Translational invariance in bag model

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TRANSLATIONAL INVARIANCE

IN

BAG MODEL

by

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PREFACE

The work presented in this thesis was carried out at the Department of Mathematics, University of Durham, Durham, U.K., between October 1977 and October 1981 under the supervision of Professor E.J.Squires.

This thesis has not been submitted for any degree in this or any other university. It is claimed to be original except for chapter one, sections one and two of chapter two, section one of chapter three and other places where explicitly referenced. Some of the material in chapter two is available as a Durham University pre-print and chapter three contains unpublished work by the author. Chapters four and five are based on published works by the author in collaboration with E.J.Squires and with M.R.Pennington and E.J.Squires respectively.

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ABSTRACT

In this thesis, we investigate the effect of restoring the translational invariance to an approximation to the MIT bag model on the calculation of deep inelastic structure functions. In chapter one, we review the model, its major problems and we outline Dirac's method of quantisation. This method is used in chapter two to quantise a two-dimensional complex scalar bag and formal expressions for the form factor and the structure functions are obtained. In chapter three, we try to study the expression for the structure function away from the Bjorken limit. The corrections to the L_o - approximation to the structure function is calculated in chapter four and it is shown to be large. Finally, in chapter five, we introduce a bag-like model for kinematic corrections to structure functions and obtain agreement with data between 2 and 6 $(GeV/C)^2$.

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INTRODUCTION

This thesis is concerned with the calculation of currents, in particular as applied to deep-inelastic structure functions, in the MIT bag. Most of the calculations were done in the $1+1$ dimensional bag where the MIT Bag model is completely soluble (section 1.1). Nevertheless even in this model it is not possible to calculate an exact expression for the current operator (section 1.2). Several attemps to solve this problem have been made and, after a brief outline of the Dirac method of quantising constraint systems (section 1.3), I discuss the most ambitious of these due to Shalloway (Chapter 2). In particular, I extend Shalloway's work and obtain formal expressions for the form factor and the structure function. I evalute these in the Bjorken limit using the "L_o-approximation" of Krapchev and it turns out that, I then reproduce Krapchev's results for these quantities, although Krapchev originally obtained them with a simpler formal quantization procedure.

Since my ultimate aim is to obtain expressions that can be compared with experiment, it is necessary to go beyond the scalar field in 1+1 dimensions and also to study the situation away from the Bjorken limit. Some considerations relevant to this were given in Chapter three. First, I treat the fermion field in 1+1 dimensions, using essentially Krapchev's method (sections 3.1 and 3.2). Then, since the momentum distribution associated with the bag model does not actually give a very good fit to the structure functions data and to allow discussion of alternatives, I calculate directly from QED the structure function associated with a free quark with an arbitrary distribution of momentum and mass. Here I do not take the Bjorken limit, it is an exact calculation. A calculation of this type does not contain the information on the final hardon mass which in fact makes the structure function go to zero at $x = 1$. However the comparison of the results obtained in the L_o approximation to the 1+1 dimensional bag with these determined from fixed non-recoiling cavity suggest that the effect of this, in the Bjorken limit, is to replace x, in the QED calculation, with $-\log(1-x)$. I use this trick in Chapter five. In (section 3.4), I endeavour to study the approximate trick when I go beyond the Bjorken limit but this calculation produced little useful information.

In Chapter four, I consider the higher order corrections to the L_o approximation. This cannot be done in a systematic way, since the approximation is not obviously the lowest order term in an expansion parameter. A simple procedure for calculating a correction term is used but unfortunately it suggests very large corrections. Attempts to understand the origin of these have not been successful.

Finally, in Chapter five, I put together the positive results of this work, ignoring the uncertainties, and show that a suitable form for the initial quark momentum distribution gives a good description of the structure function and, in particular, of its dependence on Q^2, at least over a region where the (short distance) QCD corrections are likely to be unimportant.

CHAPTER 1

THE MIT BAG MODEL

Seventeen years ago, Gell-Mann[13] and Zweig[14], indepdently postulated quarks as the main building blocks of hadrons. Since then the quark ideas have grown steadily to dominate entirely the study of elementary particles and their interactions (strong, weak and electromagnetic). Theorists have had great success in building quark models and using them to calculate the properties of hadrons. In the meantime experimentalists have not been so fortunate because, in spite of much ingenuity and persistence, they have had no success in producing any significant evidence for the observation of free quarks. This negative experimental result has motivated the idea of quark confinement and accordingly quarks are assumed to be permanently bound inside hadrons. The final microscopic theory for describing this situation in hadron physics is not known yet.

A popular candidate is the elegant and simply formulated quantum chromodynamics[24] (QCD). In this theory, quarks possess an extra degree of freedom called "colour" described by the colour group $SU(3)_c$ and the strong interactions between these coloured quarks are mediated by an octet of coloured vector fields called "gluons". The main idea of QCD is to make the $SU(3)_c$ colour symmetry a local, rather than a global symmetry. Thus, QCD is a non-abelian gauge theory. It has many important features, for instance, the theory is renormalisable, it is asymptotically free and it predicts that there will be violation of Bjorken scaling and allows this to be calculated exactly. On the otherhand, the theory has the possibility that it might confine colour. There is a conjecture in certain theoretical circles that the colour non-singlet degrees of freedom cannot be excited in the theory and only colour singlet hadrons are physically observable. This is a pure conjecture and it has not been proved yet.

Alternative models, which also confine quarks, are the bag models in which the space is divided into exterior and interior regions. An early version of this idea is due to Bogoliubov[20] in which the bags are fixed, infinite square-well, scalar potentials. A more advanced version, which is at present the most successful, is the MIT bag model[4,15,17,18]. In this model, the fields are those of conventional QCD i.e. coloured quarks and gluons. The coloured quanta are confined to the interior of the bags, which are colourless hadrons. Thus rather than starting with quark and gluon fields defined throughout space, and then trying to derive confinement, we start by assigning coloured fields only to the inside of hadrons. The outward pressure due to the quark fields is balanced by an inward pressure which is the product of a volume energy density. This radical departure from conventional local field theory is the essential hyptothesis of the MIT bag theory. A similar model is the Budapest bag model - reviewed in ref.9 - in which the volume energy term is replaced by a surface term. Bardeen[21] et al developed another model - known as "SLAC" bag - which effectively contains a surface tension term. Calculational problems - for instance with Bjorken scaling due to the fact that quarks inside the bag do not appear to be even approximately either light or free- have prevented its being used to the same extent as the MIT bag.

In the next section the solution to the problem of a two-dimensional scalar field confined in an MIT bag will be reviewed. It will be followed in section II, by a discussion of some of the difficulties of the model. The third section will be devoted to a brief review of the Dirac method of quantisation.

1.1 Review of the scalar field theory of the bag model in two-dimensions

A confined complex scalar field ϕ to a two-dimensional bag is described by the Langrangian

$$L = \int_{z_0(t)}^{z_1(t)} dz \, (\partial_\mu \phi \, \partial^\mu \phi^* - B) \quad , \tag{1.1}$$

where B is the positive energy density to be determined from the data and $z_0(t)$, $z_1(t)$ are the two points which bound the bag. Let

$$\phi = \phi_1 + i \, \phi_2 \quad , \tag{1.2}$$

where ϕ_1 and ϕ_2 are two Hermitian fields which, inside the bag, satisfy the equation of motion

$$\partial^\mu \partial_\mu \phi_i = 0 \quad , \quad i = 1, 2 \quad , \tag{1.3}$$

together with the boundary conditions

$$\phi_i = 0 \quad , \quad i = 1, 2 \quad , \tag{1.4}$$

$$\partial_\mu \phi_i \, \partial^\mu \phi_i = 2B \quad , \quad i = 1, 2 \quad . \tag{1.5}$$

Define the light-cone variables (t,x) as

$$\tau \equiv \frac{1}{\sqrt{2}} (t + z) \quad , \tag{1.6}$$

$$x \equiv \frac{1}{\sqrt{2}} (t - z) \quad . \tag{1.7}$$

Equation (1.3) is satisfied by any function of x and t of the form

$$\phi_i (x, \tau) = f_i(\tau) + g_i(x) \quad , \quad i = 1, 2 \quad . \tag{1.8}$$

Rewritting the boundary conditions in terms of $f_i(t)$ and $g_i(x)$

$$\dot{f}_i(\tau) + \dot{x}_k \, g_i'(x_k(\tau)) = 0 \quad , \quad i = 1, 2 \quad , \tag{1.9}$$

$$\sum_{i=1}^{2} \dot{f}_i(\tau) \, g_i'(x_k(\tau)) = -B \quad , \tag{1.10}$$

where $x_k(t)$, $k = o,1$, are the two end points of the bag and the derivatives with respect to τ are denoted by overdots and with respect to x by primes.

It is clear from (1.7) and (1.8) that the bag has a nonlinear boundary conditionswhich make any attempt to solve the problem in higher dimensions extremely difficult. In two dimensions the bag boundary could be linearised by introducing a new space parameter $\sigma = \sigma(x)$ according to the differential equation

$$\frac{dx}{d\sigma} = \frac{1}{P} \sum_{i=1}^{2} \left[\tilde{g}_i'(\sigma) \right]^2 , \tag{1.11}$$

where a new field $\tilde{g}_i(\sigma)$ has been introduced $\tilde{g}_i(\sigma) \equiv g_i(x(\sigma))$ and P is the light-cone momentum

$$P = \int_{x_0(\tau)}^{x_1(\tau)} dx \sum_i \left[g_i'(x) \right]^2 . \tag{1.12}$$

The boundaries of the bag are $\sigma_k(\tau) \equiv \sigma(x_k(\tau))$, $k = o,1$. When described in terms of σ, the boundary conditions (1.9) and (1.10) become

$$\dot{f}_i(\tau) + \dot{\sigma}_k(\tau)\, \tilde{g}_i'(\sigma_k(\tau)) = 0 , \tag{1.13}$$

$$\sum_{i=1}^{2} \dot{f}_i(\tau)\, \tilde{g}_i'(\sigma_k(\tau)) = \frac{-B}{P} \sum_{j=1}^{2} \left[\tilde{g}_i'(\sigma_k(\tau)) \right]^2 . \tag{1.14}$$

From the light-cone momentum expression (1.12) and (1.13), (1.14) the boundary in σ space is

$$\begin{aligned} \sigma_0(\tau) &= \frac{B\tau}{P} , \\ \sigma_1(\tau) &= 1 + \frac{B\tau}{P} . \end{aligned} \tag{1.15}$$

It follows from (1.13) that $\tilde{g}_i(\sigma)$ is periodic in the interval $[\sigma_0, \sigma_1]$ and the solution for the fields in (σ, τ) space is now immediate

$$\phi = \frac{-i}{(4\pi)^{1/2}} \sum_{n=1}^{\infty} \left[\frac{b_n}{n} \left(e^{-2\pi i n \frac{\beta\tau}{P}} - e^{-2\pi i n \sigma} \right) - \frac{a_n^{\dagger}}{n} \left(e^{2\pi i n \frac{\beta\tau}{P}} - e^{2\pi i n \sigma} \right) \right] , \quad (1.16)$$

$$\phi^{*} = \frac{-i}{(4\pi)^{1/2}} \sum_{n=1}^{\infty} \left[\frac{a_n}{n} \left(e^{-2\pi i n \frac{\beta\tau}{P}} - e^{-2\pi i n \sigma} \right) - \frac{b_n^{\dagger}}{n} \left(e^{2\pi i n \frac{\beta\tau}{P}} - e^{2\pi i n \sigma} \right) \right] . \quad (1.17)$$

Upon quantisation b_n, a_n become the annihilation operators of charge $\pm$ particles in state n. The light-cone coordinate x is given by

$$x(\sigma) = \bar{x}_0 + \frac{1}{P} \sum_{n=-\infty}^{\infty} c_n L_n , \quad (1.18)$$

where

$$c_0 = 2\pi(\sigma - \tfrac{1}{2}) , \qquad c_n = \frac{i}{n} e^{-2\pi i n \sigma} , \quad (1.19)$$

and

$$L_n = \frac{1}{2} \sum_{m=-\infty}^{\infty} \left(a_m^{\dagger} a_{m+n} + b_m^{\dagger} b_{m+n} \right) . \quad (1.20)$$

The creation and annihilation operators obey the commutation relations

$$[a_k , a_n^{\dagger}] = [b_k , b_n^{\dagger}] = k\, \delta_{k,n} , \quad (1.21)$$

and otherwise zero. Thus, the operators L_n satisfy the algebra

$$[L_n , L_m] = (n-m) L_{n+m} + \frac{1}{12} n(n^2-1)\, \delta_{n,-m} . \quad (1.22)$$

1.2 Problems with the MIT bag

The quantised version of the two-dimensional MIT bag model described in section I is the only solvable, relativistic, quantised model of a confined system known. However, there are major difficulties in using the model for realistic calculations, and these have led to consideration of other methods of quantisation[7,8,22,23]. The most serious and difficult problem is the construction of local field operators which is essential to calculate local interactions. In the two-dimensional classical bag, the field $\phi(x,\tau)$ is given in the region $x_0(\tau) < x < x_1(\tau)$ and is zero outside this region. So the physical field is defined by[5]

$$\Phi(x,\tau) = \int_{x_0(\tau)}^{x_1(\tau)} dx' \, \delta(x-x') \, \phi(x',\tau) \quad . \tag{1.23}$$

In (1.23), $x_0(\tau)$ and $x_1(\tau)$ are bag variables i.e. operators while x is the space coordinate. To resolve this problem the parametrisation $x' = x'(\sigma)$ is used in (1.23) to obtain

$$\Phi(x,\tau) = \int_{\frac{B\tau}{P}}^{1+\frac{B\tau}{P}} d\sigma \, \delta(x - x'(\sigma)) \, \frac{dx'(\sigma)}{d\sigma} \, \phi(x'(\sigma),\tau) \, . \tag{1.24}$$

To remove the operator P from the limits we introduce $\hat{\sigma} = \sigma - \frac{B\tau}{P}$ and then we replace δ function by an integral to obtain

$$\Phi(x,\tau) = \frac{1}{2\pi} \int_0^1 d\sigma \int_{-\infty}^{\infty} dq \, \exp\left\{iq\left[x - x'\left(\hat{\sigma} + \frac{B\tau}{P}\right)\right]\right\} \tag{1.25}$$

$$\frac{dx'(\sigma)}{d\sigma} \, \phi\left(x'\left(\hat{\sigma} + \frac{B\tau}{P}\right), \tau\right) \quad .$$

The construction of a formal local field operators is now completed by obtaining (1.25). However, a problem arises here because when we insert the operator expressions x_0, P, a_n, $a_n^\dagger$, b_n and $b_n^\dagger$ the various factors in the integrand do not commute and hence the problem of ordering ambiguity appears.

