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## DISTANCE SCALE

by

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## (B.Sc. Physics)

A thesis presented in candidature for the degree of

Master of Science in the Department of Physics, University of Durham.

APRIL 1982

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"We find them smaller and fainter, in constantly increasing numbers, and we know that we are reaching out into space, farther and ever farther, until with the faintest nebulae that can be detected with the greatest telescope, we arrive at the frontiers of the known Universe".

Edwin Hubble

## INTRODUCTION

Present cosmological theories are all based on the cosmological principle : the assumption that all points in the Universe are equivalent and that the large scale Universe is homogeneous and isotropic.

It can be shown that the general form for a line element in a spatially homogeneous and isotropic Universe is expressed by the RobertsonWalker metric (Weinberg 1972) :

$$
d \tau^{2}=d t^{2}-R^{2}(t)\left\{\frac{d r^{2}}{1-k r^{2}}+r^{2} d \theta^{2}+r^{2} \sin ^{2} \theta d \phi^{2}\right\}
$$

where $R(t)$ is the cosmic scale factor, representing the geometry of space and $k$ is the curvature constant which can be chosen to have the values $+1,0,-1$.

It would be more useful to expand the cosmic scale factor $R(t)$ as a power series :

$$
\begin{aligned}
& R(t)=R\left(t_{0}\right)+\dot{R}\left(t_{0}\right)\left(t-t_{0}\right)+\frac{1}{2} \ddot{R}\left(t_{0}\right)\left(t-t_{0}\right)^{2}+\ldots \\
& R(t)=R\left(t_{0}\right)\left[1+H\left(t_{0}\right)\left(t-t_{0}\right)-\frac{1}{2} H^{2}\left(t_{0}\right) q_{0}\left(t-t_{0}\right)^{2}+\ldots . .\right]
\end{aligned}
$$

where

$$
H\left(t_{0}\right)=\frac{\dot{R}\left(t_{0}\right)}{R\left(t_{0}\right)} \quad, q_{0}=-\frac{\ddot{R}\left(t_{0}\right) R\left(t_{0}\right)}{\dot{R}^{2}\left(t_{0}\right)}
$$

Note that $H\left(t_{o}\right)$ and $q_{0}$ are just the present values of two parameters which are variable with time. $H\left(t_{0}\right)$ is called the Hubble constant and $q_{0}$ the deceleration parameter.

It has been observed (Hubble, 1926) that the apparent recession velocity of a galaxy is directly proportional to the distance of the galaxy ( $v=H 1$ ). This so-called Hubble's law, confirms the linear
relation between redshift and distance, anticipated theoretically through the Robertson-Walker metric.

Introducing the Robertson-Walker metric into the Einstein field equations, it can be shown that the whole of the cosmic scale factor can be calculated if we know the two parameters $H_{o}$ and $q_{0}$.

From these two parameters, the large scale distribution of matter in the Universe and its evolution with time can be determined. Moreover, it is possible to put constraints on the age of the Universe and derive the mean mass density of the Universe. Evidently it is extremely important to determine the numerical values of $H_{o}$ and $q_{0}$ observationally.

In order to find an empirical value for $H_{0}$, it is necessary to derive independent distances to galaxies. This is a very difficult problem because of the limited number of accurate distance indicators and the various sources of error in applying them.

The first attempt to find the value of $H_{0}$ was made by Hubble in 1936, he derived a value of $530 \mathrm{~km} \mathrm{sec}^{-1} \mathrm{~m} \mathrm{pc}^{-1}$. We now know that the size of the Universe is five or even ten times larger than Hubble believed. However, the errors in Hubble's distance scale have now been well understood :
(I) The zero point of Hubble's Period-Luminosity relation for cepheids was too faint.
(II) Beyond the range of cepheids, Hubble's main indicators were the brightest stars in spiral galaxies and he misclassified some compact HII regions as very bright stars.
(III)It is now thought that some distance indicators change their properties with the luminosity of the parent galaxy. Hubble was not aware of this effect.

All the above points affect the derived value of $H_{o}$ in the same sense, making distances too small and consequently $H_{o}$ too high. Most modern estimates of the value of the Hubble Constant lie between 50 and $100 \mathrm{~km} \mathrm{sec}{ }^{-1} \mathrm{M} \mathrm{pc}^{-1}$.

The determination of the extragalactic distance scale is essentially a multiple step process. First the distances to nearby galaxies which are members of the Local group have to be determined. Then, by using these distances, new correlations between the observable parameters of galaxies must be defined. These relationships can then be extended to distant galaxies where the recession velocities are large compared with random peculiar motions.

In order to find distances to Local group galaxies we can exploit stellar standard candles (cepheids, R.R Lyrae, Novae) which have a range of $4 \mathrm{Mpc}((\mathrm{m}-\mathrm{M})=28.0)$. The expansion velocity at this distance is less than $400 \mathrm{~km} \mathrm{sec}{ }^{-1}$ and the Hubble constant cannot be determined by using such nearby galaxies. New (secondary) distance indicators must then be defined to bridge the gap between the Local group galaxies and the region where a pure Hubble flow dominates ( $C Z>4000 \mathrm{~km} \mathrm{sec}^{-1}$ ).

Secondary indicators (HII regions, brightest stars, globular clusters) have a useful range of 10 Mpc . Since they cannot be calibrated directly in our own galactic neighbourhood, their calibration is inevitably based on the primary (stellar) indicators. Using distances from primary and secondary indicators the global properties (absolute magnitudes and diameters) of galaxies can be calibrated. These Tertiary indicators can be used beyond a distance of $100 \mathrm{Mpc}((\mathrm{m}-\mathrm{M})=35 . \mathrm{O})$, where there is evidence that a pure Hubble flow dominates. It is in this region that the determination of $H_{o}$ must be made.

In recent years there has been a great effort made to find the accurate value for $\mathrm{H}_{\mathrm{o}}$.

The main aim of this thesis is to give a general review of the work which is being done on the distance scale problem and to present some of the results which have been obtained.

The first chapter contains a general review of different standard candles, the way in which each of them can be used and the range and accuracy
of each distance indicator. In the second chapter we will explain two of the most important and divergent attempts, Sandage and Tammann (throughout this thesis abbreviated by $S T$ ) and G.de Vaucouleurs (GdV) to the extragalactic distance scale. Since no discussion is complete without comparing these two scales a critique of the $S T$ and GdV methods is presented in the third chapter. In the fourth chapter some new methods that are more or less independent of the conventional chain of measurements to determine the distance scale will be discussed.

## CHAPTER 1

## CONVENTIONAL STANDARD CANDLES

## 1.1 <br> INTRODUCTION

Standard candles must be easily identifiable objects with a small dispersion in absolute luminosity. Their distances can then be determined directly from their apparent magnitude. All determinations of distances beyond the Local group are ultimately based on the assumption that recognizable types of object are similar in nearby and distant galaxies of the same type. This assumption may not be correct for any particular standard-candle, because of possible differences in age, evolutionary history and element abundances. Systematic errors due to such differences will be minimised if the maximum number of independent methods of distance determination are used.

In this chapter, we begin with a discussion of the sources of error and the necessary corrections in the extragalactic distance scale problem. A discussion on the primary distance indicators (RR Lyrae variables, Novae, cepheids) is given in sections 1.4 to 1.6 . In sections 1.7 to 1.10 the secondary indicators (H II regions, brightest stars, Globular clusters) will be explained, followed by a discussion in section 1.11 on the Tertiary indicators (the absolute magnitudes and diameters of galaxies).

### 1.2 SYSTEMATIC EFFECTS IN DISTANCE DETERMINATION

1.2.1 Interstellar absorption :

Anticipating the importance of the interstellar absorption to the extragalactic distance scale, we must establish its wavelength dependence (the reddening curve). This is conveniently expressed in a plot of absorption ( in magnitude ) as a function of inverse wavelength (Fig 1.1).

The reddening curve can be established empirically through a spectrophotometric comparison of two stars, one highly reddened and one little reddened. It is essential to select pairs of stars having the same spectral type. In this case the two stars can be assumed to have the same spectral energy distribution and the difference in magnitude as a function of wavelength, as entirely due to interstellar absorption except for a constant difference depending on the relative distances and absolute magnitude of the two stars. The constant can be calculated by extrapolating the observed reddening curve to $\frac{1}{\lambda}=0$ beyond the last observed point in the infrared.
 Absorption in magnitudes against inverse wavelength for stars with the same spectral type. The curve is extrapolated towards infrared.

If the magnitude of a star is observed at two wavelengths $\lambda_{1}$ and $\lambda_{2}$, then the corresponding colour excess resulting from interstellar reddening is :

$$
A\left(\lambda_{1}\right)-A\left(\lambda_{2}\right)=E\left(\lambda_{1}, \lambda_{2}\right)
$$

E is the colour excess. The corresponding ratio of total to selective absorption is then :

$$
R\left(\lambda_{1}, \lambda_{2}\right)=\frac{A\left(\lambda_{1}\right)}{A\left(\lambda_{1}\right)-A\left(\lambda_{2}\right)}=\frac{A\left(\lambda_{1}\right)}{E\left(\lambda_{1}, \lambda_{2}\right)}
$$

The total absorption can be found from the measured colour excess given a knowledge of $R$. The magnitude of a star measured at effective wavelength $\lambda_{1}$ can thus be corrected for the effects of interstellar absorption. The ratio $R$ of the total to selective absorption for any two colour bands for which the effective inverse wavelengths are known, can be obtained directly from the observed reddening curve.

Error in the determination of $R$ arise mainly from differences in the intrinsic spectral energy distributions of the two stars being compared, and in the extrapolation of the reddening curve.

The ratio of the total to selective absorption on the BV system has been the subject of extensive study, with most values being around

$$
R \equiv \frac{A v}{E}=3.1 \pm 0.2
$$

The possibility of variation of $R$ in different regions of the sky must also be investigated. Johnson (1968) concludes that interstellar extinction law is not the same everywhere and that $R=3$ is the minimum value with local deviations to higher values near regions of hot stars.

In order to derive the absorption from this relation, we must be able to determine the amount of the reddening in particular cases. One method for determining stellar reddening is to use three colour (e.g.UBV)
photometry. The locus of the intrinsic colours of main sequence stars in the [ U-B, B-V ] plane is well defined. Reddening of a cluster main sequence shifts this locus along the "reddening line" (Fig 1.2) and the colour excess can be estimated from the amount of this shift.


Figure 1.2 : Reddening vectors in the UBV plane. The dotted line represents the position of un-reddened main sequence stars in the ( $U-B$ ), $(B-V)$ diagram. The solid line represents the position of a reddened main sequence.

### 1.2.2 Galactic absorption :

Galactic absorption can systematically affect the observed photometric properties of extragalactic objects and so it must be carefully accounted for in establishing the extragalactic distance scale.

A model commonly adopted for galactic absorption is to assume that it changes with galactic latitude $b$ as

$$
\begin{equation*}
A=a \csc |b| \tag{1,2.1}
\end{equation*}
$$

This so-called cosecant law is based on the assumption that the absorption in our galaxy is caused by a thin homogeneous absorbing layer near the galactic plane.

The constant ' $a$ ' can,in principle, be determined from number counts of extragalactic objects.

From galaxy counts at different latitudes one finds a linear relation (Shane, Wirtanen, 1967)

$$
\begin{equation*}
\log N=A_{1}+B_{1} \mid \csc b \tag{1.2.2}
\end{equation*}
$$

where $A_{1}$ is the logarithmic number of galaxies, at a given apparent magnitude, per square degree outside the absorbing layer and $B_{1}$ is the reduction in $\log N$ caused by interstellar extinction. Differentiating equation (1.2.2) gives :

$$
\begin{equation*}
d(\log N(m))=B_{1} d(\csc b)=\gamma d A=\gamma \operatorname{lad}(\operatorname{cscb}) \tag{1.2.3}
\end{equation*}
$$

where $\gamma=\frac{d(\log N(m))}{d m}$ and $d A=a d(\csc b)$ from (1.2.1)
From this equation we see that $a=\frac{B_{1}}{\gamma}$, so the constant " $a$ " can be detexmined from the observed values of $B_{1}$ and $\gamma . \quad B_{1}$ can be obtained from a plot of $\log N(m)$ versus $c s c|b|$ and $\gamma$ from the slope of the number counts at a fixed galactic latitude.

Relatively small errors in the magnitude scale translate into incorrect absorption values (Phillipps et al, 1981). In fact "a" is poorly determined from galaxy counts because of the uncertainty in $\gamma$.

Since csc $b$ varies much more rapidly with $|\mathrm{b}|$ at low latitudes and since more data points are available there, the solution is influenced mainly by the lower latitude data (Galaxy counts show the steep gradient
$\frac{d \log N(m)}{d(\csc b)} \geqslant 0.15$ for galactic latitudes below $\left.b \sim 30^{\circ}\right)$. In the range $50^{\circ}<|\mathrm{b}| \leqslant 90^{\circ} \mathrm{csc}|\mathrm{b}|$ changes only from 1.3 to 1 requiring large numbers of galaxies to define the csc law at high latitudes. The statistical fluctuations and non-randomness in the angular distribution of galaxies can also affect estimate of "a" from this method.

A second method to determine the galactic reddening law is to use the colours of brightest galaxies in clusters at low galactic latitudes ( $\mathrm{b} \leqslant 30^{\circ}$ ). The absolute magnitudes of brightest members of groups and clusters are approximately the same (Sandage, 1973, papers II, VI), so their intrinsic colours $(\mathrm{B}-\mathrm{V})_{\mathrm{O}}$ should also be similar.

Sandage (1975, paper VIII) uses a sample of first ranked E/SO galaxies and plots ( $B-V$ ) colours versus csc b. It is clear from Figure (1.3) that a correlation exists between colour and galactic latitude. The linear regression has a slope of 0.033 .


Figure 1.3 : Variation of observed ( $B-V$ ) colour of first ranked $E$ galaxies in groups and clusters with csc of the galactic latitude, (after A. Sandage, 1975 VIII).

This diagram is also consistent with zero reddening at the polar caps $\left(|b|>50^{\circ}\right)$ and a cosecant law for $\left(|b|<50^{\circ}\right)$ :

$$
\begin{array}{ll}
E_{B-V}=0.033(\csc b-1) & \text { for }|b|<50^{\circ} \\
E_{B-V}=0 & \text { for }|b|>50^{\circ}
\end{array}
$$

De Vaucouleurs and Malik (1969) suggest that ignoring the longitude dependence in the number counts-latitude relation leads to a bias in the results. According to their argument the absorbing clouds which are responsible for the zone of partial absorption are not necessarily
distributed symmetrically about the galactic centre. Taking this point we should include a longitude dependent term to the number count-latitude relation.

The expression they derive for $A_{B}(l, b)$ is consistent with $\left\langle A_{B}=0.2\right.$ and $E_{B-V}=0.046$ at the galactic pole. These values are used by de Vaucouleurs to correct his magnitudes for galactic absorption at the polar caps.

This value of 0.2 , absorption in the polar caps was dismissed by Tammann et al (1979) as due to the misintepretation of abundance anomalies in terms of too large a colour excess. Also, the polarization of starlight has been used to support the polar window picture. Low interstellar polarizations are indeed seen at the Galactic poles but the conversion of polarization into extinction requires a precise knowledge of the dust properties and these results have not been generally considered as convincing.
1.2.3 Internal absorption in external galaxies :

Holmberg (1958) has shown that spiral galaxies suffer intrinsic absorption as a function of their inclination, in the sense that edge-on galaxies seem fainter than the face-on galaxies with the same luminosity.

Holmberg (1958) has determined the intrinsic absorption correction to face-on orientation as a function of inclination :

$$
A_{p g}^{i}=0.28(\csc i-1) \quad\left(i \sim 90^{\circ}\right. \text { for face-on galaxies) }
$$

where the inclination angle $i$ between the equatorial plane and the line of sight can be calculated from the Hubble (1926) equation

$$
\cos ^{2} i=\frac{\left(q^{2}-q_{0}^{2}\right)}{\left(1-q_{0}^{2}\right)}
$$

where $q_{0}$ is the intrinsic axial ratio (taken to be 0.2 for all spirals
regardless of type). $q\left(=\frac{b}{a}\right)$ is the apparent ratio of minor to major axes. Sandage and Tammann (1976) apply a morphological type (T) dependent inclination correction to their magnitudes of the form :

$$
A_{B}=\alpha\left(\frac{a}{b}\right), \text { where } \alpha=\alpha(T)
$$

### 1.2.4 Malmquist bias :

The mean absolute magnitude in an apparent magnitude limited sample increases with distance. Since we attribute an average absolute magnitude to all the galaxies with the same luminosity class in the sample, this average value will be less than the true absolute magnitude of the remotest galaxies, in this case one always underestimates the distances of the remote (brighter) galaxies and consequently finds a value of $H_{o}$ which increases with distance. This effect is known as the Malmquist bias. The importance of the Malmquist bias diminishes with the width of the luminosity function of a distance indicator under consideration. This explains the value of those distance indicators which have well defined luminosities with little scatter, such as the RR Lyrae stars $(\sigma(M)=0.2$ ) and the cepheid variables $(\sigma(M)=0.08$ ) the way to remove the effect of a Malmquist bias is to impose a velocity cut-off to the sample (Fig l.4), to convert the magnitude limited sample to a distance limited one.


Figure 1.4 : The redshift-magnitude relation showing the bias caused by incompleteness in a sample that is magnitude limited compared to a distance limited sample (chosen by velocity).

### 1.3 THE GALACTIC DISTANCE SCATE

The extragalactic distance scale is firmly based on the determination of distances within our own Galaxy. There are two main approaches to this problem.
1.3.1 The method of moving groups :

Proper motion and parallax methods are a fundamental starting point for the cosmic distance scale.

Galactic open clusters are found in or near spiral arms and are therefore population I objects. In some open clusters there is a convergent point where the proper motions of individual stars appear to intersect. This effect is interpreted as the projection to infinity of the common motion of the group. Thus the angular distance on the sky between a given star in the cluster and the convergence point is also the angle between the space-velocity vector of the star and the line of sight. The stars tangential velocity can then be determined directly from its obsexved radial velocity.

