree of correlation depending on the particular concepts, skills and operations involved.

The question as to what mathematical skills are involved in primary school books used in Egypt is one that this chapter attempts to answer. Towards this end a practical or operational definition of the various skills has been made , as well as identifying the mathematical skills involved in contemporary Egyptian school books.

The practical definitions came as a result of:
Studying the theoretical definitions of mathematical skills.
Previous studies and articles in that field.
The long personal experiences of studying and teaching mathematics for all age ranges starting from

six years old up to university students who are aged 24 years old.

Attending some lessons of mathematical teachers for both in-service and teaching practice students.

Analysis of some school books' drill examinations and some children's answers to these exercises.

The point of view of the experts on teaching mathematics in general, such as professors of the teaching of mathematics; from both faculties of education, and sciences as well as those experts on the mathematics curriculum, and the primary mathematics teaching in particular, such as inspectors, advisers, head teachers and teachers in primary schools. These points of view developed as a result of an analysis of the replies to a questionnaire from the previously-mentioned experts.

The final list of the definitions of mathematical skills for primary school children was the outcome of the investigation of the above-mentioned points. These definitions were modified, revised, adjusted and adapted for primary school children in the light of Egyptian and English questionnaire results, which reflected the point of view of experts in that field.

Practical definitions and classifications of the mathematical skills which should be acquired.

It is argued in this thesis that the mathematical skills form a hierarchy.

The final classification of the practical definition of skills was transference, logical, procedural, applied, and problem solving.
Transference skill is taken as the ability to transfer mathematical information from one form into another form. (for example, the ability in geometry to read measurements of a triangle and to draw that triangle, or being able to read a mathematical sentence and being able to express its meaning in the children's own language, reading a table of data and being able to draw a graph of that data, or reading a graph and being able to understand its meaning and significance etc.). Clearly, this is essential to all other mathematics.

The transference skills can be represented as follows:
Practical skills, as in practical geometry; construction of shapes in geometry, which means dealing with the different geometrical tools, various

measurements, constructing and drawing geometrical shapes when provided with sufficient data, e.g given data for a triangle, being able to draw and construct the figure.

Reading skills of mathematical language, as in reading mathematical symbols and terms and being aware of their meaning, or being able to read a mathematical sentence and being able to express its meaning in children's language. This includes an ability to understand the expressions used in stating a problem.

Transferring a table of data to a graph, transferring a verbal problem or a geometrical exercise to a shape. Reading and expressing a geometrical shape verbally.

Expressing a mathematical law, or relation, in words; it also means transferring a verbal problem to symbolic relations.

Procedural skills are considered the ability to perform a fundamental mathematical process (e.g the ability to do addition, subtraction, division, multiplication). Procedural skills include transference skills. For example, a child doing any four rule operation must be able to interpret the symbols used, whether they are words, numbers, or even concrete objects.

Some examples of procedural skills are as follows:

Being able to perform addition, subtraction, multiplication, and division.

Being able to calculate the square or cube of a number.

Finding a common divisor, denominator or common multiple of two numbers.

Finding the union or intersection of two sets.

Being able to change a number from one base to another (e.g being able to change 12 base ten to a binary number).

Being able to do simple four rule examples in multi-base arithmetic.

Logical skills are the ability to draw a conclusion from given data so that procedural skills may then be used to arrive at the solution.

Some examples of logical skills are as follows:

Simple understanding of some geometrical or algebraic inferences (e.g if a=b and b=c then a=c).

Being able to see that there are different ways of arriving at a solution (e.g being able to decide the longest side of a triangle can be found by measurement or realizing it is opposite the largest angle).

Recognising that the solution of 5 - ? =3 may be found by calculating 5-3.

Applied skills are the ability to apply simple formulae, laws or relations. (For example, knowing or having been given the formula for the area of a rectangle, being able to use the formula to calculate one variable given the other two).

Other examples of applied skills may be represented as follows:

Direct application of mathematical laws or relations.

Expressing a variable as a combination of other variables (e.g the perimeter of a rectangle is the sum of the lengths).

Problem solving skills are the ability to discover the mathematical relationships in a situation and organise these relationships in such a way that a solution may be computed.

Examples of problem solving skills are:

Being able to establish the relationship between given data and the solution or result.

Being able to postulate general results from particular examples.

Solving the non-standard, unusual or unexpected case.

Being able to formulate a simple scientific problem in terms of mathematics (e.g establishing the relation between the height a ball which is dropped and the rebound height).

Being able to use mathematical methods in solving what seems at first sight to be a non-mathematical situation.

Creating a mathematical model of a physical situation in terms of symbols, equations or formulae and arranging these so that results can be calculated.

The mathematical skills involved in primary school books in Egypt

In Egypt, schools use sets of text books recommended by the government. All children use these books in their schools but they may supplement these by the use of other available text books supplied by their parents.

To investigate and classify the mathematical skills involved in the recommended school books, a content analysis of these books was carried out. The content of the books in every year was classified into the categories of the practical definitions explained above. The frame of that content is given in appendix XIV. This analysis was shown to mathematical experts in Egypt so that the content analysis could be checked, revised, justified and modified.

A precise content analysis of fourteen Egyptian primary school mathematical books was made in order to produce a concise review of the mathematical skills contained therein. These fourteen books covered both the traditional and modern syllabuses currently used in primary education.

The content of the books was divided into two age ranges, 7-9 year old and 9-11 year old, so as to correspond with the tests which were eventually applied to Egyptian and English children. The content was also classified under the practical definition of skills given earlier in this chapter and traditional and modern books were treated separately.

The mathematical skills involved in the traditional mathematics syllabuses in Egypt

AGE RANGE 7-9

FIRST UNITS: NUMBERS

Transference skills

Being able to recognize and read one, two, three, four, five, and six figure numbers.

Being aware of the meaning of numbers up to six figures.

Being able to understand a graph which represents data up to six figure numbers and being able to transfer graphical representation to numbers or vice versa.

Procedural skills

Being able to do addition and subtraction without and with carrying or borrowing up to six figure numbers.

Being able to do simple multiplication and division.

Being able to calculate the square or cube of a number.

Finding a common divisor, denominator or common multiple of two numbers.

Being able to do simple addition and subtraction on common and decimal fractions, and cancelling fractions.

Logical skills

Being able to use inversion processes (e.g solving 15- ? =5 by calculating 15-5).

Recognizing ascending and descending order on sets of numbers and being able to put sets of numbers in order using $<$ (Less than) or $>$ (Bigger than).

<u>Applied skills</u>

Being able to apply the properties of numbers, such as distributive, associative and commutative properties.

<u>Problem solving skills</u>

Being able to establish relationships between numbers (e.g $15>14$, $15=3*5=14+1=15*1=75/3$ etc).

Being able to investigate and arrive at a general result from particular examples of numbers.

Solving a problem which requires the preceding skills with numbers.

Being able to differentiate and distinguish between numbers which imply full understanding of place value, composition, equivalent, odd, even, bigger than, smaller than, and inequalities.

SECOND UNITS: SURFACES AND SOLIDS

Transference skills

Being able to recognize and identify angles, straight and curved lines, triangles, rectangles, squares, circles, parallelogram, cube, sphere, pyramid, cylinder and parallelepiped.

Being able to read the geometrical terms mentioned above. Geometrical concepts, symbols and shapes, and knowing their significance.

Being able to transfer a verbal geometrical question to geometrical symbols or shapes and vice versa.

Procedural skills

Being able to do calculations among the different units of measurement.

Logical skills

Being able to rationalize the methods used in solving a particular geometrical problem.

Being able to recognize the shortest and easiest way to solve a geometrical problem.

Being able to deduce and generalize the various properties of the previous shapes.

Being able to understand and rationalize a problem by different methods (e.g being able to decide the longest side of a triangle by measurement of length or realizing it is opposite the largest angle).

Being able to predict and distinguish reasonable answers from the obviously incorrect.

Applied skills

Being able to draw and measure different types of lines and angles.

Being able to calculate the perimeters and areas of the shapes in the syllabus.

Being able to find the lengths of sides and heights by knowing the area or perimeter in particular cases, such as a square.

Calculating a variable as a combination of other variables (e.g the perimeter of a rectangle is the sum of the lengths).

Constructing shapes according to provided data.

Problem solving skills

Being able to establish the relationship and distinguish between the various types of curves and geometrical shapes. (e.g identifying the differences between triangle and rectangle, recognizing the main features of each shape).

Being able to give examples of geometrical shapes and solids from real life.

AGE RANGE 9-11

FIRST UNITS: NUMBERS

Transference skills

Being able to recognize and read ratio, proportion and percentage.

Being able to transfer a table of data to a graph or vice versa.

Being aware of vulgar fractions, decimals and percentages.

Procedural skills

Being able to do approximations to the nearest whole, ten, hundred, thousand and to one, two, and three decimal places.

Being able to calculate the square or cube of a number.

Finding a common divisor, denominator or common multiple of two numbers.

Finding the scale of a graph, and finding the real length for a graph length.

Being able to calculate means and interest, rate, time and principal in simple interest problems.

Performing long division of two, three and four figure numbers.

Performing with addition and subtraction on proportional numbers.

Logical skills

As in age 7-9 with extension in the complexity of the numbers used.

Applied skills

In addition to the ones in age 7-9,

being able to change vulgar fractions into percentages.

Being able to tabulate graphical data.

Being able to convert percentages into decimals and vice versa.

Problem solving skills

As in age 7-9 with extension to include more complex numbers.

SECOND UNITS: GEOMETRY

Transference skills

Being able to recognize and identify parallel lines, perpendicular lines and the intersection of lines.

Being able to read the geometrical terms and symbols of the geometrical shapes, surfaces and solids.

Procedural skills

As in age 7-9 with more complex cases .

Logical skills

In addition to those points mentioned in the age 7-9 there are the following.

Being able to realize the relationship between square units.

Being able to realize the relationship between Egyptian measurement units of the agricultural area such as Fidan, Kirat, Sahm etc.

Applied skills

Being able to draw, measure and construct regular shapes.

Being able to calculate the perimeters and areas of the square, rectangle, circle, parallelogram, triangle.

Being able to construct triangles given one side and two angles, three sides, and two sides and the included angle.

Constructing a circle with a given radius.

Being able to calculate the volume of a parallelepiped.

Problem solving skills

As in age 7-9 plus being able to explain and describe perpendiculars, reflex angles and rectangles etc.

The mathematical skills which are involved in the modern mathematics syllabuses in Egypt

CHILDREN AGED FROM 7-9 YEARS

FIRST UNITS: SETS, NUMBERS, GROUPS, EQUATIONS AND INEQUALITIES

Transference skills

Being able to read and write terms of sets, i.e element, membership, subset, equivalence of sets, empty set, universal set and disjoint sets; and being aware of their meaning.

Being able to read numbers up to six figures.

Being able to read and understand the term natural numbers.

Being able to read and identify the identity element for addition and multiplication.

Being able to read and know the significance of even, odd, ascending and descending order of numbers.

Being able to transfer a variable situation into an arithmetic relation.

Being able to understand a graph which represents data and being able to transfer graphical data to numbers or vice versa.

<u>Procedural skills</u>

Finding the union or intersection of two sets by using Venn diagrams.

Finding the composition of numbers, e.g 5=3+2 =1+4.

Being able to do the four rules on numbers.

Being able to double numbers.

Performing the four rules on vulgar fractions and decimals.

Being able to calculate the numbers of elements in a particular set.

Logical skills

The realization of common characteristic of a set of elements, classification of sets and breaking down a set into subsets.

Arranging sets of numbers into ascending and descending order.

Being able to use an inversion process (e.g solving 20- ? =17 by calculating 20-17).

Applied skills

Being able to apply the properties of numbers and union and intersection of sets.

Being able to apply the knowledge of the relationships between Egyptian money units and place value of numbers.

Problem solving skills

Being able to give examples of empty sets, equivalent sets, prime numbers, the lowest common multiple.

Being able to distinguish between the different terms above.

Solving linear equations with one variable unknown.

Solving inequalities with one variable unknown.
Being able to distinguish between a prime number and a non-prime number.
Being able to test for divisibility by 2, 5, 3 and 10 without performing the operation of division.

SECOND UNITS: GEOMETRY

Transference skills

Being able to identify two dimensional geometrical shapes such as square, rectangle, triangle, circle, parallelogram and trapezium.
Being able to identify the terms point, ray, sector, segment, straight line, diameter, radius, parallelism, intersection of lines, perpendicular lines or lines at right angles, curved, surface, vector and displacement.
Being able to read graphical representation of data.
Being able to identify 3-D shapes such as the cube, cylinder, cone, pyramid, ball and parallelepiped.

Procedural skills

Being able to calculate with the measurement units.

Logical skills

Being able to deduce the properties of geometrical shapes.

Being able to give an approximation of a particular length, area or volume.

Applied skills

Drawing staight lines, parallel lines, angles and perpendicular lines.

Construction of a triangle (given three sides; two angles and one side; two sides and the included angle), a circle given its radius, a parallelogram, and a square.

Finding the perimeter of the above mentioned shapes.

Finding the result of two displacements.

Finding the area of the two-dimensional shapes above and the volume of the solids previously mentioned.

Problem solving skills

Using given information to identify the properties and distinguish the different types of the geometrical shapes.

Being able to give different examples of the geometrical shapes and solids, given particular properties.

CHILDREN AGED 9 TO 11 YEARS

FIRST UNITS: NUMBERS

Transference skills

Those skills which are included in age 7-9, plus recognition of properties of numbers, means and approximation of decimals to one, two and three places.

Procedural skills

Calculating the mean plus those skills mentioned in 7-9 year age range.

Logical skills

The same skills as mentioned in the 7-9 year range.

Applied skills

Those mentioned previously in the 7-9 year range.

Problem solving skills

As in the 7-9 year range.

SECOND UNITS: GEOMETRY AND GEOMETRICAL TRANSFORMATIONS

Transference skills

In addition to those mentioned in the age range 7-9 years, being able to read and understand the terms of rotation, centre of rotation, reflection, enlargement, line of symmetry, axis of symmetry, vertical axis, horizontal axis and displacement.

Procedural skills

As in age range 7-9 years.

Logical skills

Being able to deduce the properties of rotation, reflection and displacement along with the skills of the age range 7-9.

Applied skills

Drawing and constructing 2-D and 3-D geometrical shapes.

Perimeter and area of 2-D geometrical figures.

Finding centres of rotation.

Those skills mentioned in age range 7-9.

Problem solving skills

In addition to those skills of the 7-9 age range, being able to establish and use the relationships between rotation, reflection and displacement.

The validity of these frames were examined by the Egyptian mathematical experts and were modified, justified, adapted, and developed in the light of their views and recommendations.

In this chapter, a final working definition and classification of mathematical skills has been set out. These were developed by considering the views of experts consulted by means of a questionnaire; by reading the theoretical literature on the subject; by the author's own experience of teaching, and observations of children' performance, both informally and through tests; and by a precise analysis of school textbooks, identifying the skills covered in each unit.

It is important to realise the difficulties involved in the tasks outlined above. Previous work concentrated on the definition of general, not just mathematical, skills; and did not give teachers and researchers clear examples appropriate to particular age ranges. It is hoped that this research will focus a spotlight on these neglected areas, and lead to further work being undertaken.

CHAPTER EIGHT

DESIGN OF THE EXPERIMENT:

PREPARATION, DESCRIPTION, SAMPLING AND ADMINISTRATION OF THE TESTS.

PREPARATION AND DESCRIPTION OF THE TESTS

Two tests, including evaluation of the five main classifications of mathematical skills, which were illustrated in the previous chapter with more detail, were prepared and tried on a small sample of 146 children in both Egyptian and English primary schools. The first test, which is designed for the 7-9 age range, includes 52 items, and the second test, which is designed for the 9-11 age range, includes 42 items. These tests cover the primary school age, and the syllabuses used in that stage were taken into account when test items were designed. The content of these tests includes items evaluating and developing the various mathematical skills in that age. The total score of the items in each category is 20 although the number of items in each classification is different.

The pilot study was intended to discover whether the test items were suitable for the age range and types of children who were to be tested and whether the questions put to the children were couched in quantifying terms which they themselves used and understood. The pilot study was for both Egyptian and English children.

These tests were administered to Egyptian and English primary school children in EL MINIA in EGYPT and DURHAM in ENGLAND, so the result would be defined in relation to those children. The test items which covered the various skills for both age range are shown below:

Table 2: ITEM NUMBER FOR AGE 7-9

TRANSFERENCE SKILLS	1(1), 2(1), 3(1), 4(1),5(1), 6(1), 7(1), 8(4), 9(2),10(1),13(6).	20.
PROCEDURAL SKILLS	11(1),12(1),14(1),15(1),16(1),17(2), 18(3), 19(5), 20(3), 21(2).	20.
LOGICAL SKILLS	22(2), 23(2), 24(2), 25(1), 26(2), 27(1), 28(1), 29(3), 30(1), 31(1), 30(1), 31(1), 32(1), 33(1), 34(2).	20.
APPLIED SKILLS	35(1),36(3),37(2),38(4),39(1),40(1), 41(1), 42(1), 43(2), 44(4).	20.
PROBLEM SOLVING SKILLS	45(1), 46(1), 47(3), 48(3), 49(3), 50(3), 51(3), 52(3).	20.

Table 3: ITEM NUMBER FOR AGE 9-11

TRANSFERENCE SKILLS	1(4),2(4),3(12),	20.
PROCEDURAL SKILLS	4(1), 5(2), 6(7), 7(2), 8(1), 9(1), 10(1), 11(1), 12(1), 13(1), 14(1), 15(1).	20.
LOGICAL SKILLS	16(1), 17(1), 18(1), 19(3), 20(1), 21(2), 22(1), 23(6), 24(2), 25(2).	20.
APPLIED SKILLS	26(1), 27(1), 28(2), 29(2), 30(4), 31(2), 32(1), 33(1), 34(2), 35(2), 36(1), 37(1).	20.
PROBLEM SOLVING SKILLS	38(1), 39(4), 40(5), 41(5),42(5). 40(5), 41(5), 42(5).	20.

Tables 2 and 3 show the number and the score of items in the tests in each classification which were previously described. Items were selected for the final test according to an analysis carried out on the results of the pilot study in that context. The difficult items were excluded. An items' difficulty is expressed as the percentage of those who do not succeed in answering the item correctly in the group tested. This percentage is termed the item's facility value (F I). High facility values show the item as being easy; low values as being difficult. Items with facility values at, or near, 0.5 were favoured for inclusion in the final test, and items with extreme values (0.8 or over, and 0.2 or less) were rejected. Item discrimination shows the extent to which the able children succeed in answering the item correctly more often than do the less able. Ideally an external criterion of ability is desirable, but in the absence of one the total score on the test may be used. Items will then be considered to have a satisfactory discrimination if markedly more of the children with high total scores answer the item correctly than do those with low total scores. The discrimination index used was one based on successive quarters of total scores. It is given by the formulae.

$$D = \frac{2a + b - c - 2d}{110}$$

Where 'a' is the number in the top quarter (37 children) answering the item correctly, 'b' is the number in the second quarter (36 children) answering the item correctly, 'c' is the third quarter(36 children) answering the item correctly, and 'd' is the number in the bottom quarter (37 children) answering the item correctly. The denominator, 110, is obtained as the weighted sum of the top and second quarter totals, i.e, 2x37+36. Thus if all in the top two quarters answered correctly and none in the bottom two quarters answered correctly, the value of D would be 1. The index is similar to the better known upper and lower thirds index, but has the advantage of using all the information available and at the same time giving greater weight to the extremes. A minimum value of 0.20 was necessary for the inclusion of an item in the final test.

In summary then, items with F values between 0.2 and 0.8 and with D values not less than 0.20 were considered suitable for the final test. A few items satisfying these criteria were nevertheless not included as it was thought that sufficient items of the same type had already been selected. Also a few items with F and D statistics outside the accepted ranges were included to balance a set of otherwise acceptable items, or for some other special reason. A table of the F and D values of all items, and showing also the accept-reject decision made, appears in appendix (III). In all, 52 items for the first test, and 42 items for the second test, were selected for the final test, these being made up from the various classifications of the practical definition of mathematical skills as follows:

FOR THE FIRST TEST 7-9 AGE

TRANFERENCE SKILLS have 11 items, PROCEDURAL SKILLS have 10 items, LOGICAL SKILLS have 13 items, APPLIED SKILLS have 10 items, PROBLEM SOLVING SKILLS have 8 items.

FOR THE SECOND TEST 9-11 AGE

TRANSFERENCE SKILLS have 3 items, PROCEDURAL SKILLS have 12 items, LOGICAL SKILLS have 10 items, APPLIED SKILLS have 12 items, PROBLEM SOLVING SKILLS have 5 items.
The number of these items was previously illustrated in the tables for both age ranges.

The complete test appears in the appendix (IV) (The Egyptian and English version). The scoring of the test was done according to the above sub-division of skills. The score of each item appeared between brackets in the tables 2 and 3. The distribution of these items among the various skills was finally adjusted, modified, adapted, and fitted according to the opinion of Egyptian and English experts in that field, and was decided upon after showing them matrices of these items and their different distributions in these skills. Six experts were chosen for both Egyptian and English tests, and the final distribution was in the light of their views.

This material allows an assessment of the way in which pupils of age range 7-9, 9-11 years old organise their knowledge and experience in the face of the items presented to them. This requires a standard of ease in abstraction and generalisation of mathematical concepts. In all the items in this area the pupils had to identify and understand the ideas contained in the item in order to be able to give the correct answers. This also requires a certain skill in distinguishing what is correct in a series of questions, which might cause hesitation. It is necessary therefore, that the pupil should have attained a certain amount of skill in the interpretation of data as much as in the comprehension of the relationships found within the concepts.

The evaluation of the contents of the tests in this context are as follows: the ability to interpret a graphical presentation and to write the corresponding number; the ability to interpret the meaning of mathematical symbols; the ability to recognize in given illustrations those which represent the fractions, a third, quarter, and half; the ability to indicate and identify the methods used in measuring length, weight, and time. 'Teachers frequently say that if a pupil really comprehends something, then he can apply it' (*). Despite this affirmation, the area of "application" in traditional teaching has been relegated to a lower plane in elementary schools. In the first grades, when the pupils begin to acquire experience of problem-solving, teachers ought to encourage them by presenting them with increasingly difficult problems so that the pupils become more mature. These ideas were taken into account when the tests were designed. Faced with a problem, children generally try to find in it the elements with which they are familiar as well as the aspects which call their attention, so that they can later use these things as a guide to the solution. When all the elements of a problem are clearly presented to them, the complex process of

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(*) B. S. Bloom, <u>TAXONOMY OF EDUCATIONAL OBJECTIVES</u>: <u>THE CLASSIFICATION OF EDUCATIONAL GOALS</u>, <u>HAND BOOK 1</u>: <u>COGNITIVE DOMAIN</u>, Longmane, 1956, 120.

reasoning comes into play, as they remember and organise the necessary concepts which they must use. On other occasions they will try to find a method suitable for reaching their answer or they may even use an abstract element in solving the problem. These aspects of "application" are synthesised in the fourth area of the tests.

THE TESTING SCHEDULE

The length of the tests and the possibility of results being affected by fatigue and boredom, on the part of those tested, made it necessary to consider very carefully the order of the presentation of the tests. It was hoped that whenever possible, the tests could be administered in two sessions on a single day with the individual tests at some other time convenient to the relevant school. Teachers of the chosen schools helped in the application and administration of these tests. To complete the answers to all the tests the average child took one and a half hours, it having been decided that two sessions per day would be less tiring for the child. The tests were proceeded by some personal information, such as the name of the school, the name of the child, the sex, and the date of birth. In addition to these there are sheets which have the scores of every child in mathematics during the year, representing the teachers' assessments and observation of their children. The aim of having these scores is to determine the validity of the tests externally by calculating the coefficient of correlation between these scores and the tests. If the correlations are high, that suggests that the tests have a high validity, assuming that the teachers' assessments will be more valid as they know their pupils better. The validity was also checked by other means, as mentioned later in this chapter.

