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# Rolling Mill Roll Design 

By

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## 27 JAM 2003

The thesis is submitted to the University of Durham in accordance with the regulation for admittance to the Degree of Masters of Science


#### Abstract

In this thesis, some previously published experimental and theoretical studies of hot rolling are reviewed. A thorough understanding of the available roll design methods, and conditions of their application is extremely important in order to achieve the objective of producing high quality rolled products.

Successful hot roll design is dominated by the calculations of some important parameters, which describe two-dimensional (2D) or three-dimensional (3D) deformation in the workpiece. These parameters, such as roll separation force, torque, elongation, spread and draft, are discussed in detail. The method or formula for the calculation of each parameter is different for each set of different application conditions. A thorough study of these methods in different application cases will lead to the optimised design of hot rolled products.

Finite Element (FE) is an important method which has been employed in the study of hot rolling. Design theory, commercial software and application cases have been described. 2-D and 3-D Finite Element Methods (FEM) for hot rolling simulation have also been discussed within the work. The current techniques and the problems of using the Finite Element system in hot roll design have been presented briefly. Possible solutions to these problems have also been discussed and there need to be considered in order to successfully apply Finite Element theory in hot roll design.

An important alternative approach for hot roll design has been introduced in this thesis. A Matrix-based roll design system has been developed. It includes a Matrix-based system for flat and section roll designs. The realisation of the Matrix-based system is discussed. All the methods and formulae considered previously can be integrated in the proposed roll design system. The approach emphasizes the need for teamwork. The design procedure allows both less experienced designers and senior designers to benefit from participation. It is suggested that high quality rolled products could be achieved from optimised designs produced using this systematised the approach compared to the ad-hoc use of existing techniques, formulae and methods.


## Acknowledgement

It is hard to believe that two years have passed. In this time, I have learnt so much from so many people. I would like to thank the people for their helps. I want to thank the CARD project team and here every one is so kindly and ready to help.

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During this time, I always have the supports and encouragements from my family and this is important to keep my study moving forward.

Finally, I will like to thank the people at Durham, a wonderful city. Good memory cannot be forgotten.

## Symbols

| $l_{0}$ | Entry length of the workpiece |
| :---: | :---: |
| $l_{1}$ | Exit length of the workpiece |
| W | Workpiece mean width (Breadth) |
| $W_{0}$ | Entry workpiece width |
| $W_{1}$ | Exit workpiece width |
| $h_{0}$ | Entry thickness of the workpiece |
| $h_{1}$ | Exit thickness of the workpiece |
| R | Roll radius |
| $\alpha$ | Bite angle |
| $\theta$ | Roll contact angle |
| n | Roll revolutions |
| $D_{w}$ | Working roll diameter |
| $\varepsilon_{1}$ | Strain in the rolling direction |
| $\varepsilon_{2}$ | Strain in the thickness direction |
| $\varepsilon_{3}$ | Strain in the width direction |
| F | Cross sectional area |
| $F_{0}$ | Section area after roll pass |
| $F_{1}$ | Section area after roll pass |
| V | Speed of deformation |
| $\varepsilon$ | Engineering strain rate |
| N | Roll peripheral speed |
| R' | Flattened roll radius |
| $h_{n}$ | Strip thickness at neutral point |
| $v_{0}$ | Entry speed |
| $V_{r}$. | Surface speed of the roll |
| $v_{f}$ | Exit speed |

Mean pass elongation coefficient Elongation coefficient of the whole section

The lever arm Projected arc of contact Projected length of contact Coefficient of draft Coefficient of elongation Coefficient of spread Coefficient of friction Efficiency of process

Resistance to deformation in rolling Yield stress of specific resistance to deformation Resistance to friction Roll separation force Total roll force

Torque
Total driving torque

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## Chapter 1

## Introduction

### 1.1 Introduction of Rolling Products

Rolling is the process of reducing the thickness or changing the cross-section of a long workpiece by compressive forces applied through a set of two rolls that revolve in opposite directions, the space between the rolls being less than the thickness of the entering material. In the rolling process, material passed between rolls is plastically deformed.

The rolling process is a widely used industrial process because it makes possible high production and close control of the final product shape and properties. It accounts for about 90 percent of all metals produced by a metal working process. Rolling is one of the oldest processes used in the metal working industry. In view of the tremendous volume and wide variety of rolled products manufactured each year, rolling can be considered to be one of the most important forming processes.

Rolling processes are classified as cold or hot rolling according to whether work hardening occurs. Cold rolling is usually associated with operations performed at room temperature or below the recrystallization temperature at the rolled material. However, hot rolling is usually terminated when the temperature falls to about $50{ }^{\circ} \mathrm{C}$ to $100{ }^{\circ} \mathrm{C}$ above the recrystallization temperature of the material. There is no work hardening during hot rolling. In this thesis, only hot rolling is considered.

The ultimate goal in hot roll pass design is to manufacture the correct size and shape of a rolled product with a defect free surface and the required mechanical properties. In addition, economic condition must be achieved, for example, maximum output and lowest cost, easy working conditions for the rolling crew and minimum roll wear.

Improvement in the quality and reliability of rolled products can only be achieved through a thorough understanding of the rolling process. This has usually been acquired through practical observations over many years. With the introduction of sophisticated technology, information flows faster than ever. A better understanding of the behaviour of the rolled product in the rolling process is considered a key point towards improved quality. This explains why more and more work is being devoted to rolling modelling by both academic and industrial research groups.

### 1.2 Rolling Design

As in all metalworking processes, rolling involves a number of process and material variables that should be controlled in order to roll products having high quality, properties, surface finish, and dimensional accuracy. These variables include rolling deformation, temperature and speed, lubrication, and the condition of the rolls.

The ability to predict and control these interdependent variables can be obtained in two distinct ways: (1) empirical or experimental methods and (2) theoretical methods. Empirical methods have a long history and are an important approach for hot rolling process design. Empirical investigation in rolling consists of careful observation of the obtained data, for the purpose of inducing some useful conclusions, or confirmation of preconceived theories. However, empirical techniques develop restricted information and accurate data is not always easy to obtain.

There are a number of theoretical methods based on different analytical approaches. Using a theoretical method it is difficult to get the analytical solution for section rolling and the approach is only suitable for the study of flat rolling in most cases. Section rolling are used for the production of complex cross-section shapes such as beams, channels etc. For flat rolling design such as sheet, bar etc, the theoretical method can now achieve the calculation of rolling force, rolling torque and some deformation parameters with a great degree of accuracy.

More details about empirical and theoretical methods will be presented in Chapter 2.

Due to the development and availability of digital computers over the past few decades, Finite Element Method (FEM) has been extensively used for rolling analysis. Currently two and three-dimensional simulation of rolling is possible. Computer simulation of industrial processes is an important alternative to complement or to replace the expensive experimental procedures associated with innovative development. Computer programs have been developed for optimising passes but such programs are largely based on empirical data and practical experience. The disadvantages of FEM includes time consuming calculation and that the computer simulation can only be carried out for a specific design with a specific shape. When the rolling product shape and loading conditions change, the FE simulation must be repeated.

Correct applications of all of the above techniques are important to ensure high quality hot rolled products and this can only be achieved by optimised design according to the specific design problem.

### 1.3 The Motivation and Thesis Structure

Roll design involves preparing analytical and physical models of the rolled products, as an aid to analysing factors such as forces, stresses, and optimal rolled product shapes.

Although many investigators are working on hot rolling and many methods have been published, some fields remain practically unexplored. Each technique has its own merits and limitations. Improvements in the quality and reliability of roll design can only be achieved through a through understanding of the rolling process. There is still much work that needs to be carried out on roll design, especially for section rolling. A common problem for the existing hot roll design system and method is that each method or formula can only be applied for a specific application case and this is especially true for section rolling design. A methodology which could integrate all of the existing formulae, methods and approaches, would be very useful and powerful for rolling design, specially for inexperienced designer. It is suggested that high quality rolling product can be achieved by comparing existing formulae or methods to find the optimised design approach, which gives a best fit for a specific design problem.

The work presented in this thesis aims to integrate the knowledge obtained from the extant literatures into a Matrix-based roll design system. The contents of this thesis are organized as followings:

Chapter 2: Because of the complex nature of hot rolling design, a thorough understanding of available design and analysis methods and the conditions of their applications is extremely important in order to achieve high quality rolled products. This chapter gives a review of the existing theory and experimental work in rolling. It provides a comprehensive discussion of a number of different empirical and theoretical methods for roll pass design. Finite Element (FE) methods for roll design are also discussed.

Chapter 3: Roll separation force and torque influence the process of selecting a rolling mill, motor and the whole process of roll pass design. The roll force is imposed on the rolls by the processed metal during hot rolling. The roll force and torque are the most important design issues in rolling. Several important formulae and their conditions of applications are discussed which include experimental methods, and the slab and upperbound methods. Prediction of roll separation force and torque in section rolling are also described.

Chapter 4: The deformation of a hot rolled workpiece can be described by the three important design issues: elongation, spread and draft. In this chapter, fundamental concepts of elongation, spread and draft will be discussed. Several existing formulae for the calculation of spread are also presented.

Chapter 5: In recent years, with the development of computer hardware and software, more sophisticated models of hot rolling simulation became possible by using Finite Element Methods (FEM). FEM is an important tool to simulate and study hot rolling. The principle of FE and its application to hot rolling are described in this chapter.

Chapter 6: Traditionally, roll design is done by a designer working in isolation. With the creation of Concurrent Engineering (CE), this method has broken down and been replaced by designers working in a team for a common goal. Nowadays, roll designers face more challenges because of rapidly changing situations. The design of rolls by a single person is no longer practical.

A Matrix-based methodology for hot roll design system is introduced in this work. It is based on the economic leverage of addressing all aspects of the design of a product as early as possible in the design process using a systematic approach and structured method. The Matrix based design method was first used in the assembly industry by Durham University. In this chapter, the concept of the matrix methodology is extended to design for hot rolling.

Chapter 7: This chapter continues the work of chapter 6. In this chapter, the Matrixbased hot roll design system is realised by integrating of the possible methods and formulae for hot rolling into a Matrix-based roll design system. The features of the Matrix based hot roll design system are also discussed.

Chapter 8: Finally, Chapter 8 contains a concluding discussion of features of Matrixbased hot roll design and suggestions for future work.

## Chapter 2

## Review of Rolling process Design

### 2.1 Introduction

The purpose of rolling is to convert material of large cross-sections into smaller sections of various shapes. This deformation is accomplished by applying compressive force through a set of rolls. Although hot rolling has been extensively studied by a large number of researchers only few have totally tackled the hot rolling roll design problem. Hartley et al. (1989) reviewed the previous experimental and theoretical studies of flat rolling. Montmitonnet et al. (1991) on the other hand has reviewed the theoretical analysis of flat rolling in Europe. However, these reviews do not reflect the complex nature of the problem in hand as they only focus on either flat or special application design issues.

Because of the complex nature of rolling mill roll design, a thorough understanding of the available methods and conditions of their application is extremely important in order to achieve high quality rolled products. The study of section rolling is still at a very low level of technological development (Lundberg 1997). Therefore, a further investigation of existing rolling methods in general is necessary for further contribution to the solution of the roll design problem.

The remaining part of this chapter provides a comprehensive discussion of a number of different empirical and theoretical methods for roll pass design. However, in studying the section rolling design problem a number of different Finite Element (FE) approaches are also available. Hence, some FE approaches are also discussed in this chapter. Finally, an introduction of a novel approach, namely the " Durham Matrix method " is presented.

### 2.2 Empirical Methods

The early development of rolling mill design was based upon individual experience and craft skills. The first attempt to investigate metal flow during rolling was made by Hollenberg in 1883. His method consisted of drilling transverse holes in the bar to be rolled and inserting rivets (Puppe 1929). A number of similar investigations into both hot and cold rolling followed (Unckel 1936, Weiss 1928, Siebel 1925).

Deformation was investigated by Orowan (1943) who used plasticine bars for this purpose. This investigation of deformation also gave some information on internal flow. Studies on spread in strip and slab rolling also received attention. There are many publications in this area of work. The most notable work includes Ekelund (1933), Wusatowski (1969), McCrum (1956) and Sparling (1961).

Empirical investigation in science and engineering consists of careful observation of the obtained data, for the purpose of inducing some useful conclusions, or confirmation of preconceived theories. This is also the case here and numerous analytical and experimental investigations on hot rolling have been carried out. However, the empirical roll design techniques that have been developed also have restrictions. Sufficiently conclusive or accurate data is not always easy to obtain practically, for complex processes such as section rolling.

### 2.3 Theoretical analyses

The following section reviews theoretical analyses based upon different analytical approaches.

### 2.3.1 Homogeneous Deformation Method

The Homogeneous deformation method requires the selection of an element in the workpiece and identifies all normal and frictional forces acting on this element. The principal assumption is that the deformation of metal is homogenous throughout the whole deformation zone.

This method was first introduced by Karman (1925). This theoretical treatment allowed the inclusion of a horizontal stress component in the analysis. Von Karman's theory was further developed by Trinks (1937), Nadai (1939) and Orowan (1943). Von Karman's equation was established from a consideration of the equilibrium of forces acting on the elemental slab in the zone of deformation. Consequently, the method is sometimes referred to as the " slab method". Alexander (1972) greatly simplified the use of the 'Von Karman' equation by presenting a computer-aided solution. He obtained more accurate results than the existing homogeneous theories. Kalpakjian (2001) researched to determine the pressure distribution at the roll and workpiece interfaces, allowing the calculation of the total compression (rolling force).

The theories of homogeneous deformation are based on the common assumptions that a plane vertical section of the rolled material remains plane during rolling. Many such assumptions are used to derive force models for strip hot or cold rolling. However, the theoretical results generally show poor agreement with experimental results.

### 2.3.2 Inhomogeneous deformation

An attempt to overcome the restriction of the assumption of homogeneous deformation was made by Orowan (1943) in his theory of inhomogeneous deformation. The method assumed that the stress distribution in the vertical plane is not homogeneous. Following Orowan's original method, Sims (1954) further assumed that sticking friction exists along the arc of contact in hot rolling. In order to test the accuracy of this approach, a series of experiments were conducted using lead strip and bar. Good agreement was found between the results of the experiments and the calculation. Stewartson (1954) measured the roll forces during a number of rolling tests, and compared these measured values with those predicted by the formulae of Ekelund (1933), Orowan and Pascoe (1946), and Sims (1954). Stewartson concluded that the Sims' formula was the most accurate. Venter and Abo-Rabbo (1980) developed a computer-aided model, which incorporated an inhomogeneous parameter.

The inhomogeneous theory is certainly more realistic than the homogeneous theory but it still gives little information about the internal stress state of the workpiece. Both
homogeneous and non-homogeneous theories of rolling give a simplistic description of the deformation zone.

### 2.3.3 Slip Line Analysis

Slip-line field analysis allows the introduction of a more realistic model of metal flow. It utilizes a graphical approach, which presents the flow pattern in the deformed metal in a point by point basis.

Alexander (1955) presented the first slip line field solution for hot rolling of wide sheets. He only offered a single and simple geometry of rolling. Crane and Alexander (1966) used new slip line fields for hot rolling to predict the deformation of the metal for a wide range of geometries. Dewhurst et al. (1973) presented a series of slip line field solutions for hot rolling of wide strip, which was further discussed by Druyanov (1973).

Slip line field solutions permit the determination of stress and velocity distributions in the plastic deformation zone. However, this method is only valid for assumed rigidperfectly plastic non-hardening materials (Shabaik 1968).

### 2.3.4 Energy method

Energy methods are composed of lower-bound analysis and upper-bound analysis. The two methods are used for the approximate evaluation of force that causes plastic flow. The energy method was first used in rolling by Siebel (1925). A much later energy method used in metal forming is the upper bound approach. The first upper bound solution for rolling was proposed by Johnson and Kudo (1960).

The upper bound approach has been used more recently by Pawelski and Okado (1984). Zimmerman (1985) has also used an upper bound approach to study the rolling of square bar. Zhang et al. (1995) have proposed a formulation for forward slip in the process of the continuous rolling of H -beam and in this study, the web and flange are modelled separately as plane problems. Martins (1999) presented an innovative approach for analysing plane strain rolling, based on a solution resulting from the combination of the upper-bound method with the weighted residuals method. This method is utilized for obtaining the distribution of stress, the roll separation force and the normal contact pressure along the surface of the rolls.

Energy methods can achieve the calculation of rolling force, rolling torque and some deformation parameters with a good degree of accuracy.

### 2.4 Finite Element Method (FEM)

Finite Element (FE) techniques are able to provide extensive information about material flow, strain and temperature distribution, which are key to estimating rolling parameters and predicting microstructure development. For hot rolling, numerical simulation provides information about forces and power, deformation levels, stress, thermal evolution, etc. At the present time the FEM is probably the most common technique used in the investigation of rolling problems (Anza 1997). It has positive influence on the design of hot rolling process equipment, the saving of material and reduction in the cost of the process.

Extensive reviews of current Finite Element work have been published recently. Hartley et al (1989) gives over 130 references to papers covering experimental and FE analysis of rolling. Montmitonnet and Buessler (1991) give further details of the work of current European research groups, which also mainly applying Finite Element models. Mackerle (1997) has listed over 120 papers that have been published between 1994 and 1996 on the application of FE to rolling subjects.

### 2.4.1 2D FE Method

Viscoplastic Finite Element solutions for rolling were presented by Zienkiewicz et al. (1978), and also by Dawson (1978). In these methods, the flow of the metal has been treated as incompressible and the elastic strain of the deformed metal was ignored.

Thompson (1982) has made an attempt to include elastic effects in the viscoplastic approach by adding an extra term in the constitutive relation between strain rate and stress. Grober (1986) has also used the elasto-viscoplastic approach for hot flat rolling and Bertrand et al. (1986) for edge rolling.

A rigid-plastic Finite Element method was used by Li and Kobayashi (1982) to study plane strain rolling, which was also studied by Mori et al.(1982), using the same method.

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The Elastic-plastic Finite Element method was also used for rolling process design. Early use of this approach was suitable for small reduction rolling. Further studies of plane strain rolling using this technique have been undertaken by Key et al. (1974). An existing elastic-plastic FE program was "epfep3" which was used for hot rolling design for two special sections and much information about rolling processes was obtained (Wen 1994).

