Higher dimensional theories in physics, following the Kaluza model of unification

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ABSTRACT

This thesis traces the origins and evolution of higher dimensional models in physics, with particular reference to the five-dimensional Kaluza-Klein unification. It includes the motivation needed, and the increasing status and significance of the multidimensional description of reality for the 1990's. The differing conceptualisations are analysed, from the mathematical, via Kasner's embedding dimensions and Schrödinger's waves, to the high status of Kaluza-Klein dimensions in physics today. This includes the use of models, and the metaphysical interpretations needed to translate the mathematics.

The main area of original research is the unpublished manuscripts and letters of Theodor Kaluza, some Einstein letters, further memoirs from his son Theodor Kaluza Junior and from some of his original students. Unpublished material from Helsinki concerns the Finnish physicist Nordström, the real originator of the idea that 'forces' in 4-dimensional spacetime might arise from gravity in higher dimensions. The work of the Swedish physicist Oskar Klein and the reactions of de Broglie and Einstein initiated the Kaluza-Klein connection which is traced through fifty years of neglect to its re-entry into mainstream physics.

The cosmological significance and conceptualisation through analogue models is charted by personal correspondence with key scientists across a range of theoretical physics, involving the use of aesthetic criteria where there is no direct physical verification. Qualitative models implicitly indicating multidimensions are identified in the paradoxes and enigmas of existing physics, in Quantum Mechanics and the singularities in General Relativity.

The Kaluza-Klein philosophy brings this wide range of models together in the late 1980's via supergravity, superstrings and supermanifolds. This new multidimensional paradigm wave is seen to produce a coherent and consistent metaphysics, a new perspective on reality. It may also have immense potential significance for philosophy and theology. The thesis concludes with the reality question, "Are we a four-dimensional projection of a deeper reality of many, even infinite, dimensions?".

HIGHER DIMENSIONAL THEORIES IN PHYSICS, FOLLOWING THE KALUZA MODEL OF UNIFICATION

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Preface

The research work has been carried out between January 1983 and December 1987 in the Department of Mathematical Sciences under the supervision of Professor Euan Squires, and in the Department of Philosophy under the joint supervision of Dr.David Knight.

I should like to express my deep gratitude to my Supervisors, Professor Squires and Dr.Knight for their advice, guidance and constant encouragement over the past five years.

Introduction and a Discussion of Models and Metaphysics

A. Introduction

This is an investigation into some aspects of models of space and time in twentieth century physics. In particular, it will trace the history of the development of models of more than three space dimensions. Detailed attention will be paid to the Kaluza-Klein model in five dimensions, from its origins to its current generalisation and widespread use in theories of Supergravity and Superstrings. Reference will be made to other attempts to describe reality, either with multidimensions, e.g. by Penrose, or with qualitative models containing implied extra dimensions e.g. by Wheeler. A wider objective will involve evidence of transcendence in contemporary physics, as indicated by a paradigm change to a multidimensional reality.

The practical aim is to give an account of how and why physicists have used ideas of more than four dimensions, with particular reference to Theodor Kaluza (1885-1954). To understand the physics, the motivation and where the idea came from, will lead to the questions of what "dimensions" mean, and what are their significance and physical status.

The historical development of our concept of space must have its origins in the Copernican revolution. The pre-Copernican mediaeval "sandwich" universe, Heaven: Earth: Hell, still lingers in literature. However the de-centralisation of the earth may have been the first radical change since the Greeks, an overturning of the apparent commonsense idea that the sun revolves round the earth. Chapter 1 will trace the ideas of space and time from Euclid to 1900. Newtonian absolute space in physics was the counterpart to Euclidean space in mathematics - and may well represent present concepts of space and time in everyday use.

1. Setting the scene for paradigm change, from prevailing ideas of space and time

Before 1900, Newton's gravity, classical mechanics and the nineteenth century wave theory of light were three accepted theories of nature. By 1900, some of the problems had become clear. The orbit of Mercury was not in agreement with Newton's predictions. The Michelson-Morley experiment produced results which disagreed with classical mechanics, which expected light waves to vibrate in an aether. Light did not behave the way it should on the prevalent aether theory. Photons of light were explained by discrete Planck's quanta - packets of light energy which could not be explained on the existing wave theory.

Chapter 1 examines the new concepts of space and time which provided the basis for Einstein's Special Relativity in 1905, which explained the Michelson-Morley result using a four dimensional space time continuum. Why we seem to live in an apparently four dimensional world is a critical question to be answered. This involves a look at the inadequacies of our present concepts and the motivations for introducing more than four spacetime dimensions.

Concepts of space and time still held today may have stopped at this point. In Chapter 2 after looking at the origins of multidimensiona space in mathematics, we examine the second stage of the revolution in thought provided by physics in the first quarter of the twentieth century. Einstein's General Relativity provided an explanation for the orbit of the planet Mercury and was able to predict successfully the bending of light from behind a solar eclipse. Although still part of classical physics, the curvature of the four dimensional spacetime indicated the need for extra embedding dimensions. The final phase of this first revolution was Quantum Mechanics, which led to Quantum electro-dynamics. In giving extremely accurate descriptions, quantum mechanics has wide applications, although it involves the mathematical trick of renormalising infinities (see Chapter 4).

These three aspects of the early twentieth century revolution provided answers to problems in the existing Newtonian physics - but at a price. The new ways of thinking viewed nature in a very new and different way. Commonsense and intuition were no longer applicable, and the new concepts have not really entered our thinking. We shall look at what the models actually say, and their implications. In General Relativity, high curvature at very high energies produces 'singularities', where our present concepts of space and time break down in the Big Bang or in Black Holes. Quantum mechanics involves the Uncertainty Principle and a wave/particle duality. Reality is described by a multidimensional Schrödinger wave, and may indeed be created by the observer. Thus the first revolution itself throws up enigmas which themselves imply the need for a new physics, a further paradigm change.

2. The need for a new physics - the Second Revolution of the Twentieth Century:a multi-dimensional reality

The new concepts of General Relativity are very useful on a large scale, where Newton's partial laws are inadequate. However Relativity does lead to enigmas and paradoxes in classical physics, via the curvature of four dimensional spacetime to Singularities. The new ideas of Quantum Mechanics produced the final breakdown of classical ideas, leading to further paradoxes. Although mathematically correct, the interpretations, the 'metaphysics' were uncertain, and led to controversies.

Thus after the paradoxes and dilemmas in the existing twentieth century physics of General Relativity and Quantum Mechanics, there has been a search for a deeper unity. One of the ways forward has been that of increasing the dimensionality of spacetime. This need for models of a deeper kind beyond 3-space has led to attempts to know the deeper almost 'transcendent' reality beyond mere appearance. The answer from contemporary physics seems to involve many dimensions, ten, twenty-six or even an infinite number. The origins of this new paradigm lie with Theodor Kaluza's original paper of 1921. Reference is also made to a little known, apparently unsuccessful attempt at unification using five dimensions by Gunnar Nordström, 1914. We must explore in Chapter Three why the critical input was ignored for so long and why the beginnings of the new revolution seemed to pass without comment, and yet it is crucial to today's concepts of unification in physics. A resurgence of interest took place in 1926, following Oskar Klein's paper. Although Klein attempted to include quantum theory in his analysis, the interest proved to be only temporary (see Chapter 4).

The main questions to be answered are why Einstein delayed the publication of Kaluza's paper for two years, why Kaluza remained unrecognised for so long, and why there was such a history of neglect over the next forty to fifty years. In Chapter 5 we shall look at Einstein's own contribution over a number of years and in Chapter 6 at others who kept the idea alive between 1926/7 until the prophetic insights of Souriau in 1958 and 1963.

The final questions involve why the Kaluza-Klein idea came to be so useful, what tools or concepts were necessary e.g. in Chapter 6, and why it has become so essential in the 1970's and

1980's (Chapters 7 and 8). The full unification must involve all four forces, involving gauge theories as well as gravity. The link with gauge theories, supergravity and strings may have been the final catalyst on the route to Supergravity and superstrings.

3. Models and Metaphysics I - introduction

Concepts of embedding dimensions (Kasner,1921) and compacted dimensions (Kaluza 1921) are extremely difficult, if not impossible, to visualise directly. If concepts are unimaginable (except in mathematical language) they are easily rejected. Questions of the correct dimensionality, the correct topology for spacetime, the problem of the intrinsic and extrinsic view points (e.g. standing outside the surface or space)need techniques for describing the answers. We need a language to talk about extra dimensions. Our view point is inside our space, intrinsic to three dimensions. This produces a conceptualisation problem, and the need to use models.

The language of mathematics is the basic underlying foundation to all ideas and concepts in physics. It has been realised for some time that metaphysical ideas are as important as mathematics in science (e.g. J.W.N.Watkins, "Metaphysics and the Advancement of Science", 1975, p.91). Very new concepts in science are often treated as hypothetical. Berzelius' atoms of the nineteenth century and Gell-Mann's quarks in the 1960's were initially only mathematical not physically there. The next stage was to treat atoms, molecules and quarks as real physical entities. This question of physical status becomes even more challenging when dealing with current models of strings and superstrings.

Mathematical or theoretical models can provide a geometric picture where the entity described cannot be pictured. However even geometric pictures may be ambiguous in describing aspects of reality beyond the four dimensions of spacetime, where we need models of a transcendent reality.

There are clearly two parts of any description in theoretical physics. Each theory or equation consists of:

(A) The Mathematical Formalism

and (B) The Metaphysical Interpretation

The metaphysical interpretation requires a language to describe the mathematics, and physicists may differ as to the metaphysics of the given mathematics. The interpetation, the ontological description of reality, requires metaphors, models and even, on a larger scale, paradigms (Kuhn, 1962). Michael Polanyi emphasised the different levels of reality. For him, the predominant principle that has guided modern theory has been "the transition from a mechanical conception of reality to a mathematical conception of it" (Polanyi, 1967, p.177).

However we still need a true metaphysical foundation for science. Translations of the mathematics are still needed. It is possible for the metaphysics to try to keep strictly to the mathematics. This may involve often unacknowledged assumptions about the limits of reality, and may often baulk at interpreting transcendent ideas such as extra dimensions beyond four dimensional spacetime. Thomas Kuhn introduced the idea of interlocking theories being stabilised in a paradigm which resisted change (Kuhn, 1962, (The Structure of Scientific Revolutions). A scientific revolution involves the rejection of the current paradigm and the need for a new physics to produce a new paradigm (ibid.,p.156).

In the course of following the increasing acceptance of the Kaluza-Klein extra dimensions, we shall look for evidence of any major paradigm change from the traditional four-dimensions of spacetime (see further, Chapter 9).

We shall also need to look more closely at the nature of models used by physicists to describe reality. There is now uncertainty about the terms used by philosophers of science, and writings of physicists themselves are very important. The heuristic importance of models and analogies seems to be universally recognised. Modern physics gives strong indications against literalism rather than any absolute rejection of models. Symbolic representations of aspects of reality which cannot be consistently visualised, are necessary. Such analogies, or 'analogue models' are in terms of analogies with everyday experience, and only indirectly related to observable phenomena. It must not be forgotten that the only invariants are the mathematical expressions. Yet a metaphysical interpretation is essential. Models, like metaphysics, are meant to communicate, not to be a private language. Yet we <u>need</u> models, particularly analogue models to describe the transcendent many dimensional concepts of reality in contemporary physics. It is too easy to reject concepts which are not directly visualisable and have to remain fixed in existing ideas of "reality" (ref Black, 1962; Hesse, 1963 etc.)

Reality may indeed be best described by mathematical models, but technical discussions cannot do without metaphysical language e.g. analogue models. The danger is that we may "forget the origin of our metaphors and try to make them do a job they cannot do" (Hutton, 1956, p.84).

4. Methods of Approach

Three space and one time dimension may not be right at a deeper level. There is a growing feeling in the 1980's that reality *is* higher or multi-dimensional. The case of models thus leads to the reality question - perhaps also to the question of the consequences of taking our models seriously in a reappraisal of the world picture where a consensus in physics leads to a reality only described by many dimensions. We become involved in the ontological problem of what reality is, and the epistemological problem of how we investigate and describe reality. These are the underlying but subsidiary questions for this thesis.

The immediate questions to be answered in this thesis are more direct:-(a) Why does physics seem to be in 3+1 dimensions? (b) What are the paradoxes and enigmas of the existing revolutions of General Relativity and Quantum mechanics which lead to a need for a new physics, and (c) Why does physics today <u>need</u> extra dimensions beyond 3+1?

My approach to answering the questions posed will be via the original documents, to look at the origins of the 5-Dimensional Kaluza-Klein idea, and also at the way contemporary physicists use the model in the 1980's. The Kasner original papers on embedding dimension will also be examined.

I will refer to Theodor Kaluza's original paper, to letters from Einstein to Kaluza (in the possession of the son, Theodor Kaluza, Junior) and to letters from Kaluza to Einstein("The Collected Papers of Albert Einstein", Boston University). Biographical details of Theodor Kaluza have been obtained from Th. Kaluza,Junior (personal correspondence and visits to his home , Hannover) and from some of his ex-students. Reference is made to many further publications in the literature, e.g. by Oskar Klein, together with the reaction of other physicists at the time. Papers, unpublished letters and correspondence, where unacknowledged, are translated by C.H. Middleton from the German.

The earlier attempt by Gunnar Nordström is obtained from his original papers and his unpublished letters and correspondence (The University of Helsingfors Archives). These are translated from the Swedish by Mrs. D.Jowsey. Correspondence from de Broglie is translated by Mrs.A.M.Glanville.

The "Wilderness years" involve published literature in German, and increasingly in English in the post-war years. The re-entry of the Kaluza-Klein idea needs many references to papers published in the standard journals. The reasons for the wide acceptance today, the physical status for the extra dimensions, and a language for understanding the ideas, have involved personal correspondence with key scientists.

I should therefore like to thank Professor Dr.Theodor Kaluza (Junior) for all his help e.g. letters from Einstein to Kaluza, the Hebrew University in Israel for permission to use the Kaluza to Einstein letters via John Stachel of Boston University, and the Helsinki University to use Nordström's correspondence. I should also like to pay tribute to my indefatigable translators, Chris Middleton and Dagné Jowsey, and to personal contributors to the history of Kaluza's idea such as Schmuel Sambursky (pupil of Kaluza), Peter Bergmann (colleague of Einstein) and to Corporal B.H.Wheyman (British army flash spotter sharing the experiences of a gun spotter, on the 'other side' to Kaluza in 1917).

May I also pay tribute to a number of scientists currently involved with Kaluza-Klein methods who have so kindly written to me about their motivations for using the idea,the physical status which they give to the extra dimensions, and a possible language for communicating such ideas. In particular I should like to thank

Alan Chodos, Steven Detweiler, Michael Duff, Peter Freund, Michael Green, Steven Hawking, William Marciano, Roger Penrose, Chris Pope, John Schwarz, and other correspondents e.g. Louis de Broglie, David Bohm for their letters.

2. Models and Metaphysics

There has been an increasing need in the last ten to fifteen years for physicists to use solutions involving multidimensions. With the emphasis on Supergravity and Superstrings in particular, the physical status of these extra dimensions has become more obvious. What began as a purely theoretical mathematical idea has developed into a description of physical reality - the extra dimensions are really there. This has produced a problem of the use of language and the need to translate mathematical symbols representing different levels of physical reality.

Where we need to talk about a deeper reality than four spacetime dimensions, we must watch where this involves a language shift, in describing what are no longer the visualisable and historical concepts of nineteenth century physics. Quarks, singularities and strings were once only mathematical concepts. With their increasing status as actually describing physical reality there is a need to examine our use of models.

There is a need for models and a need to look at the way we use models to describe reality. These may often be an incomplete and partial description, an interpretation of mathematical language, perhaps even "adequate", rather than "true" (Schrödinger, 1951,p.22).

Where the models are successful, they begin to prompt the reality question, the 'best candidates for reality' (Harré, 1972,p.93). We need to consider not only models, but also metaphysics. The real question behind this thesis on the development of the Kaluza-Klein five dimensional idea may well be "what is reality ?" The deeper reality beyond 4 dimensional spacetime may involve models of the transcendent reality described by contemporary physics. If we are indeed three dimensional slices or projections of a multidimensional reality, then the hermeneutics of contemporary theology may also be involved at a later stage.

1. Metaphysical problems - the deeper questions

There are three main metaphysical questions which should be asked.

(i) The ontological questions - what there is?, what really exists? - what is reality?

(ii) Epistemological questions - whether we can know? - what can be known and how we can know?

(iii) Axiological questions - what is worthwhile? what has value? what should be done? (for further details, see Open University A.381).

The ontological problem of 'being' involves the status of physical reality of the various descriptions used in physics. The nature of reality seems to be deeper than the traditional three space dimensions and one time dimension which have normally been accepted as the whole of reality. The limitation to any further investigations of reality beyond four dimensions of spacetime has often been an unconscious assumption. Yet though unrecognised it is in itself a metaphysical decision which has produced the positivist philosophy of the earlier part of the century.

The second question, of Epistemology, is the practical question to which this thesis is addressed - the ways of knowing. As the nature of reality is being examined at very high energies (e.g. at the Big Bang) or at very small distances (e.g. the Planck length of 10^{-33}cm) the results are increasingly beyond the reach of experimental verification. The criteria are no longer by direct testing, but the testing of second order predictions, e.g. the cosmological implications of a unified theory as a description of reality. Increasingly, the plausibility of new theories is judged initially by aesthetic criteria - of elegance, symmetry, simplicity and beauty.

The Axiological question is one which we must leave unanswered at the end of this thesis. The implications of taking our models seriously and the value judgements involved, may be the most important questions of all. A full metaphysical enquiry should not, however, neglect the implications for us.

2. The nature of reality

We will be concerned throughout this thesis with the interpretation of the purely theoretical physics. There are two parts to every theory:

(A) The theoretical Formalism (often Mathematical) - (B) The Metaphysical interpretation.

It is often assumed that only the mathematical formalism is correct. Yet the interpretation of the mathematics itself is essential, even if physicists themselves differ in the descriptions used, the language of ontology and epistemology. The ways of knowing involve both mathematics and models, metaphors, ways of talking about concepts which may be non-visualisable in themselves, such as dimensions beyond spacetime's traditional four.

3. The need for models, their classification and their status

Until the twentieth century, most scientists assumed that scientific theories were exact descriptions of the world. This 'naïve realism' (Barbour, 1974,p.34) corresponded to a literalistic interpretation of models. The most famous exponent was William Thomson, who gave his version in the Baltimore lectures:

> "I never satisfy myself until I can make a mechanical model of a thing. If I can make a mechanical model I can understand it." (Thomson, 1904,p.187)

This view of models led to the dismissal of models as intermediaries between theories and observations, for example in the positivist philosophy. Instrumentalists would be more concerned with the usefulness of theories, rathern than their truthfulness in representing reality. Ian Barbour follows the most helpful view, taking theories to be 'representatives of the world' but recognising the importance of creative imagination in the use of models. This 'critical realism' is the most frequent description of the way scientists use models today. "Models are limited and inadequate ways of imagining what is not observable" (Barbour,1974, p.38).

It is important to describe "the way the term model is actually used by physicists"(Redhead, 1980, p.145). This may "avoid forcing science into a preconceived scheme, as philosophers have so often done". (Hutten, 1956, p.81).

For pragmatic scientists at the sharp end, a model is used

> "to restate a complex problem in some simpler terms,
> to highlight key factors, and to display the linkages
> which exist between the parts" (I.C.I.,(D.Brown), 1972).

Although the model is acknowledged as the major technique in analytical problem solving, in practice there is no rigid model making procedure. "Models should be devised to meet the needs of the problem and in accordance with the temperament of the user" (ibid.,p.1). Nevertheless models are classified as (i)'Pictorial',a two-dimensional representation to show a particular characteristic of reality e.g. spatial, mechanical or activity relationships;

(ii) Physical models, e.g. of plant, aircraft;

(iii) Numerical models, e.g. equations, formulae or graphs;

(iv) Descriptive models, e.g. word models or a logic tree

For Einstein, even quantum mechanics, with its complete mathematical correspondence to physical observation, does not "provide a complete description of the physical reality" (Einstein, Podolsky and Rosen, 1935, p777). Bohr agreed in emphasising "how far, in quantum theory, we are beyond the reach of pictorial visualisation", while believing that the apparent inconsistencies could be resolved from the point of view of complementarity. (Bohr, "Discussion with Einstein", 1949, p.59). In 1935 Bohr himself called for "a radical revision of attitude towards the problems of physical reality" (Bohr, 1935, p.696). Both physicists criticised one another's opposing view points for their underlying ambiguity when applied to actual problems - which for Bohr, included

> "the outstanding simplicity of the generalisation of classical physical theories, which are obtained by the use of multidimensional geometry and non-commutative algebra, respectively, rests in both cases essentially on the introduction of the conventional symbol $\sqrt{-1}$".

Physicists were concerned about these problems of non-concrete mathematical models. Max Planck was compelled

> "to assume the existence of another world of reality behind the world of the senses; a world which has existence independent of man, and which can only be perceived indirectly through the medium of the world of the senses, and by means of certain symbols which our senses allow us to appreciate" (Planck, 1931, p.8).

He recommended that

> "our view of the world must be purged progressively of

The job of a model is thus to condense by displaying the essentials in an acceptable language, so that the problem can be "confronted, manipulated, modified or communicated more effectively" (ibid.,p.2).

However, for scientists dealing in the deeper paradoxes of contemporary physics, the real problem is how to imagine things we have never, or may never, experience directly,such as extra dimensions of either the Kaluza-Klein model or the Kasner embedding model.

As Sir Arthur Eddington wrote in his Gifford Lectures of 1927, in Bohr's semi-classical model of the hydrogen atom there is an electron describing a circular or elliptic orbit:

> "this is only a model, the real atom contains nothing of the sort .... The real atom contains something which it has not entered into the mind of man to conceive, which has, however, been described symbolically by Schrödinger .... The electron, as it leaves the atom, crystallises out of Schrödinger's (multidimensional) mist like a genie emerging from his bottle" (Eddington, **1935**,pp.196,197).

For Eddington, metaphor was the alternative to the symbolic world of mathematics for describing reality (ibid,p.207). The physicist regarded his own external world

> "in a way which I can only describe as more mystical, though no less exact and practical, than that which prevailed some years ago, when it was taken for granted that nothing could be true unless an engineer could make a model of it." (ibid.,p.330).

Although in common usage, "concrete and real are almost synonymous", the scientific world "often shocks us by its <u>appearance</u> of reality." (ibid.,p.265)

> all anthropomorphic elements" as "the structure of the physical world view moves further and further away from the world of the senses, and correspondingly approaches the real world (which, as we saw, cannot be appreciated at all)". (ibid,p.49).

Max Born refers to "the mysterious equation" of Heisenberg's ideas on quantum uncertainty which produces such diverse interpretations as the models of both wave and particle (Born, "Physics and Metaphysics", 1950, p27). Born continues to emphasise that a scientist 'must be a realist, he must accept his sense impressions", despite using ideas "of a very abstract kind, group theory in spaces of many or even infinitely many dimensions", (ibid,p.26). He recommended the wholeness of Bohr's "Complementarity model", where even in restricted fields, "a description of the whole of a system in one picture is impossible" (Born, 1950, p.27).

Einstein agreed with the difficulty of providing a model, a metaphysical description of "ψ", the wave function in quantum mechanics as 'the complete description' of the individual system, which is "very complex", and where "its configuration space is of very high dimension" (Einstein, "My Attitude to Quantum Theory", 1950,p.32). Only an ensemble description, a statistical interpretation or model, would do for Einstein, where "there is no such thing as a complete description of the individual system" (ibid.,p.34). Schrödinger himself, the author of the complex multidimensional equation describing reality, wrote a chapter on "The Nature of our Models" in his "<u>Science and Humanism : Physics in our Time</u>" (Schrödinger, 1951). He admitted that in thinking about an atom, etc., geometrical pictures are very often drawn ("more often just only in our mind") where the details of the picture are

> "given by a mathematical formula with much greater precision

> and in much handier fashion than pencil or pen could ever give." (Schrödinger , 1951, p.22).

Nevertheless he warned that geometrical shapes are not observable in real atoms.

> "The pictures are only a mental help, a tool of thought, an intermediary means for deducing reasonable expectations about new experiments to be planned."

This is to see "whether the pictures or models we use are adequate" - adequate, rather than true.

> "For in order that a description be capable of being true, it must be capable of being compared directly with actual facts. That is usually not the case with our models." (ibid.,p.22)

Analogue Models

Thus we have come to the central problem in twentieth century physics, and which the ICI range of models did not see. Either we speak in purely mathematical language, or we must argue from analogy, using models and metaphors from what we do know, to describe the indescribable. Otherwise there is a real danger of rejecting whole concepts if we are unable to visualise them directly. We may need to use new models, to change out-of-date models. Because models can never tell the whole truth, we may need several different models - "Analogue Models".

4. Classification of Models

Despite the firm views on models by scientists such as Bohr, Einstein, Schrödinger, etc., it was left to philosophers of science to attempt a classification. Despite Hutten's own caveat, he was one of the first to classify models in the 1950's, following scientists as closely as possible. The term 'model' was first used in science in the nineteenth century, having been used since the seventeenth century to denote what we refer to as an architectural blue-print (Hutten, The Language of Modern Physics, 1956, p.82). Apart from its heuristic or pragmatic use, Hutten emphasised that the model had a logical function which was indispensable, in the interpretation of a theory in simpler terms. "Models thus resemble metaphors in ordinary language" but they are often too simple and "we forget their limitations" (ibid., p.84). Hutten was careful in advocating the metaphysical use of models as a

> " simple and simplified situation used as a standard of comparison for other more complex situations", and "as a basis for building up a technical language".

It could therefore be used to provide both syntactic rules for an equation and as an interpretation for the equation. When words fail us, we have recourse to analogy and metaphor" (ibid., p.201). In suggesting that the model functions as a more general kind of metaphor, Hutten insisted that there were no mathematical models in physics. "The equation by itself is not the model, but the interpretation of the equation is." (ibid., p.289).

Philosophers such as Stephen Toulmin criticised the frequent introduction of models without classifying them. Certainly the use of language had to be analysed, particularly where metaphors were involved (Toulmin, 1953). Mary Hesse was one of the most persistent philosophers in attempting a classification, like Hutten emphasising the predictive open ended qualities of a good model, and suggesting the use of analogy. However from her article "Models in Physics" (Hesse, 1953), she varied in her use of analogy and analogue model. By 1963, she settled on Model_1, the actual representation in perfect correspondence with the theory, and Model_2, other natural processes from which the analogy is first drawn.

An interesting colloquium took place in 1960 on "The Concept and the rôle of the model in mathematics and natural and social sciences" (Ed.Freudenthal, 1961). Leo Apostell identified the relation between a model and its prototype as "a relation between two languages" (ibid.,p28). Groenewold enumerated the representational model, the substitute model (varying from the pictorial to the more abstract) the study model and the picture model, noting the shift to increasingly abstract models, so that the particle and wave pictures for example are inadequate approximations: "the explanatory function of models is becoming obsolete in present day physics" (ibid.,p.123). Others also referred to the increasingly abstract model and the need for the mathematical formulism.

R.Harré identified the scale model (a 'micromorph') on the analogy of Hesse which he called the 'paramorph': "the analogy is the simplest form of conceptual paramorph' (Harré, 1960,p82). E.Nagel outlined careful "rules of correspondence" in order to define a model classifying analogies into "substantive" (parallels between one system and another) and "formal" (more exact replica) (Nagel, 1961, p.97). Like Hesse, he emphasised the heur istic values of models but warned that "the model may be confused with the theory itself" (ibid.,p.114). Nagel also pointed to the danger of adapting familiar language to new cases without being aware of the historical perspective on the meaning of the words. This was ironic in that the very problem confusing a classification of models was that each philospher of science was dissatisfied with previous attempts, and invented his or her own words, announcing their new and exact meanings.

Max Black in 1962 took a wider view of the meaning of a model, proceeding from the construction of miniatures to the making of scale models in a more generalised way; then from 'analogous models' and 'mathematical models' up to 'Theoretical models' with an "imaginary but feasible structure". (Black, 1962, pp219,239). In what became a classical account of models, Black went one step further and considered cases where there is an implicit or submerged model not immediately obvious. These roots or "archetypes" were very useful in analysing thought forms.

> "Perhaps every science must start with metaphor and end with algebra; and perhaps without the metaphor there would never have been any algebra" (ibid.,p.242). The danger for Black was that the archetype "would be used metaphysically, so that its consequences will be permanently insulated from empirical proof" and it could become a"self-certifying myth" (ibid.,p.242). Black's own perceptive use of metaphor is seen in his sentence:
>
> > "A memorable metaphor has the power to bring the separate domains into competitive and emotional relation by using language directly appropriate to one as a lens for seeing the other" (ibid.,p.242)

This proved to be an important link between model and metaphor.

P.Achinstein argued a cogent case for his categories of 'model'. "Theoretical models" were Achinstein's key category, becoming "Models" for short (e.g. the Bohr model, the billiard ball model of gases etc.) in physics, biology, psychology and economics. He described four categories of theoretical models, including the basis of an analogy. Achinstein rightly criticised Nagel (in Structures of Science, 1961) for using 'model' and 'analogy' interchangeably, confusing model and theory like so many other philosophers. Achinstein himself appears to follow Hesse's two uses which he describes as 'theoretical model' (Model_1) and 'analogy' (Model_2) (Achinstein,1969).

Philosophers such as R.B.Braithwaite were wary of extending any features of a model. "Analogy can provide no more than suggestions of how the theory may be extended" (Braithwaite, 1970, p.268).

He argued against any evidence of the greater predictive power of the model over the theory itself, citing the danger of dead-ends etc. Achinstein, on the other hand, became an accepted proponent of two quite different concepts, (a) 'Models' or 'theoretical models' and (b) Analogies. Others tried to separate these out further into e.g. (i) Positivist formal models, (ii)Achinstein's theoretical models, (iii) Achinstein's representational models and (iv) physical analogies. (Girill, 1972, p.241 in his "Analogies and Models Revisited"). For Achinstein, only two types were acceptable, and he would probably have equated (i) with (ii), and (iii) with (iv).

This would seem to be the most accepted division. Achinstein confirmed N.R.Campbell's original ideas of 1920,

> "In order that a theory may be valuable it must have a second characteristic; it must display analogy. Analogies are not aids, but.... utterly essential part of theories." (Campbell, 1920,e.g.Ed.B.A.Brody 1970,p.251).

The danger of successive, individually interpreted definitions is that philosphers seldom refute one another but invent their own definitions.

5. Recent attempts at Classification of Models

For philosophers of science such as Michael Redhead, "science is the art of modelling" (Redhead, 1980, p.162) in the extended sense of models, emphasising the heuristic role of models. Thus Redhead in his "Models in Physics" follows Achinstein's 'Theoretical models', subsuming the "Analogue models" (Black, 1962; Hesse's Model$_2$, 1963). This division is also emphasised by R.Harré, who rather unnecessarily introduces the word 'Iconic' models in science, dividing them into Homeomorphs (scale models) and Paramorphs (analogue models) (Harré 1972, p.174). These have not passed into the literature, although Harré's analysis is excellent: "successful use of an iconic model begins to prompt 'reality' questions", such as the "real causal mechanism". (ibid.,p.182)

Ian Barbour also emphasised the Theoretical model, and included the Analogue model, with both positive and negative analogies, contributing to the extension of theories. His finer division of Mathematical models as intermediaries between Experimental models and Logical models (Barbour, 1974.,p.29) has however not been generally accepted.

Sir Rudolf Peierls has been one of the few well-known scientists to write in this area. In his "Model-Making in Physics" (Peierls, 1980.,p.3) Peierls writes independently of the accepted vocabulary itemising Type 1: Hypothesis ('Could be true'); 2: Phenomenological model ("Behaves as if.."p.5"); 3: Approximation ("Something is very small, or very long,"p.7); 4: Simplification ("Omit some features for clarity" p.9); 5: Instructive model ("No quantitative justification, but gives insight", p.13); 6: Analogy ("Only some features in common", p.14), and 7: Gedanken experiments ("Mainly to disprove a possibility", p.16). For Peierls, Type 2 are only metaphors, and Type 3 only roughly mathematical. He pointed out the dangers or pitfalls in working with analogies of Types 4, 5 and 6. This was an interesting analysis by a practising scientist using recent examples, rather than nineteenth and early twentieth century models.

Further work on models has been left to philosophers such as Sneed (1971) and Stegmüller who have turned further inwards by using a private language system (e.g. Stegmüller's The Structure and Dynamics of Theories , 1976), for example, following Kuhn, "a new metascientific reconstruction" (ibid.,p.iii).

The dichotomy today is that scientists themselves are increasingly using computerised language in practical analyses of their results. Because of the availability of a wide range of mathematical techniques and of computers to do the 'number crunching',

> "it is often very tempting to model only those aspects of a complex problem which are quantifiable or to reduce complex problems to a quantifiable form". (Hughes and Tait, 1984 "The Hard Systems Approach : System Models" in O.U.Technology T301, 8, p.17).

John Hughes and Joyce Tait warn against concentrating on mathematical aspects of modelling and against losing sight of unquantifiable objectives and constraints.

6. Conclusion of Models

In order to look more closely at the theories of Einstein, Schrödinger and Bohr, or Kaluza, Klein and Kasner, as well as 10-dimensional supergravity and superstrings, it is necessary to look at how we use our description of reality. Extra dimensions and strings may be our best description of a deeper reality beyond 3-space. The images suggested must be used with care.

The basic model in twentieth century physics is undoubtedly the mathematical model or equation. Each symbol corresponds to a different concept, and it is the interpretation of the equation, in terms of theoretical or analogue models, which is essential. This metaphysical interpretation may be open to different opinions, but it cannot be bypassed (as Bohr attempted to do in the 'Copenhagen' interpretation of Quantum Mechanics in 1926).

A model is an image, a description of reality, which is not the same as the thing it models, but may often argue from analogy.

Indeed, there may be no sharp dividing line between our classification of models (Osborne and Gilbert, 1980, p.60). Whether we use a liquid drop model of a nucleus or a string model for quarks, drops and strings may be scale models and analogue models as well as mathematical models. We must certainly watch the boundaries between model and reality, as models point to analogies between the known and the unknown (or imperfectly known).

Reality today : a paradigm change

In the 1980's we must accept that the understanding of the solutions of supergravity, superstrings etc. are also metaphysical. Creative thinking is an essential factor, and any agreed metaphysic requires the convergence of several different models. The use of multidimensions, even infinite dimensions appears to be such a convergence, and seems to give the most adequate description of the actual structure of the world.

Although essentially beyond the range of direct experimental testing, this range of models describing solutions requiring more than the four dimensions of traditional spacetime reality, is becoming widely accepted. This would seem to suggest that the paradigm or description of reality is changing. The word 'paradigm' in this sense was introduced by Thomas Kuhn, at first in a somewhat vague sense. In the second edition of his book '<u>The Structure of Scientific Revolutions</u>' (Kuhn, 1970), he made a clear distinction between the 'normal' science of experiment, induction and inference, and the revolutionary nature of real scientific discovery and revolution. Here a group of scientists abandon one tradition, the old paradigm, in favour of another. Any new interpretation of nature is a "candidate for paradigm change". At the start "a new candidate for paradigm may have few supporters". As further experiments confirm the paradigm, "more scientists will then be converted". "Gradually, the number of experiments, instruments, articles, and books based upon the paradigm will multiply" (Kuhn, 1970, Ch12,e.g.p.158). At first the evidence for the revolutionary new hypothesis may be far smaller than for the previous well-confirmed earlier version which it seeks to replace. Acceptance may at first represent a commitment on the part of a scientist which cannot be justified by the normal science of induction and inference, and a leap of faith is almost required. Thus did Einstein's four spacetime dimensions and General Relativity replace Newton's physics. Quantum Mechanics similarly replaced nineteenth century ideas of the atom.

Today, the evidence would clearly suggest that the Kaluza-Klein model using five (or more) dimensions has paved the way for a new paradigm. Reality is multidimensional.

A multidimensional reality - problems of interpretation of the new paradigm

The tide of scientific opinion has led to a paradigm change. The paradigm wave of many dimensions is overturning previous models of reality, as deeper ontological levels are increasingly necessary to describe the world.

To interpret the language of mathematics, we need the metaphysical questions of the ontology of multidimensions and the epistemology of both mathematical and theoretical or analogue models. A single coherent description needs a large number of models converging, in conjunction with the formalism. 'Fibre bundles', 'strings' etc. of the 1980's have become more than convenient metaphors. Many dimensions are needed to describe the "ultimate metaphysical reality" as Michael Roberts described the world in "The Modern Mind" (Roberts, 1937, p.171). They are also given high status for describing reality rather than merely as mathematical tools.

The problem in emphasising this metaphysical description of reality is that these extra dimensions are often referred to in purely mathematical symbols or equations. There are no direct scale models, only analogue models. The difficulty is probably because our investigation is based on three-dimensional sensory perception, and it can fail "when physics exceeds the sphere of our natural perception..... Our ability to imagine space fails in the face of cosmic dimensions" (Lind: 'Models in Physics, 1980, p.19). Gunter Lind referred to the problem of imagining a bent space graphically - how much greater the problem with heterotic strings in 10 and 26 dimensions!

The implications of today's answers must not be obscured by the reassuring façade of the mathematical language of "10 and 26 Dimensions", or by the difficulty in visualising such concepts as multidimensions. The truth of the metaphysical description must be able to be presented in terms which are acceptable to scientific thought patterns of today.

CHAPTER 1: Present Concepts of space and time, from Euclid to Special Relativity, 1905 and the motivations for introducing extra dimensions

Synopsis

1. What is space?
    - (a) Euclidean 'flat' space
    - (b) Newtonian space
2. What is time?
    - (a) uniform flow
    - (b) space and time at the end of the nineteenth century
3. Space, time and Special Relativity
4. The dimensionality of space
    - (a) Our apparently three dimensional world (3-space, or four dimensional spacetime)
    - (b) Against 3-space and 1-time
5. A multi-dimensional reality?
    - (a) Different uses of "dimension"
    - (b) Theoretical or physical status?
6. Motivation for using extra dimensions
    - (a) Mathematical multidimensional space as a theoretical tool
        - (i) Hilbert, Minkowski and Riemann (Chapter 2)
        - (ii) Schrödinger's equation and Quantum Mechanics (Chapter 4)
    - (b) Embedding dimensions (Chapter 2), for large scale curvature
        - (i) Kasner's mathematical treatment, to interpret General Relativity
        - (ii) As an aid to visualisation e.g. of curved spacetime
    - (c) Unification of forces by increasing the dimensionality of spacetime - the Kaluza-Klein model

- (i) Kaluza's unification of gravity and electromagnetism (Chapter 3)
- (ii) Klein's attempt to include quantum dynamics, with increasing status, developed by de Broglie, and later Einstein and Bergmann (Chapters 4 and 5 onwards)
- (iii) Attempts to include the Kaluza-Klein idea in gauge theory (de Witt), and further progress by using supersymmetry to include the weak and strong forces (Cho and Freund).
- (iv) To link with dual models using string theory rather than point particles (Scherk and Schwarz)
- (v) The search for a fully unified theory of gravitation consistent with quantum mechanics via Superstrings (Green and Schwarz)
- (vi) The alternative theory of everything using Supergravity and Kaluza-Klein (Cremmer, Julia and Scherk)
- (vii) Application to cosmology and the Big bang (Chodos, Detweiler, Applequist)
- (viii) Increasing the physical status - from Kaluza and Klein to cosmology, spontaneous compactification and the geometric interpretation of quantum numbers (e.g. Cremmer and Scherk etc.)

d) Other (non-Kaluza-Klein) methods of changing the actual dimensionality of spacetime

- (i) Pregeometry of no particular dimensionality e.g. as foam space (Wheeler, Hawking)
- (ii) Podolanski's six-dimensions to solve quantum mechanics anomalies e.g. infinities
- (iii) Penrose's Twistor space to resolve enigmas such as infinities attached to point particles.

7. Conclusion

1. What is space? (a) Euclidean 'flat' space.

Greek geometry was almost entirely confined to the plane, with space as an extension of a flat two-dimensional surface. The science of solid geometry attracted much less attention. The idea of extra dimensions beyond three certainly did not occur in Greek science, although Ptolemy wrote a study on dimensions and proved that not more than three dimensions of space were permitted by nature. (O. Neugabauer, 1975 p.848). Plato commented on the ludicrous state of research in solid geometry, with particular reference to its use in astronomy (Plato, Republic, VII p529).

Plato, in his Timaeus, identified space with matter. Aristotle in his Physics was more concerned with position in space, where space and matter were therefore finite, the sum total of all places (Jammer, 1954, Ch.1.). These ideas of absolute space on the one hand, and a relational theory of space on the other, have been held in tension ever since.

As Reichenbach suggested (Ed. Smart, 1964, eg. p.219), our common sense is convinced that real space is in fact Euclidean space of three dimensions. Euclid's Elements, Book I, begins with the concepts which are the basis for much of our thinking (eg. Kline, 1972, p.58,81). Euclid's Definitions are still standard to our thinking:

(a) A point is that which has no part (Book I, Definition 1)

(b) A line is breadthless length (Book I, Definition 2)

- The word 'line' also means 'curve' (always finite in length)

(c) A surface is that which has length and breadth only (Book I, Definition 5)

(d) A solid is that which has length, breadth and depth (Book XI, Definition 1).

Definitions and deductions from Euclid's Elements imply a flat planar surface where angles of a triangle add up to 180°, and in particular the 5th postulate holds, that parallel lines never meet. Adding a third dimension at right angles to the flat planar surface gives the intuitive idealised space-'flat' or Euclidean space - of orthodox solid geometry.

(b) Newtonian space

Newtonian space is the counterpart in physics to Euclidean space in mathematics. This is central to the commonly held world picture of space even in post-Relativity times. Such a discussion takes us away from mathematics to more empirical science, and involves the properties of the physical world. Space needs a physical description not a mathematical one. (See Smart, 1964, Introduction).

Newton's space is homogeneous and isotropic. Such a homogeneous space is presumed to be 'flat', i.e. obeying Euclidean axioms (e.g. the 5th postulate). This uniform isotropic space implies a continuum extending to infinity in all directions - a mathematical definition, difficult to conceptualise.

The position of an object in Newtonian space is defined by coordinates. Those in general use are known as Cartesian coordinates, from Descartes' original definitions using three perpendicular axes, x, y and z:-horizontal, vertical and out of the plane at right angles to both.

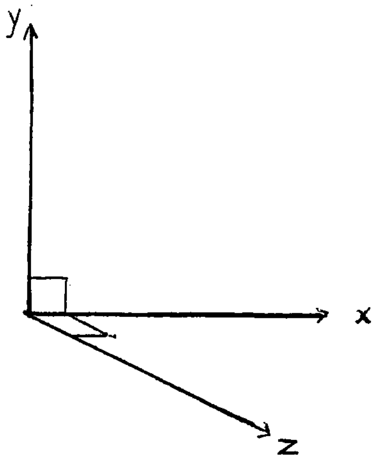


Figure 1 : Cartesian co-ordinates

Three given coordinates identify a point at any given time in Euclidean or Newtonian 3-space of three dimensions. Descartes himself hedged on absolute space, partly because of its Copernican tendency and partly because for Descartes, motion was relative, depending on the place of origin of his coordinates. Descartes' theory of place was followed by the absolute space of Kant and Newton himself. However this was really a metaphysical extension since Newton's theory of dynamics was in effect a relational theory of space and time - an inertial system with a system of axes superimposed.

A thorough-going relational theory of space, a system of particles related to one another, was championed by Leibniz and indicated by Mach. Nevertheless the standard viewpoint was to accept the notion of absolute space. The nineteenth century wave theory of light subsequently needed an aether to establish whether events at different parts of space occurred at the same point in time.

Although concepts of absolute space and the aether were later shown to be unnecessary, (e.g. from the Michelson-Morley experiment, which was explained by Einstein's Special Relativity), they were only slowly abandoned. The idea of space as a continuum, uniform, isotropic, infinite and three-dimensional, which took root when analytical geometry was invented by Descartes, has remained in common usage.

2. What is time? (a) Uniform flow

The concept of time has provided a number of variations. Although apparently quite different from space, time has also been held to be uniform and continuous. Aristotle held that time is associated with the mind, and there are many ways of conceptualising time, e.g. human time, biological time, psychological time, mathematical time and cosmic time (Whitrow, 1980), and even sacred time (Eliade, 1959, e.g. Ch 2). Kant in his Critique of Pure Reason, affirmed that time is a 'category', merely a part of our mental apparatus for imagining or visualising the world.

Our actual perception of time is a complex process. The Greeks implied at least two kinds of time in having the work 'Kairos' - creative or transcendent time, as well as 'chronos', the metronome time of physics.

Absolute, mathematical time was described by Newton himself: 'Absolute, true and mathematical time, of itself and from its own nature, flows equably without relation to anything external' (Ed. Alexander 1956, The Leibniz-Clarke correspondence, p.40). The moments of absolute time formed a continuous sequence, like the points on a geometric line, succeeding each other at a rate independent of all particular events and processes. This was the time which appeared in Newton's laws of motion. The alternative model of a discrete, discontinuous series of instants was used by Leibniz to oppose Newton's absolute theory. Leibniz' relational or relative theory, after Lucretius (Whitrow, 1980), was used to describe the successive order of things. (Ed. Alexander, 1956, Leibniz' 5th letter). This is developed in the cinematograph or film-strip model used by William James (James, 1890).

The uniformity and continuity of time have been widely accepted since Galileo, the most influential pioneer of the notion of representing time by a continuous straight line. The flow of time is indicated by metaphors of a river in literature. 'We see which way the stream of time doth run' (Shakespeare, Henry IV, Part II, Act IV,i, 1.70). In practice this is not an easy concept, and indeed in Newton's equations, there is no 'present', no quantity which measures the motion of time. That the flow of time is an illusion has also been cogently argued (Smart, 1964, eg. p.18). A qualitative interpretation involving awareness of awareness of the flux of time has also been set against the traditional metrical flow (Grünbaum, 1964 e.g. Ed. Smart). Nevertheless it was the uniform flow of time which was widely accepted.

(c) Space and time at the end of the nineteenth century

We have seen that for Newton, there was one universal time that served for the ordering of all processes in the universe, at all places in the universe and for any observer, moving or stationary. The dependence of time upon the velocity of the observer, which would in fact rotate the axis of time/direction, had been completely unthinkable from the Newtonian view point. The simultaneity of two events was completely unambiguous for all observers.

There were in fact various questions on the problem of space and time. Leibniz and Clarke in their correspondence addressed the status of space and time - what are space and time? (Ed. Alexander, 1956). Newton's arguments, outlined by Clarke, did not in fact show that space was absolute, but only that one argument for its being relative was invalid. Only a frame of reference to which the earth is rotating and the fixed stars at rest, represents an absolute inertial frame.

To use Newton's laws to explain a particle's motion, the laws must be written in terms of this inertial frame, that is at rest with respect to what he called "absolute space". This definition was criticised even in Newton's lifetime because there was no way of establishing by experiment whether the centre of the solar system is at rest or in motion (see further, Open University A381, 1981, IV, Unit 6, p.18).

It was an important part of the criticism of Newton's claim that such an absolute frame of reference existed, at rest with respect to "absolute space", that no phenomenon of motion can distinguish this special frame of reference. Indeed the distinction between absolute time and relative time, which depended on the natural solar day, led Newton himself to distinquish between these in practice.

He frequently avoided a full statement of his hypothesis in his publications, perhaps because he hoped thereby to escape any controversy. "And to us it is enough that gravity does really exist, and act according to the laws which we have explained". (Ed. Cajori, 1934, p.546). Without overturning his whole concepts of absolute space and time, Newton had no other way forward. His only explanation of action-at-a-distance would be that God caused it : "This most beautiful system of the sun, planets and comets, could only proceed from the counsel and dominion of an intelligent and powerful Being "(ibid., p.544). Newton therefore left this out of his Mathematical principles of natural philosophy (Hall and Hall, 1962 e,g. p.213).

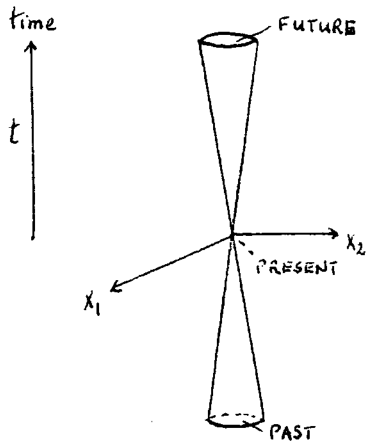
3. Time, Space and Special Relativity

In Einstein's theory of special relativity, published in 1905, the paradox of the aether was resolved. Using absolute space and time, the concept of an aether seemed to be needed for the electromagnetic field theory developed in the nineteenth century. This hypothesis of a fixed invisible stationary luminous substance in which electromagnetic waves propagated was not consistent with the results of the Michelson and Morley experiments. They failed to detect any motion with respect to such an aether. The paradox was apparently resolved by Einstein's solution: neither space nor time were absolute; they are merely co-ordinates or labels on a four dimensional space-time continuum. Different times are needed for the same event if the observers are moving. Einstein's theory automatically accounted for the Michelson-Morley results. Einstein also predicted the so-called 'clock or twin-paradox': time dilation occurs for a clock, and for one of a pair of identical twins, travelling on a long space flight at a speed which is a significant fraction of the speed of light and returning to earth at some later time. The clock appears to run slower and the twin to be younger than a clock and the other twin left on the earth. Different times have passed for each twin. The effect would not be noticeable at lesser speeds but illustrates a real difference, in the absence of any "Absolute Time".

Such a four-dimensional manifold of all possible events is nearer Leibniz' relational or relative time. Einstein's radical revision of space and time introduced a 'world line' or geodesic for the path of a particle, using a fourth co-ordinate of time. This replaced the 'Galilean' transformation (after Galileo) in three Cartesian perpendicular co-ordinates.

A lightcone:

To draw a picture of 4-dimensional space-time, one of the space co-ordinates (x_3), may be suppressed, and a cone results.

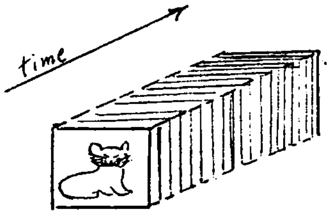


One space axis is of course suppressed (x_3) and another suggested by perspective (x_1), so that an effort of imagination is needed to supply the missing dimensions. A stationary object now follows a line path on the diagram where x_1, x_2 ($+x_3$) are constant, and only time varies.

Figure 2 Spacetime cone

Einstein's brilliant unification of the concepts of time and space into a single entity called spacetime can thus only be described by analogy. For example the fusion together of successive ciné film frames, again suppressing a dimension, as suggested by William James' "block universe".

Figure 3
"block universe"



Einstein assumed that there were no instantaneous connections between distant external events and the observers: the classical theory of time, with world-wide simultaneity for all observers, had to be abandoned.

In special relativity theory, time was regarded as a dimension, like the dimensions of space. The dimension of time was exactly analogous to space dimensions, mathematically; it had however a different "signature" with respect to the three positive space dimensions. The metric shorthand is +++-, and its full description given by the Minkowski metric:

$$ds^2 = dx^2 + dy^2 + dz^2 - c^2dt^2$$

Following the Second Law of Thermodynamics and the increase in overall disorder or entropy, the "time's arrow" of Ludwig Boltzmann is unquestionably forwards for physicists. Space itself has no such progression. The unique reality of present time (with past history not existing, merely having been real, and the future yet to exist) is an additional argument against the analogy with space. The psychological arrow of time is also forwards - a feature of consciousness with no objective counterpart (Whitrow, 1980, p.374).

The simplicity, elegance and predictive power of special relativity however, is obtained by taking time as an extra dimension and using the spacetime interval. The case for spacetime is an impressive one, although not without its detractors. In 1908 the mathematician Hermann Minkowski in his famous lecture on 'Space and Time' in his address to the Eightieth Assembly of German Natural Scientists and Physicists at Cologne, explained the idea of formal unification of space and time (presented mathematically in 1907) "Henceforth space by itself and time by itself are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality" (Minkowski; 1923, p.76).

Time and space are still distinct concepts, but fused together and no longer isolated.

4. The Dimensionality of space (a) our apparently three dimensional world -(three—space or four-dimensional spacetime).

It was probably Immanuel Kant who first wrote about the problem of the dimensionality of space. Even Newton's rival Leibniz, who worked on the idea of space in a searching manner, took very little notice of the dimensionality. Having considered different dimensionalities, Kant thought he had discovered the reason for space being three dimensional in Newton's laws of gravitation. By Newton's Inverse Square Law, the intensity of the force decreases with the square of the distance. Kant realised this intimate connection between the inverse square law and the existence of three dimensions. The three dimensions of space (as other laws of nature) were seen as 'a condition for the possibility of phenomena' (Critique of Pure Reason eg. Ed. Green ,(1929) p.47).

The reasoning of Kant has not been improved on in all subsequent references to this problem. Many have rediscovered his logic, rewriting the proof that the world has only three space dimensions, assuming that Newton's law is correct for gravitation and for electro-magnetism. Gauss and mathematicians after him,e.g.Riemann and Grassman, began to explore manifolds with arbitary numbers of dimensions; these were not given physical application at the time.