The ratio of the tangential velocity to the observed proper motion yields the distance for each star in the cluster (Fig 1.5). If $\theta$ is the angle on the sky from the star to the convergent point, $\mu$ is the proper motion and V is the radial velocity relative to the sun, the transverse component is $T=\frac{\mu}{p}$ (in units of $\frac{1}{2 \pi}$ times the orbital speed of the Earth, $4.74 \mathrm{~km} \mathrm{sec}{ }^{-1}$ ). where p is the parallax.

$$
T\left(\mathrm{~km} \mathrm{sec}{ }^{-1}\right)=4.74 \frac{\mu \frac{\mathrm{sec}}{\mathrm{yr}}}{\mathrm{p}(\mathrm{sec})} \rightarrow \mathrm{P}=4.74 \frac{\mu}{V \tan \theta}
$$

Figure 1.5 : Moving cluster

In the method of moving groups the question of cluster membership is of importance. All that can be expressed i.s the probability that a star with a given proper motion is a member of the cluster and not a field star. A clear segregation of fainter members from field stars may not be possible, but membership of the bright stars can be settled definitively with a sufficiently precise proper motion measurement.

This method has been shown to have an error of $0^{m} 18$ at the distance of the Hyades cluster.
1.3.2 The method of photometric parallaxes :

Distances to the open clusters can also be determined by a photometric comparison of stars on the lower part of the main sequence with corresponding stars in a standard cluster with known distance (i.e. from geometric methods) or with stars in the solar neighbourhood whose distances are known from direct parallax measurements.

Any comparison of the sequences of open clusters with nearby field dwarfs necessarily assumes that they have similar physical characteristics.

One application of this main sequence fitting method uses URI photometry (Upgren, 1974). The advantage of the ( $R-I, V$ ) diagram is that it is linear over a range of magnitudes which reduces the fitting errors. Complications include uncertain distances for nearby field stars and reddening inside the clusters.
1.3.3 Application to the detemanation of the Hyades distance : As an example of distances derived by proper motion and photometric methods, we may consider the Hyades cluster.

The Hyades cluster is an open cluster which is 0.6 billion years old. The existence of a convergent point makes it possible to find the distance from the method of moving groups (see Fig l.5). The distances derived for the Hyades in recent years are summarised in Table 1.1.

TABLE 1.1 : Distance modulus to the Hyades

| Source | Method | $(\mathrm{m}-\mathrm{M})$ |
| :--- | :--- | :--- |
| Van Altena 1974 | Parallax | $3.21 \pm 0.03$ |
| Upgren 1974a | Trigonometric parallaxes | $3.29 \pm 0.18$ |
| Upgren 1974b | Photometric parallaxes <br> Hanson 1975 <br> Hanson 1977 <br> Hanson 1979 | $3.22 \pm 0.04$ |
| Convergent point | $3.42 \pm 0.2$ |  |
| Weighted mean of <br> parallaxes and proper <br> motion method | $3.3 \pm 0.04$ |  |

A value of $3^{m} \cdot 3 \pm 0^{m} .04$ can be accepted with confidence for the distance modulus to the Hyades cluster. Accurate radial velocity data for faint Hyades stars and photometry of faint Hyades proper motion candidates would aid membership assessment and eventually improve this estimate.


Figure 1.6 : C-M diagram for a number of galactic clusters. The visual absolute magnitude $M$ is plotted against colour index $(B-V)$. position of variable stars and the instability strip of cepheid variables are indicated. The lines crossing the instability strip are the constant period lines of cepheids.
The age of each cluster may be read off from the vertical scale on the right by noting the position of the main sequence turn off, (after A. Sandage, 1958).

After finding the distance to the Hyades cluster by geometric methods we can use it as a measuring rule to work out distances to other open clusters. By fitting the main sequence to that of the Hyades cluster one can derive distances to several open clusters. This method assumes that all clusters have the same Zero-age-main sequence (ZAMS), or equivalently that all clusters have identical chemical composition.

The Colour-Magnitude (C-M) diagram for a number of galactic clusters (including the Hyades) is shown in Figure 1.6.

The differences in chemical compositions between the Hyades and the open clusters can produce random errors in the derived distances to the clusters. These differences can be summarised as follows :-
(1) Observed colours are affected by variations in $z$ (metallicity).
(2) The position of main sequences in the ( $\mathrm{M}_{\mathrm{bol}}, \log \mathrm{T}$ ) plane are sensitive to changes in $Y$ (Helium abundance) and $Z$.
(3) There is no guarantee that the Hyades ( $M, B-V$ ) main sequence is the correct one for other clusters.

We may estimate the consequences of the variation of $Z$ and $Y$ to the distance determination by main sequence fitting methods as follows.

Consider the change of the bolometric magnitude of a zAMS F-type star as given by :

$$
\Delta M_{b o l}=-16 \Delta Z+3 \Delta Y
$$

at constant effective temperature (P.E. Nissen, 1979). The metal abundance difference of young clusters corresponds to a variation of 0.02 in $Z(\Delta Z=0.02)$ whereas the Helium-abundance difference for clusters with the same age is $\Delta Y=0.1$ (Nissen, 1979). Using these values in the above formulae we see that the changes in metal abundance and Helium abundance tend to cancel each other out.

## 1.4 <br> R R LYRAE VARIABLES

The R R Lyraes are giant stars ; larger, hotter and more
luminous than the sun. They are found in the so-called Instability strip in the Colour-Magnitude (C-M) diagram (Fig 1.6); with periods ranging from 0.3 to 1 day, and are population II objects (old stars with low metal content).

The absolute magnitudes of some $R$ R Lyrae variables can be found by direct measurement of their parallaxes. For cluster variables, methods relying on main sequence fitting of the $C-M$ diagrams of globular clusters to those of the nearby dwarfs or sub-dwarfs can be applied, so the absolute magnitude of the $R R$ Lyrae variables are Independent of the Hyades distance modulus and provides a good test for the cepheid distances. The mean absolute magnitude of $R$ R Lyrae variables is $\left\langle M_{v}\right\rangle=O^{m} 8 \quad 0^{m} 15$ Because of the low dispersion in their absolute luminosity they are free from the Malmquist bias. The reason why all $R$ R Lyrae variables have equal absolute magnitude is because they all have about the same age, mass and Helium content and are in the Initial phase of Helium burning in their cores.

The main uncertainty in $R R$ Lyrae variables as distance Indicators is that their luminosity depends on the He abundance in addition to the metal abundance. Since Local group galaxies have different evolutionary histories, the He content of these galaxies may be different, and their variables may have different absolute magnitudes. Errors in R R Lyrae variable distances also arise because of the extinction, but as Population II objects they suffer only a small amount of absorption.

With present techniques, $R$ R Lyrae variables cannot be seen at distances greater than about $3 \times 10^{5} \mathrm{pc}$, but are extremely useful in finding distances to the globular-clusters. By using the space Telescope they will be observable at a distance modulus of $26 . \mathrm{m}$.

## 1.5 <br> NOVAE

A Nova is characterized by a sudden change in brightness. The luminosity increases by $10-12 . \mathrm{m}$ over a period of several days, followed by a return to its former level over a period which ranges from several hundred days to several decades.

From an analysis of the light curves of 35 galactic novae with known distances from expansion parallaxes, H.W. Duerbeck (1981) classifies novae into two groups :
(I) Very fast and fast novae with smooth light curves and absolute visual magnitudes at maximum light between -8.0 and -11.0
(II) Slow novae with absolute magnitudes at maximum between -6.0 and -7.0 . Novae of this type radiate for a longer time with nearly Constant absolute magnitude.

A correlation can be defined between the absolute magnitude of novae at maximum light, $M_{o}$, and the decay rate parameter $t_{3}$, which is the time required for a novae to decline from maximum light by 3 magnitudes. Absolute magnitude at maximum light can be found for galactic novae using distances derived from expansion parallaxes. The relation between absolute magnitude and the decay rate parameter $\left(t_{3}\right)$ is linear to a first approximation in the interval $1 \leqslant \log t_{3} \leqslant 2$ where $t_{3}$ is measured in days. The zero point of this relation is based on 15 galactic novae

$$
M_{o}(p g)=2.4 \log _{3}-11.3
$$

The mean error over the useful range of $\log _{3}$ is $\varepsilon=0.18$ G. de Vaucouleurs (1978a). This relation is adequate for systems such as the Magellanic clouds with a few well observed novae that are neither very fast nor slow ( 11 novae in the LMC and 4 in the SMC).

For richer systems like M31 a nonlinear relation is found over a greater range of logt (S.Van denBerg, 1976).

By plotting the $\left(\mathrm{m}_{\mathrm{o}}(\mathrm{pg}), \operatorname{logt}_{3}\right)$ relation for the Local group galaxies and fitting it to the $\left.\left(M_{o}(p g), l_{3}\right)_{3}\right)$ relation for galactic novae, the distance moduli to the Local group galaxies can be found. The apparent magnitude of novae at maximum $m_{0}$ and their decay rate parameters are difficult to determine accurately, so the distance moduli derived from the $M_{o}\left(\log t_{3}\right)$ relation are often uncertain. It has also been shown that galactic novae all have roughly the same absolute magnitude 15 days past maximum. The mean value based on 15 galactic novae is $<M_{15}>=-5 m_{5} \pm 0.18$ (G. de Vaucouleurs, 1978a).

Novae can be used as distance indicators out to about $10^{7} \mathrm{pc}$. The calibration of extragalactic novae, using galactic novae, is uncertain due to systematic observational effects and differences in chemical composition which can influence the intrinsic luminosity. The present uncertainty in distance moduli derived by novae amounts to $\pm 0.5$. More observations are required to investigate the differences between the galactic and extragalactic novae.

Cepheid variables are the primary distance indicators for late type galaxies closer than ( $\mathrm{m}-\mathrm{M}$ ) $\sim 28.0$. The pexiod-luminosity ( $\mathrm{P}-\mathrm{L}$ ) relation of the classical cepheids provides a powerful tool for the investigation of astronomical distances.

Cepheids are evolved supergiants and occupy an instability strip of finite width in the Colour-magnitude diagram (Fig. 1.6). The wider this region, the larger the intrinsic scatter of the $P-L$ relation. In ordex to minimize the uncertainty in the absolute magnitude of a cepheid we must know its exact position in the instability strip. As is clear from Figure (1.6) this can be done by using a third parameter, the mean colour (C) of an individual cepheid, to fix its position in the HR diagram. Using a P-L-C relation instead of a $\mathrm{P}-\mathrm{L}$ relation, substantially reduces the dispersion in absolute-magnitudes of cepheids. Classical cepheids have periods ranging from 2 to 40 days and are population 1 variables (young stars with high metal content).

The P-L relation can be calibrated by using cepheids in the galactic clusters and associations with known distances from the method of main sequence fitting (Sandage and Tammann, 1969). We must also correct the magnitudes of cepheids for reddening in the Galaxy. This can be done from observations of their spectral type (see also Vanden Berg, 1976).

To augment our sample of cepheids and find the shape of the $\mathrm{P}-\mathrm{L}$ relation, we can also use the best observed cepheids in the Local group galaxies. Then by shifting vertically to minimise the scatter about the ridge line of galactic cepheids we can produce a composite P-L diagram (Sandage and Tammann, 1968). In this way one obtains the first estimates of the extragalactic distances (Fig 1.7). The upper and lower envelope lines of the P-L relation are traces of the blue and red boundaries of the instability strip in the C-M diagram.


Figure 1.7 : The composite period-luminosity relation at mean intensity in $B$ and $V$ wavelengths derived from the sources indicated at the lower right. The absolute calibration was made by using the nine Cepheids of the galactic system shown as open and filled circles. The photographic data from the SMC are plotted with smaller crosses than the Cascoigne and Kron photoelectric data, (after A. Sanage and G.Tammann,1968).

The diagram illustrates three important points :-
(I) There is a linear relation between logp and $I$ for the range 2.5 days $\leqslant P \leqslant 65$ days.
(II) There is no evidence that cepheids in different galaxies follow different $\mathrm{P}-\mathrm{L}$ relations.
(III) The $\mathrm{P}-\mathrm{L}$ relation shows considerable scatter in $\mathrm{M}_{\langle\mathrm{V}\rangle}$ and even larger in $M_{<B>}$.

Referring to the first point, it is clear from the diagram that there is some curvature in the Long period range $\sim 100$ days, of the $P-I$ relation. This curvature is probably due to an underestimation of the luminosities of four long period cepheids in the Magellanic clouds. Feast (1974) corrected these cepheids for reddening by using their spectral types and concluded that they should be shifted upward (see Fig 1.7). In this case the relationship would be linear over the entire range observed. Cepheids seem to obey a single $P-L$ relation because there is no evidence for any change of slope in the composite P-L diagram (Fig 1.7), and the dispersion about the mean line is the same for each group. However, some differences between galactic and SMC cepheids have been seen, so this conclusion should be treated with caution.

The intrinsic scatter in the $P-L$ relation is 0.6 in $M_{B}$ and 0. 45 in $M_{V}$. The mean random error in using this relationship for estimating magnitudes is then set by the halfwidth of the instability strip in the C-M diagram (Fig 1.8a), and by the slope of the constant period lines in the $H$ R diagram (Fig l. $8 \mathrm{~b}, \mathrm{c}$ ). The scatter is a strict function of colour, so the addition of a colour term to the $\mathrm{P}-\mathrm{L}$ relation ( $\mathrm{P}-\mathrm{L}-\mathrm{C}$ ) can reduce the dispersion.


Figure 1.8 : (a) The cepheid instability strip, crossed by lines of constant period in the Colour-magnitude diagram. $R$ is the magnitude residual and $\Delta(B-V)$ is the Colour residual for a cepheid on $A^{\prime} B^{\prime}$.
(b) Traces of the boundary lines of the cepheid instability strip in the $\mathrm{p}-\mathrm{I}$ plane. The intrinsic scatter is related to the width of the instability strip and the slope of the constant period lines in Fig 1.8a.
(c) Ditto as (b) but in the $p-(B-V)$ plane.

The slopes of the constant period lines $\left(\frac{\Delta M}{\Delta(B-V)}\right.$ or $\left.\frac{\Delta M B}{\Delta(B-V)}\right)$ determine the coefficients of the colour term in the $\mathrm{P}-\mathrm{L}-\mathrm{C}$ relation. They may be derived from semi-theoretical calculations and are equal to 2.52 and 3.52 for $M_{V}$ and $M_{B}$ respectively (Sandage, 1972).

The largex value of slope in $M_{B}$ than that of $M_{V}$ explains the higher intrinsic dispersion observed in $M_{B}$ than that in $M_{V}$ magnitudes (Fig 1.7).

Accepting the above colour coefficients for the P-L-C relation, we can write this correlation in the following form :-

$$
\begin{aligned}
& M_{V}=m \log p+2.52(B-V)+n \\
& M_{B}=m \log p+3.52(B-V)+n
\end{aligned}
$$

* The P-L-C relation of cepheids may be written in the following form :
$-\mathrm{M}=\mathrm{alog} \mathrm{p}+\mathrm{b}(\mathrm{B}-\mathrm{V})+\mathrm{C}$

Differentiating this relation, we get

$$
-\Delta M=b \Delta(B-V) \quad \text { or } \quad b=-\frac{\Delta M}{\Delta(B-V)}
$$

i.e. $\frac{\Delta M}{\Delta(B-V)}$, the slope of the constant period lines in the $H R$ diagram (Fig 1.8a), is equal to the coefficient of the colour term (b).

The coefficients $m$ and $n$ have been found by using the data for 13 cepheids in Galactic clusters and associations (Sandage and Tammann, 1969). But the sample of Galactic cepheids is too small to derive colour coefficients. Using the intrinsic colours of galactic cepheids and their absolute magnitudes, we can get the P-L-C relation at the mean light :

$$
M_{\langle V\rangle}=-3.425 \log p+2.52\left(\langle B\rangle^{0}-\langle V\rangle^{0}\right)-2.459
$$

This is based on a distance modulus of 3 . 03 for the Hyades. The observational data of the cluster cepheids compared with the estimates using the $P-L-C$ relation gives a mean scatter in $M$ of only $\sigma(M)=0$. 08 . Thus cepheids are free of the Malmquist effect if they are treated, using the P-L-C relation.

Martin et al (1979) used Magellanic cloud cepheids corrected for reddening and derived the $\mathrm{P}-\mathrm{L}-\mathrm{C}$ relation :

$$
M_{\langle V\rangle}=-3.8 \log p+2.7\left(\langle B\rangle^{\circ}-\langle V\rangle^{\circ}\right)+\phi
$$

The agreement of the colour coefficient of this relation with the semitheoretical value (2.52) is gratifying. Using galactic cepheids with known distances, Martin et al find a value of -2.39 for the zero point, $\phi$. The closeness of the colour coefficient to the slope of the reddening line $\left(R \equiv \frac{A_{V}}{E_{B-V}}=3\right)$ makes it difficult to distinguish between changes in temperature and reddening. In the following discussion we show that the values of the distance moduli derived from the P-I-C relation require less correction for absorption. If ( $m-M$ ) is the distance modulus and
$(m-M)^{\circ}$ the corrected modulus, then have :

$$
\begin{aligned}
& (m-M)=\langle V\rangle-\alpha \log p-\beta(\langle B\rangle-\langle V\rangle)-\phi \\
& (m-M)^{\circ}=\langle V\rangle^{\circ}-\alpha \log p-\beta\left(\langle B\rangle^{\circ}-\langle V\rangle^{\circ}\right)-\phi
\end{aligned}
$$

Since $\quad R \equiv \frac{A}{E_{B-V}}$ we would get,
$(m-M)^{O}=(m-M)-(R-\beta) E_{B-V}$
$(R-\beta)$ is small hence the correction required to the $P-I-C$
'apparent' modulus is very small.
The question of the universality of the P-L-C relation is of importance. The colour coefficient is sensitive to metallicity changes through the colour-temperature relation. This is also sensitive to the photometric and reddening errors.

