Extending AdS/CFT: dual states for new geometries

Titchener, Georgina

Extending AdS/CFT: Dual States for New Geometries

Georgina Titchener

A Thesis presented for the degree of
Doctor of Philosophy

Centre for Particle Theory
Department of Mathematical Sciences
University of Durham
England

August 2006

Dedicated to

my family

Extending AdS/CFT: Dual States for New Geometries

Georgina Titchener

Submitted for the degree of Doctor of Philosophy
August 2006

Abstract

In this thesis we present research that extends our knowledge of the AdS/CFT correspondence; in particular we look at various non-supersymmetric spacetimes and their conjectured dual field theory states.

We consider known $U(1)\times U(1)$ invariant spaces and investigate the requirements for smoothness, which results in the construction of new smooth non-supersymmetric soliton solutions with D1, D5 and momentum charges. We are able to identify dual states for these geometries in the field theory describing D1-D5 systems. Also discussed are interesting aspects of these spacetimes and new orbifold solutions which are valid string backgrounds.

In addition to this, we study time-dependent spacetimes which are asymptotically locally anti-de Sitter. There are two different spacetimes with the same asymptotics: the 'bubble of nothing' solutions and higher dimensional BTZ black holes, which are both asymptotically locally anti de Sitter and whose conformal boundaries are both conformal to de Sitter space times a circle. We use the AdS/CFT correspondence to give a description of the spacetimes in the dual field theory. We are also able to relate horizons and their thermodynamic quantities in the bulk and boundary spacetimes and are able to assign entropy to non-compact horizons.

Declaration

The work in this thesis is based on research carried out at the Centre for Particle Theory, the Department of Mathematical Sciences, the University of Durham, England. No part of this thesis has been submitted elsewhere for any other degree or qualification and is all my own work unless referenced to the contrary in the text. The work in chapter 3 was performed in collaboration with Vishnu Jejjala, Owen Madden and Simon Ross. The work in chapter 4 was performed in collaboration with Simon Ross.

Acknowledgements

I am indebted to Simon Ross for his encouragement and patience throughout the past three years. His guidance and suggestions have been invaluable.

Thanks to Vishnu Jejjala and Owen Madden for their parts in the research and for their efforts in helping me at various times, even if I didn't always know what they were talking about. Owen and Wadey provided technical support.

Thanks also to all the staff in Durham, especially those involved in the CPT lecture course, and in York, in particular Niall MacKay and Tony Sudbery for all their time and effort over the years. M. Imran created the template for the thesis, saving me numerous hours. My research was funded by EPSRC.

To everyone in my office: Lisa, Jenny, Dave and Mark, but especially Anna for helping me settle in and find my way around when I arrived, and Gav for his entertaining opinions on everything from transpennine "expresses" to the ECB. And those who weren't in my office but kept me company anyway: Jane, Jo and Steve. Thanks. You all kept me going.

Immeasurable thanks to my parents and my sister for their support which allowed me to pursue my studies, and for their quiet encouragement throughout my life. And to John: I couldn't have done this without your understanding and sacrifices. Thank you.

Contents

List of Figures

Chapter 1

Introduction

String theory, whilst far from being complete, is currently our best theory of quantum gravity: it contains a spin two gauge boson which can be identified with the graviton. At long distances it reduces to perturbative quantum field theory which includes gravitational interactions and incorporates supergravity.

Originally formulated as a theory of open and closed one-dimensional objects in ten dimensions, string theory has been enlarged to include extended objects of higher, and lower, dimensionality. These additional objects, D-branes, are the first hint that string theory can not be the fundamental definition of our unified theory. Such solitons only appear as boundary conditions at the perturbative level of string theory. They were discovered when the behaviour of strings wrapped on compact dimensions was studied. If we take the radius of such a direction to 0, the physics of closed strings remains unchanged but the ends of open strings are seen to be restricted to a hyperplane. This invariance of the physics of closed strings under inversion of the radius of one direction is known as 'T-duality'. To restore the correct degrees of freedom to the system, we must treat these planes as dynamical objects in the theory [1, 2].

D-branes have become central to our understanding of string theory. They are used extensively in string models aiming to recreate a spectrum of particles similar to the minimal supersymmetric standard model, as a way of including chiral fermions and gauge fields [3]. They are sources for charges which can not be carried by our original strings. As it was already known that black hole geometries can also support these charges, we can identify these spacetimes as those sourced by the D-branes [4]. The range of parameters for which the geometrical description is the most appropriate is precisely when the string coupling is large.

The study of D-branes has also lead to a new paradigm which offers a hope of investigating strongly coupled string theory, and may even provide a definition of it. This is the Anti-de Sitter/Conformal Field Theory correspondence, suggested by Maldacena in [5]. He conjectures that Type IIB string theory on a spacetime with anti-de Sitter boundary conditions is equivalent to a state in a gauge theory in one fewer dimensions which can be thought of as living on the conformal boundary of the spacetime. The string background is required to have some compact directions for consistency and the isometries of these correspond to internal symmetries in the gauge theory.

This conjecture brings together two important directions in mathematical physics: the belief that high energy field theories with a large number of colours are describable by strings [6], and the holographic principle. Any description of a gravitating system in terms of a non-gravitational theory in one fewer dimensions obeys the holographic principle, that the number of degrees of freedom in a volume of space is limited by its area in Planck units [7]. The AdS/CFT correspondence is the most thoroughly explored example of a holographic theory.

The principle of holography finds its greatest support from research into the nature of black holes. Classical laws are known for black holes which mimic every law of thermodynamics [8,9]. They have constant surface gravity κ over their horizons; any change in mass is proportional to change in area plus change in momentum; area increases in any physical process; and it is impossible to reach $\kappa = 0$ classically. These can be contrasted [10] with the usual laws of statistical mechanics: that temperature T is constant in a body in thermal equilibrium; any change in energy is proportional to change in entropy plus work done to the system; entropy increases in any process; and it is impossible to reach $T = 0$. Semiclassical considerations show a black hole also has a temperature T due to its Hawking radiation [11] : pair creation in the strong gravitational fields at the horizon causes the hole to appear as a radiating body with a temperature proportional to the surface acceleration κ

of the black hole, $\kappa/2\pi = T$ [8]. By comparison with the first law for a non-rotating statistical mechanical system and using the result that for a black hole any change in energy results in an increase in its horizon area via $dM = \kappa dA/8\pi G$, we are lead to the Bekenstein formula [12]

$$S = A/4G. \tag{1.0.1}$$

A microscopic explanation for the origin of the entropy of extremal 5-dimensional black holes given by Strominger and Vafa [13] was one of the early successes of D-brane theory. They were able to follow contributing states from weak coupling to strong, which is the same as going from a description in terms of gauge theory on D-branes to one in terms of a gravitational system, namely a black hole. This was possible because D-branes in certain configurations preserve precisely half the number of supersymmetries of the vacuum. They are individually BPS states, in that they saturate a relationship that states that the mass of a charged object is always greater than its charge in some units, $M \geq Z$. Parallel branes of the same type continue to satisfy the equality. Such states form short multiplets and these are believed to be protected against combining into longer representations as the coupling changes. So by counting sufficient states in the gauge theory on the D-branes at weak coupling, they could explain the entropy of the resulting black hole. Entropy of non-extremal branes was subsequently given such a derivation as well, as were absorption cross sections [14].

In retrospect, these calculations can be seen as evidence for the validity of the AdS/CFT correspondence. Closely related to these successes is the ability of the gravitational description to reproduce the correlation functions of the conformal theory [15]. The form of these are highly constrained in conformal field theories, so this is a test of the correspondence. Other such checks of the conjecture include the fact that the spectrum of chiral primaries are, for some limits of parameters, identical in the two theories. More simple checks include the symmetries shared by the two sides of the correspondence and the number of supersymmetries in each. Anti-de Sitter space naturally emerges in field theories on flat space where the energy scale is included as a dimension in the theory [16].

One of the reasons that further such verifications of the conjecture are difficult to find is that the correspondence relates strongly coupled theories to weakly curved spacetimes, or vice versa. That is, one side of the correspondence is always nearly impenetrable and we must restrict calculations to coupling-independent quantities in general. On the other hand, this gives the duality its power to offer insights into strongly coupled field theories and possibly gives us our first definition of strongly coupled string theory.

Challenges remain in fleshing out the correspondence. One question that has received a lot of attention but has not been fully answered is, what is the effect of perturbing the 'boundary' field theory on the bulk geometry, and vice versa. This followed seminal work by Witten in [17], who wrote down an equivalence between the partition functions of the two theories. In the field theory, we can add new terms to the Lagrangian for the conformal theory such that an operator $\mathcal{O}$ has the total term $\int d^n x \phi_0(x) \mathcal{O}$ [2] (where the bulk geometry is asymptotically $AdS_{n+1} \times \mathcal{M}$.) The scalar field $\phi_0(x)$ acts as a source for the operator $\mathcal{O}$ but is also the large radius or $z \longrightarrow 0$ boundary condition for the supergravity field $\phi(x, z)$ in Poincaré co-ordinates A.1.14. In a Lorentzian spacetime there exist two solutions to the wave equation for $\phi(x, z)$ in the bulk: one normalizable and one not. They behave near the boundary as $z^{\Delta}\phi_0(x)$ and $z^{(n-\Delta)}\phi_0(x)$ for some Δ where the dimension of $\mathcal{O}$ is Δ [2, 17, 18].

The interest in this coupling of bulk fields and boundary operators comes from the resulting deformations of the field theory when the non-normalizable solution for ϕ is included. This 'switches on' the operator [2] and changes the field theory in some way. If the effect of the operator included becomes stronger (termed 'relevant') or remains the same (marginal) [19] as we decrease the energy, we may move towards a new conformal theory or a non-conformal one. Operators which get weaker at low energies are irrelevant; adding these to the action would require a new field theory at high energies. For relevant and marginal operators we start with a known conformal theory at the ultra-violet scale and move towards an unknown conformal or non-conformal theory in the infra-red [20]. This is of phenomenological interest as exact conformal symmetry is not observed in QCD. Thus through the AdS/CFT correspondence, the study of renormalization group flow of CFTs may be informed by geometrical considerations. In addition to deformations of the boundary conditions which change the field theory under consideration, in a Lorentzian spacetime we can change the geometry in a way that only changes the state within the field theory, by including a normalizable mode [21, 22] . This method of formalizing the correspondence also encompasses the observation that regularizations performed in the gravity theory to remove "infra-red" divergences, such as the infinite volume of the space, are in fact related to renormalizations of ultra-violet divergences in the field theory. It has been established [7, 23, 24] that cut-offs introduced at the upper end of the length scale in AdS are related to cut-offs to remove the high energy range of the CFT. The distance of a field from the boundary in some co-ordinates is proportional to the size of the dual excitation in the dual field theory.

The aim of this thesis is to extend knowledge of the mapping from geometries to dual field theory states in the AdS/CFT correspondence. We do this by finding new non-supersymmetric string backgrounds and, by identifying their charges, finding their dual descriptions. We are also able to relate thermodynamic quantities in bulk and boundary spacetimes.

Perhaps the most important unresolved problem in the literature on AdS/CFT is the need for an explicit map from a given geometry to a field theory state and vice versa. One area of success in this direction has been work by Lin, Lunin and Maldacena [25] on 1/2 BPS chiral states; that is, ones that preserve half the number of supersymmetries. They have found asymptotically $AdS_5 \times S^5$ IIB solutions where the topology of the geometry is determined by a planar curve, and been able to identify them with a sea of fermions whose state depends on the same curve in a quantum hall effect. These are very special metrics with particular symmetries and high supersymmetry. Whilst the results are very interesting and illuminating, the method is only transferable to similarly special spacetimes. The original paper extended the method to spaces with asymptotics of the form $AdS_4 \times S^7$ and $AdS_7 \times S^4$; subsequent work [26, 27] incorporated $AdS_3 \times S^3 \times T^4$.

One area where such a precise map would be invaluable is in the efforts to repeat the success of deriving the entropy of two charge black holes with three charge ones. Some of the most active research into this is due to Mathur. Mathur has proposed [28] an unorthodox description of black holes which explains the origin of entropy of black holes and may resolve the information paradox [1] although details remain unclear.

The Mathur programme takes as its starting point the identification of a black hole in anti-de Sitter space with a thermal ensemble in the related conformal field theory. In the CFT we can count contributing states in the ensemble. We should then presumably be able to use the correspondence to find their dual geometries. If these geometries are also themselves black holes, they require an explanation of their entropy in similar terms. Clearly this is a circular argument and explains nothing. So Mathur argues that these states must be smooth, horizon-free metrics and it is only when we coarse grain over all suitable geometries that we observe a black hole. A large number of suitable two-charge metrics have been constructed. However, these holes' horizons have a zero-size classically, so the three-charge hole which has a sizable horizon is now the subject of investigation [29]. Despite numerical matches and strong motivation for the conjecture, the proposal is subject to much scepticism; in particular, it is hard to see precisely how smooth spacetimes recreate the effect of singularities [30].

Part of the research presented here looks for further three-charge metrics which were not previously discovered. The spaces found are new smooth non-supersymmetric solitons with $D1$ and $D5$ and momentum charges. These have some relevance to the work outlined above, but the spacetimes are also interesting in their own right as novel, generally non-supersymmetric, string backgrounds. They can also be generalized to allow orbifold singularities, which are acceptable in string theory. We are also able to find the dual field theory states for the non-orbifolded spaces and thus expand the AdS/CFT dictionary.

[1]This is the conflict between the unitarity of quantum mechanics and the calculations in semi-classical gravity showing that detailed information about the matter that forms a black hole is not present either in the Hawking radiation from or the final endstate after decay of a black hole. The AdS/CFT correspondence is strong evidence that the information should in fact not be destroyed and that we need a complete theory of quantum gravity to resolve the problem. The semiclassical approach still supports the 'no-hair' theorem of black holes, which is the statement that the geometry of a black hole is entirely determined by its mass, electro-magnetic charges and angular momentum.

Another topic of great interest where the AdS/CFT correspondence may prove useful and illuminating are time-dependent metrics. These are ones without symmetry under translations in the time co-ordinate, and are of great phenomenological importance. With all realistic cosmological models starting from a big bang-like singularity and many ending in a future singularity, understanding issues arising from time-dependence is clearly essential to understanding the universe. We live in a spacetime with a small positive cosmological constant, making it close to de Sitter space. Unfortunately it is difficult to embed de Sitter vacua in string theory (see for example [31]); however, we can study other time-dependent metrics and they may teach us about aspects such as particle detection and vacuum ambiguity. In curved spacetimes, the asymptotic past and future definitions of particles are inequivalent due to different concepts of time: there is no preferred timelike co-ordinate, unlike in flat space, and hence no preferred vacuum above which we can measure excitations. The subject of quantum field theories in curved spacetimes has been extensively studied, but questions remain over objects such as the definition of the stress-energy tensor. The AdS/CFT correspondence gives us a new window on time-dependent string backgrounds via their dual time-dependent field theories. In this work, we study 'bubble of nothing' solutions in AdS [32] and also higher dimension generalizations of 'BTZ' [33] black holes which share the same locally anti-de Sitter asymptotic behaviour. These are higher dimensional analogues of 2+1 dimensional BTZ black holes, the only permissible black holes in three-dimensions where gravity is a non-propagating force.

Both the spaces under consideration are time-dependent asymptotically locally anti de Sitter, with conformal boundaries (see chapter 2) of the form $dS_3 \times S^1$ where dS_3 is three dimensional de Sitter space. Thus we believe them to be different states in the same gauge theory on the time-dependent background $dS_3 \times S^1$. Our work explores aspects of the correspondence including the relationship between horizons in the bulk and those in the boundary, and the suitability of different vacua in de Sitter space. Our results have implications for such disparate topics as field theory on dS_3 and attempts to give BTZ-type black holes a thermodynamical interpretation [34, 35].

The progress achieved in this research was to develop specific examples within the anti-de Sitter/conformal field theory correspondence, particularly time-dependent and non-supersymmetric spacetimes and their duals. There remain open questions about the duality, and our understanding of the map between general geometries and gauge theory states is still limited: improving it is an active area of research. Some questions and possible directions for future research are discussed in chapter 5.

Chapter 2

Background

In this chapter, we review the Anti-de Sitter/ Conformal Field Theory (AdS/CFT) correspondence and explain its motivation in more mathematical terms, with emphasis on D1-D5 branes. A comprehensive review of all aspects of AdS/CFT can be found in [20]. We introduce parametrizations of anti-de Sitter space and elements of field theories which are used in other chapters. The reader is assumed to have some knowledge of string theory and general relativity.

2.1 AdS/CFT Correspondence

The AdS/CFT correspondence is concerned with different ways of describing the low energy physics of D-branes. The motivation for Maldacena's conjecture [5] takes as its starting point Type IIB string theory in flat ten-dimensional space, $\mathbb{R}^{1,9}$ with Dp-branes, p odd.

One side of the correspondence takes N Dp-branes and considers the low energy description of their dynamics when the string coupling g_s is very small. Small fluctuations of a single Dp-brane are described by the transverse zero-mass excitations of open strings which couple to it, $\alpha^b_{-1}|k\rangle$, $b = p+1 \ldots D-1$. The other massless excitations form a vector, $\alpha^m_{-1}|k\rangle$ $m = 0 \ldots p$ tangential to the brane. If there is more than one brane, the ends of the strings may couple to different ones and we must include indices $|ij\rangle$ when describing a string to specify which branes it ends on. However, the stretching between branes contributes to the mass of the string, so we generally consider only $\alpha_{-1}^{m}|k;ii\rangle$, giving N $U(1)$ gauge groups. As the separation of the branes and the string length goes to zero, such that the ratio of the two is constant, the massless strings are allowed to end on any brane so the branes are well-described by a $U(N)$ gauge theory.

Let us take co-ordinates ξ^a, $a = 0 \ldots p$ to describe the worldvolume of one Dp-brane. The spacetime co-ordinates $X^\mu(\xi^a)$ $\mu = 0 \ldots D-1$ and the gauge field tangent to the p-brane $A^m(\xi^a)$ $m = 0 \ldots p$ are the fields on the brane. The pull-back of the spacetime metric and anti-symmetric tensor to the $p+1$ dimensional worldvolume are

$$G_{ab} = \frac{\partial X^\mu}{\partial \xi^a}\frac{\partial X^\nu}{\partial \xi^b} G_{\mu\nu} \tag{2.1.1}$$

$$B_{ab} = \frac{\partial X^\mu}{\partial \xi^a}\frac{\partial X^\nu}{\partial \xi^b} B_{\mu\nu}. \tag{2.1.2}$$

There is also the field $F_{ab} = \partial_a A_b - \partial_b A_a$ due to the gauge potential and open string coupling requires one power of g_s^{-1}. It can be deduced via the β function for the open string that a suitable action for low energy Dp-branes is the Dirac-Born-Infeld bosonic action [10, 36, 37]

$$S_{D_p} = -T_p \int d^{p+1}\sigma e^{-\phi} \sqrt{-\det(G_{ab} + 2\pi\alpha' F_{ab} - B_{ab})}. \tag{2.1.3}$$

For multiple coincident D-branes, we must incorporate the matrix nature of X^μ and A^a which will not in general be commutative. The invariant quantity we can form from a matrix is the trace, in this case in the fundamental representation of our group $U(N)$. It is proposed [2] that we should in fact take the trace after symmetrizing over the gauge indices.

By assuming that the size of the field-strength is small and that the branes are close to flat, and by working in the static gauge where the spacetime co-ordinates are taken to be the same as the worldvolume ones, $X^a(\xi^a) = \xi^a$ for $a = 0 \ldots p$, we can approximate this action when $B = 0$ by an expansion in α' [10]

$$S_{(bosonic)} \sim \frac{1}{g_s} \int d^{p+1}\sigma Tr \left(-\frac{1}{4} F^{ab} F_{ab} + \frac{1}{2} D^a X^i D_a X^i + \frac{1}{4} [X^i, X^j]^2 \right) \tag{2.1.4}$$

where X^i $(i = p+1 \ldots D-1)$ are the co-ordinates transverse to the branes. Note that the derivative D^a is now the covariant derivative required by non-commutative fields, $D^a X^i = \partial^a X^i + [A^a, X^i]$. The string coupling g_s is $e^{-\phi}$, where ϕ is the dilaton, taken to be constant. This action is recognizable as the bosonic part of the Lagrangian of supersymmetric Yang-Mills theory reduced to $p+1$ spacetime dimensions with $g_s \propto g_{YM}^2$ [38].

We will focus our attention on one version of the correspondence. Let us take N_1 D1-branes [1] and N_5 D5-branes placed in flat ten-dimensional space with four of its directions compactified and one dimension finite but sizable, that is $\mathbb{R}^{1,4} \times S^1 \times T^4$. We wrap four of the five spatial directions of the D5-branes around the T^4 and orient the remaining direction parallel to the D1-branes along the S^1. This gives them a finite energy. The spatial direction is along the branes which appear as effective strings in the five remaining extensive directions. Then in the low energy limit, the branes are described by a sigma model on $1+1$ dimensions with a target space believed to be the symmetrized product of $N_1 N_5$ copies of T_4 [39].

By virtue of the fact that the branes are taken to be on top of each other, we are expecting to be in the Higgs branch of the CFT [40]. The Coulomb branch can be removed only if the charges Q_1, Q_5 are co-prime [20].

Away from the branes closed strings propagate freely. The two systems interact via closed strings which couple to the branes. As the coupling reaches 0, we have two separate theories: the D-branes, described by gauge theory, and perturbative gravity in flat space. An 'effective potential' in the wave equation which describes the propagation of particles prevents the two from talking to each other.

The system to which this is supposed to be dual is again Q_1 D1-branes and Q_5 D5-branes where now we bear in mind the fact that tension of a D-brane varies with g_s like $1/g_s$. Clearly as the coupling goes to zero here, the branes become infinitely massive. The presence of very heavy objects warps the surrounding space via Einstein's equation (A.1.1), and flux sourced by the branes affects the geometry as well. As we are in a low-energy regime, supergravity is a good approximation to string theory so we look for solutions to type IIB supergravity action with the same fluxes.

[1] These have the appearance of strings but are different to the original strings of string theory, F-strings, as they have different tensions.

It has been found that a solution is to replace the branes with an effective geometry of the form [41]

$$ds^2 = f_1(r)^{-1/2} f_5(r)^{-1/2}(-dt^2+dx^2)+f_1(r)^{1/2} f_5(r)^{1/2}(dr^2+r^2d\Omega^2)+\sqrt{\frac{f_1}{f_5}}\sum_{i=1}^{4} dz_i dz^i. \tag{2.1.5}$$

Here r is the distance perpendicular to the branes, x is the co-ordinate along the shared direction of the branes, Ω is a parametrization of a 3 sphere, the z_i's parametrize the torus and the functions f_i are

$$f_1 = 1 + \frac{g\alpha' Q_1}{vr^2} \quad f_5 = 1 + \frac{g\alpha' Q_5}{r^2}. \tag{2.1.6}$$

The quantity v is proportional to the volume of the T^4 which may vary with r; all other symbols carry their usual meanings.

In the region where $g\alpha' Q_i/r^2 \gg 1$, the functions f_i are well approximated by $f_i - 1$ and v tends to a constant value by the attractor mechanism [42]. This is the near-horizon limit where we are focussing on the centre of the spacetime. The six dimensional metric in 2.1.5 may then be written

$$ds^2 = \frac{r^2}{g_6\alpha'\sqrt{Q_1Q_5}}(-dt^2+dx^2) + \frac{g_6\alpha'\sqrt{Q_1Q_5}}{r^2}(dr^2+r^2d\Omega^2) \tag{2.1.7}$$

$$= \frac{r^2}{g_6\alpha'\sqrt{Q_1Q_5}}(-dt^2+dx^2) + \frac{g_6\alpha'\sqrt{Q_1Q_5}}{r^2}dr^2 + g_6\alpha'\sqrt{Q_1Q_5}d\Omega^2. \tag{2.1.8}$$

Here we have implicitly defined $g_6 = g/v^{1/2}$, the six-dimensional string coupling. If we rescale the radial co-ordinate to $u = r/\alpha'$, we get the metric of three-dimensional Anti de-Sitter (AdS) space, times a three sphere, both with a radius of curvature of $R^2 = g_6\sqrt{Q_1Q_5}l_s$,

$$ds^2 = \alpha'\left(\frac{u^2}{g_6\sqrt{Q_1Q_5}}(-dt^2+dx^2) + g_6\sqrt{Q_1Q_5}\frac{du^2}{u^2} + g_6\sqrt{Q_1Q_5}d\Omega^2\right). \tag{2.1.9}$$

In the opposite limit, when r is very large, the f_i are approximately 1, and we have 5-dimensional flat Minkowski space, times a macroscopic circle and a small 4-torus. So there are two different regions of spacetime: near $r = 0$ there is curved space of the form $AdS_3 \times S^3$, whilst the asymptotic behaviour is 10-dimensional flat space with some of them compactified. The AdS space appears as a throat with constant radius until it ends at $r = 0$.

As these two situations are supposed to describe the same systems from different perspectives, and because they both contain gravity in flat space, the AdS/CFT correspondence was proposed, which conjectures that the other two components of the theories are also identified. That is, Type IIB string theory, on a background which is asymptotically Anti-de Sitter space, and supersymmetric Yang-Mills in one fewer dimensions are equivalent or dual to each other. The isometries of the compact factors of the background spacetime are identified with symmetries of the field theory. In the specific case of D1 and D5 branes, we claim that string theory on $AdS_3 \times S^3 \times T^4$ is equivalent to a superconformal field theory on the $1+1$ dimensional space which is the IR limit of a number of parallel D1-D5 branes [43], and the $SU(2)_L \times SU(2)_R$ R-symmetry in the CFT is identified with the $SO(4) = SU(2) \times SU(2)$ symmetry of the S^3 factor of the near-horizon geometry [44]. In general, we can identify the partition functions of the 'bulk' and 'boundary' theories via [17]

$$\langle e^{\int d^2x \phi_0(x_i) \mathcal{O}(x_i)} \rangle_{CFT} = \mathcal{Z}\left(\phi(x_i, z)|_{z=0} = \phi_0\right) \tag{2.1.10}$$

in the co-ordinates A.1.14. The scalar field ϕ is a supergravity field in the bulk that is constrained to have the value ϕ_0 on the boundary. The right hand side is given by [18]

$$\mathcal{Z}\left(\phi_0\right) = \int_{\phi_0} \mathcal{D}\phi e^{-I[\phi]} \tag{2.1.11}$$

and this is well approximated by e^{I_S}, where I_S is the supergravity action, for certain values of the parameters of the theory. In Lorentzian spacetimes, there will be two types of solutions to the wave equation, which in Poincaré co-ordinates (A.1.14) is [21]

$$(-\nabla^2 + m^2)\phi(t, x^i, z) = \left(-\frac{z^2}{R^2}\partial_z^2 + (d-1)\frac{z}{R^2}\partial_z + \frac{z^2}{R^2}\partial_t^2 - \frac{z^2}{R^2}\nabla_\Omega^2 + m^2\right)\phi. \tag{2.1.12}$$

There are non-normalizable functions which are regular in the bulk which behave at small z like

$$\phi(x, z) = z^{d-\Delta}\phi(x) \tag{2.1.13}$$

where

$$\Delta = \frac{d}{2} + \sqrt{\frac{d^2}{4} + R^2 m^2}. \tag{2.1.14}$$

There will also be normalizable modes which do not affect the behaviour of ϕ at small z. In the correspondence, these only change the geometry in a way that affects to which state in the theory the geometry is dual, whereas the non-normalizable ones affect what the boundary field theory is.

