Working Memory and Children's Mathematical Skills

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Declaration

None of the data or material contained in this thesis has been submitted for previous or simultaneous consideration for a degree in this or any other university.

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Abstract

Previous studies which examined the relationship between working memory (WM) ability and children's mathematics performance typically measured mathematics ability as a general skill (e.g. Gathercole & Pickering, 2000a) or mental arithmetic ability (e.g. Adams & Hitch, 1997), used number- or digit-based WM assessments and did not control for individual differences in a child's general ability (e.g. intelligence). The aim of this thesis was to extend this research to investigate the associations between the components of the tripartite WM model (e.g. Baddeley, 1986) and a range of mathematical skills in 7-/8- and 9-/10-year-olds using non-digit-based WM assessments, controlling for a measure of general ability.

The relationship between WM ability and children's curriculum-based mathematics performance was investigated using a correlational design in Chapters 3 and 4. Assessments, developed in Chapter 2, were used to measure four mathematical skills outlined in the National Curriculum for England. The results indicated that central executive and visuo-spatial sketchpad, but not phonological loop, scores predicted unique variance in performance across all four mathematical skills, even when controlling for NVIQ. Furthermore, both WM abilities were found to predict Key Stage 2 mathematics achievement one year after initial testing (Chapter 8).

The same methodology was used in Chapters 6 and 7 to explore the relationship between WM ability and children's performance-related mathematics abilities (see chapter 5). All three components of WM predicted unique variance in these mathematical skills, but a markedly distinct pattern of associations was revealed between the two age groups. In particular, the data implicated a stronger role for the visuo-spatial sketchpad in the younger children's mathematics.

The role of visuo-spatial WM in children's mathematics was explored further in Chapter 9 where a discrepancy definition was applied to identify children with poor mathematics or poor visuo-spatial abilities. The data provided an initial indication that normal visuo-spatial sketchpad development may be important for normal mathematics development.

The overarching conclusion is that WM, and the central executive and visuo-spatial sketchpad in particular, may support the development of early mathematical ability. The practical and theoretical implications of these findings are considered.

Chapter One

Introduction

Working memory is a limited capacity system responsible for the manipulation and storage of information during the performance of cognitive tasks (Baddeley, 1986). Since its conception, the multi-component model of working memory has been particularly valuable in advancing our understanding of how children learn. It has been implicated in general scholastic attainment at 7- (Gathercole & Pickering, 2000a; 2000b), 11- (Jarvis & Gathercole, 2003) and 14-years (Jarvis & Gathercole, 2003) and is thought to play an important role in the acquisition of language skills. In particular, the phonological loop is thought to support vocabulary acquisition in childhood (e.g. Gathercole & Baddeley, 1989), while the central executive is thought to be important for language and text comprehension (e.g. Leather & Henry, 1994; Yuill, Oakhill & Parkin, 1989). More recently, research has begun to define a role for working memory in children's mathematical development. Therefore, the aim of this thesis was to systematically examine the contributions of three different components of the working memory model (Baddeley, 1986) to a range of mathematical skills in children.

To introduce this thesis, a review of the relevant literature is presented. Section 1.1 introduces the concept of working memory. Section 1.2 details the development of working memory throughout childhood. Section 1.3 outlines the major developmental changes that occur in mathematical cognition throughout the lifespan, while Section 1.4 provides an overview of the research that implicates a role for working memory in mathematics performance and mathematical development. Finally, the aims of this thesis are established in Section 1.5.

Section 1.1

Working Memory

This section will provide an overview of Baddeley and Hitch's (1974) concept of working memory and detail the major revisions that have been made to the original tripartite model over recent years.

1.1.1 Working memory – an introduction

Since its conception in the early 1970's, the multi-component model of working memory (Baddeley & Hitch, 1974) has become the focus of research in both theoretical and applied fields of cognitive psychology. It is a limited capacity system responsible for the manipulation and storage of information during the performance of cognitive tasks (Baddeley, 1986). Its origins lie in the early componential models of memory proposed by Broadbent (1958) and Atkinson and Shiffrin (1968).

During the 1950's it became widely acknowledged that the human memory system was not unitary. In its simplest form distinctions could be made between long-term and short-term memory processes (e.g. Broadbent, 1958; Brown, 1958; Peterson & Peterson, 1959). At this time, Broadbent (1958) introduced an information processing model of short-term memory. His theory assumed short-term memory consisted of two subcomponents; the first, a store to temporarily hold sensory information and feed into the second, a limited capacity system for processing information. Broadbent's (1958) model was the first to suggest an active, limited capacity short-term memory system capable of both processing and temporarily storing information.

Atkinson and Shiffrin (1968) proposed a similar model of human memory, which incorporated sensory stores that acted as an input system encoding information from different modalities. It also included a unitary, limited capacity short-term store (STS) and an enduring, unitary long-term store (LTS). This model was important to the development of the working memory model (Baddeley & Hitch, 1974) as it emphasised the function of the STS and the processes between the STS and the LTS. In essence, the STS was an active, working memory responsible for encoding, temporarily storing and processing information before transferring it to the LTS. It was assumed that information could be maintained in the STS through rehearsal processes or retrieved from the LTS through retrieval processes. Atkinson and Shiffrin's (1968) model was somewhat oversimplified.

Consequently, researchers offered alternate models of memory that focussed on either the processes (e.g. Craik & Lockhart, 1972) or the structure and the processes of the human memory system (e.g. Baddeley & Hitch, 1974). The most significant of these was Baddeley and Hitch's (1974) model of working memory, which fractionated the once unitary, limited capacity STS of Atkinson and Shiffrin's model into three subcomponents.

Over recent years researchers have become increasingly interested in the nature, structure and function of the working memory system. There are now a number of competing theoretical models available. In a comprehensive review, Miyake and Shah (1999) presented 10 theoretical models. Arguably the clearest distinction between the available models is between those that view working memory as a unitary, limited capacity system where processing and storage operations compete for a limited pool of resources (e.g. Case, Kurland & Goldberg, 1982), those that conceptualise working memory as a multi-component system comprised of specialised subsystems (e.g. Baddeley & Hitch, 1974; Baddeley, 1986) and those that view working memory as part of a broader information processing framework or an activated subset of long-term memory closely related to attention (e.g. Engle, Kane & Tuholski's (1999) Controlled Attention framework or Cowan's (1988) Embedded Processes model). Other models presented in Shah and Miyake's review include; those that are based on computational architectures (e.g. Lovett, Reder & Lebiere's (Anderson & Lebiere, 1998) ACT-R model of working memory); those that propose a distributed framework (e.g. Barnard's ICS architecture (Barnard, 1985) and those that emphasise the neural basis of working memory (e.g. O'Reilly, Braver & Cohen's (1999) connectionist framework).

Although a number of alternate models of working memory are available, a revision of the early working memory model (Baddeley, 1986) remains both prominent and widely acknowledged in contemporary literature and continues to generate a mass of empirical research, especially within the UK. Therefore, this thesis is guided by the Baddeley and Hitch (1974) model.

1.1.2 Working Memory – The Baddeley and Hitch model

The original model, proposed by Baddeley and Hitch (1974), comprised of three components; the central executive, which acts as a control system, and two slave systems, the phonological loop and the visuo-spatial sketchpad, which temporarily maintain and process verbal and visual and/or spatial information. A number of revisions have since been made to this model, including a change to the original phonological loop (Baddeley, 1986) and the recent addition of a fourth component, the episodic buffer (Baddeley, 2000). A widely cited version of the model often used in empirical research is presented in figure 1.1.

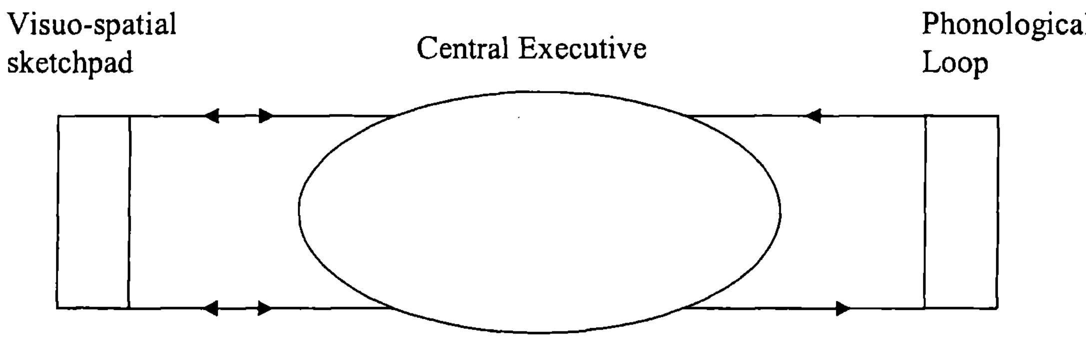


Figure 1.1

A simplified representation of the tripartite working memory model (Baddeley & Hitch, 1974)

1.1.2.1 The Phonological Loop

The phonological loop, in its current form, serves to hold and manipulate phonological / verbal information. It is comprised of two subcomponents; the first, a phonological store, which holds speech based information for approximately 2 seconds, but that can be maintained by sub-vocal rehearsal in the second, an articulatory control process (Baddeley, 1986; 1990).

In its earliest form (Baddeley & Hitch, 1974), the phonological loop was better known as the articulatory loop. It was assumed to be a single, limited capacity component responsible for speech (phonological) coding and maintenance within the short-term memory system. Early evidence for phonological coding in short-term memory comes from studies of the phonological similarity effect (e.g. Conrad, 1964; Conrad & Hull, 1964). That is, when more errors are made in the immediate recall of visually presented sequences of letters that are phonologically similar, compared to sequences of letters that are phonologically dissimilar (Conrad & Hull, 1964). It is still thought that the articulatory loop, or phonological loop as it is now known, is responsible for phonological coding.

Baddeley and Hitch (1974) proposed that the limited capacity of the articulatory loop was determined by temporal duration, rather than the number of chunks of items as had been previously suggested (Miller, 1956; Simon, 1974). This was demonstrated by the word length effect. The word length effect is best explained as the superior immediate serial recall for a list of short words over a list of long words where both lists contain the same number of words. Baddeley, Thomson & Buchanan (1975) demonstrated the word length effect where memory for a five-word sequence dropped from 90% with monosyllabic words to 50% with five syllable words. They suggested that it reflected a person's ability to sub-vocally articulate and subsequently proposed that immediate serial recall for words was determined by the spoken duration (rate of sub-vocal rehearsal) of the words. Any information that took longer than approximately 2 seconds to rehearse was lost through a process of temporal decay (Baddeley, 1986).

The evolution of the phonological loop from the articulatory loop owes itself to studies of articulatory suppression (the process of repeatedly speaking aloud an irrelevant word, while concurrently performing an immediate serial recall task). Whilst investigating the word length effect, Baddeley et al. (1975), found that under conditions of concurrent articulatory suppression, the effect was eliminated for visually presented stimuli. Likewise, the phonological similarity effect was eliminated for visually presented stimuli with concurrent articulatory suppression (Estes 1973; Peterson & Johnson, 1971). Similarly, the irrelevant speech effect (which occurs when immediate serial recall for words is impaired by concurrent presentation of irrelevant spoken information) was eradicated for visually presented words (Salame &

Baddeley, 1982; 1983; 1987; 1989). Collectively these studies suggested that articulatory suppression prevented phonological encoding of visual stimuli, but that spoken information could be stored directly without articulation. A revised account of the phonological loop was proposed in light of these empirical findings.

In the revised model (Baddeley, 1986), the phonological loop comprised of two subcomponents. A temporary storage system, which served to hold memory traces, and an articulatory control process, which maintained information and registered visual information within the store providing it could be phonologically recoded. It was assumed that verbal / auditory stimuli could directly access the phonological store, by-passing the articulatory control process, whereas visually presented material entered the store via the articulatory control process (Baddeley, 1986).

Neuropsychological evidence, from patients with lesions that have resulted in phonological loop deficits, and neuroimaging studies (e.g. Smith & Jonides, 1997) support the fractionation of the phonological loop into separate components for storage and subvocal rehearsal. In a review of the data from patients with phonological short-term memory deficits, Vallar and Papagno (2002) proposed that the neuroanatomy of the phonological loop incorporated separate storage and processing systems. They suggested that with auditory presentation speech streams fed directly into a phonological storage system in the inferior parietal lobe. Here the speech streams were coded into a phonological format before being fed into an articulatory control system in Broca's area-premotor cortex for rehearsal or direct recall. Indeed, neurpsychological studies suggest that different brain regions are associated with the two subcomponents. In particular, Brodmann's area has been associated with the phonological store, while Broca's area has been associated with subvocal rehearsal (Vallar, Betta & Silveri, 1997). Anatomically, it is suggested that white matter tracts support the interaction between rehearsal and the phonological store.

The revised model of the phonological loop (Baddeley, 1986), with minor revisions, has generated a mass of empirical research over recent years. Since the suggestion that it evolved to facilitate the acquisition of language (Baddeley, Papagno & Vallar, 1988), it has been particularly valuable in advancing our understanding of language learning.

The phonological loop was first implicated in second language learning following a study of patient PV who had a pure phonological STM deficit. Patient PV was tested on her ability to acquire the vocabulary of a second language, in comparison to her ability to learn associate pairs of words in her native language. PV's ability to learn the native word pairs was comparable to normal control participants, but she failed to learn any of the new foreign words (Baddeley, Papagno & Vallar, 1988). As such, it was suggested that the phonological loop assisted the acquisition of new words. Supporting this, variables that disrupted performance on phonological loop tasks also disrupted the learning of new words. Specifically, articulatory suppression disrupted foreign but not native language learning in Italian and English participants (Papagno, Valentine & Baddeley, 1991). Furthermore, phonological similarity among the items, and increases in the length of the novel items to be learned, disrupted the acquisition of new words (Papagno & Vallar, 1992). Service (1992) reported similar results for children, where the acquisition of English as a second language was studied. She found that children with longer verbal short-term memory spans were better at language learning than children with shorter memory spans, concluding that the ability to represent unfamiliar phonological material in working memory supports foreign language vocabulary acquisition.

The role of the phonological loop in language learning has been extended to native language acquisition. Initially the phonological loop was implicated in native language learning in children with specific language impairments (SLI). Gathercole and Baddeley (1989) compared the performance of 8 year-olds with SLI (who had a 2-year delay in language development) to normal children matched for age and nonverbal intelligence, and younger children matched for language ability, on a test of nonword repetition. The nonword repetition measure, used as an index of phonological loop capacity, required the children to repeat unfamiliar sequences of phonemes. The SLI children performed significantly worse than both the age- and language-matched controls, which suggested they had a deficit in the phonological loop. Following this, significant associations were found between vocabulary scores and nonword repetition ability in groups of normal children when other variables, such as nonverbal intelligence, were controlled (Gathercole & Baddeley, 1990; Gathercole, Willis, Emslie & Baddeley, 1992). In one study, Gathercole et al. (1992) applied cross-lagged correlational analysis to longitudinal data collected from 80 children across three testing sessions between 4- and 8-years-of-age. They found that nonword repetition was significantly associated with vocabulary scores one year later, but that vocabulary scores were not predictive of nonword scores one year later. Gathercole et al. (1992) suggested that this implied some direction to the relationship between vocabulary and nonword repetition; that the ability to repeat nonwords influences the learning of new words. These findings have been replicated with groups of children between the ages of 4- to 13-years (e.g. Baddeley, Gathercole & Papagno, 1998). For example, Gathercole, Hitch, Service and Martin (1997) reported that phonological loop ability was related to the rate of learning word-nonword pairs, but not word pairs, after controlling for nonverbal intelligence in 5-year-olds. Michas and Henry (1994) reported similar findings when they conducted a similar study, which also controlled for spatial abilities. These studies demonstrate that the learning of new words is constrained by the capacity of the phonological loop.

Baddeley et al., (1998) proposed that nonword repetition provides a measure of the phonological store, not phonological rehearsal. The task typically requires the immediate repetition of an unfamiliar item within 1-second of presentation, which itself has a spoken duration of less than 1-second. The temporal capacity of the phonological store is estimated to be 2-seconds (Baddeley et al., 1975), meaning it is unlikely that rehearsal is contributes to nonword repetition. Furthermore, the phonological loop has been associated with vocabulary learning in children as young as 3-years (Gathercole & Adams, 1993), prior to the onset of subvocal rehearsal. It is therefore suggested that it is the phonological store, not the articulatory control process, which mediates vocabulary acquisition in the native language (Baddeley et al., 1998). However, it has been suggested that the rehearsal process plays a role in second language learning, as demonstrated by the disruptive effect of articulatory suppression (e.g. Papagno et al., 1991).

Brown and Hulme (1996) have offered an alternate explanation for the involvement of phonological processes in vocabulary acquisition. While Gathercole and colleagues suggested that nonword repetition predicted vocabulary learning, Brown and Hulme (1996) suggested that vocabulary growth and the ability to repeat nonwords shared a reciprocal relationship. Their model did not specify a role for phonological short-term memory in vocabulary acquisition; rather it suggested that vocabulary growth was associated with other variables that involve both lexical and phonological development. They suggested that phonological storage reflects deeper phonological processing (e.g. Snowling, Chiat & Hulme, 1991). While this issue remains a topic of debate, Gathercole and colleagues suggest their account provides a better understanding of the early stages of development as it can explain why some children develop vocabulary quicker than others in terms of the phonological store (Gathercole et al., 1992).

Overall, few changes have been made to the model of the phonological loop proposed by Baddeley in 1986, possibly for the reason that it has proven to be particularly productive in advancing our knowledge of language learning.

1.1.2.2 The Visuo-Spatial Sketchpad

The visuo-spatial sketchpad is responsible for generating and manipulating visuo-spatial images. Like the phonological loop it has been subject to a number of revisions since its conception (e.g. Logie, 1986).

Early distinctions between verbal and visual processing (e.g. Milner, 1971) influenced the development of separate verbal and visual slave systems within the working memory model (Baddeley & Hitch, 1974). Initially the visuo-spatial sketchpad (known then as the visuo-spatial scratchpad) was assumed to be a limited capacity system for the temporary storage of spatial information; the visual aspect of the visuo-spatial system did not receive much attention until Logie's work in the 1980's.

The development of an independent visuo-spatial system began through the work of Baddeley, Grant, Wight and Thomson (1975) who speculated that processing spatial information might require a specialised system. Baddeley et al. (1975) found that a concurrent spatial tracking task disrupted the serial recall of a visuo-spatial sequence. Participants were asked to listen to and recall either visuo-spatial (easily visualised) or nonsense (difficult to visualise) sequences of digits in a matrix, in a similar fashion to the early memory span procedures employed by Brooks (1967). Half of the participants were required to undertake a concurrent tracking task, which involved tracking a light that moved along a circular track. The concurrent tracking task impaired recall of the visuo-spatial sequences, but had no effect on recall of the nonsense sequences. Unsure whether the decrement in performance was due to a disruption in visual perception or spatial coding, they conducted a second study. They found that the concurrent tracking task did not reduce the normal advantage for concrete/imageable word pairs over abstract word pairs in a paired-associate learning task. This led them to suggest that spatial coding, required for the recall of spatial locations in the matrix task, may involve a specialised system (the visuo-spatial sketchpad). However, they suggested that the concrete words in the latter task did not depend on such a system, as they contained characteristics that are directly accessible from long-term semantic memory.

Baddeley and Lieberman (1980) investigated whether disruption to visual perception, rather than spatial coding, caused the impairment in immediate serial recall of visuo-spatial sequences in the earlier study (Baddeley et al., 1975). Initially they employed the matrix procedure, but refined the study by asking participants to complete one of two concurrent tasks. One was a spatial task with no visual component, in which participants, while blindfolded, were required to point to a moving pendulum that emitted a steady sound. The alternate was a visual task, with minimal spatial requirements, in which participants were asked to make a series of brightness judgements. The concurrent spatial task disrupted the recall of visuo-spatial sequences, implying that they require spatial coding in a visuo-spatial sketchpad system.

In subsequent studies Baddeley and Lieberman (1980) investigated the nature of spatial information processing within the visuo-spatial component. In the first of two peg-word studies, they demonstrated that the concurrent tracking task reduced the normal advantage for recalling words learned through an imagery mnemonic compared to rote rehearsal. Although significant, the effect was relatively small in comparison to previous results using the matrix task. They speculated that this occurred because the peg-word stimuli were less spatial in nature than the sequences in the matrix studies. Subsequently they replicated the study using a spatial mnemonic. Participants were required to learn ten peg-words, either using a location mnemonic whereby the words were associated with locations along a familiar walk or through rote rehearsal. As expected, when participants were asked to complete the concurrent tracking task, the normal advantage for learning words using a spatial mnemonic was lost. Baddeley and colleagues (1975; 1980) demonstrated that tasks with a spatial component could be disrupted by a concurrent tracking task. Collectively, they showed that the stronger the spatial component of the task, the easier it was to disrupt performance. This was taken as evidence that there was a specialised system for storing and processing spatial information. At this time visual processing was considered to be less dependent on such a system.

The mid 1980's saw a revision of the visuo-spatial sketchpad. Prior to Logie's (1986) work, Phillips and Christie (1977a; 1977b) postulated that the short-term storage and manipulation of visual information may be part of a general processing system. In a series of studies, Logie (1986) investigated visual information processing using a dual-task methodology. In the first study, he found that a concurrent visual task (a pattern-matching task) disrupted the recall of peg-words that were learned using a visual imagery mnemonic more so than words learned using rote rehearsal. These findings suggested that visual processing might demand a specialised system. However, the picture-matching task required participants to make decisions about a succession of patterns (whether the current pattern was the "same" or "different" to the previous pattern), which it was feared could recruit executive skills. Therefore, the decision-making component of the secondary task was dropped and the study was replicated using complex and simple, repetitive patterns. In both studies participants were instructed to try and ignore the patterns. Once more the concurrent visual task disrupted performance for words learned using the visual imagery mnemonic. Logie (1986) took this as evidence to show that visual information had direct access to a visual store within the visuo-spatial system. He suggested that visual information in the concurrent task was interfering with performance on visual memory tasks as it gained obligatory access to the store, even though it was unattended to. To further investigate this, Logie (1986) replicated the study, this time using unattended irrelevant pictures and unattended irrelevant speech as concurrent tasks. As expected, irrelevant pictures disrupted performance for words learned using the visual imagery mnemonic (the "irrelevant picture" effect) and irrelevant speech disrupted performance for words learned using rote rehearsal.

The "irrelevant picture effect" was replicated in a number of ways in the 1990's. Logie, Zucco and Baddeley (1990) demonstrated that irrelevant pictures disrupted recognition memory for visual matrix patterns, but not for letter span (which was disrupted by a concurrent verbal arithmetic task). Quinn and McConnell (1994; 1996) reported similar findings using dynamic visual noise (DVN) as a concurrent visual interference task, which again disrupted the recall of words learned using a visual imagery mnemonic, but did not disrupt performance of words learned using rote rehearsal.

The evidence thus far implied that there was a specialised visuo-spatial system within the working memory framework, independent of the verbal system, responsible for retaining visual information (Logie, 1986; Logie, et al., 1990; Quinn & McConnell, 1994; 1996) and manipulating spatial information (Baddeley, Grant et al., 1975; Baddeley et al., 1980). In 1995, Logie reviewed the converging evidence for a specialised visuo-spatial system and speculated that the system may comprise of two sub-components; a temporary visual store and a temporary spatial store. The temporary visual store (or visual cache), presumed to store information about visual form and colour, was closely linked to the visual perceptual system, while the temporary spatial store (or inner scribe), presumed to store information about movement sequences, was closely linked to planning and movement. Logie (1995) proposed that the visual cache was subject to both decay and interference, but that the inner scribe could rehearse and maintain the contents of the visual cache and extract items from it.

Evidence of double dissociations in neuropsychological patients and clinical populations (De Renzi & Nichelli, 1975; Luzzatti, Vecchi, Agazzi, Cesa-Bianchi & Vergani, 1998; Milner 1971) and of developmental fractionation (Logie & Pearson, 1997) supports the fractionation of the visuo-spatial system. For example, Logie and Pearson (1997) found that visual and spatial memory abilities develop at different rates in childhood when they examined 5-year-olds, 8-year-olds and 11-year-olds recall and recognition performance on a Corsi blocks task (to measure spatial abilities) and a visual patterns task (to measure visual abilities). Although the distinction between the visual and spatial subsystems is widely accepted, some have argued that the fractionation of visuo-spatial memory is better interpreted as separate subsystems for static and dynamic information (e.g. Pickering, Gathercole, Hall & Lloyd, 2001). This issue will be discussed in greater detail in Chapter 4. Whatever the interpretation, many researchers attempt to address separate subsystems when investigating the visuo-spatial sketchpad (e.g. in the development of the Working Memory Test Battery For Children, Pickering & Gathercole, 2001).

The visuo-spatial sketchpad and the central executive are closely associated. At present there are two explanations for this. The first, offered by Shah and colleagues (Shah & Miyake, 1996; Miyake, Friedman, Shah, Rettinger & Hegarty, 2001), suggests that the visuo-spatial subsystem does not mirror the phonological. They argue for a visuo-spatial system that is closely linked to the central executive system due to the nature of visuo-spatial functions. Shah and Miyake (1996) compared performance across a simple spatial span task (keeping track of spatial orientations) and a complex spatial span task (keeping track of spatial orientations while simultaneously performing mental rotation) to investigate how they predicted complex spatial cognitive abilities. They believed that this procedure mirrored the verbal, phonological loop domain where researchers have traditionally compared performance across simple verbal span tasks (i.e. word and digit span) and complex verbal span tasks (i.e. reading and operation span). These studies typically conclude that complex verbal span tasks are better predictors of complex verbal cognitive task performance (e.g. Daneman & Carpenter, 1980), therefore differentiating between the phonological loop and the central executive. In their study Shah and Miyake (1996) found that both complex and simple spatial tasks predicted complex spatial abilities equally in the visuo-spatial domain, implying that it is asymmetrical to the verbal domain. More recently, Miyake, et al., (2001), replicated this in a latent variable analysis study. They examined performance on simple visuo-spatial span tasks (Dot Memory (Ichikawa, 1983) and Corsi blocks (Milner, 1971)), complex visuo-spatial span tasks (Letter Rotation (Shah & Miyake, 1996) and Dot Matrix (Law, Morrin & Pellegrino, 1995)), executive functioning tasks (Tower of Hanoi and Randon number Generation) and three spatial ability tasks. Once again they reported close links between the central executive and visuo-spatial components of working memory (as both the simple and complex spatial tasks equally implicated executive functioning) that were asymmetrical to the findings in the verbal domain. Shah, Miyake and colleagues conclude that this is not surprising given the idea that visuo-spatial working memory functions, such as maintaining mental representations of visual stimuli, are effortful and demanding of executive resources (Baddeley, Cocchini, Della Salla, Logie & Spinnler, 1999).

The second explanation for the links between the visuo-spatial sketchpad and the central executive relates to task demands. That is, although the structure of the visuo-spatial sketchpad may mirror the structure of the phonological loop, as Logie's model (1995) suggests, it is not shown empirically due to the complex nature of visuo-spatial working memory tasks, which place heavy demands upon the processing and storage resources of the central executive (Chuah & Maybery, 2000; Gathercole & Pickering, 2000a; Hamilton, Coates & Heffernan, 2003; Phillips & Christie, 1977a; Wilson, Scott & Power, 1987). Recently Hamilton et al. (2003) suggested that serial order demands in traditional spatial working memory tasks, such as Corsi blocks, and implicit spatial demands in traditional visual working memory tasks (e.g. spatial rehearsal) might draw upon central executive resources. They suggest that new tasks, tailored to the hypothetical characteristics of a componential visuo-spatial working memory that are independent of executive involvement, would prove useful tools with which to resolve this debate.

Despite the current disagreement about the structure of the visuo-spatial working memory system, many researchers follow a model close to that proposed by Logie (1995), with an awareness of the close links between the visuo-spatial and executive systems.

1.1.2.3 The Central Executive

The central executive is typically viewed as a domain general control system within the working memory framework.

Until the 1980's the central executive had been an "area of residual ignorance" (Baddeley, 1986. pp. 225). At this time, Baddeley described the component as a supervisor, responsible for the integration of information and strategy selection, which was closely related to the control of attention. He suggested (1986; 1990) that it may resemble a component of Norman and Shallice's model of attentional control, the Supervisory Activating System (SAS) (Norman & Shallice, 1980; Shallice, 1982).

According to the model of attentional control, schemata control actions. At any point, several schemata may be active. The schemata are controlled by two systems, an automatic conflict resolution process and the SAS, to prevent conflict between simultaneously activated schemas. The conflict resolution process selects the appropriate schemata, while the SAS acts as a controller, which overrides the resolution process to give priority to a schema based on external factors. Shallice (1982) described the SAS as a limited capacity system that was important for planning and decision making, trouble shooting in automatic processes, novel and poorly learned sequences, dangerous situations and when habitual responses were involved.

Baddeley (1986) likened the central executive to the SAS. There were two reasons for this. Firstly, it explained a pattern in random generation data that previously had no explanation. The data, collected in a series of experiments, showed a lawful pattern. When participants were asked to produce a random stream of letters or digits the degree of randomness decreased with the increased rate of production (Baddeley, 1966). Baddeley's (1986) explanation for this pattern was that the SAS was attempting to control a set of retrieval processes. That is, randomness was achieved by the SAS overriding the strong schemata for generating letter and number sequences, such as the alphabet. Baddeley proposed that the increased rate of production overloaded the capacity of the supervisor, which caused a decrease in randomness. Secondly, the SAS model explained the disruptive effect of a concurrent card-sorting task on randomness. Baddeley (1966) reported a decrease in randomness as the demands of the concurrent card-sorting task increased.

In an attempt to better understand its control functions, Baddeley endeavoured to fractionate the central executive. He presented evidence for the involvement of four areas of executive function: the co-ordination of two concurrent tasks, attentional control / switching retrieval strategies, selective attention (filtering out irrelevant information) and activating, holding and manipulating areas of long-term memory (Baddeley, 1996). Following this relatively early attempt to fractionate the executive component of working memory, Baddeley (1996) concluded that he still thought of it as a unitary system. However, he did acknowledge that it might become an executive committee of control processes.

Indeed, the idea of a unitary executive has been somewhat thwarted over the years. Although there is still a belief that there may be something unitary about the system, such as a common mechanism that characterises the deficits of frontal lobe patients and the functions of the frontal lobes (e.g. Engle et al., 1999), it is more widely accepted that the central executive represents a fractionated system. Lehto (1996) explored the relationship between working memory capacity and three tests designed to measure executive function (Tower of Hanoi (TOH), Wisconsin Card Sort Task (WCST) and Goal Search Task) in young adults. He found that the WCST was the only executive function test correlated with working memory. Furthermore, he reported that there were no intercorrelations between the executive function tests. He suggested that this evidence supported the existence of separate executive functions rather than a unitary, limited capacity central executive. Similar findings have since been reported for elderly adults (Lowe & Rabbit, 1997) and brain-damaged patients (Shallice & Burgess, 1991).

Arguably the most compelling evidence for a fractionated executive system comes from the work of Miyake and colleagues. They suggested that the low intercorrelations reported between executive tasks, and the separable factors alluded to, in previous studies could be due to the following: the nature of the processing of executive tasks (visuo-spatial versus language); the separate cognitive systems they operate upon; the low test re-test reliability and poorly established construct validities of executive measures; and the uncertainty about what the tasks are actually measuring (Miyake, Friedman, Emerson, Witzki, Howerter & Wager, 2000). They conducted a large-scale latent variable analysis in an attempt to clarify some of the issues. They explored the separability of three executive functions (shifting, updating and inhibition) and their role in five commonly used executive tasks (WCST, TOH, Random Number Generation (RNG), operation span and dual tasking). Their results suggested that although the three executive functions were correlated, they formed separable factors in confirmatory factor analyses (CFA). Furthermore, structural equation modelling (SEM) showed they contributed differentially to performance on the executive tasks. In short, performance on the WCST was predicted by shifting, the TOH was predicted by inhibition, RNG was predicted by inhibition and updating, operation span was predicted by updating and dual tasking was predicted by inhibition and updating.

In addition to fractionating executive functions, Miyake and colleagues suggested that there might be separable spatial and verbal working memory resources within the executive domain (Shah & Miyake, 1996). They explored the separability of working memory in spatial thinking and language comprehension using a reading span task to measure verbal processing and storage, and an analogous spatial span task to measure spatial processing and storage. Their results supported a separability hypothesis similar to that suggested by Jurden (1995). They found that spatial span correlated with other spatial measures and predicted performance on complex spatial thinking tasks, while reading span correlated with other verbal measures and predicted performance on complex language processing tasks. Furthermore, their factor analysis yielded a clear two-factor solution. Interpreting these results in terms of the working memory model, Shah and Miyake (1996) suggested that there might be distinct spatial and verbal aspects in working memory beyond the slave systems (which may only exist as relatively passive storage buffers).

Although recent work has begun to adopt this approach (e.g. Jarvis & Gathercole, 2003), the separability of verbal and non-verbal domains within the executive remains questionable. For example, Kane, Hambrick, Tuholski, Wilhelm, Payne and Engle (2004) recently reported the results of a large scale latent variable study, which suggested verbal and visuo-spatial working memory span tasks reflected a domain general factor, while verbal and visuo-spatial short-term memory tasks were domain-specific.

The central executive has often been associated with human intelligence, or Spearman's *g*. A number of studies suggest that working memory, and in particular the central executive, are related to reasoning, fluid intelligence or *g*. Kyllonen and Christal (1990) reported correlations as high as .8 between working memory and reasoning tasks. Carpenter, Just and Shell (1990) suggested that working memory capacity may be a main factor underpinning individual differences on the Raven's Progressive Matrices tests, a commonly used intelligence test. In a re-analysis of Kyllonen and Christal's (1990) data, Jurden (1995) derived 2 working memory factors, one verbal and non-verbal, but reported that both shared approximately two-thirds of their variance with a second order factor, g. Furthermore, complex working memory has been reported to predict performance on tests of general intelligence (Engle et al., 1999; Kane et al., 2004). As such, the shared variance between the two components of working memory identified by Jurden (1995) is purported to demonstrate that working memory may underpin general intelligence. Indeed, more recent studies have acknowledged the close association between executive function and human intelligence (e.g. Miyake et al., 2001) and the results of one study suggest that working memory is almost perfectly predicted by g (Colom, Rebello, Palacios, Juan-Espinosa & Kyllonen, 2004).

To summarise, the central executive lies at the heart of the working memory system (Baddeley, 1986). It is thought to command a number of functions (that may or may not be separable) including planning, switching attention, shifting, inhibition and updating (Baddeley, 1986, 1996; Baddeley & Logie, 1999; Miyake et al., 2000). The idea that it is a limited-capacity system has since been rejected in favour of the view that it supports on-line processing, while the phonological loop (Baddeley & Logie, 1999) or the episodic buffer (Baddeley, 2000) support storage.

1.1.2.4 The Episodic Buffer

The recently added fourth subcomponent, the episodic buffer (Baddeley, 2000), has been fractionated from the central executive. It is considered responsible for combining information from the slave systems and long-term memory into unitary episodes. It was first alluded to in Baddeley's (1992) Bartlett lecture, where he discussed the associations between the central executive and conscious awareness, and finally added to the original tripartite model in 2000. The revised working memory model is presented in Figure 1.2.

Baddeley (2000) describes the episodic buffer as a limited-capacity temporary storage system, responsible for integrating information from a variety of sources into a multimodal code and retrieving information from long-term memory. He suggests that it is controlled by the central executive, which retrieves information from it in the form of conscious awareness and reflects upon on, modifies, manipulates and controls its contents by attending to a certain source of information.

The episodic buffer was introduced to address some of the theoretical issues that could not be explained by the original model (Baddeley, 1986), namely the integration of information within the working memory system.

The first problem it addressed related to the integration of information between the slave systems. Baddeley, Lewis and Vallar (1984) reported that participants were able to repeat back visually presented lists of words under conditions of articulatory suppression. According to the original model this should not have been possible, as articulatory suppression should have prevented subvocal rehearsal. As such, Baddeley suggested that the items to be remembered were being stored in an alternative system, possibly a "back-up store" that integrated information from both slave systems, thus allowing the visuo-spatial sketchpad to support the retention of visually presented items under conditions of articulatory suppression.

The second issue explained by the episodic buffer related to the integration of information between the slave systems and long-term memory. In a typical word span task participants can recall lists of 5/6 words when the words are unrelated. However, if the words form a meaningful sentence, participants can recall up to 16 words (Baddeley, Vallar & Wilson, 1987). Baddeley (2000) suggested that this demonstrated the integration of information between the phonological loop and long- term memory, which may also occur in a "back-up" store.

The third integration problem addressed related to the combination of information between all three subcomponents of working memory and long-term memory in consciousness. Baddeley and Andrade (2000) showed that the slave systems were involved in conscious awareness, and also found that the central executive and long-term memory played important roles. As such, Baddeley (2000) suggested that a component was needed to integrate information within the working memory system.

Finally, the episodic buffer was introduced to explain the temporary activation, manipulation and maintenance of information from long-term memory in working memory. Amnesic patients typically perform poorly on tests of immediate recall. However, some are able to recall passages of prose immediately after presentation, which suggests that they have a normally functioning executive system. Although their performance was initially explained as the temporary activation of long-term memory, Baddeley (2000) suggested that the immediate recall of a passage of prose could involve the manipulation and maintenance of information in addition to activation. Subsequently he speculated that their performance might be explained by attributing such a role to the episodic buffer.

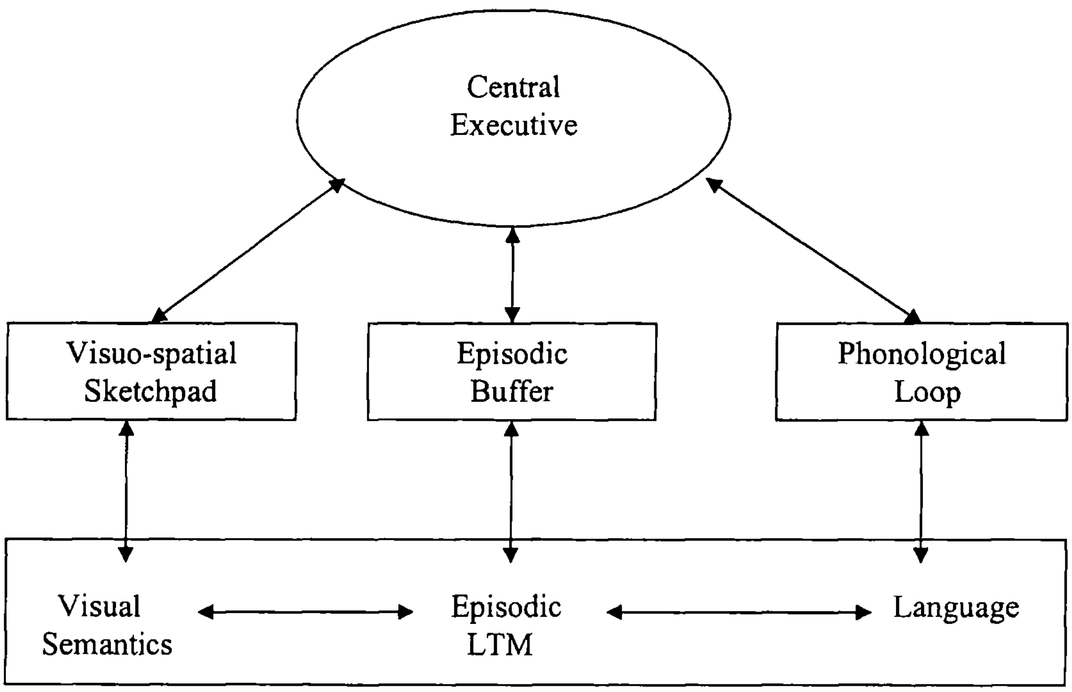


Figure 1.2

A representation of the revised working memory model (Baddeley, 2000)

The episodic buffer (Baddeley, 2000) is a useful hypothetical addition to the working memory framework, which resolves some inconsistencies in the original model. However, it was not incorporated within this thesis, as there were no standardized measures available to assess it at the beginning of this project. However, a recent study, has attempted to investigate the episodic buffer in children, developing a measure based upon the spoken recall of sentences (Alloway, Gathercole, Willis & Adams, 2004). The rationale was that repeating sentences involves the integration of information from different sources (namely the integration of information from temporary memory systems to support the recall of the words and the products of the language processing system). Alloway et al.'s findings supported the usefulness of the task, as structural analyses revealed that the best fitting measurement model incorporated a construct corresponding to the episodic buffer (Alloway et al., 2004). However, the sentence recall task has not been standardized for use as a measure of the episodic buffer. Furthermore, there are no measures of reliability or validity to support its use as a measure of the capacity of the episodic buffer.

Section Summary

1. The notion of a "working" memory system was developed from early information processing models (e.g. Broadbent, 1958).
2. Currently, a number of theoretical models of working memory are available. The primary distinction between the models is that some view working memory as a unitary, limited capacity system, while others conceptualise working memory as a multi-component system comprised of specialised subsystems.
3. Baddeley and Hitch's (1974) multi-component model is arguably the most widely accepted model. In its original form their model comprised of three subcomponents: the central executive, the phonological loop and the visuo-spatial sketchpad. A fourth component, the episodic buffer, is a recent addition to the model.
4. The phonological loop is responsible for the temporary storage and manipulation of phonological information. In its earliest form it was a single, limited capacity system. It has since been revised to comprise two components; a phonological store which holds speech based information and

an articulatory control process, which maintains verbal information and recodes visually presented information.

5. The visuo-spatial sketchpad is responsible for the temporary storage and manipulation of visuo-spatial information. Early evidence suggested that there might be a specialised system for storing and processing spatial information, but it was not until the 1980's that a similar specialised system was proposed for the processing of visual information. Logie (1995) defined these systems as a visual cache, which stores visual information, and an inner scribe, which stores information about movement sequences and maintains the contents of the visual store. The visuo-spatial sketchpad has been linked with the central executive.
6. The central executive lies at the heart of the working memory system acting as a control. Once thought to be a limited capacity system, it is now thought to support on-line processing. It performs a number of functions, including planning, switching attention, shifting, inhibition and updating and is closely related to human intelligence.
7. The episodic buffer was recently introduced to resolve some of the theoretical inconsistencies in the original tripartite model. Controlled by the central executive, it is responsible for combining information from the slave systems and long-term memory into unitary representations. At present it is somewhat under specified and there are no current standardized measures available to assess it. For this reason the original tripartite model guides this thesis.

Section 1.2

The Development of Working Memory

Performance on working memory tasks follows a broadly linear increase as a function of age (e.g. Gathercole, Pickering, Ambridge & Wearing, 2004). This development, seen throughout childhood, can be explored one of three ways. Firstly, the development of the entire processing and storage system can be considered. Secondly, developmental changes within each component of the model can be explored. Finally, changes to the structure of working memory throughout childhood can be examined. These three lines of research are discussed in turn in the subsequent sections.

1.2.1 Development of the working memory processing and storage system

The overall view of working memory is that it is a system responsible for the temporary storage and processing of information. Its capacity is typically assessed by tasks designed to measure concurrent processing and storage (e.g. complex span tasks). Performance on these tasks improves throughout childhood, demonstrating an increase in memory span. There are broadly two competing accounts of this developmental increase; the resource-sharing (trade-off) hypothesis and the resource-switching hypothesis. The first account, provided by Case (1985), was based upon Piagetian principles and the work of Pascual-Leone (1970). Case's resource-sharing hypothesis suggested that processing space, or memory span, did not increase throughout childhood. Rather, he explained that processing space could be deployed as processing or storage, and over time processing efficiency increased, allowing more resources to be employed in storage. This was demonstrated in a series of studies in which processing efficiency and storage were found to have a linear relationship (Case, et al., 1982). In the first study, 3- to 6-year-olds word span (storage) and word repetition speed (processing efficiency) increased with age in a linear fashion. These findings were replicated using a counting span task with 6- to 12-year-olds and both a word span and counting span task with adult populations. Furthermore, Case et al. (1982) showed that when adults' processing efficiency was controlled, by manipulating the familiarity of the words presented, storage capacity decreased in a linear manner. These findings are consistent with the view that processing demands decrease with age, freeing up space for storage.

The primary problem with Case's (1985) theory was that the findings (Case et al., 1982) could be accounted for by articulation rate in the phonological loop. That is, due to fixed decay in the phonological loop, the faster a child can articulate a word/number through subvocal rehearsal, the greater their memory span will be due to less items being lost through decay. For this reason, Towse and Hitch (1995) offered an alternate, resource-switching account of the developmental increase in children's memory span. Their hypothesis suggested that processing does not determine performance on complex span tasks, but rather that time-based forgetting does. That is, the time elapsed between the presentation of a stimuli and its subsequent retrieval determines memory span.

Initially Towse and Hitch (1995) compared their memory decay hypothesis to Case's (1985) cognitive space hypothesis through an evaluation of the effect of time and processing difficulty on counting span performance in 6- to 11-year-olds. They incorporated a visual search paradigm within the counting phase of the task in order to impose the relevant manipulations. They varied processing difficulty by asking the children to perform either a single feature or a feature conjunction search. Asking the children to count single feature stimuli in one of two conditions varied processing time. In the first condition, the children had to count the target stimuli as in the single feature condition of the processing difficulty paradigm. In the second, "feature-slow", condition the number of target stimuli were increased so that they would take the same time to count as the feature conjunction targets in the processing difficulty paradigm. Case's hypothesis would predict lower performance for the conjunction feature condition than the single feature or feature-slow conditions, due to increased processing demands reducing the space for storage. Whereas Towse and Hitch's (1995) hypothesis would predict better performance on the single feature condition (due to shorter delay periods), with equivalent performance on the conjunction feature and feature-slow conditions, where interval times were matched. Their results supported the memory decay hypothesis, leading to the proposal of the resource-switching hypothesis (Towse & Hitch, 1995), which suggested that at any one time children were either performing processing operations *or* remembering their products so that the more time spent processing (counting), the greater the time spent "switched-out" of remembering, thus resulting in greater decay for the memory traces. Subsequent experiments (Towse, Hitch & Hutton, 1998) extended this finding to other complex working memory span tasks (reading, operation and counting span tasks) in 6-to 11-year-olds, where span decreased as a function of increased retention intervals.

One concern about this hypothesis in relation to the current review relates to a recent finding that it may not explain developmental differences in recall. An investigation of the time-based forgetting hypothesis in 8- to 17- year-olds revealed developmental differences in recall when the interval duration was held constant (Towse, Hitch & Hutton, 2002), suggesting that other factors may be influencing developmental change. As Towse et al., (2002) suggest, processing speed is the obvious contender as increased processing speed, which occurs with age, reduces interval time meaning there is less time for decay.

The resource-sharing versus resource-switching issue remains a topic of current debate. Recent attempts to resolve this debate typically conclude that no single factor constrains children's working memory span development. For example, both time and attentional resources have been found to constrain children's performance on working memory tasks (Barrouillet & Camos, 2001; Bayliss, Jarrold, Gunn & Baddeley, 2003).

1.2.2 Development of the components of working memory

Developmental changes occur throughout childhood within each component of the tripartite working memory model. The capacity of the phonological loop is thought to increase from 2-3 items at 4 years to 6 items at 12 years of age (Hulme, Thompson, Muir & Lawrence, 1984). This indicates that children are able to store increasing amounts of verbal information in the phonological store with age. The primary reason for this developmental increase is the onset of subvocal rehearsal. Auditory / verbal information has automatic access to the phonological store, but it is maintained via subvocal rehearsal, which does not develop until the age of 7 years (e.g.Gathercole & Hitch, 1993). Prior to this age recall is mediated by the phonological store, which is subject to rapid decay (see section 1.1.2.1). When rehearsal begins at 7 years, the retention of verbal information is maximised. Furthermore, as articulation rate increases, memory span continues to increase beyond 7-years due to an increase the effectiveness of subvocal rehearsal (e.g. Gathercole, Adams & Hitch, 1994). Phonological loop capacity also increases when children begin to recode visual information into phonological codes. Visual information enters the phonological store when it is recoded into a phonological form via subvocal rehearsal (Baddeley, et al., 1984). As such, it does not enter the phonological store until the onset of subvocal rehearsal at 7-years. Prior to this age, the visuo-spatial sketchpad supports the retention of visual stimuli (Hitch, Halliday, Schaafstal & Schraagen, 1988). Although visual information is recoded in a phonological code wherever possible after this age (Hitch & Halliday, 1983), it has been suggested that children progress through a period of dual visual-verbal coding before they begin to use the adult-like verbal recoding strategies (e.g. Palmer, 2000). Other factors have also been associated with a developmental increase in verbal short-term memory, which may influence the increase in phonological loop capacity. These include changes in the speed of memory scanning during retrieval (Cowan, Wood, Wood, Keller, Nugent & Keller, 1998), increases in the availability of phonological representations of words in long-term memory (Roodenrys, Hulme & Brown, 1993) and increases in knowledge about the structure of language (e.g. Gathercole, Frankish, Pickering & Peaker, 1999).

The capacity of the visuo-spatial sketchpad is thought to follow a steady developmental increase. The visual aspect, as measured by the ability to recall a two dimensional pattern of squares, is thought to follow an increase from 4 blocks (squares) at 5-years to 14 blocks (squares) at 11-years (Wilson et al., 1987). Although this may represent a developmental increase in the visuo-spatial sketchpad per se, concurrent phonological loop and central executive tasks disrupted performance on this task, causing speculation that the development of the visuo-spatial sketchpad may be supported by the development of the other components. As previously discussed, visual information is phonologically recoded wherever possible with the onset of subvocal rehearsal, and as such it may support performance on visuo-spatial span tasks. The spatial aspect of visuo-spatial working memory, as measured by a Corsi span, is also thought to follow a regular developmental increase between 7- and 15-years (e.g. Isaacs & Varga-Khadem, 1989). It has been suggested that the visual and spatial aspects may reflect independent subsystems that follow independent developmental trajectories. One study that explored a dissociation between visual and spatial working memory in children suggested that the two subcomponents were fractionated (Logie & Pearson, 1997). Children aged 5-/6-, 8-/9- and 11-/12- years were given recall and recognition tests for visual patterns and sequences of movements. The results suggested that children had better memory for patterns than sequences of movements, and that this difference between visual and spatial memory was more pronounced in the older children. Logie and Pearson (1997) suggest that this supports the idea of a visual cache for storing visual information and an inner scribe for storing spatial information. They reported a more rapid developmental change in children's ability to retain visual information than spatial information, which they suggest may reflect the use of retention strategies (such as generating and retaining an image of stimulus) that may be more effective for visual than spatial stimuli (Logie & Pearson, 1997). Their interpretation is somewhat speculative, and more recent studies have suggested that there may be alternate, fractionated developmental trajectories within the visuo-spatial system (see section 1.1.2.2). At present it is not yet fully understood how the capacity of the visuo-spatial sketchpad increases for items that cannot be phonologically recoded.

The capacity of the central executive is typically assessed using complex span tasks. It has been suggested that children show a regular increase in performance on such tasks, increasing from a span of 1.5 at 5-years to 6.5 at 16-years (Siegel, 1994). The reason for this developmental increase has been attributed to either an increase in processing efficiency and use of cognitive resources (e.g. Case, 1985) or an increase in efficiency switching between processing and storage (e.g. Towse & Hitch, 1995). Both accounts have been discussed previously in relation to the development of the whole working memory system (see section 1.2.1).

Another development within the central executive relates to the diversity of its functions. Miyake et al. (2000) suggested that executive functions are fractionated in adults. Using confirmatory factor analysis they identified three separate functions; shifting, updating and inhibition. When exploring the diversity of executive functions in children aged 6- to 8-years, Bull and Scerif (2001) suggested that the Miyake et al. (2000) model may be usefully applied to children. Indeed, Lehto, Juujarvi, Koistra and Pulkkinen (2003) obtained three factors resembling those identified by Miyake et al. (2000) in 8- to 13-year-olds. However, when Jarvis and Gathercole (submitted) recently investigated this in 11- and 12-year-olds, using a battery of tasks analogous to those used by Miyake et al. (2000), they could only identify two separate functions; updating and inhibition. Jarvis and Gathercole (submitted) did not identify a separate shifting ability, which may reflect a fundamental difference in the organisation of executive functions between adults and children. Alternatively, the unity among some executive functions in their sample may be accounted for by inhibition. That is, it has been suggested that all executive functions involve inhibitory processes for proper operation. As Miyake et al. (2000) suggest, updating may involve ignoring previous and incoming irrelevant information and shifting may involve suppressing a redundant mental set to shift to a new set. In this context, where the unity and diversity of executive functions is debated (e.g. Miyake et al., 2000), further research is needed to understand the development of executive functions throughout childhood. For example, there are several diverse executive functions, which are dissociable from inhibition in children (e.g. Espy, 1997), which highlights the need for this further research.

1.2.3 Development of the structure of working memory

A better understanding of the structure of working memory throughout childhood has been achieved with the increased use of latent variable techniques, such as factor analysis. Overall the evidence leans towards a fractionated working memory system. Early studies suggested that the phonological loop and the visuo-spatial sketchpad were independent of one another in 5- and 8-year-olds (Pickering, Gathercole & Peaker, 1998), and that the phonological loop and the central executive were separable but associated factors in 6- and 7-year-olds (Gathercole & Pickering, 2000b). Recent work has replicated the dissociation between the phonological loop and the central executive in children as young as 4-/6-years (Alloway, et al., 2004).

Initially there did not appear to be such a clear distinction between the central executive and visuo-spatial sketchpad in children. For example, Gathercole and Pickering (2000b) identified a two- factor solution in the design of the Working Memory Test Battery for Children (WMTB-C), with visuo-spatial sketchpad measures loading on the executive factor. Although this appears to mirror the adult literature, where it has been suggested that the visuo-spatial sketchpad is dependent upon support from other resources such as the central executive (e.g. Phillips & Christie, 1977a), more recent studies have reported that an independent visuo-spatial sketchpad factor may in fact exist (e.g. Jarvis & Gathercole, 2003). Indeed, one recent study drew a parallel between the structure of children's working memory at 11- and 14-years-of-age and a contemporary model of adult's working memory. Jarvis and Gathercole (2003) suggested that there are distinct verbal and visuo-spatial aspects in children for both complex and storage only systems, analogous to the adult model proposed by Shah and Miyake (1996).

Gathercole and colleagues recently conducted one of the most comprehensive and informative investigations of the structure of working memory in children (Gathercole, Pickering, Ambridge et al., 2004). They administered a battery of tests designed to tap the three different components of the original tripartite model to over 700 children aged between 4 and 15 years of age and explored the factor structure of working memory for both the group as a whole and for different age groups (6-/7-, 8-/9-, 10-/11-, 13-/15-year-olds). Their results suggested a model corresponding to the Baddeley and Hitch (1974) model, with three distinct but correlated factors, was the best fit to the data for all age groups. This indicates that the tripartite, adult –based model of working memory is in place by 6-years-of-age (Gathercole, Pickering, Ambridge et al., 2004). Although this study did not incorporate a measure of the recently added episodic buffer component, a recent study conducted by Alloway et al. (2004), which incorporated measures of the central executive, phonological loop and episodic buffer in a large scale study exploring the organisation of working memory and related cognitive abilities, suggested that there a distinct construct corresponding to the episodic buffer may be in place by 4-/6-years-of-age.

In summary, there are two competing theoretical accounts of the development of working memory throughout childhood; the resource-sharing and the resource-switching hypotheses. While the adult tripartite structure of working memory is proposed to be in place by 6-years-of-age (e.g. Gathercole, Pickering, Ambridge et al., 2004), each component of the Baddeley and Hitch (1974) model appears to follow its own developmental trajectory. Typically, the capacity of each component increases between childhood and adolescence.

Section Summary

1. Performance on working memory tasks follows a linear increase as a function of age.
2. Both processing efficiency and storage capacity increase throughout childhood. Two accounts of this developmental increase are offered; a resource-sharing hypothesis (e.g. Case et al., 1982) and a resource-switching hypothesis (e.g. Towse & Hitch, 1995). At present it is thought that both time and cognitive resources are important factors in children's working memory development.
3. Developmental changes occur within each component of the tripartite working memory model.
4. The capacity of the phonological loop typically increases from 2-3 items at 4-years to 6 items at 12-years. The onset of subvocal rehearsal and phonological recoding at about 7-years are thought to be the primary reasons for this developmental increase.
5. The capacity of the visuo-spatial sketchpad increases from 4 items at 5-years to 14 items at 11-years. Little is known about its development, although some have speculated that it may be supported by the development of the phonological loop and the central executive and that the visual and spatial aspects may follow independent developmental trajectories.
6. Typically the capacity of the central executive increases from 1.5 items at 5-years to 6.5 items at 16-years. This increase may reflect the increased processing efficiency and storage capacity of the entire working memory system. It is unclear whether executive functions are distinct in childhood.

7. Overall, latent variable studies suggest that the adult-like structure of working memory proposed by Baddeley and Hitch (1974), with three distinct but correlated factors, is in place by 6-years.

Section 1.3

Mathematical Cognition

This section will provide an overview of the development of mathematical cognition throughout the lifespan. Due to the vast literature on this topic, which extends across studies of normal adult (e.g. Ashcraft & Battaglia, 1978; LeFevre, Sadesky & Bisanz, 1996) and child populations (e.g. Siegler, 1987), brain-lesioned patients (e.g. Warrington, 1982), bilinguals (e.g. Geary, Cormier, Goggin, Estrada & Lunn, 1993; Jensen & Whang, 1994), mathematically gifted (e.g. Dark & Benbow, 1991) and mathematically impaired populations (e.g. Geary, Bow-Thomas & Yao, 1992), this introduction offers an overview of the major changes that occur between preschool and adulthood.

1.3.1 Preschool

Piaget's (1952) early suggestion that children's abstract knowledge of arithmetic does not emerge until 4- or 5-years-of-age has since been contended. Evidence now suggests that children possess some numerical skills before formal schooling begins at around the age of 5-years. A number of studies have shown that children younger than 4-years have mastered less demanding, non-verbal versions of the tests of numerical conservation used by Piaget (Gelman & Gallistel, 1978; McGarrigle & Donaldson, 1974). Furthermore, it has been proposed that the emergence of visual number forms occurs during infancy, independent of formal number or calculation teaching (Seron, Pesenti, Noel, Deloche & Cornet, 1992).

Wynn (1992; 2000) demonstrated, using the violation of expectation model, that preverbal infants as young as 4 days old could perform simple arithmetic operations. She found that infants were able to discriminate visual numerosity, and suggested that these skills provide the foundation for later, complex mathematical abilities. Using the same paradigm with 2- and 3-year-olds, where language skills had begun to emerge, Houdé (1997) reported that children's mathematical abilities for small numbers was comparable to that of preverbal infants (Wynn, 1992) and monkeys (Hauser, MacNeilage & Ware, 1996), demonstrating that early preverbal skills may indeed underpin later number development. Similar findings have been reported for auditory stimuli, where newborn babies have been shown to discriminate two- and three-syllable words (Bijeljac-Babic, Bertoncini & Mehler, 1991). Furthermore, infants, as young as 6 months, have been shown to perform cross-modal numerosity matching. When they hear two or three drumbeats, and are then shown visual displays of two or three objects, they spend longer looking at the slide with the numerosity that matches the number of sounds they hear (Starkey, Spelke & Gelman, 1983). More recently, it has been suggested infants' numerical abilities are reasonably sophisticated by 11 months. At this age babies are able to recognise different numerosities, and judge which of two numerosities is larger (Brannon, 2002).

Studies such as Wynn's (1992; 2000) suggest that humans possess an innate capacity to perform simple arithmetical operations. Specifically, Wynn (1995) claims that infants understand the true numerical value of a set of objects, make distinctions between them based on numerosity, have a mechanism for non-verbal counting called the accumulator, and understand the ordinal relations between different collections. In line with Wynn (1992), Meck and Church (1983) proposed that animals represent numbers internally by means of an analogue accumulator. Gallistel and Gelman (1992) suggest that human infants may possess a similar accumulator, which could underpin the acquisition of a verbal number system. The notion of an analogue number representation system is akin to Dehaene's (1992) notion of a "mental number line", which is discussed in section 1.4.2.2.

However, an alternate interpretation suggests that infants' ability to make numerical comparisons is based on a perceptual process known as subitizing. Subitizing provides information about very small numerosities faster than counting, as items are processed simultaneously rather than in succession. Evidence suggests that infants' numerical discriminations were limited to the same, small numerical values of three versus four items (Strauss & Curtis, 1981), as subitizing is in older children and adults (e.g. Dehaene, 1992). Cooper (1984) demonstrated that infants up to the age of 12 months were not able to detect changes in numerical relation. That is, they did not react when the numerical relation between successive collections switched between more-than and less-than relations. Contrary to the accumulator theory, which proposes that infants know the basic arithmetical relations among small numbers (such as one plus one makes two), this suggests that relational information is not inherent in infants' representation of numerical values. In a study which replicated Wynn's (1992) original work and extended it to incorporate subtraction, Wakeley, Rivera and Langer (2000a; 2000b), both failed to replicate the original findings with addition trials and, more importantly, found that infants' looking times were not significantly different for trials with correct and incorrect outcomes in the subtraction conditions. This finding further supports the idea that infants do not understand arithmetical relations. In summary, the numerical knowledge attributed to infants by this alternate interpretation is far less than suggested by Wynn and colleagues.

Whether infants' ability to make numerical judgements reflects an innate understanding of arithmetical relations and the existence of a nonverbal counting mechanism (e.g. Wynn, 1992) or a more basic numerical ability such as subitizing (e.g. Strauss & Curtis, 1981), the overriding conclusion is that humans possess some form of innate ability to process number.

This notion is strongly supported by Dehaene, Dehaene-Lambertz and Cohen (1998) and Butterworth (1999). Dehaene et al. (1998) propose that we are born with an innate "number sense". That is, although higher-level arithmetic is culturally achieved, animals, young infants and adults possess a biologically determined, domain-specific representation of number and elementary arithmetic operations. They speculate that animal and human number processing reflect the operation of similar biological neural systems that are anatomically located in the parietal cortex, a visuo-spatial area of the brain. They cite numerical distance effect and number size effect studies as support for this suggestion. To elaborate, the numerical distance effect, which refers to the notion that the ability to discriminate between two numbers improves as the numerical distance between them increases, has been found in animals (Gallistel & Gelman, 1992) and humans (e.g. Dehaene, Dupoux & Mehler, 1990). Similarly, number size effects, which refer to the notion that the ability to discriminate between two numbers of equal numerical distance worsens as their numerical size increases, have been found in animals (Gallistel & Gelman, 1992) and humans (e.g. van Oeffelen & Vos, 1982). The association between animal and preverbal infants' innate ability to deal with numerosity has been extended through models of animal counting (e.g. Meck & Church, 1983).

The idea that humans possess an innate ability to deal with number is one echoed by Butterworth's (1999) theory of the mathematical brain. He suggests that a "number module", which is genetically determined by the human genome, exists in the parietal lobe (Butterworth, 1999). He believes that this module is specialised for dealing with numerical representations and that it is responsible for categorising the world in terms of numerosities. In particular, he suggests that three basic biological numerical capacities present in infants are embedded within the number module. These are the capacity to recognise numerosities, the capacity to detect changes in numerosity, and the capacity to order numbers by size (Butterworth, 1999). Although a specific gene is yet to be linked to the number module, Butterworth argues that infants' apparent innate abilities, such as those reported by Wynn (1992; 2000), the selective impairments caused to number abilities following neurological damage (e.g. the case of DRC reported in Warrington, 1982) and cases of developmental dyscalculia (e.g. Shalev & Gross-Tsur, 2001) support the idea of a genetically-determined ability to deal with number. Additional support for the existence of a number module is taken from the animal literature. As Butterworth (1999) points out animals can, and do, use numerosity skills. For example, when foraging for food animals demonstrate that they can estimate and compare quantities as they will chose a patch with more food, and return to it more often than a patch with little food (e.g. Gallistel, 1990). Similarly, lions demonstrate the ability to judge numerosity when defending their territory. They will only attack intruders when the number of defenders is greater than the number of attackers (e.g. McComb, Packer & Pusey, 1994). Butterworth (1999) speculates that animals with a greater capacity to deal with number may have an adaptive advantage, and as such, he reasons that an evolutionary theory for the existence of a number module in animals lends support to its existence in humans.

By the time children begin school, it is argued that their innate numerical abilities have developed into an informal knowledge of simple arithmetic tasks when they are set in a familiar, concrete context (e.g. Hughes, 1986). Hughes (1986) found that pre-school children could more readily solve concrete problems (those that refer to specific objects, people or events), than abstract problems that do not have a concrete referent. Likewise, Ginsburg (1989) proposed that children are intuitive mathematicians, who spontaneously and frequently engage in "everyday mathematics" (Ginsburg, Pappas & Seo, 2001). He suggested that by about 4- or 5-years-of-age children are able to perform practical arithmetic, which is to say that they can deal informally with real world, concrete mathematical problems. It is argued that through building upon these concrete experiences with formal schooling children acquire conceptual principles necessary for abstract mathematics (e.g. Fuson, 1988).

Contrary to this, Gelman and Gallistel (1978) argue for a "principles-before-skills" hypothesis. They suggest that any functioning number system must have five implicit principles that guide learning (rather than principles being learned through concrete experiences). Three of these principles, known as the "how to count" principles, are thought to guide the acquisition of counting procedures. These include one-to-one correspondence (understanding that one word tag belongs to one number), stable order (understanding that word tags have a fixed sequence, e.g. one, two, three), and cardinality (understanding that a final word tag represents the size of a set). The other two principles, abstraction (understanding that any objects can be counted) and order irrelevance (understanding that objects can be counted in any order), govern counting. In a study designed to investigate how well children count, Gelman and Gallistel (1978) gave 2- to 5-year olds sets that varied in number from 2 to 19 and asked them to count them aloud. They observed that although the older children performed better than the younger children, children of all ages respected the five principles. From this Gelman and Gallistel (1978) claimed that children start with the right principles and develop the skills to apply them over time. In particular, they suggested that pre-school children understand the three "how to count" principles before they have learned the correct counting sequence and long before they are formally taught mathematics. While there is evidence to support this (e.g. Gelman & Meck, 1983; Gelman, Meck & Merkin, 1986), critics suggest that children younger than 5-years often violate one counting principle to satisfy another (e.g. Wagner & Walters, 1982) and that they may be remembering lists that were presented to them (Fuson, Richards & Briars, 1982) rather than fulfilling the principles of counting. Despite this debate, it is clear that children enter formal schooling at about 5-years of age with at least some knowledge of how to count and solve everyday, concrete mathematical problems.

In summary, it appears that children are born with some innate capacity to deal with numerosity (e.g. Dehaene et al., 1998; Butterworth, 1999), which is demonstrated by young infants' ability to perform simple mathematical operations (e.g. Wynn, 1992). Throughout the early preschool years these innate capacities develop into a basic, informal concrete understanding of number. At this age children are beginning to develop the skills to apply to mathematical principles (e.g. Gelman & Gallistel 1978). However, they may not yet fully understand the implications of these principles until formal schooling begins (e.g. Geary, 1994).

1.3.2 The School Years

The key developmental shift in children's mathematical cognition occurs during the school years, particularly during the primary school years (5-years to 11-years in the UK). The general consensus is that children advance from using slow, procedural counting-based strategies for the solution to basic arithmetic problems, to more efficient retrieval-based strategies analogous to those used in adulthood (e.g. Hamann & Ashcraft, 1985; Kaye, 1986).

Typically young children begin solving mathematical problems using counting strategies, such as count-all. The most sophisticated counting strategy, the "min" model (Groen & Parkman, 1972), offers perhaps the most widely adopted account of children's counting procedures. In a comprehensive study designed to explore children's counting algorithms in 6-year-olds, Groen and Parkman (1972) compared children's solution times (reaction times) on all 55 single digit additions with a solution equal to, or less than 9, against five different counting models. Each model specified the existence of a mental counter. The "min" (for minimum addend) model provided the best fit to the data. According to this model, the mental counter is set to the larger of the two addends, and is then incremented by steps of one equal to the value of the smaller addend, ending at a position (number) equal to the solution to the problem. The time to set the counter to the larger addend is thought to be constant, meaning children's solution times are assumed to be the time taken for incrementing the counter. However, studies that have investigated numerical inequality suggest that the time taken to set the counter may not be constant given that the time taken to judge pairs of numbers differs. For example, Sekuler and Mierkiewicz (1977) reported that numbers of greater disparity (such as 1 and 9) were responded to faster than numbers of greater parity (such as 6 and 7). Although the "min" model has weaknesses, its principles remain widely accepted and its influence can be seen in many theories of mathematical cognition. For example, in Dehaene's (1992) suggestion of a "mental number line" (see section 1.4.2.2).

Throughout the school years, children begin to adopt more adult, retrieval-based solution strategies. Evidence for this developmental shift in children's choice of solution strategies can be drawn from studies of children with mathematical difficulties who do not successfully achieve this transition. Children with mathematical difficulties (MD) are less likely to use direct memory retrieval to solve arithmetic questions (Bull & Johnston, 1997; Geary & Brown, 1991), and count more slowly and inaccurately than children with normal abilities (Bull & Johnston, 1997; Geary, et al., 1992). Furthermore MD children have weak, or incomplete, networks of number facts in long-term memory (Geary, Brown & Samanayake, 1991; Hitch & McAuley, 1991). This implies that poor counting skills impair the acquisition of number facts in early childhood, meaning incomplete networks of learned number facts are formed in long-term memory preventing the use of direct retrieval strategies.

Siegler (1987) proposed that the developmental shift occurred through the systematic exposure to arithmetic facts in the classroom. He suggested that this fostered the development of a complete network of arithmetic facts, which subsequently encouraged the use of more efficient retrieval strategies over slower procedural strategies. Siegler and Shrager (1984) proposed a model of the development of arithmetic facts, which accounted for the frequency with which problems were presented at school. Children often derive correct and incorrect solutions to problems when using a counting procedure. According to the Distribution of Associations model, addition pairs and correct and incorrect solutions are stored in an interconnected network of number facts. With increased presentations of problems in the classroom, children become more efficient at using counting procedures, and reach the correct solution more often. This strengthens the problem-correct solution association to a point where the correct solution shows "peakedness" over the incorrect solutions. At this point a threshold, known as the confidence criterion, is reached and children begin to retrieve answers. The overall idea behind the model was that the probability of an answer being retrieved was based upon its associative strength with the correct solution, which was strengthened by frequent exposures to the problem. Supporting this, Ashcraft (1987) reported that children were more likely to retrieve answers to small problems than large problems following a higher frequency of exposure to the smaller problems.

The shift from the use of procedural to retrieval strategies in childhood does not follow a smooth developmental curve, nor does it undergo a sudden upward shift as suggested by Case (1992). According to Case's staircase model of development, children's thinking remains at a certain level for an extended period of time (a tread on a staircase), then undergoes a sudden transition to a new, higher level of thinking (a riser). It has, however, been shown that children use a variety of strategies and levels of thinking at any given time during development. For example, Groen and Parkman's (1972) data suggested that children were using a mixture of retrieval and counting on procedures. They observed that children's reaction times were faster for ties (e.g. 1+1, 2+2, 3+3) than problems with the same minimum addend (e.g. 2+1, 3 + 2, 2 + 3 respectively), which led them to suggest that the children had memorised the correct responses to ties and were retrieving the answers rather than computing them.

