Locality, Lorentz invariance and the Bohm model

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Locality, Lorentz Invariance and the Bohm Model

A thesis submitted by

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To my wife, two sons and a daughter

Abstract

Non-local forces exist in nature for two reasons. First that the recent experiments on locality are supposed to be accurate enough. Second that there is no local theory that can reproduce all the predictions of orthodox quantum theory which, almost for about a century, have been proved to be correct experimentally again and again. This thesis concerns both of these.

A brief discussion of the measurement in quantum theory is followed by two comments which show that the quantum description is frame dependent and that the collapse of the wave-function of a system may occur without the relevant measurement being performed. After this the Bohm model and a modified version of the Bohm model are described.

Next we introduce a new method for obtaining the Bell-type inequalities which can be used for testing locality. We derive more inequalities by this method than obtained by other existing procedures. Using Projection Valued(PV) and Positive Operator Valued Measures(POVM) measurements we have designed experiments which violates one of the Bell inequalities by a larger factor than existing violations which in turn could increase the accuracy of experiments to test for non-locality. This is our first result.

After discussing the non-locality and non-Lorentz invariant features of the Bohm model, its retarded version, namely Squires' model - which is local and Lorentz invariant - is introduced. A problem with this model, that is the ambiguity in the cases where the wave-function depends on time, is removed by using the multiple-time wave-function. Finally, we apply the model to one of the experiments of locality and prove that it is in good agreement with the orthodox quantum theory.

Preface

During my research studies in the Department of Mathematical Sciences at the University of Durham this thesis was prepared. Everything I have written is in my own words. My original work described in this thesis has not been published anywhere and the work of other authors is referred to whenever it is used.

Part of this work has been done under the supervision of Professor Euan James Squires. I owe him much more than words can describe for his assistance and kindness which were beyond academic purposes. It was one of the saddest days for me when he tragically died on 6*th* of June, 1996. The reminiscences of my days working with him will be with me forever.

I sincerely thank Professor Ed Corrigan and Dr Lucien Hardy, who took me over for the rest of my studies. Although, as an official supervisor, Professor Corrigan emphasized that he was not an expert on the subject, his comments and the questions he raised led me to a deeper understanding of the material. Dr Hardy (Rome and Oxford) left Durham in September 1996 and took over the difficult job of supervising me via email. I found this most effective and I was also able to visit him occasionally.

Over the time working on this subject, I have always enjoyed and greatly

appreciated the invaluable advice, encouragement and help of all of my supervisors.

I am most grateful to my wife and children who showed remarkable powers of endurance and supported me to the end. Especially my wife's continual encouragement was of vital importance.

I acknowledge with gratitude the Sabzevar Teacher Training University and the Ministry of Culture and Higher Education of Iran for financial support.

Within the years staying in this historical and pretty nice city of Durham, I took great pleasure and delight in associating with many people in and out of the department and the university. I feel it my duty to thank them and to wish them all a happy and successful future.

Statement of Copyright

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Chapter 1

Introduction

In spite of its great success, orthodox quantum theory has suffered from two difficulties – the first, which is as old as the theory itself, is the *measurement problem* in which a pure state under the time dependent Shrödinger equation seems to change to a mixed one; and the second one, developed later in 1935, is the *Einstein, Podolsky, Rosen (EPR) theorem* which suggests that the wave-function does not give the complete description of the state of a physical system in quantum theory, or otherwise, without performing any measurement on a system, the values of two observables with non-commuting operators simultaneously can be predicted with certainty. Since then many physicists have studied these issues.

In this thesis, we first review the quantum measurement problem and the consequences of the collapse of the wave-function which lead to some well-known paradoxes in quantum theory. The hidden-variable theory of David Bohm, developed in 1952 as a solution to the measurement problem, and its contextual dependent feature which blocks the existing no hidden-variable theorems, will be discussed next. In Bohm model each particle moves on a given trajectory, which is determined by the initial conditions of the system. However in most versions of this model no trajectory can be defined for bosons. Furthermore, there is no hidden-variable for the spin of a particle. This was the motivation for generalizing the Bohm model by Squires and Mackman in 1995. With application of this generalized model to a simple case we have ended chapter 2.

In chapter 3 the EPR argument and the suggestion of introducing an extra hidden-variable along with the wave-function for complete description of a system are explained. However John S. Bell in 1964, based on local deterministic hidden-variable (and later any local) theories deduced an inequality, and proved that this inequality is violated by the predictions of orthodox quantum theory. But Bell's original inequality cannot be used to test non-locality. In the rest of the chapter a method is introduced for obtaining Bell-type inequalities. The method is especially applied to the experiments with two arms and two local variables in each arm. Here the inequalities for two different cases are deduced: in the first case for each local variable there are two outputs which we call it 2,2:2,2 case and in the other in each arm for one of the local variable setting there are three outputs which we call it 2,3:2,3 case.

Two types of measurements, that is, Projection Valued(PV) measurement and Positive Operator Valued Measure(POVM) measurement are discussed in chapter 4. Based on the predictions of orthodox quantum theory, the values of some of the 2,3:2,3 inequalities are calculated using PV and POVM measurements where these are used for 2 and 3 outputs respectively.

Hardy's proof according to which any local hidden-variable theory is essentially non-Lorentz invariant is discussed in chapter 5. That the Bohm model is generally a non-local theory and at the level of an individual system is not Lorentz invariant is explained later. The Squires model which is local and Lorentz invariant is the core of this chapter. This model which is in fact a retarded version of the Bohm model is applied to one of the experiments of Aspect *et.al.* to see to what extent does the model agree with the predictions of quantum theory.

Finally chapter 6 is a summary of the results obtained.

Chapter 2

Quantum Measurement and the Bohm model

2.1 The quantum measurement problem

Measurement, even if not one of the fundamental concepts, has a basic role in physics whose theories are tested via experiments. Two important features that we might expect of a measurement are: firstly, if two systems are identical, that is their initial wave-function is the same and therefore the same wave-function for all future times, the results of the measurement on both systems should be the same, secondly, if the state of a system does not change with time or in a very short time such that the changes of the system can be neglected, repeated measurements on the system should give the same results. It should be noted that here we are assuming error-free measurements. So, for example, if the predicted probability for a measurement is zero, then the measurement would give no result.

Quantum theory, which is statistical in nature, when applied to a statistical ensemble of identical systems, predicts exact results in accordance with the above requirements. If the (error-free) measurements are done on two statistical ensembles of identical systems, or on one statistical ensemble of identical systems in such a short time that the time evolution of the systems can be neglected, then exactly the same results are obtained. However in the case of an individual system, *e.g.* a one particle system itself, quantum theory does not satisfy the first of these requirements.

According to quantum theory, at any given time there corresponds a *state* to any physical system. All properties of this physical system which are called *observables* are determined from this state via a corresponding hermitian operator. Orthodox quantum theory says that the wave-function, which satisfies the Schrödinger equation in non-relativistic quantum mechanics, gives the complete description of the state of a physical system. In fact von Neumann [vN55] claims that " *One never needs more information than this: if both system and state are known, then the theory gives unambiguous directions for answering all questions by calculation.* " However in the case of an individual system, if we do a measurement on the system, the result of the experiment cannot be determined exactly in advance. Here the wave-function gives the probability of the outcome of the measurement.

To explain the process of measurement in quantum mechanics, suppose that before the measurement is done, the wave-function of the system, S, is $\psi_0^S(\boldsymbol{x})$ and the wave-function of the measurement apparatus is $\psi_0^M(\boldsymbol{y})$ and that we are going to measure the observable $\mathcal{O}$ with eigen-functions u_n and eigen-values O_n. As there is no interaction between the system and the apparatus before measurement, initially the wave-function of the whole system is $\Psi(\boldsymbol{x},\boldsymbol{y},t_0) = \psi_0^S(\boldsymbol{x})\psi_0^M(\boldsymbol{y})$; where $\boldsymbol{x}$, $\boldsymbol{y}$ represent the coordinates of configuration space of the system and the measurement apparatus respectively. As the interaction between the system and the measurement apparatus occurs, the total wave-function evolves according to Schrödinger equation. Clearly before the measurement is done there is a probability of $|a_n|^2$ to get the value O_n for the observable $\mathcal{O}$ where $a_n = \langle u_n|\psi_0^S\rangle$.

Now suppose that we do the same measurement on this system in such a short time that the time evolution of the system is ignored. For such successive measurements von Neumann's interpretation gives a mathematical description for the cases where the result of the two successive measurement would be the same – the so called the measurement of the first kind [Jau68].

We write the system state

$$|\psi_0^S\rangle = \sum_n a_n|u_n\rangle$$

where the $|u_n\rangle$ are a complete set of eigen-states of the observable which is to be measured. Then the measuring apparatus is designed so that

$$|\psi_0^S\rangle|\psi_0^M\rangle \xrightarrow{\textbf{unitary evolution}} \sum_n a_n|u_n\rangle|v_n\rangle$$

i.e. each state $|v_n\rangle$ corresponds to a unique "result" for the system. The "*collapse*" then takes the sum into one state $|u_n\rangle|v_n\rangle$ with probability $P_n = |a_n|^2$. This collapse of the wave-function raises another problem – *measurement problem* – a pure state has changed to a mixed state, which means that the quantum evolution is not linear.

Schrödinger cat – Consider a live cat sitting in a box, that could be killed if a particle strikes it. A source of particles and a beam-splitter is provided in this box such that only the transferred beam hits the cat and will kill it. Now, the source emits one particle. When this particle reaches the beam-splitter, the wave-function splits to $\frac{1}{\sqrt{2}}(|t\rangle + i|r\rangle)$, with equal probability to find the particle in $|t\rangle$ (transfered) state or $|r\rangle$ (reflected) state. Initially the total wave-function of the system is

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|t\rangle + i|r\rangle)|L\rangle$$

And the final state is:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}|t\rangle|D\rangle + i\frac{1}{\sqrt{2}}|r\rangle|L\rangle \tag{2.1}$$

Where $|D\rangle$ is the state of dead cat and the $|L\rangle$ is the state of the live cat. As long as there is no observation the probability to find the cat live or dead is equal. The collapse of the wave-function is postponed and only occurs when an observation is made and accordingly the system collapses to the state $|t\rangle|D\rangle$ or $|r\rangle|L\rangle$, and only at this time the predestination of the cat is determined. It will be alive or dead.

Negative measurement – Consider measuring the z-component of the spin of a particle, which has only two eigen-states $|+\rangle$ and $|-\rangle$. If the initial state of the system is $|\Psi\rangle = \alpha_+|+\rangle + \alpha_-|-\rangle$, and the particle is passed through a Stern-Gerlach apparatus, then the particle is deflected according to its spin state, for example with the probability $|\alpha_+|^2$ ($|\alpha_-|^2$) it is in the $|+\rangle$ ($|-\rangle$) state and will go up (down) toward some point P (Q). At this stage the wave-function has not collapsed yet, because we have not obtained any information about the system. So the measurement is done only if the particle interacts with one of the detectors provided at the point P or Q, and according to what we said previously, after this the collapse of the wave-function happens. Now , if a detector is provided at the point Q, and we do not detect the particle, it means that it is at the point P with the state $|+\rangle$. Clearly, the wave-function should have collapsed to the state $|+\rangle$. But, really, have we made a measurement on the system? Of course, not. Because there was no interaction between the particle and the measurement apparatus - here the detector at Q. However, this is just the same as the Schrödinger cat. Instead of looking at the cat to see if it is alive or dead, we may put a detector to detect the particle in the reflected wave.

Wigner's friend – If in the Schrödinger cat experiment, the particle is the system and the cat which is conscious is the measurement apparatus, and the effect of the particle is not to kill the cat but only make him aware of its arrival. Then, from the point of view of the cat, when the particle reaches him, the collapse of the wave-function happens, say at time t_c. However for the observer outside the box the state of the system is still pure, given by equation (2.1), until at some later time, t_m, he knows the results e.g. by means of asking the cat, then the collapse occurs. It is seen that in the time interval between t_c and t_m according to the cat the collapse has happened and so the state of the system is a single term, $|t\rangle|D\rangle$ or $|r\rangle|L\rangle$, but according to the observer the state of the system is still the superposition given by the equation (2.1), because the collapse has not happened yet.

Comment 2.1.1 – One of the consequences of the collapse of the wave-function is that in quantum mechanics the probability, as a function of time, in two frames of reference is different. To show this we consider a system of two non-interacting particles.

If $\mathcal{P}$ is an observable of the first particle with eigen-functions ζ_m and eigen-values p_m then:

$$\mathcal{P}|\zeta_m\rangle = p_m|\zeta_m\rangle$$

and if $\mathcal{Q}$ is an observable of the second particle with eigen-functions ξ_n and eigen-values q_n then:

$$\mathcal{Q}|\xi_n\rangle = q_n|\xi_n\rangle$$

Now let's consider the general case where the two particles are correlated, so that their wave-functions are entangled. We assume the total state of the system is:

$$|\Psi(\boldsymbol{x},\boldsymbol{y})\rangle = |\psi_1(\boldsymbol{x})\rangle|\phi_1(\boldsymbol{y})\rangle + |\psi_2(\boldsymbol{x})\rangle|\phi_2(\boldsymbol{y})\rangle \tag{2.2}$$

where $\boldsymbol{x}$ and $\boldsymbol{y}$ represent the coordinates of configuration space of the two particles.

Now we expand the states $|\psi_{(1,2)}(\boldsymbol{x})\rangle$ and $|\phi_{(1,2)}(\boldsymbol{y})\rangle$ as follows:

$$|\psi_{(1,2)}(\boldsymbol{x})\rangle = \sum_m \alpha_{(1,2)m}|\zeta_m\rangle$$

$$|\phi_{(1,2)}(\boldsymbol{y})\rangle = \sum_n \beta_{(1,2)n}|\xi_n\rangle$$

So the state $|\Psi(\boldsymbol{x},\boldsymbol{y})\rangle$ would become:

$$\begin{aligned}
|\Psi(\boldsymbol{x},\boldsymbol{y})\rangle &= \left[\sum_m \alpha_{1m}|\zeta_m\rangle\right]|\phi_1(\boldsymbol{y})\rangle + \left[\sum_m \alpha_{2m}|\zeta_m\rangle\right]|\phi_2(\boldsymbol{y})\rangle && (2.3)\\
&= |\psi_1(\boldsymbol{x})\rangle\left[\sum_n \beta_{1n}|\xi_n\rangle\right] + |\psi_2(\boldsymbol{x})\rangle\left[\sum_n \beta_{2n}|\xi_n\rangle\right] && (2.4)\\
&= \sum_{m,n}(\alpha_{1m}\beta_{1n} + \alpha_{2m}\beta_{2n})|\zeta_m\rangle|\xi_n\rangle && (2.5)
\end{aligned}$$

It is clear that the state $|\zeta_m\rangle|\xi_n\rangle$ is an eigen-state of the operator $\mathcal{PQ}$ and we can expand the state $|\Psi(\boldsymbol{x},\boldsymbol{y})\rangle$ in term of these eigen-states.

Now if we measure the observable $\mathcal{PQ}$ of this system, then after the measurement is made, with the probability $|\alpha_{1m}\beta_{1n}+\alpha_{2m}\beta_{2n}|^2$ we obtain the value p_m for $\mathcal{P}$ and the value q_n for $\mathcal{Q}$ and with the same probability the final state of the whole system collapses to $|\zeta_m\rangle|\xi_n\rangle$. Here we see that as time goes on, the probability changes as follows:

$$|\alpha_{1m}\beta_{1n}+\alpha_{2m}\beta_{2n}|^2 \longrightarrow 1$$

However, as another possibility we can first measure the observable $\mathcal{Q}$ and then the observable $\mathcal{P}$. In this case the wave-function of the state of the whole system, with some probability which is easily calculated from equation (2.4), collapses to $\lambda(\beta_{1n}|\psi_1(\boldsymbol{x})\rangle+\beta_{2n}|\psi_2(\boldsymbol{x})\rangle)|\xi_n\rangle$ where:

$$\lambda=[|\beta_{1n}|^2\langle\psi_1|\psi_1\rangle+|\beta_{2n}|^2\langle\psi_2|\psi_2\rangle+2\Re(\beta_{1n}^*\beta_{2n}\langle\psi_1|\psi_2\rangle)]^{-\frac{1}{2}} \tag{2.6}$$

is a normalization constant and then with the probability $\lambda^2|\alpha_{1m}\beta_{1n}+\alpha_{2m}\beta_{2n}|^2$ collapses to $\zeta_m\xi_n$ so in this case as time goes on, the probability changes as:

$$|\alpha_{1m}\beta_{1n}+\alpha_{2m}\beta_{2n}|^2 \longrightarrow \lambda^2|\alpha_{1m}\beta_{1n}+\alpha_{2m}\beta_{2n}|^2 \longrightarrow 1$$

However, as the theory is assumed to be Lorentz invariant, so in any other frame of reference, if $\mathcal{P}/\mathcal{Q}$ is measured then the corresponding state should collapse to the same $|\zeta_m\rangle/|\xi_n\rangle$. But, if the two measurements of $\mathcal{P}$ and $\mathcal{Q}$ are space-like separated,[1] then there is a frame of reference in which the order of

[1] Here a measurement is considered as an event which take place at some time t and at some point x in space.

measurements is reversed, *i.e.* as time goes on, the probability changes as:

$$|\alpha_{1m}\beta_{1n} + \alpha_{2m}\beta_{2n}|^2 \longrightarrow \mu^2|\alpha_{1m}\beta_{1n} + \alpha_{2m}\beta_{2n}|^2 \longrightarrow 1$$

where:

$$\mu = [|\alpha_{1n}|^2\langle\phi_1|\phi_1\rangle + |\alpha_{2n}|^2\langle\phi_2|\phi_2\rangle + 2\Re(\alpha_{1n}^*\alpha_{2n}\langle\phi_1|\phi_2\rangle)]^{-\frac{1}{2}} \tag{2.7}$$

is again a normalization constant. It is obvious that λ and μ are not the same. So we conclude that the quantum description depends on the frame of reference. Later when we discuss the EPR experiment, we will see the same situation.

Comment 2.1.2 – We can show that in some cases if an observable of a system is measured, then the spatial part of the wave-function collapses, and if the observable can be measured by negative measurement, then the spatial part of the wave-function collapses to a Dirac delta function. As an example consider a particle whose z-component σ_z is to be measured. The wave-function of the particle consists of two parts, the spatial part $\psi(\boldsymbol{x}, t)$ and the spin part ϕ. The particle is then passed through the Stern-Gerlach apparatus which is set up properly to measure σ_z. As shown in figure 2.1 when at time t_1 the z-component of the spin, σ_z, is measured at A, the spatial part of the wave-function is the wave-packet shown by the solid-line. Here the probability to find the particle at the point A is $\mathcal{P}_A$. At the time t_2, the wave-packet, shown in the figure by a dashed-line, has moved the distance $v\Delta t$ where $\Delta t = t_2 - t_1$ and v is the velocity of the wave-packet. At this time there is a point X whose distance is $C\Delta t$ from A. At the point B in the right-hand side of X, the probability to find the particle is $\mathcal{P}_B$. Now

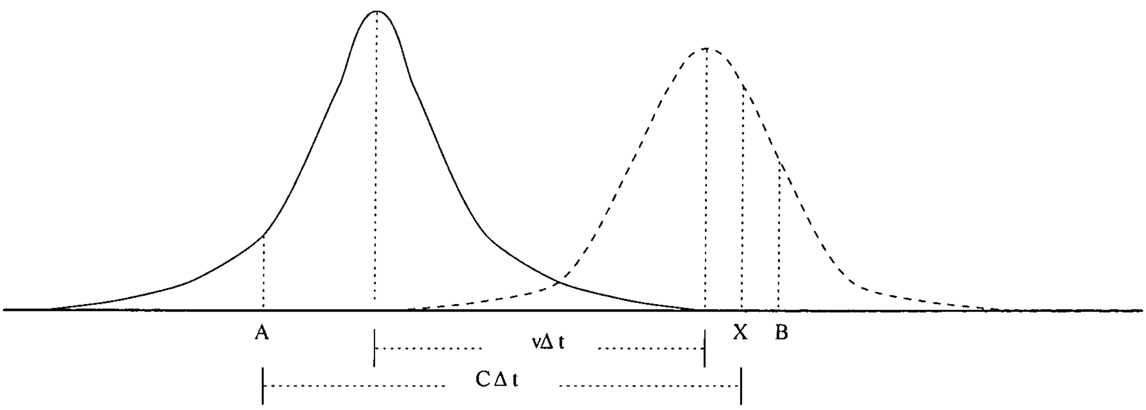

Figure 2.1: The wave-packet of the particle at time t_1 (solid-line) and at time t_2 (dashed-line).

if we measure the x-component of the spin at this point, then we have two space-like separated events: the measurement of σ_z of the particle at point A and at time t_1 which we call event $\mathcal{A}$; and the measurement of σ_x of the particle at point B and at time t_2 which we call event $\mathcal{B}$. As we said before, due to Lorentz invariance of the theory, the collapses resulting from the measurements should happen in all frames of reference. Now there is a frame of reference in which the two events $\mathcal{A}$ and $\mathcal{B}$ are simultaneous. This means that the z-component and x-component of the spin could be measured simultaneously with the probability $\mathcal{P}_A\mathcal{P}_B$ which violates the uncertainty principle. So $\mathcal{P}_B$ should be zero, which in turn means that $\psi(\boldsymbol{x}, t_2) = 0$ for all the points in the right-hand side of X. This collapse of the spatial part of the wave-function may be considered as the consequence of the measurement of the σ_z, because in fact we should make a position measurement to detect the particle in the Stern-Gerlach apparatus. But, if we do the z-component measurement by means of a *negative measurement*, then we have not detected the particle directly. In this case it is even possible to choose Δt very near to zero, so that X gets very close to A. As the same argument can be applied to the points on the left-hand side of A we conclude that the spatial part of the wave-function should collapse to a Dirac delta function.

2.2 The Bohm model

In the previous section we briefly discussed the problem with measurement in quantum mechanics. To overcome this difficulty some physicists have suggested to add an extra nonlinear term in the Schrödinger equation [see reference [GR90] and the the references therein], but some have suggested the revision of the primary concept of quantum theory; among these are the hidden-variable theory of de Broglie [dB27][2] and David Bohm [Boh52], the so called de Broglie–Bohm model, in which for the complete description of the state of the system not only the wave-function of the system, but also an extra parameter is needed. To discuss this model consider the Schrödinger equation for a one particle system

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi + V(\boldsymbol{x}, t)\Psi \tag{2.8}$$

Here the wave-function can be written as:

$$\Psi = R(\boldsymbol{x}, t)\ \exp[iS(\boldsymbol{x}, t)/\hbar] \tag{2.9}$$

[2]I have not seen this reference. It is quoted here from within other references [Boh52] for historical purposes only.

where $R(\boldsymbol{x}, t)$ and $S(\boldsymbol{x}, t)$ are both real functions of $\boldsymbol{x}$ and t. So the equation (2.8) splits into two equations of the form:

$$\frac{\partial R}{\partial t} = -\frac{1}{2m}\left[R\,\nabla^2 S + 2\boldsymbol{\nabla} R \cdot \boldsymbol{\nabla} S\right] \tag{2.10}$$

$$\frac{\partial S}{\partial t} = -\left[\frac{(\boldsymbol{\nabla} S)^2}{2m} + V(\boldsymbol{x}) - \frac{\hbar^2}{2m}\frac{\nabla^2 R}{R}\right] \tag{2.11}$$

If we define the *quantum potential* $Q(\boldsymbol{x})$ as:

$$Q(\boldsymbol{x}) = -\frac{\hbar^2}{2m}\frac{\nabla^2 R}{R} \tag{2.12}$$

and the momentum $\boldsymbol{p}(\boldsymbol{x})$ of the particle as

$$\boldsymbol{p}(\boldsymbol{x}) = \boldsymbol{\nabla} S(\boldsymbol{x}) \tag{2.13}$$

then the equation (2.11) is the Hamilton-Jacobi equation of the system with the potential

$$U(\boldsymbol{x}) = V(\boldsymbol{x}) + Q(\boldsymbol{x}) \tag{2.14}$$

From equation (2.13) one readily finds that

$$\dot{\boldsymbol{x}} = \frac{1}{m}\Re\left(\frac{\Psi^* \boldsymbol{p}_{op} \Psi}{\Psi^* \Psi}\right) \qquad \text{(assuming } \boldsymbol{p} = m\dot{\boldsymbol{x}}) \tag{2.15}$$

where $\boldsymbol{p}_{op}$ is the momentum operator $\frac{\hbar}{i}\boldsymbol{\nabla}$ and from equation (2.14) it is seen that the particle moves in the potential field $U(\boldsymbol{x})$, so from the Newton's second law we can write

$$m\frac{d^2\boldsymbol{x}}{d\,t^2} = -\boldsymbol{\nabla} U(\boldsymbol{x}) = -\boldsymbol{\nabla}\left[V(\boldsymbol{x}) - \frac{\hbar^2}{2m}\frac{\nabla^2 R}{R}\right]. \tag{2.16}$$

Moreover in orthodox quantum theory the probability density $P(\boldsymbol{x})$ is defined as:

$$P(\boldsymbol{x}) = R^2(\boldsymbol{x}) \tag{2.17}$$

so the equations (2.10) and (2.12) respectively would become

$$\frac{\partial P}{\partial t} + \boldsymbol{\nabla} \cdot \left(P \frac{\boldsymbol{\nabla} S}{m} \right) = 0 \tag{2.18}$$

$$Q(\boldsymbol{x}) = -\frac{\hbar^2}{4m} \left[\frac{\nabla^2 P}{P} - \frac{1}{2} \frac{(\boldsymbol{\nabla} P)^2}{P^2} \right] \tag{2.19}$$

From equation (2.15) one finds that the trajectory of the particle depends on the initial position of the particle – the so-called hidden-variable. So we see that if the initial wave-function of the system is Ψ_0 then there is a distribution of the initial positions of the particle and any outcome of a measurement on the system depends on what trajectory the particle is on.

Up to now we were considering a system of one particle. The generalization for a system of many particles is straightforward; but, here we use a different approach [Squ96] to derive the equations of motion.

Consider a system of N particles with the wave-function $\Psi(x_1, \cdots, x_{3N}, t)$ where $x_1, \cdots, x_{3N}$ represent the coordinates of the particles in $3N$ dimensional configuration space of the system. The probability density is:

$$\rho(x_1, \cdots, x_{3N}, t) = |\Psi(x_1, \cdots, x_{3N}, t)|^2 \tag{2.20}$$

and from Schödinger equation:

$$i\hbar \frac{\partial \Psi}{\partial t} = \left[\sum_{i=1}^{3N} \frac{-\hbar^2}{2m_i} \nabla_i^2 + V \right] \Psi \tag{2.21}$$

it can be shown that:

$$\frac{\partial}{\partial t}\rho(x_1, \cdots, x_{3N}, t) = \nabla \cdot \mathbf{J}(x_1, \cdots, x_{3N}, t) = 0 \tag{2.22}$$

where

$$\mathbf{J}(x_1, \cdots, x_{3N}, t) = \sum_{i=1}^{N} \frac{1}{m_i} \Re \left[\Psi^* \frac{\hbar}{i} \nabla_i \Psi \right] \tag{2.23}$$

is the probability current. Note that ∇_i is a 3 dimensional vector which operates on the subspace corresponding to the ith particle. If $\mathbf{X} = \sum_{i=1}^{N} \mathbf{x}_i$ is the position vector of the system in the $3N$ dimensional configuration space, then the vector $\dot{\mathbf{X}}$ is the velocity in this space and $\rho\dot{\mathbf{X}}$ is the rate of flow of probability at the point $\mathbf{X}$. Now the rate of flow of probability into the volume element $d^{3N}\mathbf{X}$ in the positive direction of x_i in one unit of time is:

$$\left[\rho\dot{x}_i\big|_{x_i+dx_i} - \rho\dot{x}_i\big|_{x_i} \right] d^{3N-1}S_{\perp x_i} = \left(\frac{\partial}{\partial x_i}\rho\dot{x}_i \right) d^{3N}\mathbf{X} \tag{2.24}$$

where

$$d^{3N-1}S_{\perp x_i} = \prod_{j\neq i}^{3N} dx_j \tag{2.25}$$

is the element of a $3N-1$ dimensional surface. and

$$d^{3N}\mathbf{X} = \prod_{j=1}^{3N} dx_j \tag{2.26}$$

is the element of 3 dimensional volume. So the total rate of flow of probability into the unit volume is:

$$\sum_{i=1}^{3N} \frac{\partial}{\partial x_i}\rho\dot{x}_i = \sum_{i=1}^{N} \nabla_i \cdot \rho\dot{\mathbf{X}} = \nabla \cdot \rho\dot{\mathbf{X}} \tag{2.27}$$

which should be equal to the change of probability density in a unit of time, that is:

$$\nabla \cdot \rho \dot{\mathbf{X}} + \frac{\partial \rho}{\partial t} = 0 \tag{2.28}$$

From equations (2.22) and (2.28) we have:

$$\rho \dot{\mathbf{X}} = \mathbf{J} + \mathbf{C} \tag{2.29}$$

where $\mathbf{C}$ is an arbitrary $3N$ dimensional vector; and

$$\sum_{i=1}^{N} \nabla_i \cdot \mathbf{c}_i = 0 \tag{2.30}$$

So the equation of motion for the ith particle is:

$$\dot{\mathbf{x}}_i = \frac{1}{m_i} \Re \left(\frac{\Psi^* \boldsymbol{p}_i \Psi}{\Psi^* \Psi} \right) + \frac{\mathbf{c}_i}{\Psi^* \Psi} \tag{2.31}$$

Neglecting the last term in the right-hand side which is an arbitrary term the well known equation of motion is obtained.

There are two important features of the Bohm theory which we would like to emphasize on here. The first is that as the equation of motion (2.15) shows the Bohmian trajectories do not intersect each other, otherwise at the point of intersection the velocity cannot be defined uniquely. And the second is that if initially the relation (2.17) holds for the probability density, then equation (2.18) guarantees that it holds for all times.

2.3 Contextuality

The question of whether the hidden variable theories can exist in quantum mechanics was first formulated mathematically by von Neumann [vN55] who, based on some assumptions, proved that hidden-variable theories are not possible. One of his assumptions was:

If $\mathcal{R}, \mathcal{S}, \cdots$ are any arbitrary observables and $r, s, \cdots$ are any real numbers, then the linearity of averages implies:

$$< r\mathcal{R} + s\mathcal{S} + \cdots >= r < \mathcal{R} > +s < \mathcal{S} > + \cdots \tag{2.32}$$

Although this holds when applied to ensembles with dispersion, it was shown by Bell [Bel66] that it encounters serious difficulties in the case of dispersion-free states which is where the hidden-variable theories come in. To see this suppose that $\mathcal{R}$ and $\mathcal{S}$ do not commute and the set of hidden-variables which determine the outcome of the measurement is $\boldsymbol{\lambda}$. As we are considering a dispersion-free ensemble, then according to the definition $\boldsymbol{\lambda}$ is the same for all members of the ensemble and so the resulting values from the measurement made on each of the members (which is of course an eigen-value of the corresponding observables) are the same. So the averages on the the right-hand side of the (2.32) are simply the eigen-values of two non-commuting observables that are measured simultaneously and this violates quantum theory. Incidentally, von Neumann uses dispersion-free states in his proof and this was the motivation for considering the commuting observables for this purpose. The work of Kochen and Specker [KS67] is the most important on the subject and two simple examples about this procedure are introduced by Mermin [Mer90]. Here nine sets of observables are considered in the Hilbert space of two spin $\frac{1}{2}$ particles as shown in table 2.1.

As it is seen the observables in each column and row commute with each other and also the value of each observable can only be ± 1. Now consider the

σ_{1x}	σ_{2x}	$\sigma_{1x}\sigma_{2x}$	1
σ_{2y}	σ_{1y}	$\sigma_{1y}\sigma_{2y}$	1
$\sigma_{1x}\sigma_{2y}$	$\sigma_{1y}\sigma_{2x}$	$\sigma_{1z}\sigma_{2z}$	1
1	1	-1	

Table 2.1: Mermin's nine sets of observables.

multiplication of the operators in the third column. Here the multiplication of the first and the second operators yields

$$\sigma_{1x}\sigma_{2x}\sigma_{1y}\sigma_{2y} = \sigma_{1x}\sigma_{1y}\sigma_{2x}\sigma_{2y} = i\sigma_{1z}i\sigma_{2z} = -\sigma_{1z}\sigma_{2z}$$

which if multiplied by the third operator, $\sigma_{1z}\sigma_{2z}$, gives -1. Similar calculations show that for all other rows and columns, the multiplication of the elements of each row or column is 1. Also as the operators in each row or column commute with each other, the above table is still valid if we replace each of the operators with one of its eigen-values. However we see that it is impossible to assign values simultaneously to the operators in the above table so that the whole table is satisfied because some of the elements do not commute. This reveals that hidden-variables which assign values to operators may not be possible. However, in the Bohm theory a hidden-variable is assigned to an operator based not only on the present state of the system but also on the state of the measurement apparatus. So if a measurement apparatus M is set up to measure the set of operators in the first column, the outcome for σ_{1x} would be a^M_{1x}. However, if one decides to measure the set of observables in the first row, one should use a different apparatus M', because the operators in the two sets of observables do not commute. As a result a new set of hidden-variables are assigned to these and in particular σ_{1x} observable which could lead to a different outcome $a_{1x}^{M'}$. This context dependence is one of the most distinctive features of Bohm theory and it is in this way that the above contradiction fails for this theory. Some examples about the contextuality of Bohm theory has been carried out by Dewdney [Dew92] and Pagonis and Clifton [PC95] using spin measurement but here we consider the one discussed by Hardy [Har96].

Figures 2.2 and 2.3 show the scheme of two experiments in which a particle is free to move in one of the three paths a, b or c. So the states $|a\rangle$, $|b\rangle$ and $|c\rangle$ are all orthogonal and supposed to be normalized. They form a complete basis of the 3 dimensional Hilbert space of the system. The initial state of the system is supposed to be $|\Psi_0\rangle = \alpha|a\rangle + \beta|b\rangle + \gamma|c\rangle$ and the reflectivity of beam-splitters BS1 and BS2 are R_1 and $R_2 (\geq R_1)$, so the transmittances would be $T_1 = 1 - R_1$ and $T_2 = 1 - R_2$ respectively. The phase shifters on the left(right) change the incoming(outgoing) states by a factor $+i(-i)$.

The incoming states in each beam-splitter are orthonormal, so in order to keep the outgoing states orthonormal the following relations must hold:

$$|I_1\rangle = \sqrt{T}|O_1\rangle + i\sqrt{R}|O_2\rangle \tag{2.33}$$

$$|I_2\rangle = \sqrt{T}|O_2\rangle + i\sqrt{R}|O_1\rangle \tag{2.34}$$

where I_1, I_2 and O_1, O_2 are incoming and outgoing states.

With the above assumptions it is seen that the states $|d\rangle$, $|e\rangle$ and $|f\rangle$ are orthonormal. The same is true for $|d\rangle$, $|e'\rangle$ and $|f'\rangle$. In fact for figure 2.2 with $R_1 = \frac{1}{3}$ ($T_1 = \frac{2}{3}$) and $R_2 = \frac{1}{2}$ ($T_2 = \frac{1}{2}$) we have:

$$|d\rangle = \frac{1}{\sqrt{3}}(|a\rangle + |b\rangle + |c\rangle) \tag{2.35}$$

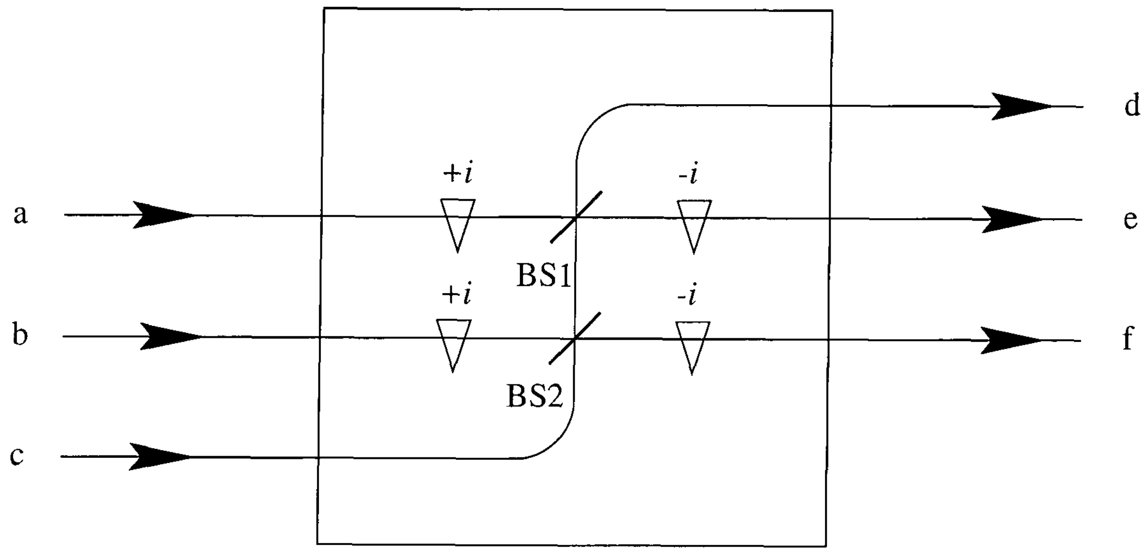

Figure 2.2: The scheme of an experiment in which 3 observables d, e and f are measured.

$$|e\rangle = \frac{1}{\sqrt{6}}(-2|a\rangle + |b\rangle + |c\rangle) \tag{2.36}$$

$$|f\rangle = \frac{1}{\sqrt{2}}(-|b\rangle + |c\rangle) \tag{2.37}$$

and for figure 2.3 we have:

$$|d\rangle = \frac{1}{\sqrt{3}}(|a\rangle + |b\rangle + |c\rangle) \tag{2.38}$$

$$|e'\rangle = \frac{1}{\sqrt{6}}(|a\rangle - 2|b\rangle + |c\rangle) \tag{2.39}$$

$$|f'\rangle = \frac{1}{\sqrt{2}}(|a\rangle - |c\rangle) \tag{2.40}$$

where due to the presence of the phase shifters all i's have been omitted from the above solutions. In both experiments the same observable d is measured, but the context of the measurement is different in the two schemes. In

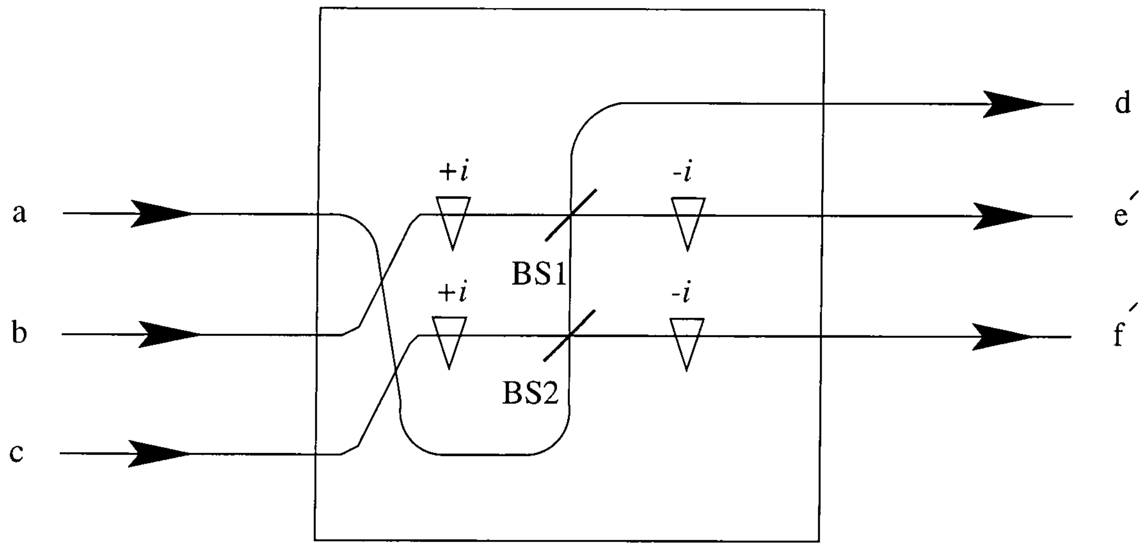

Figure 2.3: The scheme of an experiment in which 3 observables d, e' and f' are measured.

figure 2.2 d is measured along with e and f, while in the figure 2.3 it is measured along with e' and f'.

Now if the initial state of the system is $|\Psi_0\rangle = |a\rangle$, the particle would be reflected or transmitted by the beam-splitter depending on the value of the hidden-variable which is the initial position of the particle in its wave-packet in path a. Furthermore, Hardy shows that the the reflected Bohmian trajectories correspond to the initial positions in the back part and the transmitted Bohmian trajectories correspond to the initial positions in the front part of the wave-packet. So, as the reflectivity of the beam-splitter in figure 2.2 is $R_1 = \frac{1}{3}$ then for the range of the initial position of the particle in the back third of the wave-packet proportional to this, the particle is reflected and the detector in path d would fire. However in figure 2.3 as the reflectivity of the beam-splitter is greater than R_1 the particle would certainly be reflected into path f' and the detector in path d would not fire. This experiment shows that even if the hidden-variable is the same, the value measured for the same observable in the Bohmian mechanics may differ if the context of the measurement is changed, and this reveals that the Kochen and Specker theorems and the similar ones which are based on non-contextuality of the measurements cannot be applied to Bohm theory.

2.4 A generalized Bohm model

In section 2.2 it was shown that for particles which obey Schrödinger equation, the guidance relation is given by (2.15) and from the conservation equation (2.18) it is seen that the relation $\dot{\boldsymbol{x}} = \frac{\boldsymbol{j}}{\rho}$ guarantees $\rho(\boldsymbol{x}) = |\Psi(\boldsymbol{x})|^2$ at all times and the trajectories of the particles in the configuration space of the system depends on the initial positions $\boldsymbol{x}_0$ which are distributed with the probability density $\rho(\boldsymbol{x}_0) = |\Psi(\boldsymbol{x}_0)|^2$. Here of course ρ must be positive definite. So, for spin $\frac{1}{2}$ particles that obey Dirac equation still the above guidance relation is valid because the charge four current $j^\mu = (\rho, \boldsymbol{j})$ is conserved and ρ is always positive definite. However this is not the case for bosons. Here we are mainly interested in photons, so let's consider the Klein-Gordon equation which is the relativistic Schrödinger analogue for spin-less particles. With $\hbar = c = 1$ it reads:

$$(\frac{\partial^2}{\partial t^2} - \boldsymbol{\nabla}^2 + m_0^2 + V)\Psi = 0 \tag{2.41}$$

If we consider probability density ρ and current density $\boldsymbol{j}$ as following:

$$\rho = \frac{i}{2m_0}(\Psi^* \frac{\partial \Psi}{\partial t} - \Psi \frac{\partial \Psi^*}{\partial t}) \tag{2.42}$$

$$\boldsymbol{j} = -\frac{i}{2m_0}(\Psi^*\nabla\Psi - \Psi\nabla\Psi^*) \tag{2.43}$$

then the charge four current $j^\mu = (\rho, \boldsymbol{j})$ is conserved.

However, since the differential equation (2.41) is second order, then both Ψ and $\frac{\partial\Psi}{\partial t}$ can be fixed arbitrarily such that in equation (2.42) ρ becomes negative. Clearly in such a case the Bohmian trajectories can not be defined as in equation (2.15).

So for physical systems in which bosons and fermions are correlated, a model was proposed by Squires and Mackman[SM94] in which they suggested that for obtaining the fermion probability density, the total probability density be averaged over boson positions. That is:

$$\rho(\boldsymbol{x}) = \int d^3\boldsymbol{z}|\Psi(\boldsymbol{x}, \boldsymbol{z})|^2 \tag{2.44}$$

where $\boldsymbol{x}$ and $\boldsymbol{z}$ stands for all of the fermions and bosons positions respectively.

As we said, here it is supposed that there are no trajectories for bosons, so in this version of the Bohm model, the detection of the bosons depends on the the positions of the matter particles, that is, fermions. Note that this accords with the most commonly accepted version of the Bohm model [Boh52, BH93], since it is well known that it is difficult to define trajectories for bosons. In fact we would like to emphasize that in the model we are considering here, there are no beables for bosons and only the wave-function is real. This is quite different to the original model for bosons proposed by Bohm in which the beables are the field variables and the wave-functional, which depends on these field variables, is real (see the above references) . To be more specific, what we said about the particle position and its trajectories for fermions is applied in the same way to bosons but for field variable and its trajectories. However, even if it does turn out that bosons have trajectories, the discussion here is still appropriate since the main experimental tests to which we shall refer are concerned with the determination of a particle (photon) spin, where there is no spin "hidden-variable". In this case it is certain that the recorded value is a property of the hidden-variables in the detectors. In passing it is worth noting that the word "measurement" in situations of this nature is somewhat misleading, since it suggests that a previously existing value is being discovered by the procedure, whereas in fact there is no such value to be discovered; rather, a value is being *created* by the experiment.

Using the non-relativistic Schrödinger equation, the above would become:

$$\frac{\partial \rho}{\partial t} = \frac{i\hbar}{2m} \boldsymbol{\nabla}_x \cdot \int d^3\boldsymbol{z} (\Psi^* \boldsymbol{\nabla}_x \Psi - \Psi \boldsymbol{\nabla}_x \Psi^*) \tag{2.45}$$

On the other hand the equation of motion (2.15) should be modified in such a way that the relation (2.18) holds. This implies that:

$$\frac{\partial \rho}{\partial t} = -\boldsymbol{\nabla}_x \cdot \rho \dot{\boldsymbol{x}} \tag{2.46}$$

From the last two equations one obtains:

$$\dot{\boldsymbol{x}} = \frac{1}{m} \Re \left(\frac{\int d^3\boldsymbol{z} \Psi^* \boldsymbol{p}_x \Psi}{\int d^3\boldsymbol{z} \Psi^* \Psi} \right) \tag{2.47}$$

where $\boldsymbol{p}_x$ is the momentum operator and where on the right-hand side an additional term $(\boldsymbol{\nabla} \times \phi)/\rho$ with ϕ arbitrary, has been neglected. This is the generalized equation of motion for Bohmian trajectories if the wave-functions of fermions and bosons are entangled

2.5 Simple example – detection of a photon

In this section we shall apply the generalized standard Bohm model to a simple case of the detection of a photon in one dimensional space. The photon is emitted in the form of two wave-packets, one going to the left, $\phi_{\rm L}(z)$, and the other going to the right, $\phi_{\rm R}(z)$, from a source which is taken to be at the origin. The detector is a free particle, with mass m, initially in a stationary Gaussian wave-packet $\psi_{0\rm L}(X_{\rm L})$ which is centered around a point at a distance D from the photon source and at left-hand side of the source. The detection occurs by an interaction that we suppose has the effect that the particle receives some momentum from the photon and then starts moving due to this momentum transfer and after this process, as we will see in chapter 5, the particle may be accelerated and gains an average momentum p.