Where can we trust each description of the D1-D5-branes? We know, for example, that on the gravity side of the correspondence we must not have large α' which would lead to massive string states affecting the geometries. Let us define the 't Hooft parameter $\lambda = g_{YM}^2 \sqrt{Q_1 Q_5} = R^2/l_s$. For supergravity to be a good approximation to the physics of the system, we must have that the radius of curvature, R, is much larger than the string length l_s. This implies that λ is much greater than 1. On the other hand, if we want the gauge theory to be weakly coupled so that we can use the perturbative description of Yang-Mills, we need $g_{YM}^2 \sqrt{Q_1 Q_5}$, not just g_{YM}, to be much less than one. Obviously, this is when $R_{AdS} \ll l_s$; that is, the geometry is highly curved. The two ranges are completely separate; hence the use of the word dual to describe their relationship. Note that we also need $N \gg 1$ in order to perform calculations in Yang-Mills.

The strongest statement of the conjecture that these two theories are dual is that they are equivalent at every value of g_s and N, not just in the limit $g_s \to 0$ and $N \to \infty$ whilst λ is kept fixed. If g_s can take any value then this gives a strong coupling description of string theory.

2.2 Structure of Anti-de Sitter Space

It is instructive to study the conformal structure of anti-de Sitter spaces, since the duality outlined above uses the fact that the related field theory can be thought of as living on its conformal boundary to describe such interesting objects as bulk to boundary propagators. Here, the word conformal still means 'preserving angles', just as in conformal field theories: it refers to transformations we can make to the metric which leave angles between vectors unchanged. In particular, they leave the definition of a null vector unchanged and hence does not change the definition of two events being causally related.

A conformal compactification preserves the causal structure of a metric whilst bringing infinity to a finite distance in some parameter. These transformations are extremely useful for studying the asymptotic behaviour of spacetimes.

We get to the boundary of g by taking for our transformation some positive, twice differentiable function W with a first order zero [17] on the boundary. Any such function is acceptable, so the boundary does not have a natural metric, only a conformal class. Then $\tilde{g}_{ab} = W^2 g_{ab}$ gives a conformally related metric and the conformal boundary is at $W = 0$. This cancels the $r \to \infty$ pole in the metric.

To study $d+1$-dimensional anti-de Sitter space, let us take a particular co-ordinate system, A.1.7

$$\begin{aligned} x_0 &= R\frac{1}{\cos\psi}\sin\theta \\ x_{d+1} &= R\frac{1}{\cos\psi}\cos\theta \\ x_i &= R\Omega_i \tan\psi \quad i = 1...d. \end{aligned} \tag{2.2.1}$$

where the Ω parametrize a $d-1$ sphere. In this case the metric is

$$ds^2 = \frac{R^2}{\cos^2\psi}\left(-d\theta^2 + d\psi^2 + \sin^2\psi d\Omega\right). \tag{2.2.2}$$

To cover the whole space we only need to take $\theta \in (-\infty, \infty), \psi \in (0, \pi/2)$.

A candidate for our conformal factor is easily identifiable in these co-ordinates: let $W = \cos\psi/R$. Then our rescaled metric is $d\tilde{s}^2 = -d\theta^2 + d\psi^2 + \sin^2\psi d\Omega$ and a metric for the boundary is $ds^2 = -d\theta^2 + d\Omega^2$. This has the form of $\mathbb{R} \times S^{d-1}$. Note that S^{d-1} is itself a conformal compactification of $\mathbb{R}^{d-1}$ achieved by adding a point at infinity. So the conformal boundary of AdS_{d+1} is conformally compactified $\mathbb{R} \times \mathbb{R}^{d-1}$. Figure 2.1 suggests how AdS_3 may be compactified so that $r \to \infty$ is at a finite distance.

Anti-de Sitter space has the property that null rays can, with the appropriate boundary conditions, travel to the boundary and back in finite time [45]. This is most easily seen by considering the visualization of the geometry via a Penrose diagram, and remembering that conformal compactification does not change causal structure. Figure 2.2 illustrates the difference between AdS_2 and $\mathbb{R}^1 \times \mathbb{R}^1$, that is, Minkowski space.

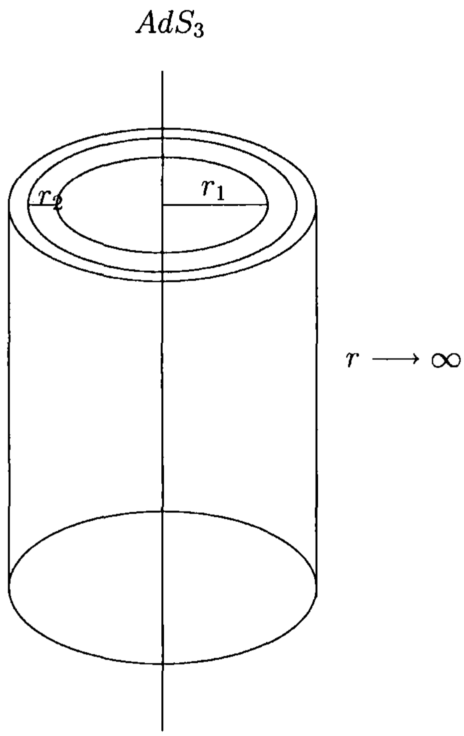

Figure 2.1: This shows how conformally compactified AdS_3 appears: its boundary is $\mathbb{R} \times S^1$. S^1 is equivalent to $\mathbb{R}$ with a point at infinity added. The distances r_1 and r_2 are of similar proper length.

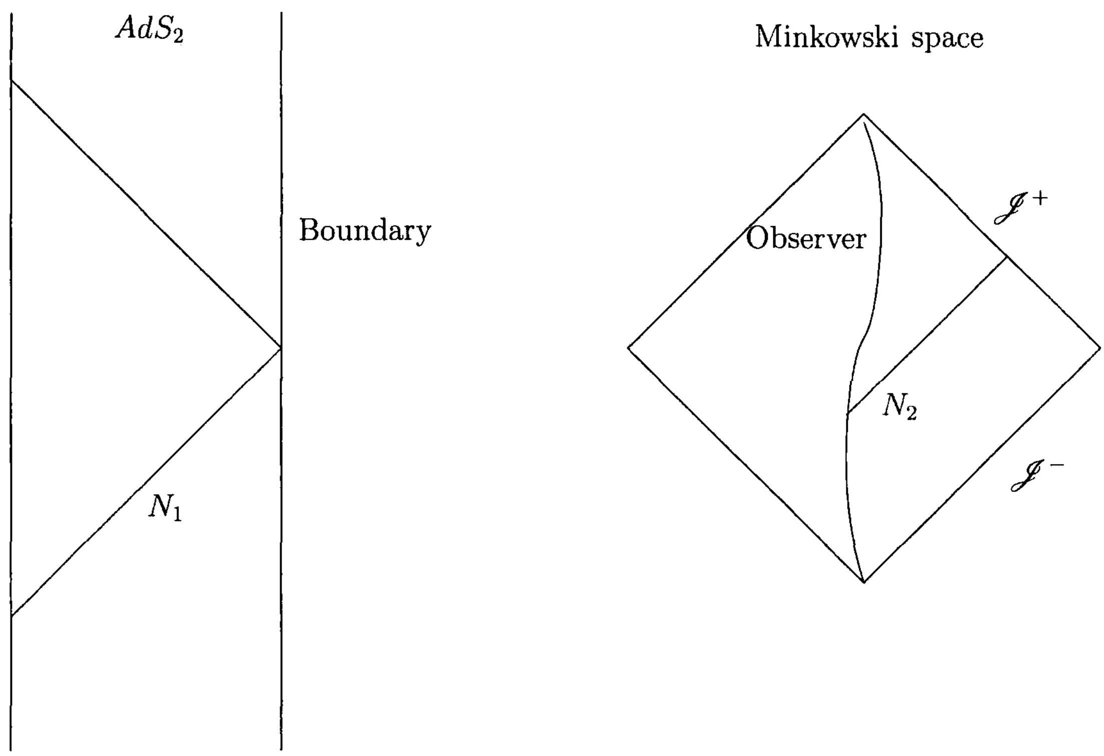

Figure 2.2: This shows how a null ray, N_1, reaches spatial infinity in a finite range of an affine parameter in anti-de Sitter space. This is to be contrasted with the causal nature of Minkowski space. Here, a lightray N_2 emitted by a timelike observer reaches $\mathscr{I}^+$ but cannot return.

Due to the time-like nature of spatial infinity of AdS, a Cauchy surface at any time does not completely determine the future behaviour of the spacetime as information enters the bulk from the boundary in finite time [46]. Anti de Sitter space is not globally hyperbolic [8].

2.3 Horizons

Horizons may occur on both sides of this correspondence, for example in black holes in gravity or field theories on curved spacetimes, so it is useful to introduce some definitions and results. An event horizon is the boundary of that section of a spacetime M which is not the past of spatial infinity: in Minkowski notation, [8]

$$B = \overline{M - J^{-}(\mathscr{I}^{+})}. \tag{2.3.1}$$

This defines horizons in the intuitive sense, as boundaries of regions from which an external observer can never receive information. In certain spacetimes, event horizons are also Killing horizons: null surfaces generated by orbits of Killing vectors [47]. A Killing vector ξ_a is a direction along which the Lie derivative of a metric g_{ab} is 0, most easily written as

$$\nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu = \nabla_{(\mu}\xi_{\nu)} = 0. \tag{2.3.2}$$

Killing vectors are essential for describing many objects in general relativity as they describe symmetries of a geometry. For example, if the components of a metric are independent of a variable θ, $(\partial/\partial\theta)^a$ will be a Killing vector.

As well as in the original context of black holes, we encounter horizons when a spacetime is accelerating such as in de Sitter space. Acceleration is a particular type of non-static behaviour, which means there is no time-like Killing vector. Horizons can then result as the speed of separation of points in the geometry becomes larger than that of light, and communication with all regions of spacetime becomes impossible. Horizons can also be caused by identification of co-ordinates along Killing vectors, so that $\xi^a \cong \xi^a + k$ for some k, as this can result in singularities which will swallow geodesics from sections of the surrounding spacetime: the boundaries of these regions are then effectively horizons. Whilst these are man-made horizons, they are useful as interesting environments which are under control and which can teach us about such issues as causality.

One topic on which they cannot shed light is thermal effects. Both accelerating observers in flat spacetimes and stationary observers in the presence of a black hole detect a thermal background. Unruh observed [48] that a detector in flat Minkowski space which moves along orbits of the Killing vector

$$\psi^\alpha = a\left((X\frac{\partial}{\partial T})^\alpha + (T\frac{\partial}{\partial X})^\alpha\right), \tag{2.3.3}$$

which describes boost isometries, detects particles with a thermal spectrum when an inertial observer detects a vacuum. This phenomenon is due to differing definitions of time and hence particle creation operators. The moving observer is accelerating with acceleration a as measured by an observer in the preferred vacuum; the temperature of the perceived bath of particles is $T = a/2\pi$ [9].

A similar effect happens when it is the ambient spacetime that is accelerating. Hawking and Gibbons [11] determined that in the unique non-singular de Sitter invariant vacuum, all observers detect a thermal background. The limit of each person's domain of influence is a horizon with an entropy. It is believed that this entropy is due to the inability of an observer to know precisely the state of matter behind the horizon.

In the presence of black holes, a different though superficially similar process takes place. Whilst an accelerated observer in flat spacetime observes particles, at an event horizon observers need to accelerate just to remain at a fixed point. However, this is not the reason event horizons are thermal bodies. Here, the strength of gravitational fields is sufficient to produce pairs of particles in the vacuum. If one of the pair is absorbed by the event horizon, the other may escape to infinity. The spread of energies of the detected particles matches a black body, with greybody factors [49] due to gravitational potentials. The observed temperature, T, of the black hole is related to the surface acceleration κ by

$$\frac{\kappa}{2\pi} = T. \tag{2.3.4}$$

κ has the interpretation of the acceleration needed to keep a particle stationary at the horizon of a non-rotating black hole. It is determined for a stationary black hole by [8] considering a Killing field χ_a which, if not proportional to the stationary Killing field ξ_a, is the sum of ξ_a and an axial Killing field ψ_a and which is normal to the horizon. Then as a horizon is a null surface, $\chi_a\chi^a = 0$ on the horizon, implying $\nabla^b\chi_a\chi^a$ is itself normal to the horizon. So there exists some constant of proportionality describing the relationship between these two vectors normal to the horizon: this constant is κ

$$\nabla^a\chi^b\chi_b = -\kappa\chi^a. \tag{2.3.5}$$

Note that the definition of κ does not specify where we are on the surface of the black hole in any way, so it is constant across the horizon. This is reminiscent of the fact that the temperature of a body in equilibrium is constant throughout it. In general, if a body has a non-zero temperature T, there is a relationship between increases in its energy and increases in its entropy, of the form

$$\delta E = T\delta S. \tag{2.3.6}$$

Direct classical calculations [50] show that a change in the mass of a (non-rotating, uncharged) black hole results in a change in its area

$$\delta M = \frac{1}{8\pi}\kappa\delta A. \tag{2.3.7}$$

The mass of a black hole is equal to its energy [8]. Thus we can use the identification between surface gravity and temperature to write a linear relationship between entropy and area [12]

$$S = A/4G \tag{2.3.8}$$

where A is the area of the horizon and G Newton's constant.

If a black hole is rotating, it will produce an ergoregion. These are areas external to the horizon where the timelike variable as determined by observers at infinity is spacelike. As a result, objects in this region cannot remain stationary with respect to these observers. This is seen by looking at the tangent of a timelike path in this region [8],

$$g_{\mu\nu}\dot{x}^\mu(\lambda)\dot{x}^\nu(\lambda) < 0 \tag{2.3.9}$$

where λ is proper time along the trajectory. Every term contributing to the sum on the left is positive due to the spacelike nature of the asymptotically defined timelike co-ordinate, with the possible exception of

$$2g_{t\theta_i}\dot{x}^t(\lambda)\dot{x}^{\theta_i}(\lambda) = 2g_{t\theta_i}\frac{dt}{d\lambda}\frac{d\theta_i}{d\lambda} \tag{2.3.10}$$

where θ_i is (one of) the direction the black hole is rotating in. Then $d\theta_i/d\lambda$ must be less than 0; in particular it cannot equal 0.

A third type of horizon, that resulting from the imposition of identifications on co-ordinates, does not necessarily have the properties used to define κ. These horizons generally arise because the act of identifying variables results in singularities which will eventually absorb incoming matter. Any regions of spacetime which are bound to coincide with the singularity must be regarded as being behind a horizon. These horizons are not boundaries of trapped surfaces, and they need not be null hypersurfaces; in particular, they are not necessarily Killing horizons. So the act of assigning to them a temperature and hence an entropy is not justified.

2.4 Conformal Field Theories

As the AdS/CFT correspondence is a relationship mapping geometries to states in conformal field theories (CFTs), we cover here some relevant aspects of CFTs.

Conformal field theories are a vast topic and have been studied in a number of applications. The most important one for the area of work we are in is that of their existence on string worldsheets; however, we will also be concerned with the field theories which exist on the conformal boundary of spacetimes, which may have different features.

Conformal field theories have been formulated in a variety of dimensions, but are particularly rich when working with 2, or 1+1, dimensions and it is on these that we focus. We will work with two independent complex variables, z and $\bar{z}$. In this number of dimensions, the group of conformal transformations, that is, ones which preserve angles, is infinite dimensional. This results in an infinite number of conserved quantities. The conserved currents of a CFT always include the stress-energy tensor, T, which is traceless by conformal invariance. The holomorphic part of it, written $T_{zz} = T(z)$, has a Laurent expansion

$$T(z) = \Sigma \frac{L_n}{z^{n+2}} \tag{2.4.1}$$

and the anti-holomorphic part likewise, with $\bar{L}_m$ and $\bar{z}$. The modes L_n form a Virasaro algebra with central charge c [51]

$$[L_n, L_m] = \frac{c}{12} m(m^2 - 1)\delta_{m,-n} + (m - n)L_{m+n}. \tag{2.4.2}$$

Other important fields in a CFT are those fields which, under a rescaling of their argument $z \longrightarrow \lambda z = z', \bar{z} \longrightarrow \lambda \bar{z} = \bar{z}'$ behave like

$$\phi(z', \bar{z}')(dz')^h (d\bar{z}')^{\bar{h}} = \phi(z, \bar{z})(dz)^h (d\bar{z})^{\bar{h}}. \tag{2.4.3}$$

They are called primary fields and are said to have scaling dimensions or conformal weights $(h, \bar{h})$. The *operator product expansion* of a primary field of weight h with the holomorphic part of the stress energy tensor is of the form [52]

$$T(z)\phi(w, \bar{w}) \sim \frac{h}{(z - w)^2}\phi(w, \bar{w}) + \frac{1}{z - w}\partial_w \phi(w, \bar{w}). \tag{2.4.4}$$

Operator product expansions (OPEs) are useful tools in the field theory literature [1, 53] which write as sums of operators the behaviour of fields as they are brought together. The coefficients may be singular, but the result must not involve any new operators that were not already present in the theory. OPEs contain information about commutators, and those involving the stress-energy tensor tell us whether or not the operators involved are primary fields and of what weight.

Again, similar expressions hold for the anti-holomorphic part of the stress tensor $\bar{T}(\bar{z})$.

In two dimensions, we have the ability to map from fields to states. We may form a state from a field, labelling it with the related scaling dimension, via

$$|h\rangle = lim_{z \to 0} \phi(z)|\text{vac}\rangle. \tag{2.4.5}$$

The modes of the stress-energy tensor, L_n, are now operators on the states. The vacuum $|vac\rangle = |0\rangle$ is invariant under the operators L_{-1}, L_0, L_1, the $SL(2)$ subgroup of the Virasaro algebra.

In this language, a primary field becomes a highest weight state; that is, annihilated by $L_n, n > 0$ [54]. The operator L_0 when applied to a state corresponding to a primary field ϕ evaluates its conformal weight:

$$L_0\phi(0)|0\rangle = L_0|h\rangle = h|h\rangle. \tag{2.4.6}$$

The full field, depending on z and $\bar{z}$, then has scaling dimension $\Delta = h + \bar{h}$. In this formulation, $L_0 + \bar{L}_0$ is equivalent to the momentum operator so Δ is the energy of the field. The angular momentum of a field is $s = h - \bar{h}$. Later we shall have cause to refer to this as q_p.

It is possible to extend conformal field theories by introducing one or more Grassman variables, θ such that $\theta^2 = 0$, so that the arguments of fields are in superspace. These are known as superconformal field theories (SCFTs.) Any string worldsheet theory requires a minimum of $N = 1$ supersymmetry, whereby the stress-energy tensor has one superpartner (generally written G), if it is to produce spacetime fermions [19]. A superstress tensor has the form

$$T(z,\theta) = T_{z\theta}(z) + \theta T_{zz}(z) \tag{2.4.7}$$

and its expansion is [55]

$$T = \Sigma_n z^{-n-3/2}\frac{1}{2}G_n + \theta\Sigma_m z^{-m-2}L_m. \tag{2.4.8}$$

The modes G_n may have $n \in \mathbb{Z}$ or $\mathbb{Z} + 1/2$. In the former case, we are said to be in the Ramond (R) sector; in the latter, the Neveu-Schwarz (NS) sector.

The coefficients of the expansion generate the super-Virasaro algebra

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{c}{12}(m^3 - m)\delta_{m,-n} \tag{2.4.9}$$

$$\{G_m, G_n\} = 2L_{m+n} + \frac{c}{3}(m^2 - \frac{1}{4})\delta_{m,-n} \tag{2.4.10}$$

$$[L_m, G_n] = (\frac{1}{2}m - n)G_{m+n}. \tag{2.4.11}$$

The SCFTs we are most interested in will have $N = 2$ supersymmetry or $N = 4$, a particular case of $N = 2$. The stress energy tensor will now have two or four superpartners, commonly written G^i. Notably, with this amount of supersymmetry a non-trivial symmetry is present amongst the supercharges G^i, known as

$R-$symmetry [56]. The 'R-charge' , j, of a state in $N = 2$ is then its value under the zero-mode of a conserved current, J which generates the R-symmetry,

$$J_0|\psi\rangle = j|\psi\rangle. \tag{2.4.12}$$

For $N = 2$, G^1 and G^2 can be arranged into $G^{\pm}$ which have charges ± 1 with respect to J.

In $N = 4$, the supercharges can be organized into two complex quantities, which form an $SU(2)$ doublet [57]. We also now have three conserved currents J^i. We usually take J^3 as playing the role of R-symmetry [20] although an additional $SU(2)$ maps the J^i amongst themselves.

An important set of states are those which have an R-charge equal (in our normalization [20]) to their scaling dimension: $h = j$ or

$$L_0\phi(0)|0\rangle = J_0^3\phi(0)|0\rangle. \tag{2.4.13}$$

Such states are called chiral primaries : they are annihilated by half the supercharges of the theory as can be seen from the relevant commutation relation

$$\{Q_r^+, Q_s^-\} = 2L_{r+s} + 2(r-s)J_{r+s}^3 + \frac{c}{3}\delta_{r,-s}(r^2 - \frac{1}{4}). \tag{2.4.14}$$

This implies that the states $Q_{1/2}|h\rangle$ with $h = j$ are of zero norm. The interest in such states stems from their invariance under changes in moduli of theories, which is due to their belonging to short multiplets.

The system with which we will be concerned in chapter 3 is that outlined in 2.1, of N_1 $D1-$branes and N_5 $D5-$branes in a space with topology $\mathbb{R}^{1,4} \times S^1 \times T^4$. The compact directions are not written T^5 as the two compact spaces play slightly different roles. The T^4 will be of very small radius, whilst the S^1 is sizable with radius R. Four of the directions of the $D5$-branes will be wrapped on the T^4 and the remaining direction lies along the S^1 parallel to the $D1-$branes. They appear as effective strings in the remaining dimensions. The 'spacetime' CFT will thus be $1+1$ dimensional. It is believed [58] that the resulting CFT is the IR limit of a strongly coupled $\mathcal{N} = 4$ non-linear sigma model on $\mathbb{R} \times S^1$ with central charge $c = 6N_1N_5$. Its target space is a member of the family of spaces given by deformations of the symmetrized product of N_1N_5 copies of T^4; that is, $T^{4N_1N_5}/S_{N_1N_5}$ [59, 60]. Note that this T^4 is not identical to that in the spacetime as this latter one may change size with position. The set of spaces includes the orbifold point itself, where we can perform field theory calculations [59,61] but the gravity description is not valid [20]. The spectrum of states of the orbifold field theory contains chiral primaries which are twist operators σ_n^{--}, linking up n of the strings together to form long strings of length nR [62] and carrying charge $h = j$ [63]. The basic twist operator is σ_2^{--}, which we can apply n times to create σ_n^{--} since any permutation of n objects can be decomposed into permutations of just two objects. So we can link n strings in any order to make a long string.

Chapter 3

Non-supersymmetric smooth geometries and D1-D5-P bound states

In this chapter we construct smooth non-supersymmetric soliton solutions with D1-brane, D5-brane and momentum charges in type IIB supergravity compactified on $T^4 \times S^1$, with the charges along the compact directions. This generalises previous studies of smooth supersymmetric solutions. The solutions are obtained by considering a known family of $U(1) \times U(1)$ invariant metrics, and studying the conditions imposed by requiring smoothness. We discuss the relation of our solutions to states in the CFT describing the D1-D5 system, and describe various interesting features of the geometry.

3.1 Introduction and Summary

String theory has made tremendous advances in understanding the microscopic origins of black hole entropy [13,64]. In the original calculations, two different dual descriptions of a supersymmetric object were considered: a weakly-coupled description in terms of perturbative strings and D-branes, and a strongly-coupled description as a classical black hole solution. The picture of this black hole, as a background for the perturbative string, is essentially the same as in semiclassical general relativity.

We have a singularity in spacetime that is shielded (censored) by a horizon. The horizon area determines the Bekenstein-Hawking entropy $S_{BH} = \frac{A}{4G_N}$. This entropy was successfully reproduced by counting the degenerate supersymmetric vacua in the dual perturbative D-brane description. This picture did not provide any understanding of where the microstates were in the strong-coupling black hole picture: smooth black hole solutions 'have no hair', so the geometry is entirely determined by the charges [65]. There was, however, a suggestion that pure states would be dual to geometries which were not smooth at the event horizon [66].

The anti-de Sitter/conformal field theory (AdS/CFT) correspondence [5, 15, 17] provided a deeper understanding of the counting of black hole entropy in string theory. The black holes in AdS are identified with the thermal ensemble in the dual CFT. The CFT was conjectured to provide a fundamental, non-perturbative description of string theory with asymptotically AdS boundary conditions, so the microstates were fundamentally thought of as states in the CFT, and it did not appear that they could be thought of as living somewhere in the black hole geometry. The evolution of the states in the CFT is unitary. Certain states can be identified with classical geometries, but as has been emphasised in e.g. [17, 67], the CFT provides a fully quantised description, and reproducing the behaviour of the CFT from a spacetime point of view will in general involve a sum over bulk geometries.

In a series of papers, Mathur and his collaborators have challenged the conventional picture of a black hole in string theory (see [28] for a review). They argue that the black hole geometry is merely a coarse grained description of the spacetime, and that each of the $e^{S_{BH}}$ microstates can be identified with a perfectly regular geometry with neither horizon nor singularity [68, 69]. The black hole entropy is a result of averaging over these different geometries, which produces an 'effective horizon', which describes the scale at which the $e^{S_{BH}}$ individual geometries start to differ from each other. They further argue that if a system in an initial pure state undergoes gravitational collapse, it will produce one of these smooth geometries, and the real spacetime does not have a global event horizon, thus avoiding the information loss paradox associated with outgoing Hawking radiation [70]. Thus, the idea is that stringy effects modify the geometry of spacetime at the event horizon, rather than, as would be expected from the classical point of view, at Planck or string distances from the singularity. This is a radical modification of the expected geometry. There are similarities with the correspondence principle ideas [71], but unlike that picture, there is no obvious sense in which the spacetime as seen by an infalling observer will be different. It is difficult to see how the singularity behind the black hole's event horizon can arise from a coarse graining over non-singular geometries.[1]

The evidence for this proposal comes from the construction of smooth asymptotically flat geometries in the D1-D5 system that can be identified with individual microstates in the CFT on the worldvolume of the branes. The theory considered is type IIB supergravity compactified on $S^1 \times T^4$ with n_5 D5-branes wrapping $S^1 \times T^4$, and n_1 D-strings wrapping the S^1. The near-horizon geometry is $AdS_3 \times S^3 \times T^4$, and has a dual $1+1$ dimensional CFT description with $c = 6n_1n_5$. The first such geometries were constructed in [73, 74], and correspond to the Ramond-Ramond (RR) ground state obtained by spectral flow from the Neveu-Schwarz–Neveu-Schwarz (NSNS) vacuum state. This was subsequently extended [68, 75, 76] to find a family of smooth geometries corresponding to the whole family of RR ground states in the CFT. The D1/D5 system via string dualities is the same as a system with n_5 units of fundamental string winding and n_1 units of fundamental string momentum on a circle. The D1/D5 bound state corresponds to a multi-wound F-string carrying momentum, and the geometries are characterised as functions of the displacement of the string in its transverse directions. As a test of whether the two-charge system indeed describes the correct physics, the collision time for left- and right-moving excitations on the component string was computed in field theory and compared to the time for graviton absorption and re-emission in supergravity; the two are found to match [30, 68]. The degeneracy of RR ground states in this theory gives a microscopic entropy which scales as $\sqrt{n_1 n_5}$; this was found to match a suitable counting in a supertube description in [77, 78]. However, this entropy is not large enough to correspond to a black hole with a macroscopic horizon. It is therefore important to extend the identification to states that carry a third charge n_p, momentum along the

[1]Although it may be that most measurements in the dual CFT find it difficult to distinguish between regular geometries and the conventional semi-classical picture of a black hole [72].

string. These states have a microscopic degeneracy $\sqrt{n_1 n_5 n_p}$, and were used in [13] in the calculation of the black hole entropy. Recently, Giusto, Mathur, and Saxena have identified smooth geometries corresponding to some of these states [79, 80], although the geometries constructed so far correspond to very special states, the spectral flows of the RR ground states studied earlier.[2] The overall evidence for the picture of black holes advanced by these authors is, in our judgement, interesting but not yet compelling.