Physicists reaffirmed that the world had only three dimensions. One of the first to do this was Ueberweg in 1882, involving internal experience as well as the inverse square law. Poincaré, despite his own insistence on the particular geometry one chooses being a matter of convention, also attempted to demonstrate that this space of experience was in fact three dimensional. However he only eliminated one and two dimensions leaving three almost by default: 'space shows itself to be three dimensional' (Poincaré, (1917) 1963, p.7B).

Poincaré was more interested in the physical and philosophical implications of dimension, yet his essay reviewing the metaphysics seems to have initiated the research into the topology of dimensionality. In using disconnecting subspaces, Poincaré emphasises the inductive character of the definition (Poincaré (1902), 1952, p.486). This was used as a base for Brouwer's accepted topological invariant definition of dimension (Brouwer,1913) Brouwer first established the proof that Euclidean spaces of different dimensionalality are 'nonhomeomorphic' (Brouwer 1911), i.e. "they cannot be mapped on each other by a continuous one-to-one correspondence" (see Jammer, 1970, p.184, and Kline, 1972, e.g. p.1178).

Kant's and Ueberweg's arguments were formulated quite clearly by Ehrenfest in his paper: 'In what way does it become manifest in the fundamental laws of physics that space has three dimensions?' (Ehrenfest, 1917). Ehrenfest's argument rested on the stability of the trajectories of the planets. If there are n dimensions, for $n>3$ there do not exist motions comparable with the elliptic motion in R_3 (3-space) - all trajectories would have the character of spirals. This argument was also applied to the orbits of electrons round the nucleus of an atom.

This argument has continued to be the basis of similar 'proofs' (Whitrow, 1955), and even showing that the apparatus used in describing our physical world shows preferences for the four dimensional spacetime world (Penney, 1965). The anthropic argument - that three dimensions of space are necessary for life to exist as we know it - appeared in Whitrow (ibid., p.13). The reasoning from stable periodic orbits as a necessity for planetary life has been extended recently by Barrow. Only in Barrow's paper has Ehrenfest's (1917) argument in terms of planetary electrons been soundly criticised in terms of atomic stability. He used Schrödinger's equation (although only the three dimensional case for one atom) in a further reductionist argument: 'the three dimensionality of the universe is a reason for the existence of chemistry and therefore, most probably, for the existence of chemists also' (Barrow, 1983 p.39).

Barrow elegantly summarised the arguments for the properties of wave equations being very strongly dependent upon spatial dimensions. Three dimensional space appears to possess a unique combination of properties which allows sharply defined transmission of electromagnetic waves, free from reverberation, and to allow information-processing.

Thus the reasons for three dimensions comprise some of the aesthetically pleasing features of space - a continuum, the inverse square law of Newton, the equations of gravitationand of electromagnetism in normal physics - appear to work in 3-space. This orthodox tradition of the universe existing in only three dimensions seems to be confirmed by our common sense and intuition.

(b) Against 3-space and 1-time

Despite the fact that space clearly appears to have only three dimensions, the arguments used to prove 3-space have not been entirely free from criticism. There are also problems with the use of the word 'dimension' if it is to be used beyond three. The space we experience seems to have three 'physical' dimensions, perhaps 'expanded' dimensions. There seems to be a conceptual discontinuity between the three of experience and any extra or higher dimensions, a discontinuity already obvious even within the well established space-time concept of four dimensions. Einstein's mathematical arguments for the similarity of time and space remained unconvincing, even to Einstein himself.

The reasoning from gravitationand electromagnetism, which follow the inverse square law, is not valid over the range of forces now known to exist. There are four fundamental forces including the two close-range nuclear:- the strong force within the nucleus and the weak force of radioactivity. These do not obey the inverse square law, so that at very small distances the dimensionality need no longer be three on the standard method of "proof."

Although the argument from the stability of the planets in their orbits does lead to three dimensions, the analogous argument from the stability of the electrons in their orbits is invalid. The Rutherford-Bohr planetary theory of the atom was pre-quantum mechanics. Electron energy levels, the uncertainty principle and the analysis by quantum numbers give an entirely different model. Barrow's paper of 1983 was perhaps the main source to point out that this model was no longer valid.

Barrow additionally used what has become known as the 'Anthropic Principle'. Three dimensions are a necessary requirement for life to exist - particularly human life. Consciousness and awareness are a philosophical and even theological precondition for these arguments to be used at all. There are implications that there are other universes - possibly where life does not exist (see the 'Many Worlds Theory' of Everett and Wheeler (Chapter 4 for further discussion). If there are more dimensions than three for us, they do not affect the arguments that space does appear actually to have three dimensions.

Newton's Inverse square law is only a good working hypothesis. It has been replaced by Quantum Mechanics and Geometrodynamics on the small scale, with the resultant enigmas and paradoxes in their interpretation within 3-space (see Chapter 4). On the large scale, General Relativity has superceded Newton's laws. The interpretation of Relativity and of its resultant singularities has also led us to the limits of physics and the need for a new physics (see Chapter 2). The implications of Schrödinger's Equation in many dimensions, of possible discontinuities in the metric, of the laws of physics breaking down at the centre of singularities, all indicate the need for a reappraisal of dimensionality.

The classical arguments for 3-space are thus open to criticism. The apparent three dimensions is certainly limited to the range of traditional physics and ignores the very small scale and the situation at high energies.

Nevertheless we do appear to live in a space of three dimensions. The reasons comprise the unique combination of properties in 3-space; our common sense and experience confirms the evidence of normal physics. Classical physics demands that there have to be three large flat dimensions.

5. A multidimensional reality?

(a) Distinguishing between different uses of "dimension"

The problem in considering dimensions beyond three has precisely the disadvantages which have been given in support of the orthodox three. Our common sense and intuition may fail, and we must resort to mathematics, (preferably where the mathematical formalism can be translated into words), and to analogy. Although only three dimensions are apparent, space may be extended without our being directly aware of it at our normal energies. It is salutary to note de Broglie's acknowledgement of the difficulties involved in the use of our accepted notions of space and time on a microscopic scale, in that there were 'no alternative known conceptual categories which could be substituted' (De Broglie, 1949 , p.814).

Kant affirmed that the proposition that space has only three dimensions cannot be experimentally tested (Kant,1781). Barrow pointed a way forward- that in the arguments involving special features in physics in three dimensions, the assumption has been made that 'the form of the underlying differential equation do not change with dimension...one might suspect the form of the laws of physics to be special in three dimensions if only because they have been constructed solely from experience in three dimensions' (Barrow,1983,p.342). Our perceptual apparatus is circumscribed in three dimensions. There is a danger in unacknowledged reductionism preventing the consideration that higher dimensions are even possible.

Although the universe appears to be in 3-space, 'this may not be right at a deeper level' (Penrose, 1980). There is a growing feeling in the 1980's that the physical world is higher dimensional (e.g, Ed. De Sabbata and Schmutzer, 1983). Despite the fact that the space we experience has three space dimensions (and one time) we may not know for example if there are other compacted dimensions (Chapter 3, 4, etc.) or extra embedded dimensions (Chapter 2).

The critical question is appearing:

Is it possible that the space we experience is only a part, a projection of a higher dimensional space?

(b) Theoretical or physical status?

We shall examine the differing reasons why phycisists have found the need to try more than three space dimensions, despite the fact that the space we live in has only three dimensions. This will vary along the spectrum from a purely theoretical or mathematical model, to the increasing status of the extra dimensions actually being physically there. Thus abstract multidimensional phase space has been used first as a tool for mathematicians such as Minkowski and Riemann. However in modern approaches to theoretical physics, extra dimensions are increasingly treated as physical rather than as merely mathematical. Extra embedding or compacted dimensions may be merely conceptually useful or they may be real, but somehow hidden from our immediate experience. This higher status to extra dimensions describing a deeper reality is not susceptible to direct proof, except under abnormal conditions, for example very high energies. Extra dimensions cannot be subjected to experimental proof but may have second order verifiable predictions. The arguments are theoretical, at least for this present moment in time.

One problem which will constantly challenge our thought will be the difficulties involved in conceptualising or visualising extra dimensions. The mathematician has used a language of multidimensions without any difficulty for over a century. For others the increase in status brings the reality problem - there seems to be a discontinuity between the use of 'dimensions' for ordinary flat physical space - and its use in describing dimensions of space beyond three.

6. Motivation for using extra dimensions

Although the world appears three-dimensional, phycisists have shown an increasing need to go beyond 3-space in recent unification of forces, particles and theories. There has been a major conceptual change in moving from the theoretical possibility of multidimensions to the need to incorporate extra dimensions in a new physics. The two great revolutions of the twentieth century were General Relativity and Quantum Mechanics. Despite their widespread usefulness, they have led to paradoxes and enigmas in their interpretation. A new revolution is necessary.

(a) Use of extra dimensions as a tool or 'mathematical convenience'

(i) Hilbert, Minkowski and Riemann

The position of a single particle is a point in 3-space, usually specified by its Cartesian components (x,y,z,) relative to some axes. For two particles, the two positions require 6 components for their specification (x_1, y_1, z_1, and x_2, y_2, z_2,). It is clearly possible to think of these two points in 3-space as one point in a space of 6 dimensions. Three particles may be thought of as corresponding to a position in 9-space etc., as used by Hilbert, Minkowski or Riemann.

This of course is merely a manner of speaking and no particular 'reality' is attached to the higher dimensional space (see Chapter 2).

(ii) Schrödinger's Equation and Quantum Mechanics

The situation changes somewhat when we involve the quantum theory. The wave function of a single particle is a (complex-valued) function of position $\psi(x_1, y_1, z_1,)$. Thus at each point of space it has a well defined value (working at a particular given time). For two particles the wave function becomes a function of two positions: $\psi(x_1, y_1, z_1; x_2, y_2, z_2)$. Thus it is a scalar field defined in a 6-dimensional space - it cannot be thought of as having a value at a particular point of 3-space. Similarly this situation extends to more particles; the wave function for N particles becomes a function of position in a 3N-dimensional space (see Chapter 4).

Here we are involved with questions of the "reality" of the wave function; questions which are still the subject of much controversy. It is interesting that Schrödinger's equation, widely used across physics, needs a complex multi-dimensional space. The status is clearly increased above mere mathematical theory. Nevertheless it is hard to describe any reality to the multidimensional space in which the wave function is defined. For the physicist the problem is normally one of understanding the meaning of the wave function, rather than that of understanding the significance of the higher dimensions!

(b) The use of Embedding Dimensions for large scale curvature

This has an ambiguous status, often regarded as merely an aid to visualisation of the curvature of space. However from an extrinsic viewpoint it is available for higher status, although this is not susceptible to experimental verification.

-Kasner's mathematical treatment and as an aid to visualisation to interpret General Relativity

We are familiar with the difference between a flat 2-dimensional surface and a curved 2-dimensional surface because we can visualise and indeed construct such surfaces in 3-space. The question of whether a surface is flat or curved may be seen however as intrinsic to the

2-dimensional surface and does not require it to be embedded in 3-space see (Chapter 2 and the concept of a "Flatlander" - Abbott, 1884).

The same thing occurs in higher dimensions, e.g. in the interpretation of General Relativity. Einstein was able to assert that gravity "curves" 3-space (more generally 4-dimensional spacetime), i.e. gives it an intrinsic curvature without having to embed it in a higher dimensional space. Nevertheless, as with a 2-surface, it is easier to visualise curvature if we do embed the curved space in a higher dimensional space. In fact (see Chapter 2) the Einstein equations of General Relativity require in general a space of at least 6 and in practice at least 10 embedding dimensions (Kasner, 1921). Whether such an embedding gives any "reality", (i.e. 'status') to the extra dimensions is of course open to doubt.

(c) Unification of forces by increasing the dimensionality of spacetime - the Kaluza-Klein model of compacted dimensions

(i) Kaluza - to unify electromagnetism and gravity in five dimensions.

After an interesting but unsuccessful earlier attempt (Nordström, 1914), Theodor Kaluza (1921) was the pioneer of the successful unification of the two then known forces using an extra fifth dimension. Kaluza himself implied a high status, although using the "cylinder condition" to explain the apparently four-dimensional real world (see Chapter 3).

(ii) Oskar Klein rediscovered Kaluza's theory in 1926, and attempted to make these five dimensions consistent with Quantum Mechanics. However, he still had to treat it mathematically in a way which distinguished it from other space dimensions (see Chapter 4). Einstein and Bergmann tried to develop this further, and increase the physical status (1938, see Chapter 5).

(iii) Attempts to include Kaluza-Klein models in gauge theory were the beginning of the revival of interest in Kaluza's idea forty years later (de Witt, 1965, see Chapter 6). This was further developed to include supersymmetry (Cho and Freund, 1975) and to unify electromagnetic, weak and strong fields.

(iv) the motivation to link Kaluza-Klein with Dual models was seen in the 1970's. This was done by Scherk and Schwarz (1975) using the string theory, which replaced point particles by extended objects called strings, in order to remove the infinities of field theory (see Chapter 7). The hope was to include the link of quantum mechanics with special relativity.

(v) This led to a search for a fully unified complete theory of gravitation consistent with quantum mechanics. This was developed by Green and Schwarz using superstrings, the supersymmetric version of strings. They also helped to give physical meaning to theories containing gravitation and gauge fields (see Chapters 7 and 8) and remove anomalies.

(vi) The search for a fully unified field theory to solve enigmas in General Relativity also led to the development of supergravity in 10 or 11 dimensions. This also brought in the Kaluza-Klein idea at a later stage (1979).

(vii) Further motivation in the 1980's has involved the attempt to explain cosmology. This involved the variation of the extra dimensions with time. The five, ten or eleven dimensions were once all co-equal in the earliest stage of the Big bang (Souriau, Chodos, Marciano etc.)

(viii) Attempts to give physical meaning to the extra dimensions and to explain why they are not observed in our apparently three dimensional world have been a continuing motivation. From Kaluza and Klein, via Einstein and Bergmann, this led to Chodos and Detweiler's link with cosmology in 1980. We must also include the change from the theoretical tool of dimensional reduction (from 11 dimensions to 4) to spontaneous compactification (e.g. Cremmer, Scherk and Julia, 1976) Luciani had a similar motivation including the spinor dual model with supergravity in 1978.

Witten's attempt to understand the geometrical meaning of superstrings using Penrose's twistor theory may also be included, together with the need to understand spontaneous symmetry breaking, e.g. to give quarks and leptons (see Chapter 8).

The geometrical interpretation of internal quantum numbers e.g. as charges, was a similar motivation from Salam and Strathdee, 1982.

(d) Other (non-Kaluza-Klein) methods of changing the dimensionality of spacetime

These are given varying status. Some do not involve any quantitative number of dimensions, and could even include the Many Worlds theory of Everett, de Witt and Wheeler (see Chapter 4).

(i) John Wheeler's Geometrodynamics. Wheeler applied General Relativity to the microscopic scale with many creative ideas, e.g. foam space, wormholes in space, etc. His idea of "pregeometry" implied no particular dimension at all (see Chapter 2). Ideas of foam space have been developed more recently by Stephen Hawking.

(ii) Podolanski's use of six dimensional space time was developed in 1950 to make field theory finite. This involved the cancellation of the infinities implicit in quantum mechanics. Podolanski in fact used a foliate spacetime with 4-space and 2-time, (see Chapter 6).

(iii) Roger Penrose attempted to resolve the enigmas and paradoxes of point particles and quantum mechanics using his Twistor space in six or eight dimensions. This description of reality implied taking six dimensional spacetime seriously. Penrose himself gives it a high status as an alternative model, with the complex manifolds providing a powerful mathematical tool eg. in quantum physics (see Appendix to Chapter 7).

7. Conclusion

These motivations for looking beyond three space dimensions have implied the need for a new physics. This thesis will trace the origins and development of the use of extra dimensions beyond the four of spacetime which we appear to experience. These will include embedding dimensions as well as the purely mathematical multi-dimensions of the nineteenth century. Particular attention will be paid to the evolution and physical status of the Kaluza-Klein model to produce realistic theories.

All models of multidimensions in fact have a range across the purely mathematical to the physical. One of the problems is why the Kaluza model has been neglected for many years when it is now widely felt to be needed. The revival of the Kaluza -Klein idea in the 1970's has paved the way for current "theories of everything".

In order to face the consequences of taking a multidimensional reality seriously, we must move from the mathematical formalisms to the metaphysical problem of the conceptualisation of such transcendent ideas. These will be explored through suitable analogue models rather than in abstract mathematical language.

Chapter 2. General Relativity, 1915: Four Dimensions of spacetime - and the need for extra embedding dimensions

Synopsis

Introduction

(1) The geometrical interpretation of spacetime in Einstein's theory

(2) The geometry of curved space

(3) The mathematical concepts needed for a geometrical approach to reality.

  (a) Ideas of Non-Euclidean mathematics - Gauss, Bolyai and Lobachevski

  (b) Geometry of more than three dimensions - multidimensions in mathematics

  (c) The unifying work of Riemann

  (d) Einstein's generalisation of Riemannian geometry - via Tensor analysis

(4) The geometrical interpretation of spacetime : "Curved" space and the need for embedding.

(5) Conceptualisation - requires embedding to visualise extrinsic curvature

(6) Embedding requires extra dimensions

  (a) By Analogy

  (b) Mathematically : Kasner (1921), Embedding theorems - the need for extra dimensions beyond four.

(7) The implications of curved spacetime.

(8) Postscript: Problems arising from the General Theory of Relativity.

  (a) The "Big Bang"

  (b) The "Big Crunch"

  (c) "Black Holes" - Singularities within the universe

  (d) The existence of Black Holes

  (e) Intense curvature on the very small scale : Foam Space and Geometrodynamics.

(9) Conclusion: Reappraisal of General Relativity - the need for a new physics.

We have seen that Einstein's Special Theory of Relativity solved a number of the problems of late nineteenth century physics. Without referring to the aether at all, Special Relativity was able to interpret wave theory phenomena and the Michelson-Morley experiment, destroying the absolute space and absolute time of Newton. All reference systems moving with constant velocity relative to each other are equally legitimate in forming the laws of physics - (1). Light always propagates with the same velocity c in every such legitimate reference system - (2). Although all physical events seemed to be described perfectly by these postulates, Einstein was not completely satisfied. He was concerned to describe not only uniformly moving systems, but arbitrarily moving systems such as accelerating systems, without any privileged reference system. The equivalence principle led him to the conclusion that a more universal principle was needed than his 1905 postulates which must break down in the presence of a gravitational field.

1. The geometrical interpretation of spacetime in Einstein's Theory

In his search for a better theory, Einstein needed more mathematics, more tools to describe his ideas. He needed to extend from the Euclidean flat space of Special Relativity and from privileged reference systems, in order to answer the problem of gravitation. He found the branch of mathematics called 'Absolute calculus' or 'Tensor Calculus', was exactly what he needed to solve the problem of arbitrary co-ordinates. A four-dimensional geometry was also required, and had been demonstrated by Minkowski. The underlying principle was in geometry, rather than in physics.

The essential feature of special relativity involves the transformation from one inertial frame to another (i.e. one observer to another moving with constant velocity), where the four-dimensional line-element or "interval":

$$(ds)^2 = (dx^1)^2 + (dx^2)^2 + (dx^3)^2 - (dx^4)^2$$

does not change. Here x^i (i = 1,2,3) are the Cartesian space co-ordinates, and x^4 = ct by definition. The quantities dx , etc, represent the difference between coordinates of two events, $dx^1 = X_1' - X_1$ etc. (This invariance for different observers is the space-time analogue of the fact that in three dimensions, the quantity $(dx^1)^2+(dx^2)^2+(dx^3)^2$, is unchanged by a rotation of the axes, as follows from Pythagorus' Theorem (see also Chapter 3).

If we use more general coordinates, then the expression for the line element takes a different form:

$$(ds)^2 = \sum_{i,k=1}^{4} g_{ik} \, dx^i \, dx^j$$

where g_{ik} is the "metric", which of course in the special case of Cartesian coordinates is given by $g_{ik} = 0$, $i \neq k$ etc.

Einstein realised that by using this general line element he could incorporate the effects of gravitation and of accelerated reference frames. In the presence of general gravitational fields, g_{ik} would be a function of position and time, and it would be possible to find coordinates such that the simple form of the line element was valid everywhere. The gravitational "force" would then disappear and instead gravity would affect space itself through the metric g_{ik}. Since all bodies would move in the same geometry, the principle of equivalence would be an automatic consequence.

2. The geometry of Curved space

The geometry developed by Riemann soon after Gauss in the mid-nineteenth century, provided the more general non-Euclidean geometry of more than three dimensions which Einstein needed and which had been recently developed by Minkowski. Minkowski's line element would then be still correct in sufficiently small (Euclidean) dimensions. However on a larger scale, g_{ik} must be seen as some function of the four coordinates x_1, x_2, x_3 and x_4. These need no longer be Cartesian, but arbitrary Gaussian - type coordinates. Riemann did not specify these, but characterised this geometry by a decisive quantity, (a tensor of the fourth order called the Riemann-Christoffel curvature tensor) R_{ijkm}.

The simplest geometry is obtained by putting the full Riemannian tensor equal to zero, giving the flat space of Minkowski geometry. The metrical tensor g_{ik} has ten components in four dimensions and only a tensor of the second order is needed, which can be obtained by contraction. In other words, only the vanishing of the contracted curvature tensor is used:

$$R_{ik} = \sum_{j,m=1}^{4} g^{jm} R_{ijkm}$$; where g^{jm} are elements of the metric, i.e. $g^{jm} g_{mk} = \delta^{j}_{k}$.

The field equations $R_{ik} = 0$ are thus the famous equations of Einstein's General Relativity. The mysterious 'force of gravity', which Newton would not elaborate in any published hypothesis (see Chapter 1) could be perfectly explained (using a matter term on the R.H.S) as a property of the Geometrical structure of the universe - Riemannian, non-Euclidean.

The second unexplained puzzle of Newton's theory, the strict proportionality of inertial and gravitational mass, could now have a different, geometrical explanation. The source of gravitational action is the curvature in space caused by the inertial mass of a body.

Thus Einstein used the relatively recent procedure of the Tensor Calculus, formulated by Ricci and Levi-Civita in their paper of 1901, to formulate the laws of physics in arbitrary coordinates ("general covariant form"). He immediately noticed however that there was a new feature in the equation which was not there when

Cartesian coordinates were used. A new field quantity is now added to the previous physical field - the coefficients g_{ik} of the metrical tensor. For Einstein this was not just a geometrical abstract parameter, but a physical field quantity. If it is true that the g_{ik} determines the geometry of the universe then it must be included in the field equations. This was Einstein's great innovation.

3. The Mathematical concepts needed for a Geometrical approach to reality - an historical review

(a) Ideas of Non-Euclidean mathematics - the historical ideas behind "curved" space

The discovery of non-Euclidean geometry paved the way for the elimination of the final traditional characteristic of space, and provided the base for the Riemannian concepts of a multidimensional manifold which Einstein needed.

The initial publications were the independent contributions of Bolyai and Lobachevski. Even before this, Carl Frederick Gauss had already explored the possibilities of non-Euclidean geometry, believing that Euclid's parallel axiom was unprovable, but did not publish his ideas. Nikolai Lobachevski's paper "On the Principles of Geometry" was published in 1829. This described a valid logical geometry, but yet apparently so contrary to common sense that even Lobachevski called it "imaginary geometry" (Boyer, 1968, p.587), although he was well aware of its significance. In 1832, James Bolyai (whose father, a friend of Gauss, also worked on the problem) reached the same conclusion in his Tentamen as had Lobachevski a few years earlier. There were other less well-known predecessors, and the possible application of the new geometry to physical space had in fact been seen by Gauss (Kline, 1972, p878).

Euclidean geometry came to be seen as one system among others, logically holding no privileged position. It also became clear that there was no 'a priori' means from the mathematical or theoretical point of view for deciding which type of geometry represented the world of physical objects. The Lobachevski world, for example, was an infinite world. What was defined only as a point in a given space may well be some more elaborate structure in another. Nevertheless, terrestrial geometry seemed to be Euclidean, as far as experience goes. To test Einstein's ultimate application to physics, experiments on a very large scale were needed, to see whether physical space was different from Euclidean space.

c) Geometry of more than three dimensions - multidimensions in mathematics

Meanwhile, the first half of the nineteenth century also saw the independent development of the rise of multidimensional geometry as a new mathematical language. Arthur Cayley (in his work on matrices) and Hermann Grassman (in his generalisation of complex numbers) independently developed the serious study of n-dimensional geometry, although not suggesting any physical implications at the time. Grassman was the initiator of a vector analysis for n-dimensions, although he only published his Die lineale Ausdehnungslehre (The Calculus of Extension) in 1844. This was the year after Hamilton announced his discovery of quaternions, numbers containing both real (scalar) and complex (vector) parts, which was to be so useful in the early twentieth century, Lectures on Quarternions, 1853. Grassman's work was scarcely recognised at the time, even after his revised and simplified edition in 1862. Cayley in England initiated the ordinary analytic geometry of n-dimensional space. He published this extension from three dimensional space, without recourse to any metaphysical notions which had made Grassman's work little understood at the time (Cambridge Mathematical Journal, 1845).

Further studies on the classification of geometries was carried out by Hermann von Helmholtz, who worked on problems of physical space. These were elaborated mathematically in the work of Sophus Lie on groups of transformations in the various possible spaces.

c) The unifying work of Riemann, anticipating Einstein's central ideas

Both these mathematical languages—of non-Euclidean geometry and of n-dimensional space - remained outside mainstream mathematics until fully integrated by Georg Bernhard Riemann (1826-1866). He generalised Gauss' work, culminating in the concept of 'curved space' and made it clear that the curvature of space may vary from point to point. Riemannian space was a continuous n-dimensional curved manifold, and a more general concept than of other contemporaries. Only three types of geometry seemed compatible with isotropic space. These spaces had indeed a special significance, as spaces of constant curvature, used by Einstein later. The space of constant positive curvature is called 'spherical', because it is the three dimensional analogue of the sphere. If the Riemannian curvature is everywhere negative, the space is that of Bolyai-Lobachevski (hyperbolic). The space of constant zero curvature is Euclidean. The analytic method of Riemann in fact led to the discovery of more types of space with varying curvature (H.Reichenbach, 1958).

Riemann, like Lobachevski, believed that astronomy would decide which geometry fits physical space. His allusions were largely ignored by his contemporary mathematicians and physicists (Jammer, 1953, p.162). His investigations were thought to be too speculative and abstract to have any relevance to physical space, the space of experience. Riemann's fundamental investigations were not even published in his lifetime. Only when they appeared posthumously did Helmholtz apply the ideas, although he did not consider the possibility that matter may influence the geometry of space.

The possibilities of a Riemannian space did however find an enthusiastic supporter in the young geometer, William K.Clifford, who in fact translated Reimann's work into English. Only Clifford saw the potential for combining geometry with physics. He anticipated, in a qualitative manner, that physical matter might be thought of as a curved ripple on a generally flat surface, describing moving particles as little hills in space, "variation of the curvature of space","... continually passed on from one portion of space to another in the manner of a wave" (W.K.Clifford, 1870"On the Space Theory of Matter"; quoted by Kline,1972,p.893). Many of Clifford's ingenious ideas were later actualised quantitatively in Einstein's theory of gravitation. Clifford himself held that space was largely Euclidean and had not grasped the full extent of the idea. He regarded the variation of space curvature as local, on a small scale.

d) Einstein's Generalisation of Riemannian geometry

The final mathematical tool which Einstein was to make such creative use of, was that of Tensor Analysis. This was the differential geometry associated primarily with Riemannian concepts. The new approach was initiated by Gregorio Ricci-Curbastro, influenced by the work of Christoffel and Bianchi. In a collaborative effort with his famous pupil Tullio Levi-Civita, they published a comprehensive paper on the Absolute differential calculus in 1901. This involved the expression of physical laws in a form invariant under change of coordinates. It became known as "Tensor analysis" after Einstein's description in 1916.

In 1908, in his address to the Eightieth Assembly of German Natural Scientists and Physicians, Hermann Minkowski gave a strikingly new interpretation of Einstein's two postulates of Special Relativity theory. He realised that they were not so much physics as geometry. The deeper significance was that time has to be added to the metric, going beyond our usual geometry of three dimensions. This formed a unified four dimensional spacetime. In the Special theory of 1905, space and time were no longer independent entities. As Minkowski said, with a sense of hyperbole, "Henceforth space by itself and time by itself are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality". (Ed.Smart, 1964, p.297).

Following Minkowski's thrust, Einstein concluded that the objective world of physics is essentially a four-dimensional geometrical structure. He combined the principles of equivalence and general covariance with Riemannian geometry using tensor analysis. Einstein thereby succeeded in absorbing gravitation into the geometry of spacetime in his General Theory of Relativity of 1915 : Einstein, 1916, "The Foundation of the General Theory of Relativity" - (in Lorentz et al., 1922). Here spacetime is no longer flat. Gravitation distorts or modifies the spacetime geometry, 'warping' or 'curving' space. Einstein thus explained gravitation in terms of the geometry, the metric structure, of spacetime, rather than in terms of Newton's mysterious 'action-at-a-distance'. (For weak gravitational fields, e.g. terrestrial physics, Einstein's theory reduces to Newton's theory of gravitation). There was no need for forces at a distance, such forces become geometry.

4. The geometric interpretation of spacetime : 'Curved' space and the need for Embedding

Besides the paradox of the effect of gravity upon time (the 'Twin Paradox'), Einstein had also predicted the unheard of effect of gravity upon electromagnetic forces. The bending of the patterns of light rays travelling very near to massive objects went completely beyond Newtonian mechanics. This new prediction led to the first public affirmation of Einstein's General Relativity. Already Einstein's Theory had successfully explained the path of the planet Mercury, which Newton's theory could not, although the Newtonian discrepancy was extremely small.

Evidence for Einstein's General Relativity was sought in the observation of the bending of light from a distant star, passing near the sun. Four years after Einstein had announced his theory, an expedition led by Arthur Eddington to observe this during a total eclipse of the sun, confirmed Einstein's prediction. Light from a distant star seen near the edge of the eclipse was deflected through a small angle by the gravitational field of the sun.
The mathematical model became more than an abstract theory. People became aware of the physical significance - they did live in a curved universe. The forces of gravity could be understood as an effect of the (internal) curvature of spacetime.

The <u>New York Times</u> for Tuesday December 27th, 1919, carried the headline: "New Einstein Theory gives a Master Key to the Universe". And even more surprisingly underneath : "$R_{ik} = 0$"..."Einstein offers the key to the universe ... etc.".
For Einstein himself, his reputation was enhanced, yet the elegance, beauty and simplicity of his equations had been evidence enough.

The generalisation of Minkowski's geometric notion of a four-dimensional spacetime manifold had led to gravitational fields being interpreted as manifestations of the curvature of the manifold (Bergmann, 1968 , The Riddle of Gravitation). The effect of modifying the geometry of spacetime produced a curvature or distortion of the geometry. The world line or geodesic of a particle was curved, not the straight line of flat spacetime. Action-at-a-distance is the result of local properties of spacetime.

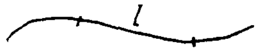
Curvature of space is not necessarily a smooth curve, but is the bending and distortion of spacetime. The physical manifestations involved in the above examples were only one type of curved-space-'intrinsic' or internal curvature, manifest from within spacetime. There is another external or "extrinsic" curvature which is evident only if the space is embedded in a higher dimensional space, if it could be viewed from outside.

5. Conceptualisation - requires "Embedding" to visualise extrinsic curvature.

There are thus two meanings to curvature. One is the intrinsic inner curvature which produces the physical effect of light bending. The other extrinsic outer curvature does not necessarily have a physical meaning. It is regarded as a purely mathematical device to aid calculations and provide a way of imagining the unimaginable, using analogue models.

"Curvature" is usually a concept applied to two dimensions curved in our 3-space as a cup or a sphere, for example. Even more fundamental is a one dimensional line curved in an arc or circle - or indeed in any curved shape - in the two dimensional plane of paper or blackboard.

A one dimensional string is 'flat' from an internal viewpoint.

Figure 4 

l = the distance on the one-dimensional string, where $ds^2 = dl^2$.

However the string is curved if embedded in our two dimensional surface, i.e. extrinsically curved from our higher viewpoint. Line-landers (Abbott, 1884) only knew the intrinsic appearance which is therefore flat for them.

A two-dimensional surface:

$$(ds)^2 = \sum_{i,j=1}^{2} g_{ij}\, dx_i\, dx_j$$

(i) Because the g_{ij} are, in general, functions of position, there can be 'genuine' curvature, i.e. internal intrinsic curvature - whether or not the surface is embedded.

(ii) If the plane is embedded in flat 3-space, then it becomes a surface with extrinsic curvature (although this plays no rôle in 2-Dimensional physics, or, by analogy, in relativity theory).

Three dimensions

In order to represent a space of three dimensions on paper, we must suppress one space dimension (as we would in drawing a cube on a blackboard). We can look at the relatively regular curvature of the earth in three dimensions. Two lines of longtitude, which we think at the equator are parallel, nevertheless converge and meet at the North Pole. This apparent mutual attraction of two aircraft flying precisely north along the lines AN and BN, appears as a force moving them gradually together. The explanation however is in the geometrical distortion due to the spherical nature of the earth's surface:

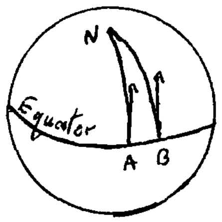


Figure 5: The apparent attractive force caused by curved geometry (Davies, The Edge of Infinity, 1981, p.16)

The apparent force of attraction felt under local condition is in fact due to the curved geometry. Similarly the attraction of bodies to the earth, or the earth to the sun, looks like a gravitational force - and feels like it to a parachutist. Thus on a large scale there appears to be instantaneous action-at-a-distance as a result of the bending of space. The path of the earth round the sun lies on the geodesic resembling an elipse. Locally the earth appears to move in a straight line. This is also true of the aircraft in the above diagram, where local conditions indicate that their paths are effectively straight lines, each starting at 90° to the equator. In fact this also illustrates the non-Euclidean nature in the intrinsic description of a two dimensional curved surface. "Parallel" lines may meet, contradicting Euclid's parallel postulate. The angles of a triangle add up to more than 180° (with spherical positive curvature). In the above example, the sum would be $90^\circ + 90^\circ + \angle ANB$. This is a useful analogue model, extending to the gravitational attraction in four dimensions of spacetime.

In General Relativity, matter itself causes curvature, bending or distortion of spacetime. Space and time are given a dynamical rôle. The curvature can be both an intrinsic and an extrinsic concept, depending on whether the world is viewed from an internal human viewpoint or from a perspective external to the world. This requires an extra embedding dimension to conceptualise ideas which cannot be directly visualisable. In order to represent a space of three dimensions on paper, we suppressed one space dimension. To represent the curvature of a spacetime of four dimensions, only one dimenion of space, together with a time coordinate, can be used.

We normally view countryside in two ways. First as a surface on which we walk and orientate ourselves, needing two coordinates to describe our position: u,v (e.g. latitude and longitude). Secondly as a surface which rises and falls and brings in a third dimension of height or depth, needing three coordinates : x, y, z (although only certain combinations would be used, since the x and y coordinates both determine the height above sea level, or the contour).

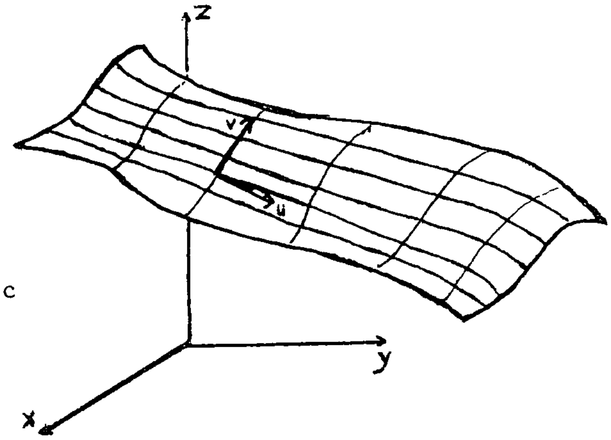


Figure 6
Intrinsic and extrinsic
curvature coordinates

(Gray, Ideas of Space, 1979, p.121)

6. Embedding requires extra dimensions

(a) By Analogy The (u,v) description is intrinsic - it is the only description available to beings constrained to live in the surface e.g. "Flatlanders" (Abbott, 1884).

The (x, y, z) description is extrinsic, and needs the extra third dimension (of height in this case) to appreciate the view. It is thus available to the 'superior' three dimensional beings who can see above and below the curved "Flatland".

This simulation is an analogue model for a three dimensional space curved in higher dimensions, or indeed for four-dimensional spacetime itself. By transposing upwards we can attempt to visualise the process of Einstein's curved Riemannian manifold, which he needed to improve on the flat spacetime of Minkowski, used in Special Relativity.

(b) Mathematically : Kasner's use of Extra Dimensions in embedding theorems.

Using embedding dimensions purely mathematically, it is easy to postulate spacetime as "curved" inward or outward, with the need for a fifth or higher dimension.

This may be pictured as if embedded in higher dimensions, and analysed as Edward Kasner first demonstrated in 1921 and 1922 volumes of the American Journal of Mathematics. In his first paper, Kasner discussed the determination of a four dimensional manifold in "Einstein's Theory of Gravitation : Determination of the field by light signals". The manifold is described by

$$(ds)^2 = \sum g_{ik} dx^i dx^k \qquad (i, k = 1, 2, 3, 4)$$

obeying Einstein's equations of Gravity $G_{\mu\nu} = 0$, when we are given merely the light equation

$$\sum g_{ik} \, dx^i dx^k = 0 \, .$$

Kasner demonstrated that the light determines the orbits, and went on also to show that "the (exact) solar field can be regarded as immersed in a flat space of 6 dimensions, but that no solution of the Einstein equations can be obtained from a flat space of 5 dimensions" (Kasner, 1921a, p.20). He used the ten functions g_{ik}, and employed flat space - either nearly-Euclidean or Euclidean.

Kasner carried on his discussion in his second paper in the same volume, "The impossibilities of Einstein fields immersed in flat space of five dimensions". Using the theory of quadratic differential forms, Kasner deduced that a general Riemannian manifold of m-dimension "can always be regarded as immersed in some flat space of n-dimension, where n does not exceed ½m (m+1)" (Kasner, 1921b,p.126).

Thus if m = 4 as in the Einstein theory, the form as before

$$(ds)^2 = \sum g_{ik}\, dx^i dx^k \quad (i, k = 1, 2, 3, 4)$$

can be immersed in an "n-flat" where the possible values of n are 4, 5, 6, 7, 8, 9 or the maximum of 10. Kasner then examined which of the values of n were actually realisable, if the manifold is required to obey Einstein's equation of gravitation $G_{ik} = o$. He noted that the case n = 4 was Euclidean and trivial, since the curvature vanished and there was no permanent gravitation. His paper then went on to demonstrate that the case n = 5 was impossible. No Einstein manifold could be regarded as embedded in a five-flat, if the ten gravitational equations for $G_{ik} = 0$ representing a permanent gravitational field were to be satisfied.

However, Kasner did show that in a flat space of six dimensions, actual Einstein manifolds did exist. He referred in particular to the solar fields which he discussed in his next paper "Finite representation of the solar gravitational field in flat space of six dimensions" (Kasner, 1921c, p.130). It could only be embedded in a flat space of more than five dimensions. Kasner demonstrated mathematically that for the solar field six dimensions are actually needed for embedding ("imbedding"), giving finite solutions in six Cartesian coordinates. "This spread may be described as a geometric model of the exact field in which we are living" (ibid.,p130).

The 1922 final fourth paper generalised the above results: "Geometric theories on Einstein's Cosmological Equations" (and had already appeared in Science Vol.54 in 1921). This time Kasner used equations of gravitation from Einstein's later introduction of "a so-called cosmological term" involving a constant λ

$$G_{\mu\nu} - \lambda g_{\mu\nu} = 0 \ .$$

Kasner used Einstein's more recent equation of 1919,

$$G_{\mu\nu} - \frac{1}{4} g_{\mu\nu} G = 0$$

where G is the scalar curvature. Following Einstein, Kasner used the ten cosmological equations involving one extra dependence as compared with $G_{\mu\nu} = 0$.

Kasner derived one solution where the four principal curvatures are equal at every (umbilical) point - a hypersphere which is actually imbedded in a 5-flat, and sometimes referred to as De Sitter's 'Spherical world' (Kasner, 1922, p.218). The second solution dealt with a 'hyperminimal spread' with every point semi-umbilical and the Riemannian curvature not constant (Theorem I). His conclusion, in Theorem 5 of that paper, was that the only solution was one which "can be imbedded in a 6-flat with cartesian coordinates $(X_1\ X_2\ X_3\ X_4\ X_5\ X_6)$. He grouped them in finite representations

$$X_1^2 + X_2^2 + X_3^2 = 1 \ ; X_4^2 + X_5^2 + X_6^2 = 1 \quad \text{(ibid., p.221).}$$

Excluding the obvious flat and spherical solutions, this was the <u>simplest</u> solution of Einstein's equations which had been obtained, and was the first case where the finite solution was an algebraic spread.

J.A.Schouten and D.J.Struik in fact gave an independent proof of one of the theorems in Kasner's final paper : Only manifolds of constant Riemannian curvature which obey the cosmological equations can be represented on a 4-flat - i.e. of spherical or pseudo-spherical character. (Schouten and Struik, 1922). There were no comments in subsequent editions of the journal in which Kasner published his work.

The significance was only seen later; Kasner's results were referred to as a fundamental paper in much later volumes (e.g. Fialkov,1938).

Kasner's was an entirely mathematical approach. Interestingly, although 6 dimensions seemed enough, Kasner noted that the maximum number of dimensions required to embed or immerse four spacetime dimensions was ten: $n = \frac{1}{2} m (m + 1)$, where $m = 4$.

Thus the four dimensional vacuum space needed six dimensions of flat Euclidean embedding space, i.e. for $G_{\mu\nu} = 0$ (or $R_{ij} = 0$ in earlier nomenclature). This helps the conceptualisation of the concept "curved", which is only an analogue model. It becomes meaningless except when one space is immersed or embedded in another. Most scientists would deny any real existence to these higher dimensions, but consider them valuable for visualising, for conceptualising the 'curved' manifold of spacetime.

(7) The implications of curved spacetime

Despite the newspaper headlines in 1919 declaring its success, and although General Relativity was recognised as a major conceptual revolution, it was of little practical significance for normal terrestrial gravitational fields. Nevertheless it made a number of predictions that were tested in the following years, confirming that as a theory of gravitation, the General Theory had strong claims to supersede Newtonian mechanics. Firstly it had cleared up an anomaly observed by nineteenth century astronomers, in the motion of the planet Mercury about the sun, where it did not conform to Newtonian mechanics. Then, as we have also noted, the prediction that the sun would deflect light rays passing close to its edge was confirmed in 1919. Einstein himself was chiefly impressed by the power of his mathematical structure to define the ultimate nature of physical theory. Nevertheless he was not completely satisfied. His General Relativity possessed two kinds of ontology. There were two ontological categories, fields and particles, both with their rôles to play in the theory. Einstein however was convinced by 1915 that reality had only one type of ontological category - the field.

Einstein was also dissatisfied that there was no unified treatment of the phenomena of gravity and electromagnetism. These two aims led to Einstein's quest for a new and better relativity, the unified field theory (see Chapter 5). Meanwhile a mathematician, Theodor Kaluza, was to initiate just such a revolution. Little known and only belatedly recognised, his breakthrough was to try to unify the two forces using a spacetime of five dimensions - in 1919, only published in 1921 (see Chapter 3).

(8) Postscript : Problems arising from the General Theory of Relativity

Although General Relativity is now one of the key topics of fundamental research, at the time it was so far in advance of any real application that it was isolated from mainstream physics and astronomy for about forty years. For terrestrial and normal astronomical purposes, Newtonian gravity gave an adequate description of most isolated astronomical systems. Only in the 1960's, in studying the cosmology of the Universe as a whole, did Einstein's theory of gravitation become extremely relevant.

Einstein's first paper on cosmology appeared in 1917 (Lorentz et al., 1923), well before Edwin Hubble discovered the expansion of the universe. In the first self-consistent cosmological model for a homogeneous unbounded universe, Einstein felt himself obliged to introduce the so-called "cosmological constant" Λ to allow a static universe. He had realised that his theory predicted an expanding universe from an initial singularity. This was the simplest solution of this equation and was very much against the prevailing ideas. In 1922, the mathematician Alexander Friedmann showed clearly that the equations of Einstein's theory had solutions that implied an expanding universe. Einstein later regretted his addition of the cosmological constant, calling it one of his major mistakes; it was certainly his greatest missed opportunity.

(a) The "Big Bang"

The present evidence in fact allows us to trace the history of our Universe back to within fractions of a second of the initial 'big bang'. Friedmann's model has remained precise and consistent with Einstein's ideas and Hubble's observations. The first evidence of the cosmological application of General Relativity came with the discovery of the red-shift by Edwin Hubble. The wavelength patterns of the light from other galaxies were found to be shifted towards its red or longer wavelength in the spectrum. The only satisfactory explanation (an approximate analogy is the Doppler effect with sound waves) was that the galaxies are moving away from us. Hubble's results showed that the redshifts of galaxies are proportional to their distance. This has now been extended and confirmed "by observations of galaxies so far away that they are receding at more than half the speed of light" (see Rees, 1980, p.109).

The commonest analogue model to describe the expansion is the two dimensional surface of a balloon being blown up. Each galaxy (represented by a dot on the surface) expands away from the others. There is no absolute centre. Although this is a useful conceptual aid to visualisation for the expansion of four dimensional spacetime from a point singularity, the space around the balloon has no definite physical meaning; the balloon is all of two dimensional space. For our universe, spacetime itself expands from an infinitely small singularity. Questions about what "surrounds" the spacetime of our universe are not physical questions, but are about the reality of the extra embedding dimensions model.

Further accepted evidence for the 'Big bang' came from observations of a background of microwave radiation, discovered by accident at the Bell Telephone Laboratories by Arno Penzias and Robert Wilson in 1964/5. This diffuse background radiation (with energy equivalent to a temperature of about 3°K) is one of the main reasons why the expanding universe model and the Big bang theory of creation has steadily become the dominant idea in cosmology. The theory of the Big bang, worked out in the 1940's by George Gamow and others, correctly predicted both the existence and the intensity of the radiation. This work was largely forgotten, however, until the discovery of the microwave radiation twenty years later.

On the Big bang theory, the Universe is expanding from an initial condition so hot and dense that the entire present day Universe was contracted into an extremely small volume of almost negligible size. The explosion from an infinitely dense, microscopically-sized universe which evolved and produced the now receding distant galaxies occurred about fifteen billion years ago. At a finite time in the past ("t = 0") "The beginning", all the matter of the observed expansion was concentrated in a (mathematical) point of infinite density. Mathematicians call the state of affairs a 'singularity', and physicists a 'big bang'. Singularities imply an end of spacetime as we know it, a breakdown in the known laws of General Relativity (Weinberg, 1977). For spacetime to have a beginning implies the creation of spacetime itself. The known laws of physics at that point are incomplete and irrelevant (Rees,1980).

(b) The 'Big Crunch'

There are three kinds of generalised models from Friedmann's solutions. Firstly the galaxies may be moving apart sufficiently slowly for the gravitational attraction between them to eventually overcome the expansion. They will then start moving closer again. The universe will thus expand to maximum size and then recollapse to a singularity again. Secondly the galaxies may be expanding too fast and there is not sufficient matter in the universe for gravity to prevent the Universe expanding for ever. Finally in a third scenario, the galaxies may be moving apart at just the critical rate to avoid collapse.

In principle we can decide which is correct by estimating the average density of the universe. In fact the mass of the visible universe is not enough to stop the expansion. The mean density of matter in the luminous visible part of the galaxies falls short of the critical density by a factor of almost 100 (Loh and Spiller, 1986, p.L1). There is much evidence from calculations based on dynamical arguments of the rotation of galaxies that there is far more 'invisible mass' which we cannot see. Spiral galaxies and clusters of galaxies move too fast for the observed visible matter (Hut and Sussman, 1987, p.141). Apart from this extra 'dark mass', there may be more material between the clusters of galaxies.

Many suggestions have been made to explain this missing or dark matter. Cosmic strings, (loops of massive one-dimensional material) neutrinos or intergalactic black holes have been suggested, but may well be too elusive to be detected. It is certainly possible that there is enough material to cause the universe to recollapse.

Einstein was himself aware of the missing mass problem (Einstein,1921d).

In his 'Meaning of Relativity', the later editions after 1923 argued that there could only be a lower estimate and that the proportion of 'dark' matter should be larger outside galaxies than within. If the universe does recollapse, there will be another cosmic singularity, the 'Big crunch', where the curvature of spacetime is again infinite and space and time come to an end. The concepts of space, time and dimensionality would cease to have any meaning. General Relativity laws of physics break down and again a new physics is needed (S.W.Hawking and W.Israel, 1979).

(c) Black Holes - Singularities within the Universe

Another application of General Relativity, testing it beyond its limits, is the intense curvature of the singularity inside a Black hole. These are usually stars which, after a supernova-type explosion, have collapsed to such small dimensions that no light or indeed any other signal can escape. The possible occurrence of black holes is in fact a consequence of almost all theories of gravity. The first theoretical description was given in 1917 by Karl Schwarzschild. There are fundamental and far-reaching paradoxes associated with the singularity at the centre of the black hole : time would stand still, and space would behave in "peculiar and non-intuitive ways". (Rees,1980, p.102).

The significance of the collapse of a star of more than a certain mass was provided by Robert Oppenheimer in 1939 (Oppenheimer and Snyder, 1939). This mass was calculated to be about two and a half times the mass of our sun, by S.Chadresekhar and L.D.Landau in the early nineteen thirties . Such a star would collapse down to a single point - a singularity - under its own gravity after an initial explosion. Most scientists at the time refused to take the extrapolation of the accepted laws seriously. Even Einstein and Eddington were adamant.

Einstein's belief in the inadmissability of singularities was so deeply rooted that it drew him to publish a paper purporting to show that the "Schwarzschild Singularity" at radius $r = \frac{2GM}{c^2}$ does not appear in nature (Einstein, 1939). His reason was that matter cannot be concentrated arbitrarily - because otherwise the constitutary particles would reach the velocity of light. (In fact Einstein allowed an exception in the two sheeted manifold for a singularity which was first introduced with Rosen (Einstein and Rosen, 1935).

This denial that such collapsed objects could exist was submitted in 1939, two months before Oppenheimer and Snyder (1939) submitted their theory on stellar collapse. It is not known how Einstein reacted to this.

Belief in the physical significance of Black holes was encouraged by the discovery of quasars (quasi-stellar objects) in the early ninteen sixties, which were thought to be similar in nature to Oppenheimer's collapsed objects. The Penzias and Wilson discovery of the background radiation in these years was interpreted as a fossil or relic of the original singularity.

The increase in physical status was strengthened by the theories of Penrose and Hawking (see Hawking and Ellis, 1973). Between 1965 and 1970, Roger Penrose and Stephen Hawking proved a number of theorems which showed fairly conclusively that there <u>must</u> have been a singularity if General Relativity was correct. These conclusions were independently proved by F.M.Lifshitz, I.M.Khalatnikov and V.A.Belinsky (in 1969). These proofs further encouraged the belief in the existence of real singularities in the universe. Such a collapse was also calculated to be true even if the star was not exactly spherical - the Kerr model (1963).

There are deeper implications of the immense curvature in the beginning (and possible end) of spacetime in these "holes in space". Such regions of spacetime, where neither light nor any other energy or matter could escape (Penrose's "cosmic censorship" phenomena) were christened "Black holes" by John Wheeler, who initiated much of the work on them in the late sixties (Wheeler, 1968).

(d) The Existence of Black holes

The search was intensified after the discovery in 1968 of rapidly pulsing radio sources or "pulsars". These were interpreted as rotating neutron stars, about the mass of the sun, but with a radius of only ten kilometers. Black holes themselves could be observed only indirectly by their gravitational effect on nearby matter, e.g. as one of a pair of twin stars, rotating round its twin (visible) star.

The first accepted identification was the X-ray source Cygnus X-1 in our galaxy, a binary star with hot matter from the visible twin sucked into the Black hole, emitting X-rays in the process. Apart from possible stellar-mass black holes such as Cassiopeia A, and LMCX-1 there is increasing evidence of super-massive Black holes at the centres of galaxies. Examples are NGC 5548, Centaurus A, elliptical galaxies NGC 6151, 3 C 449, M.87 and at the centre of our own galaxy. The central power-house for the energy of a quasar is widely believed in the 1980's to be a supermassive Black hole.

Most astronomers in 1987 agree that quasars occur in the centres of a good proportion of all galaxies, perhaps rather similar to our own Galaxy. According to some theorists, there was a delay in black hole formation of several billion years from the age of formation of galaxies, 15-18 billion years ago, representing the time required for a galaxy to build up a massive black hole in its nucleus (Miller, 1987, p.60). Such black holes, millions of times more massive than our sun, may also serve as the hubs of the Milky Way's closest neighbours, the great spiral galaxy in Andromeda and its smaller elliptical companion M32, two million light years from the earth (Rickstone, et al., 1987). Violent collisions between spiral galaxies are now thought to fuel quasars with supermassive black holes at the heart of each galaxy. The distinction may only be that of degree, including quasars, galaxies and the intermediate Seyferts (from Carl Seyfert who found the first "active" galaxies in 1943). Possibly all galaxies are centred upon black holes, very massive in the case of quasars. A recent report from astronomers at NASA in California have found gamma coming from the vicinity of a Black hole in our galaxy, Cygnus X-1. This should help to provide a new test for distinquishing black holes from neutron stars (Ling et al., 1988: "Gamma rays reveal Black Holes"). It is thought that the black hole sucks in surrounding gas, matter (and even other stars in a massive black hole). The gravitational energy released heats up the gas, thereby converting the gravitational energy into radiation. (The future detection of gravitational waves themselves would be the best clear and unambiguous evidence.)