### 2.4.2 3D FE Method

The first 3D FE analysis of rolling was presented by Mori and Osakada (1982), using a Rigid-Plastic method. Lahoti et al. (1980) developed a computer aided design system for predicting spread and stresses in rolling. Kiefer (1984) used an Elastic-plastic FE analysis method for 3D rolling. Although only sticking friction was considered, predictions of metal flow were obtained, and the effect of variation of width on the strain distributions was presented. Liu and co-workers have considered in detail the 3D flow in rolling using an elastic-plastic method developed for general metal forming analyses. Liu (1987), Hartley (1988), and Tozawa (1982) and co-workers studied the 3D deformation of the strip using a very simplified approach and have combined with the elastic roll deformation (Tozawa 1976, 1982).

Because section rolling is a 3D problem, the 3D FEM is very suitable and in many cases is necessary for the study of section rolling, however, the 3D FEM is very complex and time consuming for calculation.

### 2.5 Section Rolling Design

The deformation in section rolling is a complicated three-dimensional problem and analysis remains difficult. Compared with flat rolling, the metal flow of section rolling has particularly complex characteristics.

Empirical methods are often used in section roll design. Some of the well-known empirical formulae were derived by Wusatowski (1969), Trinks (1941), Sheffield (1960) and Roberts (1983). However, there are a lot of practical problems that must still be solved case by case.

Lendl (1948) proposed an empirical procedure for roll pass design for simple square, diamond, round and oval grooves in which the pass cross section was subdivided into vertical strips. The spread of each of these strips was then calculated by the empirically derived spread formulae developed by Ekelund (1927).

Recently, computer programs has been developed for establishing section rolling and pass schedule design. Bertrand et al (1986) used a rigid-viscoplastic model for the calculation of stress in the hot shape rolling process from round to oval. Mori and Osakada (1989) used a rigid-plastic FEM to simulate 3D steady-state deformation in section rolling. Hartley et al. (1990) presented a FE investigation of the deformation process in the finishing pass rolling of a railway rail, using an elastic-plastic FEM. Chen and Kobayashi (1990) employed a 3D rigid-plastic FE program to simulate different shape rolling processes: web rolling, full rolling.

However, the fully 3D FEM approach is highly intensive in computational resources, data preparation and interpretation. A novel pseudo 3D approach (finite-slab element method) has been adopted for the analysis of section rolling as a kinematically steadystate process (Kim et al 1991). Based on this method, a computer program called TASKS has been developed which can simulate the rolling of H -section (Kim et al 1991). TASKS has also been used by Lee et al. (1992) and Wen (1994).

Jin et al. (1998) have presented a three-dimensional analysis for a universal beam tanden rolling process. Two new torque and force models were introduced in this paper. Lapovok et al (1996) developed a methodology, which uses 3D Finite Element simulation, along with empirical procedures to arrive at an iterative scheme for reducing the number of passes and improving metal flow in the passes.

For studying section rolling, investigators have also considered using expert systems (Zhang et al. 1995, Kim et al. 1999, Xiong et al. 1997). This approach has also been used to optimise scheduling of hot rolling process to reduce pass number (Ozsoy et al. 1992). However, some fields remain practically unexplored, such as ring rolling or beam rolling on universal mills, or to a lesser extent, tube rolling (Montmitonnet et al. 1991). Ivestigators have also considered using expert systems in studying section rolling (Zhang et al. 1995, Kim et al. 1999).

### 2.5.2 Commercial FE Codes

Due to the development of digital computers over the past few decades, the Finite Element Method has been extensively used for metal-forming analysis and currently the three-dimensional Finite Element simulation of section rolling is possible by using commercial FE codes.

There are many commercial FE software package currently available. Datta (1996) used a commercial Finite Element package (ABACUS) for friction and heat transfer Modelling. Wen (1994) used I-DEAS Solid Modelling and ABACUS applied to "2D and pseudo-2D" models for section rolling. Kopp et al. (1990) used the "Barrol code" written by Mori(1982), complemented by ABAQUS for thermal aspects.

Mirabile et al. (1991) has compared eight different FE software package for the simulation of the rolling of hot flat and special sections such as beams and rails. This software includes: ABAQUS, DYNA3D, LAGAMINE, LARSTRAN, MARC, NIKE2D, PREFECT, ROLL3 These codes provide good agreement between experimental and predicted values of temperature, strain distribution, rolling forces and torques. The disadvantages of these codes are that they are difficult to use, require long calculation time and are far from reliable for industrial practice.

### 2.6 The Matrix Method

Belson (1994) has defined a new approach to the design process, which is called Concurrent Engineering (CE). It is a systematic approach to the integrated, concurrent design of products and their related processes, including manufacture and business support.

A matrix-based methodology was introduced by the University of Durham. It allows a team of designers to cooperate in the design decision-making process. It is highly interactive and visual and gives the necessary design flexibility (Appleton and Summad 1999). Appleton and Garside (2000a, 2000b) have described the methodology, which is based upon practical experience in Design for Assembly, to facilitate the teaching of these design principles to small groups. This methodology has been successfully used in other design areas such as Design for Manufacture (DFM), Maintenance Planning, and Manufacturing Strategy.

Appleton and Summad (1999) concluded with some recommendation for the future direction in the development of a new approach to roll pass design. However, there is a challenge to further develop a matrix-based hot rolling system and there are some problems still in need of solving in the detailed approaches to this design. Appleton, Jing and Summad (2000) presented possible approaches, based on the matrix methodology for roll design and further details will be discussed in a later chapter.

### 2.7 Summaries and Conclusions

In this chapter, several techniques for design and study of hot rolling have been discussed. These include: empirical methods, theoretical analyses and Finite Element methods. Section roll design and the Matrix method have also been discussed individually and briefly.

Each technique has its own merits and limitations. Improvements in the quality and reliability of roll design can only by achieved through a thorough understanding of the rolling process. Application of each technique is determined by the specific rolling conditions, the required information, the calculation time, and in some case, what computing facilities are available. For example, the slab method is simple and can give reasonable estimations of force and rolling torque. The slip line field method can illustrate metal flow and stress distribution for non-hardening plane strain flat rolling. However, for detailed study of metal flow, thermal effects and stress distribution in section rolling, the FEM seems to be necessary.

Although many investigators are working on hot rolling and many papers have been published, some fields remain practically unexplored. Therefore, there is still much work that needs to be done on rolling design, especially for section rolling. Additionally, there seems to the little coordination of the design processes with each designer working their way through an idiosyncratic process based up an ad-hoc technical foundation.

## Chapter 3

## Theory 1: Hot Rolling Force and Torque

### 3.1 Introduction

Hot rolling is a compression process. The roll force, which is the force imposed on the rolls by the processed metal during hot rolling, is an important design issue. Roll separation force and torque influence the process of selecting a rolling mill, motor and the whole process of roll pass design. The calculation of roll force is necessary to ensure that the mill is not overloaded and there is enough power available for the rolling reduction to be made.

Until recently, theoretical hot rolling analysis has been used to give a guide for roll force and torques but has not been able to provide details of the behaviour of the workpiece (Ginzburg 1989, Robert 1983 and Wusatowski 1969). Making exact calculations of forces and torque requirements are difficult because of the problems involved in determining the exact contact geometry and estimating the coefficient of friction, the strength of the material and an accurate knowledge of temperature in the roll gap.

In recent years, a number of investigators have researched in this area and many formulae have been introduced to calculate hot rolling force and torque. The slab, the slip-line and the upper-bound methods have been widely used in theoretical analyses. However, each of these methods has its own assumptions and each formula is only suitable for specific conditions of application. Therefore, it is useful to discuss these formulae and their conditions of application. In this chapter, several important formulae are discussed. Other existing formulae for calculation of force and torque are also listed and more detail can be found in the references.

### 3.2 Flow of Metal and the Neutral Point in Rolling

Rolling deformation of material is plastic deformation. Observation of metal flow phenomena gives a very valuable insight into hot rolling.

### 3.2.1. Flow of Metal

Early experiments on rolling were designed to study the internal flow of rolled strip. A great number of similar investigations into hot and cold rolling were conducted and the metal flow under various conditions of rolling was considered.

Nearly all published papers use Orowan's modes of metal rolling and his experimental work (Orowan 1943). Orowan rolled multicoloured Plasticine bars and observed the deformation, see Figure 3-1. Orowan's experiments referred to the " neutral point ", see Figure 3-2. In flowing through the roll gap the work material accelerates. Before the work material reaches the neutral point it is travelling slower then the surface velocity of the roll. After the neutral point it is moving faster than the surface velocity of the roll.


Figure 3-1. Flow of material in rolled composite Plasticine bar (Orowan 1943).


Figure 3-2. Plastic deformation distribution based on experimental observations (Orowan 1943).

### 3.2.2 Neutral Point

A schematic illustration of the flat rolling process is shown in Figure 3-3.


Figure 3-3. A schematic of the flat rolling process force

The surface speed of the roll is $V_{r}$. The initial value of velocity of the workpiece is $V_{0}$. The velocity of the workpiece is highest at the exit of the roll gap and is denoted by $V_{f}$. Because the surface speed of the roll is constant, there is relative sliding between the roll and the strip along the arc of contact in the roll gap $L_{d}$. At one point along the contact length, the velocity of the workpiece is the same as that of the roll. This is called the neutral, or no slip point. To the left of this point, the roll moves faster than the strip, and to the right of this point the workpiece moves faster than the roll. Hence the frictional forces, which oppose motion, act on the workpiece as shown in Figure 3-3.

### 3.3 Rolling Separation Force

Friction on the surface of workpiece restricts the metal flow. The pressure of the rolls must overcome the constrained yield stress and the friction.

### 3.3.1 Friction

The rolls pull the material into the roll gap. There is a net frictional force along the tangent at the point of contact between rolls and the material (See Figure 3-3). Friction
is needed since it forces material into the roll gap during rolling. However, energy is dissipated in overcoming friction, so increasing friction means increasing forces and power consumption. Furthermore, high friction could damage the surface of the rolled product. The friction is the cause of non-uniform stress distribution and disturbances in the deformation of the metal. As a result, a compromise is made, obtaining low coefficients of friction with effective lubricants.

### 3.3.2 Roll Force

There is also a force that acts in the line of the radius. At the point of entry into the rolls, the radial force is $P_{r}$ which tends to compress the workpiece. If $\mu$ is the coefficient of friction between the workpiece and the rolls, the tangential force is $\mu P_{r}$ acting towards the roll gap. The resultant of $\mu P_{r}$ and $P_{r}$ is $R_{1}$, as found by the parallelogram of force. This force itself can be resolved into (i.e. replaced by) horizontal and vertical components $H_{1}$ and $V_{1}$. The vertical components $V_{1}$ of the forces involved in this deformation constitute the roll load, which forces the rolls apart.

Roll force is a resultant along the contact arc. Descriptions of the model have been developed widely within the literature, with extensions to cover other features including lubrication, thermodynamic changes, and more complex work hardening behaviour. Its chief application has been in the prediction of roll force and roll torque to aid in mill design.

### 3.3.3 Resistance of Deformation

For roll design, not only the direction, but also the magnitude of the deformation force must be known. The magnitude of the force is determined by the specific compression resistance of the workpiece and by the area, which is being compressed.

The roll separation force can be determined if the distribution of pressure in the deformation zone is known. The resistance of deformation i.e. roll pressure distribution is a major parameter in rolling. To determine the roll pressure causing this deformation, it is necessary to know the stresses arising during the process. Friction causes nonuniform stress distribution during rolling. The friction arising in the roll gap acts in two
directions opposite to each other in roll and workpiece. The pressure across the region of contact with each roll is not constant. It increases towards the centre of the contact area. This type of pressure distribution is often referred to as a " friction hill ". The greater the coefficient of friction, the higher is the " hill ". Friction has the effect of increasing the flow stress of the workpiece. The true roll pressure diagram (see Figure 3-4) consists of two parts (Wusatowski 1952).


Figure 3-4. Roll pressure distribution (Wusatowski 1969).

The lower part ADGEC shows the work hardening curve of metal during ideal (frictionless) plastic deformation, namely the curve of roll pressure necessary to overcome the constrained yield stress $\eta K_{f}$. The upper part DFEGD shows the roll pressure necessary to overcome the additional constraint caused by the friction forces between the surface of the metal being rolled and roll, namely the so called resistance to flow in rolling $K_{r}$. Different temperatures, roll diameters and coefficients of friction will influence the pressure diagram.

The resistance of deformation of the material rolled $K_{w}$ is determined by

$$
\begin{equation*}
K_{w}=\eta K_{f}+K_{r} \tag{3.1}
\end{equation*}
$$

The theoretical roll separation force along the projected length of contact is given by
$P=\int_{0}^{y_{1}} K_{w} d x=\int_{0}^{a} R P_{\theta} d \theta$

Total roll force $P_{r}$ along the width

$$
\begin{equation*}
P_{T}=\mathrm{W} \mathrm{P} \tag{3.3}
\end{equation*}
$$

### 3.3.4 Mill Load

Forces equal in magnitude, but opposite in direction act on the rolls, so giving rise to the load on the bearings and necessitating a torque to turn the rolls see Figure 3-5.


Figure 3-5. Roll force P and torque acting on the rolls.

In hot rolling, a model to predict material flow and loads is invaluable in designing mills to minimise machine forces. This load P results in mill spring, which is made up of several components including: bending of rolls, compression of the bearings, chocks and screws, and stretching of the housing.

Roll mill equipment must be capable of withstanding the roll load i.e. the mill is not overloaded and there is enough power available for the rolling reduction to be made. A roll designer is subjected to limitations applied by the rolling load, the roll strength and torque available for rolling.

Roll force can be reduced by (Sheffield 1960): (1) reducing friction; using smaller diameter rolls to reduce the contact area; (2) taking smaller reductions per pass to reduce the contact area; (3) rolling at elevated temperature to reduce the strength of the material; (4) applying longitudinal tensions to the strip during rolling.

### 3.4 Strain and Strain Rate

Determining external pressure distribution, internal distributions of stress, strain and strain rate, is very valuable in seeking to understand hot rolling phenomena.

Stress and strain describe the distribution of forces that affect not only roll life, but also the operation of the mill and quality of the rolled product. Higher stresses may result in excessive roll wear and deterioration of the surface finish of the product. Therefore, when rolling force is calculated, stress and strain are essential parameters.

The basic quantities that may be used to describe the mechanics of deformation when a body deforms from one shape to another under an external load are the stress, strain and strain rate. Various measures of their quantities are defined depending upon how closely formulations represent actual situations.

The tri-axial deformation may be considered as a general case. The volume of metal remains constant, namely

$$
\begin{equation*}
\varepsilon_{1}+\varepsilon_{2}+\varepsilon_{3}=0 \tag{3.4}
\end{equation*}
$$

In practice, a metal is never rolled under plane strain conditions, i.e. $\varepsilon_{3}=0$. In the case of flat products, the lateral spread is limited and the metal may be assumed to be deforming under plane strain conditions.

True strain (logarithmic strain) is given by
$\varepsilon=\int_{l_{1}}^{l} \frac{d l}{l}=\ln \frac{l}{l_{0}}$

The strain rate is the time rate for straining (Vladimir 1989). For tension and compression, the engineering strain rate is defined as

$$
\begin{equation*}
\dot{\varepsilon}=\frac{d \varepsilon}{d t}=\frac{V}{l_{0}} \tag{3.6}
\end{equation*}
$$

The true strain rate is defined as (Higgins 1983)

$$
\begin{equation*}
\dot{\varepsilon}=\frac{d \varepsilon}{d t}=\frac{v}{l}=\frac{1}{l} \frac{d l}{d t} \tag{3.7}
\end{equation*}
$$

The effect of strain rate on the strength of materials was generally expressed by (Wen 1994)

$$
\begin{equation*}
\sigma=c \mathcal{E}^{m} \tag{3.8}
\end{equation*}
$$

where $\mathrm{c}=$ strength coefficient
$\mathrm{m}=$ strain-rate sensitivity exponent of the material

In an actual forming process, a workpiece may be deformed at a variety of speeds. Strain rate is a function of the geometry of the workpiece. The strain rate is affected mainly by temperature. Increasing strain rate increases the flow stress. Typical deformation rates and average strain rates employed for hot forging and rolling (The large of strain rates employed) are given in Table 3-1.

Table 3-1. (Lyndon et al. 1990)

| Process | True strain | Deformation rate/ms | Strain rate $1 / \mathrm{s}$ |
| :---: | :---: | :---: | :---: |
| Hot forging and rolling | $0.1-0.5$ | $1-30$ | $10-100$ |

### 3.5 Mathematical Models for Rolling Force

Hot rolling theory allows roll force to be evaluated. Theory has not yet advanced enough to give accurate answers to all problems and quantitative terms. Early publications in hot rolling theory tended to be application specific. Hence, some research results are not widely applied in industry.

The resistance to deformation of material depends upon its chemical composition, rolling temperature, and conditions of deformation. Till now none of the formulae provided a method for calculation of resistance to deformation with perfect accuracy, since certain simplifications and hypothesis have been used in each case. Therefore, the most acceptable research method is based on experimental investigations. Numerous mathematical models have been developed for prediction of rolling force in hot rolling
operations ranging from very simple to very complex expressions. The following sections outline important mathematical modelling approaches. Each of these models is a significant part of the theory and hence can only be briefly introduced here.

### 3.5.1 Sims' Method of Calculation of Roll Separation Force (1954)

A widely used model for the pressure distribution along the arc of contact in hot rolling was developed by Sims (1954). The Sims' method is based upon the slab analysis method and Orowan's (1943) method. Sims assumed sticking friction existed within the original method. However, in order to test the accuracy of this approach, a series of experiments were conducted where good agreement was found between the results of the experiments and the theory. Many publications have used this method for predicting separation force and torque (Sheffield 1960, Wusatowski 1969, Ginzburg, 1989, Datta 1996 and Malmgren 2000).

The Sims' equation for calculating roll separation force during flat hot rolling is
$P=W I_{d} Q_{p} S_{p}$

Equation (3.9) includes four parameters which are determined in the following stages:

1. Calculation of mean of workpiece width W . There are three methods:
(1) Arithmetic average width $(W)=W_{a}=W_{0}+W_{1}$ assuming the curve of spread is a straight line
(2) Parabolic mean width $(W)=W_{p}=\frac{W_{0}+2 W_{1}}{3}$ approximating the curve of spread with a parabola
(2) Geometric $(W)=W_{g}=\sqrt{W_{0} W_{1}}$ approximating root mean square value
2. Projected arc of contact $I_{d}$
$I_{d}=\sqrt{R \Delta h-\frac{\Delta h^{2}}{4}}$
In hot rolling, roll diameter is large compared with reduction $\Delta h$. Hence arc of contact $I_{d}$ approximates to the projected length of contact $L_{d}$.