We will extend these investigations to find more general solitonic solutions in supergravity, and to identify corresponding CFT states. We believe that whether or not the picture of black holes advanced by Mathur and collaborators proves to be correct, these solitonic solutions will remain of interest in their own right. It is particularly interesting that we can find completely smooth non-supersymmetric solitons. These are, as far as we are aware, the first explicit examples of this type.

We find these solutions by generalising an analysis previously carried out for special cases in [73, 74, 79, 80]. We consider the nonextremal rotating three-charge black holes given in [88], and systematically search for values of the parameters for which the solution is smooth and free of singularities. We find that if we allow non-supersymmetric solutions, there are two integers m, n labelling the soliton solutions. The previously studied supersymmetric solutions correspond to $m = n+1$. Thus, we find new non-supersymmetric solitons. Further solutions, some of which are smooth, can be constructed by orbifolds of this basic family. This provides another integer degree of freedom k. Some of the supersymmetric orbifolds have not been previously studied.

We identify the basic family of smooth solutions labelled by m, n with states in the CFT constructed by spectral flow from the NSNS vacuum, with $m + n$ units of spectral flow applied on the left and $m - n$ units of spectral flow applied on the right. We find a non-trivial agreement between the spacetime charges in these geometries and the expectations from the CFT point of view. This agreement extends to

[2]Three-charge states were previously studied in the supertube description [81, 82] in [83, 84]. Other supersymmetric three-charge solutions have been found in [85–87], but the regular solutions have not yet been identified or related to CFT states.

the geometries constructed as orbifolds of the basic smooth solutions. We have studied the wave equation on these geometries, and we find that as in [80], there is a mismatch between the spacetime result, $\Delta t_{sugra} = \pi R \varrho$, and the expectation from the CFT point of view, $\Delta t_{CFT} = \pi R$. We believe that understanding this mismatch is a particularly interesting issue for further development. Finally, we discuss the appearance of an ergoregion in the non-supersymmetric solutions. We find that the ergoregion does not lead to any superradiant scattering for free fields.

The existence of these non-supersymmetric solitons, and the fact that they can be identified with states in the dual CFT, might be regarded as further evidence for the proposed description of black holes. However, we would advocate caution. We still find it questionable whether we can really describe a black hole in this way. First of all, the three-charge states described so far are very special. The orbifolds we consider provide examples where the CFT state is not the spectral flow of a RR ground state, but the geometries we consider all have a $U(1) \times U(1)$ invariance. It is unclear whether the techniques used to date can be extended to obtain even the geometries corresponding to spectral flows of the more general RR ground states of [68,76], let alone to reproduce the full $e^{\sqrt{n_1 n_5 n_p}}$ states required to explain the black hole entropy. The much more difficult dynamical questions — how the appearance of a global event horizon in gravitational collapse can always be avoided, for example — have not yet been tackled. Nonetheless, the study of these smooth geometries offers a new perspective on the relation between CFT and spacetime, and it is interesting to see that their existence does not depend on supersymmetry.

This chapter is organised as follows. In the next section, we recall the metric and matter fields for the general family of solutions we consider, and discuss the near-horizon limit which relates asymptotically flat solutions to asymptotically $AdS_3 \times S^3$ ones. In section 3.3, we discuss the constraints required to obtain a smooth soliton solution. We find that there is a basic family of smooth solutions labelled by the radius R of the S^1, the D1 and D5 brane charges Q_1, Q_5, and two integers m, n. Further solutions can be constructed as $\mathbb{Z}_k$ orbifolds of these basic ones; they will be smooth if m and n are both relatively prime to k. We also discuss the asymptotically $AdS_3 \times S^3$ solutions obtained by considering the near-horizon limit.

The asymptotically $AdS_3 \times S^3$ solutions corresponding to the basic family of smooth solutions are always global $AdS_3 \times S^3$ up to some coordinate transformation. In section 3.4, we verify that the solutions are indeed smooth and free of closed timelike curves. In section 3.5, we identify the corresponding states in the CFT, identifying the coordinate shift in the global $AdS_3 \times S^3$ solutions with spectral flow. Finally in section 3.6, we discuss the massless scalar wave equation on these solutions, and show that the non-supersymmetric solutions always have an ergoregion.

3.2 General nonextremal solution

We will look for smooth solutions as special cases of the nonextremal rotating three-charge black holes given in [89] (uplifted to ten-dimensional supergravity following [90]). The original two-charge supersymmetric solutions of [73, 74] were found in this way, and the same approach was applied more recently in [79, 80] to find supersymmetric three-charge solutions. In the present work, we aim to find all the smooth solutions within this family.

In this section, we discuss this family of solutions in general, writing the metric in forms that will be useful for finding and discussing the smooth solutions. We will also discuss the relation between asymptotically flat and asymptotically $\mathrm{AdS}_3 \times S^3$ solutions. We write the metric as

$$\begin{aligned}
\mathrm{d}s^2 \;=\;& -\frac{f}{\sqrt{\tilde{H}_1\tilde{H}_5}}(\mathrm{d}t^2-\mathrm{d}y^2)+\frac{M}{\sqrt{\tilde{H}_1\tilde{H}_5}}(s_p\mathrm{d}y-c_p\mathrm{d}t)^2\\
&+\sqrt{\tilde{H}_1\tilde{H}_5}\left(\frac{r^2\mathrm{d}r^2}{(r^2+a_1^2)(r^2+a_2^2)-Mr^2}+\mathrm{d}\theta^2\right)\\
&+\left(\sqrt{\tilde{H}_1\tilde{H}_5}-(a_2^2-a_1^2)\frac{(\tilde{H}_1+\tilde{H}_5-f)\cos^2\theta}{\sqrt{\tilde{H}_1\tilde{H}_5}}\right)\cos^2\theta\mathrm{d}\psi^2\\
&+\left(\sqrt{\tilde{H}_1\tilde{H}_5}+(a_2^2-a_1^2)\frac{(\tilde{H}_1+\tilde{H}_5-f)\sin^2\theta}{\sqrt{\tilde{H}_1\tilde{H}_5}}\right)\sin^2\theta\mathrm{d}\phi^2\\
&+\frac{M}{\sqrt{\tilde{H}_1\tilde{H}_5}}(a_1\cos^2\theta\mathrm{d}\psi+a_2\sin^2\theta\mathrm{d}\phi)^2\\
&+\frac{2M\cos^2\theta}{\sqrt{\tilde{H}_1\tilde{H}_5}}[(a_1c_1c_5c_p-a_2s_1s_5s_p)\mathrm{d}t+(a_2s_1s_5c_p-a_1c_1c_5s_p)\mathrm{d}y]\mathrm{d}\psi\\
&+\frac{2M\sin^2\theta}{\sqrt{\tilde{H}_1\tilde{H}_5}}[(a_2c_1c_5c_p-a_1s_1s_5s_p)\mathrm{d}t+(a_1s_1s_5c_p-a_2c_1c_5s_p)\mathrm{d}y]\mathrm{d}\phi\\
&+\sqrt{\frac{\tilde{H}_1}{\tilde{H}_5}}\sum_{i=1}^{4}\mathrm{d}z_i^2
\end{aligned} \tag{3.2.1}$$

where

$$\tilde{H}_i=f+M\sinh^2\delta_i,\quad f=r^2+a_1^2\sin^2\theta+a_2^2\cos^2\theta, \tag{3.2.2}$$

and $c_i=\cosh\delta_i$, $s_i=\sinh\delta_i$. This metric is more usually written in terms of functions $H_i=\tilde{H}_i/f$. Writing it in this way instead makes it clear that there is no singularity at $f = 0$. As the determinant of the metric is

$$g = -r^2 \frac{\tilde{H}_1^3}{\tilde{H}_5} \cos^2\theta \sin^2\theta, \tag{3.2.3}$$

it is clear that the inverse metric is also regular when $f = 0$. The above metric is in the string frame, and the dilaton is

$$e^{2\Phi} = \frac{\tilde{H}_1}{\tilde{H}_5}. \tag{3.2.4}$$

From [79], the 2-form gauge potential which supports this configuration is

$$\begin{aligned} C_2 &= \frac{M\cos^2\theta}{\tilde{H}_1}\left[(a_2 c_1 s_5 c_p - a_1 s_1 c_5 s_p)dt + (a_1 s_1 c_5 c_p - a_2 c_1 s_5 s_p)dy\right] \wedge d\psi \\ &+ \frac{M\sin^2\theta}{\tilde{H}_1}\left[(a_1 c_1 s_5 c_p - a_2 s_1 c_5 s_p)dt + (a_2 s_1 c_5 c_p - a_1 c_1 s_5 s_p)dy\right] \wedge d\phi \\ &- \frac{M s_1 c_1}{\tilde{H}_1} dt \wedge dy - \frac{M s_5 c_5}{\tilde{H}_1}(r^2 + a_2^2 + M s_1^2)\cos^2\theta d\psi \wedge d\phi. \end{aligned} \tag{3.2.5}$$

We take the T^4 in the z_i directions to have volume V, and the y circle to have radius R, that is $y \sim y + 2\pi R$.

Compactifying on $T^4 \times S^1$ yields an asymptotically flat five-dimensional configuration. The gauge charges are determined by

$$Q_1 = M \sinh\delta_1 \cosh\delta_1, \tag{3.2.6}$$

$$Q_5 = M \sinh\delta_5 \cosh\delta_5, \tag{3.2.7}$$

$$Q_p = M \sinh\delta_p \cosh\delta_p, \tag{3.2.8}$$

where the last is the charge under the Kaluza-Klein gauge field associated with the reduction along y. The five-dimensional Newton's constant is $G^{(5)} = G^{(10)}/(2\pi RV)$; if we work in units where $4G^{(5)}/\pi = 1$, the Einstein frame ADM mass and angular momenta are

$$M_{ADM} = \frac{M}{2}(\cosh 2\delta_1 + \cosh 2\delta_5 + \cosh 2\delta_p), \tag{3.2.9}$$

$$J_\psi = -M(a_1 \cosh\delta_1 \cosh\delta_5 \cosh\delta_p - a_2 \sinh\delta_1 \sinh\delta_5 \sinh\delta_p), \tag{3.2.10}$$

$$J_\phi = -M(a_2 \cosh\delta_1 \cosh\delta_5 \cosh\delta_p - a_1 \sinh\delta_1 \sinh\delta_5 \sinh\delta_p) \tag{3.2.11}$$

(which are invariant under interchange of the δ_i). We see that the physical range of M is $M \geq 0$. We will assume without loss of generality $\delta_1 \geq 0$, $\delta_5 \geq 0$, $\delta_p \geq 0$ and $a_1 \geq a_2 \geq 0$.

We also want to rewrite this metric as a fibration over a four-dimensional base space. It has been shown in [91] that the general supersymmetric solution in minimal six-dimensional supergravity could be written as a fibration over a four-dimensional hyper-Kähler base, and writing the supersymmetric two-charge solutions in this form played an important role in understanding the relation between these solutions and supertubes in [76] and in an attempt to generate new asymptotically flat three-charge solutions by spectral flow [92]. The supersymmetric three-charge solutions were also written in this form in [93]. Of course, in the non-supersymmetric case, we do not expect the base to have any particularly special character, but we can still use the Killing symmetries ∂_t and ∂_y to rewrite the metric (3.2.1) as a fibration of these two directions over a four-dimensional base space. This gives

$$\begin{aligned} ds^2 \;=\; & \frac{1}{\sqrt{\tilde{H}_1 \tilde{H}_5}} \Big\{ -(f-M)\left[d\tilde{t} - (f-M)^{-1} M \cosh\delta_1 \cosh\delta_5 (a_1 \cos^2\theta d\psi + a_2 \sin^2\theta d\phi)\right]^2 \\ & + f\left[d\tilde{y} + f^{-1} M \sinh\delta_1 \sinh\delta_5 (a_2 \cos^2\theta d\psi + a_1 \sin^2\theta d\phi)\right]^2 \Big\} \\ & + \sqrt{\tilde{H}_1 \tilde{H}_5} \Big\{ \frac{r^2 dr^2}{(r^2+a_1^2)(r^2+a_2^2) - Mr^2} + d\theta^2 \\ & + (f(f-M))^{-1} \left[\left(f(f-M) + f a_2^2 \sin^2\theta - (f-M) a_1^2 \sin^2\theta\right) \sin^2\theta d\phi^2 \right. \\ & + 2 M a_1 a_2 \sin^2\theta \cos^2\theta d\psi d\phi \\ & \left. + \left(f(f-M) + f a_1^2 \cos^2\theta - (f-M) a_2^2 \cos^2\theta\right) \cos^2\theta d\psi^2 \right] \Big\}, \end{aligned} \tag{3.2.12}$$

where $\tilde{t} = t\cosh\delta_p - y\sinh\delta_p$, $\tilde{y} = y\cosh\delta_p - t\sinh\delta_p$.

We can see that this is still a 'natural' form of the metric, even in the non-supersymmetric case, inasmuch as the base metric in the second $\{\}$ is independent of the charges. This form of the metric is as a consequence convenient for studying the 'near-horizon' limit, as we will now see.

In addition to the asymptotically flat metrics written above, we will be interested in solutions which are asymptotically $AdS_3 \times S^3$. These asymptotically $AdS_3 \times S^3$ geometries can be thought of as describing a 'core' region in our asymptotically flat soliton solutions, but they can also be considered as geometries in their own right. It is relatively easy to identify the appropriate CFT duals when we consider the asymptotically $AdS_3 \times S^3$ geometries. To prepare the ground for this discussion, we should consider the 'near-horizon' limit in the general family of metrics.

The near-horizon limit is usually obtained by assuming that $Q_1, Q_5 \gg M, a_1^2, a_2^2$, and focusing on the region $r^2 \ll Q_1, Q_5$. This limiting procedure is easily described if we consider the metric in the form (3.2.12): it just amounts to 'dropping the 1' in the harmonic functions $\tilde{H}_1, \tilde{H}_5$, that is, replacing $\tilde{H}_1 \approx Q_1$, $\tilde{H}_5 \approx Q_5$, and also approximating $M \sinh\delta_1 \sinh\delta_5 \approx M \cosh\delta_1 \cosh\delta_5 \approx \sqrt{Q_1 Q_5}$ in the cross terms in the fibration. This gives us the asymptotically $\mathrm{AdS}_3 \times S^3$ geometry

$$\begin{aligned} ds^2 &= \frac{1}{\sqrt{Q_1 Q_5}} \Big\{ -(f-M)[d\tilde{t} - (f-M)^{-1}\sqrt{Q_1 Q_5}(a_1 \cos^2\theta d\psi + a_2 \sin^2\theta d\phi)]^2 \\ &\quad + f\,[d\tilde{y} + f^{-1}\sqrt{Q_1 Q_5}(a_2 \cos^2\theta d\psi + a_1 \sin^2\theta d\phi)]^2 \Big\} \\ &\quad + \sqrt{Q_1 Q_5} \Big\{ \frac{r^2 dr^2}{(r^2+a_1^2)(r^2+a_2^2) - Mr^2} + d\theta^2 \\ &\quad + (f(f-M))^{-1} \big[\big(f(f-M) + f a_2^2 \sin^2\theta - (f-M) a_1^2 \sin^2\theta\big) \sin^2\theta d\phi^2 \\ &\quad + 2M a_1 a_2 \sin^2\theta \cos^2\theta d\psi d\phi \\ &\quad + \big(f(f-M) + f a_1^2 \cos^2\theta - (f-M) a_2^2 \cos^2\theta\big) \cos^2\theta d\psi^2 \big] \Big\}. \end{aligned} \tag{3.2.13}$$

This can be rewritten as

$$\begin{aligned} ds^2 &= -\left(\frac{\rho^2}{\ell^2} - M_3 + \frac{J_3^2}{4\rho^2}\right) d\tau^2 + \left(\frac{\rho^2}{\ell^2} - M_3 + \frac{J_3^2}{4\rho^2}\right)^{-1} d\rho^2 + \rho^2 \left(d\varphi + \frac{J_3}{2\rho^2} d\tau\right)^2 \\ &\quad + \ell^2 d\theta^2 + \ell^2 \sin^2\theta [d\phi + \frac{R}{\ell^2}(a_1 c_p - a_2 s_p) d\varphi + \frac{R}{\ell^3}(a_2 c_p - a_1 s_p) d\tau]^2 \\ &\quad + \ell^2 \cos^2\theta [d\psi + \frac{R}{\ell^2}(a_2 c_p - a_1 s_p) d\varphi + \frac{R}{\ell^3}(a_1 c_p - a_2 s_p) d\tau]^2, \end{aligned} \tag{3.2.14}$$

where we have defined the new coordinates

$$\varphi = \frac{y}{R}, \quad \tau = \frac{t\ell}{R}, \tag{3.2.15}$$

$$\rho^2 = \frac{R^2}{\ell^2}[r^2 + (M - a_1^2 - a_2^2)\sinh^2\delta_p + a_1 a_2 \sinh 2\delta_p] \tag{3.2.16}$$

and parameters

$$\ell^2 = \sqrt{Q_1 Q_5}, \tag{3.2.17}$$

$$M_3 = \frac{R^2}{\ell^4}[(M - a_1^2 - a_2^2)\cosh 2\delta_p + 2a_1 a_2 \sinh 2\delta_p], \tag{3.2.18}$$

$$J_3 = \frac{R^2}{\ell^3}[(M - a_1^2 - a_2^2)\sinh 2\delta_p + 2a_1 a_2 \cosh 2\delta_p]. \tag{3.2.19}$$

Thus, we see that we recover the familiar observation that the near-horizon limit of the six-dimensional charged rotating black string is a twisted fibration of S^3 over the BTZ black hole solution [90].

3.3 Finding solitonic solutions

In general, these solutions will have singularities, horizons, and possibly also closed timelike curves. Let us now consider the conditions for the spacetime to be free of these features, giving a smooth solitonic solution.

Written in the form (3.2.1), the metric has coordinate singularities when $\tilde{H}_1 = 0$, $\tilde{H}_5 = 0$ or $g(r) \equiv (r^2 + a_1^2)(r^2 + a_2^2) - Mr^2 = 0$. In addition, the determinant of the metric vanishes if $\cos^2\theta = 0$, $\sin^2\theta = 0$, or $r^2 = 0$, which will therefore be singular loci for the inverse metric. The singularities at $\tilde{H}_1 = 0$ or $\tilde{H}_5 = 0$ are real curvature singularities, so we want to find solutions where $\tilde{H}_1 > 0$ and $\tilde{H}_5 > 0$ everywhere. The vanishing of the determinant at $\theta = 0$ and $\theta = \frac{\pi}{2}$ merely signals the degeneration of the polar coordinates at the north and south poles of S^3; these are known to be just coordinate singularities for arbitrary values of the parameters, so we will not consider them further.

The remaining coordinate singularities depend only on r. We can construct a smooth solution if the outermost one is the result of the degeneration of coordinates at a regular origin in some $\mathbb{R}^2$ factor; that is, of the smooth shrinking of an S^1. If this origin has a large enough value of r, we will have $\tilde{H}_1 > 0$ and $\tilde{H}_5 > 0$ there, and we will get a smooth solution. The coordinate singularity at $r^2 = 0$ cannot play this role, as we can shift it to an arbitrary position by defining a new radial coordinate by $\rho^2 = r^2 - r_0^2$. The determinant of the metric in the new coordinate system will vanish at $\rho^2 = 0$.

The interesting coordinate singularities are thus those at the roots of $g(r)$, and the first requirement for a smooth solution is that this function *have* roots. If we write

$$g(r) = (r^2 - r_+^2)(r^2 - r_-^2) \tag{3.3.1}$$

with $r_+^2 > r_-^2$, then

$$r_\pm^2 = \frac{1}{2}(M - a_1^2 - a_2^2) \pm \frac{1}{2}\sqrt{(M - a_1^2 - a_2^2)^2 - 4a_1^2 a_2^2}. \tag{3.3.2}$$

We see that this function only has real roots for

$$|M - a_1^2 - a_2^2| > 2a_1 a_2. \tag{3.3.3}$$

There are two cases: $M > (a_1 + a_2)^2$, or $M < (a_1 - a_2)^2$. Note that in the former case, $r_+^2 > 0$, whereas in the latter, $r_+^2 < 0$ (which is perfectly physical, since as noted above, we are free to define a new radial coordinate by shifting r^2 by an arbitrary constant).

Assuming one of these two cases hold, we can define a new radial coordinate by $\rho^2 = r^2 - r_+^2$. Since $r^2 dr^2 = \rho^2 d\rho^2$, in this new coordinate system

$$g_{\rho\rho} = \sqrt{\tilde{H}_1 \tilde{H}_5} \frac{d\rho^2}{\rho^2 + (r_+^2 - r_-^2)}, \tag{3.3.4}$$

and the determinant of the metric is $g = -\rho^2 \frac{\tilde{H}_1^3}{\tilde{H}_5} \cos^2\theta \sin^2\theta$. Thus, in this coordinate system, the only potential problems are at $\rho^2 = 0$ and $\rho^2 = r_-^2 - r_+^2$, that is, at the two roots of the function $g(r)$.

To see what happens at $r^2 = r_+^2$, consider the geometry of the surfaces of constant r. The determinant of the induced metric is

$$g^{(ty\theta\phi\psi)} = -\cos^2\theta \sin^2\theta \tilde{H}_1^{1/2} \tilde{H}_5^{1/2} g(r). \tag{3.3.5}$$

Thus, at $r^2 = r_+^2$, the metric in this subspace degenerates. This can signal either an event horizon, where the surface $r^2 = r_+^2$ is null, or an origin, where $r^2 = r_+^2$ is of higher codimension. We can distinguish between the two possibilities by considering the determinant of the metric at fixed r and t; that is, in the (y, θ, ϕ, ψ) subspace. This is

$$\begin{aligned} g^{(y\theta\phi\psi)} &= \cos^2\theta \sin^2\theta \left\{ g(r) \left(r^2 + a_1^2 \sin^2\theta + a_2^2 \cos^2\theta + M(1 + s_1^2 + s_5^2 + s_p^2) \right) \right. \\ &\quad + r^2 M^2 (c_1^2 c_5^2 c_p^2 - s_1^2 s_5^2 s_p^2) + M^2 (M - a_1^2 - a_2^2) s_1^2 s_5^2 s_p^2 \\ &\quad \left. + 2M^2 a_1 a_2 s_1 c_1 s_5 c_5 s_p c_p \right\}. \end{aligned} \tag{3.3.6}$$

This will be positive at $r^2 = r_+^2$ if and only if $M > (a_1 + a_2)^2$. If it is, the constant t cross-section of $r^2 = r_+^2$ will be spacelike, and $r^2 = r_+^2$ is an event horizon. Thus, we can have smooth solitonic solutions without horizons only in the other case $M < (a_1 - a_2)^2$.

To have a smooth solution, we need a circle direction to be shrinking to zero at $r^2 = r_+^2$. That is, we need some Killing vector with closed orbits to be approaching zero. Then by a suitable choice of period we could identify $\rho^2 = 0$ with the origin in polar coordinates of the space spanned by ρ and the angular coordinate corresponding to this Killing vector. The Killing vectors with closed orbits are linear combinations

$$\xi = \partial_y - \alpha \partial_\psi - \beta \partial_\phi, \tag{3.3.7}$$

so a necessary condition for a circle degeneration is that (3.3.6) vanish at $r^2 = r_+^2$, so that some linear combination of this form has zero norm there. We can satisfy this condition in two different ways.

3.3.1 Two charge solutions: $a_1 a_2 = 0$

The first possibility is to set $a_2 = 0$, so $a_1 a_2 = 0$. Then for $M < a_1^2$, $r_+^2 = 0$, and we set (3.3.6) to zero at $r^2 = 0$ by taking one of the charges to vanish. We will focus on setting $\delta_p = 0$, since these solutions will have a natural interpretation in CFT terms. Recall that in string theory, we can interchange the different charges in this solution by U-dualities.

For this choice of parameters, the metric simplifies to

$$\begin{aligned} ds^2 &= \frac{1}{\sqrt{\tilde{H}_1 \tilde{H}_5}} [-(f-M)(dt - (f-M)^{-1} M c_1 c_5 a_1 \cos^2\theta d\psi)^2 \\ &\quad + f(dy + f^{-1} M s_1 s_5 a_1 \sin^2\theta d\phi)^2] \\ &\quad + \sqrt{\tilde{H}_1 \tilde{H}_5} \left(\frac{dr^2}{r^2 + a_1^2 - M} + d\theta^2 + \frac{r^2 \sin^2\theta}{f} d\phi^2 + \frac{(r^2 + a_1^2 - M)\cos^2\theta}{f - M} d\psi^2 \right). \end{aligned} \tag{3.3.8}$$

Since (3.3.6) vanishes at $r^2 = 0$, the orbits of a Killing vector of the form (3.3.7) must degenerate there. It is easy to use the simplified metric (3.3.8) to evaluate

$$\alpha = 0, \quad \beta = \frac{a_1}{M s_1 s_5}. \tag{3.3.9}$$

That is, if we define a new coordinate

$$\tilde{\phi} = \phi + \frac{a_1}{M s_1 s_5} y, \tag{3.3.10}$$

the direction which goes to zero is y at fixed $\tilde{\phi}, \psi$. To make $y \to y + 2\pi R$ at fixed $\tilde{\phi}, \psi$ a closed orbit, we require

$$\frac{a_1}{M s_1 s_5} R = m \in \mathbb{Z}. \tag{3.3.11}$$

Around $r = 0$, we then have

$$ds^2 = \ldots + \sqrt{\tilde{H}_1 \tilde{H}_5} \left(\frac{dr^2}{a_1^2 - M} + \frac{r^2 dy^2}{M^2 s_1^2 s_5^2} \right) + \ldots \tag{3.3.12}$$

This will be regular if we choose the radius of the y circle to be

$$R = \frac{M s_1 s_5}{\sqrt{a_1^2 - M}}. \tag{3.3.13}$$

Thus, the integer quantisation condition fixes

$$m = \frac{a_1}{\sqrt{a_1^2 - M}}. \tag{3.3.14}$$

With this choice of parameters, the solution is completely smooth, and $\theta, \tilde{\phi}, \psi$ are the coordinates on a smooth S^3 at the origin $r = 0$. We recover the smooth supersymmetric solutions of [73, 74] for $m = 1$.

From the CFT point of view, it is natural to regard the charges Q_1, Q_5 and the asymptotic radius of the circle R as fixed quantities. We can then solve (3.3.13) and (3.3.14) to find the other parameters, giving us a one integer parameter family of smooth solutions for fixed Q_1, Q_5, R. The integer (3.3.14) determines a dimensionless ratio a_1^2/M, while the other condition (3.3.13) fixes the overall scale (a_1, say) in terms of Q_1, Q_5, R.

3.3.2 Three charge solutions

Solutions with all three charges non-zero can be found by considering $a_1 a_2 \neq 0$. Setting (3.3.6) to zero at $r^2 = r_+^2$ implies that

$$M = a_1^2 + a_2^2 - a_1 a_2 \frac{(c_1^2 c_5^2 c_p^2 + s_1^2 s_5^2 s_p^2)}{s_1 c_1 s_5 c_5 s_p c_p} \tag{3.3.15}$$

and hence that

$$r_+^2 = -a_1 a_2 \frac{s_1 s_5 s_p}{c_1 c_5 c_p}. \tag{3.3.16}$$

The Killing vector which degenerates is (3.3.7) with[3]

$$\alpha = -\frac{s_p c_p}{(a_1 c_1 c_5 c_p - a_2 s_1 s_5 s_p)}, \quad \beta = -\frac{s_p c_p}{(a_2 c_1 c_5 c_p - a_1 s_1 s_5 s_p)}. \tag{3.3.17}$$

[3]This choice of parameters is most easily derived by requiring $g_{ty} \to 0$ at $\rho^2 = 0$; having derived it, one can then check that it also gives $g_{yy} \to 0$ at $\rho^2 = 0$ as required.