One of the earliest documentations of children's diverse strategy use came from Siegler (1987). He conducted a study to investigate the solution strategies used by 5 to 7 year olds to solve addition problems and found that although children's solution times followed the same linear function observed by Groen and Parkman (1972), suggesting they were using the min procedure, their verbal reports revealed they were using up to five different strategies. These included count-all, retrieval, decomposition, guessing and the min strategy. Siegler (1987) examined children's solution times and errors for different strategies and found that the min model accounted for 86% of the variance in trials where the children reported using the min strategy, and only 40% of the variance in trials where children reported using an alternate strategy, which suggested that the children's verbal reports were reasonably accurate. It is now readily acknowledged that children adopt a variety of strategies to solve arithmetic problems (e.g. Chen & Siegler, 1999; Coyle & Bjorklund, 1997; Geary, Fan & Bow-Thomas, 1992).

Siegler and Jenkins (1989) subsequently revised the Distribution of Associations model to take into account the number of strategies available to children. In brief, the new Strategy Choice model worked along the same principles as the original model, including associations between problems and answers, but also incorporated associations between specific problems and strategies, types of problems and strategies and classes of problems as a whole. These new associations were strengthened or weakened based upon information about the speed and accuracy of a strategy, and each strategy was assigned novelty points based upon how recently it was discovered. Although novelty points were lost with each use of a strategy, its effectiveness increased. Siegler and Jenkins' (1989) new model proposed that the solution process involved two phases; strategy choice followed by strategy execution. If a particular strategy was chosen, but could not be executed an alternate back-up strategy would be chosen. In recent years this model has been adapted, and now the Overlapping Waves model (Siegler, 1999) is among the best contemporary depictions of children's strategical development. According to this model, which is again based on the observation that individual children use a variety of strategies to solve individual mathematical problems at all times, the relative frequency of each strategy changes with age and experience so that some strategies become less frequent, some become more frequent then less frequent, some never become very frequent, some become more frequent, some old ones cease to be used and some new ones are discovered. Chen and Siegler (2000) conceptualise this developmental trajectory in terms of five components: 1.child's acquisition of a new strategy 2. mapping the new strategy on to novel problems 3. strengthening the new strategy 4. refinement of choices among available strategies 5. successful executive of the new strategy. Shrager and Siegler (1998) modelled these ideas about children's strategy choice and discovery. Their model, known as SCADS (Strategy Choice And Discovery Simulation), combines metacognitive and associative mechanisms, where the associative processes lead to adaptive strategy choices and the metacognitive processes lead to the discovery of new strategies. This model accounts for all eight of the key characteristics of strategy development (see Shrager & Siegler, 1998), demonstrating that children's strategy choice is indeed variable.

In summary, formal schooling fosters children's mathematical development, encouraging the use of adult-like retrieval solution strategies (e.g. Siegler, 1987). Although a general developmental shift occurs throughout the school years as children move away from the use of slow-procedural strategies and become more efficient in their use of retrieval strategies, individual children use a variety of strategies to solve individual mathematical problems, including retrieval, decomposition, count-all and min procedures. Furthermore, mathematical development involves changes in the use of different strategies and strategy choice is related to problem difficulty (Siegler, 1999).

A final point worthy of note is that since the early theories of mathematical development, the emergence of arithmetic skills has been the focus. Relatively little is known about the development of other mathematical processes, such as algebraic skill and geometric abilities, in comparison to what is known about the development of children's arithmetic abilities. One assumption, that is yet to be formally substantiated, is that the development of different mathematical skills follows a similar pattern to the development of arithmetic abilities. This is suggested because basic arithmetic computation skills have been related to broader mathematical problem solving abilities. For example, Siegler (1988) found that maths fact skills were predictive of more general problem-solving abilities in 6-year-olds. Kail and Hall (1999) reported a similar relationship between basic arithmetic measures and problem-solving skills in 8- to 12-year-olds. Furthermore, factor-analytic and structural-equation-modelling studies have shown that arithmetic skills and broader mathematical competencies are closely related. Widaman, Little, Geary and Cormier (1992) reported that addition efficiency was closely related to a mathematics achievement latent variable, which predicted computational, conceptual and application skills in mathematics, in 7- to 12-year-olds. Similar findings, that arithmetic production was related to mathematics achievement in Asian-American students aged 9- to 12-years, were reported by Whang and Hancock (1997). The notion that mathematical skills develop in a similar fashion to arithmetic skills is supported by children's varied strategy use for the solution to algebraic and geometric mathematics problems. For example, children will use one of three strategies to solve algebraic problems. They either move all the letters to one side and all the numbers to the other side of an equation (the isolation strategy, Mayer, 1982), replace variables with numbers in a trial-and-error fashion in an attempt to balance the equation (the substitution strategy, Sleeman, 1984) or clear the parentheses by carrying out the necessary operations (the reduce strategy, Mayer, 1982). It could be argued that this diverse strategy use mirrors the varied use of arithmetic solution strategies. Further support for this idea is that children begin to solve geometry problems using concrete solution strategies, then move on to using abstract solution strategies (e.g. Clements, Battista, Sarama, Swaminathan & McMillen, 1997). This developmental trend follows that suggested by Hughes (1986) for children's arithmetic development. Overall, the evidence supporting the idea that different mathematical skills follow similar developmental paths is limited at best, meaning any interpretation is merely speculative. Further research into the development of different mathematical skills in children is certainly needed.

1.3.3 Through to Adulthood

Although adults, like children, use a variety of strategies for the solution of mathematical problems (e.g. LeFevre, Sadesky & Bisanz, 1996), they predominantly rely on efficient retrieval-based strategies (e.g. Campbell & Graham, 1985).

A number of models of adult fact retrieval have been suggested over the years, some of which also apply to children (e.g. Dehaene & Cohen. 1995). Ashcraft and Battaglia (1978) and Ashcraft (1982) offered one of the simplest accounts. According to the Network Retrieval Model, or the fact-retrieval model as it became (Ashcraft, 1982), adults mentally represent number facts in an addition table form where augends (first numbers) head each column and addends (second numbers) head each row. In this table, the augends and addends increase sequentially along their respective axis. Retrieval is a search process along to the augend, then down to the addend, where the solution is located. As such, retrieval time is a function of the distance travelled, meaning the larger the addends the longer the solution time.

Ashcraft (1982) reported that this model did not fit young children's performance, but that it did fit older children's and adult's performance, demonstrating that while young children rely on min type procedures, older children and adults use the fact-retrieval model. Ashcraft and Fierman (1982) determined that performance shifts from counting to retrieval at around 8-/9-years (Grade 3). They explored mental addition in Grade 3, 4 and 5 children using a true/false verification task. Their results suggested Grade 3 children process mental addition problems in a different way to Grade 4 and Grade 5 children. Indeed, they reported that half of the Grade 3 children were using immature counting type strategies, whilst the other half were solving the problems more efficiently in a manner that resembled the Grade 4 and 5 children's performance. The older children's performance was consistent with that of adults using retrieval-based strategies (e.g. Ashcraft & Battaglia, 1978), suggesting children begin to use retrieval strategies at about 8-/9-years.

Stazyk, Ashcraft and Hamaan (1982) proposed a similar fact-retrieval model for the solution of multiplication problems. Although more complex models of fact-retrieval have since been proposed, such as those that incorporate both incorrect and correct solutions in their networks (e.g. Campbell, 1994) and those that include information about numerical magnitude in their networks (e.g. Butterworth, Zorzi, Girelli & Jonckheere, 2001), a central theme to all models is the principle that associative networks of facts are formed and stored in long-term memory for the direct retrieval of answers to arithmetic problems in adulthood.

Although adults predominantly retrieve answers from an associative network of arithmetic facts, they also use a variety of alternate strategies for the solution of mental arithmetic problems (e.g. Dowker, 1998), particularly for larger problems where procedural strategies are more effective (e.g. LeFevre, Sadesky & Bisanz, 1996). Current models of numerical cognition differ as to whether the internal processing of numerical information during procedural and retrieval strategies involves one or many codes. According to McCloskey's abstract code model (McCloskey, 1992; McCloskey, Caramazza & Basili, 1985; McCloskey & Maracuso, 1995) numerical input is translated into one internal abstract code, which reflects the basic quantities in a number and the power of ten associated with it. This code is used to perform the calculation, before the solution information is translated into an output code, which is either verbal or written based on the output modality. A key feature of this model is that number comprehension and number production are independent components of the number-processing system, each responsible for translating between the abstract code and the input/output codes. In addition, this model suggests that calculation requires three processes: 1. comprehension of operands and words 2. retrieval of arithmetic facts 3. execution of the calculation process. Contrasting this, Dehaene's (1992; Dehaene, 1997; Dehaene & Cohen, 1995) triple code model suggests that the internal code used to perform a calculation depends upon the processing task. According to this model, as the name suggests, three codes are used. A visual Arabic code, in which numbers are represented as strings of digits, is used for multi-digit operations and parity judgements. An analogue quantity or magnitude code, in which numbers are represented along a "mental line" (Dehaene, 1992), is used for semantic knowledge about quantities, proximities and larger-smaller relations. Finally, a verbal code, in which numbers are represented as words, is used for the retrieval of arithmetic facts. There are two routes for the solution of arithmetic problems; a direct and an indirect route. In the direct route, the operands of the problems are transcoded into a verbal representation, which then activates the completion of the word sequence via rote verbal memory (e.g. "2 x 6" is transcoded to "two times six" and completed as "twelve"). In the indirect semantic route, the operands are encoded as quantity representations upon which semantically meaningful manipulations are performed before the resulting quantity is named by the language network (e.g. "6-3" is mentally represented as a starting quantity of "6", which is then decremented three times to reach a quantity of "3", which is then named by the language system). Dehaene's model suggests that many calculations often involve the simultaneous operation of both routes (Dehaene & Cohen, 1995). Campbell's encoding complex model (Campbell, 1994; Campbell & Clark, 1992) is more elaborate again. According to this model, multiple internal codes are activated to varying degrees depending on the presentation format of the task under the assumption that Arabic and verbal inputs may independently access number fact representations. They suggest that visual number words (e.g. "seven") activate verbal codes, while Arabic - digits (e.g. "7") activate visual codes. This model hypothesises that Arabic-digit formats activate number-representations for retrieval more efficiently than number words, but that the two processes (number reading and number-fact retrieval) interact (Campbell & Clark, 1992). According to this account operand intrusion errors, which are usually arithmetically related incorrect answers, occur because multiple responses are activated in response to a problem due to the interaction of the two processes. The current view is more in line with the latter two accounts, which suggest that adults use a variety internal codes and processes for the solution of arithmetic problems.

To summarise, adults predominantly rely on efficient retrieval strategies for the solution of arithmetic problems, but also rely on procedural strategies, which involve a variety of internal codes and processes, for the solution to more complex problems. As such, it could be argued that Siegler's Distribution of Association models (such as SCADS) (e.g. Siegler & Shrager, 1984; Siegler, 1999; Shrager & Siegler, 1998), where strategy choice is an integral part of the solution process, may provide the most comprehensive account of how older children and adults solve arithmetic problems.

Section Summary

1. Preschoolers and infants demonstrate a basic capacity for number and numerosities. One suggestion is that we are born with some form of innate mathematical ability, which may be linked to the human genome.
2. By the time children begin school they have an informal knowledge of simple arithmetic, and are beginning to display at least three of the five implicit principles proposed by Gelman and Gallistel (Gelman & Gallistel, 1978; Gallistel & Gelman, 1992).
3. Formal schooling fosters children's mathematical development. During this time children advance from using slow, procedural counting-based solution strategies to more efficient adult-like retrieval-based solution strategies.
4. Children's mathematical development does not follow a smooth developmental curve, nor does it undergo a sudden developmental shift. Rather, individual children use a variety of strategies to solve individual mathematical problems (e.g. Siegler's overlapping waves model, 1999).
5. Our knowledge of the development of children's wider mathematical skills is rather limited to the development of arithmetic problem solving.
6. Adults predominantly rely on retrieval-based solution strategies for the solution to arithmetic problems. They do, however, use a variety of alternate procedural strategies where retrieval strategies cannot be deployed.
7. Models of numerical cognition suggest that adults and children use a number of internal codes and processes for the solution to arithmetic problems.

Section 1.4

Working Memory and Mathematics

The first part of this section defines a role for working memory in cognitive abilities as a precursor to the second part, where a review of the literature is presented that suggests a role for working memory in mathematics.

1.4.1 Working Memory and Cognitive Abilities

As previously noted the working memory system assumes responsibility for the temporary storage and processing of information during cognitive tasks. It therefore follows that performance on working memory tasks has been associated with a variety of cognitive abilities, including language and reading comprehension (Daneman & Carpenter, 1980; Siegel, 1994; Yuill, et al., 1989), vocabulary acquisition (Daneman & Green, 1986; Gathercole & Baddeley, 1990; Gathercole, et al., 1992), literacy (de Jong, 1998; Swanson, 1994) and arithmetic / mathematics (Adams & Hitch, 1997; Bull, Johnston & Roy, 1997; Bull & Scerif, 2001; Hitch, 1978; Leather & Henry, 1994; Maybery & Do, 2003; Reukhala, 2001).

Early studies that defined a role for working memory in cognitive abilities focussed on individual differences in working memory as a correlate of reading comprehension in adults. In a seminal study, Daneman and Carpenter (1980) administered participants with a reading span task, a word span task and three reading comprehension measures; answering fact questions, pronoun reference questions and the Verbal Scholastic Aptitude Test. They found that reading span performance, a working memory measure, was significantly related to reading comprehension, while word span performance, a simple span task designed to measure short-term memory capacity, was not. Turner and Engle (1989) reported similar findings using operation span as the complex span measure, emphasising the generality of working memory as a predictor of cognitive ability. Although these early studies suggested that the concurrent processing and storage demands of complex working memory tasks were important for predicting cognitive abilities, more recent research, which has worked within the tripartite framework, suggests that simple span measures designed to tap the slave systems are also important predictors of cognitive abilities, especially in children.

Since Daneman and Carpenter's (1980) early work, numerous studies have explored the role of working memory in children's academic attainment. It has recently been associated with scholastic attainment, as measured by standardized tests of achievement, at 7- (Gathercole & Pickering, 2000a; 2000b), 11- (Jarvis & Gathercole, 2003) and 14-years (Jarvis & Gathercole, 2003). This will be discussed in more detail in Chapter 3. As already detailed in Section 1.1.2.1, it is thought that the concurrent processing and storage functions of the working memory system are important in supporting children's language development. In particular, the phonological loop is thought to play an integral part in vocabulary acquisition in childhood (e.g. Gathercole & Baddeley, 1989) and second language learning in both childhood and adulthood (e.g. Papagno et al., 1991). As previously discussed, it is thought that the phonological representation of an unfamiliar word is represented by the phonological store, which facilitates vocabulary acquisition in children, while the subvocal rehearsal process of maintaining the contents of the phonological store aids second language learning in adults (Baddeley, et al., 1998). Furthermore, phonological loop deficits have been reported in populations of children and adults with reading difficulties (Siegel, 1994), general learning disabilities (Henry, 2001) and syndromes associated with learning disabilities, such as Down's (Jarrold & Baddeley, 1997) and autism (Russell, Jarrold & Henry, 1996).The central executive is thought to be more important for language and text comprehension (Dixon, LeFevre, & Twilley, 1988; Hitch, Towse & Hutton, 2001; Leather & Henry, 1994; Yuill, et al., 1989).

Working memory has been implicated in mathematical competence in adults, but less is known about its possible role in children's mathematics. This topic, which is becoming an area of increased interest among contemporary researchers, is the focus of this thesis. A comprehensive review of the relevant literature is provided in the following section.

1.4.2 Working Memory and Mathematics

All three components of the tripartite model of working memory have been associated with mathematical abilities in adults and children. Evidence suggesting a link between each component and mathematics is presented in the following sections.

1.4.2.1 The Phonological Loop and Mathematics

The primary role of the phonological loop in mathematics is to encode and retain the verbal codes that children and adults use for counting (Nairne & Healy, 1983; Healy & Nairne, 1985), exact arithmetic (Dehaene, Spelke, Pinel, Stanescu & Tsivikin, 1999) and mathematical algorithms, such as addition and subtraction (Siegler & Jenkins, 1989).

Dual-task studies with adults have demonstrated that the phonological loop is responsible for maintaining interim results during addition (Heathcote, 1994; Lemaire, Abdi & Fayol, 1996; Logie, Gilhooly & Wynn, 1994; Seitz & Schumann-Hengsteler, 2002), subtraction (Seyler, Kirk & Ashcraft, 2002) and multiplication (Seitz & Schumann-Hengsteler, 2000). Across these studies, counting accuracy and arithmetic performance was disrupted under conditions of articulatory suppression, indicating that the subvocal rehearsal process, which occurs in the phonological loop, is needed to maintain interim results during the calculation of both single-digit and multi-digit problems.

Subvocal rehearsal is also important for the retention of problem information during calculation in both adults and children. Fürst and Hitch (2000) reported that concurrent articulatory suppression disrupted the addition of pairs of three-digit numbers when the digits were presented for a brief period of time. When the numbers to be added remained visible throughout the calculation, adults' performance was not affected by articulatory suppression. Adams and Hitch (1997) reported similar results with children; that their mental addition spans were lower when the numbers to be totalled were not visible, implying that the two procedures (retaining the problem information and performing mental addition) were competing for the same cognitive resource, the articulatory loop. Supporting the idea that brief presentation of the problem information leads to phonological coding and maintenance of the information in the articulatory loop in adults, Noel, Desert, Auburn and Seron (2001) reported an interference effect of phonologically similar digits that were presented briefly in multi-digit addition, which was not found for visually similar digits.

It has been suggested that the phonological loop is important for counting across different types of arithmetic in children and adults. For example, Ellis and Hennelley (1980) reported that arithmetic performance and digit span were poorer for bilingual children speaking Welsh than bilingual children speaking English. They attributed this phenomenon to the time taken to pronounce the digit words in each language. That is, the Welsh words took longer to pronounce, therefore they took longer to subvocalize and consequently placed heavier demands on the phonological loop in both tasks. Evidence for the involvement of the phonological loop in counting per se comes from a dual task study with adults where participants were required to count the total number of dots, or the total number of times a stimulus appeared, under conditions of concurrent articulatory suppression (Logie & Baddeley, 1987). As with previous findings, concurrent articulatory suppression disrupted counting accuracy, suggesting that subvocal rehearsal is important for keeping track during counting.

The retrieval of number facts from long-term memory is based on a verbal code (Dehaene & Cohen, 1995), which implicates an additional role for the phonological loop in mathematics. Verifying this assumption, researchers have reported that the storage and retrieval of multiplication facts (LeFevre, Lei, Smith-Chant & Mullins, 2001; LeFevre & Liu, 1997; Lee & Kang, 2002) and addition facts (Lemaire, et al., 1996), which rely upon phonological codes, can be disrupted by concurrent articulatory suppression in adults. However, the evidence for such a role is controversial as concurrent verbal central executive tasks (such as random letter generation) have a greater disruptive effect on the retrieval of number facts from long-term memory (Seitz & Schumann-Hengsteler, 2000; 2002), implying that this process may involve executive skills, rather than being purely phonological.

Evidence suggests that the phonological loop is important for children's mathematical development. Many studies report significant associations between children's mathematical abilities and their performance on measures of phonological loop functioning, such as digit, word or nonword recall span tasks (Dark & Benbow, 1991; Dark & Benbow, 1994; Gathercole & Pickering, 2000a; Hitch & McAuley, 1991; Jarvis & Gathercole, 2003; Maybery & Do, 2003; Passolunghi & Siegel, 2001;

Towse & Houston-Price, 2001; Wilson & Swanson, 2001). Gathercole and colleagues reported significant associations between National Curriculum mathematics attainment and phonological loop abilities at Key Stage 1 (Gathercole & Pickering, 2000a) and Key Stages 2 and 3 (Jarvis & Gathercole, 2003), implying that it supports mathematics throughout childhood.

While the phonological loop may play a role in children's mathematics analogous to that played in adults mathematics (e.g. storing problem information Adams and Hitch (1997); supporting counting (Ellis & Hennelley, 1980)), it is particularly important for mathematics development as it supports the acquisition of number facts that are stored in long-term memory. It has been suggested that impaired verbal / phonological working memory abilities may impair counting. Counting provides an important source of feedback when learning numerical / arithmetical relationships (Siegler & Robinson, 1982), which is crucial to the formation of associations between problems and answers. These associations form the base of number facts in long-term memory for direct retrieval and other solution strategies, such as decomposition, later in life. Consequently, mathematical difficulties can arise when the acquisition of arithmetic facts is impaired and weak or poorly formed networks of number facts are stored in long-term memory (Geary et al., 1991; Hitch & McAuley, 1991). As discussed in section 1.3.2., children with mathematical difficulties (MD) count more slowly than children with normal abilities (e.g. Geary, et al., 1992), have weak, or incomplete, networks of number facts in long-term memory (e.g. Geary, 1990) and are less likely to use direct memory retrieval to solve arithmetic questions (e.g. Bull & Johnston, 1997). Crucially, these children perform poorly on measures of phonological loop ability such as the digit span task (Bull & Johnston, 1997; Hitch & McAuley, 1991; Passolunghi & Siegel, 2001; Siegel &

Linder, 1984). As such, poor verbal working memory has been implicated as one of the many cognitive deficits underlying MD.

Recently McKenzie, Bull and Gray (2003) demonstrated, using a dual task design, that younger children use visuo-spatial strategies in mental arithmetic, while older children use a mixture of phonological and visuo-spatial strategies. They suggest the phonological loop might be important in older children's mathematics where subvocal rehearsal occurs spontaneously. At this age, its function may mirror the role it plays in supporting adults' mathematics.

To summarise thus far, the phonological loop has been associated with mathematical abilities in children and adults. It is thought to play a key role in the acquisition and retrieval of number facts, and is important for retaining interim results and problem information during calculation. Although this evidence seems compelling, three lines of evidence suggest that the phonological loop may not be crucial for mathematics.

The first relates to the idea that the phonological loop is only associated with mathematics when other variables, such as processing speed and reading ability, are not included in the analysis. Hulme and Roodenrys (1995) point out that there is little evidence to suggest a direct causal link between short-term memory problems and cognitive impairments. They warn that any findings should be interpreted with caution as they are rarely considered in conjunction with other cognitive deficits. Indeed, one study, which included a comprehensive battery of cognitive assessments, reported that once reading ability had been controlled for, processing speed was the best predictor of mathematics ability, while phonological working memory contributed no further unique variance (Bull & Johnston, 1997). Previous studies have not typically included measures of processing speed and rarely account for a child's reading ability. As such, Bull and Johnston (1997) propose that other studies may have detected an association between general academic abilities (such as reading ability) and phonological short-term memory that is not specific to mathematics.

A second line of argument against the involvement of the phonological loop in mathematics is that digit-based measures of phonological functioning, namely digit span, share a stronger association with mathematics than non-digit based measures (e.g. Dark & Benbow, 1990; Dark & Benbow, 1991; Passolunghi & Siegel, 2001; Siegel & Ryan, 1989). One explanation purported for this is that participants who are mathematically able may have stronger representations for digits, meaning they are identified more quickly, leading to an enhanced memory span (Dark & Benbow, 1990; 1991). As such, performance on digit-based measures of phonological short-term memory and mathematics may be related because the assessments of both involve either number processing or direct access to numerical information.

The final line of argument against the involvement of the phonological loop in mathematics concerns the central executive. Evidence suggests that tasks designed to tap the two components, which are both typically verbal in nature, are somewhat co-dependent. For example, it has been suggested that reading span is affected by the storage capacity of working memory as measured by digit span (Dixon et al., 1988) and studies that report non-significant associations between the phonological loop and cognitive abilities, but significant associations between the central executive and cognitive abilities, typically report strong associations between the tasks used to measure central executive and phonological abilities (e.g. Gathercole & Pickering, 2000b). Two explanations for this co-dependence have been offered. Firstly, the central executive may play a role in relaying and retrieving information to and from the phonological loop, thus constraining performance on phonological loop measures.

Secondly, due to the verbal nature of many central executive tasks they may recruit phonological loop resources for storage, meaning phonological loop capacity constrains performance on central executive tasks. Given the covariance of measures designed to assess the two components, it is suggested that the phonological loop may not predict unique variance in mathematics. Indeed, Leather and Henry (1994) report a study in which simple span tasks (phonological loop tasks) only predicted variance in reading accuracy, comprehension and arithmetic when they were entered first into the regression analyses, while complex span tasks (central executive tasks) shared unique variance with all three cognitive abilities.

Overall, this evidence suggests that the shared variance between phonological loop abilities and mathematical competence may be accounted for by other variables such as reading ability, processing speed, fluency with numbers and numerical information or general verbal executive skills.

1.4.2.2 The Visuo-Spatial Sketchpad and Mathematics

Although much of the evidence implicating working memory in complex cognitive activities comes from studies in the verbal domain, where the tasks used depend heavily on verbal-phonological representations, it has been suggested that the visuo-spatial sketchpad may play a key role in planning movements (Baddeley & Lieberman, 1980; Logie & Marchetti, 1991) and learning spatial routes and faces (Hanley, Young & Pearson, 1991). Furthermore, the visuo-spatial sketchpad has been associated with general measures of scholastic attainment (Jarvis & Gathercole, 2003) and reading (Brooks, 1967) and visuo-spatial working memory deficits have been observed in children with learning disabilities (Cornoldi, Rigoni, Tressoldi & Vio, 1999).

Individual differences in visual and spatial abilities have been related to individual differences in arithmetic reasoning (Geary, Saults, Liu & Hoard, 2000) and choice of advanced solution strategy (Luria, 1966; Geary & Burlingham-Dubree, 1989) in adults. Furthermore, visuo-spatial working memory abilities have been associated with mathematics performance and evidence suggests that visuo-spatial skills support mathematical processing in children and adults.

Research has suggested that we develop visuo-spatial codes for numbers, and represent them along a "mental number line" to assist in mathematical processing (Moyer & Landauer, 1967; Dehaene, 1992; Hayes, 1973). The notion of a "mental number line" suggests that we have a mental representation of numbers. In an early observation Moyer and Landauer (1967) reported that the time to judge differences between numerals (which was larger) decreased as the numerical distance between them increased. They suggested that this was analogous to comparisons of physical stimuli. Similarly, Dehaene et al. (1990) reported that numerical judgements were faster for numbers that were numerically closer, with no effect of decade boundaries. That is, when deciding which was larger of a pair of 2-digit numerals from the same and different decades (e.g. 51 and 56 from the same decade and 51 and 67 from different decades), participants' judgement time reflected the distance between the two numerals, rather than the time to compare the decades (e.g. the time to compare 5 and 6). This led them to propose that we have a digital code for numbers, which is converted into an internal magnitude code known as the "mental number line". Dehaene and colleagues provided further evidence to substantiate the existence of the "mental number line" in experiments that demonstrated the SNARC effect (spatial-numerical association of response codes). The results of these studies demonstrated that the "number line" extends from left to right; larger numbers were responded to faster with the right hand and smaller numbers were responded to faster with the left (Dehaene, Bossini & Giraux, 1993).

It has been suggested that we use the "mental number line" for quantity manipulation and approximation (Dehaene & Cohen, 1995). This type of arithmetic, in which an analogue magnitude representation is used for subitizing and estimation, invokes the use of nonverbal visuo-spatial networks in the bilateral parietal lobes (Dehaene et al., 1999). Although this research has not been directly associated with the visuo-spatial sketchpad it does suggest that visuo-spatial abilities are important for mathematics. In particular, it reflects the idea that bilateral areas of the brain are invoked for one of the three modalities of mathematics described by Dehaene's triple-code model of mathematics (see section 1.3.3). Indeed, considerable neuropsychological evidence implicates visuo-spatial areas of the brain in the representation, manipulation and processing of numbers (Dehaene et al., 1998; Dehaene et al., 1999; Pesenti, Zago, Crivello, Mellet, Samson, Duroux, Seron, Mazoyer & Tzourio-Mazoyer, 2001; Simon, 1997; Simon, 1998; Simon, 1999; Zago, Pesenti, Mellet, Crivello, Mazoyer & Tzourio-Mazoyer, 2001).

Further evidence for the involvement of the visuo-spatial sketchpad in mathematics comes from studies of children and adults with mathematical difficulties. Developmental dyscalculia, or mathematical difficulty (MD), is defined as a discrepancy between specific mathematical abilities and general intelligence (Diagnostic and Statistical Manual of Mental Disorders, 4th Edition). Contrasting this, typically developing children who are at the lower end of a normal distribution may have comorbid disorders, such as reading difficulties, which represent part of a more general learning difficulty. Therefore, it could be argued that there are potential differences in the cognitive factors underlying poor mathematics ability in these two groups. Geary (1993) defined a subtype of mathematical difficulty as characterised by visuo-spatial deficits. He reported that people with this type of mathematical difficulty often have problems with the spatial alignment of numerical information (Rourke & Finlayson, 1978), which affects their functional skills (such as the columnar alignment of numbers) and conceptual understanding of number representations (such as place value). Across the acquired and developmental dyscalculia literature subsets of adults and children are also described to have visuo-spatial deficits (Rourke, 1993; Rourke & Conway, 1997; Rourke & Finlayson, 1978; Strang & Rourke, 1985). Indeed, studies of children with specific mathematical difficulties have shown that they typically perform poorly on visuo-spatial sketchpad span measures (McLean & Hitch, 1999; White, Moffit & Silva, 1992).

Evidence, primarily from adult populations, suggests that the visuo-spatial sketchpad provides a "mental blackboard" (Heathcote, 1994) upon which visually presented mathematical problems are encoded, retained and manipulated. It is also thought to play a key role in the acquisition of mathematical skills in young children (Houdé & Tzourio-Mazoyer, 2003).

The visuo-spatial sketchpad has been implicated in adults' calculation. It is involved in encoding, retaining and transforming problem information (Heathcote, 1994; Pesenti et al., 2001), retaining interim results for approximation (Dehaene et al., 1999; Logie et al., 1994), counting procedures (Trbovich & LeFevre, 2003) and exact computation (Zago & Tzourio-Mazoyer, 2002).

Evidence that the visuo-spatial sketchpad is involved in encoding problem information comes from studies that have compared adults' solution times and accuracy under alternate presentation formats. It has been reported that participants are slower and more erroneous when problem information is presented as number words compared to digits (Campbell & Fugelsang, 2001; Noel, Fias & Brysbaert, 1997). The phonological code in the number word condition appears to cause an interference effect. This implies that solution times are faster and more accurate when an alternate, possibly visuo-spatial, code is used to encode problem information

It has been reported that adults' response times and accuracy are faster for problems that are presented vertically compared to horizontally (Heathcote, 1994; Trbovich & LeFevre, 2002), which is consistent with the notion that digits are represented in columns on the visuo-spatial sketchpad during calculation (Hayes, 1973; Heathcote, 1994). Concurrent phonological memory load disrupts performance for horizontally (Heathcote, 1994; Trbovich & LeFevre, 2003), but not vertically (Trbovich & LeFevre. 2003), presented problems, suggesting that the use of an alternate code (not phonological) promotes fast, accurate arithmetic solutions. It has been suggested that this code may be visuo-spatial given that performance on a concurrent visuo-spatial memory task was poorer when the mathematical problems to be solved were presented vertically compared to horizontally (Trbovich & LeFevre, 2003).

In addition to encoding and maintaining problem information, the visuo-spatial sketchpad stores interim results during calculation. Dual task studies with adults have shown that although concurrent passive memory tasks (such as irrelevant pictures or visual noise) do not disrupt arithmetic performance (Logie et al., 1994; Quinn & McConnell, 1999), concurrent spatial or dynamic tasks (such as hand movement) disrupt performance on visually presented arithmetic problems where interim results (running totals) are to be maintained (Heathcote, 1994; Logie et al., 1994).

Whilst neuro-imaging studies report that visuo-spatial areas are recruited for magnitude representation in approximation (Dehaene et al., 1999), dual task studies suggest that the visuo-spatial sketchpad might be involved in counting in complex arithmetic where carry operations are required. Concurrent visuo-spatial memory tasks are reported to have facilitatory effect upon carry problems (Trbovich & LeFevre, 2003), but only for participant's with poor mathematical skills. It has been suggested that the concurrent visuo-spatial task may have prevented counting, thus forcing participants to retrieve the answer, which was likely to be incorrect for participants with poor mathematical knowledge. Supporting the involvement of the visuo-spatial sketchpad in exact computation, Zago and Tzourio-Mazoyer (2002), conducted a PET study to investigate the areas of the brain activated during arithmetic fact retrieval and computation. Arithmetic fact retrieval activated parietal areas associated with visuo-spatial working memory and a naming network located in the left anterior insular and right cerebellar cortex, while computation activated the bilateral parietofrontal network which is thought to hold numbers in visuo-spatial working memory. This suggests that a concurrent visuo-spatial task may disrupt counting (computation) as it recruits the same resources, but does not disrupt retrieval as this process recruits alternate areas of the brain.

The visuo-spatial sketchpad has been associated with children's mathematics performance on standardized tests of achievement at 7-years (Gathercole & Pickering, 2000a), 10-years (Maybery & Do, 2003), 11-years (Jarvis & Gathercole, 2003), 14-years (Jarvis & Gathercole, 2003) and 15-/16-years (Reuhkala, 2001). While the phonological loop supports the construction of a network of number facts, the visuo-spatial sketchpad may be important for the development of early mathematical skills, as discussed in the subsequent paragraphs.

As discussed in section 1.3.1, Wynn (1992; 2000) suggests that humans possess a preverbal innate capacity to perform simple arithmetic operations, which provides the foundation for later mathematical abilities. This idea is captured by Dehaene's (1992) notion of an innate, preverbal "number sense" (see section 1.3.1). It has been suggested that the foundations of numerical processing are located in visuo-spatial areas of the brain (Simon, 1999) and that the emergence of visual number forms during infancy occurs independent of formal number or calculation teaching (Seron et al., 1992). These preverbal foundations, anatomically located in the parietal cortex, a visuo-spatial area of the brain (Dehaene et al, 1999), may be specifically related to visuo-spatial working memory (Feeney, Adams, Webber & Ewbank, 2004).

Houdé (1997) reported that 2-/3-year-olds mathematical ability for small numbers was comparable to that of preverbal infants (Wynn, 1992) and monkeys (Hauser et al., 1996) (see section 1.3.1.) and subsequently offered a developmental account of the involvement of visuo-spatial abilities in early mathematics. He suggested that this demonstrates a shift from the early visuo-spatial arithmetic reported by Wynn (1992) to later symbolic-linguistic arithmetic (Houdé, 1997). Supporting this, McKenzie et al. (2003) suggested that younger children use visuo-spatial strategies in mental arithmetic, while older children use a mixture of phonological and visuo-spatial strategies. Although the precise role of the visuo-spatial sketchpad in children's mathematical development is yet to be established, it appears to support early preverbal mathematics.

To summarise so far, evidence suggests that visuo-spatial working memory is involved in encoding and retaining problem information and interim results during calculation in adults. Furthermore, very young infants and children perform visuo-spatial arithmetic (Houdé, 1997; Wynn, 1992) and visuo-spatial representations underpin our number system (Dehaene, 1992) implicating a key role for visuo-spatial working memory in the development of early numeracy skills.

Despite this evidence it has been suggested that the associations between visuo-spatial working memory and mathematics may reflect the role of the central executive in mathematics. Current literature suggests that the visuo-spatial sketchpad may not be separable from the central executive (e.g. Miyake et al., 2001; Shah & Miyake, 1996. See section 1.1.2.2) and there are concerns that tasks designed to tap the visuo-spatial sketchpad place heavy demands on the central executive (Phillips & Christie, 1977a; Wilson et al., 1987). However, it could be argued that if the tasks used to assess visuo-spatial ability recruit executive resources they may actually be compromising the observation of contributions of the visuo-spatial sketchpad to mathematics. As Hamilton et al. (2003) purport, the development of new, or modification of existing, measures designed to tap a specific visuo-spatial working memory that are free of executive demands, may define a more pertinent role for visuo-spatial sketchpad in mathematics. A second line of defence for the involvement of visuo-spatial working memory in mathematics relates to the notion of a separable executive system. Studies have shown that verbal and spatial complex span tasks tap distinct resources (e.g. Jurden, 1995; Shah & Miyake, 1996), and as such Shah and Miyake (1996) suggested, in reference to the Baddeley's (1986) model of working memory, that the processing and storage component (the central executive) may be separable. If this is so, the associations observed between the visuo-spatial sketchpad and mathematics in previous studies may in fact reflect a role for visuo-spatial/ non-verbal executive skills in mathematics. Indeed recent studies that have employed measures of non-verbal / visuo-spatial executive ability have reported significant associations with mathematical competence (Jarvis & Gathercole, 2003; Maybery &

Do, 2003) in addition to significant associations between traditional visuo-spatial sketchpad measures and mathematics (e.g. Jarvis & Gathercole, 2003; Maybery & Do, 2003).

To conclude, evidence suggests a role for the visuo-spatial sketchpad in children's early mathematics and in adults and children's complex mathematical processing. However, these findings need to be interpreted with caution given the close associations between the central executive and the visuo-spatial sketchpad.

1.4.2.3 The Central Executive and Mathematics

A body of empirical work implicates the central executive in a range of cognitive abilities predominantly related to language and text comprehension (Daneman & Carpenter, 1983; Daneman & Green, 1986; Hitch et al., 2001; Yuill et al., 1989). Measures of executive function (such as the Wisconsin Card Sort Task) and central executive ability (such as operation and listening span) have also been associated with children's and adults' mathematical competence (Ashcraft & Kirk, 2001; Bull et al., 1999; Bull & Scerif, 2001; Gathercole & Pickering, 2000a; Gathercole & Pickering, 2000b; Geary, Hoard & Hamson, 1999; Hitch & McAuley, 1991; Hitch, et al., 2001; Lehto, 1995; Passolunghi & Siegel, 2001; Towse & Houston-Price, 2001; Turner & Engle, 1989; Wilson & Swanson, 2001). More recently, in line with alternate theoretical models of working memory that suggest verbal and spatial modalities may be separable within the central executive (e.g. Shah & Miyake, 1996), studies have incorporated non-verbal executive measures and reported significant associations with children's mathematics (Jarvis & Gathercole, 2003; Maybery & Do, 2003).

Studies that have explored the involvement of the slave systems in mathematics often implicate a role for the central executive. As discussed previously, these studies typically report covariance between visuo-spatial sketchpad or phonological loop measures and central executive measures (e.g. Dixon et al., 1988; Gathercole & Pickering, 2000b; Phillips & Christie, 1977a; Wilson et al., 1987). It has been suggested that the phonological loop and visuo-spatial sketchpad may not predict unique variance in mathematics, but instead reflect the contributions of the central executive.

The central executive component of the working memory model performs a number of functions, including: dual-task performance; switching sets and strategies; inhibition; activation and retrieval from long-term memory and updating information in working memory (Baddeley, 1996; Miyake et al., 2000). Many of these functions, especially control, co-ordination, switching and inhibition, are important for mathematics.

The central executive has been implicated in the control of attention during calculation (Kaye, de Winstanley, Chen & Bonnefil, 1989; Ashcraft & Kirk, 2001). Ashcraft and Kirk (2001) reported that participants with high levels of maths anxiety had significantly lower complex working memory spans than those with normal levels of anxiety. Furthermore, a smaller working memory capacity was related to slower and more erroneous answers to mental addition. They proposed that the high levels of anxiety specifically disrupted the central executive component of working memory, consuming attentional resources required for mathematics, which consequently disrupted mental addition. As such, their findings implicate a role for the central executive in controlling attention during mental arithmetic.

The central executive is also thought to control the slave systems during mathematics. The recruitment of visuo-spatial or phonological resources for encoding, retaining and manipulating problem information depends upon the presentation format of the problem. In short, it has been proposed the phonological loop is recruited in adults and children when the problems are presented briefly or auditorily, and therefore need to be maintained (Adams & Hitch, 1997; Fürst & Hitch, 2000; Heathcote, 1994; Logie et al., 1994), whereas the visuo-spatial sketchpad is recruited in adults when problems are presented visually (Logie et al., 1994). Logie et al. (1994) reported that a concurrent central executive task disrupted arithmetic performance regardless of modality, such that mental addition was disrupted when the problems were presented both auditorily and visually. Therefore, it has been suggested that the involvement of the central executive across modalities implicates a role for it in co-ordinating the recruitment of the slave systems.

Dual-task studies implicate a role for the central executive in adults' calculation. Concurrent executive tasks (such as random letter or number generation) have been reported to disrupt single-digit addition (Ashcraft, Donley, Halas & Vakali, 1992; DeRammelaere, Stuyven & Vandierendonck, 2001; Hecht, 2002; Lemaire et al., 1996; Seitz & Schumann-Hengsteler, 2000; 2002) and single-digit multiplication (DeRammelaere et al. 2001;Hecht, 2002; Lemaire et al., 1996; Seitz & Schumann-Hengsteler, 2000; 2002). The solution to single digit sums relies on direct fact retrieval in adults (Seitz & Schumann-Hengsteler, 2000). Consequently the disruptive effect of a concurrent central executive load suggests that the central executive accesses and retrieves solutions / numerical facts from long-term memory. Similar disruptive effects have been reported for multi-digit addition and multiplication problems (DeRammelarere et al., 2001; Lemaire et al., 1996; Seitz & Schumann-

Hengsteler, 2000; 2002). Hecht (2002) reported that a concurrent executive task had a greater disruptive effect on problems that required a procedural strategy, such as counting and decomposition, compared to those that could be solved via direct retrieval. Similarly, there is a greater demand for central executive resources in carry problems compared to no-carry problems, as demonstrated by the increased error rates on such problems under a concurrent executive memory load (Fürst & Hitch, 2000; Seitz & Schumann-Hengsteler, 2000;2002). It has been suggested that complex mathematical problems that require carry operations or procedural strategies recruit central executive resources to co-ordinate the various stages of the solution process (DeStefano & LeFevre, 2004) and maintain interim results (Fürst & Hitch, 2000; Logie et al., 1994).

Performance on central executive measures has been related to general scholastic attainment in English (Gathercole & Pickering, 2000a), Maths and Science (Gathercole, Pickering, Knight & Stegman, 2004; Gathercole, Brown & Pickering, 2003; Jarvis & Gathercole, 2003) and central executive deficits have been observed in children with general learning disabilities (Henry, 2001; Siegel & Ryan, 1989), reading disabilities (de Jong, 1998; Siegel, 1994) and specific language impairments (Archibald & Gathercole, submitted; Ellis Weismer, Evans & Hesketh, 1999; Montgomery, 2000). As such, it follows that associations have been found between the central executive and mathematical abilities in children.

The central executive is important for mathematical development as its inhibitory function may be important for the acquisition of new solution strategies and for switching between learned solution strategies; two key skills that are important for mathematical proficiency (Lemaire & Siegler, 1995; Rourke, 1993). In a series of studies Bull and colleagues compared the executive functioning of children with normal or high mathematical abilities to children with low mathematical abilities. Children of low mathematical abilities performed significantly worse on the Wisconsin Card Sort Task (WCST) and the Stroop task (Bull et al., 1999; Bull & Scerif, 2001). This suggests that deficits in executive functioning associated with the central executive (such as problems inhibiting learned strategies and switching to new ones) relate directly to poor mathematical abilities. Similarly, it has been proposed that the central executive may be important for evaluating, selecting and implementing the appropriate solution strategy (Bull & Scerif, 2001; Logie et al., 1994). Studies of children with mathematical learning difficulties support this suggestion. These children typically show impairments on measures of central executive ability and / or executive function (Bull et al., 1999; Bull & Scerif, 2001; Gathercole & Pickering, 2000a). However, many of these children have a general learning disability, whereby mathematical difficulties are accompanied by a reading deficit. As reported by Bull et al. (1999), controlling for a reading deficit can eliminate associations between the central executive and mathematics. Despite this, children with specific arithmetic disabilities, where reading levels are normal, do exhibit central executive deficits (e.g. McLean & Hitch, 1999; Wilson and Swanson, 2001). The types of executive deficits they typically exhibit include problems with inhibitory processes (Passolunghi & Siegel, 2001) and problems on novel, complex tasks that involve shifting psychological sets and planning actions (Rourke, 1993). These deficits can lead to problems in selecting the correct solution strategy and retrieving answers from long-term memory.

To summarise, the central executive is associated with mathematics in both normal and mathematically disabled children and adults. It is thought to be involved in controlling processes, such as attention and strategies, co-ordinating both procedures and the slave systems and inhibiting the selection of inappropriate solution strategies.

As with the slave systems, it has been suggested that the associations between the central executive and mathematics may reflect the contribution of an alternate factor. As discussed previously (section 1.1.2.3) the central executive is closely related to intelligence (e.g. Kyllonen & Christal, 1990; Jurden, 1995; Miyake et al, 2001). Therefore, the associations between the central executive and mathematics may reflect the contributions of general intelligence to mathematical competence. However, studies that have controlled for IQ report that the associations between executive function and mathematics remain significant (e.g. Bull et al., 1999). A second line of argument against the involvement of the central executive mirrors one against the involvement of the phonological loop. That is, that there is a stronger association between digit-based measures of central executive ability, namely counting or operation span, than there is between non-digit based measures and mathematics because the assessments of both involve either number processing or direct access to numerical information. For example, Bull and Scerif (2001) reported that the numerical stroop task was related to mathematical ability, while the word stroop task was not. Furthermore, people with specific arithmetic learning difficulties are impaired on counting span tasks but not on sentence span tasks (Siegel & Ryan, 1989) and children with arithmetical learning difficulties are impaired when retaining information if the concurrent working memory task is numerical, but not if it is word-based (Hitch & McAuley, 1991). As such, the associations between the central executive and mathematics may reflect a proficiency in processing numerical information. Alternatively, as suggested in section 1.4.2.1, participants who are mathematically able may have stronger representations for digits, meaning they are identified more quickly, leading to an enhanced memory span (Dark & Benbow, 1990; 1991).

In conclusion, considerable evidence implicates a role for the central executive in mathematics, but this literature should be interpreted with caution as the associations may reflect general intelligence or a general fluency with numerical information.

Section Summary

1. The phonological loop has been associated with children's and adults' mathematics.
2. It supports the solution to mathematical problems as it retains problem information and interim results during calculation. Furthermore, it maintains accuracy as it keeps track during counting.
3. The phonological loop, which is important for the retrieval of number facts from long-term memory, supports children's mathematical development. It is important for the formation of a complete network of arithmetic facts in long-term memory.
4. Recent evidence suggests that the phonological loop might be important in supporting children's mathematics following the onset of subvocal rehearsal.
5. Evidence suggesting a role for the phonological loop in mathematics needs to be interpreted with caution as the shared variance between scores on phonological loop measures and mathematics performance may be accounted for by other variables (e.g. reading ability or processing speed).
6. We use visuo-spatial codes for numbers and neuropsychological evidence suggests that visuo-spatial processing is involved in mathematics.

7. The visuo-spatial sketchpad has been associated with mathematics performance in children and adults and people with MD often have visuo-spatial deficits.
8. The visuo-spatial sketchpad has been implicated in encoding, retaining and manipulating problem information and interim results during calculation. It also supports counting and approximation.
9. The visuo-spatial sketchpad is important for mathematical development as it supports early, preverbal mathematics. It has been shown to be more important for younger children's than older children's mathematics.
10. Current literature suggests that the visuo-spatial sketchpad is closely related to the central executive. As such, associations between visuo-spatial working memory scores and mathematics performance need to be interpreted with caution.
11. The central executive has been related to a variety of cognitive abilities, including mathematics.
12. It is thought that the central executive performs a number of functions in mathematics, including; controlling attention, co-ordination of the slave systems, co-ordination of the stages of the solution process during calculation, maintaining interim results and supporting direct retrieval from long-term memory.
13. The central executive is important for mathematical development as it supports the acquisition of new solution strategies and helps switch between different solution strategies.
14. Associations between the central executive and mathematics may reflect the contribution of other factors such as intelligence.

Section 1.5

Aims

Mathematical skills are essential for higher education and employment (Department for Education and Employment (DFEE), 1998), yet up to 6.5% of the school-age population (Gross-Tsur, Manor & Shalev, 1996) have developmental dyscalculia, or mathematical difficulties. A government-backed survey in the UK reported that 25% of adults had poor numeracy skills that made it difficult to complete everyday tasks successfully (Bynner & Parsons, 1997). Over recent years researchers have become increasingly interested in delineating the etiological factors in mathematical difficulties and mathematical attainment in children and adults.

Considerable evidence suggests that working memory may support children's mathematics (e.g. Gathercole & Pickering, 2000a; McKenzie et al., 2003). From this basis, the overall aim of this thesis was to systematically assess the contributions of the different components of working memory to children's National Curriculum mathematical attainment.

Different roles have been ascribed to the different components of working memory in literacy and language development. Therefore, one aim of this thesis was to assess the contribution of the different components of working memory to children's mathematics. For pragmatic reasons, the traditional tripartite model of working memory was adopted given that there were no current standardized measures available to assess the episodic buffer at the start of this project (see section 1.1.2.4).

Hecht (2002) reported that different arithmetic solution strategies recruited different working memory resources, hence, it is suggested that different mathematical skills may place different demands on working memory. Children's working memory abilities have been associated with mental arithmetic (e.g. Adams & Hitch, 1998) or a general mathematical ability, as measured by a standardized assessment (e.g. Gathercole & Pickering, 2000a; 2000b). Relatively few studies have systematically examined the contributions of working memory to a range of mathematical skills. This is surprising given the diversity of children's mathematical abilities and range of solution strategies available to them (e.g. Siegler, 1999. See section 1.3.2). Therefore, a second aim of this thesis was to assess the contributions of the different components of working memory to a range of mathematical skills.

Previous studies that report an association between working memory and children's mathematics typically incorporate digit- or number-based measures of working memory, such as digit or operation span. It has been suggested that number-based working memory span measures are more strongly associated with mathematics than non-numerical span measures (e.g. Passolunghi & Siegel, 2001. See sections 1.4.2.1 and 1.4.2.3). Therefore, one possibility is that working memory and mathematics are linked because the assessments of both involve either number processing or direct access to numerical information. One aim of this thesis was to explore the associations between working memory and children's mathematics using measures of working memory that did not contain numerical stimuli to explore the association between mathematics and working memory ability per se.

It has been suggested that the association between working memory and mathematics may reflect the contribution of working memory to a higher order construct, such as general intelligence (e.g. Kyllonen & Christal, 1990. See section 1.4.2.3). Therefore, a further consideration of this thesis was to explore the contribution of working memory to children's mathematics after controlling for a general ability (i.e. non-verbal intelligence (NVIQ).

Section Summary

1. It has been suggested that working memory may be important for children's mathematics.
2. The overarching aim of this thesis was to assess the contribution of the different components of working memory to a range of children's mathematical skills using non-digit based working memory assessments, taking into account a measure of children's general abilities.

Chapter Two

Developing Mathematics Assessments

Aim

Mathematics is comprised of different arithmetical components, such as number knowledge and memory for arithmetic facts (e.g. Dowker, 1998), and children are taught different mathematical skills at school (e.g. National Curriculum, n.d.). It has been suggested that different arithmetical components might have diverse cognitive correlates, such as working memory (e.g. Dowker, 1998). The aim of the present study was to develop mathematics assessments for children to measure distinct mathematical skills, using the National Curriculum for England as a guide. These assessments were developed for use in subsequent studies, where the contribution of working memory to a range of mathematical skills is investigated.

Introduction

Increasing evidence suggests that mathematics is comprised of a number of arithmetic components and that these components follow different developmental trajectories that invoke the use of diverse solution strategies and cognitive resources.

Distinctions have been made between procedural, factual and conceptual arithmetic abilities. In normal populations, distinctions have been made between pre-schoolers understanding of basic number facts and counting principles (conceptual competence) and their ability to count accurately (procedural competence) (Greeno, Riley & Gelman, 1984). Similar distinctions have been reported for primary and secondary school aged children, where some children have a better conceptual than procedural knowledge (Baroody, 1987; Dowker, 1995; Russell & Ginsburg, 1984), while others are able to carry out procedures without understanding the concepts

(Bryant, 1985). Double dissociations reported in studies of children with arithmetical learning difficulties mirror this distinction. For example, Temple (1991) describes two children with developmental dyscalculia; one who has intact factual knowledge and conceptual understanding but poor procedural skills, and one who shows the reverse pattern. These profiles can be related to two of Geary's (1994) subtypes of developmental dyscalculia; a "memory" subtype who, with a low frequency of arithmetic fact retrieval, demonstrate poor factual knowledge and a "procedural" subtype who, relying on immature procedures, demonstrate poor procedural knowledge.

Studies of normal populations (e.g. Hitch, 1978) and acquired dyscalculics suggest that procedural, factual and conceptual abilities are also separable in adults. Dissociations between factual and procedural knowledge have been reported in both directions for adults with acquired dyscalculia. While some adults have impaired procedural / calculation skills and intact factual knowledge (e.g. McCloskey et al., 1985), others have intact procedural knowledge and impaired factual knowledge (e.g. Warrington, 1982).

Further evidence for the componential nature of arithmetic comes from studies where individual differences in performance have been observed for different components of arithmetic. Significant individual differences in written and spoken counting, and in transcoding between digits and written and spoken number words have been reported for adults (Deloche, Seron, Larroque, Magnien, Metz-Lotz et al., 1994). Furthermore, double dissociations have been demonstrated between oral and written presentation modes in patients with acquired dyscalculia (e.g. Campbell, 1994) and children with arithmetical difficulties have been reported to have specific difficulties either in solving word-problems (e.g. Russell & Ginsburg, 1984) or in reading and writing Arabic numbers (e.g. von Aster, 2000).

Adults and children show significant individual differences in their strategy choice for addition (Geary & Wiley, 19991; Siegler & Robinson, 1982), subtraction (Siegler, 1987; 1989) and multiplication (LeFevre et al., 1996; Lemaire & Siegler, 1995; Siegler, 1988). The nature and goals of a problem and the difficulty and novelty of a problem can influence this strategy choice. For example, children are more likely to use a "back-up" strategy for a difficult or novel problem (Siegler & Jenkins, 1989). Children use different strategies that are specific to different mathematical domains; for addition and subtraction they will use count-all, count-on, count-back, retrieval and decomposition (e.g. Carpenter & Moser, 1984), for multiplication they will use direct counting, repeated addition and multiplicative calculation (e.g. Mulligan & Mitchelmore, 1997), for fractions they will use a distribution strategy, a mark-all strategy, preserved-pieces strategy (e.g. Lamon, 1996), a parts strategy, component strategy, reference point strategy and a transform strategy (e.g. Smith, 1995) and for algebraic problems they will use the substitution strategy (Sleeman, 1984), the reduce strategy or the isolation strategy (Mayer, 1982). This evidence suggests that different mathematical problems invoke the use of different solution strategies, further demonstrating the varied nature of mathematics.

In summary, converging theoretical and empirical evidence suggests that mathematics is not unitary. Dowker (1998) proposed that mathematics was comprised of a number of arithmetical components, including; basic number knowledge (the ability to recognise numbers in different forms, such as Arabic digits and number words and place them in order), memory for arithmetical facts (category-based factual knowledge), conceptual understanding (understanding properties of, and relationships between, arithmetical operations and being able to use them to derive unknown facts, including exact answers and approximate answers) and procedural understanding (remembering learned procedures and carrying out a sequence of procedures, including keeping track and the correct spatial alignment of numbers for written calculation). Indeed, when investigating individual differences in arithmetic performance in children aged 5-10 years, Dowker (1998) found children with marked discrepancies between different components of arithmetic, similar to those reported for adults in a previous study (Dowker, 1992).

Different components of mathematics may have different cognitive correlates. For example, Hecht (2002) reported that different solution strategies recruited different working memory resources. Furthermore, studies of arithmetical learning disabilities imply that different cognitive deficits underlie different types of arithmetical difficulties. Geary (1993) defined one subtype of mathematical difficulty as characterised by visuo-spatial deficits, while another was characterised by deficits in fact-retrieval, which is based on a verbal code (Dehaene & Cohen, 1995). It is therefore important to acknowledge the componential nature of mathematics in cognitive research.

The assessments were developed from the National Curriculum for England. There were two reasons for this; firstly, children's National Curriculum test performance has been previously associated with working memory abilities (e.g. Gathercole & Pickering, 2000b) and secondly because mathematics is taught and assessed in a componential manner in England. The National Curriculum, which was introduced in 1988, stipulates what must be studied in England and Wales by state school children up to the age of 16 across core subjects such as English, Mathematics and Science. Key Stages define National Curriculum learning for specific age groups.

Key Stage 1 covers 5- to 7-year-olds, Key Stage 2 covers 8- to 11- year-olds, Key Stage 3 covers 12- to 14-year-olds and Key Stages 4 and 5 cover 15- to16- year-olds. The programme of work outlined in the mathematics National Curriculum for Key Stages 1 to 3 defines four mathematical abilities; Number and Algebra, Shape, Space and Measures, Handling Data and Mental Arithmetic. Children are taught these programmes of work, before taking Standardized Attainment Tests (SATs) at 7-, 11- and 14-years. Children's performance on these tests is compared to standardized attainment targets across the different mathematics programmes, which are measured in Levels. The current assessments were developed from the Key Stage 2 mathematics National Curriculum guidelines and past SATs examination papers. Assessments were developed for two age groups within Key Stage 2 (Year 3, aged 7-/8-years and Year 5, aged 9-/10-years). The reason for this was to provide a measure of children's mathematical competencies across Key Stage 2 to explore the associations with working memory ability, rather than focussing on mathematics ability at the end of the Key Stage as previous research has done (e.g. Jarvis & Gathercole, 2003).

Method

Participants

The participants were 72 children (38 boys and 34 girls), who attended two primary schools in England. 34 children (17 Year 3 and 17 Year 5) attended a school in the North-East of England, 38 (16 Year 3 and 22 Year 5) children attended a school in the South-East of England. There were 33 Year 3 children (15 boys and 18 girls), mean age 8 years and 3 months (*SD* = 3.5 months, range 7 years 7 months to 8 years 6 months) and 39 Year 5 children (23 boys and 16 girls), mean age 10 years and 1 month (*SD* = 3.6 months, range 9 years 7 months to 10 years 6 months).

The percentage of children achieving Level 4 attainment and above in English, Mathematics and Science was higher than the national average in one of the schools (97%, 85% and 97% respectively) and lower than the national average in the second school (57%, 47% and 72% respectively).

Design and Procedure

All children participated in a one-hour testing session. The children were administered age appropriate mathematics assessments under standardized test conditions within a classroom setting. The assessments were comprised of three 10-minute written sections followed by one 10-minute mental arithmetic test, which was presented orally with written responses.

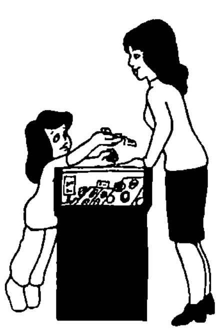
Materials

The mathematics assessments were developed from the framework of the National Curriculum for England and the Qualifications and Curriculum Authority (QCA) assessments. The National Curriculum specifies what is to be taught in mathematics across different Key Stages, while the QCA develop this curriculum and its associated assessments. The QCA are responsible for producing Standardized Attainment Tests (SATs) that every child takes at the end of Key Stages 1, 2 and 3.

Using past Key Stage 2 mathematics tests (administered by the QCA) and the programme of study outlined by the National Curriculum, age appropriate assessments were developed for 7-/ 8-year-olds (Year 3) and 9-/ 10-year-olds (Year 5). The tests were designed to assess the four mathematical skills outlined by the National Curriculum and tested by the QCA; Number and Algebra, Shape, Space and Measures, Handling Data and Mental Arithmetic (National Curriculum, n.d.). Three of the tests were presented visually as written assessments. The fourth, Mental Arithmetic, was presented verbally. Examples of the Year 3 and Year 5 mathematics tests and criteria for scoring items are provided in Appendices I and II respectively. Standardized instructions, which are the same for both tests, are provided in Appendix III.

Number and Algebra

The Number and Algebra assessments are a test of number knowledge and counting. They are presented as a written test in word and digit format. Primarily the questions require children to demonstrate understanding of the four number operations (add, subtract, multiply and divide), recognise number patterns and sequences, deal with fractions and decimals and use the related vocabulary to solve problems. This section contains 15 questions. An example question is shown is Figure 2.1.



Sarah goes to the shop. She has £2.00. She spends £1.20 on a book. How much money has she got left from the £2.00?

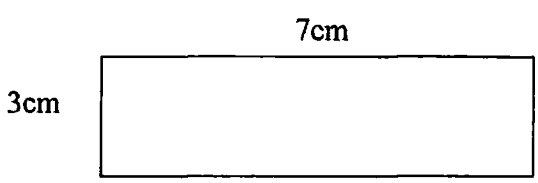
Figure 2.1

Example question taken from the Number and Algebra assessments

Shape, Space and Measures

The Shape, Space and Measures assessments are a test of geometrical abilities. Both are presented as written tests in word and digit format. Primarily the questions ask the children to demonstrate their understanding of standard units of measurement and properties of shape, position and movement. This section contains 15 questions. An example question is shown in Figure 2.2.

Find the area of the rectangle.



= cm^2

Figure 2.2

Example question taken from the Shape, Space and Measures assessments

Handling Data

The Handling Data assessment is a test of data processing, representation and interpretation. Primarily the questions require children to interpret and draw tables and graphs and understand measures of spread (e.g. range and mode). This section contains 15 questions. An example question is shown in Figure 2.3.

Read the table below:

Boat Hire	
Rowing Boat	Motor Boat
£2 for 1 hour	£1.50 for 10 minutes

Which boat is more expensive to hire?

Figure 2.3

Example question taken from the Handling Data assessments

Mental Arithmetic

The Mental Arithmetic assessment tests the children's ability to solve mathematical problems without using written working out as an aid. The questions are spoken aloud and a set time period is given (5, 10 or 15 seconds depending upon the level of difficulty of the questions) for a written response. There are 10 questions within this section. An example question is presented in Figure 2.4.

What is 88 take away 42?

Figure 2.4

Example question taken from Mental Arithmetic assessments

Results

Descriptive Statistics

Descriptive statistics for children's mathematics test performance are presented in Tables 2.1 and 2.2.

No significant differences in performance were found between boys and girls in Year 3 on the mathematics measures: Number and Algebra $t(31)=1.39, p>.05$; Shape, Space and Measures $t(31)=-.81, p>.05$; Handling Data $t(21.86)=-1.60, p>.05$; Mental Arithmetic $t(31)=1.43, p>.05$; and Total Mathematics Score $t(31)=.21, p>.05$. The only significant difference in Year 3 children's scores was between Number and Algebra and Mental Arithmetic scores ($F(3,99)=3.71, p<.05$).

Table 2.1

Descriptive Statistics of Year 3 Children's Mathematics Performance. (n=33).

Mathematics Measure	Girls (*n*=15)		Boys (*n*=18)		Total	
	M	*SD*	*M*	*SD*	*M*	*SD*
Number and Algebra	37.54	25.16	51.56	33.38	43.73	29.45
Shape, Space and Measures	54.38	20.64	48.00	25.47	51.57	22.76
Handling Data	57.19	14.79	45.78	24.41	52.16	20.13
Mental Arithmetic	52.63	30.15	66.00	22.61	58.53	27.54
Total Mathematics Score	50.23	18.52	51.63	21.02	50.86	19.37

Note. Scores shown are proportions correct.

There were no significant differences between boys' and girls' performance in Year 5 on the following mathematics measures: Shape, Space and Measures $t(37)=-.49, p>.05$; Handling Data $t(37)=.59, p>.05$; and Total Mathematics Score $t(37)=.1.70, p>.05$. Year 5 boys performed significantly better than Year 5 girls on the Number and Algebra ($t(37)=2.07, p<.05$) and Mental Arithmetic ($t(37)=2.26, p<.03$) measures.

There were no significant differences in Year 5 children's scores across the different mathematics measures ($F(3,114)=.438, p>.05$.

Table 2.2

Descriptive Statistics of Year 5 Children's Mathematics Performance. (n=39).

Mathematics Measure	Girls (*n*=23)		Boys (*n*=16)		Total	
	M	*SD*	*M*	*SD*	*M*	*SD*
Number and Algebra	43.75	21.50	57.39	19.36	51.79	21.10
Shape, Space and Measures	50.83	15.37	53.33	15.95	52.31	15.57
Handling Data	51.67	16.42	55.36	21.03	53.85	19.13
Mental Arithmetic	40.63	22.64	56.96	21.83	50.26	23.34
Total Mathematics Score	26.00	8.24	30.61	8.44	52.21	15.57

Note. Scores shown are proportions correct.

Overall, there were no significant differences in performance between Year 3 and Year 5 children across the mathematics measures: Number and Algebra $t(58.90)=-1.33, p>.05$; Shape, Space and Measures $t(57.14)=-.16, p>.05$; Handling Data $t(71)=-.37, p>.05$; Mental Arithmetic $t(71)=1.39, p>.05$; and Total Mathematics Score $t(71)=-.33, p>.05$.

Reliability Analyses

The internal reliability of the mathematics measures was assessed using an item discrimination method known as the Kuder-Richardson (*KR*-20) method (Kuder and Richardson, 1937). This method, used on nominal data where the item response is "yes/no", measures the inter-item consistency of a scale by averaging all possible split-half correlations for a set of items. It is equivalent to the Cronbach's Alpha coefficient, which is more commonly used for parametric data. The formula for this calculation is presented in Appendix IV.

Reliability coefficients for the mathematics measures for both age groups are presented in Table 2.3.

Table 2.3.

Reliability Coefficients of Mathematics Measures.

Mathematics Measure	Year 3	Year 5
Number and Algebra	.88	.76
Shape, Space and Measures	.82	.69
Handling Data	.80	.81
Mental Arithmetic	.82	.68

All *KR*-20 coefficients exceed .64 (Kuder & Richardson, 1937), demonstrating that the mathematics assessments for both age groups have acceptable internal reliability.

Validity

The assessments were designed to provide a measure of mathematics performance across different skills in children. Content validity was assured as the assessments were developed from the programme of work outlined by the National

Curriculum for England. Furthermore, all areas were assessed as recommended by the QCA, who provide standardized attainment tests for children in England. Verbatim reports, obtained from Head Teachers and teaching staff involved in the administration of the tests, assured face validity.

Power Analyses

The statistical power of a test is the probability that a statistically significant result will be found. It is the probability of not rejecting the null hypothesis given that it is false (the probability of making a Type 2 error in an experiment) (Muncer, Craigie & Holmes, 2003). The power of an experiment can be affected by sample size, effect size and significance level. It can be derived using computer packages such as G Power (Faul & Erdfelder, 1992). The acceptance criterion for statistical power is .8 or above (Cohen, 1992). If the criterion is not equalled or exceeded, an A Priori or a Compromise Power Analysis can be used to inform the researcher how many participants will be needed for the study to be statistically powerful to test for a desired effect size.

Power analyses were conducted on the mathematics assessments for each age group and for the sample as a whole. For all analyses, the difference between male and female scores was used to provide a common metric so that the effect size could be derived (*d*). The effect size was derived using Cohen's (1977) effect size index (see Appendix V). Post-Hoc power analyses were conducted using G Power (Faul & Erdfelder, 1992), into which the effect sizes and sample sizes were input. Tables 2.4, 2.5 and 2.6 show the results of the power analyses for the mathematics assessments.

Table 2.4

Post-Hoc Power Analyses for Year 3 Mathematics Assessments.

Mathematics Measure	n_1	n_2	Effect Size (*d*)	Actual Power
Number and Algebra	19	15	.12	.10
Shape, Space and Measures	19	15	.28	.19
Handling Data	19	15	.37	.28
Mental Arithmetic	19	15	.51	.42
Total Mathematics Score	19	15	.08	.08

None of the mathematics Year 3 mathematics assessments are adequately powered according to Cohen's .8 criterion (Cohen, 1988). This is probably due to the small sample and effect sizes.

Table 2.5

Post-Hoc Power Analyses for Year 5 Mathematics Assessments.

Mathematics Measure	n_1	n_2	Effect Size (*d*)	Actual Power
Number and Algebra	16	23	.55	.51
Shape, Space and Measures	16	23	.17	.12
Handling Data	16	23	.19	.14
Mental Arithmetic	16	23	.70	.69
Total Mathematics Score	16	23	.54	.49

None of the mathematics Year 5 mathematics assessments are adequately powered according to Cohen's .8 criterion (Cohen, 1988). Again, this is probably due to the small sample and effect sizes.

Table 2.6

Post-Hoc Power Analyses for Mathematics Assessments (both year groups combined).

Mathematics Measure	n_1	n_2	Effect Size (*d*)	Actual Power
Number and Algebra	35	38	.58	.79
Shape, Space and Measures	35	38	.08	.10
Handling Data	35	38	.17	.18
Mental Arithmetic	35	38	.53	.72
Total Mathematics Score	35	38	.30	.35

According to Cohen's criterion (.8), none of the mathematics assessments are adequately powered when the two age groups are combined. However, the Number and Algebra and Mental Arithmetic measures are approaching the acceptance level.

The Post-Hoc analyses suggest that the statistical power of the mathematics assessments is poor. According to Cohen's (1977) popular effect size conventions, the effect sizes were small for most of the assessments (0.2 and below), reaching medium at best (0.5 below). In addition, the sample sizes were small. Therefore, the assessments appear statistically underpowered. This is not devastating in this instance, as the mathematics assessments have not yet been used in a study. At this stage power analyses can be useful as A Priori analyses can be conducted to inform the researcher as to how many participants would be needed to give the study statistical power to detect a medium or large effect size. A Priori analyses do not take into account pragmatic constraints of research, such as access to children and testing times, and often suggest that extremely large numbers of participants are recruited. It is therefore recommended that Erdfelder's (1984) Compromise Power Analyses should be conducted. For this, the maximum sample size possible, the desired effect size and the error probabilities are input. G Power then calculates the statistical power of a test with these parameters. For example, if a maximum of 70 children could be tested and a medium effect size was needed, the input would be n_1 =35 and n_2 = 35, effect size .5. The error probability is typically the beta (detecting a false negative result) / alpha (detecting a false positive) because both types of error are considered equally serious. With this information, G Power calculates the power of the study and would therefore inform the researcher if she/he has enough participants.

Compromise Power Analyses were conducted for the mathematics assessments. In subsequent studies the mathematics assessments will be administered to a minimum of 70 children per age group (n_1 = 35, n_2 = 35 for Year 3 and Year 5) and 140 children overall (n_1=70, n_2=70). The results are presented in Table 2.7.

Table 2.7

Compromise Power Analyses for Mathematics Assessments.

Sample	Minimum n_1	Minimum n_2	Power
Year 3 children	35	35	.85
Year 5 children	35	35	.85
All children	70	70	.93

All of the mathematics assessments power values exceed Cohen's (1988) .8 criterion, meaning they will be adequately powerful to test for significance, with a medium effect size, in subsequent studies.

Discussion

Overall, the results show that reliable, valid and statistically powerful mathematics assessments have been developed. There were no significant differences in performance between the two age groups, which suggests that the assessments are age-appropriate and pitched at a suitable level for use with 7-/8-year-olds and 9-/10-year-olds. Furthermore, there were no significant differences in performance between boys and girls in Year 3, but there were significant sex differences on two of the mathematics measures in Year 5. This tentatively suggests that the assessments may be sensitive to developmental change, as they detect gender differences that emerge throughout the school years (e.g. Geary, 1996). Geary's (1996) review suggests that there are no sex differences in mathematical abilities in infancy and during the preschool years (e.g. Starkey, Spelke & Gelman, 1990), but that they emerge (in favour of boys) during the school years (e.g. Lummis & Stevenson, 1990) and become pronounced by adolescence (Hyde, Fennema & Lamon, 1990; Benbow, 1988). Although the present data appear consistent with this notion, they must be interpreted with caution due to the small sample sizes.