It seems that the theoretical concept of black holes "has been substantiated by a number of observational discoveries" and that black holes "are probably responsible for the most bizarre and energetic objects in the Universe" (Hutchings, "Observational evidence for black holes", 1985,p.59).

The mathematical concept of a "singularity" covers up the unimaginable concept of the space of our universe being "punctured" (Rees,1980,p.107) in a "black hole", a "hole in space", a "rent in spacetime", where space and time themselves come to an end, and the concepts transcend contemporary physics, even to joining "another universe" (Penrose 1968, p.222).

Stephen Hawking in 1974 discovered that black holes emit thermal radiation. The potential barrier around the hole created by the gravitational field, a barrier that could not be surmounted classically (Hawking and Israel 1979, p.18), is breached by "quantum mechanical tunnelling" (see Chapter 4). This final disappearance of a black hole is however only forecast on a small scale, and is only significant for 'mini-black holes'. This was confirmed by Hawkings in his "Quantum Mechanics of Black Holes" (Hawking 1977, p.37) when he described a black hole as "a region of spacetime from which it is possible to escape to infinity". ("Primordial evaporating black holes" have in fact been clearly demonstrated by Arnold Wolfendale and others at Durham; P.Kiraly et al., 1981,p.120).

(e) Intense curvature on the very small scale : Foam Space and Geometrodynamics

In order to avoid the Schwarzschild singularity, Einstein and Rosen represented the solution by two perfectly symmetrical spaces, instead of having one space that curves up sharply and comes to a cusp at the point $r = 0$ (Einstein and Rosen, "The particle problem in the general theory of relativity", 1935). Both of these symmetric spaces asymptotically approach Euclidean space at great distances, joined together by what they called a "bridge" (the 'Einstein-Rosen bridge') centred at $r = 2m$ (where $r = 2m + \frac{W^2}{8m}$). This value was the radius of the largest sphere that could fit into the narrowest part of the bridge at its centre. In trying to go beyond this value, one simply moved on to the other sheet of the total space, and $r = 0$ corresponded to the point at infinity on this other sheet.

John Wheeler took over this idea of a multiply-connected topology and put it to more general use. By allowing the two Einstein-Rosen sheets to be part of a single space, but very far removed from each other, he interpreted the "bridge" as a "handle" on the space, or a 'wormhole'. Einstein and Rosen's bridge between two identical spaces had seemed to introduce a separate 'mirror-space' for each particle, proliferating these unrelated and apparently uninterpreted spaces.

There was a way of removing singularities, by giving up the requirement that spacetime should have a Euclidean topology and by allowing multiple connections within the space. This modification of Relativity Theory became known as Geometrodynamics. This is the study of curved, empty, multiply-connected space and its evolution in time according to the equations of General Relativity.

The idea was first proposed by G.Y.Rainich (1925), but received little attention until rediscovered by C.W.Misner, who developed it further with Wheeler (Wheeler and Misner, 1951). Here the electromagnetic field was viewed as a particular distortion of the spacetime metric - "lines of force trapped in the topology of space", and Wheeler suggested a "foam-like" structure on the Planck scale of length (Wheeler, 1964).

Hermann Weyl following Riemann's description of multiply-connected topologies, had in fact also used this model. He described it as an elementary piece of reality which has "tiny handles attached which change the connectivity of the piece" (Weyl(1927) 1949,p.91 quoted in C.W.Misner et al.,1973,p.221). Wheeler's analogy was of a wave evolving continuously until it crests and breaks up into a foam, where we need more than the normal physical laws of wave motion for a complete explanation of the phenomenon. As Graves pointed out, as in the case of singularities in classical General Relativity, 'elements of mystery' are admitted in the hope that they will somehow be clarified once the theory has progressed to a higher stage (see Chapter 5, Graves, 1971).

Geometrodynamics was a very interesting model on a qualitative basis, but was never completely accepted. It lacked the conceptual strength of a clear multidimensional approach. Wormholes as a model has not passed into current use. However it has not been an abandoned model, but has been developed as a foam space model of spacetime by Hawking and others (Atiyah,1982).

The wormhole model for electric charge implies extra dimensions. Conceptually it can be viewed as embedded in higher dimensions, although no physical meaning is necessarily to be attached (Penrose, 1978). Quantum fluctuations of geometry are also involved. Quantum jumps of topology are said to "pervade all space at the Planck scale of distances to give it a foam-like structure" (Wheeler, 1980, Ch.22 "Beyond the Black Hole").

(9) Conclusion : Reappraisal of General Relativity - the need for a new physics

Thus ideas of space and time are breaking down at singularities both on the large scale and micro scale. For Wheeler, the concept of a continuum breaks down. "Space" and "dimensionality" are only approximate words for an underpinning substrate, a "pregeometry" that has no such property as dimension, whether in the big bang or in the black holes or in foam space (Wheeler, 1980,p.351).

Four dimensional space begins to break down at the Planck length, when ideas of quantum mechanics are applied to general relativity, to give violent fluctuations in a foam-like character. The concept of dimensionality itself ceases to have any meaning. The laws of physics break down at " singularities in spacetime " (Misner, et al.,1973,p.613). For Wheeler, three dimensional geometrodynamics, both classical and quantum, "unrolls in the area of superspace" (ibid.,p.740).

Developments in quantum gravity involve using n-dimensions to make the theory work, then "transposing back to fit the conventional four dimensions" - but gravity is not renormalisable (i.e. the presence of infinite terms in the theory cannot be removed by adjusting the zero point on the scale by an infinite amount, as in Quantum electro-dynamics). "We need a new physics" (G.t'Hooft, 1973, ibid.,p.336). t'Hooft suggested removing the idea of continuous spacetime and replacing the continuum with a discrete discontinuous spacetime, "a totally new physics is to be expected in the region of the Planck length for a start" (ibid.,p.344).

As Hawking and Israel noted, classical General relativity was very complete, but failed to give a satisfactory description of the observed universe. By taking the model seriously, it leads inevitably to singularities in spacetime, where the theory itself breaks down. It does not provide boundary conditions for the field equations at singularities (Hawking and Ellis, 1973, Chapter 15, Ed.Misner et al.). The singularities are predicted to occur at the beginning of the universe and in the collapse of stars to form black holes, as well as in the foam-like structure of space on the Planck scale of length, where Hawking and Israel suggest the use of higher dimensions (ibid.,p.789). Even the topological structure itself may be too conservative, a totally new physics is to be expected.

Roger Penrose was also trying to reformulate the basic concepts of space and time with his twistor calculus (see Chapter 8). "One needs a deeper understanding of the structure of space" (Penrose, 1984,p.8) - a new mathematical language and a new physics". Singularities in spacetime tell us that our present approach to spacetime geometry is really inadequate for handling all circumstances in physics (ibid.,p.8).

The presence of singularities is usually taken as a sign that the theory is incomplete and needs a more consistent explanation. The astronomer Martin Rees commented that "near the singularity naïve ideas of space and time become very inadequate" (Rees, 1980, personal communication). He also described the paradoxes associated with the singularity as far reaching in their implications. He believed that such physical uncertainties may involve something fundamentally new.

Even in the early 1970's, physicists such as John Wheeler and Dennis Sciama saw the need for a new approach. "General relativity itself must breakdown in the occurrence of physical singularities" (Sciama,1973, Ed.Mehra; p.19). We therefore face a crisis in theoretical physics" (ibid., 1973,p.32). Physicists such as Sciama and Rees hoped that quantizing General relativity might resolve the crisis. The Big bang origin of the universe and the existence of Black holes in the universe are widely accepted examples of singularities. Although cosmic strings may provide an alternative model for quasars (e.g. Superconducting cosmic strings, Hogan, 1987,p.742), Black holes are a part of the well-accepted scenario of contemporary physics.

The 'Big crunch', indicating the way the universe ends, is less widely accepted as the standard model. Current estimates of Λ, the cosmological constant, are so close to zero that the result is uncertain, although theorists imply there is about 100 times more dark matter in the Universe than all the visible matter we can observe (Loh and Spiller, 1986). John Barrow and Frank Tipler argued for a spherical universe, closed in space and time. Located in a singularity, the universe will go through a cycle of expansion and collapse to end in a singularity - real physical events which crush matter out of existence (Barrow and Tipler, 1985,p.395) (- or perhaps leave this universe altogether). However an inflationary theory such as Alan Guth's proposal in 1981, that the galaxies fly apart, but decelerate to an equilibrium, is still a possibility. In any case, the universe may "bounce" at a possible Big crunch, thereby avoiding the singularity.

Nevertheless singularities of the Big bang and in Black holes are widely accepted. Some physicists would even equate particles with black hole type singularities (Green, 1987). The joining of cosmology and high energy particle physics may be essential.

Certainly physicists such as Steven Weinberg think the "absurd features" of General relativity cannot be corrected. On the small scale "I think that general relativity is wrong" (Weinberg, 1979 "Einstein and Space-time. Then and Now", p.42). Steven Hawking accepts the probability of the singularity at the end of the recollapse of the universe. "Singularities are places where the curvature of spacetime is infinite, and the concepts of space and time cease to have any meaning (Hawking, 1984 "The Edge of spacetime",p.12). The need for a new physics is paramount. There is even an acknowledgement that a "purely metaphysical" approach is implied before the Big bang (Hawking, ibid.,p.12).

<u>The way ahead</u>

There are problems and paradoxes even in the first major revolution of the twentieth century, Einstein's theory of General Relativity, mainly centered on the existence of singularities. There is a need for a theory relating quantum theory to general relativity, a need for a unified treatment of Gravity and electromagnetism (and also the two nuclear forces) - a unified field theory.

"We don't yet know the exact form of the correct quantum theory of gravity. It may be some theory we have not thought of"..."It may be some version of supergravity or it may be the novel theory of superstrings" (Hawking, July 1987, p.48).

Chapters 7 and 8 will explore these possibilities. There are many attempts to achieve a unified field theory, many of which involve increasing the dimensionality of spacetime. The curved spacetime of General relativity produced the need for higher embedding dimensions to conceptualise the extrinsic curvature. This was needed both mathematically and conceptually, although no physical interpretation of these dimensions was implied.

In supergravity and strings, extra dimensions are also needed, which are increasingly given high physical status. The basic idea was entirely due to a little known physicist, Theodor Kaluza, who published his unified field theory involving five dimensions of spacetime in 1921. Chapter 3 will explore the origins and the effect of this unique creative idea which was to revolutionise physics half-a century later. Why was the idea neglected for so long, and why is it now so widely used?

CHAPTER 3 Theodor Kaluza's unification of gravity and electromagnetism in five dimensions

Synopsis

Introduction

1. Kaluza's 1921 paper - the mathematics
2. Precursors:
    (i) Thirring and Weyl - acknowledged in Kaluza's paper
    (ii) Nordström, 1914, a little known earlier attempt at unification in five dimensions.
    1914 Paper: Biographical details, and reactions to his paper; Conclusion
3. Why Kaluza's paper was almost completely neglected.
    (i) The two year delay in publication
    (ii) The delay in Kaluza's own promotion
    (iii) Kaluza's personality; teaching and publications
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4. Sources of inspiration.
5. Reactions to Kaluza's paper.
6. Conclusion.

Introduction

Although we do seem to live in three dimensions of space and one of time, combined together in Einstein's four dimensions of spacetime, there is evidence today of the need for a deeper physics.

The first attempts to introduce extra dimensions into our description of spacetime seem however to have been largely ignored until the last decade or so. The real origins lay in a paper by Theodor Franz Edward Kaluza (1885-1954), an almost unknown privatdocent at the University of Königsberg, now Kaliningrad in the USSR.

In 1919, Theodor Kaluza arrived at his now celebrated unification of the forces of gravity and electromagnetism. Instead of the four dimensions of spacetime which Einstein had used, Kaluza extended the dimensionality to five and showed that this led to a remarkable fusion of gravity and electromagnetism. For Kaluza the resultant five dimensional metric was a description of the world, not a mere mathematical device. His theory has until relatively recently, however, suffered consistent neglect. The problem which needs to be solved is why his idea was ignored, when it is today widely felt to be very important.

(1) Kaluza's 1921 paper

Theodor Kaluza's Unification of Gravitation and Electromagnetism in Five Dimensions - the mathematics

- Kaluza (1921) "Zum Unitätsproblem der Physik" ("On the Unity Problem of Physics").

Einstein had used a tensor calculus to describe the metric of a four dimensional spacetime continuum. Kaluza combined the ten gravitation potentials which arose in Einstein's General Relativity theory with the four components of the electromagnetic potential of Maxwell's theory. He did this by means of his fifth dimension.

The essential mathematics can be stated quite simply. In Einstein's theory the gravitational field is contained within the "metric tensor" $g_{\mu\nu}$ which expresses the interval (ds) as

$$(ds)^2 = \sum_{\mu,\nu=1}^{4} g_{\mu\nu}\, dx^\mu\, dx^\nu$$

where dx^μ $(\mu = 1,2,3,4)$ is the change in the x^μ coordinate.

This formula generalises the familiar (Pythagorus' Theorum) result in two flat dimensions $(ds)^2 = (dx)^2 + (dy)^2$:-

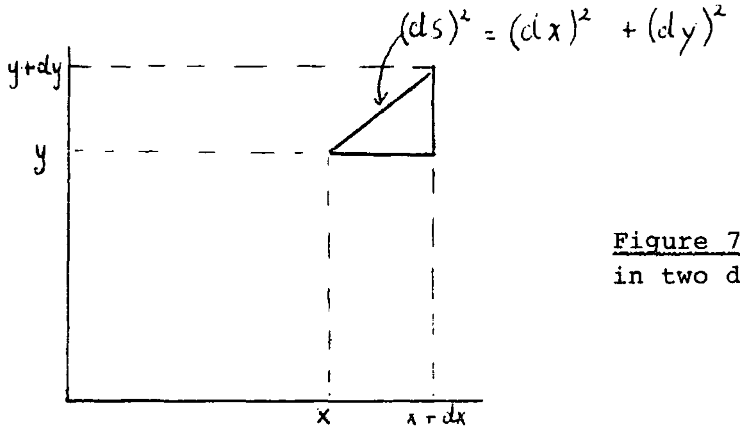

Figure 7 The line element $(ds)^2$ in two dimensions (Pythagorus' Theorum)

In the absence of gravitational fields the coordinates can always be chosen such that

$$g_{\mu\nu} = \begin{pmatrix} +1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

hence,

$$(ds)^2 = (dx^1)^2 - (dx^2)^2 - (dx^3)^2 - (dx^4)^2$$

Here, $x^1 = ct$, is the "time coordinate". The interval given in this last equation is appropriate to special relativity (inertial frames, no gravitational field). More generally, $g_{\mu\nu}$ is a symmetric tensor which has 10 (=4+3+2+1) entries

The generalisation to 5-dimensions in then:

$$(ds)^2 = \sum_{m,n=1}^{5} g_{mn}\, dx^m\, dx^n$$

$$= \sum_{\mu,\nu=1}^{4} g_{\mu\nu}\, dx^\mu\, dx^\nu + 2\sum_{\mu=1}^{4} g_{5\mu}\, dx^5\, dx^\mu + g_{55}\,(dx^5)^2$$

The enlarged tensor now has 15 entries. Ten of these are the original $g_{\mu\nu}$ describing the gravitational field. Four of them, $g_{\mu 5} \equiv g_{5\mu}$ are a vector (one index) in the physical space of 4 dimensions. Kaluza identified this with the electromagnetic vector potential:

$$A_\mu = (\text{constant})\, g_{\mu 5}$$

The remaining entry g_{55} is a scalar (it has no indice in the physical space).

In general of course, all g_{55} are functions of the $x^1 \ldots x^5, \ldots$ Other assumptions have to be made:

(a) g_{55} = constant (This gets rid of the scalar).

(b) All $g_{\mu\nu}$ are independent of the newly introduced fifth coordinate x^5 - a key assumption. Einstein's equation of pure gravity in five dimensions thus gave not only the correct gravity equations for $g_{\mu\nu}$ in four dimensions, but also the correct Maxwell equations of electromagnetism for A_μ, (-and also a Poisson equation, although this was made constant by Kaluza, who identified it at the time as a "negative gravitational potential"). Kaluza's idea thus produced the symmetry of the combined Einstein-Maxwell equations in one Lagrangian. In other words, Maxwell's theory of electromagnetic fields can be seen to be a consequence of Einstein's theory of gravitation restated in five dimensions.

Notes 1. The positive sign of g_{55} implies that the fifth dimension is metrically space-like.

2. The condition where $g_{\mu\nu}$ are independent of x^5 is called the "cylinder" condition (condition of cylindricity), i.e.

$$\frac{\partial g_{\mu\nu}}{\partial x^5} = 0$$

3. A geodesic in this cylinder world can be identified with the motion of a charged particle moving in a combined gravitational electromagnetic field. Kaluza could thus correctly deduce that the charge/mass ratio for an electron is a constant in time.

2. Precursors of Kaluza's Unification in five dimensions

(i) Two acknowledged pre-cursors: Hans Thirring and Hermann Weyl

Thirring and Weyl were referred to by Kaluza himself in his 1921 paper. Kaluza had written some earlier papers e.g. on the rotation of a rigid body and the higher geometry that applies to it (Kaluza, 1910) so as to represent the phenomena on the Special Relativity theory. However his interest in the potential similarities between the formulation of General Relativity and of Electromagnetism was aroused by a paper by Hans Thirring.

(a) Thirring had already noted the formal unity of the equations of gravitation and electromagnetism. His paper (Thirring 1918) derives a "formal analogy" between the Maxwell-Lorentz equations for electromagnetism, and those which express the motion of a point in a weak gravitational field. Thirring notes (ibid., p.205) that "it seems to be very unlikely that mathematical laws which represent one area of appearance ..... should also exactly describe the formulae of a different area of appearance." Although Thirring thought that it was indeed no coincidence, he did not himself explore the significance. His paper describes only the spacetime of four dimensions.

(b) An attempt at the unification of gravitation and electromagnetism by Hermann Weyl (1918) also made a great impression on Kaluza. This was regarded at the time as the first attempt at a unification of Einstein's and Maxwell's theories, although Weyl restricted himself to the four classical dimensions, based on Einstein's spacetime dimensions. Weyl used a generalisation of Riemannian geometry in the usual four dimensions. He associated an additional gauge vector field with the Einstein metric tensor. Weyl thus proposed to modify the geometric structure of spacetime by abandoning the assumption that the length of vector is unchanged by parallel displacement - a "gauge transformation".

The implications of Weyl's gauge theory were that sizes, e.g. of atoms, could vary in different coordinate positions. This produced the difficulty that the varying history of individual atoms was difficult to reconcile with their experimental identity - all atoms of a given element emit the same frequency of spectral lines. The possibility of linking this with the red-shift was ignored. Although he arrived at a non-Riemannian spacetime, with the same ten metric tensors (potentials of the gravitational field) as in General Relativity, together with an electromagnetic four-vector potential, Weyl's theory was still in four dimensions. Einstein's criticism of the varying history of atoms, together with the lack of predictive power, led to the theory being abandoned, e.g. by Weyl himself within a few years of publication.

Nevertheless, Weyl's principle of gauge invariance was a brilliant conception and laid the foundation of the later success of the gauge theory (used later by Yang & Mills, Weinberg etc.) Weyl's theory, as found also in the first German edition of his Raum-Zeit-Materie of 1918, contained many other creative ideas. He regarded the electron as a sort of 'gap' or 'hole' in the non-Euclidean spectrum, as a local wrinkling of spacetime. This was developed in the next year or two by Weyl in his n-dimensional geometry, embedding the Riemann space in a Euclidean space of higher dimensions (Weyl, 1922, p.23). He developed other creative ideas, e.g. that "particles of matter are nothing more than singularities of the field" (ibid.,p.169). He was also to analyze space as "multiply connected" (Weyl, 1924, p. 56) to describe lines of force "trapped in the topology" of multiply connected space.

Weyl's powerful but prematurely abandoned effort to generalise Einstein's new general relativity made a great impression on Kaluza. As Kaluza uniquely noticed, if Weyl is taken seriously the theory needs extra dimensions of space. This was one of the reasons for Kaluza going in this direction and abandoning the limitations of four dimensions. Incomplete yet suggestive, Weyl's theory lacked the further originality of breaking the classical four-dimensional model which was to be the necessary innovation.

(ii) A little-known earlier attempt at unification in five dimensions : Nordström, 1914.

In 1914 the Finnish Physicist Gunnar Nordström of Helsingfors (now Helsinki) University had attempted to give a unified description of the two known forces of electromagnetism and gravity using a five dimensional space. Kaluza appears not to have known (Th. Kaluza, Jun. 1984) of this one previous attempt at unification in more than four dimensions. Certainly Kaluza made no reference to this proposal. Although Hermann Weyl does draw attention to Nordström's paper in the notes after his fourth chapter in Space, Time and Matter (1922) which is based on his earlier article, this was not mentioned in the original paper (Weyl 1918) nor in the footnotes. It appears that neither Kaluza nor Weyl (Kaluza's main reference) knew of Nordström's theory in 1918/1919, although it was drawn to Weyl's notice by the time of the fourth edition of his book (1922, Note 4 and 33).

Nordström's paper (written in German for the Physik Zeitschrift, 1914) was called "On the possibility of uniting the electromagnetic field and the gravitational field." He based his unification on the need to introduce a fifth world dimension. "The five dimensional world has a singular axis, the w-axis"where "the four dimensional spatio-temporal world stands vertical to the axis, and in all its points the derivation of all its components in relation to w equals zero" (Nordström, 1914, p.505). This in fact is the cylinder condition, again anticipating Kaluza.

Nordström's remarkable but little known attempt at unification in five dimensions poses the questions of why this was not recognised, and why Nordström was given no credit for the five dimensional idea.

Nordström's 1914 paper

Certainly Nordström's was the first unification of electromagnetic fields with the gravitational field. He was the first to point out the "formal advantages" (Nordström, 1914, p. 506) in understanding these as one field. While admitting that " a new physical content, however, is not given to the equations by this", Nordström nevertheless thought it not impossible that "the found formal symmetry could have an underlying reason" (ibid., p.506). However he did not want to enter into the implications of this.

No references are given by Nordström to any other scientist with regard to his five dimensional theory. Apart from acknowledging his work with Mie on his purely gravitational theory of 1913, and Minkowski's 1908 theory which uses a 6-potential vector to describe electromagnetism, Nordström gives references only to his own earlier works (1912 and two papers in 1913). Minkowski's work in any case does not apply when a gravitational field is added to the electromagnetic field, whereas Nordström's approach in five dimensions does show a possible way forward.

Nordström's interpretation of the electromagnetic equation in five dimensions shows that it is

> "legitimate to understand the four dimensional spatio-temporal world as a plane laid through a five-dimensional world" (ibid., p.504).In this five dimensional world, the four-potentials of gravitation and the six potentials of electromagnetism can be combined using the ten vectors of a five-dimensional world.

<u>Biographical details of Nordström, and reactions to his paper</u>

Gunnar Nordström was born in Helsinki on March 12th, 1881. His father Ernst Samuel Nordström was the director of the Arts and Crafts School and curator of the Finnish Society's museum (Helsinki Archives - E. Vallisaari,1986). Gunnar was taught at school in Swedish and left in 1899, graduating in 1903 with a degree in mechanical engineering from the Helsinki Polytechnic Institute. Nordström made exceptionally rapid progress to complete the Masters degree at the highest possible grade in 1907 under Professor Hjalmar Tallqvist at the University of Helsinki. He continued studying science at Göttingen University for his Licentiates degree in 1909, and on the basis of this, the degree of Doctorate was conferred upon Nordström in 1910.

From being a privat-docent in Theoretical Physics at Helsinki, he was appointed Professor of Physics in 1918 and of Mechanics in 1920. Nordström lectured on theoretical physics (mostly in Swedish).

Nordström's five-dimensional theory passed almost without comment. It was his better known 1913 paper on gravitation which won the support of Einstein at the time. Although it did not survive, it "deserves to be remembered as the first logically consistant relativistic field theory of gravitation ever formulated" (Pais, 1982, p.232). Nordström owed some of these ideas to von Laue, Abraham Mie and Einstein, although the physical conclusions were those of Nordström himself. In a letter to E. Freundlich, early 1914 but undated, Einstein found Nordström's 1913 theory very plausible, but criticised it for being built on the a priori Euclidean four-dimensional space. His approval was noted in a paper (Einstein and Fokker, 1914).

In 1915, Freundlich also referred with approval to Nordström's Relativity theory (in four dimensions). Nordström's unique five-dimensional theory of 1914 found only one champion in J. Ishiwara: "On the five fold variety in the physical universe" (Ishiwara, 1916). Interestingly Ishiwara stressed the physical significance where the differentials of similar quantities with respect to "w" are equated to zero. It followed from Ishiwara however that no physical change takes place in this direction. Ishiwara used a multidimensional general analysis, giving his own physical interpretation. He postulated that at every point in space, there is a direction "w" along which the universal potential remains always constant. The four dimensional space perpendicular to this direction was called "Minkowski's Universe." There were no further references to Nordström's five dimensional theory in the following decade, apart from a critical comment by Von Laue in 1917.

No biography of Nordström seems to have been written. Further details can only be obtained from his own work and letters (either from Swedish or German), and from a speech of commemoration given in 1924 after his death. He was married in 1917, aged 36 and had three children. The last one, a daughter, was born in 1922. Nordström died on Christmas Eve the following year.

In 1915, the year after his five dimensional paper, Nordström applied for the Rosenberg travelling Scholarship. In support of his application, he wrote that the "most important and the most comprehensive task" during his study travels would be "to develop my method of coordinating the electromagnetic field and the field of gravity to bring about a five dimensional field" (letter to the Academic Council, 1915 translated from the Swedish by D. Jowsey). His reports on his travelling scholarships, (all written in Swedish) show that, although he still worked on a five dimensional symmetry, his task remained unfulfilled, and was in fact overtaken by Einstein's 1915 gravitational theory of General Relativity. Nordström applied to go first to Leiden in Holland, "the most suitable for study in time of war" (Nordström, 1915). There he stayed, exploring further Einstein's theory, discussing the progress of the quantum theory (Nordström 1917) writing his book *The Theory of Electricity* (1917c),publishing two papers on Einstein's theory (Science Academy in Amsterdam, 1918), keeping up with other physics topics e.g. radioactivity (and incidentally getting married in August 1917 in Leiden).

Some ideas of Nordström's personality may be gained from the speech (in Swedish) given in commemoration after his death. This was delivered by his old Professor, Hj. Tallqvist at the Conference of the Finnish Science Society (1924). Nordström had born the sufferings of his final illness bravely, still hoping to return eventually to work. Born into a home with idealistic standards, where both artistic and scientific interests prevailed, Gunnar was influenced by other areas besides science and mathematics. His scientific studies included astronomy and chemistry besides physical sciences, and he later published books e.g. on Maxwell's Theory of Electromagnetic Phenomena (1907) as well as on his own speciality, The Theory of Relativity (1910): Space and Time according to Einstein and Minkowski. His main life's work in the area of relativity and gravitation was overshadowed by the work of Einstein, although he won a reputation for himself in Europe. His works were published in German, Dutch, Finnish and Swedish. His work

> "remoulds such hallowed ideas of time, space, mass and energy" so that "some phycisists have felt an instinctive enmity towards it, certainly partly because they have not been able to grasp its full import" (Tallqvist p.8). An additional factor must be noted, that many of Nordström's papers, including the commemoration speech by Tallqvist, were not in English or German, the more common languages of scientific papers.

Nordström's international reputation led to his election as a member of the Finnish Science Society in 1922, but he did not live long enough to lecture at any of their meetings. Not one-sided in any way, Nordström "thought generously and well of his fellow men and was by nature an optimist"

> (Tallqvist, p.12) "his spirit constantly searching, looking for truth and striving to clothe it in clear acceptable forms." His Professor's eulogy ends: "his lofty spirit has found peace and passed from these dimensions which are so relative, to another higher realm - a higher plane in the time and space-less world of eternity."

Despite these words there was no reference to Nordström's own paper in extra dimensions. His idea had not been recognised. He himself fell ill and died in the year following Kaluza's paper (itself unrecognised at the time) without the chance to see Kaluza's version of five dimensions. It was perhaps Von Laue's article which was a critical factor for Nordström's five dimensional idea. In a paper on Nordström's 1913 Gravitational Theory (noted with satisfaction by Nordström, 1917) Von Laue has a short section on Nordström's five dimensional theory, "Beginnings of the Continuation of the

Theory" (Von Laue, 1917). Describing Nordström's 1914 theory of unification through the introduction of a five dimensional world expansion, von Laue noted the appearance of a fifth coordinate w in addition to x,y,z and t. For von Laue as well as for Nordström, "this is for all intents and purposes a purely mathematical question" (ibid., p.310). The extra hypotheses are within the fifth dimensional portrayal but "whose physical meaning comes out less clearly..". ."the consequences coming from these have not yet been followed through." Von Laue pays tribute to Nordström's unusual attempt to unify gravitational and electromagnetism by adding a fifth coordinate, but his criticism that the attempt is not particularly clear, in that it does not solve any problem, marked the end of its serious consideration.

Nordström's approach had to be abandoned because it did not contain general relativity and could not explain the bending of light near the sun, the test (by Eddington in 1919 of the sun's eclipse) which was to mark the first positive test of Einstein's theory.

Nordström meanwhile probed the paradoxes of the Rutherford-Bohr model (Nordström, 1918,1919 in Dutch) with ideas such as that the three dimensional space of an atomic nucleus crosses itself at a certain point - solutions which needed the full development of quantum mechanics. He probed other problem areas, even "waves of gravitation" (Nordström, 1917 a) and remained convinced that a "five dimensional symmetry" would provide the answer, but delayed publishing any further because of the complicated mathematics needed in the solutions, (Nordström, 1917). Nordström's papers of 1917 and 1918 left behind his own five dimensional theory without further comment.

Only Einstein, of all other physicists, including Abraham Mie as well as Nordström, was ready to follow a tensor theory of gravitation (a summation or mapping of a field of vectors.) A curved space was essential, unlike Nordström's dependence on Euclidean space. Einstein's great theory of General Relativity, 1915,involving a Riemannian curved four dimensional space-time continuum, was published in 1916. Its astounding depth, beauty and elegance, combined with its potential predictive power, took the full attention of the scientific world.

Nordström's unification in five dimensions involved only a scalar gravitational field (a scalar is a one-component object, e.g. the temperature of a room, whose value is independent of any coordinate transformation such as position within the room). This was inadequate for the purpose, and it was Kaluza who later built his unification in five dimensions on the essential tool of the tensor field analysis.

Conclusion

Nordström was certainly the first to show that a single treatment of the electromagnetic and gravitational field was possible in five dimensions. Nordström had the basic idea which Kaluza was to use, but his method needed further tools - a proper theory of gravitation using tensor field theory, rather than only a scalar field with limited potential available.

Nordström was celebrated more for his earlier theory of gravitation. Both this and his five dimensional idea were overtaken by Einstein's theory of General Relativity in four-dimensional curved space-time. Von Laue's demolition of Nordström's five dimensional theory brought the concept to an apparent end, and Nordström's further work was often in Finnish, Swedish or Dutch. The most important reason, however for the lack of recognition of both Nordström and his unique idea was the use of a scalar, not a tensor field.

Nevertheless Nordström's attempt has occasionally been given some credit in recent years (e.g. Pais, 1982, p.329) but without any real analysis. Although never a physical interpretation, he was certainly prophetic in his treating the four dimensional world of spacetime as a "surface (plane) laid through a five dimensional world" (Nordström, 1914, p. 504).

gure 8 from Tallqvist, Hj., Helsingfors, 1924.

Gunnar Nordström

1916

Although superficially similar, Kaluza's approach was completely independent of Nordström's attempt, and did break completely with earlier ideas. Extending the dimensions from four to five using a tensor gravitational field enabled Kaluza to leave room for the extra electromagnetic potentials (and provide a spare scalar).

This is usually said to have established Kaluza's primacy but it was in fact clearly shared with Nordström. Sadly, Nordström did not see Kaluza's work, and died the year following the actual publication of Kaluza's paper. The time was not ripe, the tools only became available in 1915, and even Kaluza was only to be given belated recognition.

Note: It was of course true that Maxwell was in a sense a precursor of Nordström and Kaluza in noting the similarity between magnetism and electricity being proportional to the inverse of the distance squared - as well as gravitation. His vector theory of gravitation meant however that electrical forces could repel and gravitation was always an attraction - noted by Maxwell as a paradox (Maxwell, 1864).

Kaluza saw, together with the symmetry noted by Thirring, that if he was to take Weyl seriously, an extra dimension of space was needed. Four dimensions was uninviting with no spare potentials, and so this pointed Kaluza in the direction of using one universal tensor to unify the forces in five dimensions. Kaluza was able to build on the correct structure of Einstein's General Relativity Theory of 1915 using a tensor, a spatially directed field, to describe the metric.

3. The problem of why Kaluza's paper was almost completely neglected for fifty years

The first question must be why publication was delayed for over two years until 1921, with even Einstein withholding his approval. A subsidiary question hangs on the many years delay before Kaluza's own promotion to Professor level, and the apparent lack of personal recognition.

Although Oskar Klein republished Kaluza's idea five years later in 1926, giving a major impetus to the five dimensional idea, interest was not sustained. This leads to the related problem of the history of continuing neglect, despite attempts at renewal by Einstein himself. Certainly when Kaluza's paper was published in 1921, there was no reaction in the scientific journals. It is surprising that there were no references at all, even in the journal of publication, Sitzungsberichte der Preussischen Akademie der Wissenschaften Berlin, over the next few years, either to Kaluza or to five dimensions.

Reasons for the neglect : (i) The two year delay in publication

Kaluza had already achieved his synthesis in the early months of 1919, as can be seen from the letter which Einstein wrote to Kaluza on 21st April 1919. Referring to the unification, Einstein wrote:

> "The thought of achieving this, through a five-dimensional cylinder world, has never occurred to me and may be completely new. Your idea is extremely pleasing to me" (Einstein, 1919a)

He regarded Kaluza's idea as "more promising" than the more mathematical theory of Weyl, but in fact was discouraging to Kaluza in his letters.

In this first letter, Einstein had only a minor mathematical quibble, and a request to limit the paper to the eight printed pages required as the maximum by the Prussian Academy: "You would however, have to arrange that the paper does not exceed eight printed pages, as the academy does not accept longer papers from non-members any more due to the enormous cost of printing." Einstein's great interest in Kaluza's idea is seen in his apparent happiness to present Kaluza's paper to the Academy in Berlin for publication.

A week later (28th April) Einstein wrote that he found Kaluza's paper "really interesting", but had some suggestions to make before the paper was published, and asked that some experimental verifications could be found "with the accuracy guaranteed by our own empirical knowledge" to make the theory fully convincing (Einstein,1919 b). The length of the paper was mentioned again as being too long for the Academy, "there is a resolution on this matter from which exceptions are not made," and Einstein even suggested that Kaluza arrange for the new 'mathematische Zeitschrift' to publish it speedily. The required experimental tests would be difficult, even today— perhaps Einstein took pride in the recent Eddington experiment confirming his own theory.

Within a few weeks, in a letter of 5 May, Einstein confirmed that he was "most willing" to present an extract of Kaluza's work to the Akademie for the Sitzungsberichte, but continued "also to advise you to publish in another Journal," either the previously mentioned mathematical 'Zeitschrift or the physics-orientated 'Annalen der Physik'. Einstein guarantees his support,

> "I shall gladly send it in your name wherever you wish, and add to it a few words of recommendation" (Einstein, 1919 c, unpublished letter).

In fact Einstein had now cleared up the earlier difficulty of $\frac{dx_0}{ds}$ being constant on a geodesic line (21st April), "I have been able to explain it for myself" he wrote acknowledging a letter from Kaluza of 1st May and helping to explain further points (while finding a new minor problem). He stated that from the standpoint of recent experimental discoveries, "your theory has nothing to fear".

Ten days later, on 14 May 1919, Einstein wrote again to his "highly revered colleague" Kaluza, acknowledging receipt of his manuscript ready for the Academy. Einstein however brought to Kaluza's notice a further mathematical difficulty concerning the differential $\frac{dx_0}{ds}$ being too large which he had expanded at some length, hoping that Kaluza "will find a way out". Einstein returned the manuscript until the problems were settled: "I will wait to hand it in until I receive notification from you that we are clear about this point" (Einstein, 1919 d, unpublished letter).

In a further communication that month dated 29th May, Einstein now admitted a mathematical blunder in his latest correction, and acknowledged Kaluza's careful and considered response. Despite Einstein's continuing insistence that "I have great respect for the beauty and audacity of your thought", the remaining difficulties (as Einstein saw them) still gave him doubts about publishing. He did however again press the publication in the alternative new mathematical journal. Einstein in fact sent his own unification attempt to Kaluza. This however still suffered from the separate dualistic treatment of electromagnetic and gravitational forces in four dimensions, "by lack of anything better" (Einstein,1919 d).

Over two years were to pass before Einstein again wrote to Dr. Kaluza on a postcard dated 14th October 1921. Einstein now admitted, "I am having second thoughts about having restrained you from publishing your idea on a unification of gravitation and electricity two years ago" (Einstein,1921a). In any case, Einstein acknowledged that he still judged Kaluza's unification to be a better approach than that of Hermann Weyl. At last Einstein offered to present Kaluza's paper to the Academy.

Kaluza replied immediately on 24th October, receiving Einstein's news "with great joy". He noted Einstein's slight quibble, and offered to include a note on this inconsistency in the abstract of his ideas which Einstein had requested. Kaluza admitted that he was too busy with his teaching duties to provide a firm solution: "for local reasons I had to spend what little time I have because of my teaching duties on pure mathematical thoughts". He stated however that the difficulties did not in fact seem to him so unsurmountable as before:

"It does not impress me"! (Kaluza , an unpublished letter,1921 b). Within a month, on 28th November, Kaluza sent off a short abstract of his paper, with further notes about the difficulties and a possible solution in the treatment of electrons and protons.

If Einstein still had any doubts, Kaluza said "he did not mind at all omitting the paragraph in question for the time being", no doubt to expedite publication (Kaluza, 1921 c, unpublished letter). However he was confident enough to suggest that it may lead to further ideas for someone else if it were left in. This seemed to satisfy Einstein completely. Kaluza in fact also hinted that a proportionality constant K required for the scalar of the energy tensor (Too and T44) "should be a statistical quantity" (ibid., 1921). This difference effect provided a possible way in for a quantum mechanics interpretation (see Klein, 1926).

Then in a postcard dated 9th December (postmarked 8th December), Einstein finally stated that he had handed in Herr Dr. Kaluza's work to the Academy. He advised that corrections were expensive and insisted:

"Your thought is really fascinating. There must be something true in it" (Einstein, 1921 b, unpublished postcard). He even suggested that Kaluza's latest explanation of his (Einstein's) final quibble was unnecessary! The paper was accepted and published, December 1921.

This delay in publication of Kaluza's work, from 1919 to 1921, which appeared to be due to Einstein himself, has caused some surprise. Even so thorough an analyst as Abraham Pais admitted that he did not know why the publication was delayed so long (Pais, 1982 p.330). Kaluza's son writes,

"I believe the delay was caused in the first place by Einstein's additional questions about certain minor problems, and also by his statement that owing to financial problems he could concede no author more than 8 pages .....

Despite Einstein's private approval in 1919, the paper needed to be officially endorsed by a well-known physicist" (Kaluza, Jun., 1984). Einstein himself seems to have regretted discouraging Kaluza for over two years. Einstein in his rather ambiguous correspondence with Kaluza, certainly showed his thorough and painstaking character, and did not lightly alter course - a clear impression left on the Kaluza family, although the two men never actually met. The idea of five dimensions always remained outside Einstein's concepts of reality, despite approaching the idea later with different students. In 1922 Einstein, with a colleague, wrote a paper denying the truth of Kaluza's theory because of the absence of singularity-free solutions (Einstein and Grommer, 1923), only returning to the idea after Oskar Klein had championed Kaluza's ideas in 1926. This was despite constantly maintaining his high regard for Kaluza's theory in their private correspondence. Einstein spoke in his final postcard to Kaluza, on 27th February 1925, of Kaluza's great originality and of meriting the serious interest of his academic colleagues. He again acknowledged that it was the only attempt to take unification seriously (see further, Chapter 5).

A point of some academic interest was Einstein's insistence that only eight printed pages are allowed for non-members. This was one of the initial reasons for Einstein's refusal to publish. The Journal rules were published in the brown pages at the back of each volume, e.g. 1st January 1921, with a list of Members who were allowed 32 pages. It was further stated that the limit of eight sides could only be exceeded if everyone in the Academy agreed. Nevertheless in the intervening years before Kaluza's article was published, i.e. 1920 to 1921, there were articles published of more than eight pages from "Associate Members" who were supported by Full Members (such as Planck, von Laue, etc.) It would seem that this limit could have been exceeded with Einstein's personal backing, and that Einstein was not ready to give this public endorsement until December 1921. This is in fact confirmed by Einstein's remarks to Kaluza, "You must not be offended by this because if I present your work I am backing it up with my name" (Einstein, 1919 b).

Letters to Einstein from Kaluza in 1919 have not been preserved. The first to be kept by Einstein was the postcard of October 24th, 1921, acknowledging joyfully Einstein's decision to publish his paper at last.

There was presumably no indication that Kaluza was interested in being published in the new and less prestigious mathematical Zeitschrift. He did however publish his later pure mathematical research findings in this journal.

(ii) The delay in Kaluza's own promotion

Kaluza remained a little known and poorly paid assistant lecturer ('privat-dozent') for some eight years after the publication of his five dimensional unification idea. This comparative obscurity, together with the fact that he did not get a University chair, became a matter of great concern to Kaluza for family reasons.

Although a pleasant, encouraging postcard of 27th February 1925, this last postcard from Einstein to Kaluza does not seem to respond in any immediate way to Kaluza's own letter, earlier in that month (6th February) asking for a reference. Kaluza had continued in his poorly paid position for the four years after his paper was published when he wrote this appeal for help. It appeared that Einstein was the only person who might know of his worth. Kaluza offered to put one of his students to do further work on the five dimensional idea, remarking that he himself could only very occasionally dedicate himself to physics, because his mathematical teaching and research absorbed too much of his energies. He had to try to become better known by publishing intensively,

> "and thus perhaps end my unsatisfactory Cinderella-existence here"

(Kaluza, 1925). Kaluza mentioned that he would be appealing to Professor Richter to obtain "a better economic security for my family" than his existing teaching assignment.

Kaluza was too proud lightly to ask anyone for help, and had delayed writing to Einstein for a short reference concerning

> "his understanding of questions on the mathematical-physical borderline (interface)" (ibid., 1925)

It must not have occurred to Einstein that Kaluza was in a position far below his merit. Einstein did respond to this request for a reference although there is no evidence of any urgent action. While offering his high regard in the 1925 postcard for "the great originality of your idea ", Einstein urged Kaluza to look at the matter again, admitting that he himself had so far struggled with the problem in vain (Einstein, 1925).

In the only letter we have evidence of; "to a colleague" - perhaps at the University of Kiel and dated 7th November 1926 (now in the possession of Kaluza's son) Einstein recommended Kaluza for recognition and promotion. This letter, eighteen months after Kaluza's request, may well have been catalysed by Klein's rejuvenation of Kaluza's theory. Klein had brought the five dimensional idea more forcibly to the attention of the scientific world, with his own modifications to bring in quantum ideas, both in German and in English (Klein, 1926, 1927). Whatever the motivation, Einstein in his letter acknowledged Klein's recent acceptance of Kaluza's idea of the world "as a continuum of five dimensions, but whose metric tensor is not

> dependent on the fifth coordinate. This restricting condition forces the actual 4-dimensionality, but has the disadvantage ... of being less natural."
>
> ... Einstein's testimonial is clear:
>
> "but after all efforts to bring gravitation and electricity into a unifying aspect have collapsed, Kaluza's idea appears, of all those which have emerged up till now, to be the only one which is not completely without some possibility."
>
> ... He acknowledges further:
>
> "However the final truth may be, Kaluza's thought is of such a kind which shows creative talent and strength of concept. This achievement is all the more remarkable as Kaluza works under difficult external conditions. It will please me very much if he could acquire a suitable sphere of effectiveness" (Einstein, 1926).

At last, aged 44, Kaluza obtained an ordinary professorship ('ordentliche') at the University of Kiel in 1929. He was invited to the University of Göttingen in 1935 "with the known support of Einstein behind him" (Laugwitz, 1986), where he became a full professor (lehrstuhle)-despite his having courageously omitted all the officially prescribed references to "the glorious Nazi regime" by the Nazi-Rectors in 1933, who asked their colleagues to speak about the "right" way to think scientifically (Kaluza Jun., 1987). He stressed instead the share of Jewish mathematicians in fundamental research (Sambursky, 1986). Kaluza emphasised that mathematical facts and proofs concerned "an immaterial reality independent even of the existence of mankind." He continued to work on purely mathematical treatises e.g. Fourieranalyses.

It is surprising that Kaluza had no patron at his home University during all this time. In fact Kaluza had been called up to serve his country as a scientist on the Western Front in 1916. He had been invalided out in 1918 with suspected tuberculosis, which proved later to be only pneumonia and needed a long period of rest. Why his University did not promote him to a Professorship after his decisive paper, Kaluza never understood. An older Mathematics Professor told him later (Kaluza, Jun., 1986) with sadness that everyone had assumed he had T.B. They thought he was terminally ill and so ignored him for promotion. However his pupil Schmuel Sambursky recounts that, from student gossip, Kaluza's Professor, Franz Meyer (1856-1924), a rather ill-humoured and always grumbling "Old Ordinarius", was not interested "to put it mildly" in young Kaluza's promotion. Sambursky himself describes Kaluza in his professional work as

> "a brilliant teacher, clear and lucid even when the subject was difficult" (Sambursky, 1985).

Thus it was not until Einstein's reference and Klein's re-appraisal that Kaluza was promoted. It does appear that Einstein wrote another reference, perhaps on request, to the mathematician Abraham Fraenkel, from Berlin in October 1928. He speaks of Kaluza "making a good impression" in his letters. Not over enthusiastic, Einstein writes that from the publications, "no great formal gift is shown", but defended the attractiveness (Genialität) of the five dimensional idea, and remarked on Kaluza having worked under very difficult external circumstances, Surprisingly Einstein can give "no judgement about the extent of his mathematical knowledge and ability" and refers Fraenkel instead to another colleague Kowalewski in the University of Leipzig (Einstein, 1928). However Einstein's letter must have helped to secure Kaluza's appointment to the professorship at Kiel in April 1929. Gerhard Kowalewski, a Professor of mathematics, had in fact been present at the long discussion after the lecture in which Kaluza read his 1921 paper. Thus Kaluza remained a privat-docent in particular difficult material conditions during the galloping inflation of the 1920's. Interestingly the other professorship at Kiel was in fact held by Fraenkel, who held strong Zionist views. He emigrated to Jerusalem in 1933.

It must be said that Einstein would have had many scientists (Stachel, 1988) sending their papers to him for approval. He was widely respected as kind and considerate, yet remained ambiguous in his support for Kaluza's idea (see Chapter 5).

(iii) Kaluza's own personality - the deeper reason

The main reasons for the lack of recognition of Kaluza and his five-dimensional theory may well lie in Kaluza's own character. Modest and unassuming, he sought neither personal prestige nor patronage.

Theodor Franz Eduard Kaluza was born on 9th November 1885 at Ratibor, near Oppeln in East Prussia, now Poland. He was the only child of the Anglicist Max Kaluza, whose works on phonetics and Chaucer were classics in his day. The Kaluza family may be traced back continuously in Oppeln to the end of the sixteenth century. It has been in Austria, Upper Silesia, alternating from Polish to East Prussian with the outcome of wars. Traditionally in the family there had been one pastor and one teacher in each generation. 'Kaluza' was never used as a surname in Poland, but a similar name used in the sixteenth century by Hungarian and even Italian families was turned into Kaluza by the inhabitants of the Oppeln region. (See Kaluza, Jun., 1984, 1985). In fact they were a Roman Catholic family for many generations which was exceptional for the Lutherans and Calvinists in Silesia.

Theodor was two years old when his father, Max, came to Königsberg (now Kaliningrad) in East Prussia as Professor of English in 1887. He grew up in Königsberg, attended the Gymnasium/Grammar School "Friedrichs Kolleg" and began his mathematical studies at the University, where in 1909 he gained his doctorate on the "Tscirnhaus transformation" under Professor F.W.F. Meyer. This qualified him to become a 'privatdocent', a private lecturer at the University - unpaid but with the right to give lectures which earned some money. He was married in the same year to Fräulein Anna Helene Beyer, and remained as a poorly paid privatdocent for some twenty years.

Apart from being a brilliant mathematician, his son notes that he had many outstanding gifts as a musician and linguist in fifteen languages (including being able to read the Bible and the Koran in the original texts as a schoolboy) although he did everything in a very unobtrusive way. Kaluza was a man of wide interests and a good sense of humour. From the age of ten he accompanied the choir on the organ in his holidays.

Kaluza's pride and reticence can be seen in his unobtrusive rejection of a free scholarship for his son (despite their straitened circumstances) in favour of another very able pupil, whose mother was even more poverty-stricken. The Kaluza's brought up their son and daughter according to the inspiration of Rousseau and Pestalozzi - to learn for themselves, not taught in a didactic manner (e.g. Rousseaus' Emile).

Kaluza was liked and respected by his students and was on extremely good terms with his colleagues. His son's appraisal is confirmed by a pupil, Schmuel Sambursky, now a Professor at the Israel Academy of Sciences and Humanities in Jerusalem. Dr. Kaluza, he writes,

> "was an extremely kind, charming and witty man, always encouraging, and always too modest to talk about himself or his famous paper"

(Sambursky, 1985). A later pupil, D. Laugwitz now also a professor described him as "always shy and modest in his presentations" (Laugwitz, 1986), who would "never deliberately put himself in the limelight."
For Sambursky, he was his 'Doctor Father', always helpful in discussion on his thesis, and was outstanding even among his great academic teachers: Planck, Rubens and Erhard Schmidt in Berlin, and Knopp, Volkman and Kaluza in Königsberg.

There was little discussion of any science at home, and no talk of his own paper. Frau Kaluza's education gave her no insight into mathematics or science. At the time when Einstein wrote to Kaluza, his letters "were of course a sensation", but Theodor (Junior) born in 1910, and his sister born six years later, were not interested at the time. In any case, as far as any discussion of his paper with anyone, as his son comments

> "my father was most adverse to any form of nebulous explanations"

(Kaluza, Jun., 1984). Although originally from a Catholic family, Kaluza was not a Catholic himself. In the 1920's, however, he accepted Christianity. He remained a Christian, his son also writes, in the same sense as Albert Schweitzer, bringing the same "reverence for life". His son quotes from a book of Schweitzer's which his father gave him as a present, that

> "it is good to preserve and to encourage life, it is evil to destroy life or to restrict it," (Schweitzer, 1923).

Kaluza himself was a very private person and never commented openly about such spiritual matters, although in acknowledging the spiritual force of the religion of love, "there were many other indications that this was spoken from the heart" (Kaluza, Jun., 1985a).

Kaluza found the Schweitzer idea of awakening self-understanding and self-revelation in himself, and agreed with it as something that cannot be proved, but also which does not need any proof. His son also confirms Kaluza as being full of understanding and tender-hearted. He once overheard two students talking about Kaluza. One said:

"Kaluza never humiliates you, as other lecturers do" (Kaluza, Jun., 1985a).

Everyone who met him experienced this modesty and concern for others. In fact about eight hundred students were present to show their respect at his graveside. As an older colleague once said to Theodor Junior, "people were happy if he only said Good day to them!"

Frau Kaluza later told her son of times when his father would respond to any cry for help. In 1919 he organised night watches round the groups of houses where they lived, so that many burglaries and attacks were prevented. His compassion was seen for example in running with his friend Herr Szego in response to cries of help from the nearby park, to drive off young men who had tried to attack two young women. Kaluza later advocated unhesitating defence, "if one is not totally terrified". This compassion was seen further in his great liking for children and sensitivity to animals.

Another interesting aspect of Kaluza's philosophy and also the wish sometimes to be alone, is seen from an incident recounted by his son from his father's personal letters from the trenches in 1917. He was stationed behind the front with a small contingent (Schallmesstrupp). During these gun-location exercises, Kaluza often remained outside their blockhouse when the troop was under fire. Questioned by a fellow soldier, Kaluza commented on the probability of being hit being equal - but in addition his real reason was "to be alone with danger" (Kaluza, 1986 a).

His physical youthfulness and unassuming nature may also be seen in that he was asked as a Junior lecturer

Figure 9

Theodor Kaluza

In 1920 (aged 35)

produced by kind permission of Theodor Kaluza (Junior) "to differ from the students in their appearance : 'would you mind growing a beard' - to which he agreed (rapidly calculating the saving!)" (Kaluza, Jun., 1985). He wore the beard until 1933, when Kaluza was openly threatened in the streets several times, because of his Jewish appearance.