The difficulty in using (1.25) in a realistic calculation motivated Krapchev[5] to consider two approximations - the L_o approximation and the lowest mode - and use them to calculate the electromagnetic form factor. The L_o approximation was also used by Davis and Squires to calculate the deep inelastic structure function[6]. On the other hand Shalloway[8] suggested that $x(\sigma)$ in (1.24) should be regarded as an additional set of field variables which means further additional constraints on the physical solution.

1.3 Dirac Method of Quantisation

Starting with any Langrangian, there are well-established rules to set up the corresponding quantum theory provided that the momenta are independent functions of the velocities. Dirac was motivated by this lack of generality to produce a new theory[9] which is also applicable to systems with constraint equations i.e. when the momenta are not independent functions of the velocities. In this section, the Dirac method of quantisation will be outlined.

For the Lagrangian $L(q,\dot{q})$ of any dynamical system of N degrees of freedom, the primary constraints are defined as the set of all independent functions $\phi_m(q,p)$, $m=1, \ldots M$; $M \leqslant N$, which arise from dependence among the coordinates and canonical momenta as originally defined and those which are specified initially for physical reasons - such as boundary conditions-. The primary constraints are weakly equal to zero, i.e. $\phi_m(q,p) \approx 0$, which means that those equations will be violated if the quantities q's and p's are variated by a first order variation.

According to this method, the Hamiltonian can be formed in the usual way, then, the primary constraints multiplied by Lagrange multipliers - which are functions of q's and p's - have to be inserted into the Hamiltonian H.

$$H \longrightarrow H + C_m(p,q)\, \phi_m(p,q) \quad . \qquad (1.26)$$

Next step is to define the canonical Poisson brackets, usually taken as

$$[A,B] = \frac{\partial A}{\partial q_n}\frac{\partial B}{\partial p_n} - \frac{\partial A}{\partial p_n}\frac{\partial B}{\partial q_n} \quad , \tag{1.27}$$

for any two functions A, B of p's and q's. Doing that, it is now possible to calculate the time development of any function D(p,q) through

$$\dot{D} = [D,H] + U_m[D,\phi_m] \quad . \tag{1.28}$$

Since, the primary constraints are weakly equal to zero so, its time development must be weakly equal to zero also. This requirement may lead to some relations between the q's and the p's only - independent of the ϕ equations-. This additional constraints $\psi_k(q,p)$, k =1,2..... - which will be called secondary constraints - are also weakly equal to zero and its time development may lead to further relations between the q's and p's. This procedure has to be carried on as far as it goes until a closed set - under time development - of secondary constraints $\psi_k(q,p)$, k=1,2...K, is obtained.

The primary and secondary constraints will be clasified to two kinds, the first-class and the second class ones. A function $\chi(q,p)$ is defined to be first-class if it satisfies

$$\begin{aligned} [\chi,\phi_m] &\approx 0 \quad , \\ [\chi,\psi_k] &\approx 0 \quad , \\ [\chi,H] &\approx 0 \quad . \end{aligned} \tag{1.29}$$

A function of q's and p's that does not satisfy these conditions is called second-class.

The second-class constraints can be eliminated by taking them to be strong equations or definitions and reduce the number of degrees of freedom. The cannonical Poisson brackets are then suitably modified to reflect the elimination of same degrees of freedom. The time development of the first-class constraints will not be affected by the modification in the Poisson brackets which means that the set of the first-class constraints will remain a closed system. The first-class constraints are then imposed as supplementary conditions on the values taken by the dynamical variables.

Having done this quantisation then can easily be achieved by passing from the modified Poisson brackets to commutators.

CHAPTER 2

EVALUATION OF CURRENT OPERATORS IN A QUANTISED VERSION OF THE TWO-DIMENSIONAL MIT BAG MODEL

The failure of attempts to generalise the previous discussion of the quantised MIT bag to more than two dimensions, together with the fact that quantisation, though a unique procedure for simple systems, is not necessarily so for complex systems like the bag model where different procedures might give different results, have led to many alternative discussions of the quantisation problem for the two-dimensional bag[7,8,22,23,25,26].

A very general discussion of the problem is given by Shalloway[7,8] in which Dirac's method of quantisation in the presence of constraints is applied to the two-dimensional scalar bag. He argues that the CJJTW[4] quantised bag theory is inappropriate for consideration of locality. He introduced a boundary regularisation procedure and a local field operator having cut off independent matrix element is defined. Also, the CJJTW[4] operator algebra is reproduced as a subalgebra of an extended system. However, the difficulties found in attempting to construct local field operators are partially removed and the ordering ambiguities involving some of the operators are still unresolved.

In this chapter we apply Dirac's method to a two-dimensional charged scalar field confined to a bag. Formal expressions for the form factor and the structure function are calculated. A comparison between our results and other works[5,6] is presented.

2.1 The Classical Solution

The two-dimensional complex scalar bag is defined by the Lagrangian

$$L = \int_{z_0(t)}^{z_1(t)} dz \left[\frac{\partial \phi^*}{\partial t} \frac{\partial \phi}{\partial t} - \frac{\partial \phi^*}{\partial z} \frac{\partial \phi}{\partial z} - B \right] , \qquad (2.1)$$

where $\emptyset(z,t)$ is the complex scalar wave function and $z_0(t), z_1(t)$ are the end points of the bag. We define the light-cone variables (τ,x) by

$$\tau \equiv \frac{1}{\sqrt{2}}(t+z) \qquad , \qquad x \equiv \frac{1}{\sqrt{2}}(t-z) \qquad . \tag{2.2}$$

In order to apply Dirac's method, we require a fixed number of variables, $\emptyset(z,t)$, in the Lagrangian, but that is not the case with the Lagrangian given by (2.1), because the number of variables changes as the end points move. A space parameter $\sigma(x)$ is introudced[4] to counter the problem, with $0 \leqslant \sigma \leqslant 1$. This will linearize the boundary condition and allow us to rewrite equation (2.1) in the following reparametrized form

$$L = \int_0^1 d\sigma \; x' \left\{ \frac{\phi_1'}{2x'}\left(\dot{\phi}_1 - \phi_1' \frac{\dot{x}}{x'}\right) + \frac{\phi_2'}{2x'}\left(\dot{\phi}_2 - \phi_2' \frac{\dot{x}}{x'}\right) - B \right\} , \tag{2.3}$$

where

$$\phi_1 = \frac{1}{\sqrt{2}}(\phi + \phi^*) \qquad , \qquad \phi_2 = \frac{i}{\sqrt{2}}(\phi - \phi^*) \quad , \tag{2.4}$$

and we denote derivaties with respect to σ by primes and with respect to τ by dots. We take $\emptyset_1(\sigma)$, $\emptyset_2(\sigma)$ and $x(\sigma)$ as the dynamical variables of the theory. The conjugate momenta $P_{\emptyset_1}(\sigma)$, $P_{\emptyset_2}(\sigma)$ and $P_x(\sigma)$ are not independent of the dynamical variables. However, we treat them as independent and use them to define the sets of primary constraints χ_1, χ_2 and χ_3 respectively. These are

$$P_{\phi_1}(\sigma) \approx \frac{\phi_1'}{2} \quad , \tag{2.5}$$

$$P_{\phi_2}(\sigma) \approx \frac{\phi_2'}{2} \quad , \tag{2.6}$$

and

$$P_x x' + P_{\phi_1} \phi_1' + P_{\phi_2} \phi_2' \approx 0 \quad , \tag{2.7}$$

where $\approx$ denotes "weakly" equal .

The Hamiltonian, H, is formed from the Lagrangian in the usual way and then modified by adding to it the constraints χ_1, χ_2 and χ_3 multiplied by Lagrange multipliers $w_1(\sigma,\tau)$, $w_2(\sigma,\tau)$ and $w_3(\sigma,\tau)$

$$H = \int_0^1 d\sigma \left\{ \left(P_x x' + P_{\phi_1} \phi_1' + P_{\phi_2} \phi_2' \right) \omega_3 + \left(P_{\phi_2} - \frac{\phi_2'}{2} \right) \omega_2 + \left(P_{\phi_1} - \frac{\phi_1'}{2} \right) \omega_1 + B x' \right\} . \tag{2.8}$$

Then, we check for secondary constraints by requiring that

$$\left\{ \chi_i , H \right\}_{PB} \approx 0 \qquad , \quad i = 1,2,3 \quad . \tag{2.9}$$

We finally get an infinite sequence of secondary constraints

$$\left. \frac{\partial^n \phi_1'}{\partial \sigma^n} \right|_0^1 \approx 0 \qquad , \quad n = 0,1,2, ---- \quad , \tag{2.10}$$

$$\left. \frac{\partial^n \phi_2'}{\partial \sigma^n} \right|_0^1 \approx 0 \qquad , \quad n = 0,1,2, ---- \quad , \tag{2.11}$$

$$\left. \frac{\partial^n x'}{\partial \sigma^n} \right|_0^1 \approx 0 \qquad , \quad n = 0,1,2 ---- \quad . \tag{2.12}$$

The three sets of secondary constraints above give the periodicity requirements on the derivaties of $\phi_1(\sigma)$, $\phi_2(\sigma)$ and $x(\sigma)$. However, in our quasicanonical formulation, we must work with the nonperiodic $\phi_1(\sigma)$, $\phi_2(\sigma)$ and $x(\sigma)$, hence, a special basis function $\lambda(\sigma)$[7,8] is introduced. It is defined by

$$\lambda(\sigma) \equiv (\sigma - \tfrac{1}{2}) + \frac{i}{2\pi} \sum_{n \neq 0}^{N} \frac{e^{-2\pi i n \sigma}}{n} \quad . \tag{2.13}$$

This will enable us to expand ϕ_j, P_{ϕ_j} (j=1,2), x, p_x, χ_1, χ_2 and χ_3 in a uniformally convergent Forier series expansions. Doing this we get

$$\phi_j(\sigma,\tau) \equiv \sqrt{\pi}\, a_\lambda^{(j)}(\tau)\, \lambda(\sigma) + a_0(\tau) + \frac{i}{(4\pi)^{1/2}} \sum_{n\neq 0}^{\pm N_1} \frac{a_n^{(j)}(\tau)}{n} e^{-i2\pi n\sigma} \quad, j=1,2 \quad (2.14)$$

$$P_{\phi_j}(\sigma,\tau) \equiv \frac{b_\lambda^{(j)}(\tau)\,\lambda(\sigma)}{\sqrt{\pi}\,|\lambda|^2} + b_0^{(j)}(\tau) + i(4\pi)^{1/2} \sum_{n\neq 0}^{\pm N_1} n\, b_n^{(j)}(\tau)\, e^{-i2\pi n\sigma} \quad, j=1,2 \quad (2.15)$$

$$x(\sigma,\tau) \equiv \sqrt{\pi}\, C_\lambda(\tau)\, \lambda(\sigma) + C_0(\tau) + \frac{i}{(4\pi)^{1/2}} \sum_{n\neq 0}^{\pm N} \frac{C_n(\tau)}{n} e^{-i2\pi n\sigma} \quad, \quad (2.16)$$

$$P_x(\sigma,\tau) \equiv \frac{d_\lambda(\tau)}{\sqrt{\pi}\,|\lambda|^2}\lambda(\sigma) + d_0(\tau) + i(4\pi)^{1/2} \sum_{n\neq 0}^{\pm N} n\, d_n(\tau)\, e^{-i2\pi n\sigma} \quad, \quad (2.17)$$

$$\chi_{j,m} \equiv \frac{1}{\sqrt{\pi}} \int_0^1 e^{i2\pi m\sigma} (2P_{\phi_j} - \phi_j')\, d\sigma \quad, \quad j=1,2 \quad, \quad (2.18)$$

$$\chi_{j,\lambda} \equiv \frac{1}{\sqrt{\pi}} \int_0^1 \lambda(\sigma)\, (2P_{\phi_j} - \phi_j')\, d\sigma \quad, \quad j=1,2 \quad, \quad (2.19)$$

$$\chi_{3,m} \equiv \frac{1}{\sqrt{\pi}} \int_0^1 e^{i2\pi m\sigma} (P_{\phi_1}\phi_1' + P_{\phi_2}\phi_2' + P_x x')\, d\sigma \quad, \quad (2.20)$$

$$\chi_{3,\lambda} \equiv \frac{1}{\sqrt{\pi}} \int_0^1 \lambda(\sigma) (P_{\phi_1}\phi_1' + P_{\phi_2}\phi_2' + P_x x')\, d\sigma \quad, \quad (2.21)$$

the cutoffs on the ϕ_j, P_{ϕ_j} and x, P_x fields (N_1 and N) respectively are kept independent[7].