The associated shifts in the ϕ, ψ coordinates are hence

$$\tilde{\psi} = \psi - \frac{s_p c_p}{(a_1 c_1 c_5 c_p - a_2 s_1 s_5 s_p)} y, \quad \tilde{\phi} = \phi - \frac{s_p c_p}{(a_2 c_1 c_5 c_p - a_1 s_1 s_5 s_p)} y, \tag{3.3.18}$$

and $y \to y + 2\pi R$ at fixed $\tilde{\phi}, \tilde{\psi}$ will be a closed orbit if

$$\frac{s_p c_p}{(a_1 c_1 c_5 c_p - a_2 s_1 s_5 s_p)} R = n, \quad \frac{s_p c_p}{(a_2 c_1 c_5 c_p - a_1 s_1 s_5 s_p)} R = -m \tag{3.3.19}$$

for some integers n, m. As in the two-charge case, requiring regularity of the metric at the origin fixes the radius of the y circle. We do not give details of the calculation, but simply quote the result,

$$R = \frac{M s_1 c_1 s_5 c_5 (s_1 c_1 s_5 c_5 s_p c_p)^{1/2}}{\sqrt{a_1 a_2}(c_1^2 c_5^2 c_p^2 - s_1^2 s_5^2 s_p^2)}. \tag{3.3.20}$$

If we introduce dimensionless parameters

$$j = \left(\frac{a_2}{a_1}\right)^{1/2} \leq 1, \quad s = \left(\frac{s_1 s_5 s_p}{c_1 c_5 c_p}\right)^{1/2} \leq 1, \tag{3.3.21}$$

then the integer quantisation conditions determine these via

$$\frac{j + j^{-1}}{s + s^{-1}} = m - n, \quad \frac{j - j^{-1}}{s - s^{-1}} = m + n. \tag{3.3.22}$$

Note that this constraint is invariant under the permutation of the three charges. We note that we can rewrite the mass (3.3.15) as

$$M = a_1 a_2 (s^2 - j^2)(j^{-2} s^{-2} - 1) = a_1 a_2 nm (s^{-2} - s^2)^2, \tag{3.3.23}$$

so $M \geq 0$ implies $s^2 \geq j^2$ and $nm \geq 0$. Our assumption that $a_1 > a_2$ implies $n \geq 0$, so $m \geq 0$, and (3.3.22) implies $m > n$.

Thus, in this case, for given Q_1, Q_5, R, we have a two integer parameter family of smooth solutions. It is a little more difficult to make direct contact with the supersymmetric solutions of [79] in this case, since one needs to take a limit $a_1, a_2 \to \infty$, but these would correspond to $m = n+1$, as it turns out that $s = 1$ and $M = 0$ if and only if $m = n + 1$. We can also think of the two-charge solutions in the previous subsection as corresponding to the case $n = 0$. To gain some insight into the values of the parameters for other choices of m, n, we have plotted the dimensionless quantities $a_1/\sqrt{M}, a_2/\sqrt{M}$ for some representative values in figure 3.1.

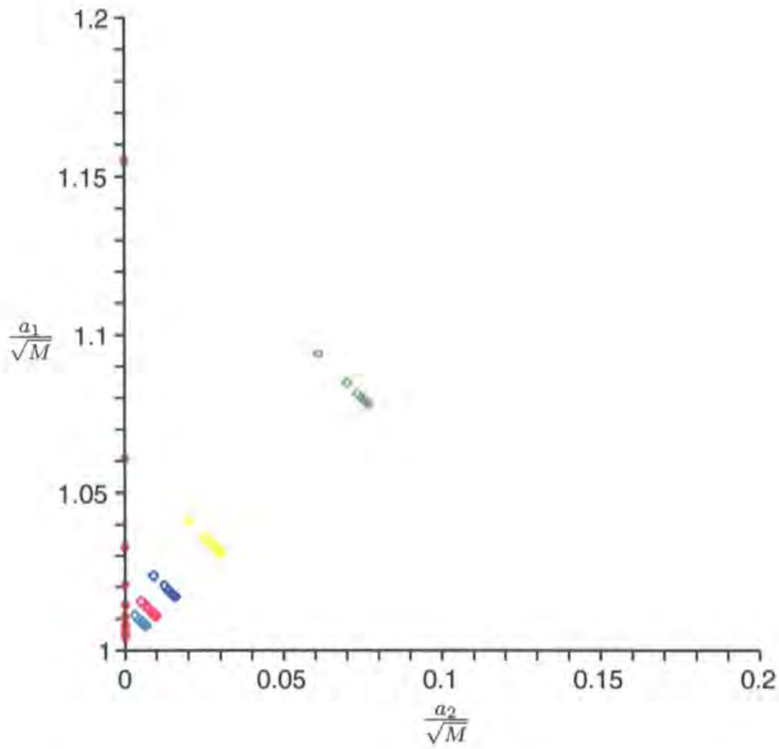

Figure 3.1: The values of the dimensionless quantities $a_2/\sqrt{M}, a_1/\sqrt{M}$ for which smooth solitons are obtained are indicated by points. The highest point on the figure corresponds to $m = 2, n = 0$. Increasing n moves diagonally downwards towards the diagonal, and increasing $m - n$ moves down towards $(0, 1)$. For each point, there is a set of orbifolds labelled by k. Solutions with event horizons exist in the region $a_1/\sqrt{M} + a_2/\sqrt{M} < 1$ (off the bottom of the plotted region).

3.3.3 Orbifolds & more general smooth three-charge solution

So far, we have insisted that the solution be smooth. However, in the context of string theory, we may also consider solutions with orbifold singularities, since the corresponding worldsheet conformal field theory is completely well-defined. In the context of the above smooth solutions, a particularly interesting class of orbifolds is the $\mathbb{Z}_k$ quotient by the discrete isometry $(y, \psi, \phi) \sim (y + 2\pi R/k, \psi, \phi)$.

In the two-charge case, the quotient acts as $(y, \psi, \tilde{\phi}) \sim (y+2\pi R/k, \psi, \tilde{\phi}+2\pi m/k)$ in the coordinates appropriate near $r = 0$. This isometry has a fixed point at $r = 0, \theta = 0$, so the resulting orbifold has a $\mathbb{Z}_k$ orbifold singularity there. In addition, if k and m are not relatively prime, there will be a $\mathbb{Z}_j$ orbifold singularity at $r = 0$ for all θ, where $j = \gcd(k, m)$. The supersymmetric orbifolds corresponding to $m = 1$ have previously been studied [30, 68, 73].

In the three-charge case, the discrete isometry becomes $(y, \tilde{\psi}, \tilde{\phi}) \sim (y+2\pi R/k, \tilde{\psi}-2\pi n/k, \tilde{\phi} + 2\pi m/k)$, and the $\mathbb{Z}_k$ will be freely acting if m and n are relatively prime to k. Thus, we get new smooth three-charged solutions by orbifolding by a k which is relatively prime to m and n. We could have found such solutions directly if we had allowed for the possibility that $y \to y + 2\pi Rk$ is the closed circle at $\rho = 0$, instead of insisting that it be $y \to y + 2\pi R$. We also have orbifolds similar to the two-charged ones if one or both of m and n are not relatively prime to k. In particular, the simple supersymmetric orbifolds studied in [80] correspond to taking $m = kn' + 1$, $n = kn'$ for some integer n'.[4] The preserved supersymmetries in the solutions with $m = n + 1$ correspond to Killing spinors which are invariant under translation in y at fixed ϕ, ψ, so all the orbifolds of cases with $m = n + 1$ will be supersymmetric. In particular, orbifolds where k is relatively prime to n and $n + 1$ will give new smooth supersymmetric solutions.

[4] In [80], other examples where $n \neq kn'$ are obtained by applying STTS duality to these ones. This is possible because while (m, n) are U-duality invariant, k is not, so this transformation can map us to new solutions.

3.3.4 Asymptotically AdS solutions

In order to understand the dual CFT interpretation of these solutions, it is interesting to see the effect of the constraints (3.3.20, 3.3.22) on the asymptotically AdS solution (3.2.14). Consider first the two-charge case. If we set $a_2 = 0$, $\delta_p = 0$ and insert (3.3.13, 3.3.14) in (3.2.14), we will have

$$\begin{aligned} ds^2 &= -\left(\frac{\rho^2}{\ell^2}+1\right)d\tau^2 + \left(\frac{\rho^2}{\ell^2}+1\right)^{-1} d\rho^2 + \rho^2 d\varphi^2 \\ &\quad +\ell^2\left[d\theta^2 + \sin^2\theta(d\phi + m d\varphi)^2 + \cos^2\theta(d\psi + m d\tau/\ell)^2\right]. \end{aligned} \tag{3.3.24}$$

Thus, the asymptotically AdS version of the soliton is just global $AdS_3 \times S^3$, with a shift of the angular coordinates on the sphere determined by m.

In the general three-charge case, the interpretation of the dimensionless parameter s changes in the asymptotically AdS solutions: it is now $s = \sqrt{\tanh\delta_p}$. The conditions (3.3.22) are unaffected, however, and inserting these and the value of the period (3.3.20) in (3.2.14), we will have

$$\begin{aligned} ds^2 &= -\left(\frac{\rho^2}{\ell^2}+1\right)d\tau^2 + \left(\frac{\rho^2}{\ell^2}+1\right)^{-1} d\rho^2 + \rho^2 d\varphi^2 \\ &\quad +\ell^2\left[d\theta^2 + \sin^2\theta(d\phi + m d\varphi - n d\tau/\ell)^2 + \cos^2\theta(d\psi - n d\varphi + m d\tau/\ell)^2\right]. \end{aligned} \tag{3.3.25}$$

Thus, again, the asymptotically AdS version of the soliton is just global $AdS_3 \times S^3$, with shifts of the angular coordinates on the sphere determined by m, n.

Thus, in the cases where they have a large 'core' region described by an asymptotically AdS geometry, the smooth solitons studied in the first two subsections above approach global $AdS_3 \times S^3$ in this region. As a consequence, the orbifolds studied in the previous section will have corresponding orbifolds of $AdS_3 \times S^3$; some of these orbifolds were discussed in [94, 95]. The resulting quotient geometry is still asymptotically $AdS_3 \times S^3$, as can be seen by introducing new coordinates $\varphi' = k\varphi$, $\tau' = k\tau$, $\rho' = \rho/k$. The metric on the orbifold in these coordinates is then

$$\begin{aligned} ds^2 &= -\left(\frac{\rho'^2}{\ell^2}+\frac{1}{k^2}\right)d\tau'^2 + \left(\frac{\rho'^2}{\ell^2}+\frac{1}{k^2}\right)^{-1} d\rho'^2 + \rho'^2 d\varphi'^2 \\ &\quad +\ell^2\left[d\theta^2 + \sin^2\theta\left(d\phi + \frac{m}{k}d\varphi' - \frac{n}{k\ell}d\tau'\right)^2\right. \\ &\quad \left. + \cos^2\theta\left(d\psi - \frac{n}{k}d\varphi' + \frac{m}{k\ell}d\tau'\right)^2\right]. \end{aligned} \tag{3.3.26}$$

The redefined angular coordinate φ' will have period 2π on the orbifold.

3.4 Verifying regularity

In the previous section, we claim to have found a family of smooth solitonic solutions, by imposing three conditions on the parameters of the general metric. We should now verify that these solutions have no pathologies. In this section, we will use the radial coordinate $\rho^2 = r^2 - r_+^2$ (for the two-charge solutions, $\rho^2 = r^2$), which runs over $\rho \geq 0$.

The first step is to check that $\tilde{H}_1 > 0$, $\tilde{H}_5 > 0$ for all $\rho \geq 0$, as desired. In these coordinates,

$$f = \rho^2 + (a_1^2 - a_2^2)\sin^2\theta + (a_2^2 - a_1 a_2 s^2). \tag{3.4.1}$$

In the two-charge case, where $a_2 = 0$, the last term vanishes, so $f \geq 0$, and hence $\tilde{H}_1 > 0$, $\tilde{H}_5 > 0$ everywhere. In the more general case, however, the last term is

$$a_2^2 - a_1 a_2 s^2 = -a_1 a_2 (s^2 - j^2) < 0, \tag{3.4.2}$$

so we do not have $f \geq 0$. Examining $\tilde{H}_1$ directly,

$$\tilde{H}_1 = \rho^2 + (a_1^2 - a_2^2)\sin^2\theta + a_1 a_2 (s^2 - j^2)(s^{-2} j^{-2} s_1^2 - c_1^2), \tag{3.4.3}$$

so for $\tilde{H}_1 > 0$ everywhere, we need the last factor to be positive. We know $s^2 > j^2$, and we can rewrite the last bracket as

$$(s^{-2} j^{-2} s_1^2 - c_1^2) = \frac{c_1^2}{j^2}\left(s^2 \frac{c_5^2 c_p^2}{s_5^2 s_p^2} - j^2\right) > 0, \tag{3.4.4}$$

so we indeed have $\tilde{H}_1 > 0$. We can similarly show $\tilde{H}_5 > 0$. Thus, the metric in the $(t, \rho, \theta, \tilde{\phi}, \tilde{\psi}, z^i)$ coordinates is regular for all $\rho > 0$, apart from the coordinate singularities associated with the poles of the S^3 at $\theta = 0, \pi/2$, so the local geometry is smooth.

Next we check for global pathologies. We can easily see that these solutions have no event horizons. The determinant of the metric of a surface of constant ρ, (3.3.5), is negative for $\rho > 0$. That is, there is a timelike direction of constant ρ for all $\rho > 0$, and hence by continuity there must be a timelike curve which reaches the asymptotic region from any fixed ρ. We will demonstrate the absence of closed timelike curves by proving a stronger statement, that the soliton solutions are stably causal. Using the expression for the inverse metric in appendix A.2, we can evaluate

$$\partial_\mu t \partial_\nu t g^{\mu\nu} = -\frac{1}{\sqrt{\tilde{H}_1 \tilde{H}_5}} \left[f + M(1 + s_1^2 + s_5^2 + s_p^2) + \frac{M^2(c_1^2 c_5^2 c_p^2 - s_1^2 s_5^2 s_p^2)}{\rho^2 + r_+^2 - r_-^2} \right] < 0, \tag{3.4.5}$$

so $\partial_\mu t$ is a timelike covector, and t is a global time function for the solitons. Hence, since any spacetime is stably causal if and only if there exists in it a function whose covariant derivative is a past-directed timelike vector field [8], the solitons are stably causal, and in particular free of closed timelike curves.

Finally, we should check regularity at $\rho = 0$. In the previous section, we chose R so that the ρ, y coordinates were the polar coordinates in a smooth $\mathbb{R}^2$. If we define new coordinates on this $\mathbb{R}^2$ regular at $\rho = 0$ by

$$x^1 = \rho \cos(y/R), \quad x^2 = \rho \sin(y/R), \tag{3.4.6}$$

then

$$dy = \frac{1}{(x_1^2 + x_2^2)}(x^1 dx^2 - x^2 dx^1), \tag{3.4.7}$$

and we need the other $g_{\mu y}$ components in the metric to go to zero at least linearly in ρ for the whole metric to be smooth at $\rho = 0$ once we pass to the Cartesian coordinates x^1, x^2. In fact, we find that the $g_{\mu y}$ go like ρ^2 for small ρ in the $(t, \rho, \theta, \tilde{\phi}, \tilde{\psi}, z^i)$ coordinates.

We also need to verify the regularity of the matter fields. The dilaton is trivially regular, since $\tilde{H}_1 > 0$, $\tilde{H}_5 > 0$, but the Ramond-Ramond two-form requires checking. The non-trivial question is whether the $C_{y\mu}$ go to zero at $\rho^2 = 0$. In fact, in the gauge we used in (3.2.5), they don't: we find

$$\begin{aligned} C_{y\tilde{\phi}} &= \frac{M s_p c_p s_5 c_5}{a_1 c_1 c_5 c_p - a_2 s_1 s_5 s_p} + O(\rho^2), \\ C_{y\tilde{\psi}} &= \frac{M s_p c_p s_5 c_5}{a_2 c_1 c_5 c_p - a_1 s_1 s_5 s_p} + O(\rho^2), \\ C_{yt} &= \frac{1 + s_1^2 + s_p^2}{s_1 c_1} + O(\rho^2). \end{aligned} \tag{3.4.8}$$

We can remove these constant terms by a gauge transformation, so the Ramond-Ramond fields are regular at $\rho = 0$. The physical importance of the constant terms is that they correspond to electromagnetic potentials dual to the charges carried by the geometry, and their presence is presumably related to the first law satisfied by these soliton solutions, as in [96].

In summary, we have shown that the two integer parameter family of solutions identified in the previous section are all smooth solutions without CTCs. In the next section, we will explore their relation to the CFT description of the D1-D5-P system.

3.5 Relation to CFT

We have found new smooth solutions by considering the general family of charged rotating black hole solutions (3.2.1). These are labelled by the radius R, charges (Q_1, Q_5) and three integers (m, n, k). They include the previously known supersymmetric solutions as special cases, and add non-supersolutions and new supersymmetric orbifold solutions. We would like to see if we can relate these solutions to the CFT description, as was done for the earlier supersymmetric cases in [73, 74, 79].

If we consider the asymptotically $AdS_3 \times S^3$ solutions constructed in section 3.3.4, which describe the 'core' region of the asymptotically flat solitons, we can use the powerful AdS/CFT correspondence machinery to identify the corresponding states in the CFT. The dual CFT for the asymptotically $AdS_3 \times S^3 \times T^4$ spaces with radius $\ell = (Q_1 Q_5)^{1/4}$ is a sigma model [97] with target space a deformation of the orbifold $(T^4)^N/S_N$ [40, 44, 98], where

$$N = n_1 n_5 = \frac{\ell^4 V}{g^2 l_s^8}, \tag{3.5.1}$$

where V is the volume of the T^4. This theory has $c = 6n_1 n_5$. In section 3.3.4, we showed that the corresponding asymptotically AdS solutions for a basic family of solitons were always global $AdS_3 \times S^3$, with a shift on the angular coordinates on the sphere specified by n, m. Following the proposal outlined in [73], we identify the geometries (3.3.25) with CFT states with charges

$$\begin{aligned} h &= \frac{c}{24}(m+n)^2, \quad & j &= \frac{c}{12}(m+n) \\ \bar{h} &= \frac{c}{24}(m-n)^2, \quad & \bar{j} &= \frac{c}{12}(m-n), \end{aligned} \tag{3.5.2}$$

where h, j are defined in 2.4.6 , 2.4.12. Thus, these states have energy

$$E = h + \bar{h} = 2(m^2 + n^2)\frac{c}{24} = \frac{1}{2}(m^2 + n^2)n_1 n_5, \tag{3.5.3}$$

and momentum

$$q_p = h - \bar{h} = 4mn\frac{c}{24} = nmn_1n_5. \tag{3.5.4}$$

Since the non-compact geometry is global AdS_3, there is a single spin structure on the spacetime. Because of the shifts in the angular coordinates, this spin structure can be either periodic or antiperiodic around φ at fixed ϕ, ψ: it will be periodic if $m + n$ is odd, and antiperiodic if $m + n$ is even. Thus, the geometry is identified with a RR state with the above charges if $m+n$ is odd, and with a NSNS state with these same charges if $m + n$ is even.

These states can be interpreted in terms of spectral flow. Spectral flow is an automorphism acting amongst two or more superymmetry charges [56,57]. Recalling that spectral flow shifts the CFT charges by [57]

$$h' = h + \alpha j + \alpha^2\frac{c}{24}, \quad j' = j + \alpha\frac{c}{12}, \tag{3.5.5}$$

$$\bar{h}' = \bar{h} + \beta\bar{j} + \beta^2\frac{c}{24}, \quad \bar{j}' = \bar{j} + \beta\frac{c}{12}, \tag{3.5.6}$$

we can see that the required states can be obtained by spectral flow with $\alpha = m+n$, $\beta = m - n$ acting on the NSNS ground state (for which $h = j = 0$, $\bar{h} = \bar{j} = 0$). This spectral flow can be identified with the coordinate transformation in spacetime which relates the (φ, ϕ, ψ) coordinates to the $(\varphi, \tilde{\phi}, \tilde{\psi})$ coordinates. Thus, we see that the non-supersymmetric states corresponding to all the geometries labelled by m, n are constructed by starting with the maximally supersymmetric NSNS vacuum and applying different amounts of spectral flow.

In [73], the special case $m = 1, n = 0$ was discussed. In this case, the spectral flow is by one unit on both the left and the right, and maps the NS vacuum to a R ground state both on the left and the right. We can see the supersymmetry of this state from the spacetime point of view: the covariantly constant Killing spinors in global AdS have the form

$$\epsilon_L^{\pm} = e^{\pm i\frac{\tilde{\phi}_L}{2}}e^{-i\frac{\varphi}{2}}\epsilon_0, \quad \epsilon_R^{\pm} = e^{\pm i\frac{\tilde{\phi}_R}{2}}e^{-i\frac{\varphi}{2}}\epsilon_0, \tag{3.5.7}$$

so when we shift $\tilde{\phi}_L = \phi_L + \varphi$, $\tilde{\phi}_R = \phi_R + \varphi$, the Killing spinors ϵ_L^+, ϵ_R^+ become independent of φ, corresponding to the preserved Killing symmetries in the R ground state. If we consider $m = n+1$, the spectral flow on the right is by one unit, so ϵ_R^+ is still independent of φ. These are the supersymmetric states considered in [79], which are R ground states on the right, but the more general R states obtained by spectral flowing by $2n+1$ units on the left. Our non-supersymmetric solitons correspond to the more general non-supersymmetric states obtained by spectral flowing the NSNS vacuum by $m-n$ units on the right and $m+n$ units on the left. In [79], an explicit representation for the R sector state obtained by spectral flow by $2r+1$ units was given,[5]

$$|2r+1\rangle_R = (J^+_{-(2r)})^{n_1 n_5} (J^+_{-(2r-2)})^{n_1 n_5} \dots (J^+_{-2})^{n_1 n_5} |1\rangle, \tag{3.5.8}$$

where J^+_{-k} is a mode of the $su(2)$ current of the full CFT which raises h and j by $\Delta h = k$, $\Delta j = 1$, and $|1\rangle$ is the R ground state with $j = +c/12$ obtained by spectral flow from the NS ground state. Similarly, one can give an explicit representation of the NS sector state obtained by spectral flow by $2r$ units, following [58],

$$|2r\rangle_{NS} = (J^+_{-(2r-1)})^{n_1 n_5} (J^+_{-(2r-3)})^{n_1 n_5} \dots (J^+_{-1})^{n_1 n_5} |0\rangle_{NS}. \tag{3.5.9}$$

The CFT state corresponding to the geometry (3.3.25) is then $|m+n\rangle_R \times |m-n\rangle_R$ or $|m+n\rangle_{NS} \times |m-n\rangle_{NS}$, depending on the parity of $m+n$.

The situation is more interesting when we consider the orbifolds. The geometries (3.3.26) should be identified with CFT states with charges

$$\begin{aligned} h &= \frac{c}{24}\left(1 + \frac{(m+n)^2 - 1}{k^2}\right), \quad & j &= \frac{c}{12}\frac{m+n}{k}, \\ \bar{h} &= \frac{c}{24}\left(1 + \frac{(m-n)^2 - 1}{k^2}\right), \quad & \bar{j} &= \frac{c}{12}\frac{m-n}{k}. \end{aligned} \tag{3.5.10}$$

In the supersymmetric case, when $m = n+1$, $\bar{h} = \frac{c}{24}$, $\bar{j} = \frac{c}{12}\frac{1}{k}$, so these geometries still have the charges of R ground states on the right. This particular R ground state corresponds to the spectral flow of the NS chiral primary state with $\bar{h} = \bar{j} = \frac{c}{24}\frac{k-1}{k}$. However, the charges of the state in the left-moving sector are, in general, not those of a R ground state or even the result of spectral flow on a R ground state. For

[5] We use a slightly different notation than [79].

general m, n, neither sector is the spectral flow of a ground state. Thus, these provide examples of geometries dual to more general CFT states.

To specify the CFT state completely, we need to say if (3.5.10) are the charges of a RR or a NSNS state. To do so, let us consider the spin structure on spacetime. When m or n is relatively prime to k, there is a contractible circle in the spacetime, and as a result the spin structure is fixed. The contractible circle is $(\varphi', \phi, \psi) \to (\varphi' + 2\pi k, \phi - 2\pi m, \psi - 2\pi n)$. The fermions must be antiperiodic around this circle. For the case where neither m nor n is relatively prime to k, we are not forced to make this choice, but we will assume that we still choose a spin structure such that the fermions are antiperiodic around this circle; this would correspond to the spin structure inherited from the covering space of the orbifold.

In the supersymmetric case $m = n + 1$, and more generally for $m + n$ odd, this implies that the fermions are periodic under $\varphi' \to \varphi' + 2\pi k$ at fixed ϕ, ψ. For k odd, this implies the fermions must be periodic under $\varphi' \to \varphi' + 2\pi$, while for k even, they may be either periodic or antiperiodic. Thus, for $m = n + 1$, we can always choose the periodic spin structure for the fermions on spacetime. This spacetime will then be identified with the supersymmetric RR state with the charges (3.5.10). However, for k even, we can choose the antiperiodic spin structure for the fermions on spacetime; this spacetime will then be identified with a NSNS state with the same charges (3.5.10). In this latter case, neither the spacetime solution nor the CFT state is supersymmetric.

The situation becomes stranger for $m + n$ even. The antiperiodicity around the contractible cycle implies that the fermions will be antiperiodic under $\varphi' \to \varphi' + 2\pi k$ at fixed ϕ, ψ. If k is odd, this is compatible with a spin structure antiperiodic in φ', but if k is even, there is no spin structure on the orbifold which satisfies this condition. The orbifold cannot be made into a spin manifold. The general conditions for such orbifolds M/Γ to inherit a spin structure from the spin manifold M were discussed in [99]; see also [100] for further discussion relevant to the case at hand. It will be interesting to see how this obstruction for k even, $m + n$ even is reflected in the CFT dual.

In the other cases, we can unambiguously identify the CFT state corresponding to the geometry as the state with charges (3.5.10) in the sector with the same periodicity conditions on the fermions as in the spacetime (choosing one of the two possible spin structures on spacetime in the case k even, $m+n$ odd). It would be interesting to construct an explicit description of these states, as in the discussion in [79, 80].

Thus, there is a clear CFT interpretation of the asymptotically $AdS_3 \times S^3$ geometries. However, the interesting discovery in this paper is that there are non-supersymmetric asymptotically flat geometries, and we want to ask to what extent these can also be identified with individual microstates in the CFT. Clearly the appropriate CFT states to consider are the ones described above, but does the identification between state and geometry extend to the asymptotically flat spacetimes? In particular, does it make sense to identify the asymptotically flat spacetime with a CFT state in the general case where it does not have a large approximately $AdS_3 \times S^3$ core region, and there is no supersymmetry?[6] We would not in general expect the match to asymptotically flat geometries to be perfect, but there is one non-trivial piece of evidence for the identification of the full asymptotically flat geometries with the CFT states: the form of the charges still reflects the CFT structure. Plugging our parameters into (3.2.8, 3.2.10, 3.2.11) gives

$$Q_p = nm\frac{Q_1 Q_5}{R^2}, \tag{3.5.11}$$

$$J_\phi = -m\frac{Q_1 Q_5}{R}, \tag{3.5.12}$$

$$J_\psi = n\frac{Q_1 Q_5}{R}. \tag{3.5.13}$$

These reproduce the quantisation of the CFT charges in (3.5.2). In the orbifold case, we replace R by kR, as the physical period of the asymptotic circle is k times smaller, and these values now agree with the charges in (3.5.10). This seems to us like a very non-trivial consistency check, as it is very difficult to even express the

[6] The CFT state for some of the geometries is in the NSNS sector. We do not regard this as a serious obstruction to an identification at the classical level: we are considering non-supersymmetric geometries, so we can allow the fermions to be antiperiodic around the asymptotic circle in spacetime. At the quantum level, one might worry that these antiperiodic boundary conditions lead to a constant energy density inconsistent with the assumed asymptotic flatness.

parameters M, a_1, a_2 appearing in the metric (3.2.1) in terms of Q_1, Q_5 and R and the integers m, n, so there is no reason why we would have expected to get such a simple result automatically. So this appears a good reason to believe properties of the full asymptotically flat geometries are connected to the CFT states. Note, however, that it does not seem to be possible to cast the ADM mass in such a simple form. In the next section, we will also see that the predicted time delay involved in scattering of probes does not quite match CFT expectations.