The testing programme occupied a period of eighteen weeks, for both Egypt and England. The starting time each morning varied, depending on the environment of each school. In order to present the tests so that conditions would be as uniform as possible, each of the tests was administered with help from school teachers. Each test was administered according to the instructions prepared for it. In general, the description of the task and explanation of the work were given beforehand. In the introductory remarks at the beginning of the first session, the purpose of the investigation was explained and the co-operation of the teachers enjoined. The schedule was explained and a progress report of the project was given where possible.

THE RELIABILITY AND VALIDITY OF THE TESTS

RELIABILITY

The statistics related to the reliability of the tests in the investigation were obtained by different methods:

(I) Coefficient of correlation between the score of odd items and even items of the test.

(II) Coefficient of correlation between the scores of the same test for the same individual by applying the same test once more, five weeks later.

The tables below show the reliability coefficients for both tests 7-9, 9-11, and for both countries Egypt and England.

Table 4 shows the correlation coefficient and its significance for both tests, 7-9 and 9-11 age range, for Egyptian children.

No of children	Age range	Methods used	Coefficient of correlation	Significance
300	7-9	odd & even items	0.880434159	32.0532735
307	9-11	odd & even items	0.832852378	26.2787138
300	7-9	re-apply test	0.870213456	
307	9-11	re-apply test	0.821034523	

Table 5 shows the correlation coefficient and its significance for both tests, 7-9 and 9-11 age range, for English children.

No of children	Age range	Methods used	Coefficient of correlation	Significance
223	7-9	odd & even items	0.77894482	18.4659362
202	9-11	odd & even items	0.911511123	31.3432635
223	7-9	re-apply test	0.777653321	
202	9-11	re-apply test	0.901032111	

These figures were obtained by computer. The estimated reliability coefficients for both tests in each age range in Egypt and England were high, being greater than 0.77, so they have a level of confidence of 99% or 95%, which in both cases constitutes a high and acceptable level of reliability.

VALIDITY

Since (*) 'reliability is a necessary, but not sufficient, condition for validity' , and since the estimated reliabilities shown in tables 4 & 5 are all 0.77 (to the second decimal place) or higher, it is assumed that the test instruments are reliable. Consequently, an investigation of the validity of the tests is now possible. This examination (of the validity of the tests) is presented in three ways:

(i) Content validity in relation to the Mathematical Skills Tests. This is known as jury validity.

(ii) Predictive validity of the Mathematical Skills Tests for the scores of the pupils in Mathematics at the end of the year.

(iii) A congruence validity, by using an external criterion, which has a high validity and is known as an acceptable measurement.

(iv) Square roots of the reliability.

(I) CONTENT VALIDITY IN RELATION TO THE MATHEMATICAL SKILLS TESTS WHICH IS KNOWN AS JURY VALIDITY

(*) D. L. Nuttall, A. S. Willmott, BRITISH EXAMINATIONS-TECHNIQUES OF ANALYSIS, N. F. E. R, Great Britain, 1972, 14.

Since a valid test is one which achieves the objective for which it has been designed, the purpose of the tests must be considered in relation to content validity. The aims and the practical definition of the Mathematical Skills Tests were determined, and the distribution of test items among the different types of skills were displayed and illustrated clearly in matrix form. These matrices were accompanied by a letter explaining what to do, a sheet containing the aims of the Mathematical Skills Tests, and a sheet containing the practical definitions of these Mathematical Skills which have been covered by the items of these tests. These matrices and sheets together with the tests and their instruction were given to experts in that field as a jury, to see whether the items were distributed among skills and if they were evaluating a certain skill or not. They were given the opportunity to explain their point of view in that context by putting the items of the tests in the correct area to evaluate every skill correctly and precisely. These juries were considered experts in the field of teaching mathematics in general and concerned with mathematical skills in particular. These experts decided whether the content of these items achieved the aims of the tests, and whether they were well distributed according to the practical definitions or not. Those experts in both countries, Egypt and England, were chosen very carefully according to their specialisms, which related in someway to the field of the Mathematical Skills Tests. Their points of view were taken into account in the final version of these tests.

(II) PREDICTIVE VALIDITY OF MATHEMATICS SKILLS TESTS FOR THE SCORES OF THE PUPILS IN MATHEMATICS, AT THE END OF THE YEAR.

Since there is some interest in using other ways to check the validity of the tests, such as comparison of scores with the results of equivalent tests in standardised form, their predictive reliability is important. The primary school subjects tested by the end of 1984 all sat for their primary final examination in Mathematics that same year. If the Mathematics Skills Tests have predictive validity, there should be a significant correlation between the scores on them and the primary final. The available scores on all tests were obtained and the Pearson Product-Moment Correlation Coefficient computed. The correlation coefficient of 0.998479423 for the first test aged 7-9, and 0.997141664 for the second test aged 9-11 are high in the view of the fact that the 1984 Primary Final Examination was intended mostly as a test of achievement at the end of a Mathematics course rather than a test of Skills. The previous coefficients were for Egyptian children as these scores were available. The significance of the correlations were 312.675047 for 300 children in the first test, and 230.486628 for 307 children in the second test.

(III) <u>A CRITERIA VALIDITY</u>

The validity was checked by other means, by comparing the scores of these tests with other measurements which had a high and acceptable validity. The measurment which was used as a valid criterion for these tests is the evaluation of the teachers of their pupils through the year and through the long-term observation of their pupils. The scores which were obtained from teachers for the pupils were considered as a valid measurement of those pupils, because the teacher is the best person to know his pupils, being in contact and dealing with them most of their time even more than their parents. The coefficient of correlation between these tests and the scores of the pupils given by the teachers were high and of an acceptable degree. The correlation of the tests were 0.88, for the first test and 0.94, for the second test. The pupils' scores which were used as an criterion for the validity test were from the previous year's assessments.

(IV) THE SQUARE ROOTS OF THE COEFFICIENT CORRELATIONS OF THE RELIABILITY.

The square roots of the reliability of the tests were high, being 0.938314434, for the first test and 0.912607134 for the second test for the Egyptian children. For the English children the score was 0.882578423, for the first test and 0.95473094, for the second test. Generally it could be said that reliability and validity, important factors in these tests, which were being applied for the first time, were acceptable.

THE SAMPLE OF THE RESEARCH

(I) THE SAMPLE FOR THE PILOT STUDY

The sample comprised 146 children in Egyptian primary schools, chosen randomely in EL-MINIA, to represent the main sample which in every respect had all the dimentions of the pilot sample. 146 children were also chosen in DURHAM primary schools, again representing the main sample in every respect and dimension. The main aim of the sample in the pilot study was to examine the research tools experimentally, so that modification, justification, development and evaluation of the research tools would be possible, in the light of the results of the pilot study. This procedure aimed to arrange the tools employed in the final forms to be ready for the main sample and the main experiment, after alterations and modifications.

(II) THE MAIN SAMPLE

The populations from which the samples were selected.

The children were from government schools in the primary stage. The sample of the experiment was chosen, taking into consideration the need to be representative of the main population and homogeneous in type of accommodation and socio-economic status, by having children from different schools with different type of accommodation, different backgrounds and socio-economic status. This was achieved by having schools in different locations. It then became apparent that children would need to be selected from several districts. Six districts were chosen in EL MINIA in EGYPT using the following criteria:

(i) The geographical location and ease of travel.

(ii) The size and density of population of the district. It was decided that the size of the district should be of an order sufficient to provide a significant number of children with varied levels of socio-economic status and different types of teachers. It was also conidered important for the purpose of this study to ensure that the sample included both sexes, the age ranges 7-9, 9-11 and rural and urban children from as many districts as possible.

(iii) Schools following "modern" mathematics,and others following "traditional" mathematics.

The number of children for the first test aged 7-9, was 300 , and 307 for the second test aged 9-11, in the schools, as previously illustrated, in the various districts in EL-MINIA in Egypt.

The sample in the pilot study in England was 146 children chosen in different districts in DURHAM.

The number of children in the sample in the main study in England was 223 for the first test aged 7-9, and 202 for the second test aged 9-11. These numbers were selected from different schools in different districts in DURHAM.

The selection of the children was in terms of the age in the school year, which presumably would be located in the second, third and fourth primary school year children for the first test aged 7-9 in Egypt, where children start school at the age of six, and in the fifth and sixth primary school year for the second test aged 9-11.

ADMINISTRATION

The administration of the tests in the primary schools in EL-MINIA in Egypt took place at the beginning of the 1984 academic year, and the administration of the tests in the primary schools in DURHAM in England took place in the middle of 1984. Every test took place on the same day in order that the entire application of the test for the individual was completed in a day. However, the test was administered in two sessions to allow a break for the children. In the first session the pupils worked the first half of the items of the tests, which took about half an hour. In the second session the pupils worked through the second half of the items of the tests, which also took half an hour.
Full consultation took place with the teachers of each class as to the procedure and supervision which they should carry out during the testing, since the success of the results depended indirectly on the

"rapport" which the teachers could achieve. As far as possible, any distraction was avoided and it was ensured that the elements inherent in a natural school atmosphere, such as the arrangement of seats, the lighting, the suitable number of pupils and the materials necessary for carrying out the tests were properly planned and carried out before the tests were begun. The teachers tried throughout to create an atmosphere of confidence and interest in the tests so that it could be worked under optimal conditions.

CHAPTER NINE

THE RESULTS OF THE RESEARCH

The purpose of this research was to investigate mathematical skills, the design characteristics of which suggest that it may be applicable to certain stages in the learning of mathematics in the classroom situation. Since however the technique to deal with this problem is believed to be new, it was felt that the first duty of this thesis should be to present a clear picture of the skills.

The work described was exploratory in character, in the belief that the task must be to answer all the important questions broadly rather than to answer one with absolute precision. Looking at the investigation broadly, the questions that must be answered are:

To what extent do the children acquire mathematical skills?.

To what extent are children different in acquiring skills?.

To what extent do children acquire skills involved in scholastic units?.

To what extent are skills different in their acquisition?.

Are there any differences in acquiring skills according to:

Sex (boys, girls) , countries.

Does the acquisition of one skill affect the acquisition of another?.

In the context of these questions the results are now described. It will be appreciated that the first five chapters of this work do not present results in the experimental sense; rather they combine to present a proposition. The first chapter states in detail the problems of the research. Chapters two and three, in general, relate a research problem to an historical background of learning primary mathematics in both countries, Egypt and England, while chapter four describes the theory of children learning mathematics. An analysis of previous research and its relation to this research is one of the procedures required to help in presentation, and this is dealt with in chapter five. Chapters six and seven form a bridge between the propositional first half of the thesis and the experimental second half. These chapters report an investigation of responses from a sample of experts from both Egypt and England, who were involved in various ways with the teaching of mathematics. Their main conclusions are stated in the list of mathematical skills which are described in detail in chapter seven.

Two age ranges of children at the primary stage in various schools were involved in the study. The total number of groups for the two age ranges in both countries was four. The groups were 7-9 age range and 9-11 age range for Egypt as well as for England. Each age range is discussed in order to bring out a pattern of development for all the skills throughout the years and within different skills. In this chapter the results of children in the tests are discussed. The main criterion for the formation of the tests was to examine the classification and the acquisition of mathematical skills. The other criteria were the characteristics of the children in the 7-9 and 9-11 age ranges. The principle behind the formation of the tests was that of the development of the children in acquiring skills.
Test items were categorised by "content" and by "learning outcome". Three main content categories were used - geometry, mensuration and numbers. Skill in these areas was identified as one of the learning

outcomes. Data about sex and age was also collected. For all scores on the variables, the descriptive statistics of means M and standard deviation were computed for sex and age groups. Both mean and standard deviation were separately calculated for sets in the four groups, sets in the different skills, sets in the various variables, and sets in the whole test. Significance of differences between the means of different sets was also found by the technique of the T-Test at 95% and 99% levels. This was done, in fact, solely to describe the sets better. Besides the absolute characteristics of a group, the relative or comparative standing also helped to know and understand the group better. When it was known whether a particular set was better or worse than another, and whether its difference from others was or was not really significant, then these sets were better understood. The means and standard deviations of the variables described the sets in respect of the variables. Graphs were plotted by computer for each group in these variables for each country separately and for both together, and these compare the responses of children learning skills in both countries. In taking the groups as they were, there was an assumption that:

The groups were homogeneous.

They were similar as regards age for Egyptian and English children.

They were different as regards skills and understanding in mathematics.

They were different in the development of acquiring skills as regards age.

The significance or otherwise of the differences were computed merely for descriptive purposes.

It might be thought that an ideal way of studying the development of a mathematical skill would be to carry out a longitudinal study extending from a child at the beginning of school to the end of school and in which the individual mathematical acquirements at particular points in time would be related to those of his experiences thought to be relevant. Such an approach would require a large investment in terms of time and resources, and those available were very limited. Even if this were not the case there would still be objections to this approach, for example the number of significant events in the individual's learning experience would still be so great that to record them would be an almost impossible task.

THE RESULTS FOR CHILDREN IN THE 7-9 AGE RANGE

(A) EGYPTIAN RESULTS

In the Egyption test there were 300 pupils, 139 girls and 161 boys who came from eight schools.

(I) The whole test result

The Mean and Standard deviation of children's scores for the whole test were computed by a computer programme. It was found that the Mean for all children in the whole test was 63.51 and standard deviation was 20.145. The maximum score for a whole test is 100. Considering the fact that the aim of teaching mathematics is to reach a high level of understanding and acquiring skills, the mean may be thought to be low. This desirable aim is far from being achieved with large numbers of children. The standard deviation was high, indicating the large range of scores which show the differences among children in the level of acquiring skills.

Distribution of scores for children in the whole test is given below:

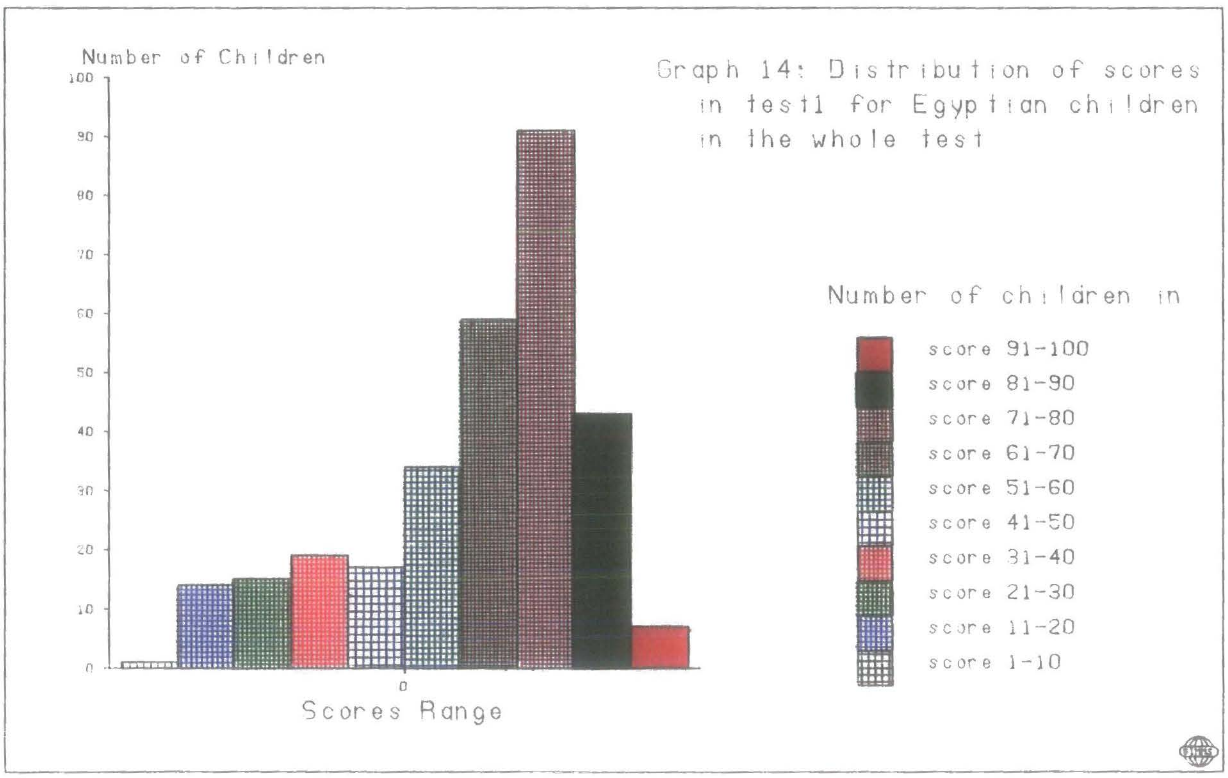
Table 6: Distribution of scores in test 1 for Egyptian children in the whole test

Scores Range	1-10	11-20	21-30	31-40	41-50	51-60
Number of Children	1	14	15	19	17	34

Scores Range	61-70	71-80	81-90	91-100	Total Child.
Number of Children	59	91	43	7	300

It appears from table 6 and graph 14 that the number of children who have scored more than 60 in the whole test is high, 200 out of 300. The number who got lower than 61 was 100.

There were 22.0% of the children lower than 51, 31% achieved from 51 to 70, 44.67% achieved from 71 to 90, and only 2.33% had from 91 to 100.


Number of Children
100
90
80
70
60
50
40
30
20
10
0
0
Scores Range
Graph 14: Distribution of scores in test1 for Egyptian children in the whole test
Number of children in
score 91-100
score 81-90
score 71-80
score 61-70
score 51-60
score 41-50
score 31-40
score 21-30
score 11-20
score 1-10

(II) The whole test result for different age ranges

The overall results have been classified according to their age groupings as shown in table below.

Table 7: Mean score and standard deviation of Egyptian pupils classified according to age range

age range	7.0& 7.1& 7.2	7.3& 7.4& 7.5	7.6& 7.7& 7.8	7.9 & 7.10& 7.11	8.0& 8.1& 8.2	8.3& 8.4& 8.5	8.6& 8.7& 8.8	8.9 & 8.10& 8.11
Mean	35.5	38.6	46.9	66.67	66.0	65.4	66.7	71.2
S. D	18.9	17.4	23.6	24.4	18.4	17.6	19.6	26.6
Number of Children	17	36	24	12	32	34	17	10

age range	9.0& 9.1& 9.2	9.3& 9.4& 9.5	9.6& 9.7& 9.8	9.9 & 9.10& 9.11
Mean	73.8	78.3	78.7	74.2
S. D	15.3	16.7	20.2	21.7
No. of ch.	42	31	21	24

One would expect an increase in score with age and indeed the general trend is just this. There are however some differences, for example the 8.3 to 8.5 group scores less than the 8.0 to 8.2 group. Differences of this sort can be as a result of earlier school teaching and learning of the particular group and it would be interesting to have had available long term research results for each group, but this is beyond the range of this thesis. Other reasons for the differences may be explained by the small number of children in each group and the possibility that the mean IQ of one group differs considerably from another.

Generally, there is progress in acquiring skills as age increases. The operations of addition and subtraction of numbers cause little difficulty for the majority of pupils. Where failure does occur it can usually be ascribed to a lack of understanding of the concept of place value. Pupils were less competent when dealing with multiplication or division and the cause here appears to be lack of-practice in the operation of multiplication-tables, lack of understanding of place value and other concepts upon which skills are based. Many children have problems in dealing with order of quantities. One would suggest that practice in arithmetical operations in both row and column form would be beneficial in a number of respects, for example in learning place value. It was evident that the conventions regarding the order of operations and the use of brackets had not been learnt and this will surely became a source of difficulty as the children become older.

So far as the concepts of numbers are concerned, it seems likely that the pupils in this study who are well below average for their groups, would be prevented from generalising such concepts to other fields. This will have the effect of restricting them to particular concrete instances of the concepts.

The numeric section of the test consisted of those concepts thought to be part of the hierarchy of "what must be known" in order to solve an exercise. Solving such problems requires the pupil not only to know the individual concepts but also to integrate and use them together. For example, to form mathematical skills, if the pupils have not been able to use their concepts in this way then their knowledge will be fragmented and uncoordinated. From the findings already obtained it might be anticipated that, as the pupils get older, the earlier items will became "easier" for them.

(III) Different skills

In this section the results are disscussed for each category of skills.

The result of the different skills for all children.

The result of the different skills through the age ranges.

(i) The results of all children in different skills for the whole test are stated below in table 8:

Table 8: The mean and standard deviation of all children for different skills

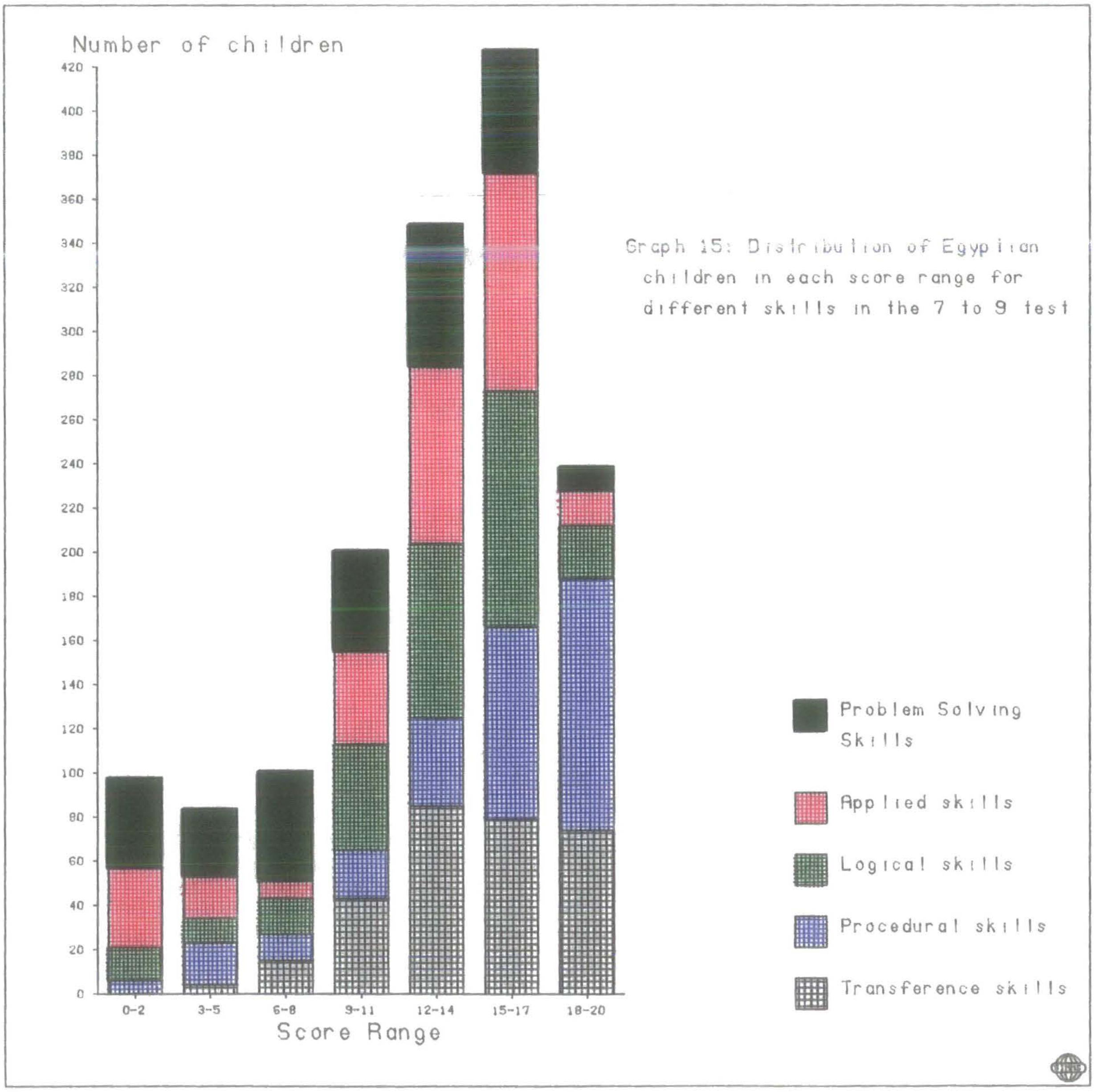
Different Skills	Transf--erence	Procedural	Logical	Applied	Problem Solving
Mean	14.46	14.83	12.84	11.60	9.79
Standard Deviation	3.60	4.87	4.42	5.35	5.43

As can be seen from the results in table 8, the order of acquiring skills according to the mean score is: Procedural, Transference, Logical, Applied, and Problem solving, but it should be noted that the difference between transference and procedural is small. The Problem Solving proved difficult for children in this sample indicating that it is unlikely that they have had the experience that would bring about coordination, and this argument is supported by the findings of the test analysis which indicates the presence of several weaknesses in other skills. The mean for problem solving skills was 9.79 which is below 50% of the total possible. The mean scores for other skills were all above half the possible score, although the logical and applied means were only just above. The means for the procedural and transference skills were the highest, as would be expected, and at 14.83 and 14.46 were 74.15% and 72.3% of the total possible.