Here, we have departed a little from Dirac's method, because instead of inserting the secondary constraints (2.10), (2.11) and (2.12) into the Hamiltonian, we used them - since they are second-class - to define the fields ϕ_i' and x' in terms of periodic basis functions and canonically define a restricted set of P_x and P_{ϕ_j} expansion coefficients. Now, we are left with the set of constraints (2.18), (2.19), (2.20) and (2.21) which will be classified into first- and second-class constraints. To simplify this procedure we introduce the following relations

$$
\begin{aligned}
\bar{a}_n^{(j)} &\equiv \bar{a}_n^{(j)} + a_\lambda^{(j)} , && j=1,2 , \quad n \neq 0 , \\
\bar{a}_0^{(j)} &\equiv a_\lambda^{(j)} , && j=1,2 , \\
\bar{C}_n &\equiv C_n + C_\lambda , && \\
\bar{C}_0 &\equiv C_\lambda , && \\
\bar{b}_n^{(j)} &\equiv 2i\left[n\, b_n^{(j)} - \delta_{n,0} \frac{b_0^{(j)}}{(4\pi)^{1/2}}\right] , && j=1,2 , \\
\bar{d}_n &\equiv 2i\left[n\, d_n - \delta_{n,0} \frac{i d_0}{(4\pi)^{1/2}}\right] .
\end{aligned}
\tag{2.22}
$$

These coefficients satisfy the following equal $-\tau$ PB's

$$
\begin{aligned}
\left\{ \bar{a}_n^{(j)}(\tau) , \bar{b}_{n'}^{(j)}(\tau) \right\}_{PB} &= -2in\, \delta_{n,-n'} , && j=1,2 , \\
\left\{ \bar{a}_n^{(j)}(\tau) , b_\lambda^{(j)}(\tau) \right\}_{PB} &= 1 , && j=1,2 , \\
\left\{ a_0^{(j)}(\tau) , \bar{b}_0^{(j)}(\tau) \right\}_{PB} &= \frac{1}{\sqrt{\pi}} , && j=1,2 , \\
\left\{ \bar{C}_n(\tau) , \bar{d}_{n'}(\tau) \right\}_{PB} &= -2in\, \delta_{n,-n'} , && \\
\left\{ \bar{C}_n(\tau) , d_\lambda(\tau) \right\}_{PB} &= 1 , && \\
\left\{ C_0(\tau) , \bar{d}_0(\tau) \right\}_{PB} &= \frac{1}{\sqrt{\pi}}
\end{aligned}
\tag{2.23}
$$

All other PB's = 0.

Using equations (2.14) - (2.17) and (2.22) we can write equations (2.18), (2.19), (2.20) and (2.21) as

$$\chi_{j,m} = 2\,\bar{b}_m^{(j)} - \bar{a}_m^{(j)} \qquad , \qquad j = 1,2 \quad , \tag{2.24}$$

$$\chi_{j,\lambda} = \frac{2 b_\lambda^{(j)}}{\pi} \qquad , \qquad j = 1,2 \quad , \tag{2.25}$$

$$\chi_{3,m} = \sqrt{\pi}\left[\sum_{j=1}^{2} \sum_{n}^{\pm N_1} \bar{b}_n^{(j)}\, \bar{a}_{m-n}^{(j)} + \sum_{n}^{\pm N} \bar{d}_n\, \bar{C}_{m-n} \right] , \tag{2.26}$$

$$\chi_{3,\lambda} = \frac{1}{\sqrt{\pi}}\left[\sum_{j=1}^{2} \sum_{n}^{\pm N_1} b_\lambda^{(j)}\, \bar{a}_n^{(j)} + d_\lambda \sum_{n}^{\pm N} \bar{C}_n \right] . \tag{2.27}$$

We define also the quantities

$$L_m \equiv \tfrac{1}{2} \sum_{j=1}^{2} \sum_{n}^{\pm N_1} \bar{b}_n^{(j)}\, \bar{a}_{m-n}^{(j)} \quad , \tag{2.28}$$

$$K_m \equiv \tfrac{1}{2} \sum_{n}^{\pm N} \bar{d}_n\, \bar{C}_{m-n} \quad , \tag{2.29}$$

such that (2.26) takes the form

$$\chi_{3,m} = (4\pi)^{1/2} \left(L_m + K_m \right) . \tag{2.30}$$

The L_m and K_m satisfy the following equal - τ PB's

$$\left\{ L_m(\tau) \, , \, L_{m'}(\tau) \right\}_{PB} = -i(m - m')\, L_{m+m'}(\tau) \quad , \tag{2.31}$$

$$\left\{ K_m(\tau) \, , \, K_{m'}(\tau) \right\}_{PB} = -i(m - m')\, K_{m+m'}(\tau) \quad , \tag{2.32}$$

$$\left\{ L_m(\tau) \, , \, K_{m'}(\tau) \right\}_{PB} = 0 \quad . \tag{2.33}$$

We also introduce the following combinations of the constraints

$$\bar{\chi}_{3,m} \equiv \chi_{3,m} - \chi_{1,0} \qquad m \neq 0 \quad , \tag{2.34}$$

which is a first-class constraint and

$$\bar{\chi}_{3,0} \equiv \chi_{3,0} + \sqrt{\pi} \sum_{j=1}^{2} \sum_{n} \chi_{j,0} \, \bar{b}_n^{(j)} \quad , \tag{2.35}$$

$$\bar{\chi}_{3,\lambda} \equiv \frac{\sqrt{\pi}\left(\chi_{3,\lambda} - \frac{\sqrt{\pi}}{2} \sum_{j=1}^{2} \sum_{n} \chi_{j,\lambda} \, \bar{a}_n^{(j)} \right)}{\sum_n \bar{C}_n} = d_\lambda \quad , \tag{2.36}$$

$$\bar{\chi}_{j,m} \equiv \chi_{j,m} - \chi_{j,0} \quad , \quad m \neq 0 \, , \quad j = 1,2 \, , \tag{2.37}$$

$$\bar{\chi}_{j,0} \equiv \chi_{j,0} \quad , \quad j = 1,2 \quad , \tag{2.38}$$

$$\bar{\chi}_{j,\lambda} \equiv \chi_{j,\lambda} = \frac{2\bar{b}_\lambda^{(j)}}{\pi} \quad , \quad j = 1,2 \quad , \tag{2.39}$$

which are second-class contraints.

These constraints satisfy the following equal-t PB's

$$\left\{ \bar{\chi}_{j,m}(\tau) \, , \, \bar{\chi}_{j,m'}(\tau) \right\}_{PB} = 8 i m \, \delta_{m,-m'} \quad , \quad j = 1,2 \quad , \tag{2.40}$$

$$\left\{ \bar{\chi}_{3,\lambda}(\tau) \, , \, \bar{\chi}_{3,0}(\tau) \right\}_{PB} = -\sqrt{\pi} \sum_{n} d_n(\tau) \quad , \tag{2.41}$$

$$\left\{ \bar{\chi}_{j,\lambda}(\tau) \, , \, \bar{\chi}_{j,0}(\tau) \right\}_{PB} = \frac{2}{\pi} \quad , \quad j = 1,2 \, , \tag{2.42}$$

$$\left\{ \bar{\chi}_{j,m}(\tau) \, , \, \bar{\chi}_{3,0}(\tau) \right\}_{PB} = -i (4\pi)^{1/2} m \, \bar{\chi}_{j,m}(\tau) \approx 0 \, , \quad j = 1,2 \, , \tag{2.43}$$

and all others are strongly zero. The second-class contraints can be used to eliminate the variables $\bar{b}_n^{(j)}$, $b_\lambda^{(i)}$ and d_λ by setting them identically equal to zero. Doing this we now redefine L_m as

$$L_m \equiv \frac{1}{2} \sum_{j=1}^{2} \sum_{n \neq 0}^{\pm N_1} \bar{a}_n^{(j)} \, \bar{a}_{m-n}^{(j)} \quad . \tag{2.44}$$

The canonical PB's have to be modified to reflect the elimination of some degrees of freedom which resulted from setting the second-class constraints identically zero. This can be done as follows

$$\{\xi, \eta\}_{PB^*} \equiv \{\xi, \eta\}_{PB} - \sum_{a,b} \{\xi, \chi_a\}_{PB} \{\chi_a, \chi_b\}^{-1}_{PB} \{\chi_b, \eta\}_{PB} , \tag{2.45}$$

where χ_a, χ_b represent all the second-class contraints.

We impose Dirichlet boundary conditions $\phi_j(o,\tau) = \phi_j(1,\tau) = o$ these are equivalent to

$$\begin{aligned} &\overset{(j)}{a}_{\lambda}(\tau) = o , \\ &\overset{(j)}{a}_{0}(\tau) + \frac{i}{(4\pi)^{1/2}} \sum_{n\neq o}^{\pm N_1} \frac{\overset{(j)}{a}_{n}(\tau)}{n} = o . \end{aligned} \tag{2.46}$$

These new constraints have zero PB* with H, so there are no secondary constraints resulting from imposing them. The field functions, which satisfy the new condition, take the form

$$\phi_j(\sigma,\tau) = i(4\pi)^{-1/2} \sum_{n\neq o}^{\pm N_1} \frac{\overset{-(j)}{a}_{n}(\tau)}{n} \left(e^{-i2\pi n\sigma} - 1\right) , \quad j=1,2 , \tag{2.47}$$

$$\chi(\sigma,\tau) = \sqrt{\pi}\, c_\lambda(\tau)\,\lambda(\sigma) + c_0(\tau) + \frac{i}{(4\pi)^{1/2}} \sum_{n\neq o}^{\pm N} \frac{c_n(\tau)}{n} e^{-i2\pi n\sigma} , \tag{2.48}$$

$$P_\chi(\sigma,\tau) = i(4\pi)^{1/2} \sum_{n}^{\pm N} \bar{d}_n(\tau)\, e^{-i2\pi n\sigma} . \tag{2.49}$$

The first-class constraint (2.34) now takes the form

$$\bar{\chi}_{3,m} = (4\pi)^{1/2}\left(L_m + K_m - L_0 - K_0\right) , \quad m \neq o . \tag{2.50}$$

The field coefficients obey the following PB*'s

$$\left\{ \bar{a}_n^{(j)}(\tau) , \bar{a}_{n'}^{(j)}(\tau) \right\}_{PB^*} = -i n \delta_{n,-n'} \quad , \quad j = 1, 2 \quad , \tag{2.51}$$

$$\left\{ \bar{a}_n^{(j)} , c_{n'} \right\}_{PB^*} = \left\{ \bar{a}_n^{(j)} , \bar{d}_{n'} \right\} = 0 \quad , \tag{2.52}$$

$$\left\{ c_n , \bar{d}_{n'} \right\}_{PB^*} = -2in \delta_{n,-n'} + \frac{1}{\sqrt{\pi}} \delta_{n,0} \delta_{n',0} \quad , \tag{2.53}$$

$$\left\{ \bar{a}_n^{(j)} , c_\lambda \right\}_{PB^*} = \frac{2in \bar{a}_n^{(j)}}{\sum_n \bar{d}_n} \quad , \tag{2.54}$$

$$\left\{ \bar{d}_n , c_\lambda \right\}_{PB^*} = \frac{2in \bar{d}_n}{\sum_n \bar{d}_n} \quad , \tag{2.55}$$

$$\left\{ c_0 , c_\lambda \right\}_{PB^*} = \frac{-c_\lambda}{\sqrt{\pi} \sum_n \bar{d}_n} \quad , \tag{2.56}$$

$$\left\{ d_0 , c_\lambda \right\}_{PB^*} = 0 \quad , \tag{2.57}$$

$$\left\{ \bar{c}_n , c_\lambda \right\}_{PB^*} = \frac{2in \bar{c}_n}{\sum_n \bar{d}_n} \quad . \tag{2.58}$$

To apply Dirac's procedure, one has to work within the physical subspace of functions of dynamical variables i.e. the time development of these functions remains within the physical subspace. This could be done by using dynamical variables which have zero PB* with the first-class constraints $\bar{\chi}_{3,m}$. The variables $a_n^{(j)}$, c_n and d_n are not physical variables. One can proceed by imposing additional constraints[7] which have non zero PB*'s with $\bar{\chi}_{3,m}$. This will make both-the additional and $\bar{\chi}_{3,m}$ constraints-second-class constraints and hence all the dynamical variables will be physical variables. Such a parameterisation of constraints will generate far more complicated PB* relations than the one which we already have. In the next section we shall see how this problem is cured[8] by constructing states using constraint consistent operators.

2.2 Quantisation

We pass to the quantum theory by the substitution $i\{A,B\}_{PB^*} \longrightarrow [A,B]$. If we require the empty bag of momentum p to be a physical state, we demand

$$\bar{\chi}_{3,m}|\Omega_p\rangle = 0 \qquad ; \quad \langle\Omega_p|\bar{\chi}^*_{3,m} = 0 \quad ; \; m>0 \; , \tag{2.59}$$

where the normalisation of the states is

$$\langle\Omega_{p'}|\Omega_p\rangle = \delta(p-p') \qquad . \tag{2.60}$$

This implies the requirement for both L_o and K_o to be normal ordered. So, we get the following equal-τ commutation relations

$$\left[L_m(\tau), L_{m'}(\tau)\right] = (m-m')\,L_{m+m'}(\tau) + \frac{1}{12}\,\delta_{m,-m'}(m^3-m) \quad , \tag{2.61}$$

$$\left[K_m(\tau), K_{m'}(\tau)\right] = (m-m')\,K_{m+m'}(\tau) + \frac{1}{6}\,\delta_{m,-m'}(m^3-m) \quad , \tag{2.62}$$

$$\left[\bar{\chi}_{3,m}(\tau), \bar{\chi}_{3,m'}(\tau)\right] = (4\pi)^{1/2}\left\{(m-m')\,\bar{\chi}_{m+m'}(\tau) - m\,\bar{\chi}_m(\tau) + m'\,\bar{\chi}_{m'}(\tau) + \frac{1}{4}\,\delta_{m,-m'}(m^3-m)\right\} . \tag{2.63}$$

The c-number Schwinger terms in the commutation relations make $\bar{\chi}_{3,m}$ a second-class constraints and break the reparametrisation invariance of the theory. Although $\bar{\chi}_{3,m}$ are now second-class constraints, we can still proceed by imposing them as weak conditions and construct states[8] using constraint consistent operators θ which satisfy

$$\left[\bar{\chi}_{3,m}, \theta\right] = 0 \qquad ; \quad |m| < N_1 \qquad . \tag{2.64}$$

We define the dimensionless variable ρ as

$$\rho(\sigma) \equiv \frac{P(\sigma) - P(0)}{P} = \sigma + \frac{i}{\sqrt{\pi}\,d_0}\sum_{n\neq 0}^{\pm N}\frac{\bar{d}_n}{n}\left(e^{-i2\pi n\sigma} - 1\right) \quad , \tag{2.65}$$

where

$$P(\sigma) \equiv -\int^{\sigma} P_x(\sigma')\, d\sigma' =$$

$$-d_0(\sigma - \tfrac{1}{2}) - \frac{i}{\sqrt{\pi}} \sum_{n \neq 0}^{\pm N} \frac{\bar{d}_n}{n}\, e^{-2\pi i n \sigma} . \tag{2.66}$$

[Any variables without arguments (such as ρ) denote c-number parameters while variables with arguments ($\rho(\sigma)$) denote the corresponding operators .]