The reliability, validity and statistical power of the Year 3 and Year 5 mathematics tests were assessed. This was important, as no such reliability, validity or power statistics have been recently published for the existing National Curriculum Key Stage tests (SATs) from which the assessments were developed. In 1999, a report was produced that validated the Key Stage 2 test development procedures and demonstrated that the test data was a reliable measure of pupil attainment (Rose, 1999). This report, produced by an Independent Scrutiny Panel that was appointed by the Department for Education and Employment and chaired by Jim Rose (the then HMI Director of Inspection at OFSTED), also recommended that the test development and assessment arrangements be subject to periodic scrutiny. Since this report, the QCA have ensured that the setting of standards relies on empirical evidence and statistical methods (so that standards remain consistent), but they have not published reliability and validity statistics. Robust reliability (*KR*-20>.64) coefficients were produced by both assessments in this study. Furthermore, the power analyses revealed that both assessments would be statistically powerful to test for medium effect sizes if they are administered to a minimum sample size of 70 children per age group. These results demonstrate that the mathematics assessments developed in this study can be used confidently in subsequent studies as powerful scientific instruments. Arguably, they can be used with more confidence than Key Stage 2 SATs data, for which the same statistics are not available.

Each mathematics measure (Number and Algebra, Shape, Space and Measures, Handling Data and Mental Arithmetic) within each assessment produced a good reliability coefficient (*KR*-20>.64). This is important as existing mathematics assessments, such as SATs, typically provide a global score as a measure of a child's ability. Relatively few mathematics tests provide a breakdown of children's performance across a range of mathematics skills. Uniquely, the assessments developed in this study provide reliable measures of the mathematics skills taught at Key Stage 2 of the National Curriculum.

The mathematics assessments developed in this study may be valuable tools for use with children in both cognitive research and in the classroom. They provide reliable measures of different mathematical abilities, which not only reflect the componential nature of mathematics (e.g. Dowker, 1998), but also provide a sensitive measure of ability. They will be useful for research, where it is considered important to measure different mathematical skills as they may have different cognitive correlates (e.g. Dowker, 1998), and for education, where it could be argued that more fine-grained measures of mathematics achievement are needed. Currently, children in England are tested at the end of each Key Stage (SATs), and provided with a Level of attainment. This Level refers to a target, which describes what children at a particular age should be able to do and know. Level 4 is the target for children at the end of Key Stage 2. Under the existing system, most children are awarded this Level, meaning there is little differentiation between children's scores. Furthermore, the Level awarded is for "mathematics"; it is not broken down into Levels for the different programmes of work that are taught under the National Curriculum. It is therefore suggested that tests analogous to those developed in the present study may be more informative for educators. They could provide a more sensitive measure of attainment and be used to depict individual strengths and weaknesses (e.g. poor performance on the mental arithmetic component).

In summary, two reliable, valid and statistically powerful mathematics assessments, which measure the four mathematics skills defined by the National Curriculum for England, have been developed for use with children. The assessments will be important for use in subsequent studies, where the contribution of working memory to different mathematical skills will be explored. It is suggested that the mathematics tests may be of value to the wider academic community, in other areas of cognitive developmental research, and to educators where they may prove more informative than existing assessments.

Chapter Summary

1. Mathematics is not unitary. It is comprised of a number of arithmetical components, such as procedural, factual and conceptual number abilities. Different strategies are used for the solution of different mathematical problems.
2. It is important to acknowledge the componential nature of mathematics in research. Different mathematical abilities may draw upon different cognitive resources.
3. The aim of the present study was to develop tests for use with children that assessed different mathematical abilities. These assessments were based upon the programme of work outlined by the National Curriculum for England and previous SATs examination papers.
4. Two tests were developed, one for children aged 7-/8-years and one for children aged 9-/10-years. The assessments were piloted on a group of children. Analyses revealed that both assessments were reliable measures that would have statistical power to test for significant results in subsequent studies if the sample size exceeds 70 children.
5. Importantly, the tests provide reliable measures of different mathematical abilities. They will be central to the research conducted in this thesis and may be valuable tools for further cognitive developmental research and for educators.

Chapter Three

Working Memory and Children's Mathematical Skills

Aim

As noted in Chapter 1, few studies have examined the contributions of working memory to a range of children's mathematical skills. The first aim of the present study was to explore the contribution of the three components of working memory to a range of children's mathematical skills, using the mathematics assessments developed in Chapter 2 and measures from the Working Memory Test Battery for Children (WMTB-C) (Pickering & Gathercole, 2001). A second aim was to explore the effect of controlling for individual differences in non-verbal IQ on the relationship between working memory abilities and mathematics performance.

Introduction

Children's working memory abilities have been associated with performance on National Curriculum assessments in English, Science and Mathematics in the UK. They have also been related specifically to children's mathematical abilities as measured by National Curriculum tests and other standardized tests.

Gathercole and Pickering (2000b) reported associations between working memory abilities and performance on standardized measures of scholastic attainment when validating the WMTB-C (Pickering & Gathercole, 2001). They administered the test battery, which was designed to tap the capacity of the 3 components of Baddeley and Hitch's (1974) working memory model, to 6- and 7-year-olds and obtained measures of achievement on standardized attainment tests in vocabulary, literacy and mathematics. Phonological loop scores were significantly associated with performance on all three measures of scholastic attainment at 7-years-of age, but only with performance on the vocabulary measure at 8-years-of-age. After controlling for age and individual differences in central executive ability, phonological loop scores were only uniquely associated with vocabulary scores at 7- and 8-years. Conversely, central executive scores, which were also significantly related to all three measures of attainment at 7-years, were significantly related to literacy and arithmetic scores at 8-years. These associations persisted after age and individual differences in phonological loop ability were controlled for. The associations between visuo-spatial sketchpad scores and attainment were not explored, as the higher-level factor structure of the visuo-spatial sketchpad measures was unclear. Overall, Gathercole and Pickering's (2000b) results suggested that working memory was related to performance on standardized measures of attainment. In particular, central executive scores uniquely predicted arithmetic scores at 8-years.

In a subsequent study, Gathercole and Pickering (2000a) reported an association between working memory abilities and National Curriculum attainment at 7-years-of-age. Children were assigned to normal and low achievement groups based on their Key Stage 1 National Curriculum test scores in English and Mathematics. Their working memory abilities were assessed using an early version of the WMTB-C (Pickering & Gathercole, 2001), which consisted of thirteen tests designed to tap the three components of working memory (Baddeley & Hitch, 1974). Overall, children who were judged to be failing to achieve normal levels of curriculum attainment showed marked impairments on tests of central executive and visuo-spatial working memory skills. In particular, children with low achievement specifically in Mathematics or in both curriculum areas exhibited stronger working memory deficits than children with low achievement in English. Additionally, scores on visuo-spatial sketchpad and central executive measures were used to identify children with at least one area of low achievement. These results suggest that working memory (in particular the visuo-spatial sketchpad and central executive components) supports curricular progress at 7-years, particularly in mathematics (Gathercole & Pickering, 2000a). More recently, Gathercole, Pickering, Knight et al., (2004) reported that 7-/8-year-olds performance on National Curriculum assessments in English and Mathematics was significantly associated with phonological loop and central executive working memory scores. Using a similar methodology to the previous study children were split into groups based on their National Curriculum achievements. Gathercole, Pickering, Knight et al. (2004) found that children with high abilities in English and Mathematics scored significantly better on working memory measures than children of average or low abilities. Furthermore, working memory scores effectively discriminated children of low abilities, who were failing to achieve expected levels of attainment, from the rest of the group.

Working memory abilities have also been associated with achievements on National Curriculum tests at 11-years (Key Stage 2) and 14-years (Key Stage 3). Jarvis and Gathercole (2003) explored the relationships between verbal and non-verbal working memory abilities and National Curriculum attainment in these age groups and reported significant associations between working memory scores and achievements in Mathematics, English and Science. They found particularly strong associations between central executive scores and Key Stage 2 achievements in all curriculum areas, with a strong association between visuo-spatial sketchpad scores and Science performance at this age. They reported similarly strong associations between Key Stage 3 achievements in all curriculum areas and central executive scores, with a strong association between visuo-spatial sketchpad scores and Mathematics performance. In subsequent analyses, Jarvis and Gathercole (2003) explored the relationship between verbal (phonological loop and verbal central executive) and nonverbal (visuo-spatial sketchpad and nonverbal central executive) working memory skills and curriculum attainment. Their results suggested that verbal working memory ability was related to performance in English and Mathematics and that nonverbal working memory ability was related to performance in Mathematics and Science. Despite these distinctions, the results of their structural equation modelling suggested that separate verbal and nonverbal working memory constructs predicted a single National Curriculum attainment score, which was comprised of all three areas. They therefore concluded that working memory ability predicted National Curriculum attainment at 11- and 14-years.

Gathercole, Pickering, Knight et al.'s (2004) recent study supports the idea that working memory ability predicts achievement across the three National Curriculum areas at 14-years. They found that attainment levels in Mathematics and Science were highly significantly related to scores on phonological loop and central executive tasks, while attainment levels in English were more moderately, but still significantly, related to working memory scores. As described earlier, children were split into three groups based on their attainment levels. There were significant differences in performance on the working memory tasks between children of low and average abilities, and between children of average and high abilities in Mathematics and Science, but there were no significant differences in performance of children with different attainment levels in English. These findings suggest that working memory abilities are related to different areas of curriculum attainment.

In summary, converging evidence supports an association between working memory skill and performance on National Curriculum tests. Importantly, many of these studies report associations between working memory abilities and National

Curriculum mathematics achievement. Central executive scores have been related to National Curriculum mathematics attainment at 7- (Gathercole & Pickering, 2000a; 2000b; Gathercole, Pickering, Knight et al., 2004), 11- (Jarvis & Gathercole, 2003) and 14-years (Jarvis & Gathercole, 2003; Gathercole, Pickering, Knight et al., 2004), visuo-spatial sketchpad scores have been related to National Curriculum mathematics attainment at 7- (Gathercole & Pickering, 2000a), 11- and 14-years (Jarvis & Gathercole, 2003) and phonological loop scores have been related to mathematics performance at 7- and 11-years (Gathercole, Pickering, Knight et al., 2004).

Other studies that have explored the relationship between children's working memory abilities and their performance on standardized mathematics tests report similar associations. For example, Maybery and Do (2003) reported significant associations between simple and complex verbal and visuo-spatial span performance and mathematics ability in a sample of 10-year-old Australian children. They administered children with a curriculum-based mathematics test, which measured performance across number, space and measurement skills. Each child also completed four working memory tasks: an auditory-verbal fixed span task to measure phonological loop ability; a visual-spatial fixed span task to measure visuo-spatial sketchpad ability; an auditory-verbal running span task to measure verbal executive ability; and a visual-spatial running span task to measure nonverbal executive ability. Overall, they found significant associations between all working memory measures and mathematics performance, excluding the visual-spatial running span task. When controlling for individual differences in reading ability and performance on the other working memory tasks, they found that only the fixed auditory-verbal and fixed visual-spatial span tasks accounted for unique variance in performance on the mathematics tasks. These findings suggest that phonological loop and visuo-spatial sketchpad abilities are related to children's mathematics. Reuhkala (2001) also reported significant associations between visuo-spatial sketchpad abilities and mathematics in a study of 15- and 16-year-olds mathematical skills. She found that scores on visuo-spatial tasks were significantly related to mathematics performance, even after individual differences in other working memory abilities were controlled for. Central executive and phonological loop scores were not related to mathematics ability in this study. However, other studies that have used the same correlational techniques have reported significant associations between central executive (e.g. Gathercole et al., 2003) and phonological loop (e.g. Dark & Benbow, 1991; Passolunghi & Siegel, 2001; Wilson & Swanson, 2001) abilities and mathematics.

Overall, the evidence suggests that children's working memory skills are significantly related to their mathematics performance. Furthermore, each component of the working memory model (Baddeley & Hitch, 1974) has been ascribed a different role in supporting children's mathematics (see Chapter 1, Sections 1.4.2.1., 1.4.2.2. and 1.4.2.3.). However, relatively few of the studies supporting an association between working memory and mathematics have controlled for individual differences in other cognitive abilities. As Hulme and Roodenrys (1995) point out, they should therefore be interpreted with caution as these associations may be mediated by other cognitive abilities, such as IQ, processing speed or an ability to process numerical information. Indeed, Bull and Johnston (1997) have shown that processing speed may mediate the relationship between phonological loop ability and mathematics (see Chapter 1, section 1.4.2.1.). Furthermore, it has been shown that digit-based measures of working memory are more closely associated with mathematics proficiency than non-digit based measures of working memory (e.g. Passolunghi & Siegel, 2001), suggesting that working memory and mathematics may be linked as the assessments of both involve either number processing or direct access to number representations. A final possibility is that IQ or nonverbal IQ (NVIQ) may mediate the association between working memory and mathematics (see Chapter 1, sections 1.4.2.3).

Many of the studies, excluding Maybery and Do's (2003), that report an association between children's mathematical abilities and working memory skills have measured mathematics as a general ability, which is assessed by a standardized test to give a single score (e.g. Gathercole & Pickering, 2000b). Others have focussed on mental arithmetic (e.g. Adams & Hitch, 1998). Few studies have explored the associations between working memory abilities and different mathematical skills in children.

The aim of the present study was to extend the work of Gathercole and colleagues, who have found an association between working memory abilities and National Curriculum test performance, to specifically explore the associations between working memory skills and National Curriculum mathematics performance in 7-/8-year-olds and 9-/10-year-olds. National Curriculum mathematics test performance was measured using the mathematics tests developed in Chapter 2 to provide an index of different mathematical competencies. Working memory abilities were assessed using non-digit based measures to eliminate the chance of detecting a general ability to process number or numerical information across the working memory and mathematics tasks. Finally, a measure of NVIQ was included to explore the effect of controlling for individual differences in NVIQ on the relationship between working memory abilities and mathematics performance.

Method

Participants

The participants were 148 primary school children (79 boys and 69 girls), who attended three schools in the North-East of England. These were 78 Year 3 children (46 boys and 32 girls), mean age 8 years and 1 month (*SD* = 5.6 months, range 7 years and 1 month to 8; years and 9 months), and 70 Year 5 children (33 boys and 37 girls), mean age 9 years and 10 months (*SD* = 5.7 months, range 9 years and 1 month to 10 years and 9 months). 48 children (24 Year 3 and 24 Year 5) attended one school, 56 children (31 Year 3 and 25 Year 5) attended a second school and 44 children (23 Year 3 and 21 Year 5) attended a third school.

The percentage of children achieving Level 4 attainment and above in English, Mathematics and Science was higher than the national average (75%, 72% and 85% respectively) in two of the schools (English 83% and 96%; Mathematics 90% and 93%; Science 95% and 96%) and lower than the national average in the third school (55%, 48% and 65% respectively).

Design and Procedure

All children participated in three testing sessions. In the first session, each child was administered three working memory tasks in a counterbalanced order. Each child was tested individually in a quiet area of the school. In the second session, children were administered age appropriate mathematics assessments under standardized test conditions within a classroom setting. In the final session, children were administered the non-verbal IQ test, again under standardized test conditions in a classroom setting.

Materials

Working Memory Tasks

Due to the time constraints associated with working in schools, only one measure was used to assess each component of the working memory model. The three working memory tasks were taken from the Working Memory Test Battery for Children (WMTB-C, Pickering & Gathercole, 2001). The tasks were selected as non-digit based measures of the components of working memory.

Phonological loop task

The Nonword List Recall task (Pickering & Gathercole, 2001) involved the spoken presentation of monosyllabic nonsense words for immediate serial recall. The nonwords were created using the phonemes of real words used a Word List Recall subtest (e.g. lotch was created from scotch, meck was created from peck, targ was created from target) (WMTB-C, Pickering & Gathercole, 2001). The nonsense words were presented at a rate of one per second. Participants were asked to recall the sequence of words in exactly the same order as they were presented. Testing began with a block of six trials, in which each sequence contains a single nonword. The sequence length increased at a rate of one nonword every block of six trials. If 4 correct responses were given within a block, the experimenter proceeded to the next block, giving credit for the omitted trials. Testing continued until 3 incorrect responses were given in a block.

The score given was the Trials Correct score. Responses for each trial were scored as 0 or 1. The sum of the correct responses provided the Trials Correct score. The maximum score was 36. Test-retest reliability for this task was .43 for Year 5 and Year 6 children (Pickering & Gathercole, 2001). Coefficients for Year 3 children were not available, although the test-retest for younger children (Year 1 and Year 2) was .68.

Visuo-spatial sketchpad task

The Mazes memory task (Pickering & Gathercole, 2001) involved the presentation of two-dimensional mazes. A route, presented in red, travelled from the middle of the maze to the outside. Each maze was presented for approximately 3 seconds, in which time the experimenter traced the route with his/her finger. Immediately after the route was traced, the participant was asked to recall it by drawing in pencil in a response booklet that contained a blank maze. Participants were asked to recall the exact route that had been traced. Testing began with a block of six trials, in which each trial contained a simple, small maze with two walls. The complexity of the mazes increased every block of six trials. At this stage mazes increased in size by one wall and consequently became more complex.

The testing and scoring procedures were identical to that of the Nonword List recall task. The maximum score was 42. Test-retest reliability for this task was .43 for Year 5 and Year 6 children (Pickering & Gathercole, 2001). Coefficients for Year 3 children were not available, although the test-retest reliability for younger children (Year 1 and Year 2) was .68.

Central Executive task

The Listening Recall task (Pickering & Gathercole, 2001) involved the spoken presentation of short sentences, some of which were true and some that were false. The spoken duration of each sentence was approximately 1-2 seconds. Immediately after a sentence was presented, the participant was asked to judge whether the statement was "true" or "false". Once all sentences within a trial had been presented, the participant was asked to recall the final word of each sentence in the exact order they heard them. Testing began with a block of six trials, in which each trial contained one sentence. The number of sentences then increased by one every block of six trials.

The testing and scoring procedure was identical to that of the previous working memory tasks. The maximum score was 36. Test-retest reliability for this task was .38 for Year 5 and Year 6 children (Pickering & Gathercole, 2001). Coefficients for Year 3 children were not available, although the test-retest for younger children (Year 1 and Year 2) was .83.

Mathematics Tasks

The mathematics assessments administered were the age appropriate assessments previously developed by the author (see Chapter 2). They were designed to measure children's performance across four mathematical skills defined by the National Curriculum for England: Number and Algebra; Shape, Space and Measures; Handling Data; and Mental Arithmetic.

Non-verbal IQ Task

The Matrix Analogies Test Short Form (MAT-SF) (Naglieri, 1985) is a standardized test of non-verbal reasoning intended for group administration. It contains abstract items (in black, white, blue and yellow) in a matrix format similar to Raven's Progressive Matrices (1956) test. For each item, participants are required to look at a set of three pictures of shapes and chose a missing piece for each picture from four alternatives. They are required to look at the four missing pieces and then circle their answer.

The MAT-SF contains 34 items. The complexity of the items increases as the test progresses. Testing lasts 25 minutes and participants are asked to complete as many items as possible in this time. The score given was the raw score (because age was controlled for). This is the total number of correct answers across the 34 scorable items. Test-retest reliability for this task is .51 for Grade 2 children (Year 3 children in the UK) and .91 for Grade 4 children (Year 5 children in the UK).

Results

Power Analysis

Erdfelder's (1984) compromise power analysis was conducted prior to further analyses to determine the statistical power of this study (refer to Chapter 2 for details on power analysis). The results of the power analyses, conducted using Faul and Erdfelder's (1992) G Power programme, are presented in Table 3.1.

Table 3.1

Compromise Power Analysis for Working Memory and Children's Curriculum-Based Mathematics Study.

Effect Size	n_1	n_2	Power
0.5 (medium)	78	70	.93

The power of this study to test for significance with a medium effect size is .93. This exceeds Cohen's (1988) criterion of .8, meaning this study is statistically powerful.

Descriptive Statistics

Descriptive statistics for working memory measures, mathematics test performance and NVIQ scores are presented in Table 3.2.

Table 3.2

Descriptive Statistics of Working Memory and Mathematics Measures for Year 3 (maximum score for each measure shown in brackets). (n = 78).

Measures	Girls (n= 32)		Boys (n = 46)		Total	
	M	*SD*	*M*	*SD*	*M*	*SD*
Working Memory Measures						
Phonological Loop (36)	14.15	2.34	13.86	2.57	13.97	2.47
Visuo-Spatial Sketchpad (42)	8.14	4.86	9.86	6.33	9.20	5.82
Central Executive (36)	10.25	3.21	8.53	3.73	9.20	3.61
Mathematics Measures						
Number and Algebra	54.81	27.87	53.67	28.16	54.13	27.84
Shape Space and Measures	59.25	27.13	52.83	29.13	55.42	28.31
Handling Data	62.72	20.86	61.00	19.48	61.69	19.91
Mental Arithmetic	65.92	30.54	63.25	30.41	64.94	30.41
NVIQ (34)	16.51	6.07	14.85	6.19	15.55	6.16

Note. Mathematics scores shown are proportions correct.

There were no significant differences between Year 3 boys and Year 3 girls performance on either the working memory measures, mathematics assessments or the NVIQ measure (all p>.05).

Table 3.3

Descriptive Statistics of Working Memory and Mathematics Measures for Year 5 (maximum score for each measure shown in brackets). (n = 70).

Measures	Girls (n= 32)		Boys (n = 46)		Total	
	M	*SD*	*M*	*SD*	*M*	*SD*
Working Memory Measures						
Phonological Loop (36)	17.15	2.54	17.10	2.68	17.12	2.58
Visuo-Spatial Sketchpad (42)	16.87	5.98	17.30	8.25	16.94	7.45
Central Executive (36)	12.93	2.78	13.41	3.41	13.01	3.06
Mathematics Measures						
Number and Algebra	56.11	20.31	54.00	24.28	55.16	22.05
Shape Space and Measures	61.11	13.28	57.22	17.39	58.88	15.36
Handling Data	55.37	12.12	54.22	17.21	54.85	14.54
Mental Arithmetic	65.38	20.66	66.33	23.14	65.78	22.18
NVIQ (34)	21.94	6.01	22.93	5.96	22.39	5.97

Note. Mathematics scores shown are proportions correct.

Year 5 boys and Year 5 girls performance did not differ significantly on the working memory measures, mathematics assessments or the NVIQ measure (all p>.05).

Overall, Year 5 performed significantly better than Year 3 on all three working memory measures (p<.05). Across both age groups, children performed better on the phonological loop measure than the other two working memory component measures. There was greater variability on the visuo-spatial sketchpad measure than the other two measures of working memory ability across both age groups. Children's performance did not differ significantly across the mathematics assessments, but Year 5 performed significantly better on the NVIQ measure than Year 3 children ($p<.01$). These raw scores were consistent with the MAT-SF age norms (Naglieri, 1985). Across both age groups, there were no significant gender differences on any of the measures ($p>.05$).

Correlation Analyses

Associations between working memory measures and mathematical abilities are presented in the correlation matrix (Table 3.4). Simple correlations are displayed in the upper triangle; partial correlation coefficients controlling for age are displayed in the lower triangle. The data was collapsed across all children, as there were no significant differences in performance ($p>.05$) between the two age groups on the age appropriate mathematics assessments.

Scores on the working memory measures were intercorrelated (all *r*s > .30, $p<.05$), even when the variance related to age was eliminated (all *pr*s >.20, p<.05, except phonological loop and visuo-spatial sketchpad scores *pr* > .30, $p>.05$). Similarly performance on the mathematics assessments was intercorrelated, even when the variance related to age was eliminated (all *r*s >.30, $p<.01$, all *pr*s >.30, $p<.01$).

Central executive scores were significantly related to mathematical abilities (all *r*s >.20, $p<.01$) and remained so after age related variance was eliminated (all *pr*s >.30, $p<.01$). Visuo-spatial sketchpad scores were significantly related to mathematics performance (all *r*s >.20, $p<.01$) and remained significantly related to all mathematical abilities when age related variance was controlled for (all *pr*s >.20,

Table 3.4

Correlation Matrix for Working Memory and Mathematics Measures. Simple coefficients are displayed in the upper triangle; partial coefficients are displayed in the lower triangle. (*N*=148).

	Phonological Loop	Visuo-Spatial Sketchpad	Central Executive	Number and Algebra	Shape, Space and Measures	Handling Data	Mental Arithmetic
Phonological Loop	-	.36*	.44**	.12	.14	-.01	.21**
Visuo-Spatial Sketchpad	.11	-	.42**	.28**	.26**	.13	.26**
Central Executive	.24*	.26**	-	.26**	.51**	.28**	.51**
Number and Algebra	.09	.28**	.42**	-	.61**	.60**	.72**
Shape, Space and Measures	.10	.24*	.51**	.61**	-	.48**	.60**
Handling Data	.09	.28**	.43**	.65**	.55**	-	.46**
Mental Arithmetic	.15	.21*	.49**	.71**	.60**	.53**	-

Note. *$p<.05$ **$p<.01$

p<.05). Phonological loop scores were only related to Mental Arithmetic ability (r = .21, p<. 01), but this was accounted for by age related variance (pr =.15, p>.05).

Associations between working memory scores and mathematical skills, controlling for NVIQ are presented in Table 3.5. Chronological age was controlled to eliminate age–related variance on the NVIQ measure.

Central executive scores were significantly related to all mathematical abilities (all rs > .30, pr<.01, except Number and Algebra r = .29, pr<.05) when individual differences in NVIQ were controlled for. Visuo-spatial sketchpad and phonological loop scores were not significantly related to any mathematical abilities when NVIQ scores were controlled (all rs<.30, pr>.05).

Table 3.5

Correlation Matrix for Working Memory Measures and Mathematics Assessments, Controlling for Age and NVIQ Scores. (N = 148).

	Phonological Loop	Visuo-spatial Sketchpad	Central Executive
Number and Algebra	.03	.18	.29*
Shape, Space and Measures	.01	.16	.45**
Handling Data	.05	.16	.34**
Mental Arithmetic	.13	.14	.41**

Correlation Analyses Corrected for Attenuation

Low reliability of the measures used in a study can cause underestimation of the correlation coefficients. This error can be corrected using a technique known as attenuation, which takes into account the reliability of the measures used.

Due to the low test re-test reliability coefficients for the working memory measures, the correlation coefficients between working memory scores and mathematics performance controlling for age were corrected for attenuation. Correlation coefficients corrected for the reliability statistics of the younger groups' measures are presented in Table 3.6, with the corrections for the reliability statistics of the older age groups' measures presented in parentheses.

Table 3.6

Attenuated Correlation Matrix for Working Memory Measures and Mathematics Assessments, controlling for age. Corrections for reliability of Year 3 measures shown with corrections for reliability of Year 5 measures in parentheses. (*N* = 148).

	Phonological Loop	Visuo-spatial Sketchpad	Central Executive
Number and Algebra	.11 (.16)	.36** (.44**)	.49** (.78**)
Shape, Space and Measures	.13 (.18)	.32** (.39**)	.61** (.98**)
Handling Data	.11 (.13)	.38** (.42**)	.53** (.54**)
Mental Arithmetic	.19* (.27**)	.27** (.34**)	.56** (.89**)

Central executive and visuo-spatial sketchpad scores were significantly associated with performance across all mathematical skills when the reliability of the measures was considered (all *rs* > .30, $p<.01$), while phonological loop scores were only significantly related to mental arithmetic performance ($r<.30$, $p<.05$, corrected for Year 3 reliability and $r<.30$, $p<.01$, corrected for Year 5 reliability).

Corrections for attenuation inflated the correlation coefficients between working memory scores and mathematics scores. It was feared that these inflations did not accurately reflect the data (e.g. a coefficient of .98 between Shape, Space and Measures scores and central executive scores seems unlikely). Therefore, no further corrections for attenuation were made.

Regression Analyses

A simple linear regression analysis revealed that the working memory measures predicted 27.7% of the variance in overall mathematics performance. Subsequently, a series of fixed-order unique variance regression analyses were used to assess the amount of unique variance in mathematics scores predicted by each of the measures. For each analysis the mathematics assessment was the regressor and the unique contribution (measured as r^2) of each working memory measure was assessed as a predictor entered into the regression equation after the other predictors. The data was collapsed across all children, as there were no significant differences in performance ($p>.05$) between the two age groups on the age appropriate mathematics assessments. Age was entered as the first variable into each regression equation to control for age-related variance. See Table 3.7 (Appendix VI) for results.

Models A_1, A_2, A_3, A_4 and A_5 show that phonological loop scores do not account for any unique variance in mathematics scores above and beyond that accounted for by age and the other two working memory constructs.

Models B_1, B_2, B_3, B_4 and B_5 show the amount of unique variance in mathematics scores predicted by visuo-spatial sketchpad scores when the variance attributable to age and central executive and phonological loop scores is accounted for. These models indicate that visuo-spatial ability accounts for a small, but significant, amount of variance in overall mathematical ability (3%), Number and Algebra scores (3%), Shape, Space and Measures scores (1%), Handling Data scores (3%) and Mental Arithmetic scores (1%).

Models C_1, C_2, C_3, C_4 and C_5 show that of the working memory measures central executive scores account for the greatest amount of unique variance in mathematics scores. After the variance contributed by age and visuo-spatial and phonological scores is accounted for central executive scores account for 23% of variance in overall mathematical ability, 12% of variance in Number and Algebra scores, 21% of variance in Shape, Space and Measures scores, 13% of variance in Handling Data scores and 18% of variance in Mental Arithmetic scores.

A second series of fixed-order unique variance regression analyses were conducted to assess the amount of unique variance in mathematics scores predicted by each of the measures of working memory after the variance accounted for by NVIQ was considered. Again, for each analysis the mathematics assessment was the regressor and the unique contribution (measured as r^2) of each working memory measure was assessed as a predictor entered into the regression equation after the other predictors, which included age, NVIQ and performance on the other working memory measures. As before, the data was collapsed across all children. The results are presented in Table 3.8 (Appendix VII).

Models D_1, D_2, D_3, D_4 and D_5 show the amount of unique variance in mathematics skills predicted by central executive ability when the variance predicted by age, non-verbal IQ and visuo-spatial and phonological abilities is accounted for. These models indicate that central executive ability predicts a significant amount of unique variance in overall mathematics ability (15%), Number and Algebra skills (6%), Shape, Space and Measures skills (17%), Handling Data skills (8%) and Mental Arithmetic ability (13%).

Models E_1, E_2, E_3, E_4 and E_5 show the amount of unique variance in mathematics skills predicted by visuo-spatial ability when the variance predicted by age, non-verbal IQ and central executive and phonological abilities is accounted for. These models indicate that visuo-spatial ability predicts a significant amount of unique variance in overall mathematics ability (1%), Number and Algebra skills (2%), Shape, Space and Measures skills (1%) and Handling Data skills (1%). Visuo-spatial ability did not predict unique variance in Mental Arithmetic performance.

Models F_1, F_2, F_3, F_4 and F_5 show that phonological loop ability contributes no unique variance to mathematics ability above and beyond that predicted by age, non-verbal IQ and visuo-spatial and central executive abilities.

Exploratory Factor Analyses

Principal components analyses were conducted to determine the higher-order factor structure underpinning variations in scores on all measures as a descriptive, summative method prior to conducting model-based techniques.

Two factors emerged with eigenvalues in excess of 1.00 in the first analysis, which included all measures except the NVIQ measure. Factor loadings greater than .30 on the rotated component matrix are presented in Table 3.9.

Table 3.9.

Factor Loadings of Working Memory and Mathematics Measures, Excluding NVIQ, on Rotated Component Matrix.

	Factor	
Measure	1	2
Working Memory measures		
Phonological Loop		.81
Visuo-Spatial Sketchpad		.71
Central Executive	.44	.68
Mathematics Measures		
Number and Algebra	.87	
Shape, Space and Measures	.78	
Handling Data	.82	
Mental Arithmetic	.79	

Note. Only loadings greater than .3 are shown.

Kaiser-Meyer-Olkin (KMO) measure of sampling adequacy = .83.

All four measures of mathematics ability loaded on Factor 1, all with loadings in excess of .75. The three working memory measures loaded on Factor 2, which corresponds with the tripartite model of working memory proposed by Baddeley and Hitch (1974). The central executive loaded on both factors. A possible explanation for this may be the demands that both central executive and mathematics measures place upon general intelligence (e.g. Fry & Hale, 1996; Kyllonen & Christal, 1990). That is, both may share variance with general intelligence. Despite this, it should be noted that the weight of the loading on the working memory factor (Factor 2 = .68) was greater than the loading on the mathematics factor

(Factor 1 = .44), consistent with the notion that it is primarily a measure of working memory ability.

Two factors with eigenvalues in excess of 1.00 emerged in the second analysis, where the NVIQ measure was included. Factor loadings greater than .30 on the rotated component matrix are presented in Table 3.10.

Table 3.10

Factor Loadings of Working Memory, Mathematics and NVIQ Measures on Rotated Component Matrix.

	Factor	
Measure	1	2
Working Memory measures		
Phonological Loop		.77
Visuo-Spatial Sketchpad		.72
Central Executive	.38	.70
Mathematics Measures		
Number and Algebra	.86	
Shape, Space and Measures	.78	
Handling Data	.81	
Mental Arithmetic	.79	
NVIQ		.70

Note. Only loadings greater than .3 are shown.

KMO measure of sampling adequacy = .85.

The same factor structure emerged for the working memory measures and mathematics assessments. The NVIQ loaded on Factor 2, the working memory factor, suggesting the tasks may have similar demands or that both NVIQ and working memory share variance.

Confirmatory Factor Analysis

To further investigate the relationship between mathematical abilities and working memory and to test the higher order factor structure suggested by the principal components analysis, confirmatory factor analysis was conducted using EQS 6 (Bentler, 2001). The reason for this approach was to find the best theoretical account of the data through formally testing a number of competing theoretical models. For each model assessed, coefficients for the paths between latent constructs and variables were produced, indicating the strength of the relationships between them. A variety of fit statistics were produced to indicate the goodness of fit of the model to the data. The most commonly used fit index chi squared (χ^2), was used alongside other fit indices including the Comparative Fit Index (*CFI*), Bollen's Incremental Fit Index (*IFI*), the Goodness of Fit Index (*GFI*), the standardized root mean square of the model residuals (*SRMR*) and the root mean square of approximation (*RMSEA*). These fit indices were compared across competing models to find the best theoretical model of the data.

It is important to note that the confirmatory factor analysis was used as an exploratory procedure to show the relationship between the two constructs suggested by the principal components analysis.

The first principal components analysis, which did not include the NVIQ measure, yielded a two-factor solution (Table 3.9.), but it did not indicate the correlation between the two latent constructs. Therefore, three two-factor models of the relationship between working memory measures and mathematics were tested using confirmatory factor analysis. The input to the program was the raw data, from which the programme computed a covariance matrix for the analyses. For all models, the mathematics factor corresponded to the structure of the National Curriculum, comprising of four measures of mathematics ability; Number and Algebra, Shape, Space and Measures, Handling Data and Mental Arithmetic. The working memory factor differed across the models. The first model (CFA1) corresponded to the Baddeley and Hitch (1974) working memory model, with measures representing the phonological loop, visuo-spatial sketchpad and central executive components. The second (CFA2) model included only the verbal working memory measures (central executive and phonological loop). In the third model tested (CFA3), the working memory factor comprised of the visuo-spatial sketchpad and central executive measures, corresponding to the notion that visuo-spatial sketchpad measures share variance with central executive measures as they place heavy demands on the general processing and storage functions of the central executive (Gathercole & Pickering, 2000a; Miyake et al., 2001; Phillips & Christie, 1977a; Wilson et al.,). Fit statistics for the three models are presented in Table 3.11.

All models yielded fit indices in excess of .95 (*CFI* and *IFI*) and .9 (*GFI*) indicating good fit. However, the fit of models CFA2 and CFA3 was not ideal as the *SRMR* and *RMSEA* values were in excess of .08, indicating poor fit. Furthermore, both models yielded significant χ^2 values, meaning they differed significantly to the data. In addition, examination of the diagram for CFA2 revealed that the path between the central executive measure and factor 1 (working memory) was 1.00. This indicates that Factor 1 accounted for all of the variance in the central executive measure. Therefore, its communality with the other variable loading on Factor 1 (phonological loop measure) is 1.00, meaning the other variable has no uniqueness (Loehlin, 1987). In short, the central executive measure shares a large amount of variance with the phonological measure, indicating that this diagram is probably not a good

Table 3.11

Goodness of fit statistics for CFA models (working memory and curriculum-based mathematics).

Model	*df*	χ^2	*p*	*CFI*	*GFI*	*IFI*	*SRMR*	*RMSEA*
CFA1	13	21.07	.071	.969	.946	.970	.058	.076
CFA2	8	16.59	.035	.964	.949	.965	.053	.100
CFA3	8	15.52	.049	.969	.954	.970	.043	.094

Note. *CFI* = Bentler's Comparative Fit Index. *GFI* = Goodness of Fit Index. *IFI* = Bollen's Incremental Fit Index. *SRMR* = Standardized Root Mean Squared Residual. *RMSEA* = Root Mean Square Error of Approximation.

description of the data. For these reasons, models CFA2 and CFA3 were discarded. Model CFA1, the traditional three-factor working memory model, provided the best fit to the data across all fit indices. In this model, all fit indices (*CFI, GFI* and *IFI*) were good, the χ^2 value for the model, with 13 degrees of freedom, was 21.07, $p = .071$, and the *SRMR* and *RMSEA* values were below .08 meaning the model did not differ significantly to the data. A significant path existed between the working memory construct and mathematics performance; the path covariance coefficient was .606, $p<.05$. A diagrammatic representation of this model is shown in figure 3.1.

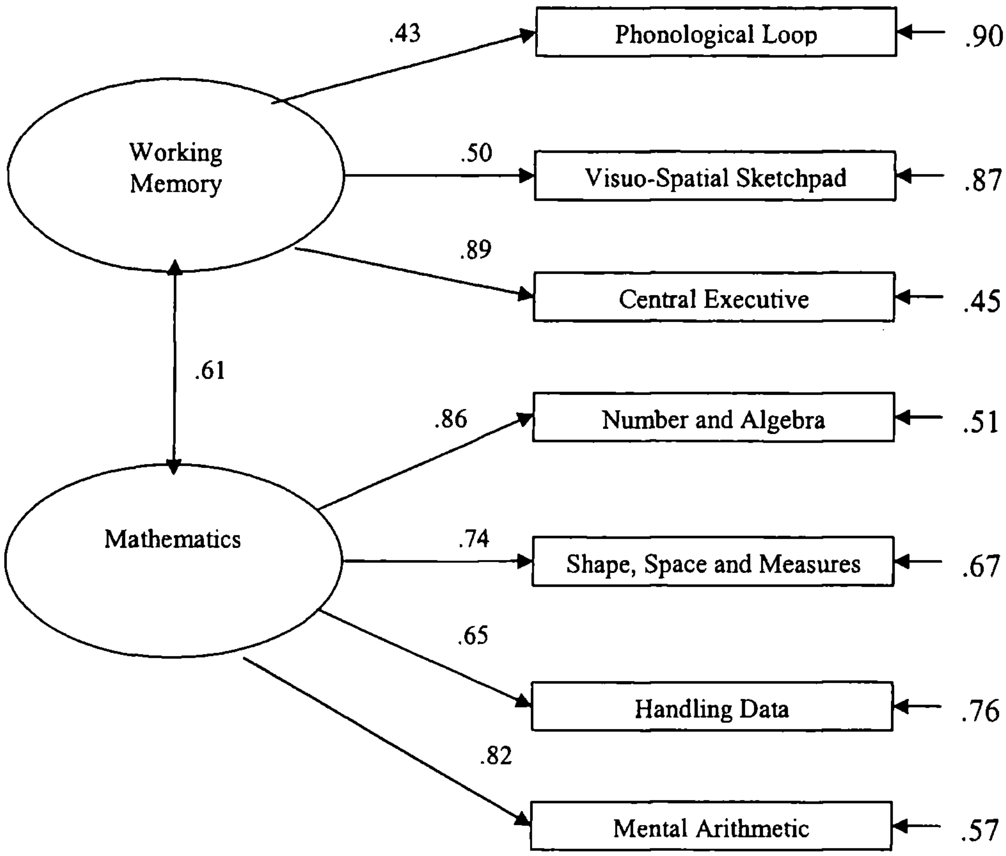


Figure 3.1

Diagrammatic Representation of the Best Fitting Factor Model (CFA1) for Working Memory and Curriculum-Based Mathematics

The second principal components analysis, which included the NVIQ measure, also yielded a two-factor solution (Table 3.10). Again, various models of the relationship between the two factors were tested using confirmatory factor analysis. As before, the input to the programme was the raw data, from which the programme (EQS 6, Bentler, 2001) computed a covariance matrix for the analyses. As before, the mathematics factor corresponded to the structure of the National Curriculum. The working memory factor and the NVIQ variable varied across the different models tested. The first model (CFA4) corresponded to the results of the principal components analysis, where NVIQ loaded on the working memory factor, which itself corresponded to the Baddeley and Hitch (1974) tripartite model. The second

(CFA 5) and third (CFA 6) models included a factor corresponding to the tripartite working memory model, with NVIQ as an independent variable. In the second model (CFA 5) NVIQ shared variance with the working memory factor, and in the third (CFA 6) it shared variance with both the working memory and mathematics factors. Fit statistics for the three models are presented in Table 3.12.

Table 3.12

Goodness of fit statistics for CFA models (working memory, curriculum-based mathematics and NVIQ).

Model	*df*	χ^2	*p*	*CFI*	*GFI*	*IFI*	*SRMR*	*RMSEA*
CFA4	19	28.03	.082	.968	.936	.969	.058	.069
CFA5	19	46.91	.000	.902	.905	.905	.146	.121
CFA6	18	4103.9	.000	.000	.046	.000	.273	1.507

Note. *CFI* = Bentler's Comparative Fit Index. *GFI* = Goodness of Fit Index. *IFI* = Bollen's Incremental Fit Index. *SRMR* = Standardized Root Mean Squared Residual. *RMSEA* = Root Mean Square Error of Approximation.

Models CFA 5 and CFA 6 yielded poor fit indices below .95 (*CFI, GFI* and *IFI*). Furthermore, both models yielded significant χ^2 values, meaning they differed significantly to the data. Model CFA 4, which comprised of two-factors as specified by the results of the principal components analysis (see Table 3.10), provided the best fit to the data across all fit indices. All fit indices were good (*CFI, IFI* and *GFI*) for this model. The χ^2 value for the model, with 19 degrees of freedom, was 28.03, $p = .08$, and the *SRMR* and *RMSEA* values were below .08, meaning the model did not differ significantly to the data. A significant path existed between the working memory and NVIQ factor and the mathematics construct; the path covariance coefficient was .623, $p<.05$. A diagrammatic representation of this model is shown in figure 3.2.

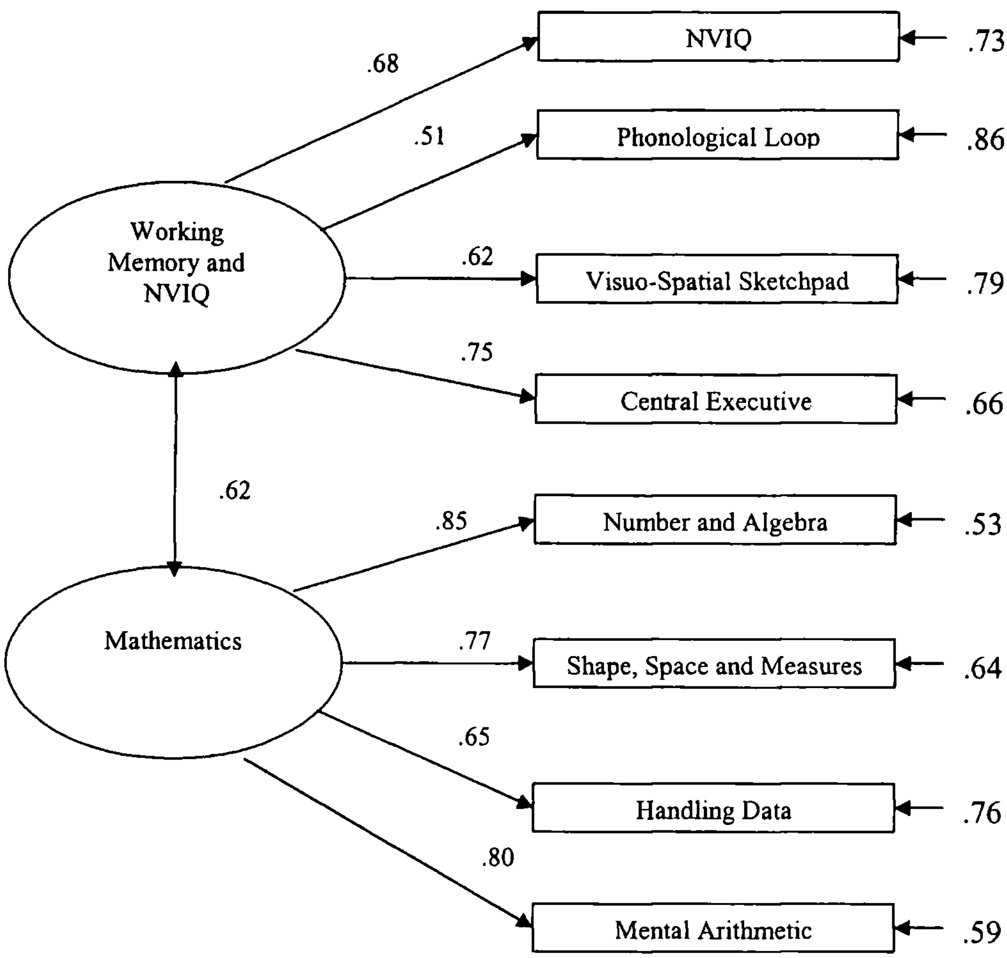


Figure 3.2

Diagrammatic Representation of the Best Fitting Factor Model (CFA 4) for Working Memory, Curriculum-Based Mathematics and NVIQ

There was no significant difference between the fit of two best fitting models from each of the confirmatory factor analyses (models CFA 1 and CFA 4, $p>.05$). Model CFA 1 yielded marginally better fit indices (*CFI, IFI* and *GFI*), but model CFA 4 yielded better χ^2, *SRMR* and *RMSEA* values, meaning it was less different to the data set. Structurally these two models differed due to the inclusion of the NVIQ measure (see figures 3.1 and 3.2). Therefore, the results suggest that including the NVIQ measure did not significantly alter the results of the factor analyses. Inclusion of the NVIQ measure only increased the path covariance coefficient between the working memory and mathematics factors by .01. This indicates that working memory per se is associated with mathematics.

Discussion

Overall, the results show a significant association between children's working memory ability and their mathematics attainment. The results of the first confirmatory factor analysis revealed that a significant path existed between a working memory construct corresponding to the tripartite model (Baddeley & Hitch, 1974), and mathematics performance. Furthermore, a simple regression analysis revealed that the tripartite model of working memory (Baddeley & Hitch, 1974; Baddeley, 1986) predicted 27.7% of the variance in the children's mathematics scores, suggesting that it is a significant predictor of performance. Although this association may reflect the contribution of working memory to a higher order construct, such as general intelligence (Kyllonen & Christal, 1990), the findings support the notion that working memory is involved in children's mathematics (e.g. Adams & Hitch, 1997; 1998).

The contributions of the different components of the working memory model to a range of children's mathematical skills were assessed using measures of working memory function from the WMTB-C. Importantly, these standardized measures did not involve numerical stimuli, thus controlling for potential interference from general number fluency. Initially the associations between working memory and curriculum-based mathematics skills were explored without consideration for the contribution of NVIQ. The effect of controlling for individual differences in NVIQ on the relationship between working memory abilities and mathematics performance was later explored. The relationship between National Curriculum mathematics performance and working memory abilities, and the subsequent impact of controlling for NVIQ will be discussed in turn.

The results of this study found that working memory predicted National Curriculum-based mathematical skills. This is consistent with previous findings that working memory predicts wider aspects of National Curriculum attainment (e.g. Gathercole & Pickering 2000a; 2000b), and that working memory assessments may be useful as early predictors of scholastic attainment. Furthermore, it extends these findings to suggest that working memory supports different aspects of a particular curricula area (mathematics). Different components of the working memory model, namely the visuo-spatial sketchpad and the central executive, were found to predict all four areas of mathematics defined by the National Curriculum for England. However, there was little difference between the working memory demands of each mathematical skill (e.g. visuo-spatial sketchpad scores predicted between 1% and 3% of unique variance across all four skills before NVIQ was controlled, and between 1% and 2% across all four areas after NVIQ was controlled). Contrary to Maybery and Do (2003) this implies that different mathematical skills do not recruit different working memory resources.

Detailed analyses indicated that both central executive and visuo-spatial sketchpad scores, but not phonological loop scores, predicted unique variance in children's National Curriculum mathematics skills.

The results confirmed previous findings that the central executive is an important predictor of children's mathematics (e.g. Bull et al., 1999; Bull & Scerif, 2001). It predicted a significant amount of unique variance on all curriculum-based skills (12-22%). The proportion of unique variance predicted by the central executive was greater than that reported in previous findings. For example, Bull and Scerif (2001) report more conservative values of 2% and 3%. A possible explanation for this discrepancy relates to the assessment of children's mathematics performance. In the current study mathematics ability was assessed using written tests, which involved the children reading the questions. The central executive is thought to be important for language and text comprehension (e.g. Yuill et al., 1989) and reading (e.g. Daneman & Carpenter, 1980). Therefore, it may have accounted for a large proportion of unique variance in the current study because it supported children's reading and understanding of the questions in addition to supporting their mathematics.

The results of the first principal components analysis found that the central executive loaded on both the working memory and mathematics factors. This could be indicative that the central executive measure is related to a more general resource such as intelligence. Working memory, and in particular the central executive, have been associated with human intelligence (e.g. Colom, et al., 2004; Kyllonen & Christal, 1990). Furthermore, working memory abilities have been found to predict performance on tests of general intelligence (e.g. Engle et al., 1999) (see Chapter 1, section 1.1.2.3). The loading of the NVIQ measure on the working memory factor in the second principal components analysis supports the suggestion that working memory and intelligence are closely associated. Therefore, the significant associations between the central executive measure and a range of mathematical abilities may in part reflect the contribution of general intelligence to mathematical competence. Alternatively, as suggested by Bull and colleagues, the central executive may be important for supporting the acquisition and selections of appropriate solution strategies in children's mathematics (see Chapter 1, section 1.4.2.3). Alternatively, it may be involved in controlling and co-ordinating the solution to mathematical problems as has been suggested for adult populations (e.g. Seitz & Schumman-Hengsteler, 2002. See Chapter 1, Section 1.4.2.3).

The visuo-spatial sketchpad predicted a small, but significant, amount of unique variance in children's curriculum-based mathematics scores (between 1% and 3%). This supports previous findings that visuo-spatial working memory is related to children's

National Curriculum mathematics attainment (e.g. Jarvis & Gathercole, 2003) and performance on other standardized mathematics tests (e.g. Maybery & Do, 2003). Furthermore, finding an independent role for the visuo-spatial sketchpad in children's mathematics supports neuropsychological evidence that suggests the nature of visuo-spatial cognition important for mathematical cognition specifically incorporates a visuo-spatial working memory system (e.g. Zago & Tzourio-Mazoyer, 2002). Previous studies have typically focussed on the associations between phonological loop (e.g. Adams & Hitch, 1997) and central executive (e.g. Bull et al., 1999) skills and children's mathematics. As such, the role of the visuo-spatial sketchpad in children's mathematics is somewhat undefined. It is tentatively suggested that it may act as "mental blackboard" for encoding, storing and manipulating problem information during calculation, as has been suggested with adult populations (e.g. Heathcote, 1994; Logie et al., 1994. See Chapter 1, section 1.4.2.2). Alternatively, it may support the use of a "mental number line" or be involved in approximation as suggested by Dehaene (1992) (see Chapter 1, section 1.4.2.2).

Contrary to previous findings phonological loop scores did not predict unique variance in children's curriculum-based mathematics performance. Scores on the phonological loop measure were, however, significantly associated with mental arithmetic performance before the variance associated with age was controlled for. Of the four curriculum-based mathematics skills assessed, mental arithmetic was the only skill that involved auditory presentation of the problems. As such, the data may tentatively suggest that the children were able to use subvocal rehearsal processes to support the retention of problem information (e.g. Adams & Hitch, 1997) and the direct retrieval of arithmetic facts from long-term memory, which is based on a verbal code (e.g. Dehaene & Cohen, 1995). An alternate explanation is that phonological loop scores did not explain any further variance in children's mathematics performance once the variance attributed to central executive abilities had been controlled for due to the co-dependence of the tasks used to measure the two components (see Chapter 1, section 1.4.2.1).

When controlling for individual differences in NVIQ, the amount of unique variance predicted in mathematics performance by central executive and visuo-spatial scores was reduced. Both, however, remained significant predictors of performance across all four mathematical skills. The results of the confirmatory factor analyses suggested that including NVIQ as a variable alongside the three working memory measures added little to the fit of the model, nor did it increase the covariance coefficient between the two latent constructs, working memory and mathematics. These results extend previous findings (e.g. Gathercole & Pickering, 2000b) to suggest that working memory ability predicts National Curriculum mathematics performance above and beyond what is predicted by NVIQ. This has implications for educational practice as it further supports the suggestion that working memory assessments may be useful predictors of later academic achievement (e.g. Gathercole & Pickering, 2000b), and that screening for impaired working memory in young children may help to identify those at risk of maths difficulties (Gathercole & Pickering, 2001).

Exploration of the higher-order factor structure suggested that the working memory measures and the NVIQ measure were closely related. All measures loaded on a single factor in the second principal components analysis. Furthermore, in the confirmatory factor analyses where NVIQ was included, the same four measures were grouped as a latent construct in the best-fitting factor model. One reason for this may be, as discussed earlier, that working memory and intelligence are closely related constructs (e.g. Colom, et al., 2004). The NVIQ measure may have grouped with the working memory measures in the factor analyses due to the similarity of the task demands. If another measure of intelligence, such as a measure of verbal IQ, had been included the data may have yielded a separate factor corresponding to intelligence.

In conclusion this study provides additional evidence for the involvement of working memory in children's mathematics and supports the usefulness of working memory assessments as predictors of National Curriculum test performance. In particular, it provides evidence to support the notion that children's mathematics may involve executive functions (e.g. Bull et al., 1999) and visuo-spatial cognition (e.g. Houdé & Tzourio-Mazoyer, 2003; Maybery & Do, 2003). Due to time constraints only one measure of each component of working memory was administered, which rather limits the scope of this study. However, the suggestion that visuo-spatial working memory may play an important role in children's mathematics, which is a relatively new finding, certainly warrants further investigation. For example, future research might explore the potentially different roles of visual and spatial working memory in children's mathematics. One approach to this would be to adopt Logie's (1995) idea of separate cache and scribe processes within the visuo-spatial sketchpad.

Chapter Summary

1. Children's working memory abilities have been related to National Curriculum attainment across the three Key Stages. The aim of the present study was to extend these findings to explore the relationship between working memory abilities and mathematical skills defined by the National Curriculum at Key Stage 2. A further aim was to explore the effect of controlling for individual differences in NVIQ on these relationships.
2. Phonological loop scores predicted unique variance in children's mental arithmetic scores, suggesting it may support the retention of problem information for auditorily

presented problems. However, this relationship did not persist once age-related variance had been removed.

3. Both visuo-spatial and central executive scores predicted unique variance in children's mathematical skills beyond that predicted by NVIQ.
4. It was suggested that the central executive may support the acquisition and selection of appropriate solution strategies in children's mathematics.
5. Finding an independent role for the visuo-spatial sketchpad supports the suggestion that visuo-spatial cognition may be important for mathematics. As this is a relatively new finding the precise role of the visuo-spatial sketchpad is unclear.
6. The association between visuo-spatial sketchpad scores and children's mathematics performance certainly warrants further investigation.

Chapter Four

Visuo-spatial working memory and children's mathematical abilities

Aim

An association was found between children's scores on a single visuo-spatial working memory measure and performance on tests of National Curriculum mathematical abilities in Chapter 3. The aim of the present study was to further investigate this finding by including several measures of visuo-spatial working memory ability. Based on the suggestion that the visuo-spatial working memory system may comprise of two subsystems (e.g. Logie, 1995) the tasks used were categorised into those that measured a child's ability to maintain visual information and those that measured a child's ability to maintain spatial information. A second aim of the present study was to explore the patterns of associations between these visual and spatial measures and children's mathematics performance. Related to this, a further aim was to explore the separability of visual and spatial subcomponents of working memory in children. As in Chapter 3, a measure of NVIQ was included to explore the effect of controlling for individual differences in NVIQ on the relationship between visuo-spatial working memory abilities and mathematics performance.

Introduction

Children's visuo-spatial working memory abilities have been associated with performance on National Curriculum assessments in English, Mathematics and Science in the UK. More recently, they have been associated specifically with children's mathematics competency across a range of skills.

As discussed in Chapter 3, Gathercole and colleagues have conducted extensive investigations into the relationship between children's working memory abilities and their National Curriculum performance (e.g. Gathercole & Pickering, 2000b; Gathercole, Pickering, Knight et al., 2004). Although several of their investigations did not incorporate measures of visuo-spatial ability (e.g. Gathercole, Pickering, Knight et al., 2004), studies where such measures were included suggested that visuo-spatial abilities were related to National Curriculum test performance in 7- (Gathercole & Pickering, 2000b), 11- and 14-year-olds (Jarvis & Gathercole, 2003). Importantly, visuo-spatial sketchpad scores were significantly associated with National Curriculum mathematics performance across these age groups.

Other studies that have explored the relationship between visuo-spatial working memory ability and mathematics performance in children report similar findings. For example, Maybery and Do (2003) reported significant associations between simple and complex visuo-spatial span performance and curriculum-based mathematics performance in 10-year-old Australian children. Furthermore, Reuhkala (2001) reported significant associations between scores on visuo-spatial span tasks and mathematics performance in 15-/16-year-olds, even after individual differences in verbal working memory abilities were controlled for.

Significant associations between visuo-spatial sketchpad scores and children's curriculum-based mathematics performance were reported in Chapter 3. Specifically, visuo-spatial sketchpad scores predicted unique variance in performance across different mathematics skills, even after individual differences in NVIQ and other working memory abilities were controlled for.

In summary, converging evidence suggests children's visuo-spatial sketchpad skills are related to their mathematics performance. Furthermore, it has been suggested that the visuo-spatial sketchpad may support mathematical processing in adults (e.g. Heathcote, 1994) and young children (e.g. McKenzie et al., 2003) (see Chapter 1, section 1.4.2.2). However, relatively few of the studies that report significant associations between children's visuo-spatial sketchpad scores and their mathematics performance have measured different mathematical skills (e.g. Gathercole & Pickering, 2000b). Furthermore, those that have measured different mathematical skills have typically included only one measure of visuo-spatial sketchpad functioning (e.g. Maybery & Do, 2003; the author's previous study). This is surprising given that the visuo-spatial sketchpad may be comprised of two subsystems; one for maintaining spatial information and one for maintaining visual information (e.g. Logie, 1995).

Logie's (Logie, 1995; Reisberg & Logie, 1993; Salway & Logie, 1995) model of the visuo-spatial sketchpad system suggests that it is comprised of two subsystems; a temporary visual store (visual cache), presumed to store information about visual form and colour, and a temporary spatial store (inner scribe), presumed to store information about movement sequences (see Chapter 1, section 1.1.2.2). Studies with neurpsychological patients, adults and children and neuroanatomical studies support this fractionation.

Experimental studies with adults support a dissociation between visual and spatial subcomponents within the visuo-spatial working memory framework. Many of these studies have used a selective interference paradigm to demonstrate that visual interference tasks only disrupt performance on primary tasks that are visual in nature, while spatial interference tasks cause selective interference in primary tasks that are spatial in nature (e.g. Logie & Marchetti, 1991; Tresch, Sinnamon & Seaman, 1993). As discussed in Chapter 1 (section 1.1.2.2) Quinn and McConnell (1996; 2000) reported that visual noise disrupted performance on an immediate visual memory task, but did not affect performance on a spatial task. Similarly, Della Sala, Gray, Baddeley, Allamano & Wilson (1999) reported that a visual interference task (viewing abstract pictures) caused a decrement in performance on an immediate visual memory, but not spatial memory, task. Smyth and colleagues reported the opposite pattern, where spatial interference tasks such as spatial tapping (Smyth & Pendleton, 1989) or listening to tones in different locations (Smyth & Scholey, 1994) selectively disrupted performance on spatially loaded primary tasks, such as memory for spatial locations. In addition, other studies that have compared adults' performance on immediate memory tasks that are either spatial or visual in nature have reported that performance is unrelated. For example, Wilson, Brodie, Reinink, Wiedman and Brooks (1968) found that performance was unrelated on a visual memory task (Visual Patterns Task) and a spatial memory task (similar to a Corsi block task) in a sample of adults' with senile dementia. Similarly, Smyth and Scholey (1996) reported that while recall and recognition versions of the Corsi blocks tasks (spatial) and the Visual Patterns Test (visual) were related, performance on the Corsi blocks recognition task was less well correlated with the recognition version of the Visual Patterns Test than the recall and recognition versions of the Visual Patterns Test were with each other. This evidence suggests that there might be a dissociation between immediate memory for visual and spatial information in adults.

Double dissociations in neuropsychological patients and clinical populations provide further support for a fractionated visuo-spatial system in adults. Della Sala et al. (1999) identified two brain-damaged adults who were significantly impaired on a spatial memory task (Corsi blocks), but relatively unimpaired (performing above the median) on a visual memory task (Visual Patterns Task). A third adult showed the opposite pattern, demonstrating a double dissociation. Furthermore, when taken together case studies provide evidence of neurpsychological double dissociations in spatial and visual working memory (Luzzatti et al., 1998). Patient LH was reported to have impaired visual memory, but spared memory for spatial locations (Farah, Hammond, Levine & Calvanio, 1988). Conversely, patients EP (Luzzatti et al., 1998) and MV (Carlesimo, Perri, Turriziani, Tomaiuolo & Caltagirone, 2001) showed impairments in spatial, but not visual working memory.

Neuroanatomical findings suggest that different anatomical brain locations may underpin immediate memory for visual and spatial information. For example, De Renzi (1982) reported that patients with parietal occipital lesions experienced problems in processing spatial information, while patients with inferior temporal lesions showed deficits in visual processing. Furthermore, neuroimaging studies with normal adults support a distinction between visual and spatial working memory. That is, performance on a spatial working memory task activated an area of the right premotor cortex, while an object (visual) working memory task activated a region in the right dorsolateral prefrontal cortex (DLPFC) (Courtney, Ungerleider, Keil & Haxby, 1997). In a follow-up study, the DLPFC remained activated during a delay in the visual memory task, and the premotor cortex remained activated during a delay in the spatial memory task. These findings strengthen the argument that these brain areas mediate storage (Courtney, Petit, Maisog, Ungerleider & Haxby, 1998). Work with nonhuman primates provides evidence to support the notion that spatial and visual working memory have different neural bases (Wilson, O' Scalaidhe & Goldman-Rakic, 1993).

Converging neuropsychological, neuroanatomical and experimental evidence from adult populations suggests that two different components of the visuo-spatial sketchpad support the temporary retention of visual and spatial information. Evidence for a fractionated visuo-spatial system in children can be drawn from studies of developmental fractionation (Hitch, 1990). As discussed in Chapter 1 (section 1.2.2), Logie and Pearson (1997) found that visual and spatial working memory abilities develop at different rates in childhood when they examined 5-, 8- and 11-year-olds. Hamilton et al. (2003) replicated these findings in a study of 5- to 7-, 8- to 10-, 11- to 13- and 18- to 25-year-olds. The pattern of development evident in their data suggested that visual working memory developed relatively rapidly between 5-years to adulthood, while spatial working memory showed a slower, steadier increase from childhood to adulthood. These studies provide evidence for separate visual and spatial subsystems in children's visuo-spatial working memory, as they appear to follow different developmental trajectories. Pickering et al. (2001) conducted a similar study to Logie and Pearson's (1997) and Hamilton et al.'s (2003) with 5-, 8- and 10-year-old children. Again the results supported the idea of a fractionated visuo-spatial system, where memory for visual patterns and movement sequences are handled differently. However, Pickering et al. (2001) argued that the separable subsystems reflected a static/dynamic distinction rather than a visual/spatial one.

In summary, converging evidence supports Logie's (1995) idea of a fractionated visuo-spatial working memory system in both adults and children. Based on the assumption of developmental fractionation within the visuo-spatial working memory system, it is suggested that the visual cache (visual temporary store) and the inner scribe (temporary spatial store) might be differentially related to children's mathematics performance.

The aim of the present study was to further investigate the association between visuo-spatial sketchpad scores and children's National Curriculum mathematics performance reported in Chapter 3. Importantly, several measures of visuo-spatial working memory were administered. The tasks were selected on the basis that they were presumed to measure the two subcomponents of visuo-spatial working memory. The Visual Patterns Test (Della Sala, Gray, Baddeley & Wilson, 1997) was selected as a standardized measure of the visual subcomponent and the Block Recall (WMTB-C, Pickering & Gathercole, 2001), a version of the Corsi block task, was selected a standardized measure of the spatial subcomponent. These tasks have been used in previous studies where a dissociation between the two subcomponents has been found with children (e.g. Logie & Pearson, 1997). Furthermore, spatial interference tasks cause selective interference in performance on the Corsi task (e.g. Smyth & Pendleton, 1989) and visual interference tasks selectively disrupt performance on the Visual Patterns Test (Della Sala et al., 1999), suggesting the two tasks tap distinct abilities. Two relatively new tasks, Blobby Spatial (Phillips & Hamilton, 2001) and Blobby Visual (Phillips & Hamilton, 2001), were included as additional measures of separate visual and spatial working memory abilities. These were included as they were designed to isolate visual and spatial components from one another and from verbal and executive working memory resources (Phillips & Hamilton, 2001). As discussed in Chapter 1 (section 1.4.2.2) it has been suggested that existing visuo-spatial sketchpad measures may recruit executive resources, and subsequently compromise the observation of the contribution of visuo-spatial working memory skills to mathematics performance. Therefore, it was important to measure visual and spatial abilities that were free from executive demands to measure the contribution of visuo-spatial working memory to mathematics per se. As Hamilton et al., (2003) suggest, using such measures may define a more pertinent role for visuo-spatial working memory in mathematics. A fifth visuo-spatial working memory task, Mazes Memory (Pickering & Gathercole, 2001), was included for two reasons. Firstly, because it was found to predict unique variance in children's mathematics performance in the previous study (Chapter 3) and secondly, because it is arguably both visual and spatial in nature.

In summary, the aim of the present study was to further investigate the association between children's visuo-spatial working memory scores and their National Curriculum mathematics abilities using multiple measures of visuo-spatial working memory ability. A second aim was to explore the structure of visuo-spatial working memory in children, and subsequently explore the contributions of the subcomponents of visuo-spatial working memory to children's mathematics performance. As in Chapter 3, none of the working memory measures were digit-based so as to eliminate the chances of detecting a general ability to process number or numerical information across the working memory and mathematics tasks. As before, children's mathematics ability was assessed using the tests developed in Chapter 2. A measure of NVIQ was also included to explore the effect of controlling for individual differences in NVIQ on the relationship between visuo-spatial working memory abilities and mathematics performance.

Method

Participants

The participants were 107 children (55 boys and 52 girls), who attended a primary school in the North-East of England. There were 51 Year 3 children (28 boys and 23 girls), mean age 7 years and 7 months (*SD* = 3.7 months, range 7 years 1 month to 8 years 3 months), and 56 Year 5 children (27 boys and 29 girls), mean age 9 years and 7 months (*SD* = 3.8 months, range 9 years 3 months to 10 years 3 months). No children were excluded due to any intellectual or behavioural difficulties.

The percentage of children achieving Level 4 attainment and above in English, Mathematics and Science was 87%, 89% and 90% respectively. This was higher than the national average of 75% in English, 72% in Mathematics and 85% in Science.

Design and Procedure

All children participated in three testing sessions. In the first session, each child was administered five visuo-spatial sketchpad tasks in a counterbalanced order. Each child was tested individually in a quiet area of the school. In the second session, the children were administered age appropriate mathematics assessments under standardized test conditions within a classroom setting. In the final session, the NV IQ test was administered, again under standardized test conditions within a classroom setting.

Materials

Visuo-spatial Sketchpad Tasks

Five non-digit based visuo-spatial sketchpad tasks were used to assess the visual and spatial subcomponents of visuo-spatial working memory. The Visual Patterns Test (Della Sala et al., 1997) and Blobby Visual (Phillips & Hamilton, 2001) were used as measures of the visual subcomponent. Block Recall (Pickering & Gathercole, 2001) and Blobby Spatial (Phillips & Hamilton, 2001) were used as measures of the spatial subcomponent. The Mazes Memory task (Pickering & Gathercole, 2001) was used as a measure of both visual and spatial working memory abilities.

Visual Patterns Test

The Visual Patterns Test (Della Sala et al., 1997) involves the presentation of matrices in which some of the squares are filled black and some are unfilled. Each matrix is presented for 2 seconds. Participants are asked to look carefully at each matrix and try to remember where the black squares are. After a half-second delay participants are asked to recall the black squares in a blank matrix, which they fill in. Testing begins with a block of three trials, each of which is a 2 x 2 matrix in which two of the square cells are filled. The size of the matrix increases by 2 cells every three trials (or block), while the number of filled cells increases by one every three trials. If one or more correct responses are given within a block of three trials, the experimenter proceeds to the next block. Credit is not given for incorrect trials. Testing continues until 3 incorrect responses are given in a block.

The score given was the Trials Correct. Reponses for each trial are scored as 0 or 1. The sum of the correct responses provides the Trials Correct score. The maximum score is 42. The test-retest reliability of this task with British adults is .75 for Version A and .73 for Version B. Coefficients are not available for children.

Mazes Memory

See Chapter 3 for details of the Mazes Memory task (Pickering & Gathercole, 2001).

Block Recall

The Block Recall task (Pickering & Gathercole, 2001) involves the presentation of sequences tapped out on blocks on a board of 9 randomly distributed blocks for immediate serial recall. The blocks are tapped at a rate of one per second. Participants are asked to recall the tapped sequence in exactly the same order as it was presented. Testing begins with a block of six trials, in which each sequence contains only one tap on a single block. The sequence length increases at a rate of one block (or tap) every six trials. Within each trial any block is only tapped once. If 4 correct responses are given within in a block, the experimenter proceeds to the next block, giving credit for any omitted trials. Testing continues until 3 incorrect responses are given in a block.

The score given was the Trials Correct score. Responses for each trial are scored as 0 or 1. The sum of the correct responses provides the Trials Correct score. The maximum score is 54. Test-retest reliability for this task is .43 for Year 5 and Year 6 children (Pickering & Gathercole, 2001). Coefficients are not available for Year 3 children, although the test-retest reliability for younger children (Year 1 and Year 2) is .63 (Pickering & Gathercole, 2001).

Blobby Visual

The Blobby Visual task (Phillips & Hamilton, 2001) requires participants to remember the size of squares presented on a computer screen on the stomach of a Mr Blobby figure. Each square is presented for 2 seconds, followed by a delay of 4 seconds (to ensure no perceptual image remains), and then a second square is presented. Participants are asked to judge whether the second square was the same size or a different size to the first square presented. Testing begins with a size step of 50% between the "different" squares and reduces every block of twenty trials to 40%, 30%, 20%, 10% and finally 5%. If more than 10 consecutive correct responses are given within a block, the experimenter proceeds to the next block of trials, giving credit for the omitted trials. Testing continues until 5 incorrect responses are given in a block.

The score given was the Trials Correct score. The computer programme produced the number of errors within each block of twenty trials. The number of errors was subtracted from twenty to give the number of correct trials per block. The sum of the trials correct for each block provides the total Trials Correct score. The maximum score is 120 trials (6 blocks of twenty trials). The task is unstandardized meaning no reliability coefficients are available.