It may be deduced from the outline of Kaluza's character, that his integrity, modesty and unassuming nature would not lead to his seeking personal promotion or patronage. He did not make a case for his discovery, either in writing or verbally to impress his colleagues, and he would not lightly expound on the meaning of his mathematically-worded solutions. Kaluza would not fight for himself (or for his son's scholarship), although he was prepared to exert himself for others. He was determined not to enthuse openly about his work even to single postgraduate students bright enough to cope with Kaluza's lectures. This wariness of boasting, although he was certain that he was right and that his work was important, no doubt contributed greatly to the neglect of his ideas. Kaluza was bitterly disappointed when the world of physics did not acknowledge his work.

It must further be admitted that his work was perhaps too brief. While Kaluza clearly saw the importance of what he had done, the beauty and elegance of his solution, he did not take it further, despite Einstein's urging. There probably was no clear way ahead at the time, and Kaluza needed to establish a reputation by writing papers, and pure mathematics was his professional brief. His aim - to achieve the unification of gravity and electromagnetism in five dimensions - had certainly been achieved.

Teaching and Publications

Besides his famous paper of 1921, Kaluza worked on models of the atomic nucleus, applying the general principles of energetics (Kaluza, 1922). Interestingly, he used here only the one - dimensional case, to simplify the difficulties of the spatial problem. In the lateral thinking employed by Kaluza, this was no doubt an early type of dimensional reduction. He also wrote on the epistemological aspects of relativity, and was sole author of, or collaborator on, several mathematical papers.

Kaluza's main interests in the 1920's, diverging completely from his five dimensional paper in physics, centred on infinite series, of use in both mathematics and physics. He was in 1928 the first person to give the necessary and sufficient conditions for the presentation of a function via the Dirichlet series in the Mathematical Zeitschrift and in Schriften Königsberg (Kaluza 1928 a,b). The analogous question for the Fourier Series appeared to have occupied him much further. The consequences from his work on coefficients of reciprocal potential series (Kaluza 1928 c) were named the "Kaluza equation" or "Kaluza series" (Laugwitz, 1986, p. 180). Kaluza's colleagues in the 1920's in Königsberg included Konrad Knopp, Gabor Szego and Werner Rogosinski.

In his later years, Kaluza continued to rely on his prodigious memory and gave all his lectures without notes. He was often requested to publish certain lectures but was of the opinion that something would be lost from that which his listeners treasured. It is confirmed by Laugwitz as a student in the late 1940's that Kaluza until the last, held lectures on many new ideas in mathematics, in addition to the regular basic lectures about complex analyses. Sadly, Kaluza left no notes about his considerations, "everything was read freely from the lecture position and was so fascinating that one often forgot to take notes" (Laugwitz, 1986 p.181). It was noticed that Kaluza had a complete grasp of a wide range of mathematics, and could discuss and argue with any specialist in seminars and colloquia. In fact he did not like publishing, and thus some ideas disappeared in the works of his students without their being aware of this. Particular mention is made by his student Laugwitz that it would be profitable to resurrect Kaluza's work of 1916, "The relationship of the Transfinite Cardinal Theory to the Finite" (Kaluza, 1916).

As a teacher, he was obviously outstanding and delivered exemplary lessons for beginners and lectures for natural scientists with a fine feeling for the level of understanding of his listeners. His 1938 completed book with the physicist Joos in Göttingen, the "Joos-Kaluza", was, until far into the post-war period, the teaching book of mathematics for Natural Scientists.

(iv) Kaluza's idea: ahead of its time

The world was not yet ready to accept more than three dimensions of space (four dimensions of space time). There was clearly the zeitgeist for change in the early quarter of the twentieth century. Although the actual incentive to use an extra fifth dimension probably came from Einstein's seminal papers on the four dimensional continuum of Relativity Theory, Kaluza himself was certainly very aware of the contemporary cultural revolutions. The zeitgeist which involved the break-up of the classical tradition was seen in science and the arts. The pattern breaking was seen also in the change from national idealism to disillusionment in the course of the First World War, as Kaluza emphasised to his son. His son remembers Königsberg's reputation for modern plays and music, and his father's avant-garde furnishing and decoration after his marriage in 1909. Art Nouveau style ('Jugendstil') of the new realism ('Sachlichkeit'), and contemporary artists and literature were evident in the home (Kaluza, 1986 b). He was interested also in both contemporary technology and music. Pictures by contemporary artists such as Emil Nolde and Ernst Barlach (who was to influence Otto Flath) were hung on the walls.

(a) Despite the favourable cultural climate, there was no clear evidence forthcoming to support Kaluza's theory, whereas the bending of light from an eclipse of the sun had been used in 1919 to support Einstein's General Relativity. The other current theory being developed in Quantum Mechanics was soon to find practical applications. The significance of a five dimensional world still lay in the future.

(b) Indirect evidence of the need for a completely new physics was to emerge only much later in the paradoxes and enigmas of Relativity (see Chapter 2) and of Quantum Mechanics (Chapter 4). No evidence had emerged at the time however against these very recent and very complex mathematical themes.

Singularities of the Big Bang and Black Holes were not yet investigated to disturb General Relativity. Neils Bohr's orthodox Copenhagen interpretation of the Quantum theory in 1926 papered over the cracks in the interpretation, hiding the paradoxes of wave/ particle duality, observer-centred reality and non-locality.

(c) Kaluza, while acknowledging the threat of "the sphinx of modern physics, the quantum theory" in his conclusion to his paper, (Kaluza, 1921, p. 972) did not himself include the theory of Quantum mechanics. It was only being developed in the 1920's and even Klein's attempt in 1926 to incorporate Quantum theory into Kaluza's work was not a success (see Chapter 4). Kaluza in fact took up Weyl's idea and elaborated the restlessness of space on the micro scale, compared with the smoothness of the macro scale, perhaps anticipating the ideas of foam space developed much later by John Wheeler. Kaluza also hinted at the rôle of a "statistical quantity" (Kaluza, 1921, p.972; 1921 b) that may be assigned to the fifth dimension - the role which Klein took up more strongly.

(d) The extra tools which were needed were not then available to Kaluza, Klein and Einstein. As these appeared in the 1960's, the re-entry of the Kaluza-Klein model was to be of critical importance to the progress of unification of forces and particles - gauge theory, strings and supersymmetry, leading to supergravity and superstrings.

(v) Problems of communication and of metaphysics - a challenging concept

Kaluza's conceptual challenge of five dimensions, besides being ahead of its time, lay on an awkward boundary between mathematics and science. This dividing line was between abstract pure mathematics as a tool and the 'reality' of physics which Kaluza was at pains to emphasise.

In his mathematical thoughts, his son (Kaluza Jun., 1985) emphasised the quotations from Kaluza's own published paper of 1921. His mathematical searches speak for the fact that he saw his iconoclastic use of five dimensions in the framework of existing mathematics and Kaluza referred to both Weyl's unification and to Thirring.

Kaluza had an impression of the "mathematical zeitgeist" as being ready for a change, his son affirms. Perhaps the particular impression made on him by Hermann Minkowski of Göttingen was also a catalyst (Laugwitz, 1986, p. 179.).

Kaluza's theory was often criticised as a purely mathematical artifice with no physical meaning and of only formalistic significance. This is untrue to Kaluza's own intention. After referring to the 'formal correlation' of Thirring, Kaluza himself does <u>not</u> use the expressions of the earlier, nineteenth century mathematicians working on non-Euclidean space or on extra dimensions. Kaluza clearly describes in his published paper how he is "forced into a particularly uninviting path", a "terrifyingly strange and surprising conclusion" to call in a new fifth dimension to help understand these correlations, which cannot be done in a world of four dimensions. He has to "stoke himself up for a rather uncomfortable approach", (ibid., p. 967) (literally) for this surprising decision to ask for help from a new fifth dimension of the world. These are hardly the words of a pure mathematician, and are clearly distinct from Kaluza's other papers. For Kaluza there is certainly more behind the presumed connections that just an empty formalism. He is fully aware of the practical problems of why we cannot see this extra dimension, but is nevertheless convinced of its full physical status. That Kaluza assigned a physical status to the fifth dimension is confirmed by his student Sambursky,

"It is clear that the fifth dimension - although of very small extension in comparison with the four classical ones - was regarded by Kaluza as a <u>reality</u> and not as a mathematical device" (Sambursky, 1986).

Kaluza concludes:

> "In spite of the full recognition of the physical and epistemological difficulties outlined which tower in front of our understanding ... it is difficult for one to believe that in all these relations which in their formal unity are scarcely to be surpassed, there is but a capricious chance performing an alluring play" (Kaluza, 1921, p.972).

Kaluza confronts the problem of why we never notice or realise any spacetime changes in the state vector:

> "Although our previous physical vocabulary of experience does not uncover any hint of such a supernumerary world parameter ...we must keep open the consideration (of the extra dimension)" (ibid.,p.967).

Because the fifth dimensional deviations are not noticeable in four dimensions, Kaluza therefore put the derivation of this new parameter equal to zero, treating it as "very tiny but of higher order", which he called the "cylinder condition." this implies that the fifth dimension is wrapped up into a small circle of cylinder with a high energy of excitation. We cannot enter the fifth dimension, he notes, due to

> "the close linked enchainment of the three spatial coordinates in 4-dimensional spacetime" (ibid., p 971).

Thus Kaluza set out "to characterise the phenomena of the world" with the unusual aim of combining gravitational and electromagnetic fields by establishing the reality of the fifth dimension. Beauty and elegance are the best guides, as both Einstein and recent physicists agree. Kaluza's perspicacity is nowhere better seen than in his description of our spacetime as

> "a four dimensional part of a five dimensional R_5 world" (ibid.,p.967)

a projection or cross section of a five dimensional reality. In Kaluza's conclusion, he acknowledges that Einstein's General Theory will be the base, a subset of Kaluza's more general five dimensional world, and that the "analogous application to a five dimensional world" would in fact be a triumph for Einstein's theory. It was Kaluza's hope that his theory would recognise gravitation and electricity as "manifestations of a universal field."

These words of Kaluza clearly demonstrate that he is on the physics side of the maths/physics interface - but the boundary line was not perhaps clear enough to his contemporaries. The earlier little known and abortive attempt by Nordström to use five dimensions did remain purely mathematical.

If Kaluza's theory is true, then there is a further boundary which his idea crosses, and which lies deep within the paradox of the continued neglect of the idea of an extra dimension. While his contemporary Kasner was able to use a fifth, sixth or even tenth embedding dimension as a mathematical tool, Kaluza's concept lies on the interface between physics and 'beyond traditional three dimensional physics'. Whether this is described in terms of transcendence or of metaphysics, the extra dimension certainly seemed to be beyond the physics of the time, the classical space of three dimensions. These overtones deterred traditional physicists, even such men as Einstein and de Broglie. Like Arrhenius' particles or Copernicus' sun-centred universe, extra dimensions also seemed to be against common sense and intuition.

4. Sources of inspiration

For Kaluza, music held a key place in the arts, and in music, where classical composers from Bach onwards were still the favourite:

> "The Creator would do nothing which contradicted mathematical tenets and order, for a framework of the possible, for structures which can be considered without contradiction" (Kaluza, Jun., 1986 b).

His son affirms the literal quotations from memory, and emphasises that like composers, mathematicians

"normally start from reality as it appeared to them,.. although for at least a century, the imagination of mathematicians has played an equally large rôle. I believe that the reality for everything which our imagination conjures up does indeed exist."

Like music, mathematics can go 'beyond the boundaries' of what had previously been thought to exist.

Kaluza had been sure that his own discovery could not just be a coincidence, and that some secret of nature had been revealed. Like Einstein with his own theory, Kaluza thought it "too beautiful to be false". Dr. Kaluza (Junior) remembers the moment of inspiration while reading in his father's study as an eight year old. One day, his father

"was still for several seconds, whistled sharply and banged the table: he stood up, motionless for several seconds - then hummed the aria of the last movement of Mozart's Figaro" (Kaluza Jun., 1985, BBC2).

The five dimensional unification had been achieved. Whether the idea of unifying gravitation and electromagnetism was perhaps germinated while serving as a 'Flash Spotter' observer on the Western Front, we cannot be sure. Sound ranging focussed on the flash of gunfire, working out the position using ballistics theory, and communicating with field headquarters using a telephone system cranked by hand (Whayman, 1986). No doubt such vivid memories of 1917/1918, perhaps even of electricity generated by German soldiers riding static bicycles (Imperial War Museum, Q.23; 701) helped to fertilise Kaluza's thinking during the year's convalescence prior to his famous paper on unifying gravity and electromagnetism.

Figure 10

Generation of electricity by German soldiers on static bicycles, 1917

German Tandem Generator (Q23,701 - Imperial War Museum; ref. in Taylor, A.J.P., 1963, p.35).

Despite his weak heart, Kaluza had been called up as a scientist to serve his country in 1915. First conscripted to measure tonnage on railway lines, to gauge how the war machine was working : on newly laid rails into France, Kaluza was involved in the Schliefer plan to speed up occupation. Then he was used as an engineer on the Western Front in Rheims (Champagne) in 1916. Essential equipment included instruments like telescopes, telephone, chronometers etc., issued to Sound Rangers and Flash Spotters. As an Artillery Officer, Kaluza was therefore having to face the emotional strain of war at a peak time in his creativity as a mathematician. Kaluza was invalided out in 1918. During his invalid period and convalescence, his brilliant idea of unifying gravity with electromagnetism came to fruition. Perhaps this combination of the mathematical and cultural zeitgeist and the war experience involving practical physics, provided the fertile ground for Kaluza to develop his theory in five dimensions. (McCormach, 1982, Night Thoughts of a Classical Physicist) .

The gestation period certainly ended in inspired mathematics. The difficulties of interpreting the extra dimension still lay in the future.

5. Reaction to Kaluza's paper of 1921

Apart from the private correspondence between Einstein and Kaluza (even today largely unpublished) there was no reaction in the literature. Certainly there are no references in the Prussian Akademie's Journal of publications of his paper, nor in any other major scientific journal. Einstein himself wrote frequent articles on gravitation and on a possible solution to quantum problem in the Sitzungsberichte der Preussichen Akademie der Wissenschaften (P.A.W.). In 1923, articles by Einstein made references to Weyl's theory and to Eddington's theory but, with one negative exception, there was no reference to Kaluza on five dimensions up until 1927 despite his private encouragement in his letters to Kaluza. The one response was with J. Grommer (Einstein and Grommer, 1923) rejecting Kaluza's idea. As already mentioned, Einstein still insisted on singularity-free solutions although this criteria is no longer accepted. Not until 1927, after Kaluza's paper, did Einstein himself take up Kaluza's article from a positive standpoint in the journals.

In fact no positive reaction was found anywhere until Oskar Klein's famous paper of 1926. Klein rediscovered Kaluza's paper, extending the ideas to try to incorporate the new Quantum Mechanics, and making additional references to the work of de Broglie in 1925 and of Schrödinger in 1926 (see Chapter 4).

6. Conclusion

We have seen that despite the zeitgeist in favour of breaking the classical mould in sciences and the arts, Kaluza's paper and his own promotion were delayed, and the idea neglected over the succeeding years. The solution of the problem has been seen to lie in two areas.

The conceptual challenge of the non-visualisable fifth dimension needed a new world picture. It was to be over fifty years before scientists really perceived the need to go beyond the four dimensions of spacetime. (Einstein himself was in fact against the implications of Quantum theory, despite his own work on quanta in the early years of the century. He also never accepted the possible existence of singularities (- paradoxes at the heart of his own General Relativity). Even now there is a communication problem for non-mathematicians in beginning to think about the extra dimensions which seem to be needed in theoretical physics today to resolve these dilemmas.

The second answer we have seen lies in Kaluza's modest and unassuming personality. Not given to self-praise, he was unfortunate in the lack of patronage from his supervisor, and Einstein's tepid support did not reinforce the importance of his discovery. It is interesting to note that in his later years, Professor Kaluza's personal integrity was so highly regarded, and he was so gifted in languages, that he was appointed as Göttingen University's liaison with the British Occupational forces. This was to ensure the de-Nazification procedure, "to let an old German University return to scientific work without any ideology" (Kaluza, Jun., 1986 a).

As we have seen, Kaluza did not have the combative personality of a Galileo, nor the right mathematical practical tools (gauge theory and supersymmetry, rather than a telescope); he did not have the rumbustious iconoclastic personality of a Luther. Perhaps above all, the scientific world was not ready for such a creative idea as a fifth dimension, which may still need to be put into an understandable language and not remain in mathematics. The scholarly truths of Erasmus' Latin needed Luther's German (the language of the people) to start the Reformation. Galileo's book in his native Italian served to spark off the real controversy behind the Latin of Copernicus' 'De Revolutionibus'.

The delay in recognition of Kaluza's paper was thus due to many contributory factors. His character, circumstances and the mould breaking nature of a non-visualisable extra dimension lay behind the neglect which lasted until the nineteen seventies.

The Kaluza-Klein model is widely used today. Theodor Kaluza died in Göttingen on 19 January 1954 after a brief illness, two months before he was to be named Professor Emeritus.

Figure 11

Theodor Kaluza with Gabor Szego, 1946

Göttingen, 1946 (reproduced by kind permission of Theodor Kaluza, Junior).

Chapter 4 Oskar Klein's Revival : Quantum Theory and Five Dimensions

Synopsis

Introduction

1. Klein's first paper, "Quantum Theory and Five-dimensional Relativity", 1926.
2. Precursors of Klein's paper (apart from Kaluza)
   - (i) Erwin Schrödinger's Wave Mechanics, in multidimensional configuration space
   - (ii) Louis de Broglie's "associated waves" of matter
3. Further developments from Klein's paper - the immediate effect.
4. Klein's rejuvenation of Kaluza's paper met with temporary success:
   - (i) Reactions of other scientists were initially very favourable
   - (ii) Further strengthening by Klein
   - (iii) The use of five dimensions was adopted by Einstein, de Broglie and others, e.g. Louis de Broglie's paper on five dimensions (1927)

     Postscript to de Broglie
5. Reasons why Klein's attempted synthesis of Quantum Mechanics with Kaluza's five dimensional unification did not become accepted, after its initial success; Quantum mechanics - the orthodox view leads to enigmas and paradoxes in interpretation, although very successful mathematically e.g. the two slit paradox and non-locality.
6. Postscript on Quantum Mechanics today e.g. the Many Worlds theory
7. Metaphysics and Paradoxes
8. Conclusion
9. The Way forward

Introduction

Oskar Benjamin Klein, the theoretical physicist, was born on 15th September 1894 in Mörby, Sweden. He gained his degree in 1915 after three years study at the University of Stockholm, and remained as an Assistant in the Physical Chemistry department of the Nobel Institute at the University. Klein was a junior lecturer at the Universities of Copenhagen, Stockholm and also Michigan where he was an Assistant Professor 1924-25. He returned to Copenhagen University in the summer of 1925 where he was a lektor in the Institute of Theoretical Physics until 1931, when Klein was offered a chair at his old University of Stockholm. He remained there as Professor and Director of the Institute of Mechanics, lecturing and writing across a wide range of theoretical physics. Klein was later awarded the 1957 Nobel Prize for Physics, the Max-Planck Medal (1959) and was honoured as Professor Emeritus in 1962 at the University of Stockholm.

At Copenhagen in 1926, Oskar Klein frequently took part in the discussions between Neils Bohr and Werner Heisenberg on the new quantum mechanics. He was undoubtedly influenced by the Bohr-Heisenberg-Einstein controversy and devoted himself to attempting to solve the problems. Klein rejuvenated Kaluza's unification theory involving five dimensions. There had in fact been no positive reference to Kaluza in the literature since the original paper in 1921. Klein's aim was to combine the new quantum theory with the unification of electromagetism and gravity, using five dimensions.

1. Klein's first paper, "Quantum Theory and Five Dimensional Relativity"

(1926) "Quantentheorie und fünfdimensionale Relativitäts-theorie"). This was received in April 1926, and published in that year in the Zeitschrift für Physik (Klein, 1926a).

Klein attempted to achieve his aim by linking Kaluza's unification theory with de Broglie's and Schrödinger's treatments of quantum problems. He regarded the electromagnetic equations as describing the motion of matter as "a kind of wave propagation". Klein considered solutions in which the fifth dimension is "purely periodic or harmonic, with a definite period related to the Planck constant" (Klein, 1926a, p.895) - the entry point to the quantum theoretical method.

Oskar Klein started from the five dimensional Relativity theory in a Riemannian space, similar to Kaluza's paper. However he left the measurement of the fifth coordinate tentatively undetermined, rather than restrict g_{55} to unity as Kaluza did. For Klein this value of unity was not essential, and led him to describe spacetime as periodic in the fifth dimension. De Broglie's theory where one part of the wave oscillates periodically with time as a standing wave provided one idea. Schrödinger's equation was the other inspiration. Klein wrote down a version having five variables instead of four, and showed that the solutions of the equation could be interpreted as waves moving in gravitational and electromagnetic fields of ordinary four dimensional spacetime. Klein was able to interpret these waves as particles, according to quantum theory. For him, Kaluza's two constraints of small velocity and weak field were irrelevant.

Klein's wish was to use the analogy between mechanics and optics to provide a deeper understanding of the quantum phenomena. He claimed to give "a real physical meaning to the analogy" in using the fifth dimension - "the analogy is congruent in a real physical sense" (ibid.,p.905). However Klein pertinently pointed out that concepts like point charge and material point are alien to classified field theory, a rare criticism at the time. In his concluding remarks Klein noted that the matter particles should be regarded as special solutions of the unified field equations, since "the movement of the material particles has similarities with the properties of waves" (ibid.,p.905). The analogy however was incomplete in a spacetime of only four dimensions. It can be made complete if the observed motion is regarded as "a kind of projection on to spacetime of the wave propagation which takes place in a space of five dimensions" (ibid.,p.905). Using the Hamilton-Jacobi equation in five dimensions leads to the theory of Kaluza.

Klein attempted to strengthen further the physical status which Kaluza gave to the extra dimension, like Kaluza acknowledging that it may be strange or surprising in our physical thoughts. In addition, Klein insisted that the possibility of describing quantum phenomena via five dimensional field equations could not be denied *a priori*, Charged particles would move on five-dimensional geodesic lines. Klein admitted in his conclusion that "only the future would show whether reality lies behind these hints to possibilities" (ibid.,p.906). He also showed remarkable foresight in his final sentence in wondering whether, in the description of physical events, even the 14 potentials were enough, or whether Schrödinger's method would lead to the introduction of new quantities of state, new variables ("zustandsgrosse").

Oskar Klein was therefore the first to try to use the extra fifth dimension not only to unify electromagnetism and gravity (after Kaluza) but also to try to understand quantum theory.

2. Precursors of Klein's 1926 paper

Apart from Kaluza's original paper of 1921, Klein referred to papers by Schrödinger (1926a and 1926b) and by de Broglie (1924 and 1925).

(i) Schrödinger's Wave Mechanics

Erwin Schrödinger, in the development of his own theory of wave mechanics, also made particular reference to the 1925 paper of de Broglie. His crucial paper showed the wave to be a better model than the particle. For more than one particle, his equation in fact involved waves in an abstract multidimensional space. This was actually an infinite dimensional Hilbert or configuration space - a purely mathematical concept for Schrödinger, to be established as the basis of Quantum Mechanics.

In the preliminary paper (Schrödinger, 1926a) he started to take seriously de Broglie's wave theory of moving particles of matter, and superimposed on this a quantisation condition. This led to his key paper (1926b). This contained his equation for a Hydrogen atom, and marked the birth of Wave Mechanics. Schrödinger used the concept of standing waves, where the wave function ψ is everywhere real and finite. He discussed the possible physical significance of ψ in describing the characteristic periodic processes in the system. Schrödinger took a similar point of view in his third paper in the journal 'Physical Review' written in English: "material points consist of, or are nothing but, wave systems" (Schrödinger, 1926e,p.1049). This in turn was based on de Broglie's "phase waves" ("ondes des phase" - De Broglie, 1925, p.22). Schrödinger admitted however that only a harmonic union of the two extremes, material points and wave systems, would provide a thorough correlation of all features of physical phenomena. He pictured the motion in its configuration (or "coordinate") space, giving the propagation of a stationary wave system:

> "In the simple case of one material point moving in an external field of force, the wave phenomenon may be thought of as taking place in the ordinary three dimensional space; in the case of a more general mechanical system it will primarily be

located in the coordinate space, and will have to be projected somehow into ordinary space" (Schrödinger, 1926e, p.1054).

This was a dilemma which was never satisfactorily interpreted. The other interesting factor, beside multidimensional space, is the imaginary as well as the real value which has to be given to the wave function ψ, only $\psi\overline{\psi}$ is real : "What does this imply?" (ibid.,p.1060). Schrödinger then attempted to attach a definite physical meaning to the wave function ψ, "a certain electro-dynamical meaning" (ibid.,p1062). He did not develop these issues further, leaving ψ as a purely mathematical solution to the Schrödinger Equation. The Eigenstate has a constant potential - for example in the simplest one dimensional case,

$$\psi = Ae^{\frac{2\pi i}{h}} \sqrt{2m\,(E-Vo)}$$

This is the eigenstate of energy $e^{\pm\frac{2\pi i\ Et}{h}}$ where E is the energy constant, h = Planck's constant, V = the potential energy.

Schrödinger's brilliance led him to emphasise that he had later noticed that his Wave Mechanics was "in complete mathematical agreement with the theory of matrices put forward by Heisenberg, Born and Jordan" (ibid.,p.1063).

Schrödinger gave his full equation in 3 dimensional Euclidean space, written for the hydrogen atom (one particle in three dimensions):

$$\frac{d^2\psi}{dx^2}+\frac{d^2\psi}{dy^2}+\frac{d^2\psi}{dz^2}+\frac{8\pi^2 m}{h^2}\left(E+\frac{e^2}{r}\right)\psi = 0$$

\- where for the Hydrogen atom, m = mass, e = charge, and r = radius.

Schrödinger admitted at this point that ψ is not a function of ordinary space and time, except in the (one body) Hydrogen atom (ibid., p.1066). For N electrons, the integrals are 3N-fold, extending over the whole coordinate space. He attached a clear physical meaning only to the *product* $\psi\bar{\psi}$. The equation for 2 or more particles:

$$\left(\frac{d^2\psi}{dx_1^2}+\frac{d^2\psi}{dx_2^2}+\ldots\right)+\left(\frac{d^2\psi}{dy_1^2}+\frac{d^2\psi}{dy_2^2}+\ldots\right)+\left(\frac{d^2\psi}{dz_1^2}+\frac{d^2\psi}{dz_2^2}+\ldots\right)+\frac{8\pi^2 m}{h^2}\left(E+\frac{e^2}{r}\right)\psi=0$$

Postscript

Schrödinger never really resolved the problem. He insisted for many years on the ontology of the wave - that particles should be described in terms of the wave model. As Einstein later wrote, Schrödinger had "an emotional commitment" to the objectivity or reality of waves in multidimensional phase space, while admitting they are "less real and less concrete than ordinary waves" (physical, three dimensional waves, in position space) - (Einstein, 1950.p32). Nevertheless the paradox of Young's two slit interference experiment led Schrödinger to affirm later "that we must think in terms of waves through the two slit experiment", but that the interference pattern "manifests itself to observation in the form of single particles" (Schrödinger, 1951, p.47). Schrödinger remained ambiguous, affirming that "reality is *neither* classical particles *nor* the so-called wave picture" (ibid.,p.40), with the caveat that "no model shaped after our large-scale experiments can ever be true" (ibid.,p.25).

(ii) Louis de Broglie's matter-waves and "guiding-wave"

In his papers written in the 1920's, de Broglie also probed to the heart of the paradox of waves and particles, influencing both Schrödinger and Klein.

In an early paper, de Broglie was already talking of an "integral taken over the whole phase extension of 6N dimensions" (de Broglie, 1922, p.422). In September 1923 he enunciated his pivotal new principle : that particle-wave duality should apply not only to radiation but also to matter. In his preface to his re-edited 1924 Ph.D. thesis, de Broglie wrote,

> "After long reflection in solitude and meditation, I suddenly had the idea during the year 1923, that the discovery made by Einstein in 1905 should be generalised in extending it to all material particles, and notably to electrons" (de Broglie, 1963 edition, p.4).

Thus he made the "paradigm change" (see Kuhn, 1962) in his 1923 paper, that $E = h\nu$ should hold not only for photons but also for electrons, to which he assigned his famous "fictitious associated wave" (de Broglie, 1923, pp.507-508). In the equation, E is the energy, ν is the frequency of the wave, and h = Planck's constant.

In his paper of 1923, de Broglie tried to save both the corpuscular and the undulatory characters of light, using "energyless light phase waves" (de Broglie, 1926 edition,p.456). He also used such terms as "spherical phase wave", "non - material phase wave" etc., while acknowledging that these "cannot carry energy, according to Einstein's ideas" (ibid.,p.449.).

The dilemma of particle-waves spreading out over the whole space was pursued unremittingly by de Broglie, never accepting a compromise as did Niels Bohr, nor permanently happy with any given solution. His original thesis on "matter waves" made reference to "periodic internal phenomena" (de Broglie, 1923, p.507) and the real existence of light quanta, in his attempt to save both particle and wave phenomena. This "periodic phenomena" undoubtedly influenced Klein's ideas, and was expanded in a 1925 paper. De Broglie wrote of an association between a uniform motion of a particle and the propagation of a certain wave, "of which the phase advances in space with a speed exceeding that of light" (de Broglie, 1925,p.22).

This proved unsatisfactory, and in a 1926 paper, de Broglie used Schrödinger's equation to derive the equations of propagation of this wave associated with a universal potential vector (de Broglie, 1926b). In another paper the same year, he wrote further of the propagation of the "non-physical wave" associated with the motion of a material particle, linking it with light and optics (de Broglie, 1926c,p.1). The basic idea of his original doctorate thesis was again used in the same Journal, involving a "generally imaginary function" of x, y and z coordinates (de Broglie, 1926d, p.321). De Broglie was clear that Schrödinger's equation had a meaning only in abstract mathematical or configuration space (which included complex numbers in the description). This was not really a physical equation of propagation, although $\psi \bar{\psi}$, the amplitude squared, gave a probability description. In a 1927 paper, de Broglie argued that this "non-physical equation", this "fictitious wave" with a complex or imaginary base, provided the information for the amplitude (de Broglie,1927a - Selected papers 1928, pp.132, 134). This became the accepted interpretation, yet its ambiguities and 'non-physical' description have rarely been stated so clearly.

De Broglie thought of the waves as being associated with the particles, and suggested that a particle such as a photon or electron is in fact guided on its way by the associated wave, to which it is tied. De Broglie's summary as a "Guiding Wave" or "Pilot-Wave" retained the problem without accepting the Copenhagen compromise of Bohr. He affirmed that it was

> "permissible to adopt the following point of view : assume the existence of the material particles and of the continuous wave represented by the function ψ as distinct realities" (ibid.,p.138).

He postulated that the motion of the particle was determined as a function of the phase of the wave. The continuous wave spreading out throughout space is then thought of as "directing the motion of the particle : it is the guiding wave". So de Broglie reached the centre of the paradox, although he back-tracked immediately:

> "the corpuscle will doubtless have to be 're-incorporated' into the wave phenomena, and we shall probably be led back to ideas analogous to those developed above ... a sort of average density" (ibid.,p.135).

This was further diluted (and nearer to Born's probability ideas) in an appendix added by the author, de Broglie, for this 1928 edition : the ψ wave is a "guiding wave" by which the motion of the particle in controlled, however "ψ is also a probability wave" (ibid.,p.138).

The dilemma has often been glossed over, yet never really resolved. Born's paper in 1926 interpreted the wave as a probability wave in order to explain Schrödinger's theory. Heisenberg epitomised the paradox in an unambiguous way, pointing out that

> "in considering 'probability waves', we are concerned with processes not in ordinary three-dimensional space, but in an abstract configuration space (a fact which is, unfortunately, sometimes overlooked even today) ... the probability wave is related to an individual process". (Heisenberg, in Ed. Pauli, 1955, p.13).

At this point in de Broglie's thinking, he became very excited and influenced for some time by Klein's seminal papers of 1926.

His own thinking in 1924 and 1925 had itself helped to set Klein on the original Kaluza path of five dimensions.

3. Further developments from Klein's original paper - the five dimensional theory spreads.

It has been shown that Klein's 1926 article in the Zeitschrift für Physik was the first paper to make positive reference to Theodor Kaluza's paper, five years previously. Oskar Klein had published other papers, e.g. an energy perturbation of the atom (Klein, 1924), but the 1926 paper on Quantum theory and five-dimensional Relativity theory was new ground for him. As we have seen, Klein built on both Schrödinger's equation in multidimensional space and on de Broglie's associated pilot wave, with Kaluza's unification as foundation.

Klein's second paper in 1926 was published in English in the journal "Nature" (Klein, 1926b) and gave only his own fundamental paper and that of Kaluza as base references. It was Klein's aim to link the fifth dimension with quantisation, seen as electric charge. The fifth dimension was assumed to be closed in that direction, with a very small period of oscillation "ℓ". This smallness of 'ℓ' helped to explain "the non-appearance of the extra dimension in ordinary experiments, as a result of the averaging over the fifth dimension" (Klein, 1926b,p.516).

The clear implication is that the fifth coordinate is periodic, hence the fifth dimension should have a different "topology" from the other four. The fifth dimension has been compactified to a circle of radius r. Mathematically this implies that spacetime has the topology $R^4 \times S^1$ (where S^1 is a circle; if we set out in the fifth direction we would always return to our starting point).

"Quantisation" required a number of wavelengths 'λ' to fit on to the circumference of the five dimensional circle:

$$n\lambda = 2\pi r$$

$$\text{and} \quad \lambda = \frac{2\pi r}{n}$$

The momentum

$$p = \frac{h}{\lambda}$$, where h is Planck's constant

hence

$$p = \frac{nh}{2\pi r}$$

and $$p^2 = \frac{n^2 h^2}{(2\pi r)^2}$$

This is large if r is sufficiently small, and $n \neq 0$.

Thus only the $n = 0$ states of zero excitation are observed in the "low energy" domain of normal physics. This is the extra idea that the quantum effects produce. The electric charges of the elementary particles are quantized in units of a fundamental charge (a well-known, but hitherto unexplained fact).

(Note: The idea is much used today, where $\frac{h}{r^2}$ is the "Planck Mass", where r is the radius of the Planck size.)

Klein in fact found this to be 0.8×10^{-30}cm. He noted that this small value, together with the periodicity

> "may perhaps be taken as a support of the theory of Kaluza in the sense that they may explain the non-appearance of the fifth dimension". (Klein, 1926b, p.516)

In the following year, 1927, Klein elaborated further on his five-dimensional thesis, giving as additional reference V.Fock (1926), who published his own five dimensional version a month or two after Klein's first seminal paper. In this lesser-known paper, received in December 1926, published early 1927 (1927a), Klein repeated this reference, de Broglie's as before, and extra Schrödinger papers

(1926c and 1926d). The fifth dimension appeared as a pure harmonic component. Klein emphasised that it had a period conforming with the value of Planck's constant, which effected the transition to the Schrödinger theory of quantum mechanics. Klein also emphasised the basic oscillation of the fifth dimension x_0 and the fact that the fifth dimension is "closed in the direction of x_0" (Klein, 1927a, p.441). A more comprehensive summary was produced by Klein in his better known paper of October 1927 : "Five-dimensional Representation of the Theory of Relativity" (1927b).

Note: Klein maintained his belief that the fifth dimension was somehow linked with quantisation for many years e.g. Klein, 1956 (See Chapter 6 - and also Chapter 8 to find his basic principle reemerging in Superstrings).

4. Klein's rejuvenation of Kaluza's theory met with temporary success

Klein thus took Kaluza's idea of an extra dimension and tried to elevate further the fifth dimension to the physical status of the others, while retaining an apparent four dimensions of spacetime. While he regarded it as physically real, Klein did treat it differently from the other four, picturing the fifth dimension as too small to be directly observable. However the description was still not convincing enough to gain later acceptance for the actual physical reality. Klein, like Kaluza, noted that the use of an extra fifth dimension might well appear surprising, but was himself convinced of its importance.

(i) Reactions of other scientists were initially very favourable

At the time, in 1926, the five dimensional theory took the scientific world by storm. George Uhlenbeck reported later to Abraham Pais, "I remember in the summer of 1926, when Oskar Klein told us of his ideas which would not only unify the Maxwell with the Einstein equations, but also bring in the quantum theory, I felt a kind of ecstasy! Now one understands the world!" (Pais, 1982,p.332).

In 1926 the popularity of the five dimensional theory was increasing rapidly. Only two days after Oskar Klein's first article was published in Zeitschrift für Physik on 10 July 1926, Heinrich Mandel's article was received for publication. Mandel claimed independent discovery of Kaluza's theory, but made reference to Klein's article, presumably after it was received in April, prior to publication. Mandel tried to explain non-Euclidean measurement

> "by imagining the world as a four dimensional hyperplane in a superior five dimensional (4+1) Euclidean space. A five dimensional point of view seems to be essential for the understanding of the electromagnetic properties of matter". (Mandel, 1926, p.136).

Mandel claimed that the fact that this had been noticed previously by Kaluza in 1921 and developed in the same way was only made known to him by a reference of Klein in his 1926 paper!. Mandel intended "a certain physical meaning"(ibid.,p.139) to be ascribed to the five-dimensional manifold. His analogue of the four/five dimensions was similar to interpreting a two dimensional non-Euclidean surface by reference to "a superior three-dimensional Euclidean space", and where"geodesics are lines of curvature in the universe" (ibid.,p.136).

Within two weeks of Klein's published article, the same journal received an article for publication by the Soviet physicist V.Fock from Lenningrad, and published in the same volume as Mandel's paper. He confirmed that while Mandel's note was being printed, having been lent in manuscript form to Fock, "the nice work of Oskar Klein" was published, "in which the author reached results which are principally identical." "The introduction of a fifth coordinate parameter appears to us to be very suitable for the setting up of the Schrödinger wave equation" (Fock, 1926,p.226), i.e. in five dimensional space. Einstein was to give Fock credit for his contemporaneous attempt at unification (Einstein,1927, p.30). No one seems to have recognised an equal claim to primacy by Mandel who not only used the Kaluza-type approach but also the understanding of curvature by embedding.

In the same volume of the Journal, Ehrenfest and Uhlenbeck used a graphical illustration of de Broglie's phase wave in the five dimensional Klein theory. (This was received in September, before the publication of Mandel or Fock's papers). They attempted to link de Broglie's pilot wave even more firmly into five dimensional theory. The idea of "the movement of an electron being in reality the spreading out of wave groups in a dispersing aether, situated in the usual 4-dimensional world" (Ehrenfest, Uhlenbeck, 1926, p.495) was of course developed further by Schrödinger. They acknowledged the same conclusions reached by Klein, adding explicitly that the de Broglie phase waves are in five dimensions, seen as "traces" in the usual four dimensional space. Their paper also confirmed that the world is periodical in the fifth dimension, with a period connected with the Planck constant. They used the two dimensional analogy effectively to picture the four dimensional world.

Still in volume 39 of that year, the Journal carried an article by Gamow and Iwanenko. They noted that Klein and Fock had shown that the idea of de Broglie's wave, together with the wave equation of Schrödinger, could be put into a simple form if a fifth coordinate is introduced. The waves in five dimension are again seen to be identical with the phase waves, the "inner process" of de Broglie (Gamow and Iwanenko, 1926, p.867).

A flurry of articles on five dimensional unification came in the next volume in 1927. Iwanenko, this time with Landau, began the withdrawal from a fifth dimension with any physical significance. They reached a generalisation of the Schrödinger equation to coincide with the "Klein-Fock equation", but without the "somewhat artificial introduction of the fifth coordinate" (Iwanenko and Landau, 1927,p.162). A similar trend appeared in an article by Guth who treated the solutions in a purely mathematical way. (Guth, 1927). Jordan, writing at the same time, referred also to Klein and Fock's attempt to make the wave equation real by introducing the fifth dimension, preferring himself a mathematical, theoretical and statistical analysis (Jordan, 1927).

(ii) Further strengthening by Klein

As we have seen, Klein returned twice to his theme in the same Journal in 1927, having already elaborated his ideas in Nature. His first paper was mainly mathematical, emphasising that the fifth dimensional space is closed in the direction of x_0, where Planck's constant is related to the basic oscillation of x_0. The smallness of this extra dimension accounts for the "non-appearance of the fifth coordinate in our usual physical equations" (Klein,1927a,p.441), i.e. it leads directly to the four dimensional correspondence presentation. The second paper emphasised the physical status of the extra dimension, the fifth dimension being portrayed in a mathematical way "which appears in a natural light". (Klein, 1927b,p.194). Klein himself however hoped to replace the g_{ik} being merely independent of x_0 by a "more rational" derivation from quantum mechanics (ibid.,p.208).

In the following volume of 1927, references were made to all the above articles in a paper by London. He admired the boldness of Weyl's theory using variable curvatures of Riemannian space (a gauge theory ahead of its time) although Weyl needed "a strong and clear metaphysical conviction" (London, 1927, p.377) in the face of everyday experience. Weyl's scalar is numerically identical with de Broglie's field scalar, which London tried to simplify by bringing in the five dimensional wave function. London pointed out the "complex amplitude" of the de Broglie wave, which "as a useless part of contemporary physics, he had to supply with a metaphysical existence" (ibid.,p.380) - a trenchant appraisal. This fifth coordinate was supported as the quantum mechanics link by London, although he raised the problem that this fifth coordinate involved an unknown factor which still had to be defined in contrast to the other four coordinates, and was orthogonal to them.

Only very occasional references to the Kaluza-Klein idea were made after this in the Zeitschrift für Physik, the main journal to carry articles on the subject. These became purely mathematically based (e.g. Landé, 1927) with a declining physical status to the reality of the fifth dimension. Meanwhile, Klein's article in Nature (1926b) had produced varying responses. Klein himself had used the small value for the radius of the curves in the fifth dimension, together with the periodicity in this dimension, to explain the non-appearance of the fifth dimension in ordinary experiments. After this there were very few references to Kaluza-Klein. Schott gave an excellent summary of Schrödinger's papers and of the views of his predecessor, de Broglie. He made only a passing reference to Klein, without details (and even then a reference to Klein's less important paper - 1927a). Guth (1927) also referred to this paper of Klein's, rather than the articles of 1926, or particularly the article in Nature itself, and the emphasis on five dimensions was disappearing. Wiener and Struik wrote to Nature that year, referring to Klein's original article (1926a), and claiming an analogous treatment. It is interesting to see the decline in the possible physical significance of the extra dimension "the fifth dimension turns out to be a mere mathematical convention..." (Wiener and Struik, 1927, p.854).

(iii) The use of five dimensions was adopted by Einstein, de Broglie and others

Despite the lack of interest in the columns of Nature, solid contributions to physics involving the idea of a five-dimensional universe were being made independently in 1927 in some other journals. Klein's rejuvenation of Kaluza's idea may well have provoked Einstein's attempts to unify gravitation and electromagnetism in terms of a single metric in a five dimensional spacetime (e.g. Einstein, 1927 - see Chapter 5). This was to be a recurring theme at occasional intervals in Einstein's work.

Other prominent physicists to explore such ideas mathematically included de Broglie himself, Rosenfeld's "The universe in five dimensions and mechanical wave theory" (Rosenfeld, 1927a) and also Gonseth and Juvet - "The space metric of five dimensions of electromagnetism and gravitation" (1927). Klein himself with Jordan explored the particle/wave dilemma, "the many-body problem and the Quantum theory" (1927). This in fact led to the Klein-Jordan-Wigner mathematical expression of the wave-particle duality (Jammer, 1966, p.68).

A masterly survey was given by Struik and Wiener (following their own article in Nature on five dimensions) in the Journal of Mathematics and Physics. This traced the Weyl-de Broglie-Schrödinger development, to the "Kaluza-Fock-Klein five dimensional quantum theory "developed by Einstein, de Broglie and themselves. Struik and Wiener noted that in the five-dimensional theory, the notion of an electron in an electromagnetic field may be represented as a projection on the 4-dimensional manifold of a geodesic line of the five dimensional manifold (Struik and Wiener, 1927,p21). This is a considerable advantage in interpreting the extra dimension. Interestingly they refer to classical point mechanics where each body traces a locus in a four-dimensional spacetime, and in the wave mechanics where a body is a phenomemon pervading the whole of spacetime. In order to

> "preserve the identity of different bodies, it is apparently necessary to attribute to each a set of space dimensions of its own ... and a time of its own as well". Hence "the world of the problem of two bodies is an eight dimensional world" (ibid.,p.22).

Thus one matter of considerable importance is that of "forming some sort of a well-defined four dimensional spacetime from the multidimensional world of the problem of several bodies" (ibid.,p.23). Struick and Wiener thus clearly demonstrated the inner paradox of the ontology of multidimensions.

In an interesting and little recognised insight, Gonseth and Juvet suggested in their 1927 paper that g_{55} should be taken as a scalar field (as Kaluza had originally seen) which however might play the role of the Schrödinger wave field. Although in the standard Kaluza ansatz, $g_{55} = 1$, this does not satisfy the five dimensional Einstein equation : g_{55} cannot be a constant and therefore has to be a scalar field.

Louis de Brolie's temporary espousal of a five dimensional reality (1927)

The problem of why Klein's rejuvenation of Kaluza's theory seemed to be only a temporary mini-explosion is epitomised in the work of de Broglie. Although Einstein and Klein himself made further attempts at a unification (with only limited success), it is not entirely clear why de Broglie did not follow up the five-dimensional idea. He had adopted it fervently in his paper, "The Universe of five dimensions and the wave mechanics" (de Broglie, 1927b, or 1928 Edition p.101). He believed it would solve the wave/particle dilemma, with matter being the periodic phenomena in the five-dimensional universe. Klein's idea thus brought together his own ideas of matter as waves (and therefore periodic as stacking waves) and also an associated wave or guiding wave in the fifth dimension.

De Broglie in fact went back to Kaluza's original paper. He thought the dilemma of the associated wave not being in three-space dimensions was solved in the extra space dimension, which was

> "quite beyond our senses, so that two points of the Universe corresponding to the same values of the four variables of space time but to different values of the variable x^0 are indistinguishable. We are, as it were, shut up in our space - time manifold of four dimensions and we perceive only the projections on this space-time of points in the Universe of five dimensions" (de Broglie, 1927b,p.104).

However he did not advance the mathematics materially further than Klein, and concluded:

> "In order to get to the bottom of the problem of matter and its atomic structure, it will no doubt be necessary to study the question systematically from the viewpoint of the five-dimensional Universe, which seems more fertile than M.Weyl's point of view.....If we succeed in interpreting...(the)

> equation, we shall be very close to understanding some of the most perplexing secrets of Nature." (ibid.,p.111)

Although retaining the ambiguities of particle and phase wave throughout his life, de Broglie was convinced in 1927 that Kaluza's original approach was the correct one. His stated aim was

> "to show how remarkably simple an aspect mechanics assumes, in its old form as well as in its new wave form, when the idea of a Universe of five dimensions, which has been brought forward by Monsieur Kaluza, is adopted" (de Broglie, 1927b, p.101 in 1928 Edition - p.65 in original French).

Force is replaced by geometric conceptions:

> "thanks to the theory of the Universe of five dimensions, it is possible to put the laws of propagation in the new wave mechanics in a very satisfactory form" (ibid.,p.101)

De Broglie paid tribute to Kaluza's "bold but very elegant theory" and emphasised that "in the five dimensional universe, the world line of every material particle is a geodesic".(ibid.,p.106).

Postscript to de Broglie

Despite his full approval in 1927 of the Kaluza-Klein approach, de Broglie was to remain ambiguous about five dimensions as an ultimate answer in his later writings.

In a book published in 1930, An introduction to the study of wave mechanics, de Broglie was still agonising over the wave particle duality. He saw that if particles were simply "wave packets", they would have no stable existence, and he reluctantly accepted that it appeared impossible to maintain Schrödinger's wave ontology. De Broglie admitted that it was no easier to accept his own concept, that the particle is a singularity in a wave phenomena. He preferred to consider the "matter wave" as the reality, and came to the position that "the particle is guided by the wave which plays the part of a pilot wave". He also admitted that this was still unsatisfactory, nevertheless he wished "to preserve some of the consequences" (de Broglie, 1930,p.7).

De Broglie however tended to lean towards Heisenberg and Bohr in that "the wave is not a physical phenomena" taking place in a region of space - "it is the nature of a symbolic representation of a probability" (ibid.,p.120). He was also attracted to Schrödinger's multidimensional space, "a single wave travelling in the generalised space" (ibid.,p.177). The difficulty of the "fictitious" space "seem to strengthen the view that no physical reality is to be attached to the associated wave" (ibid.,p.187).

The inherent paradoxes were never hidden by de Broglie, and were later to be explored by David Bohm (1952), J.S.Bell (1964) and others. The symbolic representation by a wave, without representing a physical phenomenon, makes interference phenomena hard to understand. De Broglie now clearly saw that the orthodox wave/particle Copenhagen solution of Niels Bohr was inadequate: "they exclude each other because the better one of them is adapted to Reality, the worse is the other and conversely" (de Broglie,1939, p.278). De Broglie's non-material "phase", "pilot", "guiding" or "associated wave" wasnever a clear cut model. It was more an analogue model of the mathematics, as was his insight in describing particles as "point singularities". Although at the time this was interpreted as no more than singular solutions, de Broglie used it frequently after 1927: "each particle constitutes a singularity in a wave phenomena in space" (e.g. de Broglie,1927a, pp.114,131; 1930, p.7).

The reason for de Broglie's abandoning his use of five dimensions will never be quite clear. He was torn between the concept of extra dimensions and the prevailing idea that reality was limited to three space dimensions : "Having a very 'realist' conception of the nature of the physical world", de Broglie later explained how he himself was concerned with concrete physical ideas (de Broglie, 1973, p.12). He could only see that the wave function λ of configuration space "cannot be considered as a real wave, being propagated in physical space" (ibid.,p.14). Yet he was "disturbed to see the clear and concrete physical image completely disappear" in the representation as probabilities (ibid.,p.15), and later came back to the ambiguities of his own theory of the "double solution", containing both physical and abstract interpretations in the conclusion to his article written for Wave Mechanics, the first fifty years (Ed. Price, et al.,1973).

Indeed, only a year before his death in 1987, de Broglie explained his final thoughts to me through his amanuensis, Georges Lochak, Director of the Louis de Broglie Foundation in Paris. I had written to Monsieur Louis de Broglie about the wave/particle paradox and his original paper in 1927 using five dimensions. M. de Broglie

> "remains convinced that you have touched on something absolutely vital in the co-existence of waves and particles in his theory of the double solution and the idea of the guiding of particles by the waves; he is convinced of this, but the real problem is to reach the point of making this a general theory, and one having heuristic power sufficient to predict new effects. On the other hand, M.de Broglie has abandoned the penta-dimensional theory completely, above all since he is convinced of the

necessity of a return of the theory with a more concrete physical manifestation (la nécessité d'un retour de la théorie à des représentations physiques plus concrètes) than is the case in present day physics" (de Broglie, 23rd January 1986, private correspondence).

5. Reasons why Klein's attempted synthesis of Quantum Mechanics with Kaluza's five dimensional unification did not become accepted after its initial success

We have seen, in the case of Kaluza's theory, that for a number of reasons his idea was ahead of its time. Although Klein's revival of Kaluza's theory was more widely noticed after its publication, the lack of permanent success was again due to a lack of the mathematical concepts which were to become available much later, and to the concentration on uniting only the two forces known at the time. In addition Klein had made the ambitious attempt to link his five dimensional concept with Quantum mechanics, where the concepts often seem non-intuitive and against common sense.

Enigmas and paradoxes in Quantum Mechanics

Despite its extraordinary success mathematically, the orthodox interpretation of Quantum Mechanics led to a number of enigmas and paradoxes. Quantum Mechanics in fact became the conceptual basis for many later technological developments such as lasers and computer chips. It has been completely successful at all levels accessible to measurement. Nevertheless, despite the widespread agreement on its use, physicists have always disagreed profoundly on how to describe the quantum nature of reality which underlies the everyday world. The abstract mathematical formalism therefore seems to represent correctly particles as waves, described by the state vector ψ, in a multidimensional abstract mathematical space.

Quantum Mechanics replaces Newtonian deterministic laws by an equation which describes the probability of finding a particle at a particular point in this infinite dimensional Hilbert space.

The interpretation of this is the metaphysical framework ascribing physical meaning to the theoretical formalism. When we measure a particle at a particular point, the probability of finding the particle becomes certain, the wave function is said to "collapse". The conscious observer therefore plays a central and fundamental role in quantum theory. That particles and atoms exist only when they are observed, is the most usual interpretation, although in conflict with the realistic approach which many physicists adopt in practice.

De Broglie and Schrödinger had both attempted to tackle the problem, without convincing or universal approval. As a result of deliberations with Schrödinger in Copenhagen in 1926, Bohr affirmed that both the theoretical pictures - particle physics and wave physics - are equally valid, providing complementary descriptions or models of the same reality. Yet the waves were not real waves, but a complex form of vibration in an imaginary mathematical space (multidimensional and including complex or imaginary numbers). Also each particle, e.g. an electron, needed its own three dimensions in this space.