Hence for hot rolling $I_{d}=L_{d}=\sqrt{R \Delta h}$
Where Draft $\Delta h=h_{0}-h_{1}$
If the rolls are of unequal diameters the mean radius is used $(R)=R_{m}=\frac{2 R_{1} R_{2}}{R_{1}+R_{2}}$
3. Geometrical coefficient $Q_{p}$

A number of geometrical factor are collected together into the coefficients $Q_{p}$.
$Q_{p}=\left[\frac{\pi}{2 a} \tan ^{-1} a-\frac{\pi}{4}-\frac{1}{a} \sqrt{\frac{R^{\prime}}{h_{1}}}\left(\ln \frac{h_{n}}{h_{1}}+\frac{1}{2} \ln \frac{a^{2}}{r}\right)\right]$
Where Relative reduction $r=\frac{\Delta h}{h_{0}}=1-\frac{h_{1}}{h_{0}}$

$$
\begin{aligned}
& a=r(1-r) \\
& h_{n}=\text { strip thickness at neutral point } \\
& R^{\prime}=\text { flattened roll radius. When hot rolling } R^{\prime}=\mathrm{R}
\end{aligned}
$$

Sample values for geometrical factor $Q_{p}$ are shown in Figure 3-6. In this figure the application range for the parameters are given by

$$
\frac{R}{h_{1}}=5-300 \quad \mathrm{r}=0-0.6 \quad Q_{p}=1-5
$$

4. Mean constrained yield stress $S_{p}$

The mean constrained yield stress $S_{p}$ is obtained from the equation
$S_{p}=\frac{1}{\alpha} \int_{0}^{\alpha} S d \theta$
Where bite angle $\alpha=\arccos [1-\Delta h /(2 R)]$
$S=$ constrained yield stress determined by plane strain compression
Roll contact angle $\theta=\alpha / 2$
The conditions for application are:(1) dry slipping friction. (2) bite angle is small The values for the mean constrained yield stress $S_{p}$ are shown in Figure 3-7.For this figure the conditions of application were:

Temperature: $1000^{\circ} \mathrm{C} \quad$ r: 0-0.5. $\quad \lambda=1-100 \mathrm{sec}^{-1} \quad S_{p}: 2-16$ ton $/ \mathrm{in}^{2}$

In the Figure 3-7 $\lambda=$ mean strain rate which can be determined by several methods

Orowan and Pascoe' solution (1946).
$\lambda=\frac{\pi N}{30} \sqrt{\frac{R}{h_{0}}}\left(\frac{1-0.75 r}{1-r}\right) \sqrt{r}$

Sims' solution (1954).
$\lambda=\frac{\pi N}{30} \sqrt{\frac{R}{h_{0}}} \frac{1}{\sqrt{r}} \ln \left(\frac{1}{1-r}\right)$

Wusatowski' solution (1969)

$$
\begin{equation*}
\lambda=\frac{\pi N}{30} \sqrt{\frac{R}{h_{0}}} \sqrt{\frac{r}{1-r}} \tag{3.14}
\end{equation*}
$$

Where roll peripheral speed $N=\frac{\pi D_{w} n}{60} \mathrm{~m} / \mathrm{sec}$


Figure 3-6. Sims' geometrical Factor Figure 3-7. Mean constrained yield stress used (Sims 1954). for force $(0.17 \% \mathrm{C})$ at $1000^{\circ} \mathrm{C}$ (Sims 1954).

### 3.5.2 Cook and McCrum 's Method of Calculation of Roll Separation Force (1958)

This is a graphical method for determination of roll separating force. It has been developed by Cook and McCrum and is based on Sims's formula.

The graphical method can be explained by the following formulae:
$P=R W C_{p} I_{p}$
Where

$$
\begin{aligned}
& C_{p}=Q_{p} \sqrt{\frac{h_{1} \lambda}{R(1+\lambda)}} \\
& I_{p}=k_{p} \sqrt{\frac{1+\lambda}{1-\lambda}}
\end{aligned}
$$

$k_{p}$ depends on the yield stress.
$C_{p}$ and $I_{p}$ are geometrical functions

The values of geometrical functions $C_{p}, I_{p}$ are based on the Cook and McCrum method. The diagrams published by Cook and McCrum provide a simple method for calculating the forces (Cook and McCrum 1958).

### 3.5.3 Roberts' Method (1988)

For the computer control of flat mills, a simpler model has been proposed. A method by Roberts gives one of the simplest models for predicting rolling force:

A model for predicting rolling force for low-carbon steels on an approximate basis is discussed below. It assumes that the neutral point occurs at the centre of the arc of contact and that the pressure distribution along the arc is a " friction hill ". Before calculating the roll separation force, the following parameters must be known.
1.Incoming workpiece thickness, $h_{0}$ (in.)
2. Workpiece width, W (in.)
3. Workpiece temperature, $\mathrm{T}\left({ }^{\circ} \mathrm{F}\right)$
4.Entry and exit strip tensile stresses,
5.Coefficient of friction, $\mu$
(1) In hot rolling of steel Ekelund's empirical formulae are used for determining Coefficient of friction. The coefficient of friction will be dependent upon the rolling temperature (Underwood 1952), as indicated by the figures below.

- for cast iron and rough steel rolls

$$
\mu=1.05-0.0005 \mathrm{~T}
$$

-for chilled and smooth steel rolls

$$
\mu=0.8(1.05-0.0005 \mathrm{~T})
$$

-for ground steel rolls

$$
\mu=0.55(1.05-0.0005 \mathrm{~T})
$$

(2)Bachtinov proposed a modification to Ekelund' formulae (Bachtinov 1960). It allows for the influence of rolling speed:

$$
\mu=a \mathrm{~K}(1.05-0.0005 \mathrm{~T})
$$

Where $\mathrm{a}=$ depends on roll quality
$\mathrm{K}=$ depends on rolling speed $V_{r}$ The Table belowshow K and $V_{r}$ values.

Table3-2. (Bachtinov 1960).

| $V_{r}$, <br> $\mathrm{m} / \mathrm{sec}$ | Up <br> to 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | $\cdots$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| K | 1 | 0.9 | 0.8 | 0.72 | 0.66 | 0.6 | 0.57 | 0.55 | 0.52 | $\cdots$ |

6.Work roll diameter, D (in.)
7.Rolling speed, $\mathrm{V}(\mathrm{in} / \mathrm{sec})$
8.Relative reduction, r

From the above information, the following calculation may be made.

1. The length of the arc of contact, assuming rigid work rolls
$L_{d}=\sqrt{\frac{D h r}{2}}$
2. The approximate strain rate $\left(\mathrm{sec}^{-1}\right)$

$$
\dot{\varepsilon}=\frac{V}{L_{d}}
$$

3. The dynamic constrained yield strength (psi)

$$
\sigma_{c}=238.5 \exp \left(\frac{8.53 \times 10^{3}}{459+T}\right)+2600 \ln \dot{\varepsilon}
$$

4. Average tensile stress in the bite, (psi)
$\sigma_{a}=\frac{\sigma_{1}+\sigma_{2}}{2}$
5. Average strip thickness in the bite, (in.)
$T_{a}=h\left(1-\frac{r}{2}\right)$
6. Effective coefficient of friction in the bite
$\mu=2.7 \times 10^{4} T-0.08$

7 Roll separation force can be calculated by (lb/in.)
$P=\frac{\left(\sigma_{c}-\sigma_{a}\right) T_{a}}{\mu} Q$
Where $Q=\frac{\exp \left(\mu L_{d} / T_{a}\right)-1}{\mu L_{d} / T_{a}}$
$\mathrm{E}=$ elastic modulus of the roll material

$$
L_{d}=(D(1+4.62 P /(E h r)) / 2)^{1 / 2}
$$

Finally, the rolling force is calculated by

$$
\begin{equation*}
P=\frac{\left(\sigma_{c}-\sigma_{a}\right) T_{a}}{\mu} Q W \tag{3.16}
\end{equation*}
$$

### 3.5.4 Lower and Upper Bound Analysis

In predicting loads for metal forming modelling, the main purpose is to predict exactly how and where the material will flow. The traditional empirical analysis models are based on flat rolling theory and cannot meet the modern requirements of high precision, high quality and high efficiency. Till now, the exact solutions for hot rolling load have not been found, however, a number of different methods have been developed.

Two methods are used for approximate evaluation of forces. The first is called the lower-bound analysis while the second is called the upper-bound analysis. These are underestimate or overestimate of the actual forming load. The real load will lie between these lower and upper bounds.

The lower bound analysis predicts loads that are less than or equal to the exact load needed to produce full plastic deformation. The method can take into account frictional forces and always gives a lower bound (underestimate) of the actual forming load. It is particularly useful in assessing the affects of friction on a given forming process. For most design or operational purposes it is more important to know the upper bound solution since this will ensure that the calculated load is sufficient to complete the forming operation.

## Upper bound analysis

The basic assumption: is that strain is continuous and the volume of the material remains constant when its shape changes. The shape change must correspond to the flow velocity of the material. The method is more valuable in metalworking, where it is important to know the upper bound solution, to ensure the calculation of the load necessary to complete the forming operation.

A detailed description of the technique would be lengthy. However, Figure 3-8. Illustrates a simple upper bound solution for plane strain extrusion. It has a reduction in area of $2: 1$. Complex shape change will not usually be plane strain and hence the deformation will only be approximated by this calculation. The simple example demonstrates the principles.


Figure 3-8. Upper bound solution for strain extrusion (Lyndon 1990)

The region of deformation is divided into a numbers of zones, they are separated by a series of shear planes, which provide the deformation required.

Fig. (a) (b) (c) are relative velocity diagrams known as hodographs, which describe the shear plane geometry. There are an infinite number of possible hodographs and the most suitable configuration is that which gives the lowest solution for deformation force.

In this analysis material enters at velocity $V_{0}$ and leaves at velocity $V_{1}$, In this case with a reduction ratio of 2:1, $V_{1}=2 V_{0}$

$$
\begin{aligned}
& V_{A C}=V_{0}+V_{A B} \\
& V_{1}=V_{A C}+V_{B C}
\end{aligned}
$$

Fig. (d) is used to estimate the total change in shear velocity of an element of material passing through the region ABC see Figure 3-8 (a).

For each shear plane, the work done per unit time is:

$$
\frac{d w}{d t}=\bar{K} V S
$$

Where $\bar{K}=$ the average shear yield stress.
$V=$ velocity of the material along that shear plane.
$S=$ length of the shear plane.

V is calculated from the hodograph and S is obtained from the physical diagram, see Figure 3-8. Work is done on three shear planes and the total work per unit time is given by

$$
\begin{equation*}
\frac{d w}{d t}=\bar{K}\left(V_{A B} S_{A B}+V_{B C} S_{B C}+V_{A C} S_{A C}\right) \tag{3.17}
\end{equation*}
$$

Using simple trigonometry from Figure 3-8 (c) and (d), the total work done per unit time on internal shear is
$\frac{d w}{d t}=\bar{K}\left(V_{0} \frac{h_{0}}{2}+\sqrt{2} V_{0} \frac{h_{0}}{2 \sqrt{2}}+\sqrt{2} V_{0} \frac{h_{0}}{2 \sqrt{2}}\right)=\frac{3}{2} \bar{K} V_{0} h_{0}$

For half of the extrusion, total internal work done is $3 \bar{K} V_{0} h_{0}$. This must be equal to the work done by pressure $P$, which is used to produce the initial velocity $V_{0}$. The work done per unit time by the external extrusion pressure is
$\frac{d w}{d t}=P V_{0} h_{0}$
Hence $\mathrm{P}=3 \bar{K}=1.5 \bar{Y}$
Where $\bar{Y}=$ average flow stress, table 3-2 gives some values of $\bar{Y}$ from a range of methods.

Table 3-2. (Lyndon 1990)

| Ally | Flow stress $\overline{\bar{Y}} / \mathrm{MPa}$ |
| :---: | :---: |
| Lead | 20 |
| Aluminium | 80 |
| Copper | 140 |
| Mild steel | 200 |

Jin et al. has analysed the rolling of a universal beam based on the upper bound theorem and 3D continuous velocity. They predicted metal flow, flange spread, web forward-slip and compared them with experimental results (Jin et al. 2000).

### 3.5.5 Visioplasticity Method

Lower and upper bound analyses are useful in predicting forming loads. However, both methods require quite complicated mathematical modelling if accurate answers are to be obtained.

The Visioplasticity method is a more empirical approach. It predicts both material flow and forming loads using materials which have the same flow characteristics but lower flow stresses than the real workpiece material. This technique is particularly useful in modelling complex forming processes (Lyndon 1990 and Shin 1992).

### 3.6 Rolling Torque

The roll torque is another design parameter for roll design. It can be calculated by several methods.

### 3.6.1 The General Equation for Roll Torque

The roll torque is equal to the total torque required to drive both rolls. When rolls of equal diameter are used, the general equation for rolling deformation torque is given by

$$
\begin{equation*}
M=2 P a \tag{3.20}
\end{equation*}
$$

Where. the lever arm $a=m I_{d}=m \sqrt{R \Delta h}$ (see Figure 3-4)
$\mathrm{m}=$ lever arm coefficient

Defining the lever arm coefficient m presents the most difficult part in the calculation of roll torque.
3.6.2 Sims' Formulae for Torque (1954)

The Sims' formulae can be obtained by

$$
\begin{equation*}
M=2 R R ' W Q_{g} S_{g} \tag{3.21}
\end{equation*}
$$

Where $S_{g}=\frac{1}{r} \int_{0}^{r} s d r x$
$S_{g}=$ mean constrained yield stress torque (see Figure 3-10).
$Q_{g}=$ Torque geometrical factor (see Figure 3-9)
for hot rolling, $R=\mathrm{R}$


Figure 3-9. Torque geometrical factor $\left(Q_{g}\right)($ Sims 1954)


Figure 3-10. Mean constrained yield stress torque $\left(S_{g}\right)$ (Sims 1954)

### 3.6.3 Total Torque

The total driving torque $M_{D}$ consists of the following component torques.
$M_{D}=M_{W}+M_{f}+M_{d}$
Where $M_{w}$ is overcome resistance of deformation.
$M_{f}$ is frictional torque
$M_{d}$ is dynamic torque

### 3.7 Prediction of Roll Separation Force in Hot Section Rolling

Compared with flat rolling, the metal flow of section rolling has its own complex characteristics. First, the reduction along the width direction of the stock is non-uniform. Secondly, the angle of entry is unequal i.e. all the material along the width direction of the workpiece does not contact the rolls at the same time. Thirdly, the shape of the roll pass has a great effect on the deformation of the material. Fourthly, the peripheral velocity along the profile of the roll pass varies from point to point, because of the difference in diameter of the roll.

In hot rolling, none of the formulae discussed above provide a method for calculating roll load with perfect accuracy in flat rolling. At present, there is obviously no theoretical formula to directly calculate the roll separating force in section rolling since
this deformation is far more complicated. In practice, designers apply roll separation force formula for non-flat products, which requires some artifice to reduce the section to an equivalent rectangular workpiece. However, all of these methods only give a very rough approximation for calculating roll separation force.
3.7.1 Prediction of Roll Separation Force and Torque in Regular Sections (square, round etc.)

In practice, a rough approximation of roll separation force can be made, for regular section transformations such as diamond-square, oval-round, square-oval, etc., using the following equation (Wusatowski 1969):

$$
\begin{equation*}
P=F_{d m} P_{m} \tag{3.23}
\end{equation*}
$$

Where $P_{m}$ is the mean specific roll pressure.
$F_{d m}$ is the projected area of contact between the roll and the workpiece.

The mean specific roll pressure $P_{m}$ is calculated by using the equivalent rectangle method (Wusatowski 1969) and flat rolling formulae. The projected contact area $F_{d m}$ is calculated either by the equivalent rectangle method or by a graphical method (Trinks 1943).

In practice the roll torque is mostly calculated using the following simplified approximation:

$$
\begin{equation*}
M_{W}=P a \tag{3.24}
\end{equation*}
$$

Where: $a=\lambda l_{d}$

$$
\lambda=0.44-0.64 \text { (Muller 1957). }
$$

Souresrafil has introduced the method for square-diamond transformation force and torque; the method is also based on the flat rolling formulae (Souresrafil 1970). Labuda et al. presented relationships between geometrical parameters in the design of modified shapes at oval-vertical, oval-oval system grooves (Labuda et al. 1998).

### 3.7.2 Prediction of Rolling Separation Force in Complex Sections

Early studies were focused on establishing independent sub-models of the main factors such as beam web forward-slip, flange spread, roll separating force and torque of rolls and so on. The application of roll load formulae to non-flat products requires some artifice to reduce the sections to equivalent a rectangular workpiece entering and leaving the rolls (Sheffield 1960). The equivalent rectangle method may be used for simple shapes such as diamonds, squares, ovals, etc. In the case of complex sections, an attempt can still be made to obtain an estimate of the rolling load by splitting the section area into elements.

For example, Figure 3-11 shows a beam being rolled on the diagonal.

The total force $P=P_{a}+P_{b} \cos \alpha+P_{c} \cos \beta$
Where $P_{a}=$ closed flanges force
$P_{b}=$ open flanges force
$P_{c}=w e b$ force


Figure 3-11. Components of rolling load in a beam pass (Sheffield 1960).

### 3.7.3 A Experimental Method in Section Rolling

Visioplasticity described a more empirical method of predicting both material flow and forming loads. This is an experimental technique which physically models the
deformation process using materials which have the same flow characteristics but lower flow stresses than the real workpiece material. The lower forming forces allow the use of wooden tools so that design changes can be investigated quickly and cheaply. In particular can be used in modelling complex hot rolling processes. Shin et al. (1994) presents the method for I-beam rolling. For example, Figure 3-12 shows the flow pattern of the model material for pass three and pass four.


Figure 3-12. Grid distortions of the billets (experiment: plasticine) (Shin et al. 1994).

