Market efficiency & arbitrage opportunities in the ftse-100 option market: an application on the put-call parity with high frequency data

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"Market Efficiency & Arbitrage opportunities in the FTSE-100 option market : An Application on the Put-Call Parity with High Frequency Data."

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ABSTRACT

"Market Efficiency & Arbitrage opportunities in the FTSE-100 option market : An Application on the Put-Call Parity with High Frequency Data."

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This thesis examines Put Call Parity (PCP) deviations in the LIFFE FTSE-100 Options quoting system and tests the two competing hypotheses put forward in the literature. Our dataset covers the period of July 1994 to March 1997 and contains 357,985 and 431,145 observations (for the European and the American types) resulting in 40,124 and 57,382 PCP deviations respectively.

We calculate PCP misspricings using the model proposed in Kamara and Miller (1995). The model used here accommodates market imperfections but does not include taxes. The model also allows for the immediacy risk and the early exercise risk associated with evidence of Put-Call Parity deviations documented in the literature. We find evidence of significant deviations, net of costs, throughout the period.

We test misspricings in both American and European contracts for the same period and equal contract parameters and find evidence supporting both hypotheses where appropriate. The level of deviations found suggest that other factors could attribute to their identification, we propose liquidity-related factors such as inventory constraints.

We assume that persistent deviations from the PCP, which are not supported by the option pricing theory are indications of market inefficiency. In well functioning markets we expect that larger PCP deviations will be removed from the system first. We fit a Cox Proportional Hazard model and test the significance of the level of deviations as a covariate. We find the degree of deviations to be a significant factor in the duration of the misspricings for the majority of the observations. We conclude that under these evidence the market is not characterised as inefficient.

The last part of this thesis models the PCP deviation series as a sequential stochastic process. We fit around the process the Autoregressive Conditional Duration Model, as proposed by Engle and Russell (1995) and modified by Bauwens and Giot (1997). We conclude that the model offers an adequate representation for this high frequency, irregularly spaced series.

Keywords: Put-Call Parity, High Frequency, Duration, Cox (AND) Proportional (AND) Hazards, Irregularly Spaced Observations, Autoregressive (AND) Conditional (AND) Duration (AND) Model.

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CHAPTER 1

1. INTRODUCTION

1.1 The Importance of the Economic Benefits and Risks of Derivatives.

In recent years and especially after the market turbulence of the late eighties, the financial world has become more sceptic about the general issue of market volatility. Topics regarding the economic benefits, arising from the use of derivative markets and any risks stemming from it, have become increasingly important. The fact that most derivative instruments appear to have close substitutes in the underlying markets, poses the question of whether the overall effect of the derivative markets is beneficial, ceteris paribus, or unduly adds strains to the financial system. As the academic society has struggled, in the early days, to reach a common verdict on the above question there have been increasing suggestions, from members of the financial world, calling for increased monitoring of the derivative markets and the imposition of stronger trading restrictions[1]. Any such moves however, if not fully documented, may result in the introduction of trading anomalies causing a disruption in the efficient wealth allocation.

As Merton (1992, p.263) points out "The core function of the financial system is to facilitate the allocation and development of economic resources,

[1] The majority of the empirical studies examining the imposition of additional margin requirements in order to dampen excess volatility, overwhelmingly oppose

both spatially and across time, in an uncertain environment". To fully appreciate and assess the impact of derivative markets, perhaps it would be very useful to put the above into perspective. In the absence of capital markets members of the society are required to balance earnings and spending over each period. The presence of a financial institution, which is an integral part of the financial system, will enable individuals to reach this equilibrium across time[2]. By doing so however, they introduce a multidimensional problem as market participants start to face the risk of deferring spending, or saving, into a less favourable future and having to assess the available information. Nevertheless, capital markets potentially should enable the elimination or re-allocation of uncertainty among market participants. As Gibson and Zimmermann (1994) state "In order to achieve an unconstrained Pareto-efficient allocation of these risks within a market system, capital markets must provide sufficient opportunities to trade and price the various kinds of risk." It is obvious that an integral role of financial markets is the provision of sufficiently expanded opportunity sets for investors, the efficient dissipation of information and under certain circumstances, the facilitation of a better understanding of financial markets functioning.

such measures. For a detailed analysis see Chance (1990) and also Kupiec (1991).

[2] According to Gibson and Zimmermann (1994), if we substitute individuals with firms which are able to separate, in the presence of financial markets, investments and financing decisions, it is apparent that ownership is separated from management. However, the importance of the above may be greater to the extent that this decreases the cost of capital.

It is within this framework that the presence of financial derivatives should be considered. First if the derivative markets serve the socially justified requirements of a financial system and second whether or not their informational role promotes the efficiency of the related markets. This thesis deals mainly with the last.

1.2. Option Markets.

Option markets offer substantial trading differences compared to the spot market. The absence of short sales restrictions, markedly lower transaction costs, the considerable leverage effect and the limited downside liability offer to investors new trading potential compared to the cash market. Additionally hedging requirements and arbitrage activities create a trading feedback between the two markets[3]. On the other hand the absence of suitable contracts for long-term investment horizons, position limits and different non-trading restrictions (legal etc.) make, for various kinds of investors, trading in options markets less favourable[4]. Intuitively this may have resulted in capital transfers from the underlying to the derivative market (or else) with obvious effects on the functioning, and linkage, of the two environments.

According to Black and Scholes and their seminal article on the "pricing of options and corporate liabilities" (1973), options should be treated as redundant securities; in an efficient theoretical setting the implications are that option trading initiation should have no effects, or at least no permanent effects, on the return characteristics of the underlying market. Both theoretically and empirically, however, the redundancy of options is questionable. As Gibson and Zimmermann (1994) point out completeness of

[3] Indeed Poon (1994) documents a contemporaneous (positive) relationship between spot and options trading volume.

a market requires that "...the entire set of state securities can be constructed with portfolios of existing assets." However the introduction and subsequent trading of an option contract[4], although it may appear "physically" redundant (i.e. dynamically replicated), in reality could induce significant changes in the pricing evolution of the underlying. By definition trading in option contracts will emit investors' sentiments or information otherwise unobservable, see Grossman (1988). The informational content of options is classified on the basis of the statistical inferences drawn from the relevant option contract prices, with respect to the underlying's expected returns. In accordance with the relevant literature, see O'Brien and Selby (1986), this study adopts the view that put call parity deviations could provide information on investors' expectations of future returns while implied standard deviations could reveal the market's anticipated measure of risk. Here, the properties of put call parity deviations are identified and analysed.

[4] Cox and Rubinstein (1985, chapters 2-3) present a detailed analysis of issues affecting trading in options.

1.3 Importance of Put Call Parity Deviations

The theory of Option pricing, as developed by Black and Scholes (1973) and Merton (1973), addresses an important issue in finance, viz. the pricing of contingent claims. Although it is true that option type contracts have been in use from as early as in the 16^{th} century, it was not until the beginning of the 20^{th} century when the first attempt to provide the first analytical option pricing model was made public in the form of a Sorbonne's doctoral dissertation by Luis Bachelier (1900). Subsequent research work by Sprenkle (1964), Bones (1964) and Samuelson (1965) bridged the gap between Bachelier's work and the Black and Scholes model.

During this period of evolution of the basic contingent claims pricing models, it had been apparent that there exists a pricing link between the two main types of contingent claims contracts, the put and the call options. A formal manifestation of this link came with the works of Stoll (1969) and Merton (1973) in the form of the Put-Call Parity, a simultaneous trading in a put and a call contract which theoretically should result in parity conditions.

The paper by Merton (1973) on intrinsic values of American contracts finalised a set of Put Call Parity conditions for American and European contracts. However, the advent of option pricing models and especially of the B&S model, is not a prerequisite of the Put-Call Parity model. The importance of this is that the assumptions and limitations of the B&S model do not extend to Put-Call Parity theory.

The derivation of the Put-Call Parity uses what is the equivalent of the Second Thermodynamic Law of engineering in Finance, viz. that wealth cannot be created, it merely changes forms through production methods. In engineering, nature ensures the validity of the 2^{nd} thermodynamic law under any conditions. In finance, this most important role is relegated to arbitrageurs. It is the construction of arbitrage portfolios, which is the prerequisite of Put-Call Parity. With such a strong theoretical background, Put-Call Parity forms a very strong condition in Finance; risk preferences are not important, Put-Call Parity deviations should be absent under any conditions.

In the empirical literature, however, there is a series of studies documenting significant Put-Call Parity deviations. Among them Stoll (1969), Gould and Galai (1974), Klemkosky and Resnick (1979) and more recent Finucane (1991) and Kamara and Miller (1995) have all identified statistically significant deviations.

Occurrence of Put-Call Parity deviations, other than being a potential signal of market expectations, presents opportunities for excess returns. As explained in chapter 2 a deviation of the put-call parity is based on simultaneous but opposing trading on an outright option and a synthetically constructed one. The synthetically constructed option uses a position on the underlying market plus money market instruments to replicate the contingent claim contract.

Intuitively both the synthetic and the outright option should be valued at the same price. When the relationship deviates from equality, selling the expensive and buying the cheaper of the two positions will result in a profit. As this entails no risk the required return should be the risk-free one.

In a critique on Stoll's representation of the equivalence between a put and a call, Merton (1973) has shown that the construction of Put-Call parity conditions for American contracts is not riskless as in the case of European options. As Merton points out an American option could be subject to early exercise. As the value of a plain option is given by the difference between the underlying's terminal value and the strike price, significant fluctuations of the spot price could make early exercise desirable, especially surrounding dividend announcement days. However, early exercise is not permitted in the Put-Call Parity derivation; all positions should be held until expiration of the option contract. As the trading strategies involved in the Put-Call Parity may require that the counterparty will hold an American contract, possible early exercise presents a potential risk.

Risk return trade-off suggests that American trading strategies according to Put-Call Parity will demand higher returns than the risk-free one. Among others Klemkosky and Resnick (1979) suggest this as a possible explanation for the deviations documented on American contracts.

Indeed until Kamara and Miller (1995) all empirical analysis of Put-Call Parity deviations were done using data based on American contracts. In both American and European cases, however, a successful reconstruction of the Put-Call Parity will require the simultaneous replication of the option position using the underlying market. Non-synchronicity in prices observed, or liquidity constraints, which force quoted prices to change during the construction of the option and underlying trades could make the exact replication of the option unattainable. In this case Put-Call Parity ceases to be a riskless statement and thus commands a higher than risk-free rate.

In their analysis of European options (the first on European contracts), Kamara and Miller argue that since the early exercise arguments are not applicable, liquidity problems should account for all documented deviations. Indeed by delaying trade execution by a short period, thus accounting for immediacy risks, they are able to reduce significantly the level and amount of deviations. *Although this seems to be a plausible explanation, the question remains to what extent these are market specific results and more importantly what percentage of American observed deviations are indeed due* *to liquidity, early exercise or market inefficiencies. Depending on the answer then, one could possibly gain some additional explanation on reasons behind the occurrence of European deviations, perhaps rejecting or offering further support to the liquidity explanation.*

The issue is by no means a trivial one. Deviations due to early exercise premia are well within the theoretical framework of option pricing. Severe liquidity problems, though, giving rise to substantial arbitrage opportunities could perhaps require a closer examination of the market structure, or trading rules by the exchange authorities so as to facilitate price corrections.

The simultaneous trading of both European and American contracts on the same underlying in the London International Financial Futures Exchange, offers an ideal area of research between these two proposed explanatory hypotheses. A parallel analysis of these two related markets could identify and account for both types of Put-Call Parity deviations. Indeed McMurray and Yadav (1993) and Dawson (1994) examine the early exercise premia of American contracts. In their respective analyses the first find significant overpricing of the American contracts and the second finds evidence of overall deviations which are attributed to market inefficiencies.

It is apparent from the above that the evidence presented in the literature thus far do not support a common explanatory hypothesis. From the results of previous studies it is not clear whether or not markets price correctly the early exercise risk in American put contracts and whether or not liquidity in the form of immediacy risk can be linked to PCP deviations. Subsequently and more importantly what proportion of the identified deviations remain beyond these two proposed hypotheses.

If indeed as in the case of Dawson (1994) deviations exist that are not attributed to either of these hypotheses, or to an alternative one, then this raises the issue of market inefficiencies. As it is argued in the following section this should be examined within the context of high frequency processes and requires further research. A prerequisite, however, of this is the identification of deviations using a methodology that can distinguish between cases of excess returns that are, or are not explained by the theory of option pricing or that of market microstructure.

In the current study, a substantial part of the immediacy-related deviations is removed from the sample by allowing a delayed execution time of two minutes between identification of the parity and trading. Still, using the frequency of quotes as a proxy for liquidity, see Kamara and Miller (1995), the level of deviations is compared with market liquidity.

Additionally, by using a suitable representation of the Put-Call Parity, suggested by Kamara and Miller[5], the analysis separates those deviations for American contracts, which are subject to early exercise, thus offering a direct identification of "early exercise" related Put-Call Parity deviations.

Trading on these instruments and especially on the underlying, however, incurs significant costs. Additionally market imperfections could reduce or even eliminate trading profits. In general borrowers will face different rates than lenders[6], trading on the underlying is usually subject to significant restrictions and finally buy or sell orders will differ substantially by the bid-ask spread.

Indeed all previous empirical work cited here, either acknowledge the importance of transaction costs, or in the case of Kamara and Miller (1995) explicitly account for them and demonstrate their significance in the reduction of risk-free opportunities.

As in Kamara and Miller this thesis offers results on the progressive effect of transaction costs on the level of Put-Call Parity deviations. It also presents evidence on the progressive influence of Market restrictions, and specifically

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[5] Kamara and Miller (1995) are examining exclusively European contracts. Thus they are unable to
examine the particular attribute of their model.

[6] Going short or long in any trade will require investing or financing on money markets to satisfy the arbitrage condition of no initial wealth.

1.4 The Issue of Market Efficiency

The existence of risk free arbitrage opportunities in the form of PCP deviations could indicate a form of market inefficiency if these deviations cannot be explained by theoretical arguments, or are not removed from the quoting system by market forces. Any PCP deviations that may have been identified in the first part of this study present arbitrage opportunities for the market participants. As has been suggested in the relevant literature these could be attributed either to early exercise or liquidity premia or both.

As argued above, deviations due to early exercise are acceptable according to option pricing theory. As arbitrageurs cannot construct riskless trading positions in order to exploit them, they are not required to intervene and remove these deviations. Consequently one should not question the efficiency of the option market, and indeed its link with the corresponding underlying market, on the ground of findings of "early exercise" PCP deviations.

However, liquidity premia indicate the inability of the market to absorb directly trading orders. The derivation of the Put-Call Parity requires the construction of a synthetic rate of return, which in turn consists of simultaneous positions in the option and the underlying market. It is possible that continuously updated prices and quotes for both assets will change over the duration of the trading set-up. This represents a considerable risk for arbitrageurs wishing to exploit the deviation opportunities.

Working with daily data, Kamara and Miller (1995) use the moneyness of the option contract and the volatility of the underlying security as proxies for market liquidity. The authors find that immediacy (liquidity) risk in the form of uncertain prices over the duration of the trade, account for most of the documented PCP deviations for European contracts. In their same study but using intradaily data, KM proxy for liquidity on intradaily values by allowing different times between posting of a quote and execution of the deal. Thus they account for any changes in the prices of the underlying or derivative assets. They show that delay of the execution reduces substantially the occurrence of the deviations.

Although the above will evaluate the immediacy risk as suggested by Garbade and Silber (1979) and Kamara (1988), depending on the trading system in place other factors could give rise to deviations. The quoting system in the FTSE 100 options LIFFE market consists of a number of competing market makers. Market makers will post their quotes and collectively these will form the strike by strike series of quotes available to investors. It is possible that liquidity constraints such as inventory imbalances force market makers to post quotes, which give rise to PCP deviations. In efficient settings market forces should intervene and reinstate trade equilibrium thus removing the observed deviations.

Market efficiency studies using end of period data, concentrate on whether or not the market can yield efficient prices sampling from the end of each period under consideration, consequently they offer an infrequent point analysis of the market. As it is acknowledged in Engle (1996) and Goodhart and O'Hara (1997), among other researchers, the study of similar issues in a high frequency framework will not be sufficient with the simple estimation of end of period effects. Indeed O'Hara (1995) accepts that market values will only gradually tend to their equilibrium values. As the author refers specifically to the issue of market efficiency she acknowledges that inefficiencies in the market will progressively be removed from prices. Theoretically, market microstructure models, see for example Glosten and Milgrom (1985), have allowed a sequential progression in the elimination of market inefficiencies within the context of real time, as opposed to end of period, pricing processes.

The question then becomes: "are the documented abnormal prices in high frequency markets a statement of market inefficiency?" By allowing a progressive eradication of inefficiencies, and thus accepting their occurrence in the first place, the research community has argued against the above statement. The notion of progressive eradication of inefficiencies, however, lower deviations. The level of deviations targeted, should be a function of the risk aversion of the traders and their view on the depth of the market.

To examine the relationship between the level of inefficiency and the time it spends in the system it is proposed to examine the hazard for the duration of each deviation. This will give us the probability of survival of each PCP deviation using as the single covariate the level of misspricing.

Chapter 2 of this study identifies all PCP deviations discarding those where prices for either the option or the underlying are not stable, thus it removes most of the liquidity related deviations. The majority of the remaining deviations are treated as potential market inefficiencies and it is examined whether or not they are effectively removed from the quoting mechanism given their level. The methodology employed is the Cox Proportional Hazard Model as developed by Cox (1972) and later extended by Fleming and Harrington (1991).

With respect to the relevant literature of Put-Call Parity deviations, chapter 3 adopts a new way of assessing market efficiency in the options market. It allows for a gradual learning process in the identification and removal of excess return opportunities. It explicitly makes the assumption that overall efficiency requires not the non-existence of these opportunities but their gradual elimination based on rational market factors such as the level of each deviation. To the best of our knowledge, the use of the Cox Proportional Hazard model offers the first application of transition data analysis models in the particular area of finance. The use of transition data analysis models is important because they offer an efficient analysis of the effect of particular factors on the hazard of a variable while relegating and in common parameter influences of variables not central to the analysis.

1.5 Modelling the Occurrence of Put-Call Parity Deviations.

Assuming that the PCP deviations form a process of arbitrage opportunities (which most importantly can give rise to subsequent trading), the first part of this study identifies the process and examines its significance with respect to the important issue of market efficiency in finance. *In this case, a natural research progress is to model the properties and evolution of this process. However, the methodology used should be of sufficient complexity so as to account for all the factors, which can characterise high frequency, irregularly spaced sequential processes.*

The previous section has indicated that the microstructure literature has acknowledged the role of time in the pricing formation process. Indeed among the theoretical pricing models suggested by the research community, a plethora of them assume a specific sequential pricing process where time affects the outcome through the arrival rate of information and the nature of market participants (informed, non-informed, liquidity traders), see for example Admati and Pfeiderer (1988), Diamond and Verrechia (1987) and Easly and O'Hara (1992). Additionally as O'Hara (1995) suggests such pricing processes are dependent on successive prices, as they are not Markovian. *Consequently the modelling of the PCP deviations should specifically address the sequential influence of time.*

An additional feature of the above theoretical models is the acknowledgement of areas of clusters in pricing series. Among the very few empirical examinations of intradaily high frequency data, this has been documented by Engle and Russell (1995) and Bauwens and Giot (1997). The authors report on the significant presence of serial correlation, which is typical in financial markets data. *Consequently the methodology used for the modelling of the PCP deviation series should be able to explicitly account for the serial correlation in the data. If such, it will be able to represent successfully the evolution of the PCP deviation as a sequential process in time.*

This study uses the Autoregressive Conditional Model, henceforth the ACD model, for the representation of high frequency irregularly spaced data as suggested by Engle and Russell (1995) and later modified by Bauwens and Giot (1997), to model the PCP series.

To the best of our knowledge, chapter 3 offers the first representation of Put-Call Parity deviations as a series of high frequency mode, explicitly accounting for the arrival of each deviation in the system. Additionally we offer the first application of the promising **ACD** model for irregularly spaced data in the area of derivative trading. For practitioners this proposes a clear representation of the stochastic evolution of a series of arbitrage opportunities based on the Put-Call Parity theory.

Chapter 2 sets the empirical framework for the identification of the PCP deviations, discusses the methodological issues related to option pricing theory and comments on the results. Chapter 3 introduces models of transition data analysis and presents the Cox Proportional Hazard Model as a suitable method for the impact of the PCP deviations on market efficiency. Subsequently it models each PCP deviation as an event influenced by the level of the deviation and presents and discusses the results. Chapter 4 refers briefly to theoretical models of sequential processes in market microstructure and offers an extensive discussion on the modelling aspects of the associated random arrival point processes. It introduces the Autoregressive Conditional Duration Model as a suitable model and presents and comments on the results. Specifically, it discusses the specification of the resulting models as well as the ability of the representations to account for the interdependency in the process. Chapter 5 concludes.

CHAPTER 2

Identification and Analysis of Put-Call Parity Deviations.

2 Introduction

The identification and analysis of deviations of the Put-Call Parity is of particular importance in the study of financial markets.

The Put-Call Parity statement links in a risk free relationship a European option with an identical American contract. Any deviations from this relationship which are inadequately explained by option valuation or market microstructure theory constitute cases of market inefficiencies.

Previous research on this area has identified significant deviations from the PCP conditions. Among the explanatory hypotheses put forward, these of "immediacy" and "early exercise" risk appear to receive the stronger support from the Academic community, see Gould and Galai (1976), Klemkosky and Resnick (1979) and Kamara and Miller (1995).

"Immediacy risk" attributes deviation to unstable market prices during the execution of the Put-Call Parity replication. It has been put forward in Kamara and Miller (1995) and essentially refers to the inability of traders (or arbitrageurs) to follow quotes with firm trades at the specified prices. The "early exercise" hypothesis concentrates on the study of American contracts, for which the Put-Call Parity is a boundary condition, and attributes deviations to the risk associated with the early exercise of part of the Put-Call Parity replication strategy. Two of the main advocates of the "early exercise" hypothesis are Klemkosky and Resnick (1979, 1980).

Deviations that cannot be explained by the above could possibly reveal instantaneous market inefficiencies such as inventory problems or other liquidity related market imbalances.

Whereas Klemkosky and Resnick (1979, 1980) have identified significant deviations on American type contracts, which have attributed to "early exercise" premia, Kamara and Miller (1995) offer the first analysis of European contracts, on which early exercise is not applicable. They argue that the majority of deviations are due to immediacy risks with some deviations remaining after unstable quotes have been removed from the system. In an analysis of early exercise risk premia of American contracts McMurray and Yadav (1993) offer evidence of significant premia for American over European contracts. However, in a similar study Dawson (1994) offers evidence against the early exercise hypothesis by the use of a suitable trading strategy. Dawson (1994) attributes the documented deviations to market inefficiencies.

Hence, the research findings are contradictory. Research on American markets has shown significant early exercise premia whereas European studies have resulted in a substantial degree of immediacy related deviations. Yet a study on a market which trades both contracts rejects both hypotheses in favour of market inefficiencies. To the research community this represents an interesting issue, viz. whether or not the results are market specific and consequently what are the implications for the evaluation of the market efficiency.

In the light of recent theoretical suggestions in the analysis of high frequency markets, see Goodhart and O'Hara (1997), it is proposed here that the identification of PCP deviations, which are not adequately explained by theoretical arguments, does not constitute direct evidence against efficiency. It is merely the starting point for such an analysis (i.e. of market efficiency). Essentially the research should concentrate on the evolution and not on the identification of the deviations.

The current chapter incorporates all the findings and suggestion in the Put-Call Parity literature to construct an efficient dataset containing all PCP deviations which are not explained by the theory of option pricing and thus far have constituted evidence of market inefficiencies. As such it prepares the dataset used in chapters 3 and 4.

2.1 Option Pricing Theory and Put-Call Parity Deviations.

The put call parity relationship is an important *boundary statement* in the theory of option valuation because any systematic deviations from it could reveal market expectations, which are difficult to observe otherwise. Option pricing theory, as expressed in Black and Scholes (1973) and Merton (1973b), does not require any knowledge of the expected return of the underlying security, the risk characteristics of the market participants and the total supply of assets in the market. Indeed the majority of option pricing models assume independence of option prices from the expected returns of the underlying assets. Hence, option prices may not be used as expectations predictors but only as predictors of the total variance (*as opposed to the beta risk*) of the return on the underlying. Consequently Put-Call Parity deviations are ruled out by the traditional theory of option pricing, as this is expressed in the Black and Scholes model, even when expectations among counterparties differ.

According to Samuelson (1965), however, option prices could reveal information on market expected returns. Pointing at the same direction Cox and Rubinstein (1985) suggest that, although the consensus is that expected returns do not affect directly option prices their effect could be significant, albeit a subtle one. Indeed some recent theoretical models are explicitly dependent on expectations. Lee, Rao and Auchmuty (1981) derive option prices in a CAPM framework assuming given expectations, whereas Constantinides (1978) assumes imperfect capital markets and uses *an expectations term* other than the risk free rate.

The significance then of PCP deviations as indicators of expected market returns is not unambiguous. The discussion of put call parity deviations is not confined to its strict "expectations" role however. The parity links in a riskless arbitrage relationship the price of a put with that of the equivalent call thus making obvious any misspricings due to market inefficiencies. The risk free nature of the relationship stems from the fact that the two options and the underlying asset can be combined in a synthetic instrument, thus replicating with certainty the pay-out of either of the options. Table 2.1, details the two contracts accompanied by the respective synthetic instruments. Interestingly enough, the synthetic instrument replicating, for example, the call will only contain the put and the underlying, thus enabling investors to explore any economically significant misspricings between the two option contracts. Any such profitable deviations could be observed directly thus giving rise to a suitable risk free trading strategy.

Table 2.1

Replication of an underlying security from a simultaneous position in the spot and derivatives markets with the appropriate use of a risk free borrowing and lending instrument. The underlying has no form of pay-outs during the life of the contracts and markets are assumed perfect. In the table f (S, t; E) denotes a European Put on underlying S, with maturity t and strike E. EP(t) denotes the present value of the strike.

Synthetic Instrument			Actual Instrument		
Position	Pay-off		Pay-off		Position
-f(S, t; E) -S+EP(t)	E > S	E - S	E – S	E > S	Put
-f(S, t; E) -S+EP(t)	E < S	0	0	E < S	Put
-g(S, t; E) +S-EP(t)	E > S	0	0	E > S	Call
-g(S, t; E) +S-EP(t)	E < S	S - E	S – E	E < S	Call

Indeed, as long as trading over a PCP strategy is risk free, deviations should be attributed to market inefficiencies. In a high frequency context though these deviations could be acceptable but they should be short-lived. To see why, we should note the following. It is true that market completeness will enable the replication of any additional security introduced. It is also true that this holds in equilibrium. In disequilibrium, however, the replication strategy will not yield equilibrium prices. Existing securities will indeed span the same space (based on their pay-out matrix) but replication prices will not necessarily reflect those for which market clears and consequently the theoretically derived prices. Assuming that vector $\theta = (\theta_1, \theta_2, \dots \theta_n)$ denotes the trading strategy over the set n of securities, the market is clear when $\sum_{1}^{I} \theta_n^i = 0$, with i = 1, ...I denoting individual traders. When the condition does not hold traders are not willing to purchase a certain security for certain reasons thus giving rise to market disequilibrium. In efficient markets it is assumed that disequilibrium periods are short-lived and are always followed by price correction processes, see O'Hara (1995).

In a dynamic context then, risk-free[1] PCP deviations will reveal market constraints during disequilibrium periods. It is over these conditions that the current study is of particular importance. Under these conditions, deviations will reveal momentarily formed expectations or instantaneous market imbalances

[1] These deviations refer to parities excluding early exercise and immediacy risks.

such as liquidity, inventory problems etc. The identification and study of these deviations, from the moment they are formed until they leave the quoting system, can reveal important information with respond to the market functioning and particularly the price adjustment mechanisms.

2.2 Theoretical Analysis of Put-Call Parity

It is commonly believed that the price of a call option can be uniquely determined from the corresponding theoretical value of the put. This equality is expressed in the put - call parity theorem, first introduced by Stoll (1969). Stoll has shown that given two suitable trading strategies, one can express the value of a put as a function of an identical call, the risk free rate and the value of the underlying asset as follows[2]:

$$f(S, t; E) = g(S, t; E) + S - EP(t) \qquad \text{Eq 2.1}$$

where

f(S, t; E) is the price of a put with a strike price of E, t time periods to expiration written on asset with a value of S at the time of contract formation,

g(S, t; E) is the corresponding price of the call and

EP(t) is the exercise price today in a world of certain and equal borrowing and lending rates.

To see that Eq. 2.1 holds construct two portfolios. For the first strategy buy a unit of the underlying asset, one put and write one call. Construct the second portfolio by shorting the underlying asset, write one put and purchase a call. To finance the first strategy, borrow at the prevailing rate and invest the proceeds of the second strategy at the same rate. It is easily shown that both of the strategies will have a zero yield at expiration and consequently, in order to avoid riskless profits, both should have zero initial cost.

In his analysis of the implications of institutional restrictions on the theoretical validity of the put call parity relationship, Stoll (1969) identifies three major sources of deviation. He treats transaction costs, in the form of trading spreads as well as borrowing and lending costs, as the most important market friction sources. He also identifies short sale restrictions and tax effects as further causes of potential parity deviations and produces a band of admissible put call equilibria points. He acknowledges the fact, however, that the whole issue of tax effects is not clear even if tax levies are completely known; tax payments are constrained on individual tax positions and these can differ widely between similar cases, ceteris paribus.

Gould and Galai (1974), however adopt the opposing view that the treatment of tax effects need not be so complicated. By suggesting that a suitable investment in the risk free asset will dominate a "put call parity portfolio"[3] they put forward a parity relationship, in a tax inclusive world, where tax rates are not present. They argue that this is intuitively acceptable because tax effects are already included in the price of the opportunity cost. Transaction costs, however, do enter the final model. The authors suggest that the effect would be to increase the upper and decrease the lower bound of the put call parity, with changes being more significant for non institutional members.

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[2] Stoll assumes frictionless markets and pay-out protected options.

In an indirect criticism of Stoll's paper, Merton (1973a) has shown that the adoption of the Black and Scholes model for pricing American options can be inappropriate as it involves terminal boundary conditions. For the same reason the original put call parity analysis, as suggested by Stoll (1969), is inadequate as it does not examine the possibility of an early exercise, which will violate the risk free condition of the hedging strategies.

For an American call, early exercise cannot be ruled out with certainty unless the level of dividends paid throughout the duration of the contract do not exceed its intrinsic value. Consequently for a payout protected American contract early exercise will never be optimal[4]. On the contrary, for an American call facing dividend payments Merton (1973a) has shown that early exercise should be optimal whenever the present value of the future sum of dividends is greater or equal than the present value of the exercise price discounted at the risk free rate. Formally to rule out, with certainty, early exercise:

$$E > \sum_{\tau=0}^{\tau} \frac{d(t)P(\tau - t)}{1 - P(\tau)} \qquad \text{Eq 2.2}$$

Where

E denotes the exercise price,

d(t) denotes dividend payment on t,

P denotes present value and

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[3] The term suggests a similar trading strategy as the one used in the derivation of the put call parity relationship, i.e. long in the asset and the put and short in the call.

τ denotes maturity.

Eq. 2.2 implies that when the dividend payments and dates are known and obey the above inequality, the use of an American call contract in the formation of a "put call parity portfolio" should not violate the notion of a riskless hedging strategy.

Early exercise for a put, however, cannot be ruled out with certainty, even if it is pay-out protected. This limitation invalidates the put call parity for an American option. Merton (1973a) has shown that the possibility of early exercise, meaning that the "put call parity portfolio" cannot be fully described by its two terminal states, prevents the inequality from holding.

Intuitively the argument goes as follow. Essentially the put call parity requires that two different hedging strategies be set up, resulting in the same pay out states on maturity. Both of these make use of either a put or a call contract. For a totally riskless hedge it is assumed that both are held to expiration. A possible early exercise of the put[5], however, will introduce a potential risk for the hedger as it will expose him to unfavourable market prices (but favourable prices for his trading partner). It is obvious that a risk free position cannot be set up with certainty thus violating the put call equality.

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[4] For a proof see Merton (1973,bell Theorems 1 and 2, pg 144).

[5] As Merton has shown it is possible for some small enough prices of S that the put will be worth more than the corresponding value of the synthesised portfolio (which is a European put) which in effect reduces to theorem 8.13 , Merton (1992, Chapter 8, pg 281) for the relative prices of two identical American and European puts.

In contrast to the PCP Parity for European contracts then, the following set of bounding inequalities prevails:

$$G(S, \tau, E) \leq E - S + F(S, \tau, E) \qquad \text{Eq 2.3}$$

Where

$G(S, \tau, E)$ is an American call on underlying S, with expiration E and time to maturity τ and $F(S, \tau, E)$ is the identical American put.

We should note that eq. 2.3 holds for American options on stocks paying no dividends. Klemkosky and Resnick (1979) have extended the bounding parity to include unprotected American contracts. For an asset paying a known non stochastic dividend, D, during the life of the hedging portfolio it holds (in order to prevent dominance of the call or put) that, (from Klemkosky and Resnick (1979)),:

$$F(S, \tau, E) \leq G(S, \tau, E) + S - [E + D\,TV(\tau_d)]\,P(\tau) \qquad \text{Eq 2.4 and}$$

$$G(S, \tau, E) \leq F(S, \tau, E) - S + [E + D\,TV(\tau_d)]\,P(\tau) \qquad \text{Eq 2.5}$$

where

TV denotes terminal values

T_d denotes the time of the dividend payment and

τ_d denotes the time to maturity after the dividend payment for a single dividend payment. For n multiple dividend payments eq 2.4 and 2.5 are modified to

$$F(S, \tau, E) \leq G(S, \tau, E) + S - [E + \sum DTV(\tau_{dj})]\, P(\tau) \qquad \text{Eq 2.6}$$

$$G(S, \tau, E) \leq F(S, \tau, E) - S + [E + \sum DTV(\tau_{dj})]\, P(\tau) \qquad \text{Eq 2.7}$$

where

$$\sum DTV(\tau_{dj}) \qquad \text{Eq 2.8}$$

denotes the sum over the dividend payments.

Eq 2.6 has been modified by Roll (1977) to accommodate different ex-dividend and payment dates.

Klemkosky and Resnick (1979) also produce an initial boundary condition which must hold in order to ensure no early exercise of the American put at the inception of the "put call parity portfolio". This obtains straight from inequality (2.7) by observing that :

$$F(S, \tau, E) - S + [E + \sum DTV(\tau_{dj})]\, P(\tau) < E - S \qquad \text{Eq 2.9},$$

if

$$F(S, \tau, E) < [E(TV-1) - \sum DTV(\tau_{dj})]\, P(\tau) \qquad \text{Eq 2.10}$$

Hence violation of equation 2.10 indicates early exercise of the put contract and termination of the arbitrage strategy. Condition 2.9, though, ensures that the hedger is insured against early exercise of the put only at the inception. For certainty during the life of the contracts, condition 2.9 must be violated at every instance, thus making the put call parity relationship for American contracts, as expressed by Stoll (1969) subject to a degree of risk. Consequently Klemkosky and Resnick (1979) argue that condition 2.10 would imply more often and greater violations than condition 2.9 due to the higher risk associated with the hedged position.