Max Born's interpretation of the wave as a measure of the probability of finding a particle at any particular point was followed by Heisenberg's discovery (working at Bohr's Institute later in 1926) that uncertainty is indeed inherent in quantum mechanics. Because of the wave/particle dilemma, it is impossible to define the position and the momentum of a particle such as an electron at the same time. Heisenberg's "Uncertainty Principle", complementarity, probability and the disturbance of the system by the observer (the "collapse of the wave function or quantum state") became known as the "Copenhagen interpretation" of quantum mechanics.

This allowed physicists to accept the Bohr proposals as the orthodox interpretation and to get on with the mathematics, and thereby ignore the enigmas and paradoxes inherent in the description of the theory. In particular, as Bohr was the first to point out, quantum systems have a certain "wholeness". Because of this irreducibility, it is impossible to give a complete description of a system by breaking it down into its parts, as could be done in classical physics.

The two-slit paradox

One illustration of the wave/particle paradox is given in the two slit experiment. Electrons or photons from a source pass through two nearby slits in a screen A and travel on to strike a second screen B where their rate of arrival can be monitored. A pattern of peaks and troughs on screen B indicates a wave interference phenomenon. If the experiment is performed with single photons and repeated frequently, as was found by G.I.Taylor (Abramsky,1975, p.4) the statistical ensemble of photons produces such a pattern. Even though a single photon passing through one of the slits could arrive on the screen or photographic plate at a point midway between the bright bands, i.e. in the interference shadow band, there is no evidence of this.

Schrödinger and Einstein (e.g. Einstein et al, 1935) recognised the crucial importance of the double slit experiment, in which are embodied all the essential features and paradoxes of quantum mechanics. The patterns of interference seem to be caused by the two waves, one from each slit, interfering with one another. Light scintillations can be picked up on a sensitive screen from individual photons or electrons. One electron still produces interference patterns as if it "knew" the other slit existed and adjusted accordingly - or as if it went through both slits at once. It seems as if we must

> "assume that a particle flying through the opening of the first slit is influenced also by the opening of the second slit ..... and that in an extremely mysterious fashion" (Schrödinger, 1951, pp.46,47).

Schrödinger described this as the only solution if effectively one particle at intervals of time passed through one or other slits.

This independence takes place without another particle to gauge its "step" or "interference" position. This quantum theory explanation was rejected as bizarre by Einstein and his colleagues in his thought-experiment (Einstein, Podolsky and Rosen, 1935). Schrödinger insisted that

> "we must think in terms of spherical waves emitted by the source, parts of each wave front passing through both openings, and producing our interference on the plate - but this pattern manifests itself to observation in the form of single particles" (Schrödinger, 1951, p.47).

The non-locality paradox

Another peculiar aspect of quantum theory is the fact that when two photons (quantumentities), A and B, briefly interact and then separate beyond the range of interaction, quantum theory describes them as a single entity-"quantum inseparabiltiy". All objects which have once interacted are in some sense still connected to one another. This is a 'non-local' connection, not subject to normal force fields. Schrödinger and Einstein always opposed this interpretation, although granting it the quantum formalism.

This is an elaboration of Bohr's original "wholeness" of quantum systems. It was to be further elucidated by Bell's Theorem (J.S.Bell, 1964). That quantum theory is correct and the correlations are inevitable was confirmed even more recently by Alain Aspect and colleagues in Paris (in 1981 and 1982). This verified the quantum mechanics prediction that particles originally paired then widely separated have their spins related. This "action-at-a-distance" cannot be explained on existing laws of physics.

6. Postscript : Quantum Mechanics today

The paradoxes have become more apparent since 1926. Alternative interpretations have included an even more bizarre interpretation such as Everett's Many World Theory in 1955, advocated initially by Bryce De Witt, John Wheeler and others (Everett, 1955).

As Werner Heisenberg described, the criticism of the Copenhagen interpretation of quantum theory

> "came at first from the older physicists, who were not prepared to sacrifice so much of the edifice of ideas of classical physics as was here demanded of them.....

Einstein, Schrödinger and von Laue did not regard the new interpretation as conclusive or convincing. In recent years, however, various younger physicists have also taken their stand against the "orthodox" interpretation, and some have made counterproposals". (Heisenberg, 1955 p.16).

Heisenberg notedsome who are dissatisfied with the language used - i.e. the underlying metaphysical philosophy, and who tried to replace it with another, e.g. David Bohm and de Broglie. Others expressed general dissatisfaction. Einstein originally advocated a statistical interpretation, because quantum mechanics gave an incomplete picture of physical reality. This implied that a deeper theory was possible, and led to the "hidden variable" theory (Bohm,1952). Einstein described it as an "Ensemble Interpretation" awaiting a deeper theory, a completely deterministic theory parallel to the realism of his own philosophy (Einstein, 1950, p.31 - see Chapter 5). For Einstein, "the essentially statistical character of contemporary quantum theory is solely to be ascribed to the fact that this (theory) operates with an incomplete description of physical systems (P.A.Schilp, Ed., 1949, p.666).

David Bohm revived the Hidden Variable theories as early as 1951 in his Quantum Mechanics. He affirmed that

> "the basic criticism of quantum mechanics is not, as Einstein insisted, its lack of determinism, but rather its lack of conceiving the structure of the world in any way at all" (Bohm, 1982, p.362).

Bohm's original concept of hidden variables changed from being potentially physically verifiable to being beyond the reach of experimental search. As David Bohm wrote in reply to my questions, "My ideas of hidden variables change from taking 10^{-13}cm. as a limit, to a gravitational radius of 10^{-33}cm. within the past ten years". (Bohm, 9 January 1984, private correspondence).

Even for Max Born, the Uncertainty Principle led to "a paradoxical situation". Physical quantities were represented by non-commuting symbols. He described the thrill he experienced in condensing Heisenberg's ideas on quantum conditions for momentum of particles in "the mysterious equation $pq - qp = \frac{h}{2\pi i}$". This was in fact the centre of quantum mechanics

> "and was later found to imply the uncertainty relations"

as he described in "Physics and Metaphysics" (Born 1950, p.17).

Schrödinger tried to pour scorn on the dilemma of observer-centred reality with the paradox of a cat in suspended animation - dead and alive - (after possible death in a thought experiment) until actually observed. Only then does the wave function collapse and the cat exhibit death or life. Either the hybrid state of being alive and dead was true, or the cat was not real at all until seen by an observer. The Schrödinger cat paradox epitomises the strange though orthodox interpretation of quantum theory.

The Many Worlds Theory

The incompleteness of quantum mechanics either in describing Schrödinger's cat , or in the "non-local interaction between separated systems"(Bell, 1965, p.195), is of a totally different nature from the incompleteness that could be solved by introducing physical hidden variables. Either one must totally abandon the realistic working philosophy of most scientists, or completely and dramatically revise our concepts of spacetime. Many scientists do accept the Many Worlds Theory of Hugh Everett III. The problem which seems to have motivated Everett, supported by De Witt and later Wheeler, was that if they wished to describe the whole universe in terms of quantum state, "there cannot be any observers outside the universe to make measurements on it" (Smolin, 1985, p.42). The Many Worlds interpretation avoids the "collapse of the quantum state" by taking Schrödinger's equation literally (Everett, 1957).

Wheeler and De Witt went further and proposed that physical reality contains all the probability possibilities, all the possible worlds in which a particle (e.g. an electron) could move, although we ourselves only experience one outcome, one part of reality. Smolin noted that at a 1985 symposium at Oxford, physicists interested in quantum gravity voted on whether they took the Many Worlds theory seriously, and the result was about even, for and against (ibid.,p.43). The wave function ψ from Schrödinger's equation is linear and should not collapse. Everett's logical conclusion was to take the multidimensional reality of the equation seriously. Schrödinger himself remained quite firm about the mind of the observer not collapsing the wave function, not affecting the physics of quantum theory: "the observing mind is not a physical system, it cannot interact with any physical system" (Schrödinger, 1951, p.53). Schrödinger did not espouse the Many Worlds theory, although he was sure that "the 3-dimensional continuum is an incomplete description" (ibid.,p.40).

John Wheeler, as he explained in a discussion following a lecture "Beyond the Black Hole", has abandoned the idea of many worlds.

> "I confess that I have reluctantly had to give up my support of that point of view in the end - much as I advocated it in the beginning, because I am afraid it creates too great a load of metaphysical baggage to carry along". (Wheeler, in Ed.Woolf, 1980, p.385).

Wheeler himself abandoned any idea of dimensionality for the "pregeometry" of a foam-like spacetime structure, but also retained metaphors like "leaves of history to describe reality". (ibid.,p.351).

7. Metaphysics and Paradoxes

Niels Bohr's Complementary interpretation, the orthodox "Copenhagen", has ignored the metaphysics. In his later book *Atomic Physics and Human Knowledge*, he was to admit that quantum mechanics does not "provide a complete description" of physical reality, and emphasised "how far, in quantum theory, we are beyond the reach of pictorial visualisation". (Bohr, 1958, p.59).

Other interpretations still include the Many World's interpretation of Everett. This branching-universe or many-universes theory has been developed more recently by David Deutsch in an infinite number of parallel universes (Deutsch, 1986, pp.84,85) with reference to "the very inadequacy of the conventional interpretation of quantum theory"(Deutsch, 1985, p.2).

A further interpretation was originally advocated by Einstein - the Statistical interpretation, following his criticism of the quantum theory for its "incomplete representation of real things" (Einstein, 1936, reprinted 1954, "Physics and Reality" p.325, and quoted in Feyerabend, 1981, p.10). This was developed by David Bohm to imply a possible deeper theory of "hidden variables" (Bohm, 1952) and more recently as his "implicate order", a deeper order "unfolding" the explicate order of possible, phenomenal reality (Bohm, 1986, p.121 and in Wholeness and the Implicate Order, 1980). Bohm developed the idea of a "quantum potential" to explain the two-slit paradox, and which has been championed by Basil Hiley, e.g. "On a new mode of description in physics" (Bohm & Hiley, 1970,p.171).

The more straightforward version of the Ensemble interpretation has been consistently put forward by John G.Taylor. This eliminates any involvement of a conscious observer, emphasising the overall probability distribution. It is a statistical interpretation which makes no attempt at all to describe what is going on in an individual system and thereby avoids the problems or any discussion of the paradoxes involved (Taylor, 1986, pp.106,107).

The enigmas and paradoxes of Quantum Mechanics still remain today. In the opinion of de Broglie, the wave in many dimensions which describes the particle in three dimensions is "the deep mystery which has to be solved in the first place if one is to understand quantum mechanics" - quoted by Lochak in The Wave Particle Dualism:

A tribute to Louis de Broglie on his 90th Birthday (Ed.S.Diner,1984,p.4). De Broglie was still hoping that "one day, somebody will explain the profound nature of this strange link between waves and particles" (ibid.,p.8) which he discovered sixty years ago. Einstein, de Broglie and Schrödinger all ultimately rejected the prevalent Copenhagen orthodox representation of quantum mechanics.

More recent critics demonstrate that for them also, Quantum Mechanics is incomplete, or at least inexplicable.

> "Nobody understands quantum mechanics" (Feynman,1978,p.129).
>
> "It is all quite mysterious. And the more you look at it, the more mysterious it is" (Feynman, 1972,pp.8,13).

With reference to the crucial importance of the double slit experiment, which embraces all the essential features and paradoxes of quantum mechanics, "in reality it contains the only mystery" (Feynman,1965,p.1). The central role of the conscious observer, non-locality and a rejection of the Copenhagen Interpretation which conveniently removes the need to ask awkward question is described by Euan Squires in The Mystery of the Quantum World (Squires 1986). Quantum mechanics contains

> "many conceptual difficulties and ambiguities"; "it is no more than a set of rules .... something more is generally demanded of a theory" (d'Espagnat,1979,p.128), in "The Quantum Theory and Reality").
>
> "I'm quite convinced of that: quantum theory is only a temporary expedient" (J.S.Bell, 1986, p.51).

We need "a radical revision in our concepts of space" especially to cope with non-locality, although Quantum Mechanics predictions have been confirmed mathematically (Smolin, 1985, pp.40-43). Wheeler is careful to emphasise that

> "quantum theory in an everyday context is unshakeable, and unchallenged, undefeatable- it's battle tested" (Wheeler 1986,p.60).

Yet he insists that

> "if we are ever going to find an element of nature that explains

> space and time, we surely have to find something that is deeper than space and time ... I would rather hope that we shall still find a deeper conceptual foundation from which we can derive quantum theory" -

conceptual rather than experimental (Wheeler, 1986, p.66,69). A further reference is given by a pragmatic physicist in this 1986 "A discussion of the mysteries of quantum physics" (The Ghost in the Atom, Ed. Davies and Brown). Sir Rudolf Peierls is happy with the Copenhagen interpretation, yet sees the connection between biology and quantum mechanics:

> "we won't be finished with the fundamentals of biology until we have enriched our knowledge of physics with some new concepts" (Peierls, 1986, p.81).

The mathematics is not in question, but a new language, new concepts are required to interpret quantum mechanics. Richard Feynman

> "does not know any other way than mathematical to appreciate deeper aspects of reality of the physical world... one must know mathematics in understanding the world". (Feynman, 1981).

The full theory of elementary particles involves the relativistic equation of Quantum Mechanics as developed by Dirac in 1928 and other workers. The theory has been highly successful in many ways, correctly assigning the existence of an intrinsic quantised angular momentum or spin to each particle, and also predicting the existence of anti-particles. The theory of elementary particles is not complete, but Quantum Mechanics underlies the entire theory. There is the constant problem of infinities in quantum field theory: "we evade by 'renormalisation' .... a stop gap procedure that reflects our own ignorance" (Penrose, 1979, p.734). The problem is also the use of non-visualisable mathematical models, which if based only on the use of mathematics have long lost their surprise element of shock (e.g. Bohm and Hiley, 1975). We need a new consistent metaphysics.

A large part of observable physics, quantum electrodynamics and electromagnetism, is derived from the phase of a complex wave function in multidimensional space. The phase itself has no meaning and is unobservable. J.S.Bell, for example, confronts the dilemma:

> "The wave ψ is.... just as 'real' and 'objective' as, say, the fields of classical Maxwell theory...". "No one can understand this theory until he is willing to think of ψ as a real objective field, rather than just a 'probability amplitude', even though it propagates not in 3-space but in 3N-space" (see "Quantum Mechanics for Cosmologists" in Isham et al. (1981) p.625).

8. Conclusion

In chapter 4 we have seen how Klein tried to strengthen the physical reality of the fifth dimension originally introduced by Kaluza. He also attempted to incorporate quantum mechanics, following the inspiration of de Broglie and of Schrödinger. However Klein still had to treat the fifth dimension differently from the other four. He made a clear attempt to reply to the criticism that the fifth dimension was so small. Klein tried to link its periodic nature with the new quantum mechanics, using a different topology - that of a tiny circle within the four dimensions of normal physics. He successfully explained why the fundamental charges of elementary particles such as electrons were quantised, and linked them with the gravitational constant in a ratio of the size of the extra dimensions. Klein's calculations showed that these extra dimensions must be of very tiny radius, near the Planck size (10^{-33}cm) and therefore beyond the reach of standard physics.

A second way of using extra dimensions, besides the Kaluza-Klein model, has been seen in the use of multidimensional configuration or mathematical space in the Schrödinger equation. This complex, even infinite dimensional space is necessary in describing particles by the wave function ψ - an interesting feature of quantum mechanics which has no direct equivalent to the physical three dimensional world, although the square, $\psi \bar{\psi}$ is widely interpreted as predicting the probability of finding a particle at any particular point.

The Way Forward

There were to be problems with General Relativity at intense curvatures, and paradoxes within quantum mechanics were not satisfactorily resolved (although many physicists accepted the Copenhagen interpretation as a working compromise).

A new physics seemed to be needed, a deeper theory than these first two revolutions in the first quarter of the twentieth century. However, although widely used in present day theories of unification, Klein's exposition of Kaluza's theory was in advance of his time. Physicists and mathematicians needed the extra mathematical concepts which were only to become available in the last quarter of the twentieth century.

Even de Broglie and Einstein only gave temporary support. Only Einstein made intermittent efforts, with the support of one or two of his colleagues , to go beyond the four spacetime dimensions of General Relativity in search of a deeper, more consistent unified theory of gravity and electromagnetism (see Chapter 5).

CHAPTER 5

Einstein - intermittent flag-carrier of the five-dimensional universe

Synopsis

1. Einstein in the 1920's
2. Einstein returns to five dimensions in the 1930's
3. Einstein's final attempts at five dimensional theory, with collaborators
4. A critique of Einstein's 1938 high status for the fifth dimension
5. Einstein in the 1940's
6. Conclusion: Why Einstein was not successful in his search for unification using the Kaluza model.

The flurry of articles in the scientific journals on the Kaluza-Klein unification in five dimensions was to fade from 1928 and thereafter. The brilliance of the conception of five dimensions was perhaps plagued by the apparent three-space of the everyday physical world. Surprisingly, in view of his reservations in 1919 and his opposition in his 1923 paper, the most persistent renewals and inspiration attached to the five-dimensional theory came from the creator of the four dimensional spacetime concepts - Einstein himself.

1. Einstein in the 1920's

As has been noted previously (Chapter 3), it was Einstein who encouraged Kaluza, although the publication of the original theory was delayed until 1921. It was Einstein who discussed the theory two years later (although dismissively in an obscure publication, Einstein and Grommer, 1923). But it was also Einstein who entered the field himself, inspired by Klein's rejuvenation of Kaluza's ideas in 1926.

Einstein first entered the literature about Kaluza with Jakob Grommer in their 1922 paper, published in 1923, "Proof of the non-existence of an overall regular central-symmetric field from the Field theory of Theodor Kaluza". They acknowledged the "unsolved dualism" between the characters of the gravitational field and the electromagnetic field. Weyl's theory had been the previous (flawed) best attempt. Kaluza "avoids all the flaws, and is of amazing formal simplicity" (Einstein and Grommer, 1923, p.1). Einstein's view was that if the five dimensional manifold (called 'cylindrical') was equivalent to the four-dimensional spatio-temporal manifold, then it did not represent a particularly physical hypothesis. Kaluza however assumed the physical reality of this five-dimensional continuum, which for Einstein became completely unjustified from a physical point of view. Einstein also criticised the considerable symmetry that the demand for cylindricity prefers one dimension over the others whereas "in relation to the structure of the equation, all five dimensions should be equal" (ibid.,p.5) (a trenchant remark which was only answered fifty years later, e.g. Souriau 1959, 1963; Chodos, Detweiler 1981 - see Chapters 6 and 7). On Einstein and Grommer's calculations, moreover, there was no spatial variable for electric potential in four dimensions, i.e. no solution for an electron, free of singularities. (Only de Broglie was brave enough to regard photons as singularities of a field of waves, even "Mobile singularities" in his 1927 paper in Comptes Rendus - although the interpretation was ahead of his time - see Chapter 4, and also Chapter 2 for the appearance of singularities from the General Theory of Relativity itself).

In 1925, Einstein tried a different unified field theory, establishing the essential identity of the gravitational and electric fields mathematically, without extending space to more than four dimensions (Einstein,1925). From 1925 Einstein was concerned not merely for the search for a unified theory of forces, but "to conjure the quantum graininess out of the flowing field work" (Pais, 1982,p.333).

At this period of time, Einstein was playing with similar ideas to de Broglie. To explain the duality of wave/particle behaviour of light (and other particles), Einstein proposed the idea of a "guiding field". ("Führungsfeld"-Wigner, Ed.Woolf,1980,p.463). This field obeys the field equation for light, i.e. Maxwell's equation. However the field only serves to guide the light quanta or particles. Yet Einstein, although in a way he was fond of it, never published it (as he related it to his friend Eugene Wigner, ibid.,p.463). The momentum and energy conservation laws would be obeyed only statistically. Einstein could not accept this and hence never took his idea of the guiding wave quite seriously. In fact he also spoke of a "ghost field"("Gespensterfeld") although only quoted indirectly by Born (1926, p.803) in support of Born's own idea. The problem was solved, as Wigner put it, by Schrödinger's theory, "in which the guiding field progresses in configuration space (Wigner,in Ed.Woolfe 1980,p.463) so that the joint configuration of the colliding particles is "guided", rather than the two separately and independently. In Einstein's view, Schrödinger's great accomplishment was this idea of a guiding field in configuration space - "surely much less picturesque", said Wigner, "than separate guiding fields in our ordinary space for separate particles" (Ed.Woolf,1980, p.464). In a letter to Einstein, Born noted that the wave field in phase space was "merely mathematical" (Born, Nov.1926).

Despite ignoring these ideas, and without the slightest indication that the two might indeed be talkingof the same mode of a deeper reality, Einstein himself entered the literature with a theory involving five dimensions, referring back to Kaluza's theory: "On Kaluza's Theory of the correlation of Gravity and Electricity" (Einstein 1927a,b). Strangely, he did not refer to Klein, although those two papers were published after Klein's (April 1926) improved version of Kaluza's theory. From the evidence of Einstein's own letter to Ehrenfest, "Herr Grommer has drawn my attention to the work of Klein" (Einstein, August 1926). Indeed, in a second letter a few days later, Einstein wrote, "Klein's paper is beautiful and impressive, but I think that Kaluza's is entirely too unnatural" - remarking on the difficult idea of the the cylinder condition (Einstein, Sept.1926). Einstein appeared to change his opinion somewhat in a letter to H.A.Lorentz, just after Einstein's two papers were published : "It appears that the union of gravitation and Maxwell's theory is fulfilled completely through the five-dimensional theory of Kaluza - Klein - Fock... I am curious as to what you will say about it" (Einstein to Lorentz, Feb.1927).

Einstein's paper of 1927 noted Kaluza's idea of a continuum of five-dimensions which "by the 'cylinder condition' is somehow reduced to a continuum of 4-dimensions"(Einstein, 1927a,p.23). He showed that besides the symmetric tensor of the metric, only the antisymmetric tensor, derivable from a potential function, is significant as regards the electromagnetic field. It almost seemed as though the "tensor of curvature in R_5" is to be compacted ("narrowed") and equal to zero (ibid.,p.24). In the second part of the paper, Einstein gives the result of his "further thinking .... seems to speak very much in favour of Kaluza's idea"(ibid.,p.26) and expands the ideas, not unlike Klein's development and already described by Klein. There is a most surprising postscript to the above article:

"Mandel pointed out to me, that the results of my review of Kaluza are not new. The entire contents are to be found in the work of O.Klein... Compare furthermore Fock's work" (ibid.,p.30) (also predating Einstein's!) — an explicit reference to Klein's 1926 paper, as if Einstein had rediscovered Kaluza's work independently of Klein, and contrary to his own admissions to Ehrenfest. One wonders why Einstein waited for Mandel when from the evidence he knew already. In which case it is very surprising that Einstein published in the first place and provides further evidence that he readily concentrated on what he himself had created.

2. Einstein returns to Five-Dimensions in the 1930's

Einstein continued to write papers on the General Theory of Relativity, making no reference to the five-dimensional ideas, in that same year (Einstein and Grommer, 1927; Einstein, 1927b). Indeed, he made no references to five dimensions in the literature until 1931, when with W.Mayer he presented a new formalism which "runs psychologically on to the well-known theory of Kaluza", and even here "avoiding, however, the extension of the physical continuum into one of five-dimensions". (Einstein and Mayer, 1931,p.541). For Einstein and Mayer, at this time, it is "not quite satisfying" that a five-dimensional continuum has to be suggested, while the world is "apparently 4-dimensional in our reception" (ibid.,p.542). They also argued that the cylindrical condition is formally unnatural. Einstein and Mayer introduced their own theory by "holding on to the 4-dimensional continuum, but introducing into it vectors with five-components" (ibid.,p.542). In other words Einstein claimed to avoid five dimensions as artificial. They needed it, but then tied it up so that it did not manifest itself - embedded in a "local (M_5) five-vector space", but not the embedding of the whole Riemannian manifold in a five-space (Einstein and Mayer, 1931,p.549).

In introducing the 5-vectors into 4-dimensions, Einstein hoped to dispense with Heisenberg's indeterminism, which under a unified field theory could then be regarded as merely a projection on to a world of 4-vectors. Their statistical implication could then be regarded as the result of the suppression of the fifth cooordinate of a five-dimensional physical process. If so, the Bohr-Heisenberg formulation of quantum theory would seem to offer an incomplete description of physical reality, yet successful as an approximation (see Chapter 4).

The Einstein and Mayer paper of 1931 did not provide a lasting solution. This was despite writing enthusiastically to Ehrenfest that this theory "in my opinion definitely solves the problem in the macroscopic domain" ("excluding quantum phenomena" interprets Pais - Pais 1982, p.333, quoting Einstein to Ehrenfest, Sept.1931). In the <u>Science</u> article of 1931, Einstein stated as a prelude that he had been "striving in the wrong direction, and that the theory of Kaluza, while not acceptable, was nevertheless nearer the truth than the other theoretical approaches." He thought that Mayer and he had removed the anomaly of a fifth dimension, subsequently tied up, by using "an entirely new mathematical concept" (Einstein, 1931,p.550).

(Other attempts, before or soon after the Einstein-Mayer theory, which assumed four-dimensions but used a projective 5-space, are known as Projective Field Theories, but did not follow the Kaluza idea, e.g. Veblen and Hofmann, 1930; Pauli, 1933).

In the extension of spacetime to a five-dimensional manifold, Einstein made one last try at a five-dimensional unification. This attempt also failed because more mathematical concepts were not yet available, and Einstein ignored the further effects of the strong and weak forces. This last version seemed to discourage the vast majority of physicists from taking seriously the Kaluza-Klein idea for over thirty years.

Einstein had tried to remove the necessity for quantum uncertainty. He had tried to build on Eddington's programme (depending on Weyl's theory) for a unified field theory (Einstein, 1918, 1921, 1923a), but found no singularity - free solution (Einstein 1923b,p.448) and was unable to bring progress in physical knowledge (Einstein, in Eddington, 1925, appendix): "It brings us no enlightenment on the structure of electrons" (Einstein 1923b,p.449). Einstein himself remained loyal to the reality of the photon which he perhaps more than anyone established in his 1905 photo-electric effect:

- a new entity, at once a wave and a particle. He hoped for a fusion of the wave and emission theories which were for him to be somehow compatible. Yet he found the 1926/1927 version of this idea repugnant when it appeared in full. Perhaps he needed to take the possibility of the extra-dimensional concepts more seriously to do justice to their manifestation in four dimensions.

3. Einstein's final attempts at Five-dimensional theory, with collaborators

In his 1938 attempt, Einstein had in mind not to make the fifth dimension less real than Kaluza-Klein, but more real. He first worked with Peter Bergmann. Their field equation in five dimensions looks exactly like the Einstein-Maxwell system in four dimensions (Einstein and Bergmann, 1938,p.683). Their great difficulty was that Kaluza's theory is actually a five dimensional representation of the four dimensional space, and the restrictions imposed are a necessary consequence of this. We learn the motivation from Bergmann's own book. It appeared impossible for an "iron-clad" four-dimensional theory ever to account for the results of quantum theory, in particular for Heisenberg's indeterminacy (Bergmann, 1942,p.272). Since the description of a five dimensional world in terms of a four-dimensional formalism would be incomplete, it was hoped that the quantum phenomena would, after all, "be explained by a (classical) field theory"(Bergmann, 1942,p.272) where the 5-space is closed in the fifth dimension with a fixed period, following Klein. The possibility of averaging over the fifth dimension to account for its non-appearance gave an implicit high status to the reality for both Klein and Einstein.

A second version was published, with Valentine Bargmann joining them three years later (Einstein, Bargmann and Bergmann, 1941,e.g.p.212). With Bargmann and Bergmann, Einstein thought that quantum fields could be interpreted using the theory, and when these hopes did not materialise, he gave up the five dimensional approach for good. His search had continued for more than thirty years. Einstein had sought for a deeper-lying theoretical framework that would permit the description of phenomena independent of quantum conditions. This is what he meant by "objective reality". By the early 1930's, it was Einstein's personal thrust that quantum mechanics is logically consistent, but that (e.g. with Rosen and Podolsky 1935) it is an <u>incomplete</u> manifestation of an underlying theory in which an objectively real description is possible. Indeterminacy may be a consequence of our incomplete four-dimensional world. However the 1938 theory proved unequal to the task.

The 1938 "Scalar Tensor" theory of Einstein and Bergmann was developed independently by two others, P.Jordan and also Y.R.Thiry, modifying Kaluza's attempt by adding to the gravitational and electromagnetic fields one extra variable quantity. In fact Jordan attempted to turn this extra mathematical quantity to advantage in cosmology by relating it to Dirac's special cosmological variable (see Bergmann, 1969,p.187).

This renewal in the late 1930's e.g. from Einstein, Leopold Infeld and B.Hoffmann had its roots in the work of twelve years before with J.Grommer (Hoffmann, "Albert Einstein", 1975,p.228). Einstein and Infeld (1938), insisted, but did not follow up, that if a probability wave in thirty dimensions (3N) is needed for the quantum description of ten particles, then a probability wave with an infinite number of dimensions is needed for the quantum description of a field! For them the 6, 9, 12 or more dimensional-continuum for 2, 3, 4 or more particles indicates that those waves are more abstract than the electromagnetic and gravitational fields existing in a three dimensional space. However in a striking analysis of de Broglie's "new and courageous idea" of 1927, they equated the vibration at rest of a standing wave with x_0, equivalent to nodes of the fifth dimension, with the "held oscillations of Klein (Einstein and Infeld, 1938,p.235) as a model to help to imagine these extra dimensions.

4. A critique of Einstein's 1938 high status for the fifth dimension

Einstein and Bergmann made a scholarly analysis in 1938 of the two attempts to connect gravitation and electricity by a unitary field theory by Weyl and by Kaluza, explaining that the Kaluza theory is "contained in part" (Einstein and Bergmann, 1938 ,p.683) in Klein's 1926 paper and also in Einstein's 1927 paper. They noted some attempts to represent Kaluza's theory formally so as to avoid the introduction of the fifth dimension of the physical continuum. In their paper they went on to argue that this would differ from Kaluza's in one essential point:

> "we ascribe physical reality to the fifth dimension whereas in Kaluza's theory this fifth dimension was introduced only

> in order to obtain new components of the metric tensors representing the electromagnetic field. Kaluza assumes the dependence of the field variables on the four coordinates, x^1, x^2, x^3, x^4 and not on the fifth coordinate x^0 when a suitable coordinate system is chosen" (Einstein and Bergmann, 1938, p.683) -
> "It is clear that this is due to the fact that the physical continuum is, according to our experience, a four dimensional one".

They went on to attempt to prove that it is possible to assign some meaning to the fifth coordinate without contradicting the four-dimensional character of the physical continuum. They considered a five dimensional space where the arbitrary physical vector is replaced by the Klein assumption that the space is closed or periodic in the fifth dimension. They further assumed that through every point in space passes a geodesic line closed in itself and free from singularities.

For Einstein and Bergmann (ibid.,p.687),

> "Kaluza's roundabout way of introducing the five dimensional continuum allows us to regard the gravitational and electromagnetic fields as a unitary space structure".

The only arbitrary step (to be fair to Kaluza's theory) is taken when the five dimensional representation of the four dimensional space is assumed. They affirmed that although Kaluza's aim "was undoubtedly to obtain some new physical aspect for gravitation and electricity", by introducing a unitary field structure, "this end was, however, not achieved" (Einstein and Bergmann ibid.,p.687). Many fruitless efforts to find a field representation of matter free from singularities based on this theory "have convinced us, however, that such a solution does not exist" (ibid.,p.688). Their investigation was in fact based on the theory of "bridges" (Einstein and Rosen, 1935,p.73), but appeared not to lead anywhere: "we convinced ourselves, however, that no solution of this character exists" (Einstein and Bergmann, 1938,p.688).

Perhaps if they had been able to see the later evidence of singularities, (see Chapter Two), this line of enquiry would indeed have been extremely fruitful.

At the time (1938) the need to refer back to four dimensions, "without sacrificing the four dimensional character of the physical space" ,... "shows distinctly how vividly our physical intuition resists the introduction of the fifth dimension"(Einstein and Bergmann, 1938,p.688). It is easy to forget or ignore Einstein and Bergmann's conclusions:

> "It seems impossible to formulate Kaluza's idea in a simple way without introducing the fifth dimension. We have therefore to take the fifth dimension seriously although we are not encouraged to do so by plain experience" (ibid.,p.688).

They argued that if the space structure seemed to force acceptance of the five-dimensional space theory upon us, "we must ask whether it is sensible to assume the rigorous reducability to four-dimensional space (ibid.,p.688). Their answer was "no", but they hoped to understand in another way "the quasi-four dimensional character of the physical space by taking as a basis the five dimensional continuum" (Einstein and Bergmann, 1938, p.688).

It may well be that one of the first arguments by analogy "by reduction of dimension" occurs in this paper. Their argument was unusual in considering a two-dimensional space (x^o, x^1) instead of the five dimensional one, which approximates to a one-dimensional space continuum (instead of a four dimensional one). They imagined the strip curved into a tube to form a cylindrical surface with a small circumference, where ST coincides with S^1 T^1.

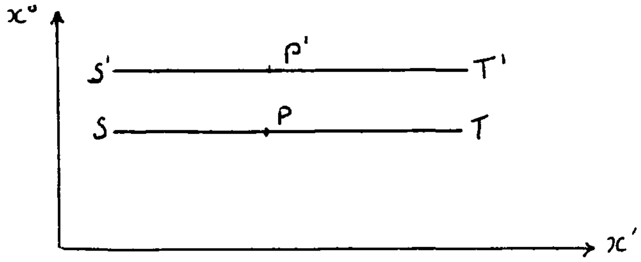

Figure 12: "A two dimensional space that is approximately a one dimensional continuum" (Einstein and Bergmann, 1938,p.688).

Every point P on ST coincides in this way with a certain point P^1 on S^1T^1. (ibid.,p.688).

"This reduction in the number of dimensions of the space" was achieved because, as in Klein's idea, the space is closed in the fifth dimension (x^o) and the characteristic width is very small (ibid.,p.689) - too small to be detected in ordinary experiments. Interestingly, this gives it "a continuous and slowly changing function" whereas this quasi-one dimensional character does not exist if the function χ (x^o,x^1) varies too rapidly" (ibid.,p.689). They therefore argued that instead of a space "closed" in the x^o-direction, a space "periodic" in the x^o-direction may be equivalent. The authors admitted (ibid.,p.689) that "the expression 'closed' is not quite clear". The 'periodic' and 'closed' character become equivalent if the corresponding points P, P^1, P^{11}... are regarded as 'the same' point.

This analogue model becomes explicit by replacing the one dimensional continuum with the four dimensional continuum to obtain a picture of physical space. In technical terms, the 'rigorous cylindricity' hypothesis has been replaced by the assumption that "sphere is closed, or periodic" (after Klein), in the x^o direction, or fifth dimension. It seemed that for a given point P in the four-dimensional physical space, P can be represented by an infinite number of points P, P^{1}, P^{11}..., all open and periodic in the extra dimension, and by five coordinates corresponding to every space point. "This postulate replaces the cylindricity condition in Kaluza's original theory" (ibid.,p.689). The authors argued that it was much more satisfactory to introduce the fifth dimension "not only formally, but to assign to it some physical meaning" (ibid.,p.696). Strikingly, they confirm: "nevertheless there is no contradiction with the empirical four-dimensional character of physical space" (ibid.,p.696). Einstein and Bergmann seem to be reiterating Klein's view without conscious realisation that they were going over old ground.

Einstein and Bergmann may well have reached the ultimate point, given their lack of further mathematical tools (such as gauge theory, supersymmetry, etc.) and their disregard of the other force fields - the 'strong' nuclear force and the 'weak force' of radioactivity. Certainly there were no references made to their work in the literature, even in the years immediately following the period 1938-1942.

5. Einstein in the 1940's

Within two or three years, Einstein and Bergmann (joined also by V.Bargmann) elaborated their 1938 paper but back-tracked on the high status of the fifth dimension. Because the equations now derived are uniquely determined, the extra dimension "causes serious difficulties for the physical interpretation of the theory" (Einstein, Bargmann and Bergmann, "On the five dimensional representation of gravitation", 1941,p.224) - no consistent theory of matter with non-singular solutions of the field equation was possible.

In a highly mathematical paper, one of the three authors analysed the Kaluza and Projective field theories (Bergmann,1942). The attempts to generalise Kaluza's theory (Einstein and Mayer, 1931 etc.)

include the recent attempts by Einstein et al. (1938 and 1941) "to give the fifth dimension a stronger physical significance" (Bergmann, 1942, p.272). It appeared impossible for four dimensional field theory ever to incorporate the results of the quantum theory. Bergmann affirmed that these high hopes of a five dimensional world appeared unjustified, although parts of this approach may stand the test of time. He himself described "the cutting out from a five dimensional continuum a thin slice of infinite extension and identifying the two open (four dimensional) faces of the slice", as a model of such a closed five dimensional space (Bergmann, 1942, p.273). The 'cylindrical' fifth dimension is proved to have "a circumference everywhere the same" (ibid., p.273).

Einstein abandoned a higher dimensional space for "bivector fields" within another year or so in two papers, the first in collaboration with Bargmann (Einstein and Bargmann, 1944). Peter Bergmann in 1948 published Jordan's attempt to generalise Kaluza's theory (given over by Pauli in 1946 after Physikalische Zeitschrift had ceased to publish). It was similar to Bergmann's own theory, first presented in Bergmann's book, Introduction to the Theory of Relativity, of 1942 - perhaps " 'true' only in a restricted sense" and preserved for future evaluation (Bergmann, 1948, p.264). In fact Jordan had attempted to generalise the five dimensional unified field theory of Kaluza by keeping g_{55} as a fifteenth field variable. Although rejected earlier by both Bergmann and Einstein, it was to be an abortive attempt at the theories rejuvenated in the 1980's which "vary the constant of gravitation" (Bergmann, 1948, p.255), when the extra tools of supergravity etc. would be available.

6. Conclusion : why Einstein was not successful in his search for unification using the Kaluza model

In his prolonged search for a unified field theory, Einstein was not consistent in his approach to Kaluza's theory, varying from being unconvinced in 1922 to high approval in 1927, 1931 and 1938. Indeed in the late thirties he, and by inference his colleague Bergmann, assumed that "at least some of the field variables were in fact functions of all five coordinates" and "took the fifth dimension quite seriously" (P.G.Bergmann, 1985, Private Correspondence to E.W.Middleton).

In his autobiographical notes, Einstein admits that all such endeavours had been unsuccessful, and that he "gave up an open or concealed raising of the number of dimensions of space, an endeavour which was originally undertaken by Kaluza and which, with its projective variant, even today has its adherents" (Einstein, in Ed.,Schilpp,1949, p.91). After a period he described as "many years of fruitless searching" over twenty years, he was still searching for a deeper unity. For Einstein, a theory could be tested by experience, but "there is no way from experience to the setting up of a theory" (Schilpp,Ed.,1949,p.81). This of course was not the 'normal science' or inductive method, but the creative shift, which for an extra dimension could approach a "paradigm change" for new conceptual frameworks (Kuhn, 1977,p.495). Additionally, of course, the nature of the electromagnetic field is so bound to the existence of quantum phenomena that any non-quantum theory is necessarily incomplete. Einstein himself was always looking for such a deeper theory than the incomplete description of physical reality offered by quantum theory. He had advocated a statistical or ensemble interpretation and came to the conclusion that "one must look elsewhere for a complete description of the individual system" in "My Attitude to the Quantum Theory" (Einstein, 1950,p.31).

As we have seen in Chapter Two, Einstein came to his theory of Gravitation, of General Relativity, in 1915 ahead of David Hilbert and others, with a new structure of space and time. In a sense this is still classical physics, without any concessions to the developing Quantum Mechanics, yet also with implications of higher dimensions. These are seen in the embedding dimensions, from a minimum of six to the ten which could be required (Kasner,1922)- see Chapter Two. It has been much less obvious that this 1915 theory of Einstein's "applies to <u>any</u> number of dimensions" (Schrödinger, 1950, p.3 - my emphasis). Schrödinger also noted in his introduction however that, "of most interest and importance is the case when a theory is restricted to n = 4; therefore this fact will usually be stressed explicitly ." The implicit multidimensions was never used by Einstein in further work. Indeed, in his letter to Lorentz concerning the unification in five dimensions of Kaluza-Klein, he wrote "But this cannot be the description of the real proceeding - reality. It is a mystery". (Einsten to Lorentz, Feb.1927).

That Relativity itself might not be a complete theory was of course never acknowledged by Einstein. This, and the lack of tools to take the five dimensional unification further, explains why even Einstein did not succeed on the Kaluza-Klein basic theory. John Wheeler later spoke against taking General Relativity seriously at small distances (Wheeler, 1968 , p.300, Note 33). He quoted most aptly about Einstein from Robert Oppenheimer's article in the <u>New York Review</u> (1966)

> "He also worked on a very ambitious programme, to combine the understanding of electricity and gravitation .... I think that it was clear then, and believe it to be obviously clear today, that the things that this theory worked with were too meagre, left out too much that was known to physicistsbut

> had not been known much in Einstein's student days. Thus it looked like a hopelessly limited and historically rather accidentally conditioned approach." (Oppenheimer, 1966, pp.4,5).

Meanwhile in the late sixties, Physics was veering more towards quantum field theory and even towards the string model (via dual resonance), with new descriptions, new particles and new forces. As Abraham Pais also noted, Einstein "grew apart from the mainstream", and this work of his "did not produce any results of physical significance" (Pais, 1982, p.327). He had looked in two areas - the extension of spacetime to a five dimensional manifold, based on Kaluza's paper of 1921, - and on the generalisation of the geometry of Riemann. He had sought solutions of pure field equations, free of singularities. He knew no standard practicable method for achieving these solutions. "Supergravity in particular draws much of its inspiration from elementary particle physics. In his own time Einstein could not have been aware of this source", explains one of his colleagues, Peter Bergmann (Bergmann, to Middleton, Private Correspondence, 1985). Yet Einstein had "struggled on despairingly", knowing himself what was necessary: "I need more mathematics" (Einstein, quoted by B.Hoffmann, 1975, p.240). Supersymmetry, gauge theory and the dual resonance model were needed on the route to Supergravity and superstrings.

Einstein was unaware that such concepts would become available in the years to come. He had originally tried to build on Eddington's programme (which depended on Weyl's theory) for a unified field theory, but found no singularity free solutions (<u>Nature</u>, 1923,p.448). He always looked for a physical reality that existed independently of the observer or any particular set of coordinates. Einstein consistently rejected quantum mechanics in his belief that any satisfactory theory, like his own General Relativity, must be constructed from a single ontological entity, the field. His quest for a theory without the particle ontology was for a unified treatment of gravity and electromagnetism, often trying five dimensions. A new and greater relativity theory, a unified field theory, would always have a logical mathematical and simple structure. The fact that the masses of particles "appear as singularities", indicates that "these masses cannot be explained by gravitational fields" (Einstein, 1950,p.16 "On the Generalised Theory of Gravitation").

Einstein's ambition to achieve a unified field theory drew him again and again to Kaluza's original idea. In 1931 he did in fact prepare a statement in German to be published in Science, with the publication in English authorised by him. Referring to his work with Walter Mayer,

> "we reached the conclusion that we were striving in the wrong direction and that the theory of Kaluza, while not acceptable, was nevertheless nearer the truth than the other theoretical approaches" (Einstein, 1931a,p.438).

In his lucid discussion in his 1938 paper with Bergmann, on how the world appears to be four-dimensional, Einstein's exposition was near the modern idea in that the ground state of five dimensional General Relativity is not five-dimensional Minkowski space M^5, but the product $M^4 \times S^1$. Such a four-dimensional Minkowski space with a circle S_1 had already been outlined by Klein (1926). The assumption was that the radius of the circle was so tiny that in everyday experience observed phenomena would always involve averaging over the position in S_1, so that the world appears to be four dimensional. Einstein and Bergmann also predicted that g_{44} would behave as a massless scalar, a prediction copied from Kaluza, which had not been accepted, but which was to reappear in the form of a dilaton field in the dual models of the early seventies.

Looking back in his chapter "Thirty years of knowing Einstein", his friend Eugene Wigner talked about the search for a general law representing the unity of all theoretical physics. Einstein had "always hoped that such a theory would eventually be established, at least for physical phenomena" (Wigner, in Ed.H.Woolf,1980,p.464). He also quoted Peter Bergmann, "the effort was premature, it was undertaken at a time when no full theory of the other interactions, strong and weak, was available". Wigner went further,

> "even if a physics of the limiting situation in which life and consciousness play no rôle is possible, physics is as yet very far from perfection, and some of Einstein's assumptions, and those of present day physics, may have to be revised". (Ed.H.Woolf, 1980,p.466).

Other physicists, besides Einstein, kept alive the 5-dimensional Kaluza-Klein idea until Souriau in 1959 and 1963 published his creative and indeed catalytic approach (see Chapter 6). However Einstein himself had by then abandoned his own dream of a geometrical unification of all the forces of nature (Einstein (1949),'Autobiographical Notes'. Ed.Schlipp pp.89-95).

Chapter 6 : Other Attempts at Higher Dimensional Theories, 1928-1960, including Klein himself (apart from Einstein - See Chapter Five)

Synopsis

1. Eddington's use of extra dimensions, a purely mathematical concept
2. Five dimensional theories on the Kaluza pattern
3. Other five dimensional attempts at unified theory e.g. Projective Geometry - apparently an alternative path
4. Keeping the flame burning : Klein, Thiry, Bergmann and Souriau
5. A new approach in six dimensions : J. Podolanski's unified field theory
6. Other papers in the 1950's referring to the Kaluza-Klein idea
7. Intimations of physical relevance : J.M. Souriau (1958, 1963)
   - five dimensions observable in the initial seconds of the big bang
   - a very large symmetry is needed between the five dimensions giving a complex wave function for the charged particle.
8. Conclusion

Since 1927, there have been a few scientists, apart from Einstein and his collaborators, who also kept alive the conception of a five dimensional world through the wilderness years. It was hardly surprising that without the tools which are now available, there was little chance of any real growth from the original conception of Kaluza. There were occasional attempts at extra dimensional theories e.g. in ten dimensions (Eddington 1928, 1936) or in six dimensions (Podolanski, 1950) outside the Kaluza-Klein concept, but these usually proved to be blind alleys.

1. Eddington's use of extra dimensions - a purely mathematical concept

In his 1928 paper for example, Eddington suggested attention had been so concentrated on four dimensions that "we have missed the short cuts through the regions beyond" - six or ten dimensions (Eddington, 1928, p.156). Using six extra dimensions he described how to "bend the world in a superworld of ten dimensions." Eddington did not have the gauge theory and supersymmetry ideas, nor the mathematical vocabulary to strengthen this prophetic idea and his ideas were not taken seriously. However he admitted that it at least helped him to form a picture which "suggests a useful vocabulary" (pp.158,214). Eddington's two alternatives are posed as either a curved manifold in a Euclidean space of ten dimensions or a manifold of non-Euclidean geometry and no extra dimensions. It is not surprising that Eddington did not take the ten dimensions seriously as a physical reality although he supported Poincaré's idea that space is neither Euclidean nor non-Euclidean, but a matter of convention. Eddington gave a low status to the configurational space corresponding to Schrödinger "generously allowing three dimensions for each electron" (Eddington, 1928, p.215). This paper was an account of his 1927 Gifford lectures of Edinburgh, whilst presumably unaware of Klein's paper.

Eddington returned to the mathematical analogy of extra dimensions using embedding ideas of six dimensions or, "when we extend the same ideas from space to spacetime, ten dimensions are needed" (Eddington, 1940, p.37), but "no metaphysical implications of actual bending in new dimensions is intended" (Eddington, 1940, p.99). He had also worked on a 16-dimensional space, but found that by limiting himself to a sub-space of five dimensions, there were fewer conceptual problems. However, his examination of why the actual world is four dimensional (although his attempt at unification of relativity and quantum mechanics needed at least five dimensions "which we have reason to think is appropriate to the physical world") led him from a wave tensor idea to the embedding concept in ten dimensions (Eddington, 1936, p.55). Eddington's theory involving five independent coordinate E numbers was never taken very seriously. He used locally orthogonal components of a point using a Riemannian geometry defined in ten dimensional phase space. Eddington's speculation regarding the ratio of masses of proton and electron, and other fundamental constants of nature, attracted wide interest, but were seen as very daring and were often viewed with incredulity. Nevertheless it could be said that Eddington actually started the idea of superspace. He gave a geometric description in an extended spacetime, in which every point has not only the usual four spacetime coordinates, but also an additional set of coordinates identified by anti-commuting numbers. This may correspond to the flat superspace of the 1970's.

2. Five Dimensional theories on the Kaluza pattern

Five dimensional theories directed primarily to the removal of contradictions in wave mechanics and quantum theory were developed by H.T. Flint. In these theories, the fifth dimension is related to the wave function. Einstein-Riemannian space is the base and Flint developed the use of the harmonic possibilities of a fifth dimension. In fact one of the spring-boards for Flint was the de Broglie phase wave in generalised spacetime, although at the time, February 1927, the fifth dimensional solution had not made an impact (Flint & Fisher, 1927). Flint continued to develop his ideas (Flint, 1931, 1938). His research was published in 1940-42 and included Kaluza's conception in his five dimensional system. This provided a convenient mode of description for expressing the notation of the quantum theory in relativistic form, and "is indeed forced upon us by the requirements of the quantum theory" (Flint, 1942, p.369). He incorporated quantisation of electric charge into his theory, stating that the character of the restriction on the use of the fifth coordinate is controlled by the application to the quantum theory. Flint suggested that we must look for "some new source or sink of electric charge if the fifth dimension is involved" (Flint, 1942, p.380) foreshadowing some of Wheeler's later ideas in geometrodynamics.

A further attempt by Flint in 1945 regarded the fifth dimension as "a new degree of freedom" for an electrically charged particle (Flint, 1945, p.635). A further innovation in the same paper was to try to take account of "other fields, such as are considered in nuclear theory" (ibid., p.636) -seemingly the first time these ideas of forces beyond gravity and electromagnetism were raised in the literature. At the same time, P. Caldirola was bringing in considerations of energy and entropy in a further attempt to strengthen the physical significance of the fifth dimension (Caldirola, 1942, p.25).

3. <u>Other five dimensional attempts at unified theory</u> e.g. Projective geometry, apparently an alternative path.

In 1933, Wolfgang Pauli wrote his most comprehensive paper on general relativity "with five homogeneous coordinates" (Pauli, 1933, p.305). In Klein's improved version of Kaluza's theory, the metric tensor of the five dimensional Reimannian space was assumed to be independent of the fifth coordinate. This was, however, felt by many physicists to be quite artificial from the point of view of a truly five-dimensional geometry. Several mathematicians (Veblen and Hoffmann, 1930; van Dantzig, 1932; Schouten, 1935) suggested therefore the introduction of five projective coordinates, $x' \ldots x^5$. This meant that on the one hand the symmetry in the five coordinates would be maintained, and yet on the other, these coordinates would describe a four dimensional manifold because only the <u>ratios</u>, $x^1:x^5$ would have a geometric meaning.

Pauli's paper gave a clear survey of five-fold projective geometry applicable to a Riemannian space of four dimensions. He introduced a new calculus of spinors - by far the most satisfactory expression, in the later opinion of Bargmann (Ed. Fierz, 1960). (Their fundamental property is that spinors transform conventionally with the matrices defining the metric.) Pauli was able to show that the projective formulation was mathematically equivalent to the original Kaluza-Klein theory. Jordan later produced a generalisation of projective Relativity using a scalar field in five dimensions (Jordan, 1955). Although the <u>geometry</u> is truly five dimensional, a projection is always made from the 5-space to 4-space in these theories, which Bergmann and Einstein also experimented with (Bergmann, 1948, p.255). The mathematical connection between Projective Relativity and Kaluza-Klein theory was most clearly stated by Bergmann in his Introduction to the Theory of Relativity (Bergmann, 1942, p.272). According to Bergmann, his Scalar Tensor Theory of 1948 was definitive. It was subsequently re-invented by Jordan, Thiry and Schmutzer in later years (Ed. de Sabbata et. al. 1983, p.8).

Although mathematically interesting, this work on projective field theories kept the symmetry of five dimensions but the clear physical substance of four dimensions, and was of little physical importance. The interest in Kaluza-Klein theories had decreased progressively. Only very occasional papers on the topic were published (e.g. J.G. Bennett et al, 1949). (This developed into his metaphysical concepts of 1956, using a private language leading beyond that which his contemporary physicists were ready to accept).

4. Keeping the flame burning: - Oskar Klein and others e.g. Thiry, Bergmann and Souriau

In 1946 Oskar Klein himself returned to the scene and attacked the problem of nuclear interaction as well as the original electromagnetism and gravitation. Klein himself acknowledged H.T. Flint's pioneer work in extending five dimensional theory beyond the original two forces (Flint, 1946, p.14), although he also mentions the promising attempts made by Yukawa to consider these forces some years earlier(although these were in four dimensions) (Yukawa, 1935, p.48/ 1937, pp. 91 - 95). Klein argued that "the quantum theoretical wave functions of any electric particle will in the five – dimensional representation be periodic functions of x^0 with period ℓ_0 " (Klein, 1946, p.3) restating the findings of his 1926 paper. This assumption would on general quantum principles imply "an indeterminacy of x^0 corresponding to a whole period where the charges of the particles used are quite fixed." In practice, without the use of a fifth dimension in any classical geometric sense this meant that "particles of given charges have mutually incoherent wave functions" (Klein, 1946, p.3) as Klein had always assumed (Klein, 1926 b; 1927) in his early papers. Klein admitted however that it was very doubtful whether such a theory could be regarded as more than "a guiding physical analogy"...