Table 9: Number of Egyptian children in each score range for different skills in the 7 to 9 test

\Score Skills\Range	0-2	3-5	6-8	9-11	12-14	15-17	18-20
Transference Skills	0	4	15	43	85	79	74
Procedural Skills	6	19	12	22	40	87	114
Logical Skills	15	11	16	48	79	107	24
Applied Skills	36	19	8	42	80	99	16
Problem Solving Skills	41	31	50	46	65	56	11

Graph 15: shows the scores of all children for the different skills.



Graph 15: Distribution of Egyptian children in each score range for different skills in the 7 to 9 test

In transference skills, 20.67% of the children scored lower than 12, 54.67% achieved from 12 to 17, and 24.67% scored a high score of 18 to 20.
With procedural skills, 19.67% had lower than 12, 42.33% scored from 12 to 17, and 38% had the high score of 18 to 20.
In logical skills, 30% of them scored from 0 to 11, 62% achieved from 12 to 17, and only 8% had a high score from 18 to 20.
For applied skills, 35% had from 0 to 11, 59.67% scored from 12 to 17, and again only 5.33% had from 18 to 20.
In problem solving skills, a high percentage of them had from 0 to 11 (56%), 40.33% achieved from 12 to 17, and only 3.67% scored a high score of 18 to 20.

It should be pointed out that 38% of the children had from eighteen to twenty for procedural skills, and 24.67% scored in that range for transference skills. In contrast a lower percentage of children scored in that range for logical (8%), applied (5.33%), and problem skills (3.67%). With logical, applied and transference skills, the majority scored from twelve to seventeen, since 62% of children fell in this range for logical, 59.67% for applied and 54.67% for transference skills. The low percentages of scores for problem solving skills with high scores in procedural skills indicate a concentration of teaching on mechanical arithmetical processes. The number of children who scored lower than 9 tend to increase steadily as the skills become more complex from the transference to problem solving skills, and this is as would be expected. The actual figures were 6.33%, 12.33%, 14%, 21%, and 40.67% for transference, procedural, logical, applied, and problem solving skills respectively. In contrast, the percentages of the number of children who scored 18 and more tends to decrease from transference to problem solving skills. There were 24.67%, 38%, 8%, 5.35%, and 3.67% for transference, procedural, logical applied, and problem solving skills respectively. With the 9-11 score range, the percentages were 14.33% for transference, 7.33% for procedural, 16% for logical, 14% for applied, and 15.33% for problem solving skills. The significances of the differences between the mean scores were examined by a T-Test, the results of which are given below:

Table 10: T-Test for the mean of different skills in the whole test

	Transf-erence	Procedural	Logical	Applied	Problem Solving
Trans.		-1.0564	*4.9140	*7.6691	*12.395
Proc.			*5.2321	*7.7201	*11.948
Log.				*3.0897	*7.8199
Appl.					*4.1058

*Indicates significance at level 99% of confidence.

In the different skills, children's performance varied appreciably according to the methods by which they had been taught and to the intrinsic difficulty of the skills. As might be expected, pupils' scores varied widely across the range of questions between the different categories of skills. Children appeared to perform very much as might have been expected from the standpoint taken in this research and assumptions of the hierarchy of skills. The T values marked with an asterisk in the above table are all significant at the 0.01 level. The results combine to show the difficulty experienced by many children in logical, applied, and problem solving skills and these difficulties are well illustrated in the graphs and bar charts. These results reinforce the impression given by the answers to many questions. They indicate that while most primary school children can do mathematics involving the more mechanical and fundamental concepts and skills to which they have been introduced, and can cope with simple applications of them, there is a fairly sharp decline in performance when these concepts have to be applied in more complex settings. To perform well in problem solving skills children need to perform well in transference, procedural, logical, and applied skills in that order. Deficiency in an earlier skill leads to difficulties with later skills.

(ii) The result of the different skills through the age range ranges

As well as the overall mean scores for each category, the mean score was calculated for the pupils in each skill in three month age ranges from 7 to 9 years. These results are in table 11 below:

Table 11: Mean scores and standard deviations of Egyptian pupils for different skills through the ages range in test 1.

age range		7.0& 7.1& 7.2	7.3& 7.4& 7.5	7.6& 7.7& 7.8	7.9 & 7.10& 7.11	8.0& 8.1& 8.2	8.3& 8.4& 8.5	8.6& 8.7& 8.8	8.9 & 8.10& 8.11
Mean	TRA	12.4	11.3	11.9	12.9	14.5	14.4	15.4	16.2
S. D		5.30	3.83	4.54	4.93	4.07	3.63	4.47	5.62
Mean	PRO	8.7	9.3	11.6	17.3	15.4	15.3	15.4	16.1
S. D		4.90	5.46	5.92	5.59	4.66	4.31	5.00	6.77
Mean	LOG	7.7	7.9	9.4	14.4	13.9	13.4	14.2	15.4
S. D		4.56	4.71	5.29	5.28	4.14	4.13	4.87	5.57
Mean	APP	4.6	5.9	7.4	11.8	12.3	12.6	13.4	13.0
S. D		5.12	5.48	6.02	5.02	4.59	4.77	5.17	6.57
Mean	SOL	2.2	4.2	6.7	10.3	10.3	9.6	8.4	10.5
S. D		3.50	4.04	5.28	5.20	5.36	4.99	3.35	6.90

Age Range		9.0& 9.1& 9.2	9.3& 9.4& 9.5	9.6& 9.7& 9.8	9.9 & 9.10& 9.11
Mean	TRA	15.3	17.2	16.4	16.0
S. D		3.45	3.95	4.76	4.68
Mean	PRO	17.3	17.4	18.2	16.7
S. D		3.56	3.86	4.75	5.04
Mean	LOG	14.1	15.4	15.3	15.2
S. D		3.52	3.72	4.01	4.37
Mean	APP	14.2	14.7	14.8	13.7
S. D		3.31	4.12	4.02	5.16
Mean	SOL	12.8	13.7	13.9	12.7
S. D		4.32	4.07	4.60	4.66

It can be seen that some of the pupils are still unable to deal with some concepts. As was the case with total scores through different ages, here again it can be seen that skills tend generally to be more developed as the children become older. The scores for some children in particular age ranges do not seem to have developed as would be expected, but it must be remembered that the results are not for the same group followed through, but for different groups in each age range. All sorts of differences can occur between groups. Reasons can include prolonged absences causing some pupils to miss important segments of instruction in the preceding grade, or pupils sometimes failing to grasp explanations of a process because they may be mentally less mature than their fellows. Explanations which were not understood or only partially understood last year may be grasped more successfully this year. Even though a pupil gains a satisfactory understanding of what is taught at one grade level, he may gain a richer and fuller understanding of the same topic in a later grade level when he is mentally more mature and provided with appropriate teaching. Whatever the reasons are for the deficiencies found in pupils, what such pupils need is not drill but chance to learn. Practice comes later to fix what has been learned and to maintain it at a high level of working efficiency.

Serious mistakes can sometimes be made in pitting one pupil against another with whom he cannot be expected to compete successfully. Again, pupils should be judged by their own progress, not by that of others. The pupil is in school to learn, not to compete or to work out assignments to please the teacher.

One of the objectives of arithmetic teaching is to help the pupil become aware of his own progress. He will, moreover, become aware of the specific things he can do to make greater progress.

(IV) Differences between boys and girls

The section contains separate details of the performance in skills of boys and girls. The results give details :

(i) In the whole test

Mean and standard deviation for boys and girls have been computed for a whole test. The mean for boys was 65.34 and the S.D was 18.19, while for the girls they were 61.40 and 22.70 respectively. The significances of the differences between boys and girls was examined by T-Test, the result of which was 1.6588. Since this is only significant at the 10% level, this is considered as being no difference between boys and girls in acquiring skills. These results are supported by other findings on the achievement of mathematics as a whole (*).

Table 12: The scores of Egyptian boys and girls for the whole test

Scores Range	1-10	11-20	21-30	31-40	41-50	51-60
No. Boys	0	2	6	14	10	15
No. Girls	1	12	9	5	7	19

Scores Range	61-70	71-80	81-90	91-100	Tot.No of ch.
No. Boys	41	43	28	2	161
No. Girls	18	48	15	5	139

The mean score of boys seemed to be better than that of the girls, while the standard deviation of girls at 22.70 was higher than boys which is 18.19. This indicates that the differences between scores

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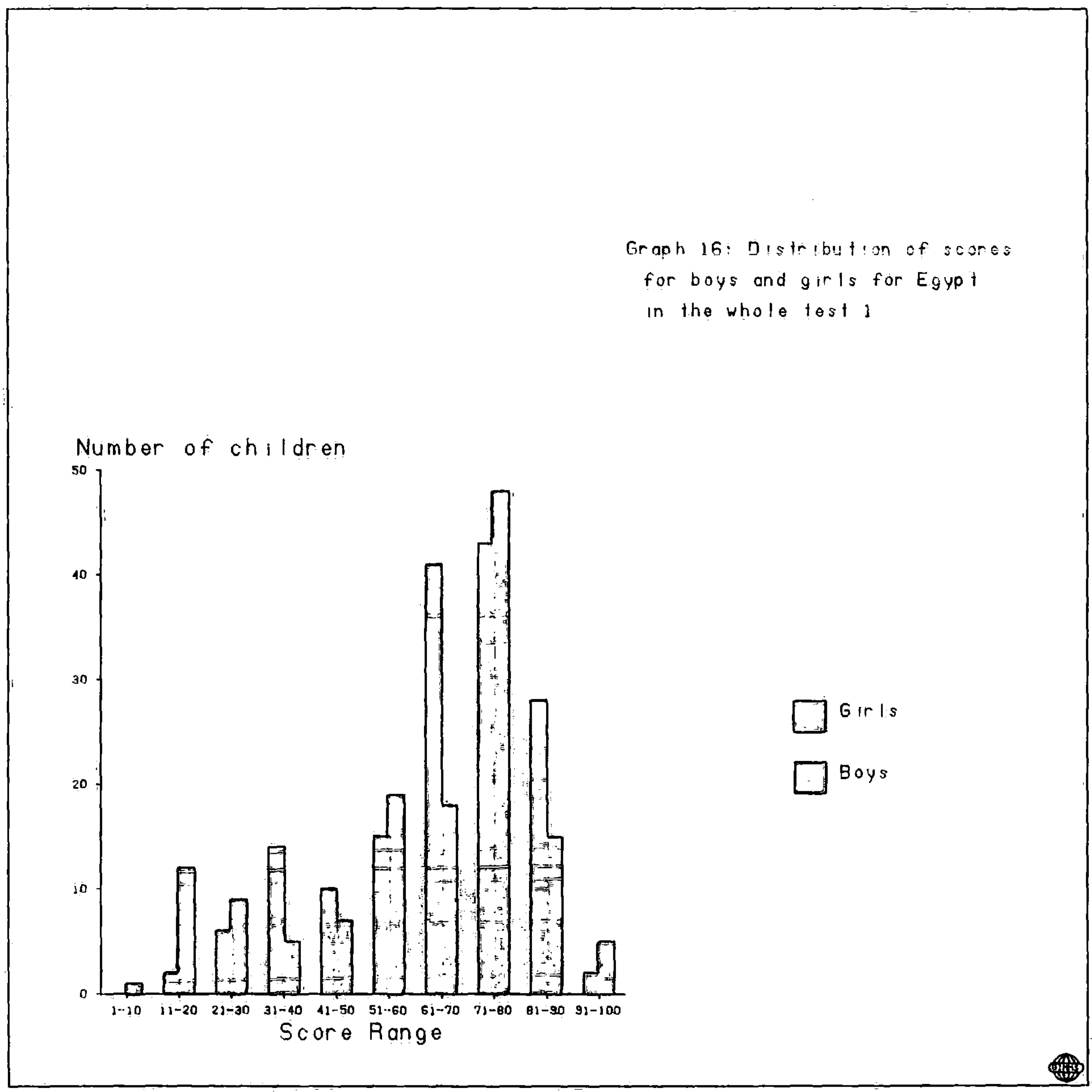
(*) Department of Education and Science, Assessment of Performance Unit, MATHEMATICAL DEVELOPMENT, Second Survey Report No.2, , H. M. S. O, London, 1981.

and the mean are higher in the case of girls than boys i.e the range of the boys' marks was less than the range of girls' marks. Despite the scores being higher for the boys than the girls, 70.81% of girls obtained a score higher than 60 as opposed to 61.78% of the boys. 19.88% boys, and 24.46% girls scored lower than 51. The percentage in 51 to 60 score range is higher in the case of girls than boys, 13.67% for girls and 9.32% for boys. In contrast 29.19% boys scored lower than 61, while 38.13% girls were in the same range.

34.78% of boys scored more than 50 and less than 71, and 26.62% of girls were in the same range, while 44.10% of boys and 45.32% of girls achieved more than 70 and less than 91. With scores of more than 90 there were only 1.24% boys and 3.60% girls.

Graph 16: shows the distribution of scores for boys and girls for Egypt in the whole test 1.



Graph 16: Distribution of scores
for boys and girls for Egypt
in the whole test 1
Number of children
50
40
30
20
10
0
1-10
11-20
21-30
31-40
41-50
51-60
61-70
71-80
81-90
91-100
Score Range
Girls
Boys

105 girls out of 139 (75.54%) obtained scores higher than 50, while 129 boys out of 161 (80.12%) scored above 50. On the other hand, 34 girls (24.46%) and 32 boys (19.88%) scored less than 51.

(ii) In the different skills

Table 13: The mean and standard deviation for different skills for boys and girls in Egypt in test 1

Different Skills		Transf- -erence	Procedural	Logical	Applied	Problem Solving
Mean	BOYS	14.56	15.24	13.52	12.04	9.98
S. D		3.46	4.38	4.05	5.27	5.22
Mean	GIRLS	14.35	14.36	12.04	11.09	9.56
S. D		3.96	5.50	4.85	5.51	5.74

Table 14 shows the T-Test result of different skills for boys in test 1.

Table 14: T-Test for the mean of different skills for Egyptian boys in test 1

	Transf-erence	Procedural	Logical	Applied	Problem Solving
Trans.		-1.54	*2.47	*5.06	*9.25
Proc.	1.54		*3.65	*5.91	*9.76
Log.	-2.47	-3.65		*2.82	*6.78
Appl.	-5.06	-5.91	-2.82		*3.51
Pr.So.	-9.25	-9.76	-6.78	-3.51	

*Indicates significance at level 99% of confidence.

Table 15 shows the T-Test result of different skills for girls in test 1.

Table 15: T-Test for the mean of different skills for Egyptian girls in test 1

	Transf-erence	Procedural	Logical	Applied	Problem Solving
Trans.		-0.02	*4.33	*5.64	*8.07
Proc.	0.02		*3.72	*4.93	*7.09
Log.	-4.33	-3.72		1.52	*3.88
Appl.	-5.64	-4.93	-1.52		*2.26
Pr.So.	-8.07	-7.09	-3.88	-2.26	

*Indicates significance at level 99% of confidence.

Table 16 shows T-Test result of different skills among boys and girls.

Table 16: T-Test for the mean of different skills between boys and girls for Egypt in test 1

BOYS AND GIRLS	Transf- -erence	Procedural	Logical	Applied	Problem Solving
	0.49	1.54	*2.87	1.52	0.66

*Indicates significance at level 95% of confidence.

In general the order of acquiring skills of boys and girls was; transference; procedural; logical; applied and problem solving skills respectively, which agrees with the research hierarchy. However the boys mean score for procedural was higher than for transference. The differences between different skills were, with one exception, significant both for boys and girls, although it was more significant in the case of girls. Differences between boys and girls for different skills were significant only on logical skills, with boys better than girls, although the other mean scores of the different skills were slightly higher for boys than girls.

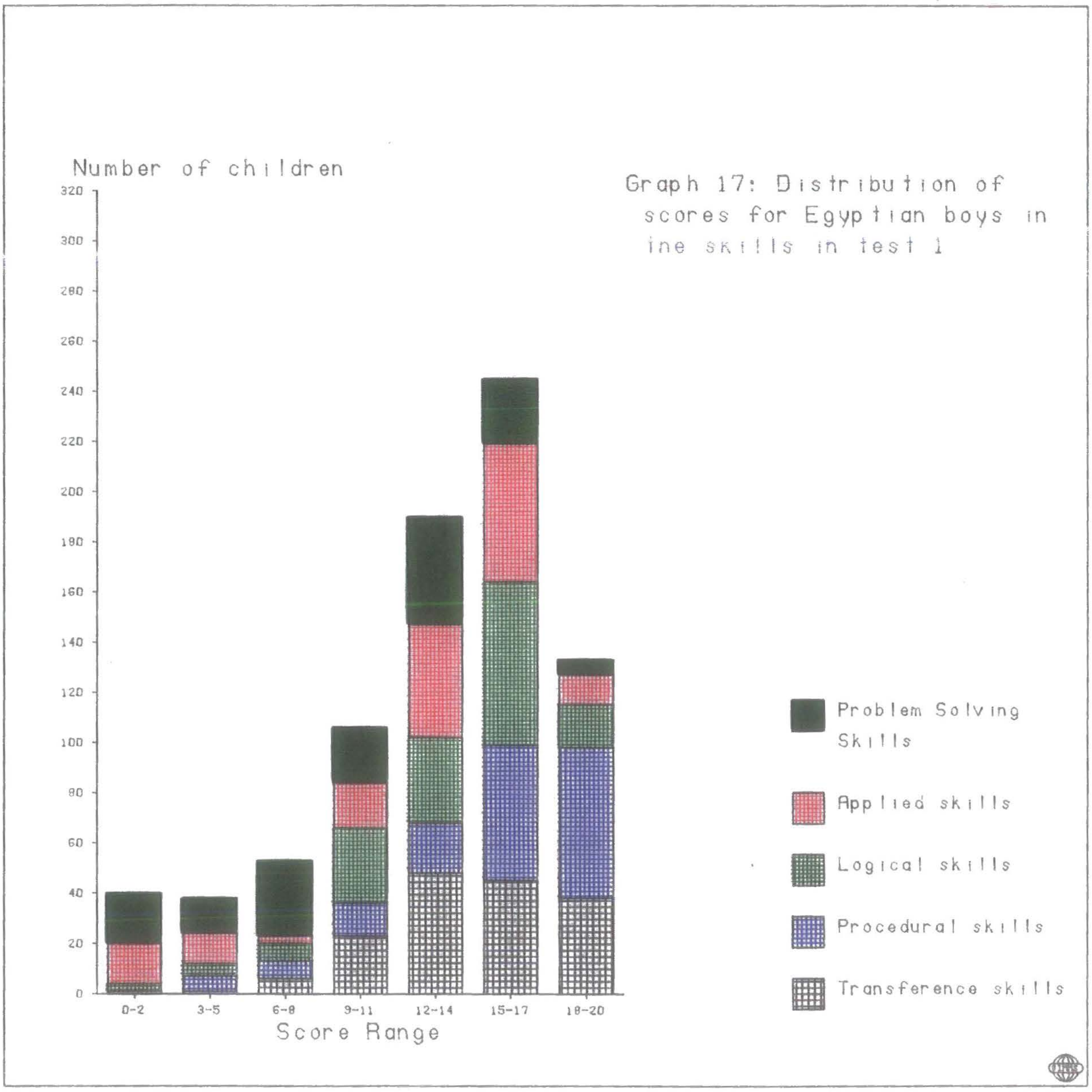
Table 17 shows the scores for boys and girls for different skills in Egypt in test 1.