For example, there are three differences between the cepheids in SMC and those in the Galaxy :
(I) SMC has many 2 day period cepheids, while the Galaxy has only a few.
(II) The short period variables have very laxge amplitudes whereas the amplitudes of the few galactic cepheids in the same period range are small.
(III) SMC cepheids are bluer than the galactic variables by $\Delta(B-V)=0 . m^{m} \quad$ for periods shorter than 10 days.

We showed above that colour can be a second parameter leading to a p-L-C relation. If now the relative amplitude is considered as a second parameter, then a P-L-Amplitude relation would be a suitable correlation
which is independent of reddening, (Sandage and Tammann, 1971). We define the "amplitude defect" $f_{B}$ of a star by the equation :

$$
2.5 f_{B}=(\Delta B)-(\Delta B)_{\max }
$$

where $\Delta B$ is the observed amplitude in magnitudes, $(\Delta B)_{\max }$ is the maximum amplitude reached by any cepheid at a given period. The quantity $f_{B}$ measures the amount by which the amplitude of a cepheid is less than the maximum amplitude known for that period. So, $f_{B}$ is free from reddening. The error in $f_{B}$ depends on the accuracy of the upper envelope curve and the photometry of individual cepheids.

The three parameter amplitude relation (Sandage and Tammann, 1971) has a dispersion of $\pm 0.17$ and the observed difference between the SMC and galactic cepheids is removed.

Cepheids can be used as distance indicators from the Local group galaxies up to the NGC2403 Galaxy.

Assuming the space Telescope could reach to $27 . \mathrm{m}_{\mathrm{o}}$, cepheids would be observable at a distance modulus of 29 . 8. Obtaining photometry of a large sample of cepheids, we could thus improve the $\mathrm{P}-\mathrm{L}-\mathrm{C}$ and $\mathrm{P}-\mathrm{L}-\mathrm{Amplitude}$ relations and test the universality of the relations.

The sources of errors in the cepheid calibration can be summarised as follows :
(I) the derived P-I-C relation is based on a distance modulus of 3.03 for the Hyades cluster. Recently the distance to the Hyades has been raised to $3.3 \pm 0 . \mathrm{m}^{m}$ (Hanson, 1979), which would affect the absolute magnitude of cepheids and would make them brighter by 0.27 and consequently increase all distances derived from the P-L-C relation by $10 \%$.
(II) The difference in metallicity and the chemical composition between the Hyades and other open clusters which are used in determining distances by main sequence fitting and estimating the absolute reddening for each cluster. Alternatively, because the main sequence is steep at this point the reddening error transforms to an uncertainty of $\pm 0 . \mathrm{m}_{1}$ in $M_{v}$. (III) The fit of the calibrators to the P-L-C relation (ST 1969): The systematic error this causes is $\pm 0.05$ (Sandage, 1972). (IV) The absorption in cepheids in Local group galaxies and NGC2403 can cause a systematic error in cepheid distances : the error could be as large as 0.4 (Madore, 1976). The reddening of cepheids in Local group galaxies can be found from the colourcolour diagram or the spectral type of variables, but errors in the value of $R$ (ratio of total absorption to reddening) affect the extinction. Moreover, there is some evidence that long period cepheids (the only ones observable in distant galaxies) are more reddened than short period ones.
(V) Non-uniform filling of the instability strip. It is difficult to get photometric data for cepheids in distant galaxies and the small sample of cepheids in these galaxies do not fill the instability strip uniformly, causing systematic errors in the fitting procedure. Combining the above errors suggest that the accuracy of the present calibration of the P-L-C relation is $\pm 0.13$. So, the photometric distance to galaxies containing cepheids can only be obtained to within $\pm 10$ percent. This estimate, however, does not include the random error due to distance of the Hyades cluster.

The following table (Table l.2) summarises the distances to the nearby galaxies obtained using primary indicators, together with the amount of absorption in each Galaxy.

TABLE 1.2 : Distances to the Local Group and NGC2403 galaxy found using the primary distance indicator.

|  | ST |  |  | Gdv |  |  |  | V dB. |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Cepheid | ${ }^{\text {A }}$ B | Cepheid | Novae | R R Lyrae | ${ }^{\text {A }}$ B | $\begin{aligned} & \text { Adopted } \\ & \text { by GM } \end{aligned}$ | Cepheid | Novae | R R Lyrae | ${ }_{\text {A }}^{\text {B }}$ | $\begin{aligned} & \text { Adopted }{ }^{\text {DM }} \\ & \text { by VdB } \end{aligned}$ |
| LMC | 18.59 | 0.32 | 18.27 | 18.46 | 18.17 | 0.43 | 18.31 | 18.56 | 19.08 | 18.28 | 0.2 | 18.43 |
| SMC | 19.27 | 0.08 | 18.7 | 18.59 | 18.49 | 0.31 | 18.62 | 18.81 | 19.27 | 18.8 | 0.12 | 18.72 |
| M31 | 24.12 | 0.64 | 24.09 | 23.95 | 24.16 | 0.41 | 24.07 | 24.28 | 24.03 |  | 0.4 | 24.02 |
| M33 | 24.56 | 0.12 | 24.12 |  |  | 0.31 | 24.3 | 24.3 .1 | 23.31 |  | 0.24 | 24.06 |
| N6822 | 23.95 | 1.08 | 23.89 |  |  | 0.79 | 23.73 | 23.64 |  |  | 1.08 | 23.61 |
| IC1613 | 24.43 | 0.12 | 24.11 |  |  | 0.21 | 24.02 | 24.41 |  |  | 0.12 | 24.4 |
| N2403 | 27.56 | 0.24 | 26.94 |  |  | 0.38 | 27.1 |  |  |  |  |  |
|  | $(\mathrm{m}-\mathrm{M})_{\circ}$ | ${ }^{\text {A }}$ B | $(\mathrm{m}-\mathrm{M})$ 。 |  |  | $A_{B}$ | ${ }^{(m-M)} \mathrm{O}$ | ${ }^{(m-M)} \mathrm{C}$ |  |  | ${ }^{\text {A }}$ B | $(\mathrm{m}-\mathrm{M}) \mathrm{O}$ |

## N.B. ST Sandage and Tammann (1974) <br> GdV G.de Vaucouleurs (1978) <br> VdB S.Vandenberg (1976)

ST and VdB cepheid distances are based on a value of $3.03^{\text {mag }}$ for the Hyades distance modulus but GdV cepheid distances are based on a distance modulus of 3.29 mag for the Hyades.

GdV and VdB adopted distances (Columns 8 and 13) are based on other primary indicators in addition to three indicators mentioned in the table.

## 1. 7 H II REGIONS

Linear diameters of $H$ II regions in galaxies can be used as extragalactic distance indicators for the late type galaxies of classes $S C, S d$, Sm , Ir to a distance modulus of $32^{\mathrm{m}}$. O, which is too distant for the stellar distance indicators such as cepheids to be of practical use. H II regions can thus be used as fundamental calibrators of the distance scale from the M 81 group out to the virgo cluster.

By using ll calibrating galaxies with distances known from cepheids, Sandage and Tammann (1974a, hereafter STI) obtained the relation between the linear diameter of $H$ II regions and the absolute magnitude of the parent galaxy. The galaxian luminosity sand the linear size of $H$ II regions are unknown as long as the distance is undetermined. In order to overcome this difficulty, STI employed the luminosity class , $L_{c}$, of the galaxies and derived a relation between the linear diameters of $H$ II regions and the luminosity classes of the parent glaxies (Fig 1.9)


Figure 1.9 : Relation between the linear diameter of $H$ II regions and luminosity class of parent galaxies.

- Local group galaxies. OM81-NGC2403 Group.
Separation of Jocal group and NGC2403 group galaxies are clear.

As is clear from figure 1.9 there is a strong systematic separation in the linear diameter-luminosity class relation. At a given luminosity class the HII regions in the M81-NGC2403 group galaxies tend to be larger than their Iocal group counterparts.

The discrepancy could be due to three factors :
(I) It may reflect real differences between the $H$ II regions in the two groups of galaxies due to a small sample effect. (This possibility is preferred by ST, 1974b, hereafter STII).
(II) The luminosity classes of some of the galaxies may be incorrectly assigned.
(III) The distance to the M81-NGC2403 group may be over-estimated.

To investigate the first possibility we find the deviations of the observed linear diameters from their regression line in the Linear diameterIuminosity class plane (Fig 1.9) for the 11 calibrating galaxies. The results are summarized in the following table.

TABLE $1.3:$

| Galaxy | Dgal/DFit | Galaxy | Dgal/DFit |
| :--- | :--- | :--- | :---: |
| LMC | 1.02 | NGC2403 | 1.02 |
| SMC | 0.7 | N 2366 | 1.25 |
| M33 | 0.95 | N 4236 | 1.33 |
| N6822 | 0.59 | IC2574 | 1.07 |
| ICl613 | 0.78 | HO II <br> HO I | 1.16 |
| Mean for <br> Local <br> group <br> galaxies | 0.8 | Mean of <br> NGC2403 <br> group | 1.19 |

$$
\frac{\mathrm{D}_{\mathrm{N} 2403}}{\mathrm{D}_{\text {Local group }}}=1.46
$$

As is clear from Table 1.3, H II regions in M81 - NGC24O3 group galaxies are larger by $46 \%$ than Local group galaxies. In fact the observed disagreement is high and could hardly be due to the small sample as is suggested by STI.

Considering the second possibility, we should examine the 11 calibrating galaxies and reclassify some of them. We should then recalibrate the $H$ II region correlation. We reclassify three galaxies M33, the LMC and the SMC, $1,1 / 2,1 / 4$ classes fainter (the luminosity classification of these galaxies were doubtful). After applying the regression line and comparing the $H$ II region diameters in the M81 and Local group galaxies, a discrepancy of $19 \%$ still remains. Thus errors in assigned luminosity classes may account for part of the inconsistency, but they cannot account for all of the difference.

At large distances, galaxies of a given $L_{c}$ are, on average, intrinstcally brighter than their nearly counterparts (Malmquist bias), since the diameters of H II regions are a function of the brightness of the parent galaxy the observed average diameters would also suffer from the Malmquist bias. There remains two alternatives to explain the observed segregation in Fig 1.9 between the $H$ II regions in the Local group and M 81-NGC2403 group galaxies. One is that the sizes of $H$ II regions only weakly depend on the magnitude of the parent galaxy, and their use as distance indicators would be inappropriate. The second alternative is that the distance to the NGC2403 galaxy is over-estimated. Before using H II regions as extragalactic distance indicators, it is therefore vital that we know the accurate distance to the NGC2403 galaxy and the M 81 group (see section 3.4) and the reddening of cepheids inside the NGC2 403 galaxy must also be well understood.
plotting the $H$ II region correlation in a linear plane (STI), equal deviations of the diameters from the mean line for all luminosity classes
enter with the same weight into the H II region correlation. Since we know that H II regions in luminosity class I galaxies are larger than those in $L_{c}=V$ galaxies, the deviations (errors in diameters) are more significant in $L_{c}=V$ galaxies than for those in luminosity class one. Therefore, we must give the diameters different weights. Because of this reason the H II region correlation should be plotted in a logarithmic plane, (D. Hanes, 1980).

Kennicutt (1979 a, b, c) measures the isophotal diameters of HII regions. Comparing the isophotal diameters with those of STI (core/halo) we get good agreement between the two diameters for the Local group and M 81-NGC2403 group galaxies (Kennicutt, 1979b, Fig 1), but between the isophotal diameters and core/halo diameters for a sample of field galaxies that are more distant than the local calibrators the agreement is poor (Fig 1.10).


Figure 1.10 : Comparison of isophotal diameters with mean core/halo diameters as measured by $S-T$, for $H$ II regions in field galaxies, (after R.C.Kennicutt, 1979 b).

Since the Local group and M 81 group calibrators only span the lower range of luminosity classes ( $\operatorname{Ir} \mathrm{V}$ to SC III) the diameter calibration must be extrapolated to brighter galaxies. M 101 is the nearest super giant SCI galaxy. This galaxy provides the prime calibration of H II region correlation for this luminosity class. If we extrapolate the $H$ II region correlation towards the luminosity class I galaxies, the M 101 galaxy lies above the extrapolated line (STI). ST attribute the difference between the M lOl HII regions and the extrapolated value to the extraordinarily large $H$ II regions in M lol. However, ST's argument about the abnormality of $H$ II regions in $M 101$ is based on a circular argument (see section 3.6).

Kennicutt has calibrated a weighted H II region correlation and concluded that one of the primary uncertainties is the calibration procedure itself whether one includes M 101 or not, extrapolated diameters for SCI galaxies range from 460 to 710 pc which corresponds to an uncertainty of $40 \%$ in the derived distances. If we are going to use H II regions as distance indicators at large distances, the behaviour of regions in the M lol galaxy must be well understood.

From the above discussion we conclude that it is unlikely that $H$ II regions could be powerful distance indicators until the problems with their definition and measurement, their unknown nature in SCI galaxies, and systematic effects in seeing,have been resolved.

One alternative is to use the fluxes of $H$ II regions as a distance indicator. This has been investigated by Kennicutt (1981) and Kennicutt and Hodge (1980). Finding the fluxes of the 3 largest $H$ II regions in an unbiased sample of 21 SC galaxies in the Virgo cluster and plotting them against the apparent magnitudes and luminosity classes of parent galaxies (Kennicutt, 1981) produced a scatter diagram, showing only a slight trend between the luminosity classes and the fluxes of the $H$ II regions.

### 1.8 VELOCITY DISPERSION INSIDE H II REGIONS

A correlation has been found between the average turbulent velocity, <v>, of the three largest H II regions in late type spiral and irregular galaxies, and the mean linear diameter of the regions (Melnick, 1977). The velocity dispersion inside $H$ II regions can be found from the measurement of the spectrum line profile.

Using the linear diameter of the $H$ II regions in the 5 Local group and NGC2403 galaxy and plotting them against the velocity dispersion inside the regionswe get a (log < D> , log <v> ) relation (Melnick, 1977). Because of the uncertainty in the diameters of H II regions, it is preferable to use the absolute magnitude of the parent galaxy. Using 6 nearby galaxies, the following relation was found :

$$
\mathrm{M}_{\mathrm{pg}}^{\mathrm{O}, \mathrm{i}}=(-0.29 \pm 0.01) \quad\langle\mathrm{v}\rangle-(12.17 \pm 0.16)
$$

with $\mathrm{m}_{\mathrm{pg}}^{\mathrm{o}, i}$ in magnitudes (corrected for galactic and internal absorption) and $\langle v\rangle$ in $\mathrm{km} \mathrm{sec}{ }^{-1}$. The dispersion at constant velocity for the above relation is 0.08 .

The relation can thus be used to estimate distances to galaxies from measurements of their apparent magnitudes. It should be pointed out that the use of this method for M1Ol and other supergiants is based on an extrapolation of the relation far beyong the region for which the equation has been observed to hold true. Therefore it may be subject to large systematic errors. Melnick,(1978), also showed that the absolute magnitudevelocity dispersion relation only weakly depends on the inclination corrections to the absolute magnitudes of the calibrators.

There is as yet no physical explanation for the origin of the (magnitude-velocity) relation. It is possible that more massive galaxies could have more turbulent interstellar media, giving rise to giant H II regions by the aggregation of large numbers of smaller nebulae in a region of strong star formation.

From the above discussion on $H$ II regions, we conclude that :
(1) The accurate distance to the M81-NGC2403 group is of importance, and the presence or absence of absorption inside this galaxy must be well understood.
(2) Uncertainty in the definition of diameters of $H$ II regions, depending on whether we use core + halo diameters or isophotal diameters, the distances derived by this method are different, isophotal diameter distances being smaller than core + halo distances.
(3) An independent distance to Mlol is urgently needed.
(4) The calibration of $H$ II region correlation itself is uncertain for the very bright galaxies.
(5) The fluxes of $H$ II regions are also unreliable for distance estimation.
(6) The velocity dispersion inside H II regions seem very promising but it, at present, is difficult to obtain much data. Obviously more work must be done on this indicator.

The brightest stars have low weight in any distance determination procedure but they are of historical importance (Hubble, 1936).

The danger in trying to identify the brightest stars in a distant galaxy is the possibility of contamination by stars from our own galaxy. The problem of confusion by field stars can be reduced by selecting the candidates to have $(B-V) \leqslant 0.4$ and $(B-V)>2.0$, because the galactic contamination is almost confined to the colour range $0.4<(B-V)<2^{m} .0$ As we shall see the brightest blue stars in the brightest galaxies do not exceed $M_{v}=-10.0$, so the range of the method is limited to distances of less than $10 \mathrm{Mpc}\left((\mathrm{m}-\mathrm{M})<30^{\mathrm{m}} .0\right)$ and the method is restricted to SC type galaxies or later, since the brightest blue stars are systematically fainter and more difficult to observe in galaxy types Sb and earlier.

The absolute magnitudes of the brightest blue stars in SC or later type galaxies become brighter with increasing luminosity of the parent galaxy (the brightest star correlation, STII). It has been shown (Hanes, 1980, Sandage and Tammann, 1982, ST VIII) that the absolute magnitude of the brightest blue supergiants varies nearly in step with the absolute magnitude of the parent galaxy (the correlation line is almost $45^{\circ}$ ). In this case blue stars are useless as distance indicators. Bearing the above point in mind, Humphrey, 1980 (Papers V, VI) has identified the brightest stars in the Local group galaxies (NGC6822, ICl613, M33). The brightest blue stars in these galaxies confirm the brightest star relation. G. de Vaucouleurs (1978c) also supports this brightest blue star correlation using a sample of 7 galaxies in the Local group and the NGC2403 galaxy. The absolute magnitudes of the brightest stars depend only weakly on the magnitudes of parent galaxies and the observed relation might just arise from the smallness of the calibrating sample or from the uncertain absorption correction inside the Local group and M 81-NGC2403 group galaxies. These
selection effects give false weight to the brightest star correlation (for a detailed discussion, see Hanes, 1980).