The initial wave-function of the system is:

$$|\Psi\rangle = 2^{-1/2}[\phi_{\rm L}(z) + \phi_{\rm R}(z)]\psi_{0\rm L}(X) \tag{2.48}$$

where

$$\psi_{0\rm L}(X) = \left(\frac{a}{\pi}\right)^{1/4} \exp[-\frac{1}{2}aX^2] \tag{2.49}$$

and

$$X = X_{\rm L} - D \tag{2.50}$$

Here $X_{\rm L}$ is the initial position of the center of the wave-packet with respect to the origin and $\frac{1}{\sqrt{a}}$ is the width of the wave-packet. After the interaction of the photon and the particle, the wave-function of the system becomes:

$$|\Psi\rangle = 2^{-1/2}[\phi_{\mathrm{L}}(z)\psi_{0\mathrm{L}}^{-p}(X) + \phi_{\mathrm{R}}(z)\psi_{0\mathrm{L}}(X)] \,, \tag{2.51}$$

where

$$\psi_{0\mathrm{L}}^{-p}(X) = \left(\frac{a}{\pi}\right)^{1/4} \exp\left[i\left(\frac{p}{\hbar}X - \frac{p^2}{2\hbar m}t\right) - \frac{a}{2}\left(X - \frac{p}{m}t\right)^2\right] . \tag{2.52}$$

This is the equation of a wave-packet moving in the positive direction of X in which we have ignored the quantum spreading. The superscript $-p$ means that the momentum p transferred to the detector is in the left direction.

It is worth to note that due to neglecting the quantum spreading, the wave-function $\psi_{\mathrm{R}}^{-p}(X)$ given by (2.52) does not satisfy the free particle Schrödinger equation but corresponds to a particle moving in the potential field of the form

$$V = -\frac{\hbar^2 a}{2m} + \frac{\hbar^2 a^2}{2m}\left(X - \frac{p}{m}t\right)^2 \tag{2.53}$$

and is not free in the classical sense. However, in this case, the quantum potential is

$$Q = \frac{\hbar^2 a}{2m} - \frac{\hbar^2 a^2}{2m}\left(X - \frac{p}{m}t\right)^2 \tag{2.54}$$

which exactly cancels the classical potential V. Here the particle is not free classically, but it is free in quantum sense.

However, the criterion for the approximation made in equation (2.52) to be good, is determined by inserting this equation in the Schrödinger equation which yields:

$$\frac{\hbar^2 a}{2m}\left|1 - a\left(X - \frac{p}{m}t\right)^2\right| \ll 1 \tag{2.55}$$

In the examples that we are considering throughout, the particle is an electron initially in a Gaussian wave-packet with the width of the order of Bohr radius ($R_{Bohr} = 5.29 \times 10^{-9}$ cm), so $a \approx 10^{16}$ cm^{-2} and condition (2.55) is fulfilled if $(X - \frac{p}{m}t)$ remains less than 10^{-3} cm. As X is measured from the initial position of the center of the wave-packet and $\frac{p}{m}$ is the velocity of the center of the wave-packet, this means that as long as our working domain is within the distances of 10^{-3} cm around the center of the wave-packet the approximation is good. Later when we discuss the retarded Bohm model, we will see that this indeed happens in the experiments that we are considering.

A similar situation applies if the detector was in the right-hand side of the photon source, at a distance D from the source, with the initial wave-function $\psi_{0\mathrm{R}}(Y_{\mathrm{R}})$. Then we would have

$$|\Psi\rangle = 2^{-1/2}[\phi_{\mathrm{L}}(z) + \phi_{\mathrm{R}}(z)]\psi_{0\mathrm{R}}(Y) \tag{2.56}$$

where

$$\psi_{0\mathrm{R}}(Y) = \left(\frac{a}{\pi}\right)^{1/4} \exp[-\frac{1}{2}aY^2] \tag{2.57}$$

and

$$Y = Y_{\mathrm{R}} - D \tag{2.58}$$

After the interaction of the photon with the particle the wave-function of the system would be

$$|\Psi\rangle = 2^{-1/2}[\phi_{\mathrm{L}}(z)\psi_{0\mathrm{R}}(Y) + \phi_{\mathrm{R}}(z)\psi_{0\mathrm{R}}^{+p}(Y)]\,, \tag{2.59}$$

where

$$\psi_{0\mathrm{R}}^{+p}(Y) = \left(\frac{a}{\pi}\right)^{1/4} \exp\left[i\left(\frac{p}{\hbar}Y - \frac{p^2}{2\hbar m}t\right) - \frac{a}{2}\left(Y - \frac{p}{m}t\right)^2\right]\,. \tag{2.60}$$

We return now to the case where there is one detector on the left-hand side of the photon source. From the equation of motion [SM94] we have

$$\dot{X} = \frac{1}{m}\Re\left[\frac{\int d^3z\Psi^*\boldsymbol{p}_X\Psi}{\int d^3z\Psi^*\Psi}\right] = \frac{1}{m}\Re\left[\frac{\psi_{\rm L}^{-p^*}\boldsymbol{p}_X\psi_{\rm L}^{-p} + \psi_{\rm L}^*\boldsymbol{p}_X\psi_{\rm L}}{\left|\psi_{\rm L}^{-p}\right|^2 + |\psi_{\rm L}|^2}\right] , \tag{2.61}$$

where

$$\boldsymbol{p}_X = -i\hbar\frac{\partial}{\partial X} \ . \tag{2.62}$$

This leads to

$$\dot{X} = \frac{\frac{p}{m}}{1 + \exp\left[a\frac{p}{m}t\left(-2X + \frac{p}{m}t\right)\right]} \ . \tag{2.63}$$

With the same procedure, for the detector in the right-hand side of the photon source we have,

$$\dot{Y} = \frac{\frac{p}{m}}{1 + \exp\left[a\frac{p}{m}t\left(-2Y + \frac{p}{m}t\right)\right]} \ . \tag{2.64}$$

From equation (2.63) it is seen that after the interaction, the detector starts moving; but the asymptotic behavior of this equation determines if it will continue or will stop recording. The position of the particle, X, at time t can be found by dividing the time interval $[t_0, t]$ to N equal intervals Δt and calculating X at each interval as followings:

$$t = 0\left\{\begin{array}{l} X = X_0 \\ \dot{X} = \frac{1}{2}\frac{p}{m} \end{array}\right.$$

$$t = \Delta t\left\{\begin{array}{l} X = X_0 + \frac{1}{2}\frac{p}{m}\Delta t \\ \dot{X} = \frac{\frac{p}{m}}{1+\exp[a\frac{p}{m}\Delta t(-2X_0)]} = \left(\frac{1}{2} + \alpha_1\right)\frac{p}{m} \end{array}\right.$$

$$t = 2\Delta t\left\{\begin{array}{l} X = X_0 + (\frac{2}{2} + \alpha_1)\frac{p}{m}\Delta t \\ \dot{X} = \frac{\frac{p}{m}}{1+\exp\left[2a\frac{p}{m}\Delta t(-2X_0-\alpha_1\frac{p}{m}\Delta t)\right]} = \left(\frac{1}{2} + \alpha_1 + \alpha_2\right)\frac{p}{m} \end{array}\right.$$

$$t = 3\Delta t \begin{cases} X = X_0 + (\frac{3}{2} + 2\alpha_1 + \alpha_2)\frac{p}{m}\Delta t \\ \dot{X} = \frac{\frac{p}{m}}{1+\exp[3a\frac{p}{m}\Delta t(-2X_0-(4\alpha_1+2\alpha_2)\frac{p}{m}\Delta t)]} = \left(\frac{1}{2} + \alpha_1 + \alpha_2 + \alpha_3\right)\frac{p}{m} \end{cases}$$

$$\vdots$$

$$t = N\Delta t \begin{cases} X = X_0 + \left(\frac{N}{2} + \sum_{i=1}^{N}(N-i)\alpha_i\right)\frac{p}{m}\Delta t \\ \dot{X} = \frac{\frac{p}{m}}{1+\exp\left[Na\frac{p}{m}\Delta t\left(-2X_0-2\sum_{i=1}^{N}(N-i)\alpha_i\frac{p}{m}\Delta t\right)\right]} = \left(\frac{1}{2} + \sum_{i=1}^{N}\alpha_i\right)\frac{p}{m} \end{cases}$$

In the above equations α_i's are auxiliary parameters used so that the equalities hold. Here we have assumed that N is very large (in fact $N \to \infty$). It is seen that if X_0 is positive/negative then α_i's are always positive/negative, so $\dot{X}$ is strictly increasing/decreasing. And since $\dot{X}$ is always positive or at most zero, if it is decreasing then it must approach to zero.[3]

Thus we see that the detector will record the photon if $X_0 > 0$ and will not record it if $X_0 < 0$. This means that the "measurement" is actually not measuring anything about the "photon". It is really measuring something about the detector. There is no property of the actual photon wave which is being measured. Of course, provided the distribution of initial particles (X_0) in a set of repeated experiments agrees with the quantum probability rule, so that, according to equation (2.49) there will be an equal number of cases with $X_0 < 0$ and $X_0 > 0$, then the photon will be recorded as going to the right in exactly half of the events.

[3]In chapter 5 we will see that the numerical calculations also confirm these somewhat unreliable calculations.

If the two portions of the photon wave-function going to the left and to the right are not equal; that is if the initial wave-function of the system is:

$$|\Psi\rangle = (\alpha^2+\beta^2)^{-1/2}[\alpha\phi_{\mathrm{L}}(z)+\beta\phi_{\mathrm{R}}(z)]\psi_{\mathrm{L}}(X) \tag{2.65}$$

then the velocity of the left detector becomes

$$\dot{X} = \frac{\frac{p}{m}}{1+\frac{\alpha^2}{\beta^2}\exp\left[a\frac{p}{m}t\left(-2X+\frac{p}{m}t\right)\right]} \,. \tag{2.66}$$

Here as it is expected, if the whole wave-function of the photon is going to the left/right (that is $\beta = 0$ / $\alpha = 0$) the detector will/will not record the photon irrespective of the initial position of the detector.

The same argument is applied to the case where the detector is in the right with the equation (2.64).

We shall now consider the case where both of the detectors are present. Here the equations of motion are:

$$\begin{aligned} \dot{X} &= \frac{1}{m}\Re\left[\frac{|\psi_R|^2\,\psi_{\mathrm{L}}^{-p^*}\boldsymbol{p}_X\psi_{\mathrm{L}}^{-p}+|\psi_R^{+p}|^2\,\psi_{\mathrm{L}}^*\boldsymbol{p}_X\psi_{\mathrm{L}}}{|\psi_R|^2\,|\psi_L^{-p}|^2+|\psi_R^{+p}|^2\,|\psi_L|^2}\right] \\ &= \frac{\frac{p}{m}}{1+\exp\left[-2a\frac{p}{m}t(X-Y)\right]} \end{aligned} \tag{2.67}$$

and

$$\begin{aligned} \dot{Y} &= \frac{1}{m}\Re\left[\frac{|\psi_L|^2\,\psi_{\mathrm{R}}^{+p^*}\boldsymbol{p}_Y\psi_{\mathrm{R}}^{+p}+|\psi_L^{-p}|^2\,\psi_{\mathrm{R}}^*\boldsymbol{p}_Y\psi_{\mathrm{R}}}{|\psi_L|^2\,|\psi_R^{+p}|^2+|\psi_L^{-p}|^2\,|\psi_R|^2}\right] \\ &= \frac{\frac{p}{m}}{1+\exp\left[-2a\frac{p}{m}t(Y-X)\right]} \end{aligned} \tag{2.68}$$

Again from equations (2.67) and (2.68) it is seen that after the interaction, both of the detectors start moving; but as before the asymptotic behavior of these equations determines if they will continue or will stop recording.

The positions of the particles, X and Y, at time t can be found by a similar procedure; that is by dividing the time interval $[t_0, t]$ to N equal intervals Δt and calculating X and Y at

$$t=0\begin{cases} X = X_0 \\ Y = Y_0 \\ \dot{X} = \frac{1}{2}\frac{p}{m} \\ \dot{Y} = \frac{1}{2}\frac{p}{m} \end{cases}$$

$$t=\Delta t\begin{cases} X = X_0 + \frac{1}{2}\frac{p}{m}\Delta t \\ Y = Y_0 + \frac{1}{2}\frac{p}{m}\Delta t \\ \dot{X} = \frac{\frac{p}{m}}{1+\exp[-2a\frac{p}{m}\Delta t(X_0-Y_0)]} = \left(\frac{1}{2}+\alpha_1\right)\frac{p}{m} \\ \dot{Y} = \frac{\frac{p}{m}}{1+\exp[-2a\frac{p}{m}\Delta t(Y_0-X_0)]} = \left(\frac{1}{2}-\alpha_1\right)\frac{p}{m} \end{cases}$$

$$t=2\Delta t\begin{cases} X = X_0 + (\frac{2}{2}+\alpha_1)\frac{p}{m}\Delta t \\ Y = Y_0 + (\frac{2}{2}-\alpha_1)\frac{p}{m}\Delta t \\ \dot{X} = \frac{\frac{p}{m}}{1+\exp\left[-4a\frac{p}{m}\Delta t(X_0-Y_0+2\alpha_1\frac{p}{m}\Delta t)\right]} = \left(\frac{1}{2}+\alpha_1+\alpha_2\right)\frac{p}{m} \\ \dot{Y} = \frac{\frac{p}{m}}{1+\exp\left[-4a\frac{p}{m}\Delta t(Y_0-X_0-2\alpha_1\frac{p}{m}\Delta t)\right]} = \left(\frac{1}{2}-\alpha_1-\alpha_2\right)\frac{p}{m} \end{cases}$$

$$t=3\Delta t\begin{cases} X = X_0 + (\frac{3}{2}+2\alpha_1+\alpha_2)\frac{p}{m}\Delta t \\ Y = Y_0 + (\frac{3}{2}-2\alpha_1-\alpha_2)\frac{p}{m}\Delta t \\ \dot{X} = \frac{\frac{p}{m}}{1+\exp[-6a\frac{p}{m}\Delta t(X_0-Y_0+(4\alpha_1+2\alpha_2)\frac{p}{m}\Delta t)]} = \left(\frac{1}{2}+\alpha_1+\alpha_2+\alpha_3\right)\frac{p}{m} \\ \dot{Y} = \frac{\frac{p}{m}}{1+\exp[-6a\frac{p}{m}\Delta t(Y_0-X_0-(4\alpha_1+2\alpha_2)\frac{p}{m}\Delta t)]} = \left(\frac{1}{2}-\alpha_1-\alpha_2-\alpha_3\right)\frac{p}{m} \end{cases}$$

$$\vdots$$

$$t = N\Delta t \begin{cases} X = X_0 + \left(\frac{N}{2} + \sum_{i=1}^{N}(N-i)\alpha_i\right)\frac{p}{m}\Delta t \\ Y = Y_0 + \left(\frac{N}{2} - \sum_{i=1}^{N}(N-i)\alpha_i\right)\frac{p}{m}\Delta t \\ \dot{X} = \dfrac{\frac{p}{m}}{1+\exp\left[-2Na\frac{p}{m}\Delta t\left(X_0-Y_0+2\sum_{i=1}^{N}(N-i)\alpha_i\frac{p}{m}\Delta t\right)\right]} = \left(\frac{1}{2}+\sum_{i=1}^{N}\alpha_i\right)\frac{p}{m} \\ \dot{Y} = \dfrac{\frac{p}{m}}{1+\exp\left[-2Na\frac{p}{m}\Delta t\left(Y_0-X_0-2\sum_{i=1}^{N}(N-i)\alpha_i\frac{p}{m}\Delta t\right)\right]} = \left(\frac{1}{2}-\sum_{i=1}^{N}\alpha_i\right)\frac{p}{m} \end{cases}$$

In the above equations we have used the equality $\dot{X} + \dot{Y} = \frac{p}{m}$. Also as before it is assumed that N is very large (in fact $N \to \infty$). It is seen that if $X_0 - Y_0$ is positive/negative then α's are always positive/negative, so $\dot{X}$ is strictly increasing/decreasing and $\dot{Y}$ is strictly decreasing/increasing. However, $\dot{X}$ and $\dot{Y}$ are always positive or at most zero, so if $\dot{X}$ or $\dot{Y}$ is decreasing then it must approach to zero. The overall result is that if $X_0 - Y_0 < 0$ then equation (2.67) shows that the left detector stops moving while the right detector according to equation (2.68) continues moving. On the other hand if $X_0 - Y_0 > 0$, the left detector continues moving while the right detector stops moving. These theoretical predictions are in agreement with the experimental results, where only one detector will record the photon.

The case where $X_0 - Y_0 = 0$ of course remains ambiguous, as both of the detectors continue to record the photon.

Chapter 3

Non-locality

3.1 EPR and Bell's theorem

The completeness of quantum theory was seriously criticized by A. Einstein, B. Podolsky and N. Rosen in 1935 [EPR35] in the form of the so called EPR theorem. There is no way to escape this conclusion that quantum mechanics is not complete, unless one abandon *local realism*, according to which the probability of the outcome of a measurement on one system does not depend by any means on any measurement that is done on the other system which has interacted with it sometime in the past.

The EPR theorem is based on two assumptions, the first is the necessary condition for completeness of a physical theory which states that *"every element of the physical reality, must have a counterpart in the physical theory"*. Here, by physical reality we mean *objective reality* which exists independent of our mind. The second assumption is the sufficient condition of physical reality which states that *"If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a phys-*

ical quantity, then there exists an element of physical reality corresponding to this physical quantity". From this assumption one immediately concludes that in quantum theory only the eigen-states of an observable are elements of physical reality, and of course the eigen-values are the values assigned to these elements.

The EPR theorem starts with consideration of two systems 1 and 2 which had interacted in the past. Due to this interaction the wave-function of the composite system is, in general, entangled and the wave-function of the correlated systems may be such that it could be expanded in the following two ways:

$$\Psi(1,2) = \sum_n \psi_n^{A_1}(1)\varphi_n^{A_2}(2) \tag{3.1}$$

$$\Psi(1,2) = \sum_m \psi_m^{B_1}(1)\varphi_m^{B_2}(2) \tag{3.2}$$

where $\psi_n^{A_1}(1)$ and $\psi_m^{B_1}(1)$ are eigen-functions of two non-commuting hermitian operators A_1 and B_1 on subspace 1 and $\varphi_n^{A_2}(2)$ and $\varphi_m^{B_2}(2)$ are eigen-functions of two hermitian operators A_2 and B_2 on subspace 2 respectively. For an example of this situation the interested reader is referred to [EPR35]. It should be noted that in the example given by EPR A_2 and B_2 do not commute either, however it is not a necessary condition for this argument.

One would make the following two different measurements on system 2:

I– Make a measurement of the observable $\mathcal{A}_2$ (corresponding to operator A_2). The wave-function $\Psi(1,2)$ then collapses to $\psi_n^{A_1}(1)\varphi_n^{A_2}(2)$. So system 1 is now left in an eigen-state whose eigen-function is $\psi_n^{A_1}(1)$

and the eigen-value is, say, $\lambda_n^{A_1}$. This means that the observable $\mathcal{A}_1$ is an element of reality because, in accordance with the sufficient condition of the physical reality, without disturbing the system and with probability equal to unity we have predicted its value, that is $\lambda_n^{A_1}$.

II- Make a measurement of the observable $\mathcal{B}_2$ (corresponding to operator B_2). The wave-function $\Psi(1,2)$ then collapses to $\psi_m^{B_1}(1)\varphi_m^{B_2}(2)$. With the same argument as in part (I) it is seen that the observable $\mathcal{B}_1$ is an element of physical reality with value , say, $\lambda_m^{B_1}$.

Now let's suppose that the two non-commuting observables $\mathcal{A}_1$ and $\mathcal{B}_1$ are simultaneous elements of physical reality and do have simultaneous values. As in orthodox quantum theory the prediction of these two values is not possible simultaneously, so in accordance with the necessary condition for completeness of a physical theory we conclude that the wave-function does not give a complete description of the physical reality and that beside the wave-function there must be some other parameters which enables the description of two simultaneous reality corresponding to two non-commuting observables. That is the quantum theory in the present form does not give a complete description of reality.

On the other hand, if two non-commuting observables $\mathcal{A}_1$ and $\mathcal{B}_1$ are not simultaneous elements of physical reality then the above argument that we made about the incompleteness of the quantum theory does not apply. However, in this case at any time only one of the observables $\mathcal{A}_1$ or $\mathcal{B}_1$ is an element of reality, but as we saw this depends on the measurement done on system 2. That is, had we measured the observable $\mathcal{A}_2$ ($\mathcal{B}_2$) then the value of the observable $\mathcal{A}_1$ ($\mathcal{B}_1$) would be $\lambda_n^{A_1}$ ($\lambda_m^{B_1}$) with probability equal to unity and there would be a probability less than 1 to know the value of the observable $\mathcal{B}_1$ ($\mathcal{A}_1$). So the overall result is that the probability of outcome of a measurement on system 1 depends on the measurement that has been done on system 2. But how does system 1 know really which measurement is done on system 2? This is only possible through a non-local interaction between the two system which must of course act simultaneously. Thus we are left with non-locality in quantum theory.

Inspired by the work of EPR, in 1964 John S. Bell showed that local deterministic hidden-variable theories are inconsistent with orthodox quantum theory [Bel64]. He considered a system of two spin $\frac{1}{2}$ particles in the singlet state which had interacted with each other sometime in the past and then separated by a large distance needed to prevent the interaction between these two particles. Denote the component of the spin of particle 1(2) in a given direction $\boldsymbol{u}$ by σ_1^u (σ_2^u) and $M_1^u(M_2^u)$ the outcome of the measurement of spin of particle 1(2) in the direction $\boldsymbol{u}$. If λ is a set of hidden-variables which (including the wave-function) give a complete description of the state of the system then from locality definition we have:

$$M_1^u(\lambda) = \pm 1 \qquad \text{and} \qquad M_2^u(\lambda) = \pm 1 \tag{3.3}$$

$$\int d\lambda \rho(\lambda) = 1 \tag{3.4}$$

where $\rho(\lambda)$ is the probability distribution of λ. So the expectation value of the observable $\sigma_1^a \sigma_2^b$ would be

$$\mathcal{E}_{12}^{ab} = \int d\lambda \rho(\lambda) M_1^a(\lambda) M_2^b(\lambda) \tag{3.5}$$

Here Bell's assumption is that there is a perfect correlation between two particles, so if for both system we use the same direction $\boldsymbol{a}$ (or if the polarizers on both sides are parallel) then

$$M_1^a(\lambda) = -M_2^a(\lambda) \tag{3.6}$$

Note that this ideal condition is impossible to fulfill in the real experiments in which for example the detector efficiency is less than 100% and this makes Bell's original theorem untestable.

With the help of (3.5) and (3.6) one can write

$$\begin{aligned} \mathcal{E}_{12}^{ab} - \mathcal{E}_{12}^{ac} &= -\int d\lambda \rho(\lambda)[M_1^a(\lambda)M_1^b(\lambda) - M_1^a(\lambda)M_1^c(\lambda)] \\ &= \int d\lambda \rho(\lambda) M_1^a(\lambda)M_1^b(\lambda)[M_1^b(\lambda)M_1^c(\lambda) - 1] \end{aligned}$$

The upper(lower) bound of the left-hand side is obtained if on the right-hand side we put $M_1^a(\lambda)M_1^b(\lambda) = +1(-1)$ so

$$|\mathcal{E}_{12}^{ab} - \mathcal{E}_{12}^{ac}| \leq \int d\lambda \rho(\lambda)[1 - M_1^b(\lambda)M_1^c(\lambda)] \tag{3.7}$$

or

$$|\mathcal{E}_{12}^{ab} - \mathcal{E}_{12}^{ac}| \leq 1 + \mathcal{E}_{12}^{bc} \tag{3.8}$$

This is the well known Bell inequality. He then shows that this inequality is violated under some especial conditions by statistical predictions of quantum theory. After the work of Bell, some other Bell-type inequalities were derived which were, just as Bell inequality, violated by statistical prediction of quantum theory but didn't make use of equation (3.6) so can be tested in real experiments [CHSH69, FC72, CH74, CS78]. Among these are Clauser, Horne, Shimony and Holt (CHSH) inequality which reads [CHSH69]:

$$|\mathcal{E}_{12}^{ab} + \mathcal{E}_{12}^{ab'} + \mathcal{E}_{12}^{a'b'} - \mathcal{E}_{12}^{a'b}| \leq 2 \tag{3.9}$$

Although this inequality was originally derived based on the local deterministic hidden-variable theories but later it was derived with only locality assumption by Bell.

In the next two sections we develop a new method for obtaining Bell-type inequalities by which not only the total number of inequalities are increased but in the next chapter we will show how and the conditions under which some of them may be violated by orthodox quantum theory.

3.2 Bell-type inequalities – case 2,2:2,2 outputs

As we pointed out in the previous section the Bell inequalities enable us to test locality, and this is the motivation for deriving more inequalities which can be tested and can be violated by quantum theory by a stronger factor, that is, for example in equation (3.9) the ratio of the value of the left-hand side (predicted by quantum theory) to its upper bound (2) would become as large as possible. Our aim in this section is to introduce a method for deducing a number of Bell-type inequalities some of which were obtained by Clauser-Horne [CH74].

Consider a system which consists of two parts far from each other such that there is no (known) interaction between them. A set of experiments are done on each part independent of the other. The scheme of such experiments are shown in figure 3.1. On the left(right) arm, an experiment is specified by the parameter setting $i(j)$ which we call local variables and for each setting $i(j)$ there are $M_i(N_j)$ outcomes.

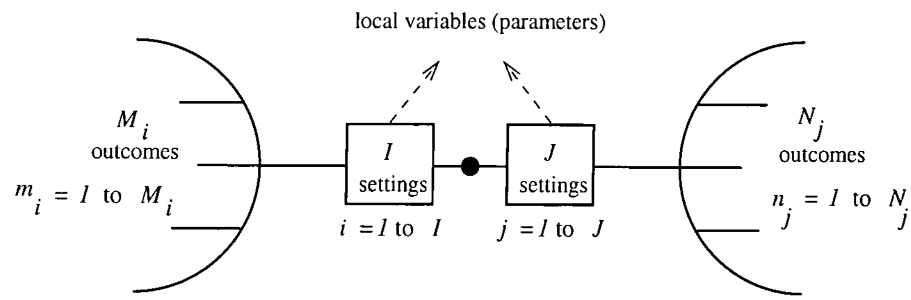

Figure 3.1: The scheme of the experiments carried out on two parts of a system with variable settings and outcome on each arm, for deducing Bell-type inequalities.

Now we define the joint probability $p_{ij}^{m_i n_j}$ as:

$p_{ij}^{m_i n_j}$ $\equiv$ The probability that if the local variable on the left arm is set to i and the local variable on the right arm is set to j, then the outcome on the left would be m_i and the outcome on the right would be n_j.

In this section we consider the case where $I = J = M_1 = N_1 = M_2 = N_2 = 2$.

There are totally 16 possible combinations as:

$$\begin{array}{cccccccc} p_{11}^{11} & p_{11}^{12} & p_{11}^{21} & p_{11}^{22} & p_{12}^{11} & p_{12}^{12} & p_{12}^{21} & p_{12}^{22} \\ p_{21}^{11} & p_{21}^{12} & p_{21}^{21} & p_{21}^{22} & p_{22}^{11} & p_{22}^{12} & p_{22}^{21} & p_{22}^{22} \end{array}$$

These are not all independent. To reduce the number of independent joint probabilities, we impose two types of constraints:

I– As the individual probability distribution on each arm is normalized then for each setting on two arms, the sum of all joint probabilities add up to unity:

$$\sum_{m,n} p_{ij}^{mn} = 1 \qquad i = 1,2 \;\; j = 1,2 \tag{3.10}$$

Note that although the above equation may be derived simply by defining the joint probability as the product of individual probabilities, we are not assuming this here. This is the assumption of Clauser-Horne for objective local theories. We deduce this relation from total possible combinations as stated above.

The above constraint results in 4 equations:

$$p_{11}^{11} + p_{11}^{12} + p_{11}^{21} + p_{11}^{22} \quad = 1 \tag{3.11}$$

$$p_{12}^{11} + p_{12}^{12} + p_{12}^{21} + p_{12}^{22} \quad = 1 \tag{3.12}$$

$$p_{21}^{21} + p_{21}^{12} + p_{21}^{21} + p_{21}^{22} \quad = 1 \tag{3.13}$$

$$p_{22}^{11} + p_{22}^{12} + p_{22}^{21} + p_{22}^{22} \quad = 1 \tag{3.14}$$

II– If the sum of all joint probabilities on one arm, for a particular setting, e.g. $j = 1$ on the right, is p, the settings and outcomes on the other arm, e.g. $i = 1, m = 1$ on the left, does not affect this.[1] This is due to the assumption of signal locality, that is the experiment on one arm does not disturb the experiment on the arm instantaneously otherwise

[1]If the two parts of the system are not correlated then p is simply the probability of the outcome on the other arm.

the signals would be sent faster than the speed of light. So we have:

$$\sum_n p_{ij}^{mn} = \sum_n p_{ik}^{mn} \quad i = 1,2 \;\; j = 1,2 \;\; k = 1,2 \tag{3.15}$$

$$\sum_m p_{ij}^{mn} = \sum_m p_{lj}^{mn} \quad i = 1,2 \;\; j = 1,2 \;\; l = 1,2 \tag{3.16}$$

The above constraints result in 2 groups of equations.

From equation (3.15) we have:

$$p_{11}^{11} + p_{11}^{12} = p_{12}^{11} + p_{12}^{12} \tag{3.17}$$

$$p_{11}^{21} + p_{11}^{22} = p_{12}^{21} + p_{12}^{22} \tag{3.18}$$

$$p_{21}^{11} + p_{21}^{12} = p_{22}^{11} + p_{22}^{12} \tag{3.19}$$

$$p_{21}^{21} + p_{21}^{22} = p_{22}^{21} + p_{22}^{22} \tag{3.20}$$

And from equation (3.16) we have:

$$p_{11}^{11} + p_{11}^{21} = p_{21}^{11} + p_{21}^{21} \tag{3.21}$$

$$p_{11}^{12} + p_{11}^{22} = p_{21}^{12} + p_{21}^{22} \tag{3.22}$$

$$p_{12}^{11} + p_{12}^{21} = p_{22}^{11} + p_{22}^{21} \tag{3.23}$$

$$p_{12}^{12} + p_{12}^{22} = p_{22}^{12} + p_{22}^{22} \tag{3.24}$$

Equations (3.11) through (3.14) and (3.17) through (3.24) are not all independent. The latter 4 equations (3.21 – 3.24) can be written in terms of the others.

For example, addition of equations (3.17) and (3.18) results:

$$p_{11}^{11} + p_{11}^{12} + p_{11}^{21} + p_{11}^{22} = p_{12}^{11} + p_{12}^{12} + p_{12}^{21} + p_{12}^{22}$$

According to equation (3.12) the right-hand side is 1, and by use of (3.13) and (3.22) we get:

$$p_{11}^{11} + p_{11}^{21} = p_{21}^{11} + p_{21}^{21}$$

which is equation (3.21). So we expect 8 independent joint probabilities and 8 independent quantities can be defined in terms of these[2].

To do this let's define p_{ij}, p_i^L and p_j^R as followings:

p_{ij} $\equiv$ The joint probability that the outcome on both arms is 1 – for settings i on the left arm and j on the right arm.That is:

$$p_{ij} = p_{ij}^{11} \qquad i,j = 1,2 \qquad \text{(4 equations)} \tag{3.25}$$

p_i^L $\equiv$ The joint probability that the outcome on the left arm is 1, whatever the outcome on the right is – for settings i on the left arm and j on the right arm.That is:

$$p_i^L = p_{ij}^{11} + p_{ij}^{12} \qquad i,j = 1,2 \qquad \text{(2 equations)} \tag{3.26}$$

Similarly,

$$p_j^R = p_{ij}^{11} + p_{ij}^{21} \qquad i,j = 1,2 \qquad \text{(2 equations)} \tag{3.27}$$

From equation (3.15) and (3.16) it is seen that p_i^L is independent of j, and p_j^R is independent of i.

So 4 of the independent quantities are defined from (3.25) as:

$$p_{11} = p_{11}^{11}; \quad p_{12} = p_{12}^{11}; \quad p_{21} = p_{21}^{11}; \quad p_{22} = p_{22}^{11} \tag{3.28}$$

[2]At this stage we only claim (with no proof) that there are exactly 8 independent joint probabilities because no other constraints can be found and that none of the 8 quantities that we define below can be written in terms of the others, however this is verified later when we find non-singular matrices whose dimensions are equal to the number of independent joint probabilities. Although this type of reasoning seems weak, it is enough for our purpose here

and 2 of them from (3.26) as:

$$\begin{aligned} p_1^L &= p_{11}^{11} + p_{11}^{12} && i=1,\ \ j=1 \\ &= p_{12}^{11} + p_{12}^{12} && i=1,\ \ j=2 \qquad (3.29) \\ p_2^L &= p_{21}^{11} + p_{21}^{12} && i=2,\ \ j=1 \\ &= p_{22}^{11} + p_{22}^{12} && i=2,\ \ j=2 \qquad (3.30) \end{aligned}$$

and the last 2 from (3.27) as:

$$\begin{aligned} p_1^R &= p_{11}^{11} + p_{11}^{21} && i=1,\ \ j=1 \\ &= p_{21}^{11} + p_{21}^{21} && i=2,\ \ j=1 \qquad (3.31) \\ p_2^R &= p_{12}^{11} + p_{12}^{21} && i=1,\ \ j=2 \\ &= p_{22}^{11} + p_{22}^{21} && i=2,\ \ j=2 \qquad (3.32) \end{aligned}$$

Now we use an algorithm to find the governing relations among p's. At this point let's define yet another double joint probability $\gamma^{\alpha\beta\eta\zeta}$ as:

$\gamma^{\alpha\beta\eta\zeta} \equiv$ The probability that

if the setting on the left is set to $i = 1$, then the outcome on that arm is α where $\alpha = \begin{cases} 1 & if \quad m=1 \\ 0 & if \quad m=2 \end{cases}$ and

if the setting on the left is set to $i = 2$, then the outcome on that arm is β where $\beta = \begin{cases} 1 & if \quad m=1 \\ 0 & if \quad m=2 \end{cases}$ and

if the setting on the right is set to $j = 1$, then the outcome on that arm is η where $\eta = \begin{cases} 1 & if \quad n=1 \\ 0 & if \quad n=2 \end{cases}$ and finally

if the setting on the right is set to $j = 2$, then the outcome on that arm is ζ where $\zeta = \begin{cases} 1 & if \quad n=1 \\ 0 & if \quad n=2 \end{cases}$

So according to the above definition

$$\gamma^{0110} = \text{probability that } \begin{pmatrix} i=1 \\ m=2 \end{pmatrix} \& \begin{pmatrix} i=2 \\ m=1 \end{pmatrix} \& \begin{pmatrix} j=1 \\ n=1 \end{pmatrix} \& \begin{pmatrix} j=2 \\ n=2 \end{pmatrix}$$

Using equation (3.10) it can be shown that

$$\sum_{\alpha,\beta,\eta,\varsigma} \gamma^{\alpha\beta\eta\varsigma} = 1 \qquad \text{where} \qquad 0 \leq \gamma^{\alpha\beta\eta\varsigma} \leq 1 \tag{3.33}$$

It is worth noting that the definitions of $\gamma^{\alpha\beta\eta\varsigma}$ implicitly implies locality, as we are assuming that there is a joint probability for two measurement which are not simultaneous. In terms of γ's

$$p_{11} = \gamma^{1010} + \gamma^{1011} + \gamma^{1110} + \gamma^{1111}$$

$$p_{12} = \gamma^{1001} + \gamma^{1011} + \gamma^{1101} + \gamma^{1111}$$

$$p_{21} = \gamma^{0110} + \gamma^{1110} + \gamma^{0111} + \gamma^{1111}$$

$$p_{22} = \gamma^{0101} + \gamma^{1101} + \gamma^{0111} + \gamma^{1111}$$

$$p_1^L = \gamma^{1010} + \gamma^{1011} + \gamma^{1110} + \gamma^{1111} + \gamma^{1000} + \gamma^{1001} + \gamma^{1100} + \gamma^{1101}$$
$$\text{for} \quad i=1, \quad j=1$$

$$p_1^L = \gamma^{1001} + \gamma^{1011} + \gamma^{1101} + \gamma^{1111} + \gamma^{1000} + \gamma^{1010} + \gamma^{1100} + \gamma^{1110}$$
$$\text{for} \quad i=1, \quad j=2$$

$$p_2^L = \gamma^{0110} + \gamma^{0111} + \gamma^{1110} + \gamma^{1111} + \gamma^{0100} + \gamma^{0101} + \gamma^{1100} + \gamma^{1101}$$
$$\text{for} \quad i=2, \quad j=1$$

$$p_2^L = \gamma^{0101} + \gamma^{0111} + \gamma^{1101} + \gamma^{1111} + \gamma^{0100} + \gamma^{0110} + \gamma^{1100} + \gamma^{1110}$$
$$\text{for} \quad i=2, \quad j=2$$

$$p_1^R = \gamma^{1010} + \gamma^{1011} + \gamma^{1110} + \gamma^{1111} + \gamma^{0010} + \gamma^{0011} + \gamma^{0110} + \gamma^{0111}$$

$$\text{for} \quad i = 1, \quad j = 1$$

$$p_1^R = \gamma^{0110} + \gamma^{0111} + \gamma^{1110} + \gamma^{1111} + \gamma^{0010} + \gamma^{0011} + \gamma^{1010} + \gamma^{1011}$$

$$\text{for} \quad i = 2, \quad j = 1$$

$$p_2^R = \gamma^{1001} + \gamma^{1011} + \gamma^{1101} + \gamma^{1111} + \gamma^{0001} + \gamma^{0011} + \gamma^{0101} + \gamma^{0111}$$

$$\text{for} \quad i = 1, \quad j = 2$$

$$p_2^R = \gamma^{0101} + \gamma^{0111} + \gamma^{1101} + \gamma^{1111} + \gamma^{0001} + \gamma^{0011} + \gamma^{1001} + \gamma^{1011}$$

$$\text{for} \quad i = 2, \quad j = 2$$

In terms of matrices the above relations would be

$$\begin{pmatrix} p_{11} \\ p_{12} \\ p_{21} \\ p_{22} \\ p_1^L \\ p_2^L \\ p_1^R \\ p_2^R \end{pmatrix} = \begin{bmatrix} 0&0&0&0&0&0&0&0&0&0&1&1&0&0&1&1 \\ 0&0&0&0&0&0&0&0&0&1&0&1&0&1&0&1 \\ 0&0&0&0&0&0&1&1&0&0&0&0&0&0&1&1 \\ 0&0&0&0&0&1&0&1&0&0&0&0&0&1&0&1 \\ 0&0&0&0&0&0&0&0&1&1&1&1&1&1&1&1 \\ 0&0&0&0&1&1&1&1&0&0&0&0&1&1&1&1 \\ 0&0&1&1&0&0&1&1&0&0&1&1&0&0&1&1 \\ 0&1&0&1&0&1&0&1&0&1&0&1&0&1&0&1 \end{bmatrix} \begin{pmatrix} \gamma^{0000} \\ \gamma^{0001} \\ \gamma^{0010} \\ \gamma^{0011} \\ \gamma^{0100} \\ \gamma^{0101} \\ \gamma^{0110} \\ \gamma^{0111} \\ \gamma^{1000} \\ \gamma^{1001} \\ \gamma^{1010} \\ \gamma^{1011} \\ \gamma^{1100} \\ \gamma^{1101} \\ \gamma^{1110} \\ \gamma^{1111} \end{pmatrix} \tag{3.34}$$

Equation (3.34) can be written as $\mathrm{p} = \mathrm{x}\,\Gamma$ where p is the column vector of probabilities, and x is the conversion matrix and Γ is the column vector of γ's. Provided the matrix $\mathrm{x}^T\mathrm{x}$ is non-singular one can multiply both sides of p$=$xΓ by tranpose of x, that is x^T, and again multiply both sides by $(x^T x)^{-1}$ to find Γ in terms of p, and finally construct the possible inequalities from the condition (3.33). However, in our case $x^T x$ is singular and so in the followings we will use another procedure to find these inequalities (see [Noc95]).

To make the calculations easier let's introduce some new parametric notations. Equations (3.33) and (3.34), can be written respectively as:

$$\sum_{t=1}^{u} x_{v+1,t}\gamma^t = 1 \quad x_{v+1,t} = 1 \ \text{ for all } \ t \tag{3.35}$$

$$\sum_{t=1}^{u} x_{st}\gamma^t = p_s, \quad s = 1, \cdots, v \tag{3.36}$$

Here $u = 16$, $v = 8$. Combining the above two equations we get:

$$\begin{pmatrix} p_1 \\ \vdots \\ p_v \\ 1 \end{pmatrix} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1u} \\ x_{21} & x_{22} & \cdots & x_{2u} \\ & & \vdots & \\ x_{v1} & x_{v2} & \cdots & x_{vu} \\ x_{v+1,1} & x_{v+2,2} & \cdots & x_{v+1,u} \end{bmatrix} \begin{pmatrix} \gamma^1 \\ \gamma^2 \\ \vdots \\ \gamma^u \end{pmatrix} \tag{3.37}$$

or simply

$$P = X\Gamma \tag{3.38}$$

If only y of p's are linearly independent then we may write:

$$X\{(v+1)\times u\} = \begin{bmatrix} X_1\{y\times y\} & X_2\{y\times(u-y)\} \\ X_3\{(v+1-y)\times y\} & X_4\{(v+1-y)\times(u-y)\} \end{bmatrix} \tag{3.39}$$

$$\Gamma\{u\times 1\} = \begin{pmatrix} \sigma\{y\times 1\} \\ \tau\{(u-y)\times 1\} \end{pmatrix} \tag{3.40}$$

$$P\{(v+1)\times 1\} = \begin{pmatrix} \Pi_1\{y\times 1\} \\ \Pi_2\{(v+1-y)\times 1\} \end{pmatrix} \tag{3.41}$$

where in the above equations the size of the matrices are explicitly shown inside the braces. If $\det X_1 \neq 0$ then[3]:

$$X_1\sigma + X_2\tau = \Pi_1 \tag{3.42}$$

$$\sigma = X_1^{-1}\Pi_1 - X_1^{-1}X_2\tau \tag{3.43}$$

From equation (3.33) one can easily conclude that:

$$0 \leq \sum_{t=1}^{u} l_t\gamma^t \leq 1 \quad \text{where} \quad l_t = 0, 1 \tag{3.44}$$

The above equation can be written in the form:

$$0 \leq L\Gamma \leq 1 \tag{3.45}$$

where L is a row vector whose elements are 0 or 1 and we write it as:

$$L\{1 \times u\} = \big(r\{1 \times y\} \;\; q\{1 \times (u-y)\}\big). \tag{3.46}$$

From equation (3.45) and (3.40)we have:

$$0 \leq r\sigma + q\tau \leq 1 \tag{3.47}$$

and from (3.43) we have

$$0 \leq rX_1^{-1}\Pi_1 - rX_1^{-1}X_2\tau + q\tau \leq 1 \tag{3.48}$$

$$0 \leq rX_1^{-1}\Pi_1 + (q - rX_1^{-1}X_2)\tau \leq 1 \tag{3.49}$$

If we can find r and q such that:

$$q - rX_1^{-1}X_2 = 0 \tag{3.50}$$

[3]If $\det X_1 = 0$ then other possible $y \times y$ matrices deducible from the matrix X may be used by rearrangement of columns and rows of X (and of course accordingly that of P and Γ).

then

$$0 \leq rX_1^{-1}\Pi_1 \leq 1 \tag{3.51}$$

This gives the inequalities among p's.

For the special case that we are considering the matrix X_1 is of dimension 8×8 with $y = 8$, or 9×9 with $y = 9$. So from the 8×16 matix in equation (3.34) ($v = 8, u = 16$), we have to choose those 8×8 matrices which are non-singular such that equation (3.50) is satisfied for any arbitrary q and r. There are totally 2304 8×8 non-singular matrices which were calculated by computer. One of them, constructed from columns 2, 3, 5, 6, 7, 12, 13 and 14 is:

$$X_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix} \tag{3.52}$$

$$\det X_1 = 1 \tag{3.53}$$

and the inverse of X_1 is:

$$X_1^{-1} = \begin{bmatrix} -1 & 0 & 0 & -1 & 0 & 0 & 0 & 1 \\ -1 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & -1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{3.54}$$

The matrix X_2 is constructed from the rest of the columns 1, 4, 8, 12, 13, 14, 15 and16 that is:

$$X_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \tag{3.55}$$

It is seen that for

$$q = (\ 0\,1\,0\,0\,1\,0\,0\,0\ ) \ \text{ and } \ r = (\ 0\,1\,1\,0\,1\,1\,1\,1\ ), \tag{3.56}$$

equation (3.50) is satisfied and from equation (3.51) we have:

$$-1 \leq p_{11} - p_{12} + p_{21} + p_{22} - p_2^L - p_1^R \leq 0 \tag{3.57}$$

which is the Clauser-Horne inequality.

For the case of 8×8 and 9×9 matrices, we have derived 44 and 89 inequalities respectively which are shown in Appendix **A**. However, note that the inequality No. 1 is exactly the same as inequality No. 89, the inequality No. 2 is exactly the same as inequality No. 88, and so on. In fact the independent inequalities derived by this method are those which are obtained by considering the 8×8 matrices only. These includes all of the Clauser-Horne-type inequalities derived by Noce [Noc95] plus 4 more interesting inequalities as the following:

$$0 \leq \ -2p_{11} \ +p_1^L \ +p_1^R \ \leq 1 \tag{3.58}$$

in which a coefficient of 2 appears.

3.3 Bell-type inequalities – case 2,3:2,3 outputs

The method developed in the previous section for obtaining Bell-type inequalities can easily be extended to the higher orders. Here we consider the case where as before, on each arm there are two local variables but for one of the variables there are two outputs and for the other there are three outputs that is in figure 3.1 we put $I = J = M_1 = N_1 = 2$ and $M_2 = N_2 = 3$ which is in fact the next simplest case.

With the same definition for $p_{ij}^{m_i n_j}$ there would be 25 possible combinations which are:

$$\begin{array}{lllllllll}
p_{11}^{11} & p_{11}^{12} & p_{11}^{21} & p_{11}^{22} & & & & & \\
p_{12}^{11} & p_{12}^{12} & p_{12}^{13} & p_{12}^{21} & p_{12}^{22} & p_{12}^{23} & & & \\
p_{21}^{11} & p_{21}^{12} & p_{21}^{21} & p_{21}^{22} & p_{21}^{31} & p_{21}^{32} & & & \\
p_{22}^{11} & p_{22}^{12} & p_{22}^{13} & p_{22}^{21} & p_{22}^{22} & p_{22}^{23} & p_{22}^{31} & p_{22}^{32} & p_{22}^{33}
\end{array}$$

Imposing the two types of constraints discussed in section 3.2 the independent probabilities reduces to 16. Now we define the following new set of quantities:

$$p_{ij} = p_{ij}^{11} \qquad i,j = 1,2 \qquad \text{(4 equations)} \tag{3.59}$$

$$p_i^L = \sum_{n_j} p_{ij}^{1n_j} \qquad i,j = 1,2 \qquad \text{(2 equations)} \tag{3.60}$$

$$p_j^R = \sum_{m_i} p_{ij}^{m_i 1} \qquad i,j = 1,2 \qquad \text{(2 equations)} \tag{3.61}$$

Again it can be shown that p_i^L is independent of j, and p_j^R is independent of i.