Table 17: Number of Egyptian boys and girls in each score range for different skills in the 7 to 9 test

Skills \ Score Range		0-2	3-5	6-8	9-11	12-14	15-17	18-20
Transf- -erence Skills	B	0	1	6	23	48	45	38
	G	0	3	9	20	37	34	36
Proced- -ural Skills	B	1	6	7	13	20	54	60
	G	5	13	5	9	20	33	54
Logical Skills	B	3	5	7	30	34	65	17
	G	12	6	8	18	46	42	7
Applied Skills	B	16	12	3	18	45	55	12
	G	20	7	5	25	34	44	4
Problem Solving Skills	B	20	14	30	22	43	26	6
	G	21	17	20	24	22	30	5

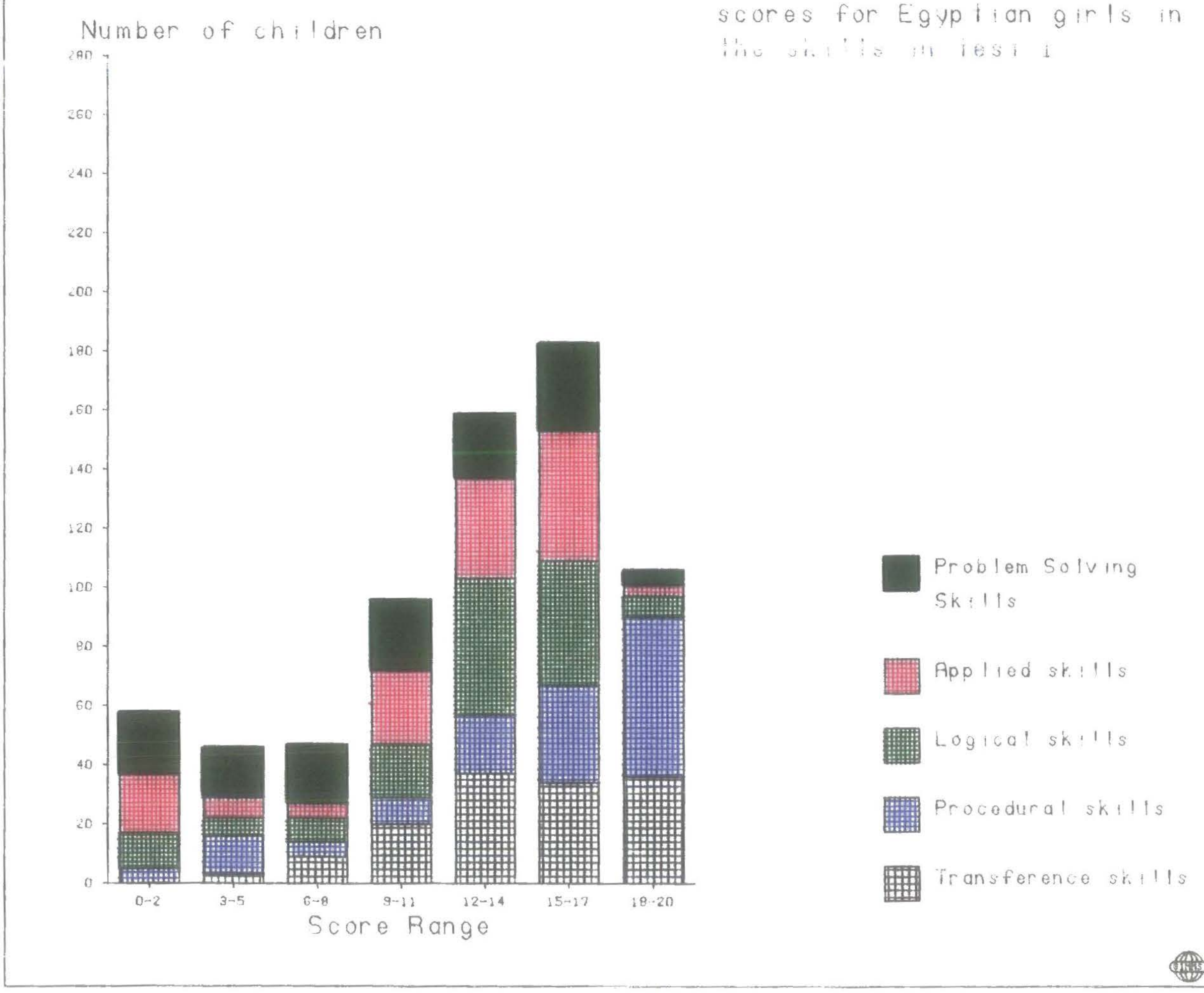
Relatively high percentages of boys and girls scored high scores from 18 to 20 on procedural skills, since 37.27% boys and 38.85% girls were in that range. Again a high percentage of children were in that range of score for transference skills, 23.60% boys and 25.90% girls. The lowest percentages in that range were in problem solving skills, 3.73% for boys and 3.60% for girls; applied skills, 7.45% for boys and 2.88% for girls; and logical skills, 10.56% for boys and 5.04% for girls. The percentage of children who scored less than 9 tends to increase as the skills become more difficult for both girls and boys. The percentages of girls who got less than 9 were 8.63%, 16.55%, 18.71%, 23.82%, and 41.73% on transference, procedural, logical, applied, and problem solving skills respectively. For boys the corresponding figures are 4.35%, 8.70%, 9.32%, 19.25%, and 39.75% (for transference, procedural, logical, applied, and problem solving skills respectively). The order of skills within the range 12 to 17 for girls was logical, applied, transference, procedural, and problem solving skills, where the percentages were 63.31%, 56.12%, 51.08%, 38.13%, and 37.41% respectively. But the order for boys was applied, logical, transference, procedural, and problem solving skills, since the figures in these skills in that range of score were 62.11%, 61.49%, 57.76%, 45.96%, and 42.86% respectively. The highest percentage for girls in that range was for logical skills, 63.31%, and the highest for boys was for applied skills 62.11%. The lowest percentages of boys and girls who scored 9 to 11 was on procedural skills, 8.07% boys and 6.47% girls.

Graph 17 shows the distribution of scores for Egyptian boys in the skills in test 1.



Graph 17: Distribution of scores for Egyptian boys in ine skills in test 1

Graph 18 shows the distribution of scores for girls in the skills for Epypt in test 1.



Graph 18: Distribution of scores for Egyptian girls in the skills in test 1

(B) ENGLISH RESULTS

The overall picture of performance which is presented by the results for English children is very similar to that obtained for Egypt. A few statistically significant differences were identified but these may have arisen by chance in the selection of the groups. In the English test there were 223 children, 107 boys and 116 girls. The pattern of results obtained was similar to that described in Egyptian results for 7 to 9 year old children. This similarity in the general picture applied to the test results as a whole and also within the different category of skills. The results have added some more detail to the picture of performance by giving some tentative indications of the different levels of understanding that pupils have of the various mathematical concepts and skills.

(I) The whole test result

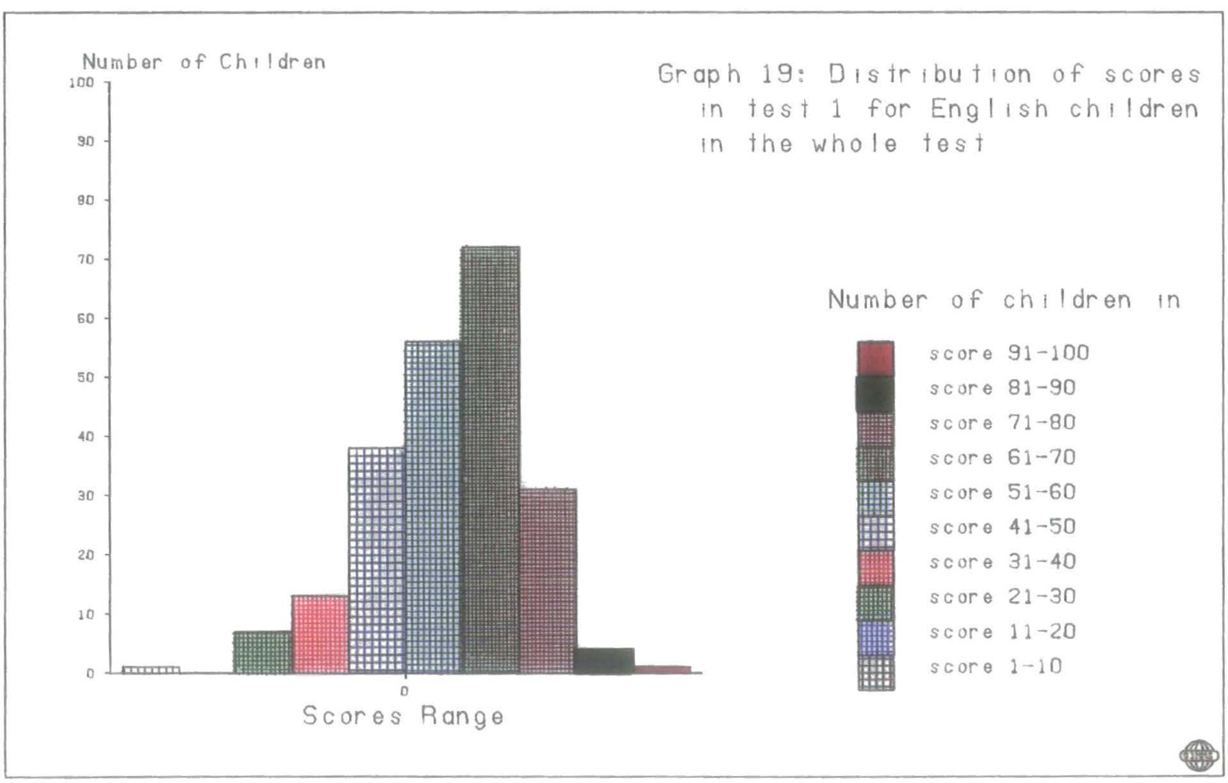
The mean score of English children in the test was 58.53 and standard deviation was 13.93. Although the mean score of the whole test of English children was lower than that of the Egyptian one, the standard deviation for Egyptian children was 20.15, higher than English result of 13.93. These results indicates the homogeneity of English scores as opposed to the divergence of the Egyptian scores. The differences between Egyptian mean and English mean scores were significant at the 5% level. The T-Test between the two means was 3.165. Ranges of scores were obtained for the 223 children in the tests as a whole and are shown in table 18.

Table 18: Distribution of scores in test 1 for English children in the whole test

Scores Range	1-10	11-20	21-30	31-40	41-50	51-60
Number of Children	1	0	7	13	38	56
Scores Range	61-70	71-80	81-90	91-100	Total Child.	
Number of Children	72	31	4	1	223	

36 English pupils out of 223, or 16.14%, scored more than 70, while 47% of the Egyptian children were in the same range. In contrast 26.46% of the English children scored less than 51, and 22.0% of the Egyptian fell into the same range. Between 50 and 71, there were 57.40% English children and 31% Egyptian children.

Graph 19 revealed distribution of scores for all pupils in test 1 for England.



Number of Children
100
90
80
70
60
50
40
30
20
10
0
0
Scores Range
Graph 19: Distribution of scores in test 1 for English children in the whole test
Number of children in
score 91-100
score 81-90
score 71-80
score 61-70
score 51-60
score 41-50
score 31-40
score 21-30
score 11-20
score 1-10

(II) The result of the whole test through age range.

The mean and standard deviation for the whole test through age ranges are indicated below:

Table 19: Mean score and standard deviation of English pupils classified according to age range

age range	7.0& 7.1& 7.2	7.3& 7.4& 7.5	7.6& 7.7& 7.8	7.9 & 7.10& 7.11	8.0& 8.1& 8.2	8.3& 8.4& 8.5	8.6& 8.7& 8.8	8.9 & 8.10& 8.11
Mean	44.3	50.1	53.1	53.6	58.1	61.7	58.6	64.6
S. D	31.8	25.2	14.8	16.7	14.6	18.8	16.1	17.6
Number of Children	3	8	34	29	28	35	19	20

age range	9.0& 9.1& 9.2	9.3& 9.4& 9.5	9.6& 9.7& 9.8
Mean	62.8	63.5	61.0
S. D	23.4	17.0	0.0
Number of Children	21	25	1

Each age range has been analysed separately and an overall indication of performance has been obtained for each by taking the mean and standard deviation of the results from their different age groups. The pattern of results obtained in this part was similar to that described in Egyptian results in relation to the mean increasing with age as would be expected.

This similarity in the general picture applied to the test as a whole. The gradual increase of scores by age is more noticeable in the English results than in the Egyptian scores. The results show some indications of the different levels of understanding that pupils have of the various arithmetical concepts and skills in mathematics.

Different skills

An analysis was made to focus on different skills of the test for English responses. The performances of English children for the different skills in the age range 7 to 9 years are discussed under two headings:

(i) The result of the different skills for all children.

(ii) The result of the different skills in age ranges, each covering three months.

Table 20: The mean and standard deviation of all children for different skills in English test 1

Different Skills	Transf-erence	Procedural	Logical	Applied	Problem Solving
Mean	17.87	12.00	11.64	8.98	8.04
Standard Deviation	3.45	4.38	3.60	2.58	4.33

Table 21: T-Test for the mean of different skills in the whole test for English children in test 1

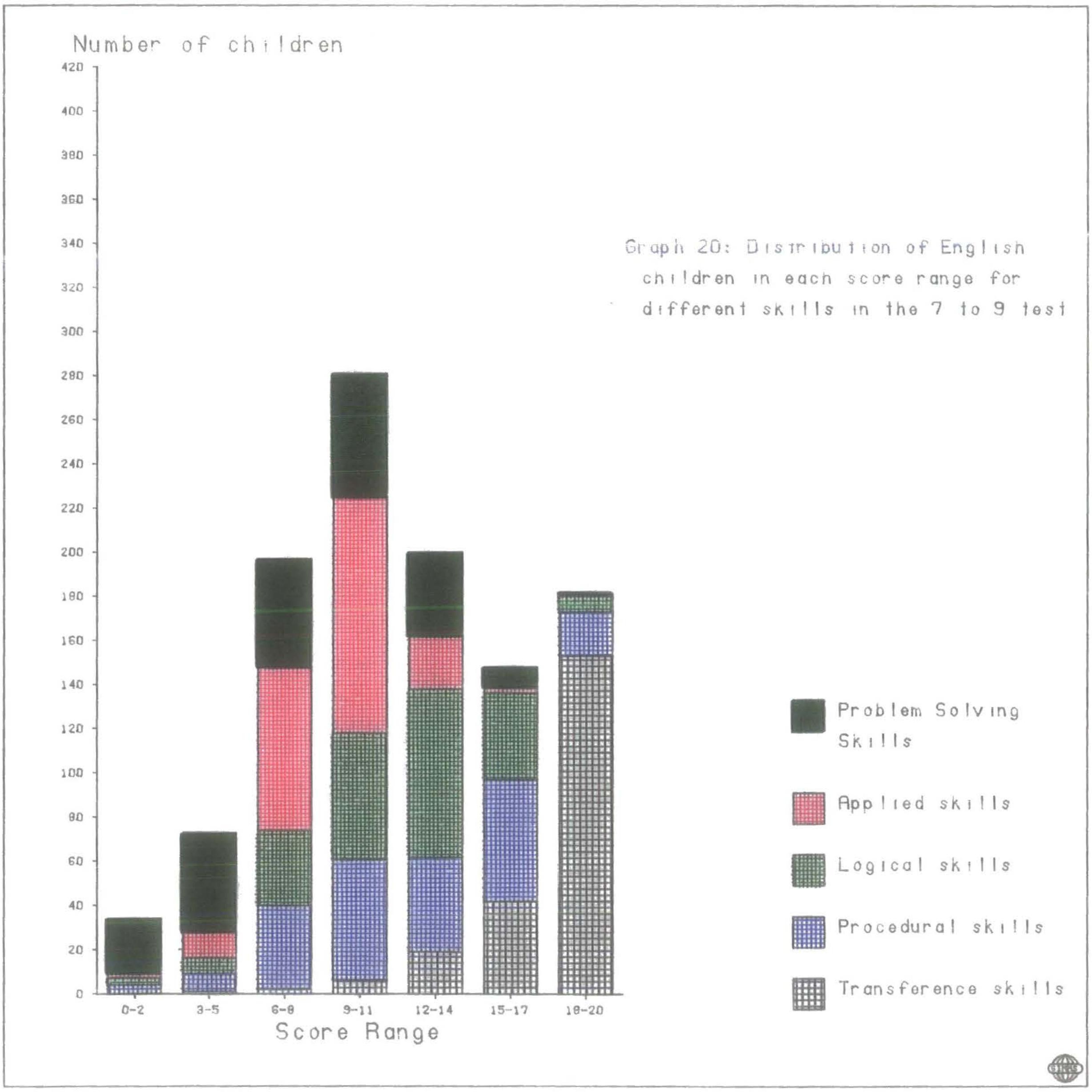
	Transf-erence	Procedural	Logical	Applied	Problem Solving
Trans.	-	*15.69	*18.62	*30.75	*26.45
Proc.			0.95	*8.85	*9.58
Log.				*8.95	*9.53
Appl.					*2.78

*Indicates significance at 0.01 level.

Each category of skill has been analysed separately and an overall indication of performance has been obtained for each by taking the mean of the results from the different parts. The mean scores as a percentage of the possible score of twenty were 89.35%, 60%, 58.2%, 44.9%, and 40.2% on transference, procedural, logical, applied, and problem solving skills respectively. The order of skills, according to chidren's mean scores is transference, procedural, logical, applied, and problem solving skills. This supports the categorisation and hierarchy of skills proposed in this thesis which is based on information gained before and through the research, as explained previously in chapter seven.

Graph 20 shows the distribution of English children in each score range for different skills in the 7 to

9 test.



Graph 20: Distribution of English children in each score range for different skills in the 7 to 9 test

Table 22: Number of Engilsh children in each score range for different skills in the 7 to 9 test

Skills\Range \Score	0-2	3-5	6-8	9-11	12-14	15-17	18-20
Transference Skills	0	1	2	6	19	42	153
Procedural Skills	4	8	38	55	43	55	20
Logical Skills	3	7	34	57	76	39	7
Applied Skills	2	12	74	107	24	3	1
Problem Solving Skills	25	45	49	56	38	9	1

With transference skills, 68.61% of the children scored from 18 to 20, 27.35% scored from 12 to 17, 2.69% scored from 9 to 11, while only 1.35% scored lower than 9.

In procedural skills, 8.97% scored from 18 to 20, 43.95% scored from 12 to 17, 24.66% scored 9 to 11, and 22.42% were lower than 9.

There were 3.14% only in the 18 to 20 range in logical skills with 51.57% in the 12 to 17 range, 25.56% scored 9 to 11, and 19.73% of the children scored lower than 9.

With applied skills, 0.45% only scored from 18 to 20, 12.11% were in the 12 to 17 range, 47.98% scored 9 to 11, and 39.46% scored from 0 to 8.

Finally, 0.45% were in the range 18 to 20, 21.08% scored 12 to 17, 25.11% scored 9 to 11, and 53.36% obtained 0 to 8 score for problem solving skills.
The children found greatest ease with transference skills since 87.44% of the children scored from 15 to 20, while the percentages of children in that range of scores for procedural, logical, applied, and problem solving skills were, 33.63%, 20.63%, 1.79% and 4.48% respectively. Logical, applied, and problem solving skills were found to be hardest with only 3.14%, 0.45%, and 0.45% of the children in the upper range of scores in logical, applied, and problem solving skills respectively. In contrast, the percentage of children who scored lower than 9 tended to increase from transference to problem solving skills, as the hierarchy of skills suggests, the percentages being 1.35%, 22.42%, 19.73%, 39.46% and 53.36% respectively. The highest percentage of children in the range 12 to 17 was 51.57% in logical skills followed by procedural at 43.95%, then transference, problem solving, and applied skills at 27.35%, 21.08%, and 12.11% respectively.

Graph 21 illustrates the distribution of the mean of scores in the different skills for English children in test 1.

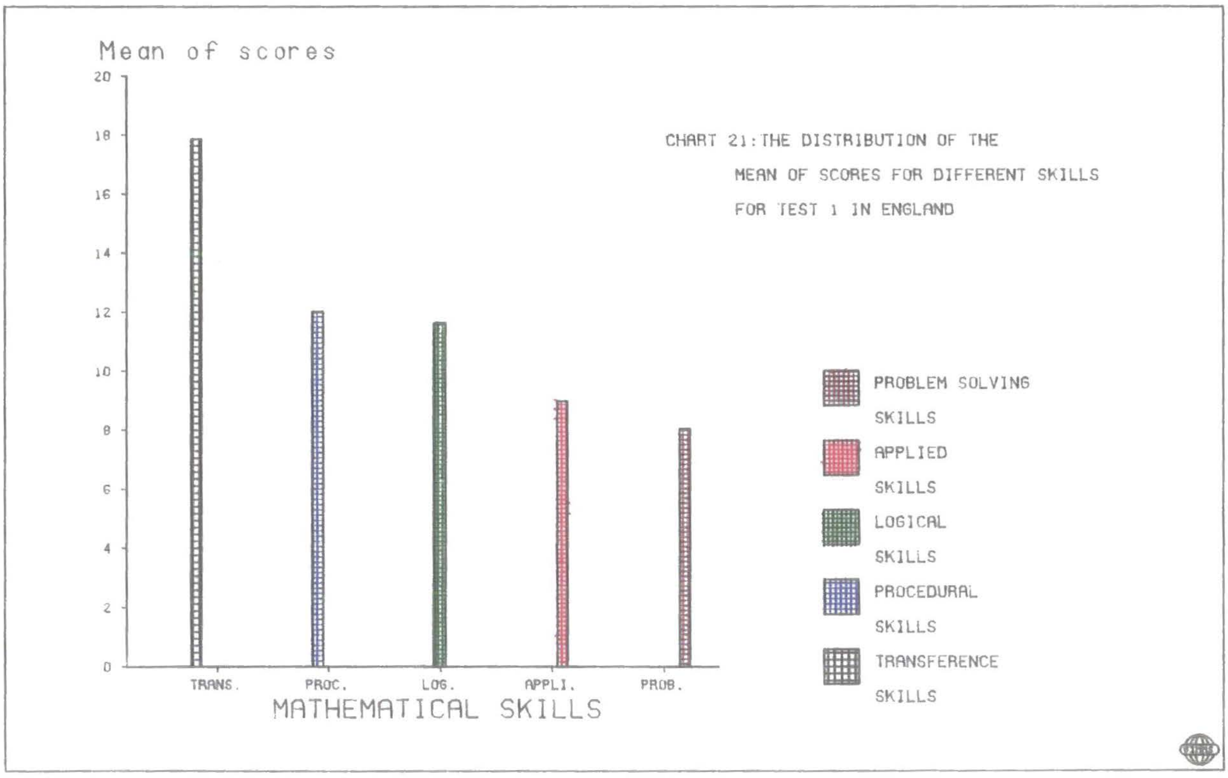


CHART 21: THE DISTRIBUTION OF THE MEAN OF SCORES FOR DIFFERENT SKILLS FOR TEST 1 IN ENGLAND

(ii) The result of the different skills through different age ranges.

Mean scores and standard deviation were obtained for age ranges, each covering three months from 7 to 9 years for the English children in different skills.

Table 23: Mean scores and standard deviations of Engish pupils for different skills through the age ranges, each covering three months, in test 1

age range		7.0& 7.1& 7.2	7.3& 7.4& 7.5	7.6& 7.7& 7.8	7.9 & 7.10& 7.11	8.0& 8.1& 8.2	8.3& 8.4& 8.5	8.6& 8.7& 8.8	8.9 & 8.10& 8.11
Mean	TRA	12.3	14.6	16.6	17.2	18.4	18.6	18.0	18.9
S. D		8.74	7.35	4.14	4.69	4.10	4.15	4.68	4.55
Mean	PRO	9.3	9.6	11.5	11.2	11.7	13.4	12.1	13.2
S. D		7.74	6.35	4.41	5.09	4.34	5.23	4.91	4.55
Mean	LOG	10.0	11.4	9.8	10.6	11.4	12.0	12.4	13.2
S. D		7.35	5.97	3.57	4.00	4.04	4.35	4.16	4.23
Mean	APP	8.3	8.0	8.6	8.3	8.9	9.7	8.5	9.7
S. D		6.09	3.82	2.58	2.57	2.55	3.56	2.59	2.86
Mean	SOL	4.3	6.5	6.6	6.3	7.8	8.1	7.7	9.7
S. D		3.12	4.00	4.16	4.24	3.88	4.53	4.10	4.90

Age Range		9.0& 9.1& 9.2	9.3& 9.4& 9.5	9.6& 9.7& 9.8
Mean	TRA	18.1	19.4	20.0
S. D		5.54	5.80	0.00
Mean	PRO	11.9	12.4	7.0
S. D		5.81	4.82	0.00
Mean	LOG	12.1	13.2	11.0
S. D		5.21	3.66	0.00
Mean	APP	9.19	9.52	9.00
S. D		4.08	3.70	0.00
Mean	SOL	11.4	9.0	14.0
S. D		5.33	4.58	0.00

The pattern of results obtained in the above table was similar to that described in Egyptian results table 11, in relation to performance according to different ages in various skills. This similarity in the general picture applied to the test of skills as a whole and also within the different skills. The general trend of performance in the skills is to improve as the children become older.

(IV) Difference in performance between boys and girls.

Data is also recorded for boys and girls separately. In the test as a whole and within the categories of skills the mean percentage scores were computed and this data is displayed in table 24. Girls had higher mean scores in the whole test, with 59.15% as opposed to 57.84% for boys. The standard deviations were 13.81% for girls and 15.19% for boys. The differences between them are not significant, since the T-Test value between girls and boys is T= 0.6692.

Table 24: The mean and standard deviation for different skills for boys and girls in England in test 1.

Different Skills		Transf-erence	Procedural	Logical	Applied	Problem Solving
Mean	BOYS	17.64	11.45	11.75	9.19	7.81
S. D	BOYS	3.85	4.59	3.96	2.88	4.51
Mean	GIRLS	18.08	12.51	11.54	8.78	8.23
S. D	GIRLS	3.46	4.29	3.42	2.43	4.22

Number of Children		
Number of Children	Boys	107
Number of Children	Girls	116

Table 25 shows the T-Test result of different skills for English boys in test 1.

Table 25: T-Test for the mean of different skills for English boys in test 1

	Transf-erence	Procedural	Logical	Applied	Problem Solving
Trans.	-	*10.64	*10.98	*18.09	*17.07
Proc.			0.51	*4.29	*5.82
Log.				*5.38	*6.76
Appl.					*2.66

* indicates significance at 0.01% level.