## 3. 8 An Example for Calculating Rolling Separation Force

The following data are based on the EUR report (Mirabile 1997).

| Radius of the rollers | $\mathrm{R}=300 \mathrm{~mm}$ |
| :--- | :--- |
| Slab Length | $\mathrm{L}=300 \mathrm{~mm}$ |
| Slab Width | $\mathrm{W}=350 \mathrm{~mm}$ |
| Slab Thickness | $\mathrm{H}=100 \mathrm{~mm}$ |
| Thickness reduction | $13.2 \%$ |
| Friction coefficient | $\mu=0.35$ |

Mechanical properties of steel

| Young modulus | $\mathrm{E}=100 \times 10^{3} M P_{a}$ |
| :---: | :---: |
| Poisson coefficient | $\mathrm{V}=0.35$ |
| Yield stress | $\sigma_{y}=33.3 M P_{a}$ |

Calculating force:
According to Sims (1954). Rolling separation force $P=W I_{d} Q_{p} S_{p}$

1. Mean workpiece width W

Where $W=350 \mathrm{~mm}=350 / 25.4=13.78 \mathrm{in}$
2. Projected arc of contact $I_{d}$
$I_{d}=L_{d}=\sqrt{R \Delta h}=\sqrt{300 \times(100-86.8)}=62.93 / 25.4=2.48 \mathrm{in}$
3. Geometrical coefficient $Q_{p}$

Assumption $R^{\prime}=R$ i.e. no flatten
$R^{\prime} / h_{2} \approx R / h_{2}=300 / 86.8=3.46$
Relative reduction $r=\Delta h / h_{0}=\frac{h_{0}-h_{1}}{h_{1}}=0.15$
From Figue 3-6.
$Q_{p}=1.1$
4. Mean constrained yield stress $S_{p}$

Mean strain rate. According to Sims' solution (1954).
$\lambda=\frac{\pi N}{30} \sqrt{\frac{R}{h_{1}}} \frac{1}{\sqrt{r}} \ln \left(\frac{1}{1-r}\right)$
Where roll peripheral speed $N=\frac{\pi D_{w} n}{60} \mathrm{~m} / \mathrm{sec}$
working roll diameter $D_{w}=2 \mathrm{R}=600 \mathrm{~mm}=0.6 \mathrm{~m}$
$\mathrm{n}=$ roll revolutions, rpm
Assumption $\mathrm{n}=300 \mathrm{rpm}$

$$
\lambda=\frac{\pi N}{30} \sqrt{\frac{R}{h_{1}}} \frac{1}{\sqrt{r}} \ln \left(\frac{1}{1-r}\right)=\frac{\pi \times 9.3 \times 1000}{30} \times \sqrt{\frac{300}{86.8}} \times \frac{1}{\sqrt{1.15}} \times 0.163 \approx 76
$$

From Figure 3-7. $S_{p}=11.1$ tons $/ \mathrm{in}^{2}$ (carbon steel $0.17 \% \mathrm{C}$ at $1000^{\circ} \mathrm{C}$ )

Hence, the roll separation force $P=W I_{d} Q_{p} S_{p}=13.78 \times 2.48 \times 1.1 \times 11.1=417.2$ tons

## Calculation Torque

According to Sims (1954).Torque $M=2 R R W Q_{g} S_{g}$
From Figure 3-9.
$Q_{g}=0.022$
Where
$R^{\prime} / h_{2} \approx R / h_{2}=300 / 86.8=3.46$
$\mathrm{r}=0.15$
From Figure 3-10
$S_{g}=4.5$ tons $/$ in $^{2}$
$\lambda=76$
Hence $M=2 R R W Q_{g} S_{g}=2 \times \frac{300}{25.4} \times \frac{300}{25.4} \times \frac{350}{25.4} \times 0.022 \times 4.5=380.6$ tons. In


Figure 3-13. Nomogram for determination of peripheral speed of rolls (Wusatowski 1969).

### 3.9 Summary and Conclusions

In this chapter, the foundation concepts of metal flow have been discussed. A few formulae for calculating flat roll separation force and torque, including experimental methods, slab method and upper-bound method have been discussed. Prediction of roll separation force and torque in section rolling has also been discussed. Other existing formulae for calculating force and torque are listed and more details can be found from the list of the references.

It is clear that most of the theories are concerned mainly with predicting the rolling pressure distribution and roll load under the assumption of plane strain deformation. A very large number of books and papers are only concerned with flat rolling. Due to the complexity of three-dimensional plastic deformation, there had been no direct theoretical solution for force and torque. In practice, applying these formulae for nonflat products requires some artifice to reduce the section to its equivalent rectangular workpiece shape.

## Chapter 4 Theory 2

Hot Rolling Spread, Elongation and Draft

### 4.1 Introduction

Elongation, spread and draft are important parameters which describe the three dimensional deformation of a workpiece in hot rolling. The early work of the theory of rolling has focused on two-dimensional deformation, where the ratio of the width to thickness of the workpiece is high i.e. $>10$ and the spread of the workpiece is considered negligible. Nowadays it has been more important to study and predict metal flow in all directions in order to accurately control the material properties and final quality of the rolled products. In this chapter fundamental concepts of elongation, spread and draft will be discussed.

### 4.2 Elongation and Draft

Elongation and draft describe the two deformations in the directions of length and height of the workpiece. They are main parameters in the rolling process.

### 4.2.1 Definition of Draft

Figure 4-1 shows a flat rolling process. For the deformation of the parallelepiped of material shown in the Figure 4-1, draft is expressed as a linear reduction in height of the workpiece under the action of the compressive force. Draft can be expressed by several methods:


Figure 4-1. Flat rolling process (Kalpakjian 1995)

Absolute draft $\Delta h=h_{0}-h_{1}$
Relative draft $\varepsilon h=\left(h_{0}-h_{1}\right) / h_{0}$
Natural draft $\phi h=\log _{e}\left(h_{1} / h_{0}\right)$
Coefficient of draft $\gamma=h_{1} / h_{0}$
In rolling non-rectangular sections, the term maximum draft is sometimes used.
Maximum percentage draft (See Figure 4-2) is given by:

$$
\begin{equation*}
\frac{h_{1 \max }-h_{2 \min }}{h_{1 \max }} \times 100 \% \tag{4.5}
\end{equation*}
$$



Figure 4-2. Methods of determining the average height of passes: (a)-three successive regular passes for rolling bars, (b)-pass for rolling a section (Wusatowski 1969)

### 4.2.2 Definition of Elongation

In the theoretical analyses of plastic deformation processes, if the deformation of a parallelepiped is assumed, the volume of metal passing though the rolls remains constant, see Figure 4-1. Elongation is defined as the increase in length of the rolled material. Elongation is expressed by different methods:

Absolue elongation $\Delta l=l_{1}-l_{0}$

Relative elongation $\varepsilon l=\frac{\left(l_{1}-l_{0}\right)}{l_{0}}=\frac{\Delta l}{l_{0}}=\frac{\left(F_{0}-F_{1}\right)}{F_{1}}$

Natural elongation $\phi l=\log _{e}\left(\frac{l_{1}}{l_{0}}\right)=\log _{e}\left(\frac{F_{0}}{F_{1}}\right)$

Coefficient of elongation $\lambda=F_{0} / F_{1}=l_{0} / l_{1}$
Percent reduction $\mathrm{U}=\frac{\lambda-1}{\lambda} \times 100 \%$

### 4.2.3. Section Rolling (Wusatowski 1969 )

In section rolling the problem of metal flow is more complicated. Section rolling presents some difficulty in term of defining elongation, as described below.

### 4.2.3.1 Regular Section

Regular sections are those that have at least one axis of symmetry. The mean height $h_{m}$ is calculated by (See Fig.4-2 (b)).

$$
\begin{equation*}
h_{m}=F / W \tag{4.11}
\end{equation*}
$$

i.e. it is calculated by dividing the cross-section area F by the maximum width W of the filled section.

In rolling, the coefficient of elongation can be expressed as:

$$
\begin{equation*}
\lambda=v_{1} / v_{0} \tag{4.12}
\end{equation*}
$$

### 4.2.3.2 Irregular Sections

In the case of an irregular section the elongation coefficients is the most important design issue, which influences the whole rolling process. Elongation is used to determine the number of passes needed. In section rolling, one of the main tasks of the roll pass designer is to distribute the total elongation between each individual pass, based upon the number of passes that have been decided. In order to ensure correct filling of the next pass the mean elongation coefficient is needed. The mean elongation coefficient is defined as the average of coefficients calculated for the various parts of the section. For calculating these part elongation coefficients, the whole section is divided into parts with approximately the same draught see Figure 4-3. Usually the various parts of the section have different elongations.


Figure 4-3. Rolling of two-part section: (a) both bars before roll pass, (b) two bars rolled separately, (c) both bars rolled together (Wusatowski 1969)

If the mean elongation coefficient of a section is composed of N parts, Wusatowski and Gorecki (1969) method can be used, in which;

$$
\begin{equation*}
\lambda_{m \mathrm{~mm}}=\frac{\sum_{i=1}^{k=N} E_{1} \lambda_{i}}{\sum_{i=1}^{k N} E_{n}} \tag{4.13}
\end{equation*}
$$

Where $\lambda_{m w}=$ the mean pass elongation coefficient
$\lambda_{i}=i$ part elongation factors
$F_{i 1}=$ The area of component parts of the section after deformation i.e. its true dimensions.

Another basic relationship is:

$$
\begin{equation*}
\lambda_{m i v}=\frac{F_{1}}{F_{0}}=\frac{F_{A 0}+F_{B 0}+F_{C 0}+\ldots+F_{N 0}}{F_{A 1}+F_{B 1}+F_{C 1}+\ldots+F_{N 1}} \tag{4.14}
\end{equation*}
$$

Where $F_{A 0}, F_{A 1}, F_{B 0}, F_{B 1}, \ldots, F_{N 0}, F_{N 1}=$ section area A, B $, \ldots, \mathrm{N}$ before and after the roll pass.

Calculation of the number of passes is by

$$
\begin{equation*}
n=\frac{\log F_{0}-\log F_{n}}{\log \lambda_{m w}}=\frac{\log \lambda_{i}}{\log \lambda_{m w}} \tag{4.15}
\end{equation*}
$$

Where $\lambda_{i}=$ total elongation coefficient

### 4.3.Spread

The prediction of spread in rolling is mostly based on experimental observations. As a consequence a number of experimental investigations on spread have been carried out. Several formulae for the calculation of spread are discussed below. Other useful formulae are also listed and more detail can be found from the list of the references.

## 4. 3.1 Definition of spread.

In the rolling process, any compression applied to the metal causes it to elongate in the direction of rolling and to spread in the transverse direction, see Figure 4.1. Spread is an increase in width of the rolled material that is being reduced in thickness. Spread is often expressed as a difference between maximum values of widths before and after rolling.

$$
\begin{align*}
& \text { Absolute spread } \Delta W=W_{1}-W_{0}  \tag{4.16}\\
& \text { Relative spread } \varepsilon_{W}=\left(W_{1}-W_{0}\right) / W_{0}  \tag{4.17}\\
& \text { Natural spread } \phi_{W}=\log _{e}\left(W_{1} / W_{0}\right)  \tag{4.18}\\
& \text { Coefficient of spread } \beta=W_{1} / W_{0} \tag{4.19}
\end{align*}
$$

Spread is often presented by the spread coefficient which is equal to:

$$
\begin{equation*}
S_{W}=\left[\ln \left(W_{1} / W_{0}\right)\right] /\left[\ln \left(h_{0} / h_{1}\right)\right] \tag{4.20}
\end{equation*}
$$

### 4.3.2 Classification of Spread and Filling

Spread can be classified into two types. The first is called free spread (unrestricted spread), for example, in the case of flat rolling where the workpiece passes through plain rolls. This unrestricted spread has a minor effect on producing elongation. The second type is restricted spread, which takes place in grooved passes. In this case the spread is smaller than for free spread. In restricting spread in section rolling, the edge of the work piece is compressed. However, if the spread is not correctly anticipated, it can result in over-filling or under-filling of the section.

On entering the rolls the workpiece must be smaller in width than the grooves in the groove passes. Spread is thus restricted within certain limits. Over-filling means
excessive wear, and side thrusts on the roll collars, which is harmful. In under-filling the spread is not sufficient to fill the section and will not form the complete across-section shape.

Incorrect estimates of spread result in under-filling on one side and over-filling or fin formation on the other side. Fin formation must be avoided in all passes except the finishing pass, since it would be overlapped in edging and cause defects known as laps or seams. In the finishing pass, fin formation is permissible and sometimes unavoidable.

### 4.3.3 Factors Which Affect Spread

In roll pass design, the dimensions of groove width is determined from known spread formulae. This obviously indicates that spread is the most important factor that affects width control of the rolled product. Spread depends on many factors:

## 1. Pressure

The workpiece is subjected not only to vertical pressure by the rolls, but also to horizontal longitudinal pressure. The greater this pressure, the greater spread.

## 2. Friction

The higher the coefficient of friction, the higher the resistance to elongation and the more spread may occur.

## 3. Chemical components and microstructures

Different chemical components and microstructures result in different coefficient of friction, which in turn influence spread. From empirical observations, spread of high carbon steel is greater than low carbon steel; the spread of alloys is greater than the spread of carbon steel.
4. Speed of rolling and temperature

Within reasonable limits, slow speed favours spread, while high speed favours elongation.

## 5. Prevention of elongation

If the elongation of a workpiece is prevented by non-uniform plastic deformation distribution, the deformation will tend to go into spread.
6. Mill diameter

Sometimes mill diameter influences spread. If bite area width increases, then spread increases.

### 4.4 Calculation of Spread

The roll designer is interested in the amount of spread and in its distribution.
When width / thickness $>10$ i.e. for two-dimensional problems, the spread of the workpiece is considered negligible. However, if width / thickness $<10$, spread cannot be neglected i.e. the deformation is considered in three dimensions.

Spread is a major research parameter for designers. It was observed as early as the beginning of theoretical analysis of rolling. Many of the early investigators attempted to consider the above factors simultaneously and to derive a general formula for spread but till now no such generally accepted formula exists. However, formulae have been developed for the determination of spread in flat rolling. Unfortunately, these formulae do not give sufficient accuracy when applied to practical rolling condition, because of the complicated nature of hot rolling i.e., their practical application is limited. Further, there is no theory of rolling to deal with 3D deformation. With few exceptions, studies on deformation in rolling have been largely experimental (Kobayashi 1983). This means that prediction of spread in rolling has until recently remained mostly based on experimental observations.

Some well-known formulae for spread in flat rolling are given below.

## Siebel's formula

In Siebel's (1933) method, the condition of application is $w_{0} / h_{0}>1$, for variable temperature and variable $w_{0} / \sqrt{R \Delta h}$. The method was used to determine free spread and has not been used any more recently.

## Ekelund Formula

The Ekelund (1933) formula considers most of the parameters affecting spread, except those of the quality or composition of the steel. The formula is complicated, making it difficult to use in practice.

$$
W_{1}^{2}-W_{0}^{2}=2\left[4 \Delta h-2 \ln \left(\frac{W_{1}}{W_{0}}\right)\left(h_{0}+h_{1}\right)\right](\sqrt{R \Delta h})\left(\frac{1.6 \mu \sqrt{R \Delta h}-1.2 \mu}{h_{0}+h_{1}}\right)
$$

The Ekelund formula was found to overestimate spread in carbon steels and to underestimate spread in alloy steels.

More recently, other methods have been used for predicting spread.

## Hill's Formula for Spread

Hill (1963) proposed a general and approximate method for metal forming processes by using the upper bound theorem. This approach provides a good approximate solution in realistic conditions, and covers useful ranges of all relevant parameters. It gives reliable information about loads and principal dimensional changes.

Hill has presented the simplest formula for the width spread coefficient using a theoretical basis.

$$
\begin{equation*}
S_{w}=0.5 \exp \left(0.707 c W_{0} / L_{d}\right) \tag{4.21}
\end{equation*}
$$

Where $\mathrm{c}=$ constant

An Original value for $c$, suggested by Hill, was 0.5 . McCrum showed that a value of 0.525 gave the best agreement with his experimental results. The equation shows that spread increases with decreases in the ratio $W_{0} / L$. In order to explain Hill's method in some detail and to assess its usefulness, Lahoti and Kobayashi (1974) carried out an analysis of ring compression. Oh and Kobayashi (1975) also used this approach for an analysis of three-dimensional deformation in rolling. To reduce the complexity involved in the above approach. Lahoti et al (1980) developed a computer aided design system for predicting spread and stresses in rolling.

In general terms, Hill's approach has provided a useful method for predicting spread in rolling but it is only suitable for rigid-perfectly plastic material where the determination of acceptable velocity fields is difficult (Hartley 1989).

## Wusatowski's Formula for Spread (1969)

Wusatowski' formula is an empirical formula that collects and analyses different results from several sources. It has a good agreement with practice. The formula applies only to flat rolling.

Wusatowski concluded the coefficient of spread could be described by

$$
\begin{equation*}
\beta=\gamma^{-W} \tag{4.22}
\end{equation*}
$$

Where W is function, $-W=f\left(\delta_{w,} \varepsilon_{w}\right)=-10^{-1.269^{9}{ }_{\varepsilon_{w}} .56} \delta_{w}$

Equation (4.18) depends on two important factors. The first is the initial shape of the rolled workpiece $\delta_{w}=W_{0} / h_{0}$. The second is the roll factor or thickness ratio $\varepsilon_{w}=h_{0} / D_{w}$. It is obvious that spread increases with decreases in $W_{0} / h_{0}$ and $h_{0} / D_{w}$ in the equation (4.18).

Substituting equation $\gamma \beta \lambda=1$ into this equation gives the coefficient of elongation

$$
\begin{equation*}
\lambda=\gamma^{-(1-W)} \tag{4.23}
\end{equation*}
$$

The coefficients of spread and elongation are so closely related, in any given pass, that by calculating -W for spread, the elongation is determined simultaneously.

Mathematical equation (4.18) is very difficult to analyse. Wusatowski produced a monogram for the values of $-W$ (Wusatowski 1969)

## El-Kaylay and Sparling's Formula for Spread (1968)

This method gives a good estimate for spread, although it has a limited range of application. It is an empirical formula for calculation of the coefficient of spread.

$$
\begin{equation*}
S_{W}=a \exp \left[-b\left(\frac{W_{0}}{h_{0}}\right) c\left(\frac{h_{\mathbf{0}}}{R}\right)^{d \gamma e}\right] \tag{4.24}
\end{equation*}
$$

Where $\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}, \mathrm{e}=\mathrm{constants}$

The method found that spread is not very sensitive to temperature. However, the effect of finish rolling on spread does depend upon geometrical parameters. The spread is higher for high reductions, smooth rolls and lower $W_{0} / h_{0}$ ratios.

### 4.5 Interactive Elongation Spread and Draft in Hot Rolling

Elongation, spread and draught will occur in the hot rolling process. The workpiece will elongate in the direction of rolling and spread in the transverse direction as a result of the draft in thickness. Hence, elongation, spread and -draft are interdependent and closely related to each other.

