Essays on the Provision of HE, Student Loans and Second Chance Policies

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Abstract

In many countries including the UK, the higher education (HE) sector is increasingly being funded by student loans while the level and methods of funding HE continues to be a hotly debated political topic. This thesis includes four theoretical papers on the provision of HE, student loans and Second Chance policies and is divided into two parts.

There is evidence of improved labour market performance for mature university students who completed HE and for workers who succeeded in the labour retraining program. It could be beneficial for the government to provide a second chance at HE to those who failed their first chance at HE and to provide a labour retraining program to those who suffered downward labour mobility. The first part explores how to design a student loan system that takes into account foreseeable potential HE and labour market failures by extending the model of Gary-Bobo and Trannoy (2015). Chapter 3 investigates how to optimally provide a HE system that offers a second chance to all failed students and analyses the impact of such an optimally provided second chance on inequality. We found the second chance policy increases the degree of ex-ante inequality and causes the students who failed twice in HE to be worse off than everyone when old. Chapter 4 investigates how to optimally provide HE with a labour retraining program and the effect of this program on inequality. Although the

labour retraining program has increased the degree of ex-ante inequality we found the degree of ex-post inequality can be decreased but only when the government knows who needs retraining. Our results suggest that if the government wants to use a labour retraining program to reduce inequality, it needs to improve its ability to correctly identify which workers are at risk of downward mobility. Our results in both chapters also support the findings in the related literature on the usefulness of ICL schemes to fund HE and suggest that the government cannot charge a separate ordinary loan for using these Second Chance policies.

Part 2 focuses on the political economy of HE, a relatively underexplored field. Chapter 5 investigates the effect of the design of the constitution on the level of HE spending and income inequality between the presidential and parliamentary regimes by modifying the model of Persson, Roland and Tabellini (2000). We show the way the level of HE spending is determined in the two regimes is fundamentally different because the identity of the residual claimant of tax revenue is different and this presents a mechanism on how the constitution of the presidential regimes itself leads to a higher level of income inequality compared to the parliamentary regimes. In the UK and the US, the rising tuition fees and the building up of student loan debt have caused concern over its potential default. Chapter 6 examines whether a government could be motivated to increase the level of student loans and tuition fees when facing political uncertainty and a polarised society in theory by modifying the model of Persson and Svensson (1989). Our results agree with the insights in the related literature on the incentives to use public investment to influence the decision of the future government but we do not predict the incumbent government must increase or decrease the level of investment under uncertain re-election prospects.

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Declarations

No part of this thesis has been submitted elsewhere for any other degree or qualification and it is the sole work of the author unless referenced contrary to the text.

Statement of Copyright

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Dedication

To my parents: Mr Shibin Jiang and Mrs Chen Yin

Chapter 1: Introduction

1.1 Introduction

In many countries including the UK, the higher education (HE) sector is increasingly funded by the university students themselves in the form of student loans. Although the concept of student loans to finance HE is not new, the issue of the optimal design of student loans has not been resolved yet. There is also a lack of papers examining the political economy dimension of the provision of HE and student loans. This thesis aims to explore the optimal provision of HE combined with Second Chance policies and examine the impact of the design of the constitution, political uncertainty and polarisation on the provision of HE and student loans.

An income-contingent loan (ICL) is a loan in which the repayment is dependent on the income of the borrower. If the HE sector is funded by mortgage-style loans, there will be an under-provision of these loans by private lenders because if the graduate defaults on the loan, there is no collateral to be sold and the graduate cannot sell part of the investment to refinance his HE. Friedman and Kuznets (1945) and Friedman (1955) recognized these capital market failures and proposed a possible solution with the graduates using their human capital as equity to finance HE which could be achieved using an ICL. In the more recent theoretical studies of ICL, papers compare the different possible funding schemes of HE, including ICL, and explore the welfare under different schemes (Del Rey and Racionero, 2010; Garcia-Penalosa and Wälde, 2000). There are also papers using the optimal taxation models with endogenous human capital to study the optimal design of student loans (Findeisen and Sachs, 2016; Gary-Bobo and Trannoy, 2015; Stantecheva, 2017).