Extrapolating the brightest star correlation towards SCI galaxies, we would expect to find early type supergiants as bright as $M_{v}=-10.0$. ST II adopt this value as the absolute magnitude of the brightest blue stars in SCI galaxies. Humphrey (1978, paper I) has found the brightest visual star in our Galaxy at $M_{v} \approx-9.9$. There are some other stars in the galaxy, LMC and M33 with luminosities close to -10 m .0 . If the brightest blue stars in supergiant galaxies are useful distance indicators, this fact should somehow be explained. The absolute magnitudes of the brightest blue stars in the supergiant galaxies will remain uncertain until the distance to Mlol (the closest SCI galaxy) is determined independently.

Wray and de Vaucouleurs (1980) have considered using the brightest super-associations in spirals and irregular galaxies as distance indicators. Their absolute magnitudes range from -10.0 to -15.0 and they could be used as distance indicators for distances up to $100 \mathrm{Mpc}(\mathrm{m}-\mathrm{M})=35 \cdot \mathrm{O})$. A relation between the absolute magnitude of the brightest super-associations, morphological type of the galaxy and the colour of the galaxy has been defined for a sample of 78 galaxies. The error in this indicator is about 0.5 in the distance modulus.

ST II have observed that the visual absolute magnitudes of the brightest supergiant red variables at maximum is constant at $\left\langle M_{v}\right\rangle(\max )=-7.9$. Recently they have revised this value to $\left\langle M_{v}\right\rangle(\max )=-7^{m} .72 \pm 0.06$ (ST VIII, 1982). Humphrey has found the upper limit to the luminosities of the $M$ supergiants for four Local group galaxies (LMC, N6822, IC1613, Milky Way) of different morphological type and luminosities. This investigation confirms the constancy of the absolute magnitude of red supergiants at $M_{v}=-8^{m} Q^{+} 0.1$. (These magnitudes are based on the old. Hyades distance modulus of 3.03 ). There is no significant difference between the supergiants in the galaxy, LMC,

M31 and M33. (Humphrey, papers IV, V, VI). Obviously, more work is required to understand the possible effects of metallicity and internal environment of the parent galaxy on the luminosities of brightest stars. At present, the distance moduli from the brightest stars cannot be found with an accuracy better than 0.5 .

The above points can be summarised as follows :

The danger in using the brightest stars in galaxies as distance indicators is the problem of local absorption inside the parent galaxy. This could cause a systematic error in the derived distances. Moreover, the slope of the brightest star correlation is close to one which suggests that they are not valuable indicators. The closeness of the absolute magnitudes of the brightest blue stars in supergiant galaxies with nearby blue supergiants seem unlikely. The absolute magnitude of the brightest blue stars in supergiant galaxies remains unknown until the distance to the M1Ol galaxy is determined.

The brightest super-associations could be used as distance indicators up to a distance of 100 Mpc . Before using it as an extragalactic distance indicator, one needs to solve the problem of local absorption inside the parent galaxy and of the field stars projected on the super-associations.
4) The brightest red supergiant variables have small dispersion in absolute magnitude. They will be seen at large distances by the space Telescope $((\mathrm{m}-\mathrm{M})=34.0)$.

As a final point $I$ should mention that the brightest stars in galaxies cannot provide a good test for cepheid distances, because both suffer from the same problem of absorption and crowding inside the parent galaxy.

Globular clusters are intrinsically very luminous objects
$\left(M_{v} \leqslant-10.0\right)$, and are seen in large numbers around the elliptical galaxies in the Virgo cluster. It is not reasonable to assume a fixed luminosity for the brightest globulars in galaxies, because of the dependence of the globular cluster's luminosity to the size of the parent galaxy and on the total number of clusters per galaxy (G. de Vaucouleurs, 1970, P. Hodge, 1974). However, G. de Vaucouleurs (1970) did find a correlation between the absolute magnitude $M_{B}(1)$ of the first ranked cluster and the total absolute magnitude $M_{B}(G)$ of the galaxy to which the cluster belongs. The correlation as calibrated from the Local group galaxies is as follows :

$$
M_{B}(1)=-10+0.2\left[-21+M_{B}(G)\right]
$$

One conventional way to calibrate the intrinsic magnitude of globular clusters in distant galaxies is to exploit their observed luminosity function $\phi(m)$ and compare it with that of the galactic globulars, $\phi(M)$. The method can be used to find a distance to Virgo that is independent of the conventional chain of the distance scale ladder and is entirely based on galactic stellar indicators (RR Lyrae variables). Before using this method to find the Virgo distance, we should demonstrate the similarity of globular clusters luminosity function between the Local group spirals and the $M 87$ which is agiant elliptical galaxy in the Virgo. Additionally, one needs also to understand the possible effect due to field contamination, in order to find the true shape of the luminosity function of M87. The shape of the luminosity function for globular clusters may, of course, not be universal. This is the biggest and most important problem in using globular clusters as extragalactic distance indicators. However, as is clear from the work of Harris and Racine (1979), the concept of a universal luminosity function for globular clusters now seems reasonable for a wide range of galaxy types and only the faint part
of the luminosity function of M31 is incomplete (Fig 1.11).


Figure 1.11 : Luminosity distribution for the globular cluster systems in the Local group galaxies. For M31, the data are progressively more incomplete for $M_{v}>-8.0$. Similarity of globulars luminosity function in different galaxies is of importance, (after W.E.Harris and R. Racine,1979).

The luminosity functions within various Virgo elliptical galaxies seem to be similar (D. Hanes, 1979) and have the same shape as the luminosity function for globulars in the Local group galaxies (Fig 1.11). However, the data for Virgo does not cover the full range seen in the Local group. Since the clusters in our own galaxy are limited in number at the bright end, the overlap between the Virgo and the galactic globulars is limited. To extend the region of overlap, we can extrapolate the luminosity function of globular clusters in our galaxy to brighter levels or, alternatively, improve the observations and try to sample the Virgo globular clusters to fainter levels.

A different method has been introduced by G. de vaucouleurs (1977). The following quantity can be measured observationally:

$$
\Delta m=M_{B}(C L)-M_{B}(G)=B(C L)-B(G)
$$

where $B(G)$ and $M_{B}(G)$ are apparent and absolute magnitudes of the galaxy and $B(C L)$ and $M_{B}(C L)$ the apparent and absolute magnitudes of the globular cluster. We can then establish observationally the relation between the absolute magnitude of the parent galaxy and $\Delta \mathrm{m}$ and use it as a distance indicator. It is also found to be valid for non-dwarf galaxies.

The globular cluster distance moduli rely only upon pure population II indicators (the cluster themselves through the $H$ R diagram and $R R$ Lyrae stars) to set the calibration in the globulars in our own galaxy. Because of this fact the use of globulars will afford an independent check on the population I distance indicators.

The important points of this section can be summarised as follows:
(1) Globular clusters can be calibrated in our own galaxy through the $H R$ diagram or $R R$ Lyrae variables. Their distancesare therefore independent of the Hyades distance modulus and cepheids.
(2) The universality of the luminosity function of globulars should be investigated fuxther. All distances derived by globular clusters are ultimately based on this assumption.
(3) Globular clusters provide a single step process towards the Virgo cluster distance modulus.
(4) Identification of globular clusters associated with the Virgo cluster galaxies is a severe problem. They are expected to be unresolved at the distance of the Virgo cluster and be contaminated with foreground stars. Moreover, the peak of the globular's luminosity function at Virgo has not been seen and observations of the turnover point is crucial. Evidently, we need to observe Virgo globulars at fainter levels.

Noting the above points, reveals that the present uncertainty in distances derived by globular clusters is high. The uncertainty in distances by this method is estimated to be 0.4 in the distance modulus.

## TYPE)

It is important to determine the expansion rate from remote galaxies beyond any possible local velocity anomaly. There is evidence that a pure Hubble flow exists beyond a distance of $(\mathrm{m}-\mathrm{M})=35^{\mathrm{m}} \mathrm{o}\left(>5000 \mathrm{kmsec}^{-1}\right)$, indicating that $H_{o}$ needs to be calibrated beyond this distance. To do this, we have to exploit new distance indicators. The most useful distance indicators here are the isophotal diameters and absolute magnitudes of galaxies. We must thus find new relationships between the isophotal diameters or absolute magnitudes of the galaxies and other distance independent parameters such as morphological type or luminosity classes.

The extended classification of the luminosity classes for all shapley-Ames spirals are presented in the Revised Shapley-Ames catalogue (RSA, Sandage and Tammann, 1981). The galaxies of the RSA have been divided into 11 various types and luminosity classes. The absolute magnitudes of the spiral galaxies in the RSA are derived by assuming a value of $\mathrm{H}_{\mathrm{O}}=50 \mathrm{~km} \mathrm{sec}{ }^{-1} \mathrm{MpC}^{-1}$. Since the Shapley-Ames catalogue is apparent magnitude limited $\left(m_{B}<13.2\right)$ there is a Malmquist bias. Neglect of this variation of <M> with $v$ will cause photometric distances to be progressively incorrect with increasing redshift. At large distances galaxies must be biased in favour of the bright galaxies and hence their absolute magnitudes are too bright by $\Delta M=<M>-M_{\text {true }}$, where $M_{\text {true }}$ is the intrinsic luminosity of a galaxy and <M> is the average absolute magnitude which we attribute to the class of the galaxy.

It is of interest to plot the luminosity function of spiral galaxies (corrected for Malmquist bias) for each luminosity class separately (Sandage et al, 1979). This plot gives us an idea of the dependence of the average absolute magnitudes on the luminosity classes for spiral galaxies (Fig 1.12). The luminosity functions for all classes are very
broad and they cover a wide range in absolute magnitude. For example, SCI galaxies exist with $M_{B}$ values as faint as -19.0 , overlapping the range of SC III-IV systems completely ; the brightest SC III-IV galaxies $\left(M_{B}=-21^{m} .5\right)$ are still brighter than the faintest $\operatorname{SCI}$ galaxies ( $M_{B}=-19.0$ ). The other important point is the large spread in absolute magnitude for SCI galaxies (a spread of about 3.0 ). If the observed dispersion is not caused by errors due to luminosity - classification, then $L_{C}$ is only weakly correlated with the absolute magnitude.


Figure 1.12 : Calculated luminosity functions for spirals of various luminosity classes, compared with E galaxies. The absolute magnitude scale is blue absolute on the system of "total" magnitudes of de Vaucouleurs, de Vaucouleurs, and Corwin (1976) but corrected for internal absorption (i.e. with CIA), (after A. Sandage et al,1979).

Sandage and Tammann (1975a,b) used a distant sample of SCI galaxies beyond $v_{o} \tilde{\sim} 4000 \mathrm{~km} \mathrm{sec}^{-1}$ to find the global value of the Hubble constant. The large scatter observed in absolute magnitude for each luminosity class necessitates a reconsideration of their result. One objection in the whole procedure of using luminosity classes as extragalactic distance indicator is that of assigning numerical values to $T$ and L (morphological type and luminosity class), because they are not measurable parameters. Measurements of the arm/disk ratio may provide a better and more physical way of explaining the intrinsic nature of a galaxy, but at the present time no simple quantitative substitute has been found to replace Van denBerg's luminosity classification system (Van denBerg, 1960 , a,b,c). An analysis of $\langle L\rangle$ at $T=$ const. as a function of $\log R$, where $R=\frac{D}{d}$, the ratio of major to minor axes, demonstrates that luminosity classifications are overestimated (the absolute magnitudes underestimated) when the inclination increases from face-on (i $\sim 0, \log R \approx 0$ ) to near edge-on (i) $\sim 90^{\circ}$ ).
G. de Vaucouleurs $(1979$, e) introduces a composite parameter of luminosity class and morphological type, $\Lambda=\frac{L_{C}+T}{10}$ called the luminosity index, as compromise between the Van denBerg's DDO system (Van denBerg,1960,a, $\mathrm{b}, \mathrm{c}$ ) and the revised Hubble system (Sandage, 1975). The range of $\Lambda$ is from 0.3 ( Sab I ) to $1.9(\mathrm{Im} \overline{\mathrm{V}})$. The luminosity index corrected for inclination $\left(\Lambda_{C}\right)$ can be correlated with the absolute magnitude and linear diameter of galaxies (G. de Vaucouleurs, 1979e). These relations are both linear for $\Lambda_{C}<1.7 . \quad$ For $\Lambda_{C}>1.7$ one cannot avoid systematic errors due to poor classification.

The above discussion can be summarised as follows :
(1) The dependence of absolute magnitude to luminosity class is weak. There is more than 3.0 dispersion in the absolute magnitude of SCI galaxies. Thus SCI galaxies cannot be good standard candles.
(2) Luminosity classification is affected by the inclination of the galaxy and this causes systematic distance dependent classification errors.
(3) The diameter of a galaxy may have an even poorer correlation with its luminosity class and morphological type.

Noting the above points we conclude that the present uncertainty in distance moduli derived by luminosity classes is about $\pm 0.5$. The amount of uncertainty increases as we go to distant galaxies, because of difficulty in luminosity classification.

We now conclude our general discussion on standard candles in this chapter. The present uncertainties in the available distance indicators, adopted in the literature, together with the range of each standard candle and the dependence of the derived distances to the Hyades modulus are summarised in Table 1.4.

TABIE 1.4 : The present uncertainties in the available standard candles, together with the range of each indicator and its dependence to the Hyades modulus are listed in this table.

| Indicator | Uncertainty (mag,in modulus) | Range <br> (Mpc) | Dependence to Hyades distance |
| :---: | :---: | :---: | :---: |
| Hyades distance | 0. 2 |  |  |
| Hyades abundance effect | 0.2 |  |  |
| Reddening inside Hyades | >0.02 |  |  |
| $R \mathrm{R}$ Lyrae variables in | 0.2 | 0.3 | NO |
| Local group |  |  |  |
| Novae | 0.5 | 10 | No |
| Cepheid P-L-C relation | 0.15 | 4 | YES |
| Cepheid P-L-A | 0.2 | 4 | ' |
| Cepheid abundance effect | 0.3 | 4 | " |
| Cepheid absorption in Local group ( $A_{B}$ ) | 0.35 | 4 | " |
| Cepheid in NGC2403 | 0.4 |  | " |
| H II region diameter | 0.4 | In SCI galaxies 25 | " |
| H II region luminosity | 0.4 | SCI 100 | " |
| Velocity dispersion inside | 0.3 | 6 | " |
| H II regions |  |  |  |
| Brightest blue star as a | 0.5 | SCI 25 | " |
| function of luminosity of galaxy |  |  |  |
| Brightest red variable | 0.2 | 10 | " |
| Brightest superassociations | 0.55 | 100 | " |
| Globular cluster luminosity | 0.4 | 20 | NO |
| Absolute magnitude of SCI galaxies | 0.4 | $>100$ | YES |
| Diameter-Morphological type relation | 0.6 | 100 | " |
| TF relation | 0.4 | 100 | " |
| IR/velocity width relation | 0.3 | 100 | " |
| Supernovae | 1.0 | 200 | NO |
| (U-V) colour-magnitude relation for E galaxies | 0.5 | 100 | YES |

## CHAPTER 2

## THE APPLICATION OF STANDARD CANDLES

TO THE EXTRAGAIACTIC DISTANCE SCALE

### 2.1 THE SANDAGE AND TAMMANN's METHOD

Using the concept that the properties of some distance indicators are strongly correlated, with the luminosity of the parent galaxy, Sandage and Tammann (1974a, b, c, d, 1975 a, b, 1976, referred hereafter as STI, STII, STIII, STIV, ST V, ST VI, ST VII respectively) followed a step by step process to extend the distance scale to distances of the order of 100 Mpc , beyond the influence of any possible velocity anisotropy of the Hubble flow. Their work resulted in a series of seven papers entitled 'Steps toward the Hubble constant'; their procedure is briefly described in this chapter.
(I) The fundamental indicator in $S T$ procedure is the $\mathrm{p}-\mathrm{L}-\mathrm{c}$ relation of cepheids (Sandage and Tammann, 1971). They use cepheids to find distances to the five Local group and the NGC2403 galaxy (Tammann and Sandage, 1968). The cepheid distance to the NGC2403 galaxy, a probable member of the M81 group, gives the distances to five additional late type galaxies. Using these eleven calibrating galaxies, a relation between the linear size of the largest H II regions and the absolute magnitude or luminosity class of the parent galaxy is obtained (the $H$ II region correlation).
(II) Using the above calibrated sample the dependence of the absolute magnitude of the brightest blue stars to the luminosity and, in particular, to the luminosity class of the parent galaxy is investigated (the brightest star correlation, ST II). For the brightest red supergiants, no significant variation with galaxy luminosity was found. The mean absolute magnitude of the brightest red supergiants is found to be $M_{v}=-7.9 \pm 0.1$.
(III) In the third step ST III determine the distance to MIOl, the nearest SC I galaxy. They apply the H II region and the brightest star correlations to the five late type galaxies of the Mlol group and by using six other methods S T III find a distance of $(\mathrm{m}-\mathrm{M})_{o}=29^{\mathrm{m}} .3 \pm 0.3 \cdot(7.2 \pm 1 \mathrm{Mpc})$ for this group. The distance to Mlol is of great importance because it allows the extension of calibrators discussed above to the SCI galaxies.
(IV) Using the H II region correlation (extrapolated for SCI galaxies), ST IV find distances to 40 late type field galaxies with known luminosity classes. The mean absolute-magnitudes for different luminosity classes are derived. ST IV use the absolute magnitude-luminosity class relation to find distance to the Virgo cluster. They get a value of ( $\mathrm{m}-\mathrm{M})_{0}=31^{\mathrm{m}} .45+0 . \mathrm{m} .09$ $(19.5 \pm 0.8 \mathrm{Mpc})$ for the distance to Virgo. Using the redshift of Virgo they come to their first estimate of the value of Hubble constant $\mathrm{H}_{\mathrm{O}}=57 \pm 6 \mathrm{~km} \mathrm{sec}{ }^{-1} \mathrm{Mpc}^{-1}$.
(V) $S T$ V use the absolute magnitude-luminosity class relation from the fourth step to find distances to a sample of about 50 nearby spiral galaxies. They derive, after correcting for Malmquist bias, a Local expansion rate of $H_{o}=57 \pm 3 \mathrm{~km} \mathrm{sec}^{-1} \mathrm{Mpc}^{-1}$ from this sample alone.
(VI) To avoid the problem of local perturbation and test the isotropy of the Hubble flow, ST VI find distances to a sample of remote SCI galaxies from the absolute magnitudes of these galaxies using Step IV. Knowing the radial velocities of these galaxies and correcting distances for the Malmquist effects, ST VI come to a global value of $56.9 \pm 3.4 \mathrm{~km} \mathrm{sec}{ }^{-1} \mathrm{mpc}^{-1}$ for the Hubble constant.