Table 26 shows the T-Test result of different skills for English girls in test 1.

Table 26: T-Test for the mean of different skills for English girls in test 1

	Transf-erence	Procedural	Logical	Applied	Problem Solving
Trans.		*10.84	*14.42	*23.59	*19.36
Proc.			1.90	*8.11	*7.63
Log.				*7.05	*6.53
Appl.					1.21

*denotes that the differences are significant at the 0.01 percent level.

Table 27 shows T-Test result of different skills among boys and girls.

Table 27: T-Test for the mean of different skills between boys and girls for England in test 1

BOYS GIRLS	Transf-erence	Procedural	Logical	Applied	Problem Solving
SD	0.89	1.77	0.42	1.14	0.72

None of the differences between boys' and girls' results are indicated as being statistically significant at either the 5 percent or 1 percent level.

Further analyses were carried out to study the differences between skills for boys as well for girls. It was found that the pattern of performance was in general similar to Egyptian one. The differences among skills are almost all significant both for boys and girls. The order of achievement for both boys and girls tends to be transference, procedural, logical, applied, and problem solving skills as would be expected.

In order to differentiate between the performance of boys and girls the number of boys and girls in each score range for the whole test is given in table 28 below.

Table 28: The scores of English boys and girls for the whole test

Scores Range	1-10	11-20	21-30	31-40	41-50	51-60
No. Boys	0	0	5	8	17	29
No. Girls	1	0	2	5	21	27

Scores Range	61-70	71-80	81-90	91-100	Tot.No of ch.
No. Boys	31	13	3	1	107
No. Girls	41	18	1	0	116

There was no pattern in the differences between boys and girls in the whole test in England. While girls' success rates were marginally higher than boys in general, boys scored 3.74% against girls with 0.86% in the 81 to 100 range. There were 25% of the girls and 28.04% of the boys lower than 51 with 74.14% girls and 68.22% boys between 50 to 81.

The whole test results for different skills were also studied for boys and girls separately as shown in table 29 below.

Table 29 show the scores for boys and girls for different skills in England in test 1.

Table 29: Number of English boys and girls in each score range for different skills in the 7 to 9 test

Skills\Range \Score		0-2	3-5	6-8	9-11	12-14	15-17	18-20
Transf-erence Skills	B	0	0	1	4	9	29	64
	G	0	1	1	2	10	13	89
Proced-ural Skills	B	3	5	19	31	17	24	8
	G	1	3	19	24	26	31	12
Logical Skills	B	2	4	16	23	36	21	5
	G	1	3	18	34	40	18	2
Applied Skills	B	0	8	31	52	12	3	1
	G	2	4	43	55	12	0	0
Problem Solving Skills	B	13	24	22	26	17	4	1
	G	12	21	27	30	21	5	0

With transference skills, 59.81% of boys and 76.72% of girls scored 18 to 20, 35.51% of boys and 19.83% of girls scored 12 to 17, 3.74% of boys and 1.72% of girls were in the 9 to 11 score, while 0.93% of boys and 1.72% of girls scored lower than 9.

In procedural skills, 7.48% of boys and 10.34% of girls scored 18 to 20, 38.32% of boys and 49.14% of girls scored 12 to 17, 28.97% of boys and 20.69% of girls scored 9 to 11, while 25.23% of boys and 19.83% of girls scored lower than 9.

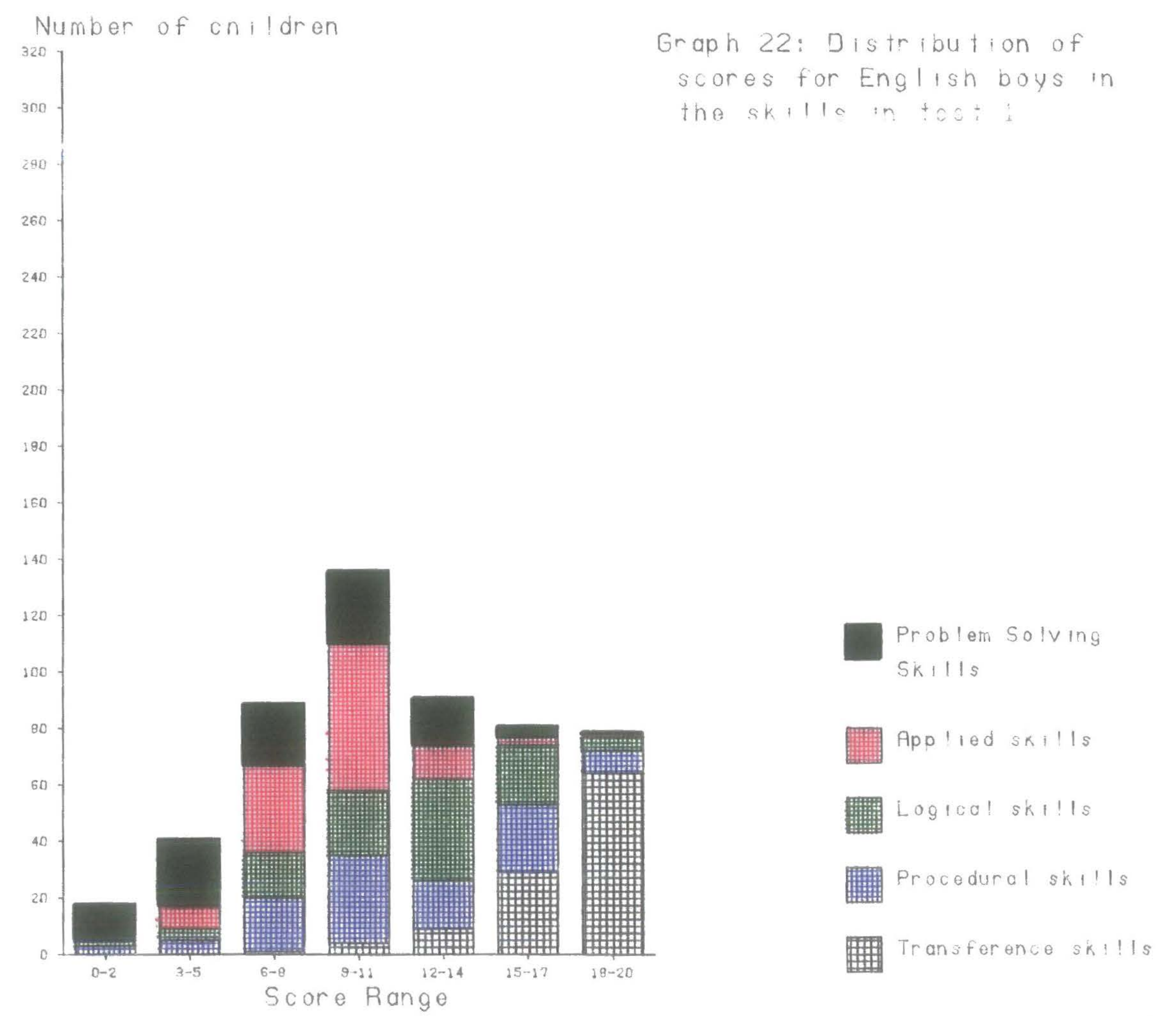
In logical skills, 4.67% of boys and only 1.72% of girls scored 18 to 20, 53.27% of boys and 50% of girls in the 12 to 17 range, 21.50% of boys and 29.31% of girls scored 9 to 11, while 20.56% of boys and 18.97% of girls scored lower than 9.

In contrast to the differences of the levels of both boys and girls in the skills mentioned above, the level of achievement for both decreases in the applied and problem solving skills. A higher percentage of boys and girls got lower scores than 9, and a lower percentage of them got higher scores on applied and problem solving skills. It was found that only 0.93% of boys and no girls scored 18 to 20, 14.02% of boys and 10.34% of girls obtained 12 to 17, 60% of boys and 47.41% of girls were in the 9 to 11 range, and 36.45% of boys and 42.24% of girls scored lower than 9 on applied skills.

In problem solving also, a higher percentage of boys and girls scored lower than 9, 55.14% of boys and 51.72% of girls scored lower than 9, while only 0.93% of boys and no girls had from 18 to 20, 19.63% of boys and 22.41% of girls obtained 12 to 17, and 24.30% of boys and 25.86% of girls were in the 9 to 11 range.

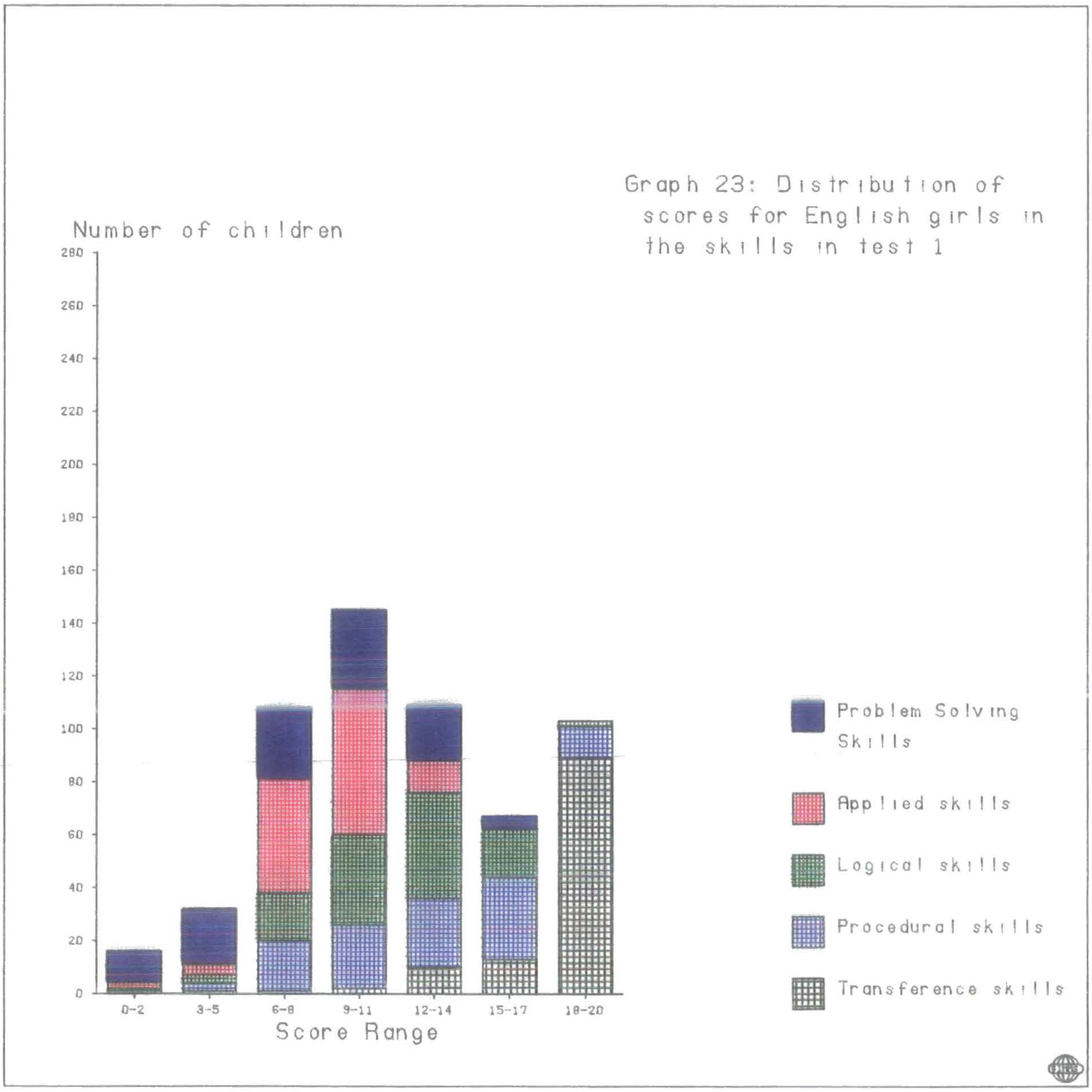
The numbers of boys and girls in each score range for the whole test and for each of the skills are shown in graphs 22, 23, and 24.

Graph 22 illustrates the distribution of scores for English boys in the skills in test 1.



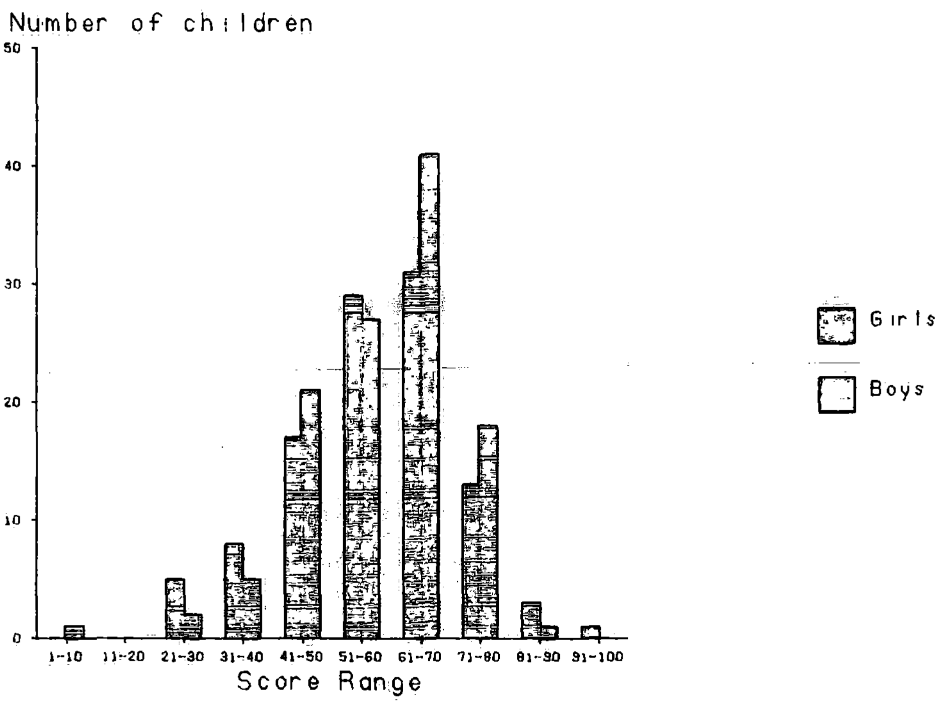
Graph 22: Distribution of scores for English boys in the skills in test 1

Graph 23 illustrates the distribution of scores for English girls in the skills in test 1.


Graph 23: Distribution of scores for English girls in the skills in test 1
Number of children
280
260
240
220
200
180
160
140
120
100
80
60
40
20
0
0-2
3-5
6-8
9-11
12-14
15-17
18-20
Score Range
Problem Solving Skills
Applied skills
Logical skills
Procedural skills
Transference skills

Graph 24 illustrates the distribution of scores for English boys and girls in the whole test in test 1.

Graph 24: Distribution of scores for boys and girls for England in the whole test 1



The picture which emerges here of the overall performance and performance in relation to the skills is very similar to that of the Egyptian children in test 1, which covers the 7 to 9 year olds. No valid or meaningful comparisons can be made between these two investigations since, even where the results differ, one research study cannot provide absolute evidence of the presence of a trend. Such evidence, if such there be, cannot be produced clearly, accurately and precisely until further research has been conducted and any differences between Egyptian and English results can be shown either to be due to chance variation or to be part of a general long-term trend. This research spotlights the need to have more investigations in this area.

THE RESULTS OF THE 9 TO 11 AGE RANGE TESTS.

The first set of results was presented with in-depth comment and detailed analyses of performances of 7 to 9 age range subjects; the second set of results relates in the same way to performances of 9 to 11 range subjects in mathematical skills tests.

Both sets of results provide, in combination, an overall view of mathematical skills performance in the primary schools. This discussion of the second test continues to aim at the presentation of a picture of the mathematical performance of primary school children in Egypt and England. It continues the explanations given in the first test for children aged 7 to 9 years old, thus covering most of the primary school age range. This analysis is of the test for children aged 9 to 11 years old.

The framework is the same as that used in the first test. Statistical methods in this thesis are concerned with measurement of performance of primary school children in tests of mathematical skills. Statistical methods are merely ways of describing the results as simply, yet as fully, as possible.

(A) THE EGYPTIAN RESULTS

(I) The results for the whole test

Mean score and standard deviation for the overall scores of test 2 were computed. Mean score was 61.52 and standard deviation was 14.14.

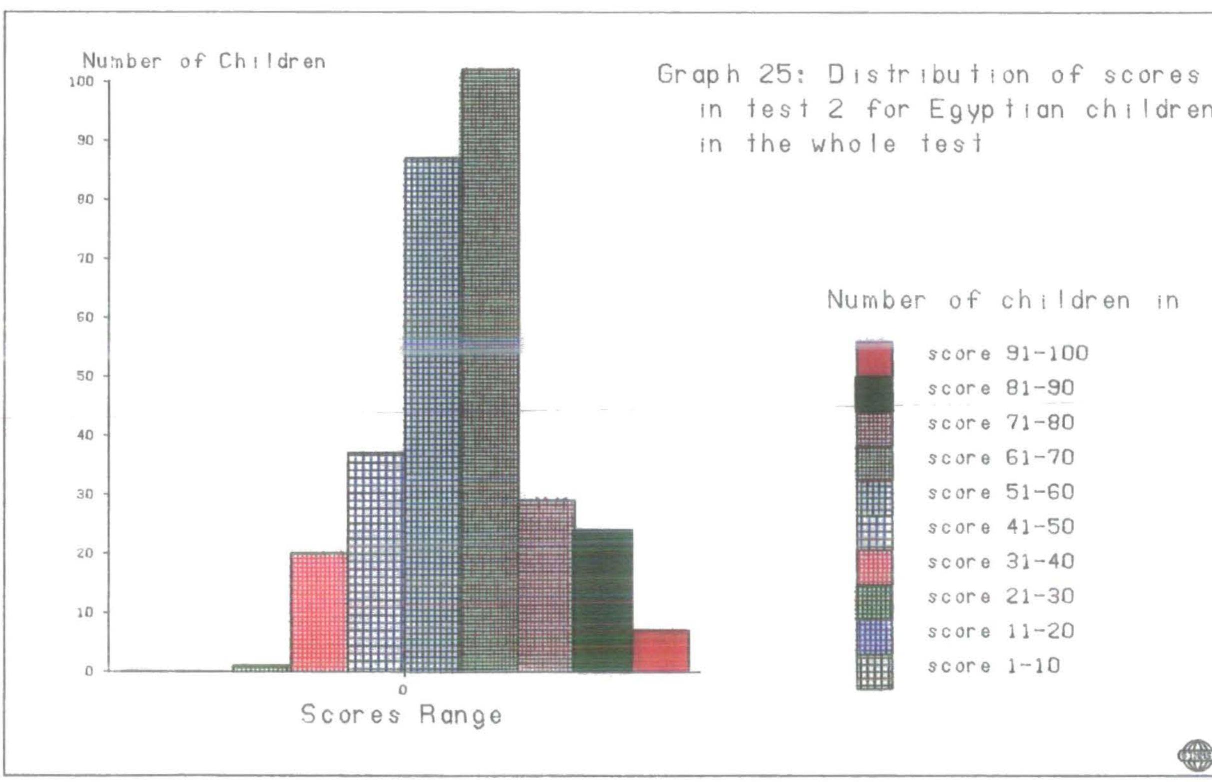
The number of children in each score range is displayed in table 30, which is below:

Table 30: Distribution of scores in test 2 for Egyptian children in the whole test

Scores Range	1-10	11-20	21-30	31-40	41-50	51-60
Number of Children	0	0	1	20	37	87

Scores Range	61-70	71-80	81-90	91-100	Total Child.
Number of Children	102	29	24	7	307

Graph 25 shows the distribution of scores for all pupils in the second Egyptian test of skills.



Number of Children
100
90
80
70
60
50
40
30
20
10
0
0
Scores Range
Graph 25: Distribution of scores in test 2 for Egyptian children in the whole test
Number of children in
score 91-100
score 81-90
score 71-80
score 61-70
score 51-60
score 41-50
score 31-40
score 21-30
score 11-20
score 1-10

This distribution is quite normal. It tends to be a humped or bell-shaped curve, symmetrical about the mean. Many points are distributed in a way that conforms closely to the normal curve. For instance, 61.56% of the children scored from 51 to 70, 17.26% recorded 71 to 90 and 2.28% scored 91 to 100. In contrast, 12.05% were in the 41 to 50 bracket, 6.51% ranged from 31 to 40, and 0.33% scored from 21 to 30.

(II) The result of the overall scores of Egyptian test 2 through age ranges, each covering three months.

The pattern of performance in age ranges, each covering three months of the mean and standard deviation of the overall Egyptian test 2 is described below in table 31.

Table 31: Mean score and standard deviation of Egyptian pupils classified according to age range in test 2

age range	8.6& 8.7& 8.8	9.0& 9.1& 9.2	9.3& 9.4& 9.5	9.6& 9.7& 9.8	9.9 & 9.10& 9.11	10.0& 10.1& 10.2	10.3& 10.4& 10.5
Mean	42.0	38.1	43.6	50.2	55.0	59.1	60.5
S. D	00.0	11.5	17.6	19.97	21.28	18.11	12.71
Number of Children	1	16	10	9	8	21	47

age range	10.6& 10.7& 10.8	10.9 & 10.10& 10.11	11.0& 11.1& 11.2	11.3& 11.4& 11.5	11.6 & 11.7 & 11.8	11.9 & 11.10& 11.11
Mean	59.88	59.94	68.11	74.73	76.85	76.21
S. D	12.38	12.92	15.42	20.89	25.80	24.45
Number of Children	51	50	45	22	13	14

Although there are some exceptions, the general trend of the mean tends to increase gradually through age ranges. For instance, in the age range 10.6-10.8 and 10.9-10.11 the mean scores dropped 59.88 and 59.94 respectively. The steady rate of increase is more noticeable and obvious than in the first test.

(III) Different skills

Dealing with the results of this test similarly to that described in test 1, the main points analysed are:

(i) The result of the different skills for all children.

(ii) The result of the different skills through age ranges, each covering three months for 9 to 11 years.

(i) The result of the different skills for all children

Table 32: The mean and standard deviation for all children in different skills for Egypt in test 2.

Different Skills	Transf- -erence	Procedural	Logical	Applied	Problem Solving
Mean	13.85	14.87	14.07	11.93	6.80
Standard Deviation	3.94	3.23	3.31	3.79	4.28

The mean scores of procedural and logical skills are slightly different from the expected hierarchy of skills in that they are higher than the mean for transference skills.

The significances of the differences between the mean scores were examined by a T-Test, the results of which are given below:

Table 33: T-Test value between skills for all children for Egyptian test of 9 to 11 years

	Transf- -erence	Procedural	Logical	Applied	Problem Solving
Trans.		*3.50	0.75	*6.14	*21.20
Proc.			*3.03	*10.33	*26.33
Log.				*7.44	*23.50
Appl.					*15.70

*Indicates the significance at 1% level.

The differences between skills, except that between transference and logical, are significant at 1% level.

The number of children in each score range for different skills for Egyptian children of 9 to 11 years are shown in table 34.

Table 34: Number of Egyptian children in each score range for different skills in the 9 to 11 test

\Score Skills\Range	0-2	3-5	6-8	9-11	12-14	15-17	18-20
Transference Skills	0	4	32	44	73	99	55
Procedural Skills	0	0	17	34	51	156	49
Logical Skills	1	2	19	38	80	132	35
Applied Skills	1	10	48	79	93	49	27
Problem Solving Skills	38	108	79	50	12	6	14

11.73% of children obtained lower than 9, 14.33% scored 9 to 11, 56.03% had 12 to 17, and 17.92% scored 18 to 20 in transference skills.