> "the unity obtained being in some way illusory since the periodicity condition places the fifth coordinate on a different footing from the space coordinates" (Klein, 1946, p.3).

With fascinating insight, Klein introduced spacetime coordinate transformations as so-called "gauge transformations" and asked to be allowed to propose the more adequate name of "phase transformation" since it changes the phases of the wave function - although this did not in fact affect the use of the standard phrase which continued in the literature. This was not taken up again until the late seventies. Klein was once again ahead of his time in developing a quantum theoretical probability wave equation for the propagation of a static (or "quasi-static ") rigorous solution which he called " a kind of singularity of the field"! (Klein, 1946, p.11). However it must be said that despite these prophetic insights (reminiscent of John Wheeler much later) Klein's main aim (of correlating a unified field theory, including nuclear fields, with quantum mechanics), although promising, fell short of a successful theory. The necessary concepts of strings and supersymmetry were not yet at hand. Interestingly, in a little known book, New Theories in Physics (Klein, 1939), Oskar Klein had in fact anticipated the extension of the Kaluza-Klein idea to non-abelian gauge theories which were to prove so essential.

The few who still worked in five dimensions (excluding the four dimensional projective theories with five coordinates) included K.C. Wang and K.C. Cheng of Chekiang University in China (1946) (who surprisingly made no mention of Kaluza or Klein) and Yves Thiry (1951). That electrodynamics in Wang and Cheng's paper was in agreement with classical theory is not surprising. Their thought was that: "as the momentum and velocity of a particle in the fifth dimension have never been observed, they are assumed to be zero." Their model was nevertheless interesting in saying that the particle in five dimensional space i.e. the geodesic," is a long line extending in the fifth dimension" (Wang and Cheng, 1946, p.516) - without necessarily being rolled up.

Yves Thiry mentioned Weyl's now discarded theory, the theories of Kaluza and Klein, and also the Veblen projection theory which Thiry noted "is not really different from the five dimensional essays" (Thiry,1951a, p.276). Thiry commented on the essential nature of the cylindricity hypothesis, but rejected keeping the fifth dimension a constant as being not very satisfactory mathematically. As in an earlier paper (Thiry 1948), Thiry aimed to give a different derivation of the fifteen equations of Kaluza's original theory, making extensive use of Cartan's exterior calculus theories (Cartan, 1946). In 1951 he went further : his unitary theory involved the fifth space variable being none other than the 'constant' of gravitation (unaware of earlier suggestions,e.g. H.T. Flint, 1942.). Thiry developed the five-dimensional unitary theory having first provided a mathematical justification. For Thiry, Kaluza's theory was not unitary from a physical point of view, a viewpoint which he acknowledged had Einstein and Pauli's support. As the fifth coordinate was treated in a very different way from the other four, the unification was only apparent. He argued that Kaluza had introduced the fifth dimension a priori without any physical significance. This and further attempts were regarded as unsatisfactory by Thiry, who agreed that

> "many wise men have been attracted by such a theory because they are persuaded that it contains some truth" ('Part de verité' - Thiry,

1951a,p.312). Thiry acknowledged that for him the fifth dimension had appeared purely mathematically, although he attributed a spatial character to the fifth variable in his Chapter II, while understanding it by the hypothesis of cylindricity in a purely mathematical way.

5. A new approach in six dimensions : J. Podolanski's Unified Field theory

In a paper written in 1949, supported by R.E. Peierls, Podolanski of Manchester University was one of the first to give a high status to the physical possibility that reality requires more than five dimensions. The mathematical necessity of six rather than five to embed Einstein's field had been demonstrated in 1921 and 1922 by Kasner in a series of papers. Podolanski started from the Dirac matrices in six dimensions. Following an earlier version (Schouten and Haantjes, 1935) with two time-like dimensions, Podolanski took the ordinary spacetime world as a subspace of a six dimensional manifold. He showed that "the six dimensional (classical) field theory avoids the difficulties with which the Kaluza-Klein theory has to contend." In addition "the possibility was gained of making the field energy of a point source finite "(Podolanski, 1950, p.234).

Podolanski in fact stated specifically that unlike the Kaluza-Klein theory his representation "may be classified as an embedding theory, the electromagnetic forces having the character of forces of constraint" (Podolanski, 1950, p.235). In contrast to Kasner and others (e.g. Dingle, 1937), the use of extra dimensions was not just seen as a mathematical exercise. Our traditional spacetime subspace was "immersed in the six dimensional space," where "each world point corresponds to a sheet of physically indistinguishable points (Podolanski, 1950, p.235). This concept of space being laminated and folded up into two-dimensional sheets may indeed be a forerunner of superstring theory and even of supermembranes. Podolanski did not explain clearly the consequences, but it would seem that the two dimensional sheet included one extra space and one extra time dimension. Certainly for him each world point corresponded to a sheet of physically indistinguishable points, a multi-sheeted reality.

Podolanski noted the Kaluza-Klein idea as a convincing unification of the conservation laws and the interpretation of the gauge transformation. He argued, however, that their formalism was too vague and that the theory had turned out to be sterile; the projective version was a more precise formulation, but showed up these shortcomings even more clearly. Podolanski nevertheless believed that a "hyperdimensional description of nature was useful" (Podolanski 1950, p.234) while referring back to the real world of four dimensions in his section, "How to get rid of two dimensions" (ibid, p.235) in his proposed unified field theory.

His paper was still classical, the interpretation of the wave function remained obscure, and Podolanski admitted that without this, his paper could not give "the whole truth" (ibid., p.236), although he helped to develop a quantum mechanical step later. It was Klein in 1926 who began the attempt to connect extra dimensions with quantum theory, and Podolanski could only confirm that this connection was not yet resolved. Podolanski did however write a six dimensional Schrödinger equation, and "took the opportunity of making one of the embedding fields complex" (ibid., p.258). Podolanski's paper was perhaps ahead of its time. Our own apparent four dimensional universe appears merely as a "subspace immersed in the six-dimensional space of the deeper reality, a projection into "the four real dimensions" (Mathematical Review, 1950, p.746). Science Abstracts, in its 1950 review used "subspace" of a higher dimensional space, without applying the word "real" to either space in Podolanski's paper.

6. Other papers in the 1950's referring to the Kaluza-Klein idea

Either Podolanski was wrong - or ahead of his time; little notice was taken of his paper in the literature. Klein himself attempted an up-to-date overview in 1956. Klein admitted that five dimensional theory, although it was "in a certain sense the most direct generalisation of relativity theory

including gauge invariance and charge conservation ...." "has such strange features that it should hardly be taken literally" (Klein, 1956, p.59). He now had similar doubts about his original idea of the similarity of the periodicity condition to "a quantum condition in classical disguise" (ibid., p.61). Klein realised that the restriction of the fifteenth tensor g_{00} to be constant was certainly not natural. He had discussed this also in a paper two years previously (Klein 1954). The mostobvious solution was to leave out this restriction altogether and let g_{00} be determined by the fifteenth field equation. Klein's calculation in the absence of matter, led to a variation of g_{00} in the presence of electromagnetic fields which, however, "is extremely weak and probably far outside the realm of experimental investigation" (Klein, 1956, p.64). Klein estimated that if matter were present a negligible average variation of g_{00} would occur.

Klein's approach was to use isotropic spin space as a potentially physical concept. He hoped that the problem of enormous particle mass terms would be overcome in the way he described. Bergmann's review article the following year summarised the existing attempts to go beyond the theory of relativity - either to produce a unified field theory or to quantize the gravitational equation. He regarded Klein's dropping of Kaluza's cylinder condition that the field quantities be independent of x^5, as leading to the development of a truly five-dimensional theory, where the fifth coordinate has a quantum theoretical significance (Bargmann, 1957, p.161).

Klein's use of isotropic spin space seemed to be independent of the Yang-Mills idea in 1954, where spin symmetries converted to a local symmetry, maintaining the invariance of the laws of physics by adding *six new* vector fields. This was to be of enormous importance, although as originally planned seemed inappropriate to describe the real world. It was regarded as an elegant mathematical curiosity - as indeed was the original Kaluza-Klein unification in five dimensions. Kaluza's theory was often criticised on the grounds that the fifth dimension was a purely mathematical device, of no meaning for the real world, despite Kaluza's personal evaluation, and that of occasional scientists since the 1921 publication.

7. *Intimations of physical relevance*: - *J-M. Souriau*

In 1958 a fresh impetus giving high physical status was provided in a paper by Jean-Marie Souriau. He used a fifth dimension in the *same* way as the standard four, but his model regarded its *present* size as unobservably small. This gave the possibility of a higher status as a true "physical dimension" (i.e. tangible and measureable, at least in principle) as a possibility within the distant past in the early stages of the Big Bang (Souriau, 1958, p.1559). This key insight by Souriau implied that in the first few seconds of the big bang, the fifth dimension was manifest or directly observable at the same time as the other more familiar dimensions. Although these ideas were to be acknowledged in the 1980's, Souriau's scholarly input from 1958 onwards was rarely recognised. In a seminal paper published in 1963, Souriau both analysed the situation to date and pointed the way to continued research in Kaluza's five dimensional model of relativity.

Souriau noted the initial motivations for adding a fifth dimension : (i) to simplify the study of spinors, (ii) to give an interpretation of the Hamiltonian action (Souriau himself) and (iii) to unify electromagnetism and

> gravitation. For Souriau, "if such a method is to be more than a simple mathematical trick, it is necessary to put forward a symmetry, as

large as possible, between the five dimensions" (Souriau, 1963, p.566).

In Kaluza's theory, as Souriau interpreted it, the symmetry was broken by the principle of "stationarity" for the fifth dimension; one of the fifteen equations produced in the unification was also modified (in a non-symmetric way). Jordan and Thiry (e.g. Thiry, 1951, p.275), for example used the fifteenth field in a symmetrical way while keeping the principle of stationarity (where the components g_{ik} of the fundamental tensor are independent of the fifth coordinate x^5).

Souriau was thus able to point out that the five dimensional universe

> "acquires a structure of <u>bundle</u> space; its base is a four dimensional Reimannian manifold" - which is naturally identified with spacetime

(Souriau, 1963, p.567). For Souriau, Klein's hypothesis to replace the condition of stationarity by the components g_{ik} being periodical functions of x^5, did not seem sufficient. Einstein and Bergmann in 1938 had added other conditions, tending towards Kaluza's idea, whereas Pauli in 1958 suggested returning to Klein's original idea, (Pauli, 1958), as did Souriau independently also in 1958, giving it more precise meaning. The fifth dimension is closed upon itself and is spacelike in Souriau's five-dimensional theory. He claimed that it subsumed the ideas of Jordan, Thiry and Kaluza as approximations.

For Souriau, these approximations were useful for the physical interpretation of the theory, "allowing one to give an approximate quadri-dimensional picture of it" (Souriau 1963, p.569). In his five dimensional wave equation, Souriau affirmed a complete explanation of classical electrodynamics, and suggested that the formulation of quantum mechanics should be renovated if five dimensions were used. Certainly he gave "a geometrical origin to the quantification of charge" in five dimensions, which has no explanation in four dimensional relativity (ibid., p.573). A complex wave function for a charged particle would then appear quite naturally in quantum dynamics.

Souriau's highly original approach already brought in both gauge transformation and fibre bundles, and he should be given credit for this. Souriau also claimed that a further consequence of the five-dimensional approach was the maximum violation of parity (ibid., p.576) - as expressed by Salam, Landau, Lee, Yang, etc. and in fact observed in experiments for weak interactions.

Souriau's paper has been unduly neglected in these connections.

Pauli had been concerned with the difficult problem of the physical interpretation of general functions periodic in x^5. He was clear in his 1958 book, the Theory of Relativity, that there must be other wave-mechanical fields, e.g. spinor fields, describing particles of low mass. He concluded that "the question of whether Kaluza's formalism has any future in physics" is thus leading to the more general unsolved main problem of accomplishing a synthesis between the general theory of relativity and quantum mechanics

(Pauli, 1958, p.226).

8. Conclusion

The Kaluza-Klein concept was kept alive in the forty year wilderness period. In the next chapter we shall look at the return of the Kaluza-Klein idea into the mainstream physics of the 1960's and 1970's. Some of these connections had already been anticipated, particularly by Souriau, but also by Oskar Klein himself. New concepts such as gauge theory, strings, fibre bundles and above all, supersymmetry were to lead to theories which did accomplish the synthesis which Pauli and others hoped for.

Chapter 7 : The return of Kaluza-Klein ideas to mainstream physics

Synopsis

1. The revival of the Kaluza-Klein model
2. Seminal papers in the 1960's

   incorporating non-Abelian Gauge Fields with the Kaluza-Klein concept

   (i) De Witt, 1964 (via Souriau and Klein who are very seldom acknowledged)

   (ii) Trautman, 1967 and 1970, Kerner 1968 and Thirring, 1972, using fibre bundles
3. The introduction of String Theory in the 1970's

   (i) via Venziano's Dual Resonance Model ; rediscovery of the importance of supersymmetry

   (ii) Nielson, Nambu; Susskind, 1970: Dual model is a string theory, in 26 dimensions

   (iii) 1971 spinning string model: Ramond; Neveu and Schwarz, in 10 dimensions (also Bardakçi and Halpern, 1971)

   (iv) Scherk and Schwarz, 1975 : string theory and unification of all four forces.
4. Kaluza-Klein enters the String Model.

   (i) Scherk and Schwarz, 1975, in a unified theory of gravity coupled to Yang-Mills matter - Spinor dual model in 10 dimensions includes a 6-dimensional compact domain (torus-shaped).

   - string on the Kaluza-Klein model is consistent with the principles of both special relativity and quantum mechanics.
   - the full 10 dimensional symmetry should be recovered at very high energies.
   - reference Ne'eman's 10 dimensional embedding solution.

   (ii) Cremmer and Scherk, 1976a - internal symmetries again - introduced in the Kaluza-Klein model by compactifying the extra dimensions, 1976b — 'spontaneous compactification' introduced as a real "physical" process, not the mere mathematical tool of 'dimensional reduction,' 1977 Internal space of compact dimensions, radius of the order of 10^{-33} cm (Planck length)

(iii) Cremmer and Julia, 1979, also with Scherk (1978) - spontaneous compactification.

5. The development of superspace and supersymmetry

Wheeler's superspace and Kaluza-Klein, via Graves.

6. Origins of Supersymmetry and supergravity.

(i) Wess and Zumino, 1974 : Spacetime Supersymmetry to link fermions and bosons and include quantum field theory via Gol'fand and Likhtman's supersymmetry ; Volkov and Akulov, 1973 and earlier ; Noether, 1918; Cartan and Cantor.

(ii) Salam and Strathdee, 1974 : Superspace in eight dimensions

(iii) Freedman, van Nieuwenhuizen and Ferrara 1976, Supergravity - gravitational theories entailing local supersymmetry - no infinities

(iv) Deser and Zumino 1976, simpler version of Supergravity (after Arnowitt et al, 1975

(a) Supersymmetric transformations imply particles such as the gravitino, slepton etc.

(b) No experimental confirmation

(v) Freedman and van Nieuwenhuizen, 1978 : Extended Supergravity

- superparticles with an arrow in auxiliary space of many dimensions unifies all particles - simplest is N=1 (1 gravitino) equivalent to original Supergravity.
- N=8, most realistic and most promising, anomalies (e.g. infinities) do cancel but more than four spacetime dimensions are needed - 10 or 11 dimensions.

7. Re-entry of the Kaluza-Klein idea from 1975 : a Review of the three strands,

(A) Non-Abelian Gauge Fields : Cho and Freund, 1975;

- the most promising avenue : supersymmetries - to enable scalar fields to become gauge fields

- extend to more than 5 dimensions

(B) Strings : Scherk and Schwarz 1975 - unified field theory

Cremmer and Scherk 1976

(C) Supergravity : Cremmer and Julia 1978 - Extended N=8

Supergravity in 11 dimensions, 7 compacted with broken symmetry

Maximum for supersymmetric strings, D=10 ; for supergravity, D=10.

(Cremmer, Julia and Scherk, 1978)

Spontaneous compactification of 7 of the 11 dimensions (Cremmer and Julia, 1979).

1. The revival of the Kaluza-Klein model

We have seen that Klein, Einstein and his collaborators, Pauli, Thiry and Souriau, for example, kept Kaluza's idea in their thinking during its "Classical" period which extended into the mid-sixties or early seventies. Quantum Mechanics only began to be connected in the mid-seventies - without this, a true unification was impossible, as indeed Kaluza himself, as well as Klein, had foreseen.

The mathematical tools and physical concepts which were necessary became available, and their appropriate usefulness was realised in stages. The original aim was to lead to the unification of gravity with electromagnetism, by assuming the necessary existence of an extra spatial dimension. This was to be extended to four forces, needing at least ten dimensions of spacetime.

2. Seminal papers in the 1960's : Incorporating non-Abelian Gauge Fields with Kaluza-Klein concepts

The relatively recent attempts to include the strong and weak forces, although already suggested by Souriau (1963), arenormally attributed to the

work of Bryce S. De Witt of the University of North Carolina, although these were also anticipated by Klein in 1939 (Gross and Perry, 1983, p. 29). Certainly it was De Witt who realised, in a paper published in 1964, that by adding *more* than one dimension, he could unify non-Abelian gauge theories, as well as gravity and electromagnetism. The non-Abelian extension of Kaluza-Klein theory was first published mathematically although presented unobtrusively as a homework exercise ("Problem 77") in the course of a lecture by B. DeWitt at the 1963 Summer School of Theoretical Physics (Les Houches, Grenoble). This "Dynamical theory of Groups and fields" was published in *Relativity Groups and Topology* (Ed. C. De Witt and B. De Witt, 1964, p. 725). This was reprinted under its lecture title as one of the Documents on Modern Physics (B. DeWitt, 1965, p.139) still less than one page long.

De Witt introduced Kaluza's paper in combining gravitational and Yang-Mills gauge fields by increasing the dimensionality of spacetime from 4 to 4+m. The result "forms the basis for the existence of a class of so-called unified field theories (originated by Kaluza) and suggests that geometry should perhaps provide the foundation for all of physics"

De Witt(1964, p.725)makes no reference to Souriau's paper of 1963. Indeed there are no references given, save the original Kaluza paper of 1921. De Witt explained the apparent four dimensionality of spacetime : "the lack of direct tactile evidence for the extra dimensions of spacetime could be regarded as due to the extreme smallness of the average volume of the cross sections" (De Witt, 1964, p.725), and affirms "the topology selected for the cross sections .... would be of fundamental importance ....." (- a prophetic remark for the 1980's).

Note: In the gauge field model developed by C.N. Yang and R.L. Mills in 1954, three new gauge fields were introduced as the solution to local symmetries. Poincaré's global symmetry is equivalent to the invariance in spacetime geometry underlying Einstein's Special Theory of Relativity. If a set of physical laws is invariant under some global symmetry, the stronger requirements of invariance under local symmetry can be met by introducing new fields which give rise to new forces. These new Gauge Fields are associated with new gauge particles .

De Witt's short exercise is referred to frequently in the 1980's as being a natural generalisation of the original Kaluza-Klein idea, and which incorporated non-Abelian gauge fields, a topic of high current interest. Thus De Witt's idea (later to be elaborated by others) considered a higher dimensional theory, with dimensions more than five, in which gauge fields became part of the metric, just as the electromagnetic field did in Kaluza's theory. He also pointed to the likelihood of a dynamical variation for the geometry of the cross sections of these dimensions, rather than their being held rigid. This interesting forecast was somewhat akin to Souriau 's independent paper of 1963. De Witt himself firmly stated in his opening sentence that his paper was a mathematical exercise, a "purely geometrical interpretation" (De Witt, 1964, p.725).

The next fundamental referral to Kaluza-Klein theories, although mentioned in later reviews as a 1970 paper, was in fact given in lectures at King's College, London in 1967 by A. Trautman of Warsaw University. Trautman's paper was to become a classic source for interpreting the gauge fields with the Kaluza-Klein idea in terms of fibre bundles. This new application provided a convenient framework not only for mathematical development but also for a visual way of conceptualising extra dimensions. The notion of a fibre bundle provided a convenient framework for discussing the concepts of relativity, invariance and gauge transformations, and "also for local problems of differential geometry and field theory" (Trautman, 1970, p. 29). He noted that the simplest non-trivial example of a fibre bundle was probably the Möbius strip, a two-dimensional bundle over the one dimensional circle, T, which is a summary of the more complicated Möbius strip. In losing a dimension however, information is of course lost as the Möbius bundle is represented over the base space of a circle.

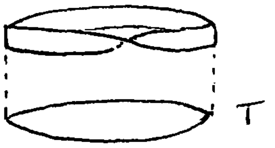

Figure 13
Möbius strip as a
Fibre Bundle

A three dimensional fibre bundle may be projected as a two dimensional circle or disc. Similarly higher dimensions can be represented mathematically and figuratively : an n-dimensional vector space is projected on an (n-1) dimensional base space. Thus Trautman extrapolated from ordinary space time as the product bundle to General Relativity and then to higher dimensions, (Trautman, 1970, p. 55) as a multidimensional Riemann space e.g. for the five dimensional Kaluza-Klein theory (ibid., p.60). Trautman noted the isomorphism between Utiyama phase space (Utiyama, 1956, p.1597) and 'Kaluza-Klein space'. He shows how one can construct a principal fibre bundle from the Kaluza-Klein space, with 4-dimensional space-time as the base manifold. (The morphisms of

Trautman and of Utiyama are "mappings", preserving the structure inherent in the theory, and based on physical hypotheses.)

In his published paper in 1970, Trautman acknowledged Kaluza's original paper, Einstein and Bergmann's 1938 paper, and also Penrose's Twistor Analysis in six or eight dimensions (Penrose 1966). Trautman in fact referred to his own 1967 original lectures and also to Kerner's paper of 1968, which elaborated Trautman's work. Kerner, a Polish phycisist from Warsaw, had independently referred to the equivalence of the Utiyama and the Kaluza-Klein approaches to the unification of the electromagnetic and gravitational fields in a five-dimensional manifold as a fibre bundle space. Ryzard Kerner, a student of Trautman, published a paper on the generalisation of the Kaluza-Klein theory for non-abelian gauge groups. His paper was almost entirely mathematical, with no indications of any physical relevance : "Generalisation of the Kaluza-Klein theory for an arbitrary non-abelian gauge group."(Kerner, 1968). Neither Trautman nor Kerner seemed to know about De Witt.

These ideas were extended in 1972 by W. Thirring of Wien University in a paper involving parity violation and the internal space of elementary particles:

"Five dimensional theories and C-P violation." Thirring tackled "the naïve argument that five dimensional theories are nonsense because nobody has seen the fifth dimension "(Thirring, 1972 p.268). Like Klein's original paper, Thirring argued that the reason why we cannot directly see the fifth coordinate is that "the manifold is periodic in the s-direction" (ibid.,p.256). The s- or fifth coordinate appears as a charge degree of freedom in the internal space of elementary particles, and behaves differently from spacetime. It was best described as a fibre space. Thirring acknowledged Kerner's work, incorporating all gauge fields; he himself hoped that the answer to the observed C-P parity violation might be obtained if the strong interactions were included in the unification. Otherwise the prediction of "insanely high bare masses" (ibid., p.270) remained a problem. This turned out to be correct; the problem disappeared in non-Abelian models.

<u>Note</u> Further attempts to include the strong and weak forces in a Kaluza-

Klein theory, e.g. by Y.M. Cho and P.G.O. Freund in 1975, were to await the development of ideas of supersymmetry and to be subsumed into supergravity theories e.g. by E. Cremmer and B. Julia in 1979 .

3. The Introduction of String Theory in the 1970's

This was initially through the Dual Resonance Model via Veneziano's original 1968 paper. The importance of supersymmetry was also rediscovered in using Dual Models. There was no connection made with Kaluza-Klein ideas in these early stages of the development of the string model. Indeed, for Neveu and Schwarz, two of the pioneers of strings, quarks themselves were 'only mathematical' rather than physical entities (Neveu and Schwarz, 1971, p.1111) as in the original invention by Gell-Mann. Gabriele Veneziano produced a formula by inspired guesswork, which was unrelated to the formulae of quantum field theory, and expressed many features of hadron interactions. The many hadron "resonances" (particles with very short lifetimes) which have a variety of properties, were found to be described best in terms of two complementary classification schemes - "dual resonance models." One described the resonances in terms of the quark model, the other used the alternative family correlation of Regge theory, "Regge trajectories". The pictures of resonance exchange between particles in a reaction was found to be complementary to the picture of a reaction as taking place entirely by the production and subsequent decay of resonances.

This dual model motivated the suggestion independently by Holger B. Nielsen (1969,1970), Yoichio Nambu (1970), and Leonard Susskind (1970) that the dual model was some kind of string theory ('old string', as it is referred to in the 1980's). Applying Veneziano's formula was equivalent to describing the hadrons as strings, which bound together the quarks that made up the proton, neutron and other hadrons. This original model could account only for bosons (whose spin is an integer in fundamental units) e.g. the pi meson. The quantum mechanical behaviour of this original string theory was found to make sense only if spacetime has 26 dimensions (25 space and one time dimension). It also requires the existence of a particle travelling faster than the speed of light (tachyon). These problems, to physicists steeped in the 4-dimensions of spacetime, produced the description of the model as "sick" or "having an illness". In the Danish school at the Niels Bohr Institute in Copenhagen, for example, this led some of the team to question the reality of the model (B. Durhuus, Private Communication to Middleton 1982) whereas Nielsen himself took the idea of 26 (or 10) dimensions realistically - "realistic .... although generally not meant to be taken seriously" (H.B. Nielsen, private correspondence to Middleton, 1980).

The classical string, developed from the dual resonance model, indicated that particles were not points but massless one dimensional strings, whose ends rotate at the speed of light. Incorporating the special theory of relativity within quantum theory led to the problem of extra space dimensions. 26 dimensions however could not account for fermions such as the electron and proton (particularly with spin =½). In 1971 a variant of the original theory, but to include fermions was developed by Pierre Ramond, closely followed by André Neveu and John Schwarz. This was known as the spinning string (or R.N.S.) model, and was the precursor of supersymmetric theories. This version, adding extra internal spins (or degrees of freedom) was only consistent in <u>10</u> dimensions (9 space + 1 time) (Neveu and Schwarz 1971; Ramond, 1971).

One of the significant motivations for this interest in dimensions beyond four was to satisfy both principles of contemporary physics - the special theory of relativity and the quantum theory - a striking unification breakthrough. Similar ideas to Ramond, Neveu and Schwarz had in fact been introduced by K. Bardakçi and M.B. Halpern in 1971. They introduced what is now called the R.N.S. model and have had no recognition in the literature for this and earlier encyclopedic writings, although their work has been recently acknowledged in a scholarly review article by Michael Green (1986, preprint, p.15) on "Strings and Superstrings". Strangely the original motivation for the Veneziano model to solve the problem of strong interactions, was unsuccessful; this problem needed the theory of quantum chromo-dynamics developed in 1973 and 1974. The interesting suggestions by K. Wilson in terms of a lattice approach to Q.C.D. was that confinement of quarks could be due to strings viewed as a tube of colour electric flux (Wilson, 1974, p. 2445.)

The idea of using string theory as a unified theory of fundamental forces including gravity, rather than merely to describe hadrons, was developed by Joël Scherk and John Schwarz in 1974. This reinterpretation however still suffered from inconsistencies for which further mathematical tools were needed. Even consistent string models still had the problem of tachyons. Their paper did also involve interesting ideas of dimensional reduction from 10 dimensions.

4. Kaluza-Klein enters the String Model

Physicists in this area had given no real thought to the origins of higher dimensional ideas. Although C. Lovelace had given a clear hint that 26 dimensions was something special - that bound states were just the expected closed-string states formed when the end points of an open string join together (Lovelace 1971) - there was no link up with the original concepts of Kaluza and Klein. That awaited a paper of central importance by Scherk and Schwarz which was to be the inspiration for others. In 1975 they developed their suggestion of interpreting string theory as a unified theory which is a generalisation of a theory of gravity coupled to Yang-Mills matter, and brought in Kaluza's paper. In their paper, "the 10-dimensional space time of the spinor dual model" was

> interpreted as "the product of ordinary 4-dimensional spacetime and a 6-dimensional compact domain, whose size is so small that it is as yet unobserved" (Scherk and Schwarz, 1975, p.463).

Strangely, in a wide ranging review by Scherk, published in January of the same year, 1975, there was still no connection made with Kaluza-Klein and strings. He noted the conventional Veneziano or bosonic string model where the critical dimension was 26, and the R.N.S. development to include fermions but in 10 dimensions. Scherk noted the further advantages of this 10 dimensional version; "although still unphysical, the model is much more realistic than the conventional model" (Scherk, 1975, p.125) - presumably as 10 is nearer to 4! After the original proposal in 1970 of string-like particles by Nambu, Nielsen and Susskind, Scherk noted that the string picture became much clearer after the work of Goddard, Goldstone, Rebbi and Thorn in 1973. Strings could break and rejoin and the "quarks" were localised at the ends of the strings. The string itself was identified with the neutral "glue" binding the quarks. Thus dual models had gone closer towards field theory. For Scherk, dual models and the transverse string picture were "two complementary

> faces of a single self-consistent mathematical structure" ... "Whether or not these mathematical structures have anything to do with the real world is still unclear" - i.e. whether it will remain a mathematical tool, or

lead to more realistic models (Scherk, 1975, p.163).

In February, a further overview this time by Schwarz, again made no reference to Kaluza in his "Dual-Resonance Models of Elementary Particles ", He noted that the model needed nine dimensions, and was then consistent with the principles of both special relativity and quantum mechanics. Schwarz added that if elegance depended on the amount of symmetry, the model rated very high. Beginning to take a more realistic view of the model, he proposed that "elegance, so defined, is closely correlated with physical relevance" (Schwarz, 1975, p.62).

In April, Michael Green, who was to play a key role with Schwarz in later developments, wrote in the New Scientist that there was the hope of a more unified scheme involving stringlike extended hadrons (Green, 1975, p.77). No reference was then made to Kaluza by Green.

In their joint paper published in August, Scherk and Schwarz finally made the connection. The extra (six) dimensions were to span a compact and spacelike N-dimensional domain after the model of Kaluza. Interestingly, the shape of that domain was taken to be a generalised *torus*, a model which was to reappear in the 1980's. In "a sharp (if tentative) break from present attitudes" they were using the spinor dual model, with six dimensions compact, as "an alternative kind of quark-gluon field theory" (Scherk and Schwarz, 1975, p.463). The input fields have colour "and presumably do not correspond to physical particles" (Scherk and Schwarz, 1975, p.466), and therefore the model lacked physical reality. Interestingly also, they referred to the 10 dimensional theory of Ne'eman to explain "internal" symmetries (Ne'eman, 1965a, and Penrose 196 in the same journal) although Ne'eman in fact used 4-dimensional spacetime *embedded* into a 10-dimensional space and Scherk and Schwarz prefer to use a product space of ordinary 4 dimensions and a 6-dimensional compact domain. With prophetic insight, they noted that the existence of the N extra spatial dimensions is unobservable at normal energies. When the energy is very high the full 9+1 dimensional symmetry of the theory should be recovered. Scherk and Schwarz (1975, p.463) assumed that the radius of the torus would be so small that the fields could be considered to be independent of the N extra coordinates at present day energies.

Interestingly, Ne'eman did see physical implications in his global embedding to which Scherk and Schwarz referred. "Unfortunately the present state of our knowledge of the cosmology does not allow us to check this result" (Ne'eman, 1965 a, p.230). For Ne'eman the actual embeddings required a maximum of ten dimensions, since even simplified local gravitational solutions require 6 to 8 or more, "and the real world is much less symmetric that that" (ibid.,p.230.)

The Kaluza connection to string theory was elaborated further soon afterwards in a paper by E. Cremmer and J. Scherk, "Dual Models in four dimensions with internal symmetries" (1976a, received in October, 1975). Internal symmetries were again introduced into dual models by "compactifying N of the spacetime dimensions - in 26 in the conventional 'scalar' model, and 10 in the 'spinor' model. The additional compact dimensions were used in the context of field theory, and reconciled with 4 dimensional experience in that they are only observed in the form of internal symmetries. Compactifying six of the 10 dual spinor model dimensions proved to be both mathematically self-consistent and "compatible with everyday experience," where four dimensions of spacetime are "non-compact" (Cremmer and Scherk, 1976a,p.399). This model deserves further study (ibid.,p.418) "because of its great physical interest" - an increase in status from Scherk and Schwarz's paper of 1975. On the basis also of Scherk and Schwarz's paper, they saw the possibility within dual models of having a completely unified theory of all interactions, including gravity.

In a further important paper the same year, Cremmer and Scherk referred to the Kaluza-Klein idea only by implication with no direct reference. They examined how their solution "breaks the symmetry spontaneously by confining N dimensions to the compact S_N sphere" (Cremmer and Scherk, 1976b p.409). They referred again to their previous conclusion that when extra dimensions are compact their existence will not lead to any contradiction with everyday experience, provided that the dimension of the compact domain is small enough. The emphasis was on how dual models may "spontaneously screen their extra dimensions" (ibid.,p.410) (and remove their tachyons at the same time) by some kind of "*spontaneous compactification*". This concept, vital to later work, entered the literature here for the first time as the title emphasised: "Spontaneous Compactification of space in an Einstein-Yang-Mills-Higgs model". This was now used as a real "physical" process of high potential status, not the mere mathematical tool of "dimensional reduction", the term used to describe the mathematical process of reducing 10 dimensions to the 'real world' of 4. Cremmer and Scherk described their "embarrassment" that the dimensions of spacetime had to be 10 for a consistent model. Reduction to 4 "seemed an arbitrary condition imposed on the model," (ibid.,p.415), until Scherk and Schwarz in 1975 proposed to compactify the extra space dimensions and use them to generate internal symmetries. If this was a correct model, it would of course lead to the remarkable conclusion that we can see the extra dimensions in the various particle states (families etc.). "Now we see that this compactification of unwanted spatial dimensions can spontaneously happen" in a very simple model which had some of the vital features of a dual model (Cremmer and Scherk, 1976b, p.415).

The idea of spontaneous compactification was so important that Cremmer and Scherk turned to it a few months later, in Autumn 1976, published in 1977: "Spontaneous compactification of extra space dimensions." In three directions spacetime was flat and did not close, but in others, "space is so strongly curved that it closes upon itself" (Cremmer and Scherk, 1976, p.61). These compact dimensions were "like an internal space", and its shape was described by a hypersphere. The very small value, of the order of Planck's length (10^{-33}cm) found for the radius of the curled up dimensions, "justifies the unobservability at today's energies", of such extra dimensions (ibid., p.62) - since exciting these "degrees of freedom" would amount to creating particles having masses of the order of Planck's mass. Flavour symmetry and topological quantum numbers could be explained; the other attractive feature was that internal symmetries could be reinterpreted as spacetime variables. Visualisation is made easier by regarding the extra dimensions as compacted on a sphere "imbedded in a fictitious N-dimensional Euclidean space" (ibid., p.62).

There was no work done on fermionic string theory in the four years after 1976, although the work by Cremmer and Scherk just described was one of the developments which was to prove important later. This "apparent impasse in string theory" (Green, 1986, p.22) was due chiefly to the problem of tachyons, and almost all research workers in string theory worked in other new areas of field theory involving supersymmetry and supergravity, etc. Only in 1980 were Michael Green and John Schwarz to bring the new range of ideas together in their work on superstrings.

5. <u>The development of superspace and supersymmetry</u> as fundamental mathematical and conceptual tools. Superspace in more than 4 dimensions (see Salam and Strathdee, 1974,p.479) was also to need Kaluza-Klein ideas for later fruition. It was elucidated in the early 1970's using the abstract symmetry "supersymmetry" into a mathematical language which was to be essential for developments in the late 1970's.

Qualitatively, John Wheeler used the 'arena of superspace' to describe the singularities involved in the Big Bang and in "Black Holes", or holes in space (Wheeler, 1973, p.739). His synthesis of higher dimensional geometries led to his finite dimensional "truncated superspace" (ibid., p.1175). Wheeler had introduced his central new concept in a chapter called "Superspace and the nature of quantum electrodynamics" (Ed. De Witt and De Witt, 1968). Where the classical concepts of spacetime have no meaning (at the Planck length or in singularities, for example - see Chapter two) and are merely the surface appearances of reality, Wheeler used concepts of foam space as well as wormhole models, which may fluctuate throughout all space. For Wheeler (Ed. De Witt and De Witt, 1968, p.1204) superspace was defined as "space resonating between one foam-like structure and another". This involved a multiple-connectiveness of space at sub-microscopic distances with the implications of a multi-dimensional concept. Wheeler's "pregeometry", far from being endowed with any definite topology, should be viewed as not even possessing any dimensionality at all. In a striking phrase, he wrote "the pursuit of reality seems always to take one away from reality," where Geometro-dynamics "unfolds in an arena so ethereal as superspace" (ibid., p.1212).

Wheeler's creative ideas of superspace, however, needed a better mathematical language to extend his qualitative inspirations. He had confined his ideas to General Relativity in four dimensional Riemann geometry, and excluded the other forces apart from Gravity.

It was J.C. Graves, whose writing on geometrodynamics went largely unacknowledged, who explicitly transferred the ideas of Wheeler (and Misner) into the Kaluza-Klein five dimensional manifold. He compared Kaluza's original assumptions with Jordon and Thiry's versions which introduced a new scalar potential (Graves, 1971, Chapter 15, e.g. p.255). Graves noted that a variable gravitational constant had been proposed earlier by Dirac, introduced by Jordon in his scalar version (1947, 1955) and followed up by Dicke and others, although <u>without</u> the five dimensional formalism. However Graves treated Klein's modification as well as Kaluza's original theory, as an incomplete mathematical coincidence, because it gave no <u>intuition</u> of even the qualitative features of a fifth dimension, and therefore could not be evaluated. Graves also forecast that other such microdimensions may be needed if strong and weak particle interaction were to be included.

Implicit throughout Graves' book was the idea that Wheeler's geometrodynamics could be explained in terms of extra dimensions, although Wheeler is never explicit. Graves' book was perhaps premature; no references were made to his ideas in the literature and his concepts were overtaken by the development of supersymmetry and supergravity.

6. Origins of Supersymmetry and Supergravity

Julius Wess and Bruno Zumino are widely credited with starting the development of supersymmetry in 1974, as an extension of spacetime Poincaré symmetry : "Supergauge transformations in four dimensions". This involved a new symmetry principle which linked fermions and bosons in a new symmetry transformation, consistent with quantum field theory. They were inspired partly by the graded Lie (-Virasoro) algebra that had already entered dual models, and conceived the idea of spacetime supersymmetry.

The origin could therefore lie in the independent developments of supersymmetry in 1971. One development was from the flat superspace, initiated quantitatively by Y.A. Gol'fand and E.P. Likhtman in Moscow - and rediscovered in 1973 by D.V. Volkov and V.P. Akulov, of the Institute of Kharkov. Another critical exposition in 1971 involving the symmetry between bosons and fermions, started with the dual model approach to particle physics by Ramond, Neveu and Schwarz, which was to develop into the string model (J.L. Gervais and B. Sakita, ref. P.C. West in Ed. Davies and Sutherland, 1986, p,126).

The work was generalised to include quantum field theory by Wess and Zumino in a systematic procedure to construct global symmetry theories, linking particle spin properties to spatial translations. This concept of supersymmetry was to prove a powerful tool in physics and had its mathematical basis in the work of Noether from 1911 to 1918. Emmy Noether of the University of Göttingen, building on the work of Hilbert, published a theorem relating the mathematical operation of symmetry to the real world of physics. Symmetries were translated into physical properties which are conserved. Also Elie Cartan, the French mathematician, building on the work of Georg Cantor, elucidated (in the 1920's) many of the geometrical properties of multidimensional spaces and gave the complete classification of all simple Lie algebras over the field of complex values for the variables and parameters (ref. M. Kline, Mathematical Thought from Ancient to Modern Times, 1972).

Global symmetry transformations link particle spin properties to spatial translations. If the supersymmetry transformation is made local, different points transform in different ways and a link with gravity is established. Gravitational theories entailing local supersymmetry are called "supergravity". This internal symmetry, supersymmetry, has the remarkable property that a repeated supersymmetry transformation, e.g. from fermions to bosons and back, moves a particle from one point in space to another. This is a physical translation of a particle, and this displacement suggests a relationship between supersymmetry and the structure of spacetime. This deeper symmetry is well hidden, but suggests there may be just one type of particle for the description of nature.

Thus the supersymmetry of the early 1970's was purely a conceptual device and enabled a unified mathematical language to be constructed to deal with concepts which cannot easily be visualised. Supergravity was used to describe

General Relativity in the language of quantum field theory. There was no apparent reason why it should not also be formulated in geometric terms, using an extended spacetime of more than four dimensions.

In the context of supersymmetry, Abdus Salam and J. Strathdee introduced a four-dimensional quantitative version of superspace, a space defined by eight coordinates (Salam and Strathdee, 1974a, p.477). Their 'space' was essentially of eight dimensions, and they noted that the superfield of the Wess-Zumino supersymmetry group in eight dimensions was equivalent to a 16-component set of ordinary fields in four dimensions. They developed the Wess-Zumino super-gauge symmetry further in the same year, to include isospin (Salam and Strathdee, 1974b).

The primary elementary development of local supersymmetry in the form of supergravity came from Daniel Freedman, P. vau Nieuwenhuizen and S. Ferrara : "Progress towards a theory of supergravity" (1976). And then shortly afterwards a simpler version, exploring the geometry of superspace, following Arnowitt et al. (1975) was formulated by S. Deser and B. Zumino. A supersymmetric transformation related the graviton (the gauge particle of gravity) to other fields. Freedman et al. predicted the supersymmetric partner e.g. to the quantum of gravity, the gravitino. These cancelled out the infinities which plagued the old theories of gravity.

Experimental confirmation is however needed. No supersymmetric partner (Slepton, squark, gluino etc.) has yet been observed. The suggestion was made that the supersymmetry is somehow 'broken', or hidden. Thus the supersymmetry route to unification has been successful, and provided an automatic link with gravity, <u>but</u> as yet has no link with the real world. A unified field theory has to have a place for <u>all</u> elementary particles, and the gravitino etc. must be added to the list.

The most useful set of theories has been found to be extended supergravity theories, introduced by Freedman and van Nieuwenhuizen :

"Supergravity and the Unification of the Laws of Physics" (1978) - still with no mention of Kaluza-Klein theories. There are only eight of these theories, involving superparticles with an arrow in an "auxiliary space of many dimensions" in a new approach to unifying gravity with the other forces. As the arrow rotates, "the particle becomes in turn a graviton, a gravitino

> a photon, a quark and so on .... This degree of unification has never before been achieved in quantum field theory (Freedman and van Nieuwenhuizen, 1978, p.140). The simplest Extended Supergravity

is N=1 (i.e. requires one gravitino) and is simply supergravity in its original form. The most realistic model was the N=8 with eight gravitinos. It was also the most promising in attempting to explain the particles known today. Anomalies(e.g. the problem of infinities in earlier theories such as Q.E.D., removed by a mathematical trick of renormalisation) do cancel in supergravity, but at the additional price of using more than four spacetime dimensions. Full unification appeared possible only in Extended Supergravity, where the infinites in fact do cancel. There were still problems, e.g. the introduction of the cosmological term in going from a global to a local symmetry, first discussed by Einstein himself, giving a finite size to the universe. Another problem was that particles seemed to be massless, and was solved by the particle acquiring a mass through the mechanism of spontaneous symmetry breaking. The cost was the need to use ten or eleven dimensions of spacetime.

Thus supergravity, involving extra dimensions beyond four, grew up entirely independently of Kaluza-Klein theories, making the connection only in the late 1970's. It was left until 1979 for Cremmer and Julia to make this connection.

We have seen that the development of superspace and supersymmetry paved the way for ideas of supergravity and extended supergravity. Freedman and van Nieuwenhuizen's theory of extended supergravity in 1978 seemed to provide the most promising development for N=8 in 11 dimensions. An avenue involving Kaluza-Klein ideas was opened by Cremmer and Julia to remove some of the still existing difficulties. They observed that supergravity theories contained a hidden symmetry which was larger than the explicit one.

7. Re-entry of the Kaluza-Klein idea from 1975 - A Review of three strands

After 1975, various strands of physics found that the Kaluza-Klein model in 5 dimensions was a most useful idea to incorporate in the different developments.

(A) We have noted that Freund with his student Cho in 1975 provided key ideas in the generalisation of the Kaluza-Klein idea to Non-Abelian Gauge Fields initiated by De Witt in 1964. The advent of the concept of supersymmetry gave the further impetus to the studies of gauge field theory involving the spontaneous breaking of a larger symmetry. They noted that the "non-observability

> of the excess dimensions (while a difficulty for theories in which these dimensions are bosonic) should cause no problems if the higher dimensions are fermionic"(Cho and Freund, 1975, p.1711).

This new concept of supersymmetry in fact removed even the bosonic problem : only the internal-space coordinates undergo spacetime dependent transformation, spacetime itself remained unaffected. Cho and Freund (ibid., p.1715) noted that the 4+N Kaluza type higher dimensional theory "may yet have its own meaning and relevance for physics" - an early sign of the increased interest in the physical status given to these dimensions from the late 1970's. Cho and Freund regarded physical 4-space as the base manifold of a fibre bundle model of the 4+N dimensional Riemannian space. They emphasised that these internal dimensions must be space-like - "hidden" internal dimensions of spacetime. They also repeated the Klein speculation about extremely rapid variation of fields in a fifth dimension (e.g. with characteristic length of 10^{-33} cm) in constructing "the full theory, scalars and all." Freund in fact used the Kaluza-Klein idea in his student days in 1954, even "infinitely many dimensions" (Freund, private communication to Middleton, 6.1 1988).

Cho and Freund thus made the link from Non-Abelian Gauge theories to supersymmetry for their own context. "The most promising avenue is that of supersymmetries .... It is only in the presense of supersymmetries that scalar fields can become gauge fields "(Cho and Freund, 1975, p.1719). Concluding their advocacy of supergauge theories, they commented that these could of course be extended to even higher dimensions than five. Although in an added footnote, the authors acknowledged that the differential geometric field theory in curved superspace by Arnowitt et al. (R. Arnowitt, P. Nath and B. Zumino, preprint, 1975) was certainly related to their own paper, it did not have any Kaluza-Klein connections. In a highly unusual paragraph, Cho and Freund held the belief that "there is a religious flavour to such

> ideas. One would rather like to benefit from the existence of higher dimensions, while at the same time not have to realise them physically at all "
>
> (Cho and Freund, 1975, p.1719) - a critical dilemma indeed!

(B) We have also seen that 1975 marked the point where physicists working in the field of string theory made the connection with Kaluza's original idea. The idea that these extra dimensions required could be thought of as curled up at any point in space, had been around since the earliest days of the string theory. At first it seemed that no one remembered the papers of Kaluza and Klein from the 1920's. There were certainly articles trading off extra dimensions for internal symmetry in 1971 and 1972, long before Scherk and Schwarz made the Kaluza-Klein connection with string theory in a unified theory of matter.

The relation between gravity and string theory had been studied by Scherk himself and also by T. Yonega (1973, 1974). They showed that the closed string was connected to Einstein's theory of gravity in the limit of large string tension. This led to an improved version by Scherk and Schwarz who suggested that the string theory could best be interpreted as a unified theory - a generalisation of General Relativity coupled to Yang-Mills theories of matter. Scherk and Schwarz finally made the Kaluza-Klein bridge in their paper "Dual Field Theory of quarks and gluons" (Scherk and Schwarz, 1975), to be developed further by Cremmer and Scherk the following year.

(C) It was Cremmer and Julia who finally made the connection between Supergravity and Kaluza-Klein, as we have mentioned. As late as 1978, Cremmer and Julia and Scherk were studying the reduction to four dimensions, and to ten dimensions, of eleven-dimensional supergravity, without reference to Kaluza. Their aim was to look for geometrical interpretations. They noted that D=10 is the highest number of dimensions in which supersymmetric representations of the string model could exist, while supergravity theories could exist in up to 11 dimensions (Cremmer, Julia and Scherk, 1978, p.144). The N=8 supergravity theory had been successfully constructed by dimensional reduction (still a mathematical tool) starting from an 11-dimensional theory. Certainly they considered 11 dimensions seriously by interpreting seven of them as compact dimensions in the spirit of Kaluza, but generalised this to more physical models with broken symmetry in the paper by Cremmer and Julia in 1979. Here they made explicit reference, for the first time in accounts of supergravity, to Kaluza and Klein.

Cremmer and Julia presented their extended Supergravity theory of 1979 by dimensional reduction of the supergravity theory in 11 dimensions to four dimensions. They first constructed the N=1 supergravity in 11 dimensions. They noted that "independently of an eventual fundamental significance of extra dimensions", the dimensional reduction technique had become popular as the more physically realistic compactification, (Cremmer and Julia, 1977, p.142)and had been used to study the supersymmetric theories. They then clearly stated that the method was originally proposed (after Kaluza and Klein) to make sense out of dual models in four dimensions. Their motivation for studying extended supergravity was,like that of Kaluza's originally, to find a true unification of all particles in a finite theory of gravitation interacting with matter. Their theory was much simpler in 11 than in 10 dimensions and they therefore missed the significance of the 10-dimensional dual string theory. Their internal space dimensions were space-like, compact and very small. They referred also to the idea, well-known since Kaluza, that higher dimensional gravitation describes also 4-vector and scalar fields (besides the normal gravitational action in four dimensions). Cremmer and Julia suggested that the fermions "live" in a tangent space, whereas "physical fields" are the fields that propogate (Cremmer and Julia, 1979, p.193).

Their beautiful, elegant approach gave a truly unified theory at the Planck energies : the N=8 Supergravity route via Dimensional Reduction from 11 dimensions. (The other route, of N=1 Supergravity, is approximate and relevant in four dimensions at present energies.) Thus Supergravity literature caught up (Cremmer and Julia, 1979) with the prior introduction of the Kaluza-Klein theory into non-Abelian Gauge Theories (De Witt, 1964), Supersymmetry (Cho and Freund, 1975) and Dual Models and Strings (Scherk and Schwarz, 1975).

Professor Julia himself wrote that his interest in the Kaluza-Klein theory goes back to 1975, and was motivated by the famous paper of J. Scherk and J. Schwarz. In fact he notes (B. Julia, private correspondence to E.W. Middleton, 1986) that John Schwarz gave a talk at Princeton University at the time which started him on that track. Julia had obtained some unpublished results on Kaluza-Klein theory applied to fermions in 1975, but only reported briefly on them in an annual report, because he was looking for some realistic consequences from a 5 or 6-dimensional theory. In his 1978 paper with Cremmer and Scherk he used these technical devices. (In particular Julia was able to solve the mystery of how to get from 10 dimensions an SO(8) type of symmetry. Julia's experience with Y-matrices "showed me right away in October 1977 - how to get SO(8) from SO(7), at least for spinors, even for a Torus compactification"). Julia was able to explore the analogy with the heterotic string model in his Cambridge talk of 1980 (Ed. Hawking and Roček).

Supergravity thus grew up entirely independently of any overt connection with Kaluza-Klein ideas until the link was made in the late 1970's (although privately the bridge was already there). The Kaluza approach seemed to have been transcended by Extended Supergravity, which appeared to be the dominant theory. Supergravity used Kaluza-Klein ideas to supply an essential ingredient by transforming them into their proper framework of 11 (or 10) dimensions rather than 5, and by involving all four forces, rather than the original two of Kaluza's day.

CHAPTER 8 From G.U.T.s to T.O.E.s - Why the Kaluza-Klein model has been such an inspiration in contemporary physics

Synopsis

I. Unification without Gravity

1. Electricity and magnetism - unified theory : Oersted, Faraday and Maxwell.
2. Unification of weak and electromagnetic interactions - Glashow, Salam, Ward, Weinberg - a partial unification.
3. Grand Unified Theories Glashow and Georgi - adding the strong nuclear force (needs very high energies, scale of the order of 10^{16}GeV).
4. Re-entry of Kaluza-Klein into Grand Unified Theories (G.U.T.s).

II. Complete Unification of all forces including Gravity, using Supersymmetry to solve problems

A. Supergravity, the natural route from Supersymmetry, includes Gravity! (Quantum Gravity - a blind alley)

1. Progress in the 1970.s Supersymmetry; local supersymmetry or supergravity

   - Problems still remained - the theory was still "infinite". Supergravity theories inconsistent unless $D > 4$: these supersymmetric theories appeared unique.
   - Various possible compactification schemes - loses uniqueness.

2. Taking the extra dimensions seriously : increasing physical status in the 1980.s

3. Kaluza-Klein ideas and Cosmology - the evolution of the Universe with time.
4. The status of the extra dimensions of the Kaluza-Klein Theory by 1983, in Supergravity Theory.
5. The variation of Fundamental Constants with time.
6. Supergravity - why are the extra dimensions not observed?
7. Conclusion : Summary of Supergravity theories.
8. An alternative unification pathway to Supergravity.
9. Summary.

B. Superstrings, the other main path to complete unification

1. Progress in the 1980.s.
2. The September 1984 Revolution in Superstrings.
3. The Kaluza-Klein model is the inspiration for a complete unification theory ('T.O.E.') via superstrings.
4. Complete Unified Theories from 1986 : the dominance of the Superstring theories, continuing to be catalysed by the work of Kaluza and Klein, with high status given to the extra dimensions.