This value of $H_{o}$ is close to that obtained using the Local sample of SC galaxies and the Virgo cluster (Steps IV and V). From the agreement of $H_{o}$ in these cases ST VI conclude that there is no measurable velocity perturbation and only a small deviation from the smooth Hubble flow. They adopt a global value of $55+5 \mathrm{~km} \mathrm{sec}{ }^{-1} \mathrm{Mpc}^{-1}$ for the Hubble constant.
(VII) By using the Tully-Fisher relationship between the absolute magnitude and the velocity width of a galaxy (see Chapter 4), with a different correction for internal absorption, ST VII rediscuss and confirm their previous distances up to the Mlol group. The result is a revised distance modulus of $(\mathrm{m}-\mathrm{M})_{0}=31^{m} .7 \pm 0.08(21.9 \pm 0.9 \mathrm{Mpc})$ for the virgo cluster. ST VII obtain a value of $\mathrm{H}_{\mathrm{O}}=50.3 \pm 4.3 \mathrm{~km} \mathrm{sec}^{-1} \mathrm{Mpc}^{-1}$ through the velocity-distance relation for Virgo. This could be considered as the principal result of ST's program.

All the above steps are drawn in Fig 2.1 as a diagrammatic representation of Sandage and Tammann's procedure.
$\begin{aligned} \text { FIG 2.1 : } & \text { Diagram of Sandage and Tammann's procedure. Dashed lines indicate extrapolation } \\ & \text { towards } \mathrm{L}_{\mathrm{C}}=1 \text { galaxies. }\end{aligned}$ towards $L_{C}=1$ galaxies.


### 2.1.1 Comments on ST IV

Using the $H$ II region correlation, ST IV find distances to a sample of spiral galaxies with known luminosity classes. They then find the mean absolute magnitude for each luminosity class separately and derive the absolute magnitude-luminosity class relation. The relation ( $\mathrm{M}_{\mathrm{pg}} \mathrm{L}_{\mathrm{c}}$ ) is thus based on the H II region correlation. Since ST V, and ST VI use the ( $M_{p g}, L_{C}$ ) relation to find distances to a sample of nearby $\operatorname{SC}(S T \mathrm{~V})$ and distant SC I (ST VI) galaxies, the $H$ II region correlation and, in particular, the extrapolated part of the relation has a crucial effect on their ultimate value for $H_{0}$. It is then important to check the validity of this extrapolation towards luminosity classes I, I-II and II.

To test the shape of their calibration in the range $I, I-I I, I I$, ST IV use Van denBerg's (1960, $a, b, c$ ) luminosity classes. Van denBerg found distances to a sample of galaxies by assuming $H_{0}=100 \mathrm{~km} \mathrm{sec}^{-1} \mathrm{Mpc}^{-1}$. The absolute magnitudes of these galaxies are then independent of the extrapolation and can provide a good test to ST's correlation. ST IV normalise Van denBerg's data to their own scale and find that Van denBerg's calibration is fainter by 0.39 for luminosity class $I$ objects. Thus the extrapolated values of the luminosity for classes I, I-II in ST IV should be reduced. However, since the van denBerg (1960 a) magnitudes are not corrected for inclination, ST IV argue that the observed disagreement is just due to inclination effects in Van denBerg's sample. But, Bottinelli and Grouguenheim (1976) used the same formulae as ST IV for galactic absorption and inclination correction and corrected Van denBerg's sample for these effects. After normalizing them to the ST IV scale in exactly the same way and with the same weights they discovered that there is still disagreement between $S T$ and Van denBerg absolute magnitudes for lower luminosity class galaxies. These authors conclude that the shape of the extrapolated part of ( $M_{p g} L_{c}$ )relation is different from that found by ST IV.

As explained above, the $H$ II region correlations $\left[\left(d_{, ~} L_{C}\right),\left(d_{p g}\right)\right]$, ST I (Eqs 1 and 7), and the absolute magnitude-luminosity class relation, ST IV (Table 5), are not independent of each other, so these relations must be self-consistent. If the extrapolation of H II region correlation is correct, then for each luminosity class less than $L_{c}=I I$, the absolute magnitudes obtained from these relations must be the same as those found from a sample of SC galaxies (ST IV, Table 5). However, the absolute magnitude for $L_{c}=I$ galaxies derived from $\left[\left(d_{i}\right),\left(d, M_{p g}\right)\right]$ relations $\left(M_{p g}=-20.47\right)$ is not in agreement with those from ST IV SC galaxies $\mathrm{M}_{\mathrm{pg}}=-21.25$ ). This shows that the relations are not self-consistent presumably because of the uncertainty in extrapolation towards supergiant galaxies.

### 2.1.2 On the Ultimate Value of the Hubble Constant Derived by Sandage and Tammann

STV, VI use the values of the absolute magnitudes (ST IV,Table 5) corresponding to each luminosity class to find distances to nearby SC and distant SCI galaxies. $\quad S T^{\prime} \mathrm{s}$ ultimate value of $H_{0}=55+5 \mathrm{~km} \mathrm{sec}^{-1} \mathrm{Mpc}^{-1}$ is based on their calibration of the absolute magnitudes of various luminosity classes. It is important to discover possible biases and sources of errors in Sandage and Tammann's procedure.

Plotting the difference, $\Delta L C$, between the luminosity class of an individual galaxy and the mean of luminosity classes for all galaxies in the group to which it belongs, against the difference between the distance to the galaxy and the mean distance to the group, the luminosity classes are found to change with distance for galaxies in Table 1 ST V. This variation is in the sense that the bright classes are calibrated too luminous and faint classes are calibrated too faint (Fig 2.1). This effect could either be due to extrapolation which gives brighter values for luminosity class I galaxies or that the dependence of absolute magnitude on luminosity class is weak. This also shows us that luminosity classification itself suffers from systematic distance dependent errors. Considering the above point, systematically smaller values for $H_{0}$ are obtained for $L_{C}=1$ than for the other classes using ST's calibration. The 11 Class $I$ galaxies in ST V corrected for Malmquist bias give a value of $H_{0}=48.1 \pm 3 \mathrm{~km} \mathrm{sec}^{-1} \mathrm{Mpc}^{-1}$, while the 7 Class II galaxies again corrected for bias give $H_{o}=63.2 \pm 8 \mathrm{~km} \mathrm{sec}^{-1} \mathrm{Mpc}^{-1}$. The difference suggests that the two calibxations are not consistent (G. Le Denmat et al, 1975). Correcting ST V distances for the above effect (Luminosity class dependence) and using the unbiased sample, we derive a value of $\mathrm{H}_{\mathrm{O}}=78 \pm 8 \mathrm{~km} \mathrm{sec}{ }^{-1} \mathrm{MpC}^{-1}$ from SC I galaxies, instead of $55+5 \mathrm{~km} \mathrm{sec}^{-1} \mathrm{Mpc}^{-1}$ (T. Jaakkola et al, 1976).


Figure 2.1 : Distances from luminosity classification as a function of luminosity class for members in systems of galaxies according to Table I of STV. Open circles are for $L_{e}=1$ (After T. Jaakkola et al, 1976).

Bottinelli and Gouguenheim (1976) used Van denBerg's absolute magnitude-luminosity class calibration and did not get such a variation. Using Van denBerg's luminosity class calibration instead of ST IV (Table 5), they found a value of $H_{0}=76 \pm 8 \mathrm{~km} \mathrm{sec}{ }^{-1} \mathrm{Mpc}^{-1}$, in excellent agreement with the above value.

From the points mentioned above and in the last section we conclude that, there is no single reason for the low value of $H_{o}$ derived by $S T$, but an
accumulation of systematic effects in the distances, almost all working in the same sense (overestimating distances) have caused such a low value $\left(55 \pm 5 \mathrm{~km} \mathrm{sec}{ }^{-1} \mathrm{Mpc}^{-1}\right.$ ) for $\mathrm{H}_{0}$. Among the important points that contribute in the large distances derived by ST are :
(1) the galactic extinction correction, and the assumption of negligible extinction at the poles.
(2) the distance to NGC2403 and the extrapolation of cepheids p-L relation towards long period objects which are the only observable cepheids beyond the Local group.
(3) systematic errors in measurement of diameters of H II regions. (4) extrapolation of the $H$ II region correlation towards luminosity class I galaxies. This would cause an under-estimation of absolute magnitude of supergiants (as described above) and consequently overestimation in distances derived in ST V, VI.

The exact values of the above corrections are not known at the moment, so one cannot fully correct ST's calibration for all the possible errors.

### 2.2 G. de VAUCOUIEURS' PROGRAMME

In a discussion on the determination of the extragalactic distance scale, G. de Vaucouleurs builds up the scale on a variety of distance indicators, (G. de Vaucouleurs 1978a, b, c, d, $1979 \mathrm{a}, \mathrm{b}, \mathrm{c}$, hereafter referred to as GdV I,GdV II,GdV III, GdV IV, GdV V, GdV VI, GdV VII, respectively). Using five primary distance indicators with revised zero points, he finds distances to six Local group galaxies. The dispersion in these distances derived from primary indicators is $\pm 0.2$ in distance modulus. GdV's distances to Local group galaxies are 0.3 smaller than $S T^{\prime}$ 's scale, presumably because of different corrections for galactic extinction.

With these calibrators, GdV then discusses the secondary indicators that could be calibrated in the Local group galaxies and be applied to more distant objects. The secondary indicators include globular clusters, the brightest non-variable blue stars, the brightest red variables and velocity dispersions inside $H$ II regions. The distances to nearby groups (M81, M1Ol, sculptor) can be found by using secondary indicators. The mean error in distances derived by secondary indicators is $\pm 0.3$. GdV IV shows that NGC2403 and NGC2366 are not at the same distance as M81 group galaxies, contrary to ST's assumption. GdV's distances are smaller than ST's by 0.25 and 0.79 for the M 81 and Mlol groups respectively.

Using the distances of 25 nearby galaxies ( $18.3<(\mathrm{m}-\mathrm{m})<29^{\mathrm{m}} 3$ ) from papers I-IV, GdV V calibrates the two tertiary indicators, the isophotal diameters and total magnitudes of galaxies, both corrected for inclination and galactic extinction. Two relations between the linear diameters, absolute magnitudes and the "luminosity index", $\Lambda=\frac{L_{C}+T}{10}$, can be found. The mean error in distance modulus derived from the Linear diameter - luminosity index is $\pm 0^{m} .6$ in distance modulus and from the absolute magnitude -luminosity index is $\pm 0.45$. The next step in GdV's process is to use these two tertiary indicators to find distances to a sample of 458 Spiral galaxies, for which
total apparent magnitudes, diameters and appropriate classifications, are available and lie in the range $\Lambda_{c} \leqslant 1.65$ (where the two tertiary indicators are linear). The derived weighted distances to these galaxies have a mean error of $\pm 0.4$. To demonstrate the linearity of his distance scale GdV VI compares distances derived from the above tertiary indicators with three more or less independent distance indicators, (i) the effective diameters of spirals (ii) the maxima of type I supernovae (iii) the magnitudes of the brightest superassociations. All three indicators verify the linearity of this scale over the $15^{\mathrm{m}}$ o range covered by the primary, secondary, and tertiary indicators. Having found the distances to this sample of galaxies and using their redshifts, values of the Hubble constant can be derived for individual galaxies. A systematic variation of the Hubble constant with distance is observed. Since the sample is magnitude limited, it suffers from the Malmquist bias. To reduce this effect GdV VII applies a velocity limit to the sample. Moreover, the values of " $\Lambda$ " change with distances in the sense that galaxies with higher " $\Lambda$ " have lower distances. In order to remove this effect $G d V$ VII confines the sample to a distance range for which the : variation of " $\Lambda$ " is less. The value of Hubble constant is seen to be smaller for galaxies inside the Local Supercluster (LSC) than for those outside. Exdluding galaxies in the LSC and applying the above restrictions GdV VII obtained a cosmic value of $H_{o}=100 \pm 10 \mathrm{~km} \mathrm{sec}^{-1} \mathrm{Mpc}^{-1}$. The error in this value is mainly due to the uncertainties in primary and secondary indicators.

A schematic representation of G. de Vaucouleur's procedure is drawn in Fig 2.2.

FIG 2.2: The diagramatic representation of $G$. de Vaucouleurs' programme. Both eclipsing binaries and $A B$ supergiant stars have weight $1 / 2$ in GdV's distance scale.


DISTANCES OF 458 SPIRAL GALAXIES IN THE RANGE $18 \cdot 0^{m}=(\mathrm{m}-\mathrm{M})=33 \cdot 7$

$$
H_{0}=100 \pm 10
$$

## 3.1

## INTRODUCTION

We now compare the Sandage and Tammann, and G. de vaucouleurs distance scales in order to understand better the uncertainties and difficulties in the determination of the extragalactic distance scale. In Fig 3.1, we have plotted ST distances against those of GdV for the galaxies common to both samples. As is clear from the diagram the two scales deviate as we go to the more distant objects. Agreement between the two scales is worst for the field galaxies beyond the Mlol group, where distances have been estimated from the H II region diameters (ST IV). For galaxies less distant than M1O1, although there are vast differences in technique between ST and GdV, (Table 3.1), the distance scales to M81-NGC2403 are discrepant by only 0.25 . At MlOl, the discrepancy jumps to 0.8 and then to about $1^{m} .0$ for distant galaxies. In our comparison we shall pay particular attention to the methods used at these distances.

In Table 3.1, Sandage and Tamman's technique and that of G. de Vaucouleurs' are summarised. It is evident that the two processes differ at every step.


FIG 3.1 : Sandage and Tammann against g.de Vaucouleur's distance moduli. The difference between the two scales in the Local group and NGC2403 group galaxies (filled dots) is not significant (about $\mathrm{om}_{3}$ in distance modulus) but the two scales diverge at larger distances, in particular the discrepancy is evident at the region where ST use H II regions as standard candles (crosses).

TABLE 3.1: A comparison of the Sandage-Tammann \& G. de Vaucouleurs Distance Scales

|  | ST | G. de Vaucouleurs |
| :---: | :---: | :---: |
|  | $A_{B}=0 \quad b>50$ | $A_{B}\left(90^{\circ}\right)=0.2$ |
|  | $A_{B}=0.13(\operatorname{cscb}-1)$ | $A_{B}=F(\ell, b)$ |
| Reddening | $A_{V}=3 E_{(B-V)}$ | $A_{V}=R \cdot E(B-V)$ |
| No. of Primary Indicators | 1 | 5 |
| No. of Secondary Indicators | 3 | 6 |
| No. of Tertiaxy <br> Indicators | 1 | 2 |
| No. of Linearity checks | 0 | 3 |
| No.of Galaxies | 100 | 300 |
| Velocity field | Assumed Linear and Isotropic | Allowance is made for Local Supercluster |
| $\mathrm{H}_{0}\left(\mathrm{~km} \mathrm{sec}^{-1} \mathrm{Mpc}^{-1}\right)$ | $55 \pm 5$ | $100 \pm 10$ |
| Supporting evidence | SN Theory | Tiully-Fisher <br> relation for infra- <br> red,superluminal <br> objects |

The zero points of the $p-L$ relation for galactic cepheids adopted by $S T$ are $\left\langle M_{v}\left(\log p_{o}\right)\right\rangle=-3.58$ and $\left\langle M_{B}\left(\log p_{o}\right)\right\rangle=-2.84$. The zero points from van denBerg's p-L relation are -3.5 and -2.85 in visual and blue respectively. All these values are based on a distance of 3.03 for the Hyades cluster. GdV I takes the unweighted mean of visual zero points from $S T$ and van denBerg $\left(\left\langle M_{v}\left(\log p_{0}\right)\right\rangle=-3^{m} 54\right)$, corrects it to a distance of 3.29 for the Hyades cluster and gets a corrected zero point of -3.8 for the galactic $p-I$ relation. Using other independent methods, he eventually adopts a mean zero point of $M_{v}(0.8)=-3.6 \pm 0.15$, at a period of $\log p=0.8$ ( $p$ in days) which is the mean period for galactic cepheids. Comparing this value $\left(M_{v}(0.8)=-3.6\right)$ with that of $S T ' s\left(M_{v}(0.8)=-3^{m} .58\right)$ we see that GdV's zero point is brighter by 0.02 than $S T \cdot \mathrm{~s}$ and by 0.06 than the mean of the $S T$ and van denBerg's zero points. We conclude that the change of 0.26 to the Hyades distance modulus applied by GdV is cancelled out by his adopted value for the zero point of the p-I relation of cepheids and the disagreement in distance to the Hyades does not contribute to the observed difference between the two scales. In Table 3.2 we summarise the difference between distances in the two scales together with the difference in absorptions.

TABIE $3.2:$
The difference between G. de Vaucouleur and SandageTammann distance moduli together with the difference between absorptions in the two procedures for the Local group, M81-N2403 group and M1O1 group galaxies.