From (3.59) we get 4 of the independent quantities as:

$$p_{11} = p_{11}^{11}\ ;\quad p_{12} = p_{12}^{11}\ ;\quad p_{21} = p_{21}^{11}\ ;\quad p_{22} = p_{22}^{11} \tag{3.62}$$

and 2 from (3.60):

$$\begin{aligned} p_1^L &= p_{11}^{11} + p_{11}^{12} \qquad i=1,\ j=1 \\ &= p_{12}^{11} + p_{12}^{12} + p_{12}^{13} \qquad i=1,\ j=2 \end{aligned} \tag{3.63}$$

$$\begin{aligned} p_2^L &= p_{21}^{11} + p_{21}^{12} \qquad i=2,\ j=1 \\ &= p_{22}^{11} + p_{22}^{12} + p_{22}^{13} \qquad i=2,\ j=2 \end{aligned} \tag{3.64}$$

and 2 from (3.61):

$$\begin{aligned} p_1^R &= p_{11}^{11} + p_{11}^{21} \qquad i=1,\ j=1 \\ &= p_{21}^{11} + p_{21}^{21} + p_{21}^{31} \qquad i=2,\ j=1 \end{aligned} \tag{3.65}$$

$$\begin{aligned} p_2^R &= p_{12}^{11} + p_{12}^{21} \qquad i=1,\ j=2 \\ &= p_{22}^{11} + p_{22}^{21} + p_{22}^{31} \qquad i=2,\ j=2 \end{aligned} \tag{3.66}$$

The other 8 quantities are defined as followings:

$$p_{12}^{(11,13)} = p_{12}^{11} + p_{12}^{13} \tag{3.67}$$

$$p_{12}^{(13,23)} = p_{12}^{13} + p_{12}^{23} \tag{3.68}$$

$$p_{21}^{(11,31)} = p_{21}^{11} + p_{21}^{31} \tag{3.69}$$

$$p_{21}^{(22,21)} = p_{21}^{22} + p_{21}^{21} \tag{3.70}$$

$$p_{21}^{(31,32)} = p_{21}^{31} + p_{21}^{32} \tag{3.71}$$

$$p_{22}^{(11,13)} = p_{22}^{11} + p_{22}^{13} \tag{3.72}$$

$$p_{22}^{(31,33)} = p_{22}^{31} + p_{22}^{33} \tag{3.73}$$

$$p_{22}^{(33)} = p_{22}^{33} \tag{3.74}$$

For this case $\gamma^{\alpha\beta\eta\zeta}$ is defined as:

$\gamma^{\alpha\beta\eta\zeta} \equiv$ The probability that

if the setting on the left is set to $i = 1$, then the outcome on that arm is α where $\alpha = \begin{cases} 1 & if \quad m_1 = 1 \\ 2 & if \quad m_1 = 2 \end{cases}$ and

if the setting on the left is set to $i = 2$, then the outcome on that arm is β where $\beta = \begin{cases} 1 & if \quad m_2 = 1 \\ 2 & if \quad m_2 = 2 \\ 3 & if \quad m_2 = 3 \end{cases}$ and

if the setting on the right is set to $j = 1$, then the outcome on that arm is η where $\eta = \begin{cases} 1 & if \quad n_1 = 1 \\ 2 & if \quad n_1 = 2 \end{cases}$ and finally

if the setting on the right is set to $j = 2$, then the outcome on that arm is ζ where $\zeta = \begin{cases} 1 & if \quad n_2 = 1 \\ 2 & if \quad n_2 = 2 \\ 3 & if \quad n_2 = 3 \end{cases}$

As an example, according to the above definition

$$\gamma^{1321} = \text{probability that} \begin{pmatrix} i = 1 \\ m_1 = 1 \end{pmatrix} \& \begin{pmatrix} i = 2 \\ m_2 = 3 \end{pmatrix} \& \begin{pmatrix} j = 1 \\ n_1 = 2 \end{pmatrix} \& \begin{pmatrix} j = 2 \\ n_2 = 1 \end{pmatrix}$$

The 36 gamma's are:

$$\begin{matrix} \gamma^{1111} & \gamma^{1112} & \gamma^{1113} & \gamma^{1121} & \gamma^{1122} & \gamma^{1123} \\ \gamma^{1211} & \gamma^{1212} & \gamma^{1213} & \gamma^{1221} & \gamma^{1222} & \gamma^{1223} \\ \gamma^{1311} & \gamma^{1312} & \gamma^{1313} & \gamma^{1321} & \gamma^{1322} & \gamma^{1323} \\ \gamma^{2111} & \gamma^{2112} & \gamma^{2113} & \gamma^{2121} & \gamma^{2122} & \gamma^{2123} \\ \gamma^{2211} & \gamma^{2212} & \gamma^{2213} & \gamma^{2221} & \gamma^{2222} & \gamma^{2223} \\ \gamma^{2311} & \gamma^{2312} & \gamma^{2313} & \gamma^{2321} & \gamma^{2322} & \gamma^{2323} \end{matrix}$$

Equation (3.33) still holds in this case and a similar matrix relation as in equation (3.34) can be written which reads:

$$
\begin{pmatrix}
p_{11} \\ p_{12} \\ p_{21} \\ p_{22} \\ p_1^L \\ p_2^L \\ p_1^R \\ p_2^R \\ p_{12}^{(11,13)} \\ p_{12}^{(13,23)} \\ p_{21}^{(11,31)} \\ p_{21}^{(22,21)} \\ p_{21}^{(31,32)} \\ p_{22}^{(11,13)} \\ p_{22}^{(31,33)} \\ p_{22}^{(33)}
\end{pmatrix}
=
\begin{bmatrix}
1&1&1&0&0&0&1&1&1&0&0&0&1&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
1&0&0&1&0&0&1&0&0&1&0&0&1&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
1&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
1&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
1&1&1&1&1&1&0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&1&1&1&0&0&0&0&0&0&0&0&0&0&0&0 \\
1&1&1&0&0&0&1&1&1&0&0&0&1&1&1&0&0&0&1&1&1&0&0&0&1&1&1&0&0&0&1&1&1&0&0&0 \\
1&0&0&1&0&0&1&0&0&1&0&0&1&0&0&1&0&0&1&0&0&1&0&0&1&0&0&1&0&0&1&0&0&1&0&0 \\
1&0&1&1&0&1&1&0&1&1&0&1&1&0&1&1&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&1&0&0&1&0&0&1&0&0&1&0&0&1&0&0&1&0&0&1&0&0&1&0&0&1&0&0&1&0&0&1&0&0&1 \\
1&1&1&0&0&0&0&0&0&0&0&0&1&1&1&0&0&0&1&1&1&0&0&0&0&0&0&0&0&0&1&1&1&0&0&0 \\
0&0&0&0&0&0&1&1&1&1&1&1&0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&1&1&1&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&1&1&1&0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&1&1&1 \\
1&0&1&1&0&1&0&0&0&0&0&0&0&0&0&0&0&0&1&0&1&1&0&1&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&1&1&0&1&0&0&0&0&0&0&0&0&0&0&0&0&1&0&1&1&0&1 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&1
\end{bmatrix}
\begin{pmatrix}
\gamma^{1111} \\ \gamma^{1112} \\ \gamma^{1113} \\ \gamma^{1121} \\ \gamma^{1122} \\ \gamma^{1123} \\ \gamma^{1211} \\ \gamma^{1212} \\ \gamma^{1213} \\ \gamma^{1221} \\ \gamma^{1222} \\ \gamma^{1223} \\ \gamma^{1311} \\ \gamma^{1312} \\ \gamma^{1313} \\ \gamma^{1321} \\ \gamma^{1322} \\ \gamma^{1323} \\ \gamma^{2111} \\ \gamma^{2112} \\ \gamma^{2113} \\ \gamma^{2121} \\ \gamma^{2122} \\ \gamma^{2123} \\ \gamma^{2211} \\ \gamma^{2212} \\ \gamma^{2213} \\ \gamma^{2221} \\ \gamma^{2222} \\ \gamma^{2223} \\ \gamma^{2311} \\ \gamma^{2312} \\ \gamma^{2313} \\ \gamma^{2321} \\ \gamma^{2322} \\ \gamma^{2323}
\end{pmatrix}
$$

The above relation can be written as:

$$P = X\Gamma \tag{3.75}$$

where P is the column vector of p's, Γ is the column vector of γ's and X is the 16×25 matrix. Now we can use the same procedure used in the previous section to find Bell-type inequalities for this case. The total number of independent inequalities in this case add up to 1617 which are listed in Appendix **B**. Obviously the case $2,2:2,2$ is an special case of $2,3:2,3$ so the inequalities of $2,2:2,2$ must be included in the inequalities of $2,3:2,3$. To see this let's suppose that a $2,2:2,2$ experiment in which the output 1 for setting 2 on each arm is split into two parts which we label them 1 and 3. Table 3.1 shows how the p's in the two cases are related to each other.

$$\begin{array}{llllll}
\text{Case: } 2,2:2,2 & & & & & \text{Case: } 2,3:2,3 \\
p_{11} & = p_{11}^{11} & & & \longrightarrow & p_{11} \\
p_{12} & = p_{12}^{11} & = & p_{12}^{11}+p_{12}^{13} & \longrightarrow & p_{12}^{(11,13)} \\
p_{21} & = p_{21}^{11} & = & p_{21}^{11}+p_{21}^{31} & \longrightarrow & p_{21}^{(11,31)} \\
p_{22} & = p_{22}^{11} & = & p_{22}^{11}+p_{22}^{31}+p_{22}^{13}+p_{22}^{33} & \longrightarrow & p_{22}^{(11,13)}+p_{22}^{(31,33)} \\
p_1^L & = p_{11}^{11}+p_{11}^{12} & & & \longrightarrow & p_1^L \\
p_2^L & = p_{21}^{11}+p_{21}^{12} & = & p_{21}^{11}+p_{21}^{31}+p_{21}^{12}+p_{21}^{32} & \longrightarrow & p_2^L+p_{21}^{(31,32)} \\
p_1^R & = p_{11}^{11}+p_{11}^{21} & & & \longrightarrow & p_1^R \\
p_2^R & = p_{12}^{11}+p_{12}^{21} & = & p_{12}^{11}+p_{12}^{13}+p_{12}^{21}+p_{12}^{23} & \longrightarrow & p_2^R+p_{12}^{(13,23)}
\end{array}$$

Table 3.1: Relation between the quantities of the case 2,2:2,2 (left) and the case 2,3:2,3 cases (right).

With these relations the equivalent inequalities in 2,2:2,2 and 2,3:2,3 cases of Appendices **A** and **B** are:

Case 2,3:2,3		Case 2,2:2,2	Case 2,3:2,3		Case 2,2:2,2
No. 1	$\equiv$	No. 1	No. 28	$\equiv$	No. 2
No. 29	$\equiv$	No. 3	No. 32	$\equiv$	No. 4
No. 33	$\equiv$	No. 5	No. 34	$\equiv$	No. 6
No. 35	$\equiv$	No. 7	No. 36	$\equiv$	No. 8
No. 37	$\equiv$	No. 9	No. 38	$\equiv$	No. 10
No. 39	$\equiv$	No. 11	No. 40	$\equiv$	No. 12
No. 41	$\equiv$	No. 13	No. 728	$\equiv$	No. 14
No. 753	$\equiv$	No. 15	No. 763	$\equiv$	No. 16
No. 765	$\equiv$	No. 17	No. 775	$\equiv$	No. 18
No. 782	$\equiv$	No. 19	No. 791	$\equiv$	No. 20
No. 792	$\equiv$	No. 21	No. 799	$\equiv$	No. 22
No. 808	$\equiv$	No. 23	No. 863	$\equiv$	No. 24
No. 873	$\equiv$	No. 25	No. 880	$\equiv$	No. 26
No. 906	$\equiv$	No. 27	No. 908	$\equiv$	No. 28
No. 917	$\equiv$	No. 29	No. 926	$\equiv$	No. 30
No. 933	$\equiv$	No. 31	No. 934	$\equiv$	No. 32
No. 943	$\equiv$	No. 33	No. 950	$\equiv$	No. 34
No. 951	$\equiv$	No. 35	No. 958	$\equiv$	No. 36
No. 963	$\equiv$	No. 37	No. 964	$\equiv$	No. 38
No. 969	$\equiv$	No. 39	No. 976	$\equiv$	No. 40
No. 1587	$\equiv$	No. 41	No. 1588	$\equiv$	No. 42
No. 1591	$\equiv$	No. 43	No. 1592	$\equiv$	No. 44

The extension of the method discussed in the last two sections to the case 3,3:3,3 is especially applicable to the experiments where the detectors are not perfect but there are some detector inefficiencies. Here, for example, for each setting the detection of the particle is labeled 1, the non-detection is labeled 2 and the non-detection of the particle due to the inefficiency of the detector is labeled 3. Of course in 3,3:3,3 case the problem of extracting the non-singular matrices by computer is the weak point of this method because it takes a long time and especially for higher orders perhaps it may be impractical, though we have never tested that. However, as it was mentioned in the previous section one of the interesting features of the inequalities obtained by our method and listed in appendices A and B is that in some of them a coefficient of 2 appears. Also the independent inequalities deduced by our method is much more than the other methods. In fact as we said in 2,3:2,3 case we obtain 1617 independent inequalities, whereas the total number of inequalities obtained by other methods even in 4×4 case is less than this (see [Noc95]). This increase in the number of inequalities in turn makes it easier to design experiments in which some of these are violated, hence testing the non-locality in nature easier.

Although 2,3:2,3 case may be applied to the experiments where on each arm only one of the detectors is perfect, in the next chapter we will use Projection Valued (PV) measurements and Positive Operator Valued Measure (POVM) measurements to show that some of the 2,3:2,3 inequalities are violated by orthodox quantum theory predictions.

Chapter 4

PV and POVM measurements

The reduction of the state $|\psi\rangle$ of a system S, in a measurement of one of its observables, $\mathcal{O}$, is mathematically interpreted as the projection of $|\Psi\rangle$ onto one of the basis vectors, $|u_n\rangle$, of the N dimensional Hilbert-space of the system, $\mathcal{H}(\mathcal{N})$. Clearly, as we are measuring the observable $\mathcal{O}$, $|u_n\rangle$ is an eigen-state of $\mathcal{O}$ and the result of the measurement is the eigen-value o_n corresponding to $|u_n\rangle$. However, the probability of getting a specific result depends on the type of measurement used. In the following two sections we will discuss two types of measurements: Projection Valued measurements (PV) and Positive Operator Valued Measure measurements (POVM), and in the last section we use POVM measurements to test Bell inequalities.

4.1 PV measurements

In a PV type measurement, which is also called von Neumann type, only the system under consideration is involved in the measuring process. Obviously, here if there is no degeneracy the total number of outcomes of the measurement is N, the projection operator is

$$P_n = |u_n\rangle\langle u_n|, \tag{4.1}$$

the probability of the result o_n is

$$p_n = |\langle u_n|\phi\rangle|^2 \qquad \text{with} \qquad \sum_n^N p_n = 1 \tag{4.2}$$

and as the eigen-states $|u_n\rangle$ form a complete orthonormal basis we have

$$\sum_n^N |u_n\rangle\langle u_n| = \mathbb{1}. \tag{4.3}$$

If there is degeneracy, then there are totally $M(< N)$ possible outcomes and the above equations would become respectively:

$$P_m = \sum_{k=1}^{km} |u_m^k\rangle\langle u_m^k|, \tag{4.4}$$

$$p_m = \sum_{k=1}^{km} |\langle u_m^{km}|\phi\rangle|^2 \qquad \text{with} \qquad \sum_{m=1}^{M} p_m = 1, \tag{4.5}$$

$$\sum_{m=1}^{M} \sum_{k=1}^{k_m} |u_m^k\rangle\langle u_m^k| = \mathbb{1}. \tag{4.6}$$

As equations (4.2) and (4.5) show in both cases the mean value of the projection operators determines the probability of the outcome of a measurement, hence the name Projection Valued measurement.

4.2 POVM measurements

In a POVM measurement not only the system S but also an auxiliary system A is involved in the measurement. The detailed discussion of such a measurement can be found in [Per93] and [Har97], however we briefly explain some of the main points here. The state $|\phi\rangle$ of the system A with an $\mathcal{R}$ dimensional Hilbert-space is known before the measurement and there is no correlation between systems S and A. The state of the combined system before measurement is:

$$|\Psi\rangle = |\psi\rangle|\phi\rangle \tag{4.7}$$

If we expand the state $|\psi\rangle$ in terms of the eigen-states, $|u_n\rangle$, of the observable $\mathcal{O}$, and the state $|\phi\rangle$ in terms of the eigen-states, $|v_r\rangle$ of the observable of the system A and denote the unitary evolution of the combined system during the measurement process with $\hat{\mathsf{U}}$ we get:

$$\begin{aligned}
|\Psi\rangle &= |\psi\rangle|\phi\rangle \\
&= \sum_{n,r} a_n b_r |u_n\rangle|v_r\rangle \\
&\overset{\hat{\mathsf{U}}}{\Longrightarrow} \sum_{n,r} a_n b_r \hat{\mathsf{U}}[\ |u_n\rangle|v_r\rangle\ ] \\
&= \sum_{n,r} a_n b_r \sum_{m,q} \mathsf{u}_{mqnr} |u_m\rangle|v_q\rangle \\
&= \sum_{m,q} \left(\sum_{n,r} a_n b_r \mathsf{u}_{mqnr} \right) |u_m\rangle|v_q\rangle
\end{aligned}$$

where

$$\mathsf{u}_{mqnr} = \langle v_q|\langle u_m|\hat{\mathsf{U}}|u_n\rangle|v_r\rangle \tag{4.8}$$

If we make a PV measurement on the combined system at this stage, the probability that the system be found in the state $|u_n\rangle|v_r\rangle$ is:

$$p_{\{m,q\}} = \left| \sum_{n,r} a_n b_r \mathsf{u}_{mqnr} \right|^2 \tag{4.9}$$

This equation can be written as:

$$p_{\{m,q\}} = \mathcal{E}_{\{m,q\}} |\langle h_{\{m,q\}}|\psi\rangle|^2 \tag{4.10}$$

where the normalized vector $|h_{\{m,q\}}\rangle$ is a linear combination of $|u_m\rangle$'s and $\mathcal{E}_{\{m,q\}}$ can be found from the last two equations as below:

$$\begin{aligned}
\left|\sum_{n,r} a_n b_r \mathsf{u}_{mqnr}\right|^2 &= \mathcal{E}_{\{m,q\}} \, |\langle h_{\{m,q\}}|\psi\rangle|^2 \implies \\
\left|\sum_n a_n \sum_r b_r \mathsf{u}_{mqnr}\right|^2 &= \mathcal{E}_{\{m,q\}} \left|\langle h_{\{m,q\}}| \sum_n a_n |u_n\rangle\right|^2 \implies \\
\left|\sum_n a_n \sum_r b_r \mathsf{u}_{mqnr}\right|^2 &= \mathcal{E}_{\{m,q\}} \left|\sum_n a_n \langle h_{\{m,q\}}|u_n\rangle\right|^2
\end{aligned} \tag{4.11}$$

The above equality is fulfilled if we choose[1]

$$\begin{aligned}
&\sum_r b_r \mathsf{u}_{mqnr} = \sqrt{\mathcal{E}_{\{m,q\}}} \langle h_{\{m,q\}}|u_n\rangle \implies \\
&\sum_r b_r \mathsf{u}_{mqnr} \langle u_n| = \sqrt{\mathcal{E}_{\{m,q\}}} \langle h_{\{m,q\}}|u_n\rangle\langle u_n| \implies \\
&\sum_n \sum_r b_r \mathsf{u}_{mqnr} \langle u_n| = \sqrt{\mathcal{E}_{\{m,q\}}} \langle h_{\{m,q\}}| \sum_n |u_n\rangle\langle u_n| \implies \\
&\sum_n \sum_r b_r \mathsf{u}_{mqnr} \langle u_n| = \sqrt{\mathcal{E}_{\{m,q\}}} \langle h_{\{m,q\}}| \, \mathbb{1} \implies \\
&\sum_{n,r} (b_r \mathsf{u}_{mqnr})^* |u_n\rangle = \sqrt{\mathcal{E}_{\{m,q\}}} |h_{\{m,q\}}\rangle
\end{aligned}$$

If we multiply each side of the above equation by its complex conjugate we get:

$$\mathcal{E}_{\{m,q\}} = \sum_n \left|\sum_r (b_r \mathsf{u}_{mqnr})^*\right|^2 \tag{4.12}$$

[1]This is only a choice, the general solution would be obtained by equating the real part of the left-hand side with the real or imaginary part of the right-hand side of equation (4.8)and vice versa.

From equation (4.10) it is seen that the probability that the system S to be found in the state $|u_m\rangle$ after the measurement, is given by the mean value of the operator:

$$\mathcal{P} = \mathcal{E}_{\{m,q\}} |h_{\{m,q\}}\rangle\langle h_{\{m,q\}}| \tag{4.13}$$

According to equation (4.12), $\mathcal{E}_{\{m,q\}}$ is a positive number and so the eigenvalues of the operator $\mathcal{P}$ are always positive, hence the name *positive operator* and Positive Operator Valued Measure measurement. Using equation (4.9) one can show that $0 < \mathcal{E}_{\{m,q\}} \leq 1$ and

$$\sum_{m,q} \mathcal{E}_{\{m,q\}} |h_{\{m,q\}}\rangle\langle h_{\{m,q\}}| = \mathbb{1} \tag{4.14}$$

Using the above equation we immediately conclude that $\sum_{m,q} \mathcal{E}_{\{m,q\}} = N$. Note that the total number of possible outcomes in this case, that is the states $|h_{\{m,q\}}\rangle$, is more than the number of possible outcomes in a PV measurement which is N.

Up to now we have supposed that there is no degeneracy, however the extension to the degenerate case is straightforward and is left to the reader. Below we have shown three schemes of POVM measurements which we will use in the next section to apply to Bell-type inequalities, case $2,3 : 2,3$ but before that we give a short explanation about two of the devices used in the corresponding experiments, a polarizing beam-splitter (PBS) and a partially polarizing beam-splitter (PPBS)

4.2.1 A Polarizing Beam-splitter

Figure 4.1 shows the scheme of a polarizing beam-splitter. The angle of the PBS relative to the horizon is θ. If a beam of particles, initially polarized

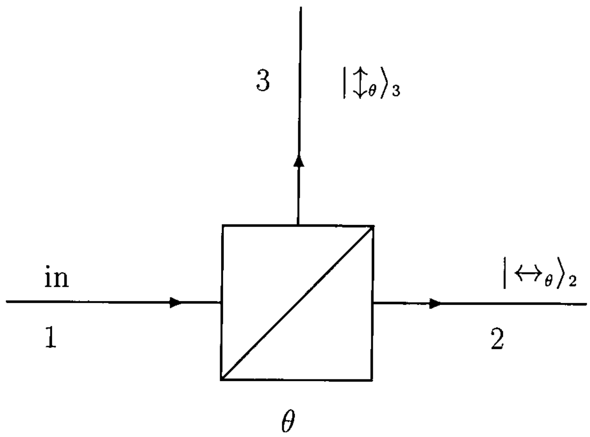

Figure 4.1: Scheme of a polarizing beam-splitter.

horizontally, with the state $|\leftrightarrow\rangle$, impinges on PBS it is split into two parts, one polarized horizontally and the other polarized vertically with the state $|\updownarrow\rangle$ according to the following relations:

$$|\leftrightarrow\rangle_1 = \cos\theta\,|\leftrightarrow_\theta\rangle_2 + \sin\theta\,|\updownarrow_\theta\rangle_3 \tag{4.15}$$

and if initially polarized vertically then

$$|\updownarrow\rangle_1 = -\sin\theta\,|\leftrightarrow_\theta\rangle_2 + \cos\theta\,|\updownarrow_\theta\rangle_3. \tag{4.16}$$

where the subscript θ means that the state depends on the polarization angle.

4.2.2 A Partially Polarizing Beam-splitter

A PPBS shown in figure 4.2, is an especial case of the PBS in the sense that the angle of polarization, θ, is zero here. However, in this case the beam is

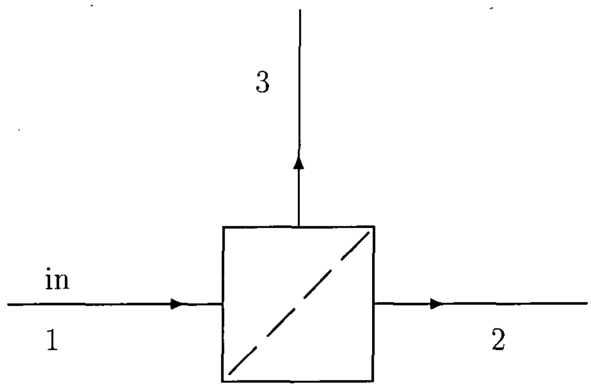

Figure 4.2: Scheme of a partially polarizing beam-splitter.

split into two parts later such that in this case we have:

$$|\leftrightarrow\rangle_1 \longrightarrow a_{\leftrightarrow}|\leftrightarrow\rangle_2 + b_{\leftrightarrow}|\leftrightarrow\rangle_3 \tag{4.17}$$

$$|\updownarrow\rangle_1 \longrightarrow a_{\updownarrow}|\updownarrow\rangle_2 + b_{\updownarrow}|\updownarrow\rangle_3. \tag{4.18}$$

4.2.3 Non-degenerate PV measurement

The scheme shown in figure 4.3 is a non-degenerate PV measurement with two outputs which uses a PBS.

In this experiment the state of the system S, which is a particle likely to be found in either horizontal or vertical polarization, evolves according to the following equation:

$$|\psi\rangle = \alpha|\leftrightarrow\rangle_1 + \beta|\updownarrow\rangle_1$$

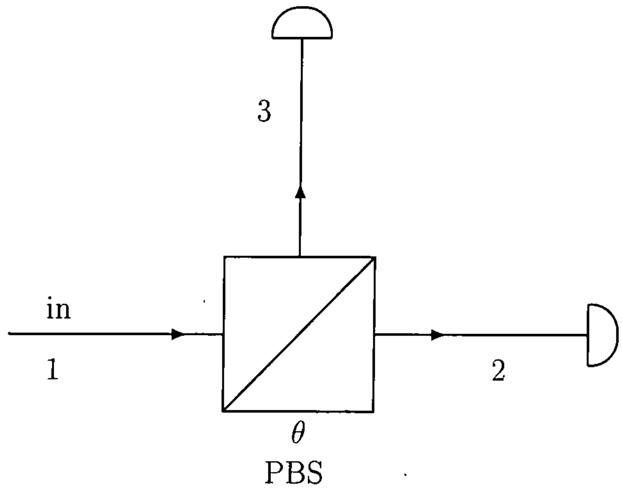

Figure 4.3: Scheme of a non-degenerate PV measurement with PBS.

$$\begin{aligned}\longrightarrow\ & \alpha\cos\theta|\leftrightarrow_\theta\rangle_2 + \alpha\sin\theta|\updownarrow_\theta\rangle_3 \\ & -\beta\sin\theta|\leftrightarrow_\theta\rangle_2 + \beta\cos\theta|\updownarrow_\theta\rangle_3 \end{aligned} \tag{4.19}$$

Here the probability to find the particle in path 2 is:

$$\begin{aligned} p_2 &= |\alpha\cos\theta - \beta\sin\theta|^2 \\ &= \mathcal{E}_2|\langle h_2|\psi\rangle|^2 \end{aligned} \tag{4.20}$$

Similarly the probability to find the particle in path 3 is:

$$\begin{aligned} p_3 &= |\alpha\sin\theta + \beta\cos\theta|^2 \\ &= \mathcal{E}_3|\langle h_3|\psi\rangle|^2 \end{aligned} \tag{4.21}$$

So for outcome 2 we have the state:

$$|h_2\rangle = \cos\theta|\leftrightarrow\rangle - \sin\theta|\updownarrow\rangle \qquad \mathcal{E}_2 = 1 \tag{4.22}$$

and for outcome 3 we have the state:

$$|h_3\rangle = \sin\theta|\leftrightarrow\rangle + \cos\theta|\updownarrow\rangle \qquad \mathcal{E}_3 = 1 \tag{4.23}$$

As expected it is seen that $\mathcal{E}_2 = \mathcal{E}_3 = 1$ which must be in a PV measurement.

4.2.4 Non-degenerate POVM, set up 1

In the experiment shown in figure 4.4 a PBS and a PPBS are used to set up a POVM measurement with the same system as before. However in this case there are three outputs. The state of the system S goes under two

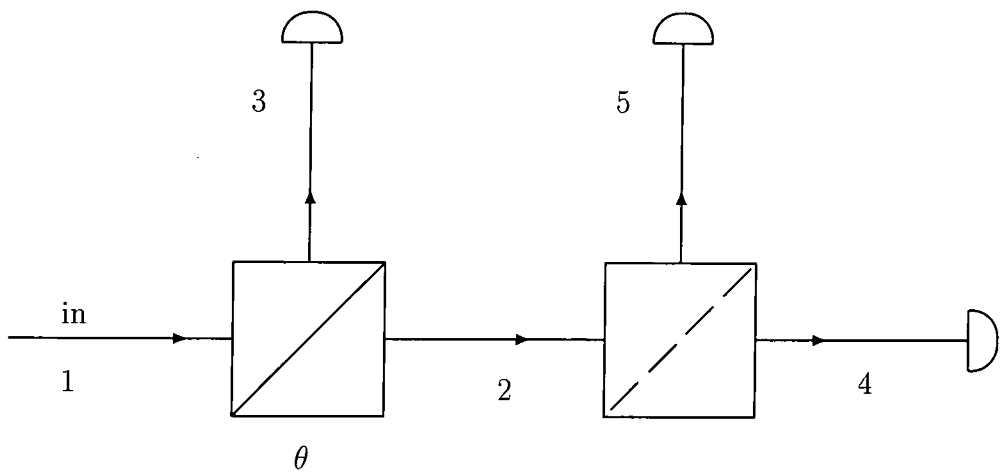

Figure 4.4: Scheme of a non-degenerate POVM with PBS and PPBS.

consecutive evolutions as below:

$$\begin{aligned}
|\psi\rangle &= \alpha|\leftrightarrow\rangle_1 + \beta|\updownarrow\rangle_1 \\
&\longrightarrow \alpha\cos\theta|\leftrightarrow_\theta\rangle_2 + \alpha\sin\theta|\updownarrow_\theta\rangle_3 - \beta\sin\theta|\leftrightarrow_\theta\rangle_2 + \beta\cos\theta|\updownarrow_\theta\rangle_3 \\
&\longrightarrow (\alpha a_\leftrightarrow \cos\theta - \beta a_\leftrightarrow \sin\theta)|\leftrightarrow\rangle_4 + \\
&\quad (\alpha b_\leftrightarrow \cos\theta - \beta b_\leftrightarrow \sin\theta)|\updownarrow\rangle_5 + \\
&\quad (\alpha\sin\theta + \beta\cos\theta)|\updownarrow_\theta\rangle_3
\end{aligned} \tag{4.24}$$

Here we have:

$$
\begin{aligned}
p_3 &= |\alpha \sin\theta + \beta \cos\theta|^2 && (4.25)\\
&= \mathcal{E}_3 |\langle h_3|\psi\rangle|^2 && (4.26)\\
p_4 &= |\alpha a_{\leftrightarrow} \cos\theta - \beta a_{\leftrightarrow} \sin\theta|^2 && (4.27)\\
&= \mathcal{E}_4 |\langle h_4|\psi\rangle|^2 && (4.28)\\
p_5 &= |\alpha b_{\leftrightarrow} \cos\theta - \beta b_{\leftrightarrow} \sin\theta|^2 && (4.29)\\
&= \mathcal{E}_5 |\langle h_5|\psi\rangle|^2 && (4.30)
\end{aligned}
$$

where:

$$
\begin{aligned}
|h_3\rangle &= \sin\theta|\leftrightarrow\rangle + \cos\theta|\updownarrow\rangle \qquad \mathcal{E}_3 = 1 && (4.31)\\
|h_4\rangle &= \cos\theta|\leftrightarrow\rangle - \sin\theta|\updownarrow\rangle \qquad \mathcal{E}_4 = |a_{\leftrightarrow}|^2 && (4.32)\\
|h_5\rangle &= \cos\theta|\leftrightarrow\rangle - \sin\theta|\updownarrow\rangle \qquad \mathcal{E}_5 = |b_{\leftrightarrow}|^2. && (4.33)
\end{aligned}
$$

4.2.5 Non-degenerate POVM, set up 2

Another non-degenerate POVM measurement that we use in the next section is shown in figure 4.5 which again uses a PPBS and a PBS. However, in this experiment the particle beam impinges first on PPBS and then on PBS. Furthermore the PPBS is modified such that for horizontal polarization the beam is not split and goes totally into path 2.

$$
\begin{aligned}
|\psi\rangle &= \alpha|\leftrightarrow\rangle_1 + \beta|\updownarrow\rangle_1\\
&\longrightarrow \alpha|\leftrightarrow\rangle_2 + \beta a_{\updownarrow}|\updownarrow\rangle_2 + \beta b_{\updownarrow}|\updownarrow\rangle_3\\
&\longrightarrow (\alpha\cos\theta - \beta a_{\updownarrow}\sin\theta)|\leftrightarrow_\theta\rangle_4 +\\
&\qquad (\alpha\sin\theta + \beta a_{\updownarrow}\cos\theta)|\updownarrow_\theta\rangle_5 +
\end{aligned}
$$

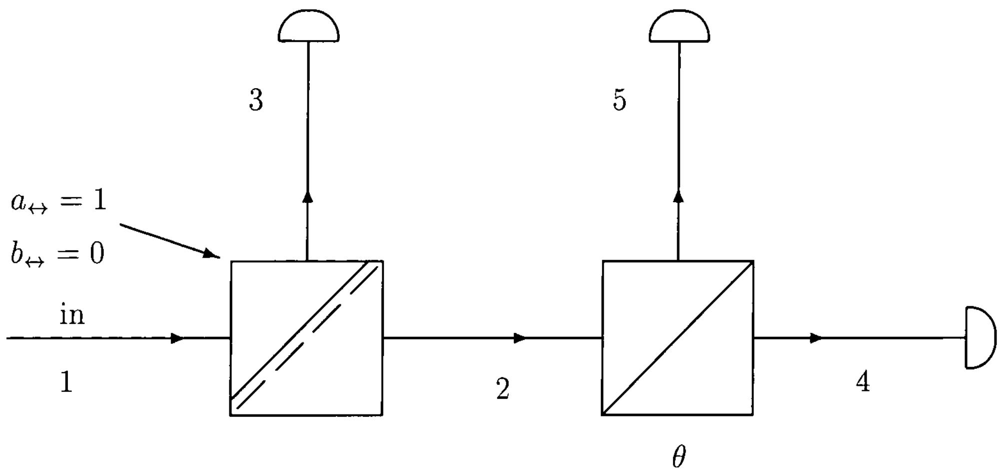

Figure 4.5: Scheme of a non-degenerate POVM with modified PPBS and PBS.

$$\beta b_{\updownarrow} |\updownarrow\rangle_3 \tag{4.34}$$

Here we have:

$$\begin{aligned} p_3 &= |\beta b_{\updownarrow}|^2 & (4.35)\\ &= \mathcal{E}_3 |\langle h_3|\psi\rangle|^2 & (4.36)\\ p_4 &= |\alpha\cos\theta - \beta a_{\updownarrow}\sin\theta|^2 & (4.37)\\ &= \mathcal{E}_4 |\langle h_4|\psi\rangle|^2 & (4.38)\\ p_5 &= |\alpha\sin\theta + \beta a_{\updownarrow}\cos\theta|^2 & (4.39)\\ &= \mathcal{E}_5 |\langle h_5|\psi\rangle|^2 & (4.40) \end{aligned}$$

where:

$$\begin{aligned} \mathcal{E}_3 &= |b_{\updownarrow}|^2 & (4.41)\\ |h_3\rangle &= |\updownarrow\rangle & (4.42) \end{aligned}$$

$$\mathcal{E}_4 = \cos^2\theta + |a_\updownarrow|^2 \sin^2\theta \tag{4.43}$$

$$|h_4\rangle = \frac{1}{\sqrt{\mathcal{E}_4}}(\cos\theta|\leftrightarrow\rangle - a_\updownarrow^* \sin\theta|\updownarrow\rangle) \tag{4.44}$$

$$\mathcal{E}_5 = \sin^2\theta + |a_\updownarrow|^2 \cos^2\theta \tag{4.45}$$

$$|h_5\rangle = \frac{1}{\sqrt{\mathcal{E}_5}}(\sin\theta|\leftrightarrow\rangle + a_\updownarrow^* \cos\theta|\updownarrow\rangle) \tag{4.46}$$

4.3 POVM measurements and Bell inequalities

Consider a two photon system with the entangled state:

$$\alpha|\leftrightarrow\rangle|\leftrightarrow\rangle + \beta|\updownarrow\rangle|\updownarrow\rangle \tag{4.47}$$

where one photon propagates to the left and the other to the right. We would like to make two series of POVM measurements on this system and verify if any of the inequalities derived in chapter 3 would be violated. In all of these experiments there are 2 outputs or three outputs which we have already discussed in the previous section.

4.3.1 Experimental set up 1

In this experimental set up two settings are available in each arm, where for setting 1 (with two outputs) a PBS, as shown in figure 4.3, and for setting 2 (with three outputs) a PBS followed by a PPBS, as shown in figure 4.4, are used. For this set up the corresponding probabilities as defined in section 3.3, would be as below.

Setting 1 on the Left and setting 1 on the Right

$$\begin{aligned}
&\alpha|\leftrightarrow\rangle|\leftrightarrow\rangle+\beta|\updownarrow\rangle|\updownarrow\rangle \quad\longrightarrow\\
&\alpha(\cos\theta_1^L|\leftrightarrow_\theta\rangle_2^L+\sin\theta_1^L|\updownarrow_\theta\rangle_3^L)(\cos\theta_1^R|\leftrightarrow_\theta\rangle_2^R+\sin\theta_1^R|\updownarrow_\theta\rangle_3^R)\\
+\;&\beta(-\sin\theta_1^L|\leftrightarrow_\theta\rangle_2^L+\cos\theta_1^L|\updownarrow_\theta\rangle_3^L)(-\sin\theta_1^R|\leftrightarrow_\theta\rangle_2^R+\cos\theta_1^R|\updownarrow_\theta\rangle_3^R)\\
=\;&(\alpha\cos\theta_1^L\cos\theta_1^R+\beta\sin\theta_1^L\sin\theta_1^R)|\leftrightarrow_\theta\rangle_2^L|\leftrightarrow_\theta\rangle_2^R\\
+\;&(\alpha\cos\theta_1^L\sin\theta_1^R-\beta\sin\theta_1^L\cos\theta_1^R)|\leftrightarrow_\theta\rangle_2^L|\updownarrow_\theta\rangle_3^R\\
+\;&(\alpha\sin\theta_1^L\cos\theta_1^R-\beta\cos\theta_1^L\sin\theta_1^R)|\updownarrow_\theta\rangle_3^L|\leftrightarrow_\theta\rangle_2^R\\
+\;&(\alpha\sin\theta_1^L\sin\theta_1^R+\beta\cos\theta_1^L\cos\theta_1^R)|\updownarrow_\theta\rangle_3^L|\updownarrow_\theta\rangle_3^R
\end{aligned}\tag{4.48}$$

The probabilities in this case are:

$$\begin{aligned}
p_{11}^{22} &= |\alpha\cos\theta_1^L\cos\theta_1^R+\beta\sin\theta_1^L\sin\theta_1^R|^2 \equiv p_{11}^{11}\\
p_{11}^{23} &= |\alpha\cos\theta_1^L\sin\theta_1^R-\beta\sin\theta_1^L\cos\theta_1^R|^2 \equiv p_{11}^{12}\\
p_{11}^{32} &= |\alpha\sin\theta_1^L\cos\theta_1^R-\beta\cos\theta_1^L\sin\theta_1^R|^2 \equiv p_{11}^{21}\\
p_{11}^{33} &= |\alpha\sin\theta_1^L\sin\theta_1^R+\beta\cos\theta_1^L\cos\theta_1^R|^2 \equiv p_{11}^{22}
\end{aligned}$$

Setting 2 on the Left and setting 2 on the Right

$$\begin{aligned}
&\alpha|\leftrightarrow\rangle|\leftrightarrow\rangle+\beta|\updownarrow\rangle|\updownarrow\rangle \quad\longrightarrow\\
&\alpha(\cos\theta_2^L|\leftrightarrow_\theta\rangle_2^L+\sin\theta_2^L|\updownarrow_\theta\rangle_3^L)(\cos\theta_2^R|\leftrightarrow_\theta\rangle_2^R+\sin\theta_2^R|\updownarrow_\theta\rangle_3^R)\\
+\;&\beta(-\sin\theta_2^L|\leftrightarrow_\theta\rangle_2^L+\cos\theta_2^L|\updownarrow_\theta\rangle_3^L)(-\sin\theta_2^R|\leftrightarrow_\theta\rangle_2^R+\cos\theta_2^R|\updownarrow_\theta\rangle_3^R) \quad\longrightarrow
\end{aligned}$$

$$
\begin{aligned}
&= (\alpha \sin\theta_2^L \sin\theta_2^R && + \beta\cos\theta_2^L\cos\theta_2^R)\ |\updownarrow_\theta\rangle_3^L|\updownarrow_\theta\rangle_3^R \\
&+ (\alpha a_{\leftrightarrow 2}^R \sin\theta_2^L \cos\theta_2^R && - \beta a_{\leftrightarrow 2}^R\cos\theta_2^L\sin\theta_2^R)\ |\updownarrow_\theta\rangle_3^L|\leftrightarrow\rangle_4^R \\
&+ (\alpha b_{\leftrightarrow 2}^R \sin\theta_2^L \cos\theta_2^R && - \beta b_{\leftrightarrow 2}^R\cos\theta_2^L\sin\theta_2^R)\ |\updownarrow_\theta\rangle_3^L|\leftrightarrow\rangle_5^R \\
&+ (\alpha a_{\leftrightarrow 2}^L \cos\theta_2^L \sin\theta_2^R && - \beta a_{\leftrightarrow 2}^L\sin\theta_2^L\cos\theta_2^R)\ |\leftrightarrow\rangle_4^L|\updownarrow_\theta\rangle_3^R \\
&+ (\alpha a_{\leftrightarrow 2}^L a_{\leftrightarrow 2}^R \cos\theta_2^L \cos\theta_2^R && + \beta a_{\leftrightarrow 2}^L a_{\leftrightarrow 2}^R\sin\theta_2^L\sin\theta_2^R)\ |\leftrightarrow\rangle_4^L|\leftrightarrow\rangle_4^R \\
&+ (\alpha a_{\leftrightarrow 2}^L b_{\leftrightarrow 2}^R \cos\theta_2^L \cos\theta_2^R && + \beta a_{\leftrightarrow 2}^L b_{\leftrightarrow 2}^R\sin\theta_2^L\sin\theta_2^R)\ |\leftrightarrow\rangle_4^L|\leftrightarrow\rangle_5^R \\
&+ (\alpha b_{\leftrightarrow 2}^L \cos\theta_2^L \sin\theta_2^R && - \beta b_{\leftrightarrow 2}^L\sin\theta_2^L\cos\theta_2^R)\ |\leftrightarrow\rangle_5^L|\updownarrow_\theta\rangle_3^R \\
&+ (\alpha b_{\leftrightarrow 2}^L a_{\leftrightarrow 2}^R \cos\theta_2^L \cos\theta_2^R && + \beta b_{\leftrightarrow 2}^L a_{\leftrightarrow 2}^R\sin\theta_2^L\sin\theta_2^R)\ |\leftrightarrow\rangle_5^L|\leftrightarrow\rangle_4^R \\
&+ (\alpha b_{\leftrightarrow 2}^L b_{\leftrightarrow 2}^R \cos\theta_2^L \cos\theta_2^R && + \beta b_{\leftrightarrow 2}^L b_{\leftrightarrow 2}^R\sin\theta_2^L\sin\theta_2^R)\ |\leftrightarrow\rangle_5^L|\leftrightarrow\rangle_5^R
\end{aligned}
$$

The probabilities in this case are:

$$
\begin{aligned}
p_{22}^{33} &= |\alpha\sin\theta_2^L\sin\theta_2^R && + \beta\cos\theta_2^L\cos\theta_2^R|^2 && \equiv p_{22}^{22} \\
p_{22}^{34} &= |\alpha a_{\leftrightarrow 2}^R\sin\theta_2^L\cos\theta_2^R && - \beta a_{\leftrightarrow 2}^R\cos\theta_2^L\sin\theta_2^R|^2 && \equiv p_{22}^{21} \\
p_{22}^{35} &= |\alpha b_{\leftrightarrow 2}^R\sin\theta_2^L\cos\theta_2^R && - \beta b_{\leftrightarrow 2}^R\cos\theta_2^L\sin\theta_2^R|^2 && \equiv p_{22}^{23} \\
p_{22}^{43} &= |\alpha a_{\leftrightarrow 2}^L\cos\theta_2^L\sin\theta_2^R && - \beta a_{\leftrightarrow 2}^L\sin\theta_2^L\cos\theta_2^R|^2 && \equiv p_{22}^{12} \\
p_{22}^{44} &= |\alpha a_{\leftrightarrow 2}^L a_{\leftrightarrow 2}^R\cos\theta_2^L\cos\theta_2^R && + \beta a_{\leftrightarrow 2}^L a_{\leftrightarrow 2}^R\sin\theta_2^L\sin\theta_2^R|^2 && \equiv p_{22}^{11} \\
p_{22}^{45} &= |\alpha a_{\leftrightarrow 2}^L b_{\leftrightarrow 2}^R\cos\theta_2^L\cos\theta_2^R && + \beta a_{\leftrightarrow 2}^L b_{\leftrightarrow 2}^R\sin\theta_2^L\sin\theta_2^R|^2 && \equiv p_{22}^{13} \\
p_{22}^{53} &= |\alpha b_{\leftrightarrow 2}^L\cos\theta_2^L\sin\theta_2^R && - \beta b_{\leftrightarrow 2}^L\sin\theta_2^L\cos\theta_2^R|^2 && \equiv p_{22}^{32} \\
p_{22}^{54} &= |\alpha b_{\leftrightarrow 2}^L a_{\leftrightarrow 2}^R\cos\theta_2^L\cos\theta_2^R && + \beta b_{\leftrightarrow 2}^L a_{\leftrightarrow 2}^R\sin\theta_2^L\sin\theta_2^R|^2 && \equiv p_{22}^{31} \\
p_{22}^{55} &= |\alpha b_{\leftrightarrow 2}^L b_{\leftrightarrow 2}^R\cos\theta_2^L\cos\theta_2^R && + \beta b_{\leftrightarrow 2}^L b_{\leftrightarrow 2}^R\sin\theta_2^L\sin\theta_2^R|^2 && \equiv p_{22}^{33}
\end{aligned}
$$