3.6 Properties of the solitons

We will briefly discuss some properties of these solutions, and their relation to the dual CFT. We first discuss the solution of the massless scalar wave equation in these geometries, following the discussion in [30, 68, 80] closely. We then consider the most significant difference between our non-supersymmetric solitons and the supersymmetric cases, the absence of an everywhere causal Killing vector.

3.6.1 Wave equation

It is interesting to study the behaviour of the massless wave equation on this geometry. This is a first step towards analysing small perturbations, and also allows us to address questions of scattering in the geometry which indicate how an exterior observer might probe the soliton. We consider the massless wave equation on the geometry,

$$\Box\Psi = 0. \tag{3.6.1}$$

It was shown in [101] that this equation is separable. Considering a separation ansatz

$$\Psi = \exp(-i\omega t/R + i\lambda y/R + im_\psi \psi + im_\phi \phi)\chi(\theta)h(r), \tag{3.6.2}$$

and using the inverse metric given in appendix A.2, we find that the wave equation reduces to

$$\frac{1}{\sin 2\theta}\frac{d}{d\theta}\left(\sin 2\theta \frac{d}{d\theta}\chi\right) + \left[\frac{(\omega^2 - \lambda^2)}{R^2}(a_1^2 \sin^2\theta + a_2^2\cos^2\theta) - \frac{m_\psi^2}{\cos^2\theta} - \frac{m_\phi^2}{\sin^2\theta}\right]\chi = -\Lambda\chi, \tag{3.6.3}$$

$$\frac{1}{r}\frac{d}{dr}\left[\frac{g(r)}{r}\frac{d}{dr}h\right] - \Lambda h + \left[\frac{(\omega^2-\lambda^2)}{R^2}(r^2+Ms_1^2+Ms_5^2) + (\omega c_p + \lambda s_p)^2\frac{M}{R^2}\right] h$$
$$- \frac{(\lambda - nm_\psi + mm_\phi)^2}{(r^2-r_+^2)}h + \frac{(\omega\varrho + \lambda\vartheta - nm_\phi + mm_\psi)}{(r^2-r_-^2)}h = 0, \tag{3.6.4}$$

where

$$\varrho = \frac{c_1^2c_5^2c_p^2 - s_1^2s_5^2s_p^2}{s_1c_1s_5c_5}, \quad \vartheta = \frac{c_1^2c_5^2 - s_1^2s_5^2}{s_1c_1s_5c_5}s_pc_p. \tag{3.6.5}$$

We see that the singularity in the wave equation at $r^2 = r_+^2$ is controlled by the frequency around the circle which is shrinking to zero there. This is a valuable check on the algebra. If we introduce a dimensionless variable

$$x = \frac{r^2 - r_+^2}{r_+^2 - r_-^2}, \tag{3.6.6}$$

we can rewrite the radial equation in the form used in [79],

$$4\frac{d}{dx}\left[x(x+1)\frac{d}{dx}h\right] + \left(\sigma^{-2}x + 1 - \nu^2 + \frac{\xi^2}{x+1} - \frac{\zeta^2}{x}\right)h = 0, \tag{3.6.7}$$

where

$$\sigma^2 = \left[(\omega^2-\lambda^2)\frac{(r_+^2-r_-^2)}{R^2}\right]^{-1}, \tag{3.6.8}$$

$$\nu = \left[1 + \Lambda - \frac{(\omega^2-\lambda^2)}{R^2}(r_+^2 + Ms_1^2 + Ms_5^2) - (\omega c_p + \lambda s_p)^2\frac{M}{R^2}\right]^{1/2}, \tag{3.6.9}$$

$$\xi = \omega\varrho + \lambda\vartheta - nm_\phi + mm_\psi, \tag{3.6.10}$$

$$\zeta = \lambda - nm_\psi + mm_\phi. \tag{3.6.11}$$

We can then use the results of [79], where the matching of solutions of this equation in an inner and outer region was carried out in detail, to determine the reflection coefficient. This reflection coefficient can be used to determine the time Δt it takes for a quantum scattering from the core region near $x = 0$ to return to the asymptotic region, by expanding $\mathcal{R} = a + b\sum_n e^{2\pi i n\frac{\omega}{R}\Delta t}$. Their matching procedure is valid when

$$\sigma^2 \gg 1 \tag{3.6.12}$$

and

$$\Delta t \gg \frac{R}{(\omega^2-\lambda^2)^{1/2}}. \tag{3.6.13}$$

Under these assumptions, their matching procedure gives

$$\Delta t = \pi R_s \varrho, \tag{3.6.14}$$

where R_s is the radius (3.3.20) for a smooth solution; in the orbifolds, $R = R_s/k$. We note that this is in agreement with their result in the supersymmetric case, as in the limit $\delta_1, \delta_5, \delta_p \to \infty$,

$$\varrho = \frac{s_1^2 s_5^2 + s_1^2 s_p^2 + s_5^2 s_p^2 + s_1^2 + s_5^2 + s_p^2 + 1}{s_1 c_1 s_5 c_5} \approx \frac{Q_1 Q_5 + Q_1 Q_p + Q_5 Q_p}{Q_1 Q_5} = \frac{1}{\eta} \tag{3.6.15}$$

in the notation of [80].

In the CFT picture, this travel time is interpreted as the time required for two CFT modes on the brane to travel around its worldvolume and meet again. Thus, from the CFT point of view, the expected value is $\Delta t_{CFT} = \pi R_s$. As in [80], there is a 'redshift factor' ϱ between our spacetime result and the expected answer from the CFT point of view. It was argued in [80] that such a factor must appear to make the spacetime result invariant under permutation of the three charges, and it was proposed that this factor could be understood as a scaling between the asymptotic time coordinate t in the asymptotically flat space and the time coordinate appropriate to the CFT. Evidence for this point of view was found by noting that in the cases where the soliton had a large $AdS_3 \times S^3$ core region, the global AdS time τ was proportional to ηt, so $\Delta\tau = \pi R_s$ in accordance with CFT expectations. In our non-supersymmetric case, for fixed m, n, the appropriate limit in which we obtain a large AdS region is the limit $\delta_1, \delta_5 \gg 1$ for fixed δ_p considered in section 3.3.4. We did not see any such scaling between the AdS and asymptotic coordinates there, but $\varrho \approx 1$ in this limit, so this is consistent with the interpretation proposed in [80]. However, we remain uncomfortable with this interpretation. It is hard to argue directly for such a redshift between the CFT and asymptotic time coordinates in the general case where the soliton does not have a large approximately $AdS_3 \times S^3$ core. Indeed, in the dual brane picture of the geometry, where we have a collection of D1 and D5 branes in a flat background, one would naïvely expect the two to be the same. A deeper understanding of this issue could shed interesting light on the limitations of the identification between CFT states and the asymptotically flat geometries.

3.6.2 Ergoregion

Although our soliton solutions are free of event horizons, they typically have ergoregions. These already appear in the supersymmetric three-charge soliton solutions studied in [79,80], where the Killing vector ∂_t, which defines time-translation in the asymptotic rest frame, becomes spacelike at $f = 0$ if $Q_p \neq 0$. However, in these supersymmetric cases, there is still a causal Killing vector (arising from the square of the covariantly constant Killing spinor), which corresponds asymptotically to the time-translation with respect to some boosted frame. A striking difference in the non-supersymmetric solitons is the absence of any such globally timelike or null Killing vector field.[7] The most general Killing vector field which is causal in the asymptotic region of the asymptotically flat solutions is

$$V = \partial_t + v^y \partial_y \tag{3.6.16}$$

for $|v^y| \leq 1$. However, when $f = 0$, the norm of this Killing vector is

$$|V|^2 = \frac{M}{\sqrt{\tilde{H}_1 \tilde{H}_5}}(c_p - v^y s_p)^2 > 0. \tag{3.6.17}$$

The best we can do is to take $v^y = \tanh \delta_p$, for which this Killing vector is timelike for $f > M$. Note that as a consequence, the two-charge non-supersymmetric solutions also have ergoregions.

In a rotating black hole solution, the existence of an ergoregion typically implies a classical instability when the black hole is coupled to massive fields [102,103]. This instability arises when we send in a wavepacket which has positive energy less than the rest mass with respect to the asymptotic Killing time, but negative energy in the ergoregion. The wavepacket will be partially absorbed by the black hole, but because the absorbed portion has negative energy, the reflected portion will have a larger amplitude. This then reflects off the potential at large distances, and repeats the process. This process causes the amplitude of the initial wavepacket to grow indefinitely, until its back-reaction on the geometry becomes significant.

[7] For the asymptotically AdS spacetimes, there is a globally timelike Killing vector field, given by ∂_t at fixed $\tilde{\psi}$, $\tilde{\phi}$. In (t, y, ψ, ϕ) coordinates, this is of the form $V' = \ell\partial_t - m\partial_\psi + n\partial_\phi$, so it cannot be extended to a globally timelike Killing vector field in the asymptotically flat geometry.

One might have thought that in the supersymmetric three-charge solitons, the instability would not appear as a consequence of the existence of a causal Killing vector, by a mechanism similar to that discussed in [104] for Kerr-AdS black holes. However, this instability is in fact absent for a different reason, which applies to both supersymmetric and non-supersymmetric solitons. The instability in black holes is a result of the existence of both an ergoregion and an event horizon, so in the solitons, the absence of an event horizon can prevent such an instability from occurring. Indeed, from the discussion of the massless wave equation in the previous section, we can see that the net flux is always zero, and the amplitude of the reflected wave is the same as that of the incident wave. That is, although there is an ergoregion, no superradiant scattering of classical waves occurs in this geometry, and the mechanism that led to the black hole bomb does not apply here. There might be an instability if we considered some interacting theory, as the interactions might convert part of an incoming wavepacket to negative-energy modes bound to the soliton, but we will not attempt to explore this issue in more detail.

Thus, for free fields, there is no stimulated emission at the classical level. We will now show that there is also no spontaneous quantum emission.[8] There is a natural basis of modes for this geometry; for the scalar field, (3.6.2). To establish which of these modes are associated with creation and which with annihilation operators, we need to consider the Klein-Gordon norm

$$(\Psi, \Psi) = \frac{i}{\hbar} \int_\Sigma d^5x \sqrt{h} n_\mu g^{\mu\nu} (\bar{\Psi} \partial_\nu \Psi - (\partial_\nu \bar{\Psi}) \Psi), \tag{3.6.18}$$

where Σ is a Cauchy surface, say for simplicity a surface $t = t_0$, and n_μ is the normal $n_\mu = \partial_\mu t$. The modes of positive norm, $(\Psi, \Psi) > 0$, correspond to creation operators, while those of negative norm, $(\Psi, \Psi) < 0$, correspond to annihilation operators. Because of the complicated form of the inverse metric (see appendix A.2), it is difficult to establish explicitly which are which. However, the main point is that we can define a vacuum state by requiring that it be annihilated by the annihilation operators corresponding to all the negative frequency modes in (3.6.2). This will then be the unique vacuum state on this geometry. Since the modes (3.6.2)

[8]Thanks to Don Marolf for explaining the following argument.

are eigenmodes of both the asymptotic time-translation ∂_t and of the timelike Killing vector in the near-core region,

$$V' = \ell\partial_t - m\partial_\psi + n\partial_\phi, \tag{3.6.19}$$

these will be the appropriate family of creation and annihilation operators for observers in both regions. That is, these observers who follow the orbits of the Killing symmetries will detect no particles in this state.

Thus, at the level of free fields, the solitons do not suffer from superradiance at either the classical or quantum level.

Chapter 4

Bubbles of Nothing

In this chapter we extend the study of time-dependent backgrounds in the AdS/CFT correspondence by examining the relation between bulk and boundary for the smooth 'bubble of nothing' solution and for the locally AdS black hole which has the same asymptotic geometry. These solutions are asymptotically locally AdS, with a conformal boundary conformal to de Sitter space cross a circle. We study the cosmological horizons and relate their thermodynamics in the bulk and boundary. We consider the α-vacuum ambiguity associated with the de Sitter space, and find that only the Euclidean vacuum is well-defined on the black hole solution. We argue that this selects the Euclidean vacuum as the preferred state in the dual strongly coupled CFT.

4.1 Introduction

The study of time-dependent backgrounds in string theory is a key area of development of the theory. An understanding of dynamical spacetimes is essential for many applications of current interest, such as cosmological evolution or black hole evaporation. Consideration of more general spacetime backgrounds can also illuminate new aspects of string theory, just as the consideration of quantum field theory on more general spacetime backgrounds brought to light new effects such as Unruh and Hawking radiation, and offered a new perspective on what the essential elements of quantum field theory are.

The Anti-de Sitter/Conformal field theory (AdS/CFT) correspondence [5,20] is a promising approach to the understanding of dynamical spacetime in string theory, since the dual field theory description is fully non-perturbative, offering a description which in principle encompasses both the dynamics of the background spacetime and the behaviour of strings or fields propagating in this dynamical background. However, our understanding of the correspondence for dynamical spacetimes remains very patchy. The aim in this chapter is to extend this dictionary by exploring aspects of the relation between bulk and boundary for two interesting time-dependent asymptotically locally AdS solutions.

The first solution we are interested in is the 'bubble of nothing' solution in anti-de Sitter space. This solution was introduced in [32,105], following the analysis of [106] of asymptotically flat bubble of nothing solutions as time-dependent backgrounds in string theory. These solutions are constructed by the double analytic continuation of black hole solutions, and describe a spacetime with a compact circle direction which shrinks to zero size on a surface which expands exponentially in the non-compact directions. This kind of solution was originally introduced in [107] to describe an instability of the Kaluza-Klein vacuum. The new observation of [106] is that they also provide nice examples of time-dependent backgrounds for string theory, as they are smooth vacuum solutions, which exhibit many of the issues we would like to address in the study of time-dependence, such as particle creation and non-trivial vacuum ambiguities for quantum fields. In [32,105], an asymptotically locally AdS bubble was constructed by double analytic continuation of the Schwarzschild-AdS black hole solution. In [32], the counterterm subtraction procedure was used to obtain the stress tensor of the dual field theory.

At large distances, this 'bubble of nothing' solution approaches a locally AdS spacetime. This locally AdS spacetime is in fact just a quotient of AdS, and was interpreted previously in [33,34] as a black hole solution. That is, this is the higher-dimensional analogue of the BTZ black hole [108,109]. This is a time-dependent solution: there is no Killing vector which is timelike everywhere outside the event horizon. Although it is not as smooth as the bubble of nothing solution, this is clearly an interesting example of a time-dependent geometry in its own right, and we will see that it has some interesting properties.

These two solutions have the same asymptotics, so they should be related to different states in the same field theory on the asymptotic boundary. This boundary is a de Sitter space cross a circle: the circle corresponds to the direction that is compactified in the locally AdS black hole, and which degenerates at the bubble in the bubble of nothing solution. Our aim is to explore the properties of the spacetimes, and relate them to the dual field theory. We will focus on understanding the relation between horizons in the spacetime and the boundary theory, and considering the question of choices of vacuum state in the bulk and the boundary.

In the next section, we give a brief review of the two solutions. We then discuss the horizons in these solutions in section 4.3. We show that the solutions have Killing horizons which can be interpreted as cosmological or acceleration horizons respectively, and which are in both cases naturally related to the de Sitter cosmological horizon in the boundary geometry. In particular, we show that the entropy of the bulk and boundary horizons agree if we introduce a cutoff at large radius. This sets up a novel correspondence relating horizons in the bulk and boundary, as opposed to relating horizons in the bulk to thermal states in the boundary. We argue that the black hole event horizon, on the other hand, does not have a thermodynamic interpretation.

Thus, the thermodynamic properties in the bulk are identified with the thermodynamic properties of the de Sitter horizon in the boundary theory. We are proposing that the time-dependence of the bulk spacetime is completely encoded in the time-dependence of the boundary spacetime, and the appropriate state in the dual CFT is simply some natural vacuum state on this curved, time-dependent background. These examples are thus a particularly simple context for further investigations of time-dependence in string theory, since we have reduced the problem to the more well-understood one of studying quantum field theory in a time-dependent background.

One of the most interesting features of time-dependent spacetimes is that they do not have unique vacuum states. We would therefore like to use our solutions as a laboratory for studying the description of vacuum ambiguities in AdS/CFT.

The natural vacuum ambiguity to consider in this context is the α-vacua in de Sitter space, since the boundary has a de Sitter factor, and there are coordinates in the bulk which write these spaces in de Sitter slicings. It has been known for some time [110–113] that de Sitter space has a one-parameter family of vacuum states invariant under the de Sitter isometry group, called α-vacua. There is a unique member of this family which has the same short-distance singularity as in flat space [112, 114], which is also the Euclidean vacuum obtained by analytic continuation from the sphere [11, 115]. There has nonetheless been considerable controversy in the literature about whether the additional de Sitter-invariant vacua are physical, particularly focusing on the definition of an interacting theory [116–127]. It is not our intention to address any of the issues raised in this literature; instead, we want to consider a different approach, using the behaviour of the analogues of the α-vacua in a free scalar field theory in the bulk spacetime to obtain information about α-vacua in the strongly-coupled dual field theory.

In section 4.4, we show that there are natural analogues of the α-vacuum ambiguity in these two bulk spacetimes, which we identify with the ability to choose α-vacua in the boundary theory. In section 4.5, we find that the propagators for the α-vacua on the locally AdS black hole have additional singularities at the event horizon of the black hole. We show that these additional singularities are reflected in a breakdown of the procedure of [128] for constructing the expectation value for the stress tensor. We also comment that the analytic continuation procedure used in [67, 129] to probe the region behind the event horizon can only be extended to the locally AdS black hole if we take the Euclidean vacuum. We therefore argue that the analogues of α-vacua are not good vacuum states for the locally AdS black hole, since they break down on the event horizon.

We interpret this as evidence that the α-vacua are not good states for the strongly-coupled CFT on the boundary, which is de Sitter space cross a circle. Although the α-vacua appear to be acceptable states at least at the free level for the bubble of nothing solution, this spacetime only exists when the size of the circle is less than a maximum value. Thus, at least for a range of parameters, there is no obvious spacetime interpretation for the α-vacua in the CFT, whereas there is a spacetime interpretation for the Euclidean vacuum. Thus, the Euclidean vacuum is selected as a preferred state in the strongly-coupled CFT.

4.2 Review of bubble & black hole solutions

The bubble of nothing solution is obtained by analytic continuation of the 5d Schwarzschild-AdS black hole[1],

$$ds^2 = -(1 + \frac{r^2}{l^2} - \frac{r_0^2}{r^2})dt^2 + (1 + \frac{r^2}{l^2} - \frac{r_0^2}{r^2})^{-1}dr^2 + r^2(d\vartheta^2 + \cos^2\vartheta d\Omega_2^2) \qquad (4.2.1)$$

where $d\Omega_2^2 = d\theta^2 + \sin^2\theta d\phi^2$ is the metric on the two-sphere. By analytically continuing two variables, $t \to i\chi$ and $\vartheta \to i\tau$, a novel solution of gravity with negative cosmological constant is found:

$$ds^2 = (1 + \frac{r^2}{l^2} - \frac{r_0^2}{r^2})d\chi^2 + (1 + \frac{r^2}{l^2} - \frac{r_0^2}{r^2})^{-1}dr^2 + r^2[-d\tau^2 + \cosh^2\tau(d\theta^2 + \sin^2\theta d\phi^2)]. \qquad (4.2.2)$$

We see that the proper length of the spacelike χ direction goes to zero at $r = r_+$, where r_+ is the root of $f(r) = l^2r^2 + r^4 - r_0^2l^2$,

$$r_+^2 = \frac{l^2}{2}\left[-1 + \sqrt{1 + \frac{4r_0^2}{l^2}}\right]. \qquad (4.2.3)$$

To make the spacetime smooth at $r = r_+$, the coordinate χ must be identified periodically, with period

$$\Delta\chi = \frac{2\pi l^2 r_+}{2r_+^2 + l^2}. \qquad (4.2.4)$$

There is no region of spacetime inside the surface $r = r_+$; this is the 'bubble of nothing'. Since the metric on this bubble is three-dimensional de Sitter space with scale r_+, we see that the bubble expands exponentially. At very early times, the region 'excised' is a very large sphere. As $\tau < 0$ increases, the size of the bubble shrinks to a minimum at $\tau = 0$. After this it grows again exponentially.

As was pointed out in [32], the solution (4.2.2) is asymptotically locally AdS: that is, at large distances, it approaches

$$ds^2 = (1 + \frac{r^2}{l^2})d\chi^2 + (1 + \frac{r^2}{l^2})^{-1}dr^2 + r^2[-d\tau^2 + \cosh^2\tau(d\theta^2 + \sin^2\theta d\phi^2)], \qquad (4.2.5)$$

[1] We focus on the case of AdS_5 for definiteness, but it is a simple exercise to extend our remarks to other AdS_d with $d \geq 4$.

which is a locally AdS spacetime. This spacetime is the result of quotienting AdS by a boost isometry to make the coordinate χ periodic. This coordinate system is related to embedding coordinates for AdS by

$$\begin{aligned}
x^1 &= (r^2+l^2)^{1/2}\cosh\chi/l,\\
x^2 &= r\sinh\tau,\\
x^3 &= (r^2+l^2)^{1/2}\sinh\chi/l,\\
x^4 &= r\cosh\tau\,\sin\theta\,\sin\phi,\\
x^5 &= r\cosh\tau\,\sin\theta\,\cos\phi,\\
x^6 &= r\cosh\tau\cos\theta,
\end{aligned} \tag{4.2.6}$$

where x^μ are embedding coordinates in terms of which AdS is defined by $-(x^1)^2-(x^2)^2+(x^3)^2+(x^4)^2+(x^5)^2+(x^6)^2=-l^2$. If χ is allowed to run over all values (4.2.6) provides a coordinatization of a part of AdS. Making χ periodic with some arbitrary period $\Delta\chi$ thus introduces discrete identifications on AdS along a boost isometry. As is evident from the form of the metric (4.2.5), the quotient preserves an $SO(1,1)\times SO(1,3)$ subgroup of the original $SO(2,4)$ isometry group. Note that we are free to choose any period $\Delta\chi$ we wish in this quotient geometry.

As was stressed in [130], this spacetime has been studied previously [33, 34], as the higher-dimensional analogue of the BTZ black hole. It was also discussed in the recent classification of quotients of anti-de Sitter spaces [100]. It describes an interesting non-stationary black hole solution with a single exterior region. The structure of this solution is more easily understood by passing to a 'Kruskal' coordinate system [33],

$$\begin{aligned}
x^1 &= l\frac{1+y^2}{1-y^2}\cosh\chi/l,\\
x^2 &= 2l\frac{y^0}{1-y^2},\\
x^3 &= l\frac{1+y^2}{1-y^2}\sinh\chi/l,\\
x^4 &= 2l\frac{y^1}{1-y^2},\\
x^5 &= 2l\frac{y^2}{1-y^2},\\
x^6 &= 2l\frac{y^3}{1-y^2},
\end{aligned} \tag{4.2.7}$$

where we have written $y^2 = y^\mu y_\mu = -(y^0)^2 + (y^1)^2 + (y^2)^2 + (y^3)^2$. In terms of these coordinates, the radial coordinate of (4.2.5) is

$$r = l\frac{2\sqrt{y^2}}{1-y^2}, \tag{4.2.8}$$

and the coordinates (τ, θ, ϕ) in (4.2.5) parametrise the hyperboloids $y^2 =$ constant. The metric in these Kruskal coordinates is

$$ds^2 = \frac{4l^2}{(1-y^2)^2}(-dy_0^2 + dy_1^2 + dy_2^2 + dy_3^2) + \frac{(1+y^2)^2}{(1-y^2)^2}d\chi^2. \tag{4.2.9}$$

One of the great advantages of this coordinate system is that it writes the metric at a constant χ as conformal to a flat space, enabling us to easily picture the causal structure of the spacetime. We see that the singularity at $r = 0$ in (4.2.5) is just a coordinate singularity, corresponding to the light cone of the origin, $y^2 = 0$ in the new coordinates. At $y^2 = -1$, the χ circle becomes null, and beyond this surface it will be timelike, so the quotient introduces closed timelike curves in this region. Following [33, 34], we assume this region of closed timelike curves is removed from the spacetime. Then $y^2 = -1$ becomes a singularity, since timelike curves end on it. The surface at $y^2 = 0$ then becomes an event horizon for the spacetime; observers who cross it will inevitably hit the singularity. The asymptotic boundary of the spacetime is at $y^2 = 1$. The geometry is depicted in figure 4.1. These Kruskal coordinates cover the whole spacetime.

This is the natural higher-dimensional analogue of the BTZ black hole, but it clearly has a somewhat different global structure: the maximally extended spacetime described by (4.2.9) has only a single exterior region, with a connected asymptotic boundary. Furthermore, the global event horizon at $y^2 = 0$ is not a Killing horizon for any Killing vector, and one can easily see that the area of its cross-sections increase with time. We will discuss the horizons in this solution and their interpretation further in the next section.

To understand the relation of these asymptotically locally AdS spacetimes to a dual field theory, we need to understand the conformal boundary of these spacetimes. Adopting a conformal factor $\Omega = l/r$, we see that the boundary metric for (4.2.2) is

$$ds_\Sigma^2 = d\chi^2 + l^2[-d\tau^2 + \cosh^2\tau(d\theta^2 + \sin^2\theta d\phi^2)]. \tag{4.2.10}$$

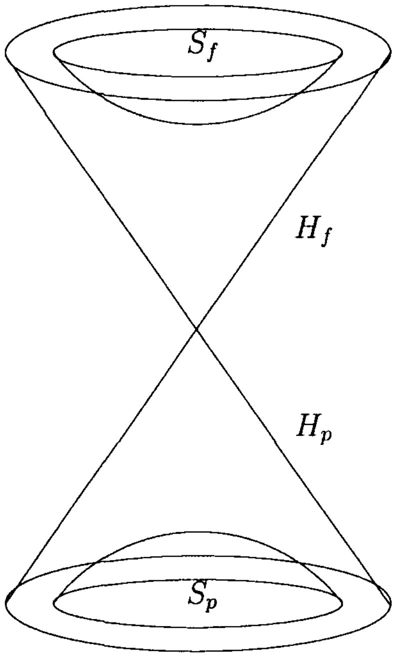

Figure 4.1: Three dimensions of the AdS black hole spacetime: one sphere direction and the S^1 factor are suppressed. The direction of increasing y_0 is up. S_f, S_p are the future and past singularities; H_f, H_p are the future and past horizons.

We can obtain the same result from (4.2.9) (in those coordinates $\Omega = (1-y^2)/2\sqrt{y^2}$). The dual CFT thus lives in a space which is three dimensional de Sitter space cross a circle, $dS_3 \times S^1$. There is a single dimensionless parameter characterising this boundary geometry, the ratio of the radius of the circle to the size of the de Sitter factor, $\Delta\chi/l$. This is thus the physical information we can specify from the field theory point of view, and it will determine the bulk geometry.