Since $0 \leq \rho \leq 1$ and $\phi(\rho = 0) = \phi(\rho = 1) = 0$, we use the classical analogy to define

$$\phi(\rho) = i(4\pi)^{-1/2} \sum_{n>0}^{N_1} \frac{1}{n}\left\{ A_n(e^{-i2\pi n\rho} - 1) - B_n^*(e^{i2\pi n\rho} - 1)\right\} , \tag{2.67}$$

where

$$A_n \equiv i(4\pi)^{1/2}\, n \int_0^1 \phi(\rho)\, e^{2\pi i n \rho}\, d\rho$$

$$\equiv \int_0^1 \sum_{m>0} \left(a_m e^{-i2\pi m\sigma} + b_m^{+} e^{2\pi i m\sigma}\right) e^{i2\pi n\rho(\sigma)}\, d\sigma , \tag{2.68}$$

and

$$B_n^* \equiv i(4\pi)^{+1/2}\, n \int_0^1 \phi(\rho)\, e^{-2\pi i n\rho}\, d\rho$$

$$\equiv \int_0^1 \sum_{m>0} \left(a_m e^{-i2\pi m\sigma} + b_m^{+} e^{2\pi i m\sigma}\right) e^{-i2\pi n\rho(\sigma)}\, d\sigma , \tag{2.69}$$

The contraint consistent condition (2.64) is satisfied in this representation

$$[\bar{\chi}_{3,m}, A_n] = 0 \quad ; \quad [\bar{\chi}_{3,m}, B_n] = 0 . \tag{2.70}$$

The A_n, B_n commutation relations are isomorphic to the $\bar{a}_n, b_n$ CR's

$$[A_n, A_{n'}] = n\,\delta_{n,-n'} \quad , \tag{2.71}$$

$$[B_n, B_{n'}] = n\,\delta_{n,-n'} \quad . \tag{2.72}$$

In analogy, $\chi(\rho)$ can be defined as

$$\chi(\rho) = \bar{x}_0 + \frac{i}{P}\sum_{n\neq 0}\frac{L^A_n}{n}\, e^{-2\pi i n\rho} + \frac{2\pi}{P}\left[L^A_0 + \frac{(m^A_0)^2}{4\pi B}\right](\rho - \tfrac{1}{2}) \quad , \tag{2.73}$$

where

$$L^A_n = \tfrac{1}{2} : \sum_m A_m B_{n-m} + B_m A_{n-m} : \quad ,$$

and

$$\bar{x}_0 \equiv C_0 + \frac{i}{2}\sum_{n\neq 0}\frac{C_n}{n} - \frac{i}{P}\sum_{n\neq 0}\frac{L^A_n}{n} \quad .$$

The quantity m_0^A is the mass of the empty bag. This completes the formalism and we now turn to the calculation of the form factor and the structure function.

2.3 The Elastic Form Factor

In terms of light-cone momenta, the elastic form factor is defined as[5]

$$F^n_- = (q_1 + q_2)\, F^n \quad , \tag{2.74}$$

where

$$F^n_- = \langle q_2, n |\, J_-(0,0)\, | q_1, n \rangle \quad ,$$

and $|q_1, n\rangle$ and $|q_2, n\rangle$ are one-particle states of light-cone momenta q_1 and q_2. We take $|q_1, n\rangle = A_n^* |\Omega_q\rangle$ with $|\Omega_q\rangle$ the state of the empty bag with momentum q and A_n^* is the creation operator for a positive particle in state n. The bag physical current $J_\mu(x,\tau)$ is defined by

$$J_\mu^{phys}(x,\tau) = \int_0^1 d\rho \; \frac{d\acute{x}(\rho)}{d\rho} \; \delta(x - \acute{x}(\rho)) \; J_\mu^B(\rho,\tau) \quad , \tag{2.75}$$

where $J_\mu^B(\rho,\tau)$ is the bag current given by

$$J_\mu^B(\rho,\tau) = \frac{i}{2}\left[\phi^*(\partial_\mu\phi) - (\partial_\mu\phi^*)\phi + (\partial_\mu\phi)\phi^* - \phi(\partial_\mu\phi^*)\right] . \tag{2.76}$$

For our simple approximation, we will keep only the L_o^A operator and ignore the L_n^A in (2.73) which will become

$$x(\rho) = \bar{x}_o + \frac{2\pi}{P}\left[L_o^A + \frac{(m_o^A)^2}{4\pi B}\right](\rho - 1/2) \quad . \tag{2.77}$$

Using equations (2.75), (2.76) and (2.77), we obtain for $x = \tau = 0$

$$J(0,0) = \frac{1}{2\pi}\int_0^1 d\rho \int_{-\infty}^{\infty} dq \; \exp\left[-iq\left(\bar{x}_o + \frac{2\pi}{P}\left(L_o^A + \frac{(m_o^A)^2}{4\pi B}\right)(\rho - 1/2)\right)\right]$$
$$(1 - \cos 2\pi\rho)(B^*B - A^*A) \quad . \tag{2.78}$$

Inserting (2.78) in (2.74), removing the q integration and performing the ρ integration, we get the following result for the one-mode elastic form factor

$$F = \frac{\sin \pi\alpha}{\pi\alpha(1-\alpha^2)} \quad , \tag{2.79}$$

where

$$\alpha = \left[1 + \frac{(m_o^A)^2}{4\pi B}\right] \log \frac{q_1}{q_2} \quad .$$

This agrees with the expression given by Krapchev[5].

2.4 The Structure Function

Here we take for the momenta of the initial and final nucleon states (M, o) and $(M+q^o, Mx+q^o)$ respectively, where we use the usual notation and work in the Bjorken limit. Then the light-cone momenta are

$$P_i^+ = P_i^- = \frac{M}{\sqrt{2}} , \tag{2.80}$$

$$P_f^+ = \sqrt{2}\, q^o , \tag{2.81}$$

and

$$P_f^- = \frac{M}{\sqrt{2}}(1-x) . \tag{2.82}$$

To calculate the structure function, we calculate first the following matrix element

$$M^\mu = \int_0^1 d\rho \, \langle f| J^\mu(0,\rho) \left[\frac{dx}{d\rho} \right] \exp\left[-i\Gamma(\rho) \log \frac{P_f^-}{P_i^-} \right] |i\rangle , \tag{2.83}$$

where $|i\rangle = A_1^* \,|\text{vaccum}\rangle$; $\langle f| = \langle \text{vaccum}| A_m$,

$$\Gamma(\rho) = 2\pi(\rho - \tfrac{1}{2})\left[L_0^A + \frac{(m_0^A)^2}{4\pi B} \right] ,$$

we are working, as before, in the L_0^A approximation to the theory. For the valence contribution, the only term in $J^+(0,\rho)$ which contributes to the calculation is $-\frac{A_m^* A_1}{2m} e^{-i2\pi\rho}$. Then using (2.79) and (2.80) in (2.83) and performing the integral we finally get

$$M^+ = \frac{-\sin \frac{M^2}{4B} \log(1-x)}{2\pi\left[1 + \frac{M^2}{4\pi B}\log(1-x)\right]} , \tag{2.84}$$

where

$$\frac{\Lambda^2}{4\pi B} = \left(1 + \frac{(m_0^A)^2}{4\pi B}\right) \qquad .$$

The valence quark structure function is then obtained by squaring (2.84)

$$F(x) = \frac{\sin^2 \frac{\Lambda^2}{4B} \log(1-x)}{4\pi^2\left[1 + \frac{\Lambda^2}{4\pi B}\log(1-x)\right]^2} \qquad . \tag{2.85}$$

As in ref.6.

2.5 Conclusion

It was hoped that by applying Dirac's method of quantization to the two-dimensional bag, many of its difficulties would be resolved. Unfortunately this did not materialise. We still have the problem of ordering the field operators which affect any attempt to calculate the exact form factor or the structure function. When the analogue of the "L_0-approximation" is used, the results are identical to those determined in ref.[5,6].

CHAPTER 3

A MOMENTUM DISTRIBUTION OF QUARKS

In two-dimensions, with a scalar field, the structure function in a translationally invariant approximation to the bag model is related to the structure function in the corresponding cavity approximation by the transformation[6] $\log(1-x) \longleftrightarrow -x$. In this chapter we will show that the same result holds for the fermionic bag. The question of how to apply the transformation to a realistic four-dimensional model, and hence to a calculation of the observed structure functions has not been answered. In ref.41 the transformation was applied only to the x's which were not of "kinematic origin". This calculation was done in the Bjorken limit and it is much less clear how to apply the transformation to the term which vanish in this limit. We have attempted to answer this question in this chapter.

Also, in this chapter, a distribution of momenta of valence quarks inside a cavity will be calculated in a simple way. We assume that a nucleon is composed of three massive valence quarks, and we make use of the differential cross-section - calculated in two-dimension - of a massless electron scattering off a free quark via one particle exchance. We write down an expression - to the leading order - for the experimental structure function in terms of a general distribution in momentum of the quarks. Then we equate this expression to the structure function of a cavity. We end up with an integral equation and by solving it we obtain the required distribution.

The chapter is organised as follows, in section 1, we introduce the notations, the solution for a fermion field confined by a bag in (τ,σ) space, and some of the commutation relations that are used throughout this chapter. In section 2, the structure function is calculated in the

L_o approximation as well as in the cavity approximation to the bag. The transformation $\log(1-x) \longleftrightarrow -x$ between the two functions is obtained. A QED calculation of the scattering cross-section of electron-quark scattering is presented in section 3. In section 4, a formula for the quark momentum distribution which fits the cavity approximation is calculated. Using this distribution, we calculate, in section 5, the experimental structure function and we also compare the result with that obtained from the light cone analysis to the bag.

3.1 A Brief Review of the Fermion Field Theory of the Bag Model

$\psi = \begin{pmatrix} ig \\ f \end{pmatrix}$ is a two-dimensional massless Dirac field confined by a bag, which - in light cone coordinates - satisfies the following equation of motion inside the bag[4]

$$\left(\gamma^+ \frac{\partial}{\partial \tau} + \gamma^- \frac{\partial}{\partial x} \right) \begin{pmatrix} ig \\ f \end{pmatrix} = 0 \quad , \tag{3.1}$$

where the Dirac matrices are

$$\beta = \gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad , \tag{3.2}$$

$$\alpha = \gamma^0 \gamma^1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad .$$

A general solution of equation (3.1) will take the form

$$\psi = \begin{pmatrix} i g(x) \\ f(\tau) \end{pmatrix} \quad . \tag{3.3}$$

The field ψ satisfies the boundary conditions

$$\sqrt{2}\, i\beta \left[\frac{\partial j}{\partial \tau} \begin{pmatrix} ig \\ 0 \end{pmatrix} + \frac{\partial j}{\partial x} \begin{pmatrix} 0 \\ f \end{pmatrix} \right] = \begin{pmatrix} ig \\ f \end{pmatrix} \quad , \quad \text{at } x = x_k(\tau) \, , \tag{3.4}$$

$$\mathrm{Im}\left(\dot{f}^* g' \frac{\partial j}{\partial \tau} + \dot{f}^* g \frac{\partial j}{\partial x} \right) \Big|_{x = x_k(\tau)} = B \quad , \tag{3.5}$$

where $x_k(\tau)$, k=o,1, are the bag end points and the locally conserved bag electromagnetic current is given by

$$j^\mu = \bar{\Psi} \gamma^\mu \psi \tag{3.6}$$

The constants of motion are

$$P^+ \equiv P = \sqrt{2} \int_{x_0(\tau)}^{x_1(\tau)} dx \, \mathrm{Im}(g'^*(x)\, g(x)) \quad , \tag{3.7}$$

$$P^- \equiv H = B\,(x_1(\tau) - x_0(\tau)) \quad , \tag{3.8}$$

$$M = \tau H - \sqrt{2} \int_{x_0(\tau)}^{x_1(\tau)} dx \; x \, \mathrm{Im}(g'^*(x)\, g(x)) \quad , \tag{3.9}$$

$$Q = \sqrt{2} \int_{x_0(\tau)}^{x_1(\tau)} dx \; g^*(x)\, g(x) \quad . \tag{3.10}$$

The problem has been linearized by defining a new space parameter $\sigma(x)$ which is given by the following equation

$$\frac{d\sigma}{dx} = \frac{\sqrt{2}}{P} \, \mathrm{Im}(g'^*(x)\, g(x)) \quad , \tag{3.11}$$

$$\sigma(x_0(0)) = 0 \quad . \tag{3.12}$$

From the light-cone momenta (3.7) we find that $\sigma(x_1(\tau)) - (x_0(\tau)) = 1$ and the boundary in σ space is

$$\begin{aligned} \sigma_0(\tau) &= \frac{B\tau}{P} \quad , \\ \sigma_1(\tau) &= \frac{B\tau}{P} + 1 \quad . \end{aligned} \tag{3.13}$$

Since $\mathrm{Im}(g'^*(x)\, g(x))$ is not an intrinsically positive definite quantity and we require that $\frac{d\sigma}{dx} > 0$ throughout the bag, a new field $\tilde{g}(\sigma) \equiv g(x(\sigma))$ has been defined by

$$\tilde{g}(\sigma) = \left(\frac{dx}{d\sigma}\right)^{1/2} g(x(\sigma)) \quad . \tag{3.14}$$