Although the workers who graduated from university enjoy a higher level of income compared to the non-graduates on average, for the individual university students, the higher income is not guaranteed as they face potential HE and labour market failures in the future. HE failure meant the student failed to successfully graduate from the university. Labour market failures meant the worker experienced a reduction in income due to being displaced which could happen because of the pressure from international trade and technological changes. Various empirical studies have found that the benefits of completing HE also exist for mature students (>25 years old) and that the labour retraining programs are effective in providing long-term help to displaced workers (Kim and Baker, 2015; Kruppe and Lang, 2018). These empirical results suggest it would be beneficial for the government to provide a second chance at HE to those students who have previously failed their first chance at HE and to provide a labour retraining program to those displaced workers who were previously earning a relatively high level of income. Despite this evidence, to our knowledge, there are no theoretical papers on financing HE that explicitly include a second chance for failed students or a labour retraining program for those who experienced downward labour mobility.

There will be two theoretical papers to fill in this literature gap. The models in the two papers will be based on Gary-Bobo and Trannoy (2015). Gary-Bobo and Trannoy (2015) is chosen because it is the only theoretical paper on the optimal provision of student loans that includes both adverse selection caused by private abilities and moral hazard with unobservable learning effort. Our first paper will examine the way to optimally provide the HE with a government-provided second chance and the effect of this second chance policy. Our first paper addresses the following two questions: "How do we optimally provide a HE system that offers a second chance to all failed students?" and "What is the impact of such an optimally provided second chance on inequality and insurance?" Our second paper will examine the way to optimally provide the HE with the labour retraining program given to those at risk of downward mobility and the effect of the labour retraining. Our second paper aims to answer the following research questions: "How do we optimally provide HE with a labour retraining program?" and "What is the impact of this optimally provided labour retraining programme on inequality and insurance?" Our first two papers go beyond a simple student loan model to explore how to design a student loan system that considers foreseeable potential HE and labour market failures. Furthermore, the empirical papers on the effect of completing HE on mature students and the effect of completing the labour retraining program only measure the effect on those who directly experienced them. Our first two papers provide theoretical predictions on the effect of Second Chance policies when they are optimally provided on the wider economy including the workers who do not need them.

In addition to the optimal provision of HE, we also analyse the problems related to the political decisions on HE and student loans. Many economics papers on the political decisions of the provision of HE often involve the unrealistic assumptions of a benevolent government and direct democracy (Del Rey and Racionero, 2012; Del Rey and Racionero, 2014). A few empirical papers have measured the effect of various political factors on HE spending, for example on the impact of interest groups and the budgetary power of the governor on the government support of HE, but the results are not based on economic models. Furthermore, there are also no attempts in the literature to model the government's decision on HE funding in a regime of student loans that takes into account the issue of the political economy of student debt.

We will use two papers focusing on the political economy dimension of the provision of HE and student loans to fill in these significant literature gaps and answer some of the most hotly debated problems related to the funding of HE using methods from political economy. Our third paper explores how the differences in the constitution between the presidential and parliamentary regimes affect the government's decision on HE spending. We model the differences between the constitutions of presidential and parliamentary regimes based on the sequence of events of the legislative bargaining game of the two regimes in Persson, Roland and Tabellini (2000). The constitution of the presidential regimes is characterized by more separation of powers and less legislative cohesion than the constitution of the parliamentary regimes and it affects the policy choices. We provide a modelling framework for HE spending to demonstrate the different decision-making processes for HE in the two regimes. Because HE spending increases the income of the beneficiary, we also analyse how the constitution of the presidential regime may result in a higher level of inequality than the parliamentary regime. This contributes to the literature examining the potential reasons for the difference in the within-nation income inequality between different countries. In our third paper we answer the following questions: “Do the different political systems such as presidential and parliamentary regimes differ in the level of HE spending?” and “Does a presidential regime lead to a higher degree of inequality compared to a parliamentary regime?” In the UK and the USA, the level of tuition fees and student loan debt have been steadily increasing over time and some US papers have examined the factors causing this increase (Belfield, 2013; Goodell, 2016). A common factor found in these papers is that attending for-profit colleges will lead to a significantly higher average level of debt after graduation. Our fourth paper examines the incentives facing an incumbent government on the decision of the student loan levels when facing political uncertainty and a polarised society with the model based on Persson and Svensson (1989). The incumbent government may increase the level of student loans even if its voters do not engage in HE. In our fourth paper, we explore the question: "Why would an incumbent government increase spending on HE and student loan levels when its voters are not directly benefitting from HE?" We contribute to the literature examining the rise in tuition fees and debt levels by presenting an additional potential factor to explain the increase that has not been studied before.