2.3 Empirical Results

Empirically the analysis of put call parity starts with Stoll (1969) and Gould and Galai (1974). Kruizenga (1964) also analyses put and call price differentials for the period of 1946 to 1956, his study, however, does not refer explicitly to the put call parity relationship.

Stoll investigates the OTC market for some 125 companies listed in the NYSE. Variably[6] the sample includes prices from the beginning of 1966 to the end of 1967. The author tests whether the implied put call parity slope approximates the theoretically suggested unit tangent. On average the results lend a strong support to the theoretical model. However Stoll acknowledges the potential problems arising from thinly traded contracts and significant non-synchronous trading between the underlying and derivative markets.

In a related article Gould and Galai (1974) examine data from a subset of companies from the beginning of 1967 to the end of 1969 and find statistically significant violations of the theoretical model. Further analysis, however, suggests that market imperfections, such as transaction and information costs, will tend to wipe out most of the arbitrage opportunities for non-institutional investors but still leave intact roughly twenty five per cent of the cases for market members.

[6] Only fifteen companies cover the whole period, the rest cover only relatively short subperiods during which they have shown significant trading activity.

Based on their theoretical analysis outlined above, Klemkosky and Resnick (1979) examine put and call contracts, not violating boundary condition 2.9, taken from the CBOE and the American and the Philadelphia Stock Exchanges for the period of July 1977 to June 1978. The analysis indicates statistically significant violations for fifty five per cent of the short positions (synthetic call positions) and forty per cent of the short (synthetically constructed put) positions. Their results also indicate higher profits for the short positions lending support to the higher risk hypothesis associated with the constructed put strategies. The authors also test the relative importance of the moneyness, dividend level and put or call overpriceness on the profitability of a long hedge. From their results it is evident that the PCP deviations identified are attributed to early exercise premia. After accounting for the early exercise the model is in accordance with the put call parity yielding no coefficients significantly different from the theoretically hypothesised values.

Finucane (1991) tests the S&P 100 OEX contracts for put call parity violations during the December 2, 1985 to November 30, 1988 period. His results reveal that there are significant parity violations. He tests and finds that options contain information for future stock returns and that indeed, option markets lead the corresponding underlying by 15 mins.

Finucane defines a put call parity deviation as

$$D = g(S, t; E)_b - f(S, t; E)_a + EP(t) - I + Div(p, t) \qquad \text{Eq 2.11}$$

where

I denotes the level of the underlying index and

Div(p, t) denotes the total dividend payment throughout the life of the contract, expressed here as a function of the dividend rate and the time of payment.

In an attempt to account for changing prices over the duration of the Put-Call trade, Klemkosky and Resnick (1980) analyse ex ante put and call contracts listed at the Chicago Board of Options Exchange, the American and the Philadelphia Stock Exchanges for the period of July 1977 to June 1978. Following Klemkosky and Resnick (1979) they examine the robustness of their previous results in an ex ante framework. The analysis of the ex post profitable long hedges (replication of call contracts for which early exercise can be ruled out with certainty) shows that "ex ante" profits are significantly lower than those implied by the ex post analysis. In their study they allow for two different time intervals between price corrections, viz. five and fifteen minutes and find that in most cases the profitability level is sensitive to the time elapsed between price changes. A cross sectional analysis of the results on the total value of the dividends and the time to expiration reveals no significant coefficients. However, a further study of the short positions (which replicate a put contract and where early exercise cannot be ruled out with certainty) reveals less profitable opportunities but a higher amount of profits in comparison to long hedges. A fact, which according to the authors is expected since the short position contains a higher degree of risk.

As noted above, a potential problem arising in the empirical analysis of the put call parity could be the improper use of American type contracts. Kamara and Miller Jr. (1995) (KM hereafter) realise the problems arising from the early exercise clause and offer an analysis of European put-call parity deviations on index contracts. Their findings suggest less frequent and smaller violations than in studies where American contracts are used. The authors attribute the deviations to premia for liquidity (immediacy) risk, i.e. the inability of the arbitrageur to construct the trading strategies at the observed prices due to changing conditions. They subsequently allow for the immediacy risk by considering a delayed execution time for the trading strategies and find that most of the deviations disappear. KM use a model which allows for transaction costs, bid-ask spreads and dividend payments hence they use an augmented empirical model.

McMurray and Yadav (1993) offer the first simultaneous analysis for American and European contracts on the same underlying. They analyse the FTSE100 options traded at LIFFE. Although they use the same dataset as we use in this study, they cover the period of October 1991 to October 1992 and most importantly they sample hourly observations as opposed to real time strike by strike quotes. The authors analyse the risk premia associated with American trading and compare three estimators, an implied risk premia estimator, risk premia derived from the theoretical Put Call Parity condition and actual risk premia between a quoted American and a European contract. It is evident from this study that far in-the-money contracts command significant risk premia which reduce as these move to out-of-the-money ratios. We should note here that a direct comparison between American and European contracts on the FTSE100 is difficult as these are quoted for strikes differing 25 index points. Although the authors offer an adjustment for this difference they acknowledge the need for a more efficient analysis.

In an analysis of the FTSE-100 Option Market covering the period of July 1, 1992 to November 12, 1992 Dawson (1994) examines the comparative pricing of American and European options. He constructs Put-Call Parity relationships between an American put and the closest identical European call (again the contracts differ by 25 index points) and examines the premia between the two contract types. The author tests parities for cases where early exercise is applicable and compares them with cases where early exercise is not an issue. He finds that in every case the American contract is overpriced. Dawson concludes that the resulting deviations represent documented market inefficiencies.

Previous research work on PCP deviations, then, identifies, in all instances significant misspricings. In some cases and especially in earlier studies, the deviations are attributed to thin and non-synchronous trading (Stoll, 1969), or market imperfections such as transaction and information costs, see Gould and Galai (1974) and Finucane (1991). Kamara and Miller (1995), however, after controlling for market imperfections attribute the deviations observed to the difficulty associated with the successful implementation of the trading strategy (of both the option and the hedging legs) at the prevailing prices after the arbitrage opportunity has been identified (immediacy risk). Klemkosky and Resnick (1979, 1980) attribute most of the misspricings to the increased risk faced by investors which relates to the possibility of early exercise of the shorted put contract (early exercise risk). Dawson (1994) documents risk-free deviations, which he attributes to market inefficiencies.

Summarising, the research evidence, thus far, suggests that the PCP is often violated, leading to significant arbitrage opportunities. Two competing explanatory hypotheses have been put forward. According to the first one (henceforth the early exercise hypothesis) deviations exist as a result of the potential risk of early exercise of the American contracts. Due to the nature of the American contract the risk associated with early exercise is more profound in the case of an American put which is in accordance to the empirical findings.

The second explanation, proposed by Kamara and Miller (1995) (henceforth the liquidity hypothesis) uses liquidity and the difficulty of replicating the option contracts as reasons behind PCP deviations. As Kamara and Miller investigate only European contracts this seems a plausible explanation for deviations in European contracts. However, even after controlling for the immediacy risk some deviations exist in their sample. What remains to be seen is whether or not these are market specific findings and, more importantly, to what extend American deviations can be attributed to the liquidity risk and not to the early exercise hypothesis. We will try to provide answers to these questions by performing a comparable analysis between American and European contracts.

The current study accounts for most of the market imperfections observed in the market such as transaction costs, borrowing and lending rates and short sales restrictions. Consequently it tests whether or not profitable trading opportunities exist in the FTSE-100 market after costs. Both American and European contracts are tested on the same market. To avoid problems arising from the existence of different strike prices and to provide clearer comparisons between liquidity and the level of PCP deviations the PCP replication models are applied separately on European or American contracts. In correspondence to Kamara and Miller (1995) the models applied account for the immediacy risk by ensuring stable prices during the replication period. The sample includes only these quotes that remain alive for at least 2 minutes and correspond to periods during which the index changes by no more than 0.5%. Thus it offers stable conditions for a minimum of 2 minutes for the construction of the trading strategies. To examine the significance of the early exercise explanatory hypothesis, the PCP replication controls for the risk of early exercise of the call position held by the counterparty by observing equations 2.6 and 2.7. Additionally it excludes early exercise of the American put at initiation of the trade by observing equation 2.9 but does not control for the early exercise of the American put *during* the life of the contract (i.e. does not test for violations of 2.9 at intermediate points). This corresponds to the exact conditions faced by arbitrageurs when intervening to exploit the documented deviations. If arbitrageurs are uncertain with respect to the early exercise of the trade by the counterparty, they will demand higher returns thus violating the risk-free nature of the Parity. Deviations subject to early exercise risk are the American deviations type two.

The following section describes the data set used and gives a detailed analysis of the methodology, problems and solutions associated with it.

2.4 Data.

This study investigates the put call parity deviations of the FTSE100 options contracts traded at the London International Financial Futures Exchange. The sample covers the period of July 4, 1994 to February 28, 1997. In total examines 357,985 and 431,145 observations (for the European and the American types) resulting in 40,124 and 57,382 PCP deviations respectively.

The data set covers actual strike by strike transactions and all the day's quotes, as these are fed into the trading system of the options exchange. The complete set of data as supplied by LIFFE includes the exact time of the transaction or the quote, the contract code, the expiration date, the strike price, the type of the contract, whether call or put, bid and ask quotes, or the transaction price and the corresponding value of the underlying. The quotes correspond to offers for trade prices advertised by market makers when faced by a trade request from brokers.

Actual trading prices are not used because their number is prohibitively small for a high frequency analysis. We should note that at any given time period the exchange trades up to sixty different puts and a similar number of call contracts. The exchange will trade contracts with delivery months: March, June, September and December and additionally any other month so that always the three nearest calendar months are available for trading for European contracts. Equally for American contracts, the three nearest months plus June and December. In general, it will also make available for trading eight to ten exercise prices with four as the theoretical minimum , two lower and two higher. It is apparent then, that a study seeking to "match" identical and recently traded contracts should draw observations from a high frequency data domain. Due to the considerably infrequent nature of transactions data, and for the purposes of this study, strike by strike quotes are treated as market prices. Although this represents a potential threat of "non real" market conditions when the results are assessed, it does have some fortunate implications. High frequency data require a careful econometric and intuitive analysis with respect to the information they convey. According to Goodhart and O'Hara (1997) connected prices (i.e. high frequency strike by strike as opposed to end of period) will not follow a Markov process but will rather depend on their position in time with respect to the rest of the prices. This property makes necessary the analysis of individual prices in conjunction with their previous history. Assuming that prices form a martingale, and consequently are semi-strong efficient, differences of prices should be uncorrelated. It is somehow difficult, though, to account for the so-called "bid-ask bounce", essentially a pattern in trade prices directly affected by the nature of trade, i.e. buy or sell. As the percentage of the bid-ask spread is highly significant compared to prices for option trading, see McMurray and Yadav (1993), the use of transaction prices could bias the results. Quotes on the other hand, supplied to the system require less strong conditions; as opposed to trades that require both parties to act, quotes are updated only by the market maker. Goodhart and O'Hara (1997) suggest that quotes constitute data of better quality as they exhibit less bias.

Additionally, LIFFE operates an open outcry trading system, which applies to the members of the exchange. The consequence is that the contract premiums reflect the prevailing market prices, which are determined through direct competition among the market participants. According to the Exchange "...the premium for a particular option at any given time is a reflection at that moment of supply and demand for the option." (LIFFE (1996), pg. 8). It is assumed here that due to the competitive nature of the exchange the quotes offered by market makers reflect market conditions. Should these quotes represent parity deviations driven by inventory imbalances this would reflect inefficiency on the part of the individual market maker and should be short lived as market forces prevail. Given that these inventory imbalances continue over a longer term, market inefficiencies should be present; the market clearly fails in its economic role to facilitate the efficient allocation of resources both in space and time.

The calculation of dividends follows the suggestion in Harvey and Whaley (1991). Actual dividends are assumed to proxy expected values. Actual dividends are calculated explicitly in the form of dividend payments, using Datastream, as opposed to average yields. Section 2.5 explains in detail the exact methodology.

The dataset as provided by LIFFE lists time stamped values for the underlying index. The PCP models employed here require the actual bid and ask values for the index. These are constructed prices. The bid ask spread calculations are based on the actual values of closing bid ask daily prices of the index constituents using the methodology suggested by Harris, Sofianos and Shapiro (1994). The data are collected from Datastream International. Borrowing and lending rates come from the appropriate bid and ask T-Bill 3 month rates, both are collected daily from Datastream. The transaction costs structure used in the models are supplied by Societe Generale Strauss Turnbull Securities Limited, LIFFE and the London Stock Exchange, section 2.5 describes in detail the methodology used in the calculation of the transaction costs.

2.5 Methodology.

2.5.1 European Contracts

The present section describes the models used to identify deviations from the put call parity on European options allowing for transaction costs, differences in borrowing and lending rates and dividend payments. The model used follows directly from Kamara and Miller (1995) and is an extension of Stoll's Put-Call Parity model, which is repeated below.

$$f(S, t; E) = g(S, t; E) + S - EP(t) \qquad \text{(Eq 2.1)}$$

where

f(S, t; E) is the price of a put with a strike price of E, t time periods to expiration written on asset with a value of S at the time of contract formation,

g(S, t; E) is the corresponding price of the call and

EP(t) is the exercise price today in a world of certain and equal borrowing and lending rates.

Kamara and Miller's model uses the same intuition of the put call parity relationship, eq 2.1. However, it extends the original work as it takes into account the transaction costs involved in buying or selling the three assets used in the portfolios formation and by allowing for a variable fraction of the short sales proceeds to be reinvested

Assuming that $f(S, t; E)_b$ and $f(S, t; E)_a$ are the current bid and ask prices of the European put option on one share S with a strike of E at time t;

$g(S, t; E)_b$ and $g(S, t; E)_a$ are the corresponding bid and ask prices for the call;

S_b and S_a are the present bid and ask prices for the underlying S;

S_t is the terminal value of the underlying;

Div is the present value of the underlying's dividend during the life of the contract;

λ_f, λ_g, λ_s are the transaction costs involved when buying or selling the put, call or underlying, η_S represents the part of the underlying's short sale proceedings available to the investor, $P_\lambda(t)^B$ and $P_\lambda(t)^L$ are the riskless borrowing and lending rates (or risk free market rates) net of transaction costs.

We derive the Put Call parity relationships by setting two similar trading strategies. The first involves buying the underlying, the put and writing the call, financing the transactions by borrowing at the market rate $P_\lambda(t)^B$ and achieving a yield of X on expiration. So

$$X - [\,(S_a - Div + \lambda_s) + (f(S, t; E)_a + \lambda_f) - (g(S, t; E)_b - \lambda_g)\,]\,(1+ P_\lambda(t)^B) \leq 0$$

Eq 2.12

In a similar way the second strategy involves shorting the underlying, writing the put and buying the call. With the cash flow at the end of period being exactly the opposite it gives

$$[\,\eta_S(S_b - Div) - \lambda_s) + (f(S, t; E)_b - \lambda_f) - (g(S, t; E)_a + \lambda_g)\,]\,(1+ P_\lambda(t)^L) - X \geq 0$$

Eq 2.13

Defining R_L and R_B, the constructed (synthetic) risk free lending and borrowing rates, as

$$R_L \equiv \{X / [(S_a - Div + \lambda_s) + (f(S, t; E)_a + \lambda_f) - (g(S, t; E)_b - \lambda_g]\} - 1,$$

Eq 2.14

and

$$R_B \equiv \{X / [\eta_S(S_b - Div) - \lambda_s) + (f(S, t; E)_b - \lambda_f) - (g(S, t; E)_a + \lambda_g)]\} - 1.$$

Eq 2.15

The put call parity can be expressed as

$$P_\lambda(t)^B - R_L \geq 0 \qquad \text{Eq 2.16}$$

$$R_B - P_\lambda(t)^L \geq 0 \qquad \text{Eq 2.17}$$

with the economic significance being that in no case at all should the market (or constructed) risk free borrowing rate be lower than the constructed (or market) risk free lending rate. Violations of boundary conditions 2.16 and 2.17 imply that investors are able to raise "cheap" capital in money markets while at the same time lend at a higher rate in the options markets without facing any risks (at least in European option markets). Equivalently they would use the options markets to raise capital at a cost lower than the cost of money.

Calculation of Dividends

In the analysis by Figlewski (1984) of the components of the basis risk for S&P500 Futures contracts, it is assumed that the ex-ante information on the dividend pay-out differs insignificantly from the actual dividends paid. Indeed throughout the study, realised dividends are used as dividend expectations for the duration of the contracts.

The exact level of dividends paid for the period of investigation is calculated. The time series contains actual payments and not the implied dividend yield over the given period, in accordance with suggestions in Harvey and Whaley (1991). This enables a more efficient calculation of the early exercise possibilities for American contracts in comparison to calculations using the dividend yield instead.

Transaction costs.

Transaction costs include both transaction and clearing fees with all costs incurred by the investor above quoted prices. The calculation of fees presented here, especially those applicable to option trading, assumes that market members engage in substantial trading in order to participate to the various volume related costs reduction schemes offered by the exchanges.

Option trading houses have faced a revised and considerably simpler transaction fees pricing schedule since February 1995. The older scheme was "charging" trades with 24 pence per lot, per side. Various "stepwise" discounts were available, linked to the trading volume figures achieved by the houses and the exact trading strategies used. The choice of the correct figure applicable to a trading strategy given by 2.12 or 2.13 is rather arbitrary. Due to the variable discounts one can only guess a suitable value representative of the average exchange member. The flat transaction and clearing fees were set to 30 pence per lot, per side. Discounts were calculated on that figure. With the new pricing schedule fees are being reduced to 24 pence per lot, per side. However, a single rebate scheme was introduced giving a reduction of 10 pence per lot, per side, for each lot traded over and above the relevant threshold number of lots per contract per month. For FTSE-100 contracts this stands at 8000 lots and is calculated on the aggregate of European and American contracts. We assume that a representative exchange member could easily satisfy the rebate criteria of the new scheme, hence a combined transaction and clearing fees figure of 14 pence per lot, per side was used for the relevant period.

Since 1986 the London Stock Exchange has replaced open outcry trading with all prices being quoted on screen by market makers. Based on market conditions and on their own positions market makers quote a "fair" price and according to supply and demand trading will take place. As far as underlying trading is concerned the following trading costs evaluation applies to market makers and not to individual investors or brokers, the last will normally face additional trading fees.

Transaction costs on underlying trading at London Stock Exchange involve three separate charge categories for the period of investigation. Starting from June 1996 the Stock Exchange has replaced the existing trading costs structure with the "CREST" system. The change was gradual involving batches of 24 shares at a time, with every batch containing no more than two FTSE 100 shares. We expect that the changes had not affected our results.

Each trade put forward will be subject to the settlement fee, which further includes the bargain input charge *ad valorum,* and the exchange bargain charge[7]. Trading according to eqs 2.12 or 2.13 requires replication of the FTSE100 index up to the value of one option FTSE100 contract (American or European) traded at LIFFE. For the period of investigation each option contract of this kind had a value of ten Sterling Pounds per index point. Hence the correct hedging, assuming an index value of 4000, would require trading in the index constituents up to a total value of

[7] The Stock Exchange classifies trading costs into separate categories to comply with the differences in the purpose of charge. Exchange Bargain Charges reflect fixed costs such as operating an adequate regulatory body, within the exchange, responsible for fair and safe trading and in general for the upkeep of the Exchange. The Settlement Cost, which includes the Bargain Input Charge, will cover the costs involved with the actual trade.

$$40000 = \sum_{i=1}^{100} 40000 * w_i \qquad (18)$$

where

w_i is the weight of security i used in the calculation of the index and

i = 1, 2, ...100.

The above replication strategy requires, according to LSE terms and regulations, one hundred separate bargain deals. These will entail a settlement fee of 50 pence per trade up to 1000 shares which includes a further bargain input charge of 25 pence per transaction and an exchange bargain charge of 15 pence per 1000 shares trade , with a ceiling of 66,666.67 and a *minimum* of 25 pence per transaction. Consequently replication of the FTSE100 index will incur total transaction fees of 100 Sterling Pounds.

From the above analysis is evident that the total value of transaction costs, per share, involved in the underlying trading is inversely proportional (or "stepwise" inversely proportional) to the number of put-call parity strategies set up. Hence the value of transaction costs used will, if at all, bias the results towards rejection of put call parity deviations as it should be expected that arbitrageurs will tend to trade on values larger than the value of one option contract. Table 2.2 below, presents an analysis of the level and nature of transaction costs used. Also, for ease of reference with previous research work and to gain some knowledge on the effect of the level of transaction costs to the magnitude of PCP deviations, in section 2.5 we present results for a series of transaction costs levels.

Table 2.2

Transaction costs involved in the replication of a put call parity trading strategy on one FTSE100 option contract. (Source: LSE)

Trade	Value of Trade	Transaction Fees per Trade	Total Transaction Fees per Trade	Total Transaction Costs
Put on one FTSE100	£10 * Index Points	15 pence	15 pence	15 pence
Call on one FTSE100	£10 * Index Points	15 pence	15 pence	15 pence
Underlying FTSE100	$\sum_{i=1}^{100} 40000 * w_i$ = £10 * Index Points	Settlement fees @ 50 pence per lot &	5000 pence	**10000 pence**
		Bargain fees @ 25 pence per lot	2500 pence	
		Exchange fees @ 25 pence minimum per lot	2500 pence	

Short Sales Restrictions.

From equation 2.13 it can be seen that construction of the synthetic borrowing rate requires finance employing the proceeds from short sales. In the case of the underlying market, however, this is not possible without the imposition of some restrictions.

For trading in the LSE the exchange allows short sales without the imposition of any restrictions for a maximum of 25 days. Deals extending beyond that time horizon are not supported or permitted by the Exchange. These will affect all the investment strategies involving options with longer than 25 days to maturity.

Investors wishing to have a longer delivery could, perhaps, either go to the over-the-counter market or consider borrowing the stock[8]. Borrowing will resemble somehow short selling; the investor holds the stock and can finance its position by selling it but at the same time will have to deliver on expiration the title to the borrower. However he would have to finance the purchase and subsequent upkeeping of the loan. In general he would have to deposit with the borrower the initial value of the loan and maintain this position for the duration of the contract. In turn the borrower should transfer back to the investor a part of the proceeds from investing the premium. We assume that these costs will be marginally below the risk free ask rate plus the associated risk with the daily up-

[8] For some stocks it can prove to be extremely difficult to achieve a loan of "low" value. Most borrowers tend to engage in substantial value deals. Consequently the higher the value of the

keep of the position. Although we will offer indicative boundary values for different assumptions on short sales restrictions we treat the resulting type two deviations as containing the above upkeep risk.

We set up trading strategies using short sales proceeds corresponding to 100, 97, 95 and 90 per cent of the value of the shares as well as the market prevailing borrowing rates.

2.5.2 American Contracts.

We treat American contracts in similar way to European ones but we test for the possibility of early exercise. We exclude from the sample contracts that violate condition 2.10. Non violation of this initial boundary condition will ensure that the trading strategy (one) requiring shorting the call will not be subject to any risk of early exercise.

We should note that condition 2.10 does not protect the investor who follows trading strategy two. This will leave the investor exposed to early exercise of the put (the shorted contract) thus entailing the risk of imperfect hedge. To ensure a perfect hedge Klemkosky and Resnick (1979) have proposed boundary condition 2.8. This will ensure against early exercise of the put at inception but it has to be tested at intermediate points (at every instance). We test for violation of condition 2.8 at the inception of the strategy. However no further testing is

PCP deal the easier it is to ensure the loan. However following the introduction of the new "CREST" system the Exchange expects that trading in smaller deals will be facilitated.

carried on through the life of the contracts. This will leave American strategies open to early exercise of the put contract. According to the early exercise hypothesis this will demand excess returns. Consequently for the validity of the early exercise hypothesis we require a statistically higher excess return for American deviation 2 contracts. Thus by leaving strategy two unprotected, we provide an efficient way (otherwise a continuous validation of Eq 2.18 is required, this will impose a substantial computational load) to control for the importance of the early exercise hypothesis.

Calculation of Dividends

Due to the nature of the underlying asset, which in our case is the arithmetic weight of 100 share components, the actual dividend paid is the linear summation of the constituents with coefficients the relative weights of the companies. Consequently the calculation of the dividend time series should involve the actual dividend payments, and the dates, of the index components. In an index, however, as broad as the FTSE100, where the calculation of the discrete dividend series is a substantial task, one may rely on a quarterly or even annualised yield.

Past studies in the broad area of finance, and specifically in the analysis of options contracts, see Harvey and Whaley (1992), have used an average yield over a certain time period. This may be a valid approximation for option contracts characterised by terminal boundary conditions, such as European style but need not be the case for contracts where early exercise, at any point or during specific points in time, is allowed. Such options will explicitly "demand" early exercise should the cash flow generated (which is directly linked to the amount of dividends paid) exceed the intrinsic value of the contract[9], in real terms. Indeed from inequality 2.2, which we repeat below, it is evident that early exercise (for the case of an American call) is directly linked to the amount of dividends paid and it is allowed only when condition 2.2 does not hold.

$$E > \sum_{\tau=0}^{\tau} \frac{d(t)P(\tau - t)}{1 - P(\tau)} \qquad \text{(Eq 2.2)}$$

As Merton (1992) argues the correct application of condition 2.2 has to be in a discrete time context (in order to track the nature of the dividend payments which occur discretely) and not continuously. Consequently the application of American option pricing with the use of a continuously calculated dividend yield may result in the false impression that early exercise is not feasible. Non feasibility of premature exercise should render the early exercise premium equal to zero thus making a European contract as valuable as its American counterpart[10,11]. The same theoretical violations will persist even if we approximate the discrete dividend pay out with a constant yield expressed over

[9] Among others Barone-Adesi and Whaley (1986) provide empirical evidence in support of the assumption that an index looses on ex dividend dates a value equal to the value of dividends paid.

[10] To the best of our knowledge the problem was first realised by Samuelson (1965). The first correct interpretation, however, most probably belongs to Merton (1973, see also reprint in Merton (1992, ch 8, pg 276)).

[11] Although the put call parity relationship, analysed here, does not involve calculation of the option premium it does depend on maintaining the long and short hedges throughout the time to maturity. Consequently the calculation of the actual dividend payments applies directly.

the exact period of the contract (as opposed to the annualised rate). For example, Figure 2.1 below, shows a hypothetical one month call contract. Conditional on a constant dividend yield the contract should never be exercised; thus it is identical to a European type. Theoretically, however, the contract contains three optimal exercise points in time, which in the case of a European contract will result in a certain loss associated with e1, e2, and e3.

More formally, if the monthly dividend yield assumes a function

g(t) and the actual dividend payments a function

f(t) with $t \in I_{(0..T]}$, then even if

$$\int_0^T g(t)dt = \int_0^T f(t)dt \text{, provided that}$$

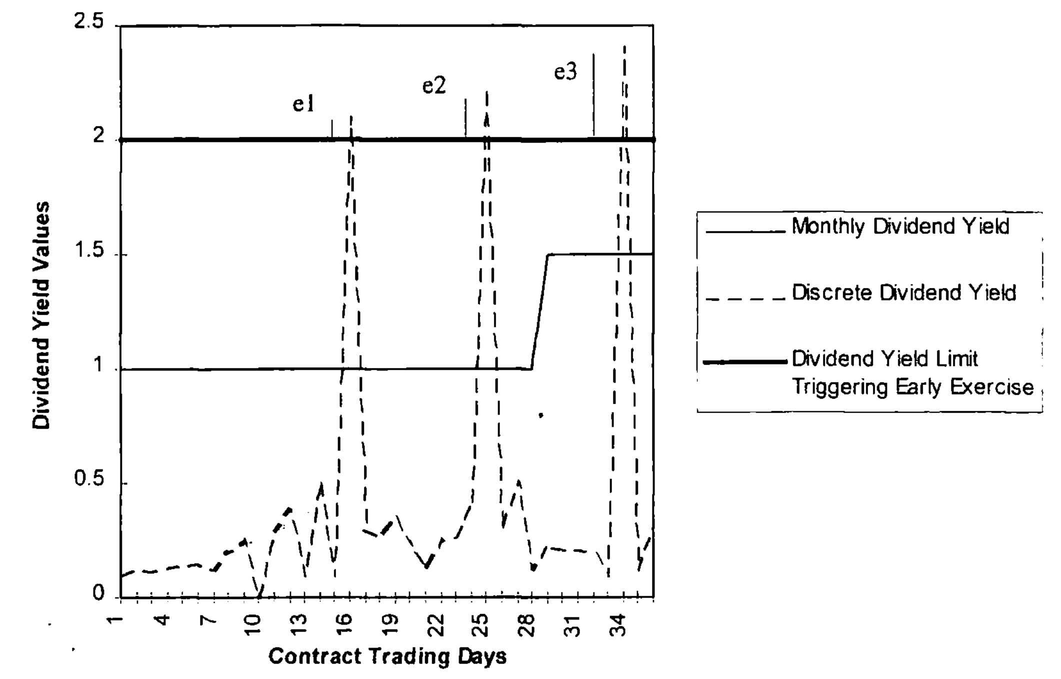
$$\exists\, T' : \max f(t)_{T'} > \max g(t)_{[0..T]}, \qquad \text{Eq 2.19}$$

there exist *possibilities* where early exercise is optimal.

Figure 2.1.

Effect of Dividend Yield on Early Exercise.

The figure shows a hypothetical example where in three instances early exercised is triggered by the amount of dividend paid



In a wide index like the FTSE100 which is comprised of one hundred components one may assume that the payment of dividends, if randomised throughout the year, will produce a smooth time series which will not differ significantly from the actual one. However, companies tend to show "preferred" dividend payment periods with two dividend payments during the financial year, see Harvey and Whaley (1990). One could expect that the presence of "lumps" in the time series might cause "misspricings", in the put call parity relationship, in the form of unanticipated early exercise.

Harvey and Whaley (1992) recognise the problem and test for the pricing errors induced by the adoption of a constant dividend yield as the expected dividend factor. Specifically they simulate option prices for the American (call and put) S&P100 index options for the period of August 1, 1988 through July 31, 1989. Using three pricing procedures, viz. a European model with constant dividend yield, an American model with constant dividend yield and an American model with discrete dividend payments, they indicate that the first two can lead to economically significant misspricings. These findings are adopted here. The dividends used are the actual dividend payments after having allowed for capital restructuring and their effect on the index value (index divisor).

The risk free lending rates are derived from the ask discount rate for Treasury Bills with maturity closest to the expiration date. Borrowing rates, however, are derived as a combination of an annualised rate calculated from the bid discount rate of the T-bill, with maturity closest to expiration, and the broker call rate. Finally transaction cost rates are obtained from a discount broker, trading in these contracts throughout the period.

2.6 Results

The previous section has presented the models and discussed the methodological issues for the identification of deviations from the put call parity both for European and American contracts. Here we present and comment on the results.

Using a suitable trading strategy involving the derivative and the underlying contract together with the fact that, at least in a static environment, an option contract is a redundant security it is possible to create synthetic borrowing and lending rates which are compared with the corresponding market rates. It is argued that under no circumstances will these rates be preferable to the market prevailing zero risk rates. As has been discussed above, previous research has identified significant misspricings in a number of cases. Even when market imperfections or issues related to early exercise premia or immediacy risk are taken into account some of these misspricings remain, thus pointing to a level of market inefficiencies. The following discussion addresses the issues of market imperfections, trading risks and thus evaluates whether or not the resulting deviations constitute potential market inefficiency cases.

European Contracts

For the period of 4th July 1994 to 28th February 1997 we have identified 41101 parity quotes, i.e. quotes for identical put and call contracts. Table 2.3 below groups parity quotes according to moneyness. At-the-money parity contracts are quoted significantly more frequently than out- or in-the-money. It is also evident, however, that deep in-the-money contracts are substantially more active than the corresponding out-of-the-money, perhaps indicating the rising nature of the market through the period of investigation. The percentage distribution of deviations by moneyness agrees with the corresponding daily trading data in KM, however, the quotes from intradaily data, presented in the same paper, give rise to a less leptokurtic distribution with a greater number of strike prices away from the underlying ones. The results also agree with the relationship between moneyness and deviations documented in McMurray and Yadav (1993) – Dawson (1994) does not report relevant values.

Table 2.3

The Distribution of parity quotes for intradaily data by moneyness.

Moneyness (S/X)	Number of matching Quotes	% of matching Quotes
0.90 > S/X	3310	8.05
0.90 < S/X < 0.92	2202	5.35
0.92 < S/X < 0.94	2740	6.70
0.94 < S/X < 0.96	3086	7.50
0.96 < S/X < 0.98	3205	7.80
0.98 < S/X < 1.00	5032	12.25
1.00 < S/X < 1.02	4412	10.70
1.02 < S/X < 1.04	3101	7.55
1.04 < S/X < 1.06	2767	6.73
1.06 < S/X < 1.08	2437	5.93
1.08 < S/X < 1.1	2213	5.38
1.10 < S/X	6653	16.20
Total Number of Quotes Percentage	41101	 100

Table 2.4, see page 85, shows the total number of PCP deviations (both types) expressed as percentage values. It is emphasised that the deviations reflect quotes for put and calls and not actual trades. Consequently they reflect the upper and lower bounds we expect to observe in the market. Nevertheless these represent trading prices offered by market makers should one wish to set up a trading position to exploit any misspricings observed.

Analysis of deviation values for zero transaction costs and no short sales restrictions show significant misspricings with many cases approaching the 100 per cent level, i.e. all cases give rise to PCP deviations. It is also evident that moneyness greatly affects put call parity relationships. Deviation one, which requires the trader to hold the put and short the call occur at 100 per cent of the cases for far out of the money contracts and gradually disappear the more we move at- and into-the money. In a similar fashion deviation two, requiring shorting of the put, increases as we move towards in-to the money contracts. The results are expected since a PCP parity is formed by identical put and call contracts, consequently one type is in the money while the rest is out of the money. Evaluation of the options on the FTSE100 market based on these results will surely indicate gross inefficiencies and significant opportunities for abnormal profits irrespective of moneyness. When we account for market imperfections, however, a substantial part of the deviations disappear.

Again from Table 2.4, full transaction costs (£100) and short sales restrictions, imposed by LSE under the deficit redemption charge scheme (λ=100-r-3), reduce the percentage of deviations significantly. At the money ($0.98<S/X<1.02$) and near the money ($0.96<S/X<0.98$ & $1.02<S/X<1.04$) deviations are close to zero for most of the cases and only deviations of type two for $1.02<S/X<1.04$ still exist for a third of the cases. Nevertheless it is apparent that even after inclusion of market imperfections market misspricings exist and at least theoretically riskless profit opportunities exist for far out- and in-to the money contracts. The results for different levels of transaction costs and restrictions are pro rata.