Setting 1 on the Left and setting 2 on the Right

$$
\begin{aligned}
& \alpha|\leftrightarrow\rangle|\leftrightarrow\rangle + \beta|\updownarrow\rangle|\updownarrow\rangle \quad \longrightarrow \\
& \alpha(\cos\theta_1^L|\leftrightarrow_\theta\rangle_2^L + \sin\theta_1^L|\updownarrow_\theta\rangle_3^L)\times \\
& \quad (a^R_{\leftrightarrow 2}\cos\theta_2^R|\leftrightarrow\rangle_4^R + b^R_{\leftrightarrow 2}\cos\theta_2^R|\leftrightarrow\rangle_5^R + \sin\theta_2^R|\updownarrow_\theta\rangle_3^R) \\
+ \;& \beta(-\sin\theta_1^L|\leftrightarrow_\theta\rangle_2^L + \cos\theta_1^L|\updownarrow_\theta\rangle_3^L)\times \\
& \quad (-a^R_{\leftrightarrow 2}\sin\theta_2^R|\leftrightarrow\rangle_4^R - b^R_{\leftrightarrow 2}\sin\theta_2^R|\leftrightarrow\rangle_5^R + \cos\theta_2^R|\updownarrow_\theta\rangle_3^R)
\end{aligned}
$$

$$
\begin{aligned}
= \;& (\alpha\cos\theta_1^L\sin\theta_2^R - \beta\sin\theta_1^L\cos\theta_2^R)\;|\leftrightarrow_\theta\rangle_2^L|\updownarrow_\theta\rangle_3^R) \\
+ \;& (\alpha a^R_{\leftrightarrow 2}\cos\theta_1^L\cos\theta_2^R + \beta a^R_{\leftrightarrow 2}\sin\theta_1^L\sin\theta_2^R)\;|\leftrightarrow_\theta\rangle_2^L|\leftrightarrow\rangle_4^R \\
+ \;& (\alpha b^R_{\leftrightarrow 2}\cos\theta_1^L\cos\theta_2^R + \beta b^R_{\leftrightarrow 2}\sin\theta_1^L\sin\theta_2^R)\;|\leftrightarrow_\theta\rangle_2^L|\leftrightarrow\rangle_5^R \\
+ \;& (\alpha\sin\theta_1^L\sin\theta_2^R + \beta\cos\theta_1^L\cos\theta_2^R)\;|\updownarrow_\theta\rangle_3^L|\updownarrow_\theta\rangle_3^R \\
+ \;& (\alpha a^R_{\leftrightarrow 2}\sin\theta_1^L\cos\theta_2^R - \beta a^R_{\leftrightarrow 2}\cos\theta_1^L\sin\theta_2^R)\;|\updownarrow_\theta\rangle_3^L|\leftrightarrow\rangle_4^R \\
+ \;& (\alpha b^R_{\leftrightarrow 2}\sin\theta_1^L\cos\theta_2^R - \beta b^R_{\leftrightarrow 2}\cos\theta_1^L\sin\theta_2^R)\;|\updownarrow_\theta\rangle_3^L|\leftrightarrow\rangle_5^R
\end{aligned}
$$

The probabilities in this case are:

$$
\begin{aligned}
p_{12}^{23} &= |\alpha\cos\theta_1^L\sin\theta_2^R - \beta\sin\theta_1^L\cos\theta_2^R|^2 &&\equiv p_{12}^{12} \\
p_{12}^{24} &= |\alpha a^R_{\leftrightarrow 2}\cos\theta_1^L\cos\theta_2^R + \beta a^R_{\leftrightarrow 2}\sin\theta_1^L\sin\theta_2^R|^2 &&\equiv p_{12}^{11} \\
p_{12}^{25} &= |\alpha b^R_{\leftrightarrow 2}\cos\theta_1^L\cos\theta_2^R + \beta b^R_{\leftrightarrow 2}\sin\theta_1^L\sin\theta_2^R|^2 &&\equiv p_{12}^{13} \\
p_{12}^{33} &= |\alpha\sin\theta_1^L\sin\theta_2^R + \beta\cos\theta_1^L\cos\theta_2^R|^2 &&\equiv p_{12}^{22} \\
p_{12}^{34} &= |\alpha a^R_{\leftrightarrow 2}\sin\theta_1^L\cos\theta_2^R - \beta a^R_{\leftrightarrow 2}\cos\theta_1^L\sin\theta_2^R|^2 &&\equiv p_{12}^{21} \\
p_{12}^{35} &= |\alpha b^R_{\leftrightarrow 2}\sin\theta_1^L\cos\theta_2^R - \beta b^R_{\leftrightarrow 2}\cos\theta_1^L\sin\theta_2^R|^2 &&\equiv p_{12}^{23}
\end{aligned}
$$

Setting 2 on the Left and setting 1 on the Right

$$
\begin{aligned}
&\alpha|\leftrightarrow\rangle|\leftrightarrow\rangle + \beta|\updownarrow\rangle|\updownarrow\rangle \quad \longrightarrow \\
&\alpha(a^L_{\leftrightarrow 2}\cos\theta^L_2|\leftrightarrow\rangle^L_4 + b^L_{\leftrightarrow 2}\cos\theta^L_2|\leftrightarrow\rangle^L_5 + \sin\theta^L_2|\updownarrow_\theta\rangle^L_3)\times \\
&\quad(\cos\theta^R_1|\leftrightarrow_\theta\rangle^R_2 + \sin\theta^R_1|\updownarrow_\theta\rangle^R_3) \\
+\ &\beta(-a^L_{\leftrightarrow 2}\sin\theta^L_2|\leftrightarrow\rangle^L_4 - b^L_{\leftrightarrow 2}\sin\theta^L_2|\leftrightarrow\rangle^L_5 + \cos\theta^L_2|\updownarrow_\theta\rangle^L_3)\times \\
&\quad(-\sin\theta^R_1|\leftrightarrow_\theta\rangle^R_2 + \cos\theta^R_1|\updownarrow_\theta\rangle^R_3) \\
=\ &(\alpha\sin\theta^L_2\cos\theta^R_1 - \beta\cos\theta^L_2\sin\theta^R_1)\ |\updownarrow_\theta\rangle^L_3|\leftrightarrow_\theta\rangle^R_2) \\
+\ &(\alpha\sin\theta^L_2\sin\theta^R_1 + \beta\cos\theta^L_2\cos\theta^R_1)\ |\updownarrow_\theta\rangle^L_3|\updownarrow_\theta\rangle^R_3 \\
+\ &(\alpha a^L_{\leftrightarrow 2}\cos\theta^L_2\cos\theta^R_1 + \beta a^L_{\leftrightarrow 2}\sin\theta^L_2\sin\theta^R_1)\ |\leftrightarrow\rangle^L_4|\leftrightarrow_\theta\rangle^R_2 \\
+\ &(\alpha a^L_{\leftrightarrow 2}\cos\theta^L_2\sin\theta^R_1 - \beta a^L_{\leftrightarrow 2}\sin\theta^L_2\cos\theta^R_1)\ |\leftrightarrow\rangle^L_4|\updownarrow_\theta\rangle^R_3 \\
+\ &(\alpha b^L_{\leftrightarrow 2}\cos\theta^L_2\cos\theta^R_1 + \beta b^L_{\leftrightarrow 2}\sin\theta^L_2\sin\theta^R_1)\ |\leftrightarrow\rangle^L_5|\leftrightarrow_\theta\rangle^R_2 \\
+\ &(\alpha b^L_{\leftrightarrow 2}\cos\theta^L_2\sin\theta^R_1 - \beta b^L_{\leftrightarrow 2}\sin\theta^L_2\cos\theta^R_1)\ |\leftrightarrow\rangle^L_5|\updownarrow_\theta\rangle^R_3
\end{aligned}
$$

The probabilities in this case are:

$$
\begin{aligned}
p^{32}_{21} &= |\alpha\sin\theta^L_2\cos\theta^R_1 - \beta\cos\theta^L_2\sin\theta^R_1|^2 &\equiv p^{21}_{21} \\
p^{33}_{21} &= |\alpha\sin\theta^L_2\sin\theta^R_1 + \beta\cos\theta^L_2\cos\theta^R_1|^2 &\equiv p^{22}_{21} \\
p^{42}_{21} &= |\alpha a^L_{\leftrightarrow 2}\cos\theta^L_2\cos\theta^R_1 + \beta a^L_{\leftrightarrow 2}\sin\theta^L_2\sin\theta^R_1|^2 &\equiv p^{11}_{21} \\
p^{43}_{21} &= |\alpha a^L_{\leftrightarrow 2}\cos\theta^L_2\sin\theta^R_1 - \beta a^L_{\leftrightarrow 2}\sin\theta^L_2\cos\theta^R_1|^2 &\equiv p^{12}_{21} \\
p^{52}_{21} &= |\alpha b^L_{\leftrightarrow 2}\cos\theta^L_2\cos\theta^R_1 + \beta b^L_{\leftrightarrow 2}\sin\theta^L_2\sin\theta^R_1|^2 &\equiv p^{31}_{21} \\
p^{53}_{21} &= |\alpha b^L_{\leftrightarrow 2}\cos\theta^L_2\sin\theta^R_1 - \beta b^L_{\leftrightarrow 2}\sin\theta^L_2\cos\theta^R_1|^2 &\equiv p^{32}_{21}
\end{aligned}
$$

The values of the inequality number 2 in appendix **B** for 3 sets of parameter settings are as followings:

$$\text{Inequality} = -1.446709 \times 10^{-2}$$

$$\begin{array}{lcllcl}
\alpha & = & 0.71 & \beta & = & 0.71 \\
a^{L}_{\leftrightarrow 2} & = & 0.33 & b^{L}_{\leftrightarrow 2} & = & 0.94 \\
a^{R}_{\leftrightarrow 2} & = & 0.50 & b^{R}_{\leftrightarrow 2} & = & 0.87 \\
\theta^{L}_{1} & = & 0.00 & \theta^{L}_{2} & = & 45.00 \\
\theta^{R}_{1} & = & 157.50 & \theta^{R}_{2} & = & 90.00
\end{array}$$

$$\text{Inequality} = 1.028356$$

$$\begin{array}{lcllcl}
\alpha & = & 0.71 & \beta & = & 0.71 \\
a^{L}_{\leftrightarrow 2} & = & 0.33 & b^{L}_{\leftrightarrow 2} & = & 0.94 \\
a^{R}_{\leftrightarrow 2} & = & 0.50 & b^{R}_{\leftrightarrow 2} & = & 0.87 \\
\theta^{L}_{1} & = & 0.00 & \theta^{L}_{2} & = & 45.00 \\
\theta^{R}_{1} & = & 67.50 & \theta^{R}_{2} & = & 180.00
\end{array}$$

$$\text{Inequality} = 1.198336819$$

$$\begin{array}{lcllcl}
\alpha & = & 0.7071067810 & \beta & = & 0.7071067810 \\
a^{L}_{\leftrightarrow 2} & = & 0.01 & b^{L}_{\leftrightarrow 2} & = & 0.9999499987 \\
a^{R}_{\leftrightarrow 2} & = & 0.01 & b^{R}_{\leftrightarrow 2} & = & 0.9999499987 \\
\theta^{L}_{1} & = & 153.00 & \theta^{L}_{2} & = & 27.00 \\
\theta^{R}_{1} & = & 45.00 & \theta^{R}_{2} & = & 0.00
\end{array}$$

The maximum violation of the Bell inequalities that we could reach for this case is 1.198336819 which is less than $\sqrt{2}$ obtained by others(see for example [CHSH69]. However, we would like to emphasize that as computer calculations take a long time to verify exactly, even for one of the inequalities in the 2,3:2,3 case, we have done calculations both by computer and by hand for some of the inequalities which obviously are not complete. So, in an exact verification we may find inequalities which are violated with a factor stronger than $\sqrt{2}$. Fortunately we have obtained this with the next set up.

4.3.2 Experimental set up 2

Again in this experimental set up two settings are available in each arm: for setting 1 (with two outputs) a PBS, as shown in figure 4.3, and for setting 2 (with three outputs) a modified PPBS followed by a PBS, as shown in figure 4.5, are used. The corresponding probabilities would be as below.

Setting 1 on the Left and setting 1 on the Right

$$\begin{aligned}
&\alpha|\leftrightarrow\rangle|\leftrightarrow\rangle+\beta|\updownarrow\rangle|\updownarrow\rangle \quad\longrightarrow \\
&\alpha(\cos\theta_1^L|\leftrightarrow_\theta\rangle_2^L+\sin\theta_1^L|\updownarrow_\theta\rangle_3^L)(\cos\theta_1^R|\leftrightarrow_\theta\rangle_2^R+\sin\theta_1^R|\updownarrow_\theta\rangle_3^R) \\
+\;&\beta(-\sin\theta_1^L|\leftrightarrow_\theta\rangle_2^L+\cos\theta_1^L|\updownarrow\rangle_3^L)(-\sin\theta_1^R|\leftrightarrow_\theta\rangle_2^R+\cos\theta_1^R|\updownarrow_\theta\rangle_3^R) \\
=\;&(\alpha\cos\theta_1^L\cos\theta_1^R + \beta\sin\theta_1^L\sin\theta_1^R)|\leftrightarrow_\theta\rangle_2^L|\leftrightarrow_\theta\rangle_2^R \\
+\;&(\alpha\cos\theta_1^L\sin\theta_1^R - \beta\sin\theta_1^L\cos\theta_1^R)|\leftrightarrow_\theta\rangle_2^L|\updownarrow_\theta\rangle_3^R \\
+\;&(\alpha\sin\theta_1^L\cos\theta_1^R - \beta\cos\theta_1^L\sin\theta_1^R)|\updownarrow_\theta\rangle_3^L|\leftrightarrow_\theta\rangle_2^R \\
+\;&(\alpha\sin\theta_1^L\sin\theta_1^R + \beta\cos\theta_1^L\cos\theta_1^R)|\updownarrow_\theta\rangle_3^L|\updownarrow_\theta\rangle_3^R
\end{aligned} \tag{4.49}$$

The probabilities in this case are:

$$\begin{aligned}
p_{11}^{22} &= |\alpha\cos\theta_1^L\cos\theta_1^R + \beta\sin\theta_1^L\sin\theta_1^R|^2 \equiv p_{11}^{11} \\
p_{11}^{23} &= |\alpha\cos\theta_1^L\sin\theta_1^R - \beta\sin\theta_1^L\cos\theta_1^R|^2 \equiv p_{11}^{12} \\
p_{11}^{32} &= |\alpha\sin\theta_1^L\cos\theta_1^R - \beta\cos\theta_1^L\sin\theta_1^R|^2 \equiv p_{11}^{21} \\
p_{11}^{33} &= |\alpha\sin\theta_1^L\sin\theta_1^R + \beta\cos\theta_1^L\cos\theta_1^R|^2 \equiv p_{11}^{22}
\end{aligned}$$

Setting 2 on the Left and setting 2 on the Right

$$
\begin{aligned}
& \alpha|\leftrightarrow\rangle|\leftrightarrow\rangle + \beta|\updownarrow\rangle|\updownarrow\rangle \quad \longrightarrow \\
& \alpha|\leftrightarrow\rangle_2^L|\leftrightarrow\rangle_2^R + \beta(a_{\updownarrow 2}^L|\updownarrow\rangle_2^L + b_{\updownarrow 2}^L|\updownarrow\rangle_3^L)(a_{\updownarrow 2}^R|\updownarrow\rangle_2^R + b_{\updownarrow 2}^R|\updownarrow\rangle_3^R) \quad \longrightarrow \\
& \alpha(\cos\theta_2^L|\leftrightarrow_\theta\rangle_4^L + \sin\theta_2^L|\updownarrow_\theta\rangle_5^L)(\cos\theta_2^R|\leftrightarrow_\theta\rangle_4^R + \sin\theta_2^R|\updownarrow_\theta\rangle_5^R) \\
+ \; & \beta a_{\updownarrow 2}^L a_{\updownarrow 2}^R(-\sin\theta_2^L|\leftrightarrow_\theta\rangle_4^L + \cos\theta_2^L|\updownarrow_\theta\rangle_5^L)(-\sin\theta_2^R|\leftrightarrow_\theta\rangle_4^R + \cos\theta_2^R|\updownarrow_\theta\rangle_5^R) \\
+ \; & \beta b_{\updownarrow 2}^L a_{\updownarrow 2}^R|\updownarrow\rangle_3^L(-\sin\theta_2^R|\leftrightarrow_\theta\rangle_4^R + \cos\theta_2^R|\updownarrow_\theta\rangle_5^R) \\
+ \; & \beta a_{\updownarrow 2}^L b_{\updownarrow 2}^R(-\sin\theta_2^L|\leftrightarrow_\theta\rangle_4^L + \cos\theta_2^L|\updownarrow_\theta\rangle_5^L)|\updownarrow\rangle_3^R \\
+ \; & \beta b_{\updownarrow 2}^L b_{\updownarrow 2}^R|\updownarrow\rangle_3^L|\updownarrow\rangle_3^L \\
\\
= \; & \beta b_{\updownarrow 2}^L b_{\updownarrow 2}^R|\updownarrow\rangle_3^L|\updownarrow\rangle_3^R \\
- \; & \beta b_{\updownarrow 2}^L a_{\updownarrow 2}^R \sin\theta_2^R|\updownarrow\rangle_3^L|\leftrightarrow_\theta\rangle_4^R \\
+ \; & \beta b_{\updownarrow 2}^L a_{\updownarrow 2}^R \cos\theta_2^R|\updownarrow\rangle_3^L|\updownarrow_\theta\rangle_5^R \\
- \; & \beta a_{\updownarrow 2}^L b_{\updownarrow 2}^R \sin\theta_2^L|\leftrightarrow_\theta\rangle_4^L|\updownarrow\rangle_3^R \\
+ \; & (\alpha\cos\theta_2^L\cos\theta_2^R + \beta a_{\updownarrow 2}^L a_{\updownarrow 2}^R \sin\theta_2^L\sin\theta_2^R)|\leftrightarrow_\theta\rangle_4^L|\leftrightarrow_\theta\rangle_4^R \\
+ \; & (\alpha\cos\theta_2^L\sin\theta_2^R - \beta a_{\updownarrow 2}^L a_{\updownarrow 2}^R \sin\theta_2^L\cos\theta_2^R)|\leftrightarrow_\theta\rangle_4^L|\updownarrow_\theta\rangle_5^R \\
+ \; & \beta a_{\updownarrow 2}^L b_{\updownarrow 2}^R \cos\theta_2^L|\updownarrow_\theta\rangle_5^L|\updownarrow\rangle_3^R \\
+ \; & (\alpha\sin\theta_2^L\cos\theta_2^R - \beta a_{\updownarrow 2}^L a_{\updownarrow 2}^R \cos\theta_2^L\sin\theta_2^R)|\updownarrow_\theta\rangle_5^L|\leftrightarrow_\theta\rangle_4^R \\
+ \; & (\alpha\sin\theta_2^L\sin\theta_2^R + \beta a_{\updownarrow 2}^L a_{\updownarrow 2}^R \cos\theta_2^L\cos\theta_2^R)|\updownarrow_\theta\rangle_5^L|\updownarrow_\theta\rangle_5^R
\end{aligned}
\tag{4.50}
$$

The probabilities in this case are:

$$
\begin{aligned}
p_{22}^{33} &= |\beta b_{\updownarrow 2}^{L} b_{\updownarrow 2}^{R}|^2 && \equiv p_{22}^{22} \\
p_{22}^{34} &= |\beta b_{\updownarrow 2}^{L} a_{\updownarrow 2}^{R} \sin\theta_2^R|^2 && \equiv p_{22}^{21} \\
p_{22}^{35} &= |\beta b_{\updownarrow 2}^{L} a_{\updownarrow 2}^{R} \cos\theta_2^R|^2 && \equiv p_{22}^{23} \\
p_{22}^{43} &= |\beta a_{\updownarrow 2}^{L} b_{\updownarrow 2}^{R} \sin\theta_2^L|^2 && \equiv p_{22}^{12} \\
p_{22}^{44} &= |\alpha \cos\theta_2^L \cos\theta_2^R + \beta a_{\updownarrow 2}^{L} a_{\updownarrow 2}^{R} \sin\theta_2^L \sin\theta_2^R|^2 && \equiv p_{22}^{11} \\
p_{22}^{45} &= |\alpha \cos\theta_2^L \sin\theta_2^R - \beta a_{\updownarrow 2}^{L} a_{\updownarrow 2}^{R} \sin\theta_2^L \cos\theta_2^R|^2 && \equiv p_{22}^{13} \\
p_{22}^{53} &= |\beta a_{\updownarrow 2}^{L} b_{\updownarrow 2}^{R} \cos\theta_2^L|^2 && \equiv p_{22}^{32} \\
p_{22}^{54} &= |\alpha \sin\theta_2^L \cos\theta_2^R - \beta a_{\updownarrow 2}^{L} a_{\updownarrow 2}^{R} \cos\theta_2^L \sin\theta_2^R|^2 && \equiv p_{22}^{31} \\
p_{22}^{55} &= |\alpha \sin\theta_2^L \sin\theta_2^R + \beta a_{\updownarrow 2}^{L} a_{\updownarrow 2}^{R} \cos\theta_2^L \cos\theta_2^R|^2 && \equiv p_{22}^{33}
\end{aligned}
$$

Setting 1 on the Left and setting 2 on the Right

$$
\begin{aligned}
& \alpha|\leftrightarrow\rangle|\leftrightarrow\rangle+\beta|\updownarrow\rangle|\updownarrow\rangle \quad \longrightarrow \\
& \alpha(\cos\theta_1^L|\leftrightarrow_\theta\rangle_2^L+\sin\theta_1^L|\updownarrow_\theta\rangle_3^L)\times \\
& \quad (\cos\theta_2^R|\leftrightarrow_\theta\rangle_4^R+\sin\theta_2^R|\updownarrow_\theta\rangle_5^R) \\
+\ & \beta(-\sin\theta_1^L|\leftrightarrow_\theta\rangle_2^L+\cos\theta_1^L|\updownarrow_\theta\rangle_3^L)\times \\
& \quad (-a_{\updownarrow 2}^R\sin\theta_2^R|\leftrightarrow_\theta\rangle_4^R+a_{\updownarrow 2}^R\cos\theta_2^R|\updownarrow_\theta\rangle_5^R+b_{\updownarrow 2}^R|\updownarrow\rangle_3^R) \\
=\ & -\beta b_{\updownarrow 2}^R\sin\theta_1^L|\leftrightarrow_\theta\rangle_2^L|\updownarrow\rangle_3^R \\
+\ & (\alpha\cos\theta_1^L\cos\theta_2^R+\beta a_{\updownarrow 2}^R\sin\theta_1^L\sin\theta_2^R)|\leftrightarrow_\theta\rangle_2^L|\leftrightarrow_\theta\rangle_4^R \\
+\ & (\alpha\cos\theta_1^L\sin\theta_2^R-\beta a_{\updownarrow 2}^R\sin\theta_1^L\cos\theta_2^R)|\leftrightarrow_\theta\rangle_2^L|\updownarrow_\theta\rangle_5^R \\
+\ & \beta b_{\updownarrow 2}^R\cos\theta_1^L|\updownarrow_\theta\rangle_3^L|\updownarrow\rangle_3^R \\
+\ & (\alpha\sin\theta_1^L\cos\theta_2^R-\beta a_{\updownarrow 2}^R\cos\theta_1^L\sin\theta_2^R)|\updownarrow_\theta\rangle_3^L|\leftrightarrow_\theta\rangle_4^R \\
+\ & (\alpha\sin\theta_1^L\sin\theta_2^R+\beta a_{\updownarrow 2}^R\cos\theta_1^L\cos\theta_2^R)|\updownarrow_\theta\rangle_3^L|\updownarrow_\theta\rangle_5^R
\end{aligned}
$$

The probabilities in this case are:

$$
\begin{aligned}
p_{12}^{23} &= |\beta b_{\updownarrow 2}^R\sin\theta_1^L|^2 &&\equiv p_{12}^{12} \\
p_{12}^{24} &= |\alpha\cos\theta_1^L\cos\theta_2^R+\beta a_{\updownarrow 2}^R\sin\theta_1^L\sin\theta_2^R|^2 &&\equiv p_{12}^{11} \\
p_{12}^{25} &= |\alpha\cos\theta_1^L\sin\theta_2^R-\beta a_{\updownarrow 2}^R\sin\theta_1^L\cos\theta_2^R|^2 &&\equiv p_{12}^{13} \\
p_{12}^{33} &= |\beta b_{\updownarrow 2}^R\cos\theta_1^L|^2 &&\equiv p_{12}^{22} \\
p_{12}^{34} &= |\alpha\sin\theta_1^L\cos\theta_2^R-\beta a_{\updownarrow 2}^R\cos\theta_1^L\sin\theta_2^R|^2 &&\equiv p_{12}^{21} \\
p_{12}^{35} &= |\alpha\sin\theta_1^L\sin\theta_2^R+\beta a_{\updownarrow 2}^R\cos\theta_1^L\cos\theta_2^R|^2 &&\equiv p_{12}^{23}
\end{aligned}
$$

Setting 2 on the Left and setting 1 on the Right

$$
\begin{aligned}
& \alpha|\leftrightarrow\rangle|\leftrightarrow\rangle + \beta|\updownarrow\rangle|\updownarrow\rangle \quad \longrightarrow \\
& \alpha(\cos\theta_2^L|\leftrightarrow_\theta\rangle_4^L + \sin\theta_2^L|\updownarrow_\theta\rangle_5^L)\times \\
& \quad (\cos\theta_1^R|\leftrightarrow_\theta\rangle_2^R + \sin\theta_1^R|\updownarrow_\theta\rangle_3^R) \\
+ \; & \beta(-a_{\updownarrow 2}^L\sin\theta_2^L|\leftrightarrow_\theta\rangle_4^L + a_{\updownarrow 2}^L\cos\theta_2^L|\updownarrow_\theta\rangle_5^L + b_{\updownarrow 2}^L|\updownarrow\rangle_3^L)\times \\
& \quad (-\sin\theta_1^R|\leftrightarrow_\theta\rangle_2^R + \cos\theta_1^R|\updownarrow_\theta\rangle_3^R) \\
= \; & -\beta b_{\updownarrow 2}^L\sin\theta_1^R|\updownarrow\rangle_3^L|\leftrightarrow_\theta\rangle_2^R \\
+ \; & \beta b_{\updownarrow 2}^L\cos\theta_1^R|\updownarrow\rangle_3^L|\updownarrow_\theta\rangle_3^R \\
+ \; & (\alpha\cos\theta_2^L\cos\theta_1^R + \beta a_{\updownarrow 2}^L\sin\theta_2^L\sin\theta_1^R)|\leftrightarrow_\theta\rangle_4^L|\leftrightarrow_\theta\rangle_2^R \\
+ \; & (\alpha\cos\theta_2^L\sin\theta_1^R - \beta a_{\updownarrow 2}^L\sin\theta_2^L\cos\theta_1^R)|\leftrightarrow_\theta\rangle_4^L|\updownarrow_\theta\rangle_3^R \\
+ \; & (\alpha\sin\theta_2^L\cos\theta_1^R - \beta a_{\updownarrow 2}^L\cos\theta_2^L\sin\theta_1^R)|\updownarrow_\theta\rangle_5^L|\leftrightarrow_\theta\rangle_2^R \\
+ \; & (\alpha\sin\theta_2^L\sin\theta_1^R + \beta a_{\updownarrow 2}^L\cos\theta_2^L\cos\theta_1^R)|\updownarrow_\theta\rangle_5^L|\updownarrow_\theta\rangle_3^R
\end{aligned}
$$

The probabilities in this case are:

$$
\begin{aligned}
p_{21}^{32} &= |\beta b_{\updownarrow 2}^L\sin\theta_1^R|^2 &\equiv p_{21}^{21} \\
p_{21}^{33} &= |\beta b_{\updownarrow 2}^L\cos\theta_1^R|^2 &\equiv p_{21}^{22} \\
p_{21}^{42} &= |\alpha\cos\theta_2^L\cos\theta_1^R + \beta a_{\updownarrow 2}^L\sin\theta_2^L\sin\theta_1^R|^2 &\equiv p_{21}^{11} \\
p_{21}^{43} &= |\alpha\cos\theta_2^L\sin\theta_1^R - \beta a_{\updownarrow 2}^L\sin\theta_2^L\cos\theta_1^R|^2 &\equiv p_{21}^{12} \\
p_{21}^{52} &= |\alpha\sin\theta_2^L\cos\theta_1^R - \beta a_{\updownarrow 2}^L\cos\theta_2^L\sin\theta_1^R|^2 &\equiv p_{21}^{31} \\
p_{21}^{53} &= |\alpha\sin\theta_2^L\sin\theta_1^R + \beta a_{\updownarrow 2}^L\cos\theta_2^L\cos\theta_1^R|^2 &\equiv p_{21}^{32}
\end{aligned}
$$

Again in this case the value of the inequality number 2 in appendix **B** for 7 sets of parameter settings are as followings:

$$\begin{array}{lllll}
\multicolumn{5}{l}{\text{Inequality} = -2.089684 \times 10^{-2}} \\
\alpha & = 0.71 & \beta & = & 0.71 \\
a^L_{\updownarrow 2} & = 0.33 & b^L_{\updownarrow 2} & = & 0.94 \\
a^R_{\updownarrow 2} & = 0.50 & b^R_{\updownarrow 2} & = & 0.87 \\
\theta^L_1 & = 67.50 & \theta^L_2 & = & 45.00 \\
\theta^R_1 & = 67.50 & \theta^R_2 & = & 67.50
\end{array}$$

$$\begin{array}{lllll}
\multicolumn{5}{l}{\text{Inequality} = 1.018246} \\
\alpha & = 0.71 & \beta & = & 0.71 \\
a^L_{\updownarrow 2} & = 0.33 & b^L_{\updownarrow 2} & = & 0.94 \\
a^R_{\updownarrow 2} & = 0.50 & b^R_{\updownarrow 2} & = & 0.87 \\
\theta^L_1 & = 0.00 & \theta^L_2 & = & 45.00 \\
\theta^R_1 & = 112.50 & \theta^R_2 & = & 90.00
\end{array}$$

$$\begin{array}{lllll}
\multicolumn{5}{l}{\text{Inequality} = 1.192572023} \\
\alpha & = 0.7071067810 & \beta & = & 0.7071067810 \\
a^L_{\updownarrow 2} & = 0.1000000000 & b^L_{\updownarrow 2} & = & 0.9949874371 \\
a^R_{\updownarrow 2} & = 0.9000000000 & b^R_{\updownarrow 2} & = & 0.4358898944 \\
\theta^L_1 & = 135.00 & \theta^L_2 & = & 99.00 \\
\theta^R_1 & = 27.00 & \theta^R_2 & = & 63.00
\end{array}$$

$$\begin{array}{lllll}
\multicolumn{5}{l}{\text{Inequality} = 1.278386383} \\
\alpha & = 0.7071067810 & \beta & = & 0.7071067810 \\
a^L_{\updownarrow 2} & = 0.8000000000 & b^L_{\updownarrow 2} & = & 0.6000000000 \\
a^R_{\updownarrow 2} & = 0.6000000000 & b^R_{\updownarrow 2} & = & 0.8000000000 \\
\theta^L_1 & = 36.00 & \theta^L_2 & = & 63.00 \\
\theta^R_1 & = 144.00 & \theta^R_2 & = & 117.00
\end{array}$$

$$\begin{array}{lllll}
\multicolumn{5}{l}{\text{Inequality} = 1.282532697} \\
\alpha & = 0.7071067810 & \beta & = & 0.7071067810 \\
a^L_{\updownarrow 2} & = 0.8000000000 & b^L_{\updownarrow 2} & = & 0.6000000000 \\
a^R_{\updownarrow 2} & = 0.6000000000 & b^R_{\updownarrow 2} & = & 0.8000000000 \\
\theta^L_1 & = 35.29411765 & \theta^L_2 & = & 59.01639345 \\
\theta^R_1 & = 138.4615385 & \theta^R_2 & = & 120.0000000
\end{array}$$

Inequality = 1.499900008

$$\begin{array}{lcllcl}
\alpha & = & 0.7071067810 & \beta & = & 0.7071067810 \\
a^L_{\updownarrow 2} & = & 0.9999 & b^L_{\updownarrow 2} & = & 0.1414178207 \\
a^R_{\updownarrow 2} & = & 0.9999 & b^R_{\updownarrow 2} & = & 0.1414178207 \\
\theta^L_1 & = & 45.00 & \theta^L_2 & = & 45.00 \\
\theta^R_1 & = & 135.00 & \theta^R_2 & = & 135.00
\end{array}$$

Inequality = 1.499999720

$$\begin{array}{lcllcl}
\alpha & = & 0.7071067810 & \beta & = & 0.7071067810 \\
a^L_{\updownarrow 2} & = & 0.99999999 & b^L_{\updownarrow 2} & = & 0.0001414213562 \\
a^R_{\updownarrow 2} & = & 0.99999999 & b^R_{\updownarrow 2} & = & 0.0001414213562 \\
\theta^L_1 & = & 44.97751125 & \theta^L_2 & = & 45.03377533 \\
\theta^R_1 & = & 135.0135014 & \theta^R_2 & = & 134.9730054
\end{array}$$

Comparing the last two sets it seems that the parameters in the last set belong to an extremum, and in fact we have tried many other sets near these values but couldn't reach higher. Furthermore, although in all the above seven sets of parameters the Bell inequality is violated by the predictions of quantum theory, the last two are more interesting as they violate the Bell inequality by a factor of ≈ 1.5 which exceeds $\sqrt{2}$. This in turn makes the experimental test of locality, in which there are always some kind of errors, easier and more accurate. Of course, the increase in the output gates can be accounted for stronger violation of Bell inequalities which may be the motivation for considering experiments with more outputs.

Chapter 5

A local and Lorentz invariant model

5.1 Hardy's proof of non-Lorentz invariance of local theories

Aimed at nonlocal hidden-variable theories Hardy [Har92, HS92] used a gedanken experiment to prove that they are not Lorentz invariant too. Although some authors have criticized Hardy's argument [CN92, BG94, CH95, CH96], with the help of non-locality and contextuality of the hidden variable theories, it is worth discussing this here as in the next sections it will be shown how Squires local model escapes this argument.

Hardy's first assumption is the same as EPR sufficient condition for reality except that the disturbance of the system under measurement is allowed. His second assumption is the *necessary condition for Lorentz invariance of the elements of reality* which states: The value of an element of reality corresponding to a Lorentz-invariant observable is itself Lorentz invariant.

The scheme of Hardy's gedanken experiment is shown in figure 5.1 in which two Mach-Zehnder-type interferometers are used. A positron e^+(electron e^-) impinges on beam splitter $BS1^+$($BS1^-$) and is reflected or transmitted through the paths u^+(u^-) or v^+(v^-) respectively and due to destructive interference goes through the path c^+(c^-) and is detected at detector C^+(C^-) only. The two interferometers are then combined such that, in the laboratory frame of reference both electron and positron reach the detectors at the same time but if the positron takes the path u^+ and the electron takes the path u^- they reach the point P at the same time, t_P, and are annihilated, that is:

$$|u^+\rangle|u^-\rangle \longrightarrow |\gamma\rangle \tag{5.1}$$

Also two removable beam splitters $BS2^+$ and $BS2^-$ are provided as shown

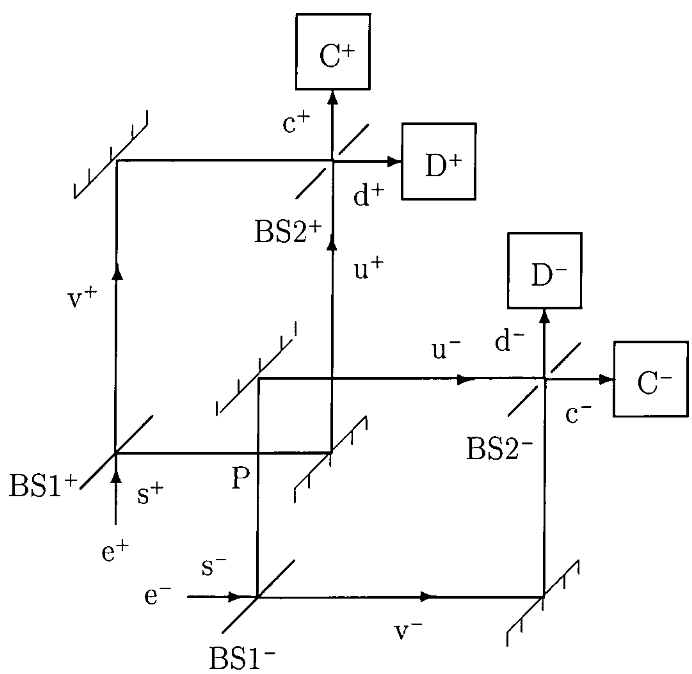

Figure 5.1: Scheme of Hardy's gedanken experiment.

in the figure. The initial state of the system is supposed to be:

$$|s^+\rangle|s^-\rangle \tag{5.2}$$

where (from now on) we are using the convention that the state of each particle is shown by the path it is in. As the only information available from the state of a particle at a given time is that if the particle exists in a given path or not, the state describing the particle does not depend on any parameter of the frame of the reference. When a beam reaches to a beam-slitter it is split as:

$$|I\rangle \longrightarrow \frac{1}{\sqrt{2}}(|T\rangle + i|R\rangle) \tag{5.3}$$

where I, T and R are incoming, transmitting and reflecting states respectively. We will see shortly that due to the configuration already discussed, the positron(electron) now will be able to reach the detector D^+(D^-) from the path d^+(d^-).

Now let's define two observables $\hat{U}^\pm$ as:

$$\hat{U}^\pm = |u^\pm\rangle\langle u^\pm| \tag{5.4}$$

$\hat{U}^+$($\hat{U}^-$) corresponds to positron(electron) being in path u^+(u^-) which is the element of reality here and its value, $[U^+]$($[U^-]$) is 1 if it is in path u^+(u^-), 0 otherwise. Hardy's conclusion from reality condition is then:

$$\hat{U}^\pm|u^\pm\rangle = |u^\pm\rangle \qquad \Longrightarrow \qquad [U^\pm] = 1 \tag{5.5}$$

$$\hat{U}^+\hat{U}^-|u^+\rangle|u^-\rangle = |u^+\rangle|u^-\rangle \qquad \Longrightarrow \qquad [U^+U^-] = 1 \tag{5.6}$$

$$\hat{U}^+\hat{U}^-|u^+,u^-\rangle_\perp = 0 \qquad \Longrightarrow \qquad [U^+U^-] = 0 \tag{5.7}$$

$$\text{if} \quad [U^+][U^-] = 1 \qquad \text{then} \quad [U^+U^-] = 1 \tag{5.8}$$

where $|u^+, u^-\rangle_\perp$ is any state orthogonal to $|u^+\rangle|u^-\rangle$.

Note that with the experiment shown in figure 5.1, the conditional statement (5.8) cannot be applied to nonlocal hidden variable theories which are context dependent. This is because U^+ and U^- cannot be measured in the same context here.

Now, in the laboratory frame of reference F, for times later than t_P but before the positron and electron reach the beam-splitters BS2$^\pm$, the state of the system evolves as:

$$
\begin{aligned}
|s^+\rangle|s^-\rangle &\longrightarrow \frac{1}{2}(i|u^+\rangle + |v^+\rangle)(i|u^-\rangle + |v^-\rangle) \\
&\longrightarrow \frac{1}{2}(-|\gamma\rangle + i|u^+\rangle|v^-\rangle + i|v^+\rangle|u^-\rangle + |v^+\rangle|v^-\rangle) \qquad (5.9)
\end{aligned}
$$

which is orthogonal to $|u^+\rangle|u^-\rangle$, as at times later than t_P with probability equal to 1 not positron nor electron exist in the corresponding path. from (5.7), we have:

$$[U^+U^-] = 0 \qquad (5.10)$$

In the F$^+$ frame of reference in which positron passes the beam-splitter BS2$^+$, but electron has not reach BS2$^-$ yet, we have:

$$|s^+\rangle|s^-\rangle \longrightarrow \frac{1}{2\sqrt{2}}(-\sqrt{2}|\gamma\rangle - |c^+\rangle|u^-\rangle + 2i|c^+\rangle|v^-\rangle + i|d^+\rangle|u^-\rangle) \qquad (5.11)$$

In this case, the state of the electron would be $|u^-\rangle$ if positron is detected at D$^+$ and from (5.5), we have:

$$[U^-] = 1 \qquad \text{if detection at D}^+ \qquad (5.12)$$

In the F$^-$ frame of reference in which electron passes the beam-splitter BS2$^-$, but positron has not reach BS2$^+$ yet, we have:

$$|s^+\rangle|s^-\rangle \quad \longrightarrow \quad \frac{1}{2\sqrt{2}}(-\sqrt{2}|\gamma\rangle - |u^+\rangle|c^-\rangle + i|u^+\rangle|d^-\rangle + 2i|v^+\rangle|c^-\rangle) \qquad (5.13)$$

In this case, the state of the positron would be $|u^+\rangle$ if electron is detected at D$^-$ and from (5.5), we have:

$$[U^+] = 1 \qquad \text{if detection at D}^- \qquad (5.14)$$

In the F$^\pm$ frame of reference in which both positron and electron pass the beam-splitters BS2$^\pm$, we have:

$$|s^+\rangle|s^-\rangle \quad \longrightarrow \quad \frac{1}{4}(-2|\gamma\rangle - 3|c^+\rangle|c^-\rangle + i|c^+\rangle|d^-\rangle + i|d^+\rangle|c^-\rangle - |d^+\rangle|d^-\rangle) \qquad (5.15)$$

where there is a probability of $\frac{1}{16}$ that positron be detected at D$^+$ and electron be detected at D$^-$. So from (5.8), (5.12) and (5.14) in these cases where [D$^+$]=1 and [D$^-$]=1, we have:

$$[U^+U^-] = 1 \qquad \text{with probability of } \frac{1}{16} \qquad (5.16)$$

which contradicts (5.10). This in turn refutes the assumption of the necessary condition for Lorentz invariance of the elements of reality that we made earlier. We end this section by emphasizing that the contexts of measurement of D$^+$ and D$^-$ are different in all the above cases, and because of this as we stated before this argument cannot be applied to nonlocal theories, for example in the case of equation (5.15) it is seen that if we measure D$^+$ and get [D$^+$] = 1 then as U$^-$ cannot be measured in the same context, if we had measured U$^-$ and got [U$^-$] = 1 this would non-locally effect the outcome of the measurement of D$^+$ which may not result the value [D$^+$] = 1, hence blocking the argument.

5.2 Non-locality, Lorentz invariance of the Bohm model

Non-locality is one of the basic and apparent features of the Bohm model when applied to many-body systems. As was shown in section 2.2, for a system with N particles the equation of motion for a particle is found from (2.31) which reads:

$$\dot{\mathbf{x}}_i = \frac{1}{m_i}\Re\left(\frac{\Psi^* \boldsymbol{p}_i \Psi}{\Psi^* \Psi}\right) \tag{5.17}$$

and the quantum potential $Q(x_1, \cdots, x_{3N}, t)$ would become:

$$Q(x_1, \cdots, x_{3N}, t) = -\frac{\hbar^2}{2m}\frac{\left[\sum_{i=1}^{3N}\nabla_i^2 R\right]}{R} \tag{5.18}$$

As it is seen from the above equations, the position of each particle depends on the other particles' positions as well. Furthermore, whatever the classical potential of the system is, the quantum interaction potential of (5.18) always depends on the positions of all particles of the system. This means that the position of each particle is *instantaneously* effected by other particles of the system, hence the non-locality feature of the theory.

Although the Bohm model reproduces all of the statistical predictions of quantum theory and still is Lorentz invariant at that level [BHK87, BH93], due to the non-locality feature, this theory, in which the particle positions play the basic role becomes non-Lorentz invariant at the level of individual systems. This can be shown by considering two particles A and B at rest in a Lorentz frame as in figure 5.2 [BH93].

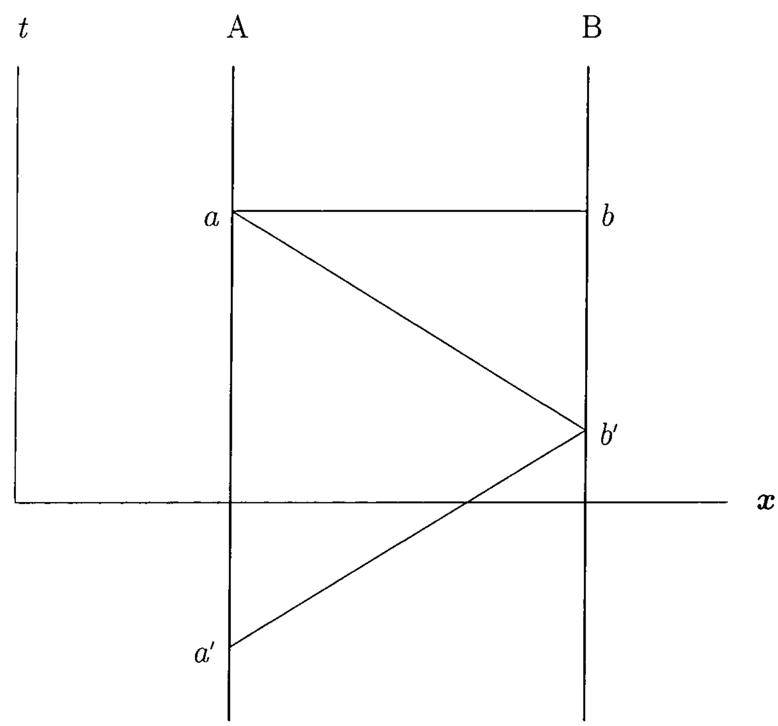

Figure 5.2: Space-like connections influencing past.

In this frame of reference there is an instantaneous interaction between the two particles at a and b respectively. Now, in another Lorentz frame where a and b' are simultaneous, still there should be an instantaneous interaction due to quantum potential. With the same argument, as a' and b' are simultaneous, an instantaneous interaction exist here. However, this is impossible because the particle a affects its past.

5.3 The Squires model

The non-locality of Bohm model, as was shown in the previous section, arises because of the instantaneous influence of particles on each other's positions, which in turn results in non-Lorentz invariance of the theory. In other words in the Bohm model the velocity of signals are infinite. Impressed by this, Squires [Squ93] proposed a model which is based on the assumption that the information each particle receives from the other one is carried by a signal moving with the velocity of light. So, in the equation of motion of each particle, the positions of the others should be determined on the backward light-cone of the particle, that is, the equation of motion for the ith particle is given by

$$\dot{\boldsymbol{x}}_i(t_i) = \frac{1}{m_i}\Re\frac{\boldsymbol{p}_i\Psi(\boldsymbol{x}_1(t_{i1}), \boldsymbol{x}_2(t_{i2}), \cdots)}{\Psi(\boldsymbol{x}_1(t_{i1}), \boldsymbol{x}_2(t_{i2}), \cdots)}, \tag{5.19}$$

where

$$t_{ij} = t_i - \frac{|\boldsymbol{x}_i - \boldsymbol{x}_j|}{c}, \tag{5.20}$$

are the retarded times. Here there is an ambiguity in the cases where the wave-function explicitly depends on time [Squ93] which can be removed by using multiple-time wave-function if the duration of the interaction between particles are zero or simply if the interaction is instantaneous.