Now, $\sigma(x)$ can be inverted and equation (3.11) will be

$$\frac{dx}{d\sigma} = \frac{\sqrt{2}}{P} \, \mathrm{Im}\left(\tilde{g}'^{*}(\sigma)\, \tilde{g}(\sigma)\right) \quad . \tag{3.15}$$

$\tilde{g}(\sigma)$ is odd periodic in the interval $[\sigma_0, \sigma_1]$ and it could be expressed in half integral modes

$$\tilde{g}(\sigma) = \frac{1}{(\frac{1}{2})^{1/4}} \sum_{m \geq 0}^{\infty} \left(b_m e^{-2\pi i \acute{m} \sigma} + d_m^{+} e^{2\pi i \acute{m} \sigma} \right) \ , \quad \acute{m} = m + \tfrac{1}{2} \ . \tag{3.16}$$

$f(\tau)$ is related to $\tilde{g}(\sigma)$ by

$$f(\tau) = (-1)^k \left(\frac{B}{P}\right)^{1/2} \tilde{g}(\sigma_k(\tau)) \quad , \tag{3.17}$$

then $f(\tau)$ can be written as

$$f(\tau) = \frac{1}{(\frac{1}{2})^{1/4}} \left(\frac{B}{P}\right)^{1/2} \sum_{m \geq 0}^{\infty} \left(b_m e^{-2\pi i \acute{m} \frac{B\tau}{P}} + d_m^{+} e^{2\pi i \acute{m} \frac{B\tau}{P}} \right) \ , \quad \acute{m} = m + \tfrac{1}{2} \ , \tag{3.18}$$

where b_m's and d_m^{+}'s in (3.16) and (3.18) are interpreted as follows $(m > 0)$

b_m annihilates a fermion,

d_m^+ creates an antifermion,

b_m^+ creates a fermion ,

d_m annihilates an antifermion

Integrating (3.15), we get the light cone coordinate operation $x(\sigma)$

$$x(\sigma) = \bar{x}_0 + \frac{2\pi}{P}\left(L_0 + \frac{m_0^2}{4\pi B}\right)(\sigma - \tfrac{1}{2}) + \frac{i}{P}\sum_{n\neq 0}\frac{L_n}{n} e^{-2\pi i n\sigma}, \quad (3.19)$$

where L_n are the fermion conformal generators

$$L_n = :\sum_{m=-\infty}^{\infty}(m' + \tfrac{1}{2}n)\, b_m^+ \, b_{n+m}: \quad , \quad m' = m + \tfrac{1}{2} \quad (3.20)$$

which obey the algebra

$$\begin{aligned} \left[L_n , L_m\right] &= (n-m)\, L_{n+m} + \frac{1}{12}\,\delta_{n,-m}(n^3 - n) , \\ \left[L_n , b_m\right] &= -\,(m + \tfrac{1}{2}n)\, b_{m+n} . \end{aligned} \quad (3.21)$$

If we take $b_m(\tau) \equiv b_m\, e^{-2\pi i m'(\frac{B\tau}{P})}$ and $d_m^+(\tau) \equiv d_m^+\, e^{2\pi i m' \frac{B\tau}{P}}$, $m' = m + \tfrac{1}{2}$

then, it obeys the anticommutation relations

$$\begin{aligned} \left\{ b_m(\tau) , b_n(\tau) \right\} &= 0 , \\ \left\{ d_m(\tau) , d_n(\tau) \right\} &= 0 , \\ \left\{ b_m(\tau) , b_n^+(\tau) \right\} &= \left\{ b_m^+(\tau) , b_n(\tau) \right\} = \delta_{m,n} , \\ \left\{ d_m(\tau) , d_n^+(\tau) \right\} &= \left\{ d_m^+(\tau) , d_n(\tau) \right\} = \delta_{m,n} . \end{aligned} \quad (3.22)$$

3.2 The Structure Function of the Fermion Bag

In this section the structure function of the two-dimensional bag will be calculated in the L_0 approximation. The transformation $\log(1-x) \leftrightarrow -x$ between the bag structure function in the Bjorken limit and its cavity approximation will be obtained.

The structure function $F(x,Q^2)$ is defined by

$$F(x,Q^2) = |M^+|^2 , \tag{3.23}$$

where M^+ is given by

$$M^+ = \int_0^1 d\sigma \langle f| \exp\left\{-i\Gamma(\sigma)\log\frac{P_f^-}{P_i^-}\right\} J^+(0,\sigma)\frac{dx}{d\sigma}|i\rangle , \tag{3.24}$$

where

$$\Gamma(\sigma) = 2\pi(\sigma-\tfrac{1}{2})\left(L_0+\frac{m_0^2}{4\pi B}\right)+i\sum_{n\neq 0}\frac{L_n}{n}e^{-2\pi i n\sigma} , \tag{3.25}$$

and $|f\rangle$, $|i\rangle$ are the final and initial one particle states of light cone momenta P_f and P_i respectively and the bag current j^μ is given by (3.6).

In calculating (3.24) only the first term in (3.25) - the L_0 approximation - will be taken into account.

For our purpose, we take $(M,0)$ and $(M+\nu,q)$ as the initial and final two momenta of the quark. Thus the light cone momenta are

$$P_i^- = P_i^+ = \frac{M}{\sqrt{2}} , \tag{3.26}$$

$$P_f^- = \frac{M+\nu-q}{\sqrt{2}} , \tag{3.27}$$

$$P_f^+ = \frac{M+\nu+q}{\sqrt{2}} . \tag{3.28}$$

To calculate (3.24), we need to know the eigenstate of some of the operators acting on the initial state $|i\rangle$ these are

$$\overset{+}{P}|i\rangle = \frac{M}{\sqrt{2}}|i\rangle \quad , \tag{3.29}$$

$$2\pi B L_0 |i\rangle = \left(\frac{m_0^2}{2} + 2\pi B\right)|i\rangle \quad , \tag{3.30}$$

$$\frac{2\pi B L_0}{P}|i\rangle = \left(\bar{P} - \frac{m_0^2}{4\pi B}\right)|i\rangle = \left(\frac{M}{\sqrt{2}} - \frac{m_0^2}{4\pi B}\right)|i\rangle \; , \tag{3.31}$$

$$\frac{2\pi B L_0}{P^2}|i\rangle = \left(1 - \frac{m_0^2}{2\sqrt{2}\pi B M}\right)|i\rangle \quad , \tag{3.32}$$

where in (3.31) we used the relation

$$H \equiv \bar{P} = \frac{2\pi B L_0}{P} + \frac{m_0^2}{4\pi B} \quad . \tag{3.33}$$

For the valence quark (3.24) will take the form

$$M^+ = \int_0^1 d\sigma \langle f| \exp\left\{-i\Gamma(\sigma) \log \frac{\bar{P}_f}{\bar{P}_i}\right\} \left\{\frac{B}{P}\sum_{m,n\geq 0}^{\infty} b_m^+ b_n\right\} \frac{2\pi L_0}{P}|i\rangle \; . \tag{3.34}$$

Doing the integral and using (3.23) we get the structure function

$$F(x,Q^2) = \frac{\left(1 - \frac{M}{2\sqrt{2}\pi B} + \frac{\sqrt{2}}{M}\right)^2 \sin^2 \frac{M^2}{4B} \log\left(1 - \frac{q-\nu}{M}\right)}{\left[\frac{M^2}{4B} \log\left(1 - \frac{q-\nu}{M}\right)\right]^2} \quad , \tag{3.35}$$

where

$$\frac{M^2}{4\pi B} = \left(1 + \frac{m_0^2}{4\pi B}\right) \quad .$$

The Bjorken limit to $F(x,Q^2)$ is

$$F_{B_j}(x) = \frac{\left(1 - \frac{M}{2\sqrt{2}\pi B} + \frac{\sqrt{2}}{M}\right)^2 \sin^2 \frac{M^2}{4B} \log(1-x)}{\left[\frac{M^2}{4B} \log(1-x)\right]^2} \quad . \tag{3.36}$$

Similarly, the structure function of a static bag of line segment ℓ will be calculated. The field $q(x, t)$ is given by[12]

$$q(x,t) = \frac{1}{\sqrt{2\ell}} \sum_{n\geq 0} \left\{ b_n \begin{pmatrix} e^{-i2\pi n' \frac{t-x}{\ell}} \\ (-1)^n \, e^{-i2\pi n' \frac{t+x}{\ell}} \end{pmatrix} + d_n^{+} \begin{pmatrix} e^{i2\pi n' \frac{t-x}{\ell}} \\ (-1)^n \, e^{i2\pi n' \frac{t+x}{\ell}} \end{pmatrix} \right\}, \quad n' = n + \tfrac{1}{2}, \tag{3.37}$$

where the b_n's and d_n^{+}'s are interpreted as l'n (3.18).

The current is given by (3.6) and in a straightforward calculation the structure function $\omega_{B_j}(x)$ in the Bjorken limit - for the cavity is

$$\omega_{B_j}(x) = \frac{\sin^2 \frac{M^2 x}{2B}}{\frac{M^4 x^2}{4B^2}} \quad . \tag{3.38}$$

We note by comparing (3.36) with (3.38) that both the structure functions of a fermion field confined either by a two-dimensional bag - in the L_o approximation - or by a cavity are relating - in general - by the transformation - $\log(1-x) \longleftrightarrow x$. This confirms the result given in ref.6.

3.3 QED Calculation of Structure Function in Two-Dimension

This is a two-dimenional calculation for a massless electron scattering off a free quark of mass m. The incident electron with laboratory momentum $P_i \equiv (E,P)$ will emit the photon $Q \equiv (\nu,q)$ which will be absorbed by the initial quark $K_i \equiv (k^o,k)$. After the collision, both the electron and the quark will emerge with the final energy - momentum vectors $P_f \equiv (E',P')$ and $K_f \equiv (k^{o'},k')$ respectively.

The differential cross section is given by[30]

$$\frac{d\sigma}{dE_2} = \frac{m^2 E' |\mathcal{M}|^2}{2(2\pi)^2 E k^o k q} , \tag{3.39}$$

where

$$|\mathcal{M}|^2 = \frac{e^2 e_q^2}{2 m^2 Q^4}\Big\{(K_f \cdot P_f)(K_i \cdot P_i) + (K_f \cdot P_i)(K_i \cdot P_f) - m^2 (P_f \cdot P_i)\Big\} \quad (3.40)$$

Using the electron and quark mass conditions and momentum conservation - law, we finally obtain for (3.39)

$$\frac{d\sigma}{dE_2} = \frac{e^2 e_q^2 E'^2 Mx\nu^3}{2(2\pi)^2 Q^4 k^0 k q^3}\Big\{1 + \frac{2}{\nu}\Big(k^0 + \frac{Mx}{2} - \frac{m^2}{2Mx}\Big) + \frac{2}{\nu^2}(k^{0\,2} - m^2)\Big\} \quad (3.41)$$

In this calculation, we assumed that the scattered quarks stays on mass shell thus

$$(k^0 + \nu)^2 - (k + q)^2 = m^2 \quad (3.42)$$

This gives us the following condition

$$k > \Big(1 + \frac{2Mx}{\nu}\Big)^{-1/2} |k^0 - Mx| \quad (3.43)$$

From (3.41), we get the structure function $F^{QED}(x, Q^2, k)$

$$F^{QED}(x, Q^2, k) = \frac{Mx\nu^3}{2k^0 k q^3}\Big\{1 + \frac{2}{\nu}\Big(k^0 + \frac{Mx}{2} - \frac{m^2}{2Mx}\Big) + \frac{2}{\nu^2}(k^{0\,2} - m^2)\Big\} \quad (3.44)$$

3.4 The Quark Momentum Distribution

We take the effect of confinement into consideration by integrating (3.44) over a momentum distribution P(k). The normalisation of P(k) is such that

$$\int_0^\infty k^2\, dk\, P(k) = 1 \quad . \tag{3.45}$$

The structure function $\mathcal{J}(x,Q^2)$ is given by

$$\mathcal{J}(x,Q^2) = \int k^2\, dk\; P(k)\; F^{QED}(x,Q^2,k) \quad , \tag{3.46}$$

where the integral is over the region given by (3.43).