Blobby Spatial

The Blobby Spatial task (Phillips & Hamilton, 2001) requires participants to remember the movement trajectory of spots across a computer screen. A spot moved obliquely across a computer screen, then disappeared for 4 seconds behind a Mr. Blobby figure, then reappeared either moving along the same or a different trajectory. Participants are asked to judge whether the spot is moving along the same trajectory or a different trajectory. Testing begins with a change in trajectory direction of 27% and reduces every 20 trials to 21%, 18%, 13%, 9% and finally 5%. If more than 10 consecutive correct responses are given within a block, the experimenter proceeds to the next block of trials, giving credit for the omitted trials. Testing continues until 5 incorrect responses are given in a block.

The score given was the Trials Correct score. The computer programme produced the number of errors within each block of twenty trials. The number of errors was subtracted from twenty to give the number of correct trials per block. The sum of the trials correct for each block provides the total Trials Correct score. The maximum score is 120 trials (6 blocks of twenty trials). The task is unstandardized meaning no reliability coefficients are available.

Mathematics Tasks

See Chapter 2 for details of the age appropriate assessments used to measure the National Curriculum mathematical skills Number and Algebra, Shape, Space and Measures, Handling Data and Mental Arithmetic.

NVIQ Task

See Chapter 3 for details of the MAT-SF (Naglieri, 1985).

Results

Power Analysis

Erdfelder's (1984) compromise power analysis was conducted to determine the statistical power of this study. The results of power analyses conducted using Faul and Erdfelder's (1992) G Power programme, are presented in Table 4.1.

Table 4.1

Compromise Power Analysis for Visuo-spatial Abilities and Curriculum-Based Mathematical Skills Study.

Effect Size	n_1	n_2	Power
0.5 (medium)	51	56	.90

The power of this study to test for significance with a medium effect size is .90. This exceeds Cohen's (1988) criterion of .8, meaning this study is statistically powerful.

Descriptive Statistics

Descriptive statistics for visuo-spatial working memory measures, mathematics test performance and NVIQ scores are presented in Table 4.2.

Table 4.2

Descriptive Statistics of Visuo-Spatial Sketchpad and Mathematics Measures (maximum score for visuo-spatial measures shown in brackets). (N = 107).

Measures	Year 3 (n = 51)		Year 5 (n = 56)	
	M	*SD*	*M*	*SD*
Visuo-spatial Sketchpad Measures				
Visual Patterns Test (42)	9.00	2.75	11.27	3.60
Mazes Memory (42)	9.52	4.10	14.00	6.58
Block Recall (54)	23.28	4.19	25.38	2.97
Blobby Visual (120)	47.06	25.47	58.14	23.50
Blobby Spatial (120)	6.84	15.24	10.61	18.17
Mathematics Measures				
Number and Algebra (15)	55.56	22.41	56.42	21.70
Shape, Space and Measures (15)	57.78	19.53	58.52	12.19
Handling Data (15)	60.39	17.55	57.53	14.55
Mental Arithmetic (15)	61.37	24.33	67.03	20.88
NVIQ (34)	15.50	5.16	20.33	5.15

Note. Mathematics scores shown are proportions correct.

Year 5 performed significantly better than Year 3 on all visuo-spatial sketchpad measures ($p<.05$), except Blobby Spatial ($p>.05$). All children performed poorly on this measure, with most failing to pass Level 1 (20 trials correct), indicating a floor effect. Across both age groups, children performed better on the Block Recall task than the other three standardized visuo-spatial measures. There was greater variability on the unstandardized visuo-spatial measures than the standardized measures. Children's performance did not differ significantly across the mathematics assessments, but Year 5 performed significantly better on the NVIQ measure than Year 3 children ($p<.01$). Separate mean scores for boys and girls are not shown as there were no significant differences performance across the measures for either age group ($p>.05$).

Correlation Analyses

Associations between visuo-spatial sketchpad measures and mathematical abilities are presented in the correlation matrix (Table 4.3). Simple correlations are displayed in the upper triangle; partial correlation coefficients controlling for age are displayed in the lower triangle. The data was collapsed across all children, as there were no significant differences in performance ($p<.05$) between the two age groups on the age appropriate mathematics assessments.

Scores on the standardized visuo-spatial sketchpad measures were intercorrelated (all rs > .30, $p<.01$), even when the variance related to age was eliminated (all prs >.20, $p<.01$). The unstandardized visual component measure (Blobby Visual) was significantly related to the standardized visuo-spatial sketchpad measure that was visual in nature (Visual Patterns Test) ($r > .30$, $p<.01$), even when the variance related to age was eliminated ($pr >.20$, $p<.01$). The unstandardized spatial component measure was significantly related to both the

Table 4.3

Correlation Matrix for Visuo-spatial Sketchpad and Mathematics Measures. Simple coefficients are displayed in the upper triangle; partial coefficients are displayed in the lower triangle. (*N*=107).

	Visual Patterns Test	Mazes Memory	Block Recall	Blobby Visual	Blobby Spatial	Number and Algebra	Shape, Space and Measures	Handling Data	Mental Arithmetic
Visual Patterns Test	-	.41**	.41**	.33**	.21*	.32**	.09	.16	.21**
Mazes Memory	.35**	-	.32**	.13	.05	.21*	-.01	.02	.16
Block Recall	.31**	.22**	-	.14	.20*	.25*	.02	-.01	.22*
Blobby Visual	.28**	.05	.07	-	.06	.34**	.21**	.15	.28**
Blobby Spatial	.18	.01	.18	.02	-	.22*	-.02	.20*	.27**
Number and Algebra	.27**	.15	.20*	.31**	.20*	-	.40**	.40**	.75**
Shape, Space and Measures	.11	.00	.03	.22*	-.01	.42**	-	.22*	.54**
Handling Data	.22*	.07	.03	.18	.22*	.44**	.22*	-	.44**
Mental Arithmetic	.17	.12	.19	.26*	.25*	.74**	.55**	.47**	-

standardized visuo-spatial measures that were visual (Visual Patterns Test) and spatial (Block Recall) in nature (all *r*s > .20, $p<.05$). However, these associations were accounted for by age-related variance (all *pr*s < .30, $p>.05$), meaning performance on the unstandardized spatial component measure was not related to performance on the other visuo-spatial sketchpad measures. Performance on the mathematics assessments was intercorrelated, even when the variance related to age was eliminated (all *r*s >.20, $p<.05$, all *pr*s >.20, $p<.01$).

Scores on all visuo-spatial sketchpad measures were significantly related to Number and Algebra abilities (Visual Patterns Test and Blobby Visual *r*s >.30, $p<.01$, Mazes Memory, Block Recall and Blobby Spatial *r*s >.20, $p<.05$) and remained so after age-related variance was eliminated (Visual Patterns Test and Blobby Visual *pr*s >.20, $p<.01$, Block Recall and Blobby Spatial *pr*s >.20, $p<.05$), excluding Mazes Memory (*pr*s <.30, $p>.05$).

Scores on all visuo-spatial sketchpad measures, excluding Mazes Memory, were significantly related to Mental Arithmetic abilities (Visual Patterns Test, Blobby Visual and Blobby Spatial *r*s >.20, $p<.01$, Block Recall *r* >.20, $p<.05$). However, only scores on the two unstandardized visuo-spatial working memory measures remained significantly associated with Mental Arithmetic ability after age-related variance was eliminated (Blobby Visual and Blobby Spatial *pr*s >.20, $p<.05$).

Shape, Space and Measures abilities were significantly related to Blobby Visual scores (*r* >.20, $p<.01$) and remained so after the variance related to age was controlled for (*pr* >.20, $p<.05$).

Handling Data abilities were significantly related to Blobby Spatial scores (*r* >.20, $p<.05$), even when the variance related to age was eliminated (*pr* >.20, $p<.05$).

Visual Patterns Test scores were also significantly related to Handling Data abilities when age-related variance was controlled for (*pr* >.20, *p*<.05).

Associations between visuo-spatial sketchpad scores and mathematical skills, controlling for NVIQ are presented in Table 4.4. Chronological age was controlled to eliminate age –related variance on the NVIQ measure.

Table 4.4

Correlation Matrix for Visuo-spatial Sketchpad Measures and Mathematics Assessments, Controlling for Age and NVIQ Scores. (N = 107).

	Visual Patterns Test	Mazes Memory	Block Recall	Visual Blobby	Spatial Blobby
Number and Algebra	.16	.08	.11	.25*	.17
Shape, Space and Measures	.00	-.05	-.07	.15	-.05
Handling Data	.11	-.00	-.09	.11	.18
Mental Arithmetic	.07	.07	.10	.18	.22*

Scores on standardized visuo-spatial sketchpad measures were not significantly related to mathematical abilities when individual differences in NVIQ were controlled for (all *prs* <.30, *p*>.05). A possible explanation for this may be that the NVIQ task and the standardized visuo-spatial sketchpad tasks are both measuring non-verbal skills. Therefore, controlling for NVIQ may eliminate the variance associated with non-verbal abilities, effectively eliminating the variance of interest associated with visuo-spatial sketchpad scores.

Controlling for individual differences in NVIQ also eliminated many of the significant associations between scores on the unstandardized visuo-spatial sketchpad measures and mathematics performance. However, Visual Blobby was significantly related to Number and Algebra ability and Spatial Blobby was significantly related to Mental Arithmetic ability (both $rs<.30$, $pr<.05$), supporting the notion that these measures may be executive-free (Phillips & Hamilton, 2001).

Regression Analyses

A simple linear regression analysis revealed that all visuo-spatial working memory measures predicted 17.4% of the variance in overall mathematics performance. Subsequently, a series of fixed-order unique variance regression analyses were used to assess the amount of unique variance in mathematics scores predicted by each of the measures. For each analysis the mathematics assessment was the regressor and the unique contribution (measured as r^2) of each working memory measure was assessed as a predictor entered into the regression equation after the other predictors. The data was collapsed across all children. Age was entered as the first variable into each regression equation to control for age-related variance. See Table 4.5 (Appendix VIII) for results.

Models A_1, A_2, A_3, A_4 and A_5 show the amount of unique variance in mathematics scores predicted by scores on the Visual Patterns Test when the variance attributed to age and performance on the other visuo-spatial working memory scores is accounted for. These models indicate that performance on the Visual Patterns Test accounts for no unique variance in overall mathematical ability, Shape Space and

Measures or Mental Arithmetic scores (0%). However, it does account for a small, but significant, amount of variance in Number and Algebra scores (1%) and Handling Data scores (2%).

Models B_1, B_2, B_3, B_4 and B_5 show the amount of unique variance in mathematics ability predicted by Mazes Memory scores when age-related variance and performance on the other visuo-spatial working memory measures is controlled for. These models indicate that Mazes Memory scores do not predict unique variance in any mathematical abilities (Number and Algebra, Shape, Space and Measures, Handling Data and overall mathematics performance) other than Mental Arithmetic scores (1%).

Models C_1, C_2, C_3, C_4 and C_5 show the amount of unique variance in mathematics scores predicted by scores on the Block Recall task when the variance attributed to age and performance on the other visuo-spatial working memory scores is accounted for. These models indicate that Block Recall scores account for no unique variance in Shape, Space and Measures or Handling Data scores, but that they account for a significant, amount of variance in Number and Algebra scores (1%), Mental Arithmetic scores (1%) and overall mathematics ability (1%).

Models D_1, D_2, D_3, D_4 and D_5 show that of the visuo-spatial working memory measures Blobby Visual scores account for the greatest amount of unique variance in mathematics scores. After the variance contributed by age and performance on the other visuo-spatial measures is accounted for Blobby Visual scores account for 7% of variance in overall mathematical ability, 6% of variance in Number and Algebra scores, 3% of variance in Shape, Space and Measures scores, 2% of variance in Handling Data scores and 3% of variance in Mental Arithmetic scores.

Models E_1, E_2, E_3, E_4 and E_5 show the amount of unique variance in mathematics skills predicted by Blobby Spatial scores when the variance predicted by age and performance on the other visuo-spatial sketchpad measures is controlled for. These models indicate that Blobby Spatial scores do not account for unique variance in Shape, Space and Measures abilities, but that they do account for unique variance in overall mathematical ability (3%), Number and Algebra scores (2%), Handling Data scores (4%) and Mental Arithmetic scores (5%).

A second series of fixed-order unique variance regression analyses were conducted to assess the amount of unique variance in mathematics scores predicted by each of the visuo-spatial working memory measures after the variance accounted for by NVIQ was considered. Again, for each analysis the mathematics assessment was the regressor and the unique contribution (measured as r^2) of each working memory measure was assessed as a predictor entered into the regression equation after the other predictors, which included age, NVIQ and performance on the other visuo-spatial working memory measures. As before, the data was collapsed across all children. See Table 4.6 (Appendix IX) for results.

Models F_1 F_2 F_3 F_4 F_5 (Visual Patterns Test) G_1 G_2 G_3 G_4 G_5 (Mazes Memory) and H_1 H_2 H_3 H_4 H_5 (Block Recall) show the amount of unique variance in mathematics skills predicted by each standardized measure of the visuo-spatial sketchpad after the variance attributed to age, individual differences in NVIQ and performance on the other visuo-spatial measures has been controlled for. Again, the standardized visuo-spatial sketchpad measures predict little (maximum 2%), if any, unique variance in mathematics performance. Visual Patterns scores only account for 1% of unique variance in Handling Data abilities, Mazes Memory scores only account for 1% of unique variance in Mental Arithmetic scores and Block Recall scores only account for a small amount of unique variance in Shape, Space and Measures (1%), Handling Data (2%) and Mental Arithmetic abilities (1%).

Models I_1 I_2 I_3 I_4 I_5 show that even when individual differences in NVIQ, age and performance on the other visuo-spatial measures are controlled for, Blobby Visual scores account for the greatest amount of unique variance in mathematics abilities. These models indicate that Blobby Visual scores account for unique variance in overall mathematics ability (5%), Number and Algebra scores (5%), Shape, Space and Measures scores (2%), Handling Data scores (1%) and Mental Arithmetic scores (3%).

Models J_1 J_2 J_3 J_4 J_5 show the amount of unique variance in mathematics scores predicted by Blobby Spatial scores when the variance attributed to individual differences in NVIQ, age and performance on the other visuo-spatial measures is controlled for. These models indicate that Blobby Spatial scores account for 2% of variance in overall mathematical ability, 1% of variance in Number and Algebra scores, 3% of variance in Shape, Space and Measures scores, 3% of variance in Handling Data scores and 3% of variance in Mental Arithmetic scores.

Overall, the regression analyses show that visuo-spatial sketchpad measures account for variance in mathematical performance. However, individual standardized visuo-spatial sketchpad measures do not predict much (maximum of 2%), if any, unique variance in mathematics performance after the variance attributed to the other visuo-spatial measures and individual differences in NVIQ are controlled for. A possible reason for this is that controlling for performance on the other visuo-spatial measures and the NVIQ measure (that may be measuring the same visuo-spatial abilities) is eliminating the variance in visuo-spatial abilities from the regression equation before the final predictor (or visuo-spatial measure) is entered. This may result in the final predictor accounting for little, or no further variance in performance. Conversely, each of the unstandardized visuo-spatial measures predicted unique variance in mathematics performance when performance on the other visuo-spatial measures and the NVIQ measure was controlled for. This suggests that they may account for additional variance in visuo-spatial abilities beyond that accounted for by existing standardized visuo-spatial sketchpad measures. Furthermore, when taken together these findings suggest that the unstandardized measures of visuo-spatial working memory may be measuring different abilities to the standardized visuo-spatial measures (given that they predict unique variance in performance over and above that predicted by the standardized measure). The results of subsequent factor analyses will address this issue.

Exploratory Factor Analyses

As in Chapter 3, principal components analyses were conducted to determine the higher-order factor structure underpinning variations in scores on all measures as a descriptive, summative method prior to conducting model-based techniques.

Three factors emerged with eigenvalues in excess of 1.00 in the first analysis, which included all measures. Factor loadings greater than .30 on the rotated component matrix are presented in Table 4.7.

All four measures of mathematics ability loaded on Factor 1, all with similar loadings in excess of .60. The three standardized visuo-spatial sketchpad measures loaded on Factor 2, as did the NVIQ measure. This suggests that all four measures may be measuring the same ability, which probably corresponds to a visuo-spatial ability.

Table 4.7

Factor Loadings of Visuo-spatial, Mathematics and NVIQ Measures on Rotated Component Matrix.

Measure	Factor		
	1	2	3
Visuo-spatial Working Memory Measures			
Visual Patterns Test		.77	
Mazes Memory		.74	
Block Recall		.66	
Blobby Visual	.37	.37	-.40
Blobby Spatial			.82
Mathematics Measures			
Number and Algebra	.78		
Shape, Space and Measures	.71		
Handling Data	.63		
Mental Arithmetic	.86		
NVIQ		.66	

Only loadings greater than .30 are shown.

KMO measure of sampling adequacy = .77.

Blobby Visual loads on both the Mathematics Factor (Factor 1) and the Visuo-spatial Factor (Factor 2), suggesting that it is measuring both visuo-spatial and mathematics abilities. It is possible that it measures visual memory in a mathematical way. That is, because the task requires children to remember the size of different visually presented squares, it is arguably requiring mathematical judgements (size judgements) to be made.

Blobby Spatial loaded positively on a third unknown factor, but it did not load on either Factor 1 or Factor 2. As such, it is difficult to infer what it may be measuring. One possibility may be that it loads on a separate factor to the other visuo-spatial measures due to a floor effect (see Table 4.2.). However, the Blobby Visual task, which did not show a floor effect, also loads on this third factor. An alternate explanation may be that both the Blobby Visual and Blobby Spatial tasks are tapping a different ability to the standardized visuo-spatial sketchpad tasks. It is unlikely that they are measuring separate visual and spatial memory abilities as, although the nature of the two tasks differs along this dimension, they both load on the same factor. One possibility is that they might be tapping a visuo-spatial memory ability that is free of executive demands (e.g. Phillips & Hamilton, 2001). Alternatively, they may have loaded on a separate factor because they were recognition, not recall, tasks. That is, the two Blobby tasks required participants to make "same / different" judgements, while the other visuo-spatial tasks required participants to recall stimuli. It is not possible to infer what these two tasks are measuring from the present data, as executive measures were not used.

Two factors with eigenvalues in excess of 1.00 emerged in the second analysis, where the unstandardized visuo-spatial measures were not included. The two unstandardized measures were not included due to the problems defining what they were measuring in the first analysis. Factor loadings greater than .30 on the rotated component matrix are presented in Table 4.8.

Table 4.8

Factor Loadings of Visuo-spatial, Mathematics and NVIQ Measures on Rotated Component Matrix, excluding Unstandardized Visuo-spatial Measures.

Measure	Factor	
	1	2
Visuo-spatial Working Memory Measures		
Visual Patterns Test		.76
Mazes Memory		.75
Block Recall		.69
Mathematics Measures		
Number and Algebra	.78	
Shape, Space and Measures	.72	
Handling Data	.64	
Mental Arithmetic	.87	
NVIQ		.67

Only loadings greater than .30 are shown.

KMO measure of sampling adequacy = .78.

Again, all four measures of mathematics ability loaded on Factor 1, all with similar loadings in excess of .60. The three standardized visuo-spatial sketchpad measures loaded on Factor 2, as did the NVIQ measure. This again suggests that all four tasks may have similar demands or that both NVIQ and working memory share variance. It is probable that all measures are tapping a visuo-spatial ability.

Confirmatory Factor Analysis

As in Chapter 3, confirmatory factor analysis was used as an exploratory procedure to show the relationship between the constructs suggested by the principal components analyses. Coefficients for the paths between latent constructs and variables were produced for each model assessed using EQS 6 (Bentler, 2001). The same fit statistics were produced to indicate the goodness of fit of the models to the data (χ^2, *CFI, IFI, GFI, SRMR* and *RMSEA*). These fit indices were compared across competing models to find the best theoretical model of the data.

The two unstandardized visuo-spatial sketchpad tasks were not included in further analyses as it was unclear what they were measuring. Simple regression analyses revealed that the three remaining visuo-spatial sketchpad tasks accounted for 8% of variance in overall mathematics ability, 12% of variance in Number and Algebra scores, 1% of variance in Shape, Space and Measures scores, 4% of variance in Handling Data scores and 8% of variance in Mental Arithmetic scores. When the NVIQ task was included in the regression equation the amount of variance accounted for in each mathematical skill increased to 13%, 16%, 4%, 8% and 9% respectively. The second principal components analysis, which included the same measures (the NVIQ, standardized visuo-spatial sketchpad and mathematics tasks), yielded a two-factor solution (see Table 4.8.), but it did not indicate the correlation between the two latent constructs. Four factor models were tested in a series of confirmatory factor analyses to further investigate the relationship between visuo-spatial skills and mathematics ability.

The input to the programme was the raw data, from which a covariance matrix was computed for the analyses. For all models, the mathematics factor corresponded to the structure of the National Curriculum, comprising of four measures of mathematics ability; Number and Algebra, Shape, Space and Measures, Handling Data and Mental Arithmetic. The visuo-spatial factor differed across the models. The first model (CFA1) corresponded to visuo-spatial working memory, with only the standardized visuo-spatial measures included. The second model (CFA2) corresponded to a general visuo-spatial ability, with measures representing visuo-spatial working memory (Visual Patterns Task, Mazes Memory and Block Recall) and non-verbal skills (NVIQ measure). In the third (CFA3) and fourth (CFA4) models tested the position of the NVIQ measure was manipulated. In the third model (CFA3) the NVIQ measure was included as an independent variable, free to share variance with a visuo-spatial working memory factor, which comprised of the three visuo-spatial sketchpad tasks. In the fourth model (CFA4), the NVIQ measure was again an independent variable, but this time it was free to share variance with both the visuo-spatial working memory and mathematics factors. Fit statistics for all four models are presented in Table 4.9.

Model CFA3 yielded low fit indices (*CFI*, *GFI* and *IFI*) and differed significantly to the data ($p<.05$), indicating that it is a poor fit. Model CFA4 yielded perfect fit indices (*CFI* and *IFI*) and did not differ significantly to the data ($p>.05$, and *SRMR* and *RMSEA* <.08). However, examination of the diagram for CFA4 revealed that the paths between the Visual Patterns Test and Factor 1 and the Mental Arithmetic task and Factor 2 were not significant. This suggests that the model is not a good description of the data.

Table 4.9

Goodness of fit statistics for CFA models (visuo-spatial working memory, Mathematics and NVIQ).

Model	*Df*	χ^2	*p*	*CFI*	*GFI*	*IFI*	*SRMR*	*RMSEA*
CFA1	13	13.50	.41	.99	.96	.99	.05	.02
CFA2	19	19.43	.43	.99	.95	.99	.06	.02
CFA3	19	30.37	.04	.95	.93	.95	.13	.08
CFA4	18	16.96	.52	1.00	.96	1.00	.05	.00

Note. *CFI* = Bentler's Comparative Fit Index. *GFI* = Goodness of Fit Index. *IFI* = Bollen's Incremental Fit Index. *SRMR* = Standardized Root Mean Squared Residual. *RMSEA* = Root Mean Square Error of Approximation.

Models CFA1 and CFA2 provided the best fit to the data. Both yielded fit indices in excess of .95 (*CFI* and *IFI)* and .9 (*GFI*) indicating good fit. Neither models differed significantly to the data, yielding *RMSEA* and *SRMR* values below .08. The χ^2 value for model CFA1, with 13 degrees of freedom, was 13.5, $p = .41$. A significant path existed between the visuo-spatial working memory construct and mathematics performance; the path covariance coefficient was .36, $p<.05$. The χ^2 value for model CFA2, with 19 degrees of freedom, was 19.43, $p = .43$. A significant path existed between the visuo-spatial ability construct and mathematics performance; the path covariance coefficient was .44, $p<.05$. Diagrammatic representations of the models are shown in Figure 4.1. and Figure 4.2.

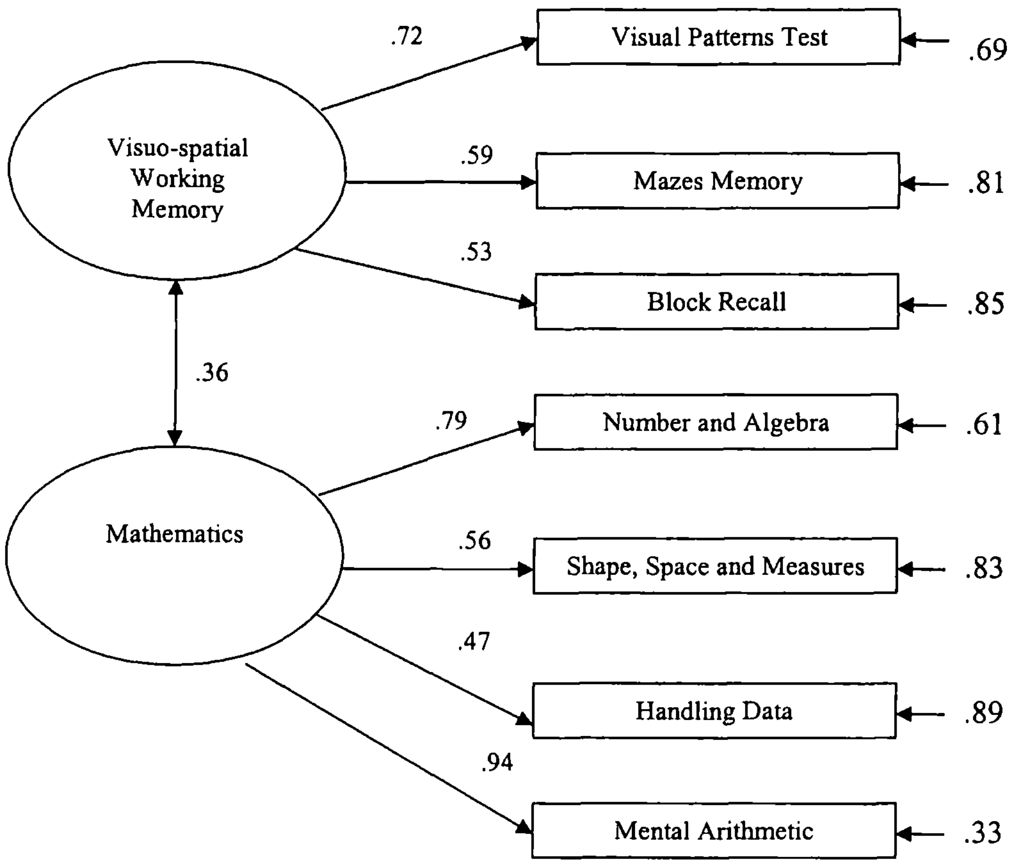


Figure 4.1

Diagrammatic Representation of the one of the Best Fitting Factor Model (CFA1) for Visuo-spatial Working Memory and Curriculum-Based Mathematics

There was no significant difference between the fit of the two best fitting models (models CFA1 and CFA2, $p<.05$). Structurally they differed due to the inclusion of the NVIQ measure (see Figures 4.1.and 4.2.). Therefore, the results suggest that while the NVIQ and visuo-spatial sketchpad tasks share variance and load on the same factor, including the NVIQ measure does not significantly alter the fit of the model. Inclusion of the NVIQ measure only increased the path covariance coefficient between the two factors, visuo-spatial working memory and mathematics, by .08. This indicates that visuo-spatial working memory is significantly associated with mathematics performance independent of NVIQ.

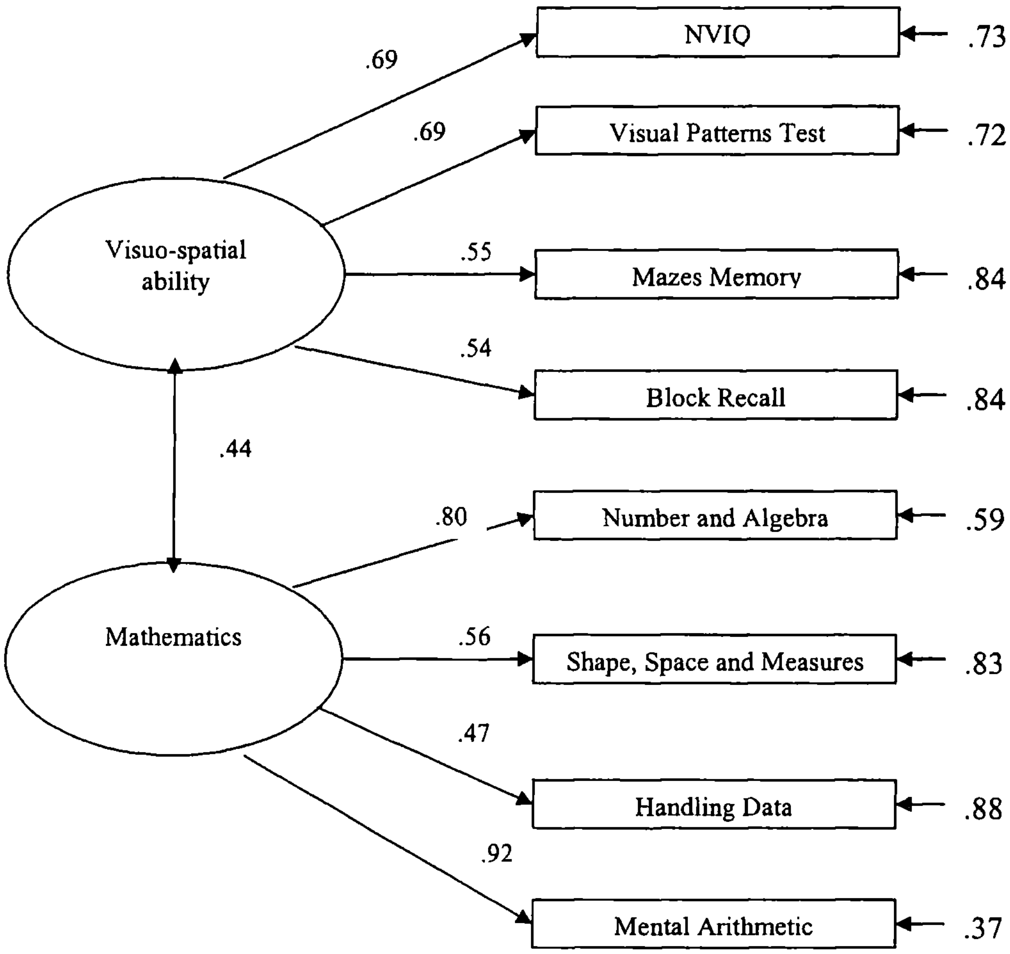


Figure 4.2

Diagrammatic Representation of the one of the Best Fitting Factor Model (CFA2) for Visuo-spatial Working Memory, Curriculum-Based Mathematics and NVIQ.

Discussion

The results provide further evidence for an association between children's visuo-spatial sketchpad ability and their mathematics attainment. A simple regression analysis revealed that the five measures of visuo-spatial ability used in the present study predicted 17.4% of the variance in children's mathematics scores. A further simple regression analysis suggested that the standardized visuo-spatial sketchpad measures alone predicted 8% of the variance in children's mathematics scores. In line with this, the results of the confirmatory factor analysis revealed that a significant path existed between a visuo-spatial sketchpad construct and mathematics performance. Although this relationship may reflect an association between visuo-spatial cognition and mathematics (e.g. Dehaene et al., 1999), the findings support the notion that visuo-spatial working memory is involved in children's mathematics per se.

The contribution of the visuo-spatial sketchpad component of Baddeley and Hitch's (1974) working memory model to a range of children's National Curriculum-based mathematical skills was explored using five non-digit based measures of visuo-spatial working memory. The results suggested that visuo-spatial working memory predicted children's mathematics performance. This is consistent with the suggestion that working memory assessments predict wider aspects of curriculum attainment (e.g. Gathercole & Pickering, 2000a; 2000b), and that working memory assessments may be useful as early indicators of scholastic attainment. Furthermore, it extends previous findings that visuo-spatial working memory assessments predict Mathematics curriculum attainment (e.g. Jarvis & Gathercole, 2003) to suggest that visuo-spatial sketchpad assessments could be used to predict attainment in different areas of the mathematics curriculum. Visuo-spatial sketchpad scores were found to predict all four areas of the mathematics curriculum defined by the National Curriculum for England. Consistent with Chapter 3, there was little difference between the working memory demands of each mathematical skill (e.g. Visual Patterns Test, Mazes Memory and Block Recall test scores predicted between 0% and

2% of unique variance across all mathematical skills). This provides further evidence to suggest that different mathematical skills do not recruit different working memory resources.

Collectively the five measures of visuo-spatial sketchpad ability predicted children's mathematics performance. This supports previous findings that the visuo-spatial sketchpad supports children's mathematics (e.g. McKenzie et al., 2003) and provides further evidence to support the suggestion from neuropsychological literature that the nature of visuo-spatial cognition important for mathematics incorporates a visuo-spatial working memory system (e.g. Zago & Tzourio-Mazoyer, 2002). However, detailed analyses revealed that each measure predicted little unique variance in mathematics performance after the variance attributed to age and performance on the other measures was controlled for. This was especially true for the standardized visuo-spatial sketchpad measures. A possible explanation for this is that controlling for performance on the other visuo-spatial measures (that are theoretically measuring the same ability) eliminated the variance in mathematics performance predicted by visuo-spatial ability before the final predictor (or measure) was entered into the regression equation. Indeed, the results of the exploratory factor analysis (Table 4.7) suggest that the standardized visuo-spatial sketchpad measures were measuring the same ability. It therefore follows that each measure predicted little or no unique variance in mathematics performance when entered after the other visuo-spatial measures.

One aim of the present study was to further investigate the role of the visuo-spatial sketchpad in children's mathematics using multiple assessments of visuo-spatial sketchpad functioning. Importantly, these measures were selected to measure the potentially distinct visual and spatial subcomponents of visuo-spatial working memory (Logie, 1995) to investigate whether one might be more important for supporting children's mathematics than the other.

The results did not highlight a differential pattern of associations between children's visual and spatial working memory abilities and mathematics. For example, two standardized tasks that have been previously found to differentiate visual and spatial subcomponents of visuo-spatial working memory in children (Visual Patterns Test and a version of Block Recall, Logie & Pearson, 1997) were both significantly associated with the same two mathematical abilities (Number and Algebra and Mental Arithmetic) and both predicted a similar amount of unique variance in all mathematics abilities (between 0% and 2%). There are two possible explanations for this pattern of results. Firstly, it may be that the visuo-spatial sketchpad is not fractionated into visual and spatial subcomponents in children (e.g. Pickering et al., 2001). Alternatively, it may be that the tasks used may not have measured distinct visual and spatial memory abilities. These two explanations will be discussed in turn.

Contrary to Logie and Pearson's (1997) finding of developmental fractionation, the present findings imply that the visuo-spatial sketchpad may not be fractionated into visual and spatial subcomponents in children. Both subcomponents defined by Logie (1995) shared similar patterns of associations with mathematics ability in children. One possibility is that the visuo-spatial sketchpad is fractionated into alternate static and dynamic (Pickering et al., 2001) or active and passive components (Vecchi, 1998; Vecchi & Cornoldi, 1999; Vecchi & Girelli, 1998; Vecchi, Monticellai & Cornoldi, 1995).

In a study investigating the fractionation of visual and spatial subcomponents in children, Pickering et al. (2001) reported a developmental dissociation in performance between static and dynamic versions of a matrices task. They compared

5-, 8- and 10-year-olds performance on the tasks and found that memory for static matrices was superior to memory for dynamic matrices and that this difference increased with age. The same pattern emerged with a Mazes Memory task where 8- and 10-year-olds performed better on static than dynamic versions of the task. Consequently, they suggested that children's visuo-spatial memory systems might be comprised of separate subsystems for dealing with static and dynamic visuo-spatial information. Related to this, Vecchi and colleagues have argued for a passive versus active distinction in visuo-spatial working memory. According to their suggestion passive memory (or processing) refers to the recall of information in the same format it was memorised (similar to traditional visuo-spatial sketchpad tasks) while active memory refers to the recall of information that has to modified, integrated or transformed (mental rotation or image subtraction tasks). They suggest that a passive-active distinction better accounts for their data from blind and sighted adults and children than a visual-spatial one (Vecchi, 1998; Vecchi & Cornoldi, 1999; Vecchi & Girelli, 1998; Vecchi et al., 1995). Cornoldi, Rigoni, Venneri and Vecchi (2000) present two children, who together, show a double dissociation of active-passive visuo-spatial working memory. However, this distinction does not arguably map onto the Baddeley and Hitch (1974) tripartite model. Rather, the passive processes appear to reflect the visuo-spatial sketchpad system (e.g. Baddeley & Hitch, 1974), while the active processes seem to reflect non-verbal executive processes (e.g. Miyake et al., 2001).

These two accounts challenge the idea that the visuo-spatial sketchpad system is fractionated into visual and spatial subcomponents in children (e.g. Logie & Pearson, 1997). This may explain why a differential pattern of associations was not found between visual and spatial memory abilities and mathematics performance in children in the present study. However, the tasks used in the present study did not load on to separate factors that would support an alternate cognitive fractionation (such as dynamic or static) within the visuo-spatial sketchpad component. Clearly a point for future investigation is to explore the potentially different patterns of associations between children's performance on static, dynamic, active, passive, visual and spatial visuo-spatial memory tasks to better understand the structure of visuo-spatial working memory in children. Within this, relations between performance on the visuo-spatial tasks and mathematics tests can be explored. This further investigation may elicit a greater understanding of the role of visuo-spatial working memory in children's mathematics.

An alternate possibility is that the visual and spatial subcomponents of the visuo-spatial sketchpad are separable in children (Logie & Pearson, 1997), but that the tasks used in the present study did not measure distinct abilities. Three standardized visuo-spatial sketchpad tasks, which were either visual or spatial in nature, were administered. It was expected that they would load on different factors according to their visual or spatial characteristics. However, all three measures loaded on a single factor in the exploratory factor analysis. This suggests that they were not measuring separable visual or spatial memory abilities. It is probable that these tasks were measuring a general visuo-spatial working memory ability and that all loaded on a single factor because each task contained elements of the other tasks. For example, the standardized spatial task, Block Recall, was presented visually. Furthermore, it has been suggested that spatial rehearsal could be employed to maintain the patterns in the standardized visual task, the Visual Patterns Test. An alternate interpretation is that these tasks may have loaded on a single factor as each makes significant demands upon executive processes. It has been suggested that participants will be able to form easily memorable representations, or "visually chunk", the patterns in the matrix task (e.g. Phillips & Hamilton, 2001; Pickering et al., 2001). This is probably also true for the routes traced in the Mazes Memory task. Furthermore, the requirement for sequential recall in the Block Recall task is believed to make significant demands upon executive processes (e.g. Smyth & Scholey, 1996). Indeed, Pickering and Gathercole (2001) reported that these three visuo-spatial tasks loaded on the same factor as the executive measures in the standardisation of the WMTB-C (2001). It is therefore suggested that the visuo-spatial sketchpad may be fractionated into visual and spatial subcomponents in children, but that the standardized tasks used in the present study were not "pure" enough measures to tap these subcomponents.

Two unstandardized measures of the visuo-spatial sketchpad were included as "pure" measures of visual and spatial working memory abilities. They were designed to isolate visual and spatial components of visuo-spatial working memory from one another and from verbal and executive demands (Phillips & Hamilton, 2001). It was expected that these tasks might share variance with the other visuo-spatial measures, but that they would also load on independent factors relating to visual and spatial working memory abilities. However, both tasks loaded on a single factor indicating that they are not measuring distinct visual and spatial abilities. Furthermore, the visual subcomponent task loaded on both the mathematics and visuo-spatial factors, while the spatial subcomponent task did not load on any other factors. It was unclear from the present data what these two tasks were measuring. Therefore, it was not possible to infer anything about the structure of the visuo-spatial sketchpad in children from these tasks.

Overall, the data provides additional evidence for an association between visuo-spatial working memory and mathematics in children. However, it does not add to the current understanding of the role or nature of visuo-spatial working memory important for supporting children's mathematics. It was not possible to determine whether the visual or spatial subcomponent (Logie, 1995) was more important for two reasons. Firstly, it was unclear whether the two subcomponents were dissociable and secondly, there were problems with the specificity of the visuo-spatial sketchpad tasks.

A final consideration of this study was to explore the effect of controlling for individual differences in NVIQ on the relationship between children's working memory abilities and their mathematics performance. When controlling for children's NVIQ scores, the unique variance predicted by the visuo-spatial measures in mathematics scores was reduced, and even eliminated for some measures. However, the results of the confirmatory factor analyses suggested that including NVIQ as a variable alongside the three standardized visuo-spatial sketchpad measures added little to the fit of the model, nor did it dramatically increase the covariance between the two latent constructs, visuo-spatial working memory and mathematics. These results suggest that visuo-spatial working memory is related to mathematics performance, but that the visuo-spatial sketchpad tasks and the NVIQ measure are also closely related. Indeed, exploration of the higher-order factor structure suggested that the NVIQ loaded on the same factor as the visuo-spatial working memory measures. As discussed in Chapter 3, this may reflect the close associations between working memory and intelligence (e.g. Colom et al., 2004). Alternatively, it is possible that the NVIQ task contained implicit visuo-spatial working memory demands. The task required participants to look at a set of pictures and choose a missing piece from a selection of four alternatives. This would have implicitly required participants to hold in mind (maintain) the visual image of the set of pictures. Furthermore, participants would have to spatially manipulate (possibly mentally rotate) the four missing pieces to choose the one that matched the original set.

In conclusion, this study provides additional evidence for the involvement of working memory, and in particular the involvement of visuo-spatial working memory, in children's mathematics. It further supports the usefulness of working memory assessments as predictors of National Curriculum test performance (e.g. Gathercole & Pickering, 2000b). Subsets of children with dyscalculia are described as having visuo-spatial deficits (e.g. Geary, 1993; Rourke & Conway, 1997) and studies of children with specific mathematics difficulties have shown that they typically perform poorly on visuo-spatial span measures. Importantly, the current data add to this literature and strengthen the argument that screening for impaired visuo-spatial working memory may help to identify those at risk of maths difficulties. It was not possible to determine the structure of visuo-spatial working memory in children, nor was it possible to identify the nature of visuo-spatial working memory important for supporting children's mathematics. Rather, the findings raised questions for future research relating to the measurement of visuo-spatial skills and the visual-spatial distinction in working memory. Importantly, the tasks used in this study did not appear to measure distinct visual, spatial and non-verbal intelligence abilities. This creates scope for a detailed investigation into the nature, specificity and processes involved in different tasks. One approach might be to conduct a factor analytic study of different visual, spatial, non-verbal executive and non-verbal intelligence measures on samples of children and adults. Another important issue arising from the current study is that although previous studies have reported dissociations between visual and spatial processes in children's visuo-spatial working memory (e.g. Logie & Pearson, 1997), no such distinctions were observed in the present data. This highlights the fact that, as yet, there is no single definitive description of the structure of visuo-spatial working memory. Other researchers (e.g. Pickering et al., 2001; Vecchi & colleagues) have offered alternative explanations, which clearly defines the need for future research. One approach to this might be to adopt a dual-task design and attempt to selectively interfere with different hypothetical subcomponents of visuo-spatial working memory.

Chapter Summary

1. An association was found between visuo-spatial sketchpad abilities and children's mathematics in Chapter 3. The aim of the present study was to further investigate this through an exploration of the relationships between the two subcomponents of the visuo-spatial sketchpad (e.g. Logie, 1995) and children's curriculum-based mathematical skills. As in Chapter 3, the effect of controlling for individual differences in NVIQ was also explored.
2. Multiple measures of visuo-spatial working memory were administered; three standardized measures and two unstandardized measures.
3. Overall the data further support an association between visuo-spatial working memory abilities and children's mathematics. Both visual subcomponent and spatial subcomponent scores predicted variance in children's mathematics performance. However, due to the similarity in the task demands of the measures used, each measure predicted little unique variance in mathematics scores after the variance attributed to performance on the other tasks had been eliminated.
4. Similarly, visuo-spatial scores predicted little unique variance once individual differences in NVIQ had been controlled for. It is possible that visuo-spatial

working memory does not contribute to mathematics performance beyond NVIQ. Alternatively, it is possible that controlling for performance on the NVIQ measure eliminated the variance of interest. That is, the NVIQ task might have contained implicit visuo-spatial working memory demands.

5. Unexpectedly there was not a differential pattern of associations between visual and spatial working memory abilities and mathematics. Two explanations were offered for this pattern of results: 1) the visuo-spatial sketchpad may not be fractionated into visual and spatial components in children 2) the tasks used to measure the two subcomponents of the visuo-spatial sketchpad may have lacked specificity.
6. These results highlight the need for further research into the measurement of and structure of the visuo-spatial sketchpad in children.

Chapter Five

Mathematics: What is being measured?

Aim

One aim of this thesis was to investigate the potential contribution of the different components of working memory to performance across a range of mathematical skills in children. While different components of working memory have been found to predict unique variance in children's mathematics performance, there was little difference between the working memory demands of each mathematical skill. It is possible that the mathematics tests, which were developed from a curriculum that is not theoretically grounded, may not be measuring distinct mathematical skills. Therefore, the aim of this study was to explore the underlying factor structure of the mathematics tests.

Introduction

The results thus far suggest that both the central executive and visuo-spatial sketchpad, but not the phonological loop, components of working memory contribute unique variance to performance across four different mathematical skills defined by the National Curriculum for England. This provides evidence to support the notion that performing mathematical operations may involve executive functions (e.g. Bull, et al., 1999) and visuo-spatial cognition (e.g. Houdé & Tzourio-Mazoyer, 2003).

Notably, the contribution of the components of working memory to mathematics has been similar across the four skills in the studies (e.g. visuo-spatial sketchpad scores predicted between 1% and 3% of unique variance across all four mathematical skills in Chapters 3 and 4). Contrary to previous studies, this implies that different mathematical skills do not recruit different working memory resources. In an investigation of the involvement of working memory in children's mathematical reasoning, Maybery and Do (2003) suggested that the different components of working memory contributed differentially to three areas of the West Australian mathematics curriculum (number, measurement and space). For example, they reported that fixed verbal span, measured by a letter recall span, was significantly associated with two of the three mathematical domains, number and space.

One explanation for the discrepancy between the results found in this thesis and other similar studies may be that the skills measured in the present investigations are not separable, distinct skills. That is, the skills outlined by the National Curriculum may not be distinct mathematical domains. Examination of the mathematics tests revealed that there was considerable overlap in the demands of the questions across the four skills. Furthermore, the questions within each skill varied in difficulty and demand. As noted in an analytical study of the National Curriculum, questions labelled within one category differ greatly in character (Shorrocks-Taylor, Curry, Swinnerton & Nelson, 2003). For example, within the Number and Algebra domain questions can be further categorised as straightforward *calculations*, which typically include the instruction "calculate the following", *rich number calculations*, which involve calculation with a problem solving aspect and non-mathematical *context calculations*, which involve solving problems set in an everyday context (e.g. buying fruit). In addition, the structure of the curriculum was the same for both age groups (and is for Key Stages 1, 2 and 3). This meant that all children were assessed on the same skills, without regard for possible developmental changes in children's mathematical abilities (e.g. the development of strategy use).

Unlike in other countries, the National Curriculum for England is not based on developmental principles. Pegg (2002) has argued that worldwide there is an absence of theoretical underpinnings in outcome-based / standards–based syllabuses and assessments, such as SATs. He proposes that outcome-based assessments focus primarily on passing and failing children, and are only partially concerned with children's mathematics learning. He continues that many curricula are developed from "personal experiences of members of writing teams…" (Pegg, 2002, pp. 236). Indeed, Brown (2001, pp:35) refers to the current National Curriculum for England as being "…dictated by "common sense" or "what works in practice" rather than being theoretically grounded.

Developmental psychology has begun to influence the mathematics curricula in some countries, including America, Australia, New Zealand and Holland. In a review of teaching practice in America, Koehler and Grouws (1992) argued that there was a clear link between research principles and teaching. In particular, they suggested that teachers were influenced by constructivism, socio-cultural and feminist perspectives and that their instruction was cognitively guided. Feldman (2002) has more recently claimed that these research findings have begun to influence state curriculum frameworks in the United States. In New South Wales, Australia and New Zealand the mathematics curricula are more directly influenced by developmental approaches. For example, the early mathematics curriculum in New Zealand is directly derived from Steffe's (1983) account of the development of children's number understanding (Wright, Martland & Stafford, 2000). In New South Wales the assessment philosophy is referred to as DBA, which stands for Developmental-based Assessment and Instruction. The basic principle of this method is that children's answers are interpreted within a framework of cognitive growth, known as the

Structure of the Observed Learning Outcome (SOLO) model proposed by Biggs and Collis (1982). SOLO, founded on the stage development ideas of Piaget, is a model for interpreting children's responses in terms of the quality of assimilation and accommodation of concepts. It is concerned with "how well" something is learned, not "how much" is learned. A child's understanding is classified as an outcome, which reflects the quality of their learning. This outcome, or SOLO level, indicates what a child knows, understands and can do (Pegg, 2002). As such, it allows teachers and educators an insight into where instruction should be directed. In short, this approach bases teaching and assessment upon a children's development. The influence of developmental psychology can also be seen in the development of teaching and assessment methods for mathematics in Holland. The main principle of the Dutch realistic mathematics education (RME) system is that formal knowledge can be developed from children's informal strategies (Treffers, 1991). As such, children contribute to their own teaching as much as possible as their own informal constructions influence their formal mathematics teaching.

It is suggested that the lack of theoretical underpinnings for the National Curriculum for England may have resulted in the development of a curriculum and associated assessments that do not measure distinct mathematical abilities. Consequently, the mathematics tests developed from these assessments in Chapter 2 may not be measuring distinct skills. This could explain the finding that different mathematical skills do not recruit different working memory resources. Therefore, the aim of this study was to statistically analyse children's performance on the mathematics tests to identify whether or not distinct mathematical abilities were being measured.

Method

Participants

The participants were 309 typically developing children (164 boys and 145 girls), who attended six primary schools in England. One school was in the South-East of England, the other five were in the North-East of England. There were 150 Year 3 children (86 boys and 64 girls), mean age 8 years and 1 month (*SD* = 4.4 months, range 7 years 3 months to 8 years 8 months) and 159 Year 5 children (78 boys and 81 girls), mean age 10 years and 1 months (*SD* = 4.6 months, range 9 years 3 months to 10 years 8 months).

34 children attended School One (17 Year 3 and 17 Year 5), 43 children attended School Two (19 Year 3 and 24 Year 5), 46 children attended School Three (25 Year 3 and 21 Year 5), 107 children attended School Four (51 Year 3 and 56 Year 5), 41 children attended School Five (22 Year 3 and 19 Year 5) and 38 children attended School Six (16 Year 3 and 22 Year 5).

The percentage of children achieving Level 4 attainment and above in English, Mathematics and Science was higher than the national average in four of the schools (School One 97%, 85% and 97%; School Two 83%, 90% and 95%; School Three 96%, 93% and 96%; School Four 87%, 89% and 90% respectively) and lower than the national average in two of the schools (School Five 55%, 48% and 65%; School Six 57%, 47% and 72% respectively).

Design and Procedure

All children participated in a one-hour testing session, which formed part of the previous studies. The children were administered age appropriate mathematics assessments under standardized conditions within a classroom setting.

Materials

The mathematics assessments were comprised of three 10 minute written sections followed by one 10-minute mental arithmetic test, which was presented orally with written responses. See Chapter 2 for details.

Results

Descriptive Statistics

Descriptive statistics for children's mathematics test performance are presented in Table 5.1.

Table 5.1.

Descriptive Statistics of Children's Mathematics Performance. (N = 309).

Mathematics Measure	Year 3 (*n*=150)		Year 5 (*n*=159)	
	M	*SD*	*M*	*SD*
Number and Algebra	50.66	26.32	51.79	21.10
Shape, Space and Measures	55.86	23.63	53.30	15.56
Handling Data	60.17	18.21	58.34	19.12
Mental Arithmetic	61.26	27.27	59.25	23.37
Total Mathematics Score	56.61	19.22	54.21	15.56

Note. Mathematics scores shown are proportions correct.

Overall, there were no significant differences between Year 3 and Year 5 children's scores across the mathematics measures (p>.05). However, Year 3 children's scores were significantly lower on the Number and Algebra measure compared to the other measures (F(3, 447=13.20, p<.01). There were no significant differences in Year 5 children's scores across the mathematics measures (F(3, 114=.44, p>.05). There were no significant gender differences across the measures (p>.05). For this reason separate scores for boys and girls are not shown.

Exploratory Factor Analysis

Principal components analyses were conducted to determine the higher-order factor structure underpinning variations in scores on the different mathematics measures for both age groups.

One factor emerged with eigenvalues in excess of 1.00 for both age groups, meaning the solutions could not be rotated. The unrotated factor solutions, showing factor loadings in excess of .30 on the component matrices are presented in Tables 5.2 and 5.3.

Table 5.2

Factor Loadings of Mathematics Measures on Component Matrix Solutions for Year 3 Data.

Mathematics Measure	Factor 1
Number and Algebra	.86
Shape, Space and Measures	.79
Handling Data	.72
Mental Arithmetic	.87

Note. KMO measure of sampling adequacy = .68.

All four measures of mathematics ability loaded on Factor 1, with loadings in excess of .70, for the Year 3 data.

Table 5.3

Factor Loadings of Mathematics Measures on Component Matrix Solutions for Year 5 Data.

Mathematics Measure	Factor 1
Number and Algebra	.79
Shape, Space and Measures	.79
Handling Data	.78
Mental Arithmetic	.84

Note. KMO measure of sampling adequacy = .78.

Similarly, all four measures of mathematics ability loaded on Factor 1, all with loadings in excess of .70, for the Year 5 data. This suggests that the mathematics assessments for both age groups are measuring the same construct, a general mathematics ability.

Cluster Analysis

Cluster analyses were conducted to explore the factor structure underpinning the mathematical skills defined by the National Curriculum. The reason for this approach was to ascertain which questions across the four mathematics skills contained aspects of the same underlying factor. Cluster analysis is a descriptive, multivariate statistical technique that aims to group variables or individuals that are close together in some way. It can be used to group individuals, e.g. those that share some of the same characteristics, or variables, e.g. those that are distributed similarly across individuals. The present analysis concerns grouping variables. Clusters are formed as distances between the cases are computed for each variable meaning different variables within the data are combined in a series of stages until all the variables have been grouped. Three methods of cluster analysis are commonly used; hierarchical (agglomerative or divisive), non-hierarchical or model-based. An agglomerative, hierarchical method was chosen for this analysis. A variety of techniques are available to measure the similarity among variables. The squared Euclidean distances measure was chosen for the present analysis because the data was binary and it is analogous to conducting a principal components analysis (Bartholomew, Steele, Moustaki, & Galbraith, 2002), which can be used to identify higher-order factor structures in interval/ratio data. Different clustering algorithms are available in cluster analysis. For the present analysis Ward's method was selected as it typically yields the clearest picture of clustering and can be used on data that is not metric to produce "meaningful" clusters (Bartholomew et al., 2002).

To explore the possibility of a developmental trend in mathematical abilities, two cluster analyses were conducted, one for each age group. A two-cluster solution was generated for the Year 3 data. One cluster (Cluster A) contained 30 variables (54.55% of questions from the mathematic test). The other cluster (Cluster B) contained 25 variables (45.45% of questions). The percentage of questions from each curriculum-based mathematics skill grouped in each cluster is presented in Table 5.4 (see Appendix X for the items comprising each cluster). A large percentage of the Number and Algebra and Mental Arithmetic questions grouped into Cluster A, while a larger portion of the Shape, Space and Measures and Handling Data questions grouped into Cluster B. Cluster A could be argued to represent a pure mathematical skill. Cluster B may represent a more applied mathematical skill.

Table 5.4

Percentage of Mathematics Questions Grouping into Clusters for Year 3.

	Cluster	
Mathematics Skill	Cluster A	Cluster B
Number and Algebra	93.33%	6.67%
Shape, Space and Measures	33.33%	66.67%
Handling Data	26.67%	73.33%
Mental Arithmetic	70%	30%

A two-cluster solution was also generated for the Year 5 data. One cluster (Cluster C) contained 37 variables (67.27% of the questions), while the other (Cluster D) contained 18 variables (32.73% of the questions). Table 5.5 presents the percentage of questions from each curriculum-based mathematics skill grouped in the two clusters (see Appendix XI for the items comprising each cluster). Cluster membership for the Year 5 data did not transpose from the original four mathematics skills with the clarity that was seen in the Year 3 data. Rather, questions from each mathematics skill were dispersed across the two clusters. The only distinction observed between the clusters was the level of difficulty of the questions (as observed by the difference between mean scores on each cluster in Table 5.6). Cluster C comprised of "easy" questions and Cluster D comprised of "difficult" questions.

Table 5.5

Percentage of Mathematics Questions Grouping into Clusters for Year 5.

	Cluster	
Mathematics Skill	Cluster C	Cluster D
Number and Algebra	60%	40%
Shape, Space and Measures	66.67%	33.33%
Handling Data	73.33%	26.67%
Mental Arithmetic	70%	30%

Overall, the original curriculum-based mathematics assessments, which comprised of four mathematical skills defined by the National Curriculum, were re-defined for both age groups. For the purpose of clarity, the original mathematics skills will be referred to as the curriculum-based skills and the re-grouped mathematics abilities will be referred to as the performance-related skills from this point forward. Descriptive statistics for the performance-related mathematics skills for Year 3 and Year 5 children are presented in Table 5.6. Year 3 children performed significantly better on Cluster B (the more applied questions) than Cluster A ($t(149)=-11.10$, $p<.01$), while Year 5 children performed significantly better on Cluster C (the easier questions) than Cluster D ($t(158)=36.63$, $p<.01$). There was no evidence of a floor effect on Cluster D questions because children scored 27% on average (*SD* 18.04) (meaning the children were scoring over a quarter of the questions correct). The significant differences in performance across the performance-related mathematics skills lend support to the idea that distinct mathematical skills are being measured.

Table 5.6

Descriptive Statistics for Children's Performance-Related Mathematics Abilities.

Mathematics Measure	*M*	*SD*
Year 3		
Cluster A	48.51	25.77
Cluster B	66.32	15.04
Year 5		
Cluster C	72.65	15.76
Cluster D	27.11	18.04

Discussion

Overall, the results suggest that the mathematics tests used to investigate the potentially different contributions of working memory to different mathematical skills were not measuring distinct, separable skills. Rather, it appears they were measuring children's general mathematical abilities. This may explain why, contrary to previous findings (e.g. Maybery & Do, 2003), the contributions of the central executive and visuo-spatial sketchpad to children's mathematics performance did not differ across the four National Curriculum-based skills in the two previous chapters.

Descriptive statistics suggested that there was little variation in children's scores between the four mathematics skills defined by the National Curriculum, for both Year 3 and Year 5 children. Indeed, the higher-order factor structure of both mathematics assessments, explored using principal components analyses, confirmed this. The four measures (Number and Algebra, Shape, Space and Measures, Handling Data and Mental Arithmetic) within each age group's assessment loaded on a single factor, which probably corresponded to a general mathematical ability. While it could be argued that this result strengthened the construct validity of the assessments (suggesting they are all measuring mathematics performance) it indicated that an alternate factor structure might underpin the tests. Indeed, in the subsequent cluster analyses a two-cluster solution was generated for both assessments. For the Year 3 data, it was suggested that these clusters might correspond to a pure mathematical skill and an applied mathematical skill. Interestingly, the Year 3 children performed significantly better on the applied questions than the pure questions. This appears consistent with Hughes's (1986) suggestion that younger children can more readily solve concrete problems (those that refer to specific objects, people and events), than abstract problems that do not have a concrete referent. A different two-cluster solution was yielded for the Year 5 data. It was suggested that these clusters might correspond to the level of difficulty of the questions, as there was no discernible difference between them other than the difference between children's mean scores. Obviously, the interpretation of these mathematics clusters is subjective and alternative interpretations are possible. For example, Cluster A (for Year 3) predominantly incorporated questions that involved computation (e.g. addition, subtraction etc.), while Cluster B incorporated questions that involved reading and manipulating numbers in a less computational way (e.g. reading from charts and graphs). Future research might want to provide a reliable objective assessment of what the clusters represent through asking educators and children to classify the questions and provide descriptions of the skills they believe the different questions and groupings of questions are measuring.

The results of this study suggest that the current QCA assessments (SATs) may not be measuring separable mathematical skills, and therefore, may not be assessing the skills defined by the National Curriculum for England. Furthermore, considering the different cluster solutions derived for the two age groups in the present study, which suggest that there may be a developmental trend in children's mathematical competencies, it could be argued that the existing structure of the mathematics curriculum may not be suitable for the wide age range at which it is aimed. While it may not be crucial that the curriculum teaches and assesses mathematical skills separately, only that it improves children's general mathematical performance, it is suggested that a more structured curriculum, tailored to specific stages in development, could facilitate and promote learning.

Using cognitive-based developmental research to guide curriculum-development could aid learning. The primary benefit of such a system would be that children could gain a deeper understanding of mathematics. Pegg (2002) suggests that this approach could provide teachers with a better understanding of the knowledge children have at certain ages. With this understanding, they could ensure that they were teaching skills at an appropriate time, thus avoiding the problems of teaching mathematics in a linear fashion (e.g. Munn, 2004). Furthermore, under a new system long-term understanding may be promoted, as there may be less pressure to raise standards, which at present can cause rote learning (Pegg, 2002). Arguably the benefit most relevant to the present research is that developing a curriculum from a developmental perspective could mean that content areas can be arranged in a developmentally justifiable way, where levels of attainment reflect phases in cognitive development. This would be beneficial for research, such as the present investigation, as it would allow a closer examination of the cognitive resources important for successful mathematical development. In due course, this research would hopefully feedback into the education system, and permit educators a better understanding of cognitive mechanisms involved in early mathematical development.

Other countries, including Australia and Holland, have adopted this approach and built mathematics curricula around developmental theories. Although the English system is yet to encompass similar ideas, recent changes aimed at improving mathematics education, including the introduction of the National Numeracy Strategy (NNS) in 1999, have been made. When the NNS was launched, the Chief Inspector for Schools Chris Woodhead, claimed that it was not based on a model of learning, but on a model of teaching (Askew, 2004). Although this suggests that the English system still has some way to go before developmental ideas are incorporated, some Local Education Authorities in England (e.g. Cumbria) have introduced training for teachers in children's conceptual development (e.g. the Mathematics Recovery approach, Wright et al., 2000) (Willey, 2004).

In light of the findings of this study, and current trends in other countries, it is suggested that the QCA review the existing Key Stage 2 mathematics curriculum to better reflect children's learning. They may want to take an analytic approach analogous to that conducted is this study to re-define areas of mathematics that directly reflect children's performance on existing measures, or re-define the curriculum in terms of children's number development, akin to the Australian or Dutch methods.

For the present purpose, this study has re-grouped the mathematical skills defined by National Curriculum for England into distinct, measurable mathematical domains for Year 3 and Year 5 children. These new performance-related mathematics skills will aid further exploration of the potentially different contributions of the components of working memory to performance across a range of mathematical abilities in children.

Chapter Summary

1. The aim of this thesis was to explore the potentially different contributions of working memory to different mathematical abilities. As a means of assessing different mathematical skills in children tests were developed from the National Curriculum guidelines.
2. Contrary to previous studies, the relative contribution of the components of working memory to mathematics was similar across the four skills. It was therefore suggested that the skills defined by National Curriculum might not be distinct, separable mathematical abilities.
3. The present study analysed children's performance on the mathematics tests. The results suggested that the tests developed from the National Curriculum guidelines were measuring one factor (general mathematics ability). Further analyses were conducted, which suggested that the mathematics tests had alternate higher-order factor structures that differed for the two age groups.
4. The results of this study imply that the Key Stage 2 mathematics curriculum in England might not be teaching and assessing distinct mathematical skills.
5. It was subsequently suggested that the QCA review the existing curriculum, and perhaps follow trends in other countries that have developed curricula from developmental perspectives to promote children's learning.
6. In relation to the present research, this study has defined separate mathematical domains that will permit subsequent investigations into the

relative contributions of the components of working memory to different mathematical skills in children.

Chapter Six

Working Memory and Children's Performance-related Mathematical Skills

Aim

A significant association was found between working memory ability and children's curriculum-based mathematics performance in Chapter 3. However, contrary to expectation, there was little difference between the working memory demands of the different curriculum-based mathematical skills. The aim of the present study was to re-analyse the data collected in Chapter 3 to explore the relationship between working memory ability and the performance-related mathematical skills derived in Chapter 5. More specifically, the aim was to explore potentially different working memory demands of the performance-related mathematical skills for each age group.

Introduction

One aim of Chapter 3 was to explore the contribution of the three components of the working memory model to performance across a range of mathematical skills defined by the National Curriculum for England. Hecht (2002) reported that different arithmetic solution strategies recruited different working memory resources. Hence, it was expected that the four mathematical skills defined by the National Curriculum (Number and Algebra, Shape, Space and Measures, Handling Data and Mental Arithmetic) would recruit different working memory resources.

The results of this earlier study suggested that visuo-spatial sketchpad and central executive scores predicted unique variance in children's curriculum-based mathematics performance when both age-related variance and individual differences in NVIQ were controlled for. However, rather unexpectedly the data suggested that the working memory demands were similar across all four skills. Phonological loop scores did not predict unique variance in any curriculum-based mathematical skills. Visuo-spatial sketchpad scores predicted a small but significant amount of unique variance in all four mathematical skills (between 1% and 3%) and central executive scores predicted a greatest amount of unique variance across all skills (between 12% and 22%).

There were two possible reasons for this finding. The first possibility was that the mathematics assessments developed in Chapter 2 were not measuring distinct mathematical skills. Indeed, the results of Chapter 5 suggested that the four mathematics skills defined by the National Curriculum for England were not distinct skills. There was little variation in children's scores between the mathematics skills and the results of the principal components analyses suggested that all four measures loaded on a single factor. Possible reasons for this are discussed in Chapter 5. In short, it appeared that the assessments were measuring a general mathematics ability. Subsequent analyses generated alternate factor structures for the mathematics assessments. Based on children's test performance new distinct and separable mathematical skills were defined for each age group (see Chapter 5 for details).

A second possible explanation for the finding that the working memory demands were similar across different mathematical skills may be that, although age-related variance was controlled for, the data from the two age groups was collapsed to form one large data set. Developmental changes occur in children's mathematics abilities during the primary school years (between 5- and 11-years-of-age). During this time, children's mathematical development is characterised by changes in the use of different strategies (e.g. Siegler, 1999). Typically, they advance from using slow procedural counting-based solution strategies to more efficient retrieval-based strategies (e.g. Kaye, 1986) (see Chapter 1, section 1.3.2 for an overview of the developmental changes in mathematical cognition during the school years). This implies that children of different primary-school ages may be using different solution strategies based on their stage of mathematical development. As such, children of different ages may recruit different working memory resources for the solution to different mathematical problems.

Potential differences in the relationship between working memory and mathematics between the two age groups (7-/8-year-olds and 9-/10-year-olds) were not explored in Chapter 3. This may have eliminated some of the variance in the working memory demands for each curriculum-based mathematical skill. Indeed, phonological loop scores were significantly associated with mental arithmetic performance before age-related variance was controlled for. Further analyses revealed that the phonological loop measure showed a stronger association with the older children's (9-/10-year-olds) mental arithmetic performance. This supports the idea that children from the two age groups may have recruited different working memory resources for the different areas of mathematics. Furthermore, children's performance-related mathematics skills differed between the two age groups (see Chapter 5). This suggests that there may be a developmental trend in children's mathematical competencies, which again may affect the working memory resources recruited.

The present study attempted to resolve some of the inconsistencies in Chapter 3. As in Chapter 3, the relationship between working memory ability and a range of mathematics skills was explored in children at Key Stage 2 of the National Curriculum. However, there were two important differences in the present study.

Firstly, the two age groups were considered separately to explore any potential developmental differences in the working memory demands of different mathematical abilities. Secondly, the mathematics skills assessed were the performance-related skills derived Chapter 5. These were used as the results of Chapter 5 indicated that they were separable abilities, unlike the mathematical skills defined by National Curriculum for England that were used in Chapter 3.

Individual differences in NVIQ were controlled for in Chapter 3. However, the NVIQ measure loaded on the same factor as the working memory measures in the factor analyses, suggesting that the task demands of the measures are similar. Furthermore, including NVIQ did not effect the overall pattern of associations between working memory ability and mathematics performance in the correlation, regression or confirmatory factor analyses (see Chapter 3). Therefore, the NVIQ measure was not included in the present study.

Method

Participants

The participants were the same 148 primary school children who participated in Chapter 3 (see Chapter 3 for details).

Design and Procedure

See Chapter 3.

Materials

Working Memory Tasks

The working memory tasks, taken from the WMTB-C (Pickering & Gathercole, 2001), were; Nonword List Recall, Mazes Memory and Listening Recall. (See Chapter 3 for details).

Mathematics Tasks

The mathematics assessments were the age appropriate tests administered in Chapter 3. Instead of scoring the tests according to performance across the four curriculum-based skills (see Chapter 3), children's mathematics abilities were measured according their performance on the performance-related skills derived in Chapter 5. This meant that two skills were assessed for each age group respectively.

Year 3 Pure Mathematics

The pure mathematics test items were those that grouped as Cluster A (see Chapter 5). These questions were predominantly from the original Number and Algebra and Mental Arithmetic curriculum areas. Primarily they require children to demonstrate their understanding of number and context-free mathematical operations. An example question is presented in Figure 6.1.

Write in the missing numbers.

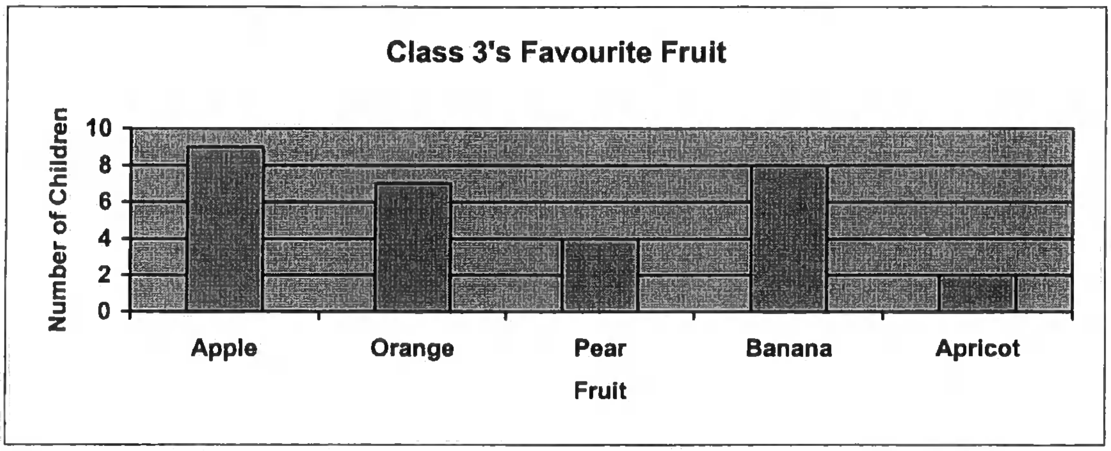
42 + □ = 73

Figure 6.1

Example Question from the Year 3 Pure Mathematics

Year 3 Applied Mathematics

The applied mathematics test consisted of items that grouped as Cluster B in Chapter 5. These questions were predominantly from the original Shape, Space and Measures and Handling Data curriculum areas. Primarily the questions require children to demonstrate their understanding of mathematical problems that are based around shapes, stories or pictures. An example question is presented in Figure 6.2.



How many children chose oranges and pears?