For analysis of elongation, spread and draft, it is usual to make the assumption that the volume of metal, for practical purposes, remains constant during hot rolling (Wusatowski 1969). Hence,
$\gamma \beta \lambda=1$
Where the draft coefficient $\gamma=h_{1} / h_{0}=\left(1-\frac{\Delta h}{h_{0}}\right)$ is always greater than zero and less than 1 , since, if $\gamma=1$, there is no draft and therefore no rolling takes place.

The elongation coefficient $\lambda=F_{0} / F_{1}=l_{0} / l_{1}=\left(1+\frac{\Delta l}{l_{0}}\right)$ is always greater than 1. If $F_{1}=F_{0}$ or $l_{1}=l_{0}$ then there is no rolling since no reduction or elongation takes place.

The spread coefficient $\beta=W_{1} / W_{0}$ is usually greater than 1 , since $W_{1}>W_{0}$. However, under specific conditions $\beta$ can take values between 0 and 1 i.e. either $W_{1}=W_{0}$ and no spread takes place, or $W_{1}<W_{0}$ i.e. the rolled workpiece becomes narrower.

In plastic flow, the boundary condition is: $\beta=1 / \gamma$ for $\lambda=1$, or $\lambda=1 / \gamma$ for $\beta=1$

For roll pass design.

- If $\lambda>\beta$, the deformation results mostly in elongation, the roll pass design and rolling process can be considered economic.
- If $\beta>\lambda$, the deformation results mostly in spread and the case is not economic.
- If $\beta=\lambda$, the case is called the theoretical limit of economic rolling


### 4.6 Spread in Section Rolling

It can be seen that there are many spread formulae for flat rolling but there is no independent formula for section rolling. In section roll design, nearly all approaches consist of sub-dividing the section into flat elements and then applying one of the existing spread formulae from flat rolling. Lendl (1948) used Ekelund's formula for this purpose, Sims's calculation was also on the same basis. Wusatowski modified and used mean values of elongation, draft and spread in the following equation.

$$
\begin{equation*}
\gamma_{m} \beta_{m} \lambda_{m}=1 \tag{4.26}
\end{equation*}
$$

### 4.6.1 Regular Sections

In simple section rolling such as diamond, oval, or square rolling, the calculation of spread, based upon the methods in flat rolling, may be used directly. In these approaches the calculation of spread uses flat formulae in the "rectangle method " (Wusatowski 1969). Another example is the rolling of angles in which the two legs are divided into two rectangles.

Since the sub-dividing section method does not consider the metal flow from one subsection to anther, and section rolling is a complicated three-dimensional problem, considerable inaccuracy could occur in calculating spread when flat spread formulae are used directly. Hence, many investigators have tried to improve the prediction of accuracy and tried to find improved formulae for section rolling. Souresrafil (1970)
focused carefully on the relation between spread and draft which takes into account all the relevant parameters that should be universally valid in square-diamond passes. The formula is obtained by taking into account the spread along the arc of contact, from the entry plane to the exit plane. It is expressed by
$S_{w}=W_{0}\left(\frac{h_{0}}{h_{0}-\delta}\right) e^{-a\left(R / h_{0}\right)^{b}\left(W_{0} / \sqrt{R \delta}\right)^{r}}$
Where $\mathrm{a}, \mathrm{b}, \mathrm{c}=$ constants, $\mathrm{a}=1.441, \mathrm{~b}=0.10, \mathrm{c}=1.072$.

The main dimensionless groups that influence spread are: $R / h_{0}$ and $W_{0} / \sqrt{R \delta}$. Both of these groups have been found previously in flat rolling spread formula. Gong et al (1991) have recently proposed a spread formula for regular section rolling.

### 4.6.2 Graphical Method

The graphical method was first introduced by Trinks in 1918, and consists of drawing sections through the workpiece and the rolls at appropriate angles to the direction of rolling (Trinks 1941). Figure 4-4 shows the graphical method.


Figure 4-3. The graphical method of studying deformation and spread (Trinks 1941)

The Wusatowski' (1969) formula gives the elongation coefficient relationship between two passes in a given roll design. Its mathematical form is very difficult to analyse and can only be used for oval-square transformations. It is suitable for the determination of initial profiles and the number of passes required.

The graphical method has the advantage of simplicity, clearness and speed, but it is less accurate in comparison with the computational methods and it also depends largely on experience.

### 4.6.3 Empirical and Semi-empirical Methods

Because all previous methods for section rolling are approximate, in practice, using both empirical and semi-empirical methods can sometimes be more favourable. For example, spread may be considered as a percentage of reduction (Zhao 1978). The percentage is obtained from large amounts of practical measurements from both rolled products and from experimental work.

It is obvious that until recently the prediction of spread in section rolling has remained mostly based on experimental observations. Roll designers usually start their attempt on a new design by working out the necessary changes on a previously existing successful design for a similar product.

### 4.7 Summaries and Conclusions

In this chapter, the foundation concepts of elongation, draft and spread have been discussed. Formulae for calculating spread in flat rolling have been discussed. Prediction of spread in section rolling has also been discussed. The calculation of these parameters is very important because they describe the deformation of the workpiece in the hot rolling process.

Elongation, draft and spread represent three directional deformation in hot rolling. Spread in rolling is usually defined as a 3D problem. 3D metal flow is difficult to analyse and most studies of this process have focussed on experimental formulae. Attempts were also made to predict elongation and spread theoretically but there is no direct formulae for these values in section rolling.

## Chapter 5

## Finite Element Method Modelling of Hot Rolling

### 5.1 Introduction

Computer simulation of industrial processes is an important alternative to complement or to replace the expensive experimental procedures associated with innovative development (Anza 1997). In the last 20 years, with the development of computing, more sophisticated simulation models have become possible. A significant contribution has been made in the area of computer simulation of rolling processes. The flow theory of plasticity, with rigid-plastic or rigid-viscoplastic material models has been found to be quite suitable for the mechanical description of hot rolling processes.

Application of the theory of plasticity requires a geometric definition of the problem with the appropriate boundary conditions. Analysis must uniquely satisfy equilibrium, compatibility and material behaviour relations. Analytically this is generally difficult, and usually impossible, so apart from simple problems, solutions are generally numerical. The Finite Element Method (FEM) has made it possible to obtain accurate approximate numerical solutions. The FEM is now probably the most common technique used to investigate rolling problems, especially for section rolling.

In this chapter, the principle of FEM is described and this is followed by a description of its applications to hot rolling.

### 5.2 The Finite Element Method (FEM)

The following sections give a briefly introduction to FEM.

### 5.2.1 Procedures and Principles of Finite Element Analysis

According to Mottram (1996): the year 1956 can be considered as the birth of the Finite Element Method; the name was first used by Clough who saw a model as consisting of a finite number of elements (or sub regions). The first FEM formulations for forming processes took place in 1974 and were based on the so-called flow formulation, which considers the material to be a Newtonian viscous fluid.

The basis of the Finite Element Method is the representation of a body or a structure by an assemblage of subdivisions called Finite Elements, which are often referred to as a mesh (Fenner 1996). These elements are considered to interconnect at joints, which are called nodes or nodal points. The domain is then an assemblage of elements connected together appropriately on their boundaries. For a 2-dimensional continuum the Finite Elements may be triangles, a group of triangles, or quadrilaterals. For 3-dimensional analysis, the Finite Elements may be in the shape of a tetrahedron, rectangular prisms, or hexahedra.

The path to the solution of a problem formulated in a Finite Element problem consists of the following processes (Kobayash 1983): (a) identification of the problem; (b) definition of the element; (c) establishment of the element equation; (d) the assembly of element equations; and (e) the numerical solution of the global equations.

In the deformation process shown in Figure 5-1, the workpiece is divided into elements without gaps or overlaps.


Figure 5-1. Finite Element mesh and nodal point specifications in a forming process. (Kobayash 1983)

A set of nodal point velocities is defined in a vector form as
$\mathbf{V}^{\mathrm{T}}=\left\{\mathrm{v}_{1}, \mathrm{v}_{2}, \ldots, \mathrm{v}_{\mathrm{N}}\right\}$
Where the superscript T denotes transposition and $\mathrm{N}=$ (total number of nodes) $\times$ (degrees of freedom per node). $\mathbf{V}$ is velocity vector. Velocity $\mathrm{v}_{1}, \mathrm{v}_{2}, \ldots, \mathrm{v}_{\mathrm{N}}$ are nodal point velocities.

A set of algebraic equations (stiffness equations) are obtained as
$\frac{\partial \pi}{\partial v_{l}}=\sum_{j}\left(\frac{\partial \pi}{\partial v_{l}}\right)_{(j)}=0$

Where (j) indicates the quantity at the jth element. The capital-letter suffix signifies that it refers to the nodal point number.

In metal forming, the stiffness equation (5.2) is nonlinear and the solution is obtained iteratively by using the Newton-Raphson method.
$\left[\frac{\partial \pi}{\partial v_{I}}\right]_{V=V_{0}}+\left[\frac{\partial^{2} \pi}{\partial v_{I}}\right]_{V=V_{0}} \Delta v_{J}=0$

Where $\Delta v_{J}$ is the first order correction of the velocity $V_{0}$. Equation (5.3) written in the form
$K \Delta v=F$

Where $\mathbf{K}$ is called the stiffness matrix and $\mathbf{F}$ is the residual of the nodal point force vector.

Solving the above equation according to the known boundary conditions, one can get the required information about the forces and then strains, deformation and so on. The deformation equations can use one of four approaches: (a) direct approach; (b) variation method; (c) method of weighted residuals; and (d) energy balance approach.

The main advantages of the Finite Element Method are: The capability of obtaining detailed solutions of the mechanics in a deforming body, namely, to determine velocities, shapes, strains stresses, temperatures, or contact pressure distributions. A computer code can be used for a large variety of problems by simply changing the input data. However, a large number of nodes are needed to model plastic flow in addition to a very large number of iterations needed to achieve a solution. Thus, significant computing capacity is required. Comparison is both iterative and slow. Most published Finite Element work on metal rolling still uses some approximation in order to reduce the problem size and hence reduce computation requirements.

FEM techniques will undoubtedly become more popular as the price of computing comes down and speed and memory increases. In the past few years FEM has been extensively used for metal rolling analysis and now it is probably the most common technique employed to investigate rolling problems (Edwards 1990, Hartley 1989 and Kobayashi 1985).

### 5.3 FE Software

In recent years, there has been substantial academic and commercial interest in making Finite Element analysis more accessible to non-specialist users.

### 5.3.1 General Software Packages Analysis

Finite Element (FE) software can meet most of the needs in industry. In application Finite Element programs use of the following processes:

## 1.Pre-processors

All of the tasks that take place before the numerical solution process are called preprocessing. The pre-processor software usually assists the analyst in carrying out the following operations: (a) Definition of geometry in computation form; (b) Definition of a mesh of nodes and elements to represent the geometry; (c) Definition of appropriate
section of boundaries of the geometry, in terms of the mesh data, at which boundary conditions will be applied; (d) Definition of the boundary conditions; (e) Definition of material and physical properties for groups of elements; (f) Application of control parameters for the solver.

The pre-processing programs tend to have a user-friendly interface. It allows various parameters to be set and resulting changes to be seen quickly. This is of particular importance when the geometry of the design is being created and when the mesh is being built. However, this is not an easy task and several approaches have been proposed.
2. Solvers

Usually this program both sets up the required numerical equations that describe the behaviour of a structure under a given set of boundary conditions and also solves the equations. The solver reads all the relevant data that has been defined by the preprocessor, usually held in files written by the pre-processor, then carries out the necessary numerical operations and writes the results to further files.

A further function of the solver is data checking. The solver checks to see if the data that is read is acceptable before attempting to produce a solution.

## 3. Post-processors

In terms of post-processing, efforts have been focused on establishing graphical facilities for displaying the results coming out of the numerical simulation (Mackerle 1983). As the solver generates large amounts of information, graphical interpretation is often the only means of assessing the results. Hence, the post-processor is devoted to the display of the results, giving a picture of the results and making extensive use of colour

With the proliferation of computers and software, many people have been used to buying a package and getting results. Unfortunately, any software that solves partial differential equations will not, by its very nature, be as mature as other simpler engineering analysis tools that are also on the market (Mottram 1996).

### 5.3.2 The ABAQUS Finite Element Solver

ABAQUS has been used to simulate hot rolling in a wide rang applications (Datta 1996, Mirabile et al. 1991 and Anza 1997). These bring out the manner in which the relevant physical quantities are calculated by ABAQS (Hibbitt 1996). ABAQUS generally uses Newton's Method as a numerical technique for solving non-linear equilibrium equations. It is available in both explicit and implicit formulations. The potential of attaching user defined subroutines makes ABAQUS a powerful tool to model problem-specific areas such as contact friction and interface heat transfer, which are vital in modelling the evolution of microstructure. However, the availability of user-defined subroutines is limited for the explicit formulation.

### 5.4 FE Simulation of Flat Rolling

To investigate the hot rolling problem, Finite Elements can be divided into three categories, according to different basic assumptions about the behaviour of the workpiece material. These are visco-plastic, rigid-plastic and elastic-plastic methods (Wen 1994). They usually focus on: (a) deformation (stress, strain, strain rate, force, torque and friction). (b) Thermal-mechanical, microstructure, hear-transfer and temperature. (c) Optimum process (pass design). There have been many FE studies of flat rolling, only a selected few are quoted here.

### 5.4.1 2D Rolling

The 2D (Plane strain) FEM simulation, under the plane deformation hypothesis, allows the behaviour of material deforming plastically under the roll bite.

Comparison between computed results and experimental values for roll separation force and torque in rolling were made by Kobayashi (1983). The results are shown in Figure 5-2. As shown, the predictions are lower than the experimental values.


Figure 5-2. Comparison of measured and computed roll separating force and roll torque (Kobayashi 1983).

Zienkiewicz et al. (1978) have presented a FE for strip rolling using a visco-plastic material model. Dvorkin et al. (1997) have presented a 2D FEM of the flat rolling process, using model results, and the correction of a slipping situation in a roughing stand. The correction for the downwards bending of the rolled plate in a roughing stand has also been demonstrated.

### 5.4.2 3D Rolling

Rolling force and torque can be estimated by 2D simulation. However, 3D simulation can estimate technological requirements such as product flatness, tail profile control, and lateral deformation efficiency.

Anza (1997) has presented the results obtained in a 3D simulation of a slab. These results are shown in Table 5 -1which gives the average, maximum, and minimum dimensions of the rolled slab. The final volume has been estimated from the average values and an acceptable variation of 0.13 per cent has been found.

Table 5-1. 3D rolling: dimensions of the rolled slab (Anza 1997)

|  | Initial <br> $(\mathrm{mm})$ | Maximum | Final (mm) <br> Minimum | Average |
| :--- | :--- | :--- | :--- | :--- |
| Length | 300 | 343.55 | 332.12 | 339.86 |
| Thickness | 50 | 43.89 | 43.36 | 43.57 |
| Width | 350 | 362.43 | 351.41 | 354.98 |
| Volume | $5,250.00$ | $5,456.80$ | $5,061.20$ | $5,256.87$ |

In Figure 5-3 the deformed mesh at the end of the process is shown. The distribution of the effective stress was obtained by the ABAQUS or other software.


Figure 5-3. The deformed mesh at the end of the process (Anza 1997)

Figure 5-4 shows the evolution of the total rolling force with time and also presents a comparison between the FE software and experimental results. The average force value in the steady rolling zone can be obtained by ABAQUS, which involves a 7.2 per cent error, compared to the experimental value.


Figure 5-4. Evolution of total rolling force along time compared with ABAQUS (Anza 1997)

Mori and Osakada $(1982,1984)$ have used a rigid-plastic method to analyse plate rolling and edge rolling in 3D. A simulation of metal flow in slab rolling has been presented by Liu et al. (1987) using an elastic-plastic FE method to simulate flow as the workpiece enters and passes through the roll gap. The spread under different geometry conditions was compared with experimental work, see Figure 5-5, and good agreement between the FE method and experimental work was achieved.


Figure 5-5. Comparison of finite element predicted spread with experimental and empirical results (Liu 1987)

### 5.5 3D FE Analysis of Section Rolling

For a successful rolling process of desired shapes, it is necessary to estimate the following parameters (Altan 1983), (a) the roll separation force and roll torque, (b) the spread and elongation, (c) the appropriate geometry of the roll grooves.

Metal flow in rolling is a 3D deformation except for strip rolling which is a 2 D deformation (Kobayashi 1983). The application of 3D deformation analysis for section rolling has been limited because of the difficulty of developing theoretical analysis for three-dimensional plastic deformation and the associated long computational times. 3D deformation analysis can be divided into two categories. The first is a 3D-FE full model and the second is a simplified FE model. Both models are described below.

### 5.5.1 Full 3D FE Modelling of Section Rolling

Bertrand et al. (1986) used a rigid-viscoplastic model for the calculation of stress in a hot shape rolling process from round to oval. This is believed to be one of the earliest appearances of FE analysis for section rolling. Shivpuri et al. (1991) presented a methodology for the optimisation of rolling passes of an existing seven pass square-toround sequence using 3D FEM. Mpri and Osakada (1989) used a rigid-plastic FEM to study 3D steady-state deformation in section rolling and two types of section rolling processes were modeled: the rolling of bars and the rolling of beams. Hartley et al. (1990) and Grober et al. (1989) have presented a 3D FE analysis for rail rolling. However, the fully 3D FEM approach is highly intensive in computational resources and has complex geometric boundary conditions.

### 5.5.2 Pseudo 3D Approach (Finite-Slab Element Method)

To overcome the difficulties of the full 3D FE method, Kiuchi and Yanagimoto (1987) developed a 3D and slab combination method to simulate shape rolling processes. Kim and Altan (1990) have developed a new simplified 3D numerical approach by modifying Kiuchi's method. The 2D rigid-plastic FEM, used for generalised plane strain condition, combined with the slab method, results in the reduction of the amount of required computation without losing much of the accuracy, compared to the complete 3D computer simulation of shape rolling processes. Recently, Shin (1990) has analysed the roll pass schedule and optimised the rolling of square to round section using a computer program called " TASKS ", which is based on the a pseudo 3D approach by Kim and Altan (1990).

Wen (1994) has employed a finite-slab method for modeling the complex 3D deformation in section rolling. This simplified FE technique has been developed by considering a single slice of the material of a finite thickness passing through the roll gap. Only one layer of a finite element mesh is generated on a slice of the material taken from the 3D workpiece. The method is applied to perform a pre- and post-rolling FE analysis for the rolling process of a special section. The deformed meshes obtained at the end of the FE analysis, for each pass, are presented in Figure 5-6. Wen (1994) concluded that a correct profile of the workpiece is required to achieve consistency and to guarantee the quality of the finished product.