To determine P(k) from 3.46, we take for $\mathcal{J}$ the value of the structure function of a cavity given by (3.38), while in the integrand we write F^{QED} - given by 3.44 - to the leading order only i.e. the zero order term of $\frac{1}{\nu}$. We also take $Mx - k^0 < k < \infty$ as the limit of integration in our particular case. Doing that we have

$$\frac{\sin^2 \frac{M^2 x}{2B}}{\frac{M^4 x^2}{4B^2}} = \frac{Mx}{2k^0} \int_{Mx-k^0}^{\infty} k\, P(k)\, dk \quad . \tag{3.47}$$

Differentiating both sides in (3.47) with respect to x, rearranging terms, we finally get

$$P(Mx - k^0) = \frac{4Bk^0}{M^6 x^4 (Mx - k^0)} \left[6B \sin^2 \frac{M^2 x}{2B} - M^2 x \sin \frac{M^2 x}{B} \right] . \tag{3.48}$$

Then P(k) could be written as

$$P(k) = \frac{4Bk^0}{M^2 k(k+k^0)^4}\left[6B\sin^2\frac{M}{2B}(k+k^0) - M(k+k^0)\sin\frac{M}{B}(k+k^0)\right]. \tag{3.49}$$

3.5 The Structure Function of Quarks with the Initial Momentum Distribution P(k)

The momentum distribution of valence quarks which is given by (3.49) will fit the experimental structure function - in the Bjorken limit - to the structure function calculated in the cavity approximation. In this section we will use the formula of P(k) given by (3.49) to calculate (3.46). In this case F^{QED} is given by (3.44) and the integral is over the region $\left(1+\frac{2Mx}{\nu}\right)^{-1/2}(Mx-k^0)<k<\infty$. So (3.46) will be

$$\mathcal{F} = \frac{2Bx\nu^3}{Mq^3}\int_{\left(1+\frac{2Mx}{\nu}\right)^{-1/2}(Mx-k^0)}^{\infty} \frac{dk}{(k+k^0)^4}\left[6B\sin^2\frac{M}{2B}(k+k^0) - M(k+k^0)\sin\frac{M}{B}(k+k^0)\right]$$

$$\left\{1+\frac{2}{\nu}\left(k^0+\frac{Mx}{2}-\frac{k^{02}-k^2}{2Mx}\right)+\frac{2k^2}{\nu^2}\right\} . \tag{3.50}$$

Doing the integral we get for the structure function

$$\mathcal{F} = \frac{2Bx\nu^3}{Mq^3}\left\{\frac{2B}{K^3}\sin^2\frac{MK}{2B}\right.$$

$$+\frac{1}{\nu}\left[\frac{6Bk^0+2BMx-\frac{6Bk^0K}{Mx}+\frac{6BK^2}{Mx}}{K^3}\sin^2\frac{MK}{2B}\right.$$

$$- \frac{k^\circ}{Mx} \sin \frac{MK}{B} - \frac{2}{x} \, Si(K) - \frac{Mk^\circ}{xB} \, Ci(K) \Big]$$

$$+ \frac{2}{\nu^2} \Big[\frac{2Bk^{\circ 2} - 6Bk^\circ K + 6BK^2}{K^3} \sin^2 \frac{MK}{2B}$$

$$- \frac{Mk^\circ}{K} \sin \frac{MK}{B} - 2M \, Si(K) - \frac{M^2 k^\circ}{B} \, Ci(K) \Big\} , \quad (3.51)$$

where

$$K = k^\circ + \left(1 + \frac{2Mx}{\nu}\right)^{-1/2} (Mx - k^\circ) ,$$

$$\int_z^\infty \frac{\sin t}{t} \, dt = - Si(z) , \quad (3.52)$$

$$\int_z^\infty \frac{\cos t}{t} \, dt = - Ci(z) .$$

Using

$$\frac{\nu^3}{q^3} = \left(1 - \frac{Q^2}{\nu^2}\right)^{-3/2} = \left(1 - \frac{3Mx}{\nu} + \frac{15}{2} \frac{M^2 x^2}{\nu^2} + \cdots \right) , \quad (3.53)$$

(3.51) will take the form.

$$J = \frac{2Bx}{M}\Bigg\{ \frac{2B}{K^3} \sin^2 \frac{MK}{2B}$$

$$+ \frac{1}{\nu}\Bigg[\frac{4MxBk^0 - 4M^2x^2B - 6Bk^0K + 6BK^2}{K^3 M x} \sin^2 \frac{MK}{2B}$$

$$- \frac{k^0}{Mx} \sin \frac{MK}{B} - \frac{2}{x} Si(K) - \frac{Mk^0}{xB} Ci(K)\Bigg]$$

$$+ \frac{1}{\nu^2}\Bigg[\frac{4Bk^{02} + 3M^2x^2B(5-2B) + 6K^2B(2+3B) - 12Mk^0B(1+x)}{K^3} \sin^2 \frac{MK}{2B}$$

$$+ (3k^0B - \frac{2Mk^0}{K}) \sin \frac{MK}{B} + 2M(3B-2) Si(K)$$

$$+ \frac{M^2k^0}{B}(3B-2)\, Ci(K)\Bigg]\Bigg\} \quad . \qquad (3.54)$$

K is dependent on ν, we expand it to the first order in $\frac{1}{\nu}$,

$$K = Mx\left[1 - \frac{1}{\nu}(Mx - k^0)\right] \quad . \qquad (3.55)$$

We rewrite (3.54) to the order of only, we have

$$J = \frac{4B^2}{M^4x^2} \sin^2 \frac{M^2x}{2B} + \frac{1}{\nu}\Bigg[\frac{8B^2}{M^4x^2}(2Mx - k^0) \sin^2 \frac{M^2x}{2B}$$

$$- \frac{2Bk^0}{M^2} \sin \frac{M^2x}{B} - \frac{4B}{M} Si\left(Mx - \frac{Mx}{\nu}(Mx-k^0)\right)$$

$$-2k^0 Ci\left(Mx - \frac{Mx}{\nu}(Mx-k^0)\right)\Bigg]\Bigg\} . \qquad (3.56)$$

In comparing (3.56) with (3.36) we first note that the model leading to (3.36) - we will call it Kapchev model - has a structure function going to zero at $q - \nu = M$. This corresponds to a final "mass" M^* [of the system proton $(M,0)$ + photon (ν,q)] which is positive.

$$M^{*2} = (M+\nu)^2 - q^2 > 0$$

$$M^{*2} - M^2 = 2M\nu + \nu^2 - q^2$$

i.e. the model does not "know" that M^* has to be greater than M. Thus the appropriate replacement for x is not (-log(1-x)) but $x \to -\log(1- \frac{q-\nu}{M})$.
Then we see that (3.36) agrees with the first term of (3.56) (with the appropriate choice of constants). However the Krapchev model shows no sign of the other terms of (3.56). We have no explanation of this but it is clear that this investigation does not help us to understand our problem.

3.6 Conclusion

1. We show that, in the Bjorken limit, the transofmration $x \leftrightarrow -\log(1-x)$ between the two structure functions of a fermion field confined first by a cavity and second by a two-dimensional bag - in the L_0 approximation - holds in this case as well as in the case of scalar field.[6]
2. The problem of using that transformation in a calculation of the observed structure function in a non-Bjorken limit is still open for further investigation.

CHAPTER 4

CORRECTION TO L_o APPROXIMATION

The valence contribution to deep ineleastic structure function was calculated by Jaffe[12] in the static cavity approximation to the bag model. The translation invariance was destroyed in this approximation as a result of ignoring the dynamical nature of the bag's boundaries. This problem was cured by Davis and Squires[6] in an approximation - now known as the L_o- approximation - to the two-dimensional MIT bag model. They calculated the structure function and showed that the effect of restoring the translation invariance to the system was to replace x in the formula calculated in the cavity by minus log (1-x). As we demonstrated in Chapter 2, the same result holds when the bag, as quantised by Shalloway, was treated in the analogous approximation. Also, it was shown in Chapter 3 that the same transformation was obtained in the case of a fermionic bag.

This result has been used by Jaffe and Ross[10] in a calculation of structure functions and their QCD scaling violations as predicted by the bag model. It has also been used by Squires[11] to predict, in a reasonable agreement with experiment, the pion structure function from that of the proton. In a phenomenilogical sense it is therefore a useful procedure. Nevertheless its justification is very incomplete since it depends on an approximation to the two-dimensional model.

In this chapter we present an attempt to assess the validity of the L_o approximation as applied to the structure function calculation in the two-dimensional MIT bag. Our procedure is to isolate the terms containing the $L_n (n \neq o)$ terms and expand the exponential involving them, keeping only the first two terms.

4.1 The Calculation of the Correction Term

We consider a two-dimensional bag containing a single massless complex scalar field. The solution to this problem is reviewed in the first chapter - section 1 - (also see ref.5). For our purpose, we start with the expression for the matrix element given in[6] (see also ref.5)

$$\mathcal{M}^{\mu} = \int_0^1 d\sigma \, \langle f | \, J^{\mu}(0,\sigma) \left(\frac{dz^{+}}{d\sigma}\right) \exp\left[-i\Gamma(\sigma) \log \frac{P_f^{-}}{P_i^{-}}\right] b_1^{\dagger} \, |0\rangle, \quad (4.1)$$

where $b_1^{\dagger}|0\rangle$ is the ground state containing one "quark", J^{μ} is the bag current and

$$\Gamma(\sigma) = 2\pi(\sigma - \tfrac{1}{2})\left[L_0 + \frac{m_0^2}{4\pi B}\right] + \sum_{n \neq 0} \frac{i}{n} e^{-2\pi i n \sigma} L_n \quad . \quad (4.2)$$

We work with light-cone variables

$$z^{-} = \tau = \frac{1}{\sqrt{2}}(t - x) \qquad ; \qquad z^{+} = \frac{1}{\sqrt{2}}(t + x) \quad (4.3)$$

In the Bjorken limit, we take $(\mathcal{M}, 0)$ and $(M + q^{0}, Mx + q^{0})$ as the initial and final two momenta of the hadron respectively. So the light-cone momenta are

$$P_i^{+} = P_i^{-} = \frac{M}{\sqrt{2}} \quad , \quad (4.4)$$

$$P_f^{+} = \sqrt{2}\, q^{0} \quad , \quad (4.5)$$

and

$$P_f^{-} = \frac{M}{\sqrt{2}}(1 - x) \quad . \quad (4.6)$$

The bag current is defined

$$J_{\mu} = \frac{i}{2}\left[\phi^{*}(\partial_{\mu}\phi) - (\partial_{\mu}\phi^{*})\phi + (\partial_{\mu}\phi)\phi^{*} - \phi(\partial_{\mu}\phi^{*})\right], \quad (4.7)$$

where ϕ and ϕ^* are given by (1.16) and (1.17). Using them we get

$$\frac{dz^+}{d\sigma} J^+(0,\sigma) = -\frac{1}{2} \sum_{n,m=1}^{\infty} \Big\{ \frac{a_m b_n - a_n b_m}{n} \left(1 - e^{-i2\pi n\sigma}\right) e^{-i2\pi m\sigma}$$

$$+ \frac{b_m^+ b_n - a_m^+ a_n}{n} \left(1 - e^{-i2\pi n\sigma}\right) e^{i2\pi m\sigma}$$

$$- \frac{a_n^+ a_m - b_n^+ b_m}{n} \left(1 - e^{i2\pi n\sigma}\right) e^{-i2\pi m\sigma}$$

$$- \frac{a_n^+ b_m^+ - b_n^+ a_m^+}{n} \left(1 - e^{i2\pi n\sigma}\right) e^{i2\pi m\sigma} \Big\}. \qquad (4.8)$$

Using (4.8), (4.4) and (4.6) in (4.1), we get for the valence contribution to the matrix element M^+

$$\mathcal{M}^+ = -\frac{1}{2} \int_0^1 d\sigma \langle f| \sum_{m,n=1}^{\infty} \Big\{ \frac{b_m^+ b_n}{n} \left(1 - e^{-2\pi i n\sigma}\right) e^{2\pi i m\sigma}$$

$$+ \frac{b_m^+ b_n}{m} \left(1 - e^{i2\pi m\sigma}\right) e^{-i2\pi n\sigma} \Big\} \exp\left[-i\alpha\Gamma(\sigma)\right] b_1^+ |0\rangle, \qquad (4.9)$$

where

$$\alpha = \log(1-x) \quad .$$

The eigen value of the formal unitary operator $e^{i\alpha\Gamma(\sigma)}$ is given by

$$e^{-i\alpha\Gamma(\sigma)} |0\rangle = e^{2\pi i\alpha\lambda(\sigma - \frac{1}{2})} |0\rangle \quad , \qquad (4.10)$$

where $4\pi\lambda$ is the mass squared of the "empty-bag" divided by the bag constant (B). $e^{i\alpha\Gamma(\sigma)}$ is a linear transformation of the creation (annihilation) operators and it satisfies[5]

$$e^{-i\alpha\Lambda(\sigma)}\, b_1^+ |0\rangle = \sum_{m>0} U_{-1,-m}\, b_m^+\, e^{-i\alpha\Lambda(\sigma)} |0\rangle$$

$$= e^{2\pi i\alpha\lambda(\sigma-\frac{1}{2})} \sum_{m>0} U_{-1,-m}\, b_m^+ |0\rangle \, , \tag{4.11}$$

where

$$U_{-1,-m} = \left[e^{-i\alpha \underset{\sim}{H}} \right]_{-1,-m} \, , \tag{4.12}$$

and the matrix $\underset{\sim}{H}$ has the components

$$H_{nm} = 2\pi(\sigma-\tfrac{1}{2})\, n\, \delta_{nm} - \frac{in}{n-m}\, e^{-2\pi i(n-m)\sigma} (1-\delta_{nm}) \, . \tag{4.13}$$

In the calculation of the L_o approximation to the structure function we only keep the first term in (4.13) i.e. (the diagonal term) and the second term is neglected. In the present calculation we keep the second term and try to develop an understanding of how far that term contributes to the structure function. To do this we write

$$\underset{\sim}{H} = \underset{\sim}{D} + \underset{\sim}{R} \, , \tag{4.14}$$

where $\underset{\sim}{D}$ and $\underset{\sim}{R}$ are respectively the diagonal and off diagonal terms in (4.13). Now, $e^{-i\alpha \underset{\sim}{H}}$ could be expanded as the following

$$e^{-i\alpha \underset{\sim}{H}} = e^{-i\alpha(\underset{\sim}{D}+\underset{\sim}{R})} = e^{-i\alpha(\underset{\sim}{D}+\underset{\sim}{R})}\, e^{i\alpha \underset{\sim}{D}}\, e^{-i\alpha \underset{\sim}{D}}$$

$$= \left\{ 1 - i\alpha \underset{\sim}{R} + \frac{\alpha^2}{2} [\underset{\sim}{R}, \underset{\sim}{D}] + \cdots \right\} e^{-i\alpha \underset{\sim}{D}} \, . \tag{4.15}$$

Taking the first order term in α in (4.15) and using (4.11) and (4.12), (4.9) will take the form

$$\mathcal{M}^+ = -\frac{1}{2}\int_0^1 d\sigma \; e^{2\pi i\alpha\lambda(\sigma-\frac{1}{2})} \langle f| \sum_{m,n=1}^{\infty} \left\{ \frac{b_m^+ b_n}{n} (1-e^{-2\pi i n\sigma})\, e^{2\pi i m\sigma} + \frac{b_m^+ b_n}{m} (1-e^{i2\pi m\sigma})\, e^{-i2\pi n\sigma} \right\}$$

$$\sum_{\ell>0} \left[(1-i\alpha \underset{\sim}{R})\, e^{-i\alpha \underline{D}} \right]_{-1,-\ell} b_\ell^+ |0\rangle. \qquad (4.16)$$

Performing the integral in (4.16) we finally obtain for the matrix element M^+

$$\mathcal{M}^+ = \frac{-\sin\pi\alpha(1+\lambda)}{2\pi(\alpha+\alpha\lambda-1)} - \frac{\alpha}{2\pi}\sum_{n>1} \frac{n\sin\pi\alpha(n+\lambda)}{(n-1)(\alpha n+\alpha\lambda-1)}, \qquad (4.17)$$

Where the first term ($\mathcal{M}_0^+$) is the L_0 approximation and the second (M_1^+) is the correction.