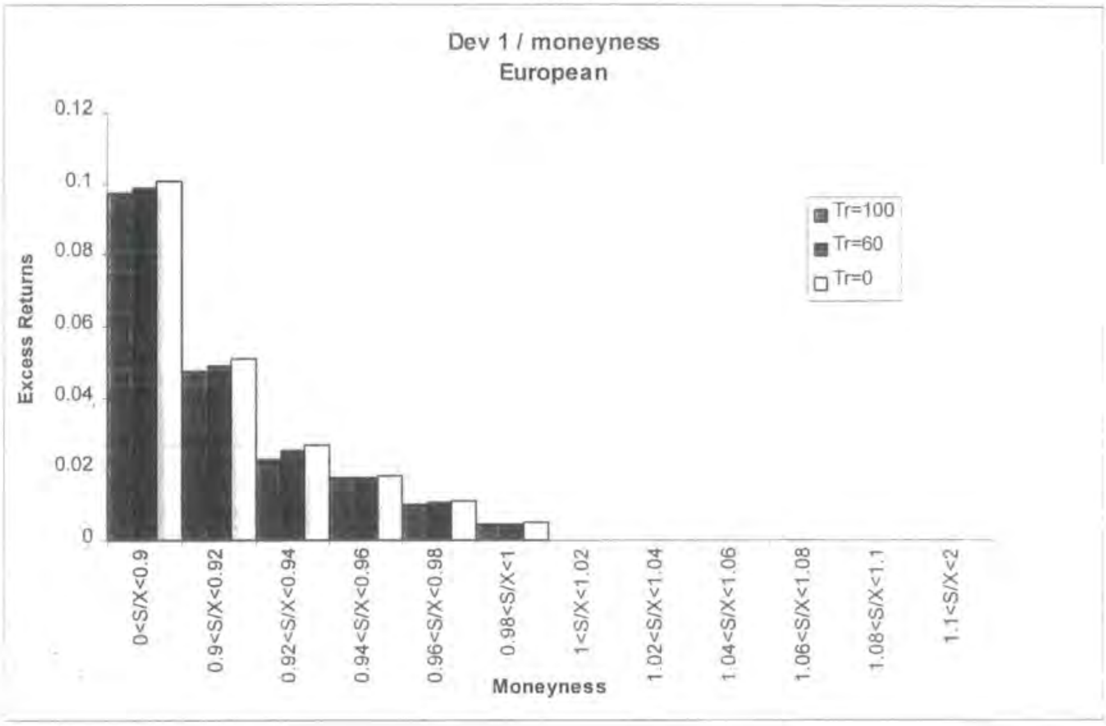
The above lent support to findings by Klemkosky and Resnick (1979, 1980), Gould and Galai (1974), Kamara and Miller (1995) and as far as the inclusion of bid ask spreads is concerned to Dawson (1994), but contradict Finucane (1991) who finds no profitable opportunities after transaction costs. The results also indicate that deviations are present even if market imperfections are taken into account.

Tables 2.5 through to 2.7 (pages 85-95) show that substantial returns can be achieved in excess of the risk free rate. Table 2.5 shows the level of deviations, expressed as excess returns, with variable transaction costs and according to moneyness. The deviations are grouped into type one and type two. Type two deviations require that the investor-arbitrageur sets-up the position using finance from short sales proceeds, hence results for this type are subject to short sales restrictions.

Assuming zero transaction costs investors trading in far-out-of-the-money contracts can achieve, on average, excess returns of 10 per cent with a standard deviation of 0.03 (table 2.5). As we move towards at the money contracts the deviations (always of type one which are not subject to short sales restrictions) decrease and become zero (negative returns) for $S/X > 1$. Imposition of transaction costs will slightly reduce the level of deviations, as can be seen from the same table and also from figure 2.2. It is obvious that the reduction in average excess returns is not substantial and does not agree with suggestions in earlier work, Gould and Galai (1974) and Finucane (1991), that transaction costs will wipe out most of the misspricings. It is possible, however, that this comes as a result of different transaction cost structures between exchanges leading to unequal fee to value ratios for the underlying. Nevertheless the level of misspricings and excess returns indicate that prices quoted in the pit floor can give rise to market inefficiencies.

Figure 2.2

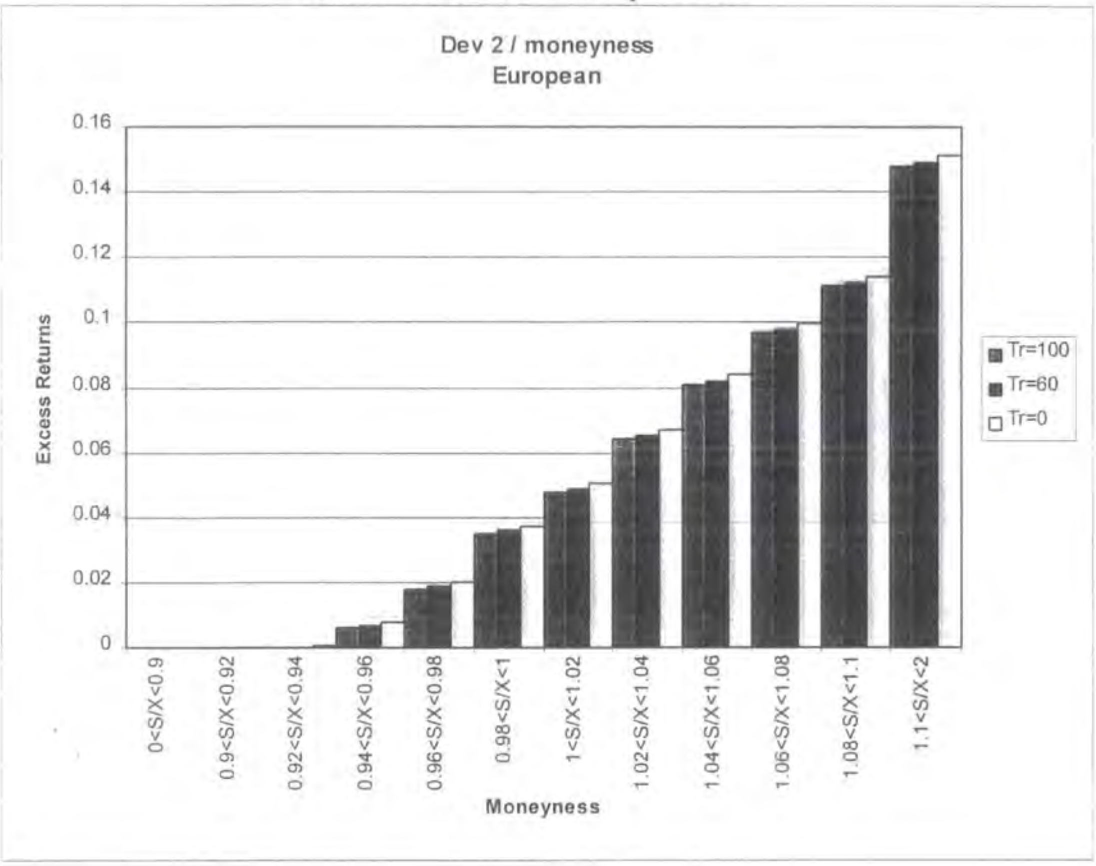
Variation in Deviations of type one, expressed in excess returns form, with respond to moneyness for Transaction costs of £100, £60 and £0.



Additionally, deviations of type two, for zero transaction costs, appear for 0.92<S/X<0.94 and reach roughly 15 per cent, on average, for far-in-the-money contracts. Again inclusion of transaction costs in the model has some effect but not a "dramatic" one (see figure 2.3). For example close to-the-money contracts give rise to excess returns of 3.73 per cent for zero transaction costs whereas for the possible minimum and maximum bounds of £60 and £100, transaction costs imply misspricings of 3.59 and 3.49 per cent, respectively.

Figure 2.3

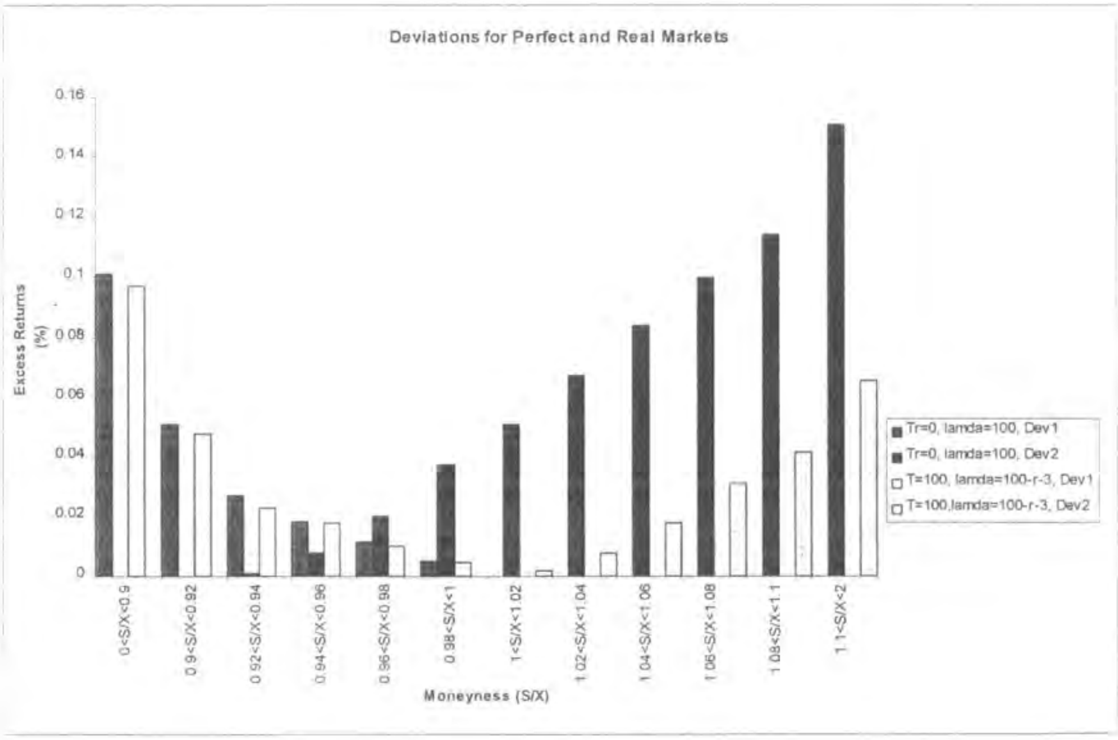
Variation in Deviations of type two expressed in excess returns form, with respect to moneyness for Transaction costs of £100, £60 and £0. Requires re-investment of short sales proceeds.



It is evident from table 2.5 (pages 85-88) that an investor trying to arbitrage between put and call misspricings at the upper band of moneyness will achieve significant excess returns even after accommodating for differences in borrowing and lending rates and transaction costs. Deviations of type two, however, are subject to short sales restrictions imposed by the primary exchange. Table 2.6 (pages 89-92) presents results for different levels of short sales restrictions and figure 2.4 gives an indication of changes induced in misspricings when the full range of market imperfections are taken into account as opposed to "near perfect" markets (differences in lending and borrowing rates included).

Figure2.4

Excess Returns for type 1 and type 2 Deviations, for 1 minute European put-call parities. The graph shows values for "Perfect", but with different borrowing and lending rates and "Real" markets. "Real" markets refer to markets with bid and ask values, short sales restrictions, differences in borrowing and lending rates and transaction costs. (Dev2 series will require short sale trading strategies).



As expected deviations of type 1, representing trading strategies which do not involve short sales, remain unchanged. For short sales restrictions of 97 per cent, corresponding to the level used in KM, misspricings for 0.94<S/X<0.96 are no longer present and indeed this reduces excess returns for far in-the-money trading from 14.8 to 12 per cent. Similar results are obtained for restrictions of 95 per cent. When we calculate the misspricings for the level of short sales restrictions imposed by London School of Economics, deviations are only present for in-the- money contracts with excess returns for far in the money reducing to only 6.5 per cent from the previous level of 14.8. When a level of restrictions equal to 90 per cent is used significant misspricings are present only for far-in-the-money contracts (4.7 per cent).

The above results indicate that even after taking into account market imperfections we can construct synthetic borrowing and lending rates which are higher and lower, respectively, than the market ones. For the duration of the PCP strategy we can achieve excess returns between 6 and 7 per cent for far-in-the-money contracts. Given that these rates are net of costs, the results indicate a discrepancy between quoted option prices and market rates. Taken together the results support the findings by KM for daily data; after accounting for costs they document excess returns between 2 and 7.5 per cent depending, however, on the period and the type of the deviation.

Kamara and Miller (1995) have suggested that the remaining misspricings vary according to moneyness and represent liquidity risk premia since the liquidity is higher for at-the-money but lower for out-of-the-money contracts. In table 2.7 (pages 93-95) we break down excess returns according to moneyness and present their distribution; the results correspond to the deficit redemption account of London School of Economics and the top boundary for transaction costs of £100.

These more analytical results confirm tables 2.5 and 2.6. The deviations vary consistently with moneyness from high values, for out-of-the-money, to low values, for in-the-money, for deviations one and vice versa for type two. Our results do agree with the findings in KM. Both studies find a smooth evolution of misspricings from high to low moneyness and vice versa.

However, the results presented here exclude cases where stable conditions do not exist for the replication of the trading strategies for at least two minutes. Consequently this should result in the removal of most deviations related to immediacy risk. It is probable then, that the deviations reported here could correspond to factors other than immediacy related ones. It is a possibility that some of these include inventory-related constraints. As such these could be market maker specific imbalances and should be gradually removed by arbitrageurs; we should return to this in the following chapter.

Moneyness	Transaction Costs	T=100					T=60	T=0
	% of Short Sales available for reinvestment	λ=100	λ=97	λ=95	λ=90	λ=100-r-3	λ=100	λ=100
0.90 > S/X	DEV1	100	100	100	100	100	100	100
	DEV2	0	0	0	0	0	0	0
0.90 < S/X < 0.92	DEV1	100	100	100	100	100	99.6	100
	DEV2	0	0	0	0	0	0	0
0.92 < S/X < 0.94	DEV1	93.87	93.87	93.87	93.87	93.87	95.4	99.74
	DEV2	0	0	0	0	0	0	0.26
0.94 < S/X < 0.96	DEV1	39.08	39.08	39.08	39.08	39.08	41.4	54.05
	DEV2	28.6	0	0	0	0	31.7	45.95
0.96 < S/X < 0.98	DEV1	13.83	13.83	13.83	13.83	13.83	15.07	19.88
	DEV2	73.7	3.06	0	0	0	74.69	80.12
0.98 < S/X < 1.00	DEV1	1.19	1.19	1.19	1.19	1.19	1.39	2.43
	DEV2	92.69	61.78	3.84	0	0	93.4	97.57
1.00 < S/X < 1.02	DEV1	0	0	0	0	0	0	0
	DEV2	99.5	82.4	44.4	0	1.27	99.7	100
1.02 < S/X < 1.04	DEV1	0	0	0	0	0	0	0
	DEV2	100	94	75.45	0	33.4	100	100
1.04 < S/X < 1.06	DEV1	0	0	0	0	0	0	0
	DEV2	100	99.8	90.7	0.3	63.4	100	100
1.06 < S/X < 1.08	DEV1	0	0	0	0	0	0	0
	DEV2	100	100	99.7	31	72.2	100	100
1.08 < S/X < 1.1	DEV1	0	0	0	0	0	0	0
	DEV2	100	100	100	66.6	79.2	100	100
1.10 < S/X	DEV1	0	0	0	0	0	0	0
	DEV2	100	100	100	94.9	95.2	100	100

Table 2.4

Violations of DEV1 and DEV2 according to moneyness. The number of violations are expressed as percentage of total cases. The table shows values for Transaction Costs = $0, $60,$100 and λ = 100%, 97%, 95%, 90% and 100-r-3%.

Moneyness	Transaction Costs	T=$0		T=$60		T=$100			
		DEV1	DEV2	DEV1	DEV2	DEV1	DEV2		
0<S\X<0.9	μ	-0.10092	-	-0.09867	-	-0.09717	-	μ	0<S\X<0.9
	S.E	0.000542	-	0.00054	-	0.000539	-	S.E	
	median	-0.09866	-	-0.0965	-	-0.09503	-	median	
	S.D	0.031172	-	0.031072	-	0.031006	-	S.D	
	kyrtosis	-0.33663	-	-0.33683	-	-0.33696	-	kyrtosis	
	skewness	-0.3858	-	-0.38506	-	-0.38455	-	skewness	
0.9<S\X<0.92	μ	-0.05099	-	-0.0489	-	0.04751	-	μ	0.9<S\X<0.92
	S.E	0.0000389	-	0.000388	-	0.000387	-	S.E	
	median	-0.04826	-	-0.04625	-	-0.04481	-	median	
	S.D	0.018271	-	0.018208	-	0.018167	-	S.D	
	kyrtosis	-0.88125	-	-0.88395	-	-0.88563	-	kyrtosis	
	skewness	-0.35098	-	-0.35083	-	-0.35072	-	skewness	
0.92<S\X<0.94	μ	-0.02679	-0.00083	-0.02529	-	-0.02266	-	μ	0.92<S\X<0.94
	S.E	0.000338	0.000222	0.000335	-	0.000338	-	S.E	
	median	-0.02291	-0.00094	-0.02137	-	-0.01879	-	median	
	S.D	0.017439	0.000588	0.01715	-	0.017698	-	S.D	
	kyrtosis	-0.59931	-1.5507	-0.60963	-	-0.59165	-	kyrtosis	
	skewness	-0.59819	0.14354	-0.61042	-	-0.56188	-	skewness	

Table 2.5.

Put-Call parity Deviations with respect to transaction costs. The deviations are expressed in the form of excess returns. Dev2 require re-investment of short sale proceeds.

Moneyness	Transaction Costs	T=$0		T=$60		T=$100			
		DEV1	DEV2	DEV1	DEV2	DEV1	DEV2		
0.94<S\X<0.96	μ	-0.01807	-0.0076	-0.01789	-0.00673	-0.01765	-0.00616	μ	0.94<S\X<0.96
	S.E	0.000355	0.000146	0.000355	0.000145	0.000353	0.000143	S.E	
	median	-0.01566	-0.00696	-0.01652	-0.00606	-0.01662	-0.00557	median	
	S.D	0.013355	0.004939	0.012686	0.00452	0.012237	0.004249	S.D	
	kyrtosis	-0.79371	-0.58493	-0.78959	-0.50825	-0.77386	-0.44253	kyrtosis	
	skewness	-0.47431	-0.4588	-0.42398	-0.52717	-0.40143	-0.5648	skewness	
0.96<S\X<0.98	μ	-0.01124	-0.02004	-0.01043	-0.01857	-0.00992	-0.01752	μ	0.96<S\X<0.98
	S.E	0.00032	0.000183	0.000311	0.000178	0.000308	0.000176	S.E	
	median	-0.01068	-0.02054	-0.00974	-0.01889	-0.00891	-0.01778	median	
	S.D	0.007444	0.00906	0.006845	0.00871	0.006453	0.008547	S.D	
	kyrtosis	-0.57006	-0.6364	-0.49976	-0.67789	-0.45362	-0.6882	kyrtosis	
	skewness	-0.45404	0.143155	-0.50657	0.067454	-0.55016	0.032073	skewness	
0.98<S\X<1	μ	-0.0049	-0.03732	-0.00458	-0.03592	-0.00459	-0.03494	μ	0.98<S\X<1
	S.E	0.000484	0.000203	0.000574	0.000198	0.000666	0.000195	S.E	
	median	-0.0035	-0.04008	-0.00276	-0.03835	-0.00271	-0.03723	median	
	S.D	0.004842	0.014013	0.004771	0.013561	0.004759	0.013314	S.D	
	kyrtosis	2.11837	-0.08742	1.308664	-0.13861	0.514731	-0.17065	kyrtosis	
	skewness	-1.58754	0.776914	-1.46232	0.736926	-1.26734	0.712293	skewness	

Table 2.5 (contd)

Put-Call parity Deviations w.r.t. transaction costs. The deviations are expressed in the form of excess returns. Dev2 require re-investment of short sale proceeds.

Moneyness	Transaction Costs	T=$0		T=$60		T=$100			
		DEV1	DEV2	DEV1	DEV2	DEV1	DEV2		
1<S\X<1.02	μ	-	-0.05054	-	-0.0488	-	-0.04765	μ	1<S\X<1.02
	S.E	-	0.000236	-	0.000235	-	0.000234	S.E	
	median	-	-0.05463	-	-0.0528	-	-0.05158	median	
	S.D	-	0.015654	-	0.015567	-	0.015512	S.D	
	kyrtosis	-	0.301528	-	0.244641	-	0.213031	kyrtosis	
	skewness	-	0.971088	-	0.955071	-	0.945551	skewness	
1.02<S\X<1.04	μ	-	-0.06726	-	-0.0654	-	-0.06415	μ	1.02<S\X<1.04
	S.E	-	0.000295	-	0.000297	-	0.000298	S.E	
	median	-	-0.07171	-	-0.06979	-	-0.06859	median	
	S.D	-	0.016432	-	0.016514	-	0.016569	S.D	
	kyrtosis	-	-0.0585	-	-0.05634	-	-0.05488	kyrtosis	
	skewness	-	0.869082	-	0.869039	-	0.868993	skewness	
1.04<S\X<1.06	μ	-	-0.084	-	-0.08215	-	-0.08091	μ	1.04<S\X<1.06
	S.E	-	0.000308	-	0.000309	-	0.00031	S.E	
	median	-	-0.08865	-	-0.08687	-	-0.08569	median	
	S.D	-	0.016198	-	0.016278	-	0.016331	S.D	
	kyrtosis	-	-0.14077	-	-0.13988	-	-0.13929	kyrtosis	
	skewness	-	0.848294	-	0.847033	-	0.846134	skewness	

Table 2.5 (contd).

Put-Call parity Deviations w.r.t. transaction costs. The deviations are expressed in the form of excess returns. Dev2 require re-investment of short sale proceeds.

Moneyness	Transaction Costs	T=$0		T=$60		T=$100			
		DEV1	DEV2	DEV1	DEV2	DEV1	DEV2		
1.06<S\X<1.08	μ	-	-0.09988	-	-0.09805	-	-0.09682	μ	1.06<S\X<1.08
	S.E	-	0.000319	-	0.00321	-	0.000322	S.E	
	median	-	-0.10474	-	-0.10287	-	-0.10167	median	
	S.D	-	0.015754	-	-0.27956	-	0.015873	S.D	
	kyrtosis	-	-0.28192	-	0.79834	-	-0.27801	kyrtosis	
	skewness	-	0.7993	-		-	0.797616	skewness	
		-							
1.08<S\X<1.1	μ	-	-0.11404	-	-0.11222	-	-0.11101	μ	1.08<S\X<1.1
	S.E	-	0.000333	-	0.000334	-	0.000336	S.E	
	median	-	-0.11842	-	-0.11667	-	-0.11554	median	
	S.D	-	0.015661	-	0.015734	-	0.015784	S.D	
	kyrtosis	-	-0.36884	-	-0.36518	-	-0.36274	kyrtosis	
	skewness	-	0.743102	-	0.743124	-	0.743064	skewness	
1.1<S\X<2	μ	-	-0.15089	-	-0.14912	-	-0.14794	μ	1.1<S\X<2
	S.E	-	0.000303	-	0.000304	-	0.000304	S.E	
	median	-	-0.14888	-	-0.14713	-	-0.14596	median	
	S.D	-	0.024722	-	0.024784	-	0.024826	S.D	
	kyrtosis	-	-0.05199	-	-0.04842	-	-0.04601	kyrtosis	
	skewness	-	0.2128	-	-0.21148	-	-0.21099	skewness	

Table 2.5 (contd).

Put-Call parity Deviations w.r.t. transaction costs. The deviations are expressed in the form of excess returns. Dev2 require re-investment of short sale proceeds.

Moneyness	Percentage of short sales available to investors	λ=100		λ=97		λ=95		λ=90		λ=100-r-0.03 (restrictions imposed by LSE under the capital redemption account)	
		DEV1	DEV2	DEV1	DEV2	DEV1	DEV2	DEV1	DEV2	DEV1	DEV2
0<S\X<0.9	μ	-0.09717	-	-0.09717	-	-0.09717	-	-0.09717	-	-0.09717	-
	S.E	0.000539	-	0.000539	-	0.000539	-	0.000539	-	0.000539	-
	med.	-0.09503	-	-0.09503	-	-0.09503	-	-0.09503	-	-0.09503	-
	S.D	0.031006	-	0.031006	-	0.031006	-	0.031006	-	0.031006	-
	kyrt.	-0.33696	-	-0.33696	-	-0.33696	-	-0.33696	-	-0.33696	-
	sk.	-0.38455	-	-0.38455	-	-0.38455	-	-0.38455	-	-0.38455	-
0.9<S\X<0.92	μ	-0.04751	-	-0.04751	-	-0.04751	-	-0.04751	-	-0.04751	-
	S.E	0.000387	-	0.000387	-	0.000387	-	0.000387	-	0.000387	-
	med.	-0.04481	-	-0.04481	-	-0.04481	-	-0.04481	-	-0.04481	-
	S.D	0.018167	-	0.018167	-	0.018167	-	0.018167	-	0.018167	-
	kyrt.	-0.88563	-	-0.88563	-	-0.88563	-	-0.88563	-	-0.88563	-
	sk.	-0.35072	-	-0.35072	-	-0.35072	-	-0.35072	-	-0.35072	-
0.92<S\X<0.94	μ	-0.02266	-	-0.02266	-	-0.02266	-	-0.02266	-	-0.02266	-
	S.E	0.000338	-	0.000338	-	0.000338	-	0.000338	-	0.000338	-
	med.	-0.01879	-	-0.01879	-	-0.01879	-	-0.01879	-	-0.01879	-
	S.D	0.017698	-	0.017698	-	0.017698	-	0.017698	-	0.017698	-
	kyrt.	-0.59165	-	-0.59165	-	-0.59165	-	-0.59165	-	-0.59165	-
	sk.	-0.56188	-	-0.56188	-	-0.56188	-	-0.56188	-	-0.56188	-

Table 2.6.

Put-Call parity Deviations w.r.t short sales restrictions. The deviations are expressed in excess returns form. Deviations of type 2 require re-investment of short sales proceeds.

Moneyness	Percentage of short sales available to investors	λ=100		λ=97		λ=95		λ=90		λ=100-r-0.03 (restrictions imposed by LSE under the capital redemption account)	
		DEV1	DEV2	DEV1	DEV2	DEV1	DEV2	DEV1	DEV2	DEV1	DEV2
0.94<S\X<0.96	μ	-0.01765	-0.00616	-0.01765	-	-0.01765	-	-0.01765	-	-0.01765	-
	S.E	0.000353	0.000143	0.000353	-	0.000353	-	0.000353	-	0.000353	-
	med.	-0.01662	-0.00557	-0.01662	-	-0.01662	-	-0.01662	-	-0.01662	-
	S.D	0.012237	0.004249	0.012237	-	0.012237	-	0.012237	-	0.012237	-
	kyrt.	-0.77386	-0.44253	-0.77386	-	-0.77386	-	-0.77386	-	-0.77386	-
	sk.	-0.40143	-0.5648	-0.40143	-	-0.40143	-	-0.40143	-	-0.40143	-
0.96<S\X<0.98	μ	-0.00992	-0.01752	-0.00992	-0.00248	-0.00992	-	-0.00992	-	-0.00992	-
	S.E	0.000308	0.000176	0.000308	0.000206	0.000308	-	0.000308	-	0.000308	-
	med.	-0.00891	-0.01778	-0.00891	-0.00206	-0.00891	-	-0.00891	-	-0.00891	-
	S.D	0.006453	0.008547	0.006453	0.002024	0.006453	-	0.006453	-	0.006453	-
	kyrt.	-0.45362	-0.6882	-0.45362	1.103426	-0.45362	-	-0.45362	-	-0.45362	-
	sk.	-0.55016	0.032073	-0.55016	-1.19088	-0.55016	-	-0.55016	-	-0.55016	-
0.98<S\X<1	μ	-0.00459	-0.03494	-0.00459	-0.01121	-0.00459	-0.00191	-0.00459	-	-0.00459	-
	S.E	0.000666	0.000195	0.000666	0.00019	0.000666	0.000106	0.000666	-	0.000666	-
	med.	-0.00271	-0.03723	-0.00271	-0.01082	-0.00271	-0.0015	-0.00271	-	-0.00271	-
	S.D	0.004759	0.013314	0.004759	0.006638	0.004759	0.001468	0.004759	-	0.004759	-
	kyrt.	0.514731	-0.17065	0.514731	-0.92733	0.514731	0.741665	0.514731	-	0.514731	-
	sk.	-1.26734	0.712293	-1.26734	-0.22459	-1.26734	-1.11137	-1.26734	-	-1.26734	-

Table 2.6 (contd).

Put-Call parity Deviations w.r.t short sales restrictions. The deviations are expressed in excess returns form. Deviations of type 2 require re-investment of short sales proceeds.

Moneyness	Percentage of short sales available to investors	λ=100		λ=97		λ=95		λ=90		λ=100-r-0.03 (restrictions imposed by LSE under the capital redemption account)	
		DEV1	DEV2	DEV1	DEV2	DEV1	DEV2	DEV1	DEV2	DEV1	DEV2
1<S\X<1.02	μ	-	-0.04765	-	-0.02219	-	-0.0074	-	-	-	-.0018
	S.E	-	0.000234	-	0.000154	-	0.000112	-	-	-	0.000198
	med.	-	-0.05158	-	-0.02275	-	-0.00679	-	-	-	-0.00138
	S.D	-	0.015512	-	0.009285	-	0.004969	-	-	-	0.001481
	kyrt.	-	0.213031	-	-0.47772	-	-0.314	-	-	-	0.028049
	sk.	-	0.945551	-	0.268654	-	-0.58937	-	-	-	-0.80615
1.02<S\X<1.04	μ	-	-0.06415	-	-0.03566	-	-0.01989	-	-	-	-0.00762
	S.E	-	0.000298	-	0.000264	-	0.000179	-	-	-	0.000161
	med.	-	-0.06859	-	-0.03902	-	-0.02052	-	-	-	-0.00699
	S.D	-	0.016569	-	0.014275	-	0.008676	-	-	-	0.005174
	kyrt.	-	-0.05488	-	-0.30273	-	-0.56719	-	-	-	-0.38313
	sk.	-	0.868993	-	0.723208	-	0.150301	-	-	-	-0.58219
1.04<S\X<1.06	μ	-	-0.08091	-	-0.05045	-	-0.03265	-	-0.00101	-	-0.01773
	S.E	-	0.00031	-	0.000318	-	0.000264	-	0.00022	-	0.000208
	med.	-	-0.08569	-	-0.05529	-	-0.03564	-	-0.00092	-	-0.01789
	S.D	-	0.016331	-	0.01672	-	0.01322	-	0.000659	-	0.008705
	kyrt.	-	-0.13929	-	-0.18852	-	-0.27177	-	1.361602	-	-0.74118
	sk.	-	0.846134	-	0.8366	-	0.688057	-	-0.718	-	0.004057

Table 2.6 (contd).

Put-Call parity Deviations w.r.t short sales restrictions. The deviations are expressed in excess returns form. Deviations of type 2 require re-investment of short sales proceeds.

Moneyness	Percentage of short sales available to investors	λ=100		λ=97		λ=95		λ=90		λ=100-r-0.03 (restrictions imposed by LSE under the capital redemption account)	
		DEV1	DEV2	DEV1	DEV2	DEV1	DEV2	DEV1	DEV2	DEV1	DEV2
1.06<S\X<1.08	μ	-	-0.09682	-	-0.06673	-	-0.04573	-	-0.00575	-	-0.03076
	S.E	-	0.000322	-	0.000332	-	0.000336	-	0.000147	-	0.000283
	med.	-	-0.10167	-	-0.07171	-	-0.0508	-	-0.00515	-	-0.03258
	S.D	-	0.015873	-	0.016366	-	0.016544	-	0.004034	-	0.011876
	kyrt.	-	-0.27801	-	-0.27846	-	-0.32387	-	-0.27893	-	-0.08551
	sk.	-	0.797616	-	0.801957	-	0.789767	-	-0.607	-	0.649499
1.08<S\X<1.1	μ	-	-0.11101	-	-0.08131	-	-0.06046	-	-0.01494	-	-0.04134
	S.E	-	0.000336	-	0.000346	-	0.000353	-	0.000195	-	0.000399
	med.	-	-0.11554	-	-0.08593	-	-0.06514	-	-0.01515	-	-0.04605
	S.D	-	0.015784	-	0.016266	-	0.016606	-	0.007502	-	0.016689
	kyrt.	-	-0.36274	-	-0.36877	-	-0.37276	-	-0.60823	-	-0.20913
	sk.	-	0.743064	-	0.744827	-	0.746005	-	-0.03386	-	0.801174
1.1<S\X<2	μ	-	-0.14794	-	-0.11925	-	-0.0991	-	-0.04755	-	-0.06543
	S.E	-	0.000304	-	0.000313	-	0.000319	-	0.000315	-	0.000366
	med.	-	-0.14596	-	-0.1172	-	-0.09699	-	-0.0447	-	-0.06731
	S.D	-	0.024826	-	0.025528	-	0.02602	-	0.025055	-	0.029113
	kyrt.	-	-0.04601	-	-0.0422	-	-0.03946	-	-0.00903	-	-0.20114
	sk.	-	-0.21099	-	-0.20814	-	-0.20611	-	-0.4608	-	0.095878

Table 2.6 (contd).

Put-Call parity Deviations w.r.t short sales restrictions. The deviations are expressed in excess returns form. Deviations of type 2 require re-investment of short sales proceeds.

Excess Returns | Moneyness	0<S\X<0.9 (sample = 3310)		0.9<S\X<0.92 (sample = 2202)		0.92<S\X<0.94 (sample = 2740)		0.94<S\X<0.96 (sample = 3086)		0.96<S\X<0.98 (sample = 3205)		0.98<S\X<1 (sample = 5032)	
	DEV1	DEV2	DEV1	DEV2	DEV1	DEV2	DEV1	DEV2	DEV1	DEV2	DEV1	DEV2
0.0-0.01	-	-		-	16.72	0.26	16.66	29.08	7.83	11.99	1.75	5.94
0.01-0.02	-	-	0.5	-	26.38	-	11.72	16.64	6.77	24.43	0.64	7.21
0.02-0.03	-	-	12.49	-	18.62	-	11.92	0.23	4.28	29.11	0.04	14.09
0.03-0.04	0.03	-	20.16	-	12.57	-	8.67	-	0.66	10.64	-	22.81
0.04-0.05	2.69	-	19.71	-	11.38	-	4.66	-	0.34	3.61	-	29.85
0.05-0.06	6.8	-	15.35	-	9.02	-	0.42	-	-	0.34	-	17.29
0.06-0.07	8.15	-	12.17	-	4.65	-	-	-	-	-	-	0.38
0.07-0.08	10.34	-	12.04	-	0.36	-	-	-	-	-	-	-
0.08-0.09	11.02	-	6.44	-	0.04	-	-	-	-	-	-	-
0.09-0.10	13.24	-	1.14	-	-	-	-	-	-	-	-	-
0.10-0.11	11.26	-	-	-	-	-	-	-	-	-	-	-
0.11-0.12	9.58	-	-	-	-	-	-	-	-	-	-	-
0.12-0.13	8.25	-	-	-	-	-	-	-	-	-	-	-
0.13-0.14	6.37	-	-	-	-	-	-	-	-	-	-	-
0.14-0.15	5.23	-	-	-	-	-	-	-	-	-	-	-
0.15-0.16	3.35	-	-	-	-	-	-	-	-	-	-	-
0.16-0.17	1.64	-	-	-	-	-	-	-	-	-	-	-
0.17-0.18	0.99	-	-	-	-	-	-	-	-	-	-	-
0.18-0.19	0.73	-	-	-	-	-	-	-	-	-	-	-
0.19-0.20	0.27	-	-	-	-	-	-	-	-	-	-	-
0.20-0.21	0.06	-	-	-	-	-	-	-	-	-	-	-

Table 2.7

Distribution of percentage deviations w.r.t. moneyness. The deviations are calculated with zero transaction costs and no short sales restrictions.