Figure 5-6. Deformed meshes and filling of the roll passes obtained from the FE analysis for the first six rolling passes of the mast section (Wen 1994)

### 5.6 FEM for I-beam

Jin et al. (1998) has presented a 3D analysis of the rolling of a the universal beam. The deformation analysis was carried out based on the upper bound theorem and a 3D continuous velocity field. The major factors that are taken into account are: (1) deformation of the web; (2) uneven distribution of the flow; (3) velocity in the lateral direction in both the flange and web deformation zones; (4) the special deformation mechanism of a wide flange beam in a universal pass; (5) flow between flange and web parts of the section; (6) and the roll separation forces.

Using a simplified 3D numerical approach, Shin et al. (1994) presented a study in which the rolling of an I-section beam in the first four passes is analysed numerically and experimentally. The effective strain rate distributions were presented for the right half of the billet as shown in Figure 5-7. Strain rates are concentrated at the center of the contact region.


Figure 5-7. Distributions of effective strain rates (Shin et al 1994)

FEM has also been used by Mirabile et al. (1997) to study plastic strain distribution and temperature distribution within an I-Beam. Figure 5-8 shows the plastic strain distribution in an I-Beam after 170 seconds.


Figure 5-8. Plastic strain distribution in I-Beam after 170 seconds steps (Mirabile et al. 1997)

### 5.7 Summaries and Conclusions

In this chapter, the principle of the FE method and its application to hot rolling has been discussed. A number of attempts have been made in order to improve hot rolling process design. Finite Element techniques are the most promising solution for the 3D analysis of section rolling. However, FE modeling of rolling is a relatively new subject and has many difficulties, such as the great size of the numerical model and the nonlinear behavior of material deforming plastically under the roll bite. Accurate solution
requires a fine mesh that includes many equations. Applications of FE have been quite restricted and many basic characteristics still need to be addressed.

Most published Finite Element (FE) work on rolling still uses some approximation to reduce the problem size. The results show that FE is especially suitable for section rolling.

## Chapter 6

## Matrix Methodology

### 6.1 Introduction

With the rapid advancement in production techniques, manufacturing processes are becoming more complex and sophisticated. The general manufacturing environment is also changing because of tough international competition. In order to be a world-class competitor, a company must bring high quality products quickly to market (Schonberger 1986). As a result, academics have been trying hard to find ways to achieve lower cost and higher productivity and quality. Practical experience has shown that success depends on the manner of using a technique as much as on the technique itself.

The matrix methodology discussed here is an approach which was originally used in design for assembly and manufacture. It is based on the economic leverage of addressing all aspects of the design of a product, as early as possible in the design process, from a systematic view. It has been successfully used in several manufacturing design areas. In this chapter, the matrix methodology is extended to be used in the design of rolling mill rolls.

### 6.2 Concurrent Engineering

Manufacturing is that organized activity devoted for the transformation of raw materials into marketable goods. It is one of the most basic and important functions of human activity in modern industrial societies. Manufacturing industry is classified as a secondary industry (Wu 1992), while primary industry deal with the industry of raw materials; mining, agriculture etc.

Belson (1994) has defined a new systematic approach that is called Concurrent Engineering (CE). CE is the development of products by integrating the design process with other tasks such as planning for manufacturing, quality, and marketing. Its primary feature is to use a team approach to the creation or improvement of new products. In the
past, each function developed its own terminology. With creation of the Concurrent Engineering concept, isolated design attitudes are broken down and replaced by designers working as a team for a common goal. In a defense analysis report, Winner et al. (1988) defined Concurrent Engineering (CE) as a systematic approach to the integrated, concurrent design of products and their related processes, including manufacture and support.

CE presents a number of opportunities for scientific research. For example, the current literature does not yet provide detail on how to integrate concurrent design of products and their related manufacturing processes. The matrix methodology has provided an approach that applies some of the features of CE, such as team working and flexible design. Efficiency and flexibility are important aims in the design of products in manufacturing industry. Wu (1992) pointed out that a combination of technology and methodology leads to a higher chance of success in modern economic conditions.

### 6.3 The Matrix Methodology for Assembly

The following sections introduce a matrix approach used in assembly in order to reduce parts and obtained lower cost.

### 6.3.1 Design for Assembly

Ensuring maximum appropriate output at lowest cost is always the aim of any manufacturing strategy. The Durham Matrix-based Methodology (Appleton and Garside 2000) introduced for the first time the matrix methodology for the design of assemblies. It addressed the application of design technique for reducing the cost of manufacture by reducing parts count and assembly time.

Compared with other design methodologies, the matrix methodology is a systematic approach for design and manufacture. It emphasizes team working and allows transfer of skills and knowledge from one designer to another. The topics of the analysis in the Matrix-based design approach are very wide, including production, planning and prototyping.

The analysis and redesign takes the form of a cascading matrix, arranging the current manufacturing process sequence in a row that represents the steps and the order of the manufacturing process. The columns include ideas, design issues and points of concern for design, manufacture and assembly.

Designer can identity changes to features that may improve the manufacturability of the product, and seek parts that can be eliminated in order to reduce part count. The designer starts by identifying essential, unnecessary or " not sure " parts. Figure 6-1 shows the analysis matrix for assembly.


Figure 6-1. The analysis matrix for an assembly (Appleton and Garside 2000a)

### 6.3.2 The Trial of Matrix-based Design

The first trial of this methods was carried out at a company manufacturing automotive locks where it was found that a considerable reduction in part count could be achieved quite quickly and easily (Appleton and Garside 2000b). In this case, a team was trained, and a product was redesigned using the Matrix-based method. The method was then tried on a wide range of products in different companies.

The first case was a transformer manufacturer in Bishop Auckland. Its main business was in the industrial sector, manufacturing power supplies. The Matrix-based redesign process reduced the part count of the transformer from 49 to 9 parts. The results are summarized in Figure 6-2.

|  | Value |
| :--- | :---: |
| Initial part count | 49 |
| Final part count | 9 |
| Initial operation count | 25 |
| Final part count | 15 |
| Estimated time reduction | $80 \%$ |

Figure 6-2. Summary of results from industrial power source design (Appleton and Garside 2000b).

The second case study was a complicated product assembly. It was a dehumidifier made by a small company in Bowburn, County Durham, England. The Matrix-based redesign achieved a 65 percent reduction in both part count and assembly time. The results are summarized in Figure 6-3.

| Initial assembly time | 13.7 minutes |
| :--- | :--- |
| Final assembly time | 4.47 minutes |
| Reduction in time | 67.4 percent |
| Initial part count | 64 |
| Initial part cost | 91.04 |
| Final part count | 23 |
| Final part cost | 81.29 |
| Part cost saving | 9.85 |
| Part count reduction | $65 \%$ percent |

Figure 6-3. Redesign results from an Aridar filter (Appleton and Garside 2000b)

Subsequently the Matrix-based method has been used in a wide range of companies and manufacture. More details can be found in Appleton and Garside (2000a). Obviously, In addition to the direct savings in assembly operation costs, several teams have found additional benefits, for example improved product quality and better interaction between design and manufacturing functions.

### 6.4 Application of a Matrix-based Approach to Hot Rolling Design

The Durham Matrix-Based Methodology has been successfully used in several areas, such as design for manufacture (DFM), maintenance planning and manufacturing strategy. Appleton and Summad (1999) suggested, for the first time, using the method for hot rolling roll design. The Durham Matrix-Based Methodology was proposed for the optimization of shape rolling technology. Appleton and Summad (1999, 2000) emphasized the importance of a systematic approach applied to such a complex design process. They reviewed the characteristics of section rolling and information relating to design parameters.

Although roll pass design for section rolling is usually based on the empirical knowledge of human experts, most of the empirical formulae only give good results within a limited range of applications. Section rolling has always represented the most complicated rolling process for optimization. Searching the recent literature, it is still difficult to decide on the optimal solution for a given roll pass design problem. Part of the problem is that there is no predetermined specific rules for roll pass design, so that when a problem occurs it is difficult to know the exact cause. Also, the problem gets more complicated when there are hierarchical competing objectives. Therefore, progress in the current and familiar research directions may only result in limited and restricted types of solutions (Appleton and Summad 1999).

Based on both existing shape rolling design and matrix methodology, Appleton and Summad (1999) speculated on the possibility of developing a system for assisting less experienced designers in the rolling industry. The method allows a team of designers to co-operate in the design decision making process.

A methodology was introduced by Appleton and Summad (Appleton and Summad 2000) based on a matrix design in which the row contains the description of the roll pass such as: Geometry, Process and Infrastructure parameters. The column lists the issues and constraints in a cascading order of importance and priority such as: Draught, Elongation, Spread, Force, etc. This process is shown in Figure 6-4. In the above paper, the possible direction and general idea of Matrix-based rolling mill design was discussed. However, detailed design work was not discussed or carried out. Therefore, further work is needed to realize the idea in a more practical system There is also need
to note that consideration of roll design in a real system requires, modification of the previous work which is necessary because of the complexity of the section rolling system.


Figure 6-4. A scheme of the Durham Matrix-Based Methodology(c) (Appleton and Summad 2000).

### 6.5 Evolution of the Matrix-based Rolling Mill Roll Design

Because of the characteristic of rolling, the initial matrix methodology applied to assembly has been modified and extended with further features.

### 6.5.1 Hierarchical Classification System

The optimization criteria for hot rolling mill roll design may vary depending upon the product requirements. However, establishing appropriate criteria requires a thorough understanding of the hot rolling process. Without the relevant knowledge of the influences of variables, such as load, stress, friction conditions, and workpiece geometry, it would not be possible to predict deformation of the workpiece and the required design of the rolling mill rolls. Figure 6-5 indicates the general process of the hot rolling design mill roll. It is obvious that this hot rolling design is a complicated process.


Figure 6-5 Schematic of hot rolling system

Figure 6-4 only gives a brief suggestion for a Matrix for rolling mill roll design. However, design detail is not discussed in this paper (Appleton and Summad 2000), therefore, more detailed work and further development is necessary.

Hot rolling mill roll design is a very complicated process. There are many parameters that affect the process as shown in Figure 6-5. According to the nature of the rolling process, designs can be divided into flats and sections. Sections can be further divided into Beams and Channel and so on. The design formulae can also be divided into theoretical methods and empirical methods. In order to make the Matrix-based design system clear and well organized, it is necessary to find a way to classify and organize the hot rolling design system. A useful approach is the Hierarchical Classification System which was discussed by Edwards (1990) for the application to material section shape. The Hierarchical Classification System is a tree classification system. The tree system consists of many classes and each class can be further divided into sub-classes. For convenience, the classes above a particular class of interest are called its superclasses and anything below it is called its sub-classes. This means that classes consist of all its sub-classes and the super-class belongs to its super-classes. There are some classes that might have the same level on the tree system. In the tree system, in fact,
each class is a branch. One key characteristic of the hierarchy is that each branch of the tree inherits the attributes and properties of the branch from which it grows.

For each class, the user can go into the rolling design issues for that class, such as force, torque, elongation. These design issues are very important in roll design. As a matter of fact the design involves the process of obtaining these values for each pass

The simplest description of a hierarchy classification system used in rolling design is shown in Figure 6-6. The root of the tree "Metal Forming " has the attributes of being able to carry things and move down to Hot Rolling. The class of Hot Rolling originally comes from Metal Forming. "Hot Rolling " is the sub-class of the Metal Forming class. Hot Rolling can be further divided into its sub-classes of section Rolling and Flat Rolling. As will be discussed in the next chapter, the section rolling can be further divided as Beam and Angle Rolling and so on. The Section Rolling and Flat Rolling are on the same class level and they are not directly connected to each other. This means that Section Rolling and Flat Rolling are different design problems and different design methods are needed to solve these problems.


Figure 6-6. A hierarchy structure of hot rolling design


#### Abstract

A successful classification system is useful in the selection and identification of design problems and it must be precise enough to be manageable within the design matrix. In Matrix-based design the sequence of matrix menus needs to be assisted by the hierarchy classification system. It is important to recognize that hot rolling is a complex manufacturing process and that a number of factors influence it. Different issues and parameters are involved in each different sub-class in the Hierarchical Classification System. Figure 6-7 shows a cascading menu system. In the matrix, each menu may contain submenus. The design information of the Hierarchical Classification System can be categorized and realized using the cascading menu system.




Figure 6-7 The Hierarchical Classification System realized by a cascading menu system for shape rolling design (Appleton, Jing and Summad 2000).

### 6.5.2 The Structure of the Matrix

For the matrix, the input of pass parameters, process parameters and infrastructure are normally arranged in the horizontal direction, i.e. in the first row. For convenience, all of these are called rolling process elements. The design issues of hot rolling are arranged in the vertical direction, i.e. in the first column. The design issues normally include the important design elements such as rolling force, torque, elongation and so on. These design issues are needed to be calculated at each rolling pass. In some instances, the relationship between the design issues and the process parameter is mathematical. In other cases design issues and rolling process elements have no clear mathematic relationship, but have undefined logical relationships, which are recognized because these rolling process elements affect design issue values.

Figure 6-8 is a schematic of the Matrix-based rolling mill roll design approach. If a design issue in the first column is chosen (by clicking on the design element with the left key of the mouse), the available formulae will appear. The formulae will also list their suitable application cases. The team of designers can discuss which formula is suitable for the specific design. In this table, the cell which has a mathematical
relationship appears as " " and will have a value only after the design decision has been made. The cell which has a logical relationship will always appear as "*". However, when this cell is chosen (by clicking on the cell with the left key of the mouse), a general rule provides a brief discussion about how this rolling process element affects the design issue. As discussed previously, the matrix is assisted by the Hierarchical Classification System, which is realized by a series of cascading menus (see Figure 67). Note that there are empty rows for the rolling process elements and empty columns for the design element. The empty row/column represent the possibility of further process element and design elements. Matrix-based roll design is an open system which allows more elements to be added if the team of designers need to extend the Matrixbased system.

| $5$ | Pass Parameter | Process <br> Parameters | Infra-Structure | $\cdots$ |
| :---: | :---: | :---: | :---: | :---: |
| Design issue 1 | * | - | * |  |
| Design issue 2 | * | * | * |  |
| $\cdots$ |  |  |  |  |
| - Mathematical relationship <br> * Logical relationship |  |  |  |  |

Figure 6-8. Schematic of a Matrix-based rolling design

In hot rolling, the process design would involve empirical methods, mathematical formulae and computer based simulation analysis. Even if the designer considers only one design issue in one rolling pass, this design issue may interact with, and involve the calculation of other design issues. This is true for both flat and section rolling design. In this case, efficient management of the design issues is important.

Design issues which are similar in nature in terms of calculation are organized into common groups. The position of each group is not fixed in the matrix and can be changed according to importance and priority. Priority could be different in a different matrix or for different design levels. An example is that priority may be different for a beam, angle or channel in section design. Figure $6-9$ is a schematic of a Matrix with grouped issues. An example of group issues could be Roll Force, Roll Torque, Spread and Draft, which are related under a " deformation" group heading.

| Process Design | Pass <br> Parameters | Infra-Structure | $\cdots$ |  |
| :--- | :---: | :---: | :---: | :--- |
| Group Issues 1 | $*$ | $*$ | $*$ |  |
| Group Issues 2 | $*$ | $*$ | $*$ |  |
| $\cdots$ |  |  |  |  |

Figure 6-9 Matrix-based rolling design with grouping issues

For the design of a new production or redesign of existing products, the team of designers first discuss the rolling process elements along the first row. After rolling process elements have been decided, the team discusses the design issues in a predetermined order or in any order that they prefer.

The design team selects the suitable formulae or empirical data using the menu system described earlier. The mark " $\downarrow$ " appears in the matrix if the relationships between the design elements and the rolling process elements are mathematical. For logical relationships, designers can get extra information about how these rolling process elements affect the design elements, by using the menu system. Further detail will be given in the next chapter.

### 6.5.3 Working in the Matrix

The team of designers may be composed of designers from various disciplines. Normally the team consists of senior designers, less experienced designers, computation experts and operations experts. Less experienced designers should be well trained in the generic design process. Senior designers will have more personal roll design experiences, however, they may also gain new knowledge and information from working with experts on new techniques such as Finite Element methods. Less experienced designers can also gain empirical experiences by working with senior roll designer. The team can quickly identify the required matrix using the cascading menu system. In each Matrix, the team can discuss the process elements and design issues in a parallel or sequential order. The method of Matrix-based roll design allows the designers to share their experiences more widely, as well as gain valuable input of the
design process. In addition the team shares databases and standards to facilitate communications. Matrix-based roll design is highly flexible, it supports team working and allows the team members to discus all ideas and concerns.

Matrix-based roll design is an open system. The designer is allowed to add new design issues to the matrix and new formulae, experimental data and any novel approach for specific cases. As a consequence, the Matrix-based roll design approach has the potential to become more and more powerful. Theoretically, both the process element and design element can be expanded along the row or column in the matrix, respectively.

### 6.6 Summaries and Conclusions

Hot rolling is a complex process. A successful classification system is useful in the solution of roll design problems and it must be precise enough to be manageable. In this chapter, the Hierarchical Classification System is first used for the classification and selection of design data for section rolling rolls. The Hierarchical Classification System is a tree-type system and the hot rolling design problems are classified and each class can be further divided into as sub-classes. It is also demonstrated that the selection of design problems in the Hierarchical Classification System could be used in a cascading menu system.

The matrix methodology was originally applied to design for mechanical assembly in the manufacture industry. Its application in the assembly design process has been demonstrated. This technique is now proposed for use for roll design. A Matrix-based rolling design system is introduced in this chapter. The team of designers normally consists of senior designers, less experienced designers and technical specialists, such as FEM experts. This ensures that everyone can make a concurrent contribution to the design process and can also benefit from participation.

Compared with the original use of the methodology, Matrix-based roll design maintains the advantages of the initial matrix methodology and complements it with new features. It is a systematic approach to integrate teamwork into roll design and is an open system
and therefore is more flexible and efficient for this more complex and technical design process.

## Chapter 7

## Matrix-based Hot Rolling Roll Design

### 7.1 Introduction

In recent years, the art of roll pass design has turned into a modern technology based on the progress of science. Preferably, pass design not only aims to improve the size, shape and mechanical properties of the rolled products but also it seeks to optimise the roll pass process itself. This work is also made more relevant by the highly increased demand for shape variety in rolled products in all branches of industry. Also, the design of rolls for hot rolled products by a single person is no longer practical.