## GdV - ST

|  | $\Delta(\mathrm{m}-\mathrm{M})_{\mathrm{B}}$ | $\Delta A_{B}$ | $\Delta(m-M)^{\circ}{ }_{B}$ |
| :---: | :---: | :---: | :---: |
| LMC | -0.17 | 0.11 | -0.28 |
| SMC | -0.42 | 0.23 | -0.65 |
| M33 | -0.07 | 0.19 | -0.26 |
| NGC6822 | -0.51 | -0.29 | -0.22 |
| IC1613 | -0.32 | 0.09 | -0.41 |
|  | -0. 3 | 0.02 | -0.32 |
| NGC2403 | -0. 33 | 0.14 | -0.47 |
| NGC2366 | -0.21 | 0.21 | -0.42 |
| NGC4236 | 0.36 | 0.27 | 0.09 |
| IC2574 | 0.41 | 0.25 | 0.16 |
| HOII | 0.48 | 0.25 | 0.23 |
| HOI | 0.37 | 0.25 | 0.12 |
| M1 Ol Group | -0.05 | 0.23 | -0.25 |
| M1O1 Group | -0. 56 | 0.23 | -0.79 |

### 3.3 LOCAL GROUP AND NGC2403 GROUP COMPARISON

The distance moduli of GdV are smaller than those of ST. The difference is of the order of $0 . \mathrm{m}_{3}$ in the Local group galaxies, 0.05 for M81-NGC2403 and 0.56 at the distance of MlO1. After applying the correction for galactic absorption to these distances the differences increase to $0.32,0.25$ and 0.79 respectively. This increase is expected because of the different treatment of galactic absorption. ST use the csc law with $A_{B}=0$ at the pole to estimate the galactic absorption, but GdV uses a latitude dependent formulae with 0.2 absorption at the pole. The difference in galactic absorption corrections certainly affects the distance moduli to M81-NGC2403 and M1O1 but does not affect the Local group moduli, because their absorption corrections are based on individually determined values of $E_{B-V}$ through the reddening law. At this stage we take the LMC as an example for the Local group galaxies and now explain the difference in the absorptions between the two scales for this particular galaxy. GdV's value for the absorption to the LMC seems to be large ( 0.43 ) this value has been found by adopting a reddening of $E_{B-V}=0.11$ and $R=4$. $S T$ assume a reddening of $E_{B-V}=0.08$ for the LMC (Gascogine 1969). However, the best available estimate of the absorption to the LMC is $E_{B-V}=0.04$ and $A_{B}=0.17$ (Martin et al, 1979). It is clear from Table 3.2 that the low value for the mean difference in absorption estimates in the Local group is due to $N 6822$. As we discussed in the cepheid section, the distance moduli derived from the p-I-c relation of cepheids are affected less by reddening than those from the p-L relation. So, applying the same corrections for absorption directly to both moduli would underestimate the distance moduli from the p-L-c relation (see section l.6). These points can explain part of the difference between the two scales at the Local group distances. However, the remaining difference can be due to the different independent distance indicators used by GdV (R R Lyrae, Novae, etc).

The main difference between the two scales in the M81-NGC2403 group is due to the galactic absorption corrections. We can remove the difference by applying the same absorption corrections to both scales. The small difference of 0.05 in the mean between the uncorrected distance moduli for M81-NGC 2403 group galaxies (Table 3.2) does not necessarily mean that the two scales are in good agreement at that distance. Since ST have found the same distance for all galaxies in the M81-NGC2403 group whereas Gd $V$ has found larger distances for 4 galaxies ( N 4236 , IC2574, HOI, HOII) and smaller ones for two others ( $\mathrm{N} 24 \mathrm{O} 3, \mathrm{~N} 2366$ ) the differences in distance moduli to group members cancel each other out. In the following section we explain the sources of errors and difficulties in the distance of the M81-NGC2403 group galaxies.

### 3.4 THE M81-NGC2403 GROUP

- An accurate distance to NGC2403 is of great importance. It is the first and the last galaxy beyond the Local group in which cepheids can be observed and is a probable member of the M81 group. Tammann and Sandage (1968) observed 17 cepheids inside NGC2403 and derived a distance modulus of 27.56 . Because of the lack of cepheids in the other M81 group members, they assume that all of the galaxies are at the same distance and attribute this cepheid distance to the other members of the group. Some authors (G. de Vaucouleurs, 1978d, Hanes 1980, Madore 1976) have found lower distances to $N 2403$ and have questioned the validity of $S T$ 's assumption about the membership of this galaxy in the M8l group. To show the validity of their assumptions, STII apply the HII region and brightest star methods to the six galaxies in the M81-NGC2403 group and find distances to the individual galaxies. They derive a mean distance of $3.09 \pm 0.12 \mathrm{Mpc}$. From the closeness of this value to the cepheid distance of $\operatorname{NGC} 2403(3.25 \pm 0.2 \mathrm{Mpc})$, they conclude that all the galaxies are at the same distance. As we mentioned in the brightest star section, these indicators are not useful tools to check the cepheid distances. If cepheids suffer from absorption inside NGC2403, the same thing may be true for the brightest stars. Moreover, the calibration of the $H$ II region and brightest star correlations relies heavily on cepheid photometry in NGC2403. The agreement between the cepheid distance and the brightest stars and H II region correlations is an artifact of the circularity of ST's argument. In order to find an accurate distance to NGC2403, the effect of obscuration inside this galaxy should be well understood. In the presence of absorption, the magnitude and colour residuals of cepheids from the ridge lines of the $p-L$ and $p-C$ relations should be correlated, with slopes 4 (in blue) and 3 (in visual). Tammann and Sandage (1968) do not apply a correction for absorption to the cepheids in NGC2403 because the slopes of relations between the magnitude and colour residuals in the blue and visual
(1.6 and 0.73 respectively) are different from the expected reddening values. Hanes (1980) used Tammann and Sandage's (1968, Table 5) data to plot magnitude residuals against colour residuals. After applying the regressions upon the colour and magnitude residuals he discoverred that in both cases the slope of the relation is close to that predicted by differential obscuration. Therefore, the scatter in the residuals plane could partly be due to internal absorption. It is not clear why this result differs from that found by ST. Tammann and Sandage (1968) also observed that the colour of cepheids in NGC2403 are 0.4 redder than the colours of galactic cepheids. They leave this question open : one simple explanation for this colour discrepancy is that the cepheids in NGC2403 are reddened by 0.4 . Madore (1976) came to the same conclusion by plotting the $p-C$ diagram for $N G C 2403$ cepheids and comparing it with the $p-C$ relation defined by Local group galaxies,(Madore, 1976, Figure 5). We conclude that Tammann and Sandage's assumption of negligible absorption internal to N2403 led to an over-estimation of the cepheid distance modulus. Correcting the cepheids in NGC2403, we can remove the observed discrepancy between the Local group and M81 group in the H II region correlation (see the H II region section).

We conciude that S'T's distance to the M81-N2403 group must be reduced by 0.4 or average. Meanwhile, the question of membership of N 2403 to the M81 group must be reconsidered. The available distances to the M81-NGC2403 group are summarised in Table 3.3).

TABLE 3.3: Distance moduli to the M81-N2403 group

| $\begin{aligned} & \text { Distance to the } N 2403 \\ & \text { galaxy } \\ & (\mathrm{m}-\mathrm{M}) \end{aligned}$ | Method | Source |
| :---: | :---: | :---: |
| $27.82 \pm 0.1$ | Cepheid | Tammann, Sandage <br> (1968) |
| 26.96 | Cepheid corrected for absorption | Madore, (1976) |
| $27.1 \pm 0.2$ | Cepheid, H II region, Brightest stars | G. dv (1978d) |
| 27.54 | TF relation in infrared | Aaronson et al, <br> (1980a) |
| $26.96 \pm 0.13{ }^{\text {G }}$ | Revised H II region Correlation | Hanes, (1980) |
| $27.34 \pm 0.11^{\text {G }}$ | $\begin{aligned} & \text { Revised (M,L) } \\ & \text { relation } \end{aligned}$ | Hanes, (1980) |
| $27.2 \pm 0.09^{G}$ | Mean | Hanes, (1980) |

Note : All distances are adjusted to $(\mathrm{m}-\mathrm{M})_{\circ}=3^{\mathrm{m}} 29$ for the Hyades. " $G$ " represents the distance to the M81 group.

### 3.5 THE MLOI GROUF COMPARISON

In the case of the MlOl group galaxies the problem becomes more difficult. The distance modulus of 29.3 found by ST is probably an upper limit since the absence of reddening inside this galaxy for the brightest stars seems unlikely (Humphrey, 1980). However, G dV's distance to this galaxy is also not free of criticism. GdV's distance to the Mlol galaxy requires that the brightest blue stars in this galaxy ( $M_{B}=-10.0$ ) be fainter than the brightest stars in the solar neighbourhood. Also the cepheids and brightest $M$ supergiants have escaped detection, which puts a lower limit to the distance of MlOl. GdV's distance indicators to the Mlol group are the brightest blue stars and the velocity dispersion inside $H$ II regions $\left(\log D, \log \sigma_{v}\right)$. GdV applies a zexo-point correction to the $\left(\log D, \log \sigma_{V}\right)$ relation to fit it with his own primary distances (GdV II,III). This decreases distances from the original Melnick (1977) relation (based on ST's primary distances) by 0.34 on average. In order to find the distance to the MlOl group galaxies, GdV gives more weight to the velocity dispersion method, so this difference has considerable effect on his final distance to Mlol. Thus 0.34 of the difference between the two scales at the distance of MlOI can be explained by differences in the primary distances between GdV and ST. Secondly, ST do not apply any correction for galactic extinction to the M1Ol group because they are at the polar cap, but GdV applies 0.23 correction for galactic absorption to this group. Considering the above points a difference of 0.23 between the distances of Ml Ol group in the two scales remains unexplained. At present we cannot say that Mlol has an accurate distance from secondary indicators. Fortunately independent distances from supernovae and the Tully-Fisher (TF) relation are available (Chapter 4). In section 3.6 the problems in the ST distance to the MIOl group are discussed.

### 3.6 ON THE DISTANCE TO MlOL

Since there is no SCI galaxy with an accurate distance from primary calibrators, the $H$ II region correlation must be extrapolated towards luminosity Class I galaxies. It is important to test both the calibration and the validity of this extrapolation, because of its crucial role in the ( $M, L_{C}$ ) relation derived in $S T$ IV. The absolute magnitudes of $S C I$ galaxies, the ultimate distance indicators at large redshifts (ST V, VI), are therefore completely based on the extrapolation of the H II region correlation. The role of MlOl in this procedure is particularly important because it is the closest supergiant galaxy. At the ST distance modulus for Mlol (ST III), the observed H II region linear diameter is 580 pc compared with a diameter of 460 pc predicted by a linear extrapolation of their relation. There are two possible explanations for this :(i). The $H$ II regions in MlOl are abnormally large, (ii) The $H$ II regions in Mlol are typical but the absolute magnitude of Mlol is high. In ST IV two arguments are presented in justifying this extrapolation. ST use the linearly extrapolated value for diameters of $H$ II regions in SCI galaxies, and derive distances to 9 field SCI galaxies. From the agreement of the mean absolute magnitude of these SCI galaxies with that of Mlol they conclude that Mlol is a typical SCI galaxy in absolute magnitude and that the inconsistency between MiOl and other SCI galaxies on the ( $\mathrm{d}, \mathrm{I}_{\mathrm{C}}$ ) plane is because of the larger H II regions in that galaxy. In fact, by adopting a linear extrapolation, they excluded the second possibility mentioned above. So, the agreement they found for the absolute magnitude of M1Ol with the mean for the 9 SCI galaxies is far from firmly established. Moreover, Kennicutt (1979b) has demonstrated that the sizes of $H$ II regions in MlOl are not anomalously large and are typical of SCI galaxies. ST IV also used the apparent magnitude-luminosity class relation for galaxies in Virgo to confirm the extrapolated part of the ( $M, L_{C}$ ) diagram. The bright tail of the ( $m, L_{C}$ ) relation for Virgo agrees
with the ST IV calibration of the absolute magnitude of 9 SCI galaxies and also with Mlol itself. Fron this agreement they confirm, once again, the typicality of MlOl galaxy in absolute magnitude. All the estimates of the distance to MlOl depend on the assumption that it is a member of the MlOl group (Hanes, 1980). If it is not, and is a foreground galaxy (Bottinelli and Gouguenheim, 1976) the whole procedure would collapse. As a final point we return to the discussion on the agreement of the Virgo ( $m, L_{c}$ ) relation and the ( $M, L_{C}$ ) for field galaxies. Kennicutt (1979b) recalibrated the ( $\mathrm{d}, \mathrm{L}_{\mathrm{c}}$ ) relation with different weights, he proposed a range of calibrations. One solution is an analogue of $S T$ and the other is forced to pass through M1OL. The shape of both the ( $M, L_{C}$ ) relations is consistent with the ( $\mathrm{m}_{\mathrm{C}} \mathrm{L}_{\mathrm{C}}$ ) relation for Virgo which degrades ST IV argument on the validity of extrapolation.

From the above discussion we conclude that the ST IV argument about the validity of extrapolation of the $H$ II region diagram is questionable and that, their distances to the Mlol group must be treated cautiously. The question of whether MlOl is a typical galaxy in terms of the sizes of its H II region or its magnitude remains open until an independent method for determining the distance to M1Ol is found. A list of the available distances to the Mlol group is presented in Table 3.4.

TABLE 3.4: All distances are adjusted to $(m-M)_{o}=3.29$ for Hyades where appropriate. "G" represents the group distance.

Distance Moduli to MlOl

| Distance to Mlol galaxy $(\mathrm{m}-\mathrm{M})_{\circ}$ | Method | Source |
| :---: | :---: | :---: |
| $29.56 \pm 0.3^{\text {G }}$ | H II region diameters, Brightest stars Luminosity classes | ST III (1974c) |
| $29.0 \pm 1$ | Supernovae | Kirshnex (1974) |
| $28.96 \pm 0.6$ | Velocity dispersion inside H II regions | Melnick (1978) |
| $28.5 \pm 0.3^{\text {G }}$ | H II region velocity <br> dispersion, <br> Brightest stars | GdV (1978d) |
| $29.32 \pm 0.3$ | Supernovae | Schurmann et al (1979) |
| $28.84 \pm 0.1^{\text {G }}$ | Isophotal diameter of H II regions | Kennicutt (1979b) |
| $29.1 \pm 0.15{ }^{\text {G }}$ | Revised H II region correlation and the luminosity class relation | D. Hanes (1980) |
| $29.23 \pm 0.21{ }^{\text {G }}$ | Revised ( $M, L_{C}$ )relation | Mould et al (1980) |
| $29.16 \pm 0.35^{\text {G }}$ | IR/velocity width relation | Aaronson et al (1980a) |
| $29.15 \pm 0.2^{\text {G }}$ | Mean of different methods | Mould et al (1980) |

As is clear from Figure 3.1, the deviation between the two scales increases for galaxies with distances obtained using H II regions. We have seen that the weakest link in ST's process is in using H II regions to bridge the gap between the Local group and more distant galaxies. Distances found using H II region diameters are subject to several systematic errors. It is difficult to estimate the size of these errors because of the variable effects of atmospheric seeing and the subjective nature of ST's diameter measurement.

In order to reduce the divergence between the two scales and the observed dispersion in the H II region domain (Fig 3.1), and to compromise the two distance scales, we use the isophotal diameters of $H$ II regions from Kennicutt (1979b, minimum calibration) to derive distances to the field galaxies in ST IV Table 3 in exactly the same way as ST. Plotting these new distances against those of GdV's for a sample of galaxies that are in common between ST IV and GdV (Fig 3.2), we see that the difference between the two scales is reduced. We should note that the isophotal diameter correlation of H II regions (Kennicutt, 1979b, minimum calibration) is based on ST's distances to the Local group galaxies. From Figure 3.2 it seems that systematic errors in diameter measurements of H II regions are indeed a source of difference between the $S T$ and Gdv scales.

Using the field galaxies (see Fig 3.2) with absolute magnitudes from the isophotal diameter correlation and fitting them with ST IV data for the Virgo cluster, we would find a distance modulus of $(\mathrm{m}-\mathrm{M})=30.65$ and consequently $\mathrm{H}_{\mathrm{O}}=84 \mathrm{~km} \mathrm{sec}^{-1} \mathrm{MpC}^{-1}$ (compared with $57 \mathrm{~km} \mathrm{sec}^{-1} \mathrm{Mpc}^{-1}$ from ST IV). This increase in the value of $H_{o}$ is entirely due to the use of isophotal diameters for H II regions. We conclude that using isophotal diameters of H II regions decreases ST's distance scale by 0.8 , on average, at the distance of Virgo.

FIG 3.2 : The same as Fig 3.1 but for isophotal diameters of H II regions instead of core/halo diameters. The dispersion in the H II region domain, (crosses), is considerably reduced.


### 3.8 COMPARISON AT THE DISTANCE OF VIRGO

In this section we use G. de Vaucouleurs (1979b) distances to the sample of galaxies listed in ST IV Table 3 and find the mean absolute magnitude for each luminosity class. Magnitudes are corrected for galactic extinction and internal absoption according to the formulae in the introduction of the second Reference Catalogue (de Vaucouleurs, deVaucouleurs and Corwin, 1976, hereafter RC2). The correlation between absolute magnitude and luminosity class is found to be :

$$
\mathrm{M}_{\mathrm{T}}^{0}=0.87 \mathrm{~L}_{\mathrm{C}}-21.18 \quad r=0.98
$$

Absolute magnitudes for different luminosity classes from the above relation together with the ST IV calibration are summarised in Table 3.5.