Excess Returns / Moneyness	1<S\X<1.02 (sample = 4412)		1.02<S\X<1.04 (sample = 3101)		1.04<S\X<1.06 (sample = 2767)		1.06<S\X<1.08 (sample = 2437)		1.08<S\X<1.1 (sample = 2213)		1.1<S\X<2 (sample = 6653)	
	DEV1	DEV2	DEV1	DEV2	DEV1	DEV2	DEV1	DEV2	DEV1	DEV2	DEV1	DEV2
0.0-0.01	-	1.4	-	0.03	-	-	-	-	-	-	-	-
0.01-0.02	-	5.21	-	0.2	-	-	-	-	-	-	-	-
0.02-0.03	-	6.83	-	2.8	-	0.04	-	-	-	-	-	-
0.03-0.04	-	7.82	-	6.45	-	0.39	-	-	-	-	-	-
0.04-0.05	-	15.29	-	7.77	-	3.76	-	-	-	-	-	-
0.05-0.06	-	32.44	-	10.32	-	7.88	-	1.07	-	-	-	-
0.06-0.07	-	26.39	-	17.19	-	8.2	-	5.04	-	0.23	-	-
0.07-0.08	-	4.62	-	32.28	-	11.21	-	8.79	-	2.35	-	-
0.08-0.09	-	-	-	21.06	-	22.37	-	10.46	-	8.17	-	0.12
0.09-0.10	-	-	-	1.9	-	33.25	-	13.91	-	10.13	-	1.28
0.10-0.11	-	-	-	-	-	12.61	-	29.75	-	11.84	-	3.32
0.11-0.12	-	-	-	-	-	0.29	-	27.49	-	20.78	-	5.86
0.12-0.13	-	-	-	-	-	-	-	3.49	-	34.12	-	8.31
0.13-0.14	-	-	-	-	-	-	-	-	-	12.11	-	15.68

Table 2.7(contd).

Distribution of percentage deviations w.r.t. moneyness. The deviations are calculated with zero transaction costs and no short sales restrictions.

Excess Returns / Moneyness	1<S\X<1.02 (sample = 4412)		1.02<S\X<1.04 (sample = 3101)		1.04<S\X<1.06 (sample = 2767)		1.06<S\X<1.08 (sample = 2437)		1.08<S\X<1.1 (sample = 2213)		1.1<S\X<2 (sample = 6653)	
	DEV1	DEV2	DEV1	DEV2	DEV1	DEV2	DEV1	DEV2	DEV1	DEV2	DEV1	DEV2
0.14-0.15	-	-	-	-	-	-	-	-	-	0.27	-	17.11
0.15-0.16	-	-	-	-	-	-	-	-	-	-	-	13.96
0.16-0.17	-	-	-	-	-	-	-	-	-	-	-	11.86
0.17-0.18	-	-	-	-	-	-	-	-	-	-	-	9.24
0.18-0.19	-	-	-	-	-	-	-	-	-	-	-	7.23
0.19-0.20	-	-	-	-	-	-	-	-	-	-	-	3.5
0.20-0.21	-	-	-	-	-	-	-	-	-	-	-	1.4
0.21-0.22	-	-	-	-	-	-	-	-	-	-	-	0.86
0.22-0.23	-	-	-	-	-	-	-	-	-	-	-	0.16
0.23-0.24	-	-	-	-	-	-	-	-	-	-	-	0.05
0.24-0.25	-	-	-	-	-	-	-	-	-	-	-	0.01
0.25-0.26	-	-	-	-	-	-	-	-	-	-	-	0.03
0.26-0.27	-	-	-	-	-	-	-	-	-	-	-	0.01

Table 2.7 (contd).

Distribution of percentage deviations w.r.t. moneyness. The deviations are calculated with zero transaction costs and no short sales restrictions.

American Contracts

KM formalise two competing hypotheses for the explanation of PCP deviations, the misspricings arising from liquidity risk and those from early exercise premia. They found that the "liquidity" hypothesis fits better their data. Given that their sample consists of European contracts they argue that the manifested misspricings should be irrelevant to early exercise risk and be attributed to difficulties in the completion of the trading strategy at the prices quoted at the initiation of the trade. Additionally Dawson (1994) found no support for the early exercise hypothesis. In an opposing fashion both Klemkosky and Resnick (1979, 1980) and McMurray and Yadav (1993) found evidence for early exercise premia. Thus the question of early exercise premia in the American market remains.

Similar to the European contracts, the synthetically constructed borrowing and lending rates are calculated for American style contracts. This study considered only those contracts which do not violate conditions for early exercise of call contracts and tests whether or not early exercise associated with the put contract could be ruled out at the inception of the trade but not at every instance. Theoretically then the put call parity is no longer a riskless statement and as such the synthetic rates can no longer be compared with their risk free market counterparts. It is obvious then that deviation two, the strategy which is prone to early exercise of the put, should command a level of risk premia over and above the risk free rate. The two groups are compared and the differences in deviations arising from the strategy prone to early exercise of the put contract are examined. It is argued here that any systematic variation should reflect the possibility of early exercise for the put contract; it is as if the market anticipates that early exercise of the put will be profitable much sooner than expiration thus compensating the investor / arbitrageur with the excess returns observed.

Table 2.8 gives the percentage of quotes with respect to moneyness relative to the total number of put call pairs in the sample. The sample is significantly larger than the European one - 58421 pairs compared to 41101 for European types- and significantly denser for close to-the-money quotes. Again deep in-the-money contracts are quoted more frequently than the corresponding out-of-the-money group. It is interesting to note, however, that for deep-in-the-money contracts, the difference between American and European contracts is reversed, i.e. European contracts are quoted more frequently than the corresponding Americans are.

Table 2.8

The Distribution of parity quotes for intradaily data by moneyness (American contracts).

Moneyness (S/X)	Number of matching Quotes	% of matching Quotes
0.90 > S/X	3674	6.3
0.90 < S/X < 0.92	2461	4.26
0.92 < S/X < 0.94	3056	5.29
0.94 < S/X < 0.96	3305	5.73
0.96 < S/X < 0.98	4378	7.59
0.98 < S/X < 1.00	11094	19.23
1.00 < S/X < 1.02	11072	19.92
1.02 < S/X < 1.04	4136	7.17
1.04 < S/X < 1.06	2981	5.17
1.06 < S/X < 1.08	2711	4.7
1.08 < S/X < 1.1	2198	3.81
1.10 < S/X	6672	11.57
Total Number of Quotes Percentage	57671	100.00

Figures 2.5 and 2.6 present the variation in the excess return according to moneyness categories for the American contracts. It is evident from both graphs that moneyness affects directly the level of arbitrage opportunities according to the PCP. Similarities exist between American and European contracts. It is more interesting to observe that the corresponding deviations for type two strategy appear to be higher and more widespread than the deviations for type one strategy. We should recall at this point that strategy two involves writing an American put, which is subject to early exercise by the counterparty. Consequently these deviations refer directly to the "early exercise" hypothesis, which appears to be significant.

Figure 2.5.

Variation in Deviations of type one, expressed in excess returns form, w.r.t moneyness for Transaction costs of £100, £60 and £0 (American contracts).

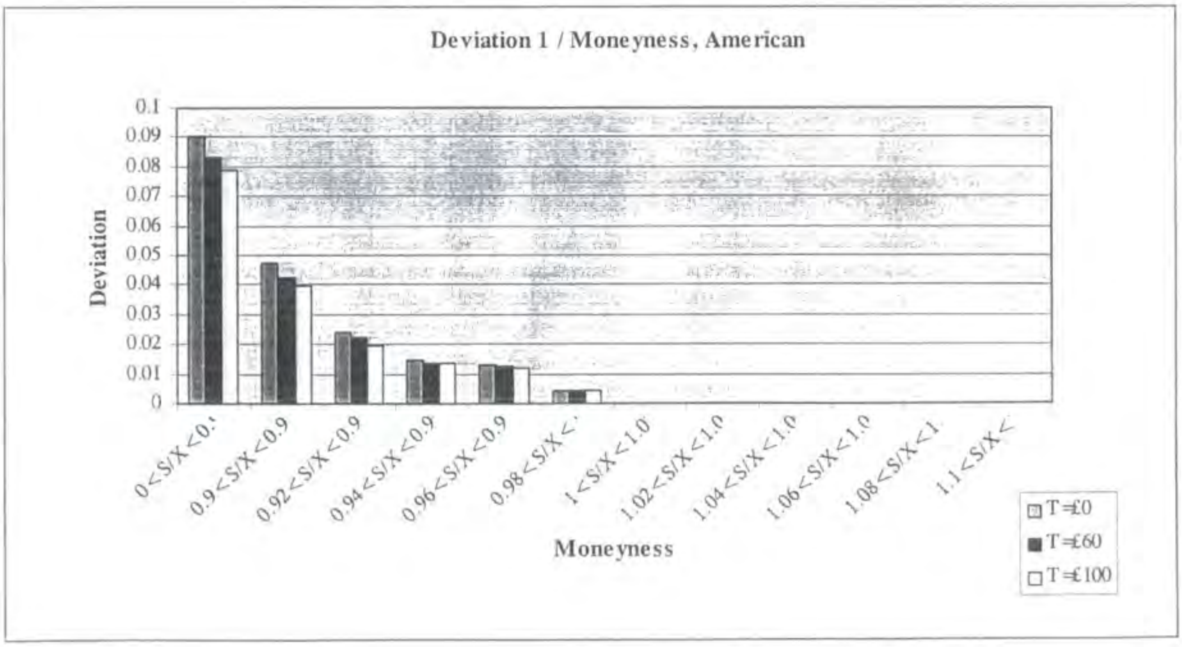


Figure 2.6.

Variation in Deviations of type two, expressed in excess returns form, w.r.t moneyness for Transaction costs of 3100, £60 and £0 (American contracts).

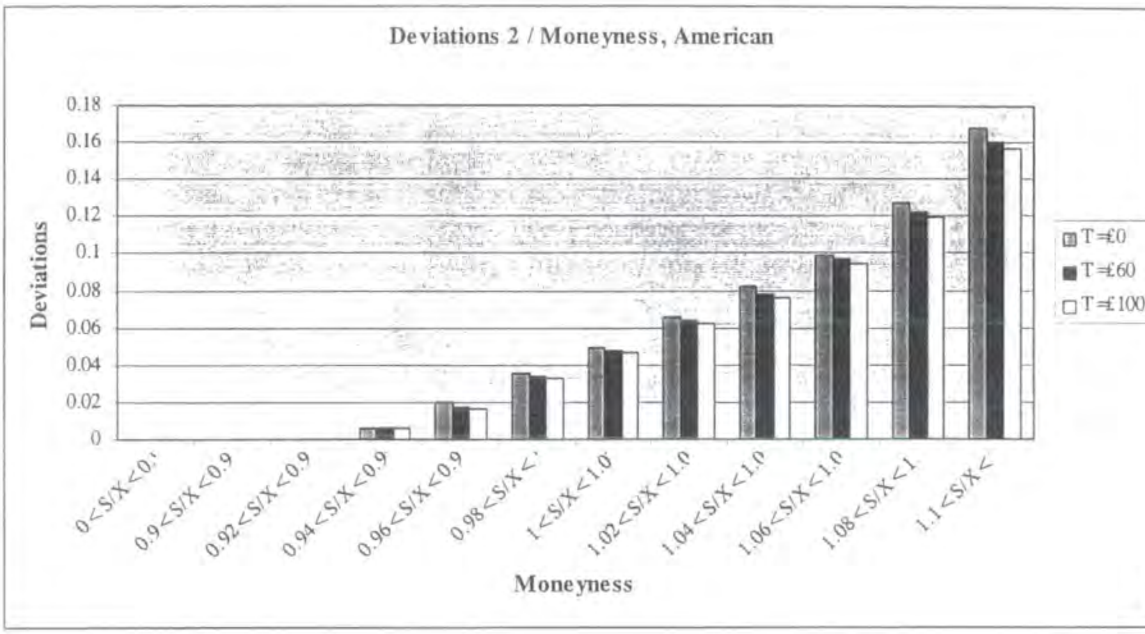
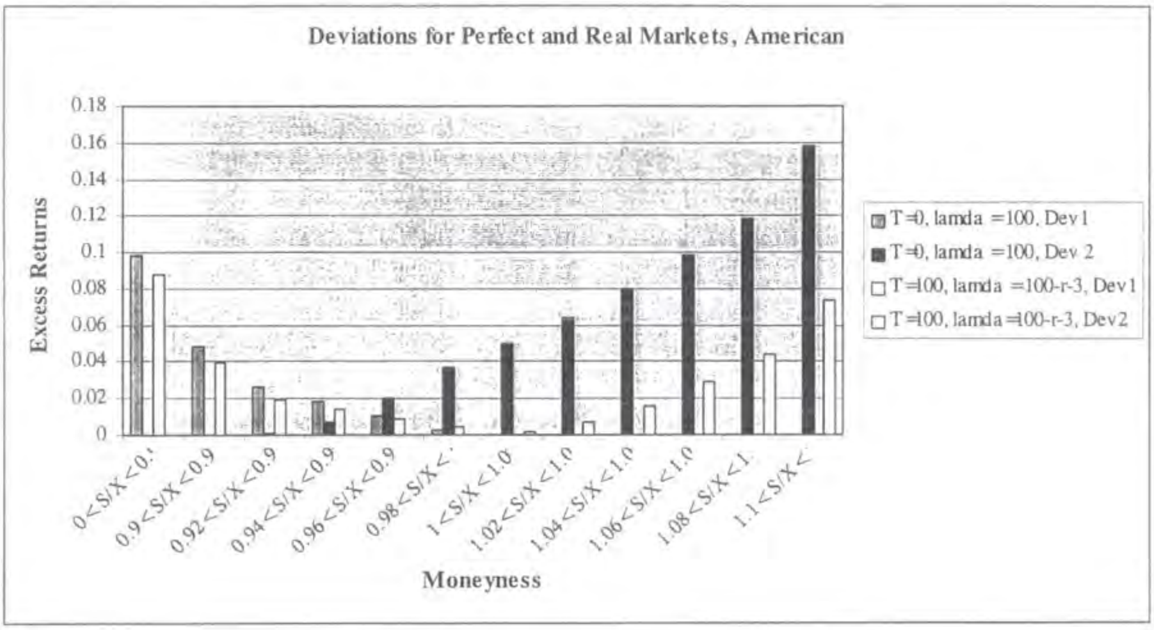


Figure 2.7 below, plots the level of deviations for both strategies but differentiates between perfect and "real" markets. The evidence conclusively confirm the fact that transaction costs alone can remove most of the otherwise documented arbitrage opportunities, the results agree with Gould and Galai (1974). It is also evident from the same graph that compared to deviation one, deviation two show a much greater reduction when market imperfections are considered. This is due to the fact that short sales restrictions are applicable only for trading strategy two. Hence it is concluded that market constraints, in the form of short sales restrictions, can greatly affect the outcome of a trading strategy and if possible should always be taken into account, the results agree with similar findings in Kamara and Miller (1995).

Figure 2.7

Comparison of level of deviations for perfect and imperfect markets, American contracts. Deviations for perfect markets have calculated after taking into account different borrowing and lending rates. Deviations two will require short sales restrictions.



Both McMurray and Yadav (1993) and Dawson (1994) have found evidence of American overpricing for earlier periods in the FTSE-100 option market. Dawson has attributed these to market inefficiencies rather than early exercise premia. Tables 2.9, 2.10 and 2.11 (pages 107-115) summarise the results which correspond to tables 2.5, 2.6 and 2.7 for the European contracts. It is evident from figure 2.8, which compares European to American deviations, that in most cases European PCP trading results in higher excess returns than the American counterparts. These results seem to lend some support to the "Liquidity Risk" hypothesis as opposed to the "Early Exercise" one. We should note, however, that the reported deviations should not be directly related with immediacy risk assuming that the delay execution time of two minutes calculated here is sufficient for trading. Consequently the liquidity hypotheses encompasses a much wider area than the immediacy risk and possibly could include wider factors such as inventory imbalances.

To test further the two hypotheses, a formal statistical inference analysis is performed on the populations of the American and European samples. Again the tests are performed on the complete period between the 4th of July 1994 and the 28th February 1997. Table 2.12 (pages 116-118) below summarises the results for a t-test for mean equality between the European and American samples. The null hypothesis is of equal first moments. The null is rejected. The results suggest statistically higher expected deviations for the European contracts.

However, the t-test is a classical statistical test in the sense that it relies on the assumptions of normally distributed and independent data. To allow for the possibility non-normality, or non-near-normality the Wilcoxon test for mean equality is performed between American and European deviations. As a nonparametric test the Wilcoxon makes no assumptions as to the underlying distribution of the observations, but rather relies on ranking order to test the hypothesis of equal first moments. The results, appearing in table 2.13 (page 120), are similar.

To test the equality of second moments the F distributed variance equality test is performed and the results are reported in the same table. The null hypothesis of equal variances is rejected.

Although the Wilcoxon test is non-parametric in the sense that no distributional assumptions are made, it is specifically adapted to encounter deviations from the normal distribution and the presence of outliers. It is not however appropriate when serial correlation is present in the sample. To avoid biasedness problems arising from serial correlation in the sample, all the above population tests are performed on a random sample (containing all observations) generated from our raw data.

Summarising, the results indicate that statistically the populations differ in the first two moments. Furthermore the level of the excess returns achieved in the case of the American contracts is lower than for the European ones. This indicates that, on average, strategies involving trades in American contracts do not attract a higher degree of risk for deviations of type one.

The above results are interesting in the sense that the supposedly riskier American trading offer less abnormal profits. Thus the results thus far do not offer support for the early exercise hypothesis. On the other hand from Table 2.8 is evident that the American quoting system is more active than the respective European. It seems that this could lend some support to the liquidity hypothesis as this was presented in KM.

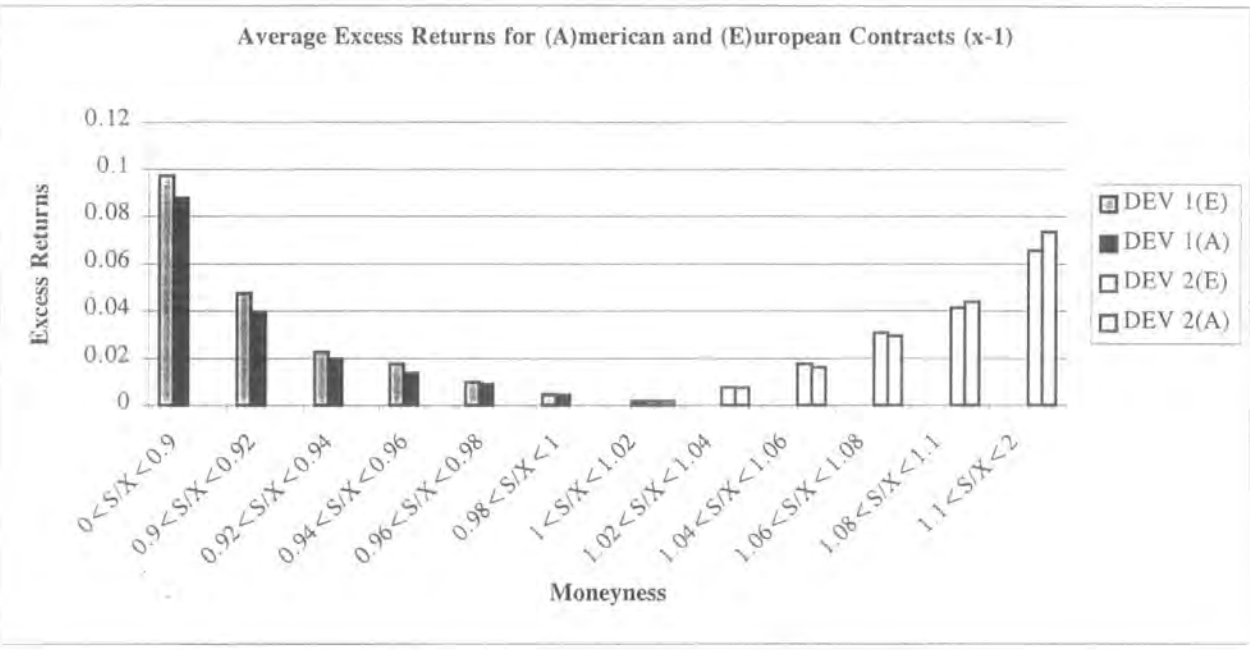
Motivated, however, by the discussion for the early exercise conditions of section 2.2 and the comparison between American and European type two deviations (summarised in figure 2.8), the Wilcoxon rank sum test is performed on the American and European contract for deviations 2 according to moneyness. A one sided test is performed between the two means for each moneyness category seeking to establish the statistical significance of our results.

Table 2.14 (page 121) summarises our findings. It is evident from the results that American trading requires higher risk premia, for deviations two, for moneyness categories $S/X>1.08$. However for different moneyness' categories the European trading offers higher returns. It is very interesting to note at this point that according to tables 2.8 and 2.3, for moneyness categories $1.08<S/X<1.1$ and $1.1<S/X<2$, American parity contracts are quoted less frequently. If one assumes frequency of quotes as a proxy for liquidity, then this implies a less liquid market. For the moneyness categories where American risk premia is lower than the European, the frequency is higher for the American sample. The results contradict the findings in Dawson (1994) where it is evident that both American and European contracts are overpriced with no apparent pattern, however the author does not examine the frequency of quotes or different moneyness categories.

It is possible then that our results conform to both hypotheses. When liquidity is restricted with thin trading, the less liquid market requires a higher return. In our case this is the European market. However, American contracts require higher risk premia for strategies subject to early exercise when the liquidity is comparable or less than the European one. Rather than accepting or rejecting one of the two hypotheses our results offer support to both of them.

Figure 2.8.

Comparison of level of Deviations for American and European contracts. Transactions are equal to the upper boundary of £100 and restrictions are calculated according to the Deficit Account of LSE.



2.7 Conclusions

This chapter examines the issue of Put-Call Parity inefficiencies in the FTSE100 LIFFE market. Previous research in the area has identified significant deviations in both American and European trading. Two explanatory hypotheses prevail, "the early exercise" which is associated with American trading and the "liquidity" one which is associated with the ability to trade at the quoted prices.

Both markets are tested and significant deviations are found for both. Excluding American deviations of type two, it is found that European trading gives rise to higher inefficiencies. Using the frequency of PCP quotes as a liquidity proxy it has been shown that the American market is more liquid. Consequently the results could be attributed to the reduced liquidity of the European market.

American deviations are calculated by a suitable use of the early exercise conditions thus allowing early exercise to dominate deviation of type two values whereas deviations one are in general free from this risk. American deviations of type two are significantly higher than deviations one or European deviations two notwithstanding the reduced liquidity of the European market. Hence the results support the early exercise hypothesis when applicable.

It is noted, however, that the PCP deviations reported here include only cases where stable prices exist for two minutes after posting of quotes. Hence it is argued that these deviations are at large free from the immediacy risk. However, other forms of liquidity premia may exist. For example the nature of trading in LIFFE makes quoted prices sensitive to individual market maker trade balance. When a trading disequilibrium exists this may be reflected in quoted prices. This has considerable effects, though, on the efficiency of the market. Even if individual members post inefficient prices, arbitrageurs should intervene and remove these within a reasonable period.

Having identified the PCP deviations present and subsequently discussed the proposed explanatory hypotheses, this chapter has raised an important question. If there exist PCP deviations in addition to those related to early exercise and immediacy risk, are these identified and rationally removed from the market system?

The following chapter answers the above question.

Moneyness	Transaction Costs	T=100					T=60	T=0
	% of Short Sales available for re	λ=100	λ=97	λ=95	λ=90	λ=100-r-3	λ=100	λ=100
0.90 > S/X	DEV1	100	100	100	100	100	100	100
	DEV2	0	0	0	0	0	0	0
0.90 < S/X < 0.92	DEV1	100	100	100	100	100	100	100
	DEV2	0	0	0	0	0	0	0
0.92 < S/X < 0.94	DEV1	91.9	91.9	91.9	91.9	91.9	92.78	93.94
	DEV2	0	0	0	0	0	0	6.06
0.94 < S/X < 0.96	DEV1	27.5	27.5	27.5	27.5	27.5	29.32	32.95
	DEV2	34.73	0	0	0	0	42.61	45.78
0.96 < S/X < 0.98	DEV1	4.6	4.6	4.6	4.6	4.6	5.1	6.01
	DEV2	81.41	8.92	0	0	0	83.57	85.82
0.98 < S/X < 1.00	DEV1	0.002	0.002	0.002	0.002	0.002	0.12	0.19
	DEV2	94.53	71.04	7.95	0	0	99.6	99.81
1.00 < S/X < 1.02	DEV1	0	0	0	0	0	0	0
	DEV2	100	92.61	54.11	0	2.8	100	100
1.02 < S/X < 1.04	DEV1	0	0	0	0	0	0	0
	DEV2	100	98.47	82.16	0	44.8	100	100
1.04 < S/X < 1.06	DEV1	0	0	0	0	0	0	0
	DEV2	100	100	98.4	12.57	75.9	100	100
1.06 < S/X < 1.08	DEV1	0	0	0	0	0	0	0
	DEV2	100	100	100	49.41	86.4	100	100
1.08 < S/X < 1.1	DEV1	0	0	0	0	0	0	0
	DEV2	100	100	100	81.36	91.3	100	100
1.10 < S/X	DEV1	0	0	0	0	0	0	0
	DEV2	100	100	100	94.7	97.7	100	100

Table 2.9

Violations of DEV1 and DEV2 according to moneyness. The number violations are expressed as percentage of total cases. The table shows values for Transaction Costs = \$0, \$60, \$100 and λ = 100%, 97%, 95%, 90% and 100-r-3%. (American contracts)

Moneyness	Transaction Costs	T=$0		T=$60		T=$100			
		DEV1	DEV2	DEV1	DEV2	DEV1	DEV2		
0<S\X<0.9	μ	-0.09034	-	-0.08294	-	-0.07865	-	μ	0<S\X<0.9
	S.E	0.000503	-	0.000517	-	0.00051	-	S.E	
	median	-0.0879	-	-0.08617	-	-0.0854	-	median	
	S.D	0.0309	-	0.0307	-	0.0306	-	S.D	
	kyrtosis	-0.3479	-	-0.3483	-	-0.3497	-	kyrtosis	
	skewness	-0.40765	-	-0.3972	-	-0.3958	-	skewness	
0.9<S\X<0.92	μ	-0.04709	-	-0.0424	-	-0.03945	-	μ	0.9<S\X<0.92
	S.E	0.00034	-	0.00034	-	0.00032	-	S.E	
	median	-0.04325	-	-0.04219	-	-0.04105	-	median	
	S.D	0.01275	-	0.01269	-	0.0126	-	S.D	
	Kyrtosis	-0.4285	-	-0.4304	-	-0.4357	-	kyrtosis	
	skewness	-0.3483	-	-0.3445	-	-0.3457	-	skewness	
0.92<S\X<0.94	μ	-0.02381	-	-0.02207	-	-0.01975	-	μ	0.92<S\X<0.94
	S.E	0.00031	-	0.00029	-	0.00029	-	S.E	
	median	-0.0256	-	-0.02316	-	-0.0207	-	median	
	S.D	0.0129	-	0.0123	-	0.0121	-	S.D	
	kyrtosis	-0.6823	-	-0.6801	-	-0.06734	-	kyrtosis	
	skewness	-0.6154	-	-0.6237	-	-0.6309	-	skewness	

Table 2.10.

Put-Call parity Deviations w.r.t. transaction costs. The deviations are expressed in the form of excess returns. Dev2 require re-investment of short sale proceeds, no restrictions are calculated (American contracts).

Moneyness	Transaction Costs	T=$0		T=$60		T=$100			
		DEV1	DEV2	DEV1	DEV2	DEV1	DEV2		
0.94<S\X<0.96	μ	-0.01451	-0.0064089	-0.01385	-0.005945	-0.01332	-0.00573	μ	0.94<S\X<0.96
	S.E	0.000278	0.000127	0.000275	0.000127	0.00027	0.000124	S.E	
	median	-0.01486	-0.006287	-0.01407	-0.005831	-0.0139	-0.00562	median	
	S.D	0.0114	0.004836	0.0109	0.00437	0.0107	0.00416	S.D	
	kyrtosis	-0.7345	-0.48698	-0.7201	-0.47590	-0.7137	-0.41507	kyrtosis	
	skewness	-0.4932	-0.57661	-0.4859	-0.48327	-0.4327	-0.53741	skewness	
0.96<S\X<0.98	μ	-0.01309	-0.01969	-0.01239	-0.01773	-0.01174	-0.01664	μ	0.96<S\X<0.98
	S.E	0.00025	0.000183	0.00024	0.000182	0.00024	0.000183	S.E	
	median	-0.01395	-0.01875	-0.01257	-0.01693	-0.01193	-0.01601	median	
	S.D	0.05912	0.008792	0.0587	0.008714	0.0583	0.00864	S.D	
	kyrtosis	-0.7152	-0.59324	-0.7403	-0.52329	-0.7347	-0.58129	kyrtosis	
	skewness	-0.5836	-0.59327	-0.5404	-0.58461	-0.5347	-0.57576	skewness	
0.98<S\X<1	μ	-0.0046	-0.03506	-0.00429	-0.03384	-0.00427	-0.03266	μ	0.98<S\X<1
	S.E	0.00044	0.00021	0.00044	0.000189	0.00042	0.000189	S.E	
	median	-0.00539	-0.03708	-0.00506	-0.03452	-0.00483	-0.03385	median	
	S.D	0.0425	0.009956	0.04142	0.009947	0.04137	0.00983	S.D	
	kyrtosis	-0.6173	-0.36973	-0.5604	-0.39471	0.5327	-0.36479	kyrtosis	
	skewness	-1.0324	-0.8695	-0.9834	-0.82841	-0.9266	-0.81327	skewness	

Table 2.10 (contd)

Put-Call parity Deviations w.r.t. transaction costs. The deviations are expressed in the form of excess returns. Dev2 require re-investment of short sale proceeds, no restrictions are calculated (American contracts).

Moneyness	Transaction Costs	T=$0		T=$60		T=$100			
		DEV1	DEV2	DEV1	DEV2	DEV1	DEV2		
1<S\X<1.02	μ	-	-0.04895	-	-0.04741	-	-0.04639	μ	1<S\X<1.02
	S.E	-	0.000218	-	0.000218	-	0.000216	S.E	
	median	-	-0.05266	-	-0.05194	-	-0.05082	median	
	S.D	-	0.011568	-	0.011497	-	0.01138	S.D	
	kyrtosis	-	-0.33583	-	-0.32683	-	-0.34681	kyrtosis	
	skewness	-	-0.45679	-	-0.45921	-	-0.58682	skewness	
1.02<S\X<1.04	μ	-	-0.06581	-	-0.06361	-	-0.06257	μ	1.02<S\X<1.04
	S.E	-	0.000243	-	0.000243	-	0.000242	S.E	
	median	-	-0.06904	-	-0.06807	-	-0.06832	median	
	S.D	-	0.013854	-	0.01377	-	0.01362	S.D	
	kyrtosis	-	-0.28689	-	-0.38597	-	-0.28596	kyrtosis	
	skewness	-	-0.68853	-	-0.69601	-	-0.68562	skewness	
1.04<S\X<1.06	μ	-	-0.08195	-	-0.07808	-	-0.07632	μ	1.04<S\X<1.06
	S.E	-	0.000274	-	0.000274	-	0.000274	S.E	
	median	-	-0.08267	-	0.08217	-	-0.08109	median	
	S.D	-	0.013501	-	0.01346	-	0.01339	S.D	
	kyrtosis	-	-0.18369	-	-0.18378	-	-0.18467	kyrtosis	
	skewness	-	-0.79367	-	-0.72708	-	-0.81893	skewness	

Table 2.10 (contd).

Put-Call parity Deviations w.r.t. transaction costs. The deviations are expressed in the form of excess returns. Dev2 require re-investment of short sale proceeds, no restrictions are calculated (American contracts).

Moneyness	Transaction Costs	T=$0		T=$60		T=$100			
		DEV1	DEV2	DEV1	DEV2	DEV1	DEV2		
1.06<S\X<1.08	μ	-	-0.09904	-	-0.09672	-	-0.09457	μ	1.06<S\X<1.08
	S.E	-	0.000302	-	0.000303	-	0.000304	S.E	
	median	-	-0.10364	-	-0.09947	-	-0.09836	median	
	S.D	-	0.014896	-	0.01483	-	0.01459	S.D	
	kyrtosis	-	-0.29376	-	-0.2917	-	-0.28375	kyrtosis	
	skewness	-	-0.96374	-	-0.88361	-	-0.82645	skewness	
1.08<S\X<1.1	μ	-	-0.1296	-	-0.1219	-	-0.11954	μ	1.08<S\X<1.1
	S.E	-	0.000336	-	0.000338	-	0.000336	S.E	
	median	-	-0.11074	-	-0.10983	-	-0.10684	median	
	S.D	-	0.01469	-	0.01466	-	0.01467	S.D	
	kyrtosis	-	-0.38752	-	-0.38761	-	-0.30751	kyrtosis	
	skewness	-	-0.11054	-	-0.11966	-	-0.10327	skewness	
1.1<S\X<2	μ	-	-0.16793	-	-0.15974	-	-0.15634	μ	1.1<S\X<2
	S.E	-	0.000329	-	0.000328	-	0.000327	S.E	
	median	-	-0.15326	-	-0.14956	-	-0.14798	median	
	S.D	-	0.02091	-	0.02085	-	0.02067	S.D	
	kyrtosis	-	-0.08874	-	-0.09879	-	-0.09173	kyrtosis	
	skewness	-	0.12654	-	-0.14329	-	-0.15629	skewness	

Table 2.10 (contd).

Put-Call parity Deviations w.r.t. transaction costs. The deviations are expressed in the form of excess returns. Dev2 require re-investment of short sale proceeds, no restrictions are calculated (American contracts).

Moneyness	Percentage of short sales available to investors	λ=100		λ=97		λ=95		λ=90		λ=100-r-0.03 (restrictions imposed by LSE under the capital redemption account)	
		DEV1	DEV2	DEV1	DEV2	DEV1	DEV2	DEV1	DEV2	DEV1	DEV2
0<S\X<0.9	μ	-0.07865	-	-0.07865	-	-0.07865	-	-0.07865	-	-0.07865	-
	S.E	0.00051	-	0.00051	-	0.00051	-	0.00051	-	0.00051	-
	med.	-0.0854	-	-0.0854	-	-0.0854	-	-0.0854	-	-0.0854	-
	S.D	0.0306	-	0.0306	-	0.0306	-	0.0306	-	0.0306	-
	kyrt.	-0.3497	-	-0.3497	-	-0.3497	-	-0.3497	-	-0.3497	-
	sk.	-0.3958	-	-0.3958	-	-0.3958	-	-0.3958	-	-0.3958	-
0.9<S\X<0.92	μ	-0.03945	-	-0.03945	-	-0.03945	-	-0.03945	-	-0.03945	-
	S.E	0.00032	-	0.00032	-	0.00032	-	0.00032	-	0.00032	-
	med.	-0.04105	-	-0.04105	-	-0.04105	-	-0.04105	-	-0.04105	-
	S.D	0.0126	-	0.0126	-	0.0126	-	0.0126	-	0.0126	-
	kyrt.	-0.4357	-	-0.4357	-	-0.4357	-	-0.4357	-	-0.4357	-
	sk.	-0.3457	-	-0.3457	-	-0.3457	-	-0.3457	-	-0.3457	-
0.92<S\X<0.94	μ	-0.01975	-	-0.01975	-	-0.01975	-	-0.01975	-	-0.01975	-
	S.E	0.00029	-	0.00029	-	0.00029	-	0.00029	-	0.00029	-
	med.	-0.0207	-	-0.0207	-	-0.0207	-	-0.0207	-	-0.0207	-
	S.D	0.0121	-	0.0121	-	0.0121	-	0.0121	-	0.0121	-
	kyrt.	-0.6734	-	-0.6734	-	-0.6734	-	-0.6734	-	-0.6734	-
	sk.	-0.6309	-	-0.6309	-	-0.6309	-	-0.6309	-	-0.6309	-

Table 2.11.