Figure 6.2

Example Question from the Year 3 Applied Mathematics

Year 5 Easy Mathematics

The easy mathematics items were those that grouped as Cluster C in Chapter 5. This group of questions had no discernible characteristics. However, the proportion correct score for Year 5 children on this cluster of questions was significantly higher than the proportion correct score for Cluster D questions. For this reason these questions were labelled "easy". An example question is presented in Figure 6.3.

Write in the missing numbers.

35 + □ = 100

Figure 6.3.

Example Question from the Year 5 Easy Mathematics

Year 5 Difficult Mathematics

The difficult mathematics test consisted of items that grouped in Cluster D in Chapter 5. Again, this group of questions had no discernible characteristics. Given that the proportion correct score for Year 5 children on this cluster of questions was significantly lower than the proportion correct score on Cluster C questions, these questions were labelled "difficult". An example question is presented in Figure 6.4.

"Calculate"

152 ÷ 8 = □

Figure 6.4.

Example Question from the Year 5 Difficult Mathematics

Results

Power Analysis

Erdfelder's (1984) compromise power analyses were conducted to determine the statistical power of this study. The results of the power analyses, conducted using Faul and Erdfelder's (1992) G Power programme, are presented in Table 6.1. Power analyses were conducted for each age group, as the data from the two samples were analysed separately

Table 6.1

Compromise Power Analysis for Working Memory and Performance-Related Mathematics Study.

Year	Effect Size	n_1 (boys)	n_2 (girls)	Power
Year 3	0.5 (medium)	46	32	.86
Year 5	0.5 (medium)	33	37	.85

The power of this study to test for significance with a medium effect size is .86 for the Year 3 sample and .85 for the Year 5 sample. Both values exceeds Cohen's (1988) criterion of .8, meaning this study is statistically powerful.

Descriptive Statistics

Descriptive statistics for working memory measures and performance-related mathematics skills are presented in Table 6.2.

Table 6.2.

Descriptive Statistics of Working Memory and Performance-Related Mathematics Measures. (Maximum scores for working memory measure shown in parentheses). (*N*=148).

Measures	Year 3. (*n* = 78).		Year 5. (*n* = 70).	
	M	*SD*	*M*	*SD*
Working Memory Measures				
Phonological Loop (36)	13.92	2.49	17.15	2.59
Visuo-Spatial Sketchpad (42)	9.37	5.79	17.05	7.43
Central Executive (36)	9.42	3.56	13.00	3.10
Mathematics Measures				
Cluster A	53.49	26.79	-	-
Cluster B	66.45	16.26	-	-
Cluster C	-	-	71.33	13.03
Cluster D	-	-	29.85	17.35

Note. Mathematics scores shown are proportion correct.

Year 3 children performed significantly better on Cluster B, the more applied questions, than on Cluster A, the purer mathematics questions ($t(77)=-5.64$, $p<.05$). Year 5 children performed significantly worse on Cluster D than Cluster C questions ($t(69)=19.83, p<.01$). As mentioned previously this indicates that the questions comprising this cluster were more difficult. As in previous chapters, there were no significant gender differences in performance ($p>.05$). For this reason boys and girls mean scores are not presented.

Table 6.3

Correlation Matrix for Working Memory and Performance-Related Mathematics Measures. Year 3 (n= 78) simple coefficients are displayed in the upper triangle; Year 5 (n=70) simple coefficients are displayed in the lower triangle.

	Phonological Loop	Visuo-Spatial Sketchpad	Central Executive	Mathematics Cluster A (Year 3)	Mathematics Cluster B (Year 3)
Phonological Loop	-	.18	.24	.15	.18
Visuo-Spatial Sketchpad	.06	-	.28*	.41**	.37**
Central Executive	.24	.37**	-	.54**	.62**
Mathematics Cluster C (Year 5)	.26	.26	.53**	-	.75**
Mathematics Cluster D (Year 5)	.07	.26	.30*	.55**	-

Correlation Analyses

Associations between working memory and performance-related mathematics skills are presented in the correlation matrix (Table 6.3). Coefficients for Year 3 are displayed in the upper triangle; coefficients for Year 5 are displayed in the lower triangle.

Scores on the visuo-spatial sketchpad and central executive working memory measures were intercorrelated for both age groups (all rs>.2, p<.05). The phonological loop measure was not correlated with the other working memory measures for either age group. Performance on the mathematics abilities was intercorrelated across both age groups (all rs>.3, p<.05).

For Year 3 children, visuo-spatial and central executive scores were significantly related to both of the performance-related mathematical abilities (all rs>.03, p<.01). For Year 5 children, only central executive scores were significantly related to the performance-related mathematics abilities (all rs>.3, p<.05). However, the associations between phonological loop scores and Cluster C mathematics performance approached significance, as did the associations between visuo-spatial scores and Cluster C and Cluster D mathematics scores.

Regression Analyses

Simple regression analyses revealed that the working memory measures predicted 45.6% of the variance in Year 3's overall mathematics performance and 19.8% in Year 5's overall mathematics performance. Subsequently, a series of fixed-order regression analyses were used to assess the amount of unique variance in the performance-related mathematics scores predicted by each of the measures of working memory for Year 3 (Table 6.4) and Year 5 (Table 6.5) children.

Table 6.4

Fixed-order multiple regression analyses predicting unique variance in performance-related mathematics performance for Year 3. (n=78).

Predictor	Mathematics Ability Predicted	Model	Order of Entry Into Equation	*r*	r^2	*Adjusted* r^2
Phonological Loop						
	Cluster A	A_1	1.CE	.54	.30	.28
			2.VSSP	.61	.37	.34
			3. PL	.61	.37	.34
	Cluster B	A_2	1. CE	.62	.38	.37
			2.VSSP	.65	.43	.40
			3.PL	.65	.43	.40
Visuo-spatial Sketchpad						
	Cluster A	B_1	1. CE	.54	.30	.28
			2. PL	.55	.30	.27
			3. VSSP	.61	.37	.33
	Cluster B	B_2	1. CE	.62	.38	.37
			2. PL	.62	.39	.36
			3.VSSP	.65	.43	.39
Central Executive						
	Cluster A	C_1	1. PL	.15	.02	.00
			2. VSSP	.42	.18	.14
			3. CE	.61	.37	.33
	Cluster B	C_2	1. PL	.18	.03	.01
			2. VSSP	.40	.16	.12
			3. CE	.65	.43	.39

The mathematics measure was the regressor for each analysis and the unique contribution (measured as r^2) of each working memory measure was assessed as a predictor entered into the equation after the other predictors.

Models A_1 and A_2 show that phonological loop scores do not account for any unique variance in mathematics scores in Year 3 beyond that predicted by the other working memory measures. Models B_1 and B_2 show that visuo-spatial sketchpad scores predicted 7% of unique variance in Cluster A (pure) scores and 4% of unique variance in Cluster B (applied) scores in Year 3. Models C_1 and C_2 show that central executive scores accounted for the largest amount of unique variance in mathematics scores in Year 3; 19% of Cluster A (pure) and 27% of Cluster B (applied) scores.

Working memory measures predicted less unique variance in mathematics performance in Year 5. Models G_1 and G_2 show that phonological loop scores predicted 2% of unique variance in Cluster C (easy) scores, but did not predict any unique variance in Cluster D (difficult) scores. Contrary to this, models H_1 and H_2 show that visuo-spatial sketchpad scores predicted 3% of unique variance in Cluster D (difficult) scores, but no unique variance in Cluster C (easy) scores. As with Year 3, central executive scores predicted the largest amount of unique variance in mathematics scores in Year 5; 18% of Cluster C (easy) and 5% of Cluster D (difficult) scores (models I_1 and I_2).

Table 6.5

Fixed-order multiple regression analyses predicting unique variance in performance-related mathematics performance for Year 5. (n=70).

Predictor	Mathematics Ability Predicted	Model	Order of Entry Into Equation	r	r^2	Adjusted r^2
Phonological Loop						
	Cluster C	G_1	1.CE	.53	.28	.27
			2.VSSP	.53	.28	.27
			3. PL	.55	.30	.27
	Cluster D	G_2	1. CE	.30	.09	.07
			2.VSSP	.34	.12	.08
			3.PL	.34	.12	.08
Visuo-spatial Sketchpad						
	Cluster C	H_1	1. CE	.53	.28	.27
			2. PL	.55	.30	.27
			3. VSSP	.55	.30	.27
	Cluster D	H_2	1. CE	.30	.09	.07
			2. PL	.30	.09	.07
			3.VSSP	.34	.12	.07
Central Executive						
	Cluster C	I_1	1. PL	.26	.07	.05
			2. VSSP	.35	.12	.09
			3. CE	.55	.30	.26
	Cluster D	I_2	1. PL	.08	.01	-.01
			2. VSSP	.27	.07	.03
			3. CE	.34	.12	.06

Discussion

Overall, the results show a significant association between working memory ability and mathematics attainment in 7-/8-year-olds and 9-/10-year-olds. Simple regression analyses revealed that the tripartite model of working memory (Baddeley & Hitch, 1974; Baddeley, 1986) predicted 45.6% of the variance in Year 3 (7-/8-year-old) children's mathematics scores and 19.8% of the variance in Year 5 (9-/10-year old) children's mathematics scores. This is consistent with previous findings that working memory assessments predict scholastic attainment (e.g. Gathercole & Pickering 2000a; 2000b), and provides additional evidence for the involvement of working memory in children's mathematics (e.g. Adams & Hitch, 1997; Bull et al., 1999). Importantly, the findings suggest that the components of working memory involved in children's mathematics may change with age.

The contributions of the different components of the working memory model to children's performance across a range of mathematical skills was assessed using non-digit based measures of working memory function from the WMTB-C (Pickering & Gathercole, 2001). Importantly, the mathematical skills assessed, which differed for each age group, were distinct abilities that were defined by children's performance (see Chapter 5). The results presented in Chapter 3 indicated that there was little difference between the working memory demands of curriculum-based mathematical skills. However, the present study revealed a developmental difference in the relationship between working memory ability and performance-related mathematical skills across the two age groups.

Overall, working memory measures predicted less variance in the older children's mathematics scores (19.8%) than the younger children's (45.6%). This suggests that working memory may support mathematics development where children are acquiring new solution strategies (e.g. Bull & Scerif, 2001) or learning mathematics facts via rote rehearsal (e.g. Hitch & McAuley, 1991). Furthermore, it suggests that young children may be more reliant upon working memory due to slow processing. Mathematics is less automatic in young children, who use slower procedural strategies for mathematics. Therefore, working memory may be important for supporting the retention of problem information during these processes (e.g. Adams & Hitch, 1998).

More specifically, the data revealed a markedly distinct pattern of associations between the different components of the tripartite working memory model (Baddeley & Hitch, 1974) and performance-related mathematical skills across the two age groups. The contribution of each component of working memory to the performance-related mathematical skills will be discussed in turn.

Consistent with earlier findings (Chapter 3) the central executive predicted unique variance in all performance-related mathematical skills (5-27% across both age groups). As discussed in Chapter 3, this may in part reflect the contribution of a more general resource such as intelligence to mathematics competency (e.g. Kyllonen & Christal, 1990). However, central executive scores predicted a greater amount of unique variance in the younger children's mathematics scores (19%-27%) than the older children's (5%-18%). The apparent lesser involvement of the central executive in the older children's mathematics supports Bull and Scerif's (2001) suggestion that executive function may be less important at a higher level of skill acquisition. That is, once a skill such as the retrieval of appropriate solution strategies has become automatic, there may be less need to establish schema, which reduces the role of executive processes.

Phonological loop scores predicted unique variance in the older children's easy questions on the performance-related mathematical skills. This was the opposite pattern to that observed between visuo-spatial sketchpad scores and children's performance-derived mathematical skills. It is suggested that the greater involvement of the phonological loop in the older children's mathematics may reflect the mastery of symbolic-linguistic arithmetic (Houdé, 1997) or mature solution strategies (such as direct retrieval) that rely on a verbal code (e.g. Dehaene & Cohen, 1995). Consistent with this notion, scores on the phonological loop measure were significantly associated with mental arithmetic performance before the variance associated with age was controlled for in Chapter 3. In other words, the phonological loop showed a stronger association with the older children's mental arithmetic performance. As such, the data may tentatively suggest that the older children were able to use subvocal rehearsal processes to support the retention of problem information (e.g. Adams & Hitch, 1997) and direct retrieval of arithmetic facts from long-term memory.

Young children use verbal solution strategies, such as counting-on, for mathematics problems. It was therefore expected that phonological loop scores would predict young children's mathematics performance. There are two possible reasons why this was not so. Firstly, it is possible that central executive supports the use of verbal solution strategies. Supporting this suggestion, the central executive predicted greater variance in younger than older children's mathematics. Secondly, it is possible that phonological loop scores did not predict unique variance in young children's mathematics due to the high degree of association between phonological loop and central executive measures (e.g. Gathercole & Pickering, 2000a). However, central executive and phonological loop scores were not significantly associated in the present study, suggesting that the first explanation may be appropriate.

Consistent with previous findings (Chapter 3), visuo-spatial sketchpad scores predicted a small but significant amount of unique variance in children's mathematics performance (between 3% and 7% across both age groups). Further examination of the relationship between the visuo-spatial sketchpad and the children's performance–related mathematics skills revealed a markedly distinct pattern of associations across age groups. Visuo-spatial sketchpad scores predicted unique variance in all of the younger children's performance-related mathematics skills (pure 7% and applied 4%), but only predicted unique variance in the older children's performance on the difficult questions (3%). This is consistent with the hypothesis that younger children have a greater dependence on the visuo-spatial sketchpad for the successful solution of mathematics problems, which may reflect the use of early visual encoding strategies (Palmer, 2000) or the use of an early visuo-spatial arithmetic (Houdé, 1997). In line with this hypothesis, it is suggested that the involvement of the visuo-spatial sketchpad in the older children's performance on the difficult questions may reflect a dependence upon, or reversion to, early visuo-spatial strategies where symbolic-linguistic arithmetic (Houdé, 1997) or direct retrieval strategies cannot be applied. As Siegler (1986) proposed, children resort to back-up strategies when the answer cannot be retrieved and it is suggested that this may be evident in the present data. In support of this, the phonological loop predicted unique variance in the older children's performance on the easy questions, where direct retrieval may have been possible.

Although this interpretation is speculative, the data provides an initial indication that the working memory processes supporting children's mathematics change with age, and consequently it defines possible independent roles for the two slave systems. These different roles demonstrate a shift from early visuo-spatial strategies to mature, verbal solution strategies, such as direct retrieval.

At approximately 7-/8-years-of-age, children appear more reliant on the visuo-spatial sketchpad to support their mathematics, which may reflect a dependence upon early visuo-spatial strategies to solve novel and complex problems where direct retrieval is not possible (e.g. McKenzie et al., 2003). Although the precise role of the visuo-spatial sketchpad is yet to be established, its involvement in young children's early numeracy development may provide a foundation for representing abstract problems in a concrete form. Hughes (1986) found that pre-school children could more readily solve concrete problems (those that refer to specific objects, people or events), than abstract problems that do not have a concrete referent. He contended that the source of children's difficulty with formally taught mathematics was the abstract, context-free nature of certain problems. Hughes proposed that when children begin school they have to learn the formal language of mathematics. As such, it is tentatively suggested that the visuo-spatial sketchpad may provide a workspace to support the development of links between informal concrete knowledge and the abstract language of mathematics necessary for children's mathematics (Tizard & Hughes, 1984).

By 9-/10-years the children are beginning to rely on the phonological loop for the solution of easy mathematical problems, which may reflect the deployment of direct retrieval strategies that typically involve verbal codes (e.g. Dehaene & Cohen, 1995). Importantly, the data reinforces the idea that the phonological loop is important for the effective use of retrieval strategies, possibly through aiding the acquisition and retrieval of number facts from long-term memory (e.g. Hitch & McAuley, 1991).

The pattern of results observed in our data are consistent with McKenzie et al. (2003) who recently demonstrated, using a dual-task design, that younger children use visuo-spatial strategies in mental arithmetic, while older children use a mixture of phonological and visuo-spatial strategies. As such, the suggestion that visuo-spatial working memory plays an important role in early mathematics certainly warrants further investigation. For example, future research might explore whether the assessment of visuo-spatial sketchpad skills at a young age could be used to identify children who may have later problems learning mathematics. Alternatively, it might explore the relative contributions of different aspects of visuo-spatial cognition (e.g. visuo-spatial working memory, visual attention or imagery) to early mathematics proficiency.

In conclusion, this study provides further support for the involvement of working memory in children's mathematics. Importantly, it provides evidence for an independent role for the visuo-spatial sketchpad in early mathematics. This is important for both cognitive theory and educational practice. Theoretically, the developmental shift in the memory processes involved in children's mathematics between 7- and 10-years advances our understanding of how children learn mathematics. This understanding relates to educational practice on two levels. Firstly, the suggestion that young children use predominantly visuo-spatial strategies until they are competent and able to deploy verbal, abstract strategies is important for the teaching of mathematics. Secondly, the importance of the visuo-spatial sketchpad in early mathematics may further our understanding of the deficits experienced by children with mathematical learning difficulties such as developmental dyscalculia (Butterworth, 2003). Visuo-spatial deficits are characteristic among children and adults with mathematical learning difficulties (e.g. Geary, 1993). Understanding that these difficulties may be specifically related to visuo-spatial working memory deficits in young children provides scope for better early screening methods and opportunities for remediation.

Chapter Summary

1. There was little difference in the working memory demands of the four mathematical skills defined by National Curriculum for England in Chapter 3. Therefore, the aim of the present study was to explore the associations between the three components of the working memory model and children's performance-related mathematical skills (as defined in Chapter 5).
2. Overall working memory scores predicted greater variance in younger than older children's mathematics. Likewise, central executive scores predicted greater variance in younger than older children's mathematics. It was suggested that younger children may be more reliant upon working memory due to slower processing or that working memory resources may support mathematics development where young children are acquiring new solution strategies and learning arithmetic facts.
3. A markedly distinct pattern of associations was revealed across the two age groups. The data indicated a stronger role for the visuo-spatial sketchpad in the younger children's mathematics performance, with phonological loop scores only predicting unique variance in the older children's performance on easy mathematics questions.
4. This finding provides an initial indication that the working memory processes supporting children's mathematics change with age, and consequently it defines possible independent roles for the slave systems.
5. These findings were discussed in terms of their implications for educational practice; for teaching practice and understanding the cognitive deficits in children with mathematical difficulties.

Chapter Seven

Visuo-spatial Working Memory and Children's Performance-related Mathematical Skills

Aim

A significant association was found between visuo-spatial working memory ability and children's curriculum-based mathematics performance in Chapter 4. However, there was little difference between the visuo-spatial sketchpad demands of the different curriculum-based mathematical skills. This same pattern emerged in Chapter 3 when the relationship between working memory ability and curriculum-based skills was explored. The data from Chapter 3 was subsequently re-analysed in Chapter 6 and a markedly distinct pattern of associations was found between the three components of the tripartite working memory model (Baddeley & Hitch, 1974) and the performance-related mathematics skills derived in Chapter 5. The aim of the present study was to replicate this procedure and re-analyse the data collected in Chapter 4 to explore the relationship between visuo-spatial working memory ability and performance-related mathematics abilities. Within this re-analysis, further aims were to explore both the structure of the visuo-spatial sketchpad and the potentially different visuo-spatial working memory demands of the performance-related mathematical skills for each age group.

Introduction

One aim of Chapter 4 was to explore the contribution of the visual and spatial subcomponents (e.g. Logie, 1995) of visuo-spatial working memory to a range of mathematical skills defined by the National Curriculum for England. It was expected that the four mathematical skills would recruit different visual or spatial memory abilities, or that each measure of visuo-spatial sketchpad ability would account for a different amount of variance in each of the curriculum-based skills.

The results of the earlier study (Chapter 4) suggested that visuo-spatial working memory, as measured by three standardized tasks, predicted children's curriculum-based mathematics performance when age-related variance was controlled for. However, it was not possible to determine whether the different mathematics abilities recruited different visual or spatial memory resources for two reasons. Firstly, it was unclear whether the visuo-spatial sketchpad could be fractionated into visual and spatial subcomponents in children. Secondly, there were potential problems with the tasks administered (Chapter 4). However, each of the three standardized visuo-spatial sketchpad measures predicted a similar amount of unique variance in all mathematical skills (between 0% and 2%) suggesting that each skill defined by the curriculum recruited similar visuo-spatial sketchpad resources.

The aim of the present study was to re-analyse the data collected in Chapter 4, following a similar method of re-analysis to that applied to the data collected in Chapter 3. The rationale behind this approach was that the re-analysis of data collected in Chapter 3 revealed a markedly different pattern of associations to that suggested by the initial analyses. It was therefore expected that the same might be true for the data collected in Chapter 4.

As in Chapter 4 the relationship between visuo-spatial working memory and performance across a range of mathematical skills was explored in children who were at Key Stage 2 of the National Curriculum. As in Chapter 6, two important changes were made to the data collected in Chapter 4. Firstly, the two age groups were considered separately to explore any potential developmental differences in the visuo-spatial working memory demands of different mathematical abilities. One reason for this was that previous findings have suggested that younger children might have a greater dependence on the visuo-spatial sketchpad for the successful solution to mathematical problems than older children (see Chapter 6). It was not possible to determine whether visual and spatial subcomponents were fractionated in children from the previous study where the two age groups were considered together. Logie and Pearson (1997) reported that the visual-spatial distinction was more pronounced in older than younger children. Therefore, a further aim was to explore the structure of the visuo-spatial sketchpad in the two age groups. Secondly, the mathematical skills assessed were the performance-related skills derived in Chapter 5.

Individual differences in NVIQ were controlled for in the initial analysis (Chapter 4). However, the NVIQ task loaded on the same factor as the standardized visuo-spatial sketchpad tasks. This suggested they may be measuring the same cognitive ability or that the NVIQ task may contain implicit visuo-spatial memory demands. Furthermore, including the NVIQ measure alongside visuo-spatial working memory measures may have eliminated the variance of interest (see Chapter 4). For this reason, the NVIQ measure was not used in the present study. Two unstandardized measures of visuo-spatial working memory, Blobby Visual and Blobby Spatial, were administered in Chapter 4. Although both tasks predicted unique variance in children's mathematics performance, it was unclear what abilities they were measuring. For this reason, neither task was included in the present study.

In summary, the aim of the present study was to re-analyse the data collected in Chapter 4 to further investigate the relationship between visuo-spatial working memory ability and children's mathematics performance. Visuo-spatial working memory ability was assessed using three non-digit based standardized visuo-spatial sketchpad measures. One task was visual in nature (Visual Patterns Test, Della Sala et al., 1999), one was spatial in nature (Block Recall, Pickering & Gathercole, 2001) and one was arguably both visual and spatial in nature (Mazes Memory, Pickering & Gathercole, 2001). Mathematics ability was assessed using the performance-related tests derived in Chapter 5.

Method

Participants

The participants were the same 107 primary school children who participated in Chapter 4.

Design and Procedure

See Chapter 4.

Materials

Visuo-spatial Sketchpad Tasks

The visuo-spatial sketchpad tasks were the standardized measures used administered in Chapter 4; Visual Patterns Test (Della Sala et al., 1999), Mazes Memory and Block Recall (Pickering & Gathercole, 2001).

Mathematics Tasks

The mathematics assessments were the age appropriate tests administered in Chapter 4. Instead of scoring the tests according to performance across the four curriculum-based skills, children's mathematical abilities were measured according to their performance on the performance-related skills derived in Chapter 5. See Chapter 6 for details of scoring procedures.

Results

Power Analyses

The results of the compromise power analyses (Erdfelder, 1984), conducted using Faul and Erdfelder's (1992) G Power programme, are presented in Table 7.1. Power analyses were conducted for each age group, as the data from the two samples were analysed separately.

Table 7.1

Compromise Power Analysis for Visuo-spatial Working Memory and Performance-Related Mathematics Study.

Year	Effect Size	n_1 (boys)	n_2 (girls)	Power
Year 3	0.5 (medium)	28	23	.81
Year 5	0.5 (medium)	27	29	.82

The power of this study to test for significance with a medium effect size is .81 for the Year 3 sample and .82 for the Year 5 sample. Both values exceeds Cohen's (1988) criterion of .8, meaning this study is statistically powerful.

Descriptive Statistics

Descriptive statistics for visuo-spatial sketchpad measures and performance-related mathematics skills are presented in Table 7.2.

Year 5 children performed better on the visuo-spatial sketchpad measures than Year 3 children. There was greater variability on the Mazes Memory task for both age groups, although Year 3 children also showed variability on the Block Recall task.

Year 3 children performed significantly better on Cluster B than Cluster A questions ($t(50) = -8.83, p < .05$). Year 5 children's performance was significantly lower on the second cluster, Cluster D ($t(55) = 22.63, p < .01$). This is consistent with the notion that the questions comprising this cluster were more difficult.

There was no significant difference between boys and girls scores on any of the measures ($p > .05$). For this reason mean scores are not shown.

Table 7.2.

Descriptive Statistics of Visuo-spatial Sketchpad and Performance-Related Mathematics Measures. Maximum scores for visuo-spatial measures shown in brackets). (N=107).

Measures	Year 3. (n = 51).		Year 5. (n = 56).	
	M	*SD*	*M*	*SD*
Visuo-spatial Sketchpad Measures				
Visual Patterns Test (42)	9.00	2.74	11.26	3.72
Mazes Memory (42)	9.52	4.10	13.69	6.49
Block Recall (54)	23.28	4.19	25.29	3.04
Mathematics Measures				
Cluster A	47.16	21.41	-	-
Cluster B	71.57	9.52	-	-
Cluster C	-	-	75.51	13.81
Cluster D	-	-	28.74	17.35

Note. Mathematics scores shown are proportion correct.

Table 7.3

Correlation Matrix for Visuo-spatial Sketchpad Scores and Performance-Related Mathematics Measures. Year 3 (n=51) simple coefficients are displayed in the upper triangle; Year 5 (n=56) simple coefficients are displayed in the lower triangle.

	Visual Patterns Test	Mazes Memory	Block Recall	Mathematics Cluster 1	Mathematics Cluster 2
Visual Patterns Test	-	.30*	.47**	.30*	.08
Mazes Memory	.39**	-	.25	.20	.01
Block Recall	.28*	.27	-	.30*	.08
Mathematics Cluster 1	.28*	.11	.00	-	.47**
Mathematics Cluster 2	.19	.11	-.02	.55**	-

Note. Cluster 1 is Cluster A for Year 3 and Cluster C for Year 5. Cluster 2 is Cluster B for Year 3 and Cluster D for Year 5.

Correlation Analyses

Associations between visuo-spatial sketchpad scores and performance-related mathematics skills are presented in Table 7.3. Coefficients for Year 3 are displayed in the upper triangle; coefficients for Year 5 are displayed in the lower triangle.

Scores on the Visual Patterns Test were significantly associated with scores on both the Mazes Memory ($p<.05$ for Year 3, $p<.01$ for Year 5) and Block Recall ($p<.01$ for Year 3, $p<.05$ for Year 5) measures for both age groups. Mazes Memory was not correlated with Block Recall for either age group ($p>.05$). Performance on the mathematics abilities was intercorrelated across both age groups (all rs>.3, $p<.01$).

For Year 3 children, Visual Patterns Test and Block Recall scores were significantly related to Cluster A (the purer mathematics questions) performance-related mathematical abilities (all rs>.3, $p<.05$). For Year 5 children, Visual Patterns Test scores were significantly related to Cluster C (the easy mathematics questions) performance-related mathematical abilities ($p<.05$). Mazes Memory scores were not related to children's mathematics test scores in either age group ($p>.05$).

Regression Analyses

Simple regression analyses revealed that visuo-spatial sketchpad measures predicted 10.8% of the variance in Year 3's overall mathematics performance and 7.8% in Year 5's overall mathematics performance. Subsequently, a series of fixed-order regression analyses were used to assess the amount of unique variance in the performance-related mathematics scores predicted by each of the visuo-spatial sketchpad measures for Year 3 (Table 7.4) and Year 5 (Table 7.5) children. The mathematics measure was the regressor for each analysis and the unique contribution

Table 7.4

Fixed-order Multiple Regression Analyses: Visuo-spatial Scores Predicting Unique Variance in Performance-related Mathematics for Year 3. (n=51).

Predictor	Mathematics Ability Predicted	Model	Order of Entry Into Equation	*r*	r^2	*Adjusted* r^2
Visual Patterns Test						
	Cluster A	A_1	1.Mazes	.20	.04	.02
			2.Block	.33	.11	.07
			3.Visual Patterns	.37	.14	.07
	Cluster B	A_2	1. Mazes	.01	.00	-.02
			2. Block	.09	.01	-.02
			3.Visual Patterns	.10	.01	-.02
Mazes Memory						
	Cluster A	B_1	1. Visual Patterns	.30	.09	.07
			2. Block	.36	.13	.09
			3. Mazes	.37	.14	.09
	Cluster B	B_2	1. Visual Patterns	.07	.01	-.02
			2. Block	.10	.01	-.02
			3. Mazes	.10	.01	-.02
Block Recall						
	Cluster A	C_1	1. Visual Patterns	.30	.09	.07
			2. Mazes	.32	.10	.07
			3. Block	.37	.14	.07
	Cluster B	C_2	1. Visual Patterns	.08	.01	-.02
			2. Mazes	.08	.01	-.02
			3. Block	.10	.01	-.02

(measured as r^2) of each working memory measure was assessed as a predictor entered into the equation after the other predictors.

Models A_1 and B_1 and C_1 show that each visuo-spatial sketchpad measure accounted for a small but significant amount of unique variance in Year 3 children's Cluster A (pure) mathematics scores (Visual Patterns Test scores accounted for 3%, Mazes Memory scores accounted for 1% and Block Recall scores accounted for 4% of unique variance). Models A_2 and B_2 and C_2 show that each visuo-spatial sketchpad measure did not account for any unique variance in Year 3 children's Cluster B (applied) mathematics scores beyond that predicted by the other visuo-spatial sketchpad measures. Visuo-spatial sketchpad measures predicted less unique variance in Year 5's mathematics performance. Models D_1 and D_2 show that Visual Patterns Test scores predicted 7% of unique variance in Year 5 children's Cluster C mathematics scores and 4% of unique variance in Cluster D mathematics scores. Neither Mazes Memory nor Block Recall scores predicted unique variance in Year 5 children's performance-related mathematics scores.

Table 7.5

Fixed-order Multiple Regression Analyses: Visuo-Spatial Scores Predicting Unique Variance in Performance-related Mathematics for Year 5. (n=56).

Predictor	Mathematics Ability Predicted	Model	Order of Entry Into Equation	r	r^2	Adjusted r^2
Visual Patterns Test						
	Cluster C	D_1	1.Mazes	.11	.01	-.01
			2.Block	.11	.01	-.01
			3.Visual Patterns	.29	.08	.02
	Cluster D	D_2	1. Mazes	.11	.01	-.01
			2. Block	.12	.01	-.01
			3.Visual Patterns	.21	.05	.03
Mazes Memory						
	Cluster C	E_1	1. Visual Patterns	.28	.08	.06
			2. Block	.29	.08	.06
			3. Mazes	.29	.08	.06
	Cluster D	E_2	1. Visual Patterns	.19	.04	.02
			2. Block	.21	.04	.02
			3. Mazes	.21	.04	.02
Block Recall						
	Cluster C	F_1	1. Visual Patterns	.28	.08	.06
			2. Mazes	.28	.08	.06
			3. Block	.29	.08	.06
	Cluster D	F_2	1. Visual Patterns	.19	.04	.02
			2. Mazes	.19	.04	.02
			3. Block	.21	.04	.02

Exploratory Factor Analyses

Principal components analyses were conducted on the data from each group to determine the higher-order factor structure underpinning variations in scores on all measures as a descriptive, summative method.

Table 7.6

Factor Loadings of Visuo-spatial and Performance-related Mathematics Measures on Rotated Component Matrix for Year 3.

Measure	*Factor*	
	1	2
Visual Patterns Test	.79	
Mazes Memory	.70	
Block Recall	.71	
Cluster A	.33	.78
Cluster B		.90

Note. Only loadings greater than .30 are shown.

KMO measure of sampling adequacy = .73.

Two factors emerged with eigenvalues in excess of 1.00 on the Year 3 data. Factor loadings greater than .30 on the rotated component matrix are presented in Table 7.6. All three visuo-spatial sketchpad measures loaded on a single factor (Factor 1). Both mathematics measures loaded on a separate factor (Factor 2). Although Cluster A mathematics loaded on both factors, the weighting of the loading on the mathematics factor (Factor 2 = .78) was greater than the loading on the visuo-spatial sketchpad factor (Factor 1 = .33), consistent with the notion that it is primarily a measure of mathematics ability.

Table 7.7

Factor Loadings of Visuo-spatial and Performance-related Mathematics Measures on Rotated Component Matrix for Year 5.

Measure	*Factor*	
	1	2
Visual Patterns Test	.70	.33
Mazes Memory	.74	
Block Recall	.74	
Cluster C		.87
Cluster D		.86

Note. Only loadings greater than .30 are shown.

KMO measure of sampling adequacy = .76

Two factors emerged with eigenvalues in excess of 1.00 in the second analysis, which was conducted on the Year 5 data. Factor loadings greater than .30 on the rotated component matrix are presented in Table 7.7. Again, all three visuo-spatial sketchpad measures loaded on a single factor (Factor 1), while both mathematics measures loaded on a separate factor (Factor 2). The Visual Patterns Test loaded on both factors. However, the weight of the loading on the visuo-spatial sketchpad factor (Factor 1 = .70) was greater than the loading on the mathematics factor (Factor 2 = .33) suggesting it is primarily a measure of visuo-spatial ability.

Discussion

Overall, the results show a significant association between visuo-spatial sketchpad ability and mathematics attainment in 7-/8-year-olds and 9-/10-year-olds. Simple regression analyses revealed that visuo-spatial sketchpad scores predicted 11% of the variance in Year 3 children's mathematics scores and 8% of the variance in Year 5 children's mathematics scores. This provides additional evidence for the involvement of visuo-spatial working memory in children's mathematics (e.g. McKenzie et al., 2003) and further supports the suggestion that visuo-spatial working memory assessments may be useful predictors of mathematics attainment (e.g. Jarvis & Gathercole, 2003). Importantly, the findings support the notion that the role of visuo-spatial working memory in children's mathematics may change with age (see Chapter 6).

The results presented in Chapter 4 suggested that there was little difference in the visuo-spatial sketchpad demands of different curriculum-based mathematical skills. However, consistent with the results of the Chapter 6, the present data revealed a developmental difference in the relationship between visuo-spatial working memory and performance-related mathematics for the two age groups. Overall, the three visuo-spatial sketchpad measures predicted less variance in the older children's mathematics scores (8%) than the younger children's mathematics scores (11%). Furthermore, all three visuo-spatial sketchpad measures predicted unique variance in the younger children's performance-related mathematics attainment, whilst only one visuo-spatial sketchpad measure (Visual Patterns Test) predicted unique variance in the older children's performance-related mathematics attainment.

Detailed analyses revealed that the three standardized visuo-spatial sketchpad measures predicted unique variance in the younger children's mathematics scores on the "pure" but not the "applied" mathematics questions. This supports the idea that young children may be relying on the visuo-spatial sketchpad to represent abstract mathematical problems that do not have concrete referents (see Chapter 6).

Consistent with previous results (Chapter 6) visuo-spatial sketchpad scores predicted less variance in older children's performance-related mathematics performance. The results of the previous study suggested that older children might rely on, or revert to, the visuo-spatial sketchpad (or visuo-spatial strategies) for the solution to "difficult" problems where symbolic-linguistic arithmetic (Houdé, 1997) or direct retrieval could not be applied. It was therefore expected that visuo-spatial sketchpad scores would predict greater variance in the difficult questions than the easy questions in the present study. However, scores on a visuo-spatial sketchpad measure (Visual Patterns Test) were found to predict older children's performance on both the easy and difficult mathematical questions. Dehaene and colleagues proposed that we develop visuo-spatial codes for numbers and represent them along an analogue "mental number line" to assist in mathematical processing (see Chapter 1, section 1.4.2.2.). Therefore, it is tentatively suggested that the involvement of the visuo-spatial sketchpad in children's mathematics may reflect the use of the "mental number line" (Dehaene, 1992). Clearly this requires further research.

One possibility would be to investigate the relationship between the strength of the SNARC effect, visuo-spatial working memory ability and mathematics performance in children. Dehaene and colleagues substantiated the existence of the "mental number line" in experiments that demonstrated the SNARC effect (e.g. Dehaene et al., 1992). Therefore, it would be expected that children who showed a large SNARC effect would perform significantly better on mathematics and visuo-spatial sketchpad tests compared to children who did showed a smaller SNARC effect. Feeney et al. (2004) conducted such an investigation with adults. They asked participants to complete graphical reasoning, number judgement (SNARC effect) and non-verbal working memory tasks. Performance on the non-verbal working memory task was significantly associated with the use of analogical representations for the graph task. In turn, the tendency to use an analogue (spatial) representation for the graph task was associated with the tendency to use an analogue representation for the number judgement task. They propose that these results provide an initial indication that people may represent concepts by analogy to space; a domain-general ability. In relation to the current discussion, Feeney et al. (2004) provide evidence to suggest that performance on a mathematical task (a graphical reasoning task) is related to the size of the SNARC effect and non-verbal working memory ability in adults. A replication of this investigation with children may prove fruitful in light of the present findings.

One aim of the present study was to explore the structure of the visuo-spatial sketchpad in children of different ages using three standardized measures visuo-spatial sketchpad ability. Consistent with findings presented in Chapter 4 the present results imply that the visuo-spatial sketchpad may not be fractionated into visual and spatial subcomponents in children. Logie and Pearson (1997) reported that the distinction between the two subcomponents was more evident in older than younger children. However, the present study, in which the same two tasks used by Logie and Pearson (1997) were administered alongside a third task to similar aged children, suggested a different pattern of results. All three measures, the Visual Patterns Test, Mazes Memory and Block Recall, loaded on a single factor in Year 3 (7-/8-year-olds) and Year 5 (9-/10-year-olds) factor analyses. This suggests that the visual and spatial subcomponents may not be distinct and that the structure of visuo-spatial sketchpad does not change with age. One possibility is that the visuo-spatial sketchpad is fractionated into alternate subcomponents (e.g. static and dynamic, Pickering et al., 2001) that become more clearly fractionated with age. An alternate explanation, as discussed in Chapter 4, is that the tasks used did not measure distinct visual and spatial memory abilities.

In conclusion, the data provides additional evidence for an association between visuo-spatial working memory and children's mathematics performance. This further supports the suggestion that visuo-spatial working memory assessments may be useful predictors of children's mathematics attainment (e.g. Jarvis & Gathercole, 2003). Importantly, this study provides evidence to support the notion that younger children may be more dependent on the visuo-spatial sketchpad for the solution to mathematical problems than older children. In particular, the present data support the idea that the visuo-spatial sketchpad may support links between concrete and abstract mathematical knowledge. With further research, this may enhance our understanding of the deficits experienced by children with mathematical learning difficulties, such as developmental dyscalculia (e.g. Butterworth, 2003).

Finally, an important theoretical issue arising from the present study relates to the structure and assessment of the visuo-spatial sketchpad in children. Consistent with the results of Chapter 4 the standardized visuo-spatial sketchpad measures used did not differentiate separable visual and spatial memory skills. This further supports the need for research into the specific nature of visuo-spatial working memory tasks (see Chapter 4).

Chapter Summary

1. The aim of the present study was to re-analyse the data collected in Chapter 4 to explore the relationship between visuo-spatial working memory ability and children's performance-related mathematical skills (as defined in Chapter 5). Further aims were to explore the structure of the visuo-spatial sketchpad and the potentially different associations between visuo-spatial ability and mathematics performance separately for each age group.
2. Overall, visuo-spatial working memory ability predicted variance in 7-/8-year-olds and 9-/10-year-olds performance-related mathematical skills.
3. Consistent with the results presented in Chapter 6, visuo-spatial sketchpad scores predicted less variance in older children's than younger children's mathematics performance. Furthermore, visuo-spatial working memory measures predicted greater variance in younger children's performance on pure questions than applied questions. These findings support the idea that the visuo-spatial sketchpad may provide a foundation upon which abstract problems can be represented in a concrete format, thus supporting young children's mathematical development.
4. Visuo-spatial sketchpad scores predicted unique variance in the older children's performance on both easy and difficult mathematics questions. It was tentatively suggested that this may reflect the use of a "mental number line". However, further research is needed to substantiate this.
5. In line with the results reported in Chapter 4, the results suggested that the visuo-spatial sketchpad may not be fractionated in children. Importantly, the present findings suggest that the structure of the visuo-spatial sketchpad does

not change with age. These results are discussed in terms of the need for further research into the nature of visuo-spatial working memory tasks.

Chapter Eight

Working Memory as a Predictor of Children's Achievements on National Curriculum Mathematics Tests at 11-years-of-age

Aim

Significant associations between working memory ability and children's mathematics performance were reported in Chapters 3, 4, 6 and 7. Consistent with previous studies (e.g. Gathercole & Pickering, 2000b) this suggests that working memory assessments may be useful predictors of academic attainment. However, none of these studies demonstrate that working memory scores at Time 1 predict mathematics attainment at Time 2. Therefore, the aim of the present study was to examine whether the three components of the tripartite working memory model (Baddeley & Hitch, 1974) share unique predictive relationships with mathematics attainment as measured by Key Stage 2 SATs performance one year after initial testing.

Introduction

An overarching aim of this thesis was to extend the work of Gathercole and colleagues, who have reported significant associations between working memory ability and National Curriculum test performance at Key Stages1 (7-years), 2 (11-tears) and 3 (14-years), to specifically explore the associations between working memory ability and mathematics performance in children at Key Stage 2 (7-/8-year-olds and 9-/10-year olds). Thus far, the results suggest that working memory ability is significantly associated with mathematics performance when assessed concurrently. However, there is no evidence to suggest that working memory test scores predict mathematics attainment at a later date.

Results of the previous studies show that children's working memory test scores, as measured by non-digit based assessments, are significantly related to their performance across a number of mathematical skills. Specifically, central executive and visuo-spatial sketchpad scores have been associated with children's curriculum-based mathematical skills (Chapters 3 and 4) and phonological loop, visuo-spatial sketchpad and central executive scores have been associated with children's performance-related mathematical skills (Chapters 6 and 7). These findings are consistent with previous studies that report significant associations between working memory scores and attainment at Time 1 (e.g. Gathercole & Pickering, 2000b; Jarvis & Gathercole, 2003).

In addition to finding significant associations between working memory test scores and attainment at Time 1, Gathercole and Pickering (2000b) found that working memory test performance at Time 1 predicted scholastic attainment one year later. They administered a battery of working memory assessments, designed to tap the three components of the tripartite working memory model (Baddeley & Hitch, 1974), to 6- and 7-year-olds and obtained scores on standardized measures of scholastic attainment at the initial time of testing and one year later. Importantly, phonological loop scores at 6- / 7-years predicted performance on a vocabulary measure one year later and central executive scores at 6- / 7-years predicted performance on both a literacy and arithmetic measure one year later (see Chapter 3).

Gathercole and Pickering's (2000b) findings suggest that associations between working memory ability and scholastic attainment persist one year after initial testing in children aged 7- / 8-years. These unique predictive relationships provide strong evidence to support the use of working memory assessments as early indicators of later academic achievement.

The studies presented in this thesis thus far have not included a longitudinal assessment. The aim of the present study was to address this weakness and explore the relationship between working memory ability at Time 1 and mathematics performance at Time 2. Working memory test scores were obtained at the initial time of testing for 9-/10-year-olds in Chapter 3. Key Stage 2 mathematics National Curriculum test performance was used as an index of mathematics attainment one year after initial testing. There were two reasons for this. Firstly, previous studies that have investigated the relationship between working memory ability and children's National Curriculum attainment at Key Stages 1 (Gathercole & Pickering, 2000a), 2 and 3 (Jarvis & Gathercole, 2003; Gathercole, Pickering, Knight et al., 2004) have not used longitudinal methodologies. Therefore, this study would extend previous research to highlight the potential usefulness of working memory assessments as prospective indicators of National Curriculum achievement. Secondly, National Curriculum test performance was deemed a suitable indicator of children's scholastic attainment as it provides an ecologically valid measure of academic achievement. As in Chapter 3 a measure of children's NVIQ (taken at Time 1) was also included. Gathercole and Pickering (2000b) reported that working memory assessments at Time 1 predicted academic attainment at Time 2, but they did not control for individual differences in general ability. Therefore, an index of children's NVIQ was included at the time of initial testing to explore whether working memory assessments at Time 1 predicted National Curriculum mathematics performance over and above NVIQ scores one year later.

Method

Participants

The participants were the 70 Year 5 primary school children who participated in the study reported in Chapter 3. At Time 1 (the Easter and summer terms of Year 5) the group consisted of 33 boys and 37 girls, mean age 9 years and 10 months (*SD* = 5.7 months, range 9 years and 1 month to 10 years and 9 months). See Chapter 3. At Time 2 (the summer of Year 6) the group consisted of the same 70 children, mean age 10 years and 10 months.

Design and Procedure

All working memory assessments were administered to the children at Time 1 (see Chapter 3). Schools supplied the mathematics test scores at Time 2.

Materials

Working Memory Tasks

Three non-digit based working memory assessments were administered. See Chapter 3 for details.

Non-Verbal IQ Task

The Matrix Analogies Test Short Form (MAT-SF) (Naglieri, 1985) was administered. See Chapter 3 for details.

Mathematics Measures

The mathematics measures were children's attainment levels in the Key Stage 2 Mathematics SATs taken in the summer term of Year 6 (11-years). At Key Stage 2 mathematics SAT scores incorporate two mathematics papers (which assess Number and Algebra, Shape, Space and Measures and Handling Data) and a mental arithmetic test. Each child is awarded an attainment level between 3 and 5. Level 4 indicates nationally expected standards.

Results

Power Analyses

The power of this study to test for significance with a medium effect size is .85. This value exceeds Cohen's (1988) criterion of .8, meaning this study is statistically powerful (see Table 6.1, Chapter 6).

Descriptive Statistics

Descriptive statistics for working memory measures and NVIQ scores were obtained in Study 2 (see Table 3.2, Chapter 3). Overall, the children performed better on the phonological loop measure than the other two working memory component measures. There was greater variability on the visuo-spatial sketchpad measure than the other two measures of working memory ability.

The mean Key Stage 2 Mathematics attainment level was 3.97, *SD* .99. The percentage of children achieving Level 4 and above in mathematics was 75.7%, with 30% achieving Level 5. This was slightly higher than the national average where 73% of children achieved Level 4 and above in mathematics, with 29% achieving Level 5.

Correlation Analyses

A simple correlation analysis revealed that Maths Time 1 scores were highly significantly related to children's maths SATs one year later (r=.76, p<.001).

Therefore, Maths Time 1 scores were not entered into further correlation or regression analyses.

Associations between working memory measures and Key Stage 2 mathematics attainment are presented in Table 8.1. Simple correlations are displayed in the upper triangle; partial correlation coefficients controlling for NVIQ are displayed in the lower triangle.

Table 8.1

Correlation Matrix for Working Memory Test Scores and Key Stage 2 Mathematics Achievements. Simple coefficients are displayed in the upper triangle; partial coefficients are displayed in the lower triangle. (*N*=70).

	Maths Achievement	Phonological Loop	Visuo-Spatial Sketchpad	Central Executive
Maths Achievement	-	.13	.35**	.45**
Phonological Loop	.09	-	.08	.26*
Visuo-Spatial Sketchpad	.30*	.05	-	.21
Central Executive	.35**	.18		-

Phonological loop and central executive scores were intercorrelated before the variance attributed to NVIQ was controlled for ($r < .30, p<.05$). Scores on the other working memory measures were not intercorrelated (all rs $< .30, p>.05$).

Central executive and visuo-spatial sketchpad scores were significantly associated with Key Stage 2 mathematics achievement (both *r*s > .30, p<.01) and remained so after the variance attributed to NVIQ scores was eliminated (visuo-spatial sketchpad $r \geq .30$, p<.05, central executive $r > .30$, p<.01). Phonological loop scores were not related to Key Stage 2 mathematics achievement (all *r*s < .30, p>.05).

Regression Analyses

Simple linear regression analyses revealed that working memory measures at Time 1 predicted 27.1% of the variance in Key Stage 2 mathematics performance at Time 2 and 16% of unique variance in Key Stage 2 mathematics attainment after the variance attributed to children's NVIQ scores had been accounted for.

A series of fixed-order unique variance regression analyses (presented in Table 8.2) were used to assess the amount of unique variance in Key Stage 2 mathematics achievement predicted by each of the working memory measures. For each analysis the mathematics assessment was the regressor and the unique contribution (measured as r^2) of each working memory measure was assessed as a predictor entered into the regression equation after the other predictors.

Table 8.2

Fixed-order Multiple Regression Analyses: Working Memory Measures Predicting Unique Variance in Key Stage 2 Mathematics Performance. (n=70)

Predictor	Mathematics Ability Predicted	Model	Order of Entry Into Equation	r	r^2	Adjusted r^2
Phonological Loop						
	Key Stage 2 Mathematics	A_1	1.VSSP	.35	.12	.11
			2.CE	.52	.27	.25
			3. PL	.52	.27	.25
Visuo-spatial Sketchpad						
	Key Stage 2 Mathematics	B_1	1. PL	.13	.02	.00
			2. CE	.45	.20	.18
			3. VSSP	.52	.27	.23
Central Executive						
	Key Stage 2 Mathematics	C_1	1. PL	.13	.02	.00
			2. VSSP	.36	.18	.10
			3. CE	.52	.27	.23

A second series of fixed-order unique variance regression analyses, presented in Table 8.3, were conducted to assess the amount of unique variance in Key Stage 2 Mathematics scores predicted by each of the measures of working memory after the variance accounted for by NVIQ was considered. Again, for each analysis the mathematics assessment was the regressor and the unique contribution (measured as r^2) of each working memory measure was assessed as a predictor entered into the regression equation after the other predictors, which included NVIQ and performance on the other working memory measures.

Table 8.3

Fixed-order Multiple Regression Analyses: Working Memory Measures Predicting Unique Variance in Key Stage 2 Mathematics Performance, controlling for NVIQ. (n=70).

Predictor	Mathematics Ability Predicted	Model	Order of Entry Into Equation	*r*	r^2	*Adjusted* r^2
Phonological Loop						
	Key Stage 2 Mathematics	D_1	1.NVIQ	.34	.11	.09
			2.CE	.47	.22	.19
			3.VSSP	.52	.27	.23
			4. PL	.52	.27	.23
Visuo-spatial Sketchpad						
	Key Stage 2 Mathematics	E_1	1.NVIQ	.34	.11	.09
			2.CE	.47	.22	.19
			3.PL	.47	.22	.19
			4. VSSP	.52	.27	.21
Central Executive						
	Key Stage 2 Mathematics	F_1	1.NVIQ	.34	.11	.10
			2.PL	.35	.12	.10
			3.VSSP	.45	.20	.15
			4. CE	.52	.27	.21

Models A_1 and D_1 show that phonological loop scores at the initial time of testing do not account for any unique variance in Key Stage 2 mathematics scores beyond that predicted by the other working memory measures one year later.

Both central executive and visuo-spatial sketchpad scores shared unique predictive relationships with Key Stage 2 mathematics achievement one year after initial testing. Models B_1 and E_1 show that visuo-spatial sketchpad scores predicted 7% of unique variance in Key Stage 2 mathematics scores beyond that predicted by scores on the other working memory measures and 5% of unique variance in Key Stage 2 mathematics scores beyond that predicted by NVIQ scores and scores on the other working memory measures. Models C_1 and F_2 show that central executive scores accounted for the largest amount of unique variance in Key Stage 2 mathematics scores; 9% of unique variance beyond that predicted by scores on the other working memory measures and 7% of unique variance that predicted by NVIQ scores and scores on the other working memory measures.

Discussion

Overall, the results show a significant association between working memory scores at Time 1 and Key Stage 2 Mathematics performance one year later at Time 2. A simple regression analysis revealed that the tripartite working memory model (Baddeley & Hitch, 1974) predicted 27.1% of the variance in Key Stage 2 mathematics scores, suggesting that it shares a predictive relationship with children's mathematics.

The results of this study suggest that working memory ability is significantly associated with children's National Curriculum attainment. This further supports an association between working memory ability and wider aspects of curriculum attainment (e.g. Gathercole & Pickering, 2000a) and specifically supports the finding that working memory ability is related to children's Key Stage 2 Mathematics attainment (e.g. Jarvis & Gathercole, 2003).

Importantly, the data extend previous findings to suggest that working memory scores at Time 1 predict later mathematics SATs attainment. Specifically, working memory scores at 9-/10-years predicted Key Stage 2 mathematics at 10-/11-years. Gathercole and Pickering (2000b) reported similar results with children at Key Stage 1 of the National Curriculum, where working memory scores at 6-/7-years predicted academic attainment at 7-/8-years. Together these findings suggest that working memory supports curricular progress throughout the early Key Stages and provide further evidence that working memory assessments may be valuable methods for predicting curriculum achievements.

Gathercole and Pickering (2000b) propose that working memory assessments may act as useful supplements to knowledge-based methods of baseline evaluation at school entry at 4 years-of-age. They suggest that working memory assessments may be useful because, unlike baseline assessments that measure a child's knowledge of a particular domain, they are relatively free from environmental and cultural experience (such as the quality and quantity of teaching in a particular domain) (Campbell, Dollaghan, Needleman & Janosky, 1997). Therefore, they may provide prospective indicators of curriculum performance independent of a child's learning experiences.

Indeed, the current study supports the usefulness of working memory assessments as prospective indicators of National Curriculum achievements. Furthermore, it adds to the existing literature to suggest that working memory ability at Time 1 predicts unique variance in mathematics attainment at Time 2 after individual differences in NVIQ have been controlled for. A simple regression analyses revealed that working memory scores at 9-/10-years predicted 16% of unique variance in Key Stage 2 Mathematics attainment after the variance attributed to children's NVIQ scores had been accounted for. Current research suggests that many working memory assessments can be administered to children as young as 4-years (e.g. Pickering & Gathercole, 2001) and that these assessments are significantly related to academic achievement at 7-, 11- and 14-years-of-age. Together with Gathercole and Pickering's (2000b) observation this finding suggests that working memory assessments may be useful prospective indicators of curriculum performance independent of a child's learning experiences over and above intelligence measures. Clearly further research is needed to investigate whether these assessments are useful supplements to baseline assessments at school entry at 4-years-of-age.

Detailed analyses revealed that visuo-spatial sketchpad and central executive scores, but not phonological loop scores, shared unique predictive relationships with children's curriculum-based mathematics performance. This consistent with previous results reported in Chapter 3 and further supports the suggestion that both the central executive (e.g. Bull et al., 1999) and visuo-spatial sketchpad (e.g. McKenzie et al., 2003) may support children's mathematics (see Chapter 3).

The failure to find an association between phonological loop scores and academic achievement replicates a result found by Gathercole and Pickering (2000a) in a study with 7-/8-year-olds. They offered two reasons why phonological loop scores were not related to scholastic attainment. Firstly, their group sizes were relatively small and secondly, they found a high degree of association between central executive and phonological loop measures. Therefore, they proposed that phonological loop scores were not related to academic achievement as they placed heavy demands on the central executive. Significant associations were found between the phonological loop and central executive measures in the present study, suggesting that Gathercole and Pickering's (2000a) latter explanation may be appropriate. Alternatively, phonological loop scores may only share significant associations with children's performance-related mathematics (Chapter 6), not their curriculum-based mathematics (Chapter 3).

In summary, this study provides additional evidence for the involvement of working memory, in particular the central executive and visuo-spatial sketchpad, in children's mathematics. It further supports an association between working memory ability and children's National Curriculum attainment and extends previous findings to suggest that working memory assessments may be useful prospective indicators of curriculum attainment above and beyond NVIQ measures. This has important implications for educational practice, where working memory assessments may be valuable tools for both predicting academic attainment in young children and screening children to identify those at risk of later academic difficulties (e.g. Gathercole & Pickering, 2000a).

Chapter Summary

1. Significant associations have been found between working memory ability and children's mathematical skills throughout this thesis. However, none of the studies conducted thus far have included a longitudinal phase.
2. The aim of this study was to explore whether working memory assessments were useful prospective indicators of mathematics attainment one year later.
3. Working memory ability at 9-/10-years predicted Key Stage 2 mathematics attainment at 10-/11 years. In particular, central executive and visuo-spatial scores, but not phonological loop scores, predicted later mathematics attainment. Furthermore, working memory scores at Time 1 predicted unique variance in mathematics attainment at Time 2 after individual differences in NVIQ had been controlled for.

4. These results extend previous cross-sectional findings to suggest that working memory assessments may be useful prospective indicators of National Curriculum test performance. It was suggested that working memory measures may prove to be useful supplements to baseline assessments, although further research is clearly needed to investigate this.

Chapter Nine

Visuo-spatial Skills and Mathematical Difficulties

Aim

Visuo-spatial deficits are characteristic among children and adults with mathematical difficulties (MD) (e.g. Geary, 1993). Studies have shown that children with MD are impaired on tests of visuo-spatial working memory (e.g. McLean & Hitch, 1999), suggesting that these deficits may be specifically related to visuo-spatial working memory. As such, visuo-spatial working memory assessments might be useful tools for screening young children to identify those with, or at risk of developing, MD. The aim of the present study was to explore this idea through examination of the mathematical abilities of children with visuo-spatial working memory deficits and examination of the visuo-spatial working memory skills of children with MD.

Introduction

Children's mathematical attainment has been associated with visuo-spatial working memory ability (e.g. Gathercole & Pickering, 2000a; Jarvis & Gathercole, 2003). Indeed, the results presented in this thesis support this association. Significant associations were found between visuo-spatial working memory abilities and children's mathematics performance in Chapters 3, 4, 6 and 7. Furthermore, visuo-spatial sketchpad scores at Time 1 shared predictive relationships with children's Key Stage 2 mathematics scores at Time 2 (see Chapter 8). Together these findings suggest that visuo-spatial working memory assessments may be useful prospective indicators of children's mathematics performance. Subsequently, it is proposed that visuo-spatial working memory assessments might be useful tools for screening young children who may be at risk of MD. To further investigate this, work must be conducted to investigate whether or not visuo-spatial working memory tasks discriminate between children with high or low mathematics abilities.

Children experience mathematical difficulties for a number of reasons, including: social and emotional problems (i.e. mathematics anxiety, Ashcraft & Faust, 1994) and cognitive, neuropsychological and cognitive neuropsychological deficits. In a review of experimental, cognitive, clinical and neuropsychological literature Geary (1993) defined three subtypes of MD. The first, mediated by a developmental delay in the acquisition of conceptual knowledge, manifests itself as procedural deficits in counting knowledge, computational skill and working memory (e.g. Geary, et al., 1992). The second, a more persistent retrieval-based deficit, manifests as memory-retrieval errors and fewer occurrences of direct fact retrieval from long term memory (e.g. Geary, 1990). The third MD subtype, characterised by visuo-spatial deficits, manifests as functional deficits (i.e. problems in the spatial alignment of numerical information) and conceptual deficits (i.e. understanding number representations such as place value) (e.g. Rourke & Finlayson, 1978). All three are identified in both the cognitive (e.g. Geary, 1990) and neuropsychological literatures (e.g. Temple, 1991). However, the third subtype characterised by visuo-spatial deficits, is less frequently identified in the cognitive literature because the visuo-spatial skills of MD children are not typically assessed (Geary, 1993).

Cognitive research suggests that the visuo-spatial deficits experienced by children with MD may be specifically related to visuo-spatial working memory. Significant associations have been found between children's visuo-spatial working memory abilities and their mathematics attainment (e.g. Jarvis & Gathercole, 2003). Furthermore, adults and children with MD perform worse on visuo-spatial working memory tasks than adults and children without MD (e.g. Wilson & Swanson, 2001). Specifically, Hitch & McLean (1999) reported that 9-year-old children with specific MD were impaired on spatial working memory tasks compared to age-matched controls, suggesting children with MD experience visuo-spatial working memory deficits. Related to this, developmental dyscalculia may be a visuo-spatial impairment. Evidence suggests that humans are born with an innate "number sense" (Dehaene, 1992) or "number module" (Butterworth, 1999), for dealing with numerical representations. Butterworth (1999) suggests that the underlying cause of dyscalculia is likely to be related to a dysfunction of this "module". Anatomically the "number module" is located in the parietal lobe (e.g. Butterworth, 1999), a brain region associated with visuo-spatial processing. Therefore, an impaired "number sense" or "number module" may be a visuo-spatial impairment.

Children who are failing to achieve expected levels of attainment show impairments on working memory tasks. For example, as discussed in Chapter 3, Gathercole and colleagues found that children who were failing to achieve normal levels of curriculum attainment showed marked impairments on working memory assessments (e.g. Gathercole & Pickering, 2000a; Gathercole, Pickering, Knight et al., 2004). More specifically, they found that children with low achievements in mathematics showed marked impairments on tests of visuo-spatial working memory (Gathercole & Pickering, 2000a).

Working memory tasks have been used to identify children who are failing to achieve expected levels of attainment. Gathercole and Pickering (2000a) used a subset of working memory measures selected from the WMTB-C (Pickering & Gathercole, 2001) to identify children who were failing to achieve expected attainment levels at 7- / 8-years. They explored the extent to which scores on individual working memory measures could be used to predict children with at least one area of low achievement. Their results suggested that working memory assessments could be used to successfully classify 82.9% of the children (83.1% of children were correctly classified as normal achievers and 82.6% were correctly classified as low achievers). Gathercole, Pickering, Knight et al. (2004) reported similar results with 7-/8-year-olds and 14-/15-year-olds. Consistent with their previous findings, 82.5% of the younger children were correctly classified as normal or low achievers across English and Mathematics based on their working memory test scores. Importantly, all of the low achieving children were correctly classified. Similar values were obtained for the older children, where working memory scores were used to correctly classify 80.5% of low or normal achievers in Mathematics and 83% of low or normal achievers in Science. Working memory scores did not successfully discriminate the older children's English achievement groups.

In summary, research suggests that visuo-spatial deficits are characteristic among a sub-group of children with MD. These deficits may be specifically related to visuo-spatial working memory. Working memory assessments have been used to correctly classify children with low levels of achievement and children with poor mathematics attainment typically perform poorly on working memory tests. It is therefore suggested that visuo-spatial working memory assessments may be useful tools with which to identify children at risk of Geary's (1993) visuo-spatial subtype of MD. Of course, visuo-spatial MD represents only a small sub-group of MD. Mathematics is a complex skill, involving language, space and quantity (Butterworth, 2003) and a number of factors contribute to good mathematics attainment. Similarly, a number of factors contribute to poor mathematics attainment, or MD. These include genetic (e.g. Butterworth's number module), environmental (e.g. inappropriate teaching methods, absence from school) and cognitive (e.g. poor working memory ability) factors. Furthermore, MD often co-occurs with other problems such as dyslexia, attention deficit hyperactivity disorder (ADHD) and specific language impairments (Butterworth, 2003). Therefore, problems arise in defining and diagnosing MD. For this reason, the present study focuses on a sub-group of MD.

The overarching aim of the present study was to investigate whether performance on visuo-spatial working memory assessments discriminated between children of high and low mathematics ability. Specifically, the visuo-spatial working memory profiles of children with different mathematical abilities were explored. This also included an exploration of the mathematical abilities of children with different visuo-spatial working memory skills. Mathematics performance was determined by the children's total scores on the assessments developed in Chapter 2. Visuo-spatial working memory ability was assessed by performance on three standardized visuo-spatial sketchpad tasks. This investigation was designed to provide an initial indication of the potential value of visuo-spatial working memory assessments as screening tools for educational practitioners.

Method

Participants

107 primary school children aged 7-/8-years and 9-/10-years participated in this study. See Study 4 for details.

Design and Procedure

The age appropriate mathematics assessments and three standardized visuo-spatial working memory tasks were administered. See Chapter 4 for details.

Materials

Visuo-spatial Sketchpad Tasks

The visuo-spatial sketchpad tasks were the standardized measures administered in Chapter 4; Visual Patterns Test (Della Sala et al., 1999), Mazes Memory and Block Recall (Pickering & Gathercole, 2001). See Study 3. The scores given were *Z* scores. These were calculated from Trials Correct Scores on each measure. *Z* scores were calculated separately for each age group. A composite visuo-spatial working memory score was also calculated for each child. Pickering and Gathercole (2001) suggested that summarising standardized scores across subtests designed to measure different components of working memory (to calculate component scores) provides a broad description of a child's working memory ability. Consistent with Pickering and Gathercole's (2001) methodology, composite visuo-spatial working memory scores were derived as a sum of the standardized scores (*Z* scores) for each of the visuo-spatial working memory tasks. The three visuo-spatial tasks were grouped because they loaded on the same factor for Year 3 and Year 5, suggesting they are measuring the same component of working memory (see Tables 7.6. and 7.7 for factor loadings).

NVIQ Task

The MAT-SF (Naglieri, 1985) was administered. See Chapter 3.

Mathematics Tasks

The mathematics assessments were the age appropriate tests developed in Chapter 2. The score given was the Proportion Correct Score. Responses for each question were scored as 1 or 0. The sum of the responses divided by the total number of questions (55) multiplied by 100 provides the Proportion Correct Score.

Results

Power Analyses

The power of this study to test for significance with a medium effect size is .81 for the Year 3 sample and .82 for the Year 5 sample. Both values exceeds Cohen's (1988) criterion of .8 (see Table 7.1, Chapter 7).

Descriptive Statistics

Descriptive statistics for visuo-spatial sketchpad measures and total mathematics scores are presented in Table 9.1.

Year 3 and Year 5 children performed similarly on the age-appropriate mathematics assessments. Year 5 children performed significantly better on the visuo-spatial sketchpad measures than Year 3 children (Visual Patterns Test $t(105)=-3.61$, $p<.01$, Mazes Memory $t(105)=-4.15$, $p<.01$, Block Recall $t(105)=-2.99$, $p<.01$). There was greater variability on the Mazes Memory task for both age groups, although Year 3 children's performance also varied on the Block Recall task.

Table 9.1

Descriptive Statistics of Visuo-spatial Sketchpad and Total Mathematics Scores (maximum score for visuo-spatial measures are shown in parentheses). (N=107).

Measures	Year 3. (n = 51).		Year 5. (n = 56).	
	M	*SD*	*M*	*SD*
Visuo-spatial Sketchpad Measures				
Visual Patterns Test (42)	9.00	2.74	11.26	3.72
Mazes Memory (42)	9.52	4.10	13.69	6.49
Block Recall (54)	23.28	4.19	25.29	3.04
Composite Visuo-spatial Working Memory	0.00	2.24	0.00	2.21
Mathematics Measure	58.25	14.21	60.20	13.29

Note. Mathematics scores shown are proportions correct. Visuo-spatial scores shown are Trials Correct Scores.

Visuo-spatial Working Memory Profiles of Children with Different Mathematical Abilities

Children were assigned to different mathematics ability groups to explore their visuo-spatial working memory profiles. They were assigned to high, average or low mathematics ability groups based on their Proportion Correct mathematics scores. Children assigned to the high ability groups were those scoring 1*SD* and above the mean proportion correct, children assigned to the low ability groups were those scoring 1*SD* below the mean proportion correct and children assigned to the average ability groups were those scoring within 1*SD* of the mean.

Year 3 children scoring below 44.04 were assigned to a low mathematics ability group. Those scoring above 72.46 were assigned to a high mathematics ability group. Those scoring between 44.04 and 72.46 were assigned to an average mathematics group. There were 9 children in the low ability group (mathematics *M*=37.17, *SD* 4.9, composite visuo-spatial *M*=-.33, *SD* 1.48), 31 children in the average ability group (mathematics *M*=58.18, *SD* 7.94, composite visuo-spatial *M*=.04, *SD* 2.33) and 11 children in the high ability group (mathematics *M*=75.70, *SD* 3.38, composite visuo-spatial *M*=1.16, *SD* 2.11).

For Year 5, children with Proportion Correct scores below 46.91 were assigned to a low mathematics ability group, children with scores above 73.49 were assigned to a high mathematics ability group and children scoring between 46.91 and 73.49 were assigned to an average mathematics ability group. 8 children were assigned to the low ability group (mathematics *M*=37.95, *SD* 5.94, composite visuo-spatial *M*= -.76, *SD* 1.88), 39 to the average ability group (mathematics *M*=60.78, *SD* 7.58, composite visuo-spatial *M*=.02, *SD* 2.02) and 9 to the high ability group (mathematics *M*=77.78, *SD* 2.69, composite visuo-spatial *M*=.61, *SD* 3.09).

Mean visuo-spatial composite scores for the three mathematics ability groups for Year 3 and Year 5 are displayed in figure 9.1.

An analysis of variance (one way ANOVA) revealed no significant differences in composite visuo-spatial sketchpad scores between the mathematics ability groups for Year 3 children ($F(2, 48)=1.44, p>.05$) or Year 5 children ($F(2, 53)=.82, p>.05$).

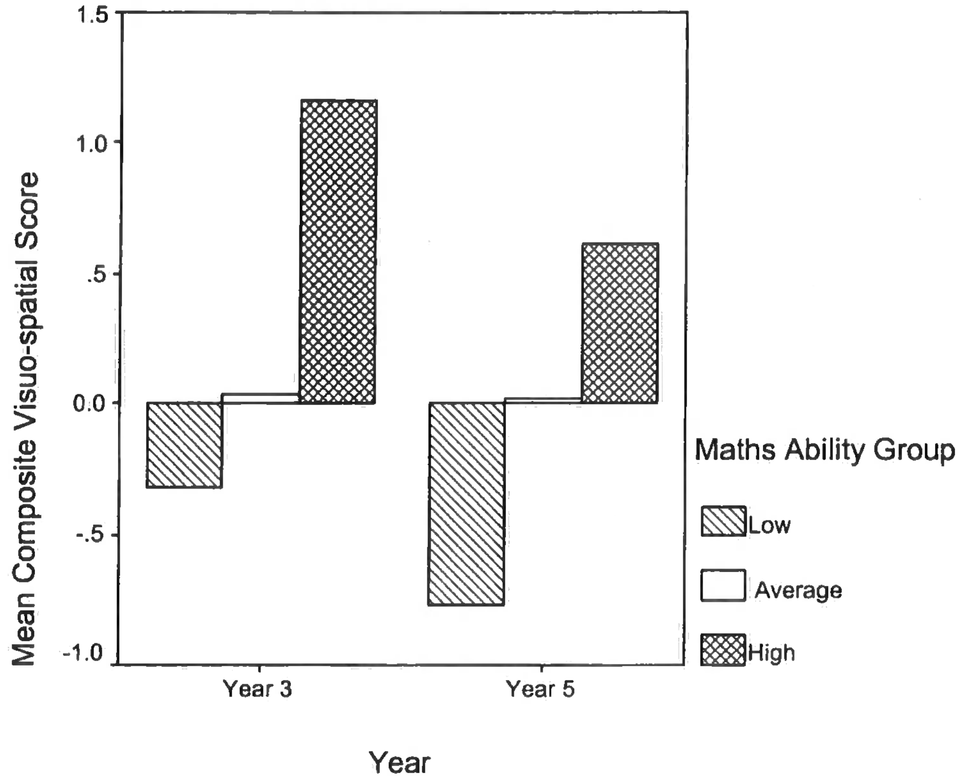


Figure 9.1

Mean Composite Visuo-spatial Scores for Children with Different Mathematical Abilities

Although the differences in mean visuo-spatial composite scores were not significant, Figure 9.1 shows that visuo-spatial scores were higher for the average mathematics ability group than the low ability group and higher again for the high mathematics ability group over the average ability group. Non-significant differences may have been found due to the relatively small samples of the low and high ability groups.