For the locally AdS black hole, there is a unique bulk geometry for each choice of $\Delta\chi$. For the bubble of nothing, on the other hand, the value of r_0 characterising the bulk geometry is determined by solving (4.2.4), which does not give a one-to-one map between $\Delta\chi$ and r_0. There is a maximum value of $\Delta\chi$ for which this equation has a solution, $\Delta\chi_{max} = \sqrt{2}\pi l$, attained when $r_+^2 = l^2/2$, that is, when $r_0 = l/2$. If we choose $\Delta\chi$ greater than this maximum value, there is no corresponding bubble of nothing solution in the bulk. If we choose $\Delta\chi$ less than the maximum, there will be two solutions, with a smaller and a larger value for r_0 [2]. It was argued in [32]

[2]Note also that the χ circle is contractible in the bulk for the bubble of nothing solution, which

that the solution with the smaller value of r_0 will be both classically and quantum mechanically unstable, and should therefore be disregarded, while we expect the other solution to be stable.

In [32], the boundary stress tensor for this solution was computed, using the counterterm subtraction procedure [131–133]. If we use the bubble of nothing as the bulk solution, we obtain a boundary energy density

$$\rho_{bubble} = -\frac{1}{16\pi G l^3}(r_0^2 + l^2/4) = -\frac{N^2}{8\pi^2 l^4}\left(\frac{r_0^2}{l^2} + \frac{1}{4}\right). \tag{4.2.11}$$

This calculation can also be applied to obtain the boundary stress tensor when we use the locally AdS black hole as the bulk solution, by setting $r_0 = 0$ in the previous result [130]. Thus, for this case

$$\rho_{bh} = -\frac{1}{64\pi G l} = -\frac{N^2}{32\pi^2 l^4}. \tag{4.2.12}$$

We note that the energy of the bubble is lower than the energy of the black hole, so it is possible for the black hole to decay into (some excited state on) the bubble of nothing. It was also pointed out in [32] that the stress tensor obtained for the black hole corresponds precisely to a geometrical contribution associated with the curvature of the background. This can presumably be interpreted as a reflection of the fact that the solution is simply a quotient of global AdS, so it corresponds to the dual field theory in a vacuum state where there is no state-dependent contribution to the stress-energy.

4.3 Horizons & thermodynamics

We now consider the relation between horizons and thermodynamics. The relation between the bulk black hole horizon and the thermal behaviour of the field theory on the boundary for the ordinary BTZ and Schwarzschild-AdS black hole solutions was one of the first things to be understood in the context of the AdS/CFT correspondence [134, 135]. The black hole solution described above has a global event

fixes the bulk spin structure to be antiperiodic around this circle. Thus, this bulk geometry will only contribute to the path integral when we take antiperiodic boundary conditions for the fermions on this circle in the boundary. There is no such restriction for the locally AdS black hole solution.

horizon, but as noted previously, this is not a Killing horizon, and it is not clear if it should have a thermodynamic interpretation.

Both these spacetimes do however have Killing horizons in them, associated with the timelike Killing vectors that generate the worldlines of comoving observers in the coordinates of (4.2.2,4.2.5). If we consider a given comoving observer, say the one at $\theta = 0$, then the corresponding Killing vector is

$$K = \cos\theta\partial_\tau + \tanh\tau\sin\theta\partial_\theta \tag{4.3.1}$$

and the Killing horizon where this Killing vector becomes null is at

$$\tanh\tau = \pm\cos\theta. \tag{4.3.2}$$

Note that the location of this Killing horizon is independent of r in these coordinates, and corresponds precisely to the usual Killing cosmological horizons in the de Sitter factor. These horizons are illustrated in figure 4.2.

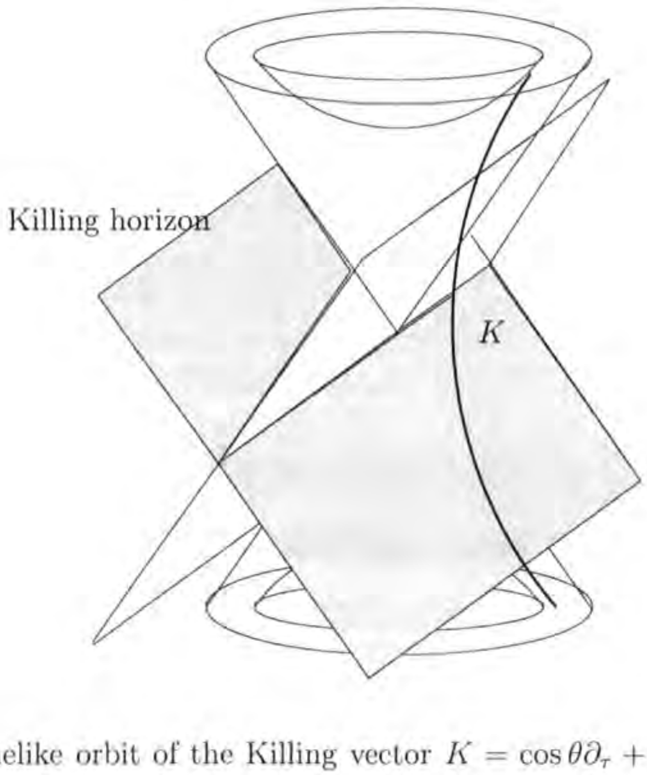

Figure 4.2: A timelike orbit of the Killing vector $K = \cos\theta\partial_\tau + \tanh\tau\sin\theta\partial_\theta$ and the plane where it becomes null, $\tanh\tau = \pm\cos\theta$.

In the bubble of nothing solutions, these Killing horizons correspond to cosmological horizons, just as they do in de Sitter space. It was noted in [106] that the bubble of nothing solution constructed from the Schwarzschild black hole has cosmological horizons, as the exponential expansion of the bubble prevents any one observer from seeing the whole of the spacetime. The same is true in the AdS bubble of nothing (4.2.2). Any observer's trajectory will asymptotically approach a constant coordinate position on the two-sphere, because of the exponential growth of the two-sphere's proper volume. The trajectory will thus lie within the region bounded by the corresponding Killing horizon, and that Killing horizon will be the cosmological horizon for this observer.

In the locally AdS black hole solution, these horizons are more naturally identified as acceleration horizons. Any observer who chooses to remain outside the black hole will asymptotically approach constant values of the angular coordinates, and the corresponding Killing horizon will be an event horizon for this observer by the same argument. However, it is clear from (4.2.9) that these observers are accelerating–uniformly accelerating if they remain at constant values of r. The Killing vector (4.3.1) is also, in these coordinates, simply a boost:

$$K = y^0 \partial_{y^3} + y^3 \partial_{y^0}. \tag{4.3.3}$$

Thus, in the locally AdS black hole solution, we can think of these horizons as analogous to the Rindler horizons in flat space.

What we would like to explain now is the interpretation of these horizons in the dual field theory. The novelty here is that the horizons are non-compact, and intersect the asymptotic boundary. This will imply a rather different relation between the horizons and the dual: the horizon in the bulk is related to a horizon in the boundary, rather than to thermal effects from considering a non-trivial mixed state in the boundary field theory. Since the structure of this cosmological/acceleration horizon is very similar in these two spacetimes, we will treat them together.

First, let us consider the thermal properties of the state. Since the horizon (4.3.2) is a Killing horizon, there is a unique regular (i.e., Hadamard) vacuum which is invariant under the action of this Killing vector. It will be a thermal state with respect to the notion of time translation defined by this Killing vector [136]. This state is clearly the usual Euclidean vacuum, obtained by analytic continuation from the Euclidean versions of (4.2.2,4.2.5). The thermal properties of this state have a natural interpretation from the boundary point of view: also in the boundary, we have a cosmological horizon at (4.3.2), and the natural CFT vacuum state defined by analytic continuation from the Euclidean version of the boundary geometry will look thermal from the point of view of comoving observers. That is, the bulk state is identified with a vacuum state in the CFT, and looks thermal simply because the CFT lives in a time-dependent background, a de Sitter universe (cross a circle).

However, there is a fly in the ointment for this very natural interpretation: we have two bulk geometries. How do we understand the difference between them from the boundary point of view? It seems that the circle plays the crucial role here. In the bubble of nothing solution, the fermions must be antiperiodic on this circle, whereas in the locally AdS black hole, we are free to choose either spin structure. We suggest that the bubble of nothing is related to a CFT vacuum with antiperiodic boundary conditions on the fermions, while the locally AdS black hole can be related to a CFT vacuum with periodic boundary conditions on the fermions[3]. This is analogous to the identification of global AdS with the NS ground state and the M=0 BTZ black hole with the RR ground state in the usual AdS_3 story [20]. Note however that in our case neither state is supersymmetric, as the background $dS_3 \times S^1$ geometry breaks all supersymmetry. As evidence in support of this suggestion, we note that the difference in energy between the bubble (4.2.11) and the black hole (4.2.12) goes like $1/\Delta\chi^4$ for small $\Delta\chi$, which is the expected behaviour for the Casimir energy associated with such a change in boundary conditions for fermions. The greatest problem with this suggestion is that the bubble of nothing solution only exists for $\Delta\chi \leq \Delta\chi_{max}$. We have no interpretation to suggest for this restriction, which seems very unnatural from the CFT point of view.

Having proposed a relation between thermal properties of the states in bulk and boundary, we would like to go on to make a more controversial suggestion, that there should also be an entropy associated with these horizons, by showing

[3]Since it admits both spin structures, the black hole can also contribute when we consider antiperiodic boundary conditions. In that context, it is presumably interpreted as an excited state above the vacuum described by the bubble solution. It is only when we consider antiperiodic boundary conditions that the black hole can decay into a bubble.

it has a natural interpretation in the boundary theory. One might think that this stands little chance of working, since the area of the bulk horizon is infinite, so the entropy $S = A/4G$ would also be infinite. How can we give an interpretation for this infinite entropy in terms of the boundary theory? However, this is precisely the right answer from the boundary point of view: the horizon in the bulk should be related to the cosmological horizon in the boundary. This has finite area, since it is just the usual de Sitter horizon, but the boundary theory is not coupled to gravity. We can formally include a gravitational term with $G = 0$, so the entropy for this horizon is indeed infinite.

To make a quantitative comparison, we introduce a cut off at $r = R$. The entropy of the bulk horizon inside this surface is

$$S_{bulk} = \frac{A}{4G} = \frac{1}{4G}\int_{r_i}^{R} r dr \int d\phi d\chi \tag{4.3.4}$$

where A is the area of the horizon's bifurcation surface $\tau = 0, \theta = \pi/2$, which we have written out explicitly in terms of an integration over (r, ϕ, χ). The lower limit of integration $r_i = r_+$ in the bubble of nothing solution and $r_i = 0$ in the locally AdS black hole, and $G = G_5$ is the gravitational constant in the bulk.

In the boundary theory, the introduction of the cut off at $r = R$ corresponds to an ultraviolet cutoff on the field theory, and introduces a coupling of the field theory to gravity, given by calculating the induced Einstein action obtained from integrating the bulk action over the radial direction. The action in the bulk is

$$I = \frac{1}{16\pi G}\int \sqrt{-g} d^5 x R. \tag{4.3.5}$$

If we set

$$ds^2 = f(r)d\chi^2 + f(r)^{-1}dr^2 + \frac{r^2}{l^2}\widehat{ds}^2 \tag{4.3.6}$$

we can rewrite this action as

$$I = \frac{1}{16\pi G}\int \sqrt{-\hat{g}}\frac{r^3}{l^3} dr d^4x \frac{l^2}{r^2}\hat{R} + \dots, \tag{4.3.7}$$

where $\hat{R}$ is the curvature of the three-dimensional metric $\widehat{ds}^2$, and $\dots$ denotes terms involving $f(r)$. This allows us to read off the Newton's constant in four dimensions as

$$\frac{1}{G_4} = \frac{1}{lG}\int_{r_i}^{R} r dr. \tag{4.3.8}$$

The entropy of the bifurcation surface $\tau = 0, \theta = \pi/2$ in the boundary theory is

$$S = \frac{A}{4G_4} = \frac{1}{4G_4} l \int d\phi d\chi, \quad (4.3.9)$$

which then agrees precisely with (4.3.4). Thus, we see that the entropy of the horizons in bulk and boundary agrees quantitatively.

Thus, the AdS/CFT correspondence identifies the entropy of the bulk cosmological/acceleration horizon with the entropy of the de Sitter horizon in the boundary. This calculation gives a powerful argument that even for non-compact horizons, the area should be regarded as an entropy, as argued in [137, 138].

One final point to note concerning the cosmological horizons is that there is a finite difference in entropy between the bubble of nothing solution and the locally AdS black hole: the entropy of the bubble of nothing is less than that of the black hole by

$$\Delta S = \frac{2\pi r_+^2 \Delta\chi}{4G}, \quad (4.3.10)$$

because of the different ranges of integration. This difference was absorbed in a change in the induced Newton's constant in the boundary from the point of view of the cutoff CFT discussed above, but it would be interesting to see if it could be related to some difference in the corresponding states, perhaps in the un-cutoff CFT.

Finally, what about the global event horizon in the locally AdS black hole? Should there be some entropy associated with this horizon as well? We will argue that the answer is no. First, we note that there is no independent temperature associated with this horizon. An observer outside the black hole will see a thermal bath, but this will come to them from the acceleration horizon that bounds the region of spacetime they can see, and not from the event horizon, which lies behind this acceleration horizon. Secondly, the event horizon is not a special surface in a spacelike slice of the spacetime: it is not, for instance, the boundary of a region of trapped surfaces. Indeed, as noticed previously, the event horizon in this spacetime is rather like the light cone of the origin in flat space. It is a purely teleological event horizon, which is a boundary of the past of infinity because of something that is going to happen in the future: the 'singularity' at $y^2 = -1$. The light cone of the origin in flat space can similarly form part of an event horizon, if a collapsing shell of matter is converging on it. However, we do not think we would associate an entropy with this horizon before the shell crossed it.

Therefore, we think the appropriate generalisation from the three-dimensional BTZ black hole, where the Killing horizon and event horizon are the same thing, is to associate an entropy with the Killing horizon in the locally AdS black hole in higher dimensions, and not to associate any entropy with the event horizon in this solution.

4.4 α-vacua in bubble and black hole

In the previous section, we have focused on relating the properties of the Euclidean vacuum state on the bulk spacetimes to the dual CFT. However, one of the most interesting features of time-dependent spacetimes is that they do not have unique vacuum states. We would therefore like to use our solutions as a laboratory for studying the description of vacuum ambiguities in AdS/CFT.

The natural vacuum ambiguity to consider in this context is the α-vacua in de Sitter space. It has been known for some time [110–113] that de Sitter space has a one-parameter family of vacuum states invariant under the de Sitter isometry group. Our boundary geometry, which is de Sitter cross a circle, will clearly inherit this ambiguity, and we would now like to relate it to the bulk spacetime.

4.4.1 Review of α-vacua in de Sitter

To begin, we review the α-vacuum ambiguity in de Sitter space. Let G_E be the Wightman function for a massive scalar field ϕ on three dimensional de Sitter space obtained by analytically continuing the unique Wightman function on the Euclidean sphere to 3d de Sitter. Then we define the Euclidean vacuum state $|E\rangle$ by

$$G_E(x, x') = \langle E|\phi(x)\phi(x')|E\rangle. \tag{4.4.1}$$

We can choose a mode expansion for a free massive scalar field ϕ

$$\phi(x) = \sum_n \left(a_n \phi_n^E(x) + a_n^\dagger \phi_n^{E*}(x)\right), \tag{4.4.2}$$

such that the vacuum state obeys

$$a_n|E\rangle = 0. \tag{4.4.3}$$

The Wightman function can be re-written in terms of these modes as

$$G_E(x, x') = \sum_n \phi_n^E(x)\phi_n^{E*}(x'). \tag{4.4.4}$$

We can choose the positive frequency Euclidean modes such that

$$\phi_n^E(x_A) = \phi_n^{E*}(x), \tag{4.4.5}$$

where x_A is the point antipodal to x in de Sitter space. If x has coordinates θ, ϕ and τ, then x_A has coordinates $\pi - \theta, \phi + \pi, -\tau$.

The α-vacua are defined by observing that for any $\alpha \in \mathbb{C}$ with Re $\alpha < 0$, we can define a new mode expansion by the Bogoliubov transform [112, 113]

$$\tilde{\phi}_n(x) = N_\alpha(\phi_n^E(x) + e^\alpha \phi_n^{E*}(x)), \quad N_\alpha = \frac{1}{\sqrt{1 - e^{\alpha+\alpha^*}}}. \tag{4.4.6}$$

The operators a_n must also be transformed, and the new operators are given by

$$\tilde{a}_n = N_\alpha(a_n^E - e^{\alpha*} a_n^{E\dagger}). \tag{4.4.7}$$

A new de Sitter invariant vacuum state $|\alpha\rangle$ is then defined by

$$\tilde{a}_n|\alpha\rangle = 0 \quad \forall \quad n > 0 \tag{4.4.8}$$

and the Wightman function in this vacuum is

$$G_\alpha(x, x') = \langle\alpha|\tilde{\phi}(x)\tilde{\phi}(x')|\alpha\rangle = \sum_n \tilde{\phi}_n(x)\tilde{\phi}_n^*(x'). \tag{4.4.9}$$

This new propagator can be expressed in terms of the original Euclidean modes and α. It is [139]

$$G_\alpha(x, x') = N_\alpha^2 \sum \left[\phi_n(x)\phi_n^*(x') + e^{\alpha+\alpha^*}\phi_n(x')\phi_n^*(x) \right. \\ \left. + e^{\alpha^*}\phi_n(x)\phi_n^*(x'_A) + e^\alpha \phi_n(x_A)\phi_n^*(x')\right]. \tag{4.4.10}$$

As a function of Euclidean Wightman propagators, it can be written as

$$G_\alpha(x, x') = N_\alpha^2 \left(G_E(x, x') + e^{\alpha+\alpha^*} G_E(x', x) + e^{\alpha^*} G_E(x, x'_A) + e^\alpha G_E(x_A, x')\right). \tag{4.4.11}$$

These new vacua are automatically invariant under the continuous $SO(1,3)$ symmetry of the de Sitter space, as can be seen from the above relation between α and Euclidean propagators. If we take α to be real, the α-vacuum is also invariant under time reversal, which interchanges the last two terms in (4.4.11).

Unlike the Euclidean vacuum, the α-vacuum is not thermal. In [139, 140], the departure from thermality was studied by considering the behaviour of a particle detector in the α-vacuum. One considers a monopole detector coupled to the scalar field ϕ by the interaction

$$g\int dt\phi(x(t))m(t), \tag{4.4.12}$$

where $x(t)$ is the path followed by the particle detector, with proper time t, and $m(t)$ is an operator acting on internal states of the detector. If the operator $m(t)$ has eigenstates $|E_i\rangle$ with energies E_i, we define the matrix elements m_{ij} as

$$m_{ij} = \langle E_i|m(0)|E_j\rangle. \tag{4.4.13}$$

Then the probability that the detector reports a change in energy from E_i to E_j, $E_j > E_i$, is

$$P(E_i \to E_j) = g^2|m_{ij}|^2\int_{-\infty}^{\infty} dtdt'e^{-i(E_j-E_i)(t'-t)}G(x(t'),x(t)), \tag{4.4.14}$$

where $G(x,x')$ is the Wightman function. Substituting in (4.4.10), [139, 140] found

$$\frac{P(E_i \to E_j)}{P(E_j \to E_i)} = e^{-2\pi\Delta E}\left|\frac{1+e^{\alpha+\pi\Delta E}}{1+e^{\alpha-\pi\Delta E}}\right|^2 \tag{4.4.15}$$

showing that the detector has a non-thermal response.

Note that in particular, at high energies the detector response becomes independent of the energy difference. This is a sign of the bad short-distance behaviour of the α-vacua. From (4.4.11), one can see that the short-distance singularity of the α-vacuum Wightman function is related to the singularity in the Euclidean Wightman function by a factor of $N_\alpha^2(1+e^{\alpha+\alpha*})$. Thus, as $x \to x'$,

$$G_\alpha(x,x') = N_\alpha^2\left[1+e^{\alpha+\alpha^*}\right]\frac{1}{(2\pi)^2\sigma(x,x')} + \ldots, \tag{4.4.16}$$

where $\sigma(x,x')$ is half the square of the geodesic distance between x and x', and $\ldots$ denotes less singular terms. The unusual coefficient of the singularity implies that the α-vacua are not Hadamard states: they have a different short-distance singularity to the flat-space vacuum propagator.

4.4.2 Vacuum ambiguity in bubble and black hole

Since the spacetimes we are interested in have a de Sitter factor, they will naturally all have a similar vacuum ambiguity. That is, if we consider a massive scalar field on either the bubble or the black hole, we can again choose a mode expansion satisfying

$$\phi_n^E(x_A) = \phi_n^{E*}(x), \tag{4.4.17}$$

where x_A is the opposite point to x only in respect to the de Sitter factor coordinates. If x has full coordinates χ, r, τ, θ and ϕ, then x_A has coordinates $\chi, r, -\tau, \theta + \pi$, and $\phi+\pi$. Then we can define Bogoliubov transforms on the Euclidean modes of this five-dimensional space in an identical way to eq.(4.4.6), to obtain new modes $\tilde{\phi}_n(x)$ and similarly from the operators on this space, a_n^E, we can define $\tilde{a}_n$. As a consequence there are also multiple vacua in this space, parametrised by α, satisfying $\tilde{a}_n|\alpha\rangle = 0$. The associated Wightman functions G_α defined using these new vacua or mode expansions can again be expressed in terms of the Euclidean Wightman function on the bubble, G_E, by (4.4.11).[4] This α-vacuum ambiguity in the bulk is clearly related to a corresponding α-vacuum ambiguity in the boundary.

For the bubble of nothing solution (and of course for the boundary spacetime), the de Sitter coordinates of (4.2.2) cover the whole spacetime, and the de Sitter factor does not degenerate anywhere. The physics of these α-vacua is hence little different from the familiar discussion in de Sitter space, and we will not elaborate on it here. The situation is more interesting for the locally AdS black hole, however, so we pursue this case in more detail.

First, we note that in the black hole background, we can find G_E explicitly. This is done by using the form of the propagator in full AdS$_d$ [141]. In AdS, the invariance under $SO(2,4)$ implies that the propagator can only be a function of the geodesic distance between the two points x, x', $2\sqrt{\sigma(x,x')}$. It is in fact convenient to express the propagator as a function of $P = X \cdot X'$, where X, X' are the corresponding points in the embedding coordinates and the inner product is with respect to the

[4]For the locally AdS black hole, it is possible to have such a vacuum ambiguity despite the fact that AdS has a unique invariant vacuum state because the quotient broke the $SO(2,4)$ isometry group to $SO(1,1) \times SO(1,3)$, which is no longer sufficient to determine a unique vacuum state.

metric of signature $(--++++)$ in the embedding space. This is related to geodesic distance through $\sigma(x, x') = -(P+1)$. In terms of P, the propagator is the solution to

$$\frac{1}{(P^2-1)^{\frac{d-2}{2}}}\partial_P\left((P^2-1)^{\frac{d}{2}}\partial_P G\right) - m^2 G = 0 \tag{4.4.18}$$

which is regular at $P = \infty$. The required function G is

$$G_E(P) = \frac{-e^{i\pi d/2}\sqrt{\pi}\Gamma(2b)}{2^{c-\frac{1}{2}}\Gamma(c)}(P^{-2})^b \ {}_2F_1\left(b+\frac{1}{2}, b\ ; c\ ; P^{-2}\right), \tag{4.4.19}$$

where

$$b = \frac{-1}{4} + \frac{1}{4}d + \frac{1}{4}\sqrt{(d-1)^2+4m^2}, \quad c = 1 + \frac{1}{2}\sqrt{(d-1)^2+4m^2}. \tag{4.4.20}$$

It is divergent for $P = 0$ and for all $|P| = 1$ [142]. Those points which are light-like separated have $P = -1$.

To write the propagator for the quotiented space, we must sum over the images due to the periodic identification in the χ direction. The Euclidean Wightman function on the locally AdS black hole can then be expressed as

$$G_E(P(x, x')) \sim \sum_{n=-\infty}^{\infty} P(x, {x'}_n)^{-2b} \ {}_2F_1\left((b+\frac{1}{2}), b\ ; c\ ; P(x, {x'}_n)^{-2}\right), \tag{4.4.21}$$

where the n dependence of x' indicates that we have included every image of x' under the identification.

This knowledge of the Euclidean Wightman function is sufficient to determine the α-vacua Wightman functions, by using the formula (4.4.11) relating it to an expression with four terms, each involving the Euclidean propagator. Recall that two of these depend only on x and x', but two involve the points antipodal to the original positions.

4.5 Singularities in the α-vacuum on the black hole

The important property of the α-vacuum Wightman function for the locally AdS black hole is that it develops new singularities on (and inside) the event horizon.

These arise because the antipodal points, which were always separated by the cosmological horizon in de Sitter space, become causally related. In the coordinates of (4.2.9), the antipode of a point y^μ, χ is the point $-y^\mu, \chi$, and on and inside the light cone $y^2 = 0$, these points are causally separated. This means that on the horizon, there are additional short-distance singularities in the propagator: in the expression

$$G_\alpha(x,x') = N_\alpha^2(G_E(x,x') + e^{\alpha+\alpha^*}G_E(x',x) + e^{\alpha^*}G_E(x,x'_A) + e^{\alpha}G_E(x_A,x')), \quad (4.5.1)$$

the last two terms can produce an additional singularity as $x \to x'$ if $x = x'$ is a point on the event horizon, as shown in figure 4.3. Because of the antipodal map involved, this additional singularity will have a completely different structure from the usual short-distance singularity: it is not simply proportional to $1/\sigma(x,x')$. It is thus potentially more dangerous than the previous failure of the α-vacuum to be Hadamard in de Sitter space.

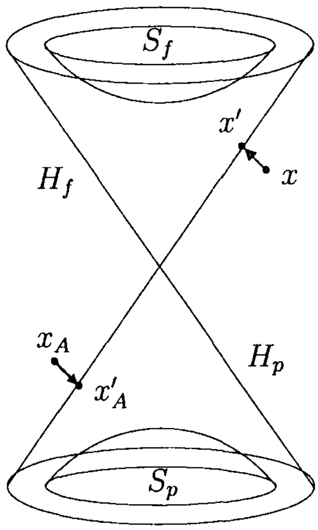

Figure 4.3: As $x \to x'$ on the event horizon, extra singularities appear due to the lightlike separation of x and x'_A.

We will argue that these singularities are a sign that the α-vacua are unphysical on this black hole spacetime. We will see below that they lead to a breakdown in the stress tensor on the event horizon, and obstruct attempts to probe the region behind the horizon by analytic continuation.

They are also interesting from the point of view of understanding the locally AdS black hole, as they are a clear sign that the event horizon is a special place in the black hole geometry. We think of these new singularities as analogous to the breakdown of a state with the wrong temperature or a non-thermal state on the event horizon of a Schwarzschild black hole, which selects the usual Hartle-Hawking state as the unique regular vacuum [136].

4.5.1 Particle detectors

We now seek signs of these new singularities in the Wightman function in physical observables. We will first examine whether a particle detector crossing the horizon sees anything special, following the calculation reviewed in section 4.4.1. As before, the probability that the detector reports a change in energy from E_i to E_j, $E_j > E_i$, is

$$P(E_i \rightarrow E_j) = g^2|m_{ij}|^2 \int_{-\infty}^{\infty} dtdt' e^{-i(E_j-E_i)(t'-t)} G_\alpha(x(t'), x(t)). \qquad (4.5.2)$$

We can consider different trajectories for the detector. If we take the detector to stay at a constant r in (4.2.5), staying outside the black hole, then the behaviour in an α-vacuum will be the same as the behaviour in the de Sitter case, and the ratio of transition probabilities will be given by (4.4.15).