It is now more desirable than at any other time in the rolling industry, to develop a system for assisting less experienced designers by allowing inexperienced roll pass designers to work closely with those with more experience (Appleton 2001).

This chapter introduces a Matrix-based roll design approach for flat and section roll design. Flat rolling is relatively simple. However, section roll design is complex but it is usually carried out on the basis of flat roll design or empirical knowledge. Therefore, the understanding of flat roll design is also very important for section roll design.

### 7.2 Matrix-based Hot Roll Design

Application of the matrix in hot roll design will be given in the following sections.

### 7.2.1 Setting up for Hot Roll Design

Similar to Figure 6-8 in Chapter 6, a typical Matrix-based roll design system is schematically shown in the table in Figure 7-1. The Matrix-based roll design system consists of rows and columns. Information can be obtained by clicking on the relevant button in the first row or the first column. For instance, the first row contains " Process

Design " which can be used to calculate the pass number. It means the pass number is calculated by clicking on the " Process Design " button. The first column contains design issues, such as force, torque, elongation, spread and draft which are important for hot roll pass design. In the matrix, there are some cells which are marked with " " and "*" (see Figure 7-1) for mathematical relationship and logical relationship, respectively. Hence, the designer decides the " Process Design ", " Pass Parameter " and " Infrastructure " in the first row, and then each design issue is calculated from the left to the right side in each column (see Figure 7-1).


- Mathematical relationship * Logical relationship

Figure 7-1. Schematic of a Matrix-based rolling design system

Hot roll design is influenced by a number of dependent and independent factors. They are sometimes non-controllable and difficult to identify. These factors influence the roll designer as the designer seeks to fulfil the purpose of the roll pass design. This allows everyone in the team to contribute to the design system and the system can become more and more sophisticated and as powerful as required. Matrix-based roll design emphasizes the value of teamwork. All members of the team work together closely and therefore may improve the design efficiency.

### 7.2.2 Classification of Components for Hot Rolling

Because of the complexity of hot roll design, a hierarchical classification system is used to classify the design process, as discussed in chapter 6. The hot rolling system is influenced by many design parameters, rolling conditions and applications, and it is important to make the shape description as close as possible to the product description
and accurate to the actual rolling process. Figure $7-2$ shows the hierarchical configuration of hot rolled products.


Figure 7-2. The hierarchical configure of hot rolled products

In Figure 7-2, the root " Hot Rolled Products " consists of sub-classes " Flat " and " Section ". The sub-classes can be further divide into their sub-classes. "Plates" and " Sheets " which are sub-classes of flat. " Section" has several sub-classes. Designers can move from super-classes to sub-classes depending upon their requirement.

In the converse, the design approach allows the designer to come back from each subclass to its super-class as soon as the designer has finished the work in the sub-class. In this way the designer can decide and choose other new design parameters quickly and efficiently.

At the present time, it is impossible to be comprehensive in terms of describing all of the work. In Figure 7-2, the solid boxes represent the work that is discussed in this chapter. More details about the configurations of flat and beam sections will be discussed in later subsections, but other section shapes are not discussed in this thesis.

The advantages of the Matrix-based roll design approach is the fact that it always begins from a systematic point of view and therefore the designer always has a systematic concept about what is the main design aim, no matter how complex the design detail procedure.

### 7.3 Design for Flat Rolling

The hierarchical configuration of flat roll design issues is shown in Figure 7-3. The main concerns in flat roll design are the "Deformation parameters" which are further divided into the issues of Force, Torque, Elongation, Draft and Spread.


Figure 7-3. The hierarchical configuration of flat rolling design issues

Matrix-based hot roll design for flat products is shown in the table in Figure 7-4. The matrix consists of several columns and rows. Information can be obtained from the buttons in the first row and first column, respectively. The first row includes " Process Design " "Pass Parameters" and " Infra-structure ". The first column contains the design issues that can be chosen to calculate and if an issue is chosen, a window will open which contains the relevant formulae or methods. The rest of the cells in the matrix are marked with " " or " *" and will be replaced by a value after the issue in the relevant pass is calculated or decided.

| $\overbrace{\text { Flat }}$ | Pass Parameter | Process <br> Parameters | Infra-Structure | $\cdots$ |
| :---: | :---: | :---: | :---: | :---: |
| Force | * | - | * |  |
| Torque | * | - | * |  |
| Elongation | * | - | * |  |
| Draft | * | - | * |  |
| Spread | * | - | * |  |
| ... |  |  |  |  |
| - Mathematical relationship <br> * Logical relationship |  |  |  |  |

### 7.3.1 The Function of the Row in Matrix-based Roll Design

The Designers start by working on the first row. In flat rolling, process design is complicated and depends upon many factors: Bite angle, Roll peripheral speed, Mean strain rate, Temperature, Material, etc. To decide these factors, there is the need for theoretical solutions, empirical data or numerical model (FEM) assistance. If " Process Design " is discussed, designers check and modify the above process parameters one by one. For example, the pass number can be calculated by using a formula and may also be decided by the senior engineer if experience is deemed to be more reliable than the calculation of the value. After the pass number is decided, the matrix could be split into several pages i.e. one page for each pass. This thesis uses a single pass to illustrate the approach. The paper by Appleton and Summad (2000) gives an indication of how each pass matrix interacts with the processes and following or preceding matrix pages.

In the Second phase, Designers discuss " Pass Parameters ". The columns under " Pass Parameters " contain workpiece Area, Length, Thickness, Width, Relative reduction, Groove size, etc. When the designers discuss a new a rolled products or redesign existing rolled products, the dimensions of the workpiece are based upon the customer's requirement. They can make the necessary geometrical changes compared with a successful existing, similar product.

The " Infrastructure" contains general information about the rolling mills and working environment conditions, which may affect hot roll design. In flat roll design, roll mill load capacity or roll drive power might be checked and decided.

To calculate design issues, most of the above parameters will be needed. It is know that the parameters of hot rolling are highly interactive and interdependent.

### 7.3.2 The Function of the Column in Matrix-based Hot Roll Design

After the pass number has been decided, the design issues can be calculated. The first column of the matrix includes Force, Torque and so on. In this stage, the team starts discussing which design issue is required to be calculated. In many cases, there will be a
number of different theoretical solutions and a list of equations could be presented for the designer to make a choice. As an example, if the design issue " Force " is chosen, a window is opened which contains several existing equations for the calculation of the force. A possible window is shown in the table in Figure 7-5. Note the window can be expanded and an insertion of further calculation methods could be accepted including new design methods or empirical data for specific applications.

| General Equation for roll Separation <br> Force, <br> Ginzburs (1989) | $P=K_{w} W_{d}$ |
| :--- | :--- | | Sims (1954) | $P=W_{d} Q_{p} S_{p}$ |
| :--- | :--- |
| Cook-McCRUM (1958) | $P=R W C_{p} C_{p}$ |

Figure 7-5. Some formulas for the calculation of force

In Figure 7-5, each equation is related to the suitable conditions for its application. Further conditions for each application and equation are also listed in specific windows. Therefore, the designer will be able to choose the best formula that is most suitable for the design and application in hand and thus an optimised design can be achieved. By clicking on the Sims' equation button, the equation window will give a further window which gives more details about this formula (See Figure 7-6).

Figure 7-6. Sims' equation

Roll separating Force, Sims' Method (1954)
1.The conditions of the application

Slab analysis of rolling deformation; Sticking friction exists in roll contact zone; Homogenous deformation
2. Calculation mean of workpiece width W

Width $W_{1} \quad$ 1) $W=W_{a}=W_{0}+W_{1}$ Assuming the curve of spread is a straight line

$$
\begin{aligned}
& W_{2} \text { 2) } W=W_{p}=\frac{W_{0}+2 W_{1}}{3} \\
& \text { 3) } W=W_{g}=\sqrt{W_{0} W_{1}}
\end{aligned}
$$

Draft $\Delta h=h_{0}-h_{1} \quad \mathrm{R}=$ roll radius
$I_{d}=\sqrt{R \Delta h-\frac{\Delta h^{2}}{4}}$ hot rolling flattening is negligible $I_{d}=\sqrt{R \Delta h}$

$$
\text { If the rolls are of unequal diameters } R_{m}=\frac{2 R_{1} R_{2}}{R_{1}+R_{2}}
$$

3. Geometrical factor $Q_{p}$

$$
Q_{p}=\left[\frac{\pi}{2 a} \tan ^{-1} a-\frac{\pi}{4}-\frac{1}{a} \sqrt{\frac{R^{\prime}}{h_{1}}}\left(\ln \frac{h_{n}}{h_{1}}+\frac{1}{2} \ln \frac{a^{2}}{r}\right)\right]
$$

Where Relative reduction $r=\frac{\Delta h}{h_{0}}=1-\frac{h_{1}}{h_{0}}$

$$
a=r(1-r)
$$

$h_{n}=$ strip thickness at neutral point

$$
R^{\prime}=\text { flattened roll radius. When hot roll } R^{\prime}=\mathrm{R}
$$

Sample values for geometrical factor $Q_{p}$ are shown in Figure 3-6. In this figure the application range for the parameters are given by
$\frac{R}{h_{1}}=5-300 \quad \mathrm{r}=0-0.6 \quad Q_{p}=1-5$
4. Mean constrained yield stress $S_{p}$

The mean constrained yield stresses obtained from the equation

$$
S_{p}=\frac{1}{\alpha} \int_{0}^{\alpha} S d \theta
$$

Where bite angle $\alpha=\arccos [1-\Delta h /(2 R)]$
$S=$ constrained yield stress determined by plane strain compression
Roll contact angle $\theta=\alpha / 2$
The conditions for application are:(1) dry slipping friction. (2) bite angle is small

The values for the mean constrained yield stress $S_{p}$ are shown in Figure 3-7.For this figure the conditions of application were:

Temperature: $1000^{\circ} \mathrm{C} \quad$ r: $0-0.5 . \quad \lambda=1-100 \mathrm{sec}^{-1} \quad S_{p}: 2-16$ ton $/ \mathrm{in}^{2}$
In the Figure 3-7 $\lambda=$ mean strain rate which can be determined by several methods
Orowan an Pascoe's solution. (1946)

$$
\lambda=\frac{\pi N}{30} \sqrt{\frac{R}{h_{0}}}\left(\frac{1-0.75 r}{1-r}\right) \sqrt{r}
$$

Sim's solution. (1954)

$$
\lambda=\frac{\pi N}{30} \sqrt{\frac{R}{h_{0}}} \frac{1}{\sqrt{r}} \ln \left(\frac{1}{1-r}\right)
$$

Wusatowskki's solution. (1969)

$$
\begin{aligned}
& \lambda=\frac{\pi N}{30} \sqrt{\frac{R}{h_{0}}} \sqrt{\frac{r}{1-r}} \\
& \text { Where } N=\frac{\pi D_{w} n}{60} \mathrm{~m} / \mathrm{sec}
\end{aligned}
$$

$$
D_{w}=2 \mathrm{R}
$$

Roll separation force

$$
P=W I_{d} Q_{p} S_{p}
$$

Because each equation is only suitable for specific cases of application, each equation must be chosen according to the specific condition of hot rolling. In some cases, it may be necessary to have hot roll theory, and empirical data in combination.

Using the above procedure, the team discusses each design issue one by one. Figure 7-7 and Figure $7-8$ show the corresponding windows for torque and spread.

| General eqution, <br> Ginzburs (1989) | $M=2 p_{a}$ |
| :---: | :---: |
| Sims (1954) | $M=2 R R^{\prime} W Q_{g} S_{g}$ |
| Cook-McCRUM (1958) | $M=2 R R^{\prime} W C_{g} I_{g}$ |
| Ford-Alexander (1963-1964) | $M=0.25 W_{d}{ }^{2}\left(\pi+Z_{a}{ }^{2} / Z_{p}\right) S$ |
| Denton-Crane (1972) | $M=W I_{d}\left(0.795+0.22 Z_{g}\right) S$ |
| $\cdots$ |  |
|  |  |

Figure 7-7. Calculation Torque

| El-Kalay and <br> $\begin{array}{c}\text { Sparling } \\ (1968)\end{array}$ | $S_{w}=a \exp \left[-b\left(W_{0} / h_{0}\right) c\left(h_{0} / R\right) d_{r} e\right]$ |
| :---: | :---: |
| Beese (1980) | $S_{w}=0.6\left(h_{0} / W_{0}\right) \exp \left[0.32\left(h_{0} / L\right)\right]$ |
| Sparling (1961) | $W_{1}{ }^{2}-W_{0}{ }^{2}=1.6 L\left[\Delta-h_{a} \ln \left(W_{1} / W_{0}\right)\right](4 \mu L-3) / h_{a}$ |
| Wusatowski (1969) | $S_{w}=\exp \left[-1.982\left(W_{0} / h_{0}\right)\left(h_{0} / R\right)^{0.56}\right]$ |
| Trinks (1933) | $S_{w}=0.35 c L\left(h_{0}-h_{1}\right) / h_{0}$ |
| Hill (1963) | $S_{W}=0.5 \exp \left(0.707 c W_{0} / L_{d}\right)$ |
| $\ldots$ | $\ldots$ |

Figure 7-8. Calculation Spread

The calculated value for each design issue is displayed in its corresponding position in each pass. As soon as the issue is calculated, the mathematical mark " $\downarrow$ " is replaced by its value. After finishing the calculation, the team can come back to the super-class level in the Matrix-based hot roll design for further work.

### 7.4 Design of Section Rolling

Section rolling represents one of the most difficult processes to model. It has a large number of parameters and the metal deformation is non-uniform (Roberts 1983). Compared with flat rolling there is very little technological understanding or development (Lundberg 1997), although many investigators are working on section rolling and numerous papers have been published. However, most of the work presents the research approaches rather than providing the practical data. Further, the research focuses upon specific cases and is not widely applicable.

Matrix-based hot roll design is an effective design methodology that considers the design of section rolling from a systematic point of view. All the available design techniques for roll design can be added in the Matrix-based approach and the designers
can choose a suitable design approach for a specific section case. The Matrix-based hot section roll design is also an open system, this means that new design methods including empirical design can be added at a later time.

In terms of classification section rolling can be further divided into Regular sections, Channels, Angles, Beams and other section shapes as shown in Figure 7-2. The Hierarchical Classification System can be illustrated by a cascading menu system in Figure 7-9 and the Matrix-based roll design menu system shown in Figure 7-10.


Figure 7-9. The Hierarchical Classification System based upon a cascading menu for section roll design


Figure 7-10, Schematic of Matrix-based section roll design menu system

### 7.4.1 Setting up the Matrix for Section Rolling Roll Design

The Hierarchical Classification of beam design issues is shown in Figure 7-11. It is shown that a number of factors influence the design for section rolling and they are interactive and interdependent. Therefore, when using a Matrix-based approach for section roll design the previous flat product matrix needs to be modified.


Figure 7-11. The Hierarchical Classification of beam design issues

Firstly, section roll design involves huge design issues. Design issues are grouped in the matrix depending upon their nature i.e. Design issues with a similar nature in terms of calculation are organized into the same groups. One group of issues could be Force, Torque, Spread, Elongation and Draft all of which are related under a "Deformation" group heading, as shown in the table in Figure 7-12. This " Deformation" group of design issues may also use FE models, using different models simultaneously.

Secondly, the order or priority of the design issues can be changed in the matrix. Section rolling is a multi-pass process, and obtaining an appropriate elongation is more important than other issues. Thus, the required elongation is calculated before the force and torque.

| Beam | Process <br> Design | Pass/part parameters | Infra-structure |
| :---: | :---: | :---: | :---: |
| Deformation | * | - | * |
| ThermalMechanics | * | - | * |
| $\ldots$ |  |  |  |

Figure 7-12. Matrix-based section roll design beam.

### 7.4.2 The Function of Rows in Matrix-based Section Roll Design

Compared to flat rolling, the rows in Matrix-based section roll design involve more design problems.

### 7.4.2.1 Process Design

In this stage, the "Process Design" is considered firstly by clicking on the "Process Design" button. When "Process Design" is chosen, a menu is opened. It contains beam division, the mean elongation coefficients, pass number and process optimisation. This process is shown in Figure 7-13. Each of the contents in Figure 7-13 can be further chosen for detailed consideration.

| Process Design | Beam division and elongation coefficients <br> Mean elongation coefficient <br> Pass number <br> Process optimisation |
| :--- | :--- |

Figure 7-13. Process Design.

By clicking on the button of "Beam division and elongation coefficients", the designers can get the relevant information in the window as shown in Figure 7-14. Then the designers split the beam into web and flanges based upon different methods and then find the elongation of both the web and flanges from Fig. E. in the window.


Figure 7-14. Beam division and elongation coefficients (Wusatowski 1969)

Selecting the button of "Mean elongation coefficient", the designer can calculate the mean elongation coefficient by using different equations, as shown in Figure 7-15.

| Mean elongation coefficient |
| :---: |
| $\lambda_{m w}=\frac{\sum_{l=1}^{l=N} F_{l 2} \lambda_{l}}{\sum_{l=1}^{l=N} F_{l 2}}$ |
| $\lambda_{m w}=\frac{F_{2}}{F_{1}}=\frac{F_{A 1}+F_{B 1}+\ldots+F_{N 1}}{F_{A 2}+F_{B 2}+\ldots+F_{N 1}}$ |

Figure 7-15. Calculation of mean elongation coefficient (Wusatowski 1969)

By selecting the button of "Pass number", the window is opened in Figure 7-16 and the pass number can be calculated. A senior designer may also decide the pass number based upon existing products or their experience.

$$
\begin{aligned}
& \text { Calculation passes number } \\
& n=\frac{\log F_{0}-\log F_{n}}{\log \lambda_{m}}=\frac{\log \lambda_{1}}{\log \lambda_{m}}
\end{aligned}
$$

Figure 7-16. Calculation of pass number (Wusatowski 1969)

Pass number can be reduced by choosing the button of "Process Optimisation" and the window is shown in Figure 7-17. In this way a designer can find information to optimise the rolling schedule.

Process Optimisation

- Ozsoy, Optimum Scheduling Of a Hot Rolling Process By Non-linear Programming, Canadian Matallurgical Quarterly, Vol.31, No.3.pp.217-224 (1992)
- Novel method and publication

Figure 7-17. Processes optimisation

### 7.4.2.2 Pass/Part Parameters

After " Process Design " is discussed and decided, designers begin to discuss " Pass/part parameters ". Usually the total rolling sequence for multi-pass products can be separated into three stages: roughing, intermediate rolling and finishing. The first passes (7-9) are called " roughing ". Some of the considerations in roughing are shown in Figure 7-18. In " Pass/Part Parameters ", every rolling stage could be further broken down into several sheets, each sheet corresponding to one pass design.