To further explore the visuo-spatial ability of children with different mathematics abilities and to increase the sample sizes of different ability groups, children were split into two equal sized mathematics ability groups based on a median split. For Year 3 children, those scoring below 60 were assigned to a low-to-average (LA) mathematics ability group and those scoring above 60 were assigned to an average-to-high (AH) ability group. Children in the LA mathematics ability group had significantly lower visuo-spatial sketchpad scores than children in the AH group ($t(36.51)=-2.07$, $p<.05$, Levene's Test for Equality of Variance $p<.05$). The median split for Year 5 children was 60.9. Children in the LA mathematics ability group did not perform significantly worse on the visuo-spatial measures than the AH group ($t(54)=-1.45$, $p>.05$).

Underachievement in Mathematics

One aim of this study was to investigate whether visuo-spatial working memory tasks could identify children with low mathematics attainment. Therefore, children were assigned to one of two mathematics ability groups using a discrepancy definition (Yule, Rutter, Berger & Thompson, 1974). This method is typically used to define underachievement by classifying children as having specific learning difficulties if their attainment (e.g. mathematics) is below the level predicted from their age and IQ. The regression equation used to predict expected mathematics attainment for Year 3 was $y = 17.70 + (.05)$ age + (.66) NVIQ. The regression equation used to predict expected mathematics attainment for Year 5 was $y = 8.81 + (.23)$ age + (.45) NVIQ.

10 of the Year 3 children were classified as having MD, with actual mathematics scores 12.87 below their predicted mathematics scores (mathematics M=39.63, SD 6.95, composite visuo-spatial M=.24, SD 1.47). The remaining 41 children were classified as AH achievers (mathematics M=63.99, SD 10.43, visuo-spatial M=.30, SD 2.33).

7 of the Year 5 children were classified as having MD with actual mathematics scores 12.84 below their predicted mathematics scores (mathematics *M*=39.22, *SD* 9.08, visuo-spatial *M*=-.45, *SD* .76). The AH group consisted of the remaining 49 children (mathematics *M*=63.47, *SD* 10.61, visuo-spatial *M*=.58, *SD* 2.04).

Mean visuo-spatial composite scores for the AH and MD groups for Year 3 and Year 5 are displayed in figure 9.2. There were no significant differences in visuo-spatial scores between the MD and AH children in Year 3 ($t(49)=-.08, p>.05$. Equal variances were assumed due to a non-significant Levene's test result $p>.05$). Children with MD in Year 5 had significantly poorer visuo-spatial abilities than the AH children ($t(23.15)=-2.50, p<.05$. Equal variances were not assumed due to a significant Levine's test result $p<.05$).

Figure 9.2 shows that AH mathematics ability children in Year 5 had better visuo-spatial working scores than AH mathematics ability children in Year 3. Between group comparisons revealed that this difference was not significant ($t(88)=-.57, p>.05$). Figure 9.2 shows that MD children in Year 5 had poorer visuo-spatial scores than MD children in Year 3. Again this difference was not significant ($t(15)=1.15, p>.05$). This may have been due to the small sample sizes of the MD groups.

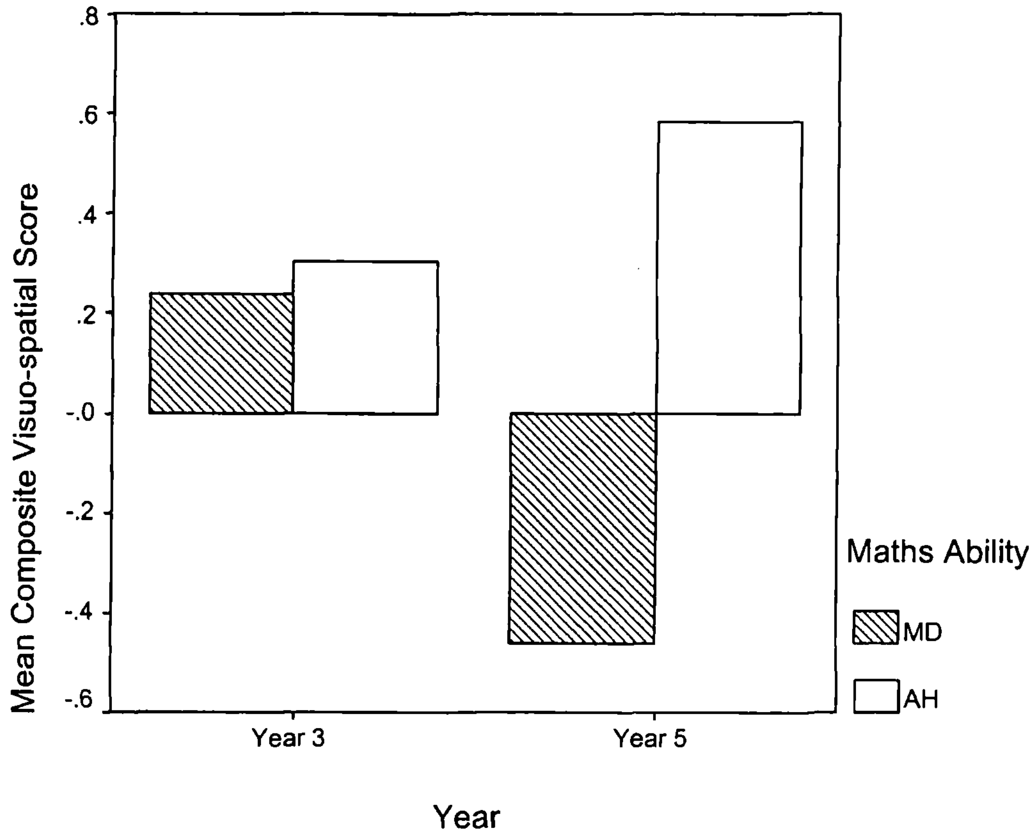


Figure 9.2

Mean Composite Visuo-spatial Scores for Underachieving (MD) and Normally Achieving (AH) Children.

The visuo-spatial working memory profiles of the MD children are presented in Figure 9.3. The mean composite visuo-spatial scores for the Year 3 group and the Year 5 group were 0.00 as the scores were standardized. Therefore, Figure 9.3 shows that MD children in Year 3 have comparable or higher visuo-spatial scores to the Year 3 group mean (Mazes Memory *M*=.00, Visual Patterns Test *M*=.15, Block Recall *M*=.10). Consistent with the pattern in Figure 9.2 this suggests that MD children in Year 3 do not have visuo-spatial deficits. MD children in Year 5 have lower Mazes Memory (*M*=-.23) and Visual Patterns Test (*M*=-.13) scores compared to the Year 5 group mean. Their Block Recall Scores are higher (*M*=2.06) than the Year 5 group mean.

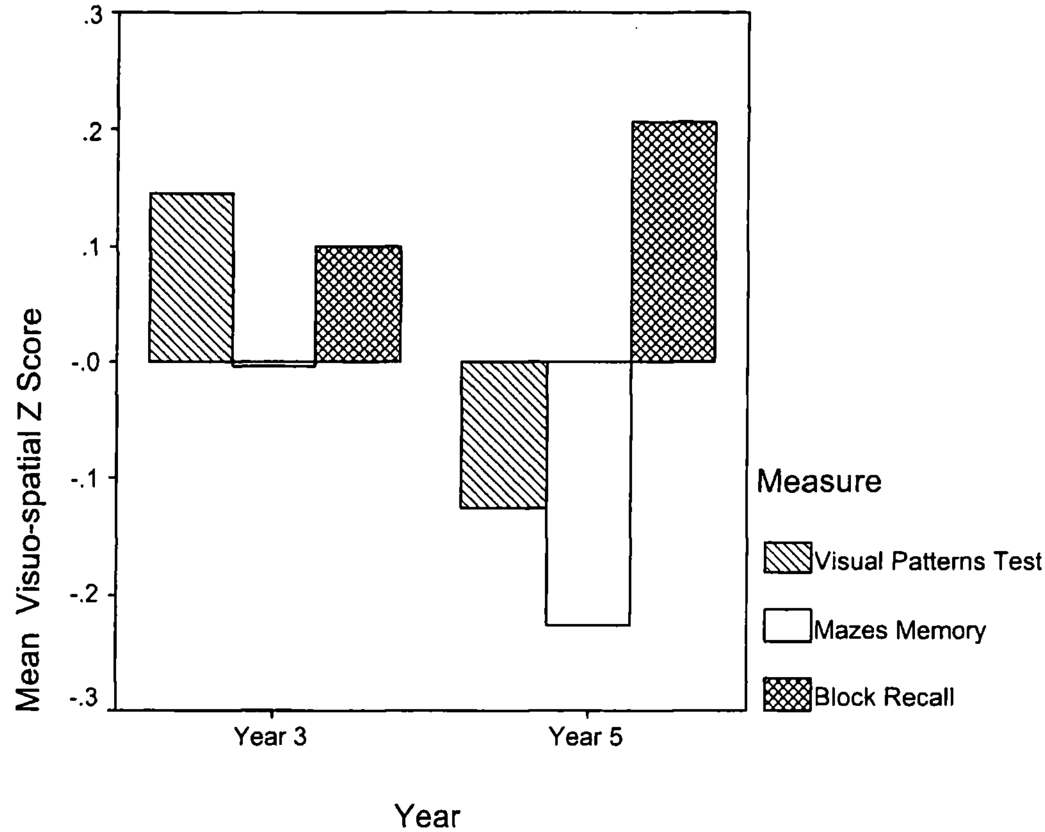


Figure 9.3

Visuo-spatial working memory profiles of children with MD

The general pattern of results thus far suggests that children with different mathematical abilities have different visuo-spatial working memory skills. However, they do not indicate whether or not visuo-spatial working memory assessments successfully discriminate between children with different mathematical abilities. Therefore, subsequent analyses were conducted to explore the mathematics abilities of children with different visuo-spatial skills.

Mathematics Abilities of Children with Different Visuo-spatial skills

To compare the mathematics abilities of children with different visuo-spatial working memory skills children were split into subgroups according to their performance on the three standardized visuo-spatial working memory tasks.

Children were assigned to one of three visuo-spatial ability groups (high, average or low) according to their composite scores. As before, children scoring 1*SD* and above the mean were assigned to a high ability group, children scoring 1*SD* and below the mean were assigned to a low ability group, and children scoring within 1*SD* of the mean were assigned to an average ability group.

For Year 3, children with scores below –2.24 were assigned to a low visuo-spatial ability group. 6 children were assigned to this group (visuo-spatial *M*=-3.35, *SD* .68, maths *M*=56.36, *SD* 8.27). Year 3 children with scores above 2.24 were assigned to a high visuo-spatial ability group. 9 children were assigned to this group (visuo-spatial *M*=3.62, *SD* .81, maths *M*=69.09, *SD* 9.79). Year 3 children with scores between –2.24 and 2.24 were assigned to an average visuo-spatial ability group. The remaining 38 children were assigned to this ability group (visuo-spatial *M*=-.35, *SD* 1.21, maths *M*=55.70, *SD* 14.79).

5 Year 5 children with scores below -2.21 were assigned to a low visuo-spatial ability group (visuo-spatial *M*=-3.85, *SD* .80, maths *M*=56.36, *SD* 11.99). 9 children in Year 5 scored above 2.21 and were assigned to a high ability group (visuo-spatial *M*=3.60, *SD* 1.04, maths *M*=65-45, *SD* 11.92). The remaining 42 children who scored between –2.21 and 2.21 were assigned to an average visuo-spatial ability group (visuo-spatial *M*=-.35, *SD* 1.07, maths *M*=59.47, *SD* 13.71). Mean mathematics scores

for the three visuo-spatial ability groups for Year 3 and Year 5 are displayed in figure 9.4.

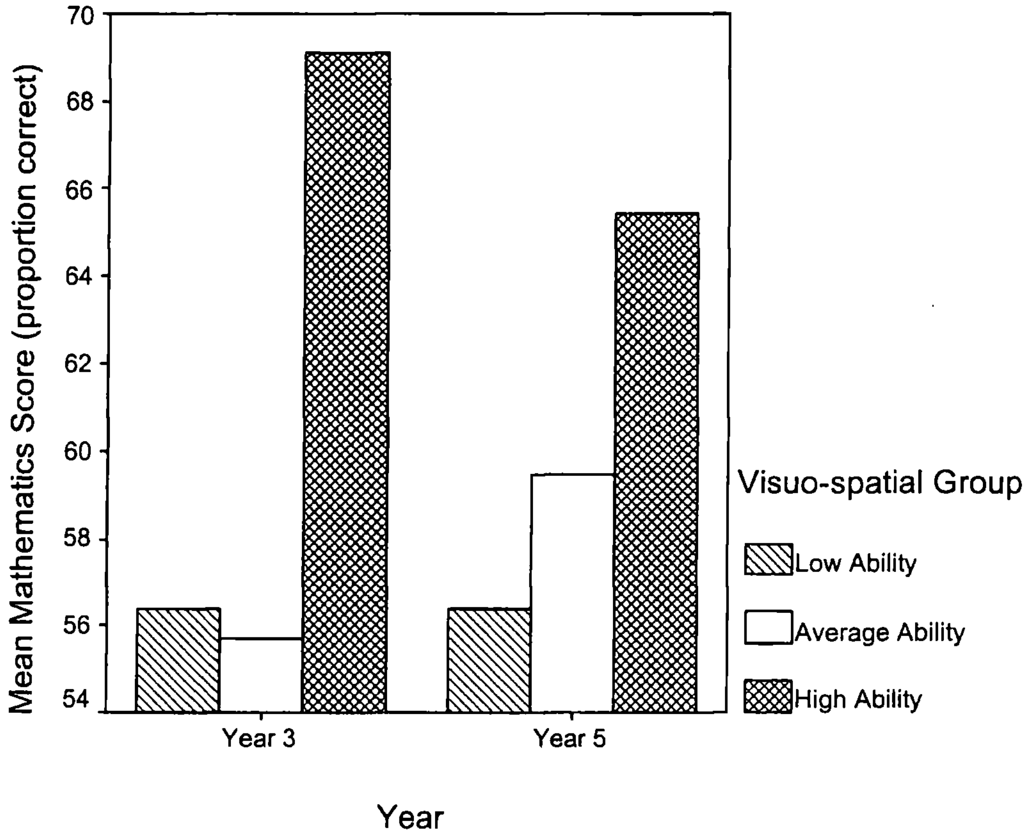


Figure 9.4

Mean Mathematics Scores for Children with Different Visuo-spatial Abilities

An analysis of variance (one way ANOVA) revealed significant differences in mathematics achievement between the visuo-spatial ability groups for Year 3 children ($F(2)=3.46, p<.05$). The three subgroups were homogeneous (Levene's test of homogeneity of variances $p>.05$). Post hoc Tukey's *HSD* tests confirmed significant differences in mathematics attainment between the average and high ability groups ($p<.05$).

There were no significant differences in mathematics achievement between the visuo-spatial ability groups for Year 5 children ($F(2)=.97, p>.05$). The three subgroups were homogeneous (Levene's test of homogeneity of variances $p>.05$). However, Figure 9.4. shows that mean mathematics scores were higher for the average visuo-spatial ability group than the low ability group and higher again for the high visuo-spatial ability group than the average ability group. These differences may have been non-significant differences due to the relatively small samples of the low and high ability groups.

Children were split into two equal sized visuo-spatial ability groups (based on a median split) to increase the sample sizes of different ability groups. Year 3 children scoring below -.31 were assigned to a LA visuo-spatial ability group. Those scoring above -.31 were assigned to an AH ability group. There were no significant differences in mathematics performance between the two groups ($t(49)=-1.41$, $p>.05$). The median split for Year 5 children was -.51. Again, there were no significant differences between the mathematics scores of the two ability groups ($t(54)=-1.26$, $p>.05$).

Visuo-spatial working memory deficits

Children were assigned to one of two visuo-spatial ability groups using the same discrepancy definition used to identify children with underachievement in mathematics (Yule, Rutter, Berger & Thompson, 1974). As before, a specific deficit in performance was defined as a discrepancy of at least 1 *S.E.* between actual and predicted scores (e.g. children's visuo-spatial working memory composite scores were below the level predicted from their age and IQ). The regression equations used to predict expected visuo-spatial composite scores were $y = -13.33 + (.11)$ age + (.19) NVIQ and $y = -21.69 + (.15)$ age + (.19) NVIQ for Year 3 and Year 5 children respectively.

3 Year 3 children with composite visuo-spatial scores at least 1.92 below their predicted scores (visuo-spatial *M*=-3.41, *SD* .43, Mathematics *M*=60, *SD* 4.81) were classified in the visuo-spatial deficit group. The remaining 48 children were assigned to the AH visuo-spatial ability group (visuo-spatial ability *M*=.55, *SD* 1.98, Mathematics *M*=58.74, *SD* 14.65).

Year 5 children classified in the visuo-spatial deficit group had composite visuo-spatial scores at least 1.99 below their predicted scores (visuo-spatial *M*=-3.27, *SD* 1.46, Mathematics *M*=54.55, *SD* 14.49). 5 children were assigned to this group. The remaining 51 children were classified in the AH ability group (visuo-spatial *M*=.35, *SD* 1.98, Mathematics *M*=60.81, *SD* 13.18).

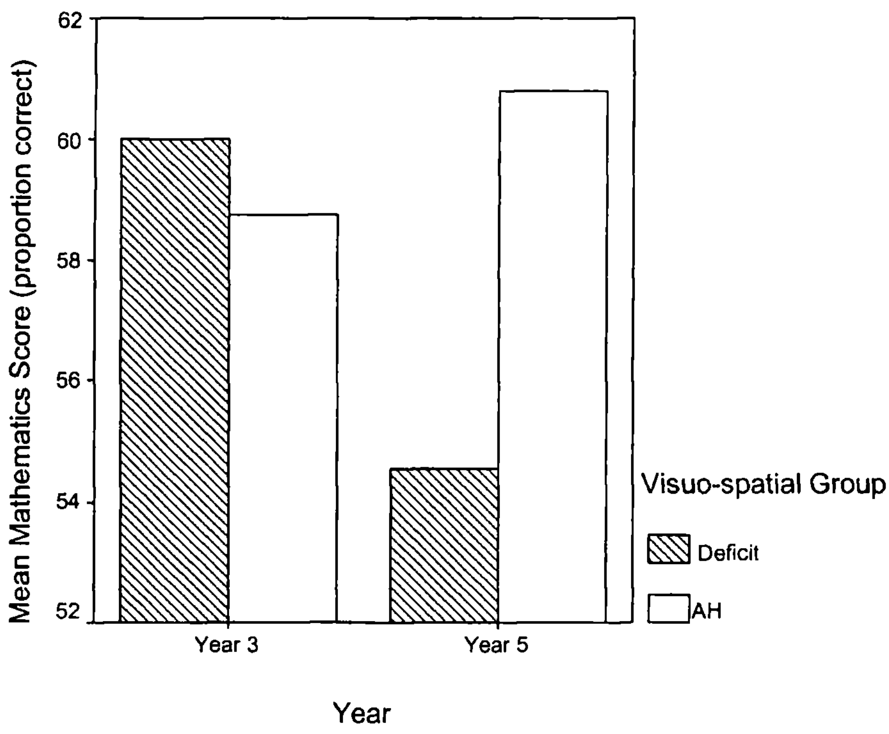


Figure 9.5

Mathematics scores of children with and without visuo-spatial deficits

Mean mathematics scores for the AH ability and visuo-spatial deficit groups for Year 3 and Year 5 are displayed in Figure 9.5. There were no significant differences in the mathematics scores of the two ability groups for Year 3 (t(5.41=-.35, p>.05. Unequal variances were assumed due to a significant Levene's result) or Year 5 (t(54)=-1.00, p>.05. Equal variances were assumed due to a non-significant Levene's result). However, Figure 9.5 shows that the Year 5 visuo-spatial deficit group had poorer mathematics scores than the AH ability group. This difference may have been non-significant due to the relatively small sample size of the low ability group.

The mathematics profiles of the visuo-spatial deficit groups across different mathematical skills are displayed in Figures 9.6 and 9.7.

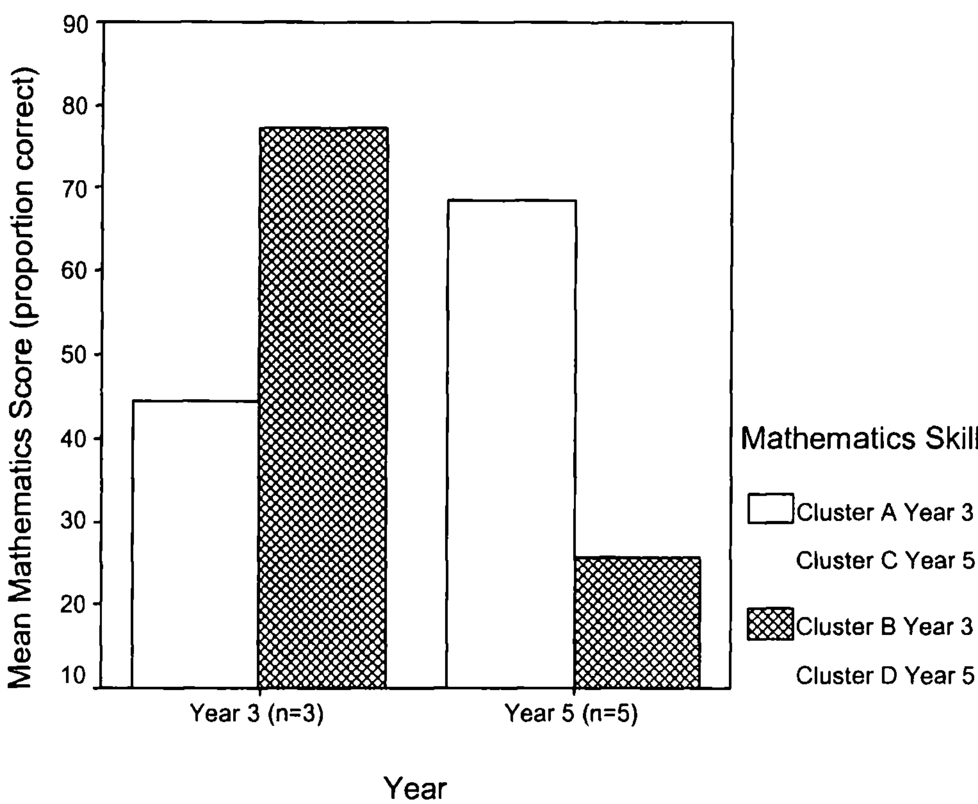


Figure 9.6

Performance-related mathematics skills of children with visuo-spatial deficits

Year 3 children with visuo-spatial deficits have comparable scores to the group mean on one of the performance-related skills (Cluster B *M*=78.66 compared to the group *M*=71.57). However, their scores on Cluster A were lower than the group mean (*M*=44.44 compared to the group *M*=47.16).

Year 5 children with visuo-spatial deficits have lower scores on both performance-related skills compared to the group means (*M*=68.64 for Cluster C compared to the group *M*=75.52; *M*=25.56 for Cluster D compared to the group *M*=28.73).

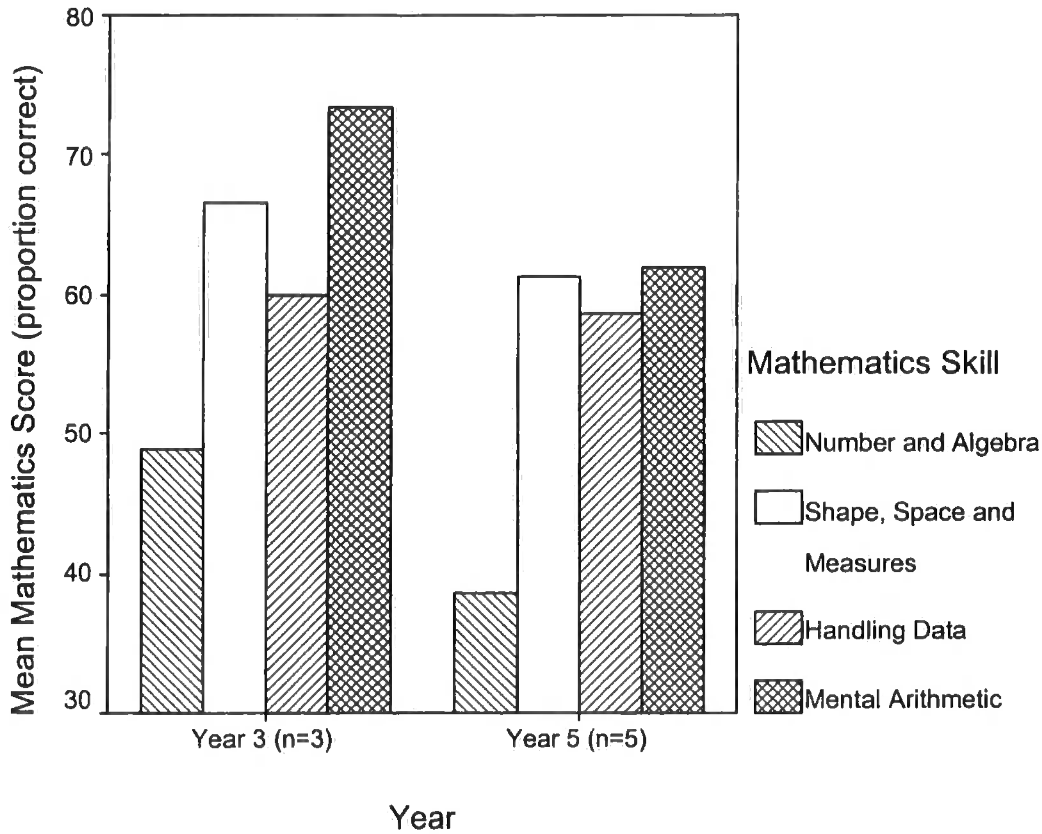


Figure 9.7

Curriculum-based mathematics skills of children with visuo-spatial deficits

Year 3 children with visuo-spatial deficits have comparable, or higher, curriculum-based mathematics scores compared to the Year 3 group mean scores on three of the measures (Number and Algebra 48.8 compared to the group *M*=47.8; Shape, Space and Measures 66.66 compared to the group *M*=60.28; Mental Arithmetic 73.33 compared to the group *M*=62.77). Their Handling Data scores are slightly lower (*M*=60) than the group mean scores (*M*=62.76).

Year 5 children with visuo-spatial deficits have comparable Shape, Space and Measures (*M*=59.23 compared to the group *M*=61.33) and Handling Data (*M*=58.67 compared to the group *M*=58.46) scores to the Year 5 group means. However, those with a visuo-spatial deficit have poor Number and Algebra scores (*M*=38.67) and Mental Arithmetic scores (*M*=62) compared to the Year 5 group means (*M*=57.69 and *M*=68.27 respectively).

Discussion

This study provides cross-sectional evidence to further support a role for the visuo-spatial sketchpad in children's mathematics development. General patterns in the data provide further support for an association between visuo-spatial working memory and children's mathematics (e.g. Jarvis & Gathercole, 2003). Children who were more able in mathematics had better visuo-spatial scores than children with poorer mathematics abilities. Specifically, children with average mathematics abilities outperformed children with low mathematics abilities and children with high mathematics abilities outperformed children with average mathematics abilities across both age groups. Similarly, children with high visuo-spatial sketchpad scores outperformed children with low visuo-spatial sketchpad scores on the mathematics assessments.

The overarching aim of this study was to explore whether visuo-spatial working memory tests could be used to identify children with low mathematics attainment. This was approached in two ways: (i) comparing the visuo-spatial skills of children with mathematical difficulties (MD) to children with AH mathematical abilities and (ii) comparing the mathematical abilities of children with visuo-spatial deficits to children with AH visuo-spatial skills.

Contrary to expectation Year 3 children with MD did not have poorer visuo-spatial skills than Year 3 children with AH mathematics abilities, nor did children with visuo-spatial deficits have poorer mathematics abilities than children with AH visuo-spatial skills. Rather, Year 3 children with above average visuo-spatial skills (defined as scores at least 1*SD* above the group mean) had significantly better mathematical abilities than children with average visuo-spatial skills (scores within 1*SD* of the group mean). This suggests that poor visuo-spatial skills do not significantly impair mathematics performance at Year 3. Instead, children with good visuo-spatial skills are boosted in mathematics at Year 3. In terms of the previous findings reported in this thesis (e.g. Chapters 3, 4, 6 and 7) the associations between visuo-spatial sketchpad scores and Year 3 children's mathematics performance may reflect this.

Poor visuo-spatial skills might not significantly impair mathematics performance at Year 3 due to the limitations of the mathematics taught and assessed at this age. Formal mathematics teaching follows a linear curriculum, which means learning is an incremental process. The Key Stage 2 curriculum begins at Year 3, where children are taught basic skills that they develop throughout Years 4, 5 and 6. Therefore, Year 3 children who are just starting this curriculum have limited mathematics knowledge. Consequently, test performance may not expose children who have MD. Arguably, the discrepancy between normally achieving and underachieving children is larger in older children who have experienced more years of formal teaching. As teaching progresses throughout their schooling children who are poor at mathematics fall further behind their normally achieving peers; hence the difference in ability becomes more apparent. Visuo-spatial resources might not support children's mathematics at Year 3 because the mathematics they are required to perform is relatively limited and simple. This would mean that a deficit in visuo-spatial ability might not impair a child's performance at this age.

Interestingly, the data suggested that children with good visuo-spatial skills have an advantage in mathematics at Year 3. Around this age children are beginning to solve abstract mathematical problems. It was suggested that the visuo-spatial sketchpad may support children's mathematics development by providing a foundation upon which abstract problems can be represented in a concrete format (see Chapters 6 and 7). As such, it is possible that Year 3 children with good visuo-spatial working memory skills are better able to forge the links between concrete and abstract knowledge than children with poorer visuo-spatial abilities. One possibility is that they have a greater meta-cognitive awareness of their visuo-spatial abilities and their application to mathematical problems. As discussed in the previous paragraph, Year 3 children with poor visuo-spatial skills are not disadvantaged in mathematics. In line with the current suggestion, visuo-spatial resources might not support the day-to-day concrete mathematics performed by most Year 3 children. This means a deficit in visuo-spatial skill would not affect mathematics test performance.

Year 3 children with visuo-spatial deficits did not have general impairments in mathematics performance (as measured by a composite score) or impairments across the curriculum-based mathematics skills. However, their scores were lower on one of the performance-related mathematics skills (Cluster A) when compared to the Year 3 group mean. Cluster A consisted of the more abstract mathematics problems (see Chapter 5), which suggests that Year 3 children with MD are impaired on "pure" but not "applied" questions. Consistent with ideas presented earlier in the thesis, this supports the idea that young children rely upon the visuo-spatial sketchpad to represent abstract mathematical problems that do not have a concrete referent (see Chapter 6).

Overall, poor visuo-spatial skills do not impair mathematics performance at Year 3. This may be because visuo-spatial resources do not support the day-to-day concrete mathematics performed by children aged 7-/8-years. Good visuo-spatial skills do, however, give children an advantage in mathematics at Year 3. This may be because visuo-spatial resources support the development between concrete and abstract processing. As such, children with good visuo-spatial skills may be better able to solve more complex mathematical problems. Alternatively, children who are more mathematically able may be doing more advanced mathematics, which could provide them with the opportunity to use and again an advantage from high visuo-spatial sketchpad abilities (e.g. Adams & Hitch, 1997).

Year 5 children with MD performed significantly worse on the visuo-spatial tasks compared to children with AH mathematical abilities. Similarly, children with visuo-spatial deficits had poorer mathematical abilities compared to children with AH visuo-spatial abilities. This pattern of results is consistent with previous research, which suggests that 9-/10-year-old children with specific MD are impaired on visuo-spatial working memory tasks (e.g. McLean & Hitch, 1999).

Visuo-spatial resources may support Year 5 children's day-to-day mathematics. Although they might use predominantly verbal solution strategies to solve mathematical problems (see Chapter 6), they rely upon visuo-spatial strategies as a back-up when they cannot deploy verbal strategies (e.g. when they encounter complex or novel mathematical problems). As such, it is possible that children with visuo-spatial deficits are unable to make use of effective back-up strategies at Year 5. Consequently, this may lead to failure and underachievement in mathematics. Indeed, Year 5 children with visuo-spatial deficits had lower scores across both performance-related mathematics skills (Cluster C and Cluster D) and two of the four curriculum-based mathematics skills (Number and Algebra and Mental Arithmetic) in comparison to the Year 5 group mean scores. In relation to the Year 3 data, visuo-spatial deficits had a larger impact on Year 5 children's mathematics performance. This may be because older children's mathematics is more complex and abstract, meaning adept visuo-spatial skills are needed to support day-to-day mathematics performance.

Year 5 children with MD had particularly low scores on both the Mazes Memory and Block Recall tasks in comparison to the Year 5 group scores. Both tasks contain a spatial element. Block Recall provides a measure of immediate spatial memory (e.g. Logie & Pearson, 1997), while Mazes Memory contains an explicit spatial component (tracing and remembering a route). Therefore, it is tentatively suggested that Year 5 children with MD may have impaired immediate spatial memory. If this is so, they may experience problems with the spatial representation and manipulation of numbers (e.g. using a mental number line), which may impair their mathematics performance. Clearly this needs further research given the problems defining the structure and assessment of visuo-spatial working memory in children (see Chapters 4 and 7).

Although this study was cross-sectional it provides an initial indication that normal visuo-spatial working memory development may be important for normal mathematics development. Logie and Pearson (1997), among others (e.g. Wilson, et al., 1987; Isaacs & Varga-Khadem, 1989), suggest that the visuo-spatial sketchpad follows a steady developmental increase between 5-years-of-age and 15-years-of-age. Between group comparisons revealed that visuo-spatial scores follow a normal developmental trajectory between Year 3 (7-/8-years-of-age) and Year 5 (9-/10-year-of-age), for children with AH mathematical abilities. AH children in Year 5 had better visuo-spatial sketchpad scores than AH children in Year 3. However, Year 5 children with MD did not have better composite visuo-spatial sketchpad scores than Year 3 children with MD. If a longitudinal interpretation is applied to cross-sectional data, this suggests that visuo-spatial working memory follows a normal developmental trajectory in children with AH mathematics ability, but that visuo-spatial working memory may not follow a normal developmental trajectory in children with MD.

It is tentatively suggested that impaired visuo-spatial working memory development between Year 3 and Year 5 may hinder normal mathematics development. As discussed earlier, visuo-spatial working memory scores of MD and AH mathematics ability children are comparable at Year 3. However, by Year 5 the visuo-spatial working memory scores of children with MD and children with AH mathematics are discrepant. It is possible that the lack of development in the visuo-spatial working memory system between Year 3 and Year 5 has impaired normal mathematics development by Year 5. In terms of the ideas presented earlier; if visuo-spatial working memory facilitates mathematics development through supporting links between concrete and abstract knowledge, an impaired visuo-spatial working memory system would intuitively impede mathematics development. Clearly this interpretation is speculative due to the limitations of using a cross-sectional design to provide an indication of the developmental trajectory between two age groups. Longitudinal research is needed to further investigate these ideas.

It has been suggested that visuo-spatial deficits are characteristic among children with MD (e.g. Geary, 1993). Therefore, this study was designed to provide an initial indication of the potential value of visuo-spatial working memory assessments as screening tools for educational practitioners. Visuo-spatial working memory deficits were not indicative of MD at Year 3, suggesting that visuo-spatial working memory assessments may not be useful tools with which to identify children with MD at Year 3. However, visuo-spatial deficits were indicative of MD at Year 5, suggesting they may be of some value for use with 9-/10-year-olds. Importantly, the data suggested that normal visuo-spatial working memory development may be important for normal mathematical development. With this is in mind, visuo-spatial working memory assessments may be useful for screening children at risk of developing MD. Although screening children for visuo-spatial deficits at an early age may not identify those with MD, it might identify children who are at risk of poor visuo-spatial working memory development. Based on the current findings, these children may be at risk of developing MD as they progress through school. Again, this suggestion is tentative as it is based on evidence from cross-sectional data. Clearly a longitudinal study, which follows the development of children's mathematical and visuo-spatial skills between the ages of 7- and 10-years, is needed.

In summary, this study provides further evidence for an association between visuo-spatial working memory ability and children's mathematics performance. Importantly, it provides an initial indication that normal visuo-spatial working memory development facilitates normal mathematics development, while impaired visuo-spatial working development might impede normal mathematics development.

This suggestion is of particular importance to educational practice and certainly warrants further investigation. If longitudinal research yields similar results, there may be scope to provide children and teachers with opportunities for remediation. For example, training children's visuo-spatial skills or encouraging the use of visuo-spatial working memory at an early age may foster normal mathematics development.

Chapter Summary

1. Visuo-spatial deficits are characteristic among a subgroup of children with MD (e.g. Geary, 1993). It has been suggested that these deficits may be specifically related to visuo-spatial working memory. The aim of this study was to explore this idea to provide an indication of the potential value of visuo-spatial working memory assessments to educational practitioners.
2. The general pattern of results suggested that visuo-spatial working memory ability was related to children's mathematics. Overall, children with good visuo-spatial skills had good mathematics scores and vice versa.
3. Contrary to expectation children with MD did not have poor visuo-spatial skills at Year 3. However, children with good visuo-spatial skills had an advantage in mathematics. It was suggested that visuo-spatial resources might not support day-to-day concrete mathematics at this age, meaning a visuo-spatial deficit would not impair test performance. The visuo-spatial sketchpad may support links between children's concrete and abstract mathematics processing. As such, children with good visuo-spatial skills may be able to solve more advanced abstract mathematical problems giving them an advantage at this age.

4. Year 5 children with MD had poor visuo-spatial skills, suggesting visuo-spatial working memory may support the development of children's complex mathematical skills.
5. Between group comparisons suggested that visuo-spatial working memory did not follow a normal developmental trajectory in children with MD. This suggests that normal visuo-spatial working memory may support normal mathematics development, while impaired visuo-spatial working memory may impede normal mathematics development.
6. These findings are discussed in terms of the value of visuo-spatial working memory assessments to educational practitioners. However, additional longitudinal research is required.

Chapter Ten

General Discussion and Conclusions

Previous studies which investigated the association between working memory ability and children's mathematics typically incorporated digit- or number-based working memory tasks and measured mathematics ability as a general skill (e.g. Gathercole & Pickering, 2000a) or performance on a mental arithmetic task (e.g. Adams & Hitch, 1998). The main aim of this thesis was to extend this work to explore the associations between the three components of the tripartite working memory model (e.g. Baddeley, 1986) and a range of mathematical skills in children using non-digit based working memory assessments, taking into account a measure of children's general ability.

Several studies were conducted with 7-/8-year-olds and 9-/10-year-olds. Overall, the results support an association between working memory ability and children's mathematics performance. The main findings and conclusions are presented in Section 10.1. These are followed by a discussion of the implications of this research for education in Section 10.2. Finally, the limitations of the current research and possible future directions are discussed in Sections 10.3.and 10.4 respectively.

Section 10.1

Conclusions

The overarching conclusion emerging from this research is that working memory ability is related to children's attainment across a range of mathematical skills. Converging evidence presented in Chapters 3, 4, 6 and 7 suggests that scores on working memory assessments are related to mathematics performance. Furthermore, data presented in Chapter 8 suggests that working memory assessments may be useful prospective indicators of children's academic attainment.

Overall, these findings support research conducted by Gathercole and colleagues, which suggests that working memory ability is significantly associated with National Curriculum attainment (e.g. Gathercole & Pickering, 2000a) and more specifically with Key Stage 2 Mathematics attainment (e.g. Jarvis & Gathercole, 2003). Importantly, the current research extends these findings to suggest that working memory assessments predict National Curriculum mathematics performance above and beyond measures of general ability (e.g. NVIQ measures). Both central executive and visuo-spatial sketchpad scores predicted unique variance in children's mathematics performance beyond that predicted by individual differences in NVIQ in Chapters 3, 4 and 8. These results provide some evidence to suggest that working memory ability may support children's mathematics independent of the contribution of working memory to a higher order construct such as IQ (e.g. Kyllonen & Christal, 1990).

Contrary to expectation phonological loop scores did not predict unique variance in children's curriculum-based mathematical skills. Previous research suggests that the phonological loop may support the retention of verbally presented problem information (e.g. Adams & Hitch, 1997). The mathematics assessments administered in the present research consisted of three written tests and one auditory test. Therefore it is tentatively suggested that phonological loop resources were not needed to support the retention of problem information for much of the mathematics tests. Consistent with this notion, scores on the phonological loop measure were significantly associated with mental arithmetic performance before the variance associated with age was controlled for. Of the four curriculum-based mathematics skills assessed, mental arithmetic was the only skill that involved auditory presentation of the problems. This suggests that the children were able to use subvocal rehearsal processes to support the retention of problem information (e.g. Adams & Hitch, 1997) and direct retrieval of arithmetic facts from LTM.

Previous research suggested that number-based working memory measures were more strongly associated with mathematics performance than non-numerical measures (e.g. Passolunghi & Siegel, 2001). Therefore, the working memory measures used throughout this research did not contain numerical stimuli. Performance on a range of these tasks was significantly correlated with children's mathematics performance, suggesting that working memory and mathematics were not simply linked in previous research because the assessments of both involved number processing or access to numerical information.

The current work further extends earlier research to suggest that working memory ability supports children's performance across a range of mathematical domains. Beyond predicting overall National Curriculum mathematics attainment (e.g. Jarvis & Gathercole, 2003), visuo-spatial sketchpad and central executive scores predicted children's performance across the four mathematical skills outlined by the National Curriculum (see Chapters 3 and 4). Although there was little difference in the working memory demands of each curriculum-based mathematical skill, the evidence advocates a role for working memory in supporting different aspects of mathematics curricula.

Evidence to support a developmental difference in the involvement of working memory in children's mathematics was reported in later chapters. Overall, working memory skill, and in particular central executive scores, predicted less variance in the older children's mathematics than the younger children's in Chapters 6 and 7. Consistent with previous research this implies that working memory resources may support mathematics development where children are learning mathematics facts (e.g. Hitch & McAuley, 1991) and acquiring new solution strategies (e.g. Bull & Scerif, 2001). Furthermore, it supports the notion that younger children may be more sensitive to working memory limitations when developing their mathematical skills (e.g. Adams & Hitch, 1998).

The working memory demands differed across the performance-related mathematical skills, suggesting there may be a developmental change in the working memory resources supporting children's mathematics. Consistent with McKenzie et al.'s (2003) findings, the younger children appeared to use visuo-spatial sketchpad resources for mathematics, while the older children appeared to use both phonological loop and visuo-spatial sketchpad resources (see Chapter 6). It was suggested that the involvement of the visuo-spatial sketchpad in the 7-/8-year-olds mathematics may reflect the use of an early visuo-spatial arithmetic (e.g. Houdé, 1997), the use of early visual encoding strategies (e.g. Palmer, 2000) or that it may provide a foundation upon which abstract mathematical problems are represented. In line with this explanation, it was proposed that the involvement of the phonological loop in the 9-/10-year-olds mathematics may reflect the deployment of more advanced solution strategies, such as direct retrieval. Although speculative, this interpretation provides an initial indication that the working memory resources supporting children's mathematics may change with age. Moreover, it defines possible independent roles for the slave systems in children's mathematics, which may map on to the developmental shift in children's mathematical cognition; from the use of early visuo-spatial solution strategies to the use of more mature verbal solution strategies.

Finding an independent role for the visuo-spatial sketchpad in children's mathematics was a relatively novel result. Previous research has typically focussed on the associations found between phonological loop (e.g. Adams & Hitch, 1998) and central executive (e.g. Bull et al., 1999) abilities and children's mathematical attainment. Of those studies that do report significant associations between visuo-spatial sketchpad scores and children's mathematics attainment (e.g. Jarvis & Gathercole, 2003), few have investigated the potential role it may play in supporting performance. The current research, however, provided evidence to suggest a role for the visuo-spatial sketchpad in children's mathematical development.

Initially a significant association was found between performance on a single visuo-spatial sketchpad measure and children's curriculum-based mathematics performance in Chapter 3. This was further investigated in Chapter 4 where several visuo-spatial working memory measures were administered to explore the nature of visuo-spatial working memory supporting children's mathematics. The tasks were selected on the basis that they were presumed to measure the two subcomponents of the visuo-spatial sketchpad (e.g. Logie, 1995). Although the results did not highlight a differential pattern of associations between children's visual and spatial working memory abilities and mathematics, they provided further evidence for a significant association between visuo-spatial sketchpad scores and mathematics attainment. Contrary to expectation there was not a significant association between performance on the Mazes Memory task and children's mathematics performance in Chapter 4, as there had been in Chapter 3. This may reflect differences between the two samples of children used in each study. Alternatively, inconsistent results may have been found due to the relatively low test-retest reliability of the Mazes Memory task (.43, WMTB-C, Pickering & Gathercole, 2001).

Paradoxically, the results of Chapter 9 suggested that 7-/8-year-olds with poor visuo-spatial skills were not disadvantaged in mathematics. Rather, children of this age with good visuo-spatial skills were more able in mathematics. At first this seems counter-intuitive as the visuo-spatial sketchpad appeared to support the younger children's mathematics in earlier chapters. However, a plausible explanation is that children with good visuo-spatial skills may be better able to forge the links between concrete and abstract processing and therefore solve more complex abstract mathematical problems, which in turn gives them an advantage. This relationship may be reciprocal. Children who are more mathematically able may be doing more advanced mathematics, which could provide them with the opportunity to use and gain an advantage from high visuo-spatial sketchpad abilities (e.g. Adams & Hitch, 1997).

An alternate explanation relates to the interaction between cognitive style and working memory in learning and attainment (Riding, Grimley, Dahraei & Banner, 2003). An individual's cognitive style describes their preferred approach to organising and representing information (Riding, 2002). Cognitive style has two dimensions; wholist-analytic (whether people view the whole or see things in parts) and verbal-imagery (whether people prefer to represent information verbally or as pictures and images). Broadly speaking, people's cognitive styles differ along these dimensions in various combinations (i.e. wholist-verbalisers, wholist-imagers etc.).

Children learn best when the information is congruent with their preferred representation mode (verbal-imagery). For example, initial reading performance in 7-year-olds was superior in verbalisers (Riding & Anstey, 1982), while 7-, 11- and 12-year-old imagers were better able to recall visually concrete information than abstract information (Riding & Taylor, 1976; Riding & Dyer, 1980; Riding & Calvey, 1981). It has been suggested that wholists initially learn faster than analytics as they are able to view the whole rather than focussing on small parts (e.g. Riding & Mathias, 1991). With this in mind, it could be argued that wholist-imagers may be at advantage when beginning to learn mathematics as concrete examples are often provided to aid understanding. Interestingly, younger children with better visuo-spatial working skills were found to have an advantage in mathematics in Chapter 9. It is possible that children with good visuo-spatial skills may be imagers, who prefer to think visually and tend to use the whole-view aspect of imagery (Riding, 2002). As such, the advantage they have when they first begin to learn mathematics may be due to the presentation of information matching their preferred style along the verbal-imagery domain. Further research is clearly needed to develop this idea beyond speculation.

Although visuo-spatial sketchpad deficits did not impair the younger children's mathematics performance, older children with visuo-spatial deficits had poor mathematics abilities. It was suggested that these children may find it difficult to make use of effective back-up strategies, which leads to failure and underachievement (see Chapter 9).

In terms of defining a role for the visuo-spatial sketchpad in children's mathematics the current research suggests that normal visuo-spatial working memory development may be important for normal mathematics development. Although a longitudinal interpretation must be applied to cross-sectional data in this instance, there is some evidence to suggest that between the ages of 7-/8-years and 9-/10-years children who have poor visuo-spatial skills fall behind in mathematics. Clearly further research is needed to progress this hypothesis beyond speculation.

The current research suggests that a variety of mathematical skills correlate with children's visuo-spatial sketchpad abilities. This could be because the visuo-spatial sketchpad is used to represent visual number form (e.g. Hayes, 1973) and spatial representations of number (e.g. Dehaene, 1992) or because it acts as mental blackboard upon which mathematical problem information is represented and manipulated (e.g. Heathcote, 1994). It is possible that the function of visuo-spatial sketchpad differs for different mathematical tasks. For example, it may provide a foundation upon which abstract algebraic symbols are represented as concrete number forms for algebraic problems, whilst supporting the mental representation and spatial re-ordering of graphical information (e.g. Webber & Feeney, 2003) for Handling Data problems. Clearly delineating the functions of the visuo-spatial sketchpad in supporting different mathematical skills is a point for future research. It is suggested that dual task studies, where visuo-spatial sketchpad functioning is selectively disrupted, may elicit a greater understanding of the mathematical processes supported by visuo-spatial skills.

A key component of this research was the development of assessments designed to measure four different mathematical skills outlined by the National Curriculum for England (see Chapter 2). However, subsequent analysis suggested that the existing curriculum structure may not be teaching and assessing separable mathematical abilities (see Chapter 5). This finding will be discussed in more detail in section 10.3.

10.2

Implications for Education

Certain aspects of the data collected have implications for educational practice. Overall, two main findings may impact on the teaching and assessment of mathematics in schools. Firstly, there are implications relevant to the structure of the mathematics curriculum in England. Secondly, there are implications related to the use of working memory assessments as prospective indicators of academic attainment. These two themes will be discussed in turn.

Developmental psychology is beginning to influence the structure of school mathematics curricula worldwide. The teaching and assessment of mathematics in countries such as America, Australia, New Zealand and Holland is theoretically grounded and reflects stages in children's number development (see Chapter 5). Conversely, the existing mathematics curriculum in England is dictated by common sense (Brown, 2001). The results presented in Chapter 5 suggest a revised approach to curriculum development and assessment in England may facilitate teaching, promote children's learning and provide a better indication of children's abilities. If the QCA were to develop a mathematics curriculum guided by cognitive-developmental ideas they could organise content areas in a developmentally justifiable way. This would enable teaching and assessment to be pitched at suitable levels for certain phases in cognitive development. Furthermore, this would benefit cognitive developmental research as it would enable better investigations into the cognitive resources supporting mathematical development. In time, this research would hopefully feedback into the education system and help teachers better understand mathematics development. Clearly there is an issue to be resolved in determining the cost and benefits of revising the mathematics National Curriculum. However, the current research indicates that a more structured curriculum, tailored to specific stages in development, may prove beneficial to researchers and educational practitioners.

Following the introduction of Key Stage assessments at 7-, 11- and 14-years there has been increasing pressure on schools to raise standards. As such, it is becoming increasingly important for teachers to monitor children's academic progress, predict their Levels of attainment and identify those at risk of failure. In recent years research has shown that working memory assessments may be useful prospective indicators of children's National Curriculum attainment and that they may be useful tools with which to identify young children at risk of low achievement in mathematics (e.g. Gathercole & Pickering, 2000a). Working memory assessments are considered useful as they provide an early indicator of performance that is independent of knowledge acquired through school and home learning experiences. That is, they measure different underlying constructs to other indicators of performance, such as baseline assessments (e.g. Gathercole et al., 2003). Performance on working memory resources is constrained by cognitive resources rather than crystallised knowledge. Furthermore, unlike baseline assessments, working memory assessments are relatively independent of background factors such as pre-school education and socio-economic factors (e.g. Alloway et al., 2004). The current research adds to this to suggest that working memory assessments (central executive and visuo-spatial measures) may also be valuable prospective indicators of National Curriculum test performance over and above intelligence measures. As such, it is suggested that working memory assessments could be used in schools to help predict attainment and consequently raise standards.

Intuitively, evidence for a significant association between working memory ability and mathematics performance implies that children with working memory deficits may be at risk of developing MD. Indeed, the results presented in Chapter 9 suggest that children with poor visuo-spatial sketchpad abilities at 7-/8-years may be at risk of falling behind in mathematics by 9-/10-years. In relation to educational practice this suggests that visuo-spatial working memory assessments may hold some value as screening tools.

Identifying children with poor working memory ability, who may be at risk of developing MD, at an early age may provide educators with opportunities for remediation. However, this is complicated as little is known about how low working memory capacity might constrain successful learning. Recently, Gathercole, Lamont and Alloway (in press) observed that children with poor working memory abilities failed in many routine classroom activities that required both memory storage and effortful processing. Such activities included carrying out numerical calculations that were embedded in everyday language, keeping their place during complex tasks and following tasks. They suggest that learning may be promoted for these children if the processing activity of heavily working memory demanding activities is simplified. For example, complex tasks could be broken down into smaller steps or external memory aids could be provided to reduce working memory loads.

Riding et al. (2003) suggested that working memory ability may interact with a child's cognitive style in learning and attainment and that poor working memory capacity reduced learning performance in analytics and verbalisers. They speculated that this may have been due to both styles demanding heavy processing of information during learning. Similar to Gathercole, Lamont et al.'s (in press) recommendation, Riding (2002) proposes that the processing load should be reduced for children with poor working memory ability, particularly if they are analytic-verbalisers. He suggests processing load could be reduced in the classroom learning situation through various methods including: providing external aids; using slow presentation, revision and sequence design when delivering material; increasing working memory capacity through reducing stress. Overall, alleviating working memory demands in the classroom may prove a useful method for improving learning and reducing the risk of failure in children with working memory impairments.

10.3

Limitations

An obvious limitation of the current research relates to the methodologies used. For the most part cross-sectional studies were conducted. While cross-sectional research provides information on different age groups (when independent groups are used) to highlight age-related changes and developmental trends within a shorter research time-frame than other developmental methodologies (i.e. longitudinal studies), they only provide a "snapshot" of ability at one moment in time. Thus the changes inferred may be confounded by variation between the groups (i.e. differences in education and socio-cultural factors). Furthermore, this approach does not bestow information on the development of individuals. The age-related differences observed between the working memory resources supporting children's mathematics at 7-/8-years and 9-/10-years within the current research may therefore be confounded by these factors. Applebaum and McCall (1983) argue that "the longitudinal method is the lifeblood of developmental science. It is the only way researchers can study change within organisms over age" (Applebaum & McCall, 1983, pp.441). Indeed, longitudinal research affords many advantages such as the study of change in individuals over time, which is arguably a truer reflection of developmental change. Although a longitudinal study was conducted in Chapter 8 to strengthen the case for a causal relationship between working memory and children's mathematics performance, additional longitudinal research is needed to substantiate the developmental differences observed between the two age groups in Chapters 6, 7 and 9.

Another related potential weakness in the current research was that correlational designs were conducted for many studies. The use of this approach provided information to support a significant association between working memory test scores and children's mathematics performance. However, it did not provide information on causality, the direction of the association or the role that working memory might play in children's mathematics. Although multiple regression procedures allowed a better predictive combination of the variables, additional research will be required to identify the nature of the relationship between working memory and children's mathematics. Again, a longitudinal approach would help resolve some of the weaknesses in correlational designs. Alternatively, dual-task designs might help to identify the role of working memory in different mathematical processes.

A general limitation of this research is the "neglect" of other important cognitive factors (e.g. verbal IQ and reading ability) and recent additions to the tripartite working memory model (e.g. the episodic buffer and non-verbal executive skills). Future research will need to focus on the recent theoretical developments to the working memory model (e.g. Baddeley, 2000) if its role in children's mathematics is to be fully understood. Furthermore, controlling for other cognitive abilities (such as verbal IQ or reading ability) may help to identify the role of working in children's mathematics. Reading and language skills often correlate highly with mathematics ability and are also affected by working memory ability. In view of the use of written mathematics assessments in the current research, part of the association between working memory ability and mathematics performance may have been influenced by the relationship between reading ability and working memory (e.g. Bull & Johnston, 1997). Controlling for reading ability in future studies would clarify this issue.

A further limitation relates to the mathematics assessments. The mathematics tests were timed, which may not have allowed all children to show their range of abilities as failure to complete or attempt a question resulted in an incorrect response. However, in each study less than 10 children failed to complete the tests, indicating that timed assessments did not pose a major problem.

Other limitations in the present research related more directly to problems within particular studies. As these issues are raised within the relevant chapters, only a brief summary is provided here. The first of these issues concerns the measures used where two problems arose. Firstly, there were problems with the NVIQ measure grouping with the working memory measures in Chapters 3 and 4. This complicated the issue of isolating the unique contribution of working memory ability to mathematics performance. It was suggested that the measures may have grouped due to the similarity of the task demands or the fact that working memory and intelligence are closely related constructs (e.g. Colom et al., 2004). The inclusion of an additional verbal IQ measure may help separate working memory and intelligence factors in future research. The second problem related to the measurement of children's visuo-spatial abilities. Several visuo-spatial sketchpad tasks were administered in Chapter 4 in an attempt to isolate visual and spatial immediate memory from one another and from executive resources. However, there were problems with the nature and specificity of the tasks (see Chapter 4). Clearly future research is needed to investigate the cognitive processes supporting performance on such tasks. The second issue arising from the current research is related to the first. Not only were there problems with the assessment of visuo-spatial sketchpad abilities, there were also problems defining its structure. Contrary to expectation there was no evidence for a fractionated visuo-spatial sketchpad system from the data presented in Chapters 4 and 7. Rather, the data highlighted the fact that, as yet, there is no definitive description of the structure of visuo-spatial working memory. These problems clearly limited the investigation into the role of the visuo-spatial sketchpad in children's mathematics.

10.4

Future Directions

In the future it would be beneficial to educational practitioners and cognitive developmental researchers to replicate and extend the current investigation into the association between visuo-spatial sketchpad ability and mathematics performance using a longitudinal methodology. In particular, it would be interesting to track the development of visuo-spatial working memory and mathematics performance between the ages of 7-years and 10-years to further investigate i) whether normal increases in visuo-spatial working memory capacity support normal mathematics development between these ages ii) whether visuo-spatial working memory impairments at 7-years predict MD at 10-years and iii) whether the association between visuo-spatial working memory and children's mathematics decreases over time .

In addition, it would be interesting to extend the current investigation to explore the associations between visuo-spatial sketchpad ability and mathematics performance in older children to see if the developmental trends suggested in the current data extend to adolescence. Based upon current observations, it would be expected that the associations between visuo-spatial sketchpad ability and mathematics performance would decrease with age. Further research could also extend this investigation to include pre-school children and explore the investigation between visuo-spatial ability and early numeracy skills in pre-school children. This type of investigation could be achieved using cross-sectional or longitudinal methodologies.

An alternate direction could be to further investigate the role of visuo-spatial working memory in children's mathematics. The current research provides additional evidence for an association between the two skills, but it does not highlight which visuo-spatial processes are involved in mathematical problem solving. One approach would be to adopt a dual-task design to selectively disrupt visuo-spatial working memory during mathematical processing.

A related avenue of research would be to explore the domain-general / domain-specific issue. Butterworth (1999) suggests that humans possess an innate "number module"; a domain-specific module for processing number. On the contrary, recent research provides evidence for a general resource related to visuo-spatial cognition and mathematical processing. For example, Feeney et al. (2004) suggest that people may represent concepts by analogy to space. Zago and Tzourio-Mazoyer (2002) report that similar cerebral networks are activated by mathematics and visuo-spatial working memory tasks in adults and the current research suggests that visuo-spatial working memory ability may constrain mathematics performance in children. Therefore, it would be interesting to investigate the associations between performance on visuo-spatial tasks (i.e. visual attention, imagery and visuo-spatial working memory tasks) and a range of mathematical tasks with children.

Another line of further investigation would be to explore the cognitive structure of the visuo-spatial sketchpad in children. Although recent theoretical developments advocate separate visual and spatial subcomponents (e.g. Logie, 1995), there was no evidence to support this in the current research. It may be beneficial to conduct a large-scale factor analytic study, using a variety of visuo-spatial sketchpad, non-verbal intelligence, non-verbal executive, immediate visual memory and immediate spatial memory tasks, to explore the relationships between different visuo-spatial skills. Within this type of investigation it would be possible to address the nature, specificity and processes involved in different visuo-spatial sketchpad tasks.

Theoretically, this type of investigation should underpin future work that aims to investigate the relationship between visuo-spatial skills and children's mathematics. A better understanding of the structure and functioning of children's visuo-spatial working memory might elicit a greater understanding of its importance in children's mathematics.

References

Adams, J.W., & Hitch, G.J. (1997). Working memory and children's mental addition. *Journal of Experimental Child Psychology, 67,* 21-38.

Adams, J.W., & Hitch, G.J. (1998). Children's mental arithmetic and working memory. In: C. Donlan (Ed.) *The development of mathematical skills,* pp. 153-173. Hove, UK; Psychology Press.

Alloway, T.P., Gathercole, S.E., Willis, C., & Adams, A.M. (2004). A structural analysis of working memory and related cognitive skills in early childhood. *Journal of Experimental Child Psychology, 87,* 85-106.

American Psychiatric Association (1994). *Diagnostic and statistical manual of mental disorders. 4th Edition.* Washington, DC: APA.

Anderson, J.R., & Lebiere, C. (1998). *The atomic components of thought.* Mahwah, New Jersey: Erlbaum.

Applebaum, M.I., & McCall, R.B. (1983). Design and analysis in developmental psychology. In P.H. Mussen (Ed.), *Handbook of child psychology,* pp.415-476. New York: Wiley.

Archibald, L.M.D., & Gathercole, S.E. (submitted). Working memory in children with specific language impairment. , , .

Ashcraft, M.H. (1982). The development of mental arithmetic: A chronometric approach. *Developmental Review, 2,* 213-236.

Ashcraft, M.J. (1987). Children's knowledge of simple arithmetic: A developmental model and simulation. In J. Bisanz, C.J. Brainerd and R. Kail (Eds.), *Formal methods in developmental psychology: Progress in cognitive development research,* pp.302-338. New York: Springer-Verglag.

Ashcraft, M.J., & Battaglia, J. (1978). Cognitive arithmetic: Evidence for retrieval and decision processes in mental addition. *Journal of Experimental Psychology, Human Learning and Memory, 4,* 527-538.

Ashcraft, M.H., Donley, R.D., Halas, M.A., & Vakali, M. (1992). Working memory, automaticity, and problem difficulty. In J.I.D. Campbell (Ed.), *The nature and origins of mathematical skills,* pp. 309-329. Amsterdam: Elsevier Science Publishers.

Ashcraft, M.H., & Faust, (1994). Mathematics anxiety and mental arithmetic performance: An exploratory investigation. *Cognition and Emotion, 8,* 97-125.

Ashcraft, M.H., & Fierman, B.A. (1982). Mental addition in third, fourth and sixth graders. *Journal of Experimental Child Psychology, 33,* 216-234.

Ashcraft, M.H., & Kirk, E.P. (2001). The relationships among working memory, math anxiety and performance. *Journal of Experimental Psychology, General, 130,* 224-237.

Askew, M. (2004). The psychology of maths education in the early primary years: A response to Munn. *The Psychology of Education Review, 28*, 9-13.

Atkinson, R.C., & Shiffrin, R.M. (1968). Human Memory: A proposed system and its control processes. In K.W. Spence (Ed.), *The psychology of learning and motivation: Advances in research and theory*, Vol. 2., pp. 89-195. New York: Academic Press.

Baddeley, A.D. (1966). The capacity for generating information by randomization. *Quarterly Journal of Experimental Psychology, 18,* 119-120.

Baddeley, A.D. (1986). *Working memory.* Oxford: Oxford University Press. *Experimental Psychology, 49A,* 5-28.

Baddeley, A.D. (1990). *Human Memory: Theory and Practice.* Oxford: Clarendon

Press.

Baddeley, A.D. (1996). Exploring the central executive. *Quarterly Journal of Experimental Psychology, Human Experimental Psychology, 49A*, 5-28.

Baddeley, A.D. (2000). The episodic buffer: a new component of working memory? *Trends in Cognitive Sciences, 4*, 417-423.

Baddeley, A.D., & Andrade, J. (2000). Working memory and the vividness of imagery. *Journal of Experimental Psychology, General, 129*, 126-145.

Baddeley, A.D., Cocchini, G., Della Sala, S., Logie, R.H., & Spinnler, H. (1999). Working memory and vigilance: Evidence from normal aging and Alzheimer's disease. *Brain and Cognition, 41*, 87-108.

Baddeley, A.D., Gathercole, S.E., & Papagno, C. (1998). The phonological loop as a language learning device. *Psychological Review, 105,* 158-173.

Baddeley, A.D., Grant, W., Wight, E., & Thompson, N. (1975). Imagery and visual working memory. In P.M.A. Rabbitt & S. Dornic (Eds.), *Attention and performance V,* pp.205-217. London: Academic Press.

Baddeley, A.D., & Hitch, G.J. (1974). Working memory. In: G.H. Bower (Ed.), *The psychology of learning and motivation*, Vol. 8, pp.47-89. New York: Academic Press.

Baddeley, A.D., Lewis, V.J., & Vallar, G. (1984). Exploring the articulatory loop. *The Quarterly Journal of Experimental Psychology, 36A,* 233-252.

Baddeley, A.D., & Lieberman, K. (1980). Spatial working memory. In R.S. Nickerson (Ed.), *Attention and performance VIII,* pp.521-539. Hillsdale, New Jersey: Lawrence Erlbaum Associates Inc.

Baddeley, A.D., & Logie, R.H. (1999). Working memory : The multicomponent model. In A. Miyake and P. Shah (Eds.), *Models of working memory:*

Mechanisms of active maintenance and control. Cambridge: Cambridge University Press.

Baddeley, A.D., Papagno, C., & Vallar, G. (1988). When long-term learning depends on short-term storage. *Journal of Memory and Language, 27,* 586-595.

Baddeley, A.D., Thompson, N., & Buchannan, M. (1975). Word length and the structure of short-term memory. *Journal of Verbal Learning and Verbal Behavior, 14,* 575-589.

Baddeley, A.D., Vallar, G., & Wilson, B. (1987). Sentence comprehension and phonological memory: Some neuropsychological evidence. In M. Coltheart (Ed). *Attention and Performance Vol XII: The Psychology of Reading,* pp. 509-529.

Barnard, P.J. (1985). Interacting cognitive subsystems: A psycholinguistic approach to short-term memory. In A.W. Young (Ed.), *Progress in the psychology of language*, Vol. 2, pp.197-258. London: Erlbaum.

Baroody, A.J. (1987). *Children's mathematical thinking: A developmental framework for preschool, primary and special education teachers.* New York: Teachers College.

Barrouillet, P., Bernadin, S., & Camos, V. (2004). Time constraints and resource sharing in adult's working memory spans. *Journal of Experimental Psychology, General, 133,* 83-100.

Barrouillet, P., & Camos, V. (2001). Developmental increases in working memory span: Resource sharing or temporal decay? *Journal of Memory and Language, 45,* 1-20.

Bartholomew, D.J., Steele, F., Moustaki, I., & Galbraith, J.I. (2002). *The analysis and interpretation of multivariate data for social scientists.* Boca Raton, Florida: Chapman and Hall/CRC.

Benbow, C. P. (1988). Sex-related differences in precocious mathematical reasoning ability: Not illusory, not easily explained. *Behavioral and Brain Sciences, 11,* 217-232.

Bentler, P.M. (2001). EQS *structural equation modelling software (Version 6.0) [Computer software].* USA:mvsoft.

Biggs, J., & Collis, K. (1982). *Evaluating the quality of learning: The SOLO taxonomy.* New York: Academic Press.

Bijeljac-Babic, R., Bertoncini, J., & Mehler, J., (1991). How do 4-day old infants categorize multi-syllabic utterances? *Developmental Psychology, 29,* 711-721.

Brannon, E. (2002). The development of ordinal numerical knowledge in infancy. *Cognition, 83, 223-240.*

Broadbent, D.E. (1958). *Perception and communication.* Oxford: Pergamon.

Brooks, L.R. (1967). The suppression of visualisation by reading. *Quarterly Journal of Experimental Psychology, 19,* 289-299.

Brown, J. (1958). Some tests of the decay theory of immediate memory. *Quarterly Journal of Experimental Psychology, 10,* 12-21.

Brown, M. (2001). Influences on the teaching of number in England. In J. Anghileri (Ed.), *Principles and practices in arithmetic teaching.* Buckingham: Open University Press.

Brown, G.D.A., & Hulme, C. (1996). Nonword repetition, short-term memory and word age-of-acquisition: A computational model. In S.E. Gathercole (Ed.), *Models of short-term memory,* pp. 129-148. Hove: Psychology Press.

Bryant, P.E. (1985). The distinction between knowing when to do a sum and knowing how to do it. *Educational Psychology, 5,* 207-215.

Bull. R., & Johnston, R.S. (1997). Children's arithmetical difficulties: Contributions from processing speed, item identification and short-term memory. *Journal of Experimental Child Psychology, 65*, 1-24.

Bull, R., Johnston, R. S., & Roy, J. A. (1999). Exploring the roles of the visual-spatial sketchpad and central executive in children's arithmetical skills: Views from cognition and developmental neuropsychology. *Developmental Neuropsychology, 15*, 421-442.

Bull, R., & Scerif, G. (2001). Executive functioning as a predictor of children's mathematics ability: Inhibition, switching, and working memory. *Developmental Neuropsychology, 19*, 273-293.

Butterworth, B. (1999). *The mathematical brain.* London: Papermac.

Butterworth, B. (2003). *Dyscalculia screener: Highlighting pupils with specific learning difficulties in maths*. London: nferNelson Publishing Company Limited.

Butterworth, B., Zorzi, M., Girelli, L., & Jonckheere, A.R. (2001). Storage and retrieval of addition facts: The role of number comparison. *Quarterly Journal of Experimental Psychology, 54A,* 1005-1029.

Bynner, J., & Parsons, S. (1997). *Does numeracy matter?* UK: Basic Skills Agency.

Campbell, J.I.D. (1994). Architectures for numerical cognition. *Cognition, 53*, 1-44.

Campbell, J.I.D., & Clark, J.M. (1992). Cognitive number processing: An encoding – complex perspective. In J.I.D. Campbell (Ed.), *The nature and origin of numerical skills,* pp.457-492. Amsterdam: Elsevier Science Publishers.

Campbell, J.I.D., & Fuselgang, J. (2001). Strategy choice for arithmetic verification: Effects of numerical surface form. *Cognition, 80*, 21-30.

Campbell, J.I.D., & Graham, D.J. (1985). Mental multiplication skill: Structure, process and acquisition. *Canadian Journal of Psychology, 39,* 338-366.

Campbell, T.F., Dollaghan, C., Needlemen, H., & Jonosky, J. (1997). Reducing bias in language assessment: Processing dependent measures. *Journal of Speech, Language, and Hearing Research, 40*, 519-525.

Carlesimo, G.A., Perri, R., Turriziani, P., Tomaiuolo, F., & Caltagirone, C. (2001). Remembering what but not where: Independence of spatial and visual working memory in human brain. *Cortex, 37,* 519-537.

Carpenter, P.A., Just, M.A., & Shell, P. (1990). What one intelligence test measures: A theoretical account of the processing in the Raven Matrices test. *Psychological Review, 97,* 404-431.

Carpenter, T.P., & Moser, J.M. (1984).The acquisition of addition and subtraction concepts in grades one through three. *Journal for Research in Mathematics Education, 15*, 179-202.

Case, R. (1982). General developmental influences on the acquisition of elementary concepts and algorithms in arithmetic. In T.P. Carpenter, J.M. Moser & T.A. Romberg (Eds.), *Addition and subtraction: A cognitive perspective,* pp.156-170. Hillsdale, New Jersey: Erlbaum.

Case, R. (1985). *Intellectual development: Birth to adulthood.* New York: Academic Press.

Case, R.D., Kurland, M., & Goldberg, J. (1982). Operational efficiency and the growth of short-term memory span. *Journal of Experimental Child Psychology, 33,* 386-404.

Chen, Z., & Siegler, R.S. (2000). Across the great divide: Bridging the gap between understanding of toddlers' and older children's thinking. *Monographs of the Society for Research in Child Development, 65,* 2, Serial 261.