1.2 Organisation of the thesis

The thesis is composed of Chapter 2 detailing and reviewing the related literature and the research gaps the thesis intends to address while Chapters 3 to 6 are four theoretical papers to address the research questions on HE, student loans and Second Chance policies.

Chapter 2 will first define the concepts used in the thesis, including the definition of HE, student loans, mature students, political regimes, political uncertainty and polarisation as well as the differences between the Second Chance policies used in Chapter 3 and Chapter 4. We then survey the related empirical and theoretical literature for the four papers, the respective literature gaps and the contribution each paper made. We also focus on the three key theoretical papers that the thesis is based on: Gary-Bobo and Trannoy (2015), Persson, Roland and Tabellini (2000) and Persson and Svensson (1989).

Chapter 3 presents our first paper which investigates how to optimally provide a HE system that offers a second chance to all failed students and primarily analyses the impact of such an optimally provided second chance on inequality. We extend the model of Gary-Bobo and Trannoy (2015) by adding a second chance at HE to all failed students and comparing the degree of ex-ante inequality to the “equal treatment” second-best optimum found in Gary-Bobo and Trannoy (2015). We examine the optimal repayment schedules and the relations between different utilities in the second-best optimum. Our results agree with other papers on the necessity of the ICL to implement the second-best optimum. Furthermore, we found the introduction of a second chance has increased the degree of ex-ante inequality.

Chapter 4 presents our second paper which investigates how to optimally provide HE with a labour retraining program and primarily analyses the effect of this retraining program on inequality. We extend the model in Gary-Bobo and Trannoy (2015) by adding the possibility of experiencing downward labour mobility after succeeding in HE and a labour retraining program to help those at risk to maintain their earning levels. The degree of inequality is also compared to the results found in the “equal treatment” second-best optimum of Gary-Bobo and Trannoy (2015). Similar to our first paper, our results show that the ICL is needed to implement the second-best optimum. We also found the introduction of labour retraining has increased the degree of ex-ante inequality. Importantly, the government’s ability to identify which workers are at risk of downward mobility is important for the effect of the retraining program on ex-post inequality. Ex-post inequality can only be reduced when the government can identify who needs retraining.

Chapter 5 presents our third paper which investigates whether the design of a country's constitution would affect its level of HE spending and the level of income inequality. Persson, Roland and Tabellini (2000) created the separation of powers by separating the voting of the tax and spending proposals, each made by a different agenda setter and created legislative cohesion by giving veto rights to the coalition's junior partner which gave this junior partner and her voters more bargaining power and induces legislative cohesion. We use an infinite horizon model based on Persson, Roland and Tabellini (2000). We modified the original model by assuming one group requires income-increasing HE spending instead of a direct transfer and allowing the tax rates on different groups' income to differ as long as the tax revenue from each group is equal to each other. Our most important result is to show the different ways HE spending is determined in the presidential and parliamentary regimes. Additionally, we show the parliamentary regime generally provides more public goods and results in more political rents than the presidential regime but in certain cases the opposite is true.

Chapter 6 presents our fourth paper which examines whether an incumbent government could in theory be motivated to increase the level of student loans and tuition fees when facing political uncertainty and a polarised society. We modify the model of Persson and Svensson (1989) which examined a similar question about public borrowing by adding a student loan level decision representing an investment into funding HE. We assume the country's HE is completely funded by government-sanctioned student loans. Similar to the insights of other theoretical papers analysing the incentives on public investment, our results show that the incumbent government is incentivised to use student loans level to influence the decisions of the future government to be closer to its preferred policies when the incumbent's government re-election prospect decreases. However, we do not predict the incumbent government must increase or decrease the level of investment when facing uncertain re-election prospects. Furthermore, our computational results support our theoretical analysis in indicating that the incumbent government will change the level of student loans in response to shifts in its re-election prospects.