## Roughing principles

- The difficulties of roughing are to control the rolling velocity.
- The roll pass design can be chosen quite freely in this stage.
$\bullet$ It is most common to take the $7-9$ passes in a reversing roughing stand.
- The temperature loss of the rolled bar also increases with the number of passes and to save energy the number of passes should be limited.

Figure 7-18. Roughing principles (Lundberg 1997)

Design in the roughing stage of rolling is mostly determined by the choice of billet size and mill layout, including the mechanization system. The billets size can be calculated from the required finished products size, as show in Figure 7-19.

| Billet Size |
| :--- |
| For Tongue and groove rolling: |
| Billet height $=2 \times$ finished flange height + lin. |
| Billet area 1.47 A. B. (Where A, B are width and Length) |
| Billet width depends on the espread, the width<finish web width. |
| Maximum reduction $=22 \%-25 \%$ |
| For Diagonal rolling |
| Billet<2xininshed flange |
| Billet width>web width. |
| Maximum reduction $\cong 30 \%$ |

Figure 7-19. Billet Size (Sheffield 1960)

Similarly, the designers can also obtain some general information or plant specific information about rolling mills and the working environment by clicking on the " Infrastructure " button in the Matrix.

### 7.4.3 The Function of Columns in Matrix-based Roll Design

The section rolling mill roll analysis design matrix is similar to that for flat rolling. The first column contains the design issues. The design issues are grouped in the column under headings such as " Deformation group ", "Thermal-mechanical group " etc. The column is as shown in the table in Figure 7-12.

In this phase, designers start to discuss each of the group's design issues in the column. By clicking on the " Deformation " button, a menu is opened which shows deformation detail and different methods to calculate values in the group. The methods include: theory, Empirical data, FEM models and other methods. Designers can choose any method as the basis of discussion of any of these issues. The process is shown in Figure 7-20. Note here, Elongation is the priority in this group of issues, as has been identified in the process design phase.


Figure 7-20.Deformation group

In Figure 7-20, if the method " Theory " is chosen, several windows are opened which display how the methods can be used to calculate Elongation, Spread, Draft, Force and Torque. These methods are shown in Figure 7-21 and Figure 7-22. Elongation has been discussed in the "Process Design " stage.

## Spread and draft

- Normally the spread in section rolling is described proportion the of elongation or appropriated results of an depend on the experiment.
- The " mean height " of a section or part of a section is determined by replacing the section with an equivalent rectangle.
- The mean draft coefficient is given by

$$
\gamma_{m}=\frac{h_{1 m}}{h_{0 m}}
$$

Figure 7-21. Spread and draft (Wusatowski 1969, Trinks 1943 and Wen 1994)

> Roll strength
> 1. Force
> In modern high-efficiency roll pass design roll strength:
> $P_{1} \approx P_{2} \approx P_{3} \approx \ldots \approx P_{n} \leq P_{p e r}$
> 2.Torque
> $M_{1} \approx M_{2} \approx M_{3} \approx \ldots \approx M_{n} \leq M_{p e r}$
> The application of roll load formulae to non-flat products requires some artifice to reduce the section to equivalent rectangular stock entering and leaving the rolls. In the case of complex sections, an attempt can still be made to obtain an estimate of the rolling load by splitting the section area into elements. After sections are divided into rectangles, their strength can be calculated using the flat rolling equations.

Figure 7-22. Force and Torque (Wusatowski 1969)

Similarly, if the "Empirical " option is chosen in Figure 7-20, a window is opened which includes some existing methods or relevant information as shown in the example in Figure 7-23.

```
Experimental Method
- Visio plasticity
- Measurement
- Geoffrey, M. HARDY et al. Hot Rolling of I-Section steel Beams Using
Lead as A Model Material, M.Sc dissertation (1975).
```



- Shin et al. A study the rolling of I-section beams, Int. J. Mach. Tools Manufact. Vol. 34. No. 2. pp. 147-160 (1994).

Figure 7-23. Empirical method

If the "FEM" is chosen in Figure 7-20, a window is opened which includes some FE methods and publications for beam design, as shown in Figure 7-24.

FEM for hot rolling
$\bullet$ FE method
$\begin{array}{llll}\text { - Visco-plastic } & \text {-Slab-FE } & \text { - 3D } & \bullet \text { Rigid-plastic } \\ \text { - 2D } & \text {-Elastic-plastic } & \bullet \text { Pseudo-2D } & \end{array}$

- FE Commercial software:
- ABACUS •I-DEAS - QFORM
- FE research in hot rolling area:
- Deformation (stress, strain, strain rate, force, torque, friction); - Thermal-mechanical, microstructure, heat-transfer, temperature; • Optimum process (pass design)
- Some FE work for beam:

1. 2D, Rigid-plastic FE, Slab.

Kim, N. and Altan, T. Three Dimensional Analysis and Computer simulation of Shape Rolling by the Finite and Slab Element Method. ERC/NSM-B-90-10, Ohio State Univ (1990).
2. Elastic-plastic FE, 3D,Pseudo-2d, Section rolling

Wen, Elastic-plastic Finite-Element Modelling of Section Rolling, Ph.D thesis, The university of Birmingham (1994)
3.Rigid-plastic FE, 3D, Slab.

Kiuchi and Yanagimoto, Computer aided simulation of universal rolling process, ISIJ International, 30, No. 2, pp. 142-149 (1990)
4. Zhang, Determination of forward slip in H-beam rolling (1994)

Forward slip. Equilibrium equations method. Web, Flange separately, as plane problems.
5. Jin, Three-dimensional analysis of the universal beam tandem rolling process par1: Deformation analysis. Part2: Stress analysis (1998)
Universal beam rolling process, prediction of metal flow, flange spread, web forward-slip. 3D and upper bound method.
6. Shin, A study on the rolling of I-section beams (1992)

I-beam, four passes, separating forces and roll torques optimally designing the roll pass schedule.
Methods: (1) 3D, FE. (2) Experiments (plasticine)
Any other shape rolling is possible.
7. Zhang, Calculation of temperature and determination of deformation resistance in H-beam (1999)

H-beam rolling. (1) Temperature variation in a stand; (2) Total change of temperature and deformation resistance
8. Ozsoy, Optimum Scheduling of a Hot Rolling Process By Nonlinear Programming (1992) Optimum scheduling of shape hot rolling
9. Datta, Finite element modeling of hot rolling, Ph.D. Thesis, Sheffield (1996)

Figure 7-24. FE methods for Beam

If "Other methods" is chosen in Figure 7-20, a window is opened which includes other existing methods such as upper bound energy method, fuzzy method and expert system, as shown in Figure 7-25.

## Other methods

- Mathematical modeling (computer program, upper bound energy)

Jin et al. A three-dimensional analysis of the universal beam tandem rolling process. Part (1): Deformation analysis, part (2): Stress analysis. Journal of Materials processing technology 102 (2000) 59-64 (2000)

- Zhang et al. Calculation of temperature and determination of deformation resistance in H-beam rolling (1) Temperature variation in a stand. (2) Total change of temperature and deformation resistance, Journal of Materials Processing Technology 94,128-132 (1999)
- Fuzzy

Jung et al. Fuzzy control algorithm for the prediction of tension variations in hot rolling,
Journal of Materials Processing Technology, Volume 92,Issue 93,30 August 1999,
Pages 463-467 (1999)

- Expert system

Kim et al. A Knowledge-based expert system for roll pass and profile design for shape rolling of round and square bars, Journal of Materials Processing Technology 89-90, 145-151(1999)

Figure 7-25. Other methods

During the design process, mathematical relationships or logical relationships will be given a mark " " or "*" in the cells, respectively. For example, Process Design and Deformation design issues influence each other, but their relationship cannot be exactly determined. Mark "*" expresses a logical relationship. From a logical relationship, designers may find design information such as that in Figure 7-12. The relationship between Pass/Part parameters and Deformation needs to be calculated because each different pass has different value of force, torque etc. Mark " $\downarrow$ " expresses a mathematical relationship. From mathematical relationships, designers may find more formulae such as given in Figure 7-5 and Figure 7-6. At the present stage of development, this kind of relationship is only given in outline in the matrix.

Once one of the group of design issues have been discussed, the team of designers will have decided suitable equations and optimisations of the process. They can discuss other design issues using the Matrix methodology sequentially. Figure 7-10 shows how the matrix can add more columns, rows and pages to cover additional design considerations.

### 7.5 Conclusions and Summaries

In this chapter, a Matrix-based roll design system is introduced. It includes both Matrixbased flat roll design and section roll design.

The hierarchical configuration has been successfully used to classify hot rolling systems. It is also demonstrated that a Hierarchical Classification could be integrated with a cascading menu system.

The complicated design process of roll design is integrated into the Matrix-based hot roll design system. The available design methods and information can be found in the matrix. It is suggested that higher quality rolled products could be achieved by using the optimized design method. An inexperienced designer could also find hot roll design information quickly and accurately.

Matrix-based hot roll design differs from other design systems, such as expert systems or databases. It emphasizes team work. The design procedure allows the less
experienced designers to gain experience. It shows that senior designers and technical specialists can also benefit from participation in a team.

Matrix-based roll design is potentially applicable to a wide range of rolling processes and is highly flexible and suitable for team working.

## Chapter 8

## Discussion and Conclusions

### 8.1 Traditional Hot Roll Design

Hot rolled products have been successfully applied in various industries. Hot roll design is a multi-discipline subject and producing rolls involves complex design and manufacturing processes. Fundamental hot roll design theory has taken several decades to develop. Attempts have been made to predict hot rolling deformations parameters, such as force, torque, elongation, spread and draft. A very large number of books and papers have been presented on flat rolling, and also on section roll design, Section rolling analysis involves 3D flow and as such is difficult to analyse.

Because of the numerous factors influencing hot roll design, at present none of the developed formulae allows for the full integration of all the factors necessary to accurately predict condition in flat rolling. Further complications associated with section rolling means that most of the formulae developed have only a limited application range. At the present stage of technology, hot roll design is achieved by using a combination of empirical knowledge, calculations and educated guesses and design of hot rolled products by a single person is no longer a feasible practice.

### 8.2 Numerical Techniques

With the rapid development of computer techniques, numerical methods are being developed in an attempt to simulate metal flow, such as in a complex hot rolling process. FEM based computer codes offer excellent potentiality for predicting metal flow in 2D and 3D hot rolling.

It is now clear that the FE technique is the most promising solution for the 3D analysis of section rolling. However, the application of FEM is quite restricted because of difficulty of setting up models, and the associated long computational times. In most cases, the results presented aim at illustrating the method proposed rather than calculation data. Some fields of application still remain unexplored.

### 8.3 Sharing Resources

Traditional hot roll design is based on specialized and isolated factors such as force and wear. Although FEM can take into consideration several parameters simultaneously, it does not sufficiently cover every aspect of the whole design process. Actually, if hot roll design is done from a systematic point of view, it involves integrating a combination of processes involving machinery, computers and design skills.

When the market only demanded a high volume of relatively unvaried, medium quality products, the use of this kind of approach could be deemed reasonable. However, this method can no longer cope with the requirements of the new market (rapid technological change), because the demand is for high quality of hot rolled products, in varying styles, with only a short-term life cycle.

The old art of roll pass design is turning into a modern technology based on scientific progress. The task of hot roll design nowadays requires using modern design tools and methodology, and Matrix-based hot roll design has sprung from studies in this field.

Matrix-based hot roll design inherits the primary feature of Concurrent Engineering (CE), which is based on teamwork, sharing resources to create and improve new and exciting products, to develop and adopt its manufacturing strategy from a broader viewpoint. It integrates hot roll analysis, design, technology and management. This means that the hot roll designers still needs to have practical knowledge and skills in relation to hot rolling. In addition, they must also have a thorough understanding of hot roll design from a systematic viewpoint. The bringing together of these two aspects, means that optimal quality hot rolled products can be achieved.

### 8.4 Features of Matrix-based Hot Roll Design

Matrix-based hot roll design presents design information required to make design decisions in a structured manner. It describes aspects in a row of the matrix such as:

- process design,
- pass parameters,
- infrastructure
and design issues in a column of the matrix such as :
- deformation,
- thermal-mechanical,
- friction etc

Other elements can be included in the matrix such as:

- size,
- shape,
- materials,
- quality,
- cost,
- schedule, etc.

Matrix-based hot roll design provides a hot roll design methodology with a systematic approach. It includes the existing formulae, empirical data, experimental graphics, and technical methods for hot roll pass design. This study has included a lot of existing empirical knowledge of experts as well as the results of research into new approaches.

For specific hot rolled products, by comparing the different formulae conditions of application, graphics data and computer simulation results, the optimum formulae and data can be chosen after discussion between the designers using the matrix. The optimum manufacturability of the rolled product is thus ensured from the optimum design, as early as possible in the design stage.

In order to classify the complexity of roll design problems, which must be precise enough to be manageable, the Hierarchical Classification System is successfully employed to assist the Matrix-based hot roll design process.

An indication that a number of design problems included in the Hierarchical Classification System have been taken into consideration is shown using a cascading menu system. Therefore, it is proposed that Matrix-based hot roll design consists of a multi-layer matrix and multi-cascading menu system.

Teamwork is the main feature of Matrix-based hot roll design, requiring the cooperation of several levels of designers. In a typical hot rolling matrix activity, the team may consist of senior designers, inexperienced designers and researchers into new methods of design. There may be a hierarchy of teams, with individual teams concerned with one part of a product, for example, a section roll group or deformation design group.

Flexibility is another main feature of the matrix. Matrix-based hot roll design is an open system. The matrix has a multi-layer menu, the Hierarchical Classification System and has the potential of extending the rows and columns. The designer is allowed to add new design issues to the matrix and new formulae, experimental data, and any technical methods for specific cases can be added to each set of design issues, according to the hot rolled products required.

All of these features are applicable in a wide range of hot roll designs, and both the design processes and design issues can be expanded along the row or column in the matrix, respectively.

At the study stage, the matrix provides a methodology to support inexperienced designers to learn about hot roll design quickly and effectively. It can be used to guide inexperienced designers in becoming acquainted with the key skills of hot roll design, without having to rely on reading about the subject in the numerous and often aged books.

The matrix also provides a systematic viewpoint and facilitates the consideration of hot roll design, not only based on the traditional methods, but also from the more modern techniques such as: Concurrent Engineering (CE), The planning of manufacturing, Computer-aided Design (CAD), Finite Element Method (FEM) and other methods.

This study only provides a suggestion of the matrix methodology, which integrates the parts of the roll design method, parts of design parameters and rolling information to the matrix. The matrix does not include the whole roll design process or all of the design issues. It also does not give information to compare different methods for specific cases. All of building up the matrix system is in hand. The matrix-based roll design
system has provided a more systematic viewpoint than specific design cases. Therefore, the matrix methodology is more suitable to demonstrate to inexperienced roll designers.

### 8.5 Further Work

The Matrix-based roll design methodology has been successfully used in assembly and its features are extended here to hot roll design. It could be used by other forms of complex manufacturing, utilizing a systematic method by which workpieces can be manufactured economically and competitively from initial stages, through to the finishing stages. In this thesis, the direction of future trends has been indicated. The need for further effort in research and development of Matrix-based hot roll design is obvious. The following can be done to further improve Matrix-based roll design.

Only the basic principles of the matrix approach have been discussed at this stage. Full hot roll design includes thermal-mechanical elements and so on. So, more sections need to be added to the matrix to encompass these aspects. This should be addressed in further research.

Training is a prominent concern linked with successful Matrix-based hot roll design. The team of designers play a key role in the matrix but these people often have little training in organization change and group motivation, the result often being that only isolated factors are focused upon.

Implementation of CAD/CAM tools, rather than using paper drawings for communicating designs in the Matrix-based hot roll design methodology, will make the task of interpreting ideas easier for the designers of the team. Improvements can be made by standardizing the databases used, facilitating more effective communication. In addition the approach should be more user friendly. Matrix-based hot roll design requires the integration of designers' skills as well as the integration of computer skills and related technology. In the long run, effective use of CAD/CAM will allow Matrixbased hot roll design to be both attractive visually and thus, more marketable.

### 8.6 Conclusions

In this thesis, the previous experimental and theoretical studies of hot rolling are reviewed. A thorough understanding of the available methods and conditions of their application is extremely important in order to achieve high quality rolled products.

Metal flow and deformation has been described. Some important design parameters, such as rolling force, torque, spread, elongation and draft, have been discussed in detail. It has been seen that rolling pass design is dominated by the calculation of these parameters. These parameters can be calculated by using different formulae or methods. However, each formula or method is only suitable for its specific application. A lot of formulae and methods have been developed and this work indicates the requirement for the designer to choose a suitable formula, according to the design and application requirements. This should result in high efficiency and high quality for the hot rolled products manufactured.

With the rapid development of computer hardware and software, FEM is becoming more and more practical for hot roll design. FEM has been developed to research and design rolled products, especially for section rolling design. Several parts of the thesis have discussed the theory and design processes of FEM. It is predicted that this method will increasingly become an essential tool in the study of hot rolling section production.

A Matrix-based roll design system has been introduced in this thesis. All the formulae and methods discussed in the previous chapters were integrated in the Matrix-based roll design, including Matrix-based flat rolling design and section rolling design. The approach is based on a matrix used for mechanical assembly design. However, the features within the present work have been further developed and adapted to make it suitable for the far more technically demanding application.

Hierarchical shape configuration has been successfully used to classify and highlight design problems in roll design. It has been demonstrated that a Hierarchical Classification System could be realised by using a cascading menu system.

The complicated design process of hot rolling has been integrated in the Matrix-based roll design system. Because the available design methods and information, such as empirical, experimental, theoretical and FE methods can be found in the matrix, the approach has the potential to produce higher quality rolled products by using the
optimized design method compared those using the application conditions of the existing ad- hoc methods.

In this approach, the importance of teamwork is emphasized. This ensures that everyone can make a contribution and can also benefit from the design process. New designers will be trained by their participation.

In comparison with the assembly matrix methodology, Matrix-based roll design inherits the primary features of Concurrent Engineering (CE). It maintains the advantage of the initial matrix methodology for assembly and complements these with new features. The Matrix-based roll design methodology is a systematic approach that integrates teamwork into the roll design process and is, in addition, an open system. Therefore this approach is more flexible and efficient for hot roll design. This Matrix-based roll design system is potentially applicable for a wide range of hot roll design. However, in this study only brief principles of Matrix-based roll design is introduced, and the methodology has not yet been developed sufficiently to use in practice.

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