TABLE $3.5: \quad M_{p g}$ and $M_{T}^{O}$ versus luminosity class derived from $S T$ and Gdv distances.

| $L_{C}$ | $\mathrm{M}_{\mathrm{Pg}}$ (ST IV) | $\mathrm{M}_{\mathrm{T}}^{\mathrm{O}}$ |
| :---: | :---: | :---: |
| IT |  |  |
| I-II | -21.25 | -20.31 |
| II | -20.74 | -19.88 |
| II-III | -20.23 | -19.44 |
| III | -19.21 | -19 |

Note that the absolute magnitude of SCI galaxies, whïch is derived by using GdV distances $\left(M_{T}^{\circ}\right)$, does not require any extrapolation, since it is not based on the use of $H$ II region diameters as standard candles. Using the sample of Virgo galaxies from ST IV Table 4 and total apparent magnitude
corrected for galactic and internal absorption from $R C 2$, we can derive a distance for the Virgo cluster. From the SCI galaxies we get a distance of $30^{m} .37 \pm 0.13$, whilst SC II galaxies give a distance modulus of $30^{m} .65 \pm 0^{m} .2$. A mean distance of $30^{m} .45 \pm 0^{m} .2$ can be derived by using all luminosity classes, giving weight 2 to luminosity class I galaxies and weight 1 to other classes. Assuming a recession velocity of $\langle\mathrm{V}\rangle=1,111 \mathrm{~km} \mathrm{sec}{ }^{-1}$ (ST IV) for Virgo, we would then get a value of $90 \pm 8 \mathrm{~km} \mathrm{sec}{ }^{-1} \mathrm{Mpc}^{-1}$ for the Hubble constant. The distance to Virgo dexived in this section from GdV's data is thus in close agreement with that found in the last section using isophotal diameters of $H$ II regions and ST primary distances $\left((\mathrm{m}-\mathrm{M})_{\mathrm{O}}=30.65\right)$.

### 3.9 SOURCES OF DISCREPANCY OTHER THAN IN THE DISTANCE SCALE

So far, we have compared the distance scales using common galaxies. However, apart from the difference in distances between the two methods, there are some other facts that contribute to the difference between the final values of the Hubble constant. In this section we briefly discuss the other possible sources of discrepancy. One such source arises from the different galaxy samples used by ST and GdV to find $H_{0}$. Specifically, ST include Virgo and galaxies in regions of the sky dominated by the Local supercluster, (LSC), whereas, GdV excludes these regions. ST V consider a linear Hubble flow inside the Local supercluster and do not find any variation of the Hubble constant with distance inside the LSC. They attribute (almost) all the scatter in the observed velocity-distance relation to the errors in the distance estimates. However, GdV VII finds a deviation of the Hubble law from linearity for galaxies inside the LSC. A systematic trend of the Hubble constant with the supergalactic longitude $L$ is observed, in the sense that the values of $H_{o}$ are smaller inside than outside the LSC. He concludes that ".... Here it will be sufficient to observe that the longitude dependence is not just a simple harmonic variation across the sky such as could result from a systematic motion of the Local group, but indicates a strong effect of the density enhancement in the LSC in a manner generally consistent with the kinematical model." Microwave background measurement tends to support this, although it implies that our galaxy's motion is the cause, and not the rotation-expansion model of Virgo suggested by GdV.

### 3.10 <br> CONCLUDING REMARKS

The distances to the Local group and M81-NGC2403 group galaxies on the $S T$ and GdV scales are different by 0.32 and 0.25 respectively. This is due to a combination of different corrections for galactic absorption, local reddening inside the galaxies, a zero point shift of the cepheid p-L relation applied by GdV, and additional distance indicators used by Gdv. The scales diverge at the distance of Mlol group galaxies by 0.79 . The distances to this group in both scales are found via the brightest stars and H II regions. Different treatments for galactic absorption and the differences in distances to primary calibrators (Local group and M81-NGC2403 group galaxies) are the main points which make the two scales divergent at this distance.

Systematic effects in estimating H II region diameters leading to a low value for the absolute magnitude of SCI galaxies in ST's procedure, can partly explain the observed discrepancy at large distances. Using isophotal diameters of $H$ II regions instead of the core/halo diameters used by ST brings the two scales in closer agreement but the discrepancy is still fairly large. Apart from the above points, the use of different galaxy samples (resulting from the inclusion of galaxies inside the Local supercluster by ST and the exclusion of these galaxies by GdV) can make the results divergent.

## CHAPTER 4

## NEW ST'ANDARD CANDLES

In the previous chapters we discussed the conventional standard candles and problems associated with each indicator. However, the uncertainty in each indicator and the present disagreement on the distances to even the nearby galaxies are discouraging. The difficulty in the extragalactic distance determination is partly because of the limited number of distance independent parameters in galaxies. Fortunately, new distanc independent parameters have been recognized and the attempt has already begun to improve the accuracy of these standard candles.

In this chapter, we discuss these indicators that are the future prospects of the distance scale problem. A discussion on the velocity width-luminosity correlation (the Tully-Fisher relation) followed by a brief description of two other methods (colour-magnitude diagram and supernovae) are subjects of this chapter.

### 4.1 LUMINOSITY-VELOCITY WIDTH RELATION

The mass of a spiral galaxy can be found in terms of the maximum rotation velocity ( $v_{\max }$ ) through the relation $M \alpha R v_{\max }^{2}$ (for circular orbits) where $R$ is the radius. Since $\frac{M}{L}$ is expected to be similar for all spirals of a certain type, there should be a relation between luminosity and $v_{\text {max }}$ for spiral galaxies. Rotation curves are difficult to measure for large numbers of galaxies. Since most spiral galaxies have an extended distribution of neutral hydrogen a convenient measure of $v_{\text {max }}$ is provided by the width $\Delta v$ of the $21^{\mathrm{cm}}$ emission line, where $\Delta V \simeq 2 v_{\max }$, the correlation between $\Delta v$ and luminosity has been successfully used as a distance indicator (Tully \& Fisher, 1977, Sandage and Tammann, 1976, ST VII). Local group galaxies show a tight correlation between absolute magnitude (M) and the velocity width $(\Delta v)$. An apparent magnitude - $\Delta v$ plot constructed from spiral galaxies in a cluster can be shifted vertically to fit the calibrators, to give the distance modulus to the cluster. The luminosity - velocity width relation (the Tully-Fisher relation hereafter $T F$ ) will be tighter if two galaxies with a given total $L$ have a similar mass distribution. The correlation found between the intrinsic luminosity and diameter with no type dependence supports this idea - (Heidmann, 1969). Before using the TF relation as an extragalactic distance indicator, the following points must be considered :
(I) The observed $21^{\mathrm{cm}}$ width $\Delta \mathrm{v}_{21}$ should be corrected to an edge-on orientation $\left(\Delta v_{21}^{i}\right)$ to give a true estimate of the maximum rotation velocity $v_{\max }$. This requires an accurate knowledge of the inclination angle $i$
$\left(\Delta v_{21}^{i}=\frac{\Delta v_{21}}{\sin i}\right)$.
(II) The apparent magnitude of a galaxy $\mathrm{m}^{\circ}$, must be corrected to a face-on orientation ( $\mathrm{m}^{0, i}$ ) to minimize the effects of internal absorption. This requires a good knowledge of inclination $i$ and also an accurate formula for the internal absorption correction.
(III) Morphological type dependence. Unfortunately the Local group and Virgo samples are small and do not allow us to test the type dependence of $T F$ relation directly. There is some evidence from field samples that spirals of different types do not follow a unique TF relation (Roberts, 1978).
(IV) Definition of the width of the $21^{\mathrm{cm}}$ profile. The $21^{\mathrm{cm}}$ width must be determined in some way to minimize the effects of noise whilst measuring as close as possible the full-width at zero intensity. In practice the width at $20 \%$ of the peak value is usually taken as a compromise.
(V) Intrinsic dispersion. As mentioned earlier the TF relation is tight for Local group galaxies, but the scatter increases at larger distances. It is important to find the range of absolute magnitude for a fixed value of $\Delta v$. A magnitude limited sample favours the intrinsically bright galaxies, since absolute magnitude is correlated with linewidth, a magnitude limited sample must be similarly biased towards large ( $\Delta \mathrm{V}$ ). So, because of the dispersion in the $(M, \Delta V)$ relation, it will suffer from the Malmquist bias.

Different investigators have derived discrepant distances to the Virgo cluster using the TF relation. At this point we briefly compare the works of TF and ST VII to test the sensitivity of the velocity width absolute magnitude relation to inclination correction and sample size. The TF and ST VII procedures vary in a number of important aspects :
(1) Different inclination corrections have been applied to magnitudes. TF adopt the cosecant law $A_{B}=0.28(a / b-1)$ whilst ST VII use a type dependent inclination correction $A_{B}=\alpha\left(\frac{a}{b}\right)$, where $\alpha$ depends
on the moxphological type of galaxies. They also adopt different values of the inclination angle i. TF's inclinations were weighted by the optical appearance of the spiral structure but $S T$ use only the apparent axial ratio $\mathrm{a} / \mathrm{b}$. For the 8 galaxies in common between TF and ST there is a $3^{\circ}$ systematic difference in inclinations. Aaronson, Huchra and Mould (1979, Fig 2.hereafter $A H M$ ) have shown that the inclination correction of ST VII for total internal absorption leads to a poorer correlation between $M$ and $\Delta V_{o}$ than the correction procedure used by TF.
(2) TF and ST VII's absolute magnitude-velocity width relation are in rough agreement for the nearby calibrating galaxies, although they apply different absorption corrections. The disagreement for the Virgo cluster comes from the fact that ST VII include 12 galaxies, nine of which are more face-on than $45^{\circ}$. As we mentioned earlier, the correction of velocity widths to edge-on is difficult and particularly uncertain for face-on galaxies. The uncertainty in the velocity width corrections is clear from Fisher and Tully (1977, Fig 4), where there is a separation between edge-on and face-on galaxies in the ( $M, \Delta V$ ) plane, in the sense that face-on galaxies ( $i<30^{\circ}$ ) give higher distances than edge-on (i>45 ) galaxies. We conclude that inclusion of face-on galaxies to the original $T F$ sample can partly explain the difference between $S T$ and $T F$ and can also explain the higher distances derived by $S T$.
(3) Selecting values of the $21^{\mathrm{cm}}$ line width from the literature. ST adopt systematically larger values of the observed 21 -cm linewidth, which leads to an increase in the rirgo cluster distance modulus.
$S T$ and TF have attempted to reduce the difference in their procedures and to reconcile the results. The results of this attempt are summarised in Table 4.1. All of the following distances are ultimately based on a distance modulus of 3.03 for the Hyades and on the ST distances to the Local group galaxies (STI).

TABLE 4.1 : A comparison of the luminosity-velocity width relation as implicited by ST and TF. The sensitivity of the relation to inclination corrections is clear.

| Author | Distance moduli to the Virgo | Sample, corrections |
| :---: | :---: | :---: |
| original <br> TF' <br> relation | $30.6+0.2$ | Inclination correction $A_{i}=0.15$ (csc $i-1$ ); 8 Virgo galaxies more edge-on than $45^{\circ}$. Inclination angle defined by the separation of spiral arms. |
| ST | $30^{m} \cdot 9+0^{m} .18$ | Same sample as $\mathrm{TF} ; \mathrm{A}_{\mathrm{i}}=\alpha\left(\frac{\mathrm{a}}{\mathrm{b}}\right)$; inclination angle defined by $a / b$ ratio. |
| TF | $30 . \mathrm{m}^{\mathrm{m}}+0 . \mathrm{m} .2$ | Accepting ST Local calibration which was slightly different from TF because of different apparent magnitude and profile width correction. $A_{i}=\alpha(a / b)$; using 8 galaxies, 3 galaxies more edge-on than $45^{\circ}$ are included from ST sample ; accepting ST velocity profiles and inclinations for these 3 galaxies. |
| ST | $31 . \mathrm{m}_{33+0} \mathrm{~m} .16$ | $A_{i}=\alpha\left(\frac{a}{b}\right) ; 20$ Virgo galaxies ; $\Delta V$ velocity widths from sources different from TF ; accepting 12 face-on galaxies ; inclination angle based on the ratio $a / b$. |
| TF | $31.3+0.4$ | Accepting ST HI profile and inclination correction ; Adding more face-on galaxies from ST and augmenting the sample to 1.6 galaxies. The scatter in TF relation in this case is $\pm 0.9$ mag. |
| Result |  | The main source of the difference between ST and TF distances to Virgo (regardless of the difference in sample) is the correction of apparent magnitude and 21 cm width of galaxies as a function of inclination, and different ways for estimating the inclination angle in the sense that ST get higher values for $i$ and high values for $V_{\max }^{i}$. |

The observed linewidth is the sum of two components: the projected maximum circular velocity, $V_{M}$ sin $i$, and the line of sight component $v_{t}$ of any noncircular or turbulent velocities $\Delta V=2\left(V_{M} \sin i+V_{t}\right)$. The rotational component of the linewidth is $V_{C}=\left(\Delta V-2 v_{t}\right)$ where $2 V_{M}=\frac{V_{c}}{\sin }$. . Neglecting the turbulent component affects mainly the low inclination galaxies ( $i<45^{\circ}$ ) and consequently result is a systematic error in the $M-\Delta V$ relation. No consistent definition of velocity width has been adopted in the literature. The width at $20 \%$ of peak is difficult to determine when the signal-to-noise ratio is poor. A width at $50 \%$ of peak is less affected by turbulence and is better determined when the signal-tonoise is poor. To reduce the errors in measurement we can find width at $20 \%, 40 \%, 50 \%$ and $100 \%$ of peak and then take the mean of these values. Improvement in the velocity width estimates can also be obtained by including only galaxies with inclinations greater than $45^{\circ}$, and excluding galaxies without a well defined nucleus and disk structure.

One of the major sources of scatter in the TF relation is the internal absorption. It is clear from Table 4.1 that uncertain inclination corrections lead to discrepant results. Since the amount of absorption in the infrared is less by a factor of $n$ lo (Johnson, 1968), The use of infrared magnitudes instead of optical ones may reduce this uncertainty. Aaronson, Huchra and Mould have carried out such a program and found the shape of the TF relation in infrared region. Two interesting results emerge from the AHM work :
(i) A decrease in the dispersion of the TF relation.
(ii) An increase in the slope of the relation over that for blue magnitudes.

The tight correlation found for the Virgo and Ursa major clusters by using infrared magnitudes uncorrected for inclination, confirms that internal extinction is indeed a major source of scatter in the TF relation.

The slope of the infrared relation is $\sim 9.5$ which may be compared with the lesser values of 6.25 from $T F$ and 6.88 from ST VII. Approximating the observed slope by a value of 10 , we would get :

$$
M_{H} \propto \operatorname{lo} \log V_{\max } \longrightarrow L_{H} \propto V_{\max }^{4} \quad \text { in infrared. }
$$

From the similarity of this relation to that between luminosity and velocity dispersion in ellipticals ( $L \alpha \sigma^{4}$ ) and the fact that $L_{B}$ measures the contribution of the young population $I$ component whilst $L_{H}$ measures the old population of red giants, AHM conclude that $\frac{M}{L_{H}}$ is constant but $\frac{M}{L_{B}}$ is not constant, for spirals of differing mass. According to these authors, this is the reason that an $L \propto V_{\max }^{4}$ relation is seen in the infrared but not in the blue. Moreover, Faber and Gallagher (1979), by using a large sample of field spiral galaxies, showed that there is a correlation between $\frac{M}{I_{B}}$ and morphological type for galaxies, but there is no correlation between $\frac{M}{L_{K}}$ and type. Since $H-k$ is independent of type, this result is also valid for $\frac{M}{L_{H}}$. Observations in Ursa Major (AHM 1979), M81-NGC2403 (Aaronson et al, 1980a , Paper 1), Virgo (Mould et al, 1980, Paper II) and more distant clusters (Aaronson et al, 1980b, Paper III) confirm a slope of 10 for the infrared magnitude-velocity width relation. From this observational evidence and the apparent constancy of the mass to light ratio in the infrared, Aaronson et al (1980b) conclude that the slope of the TF relation is universal in the infrared.

1. One objection which can be raised to this point is that the steep $T \mathrm{~F}$ relation found by these authors might be the result of the adopted $21^{\mathrm{cm}}$ velocities and not only due to the use of infrared magnitudes. However, the studies of AHM(1979) and Aaronson et al (1980a) give the same slopes, although the definitions for $V_{\text {max }}$ differ (in AHM, 'TF's velocity widths are used while in Aaronson et al, a new definition for $\Delta V$ is adopted). One can conclude that
the steep relation in the infrared region is not due to the adopted values for $\Delta \mathrm{V}$, but is entirely due to the use of infrared magnitudes.

The main conclusion from the above discussion is that the shape of TF relation is wavelength dependent. Bottinelli et al (1980) have confirmed the wavelength dependence of the slope of the TF relation. They show that the slope is an increasing function of wavelength, varying from 5 in the blue to 10 in the infrared. It is worth mentioning that because of the high slope of $T F$ relation in the infrared, the errors in $21^{\mathrm{cm}}$ velocity measurements propagate through and make the absolute magnitudes more uncertain.

It should be mentioned briefly that the implication of the difference in slopes between the blue and infrared relationships is that there is a strong correlation between ( $B-H$ ) colour and the $H I$ line profile width. Consequently a correlation can be found between ( $B-H$ ) colour and intrinsic luminosity (B. Tully et al, 1982), which could also be used to estimate distances to galaxies. Table 4.2 summarises slopes and zero points of the family of TF relations together with the supporting evidence for each relation and the methods by which they have been established.