Put-Call parity Deviations w.r.t short sales restrictions. The deviations are expressed in excess returns form. Deviations of type 2 require re-investment of short sales proceeds (American contracts).

Moneyness	Percentage of short sales available to investors	λ=100		λ=97		λ=95		λ=90		λ=100-r-0.03 (restrictions imposed by LSE under the capital redemption account)	
		DEV1	DEV2	DEV1	DEV2	DEV1	DEV2	DEV1	DEV2	DEV1	DEV2
0.94<S\X<0.96	μ	-0.01332	-0.00573	-0.01332	-	-0.01332	-	-0.01332	-	-0.01332	-
	S.E	0.00027	0.000124	0.00027	-	0.00027	-	0.00027	-	0.00027	-
	med.	-0.0139	-0.00562	-0.0139	-	-0.0139	-	-0.0139	-	-0.0139	-
	S.D	0.0107	0.00416	0.0107	-	0.0107	-	0.0107	-	0.0107	-
	kyrt.	-0.7137	-0.41507	-0.7137	-	-0.7137	-	-0.7137	-	-0.7137	-
	sk.	-0.4327	-0.53741	-0.4327	-	-0.4327	-	-0.4327	-	-0.4327	-
0.96<S\X<0.98	μ	-0.00897	-0.01664	-0.00897	-0.00201	-0.00897	-	-0.00897	-	-0.00897	-
	S.E	0.00024	0.000183	0.00024	0.000175	0.00024	-	0.00024	-	0.00024	-
	med.	-0.01193	-0.01601	-0.01193	-0.00213	-0.01193	-	-0.01193	-	-0.01193	-
	S.D	0.0583	0.00864	0.0583	0.008731	0.0583	-	0.0583	-	0.0583	-
	kyrt.	-0.7347	-0.58129	-0.7347	-0.59456	-0.7347	-	-0.7347	-	-0.7347	-
	sk.	-0.5347	-0.57576	-0.5347	-0.62368	-0.5347	-	-0.5347	-	-0.5347	-
0.98<S\X<1	μ	-0.00427	-0.03266	-0.00427	-0.01036	-0.00427	-0.00163	-0.00427	-	-0.00427	-
	S.E	0.00042	0.000189	0.00042	0.000176	0.00042	0.000102	0.00042	-	0.00042	-
	med.	-0.00483	-0.03385	-0.00483	-0.01345	-0.00483	-0.00171	-0.00483	-	-0.00483	-
	S.D	0.04137	0.00983	0.04137	0.00906	0.04137	0.002641	0.04137	-	0.04137	-
	kyrt.	0.5327	-0.36479	0.5327	-0.39527	0.5327	-0.38576	0.5327	-	0.5327	-
	sk.	-0.9266	-0.81327	-0.9266	-0.80364	-0.9266	-0.896215	-0.9266	-	-0.9266	-

Table 2.11 (contd).

Put-Call parity Deviations w.r.t short sales restrictions. The deviations are expressed in excess returns form. Deviations of type 2 require re-investment of short sales proceeds (American contracts).

Moneyness	Percentage of short sales available to investors	λ=100		λ=97		λ=95		λ=90		λ=100-r-0.03 (restrictions imposed by LSE under the capital redemption account)	
		DEV1	DEV2	DEV1	DEV2	DEV1	DEV2	DEV1	DEV2	DEV1	DEV2
1<S\X<1.02	μ	-	-0.04639	-	-0.02106	-	-0.00693	-	-	-	-0.00179
	S.E	-	0.000216	-	0.000143	-	0.000113	-	-	-	7.82*E-5
	med.	-	-0.05082	-	-0.02117	-	-0.00698	-	-	-	-0.00147
	S.D	-	0.01138	-	0.000903	-	0.009339	-	-	-	0.001425
	kyrt.	-	-0.34681	-	-0.39876	-	-0.45897	-	-	-	0.961021
	sk.	-	-0.58682	-	-0.66571	-	-0.80699	-	-	-	-1.06711
1.02<S\X<1.04	μ	-	-0.06257	-	-0.033871	-	-0.01831	-	-	-	-0.00744
	S.E	-	0.000242	-	0.00021	-	0.00014	-	-	-	0.000119
	med.	-	-0.06832	-	-0.03396	-	-0.01857	-	-	-	-0.00649
	S.D	-	0.01362	-	0.009986	-	0.00643	-	-	-	0.005108
	kyrt.	-	-0.28596	-	-0.37624	-	-0.39664	-	-	-	-0.39144
	sk.	-	-0.68562	-	-0.78589	-	-0.95798	-	-	-	-0.64189
1.04<S\X<1.06	μ	-	-0.07632	-	-0.047643	-	-0.03036	-	-0.00094	-	-0.0162
	S.E	-	0.000274	-	0.000206	-	0.000213	-	0.00017	-	0.000193
	med.	-	-0.08109	-	-0.048657	-	-0.03098	-	-0.00098	-	-0.0163
	S.D	-	0.01339	-	0.010695	-	0.011754	-	0.010663	-	0.009018
	kyrt.	-	-0.18467	-	-0.57864	-	-0.63614	-	-0.96478	-	-0.81321
	sk.	-	-0.81893	-	-0.98331	-	-0.81669	-	-0.56384	-	-0.17101

Table 2.11 (contd).

Put-Call parity Deviations w.r.t short sales restrictions. The deviations are expressed in excess returns form. Deviations of type 2 require re-investment of short sales proceeds (American contracts).

Moneyness	Percentage of short sales available to investors	λ=100		λ=97		λ=95		λ=90		λ=100-r-0.03 (restrictions imposed by LSE under the capital redemption account)	
		DEV1	DEV2	DEV1	DEV2	DEV1	DEV2	DEV1	DEV2	DEV1	DEV2
1.06<S\X<1.08	μ	-	-0.09457	-	-0.06457	-	-0.04214	-	-0.00483	-	-0.02939
	S.E	-	0.000304	-	0.000276	-	0.000413	-	0.000106	-	0.000233
	med.	-	-0.09836	-	-0.06359	-	-0.04137	-	-0.00469	-	-0.02992
	S.D	-	0.01459	-	0.01357	-	0.01039	-	0.009675	-	0.011056
	kyrt.	-	-0.28375	-	-0.44576	-	-0.34376	-	-0.383961	-	-0.5079
	sk.	-	-0.82645	-	-0.36564	-	-0.58617	-	-0.89883	-	0.184896
1.08<S\X<1.1	μ	-	-0.11954	-	-0.08462	-	-0.06432	-	-0.01561	-	-0.04369
	S.E	-	0.000336	-	0.000331	-	0.0004132	-	0.000198	-	0.000286
	med.	-	-0.12563	-	-0.08734	-	-0.06725	-	-0.01566	-	-0.0451
	S.D	-	0.01467	-	0.01987	-	0.016437	-	0.018634	-	0.012796
	kyrt.	-	-0.30751	-	-0.39854	-	-0.45488	-	-0.474134	-	0.916619
	sk.	-	-0.10327	-	-0.11492	-	-0.09899	-	-0.09645	-	0.804267
1.1<S\X<2	μ	-	-0.15634	-	-0.12807	-	-0.10307	-	-0.04101	-	-0.07343
	S.E	-	0.000327	-	0.000287	-	0.000643	-	0.000349	-	0.000329
	med.	-	-0.15798	-	-0.12076	-	-0.10001	-	-0.04876	-	-0.07336
	S.D	-	0.02067	-	0.02647	-	0.02148	-	0.037643	-	0.026571
	kyrt.	-	-0.09173	-	-0.10754	-	-0.19837	-	-0.32578	-	0.320592
	sk.	-	-0.15629	-	-0.27543	-	-0.34386	-	-0.88512	-	0.156893

Table 2.11 (contd).

Put-Call parity Deviations w.r.t short sales restrictions. The deviations are expressed in excess returns form. Deviations of type 2 require re-investment of short sales proceeds (American contracts).

Excess Returns / Moneyness	0<S\X<0.9 (sample = 3674)		0.9<S\X<0.92 (sample = 2461)		0.92<S\X<0.94 (sample = 3056)		0.94<S\X<0.96 (sample = 3305)		0.96<S\X<0.98 (sample = 4378)		0.98<S\X<1 (sample = 11094)	
	DEV1	DEV2	DEV1	DEV2	DEV1	DEV2	DEV1	DEV2	DEV1	DEV2	DEV1	DEV2
0.0-0.01	-	-	-	-	28.04	4.47	16.97	18.21	3.26	6.38	0.18	1.62
0.01-0.02	-	-	3.52	-	32.7	0.53	5.67	9.72	2.36	9.07	0.01	3.97
0.02-0.03	-	-	23.6	-	16.72	0.96	4.44	10.01	0.37	14.46	-	8.45
0.03-0.04	0.37	-	33.31	-	8.1	0.12	3.29	5.36	0.02	25.94	-	18.12
0.04-0.05	9.24	-	21.25	-	4.79	-	0.58	2.24	-	17.03	-	29.07
0.05-0.06	15.18	-	8.09	-	2.83	-	-	0.42	-	11.08	-	23.46
0.06-0.07	15	-	5.56	-	0.65	-	-	-	-	2.13	-	11.01
0.07-0.08	12.98	-	3.37	-	0.11	-	-	-	-	0.73	-	2.61
0.08-0.09	11.55	-	1.21	-	-	-	-	-	-	-	-	1.12
0.09-0.10	9.58	-	0.09	-	-	-	-	-	-	-	-	0.18
0.10-0.11	6.95	-	-	-	-	-	-	-	-	-	-	0.2
0.11-0.12	4.38	-	-	-	-	-	-	-	-	-	-	-
0.12-0.13	3.63	-	-	-	-	-	-	-	-	-	-	-
0.13-0.14	3.1	-	-	-	-	-	-	-	-	-	-	-
0.14-0.15	1.68	-	-	-	-	-	-	-	-	-	-	-
0.15-0.16	1.61	-	-	-	-	-	-	-	-	-	-	-
0.16-0.17	1.12	-	-	-	-	-	-	-	-	-	-	-
0.17-0.18	1.01	-	-	-	-	-	-	-	-	-	-	-
0.18-0.19	0.75	-	-	-	-	-	-	-	-	-	-	-
0.19-0.20	0.75	-	-	-	-	-	-	-	-	-	-	-
0.20-0.21	0.45	-	-	-	-	-	-	-	-	-	-	-

Table 2.12.

Distribution of percentage deviations w.r.t. moneyness. The deviations are calculated with zero transaction costs and no short sales restrictions (American contracts).

Excess Returns	Moneyness	0<S\X<0.9 (sample = 3674)		0.9<S\X<0.92 (sample = 2461)		0.92<S\X<0.94 (sample = 3056)		0.94<S\X<0.96 (sample = 3305)		0.96<S\X<0.98 (sample = 4378)		0.98<S\X<1 (sample = 11094)	
		DEV1	DEV2	DEV1	DEV2	DEV1	DEV2	DEV1	DEV2	DEV1	DEV2	DEV1	DEV2
0.21-0.22		0.18	-	-	-	-	-	-	-	-	-	-	-
0.22-0.23		0.27	-	-	-	-	-	-	-	-	-	-	-
0.23-0.24		0.18	-	-	-	-	-	-	-	-	-	-	-
0.24-0.25		0.04	-	-	-	-	-	-	-	-	-	-	-

Table 2.12 (contd)

Distribution of percentage deviations w.r.t. moneyness. The deviations are calculated with zero transaction costs and no short sales restrictions (American contracts).

Excess Returns / Moneyness	1<S\X<1.02 (sample = 11072)		1.02<S\X<1.04 (sample = 4136)		1.04<S\X<1.06 (sample = 2981)		1.06<S\X<1.08 (sample = 2711)		1.08<S\X<1.1 (sample = 2198)		1.1<S\X<2 (sample = 6672)	
	DEV1	DEV2	DEV1	DEV2	DEV1	DEV2	DEV1	DEV2	DEV1	DEV2	DEV1	DEV2
0.0-0.01	-	0.51	-	0.41	-	0.32	-	-	-		-	1.68
0.01-0.02	-	1.77	-	0.92	-	0.4	-	0.41	-		-	2.5
0.02-0.03	-	4.32	-	3.03	-	4.23	-	0.57	-	0.05	-	2.14
0.03-0.04	-	9.12	-	7.96	-	8.51	-	3.6	-	0.08	-	3.24
0.04-0.05	-	14.55	-	12.04	-	11.26	-	7.03	-	2.09	-	6
0.05-0.06	-	27.21	-	17.64	-	16.01	-	9.21	-	4.96	-	11.67
0.06-0.07	-	22.03	-	23.09	-	22.39	-	14.14	-	8.86	-	15.78
0.07-0.08	-	13.97	-	16.31	-	22.01	-	25.77	-	20.11	-	16
0.08-0.09	-	3.42	-	11.88	-	12.77	-	18.7	-	25.21	-	13.5
0.09-0.10	-	2.11	-	3.25	-	1.25	-	9.21	-	23.16	-	10.12
0.10-0.11	-	0.69	-	2.01	-	0.46	-	7.32	-	10.78	-	6.91
0.11-0.12	-	0.3	-	1.09	-	0.18	-	3.2	-	3.09	-	4.58
0.12-0.13	-		-	0.26	-	0.05	-	0.51	-	1.12	-	2.4
0.13-0.14			-	0.11	-	0.12	-	0.32	-	0.22	-	0.84
0.14-0.15			-		-	0.03	-	0.01	-	0.17	-	0.15
0.15-0.16			-		-		-		-	0.08	-	0.05
0.16-0.17			-		-		-		-	0.02	-	0.09
0.17-0.18			-		-		-		-		-	0.01
0.18-0.19			-		-		-		-		-	0.02
0.19-0.20			-		-		-		-		-	0

Table 2.12(contd).

Distribution of percentage deviations w.r.t. moneyness. The deviations are calculated with zero transaction costs and no short sales restrictions (American contracts).

Wilcoxon				t-Test				Variance-Test			
Sample Estimates for uncensored sample											
Am	Eur	Am	Eur	Am	Eur	Am	Eur	Am	Eur	Am	Eur
DEV1	DEV1	DEV2	DEV2	DEV1	DEV1	DEV2	DEV2	DEV1	DEV1	DEV2	DEV2
-	-	-	-	-0.0716	-0.0801	-0.0459	-0.054	0.001467	0.001699	0.001859	0.002469
Z = 13.02 p-value=0		Z=21.15 p-value = 0		t =13.53 p-value=0		t=25.29 p-value = 0		F=0.8637 p-value=0		F=0.7532 p-value = 0	
Null: equal means (95%)				Null: equal means (95%)				Null: equal variances (95%)			

Table 2.13

Population analysis for American and European deviations. The table refers to uncensored observations, i.e. contains the last observation every day.

Table 2.14

Wilcoxon rank sum test (one sided) comparing expected risk premia for American and European type two deviations.

Moneyness	0<S\X<0.9 Eur>Amer	0.9<S\X<0.92 Eur>Amer	0.92<S\X<0.94 Eur>Amer	0.94<S\X<0.96 Eur>Amer	0.96<S\X<0.98 Eur>Amer	0.98<S\X<1 Eur>Amer
Z value	21.52	12.7	6.32	5.47	2.92	2.89
P value	0	0	0	0	0.0018	0.0019

Table 2.14 (contd)

Wilcoxon rank sum test (one sided) comparing expected risk premia for American and European type two deviations.

Moneyness	1<S\X<1.02 Eur>Amer	1.02<S\X<1.04 Eur>Amer	1.04<S\X<1.06 Eur>Amer	1.06<S\X<1.08 Eur>Amer	1.08<S\X<1.1 Eur<Amer	1.1<S\X<2 Eur<Amer
Z value	1.98	2.55	3.05	2.68	8.21	13.44
P value	0.0239	0.0054	0.0011	0.0037	0	0

CHAPTER 3

Testing for the Efficiency of the FTSE 100 LIFFE Market

Chapter 3

3.1 Introduction

The first part of this study has resulted in the identification of some significant arbitrage opportunities, net of transaction and other costs. It has been shown that in addition to deviations directly related to the early exercise risk, and consequently not subject to arbitrage trading, and the immediacy risk that we have reduced by delaying execution time, further deviations remain. These may or may not be due to further liquidity constraints such as inventory problems of individual market makers. In any case because until now these have not been rationally explained they can only be treated as inefficiencies. *The question then remains if these deviations prove that the FTSE100 LIFFE market is inefficient.*

In the current chapter we argue that this does not constitute sufficient findings to characterise the market as inefficient. Whether or not, constraints such as liquidity and inventory problems manifest themselves as gross PCP deviations this does not constitute a fair test for market efficiency; in a high frequency framework such imbalances could be unavoidable in the short term. In a well functioning market it would be expected that these misspricings would gradually be erased from the quoting system.

The progressive eradication of market inefficiencies is acknowledged in O'Hara (1995). According to the author, in a Bayesian framework of learning, prices will progressively tend to the fair value thus establishing strong form market efficiency. As the author quickly accepts though, it is not at all clear that the learning process will be instantaneous. In the sequential trading models of Glosten and Milgrom (1985) and Easley and O'Hara (1992) it is theoretically demonstrated that prices can tend to their informational values gradually. Although not specific about the actual time process nevertheless the models accept a progressive price correction.

It is evident then, that the relevant literature acknowledges price adjustment processes and therefore time as a variable in microstructure models. Furthermore, option markets offer reduced transaction costs for some trade strategies (e.g. index trading), are highly leveraged, pose fewer restrictions and thus are preferable to the spot exchange for some investors. Assuming that market prices reflect the flow of information we would expect to see an intraday variation of prices reflecting news arrival. Hence the time as a variable will contain information with regards to the dynamics of the market and more precisely of the adjustment speed factor. Assuming that the duration between the information arrival and its manifestation in price changes can be identified the speed of the market adjustment to new information signals can be critically assessed. Here, the arrival of a new PCP deviation is treated as a "news signal" reaching the market and the time it stays in the system is examined. It is assumed that this time denotes the reaction speed of the market to the new signal. It is proposed that this adjustment speed constitutes a test for market efficiency to new information release.

It may be logical, though, that in our case we need not differentiate between arbitrageurs and informed traders[1]. In this study the trading signals arriving in the market are the PCP deviations and these constitute information on which the arbitrageur trades with certainty. In an efficient market it would be expected that all these deviations would be short lived. Market forces should ensure that risk free opportunities do not persist and are replaced by efficient quotes, thus initiating the price correcting mechanism starting progressively from the larger deviations.

However, in a more careful analysis it may observed that market frictions such as capital and liquidity constraints and perhaps other unobserved market factors impede the price correcting forces. Even in such an environment it should be expected that some more limited arbitrage trading would take place targeting the larger inefficiencies first. With respect to the above analysis we state the following formal test for market efficiency in a high frequency context:

> "in a high frequency analysis, prolonged deviations from equilibrium values constitute evidence against market efficiency. In efficient markets deviations from equilibrium prices should be quickly identified. Corrective market forces should target the causal factors of these deviations and gradually *tend* to re-introduce equilibrium prices as reflected in Put-Call Parity."

[1] This is not as strong as it suggests. We do not wish to describe the specific market participants, neither we do wish to exclude noise traders from such description. We merely suggest that our analysis need not differentiate between

In these "resources constrained" settings it is assumed that the level of deviation would constitute a dominant factor in the duration of the quote. Hence the current study tests the hypothesis of an efficient market by evaluating for the significance of the PCP deviations as a duration covariate, i.e. the ability of market participants to identify inefficiencies and eradicate them according to their significance.

We should note here, however, that in correspondence to Chapter 2, Chapter 3 makes the explicit assumption that all the arbitrage opportunities are exploitable. To allow for this assumption the present study considers put or call quotes allowing a time interval of 2 minutes until the next revised quote. This should ensure that, at least under equilibrium, there is enough time to complete both required sides of the arbitrage trade. Essentially the arbitrage trade involves positions in the option market and the replication of the option in the underlying market. Assuming that the option is a redundant security this should not be difficult to implement. If we take the view, though, that an option is not a redundant security under price disequilibrium the following are noted. A redundant security should not increase the opportunity set of investors. In periods of disequilibrium[2] differences in supply and demand should be expected, and hence there will be some inability of the market to absorb all trading requests (either sell or buy). Consequently the ability to replicate *every* option contract should break down. Hence during that period an option contract will indeed augment the opportunity set of investors, but only because this has been

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them. Deviations is manifested information on market prices.

[2] These could refer either to periods of significant market stress as is the case in October 1987, or temporary intra-day periods of disequilibrium.

diminished by disequilibrium forces in the market. Hence there exist a possibility, which we cannot control for, that the results presented in the next section are biased, ceteris paribus, towards rejection of the market efficiency hypothesis.

In what follows, the widely used Cox Proportional Hazards model (CPH) as proposed by Cox (1972) and later extended by Fleming and Harrington (1991) as the multiplicative hazards model, is applied. The following sections give a critical description of some models of transition data analysis. The objective is to model the duration process of the PCP deviations as an event history model and choose a specification capable of capturing the dependence of durations with PCP deviations as covariates. Treating the sample as an event history process enables the examination of each deviation individually and the assessment of its Hazard. The theoretical analysis that follows is very detailed and extensive. It is so because the topic of transitional data analysis has found very little application in the area of finance in the past. Thus a critical analysis examining the general theoretical framework is of importance.

3.2 Models of transition data analysis.

The Econometric study of event duration or of its time of arrival falls into the broader area of Transition Data Analysis (TDA). As the name suggests the models in this category examine the time before a variable changes state or the time it spends in a transitory state. Initially scientists have employed TDA theory to find solutions in engineering or biomedical research problems and only recently have econometricians shown interest.

In Economics, TDA has mainly evolved around the area of labour economics and more profoundly in the measurement of unemployment periods for individuals. To the best of our knowledge, no applications exist on the analysis of derivative markets or generally in finance.

Suppose that we are interested in the likelihood of an unemployment spell, which already occurs for a time period T, being terminated within time interval dt. Ideally, to do so, all the variables, which are likely to influence the probability of employment have to be grouped in a single vector. After constructing such $x(t)$ it can be assumed that a probability of a job offer being made within dt exists and is denoted by $\lambda(x(t))dt$. If $P(x(t))$ is the probability measure of such an offer being accepted then

$$\theta(x(t))\,dt = \lambda(x(t))\,P(x(t))\,dt \qquad \text{Eq 3.1}$$

exists and denotes the transitional probability of exit to employment.

As it appears above, Eq 3.1 (in it θ (x(t)) is well known in the duration literature as *hazard function*) describes the probability of a single state occupation or similarly the probability of transition between two states. Transition data analysis, however, does not limit itself to two state problems but extends to cover multiple destinations. Having identified ζ destinations, which are exhaustive and mutually exclusive, the probability of departure from the current zero state[3] to state ζ within dt conditional on survival to t can be calculated as

$$\theta_\zeta (t) = \lim_{dt \to 0}[\, P\, (t \leq T < t+dt,\ D_\zeta = 1 \mid T \geq t)] \,/\, dt \qquad \textbf{Eq 3.2}$$

which often is called the transition intensity to state ζ. It follows then that the hazard function is the sum of Eq 3.2 over the identified destination states,

$$\theta\ (t) = \sum_{\zeta=1}^{Z} \theta_\zeta (t) \qquad \textbf{Eq 3.3}$$

or

$$\theta(t) = \int_{\zeta=1}^{Z} \theta_\zeta\, dt \qquad \textbf{Eq 3.4}$$

[3] We will call zero state the situation of survival thus far; unemployment, no Put Call Parity deviation etc. Termination of unemployment or eradication of the

In traditional models of unemployment spells, hazard functions pose some serious design problems, which sometimes extend to neighbouring applications. Transition models are considered stochastic "at birth" as they are trying to model and forecast a transition to a destination state of which the occurrence is not certain. In comparison, for example, to standard econometric models which are trying to measure the intensity of the endogenous variable, measurable at $t+1$ with complete certainty, transition models are required to predict, first of all, whether or not a destination state will occur and subsequently and wherever applicable, the intensity. Also in a similar fashion to other related econometric models, stochastic variation of elements of $x\ (t)$ over individuals introduces what is known as *neglected heterogeneity* which may create some intrinsic problems associated with the observation of data.

Even in the case where the stochastic model, which can describe the transition probabilities over a part of the population, has been established, neglected heterogeneity will result in the inferences being valid for just a sub-sample of the population and invalid for the rest[4].

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deviation denotes change to state one.

[4] It is possible that one may argue that neglected heterogeneity arises in studies of labour economics, or more generally in panel data situations and not in time series samples like the current one. Later on we will make the assumption that due to the number and competition among the market makers heterogeneity does not arise due to the, possibly, different sourcing of quotes. It is possible though that a form of heterogeneity in our data exists simply because there is a strong dependence between the level of deviation, or the length of durations and the moneyness or the time of the day the quotes are entered into the system. This may result in a complex distributional problem where the overall sample is drawn from different individual overlapping distributions. We address the problem by removing the expected component of the dependent values

Heterogeneity in observations, in some instances means that the sample population is a construction of sub-populations where each subset can be successfully characterised by individual distributions. In such cases assuming that the hazard function can be expressed as

$$\varphi\ (t; x, z) \qquad \textbf{Eq 3.5}$$

where z is an individual or sub-sample specific realisation of the heterogeneity measure z and also time independent in nature[5]. Eq. 3.5 can be used to evaluate a probability φ (t; x, z) of transition between the two states *conditional on z* (i.e. for homogeneous in z sub-samples).

Inferences on the whole sample though, or wherever z is not measurable for sub-samples or individuals, are not valid. This is because Eq 3.5 gives us a hazard function (call it H) which is drawn on distributions unconditional on z. This failure to capture the real variability of z can be addressed if it is assumed that the real distribution is a mixture of distributions of z, where z is homogeneous across the sub-sample. Assuming that such realisations are governed by a function Q (z; x) the real hazard function, conditional on z variation, can be found by integrating the joint distribution of z and H over z

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conditional on the past history of the process.

[5] As suggested by Lancaster (1979) this could be augmented to a stochastically driven process u(t) in order to accommodate time variation of the exogenous parameters within the sub-sample.

(i.e. Q (z; x)). Such a model is called a mixture model and overcomes the problem of heterogeneity conditional on knowledge of function Q.

Considering Eq. 3.2 again, it is seen that it gives a measure of the transitional probability between two states, during time dt, conditional on having occupied the zero state for a time period of t. However, as mentioned above occurrence problems and neglected heterogeneity, which are often present in the models, can make statistical inferences biased and inaccurate.

An alternative solution may lie with fully parametric inference models, which provide unbiased and more robust techniques. As the name suggests the distributional specification of the data is totally unrelated to what is assumed to be a given number of unknown model parameters. In such an approach the assumptions and construction of the transitional probabilities are drawn conditional on the heterogeneity terms, which still remain unknown. We should note that up to this stage the specification of the transition intensities assumes the observation of the vector parameters x and does not contain any unmeasured heterogeneity. Following specification of the transition intensities fully parametric inference requires construction of the heterogeneity factors conditional on a specific number of regressors. Using the above, the likelihood which describes the data and hence its maximum value estimates can be calculated. For the purposes of correcting neglected heterogeneity this methodology offers a superior approach to other models discussed in this section, under the above specification though it may not be very appropriate for a multiple event study.

Thus far we have considered and drew our theoretical paradigms from the area of labour economics. Within that area of interest individuals will mainly consider a transition between unemployment and employment very few times throughout their labour active life. It is natural then that the history on an individual will not contain sufficiently enough information to be utilised in an event history type model. In this case the analysis rely on information relegated to the covariate matrix to provide statistical inferences. Generally speaking though, duration models have allowed inferences on models by collecting a substantial degree, or the complete set, of the information from past observations of the particular event. In such cases, which are also known as event history models, the transition probabilities are given by the following hazard function type:

$$\varphi_{kl}\ (t,\ S\ \Omega)\ ds \qquad \textbf{Eq 3.6}$$

where,

k,l denote the departure and destination states,

t denotes the entry time,

S denotes the time spent in the particular state and

Ω is the history vector containing information for the past behaviour of the model. Ω could depend on the number of entries/ exits to/from the state under consideration, a feature called "occurrence dependence" and the length of time spent in the state of interest which is known as "lagged duration dependence". For a specific event study comprising of multiple cycles (C) Eq. 3.6 becomes

$$\varphi^{c}_{kl}(t_c, S) \qquad \textbf{Eq 3.7}$$

where the superscript identifies a specific cycle and t_c the calendar time of entry in the cycle. The equation itself gives the instantaneous rate of exit to state l per unit time for occupancy of k at t_c. For that particular model if it is assumed that u is the specific time of stay in the state since arrival and integrate over 0 to S, where again S is the calendar time spent in the state, the integrated transition intensity is

$$Z^{c}_{kl}(t_c, S) = \int_0^S \varphi^{c}_{kl}(t_c, u)\, du \qquad \textbf{Eq 3.8}$$

where

Z^{c}_{kl} is the integrated transition intensity, which can be thought of as an indication of the probability of exit during time 0 to S.

3.3 High Frequency Financial Markets and Random Point Processes

The previous section has referred to the family of Transition Data Models. It presented the general framework and the intuition behind the econometric models of duration analysis used predominantly in the area of labour economics, and briefly introduced the theoretical framework for the methodology used here. The following part concentrates on duration models more suitable to financial market data, eventually leading to the discussion to the Cox Proportional Hazard Model, as proposed in Cox (1972) and the Autoregressive Conditional Duration model as proposed in Engle & Russell (1997). Again, applications of the two methodologies (and especially of the Cox model) are very scarce in the area of Finance. As we seek to provide some evidence of the suitability of the two models in the current research framework the analysis that follows is quite detailed.

Equation 3.1 (repeated below) gives the probability of transition between states one and zero during time dt. To do so it makes use of the hazard function of an individual, or more generally of a process, which can be thought of as *the instantaneous rate of transition* between states. More formally the hazard function can be expressed as

$$\theta(t) = \lim_{dt \to 0} \frac{P(t \leq T < t + dt | T \geq t)}{dt} \qquad \textbf{Eq 3.9}$$

(And also

$$\theta\,(x(t))\,dt = \lambda\,(x(t))\,P(x(t))\,dt \qquad \text{(Eq 3.1))}.$$

In the case of the present study the hazard function is used to evaluate the probability of having an economically significant PCP deviation quote within an increasingly short period of time dt. A valid analysis of the problem will require correct specification of the conditional intensity process (the hazard function) together with some form of suitably chosen statistical assumptions in order to derive the appropriate parametric or nonparametric statistical inferences. Correct specification of the conditional intensity can be done either through Eq 3.9, or through the conditional density function of durations or through specification of the conditional survivor function[6]. Indeed as it is shown below[7] the three expressions are equivalent. Assuming f(t) and F(t) as the values of the probability density and the distribution function of T at t, with

$$\mathbf{F(t) = P(T < t)\ and\ f(t)} = \frac{dF}{dt} \qquad \textbf{Eq 3.10}$$

From Eq 3.9 it is seen that the conditional intensity is a function of $P(t \leq T < t+dt \mid T \geq t)$ and, by the law of conditional probability, it follows that

[6] The survivor function is simply one minus the distribution function, basically it gives us the proportion of sample which will *survive* the experiment.
[7] The following steps appear in Lancaster (1990) and Snyder and Miller (1991).

$$\mathbf{P(t \leq T < t+dt \mid T \geq t)} = \frac{P(t \leq T < t+dt, T \geq t)}{P(T \geq t)} \qquad \textbf{Eq 3.11}$$

which since $t \leq T < t+dt \cap T \geq t = t \leq T < t+dt$ gives

$$\mathbf{P(t \leq T < t+dt \mid T \geq t)} = \frac{P(t \leq T < t+dt)}{P(T \geq t)}. \qquad \textbf{Eq 3.12}$$

Writing the probabilities in terms of the distribution function (F(t)) then gives

$$\mathbf{P(t \leq T < t+dt \mid T \geq t)} = \frac{F(t+dt) - F(t)}{1 - F(t)} \qquad \textbf{Eq 3.13}$$

dividing by dt as dt tends to zero, Eqs 3.9 and 3.13 then yield

$$\theta\ \mathbf{(t)} = \lim_{dt \to 0} \frac{F(t+dt) - F(t)}{dt} \frac{1}{1 - F(t)} \qquad \textbf{Eq 3.14}$$

which since $\frac{F(t+dt) - F(t)}{dt}$ equals to f(t) gives

$$\theta\ \mathbf{(t)} = \frac{f(t)}{1 - F(t)} \qquad \textbf{Eq 3.15}$$

(or $\theta(t) = \frac{f(t)}{1-F(t)}$ when expressed in terms of the survivor function Eq 3.15a)

Eqs 3.9 and 3.15 (& 3.15a) are both valid and equivalent expressions of the hazard function for a continuous random variable T (the duration).

Based on this result a valid representation of the expected duration of a given PCP deviation, conditional on the history of the process, should emerge as the global maximum of the log likelihood of the conditional intensity process expressed in any of the above forms. Indeed as it appears in Engle and Russell (1997) the log likelihood of the process can be stated both in terms of the conditional density of the process or the conditional intensities themselves; thus the log-likelihood in terms of the conditional densities is given by

$$L(\theta) = \sum_{i=1}^{N(T)} \log f_i(t \mid t_0,\ldots,t_{i-1}) \qquad \textbf{Eq 3.16}$$

and in terms of the conditional intensities

$$L(\theta) = \sum_{i=1}^{N(T)} \log \theta(t_i \mid i-1, t_0,\ldots,t_{i-1}) - \int_{t_0}^{T} \theta(u \mid N(u), t_0,\ldots,t_{N(u)})\,du$$

Eq 3.17

where the second term in the RHS of Eq 3.17 can be thought of as the conditional probability of no points in the interval, or equivalently an expression for the survivor measure.

It seems then, that a successful analysis will require the proper parameterisation of Eq 3.17 to capture the functioning of the market in each case. In the corresponding literature, thus far, there has been a host of different parameterisations. Collectively most of them, certainly those referred below, are known as *Self Exciting Processes*.

Perhaps the simplest and the most widely used is the Poisson process where the observations in time occur in a way best described as random. This random evolution yields the following intensity specification in terms of the number of points in the interval for a Poisson process of rate p,

$$\mathbf{Pr\ \{N(t,t+dt) = 1 \mid h_t\} = p\ \Delta t + o(\Delta t)} \qquad \textbf{Eq 3.18}$$

$$\Pr\ \{N(t,t+dt) > 1 \mid h_t\} = o(\Delta t) \qquad \textbf{Eq 3.19}$$

where

h_t represents the past history up to t and according to the usual notation

$o(\Delta t)$ is a function approaching zero at a higher rate than Δt (often this type of Poisson process is referred to as *homogeneous Poisson process*).