Chapter 7 concluded the thesis with a summary of results, a discussion on the contributions and limitations of the thesis and directions for future research.

Chapter 2: Background and Literature Review

2.1 Introduction

The thesis will include four theoretical papers on the provisions of higher education (HE), student loans and Second Chance policies. It is divided into two main parts. The first part, including Chapter 3 and Chapter 4, focuses on the optimal design of student loans by combining Second Chance policies with HE. It builds on the literature analysing the optimal design of student loans, especially on Gary-Bobo and Trannoy (2015) and explores how to design a student loan system that takes into account foreseeable potential HE and labour market failures. The second part, including Chapter 5 and Chapter 6, will analyse the political economy dimension of HE spending and student loan problems. It will mainly draw from Persson, Roland and Tabellini (2000) and Persson and Svensson (1989). We will analyse the effect of the design of the constitution, political uncertainty and polarisation on the level of HE spending and student loans using methods from the political economy. In this chapter, we will review the empirical and theoretical literature background as well as the key papers for each subsequent chapter. Moreover, we will examine the literature gaps they filled.

The structure of this literature review will be as follows. Section 2.2 will define the fundamental concepts used in the thesis. Section 2.3 presents the literature review for Chapter 3 and Chapter 4 and includes one section on stylized facts about the benefits of Second Chance policies and another section exploring the related theoretical field of study of funding HE using student loans. Section 2.4 offers the literature review for Chapter 5. It includes a detailed description of the studies on which Chapter 5 is based and highlights the literature gaps that it fills. Section 2.5 contains the literature review for Chapter 6. It also includes stylized facts on the growth of student loan debt and explains how Chapter 6 contributes to this literature.

2.2 Definitions of Main Concepts

For the definition of HE, we refer to the UK experience. In the UK HE is distinct from further education (FE). The HE is taught in universities with Undergraduate and Postgraduate level courses while FE refers to any study after secondary education that is not HE. FE is provided in FE colleges and it is at a lower level than HE. However, this distinction between different education systems is ignored in the theoretical literature on student loans, especially within the optimal taxation literature with endogenous human capital. In these papers, the content of education is not relevant in designing the optimal student loan, it is the effect of education on labour market outcome that's important. The only difference between HE and FE could be a difference in the level of human capital investment. In Gary-Bobo and Trannoy (2015) and my model, the quality of education has two different levels. The higher quality could be seen as HE while the lower quality could be seen as FE. This difference is not relevant to the real-life difference in definition between HE and FE and they only differ in their effect on students' labour market outcomes.

The student loans in this thesis are defined as government-provided loans for students to fund a specific education program. In the UK, student loans are provided by the Student Loans Company, a non-profit government-owned organisation. The key type of student loan in question is the income-contingent loan (ICL), these are loans that the students only repay once their income is above a certain threshold. In my models in Chapters 3 and 4, the student loans are offered by the government which cover the cost of HE and the Second Chance policies. The students will repay these loans once they are in the labour market. The levels of repayment are dependent on the quality of education, the current earnings and potentially on the past earnings as well.

The second chance explored in Chapter 3 refers to policies that support adult HE and is defined as a measure implemented in adult HE that grants students who did not succeed in their first attempt at HE the opportunity to enrol again and complete their studies. Chapter 4 deals with a general labour retraining program that is provided to all workers in danger of downward labour mobility. Labour retraining programs could be seen as having another chance in the labour market. Together they make up the two Second Chance policies I analysed in the thesis.