Chuah, Y.M.L., & Maybery, M.T. (2000). Verbal and spatial short-term memory: Two sources of developmental evidence consistent with common underlying processes. *International Journal of Psychology, 35,* 374-377.

Clements, D.H., Battista, M.T., Sarama, J., Swaminathan, S., & McMillen, S. (1997). Students' development of length concepts in a logo-based unit on geometric paths. *Journal for Research in Mathematics Education, 28,* 70-95.

Cohen, J. (1977). *Statistical power for the behavioural sciences.* New York: Academic Press.

Cohen, J. (1988). *Statistical power analysis for the behavioral sciences* (2nd edition). Hillsdale, NJ: Erlbaum.

Cohen, J. (1992). Quantitative methods in psychology: A power primer. *Psychological Bulletin, 112,* 155-159.

Colom, R., Rebello, I., Placios, A., Juan-Espinosa, M., & Kyllonen, P.C. (2004). Working memory is (almost) perfectly predicted by g. *Intelligence, 32*, 277-296.

Conrad, R. (1964). Acoustic confusion in immediate memory. *British Journal of Psychology, 55*, 75-84.

Conrad, R., & Hull, A.J. (1964). Information, acoustic confusion and memory span. *British Journal of Psychology, 55,* 429-432.

Cooper, R.G. (1984). Early number development: Discovering number space with addition and subtraction. In C. Sophian (Ed.), *Origins of cognitive skills*, pp. 157-192. Hillsdale, New Jersey: Erlbaum.

Cornoldi, C., Rigoni, F., Tressoldi, P.E., & Vio, C. (1999). Imagery deficits in nonverbal learning disabilities. *Journal of Learning Disabilities, 32,* 48-57.

Cornoldi, C., Rigoni, F., Venneri, A., & Vecchi, T. (2000). Passive and active processes in visuo-spatial memory: Double dissociation in developmental learning disabilities. *Brain and Cognition, 43,* 117-120.

Courtney, S.M., Petit, L., Maisog, J.M., Ungerleider, L.G., & Haxby, J.V. (1998). Object and spatial visual working memory activate separate neural systems in human cortex. *Cerebral Cortex, 6,* 39-49.

Courtney, S.M., Ungerleider, L.G., Keil, K., & Haxby, J.V. (1997). Transient and sustained activity in a distributed neural system for human working memory. *Nature, 386,* 608-611.

Cowan, N. (1988). Evolving conceptions of memory storage, selective attention, and their mutual constraints within the human information processing system. *Psychological Bulletin, 104,* 163-191.

Cowan, N., Wood, N.L., Wood, P.K., Keller, T.A., Nugent, L.D., & Keller, C.V. (1998). Two separate verbal processing speeds contributing to verbal short-term memory span. *Journal of Experimental Psychology, General, 127*, 141-160.

Coyle, T.R., & Bjorklund, D.F. (1997). Age differences in, and consequences of, multiple- and variable-strategy use on a multi-trial sort recall task. *Developmental Psychology, 33*, 372-380.

Craik, F.I.M., & Lockhart, R.S. (1972). Levels of processing: A framework for memory research. *Journal of Verbal Learning and Verbal Behavior, 11,* 671-684.

Daneman, M., & Carpenter, P.A. (1980). Individual differences in working memory and reading. *Journal of Verbal Learning and Verbal Behavior, 19,* 450-466.

Daneman, M., & Green, I. (1986). Individual differences in comprehending and producing words in context. *Journal of Memory and Language, 25,* 1-18.

Dark, V.J., & Benbow, C.P. (1990). Mathematically talented students show enhanced problem translation and enhanced short-term memory for digit and spatial information. *Journal of Educational Psychology, 82*, 420-429.

Dark, V.J., & Benbow, C.P. (1991). Differential enhancement of working memory with mathematical versus verbal precocity. *Journal of Educational Psychology, 83,* 48-60.

Dark, V.J., & Benbow, C.P. (1994). Type of stimulus mediates the relationship between working memory performance and type of precocity. *Intelligence, 19,* 337-357.

Dehaene, S. (1992). Varieties of numerical abilities. *Cognition, 44,* 1-42.

Dehaene, S. (1997). *The number sense: How the mind creates mathematics*. New York: Oxford University Press.

Dehaene, S., Bossini, S., & Giraux, P. (1993). The mental representation of parity and numerical magnitude. *Journal of Experimental Psychology, General,* 122, 371-376.

Dehaene, S., & Cohen, L. (1995). Cerebral pathways for calculation: Double dissociation between rote verbal and quantitative knowledge of arithmetic. *Cortex, 33*, 219-250.

Dehaene, S., Dehaene-Lambertz, G., & Cohen, L. (1998). Abstract representations of numbers in the animal and human brain. *Trends in Neurosciences, 21,* 355-361.

Dehaene, S., Dupoux, E., & Mehler, J. (1990). Is numerical comparison digital: Analogical and symbolic effects in two-digit number comparison. *Journal of Experimental Psychology, Human Perception and Performance, 16*, 626-641.

Dehaene, S., Spelke, E., Stanescu, R., & Tsivikin, S. (1999). Sources of mathematical thinking: Behavioral and brain-imaging evidence. *Science, 284*, 970-974.

De Jong, P. (1998). Working memory deficits of reading disabled children. *Journal of Experimental Child Psychology, 70,* 75-96.

Della Sala, S., Gray, C., Baddeley, A.D., Allamano, N., & Wilson, L. (1999). Pattern span: A tool for unwelding visuo-spatial memory. *Neuropsychologia, 37,* 1189-1199.

Della, Sala, S., Gray, C., Baddeley, A.D., & Wilson, L. (1997). *Visual Patterns Test.* Bury St. Edmunds: Thames valley Test Company.

Deloche, G., Seron, X., Larroque, C., Magnien, C., Metz-Lots, M.N., et al. (1994). Calculation and number processing battery: The role of demographic factors. *Journal of Clinical and Experimental Neuropsychology, 16,* 915-208.

Department for Education and Employment (DFEE). *Numeracy Matters: The Preliminary Report of the Numeracy Task Force.* Published for consultation, January 1998. Crown Copyright.

De Rammelaere, S., Stuyven, E., & Vandierendonck, A. (2001). Verifying simple arithmetic sums and products: Are the phonological loop and the central executive involved? *Memory and Cognition, 29,* 267-273.

De Renzi, E. (1982). *Disorders of space exploration and cognition*. New York: Wiley.

De Renzi, E., & Nichelli, P. (1975). Verbal and nonverbal short term memory impairment following hemispheric damage. *Cortex, 11,* 341-353.

DeStefano, D., & LeFevre, J. (2004). The role of working memory in mental arithmetic. *European Journal of Cognitive Psychology, 16*, 353-386.

Dixon, P., LeFevre, J., & Twilley, L.C. (1988). Word knowledge and working memory as predictors of reading skill. *Journal of Educational Psychology, 80*, 465-472.

Dowker, A. (1992). Computational estimation strategies of professional mathematicians. *Journal for Research in Mathematics Education, 23*, 45-55.

Dowker, A. (1995). Children with specific calculation difficulties. *Links, 2*, 7-12.

Dowker, A. (1998). Individual differences in arithmetical development. In C. Donlan (Ed.), *The development of mathematical thinking.* London: Taylor Francis.

Ellis, N.C., & Hennelley, R.A. (1980). A bilingual word-length effect: Implications for intelligence testing and the relative ease of mental calculation in English and Welsh. *British Journal of Psychology, 71*, 43-52.

Ellis Weismer, S., Evans, J., & Hesketh, L. (1999). An examination of verbal working memory capacity in children with specific language impairment. *Journal of Speech, Language and Hearing Research, 42,* 1249-1260.

Engle, R. W., Kane, M. J., & Tuholski, S. W. (1999). Individual differences in working memory capacity and what they tell us about controlled attention, general fluid intelligence and functions of the prefrontal cortex. In A. Miyake & P. Shah (Eds.), *Models of working memory: Mechanisms of active* *maintenance and executive control.* pp. 102-134. New York: Cambridge University Press.

Erdfelder, E. (1984). Zur Bedeutung und Kontrolle des ß-Fehlers bei der inferenzstatistischen Prüfung log-linearer Modelle. [On significance and control of the ß error in statistical tests of log-linear models]. *Zeitschrift für Sozialpsychologie, 15,* 18-32.

Espy, K.A. (1997). The shape school: Assessing executive function in preschool children. *Developmental Neuropsychology, 13,* 495-499.

Estes, W.K. (1973). Phonemic coding and rehearsal in short-term memory for letter strings. *Journal of Verbal Learning and Verbal Behavior, 12,* 360-372.

Farah, M.J., Hammond, K.M., Levine, C.N., & Calvanio, R. (1988). Visual and spatial mental imagery: Dissociable systems of representation. *Cognitive Psychology, 20,* 439-462.

Faul, F., & Erdfelder, E. (1992). GPOWER: *A priori-, post hoc-, and compromise power analyses for MS-DOS [computer software].* Bonn, Germany: Bonn University.

Feeney, A., Adams, J.W., Webber, L. & Ewbank, M. (2004). Individual differences in graphical reasoning. In A. Blackwell, K. Marriott and A. Shimojima (Eds.) *Diagrams 2004: Third International Conference on Theory and Application of Diagrams, LNAI Vol. 2980,* 271-285.

Feldman, A. (2002).Mathematics instruction: Cognitive, affective and existential perspectives. In J.M. Royer (Ed.), *Mathematical cognition,* pp. 147-174. Greenwich, Connecticut: Information Age Publishing.

Fry, A.F., & Hale, S. (1996). Processing speed, working memory and fluid intelligence. *Psychological Science, 7,* 237-241.

Fürst, A., & Hitch, G. (2000). Separate roles for executive and phonological components of working memory in mental arithmetic. *Memory and Cognition, 28*, 774-782.

Fuson, K.C. (1988). *Children's counting and concepts of number.* New York: Springer-Verlag.

Fuson, K.C., Richards, J., & Briars, D.C. (1982). The elaboration of the number-word sequence. In C.J. Brainerd (Ed.), *Children's logical and mathematical cognition.* New York: Springer-Verlag.

Gallistel, C. R., & Gelman, R. (1992). Preverbal and verbal counting and computation. *Cognition, 44,* 43-74.

Gathercole, S.E., & Adams, A. (1993). Phonological working memory in very young children. *Developmental Psychology, 29,* 770-778.

Gathercole, S.E., Adams, A., & Hitch, G.J. (1994). Do young children rehearse? An individual differences analysis. *Memory and Cognition, 22*, 201-207.

Gathercole, S.E., & Baddeley, A.D. (1989). Evaluation of the role of phonological STM in the development of vocabulary in children: A longitudinal study. *Journal of Memory and Language, 8*, 200-213.

Gathercole, S.E., & Baddeley, A.D. (1990). The role of phonological working memory in vocabulary acquisition: A study of young children learning new names. *British Journal of Psychology, 81*, 439-454.

Gathercole, S.E., Brown, L., & Pickering, S.J. (2003).Working memory assessments at school entry as longitudinal predictors of National Curriculum attainment levels. *Educational Psychology, 70,* 177-194.

Gathercole, S.E., Frankish, C.R., Pickering, S.J., & Peaker, S.H. (1999). Phonotactic influences on short-term memory. *Journal of Experimental Psychology: Learning, Memory and Cognition, 25,* 1-13.

Gathercole, S.E., & Hitch, G.J. (1993). Developmental changes in short-term memory: A revised working memory perspective. In A. Collins, S.E. Gathercole, M.A. Conway, and P.E. Morris (Eds.), *Theories of memory*, pp. 189-210. Hove, UK: Erlbaum.

Gathercole, S.E., Hitch, G.J., Service, E., & Martin, A.J. (1997). Short-term memory and new word learning in children. *Developmental Psychology, 33,* 966-979.

Gathercole, S. E, Lamont, E., & Alloway, T.P. (in press). Working memory in the classroom. In S. Pickering (Ed.). *Working memory and education*. Elsevier Press.

Gathercole, S.E., & Pickering, S.J. (2000a). Working memory deficits in children with low achievements in the national curriculum at 7 years of age. *British Journal of Educational Psychology, 70*, 177-194.

Gathercole, S.E., & Pickering, S.J. (2000b). Assessment of working memory in six- and seven-year old children. *Journal of Educational Psychology, 92*, 377-390.

Gathercole, S.E., Pickering, S.J., Ambridge, B., & Wearing, H. (2004). The structure of working memory from 4 to 25 years of age. *Developmental Psychology, 40,* 177-190.

Gathercole, S.E., Pickering, S.J., Knight, C., & Stegmann, Z. (2004). Working memory skills and educational attainment: Evidence from National Curriculum assessments at 7 and 15 years of age. *Applied Cognitive Psychology, 18,* 1-16.

Gathercole, S.E., Willis, C., Emslie, H., & Baddeley, A.D. (1992). Phonological memory and vocabulary development during the early school years: A longitudinal study. *Developmental Psychology, 28,* 887-898.

Geary, D.C. (1990). A componential analysis of an early learning deficit in mathematics. *Journal of Experimental Child Psychology, 49,* 363-383.

Geary, D.C. (1993). Mathematical Disabilities: Cognitive, neuropsychological, and genetic components. *Psychological Bulletin, 114*, 345-362.

Geary, D.C. (1994). *Children's mathematical development: Research and practical applications.* Washington, DC: American Psychological Association.

Geary, D.C. (1996). The problem-size effect in mental addition: Developmental and cross-sectional trends. *Mathematical Cognition, 2,* 63-93.

Geary, D.C., Bow-Thomas, C.C., & Yao, Y. (1992). Counting knowledge and skill in cognitive addition: A comparison of normal and mathematically disabled children. *Journal of Experimental Child Psychology, 54*, 372-391.

Geary, D.C., & Brown, S.C. (1991). Cognitive addition: Strategy choice and speed-of-processing differences in gifted, normal and mathematically disabled children. *Journal of Experimental Child Psychology, 54,* 372-391.

Geary, D.C., Brown, S.C., & Samaranayake, V.A. (1991). Cognitive addition: A short longitudinal study of strategy choice and speed-of-processing differences in normal and mathematically disabled children. *Developmental Psychology, 27*, 787-797.

Geary, D.C., & Burlingham-Dubree, M. (1989). External Validation of the strategy choice model for addition. *Journal of Experimental Child Psychology, 47*, 175-192.

Geary, D.C., Cormier, P., Goggin, J.P., Estrada, P., & Lunn, M.C.E. (1993). Mental arithmetic: A componential analysis of speed-of-processing across monolingual, weak bilingual and strong bilingual adults. *International Journal of Psychology, 28,* 185-201.

Geary, D.C., Fan, L., & Bow-Thomas, C.C. (1992). Numerical cognition: Loci of ability differences comparing children from China and the United States. *Psychological Science, 3,* 180-185.

Geary, D.C., Hoard, M.K., & Hamson, C.O. (1999). Numerical and arithmetical cognition: Patterns of functions and deficits in children at risk for a mathematical disability. *Journal of Experimental Child Psychology, 74,* 213-239.

Geary, D.C., Saults, S.J., Liu, F., & Hoard, M.K. (2000). Sex differences in spatial cognition, computational fluency, and arithmetic reasoning. *Journal of Experimental Child Psychology, 74,* 213-239.

Geary, D.C., & Wiley, J.G. (1991). Cognitive addition: Strategy choice and speed-of-processing differences in young and elderly adults. *Psychology and Aging, 6,* 474-483.

Gelman, R. & Gallistel, C.R. (1978). The *child's understanding of number.* Cambridge, M.A.: Harvard University Press.

Gelman, R., & Meck, E. (1983). Preschoolers' counting: Principles before skill. *Cognition, 13,* 343-359.

Gelman, R., Meck, E., & Merkin, S. (1986). Young children's numerical competence. *Cognitive Development, 1,* 1-29.

Ginsburg, H. (1989). *Children's arithmetic: How they learn and how you teach it.* Austin, Texas: Pro-Ed.

Ginsburg, H., Papas, S., & Seo, K.H. (2001). Everyday mathematical knowledge: Asking young children what is developmentally appropriate. In S.L. Goldbeck (Ed.), *Psychological perspectives on early childhood education: reframing dilemmas in research and practice*, pp.181-219. Mahweh, New Jersey: Lawrence Erlbaum Associates.

Greeno, T.G., Riley, M.S., & Gelman, R. (1984). Conceptual competence and children's counting. *Cognitive Psychology, 16,* 94-134.

Gross-Tsur, V., Manor, O., & Shalev, R.S. (1996). Developmental dyscalculia: Prevalence and demographic features. *Developmental Medicine and Child Neurology, 1,* 25-33.

Groen, G.J., & Parkman, J.M. (1972). A chronometric analysis of simple addition. *Psychological Review, 79,* 329-343.

Hamann, M.S., & Ashcraft, M.H. (1985). Simple and complex addition across development. *Journal of Experimental Child Psychology, 40*, 49-72.

Hamilton, C.J., Coates, R.O., & Heffernan, T. (2003). What develops in visuo-spatial working memory development? *European Journal of Cognitive Psychology, 15,* 43-69.

Hanley, J.R., Young, A.W. and Pearson, N.A., (1991), Impairment of the visuo-spatial scratchpad. *Quarterly Journal of Experimental Psychology*, *43,* 101-126.

Hauser, M.D., MacNeilage, P., & Ware, M. (1996). Numerical representations in primates. *Proceedings of the National Academy of Science, USA, 93,* 1514-1517.

Hayes, J.R. (1973). On the function of visual imagery in elementary mathematics. In W. Chase (Ed.), *Visual information processing*, pp. 177-214. New York : Academic Press.

Healy, A.F., & Nairne, J.S. (1985). Short-term memory processes in counting. *Cognitive Psychology, 17,* 417-444.

Heathcote, D. (1994). The role of visuo-spatial working memory in the mental addition of multi-digit addends. *Current Psychology of Cognition, 13*, 207-245.

Hecht, S.A. (2002). Counting on working memory in simple arithmetic when counting is used for problem solving. *Memory and Cognition, 30*, 447-455.

Henry, L.A. (2001). How does the severity of a learning disability affect working memory performance? *Memory, 9,* 233-247.

Hitch, G.J. (1978) The role of short-term memory in mental arithmetic. *Cognitive Psychology, 10*, 302-323.

Hitch, G.J. (1990). Developmental fractionation of working memory. In G. Vallar and T. Shallice (Eds.), *Neuropsychological impairment of short-term memory,* pp.221-246. Cambridge: Cambridge University Press.

Hitch, G.J., & Halliday, M.S. (1983). Working memory in children. *Philosophical Transactions of the Royal Society of London, B302,* 324-340.

Hitch, G.J., Halliday, M.S., Schaafstal, A.m., Schraagen, J.M.C. (1988). Visual working memory in young children. *Memory and Cognition, 16,* 120-132.

Hitch, G.J., & McAuley, E. (1991) Working memory in children with specific arithmetical learning difficulties. *British Journal of Psychology, 82*, 375-386.

Hitch, G.J., Towse, J.N., & Hutton, U. (2001). What limits children's working memory span? Theoretical accounts and applications for scholastic development. *Journal of Experimental Psychology, General, 130*, 184-198.

Houdé, O. (1997). Numerical development: From the infant to the child. Wynn's (1992) paradigm in 2- and 3-year olds. *Cognitive Development, 12*, 373-391.

Houdé, O., & Tzourio-Mazoyer, N. (2003). Neural foundations of logical and mathematical cognition. *Nature Reviews Neuroscience, 4,* 507-514.

Hughes, M. (1986). *Children and number: Difficulties in learning mathematics.* Oxford, UK: Blackwell Publishers, Ltd.

Hulme, C., & Roodenrys, S. (1995). Practitioner review- verbal working memory development and its disorders. *Journal of Child Psychology and Psychiatry, 36,* 373-398.

Hulme, C., Thompson, N., Muir, C., & Lawrence, A. (1984). Speech rate and the development of short-term memory span. *Journal of Experimental Child Psychology, 38,* 241-253.

Hyde, J.S., Fennema, E., & Lamon, S.J. (1990). Gender differences in mathematics performance: A meta-analysis. *Psychological Bulletin, 107,* 139-155.

Ichikawa, S. (1983). Verbal memory span, visual memory span, and their correlations with cognitive tasks. *Japanese Psychological Research, 25*, 173-180.

Isaacs, E.B., & Vargha-Khadem, F. (1989). Differential course of development of spatial and verbal memory span. *Journal of Experimental Child Psychology, 38,* 241-253.

Jarrold, C. & Baddeley, A.D. (1997). Short-term memory for verbal and visuo-spatial information in Down's syndrome. *Cognitive Neuropsychiatry, 2,* 102-122.

Jarvis, H.L., & Gathercole, S.E. (2003). Verbal and nonverbal working memory and achievements on national curriculum tests at 7 and 14 years of age. *Educational and Child Psychology*, *20,* 123-140.

Jarvis, H.L., & Gathercole, S.E. (submitted). Executive functions and achievements in school: Shifting, updating, inhibition and working memory. , , .

Jensen, A.R., & Whang, P.A. (1994). Speed of accessing arithmetic facts in long-term memory: A comparison of Chinese-American and Anglo-American children. *Contemporary Educational Psychology, 19,* 1-12.

Jurden, F.H. (1995). Individual differences in working memory and complex cognition. *Journal of Educational Psychology, 87*, 93-102.

Kail, R., & Hall, L.K. (1999). Sources of developmental change in children's word-problem performance. *Journal of Educational Psychology, 91,* 660-668.

Kane, M.J., Hambrick, D.Z., Tuholski, S.W., Wilhelm, O., Payne, T.W., & Engle, R.W. (2004). The generality of working memory capacity: A latent variable approach to verbal and visuo-spatial memory span and reasoning. *Journal of Experimental Psychology, General, 133,* 189-217.

Kaye, D.B. (1986). The development of mathematical cognition. *Cognitive Development, 1,* 157-170.

Kaye, D.B., de Winstanley, P., Chen, Q., & Bonnefil, V. (1989). Development of efficient arithmetic computation. *Journal of Educational Psychology, 81*, 467-480.

Koehler, M.S., & Grouws, D.A. (1992). Mathematics teaching practices and their effects. In D.A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning,* pp.115-126. New York: Macmillan.

Kuder, G.F., & Richardson, M.W. (1937). The theory of the estimation of test reliability. *Psychometrika, 2*, 151-160.

Kyllonen, P.C., & Christal, R.E. (1990). Reasoning ability is (little more than) working memory capacity?! *Intelligence, 14*, 389-433.

Lamont, S.J. (1996). The development of unitizing: Its role in children's partitioning strategies. *Journal for Research in Mathematics Education, 27*, 170-193.

Law, D.J., Morrin, K.A., & Pellegrino, J.W. (1995). Training effects and working memory contribution to skill acquisition in a complex coordination task. *Learning and Individual Differences, 7,* 207-234.

Leather, C.V., & Henry, L. (1994). Working memory span and phonological awareness tasks as predictors of early reading ability. *Journal of Experimental Child Psychology, 58,* 88-111.

Lee, K., & Kang, S. (2002). Arithmetic operation and working memory: Differential suppression in dual tasks. *Cognition, 83,* 63-68.

LeFevre, J., Lei, Q., Smith-Chant, B., & Mullins, D. (2001). Multiplication by eye and by ear for Chinese-speaking and English-speaking adults. *Canadian Journal of Experimental Psychology, 55,* 277-284.

LeFevre, J., & Lui, J. (1997). Numerical cognition: Single-digit multiplication skills of adults from China and Canada. *Mathematical Cognition, 3,* 31-62.

LeFevre, J., Sadesky, G.S., & Bisanz, J. (1996). Selection of procedures in mental addition: Reassessing the problem-size effect in adults. *Journal of Experimental Psychology, Learning, Memory and Cognition, 22,* 216-230.

Lehto, J. (1995). Working memory and school achievement in ninth form. *Educational Psychology, 15,* 271-281.

Lehto, J. (1996). Are executive function tests dependent on working memory capacity? *Quarterly Journal of Experimental Psychology, 49A,* 29-50.

Lehto, J., Juujarvi, P., Koistra, L., & Pulkkinen, L. (2003). Dimensions of executive functioning: Evidence from children. *British Journal of Developmental Psychology, 21,* 59-80.

Lemaire, P., Abdi, H., & Fayol, M. (1996). The role of working memory resources in simple cognitive arithmetic. *European Journal of Cognitive Psychology, 8*, 73-103.

Lemaire, P., & Siegler, R.S. (1995). Four aspects of strategic change: Contributions to children's learning of multiplication. *Journal of Experimental Psychology, General, 124,* 83-97.

Loehlin, J.C. (1987). Latent variable models: An introduction to factor, path and structural analysis. Hillsdale, New Jersey; London: Erlbaum.

Logie, R.H. (1986). Visuo-spatial processing in working memory. *The Quarterly Journal of Experimental Psychology, 38A,* 229-247.

Logie, R.H. (1995). *Visuo-spatial working memory.* Hove, UK: Lawrence Erlbaum Associates, Ltd.

Logie, R.H., & Baddeley, A.D. (1987). Cognitive processes in counting. *Journal of Experimental Psychology, Learning, Memory and Cognition, 13*, 310-326.

Logie, R.H., Gilhooly, K.J., and Wynn, V. (1994) Counting on working memory in arithmetic problem solving. *Memory and Cognition, 22*, 395-410.

Logie, R.H., & Marchetti, C. (1991). Visuo-spatial working memory: Visual, spatial or central executive? In R.H. Logie and M. Denis (Eds.) *Mental images in human cognition,* pp.105-115. Amsterdam: North Holland Press.

Logie, R.H., & Pearson, D.G. (1997). The inner eye and the inner scribe of visuo-spatial working memory: Evidence from developmental fractionation. *European Journal of Cognitive Psychology, 9,* 241-257.

Logie, R.H., Zucco, G., & Baddeley, A.D. (1990). Interference with visual short-term memory. *Acta Psychologia, 75,* 55-74.

Lowe, C., & Rabbitt, P. (1997). Cognitive models of aging and frontal lobe deficits. In P. Rabbitt (Ed.), *Methodology of frontal and executive functions*, pp. 39-59. Hove, UK: Psychology Press.

Lummis, M., & Stevenson, H.W. (1990). Gender differences in beliefs and achievement. A cross-cultural study. *Developmental Psychology, 26*, 254-263.

Luria, A.R. (1966). *Higher cortical functions in man.* New York: Basic Books.

Luzzatti, C., Vecchi, T., Agazzi, D., Cesa-Bianchi, M., & Vergani, C.A. (1998). Neurological dissociation between preserved visual and impaired spatial processing in mental imagery. *Cortex, 34,* 461-469.

Maybery, M.T., & Do, N. (2003). Relationships between facets of working memory and performance on a curriculum-based mathematics test in children. *Educational and Child Psychology*, *20,* 77-92.

Mayer, R.E. (1982). Different problem-solving strategies for algebra and word equation problems. *Journal of Experimental Psychology, Learning, Memory and Cognition, 8*, 448-462.

McCloskey, M. (1992). Cognitive mechanisms in numerical processing: Evidence from acquired dyscalculia. *Cognition, 44,* 107-157.

McCloskey, M., Caramazza, A., & Basili, A. (1985). Cognitive mechanism in number processing and calculation: Evidence from dyscalculia. *Brain and Cognition, 4,* 171-196.

McCloskey, M., & Maracuso, P. (1995). Representing and using numerical information. *American Psychologist, 50,* 351-363.

McComb, K., Packer, C., & Pusey, A. (1994). Roaring and numerical assessments in contests between groups of female lions, panthera leo. *Animal Behavior, 47*, 379-387.

McGarrigle, J., & Donaldson, M. (1974). Conservation accidents. *Cognition, 3,* 341-350.

McKenzie, B., Bull, R., & Gray, C. (2003). The effects of phonological and visual-spatial interference on children's arithmetical performance. *Educational and Child Psychology, 20,* 93-108.

McLean, J.F., & Hitch, G.J. (1999). Working memory impairments in children with specific arithmetic learning difficulties. *Journal of Experimental Child Psychology, 74,* 240-260.

Meck, W.H., & Church, R.M. (1983). A mode control model of counting and timing processes. *Journal of Experimental Psychology, Animal Behavior Processes, 9,* 320-334.

Michas, I.C., & Henry, L.A. (1994). The link between phonological memory and vocabulary acquisition. *British Journal of Developmental Psychology, 12,* 147-164.

Miller, G.A. (1956). The magical number seven, plus or minus two: Some limits on our capacity for processing information. *Psychological Review, 63,* 81-97.

Milner, B. (1971). Interhemispheric differences and psychological processes. *British Medical Bulletin, 27,* 272-277.

Miyake, A., Friedman, N.P., Shah, P., Rettinger, D.A., & Hegarty, M. (2001). How are visuo-spatial working memory, executive functioning, and spatial abilities related? A latent-variable analysis. *Journal of Experimental Psychology, General, 130,* 621-640.

Miyake, A., Friedman, N.P., Emerson, M.J., Witzki, A.H., Howerter, A., & Wager, T.D. (2000). The unity and diversity of executive functions and their contributions to complex "frontal lobe" tasks: A latent variable analysis. *Cognitive Psychology, 41,* 49-100.

Miyake, A., & Shah, P. (Eds.) (1999). *Models of working memory: Mechanisms of active maintenance and executive control.* New York: Cambridge University Press.

Montgomery, J. (2000). Relation of working memory to off-line and real-time sentence processing in children with specific language impairment. *Applied Psycholinguistics, 21,* 117-148.

Moyer, R.S., & Landauer, T.K. (1967). Time required for judgements of numerical inequality. *Nature, 215,* 1519-1520.

Mulligan, J.T., & Mitchelmore, M.C. (1997). Young children's intuitive models of multiplication and division. *Journal for Research in Mathematics Education, 28,* 309-330.

Muncer, S.J., Craigie, M., & Holmes, J. (2003). Some suggestions for the use of power in research synthesis. *Understanding Statistics, 2,* 1-12.

Munn, P. (2004). The psychology of maths education in the early primary years. *The Psychology of Education Review, 28,* 4-8.

Naglieri, J. A. (1985). *Matrix Analogies Test - Short Form.* San Antonio: The Psychological Corporation.

Nairne, J.S., & Healy, A.F. (1983). Counting backwards produces systematic errors. *Journal of Experimental Psychology, General, 112,* 37-40.

National Curriculum (n.d.). Retrieved on 8th December, 2001 from http://www.nc.uk. net/webdav/servlet/XRM?Page/@id=6004&Subject/@id=22&Session/@id=D _yis3e4CTrLs7ag596PwI

Noël, M., Desert, M., Auburn, A., & Seron, X. (2001). Involvement of short-term memory in complex mental calculation. *Memory and Cognition, 29,* 34-42.

Noël, M., Fias, W., & Brysbaert, M. (1997). About the influence of the presentation format on arithmetic-fact retrieval processes. *Cognition, 63*, 335-374.

Norman, D.A., & Shallice, T. (1980). Attention into action. Willed and automatic control of behaviour. *University of California San Diego CHIP Report 99.*

O'Reilly, R.C., Braver, T.S., & Cohen, J.D. (1999). A biologically based computational model of working memory. In A. Miyake, & P. Shah (Eds.) (1999). *Models of working memory: Mechanisms of active maintenance and executive control.* New York: Cambridge University Press.

Palmer, S. (2000). Working Memory: A developmental study of phonological recoding. *Memory, 8,* 179-193.

Papagno, C., & Vallar, G. (1992). Phonological short-term memory and the learning of novel words: the effect of phonological similarity and item length. *Quarterly Journal of Experimental Psychology, 44A,* 47-67.

Papagno, C., Valentine, T., & Baddeley, A.D. (1991). Phonological short-term memory and foreign language vocabulary learning. *Journal of Memory and Language, 30,* 331-347.

Pascual-Leone, J. (1970). A mathematical model for the transition rule in Piaget's development stages. *Acta Psychologica, 32,* 301-345.

Passolunghi, M.C., & Siegel, L.S. (2001). Short-term memory, working memory and inhibitory control in children with difficulties in arithmetic problem solving. *Journal of Experimental Child Psychology, 80*, 44-57.

Pegg, J. (2002). Assessment in mathematics: A developmental approach. In J.M. Royer (Ed.), *Mathematical Cognition*, pp.227-259. Greenwich, Connecticut: Information Age Publishing.

Pesenti, M., Zago, L., Crivello, F., Mellet, E., Samson, D., Duroux, B., Seron. X., Mazoyer, B., & Tzourio-Mazoyer, N. (2001). Mental calculation in a prodigy is sustained by right prefrontal and medial temporal areas. *Nature Neuroscience, 4*, 1-6.

Peterson, L.R., & Johnson, S.T. (1971). Some effects of minimizing articulation on short-term retention. Journal *of Verbal Learning and Verbal Behavior, 10,* 346-354.

Peterson, L.R., & Peterson, M.J. (1959). Short-term retention of individual verbal items. *Journal of Experimental Psychology, 53,* 193-198.

Phillips, W.A., & Christie, D.F.M. (1977a). Components of visual memory. *Quarterly Journal of Experimental Psychology, 29,* 177-133.

Phillips, W.A., & Christie, D.F.M. (1977b). Interference with visualization. *Quarterly Journal of Experimental Psychology, 29,* 637-650.

Phillips, L.H., & Hamilton, C. (2001). The working memory model in adult aging research. In J. Andrade (Ed.) *Working Memory in Perspective*. Hove, UK: Psychology Press.

Piaget, J. (1952). *The origins of intelligence in children* (translated by M Cook). New York: International Universities Press.

Pickering, S.J., & Gathercole, S.E. (2001). *Working memory test battery for children (WMTB-C).* London: Psychological Corporation.

Pickering, S.J., Gathercole, S.E., Hall, M., & Lloyd, S.A. (2001). Development of memory for pattern and path: Further evidence for the fractionation of visual and spatial short-term memory. *Quarterly Journal of Experimental Psychology, 54A,* 397-420.

Pickering, S.J., Gathercole, S.E., & Peaker, M. (1998). Verbal and visuo-spatial short-term memory in children. Evidence for common and distinct mechanisms. *Memory and Cognition, 26,* 1117-1130.

Quinn, J.G., & McConnell, J. (1994). The irrelevant picture effect in the visuo-spatial sketchpad. Paper presented at the International Conference on Working Memory, Cambridge, U.K.

Quinn, J.G., & McConnell, J. (1996). Irrelevant pictures in visual working memory. *Quarterly Journal of Experimental Psychology, 49A,* 200-215.

Quinn, J.G., & McConnell, J. (1999). Manipulation of interference in the passive visual store. *European Journal of Cognitive Psychology, 11,* 313-389.

Raven, J. (1956). *Manual for Raven's progressive matrices and vocabulary scales.* Oxford: Oxford Psychologists Press.

Reisberg, D., & Logie, R.H. (1993). The in's and out's of visual working memory: Overcoming the limits on learning from imagery. In M. Intons-Peterson, B. Roskos-Ewdolson and R. Anderson (Eds.), *Imagery, creativity, and discovery: A cognitive approach.* Amsterdam: Elsevier.

Reuhkala. M. (2001). Mathematical skills in ninth-graders: Relationship with visuo-spatial abilities and working memory. *Educational Psychology, 21,* 387-399.

Riding, R.J. (2002). *School learning and cognitive style.* London: David Fulton Publishers.

Riding, R.J., & Anstey, L. (1982). Verbal-imagery learning style and reading attainment in eight-year-old children. *Journal of Research in Reading, 5,* 57-66.

Riding, R.J., & Calvey, I. (1981). The assessment of verbal-imagery learning styles and their effect on the recall of concrete and abstract prose passages by eleven year old children. *British Journal of Psychology, 72*, 59-64.

Riding, R.J., & Dyer, V.A. (1980). The relationship between extraversion and verbal-imagery learning style in twelve-year-old children. *Personality and Individual Differences, 1*, 273-279.

Riding, R.J., Grimley, M., Dahraei, H., & Banner, G. (2003). Cognitive style, working memory and learning behaviour in school subjects. *British Journal of Educational Psychology, 73*, 149-169.

Riding R.J., & Mathias, D. (1991). Cognitive styles and preferred learning mode, reading attainment and cognitive ability in eleven year old children. *Educational Psychology, 11*, 383-393.

Riding, R.J., & Taylor, E.M. (1976). Imagery performance and prose comprehension in seven-year-old children. *Educational Studies, 2*, 21-7.

Roodenrys, S., Hulme, C., & Brown, G. (1993). The development of short-term memory span: Separable effects of speech rate and long-term memory. *Journal of Experimental Child Psychology, 56*, 431-442.

Rose, J. (1999). *Weighing the baby: The report of the independent scrutiny panel on the 1999 Key Stage 2 National Curriculum tests in English and Mathematics.* London: DfEE.

Rourke, B.P. (1993). Arithmetic disabilities, specific and otherwise: A neuropsychological perspective. *Journal of Learning Disabilities, 26*, 214-226.

Rourke, B.P., & Conway, J.A. (1997). Disabilities of arithmetic and mathematical reasoning: Perspectives from neurology and neurophysiology. *Journal of Learning Disabilities, 1,* 34-36.

Rourke, B.P., & Finlayson, M.A.J. (1978). Neuropsychological significance of variations in patterns of academic performance: Verbal and visuo-spatial abilities. *Journal of Abnormal Child Psychology, 6,* 121-133.

Russell, J., Jarrold, C., & Henry, L.A. (1996). Working memory in children with autism and moderate learning difficulties. *Journal of Child Psychology and Psychiatry, 37,* 673-686.

Russell, R.L., & Ginsburg, H.P. (1984). Cognitive analysis of children's mathematics difficulties. *Cognition and Instruction, 1,* 217-244.

Salame, P., & Baddeley, A.D. (1982). Disruption of short-term memory by unattended speech: Implications for the structure of working memory. *Journal of Verbal Learning and Verbal Behavior, 21,* 150-164.

Salame, P., & Baddeley, A.D. (1983). Differential effects of noise and speech on short-term memory. In *The proceedings of the Fourth International Congress on Noise as a Public Health Problem, Turin, Italy,* pp. 751-758. Edizioni Techniche a cura del Centro Ricerche e Studi Amplifon.

Salame, P., & Baddeley, A.D. (1987). Noise, unattended speech and short-term memory. *Ergonomics, 30,* 1185-1194.

Salame, P., & Baddeley, A.D. (1989). Effects of background music on phonological short-term memory. *The Quarterly Journal of Experimental Psychology, 41(A),* 107-122.

Salway, A.F.S., & Logie, R.H. (1995). Visuo-spatial working memory, movement control and executive demands. British Journal of Psychology, 86, 253-269.

Seitz, K., & Schumann-Hengsteler, R. (2000). Mental multiplication and working memory. *European Journal of Cognitive Psychology, 12*, 552-570.

Seitz, K., & Schumann-Hengsteler, R. (2002). Phonological loop and central executive processes in mental addition and multiplication. *Psychologische Beiträge, 44*, 275-302.

Sekuler, R., & Mierkiewicz, D. (1977). Children's judgements of numerical inequality. *Child Development, 48,* 630-633.

Seron, X., Pesenti, M., Noel, M.P., Deloche, G., & Cornet, J.A. (1992). Images of numbers, or "When 98 is upper left and 6 sky blue". *Cognition 44,* 159-196.

Service, E. (1992). Phonology, working memory and foreign-language learning. *Quarterly Journal of Experimental Psychology, 45A,* 21-50.

Seyler, D.J., Kirk, E.P., & Ashcraft, M.H. (2003). Elementary subtraction. *Journal of Experimental Psychology: Learning, Memory and Cognition, 29*, 1339-1352.

Shah, P., & Miyake, A. (1996). The separability of working memory resources for spatial thinking and language processing: An individual differences approach. *Journal of Experimental Psychology, General, 125*, 4-27.

Shalev, R.S., & Gross-Tsur, V. (2001). Developmental dyscalculia. *Paediatric Neurology, 24*, 337-342.

Shallice, T. (1982). Specific impairments of planning. *Philosophical Transactions of the Royal Society of London B, 298,* 199-209.

Shallice, T., & Burgess, P.W. (1991). Deficits in strategy application following frontal lobe damage in man. *Brain, 114*, 727-741.

Shorrocks-Taylor, D., Curry, J., Swinnerton, B., & Nelson, N. (2003). National Curriculum tests in England at Key Stage 2: Weights and measures? *Oxford Review of Education, 29,* 51-66.

Shrager, J., & Siegler, R.S. (1998). SCADS: A model of children's strategy choices and strategy discoveries. *Psychological Sciences, 9,* 405-410.

Siegel, L.S. (1994). Working memory and reading: A life-span perspective. *International Journal of Behavioral Development, 17,* 109-124.

Siegel, L.S., & Linder, B.A. (1984). Short-term memory processes in children with reading and learning disabilities. *Developmental Psychology, 20,* 200-207.

Siegel, L.S., & Ryan, E.B. (1989). The development of working memory in normally achieving and subtypes of learning disabled children. *Child Development, 60,* 973-980.

Siegler, R.S. (1987). The perils of averaging data over strategies: An example from children's addition. *Journal of Experimental Psychology, General, 116,* 250-264.

Siegler, R.S. (1988). Strategy choice procedures and the development of multiplication skill. *Journal of Experimental Psychology, General, 117,* 258-275.

Siegler, R.S. (1999). Strategic development. *Trends in Cognitive Sciences, 3,* 430-435.

Siegler, R.S., & Jenkins, E. (1989). *How children discover new strategies.* Hillsdale, New Jersey: Erlbaum.

Siegler, R.S., & Robinson, M. (1982). The development of numerical understandings. In H. Reese and L.P. Lipsett (Eds.), *Advances in child development and behavior, Vol. 16,* pp.241-312. New York: Academic Press.

Siegler, R.S., & Shrager, J. (1984). A model of strategy choice. In C. Sophian (Ed.). *Origins of cognitive skills,* pp.229-293. Hillsdale, New Jersey: Erlbaum.

Simon, H.A. (1974). How big is a chunk? *Science, 183,* 482-488.

Simon, T.J. (1997). Reconceptualizing the origins of number knowledge: a "non-numerical" account. *Cognitive Development, 12,* 349-372.

Simon, T.J. (1998). Computational evidence for the foundations of numerical competence. *Developmental Science, 1,* 71-78.

Simon, T.J. (1999). The foundations of numerical thinking in a brain without numbers. *Trends in Cognitive Sciences, 3,* 363-365.

Sleeman, D. (1984). An attempt to understand students' understanding of basic algebra. *Cognitive Science, 8,* 387-412.

Smith, J.P. (1995). Competent reasoning with rational numbers. *Cognition and Instruction, 13,* 3-50.

Smith, E.E., & Jonides, J. (1997). Working memory: A view from neuroimaging. *Cognitive Psychology, 33,* 5-42.

Smyth, M.M., & Pendleton, L.R. (1989). Working memory for movements. *Quarterly Journal of Experimental Psychology, 41A,* 235-250.

Smyth, M.M. & Scholey, K.A. (1994). Interference in spatial immediate memory. *Memory and Cognition, 22,* 1-13.

Smyth, M.M. & Scholey, K.A. (1996). The relationship between articulation time and memory performance in verbal and visuospatial tasks. *British Journal of Psychology, 87,* 179-191.

Snowling, M., Chiat, S., & Hulme, C. (1991). Words, nonwords and phonological processes: Some comments on Gathercole, Willis, Emslie and Baddeley. *Applied Psycholinguistics, 12,* 369-373.

Steffe, L. (1983). *Children's counting types.* Westport, CT: Greenwood Press.

Strang, J.D., & Rourke, B.P. (1985). Adaptive behavior of children who exhibit specific arithmetic disabilities and associated neuropsychological abilities and deficits. In B.P. Rourke (Ed.), *Neuropsychology of learning disabilities*. New York: The Guilford Press.

Starkey, P., Spelke, E.S., & Gelman, R. (1983). Detection of intermodal numerical correspondence by human infants. *Cognition, 222,* 179-181.

Starkey, P., Spelke, E.S., & Gelman, R. (1990). Numerical abstraction by human infants. *Cognition, 36,* 97-128.

Stazyk, E.H., Ashcraft, M.H., & Hamaan, M.S. (1982). A network approach to multiplication. *Journal of Experimental Psychology, Learning, Memory and Cognition,* 8, 320-335.

Strauss, M.S., & Curtis, L.E. (1981). Infant perception of numerosity. *Child Development, 52,* 1146-1152.

Swanson, H.L. (1994). Short-term and working memory: Do both contribute to our understanding of academic achievement in children and adults with learning disabilities? *Journal of Learning Disabilities, 27*, 34-50.

Temple, C.M. (1991). Procedural dyscalculia and number fact dyscalculia: Double dissociation in developmental dyscalculia. *Cognitive Neuropsychology, 8,* 155-176.

Tizard, B., & Hughes, M. (1984). *Young children learning*. London: Fontana.

Towse, J.N., & Hitch, G.J. (1995). Is there a relationship between task demand and storage in tests of working memory capacity? *Quarterly Journal of Experimental Psychology, 48A,* 108-124.

Towse, J.N., Hitch, G.J., & Hutton, U. (1998). A re-evaluation of working memory capacity in children. Journal *of Memory and Language, 39,* 195-217.

Towse, J.N., Hitch, G.J., & Hutton, U. (2002). On the nature of the relationship between processing activity and item retention in *children. Journal of Experimental Child Psychology, 82,* 156-184.

Towse, J.N., & Houston-Price, C.M.T. (2001). Combining representations in working memory: A brief report. *British Journal of Developmental Psychology, 19,* 319-324.

Trbovich, P.L., & LeFevre, J. (2003). Phonological and visual-spatial working memory in mental addition. *Memory and Cognition, 31,* 738-745.

Treffers, A. (1991). Didactical background of a mathematics programme for primary education. In L. Streefland (Ed.), *Realistic mathematics education in primary school.* Utrecht: ß Press.

Tresch, M.C., Sinnamon, H.M., & Seamon, J.G. (1993). Double dissociation of spatial and object visual memory: Evidence from selective interference in intact human subjects. *Neuropsychologia, 31,* 211-219.

Turner, M.L., & Engle, R.W. (1989). Is working memory capacity task dependent? *Journal of Memory and Language, 28,* 127-154.

Vallar, G., Betta, A.M., & Silveri, M.C. (1997). The phonological short-term store rehearsal system: Patterns of impairment and neural correlates. *Neuropsychologia, 35,* 975-812.

Vallar, G., & Papagno, C. (2002). Neuropsychological impairments of verbal short-term memory. In A.D. Baddeley, B. Wilson, & M. Kopelman (Eds.), *Handbook of Memory Disorders (2^{nd} Ed.)* Chichester, UK: Wiley.

van Oeffelen, M.P., & Vos, P.G. (1982). A probabilistic model for the discrimination of visual number. *Perception and Psychophysiology, 32,* 163-170.

Vecchi, T. (1998). Visuo-spatial imagery in congenitally totally blind people. *Memory, 6,* 91-102.

Vecchi, T., & Cornoldi, C. (1999). Passive and active manipulation in visuo-spatial working memory: Further evidence from the study of age differences. *European Journal of Cognitive Psychology, 11,* 391-406.

Vecchi, T., & Girelli, L. (1998). Gender differences in visuo-spatial processing: The importance of distinguishing between passive storage and active manipulation. *Acta Psychologia, 99,* 1-16.

Vecchi, T., Monticellai, M.L., & Cornoldi, C. (1995). Visuo-spatial variables affecting a capacity measure. *Neuropsychologia, 33,* 1549-1564.

Von Aster, M. (2000). Developmental cognitive neuropsychology of number processing and calculation: Varieties of developmental dyscalculia. *European Child and Adolescent Psychiatry, 9,* 41-57.

Wagner, M.C., & Walters, J.A. (1982). A longitudinal analysis of number concepts: from numbers to number. In G. Foreman (Ed.), *Action and thought.* New York: Academic.

Wakeley, A., Rivera, S., & Langer, J. (2000a). Can young infants add and subtract? *Child Development, 71,* 1525-1534.

Wakeley, A., Rivera, S., & Langer, J. (2000b). Not proved: Reply to Wynn. *Child Development, 71,* 1537-1539.

Warrington, E. K. (1982). The fractionation of arithmetic skills A single case study. *Quarterly Journal of Experimental Psychology, 34A,* 31-51.

Webber, L.J., & Feeney, A.F. (2003). How people represent and reason from graphs. In *Proceedings of the 25th Annual Conference of the Cognitive Society.* Mahwah, New Jersey: LEA.

Whang, P.A., & Hancock, G.R. (1997). Modelling the mathematics achievement of Asian-American elementary students. *Learning and Individual Differences, 4,* 167-213.

White, J.L., Moffitt, T.E., & Silva, P.A. (1992). Neuropsychological and socio-emotional correlates of specific-arithmetic disability. *Archives of Clinical Neuropsychology, 7,* 1-16.

Widaman, K.F., Little, T.D., Geary, D.C., & Cormier, P. (1992). Individual differences in the development of skill in mental addition: Internal and external validation of chronometric models. *Learning and Individual Differences, 16,* 83-97.

Willey, R. (2004). The psychology of maths education in the early primary years: A response to Munn. *The Psychology of Education Review, 28*, 20-22.

Wilson, J.T.L., Brodie, E.E., Reinink, E., Weidman, K.D., & Brooks, D.N. (1968). *Memory for pattern and path in senile dementia.* Paper presented at the 16th Annual Meeting of International Neuropsychological Society, New Orleans, USA.

Wilson, F.A.W., O'Scalaidhe, S.P., & Goldman-Rakic, P.S. (1993). Dissociations of object and spatial processing domains in primate prefrontal cortex. *Science, 260,* 1955-1958.

Wilson, J.T.L., Scott, J.H., & Power, K.G. (1987). Developmental differences in the span of visual memory for pattern. *British Journal of Developmental Psychology, 5,* 249-255.

Wilson, K.M., & Swanson, H.L., (2001). Are mathematics disabilities due to a domain-general or a domain-specific working memory deficit? *Journal of Learning Disabilities, 34*, 237-248.

Wright, R., Martland, J., & Stafford, A. (2000). *Early numeracy: Assessment for teaching and intervention.* London: Paul Chapman Publishing.

Wynn, K. (1992). Addition and subtraction by human infants. *Nature, 358*, 749-750.

Wynn, K. (2000). Findings of addition and subtraction in infants are robust and consistent: Reply to Wakeley, Rivera and Langer. *Child Development, 71*, 1535-1536.

Yuill, N., Oakhill., J., & Parkin, A. (1989). Working memory, comprehension ability and the resolution of text anomaly. *British Journal of Psychology, 80*, 351-361.

Yule, W., Rutter, M., Berger, M., & Thompson, J. (1974). Over and under achievement in reading: distribution in the general population. *British Journal of Educational Psychology, 44*, 1-12.

Zago, L., Pesenti, M., Mellet, E., Crivello, F., Mazoyer, B., Tzourio-Mazoyer, N. (2001). Neural correlates of simple and complex mental calculation. *NeuroImage, 13*, 314-327.

Zago, L., & Tzourio-Mazoyer, N. (2002). Distinguishing visuo-spatial working memory and complex mental calculation areas within the parietal lobes. *NeuroScience Letters, 331*, 45-49.

List of Appendices

Appendix I

Year 3 Mathematics Assessment

Year Three

NO CALCULATOR ALLOWED

Section A

Number and Algebra

You should spend approximately 10 minutes on this section.

1. Write in the missing numbers.

a) 42 + ☐ = 73

b) 6 x 2 - ☐ = 10

c) 9 x 5 = ☐

2. Put these numbers in order with the **biggest** first.

410 267 384

543 621

☐ ☐ ☐ ☐ ☐

3. Sarah goes to the shop. She has £2.00. She spends £1.20 on a book.

How much money has she got left from the £2.00?

4. Finish these sentences.

a) Lewis held his breath for 24 seconds, which is [ ] seconds rounded to the nearest 10 seconds.

b) Michael opened a book on page 87, which is page [ ] rounded to the nearest 10 pages.

c) Anita brushes her teeth in 12 seconds, which is [ ] seconds rounded to the nearest 10 seconds.

5. Calculate

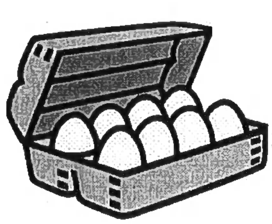
a) 27 ÷ 3 =

b) 16 ÷ 4 =

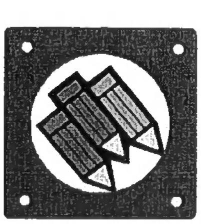
6. Answer the following questions.

a) One cat has four legs. How many legs would eight cats have?

b) One egg box holds six eggs. How many eggs would there be in four egg boxes?



c) A packet of pencils has eight pencils in. How many would there be in three packets?



7. Fill in the gaps in these number sequences.

a) −1 [ ] 1 2 [ ] 4

b) 8 10 [ ] 14 [ ] 18

Year Three

NO CALCULATOR ALLOWED

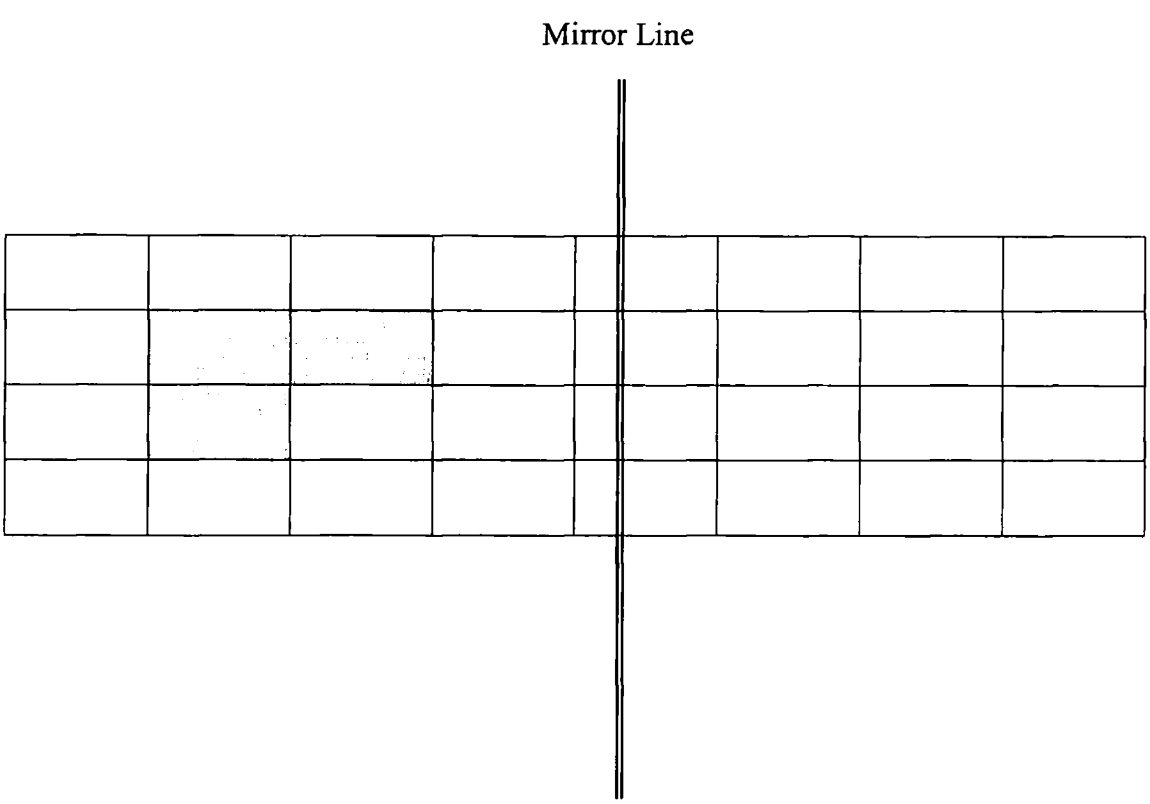
Section B

Shape, Space and Measures

You should spend approximately 10 minutes on this section.

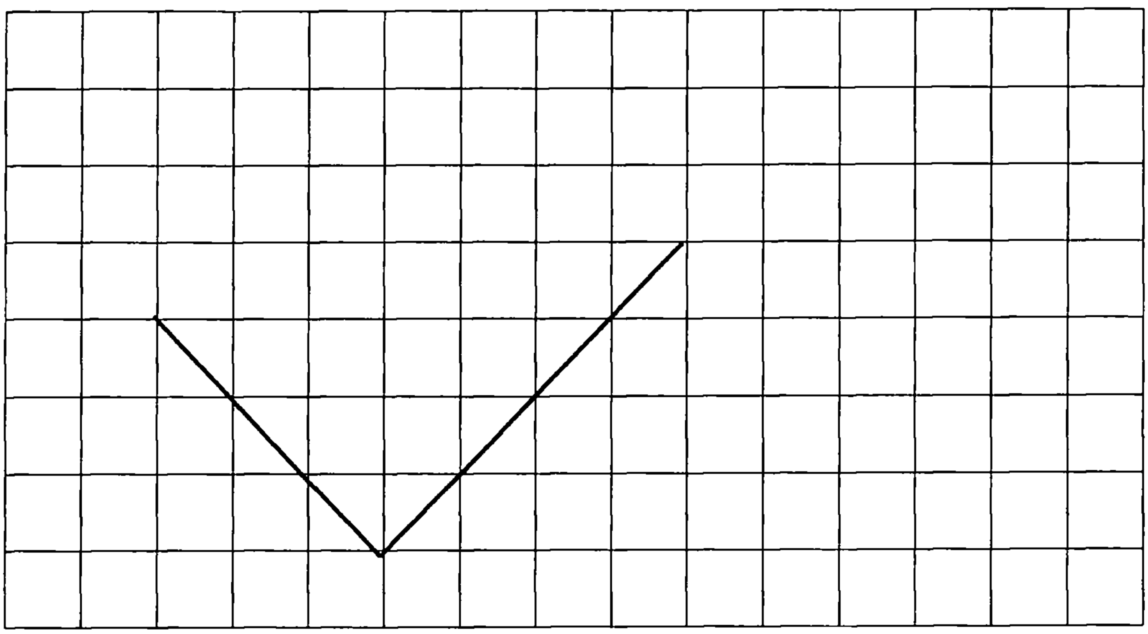
1. Draw the reflection of the shaded shape on the other side of the mirror line.

You may use a mirror or tracing paper.



2. Draw two more straight lines to make a rectangle.

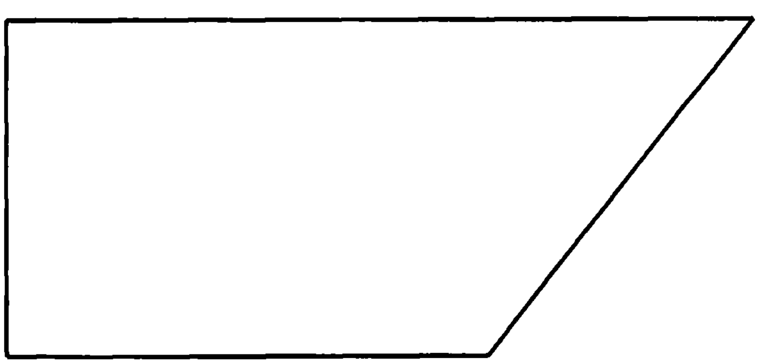
Use a ruler.



3.

N

P



a) Measure accurately the longest side of this shape. This is side N.

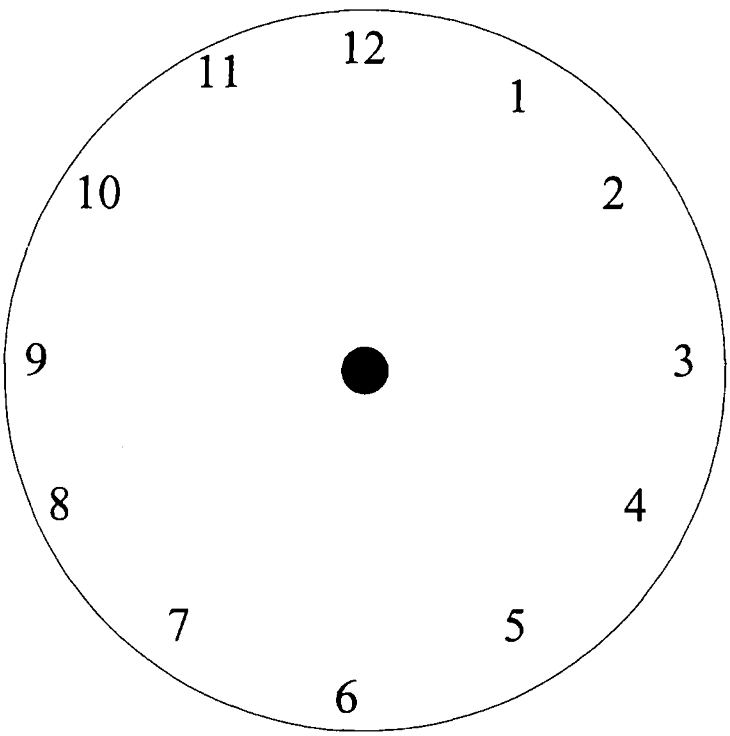
Give your answer in millimetres.

b) Measure accurately the shortest side of this shape. This is side P.

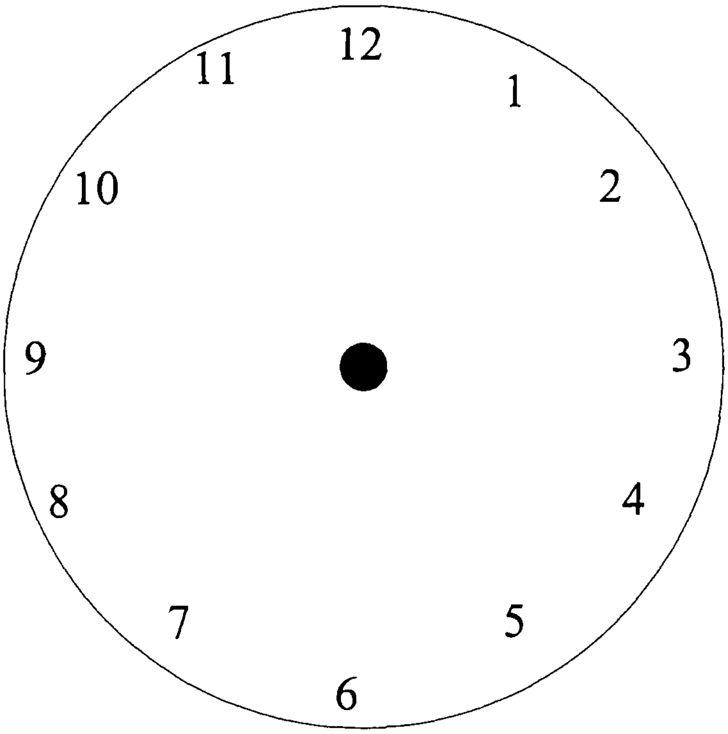
Give your answer in millimetres. [ ]

4. Draw the correct time on the clocks.

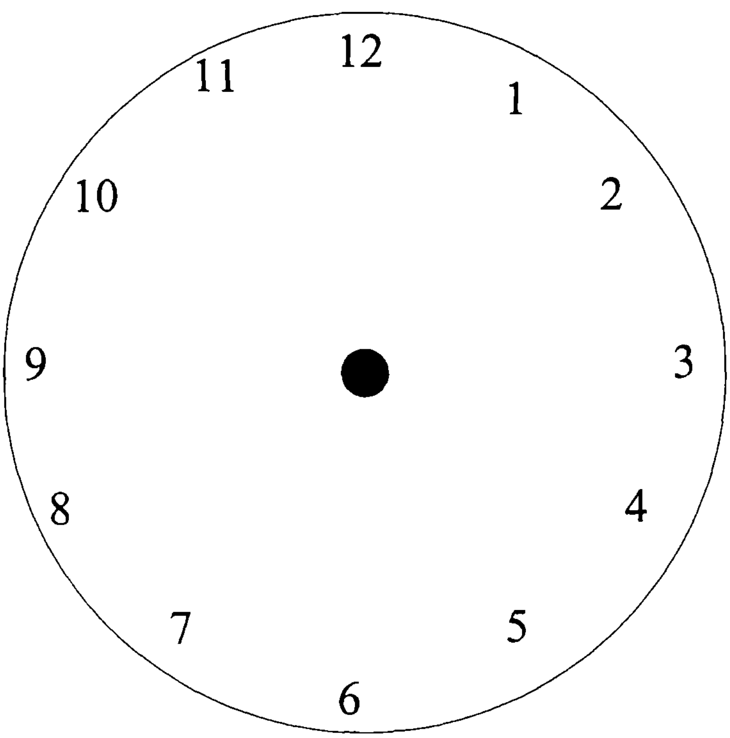
a) 25 minutes past 7



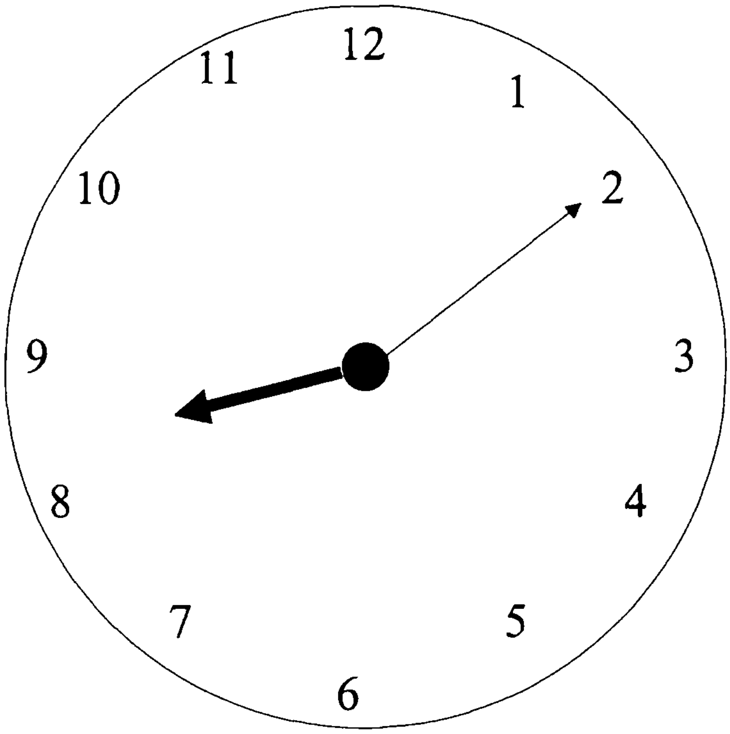
b) 35 minutes past 3



c) 5 minutes past 2



5. Look at the clock below and answer the questions.

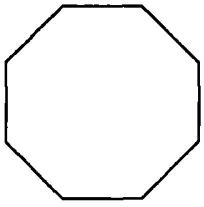


a) What will the time be in 10 minutes?

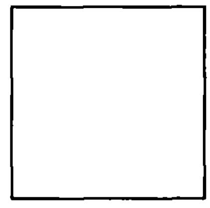
b) What will the time be in half an hour?

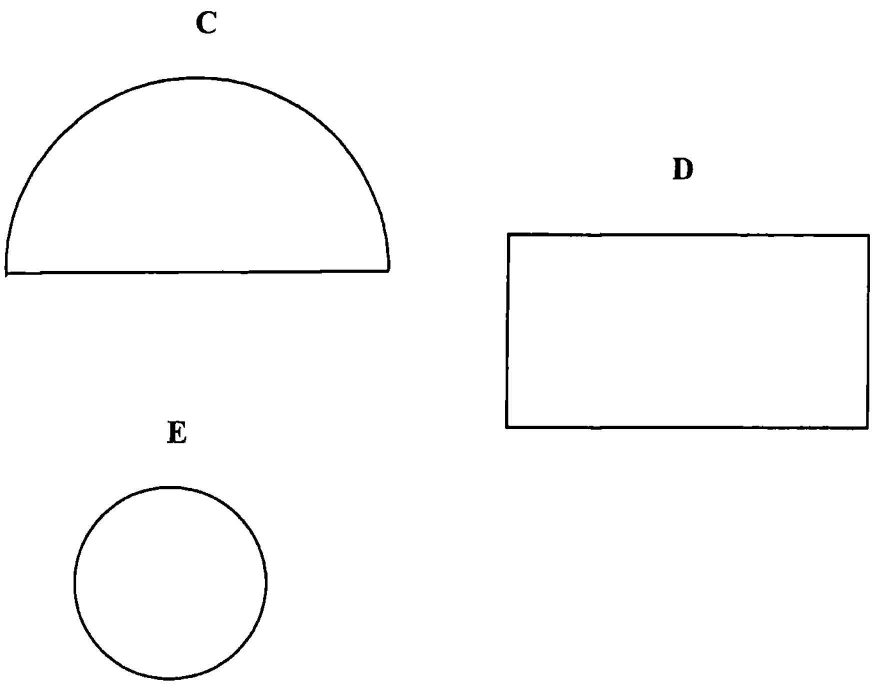
6. Here are 5 shapes.

A



B





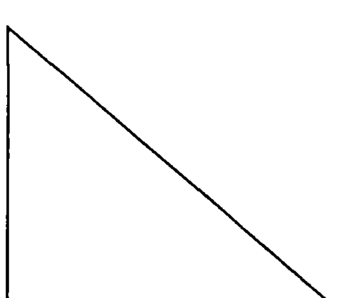
a) Which shapes have four sides?

b) Which shape is a semi circle?

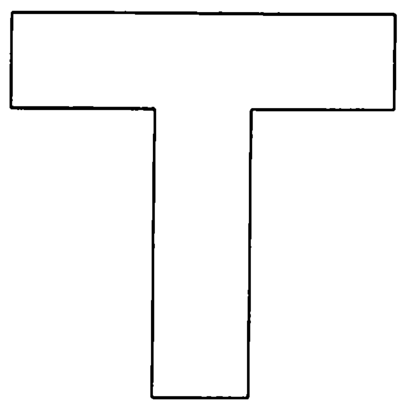
c) Which shape is an octagon?

7. Mark the right angles in these shapes.

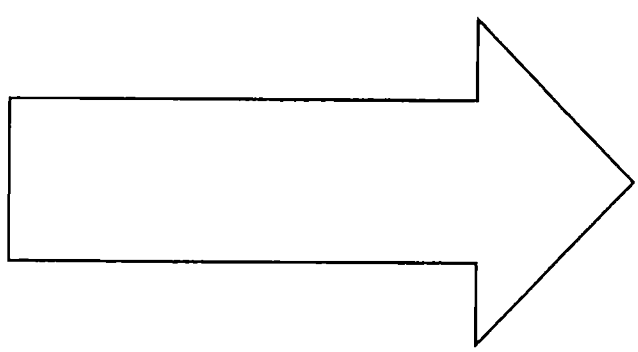
a)



b)



c)



Year Three

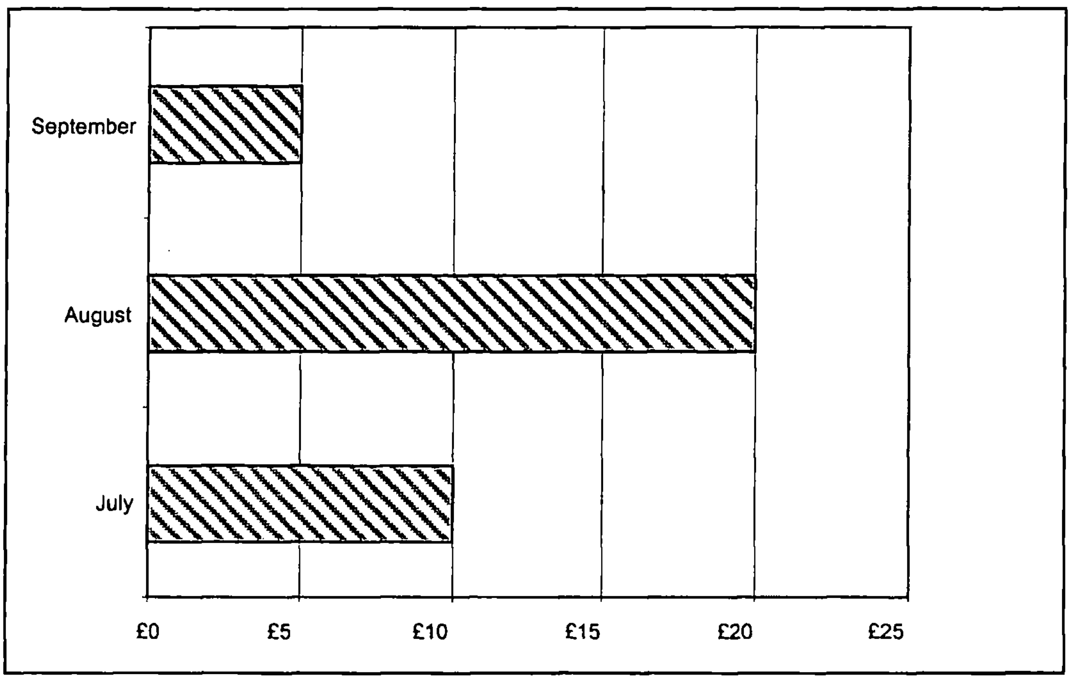
NO CALCULATOR ALLOWED

Section C

Handling Data

You should spend approximately 10 minutes on this section.