Although Eqs 3.18, 3.19 can, in most cases, define successfully point processes where the rate of arrival can be summarised in p, simulations of time varying evolutions are inappropriate. Indeed the intensities defining the process do not depend on the history h_t thus any current description of the likelihood of a point within dt makes no use of the previous information. Additionally p is required to be constant thus processes involving trends or cycles are not captured adequately. For such type of processes, however, the model is augmented to what is known as the *non-homogeneous Poisson process* where variation of p with time -p(t)- is allowed. To capture, though, a seemingly random point process which is evidently dependent on a variable z(t) a Poisson process with rate p dependent on that variable z(t), i.e. where p(t) = pz(t), has been proposed.

A natural extension, more successfully suited to processes exhibiting a degree of clustering can be used in the form of the *renewal process*, which mostly defines the process as having intervals between successive points being independently gamma distributed. It can be shown[8] that for different levels of dispersion of the density gamma, data-sets with different degrees of clustering can be modelled. For example a gamma distribution with coefficient of variation greater than one could be used for simulation of a point process exhibiting excess clustering. This ability to model heavily "clustered" series is extremely desirable when analysing market data of such high frequency but unfortunately the requirement of independent intervals shown to be extremely limiting or even unrealistic. Such independence limits the memory of h_t to just the previous occurrence.

By removing the independence requirement, dependence of current observations to past ones can yield some interesting point process models. Such a process can be fully characterised by its intensity $\theta\ (t,h_t)$. It follows, then, that the process is governed by the structure of the historical values' vector. It is obvious then that this can be constructed with the suitable "amount" of memory built into it as the data require. Hawkes (1971a, b, 1972) has proposed a model where the influence of past events is proportional to their position on the axis assuming a one dimensional time model; this family of models is known as *linear self-exciting processes.* Letting w(t) be a weight structure defined for the possible points of occurrence of elapsed points, the conditional intensity which fully characterises the point process and has a given degree of memory as expressed in w(t) is

$$\theta(t\,;\,h_t) = a + \int_{-\infty}^{t} w(t-z)dN(z) \qquad \textbf{Eq 3.20}$$

where

a is a constant greater than zero and

d N(z) is a stochastic process adding the number of points from t until the point of zero memory in the past. It follows then from the above that Eq 3.20

[8] For a detailed discussion of the renewal processes see Cox (1962).

expresses the intensity of a point process belonging to the family of self exciting processes as the weighting function w(t) is itself just a function of time.

There are numerous ways in which one can adjust the model above by using different specifications for the weighting function. This can be proportional if the dependence of the past events is a function of time, monotonically decreasing as the remote past is approached. Alternatively, as it has been proposed by Wold (1948) and later by Cox (1955), the specification of w(t) could be a function of the number of past occurrences within intervals Δt. In this context Gaver and Lewis (1980), Lawrence and Lewis (1980) and Jacobs and Lewis (1977) have suggested a family of models with correlated intervals where the likelihood of an event within dt is based on an *exponential autoregressive moving average EARMA (p, q)*. For this model, however, the calculation of the maximum likelihood estimators can be extremely complex, perhaps a factor contributing to their limited application for the study of financial markets thus far.

Closely related to the linear self exciting process but based on a different intuition, the *doubly stochastic process* has been proposed by Cox (1955). It relates to the linear self exciting one as it adopts a driving process similar to the weighting function for the occurrence of points. However, as the name suggests the process is a real valued non-negative *stochastic* one of a defined structure but usually not observable. Assuming then that the history of this stochastic process, S(t), is summarised by the vector h^s_t and that, as usual, h_t captures the history of the process at t, the conditional intensity is given by

$$\theta\ (t;\ \mathbf{h_t},\ \mathbf{h}^s_t) = \mathbf{lim}_{\delta\to 0^+}\ \delta^{-1}\ \mathbf{pr}\{\mathbf{N(t,\ t+}\delta)\ \mathbf{>0}\ )\ |\ \mathbf{h_t},\ \mathbf{S(s)}\} \qquad \textbf{Eq 3.21}$$

Thus the process is conditional on both the h_t and h^s_t. Though it may not be clear from the beginning it too belongs to the self exciting family as it can be shown that the process reduces to the expectation over h^s_t given h_t, for a proof see Snyder and Miller (1991, Th. 7.2.2), Eq 3.21 then reduces to

$$\theta\ (t;\ \mathbf{h_t},) = \mathbf{E}\{\mathbf{S(t)}\ |\ \mathbf{h_t}) \qquad \textbf{Eq 3.22}$$

It is seen from Eqs 3.21 and 3.22 that the doubly stochastic process offers a more general representation allowing a stochastic dependence on past events. Unfortunately even in very simple cases the evaluation of the expectation in Eq 3.22 is again extremely complicated.

Generalising the argument further, Cox (1955, 1972b) introduced into the calculation of the intensity function an additional dependence on observed exogenous variables summarised by the vector, say $\psi(t)$. Obviously the set of variables can include factors which the economic theory or previous statistical analysis have proved as being significant. Adopting the usual notation, the conditional intensity function is then expressed by

$$\boldsymbol{\theta\ (t;\ h_t,\ h^{\Psi}_{t}) = \lim_{\delta \to 0^+} \delta^{-1}\ pr\{N(t,\ t+\delta) > 0)\ |\ h_t,\ h^{\Psi}_{t}\}} \qquad \textbf{Eq 3.23}$$

in what is known as the *proportional hazard model.* Combining Eqs 3.22 and 3.23, however, one can yield a very general *theoretical* framework for the specification of the intensity function of point processes. In doing so it is assumed that the evolution of the process is driven by a stochastic unobserved process h^s_t, as well as by an observed explanatory process h^{Ψ}_t. Assuming a single explanatory process, although extensions to multivariate cases can easily be accommodated -at least in theory-, the conditional intensity becomes

$$\theta\ (t;\ h_t,\ h^{\Psi}_t,\ h^s_t) = \lim_{\delta \to 0^+} \delta^{-1}\ pr\{N(t,\ t+\delta) > 0)\ |\ h_t,\ h^{\Psi}_t,\ h^s_t\}$$

Eq 3.24

Although the Cox Proportional Hazard model (henceforth CPH), and its generalisation, have found extensive application in the field of labour economics their application in modelling financial markets is not widespread. The model, however, serves well our analysis. It0 accommodates a stochastically driven process and can examine the dependence of the stochastic unobserved duration on the PCP deviations.

3.4 The Cox Proportional Hazard (CPH) Model.

The CPH is a semi-parametric survival model. It does not make any strong assumptions on the distribution of the observed times but it is based on the ranks between the survival times from which it constructs its likelihood functions.

Although the use of a likelihood function assumes knowledge of the full joint distribution function, the CPH uses a limited information matrix to derive a *partial* likelihood function and consequently draw inferences on the sample. As it will be shown later on, given a set of straightforward assumptions, incompleteness of the joint data distribution still allows valid conclusions to be drawn. Hence the model is naturally suited to the present analysis, as our hypothesis constitutes a test for the inclusion and direction of the PCP deviation as a covariate and not of the full specification of the coefficients vector.

The partial likelihood function is based on the probability function, or the joint probability function, which are derived using a set of available information rather than the global information set. As such it can be used to address selective inference issues. In the present analysis the information set is limited, as it does not include the full matrix of parameters likely to affect the duration of PCP deviations. It does include, however, the necessary information to construct the joint probability function of the duration and level of deviations.

Assuming that $Z_i(t)$ denotes the vector of covariates, in the current case this refers only to the PCP deviations, for the ith contract at t, the hazard $\lambda(t, Z_i)$ is of the form

$$\lambda(t, Z_i) = \lambda_o(t)\, r_i(t) \qquad \textbf{Eq 3.25}$$

where

$$r_i(t) = e^{bZi(t)} \qquad \textbf{Eq 3.26}$$

and is referred to as the ith risk score with b a vector of regression parameters. In correspondence with the prevailing notation, $\lambda_o(t)$ denotes the baseline hazard function, which is a function of time common to all subjects, and the exponential assures positiveness of λ (probability measure).

The model $\lambda(t, Z_i)$ of the form given in eq 3.25 belongs to the family of proportional hazard models. In general, the specification of a proportional hazard model will require that the covariates are time invariant. A model similar to the one given above, where the covariates are a function of the time, should not be considered as belonging to the proportional hazard family. If one can ensure, however, that for every different observation the time varying covariates have the same function, then the assumption of proportional hazards is not invalidated. One can assume then, that the effect of this constant variability will be absorbed by the baseline hazard. In a similar fashion, a model of time invariant, or observation-constant covariates will fall into the proportional category since the hazards for two different events will form the same ratio irrespective of time.

The concept of proportionality is important in the current analysis. If this is ensured, valid inferences for the model can be drawn without specific reference to the time variability in the regressors as well as to missing variables which affect events in the same way. As it will be seen in the remaining parts of the chapter, this will greatly simplify the inference stage.

3.4.1 Estimation and Inference

To derive the statistical inferences the model maximises the likelihood of having a particular event, given that an event has occurred at time t. As noted in the introduction of this chapter, the model makes use of the partial likelihood of events. As such it ensures that the baseline hazard function, to which all the missing information contributed by omitted covariates is assumed to be relegated, does not have to be calculated.

As said at the beginning, the CPH model makes use only of the rank information among events. Usually in a duration analysis, the model is based on a duration vector giving information for the duration periods over observations. The partial likelihood methodology, suggested by Cox (1972), breaks this information set into the rank and order statistic sets respectively. Conceptually the first of the three is a joint probability statement of the other two.

In the current research the global information set (the first set as discussed above), requires ordered data for the observation and the duration as well as the full vector of explanatory variables drawn from a theoretical framework. This, however, requires knowledge of the complete theoretical framework. This is not applicable in the current discussion as we do not seek to establish such a framework. We merely seek to provide statistical inferences on the hypothesis being that the level of PCP deviations is a factor affecting the duration of inefficiencies. As such we can use the rank statistics to draw a partial likelihood without having to specify theoretical conditions necessary to estimate the baseline hazard.

Assuming that the global information set vector can be decomposed into the rank and order statistic vectors, say α and β, the full likelihood function can be expressed in terms of vectors α and β and hence derive the partial likelihood. The full likelihood written in terms of α and β is

$$L = f(\alpha_1, \beta_1, \alpha_2, \beta_2, \ldots, \alpha_n, \beta_n) \qquad \textbf{Eq 3.28}$$

It follows by the product law on conditional observations that

$$L = f(\alpha_1, \beta_1)\, f(\alpha_2, \beta_2 | \alpha_1, \beta_1)\, f(\alpha_3, \beta_3 | \alpha_1, \beta_1, \alpha_2, \beta_2)$$

$$f(\alpha_n, \beta_n | \alpha_1, \beta_1, \ldots, \alpha_{n-1}, \beta_{n-1})$$

Eq 3.29

or in a product form notation

$$L = \prod_{i=1}^{n} f(\alpha_i, \beta_i | \mathrm{A}^{i-1}, \mathrm{B}^{i-1}) \qquad \textbf{Eq 3.30}$$

where in this case only capital letters denote vectors of the series α_1 to α_{i-1}. Noting that A^i is vector A^{i-1} with point α_i, eq.3.30 can be expressed as

$$L = \prod_{i=1}^{n} f(\alpha_i | \mathrm{A}^{i-1}, \mathrm{B}^{i-1}) \times \prod_{i=1}^{n} f(\beta_i | \mathrm{A}^{i}, \mathrm{B}^{i-1}) \qquad \textbf{Eq 3.31}$$

Where the second term is the partial likelihood based on the order statistic vector. If the order statistic comprises of a sequence of values

(exit time of subject i - entry time of subject i), for i = 1 to n,

then in this case the partial likelihood is given by the sum of the subject specific terms, with the first term being

$L_i(b) = P(\text{subject i exits at t})/P(\text{any subject exits at t})$

Eq 3.32

which is equal to

$$L_i(\mathbf{b}) = \frac{\lambda_o(t) * r_i(t) * dt}{\sum_{k=1}^{N} \lambda_o(t) * r_k * dt} P_i \qquad \textbf{Eq 3.33}$$

giving for the full partial likelihood eq. 3.34

$$L_p(\mathbf{b}) = \prod_{k=1}^{N} \frac{r_k}{\sum_i r_i} \qquad \textbf{Eq 3.34}$$

As expected eq.3.33 does not make any use of the exit times. It offers, though, a quite interesting representation for the current analysis as it does not rely on the calculation of the common baseline hazard function. This in turn means that even if a complete specification is not identifiable through the economic theory, provided that all the missing terms are equally applicable to all subjects[9] we can still rely on inferences from the partial likelihood. On that ground it ensures that a partial, valid representation of the duration of observations as a Proportional Hazard model expresses the PCP durations as a function of the level of deviations. It is further assumed that given other factors affect the length of durations this is done uniformly across the observations.

[9] According to the above algebra we would additionally require that all missing terms in the specification enter the probability representation multiplicative.

Compared to a fully parametric proportional hazards model, it has been shown by Efron (1977) and Oakes (1977) that, the CPH model gives very efficient estimates even when the parametric model assumes the correct distributional form. In its extended form, as was presented in Fleming and Harrington (1991) and used here, it has been shown to accommodate left truncation and censored observation. In the current work left truncation is of significant importance as observations almost invariably enter the risk set at times different from zero.

3.5 Methodology Results

3.5.1 Methodological issues.

Censored Observations

Although the inclusion of censored observations is easily accommodated by the model, care should be taken to differentiate between forced and unforced censoring. In the relevant literature an observation is said to be censored when the experiment, measuring subjects who are already in the risk set, is abruptly terminated. Theoretically there should not be any factors giving rise to a change in the probability value of such a censored experiment, this is an unforced censoring. Only such a randomly generated censorship could be accommodated by the CPH model.

In the present analysis data are collected each day for the complete trading period, i.e. from market opening to market closure. Our data set, however, includes "alive" subjects, which are already in the risk set. By terminating the data collection mechanism at a particular time each day an upper limit on the duration of each subject is imposed, this is forced censoring.

Assuming that the majority of the last observations each day occur close (far away) to the end of trading period these censored values will have an artificially short (long) duration. It is seen then that, perhaps the process of obtaining the unexpected values, forces the final sample to contain smaller or larger than normal values. On average, the censored group contains between 50% and 67% observations terminated abnormally at the market closure. Thus the information obtained by the vector of censored values is contaminated by other than economy- or market-wide factors. Even more importantly, the inclusion of observations subject to external factors not equally affecting the dataset, could invalidate the proportionality assumption as the baseline hazard ceases to be common for all observations. In this chapter both censored (sets containing censored observations) and uncensored groups are considered.

It is difficult to know with certainty the effect of censoring on the distribution of observations with no further analysis. Intuitively it is known that the censored sample will have mass shifted either to the left or to the right of the mean. Since in this case censoring takes place at the market closure the shorter resulting durations will shift the mass of the distribution to the left of the mean. Later on the raw durations will be transformed so as to extract the stochastic part of the series and analyse deterministically free series. This may complicate further the composition of censored group. The transformation is according to equations 3.36 and 3.37, which are stated below

$$E(x_i \mid x_{i-1}, x_{i-2}, \ldots, x_1) = \Omega_i (x_i \mid x_{i-1}, x_{i-2}, \ldots, x_1 ; \theta) \equiv \Omega_i \qquad \textbf{Eq 3.36}$$

$$x_i = \Omega_i \, \varepsilon_i \qquad \textbf{Eq 3.37}$$

where $\{\varepsilon_i\}$ ~ i.i.d. with density p.

Three vectors are of importance: x (the observed duration), Ω (the fitted values) and ε (the unexpected component). According to Eq 3.37, if the inclusion of censored observations shifts the distribution of x to the left then the distributional distortion of ε will be dictated by the relative speed of shifts in Ω to shifts in x. The relative speed is determined by the smoothing technique, the filters applied and the information set. In our case it seems that there is a shift of ε to the left after inclusion of censored observations.

In contrast, the uncensored sample contains only these observations which are rationally and with certainty, updated by the market makers. It could be argued, though, that the vector of censored observations contains as well, a significant amount of information regarding valid, risk free opportunities. Being located near the market closure, it could also provide us with information specific to that period. Perhaps, it would have been ideal to extract any biasedness due to censorship and utilise the remaining information. Unfortunately we are not aware of any suitable method of quantifying the effect of censorship. Hence the CPH analysis is performed both on a sample excluding censored observations (group a, uncensored) and on a sample including censored observations (group b, censored).

3.5.2 Proportionality and Time Varying Covariates

The significance of the risk score is estimated and tested by regressing the unexpected durations on the level of deviations. The duration and the unexpected duration are defined as follows.

If the duration between two observed events is given by

$$x_i = t_i - t_{i-1} \qquad \textbf{Eq 3.35}$$

and the current expected duration is defined as

$$E(x_i \,|\, x_{i-1}, x_{i-2}, \ldots, x_1) = \Omega_i (x_i \,|\, x_{i-1}, x_{i-2}, \ldots, x_1 ; \theta) \equiv \Omega_i \qquad \textbf{Eq 3.36}$$

If Ω_i is the expected value of the current duration given the history of observations then a systematic, variation free, multiplicative[10] function between x_i and Ω_i is assumed by having

$$x_i = \Omega_i \, \varepsilon_i \qquad \textbf{Eq 3.37}$$

[10] The multiplicative function ensures positivity of parameters as these measure the time between observations.

where $\{\varepsilon_i\}$ ~ i.i.d. with density p.

It is evident from previous references to the theory of Transition Data Analysis and in particular from the discussion of the CPH and its baseline hazard function, that ε_i should represent the stochastic behaviour of durations. In any other case the presence of deterministic effects could make the use of the CPH model problematic. Information omitted in the specification process of the partial log likelihood would invalidate the results as there would exist missing variables affecting non-uniformly the sample. By removing the expected part of the duration and deviation values robustness of the methodology w.r.t the above issues is ensured.

To obtain vector ε_i any cyclical or trend parts are removed from the durations time series. It is expected that these will include any time of the day significant effects. Market makers could well be driven by inventory/liquidity problems in specific time intervals (e.g. opening or closing of the market) and feed quotes into the system accordingly. If these problems are serious enough it is possible that they take precedence over, or compete with, the stochastic arrival of information. As such they could manifest themselves as daily repeated patterns with only some correspondence to economic news, or result in the superposition of the two patterns, viz. the liquidity driven price fluctuations and effects arising from the flow of information to the market.

Evidence of intradaily patterns

Figure 3.1 shows fitted duration values (Ω_i) versus the time of the day at which each spell initiates. These values are the expected duration for the specific time period, given the past history of observations. Consequently they represent an averaged-out sequence of durations throughout the period of investigation. As such, the pattern in Figure 3.1 closely resembles that of Figure 3.2 which presents interpolated durations for a typical middle of the week day.

Figure 3.1

The Graph shows expected duration values (Ω) versus the time of the day at which each spell initiates. Durations are counted after the 2 minutes delayed execution time.

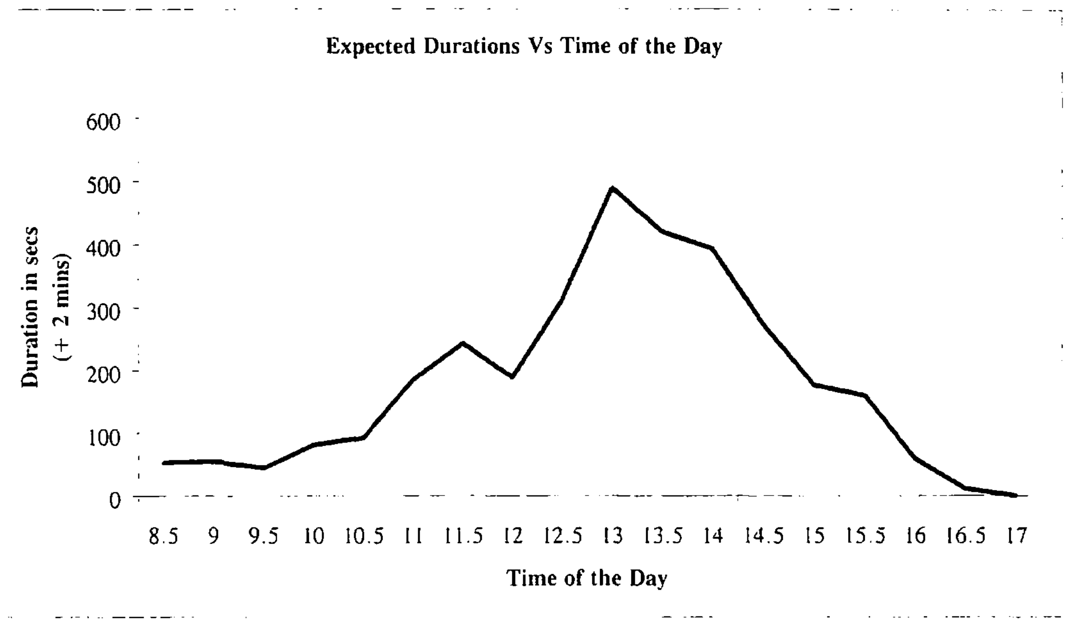
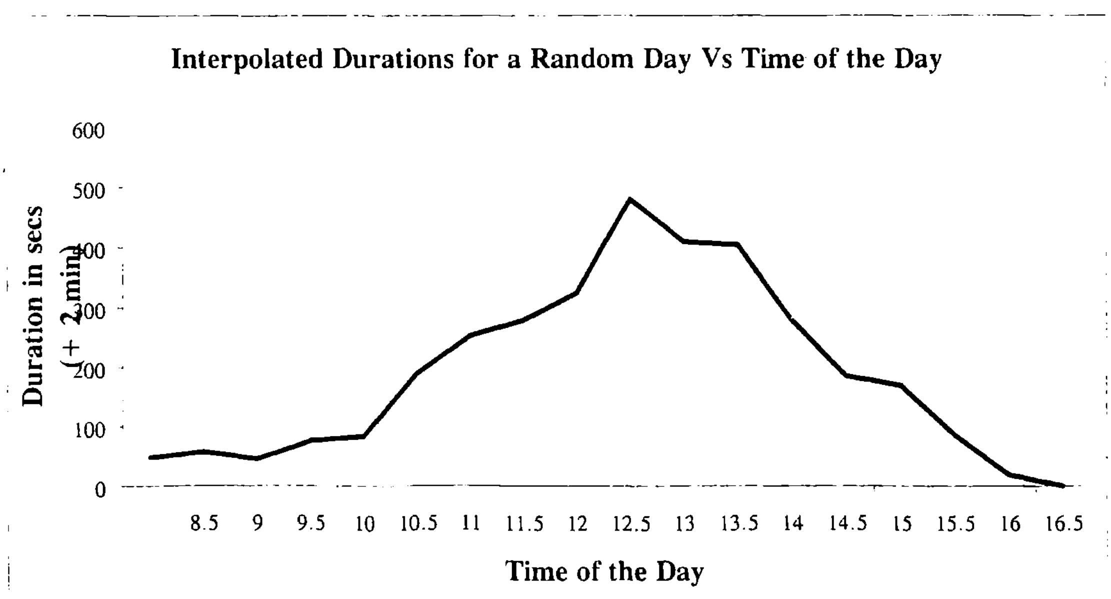


Figure 3.2

The Graph shows interpolated durations for a typical day of the week. Durations are counted after the 2 minutes delayed execution time.



From Figure 3.1 and 3.2 it is evident that lower durations are present around the market opening, indeed between the hours of 8:30 and 9:30 durations from a typical day will be of the order of 50-60 seconds. Following the market opening is a period of progressively longer durations, roughly until 1-2 pm, for a typical day, deviations can last for up to 7-9 minutes. Assuming that the intraday frequency of quotes can proxy for trade volume, see Kamara and Miller (1995), these represent a high / low activity pattern in the market.

In addition to the theoretical references made, in the introductions of chapters 3 and 4, to the volume/information association, an extensive discussion can be found in a survey by Karpoff (1987), who argues that there is a strong relationship linking both price and volume levels to information. Additionally Blume et al. (1994) argue that there is a dual dependence of volume to information and price formation. By adopting a statistical reasoning, similar to the one used in the MOD models, they argue that information is drawn from different distributions of varying "quality". As such, efficient inferences on valid asset prices cannot be made as the traders are unable to observe the true quality of the information (the Central Limit Theorem should cover the true source of the signal). However because volume does not follow a normal distribution it is observed by traders who use it as a price formation variable.

Progressively shorter durations are evident towards market closure, representing trading (or price formation) in anticipation of information flow between the close to opening time period. As it appears in Merton (1971) and

Brock and Kleidon (1992) there is a gathering of information prior to market opening. This information set refers to variables on which the price formation process depends. Consequently the buy and sell prices of traders and optimal trading positions (e.g. optimal portfolio conditions) would be considerably revised and reflected in increased volume at market opening. With a similar rational one should expect an equally active trading prior to overnight market closure periods.

Additionally, it seems that the duration patterns presented in figures 3.1 and 3.2 are in accordance with empirical findings of intradaily price analysis at the NYSE, as these are reported in Foster and Viswanthan (1990), Lockwood and Linn (1990), McInish and Wood (1990, 1991, 1992) and Lee et al (1993). Furthermore, the same pattern is evident on options traded at the CBOE as it is evident in Sheikh and Ronn (1994). Somewhat different are the results, however, presented in Kleidon and Werner (1994) who examine opening and closure volumes at the London Stock Exchange and find a double-U shape, and in Demos and Goodhart (1992) who examine the FX market and document a low-high-low pattern. If one, however, subscribes to the Brock-Kleidon, Merton model then the evidence from the FX markets do not come in direct disagreement with the results presented here. This is due to the fact that the forex is an around the clock market.

The very short durations towards the market closure are possibly due to censoring. At the closure of the market all valid quotes are terminated and assigned a value of duration between the starting of the quote and the closure of the market. Consequently there is an artificial shortening of the durations neighbouring the market closure; ceteris paribus these quotes could have lengthier durations.

The duration part that is deterministically known to depend on time is removed from the sample by fitting a smoothed function explaining the observed duration on the time of the first observation (put or call).

When discussing eq.3.26 it was noted that the baseline hazard function is a function common to all subjects. However, a prerequisite is the assumption of proportional hazards across the sample. This is because proportionality in the hazards denotes that the hazards for two different observations can be expressed as a ratio.

Figure 3.3 plots the level of the PCP deviations on the time of the day factor. The plot shows a dependence of the deviations on the exact time period of the observation. Thus a significant time variation in the covariates exists. As such, proportionality in the hazards can only be assumed if this dependence is constant across observations. Two ways of ensuring consistency in the covariates are proposed below.

Assuming that the variation in time of the magnitude of PCP deviation is in levels, calculating the correct values for each level can derive piecewise-constant hazards. As given in Lancaster (1992), if a piecewise-constant function θ(t) exists with

$$\theta(t) = \begin{cases} \theta_1, & 0 \le t \le \tau_1 \\ \theta_2, & \tau_1 \le t \le \tau_2 \\ \cdots & \cdot \\ \cdots & \cdot \\ \cdots & \cdot \\ \theta_k, & \tau_{k-1} \le t \le \tau_{k,\xrightarrow{k}\infty} \end{cases} \qquad \textbf{Eq 3.38}$$

where θ's are values constant over time periods specified by τ's, then a series of such values will form a set of descriptive statistics which will capture variation of the model with respect to time and have a survivor function given by

$$\bar{F} = \exp\{ \int_0^t \theta(s)\, ds \qquad \textbf{Eq 3.39}$$

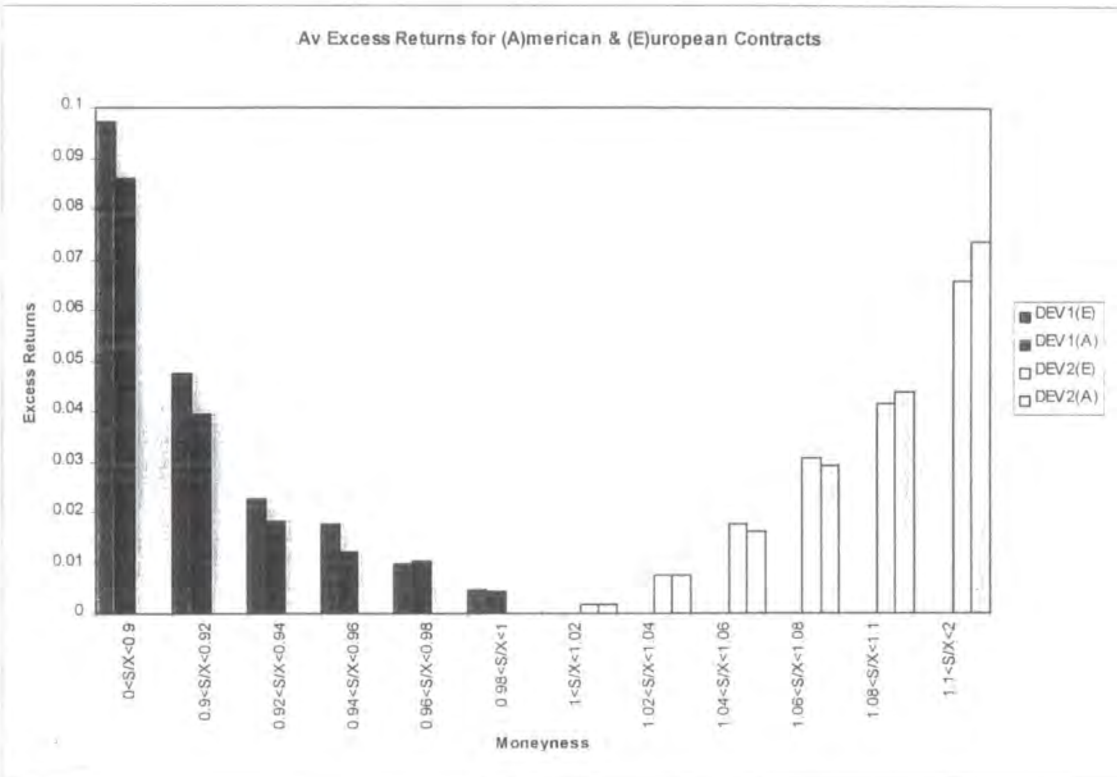
If within the time periods time invariant covariates can be assumed, then the model can be extended to cover general time variation in the covariates. By applying a suitable smoothing technique, the piecewise properties of the intervals can be extended to an asymptotically finer spacing.

A second, simpler methodology is based on the assumption that the time dependence of the deviations can successfully be removed from the sample by the use of a suitable signal-extraction technique, similar to eq 3.38. If this is valid the necessary conditions for proportionality in the hazards can be established. To avoid dependence on distributional assumptions, the use of a nonparametric technique, such as a locally fitted regression model is preferred. The application of this methodology results in a simpler algorithm for the CPH model. To the extend that the nonparametric localised regression can yield sufficiently time-independent PCP deviations, the signal-extraction methodology is preferred to the piecewise-constant hazard model.

In addition, as it stems from the analysis of PCP deviations in chapter 2, there is a strong relationship between the level of deviations and the degree of moneyness in the contracts. It is evident from Figure 2.5, which is repeated below, that far out- and in-the money contracts (contracts with extreme strike to underlying values) yield excessively large deviations. Theoretically this could be attributed to the fact that widely used option pricing models, like the B&S for example, it is known to over or under price far out- and in-the money contracts (see for example Rubinstein, 1985).

(Figure 2.5).

(Comparison of level of Deviations for American and European contracts. Transactions are equal to the upper boundary of £100 and restrictions are calculated according to the Deficit Account of LSE.)



Again extracting the expected values according to eq 3.38, but this time between deviation and moneyness, removes this dependency. In doing so it is ensured that the final results represent the adjustment time of the market to inefficiencies (deviations) due to pure market factors, i.e. other than pricing model induced biasedness.

The presence of heterogeneity

Viewed under a different perspective, this will result the avoidance of another possible source of heterogeneity in our sample. Due to the fact that our observations come from a quoting mechanism of competing market makers it is possible that the quoted prices from different market makers will be evaluated using different pricing models. To the extend that these are behaving non-uniformly with respect to moneyness they represent a source of heterogeneity in the sample. Clearly if different groups within our sample have a Data Generating Process affected each time in a different way by these factors this will result in problems of neglected heterogeneity.

Hence, one could argue that given that there is a highly dependent relationship between periods of time of the day and PCP inefficiencies, a mixture model similar to the one outlined in section 3.2 could be applicable here if Q(z; x) was known. Having identified, though, the sources of heterogeneity and by applying a suitable signal extraction technique it is assumed that the sample is homogeneous.

Smoothing, The Supersmoother.

This section describes the methodology applied to extract and use purely stochastic components of the variables entering the CPH model.

The fitted values are obtained by employing a supersmoother with local cross-validation as proposed by Friedman (1984). With this approach what is essentially a non-linear dependence can be analysed relying only on the data to specify the form of the model. The analysis fits a curve to the data points locally i.e. at the point of interest plus a specified neighbourhood of points. Although it works in a similar fashion to the simpler locally weighted regression smoothing technique, it uses a variable span to calculate the neighbourhood of points used in the analysis each time and thus offers better results for datasets with increased curvature. In particular, the application of a supersmoother model can accommodate points with variable density and extremes more satisfactorily than other simpler nonparametric methods.

Essentially the supersmoother is a smoothing technique belonging the general group of k-nearest neighbour estimates. As opposed to the simpler and widely known kernel estimate, these smoothers work in a variable region around the value of x but again yielding a weighted average estimate of the response variables. In general, as was specified by Loftsgaarden and Quesenberry (1965), a k-nearest neighbour estimate smoother, denoted as $\hat{f}(x)$ where $\hat{f}$ is the fitted smoothed function of x, as

$$\hat{f}(\mathbf{x}) = \frac{1}{n}\sum_{i=1}^{n} W_i(x) Y_i \qquad \textbf{Eq 3.40}$$

where

$W_i(x)$ denotes the weight series applied to the k nearest observations[11]. In the simpler case the weights denote the fraction of the points in the neighbourhood over the number of points in the sample. However, in the supersmoother case Friedman (1984) shows that the weight sequence is a function of the changes between the empirical distributions of the sample for x and the near-area of x. Hence, the sequence of weights adopts a functional form

$$W_{hi}(x) = K_h \propto \Delta F(X_i, x) \qquad \textbf{Eq 3.41}$$

where

the second term at the LHS denotes proportionality to the changes in the empirical distribution and K_h denotes the variable kernel function. Additionally h according to the established notation denotes the smoothing parameter. It is crucial to note here a difference between the supersmoother and other smoothing techniques. All of these nonparametric regression methods adopt an optimisation technique for establishing the smoothing parameter, which in a way defines the degree of smoothing. Usually the methods try to minimise a global error criterion. It is easy to see that this will not always offer the best local optimisation, and consequently smoothing. Mathematically, the upper lower limit of the sum of the errors is not smaller than the sum of the upper lower limits of the errors. As Hardle (1993) shows

[11] The neighbourhood comprises of the k nearest points in the Euclidean sense.

$$\inf_h \int E(\hat{f}_h - f)^2 \geq \int \inf_h E(\hat{f} - f)^2 \qquad \textbf{Eq 3.42}$$

In the current case this is not suitable as we seek to fit a fine curve extracting the local behaviour, on time, of the CPH parameters; a locally minimised error criterion is preferred. The supersmoother technique, employed here, applies what is known as a "local cross validation" to calculate the locally optimum smoothing parameter.