We consider N particles which interact with each other only at time $t = t_0$. The Hamiltonian can be written as the sum of the Hamiltonians of the individual particles. So,

$$H = \sum_{j=1}^{N} H_j\,. \tag{5.21}$$

The wave-function of the system before the interaction is

$$\Psi_b(\boldsymbol{x}_1, \cdots, \boldsymbol{x}_N, t) = \prod_{j=1}^{N} U_j(0,t)\psi_j(\boldsymbol{x}_j, 0)\,, \tag{5.22}$$

where $\psi_j(\boldsymbol{x}_j, 0)$ is the initial wave-function of the jth particle and,

$$U_j(0,t) = T\exp\left[-\frac{i}{\hbar}\int_0^t d\tau H_j(\tau)\right] \tag{5.23}$$

is the unitary time evolution operator of the jth system in which T stands for time ordered product.

At time $t = t_0$ particles interact with each other instantaneously and in general at the end of the interaction the wave-functions are entangled. So after the interaction the wave-function of the system becomes

$$\Psi_a(\boldsymbol{x}_1, \cdots, \boldsymbol{x}_N, t) = \sum_k c_k \prod_{j=1}^{N} U_j(t_0, t)\psi_{jk}(\boldsymbol{x}_j, t_0)\,. \tag{5.24}$$

Now we define the retarded wave-function corresponding to ith particle as follows:

$$\Psi_i^{ret} = \sum_k c_k \prod_{j=1}^{N} U_j(t_{ij}^0, t_{ij})\psi_{jk}(\boldsymbol{x}_j, t_{ij}^0)\,, \tag{5.25}$$

where t_{ij} is the retarded time as defined in equation (5.20), and $t_{ij}^0 = 0$ if t_{ij} maps into the region before interaction, and $t_{ij}^0 = t_0$ otherwise. Note that before the interaction the state of the jth particle is given by ψ_j but after the interaction, in general, there are many different states, ψ_{jk}, corresponding to this particle. So the evolution of ψ_j cannot be determined uniquely and this is the motivation for defining t_{ij}^0 as above.

Now the equation of motion for the ith particle becomes

$$\dot{\boldsymbol{x}}_i(t_i) = \frac{1}{m_i}\Re\left(\frac{\Psi_i^{ret*}(\boldsymbol{x}_1(t_{i1}), \boldsymbol{x}_2(t_{i2}), \cdots)\,\boldsymbol{p}_i\,\Psi_i^{ret}(\boldsymbol{x}_1(t_{i1}), \boldsymbol{x}_2(t_{i2}), \cdots)}{\Psi_i^{ret*}(\boldsymbol{x}_1(t_{i1}), \boldsymbol{x}_2(t_{i2}), \cdots)\,\Psi_i^{ret}(\boldsymbol{x}_1(t_{i1}), \boldsymbol{x}_2(t_{i2}), \cdots)}\right) \tag{5.26}$$

and if there is correlation between bosons and fermions, as was discussed in section 2.4 in the case of the Bohm model, integration over the positions of bosons is essential, that is:

$$\dot{x}_i(t_i) = \frac{1}{m_i}\Re\left(\frac{\int d^3z\ \Psi_i^{ret^*}(x_1(t_{i1}), x_2(t_{i2}), \cdots)\, p_i\, \Psi_i^{ret}(x_1(t_{i1}), x_2(t_{i2}), \cdots)}{\int d^3z\ \Psi_i^{ret^*}(x_1(t_{i1}), x_2(t_{i2}), \cdots)\, \Psi_i^{ret}(x_1(t_{i1}), x_2(t_{i2}), \cdots)}\right) \quad (5.27)$$

That the Squires model does not encounter the Bohm model difficulties is clear. In the case of Hardy's gedanken experiment as we saw in section 5.1 the contradiction arises because in the frame of reference $F^{\pm}$, where both positron and electron reach the beam-splitters BS2$^{\pm}$ at the same time, there is a probability of detection of positron in D^{+} and electron at D^{-}. However, this does not happen in Squires model, since if in $F^{\pm}$ the events BS2^{+} and BS2^{-} are space-like separated,[1] which is a must in Hardy's argument, each particle when it reaches the corresponding beam-splitter sees the other one in its backward light cone and so acts as if the other has not reached its beam-splitter, thus avoiding the joint detection at D^{+} and D^{-} (See [MS95] for a more detailed explanation on this).

5.4 Photon detection and the Squires model – type 1 detector

In a system of N particles which interact instantaneously with each other only at time t_0, the distance between each pair of particles depends on time. Suppose that we are considering the equation of motion for the ith particle. At time t_0, if the jth particle is the nearest one to this particle, then the

[1] Obviously, here by events BS2$^{\pm}$ we mean when the positron/electron reaches the beam-splitters BS2$^{\pm}$

time $\frac{|\boldsymbol{x}_i - \boldsymbol{x}_j|}{c}$ is the minimum among the others and we denote it by t_{ij}^{min}. Now from the definition of the retarded time and the retarded wave-function in the previous section, it is clear that, for times $t_i < t_0 + t_{ij}^{min}$ the retarded times for all particles would be mapped into the region before the interaction and it is seen from the equation (5.26) that the equation of motion of ith particle does not depend on the positions of the other particles. This means that in this time interval, effectively there is only one particle. Physically, this can be interpreted in the sense that the particle does not receive information from the others.[2]

With this in mind, we now reconsider the detection of a photon whose wave-function consists of two parts, one going to the left and the other to the right with one detector provided on either side. This was already discussed in section 2.5, but this time we discuss it in the context of the Squires model. If the distance between each detector and the photon source, that is D, is of the order of 1 meter, as in the Aspect *et.al.* experiments[3], then initially the retarded time, T, is about 10^{-9}. The initial wave-function of the system is:

$$|\Psi\rangle = 2^{-1/2}[\phi_{\mathrm{L}}(z) + \phi_{\mathrm{R}}(z)]\psi_{0\mathrm{L}}(X)\psi_{0\mathrm{R}}(Y) \tag{5.28}$$

where as before $\psi_{0\mathrm{L}}(X)$ and $\psi_{0\mathrm{R}}(Y)$ are the initial wave-functions of the left and right detectors and given by equations (2.49, 2.57) respectively. If the interaction of the photon and the detectors occurs at $t_0 = 0$, then for the left

[2]Obviously if $t_{ij}^{min} = 0$ this situation would never happen if we assume that the velocity of a particle is always less than that of light

[3]In most of the experiments of Aspect *et.al.* two photons are involved (see [ADR82] and the references therein), however, from now on we are considering the one discussed in [AG90] in which only one photon is used

detector from equation (5.25)we have:

$$|\Psi\rangle = 2^{-1/2}[\phi_{\mathrm{L}}(z)\psi_{0\mathrm{L}}^{-p}(X) + \phi_{\mathrm{R}}(z)\psi_{0\mathrm{L}}(X)]\psi_{0\mathrm{R}}(Y), \tag{5.29}$$

and from equation of motion (5.27) the velocity of the left detector would become:

$$\dot{X} = \frac{1}{m}\Re\left[\frac{\psi_{\mathrm{L}}^{-p^*}\boldsymbol{p}_X\psi_{\mathrm{L}}^{-p} + \psi_{\mathrm{L}}^*\boldsymbol{p}_X\psi_{\mathrm{L}}}{\left|\psi_{\mathrm{L}}^{-p}\right|^2 + |\psi_{\mathrm{L}}|^2}\right], \tag{5.30}$$

which is exactly the same as equation (2.61) where there was only one detector present on the left. A similar equation holds for the right detector. So, if as in section 2.5 we use the wave-function $\psi_{\mathrm{L}}^{-p}/\psi_{\mathrm{R}}^{+p}$ given by equations (2.52/2.60), then the whole analytic arguments we made about these equations in that section are still valid here. That is, if initially X_0 is positive/negative then the left detector will/will not record the photon. Note that equations(2.52/2.60) differ from the corresponding ones in [Squ93]. This is due to the fact that the photon wave-function is not contained in the total wave-function therein.

Before going any further, there is a point about the Squires model which should be made clear here. In the above example consider the case where both X_0 and Y_0 are positive. So both of the detectors will record the photon. On the other hand if X_0 and Y_0 are both negative, then neither of the detectors will record the photon. The probability for this to happen is $\frac{1}{4}$ in each case, and the total probability adds up to $\frac{1}{2}$. The reason for this is that for $0 < t < T$, practically we are treating the two detectors independently of each other. So if the duration of the experiment is less than the retarded time then, for 50% of the events, quantum theory is violated which is quite significant here. However, in the next sections we will see that after the retarded time elapses the wrong events disappear in a realistic experiment.

5.5 Photon detection and the Squires model – type 2 detector

In this section we will consider the experiment of the previous section but for times $t > T$. As we said before, generally the retarded time T is not constant. However, in the experiment of Aspect *et.al.* that we are considering here, the distance that each detector particle moves during the experiment is of the order of the Bohr radius which is much less than the initial distance between the two detectors which is of the order of a meter. So with a good approximation we can assume that $T \sim 10^{-9}$ is constant during the experiment.

The wave-function of the system after the interaction is given by:

$$|\Psi\rangle = 2^{-1/2}[\phi_{\rm L}(z)\psi_{0\rm L}^{-p}(X) + \phi_{\rm R}(z)\psi_{0\rm R}^{+p}(X)] \tag{5.31}$$

The equation of motion of the left detector from equations (5.27,5.25) would be:

$$\begin{aligned} \dot{X} &= \frac{1}{m}\Re\left[\frac{|\psi_R(t-T)|^2\,\psi_{\rm L}^{-p^*}(t)\boldsymbol{p}_X\psi_{\rm L}^{-p}(t) + |\psi_R^{+p}(t-T)|^2\,\psi_{\rm L}^*(t)\boldsymbol{p}_X\psi_{\rm L}(t)}{|\psi_R(t-T)|^2\,|\psi_L^{-p}(t)|^2 + |\psi_R^{+p}(t-T)|^2\,|\psi_L(t)|^2}\right] \\ &= \frac{\frac{p}{m}}{1+\exp\left[-2a\frac{p}{m}tX(t) + 2a\frac{p}{m}(t-T)Y(t-T) + a\frac{p^2}{m^2}T(2t-T)\right]} \end{aligned} \tag{5.32}$$

where the time dependence of the functions X and Y are explicitly shown. Similarly for the right detector we have:

$$\begin{aligned} \dot{Y} &= \frac{1}{m}\Re\left[\frac{|\psi_L(t-T)|^2\,\psi_{\mathrm{R}}^{+p^*}(t)\boldsymbol{p}_Y\psi_{\mathrm{R}}^{+p}(t) + |\psi_L^{-p}(t-T)|^2\,\psi_{\mathrm{R}}^*(t)\boldsymbol{p}_Y\psi_{\mathrm{R}}(t)}{|\psi_L(t-T)|^2\,|\psi_R^{+p}(t)|^2 + |\psi_L^{-p}(t-T)|^2\,|\psi_R(t)|^2}\right] \\ &= \frac{\frac{p}{m}}{1+\exp\left[-2a\frac{p}{m}tY(t) + 2a\frac{p}{m}(t-T)X(t-T) + a\frac{p^2}{m^2}T(2t-T)\right]} \end{aligned} \tag{5.33}$$

If there is no retardation, that is $T = 0$, then the above equations reduce to equations (2.67) and (2.68) as expected.

As analytic solutions are not possible for the above equations, we have solved them numerically for a hundred pairs of initial positions which were chosen randomly with normal distribution consistent with the Gaussian wave-packet given in equation (2.49) (see appendix **C**).

In figures 5.3 and 5.4 we have shown the positions of the left and right detectors as a function of time for a given pair respectively. In the time interval $0 < t < T$, in all 200 figures obtained, the velocity of the detector particle tends to zero if the initial position is negative and remains finite if it is positive. This is in agreement with the analytic argument that we used in sections 2.5 and 5.4.

At times later than the retarded time, for 99 out of 100 pairs only one of the detectors (in 52 events, the left one and in 47 events, the right one) will record the photon and the other one stops recording. For 1 pair of initial values (X_0, Y_0) (No. 76 in appendix **C**), none of the detectors will record the photon which may be treated as a wrong result because quantum theory is violated. However, it is interesting to note that if X_0 and Y_0 are

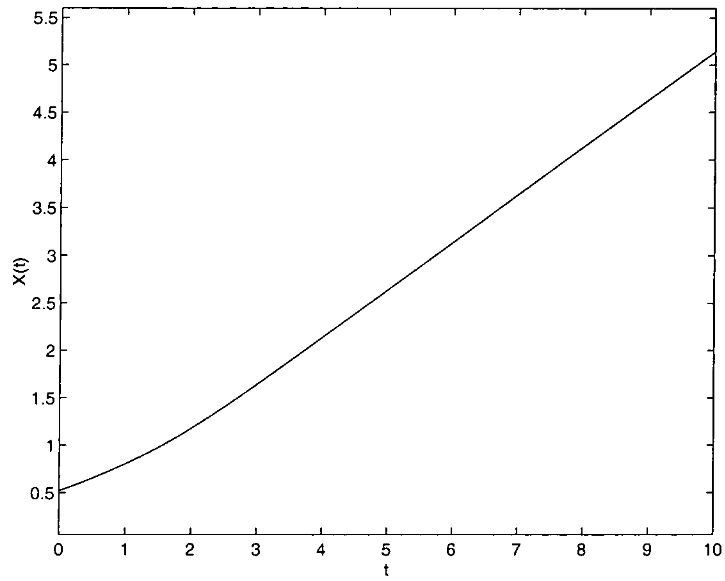

Figure 5.3: The position of the left detector particle as a function of time in Squires model with $X_0 = 0.5197$ and $Y_0 = -0.4113$.

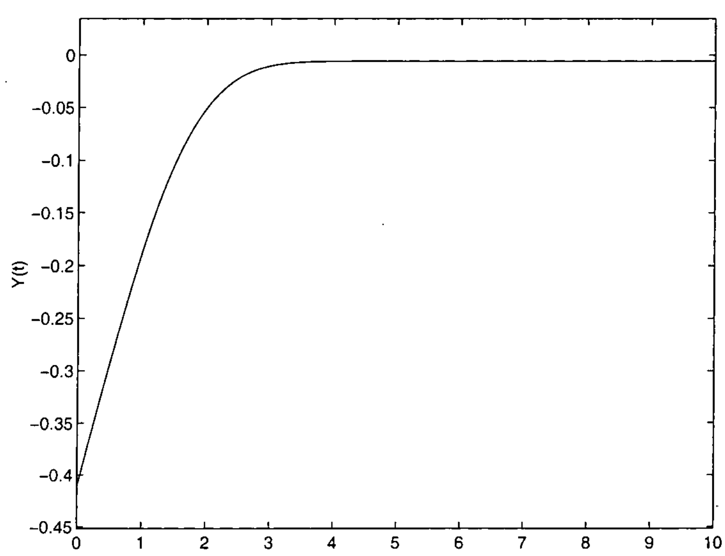

Figure 5.4: The position of the right detector particle as a function of time in Squires model with $X_0 = 0.5197$ and $Y_0 = -0.4113$.

the same then according to the symmetry of the problem it is not possible to determine which of the detectors should detect the photon and incidently in this case X_0 and Y_0 are nearly the same and both negative. Considering the limited precisions of the numerical calculations which is inevitable in computer packages and that these differential equations are very sensitive to tiny changes, one may wonder that this wrong result may be due to the errors in calculations. It should be noted that in a symmetric situation like this, the Bohm model is ambiguous too. For example as we said in section 2.3 according to the Bohm model the initial position of a particle in its wave-packet determines if the particle is transmitted or reflected by a beam-splitter. But if the reflectivity and the transmittance are the same and the wave-packet is Gaussian, then for the initial position in the center of the wave-packet the trajectory of the particle is not known. So we may conclude that here the Squires model is in exact agreement with quantum theory.

Finally, when the particle stops recording, as figure 5.4 shows, the maximum distance that it moves is of the order of the width of the wave-packet, so if the particle is bound in an atom it will remain inside that and cannot get out. But if the particle continues recording, as in figure 5.3, it would be able to leave the atom and can be accelerated in an electric field. And of course, as we said before, the time needed for this to happen is more than the retarded time. In the next section we will make use of this when applying the Squires model to a realistic experiment.

5.6 A realistic photo-multiplier and the Squires model

The experiments of Aspect *et.al.* are the most important experimental evidence of non-locality. So it seems necessary to discuss the predictions of the Squires model about these experiments in which photo-multipliers are used for detection of photons. Figure 5.5 shows the scheme of such an experiment that we are considering here.

A photon is emitted from the source S. Its wave-function consists of two parts: one going to the right $\phi_R(z)$ and the other to the left $\phi_L(z)$. At the time $t = 0$ these interact with primary detectors at the same distance $2D \sim 1$ meter from the source.

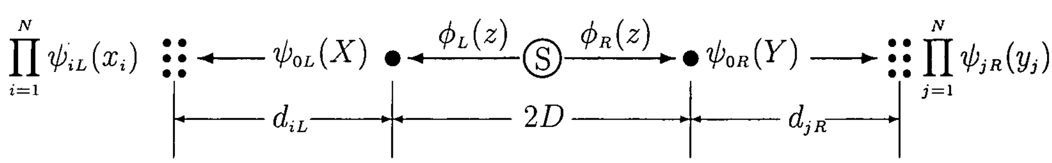

Figure 5.5: The schematic shape of photon detection in a photo-multiplier.

We model the detectors in the following way. The photon strikes a primary particle detector and gives a very small momentum to a single electron, initially in a Gaussian wave-packet. The electron will be knocked out of the atom and will then be accelerated by an electric field, of the order of 150 V/cm [Kle86]. This will give it an acceleration and will reach the secondary detectors in a time $\tau = 4 \times 10^{-8}$ *sec*. In fact we may approximate this situation by supposing that the electron wave-packets after interaction with the photon move with a constant velocity of $\frac{p}{m} \sim \frac{1}{10}c$ ($E_k \sim 1\ KeV$).

When the electron reaches the secondary detectors we assume it is brought to rest and that its energy excites some "atoms". These atoms are modeled by electrons, again initially at rest in Gaussian wave-packets, which are assumed to be bound in some potential. After the excitation they spread (decay) by usual quantum evolution, and N electrons are released.

At $t < 0$, the initial wave-functions of the primary detectors on the left and right of the photon source are respectively $\psi_{0L}(X)$, $\psi_{0R}(Y)$ given by equations (2.49 and 2.57) and those of secondary detectors are $\psi_{iL}(x_i)$ and $\psi_{jR}(y_j)$, where:

$$\psi_{iL}(x_i) = \left(\frac{a}{\pi}\right)^{1/4} \exp\left[-\frac{1}{2}ax_i^2\right] , \tag{5.34}$$

A similar equation holds for $\psi_{jR}(y_j)$. In the above equations (X, Y, x_i, y_j) are the positions of the electron that are struck in the (left primary, right primary, left secondary, right secondary) particle detector, measured from the center of their initial wave-function and as before $1/\sqrt{a} = R_{Bohr}$ is the width of the Gaussian wave-packets. Also we have assumed that all of the secondary detectors to be at the same position:

$$d_{iL} = d_{jR} = d = 30\ cm \qquad \text{for all } i, j\,. \tag{5.35}$$

Before the photon wave reaches the primary detectors, that is at times $t < 0$ the total wave-function of the system is:

$$|\Psi\rangle = 2^{-1/2}\left[\phi_L(z) + \phi_R(z)\right]\left[\psi_{0L}(X)\psi_{0R}(Y)\right]\left[\prod_{i=1}^{N}\psi_{iL}(x_i)\prod_{j=1}^{N}\psi_{jR}(y_j)\right] \tag{5.36}$$

After the photon has interacted with the primary detectors but before the secondary detectors are affected, that is at times $0 < t < \tau$ total wave-function of the system becomes:

$$|\Psi\rangle = 2^{-1/2}\left[\phi_L(z)\psi_{0L}^{-p}(X)\psi_{0R}(Y) + \phi_R(z)\psi_{0L}(X)\psi_{0R}^{+p}(Y)\right]\left[\prod_{i=1}^{N}\psi_{iL}(x_i)\prod_{j=1}^{N}\psi_{jR}(y_j)\right], \quad (5.37)$$

where $\psi_{0R}^{-p}(X)$ and $\psi_{0R}^{+p}(Y)$ are given by equations (2.52) and (2.60) where as we said we have neglected the quantum spreadings here. Also it should be noted that these states are not exactly orthogonal to their initial states (2.49) and (2.57) at $t = 0$, but their overlap is $\exp[-m^2v^2/4\hbar^2 a]$ which has the value $\exp(-47.31) \approx 0$ with the parameters used above.

For $t > \tau$ *i.e.* when the wave-packet from the primary detectors has reached the secondary detectors, we have:

$$\begin{aligned}|\Psi\rangle = 2^{-1/2}\Bigg[&\phi_L(z)\psi_{0L}(X-d)\psi_{0R}(Y)\prod_{i=1}^{N}\psi_{iL}^{e}(x_i)\prod_{j=1}^{N}\psi_{jR}(y_j) + \\ &\phi_R(z)\psi_{0L}(X)\psi_{0R}(Y-d)\prod_{i=1}^{N}\psi_{iL}(x_i)\prod_{j=1}^{N}\psi_{jR}^{e}(y_j)\Bigg], \quad (5.38)\end{aligned}$$

Here we have supposed that the primary detectors are at rest without quantum spreading and

$$\psi_{iL}^{e}(x_i) = \frac{\left(\frac{a}{\pi}\right)^{1/4}}{B}\exp\left[-\frac{a}{2B^2}x_i^2\right], \quad (5.39)$$

with

$$B = \left(1 + i\frac{\hbar(t-\tau)a}{m}\right)^{1/2}. \quad (5.40)$$

Now let's first examine the prediction of the generalized Bohm model discussed in section 2.4.

In the region $0 < t < \tau$, the wave-function of the system is given by equation (5.37), and from equation (2.47) or directly from equation (2.67) and (2.68) we get:

$$\dot{X} = \frac{\frac{p}{m}}{1 + \exp[-2a\frac{p}{m}t(X - Y)]}, \tag{5.41}$$

and

$$\dot{Y} = \frac{\frac{p}{m}}{1 + \exp[-2a\frac{p}{m}t(Y - X)]}. \tag{5.42}$$

The form of the solution to these equations depends sensitively upon the initial values, X_0 and Y_0. If for example, $X_0 - Y_0 > 0$ then X becomes large and Y remains close to Y_0. Thus the photon is "observed" at the left detector. In fact, to a good approximation, in this case,

$$X = X_0 + \frac{p}{m}t, \tag{5.43}$$

and

$$Y = Y_0. \tag{5.44}$$

We now consider the second stage of the detection. At times $t > \tau$ the wave-function of the system is given by equation (5.38), and we find for the trajectories

$$\dot{x}_k = \frac{\Gamma(t-\tau)x_k}{1 + \Gamma(t-\tau)^2}\,\frac{1}{1+E}, \tag{5.45}$$

where

$$E = \exp\left(a\left[Y^2 - (Y-d)^2\right]\right)\exp\left(a\left[(X-d)^2 - X^2\right]\right)$$
$$\exp\left(\frac{\Gamma(t-\tau)^2}{1+\Gamma(t-\tau)^2}\left[\sum_{j=1}^{N} ay_j^2 - \sum_{i=1}^{N} ax_i^2\right]\right), \tag{5.46}$$

and

$$\Gamma = \frac{\hbar^2 a^2}{m^2} = 1.7 \times 10^{33} \quad sec^{-2}. \tag{5.47}$$

A similar equation holds for y_j with E replaced by E^{-1}.

Let us consider the situation as before where $X_0 - Y_0 > 0$ and the left detector begins to record the photon. At times just greater than τ, the first two exponentials in (5.46) give a factor which is approximately $\exp[-2ad^2]$ where we have used the fact that aX_0^2, etc., are of the order of unity. It follows that E will be very small unless

$$\frac{\Gamma t^2}{1+\Gamma t^2}\left[\sum ay_j^2 - \sum ax_i^2\right] \sim 2ad^2. \tag{5.48}$$

The first factor here is always smaller than one, so since again ax_i^2 and ay_j^2 are initially all of the order of unity, and as the kinetic energy of the primary particle detector is $\sim 1\ KeV$, in an actual photo-multiplier, N is of the order of 100. It follows then that E will remain very small, so that the x_i will become large and the y_j will remain near to their original values. Thus, which detector "flashes" (in this case the left one) will be determined by just the subtraction of the initial values of the positions of the primary detectors $(X_0 - Y_0)$.

We now turn to the prediction of Squires model. Here the retarded time between the primary detectors is $T_p \sim \times 10^{-9}\ sec$.

For $t < 0$ the wave-function for the left primary detector is given by (2.49) and the wave-function of the system Ψ is given by (5.36). According to the equation of motion (5.27) the velocity of this particle detector and similarly all of the other detector particles are zero, as expected.

For $0 < t < T_p$ the wave-function of the system is given by equation (5.37) and the wave-function of the primary detector on the left is given by (2.52). From equation (5.27) we have for the trajectories:

$$m\dot{X} = \Re\left(\frac{\psi^*_{0L}(X)p^{op}_{0L}\psi_{0L}(X) + \psi^{-p*}_{0L}(X)p^{op}_{0L}\psi^{-p}_{0L}(X)}{|\psi_{0L}(X)|^2 + |\psi^{-p}_{0L}(X)|^2}\right), \tag{5.49}$$

or

$$\dot{X} = \frac{\frac{p}{m}}{\exp\left(-2a\frac{p}{m}Xt + a\frac{p^2}{m^2}t^2\right) + 1}, \tag{5.50}$$

with a similar equation for $\dot{Y}$. For the left secondary detectors we have,

$$m\dot{x_i} = \Re\left(\frac{\psi^*_{iL}(x_i)p^{op}_{iL}\psi_{iL}(x_i) + \psi^*_{iL}(x_i)p^{op}_{iL}\psi_{iL}(x_i)}{|\psi_{iL}(x_i)|^2 + |\psi_{iL}(x_i)|^2}\right) = 0\,, \tag{5.51}$$

and similarly $\dot{y_j} = 0$. From the above equations again it is seen that each of the detectors behaves as if the others are not present. After this, as we said in the previous section only one of the detector particles can leave the atom and be accelerated in the electric field which we suppose to be the left one.

The velocity of the primary particle detector is very small when it leaves the atom and it is about $\frac{1}{10}c$ when it reaches the secondary ones. In this time interval again the wave-function of the system is given by equation (5.37) and the equations of motions for primary detectors are given by (5.32) and (5.33). Using these equations, with the same method used in section 2.5, it can be shown that still the left detector will continue moving and the right one stops recording. So from now on we assume $X \approx d$ and $Y \approx Y_0$ (about the width of the wave-packet) in accordance with the numerical caculations mentioned in section 5.5. Of course the secondary detectors will remain at rest as before.

At time $t > \tau$ the retarded time between the secondary detectors is $T_s = 2(D+d)/c$ which is still about 10^{-9} *sec.* So for times $\tau \leq t < \tau + T_s$, where the wave-packet from the primary detectors has reached the secondary detectors but the secondary detectors do not receive this information from each other, the wave-function of the system is given by (5.38) and the wave-functions of the left secondary detectors are given by (5.39). Again using equation (5.27) we have:

$$
\begin{aligned}
m\dot{X} &= \Re\left(\frac{\prod_{i=1}^{N}|\psi_{iL}(x_i)|^2\,\psi_{0L}^*(X)p_{0L}^{op}\psi_{0L}(X)+\prod_{i=1}^{N}\left|\psi_{iL}^{e}(x_i)\right|^2\psi_{0L}^*(X-d)p_{0L}^{op}\psi_{0L}(X-d)}{\prod_{i=1}^{N}|\psi_{iL}(x_i)|^2\,|\psi_{0L}(X)|^2+\prod_{i=1}^{N}\left|\psi_{iL}^{e}(x_i)\right|^2|\psi_{0L}(X-d)|}\right) \\
&= 0\,,
\end{aligned}
\tag{5.52}
$$

and similarly for $\dot{Y}$. As the wave-functions of the primary detectors will remain real, $\dot{Y} = \dot{X} = 0$ in later times. However in this time interval, for secondary detectors we have:

$$
\dot{x_i} = \frac{\frac{\Gamma(t-\tau)}{1+\Gamma(t-\tau)^2}x_i}{(1+\Gamma(t-\tau)^2)^{N/2}\exp\left[-2aXd+ad^2-\sum_{k=1}^{N}\frac{a\Gamma(t-\tau)^2}{1+\Gamma(t-\tau)^2}x_k^2\right]+1}\,, \tag{5.53}
$$

with a similar equation for $\dot{y_j}$.

Here since x_{k0} and y_{k0} are randomly distributed then clearly

$$
\sum_{k=1}^{N}\frac{a\Gamma(t-\tau)^2}{1+\Gamma(t-\tau)^2}x_{k0}^2 \approx N \tag{5.54}
$$

$$
\sum_{k=1}^{N}\frac{a\Gamma(t-\tau)^2}{1+\Gamma(t-\tau)^2}y_{k0}^2 \approx N\,, \tag{5.55}
$$

and since $X(\tau) \approx d$, so $-2aXd + ad^2 \approx -ad^2$ and as $Y(\tau) \approx 10^{-9}$ then $-2aYd + ad^2 \approx ad^2$ and the exponential goes to zero for the left secondary detector but goes to infinity for the right one, so $\dot{x}_i$ remains finite while $\dot{y}_j$ approaches zero. So in this case the equation of motion for the secondary detectors would be:

$$\dot{x}_i = \frac{\Gamma t_1}{1+\Gamma t_1^2} x_i \implies x_i = x_{i0}\left(1+\Gamma(t-\tau)^2)\right)^{1/2} \tag{5.56}$$

$$\dot{y}_j = 0 \implies y_j = y_{j0}\,. \tag{5.57}$$

At times $t > \tau + T_s$ for the left secondary detectors we have:

$$\dot{x}_i = \frac{\frac{(t-\tau)\Gamma}{1+(t-\tau)^2\Gamma} x_i}{\frac{|\psi_{0R}(Y-d)|^2|\psi_{0L}(X)|^2 \prod_{k=1}^{N} \left|\psi^e_{kR}(t-T_s)\right|^2 \prod_{k=1}^{N} |\psi_{kL}(x_k)|^2}{|\psi_{0R}(Y)|^2|\psi_{0L}(X-d)|^2 \prod_{k=1}^{N} |\psi_{kR}(t-T_s)|^2 \prod_{k=1}^{N} \left|\psi^e_{kL}(x_k)\right|^2} + 1}\,, \tag{5.58}$$

which after some easy calculations become,

$$\dot{x}_i = \frac{\frac{\Gamma t_1}{1+\Gamma t_1^2} x_i}{\left(\frac{1+\Gamma t_1^2}{1+\Gamma t_2^2}\right)^{N/2} \exp\left[-2ad[X(\tau)-Y(\tau)] + \sum_{k=1}^{N} \frac{a\Gamma t_2^2}{1+\Gamma t_2^2}[y_k(t-T_s)]^2 - \frac{a\Gamma t_1^2}{1+\Gamma t_1^2} x_k^2\right] + 1}\,. \tag{5.59}$$

with $t_1 = t - \tau$ and $t_2 = t - \tau - T_s$. Similar equations hold for $\dot{y}_j$. Again with the same argument as before it is clear that the left secondary detectors would continue recording the photon, where the right ones have stopped recording.

In brief, in this section we found that according to Squires model, in the experiment of Aspect *et.al.* in the time interval after the interaction of the photon with the primary detectors but before the secondary detectors are affected, it is the hidden-variables of the primary detectors which determine which of them would record the photon. After the interaction of the primary detectors with secondary ones, it was shown that still the outcome of the experiment depends on the hidden-variables of the primary detectors. However, as it was shown in the previous section for each pair of the initial positions only one of the primary detectors will record the photon and the probability for this is $\frac{1}{2}$, this means that there is an equal probability that the photon either be detected by the left secondary detectors or the right one. So for this experiment the prediction of the Squires model is in complete agreement with quantum theory.

At the end, it would be nice to give a brief summary of this chapter. We started with Hardy's theorem, according to which local hidden-variable theories should be non-Lorentz invariant too, and that not only the Bohm model is non-local but also non-Lorentz invariant at the level of individual systems. It was shown that the Squires model circumvents all of these. We showed how, if in a system of particles the interaction between the particles is instantaneous, the ambiguity of the model is removed in the cases where the wave-function of the system explicitly depends on time and that the inconsistency of the model with orthodox quantum theory, considered by Squires, can be removed with the use of multiple-time wave-function. Finally, that the model is capable of giving a correct description of one of the experiments of non-locality was shown in the last two sections.

Chapter 6

Conclusion

Orthodox quantum theory encounters serious difficulties in the description of an individual system. The origin of these problems lies in the fact that the collapse of the wave-function happens only when the observer gains some relevant information about the system as we saw in the Schrödinger cat and the Wigner's friend paradoxes. It is interesting that the sufficient condition of physical reality which is the backbone of the EPR theorem is in fact the negation of this, that is as long as the the wave-function collapses all observers, at all times after this collapse, should be able to predict the relevant information. Furthermore, we proved in comment 2.1.1 that the quantum description is frame dependent and also in comment 2.1.2 we showed that, even if we are not measuring the position of the particle, the spatial part of the wave-function collapses. This reminds me of a sentence from Euan Squires that all measurements are really position measurements and, incidentally, the hidden-variable model of Bohm is primarily based on this fact that each particle moves on a trajectory which is determined by the value of hidden-variable and the outcome of a measurement in turn depends on this trajectory, that is, the position of the particle.

However, the Bohm model is non-local and indeed, Bell in the form of an inequality proved that orthodox quantum theory violates the predictions of any local theory. Unfortunately, the original Bell inequality cannot be used in experiments and since then some other Bell-type inequalities are deduced with some extra assumptions. We introduced a new method for obtaining Bell-type inequalities which is based only on the joint probabilities of measurements at different times which in turn implies locality. This method can easily be extended to the system with any number of local hidden variables or parameters and any number of outcomes for each parameter, though at the moment, the time needed for computer calculations is a problem for higher orders here. The inequalities for two especial cases, 2,2:2,2 and 2,3:2,3, deduced by this method, which are more than those obtained by the others, are listed in appendices. We have also used the PV and POVM measurements to test the predictions of orthodox quantum theory for the case 2,3:2,3 inequalities and it was found that one of these is violated by a factor of 1.5 which exceed that of $\sqrt{2}$ obtained by the other experiments suggested for testing locality and consequently the experiment can be done more easily and accurately. Many other 2,3:2,3 experiments can be designed and tested for violation of any of the inequalities of appendix **B**.

The Squires model is a local version of the Bohm model. By using the multiple time wave-function we showed how the ambiguity of the model, in the cases where the wave-function depends explicitly on time, can be removed if the interaction between the particles are instantaneous, which is most likely in the experiments of non-locality. To see to what extent the theory is able to predict the outcomes of the experiments in practice, we have applied the model to a situation like one of the experiments of Aspect *et.al.* which are mainly used to test non-locality. The results obtained confirm that in a sufficiently realistic condition the predictions of the Squires model is in complete agreement with orthodox quantum theory. However, there is a criticism about this model which is worth mentioning here. In the example considered in section 5.5, when a photon is emitted from the source in the form of two wave-packets moving to the right and the left, the initial position of the detectors act as the hidden-variables which determine whether the 'photon' goes to the left or the right. In answer to this, we recall that in that example we assumed that there are no trajectories for bosons and we averaged over the positions of the photon. This means that indeed we have treated the photon just the same as in orthodox quantum theory and it is due to this fact that both of the detectors initially start recording the photon, but after a short distance[1] one of them stops recording. Also, we would like to emphasize that even if it does turn out that bosons have trajectories, the discussion we made about the Aspect *et.al.* experiment is still appropriate since in such experiments the spin of the particle (photon) is determined, where there is no spin "hidden-variable". In this case it is certain that the recorded value is a property of the hidden-variable in the detectors.

[1] In the example considered in section 5.5, this distance is of the order of the width of the wave-packet.

Appendix A

Case 2,2:2,2 Bell-type inequalities

$$0 \leq -2p_{11} + p_1^L + p_1^R \leq 1$$
$$0 \leq -p_{11} + p_1^R \leq 1$$
$$0 \leq -p_{11} + p_{12} + p_1^R \leq 1$$
$$0 \leq -p_{11} - p_{21} + p_2^L + p_1^R \leq 1$$
$$0 \leq -p_{11} + p_{12} - p_{21} - p_{22} + p_2^L + p_1^R \leq 1$$
$$0 \leq -p_{11} + p_1^L \leq 1$$
$$0 \leq -p_{11} + p_{21} + p_1^L \leq 1$$
$$0 \leq -p_{11} - p_{12} + p_1^L + p_2^R \leq 1$$
$$0 \leq -p_{11} - p_{12} + p_{21} - p_{22} + p_1^L + p_2^R \leq 1$$
$$0 \leq -p_{11} - p_{12} - p_{21} + p_{22} + p_1^L + p_1^R \leq 1$$
$$0 \leq -p_{11} - p_{12} + p_1^L + p_1^R \leq 1$$
$$0 \leq -p_{11} - p_{21} + p_1^L + p_1^R \leq 1$$
$$0 \leq -p_{11} + p_1^L + p_1^R \leq 1$$
$$0 \leq +p_{22} \leq 1$$
$$0 \leq +p_{21} \leq 1$$
$$0 \leq +p_{12} \leq 1$$
$$0 \leq -p_{12} + p_2^R \leq 1$$
$$0 \leq -p_{22} + p_2^R \leq 1$$
$$0 \leq +p_2^R \leq 1$$
$$0 \leq +p_{21} - p_{22} + p_2^R \leq 1$$

$0 \leq -p_{21} + p_1^R \leq 1$
$0 \leq -p_{21} + p_{22} + p_1^R \leq 1$
$0 \leq +p_1^R \leq 1$
$0 \leq -p_{21} + p_2^L \leq 1$
$0 \leq -p_{22} + p_2^L \leq 1$
$0 \leq +p_2^L \leq 1$
$0 \leq +p_{12} - p_{22} + p_2^L \leq 1$
$0 \leq -p_{12} - p_{22} + p_2^L + p_2^R \leq 1$
$0 \leq -p_{21} - p_{22} + p_2^L + p_2^R \leq 1$
$0 \leq -2p_{22} + p_2^L + p_2^R \leq 1$
$0 \leq -p_{22} + p_2^L + p_2^R \leq 1$
$0 \leq -2p_{21} + p_2^L + p_1^R \leq 1$
$0 \leq -p_{21} - p_{22} + p_2^L + p_1^R \leq 1$
$0 \leq -p_{21} + p_2^L + p_1^R \leq 1$
$0 \leq -p_{12} + p_1^L \leq 1$
$0 \leq -p_{12} + p_{22} + p_1^L \leq 1$
$0 \leq +p_1^L \leq 1$
$0 \leq -2p_{12} + p_1^L + p_2^R \leq 1$
$0 \leq -p_{12} - p_{22} + p_1^L + p_2^R \leq 1$
$0 \leq -p_{12} + p_1^L + p_2^R \leq 1$
$0 \leq +p_{11} \leq 1$
$0 \leq +p_{11} - p_{12} + p_2^R \leq 1$
$0 \leq +p_{11} - p_{21} + p_2^L \leq 1$
$0 \leq +p_{11} - p_{12} - p_{21} - p_{22} + p_2^L + p_2^R \leq 1$
$0 \leq +1 \leq 1$
$0 \leq -p_{11} + p_{12} + p_{21} + p_{22} - p_2^L - p_2^R + 1 \leq 1$
$0 \leq -p_{11} + p_{21} - p_2^L + 1 \leq 1$
$0 \leq -p_{11} + p_{12} - p_2^R + 1 \leq 1$
$0 \leq -p_{11} + 1 \leq 1$
$0 \leq +p_{12} - p_1^L - p_2^R + 1 \leq 1$
$0 \leq +p_{12} + p_{22} - p_1^L - p_2^R + 1 \leq 1$
$0 \leq +2p_{12} - p_1^L - p_2^R + 1 \leq 1$

$0 \leq -p_1^L + 1 \leq 1$
$0 \leq +p_{12} - p_{22} - p_1^L + 1 \leq 1$
$0 \leq +p_{12} - p_1^L + 1 \leq 1$
$0 \leq +p_{21} - p_2^L - p_1^R + 1 \leq 1$
$0 \leq +p_{21} + p_{22} - p_2^L - p_1^R + 1 \leq 1$
$0 \leq +2p_{21} - p_2^L - p_1^R + 1 \leq 1$
$0 \leq +p_{22} - p_2^L - p_2^R + 1 \leq 1$
$0 \leq +2p_{22} - p_2^L - p_2^R + 1 \leq 1$
$0 \leq +p_{21} + p_{22} - p_2^L - p_2^R + 1 \leq 1$
$0 \leq +p_{12} + p_{22} - p_2^L - p_2^R + 1 \leq 1$
$0 \leq -p_{12} + p_{22} - p_2^L + 1 \leq 1$
$0 \leq -p_2^L + 1 \leq 1$
$0 \leq +p_{22} - p_2^L + 1 \leq 1$
$0 \leq +p_{21} - p_2^L + 1 \leq 1$
$0 \leq -p_1^R + 1 \leq 1$
$0 \leq +p_{21} - p_{22} - p_1^R + 1 \leq 1$
$0 \leq +p_{21} - p_1^R + 1 \leq 1$
$0 \leq -p_{21} + p_{22} - p_2^R + 1 \leq 1$
$0 \leq -p_2^R + 1 \leq 1$
$0 \leq +p_{22} - p_2^R + 1 \leq 1$
$0 \leq +p_{12} - p_2^R + 1 \leq 1$
$0 \leq -p_{12} + 1 \leq 1$
$0 \leq -p_{21} + 1 \leq 1$
$0 \leq -p_{22} + 1 \leq 1$
$0 \leq +p_{11} - p_1^L - p_1^R + 1 \leq 1$
$0 \leq +p_{11} + p_{21} - p_1^L - p_1^R + 1 \leq 1$
$0 \leq +p_{11} + p_{12} - p_1^L - p_1^R + 1 \leq 1$
$0 \leq +p_{11} + p_{12} + p_{21} - p_{22} - p_1^L - p_1^R + 1 \leq 1$
$0 \leq +p_{11} + p_{12} - p_{21} + p_{22} - p_1^L - p_2^R + 1 \leq 1$
$0 \leq +p_{11} + p_{12} - p_1^L - p_2^R + 1 \leq 1$
$0 \leq +p_{11} - p_{21} - p_1^L + 1 \leq 1$
$0 \leq +p_{11} - p_1^L + 1 \leq 1$

$$0 \leq +p_{11} - p_{12} + p_{21} + p_{22} - p_2^L - p_1^R + 1 \leq 1$$
$$0 \leq +p_{11} + p_{21} - p_2^L - p_1^R + 1 \leq 1$$
$$0 \leq +p_{11} - p_{12} - p_1^R + 1 \leq 1$$
$$0 \leq +p_{11} - p_1^R + 1 \leq 1$$
$$0 \leq +2p_{11} - p_1^L - p_1^R + 1 \leq 1$$

Appendix B

Case 2,3:2,3 Bell-type inequalities

$$0 \le -2p_{11} + p_1^L + p_1^R \le 1$$
$$0 \le -p_{11} - p_{12} - p_{21} + p_2^L + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{11} - p_{12} - p_{21} + p_1^L + p_2^R + p_{21}^{(11,31)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{11} - p_{12} - p_{21} + p_{22} + p_2^L + p_1^R + p_{12}^{(11,13)} - p_{22}^{(11,13)} \le 1$$
$$0 \le -p_{11} - p_{12} - p_{21} + p_{22} + p_1^L + p_1^R \le 1$$
$$0 \le -p_{11} - p_{12} - p_{22} + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{11} - p_{12} - p_{22} + p_1^L + p_2^R + p_{21}^{(11,31)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{11} - p_{12} + p_1^R + p_{12}^{(11,13)} \le 1$$
$$0 \le -p_{11} - p_{12} + p_2^L + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \le 1$$
$$0 \le -p_{11} - p_{12} + p_1^L + p_2^R \le 1$$
$$0 \le -p_{11} - p_{12} + p_1^L + p_1^R \le 1$$
$$0 \le -p_{11} - p_{12} + p_{22} + p_2^L + p_1^R + p_{12}^{(11,13)} - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{11} - p_{12} + p_{22} + p_1^L + p_1^R - p_{21}^{(11,31)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{11} - p_{12} + p_{21} - p_{22} + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \le 1$$
$$0 \le -p_{11} - p_{12} + p_{21} - p_{22} + p_1^L + p_2^R \le 1$$
$$0 \le -p_{11} - p_{12} + p_{21} + p_1^R + p_{12}^{(11,13)} - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{11} - p_{12} + p_{21} + p_1^L + p_1^R - p_{21}^{(11,31)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{11} - p_{21} + p_2^L - p_2^R + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{11} - p_{21} + p_2^L + p_{21}^{(11,31)} + p_{21}^{(22,21)} \le 1$$