1. This chart show the amount of money Jane spent in a toy shop in three months.



a) How much money did Jane spend in August?

b) How much more money did she spend in July than September?

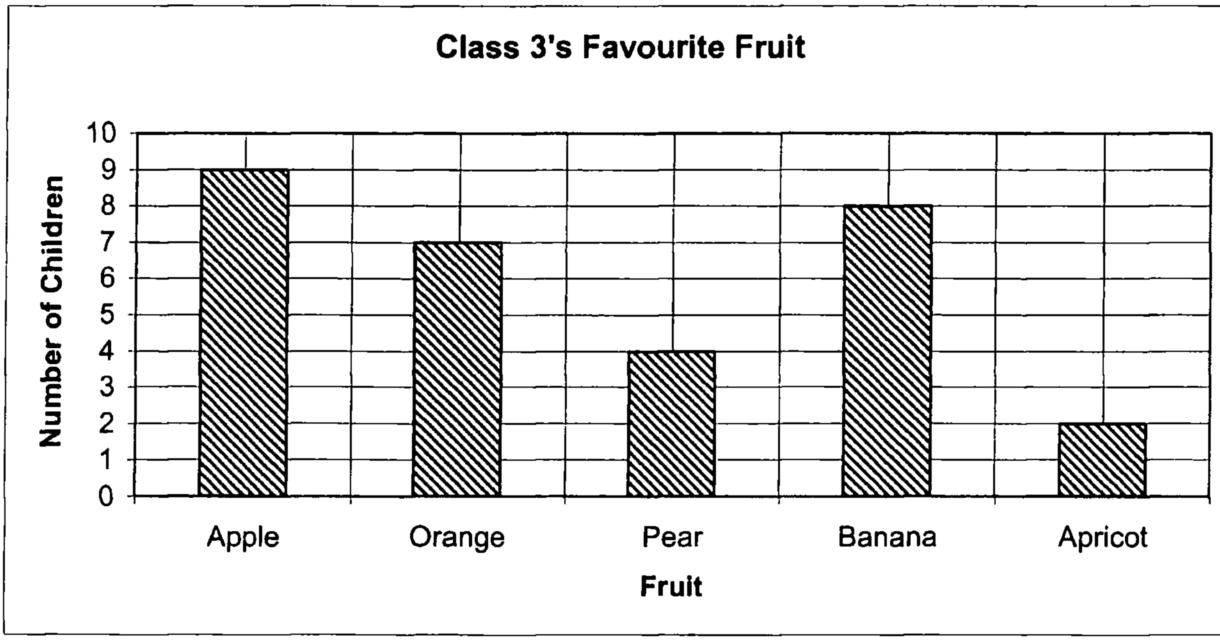
2. This chart shows some children's favourite sports.

	Lynn	Matthew	Nicola	James	Sue
Football		√		√	
Netball	√		√		
Tennis			√		
Hockey	√		√		√
Cricket				√	

a) Whose favourite sport is tennis? [ ]

b) How many children play more than one sport? [ ]

3. Answer the questions by looking at the information that the bar chart provides.



a) How many children chose pears?

b) How many children chose oranges and pears?

c) What is the most popular fruit?

d) How many children are there in Class 3 altogether?

4.

Boat Hire

Rowing Boat	Motor Boat
£2 for 1 hour	£1.50 for 10 minutes

a) How much does it cost to hire a rowing boat for 2 hours?

b) Which boat is more expensive to hire?

5. Below is a table of the 1st and 2nd innings cricket scores of some children.

Name	1st	2nd	Total
Andrew	33	20	53
Katie	20	22	44
Aman	41	46	87
Emma	34	31	65
Ian	60	53	113
Sarah	12	27	39

a) Which child has the highest total?

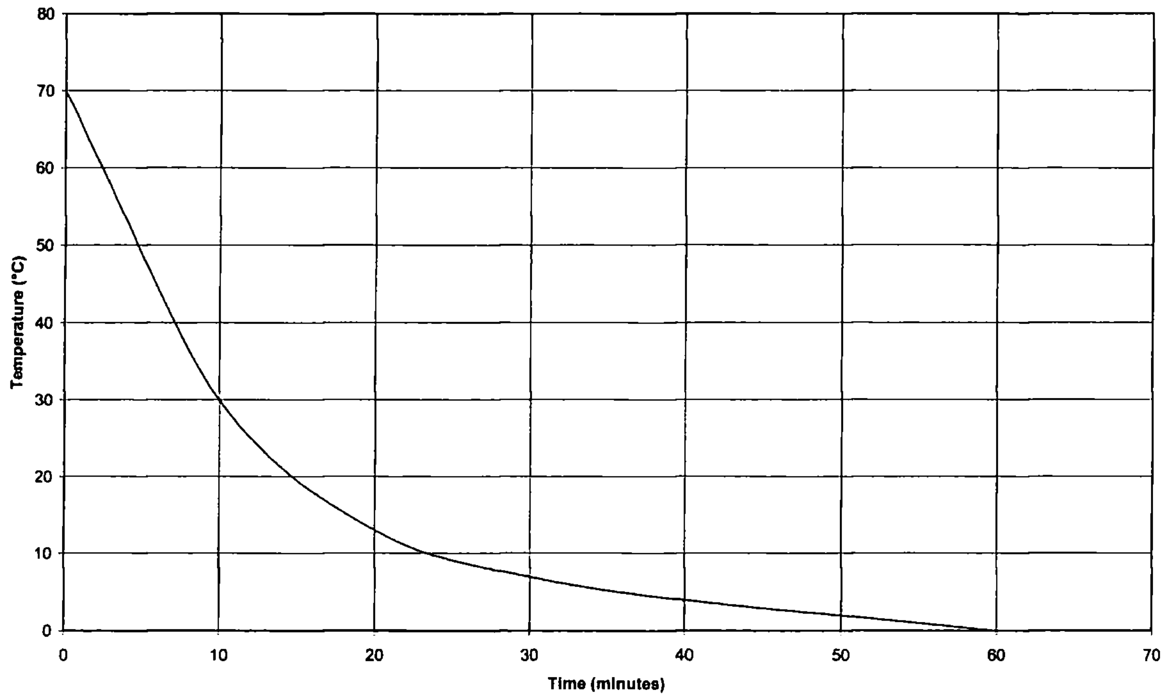
b) Which child has the lowest total?

c) What is Emma's total score?

6. Answer the questions by looking at the information that the graph provides.

A hot liquid is left to cool in a science experiment.

This graph shows how the temperature of the liquid changes as it cools.

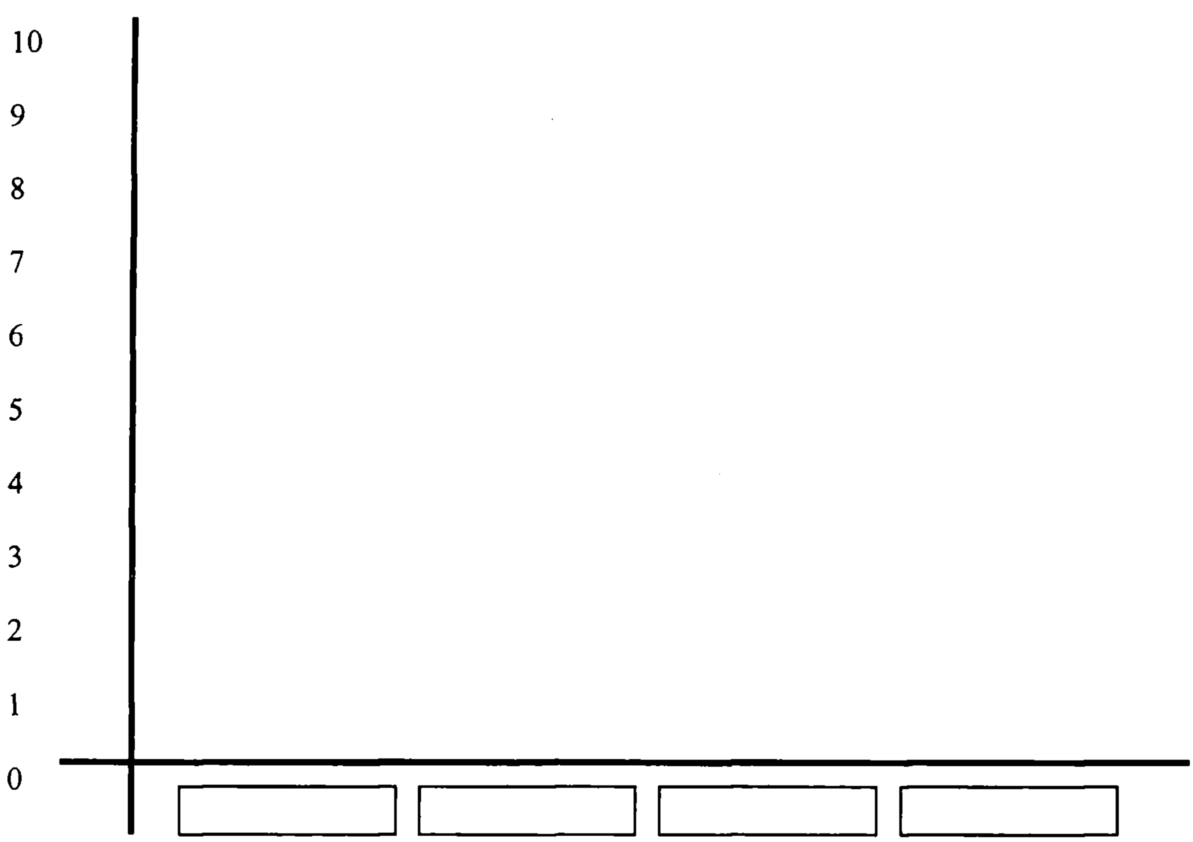


Read from the graph how many minutes it takes for the temperature to reach 30° ☐

7. Class 3 did a survey on the musical instruments that they play.

Draw the information below on the bar chart.

Instrument	Number of Children
Drums	8
Recorder	9
Keyboard	5
Trumpet	7



Year Three

Mental Arithmetic Test

TEST QUESTIONS

For this group you have 5 seconds to work out each answer and write it down.

1. How many £s is 300p?

2. Multiply 4 by 5.

3. Divide 440 by 10.

For this group you have 10 seconds to work out each answer and write it down.

4. What is a quarter of eight?

5. What is 100 take away 60?

6. My watch shows 2.20pm. What will the time show in half an hour?

For this group you have 15 seconds to work out each answer and write it down.

7. Add together 10 and 15 and 20.

8. Look at your answer sheet. Put a ring around the smallest number.

322 232 333 323 222 223

9. What is 88 take away 42?

10. Which sum has the largest total 5 x 5 or 3 x 9 ?

Put down your pen / pencil. The test is now finished.

Year 3
Criteria for Scoring Items

All correct questions are scored as 1, all incorrect as 0.

Section	Question	Correct or Accepted and score as correct
Number and Algebra	1a	31
	1b	2
	1c	45
	2	621, 543, 410, 384, 267
	3	80p or 80 or 80 pence
	4a	20
	4b	90
	4c	10
	5a	9
	5b	4
	6a	32 or 32 legs
	6b	24 or 24 eggs
	6c	24 or 24 pencils
	7a	0, 3
	7b	12, 16
Shape, Space and Measures	1	If exact reflection is depicted through shading or crosses in boxes
	2	Shape is an exact rectangle or resembles a rectangle (i.e. wiggly lines are accepted!)
	3a	77 to 83 mm is acceptable
	3b	37 to 43 mm is acceptable
	4a	Clock must show small hand to the 7 and big hand to the 25 (allow for degree of error, but score incorrect if hands point to wrong numbers)
	4b	Clock must show small hand to the 3 and big hand to the 35 (allow for degree of error, but score incorrect if hands point to wrong numbers)
	4c	Clock must show small hand to the 2 and big hand to the 5 (allow for degree of error, but score incorrect if hands point to wrong numbers)
	5a	8.20 or 20.20 or twenty minutes past eight (pm, am or neither is acceptable)
	5b	8.40 or 20.40 or twenty to nine (pm, am or neither is acceptable)
	6a	B and D (both must be given)
	6b	C
	6c	A
	7a	One right angle should be marked
	7b	Six right angles should be marked

	7c	3 right angles should be marked
Section	Question	Correct or Accepted and score as correct
Handling Data	1a	20 or £20 or 20 pounds
	1b	5 or £5 or 5 pounds
	2a	Nicola
	2b	3 or 3 children or Lynn, Nicola and James
	3a	4 or four (children)
	3b	11 or eleven (children)
	3c	Apple
	3d	30 or thirty (children)
	4a	£4 or four pounds
	4b	Motor or motor boat
	5a	Ian
	5b	Sarah
	5c	65
	6	10 or ten (minutes)
	7	Accept any pictorial representation of the correct information (bar or line chart or pictures representing instruments with correct number of pictures etc)
Mental Arithmetic	1	3 or £3 or three
	2	20
	3	44
	4	2
	5	40
	6	2.50pm (accept if pm not written)
	7	45
	8	222
	9	46
	10	3x9

Appendix II

Year 5 Mathematics Assessment

Year Five

NO CALCULATOR ALLOWED

Section A

Number and Algebra

You should spend approximately 10 minutes on this section.

1. Write in the missing numbers.

a) 35 + ☐ = 110

b) (6 x 3) - ☐ = 12

c) 40 x 3 = ☐

2. Place these numbers in order with the largest first.

0 -1 7

-5 3

☐ ☐ ☐ ☐ ☐

3. Andrea went to the cinema. She bought cinema tickets for herself and six of her friends. In total she bought seven tickets for £4.00 each. How much change was given when £30.00 was handed to the attendant?

4. Use the clues to find the numbers.

a) Find a number that is a factor of 16 but that is greater than 4.

b) List the two multiples of 3 between 5 and 10.

c) What is the first prime number after 45?

5. Calculate

a) $140 \div 6 =$

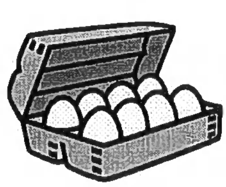
b) $152 \div 8 =$

6. Answer the following questions.

a) Jane bought a bag of 12 oranges. When she got home she discovered that one third of the oranges were bad and threw them away. How many oranges did she have to throw away?

b) One fifth of a class got all their spelling test correct. There were thirty children in the class. How many children got full marks?

c) Three boxes of six eggs were accidentally dropped on the floor. Two thirds of them were broken. How many eggs could still be used?



7. Fill in the gaps in these number sequences.

a)	–9	-7	☐	-3	☐	1
b)	-22	-12	☐	8	☐	28

Year Five

NO CALCULATOR ALLOWED

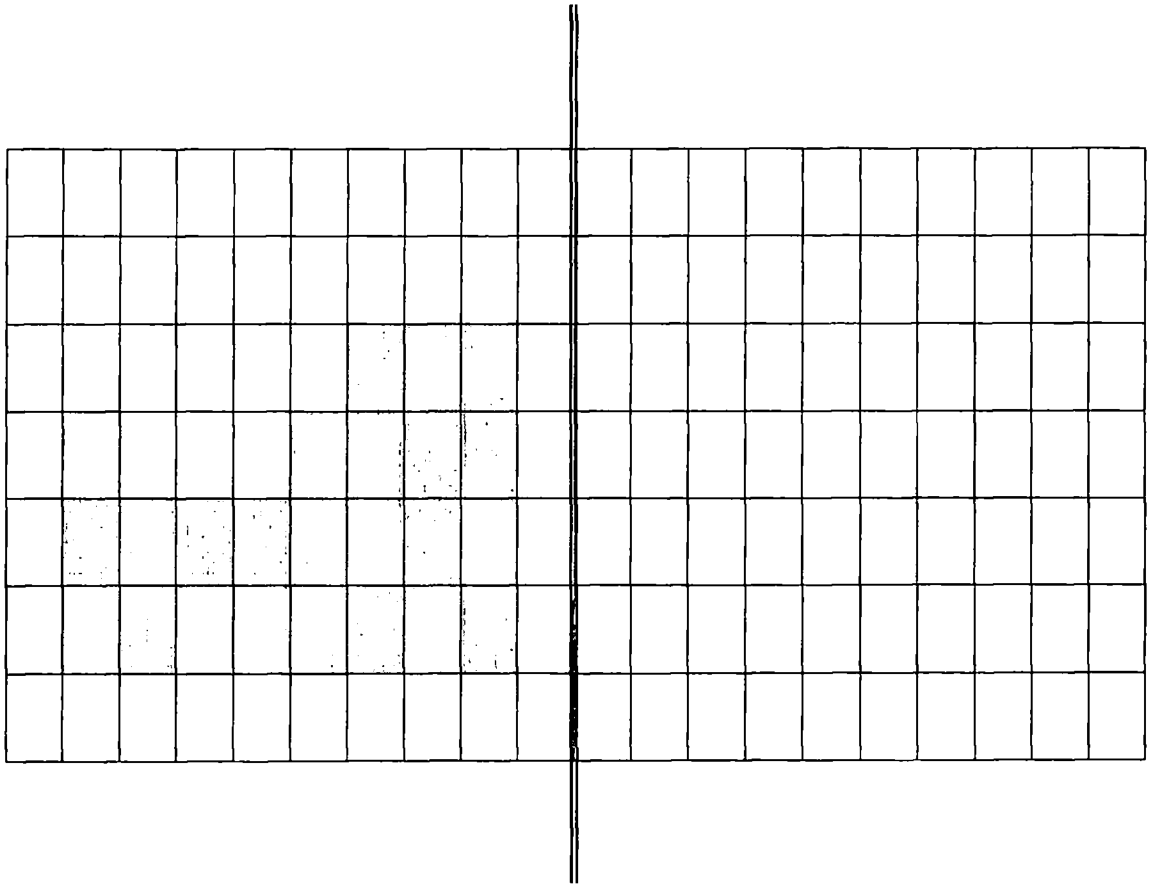
Section B

Shape, Space and Measures

You should spend approximately 10 minutes on this section.

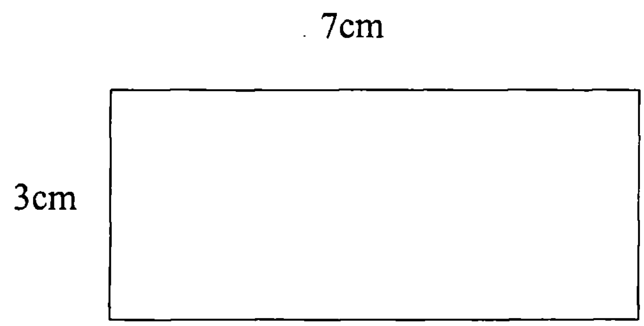
1. Draw the reflection of the shaded shape in the mirror line.

    You may use a mirror or tracing paper.



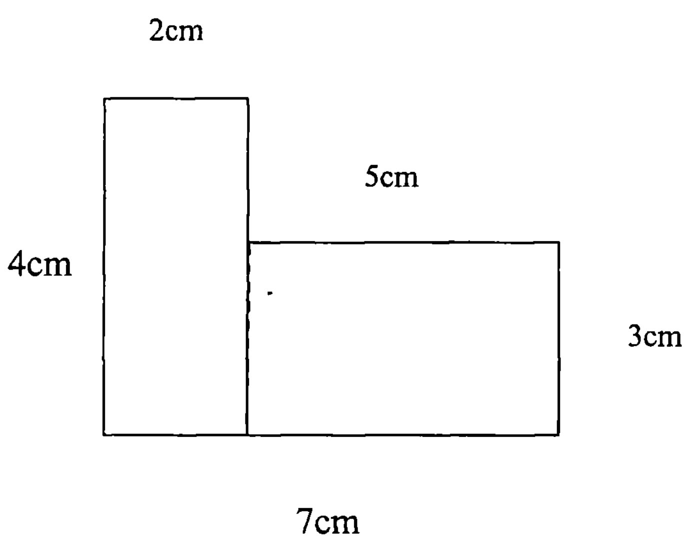
Mirror Line

2. Find the area of the rectangle.



= ☐ cm²

3. Look carefully at the shape below and answer the questions.



a) Find the area of the whole shape. ☐ cm²

b) Find the perimeter of the whole shape. ☐ cm

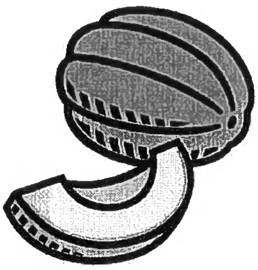
4.

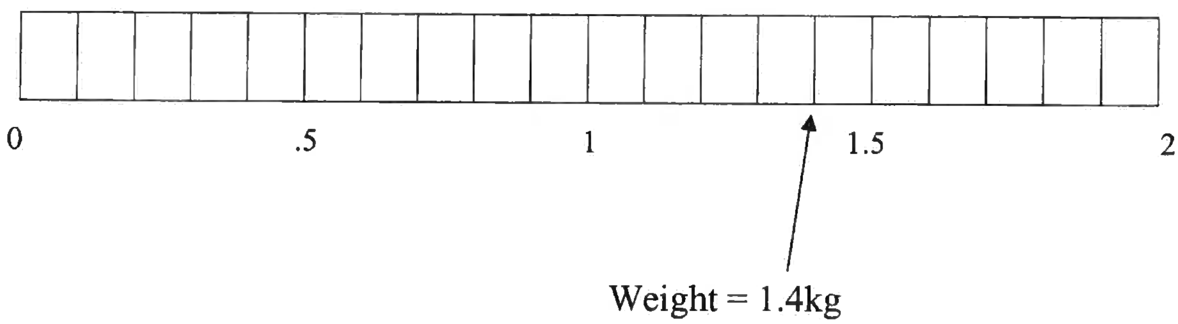
a) Name something you would measure in cm's.

b) Name something you would measure in grams.

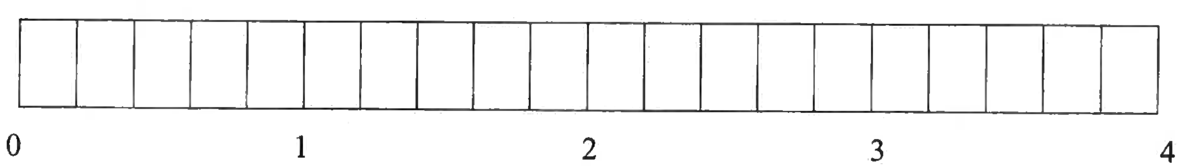
c) Name something you would measure in litres.

5. On this scale the arrow (↑) shows the weight of this melon.





Here is a different scale.



Mark with an arrow (↑) the weight of the same melon.

6. Draw the following shapes.

a) A shape with four right angles.

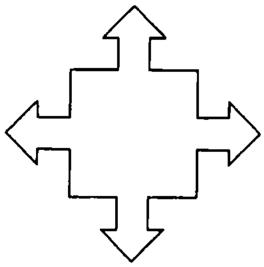
b) A shape with three acute angles.

c) A shape with six obtuse angles.

d) A shape with five angles.

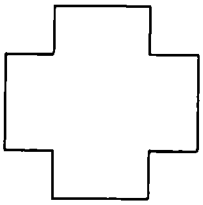
7. How many right angles can you find inside each shape?

a)



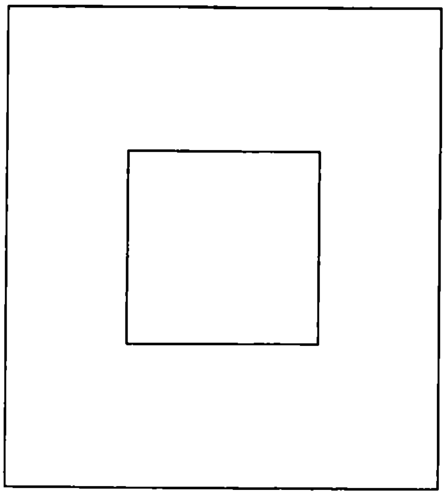
= ☐ right angles

b)



= ☐ right angles

c)



= ☐ right angles

Year Five

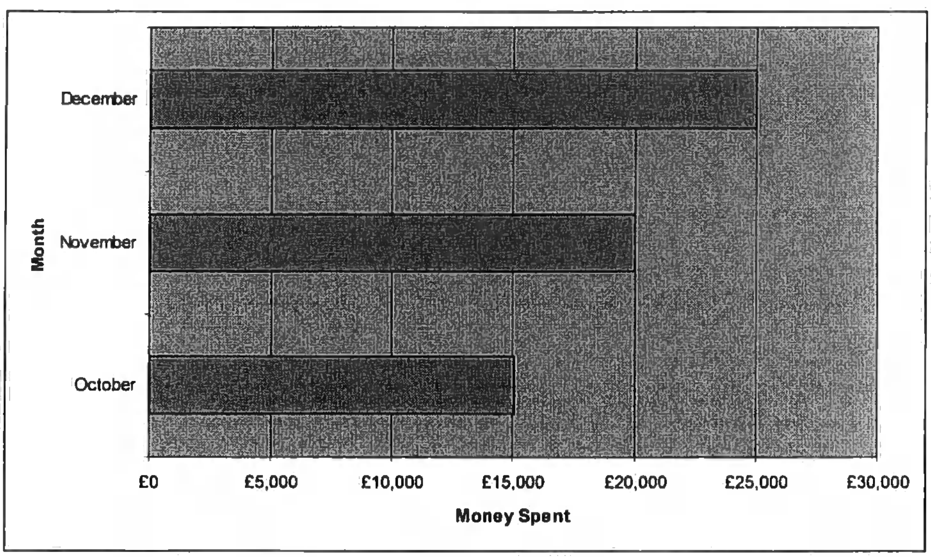
NO CALCULATOR

Section C

Handling Data

You should spend approximately 10 minutes on this section.

1. This chart shows the amount of money spent on train fares for journeys to London from Newcastle in 3 months.



a) How much money was spent in November?

b) Julie says "In December there was twice as much money spent on train fares than in October."

Is she correct? Yes / No

Explain how you can tell from the chart.

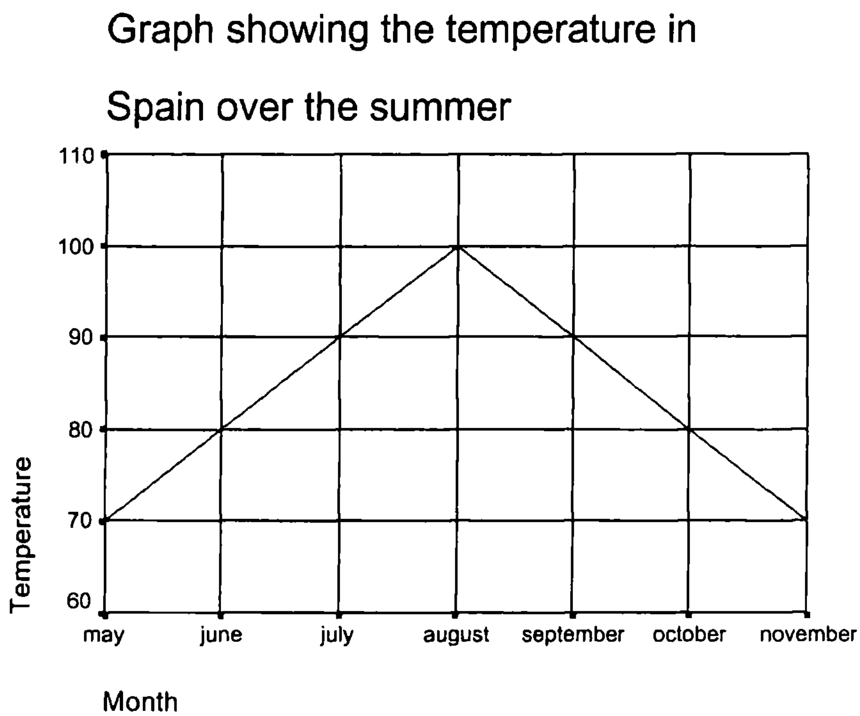
2. This chart shows some children's favourite sports.

	Lynn	Matthew	Nicola	James	Sue
Football		✓		✓	
Netball	✓		✓		
Tennis			✓		
Hockey	✓		✓		✓
Cricket				✓	

a) How many people's favourite sport is netball? ☐

b) How many more people like hockey than football? ☐

3. Answer the questions by looking at the information that the graph provides.



a) Which was the hottest month?

b) Between which months was the temperature increasing?

c) By how much did the temperature decrease between August and October?

d) Simon likes to go on holiday when the temperature is between 70 and 80 degrees. Suggest one month when it would be best for him to go on holiday to Spain.

4. Find the mode for each set of numbers.

a) 7 4 9 7 5 2 2 4 3 7

Mode ☐

b) 2 1 3 5 2 1 2 2 3 1

Mode ☐

5. Below is a table of the goals scored by some children in two football matches.

Number of Goals Scored			
Name	Match 1	Match 2	Total
Andrew	0	2	
Katie	0	1	
Aman	3	2	
Emma	4	0	
Ian	2	1	
Sarah	1	0	
Michael	0	3	
Sally	5	2	

a) Who scored the highest number of goals across the two matches?

☐

b) Who scored the lowest number of goals across the two matches?

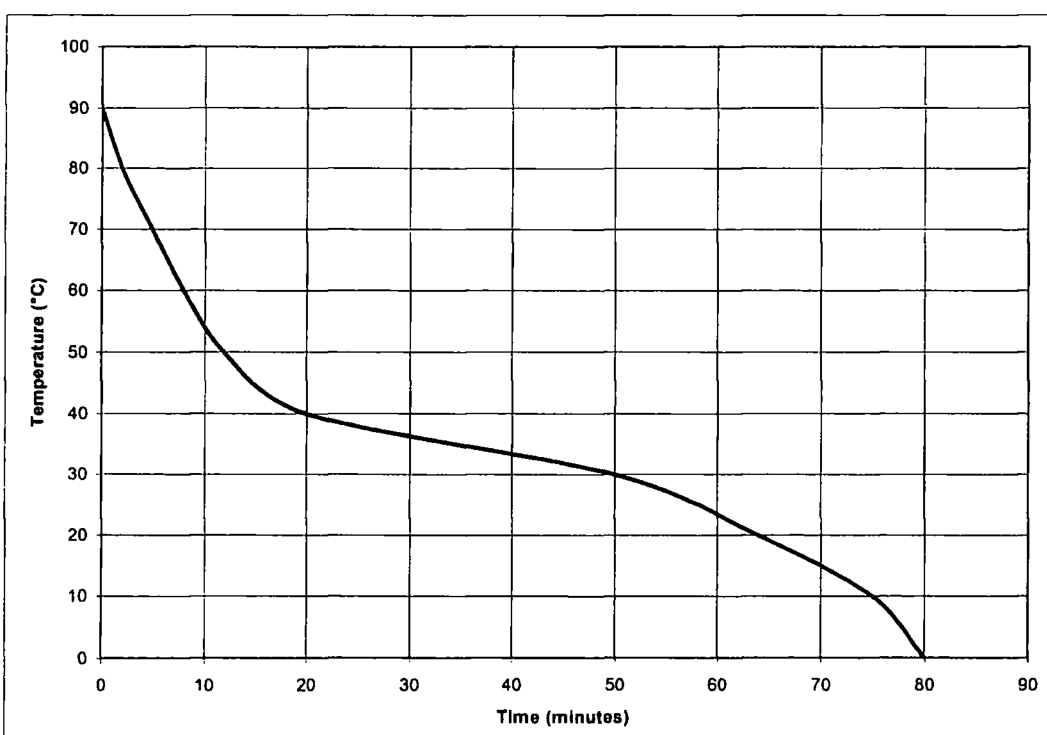
☐

c) What is the total number of goals scored by all the children across the two matches?

6. Answer the question by looking at the information that the graph provides.

A hot liquid is left to cool in a science experiment.

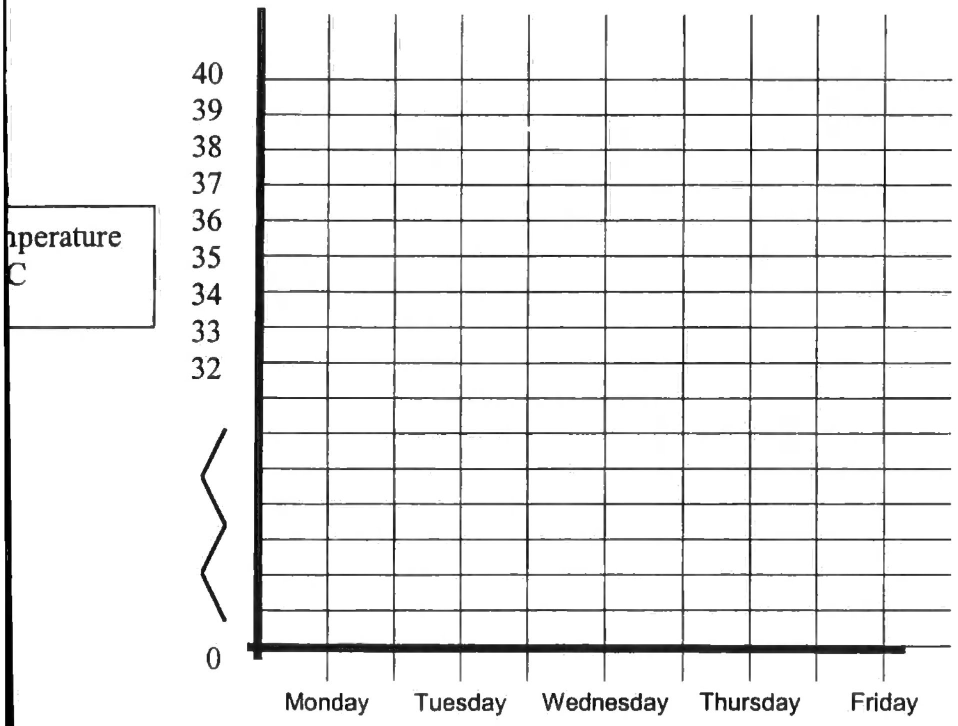
This graph shows how the temperature of the liquid changes as it cools.



Read from the graph how many minutes it takes the temperature to reach 40 degrees.

7. Lucy wasn't feeling very well. Her temperature was taken every day for a week. Draw the information below on the line graph.

Lucy's Temperature	
Day	Temperature
Tuesday	37.5
Wednesday	39
Thursday	39.5
Friday	38.5
Saturday	38
Sunday	37.5
Monday	37



Year Five

Mental Arithmetic Test

TEST QUESTIONS

For this group you have 5 seconds to work out each answer and write it down.

1. How many fifty pences are there in £7.00?

2. Multiply 5 x 8.

3. Divide 580 by 10.

For this group you have 10 seconds to work out each answer and write it down.

4. What is half of 680?

5. What is 68 – 27?

6. My watch shows the time 2.45pm. What time will it show in 45minutes?

For this group you have 15 seconds to work out each answer and write it down.

7. Add together 13, 24 and 31.

8. Look at your answer sheet. Put a ring around the number that is a multiple of 25.

380	36	120	100	47	260

9. Calculate 10 take away 4.35.

10. Look at your answer sheet. Put a ring around the smallest number.

0.37	0.307	0.037
3.07	3.7	

Read aloud the following:

Put down your pen / pencil. The test is now finished.

Year 5
Criteria for Scoring Items

All correct questions are scored as 1, all incorrect as 0.

Section	Question	Correct or Accepted and score as correct
Number and Algebra	1a	75
	1b	6
	1c	120
	2	7, 3, 0, -1, -5
	3	£2 or two pounds or 2 or two
	4a	8
	4b	6 and 9
	4c	47
	5a	23.33 or 23 and a third
	5b	19
	6a	4 or 4 oranges
	6b	6 or 6 children
	6c	6 eggs or 6 or 1 box
	7a	-5, 0
	7b	-2, 18
Shape, Space and Measures	1	If exact reflection is depicted through shading or crosses in boxes
	2	21
	3a	23
	3b	21
	4a	Anything you can sensibly measure in cms (typical answers include height or length or objects such as book, pencil)
	4b	Anything you can sensibly measure in grams (typical answers include weight or things like flour)
	4c	Anything you can sensibly measure in litres (typical answers include water, petrol)
	5a	Arrow must point to the second increment after the 1
	6a	Any shape with 4 right angles (rectangle or square – doesn't have to be exact)
	6b	A triangle with 3 small angles (doesn't have to be exact)
	6c	A hexagon or similar shape (doesn't have to be exact but must have 6 angles)
	6d	A pentagon or similar shape (doesn't have to be exact but must have 5 angles)
	7a	Eight right angles should be marked
	7b	Eight right angles should be marked
	7c	Eight right angles should be marked

Section	Question	Correct or Accepted and score as correct
Handling Data	1a	20, 000 or £20, 000 or twenty thousand pounds
	1b	No (explanation not needed for correct answer)
	2a	2 or 2 children
	2b	1 or 1 child
	3a	August
	3b	May and August (accept if all months between are listed)
	3c	20
	3d	Any of the following: May, June, October or November
	4a	7
	4b	2
	5a	Sally
	5b	Katie OR Sarah
	5c	26
	6	20 or twenty (minutes)
	7	Accept any pictorial representation of the correct information (bar or line chart or pictures representing information) Children may extend grid. This is ok.
Mental Arithmetic	1	14
	2	40
	3	58
	4	340
	5	41
	6	3.30pm (accept if pm not written)
	7	68
	8	100
	9	5.65
	10	.037

Appendix III

Mathematics Tests: Standardized Instructions

Maths Booklet

Sections A, B, and C

Administrator's Copy

Instructions:

1. Children should have pens or pencils, a maths booklet, a ruler, a mirror and tracing paper. They should not have any other mathematical equipment such as a calculator. They SHOULD NOT have access to paper for working out answers.

2. Ensure that each child has an answer sheet and tell the children to write their name and school in the box on the front.

3. Ensure that the children understand the following instructions on their sheets.

    1. Do the test on your own. Do not copy or talk to anyone else.
    2. Do not use a calculator.
    3. If you want to change an answer put a cross through your first answer.
    4. Answer as many questions as you can.
    5. You cannot ask any questions once the test has started.
    6. You should spend approximately 10 minutes on each section.
    7. The mental arithmetic test will be run separately.

4. Read out the following script, using exactly these words:

Do you have any questions? You will not be able to ask any questions once the test has begun.

Read each question carefully and try to answer it. On your sheet there is an answer box for each question, where you should write the answer to the question. You can show your working out here too.

If you make a mistake, cross out the wrong answer and write down the correct answer next to it. There are some easy questions and some harder questions, so don't be put off if you cannot answer a question..

5. Remind the children that you cannot answer any questions during the test.

6. Tell the children to begin the test.

7. After each ten minutes, remind the children that they should move on to the next section.

8. At the end of the test, tell the children to put down their pens or pencils, then collect their answer sheets.

Mental Arithmetic Test

Administrator's Copy

Instructions:

1. Children should have only pens or pencils and an answer sheet. They should not have rubbers, rulers, or any other mathematical equipment. They SHOULD NOT have access to paper for working out answers.

2. Ensure that each child has an answer sheet and tell the children to write their name and school in the box at the top.

3. Ensure that the children understand the following instructions on their sheets.

    1. Do the test on your own. Do not copy or talk to anyone else.
    2. Do not use a calculator or any other mathematical equipment.
    3. If you want to change an answer put a cross through your first answer.
    4. Answer as many questions as you can.
    5. You cannot ask any questions once the test has started.

4. Read out the following script, using exactly these words:

Listen carefully to the instructions I am going to give you. When I have finished reading them, I will answer any questions. However, you will not be able to ask any questions once the test has begun.

I will start by reading a practice question. Then I am going to ask you 10 questions for the test. On your sheet there is an answer box for each question, where you should write the answer to the question and nothing else. You should work out the answer to *each question in your head, but you may jot things down outside the answer box if this helps you. Do not try to write down your calculations because this will waste time and you may miss the next question.*

I will read out each question twice. Listen carefully both times. You will then have time to work out your answer. If you cannot work out an answer, put a cross in the answer box. If you make a mistake, cross out the wrong answer and write down the correct answer next to it. There are some easy questions and some harder questions, so don't be put off if you cannot answer a question.

5. Stop and answer any questions that the children may have.

6. Read out the following:

Here is the practice question I want you to do:

Appendix IV

Formula to calculate Kuder-Richardson Reliability Coefficient

$$KR20 = \frac{N}{N-1} \times \frac{s^2 - \sum pq}{s^2}$$

Where: N = number of items

s = standard deviation

p = % of yes or correct responses to an item

q = % of no or incorrect responses to an item

Appendix V

Cohen's (1977) Formula to Calculate Effect Sizes

$$d = \frac{[mu_1 - mu_2]}{sd \text{ of the control group}}$$

Where: mu_1 = mean in group 1

mu_2 = mean in group 2

sd = standard deviation

Appendix VI

Table 3.7

Fixed-order multiple regression analyses. Working memory scores predicting unique variance in curriculum-based mathematics performance, controlling for age-related variance. (*N*=148).

Predictor	Mathematics Ability Predicted	Model	Order of Entry Into Equation	*r*	r^2	*Adjusted* r^2
Phonological Loop						
	Number and Algebra	A_1	1. Age	.09	.01	-.00
			2. VSSP	.29	.09	.07
			3. CE	.46	.21	.19
			4. PL	.46	.21	.19
	Shape Space and Measures	A_2	1. Age	.11	.01	.00
			2. VSSP	.27	.07	.05
			3. CE	.54	.29	.27
			4. PL	.54	.29	.27
	Handling Data	A_3	1. Age	.2	.04	.03
			2. VSSP	.34	.11	.10
			3. CE	.50	.25	.22
			4. PL	.50	.25	.22
	Mental Arithmetic	A_4	1. Age	.17	.03	.02
			2. VSSP	.27	.07	.05
			3. CE	.52	.27	.25
			4. PL	.52	.27	.25
	Total Mathematics Score	A_5	1. Age	.07	.01	-.01
			2. VSSP	.31	.09	.08
			3. CE	.57	.33	.31
			4. PL	.57	.33	.31
Visuo-spatial Sketchpad						
	Number and Algebra	B_1	1. Age	.09	.01	-.00
			2. PL	.12	.01	-.00
			3. CE	.42	.18	.16
			4. VSSP	.46	.21	.18
	Shape Space and Measures	B_2	1. Age	.11	.01	.00
			2. PL	.15	.02	.00
			3. CE	.52	.28	.25
			4. VSSP	.54	.29	.26

Predictor	Mathematics Ability Predicted	Model	Order of Entry Into Equation	*r*	r^2	*Adjusted* r^2
Visuo-spatial sketchpad	Handling Data	B_3	1. Age	.20	.04	.03
			2. PL	.22	.05	.03
			3. CE	.46	.22	.20
			4. VSSP	.50	.25	.22
	Mental Arithmetic	B_4	1. Age	.17	.03	.02
			2. PL	.23	.05	.03
			3. CE	.51	.26	.24
			4. VSSP	.52	.27	.24
	Total Mathematics Score	B_5	1. Age	.07	.01	-.01
			2. PL	.14	.02	.00
			3. CE	.55	.30	.28
			4. VSSP	.57	.33	.30
Central Executive						
	Number and Algebra	C_1	1. Age	.09	.01	-.00
			2. PL	.12	.02	-.00
			3. VSSP	.30	.09	.06
			4. CE	.46	.21	.18
	Shape Space and Measures	C_2	1. Age	.11	.01	.00
			2. PL	.15	.02	.00
			3. VSSP	.28	.08	.05
			4. CE	.54	.29	.26
	Handling Data	C_3	1. Age	.20	.04	.03
			2. PL	.22	.05	.03
			3. VSSP	.34	.12	.09
			4. CE	.50	.25	.22
	Mental Arithmetic	C_4	1. Age	.17	.03	.02
			2. PL	.23	.05	.03
			3. VSSP	.30	.09	.06
			4. CE	.52	.27	.24
	Total Mathematics Score	C_5	1. Age	.07	.01	-.01
			2. PL	.14	.02	.00
			3. VSSP	.32	.10	.08
			4. CE	.57	.33	.30

Appendix VII

Table 3.8.

Fixed-order multiple regression analyses predicting unique variance in curriculum-based mathematics performance, controlling for age-related variance and NVIQ.

(*N*=148).

Predictor	Mathematics Ability Predicted	Model	Order of Entry Into Regression	R	R^2	Adjusted R^2
CE	Total Mathematics Score	D_1	1. AGE	.02	.00	.01
			2. NVIQ	.46	.21	.20
			3. VSSP	.50	.25	.23
			4. PL	.50	.26	.23
			5. CE	.64	.41	.38
CE	Number and Algebra	D_2	1. AGE	.03	.00	.01
			2. NVIQ	.44	.19	.17
			3. VSSP	.47	.22	.19
			4. PL	.48	.23	.19
			5. CE	.53	.29	.25
CE	Shape Space and Measures	D_3	1. AGE	.09	.01	.00
			2. NVIQ	.41	.17	.15
			3. VSSP	.44	.20	.17
			4. PL	.45	.20	.17
			5. CE	.61	.37	.33
CE	Handling Data	D_4	1. AGE	.25	.06	.05
			2. NVIQ	.49	.23	.22
			3. VSSP	.52	.27	.24
			4. PL	.52	.27	.24
			5. CE	.60	.35	.32
CE	Mental Arithmetic	D_5	1. AGE	.15	.02	.01
			2. NVIQ	.30	.09	.07
			3. VSSP	.34	.11	.09
			4. PL	.36	.13	.09
			5. CE	.51	.26	.22
VSSP	Total Mathematics Score	E_1	1. AGE	.02	.00	.01
			2. NVIQ	.46	.21	.20
			3. CE	.63	.40	.38
			4. PL	.63	.40	.38
			5. VSSP	.64	.41	.38

Predictor	Mathematics Ability Predicted	Model	Order of Entry Into Regression	R	R^2	Adjusted R^2
VSSP	Number and Algebra	E_2	1. AGE	.03	.00	.01
			2. NVIQ	.44	.19	.17
			3. CE	.52	.27	.25
			4. PL	.52	.27	.25
			5. VSSP	.53	.29	.25
VSSP	Shape Space and Measures	E_3	1. AGE	.09	.01	.00
			2. NVIQ	.41	.17	.15
			3. CE	.60	.36	.34
			4. PL	.60	.36	.34
			5. VSSP	.61	.37	.34
VSSP	Handling Data	E_4	1. AGE	.25	.06	.05
			2. NVIQ	.48	.23	.22
			3. CE	.58	.34	.32
			4. PL	.59	.34	.32
			5. VSSP	.60	.35	.32
VSSP	Mental Arithmetic	E_5	1. AGE	.15	.02	.01
			2. NVIQ	.30	.09	.07
			3. CE	.50	.25	.23
			4. PL	.51	.26	.23
			5. VSSP	.51	.26	.23
PL	Total Mathematics Score	F_1	1. AGE	.02	.00	.01
			2. NVIQ	.46	.21	.20
			3. CE	.63	.40	.38
			4. VSSP	.64	.41	.38
			5. PL	.64	.41	.38
PL	Number and Algebra	F_2	1. AGE	.03	.00	.01
			2. NVIQ	.44	.19	.17
			3. CE	.52	.27	.25
			4. VSSP	.53	.29	.25
			5. PL	.53	.29	.25
PL	Shape Space and Measures	F_3	1. AGE	.09	.01	.00
			2. NVIQ	.41	.17	.15
			3. CE	.60	.36	.34
			4. VSSP	.61	.37	.34
			5. PL	.61	.37	.34

Predictor	Mathematics Ability Predicted	Model	Order of Entry Into Regression	R	R^2	Adjusted R^2
PL	Handling Data	F_4	1. AGE	.25	.06	.05
			2. NVIQ	.48	.23	.22
			3. CE	.58	.34	.32
			4. VSSP	.59	.35	.33
			5. PL	.60	.35	.33
PL	Mental Arithmetic	F_5	1. AGE	.15	.02	.01
			2. NVIQ	.30	.09	.07
			3. CE	.50	.25	.23
			4. VSSP	.51	.26	.23
			5. PL	.51	.26	.23

Appendix VIII

Table 4.5

*Fixed-order multiple regression analyses. Visuo-spatial measures predicting unique variance in curriculum-based mathematics performance, controlling for age-related variance. (N=*107).

Predictor	Mathematics Ability Predicted	Model	Order of Entry Into Equation	*r*	r^2	*Adjusted* r^2
Visual Patterns Test						
	Number and Algebra	A_1	1. Age	.21	.04	.03
			2. Mazes Memory	.25	.06	.04
			3. Block Recall	.31	.09	.06
			4. Blobby Visual	.42	.18	.14
			5. Blobby Spatial	.45	.20	.16
			6. Visual Patterns	.46	.21	.16
	Shape Space and Measures	A_2	1. Age	.03	.00	.00
			2. Mazes Memory	.03	.00	.00
			3. Block Recall	.04	.00	.03
			4. Blobby Visual	.22	.05	.03
			5. Blobby Spatial	.22	.05	.03
			6. Visual Patterns	.23	.05	.03
	Handling Data	A_3	1. Age	.14	.02.02	.00
			2. Mazes Memory	.15	.02	.00
			3. Block Recall	.15	.06	.01
			4. Blobby Visual	.24	.10	.02
			5. Blobby Spatial	.32	.12	.05
			6. Visual Patterns	.35		.06

Predictor	Mathematics Ability Predicted	Model	Order of Entry Into Equation	*r*	*r*²	*Adjusted r*²
Visual Patterns Test	Mental Arithmetic	A_4	1. Age	.15	.02	.01
			2. Mazes Memory	.19	.04	.02
			3. Block Recall	.25	.06	.03
			4. Blobby Visual	.35	.12	.08
			5. Blobby Spatial	.41	.17	.12
			6. Visual Patterns	.41	.17	.12
	Total Mathematics Score	A_5	1. Age	.09	.01	.00
			2. Mazes Memory	.15	.02	.00
			3. Block Recall	.20	.04	.01
			4. Blobby Visual	.36	.13	.09
			5. Blobby Spatial	.41	.17	.12
			6. Visual Patterns	.42	.17	.12
Mazes Memory						
	Number and Algebra	B_1	1. Age	.21	.04	.03
			2. Visual Patterns	.33	.11	.09
			3. Block Recall	.36	.13	.10
			4. Blobby Visual	.43	.19	.15
			5. Blobby Spatial	.46	.21	.16
			6. Mazes Memory	.46	.21	.16
	Shape Space and Measures	B_2	1. Age	.03	.00	.01
			2. Visual Patterns	.11	.01	.01
			3. Block Recall	.11	.01	.01
			4. Blobby Visual	.23	.05	.01
			5. Blobby Spatial	.23	.05	.01
			6. Mazes Memory	.23	.05	.01

Predictor	Mathematics Ability Predicted	Model	Order of Entry Into Equation	*r*	*r²*	*Adjusted r²*
Mazes Memory	Handling Data	B_3	1. Age	.14	.02.07	.01
			2. Visual Patterns	.26	.07	.05
			3. Block Recall	.26	.08	.05
			4. Blobby Visual	.29	.12	.05
			5. Blobby Spatial	.35	.12	.07
			6.Mazes Memory	.35		.07
	Mental Arithmetic	B_4	1. Age	.15	.02	.01
			2. Visual Patterns	.23	.05	.03
			3. Block Recall	.27	.07	.04
			4. Blobby Visual	.36	.12	.08
			5. Blobby Spatial	.41	.16	.12
			6. Mazes Memory	.41	.17	.12
	Total Mathematics Score	B_5	1. Age	.09	.01	-.00
			2. Visual Patterns	.27	.07	.05
			3. Block Recall	.28	.08	.05
			4. Blobby Visual	.38	.15	.11
			5. Blobby Spatial	.42	.18	.13
			6. Mazes Memory	.42	.18	.13
Block Recall						
	Number and Algebra	C_1	1. Age	.21	.04	.03
			2. Visual Patterns	.33	.11	.09
			3. Mazes Memory	.33	.12	.09
			4. Blobby Visual	.42	.18	.14
			5. Blobby Spatial	.45	.20	.16
			6. Block Recall	.46	.21	.16

Predictor	Mathematics Ability Predicted	Model	Order of Entry Into Equation	*r*	*r*²	*Adjusted r*²
Block Recall	Shape Space and Measures	C_2	1. Age	.03	.00	.01
			2. Visual Patterns	.11	.01	.01
			3. Mazes Memory	.12	.01	.01
			4. Blobby Visual	.23	.05	.01
			5. Blobby Spatial	.23	.05	.01
			6. Block Recall	.23	.05	.01
	Handling Data	C_3	1. Age	.14	.02.07	.01
			2. Visual Patterns	.26	.07	.05
			3. Mazes Memory	.26	.08	.05
			4. Blobby Visual	.29	.12	.05
			5. Blobby Spatial	.34	.12	.07
			6.Block Recall	.35		.07
	Mental Arithmetic	C_4	1. Age	.15	.02	.01
			2. Visual Patterns	.23	.05	.03
			3. Mazes Memory	.23	.06	.03
			4. Blobby Visual	.32	.10	.07
			5. Blobby Spatial	.39	.16	.12
			6. Block Recall	.41	.17	.12
	Total Mathematics Score	C_5	1. Age	.09	.01	.00
			2. Visual Patterns	.27	.07	.05
			3. Mazes Memory	.27	.07	.05
			4. Blobby Visual	.38	.14	.10
			5. Blobby Spatial	.42	.17	.13
			6. Block Recall	.42	.18	.13

Predictor	Mathematics Ability Predicted	Model	Order of Entry Into Equation	*r*	r^2	*Adjusted* r^2
Blobby Visual						
	Number and Algebra	D_1	1. Age	.21	.04	.03
			2. Visual Patterns	.33	.11	.09
			3. Mazes Memory	.34	.12	.09
			4. Block Recall	.36	.13	.09
			5. Blobby Spatial	.39	.15	.10
			6. Blobby Visual	.46	.21	.16
	Shape Space and Measures	D_2	1. Age	.03	.00	.01
			2. Visual Patterns	.11	.01	.01
			3. Mazes Memory	.12	.01	.01
			4. Block Recall	.12	.01	.01
			5. Blobby Spatial	.12	.02	.01
			6. Blobby Visual	.23	.05	.04
	Handling Data	D_3	1. Age	.14	.02.	.01
			2. Visual Patterns	.26	07	.05
			3. Mazes Memory	.26	.07	.05
			4. Block Recall	.26	.07	.05
			5. Blobby Spatial	.32	.10	.06
			6. Blobby Visual	.35	.12	.06
	Mental Arithmetic	D_4	1. Age	.15	.02	.01
			2. Visual Patterns	.23	.05	.03
			3. Mazes Memory	.24	.06	.03
			4. Block Recall	.27	.07	.03
			5. Blobby Spatial	.34	.12	.07
			6. Blobby Visual	.41	.17	.11

Predictor	Mathematics Ability Predicted	Model	Order of Entry Into Equation	*r*	*r²*	*Adjusted r²*
Blobby Visual	Total Mathematics Score	D_5	1. Age	.09	.01	.00
			2. Visual Patterns	.27	.07	.05
			3. Mazes Memory	.27	.07	.05
			4. Block Recall	.28	.08	.05
			5. Blobby Spatial	.32	.11	.06
			6. Blobby Visual	.42	.18	.12
Blobby Spatial						
	Number and Algebra	E_1	1. Age	.21	.04	.03
			2. Visual Patterns	.33	.11	.09
			3. Mazes Memory	.34	.12	.09
			4. Block Recall	.36	.13	.09
			5. Blobby Visual	.44	.19	.15
			6. Blobby Spatial	.46	.21	.16
	Shape Space and Measures	E_2	1. Age	.03	.00	.01
			2. Visual Patterns	.11	.01	.01
			3. Mazes Memory	.12	.01	.01
			4. Block Recall	.12	.01	.01
			5. Blobby Visual	.23	.05	.01
			6. Blobby Spatial	.23	.05	.01
	Handling Data	E_3	1. Age	.14	.02.	.01
			2. Visual Patterns	.26	07	.05
			3. Mazes Memory	.26	.07	.05
			4. Block Recall	.26	.07	.05
			5. Blobby Visual	.29	.08	.06
			6. Blobby Spatial	.35	.12	.06

Predictor	Mathematics Ability Predicted	Model	Order of Entry Into Equation	*r*	r^2	*Adjusted* r^2
	Mental Arithmetic	E_4	1. Age	.15	.02	.01
			2. Visual Patterns	.23	.05	.03
			3. Mazes Memory	.24	.06	.03
			4. Block Recall	.27	.07	.03
			5. Blobby Visual	.35	.12	.07
			6. Blobby Spatial	.41	.17	.11
	Total Mathematics Score	E_5	1. Age	.09	.01	-.00
			2. Visual Patterns	.27	.07	.05
			3. Mazes Memory	.27	.07	.05
			4. Block Recall	.28	.08	.05
			5. Blobby Visual	.38	.15	.10
			6. Blobby Spatial	.42	.18	.12

Appendix IX

Table 4.6.

Fixed-order multiple regression analyses. Visuo-spatial scores predicting unique variance in curriculum-based mathematics performance, controlling for age-related variance and NVIQ. (N=107).

Predictor	Mathematics Ability Predicted	Model	Order of Entry Into Equation	r	r^2	*Adjusted* r^2
Visual Patterns Test						
	Number and Algebra	F_1	1. Age	.19	.04	.03
			2. NVIQ	.35	.13	.11
			3. Mazes Memory	.36	.13	.11
			4. Block Recall	.38	.14	.11
			5. Blobby Visual	.44	.20	.15
			6. Blobby Spatial	.47	.22	.17
			7. Visual Patterns	.47	.22	.17
	Shape Space and Measures	F_2	1. Age	.05	.00	.00
			2.NVIQ	.22	.05	.03
			3.Mazes Memory	.22	.05	.03
			4. Block Recall	.23	.05	.03
			5. Blobby Visual	.27	.07	.03
			6. Blobby Spatial	.28	.08	.03
			7. Visual Patterns	.28	.08	.03

Predictor	Mathematics Ability Predicted	Model	Order of Entry Into Equation	*r*	r^2	*Adjusted* r^2
Visual Patterns Test	Handling Data	F_3	1. Age	.15	.02.12	.01
			2.NVIQ	.35	.12	.10
			3. Mazes Memory	.35	.13	.10
			4. Block Recall	.36	.14	.10
			5. Blobby Visual	.37	.17	.10
			6. Blobby Spatial	.41	.18	.11
			7. Visual Patterns	.42		.11
	Mental Arithmetic	F_4	1. Age	.12	.02	.01
			2.NVIQ	.28	.08	.06
			3. Mazes Memory	.29	.09	.06
			4. Block Recall	.30	.09	.06
			5. Blobby Visual	.35	.12	.08
			6. Blobby Spatial	.41	.17	.11
			7. Visual Patterns	.41	.17	.11
	Total Mathematics Score	F_5	1. Age	06	00	.00
			2.NVIQ	.36	.13	.11
			3. Mazes Memory	.36	.13	.11
			4. Block Recall	.36	.13	.11
			5. Blobby Visual	.42	.18	.14
			6. Blobby Spatial	.46	.21	.15
			7. Visual Patterns	.46	.21	.15

Predictor	Mathematics Ability Predicted	Model	Order of Entry Into Equation	r	r^2	*Adjusted* r^2
Mazes Memory						
	Number and Algebra	G_1	1. Age	.19	.04	.03
			2. NVIQ	.35	.13	.11
			3.Visual Patterns	.39	.15	.12
			4. Block Recall	.39	.16	.12
			5. Blobby Visual	.45	.20	.16
			6. Blobby Spatial	.47	.22	.17
			7. Mazes Memory	.47	.22	.17
	Shape Space and Measures	G_2	1. Age	.05	.00	.01
			2.NVIQ	.22	.05	.03
			3. Visual Patterns	.22	.05	.03
			4. Block Recall	.23	.05	.03
			5. Blobby Visual	.27	.07	.03
			6. Blobby Spatial	.27	.08	.03
			7. Mazes Memory	.28	.08	.03
	Handling Data	G_3	1. Age	.15	.02.12	.01
			2.NVIQ	.35	.13	.10
			3. Visual Patterns	.36	.14	.10
			4. Block Recall	.38	.15	.10
			5. Blobby Visual	.38	.18	.10
			6. Blobby Spatial	.42	.18	.12
			7.Mazes Memory	.42		.12

Predictor	Mathematics Ability Predicted	Model	Order of Entry Into Equation	r	r^2	*Adjusted* r^2
	Mental Arithmetic	G_4	1. Age	.12	.02	.01
			2.NVIQ	.28	.08	.06
			3. Visual Patterns	.29	.09	.06
			4. Block Recall	.30	.09	.06
			5. Blobby Visual	.35	.12	.07
			6. Blobby Spatial	.40	.16	.11
			7. Mazes Memory	.41	.17	.11
	Total Mathematics Score	G_5	1. Age	.06	.00	.01
			2.NVIQ	.36	.13.14	.11
			3. Visual Patterns	.38	.14	.11
			4. Block Recall	.38	.18	.11
			5. Blobby Visual	.43	.21	.14
			6. Blobby Spatial	.46	.21	.16
			7. Mazes Memory	.46		.16
Block Recall						
	Number and Algebra	H_1	1. Age	.19	.04	.03
			2.NVIQ	.35	.13	.11
			3.Visual Patterns	.39	.15	.12
			4. Mazes Memory	.39	.15	.12
			5. Blobby Visual	.44	.19	.15
			6. Blobby Spatial	.46	.22	.16
			7. Block Recall	.47	.22	.16

Predictor	Mathematics Ability Predicted	Model	Order of Entry Into Equation	*r*	r^2	*Adjusted* r^2
Block Recall	Shape Space and Measures	H_2	1. Age	.05	.00	.01
			2. NVIQ	.22	.05	.03
			3.Visual Patterns	.22	.05	.03
			4. Mazes Memory	.22	.05	.03
			5. Blobby Visual	.27	.07	.03
			6. Blobby Spatial	.27	.07	.03
			7. Block Recall	.28	.08	.03
	Handling Data	H_3	1. Age	.15	.02	.01
			2.NVIQ	.35	.12	.10
			3. Visual Patterns	.36	.13	.10
			4. Mazes Memory	.36	.13	.10
			5. Blobby Visual	.37	.14	.10
			6. Blobby Spatial	.40	.16	.11
			7.Block Recall	.42	.18	.11
	Mental Arithmetic	H_4	1. Age	.12	.02	.01
			2.NVIQ	.28	.08	.06
			3. Visual Patterns	.29	.09	.06
			4. Mazes Memory	.30	.09	.06
			5. Blobby Visual	.34	.12	.07
			6. Blobby Spatial	.41	.16	.11
			7. Block Recall	.41	.17	.11

Predictor	Mathematics Ability Predicted	Model	Order of Entry Into Equation	*r*	r^2	*Adjusted* r^2
Block Recall	Total Mathematics Score	H_5	1. Age	.06	.00	.01
			2. NVIQ	.36	.13	.11
			3. Visual Patterns	.38	.14	.11
			4. Mazes Memory	.38	.14	.11
			5. Blobby Visual	.43	.18	.14
			6. Blobby Spatial	.46	.21	.16
			7. Block Recall	.46	.21	.16
Blobby Visual						
	Number and Algebra	I_1	1. Age	.19	.04	.03
			2.NVIQ	.35	.13	.11
			3. Visual Patterns	.39	.15	.12
			4. Mazes Memory	.39	.15	.12
			5. Block Recall	.39	.16	.12
			6. Blobby Spatial	.42	.17	.12
			7. Blobby Visual	.47	.22	.16
	Shape Space and Measures	I_2	1. Age	.05	.00	.01
			2. NVIQ	.22	.05	.03
			3. Visual Patterns	.22	.05	.03
			4. Mazes Memory	.22	.05	.03
			5. Block Recall	.23	.05	.03
			6. Blobby Spatial	.24	.06	.03
			7. Blobby Visual	.28	.08	.03

Predictor	Mathematics Ability Predicted	Model	Order of Entry Into Equation	*r*	*r²*	*Adjusted r²*
Blobby Visual	Handling Data	I_3	1. Age	.15	.02.12	.01
			2. NVIQ	.35	.13	.10
			3.Visual Patterns	.36	.13	.10
			4. Mazes Memory	.36	.14	.10
			5. Block Recall	.38	.17	.10
			6. Blobby Spatial	.41	.18	.12
			7. Blobby Visual	.42		.12
	Mental Arithmetic	I_4	1. Age	.12	.02	.01
			2. NVIQ	.28	.08	.06
			3. Visual Patterns	.29	.09	.06
			4. Mazes Memory	.30	.09	.06
			5. Block Recall	.31	.09	.06
			6. Blobby Spatial	.37	.13	.08
			7. Blobby Visual	.41	.17	.10
	Total Mathematics Score	I_5	1. Age	.06	.00	.01
			2. NVIQ	.36	.13	.11
			3. Visual Patterns	.38	.14	.11
			4. Mazes Memory	.38	.14	.11
			5. Block Recall	.38	.14	.11
			6. Blobby Spatial	.41	.16	.11
			7. Blobby Visual	.46	.21	.15

Predictor	Mathematics Ability Predicted	Model	Order of Entry Into Equation	r	r^2	*Adjusted* r^2
Blobby Spatial						
	Number and Algebra	J_1	1. Age	.19	.04	.03
			2. NVIQ	.35	.13	.11
			3. Visual Patterns	.38	.15	.12
			4. Mazes Memory	.39	.15	.12
			5. Block Recall	.39	.16	.12
			6. Blobby Visual	.45	.20	.15
			7. Blobby Spatial	.47	.22	.16
	Shape Space and Measures	J_2	1. Age	.05	.00	.01
			2. NVIQ	.22	.05	.03
			3.Visual Patterns	.22	.05	.03
			4. Mazes Memory	.22	.05	.03
			5. Block Recall	.23	.05	.03
			6. Blobby Visual	.27	.07	.03
			7. Blobby Spatial	.28	.08	.03
	Handling Data	J_3	1. Age	.15	.02.12	.01
			2. NVIQ	.35	.13	.10
			3. Visual Patterns	.36	.13	.10
			4. Mazes Memory	.36	.14	.10
			5. Block Recall	.38	.15	.10
			6. Blobby Visual	.38	.18	.10
			7. Blobby Spatial	.42		.11

Predictor	Mathematics Ability Predicted	Model	Order of Entry Into Equation	*r*	r^2	*Adjusted* r^2
	Mental Arithmetic	J_4	1. Age	.12	.02	.01
			2.NVIQ	.28	.08	.06
			3. Visual Patterns	.29	.09	.06
			4. Mazes Memory	.30	.09	.06
			5. Block Recall	.31	.09	.06
			6. Blobby Visual	.35	.12	.06
			7. Blobby Spatial	.41	.17	.10
	Total Mathematics Score	J_5	1. Age	.06	.00	.01
			2.NVIQ	.36	.13	.11
			3.Visual Patterns	.38	.14	.11
			4. Mazes Memory	.38	.14	.11
			5. Block Recall	.38	.14	.11
			6. Blobby Visual	.43	.18	.13
			7. Blobby Spatial	.46	.21	.15

Appendix X

Items from Year 3 Curriculum-Based Mathematics Assessments Comprising Clusters A and B (Performance-Related Mathematical Domains)

Cluster A	Cluster B
Number and Algebra 1a	Number and Algebra 2
Number and Algebra 1b	Shape, Space and Measures 1
Number and Algebra 1c	Shape, Space and Measures 2
Number and Algebra 3	Shape, Space and Measures 3a
Number and Algebra 4a	Shape, Space and Measures 3b
Number and Algebra 4b	Shape, Space and Measures 4a
Number and Algebra 4c	Shape, Space and Measures 4b
Number and Algebra 5a	Shape, Space and Measures 4c
Number and Algebra 5b	Shape, Space and Measures 6a
Number and Algebra 6a	Shape, Space and Measures 6b
Number and Algebra 6b	Shape, Space and Measures 6c
Number and Algebra 6c	Handling Data 1a
Number and Algebra 7a	Handling Data 1b
Number and Algebra 7b	Handling Data 2a
Shape, Space and Measures 5a	Handling Data 2b
Shape, Space and Measures 5b	Handling Data 3a
Shape, Space and Measures 7a	Handling Data 3c
Shape, Space and Measures 7b	Handling Data 4a
Shape, Space and Measures 7c	Handling Data 5a
Handling Data 3b	Handling Data 5b
Handling Data 3d	Handling Data 5c
Handling Data 4b	Handling Data 7
Handling Data 6	Mental Arithmetic 1
Mental Arithmetic 2	Mental Arithmetic 8
Mental Arithmetic 3	Mental Arithmetic 10
Mental Arithmetic 4	
Mental Arithmetic 5	
Mental Arithmetic 6	
Mental Arithmetic 7	
Mental Arithmetic 9	

Appendix XI

Items from Year 5 Curriculum-Based Mathematics Assessments Comprising Clusters C and D (Performance-Related Mathematical Domains)

Cluster C	Cluster D
Number and Algebra 1a	Number and Algebra 4c
Number and Algebra 1b	Number and Algebra 5a
Number and Algebra 1c	Number and Algebra 5b
Number and Algebra 2	Number and Algebra 6a
Number and Algebra 3	Number and Algebra 6b
Number and Algebra 4a	Number and Algebra 6c
Number and Algebra 4b	Shape, Space and Measures 2
Number and Algebra 7a	Shape, Space and Measures 3a
Number and Algebra 7b	Shape, Space and Measures 3b
Shape, Space and Measures 1	Shape, Space and Measures 5
Shape, Space and Measures 4a	Shape, Space and Measures 6d
Shape, Space and Measures 4b	Handling Data 3b
Shape, Space and Measures 4c	Handling Data 4a
Shape, Space and Measures 6a	Handling Data 4b
Shape, Space and Measures 6b	Handling Data 5c
Shape, Space and Measures 6c	Mental Arithmetic 6
Shape, Space and Measures 7a	Mental Arithmetic 9
Shape, Space and Measures 7b	Mental Arithmetic 10
Shape, Space and Measures 7c	
Handling Data 1a	
Handling Data 1b	
Handling Data 2a	
Handling Data 2b	
Handling Data 3a	
Handling Data 3c	
Handling Data 3d	
Handling Data 5a	
Handling Data 5b	
Handling Data 6	
Handling Data 7	
Mental Arithmetic 1	
Mental Arithmetic 2	
Mental Arithmetic 3	
Mental Arithmetic 4	
Mental Arithmetic 5	
Mental Arithmetic 7	
Mental Arithmetic 8	