Since luminosity is correlated with $21^{\mathrm{cm}}$ width $\left(M_{\text {tot }} \alpha R_{\max }\left(\Delta V_{o}\right)^{2}\right.$ ) and luminosity is correlated with diameter ( $L a R^{2.8}$, Heidmann (1969)), there should also be a relation between the diameter and $21^{\mathrm{cm}}$ width. TF (1977) have shown that this relation is not as tight as the luminosity $-21^{\mathrm{cm}}$ width, presumably because of more uncertain corrections to diameters for extinction and inclination. Mean luminosity and Hubble type of galaxies are related to each other, hence, even if there is no intrinsic dependence of velocity width on Hubble type, a correlation will appear through the luminosity dependence (Roberts, 1978). Rubin et al, (1980) have also suggested that the slope of TF relation depends on the Hubble type of galaxies. This dependence could have a significant effect on the distances derived from TF relation. However, Mould et al, (1980, Paper II, Table 4) have looked for this effect in the infrared-TF relation and found less evidence for it.
$-M=a \log \left(\frac{V_{c}}{\sin i}\right)+b$

|  | a | b | Supporting Evidence | Sample, Method |
| :---: | :---: | :---: | :---: | :---: |
| TF | 6.25 | 3.76 | Heidmann, Luminosity-radius relation L $\alpha R^{2.8}$. IAU Symposium, Paris, | 10 Local group galaxies with distances Known from STI, ST III 8 Virgo spirals with $i>45^{\circ}$ Inclination angles from literature and for some galaxies from the optical appearance and spiral structure. |
| ST | 6.88 | 2.62 | ```(M, 工_) relation STIV. (Nearby SC and distant SCI galaxies STV,VI). Distance to Virgo from ( }M,\mp@subsup{I}{C}{\prime relation.``` | Using 20 galaxies in Virgo ( 8 from TF sample and 12 additional galaxies with i< 45 ${ }^{\circ}$ ) Distances to Local group from ST. |
| AHM (1979) | 9.47 | $-2.39$ | Constancy of $\frac{M}{L_{H}}$ ratio for all spirals with differing mass supports universality of slope of the TF relation in infrared. | Galaxies more edge-on than $45^{\circ}$, but less than $85^{\circ}$. Using infrared magnitudes. Distance to Local group galaxies from ST. $\hat{0}$ galaxies in Local group only. Source of $V(0)$ from TF. |
| Aaronson et al (1980a) | 10 | $3.77$ $4.48$ | I $\alpha V^{4}$ relation can be derived by accepting slope of 10 . This relation is similar to that of (elliptical galaxies. This is because of the constancy of $\frac{M}{L}$. | Five galaxies in N2403-M81 group. Zero point is fixed by ST7 distances to M3I and M33 galaxies, Using infrared magnitudes. Profile widths from STVII and TF . |
| Mould,Aaronson and Huchra(1980) | 110.19 | $\left\{\begin{array}{l}\text { (224 } \\ \text { mean } \\ \mathrm{b}_{\mathrm{N} 598}=4.25\end{array}\right.$ | AHM (1979) | Eighteen galaxies in Virgo. The zero point can be fixed by STVII distances to M31 and M33 galaxies in Local group using infrared magnitudes. |
| $\begin{aligned} & \text { Aaronson et al } \\ & \text { (1980b) } \end{aligned}$ | 11.0 | $\left\{\begin{array}{l}b_{N 224}=6.69 \\ \text { mean }=6.34 \\ b_{N 598}=5.99\end{array}\right.$ | AHM (1979) | Mean of distant clusters. Using infrared photometry. The zero point in this case can also be fixed with STVII distances to M31 and M33 galaxies. In all cases a zero point of 3.77 is used in the original papers. |

TABLE 4.2 Cont'd...

|  | a | b | Supporting Evidence | Sample, Method |
| :---: | :---: | :---: | :---: | :---: |
| Adopted slope and zero point from IR/velocity width relation | $10 \pm 0.5$ | 3.77 | Fixing slope to a value of 10 which is theoretically expected and observationally confirmed the zero point can be found through distances to M31 and M33. <br> These values are used by AHM to find distances to Virgo and more distant clusters, (Papers II and III). |  |
| Bottinelli et al (1980) | $5.0 \pm 0.4$ | $19.4 \pm 0.15$ | A sample of 157 spiral galaxies with distances known from Independent method $\left(M_{B}, \Lambda_{C}\right) ;\left(D_{\ell}, \Lambda_{C}\right)$ supports these values. Gav(1979f) <br> Maximum rotation velocity and diameter of our galaxy correspond to the values derived by $T F$ relation accepting Bottinelli et alcalibration (Bottinelli, 1980). | Using 11 galaxies as our calibrators with distances known from $\operatorname{GdV}(1978 \mathrm{~b}, \mathrm{~d})$ by using primary and secondary indicators. Correcting rotational velocity for turbulent effect. |
| Rubin et al (1980) | $13+1.5$ |  |  | 21 SC galaxies with distances known by redshift ( $H_{0}=50 \mathrm{kmsec}^{-1} \mathrm{Mpc}^{-1}$ ) |
| Shostak (1978) | $5.3+0.2$ | 6.06 |  | Distances to a sample of late type galaxies are determined by assuming $\mathrm{H}_{\mathrm{O}}=50 \mathrm{~km} \mathrm{sec}-1 \mathrm{Mpc}^{-1}$ and $\mathrm{v}>500 \mathrm{~km} \mathrm{sec}{ }^{-1}$ $i>45^{\circ}$ are adopted. Distance to 10 Local group galaxies from ST. A segregation between Local group and redshift sample in the ( $M_{B}, \Delta V_{0}$ ) plane is found. The slope is the result of a fit to Local group calibrators. Low slope in this case with respect to TF and $S T$ is attributed to the absorption correction to magnitudes adopted in RC2. |

N.B. Slopes and zero points of the family of Tully-Fisher relations available in the literature, together with the supporting evidence for each relation and the methods by which these relations have been established.
All zero points are based on a distance of 3.29 for the Hyades.

### 4.2 THE COLOUR-MAGNITUDE (C-M) RELATION

Most of the problems in the determination of the extragalactic
distance scale are concerned with the luminosity classification of spiral galaxies. As an independent method, the possibility of using the correlation between the absolute magnitudes and colours of $E$ and $S O$ galaxies has been considered (Visvanathan and Sandage 1977, Visvanathan and Griersmith 1977, Sandage and Visvanathan 1978a, b, Aaronson et al, 1981). Before using the C-M relation for $E$ and So galaxies as a distance indicator, the universality of the correlation for early type galaxies and the wavelength dependence of the $C-M$ relation must be investigated. It has been shown by Visvanathan and Sandage (1977) and Aaronson et al (1981) that the slope of the $C-M$ relation in the optical region is universal. Moreover, it seems to hold equally well for galaxies in clusters, groups and field samples. Using the IR photometry of E and So galaxies in Virgo and Coma, Aaronson et al (1981) have shown that the C-M relations for optical-infrared ( $\mathrm{U}-\mathrm{K}$ ) and ( $\mathrm{V}-\mathrm{K}$ ) colours are not universal. They argue further that another parameter in the $C-M$ relation is required.

The application of the C-M relation to the determination of relative distance moduli depends on the universality of the C-M relation. Before using the colours of early type galaxies as standard candles, corrections must be made for the colour gradients within a galaxy and also " K " correction must be applied for the effects caused by redshift. The present uncertainty in distance moduli derived from the $C-M$ relation is about $0^{m} .5$.

## 4.3

 SUPERNOVAEThe conventional extragalactic distance scale process depends ultimately on the distances within the Local group and our own galaxy. Supernovae can provide an independent method of extragalactic distance measurement, which does not depend on Local-group and galactic distances (see (2), below). Two methods can be used to find distances from supernovae :
(1) Determining the average absolute magnitude at the maximum light for the class of supernovae.
(2) A generalization of Baad's method for variable stars. This method has the advantage that it can be applied to individual supernovae through a purely physical method and consequently does not require a large sample of supernovae. In this method we find the angular size of supernovae from its received flux density and temperature, then by estimating the radius from the expansion velocity the distance to supernovae can be derived.

Using a sample of 19 type II supernovae at maximum, with known distances, GdV VI derived a mean absolute magnitude of $\left\langle\mathrm{M}_{0}\right\rangle=-16.13+0.15$. This value is fainter than $\left\langle M_{p g}>=-18.7+0.2\right.$ derived by Tammann (1978) from a sample of 23 type II supernovae at maximum (with magnitudes corrected for galactic and intrinsic absorption). Tammann (1978) argues that SNeI in E/SO galaxies are nearly standard. Standard candles with an absolute magnitude (corrected for galactic and internal absorption) of $\left\langle\mathrm{M}_{\mathrm{pg}}\right\rangle=-19.8+0.2$. However, Branch (1981) has shown that the peak absolute magnitude is correlated with the rate of the early post peak decline of the light curve (100 days after maximum) and that, consequently the peak absolute magnitude changes from supernova to supernova.

Errors due to Local absorption inside the parent galaxy, photometric and observational errors and the possibility of a dependence of the absolute magnitude of supernovae on the type of the parent galaxy would make the
above magnitudes uncertain by as much as $\pm 1.0$. Clearly, the only hope of using supernovae as standard candles (method 1) rests on the Space Telescope to augment our sample and improve statistics. Meanwhile, the problem of local absorption inside the parent galaxy must be solved. one way to sort out the local absorption is to compare magnitudes of type $I$ supernovae in elliptical galaxies with those in spirals. The difference would be due to local extinction, because SNe in elliptical galaxies do not suffer from the absorption. (For more detail see Branch 1982).

Kirshner and Kwan (1974) have used the expansion velocities and the black body estimate of emission by the photosphere to get absolute magnitudes and consequently distances to the two supernovae inside the Mlol galaxy. The difficulties and uncertainties in this method are described by Schurmann et al (1979).

From an analysis of the composite light and colour curves of type I supernovae using Baad's method, Branch (1977) found the absolute magnitudes of type I supernovae. Using these values he obtained $\mathrm{H}_{\mathrm{O}}=49+9 \mathrm{~km} \mathrm{sec}{ }^{-1} \mathrm{MpC}^{-1}$. The main sources of systematic error in this procedure, as suggested by the author were associated with the redshiftmagnitude relation and the black body expressions used to relate magnitudes and colours to temperatures and radii. For high redshift, the effect of redshift on the apparent magnitude of a supernova needs to be taken into account. The uncertainties in distances derived from Baad's method can be reduced by finding the true slopes of the radius and colour curves of supernovae (the time variations of the radius and of the colour of supernovae). In a recent paper Branch (1979) has revised the radius curve of SNeI. By using SNeI in elliptical galaxies to fix the zero point of the apparent magnitude-redshift relation he found a value of $56 \pm 15 \mathrm{~km} \mathrm{sec}^{-1} \mathrm{Mpc}^{-1}$ for $\mathrm{H}_{0}$.

Schurmann et al (1979) have proposed a new method (revised Baad's
method) in which the supernovae need not be observed at maximum and
basically relies on the observed quantities. In this method they make models in order to convert bolometric luminosities and temperatures to $(B-V)$ colours and absolute magnitudes. By fitting these models to the observed data they derive distances to the extragalactic supernovae to an accuracy of $20-15 \%$.

Supernovae have a potential range of 100 Mpc and can provide a "one step process" towards extragalactic distances. Clearly more work needs to be done on supernovae to confirm its ability as a distance indicator. The most serious problem is the reddening and extinction inside the parent galaxy. This can affect both the Baad's method and the assumption of constancy of the mean absolute magnitude of SNe at maximum. Moreover, as we go further away to distant galaxies, the accuracy of the derived distances will be badly affected by Malmquist bias. Since supernovae are point sources and not extended objects, they are to be preferred as indicators at large distances than the first ranked galaxies in clusters. In Table 4.3, we compare the SNeI in E/SO galaxies as distance indicator with that using first ranked cluster galaxies. Generally speaking, the present uncertainty in distances derived by supernovae amounts to $\pm 1.0$. However, it can be used as a tool to find the deceleration parameter, $q_{0}(T a m m a n n$ et $a l, 1979)$, and consequently place constraints on the possible world models.

SNI in E/SO galaxies as distance indicator is compared with the first ranked galaxies as standard candles.

|  | First ranked galaxy | SNI (in E/SO) |
| :---: | :---: | :---: |
| Difficulty of surface photometry | Yes | No |
| Luminosity Evolution | Yes | No |
| Dynamical Evolution | Yes | No |
| Local Absorption | No | No |
| Intergalactic Absorption | Yes | Yes |
| $\mathrm{M}_{\mathrm{V}}$ | $-23.2$ | $-19.7$ |
| ```Intrinsic Dispersion, \sigma, in M``` | $>1.0$ | 1.0 |

## CHAPTER 5

## CONCLUSION

We have now finished our review of the "steps towards the Hubble constant", a description of available standard candles, the problems associated with each indicator, the Sandage-Tammann and G. de Vaucouleurs methods and a comparison between the two scales were subjects of the previous chapters. In this chapter we list the major conclusions of our discussion.

The application of cepheids, H II regions, brightest stars, globular clusters, luminosity classes, $21^{\mathrm{cm}}$ linewidth, supernovae and other indicators have so far led to a value of $H_{o}$ between 50 and $100 \mathrm{~km} \mathrm{sec}^{-1} \mathrm{Mpc}^{-1}$.

The main sources of error in determination of $H_{o}$ can be summarised as follows :
(1) Uncertainties of the local calibration. Since even the Hyades modulus is still uncertain by about lo\%, the calibration will be in no case better than $10 \%$.
(2) Galactic absorption and the additional problem for some distanceindicators of absorption inside the parent galaxy.
(3) Sampling errors (Malmquist bias and similar selection effects).
(4) Measuring errors and the intrinsic scatter of the distance indicators used.
(5) Deviation from an ideal Hubble flow (due to the random motions of field galaxies) and the large scale anisotropy.

From these points it is clear that presently known distance indicators cannot find individual distances to galaxies with an accuracy better than $20 \%$ (because of 1,2 and 4). This uncertainty increases further
in the determination of $H_{0}$ (because of 3 and 5). So, with the present knowledge, $H_{o}$ cannot be found with an accuracy bettex than $20 \%$.

The uncertainty in the value of $H_{o}$ may be decreased by measuring distances to a large sample of galaxies or by exploiting several kinds of distance indicators and obtaining the mean value of $H_{0}$. However, it is also important to refine our standard candles and reduce the errors in individual distances to the galaxies. In what follows, the conclusions on standard candles which have been used to find individual distances to galaxies are presented.

Noting the first point mentioned above and remembering the uncertainties listed in Table 1.4 , it seems that the present uncertainties in the extragalactic distance scale are high. Cepheids are without doubt the best available standard candles. Attempts must be made to observe them at larger distances. Meanwhile the problem of absorption and the effect of metallicity on the absolute magnitude of cepheids must be resolved.

Some other indicators like H II regions and brightest stars suffer from both systematic and sampling effects. The Local group in which our calibrators are located is a very small cluster. As a result there is a statistical uncertainty in the H II region and brightest star correlation. A small scatter amongst the limited number of Local group galaxies does not guarantee their validity as a standard candle for distant galaxies. Apart from cepheids, the luminosity function of globular clusters, could provide one of the most valuable tools for distance determination, although there is still a need to establish its universality. The Tully-Fisher relation in the infrared and supernovae have difficulties and weaknesses, but are good prospects for the future of the distance scale problem.

The launch of the Space Telescope will, evidently, herald a new era in the determination of the size of the Universe. Using the Space Telescope, it would be possible to observe RR Lyrae variables and cepheids up to a distance modulus of 26.0 and 29.8 , respectively. However, a good cepheid
modulus for the Virgo cluster may exceed the possibilities of even the Space Telescope. The absolute magnitude of RR Lyrae variables and the p-L-C relation of cepheids could be improved and hence the uncertainty in the secondary indicators could be reduced. The Space Telescope then allows each method to be applied over a greater range of distances and the links between them to be made stronger. Because of the difficulties and problems discussed for the $H$ II regions and brightest star methods, the valuable observing time on the Space Telescope should be spent on standard candles with the smaller cosmic dispersion.

In the third chapter the Sandage-Tammann and G. de Vaucouleurs distance scales have been compared. The main cause of the discrepancy between these authors is the use of H II regions by Sandage-Tammann to bridge the gap between the Local group and Virgo galaxies. Systematic errors in the diameters of $H$ II regions could change the derived value of $H_{0}$ by 40\%. However, the difference in the Local group distances could have an effect of $20 \%$ on the result. Using the isophotal diameters of . H II region and accepting Sandage-Tamman's distances to the local calibrators increases their estimate of $\mathrm{H}_{\mathrm{o}}$ to $84 \mathrm{~km} \mathrm{sec}^{-1} \mathrm{Mpc}^{-1}$.

A considerable amount of interest has recently been generated by the observation of Local perturbation in the expansion of the Universe.
G. de Vaucouleurs VII (1979c) found a peculiar velocity of $350 \pm 50 \mathrm{~km} \mathrm{sec}{ }^{-1}$ for the Local group inside the Local supercluster. He does not interpret his result as a simple motion of the Local group towards Virgo but considers a complicated rotating and expanding supercluster model. Using the galaxies inside the Local supercluster he derived a value of $H_{0}=82 \mathrm{~km} \mathrm{sec}^{-1} \mathrm{Mpc}^{-1}$. The value of the Hubble constant, obtained from the galaxies outside the Local supercluster is equal to $100 \mathrm{~km} \mathrm{sec}{ }^{-1} \mathrm{Mpc}^{-1}$ (the Cosmic value). The difference is assumed to be due to the high density contrast inside the LSC which affects the local Hubble flow.

Other independent measurements have led to similar but larger peculiar velocities. The weighted mean of recent measurements of dipole anisotropy in the microwave background radiation (White (1979) ) predicts a peculiar motion of $440 \pm 50 \mathrm{~km} \mathrm{sec}{ }^{-1}$ for Local group galaxies towards Virgo. The recent $21^{\mathrm{Cm}}$-infrared magnitude studies of Aaronson et al (1980b) indicate a peculiar velocity component of $480 \pm 75 \mathrm{~km} \mathrm{sec}^{-1}$ in the Virgo direction, which is in close agreement with that from the microwave background experiments. At present, there is no satisfactory way to reconcile the results from microwave background measurements with those found by Sandage and Tammann (1975a), who demonstrated a linear Expansion flow inside and outside the LSC. However, we note that this result is based on ST's distances to a sample of nearby SC and distant SCI galaxies, and in Chapter 3 we questioned the validity of Sandage and Tammann's distance scale.

Taking a value for the peculiar motion from the microwave background anisotropy $\left(v_{p}=440 \mathrm{~km} \mathrm{sec}^{-1}\right.$ ) and correcting the observed virgo redshift for this motion, even if the Sandage and Tammann IV distance to Virgo is adopted $((m-M)=31.45)$ the derived value of $H_{o}$ is $80 \mathrm{~km} \mathrm{sec}^{-1} \mathrm{mpc}^{-1}$ (using the Virgo redshift alone). In fact, the value of the Hubble constant will be higher if either Sandage and Tammann's distance scale is wrong (Chapter 3) or there exists a peculiar motion of Local group towards the Virgo.

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