Appendix to Chapter 8 : 6- and 8-Dimensional Spinor and Twistor Space of Roger Penrose - linked with Kaluza-Klein by Witten, 1986.

I. Unification theories without Gravity

1. Electricity and magnetism - unified theory

The first real unification in physics depended on two discoveries early in the nineteenth century. Hans Christian Oersted in 1819 showed that a steady electric current generated a magnetic field, and in 1831 Michael Faraday showed that a time-varying magnetic field would generate an electric current in a conductor. Oersted and Faraday thus unified magnetism and electricity, two previously independent forces. Building on these experiments, James Clark Maxwell wrote his famous paper in the Philosophical Magazine (1864). He concluded "we can scarcely avoid the inference that light consists of transverse undulations of the same medium which is the cause of electric and magnetic phenomena". He predicted that electromagnetic waves existed and would propagate at a velocity c - the ratio of electromagnetic to electrostatic units of measurement, - which turned out to be remarkably close to the velocity of light. Maxwell was able to show that the unified theory explained the behaviour of light, although it took another thirty years before Heinrich Hertz was able to demonstrate positively that the predicted electromagnetic phenomena exhibit some of the same wave properties that had been used to prove the existence of light waves.

2. Weak and electromagnetic forces

Unification of the weak (involved in radioactive decay) and electromagnetic interactions was proposed in 1959 by Sheldon Glashow of Harvard University, and Abdus Salam and John Ward independently at Imperial College, London. Gauge theory had interpreted the electromagnetic force as acting via the exchange of a photon. New messenger particles W^+ and W^- were therefore introduced, to make the weak interactions look the same as the electromagnetic. In 1961 Glashow with Steven Weinberg later, predicted a neutral counterpart W_0, not in its own right, but with the photon giving Z_0, and predicted a neutral weak interaction involving exchange of Z_o particles. This was confirmed in many experiments from 1973, emphasising also the 'standard electro-weak model'. In 1979, the Nobel Prize for this work was awarded to Glashow, Salam and Weinberg. Glashow, for one, seemed surprised, since "nobody has yet built a machine to check" the new particles predicted (Glashow, 1979). In fact the existence of the predicted particles was not demonstrated until more than twenty years later. Z and W particles were discovered at CERN in 1983 (New Scientist, 27 January 1983, p.221).

The weak and electromagnetic interactions observed in the universe are therefore in fact the visible manifestations of two unseen underlying forces. We do not seem to perceive any unified electro-weak interaction because some mechanism breaks the symmetry between "weak-like" and "electromagnetic-like" interactions,and gives mass to the field quanta associated with the observed weak force (the neutral Z heavy boson).

3. Grand Unified Theories - adding the strong nuclear force (G.U.T.s)

To the electro-weak force, the strong nuclear force needs to be added. This is the force responsible for holding protons and neutrons together. It is basically a force between quarks, arising from the exchange of field quanta known as gluons, which carry 'colour' and change the colour of quarks. To combine electroweak and strong forces is to unite the forces involving both leptons and quarks as a manifestation of one basic interaction. Although such a unity seemed improbable, it was possible to conceive the strengths or the coupling constants being equal at extraordinary high temperatures. This would involve symmetry breaking, e.g. as the Big Bang temperature cooled, in a phase transition (something like the analogy of steam cooling to water then ice). One prediction from some grand unified theories was that protons would decay very very slowly. No definite results have however been obtained from a number of experiments set up to test the 1974 prediction of Sheldon Glashow and Howard Georgi, following the work of Pati and Salam in 1973. Glashow and Georgi published their theory (1973) in which the new electroweak force was unified with the strong gluon force. Gluon fields are needed in the gauge symmetry involved in the strong force. Under this abstract symmetry, hadrons remain "white" while quarks change their (non-physical) property of colour. The quantum theory of colour (Quantum chromodynamics, Q.C.D.) readily explains the rules of quark combination (which were worked out ad hoc in the 1960.s). Although there is no direct proof of quarks, because they seem permanently confined and exist only inside hadrons, Q.C.D. is as widely accepted as the earlier theory of quantum electro-dynamics, Q.E.D. Glashow and Georgi suggested a 'grand unified force' - the first Grand Unified Theory (G.U.T.). However there is no one unique theory and the unification scale is too remote for any direct experimental proof of G.U.T.s.

The postulated symmetry only holds at very high energies. Different strengths imply unification at high energies, of the order 10^{15} or 10^{16} M proton, which is getting close to M(Planck) (about 10^{19}M proton). This produces new forces, including those giving proton decay. But the proton decay is very slow (about 10^{32} years). Experiments have shown that the proton is even more stable.

Grand Unified theories developed in the early 1970.s, but at first took no account either of gravity or of the potential for unification via Kaluza-Klein theories. In 1974 Weinberg was also involved, with Georgi and H.Quinn, and brought in the new supersymmetry to unify two, and perhaps three, of the four forces. Although the Kaluza-Klein idea again remained outside this thrust, it was to converge in the mid-seventies with Supergravity ideas.

4. Re-entry of Kaluza-Klein

In 1978 J.F. Luciani brought back Kaluza-Klein theories, acknowledging a much increased status to the extra dimensions in a link between Grand Unified Theories and Supergravity via the spinor dual model. Luciani referred to Kaluza's idea of using an internal space to generate symmetries, and the more recent generalisation (Cho and Freund, 1975) to an arbitrary gauge group. However this required the introduction of many extra dimensions (using a fibre bundle to represent a specific structure for space time) : "Thus the extra dimensions have lost their physical sense as real space-time dimensions" (Luciani,1978,p.111). However Luciani's own paper - "Spacetime geometry and symmetry breaking" developed ideas of compact extra dimensional internal space for two purposes. First, "to give a physical meaning to theories containing gravitation and gauge fields in a 4 + D dimensional space" - such as the 10-Dimensional spinor dual model, or supergravity. Secondly, to provide a realistic model for the spontaneous symmetry-breaking of quarks and leptons needed in unified gauge theories. Luciani showed how this could arise out of spontaneous compactification and extended supergravity theories, bringing in string theory and anticipating the rise of Supergravity theories to supercede Grand Unified theories.

Thus a supersymmetric grand unification was initiated which was to be developed further, e.g. "Grand Unification near the Kaluza-Klein Scale" (P.G.O.Freund, 1983). In the 1980.s there was further contact between the rather ad hoc G.U.T.s and the symmetries obtained from a consistent treatment of superstring theories as well as supergravity theories.

II The complete unification of all forces, including Gravity - using Supersymmetry

Introduction : Quantum Gravity - a blind alley?

In the late 1970.s, G.U.T.S seemed to evolve into a complete unification of all four forces in the Theory of Quantum Gravity. However, according to quantum theory, gravitational fluctuations will become significant at dimensions of about 10^{-33}cm. At this size, of the order of the Planck length, the four dimensionality of space begins to break down. There are violent fluctuations and space appears multiply-connected or foam-like, according to Quantum Geometrodynamics.

It seems unlikely that a final theory could be obtained merely by adding on gravity, almost as an afterthought, to any particular G.U.T. The success of combining the three forces of strong, weak and electromagnetic interactions depended on the criterion of renormalisability - removing the problem of infinities by a mathematical device. Einstein's General Theory of Relativity is itself non-renormalisable at the quantum level. As t'Hooft pointed out, at this level, "gravity is not renormalisable ...

we need a new physics" (Ed.C.W.Misner,et al.,1973,p.336). The quantum fluctuations of spacetime itself, around the Planck length, question the very meaning of a spacetime continuum of four dimensions . Supersymmetry was needed for supergravity or superstrings to remove the G.U.T. problem of infinities.

In the 1980.s, there was still no solution of the combining of gravity with quantum mechanics in a unified four dimensional field theory. Such a unification led to the need for some Supergravity theory; higher or extra dimensions are necessary to solve the problem using a gauge theory based on supersymmetry.

Note: The crucial step in discussing the idea of gravity as a gauge theory was taken by Ryoyu Utiyama in 1956 (see further Kibble and Stelle, 1986; Kibble 1987 - private correspondence to Middleton). For over twenty years there was no connection made with Kaluza-Klein theories.

Although in the late 1980.s supergravity has had some success in solving the problems of quantum gravity, "in itself (it) does not lead to an acceptable quantum theory".

Local supersymmetry however will be a crucial involvement and it seems likely that

> "spacetime and internal symmetries must in the end be united in a future 'super' grand unification"... "The answer may entail revising our concepts both of spacetime and of quantisation of such a highly non-linear theory as perturbative quantum gravity" (Kibble and Stelle, 1986, p.80).

In particular, the higher dimensional theory of Kaluza and Klein has been "one of the most interesting and attractive ways of unifying gauge theories and gravitation" (Appelquist and Chodos,1983a,p.141).

Their paper, "Quantum effects in Kaluza-Klein theories", building on the work of Witten (1981) on quantum theories of gravity, had already moved the solution away from the unproductive G.U.T.s or the standard Quantum Gravity theories. Certainly in their original form, "existing models for grand unification...have shortcomings which suggest that they are incomplete" wrote Zumino who recommended trying supergravity (Zumino, 1980, Cambridge Nuffield Workshop - "Supergravity and Grand Unification").

A. Supergravity, the natural route from Supersymmetry, includes Gravity!

1. Progress in the 1970.s

Supersymmetry was the basis for all the developments in supergravity. It was a new symmetry principle linking particle spin properties to spatial translation. The theory imposed a new condition on quantum field theory, the language of particle physics. Supersymmetry removed the sharp demarcation between fermions and bosons, which have strong physical differences. This unification involved the theoretical interchange between fermions and bosons into a single theory, using the powerful symmetry which is at the heart of Relativity (Lorentz-Poincaré). Supersymmetry is closely related to geometry and is built on the mathematical theory whereby two supersymmetry operations in succession produce a shift in spatial position. This brings out the gauge field nature of supersymmetry and incorporates particles of different spins within the same supersymmetric family, e.g. the graviton requires the $\frac{3}{2}$ spin gravitino, etc.

This was put on a firm basis in 1974 by Wess and Zumino and is the best model today on which to base unification. The different varieties need firm predictions which can be tested, before the theory can be entirely accepted. As Zumino himself said,

> "Considering that there is no experimental evidence whatsoever that supersymmetry is relevant to the world of elementary particles, it is remarkable that there is so much interest in the ideas" (Zumino,1983,p.18).

Extra particles, e.g. "squarks" and "gluinos" etc. are required, and gravity itself is automatically involved.

In 1976, Freedman, van Nieuwenhuizen and Ferrara produced the simplest example. Local supersymmetry or supergravity, which involves the way space changes from one point to another, involves General Relativity. This led to the development of Extended supersymmetry as Extended Supergravity by Freedman and van Nieuwenhuizen in 1978. There are many forms of extended supergravity, all of which involve the need for more than four spacetime dimensions. Ten or eleven dimensions are the most useful in leading to an overall unification and the cancellation of anomalies, e.g. infinities. Supergravity equations look simpler and more natural when written in higher dimensions. This obviously suggests a link between supergravity and Kaluza-Klein theory, which was not given explicit reference until 1979, by Cremmer and Julia. However, as already pointed out, in 1978 Luciani had in fact brought back the Kaluza-Klein theory with much increased physical status, to link Grand Unified Theories and supergravity via the spinor dual model.

Although some supergravity theories are better in dimensions higher than four, problems still remain. Supergravity is in fact inconsistent unless in more than four dimensions, or the theory is still 'infinite'. These consistent theories must be supersymmetric, and then Supergravity seemed to be unique. However turned out that there are various possible schemes for compactifying se extra dimensions, and Supergravity loses its uniqueness.

Nevertheless the N=8 extended supergravity in 11 dimensions med to be the most promising theory for a complete unification. 8 implies the number of steps in the supersymmetric transformations t connect particles with the complete range of half and integer ns from +2 to -2, and is also equal to the number of gravitinos ired.) There also seems to be a deep connection with this n of Supergravity and the resurrected Kaluza-Klein theory which o suggested 11 dimensions, with 7 dimensions compactified. act there must be at least 11 dimensions to get the 'standard el' from a purely Kaluza mechanism.

2. Taking the extra dimensions seriously : increasing physical status in the 1980.s

In the 1980.s physicists have given a steadily increasing sical status to the extra Kaluza-Klein dimensions, rather than garding them as just an intermediate mathematical device.

> "In order to include other interactions besides the gravitational and electromagnetic in the scheme, it is necessary to generalise our picture to more dimensions". (Chodos and Detweiler, 1980 p.2169).

dos and Detweiler were convinced of the possibility that extra ensions of space, which have appeared for technical reasons he literature from time to time, "may possess a hitherto unsuspected torical reality" (ibid.,p.2169).

We have seen that the change from the mathematical device dimensional reduction to the more physical status of spontaneous pactification was indicated in the 1970.s (Cremmer and Scherk 1976; mmer and Julia, 1977). This physically significant concept led to the possibility, developed in the early 1980.s, that the extra dimensions really were there, at the enormously high energy of the Big Bang, although unobservably small at present times. Supergravity was still the dominant model for unification, usually in 11 Dimensions, with 10 Dimensions as an alternative model, little regarded at first.

3. Kaluza-Klein ideas and Cosmology : the evolution of the Universe with time

The earliest study of time-dependent solutions to the equations of motion describing our expanding universe was in 1980 by Alan Chodos and Steven Detweiler. They produced a solution of the Kaluza-Klein five dimensional model in which one dimension would contract while the other three spatial dimensions expanded to form our effective four spacetime dimensional universe.

The first attempt to look seriously at the status of dimensions beyond four to describe reality (rather than being merely a mathematical technique) was this 1980 paper by Chodos and Detweiler "Where has the fifth dimension gone?". They improved the physical status of the fifth dimension, not by immediately answering where it is <u>now</u>, but by analysing a model of a five dimensional universe. They showed that

> "a simple solution to the vacuum field equations of general relativity in 4 + 1 spacetime dimensions leads to a cosmology which at the present epoch has 3 + 1 observable dimensions in which the Einstein-Maxwell equations are obeyed" (Chodos and Detweiler, 1980,p.2167).

They noted that of the fifteen degrees of freedom, ten are needed for gravitation, four for the electromagnetic potential and the fifteenth either set to one (as in Kaluza,1921) or allowed to vary (Klein, 1926; Bergmann,1948) "thereby introducing a scalar field into the problem"(Chodos and Detweiler, 1980, p.2167). Their model treated all four spatial dimensions symmetrically in the field equation, and described a model which naturally evolved into an effectively three-space. They believed there were many homogeneous cosmologies, but chose to concentrate on the Kasner solution involving five (or six) embedding dimensions (Kasner, 1921).

In their scenario, at time 't' (much greater than the initial time t_o of the Big Bang when all dimensions were infinitely small, the distance around the originally co-equal fifth dimension had shrunk, while the other three spatial dimensions had grown. Thus if the universe is sufficiently old, the fifth dimension will not be observed due to the "evolution of the cosmos". This is in preference to the previous alternative idea of spontaneous compactification at some time (Cremmer and Scherk,1976) - or of the extra dimensions always being rolled up. Chodos and Detweiler chose to follow Souriau's original idea (1958, 1963). This was by considering a quantum field coupled to a five dimensional metric, where at time t_o the four dimensions of space were equally large, thereby heightening the status of the fifth dimension as being <u>really there</u>, even if so early in the history of the cosmos.

"Where the fifth dimension has been shrinking, the other three spatial dimensions have been expanding", (Chodos and Detweiler,1980,p.2168). They also pointed out that in order to include other interactions beside the gravitational and electromagnetic, it would be necessary to generalise their picture to involve further dimensions. They themselves were convinced of the possibility that extra dimensions of space, which had appeared in the literature, therefore possessed at least an historical reality, even if unseen at present, where, at less than 10^{-30}cm, they are "hopelessly beyond direct experimental detection" (Chodos and Detweiler, private correspondence with Middleton, 1982).

> Extrapolating to the future, Alan Chodos pointed out that "the mathematics tells us that, whereas the usual three spatial dimensions expand monotonically with time, the extra dimensions first contract and then, after a certain critical time related to the magnitude of the cosmological constant, begin to expand". (A.Chodos, private correspondence with Middleton,1986).

Thus in this particular model, "the extra dimensions do not remain small forever but may become detectable if one waits long enough". (No evidence, however, is available to strengthen this hypothetical future scenario.)

In December of 1980, Freund and Rubin published a critical paper pointing out that eleven dimensional supergravity admits classical solutions in which the crucial step of spontaneous compactification can take place into only two preferred values. Noting that eleven dimensional supergravity seemed at the time the best solution, they found that "<u>either</u>7 <u>or</u> 4 space-like dimensions compactify (Freund and Rubin, 1980,p.233). In the first case, ordinary "large" spacetime would therefore have 1 time and 3 space dimensions; "a pleasing result", they noted. Their definition of ordinary spacetime as "large" is interesting. Physical spacetime could well have been seven dimensional, as in the second alternative. Not only were the seven dimensions once real, and therefore of high status, but on their model could have been (and again perhaps will be) <u>all</u> of physical spacetime reality. Freund and Rubin had shown that "preferential compactification" occurred automatically in an interesting setting without the addition of any ad hoc set of unwanted scalar fields (Freund, private correspondence to Middleton, January 1988).

E. Witten, in his celebrated paper of 1981, further raised the status of the Kaluza fifth dimension, "Search for a realistic Kaluza-Klein theory". He noted that the apparently four dimensional worldwas because of the microscopically small size of the radius of the circle of the Kaluza-Klein fifth dimension, of the order of the Planck length (10^{-33}cm). Witten was convinced at the time that 11 dimensions was correct, because of the coincidence that at least seven extra dimensions are needed in his Kaluza-Klein approach (using SU(3) x SU(2) x U(1) gauge fields) and that 11 is also the maximum for supergravity. He answered the problem of flavour quarks by giving the extra dimensions sufficient complex topology. The high status of his model does however depend on a very long nuclear lifetime which he forecast at 10^{45} years (too long to be experimentally observed). This was Witten's first attempt in the area of reviving Kaluza-Klein theories: "Kaluza's ideas were relevant, in conjunction with insights of more modern flavour" (Witten, private correspondence to Middleton, February 1988).

In another paper, Witten described the Kaluza-Klein vacuum decay, where the fifthdimension is a hole which spontaneously forms in space, and "expands to infinity with the speed of light" pushing any object ahead of it "unless massive enough to stop the expansion of the hole" (Witten, "Instability of the Kaluza-Klein Vacuum, 1982,p.486). He allowed the fifth dimension high status, and noted that quantum correctionswill give an "effective potential" that will determine the radius of the fifth dimension, an idea to be elaborated later.

In 1982 Freund's paper "Kaluza-Klein cosmologies", found that in generalised Kaluza-Klein theories, the size of the extra space dimensions was close to the grand unification scale of supersymmetric G.U.T.s. This finally brought Kaluza-Klein and supergravity to the aid of the outmoded Grant Unified Theories. He continued the increased status of the extra dimensions in exploring cosmologies where the effective dimensionality depended on time. Freund used higher dimensional Jordan-Brans-Dicke theories linked to 10- or 11-Dimensional Supergravity, noting the "preferential expansion" of three space-like dimensions. (This is another reason for the non-observation of the extra dimensions, besides Chodos and Detweiler's discussions of cosmic evolution using pure higher dimensional Einstein theory). The increase in dimensionality to an 'effective 4-dimensional' description sets in before quantum gravity effects become relevant i.e. close to the "dimensional transition".

Freund, in a critical section, tried to make the link with strings, motivated by Scherk and Schwarz' paper on fermionic string theory in 10 dimensions(1974). However in discussing cosmological solutions of ten dimensional N = 1 supergravity, he found that, unlike the eleven dimensional case, ten dimensions did not seem to preferentially expand to 3 space dimensions (Freund, 1982,p.154). He found that the strength of gravity may then vary, and this would alter the basis of his calculations. (Freund was not ready to take this variation as a possibility).

Thus Freund generalised Chodos and Detweiler's idea using 5 Dimensions, to the case of 11-Dimensional Supergravity. This also had the advantage of explaining in a natural way why 3 dimensions expanded while 7 contracted.

In 1982 also, considerable emphasis was given to taking the extra Kaluza-Klein dimensions seriously with high status in a paper by Abdus Salam and John Strathdee, "On Kaluza-Klein theory". Assuming the extra dimensions are compactified, this involved the understanding of the electric charge in terms of the radius of the extra dimension, taken as a circle (Salam and Strathdee,1982,p.318). The metric field here carries 'an infinite number of new degrees of freedom corresponding to the propagations of excitations in the new dimensions" (ibid.,p.319). Salam in fact appeared on Television to describe this unification, only achieved at the time of the Big Bang. "We believe that the final step to unite (the three forces) with gravity occurred when the universe was 10^{-43} secs.old" (Salam, BBC2,1982,p.10, 25th March in The Listener). He likened the transition, to 4 dimensions from 11, to the analogy of a phase transition. (T.Applequist had suggested earlier the possibility of a phase transition to "a qualitatively different medium" at a critical, very high, temperature (T.Appelquist and R.D.Pisarski,1981,p.2305). In his talk, Salam popularised the idea of spacetime being eleven dimensional, with seven compactified into a very small size of the order of 10^{-33}cm, admitting that this was very speculative. "We shall never apprehend them by direct measurement" he said, although their indirect effect may be seen as a "granularity" in the small scale structure of spacetime, now seen as electromagnetic charges in an overall four dimensional spacetime.

Steven Unwin also noted that physicists are beginning to "reappraise the dimensionality of the universe" (Unwin,1982,p.296). "Living in a five dimensional world" was a fairly popular article in the New Scientist, typical of the increasing interest in higher dimensions and their physical significance, certainly in the first fraction of a second of the Big Bang.

The 1982 International Conference at Sicily provided further evidence of intensified scientific interest in the Kaluza model, at least for supergravity theories. The Proceedings were published in 1983, "Unified Theories of more than 4 Dimensions - including exact solutions". In the preface, the Editors noted the generalisation of Einstein's General Relativity as a unified theory by geometrisation, through the 5-dimensional Kaluza approach and projective field theory, to "multidimensional field theories" and the modern supergravity theories (Ed. V. De Sabbata and E.Schmutzer, 1983). In the first chapter, Peter Bergmann provided an historical overview. However he maintained a low status approach, emphasising the tools of embedding and fibre bundles etc., as mathematical devices to relate manifolds of different dimensionality.

In January 1983, Peter Freund again referred to supersymmetric Grand Unification theories where the scale is close to the Kaluza-Klein calculated value of the extra dimensions. At this scale, spacetime "ceases to be well approximated by a four dimensional manifold". Looking again at the cosmological model ,"the effective dimension of the world manifold changes with time" (Freund, 1983, p.33). He added that if the seven extra dimensions do contract, there may well exist an earlier régime, even before the eleven dimensional universe. In this model, space would be effectively seven dimensional at this time ("Grand Unification near the Kaluza Klein Scale").

Michael Duff confirmed in the same year that supersymmetric models were unique among field theories in that "they are formulated most naturally in spacetime dimension d > 4" (Duff,1983,p.390). There would be a maximum of 10 dimensions for rigid supersymmetry and 11 for local supersymmetry. He emphasised the increase in status of these extra dimensions : "Up until recently, the predominant interpretation has been merely one of a mathematical device" whereby the standard four dimensional theories are obtained via "dimensional reduction", independently of these extra coordinates. "No physical significance need be attributed to these extra dimensions" (Duff, 1983,p.390). By contrast, Duff here explores "the consequences of taking the extra dimensions seriously". He looked for a solution to the d = 11 field equations in which the extra dimensions are 'spontaneously compactified' - a much more physically real process. Duff also used the vitally important scalar fields in his description of the compactification (to a squashed 7-space) which are commonly ignored in the traditional Kaluza-Klein literature. Duff's search for a "realistic Kaluza Klein theory" (ibid.,p.399) involved a higher dimensional geometric origin for the symmetry-breaking by compactifying on a space which deviated slightly from the standard 7-sphere,and is "more in keeping with the spirit of Kaluza-Klein".

The Kaluza-Klein model continued to be used in higher dimensional cosmology, for example by Shafi and Wetterich in the same year. The extra space-like dimensions were considered to be spontaneously compactified; the symmetries of this 'internal space' appeared as gauge symmetries of the "effective four dimensional theory". Increased status was again given by regarding the characteristic length scales of the internal space as of the same order of magnitude as the traditional three dimensional space at very early times in the primordial inflation of the Big Bang - both of the order of the Planck length. They described the internal D-dimensional hypersphere using a de Sitter solution to provide sufficient inflation. (Shafi and Wetterich,1983).

Duff expanded his theory of the importance of N = 8 supergravity, with his colleagues B.Nilsson and Chris Pope. This is by the spontaneous compactification of d = 11 Supergravity on the S7 squashed sphere (Duff and Pope,1982). In their 1984 paper, Duff, Nilsson and Pope argued that the only viable Kaluza-Klein theory was supergravity and that "the only way to do supergravity is via Kaluza-Klein" a pre-eminence seldom acknowledged. They gave increased status to

Kaluza and Klein's ideas that what we perceive to be internal symmetries in four dimensions are "really space-symmetries in the extra dimensions". This was why Kaluza-Klein "could be realistic despite the science fiction overtones of extra dimensions", (Duff, Nilsson and Pope,1984,p.434).

Chris Pope confirmed that they did take the extra dimensions "fairly seriously". He acknowledged that at first physicists used dimensional reduction really as a mathematical trick, and did not take the extra dimensions seriously. For Pope, there were "two rival ideas", the powerful 11 Dimensional theories of Supergravity, and also the 10-dimensional ideas based on Superstrings. At that time, in 1984, "only a few were working on string theories", mainly because of the problem of getting compactification, "which makes it seem somehow unattractive" (Pope 1984, private communication to Middleton). Like Salam, Pope in fact thought that both 11 Dimensions were needed, and the traditional four dimensions coupled to a small scale foaminess - the spacetime foam of Stephen Hawking and John Wheeler. In the higher dimensional case Pope confirmed Duff's thinking that "the extra dimensions are physical, not just a mathematical tool". However there were others who were not committed, and had reservations about the status of the dimensions.

4. The Status of the extra dimensions of the Kaluza-Klein Theory by 1983, in Supergravity Theory

In an excellent review of a 1983 Conference, "An Introduction to Kaluza-Klein Theories", the Editor, H.C.Lee showed that spontaneous compactification was "a crucial and necessary step towards making the Kaluza-Klein theory realistic" (Ed.H.C.Lee,1984,p.116). Lee was concerned to realise the "very rich physical contents" of the Kaluza-Klein theory. All interactions, (other than gravity) he attributed to the structure of the internal manifold, on the Kaluza-

Klein point of view in its present form. 11-Dimensional Supergravity was Lee's best model for unification, this internal space "manifests itself in the spectrum of elementary particles and their quantum numbers" (H.C.Lee,1984,p.126).

At the same conference, K.S.Viswanathan also noted the enthusiastic revival of the Kaluza-Klein philosophy in the previous few years. The commonest model was again via 11-dimensional supergravity, with the emphasis on spontaneous compactification (Ed.H.C.Lee,p.159). Fibre bundle language is extensively used. Alan Chodos, in his chapter on "Quantum Aspects of Kaluza-Klein theories", expanded his ideas published in 1983 with Appelquist, and hedged his opinion on the status. "Whether there is some underlying truth to this stabilisation mechanism", (thermal pressure versus Casimir attraction - see later section)", or whether it is merely a clever device, remains to be seen" (Ed.H.C.Lee,p.274). Chodos regarded his results as "an existent proof for the model, rather than as an attempt to reproduce the real world" (Ed.H.C.Lee,p.276). Problems for Supergravity with quantum corrections were being recognised, however.

In this Conference report, only M.J.Duff brought in the alternative model of Superstrings in 10 dimensions. He noted that in the 1980.s, physicists had been more ambitious in their unification schemes to involve four forces, using the Kaluza-Klein model. He repeated his assertion that the unique 11-Dimensional Supergravity (following Witten,1981) favoured traditional Kaluza and Klein ideas. Duff himself favoured the N=8 Supergravity theories in four dimensions, which also find their most natural setting within the framework of Kaluza-Klein (Ed.H.C.Lee,1984,p.280).

For Duff, however, no one route could claim complete success as yet. He noted that within the Kaluza-Klein framework, "those somewhat abstract geometrical concepts translate into something concrete and familiar in the effective four-dimensional theory". (Ed.H.C.Lee,p.283). He commented however that these extra dimensions, in spontaneous compactification, "do not conflict with one's everyday sensations of inhabiting a four-dimensional world (with its inverse square law of gravitational attraction) provided R is small" (Ed.H.C.Lee,p.288). Duff's paper did point to the emerging string development. He divided Kaluza Klein theories into (a) 10 or 11 dimensional supergravity (still his favourite, with a squashed 7-sphere), and (b) 10 dimensional string models.

This "recent renaissance" of Kaluza Klein theories was also discussed in a paper by John Barrow, in which he also brought in the Anthropic Principle: "Dimensionality" (Barrow,1983). He examined the development of the increased status given to the idea that the Universe really does possess more than three spatial dimensions. Barrow did not mention the increased physical reality given to spontaneous compactification, rather than the mathematical device of dimensional reduction. He did however emphasise the higher status of the additional dimensions as a set of internal symmetries : "We perceive them as electromagnetic, weak and strong charges"- compactified to the Planck length of 10^{-33}cm (Barrow, 1983,p.344). Barrow also stressed the further status in the 1980.s in the initial 10^{-40} seconds of the Big Bang, when the Universe is now widely regarded as fully multidimensional ($N>5$), compactified on cooling. Barrow added his own level of increased status by his adherence to the Anthropic Principle. The only reason why just three dimensions are left expanding is that this is the only possible dimensionality for observers to exist - a critical fine tuning idea!

As Alan Chodos was to point out, one limiting feature of the eleven dimensional supergravity model for cosmology was that "as the size of the internal dimensions changes with time, so do the gauge coupling constants" (Chodos,1984,p.178). He also pointed out other problems involved with increased status of the extra dimensions in this paper, "Kaluza-Klein Theories : An Overview". There was the problem of dimensional reduction, whether the solutions are also solutions of the equations of motion in these higher dimensions. Chodos pointed out that they were not, "and adding a cosmological constant or simple conformal factor will not help either" (Chodos,1984, p.176). There are three possible approaches. It can be continued in the previous tradition of a mathematical device, although no real unification is then possible. An alternative was to say the extra dimensions do exist, but involve matter fields to achieve spontaneous compactification. This had been a developing idea, but seemed to Chodos to introduce matter fields *ad hoc*. His final suggestion involved taking the extra dimensions "completely seriously". Supergravity in 11 dimensions with spontaneous compactification had seemed to work, but "only if the spacetime part of the manifold is not Minkowski space but anti-de Sitter space" (Chodos,1984,p.176). This curvature however does not correspond to the real world.

5. Variation of Fundamental Constants with time

It was William Marciano who issued some challenging questions before suggesting, in his 1984 paper "Time Variation of the Fundamental 'Constants' and Kaluza Klein", that such a variation might in fact provide evidence for extra space dimensions: "Are extra dimensions a physical reality or merely a model-building mathematical tool?" , and, "if they are real, can we find evidence for their existence?".

(Marciano, 1984,p.489). Marciano reviewed variations of mass units of the proton and of the constant of gravitation and asked for a clear scrutiny to be made. If a time variation is detected, "it could be our window to the extra dimensions, an exciting possibility" (Marciano,1984,p.491). However, little evidence of this way out for the supergravity model limitation has been found. No papers have been written on the time variation, even by Marciano himself, although he has"made a reexamination of experimental constraints on time variation of the fundamental constants from a phenomenological perspective" (Marciano,December 1987, private correspondence to Middleton).

A possible alternative escape route would be to find a model in which the extra dimensions remained fixed at some very small scale. The idea of an internal space where symmetries "correspond to the observed internal symmetries of low energy physics" was taken furtherby S.Randjbar-Daemi, Salam and Strathdee (1984,p.388). Their paper "On Kaluza-Klein Cosmology", admitted that the equations for the extra highly curved and compactified dimensions were unsolvable with the energies available at present. It therefore seemed appropriate to the authors to look for cosmological implications. They were able to confirm that Kaluza-Klein cosmology does admit of a time-independent internal radius "consistent with lack of variability of gauge couplings with time" (Randjbar-Daemi,et al., 1984,p.392). Above the temperature of phase transitions, at any rate, the internal space should have a constant radius, while the external expanding dimensions evolve in the usual manner.

Another way out was emerging in the literature. It was possible that as the contracting dimensions, after t=o, approach the Planck scale, quantum effects became the dominant force, fixing or 'freezing' the extra dimensions at some fixed size, near the

Planck length. This work was pioneered by Applequist and Chodos in "Quantum effects in Kaluza Klein theories" (1983). Their results postulated a force "tending to make the fifth dimension contract to a size of the order of the Planck length"(by a gravitational version of the Casimir effect in electrodynamics). They raised the fundamental status question - an intermediate mathematical device - or real existence i.e. where the four dimensional theory is to be regarded as an approximation to the full D-dimensional universe. One of their motivations was to explain, if the extra dimensions are given high status and really exist, how it is that they are not observed. They argued that the degrees of freedom or internal dimensions which have been compactified or frozen out can still affect low energy four dimensional physics,"because of their appearance as virtual particles in quantum loops" (Applequist and Chodos,1983,p.141). These internal dimensions would thus contribute to a "quantum effective potential". Thus (as Klein himself hoped in 1926) such quantum effects associated with the extra dimension may be the real cause of the smallness of these dimensions.

Applequist and Chodos did not restrict their analysis to five dimensions. They proposed to explore the extension to "more realistic Kaluza Klein theories", and noted, although only qualitatively, that "the resulting more complicated topology could also influence the sign of the Casimir effect, as happens in the electromagnetic case" (Applequist and Chodos, 1983,p.144). They also studied the case where the compact manifold is a d-dimensional torus. (Applequist, Chodos and Myers,1983, p.51). Their second 1983 paper on quantum properties firmly took the view that any implementation of the Kaluza-Klein idea should regard the extra dimensions as actually existing with some physical size (Applequist and Chodos,1983b,p.772.).

Others took up this application of Kaluza-Klein theories with a torus in the compact space. Again it was found that some physical circumferences tend to contract to sizes of the order of the Planck length. Contraction or expansion of the compact dimension was found to depend on other initial values (Inami and Yasudu,1983, "Quantum effects in generalised Kaluza-Klein theories",p.180).

A more recent link between Kaluza-Klein cosmology and the variation of the Gravitational Constant G with time has been made by Paul Wesson. A leading protagonist of the idea that G may be changing as time passes, Wesson introduced a new gravitational parameter into the Kaluza Klein model. This "coordinate" was treated as an extra fifth space dimension $(\frac{Gm}{C^2})$ where G and m can vary (in fact without the need for a big bang of the conventional type). If this parameter is either a constant or proportional to the age of the Universe, Wesson got a good agreement with astrophysical observations, from the Earth-Moon dynamics to the evolutionary history of stars (Wesson,1986,p.1). Such a variable gravitational constant was in fact proposed earlier by Dirac and introduced by Jordan in his scalar version, followed up by Dicke and others, but without any Kaluza-Klein formalism.

6. Supergravity - why are the extra dimensions not observed?

By the mid-1980.s, Supergravity theory in 10 or 11 Dimensions had become widely recognised as a strong candidate to achieve a unification of forces and particles to describe reality. Popular books were written, e.g. P.C.W.Davies, Superforce:the search for a grand unified theory of nature (1984), television programmes seen, e.g. by Stephen Hawking, for whom Supergravity (N=8) was a "definite candidate" for describing everything in a completely unified theory, (BBC2, October 18,1984). Broadcasts e.g. by Martin Rees and Steven Weinberg noted that classical beliefs that time has a direction and space has three dimensions may have to go. They proposed "a higher dimensional space time; the most popular candidate these days is eleven dimensional supergravity", see M.Rees "Close encounters with eleven-dimensional spacetime", March 1984 (reprinted in The Listener, 8 March 1984,p.10). There was certainly a rapid expansion in popular awareness of 10 or 11 Dimensional Supergravity theories by the end of 1984.

Nevertheless, some questions on the applicability of Supergravity theory to the real world still remained. The chief problem of Kaluza-Klein cosmology remained as to why the characteristic length scales of the unobserved internal dimensions are now so very small, while the usual three space dimensions are so large. The solution of how to compactify the scale of the extra dimensions near the Planck length received a new impulse within the framework of cosmological inflation. From 1980 onwards, physicists have given various explanations, involving the actual historical reality of the extra dimensions. The more physical approach came via spontaneous compactification (Cremmer and Scherk, 1976; Luciani,1978; Chodos and Detweiler,1980; Witten,1981,1982; Wetterich,1985).

As we have seen, reasons included (1) The spontaneous compactification at some time: (2) The evolution of the cosmos causing the fifth dimension to shrink (Chodos and Detweiler,1980) i.e. rolled up with the evolution in time. (3) Preferential expansion (Freund,1982). (4) The extra dimensions were always rolled up (i.e. of constant radius) (Randjbar-Daemi et.al.,1984). (5) A quantum potential, a force causing the fifth dimension to shrink (Applequist and Chodos,1983). This Casimir force was developed by M.A.Rubin and B.D.Roth. "Fermions and Stability in Five Dimensional Kaluza-Klein Theory". They looked to the inclusion of massive fermions, as well as massive twisted bosons, to stabilize the compact fifth dimension (Rubin and Roth, 1983, p.55). It was Chodos himself who noted that any quantum gravitation effects "must be viewed with suspicion because of the absence of a consistent theory of quantum gravity". Nevertheless he asserted that the Casimir effect in Kaluza-Klein theories "does represent a rare example where quantum gravity is expected to play a physically important role" (Chodos, 1984, p.178). (6) The attempt to quantise gravity (outside string theory) led to a sixth account of the compactification in "Primordial Kaluza-Klein inflation" (P.F.Gonzalez-Diaz, 1986, p.29).

C.Wetterich was quite clear in his paper "Kaluza-Klein cosmology and the inflationary universe", that Kaluza-Klein theory gave realistic models in higher dimensions which "may be a clue for a natural understanding of inflationary cosmology", (Wetterich, 1985, p.319). Cosmological compactification of the Kaluza-Klein extra dimensions was taken a stage further by A.Davidson and colleagues (7). Their motivation was to explain the expanding universe by bringing in the theoretical role played by Grand unified theories in the evolution of compactification. For them, this required a "positive cosmological constant, while supporting both the big bang singularity and the open character of ordinary space" (A.Davidson, J.Sonnenschein and A.A.Vozmediano "Cosmological Campactification", 1985, p.1330). Other authors extended their thinking to entropy production, thereby linking the inflation of external (ordinary) space with the collapse of the internal (compact) space. The internal space was assumed to be decoupled from the external space and "the role of viscosity due to the transport of gravitational radiation in a Kaluza-Klein multidimensional universe" was considered by Kenji Tomita and Hideki Ishihara (1985). Thus entropy production is a further explanation (8).

(9) A more unusual explanation for the non-observability of the extra dimensions came from M.Visser, "An exotic class of Kaluza-Klein models" (1985). Rather than the usual idea of the internal space being compact, Visser suggested that the particles were "gravitationally trapped near a four dimensional sub-manifold of the higher dimensional spacetime", using a five dimensional model (Visser 1985,p.22). "This four dimensional submanifold of the 'real world'," implied that higher dimensional spacetime is the real world. His method of dimensional reduction effectively removed that particular variable from low energy physics, although Visser admitted that there was no need for the five dimensional "electromagnetism", which he had considered, "to have anything to do with ordinary electromagnetism", (Visser,1985,p.24) - a low status approach to the problem.

In an interesting follow up to this alternative model to spontaneous compactification as a means of explaining the non-observability of the extra dimensions, E.J.Squires took as a base line the paper by V.A.Rubakov and M.E.Shapashnikov (1983). This had the implication that normal physical spacetime is folded up in some manner inside a larger space. Squires noted that this possibility might imply that the world was folded up inside a higher dimensional reality, so that distances which may appear large when measured within our apparently four-dimensional "physical" space, "might in fact be much smaller when measured in a flat metric in the space of higher dimension". The surprising but creative suggestion (motivated by the key paradox of quantum theory) was made: "this in turn might allow the even wilder speculation that the non-locality problems of quantum theory might be resolved in the larger space"(Squires,1985,p.1).

This daring solution did not provoke other physicists to risk a reaction. The article in fact analysed dimensional reduction from 5 to 4 by a large cosmological constant using a generalisation from the case of 4 dimensions reducing to three.

Further work on the importance of the Kaluza-Klein model in cosmology was presented at a conference on "Phase transitions in the very early universe". (Particle Physics, B252,No.1 & 2, March 1985). A multidimensional view of reality had by then clearly emerged. The dimensional reduction transition was a key theme. "The basic assumption is that the true dimensionality of spacetime is more than four, and that at present the extra dimensions are compact and too small to be observable" (E.Kolb, "The Dimensional Reduction Transition,1985,p.321). It was assumed that initially all spatial dimensions were small, and that in fact the universe had 3 + D spatial dimensions. In what had become the Standard Model, when the temperature of the Big Bang began to fall, the spacetime dimensionability of the universe underwent a reduction to effectively a 4 spacetime dimensional universe. Kolb assumed that the extra dimensions, although small today, were dynamically important in the evolution of the early universe. Then the transition to four spacetime dimensions "may have produced physically significant phenomena observable today" (Kolb,1985,p.321).

Three possible physical consequences resulting from such a cosmological dimensional reduction, Kolb suggested, were entropy production (producing inflationary cosmologies), magnetic monopole production, and massive particle production. Kaluza-Klein monopoles were massive topological defects in the geometry of compactification, "frozen in as space is split into 3 large spatial dimensions and D small compact dimensions" (J.A. Harvey, E.W.Kolb, M.Perry, Preprint, 1985). (These appear in fact in the initial conditions, whereas G.U.T. monopoles first appear during the phase transition). This paper provided an explanation for inflation (assumed by most cosmologists), magnetic monopoles (for which experimental tests are in progress) and for massive stable "pyrgons" (hypothetical towers of particles, originally noticed by Klein in his article in Nature,1926, on five dimensions).

7. Summary of Supergravity Theories

Kaluza-Klein theories with local supersymmetry have thus been seen to have a key role in the general search for a unified field theory, where Supergravity superceded Grand Unified Theories (which excluded gravitation). The literature focussed first on 11 and then also on 10-Dimensional Supergravity with spontaneous compactification. A multidimensional gravitational theory is interpreted as a four dimensional spacetime theory which "brings back to the landscape of modern theoretical physics the old, time-honoured Kaluza-Klein idea" (P.Fré, "Prospects and problems of locally supersymmetric Kaluza-Klein theories", 1985, p.331). The Journal "Classical Quantum Gravity" contained many similar conclusions, e.g. "Kaluza-Klein Supergravity in ten dimensions" as the "Theory of Everything", - by compactification of the eleven-dimensional N=1 theory, (M.Huq and M.A.Namazie,1985,p.293).

The question of how the hidden dimensions, although unobservable, were manifest today, has led a number of physicists to suggest concrete testable possibilities (Marciano,1984; Kolb,1985). The increased physical status is seen in the cosmological implications of Kaluza-Klein theory. The extra dimensions are widely seen today as being internal symmetries, symmetries of the internal space which appear as gauge symmetries of our effective four dimensional universe. Thus the structure of the internal manifold causes all the interactions, forces of nature and fundamental charges, from electric to colour and charge conjugation, flavour etc. This internal symmetry is therefore perceived as electromagnetic, weak and strong forces, often regarded as degrees of freedom.

The cosmological implications have even been carried into future events. Following the ideas of Chodos and Detweiler (1980), Applequist and Chodos assumed that the extra dimensions really exist even though we cannot detect them. They also considered the possibility of the fifth dimension evolution changing over from contraction to expansion at a certain energy (using Kasner-type embedding behaviour) "and will ultimately re-emerge from the obscurity of the submicro world" (Applequist and Chodos,1983,p.780). Physicists have developed further the reversal of the usual spontaneous compactification scenario, and even developed a new expansion of our cosmos after a possible collapse to a "Big Crunch". This 'new creation' avoids a final singularity e.g. Recami and Zanchin; "Does Thermodynamics require a new expansion after the "Big Crunch" of our cosmos?" (1986,p.304). However this seems rather fanciful and presupposes a number of arbitrary hypotheses.

8. An alternative unification pathway to Supergravity

We have seen that in a wide ranging survey of Kaluza-Klein theories - 1983 (E.H.C.Lee,1984) only M.J.Duff introduced the possible alternative of superstrings into the prevalant accepted unification by Supergravity. In 1984, E.W.Kolb and R.Slansky also looked at the application of Kaluza Klein theories in their paper "Dimensional Reduction in the early Universe". They considered both N=8 supergravity

in 11 dimensions and also the quantum superstring, which must be formulated in 10 dimensions. They looked at the evolution of the universe before the time of compactification, where the extra dimensions are 'large' (ref.Chodos and Detweiler,1980) and searched for more realistic theories with three-dimensions. Kolb and Slansky, as we have seen, postulated massive particle called pyrgons (elaborated, Kolb 1985), with resulting cosmological implications. "If there are stable pyrgons, then they become (yet further) candidates to dominate the dark matter of the universe". (Kolb and Slansky,1984,p.382).
In a footnote, John Schwarz was cited for the observation that "massive stable string configurations are expected in some versions of type II Superstrings" (Kolb and Slansky, 1984,p.381). Thus the alternative to Supergravity is again mentioned. The ripples of the 1984 Superstring revolution were spreading, even to supporters of Supergravity, hitherto the best candidate for a unified theory.

9. Conclusion

It is necessary to point out that whereas the unification of electricity and magnetism predicted a theory of electromagnetic radiation, and the unification of the Weak force and Electromagnetism predicted neutral currents, $W^{\pm}$ and Zo, all of which have been observed, the G.U.T. unification produced one striking prediction (proton decay) which has not been observed. More importantly, supersymmetry and supergravity have so far produced no successful predictions.

B. Superstrings - the other main path to complete unification

1. Progress in the 1980.s

As we have seen in the previous chapter, String theory developed as the Bosonic string with a solution in 26 Dimensions from the Veneziano Dual Resonance Model. It was seen as a model of a relativistic string in 1970 by Nambu, Nielsen and Susskind, independently, and developed as a supersymmetric string in 10 Dimensions by Ramand, Neveu and Schwarz in 1971, to include both bosons and fermions.

There had been other important developments in the early 1970.s such as the development of quantum chromodynamics as a theory of strong interactions (without the need of string theory). The lattice approach to Q. C .D (Wilson,1974) did nevertheless suggest that the string could be seen as a tube of colour electric flux which would be responsible for quark confinement. The linking of strings with Yang-Mills theory was suggested by Nielsen and Olesen (1973) in their work on string-like solitons (relativistic versions of confined types of magnetic flux in superconductors). There was also the development of Grand Unified Theories via Georgi and Glashow (1974). Only recently have the links been made between these rather ad hoc proposals for unification and sueprstring theory.

The most important development was probably the work on supersymmetry as an extension of standard Poincaré spacetime symmetry by Wess and Zumino (1974). They generalised the algebra of the Ramond, Neveu and Schwarz string model to four dimensions.

Soon afterwards, Scherk realised that field theory came out in low energy strings and with Schwarz made the connection with Kaluza-Klein ideas in 1975. No one at all was pursuing the idea of bringing in gravity, and closed strings (which contain gravity) were not mentioned. The connection with gravity was in fact first made by F.Gliozzi, J.Scherk and D.Olive in 1977. Although string models seemed to be receding in usefulness from 1976, this major development by Gliozzi et al. was to catalyse the renewal of strings as superstrings in the 1980.s. They discovered that a spectrum free of tachyons (theoretical particles which should travel faster than light) could be obtained from the Dual spinor model by making the spectrum supersymmetric in the spacetime sense. With extra dimensions compactified, Gliozzi, Scherk and Olive showed that dual models were in correpondence with supergravity. They followed a hierarchical development leading to theories of supergravity in 10 dimensions and made the correpondence with the dual model of closed strings (Gliozzi et al.,1977,p.283). However their main interest at the time was the construction of higher dimensional supergravity theories rather than in developing string theories, which were not followed up, although a strong connection was made.

A Summer school on Quark Models at St.Andrews in August 1976 (published in 1977, Ed.Barbour and Davies), produced two articles on strings. Both H.B.Nielsen "Dual Strings" (Ed.I.T.Barber,1977,p.465) and B.Zumino, "Super gravity, spinning particles and spinning strings" (ibid.,p.549), looked for the connections with supergravity, although without any mention of Kaluza-Klein. Other authors followed Cho and Freund in linking local gauge theories with supersymmetric strings. Parallels were drawn between gravitation, local gauge theories and quark-like supersymmetric strings based on superspace (L.N.Chang, K.I.Macrae and F.Mansouri,1976,p.235).

From 1976, almost all theoretical physicists turned away from the apparent blind alley of string theory, due mainly to the apparent inconsistency of theories with tachyons. Even the major development by Gliozzi et al., and the work on spacegeometry by W.Nahm "Supersymmetries and their representations" (1978) were not seen as significant at the time. Nahm was able to build on the work of Cremmer and Scherk (1976) on spontaneous compactification. Cremmer and Scherk (1977) also studied the compactification of the bosonic string on a torus, with closed strings winding round the compact dimensions. However, like most other physicists, they concentrated almost entirely on Supergravity as a model for complete unification; some with later regret, e.g. B.Zumino (1980 - private correspondence to E.W.Middleton). However, in Nahm's work on the classification of higher dimensional supersymmetric theories, he noted the possibility of there being two theories in ten dimensions as well as the standard 11-Dimensional theory (Nahm,1978,p.165) of supergravity.

In the early 1980.s, nevertheless, Michael Green and John Schwarz who had continued working on string theory, proved the connection (suggested by Gliozzi et al.) between superstrings and supergravity in a manifestly supersymmetric way. They described the supersymmetric form of the superstring action for the first time. This completely consistent theory of dual models in the form of supersymmetric string theories was renamed Superstring theories. The open-string and closed-string models were formulated in 1982 for theories which were named type I, type II_A, and type II_B (Green M.B. and Schwarz,J.H. 1981, 1982a, 1982b, 1982c). As Michael Green himself notes in a marvellously concise review article, "this was a striking result since the theory is defined in ten dimensions, which would lead to highly divergent amplitudes for ordinary field theories" (M.B.Green, 1986,p.25). These models in fact gave a very geometric interpretation of strings in superspace.

Type I Superstrings describes the dynamics of open strings that have free end points. Their effective field theory is Yang

Mills coupled to N=1 Supergravity in a unification, with only one symmetry group, SO(32) and in particular the $E_8 \times E_8$ version.

Type II theories only apply to closed strings. There are two orientations in 10 Dimensional N=2 Supersymmetry. Open strings may interact to form another open string, or two, or to form a single closed string. Hence all Type I theories in fact contain Type II.

Type III Superstrings or Heterotic Strings (Gross et al.,1985) are closed strings only, Instead of the Yang-Mills gauge charges residing at the ends of the string, there is a charge density along the string. This combines some aspects of the original 26-Dimensional bosonic string, with 16 Dimensions as a torus, leaving a space time of 10 Dimensions.

It was interesting to see that in the 1980 Cambridge Nuffield Workshop on <u>Superspace and Supergravity</u> (Ed.S.W.Hawking and P.Roček,1981) strings were hardly mentioned. For P.van Nieuwenhuizen, in his physically motivated approach, supergravity was the gauge theory of supersymmetry. M.J.Duff also emphasised the physical significance of supergravity in the change from a purely mathematical model. Only B.Julia took the broader view. He brought in the link with Kaluza Klein theories in the time evolution of symmetries in 11-Dimensional supergravity (Ed.Hawking,1981,p.332). In a fascinating link-up with the dual resonance model, Julia noted that the supergravity model in 10 dimensions was connected to the limit of a closed string dual model in 10 dimensions, and was also closely connected with supersymmetry. He also used the model of 9 transverse dimensions of the "Kaluza torus" (ibid,p.335). The higher dimensions of Supergravity, Julia concluded,ought to appear in the dual string models "and indeed they do". Julia had just begun to bridge the gap between supergravity and superstrings which he had started to investigate earlier: "At present the only interacting theories that include particles of higher spin are the string models" (ibid.,p.345).

Green and Schwarz had been developing their Superstring model quite independently of the vast literature on supergravity. The only other interesting work was by A.M.Polyakov, "Quantum Geometry of bosonic Strings" (1981a) and "Quantum Geometry of fermionic strings" (1981b). These were to transform the treatment of string theory. His method of quantising string theory also led to a better understanding of the rôle of world sheet topology, although his ideas were outside the main thrust of superstrings. He used d=26 as well as d=10 supersymmetric strings, with the "language of superspace"(Polyakov,1981b,p.211).

By December 1980, Michael Green was looking at the "tremendous mathematical elegance" of the string model, and was involved in interpreting the rolled up dimensions in a new way, but still based on the Kaluza Klein idea of unifying gravity with other forces. Green was already working on the new Superstring ideas, which as we have seen, became type I, $II_{A \& B}$ in 1981. The new and creative approach, which he was developing with John Schwarz was to take the 10 dimensional string theory and treat it as a quantum theory first (instead of compactifying first and then bringing in quantisation). He was not then sure what meaning it would have, except that on the small scale of Planck size,"the whole notion of space time breaks down" (1980 Private conversation with Middleton) "and extra dimensions are needed". This developed into the Green-Schwarz superstring and paved the way for their 1984 revolution. Even the Supergravity in 10 dimensions was beginning to fail as the best model available: superstrings were now overtaking the attention of physicists. Supergravity did not solve three main problems: The Chirality problem, because in nature neutrinos are always left handed; the cosmological problem, because the curvature of the physical universe is zero or close to zero; and the problem of quantum infinities.