In procedural skills, 5.54% scored lower than 9, 11.07% scored 9 to 11, 67.43% scored from 12 to 17 and 15.96% scored from 18 to 20.

7.17% scored lower than 9, 12.38% scored 9 to 11, 69.06% scored between 12 and 17, and 11.40% scored 18 to 20 in logical skills.

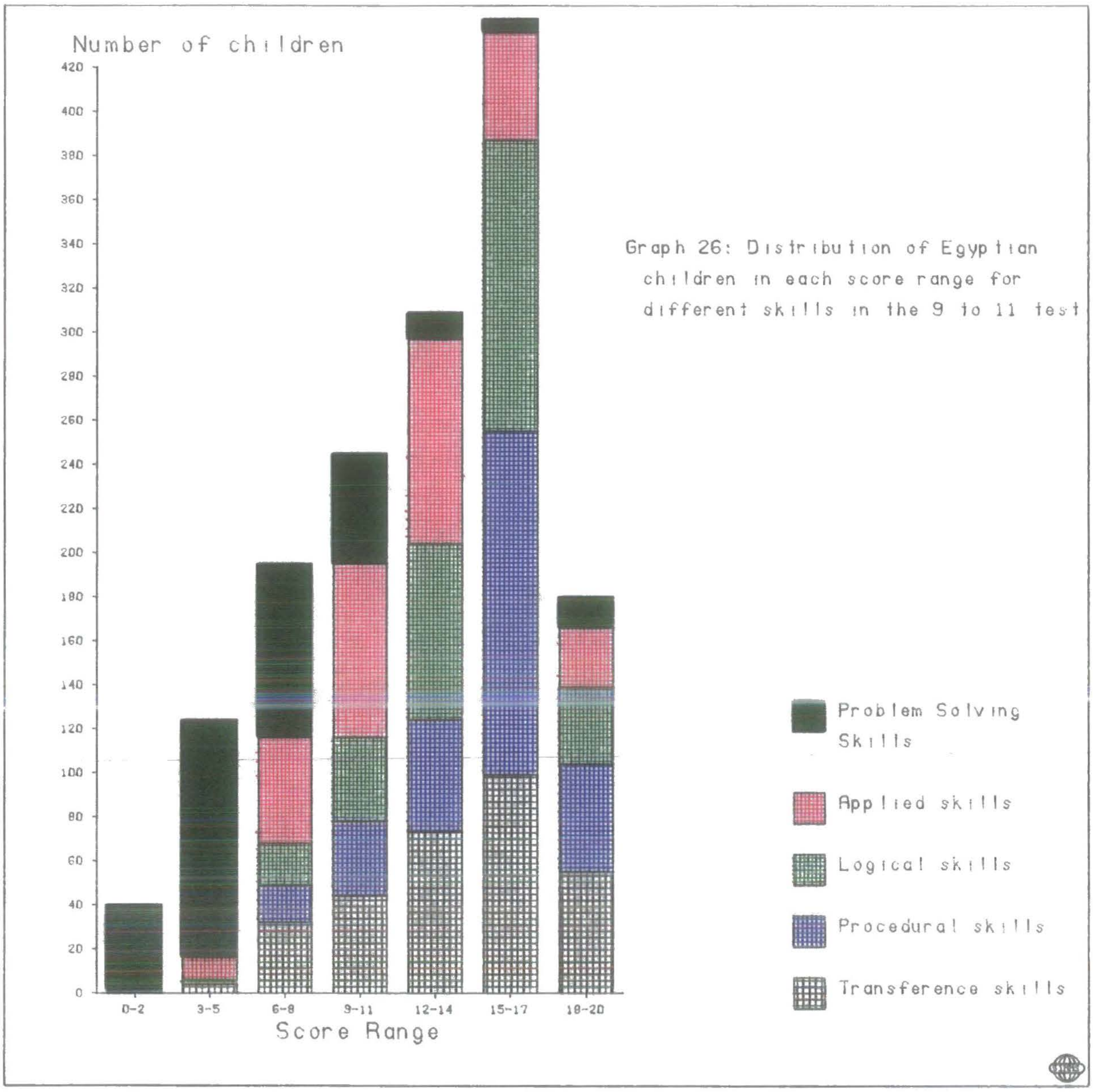
In applied skills, 19.22% scored lower than 9, 25.73% scored 9 to 11, 46.25% scored 12 to 17, and 8.79% scored between 18 to 20.

73.29% scored from 0 to 8, 16.29% scored 9 to 11, 5.86% scored 12 to 17, and 4.56% scored from 18 to 20 in problem solving skills.

In fact, a high percentage of children scored 15 to 20 on procedural skills compared to the other skills. This reflects the methods of teaching in schools, which mostly concentrate on mechanical operations on learning mathematics more than being concerned with understanding mathematical concepts.

Graph 26 shows the scores of all children for the different skills.

Graph 26: Distribution of scores for all pupils in the second Egyptian test for different skills.



Graph 26: Distribution of Egyptian children in each score range for different skills in the 9 to 11 test

<u>(ii) The result of the different skills for all children through the age ranges, each covering three months for 9 to 11 years</u>

Table 35: Mean scores and standard deviations of Egyptian pupils for different skills through the ages ranges in test 2.

age range		8.6& 8.7& 8.8	9.0& 9.1& 9.2	9.3& 9.4& 9.5	9.6& 9.7& 9.8	9.9 & 9.10& 9.11	10.0& 10.1& 10.2	10.3& 10.4& 10.5
Mean	T R	6.00	6.56	7.30	9.78	10.62	12.24	13.70
S. D	A	0.00	2.27	2.82	4.71	4.64	3.91	3.33
Mean	P R	15.0	8.81	11.4	14.2	16.87	14.33	14.72
S. D	O	0.00	2.71	4.77	5.79	6.56	4.53	3.66
Mean	L O	11.0	8.75	9.80	11.3	13.87	14.10	14.45
S. D	G	0.00	2.82	5.39	5.30	5.31	4.27	3.42
Mean	A P	8.00	9.12	10.0	9.67	8.00	11.38	11.70
S. D	P	0.00	3.49	4.52	5.12	3.70	4.11	3.60
Mean	S O	2.00	4.81	5.10	5.22	5.62	7.00	5.89
S. D	L	0.00	2.15	2.42	3.98	3.15	4.79	3.52

Age Range		10.6& 10.7& 10.8	10.9 & 10.10& 10.11	11.0& 11.1& 11.2	11.3& 11.4& 11.5&	11.6& 11.7& 11.8	11.9 & 11.10& 11.11
Mean	T R	13.16	14.46	15.73	17.77	18.62	18.57
S. D	A	3.41	2.91	3.94	4.10	5.55	5.38
Mean	P R	15.12	15.12	16.16	15.77	15.69	16.71
S. D	O	3.35	3.14	3.40	4.74	5.78	5.09
Mean	L O	13.88	13.52	15.09	16.14	17.62	16.79
S. D	G	3.26	3.27	3.38	4.25	5.61	5.22
Mean	A P	11.10	11.22	13.02	15.18	15.23	15.93
S. D	P	3.06	4.00	3.98	5.14	5.63	5.81
Mean	S O	6.63	5.62	8.11	9.86	9.69	8.21
S. D	L	3.64	3.61	4.67	6.28	6.47	6.34

Generally, as might be expected, means tend to increase with age. Again, the illustration from the results of different skills through the age ranges, each covering three months, produced in table 35 demonstrated the difficulty for many children in problem solving skills. The same difficulty is even more apparent in the results obtained for test one, as was illustrated on the graphs and bar charts.

(IV) Differences between boys and girls

As well as the overall mean scores for both the whole test and individual skills, the mean score and standard deviation were calculated for boys and girls separately.

(i) The mean scores of the whole test

The mean score of 150 Egyptian boys was 61.01 while the standard deviation was 13.05, and 62.01 for 157 girls with 14.22 as standard deviation. The T-Test value of the difference between the mean score of boys and girls was 0.6428 which is not significant at acceptable levels of confidence. Accordingly there are no significant differences between boys and girls in acquiring skills.

Table 36: Number of boys and girls for Egyptian test 2 in the test as a whole.

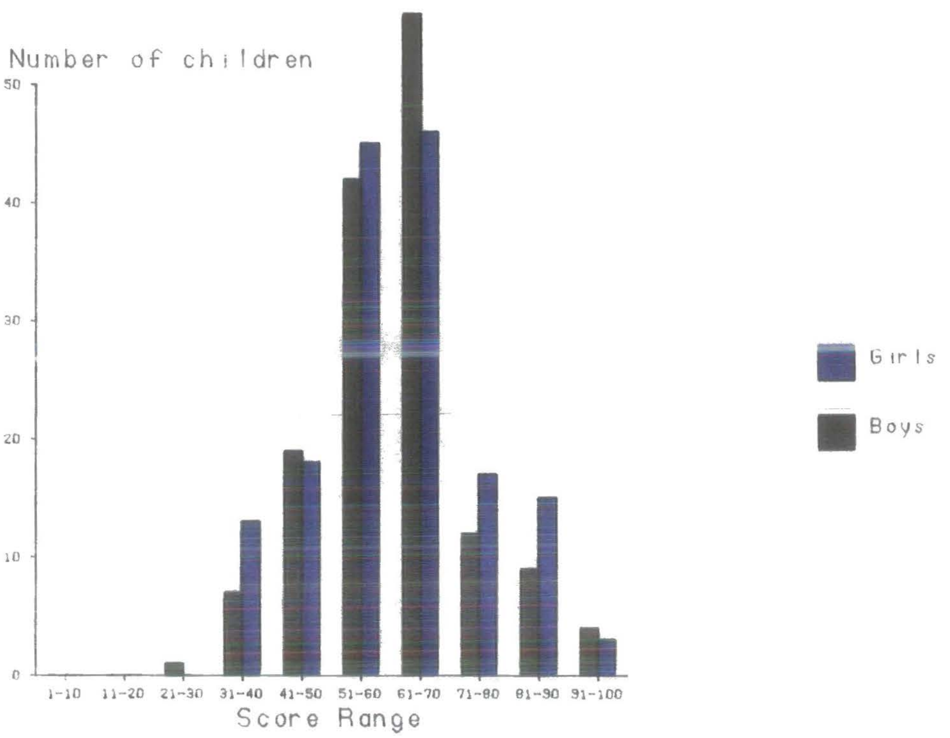
Scores Range	1-10	11-20	21-30	31-40	41-50	51-60
No. Boys	0	0	1	7	19	42
No. Girls	0	0	0	13	18	45

Scores Range	61-70	71-80	81-90	91-100	Tot.No of ch.
No. Boys	56	12	9	4	150
No. Girls	46	17	15	3	157

A concise description of boys and girls performances is given according to range of scores in the above table. 18% of boys and 19.75% of girls scored lower than 51, while 65.33% of boys and 57.96% of girls obtained 51 to 70, 8% of boys and 10.83% of girls scored 71 to 80, and 8.67% of boys and 11.46% of girls scored 81 to 100. A high percentage of boys and girls is concentrated in 51 to 70 range of scores.

Graph 27 shows the distribution of Egyptian boys and girls for the whole of test 2.

Graph 27: Distribution of scores for boys and girls for Egypt in the whole test 2



(ii) The mean score for the different skills

The mean and standard deviation of boys and girls for Egyptian test 2 in different skills were calculated.

Table 37: The mean and standard deviation for different skills for boys and girls in Egypt in test 2.
T-Test values for the differences between them

Different Skills		Transf--erence	Procedural	Logical	Applied	Problem Solving
Mean	BOYS	13.79	14.80	14.01	11.82	6.59
S. D		3.93	3.31	3.17	3.75	4.17
Mean	GIRLS	13.91	14.94	14.13	12.03	7.01
S. D		4.12	3.37	3.63	3.96	4.41
T*-Test Value		0.26	0.37	0.31	0.48	0.85

Number of Children		
	Boys	150
	Girls	157

None of the differences between boys and girls of the mean score for different skills was significant.

Table 38 shows the scores for boys and girls for different skills in Egypt in test 2.

Table 38: Number of Egyptian boys and girls in each score range for different skills in the 9 to 11 years test

Skills\Range		0-2	3-5	6-8	9-11	12-14	15-17	18-20
Transf--erence Skills	B	0	2	17	16	43	49	23
	G	0	2	15	28	30	50	32
Proced--ural Skills	B	0	0	7	17	27	77	22
	G	0	0	10	17	24	79	27
Logical Skills	B	0	1	8	19	43	68	11
	G	1	1	11	19	37	64	24
Applied Skills	B	1	5	18	45	52	16	13
	G	0	5	30	34	41	33	14
Problem Solving Skills	B	17	55	41	23	6	3	5
	G	21	53	38	27	6	3	9

In transference skills, 12.67% of boys and 10.83% of girls scored lower than 9, 10.67% of boys and 17.83% of girls scored 9 to 11, while 61.33 of boys and 50.96% of girls achieved 12 to 17, 15.33% of boys and 20.38% of girls scored 18 to 20.

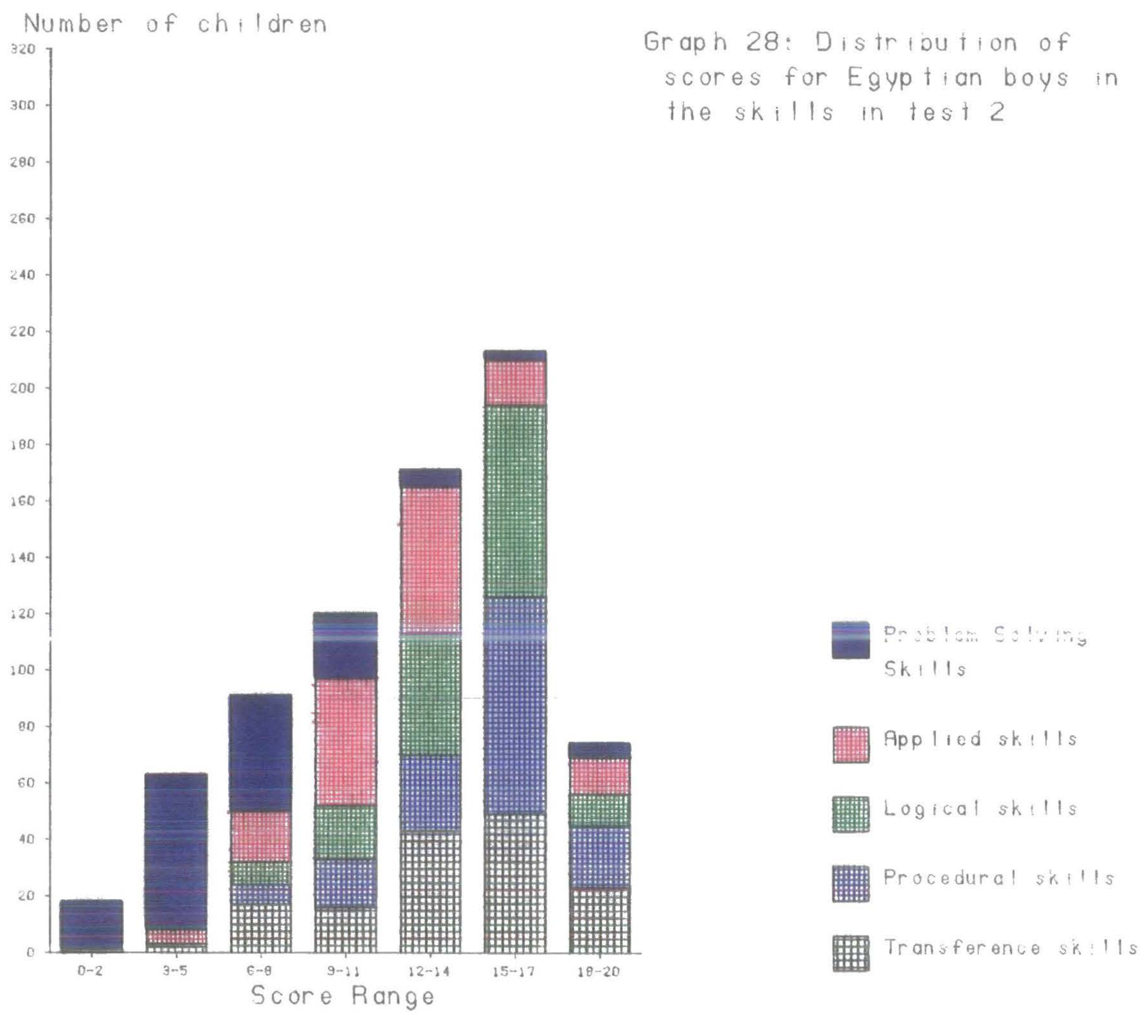
4.67% of boys and 6.37% of girls scored lower than 9, 11.33% of boys and 10.83% of girls scored 9 to 11, while 69.33% of boys and 65.61% of girls scored 12 to 17, 14.67% of boys and 17.20% of girls achieved 18 to 20 in procedural skills.

6% of boys and 8.28% of girls scored 0 to 8, 12.67% of boys and 12.10% of girls scored 9 to 11, whereas 74% of boys and 64.33% of girls scored 12 to 17, while 7.33% of boys and 15.29% of girls scored 18 to 20 in logical skills.

In applied skills, 16% of boys and 22.29% of girls scored 0 to 8, 30% of boys and 21.66% of girls scored 9 to 11, while 45.33% of boys and 47.13% of girls scored 12 to 17, 8.67% of boys and 8.92% of girls scored 18 to 20.

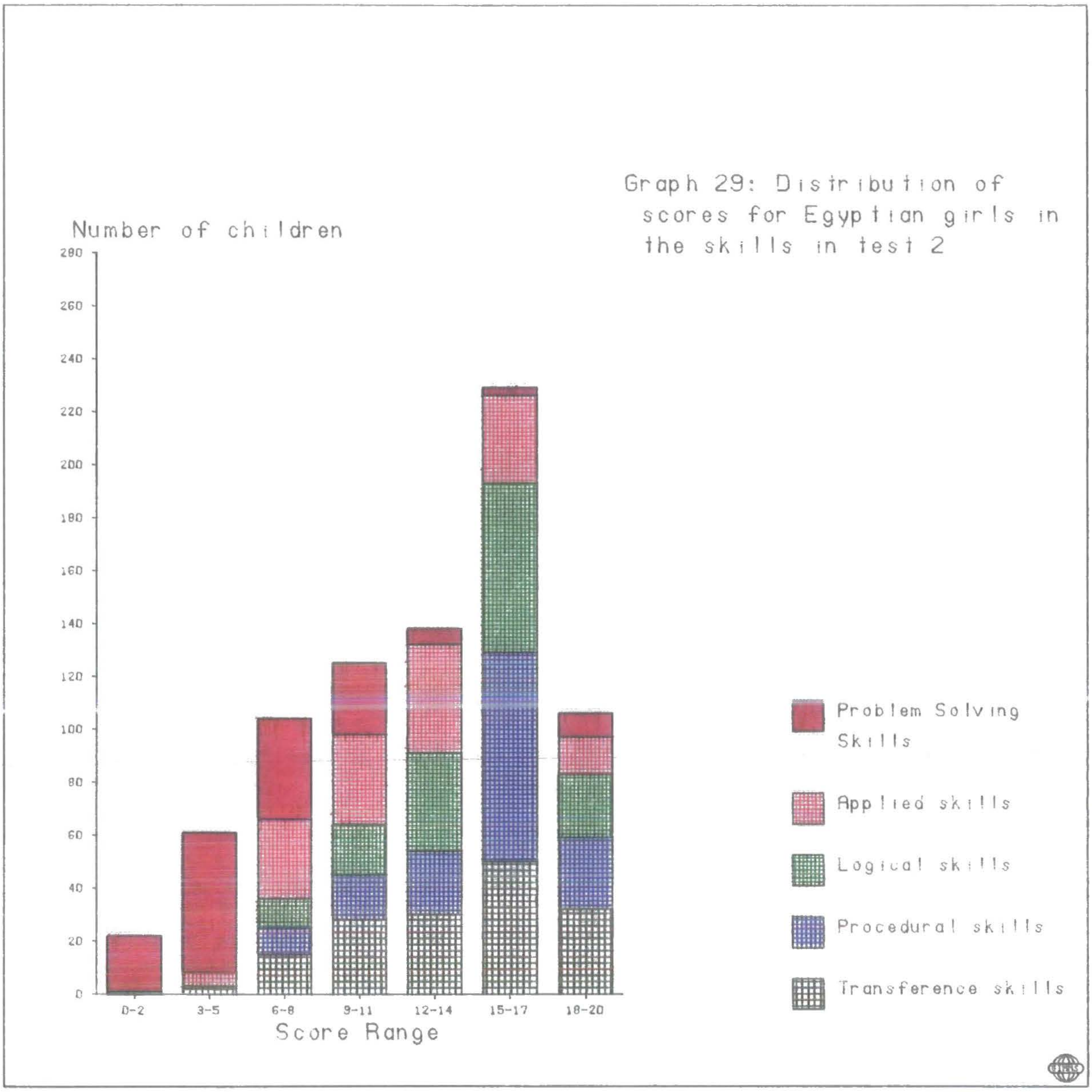
75.33% of boys and 71.34% of girls scored less than 9, 15.33% of boys and 17.20% of girls scored 9 to 11, while 6% of boys and 5.73% of girls scored 12 to 17, 3.33% of boys and 5.73% of girls obtained 18 to 20 in problem solving skills.
The different score ranges for boys and girls for different skills are illustrated in graph 28.

Graph 28 shows the distribution of scores for Egyptian boys in the skills in test 2.



Graph 28: Distribution of scores for Egyptian boys in the skills in test 2

Graph 29 shows the distribution of scores for Egyptian girls in the skills in test 2.



Graph 29: Distribution of scores for Egyptian girls in the skills in test 2
Number of children
280
260
240
220
200
180
160
140
120
100
80
60
40
20
0
0-2
3-5
6-8
9-11
12-14
15-17
18-20
Score Range
Problem Solving Skills
Applied skills
Logical skills
Procedural skills
Transference skills

(B) THE ENGLISH RESULTS

(I) The results for the whole test

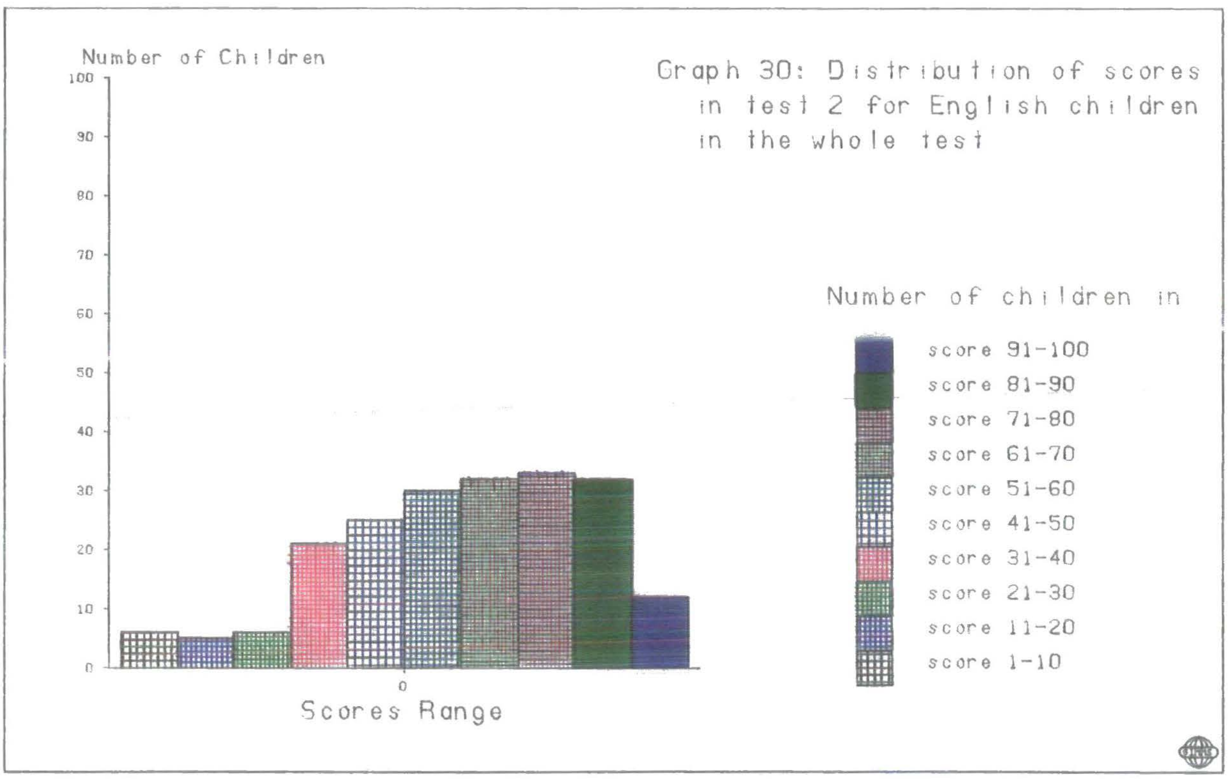
The mean score and standard deviation for the overall scores of English test 2 for 202 children were 60.85 and 22.39 respectively. The number of children in each score range is provided in table 39 below:

Table 39: Distribution of scores in test 2 for English children in the whole test

Scores Range	1-10	11-20	21-30	31-40	41-50	51-60
Number of Children	6	5	6	21	25	30

Scores Range	61-70	71-80	81-90	91-100	Total Child.
Number of Children	32	33	32	12	202

Graph 30: distribution of scores for all pupils in the second English test.