$0 \le -p_{11} - p_{21} + p_2^L + p_1^R \le 1$
$0 \le -p_{11} - p_{21} + p_2^L + p_1^R + p_{12}^{(11,13)} - p_{22}^{(11,13)} \le 1$
$0 \le -p_{11} - p_{21} + p_1^L + p_{21}^{(11,31)} \le 1$
$0 \le -p_{11} - p_{21} + p_1^L + p_2^R - p_{12}^{(11,13)} + p_{12}^{(13,23)} + p_{21}^{(11,31)} - p_{22}^{(31,33)} \le 1$
$0 \le -p_{11} - p_{21} + p_1^L + p_1^R - p_{12}^{(11,13)} + p_{22}^{(11,13)} \le 1$
$0 \le -p_{11} - p_{21} + p_1^L + p_1^R \le 1$
$0 \le -p_{11} - p_2^R + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$
$0 \le -p_{11} + p_{21}^{(11,31)} + p_{21}^{(22,21)} \le 1$
$0 \le -p_{11} + p_1^R \le 1$
$0 \le -p_{11} + p_1^R + p_{12}^{(11,13)} \le 1$
$0 \le -p_{11} + p_2^L - p_2^R + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \le 1$
$0 \le -p_{11} + p_2^L + p_{21}^{(22,21)} + p_{21}^{(31,32)} \le 1$
$0 \le -p_{11} + p_2^L + p_1^R - p_{21}^{(11,31)} + p_{21}^{(31,32)} \le 1$
$0 \le -p_{11} + p_2^L + p_1^R + p_{12}^{(11,13)} - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \le 1$
$0 \le -p_{11} + p_1^L \le 1$
$0 \le -p_{11} + p_1^L + p_{21}^{(11,31)} \le 1$
$0 \le -p_{11} + p_1^L + p_2^R - p_{12}^{(11,13)} + p_{12}^{(13,23)} \le 1$
$0 \le -p_{11} + p_1^L + p_2^R - p_{12}^{(11,13)} + p_{12}^{(13,23)} + p_{21}^{(11,31)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \le 1$
$0 \le -p_{11} + p_1^L + p_1^R - p_{12}^{(11,13)} - p_{21}^{(11,31)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$
$0 \le -p_{11} + p_1^L + p_1^R - p_{12}^{(11,13)} \le 1$
$0 \le -p_{11} + p_1^L + p_1^R - p_{21}^{(11,31)} \le 1$
$0 \le -p_{11} + p_1^L + p_1^R \le 1$
$0 \le -p_{11} + p_{21} - p_2^R + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \le 1$
$0 \le -p_{11} + p_{21} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \le 1$
$0 \le -p_{11} + p_{21} + p_1^R - p_{21}^{(11,31)} + p_{21}^{(31,32)} \le 1$
$0 \le -p_{11} + p_{21} + p_1^R + p_{12}^{(11,13)} - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} \le 1$
$0 \le -p_{11} + p_{21} + p_1^L \le 1$
$0 \le -p_{11} + p_{21} + p_1^L + p_2^R - p_{12}^{(11,13)} + p_{12}^{(13,23)} - p_{22}^{(11,13)} \le 1$
$0 \le -p_{11} + p_{21} + p_1^L + p_1^R - p_{12}^{(11,13)} - p_{21}^{(11,31)} + p_{22}^{(31,33)} \le 1$
$0 \le -p_{11} + p_{21} + p_1^L + p_1^R - p_{21}^{(11,31)} \le 1$

$$0 \leq -p_{11} + p_{12} - p_{21} - p_{22} + p_2^L + p_1^R \leq 1$$
$$0 \leq -p_{11} + p_{12} - p_{21} - p_{22} + p_1^L + p_1^R - p_{12}^{(11,13)} + p_{22}^{(11,13)} \leq 1$$
$$0 \leq -p_{11} + p_{12} - p_{21} + p_2^L - p_2^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{11} + p_{12} - p_{21} + p_1^L - p_{12}^{(11,13)} + p_{12}^{(13,23)} + p_{21}^{(11,31)} - p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{11} + p_{12} - p_{22} + p_2^L + p_1^R - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{11} + p_{12} - p_{22} + p_1^L + p_1^R - p_{12}^{(11,13)} - p_{21}^{(11,31)} + p_{22}^{(11,13)} + p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{11} + p_{12} + p_1^R \leq 1$$
$$0 \leq -p_{11} + p_{12} + p_2^L - p_2^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} \leq 1$$
$$0 \leq -p_{11} + p_{12} + p_1^L - p_{12}^{(11,13)} + p_{12}^{(13,23)} \leq 1$$
$$0 \leq -p_{11} + p_{12} + p_1^L + p_1^R - p_{12}^{(11,13)} \leq 1$$
$$0 \leq -p_{11} + p_{12} + p_{22} - p_2^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{11} + p_{12} + p_{22} + p_1^L - p_{12}^{(11,13)} + p_{12}^{(13,23)} + p_{21}^{(11,31)} - p_{22}^{(11,13)} - p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{11} + p_{12} + p_{21} + p_1^R - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{11} + p_{12} + p_{21} + p_1^L + p_1^R - p_{12}^{(11,13)} - p_{21}^{(11,31)} + p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{11} + p_{12} + p_{21} + p_{22} - p_2^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} \leq 1$$
$$0 \leq -p_{11} + p_{12} + p_{21} + p_{22} + p_1^L - p_{12}^{(11,13)} + p_{12}^{(13,23)} - p_{22}^{(11,13)} \leq 1$$
$$0 \leq -2p_{12} - p_1^L + p_2^L + 2p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \leq 1$$
$$0 \leq -2p_{12} + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \leq 1$$
$$0 \leq -2p_{12} + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \leq 1$$
$$0 \leq -2p_{12} + p_2^R + p_{12}^{(11,13)} \leq 1$$
$$0 \leq -2p_{12} + p_2^R + p_{12}^{(11,13)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} \leq 1$$
$$0 \leq -2p_{12} + p_2^L + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(31,33)} \leq 1$$
$$0 \leq -2p_{12} + p_2^L + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \leq 1$$
$$0 \leq -2p_{12} + p_2^L + p_2^R + p_{12}^{(11,13)} - p_{22}^{(11,13)} \leq 1$$
$$0 \leq -2p_{12} + p_2^L + p_2^R + p_{12}^{(11,13)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \leq 1$$
$$0 \leq -2p_{12} + p_1^L + p_2^R \leq 1$$
$$0 \leq -p_{12} - p_{22} - p_1^L + p_2^L + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \leq 1$$
$$0 \leq -p_{12} - p_{22} - p_1^L + p_2^L + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{12} - p_{22} - p_1^L + p_2^L + 2p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{12} - p_{22} - p_1^L + p_2^L + 2p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \leq 1$$

$$0 \le -p_{12} - p_{22} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + 2p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{12} - p_{22} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + 2p_{22}^{(11,13)} + p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} - p_{22} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + 2p_{22}^{(11,13)} \le 1$$
$$0 \le -p_{12} - p_{22} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + 2p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} - p_{22} + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} - p_{22} + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{12} - p_{22} + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} - p_{22} + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \le 1$$
$$0 \le -p_{12} - p_{22} + p_2^R + p_{22}^{(11,13)} \le 1$$
$$0 \le -p_{12} - p_{22} + p_2^R + p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} - p_{22} + p_2^R + p_{21}^{(31,32)} + p_{22}^{(11,13)} - p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{12} - p_{22} + p_2^R + p_{21}^{(31,32)} + p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} - p_{22} + p_2^R + p_{12}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} - p_{22} + p_2^R + p_{12}^{(11,13)} \le 1$$
$$0 \le -p_{12} - p_{22} + p_2^R + p_{12}^{(11,13)} + p_{21}^{(31,32)} - 2p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} - p_{22} + p_2^R + p_{12}^{(11,13)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{12} - p_{22} + p_2^L - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{12} - p_{22} + p_2^L - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} - p_{22} + p_2^L - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \le 1$$
$$0 \le -p_{12} - p_{22} + p_2^L - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} - p_{22} + p_2^L + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} - p_{22} + p_2^L + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{12} - p_{22} + p_2^L + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} - p_{22} + p_2^L + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \le 1$$
$$0 \le -p_{12} - p_{22} + p_2^L + p_2^R \le 1$$
$$0 \le -p_{12} - p_{22} + p_2^L + p_2^R + p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} - p_{22} + p_2^L + p_2^R + p_{21}^{(31,32)} - p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{12} - p_{22} + p_2^L + p_2^R + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} - p_{22} + p_2^L + p_2^R + p_{12}^{(11,13)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} - p_{22} + p_2^L + p_2^R + p_{12}^{(11,13)} - p_{22}^{(11,13)} \le 1$$

$$0 \leq -p_{12} - p_{22} + p_2^L + p_2^R + p_{12}^{(11,13)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - 2p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{12} - p_{22} + p_2^L + p_2^R + p_{12}^{(11,13)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \leq 1$$
$$0 \leq -p_{12} - p_{22} + p_1^L + p_2^R - p_{12}^{(11,13)} + p_{22}^{(11,13)} \leq 1$$
$$0 \leq -p_{12} - p_{22} + p_1^L + p_2^R - p_{12}^{(11,13)} + p_{22}^{(11,13)} + p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{12} - p_{22} + p_1^L + p_2^R - p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{12} - p_{22} + p_1^L + p_2^R \leq 1$$
$$0 \leq -p_{12} - p_1^L + p_2^L - p_2^R + 2p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \leq 1$$
$$0 \leq -p_{12} - p_1^L + p_2^L - p_2^R + 2p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{12} - p_1^L + p_2^L + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \leq 1$$
$$0 \leq -p_{12} - p_1^L + p_2^L + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{12} - p_1^L + p_2^L + p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{12} - p_1^L + p_2^L + p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \leq 1$$
$$0 \leq -p_{12} - p_1^L + p_2^L + 2p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{12} - p_1^L + p_2^L + 2p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \leq 1$$
$$0 \leq -p_{12} - p_2^R + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \leq 1$$
$$0 \leq -p_{12} - p_2^R + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + 2p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{12} - p_2^R + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \leq 1$$
$$0 \leq -p_{12} - p_2^R + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{12} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \leq 1$$
$$0 \leq -p_{12} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{12} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \leq 1$$
$$0 \leq -p_{12} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{12} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{12} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \leq 1$$
$$0 \leq -p_{12} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} - p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{12} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \leq 1$$
$$0 \leq -p_{12} + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{12} + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \leq 1$$
$$0 \leq -p_{12} + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{12} + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \leq 1$$

$$0 \le -p_{12} + p_{12}^{(11,13)} \le 1$$
$$0 \le -p_{12} + p_{12}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} + p_{12}^{(11,13)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{12} + p_{12}^{(11,13)} + p_{21}^{(31,32)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} + p_2^R \le 1$$
$$0 \le -p_{12} + p_2^R + p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} + p_2^R + p_{21}^{(31,32)} - p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{12} + p_2^R + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} + p_2^R + p_{12}^{(13,23)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} + p_2^R + p_{12}^{(13,23)} \le 1$$
$$0 \le -p_{12} + p_2^R + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} + p_2^R + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{12} + p_2^R + p_{12}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} + p_2^R + p_{12}^{(11,13)} \le 1$$
$$0 \le -p_{12} + p_2^R + p_{12}^{(11,13)} + p_{21}^{(31,32)} - 2p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} + p_2^R + p_{12}^{(11,13)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{12} + p_2^L - p_2^R + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{12} + p_2^L - p_2^R + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + 2p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} + p_2^L - p_2^R + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \le 1$$
$$0 \le -p_{12} + p_2^L - p_2^R + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} + p_2^L - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{12} + p_2^L - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} + p_2^L - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \le 1$$
$$0 \le -p_{12} + p_2^L - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} + p_2^L + p_{21}^{(22,21)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} + p_2^L + p_{21}^{(22,21)} + p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{12} + p_2^L + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} + p_2^L + p_{21}^{(22,21)} + p_{21}^{(31,32)} \le 1$$
$$0 \le -p_{12} + p_2^L + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} + p_2^L + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(31,33)} \le 1$$

$$0 \le -p_{12} + p_2^L + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} + p_2^L + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \le 1$$
$$0 \le -p_{12} + p_2^L + p_{12}^{(11,13)} - p_{22}^{(11,13)} \le 1$$
$$0 \le -p_{12} + p_2^L + p_{12}^{(11,13)} - p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} + p_2^L + p_{12}^{(11,13)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{12} + p_2^L + p_{12}^{(11,13)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} + p_2^L + p_2^R - p_{22}^{(11,13)} \le 1$$
$$0 \le -p_{12} + p_2^L + p_2^R - p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} + p_2^L + p_2^R + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{12} + p_2^L + p_2^R + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} + p_2^L + p_2^R + p_{12}^{(13,23)} - p_{22}^{(11,13)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} + p_2^L + p_2^R + p_{12}^{(13,23)} - p_{22}^{(11,13)} \le 1$$
$$0 \le -p_{12} + p_2^L + p_2^R + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} + p_2^L + p_2^R + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{12} + p_2^L + p_2^R + p_{12}^{(11,13)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} + p_2^L + p_2^R + p_{12}^{(11,13)} - p_{22}^{(11,13)} \le 1$$
$$0 \le -p_{12} + p_2^L + p_2^R + p_{12}^{(11,13)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - 2p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} + p_2^L + p_2^R + p_{12}^{(11,13)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{12} + p_1^L \le 1$$
$$0 \le -p_{12} + p_1^L + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} + p_1^L + p_2^R - p_{12}^{(11,13)} \le 1$$
$$0 \le -p_{12} + p_1^L + p_2^R - p_{12}^{(11,13)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} + p_1^L + p_2^R - p_{12}^{(11,13)} + p_{12}^{(13,23)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} + p_1^L + p_2^R - p_{12}^{(11,13)} + p_{12}^{(13,23)} \le 1$$
$$0 \le -p_{12} + p_1^L + p_2^R - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} + p_1^L + p_2^R \le 1$$
$$0 \le -p_{12} + p_{22} - p_1^L + p_2^L - p_2^R + 2p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \le 1$$
$$0 \le -p_{12} + p_{22} - p_1^L + p_2^L - p_2^R + 2p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} + p_{22} - p_1^L + p_2^L + p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} + p_{22} - p_1^L + p_2^L + p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} \le 1$$

$$0 \le -p_{12} + p_{22} - p_2^R + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{12} + p_{22} - p_2^R + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + 2p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} + p_{22} - p_2^R + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \le 1$$
$$0 \le -p_{12} + p_{22} - p_2^R + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} + p_{22} + p_{21}^{(22,21)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} + p_{22} + p_{21}^{(22,21)} + p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{12} + p_{22} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} + p_{22} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \le 1$$
$$0 \le -p_{12} + p_{22} + p_{12}^{(11,13)} \le 1$$
$$0 \le -p_{12} + p_{22} + p_{12}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} + p_{22} + p_{12}^{(11,13)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{12} + p_{22} + p_{12}^{(11,13)} + p_{21}^{(31,32)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} + p_{22} + p_2^R + p_{12}^{(13,23)} - p_{22}^{(11,13)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} + p_{22} + p_2^R + p_{12}^{(13,23)} - p_{22}^{(11,13)} \le 1$$
$$0 \le -p_{12} + p_{22} + p_2^R + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} + p_{22} + p_2^R + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{12} + p_{22} + p_2^L - p_2^R + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{12} + p_{22} + p_2^L - p_2^R + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + 2p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} + p_{22} + p_2^L - p_2^R + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \le 1$$
$$0 \le -p_{12} + p_{22} + p_2^L - p_2^R + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} + p_{22} + p_2^L + p_{21}^{(22,21)} - p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} + p_{22} + p_2^L + p_{21}^{(22,21)} - p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{12} + p_{22} + p_2^L + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} + p_{22} + p_2^L + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} \le 1$$
$$0 \le -p_{12} + p_{22} + p_2^L + p_{12}^{(11,13)} - p_{22}^{(11,13)} \le 1$$
$$0 \le -p_{12} + p_{22} + p_2^L + p_{12}^{(11,13)} - p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} + p_{22} + p_2^L + p_{12}^{(11,13)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{12} + p_{22} + p_2^L + p_{12}^{(11,13)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} + p_{22} + p_2^L + p_2^R + p_{12}^{(13,23)} - 2p_{22}^{(11,13)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} + p_{22} + p_2^L + p_2^R + p_{12}^{(13,23)} - 2p_{22}^{(11,13)} \le 1$$

$$0 \le -p_{12} + p_{22} + p_2^L + p_2^R + p_{12}^{(13,23)} + p_{21}^{(31,32)} - 2p_{22}^{(11,13)} - p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} + p_{22} + p_2^L + p_2^R + p_{12}^{(13,23)} + p_{21}^{(31,32)} - 2p_{22}^{(11,13)} - p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{12} + p_{22} + p_1^L \le 1$$
$$0 \le -p_{12} + p_{22} + p_1^L + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} + p_{22} + p_1^L + p_2^R - p_{12}^{(11,13)} + p_{12}^{(13,23)} - p_{22}^{(11,13)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{12} + p_{22} + p_1^L + p_2^R - p_{12}^{(11,13)} + p_{12}^{(13,23)} - p_{22}^{(11,13)} \le 1$$
$$0 \le -2p_{21} - p_{22} + p_2^L - p_{12}^{(13,23)} + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$$
$$0 \le -2p_{21} - p_{22} + p_2^L + p_2^R + p_{21}^{(11,31)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -2p_{21} + p_2^L - p_1^R + 2p_{21}^{(11,31)} + p_{21}^{(22,21)} \le 1$$
$$0 \le -2p_{21} + p_2^L - p_2^R - p_{12}^{(13,23)} + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$$
$$0 \le -2p_{21} + p_2^L + p_{21}^{(11,31)} \le 1$$
$$0 \le -2p_{21} + p_2^L + p_{21}^{(11,31)} + p_{21}^{(22,21)} \le 1$$
$$0 \le -2p_{21} + p_2^L + p_2^R + p_{12}^{(13,23)} + p_{21}^{(11,31)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \le 1$$
$$0 \le -2p_{21} + p_2^L + p_1^R \le 1$$
$$0 \le -2p_{21} + p_{22} + p_2^L - p_2^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le -2p_{21} + p_{22} + p_2^L + p_{12}^{(13,23)} + p_{21}^{(11,31)} - p_{22}^{(11,13)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{21} - 2p_{22} - p_{12}^{(13,23)} + p_{21}^{(11,31)} + p_{21}^{(22,21)} + 2p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{21} - 2p_{22} + p_2^R + p_{21}^{(11,31)} + p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{21} - 2p_{22} + p_2^L - p_{12}^{(13,23)} + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{21} - 2p_{22} + p_2^L + p_2^R + p_{21}^{(11,31)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{21} - p_{22} - p_1^R + 2p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} \le 1$$
$$0 \le -p_{21} - p_{22} - p_2^R - p_{12}^{(13,23)} + p_{21}^{(11,31)} + p_{21}^{(22,21)} + 2p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{21} - p_{22} - p_{12}^{(13,23)} + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{21} - p_{22} - p_{12}^{(13,23)} + p_{21}^{(11,31)} + p_{21}^{(22,21)} + 2p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{21} - p_{22} + p_{21}^{(11,31)} + p_{22}^{(11,13)} \le 1$$
$$0 \le -p_{21} - p_{22} + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} \le 1$$
$$0 \le -p_{21} - p_{22} + p_2^R + p_{21}^{(11,31)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{21} - p_{22} + p_2^R + p_{21}^{(11,31)} + p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{21} - p_{22} + p_2^R + p_{12}^{(13,23)} + p_{21}^{(11,31)} - p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{21} - p_{22} + p_1^R + p_{22}^{(11,13)} \le 1$$

$$0 \le -p_{21} - p_{22} + p_2^L - p_1^R + 2p_{21}^{(11,31)} + p_{21}^{(22,21)} \le 1$$
$$0 \le -p_{21} - p_{22} + p_2^L - p_2^R - p_{12}^{(13,23)} + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{21} - p_{22} + p_2^L - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{21} - p_{22} + p_2^L - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + 2p_{22}^{(33)} \le 1$$
$$0 \le -p_{21} - p_{22} + p_2^L - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{21} - p_{22} + p_2^L - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{21} - p_{22} + p_2^L - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{21} - p_{22} + p_2^L - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} - p_{22}^{(31,33)} + 2p_{22}^{(33)} \le 1$$
$$0 \le -p_{21} - p_{22} + p_2^L - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \le 1$$
$$0 \le -p_{21} - p_{22} + p_2^L - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{21} - p_{22} + p_2^L - p_{12}^{(13,23)} + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{21} - p_{22} + p_2^L - p_{12}^{(13,23)} + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{21} - p_{22} + p_2^L + p_{21}^{(11,31)} \le 1$$
$$0 \le -p_{21} - p_{22} + p_2^L + p_{21}^{(11,31)} + p_{21}^{(22,21)} \le 1$$
$$0 \le -p_{21} - p_{22} + p_2^L + p_2^R - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{21} - p_{22} + p_2^L + p_2^R - p_{22}^{(31,33)} + 2p_{22}^{(33)} \le 1$$
$$0 \le -p_{21} - p_{22} + p_2^L + p_2^R \le 1$$
$$0 \le -p_{21} - p_{22} + p_2^L + p_2^R + p_{22}^{(33)} \le 1$$
$$0 \le -p_{21} - p_{22} + p_2^L + p_2^R + p_{21}^{(31,32)} - 2p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{21} - p_{22} + p_2^L + p_2^R + p_{21}^{(31,32)} - 2p_{22}^{(31,33)} + 2p_{22}^{(33)} \le 1$$
$$0 \le -p_{21} - p_{22} + p_2^L + p_2^R + p_{21}^{(31,32)} - p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{21} - p_{22} + p_2^L + p_2^R + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{21} - p_{22} + p_2^L + p_2^R + p_{21}^{(11,31)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{21} - p_{22} + p_2^L + p_2^R + p_{21}^{(11,31)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{21} - p_{22} + p_2^L + p_2^R + p_{12}^{(13,23)} + p_{21}^{(11,31)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{21} - p_{22} + p_2^L + p_1^R \le 1$$
$$0 \le -p_{21} - p_1^R + 2p_{21}^{(11,31)} + p_{21}^{(22,21)} \le 1$$
$$0 \le -p_{21} - p_1^R + 2p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} \le 1$$
$$0 \le -p_{21} - p_2^R - p_{12}^{(13,23)} + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{21} - p_2^R - p_{12}^{(13,23)} + p_{21}^{(11,31)} + p_{21}^{(22,21)} + 2p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$$

$0 \leq -p_{21} - p_2^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$

$0 \leq -p_{21} - p_{12}^{(13,23)} + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(33)} \leq 1$

$0 \leq -p_{21} + p_{21}^{(11,31)} \leq 1$

$0 \leq -p_{21} + p_{21}^{(11,31)} + p_{22}^{(11,13)} \leq 1$

$0 \leq -p_{21} + p_{21}^{(11,31)} + p_{21}^{(22,21)} \leq 1$

$0 \leq -p_{21} + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} \leq 1$

$0 \leq -p_{21} + p_{12}^{(13,23)} + p_{21}^{(11,31)} - p_{22}^{(33)} \leq 1$

$0 \leq -p_{21} + p_2^R + p_{21}^{(11,31)} - p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$

$0 \leq -p_{21} + p_2^R + p_{12}^{(13,23)} + p_{21}^{(11,31)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \leq 1$

$0 \leq -p_{21} + p_2^R + p_{12}^{(13,23)} + p_{21}^{(11,31)} - p_{22}^{(31,33)} \leq 1$

$0 \leq -p_{21} + p_1^R \leq 1$

$0 \leq -p_{21} + p_1^R + p_{22}^{(11,13)} \leq 1$

$0 \leq -p_{21} + p_2^L - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} \leq 1$

$0 \leq -p_{21} + p_2^L - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(33)} \leq 1$

$0 \leq -p_{21} + p_2^L - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$

$0 \leq -p_{21} + p_2^L - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(31,33)} \leq 1$

$0 \leq -p_{21} + p_2^L - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} \leq 1$

$0 \leq -p_{21} + p_2^L - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$

$0 \leq -p_{21} + p_2^L - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(33)} \leq 1$

$0 \leq -p_{21} + p_2^L - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \leq 1$

$0 \leq -p_{21} + p_2^L - p_1^R + 2p_{21}^{(11,31)} + p_{21}^{(22,21)} - p_{22}^{(11,13)} \leq 1$

$0 \leq -p_{21} + p_2^L - p_1^R + 2p_{21}^{(11,31)} + p_{21}^{(22,21)} \leq 1$

$0 \leq -p_{21} + p_2^L - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \leq 1$

$0 \leq -p_{21} + p_2^L - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$

$0 \leq -p_{21} + p_2^L - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + 2p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$

$0 \leq -p_{21} + p_2^L - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + 2p_{22}^{(31,33)} \leq 1$

$0 \leq -p_{21} + p_2^L - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \leq 1$

$0 \leq -p_{21} + p_2^L - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(33)} \leq 1$

$0 \leq -p_{21} + p_2^L - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$

$0 \leq -p_{21} + p_2^L - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \leq 1$

$$0 \le -p_{21} + p_2^L - p_2^R - p_{12}^{(13,23)} + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{21} + p_2^L - p_2^R - p_{12}^{(13,23)} + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{21} + p_2^L - p_2^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{21} + p_2^L - p_{12}^{(13,23)} + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{21} + p_2^L \le 1$$
$$0 \le -p_{21} + p_2^L + p_{22}^{(33)} \le 1$$
$$0 \le -p_{21} + p_2^L + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{21} + p_2^L + p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{21} + p_2^L + p_{21}^{(31,32)} - p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{21} + p_2^L + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{21} + p_2^L + p_{21}^{(31,32)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{21} + p_2^L + p_{21}^{(31,32)} \le 1$$
$$0 \le -p_{21} + p_2^L + p_{21}^{(22,21)} \le 1$$
$$0 \le -p_{21} + p_2^L + p_{21}^{(22,21)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{21} + p_2^L + p_{21}^{(22,21)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{21} + p_2^L + p_{21}^{(22,21)} + p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{21} + p_2^L + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{21} + p_2^L + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{21} + p_2^L + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{21} + p_2^L + p_{21}^{(22,21)} + p_{21}^{(31,32)} \le 1$$
$$0 \le -p_{21} + p_2^L + p_{21}^{(11,31)} - p_{22}^{(11,13)} \le 1$$
$$0 \le -p_{21} + p_2^L + p_{21}^{(11,31)} \le 1$$
$$0 \le -p_{21} + p_2^L + p_{21}^{(11,31)} + p_{21}^{(22,21)} - p_{22}^{(11,13)} \le 1$$
$$0 \le -p_{21} + p_2^L + p_{21}^{(11,31)} + p_{21}^{(22,21)} \le 1$$
$$0 \le -p_{21} + p_2^L + p_{12}^{(13,23)} + p_{21}^{(11,31)} - p_{22}^{(11,13)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{21} + p_2^L + p_2^R + p_{21}^{(11,31)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{21} + p_2^L + p_2^R + p_{12}^{(13,23)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{21} + p_2^L + p_2^R + p_{12}^{(13,23)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{21} + p_2^L + p_2^R + p_{12}^{(13,23)} - p_{22}^{(11,13)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{21} + p_2^L + p_2^R + p_{12}^{(13,23)} - p_{22}^{(11,13)} \le 1$$

$$0 \leq -p_{21} + p_2^L + p_2^R + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - 2p_{22}^{(31,33)} \leq 1$$
$$0 \leq -p_{21} + p_2^L + p_2^R + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - 2p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{21} + p_2^L + p_2^R + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{21} + p_2^L + p_2^R + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \leq 1$$
$$0 \leq -p_{21} + p_2^L + p_2^R + p_{12}^{(13,23)} + p_{21}^{(11,31)} - 2p_{22}^{(11,13)} - p_{22}^{(31,33)} \leq 1$$
$$0 \leq -p_{21} + p_2^L + p_2^R + p_{12}^{(13,23)} + p_{21}^{(11,31)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \leq 1$$
$$0 \leq -p_{21} + p_2^L + p_1^R - p_{21}^{(11,31)} \leq 1$$
$$0 \leq -p_{21} + p_2^L + p_1^R - p_{21}^{(11,31)} + p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{21} + p_2^L + p_1^R - p_{21}^{(11,31)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{21} + p_2^L + p_1^R - p_{21}^{(11,31)} + p_{22}^{(31,33)} \leq 1$$
$$0 \leq -p_{21} + p_2^L + p_1^R - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} \leq 1$$
$$0 \leq -p_{21} + p_2^L + p_1^R - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{21} + p_2^L + p_1^R - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{21} + p_2^L + p_1^R - p_{21}^{(11,31)} + p_{21}^{(31,32)} \leq 1$$
$$0 \leq -p_{21} + p_2^L + p_1^R - p_{22}^{(11,13)} \leq 1$$
$$0 \leq -p_{21} + p_2^L + p_1^R \leq 1$$
$$0 \leq -p_{21} + p_{22} - p_1^R + 2p_{21}^{(11,31)} + p_{21}^{(22,21)} \leq 1$$
$$0 \leq -p_{21} + p_{22} - p_2^R - p_{12}^{(13,23)} + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \leq 1$$
$$0 \leq -p_{21} + p_{22} - p_2^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{21} + p_{22} - p_2^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{21} + p_{22} + p_{21}^{(11,31)} \leq 1$$
$$0 \leq -p_{21} + p_{22} + p_{21}^{(11,31)} + p_{21}^{(22,21)} \leq 1$$
$$0 \leq -p_{21} + p_{22} + p_{12}^{(13,23)} + p_{21}^{(11,31)} - p_{22}^{(11,13)} - p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{21} + p_{22} + p_{12}^{(13,23)} + p_{21}^{(11,31)} - p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{21} + p_{22} + p_2^R + p_{12}^{(13,23)} + p_{21}^{(11,31)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \leq 1$$
$$0 \leq -p_{21} + p_{22} + p_1^R \leq 1$$
$$0 \leq -p_{21} + p_{22} + p_2^L - p_1^R + 2p_{21}^{(11,31)} + p_{21}^{(22,21)} - p_{22}^{(11,13)} \leq 1$$
$$0 \leq -p_{21} + p_{22} + p_2^L - p_2^R - p_{12}^{(13,23)} + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(31,33)} \leq 1$$
$$0 \leq -p_{21} + p_{22} + p_2^L - p_2^R + p_{21}^{(22,21)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{21} + p_{22} + p_2^L - p_2^R + p_{21}^{(22,21)} + p_{22}^{(31,33)} \leq 1$$

$$0 \le -p_{21} + p_{22} + p_2^L - p_2^R + p_{21}^{(22,21)} + 2p_{22}^{(31,33)} - 2p_{22}^{(33)} \le 1$$
$$0 \le -p_{21} + p_{22} + p_2^L - p_2^R + p_{21}^{(22,21)} + 2p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{21} + p_{22} + p_2^L - p_2^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{21} + p_{22} + p_2^L - p_2^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} \le 1$$
$$0 \le -p_{21} + p_{22} + p_2^L - p_2^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(31,33)} - 2p_{22}^{(33)} \le 1$$
$$0 \le -p_{21} + p_{22} + p_2^L - p_2^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{21} + p_{22} + p_2^L - p_2^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} - p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{21} + p_{22} + p_2^L - p_2^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{21} + p_{22} + p_2^L + p_{21}^{(11,31)} - p_{22}^{(11,13)} \le 1$$
$$0 \le -p_{21} + p_{22} + p_2^L + p_{21}^{(11,31)} + p_{21}^{(22,21)} - p_{22}^{(11,13)} \le 1$$
$$0 \le -p_{21} + p_{22} + p_2^L + p_{12}^{(13,23)} - p_{22}^{(11,13)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{21} + p_{22} + p_2^L + p_{12}^{(13,23)} - p_{22}^{(11,13)} \le 1$$
$$0 \le -p_{21} + p_{22} + p_2^L + p_{12}^{(13,23)} - p_{22}^{(11,13)} + p_{22}^{(31,33)} - 2p_{22}^{(33)} \le 1$$
$$0 \le -p_{21} + p_{22} + p_2^L + p_{12}^{(13,23)} - p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{21} + p_{22} + p_2^L + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{21} + p_{22} + p_2^L + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{21} + p_{22} + p_2^L + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - 2p_{22}^{(33)} \le 1$$
$$0 \le -p_{21} + p_{22} + p_2^L + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{21} + p_{22} + p_2^L + p_{12}^{(13,23)} + p_{21}^{(11,31)} - 2p_{22}^{(11,13)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{21} + p_{22} + p_2^L + p_{12}^{(13,23)} + p_{21}^{(11,31)} - p_{22}^{(11,13)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{21} + p_{22} + p_2^L + p_2^R + p_{12}^{(13,23)} + p_{21}^{(11,31)} - 2p_{22}^{(11,13)} - p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{21} + p_{22} + p_2^L + p_1^R - p_{22}^{(11,13)} \le 1$$
$$0 \le -p_{21} + 2p_{22} - p_2^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{21} + 2p_{22} + p_{12}^{(13,23)} + p_{21}^{(11,31)} - p_{22}^{(11,13)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{21} + 2p_{22} + p_2^L - p_2^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} - p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{21} + 2p_{22} + p_2^L + p_{12}^{(13,23)} + p_{21}^{(11,31)} - 2p_{22}^{(11,13)} - p_{22}^{(33)} \le 1$$
$$0 \le -2p_{22} - p_1^L + p_2^L + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -2p_{22} - p_1^L + p_2^L + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} - p_{22}^{(31,33)} + 2p_{22}^{(33)} \le 1$$
$$0 \le -2p_{22} - p_1^L + p_2^L + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \le 1$$
$$0 \le -2p_{22} - p_1^L + p_2^L + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$$

$$0 \le -2p_{22} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + 2p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$$
$$0 \le -2p_{22} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + 2p_{22}^{(11,13)} + 2p_{22}^{(33)} \le 1$$
$$0 \le -2p_{22} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + 2p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$$
$$0 \le -2p_{22} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + 2p_{22}^{(11,13)} + p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -2p_{22} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + 2p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -2p_{22} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + 2p_{22}^{(11,13)} - p_{22}^{(31,33)} + 2p_{22}^{(33)} \le 1$$
$$0 \le -2p_{22} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + 2p_{22}^{(11,13)} \le 1$$
$$0 \le -2p_{22} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + 2p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$$
$$0 \le -2p_{22} + p_2^R + p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -2p_{22} + p_2^R + p_{22}^{(11,13)} - p_{22}^{(31,33)} + 2p_{22}^{(33)} \le 1$$
$$0 \le -2p_{22} + p_2^R + p_{22}^{(11,13)} \le 1$$
$$0 \le -2p_{22} + p_2^R + p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$$
$$0 \le -2p_{22} + p_2^R + p_{21}^{(31,32)} + p_{22}^{(11,13)} - 2p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -2p_{22} + p_2^R + p_{21}^{(31,32)} + p_{22}^{(11,13)} - 2p_{22}^{(31,33)} + 2p_{22}^{(33)} \le 1$$
$$0 \le -2p_{22} + p_2^R + p_{21}^{(31,32)} + p_{22}^{(11,13)} - p_{22}^{(31,33)} \le 1$$
$$0 \le -2p_{22} + p_2^R + p_{21}^{(31,32)} + p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -2p_{22} + p_2^L - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$$
$$0 \le -2p_{22} + p_2^L - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + 2p_{22}^{(33)} \le 1$$
$$0 \le -2p_{22} + p_2^L - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$$
$$0 \le -2p_{22} + p_2^L - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -2p_{22} + p_2^L - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -2p_{22} + p_2^L - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} - p_{22}^{(31,33)} + 2p_{22}^{(33)} \le 1$$
$$0 \le -2p_{22} + p_2^L - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \le 1$$
$$0 \le -2p_{22} + p_2^L - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$$
$$0 \le -2p_{22} + p_2^L + p_2^R - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -2p_{22} + p_2^L + p_2^R - p_{22}^{(31,33)} + 2p_{22}^{(33)} \le 1$$
$$0 \le -2p_{22} + p_2^L + p_2^R \le 1$$
$$0 \le -2p_{22} + p_2^L + p_2^R + p_{22}^{(33)} \le 1$$
$$0 \le -2p_{22} + p_2^L + p_2^R + p_{21}^{(31,32)} - 2p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -2p_{22} + p_2^L + p_2^R + p_{21}^{(31,32)} - 2p_{22}^{(31,33)} + 2p_{22}^{(33)} \le 1$$

$0 \le -2p_{22} + p_2^L + p_2^R + p_{21}^{(31,32)} - p_{22}^{(31,33)} \le 1$

$0 \le -2p_{22} + p_2^L + p_2^R + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$

$0 \le -2p_{22} + p_1^L + p_2^R - p_{12}^{(11,13)} + p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$

$0 \le -2p_{22} + p_1^L + p_2^R - p_{12}^{(11,13)} + p_{22}^{(11,13)} - p_{22}^{(31,33)} + 2p_{22}^{(33)} \le 1$

$0 \le -2p_{22} + p_1^L + p_2^R - p_{12}^{(11,13)} + p_{22}^{(11,13)} \le 1$

$0 \le -2p_{22} + p_1^L + p_2^R - p_{12}^{(11,13)} + p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$

$0 \le -p_{22} - p_1^L + p_2^L - p_2^R + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \le 1$

$0 \le -p_{22} - p_1^L + p_2^L - p_2^R + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$

$0 \le -p_{22} - p_1^L + p_2^L - p_2^R + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$

$0 \le -p_{22} - p_1^L + p_2^L - p_2^R + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$

$0 \le -p_{22} - p_1^L + p_2^L + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$

$0 \le -p_{22} - p_1^L + p_2^L + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} + 2p_{22}^{(33)} \le 1$

$0 \le -p_{22} - p_1^L + p_2^L + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \le 1$

$0 \le -p_{22} - p_1^L + p_2^L + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(33)} \le 1$

$0 \le -p_{22} - p_1^L + p_2^L + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$

$0 \le -p_{22} - p_1^L + p_2^L + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} - p_{22}^{(31,33)} + 2p_{22}^{(33)} \le 1$

$0 \le -p_{22} - p_1^L + p_2^L + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \le 1$

$0 \le -p_{22} - p_1^L + p_2^L + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$

$0 \le -p_{22} - p_1^L + p_2^L + p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} \le 1$

$0 \le -p_{22} - p_1^L + p_2^L + p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$

$0 \le -p_{22} - p_1^L + p_2^L + p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(33)} \le 1$

$0 \le -p_{22} - p_1^L + p_2^L + p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \le 1$

$0 \le -p_{22} - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} \le 1$

$0 \le -p_{22} - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$

$0 \le -p_{22} - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$

$0 \le -p_{22} - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$

$0 \le -p_{22} - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} - p_{22}^{(31,33)} \le 1$

$0 \le -p_{22} - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$

$0 \le -p_{22} - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} - p_{22}^{(33)} \le 1$

$0 \le -p_{22} - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \le 1$

$$0 \le -p_{22} - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + 2p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{22} - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + 2p_{22}^{(11,13)} + p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + 2p_{22}^{(11,13)} + 2p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + 2p_{22}^{(11,13)} + 2p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{22} - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + 2p_{22}^{(11,13)} \le 1$$
$$0 \le -p_{22} - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + 2p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + 2p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + 2p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{22} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + 2p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{22} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + 2p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + 2p_{22}^{(11,13)} + 2p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + 2p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{22} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + 2p_{22}^{(11,13)} + p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} - p_{22}^{(31,33)} + 2p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \le 1$$
$$0 \le -p_{22} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + 2p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + 2p_{22}^{(11,13)} - p_{22}^{(31,33)} + 2p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + 2p_{22}^{(11,13)} \le 1$$
$$0 \le -p_{22} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + 2p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} - p_{12}^{(13,23)} + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} + p_{22}^{(11,13)} \le 1$$
$$0 \le -p_{22} + p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} + p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{22} + p_{21}^{(31,32)} + p_{22}^{(11,13)} - p_{22}^{(31,33)} \le 1$$

$$0 \leq -p_{22} + p_{21}^{(31,32)} + p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{22} + p_{21}^{(31,32)} + p_{22}^{(11,13)} - p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{22} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \leq 1$$
$$0 \leq -p_{22} + p_{21}^{(22,21)} + p_{22}^{(11,13)} \leq 1$$
$$0 \leq -p_{22} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{22} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{22} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \leq 1$$
$$0 \leq -p_{22} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} - p_{22}^{(31,33)} \leq 1$$
$$0 \leq -p_{22} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{22} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} - p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{22} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \leq 1$$
$$0 \leq -p_{22} + p_2^R - p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{22} + p_2^R - p_{22}^{(31,33)} + 2p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{22} + p_2^R \leq 1$$
$$0 \leq -p_{22} + p_2^R + p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{22} + p_2^R + p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{22} + p_2^R + p_{22}^{(11,13)} - p_{22}^{(31,33)} + 2p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{22} + p_2^R + p_{22}^{(11,13)} \leq 1$$
$$0 \leq -p_{22} + p_2^R + p_{22}^{(11,13)} + p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{22} + p_2^R + p_{21}^{(31,32)} - 2p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{22} + p_2^R + p_{21}^{(31,32)} - 2p_{22}^{(31,33)} + 2p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{22} + p_2^R + p_{21}^{(31,32)} - p_{22}^{(31,33)} \leq 1$$
$$0 \leq -p_{22} + p_2^R + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{22} + p_2^R + p_{21}^{(31,32)} + p_{22}^{(11,13)} - 2p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{22} + p_2^R + p_{21}^{(31,32)} + p_{22}^{(11,13)} - 2p_{22}^{(31,33)} + 2p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{22} + p_2^R + p_{21}^{(31,32)} + p_{22}^{(11,13)} - p_{22}^{(31,33)} \leq 1$$
$$0 \leq -p_{22} + p_2^R + p_{21}^{(31,32)} + p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{22} + p_2^R + p_{21}^{(11,31)} - p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{22} + p_2^R + p_{12}^{(13,23)} - p_{22}^{(31,33)} \leq 1$$
$$0 \leq -p_{22} + p_2^R + p_{12}^{(13,23)} - p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$$

$$0 \le -p_{22} + p_2^R + p_{12}^{(13,23)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} + p_2^R + p_{12}^{(13,23)} \le 1$$
$$0 \le -p_{22} + p_2^R + p_{12}^{(13,23)} + p_{21}^{(31,32)} - 2p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{22} + p_2^R + p_{12}^{(13,23)} + p_{21}^{(31,32)} - 2p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} + p_2^R + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} + p_2^R + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{22} + p_1^R - p_{21}^{(11,31)} + p_{22}^{(11,13)} \le 1$$
$$0 \le -p_{22} + p_1^R - p_{21}^{(11,31)} + p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} + p_1^R - p_{21}^{(11,31)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} + p_1^R - p_{21}^{(11,31)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{22} + p_1^R - p_{21}^{(11,31)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} - p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{22} + p_1^R - p_{21}^{(11,31)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} + p_1^R - p_{21}^{(11,31)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} + p_1^R - p_{21}^{(11,31)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \le 1$$
$$0 \le -p_{22} + p_2^L - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} \le 1$$
$$0 \le -p_{22} + p_2^L - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} + p_2^L - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} + p_2^L - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{22} + p_2^L - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{22} + p_2^L - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} + p_2^L - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} + p_2^L - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \le 1$$
$$0 \le -p_{22} + p_2^L - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{22} + p_2^L - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} + p_2^L - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + 2p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} + p_2^L - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + 2p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{22} + p_2^L - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \le 1$$
$$0 \le -p_{22} + p_2^L - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} + p_2^L - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} + p_2^L - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$$

$$0 \le -p_{22} + p_2^L - p_{12}^{(13,23)} - p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} + p_2^L - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} + p_2^L - p_{12}^{(13,23)} + p_{21}^{(22,21)} + 2p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} + p_2^L - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{22} + p_2^L - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} + p_2^L - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} + p_2^L - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + 2p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} + p_2^L - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{22} + p_2^L - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} + p_2^L - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} + p_2^L - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} + 2p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} + p_2^L - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \le 1$$
$$0 \le -p_{22} + p_2^L - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} + p_2^L - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} + p_2^L - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} - p_{22}^{(31,33)} + 2p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} + p_2^L - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \le 1$$
$$0 \le -p_{22} + p_2^L - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} + p_2^L \le 1$$
$$0 \le -p_{22} + p_2^L + p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} + p_2^L + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} + p_2^L + p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{22} + p_2^L + p_{21}^{(31,32)} - p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{22} + p_2^L + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} + p_2^L + p_{21}^{(31,32)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} + p_2^L + p_{21}^{(31,32)} \le 1$$
$$0 \le -p_{22} + p_2^L + p_{21}^{(22,21)} \le 1$$
$$0 \le -p_{22} + p_2^L + p_{21}^{(22,21)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} + p_2^L + p_{21}^{(22,21)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} + p_2^L + p_{21}^{(22,21)} + p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{22} + p_2^L + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} \le 1$$

$$0 \leq -p_{22} + p_2^L + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{22} + p_2^L + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{22} + p_2^L + p_{21}^{(22,21)} + p_{21}^{(31,32)} \leq 1$$
$$0 \leq -p_{22} + p_2^L + p_2^R - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{22} + p_2^L + p_2^R - p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{22} + p_2^L + p_2^R - p_{22}^{(11,13)} - p_{22}^{(31,33)} + 2p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{22} + p_2^L + p_2^R - p_{22}^{(11,13)} \leq 1$$
$$0 \leq -p_{22} + p_2^L + p_2^R - p_{22}^{(11,13)} + p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{22} + p_2^L + p_2^R - p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{22} + p_2^L + p_2^R - p_{22}^{(31,33)} + 2p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{22} + p_2^L + p_2^R \leq 1$$
$$0 \leq -p_{22} + p_2^L + p_2^R + p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{22} + p_2^L + p_2^R + p_{21}^{(31,32)} - p_{22}^{(11,13)} - 2p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{22} + p_2^L + p_2^R + p_{21}^{(31,32)} - p_{22}^{(11,13)} - 2p_{22}^{(31,33)} + 2p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{22} + p_2^L + p_2^R + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \leq 1$$
$$0 \leq -p_{22} + p_2^L + p_2^R + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{22} + p_2^L + p_2^R + p_{21}^{(31,32)} - 2p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{22} + p_2^L + p_2^R + p_{21}^{(31,32)} - 2p_{22}^{(31,33)} + 2p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{22} + p_2^L + p_2^R + p_{21}^{(31,32)} - p_{22}^{(31,33)} \leq 1$$
$$0 \leq -p_{22} + p_2^L + p_2^R + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{22} + p_2^L + p_2^R + p_{12}^{(13,23)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \leq 1$$
$$0 \leq -p_{22} + p_2^L + p_2^R + p_{12}^{(13,23)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{22} + p_2^L + p_2^R + p_{12}^{(13,23)} - p_{22}^{(11,13)} - p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{22} + p_2^L + p_2^R + p_{12}^{(13,23)} - p_{22}^{(11,13)} \leq 1$$
$$0 \leq -p_{22} + p_2^L + p_2^R + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - 2p_{22}^{(31,33)} \leq 1$$
$$0 \leq -p_{22} + p_2^L + p_2^R + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - 2p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{22} + p_2^L + p_2^R + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$$
$$0 \leq -p_{22} + p_2^L + p_2^R + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \leq 1$$
$$0 \leq -p_{22} + p_2^L + p_1^R - p_{21}^{(11,31)} \leq 1$$
$$0 \leq -p_{22} + p_2^L + p_1^R - p_{21}^{(11,31)} + p_{22}^{(33)} \leq 1$$

$$0 \le -p_{22} + p_2^L + p_1^R - p_{21}^{(11,31)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} + p_2^L + p_1^R - p_{21}^{(11,31)} + p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{22} + p_2^L + p_1^R - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{22} + p_2^L + p_1^R - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} + p_2^L + p_1^R - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} + p_2^L + p_1^R - p_{21}^{(11,31)} + p_{21}^{(31,32)} \le 1$$
$$0 \le -p_{22} + p_1^L - p_{12}^{(11,13)} + p_{22}^{(11,13)} \le 1$$
$$0 \le -p_{22} + p_1^L - p_{12}^{(11,13)} + p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} + p_1^L - p_{12}^{(11,13)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} + p_1^L - p_{12}^{(11,13)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{22} + p_1^L + p_2^R - p_{12}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} + p_1^L + p_2^R - p_{12}^{(11,13)} - p_{22}^{(31,33)} + 2p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} + p_1^L + p_2^R - p_{12}^{(11,13)} \le 1$$
$$0 \le -p_{22} + p_1^L + p_2^R - p_{12}^{(11,13)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} + p_1^L + p_2^R - p_{12}^{(11,13)} + p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} + p_1^L + p_2^R - p_{12}^{(11,13)} + p_{22}^{(11,13)} - p_{22}^{(31,33)} + 2p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} + p_1^L + p_2^R - p_{12}^{(11,13)} + p_{22}^{(11,13)} \le 1$$
$$0 \le -p_{22} + p_1^L + p_2^R - p_{12}^{(11,13)} + p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} + p_1^L + p_2^R - p_{12}^{(11,13)} + p_{12}^{(13,23)} - p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{22} + p_1^L + p_2^R - p_{12}^{(11,13)} + p_{12}^{(13,23)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} + p_1^L + p_2^R - p_{12}^{(11,13)} + p_{12}^{(13,23)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_{22} + p_1^L + p_2^R - p_{12}^{(11,13)} + p_{12}^{(13,23)} \le 1$$
$$0 \le -p_1^L + p_2^L - p_2^R + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \le 1$$
$$0 \le -p_1^L + p_2^L - p_2^R + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_1^L + p_2^L - p_2^R + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_1^L + p_2^L - p_2^R + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(31,33)} \le 1$$
$$0 \le -p_1^L + p_2^L - p_2^R + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \le 1$$
$$0 \le -p_1^L + p_2^L - p_2^R + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_1^L + p_2^L - p_2^R + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_1^L + p_2^L - p_2^R + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$$

$$0 \le -p_1^L + p_2^L - p_2^R + p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_1^L + p_2^L - p_2^R + p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \le 1$$
$$0 \le -p_1^L + p_2^L - p_2^R + p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(31,33)} - 2p_{22}^{(33)} \le 1$$
$$0 \le -p_1^L + p_2^L - p_2^R + p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_1^L + p_2^L - p_2^R + 2p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \le 1$$
$$0 \le -p_1^L + p_2^L + p_{21}^{(22,21)} + p_{21}^{(31,32)} \le 1$$
$$0 \le -p_1^L + p_2^L + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_1^L + p_2^L + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} + 2p_{22}^{(33)} \le 1$$
$$0 \le -p_1^L + p_2^L + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \le 1$$
$$0 \le -p_1^L + p_2^L + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_1^L + p_2^L + p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \le 1$$
$$0 \le -p_1^L + p_2^L + p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_1^L + p_2^L + p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_1^L + p_2^L + p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} \le 1$$
$$0 \le -p_1^L + p_2^L + p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} \le 1$$
$$0 \le -p_1^L + p_2^L + p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_1^L + p_2^L + p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_1^L + p_2^L + p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \le 1$$
$$0 \le -p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} \le 1$$
$$0 \le -p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(31,33)} \le 1$$
$$0 \le -p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} \le 1$$
$$0 \le -p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$$
$$0 \le -p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} \le 1$$
$$0 \le -p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \le 1$$

$$0 \le -p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} - p_{22}^{(31,33)} \le 1$$
$$0 \le -p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \le 1$$
$$0 \le -p_1^R + 2p_{21}^{(11,31)} + p_{21}^{(22,21)} \le 1$$
$$0 \le -p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$$
$$0 \le -p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + 2p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + 2p_{22}^{(31,33)} \le 1$$
$$0 \le -p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + 2p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$$
$$0 \le -p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + 2p_{22}^{(11,13)} + p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + 2p_{22}^{(11,13)} + 2p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + 2p_{22}^{(11,13)} + 2p_{22}^{(31,33)} \le 1$$
$$0 \le -p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \le 1$$
$$0 \le -p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$$
$$0 \le -p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + 2p_{22}^{(11,13)} \le 1$$
$$0 \le -p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + 2p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + 2p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + 2p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$$
$$0 \le -p_2^R - p_{12}^{(13,23)} + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$$
$$0 \le -p_2^R + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_2^R + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$$
$$0 \le -p_2^R + p_{21}^{(22,21)} + p_{22}^{(11,13)} + 2p_{22}^{(31,33)} - 2p_{22}^{(33)} \le 1$$
$$0 \le -p_2^R + p_{21}^{(22,21)} + p_{22}^{(11,13)} + 2p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_2^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} - p_{22}^{(33)} \le 1$$
$$0 \le -p_2^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \le 1$$
$$0 \le -p_2^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} - 2p_{22}^{(33)} \le 1$$
$$0 \le -p_2^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$

$$0 \le -p_2^R + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$$
$$0 \le -p_2^R + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \le 1$$
$$0 \le -p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \le 1$$
$$0 \le -p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + 2p_{22}^{(33)} \le 1$$
$$0 \le -p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$$
$$0 \le -p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le -p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} - p_{22}^{(31,33)} + 2p_{22}^{(33)} \le 1$$
$$0 \le -p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \le 1$$
$$0 \le -p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$$
$$0 \le +p_{22}^{(33)} \le 1$$
$$0 \le +p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_{22}^{(31,33)} \le 1$$
$$0 \le +p_{22}^{(11,13)} \le 1$$
$$0 \le +p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$$
$$0 \le +p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$$
$$0 \le +p_{21}^{(31,32)} - p_{22}^{(31,33)} \le 1$$
$$0 \le +p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le +p_{21}^{(31,32)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_{21}^{(31,32)} \le 1$$
$$0 \le +p_{21}^{(31,32)} + p_{22}^{(11,13)} - p_{22}^{(31,33)} \le 1$$
$$0 \le +p_{21}^{(31,32)} + p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le +p_{21}^{(31,32)} + p_{22}^{(11,13)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_{21}^{(31,32)} + p_{22}^{(11,13)} \le 1$$
$$0 \le +p_{21}^{(22,21)} \le 1$$
$$0 \le +p_{21}^{(22,21)} + p_{22}^{(33)} \le 1$$
$$0 \le +p_{21}^{(22,21)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$