2. The September 1984 Revolution in Superstrings

In their 1984 paper, Green and Schwarz provided some remarkable new insights. Choosing a special gauge group (SO(32) or $E_8 \times E_8$), they were able to show that the potentially hopeless gravitational and Yang-Mills anomalies exactly cancel. SO(32) is the rotation group in 32 dimensions, and E_8 is the largest of the exceptional groups in Cartan's classification of Lie groups. Both groups in fact have 496 dimensions. The Green-Schwarz anomaly cancellation mechanism also meant modifying the conventional supergravity model. The 10-dimensional variety of supergravity had not been under intensive study because of the problems of curling up the extra dimensions and the inconsistencies at the quantum level. "The 10-dimensional version of supergravity, and consequently the mutual interaction of the massless particles described by the superstring theory, did not seem relevant for the Kaluza-Klein programme" (D.Z.Freedman and P.van Nieuwenhuizen, "The Hidden Dimensions of spacetime" 1985,p.67). Green and Schwarz had been able to show that the interaction of massless particles in superstring theory differed slightly but significantly from the supergravity version. The other problems, the Chirality problem and the cosmological problem, also seemed to be solved by the new superstring which additionally resolved the problem of quantum infinities. Superstrings satisfied both relativity and quantum mechanics. This Type I Superstring theory appeared very likely to be a "consistent quantum theory" (Green and Schwarz, 1984,p.122).

Superstrings seem to provide the solutions for the unification of gravity and other forces. The gaugeinteractions (strong, weak and electromagnetic forces) were carried by 'open' strings, and gravitational interactions by closed strings. Only in 10 dimensions was the theory consistent. The early string theories had been inconsistent as they contained tachyons. Incorporating supergravity enabled Green and Schwarz to allow their 1984 unique version of Superstring "Anomaly cancellation in Supersymmetric D=10 Gauge Theory and Superstring Theory" for "Type I Superstring Theory" of unorientated open and closed strings (Green and Schwarz,1984,p.117).

Following the discovery of anomaly cancellation, the search began for an E_8 x E_8 Superstring Theory. In an unorthodox approach, P.G.O.Freund suggested that it could be derived by compactification of a Superstring in 26 dimensions (the old non-supersymmetric Veneziano bosonic string), "Phenomenologically the most promising as a 'theory of the world'" (Freund,1985,p.387), these dimensions could be regarded as 10 large and 16 compactified. For Freund, there was a 2-Dimensional string world-sheet and a 10-Dimensional 'host space'. The 'true' dimensions of spacetime might then be 26 or 506.

This in fact turned out to be partially correct in the Heterotic String theory. This was developed from Green and Schwarz Type I Superstring Theory by David Gross, Jeff Harvey, Emil Martinec and Ryan Rohm from Princeton University: "Heterotic String" (1985). The Heterotic String or new Type III is a closed string theory, called 'Heterotic' (or Hybrid) because it combined features of the d=26 Bosonic strong and the d=10 Type II_B string, while preserving the appealing features of both. This necessitated "the compactification of the extra sixteen bosonic coordinates of the heterotic string on a maximal torus of determined radius "to produce E_8 x E_8 symmetry" (Gross et al.,1985, p.502). The string coordinate winds N times around the manifold. Thus the 'Princeton Quartet' established

> "the existence of two new consistent closed string theories, which naturally lead, by a string Kaluza-Klein mechanism, to the gauge symmetries of SO(32) or E_8 x E_8" (ibid.,p.504).

They concluded that the heterotic E_8 x E_8 string was "perhaps the most promising candidate" for a unified field theory. In an unusual extrapolation, they affirmed physically interesting compactifications of their theory to four dimensions, "including the possibility that the E_8 x E_8 symmetry is unbroken, thereby implying the existence of a 'shadow world', consisting of E_8 matter which interacts with us (E_8 matter) only gravitationally.

This speculation that there may exist another form of matter ("shadow matter") in the Universe, which only interacts with 'ordinary' matter (e.g. quarks, leptons) through gravity, has been explored theoretically, with no firm results. Such a parallel shadow world was investigated for cosmological implications by Edward Kolb, David Seckel and Michael Turner, "The shadow world of superstring theories" (1985). They noted the effect would be hard to detect in everyday life, but would have many effects in the early and the contemporary universe. They showed that an exact mirror Universe "is precluded by primordial nucleosynthesis" but that shadow matter may nevertheless "have played an interesting role in the evolution of the Universe" (Kolb, Seckel and Turner, 1985,p.419). If true, it would certainly provide an explanation for the "missing mass" problem in cosmology.

In a minor revolution to suggest how four-dimensional physics might emerge, Philip Candelas, Gary Horowitz, Andy Strominger and Ed Witten described the extra six dimensions as a Calabi-Yau space. Eugenia Calabi and Shing-Tung Yau were the names of distinguished mathematicians. Compactification from ten Dimensions to four could now be overcome on such a compact six dimensional Calabi-Yau manifold- a valuable mathematical space with interesting geometrical properties for a 'phenomenally realistic' as well as mathematically consistent theory. In particular they noted the Kaluza-Klein theory, "with its now widely accepted interpretation that all dimensions are on the same logical footing" was first proposed (by Scherk and Schwarz, 1975, and also Cremmer, 1976) to make sense out of higher dimensional string theories , (Candelas, Horowitz, Strominger and Witten, 1985,p.47). In all these papers on Superstrings, the status of the Kaluza-Klein idea was being steadily reinforced and consolidated, sometimes directly, sometimes by implication, underpinning the concept of superstrings.

3. The Kaluza-Klein model is the inspiration for a complete unification theory ("T.O.E.") via Superstrings

In a review article in Nature in 1985, "Unification of forces and particles in superstring theories", Michael Green proposed superstring field theory as a profound generalisation of the conventional framework. The basis was

> "the dynamics of string-like fundamental quanta rather than the point like quanta of more familiar relativistic 'point field theories' such as Yang Mills gauge theory or general relativity"(Green, 1985,p.409).

In these field theories, leptons and quarks may exist as the ground states of a string. With regard to existing supergravity theories (point field theories) which incorporate local gauged supersymmetry and extend Einstein's General Relativity, Green noted that despite early optimism, a consistent quantum theory does not seem to be produced. He hoped that a replacement would be the consistent superstring theory with an "almost unique unified theory" as a low energy approximation. Whereas the original (bosonic) string theory needed 26 dimensions, superstring theories require 10-dimensional space time (something like the arena in superspace). No unwanted infinities are present. The observed Chirality of our approximately four dimensional world is still present when the extra six dimensions compactify, "if the gauge fields twist up in a topologically non-trivial manner in the internal compact space" (Green, 1985,p.410). In the construction of the preferred heterotic string, some aspects of the unique 26 dimensional bosonic string are combined with 16 of the dimensions as the maximal torus, leaving 10 spacetime dimensions. In these 10 dimensions, the extra six must curl up or "compactify" to very tiny size. Green's method is different from the original Kaluza ideas in that the chirality and gauge fields are already present in the ten dimensions before compactification, rather than be produced afterwards. Nevertheless, "This is analogous to the idea originally proposed by Kaluza-Klein" (Green, 1985,p.410). The fact that the Yang-Mills gauge group in the 10 dimensions can provide all the internal symmetries needed for experimental physics, "distinquishes it from the usual Kaluza Klein schemes" (ibid.,p.413).

Thus particles are associated with the vibrational motions of one-dimensional strings in a higher dimensional space. Only 10-dimensions provide a consistent anomaly-free theory, with 6 extra dimensions curled up, e.g. in Calabi-Yau space. (Gauge interactions are carried by open strings and gravitational interactions by closed strings). The unique heterotic string combines both with the supersymmetry group $E_8 \times E_8$. Thus a consistent superstring theory provides potentially consistent quantum field theories which unify gravity with the other fundamental forces in a unique manner.

Michael Duff is another physicist who goes beyond the standard model, now favouring superstring, rather than supergravity. His plenary talk to the July conference at Bari in Italy emphasised his commitment to the Kaluza Klein philosophy, "Kaluza Klein theories and Superstrings" (Duff,1985, preprint). He elaborated the Kaluza-Klein idea in its original notation, the combined equations for gravity and electromagnetism in five dimensions being "the Kaluza-Klein miracle at work" (Duff,1985,p.5). His summary of the Kaluza Klein philosophy was that "what we perceive to be internal symmetries in d=4 (electric charge, colour, charge conjugation, etc.) are really spacetime symmetries in d=10 (general covariance, parity etc.) (ibid.,p.9). Duff pointed out the striking similarities between the equations for the heterotic string and the Kaluza-Klein equation, explaining that it was no coincidence, in Section 8, "Kaluza-Klein lives!". Duff follows the traditional Kaluza-Klein philosophy, noting however that "it is ironic therefore, that the recent spectacular successes of superstrings seem to ignore this beautiful concept", (ibid,p.20).

Although Duff agreed in October 1985 that "until a few weeks ago", the majority verdict may still have to be against the details of Kaluza Klein (while still acknowledging the catalytic value of the philosophy), he could now affirm the "old" Kaluza-Klein theory. The basis for this affirmation was the recent paper (Duff, Nilsson and Pope, CERN preprint,1985). Here the authors established that

> "the gauge bosons of the heterotic string in d=10 have a traditional Kaluza-Klein origin in the <u>bosonic string</u> in d=506" (Duff,1985,p.21).

This came from a spontaneous compactification on the 496-dimensional group manifold G (where G = E_8 x E_8 or SO(32)). Duff postulated that though the critical dimension was 26, moving through a flat spacetime, 506 dimensions were needed if space time is allowed to be curved! Duff then used the "traditional Kaluza-Klein ansatz" and arrived at the "bizarre picture of a three-in-one world" that could be described equivalently in 10, 26 <u>or</u> 506 Dimensions. This involved 496 Kaluza-Klein elementary gauge fields. In his rather flag-flying manner, Duff encapsulated the renewal of his basic philosophy, "Kaluza-Klein is dead:Long Live Kaluza Klein!" (Duff,1985,p.23),- sentiments no doubt Green and Schwarz would agree with, but that theirs is now a more radical revision of Kaluza-Klein.

In another Summer School, of the Scottish Universities in 1985, a wide ranging review was undertaken, "Superstrings and Supergravity", Ed.A.T.Davies and D.G.Sutherland (published 1986). John Schwarz noted that both G.U.T.s and Supergravity theories had a number of problems (such as renormalisation of infinities) which were likely to be resolved if particles were allowed to be represented as one dimensional curves called strings of characteristic scale 10^{-33}cm (the Planck length). Supersymmetry and ten dimensional space time were extra ingredients described in his "Introduction to Supersymmetry" (Ed.Davies and Sutherland, 1986,p.96). P.van Nieuwenhuizen also noted the problems of Supergravity (d=11 cannot have a cosmological constant), and the 'Kaluza-Klein programme' was unable to help (ibid., p.274). John Schwarz had however pointed out that there were three possible supergravity theories in D=10, "each of which can be incorporated in a superstring theory (ibid.,p.120). (There was no consistent quantum theory of gravity based on point particles.)

However, in his second paper, Schwarz pointed out that not only does string theory allow gravity to be included, the "construction of a consistent quantum theory actually <u>requires</u> it" (ibid.,p.302). Schwarz also noted the Kaluza-Klein basic philosophy on superstrings, e.g. sixteen of the massless gauge fields arising from "isometries of the torus à la Kaluza-Klein". Following the 496-dimensional model, the other 480 "correspond to strings that wrap non-trivially on the torus" (Ed.Davies and Sutherland 1986,p.351).

It was Mike Duff who emphasised the "Kaluza Klein Recipe" and the "Consistency of the Kaluza-Klein Ansatz" in the first two papers (in fact available separately in Ed.H.Sato and I.Inami, 1986 - CERN Preprint,1985b, "Recent Results in Extra Dimensions".) He used the traditional Kaluza-Klein route in his analysis both of d=11 Supergravity and d =10 Superstrings. ("Old and New Testaments" respectively in Duff's colourful language.). Duff admitted that we do not know whether the round sphere S^7 compactification of d=11 supergravity (on which he and Chris Pope had worked) will ever have any physical relevance. He used it however as "a concrete example of how the Kaluza-Klein recipe can be carried through to the bitter end" (Duff, 1985b,p.43).

In all his work, Duff prefers to be guided by the mathematical consistency of the given Kaluza-Klein models, hoping it will lead to the correct physical theory. He himself, in lecture 3, "Consistency of the Kaluza-Klein Ansatz" (in Ed.Davies and Sutherland 1986,p.519) emphasised the Kaluza-Klein approach to the heterotic string, re-emphasising his use of 506 dimensions, as well as the d=10 + d=26 string, compactified on a torus. Duff in fact started his lecture with his belief in the high physical status of Kaluza-Klein dimensions:

> "let us begin by recalling that in modern approaches to Kaluza-Klein theories, the extra (k) dimensions are treated as physical and are not to be regarded as a mathematical device".

4. Complete Unified Theories from 1986 : the dominance of the Superstring theories, continuing to be catalysed by the work of Kaluza and Klein, with high status given to the extra dimensions

Continued work on ten dimensional supergravity theories is motivated mainly by the fact that they are closely related to supersymmetric string theories (e.g. P.S.Howe and A.Umerski, 1986,p.163) Any work on Grand Unified Theories has a similar motivation (e.g. J.Okada, "Symmetry breakings in the Kaluza-Klein theory",1986). The common theme referred to is the 'recent revival' of interest in the original work of Kaluza and Klein, and the growing paradigm that $E_8 \times E_8$ Heterotic superstring theories have become the leading candidates for a finite theory unifying all interactions.

The 'Princeton String Quartet' produced a second paper on the interacting "Heterotic String II" (Gross, Harvey, Martinec and Rohm,1986,p.75). The geometric nature of the interactions, the "full beauty of the heterotic string" becomes apparent. Supersymmetric closed string theories, type II theories, and the heterotic string are "the healthiest yet" (ibid.,p.109), as they claimed to have brought the heterotic string to the same state of development as the older, consistent superstring theories. The Kaluza-Klein mechanism is still invoked, with strings winding round a 16-dimensional torus (ibid.,p.75).

M.J.Duff, B.E.W.Nilsson and N.P.Warner realised that this ran counter to the traditional Kaluza Klein philosophy, but reaffirmed their own use of the conventional or traditional Kaluza-Klein origin of the gauge bosons of the heterotic string - in 506 dimensions. "Kaluza-Klein approach to the Heterotic String II" emphasised the "ultimate utility of our Kaluza-Klein approach to throw light on the correct compactification from 10 to 4" (Duff, Nilsson and Pope, 1986,pp.170,176).

There has been an enormous proliferation of papers presenting Superstring theories as the most promising candidates for "Theories of Everything". These included an analysis of the Heterotic String "removing the shadow world from the original model" (Bennett, Brene, Mizrachi and Nielsen "Confusing the heterotic string",1986,p.179).

The shadow matter was present in the Candelas et al. version of superstrings. Whether it was ever generated and also survived in the Big Bang Creation, other physicists have questioned whether it will have already decayed - and indeed whether it may conceivably be detected in any case. Michael Green gave a fascinating summary of Superstrings in 1986, when he reviewed the history of the theory. He emphasised that for energies below the Planck energy, "the massless particles of superstring theories are the same ones found in supergravity theories" (Green 1986,p.52). Superstring theory was originally in flat 10 dimensional superspace. However to make sense of physical observations six must be highly curved to form a Calabi-Yau space. This may also be as a generalisation of such a space called an orbifold, which is simpler to handle and which leads to promising results for the physics of the four observable dimensions. Orbifolds were introduced by Dixon, Harvey, Vafa and Witten (1985). Michael Green hoped to extend the idea of ordinary spacetime to the space of all possible configurations of a string. An even more radical suggestion was that the theory should be studied in its two dimensional formulation. "No reference at all would then be made to the coordinates of space and time in which we live" (Green,1986,p.56).

These ideas were finally brought together in the prescriptive two volume book, Superstring Theory by Michael Green, John Schwarz and Edward Witten (Cambridge University Press, 1987). The most promising superstring theory is given as the heterotic string of Gross, Harvey, Martinec and Rohm. The charges on the Yang-Mills forces are included in the construction by smearing them out over the whole of the heterotic string. Waves can of course travel around any closed string in two directions. However on the heterotic closed string, the waves travelling to the right, or clockwise, are waves of the 10-dimensional fermionic superstring theory, and the waves travelling to the left, or counter-clockwise, are waves of the original bosonic (or Veneziano) 26-dimensional string theory. The extra 16 dimensions are then interpreted as internal dimensions responsible for the symmetries of the Yang-Mills forces. The toroidal compactification of superstring theories (Green, Schwarz and Brink,1982) was in fact anticipated in principle in Cremmer and Scherk's 1976 paper. Compactification on 16-Dimensional tori led to E_8 x E_8 or SO(32) symmetry groups.

In their book, the authors acknowledge the historical debt to the invention of the Kaluza-Klein theory (Green, Schwarz and Witten,1987, pp.399,444, 537 etc.) and give many references to the Kaluza-Klein idea and its application in string theory at the end of chapters 1 and 14. They have shown how most unsolved problems of elementary particles can be solved in terms of compactification of ten-dimensional string theory. However in a final section, they note the lack of understanding of why the cosmological constant vanishes after supersymmetry breaking. This may well decide the future development of string theory. In fact the authors acknowledge that the roots, the basic principles, are still mysteries and "may lie in directions not yet contemplated"(Green, Schwarz and Witten, 1987, p.552).

Appendix to Chapter 8

6- and 8-Dimensional Spinor and Twistor Space of Roger Penrose - linked with Kaluza Klein by Witten, 1986.

This is an alternative model in more than four dimensions, independent of strings or supergravity, but eventually linked with Kaluza-Klein ideas by E.Witten (1986).

A highly original alternative way of looking at space and particles was develped by Roger Penrose, quite independently of the 5-Dimensional Kaluza-Klein concept. Penrose started by looking at paradoxes, e.g. that matter is largely composed of empty space, or that an electron is a point particle of no dimensionality. Standard quantum theory however describes empty space on a small scale as seething with activity. Geometrodynamics indicated a constantly changing foam space, and quantum electrodynamics, although mathematically precise, is plagued with infinities. Localisation of particles in space is limited by the Heisenberg Uncertainty Principle.

Penrose was looking for a way out. "Apparently we must relinquish geometrical pictures and rely instead on equations, if we are to retain a reliable description of reality", wrote Penrose in "Twisting round spacetime" (Penrose, 1977,p.734). Penrose's insight found the fault not in geometry itself, but in the specific spacetime geometry to which we have become accustomed on the macroscale. Without necessarily abandoning four dimensional spacetime, Penrose looked for a new geometry which would subsume the old. Some geometrical reformulation seemed to be necessary which would incorporate both quantum mechanics and flat Minkowski geometry of special relativity, and also accommodate the current geometry of Einstein's General Relativity. Penrose started by facing the paradoxes of wave/particle at sub-atomic level, and of the essential rôle played by complex numbers e.g. particles as rays in a complex vector space.

Penrose developed an abstract 6-dimensional space whose points represented spinning photons. It turned out, quite remarkably, that this space could indeed be regarded as a complex 3-dimensional space, a projective twistor space. It was a higher dimensional version of the Riemannian space. Penrose gave a very physical description of twistor space, and in fact gives a high status to his view of space:

> "In my own twistor approach, one is required to consider geometrical spaces of real dimension six or eight, and one takes the view that the twistor space is 'more real' than the normal spacetime. But to a large extent this is merely a mathematical transcription. It is, however, possible", he admitted "that I take a stronger view with regard to the relation between mathematics and 'reality' than do most people" (Penrose,1980,private correspondence with Middleton).

This produced a more basic alternative way of viewing the geometry of spacetime at a fundamental level, emphasising the twistor description as more relevant than a four-dimensional space time (Penrose,1977,p.737). Certainly

> "our present approach to spacetime geometry is really inadequate for handling all circumstances in physics" (Penrose,1984,p.8).

For Penrose the spacetime point was completely taken over by a different object - six dimensional space (Penrose and Rindler,1985, from 1961). A line in twistor space corresponds to a single point in spacetime, giving a complex deeper reality to spacetime:

> "what is defined as a 'point' in one space may just be some more elaborate structure in another" (Penrose,1978,p.87).

He writes:

> "it would not be correct to think of spacetime as a 'part' of the larger eight-real-dimensional twistor space. The points of twistor space have a quite different interpretation from those of space time. Each point of twistor space represents, in effect, the entire history of a freely moving massless spinning particle". (Penrose,1980b, Private correspondence with Middleton).

Although Twistor theory developed quite independently of Kaluza-Klein ideas, the connection with superstrings was made in 1986 by Edward Witten. His motivation was that "the possibility that the twistor transform of ten dimensional supersymmetric field theory is the proper starting point for understanding the geometrical meaning of superstring theory" (Witten,1986,p.245). He referred to the twistor transformation of the self-dual Einstein and Yang Mills equations as one of the most striking developments in mathematical physics in recent years (Penrose,1976; Atiyah and Ward, 1977). This developed via the concept of 'supertwistors' to a twistorial formulation of the field theories which is the right starting point for generalisation to superstrings. Witten noted that either

> "twistor space N must be replaced by an infinite dimensional space, perhaps the space of orbits of a classical string" or preferably that
>
> "one must consider infinite dimensional structures over a finite dimensional twistor space N" (Witten,1986,p.263). A suitable prophesy for the late 1980's.

Whatever the exact formulation, Penrose's search was for a much more unified approach in physics, and the need to find

> "a new mathematical language for describing the universe" (Penrose, 1984, p.8).

Certainly,

"the fact that the singularities in spacetime tell us that our present approach to space-time geometry is really inadequate for handling all circumstances in physics", is now established. This is especially "where physical theory breaks down, such as in singularities, and in black holes"..... "what seems like reality all around us is deceptive; the deeper reality is the underlying abstract mathematics" (ibid.,p.9).

Chapter 9 Summary and Conclusion: The evolution of Kaluza's original theory and its final entry as a central inspiration for supergravity and superstrings.

I. Summary

1. The use of higher dimensions

Just as the first great revolution of the twentieth century, General Relativity, was found to contain within itself enigmas and paradoxes when space is highly curved, so we have seen that the second revolution, Quantum Mechanics, is also surrounded with paradoxes in its interpretation. Both areas have suggested the need for a new physics, perhaps going more deeply behind the apparent four dimensions of spacetime; indeed a new metaphysics is a clear implication.

There are a number of independent uses of a concept of extra dimensions beyond the traditional four. As a purely mathematical idea in the nineteenth century, Cayley and Grassmann developed the concept of multidimensions, while Lobachewsky and Bolyai, following Gauss, published their work on non-Euclidean geometry. For Einstein's theory of Gravitation, he needed the synthesis of non-Euclidean multidimensional space provided by Riemann. A language had become available. By the mid-nineteenth century, absolute space had been found to be unnecessary by Mach, useless in practice by Clerk Maxwell, and devoid of meaning by Poincaré. With Einstein, physics had become identified with geometry.

In Chapter 2, we noted the use of embedding dimensions, useful both to describe the 'curvature' of spacetime in mathematical language, and also to aid visualisation by an analogue model. This is a mathematical concept, without being necessarily a description of a deeper reality. The four curved spacetime dimensions of General Relativity need at least six, and maximum ten embedding dimensions (Kasner, 1921).

In the following chapter, we described how Theodor Kaluza in 1921 used one extra dimension to unify the two known forces at the time, electromagnetism and electricity. Kaluza's idea was that the (gauge) vector fields (electromagnetism only, in his case) could be obtained from the components of the five dimensional metric. Kaluza himself regarded this extra dimension, extending the number of spacetime dimensions, as being physically present to describe reality. Gummar Nordström, a little known Finnish physicist, had in fact anticipated the idea but lacked Einstein's tensor fields. In chapter 4, we have seen how, in 1926, Oskar Klein attempted to strengthen the physical status of the extra dimensions. Inspired by de Broglie and Schrödinger, Klein tried to incorporate quantum theory as well. Whereas for Kaluza the fifth dimension was made independent of the other four using the "cylinder condition", Klein attempted to establish that its size was very tiny or zero due to the cancelling out of the oscillations of the waves in the fifth dimension.

Both Kaluza and Klein had therefore to treat the fifth dimension in a different way from the other four, and explained that the extremely minute size of the extra dimension accounted for its apparently not being observed. The criticism that the fifth dimension was so tiny as to be beyond the range of direct experimental proof was more of a deterrent then than it appears to be today. Klein explained that the fifth dimension had been compactified to a tiny circle, and linked its periodic nature with Quantum Mechanics. Although the five-dimensional Kaluza-Klein theory was only a simple model, it has incorporated properties which survived in later more realistic models. Quantised units of fundamental electric charge for elementary particles have remained. The gravitational and the electric charge for elementary particles have remained. The gravitational and the electric charge are seen to be related to one another by the size of the extra compact dimensions - which itself made the radius of these extra dimensions very small, of Planck size (10^{-33}cm), and therefore not apparent in our everyday physics.

A further important use of extra dimensions was that developed by Erwin Schrödinger, and used as the basis of Quantum Mechanics. As we have explored Quantum theory in Chapter 4, we found that it requires the use of an abstract multidimensional configuration space. The description of the wave function ψ requires the mathematical concept of a complex 3N-dimensional space as Schrödinger defined it (1926) with N being the number of particles in the system. However the paradoxes inherent in the description of reality have never been resolved. Quantum reality seems to involve a large subjective element in that what exists cannot be separated from the way we choose to observe the world. The conscious mind is involved, which is assumed to be in some sense non-physical (unless the alternative Many Worlds theory is adopted). The problems of the widely accepted quantum field theory involve infinities, and the need to include gravity as well. Quantum Mechanics had failed to achieve any reconciliation with the conventional physical intuition of Chapters 1 or 2. It had therefore failed to remove the classical ideal of physics which from 1926 it officially replaced.

2. The Way Forward : the Kaluza-Klein theory

In fact a genuine multidimensional world view seems to be necessary to answer the many problems of both General Relativity and Quantum Mechanics from the first quarter of the twentieth century. Klein's rejuvenation of Kaluza's five dimensional model, widely used today as the basis of various candidates to describe a multidimensional reality - a "theory of everything", was ahead of its time in many respects. The appropriate concepts such as gauge theory and supersymmetry, etc., were not then available. Like Kaluza, Klein was still unifying only two of the four forces of nature (the strong and weak nuclear forces were not then recognised). A quantum theory of gravity is still not accepted per se. A further factor against Kaluza and Klein's theory was that their contemporary supporters such as de Broglie and Einstein did not give consistent approval.

In the 1920.s, physicists were not ready to go beyond a reality of four spacetime dimensions, despite the problems and paradoxes of Quantum Mechanics. Apart from unsuccessful independent attempts by Eddington, only Einstein himself was willing to make further radical attempts at the Kaluza-Klein unification, following his initial half-hearted support. In acknowledging the inadequacy of current physics, Einstein later went so far as to declare that "the true theorist is a kind of tamed metaphysicist " (Einstein, 1950,p.13). With Peter Bergmann in 1938, he attempted to give a much more physical interpretation to the fifth dimension, with all field variables periodic in this extra dimension (see Chapter 5). This was also tied to two forces, and lacked the mathematical concepts to explain the physical properties of the known particles, despite the comparatively modern approach expressed.

Earliermodified versions such as projective theories, e.g. of Veblen and Pauli were shown to be basically equivalent to Kaluza's version, and were not a useful way forward. Another version, the Scalar-Tensor theories, emphasised the extra scalar field which Kaluza had in fact referred to originally. Bergmann, one of those to give increased prominence to this, thought that the physical interpretation of the scalar as a variable gravitational constant was wrong, missing one of the more recent suggestions.

We have traced the way Kaluza's use of the extra dimension was used during the forty wilderness years before it entered mainstream physics in the late sixties and seventies. A constant theme for Einstein, others including Klein himself also kept the five dimensional idea alive during its "classical" period (reference Chapter 6). The more fundamental reasons for the forty to fifty year neglect of Kaluza's idea lay in the need for more mathematical tools and physical concepts. At first, from Einstein to the 1970.s, the mathematics used was already available from nineteenth and early twentieth century work. More recently, however, the mathematicians and physicists have had to work almost in parallel, with discoveries in one area sparking off creative ideas in the other. Little was really possible before the idea of quarks was proposed by Murray Gell-Mann and George Zweig in 1964, and of gauge fields and particles by Yang and Mills in 1954. Both concepts were in fact seen as abstract mathematical ideas well before their real applications were known, taking ten years or so to be incorporated into ideas of physical reality.

3. When the time was ripe : Re-entry of the Kaluza-Klein idea as tools became available

It was thus many years after Kaluza and Klein that physicists obtained the correct mathematical and physical ideas for unification of forces and particles, to include both gravity and quantum mechanics. The Kaluza-Klein idea then became a central catalyst as the idea of extra compacted dimensions was remembered and revived.

The increase in status had been hinted at by Souriau (in 1958 and in 1963), who anticipated recent ideas by his work with the four force fields and by hints that the fifth dimension might once have been larger. This historical reality (and even future importance) was only taken seriously in 1980, by Chodos and Detweiler, with the application for cosmology. Physical spacetime dimensions were defined as "large", and the alternative dimensions of the Universe were suggested as four <u>or</u> seven, the others being compacted at the present epoch (Freund and Rubin, 1980). This was the first logical explanation for physics being in four dimensions!

The non-Abelian gauge field extension of the Kaluza-Klein theory was first noted in a purely mathematical idea by B.S.De Witt in 1964. The link with the language of fibre bundles was also made in the sixties, by A.Trautman, who pointed out De Witt's idea, and R.Kerner (1968). However real progress could only await the development of ideas of supersymmetry and of strings in the early 1970.s. Peter Freund and his student Y.M.Cho constructed the full gauge theory of De Witt, using supersymmetry and scalars in 1975. Even so,compactification of dimensions by the curling up to unobservable size was an idea prevalent in this period without any apparent connection with the vital Kaluza-Klein concept. Only in 1975 did Joel Scherk and John Schwarz make the connection between Kaluza-Klein and string theory, reinterpreted as a candidate for a unified theory of gravity and the other fundamental forces. Particles were described as strings, approximately equal to the Planck length (10^{-33}cm) and their paper was quite explicit about the physical reality of compactified dimensions (Schwarz,1988). In addition to incorporating gravity in a unified theory, the problem of the meaningless infinities seemed to be removed. Yet, "for a decade, almost none of the experts took the proposal seriously"

(Schwarz, 1987a,p.15).

The other important concept for unification of forces was supergravity. This also grew up independently of Kaluza and Klein, the link only becoming clear in 1979 in the paper by Cremmer and Julia. It was now possible to increase considerably the status of the extra dimensions needed in physics by the physical concept of "spontaneous compactification", introduced by Cremmer and Scherk (1976 and 1977), rather than the purely mathematical tool of dimensional reduction. The importance of Kaluza-Klein ideas applied to cosmology further strengthened the status via supergravity, first in 11- and then 10-dimensional forms. Although once co-equal, these extra dimensions therefore "need not conflict with one's everyday sensation of inhabiting a four dimensional world (with its inverse square law of gravitational attraction)" (M.J.Duff in Ed.H.C.Lee,1984, p.28). - provided that the radius is tiny.

There has been an increased emphasis on an experimentally-orientated approach since 1982, and a second shift in emphasis "towards (super)-Kaluza-Klein theories. Far from being a peripheral interest, these theories have now come to occupy the centre of the stage among supergravity models" (Abdus Salam, Ed.De Wit et al.,1984,p.1). The shift had been discernible since Cremmer, Julia and Scherk's work on dimensional reduction from eleven spacetime dimensions of supergravity (1979), the "extended super Kaluza-Klein miracle" (ibid.,p.2).

The more recently accepted way of describing reality has been through Superstrings, developed further by Michael Green and John Schwarz. As Schwarz reminds us, "Superstring theories are promising candidates for a supersymmetric unification of fundamental interactions including gravitation. Point-particle theories, such as N=8 supergravity, can be viewed as low-energy effective descriptions of a superstring theory" (Schwarz, ed.De Wit,1984,p.426). Physicists only became convinced of the virtue of string theory after Schwarz and Green showed how certain apparent inconsistencies, called anomalies, could be avoided - the "September Revolution" of 1984. This was followed by the now widely accepted description of the Heterotic string initiated by Gross, Harvey, Martinec and Rohm: 1985. The unification of all forces and particles initiated by Green and Schwarz in 10-dimensional Superstring theory, combined the special relativity and quantum mechanics of the older string theory *with* the General Relativity of Einstein's gravity in supergravity theory.

In the 1980's the Kaluza-Klein approach has been absorbed into supergravity and then into superstrings. Superstrings is the most promising candidate to describe reality, with supergravity as a special case. It is finite and renormalisable, and unifies all four forces in a way which contains quantum gravity. Kaluza's original theory is now an essential part of the current multidimensional view of reality. We still appear to live in 3-space, because the symmetries of the internal space appear as gauge symmetries of the effective 4-dimensional spacetime. The extra dimensions are perceived as electromagnetic, weak and strong charges (Shafi and Wetterich, 1983; Barrow,1983). What we perceive to be internal symmetries in 4-dimensions, such as electric charge, colour, charge conjugates etc.,are really spacetime symmetries in higher dimensional space (Duff,1985). In Kaluza-Klein models, gauge fields arise from extra components of the metric ($g_{\mu\nu}$). In some string models the gauge fields are put in "by hand" and no use is made of the Kaluza mechanism. (This is because, for example, in Calabi-Yau compactification, the compact manifold has no symmetries.) The latest (1987) type string models were however going back to the Kaluza mechanism.

Evidence of extra dimensions is thus seen as the manifestations in forces of nature and fundamental charges. Direct evidence through cosmological applications could be obtained by the time variation in any of the fundamental constants (Marciano,1984), although normally beyond the reach of experimentation. Criteria for unified field theories in extra dimensions are often aesthetic rather than directly testable. Concepts of beauty, simplicity and elegance have been used by Einstein himself. Although the absence of directly testable inferences is still a weakness, elegance today is often linked to the amount of symmetry, and "elegance, so defined, is closely correlated with physical relevance" (Schwarz, 1975,p.62). "Superstrings are so captivating and so elegant" (Michael Green,1988) that the theory depends on its " intrinsic worth" (Salam,1988).

By 1984, the papers in the literature mentioning Kaluza's original work had escalated enormously. Two or three references per year in the sixties and early seventies, led to about fifteen per year in the later 1970.s. This rose to over forty papers in 1982, about seventy in 1983 and to over a hundred papers referring to Kaluza and Klein in 1984 (Science Abstracts). The references have almost exponentially escalated since then, until there are even articles ceasing to need the reference to Kaluza, as General Relativity does not always need to carry Einstein's name. Kaluza is now referred to in popular science books, radio and television programmes, although here superstring theories have only recently been discussed as the most promising candidates for "Theories of Everything" (Davies, et al., 14 February 1988, "Desperately seeking Superstrings").

In modern aporoaches, therefore, the extra Kaluza-Klein dimensions are treated as physical, not just as a mathematical device. Grand Unified Theories without gravity are now seen as a sidetrack, and Superstrings are becoming the dominant theory. Superstrings

> "are not just consistent theories of quantum gravity, but consistent unified theories of all interactions", -

building on Kaluza-Klein,

> "one of the earliest and best ideas for unification".
>
> (Green, Schwarz and Witten, 1987, Vol.1, p.16).

Superstring theories "seem to be entirely free of the inconsistencies that plague quantum theories of gravity" (ibid.,p.55). Green also noted the "Kaluza-Klein revival" which motivated the studies of anomalies in higher dimensions in the 1980.s (Green, 1986,p.27). There are hints that the Kaluza-Klein philosophy provided the fundamental thrust and catalyst for the tremendous success of recent unified theories, with the method being used either in a direct manner or even as a reversal of the original approach. The Kaluza-Klein framework is still used directly for the heterotic string (Candelas, Horowitz, Strominger and Witten, 1985). However for Green, the string theory is very much deeper than that : "the whole notion of space with a finite number of dimensions, e.g. 10, is only an approximation to some much bigger structure - 'stringy space'" (Green, personal cummunication, September 22, 1987) - perhaps in infinite dimensions.

Certainly the 6 dimensions of the 10 used in heterotic strings can be curled up in certain ways, and one can discuss what is happening in the language of Kaluza. However, if one starts with the forces in ten dimensions, "the Kaluza-Klein language is used, but with the opposite meaning" (ibid.,1987). The 1984 approach of Schwarz and Green was thus using the Kaluza-Klein philosophy and getting very much richer effects than in conventional theories. For them, the conventional work on supergravity was almost trivial. They envisage a string winding round a torus (hypertorus or orbifold) giving new quantum numbers or properties. (An orbifold is flat everywhere - like a torus - except at isolated points where the curvature is infinite - i.e. with singular curvature). In these recent theories, the number of dimensions in which the string oscillates is different for the left hand or right hand direction round the torus, as if in one direction were superstrings in 10 dimensions, in the other were bosonic strings in 26 dimensions.

Green himself admits that this is very difficult to think of in a conceptual or visual way. Although it is only in four dimensions that they come together, "what you mean by dimensional spacetime is utterly obscure". "It is a generalisation from Kaluza Klein which is so different that you can't even really think about it - an 'intrinsically stringy' concept" (Green, ibid.,1987) - which may even involve 496 dimensions in addition to the 10 for spacetime as an alternative description.
Note: It is interesting to remember that the strong, short range interactions decrease in strength faster than the inverse square law, indicating that the central argument, using this law to prove that space is three dimensional, is faulty on the small scale. Furthermore there is some recent evidence that Newton's inverse square law is not correct over ranges of a few hundred metres, due to the so-called "fifth force" in addition to the usual four (e.g. the "Yukawa" term of Fujii; Stacey; Fishbach (New Scientist, 16 January 1986, p.16; 7 January 1988,p.39 etc.), and the possible involvement of anti-particles in the challenge to orthodoxy (Goldman et al.,Scientific American, 1988, pp.32-40).

Spacetime cannot even be fixed if a string is a quantum object with its Uncertainty Principle. While generally regarded as real physical objects, 'perturbation approximations' of string theories have to be used, leading to an infinite dimensional 'essentially stringy' concept.

It is interesting that the recent description of Black holes, using higher dimensional spacetime, was also firmly linked to classical

Kaluza-Klein theory by Leszek Sokolowski and Bernard Carr. Such objects "might be expected to arise rather naturally in any Kaluza-Klein type theory" (Sokolowski and Carr, 1986, p.334). Their general solution in fact corresponded either to a naked singularity or to a wormhole with no singularity. Black hole solutions are discussed in five dimensions and in higher dimensions where the internal space is curved. The assumption is that Black holes really do exist in macroscopic four dimensions "as is strongly suggested by the astrophysical evidence (ibid.,p.340).

Other cosmological references extend the unified field theory by regarding hadrons as "black-hole type" solutions of their field equations (Recami and Zanchin, 1986,p.304). Other exotic extrapolations involve the ideas of Cosmic Strings, infinite length general relativistic strings produced in a phase transition of the early universe (Kibble, 1976, and Zel'dovich,1980). These one-dimensional strings could be the seeds for galaxy formation (Vilenkin, 1987, p.52). No connection has yet been made however, with superstrings and Kaluza-Klein ideas. Nevertheless in Kaluza-Klein cosmology, superstrings are involved as the best candidate for a finite theory of quantum gravity (Weiss,1986,p.183). Kaluza-Klein models have also been used in the Hartle-Hawking 1983 concept involving the quantum state of the universe being described by a universal wave function (e.g. Halliwell,1986, p.230).

There seems to be a widespread commitment to the application of the Kaluza-Klein model to a wide variety of aspects of both particle physics and cosmology in the late 1980.s. Certainly quantum cosmology is essentialy 'stringy', and superstring theories predominate in particle physics as a "generalisation of general relativity". In this context it is widely taken as sensible to consider the possibility, indeed the reality, of extra dimensions of space, curled up into a sufficiently small space so that "the observed three dimensionality of the physical world is maintained" - on the Kaluza-Klein model (John Schwarz in "Superstrings", 1987b,p.36). Schwarz quotes Edward Witten's comment that general relativity gave rise to various predictions which "seemed quite hopeless to verify when they were made" e.g. neutron stars, black holes, gravitational radiation and lenses - and yet there is "substantial observational evidence now for all of them" (ibid.,p.38). Schwarz' hope is that various predictions from string theory should enable this also to be tested by observational evidence. In a paper which regarded charged elementary particles as higher dimensional tachyonic modes, or as mini-Black holes, Aharon Davidson and David Owen are typically committed to the Kaluza-Klein theory. Their underlying principle takes the model very seriously : "Following the Kaluza-Klein idea, the four-dimensional physical trajectories are in fact projections of higher-dimensional world lines (Davidson and Owen, 1986, p.77 - my emphasised words) - an idea taking us back to Kaluza himself.

It is interesting however to observe, as William Marciano writes, that "the community seems to be split" on the physical reality of superstring models in 10 or 26 dimensions (Marciano - personal communication, 30 December 1987). Many physicists view the extra dimensions as added degrees of freedom in our 4-dimensional world. "I like to think of them as a physical reality, since I take more of a physics rather than a mathematical perspective" (Marciano, ibid.) As Michael Duff readily affirms "I still believe in the reality of extra dimensions" (personal communication, 27 January 1988).

The lack of testable predictions remained a problem for Richard Feynman in a broadcast a few days before his death. He remained sceptical to the end about superstring ideas because they cannot be checked against experiment: "These ideas are nonsense" (Feynman in Davies, et al.,1988). Steven Weinberg admitted that the theory might be right, although he thought not, since there may be other ways to get rid of infinities. In the same broadcast, Sheldon Glashow was firmly against the theory, despite its apparent uniqueness at the time. However the other professors in the programme emphasised the beauty of the ideas - David Gross, Paul Davies, John Schwarz, Edward Witten and Michael Green - although Green cautioned that the theory still lacks a deeper level. Superstrings appear to have been invented almost by accident, explained Witten, "part of the physics of the twenty-first century which fell by chance into the twentieth century" and gave a tremendous opportunity. Later physicists would look back and say - "one of the great times to do physics" (Witten, ibid.,1988).

Michael Duff, although enthusing over superstrings, has pointed out some of the problems of superstrings, having himself come via the supersymmetry route. Although 10-dimensional superstrings are the natural extensions of the supergravity theories and Kaluza-Klein unification, he reminds us that there is "as yet no shred of experimental confirmation of superstrings" (Duff, Preprint,1987,p.1). There is as yet no proof of finiteness, and there are so many string models consistent with four dimensions, all N=1 supersymmetrical, chiral and anomaly-free, etc., that there is no longer a unique theory.

Duff also showed that there is now one other theory which can provide a consistent (finite) quantum theory of gravity: "membranes". There "now exists a supermembrane in eleven dimensions which yields a superstring in ten dimensions upon dimensional reduction" (Duff, CERN preprint 4797, 1987). This other super-extended object (see also E.Bergshoeff, et al., 1987, p.75) besides the superstring exists as a "supermembrane", requiring eleven dimensions. It "moves like a soap bubble through 11-dimensional space time" in a way determined by the equations of the old eleven-dimensional supergravity with seven curled up (<u>Newsletter</u>, Physics Department of Imperial College, January 1988, p.7). "Whether the 'Theory of Everything' will turn out to be the 10-Dimensional superstring or the recently discovered supermembrane (or neither), I cannot tell" (Duff, personal communication, 27 January 1988).

Note: This is not connected with the cosmic "membrane paradigm" - a three dimensional language to translate the general relativistic mathematics of black holes, where "curved spacetime is fundamentally incompatible with the mental images on which astrophysicists base their insight" (Price and Thorne, "The Membrane Paradigm for Black Holes", 1988,p.47).

CONCLUSION

As John Wheeler described it, the inevitability of gravitational collapse, not only at the scale of the universe, but even the collapse of a star to "a so-called black hole", is "a crisis in theoretical physics today" (Wheeler, Foreward to Graves, 1971).

Both in the singularities of General Relativity and the crises of non-locality, observer-centred reality, a wave-function of the universe, etc. in Quantum Mechanics, the standard laws seem to break down. A new physics was needed by the nineteen seventies. Yet these paradoxes have been with us for a number of years. They are easily ignored and are readily accepted as 'normal'. But for creative scientists such as Wheeler, "a larger unity must exist that includes both the quantum principle and general relativity" (ibid., Foreward).

In the nineteen eighties, several different models involving a larger unity have emerged. The construction of an ontology is now possible using a multi-dimensional description of reality with a number of appropriate models, constantly being refined or redistilled to a coherent metaphysics. The *qualitative* models of pregeometry, many worlds, foam space, superspace and spacetime foam, curved spacetime and singularities in Black holes and the Big bang are all implicitly beyond 3-space dimensions. *Quantitative* models with explicit numbers of higher dimensions have proliferated, starting from Kaluza's compacted dimensions and Kasner's embedding dimensions, via gauge theory and fibre bundles in the 1960.s, through superspace, supersymmetry and strings to twistor space, supergravity and superstrings.

A multidimensional model of a deeper reality

The signs of the paradigm wave beginning could be seen in the mathematical discoveries of the nineteenth century - multidimensional, even infinite dimensional, non-Euclidean geometry. The wave began to gather shape in the need to use such ideas in physics rather than merely in theoretical mathematics. Einstein needed such ideas in his curved four dimensional spacetime of General Relativity, with higher dimensions implicit for the conceptualisation of "curvature" and necessary for the mathematical treatment using at least six, maximum ten dimensions. Schrödinger needed a space of many dimensions for his Quantum Mechanics wave model. The paradigm of extra compacted dimensions as a part of reality has been quietly building up, initiated by Kaluza's unification using five dimensions.

The large scale curvature of spacetime in General Relativity and the small scale curvature of Kaluza-Klein extra compacted dimensions has led to revised concepts of spacetime. A critical revision of the four dimensional spacetime of accepted orthodoxy is necessary. An ontology of multi-, even infinite dimensions, has converged to a coherent metaphysic in the late 1980.s.

However even the 10 or 11 dimensions of supergravity, superstrings and now supermembranes is only one level of reality. 26 and 506 dimensional models seem to be pointers to an infinite dimensional reality, of which our 4-dimensional spacetime is a low energy apparent approximation. Solutions involving many dimensions are needed for a unification which involves special relativity and quantum theory (via strings), and also combines the gravity of General Relativity in Superstrings. A multi-dimensional model will thus remove the anomalies in Quantum electrodynamics. It has the potential for further application in other areas of physics, the physics of the very small and of very high energy. A range of models is available which describes the transcendent solution of a multidimensional reality, whether explicity of many dimensions, or the implicit transcendence of holism, many worlds, pregeometry, space bridges or superspace.

Taking our models seriously

As Steven Weinberg remarked in his preface to The First Three Minutes:

> "We must learn to take our models seriously" (Weinberg,1976).

In warning that philosophers were often

> "out of their jurisdiction in speculating about these phenomena",

Weinberg also noted that this would have

> "profound implications outside of science... we have all been making abstract mathematical models of the universe to which at least the physicists give a higher degree of reality than they accord to the ordinary world of sensation" - what he calls "the Galilean style" (Weinberg, 1976, p.28).
>
> "The scientist today usually takes his models seriously but not literally".

This is part of a critical realism concerning the models that are used today (Barbour, 1974, p.38). This poses the challenge of daring to take the range of models, the paradigm of multidimensions, as saying something important about a wider concept of reality. This is to leave behind reductionist, positivist philosophy in order to approach the reality of many dimensions, certainly beyond 3-space, perhaps even a 'transcendent' reality.

Realist and idealist metaphysics both intend to give a comprehensive account of reality. In the first, the reality of the world of 3-space and 1-time is recognised. In the second, following Plato's original thrust, the spatio-temporal world is the appearance of a timeless reality, a transcendent reality. Realist metaphysics has a much closer connection with nineteenth century natural sciences. Mind, or the act of knowing, was taken to be "one factor in reality among others", and immanence was emphasised over transcendence (see for example, John

McQuarrie, 1981,p.258). Idealist metaphysics is much closer to contemporary physics, where mind is interwoven with reality (as in the standard interpretation of Quantum mechanics). Certainly there is a transcendent reality indicated in many of the models used in twentieth century physics, rather than the reductionist insistence on 3-space as the only reality of the positivists.

A new perspective on reality

We see today a new consistent metaphysics of multidimensions in theoretical physics. Its investigation is through second order effects, manifest in forces and fields in the low energy terrestrial physics of today, and more directly only in the very high energy e.g. of the Big bang and in Black holes. New criteria are therefore involved - of aesthetics, symmetry, beauty, elegance, simplicity,etc.,... rather than the direct physical verifiability of the older physics. There may even be an infinity of physical dimensions. As de Broglie saw over fifty years ago, much of the totality of the universe may even be inaccessible to scientific analysis as a description - "such a moving and infinitely complex Reality" (De Broglie, 1937, p.275).

Such a transcendent reality can be described in terms of "levels of reality" although physicists need an apparently more mathematical language of 10, 11, 26, 506 and even an infinite number of dimensions. These need to be held in parallel with a series of analogue models, the simplest being the concepts of 'embedding', 'fibre bundles' and 'compacted dimensions'. The use of numbers is itself only a model which only highlights the multidimensional description of a deeper reality beyond our imagination, certainly beyond ready conceptualisation, <u>except</u> when coupled with a strong analogue model.

The extended analogies of Plato's "Cave" in his <u>Republic</u> and of

Abbott's Flatland are able to provide the only visualisable concepts of the process, the way two dimensions is conceptualised from the viewpoint of three. This process can lead to the implications, the parallel idea of how a multidimensional reality may be represented in a three dimensional shadow or projection. Communicating such ideas is not really difficult, but unless one is bilingual with mathematics, it is not easy to accept the notion of many dimensions as an idea which is meaningful or even conceptualisable. The mental effort required to transform the relativity of two dimensions with relation to three, into the relativity of three to higher dimensions may be one of the chief reasons for the paradigm wave not overturning. The new revolution may be parallel in importance to the Copernican revolution, and is as little recognised outside theoretical physics. The decentralisation of three space dimensions as being only part of the spectrum of a reality of many dimensions is at least as significant as the paradigm changes wrought by Copernicus and Darwin.

Perhaps by the twenty-first century we shall be clearly ready to accept what Steven Weinberg already suspects:

> "The four dimensional nature of spacetime is another one of the illusory concepts that have their origin in the nature of human evolution, but that must be relinquished as our knowledge increases" (Weinberg, 1979, "Einstein and Spacetime : Then and Now", p.46).

The real question behind this thesis has been "what is reality?". Is there a deeper, even transcendent, reality than 3-space and 1-time? The initial impact of Kaluza had been dismissed by the Copenhagen orthodox philosophy which rejected any question about "being". It was a self-imposed limitation of scientific method. We cannot eliminate metaphysics, which is not knowledge itself but "the scaffolding, without which further construction is impossible", wrote the originator of the multidimensional wave equation, Schrödinger. He added that "metaphysics turns into physics in the course of its development". For Schrödinger this implied the unquestioning acceptance of a "more than physical - that is, transcendental-significance". Metaphysics is "something that transcends what is directly accessible to experience" (Schrödinger, 1925, "Seek for the Road", in My View of the World, 1964).

The real question of ontology has produced a deeper reality than 3-space and 1-time. Models of a transcendent reality are found directly or by implication throughout theoretical physics, and indeed are urgently needed in philosophy and theology. William James' conclusion from his scholarly analysis was that there was an unseen order, that our visible universe is only part of a wider reality (James, 1901). The "wholly other" cannot be ignored (Otto, 1917)

It is easy to ignore the transcendence in one's everyday use of practical mathematics. There is a transcendence in the elements of mystery, of enigmas and paradox, in existing physics, of a deeper reality which reemphasises the urgent need for models, for metaphysics and for multidimensions. The reality of many dimensions is uncomfortable, and doubts therefore still flourish, preventing the paradigm wave from completely breaking and leaving behind the four dimensional spacetime of pre-Kaluza physics. The delay in publication of his theory, in his own promotion, and in the general acceptance of five (or more) dimensions, encapsulates the dilemma of the implicit transcendent reality, despite the now widespread use of the Kaluza-Klein model.

As A.Polyakov wrote so prophetically about his own superstring model,

> "We can say that, in some sense, strings lead not only to unification of interactions but to the

unification of ideas" (Polyakov, 1968,p.406).

Our models suggest that we, and the physical three dimensional universe of our perception, may be but a part, a projection, even a cross-section of a deeper infinite multidimensional reality. It may well be a most useful language, a vocabulary to talk about the transcendent. Yet even Darwin warned that "analogy would lead me one step further", but should be taken with care. He left us at the end of his Origin of Species only with the hint: "Light will be thrown on the origin of man and his history" (Darwin, 1859 "Much light.... p.462 in the 1928 Dent Edition).

Many physicists today believe that a "Theory of Everything" is at hand. The best candidates involve a multidimensional description of reality, and owe their inspiration to Theodor Kaluza, a little known privat-docent, now a household name in theoretical physics.

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