Graph 30: Distribution of scores in test 2 for English children in the whole test

This graph illustrates the performance of pupils with mathematical skills and their knowledge of the mathematical concepts.

31.19% of pupils scored 1 to 50, 30.69% scored 51 to 70, 32.18% obtained 71 to 90, and 5.94% scored 91 to 100.

It should be mentioned here, that the T-Test value between the mean score of Egyptian children and English children in the second test is 0.41, which is not significant.

<u>(II) The result of the English children in test 2 through age ranges, each covering three months.</u>

This second discussion focuses on the results obtained with 9 to 11 years olds. Whereas the first one analysed Egyptian results, the second investigates English results for pupils in the same 9 to 11 years age range.

The results of the second test are given in the table below:

<u>Table 40: Mean score and standard deviation of English pupils classified according to age range in test 2</u>

age range	8.9 & 8.10& 8.11&	9.3& 9.4& 9.5	9.6& 9.7& 9.8	9.9 & 9.10& 9.11	10.0& 10.1& 10.2	10.3& 10.4& 10.5	10.6& 10.7& 10.8
Mean	48.0	60.7	41.2	55.69	50.35	64.55	57.67
S. D	00.0	48.3	17.3	19.35	24.75	22.40	26.10
Number of Children	1	3	12	16	26	20	27

age range	10.9 & 10.10& 10.11	11.0& 11.1& 11.2	11.3& 11.4& 11.5	11.6 & 11.7 & 11.8	11.9 & 11.10& 11.11
Mean	59.88	67.06	73.5	70.83	74.0
S. D	26.11	27.63	24.87	33.91	0.0
Number of Children	25	35	30	6	1

Again, there is no gradual increase of scores with age as would be expected. This might be due to the small number of children in each range. However, it can be said that there is generally an increase of mean score with age.

<u>(III) Different skills</u>

This result dealt with pupils' understanding of the skills. Pupils were first asked to answer the transference skill items followed by procedural, logical, applied, and problem solving skill items. In order to demonstrate the results, the performance of English children for the different skills in the age range 9 to 11 are discussed under two headings then followed by a table.

(i) The result of the different skills for all children.

(ii) The result of the different skills in age ranges, each covering three months.

Table 41: The mean and standard deviation of all children for different skills in English test 2

Different Skills	Transf- -erence	Procedural	Logical	Applied	Problem Solving
Mean	16.39	13.34	12.91	13.30	4.91
Standard Deviation	4.96	5.41	5.47	6.08	5.52

Table 42: T-Test for the mean of different skills in the whole test for English children in test 2

	Transf- -erence	Procedural	Logical	Applied	Problem Solving
Trans.		* 5.89	* 6.68	* 5.58	*21.93
Proc.			0.79	0.07	*15.46
Log.				0.68	*14.59
Appl.					*14.48

* Indicates significance at 0.01 level.

Each skill has been interpreted and calculated separately and the overall indication of trend has been summarised for each by taking the mean and standard deviation of the scores of the items of the test for each skill.

Graph 31 shows the distribution of the English children's scores in test 2 for the different skills.

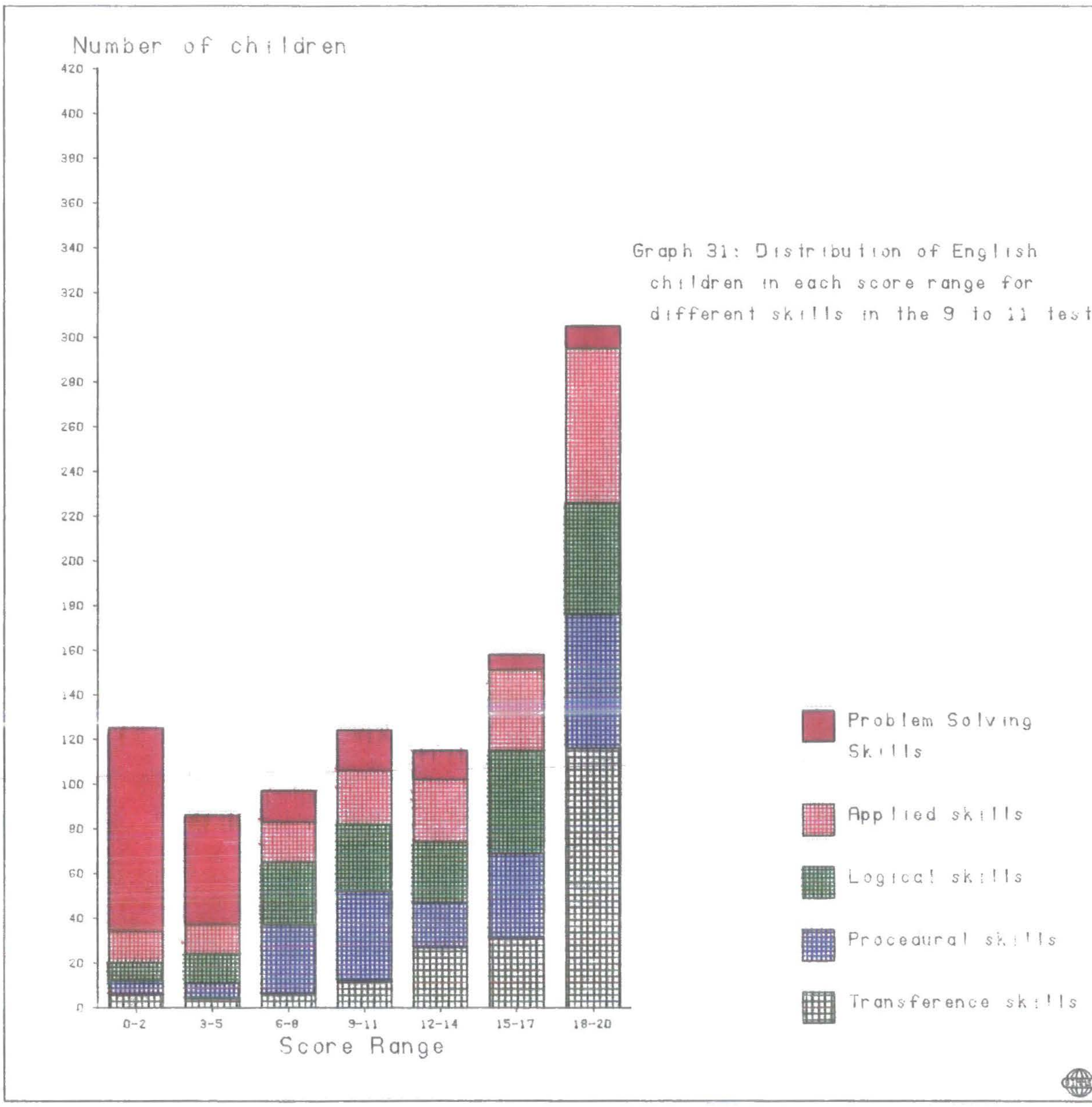

Number of children
420
400
380
360
340
320
300
280
260
240
220
200
180
160
140
120
100
80
60
40
20
0
0-2
3-5
6-8
9-11
12-14
15-17
18-20
Score Range
Graph 31: Distribution of English children in each score range for different skills in the 9 to 11 test
Problem Solving Skills
Applied skills
Logical skills
Procedural skills
Transference skills

Table 43: Number of English children in each score range for different skills in the 9 to 11 test

\Score Skills\Range	0-2	3-5	6-8	9-11	12-14	15-17	18-20
Transference Skills	6	4	6	12	27	31	116
Procedural Skills	6	7	31	40	20	38	60
Logical Skills	8	13	28	30	27	46	50
Applied Skills	14	13	18	24	28	36	69
Problem Solving Skills	91	49	14	18	13	7	10

In transference skills, 7.92% of children achieved lower than 9, 5.94% scored 9 to 11, 28.71% scored from 12 to 17, and 57.43% obtained 18 to 20.

With procedural skills, again a high percentage of children gained high scores, since 48.51% of them achieved 15 to 20, 9.90% scored from 12 to 14, 19.80% scored 9 to 11, and only 21.78% were in the 0 to 8 range.

In logical skills, similar percentages of children scored from 0 to 8, and 12 to 17, since 24.26% of them scored 0 to 8 and 36.14% scored 12 to 17, 14.85% scored 9 to 11, while 24.75% scored in the

18 to 20 range.

With applied skills, 51.98% of children scored 15 to 20, while 13.86% achieved 12 to 14, 11.88% scored 9 to 11, and 22.28% were in the 0 to 8 range.

In problem solving skills, children found difficulties as did the Egyptian children, since a high percentage of children, 76.24%, scored in the 0 to 8 range, 8.91% scored 9 to 11, while 6.44% scored 12 to 14, and 8.42% scored from 15 to 20.

(ii) The result of the different skills through different age ranges

The picture is drawn here of overall performance for the different skills in relation to the 9 to 11 year old range.

<u>Table 44: Mean scores and standard deviations of English pupils for different skills through the age ranges in test 2.</u>

age range		8.9 & 8.10 & 8.11	9.3 & 9.4 & 9.5	9.6 & 9.7 & 9.8	9.9 & 9.10 & 9.11	10.0 & 10.1 & 10.2	10.3 & 10.4 & 10.5	10.6 & 10.7 & 10.8
Mean	TRA	15.0	16.7	13.9	15.7	15.2	17.5	14.4
S. D		0.00	12.3	6.11	5.19	6.33	4.95	6.34
Mean	PRO	8.00	13.3	9.58	13.4	10.3	14.50	12.19
S. D		0.00	10.5	4.46	4.89	6.35	4.60	5.39
Mean	LOG	11.0	12.8	7.00	12.06	10.23	13.40	13.11
S. D		0.00	10.7	3.53	5.69	5.57	5.24	5.95
Mean	APP	11.0	14.7	8.92	12.12	11.08	14.05	13.19
S. D		0.00	11.6	5.53	6.38	6.44	5.66	6.23
Mean	SOL	3.00	3.33	1.75	2.44	3.50	5.10	4.78
S. D		0.00	6.24	2.72	2.36	4.16	5.52	6.41

Age Range		10.9 & 10.10 & 10.11	11.0 & 11.1 & 11.2	11.3 & 11.4 & 11.5	11.6 & 11.7 & 11.8 &	11.9 & 11.10 & 11.11
Mean	TRA	16.20	17.09	18.43	19.67	20.00
S. D		6.05	6.07	5.20	8.83	0.00
Mean	PRO	12.12	14.20	17.00	17.17	20.00
S. D		5.94	6.16	5.76	8.48	0.00
Mean	LOG	12.52	14.86	15.13	15.50	16.00
S. D		6.03	6.34	5.61	8.07	0.00
Mean	APP	13.04	14.11	16.13	14.83	15.00
S. D		6.05	7.19	6.25	11.84	0.00
Mean	SOL	5.20	6.80	6.80	3.67	3.00
S. D		5.12	6.84	6.21	4.87	0.00

The general picture of performance of English children in test 2 through ages 9 to 11 is similar to that described in the Egyptian one for age 9 to 11. Children generally found difficulties in problem solving skill items as is clear from the results in the above table.

(IV) Differences in performance between boys and girls

Comparisons between boys and girls for English children in test 2 were made in order to have a relative picture of the children's performance in primary school. Mean scores and standard deviations (for boys and girls) were calculated separately for the whole test and together for the different skills. The differences between boys and girls were discussed under two headings:

(i) The score for the whole test

(ii) The score for different skills

Boys had slightly higher mean scores in the whole test, with 61.11 as opposed to 60.596 for girls. The standard deviations were 24.137 for boys and 20.601

for girls. The value of T-Test between the mean of scores for boys and girls is 0.161 which is not significant, but the differences in standard deviation indicate that fewer girls scored extreme marks.

In order to give a clear picture of boys' and girls' performance, more detail in each score range for the whole test is produced below:

Table 45: Number of boys and girls for English test 2 in the whole test

Scores Range	1-10	11-20	21-30	31-40	41-50	51-60
No. Boys	5	1	1	12	14	13
No. Girls	1	4	5	9	11	17

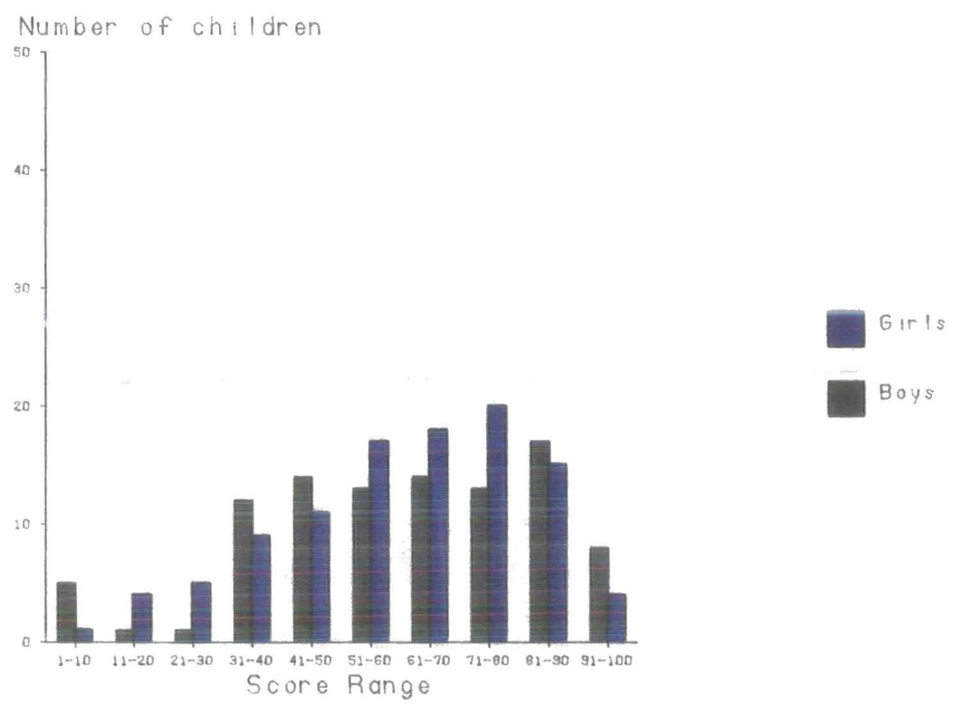
Scores Range	61-70	71-80	81-90	91-100	Tot.No of ch.
No. Boys	14	13	17	8	98
No. Girls	18	20	15	4	104

While a high percentage of boys (33.67%) relative to the percentage of girls (28.85%) achieved from 1 to 50 in the whole test, in contrast the percentage of boys who achieved from 81 to 100 (25.51%) is higher than the percentage of girls (18.27%). There were 40.82% of the boys and 52.88%

of the girls between the 50 and 81 scores.

Graph 32: distribution of scores for boys and girls in the whole of test 2 for England is shown in graph 32.

Graph 32: Distribution of scores for boys and girls for England in the whole test 2



(ii) The score for different skills

The results of the whole test for different skills were analysed for boys and girls separately as shown in tables 46 and 47 below:

Table 46: The mean and standard deviation for different skills for boys and girls in England in test 2.

T-Test values for the differences between them

Different Skills		Transf- -erence	Procedural	Logical	Applied	Problem Solving
Mean	BOYS	15.90	13.08	12.84	13.41	5.89
S. D		5.53	5.75	5.69	6.28	5.81
Mean	GIRLS	16.86	13.59	12.97	13.19	3.99
S. D		4.62	5.25	5.42	6.07	5.11
T*-Test Value		1.34	0.66	0.17	0.25	2.46

Number of Children		
	Boys	98
	Girls	104

There are only significant differences at 1% level in problem solving skills and in transference skills at the 5% level. Boys had the higher mean score for problem solving while girls had the higher mean in transference skills.

Table 47 show the scores for boys and girls for different skills in England in test 2.

Table 47: Number of English boys and girls in each score range for different skills in the 9 to 11 years test

Skills\Range	\Score	0-2	3-5	6-8	9-11	12-14	15-17	18-20
Transf- -erence Skills	B	5	2	1	8	16	10	56
	G	1	2	5	4	11	21	60
Proced- -ural Skills	B	4	3	17	20	12	11	31
	G	2	4	14	20	8	27	29
Logical Skills	B	4	8	14	13	12	21	26
	G	4	5	14	17	15	25	24
Applied Skills	B	7	5	9	15	12	15	35
	G	7	8	9	9	16	21	34
Problem Solving Skills	B	32	32	6	9	7	5	7
	G	59	17	8	9	6	2	3

The T-Test value for different skills for English boys and girls for Test 2 are illustrated in tables 47 and 48 respectively.

Table 48 shows the T-Test results of different skills for English boys in test 2.

Table 48: T-Test for the mean of different skills for English boys in test 2

	Transf-erence	Procedural	Logical	Applied	Problem Solving
Trans.		*3.4814	*3.798	*2.9307	*12.291
Proc.			0.2922	*0.3817	*8.6629
Log.				*0.6625	*8.4171
Appl.					*8.6569

* indicates significance at 0.01*% level.

Table 49 shows the T-Test result of different skills for English girls in test 2.

Table 49: T-Test for the mean of different skills for English girls in test 2

	Transf-erence	Procedural	Logical	Applied	Problem Solving
Trans.		*4.7455	*5.5433	*4.8827	*18.96
Proc.			0.8339	0.5058	*13.299
Log.				0.27437	*12.235
Appl.					11.7675

* denotes that the differences are significant at 0.01 percent.

The tables denote the interpretations of pupils' responses to test items. The analyses were carried out to study the relationships between skills for boys and girls.

In transference skills, 8.16% of English boys and 7.70% of girls obtained lower than 9, 26.53% of boys and 30.77% of girls scored from 12 to 17, 8.16% of boys and 3.85% of girls scored 9 to 11, while 57.14% of boys and 57.69% of girls scored 18 to 20.

With procedural skills, 24.49% of boys and 19.23% of girls scored lower than 9, 23.47% of boys and 33.65% of girls achieved a 12 to 17 score, 20.41% of boys and 19.23% of girls scored 9 to 11, while 31.63% of boys and 27.88% of girls scored from 18 to 20.

26.53% of boys and 22.12% of girls scored lower than 9, 13.27% of boys and 16.35% of girls scored 9 to 11, 33.67% of boys and 38.46% of girls obtained 12 to 17, while 26.53% of boys and 23.08% of girls scored from 18 to 20 in logical skills.

In applied skills, it was found that 21.43% of boys and 23.08% of girls scored lower than 9, 15.31% of boys and 8.65% of girls scored 9 to 11, 27.55% of boys and 35.58% of girls obtained from 12 to 17, while 35.71% of boys and 32.69% of girls achieved from 18 to 20.

With problem solving skills, 71.43% of boys and

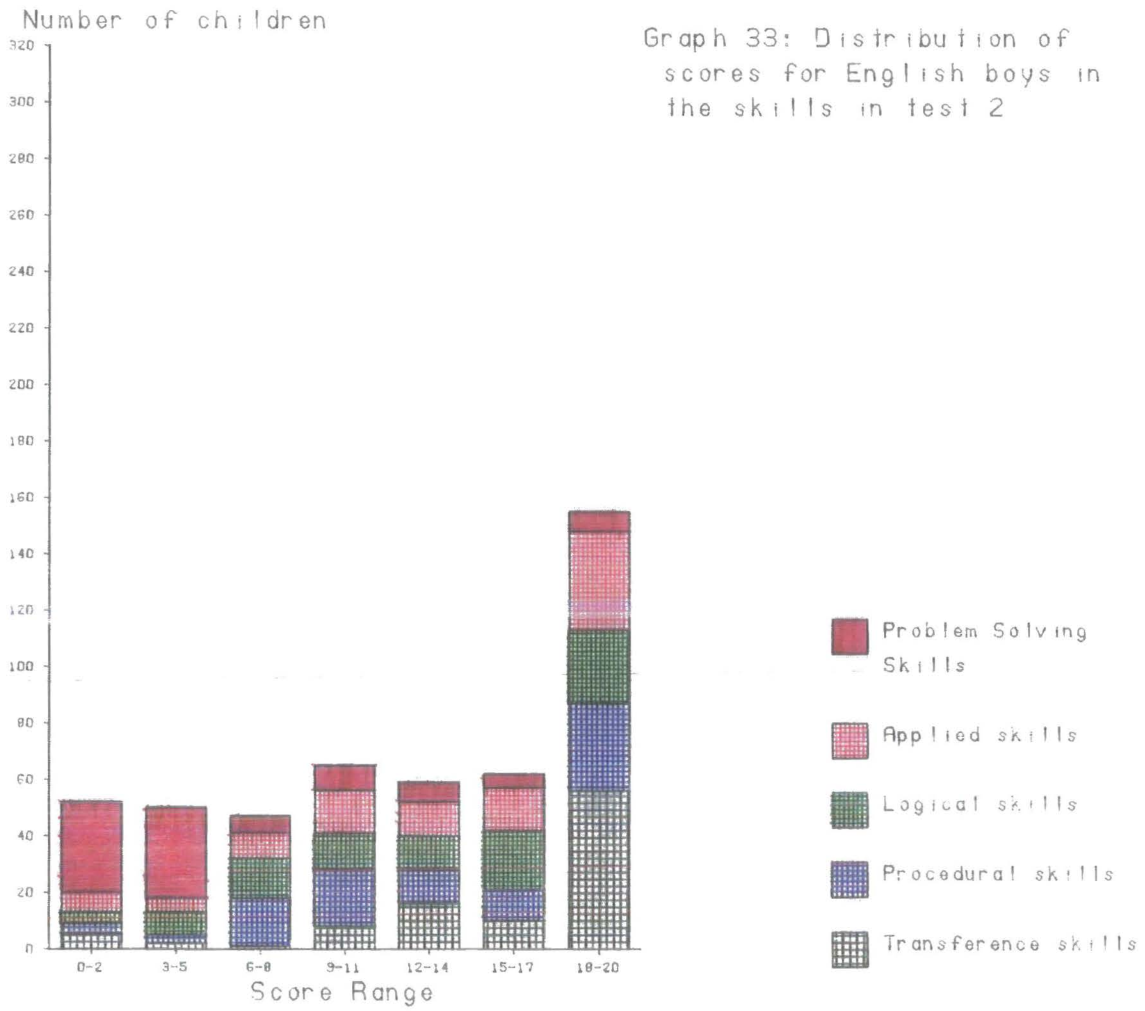
80.77% of girls were lower than 9, 9.18% of boys and 8.65% of girls scored 9 to 11, 12.24% of boys and 7.69% of girls scored from 12 to 17, while 7.14% of boys and 2.88% of girls scored from 18 to 20.