Generally speaking in the supersmoother process we assume a spectral formulation of the dependent variable. Three different frequencies, the tweeter, midrange and woofer[12] are used to construct the frequency spectrum of the variable. The smoother then is constructed from these three components. These usually take the values of 0.05n, 0.2n and 0.5n, as it is indeed done here. To optimise the process we minimise the absolute values of the residuals between the variables and the fitted values over the three different frequencies. The minimised equation of the residuals is

$$r_i(k) = [y_i - \hat{f}_k(X_i)]\{1 - \frac{1}{k} - \frac{(X_i - \bar{m}_{X_i})^2}{\mathrm{var}_{X_i}}\} \qquad \textbf{Eq 3.43}$$

[12] The terms "tweeter", "midrange" and "woofer" are used in acoustics. The human ear has a hearing spectrum of 20 Hz to 20 kHz. The tweeter, midrange and woofer are sound reproduction sources covering progressively this spectrum. In our case, though, the respective frequencies corresponding to the woofer, midrange and tweeter are the low, medium and high frequencies of the sample spectrum.

where m is the local mean and var the local variance. Here eq. 3.43 is minimised for the dependent and independent variables given in table 3.1.

Table 3.1

Lists of variables for which a simulated smoothed signal is evaluated.

Dependent (Y)	Independent (X)
Duration	Time t
PCP deviation	Time t
PCP deviation	Moneyness m

However, to reduce the variance of the resulting smoother the method suggested by Hardle (1993) is followed. Hardle (1993) proposes a smoothing of the absolute cross-validated residuals over the initial X values. A further smoothing of the span values, again following Hardle (1993), over X_i, ensures a span for reproduced (smoothed-out) observations close to the midrange values. The resulting supersmoother is a curve between the two smoothers sharing the closest spans.

Super smoothing, thus, obtains a vector of fitted values Ω. Eq 3.37 can calculate vector ε denoting the unexpected part of the durations.

3.6 Results

Following calculation of vector ε, the importance of the level of the PCP deviation for the duration of each arbitrage opportunity is examined. This is done by modelling the hazard for each duration using the CPH methodology as given in eqs 3.23 & 3.24. Thus

$$\lambda(t, Z_i) = \lambda_o(t)\, r_i(t), \qquad \text{(Equation -3.23)}$$

$$r_i(t) = e^{bZ_i(t)} \qquad \text{(Equation -3.24)}$$

with

$\lambda(t, Z_i)$	the hazard for PCP duration i,
$Zi(t)$	the vector of the PCP deviation,
b	the vector of the regression parameters and
$\lambda_o(t)$	the baseline hazard for each PCP deviation but common to all observations.

According to the definition of high frequency market efficiency given in section 3.1, the following hypothesis is tested:

" In efficient markets of the high frequency context, the duration of risk free PCP deviation will be inversely proportional to the level of the covariate deviation, i.e. the higher the level of PCP deviation the less time the particular deviation stays in the system ".

The current study examines 2864, 26241, 9024 and 48647 PCP deviations one and two for American censored and uncensored contracts. It also analyses 3243, 9880, 10309 and 30842 PCP deviations for European censored and uncensored contracts respectively. The observations correspond to zero transaction costs and cover the period of 4th July 1994 to the 27th February 1997. The sample starts with the first PCP deviation and is followed by the first available identical contracts' deviation; the sample contains all observations for a particular contract until a full price correction. The selection starts each day but the observations are not panelled. It is assumed that non-occurrence of new quotes, until the market closure, does not necessarily mean that market makers are not willing to provide corrected prices. These observations are rather treated as right censored ones. Results for both censored and uncensored groups are included.

Tables 3.2 and 3.3 below, present estimates from the CPH model applied on groups a and b. The results indicate that the level of deviations is a significant factor in the duration of quotes. In most cases an increase in the level of deviation by one unit (100%) will reduce durations between 0.015 to 0.378 times the original values. In all of these cases the results are significant at 95% level at least. There are two notable exceptions though.

Table 3.2

Duration analysis with PCP deviations as a covariate. Sample refers to group a, i.e. excludes censored values.

3 **Cox Proportional Hazard** (group a, without censored values)				
American		European		
Covariates				Statistics
DEV 1	DEV2	DEV1	DEV2	
-0.972	-4.2	-0.866	-2.9	Coefficient (b)
0.378	0.015	0.42	0.0551	Relative risk / unit change in Exp (coef)
0.446	0.189	0.926	0.216	Se(coef-b)
-2.18	0.163	-0.936	-13.4	Z (Wald's test)
0.029	**0.76**	0.042	0	P-value
Schoenfeld Residuals - **Test for Proportionality**				
American		European		
Covariates				Statistics
DEV 1	DEV2	DEV1	DEV2	
0.0131	0.143	0.103	0.006	Rho
0.44	2.207	30.9	0.367	Chi-sq
0.507	0.137	**2.71e-008**	0.545	P-value, null: Schoenfeld residuals are random walk

Table 3.3

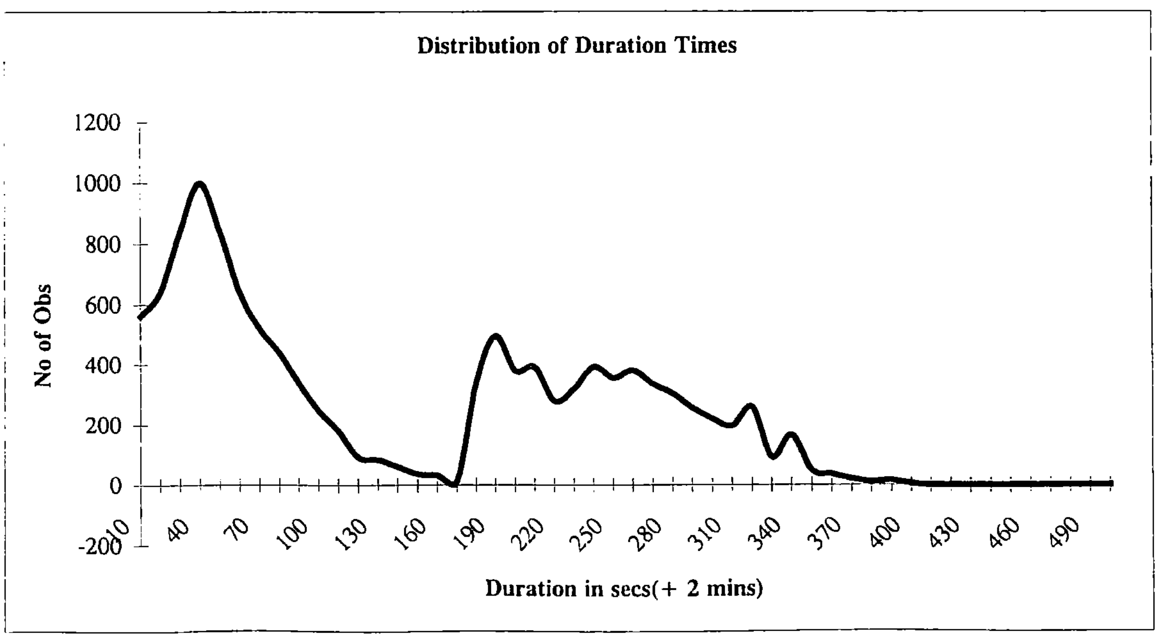
Duration analysis with PCP deviations as a covariate. Sample refers group b, i.e. includes censored values.

4 Cox Proportional Hazard (group b, with censored values)				
American		European		
Covariates				Statistics
DEV 1	DEV2	DEV1	DEV2	
-1.67	0.033	-1.33	-0.991	Coefficient (b)
0.188	0.86	0.266	0.371	Relative risk / unit change in Exp (coef)
0.445	0.177	0.402	0.213	Se(coef-b)
-3.76	0.188	-3.3	-4.66	Z (Wald's test)
0.00017	**0.85**	0.00098	0.0000032	P-value
Schoenfeld Residuals - **Test for Proportionality**				
American		European		
Covariates				Statistics
DEV 1	DEV2	DEV1	DEV2	
0.0374	0.047	0.00857	0.16912	Rho
3.58	0.3812	0.121	3.16	Chi-sq
0.059	0.537	0.728	0.075	P-value, null: Schoenfeld residuals are random walk

In group a, durations for deviations one European contracts appear to vary non-proportionally with changes in the level of deviations. The documented Schoenfeld residuals have a probability value approaching zero, i.e. reject the proportionality hypothesis. Essentially, significant Scoenfeld residuals indicate the existence of a trigger point after which the effect of the covariates changes. That does not mean to say that the effect changes in sign but merely that it changes intensity. Graph 3.3 below, presents the distribution of the durations for deviations one of European contracts. It is suggested here that a possible explanation for the change in the proportionality of covariates could be the presence of strong bimodality in the data.

Figure 3.3

Density plot for Durations of European deviation 1. The plot refers to group a data, i.e. includes censored values. Durations are counted after the elapse of the 2 min execution delay interval.



In groups a and b, durations for deviations two, American contracts, although yielding a consistent less than one coefficient, are not significant with p-value of 0.76 and 0.85; a possible explanation could be that the market realises the risk associated with deviations of type two for American contracts. Additionally the statistical estimations of the Schoenfeld residuals indicate validity of the proportional assumption for all of the cases. Appendix A.1 presents plots of the rescaled Scoenfeld residuals for the 8 groups of data.

It is indicative of the results that the market participants are able to recognise the misspricings in the quoting system. Furthermore they will act rationally towards the most profitable of the risk free PCP deviations, ignoring initially those opportunities which offer less or zero real returns. Although the quoting system gives rise to temporary misspricings, dynamically the market is efficient enough to drive them back towards the theoretically more accepted values.

3.7 Conclusions

Chapter 2 has identified a set of significant PCP deviations. This dataset contains cases where a suitable trading strategy could yield significant risk free, transactions free profits. Whether or not these are due to liquidity premia such as inventory imbalances, their persistence over time would indicate lack of arbitrage forces and possibly other market anomalies. Fortunately the data consist of time stamped, intradaily observations and thus their evolution through time can be analysed.

The notion of an efficient market would require that market participants identify these risk free opportunities. It further assumes that in a dynamically efficient context, temporary deviations should be short lived and removed from the system in a rational way, according to their significance.

Chapter 3 tests whether or not the degree of the PCP deviation is a significant factor for the length of time this stays in the system. It does so by examining the significance of the deviation as a Cox Proportional Hazard covariate.

The modelled duration vector contains the unexpected component after having adjusted for the time of the day and the degree of moneyness factors. Hence it ensures against nonproportionality of hazards. The CPH model used can explicitly account for the presence of censored data in the sample. It is obvious from the previous analysis of section 3.5.1, that the data collection mechanism forces the duration of the last PCP deviation to a maximum length determined by the market closure each day. It has been argued here that under these circumstances it is questionable to include these observations in the sample as this may result in a biased analysis since they do not conform to what is widely accepted as censored observation. Hence results both for an uncensored and a censored group of data have been offered.

The CPH analysis revealed that in the majority of the cases the degree of deviations is a significant factor in the duration of these misspricings for both groups. Higher abnormal profits stay in the system for shorter time than less significant excess returns. However, these findings should be evaluated in view of the results of chapter 2. In a strict market efficiency context there are risk free profits to be made. Nevertheless these are short lived. The results are evident of efficient arbitrage trading. Hence the results do not reject the hypothesis of efficient markets in the high frequency context.

In closing it should be noted that the current results in addition to the findings in chapter 2 point towards the existence of substantial arbitrage opportunities net of transaction costs and risk free from early exercise or immediacy premia. However these deviations are short lived. To exploit them one should act within the limited time that these stay in the system. It is very important then, to be able to model their occurrence. Specifically it is their stochastic arrival that should be analysed, as their deterministic patterns can be identified by a simple signal extraction technique. A valid representation of the arrival process of these arbitrage opportunities is discussed in the next chapter.

CHAPTER 4

Modelling the Put-Call Parity Series as a Random Point Process.

Chapter 4

4.1 Introduction

In its previous part this study has identified significant PCP deviations free from transaction costs and associated traded risks. Specifically in the concluding stages of chapter 3 it has been noted that these represent a series of short lived arbitrage opportunities. As such the complete analysis of these deviations should include a valid representation of their occurrence. It is important to note here that this occurrence could well be stochastic (in addition to any deterministic pattern observed) but most importantly it is frequent and according to irregular spacing. *The issue then is to provide a model, which can capture the stochastic component of this irregularly manifested arrival process.*

Option markets are increasingly becoming sufficiently liquid, and probably adequately "efficient"[1], so as to frequently reflect in price changes new market conditions. By the same token, financial markets can seldom, if ever, be characterised by single daily values or even by intraday series of prices without reference to time. In a quoting system where prices arrive at irregular spacing the choice of equally spaced time intervals could create significant problems. If the interval is too short, varying density in samples will inevitably create heteroscedasticity problems, on the other hand a lengthier spacing will certainly

[1] The term does not refer to market efficiency as this prevails in the literature. We rather mean that new information will be reflected in prices fast enough thus limiting the available time for practitioners to exploit parity deviations.

smooth-out most of the significant information contained in a strike by strike quoting system.

From a theoretical point of view, the reference to time in the relevant literature examining microstructure issues such as the volume, volatility and the rate of transactions in the financial markets is extensive. During the last decade or so, empirical and theoretical researchers have gradually incorporated the issue of time between quotes or trades in the analysis of financial markets. For example Jain and Joh (1986) and Wood, McInish and Ord (1985) present empirical findings on intraday patterns for the variance of price changes and the variance of returns. Also, Admati and Pfleider (1988), Diamond and Verrechia (1987), Glosten and Milgrom (1985) and Easly and O'Hara (1992) construct theoretical models on the link between time and the rate of transactions in the market.

A noticeable drawback of the above theoretical models, including the last two sequential models, is their inability to assign a specific role to the time between observables in the process. If time is an important factor in the price formation, models should specifically incorporate the stochastic behaviour of it as a form of an "explanatory variable".

Chapter 2 has derived a sequence of all the risk-free arbitrage opportunities in the FTSE-100 options market. Chapter 3 has shown that although momentarily inefficient, the market incorporates sufficient learning processes to arrive at informational correct prices. The natural extension for the researcher is to model these processes. However, due to the frequency of the data used, it is not clear if a time indifferent model can successfully capture the true underlying mechanism.

In the following section it is demonstrated that the relevant literature points towards the importance of time between quotes, or transactions, in price forming processes. Hence it is shown that empirical and theoretical evidence reject the use of time indifferent models in favour of processes where time plays a significant role in the intraday evolution of prices.

Based on this discussion the Autoregressive Conditional Duration model of Engle and Russell (1995) is used to offer a time sensitive representation of the PCP deviation process.

4.2 The Importance of Time in Intraday Learning Processes

The rate of change of transaction prices in the market is thought-off to be closely linked to information arrival. According to the mixture of distributions model, price and volume variance are reflections of the flow of information in the market. New information reaching the market, forces market participants to change their buy or sell price limits and to the extend that this leads to new trades the process continues with new revisions. Prominent in this rational is the work of Tauchen and Pitts (1983), who use the Central Limit Theorem to argue that the resulting price and volume variance is the outcome of a mixture of normal distributions, each one referring to individual market participants.

By relating the mixture of distributions model (MODM) to the widely used, and perfectly suited to account for time invariant parameters, GARCH family of models, Nelson (1990) lends some methodological support to the MODM models. He presents a MODM starting from a discrete version of the Exp-ARCH. Models similar to the MODM rely significantly on statistical grounds to provide some explanation of market microstructure effects. Consequently, they are not driven by theoretical arguments in order to explain the existence of intraday patterns.

According to the theoretical market microstructure models proposed by Kyle (1985), Admati and Pfleider (1988) and Easly and O'Hara (1992), transaction clustering should be observed as the rate of the amount of information reaching the market varies significantly with trading time. Indeed both works by Kyle (1985) and Glosten and Milgrom (1985) propose market models where new information reaching the market triggers a group of trading orders in the first case and sequential trading in the latter, and consequently price fluctuations. Given causality between the rate of information and price formation, more dense point processes could be suggested around information arrival than at other points in time. It is true, however that neither Kyle nor Glosten and Milgrom suggest the direct impact of time on price formation.

In a more progressive context, though, Diamond and Verrecchia (1987) and Easly and O'Hara (1992) treat periods of uncertain information arrival as a causal factor for the absence of trading; as such periods of non-trading could well provide further uncertain information to the market. With respect to markets where strong short sales restrictions are present, Diamond and Verrecchia have shown that restrictions of this type could prevent (adverse) information reaching the market thus prolonging periods of no-trade and perhaps intensifying high activity trading periods.

In a more general framework Admati and Pfeiderer (1988) have included, in addition to informed traders, liquidity driven traders in their microstructure model. They assume that both, albeit driven by different factors, should choose periods of high activity to proceed with trading orders, thus attenuating the intensity of trade. Also in Easley and O'Hara (1992) although liquidity traders follow a Poisson determined arrival rate, informed traders will enter the market upon observing a particular trade event - which the authors treat as a noise signal. The model, however, assumes a gradual learning period for the market specialists, given that they too observe the trade event. To maximise their profits then, it is in the interest of the informed traders to trade in the shortest possible interval following the event thus exacerbating clustering of trades.

All of these theoretical suggestions then, reject uniform trading across time in financial markets in favour of a "high and low" pattern. Furthermore it is also suggested that the arrival of new information or market signal will trigger a dense trading activity for informed traders and gradually for the whole of the market.

4.3 Introduction of time as an explanatory variable

The discussion of the previous section points to the significance of the rate of information arrival as a factor affecting, predominantly in our case, the price formation mechanism as well as other microstructure parameters. This in turn dictates that time, through its relationship to the arrival of information, will also play an important role.

Clock time will be important, as it may be able to account for intraday patterns occurring deterministically in the sample. Indeed chapter 3 uses a nonparametric technique to filter-out the deterministic part of durations, which are a direct consequence of clock time.

In addition, the above discussion of theoretical microstructure models leads to the characterisation of the market price of an asset as a reflection of a sequence of steps rather than arbitrary points at a given period. We will accept arbitrary points if as in the context of a Walrasian auctioneer the prices denote market-clearing prices and always reflect equilibrium transactions. It is, however, a process during which there is a continuous stream of quotes that are outstanding at any given point in time and as a series of steps form the observed prices. The difference is subtle but important. Examination of any point in the process should reveal a part of the price formation process. At the same time though, this is used as an input to explain further points in the process; for example, as assumed in some of the models like Admati and Pfleiderer (1988) and Easley and O'Hara (1992), updates of traders beliefs and clustering of trades will lead to serially dependent prices. The parameters assumed by microstructure theory, to affect the price formation mechanism will be time-varying. In contrast, in the Walrasian auctioneer framework prices are assumed time-insensitive.

As it is implied by trade clustering models the above constitute a trade time, in addition to clock time, dependence. In this sense, the correct examination of the time varying properties of the process should treat the vector elements of the variables involved as sequence of points.

Technically speaking, the pricing process does not follow the Markov property. Any point in the process is conditional on its previous values. Obviously the CPH used in Chapter 3 draws on the complete set of observations, but does not make the implicit assumption of a sequence in the sample.

In the following sections the Autoregressive Conditional Duration model as proposed in Engle and Russell (1995, 1997) and later modified by Bowens and Giot (1997) is used to model the time dependency of the duration values between successive PCP deviations. By removing first any dependence on clock time in the form of intraday patterns the analysis seeks to model any remaining trade-time dependence of the intensity of the risk-free opportunities occurrence. By doing so it establishes the degree of memory inherent in the process.

4.4 The Autoregressive Conditional Duration Model.

The Autoregressive Conditional Duration Model of Engle and Russell (1995, 1997) explicitly accounts for the time dimension of a process as it examines the sequential properties of durations between events. By looking at the intertemporal dependence of durations free from any deterministic effects, the model is able to identify periods of significant clustering in the observed parameter beyond any associations to clock time. Additionally, for most parameterisations of the hazard function it offers a simple calculation for the log-likelihood functions involved.

The modelling of a process requires the identification of the most important aspects of its behaviour. According to the previous section the relevant literature has pointed to the informational role of time in the pricing process. It is commonly believed that in sequential models prices do not form a Markov chain, i.e. are not independent from the recent history. Additionally researchers have questioned the martingale property, i.e. the nature of dependence of prices on past values of the explanatory vector[2]. Consequently the modelling first needs to identify as a sequential point process the arrival of observations, parameterise the intensity of arrival, either parametrically or non-parametrically and assume a specific dependency rule, i.e. the degree of history present in the model.

[2] Whether or not prices follow a martingale is open to debate, though. Certainly the majority of the theoretical models assume martingale properties in place and chapter 3 points to martingale at equilibrium.

In the introductory paper of the model, Engle and Russell (1995) identify the relationship of trade time to the pricing process in stock exchange data and the need for a methodology which is able to address serial dependence in the timing of quotes or transactions and consequently clustering of revisions. The model assumes that successive periods between points form a sequence, the intensity of which is determined by a number of lags in the model.

The authors express the ACD representation in terms of the conditional densities and specify the point process in terms of the intervals between successive points, which in turn translates in the specification of the durations among successive observations. However the simplicity of the process lies in the use of the conditional, on past observations, durations for the complete representation of the model.

According to the established notation then, for any point process a counting process $C(n|t)$ can be assigned, where n is the number of arrivals by time t. We are interested in describing such process; assign specific properties, establish expectations and finally derive a statistical representation.

The simplest model able to capture such a counting process is the Poisson. However a closer examination of the process will most certainly reveal that some Poisson properties are quite restrictive. For example the assumption of independent increments contradicts the non-Markovian property of the pricing mechanism, i.e. the dependency between successive observations.

If then

$$E[C(n|t)] = \lambda t \qquad \text{Eq 4.1}$$

represents the Poisson process and λ denotes the intensity, i.e. the instantaneous probability of an observation a valid representation of the right hand side of eq 4.1 is required assuming dependency. However, this still excludes cases where a predetermined dependency is introduced, such as in the case of a non-homogeneous Poisson where the successive arrival times are dependent in a pre specified, deterministic manner. Indeed the existence of a stochastic representation of the counting process will be in accordance with the documented evidence of serially correlated arrival times for traded assets, see Engle and Lange (1997), Engle and Russell (1997) and Bauwens and Giot (1997).

There are different specifications of λ that assume a degree of memory in the process. Among the most popular, the self-exciting process of Hawkes (1971) assumes an extension of the doubly stochastic process by Bartlett (1963) as a stationary random process with the integrated hazard function

$$\Lambda(t) = \nu + \int_{-\infty}^{t} g(t-u)dN(u) \qquad \text{Eq 4.2}$$

where u denotes history of past observations. Essentially the process assumes a current intensity determined by past events according to a function of u. However, as it does not assume any relationship between the degree of importance of past information and the current state, it is not suitable for arrival processes of intraday finance data. Such relationships are desirable if one considers the degree of serial dependency in financial data.

Another approach adopted by Wold (1948), assumes dependency and specifies the intensity as a function of the conditional hazard θ

$$\lambda\ (t|t\text{-}1\ldots) = \theta\ (t|t\text{-}1\ldots) \qquad \text{Eq 4.3}$$

and consequently as a function of the probability of observing a particular event and the survivor function. Although it is possible to represent the Wold model directly in an autoregressive form (with the desirable number of past lags), thus specifying the exact degree of memory in the process, plus the specification of the probability measures, the model is inflexible towards financial data. Specifically it assumes a minimum current duration as a function of its past value.

A more appealing approach but without the above restrictions is offered by Cox (1955) in the form of the proportional hazard model (which incidentally forms the basis of the CPH model adopted in chapter 3). The model assumes stochastic dependency on past observations and exhibits a conditional hazard

$$\theta\ (t;\ h_t,\ h^{\Psi}_t) = \lim_{\delta \to 0^+} \delta^{-1}\ pr\{C(t,\ t+\delta) > 0)\ |\ h_t,\ h^{\Psi}_t\} \quad \textbf{Eq 4.4}$$

or

$$\theta\ (t;\ h_t,\ h^{\Psi}_t) = \lim_{\delta \to 0^+} \delta^{-1}\ pr\{N(t,\ t+\delta) > 0)\ |\ h_t,\ h^{\Psi}_t\} \quad \textbf{Eq 4.5}$$

where

h_t is a vector capturing the history of events and

h^{Ψ}_t summarises the stochastic behaviour of the process.

Equation 4.4 dictates that for a successful representation of the process we should specify correctly the two vectors h_t, h^{Ψ}_t.

Ideally then, to capture correctly the complete effect of time in the observation process we should be able to break down into separate processes the effect of past observations as well as the pure stochastic component, i.e. vector h^{Ψ}_t.

Again the exact methodology of formulating vector h^{Ψ}_t is open to discussion. The stochastic behaviour of the process can be estimated either parametrically or non-parametrically. In the later case, smoothing techniques such as spline, k-

NN, orthogonal or kernel can be used to estimate semi-parametrically or non-parametrically, the hazard.

The parametric specification of the process is open to a number of suggestions in the literature. The choice of the density used could come from the simple exponential case to the more elaborate Weibull or Gamma function density families. In any case the correct specification of the density will enable the proposed model to adjust in order to capture the arrival rate of the recent events history. As such the structure of the model greatly facilitates the modelling of intraday, high-frequency market data, where periods with abnormally high or low transaction rates (or changing conditions which may manifest themselves as new quotes) are characteristic.

Representing the arrival time of events as a sequential series of duration between successive observations, with the duration, x_i, given by

$$x_i = t_i - t_{i-1} \qquad \text{Eq 4.6}$$

Engle and Russell (1995) propose that the specification of vectors h_t, h^{Ψ}_t can be constructed using the current observed duration X and the current expected duration, Ψ_i. Linking the two as

$$E(x_{i\,|} \mid x_{i-1}, x_{i-2}, \ldots, x_1) = \Psi_i \qquad \text{Eq 4.7}$$

or in a multiplicative fashion as

$$x_i = \Psi_i \varepsilon_i \qquad \text{Eq 4.8}$$

correct specification of the two vectors follows the assumption that ε_i is independently and identically distributed.

To the extend that this will lead to a successful representation of the ε_i process as an independently and identically distributed variable - which is a prerequisite for a valid ACD representation as well as the correct specification of the stochastic durations- other variables could enter Eq 4.7. Engle and Russell (1995) suggest the use of time to capture deterministic time-of-the-day effects, which should not be present in the conditional durations of Eq 4.8, hence

$$E(x_i \mid x_{i-1}, x_{i-2}, \ldots, x_1) = \Psi_i F(t_i) \qquad \text{Eq 4.7a}$$

Equation 4.7a assumes a multiplicative relationship (in order to satisfy positivity of time and duration requirements) between the function F_i and the stochastic component of the arrival time.

As can be seen from Eqs 4.7, 4.8 the currently expected duration is directly expressed in terms of the expectation process governing the evolution of the events and is directly affected by our assumptions on the distribution of ε.

However, as well as encompassing available information through the assumed parametric form of the stochastic component, the model should also assume the specific amount of past information through some parameterisation of the conditional term. Specification of the memory process can follow the suggestions in the literature for the family of Autoregressive models. Hence the process can be specified either in terms of k lags of realised durations, ACD (k)

$$\Psi_i = \omega + \sum_{j=0}^{k} \alpha_j x_{i-j} \qquad \text{Eq 4.9}$$

or, if assumed that past values of the dependent variable are significant, it can be augmented in terms of the lamda (l) most recent conditional expectations as well as the k recent lags of durations, ACD (k, l)

$$\Psi_i = \omega + \sum_{j=0}^{k} \alpha_j x_{i-j} + \sum_{j=0}^{l} \beta_j \Psi_{i-j} \qquad \text{Eq 4.10}$$

The idea behind the structure of the models in Eqs 4.7 and 4.8 invites an even more general specification with the inclusion of exogenous variables. Hence if the vector ψ_t summarises the observed independent set, the currently expected duration can be specified as a function of lagged durations and conditional expectations as well as of the history of vector ψ, H^{ψ}_{t}.

$$\Psi_i = \omega + \sum_{j=0}^{k} \alpha_j x_{i-j} + \sum_{j=0}^{l} \beta_j \Omega_{i-j} + (H^{\psi}_{t}) \qquad \text{Eq 4.11}$$

It can be seen then, that the model is specified through equations 4.7 or 4.7a and one of equations 4.9-4.11 representing the evolution of the conditional duration.

As equations 4.9-4.11 offer a wide choice for the specification of the conditional process so does the specification of the error term in equation 4.8. Looking at the pair of equations as a system capturing the time dependence, it is seen that in order to derive an expression of the conditional duration intensities it is useful to observe that realised values for the durations and the conditional expectations are related through Eq 4.8 and more specifically through ε. It is helpful then, to express, in general terms, the transition intensity of ε so as to obtain an expression for this "linking" process. A starting point is to assume that if Ψ_i is the expected value of the current duration given the history of observations, a systematic-variation free relationship between x_i and Ψ_i is established by having

$$x_i = \Psi_i \varepsilon_i \qquad \text{Eq 4.12}$$

where $\{\varepsilon_i\} \sim$ i.i.d. with exponential density.

A more widely used approach, however, is to assume the existence of a vector Φ proportional to the conditional durations, which follows a Weibull distribution. In that case the mixing process of Eq 4.8 becomes

$$x_i = \Phi_i \varepsilon_i \qquad \text{Eq 4.13}$$

and a third equation specified by the distributional assumption and links Eq 4.13 with Eqs 4.9 - 4.11

$$\Gamma(1 + 1/\gamma)\,\Phi_i = \Psi_i \qquad \text{Eq 4.14}$$

The advantage of this approach is that for a Weibull distribution of parameter γ, the conditional intensity can accommodate more successfully highly clustered or thinly spaced points depending on the value of the parameter. For this Weibull of parameter γ then, the transition intensity becomes

$$\theta(t \mid x_{N(t),\dots,}x_1) = \{\Gamma(1 + 1/\gamma)\,\xi_{N(t)+1}^{-1}\}^{\gamma}(t - t_{N(t)})^{\gamma-1}\gamma \qquad \text{Eq 4.15}$$

where according to the usual notation

$\Gamma(.)$ is the gamma function and

γ the parameter of the Weibull distribution.

For this most intuitive representation among the alternatives, Engle and Russell (1995) propose the following system of equations as the ACD model:

$$x_i = \Phi_i \varepsilon_i \qquad \text{Eq 4.13}$$

$$\Psi_i = \omega + \sum_{j=0}^{k} \alpha_j x_{i-j} + \sum_{j=0}^{l} \beta_j \Psi_{i-j} \qquad \text{Eq 4.10}$$

$$\Gamma\,(1 + 1/\gamma)\, \Phi_i = \Psi_i \qquad \text{Eq 4.14}$$

with k (number of lags) equal to 1 and with Maximum Likelihood estimates calculated through the maximisation of the following Likelihood function:

$$\text{Log}\,(\theta, \gamma) = \ln\,(\gamma/x_i) + \gamma \ln(x_i/\Phi_i) - (x_i/\Phi_i)^{\gamma} \qquad \text{Eq 4.15}$$

In a close analogy to the GARCH family of models, the ACD parameterisation defines the conditional expectation of the duration between points as vector Ψ. The unconditional expectation and variance are given in Engle and Russell (1995) as

$$E(x_i) = \mu = \omega \,/\, (1 - \alpha - \beta) \qquad \text{Eq 4.16}$$

(assuming $\alpha + \beta < 1$) and

$$\sigma^2 = \mu^2\, k\, (\,1 - 2\alpha\beta - \beta^2)\, /\, [1 - (\alpha + \beta)^2 - \alpha^2\, k] \qquad \text{Eq 4.17}$$

(assuming positivity of the denominator), where k relates to the Weibull parameters according to

$$k = \Gamma(1 + 2/\gamma) / \Gamma(1 + 1/\gamma)^2 - 1 \qquad \text{Eq 4.18}$$

In cases where the variance is greater than the estimated mean the model exhibits overdispersion in the data and vice versa. Additionally, depending on the value of gamma the model can account for a decreasing hazard function, i.e. longer durations ($\gamma < 1$) or for an increasing hazard function, i.e. shorter durations ($\gamma > 1$).

The Log-ACD Model

Since equations 4.9-4.11 model time, which by definition is positive, close inspection of the right hand side of the equations will reveal the following conditions for positivity in every case.

$\omega > 0$, $\beta \geq 0$ and $\gamma > 0$. Condition 1

Condition 1 will impose severe constraints on the inclusion of any exogenous variable in Eq 4.11 as it requires positivity of the variable under all cases; this is not always compatible with variables drawn from the microstructure theory.

Bauwens and Giot (1997) propose a simple logarithmic transformation of eq 4.8, which they call the Logarithmic ACD model (LACD). The aim of the LACD is to enable the use of exogenous variables without the positivity constraint. The LACD assumes that

$$x_i = e^{\Phi_i}\varepsilon_i \qquad \text{Eq 4.19}$$

where again ε_i are i.i.d. and distributed as a Weibull with parameters $1,\gamma$ and Φ_i is proportional to the logarithm of Ω_i according to

$$e^{\Phi_i}\Gamma(1+1/\gamma)=e^{\Psi_i} \qquad \text{Eq 4.20}$$

Equation 4.20 provides a link of the specification of observed durations with the parameterisation of the conditional values. In a logarithmic Autoregressive form the conditional duration is assumed that depends on j past values plus j lagged values of the observed duration as

$$\Psi_i = \omega + \sum_{j=0}^{k} \alpha_j f(x_{i-j}, \varepsilon_i) + \sum_{j=0}^{l} \beta_j \Psi_{i-j} \qquad \text{Eq4.21}$$

In a direct correspondence with GARCH models the exact choice of the function for x and ε is open to a large number of suggestions, perhaps according to microstructure issues for the specific market examined. Bauwens and Giot (1997) propose the following functional forms (with a single lag)

$$\Psi_i = \omega + \alpha \ln(x_{i-1}) + \beta \Psi_{i-1} \qquad \text{Eq4.22}$$

with the imposed condition of $|\alpha + \beta| < 1$ for covariance stationarity of $\ln x_i$. The logarithmic transformation of the observed duration approximates the Log-GARCH suggested in Geweke (1996) but at the same time excludes zero duration; at the limit this implies that extremely dense processes cannot be accommodated by application of eq 4.22.

In a different Autoregressive specification of the process the authors express the conditional duration on its past value and the excess value of the observed duration, i.e. ε_i,

$$\Psi_i = \omega + \alpha \ln(x_{i-1}) + \beta \Psi_{i-1} \qquad \text{Eq4.23}$$

or

$$\Psi_i = \omega + \alpha \frac{x_{i-1}\Gamma(1+1/\gamma)}{e^{\Psi_{i-1}}} + \beta \Psi_{i-1} \qquad \text{Eq4.23a}$$

which allows for zero duration and resembles the exponential GARCH suggested in Nelson (1991). Representation of the memory equation as in Eq 4.23a requires that $\beta < 1$ for covariance stationarity of Ψ. Eq 4.23a will be referred to as the ε-ACD[3].

Both the Log-ACD and the ε-ACD give for the conditional expectation and variance of the duration, expressions similar to the ones obtained by the ACD model. However neither of the two unconditional moments can be expressed analytically for these specifications.

[3] The authors refrain from calling this specification as the exponential-ACD as it clashes with the term given to the simple ACD where the excess observation is exponentially distributed.

4.4.1 Application of ACD models in the literature

The empirical use of the ACD family models in finance is not widespread. However, empirical evidence suggests that the ACD methodology successfully captures the time dependence in stochastic durations.

In Engle and Russell (1997), the authors analyse the New York Stock Exchange market between November 1, 1990 and January 31, 1991 for trade by trade transactions of IBM stock using the ACD model. Overall the method captures satisfactorily the duration dependence over the period of the investigation. To capture the Autoregressive dependence in the data they estimate parametrically the point process using a succession of Exponential ACD (1, 1), (2, 2) and Weibull ACD (1, 1) and (2, 2) models. Their results show a considerable weakness of the exponential distribution assumption to capture the observed duration and a much better performance for the WACD (1, 1) and (2, 2) models but with no significant difference between the two. In concluding, the authors refer to the promising ability of the model to capture serial correlation in the sample and thus accommodate interdependence of successive points.