$0 \le +p_{21}^{(22,21)} + p_{22}^{(31,33)} \le 1$

$0 \le +p_{21}^{(22,21)} + p_{22}^{(11,13)} \le 1$

$0 \le +p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$

$0 \le +p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$

$0 \le +p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$

$0 \le +p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} \le 1$

$0 \le +p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$

$0 \le +p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(33)} \le 1$

$0 \le +p_{21}^{(22,21)} + p_{21}^{(31,32)} \le 1$

$0 \le +p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} - p_{22}^{(31,33)} \le 1$

$0 \le +p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$

$0 \le +p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} - p_{22}^{(33)} \le 1$

$0 \le +p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \le 1$

$0 \le +p_{21}^{(11,31)} \le 1$

$0 \le +p_{21}^{(11,31)} + p_{21}^{(22,21)} \le 1$

$0 \le +p_{12}^{(13,23)} - p_{22}^{(33)} \le 1$

$0 \le +p_{12}^{(13,23)} \le 1$

$0 \le +p_{12}^{(13,23)} + p_{22}^{(31,33)} - 2p_{22}^{(33)} \le 1$

$0 \le +p_{12}^{(13,23)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$

$0 \le +p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$

$0 \le +p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} \le 1$

$0 \le +p_{12}^{(13,23)} + p_{21}^{(31,32)} - 2p_{22}^{(33)} \le 1$

$0 \le +p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(33)} \le 1$

$0 \le +p_{12}^{(11,13)} \le 1$

$0 \le +p_{12}^{(11,13)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} \le 1$

$0 \le +p_2^R - p_{12}^{(11,13)} + p_{12}^{(13,23)} \le 1$

$0 \le +p_2^R - p_{12}^{(11,13)} + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} \le 1$

$0 \le +p_2^R - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$

$0 \le +p_2^R - p_{22}^{(31,33)} + 2p_{22}^{(33)} \le 1$

$0 \le +p_2^R \le 1$

$0 \le +p_2^R + p_{22}^{(33)} \le 1$

$0 \le +p_2^R + p_{21}^{(31,32)} - 2p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$

$0 \le +p_2^R + p_{21}^{(31,32)} - 2p_{22}^{(31,33)} + 2p_{22}^{(33)} \le 1$

$0 \le +p_2^R + p_{21}^{(31,32)} - p_{22}^{(31,33)} \le 1$

$0 \le +p_2^R + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$

$0 \le +p_2^R + p_{12}^{(13,23)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \le 1$

$0 \le +p_2^R + p_{12}^{(13,23)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$

$0 \le +p_2^R + p_{12}^{(13,23)} - p_{22}^{(11,13)} - p_{22}^{(33)} \le 1$

$0 \le +p_2^R + p_{12}^{(13,23)} - p_{22}^{(11,13)} \le 1$

$0 \le +p_2^R + p_{12}^{(13,23)} - p_{22}^{(31,33)} \le 1$

$0 \le +p_2^R + p_{12}^{(13,23)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$

$0 \le +p_2^R + p_{12}^{(13,23)} - p_{22}^{(33)} \le 1$

$0 \le +p_2^R + p_{12}^{(13,23)} \le 1$

$0 \le +p_2^R + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - 2p_{22}^{(31,33)} \le 1$

$0 \le +p_2^R + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - 2p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$

$0 \le +p_2^R + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$

$0 \le +p_2^R + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \le 1$

$0 \le +p_2^R + p_{12}^{(13,23)} + p_{21}^{(31,32)} - 2p_{22}^{(31,33)} \le 1$

$0 \le +p_2^R + p_{12}^{(13,23)} + p_{21}^{(31,32)} - 2p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$

$0 \le +p_2^R + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$

$0 \le +p_2^R + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} \le 1$

$0 \le +p_2^R + p_{12}^{(13,23)} + p_{21}^{(11,31)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \le 1$

$0 \le +p_1^R - p_{21}^{(11,31)} \le 1$

$0 \le +p_1^R - p_{21}^{(11,31)} + p_{22}^{(33)} \le 1$

$0 \le +p_1^R - p_{21}^{(11,31)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$

$0 \le +p_1^R - p_{21}^{(11,31)} + p_{22}^{(31,33)} \le 1$

$0 \le +p_1^R - p_{21}^{(11,31)} + p_{22}^{(11,13)} \le 1$

$0 \le +p_1^R - p_{21}^{(11,31)} + p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$

$0 \le +p_1^R - p_{21}^{(11,31)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$

$0 \le +p_1^R - p_{21}^{(11,31)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$

$$0 \le +p_1^R - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} \le 1$$
$$0 \le +p_1^R - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le +p_1^R - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_1^R - p_{21}^{(11,31)} + p_{21}^{(31,32)} \le 1$$
$$0 \le +p_1^R - p_{21}^{(11,31)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} - p_{22}^{(31,33)} \le 1$$
$$0 \le +p_1^R - p_{21}^{(11,31)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le +p_1^R - p_{21}^{(11,31)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_1^R - p_{21}^{(11,31)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \le 1$$
$$0 \le +p_1^R \le 1$$
$$0 \le +p_2^L - p_1^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} \le 1$$
$$0 \le +p_2^L - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} - p_{22}^{(11,13)} \le 1$$
$$0 \le +p_2^L - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} - p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$$
$$0 \le +p_2^L - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} - p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_2^L - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} - p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$$
$$0 \le +p_2^L - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} \le 1$$
$$0 \le +p_2^L - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(33)} \le 1$$
$$0 \le +p_2^L - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_2^L - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(31,33)} \le 1$$
$$0 \le +p_2^L - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \le 1$$
$$0 \le +p_2^L - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le +p_2^L - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_2^L - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} \le 1$$
$$0 \le +p_2^L - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} \le 1$$
$$0 \le +p_2^L - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le +p_2^L - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_2^L - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \le 1$$
$$0 \le +p_2^L - p_2^R - p_{12}^{(13,23)} - p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$$
$$0 \le +p_2^L - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(31,33)} \le 1$$
$$0 \le +p_2^L - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le +p_2^L - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + 2p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$

$$0 \le +p_2^L - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + 2p_{22}^{(31,33)} \le 1$$
$$0 \le +p_2^L - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$$
$$0 \le +p_2^L - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le +p_2^L - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + 2p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_2^L - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + 2p_{22}^{(31,33)} \le 1$$
$$0 \le +p_2^L - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \le 1$$
$$0 \le +p_2^L - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(33)} \le 1$$
$$0 \le +p_2^L - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_2^L - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(31,33)} \le 1$$
$$0 \le +p_2^L - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \le 1$$
$$0 \le +p_2^L - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$$
$$0 \le +p_2^L - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_2^L - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$$
$$0 \le +p_2^L - p_2^R + p_{21}^{(22,21)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_2^L - p_2^R + p_{21}^{(22,21)} + p_{22}^{(31,33)} \le 1$$
$$0 \le +p_2^L - p_2^R + p_{21}^{(22,21)} + 2p_{22}^{(31,33)} - 2p_{22}^{(33)} \le 1$$
$$0 \le +p_2^L - p_2^R + p_{21}^{(22,21)} + 2p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_2^L - p_2^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_2^L - p_2^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} \le 1$$
$$0 \le +p_2^L - p_2^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(31,33)} - 2p_{22}^{(33)} \le 1$$
$$0 \le +p_2^L - p_2^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_2^L - p_2^R + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(31,33)} \le 1$$
$$0 \le +p_2^L - p_2^R + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \le 1$$
$$0 \le +p_2^L - p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{22}^{(31,33)} \le 1$$
$$0 \le +p_2^L - p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \le 1$$
$$0 \le +p_2^L - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(33)} \le 1$$
$$0 \le +p_2^L - p_{12}^{(13,23)} + p_{21}^{(22,21)} + 2p_{22}^{(33)} \le 1$$
$$0 \le +p_2^L - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(31,33)} \le 1$$
$$0 \le +p_2^L - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le +p_2^L - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$

$$0 \leq +p_2^L - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} + 2p_{22}^{(33)} \leq 1$$
$$0 \leq +p_2^L - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \leq 1$$
$$0 \leq +p_2^L - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(33)} \leq 1$$
$$0 \leq +p_2^L - p_{21}^{(11,31)} + p_{21}^{(31,32)} \leq 1$$
$$0 \leq +p_2^L - p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \leq 1$$
$$0 \leq +p_2^L - p_{22}^{(11,13)} \leq 1$$
$$0 \leq +p_2^L - p_{22}^{(11,13)} + p_{22}^{(33)} \leq 1$$
$$0 \leq +p_2^L - p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$$
$$0 \leq +p_2^L - p_{22}^{(11,13)} + p_{22}^{(31,33)} \leq 1$$
$$0 \leq +p_2^L \leq 1$$
$$0 \leq +p_2^L + p_{22}^{(33)} \leq 1$$
$$0 \leq +p_2^L + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$$
$$0 \leq +p_2^L + p_{22}^{(31,33)} \leq 1$$
$$0 \leq +p_2^L + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \leq 1$$
$$0 \leq +p_2^L + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$$
$$0 \leq +p_2^L + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(33)} \leq 1$$
$$0 \leq +p_2^L + p_{21}^{(31,32)} - p_{22}^{(11,13)} \leq 1$$
$$0 \leq +p_2^L + p_{21}^{(31,32)} - p_{22}^{(31,33)} \leq 1$$
$$0 \leq +p_2^L + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$$
$$0 \leq +p_2^L + p_{21}^{(31,32)} - p_{22}^{(33)} \leq 1$$
$$0 \leq +p_2^L + p_{21}^{(31,32)} \leq 1$$
$$0 \leq +p_2^L + p_{21}^{(22,21)} - p_{22}^{(11,13)} \leq 1$$
$$0 \leq +p_2^L + p_{21}^{(22,21)} - p_{22}^{(11,13)} + p_{22}^{(33)} \leq 1$$
$$0 \leq +p_2^L + p_{21}^{(22,21)} - p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$$
$$0 \leq +p_2^L + p_{21}^{(22,21)} - p_{22}^{(11,13)} + p_{22}^{(31,33)} \leq 1$$
$$0 \leq +p_2^L + p_{21}^{(22,21)} \leq 1$$
$$0 \leq +p_2^L + p_{21}^{(22,21)} + p_{22}^{(33)} \leq 1$$
$$0 \leq +p_2^L + p_{21}^{(22,21)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$$
$$0 \leq +p_2^L + p_{21}^{(22,21)} + p_{22}^{(31,33)} \leq 1$$
$$0 \leq +p_2^L + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \leq 1$$

$$0 \le +p_2^L + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le +p_2^L + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_2^L + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} \le 1$$
$$0 \le +p_2^L + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} \le 1$$
$$0 \le +p_2^L + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le +p_2^L + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_2^L + p_{21}^{(22,21)} + p_{21}^{(31,32)} \le 1$$
$$0 \le +p_2^L + p_{12}^{(13,23)} - p_{22}^{(11,13)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_2^L + p_{12}^{(13,23)} - p_{22}^{(11,13)} \le 1$$
$$0 \le +p_2^L + p_{12}^{(13,23)} - p_{22}^{(11,13)} + p_{22}^{(31,33)} - 2p_{22}^{(33)} \le 1$$
$$0 \le +p_2^L + p_{12}^{(13,23)} - p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_2^L + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_2^L + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \le 1$$
$$0 \le +p_2^L + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - 2p_{22}^{(33)} \le 1$$
$$0 \le +p_2^L + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_2^L + p_{12}^{(11,13)} - p_{22}^{(11,13)} \le 1$$
$$0 \le +p_2^L + p_{12}^{(11,13)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \le 1$$
$$0 \le +p_2^L + p_2^R - p_{12}^{(11,13)} + p_{12}^{(13,23)} - p_{22}^{(11,13)} \le 1$$
$$0 \le +p_2^L + p_2^R - p_{12}^{(11,13)} + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \le 1$$
$$0 \le +p_2^L + p_2^R - p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le +p_2^L + p_2^R - p_{22}^{(11,13)} - p_{22}^{(31,33)} + 2p_{22}^{(33)} \le 1$$
$$0 \le +p_2^L + p_2^R - p_{22}^{(11,13)} \le 1$$
$$0 \le +p_2^L + p_2^R - p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$$
$$0 \le +p_2^L + p_2^R + p_{21}^{(31,32)} - p_{22}^{(11,13)} - 2p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le +p_2^L + p_2^R + p_{21}^{(31,32)} - p_{22}^{(11,13)} - 2p_{22}^{(31,33)} + 2p_{22}^{(33)} \le 1$$
$$0 \le +p_2^L + p_2^R + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \le 1$$
$$0 \le +p_2^L + p_2^R + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le +p_2^L + p_2^R + p_{12}^{(13,23)} - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \le 1$$
$$0 \le +p_2^L + p_2^R + p_{12}^{(13,23)} - 2p_{22}^{(11,13)} - p_{22}^{(31,33)} \le 1$$
$$0 \le +p_2^L + p_2^R + p_{12}^{(13,23)} - 2p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$

$$0 \le +p_2^L + p_2^R + p_{12}^{(13,23)} - 2p_{22}^{(11,13)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_2^L + p_2^R + p_{12}^{(13,23)} - 2p_{22}^{(11,13)} \le 1$$
$$0 \le +p_2^L + p_2^R + p_{12}^{(13,23)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \le 1$$
$$0 \le +p_2^L + p_2^R + p_{12}^{(13,23)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le +p_2^L + p_2^R + p_{12}^{(13,23)} - p_{22}^{(11,13)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_2^L + p_2^R + p_{12}^{(13,23)} - p_{22}^{(11,13)} \le 1$$
$$0 \le +p_2^L + p_2^R + p_{12}^{(13,23)} + p_{21}^{(31,32)} - 2p_{22}^{(11,13)} - 2p_{22}^{(31,33)} \le 1$$
$$0 \le +p_2^L + p_2^R + p_{12}^{(13,23)} + p_{21}^{(31,32)} - 2p_{22}^{(11,13)} - 2p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le +p_2^L + p_2^R + p_{12}^{(13,23)} + p_{21}^{(31,32)} - 2p_{22}^{(11,13)} - p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_2^L + p_2^R + p_{12}^{(13,23)} + p_{21}^{(31,32)} - 2p_{22}^{(11,13)} - p_{22}^{(31,33)} \le 1$$
$$0 \le +p_2^L + p_2^R + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - 2p_{22}^{(31,33)} \le 1$$
$$0 \le +p_2^L + p_2^R + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - 2p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le +p_2^L + p_2^R + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_2^L + p_2^R + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \le 1$$
$$0 \le +p_2^L + p_1^R - 2p_{21}^{(11,31)} + p_{21}^{(31,32)} \le 1$$
$$0 \le +p_2^L + p_1^R - p_{21}^{(11,31)} - p_{22}^{(11,13)} \le 1$$
$$0 \le +p_2^L + p_1^R - p_{21}^{(11,31)} - p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$$
$$0 \le +p_2^L + p_1^R - p_{21}^{(11,31)} - p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_2^L + p_1^R - p_{21}^{(11,31)} - p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$$
$$0 \le +p_2^L + p_1^R - p_{21}^{(11,31)} \le 1$$
$$0 \le +p_2^L + p_1^R - p_{21}^{(11,31)} + p_{22}^{(33)} \le 1$$
$$0 \le +p_2^L + p_1^R - p_{21}^{(11,31)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_2^L + p_1^R - p_{21}^{(11,31)} + p_{22}^{(31,33)} \le 1$$
$$0 \le +p_2^L + p_1^R - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \le 1$$
$$0 \le +p_2^L + p_1^R - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le +p_2^L + p_1^R - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_2^L + p_1^R - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} \le 1$$
$$0 \le +p_2^L + p_1^R - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} \le 1$$
$$0 \le +p_2^L + p_1^R - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le +p_2^L + p_1^R - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(33)} \le 1$$

$$0 \le +p_2^L + p_1^R - p_{21}^{(11,31)} + p_{21}^{(31,32)} \le 1$$
$$0 \le +p_1^L - p_{12}^{(11,13)} \le 1$$
$$0 \le +p_1^L - p_{12}^{(11,13)} + p_{22}^{(33)} \le 1$$
$$0 \le +p_1^L - p_{12}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_1^L - p_{12}^{(11,13)} + p_{22}^{(31,33)} \le 1$$
$$0 \le +p_1^L - p_{12}^{(11,13)} + p_{22}^{(11,13)} \le 1$$
$$0 \le +p_1^L - p_{12}^{(11,13)} + p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$$
$$0 \le +p_1^L - p_{12}^{(11,13)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_1^L - p_{12}^{(11,13)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$$
$$0 \le +p_1^L - p_{12}^{(11,13)} + p_{12}^{(13,23)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_1^L - p_{12}^{(11,13)} + p_{12}^{(13,23)} \le 1$$
$$0 \le +p_1^L - p_{12}^{(11,13)} + p_{12}^{(13,23)} + p_{22}^{(31,33)} - 2p_{22}^{(33)} \le 1$$
$$0 \le +p_1^L - p_{12}^{(11,13)} + p_{12}^{(13,23)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_1^L \le 1$$
$$0 \le +p_1^L + p_2^R - 2p_{12}^{(11,13)} + p_{12}^{(13,23)} \le 1$$
$$0 \le +p_1^L + p_2^R - p_{12}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le +p_1^L + p_2^R - p_{12}^{(11,13)} - p_{22}^{(31,33)} + 2p_{22}^{(33)} \le 1$$
$$0 \le +p_1^L + p_2^R - p_{12}^{(11,13)} \le 1$$
$$0 \le +p_1^L + p_2^R - p_{12}^{(11,13)} + p_{22}^{(33)} \le 1$$
$$0 \le +p_1^L + p_2^R - p_{12}^{(11,13)} + p_{12}^{(13,23)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \le 1$$
$$0 \le +p_1^L + p_2^R - p_{12}^{(11,13)} + p_{12}^{(13,23)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le +p_1^L + p_2^R - p_{12}^{(11,13)} + p_{12}^{(13,23)} - p_{22}^{(11,13)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_1^L + p_2^R - p_{12}^{(11,13)} + p_{12}^{(13,23)} - p_{22}^{(11,13)} \le 1$$
$$0 \le +p_1^L + p_2^R - p_{12}^{(11,13)} + p_{12}^{(13,23)} - p_{22}^{(31,33)} \le 1$$
$$0 \le +p_1^L + p_2^R - p_{12}^{(11,13)} + p_{12}^{(13,23)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le +p_1^L + p_2^R - p_{12}^{(11,13)} + p_{12}^{(13,23)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_1^L + p_2^R - p_{12}^{(11,13)} + p_{12}^{(13,23)} \le 1$$
$$0 \le +p_{22} - p_1^L + p_2^L - p_2^R + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \le 1$$
$$0 \le +p_{22} - p_1^L + p_2^L - p_2^R + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(33)} \le 1$$
$$0 \le +p_{22} - p_1^L + p_2^L - p_2^R + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$

$$0 \le +p_{22} - p_1^L + p_2^L - p_2^R + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(31,33)} \le 1$$
$$0 \le +p_{22} - p_1^L + p_2^L - p_2^R + p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_{22} - p_1^L + p_2^L - p_2^R + p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} \le 1$$
$$0 \le +p_{22} - p_1^L + p_2^L - p_2^R + p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} + p_{22}^{(31,33)} - 2p_{22}^{(33)} \le 1$$
$$0 \le +p_{22} - p_1^L + p_2^L - p_2^R + p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_{22} - p_1^L + p_2^L - p_2^R + p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_{22} - p_1^L + p_2^L - p_2^R + p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \le 1$$
$$0 \le +p_{22} - p_1^L + p_2^L - p_2^R + p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(31,33)} - 2p_{22}^{(33)} \le 1$$
$$0 \le +p_{22} - p_1^L + p_2^L - p_2^R + p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_{22} - p_1^L + p_2^L + p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \le 1$$
$$0 \le +p_{22} - p_1^L + p_2^L + p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le +p_{22} - p_1^L + p_2^L + p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_{22} - p_1^L + p_2^L + p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} \le 1$$
$$0 \le +p_{22} - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} \le 1$$
$$0 \le +p_{22} - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(33)} \le 1$$
$$0 \le +p_{22} - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_{22} - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(31,33)} \le 1$$
$$0 \le +p_{22} - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} \le 1$$
$$0 \le +p_{22} - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le +p_{22} - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_{22} - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \le 1$$
$$0 \le +p_{22} - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$$
$$0 \le +p_{22} - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le +p_{22} - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + 2p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_{22} - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + 2p_{22}^{(31,33)} \le 1$$
$$0 \le +p_{22} - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \le 1$$
$$0 \le +p_{22} - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$$
$$0 \le +p_{22} - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_{22} - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$$
$$0 \le +p_{22} - p_2^R + p_{21}^{(22,21)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$

$$0 \le +p_{22} - p_2^R + p_{21}^{(22,21)} + p_{22}^{(31,33)} \le 1$$
$$0 \le +p_{22} - p_2^R + p_{21}^{(22,21)} + 2p_{22}^{(31,33)} - 2p_{22}^{(33)} \le 1$$
$$0 \le +p_{22} - p_2^R + p_{21}^{(22,21)} + 2p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_{22} - p_2^R + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_{22} - p_2^R + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$$
$$0 \le +p_{22} - p_2^R + p_{21}^{(22,21)} + p_{22}^{(11,13)} + 2p_{22}^{(31,33)} - 2p_{22}^{(33)} \le 1$$
$$0 \le +p_{22} - p_2^R + p_{21}^{(22,21)} + p_{22}^{(11,13)} + 2p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_{22} - p_2^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_{22} - p_2^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} \le 1$$
$$0 \le +p_{22} - p_2^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(31,33)} - 2p_{22}^{(33)} \le 1$$
$$0 \le +p_{22} - p_2^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_{22} - p_2^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_{22} - p_2^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \le 1$$
$$0 \le +p_{22} - p_2^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} - 2p_{22}^{(33)} \le 1$$
$$0 \le +p_{22} - p_2^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_{22} - p_2^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_{22} \le 1$$
$$0 \le +p_{22} + p_{22}^{(33)} \le 1$$
$$0 \le +p_{22} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_{22} + p_{22}^{(31,33)} \le 1$$
$$0 \le +p_{22} + p_{21}^{(31,32)} - p_{22}^{(31,33)} \le 1$$
$$0 \le +p_{22} + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le +p_{22} + p_{21}^{(31,32)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_{22} + p_{21}^{(31,32)} \le 1$$
$$0 \le +p_{22} + p_{21}^{(22,21)} \le 1$$
$$0 \le +p_{22} + p_{21}^{(22,21)} + p_{22}^{(33)} \le 1$$
$$0 \le +p_{22} + p_{21}^{(22,21)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_{22} + p_{21}^{(22,21)} + p_{22}^{(31,33)} \le 1$$
$$0 \le +p_{22} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} \le 1$$
$$0 \le +p_{22} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$

$$0 \le +p_{22} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_{22} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \le 1$$
$$0 \le +p_{22} + p_{12}^{(13,23)} - p_{22}^{(11,13)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_{22} + p_{12}^{(13,23)} - p_{22}^{(11,13)} \le 1$$
$$0 \le +p_{22} + p_{12}^{(13,23)} - p_{22}^{(11,13)} + p_{22}^{(31,33)} - 2p_{22}^{(33)} \le 1$$
$$0 \le +p_{22} + p_{12}^{(13,23)} - p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_{22} + p_{12}^{(13,23)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_{22} + p_{12}^{(13,23)} \le 1$$
$$0 \le +p_{22} + p_{12}^{(13,23)} + p_{22}^{(31,33)} - 2p_{22}^{(33)} \le 1$$
$$0 \le +p_{22} + p_{12}^{(13,23)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_{22} + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_{22} + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \le 1$$
$$0 \le +p_{22} + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - 2p_{22}^{(33)} \le 1$$
$$0 \le +p_{22} + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_{22} + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_{22} + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} \le 1$$
$$0 \le +p_{22} + p_{12}^{(13,23)} + p_{21}^{(31,32)} - 2p_{22}^{(33)} \le 1$$
$$0 \le +p_{22} + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_{22} + p_{12}^{(13,23)} + p_{21}^{(11,31)} - p_{22}^{(11,13)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_{22} + p_2^R + p_{12}^{(13,23)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \le 1$$
$$0 \le +p_{22} + p_2^R + p_{12}^{(13,23)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le +p_{22} + p_2^R + p_{12}^{(13,23)} - p_{22}^{(11,13)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_{22} + p_2^R + p_{12}^{(13,23)} - p_{22}^{(11,13)} \le 1$$
$$0 \le +p_{22} + p_2^R + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - 2p_{22}^{(31,33)} \le 1$$
$$0 \le +p_{22} + p_2^R + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - 2p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le +p_{22} + p_2^R + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_{22} + p_2^R + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \le 1$$
$$0 \le +p_{22} + p_1^R - p_{21}^{(11,31)} \le 1$$
$$0 \le +p_{22} + p_1^R - p_{21}^{(11,31)} + p_{22}^{(33)} \le 1$$
$$0 \le +p_{22} + p_1^R - p_{21}^{(11,31)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$

$$0 \leq +p_{22} + p_1^R - p_{21}^{(11,31)} + p_{22}^{(31,33)} \leq 1$$
$$0 \leq +p_{22} + p_1^R - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} \leq 1$$
$$0 \leq +p_{22} + p_1^R - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{22} + p_1^R - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{22} + p_1^R - p_{21}^{(11,31)} + p_{21}^{(31,32)} \leq 1$$
$$0 \leq +p_{22} + p_2^L - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} - p_{22}^{(11,13)} \leq 1$$
$$0 \leq +p_{22} + p_2^L - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} - p_{22}^{(11,13)} + p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{22} + p_2^L - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} - p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{22} + p_2^L - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} - p_{22}^{(11,13)} + p_{22}^{(31,33)} \leq 1$$
$$0 \leq +p_{22} + p_2^L - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \leq 1$$
$$0 \leq +p_{22} + p_2^L - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{22} + p_2^L - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{22} + p_2^L - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} \leq 1$$
$$0 \leq +p_{22} + p_2^L - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(31,33)} \leq 1$$
$$0 \leq +p_{22} + p_2^L - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{22} + p_2^L - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + 2p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{22} + p_2^L - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + 2p_{22}^{(31,33)} \leq 1$$
$$0 \leq +p_{22} + p_2^L - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \leq 1$$
$$0 \leq +p_{22} + p_2^L - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{22} + p_2^L - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{22} + p_2^L - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(31,33)} \leq 1$$
$$0 \leq +p_{22} + p_2^L - p_2^R - p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{22} + p_2^L - p_2^R + p_{21}^{(22,21)} - p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{22} + p_2^L - p_2^R + p_{21}^{(22,21)} - p_{22}^{(11,13)} + p_{22}^{(31,33)} \leq 1$$
$$0 \leq +p_{22} + p_2^L - p_2^R + p_{21}^{(22,21)} - p_{22}^{(11,13)} + 2p_{22}^{(31,33)} - 2p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{22} + p_2^L - p_2^R + p_{21}^{(22,21)} - p_{22}^{(11,13)} + 2p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{22} + p_2^L - p_2^R + p_{21}^{(22,21)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{22} + p_2^L - p_2^R + p_{21}^{(22,21)} + p_{22}^{(31,33)} \leq 1$$
$$0 \leq +p_{22} + p_2^L - p_2^R + p_{21}^{(22,21)} + 2p_{22}^{(31,33)} - 2p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{22} + p_2^L - p_2^R + p_{21}^{(22,21)} + 2p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$$

$0 \le +p_{22} + p_2^L - p_2^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(33)} \le 1$

$0 \le +p_{22} + p_2^L - p_2^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} \le 1$

$0 \le +p_{22} + p_2^L - p_2^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} + p_{22}^{(31,33)} - 2p_{22}^{(33)} \le 1$

$0 \le +p_{22} + p_2^L - p_2^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$

$0 \le +p_{22} + p_2^L - p_2^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(33)} \le 1$

$0 \le +p_{22} + p_2^L - p_2^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} \le 1$

$0 \le +p_{22} + p_2^L - p_2^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(31,33)} - 2p_{22}^{(33)} \le 1$

$0 \le +p_{22} + p_2^L - p_2^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$

$0 \le +p_{22} + p_2^L - p_{22}^{(11,13)} \le 1$

$0 \le +p_{22} + p_2^L - p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$

$0 \le +p_{22} + p_2^L - p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$

$0 \le +p_{22} + p_2^L - p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$

$0 \le +p_{22} + p_2^L + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \le 1$

$0 \le +p_{22} + p_2^L + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$

$0 \le +p_{22} + p_2^L + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(33)} \le 1$

$0 \le +p_{22} + p_2^L + p_{21}^{(31,32)} - p_{22}^{(11,13)} \le 1$

$0 \le +p_{22} + p_2^L + p_{21}^{(22,21)} - p_{22}^{(11,13)} \le 1$

$0 \le +p_{22} + p_2^L + p_{21}^{(22,21)} - p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$

$0 \le +p_{22} + p_2^L + p_{21}^{(22,21)} - p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$

$0 \le +p_{22} + p_2^L + p_{21}^{(22,21)} - p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$

$0 \le +p_{22} + p_2^L + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \le 1$

$0 \le +p_{22} + p_2^L + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$

$0 \le +p_{22} + p_2^L + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(33)} \le 1$

$0 \le +p_{22} + p_2^L + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} \le 1$

$0 \le +p_{22} + p_2^L + p_{12}^{(13,23)} - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(33)} \le 1$

$0 \le +p_{22} + p_2^L + p_{12}^{(13,23)} - 2p_{22}^{(11,13)} - p_{22}^{(33)} \le 1$

$0 \le +p_{22} + p_2^L + p_{12}^{(13,23)} - 2p_{22}^{(11,13)} \le 1$

$0 \le +p_{22} + p_2^L + p_{12}^{(13,23)} - 2p_{22}^{(11,13)} + p_{22}^{(31,33)} - 2p_{22}^{(33)} \le 1$

$0 \le +p_{22} + p_2^L + p_{12}^{(13,23)} - 2p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$

$0 \le +p_{22} + p_2^L + p_{12}^{(13,23)} - p_{22}^{(11,13)} - p_{22}^{(33)} \le 1$

$0 \leq +p_{22} + p_2^L + p_{12}^{(13,23)} - p_{22}^{(11,13)} \leq 1$

$0 \leq +p_{22} + p_2^L + p_{12}^{(13,23)} - p_{22}^{(11,13)} + p_{22}^{(31,33)} - 2p_{22}^{(33)} \leq 1$

$0 \leq +p_{22} + p_2^L + p_{12}^{(13,23)} - p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$

$0 \leq +p_{22} + p_2^L + p_{12}^{(13,23)} + p_{21}^{(31,32)} - 2p_{22}^{(11,13)} - p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$

$0 \leq +p_{22} + p_2^L + p_{12}^{(13,23)} + p_{21}^{(31,32)} - 2p_{22}^{(11,13)} - p_{22}^{(31,33)} \leq 1$

$0 \leq +p_{22} + p_2^L + p_{12}^{(13,23)} + p_{21}^{(31,32)} - 2p_{22}^{(11,13)} - 2p_{22}^{(33)} \leq 1$

$0 \leq +p_{22} + p_2^L + p_{12}^{(13,23)} + p_{21}^{(31,32)} - 2p_{22}^{(11,13)} - p_{22}^{(33)} \leq 1$

$0 \leq +p_{22} + p_2^L + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$

$0 \leq +p_{22} + p_2^L + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \leq 1$

$0 \leq +p_{22} + p_2^L + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - 2p_{22}^{(33)} \leq 1$

$0 \leq +p_{22} + p_2^L + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(33)} \leq 1$

$0 \leq +p_{22} + p_2^L + p_2^R + p_{12}^{(13,23)} - 2p_{22}^{(11,13)} - p_{22}^{(31,33)} \leq 1$

$0 \leq +p_{22} + p_2^L + p_2^R + p_{12}^{(13,23)} - 2p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$

$0 \leq +p_{22} + p_2^L + p_2^R + p_{12}^{(13,23)} - 2p_{22}^{(11,13)} - p_{22}^{(33)} \leq 1$

$0 \leq +p_{22} + p_2^L + p_2^R + p_{12}^{(13,23)} - 2p_{22}^{(11,13)} \leq 1$

$0 \leq +p_{22} + p_2^L + p_2^R + p_{12}^{(13,23)} + p_{21}^{(31,32)} - 2p_{22}^{(11,13)} - 2p_{22}^{(31,33)} \leq 1$

$0 \leq +p_{22} + p_2^L + p_2^R + p_{12}^{(13,23)} + p_{21}^{(31,32)} - 2p_{22}^{(11,13)} - 2p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$

$0 \leq +p_{22} + p_2^L + p_2^R + p_{12}^{(13,23)} + p_{21}^{(31,32)} - 2p_{22}^{(11,13)} - p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$

$0 \leq +p_{22} + p_2^L + p_2^R + p_{12}^{(13,23)} + p_{21}^{(31,32)} - 2p_{22}^{(11,13)} - p_{22}^{(31,33)} \leq 1$

$0 \leq +p_{22} + p_2^L + p_1^R - p_{21}^{(11,31)} - p_{22}^{(11,13)} \leq 1$

$0 \leq +p_{22} + p_2^L + p_1^R - p_{21}^{(11,31)} - p_{22}^{(11,13)} + p_{22}^{(33)} \leq 1$

$0 \leq +p_{22} + p_2^L + p_1^R - p_{21}^{(11,31)} - p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$

$0 \leq +p_{22} + p_2^L + p_1^R - p_{21}^{(11,31)} - p_{22}^{(11,13)} + p_{22}^{(31,33)} \leq 1$

$0 \leq +p_{22} + p_2^L + p_1^R - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \leq 1$

$0 \leq +p_{22} + p_2^L + p_1^R - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$

$0 \leq +p_{22} + p_2^L + p_1^R - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(33)} \leq 1$

$0 \leq +p_{22} + p_2^L + p_1^R - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} \leq 1$

$0 \leq +p_{22} + p_1^L - p_{12}^{(11,13)} \leq 1$

$0 \leq +p_{22} + p_1^L - p_{12}^{(11,13)} + p_{22}^{(33)} \leq 1$

$0 \leq +p_{22} + p_1^L - p_{12}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$

$0 \leq +p_{22} + p_1^L - p_{12}^{(11,13)} + p_{22}^{(31,33)} \leq 1$

$0 \leq +p_{22} + p_1^L - p_{12}^{(11,13)} + p_{12}^{(13,23)} - p_{22}^{(11,13)} - p_{22}^{(33)} \leq 1$

$0 \leq +p_{22} + p_1^L - p_{12}^{(11,13)} + p_{12}^{(13,23)} - p_{22}^{(11,13)} \leq 1$

$0 \leq +p_{22} + p_1^L - p_{12}^{(11,13)} + p_{12}^{(13,23)} - p_{22}^{(11,13)} + p_{22}^{(31,33)} - 2p_{22}^{(33)} \leq 1$

$0 \leq +p_{22} + p_1^L - p_{12}^{(11,13)} + p_{12}^{(13,23)} - p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$

$0 \leq +p_{22} + p_1^L - p_{12}^{(11,13)} + p_{12}^{(13,23)} - p_{22}^{(33)} \leq 1$

$0 \leq +p_{22} + p_1^L - p_{12}^{(11,13)} + p_{12}^{(13,23)} \leq 1$

$0 \leq +p_{22} + p_1^L - p_{12}^{(11,13)} + p_{12}^{(13,23)} + p_{22}^{(31,33)} - 2p_{22}^{(33)} \leq 1$

$0 \leq +p_{22} + p_1^L - p_{12}^{(11,13)} + p_{12}^{(13,23)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$

$0 \leq +p_{22} + p_1^L + p_2^R - p_{12}^{(11,13)} + p_{12}^{(13,23)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \leq 1$

$0 \leq +p_{22} + p_1^L + p_2^R - p_{12}^{(11,13)} + p_{12}^{(13,23)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$

$0 \leq +p_{22} + p_1^L + p_2^R - p_{12}^{(11,13)} + p_{12}^{(13,23)} - p_{22}^{(11,13)} - p_{22}^{(33)} \leq 1$

$0 \leq +p_{22} + p_1^L + p_2^R - p_{12}^{(11,13)} + p_{12}^{(13,23)} - p_{22}^{(11,13)} \leq 1$

$0 \leq +2p_{22} - p_1^L + p_2^L - p_2^R + p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(33)} \leq 1$

$0 \leq +2p_{22} - p_1^L + p_2^L - p_2^R + p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} \leq 1$

$0 \leq +2p_{22} - p_1^L + p_2^L - p_2^R + p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} + p_{22}^{(31,33)} - 2p_{22}^{(33)} \leq 1$

$0 \leq +2p_{22} - p_1^L + p_2^L - p_2^R + p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$

$0 \leq +2p_{22} - p_2^R + p_{21}^{(22,21)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$

$0 \leq +2p_{22} - p_2^R + p_{21}^{(22,21)} + p_{22}^{(31,33)} \leq 1$

$0 \leq +2p_{22} - p_2^R + p_{21}^{(22,21)} + 2p_{22}^{(31,33)} - 2p_{22}^{(33)} \leq 1$

$0 \leq +2p_{22} - p_2^R + p_{21}^{(22,21)} + 2p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$

$0 \leq +2p_{22} - p_2^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(33)} \leq 1$

$0 \leq +2p_{22} - p_2^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} \leq 1$

$0 \leq +2p_{22} - p_2^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(31,33)} - 2p_{22}^{(33)} \leq 1$

$0 \leq +2p_{22} - p_2^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$

$0 \leq +2p_{22} + p_{12}^{(13,23)} - p_{22}^{(11,13)} - p_{22}^{(33)} \leq 1$

$0 \leq +2p_{22} + p_{12}^{(13,23)} - p_{22}^{(11,13)} \leq 1$

$0 \leq +2p_{22} + p_{12}^{(13,23)} - p_{22}^{(11,13)} + p_{22}^{(31,33)} - 2p_{22}^{(33)} \leq 1$

$0 \leq +2p_{22} + p_{12}^{(13,23)} - p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$

$0 \leq +2p_{22} + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$

$0 \leq +2p_{22} + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \leq 1$

$0 \leq +2p_{22} + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - 2p_{22}^{(33)} \leq 1$

$0 \leq +2p_{22} + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(33)} \leq 1$

$0 \leq +2p_{22} + p_2^L - p_2^R + p_{21}^{(22,21)} - p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$

$0 \leq +2p_{22} + p_2^L - p_2^R + p_{21}^{(22,21)} - p_{22}^{(11,13)} + p_{22}^{(31,33)} \leq 1$

$0 \leq +2p_{22} + p_2^L - p_2^R + p_{21}^{(22,21)} - p_{22}^{(11,13)} + 2p_{22}^{(31,33)} - 2p_{22}^{(33)} \leq 1$

$0 \leq +2p_{22} + p_2^L - p_2^R + p_{21}^{(22,21)} - p_{22}^{(11,13)} + 2p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$

$0 \leq +2p_{22} + p_2^L - p_2^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(33)} \leq 1$

$0 \leq +2p_{22} + p_2^L - p_2^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} \leq 1$

$0 \leq +2p_{22} + p_2^L - p_2^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} + p_{22}^{(31,33)} - 2p_{22}^{(33)} \leq 1$

$0 \leq +2p_{22} + p_2^L - p_2^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$

$0 \leq +2p_{22} + p_2^L + p_{12}^{(13,23)} - 2p_{22}^{(11,13)} - p_{22}^{(33)} \leq 1$

$0 \leq +2p_{22} + p_2^L + p_{12}^{(13,23)} - 2p_{22}^{(11,13)} \leq 1$

$0 \leq +2p_{22} + p_2^L + p_{12}^{(13,23)} - 2p_{22}^{(11,13)} + p_{22}^{(31,33)} - 2p_{22}^{(33)} \leq 1$

$0 \leq +2p_{22} + p_2^L + p_{12}^{(13,23)} - 2p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$

$0 \leq +2p_{22} + p_2^L + p_{12}^{(13,23)} + p_{21}^{(31,32)} - 2p_{22}^{(11,13)} - p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$

$0 \leq +2p_{22} + p_2^L + p_{12}^{(13,23)} + p_{21}^{(31,32)} - 2p_{22}^{(11,13)} - p_{22}^{(31,33)} \leq 1$

$0 \leq +2p_{22} + p_2^L + p_{12}^{(13,23)} + p_{21}^{(31,32)} - 2p_{22}^{(11,13)} - 2p_{22}^{(33)} \leq 1$

$0 \leq +2p_{22} + p_2^L + p_{12}^{(13,23)} + p_{21}^{(31,32)} - 2p_{22}^{(11,13)} - p_{22}^{(33)} \leq 1$

$0 \leq +2p_{22} + p_1^L - p_{12}^{(11,13)} + p_{12}^{(13,23)} - p_{22}^{(11,13)} - p_{22}^{(33)} \leq 1$

$0 \leq +2p_{22} + p_1^L - p_{12}^{(11,13)} + p_{12}^{(13,23)} - p_{22}^{(11,13)} \leq 1$

$0 \leq +2p_{22} + p_1^L - p_{12}^{(11,13)} + p_{12}^{(13,23)} - p_{22}^{(11,13)} + p_{22}^{(31,33)} - 2p_{22}^{(33)} \leq 1$

$0 \leq +2p_{22} + p_1^L - p_{12}^{(11,13)} + p_{12}^{(13,23)} - p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$

$0 \leq +p_{21} - 2p_{22} - p_{12}^{(13,23)} - p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + 2p_{22}^{(11,13)} + p_{22}^{(33)} \leq 1$

$0 \leq +p_{21} - 2p_{22} + p_2^R - p_{21}^{(11,31)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$

$0 \leq +p_{21} - 2p_{22} + p_2^L - p_{12}^{(13,23)} - p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(33)} \leq 1$

$0 \leq +p_{21} - 2p_{22} + p_2^L + p_2^R - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$

$0 \leq +p_{21} - p_{22} - p_1^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \leq 1$

$0 \leq +p_{21} - p_{22} - p_2^R - p_{12}^{(13,23)} - p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + 2p_{22}^{(11,13)} + p_{22}^{(31,33)} \leq 1$