The summation in (4.17) can be done for particular values of α, and this is enough to show the contribution of M_1^+. In the following table, we show the results for λ=o and λ =1 (in the realistic three-dimensional models, the zero-point energy lies in this range).

α	$\lambda=0$		$\lambda=1$	
	$\mathcal{M}_0^+$	$\mathcal{M}_1^+$	$\mathcal{M}_0^+$	$\mathcal{M}_1^+$
½	.32	.69	.50	.27
1/3	.21	.75	.41	.75
¼	.15	.73	.32	.66
1/5	.12	.70	.25	.66
1/6	.10	.68	.21	.66

4.2 Conclusion

The results shown in the table are disappointing and give no confidence in the use of the L_o-approximation. They suggest that excitation of pairs, not in the ground state, by the recoiling bag makes a significant difference to the structure function. Of course it could be that an exact calculation of the correction would show that it is not as large as it appears here. So far, however, we have not found any way of obtaining the exact result.

Note that the plausibility arguments given in ref[10] for the use of the log(1-x) variable as a replacement for x do not provide any counter indications to the result described here. It is true that a translationally invariant model for which the replacement is exact - namely the MIT bag model in the L_o approximation - does exist. However, our calculation suggests that such a model is not a good approximation to the one-dimensional MIT bag.

CHAPTER 5

A BAG-LIKE MODEL FOR KINEMATIC CORRECTIONS TO STRUCTURE FUNCTIONS

Quantum chromodynamics[24] is now generally accepted as the underlying theory of the strong interactions. So far our ability to calculate with this theory is severely limited, principally because of its large coupling strength, which means that perturbation theory converges slowly. On the other hand, within a perturbative framework, only the lowest twist matrix elements are presently calculable. At high momenta, the lowest order picture of deep-inelastic scattering as predicted by the present perturbation theory, indicate the consistency of QCD with experiments[27]. However, at lower momenta, the perturbation theory, as it stands now, has not been able to confirm the experimental data. At that level, correction terms, which are due to kinematic and higher twist, have to be included. Those corrections have not been reliably calculated[28]. In this chapter, we introduce a bag-like model to evaluate those corrections which are due to target mass, quark mass (in general off-shell) and quark transverse momentum effects.

Our model is a modified cavity model in which the translation invariance is restored. That can be done, simply, by following Davis and Squires[6] suggestion that, in the Bjorken limit, x in the cavity calculation to the structure function should be replaced by -log(1-x). Supporting arguments in favour of this transformation are given by Jaffe[29]. At this point, we would point out that we have no guidance on the nature of the recoil corrections except in the Bjorken limit, so there is ambiguity in the non-scaling term.

To calculate the structure functions in this model, we first calculate the structure functions in QED for the absorption of a photon by a free Dirac quark. Then we take into account the effect of quark confinement by integrating the structure functions over a momentum distribution which will be determined by fitting to experimental data over a range of momenta.

The effect of including the "quark-mass" term in our calculation will be discussed and we will show that it is important. Also, we will see that it is unreasonable just to include the "target-mass" terms as there is considerable cancellation between these and the other terms.

5.1 The Absorption of a Photon by a Free Quark

We assume that a proton is composed of three free Dirac quarks. We work in the target proton cenre-of mass system and consider an electron scattering off a quark as shown in fig. 5.1. In this calculation we will refer to the factor $(k_o^2-k^2)$ as the square of the "effective" mass of the quark; it includes the normal quark mass and also the effect of binding in taking the quarks off their mass shell. It is necessary to assume that the effective quark mass is the same before and after the collisons in order to satisfy gauge invariance.

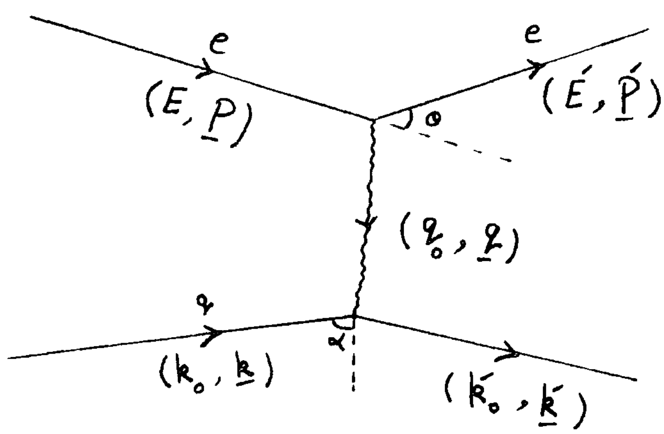


fig. 5.1

The differential cross section is given by[30]

$$d\sigma = \int \frac{d^3P' \, d^3K'}{(2\pi)^6} \; \frac{m_q^2 \, m_e^2}{E' E k_0' k_0} \, (2\pi)^4 \, \delta^4(P'+K'-P-K) \, |\mathcal{M}|^2 \, , \tag{5.1}$$

where is the matrix element for eq ⟶ eq scattering and is formally written[30]

$$|\mathcal{M}|^2 = \frac{e^4 e_q^2}{2m_e^2 m_q^2 q^4}\Big\{(K'\cdot P')(K\cdot P) + (K'\cdot P)(K\cdot P') - m_e^2(K'\cdot K) - m_q^2(P'\cdot P) + 2m_e^2 m_q^2\Big\} \, . \tag{5.2}$$

The electron mass m_e is neglected, thus $|\underline{P}'| \simeq E'$. We proceed by integrating (5.1) over d^3K', writing $d^3P' = |\underline{P}'|E'dE'd\Omega'$ and then averging over angles - assuming the spherical symmetry of the proton in its rest frame - we obtain

$$\frac{d^2\sigma}{dE'd\Omega} = \frac{e^4 e_q^2 E'}{16\pi^2 q^4 k_0 |\underline{k}| E |\underline{q}|}\Big\{(K'\cdot P')(K\cdot P) + (K'\cdot P)(K\cdot P') - m_q^2(P'\cdot P)\Big\} \, . \tag{5.3}$$

For electron-quark interaction we have

$$(P + K)^2 = (P'+K')^2$$

$$\therefore \quad K.P = K'.P' \quad ; \tag{5.4}$$

$$(P'-K)^2 = (P-K')^2$$

$$\therefore \quad K.P' = K'.P \quad ; \tag{5.5}$$

$$(P-P')^2 = q^2$$

$$\therefore \quad P.P' = \frac{-q^2}{2} \quad ; \tag{5.6}$$

$$P' = P - q$$

$$\therefore \quad K.P' = K.P - K.q \tag{5.7}$$

Substituting (5.7), (5.6) and (5.4) into (5.3) gives

$$\frac{d^2\sigma}{dE'd\Omega} = \frac{e^4 e_q^2 E'}{16\pi^2 q^4 k_0 k E q_2}\Big\{2(K\cdot P)^2 + (K\cdot q)^2 - 2(K\cdot P)(K\cdot q) + \frac{q^2}{2}m_q^2\Big\} \, , \tag{5.8}$$

where we use k for $|\underline{k}|$ and q_z for $|\underline{q}|$.

The mass condition on the struck quark gives

$$(K+q)^2 = K'^2$$

$$\text{i.e.} \quad q_0(k_0 - Mx) = k\, q_z \cos\alpha \tag{5.9}$$

$$\text{or} \quad k > |k_0 - Mx| \left(1 + \frac{2Mx}{q_0}\right)^{-1/2} .$$

In the laboratory system define

$$P \equiv (E, E\cos\theta, E\sin\theta, 0) , \tag{5.10}$$

$$K \equiv (k_0, k_0\cos\alpha, k_0\sin\alpha\cos\beta, k_0\sin\alpha\sin\beta) , \tag{5.11}$$

$$q \equiv (q_0, q_z, 0, 0) . \tag{5.12}$$

The invariant four-momentum gives us

$$(P-q)^2 = P'^2 , \tag{5.13}$$

$$(K+q)^2 = K'^2 , \tag{5.14}$$

form which we get for $\cos\theta$ and $\cos\alpha$ the values

$$\cos\theta = \frac{2Eq_0 - q^2}{2Eq_z} , \tag{5.15}$$

$$\cos\alpha = \frac{2k_0q_0 + q^2}{2k_0q_z} . \tag{5.16}$$

Also, since

$$(P-P')^2 = q^2 = -2Mxq_0$$

$$\text{i.e.} \quad -2EE' + 2|\underline{P}||\underline{P}'|\cos\theta = -2Mxq_0 \tag{5.17}$$

$$\text{or} \quad 2EE'\sin^2\frac{\theta}{2} = Mxq_0 .$$

Using (5.10), (5.11), (5.12), (5.15), (5.16) and (5.17) in (5.8) we finally get

$$\frac{d^2\sigma}{dE'd\Omega} = \frac{e^4 e_q^2 E'^2 Mx}{9\pi^2 q^4 k k_0}\left(1+\frac{2Mx}{q_0}\right)^{-5/2}\left\{\Phi_1 \sin^2\frac{\theta}{2} + \frac{Mx}{q_0}\Phi_2 \cos^2\frac{\theta}{2}\right\}, \tag{5.18}$$

where

$$\Phi_1 = 1 + \frac{5Mx}{q_0} + \frac{2}{q_0}(k_0 - Mx) + \frac{2}{q^2}(Mx+k_0)^2 + \frac{4Mxk_0^2}{q_0^3} - \frac{2(k_0^2-k^2)}{Q^2}\left(1+\frac{2Mx}{q_0}\right)^2, \tag{5.19}$$

$$\Phi_2 = 1 + \frac{5Mx}{q_0} + \frac{6}{q_0}(k^0 - Mx) + \frac{6k_0^2}{q_0^2} - \frac{2(k_0^2-k^2)}{Q^2}\left(1+\frac{2Mx}{q_0}\right), \tag{5.20}$$

and

$$Q^2 = -q_0^2 + |\underline{q}|^2 = 2Mxq_0 \,. \tag{5.21}$$

We compare (5.18) with the standard deep-inelastic electron proton cross-section formula[31], i.e.

$$\frac{d^2\sigma}{dE'd\Omega} = \frac{e^2E'^2}{(2\pi)^2 q^4}\left[\frac{F_2(x,Q^2,k)}{q_0}\cos^2\frac{\theta}{2} + \frac{2F_1(x,Q^2,k)}{M}\sin^2\frac{\theta}{2}\right]. \tag{5.22}$$

We get

$$F_1(x,Q^2,k) = \frac{e^2e_q^2}{4kk_0} M^2 x \left(1+\frac{2Mx}{q_0}\right)^{-5/2}\Phi_1 , \tag{5.23}$$

$$F_2(x,Q^2,k) = \frac{e^2e_q^2}{2kk_0} M^2 x^2 \left(1+\frac{2Mx}{q_0}\right)^{-5/2}\Phi_2 , \tag{5.24}$$

for the range of x satisfying (5.9), and zero otherwise.

5.2 The Effect of Quark Binding

In the calculation presented in the previous section, we have assumed that the initial Dirac quark is in a momentum eigenstate and hence we have ignored the effect of confinement. This can be cured by integrating the structure functions of (5.23) and (5.24) over a distribution in k. Thus

$$F_i(x,Q^2) = \int k^2\, dk\, P(k)\, F_i(x,Q^2,k) \quad , i=1,2 , \tag{5.25}$$

where the integral is over the region specified by the inequality (5.9) and P(k) is normalised by

$$\int_0^\infty k^2\, dk\, P(k) = 1 \quad . \tag{5.26}$$

In principle p(k) is determined by the dynamics of the confinement mechanism.

At this point we like to point out that our method of including the effective quark mass contribution is different to that of Barbieri et al[32]; in particular our effective mass has an explicit predicted dependence on x and Q^2. Although a similar dependence occurs in the work of Landshoff and Scott[33], these authors do not relate their structure functions to an initial momentum distribution and, since they consider scalar partons, they do not have results analogous to our eqs. (5.23) and (5.24).

The Bjorken limit of (5.26) was given by Jaffe[12] for the particular form of P(k) coming from the cavity approximation to the MIT bag model; it has been used with a general P(k) by Davis and Squires[34].

5.3 The Effect of Recoil

The expression (5.25) gives us the observed structure functions of a single quark in terms of its momentum distribution P(k). It is well known, however, that because the final hadron state must have a mass greater than, or equal to, the initial hadron mass, kinematics restricts x to the region $0<x<1$, i.e. the structure functions must be zero for $x > 1$. This result, a consequence of translational invariance, is not respected by the expression given in (5.25) which is appropriate to a quark bound in a fixed cavity, and which therefore violates translational invariance. The problem can be cured by replacing x by $-\log(1-x)$. This substitution clearly has the desired effect of making the structure functions go to zero as x tends to unity. A partial justification of this procedure has been given in an improved calculation of the structure function - by Davis and Squires[6] - for an approximation (which preserves translational invariance) to the two-dimensional MIT bag model.

Now we reach the unavoidable problem of applying the above substitution to a calculation beyond the Bjorken limit. As we mentioned at the beginning of this chapter, without a better understanding of the nature of the recoil corrections in a three-dimensional model, the ambiguity in the non-scaling terms will remain unresolved. We proceed by making the substitution $x \longrightarrow -\log(1-x)$ in (5.9) for $|k_o - Mx|$ only and leaving other parts of the expressions for F_i unaltered. Some discussions and comparisons are given on page (55).

5.4 The Determination of P(k)

One way of choosing an appropriate P(k) is to fit eq.(5.25) at one convenient momentum. A better procedure, which will be used, is to determine P(k) from data in a finite range of momenta. To do this we represent the data, presupposing that there are no logarithmic dependences, by a second order polynomial in $\frac{1}{Q^2}$ with coefficients that are simple functions of x. The leading (scaling) term then determines P(k). This leading term is shown by the dash-dot curves of fig.2.