In a related study Bauwens and Giot (1997), examine high frequency quotes and transaction prices on US Robotics and IBM stocks. Trading takes place in the NASDAQ and NYSE markets respectively and the data cover the period of October 1996 for the first and September, October and November 1996 for the second. There is evidence of statistically significant models for all three representations. However, the Log-ACD model is less successful in capturing the autocorrelation present in the sample. Both the W-ACD and the ε-ACD account for interdependencies in the process

The ACD methodology has also been successfully applied on high frequency quotes from the Foreign Exchange market. Engle and Russell (1997) examine quotes and prices, as supplied by Reuters, for the Dollar-Deutschmark foreign exchange for the period of 1 October 1992 to 30 September 1993. The authors analyse the arrival rate using the Weibull ACD (1,1). The evidence suggests that the Weibull density is a far better approximation of the true underlying distribution; the exponential hypothesis is rejected across the sample. The model captures successfully interdependence in the data, which are initially present even after conditioning on time. The authors demonstrate that the ACD representation is easily extended and test competing microstructure theories, specific to the foreign exchange market, by augmenting the model with the inclusion of market specific explanatory variables.

Although market microstructure issues have been a subject of an extensive empirical study the research on high frequency data is by far more limited. With respect to the ACD parameterisation the above represent the only available empirical testing for the analysis of intradaily, irregularly spaced high frequency data.

Here a point process is assumed, which is defined by the arrival of profitable arbitrage opportunities between the options on the FTSE-100 and the spot market on the same underlying. Our methodology employs the ACD model of Engle and Russell (1995 and 1997).

4.5 Data and Methodology

Following the results of Chapter 2, raw durations for successive Put Call parity quotes are modelled with reference to deviations of type one and two. To be consistent with the sequential requirements of the ACD family models the PCP deviations are filtered retaining quotes only for the contracts with strikes closest to the current underlying index and the nearest expiration month; throughout the sample these are by far the most heavily quoted contracts.

Although, perhaps by any other criteria, the samples studied here are of a sufficient size, the ACD models are data intensive and perform better with a denser process. Consequently, the study concentrates on deviations derived for zero transaction costs and zero short sales restrictions (stock borrowing) as these are the most frequent. Both European and American based PCP are analysed over the whole period i.e. from 7th August 1994 to 28th February 1997.

In correspondence with the previous chapter, all observations which are terminated artificially by the market closure are filtered out and separate estimates are offered for the groups containing only non-censored values. The distinction between the two groups is, perhaps, of greater importance in the ACD framework as any censored observations will tend to "upset" any patterns established in the sequential process and hence arbitrarily change the clustering and dispersion factors over the duration of the process.

In total estimations for eight groups are presented. Table 4.1 below summarises the number of observations for each different group.

Table 4.1
Description of samples, number of observations.

Type			Number of Observations
Censored	European	Dev 1	1450
		Dev 2	3257
	American	Dev 1	1927
		Dev 2	8523
Uncensored	European	Dev 1	3321
		Dev 2	6301
	American	Dev 1	3250
		Dev 2	14325

The duration between the (i-1)th and ith observation on the nth day in the sample is defined as x_{in}, so

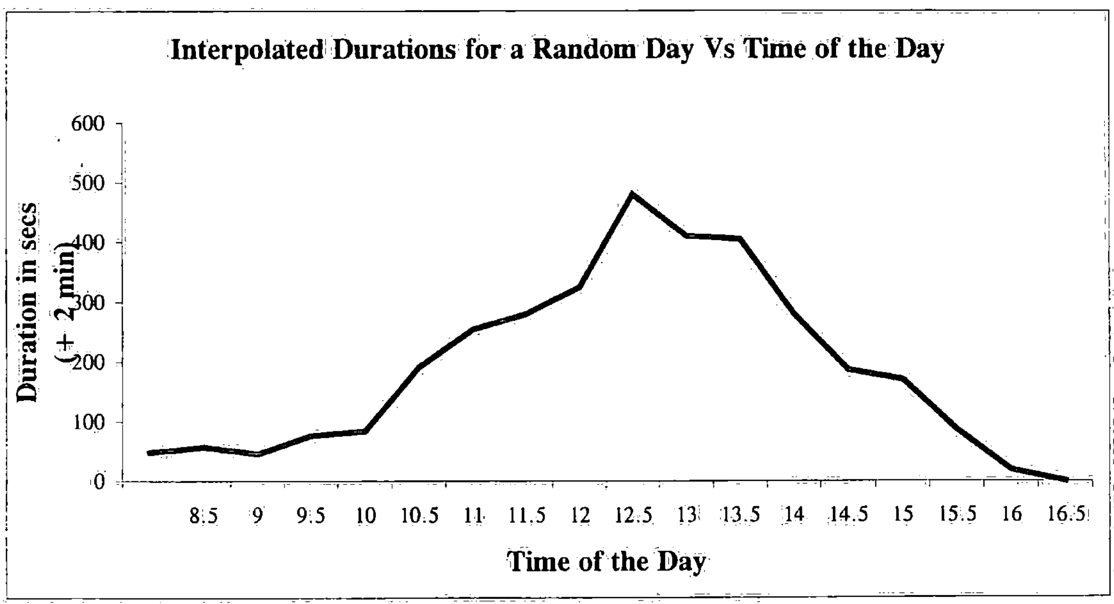
$$x_{in} = t_{in} - t_{(i-1)n} \qquad \text{Eq 4.24}$$

The durations are expressed in seconds and the count is terminated at market closure and being reset at the market opening (i.e. durations between market closure and market opening in the following day are deleted). To avoid contamination from market opening effects, and in accordance with the methodology in chapters 2 and 3, the first quote for every contract each day is removed from the sample.

According to Engle and Russell (1997) it is essential to remove any deterministic effects from the series of raw durations. Figure 3.2, repeated below, plots the durations throughout the trading period for a typical day (mid-week) of the sample.

Figure 3.2

The Graph shows interpolated durations for a typical day of the week. Durations are counted after the 2 minutes delayed execution time.



The plot provides evidence of a significant smile pattern during the day. Based on this graph, durations are expected to be extremely short lived at the opening of the market, become larger around late morning to midday and shorten again towards the closing stages of the day. In the present study, however, we are interested in the examination of the stochastic behaviour of durations. This requires that any cyclical or trend part has to be subtracted from the durations time series. Both Engle and Russell (1995, 1997) and Bauwens and Giot (1997) define the deterministic effect as a multiplicative component:

$$X_i = x_i \, \Phi(t_i) \qquad \text{Eq 4.25}$$

where X_i is the raw duration and Φ is an estimate of the expected value of the duration given the time of the day effect. The authors compute the expectations of the time of the day effect by averaging durations over a thirty minutes time interval during the day, however, they do not report on specific adjustments for the degree of density of quotes during these intervals.

The current study improves on the evaluation of the deterministic effect offered in the above papers by adjusting for the number (and hence the distance between) of observations within each interval. In direct correspondence to the signal extraction methodology followed in chapter 3, eqs 3.36 and 3.37 are used to evaluate duration values free from time of the day deterministic effects.

$$\mathbf{E}(x_i \mid x_{i-1}, x_{i-2}, \ldots, x_1) = \Omega_i (x_i \mid x_{i-1}, x_{i-2}, \ldots, x_1; \theta) \equiv \Omega_i \qquad \textbf{(Eq 3.36)}$$

$$x_i = \Omega_i \, \varepsilon_i \qquad \textbf{(Eq 3.37)}$$

where $\{\varepsilon_i\} \sim$ i.i.d. with density p.

Equation 3.37 produces the optimum deterministic values for each time of the day by applying the cross validation methodology as outlined in section 3.5.2.

Following the results in Engle and Russell (1995, 1997) and Bauwens and Giot (1997), the ACD model assuming a Weibull distribution of the error term in the mixing process of equation 4.14 (W-ACD) and the exponential representation of the expected conditional durations of equation 4.23a (ε-ACD) are applied. Thus in the W-ACD case the raw observed durations are modelled by specifying the following mixture process

$$x_i = \Phi_i \, \varepsilon_i \qquad \text{(Eq 4.13)}$$

subject to an autoregressive representation of the conditional durations given by the single lag model[4]

$$\Psi_i = \omega + \alpha \, x_{i-1} + \beta \, \Psi_{i-j} \qquad \text{Eq 4.26}$$

[4] We have tested both the single (1,1) and double (2,2) lags models, using Akaike's Information Criterion. We have found that the single model performs better. The results are reported in tables 4.2 to 4.16.

assuming a Weibull error distribution thus linking equations 4.13 and 4.26 using

$$\Gamma(1 + 1/\gamma)\ \Phi_i = \Psi_i \qquad \text{(Eq 4.14)}$$

Equivalently, in the case of the exponential representation the raw observed durations are modelled by specifying the following mixture process

$$x_i = \Phi_i\ \varepsilon_i \qquad \text{(Eq 4.13)}$$

subject to an autoregressive representation of the conditional durations given by the single lag model

$$\Psi_i = \omega + \alpha\ x_{i-1}\ \frac{\Gamma(1+\frac{1}{\gamma})}{e^{\Psi_{i-1}}} + \beta\ \Psi_{i-j} \qquad \text{(Eq 4.23a)}$$

assuming a Weibull error distribution thus linking equations and with equation

$$e^{\Phi_i}\ \Gamma(1+\frac{1}{\gamma}) = e^{\Psi_i} \qquad \text{(Eq 4.20)}$$

where, for both of the above models

x_i is the stochastic component of the ith raw duration,

Ψ_i is the ith observation of the conditional duration,

ω is the model constant,

α,β are the model coefficients,

Γ denotes the gamma function and

γ is the Weibull coefficient, note that if $\gamma = 1$ then the model reverts to the simple exponential one.

At this stage, perhaps it is useful to recall that the purpose of the current analysis is to offer a valid specification of a sequential signalling process (where the signal takes the form of the PCP deviations). This however, is conditional on the explicit assumption that the time interval between successive signals characterises the process itself. Thus our hypothesis tests whether or not the proposed models can successfully represent the duration time series of our raw data, or equivalently if the estimated duration values resemble the raw duration series.

For a successful modelling of the raw durations it is required that a specific set of conditions is met. At the coefficient estimation stage it is required that for the W-ACD model coefficients

$$\alpha + \beta < 1$$

to ensure existence of the unconditional mean of the duration. Equivalently for the ε - ACD model it is required that $\beta < 1$ for covariance stationarity of Ψ.

Following the satisfaction of these criteria the unconditional means for the W-ACD model can be computed as

$$\mu = \omega / (1-\alpha-\beta) \qquad \text{(Eq 4.16)}$$

and the unconditional standard deviations for the W-ACD model as

$$\sigma = \sqrt{(\mu^2 k \frac{1-2\alpha\beta-\beta^2}{1-(\alpha+\beta)^2-\alpha^2 k})} \qquad \text{(Eq 4.17)}$$

Unfortunately no analytical expressions for the ε-ACD model exist (see Bauwens and Giot (1997)).

The unconditional means should approach the values of the expected means, which by construction equal 1 in our case. However, even if the model yields the correct values for the unconditional means correct specification requires that the representation of the durations captures any serial dependence in the error term, i.e. it should successfully remove any serial correlation present in the data.

In accordance to the related literature the estimated values of the model coefficients are presented, including the Weibull coefficient which denotes over- or underdispersion and the calculated unconditional means and unconditional standard deviations, where appropriate. The Ljung-Box statistics for the first ten lags for the raw durations and the first ten lags for the residuals are also reported.

4.6 Results

Results are reported separately for groups containing the censored observations and the filtered group.

Censored observations

It is evident from the results covering the censored sub-sample that both representations used here are not able to capture successfully the process. Although, with the exemption of the ω constant for ε-ACD European deviations 1 (see table 4.7), all coefficient estimates are significant, as referred to in the previous section, correct specification of the model requires the validity of a number of conditions for the coefficients α and β. In particular, the ε-ACD representation requires that for covariance stationarity of Ψ b should be less than one. In both deviations for American contracts (tables 4.15 & 4.17), the value of β is significantly greater than one for the ε-ACD model. The value is still marginally higher than one for the European deviations one sample (table 4.7) and only marginally smaller for deviations two (table 4.9). These results, as well as all the estimates presented here, are robust to initial values.

Essentially, non-existence of covariance stationarity means that the mean or the autocovariance of the process will depend on the particular time of the event. For a correct representation we require that the autocovariance, for example, is a function between values but not of time. Clearly in this case the ε-ACD representation cannot approximate the standardised constant variance of the unconditional durations.

Although the W-ACD model yields coefficients α and β with a sum less than 1, these are very close to be integrated (in most cases the values range between 0.9-1.0). Integration of the model will seriously challenge the theoretical assumptions required for the existence of the unconditional mean, see Eq 4.16. Results from tables 4.6, 4.8, 4.14 and 4.16 reveal that the calculated unconditional means are significantly larger than the value of one, which is the standardised mean for the raw durations. Unconditional standard deviations, though, exhibit the expected overdispersion in the data (standard deviation greater than the mean value). Additionally both models yield a decreasing hazard function through the estimates of the gamma coefficient. Essentially this denotes that longer durations are less likely in the data, a fact which is compatible with the presence of a cluster of very short end of day censored durations.

However, even if the violations of the permissible range for the β coefficient and the excess unconditional means are ignored, it is evident that both of the models are unable to account for the serial correlation in the data. From all the censored results tables (4.6-4.9 & 4.14-4.17) it is evident that serial correlation is evident in the first 12 lags of the error term for all sub-samples.

It is clear from the results across different samples that the removal of the time-of -the-day effect cannot account for the presence of autocorrelation (values for Ljung-Box statistic for x_i). Correct specification of the model, though, would require that the imposition of the ACD structure would be able to remove any remaining serial correlation; this is not the case. It is clear that either specification cannot model successfully the arrival of PCP deviations as a sequential process.

Uncensored Observations

This section presents results for sub-samples where all artificially terminated event durations have been filtered-out. With the exemption of α for American deviation 1 (table 4.10), all estimated values for the model coefficients in tables 4.2-4.5 and 4.10-4.13 are significant, it is interesting to note, however, that the sample for American deviation 1 is the thinnest of all the uncensored groups. Closer inspection of the results reveals that all the conditions imposed in the specification stages of the model are met for both representations. In all cases either $\alpha+\beta<1$ (for the W-ACD model), or $\beta<1$ (for the ε-ACD model). Essentially this ensures that the representation of the memory of the process as the lagged values of the raw and conditional durations is valid. The stochastic arrival of the next deviation is by and large explained by the arrival of the previous deviation (or its duration). Practically this could mean that the speed of the market adjustment for the deviation removal clusters and continuous to the next observation.

Non-violation of the coefficient values, enables the calculation of the unconditional mean and standard deviation for the W-ACD model. The mean values obtained should be close to one since the duration time series used as input has been standardised (it should exhibit a mean of 1 by definition). Again with the exception of the American deviation 1 sample, all unconditional means are very close to one. Calculated standard deviations are greater than the corresponding means, a fact which denotes overdispersion in the sample – a property one expects to find in clustered data, similar to the ones studied here; again for the American deviation 1 under-dispersion is evident. Again across the sample we find a value of γ greater than one, which is evident of an increasing hazard function, i.e. of more probable long durations; with the removal of most end of day observations longer durations should be expected. Unfortunately such estimates are not available for the ε-ACD model.

From the Ljung-Box statistics for the X process, it is evident that the time of the day adjustment alone cannot provide a way of modelling serial correlation in the data. All the statistics denote excessive serial correlation in the raw duration values; an empirical result which confirms the theoretical intuition of earlier sections and the findings by Engle and Russell (1997) and Bauwens and Giot (1997). If the ACD representation is a correct specification of the process it should account for the interdependency in the data. It is evident from the LB statistics for the first 12 lags of the error series that in all of the cases the serial correlation present is insignificant. Hence the two representations can successfully model the sequential interdependencies.

From the appropriate tables it is evident that either of the two representations offer a valid modelling specification for time as an explanatory variable in the pricing process. Comparing, however, the results for the censored and uncensored groups it is apparent that the inclusion of the censored observation represents significant modelling problems. The inclusion of these observations artificially shortens a large portion of the data. Furthermore, this is a systematic process, i.e. it occurs at the end of each day. It remains a puzzle for us, and probably an issue for further research, why the smoothing-out of the time of the day effect process does not account for this effect too.

4.7 Conclusions

In its concluding stages chapter 3 has identified the need to model the occurrence of the PCP deviations and specifically their stochastic arrival. We have shown in chapter 2 that these deviations are related to market inefficiencies and consequently should be removed from the price quoting system. Additionally, they represent risk free opportunities for arbitrage trading. The successful intervention of arbitrageurs is critical to the elimination of these inefficiencies. As such a successful modelling of the PCP arrival process is of importance.

The purpose of this chapter was to model the PCP deviations as a sequential process. With reference to the theoretical market microstructure literature and specifically of price formation mechanisms, it has been shown that the time variable is an important factor for intraday financial series, which are related to price adjustments.

It has been argued that the most recent theoretical models indicate that, due to the nature of the learning processes adopted, pricing processes should not be modelled as Markov series, thus current prices are not independent from past observations. Consequently it has been proposed that a successful modelling of the evolution of the PCP deviations should specifically account for the role of time and, further, assume a memory process in the representation stages.

In order to account for the trade-time factor a simple counting process has been discussed and it has been shown that the Autoregressive Conditional Model of Engle and Russell can account for both propositions. By representing the time of events as the duration between points in a sequential process and by conditioning expected durations on a specified number of lags, the ACD model accounts for serial dependency in the data and the stochastic input of time in the process.

The deviation series has been modelled using the W-ACD and the ε-ACD model as proposed by Engle and Russell and Bauwens and Giot for two separate groups of censored and uncensored observations. The results showed that both models were able to account for the serial correlation present in the data, whereas the nonparametric "extraction of the time of day effect" mechanism was not. Furthermore both models yielded valid coefficients and the expected increasing hazard functions. Additionally the W-ACD model, provided estimates for the unconditional mean close to the assumed ones and the desirable overdispersion in the data (the mechanics of the ε-ACD model does not allow direct comparisons with respect to estimates for the unconditional mean).

However, it is clear that both models were unable to provide a successful representation of the process for the censored group. For both models the coefficient estimates violated the theoretical conditions imposed for the unconditional mean or the covariance stationarity of the modelled durations. Furthermore they failed to account for the autocorrelation present in the data. We have attributed both of these results to the significant presence of short durations at the end of day. Finally both models yielded, on average, shorter durations than the uncensored group and the W-ACD representation pointed towards an excessive overdispersion in the data according to expectations.

Table 4.2

Maximum Likelihood Estimates for the **Weibull (1, 1) ACD** model. The sample refers to type one deviations of European contracts. The data do not include censored observations and are calculated after having removed the time of the day effects.

W-ACD (1, 1)		
European Contracts Dev 1		
Coefficient	Estimate	Std. Error
ω	0.0517	0.00218
α	0.0257	0.00094
β	0.9253	0.00812
γ	1.078	0.01913
Unconditional mean	1.055	
Unconditional Std dev	1.058	
Ljung-Box Statistic for X_i (12 lags)	Q(12) = 2983.6469 (0.000)	
Ljung-Box Statistic for ε_i (12 lags)	Q(12) = 16.4601 (0.171)	
AIC for W-ACD (1,1)	3128	
AIC for W-ACD (2,2)	2506	

Table 4.3

Maximum Likelihood Estimates for the **Log- exponential (1, 1) ACD** model. The sample refers to type one deviations of European contracts. The data do not include censored observations and are calculated after having removed the time of the day effects.

ε-ACD (1, 1)		
European Contracts Dev 1		
Coefficient	Estimate	Std. Error
ω	0.0623	0.00192
α	0.0167	0.00655
β	0.9369	0.04623
γ	1.051	0.03659
Ljung-Box Statistic for X_i (12 lags)	Q(12) = 3078.7183 (0.000)	
Ljung-Box Statistic for ε_i (12 lags)	Q(12) = 20.0294 (0.06653273)	
AIC for W-ACD (2,2)	2185	
AIC for W-ACD (1,1)	3609	

Table 4.4

Maximum Likelihood Estimates for the **Weibull (1, 1) ACD** model. The sample refers to type two deviations (requires re-investment of short sales proceeds) of European contracts. The data do not include censored observations and are calculated after having removed the time of the day effects.

W-ACD (1, 1)		
European Contracts Dev 2		
Coefficient	Estimate	Std. Error
ω	0.04895	0.00627
α	0.0212	0.00894
β	0.9314	0.00696
γ	1.063	0.03721
Unconditional mean	1.0327	
Unconditional Std dev	1.037	
Ljung-Box Statistic for X_i (12 lags)	Q(12) = 2913.8781 (0.000)	
Ljung-Box Statistic for ε_i (12 lags)	Q(12) = 13.1145 (0.360)	
AIC for W-ACD (2,2)	2509	
AIC for W-ACD (1,1)	3326	

Table 4.5

Maximum Likelihood Estimates for the **Log- exponential (1, 1) ACD** model. The sample refers to type two deviations (requires re-investment of short sales proceeds) of European contracts. The data do not include censored observations and are calculated after having removed the time of the day effects.

ε-ACD (1, 1)		
European Contracts Dev 2		
Coefficient	Estimate	Std. Error
ω	0.05967	0.00374
α	0.0345	0.00127
β	0.956	0.00679
γ	1.042	0.06837
Ljung-Box Statistic for X_i (12 lags)	Q(12) = 2883.0378 (0.000)	
Ljung-Box Statistic for ε_i (12 lags)	Q(12) = 13.0019 (0.368)	
AIC for W-ACD (2,2)	2985	
AIC for W-ACD (1,1)	3423	

Table 4.6

Maximum Likelihood Estimates for the **Weibull (1, 1) ACD** model. The sample refers to type one deviations of European contracts. The data include all end of day censored observations and are calculated after having removed the time of the day effects.

W-ACD (1, 1)		
European Contracts (censored observations) Dev 1		
Coefficient	Estimate	Std. Error
ω	0.00764	0.00046
α	0.069	0.00914
β	0.927	0.00838
γ	0.739	0.09765
Unconditional mean	1.91	
Unconditional Std dev	1.935	
Ljung-Box Statistic for X_i (12 lags)	Q(12) = 4161.0610 (0.000)	
Ljung-Box Statistic for ε_i (12 lags)	Q(12) = 2792.8226 (0.000)	
AIC for W-ACD (2,2)	3036	
AIC for W-ACD (1,1)	3289	

Table 4.7

Maximum Likelihood Estimates for the **Log- exponential (1, 1) ACD** model. The sample refers to type one deviations of European contracts. The data include all end of day censored observations and are calculated after having removed the time of the day effects.

ε-ACD (1, 1)		
European Contracts (censored observations) Dev 1		
Coefficient	Estimate	Std. Error
ω	**0.0327**	**0.00924**
α	0.0614	0.00217
β	1.0037	0.01027
γ	0.697	0.10064
Ljung-Box Statistic for X_i (12 lags)	Q(12) = 4139.0280 (0.000)	
Ljung-Box Statistic for ε_i (12 lags)	Q(12) = 2723.7164 (0.000)	
AIC for W-ACD (2,2)	2802	
AIC for W-ACD (1,1)	3241	

Table 4.8

Maximum Likelihood Estimates for the **Weibull (1, 1) ACD** model. The sample refers to type two deviations (requires re-investment of short sales proceeds) of European contracts. The data include all end of day censored observations and are calculated after having removed the time of the day effects.

W-ACD (1, 1)		
European Contracts (censored observations) Dev 2		
Coefficient	Estimate	Std. Error
ω	0.00721	0.00016
α	0.05231	0.00294
β	0.9437	0.00429
γ	0.8952	0.01527
Unconditional mean	1.807	
Unconditional Std dev	1.81	
Ljung-Box Statistic for X_i (12 lags)	Q(12) = 3868.5487 (0.000)	
Ljung-Box Statistic for ε_i (12 lags)	Q(12) = 1427.7607 (0.000)	
AIC for W-ACD (2,2)	3251	
AIC for W-ACD (1,1)	3134	

Table 4.9

Maximum Likelihood Estimates for the **Log- exponential (1, 1) ACD** model. The sample refers to type two deviations (requires re-investment of short sales proceeds) of European contracts. The data include all end of day censored observations and are calculated after having removed the time of the day effects.

ε-ACD (1, 1)		
European Contracts (censored observations) Dev 2		
Coefficient	Estimate	Std. Error
ω	0.04728	0.00175
α	0.0726	0.00249
β	0.9982	0.00781
γ	0.7364	0.02076
Ljung-Box Statistic for X_i (12 lags)	Q(12) = 3840.9649 (0.000)	
Ljung-Box Statistic for ε_i (12 lags)	Q(12) = 1405.6068 (0.000)	
AIC for W-ACD (2,2)	1986	
AIC for W-ACD (1,1)	3158	

Table 4.10

Maximum Likelihood Estimates for the **Weibull (1, 1) ACD** model. The sample refers to type one deviations of American contracts. The data do not include censored observations and are calculated after having removed the time of the day effects.

W-ACD (1, 1)		
American Contracts Dev 1		
Coefficient	Estimate	Std. Error
ω	0.0306	0.0034
α	**0.0317**	**0.0019**
β	0.9418	0.0375
γ	1.088	0.0568
Unconditional mean	1.15	
Unconditional Std dev	1.08	
Ljung-Box Statistic for X_i (12 lags)	Q(12) = 2523.1732 (0.000)	
Ljung-Box Statistic for ε_i (12 lags)	Q(12) = 14.0051 (0.300)	
AIC for W-ACD (2,2)	2203	
AIC for W-ACD (1,1)	2986	

Table 4.11

Maximum Likelihood Estimates for the **Log- exponential (1, 1) ACD** model. The sample refers to type one deviations of American contracts. The data do not include censored observations and are calculated after having removed the time of the day effects.

ε-ACD (1, 1)		
American Contracts Dev 1		
Coefficient	Estimate	Std. Error
ω	0.0476	0.00231
α	0.0398	0.00294
β	0.950	0.04469
γ	1.072	0.08925
Ljung-Box Statistic for X_i (12 lags)	Q(12) = 2350.0478 (0.000)	
Ljung-Box Statistic for ε_i (12 lags)	Q(12) = 16.0351 (0.189)	
AIC for W-ACD (2,2)	3024	
AIC for W-ACD (1,1)	2933	

Table 4.12

Maximum Likelihood Estimates for the **Weibull (1, 1) ACD** model. The sample refers to type two deviations (requires re-investment of short sales proceeds) of American contracts. The data do not include censored observations and are calculated after having removed the time of the day effects.

W-ACD (1, 1)		
American Contracts Dev 2		
Coefficient	Estimate	Std. Error
ω	0.0658	0.00371
α	0.0203	0.00194
β	0.9147	0.06283
γ	1.046	0.08016
Unconditional mean	1.01	
Unconditional Std dev	1.012	
Ljung-Box Statistic for X_i (12 lags)	Q(12) = 2693.5275 (0.000)	
Ljung-Box Statistic for ε_i (12 lags)	Q(12) = 10.1489 (0.602)	
AIC for W-ACD (2,2)	3055	
AIC for W-ACD (1,1)	3648	

Table 4.13

Maximum Likelihood Estimates for the **Log- exponential (1, 1) ACD** model. The sample refers to type two deviations (requires re-investment of short sales proceeds) of American contracts. The data do not include censored observations and are calculated after having removed the time of the day effects.

ε-ACD (1, 1)		
American Contracts Dev 2		
Coefficient	Estimate	Std. Error
ω	0.0697	0.00419
α	0.0187	0.00285
β	0.0924	0.09069
γ	1.049	0.04571
Ljung-Box Statistic for X_i (12 lags)	Q(12) = 2966.9203 (0.000)	
Ljung-Box Statistic for ε_i (12 lags)	Q(12) = 11.3413 (0.499)	
AIC for W-ACD (2,2)	3047	
AIC for W-ACD (1,1)	2986	

Table 4.14

Maximum Likelihood Estimates for the **Weibull (1, 1) ACD** model. The sample refers to type one deviations of American contracts. The data include all end of day censored observations and are calculated after having removed the time of the day effects.

W-ACD (1, 1)		
American Contracts (censored observations) Dev 1		
Coefficient	Estimate	Std. Error
ω	0.0039	0.00015
α	0.0598	0.00849
β	0.9387	0.01598
γ	0.596	0.07438
Unconditional mean	2.6	
Unconditional Std dev	2.76	
Ljung-Box Statistic for X_i (12 lags)	Q(12) = 3808.6401 (0.000)	
Ljung-Box Statistic for ε_i (12 lags)	Q(12) = 3047.7782 (0.000)	
AIC for W-ACD (2,2)	2568	
AIC for W-ACD (1,1)	3056	

Table 4.15

Maximum Likelihood Estimates for the **Log- exponential (1, 1) ACD** model. The sample refers to type one deviations of American contracts. The data include all end of day censored observations and are calculated after having removed the time of the day effects.

ε-ACD (1, 1)		
American Contracts (censored observations) Dev 1		
Coefficient	Estimate	Std. Error
ω	0.047	0.00621
α	0.0493	0.00798
β	1.775	0.00837
γ	0.551	0.05917
Ljung-Box Statistic for X_i (12 lags)	Q(12) = 3893.0217 (0.000)	
Ljung-Box Statistic for ε_i (12 lags)	Q(12) = 3124.9727 (0.000)	
AIC for W-ACD (2,2)	3022	
AIC for W-ACD (1,1)	3384	

Table 4.16

Maximum Likelihood Estimates for the **Weibull (1, 1) ACD** model. The sample refers to type two deviations (requires re-investment of short sales proceeds) of American contracts. The data include all end of day censored observations and are calculated after having removed the time of the day effects.

W-ACD (1, 1)		
American Contracts (censored observations) Dev 2		
Coefficient	Estimate	Std. Error
ω	0.00597	0.00078
α	0.07649	0.00392
β	0.9219	0.05873
γ	1.472	0.00394
Unconditional mean	1.99	
Unconditional Std dev	9.35	
Ljung-Box Statistic for X_i (12 lags)	Q(12) = 3906.6277 (0.000)	
Ljung-Box Statistic for ε_i (12 lags)	Q(12) = 2974.2214 (0.000)	
AIC for W-ACD (2,2)	2812	
AIC for W-ACD (1,1)	3026	

Table 4.17

Maximum Likelihood Estimates for the **Log- exponential (1, 1) ACD** model. The sample refers to type two deviations (requires re-investment of short sales proceeds) of American contracts. The data include all end of day censored observations and are calculated after having removed the time of the day effects.

ε-ACD (1, 1)		
American Contracts (censored observations) Dev 2		
Coefficient	Estimate	Std. Error
ω	0.0269	0.00591
α	0.0897	0.00609
β	1.198	0.08347
γ	1.4102	0.15927
Ljung-Box Statistic for X_i (12 lags)	Q(12) = 3748.1469 (0.000)	
Ljung-Box Statistic for ε_i (12 lags)	Q(12) = 2764.2870 (0.000)	
AIC for W-ACD (2,2)	2489	
AIC for W-ACD (1,1)	2635	

CHAPTER 5

Conclusions

5. Conclusions, Future Research

Conclusion

This study has used the option pricing theory in the form of boundary conditions between premioums for identical European and American contracts to gain some knowledge on the behaviour of the option market in real time.

The relevant literature to Put-Call Parity deviations dates back to Stoll's (1969) research on boundary conditions. Since then research has shown that in every case real market data on option prices have violated the PCP conditions. Researchers have argued either in favour of "early exercise" or "immediacy" premia. In the introduction this study has set out to analyse the LIFFE market and offer evidence in support or against these hypotheses. Furthermore it posed the question on whether or not substantial deviations exist, perhaps due to other factors.

The analysis has shown that in both American and European cases significant deviations exist. By allowing for a delayed execution time and stable underlying prices it has also been argued that these results are substantially free from immediacy risks given our assumptions.

By comparing the deviations for the two contract types, it has been shown that the probability of achieving equal deviations for American and European trading are statistically insignificant (close to zero in most cases).

Using the frequency of PCP quotes as a proxy of market liquidity it has been shown that in comparison the more inefficient European market is in addition the less liquid one and vice versa for the American. Consequently a substantial part of the deviations could remain due to liquidity related factors.

More importantly American deviations due to early exercise have been identified and it has been shown that are significantly higher than those for European contracts, notwithstanding differences in liquidity. Hence the results offer support to the early exercise hypothesis.

Chapter 2 concludes by stating that both hypotheses give rise to PCP deviations when appropriate. However, even after accounting for these, substantial inefficiencies remain.

It has been argued, though, that in a high frequency context periods of disequilibrium could exist as a result of momentary, or individual among different market makers, inventory imbalances or different expectations. In a dynamic context a market is inefficient only in the case of persistent or irrationally removed deviations.

By employing the Cox Proportional Hazard model Chapter 3 has analysed the hazard of the survival of each deviation using its magnitude as a covariate. It has been showed that the duration of each deviation is inversely proportional to its level, thus market forces remove inefficiencies in the form of PCP deviations in a rational way, i.e. starting from the more substantial ones. It has been argued that this supports the notion of efficiency for the particular market. The evidence also shows that in most cases a proportional hazard model is well specified.

Having introduced the time as an important factor in the description of the PCP deviations, the last chapter models these arbitrage opportunities as a sequential process. It has been argued, however, that a successful modelling should be able to capture the significant clustering and non-Markovian properties, which are characteristics of intradaily prices.

Two different versions of the ACD model, the W-ACD and the ε-ACD, have been employed and it has been shown that both are capable of modelling the PCP deviations as a stochastic sequential process. Both of the process account for the large serial correlation in the raw PCP values and are well specified. Finally both offer a valid representation of the memory in the evolution process, i.e. dependence on lagged values.

Future Research, Implications for Regulatory bodies.

The current study raises some important issues with respect to future research on the particular market and in general on high frequency financial markets analysis. Both in Chapters 3 and 4 it was evident that the abrupt termination of the duration of quotes has presented us with some modelling difficulties. Unfortunately the nonparametric signal extraction techniques reviewed and applied here were not satisfactory in removing the problem of censored observations. Perhaps future research could identify ways of accommodating these abrupt terminations.

Throughout this study, however, these observations have been retained and additional analysis has been carried out for the censored samples to avoid eliminating part of the information set. It can be suggested that rather than trying to remove these censored values future research could find ways of adopting the models used around the forcefully terminated observations. More specifically it may be possible to derive likelihood functions for the ACD model that assume dependence of the error term on bimodal distributions.

Additionally, future research could concentrate on the informational role of the PCP deviations identified in the second Chapter. According to the literature referred to in the introduction of this study, PCP deviations could reveal market expectations. These together with implied volatility values drawn from the same observations could constitute a vector of implied probability values from high frequency option contracts.

One of the prevailing issues of this thesis was the question of whether or not the FTSE 100 Options market is efficient. Possible evidence of inefficiencies could have important implications on the regulatory framework of the market. It was evident from the results of Chapter 2 that even after the imposition of transaction costs and allowance for immediacy risks some inefficiencies remain. However, in Chapter 3 these inefficiencies where shown to be short lived. According to the author this represents evidence of a well functioning market. Whereas an increase in the liquidity of the market should be welcome, within the context of the current study no regulatory changes are deemed necessary.

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