It is noticeable that the level of achievement for both decreases in problem solving skills. A higher percentage of boys and girls got lower scores, while a lower percentage of them got higher scores, since 12.24% of boys and only 4.81% of girls scored 15 to 20, while 71.43% of boys and 80.77% of girls scored lower than 9.

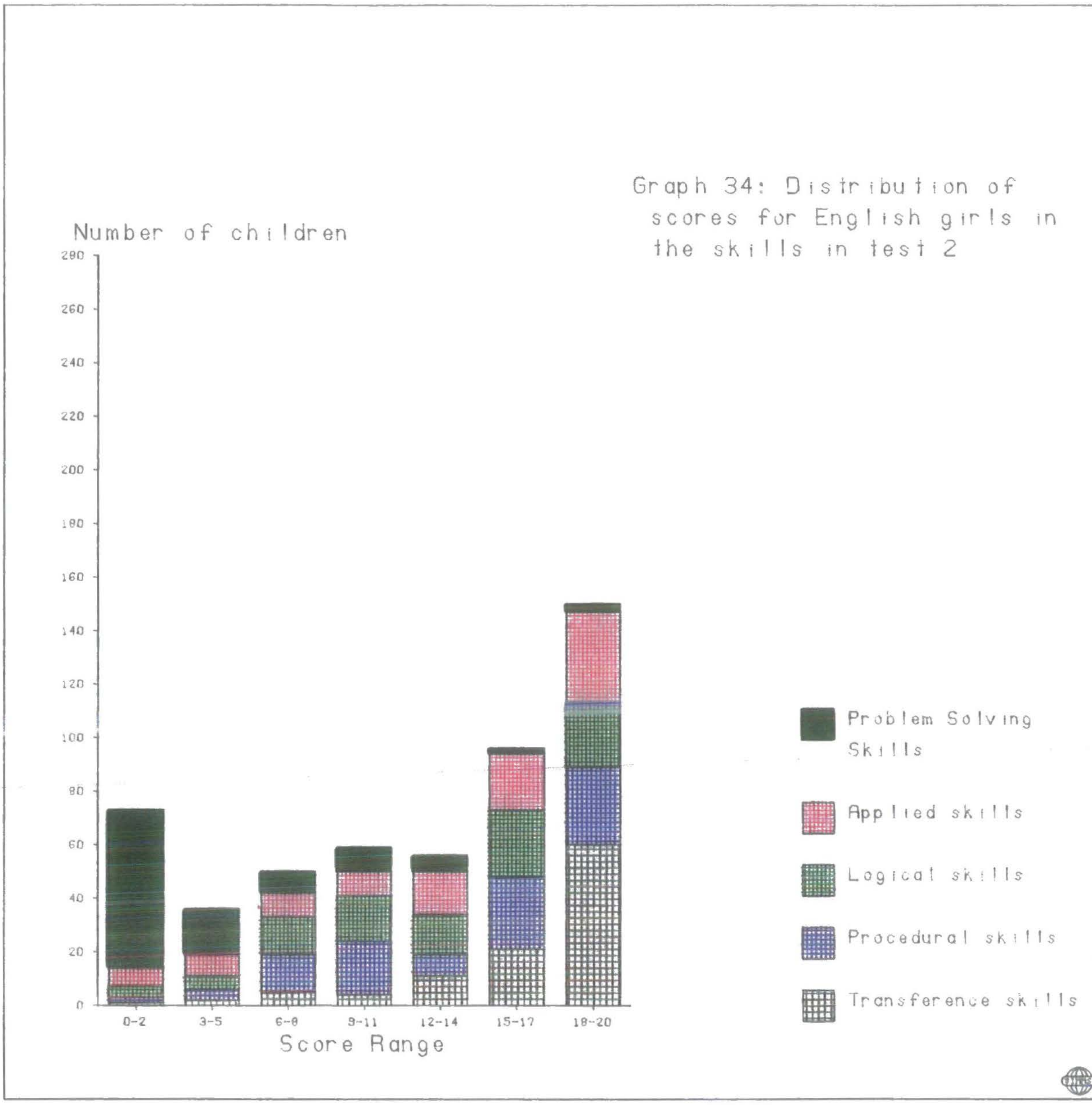
The numbers of boys and girls in each score range for each skill are shown in graphs 32 and 33.

Graph 33: distribution of scores for English boys in the skills in test 2.

Graph 33: Distribution of scores for English boys in the skills in test 2



Graph 34: distribution of scores for English girls in the skills in test 2.


Graph 34: Distribution of scores for English girls in the skills in test 2
Number of children
280
260
240
220
200
180
160
140
120
100
80
60
40
20
0
0-2
3-5
6-8
9-11
12-14
15-17
18-20
Score Range
Problem Solving Skills
Applied skills
Logical skills
Procedural skills
Transference skills

There are some similarities in performance between boys and girls for the whole test and also for different skills. Also, there are similarities between Egyptian and English children in relation to the test as a whole and their performance in different skills for age 9 to 11 year olds.

Generally speaking, there are similarities between Egyptian and English primary school performances in those tests which covered ages 7 to 11, that is, over most of the primary stage. There are other similarities drawn from tests results which revealed children's performances in mathematical skills in primary school. One of these similarities is that, while a high percentage of Egyptian and English children got lower scores in logical, applied, and problem solving skills, a high percentage of them had higher scores in transference and procedural skills. This indicates greater concern in teaching the subject through mechanical operations rather than giving more opportunity for children to discover, achieve and understand for themselves.

It appears that the children still need more care with and opportunity to use discovery methods to enable them to build mathematical concepts and skills.

In contrast, a lower percentage of Egyptian and English children had higher scores in logical, applied, and problem solving skills.

In addition, there are no patterns of differences between boys and girls in performance for both Egyptian and English children. The differences between boys and girls generally tend to be insignificant for both Egyptian and English children.

There are difficulties in building the skills hierarchy of transference, procedural, logical, applied, and problem solving skills. This research shows that there are levels of these skills which vary according to age and maturity, experience, ability and readiness, and it is claimed that it is necessary to develop this hierarchy of skills in a way which is more appropriate for the individual child.

This chapter presents a general view of the tests results as related to children's performance. Chapters 9 and 10 are a consecutive discussion of the particularly important skills that children must acquire to communicate successfully in society.

Many children, especially in the primary stage, are being "taught" the arithmetic by rote before they understand it. Systematic learning about arithmetical primary units, including the four rules and the concept of place value, should be based entirely on the children's understanding and through their own experiences of the school activities.

This discussion attempted to investigate children's acquisition of skills by having children answering the test questions. The conclusion of the investigation was that children need many experiences with number concepts based initially on concrete activities before being introduced to those based on formal ones. The skills which the children were asked to perform, described throughout this chapter, are examples of the kind of learning outcomes children should have between the ages of seven and eleven in order to begin to learn about arithmetic and build-up concepts correctly.

In summary, tests similar to those described in this chapter should be explored and mastered by children before going on to more concentrated work in the abstract arithmetic world of symbols. The results revealed that the children are "taught" the four arithmetic rules before they can understand them. So this simply causes learning by memorization rather than comprehension. Attention must be given both to methods of teaching and to stages of development of children if true understanding is to happen.

To summarise, this chapter has discussed the results of the tests for the age ranges seven to nine and nine to eleven both for Egypt and England respectively. Also the results were presented by graphs in order to draw a clear picture of children's performance. Firstly, overall results for the tests were presented; then, these were broken down into three month age intervals, calculating means and standard deviation of the test scores; next, the results were divided according to the five categories of skills showing the numbers of children in each of several score ranges, and T-Tests were performed to discover whether performances in these areas were signficantly different; after this, the results were divided into those for boys and those for girls, repeating the methods used for all children; and finally the results for Egypt and for England were compared.

CHAPTER TEN

CONCLUSIONS- WITH SOME COMMENT ON THE EDUCATIONAL IMPLICATIONS SUGGESTED BY A CONSIDERATION OF THE CHILDREN'S PERFORMANCES

The discussion may be summarised and concluded as follows:- both the modern and traditional syllabi of primary mathematics should concern themselves with the development in understanding of concepts and in skill level. This is contrasted, in some cases, with mechanical and repetitive work involving the activities of Egyptian primary schools. Nevertheless, there is no hard and fast demarcation line between the learning and development of a concept or a skill and their development of understanding. Understanding develops gradually from concrete processes to abstraction if children pass through an appropriate educational situation. Through this process it is thought that concepts are built up. To possess a concept or acquire a skill is to have an ability to make discriminative responses to the physical environment. Stress is usually placed on concept and skills as they are important elements of learning outcomes. On the other hand, the primary stage is very important in the Educational ladder, where the children's concepts are formed and skills are acquired for the first time.

As the intention of this research was to investigate the acquiring of skills in mathematics in primary schools, Egypt (which is considered to be a developing country) and England (considered to be an advanced country) were chosen to provide different ends of the scale.

A practical definition and classification of skills have been adapted, developed, and modified. Children's performance in the test of skills has been analysed. The responses of children taking part in the experimental study support the research assumption for the hierarchy and classification of a mathematical skill, the development of a skill as the children grow up, the acquisition of a skill by boys and girls and the differences between them, and the differences between acquisition of skills. It was clear that there was a gulf between the level of understanding of concepts or acquisition of skills assumed by primary school teaching goals and the actual level of understanding of concepts or of skill acquisition, as indicated by children's responses to the tests and the experimental study.

The responses of the children to such items as "logical, applied, and problem solving skills" illustrate the contrast between the primary teaching goals concerning the children's understanding of a concept or acquisition of a skill, and their actual level.

The assumed classification of skills was into five categories: transference, procedural, logical, applied and problem solving. These five divisions form a hierarchy, starting with the simplest skills and building up to the most complex, which rely on and contain all the others. The results of the test generally were that a high percentage of children achieved lower scores on logical, applied and especially problem solving skills, while most obtained good marks on questions involving transference and procedural skills. The performance of boys and girls were almost the same in the test as a whole, and in each area separately.

The children revealed in the experimental results a high level of individual achievement for some children at various ages, and at the same time a low level for others. This produced some results at variance with the expected level of acquisition of skills according to their age. Having made these comments it is clearly necessary to add that the role of the teacher is strategic. So the well trained, qualified, honest, and enthusiastic teacher is vital and necessary in the educational situation, particularly at that stage. However it would seem, for the purposes of the thesis, that it is essential to consider, in relation to the 7 to 11 group, the possibility of instituting some form of group organisation for, at least, part of the work undertaken in the field of mathematics. Furthermore, it seems reasonable to suggest that much of this group work should be organised in concrete terms via practical problem solving activities. It appears highly desirable that interaction both between individuals and materials should be encouraged in the classroom situation. Furthermore, it would probably be desirable for teachers to observe and evaluate such situations, from time to time, and use the information thus obtained to guide the planning of some aspects of the curriculum experience.

The situations evaluated in the school life could, to some extent, be observed by the teacher in an informal classroom situation, and such observations would, in the researcher's view, do three things:-

(i) Ensure that the teacher related his or her interventions to a pattern of learning that was natural to the child.

(ii) Support the use by children of a non-technical, non-precise but nevertheless "efficient" descriptive language.

(iii) Ensure an awareness, on the part of the teacher, of the vital role of concrete experience in the development of concepts, together with an appreciation that this process is a protracted one.

This emphasizes that a child returns, time and time again, to a "simple" experience in order to develop his concepts further. As Dienes (*) suggests: 'even if what they do is not entirely abstract, it is at least derived from experience and no violence has been done to the natural dynamics of their thinking'. It is very important that the concept and skill acquisition should arise as natural consequences of

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(*) Z. P. Dienes, <u>BUILDING-UP MATHEMATICS</u>, London, 1960, 98.

children's experiences.

In order that elementary school mathematical experience may serve in developing one's abilities to understand social institutions, and in equipping one to meet more effectively problems which occur in his personal life, the following principles should be taken into consideration:-

(I) The pupil needs to be aware of the quantitive aspects of the problems that he faces.

(II) The learner needs to be led to discover and comprehend the concepts which underlie the devices and procedures used in mathematical behaviour.

(III) The learner needs to discover and to master the use of devices and techniques which society has found effective in facilitating mathematical behaviour.

(IV) The teacher needs a competent mastery of devices and procedures through which pupils can be led to discover, formulate, and master the ideas and instruments necessary for serving their own and society's needs.

As has already been proved in many schools in England and a few in Egypt, the basic procedure for the successful teaching of mathematics in the primary school should be the use of discovery methods to create a "laboratory for learning" in the classroom rather than merely a rote-learning method. It seems reasonable that the active use of meaningful experiences is the way to achieve mathematical understanding and competency. The classroom becomes a learning laboratory when it produces mental and physical activity that results in experimentation; this in turn should lead to formulation of procedures and to generalizations based upon reliable and sufficient information. The materials for the laboratory are within the reach of every teacher. The materials consist of things that children and teachers bring into the classroom for the lessons under consideration. The teacher who senses the procedures for making the classroom a laboratory for learning is invaluable, and no material equipment can replace him. This type of teacher can teach without textbooks. On the other hand, the teacher who does not sense how to stimulate his learners to experiment and to formulate procedures and generalizations is not likely to do a very different job of teaching just because he has equipment in his classroom. The teacher who understands basic mathematical relationships and interprets their emergence in the learner's behaviour can challenge the learner to refine the ideas in a way that is dynamic in its effect on pupil behaviour. Instructional methods should be derived from the nature of the individual who is to be instructed, from the nature of the learning process, and from the nature of the material or behaviour that is to be achieved.

The development of understanding compares with the inculcation of simple practical skills of a utilitarian or vocational nature such as those involved in performing the "four operations" of addition, subtraction, multiplication and division in arithmetic. These operations are performed, it is alleged, mechanically and as separate unitary skills. There is no awareness on the part of the Egyptian pupils, and often enough on the part of the teachers, of the arithmetical reasoning underlying the mechanical operations. There is no appreciation of the relationship between those operations.

It is worthwhile noticing that the development of mathematical concepts and skills takes place in an apparently natural and inevitable way. It is considered by some educationists and psychologists that the primary school child could reach an understanding of some units of arithmetic, geometry and the concept of numbers and relations through an instinctive drive towards adaptation and control of the environment and appropriate for their ages. What seems so clearly lacking here is any appreciation by the children and of even by some teachers the hierarchy in mathematical skills. These skills arise as products of building up the concepts of number through appropriate teaching methods. Because of differences in the learning environment, children may have achievments at variance with expectations for that age; it was found in this thesis that children in a few age ranges had attainments which were lower than those of younger children, and below that which would be expected for their age. Children found great difficulties in dealing with logical, applied and problem solving skills and were happier with items requiring procedural skills, which may reflect the concern of teachers, parents and children to master mechanical operations needed to pass their examinations. In addition, some of the teachers are not well enough qualified or sufficiently experienced, as a result of which they are unable to lead and guide children using an appropriate approach to acquiring mathematical skills. These skills can certainly be gained if the concepts are built gradually from concrete activities to the formal ones needed by these children.

If it is agreed that concept formation, the growth of understanding and the acquisition of a skill is the aim of mathematics teaching, rather than mere learning in parrot-fashion, then it certainly does not follow that verbal teaching is absolutely excluded.

To have a concept it is to be able to use the term expressing the concept appropriately. The role of the teacher cannot be other than that of a provider of equipment and classroom manager. He presents apparatus, ensures attention, prompts concentration and tests results.

What the picture of the natural emergence of mathematical concepts through interaction with the environment has brought about in practice, has been an emphasis on the child's own exploration and discoveries, and a decline in the teacher's traditional authoritarian role.

Some procedures suggested for teaching concepts and acquiring skills concerning the arithmetical units for primary school children

(I) Lead the pupils to formulate basic concepts which underlie measurement, set, group, graphical scale, approximation, and the number relationships. Children should develop concepts concerning what and how to compare. Ideas of how and what to compare lead to techniques for comparing, measuring, evaluating, expressing and interpreting quantitative relationships. Experimentation and investigation of problems such as the following offer opportunities for developing concepts underlying measurements.

Using procedures for measuring, evolving concepts underlying measurement, and achieving intelligent interpretation of measurement are very significant parts of mathematics in the elementary school.

(II) Have the child measure with nonstandardized instruments (sticks, string, paper, boards) so that he senses the need for and appreciation of standard measures. It is well to have the child use other than standard measures for several lessons. Through such lessons, the child will meet the problem of need for agreement concerning which measure is to be used for measuring a given thing, how to divide his measuring stick to express units of it, etc.

(III) After the child has used non-standardized things for measuring, lead him to use standard measuring devices. The child should have much experience in using rulers, pint jugs, scales, and other standard measuring devices. These experiences should grow out of many of his activities throughout the day.

(IV) Have the learner develop given measurements with which he is familiar so that he can use them to compare and estimate the measurement of given things whose quantity he does not know. Developing familiar, known measurements such as the length of the classroom, the height of the classroom door, the distance of a kilometre, the width of one's hand, the distance from home to school, and the capacity of a quart jar are important references for estimation of quantities unfamiliar to the child. Many known measurements for estimating unknown quantities should be developed and used by each group of children studying mathematics in the elementary school.

(V) Lead the learner to sense, understand, and appreciate man's ingenuity in measuring through development of a brief, simple historical treatment of some familiar measures. Through a brief, simple development of the history of some common measures, the child should be led to an appreciation and understanding of measurement. Historical development can evolve through having the child sense the problems that faced his forefathers and challenging him to recommend procedures for dealing with what and how to measure.

(VI) Have the child estimate measurements of given things and then check his estimate through actual measurement. Good estimating of quantitative value is the product of many experiences of comparing known quantitative values with quantitative values that are unknown and evaluating one's estimate through actual measurement. With skill in teaching, children become astute in estimation. It has been found that children are often able to do fine estimating.

(VII) The child measures to determine basic quantitative relationships such as:- kilometre- metre relationship, pint- quart- gallon relationship, side-perimeter relationship of a square, length- width-perimeter relationship of a rectangle (other than a square), time - speed - distance relationship, size - capacity relationship (for example, width, length, and height of container as related to its capacity), and unit of measure- area relationship.

The teacher who challenges the child to sense, to discover, to understand, and to interpret such relationships releases the child to achieve a high level of competency in mathematics. Procedures such as these are the foundation of thrill of learning and thrill of teaching mathematics.

GENERAL OBSERVATION MADE FROM THE CHILDREN'S RESULTS

The general trend is an increase in score with age. Nevertheless, there are some differences in a few cases (age group) which were lower than expected. Pupils were less confident when answering multiplication or division questions. It appears that this might be due to lack of understanding of place value, practice in the operation of multiplication and other concepts upon which skills are based.

The individual needs to integrate and use concepts together in order to be able to solve a certain problem in life.

Problem solving skills proved difficult for children in the thesis'sample, indicating that it is unlikely that they have had the experience that would bring about coordination, and this argument is supported by the findings of the test analysis. This indicates the presence of several weaknesses in other skills. The number of children who had low scores tend to increase steadily as the skills become more complex from the transference to problem solving skills. In contrast the percentage of children who scored a high mark tends to decrease from transference to problem solving skills. The results combine to show the difficulty experienced by many children in logical, applied and problem solving skills. To perform well in problem solving skills children need to perform well in transference, procedural, logical, and applied skills. Deficiency in an earlier skill leads to difficulties with later skills. Whatever the reasons are for the deficiencies found in pupils, what such pupils need is not drill but a chance to learn. Practice comes later to fix what has been learned and to maintain it at a high level of working efficiency. At the same time skills tend generally to be more developed as the children become older.

The thesis result revealed no significant differences between boys and girls in acquiring skills. Neverthless, Egyptian boys were better than girls in logical skills and the mean scores for other skills were slightly higher for boys than girls.

The pattern of results was similar in Egypt and England in relation to the mean which increased with age as would be expected. However, the gradual increase of scores by age is more noticeable in the English results than in the Egyptian scores. The results show some indications of the different levels of understanding that pupils have of the various arithmetical concepts and skills in mathematics.

The order of acquiring skills of boys and girls was generally transference, procedural, logical, applied, and problem solving skills respectively. The differences generally, between different skills were significant both for boys and girls, although it was more significant in the case of Egyptian girls. Differences between Egyptian boys and girls in test 1 for different skills were significant only on logical skills, with boys better than girls, although the other mean scores of the different skills were slightly higher for boys than girls in Egyptian test 1. The results of English children have added some more detail to the picture of performance by giving some indications of the different levels of understanding that pupils have of the various mathematical concepts and skills. It also revealed how English primary schools have a variety of schemes in order to deal with the different ability of children. It is very important for children use different types of syllabi and learn through various activities which suits and attracts them. The results indicates the homogeneity of English scores as opposed to the divergence of the Eyptian scores. The general trend in England now is to train primary teachers to use and practice with computers for primary school children. Using and practicing computer systems are considered new technology in modern life and necessary too.

There was no pattern in the differences between boys and girls in the whole test 1 in England, but girls' success rates were marginally higher than boys in general.

No valid or meaningful comparisons can be made between these two investigations since, even where the results differ, one research study cannot provide absolute evidence of the presence of a trend. Such evidence, if such there be, cannot be produced clearly, accurately and precisely until further research has been conducted and any differences between Egyptian and English can be shown either to be due to chance variation or to be part of a general long-term trend. This research spotlights the need to have more investigations in this area.

Both sets of test 1 and test 2 results provide, in combination, an overall view of mathematical skills performance in the primary schools. The investigation of the second test for 9 to 11 years continues to aim at the presentation of a picture of the mathematical performance of primary school children in Egypt and England. The distribution of the results of the second Egyptian test tends to be normal. Many points are distributed in a way that conforms closely to the normal curve.

None of the differences between Egyptian boys and girls of the mean score for different skills in test 2 were significant.

There are only significant differences at 1% level in problem solving skills and transference skills at the 5% level. English boys in test 2 had the higher mean score for problem solving while girls had the higher mean in transference skills.

In conclusion, this research would like to suggest that the pattern of responses revealed by the experimental study has some significance for any discussion of appropriate ways of organising learning situations for 7 to 11 years children and it is hoped that the study has made some progress towards the need to build-up and develop a great deal of practical definitions of mathematical skills oriented towards observational material that might be applied more easily than the "theoretical" type of definitions.

The study raises some possible areas for further research:

(i) Extending this type of study to children from 6 years to 12 years in Egypt and 5 years to 11 years in England.

(ii) A similar study involving children covering most of Great Britain.

(iii) Similar definitions of skills but

involving more examples suitable to primary school age.

(iv) Similar definitions of skills adapted, modified or adjusted for other stages.

(v) More studies concerned with skill acquisition and development.

(vi) A similar study concerned with the relationships between concepts and skills.

(vii) An extensive study concerned with evaluating the level of relationships between concepts and skills.

In this chapter, a concise discussion of the main themes of the thesis has been presented. Suggestions for teaching concepts and skills were put forward, and the role of the teacher needed to ensure effective learning was explained. Finally, proposals for further study were laid down.

It is worthwile to point out some positive trends which have been noticed in England and try to convey these ideas to Egyptian schools. One of these is the use of variety of schemes which are suitable to children. Also teacher keeps progress charts for every child and he knows a great deal about the individual child. In addition, there are regular courses to train the teacher in computers and how to use them in teaching children arithmetic, along with other methods. Above all, there is a direct contact between teachers and parents, which comes through the discussion between teachers and parents at the end of each school year.