Consider instead an inertial observer, who freely falls across the black hole horizon following some geodesic. Consider first the Euclidean vacuum. Then since $G_E(t'-t)$ is holomorphic in the lower half plane, we can close the contour of integration in (4.5.2) in the LHP to find that the probability is zero. This is the expected result, since the black hole solution is locally AdS, and the Euclidean vacuum is obtained by sum over images from the usual AdS vacuum. To calculate the detector response in the α-vacuum, we consider for definiteness a geodesic through the origin, so that $x_A(t) = x(-t)$ [5]. Then, using the expression (4.4.11) for the α-vacuum propagator in terms of the Euclidean propagator, the detector response

[5]The geodesic used is not important, as the argument essentially relies only on the fact that the Euclidean propagator depends only on the difference in proper time along the path, which is true for any geodesic in AdS.

(4.5.2) is

$$P_\alpha(E_i \to E_j) = \int_{-\infty}^{\infty} dtdt' e^{-i(E_j-E_i)(t'-t)} N_\alpha^2 \times \left(G_E(t,t') + e^{\alpha+\alpha^*} G_E(t',t) + e^\alpha G_E(-t,t') + e^{\alpha^*} G_E(t,-t')\right). \quad (4.5.3)$$

In the last two terms, where the Wightman function involved is a function only of $t+t'$, we can immediately perform the integral over $t-t'$ by closing the contour in the LHP to get 0, showing that these terms make no contribution. The first term is merely proportional to the original integral in the Euclidean vacuum, so it too gives no contribution.

The remaining term involves $G_E(t',t) = G_E(t'-t) = G_E(-t,-t')$. Thus, we have

$$P_\alpha(E_i \to E_j) = \int_{-\infty}^{\infty} dtdt' e^{-i(E_j-E_i)(t'-t)} N_\alpha^2 e^{\alpha+\alpha^*} G_E(-t,-t'). \quad (4.5.4)$$

Upon changing variables from t, t' to $-t, -t'$, the exponential picks up a minus sign, and we are left with the Euclidean rate for the detector to see a change in energies from E_j to E_i,

$$P_\alpha(E_i \to E_j) = N_\alpha^2 e^{\alpha+\alpha^*} P_E\left(E_j \to E_i\right). \quad (4.5.5)$$

Similarly, if we consider the probability for the transition from $E_j \to E_i$ in the α-vacuum, the only contribution will come from the first term in (4.4.11), giving[6]

$$P_\alpha(E_j \to E_i) = \int_{-\infty}^{\infty} dtdt' e^{-i(E_i-E_j)(t'-t)} N_\alpha^2 G_E(x(t'), x(t)) \quad (4.5.6)$$

$$= N_\alpha^2 P_E(E_j \to E_i). \quad (4.5.7)$$

The ratio of the two rates is

$$\frac{P_\alpha\left(E_i \to E_j\right)}{P_\alpha\left(E_j \to E_i\right)} = e^{\alpha+\alpha^*}. \quad (4.5.8)$$

We note that this result is independent of the energies involved, which might seem a disturbing result, but this is the usual problem with the short-distance structure

[6]These rates are divergent because of the integration over $t+t'$, but this simply gives an overall factor that will cancel when considering the ratio of rates.

of the α-vacuum, corresponding precisely to the high energy behaviour of (4.4.15). There is no sign in this calculation of the additional singularities at the event horizon, because the relevant parts of the α-vacuum propagator made no contribution to the calculation. This is perhaps surprising; if there is a breakdown in the quantum state on the horizon, we would expect the behaviour of a particle detector crossing the horizon to be affected.

4.5.2 Stress-energy tensor

Another natural observable to consider in looking for reflections of this new singularity in the propagator on the horizon is the expectation value of the stress tensor in the α-vacuum, $\langle T_{\mu\nu}\rangle_\alpha$. However, the usual construction of this quantity relies on the assumption that the state is Hadamard (see [9] for a review). As we have already noted, the α-vacua are not Hadamard states. Hence, we cannot define a stress tensor by the normal procedure in an α-vacuum state on any spacetime, and it therefore does not appear to be available to us as a probe of the new singularities in the state on the horizon of the locally AdS black hole.

Fortunately, Bernard and Folacci [128] have overcome this obstacle and defined a renormalised expectation value for the stress tensor in the α-vacua of de Sitter space despite the non-Hadamard form of the short-distance singularity.

To briefly review, we usually construct the stress tensor by taking a coincidence limit of an appropriate differential operator acting on the bi-distribution:

$$\langle T_{\mu\nu}(x)\rangle_\alpha = \lim_{x\to x'} \mathcal{D}_{\mu\nu} F(x,x'), \tag{4.5.9}$$

where $\mathcal{D}_{\mu\nu}$ is a differential operator determined from the Lagrangian whose precise form is not important for the present purpose. We cannot take the bi-distribution $F(x,x')$ to be the Wightman function, as its singularity as $x \to x'$ would produce a divergent result for $\langle T_{\mu\nu}(x)\rangle$. We must first renormalize: this is done by defining

$$F(x,x') = G(x,x') - H(x,x'), \tag{4.5.10}$$

where $H(x,x')$ is the Hadamard bi-distribution. This has the form

$$H(x,x') = \frac{1}{(2\pi)^2}\frac{U(x,x')}{\sigma(x,x')} + V(x,x')\log\sigma(x,x') + W(x,x'), \tag{4.5.11}$$

where σ is half the square of the geodesic distance between x and x', and $U(x,x) = 1$. In the massive case, the functions U, V, W can be determined by requiring that H satisfy the Klein-Gordon equation in each argument. Thus, this renormalisation is state-independent, being entirely determined by the geometry of the spacetime manifold.

If $G(x,x')$ is of the Hadamard form, the subtraction in (4.5.10) will cancel the divergences in the two functions, allowing us to define a finite renormalised stress tensor via (4.5.9). However, in the case of interest, the Wightman function for an α-vacuum $G_\alpha(x,x')$ is not of Hadamard form. This subtraction will not then define a good renormalised stress tensor.

The procedure adopted in [128] to address this problem was to introduce a small element of state-dependence into the renormalisation procedure: when we are considering an α-vacuum in de Sitter space, for which

$$\lim_{x \to x'} G_\alpha(x,x') = N_\alpha^2 e^{\alpha+\alpha^*} \left(\frac{1}{(2\pi)^2} \frac{1}{\sigma(x,x')} + \bar{V}(x,x') \log \sigma(x,x') + \bar{W}(x,x') \right) \tag{4.5.12}$$

for some $\bar{V}$ and $\bar{W}$, then we define[7]

$$F(x,x') = G(x,x') - N_\alpha^2 e^{\alpha+\alpha^*} H(x,x'), \tag{4.5.13}$$

where $H(x,x')$ is given by (4.5.11), and use (4.5.9) for this F to define the stress tensor. One can also think of this as defining $\langle T_{\mu\nu}\rangle[G_\alpha] = N_\alpha^2 e^{\alpha+\alpha^*} \langle T_{\mu\nu}\rangle[(N_\alpha^2 e^{\alpha+\alpha^*})^{-1} G_\alpha]$, where $\langle T_{\mu\nu}\rangle[(N_\alpha^2 e^{\alpha+\alpha^*})^{-1} G_\alpha]$ is defined by the usual procedure. The point is that $\bar{G} = (N_\alpha^2 e^{\alpha+\alpha^*})^{-1} G_\alpha$ is of Hadamard form. Although this procedure introduces state-dependence into the renormalisation procedure, the stress tensor so defined shares all the good properties of the usual stress tensor: the difference in energy for excited states above an α-vacuum will be given by the usual point-splitting expression; the construction remains local; and $\nabla^\mu \langle T_{\mu\nu}\rangle = 0$ (since the additional factor introduced is a constant). This allows us to discuss the renormalised stress tensor in the α-vacua on de Sitter space.

[7] There is a difference in notation between our chapter and [128], so the α used there is not the same as the one used here.

We can apply this same prescription to obtain a notion of $\langle T_{\mu\nu}\rangle$ for our α-vacuum states on the bubble of nothing or locally AdS black hole spacetimes. For the bubble of nothing, this gives a finite well-defined stress tensor everywhere. For the black hole, however, this prescription breaks down on the black hole horizon: the new singularities arising from the terms involving $G_E(x, x'_A)$ and $G_E(x_A, x')$ in (4.4.11) are not cancelled by the subtraction (4.5.13), so the stress tensor becomes ill-defined on the horizon. The new singularities imply that $\bar{G} = (N_\alpha^2 e^{\alpha+\alpha^*})^{-1} G_\alpha$ fails to be of Hadamard form on the horizon.

Thus, the ill-behavedness of the α-vacua on the horizon of the locally AdS black hole is signalled through the breakdown of the procedure of [128] for defining a renormalised stress tensor.

4.5.3 Analytic continuation

Another way to see the relation between the event horizon and α-vacua is to consider the extension of the analytic continuation argument of [67, 129] to this case. In [129], it was shown that an n-point correlation function in the CFT on the two boundaries of the eternal BTZ black hole could be related either to bulk interactions integrated over the region $r \geq r_+$ in the bulk spacetime or over the region $r \geq 0$, including the region behind the event horizon (with a different $i\epsilon$ prescription for the bulk-boundary propagators). The argument used analyticity properties of the n-point function, deforming the contour integral over $r \geq r_+$ to an integral over the Euclidean spacetime by complexifying the time coordinate, and then rotating back to the real Lorentzian section in Kruskal coordinates.

Despite the somewhat different structure of the spacetime in the locally AdS black hole, we can apply a similar argument here. If we consider an integral which initially runs over the region $r \geq 0$ in the Lorentzian black hole solution

$$ds^2 = (1 + \frac{r^2}{l^2})d\chi^2 + (1 + \frac{r^2}{l^2})^{-1}dr^2 + r^2[-d\tau^2 + \cosh^2\tau(d\theta^2 + \sin^2\theta d\phi^2)], \quad (4.5.14)$$

we can continue to a Euclidean solution by $\tau \to -i\vartheta$, giving

$$ds^2 = (1 + \frac{r^2}{l^2})d\chi^2 + (1 + \frac{r^2}{l^2})^{-1}dr^2 + r^2[d\vartheta^2 + \cos^2\vartheta(d\theta^2 + \sin^2\theta d\phi^2)]. \quad (4.5.15)$$

We then define a new radial coordinate by $r = 2ly/(1-y^2)$, so

$$ds^2 = \frac{4l^2}{(1-y^2)^2}\left\{dy^2 + y^2[d\vartheta^2 + \cos^2\vartheta(d\theta^2 + \sin^2\theta d\phi^2)]\right\} + \frac{(1+y^2)^2}{(1-y^2)^2}d\chi^2. \quad (4.5.16)$$

We can recover the Kruskal coordinates of (4.2.9) by defining Cartesian coordinates y^i $i = 1,\ldots,4$ on the $(y,\vartheta,\theta,\phi)$ space, and analytically continuing $y^4 \to iy^0$. As in the BTZ case, the Cartesian coordinates are restricted to the interior of the unit ball $(y^i)^2 < 1$; however, the exterior of the unit ball is isometric to the interior, so we can take the integral to run over all y^i if we divide by a factor of two. The factor of two is then used to convert the integral over all y^μ in the Kruskal coordinates (4.2.9) to an integral over $-1 < y^\mu y_\mu < 1$, covering the full black hole spacetime.

The above procedure is possible only in the Euclidean vacuum. If we consider an α-vacuum on the black hole, it will not be possible to continue the integral in this way, as the α-vacuum does not define a regular propagator on the Euclidean spacetime when we analytically continue $\tau \to -i\vartheta$. That is, the additional pole associated with the extra divergences on the event horizon in an α-vacuum will obstruct this kind of contour deformation argument. Thus, we see again that the α-vacuum runs into trouble when we try to look inside the black hole.

4.6 Conclusions

The relation between the bulk and boundary for these time-dependent spacetimes has several new and interesting features. The identification of the cosmological/ acceleration horizons in the bulk with the de Sitter cosmological horizon in the boundary provides an example of a new way of relating thermodynamic behaviour in the bulk to the boundary. We believe this kind of relation should be very general, applying to any non-compact horizon encountered in the AdS/CFT correspondence. This identification provides new insight into the thermodynamic interpretation of horizons in spacetime, since the boundary interpretation for our bulk horizons strongly suggests that the relation $S = A/4G$ can be applied even to such non-compact horizons. This provides support for the very general connection between entropy and horizons advocated in [137, 138]. We have also argued that the area of the event horizon in the locally AdS black hole should not be interpreted thermodynamically.

We argued this from a purely spacetime point of view, but it also seems a natural result from the CFT side, since we conjectured that the CFT dual is a vacuum state, where we see no role for an increasing entropy.

A central result of this chapter was to show that the analogues of α-vacua for a free scalar field on the locally AdS black hole spacetime break down on the event horizon. Thus, these are not good quantum states on the full black hole solution. The unique regular invariant vacuum state on the locally AdS black hole is the Euclidean vacuum. We regard this as evidence that there are no α-vacua in the strongly-coupled dual field theory, which lives on de Sitter space cross a circle. This provides support, from a very different perspective, for the view taken by some authors that α-vacua are not good states in an interacting field theory from the point of view of perturbation theory [118, 119, 122, 124]. This selection of the Euclidean vacuum as a preferred state provides an interesting example of how the bulk spacetime picture can be used to study issues of quantum field theory on more general backgrounds.

It would be interesting to investigate further the interpretation of these two spacetimes from the boundary point of view. In particular, the fact that the bubble of nothing solution exists only if the radius of the χ circle is less than a maximum value seems quite mysterious from the boundary point of view.

In fact, there is also another issue of interpretation which remains open at a purely spacetime level. In the asymptotically flat case, the bubble of nothing solution is interpreted as describing a non-perturbative instability of the Kaluza-Klein vacuum, that is, flat space with one spatial direction periodically identified [107]. In the case of a negative cosmological constant, the analogous interpretation of the bubble solution would be to regard it as describing a non-perturbative decay of this quotient of AdS, the locally AdS black hole. However, the fact that this background is itself time dependent (and even has an event horizon!) may complicate this interpretation. See [143] for further discussion of the interpretation of this bubble as describing a non-perturbative instability.

Chapter 5

Conclusions and Future Directions

In this thesis we sought to explain and extend the AdS/CFT correspondence to new non-supersymmetric and time-dependent spacetimes.

In chapter 3, we generalized previously known $U(1) \times U(1)$-symmetric geometries to find novel solitonic solutions to supergravity which are asymptotically flat but, for certain charges, contain a core region which we can write in Anti-de Sitter form. By using the AdS description, we were able to identify the states in the related field theory to which they are dual. The field theory is believed to be a non-linear sigma model on the symmetric $N_1 N_5$ product of T^4 where N_1 and N_5 are the numbers of D1-branes and D5-branes respectively. The states we found were the result of asymmetric spectral flow on R-R ground states, by $m + n$ on one side and $m - n$ on the other. If these integers satisfy a certain condition, $m = n + 1$, the AdS part is one of the supersymmetric solutions found in [79, 80].

Upon making the identification between geometry and gauge theory, we found that the charges of the asymptotic flat space and the field theory also matched, which we could not have expected in advance to be the case. Recall that to have an AdS core to our spacetime, we require the charges Q_1, Q_5 to be much larger than other scales in the theory. This calculation of charges does not appear to care about the size of the charges.

One feature of the two sides of the duality which did not match was the time taken for results of probes to emerge. In the D-brane description, left and right moving portions of incoming states travel around the direction shared by both types of brane and on meeting can recombine to form closed strings which can leave the branes. In the geometrical picture, modes enter the AdS throat of the space, and travel into and out of this region in a typical time. This time does not agree with that predicted by the D-brane picture. A rigorous explanation for the cause of the disagreement is one aim for future research.

In a similar way, left and right movers already on the branes could be expected to meet and form closed strings which can leave the confining surface as gravitons. We do not see the same behaviour apparent in our geometries. Myers et al in [144] have studied the question of whether they are stable, and demonstrated that a general instability of spacetimes with ergoregions means they must decay, and to a supersymmetric state. The speed of the decay, though, is slow so we believe the spacetimes can still be studied as string backgrounds.

Also of interest is whether a Gregory - Laflamme [145] type instability can arise if we increase the radii of the 4-torus on which we have compactified. A Gregory - Laflamme instability is the tendency of black strings and branes to break up into a series of separate, lower dimension black objects if any compact directions in the spacetime are larger than wavelengths of particular modes. Clearly this could arise in these metrics. This remains a question for future research.

In chapter 3 we were also able to relax conditions on the metric to find new orbifolds of AdS$\times S^3$ in the core region. In this case, a third integer, k, is required to describe the charges h and j of the dual CFT states and these are now no longer the result of spectral flow of ground states. As they are not of this special form, they are more representative of duals of the typical three-charge state. One direction for future research is to find explicit descriptions of these field theory states. Of particular interest is the case where $m+n$ is even and k is even since then, as we noted, the geometry is not a spin manifold. We would like to see a manifestation of this fact in the allowed field theory states.

These spacetimes, both the orbifolded and unorbifolded versions, can also be treated as backgrounds for string theory. They are of interest as they are novel non-supersymmetric spacetimes and in particular have no global timelike Killing vector.

The same can be said of the geometries in chapter 4. Here we studied time-dependent asymptotically locally AdS spacetimes which were the result of analytically continuing two variables in Schwarschild-AdS black holes [32]. Their conformal boundaries are a circle × de Sitter space, so we conjecture that their duals are states in a field theory on $dS_3 \times S^1$. De Sitter space contains a cosmological horizon to which we would like to associate an entropy. The bulk space also has cosmological or acceleration horizons, and we proposed that the two can be identified. We presented evidence that these non-compact horizons still obey the Bekenstein entropy formula [12] $S = A/4G$ by introducing a cut-off in both theories. We went on to argue that the event horizon of a constant curvature black hole with the same asymptotic behaviour could not be assigned to a thermal ensemble state in the boundary theory, as is done for most black holes in the AdS/CFT correspondence, but instead that its thermal nature arises from the thermodynamic properties of the expanding background of the field theory. This is consistent with the fact that it is not a Killing horizon for any Killing vector of the spacetime. It is only a teleological horizon, interpreted as boundary of no return due to the future state of the geometry. Indeed, the dual of the geometric black hole is believed to be the vacuum in the corresponding conformal field theory, not a thermal state. It permits both periodic and anti-periodic fermions, whilst the bubble of nothing is in the NS sector, as the S^1 in the bulk corrseponding to the S^1 in the field theory is contractible. We argue that the black hole is a state of higher energy than the bubble of nothing space and it is believed it can decay to the bubble of nothing when it has anti-periodic fermions. This property was studied in [143] and is reminiscent of flat compactified spacetimes where fermions are half-integer moded on the one circle [107].

Field theories on de Sitter space are known to allow for the existence of an infinite number of possible vacuum states, known as α-vacua. Their validity as the basis for building the Hilbert space of a quantum field theory is an unanswered question. We have attempted to shed new light on the situation by showing that the stress-energy tensor in the black hole, defined according to [128], breaks down on the event horizon if we have anything other than the vacuum obtained by analytically continuing from the Euclidean vacuum. As the horizon should not be distinguishable from the rest of space by an infalling observer, this is evidence that these α-vacua are not acceptable states for perturbative field theory. This is one example of how the AdS/CFT correspondence can offer a different perspective on problems and go some way to resolving them.

At the same time, the question remains of the field theory interpretation of the maximum value of the period of the angular variable χ in equation 4.2.4 allowed by spacetime considerations. Whilst it is unsatisfying not to be able to proffer an explanation for this apparent limit in the field theory, it is a strength of the duality that it presents new problems that may not have been investigated otherwise. For example, the "stringy exclusion" principle which limits the number of single particle states allowed to move on the S^3 of $AdS_3 \times S^3$ found a natural justification in the discovery of giant gravitons [146–149].

The relation between states in gravity and gauge theory is still unclear and an exact mapping seems far from our understanding. It is likely that creating one will require much greater understanding of both sides of the conjecture. We hope our work can contribute to the formulation of a precise correspondence. When discovered, it should give us even deeper insight into the natures of string theory, conformal field theories, and ultimately quantum gravity.

Appendix A

Basic and Auxiliary Results

A.1 Spacetimes of Constant Curvature

A.1.1 Anti-de Sitter Space

Einstein's equation which describes how stress-energy and the geometry of the surrounding spacetime interact is given by

$$8\pi T_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} \tag{A.1.1}$$

where Λ is the cosmological constant. Originally this quantity Λ was thought to be, and set to, zero but for historical, experimental and mathematical reasons it has often been included with both positive and negative values. When it is negative and the space is required to be maximally symmetric, the resulting geometry is known as anti-de Sitter space. One way of picturing it is as a surface within a space of one higher dimension with two timelike directions, as follows: to obtain d-dimensional anti-de Sitter space, let us take co-ordinates $x_1, \ldots, x_{d+1}$ with flat metric $\mu_{ab} = \{diag - 1, -1, 1, \ldots 1\}$ and consider the surface defined by

$$-x_1^2 - x_2^2 + x_3^2 + \cdots + x_{d+1}^2 = -l^2 \tag{A.1.2}$$

in the flat space with metric

$$ds^2 = -dx_1^2 - dx_2^2 + \sum_{i=3}^{d+1} dx_i^2 - . \tag{A.1.3}$$

Then this space has the necessary properties of negative cosmological constant and the maximum number of spacetime symmetries. Specifically, it has $SO(2,d)$ symmetries.

There are parametrizations of this geometry that will come in particularly handy. The first, somewhat similar to polar co-ordinates, is given by

$$\begin{aligned} x_1 &= l\cosh\chi\sin\theta, \\ x_2 &= l\cosh\chi\cos\theta, \\ x_i &= l\sinh\chi\Omega_i, \quad i = 3\ldots d+1 \end{aligned} \tag{A.1.4}$$

where the Ω_i parametrize a $d-1$ sphere. The metric (A.1.3) is then

$$ds^2 = l^2(-\cosh^2\chi d\theta^2 + d\chi^2 + \sinh^2\chi d\Omega^2). \tag{A.1.5}$$

These co-ordinates with $0 \le \chi$ are known as global co-ordinates for AdS_d. The timelike variable could be taken as being between 0 and 2π and we would cover the hyperboloid once. In practise, however, it is more amenable to take it as ranging over all real values, without identifying θ and $\theta + 2\pi$ even though these give us the same geometry, so that we have a realistic range for the timelike variable. This is known as the universal covering of AdS. One may imagine a hyperboloid with many unidentified covering sheets. Clearly, spatial infinity is here described by $\chi \longrightarrow \infty$.

A common transformation is to set $r = \sinh\chi$ and $\tau = \theta$. Then our metric becomes

$$ds^2 = -(1+r^2)d\tau^2 + \frac{dr^2}{1+r^2} + r^2 d\Omega^2. \tag{A.1.6}$$

Another closely related metric is found by simply setting $\sinh\chi = \tan\psi$. ψ need only range over $[0,\pi/2]$ to describe the whole space. Then we have the metric of AdS in the form

$$ds^2 = \frac{l^2}{\cos^2\psi}(-d\theta^2 + d\psi^2 + \sin^2\psi d\Omega^2). \tag{A.1.7}$$

This has the topology of a conformal factor, $1/\cos\psi$, times half an Einstein static universe, $\lessdot \times S^{d-1}$. Here we can see that spatial infinity corresponds to $\psi = \pi/2$ and the conformal boundary can be described by the metric

$$ds^2 = -d\theta^2 + d\psi^2 + \sin^2\psi d\Omega^2. \tag{A.1.8}$$

Finally, two more relevant parametrizations are those given by 'Poincaré ' co-ordinates. Here, we take [20]

$$x_1 = \left(\frac{1}{2u}(1 + u^2(l^2 + \vec{x}^2 - t^2))\right) \quad (A.1.9)$$

$$x_2 = lut \quad (A.1.10)$$

$$x_i = lux_i \quad i = 3 \dots d \quad (A.1.11)$$

$$x_{d+1} = \left(\frac{1}{2u}(1 - u^2(l^2 - \vec{x}^2 + t^2))\right). \quad (A.1.12)$$

The metric is then

$$ds^2 = l^2\left(\frac{du^2}{u^2} + u^2(-dt^2 + dx_i dx^i)\right). \quad (A.1.13)$$

In this description, spatial infinity is at $u \to \infty$. Alternatively we can take $z = 1/u$ whereupon our description of AdS is

$$ds^2 = l^2\left(\frac{dz^2 + dx_i dx^i - dt^2}{z^2}\right). \quad (A.1.14)$$

Here, the boundary of the spacetime is at $z = 0$.

Any of these forms of anti-de Sitter space may arise in discussions according to which is most well-suited.

A.1.2 De Sitter Space

In a similar way to the previous subsection, we can consider vacuum solutions to Einstein's equation A.1.1 when $\Lambda > 0$. Then the resulting spacetime with constant positive curvature is known as de Sitter space. Here we give a few relevant ways of depicting such a space.

It can be seen as an embedding in Minkowski space of one higher dimension by taking the 'surface' defined by:

$$-X_0^2 + X_1^2 + \dots X_p^2 = l^2 \quad (A.1.15)$$

where l is the radius of curvature of the spacetime.

Another way we will be describing de Sitter space is through co-ordinates given by

$$X_0 = l \sinh t, \quad X_i = l y_i \cosh t \quad (A.1.16)$$

where the y_i parametrize an $p-1$ sphere. Then the metric in these variables is

$$ds^2 = -l^2 dt^2 + l^2 \cosh^2 t d\Omega^2 \tag{A.1.17}$$

where dΩ is the metric induced on the $p-1$ sphere.

A.2 Inverse metric

To calculate the inverse metric, it is convenient to start from the fibred form of the metric (3.2.12), construct a corresponding orthonormal frame, and invert that. For this reason, it is simpler to give the inverse metric in terms of the boosted coordinates $\tilde{t} = t\cosh\delta_p - y\sinh\delta_p$, $\tilde{y} = y\cosh\delta_p - t\sinh\delta_p$. The inverse metric is

$$g^{\tilde{t}\tilde{t}} = -\frac{1}{\sqrt{\tilde{H}_1\tilde{H}_5}}\left(f + M + M\sinh^2\delta_1 + M\sinh^2\delta_5 + \frac{M^2\cosh^2\delta_1\cosh^2\delta_5 r^2}{g(r)}\right), \tag{A.2.1}$$

$$g^{\tilde{t}\tilde{y}} = \frac{1}{\sqrt{\tilde{H}_1\tilde{H}_5}}\frac{M^2\sinh\delta_1\sinh\delta_5\cosh\delta_1\cosh\delta_5 a_1 a_2}{g(r)}, \tag{A.2.2}$$

$$g^{\tilde{t}\phi} = -\frac{1}{\sqrt{\tilde{H}_1\tilde{H}_5}}\frac{M\cosh\delta_1\cosh\delta_5 a_2(r^2+a_1^2)}{g(r)}, \tag{A.2.3}$$

$$g^{\tilde{t}\psi} = -\frac{1}{\sqrt{\tilde{H}_1\tilde{H}_5}}\frac{M\cosh\delta_1\cosh\delta_5 a_1(r^2+a_2^2)}{g(r)}, \tag{A.2.4}$$

$$g^{\tilde{y}\tilde{y}} = \frac{1}{\sqrt{\tilde{H}_1\tilde{H}_5}}\left(f + M\sinh^2\delta_1 + M\sinh^2\delta_5 + \frac{M^2\sinh^2\delta_1\sinh^2\delta_5(r^2+a_1^2+a_2^2-M)}{g(r)}\right), \tag{A.2.5}$$

$$g^{\tilde{y}\phi} = -\frac{1}{\sqrt{\tilde{H}_1\tilde{H}_5}}\frac{M\sinh\delta_1\sinh\delta_5 a_1(r^2+a_1^2-M)}{g(r)}, \tag{A.2.6}$$

$$g^{\tilde{y}\psi} = -\frac{1}{\sqrt{\tilde{H}_1\tilde{H}_5}}\frac{M\sinh\delta_1\sinh\delta_5 a_2(r^2+a_2^2-M)}{g(r)}, \tag{A.2.7}$$

$$g^{rr} = \frac{1}{\sqrt{\tilde{H}_1\tilde{H}_5}}\frac{g(r)}{r^2}, \tag{A.2.8}$$

$$g^{\theta\theta} = \frac{1}{\sqrt{\tilde{H}_1\tilde{H}_5}}, \tag{A.2.9}$$

$$g^{\phi\phi} = \frac{1}{\sqrt{\tilde{H}_1\tilde{H}_5}}\left(\frac{1}{\sin^2\theta} + \frac{r^2(a_1^2-a_2^2) - Ma_1^2}{g(r)}\right), \tag{A.2.10}$$

$$g^{\phi\psi} = -\frac{1}{\sqrt{\tilde{H}_1\tilde{H}_5}}\frac{Ma_1a_2}{g(r)}, \tag{A.2.11}$$

$$g^{\psi\psi} = \frac{1}{\sqrt{\tilde{H}_1\tilde{H}_5}}\left(\frac{1}{\cos^2\theta} + \frac{r^2(a_2^2-a_1^2) - Ma_2^2}{g(r)}\right). \tag{A.2.12}$$

Bibliography

[1] J. Polchinski, "String Theory. Vol. 1: An Introduction to the Bosonic String,". Cambridge, UK: Univ. Pr. (1998) 402 p.