The data we use are all the results on the proton structure function, $F_2^{ep}(x,Q^2)$, extracted from deep inelastic electron[35,36] and muon[37] scattering at SLAC and from the muon scattering experiment of the CHIO collaboration at Fermilab[38]. We limit the kinematic range to low momenta, $1.8 < Q^2 < 6 \quad (GEV/C)^2$, as beyond that we expect lowest twist

QCD effects which we have not included, to be important, and to $x > 0.15$, since we shall only describe the valence quark distribution. To exclude most of the effects of resonance production, we make a cut on the mass of the final state hadrons, $M_F > 1.7$ GeV.

Making the reasonable approximation that the u and d quarks in the proton have the same distributions, we find that the function

$$P(k) = \frac{1}{M^2 k} \exp\left[\frac{k_o - k}{M} - \frac{B}{A+B+1}\right]\left[1 - \exp\left(\frac{k_o - k}{M}\right)\right]^A \exp\left(\frac{-B(k_o + k)}{M}\right) , \qquad (5.27)$$

were $A = -2.4$, $B = 3.9$

represents the leading scaling term. Here we have taken k_o to be 0.15M as given in the bag model[39]. Other values in this region would give similar fits provided the parameters above were suitably adjusted.

Note that P(k) in eq.(5.27) has an arbitrary normalisation, i.e. we have not imposed eq.(5.26). The justification for this is that the normalisation, as measured by the same rule for $F_2(x)/x$, depends sensitively upon the small x region which we do not attempt to fit.

5.5 Comparison with Experiment

We can now predict the proton structure functions $F_i^{ep}(x,Q^2)$, i=1,2, at finite momenta, using eqs.519, 5.20, 5.23, 5.24 and 5.25. The results for F_2 are shown in fig.2, and it will be seen that there is excellent agreement throughout the range $1.8 < Q^2 < 6$ $(GeV/C)^2$.

The "quark-mass" effect in our model can be distinguished from the more usual target-mass correction by considering the effect of removing it, i.e. ignoring the terms depending on $(k_o^2 - k^2)$ in eqs. 5.23 and 5.24. The results, shown by the dashed curves in fig.2, are certainly not in agreement with the data.

It is interesting to note that the effect of the "quark-mass" terms is opposite in direction to that of the target-mass terms, thus they remove

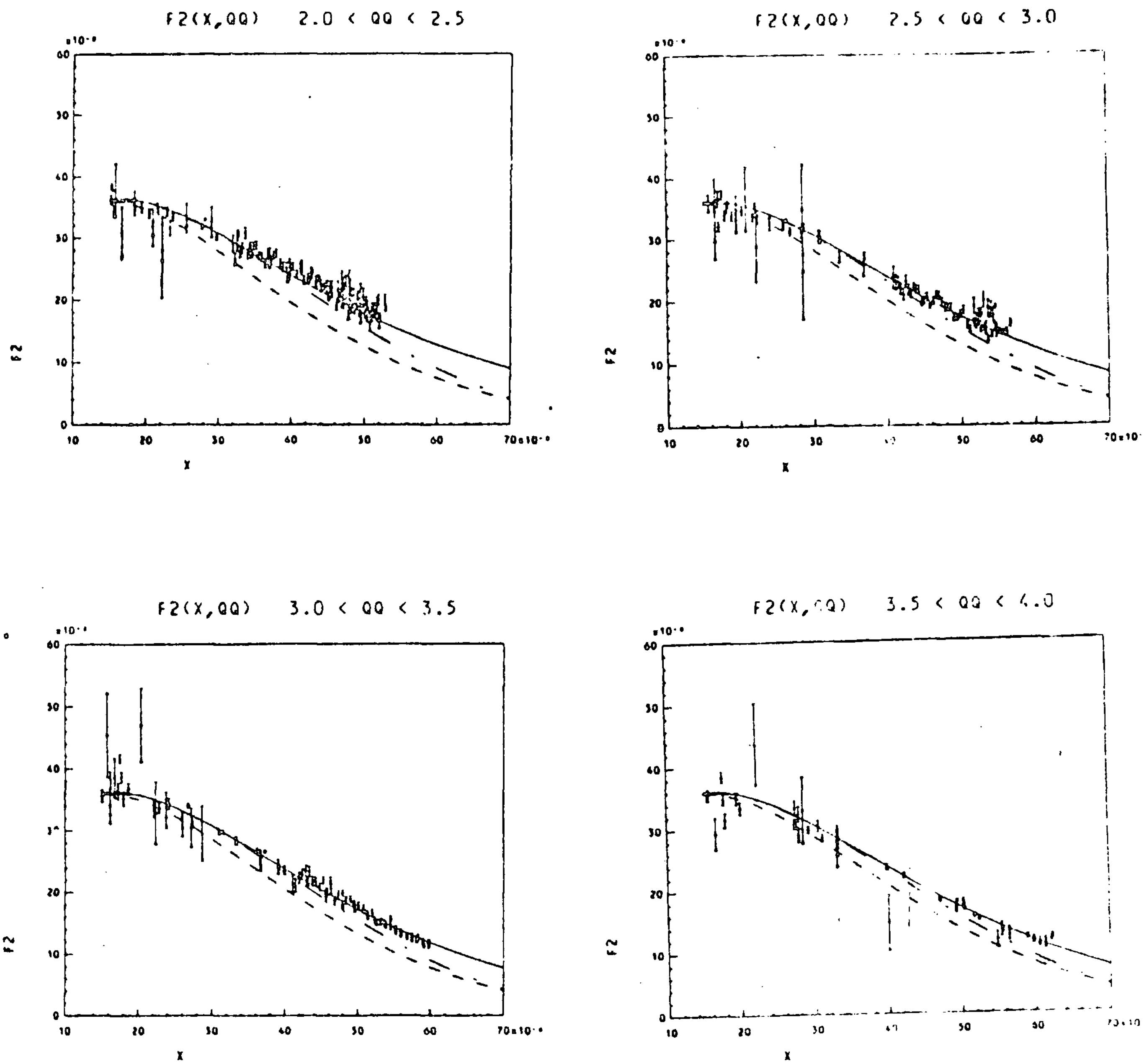
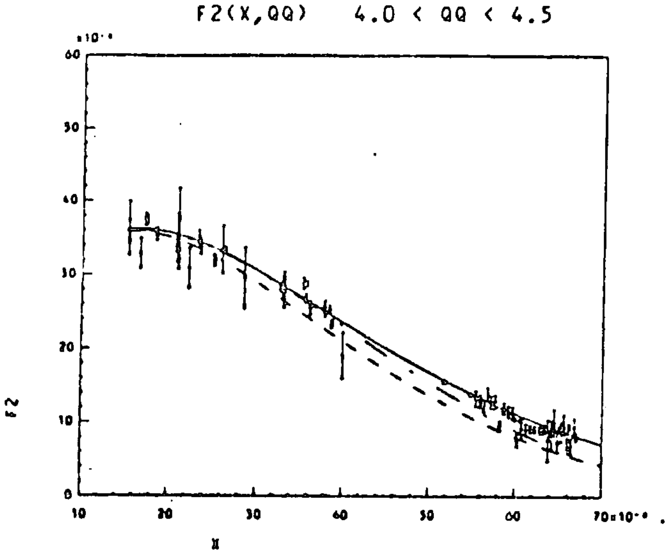


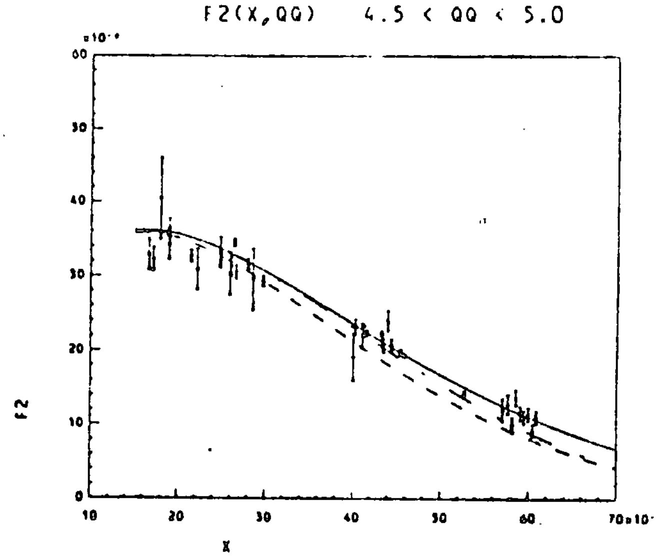
Fig. 2

The data [33-35] for F_2 is plotted as a function of x for eight equal Q^2 bins from 2 to $6GeV/c^2$. The solid line in our prediction including all the kinematic effects considered. The dashed line contains no quark mass contribution, i.e. it only has the target mass effects. The dash-dot curve is the "Bjorken limit" curve as explained in sec. 4.

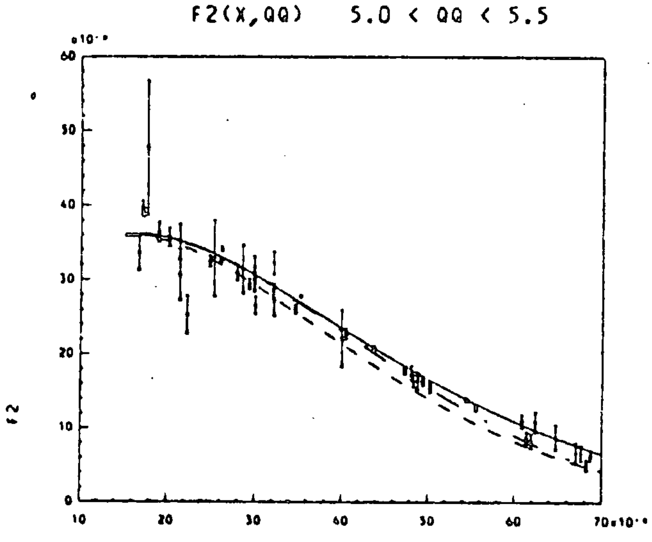
F2(X,QQ) 4.0 < QQ < 4.5



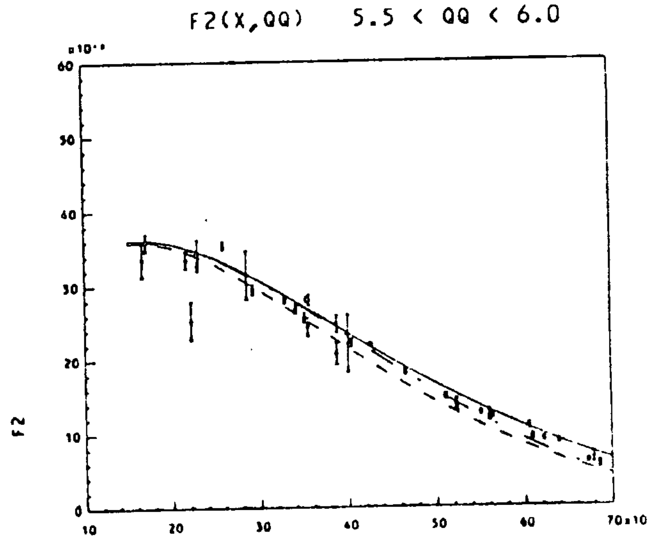
F2(X,QQ) 4.5 < QQ < 5.0



F2(X,QQ) 5.0 < QQ < 5.5



F2(X,QQ) 5.5 < QQ < 6.0



much of the, unrealistically large, scaling violation due to the latter.

The procedure we have used to take account of the recoil correction ensures that the structure functions go to zero as $x \to 1$ by making k go to infinity in this limit. This automatically takes the quarks far off mass-shell and enhances the "quark-mass" corrections in the large x region. Although the data we use does not go beyond $x \simeq 0.7$, it is nevertheless worth considering an alternative recoil correction that does not have this enhancement. Thus, rather than changing the limit of k as above, we try to effect of replacing P(k) in 5.25 by $P(\ell)$ where

$$\ell = k + \left(1 + \frac{2Mx}{q_0}\right)^{-1/2} \left[|k_0 + M \log(1-x)| - |k_0 - Mx| \right] . \qquad (5.28)$$

Clearly this gives identical results in the Bjorken limit. The predictions at finite Q^2 lie, as we expect, in between the solid and dashed curves of fig.2.

The structure function, $F_1^{ep}(x,Q^2)$ has only been determined[36] independently of F_2 at a few values of x and Q^2. Eqs. (5.19 - 5.24) predict deviations from the Callan-Gross relation, $2xF_1 = F_2$, which we may regard as "kinematic" in origin, so that the ratio

$$R = \frac{\sigma_L}{\sigma_T} = \frac{F_2}{2xF_1}\left[1 + \frac{4M^2x^2}{Q^2}\right] - 1$$

is non-zero at finite Q^2. The predictions are not inconsistent with the sparse data on R, with is very large errors[36]. However, the prediction for R, unlike that for F_2 itself, is rather sensitive to the value of k_o. Nonetheless, our results for R from just "kinematic" terms are certainly no worse than those models of higher twist contributions made up just to fit $\frac{\sigma_L}{\sigma_T}$ (for example, ref.40).

5.6 Conclusions

1. Our model for the valence quark distributions gives a satisfactory

fit to the data in the ranges. $.15 < x < \left(1+\frac{3-M_p^2}{Q^2}\right)^{-1}$ and $1.8 < Q^2 < 6\ (\mathrm{GeV/C})^2$. We do not expect agreement for smaller x, because of the sea contribution, or for large x, because of final state resonances with masses less than 1.7 GeV.

2. Our fit does not work at higher Q^2 and therefore see clear evidence at larger x for what are presumably QCD corrections.

3. Within models of this type it is misleading to include the proton mass, non-scaling, corrections without also including the "quark-mass" effects, since there is considerable cancellation between them.

4. Because of the cancellation, referred to in 3, it appears that the "higher-twist" effect we have calculated are quite small and, in agreement with data, there is little violation of scaling in the region we consider. To what extent QCD predicts other "higher-twist" effects which might be important is unclear (see, for example, Politzer[28]).

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