$0 \leq +p_{21} - p_{22} - p_{12}^{(13,23)} - p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(33)} \leq 1$

$$0 \leq +p_{21} - p_{22} - p_{12}^{(13,23)} - p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + 2p_{22}^{(11,13)} + p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{21} - p_{22} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{21} - p_{22} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + 2p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{21} - p_{22} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \leq 1$$
$$0 \leq +p_{21} - p_{22} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{21} - p_{22} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{21} - p_{22} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} - p_{22}^{(31,33)} + 2p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{21} - p_{22} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \leq 1$$
$$0 \leq +p_{21} - p_{22} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{21} - p_{22} - p_{21}^{(11,31)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \leq 1$$
$$0 \leq +p_{21} - p_{22} - p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \leq 1$$
$$0 \leq +p_{21} - p_{22} + p_2^R - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{21} - p_{22} + p_2^R - p_{21}^{(11,31)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{21} - p_{22} + p_2^R - p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{21} - p_{22} + p_2^R - p_{22}^{(31,33)} + 2p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{21} - p_{22} + p_2^R \leq 1$$
$$0 \leq +p_{21} - p_{22} + p_2^R + p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{21} - p_{22} + p_2^R + p_{21}^{(31,32)} - 2p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{21} - p_{22} + p_2^R + p_{21}^{(31,32)} - 2p_{22}^{(31,33)} + 2p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{21} - p_{22} + p_2^R + p_{21}^{(31,32)} - p_{22}^{(31,33)} \leq 1$$
$$0 \leq +p_{21} - p_{22} + p_2^R + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{21} - p_{22} + p_2^R + p_{12}^{(13,23)} - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} \leq 1$$
$$0 \leq +p_{21} - p_{22} + p_1^R - 2p_{21}^{(11,31)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \leq 1$$
$$0 \leq +p_{21} - p_{22} + p_2^L - p_1^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} \leq 1$$
$$0 \leq +p_{21} - p_{22} + p_2^L - p_2^R - p_{12}^{(13,23)} - p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \leq 1$$
$$0 \leq +p_{21} - p_{22} + p_2^L - p_{12}^{(13,23)} - p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{21} - p_{22} + p_2^L - p_{12}^{(13,23)} - p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{21} - p_{22} + p_2^L - p_{21}^{(11,31)} + p_{21}^{(31,32)} \leq 1$$
$$0 \leq +p_{21} - p_{22} + p_2^L - p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \leq 1$$
$$0 \leq +p_{21} - p_{22} + p_2^L + p_2^R - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$$

$0 \leq +p_{21} - p_{22} + p_2^L + p_2^R - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$

$0 \leq +p_{21} - p_{22} + p_2^L + p_2^R + p_{12}^{(13,23)} - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \leq 1$

$0 \leq +p_{21} - p_{22} + p_2^L + p_1^R - 2p_{21}^{(11,31)} + p_{21}^{(31,32)} \leq 1$

$0 \leq +p_{21} - p_1^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} \leq 1$

$0 \leq +p_{21} - p_1^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \leq 1$

$0 \leq +p_{21} - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} \leq 1$

$0 \leq +p_{21} - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(33)} \leq 1$

$0 \leq +p_{21} - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$

$0 \leq +p_{21} - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(31,33)} \leq 1$

$0 \leq +p_{21} - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} \leq 1$

$0 \leq +p_{21} - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$

$0 \leq +p_{21} - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(33)} \leq 1$

$0 \leq +p_{21} - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \leq 1$

$0 \leq +p_{21} - p_2^R - p_{12}^{(13,23)} - p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \leq 1$

$0 \leq +p_{21} - p_2^R - p_{12}^{(13,23)} - p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + 2p_{22}^{(11,13)} + p_{22}^{(31,33)} \leq 1$

$0 \leq +p_{21} - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \leq 1$

$0 \leq +p_{21} - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$

$0 \leq +p_{21} - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + 2p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$

$0 \leq +p_{21} - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + 2p_{22}^{(31,33)} \leq 1$

$0 \leq +p_{21} - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \leq 1$

$0 \leq +p_{21} - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(33)} \leq 1$

$0 \leq +p_{21} - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$

$0 \leq +p_{21} - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \leq 1$

$0 \leq +p_{21} - p_2^R - p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$

$0 \leq +p_{21} - p_{12}^{(13,23)} - p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(33)} \leq 1$

$0 \leq +p_{21} - p_{21}^{(11,31)} + p_{21}^{(31,32)} \leq 1$

$0 \leq +p_{21} - p_{21}^{(11,31)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \leq 1$

$0 \leq +p_{21} - p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \leq 1$

$0 \leq +p_{21} - p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \leq 1$

$0 \leq +p_{21} \leq 1$

$$0 \le +p_{21} + p_{22}^{(33)} \le 1$$
$$0 \le +p_{21} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_{21} + p_{22}^{(31,33)} \le 1$$
$$0 \le +p_{21} + p_{21}^{(31,32)} - p_{22}^{(31,33)} \le 1$$
$$0 \le +p_{21} + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le +p_{21} + p_{21}^{(31,32)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_{21} + p_{21}^{(31,32)} \le 1$$
$$0 \le +p_{21} + p_{21}^{(22,21)} \le 1$$
$$0 \le +p_{21} + p_{21}^{(22,21)} + p_{22}^{(33)} \le 1$$
$$0 \le +p_{21} + p_{21}^{(22,21)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_{21} + p_{21}^{(22,21)} + p_{22}^{(31,33)} \le 1$$
$$0 \le +p_{21} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} \le 1$$
$$0 \le +p_{21} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le +p_{21} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_{21} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \le 1$$
$$0 \le +p_{21} + p_{12}^{(13,23)} - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_{21} + p_2^R - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le +p_{21} + p_2^R + p_{12}^{(13,23)} - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \le 1$$
$$0 \le +p_{21} + p_2^R + p_{12}^{(13,23)} - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} \le 1$$
$$0 \le +p_{21} + p_2^R + p_{12}^{(13,23)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \le 1$$
$$0 \le +p_{21} + p_2^R + p_{12}^{(13,23)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le +p_{21} + p_2^R + p_{12}^{(13,23)} - p_{22}^{(11,13)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_{21} + p_2^R + p_{12}^{(13,23)} - p_{22}^{(11,13)} \le 1$$
$$0 \le +p_{21} + p_2^R + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - 2p_{22}^{(31,33)} \le 1$$
$$0 \le +p_{21} + p_2^R + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - 2p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$$
$$0 \le +p_{21} + p_2^R + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_{21} + p_2^R + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \le 1$$
$$0 \le +p_{21} + p_1^R - 2p_{21}^{(11,31)} + p_{21}^{(31,32)} \le 1$$
$$0 \le +p_{21} + p_1^R - 2p_{21}^{(11,31)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \le 1$$
$$0 \le +p_{21} + p_1^R - p_{21}^{(11,31)} \le 1$$

$$0 \leq +p_{21} + p_1^R - p_{21}^{(11,31)} + p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{21} + p_1^R - p_{21}^{(11,31)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{21} + p_1^R - p_{21}^{(11,31)} + p_{22}^{(31,33)} \leq 1$$
$$0 \leq +p_{21} + p_1^R - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} \leq 1$$
$$0 \leq +p_{21} + p_1^R - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{21} + p_1^R - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{21} + p_1^R - p_{21}^{(11,31)} + p_{21}^{(31,32)} \leq 1$$
$$0 \leq +p_{21} + p_2^L - p_1^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} \leq 1$$
$$0 \leq +p_{21} + p_2^L - p_1^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} \leq 1$$
$$0 \leq +p_{21} + p_2^L - p_2^R - p_{12}^{(13,23)} - p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(31,33)} \leq 1$$
$$0 \leq +p_{21} + p_2^L - p_2^R - p_{12}^{(13,23)} - p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \leq 1$$
$$0 \leq +p_{21} + p_2^L - p_2^R - p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{21} + p_2^L - p_{12}^{(13,23)} - p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{21} + p_2^L - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} \leq 1$$
$$0 \leq +p_{21} + p_2^L - p_{21}^{(11,31)} + p_{21}^{(31,32)} \leq 1$$
$$0 \leq +p_{21} + p_2^L - p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} \leq 1$$
$$0 \leq +p_{21} + p_2^L - p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \leq 1$$
$$0 \leq +p_{21} + p_2^L + p_{12}^{(13,23)} - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{21} + p_2^L + p_2^R - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{21} + p_2^L + p_2^R + p_{12}^{(13,23)} - p_{21}^{(11,31)} + p_{21}^{(31,32)} - 2p_{22}^{(11,13)} - p_{22}^{(31,33)} \leq 1$$
$$0 \leq +p_{21} + p_2^L + p_2^R + p_{12}^{(13,23)} - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \leq 1$$
$$0 \leq +p_{21} + p_2^L + p_1^R - 2p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} \leq 1$$
$$0 \leq +p_{21} + p_2^L + p_1^R - 2p_{21}^{(11,31)} + p_{21}^{(31,32)} \leq 1$$
$$0 \leq +p_{21} + p_{22} - p_1^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} \leq 1$$
$$0 \leq +p_{21} + p_{22} - p_2^R - p_{12}^{(13,23)} - p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \leq 1$$
$$0 \leq +p_{21} + p_{22} - p_2^R - p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{21} + p_{22} - p_2^R - p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{21} + p_{22} - p_2^R + p_{21}^{(22,21)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{21} + p_{22} - p_2^R + p_{21}^{(22,21)} + p_{22}^{(31,33)} \leq 1$$
$$0 \leq +p_{21} + p_{22} - p_2^R + p_{21}^{(22,21)} + 2p_{22}^{(31,33)} - 2p_{22}^{(33)} \leq 1$$

$0 \leq +p_{21} + p_{22} - p_2^R + p_{21}^{(22,21)} + 2p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$

$0 \leq +p_{21} + p_{22} - p_2^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(33)} \leq 1$

$0 \leq +p_{21} + p_{22} - p_2^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} \leq 1$

$0 \leq +p_{21} + p_{22} - p_2^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(31,33)} - 2p_{22}^{(33)} \leq 1$

$0 \leq +p_{21} + p_{22} - p_2^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$

$0 \leq +p_{21} + p_{22} - p_{21}^{(11,31)} + p_{21}^{(31,32)} \leq 1$

$0 \leq +p_{21} + p_{22} - p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \leq 1$

$0 \leq +p_{21} + p_{22} + p_{12}^{(13,23)} - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(33)} \leq 1$

$0 \leq +p_{21} + p_{22} + p_{12}^{(13,23)} - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(33)} \leq 1$

$0 \leq +p_{21} + p_{22} + p_{12}^{(13,23)} - p_{22}^{(11,13)} - p_{22}^{(33)} \leq 1$

$0 \leq +p_{21} + p_{22} + p_{12}^{(13,23)} - p_{22}^{(11,13)} \leq 1$

$0 \leq +p_{21} + p_{22} + p_{12}^{(13,23)} - p_{22}^{(11,13)} + p_{22}^{(31,33)} - 2p_{22}^{(33)} \leq 1$

$0 \leq +p_{21} + p_{22} + p_{12}^{(13,23)} - p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$

$0 \leq +p_{21} + p_{22} + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$

$0 \leq +p_{21} + p_{22} + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \leq 1$

$0 \leq +p_{21} + p_{22} + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - 2p_{22}^{(33)} \leq 1$

$0 \leq +p_{21} + p_{22} + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(33)} \leq 1$

$0 \leq +p_{21} + p_{22} + p_2^R + p_{12}^{(13,23)} - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \leq 1$

$0 \leq +p_{21} + p_{22} + p_1^R - 2p_{21}^{(11,31)} + p_{21}^{(31,32)} \leq 1$

$0 \leq +p_{21} + p_{22} + p_2^L - p_1^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} \leq 1$

$0 \leq +p_{21} + p_{22} + p_2^L - p_2^R - p_{12}^{(13,23)} - p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(31,33)} \leq 1$

$0 \leq +p_{21} + p_{22} + p_2^L - p_2^R - p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$

$0 \leq +p_{21} + p_{22} + p_2^L - p_2^R - p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$

$0 \leq +p_{21} + p_{22} + p_2^L - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} \leq 1$

$0 \leq +p_{21} + p_{22} + p_2^L - p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} \leq 1$

$0 \leq +p_{21} + p_{22} + p_2^L + p_{12}^{(13,23)} - p_{21}^{(11,31)} + p_{21}^{(31,32)} - 2p_{22}^{(11,13)} - p_{22}^{(33)} \leq 1$

$0 \leq +p_{21} + p_{22} + p_2^L + p_{12}^{(13,23)} - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(33)} \leq 1$

$0 \leq +p_{21} + p_{22} + p_2^L + p_2^R + p_{12}^{(13,23)} - p_{21}^{(11,31)} + p_{21}^{(31,32)} - 2p_{22}^{(11,13)} - p_{22}^{(31,33)} \leq 1$

$0 \leq +p_{21} + p_{22} + p_2^L + p_1^R - 2p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} \leq 1$

$0 \leq +p_{21} + 2p_{22} - p_2^R - p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$

$$0 \leq +p_{21} + 2p_{22} + p_{12}^{(13,23)} - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{21} + 2p_{22} + p_2^L - p_2^R - p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{21} + 2p_{22} + p_2^L + p_{12}^{(13,23)} - p_{21}^{(11,31)} + p_{21}^{(31,32)} - 2p_{22}^{(11,13)} - p_{22}^{(33)} \leq 1$$
$$0 \leq +2p_{21} - p_{22} - p_{12}^{(13,23)} - p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(33)} \leq 1$$
$$0 \leq +2p_{21} - p_{22} + p_2^R - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$$
$$0 \leq +2p_{21} - p_1^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} \leq 1$$
$$0 \leq +2p_{21} - p_2^R - p_{12}^{(13,23)} - p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \leq 1$$
$$0 \leq +2p_{21} - p_{21}^{(11,31)} + p_{21}^{(31,32)} \leq 1$$
$$0 \leq +2p_{21} - p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \leq 1$$
$$0 \leq +2p_{21} + p_2^R + p_{12}^{(13,23)} - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \leq 1$$
$$0 \leq +2p_{21} + p_1^R - 2p_{21}^{(11,31)} + p_{21}^{(31,32)} \leq 1$$
$$0 \leq +2p_{21} + p_{22} - p_2^R - p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$$
$$0 \leq +2p_{21} + p_{22} + p_{12}^{(13,23)} - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{12} - p_{22} - p_1^L + p_2^L - p_2^R + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \leq 1$$
$$0 \leq +p_{12} - p_{22} - p_1^L + p_2^L - p_2^R + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{12} - p_{22} - p_1^L + p_2^L + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{12} - p_{22} - p_1^L + p_2^L + p_{21}^{(22,21)} + p_{21}^{(31,32)} \leq 1$$
$$0 \leq +p_{12} - p_{22} - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + 2p_{22}^{(11,13)} + p_{22}^{(31,33)} \leq 1$$
$$0 \leq +p_{12} - p_{22} - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + 2p_{22}^{(11,13)} + p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{12} - p_{22} - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + 2p_{22}^{(11,13)} \leq 1$$
$$0 \leq +p_{12} - p_{22} - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + 2p_{22}^{(11,13)} + p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{12} - p_{22} - p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{12} - p_{22} - p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \leq 1$$
$$0 \leq +p_{12} - p_{22} - p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{12} - p_{22} - p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \leq 1$$
$$0 \leq +p_{12} - p_{22} + p_{22}^{(11,13)} \leq 1$$
$$0 \leq +p_{12} - p_{22} + p_{22}^{(11,13)} + p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{12} - p_{22} + p_{21}^{(31,32)} + p_{22}^{(11,13)} - p_{22}^{(31,33)} \leq 1$$
$$0 \leq +p_{12} - p_{22} + p_{21}^{(31,32)} + p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{12} - p_{22} + p_2^R - p_{12}^{(11,13)} + p_{12}^{(13,23)} - p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$$

$0 \le +p_{12} - p_{22} + p_2^R - p_{12}^{(11,13)} + p_{12}^{(13,23)} \le 1$

$0 \le +p_{12} - p_{22} + p_2^R - p_{12}^{(11,13)} + p_{12}^{(13,23)} + p_{21}^{(31,32)} - 2p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$

$0 \le +p_{12} - p_{22} + p_2^R - p_{12}^{(11,13)} + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} \le 1$

$0 \le +p_{12} - p_{22} + p_2^L - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$

$0 \le +p_{12} - p_{22} + p_2^L - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$

$0 \le +p_{12} - p_{22} + p_2^L - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \le 1$

$0 \le +p_{12} - p_{22} + p_2^L - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$

$0 \le +p_{12} - p_{22} + p_2^L - p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{22}^{(33)} \le 1$

$0 \le +p_{12} - p_{22} + p_2^L - p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{22}^{(31,33)} \le 1$

$0 \le +p_{12} - p_{22} + p_2^L - p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$

$0 \le +p_{12} - p_{22} + p_2^L - p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \le 1$

$0 \le +p_{12} - p_{22} + p_2^L \le 1$

$0 \le +p_{12} - p_{22} + p_2^L + p_{22}^{(33)} \le 1$

$0 \le +p_{12} - p_{22} + p_2^L + p_{21}^{(31,32)} - p_{22}^{(31,33)} \le 1$

$0 \le +p_{12} - p_{22} + p_2^L + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$

$0 \le +p_{12} - p_{22} + p_2^L + p_2^R - p_{12}^{(11,13)} + p_{12}^{(13,23)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$

$0 \le +p_{12} - p_{22} + p_2^L + p_2^R - p_{12}^{(11,13)} + p_{12}^{(13,23)} - p_{22}^{(11,13)} \le 1$

$0 \le +p_{12} - p_{22} + p_2^L + p_2^R - p_{12}^{(11,13)} + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - 2p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$

$0 \le +p_{12} - p_{22} + p_2^L + p_2^R - p_{12}^{(11,13)} + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \le 1$

$0 \le +p_{12} - p_{22} + p_1^L - p_{12}^{(11,13)} + p_{22}^{(11,13)} \le 1$

$0 \le +p_{12} - p_{22} + p_1^L - p_{12}^{(11,13)} + p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$

$0 \le +p_{12} - p_{22} + p_1^L + p_2^R - 2p_{12}^{(11,13)} + p_{12}^{(13,23)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$

$0 \le +p_{12} - p_{22} + p_1^L + p_2^R - 2p_{12}^{(11,13)} + p_{12}^{(13,23)} \le 1$

$0 \le +p_{12} - p_1^L + p_2^L - p_2^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} \le 1$

$0 \le +p_{12} - p_1^L + p_2^L - p_2^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$

$0 \le +p_{12} - p_1^L + p_2^L - p_2^R + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \le 1$

$0 \le +p_{12} - p_1^L + p_2^L - p_2^R + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(33)} \le 1$

$0 \le +p_{12} - p_1^L + p_2^L - p_2^R + p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(33)} \le 1$

$0 \le +p_{12} - p_1^L + p_2^L - p_2^R + p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \le 1$

$0 \le +p_{12} - p_1^L + p_2^L + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$

$0 \le +p_{12} - p_1^L + p_2^L + p_{21}^{(22,21)} + p_{21}^{(31,32)} \le 1$

$0 \le +p_{12} - p_2^R - p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$

$0 \le +p_{12} - p_2^R - p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + 2p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$

$0 \le +p_{12} - p_2^R - p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \le 1$

$0 \le +p_{12} - p_2^R - p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$

$0 \le +p_{12} - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$

$0 \le +p_{12} - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$

$0 \le +p_{12} - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \le 1$

$0 \le +p_{12} - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$

$0 \le +p_{12} - p_2^R + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$

$0 \le +p_{12} - p_2^R + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$

$0 \le +p_{12} - p_2^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} - p_{22}^{(33)} \le 1$

$0 \le +p_{12} - p_2^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \le 1$

$0 \le +p_{12} - p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(33)} \le 1$

$0 \le +p_{12} - p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$

$0 \le +p_{12} - p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$

$0 \le +p_{12} - p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \le 1$

$0 \le +p_{12} - p_{12}^{(11,13)} + p_{12}^{(13,23)} \le 1$

$0 \le +p_{12} - p_{12}^{(11,13)} + p_{12}^{(13,23)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$

$0 \le +p_{12} - p_{12}^{(11,13)} + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} \le 1$

$0 \le +p_{12} - p_{12}^{(11,13)} + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(33)} \le 1$

$0 \le +p_{12} \le 1$

$0 \le +p_{12} + p_{22}^{(33)} \le 1$

$0 \le +p_{12} + p_{21}^{(31,32)} - p_{22}^{(31,33)} \le 1$

$0 \le +p_{12} + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$

$0 \le +p_{12} + p_{12}^{(13,23)} - p_{22}^{(33)} \le 1$

$0 \le +p_{12} + p_{12}^{(13,23)} \le 1$

$0 \le +p_{12} + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$

$0 \le +p_{12} + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} \le 1$

$0 \le +p_{12} + p_2^R - p_{12}^{(11,13)} + p_{12}^{(13,23)} - p_{22}^{(31,33)} + p_{22}^{(33)} \le 1$

$$0 \leq +p_{12} + p_2^R - p_{12}^{(11,13)} + p_{12}^{(13,23)} \leq 1$$
$$0 \leq +p_{12} + p_2^R - p_{12}^{(11,13)} + p_{12}^{(13,23)} + p_{21}^{(31,32)} - 2p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{12} + p_2^R - p_{12}^{(11,13)} + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} \leq 1$$
$$0 \leq +p_{12} + p_2^L - p_2^R - p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{22}^{(31,33)} \leq 1$$
$$0 \leq +p_{12} + p_2^L - p_2^R - p_{12}^{(11,13)} + p_{21}^{(22,21)} + 2p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{12} + p_2^L - p_2^R - p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \leq 1$$
$$0 \leq +p_{12} + p_2^L - p_2^R - p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{12} + p_2^L - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(31,33)} \leq 1$$
$$0 \leq +p_{12} + p_2^L - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{12} + p_2^L - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \leq 1$$
$$0 \leq +p_{12} + p_2^L - p_2^R - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{12} + p_2^L - p_2^R + p_{21}^{(22,21)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{12} + p_2^L - p_2^R + p_{21}^{(22,21)} + p_{22}^{(31,33)} \leq 1$$
$$0 \leq +p_{12} + p_2^L - p_2^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{12} + p_2^L - p_2^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} \leq 1$$
$$0 \leq +p_{12} + p_2^L - p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{12} + p_2^L - p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{22}^{(31,33)} \leq 1$$
$$0 \leq +p_{12} + p_2^L - p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{12} + p_2^L - p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \leq 1$$
$$0 \leq +p_{12} + p_2^L - p_{12}^{(11,13)} + p_{12}^{(13,23)} - p_{22}^{(11,13)} \leq 1$$
$$0 \leq +p_{12} + p_2^L - p_{12}^{(11,13)} + p_{12}^{(13,23)} - p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{12} + p_2^L - p_{12}^{(11,13)} + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \leq 1$$
$$0 \leq +p_{12} + p_2^L - p_{12}^{(11,13)} + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{12} + p_2^L - p_{22}^{(11,13)} \leq 1$$
$$0 \leq +p_{12} + p_2^L - p_{22}^{(11,13)} + p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{12} + p_2^L + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \leq 1$$
$$0 \leq +p_{12} + p_2^L + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{12} + p_2^L + p_{12}^{(13,23)} - p_{22}^{(11,13)} - p_{22}^{(33)} \leq 1$$
$$0 \leq +p_{12} + p_2^L + p_{12}^{(13,23)} - p_{22}^{(11,13)} \leq 1$$
$$0 \leq +p_{12} + p_2^L + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$$

$0 \leq +p_{12} + p_2^L + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \leq 1$

$0 \leq +p_{12} + p_2^L + p_2^R - p_{12}^{(11,13)} + p_{12}^{(13,23)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$

$0 \leq +p_{12} + p_2^L + p_2^R - p_{12}^{(11,13)} + p_{12}^{(13,23)} - p_{22}^{(11,13)} \leq 1$

$0 \leq +p_{12} + p_2^L + p_2^R - p_{12}^{(11,13)} + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - 2p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$

$0 \leq +p_{12} + p_2^L + p_2^R - p_{12}^{(11,13)} + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \leq 1$

$0 \leq +p_{12} + p_1^L - 2p_{12}^{(11,13)} + p_{12}^{(13,23)} \leq 1$

$0 \leq +p_{12} + p_1^L - 2p_{12}^{(11,13)} + p_{12}^{(13,23)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$

$0 \leq +p_{12} + p_1^L - p_{12}^{(11,13)} \leq 1$

$0 \leq +p_{12} + p_1^L - p_{12}^{(11,13)} + p_{22}^{(33)} \leq 1$

$0 \leq +p_{12} + p_1^L - p_{12}^{(11,13)} + p_{12}^{(13,23)} - p_{22}^{(33)} \leq 1$

$0 \leq +p_{12} + p_1^L - p_{12}^{(11,13)} + p_{12}^{(13,23)} \leq 1$

$0 \leq +p_{12} + p_1^L + p_2^R - 2p_{12}^{(11,13)} + p_{12}^{(13,23)} - p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$

$0 \leq +p_{12} + p_1^L + p_2^R - 2p_{12}^{(11,13)} + p_{12}^{(13,23)} \leq 1$

$0 \leq +p_{12} + p_{22} - p_1^L + p_2^L - p_2^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} \leq 1$

$0 \leq +p_{12} + p_{22} - p_1^L + p_2^L - p_2^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$

$0 \leq +p_{12} + p_{22} - p_1^L + p_2^L - p_2^R + p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(33)} \leq 1$

$0 \leq +p_{12} + p_{22} - p_1^L + p_2^L - p_2^R + p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} \leq 1$

$0 \leq +p_{12} + p_{22} - p_2^R - p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \leq 1$

$0 \leq +p_{12} + p_{22} - p_2^R - p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + 2p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$

$0 \leq +p_{12} + p_{22} - p_2^R - p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \leq 1$

$0 \leq +p_{12} + p_{22} - p_2^R - p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$

$0 \leq +p_{12} + p_{22} - p_2^R + p_{21}^{(22,21)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$

$0 \leq +p_{12} + p_{22} - p_2^R + p_{21}^{(22,21)} + p_{22}^{(31,33)} \leq 1$

$0 \leq +p_{12} + p_{22} - p_2^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(33)} \leq 1$

$0 \leq +p_{12} + p_{22} - p_2^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} \leq 1$

$0 \leq +p_{12} + p_{22} - p_{12}^{(11,13)} + p_{12}^{(13,23)} \leq 1$

$0 \leq +p_{12} + p_{22} - p_{12}^{(11,13)} + p_{12}^{(13,23)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$

$0 \leq +p_{12} + p_{22} - p_{12}^{(11,13)} + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} \leq 1$

$0 \leq +p_{12} + p_{22} - p_{12}^{(11,13)} + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(33)} \leq 1$

$0 \leq +p_{12} + p_{22} + p_{12}^{(13,23)} - p_{22}^{(11,13)} - p_{22}^{(33)} \leq 1$

$$0 \le +p_{12} + p_{22} + p_{12}^{(13,23)} - p_{22}^{(11,13)} \le 1$$
$$0 \le +p_{12} + p_{22} + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_{12} + p_{22} + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \le 1$$
$$0 \le +p_{12} + p_{22} + p_2^L - p_2^R - p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{22}^{(31,33)} \le 1$$
$$0 \le +p_{12} + p_{22} + p_2^L - p_2^R - p_{12}^{(11,13)} + p_{21}^{(22,21)} + 2p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_{12} + p_{22} + p_2^L - p_2^R - p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \le 1$$
$$0 \le +p_{12} + p_{22} + p_2^L - p_2^R - p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_{12} + p_{22} + p_2^L - p_2^R + p_{21}^{(22,21)} - p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_{12} + p_{22} + p_2^L - p_2^R + p_{21}^{(22,21)} - p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$$
$$0 \le +p_{12} + p_{22} + p_2^L - p_2^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_{12} + p_{22} + p_2^L - p_2^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} \le 1$$
$$0 \le +p_{12} + p_{22} + p_2^L - p_{12}^{(11,13)} + p_{12}^{(13,23)} - p_{22}^{(11,13)} \le 1$$
$$0 \le +p_{12} + p_{22} + p_2^L - p_{12}^{(11,13)} + p_{12}^{(13,23)} - p_{22}^{(11,13)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_{12} + p_{22} + p_2^L - p_{12}^{(11,13)} + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \le 1$$
$$0 \le +p_{12} + p_{22} + p_2^L - p_{12}^{(11,13)} + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_{12} + p_{22} + p_2^L + p_{12}^{(13,23)} - 2p_{22}^{(11,13)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_{12} + p_{22} + p_2^L + p_{12}^{(13,23)} - 2p_{22}^{(11,13)} \le 1$$
$$0 \le +p_{12} + p_{22} + p_2^L + p_{12}^{(13,23)} + p_{21}^{(31,32)} - 2p_{22}^{(11,13)} - p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_{12} + p_{22} + p_2^L + p_{12}^{(13,23)} + p_{21}^{(31,32)} - 2p_{22}^{(11,13)} - p_{22}^{(31,33)} \le 1$$
$$0 \le +p_{12} + p_{22} + p_1^L - 2p_{12}^{(11,13)} + p_{12}^{(13,23)} \le 1$$
$$0 \le +p_{12} + p_{22} + p_1^L - 2p_{12}^{(11,13)} + p_{12}^{(13,23)} + p_{22}^{(31,33)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_{12} + p_{22} + p_1^L - p_{12}^{(11,13)} + p_{12}^{(13,23)} - p_{22}^{(11,13)} - p_{22}^{(33)} \le 1$$
$$0 \le +p_{12} + p_{22} + p_1^L - p_{12}^{(11,13)} + p_{12}^{(13,23)} - p_{22}^{(11,13)} \le 1$$
$$0 \le +2p_{12} - p_1^L + p_2^L - p_2^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} \le 1$$
$$0 \le +2p_{12} - p_2^R - p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \le 1$$
$$0 \le +2p_{12} - p_2^R - p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \le 1$$
$$0 \le +2p_{12} - p_{12}^{(11,13)} + p_{12}^{(13,23)} \le 1$$
$$0 \le +2p_{12} - p_{12}^{(11,13)} + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} \le 1$$
$$0 \le +2p_{12} + p_2^L - p_2^R - p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{22}^{(31,33)} \le 1$$
$$0 \le +2p_{12} + p_2^L - p_2^R - p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \le 1$$

$0 \leq +2p_{12} + p_2^L - p_{12}^{(11,13)} + p_{12}^{(13,23)} - p_{22}^{(11,13)} \leq 1$

$0 \leq +2p_{12} + p_2^L - p_{12}^{(11,13)} + p_{12}^{(13,23)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \leq 1$

$0 \leq +2p_{12} + p_1^L - 2p_{12}^{(11,13)} + p_{12}^{(13,23)} \leq 1$

$0 \leq +p_{11} - p_{12} - p_{21} - p_{22} - p_1^L + p_2^L + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \leq 1$

$0 \leq +p_{11} - p_{12} - p_{21} - p_{22} + p_2^L + p_2^R \leq 1$

$0 \leq +p_{11} - p_{12} - p_{21} - p_1^L + p_2^L - p_1^R + p_{12}^{(11,13)} + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(33)} \leq 1$

$0 \leq +p_{11} - p_{12} - p_{21} + p_2^L - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$

$0 \leq +p_{11} - p_{12} - p_{22} - p_1^L + p_2^L + p_{12}^{(11,13)} - p_{12}^{(13,23)} - p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(33)} \leq 1$

$0 \leq +p_{11} - p_{12} - p_{22} + p_2^L + p_2^R - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$

$0 \leq +p_{11} - p_{12} - p_1^L + p_2^L - p_1^R + p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \leq 1$

$0 \leq +p_{11} - p_{12} - p_1^L + p_2^L + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \leq 1$

$0 \leq +p_{11} - p_{12} + p_2^R \leq 1$

$0 \leq +p_{11} - p_{12} + p_2^L - p_1^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} \leq 1$

$0 \leq +p_{11} - p_{12} + p_{22} - p_1^L + p_2^L - p_1^R + p_{12}^{(11,13)} + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(33)} \leq 1$

$0 \leq +p_{11} - p_{12} + p_{22} - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$

$0 \leq +p_{11} - p_{12} + p_{21} - p_1^L + p_2^L + p_{12}^{(11,13)} - p_{12}^{(13,23)} - p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(33)} \leq 1$

$0 \leq +p_{11} - p_{12} + p_{21} + p_2^R - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$

$0 \leq +p_{11} - p_{12} + p_{21} + p_{22} - p_1^L + p_2^L - p_1^R + p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} \leq 1$

$0 \leq +p_{11} - p_{12} + p_{21} + p_{22} - p_1^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} \leq 1$

$0 \leq +p_{11} - p_{21} - p_1^L + p_2^L - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \leq 1$

$0 \leq +p_{11} - p_{21} - p_1^L + p_2^L - p_1^R + p_{12}^{(11,13)} + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} \leq 1$

$0 \leq +p_{11} - p_{21} - p_1^L + p_2^L - p_2^R + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \leq 1$

$0 \leq +p_{11} - p_{21} - p_1^L + p_2^L + p_{21}^{(22,21)} + p_{21}^{(31,32)} \leq 1$

$0 \leq +p_{11} - p_{21} + p_2^L - p_1^R - p_{12}^{(11,13)} + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(31,33)} \leq 1$

$0 \leq +p_{11} - p_{21} + p_2^L - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} \leq 1$

$0 \leq +p_{11} - p_{21} + p_2^L \leq 1$

$0 \leq +p_{11} - p_{21} + p_2^L + p_2^R - p_{12}^{(11,13)} + p_{12}^{(13,23)} - p_{22}^{(11,13)} \leq 1$

$0 \leq +p_{11} - p_1^L + p_2^L - p_1^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} \leq 1$

$0 \leq +p_{11} - p_1^L + p_2^L - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \leq 1$

$0 \leq +p_{11} - p_1^L + p_2^L - p_1^R + p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \leq 1$

$0 \leq +p_{11} - p_1^L + p_2^L - p_1^R + p_{12}^{(11,13)} + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \leq 1$

$0 \leq +p_{11} - p_1^L + p_2^L - p_2^R + p_{12}^{(11,13)} - p_{12}^{(13,23)} - p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \leq 1$

$0 \leq +p_{11} - p_1^L + p_2^L - p_2^R + p_{12}^{(11,13)} - p_{12}^{(13,23)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \leq 1$

$0 \leq +p_{11} - p_1^L + p_2^L - p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \leq 1$

$0 \leq +p_{11} - p_1^L + p_2^L + p_{21}^{(22,21)} + p_{21}^{(31,32)} \leq 1$

$0 \leq +p_{11} - p_1^R - p_{12}^{(11,13)} + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(31,33)} \leq 1$

$0 \leq +p_{11} - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} \leq 1$

$0 \leq +p_{11} \leq 1$

$0 \leq +p_{11} + p_2^R - p_{12}^{(11,13)} + p_{12}^{(13,23)} \leq 1$

$0 \leq +p_{11} + p_2^L - p_1^R - p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \leq 1$

$0 \leq +p_{11} + p_2^L - p_1^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} \leq 1$

$0 \leq +p_{11} + p_2^L - p_{21}^{(11,31)} + p_{21}^{(31,32)} \leq 1$

$0 \leq +p_{11} + p_2^L + p_2^R - p_{12}^{(11,13)} + p_{12}^{(13,23)} - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(31,33)} \leq 1$

$0 \leq +p_{11} + p_{21} - p_1^L + p_2^L - p_1^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} \leq 1$

$0 \leq +p_{11} + p_{21} - p_1^L + p_2^L - p_1^R + p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} \leq 1$

$0 \leq +p_{11} + p_{21} - p_1^L + p_2^L - p_2^R + p_{12}^{(11,13)} - p_{12}^{(13,23)} - p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(31,33)} \leq 1$

$0 \leq +p_{11} + p_{21} - p_1^L + p_2^L - p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \leq 1$

$0 \leq +p_{11} + p_{21} - p_1^R - p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \leq 1$

$0 \leq +p_{11} + p_{21} - p_1^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} \leq 1$

$0 \leq +p_{11} + p_{21} - p_{21}^{(11,31)} + p_{21}^{(31,32)} \leq 1$

$0 \leq +p_{11} + p_{21} + p_2^R - p_{12}^{(11,13)} + p_{12}^{(13,23)} - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} \leq 1$

$0 \leq +p_{11} + p_{12} - p_{21} - p_1^L + p_2^L - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$

$0 \leq +p_{11} + p_{12} - p_{21} + p_2^L - p_1^R - p_{12}^{(11,13)} + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(33)} \leq 1$

$0 \leq +p_{11} + p_{12} - p_{21} + p_{22} - p_1^L + p_2^L - p_2^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} \leq 1$

$0 \leq +p_{11} + p_{12} - p_{21} + p_{22} + p_2^L - p_{12}^{(11,13)} + p_{12}^{(13,23)} - p_{22}^{(11,13)} \leq 1$

$0 \leq +p_{11} + p_{12} - p_{22} - p_1^L + p_2^L - p_1^R + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} - p_{22}^{(31,33)} + p_{22}^{(33)} \leq 1$

$0 \leq +p_{11} + p_{12} - p_{22} - p_1^R - p_{12}^{(11,13)} + p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{22}^{(11,13)} + p_{22}^{(33)} \leq 1$

$0 \leq +p_{11} + p_{12} - p_1^L + p_2^L - p_1^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} \leq 1$

$0 \leq +p_{11} + p_{12} - p_1^L + p_2^L - p_2^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} \leq 1$

$0 \leq +p_{11} + p_{12} - p_{12}^{(11,13)} + p_{12}^{(13,23)} \leq 1$

$$0 \leq +p_{11} + p_{12} + p_2^L - p_1^R - p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} \leq 1$$

$$0 \leq +p_{11} + p_{12} + p_{22} - p_1^L + p_2^L - p_2^R - p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$$

$$0 \leq +p_{11} + p_{12} + p_{22} + p_2^L - p_{12}^{(11,13)} + p_{12}^{(13,23)} - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(11,13)} - p_{22}^{(33)} \leq 1$$

$$0 \leq +p_{11} + p_{12} + p_{21} - p_{22} - p_1^L + p_2^L - p_1^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} \leq 1$$

$$0 \leq +p_{11} + p_{12} + p_{21} - p_{22} - p_1^R - p_{12}^{(11,13)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(11,13)} \leq 1$$

$$0 \leq +p_{11} + p_{12} + p_{21} - p_1^L + p_2^L - p_2^R - p_{21}^{(11,31)} + p_{21}^{(22,21)} + p_{21}^{(31,32)} + p_{22}^{(31,33)} - p_{22}^{(33)} \leq 1$$

$$0 \leq +p_{11} + p_{12} + p_{21} - p_{12}^{(11,13)} + p_{12}^{(13,23)} - p_{21}^{(11,31)} + p_{21}^{(31,32)} - p_{22}^{(33)} \leq 1$$

$$0 \leq +2p_{11} - p_1^L + p_2^L - p_1^R + p_{21}^{(22,21)} + p_{21}^{(31,32)} \leq 1$$

Appendix C

Detectors' Path Behaviour in Squires Model

This table below shows the initial positions of the left detector, X_0, and right detector, Y_0, chosen randomly with normal distribution, and variance $\frac{1}{2}$. Here we have used a system of units in which the width of the wave-packet is $\frac{2}{\sqrt{2}}$, the retarded time is 1 and the speed of the particle is $\frac{1}{2}$. The behaviour of the functions $X(t)$ and $Y(t)$ are shown in each row where: 'S.I.' denotes that the function is Strictly Increasing and 'I.C.' denotes that the function increases and then goes to a constant. In these calculations the quantum spreading of the wave-function is neglected and the initial and final time of the experiment are $t_0 = 0$ and $t_f = 10$ respectively. The summarized results for the the behaviour of the functions $X(t)$ and $Y(t)$ are:

$X(t)$	$Y(t)$		
S.I.	S.I.	0	cases
S.I.	I.C.	52	cases
I.C.	S.I.	47	cases
I.C.	I.C.	1	cases
TOTAL		100	cases

No.	X_0	Y_0	$X(t)$	$Y(t)$
001	-0.6436218262115580	-0.8940680095447510	S.I.	I.C.
002	-0.0263130941205410	1.3691941532550930	I.C.	S.I.
003	-0.1948419303864674	0.9820635485649480	I.C.	S.I.
004	-0.2548707173430180	-0.4960825261674740	S.I.	I.C.
005	-1.3123546604722100	-0.9060091389585550	I.C.	S.I.
006	-1.2494982835959120	0.3058695209326853	I.C.	S.I.
007	-0.2243535541088562	0.1687909106383231	I.C.	S.I.
008	1.1553331203032960	-0.7124410549214800	S.I.	I.C.
009	-0.1319358834491993	0.7499717277023700	I.C.	S.I.
010	-0.2241248231619876	-0.1280303141218009	I.C.	S.I.
011	-0.2406905366020732	-0.3957036108520487	S.I.	I.C.
012	-0.8204823937367320	1.0266609834356590	I.C.	S.I.
013	1.0092209546966400	0.4837492624751250	S.I.	I.C.
014	-0.6166285820548160	0.0151493053756043	I.C.	S.I.
015	-0.3163992366835560	0.5964007078741950	I.C.	S.I.
016	-1.2623298073313570	1.0767032010780990	I.C.	S.I.
017	-1.0777137139179960	-0.1672899981425774	I.C.	S.I.
018	0.7163423358070430	-1.1556617814279840	S.I.	I.C.
019	0.5197429661313680	-0.4112767449362786	S.I.	I.C.
020	0.4128945938632177	0.5589802491021610	I.C.	S.I.
021	0.0100491639695946	-0.3427628490703934	S.I.	I.C.
022	-0.6224487246593830	-1.0916192236471960	S.I.	I.C.
023	1.0724802554447010	-0.3328989903057221	S.I.	I.C.
024	-0.1883365010985381	0.1534794663364975	I.C.	S.I.
025	-0.0104208919294711	-0.8883797022823270	S.I.	I.C.
026	0.9641907559854180	-0.3643062317601612	S.I.	I.C.
027	-0.1513441805503900	0.3021465844365968	I.C.	S.I.
028	-0.2252593531036281	-0.1554314449417189	I.C.	S.I.
029	0.4917481442666140	0.1915716510433858	S.I.	I.C.
030	-0.6935304804910670	0.3764161593022508	I.C.	S.I.
031	-0.0024352835964493	-1.6785982142710230	S.I.	I.C.
032	0.7479781817678050	-0.1697222927701411	S.I.	I.C.
033	-0.9792220080383430	-0.2564691576462179	I.C.	S.I.
034	-0.1983577361955491	0.1678074244650335	I.C.	S.I.
035	-0.5512882806615290	0.0863973674031563	I.C.	S.I.
036	-0.2224194563041156	0.6157964768246660	I.C.	S.I.
037	-0.8054916869520740	-1.1045836785428440	S.I.	I.C.
038	0.8113576010341230	-0.1147003301457300	S.I.	I.C.
039	0.9579046378237150	0.2020730651854815	S.I.	I.C.
040	-0.8642544247262060	0.2868733041060977	I.C.	S.I.
041	-0.0237943115256550	0.9888932767787440	I.C.	S.I.
042	0.2826916623842408	-0.8175488887866760	S.I.	I.C.
043	0.0186644684452462	0.3965367276065636	I.C.	S.I.
044	0.2875033573444871	0.1597889338522753	S.I.	I.C.
045	0.0899250428184499	0.0280648830978698	S.I.	I.C.

No.	X_0	Y_0	$X(t)$	$Y(t)$
046	-0.2248280222559676	1.0747040917595300	I.C.	S.I.
047	0.5844094237106640	-0.0981215511826060	S.I.	I.C.
048	0.0467138494053981	-0.3315250030442669	S.I.	I.C.
049	0.0420623941290123	-1.0597822529253390	S.I.	I.C.
050	0.2176122078363873	0.2366396344042972	I.C.	S.I.
051	-0.8434909483947080	-1.6336243237494120	S.I.	I.C.
052	-1.9378522460060230	-1.1228193554880520	I.C.	S.I.
053	0.5616781531690410	0.2888016277925924	S.I.	I.C.
054	0.9930571325305290	0.3440351359863918	S.I.	I.C.
055	0.7546797835621280	0.4015490837125702	S.I.	I.C.
056	0.2895021084148434	-0.5412052372111770	S.I.	I.C.
057	-0.6859622551976630	-0.1556443076001231	I.C.	S.I.
058	-0.3185173265554710	-0.8245724925036230	S.I.	I.C.
059	1.6941625396639140	-0.9002586229477280	S.I.	I.C.
060	-0.4760466730286680	-1.2526664621984760	S.I.	I.C.
061	-0.3431477551644192	-0.5412295137959190	S.I.	I.C.
062	0.7495910573666200	-0.3595763438328746	S.I.	I.C.
063	0.6854640794482640	0.1019451132795143	S.I.	I.C.
064	-0.7880497194580230	0.5149053087124390	I.C.	S.I.
065	1.2452953998624060	0.7310201443883560	S.I.	I.C.
066	0.3396167975632977	-0.3235404301078479	S.I.	I.C.
067	-0.4697934294088040	-0.8205886037949790	S.I.	I.C.
068	-0.0782324375652469	0.5363944273347490	I.C.	S.I.
069	0.8046280137323890	0.6734540820839940	S.I.	I.C.
070	0.7330999227749740	-1.0338157481451720	S.I.	I.C.
071	-1.5031434127310820	0.4833973362900590	I.C.	S.I.
072	1.2090472556817020	-0.0732933116085280	S.I.	I.C.
073	-0.3135762407702964	-0.2459557948725694	I.C.	S.I.
074	0.3269900232179190	0.2416569638384280	S.I.	I.C.
075	-0.0572916494420637	1.0015927122909560	I.C.	S.I.
076	-0.9931233030826920	-0.9928834032209680	I.C.	I.C.
077	0.6153614370543360	0.9929365082500910	I.C.	S.I.
078	-0.3361561830523410	0.8369111509925810	I.C.	S.I.
079	-0.6389967153496700	1.4474372487046570	I.C.	S.I.
080	-1.4060823394772830	0.3982890671271010	I.C.	S.I.
081	0.5395756714603520	-0.3632777222159499	S.I.	I.C.
082	-0.2929051924775828	0.6876757353238730	I.C.	S.I.
083	-0.5870144782702730	1.6188434239766420	I.C.	S.I.
084	-0.6119033302808660	-1.3216218252290880	S.I.	I.C.
085	-1.4459953207234020	0.4407603620778319	I.C.	S.I.
086	-0.3366765912643293	-0.8480131366525330	S.I.	I.C.
087	-1.3874793948941500	-0.5642053624994510	I.C.	S.I.
088	-1.2174088532514760	-0.0287382896383087	I.C.	S.I.
089	0.2645498464706661	0.0900530006505290	S.I.	I.C.
090	0.5015277725545360	-0.2106284118178593	S.I.	I.C.

No.	X_0	Y_0	$X(t)$	$Y(t)$
091	-0.3985141734449977	-0.9532426063225310	S.I.	I.C.
092	0.2701810447533582	-0.4230362635798486	S.I.	I.C.
093	1.1908692356252570	0.8748285632554070	S.I.	I.C.
094	0.0541517619115627	0.0462261155377179	S.I.	I.C.
095	0.5231068746984170	1.5363306841855300	I.C.	S.I.
096	-0.9430236814325800	-0.3118827382502801	I.C.	S.I.
097	-0.7328317002310510	-0.8085752346828280	S.I.	I.C.
098	-0.7304262952928370	-0.5520214672382240	I.C.	S.I.
099	-0.1598904340002017	0.6919784649415020	I.C.	S.I.
100	0.1841098254690473	1.0952933330192840	I.C.	S.I.

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