[2] C. V. Johnson, "D-Branes,". Cambridge, UK: Univ. Pr. (2003) 548 p.

[3] R. Blumenhagen, M. Cvetic, P. Langacker, and G. Shiu, "Toward Realistic Intersecting D-brane Models," *Ann. Rev. Nucl. Part. Sci.* **55** (2005) 71–139, hep-th/0502005.

[4] J. Polchinski, "Dirichlet-Branes and Ramond-Ramond Charges," *Phys. Rev. Lett.* **75** (1995) 4724–4727, hep-th/9510017.

[5] J. M. Maldacena, "The Large N Limit of Superconformal Field Theories and Supergravity," *Adv. Theor. Math. Phys.* **2** (1998) 231–252, hep-th/9711200.

[6] G. 't Hooft, "A Planar Diagram Theory for Strong Interactions," *Nucl. Phys.* **B72** (1974) 461.

[7] L. Susskind and E. Witten, "The Holographic Bound in Anti-de Sitter Space," hep-th/9805114.

[8] R. M. Wald, "General Relativity,". Chicago, Usa: Univ. Pr. (1984) 491p.

[9] R. M. Wald, *Quantum field theory in curved space-time and black hole thermodynamics.* Chicago University Press, Chicago, USA, 1994.

[10] A. W. Peet, "The Bekenstein Formula and String Theory (N-brane Theory)," *Class. Quant. Grav.* **15** (1998) 3291–3338, hep-th/9712253.

[11] G. W. Gibbons and S. W. Hawking, "Cosmological Event Horizons, Thermodynamics, and Particle Creation," *Phys. Rev.* **D15** (1977) 2738–2751.

[12] J. D. Bekenstein, "Generalized Second Law of Thermodynamics in Black Hole Physics," *Phys. Rev.* **D9** (1974) 3292–3300.

[13] A. Strominger and C. Vafa, "Microscopic Origin of the Bekenstein-Hawking Entropy," *Phys. Lett.* **B379** (1996) 99–104, hep-th/9601029.

[14] J. M. Maldacena and A. Strominger, "Black Hole Greybody Factors and D-brane Spectroscopy," *Phys. Rev.* **D55** (1997) 861–870, hep-th/9609026.

[15] S. S. Gubser, I. R. Klebanov, and A. M. Polyakov, "Gauge Theory Correlators from Non-Critical String Theory," *Phys. Lett.* **B428** (1998) 105–114, hep-th/9802109.

[16] G. T. Horowitz and J. Polchinski, "Gauge / Gravity Duality," gr-qc/0602037.

[17] E. Witten, "Anti-de Sitter Space and Holography," *Adv. Theor. Math. Phys.* **2** (1998) 253–291, hep-th/9802150.

[18] P. Minces, "On the Role of Boundary Terms in the AdS/CFT Correspondence," *Nucl. Phys. Proc. Suppl.* **127** (2004) 174–178, hep-th/0401149.

[19] J. Polchinski, "String Theory. Vol. 2: Superstring Theory and Beyond,". Cambridge, UK: Univ. Pr. (1998) 400 p.

[20] O. Aharony, S. S. Gubser, J. M. Maldacena, H. Ooguri, and Y. Oz, "Large N Field Theories, String Theory and Gravity," *Phys. Rept.* **323** (2000) 183–386, hep-th/9905111.

[21] V. Balasubramanian, P. Kraus, and A. E. Lawrence, "Bulk vs. Boundary Dynamics in Anti-de Sitter Spacetime," *Phys. Rev.* **D59** (1999) 046003, hep-th/9805171.

[22] V. Balasubramanian, P. Kraus, A. E. Lawrence, and S. P. Trivedi, "Holographic Probes of Anti-de Sitter Space-times," *Phys. Rev.* **D59** (1999) 104021, hep-th/9808017.

[23] A. W. Peet and J. Polchinski, "UV/IR Relations in AdS Dynamics," *Phys. Rev.* **D59** (1999) 065011, hep-th/9809022.

[24] A. Lawrence and A. Sever, "Holography and Renormalization in Lorentzian Signature," hep-th/0606022.

[25] H. Lin, O. Lunin, and J. M. Maldacena, "Bubbling AdS Space and 1/2 BPS Geometries," *JHEP* **10** (2004) 025, hep-th/0409174.

[26] M. Boni and P. J. Silva, "Revisiting the D1/D5 System or Bubbling in AdS(3)," *JHEP* **10** (2005) 070, hep-th/0506085.

[27] D. Martelli and J. F. Morales, "Bubbling AdS(3)," *JHEP* **02** (2005) 048, hep-th/0412136.

[28] S. D. Mathur, "The Fuzzball Proposal for Black Holes: An Elementary Review," *Fortsch. Phys.* **53** (2005) 793–827, hep-th/0502050.

[29] S. D. Mathur, A. Saxena, and Y. K. Srivastava, "Constructing 'Hair' for the Three Charge Hole," *Nucl. Phys.* **B680** (2004) 415–449, hep-th/0311092.

[30] O. Lunin and S. D. Mathur, "The Slowly Rotating Near Extremal D1-D5 System as a 'Hot Tube'," *Nucl. Phys.* **B615** (2001) 285–312, hep-th/0107113.

[31] S. Kachru, R. Kallosh, A. Linde, and S. P. Trivedi, "De Sitter Vacua in String Theory," *Phys. Rev.* **D68** (2003) 046005, hep-th/0301240.

[32] V. Balasubramanian and S. F. Ross, "The Dual of Nothing," *Phys. Rev.* **D66** (2002) 086002, hep-th/0205290.

[33] M. Banados, A. Gomberoff, and C. Martinez, "Anti-de Sitter Space and Black Holes," *Class. Quant. Grav.* **15** (1998) 3575–3598, hep-th/9805087.

[34] M. Banados, "Constant Curvature Black Holes," *Phys. Rev.* **D57** (1998) 1068–1072, gr-qc/9703040.

[35] T. Padmanabhan, "Thermodynamics and / of Horizons: A Comparison of Schwarzschild, Rindler and de Sitter Spacetimes," *Mod. Phys. Lett.* **A17** (2002) 923–942, gr-qc/0202078.

[36] M. J. Duff, R. R. Khuri, and J. X. Lu, "String Solitons," *Phys. Rept.* **259** (1995) 213–326, hep-th/9412184.

[37] I. Taylor, Washington, "Lectures on D-branes, Gauge Theory and M(atrices)," hep-th/9801182.

[38] E. Witten, "Bound States of Strings and P-branes," *Nucl. Phys.* **B460** (1996) 335–350, hep-th/9510135.

[39] E. Witten, "On the Conformal Field Theory of the Higgs Branch," *JHEP* **07** (1997) 003, hep-th/9707093.

[40] N. Seiberg and E. Witten, "The D1/D5 System and Singular CFT," *JHEP* **04** (1999) 017, hep-th/9903224.

[41] G. T. Horowitz, J. M. Maldacena, and A. Strominger, "Nonextremal Black Hole Microstates and U-duality," *Phys. Lett.* **B383** (1996) 151–159, hep-th/9603109.

[42] S. Ferrara and R. Kallosh, "Supersymmetry and Attractors," *Phys. Rev.* **D54** (1996) 1514–1524, hep-th/9602136.

[43] A. Giveon, D. Kutasov, and N. Seiberg, "Comments on String Theory on AdS(3)," *Adv. Theor. Math. Phys.* **2** (1998) 733–780, hep-th/9806194.

[44] J. de Boer, "Six-Dimensional Supergravity on S**3 x AdS(3) and 2d Conformal Field Theory," *Nucl. Phys.* **B548** (1999) 139–166, hep-th/9806104.

[45] S. J. Avis, C. J. Isham, and D. Storey, "Quantum Field Theory in Anti-De Sitter Space-Time," *Phys. Rev.* **D18** (1978) 3565.

[46] S. Aminneborg, I. Bengtsson, S. Holst, and P. Peldan, "Making Anti-de Sitter Black Holes," *Class. Quant. Grav.* **13** (1996) 2707–2714, gr-qc/9604005.

[47] S. F. Ross, "Black Hole Thermodynamics," hep-th/0502195.

[48] W. G. Unruh, "Notes on Black Hole Evaporation," *Phys. Rev.* **D14** (1976) 870.

[49] S. W. Hawking, "Black Hole Explosions," *Nature* **248** (1974) 30–31.

[50] J. M. Bardeen, B. Carter, and S. W. Hawking, "The Four Laws of Black Hole Mechanics," *Commun. Math. Phys.* **31** (1973) 161–170.

[51] J. Callan, Curtis G., J. A. Harvey, and A. Strominger, "Supersymmetric String Solitons," hep-th/9112030.

[52] M. Kaku, "Introduction to Superstrings and M-theory,". New York, USA: Springer (1999) 587 p.

[53] P. Di Francesco, P. Mathieu, and D. Senechal, "Conformal Field Theory,". New York, USA: Springer (1997) 890 p.

[54] P. Ravanini, "Informal Introduction to Extended Algebras and Conformal Field Theories with $c \geq 1$,". Lectures given at 'Program on String and Conformal Field Theory', Spring, 1989, Copenhagen, Denmark.

[55] J. Cohn, D. Friedan, Z.-a. Qiu, and S. H. Shenker, "Covariant Quantization of Supersymmetric String Theories: The Spinor Field of the Ramond-Neveu-Schwarz Model," *Nucl. Phys.* **B278** (1986) 577.

[56] D. Bailin and A. Love, "Supersymmetric Gauge Field Theory and String Theory,". Bristol, UK: IOP (1994) 322 p. (Graduate student series in physics).

[57] A. Schwimmer and N. Seiberg, "Comments on the N=2, N=3, N=4 Superconformal Algebras in Two-Dimensions," *Phys. Lett.* **B184** (1987) 191.

[58] O. Lunin and S. D. Mathur, "Three-point Functions for M(N)/S(N) Orbifolds with N = 4 Supersymmetry," *Commun. Math. Phys.* **227** (2002) 385–419, hep-th/0103169.

[59] R. Argurio, A. Giveon, and A. Shomer, "Superstrings on AdS(3) and Symmetric Products," *JHEP* **12** (2000) 003, hep-th/0009242.

[60] K. Hosomichi and Y. Sugawara, "Hilbert Space of Space-time SCFT in AdS(3) Superstring and T**(4kp)/S(kp) SCFT," *JHEP* **01** (1999) 013, hep-th/9812100.

[61] O. Lunin and S. D. Mathur, "Correlation Functions for M(N)/S(N) Orbifolds," *Commun. Math. Phys.* **219** (2001) 399–442, hep-th/0006196.

[62] O. Lunin and S. D. Mathur, "Rotating Deformations of AdS(3) x S(3), the Orbifold CFT and Strings in the pp-wave Limit," *Nucl. Phys.* **B642** (2002) 91–113, hep-th/0206107.

[63] V. Balasubramanian, P. Kraus, and M. Shigemori, "Massless Black Holes and Black Rings as Effective Geometries of the D1-D5 System," *Class. Quant. Grav.* **22** (2005) 4803–4838, hep-th/0508110.

[64] A. Sen, "Extremal Black Holes and Elementary String States," *Mod. Phys. Lett.* **A10** (1995) 2081–2094, hep-th/9504147.

[65] W. Israel, "Event Horizons in Static Vacuum Space-times," *Phys. Rev.* **164** (1967) 1776–1779.

[66] R. C. Myers, "Pure States Don't Wear Black," *Gen. Rel. Grav.* **29** (1997) 1217–1222, gr-qc/9705065.

[67] J. M. Maldacena, "Eternal Black Holes in Anti-de-Sitter," *JHEP* **04** (2003) 021, hep-th/0106112.

[68] O. Lunin and S. D. Mathur, "AdS/CFT Duality and the Black Hole Information Paradox," *Nucl. Phys.* **B623** (2002) 342–394, hep-th/0109154.

[69] O. Lunin and S. D. Mathur, "Statistical Interpretation of Bekenstein Entropy for Systems with a Stretched Horizon," *Phys. Rev. Lett.* **88** (2002) 211303, `hep-th/0202072`.

[70] S. W. Hawking, "Breakdown of Predictability in Gravitational Collapse," *Phys. Rev.* **D14** (1976) 2460–2473.

[71] L. Susskind, L. Thorlacius, and J. Uglum, "The Stretched Horizon and Black Hole Complementarity," *Phys. Rev.* **D48** (1993) 3743–3761, `hep-th/9306069`.

[72] V. Balasubramanian, J. de Boer, V. Jejjala, and J. Simon, "The Library of Babel: On the Origin of Gravitational Thermodynamics," *JHEP* **12** (2005) 006, `hep-th/0508023`.

[73] V. Balasubramanian, J. de Boer, E. Keski-Vakkuri, and S. F. Ross, "Supersymmetric Conical Defects: Towards a String Theoretic Description of Black Hole Formation," *Phys. Rev.* **D64** (2001) 064011, `hep-th/0011217`.

[74] J. M. Maldacena and L. Maoz, "De-singularization by Rotation," *JHEP* **12** (2002) 055, `hep-th/0012025`.

[75] O. Lunin and S. D. Mathur, "Metric of the Multiply Wound Rotating String," *Nucl. Phys.* **B610** (2001) 49–76, `hep-th/0105136`.

[76] O. Lunin, J. M. Maldacena, and L. Maoz, "Gravity Solutions for the D1-D5 System with Angular Momentum," `hep-th/0212210`.

[77] B. C. Palmer and D. Marolf, "Counting Supertubes," *JHEP* **06** (2004) 028, `hep-th/0403025`.

[78] D. Bak, Y. Hyakutake, S. Kim, and N. Ohta, "A Geometric Look on the Microstates of Supertubes," *Nucl. Phys.* **B712** (2005) 115–138, `hep-th/0407253`.

[79] S. Giusto, S. D. Mathur, and A. Saxena, "Dual Geometries for a Set of 3-charge Microstates," *Nucl. Phys.* **B701** (2004) 357–379, `hep-th/0405017`.

[80] S. Giusto, S. D. Mathur, and A. Saxena, "3-charge Geometries and their CFT Duals," *Nucl. Phys.* **B710** (2005) 425–463, hep-th/0406103.

[81] D. Mateos and P. K. Townsend, "Supertubes," *Phys. Rev. Lett.* **87** (2001) 011602, hep-th/0103030.

[82] R. Emparan, D. Mateos, and P. K. Townsend, "Supergravity Supertubes," *JHEP* **07** (2001) 011, hep-th/0106012.

[83] I. Bena and P. Kraus, "Three Charge Supertubes and Black Hole Hair," *Phys. Rev.* **D70** (2004) 046003, hep-th/0402144.

[84] I. Bena, "Splitting Hairs of the Three Charge Black Hole," *Phys. Rev.* **D70** (2004) 105018, hep-th/0404073.

[85] I. Bena and N. P. Warner, "One Ring to Rule Them All ... and in the Darkness Bind Them?," hep-th/0408106.

[86] H. Elvang, R. Emparan, D. Mateos, and H. S. Reall, "Supersymmetric Black Rings and Three-Charge Supertubes," *Phys. Rev.* **D71** (2005) 024033, hep-th/0408120.

[87] I. Bena, C.-W. Wang, and N. P. Warner, "Black Rings with Varying Charge Density," *JHEP* **03** (2006) 015, hep-th/0411072.

[88] M. Cvetic and D. Youm, "Rotating Intersecting M-branes," *Nucl. Phys.* **B499** (1997) 253–282, hep-th/9612229.

[89] M. Cvetic and D. Youm, "General Rotating Five Dimensional Black Holes of Toroidally Compactified Heterotic String," *Nucl. Phys.* **B476** (1996) 118–132, hep-th/9603100.

[90] M. Cvetic and F. Larsen, "Near Horizon Geometry of Rotating Black Holes in Five Dimensions," *Nucl. Phys.* **B531** (1998) 239–255, hep-th/9805097.

[91] J. B. Gutowski, D. Martelli, and H. S. Reall, "All Supersymmetric Solutions of Minimal Supergravity in Six Dimensions," *Class. Quant. Grav.* **20** (2003) 5049–5078, hep-th/0306235.

[92] O. Lunin, "Adding Momentum to D1-D5 System," *JHEP* **04** (2004) 054, hep-th/0404006.

[93] S. Giusto and S. D. Mathur, "Geometry of D1-D5-P Bound States," *Nucl. Phys.* **B729** (2005) 203–220, hep-th/0409067.

[94] E. J. Martinec and W. McElgin, "String Theory on AdS Orbifolds," *JHEP* **04** (2002) 029, hep-th/0106171.

[95] E. J. Martinec and W. McElgin, "Exciting AdS Orbifolds," *JHEP* **10** (2002) 050, hep-th/0206175.

[96] M. Cvetic, G. W. Gibbons, H. Lu, and C. N. Pope, "Rotating Black Holes in Gauged Supergravities: Thermodynamics, Supersymmetric Limits, Topological Solitons and Time Machines," hep-th/0504080.

[97] L. Paulot, "Superconformal Selfdual Sigma-Models," *JHEP* **10** (2004) 071, hep-th/0404027.

[98] F. Larsen and E. J. Martinec, "U(1) Charges and Moduli in the D1-D5 System," *JHEP* **06** (1999) 019, hep-th/9905064.

[99] J. Figueroa-O'Farrill and J. Simon, "Supersymmetric Kaluza-Klein Reductions of AdS Backgrounds," *Adv. Theor. Math. Phys.* **8** (2004) 217–317, hep-th/0401206.

[100] J. Figueroa-O'Farrill, O. Madden, S. F. Ross, and J. Simon, "Quotients of AdS(p+1) x S**q: Causally Well-Behaved Spaces and Black Holes," *Phys. Rev.* **D69** (2004) 124026, hep-th/0402094.

[101] M. Cvetic and F. Larsen, "General Rotating Black Holes in String Theory: Greybody Factors and Event Horizons," *Phys. Rev.* **D56** (1997) 4994–5007, hep-th/9705192.

[102] W. H. Press and S. A. Teukolsky, "Floating orbits, Superradiant scattering and the Black-hole bomb," *Nature* **238** (1972) 211.

[103] T. Damour, N. Deruelle, and R. Ruffini, "On Quantum Resonances in Stationary Geometries," *Lett. Nuovo Cim.* **15** (1976) 257–262.

[104] S. W. Hawking and H. S. Reall, "Charged and Rotating AdS Black Holes and their CFT duals," *Phys. Rev.* **D61** (2000) 024014, hep-th/9908109.

[105] D. Birmingham and M. Rinaldi, "Bubbles in Anti-de Sitter Space," *Phys. Lett.* **B544** (2002) 316–320, hep-th/0205246.

[106] O. , M. Fabinger, G. T. Horowitz, and E. Silverstein, "Clean Time-dependent String Backgrounds from Bubble Baths," *JHEP* **07** (2002) 007, hep-th/0204158.

[107] E. Witten, "Instability of the Kaluza-Klein Vacuum," *Nucl. Phys.* **B195** (1982) 481.

[108] M. Banados, C. Teitelboim, and J. Zanelli, "The Black Hole in Three-dimensional Space-time," *Phys. Rev. Lett.* **69** (1992) 1849–1851, hep-th/9204099.

[109] M. Banados, M. Henneaux, C. Teitelboim, and J. Zanelli, "Geometry of the (2+1) Black Hole," *Phys. Rev.* **D48** (1993) 1506–1525, gr-qc/9302012.

[110] N. A. Chernikov and E. A. Tagirov, "Quantum Theory of Scalar Fields in de Sitter Space-time," *Annales Poincare Phys. Theor.* **A9** (1968) 109.

[111] E. A. Tagirov, "Consequences of Field Quantization in de Sitter Type Cosmological Models," *Ann. Phys.* **76** (1973) 561–579.

[112] E. Mottola, "Particle Creation in De Sitter Space," *Phys. Rev.* **D31** (1985) 754.

[113] B. Allen, "Vacuum States in De Sitter Space," *Phys. Rev.* **D32** (1985) 3136.

[114] B. Allen and T. Jacobson, "Vector Two Point Functions in Maximally Symmetric Spaces," *Commun. Math. Phys.* **103** (1986) 669.

[115] I. T. Drummond, "Dimensional Regularization of Massless Theories in Spherical Space-Time," *Nucl. Phys.* **B94** (1975) 115.

[116] J. Bros and U. Moschella, "Two-point Functions and Quantum Fields in de Sitter Universe," *Rev. Math. Phys.* **8** (1996) 327–392, gr-qc/9511019.

[117] J. Bros, H. Epstein, and U. Moschella, "Analyticity Properties and Thermal Effects for General Quantum Field Theory on de Sitter Space-time," *Commun. Math. Phys.* **196** (1998) 535–570, gr-qc/9801099.

[118] T. Banks and L. Mannelli, "De Sitter Vacua, Renormalization and Locality," *Phys. Rev.* **D67** (2003) 065009, hep-th/0209113.

[119] M. B. Einhorn and F. Larsen, "Interacting Quantum Field Theory in de Sitter Vacua," *Phys. Rev.* **D67** (2003) 024001, hep-th/0209159.

[120] U. H. Danielsson, "On the Consistency of de Sitter Vacua," *JHEP* **12** (2002) 025, hep-th/0210058.

[121] K. Goldstein and D. A. Lowe, "A Note on Alpha-Vacua and Interacting Field Theory in de Sitter Space," *Nucl. Phys.* **B669** (2003) 325–340, hep-th/0302050.

[122] M. B. Einhorn and F. Larsen, "Squeezed States in the de Sitter Vacuum," *Phys. Rev.* **D68** (2003) 064002, hep-th/0305056.

[123] H. Collins, R. Holman, and M. R. Martin, "The Fate of the Alpha-Vacuum," *Phys. Rev.* **D68** (2003) 124012, hep-th/0306028.

[124] K. Goldstein and D. A. Lowe, "Real-time Perturbation Theory in de Sitter Space," *Phys. Rev.* **D69** (2004) 023507, hep-th/0308135.

[125] H. Collins and R. Holman, "Taming the Alpha Vacuum," *Phys. Rev.* **D70** (2004) 084019, hep-th/0312143.

[126] J. de Boer, V. Jejjala, and D. Minic, "Alpha-states in de Sitter Space," *Phys. Rev.* **D71** (2005) 044013, hep-th/0406217.

[127] H. Collins, "Fermionic Alpha-Vacua," *Phys. Rev.* **D71** (2005) 024002, hep-th/0410229.

[128] D. Bernard and A. Folacci, "Hadamard Function, Stress Tensor and De Sitter Space," *Phys. Rev.* **D34** (1986) 2286.

[129] P. Kraus, H. Ooguri, and S. Shenker, "Inside the Horizon with AdS/CFT," *Phys. Rev.* **D67** (2003) 124022, hep-th/0212277.

[130] R.-G. Cai, "Constant Curvature Black Hole and Dual Field Theory," *Phys. Lett.* **B544** (2002) 176–182, hep-th/0206223.

[131] V. Balasubramanian and P. Kraus, "A Stress Tensor for Anti-de Sitter Gravity," *Commun. Math. Phys.* **208** (1999) 413–428, hep-th/9902121.

[132] M. Henningson and K. Skenderis, "The Holographic Weyl Anomaly," *JHEP* **07** (1998) 023, hep-th/9806087.

[133] K. Skenderis, "Asymptotically Anti-de Sitter Spacetimes and their Stress Energy Tensor," *Int. J. Mod. Phys.* **A16** (2001) 740–749, hep-th/0010138.

[134] A. Strominger, "Black Hole Entropy from Near-Horizon Microstates," *JHEP* **02** (1998) 009, hep-th/9712251.

[135] E. Witten, "Anti-de Sitter Space, Thermal Phase Transition, and Confinement in Gauge Theories," *Adv. Theor. Math. Phys.* **2** (1998) 505–532, hep-th/9803131.

[136] B. S. Kay and R. M. Wald, "Theorems on the Uniqueness and Thermal Properties of Stationary, Nonsingular, Quasifree States on Space-Times with a Bifurcate Killing Horizon," *Phys. Rept.* **207** (1991) 49–136.

[137] T. Jacobson, "Thermodynamics of Space-time: The Einstein Equation of State," *Phys. Rev. Lett.* **75** (1995) 1260–1263, gr-qc/9504004.

[138] T. Jacobson and R. Parentani, "Horizon Entropy," *Found. Phys.* **33** (2003) 323–348, gr-qc/0302099.

[139] R. Bousso, A. Maloney, and A. Strominger, "Conformal Vacua and Entropy in de Sitter Space," *Phys. Rev.* **D65** (2002) 104039, hep-th/0112218.

[140] C. J. C. Burges, "The De Sitter Vacuum," *Nucl. Phys.* **B247** (1984) 533.

[141] C. P. Burgess and C. A. Lutken, "Propagators and Effective Potentials in Anti-De Sitter Space," *Phys. Lett.* **B153** (1985) 137.

[142] W. Magnus and F. Oberhettinger, *Special Functions of Mathematical Physics.* Chelsea Publishing Company, 1949. p.7.

[143] V. Balasubramanian, K. Larjo, and J. Simon, "Much Ado About Nothing," *Class. Quant. Grav.* **22** (2005) 4149–4170, hep-th/0502111.

[144] V. Cardoso, O. J. C. Dias, J. L. Hovdebo, and R. C. Myers, "Instability of Non-Supersymmetric Smooth Geometries," *Phys. Rev.* **D73** (2006) 064031, hep-th/0512277.

[145] R. Gregory and R. Laflamme, "Black Strings and P-branes are Unstable," *Phys. Rev. Lett.* **70** (1993) 2837–2840, hep-th/9301052.

[146] J. M. Maldacena and A. Strominger, "AdS(3) Black Holes and a Stringy Exclusion Principle," *JHEP* **12** (1998) 005, hep-th/9804085.

[147] J. McGreevy, L. Susskind, and N. Toumbas, "Invasion of the Giant Gravitons from Anti-de Sitter Space," *JHEP* **06** (2000) 008, hep-th/0003075.

[148] A. Hashimoto, S. Hirano, and N. Itzhaki, "Large Branes in AdS and their Field Theory Dual," *JHEP* **08** (2000) 051, hep-th/0008016.

[149] M. T. Grisaru, R. C. Myers, and O. Tafjord, "SUSY and Goliath," *JHEP* **08** (2000) 040, hep-th/0008015.