For mature students, UCAS defines them as anyone going to university or college after a period of time out of full-time education. The mature students are by definition the ones receiving adult HE. In much of the empirical literature that estimates the benefit of providing HE to adult learners, these adult learners are any students who are over 25 years old (Bennett, Blundell and Salvanes, 2020; Caruth, 2014; Mason, 2020). In other studies, adult learners are not specifically defined but are just referred to as students older than typical university students. Like in Carnoy et al. (2012), the data pool for adult learners is a group of older students with an average age of 32. In Chapter 3's model, this exact age for defining a mature student is not important. To capture the fact that the students are older, the second chance policy is modelled so that it is given after the students have spent time working in the labour market.

In terms of definitions of political regimes, we refer to Persson and Tabellini (2003). A regime is classified as presidential if the government's survival is not subject to a confidence procedure. If the existence of the government is subject to a confidence procedure, then the regime is classified as parliamentary. In a regime where both the president and the legislative assembly have some control over the appointment and/or dismissal of the executive, if this control primarily rests with the president, such a regime would be classified as presidential.

In Chapter 6, the term “political uncertainty” simply means that the incumbent government is uncertain of its re-election. The term “polarisation” refers to the difference in preferences for optimal policies’ levels between voter groups that are represented by partisan politicians.

2.3 Literature Review for Chapters 3 and 4

In section 2.3.1 below, we provide the motivation for the first part of this thesis. I will outline the current student loan system in the UK and current government support for Second Chance policies. Then we will draw from the papers highlighting the benefits to the individual and labour market of the labour retraining programs and of HE (and high school education) for mature students. Table 2.1 gives a general overview of some of the empirical papers that found them to be beneficial.

Section 2.3.2 gives an overview of the literature on funding HE with student loans. We especially present papers that use optimal taxation modelling with endogenous human capital to study the optimal design of student loans. Then we focus on how Gary-Bobo and Trannoy (2015) differs from the existing literature and how Chapters 3 and 4 will extend the model of Gary-Bobo and Trannoy (2015). Table 2.2 gives an overall view of the differences and findings of the most relevant theoretical papers in Chapters 3 and 4. In section 2.3.3 the contributions to the literature of Chapters 3 and 4 are highlighted.

Table 2.1: Results of the Benefits of Adult HE and Labour Retraining Summarised

Paper	Topic Examined	Questions of the Paper	Data Used	Main Findings
Bennett, Blundell and Salvanes (2020)	Adult High School Education	What is the impact of completing high school as an adult?	Adults over 25 years old that were high school dropouts.	Completing high school later in life also led to higher earnings and a higher probability of employment.
Carnoy et al. (2012)	Adult HE	What is the impact of online HE on adult learners?	Students with an average age of 32 attended the Open University of Catalonia.	Some programmes increased earnings but some did not.
Desjardins and Lee (2016)	Adult HE	Is the benefit of completing HE as an adult lower than completing it as a younger student?	Data set of OECD Survey of Adult Skills which includes adult learners between 25 to 65 years old from 23 countries.	There is no evidence that completing HE at an older age is less advantageous than completing them at a younger age.
Dorsett, Lui and Weale (2010)	Adult HE	Are there any benefits of lifelong learning?	Data on men aged 25 to 60 from the British Household Panel Survey from 1991 to 2007.	Upgrading to Level 4 qualifications beyond conventional age resulted in higher earnings.

Kim and Baker (2015)	Adult HE	What is the impact of graduating from a two-year college?	Workers aged 25 to 55 from the National Longitudinal Survey of Youth 79.	Acquiring a two-year college degree led to a 4.2% increase in hourly wage
Bailey, Chapain and de Ruyter (2012)	Labour retraining	What is the impact of a job loss on the employment trajectory of ex-workers?	Ex-workers of an MG-Rover plant in the city of Birmingham.	Strong retraining initiatives have brought disadvantaged workers who are older and less skilled back to employment.
Kruppe and Lang (2018)	Labour retraining	Is the German labour retraining policy effective?	Data includes all participants of Germany's labour retraining program from 2004 to 2007.	Retraining on average has strongly improved the employment prospects of the participants.
Dyke et al. (2006)	Labour retraining	What is the employment effect of the welfare-to-work program in the US?	The welfare recipients in Missouri and North Carolina.	The more intensive program that improved human capital resulted in long-run earning gains.
Sianesi (2008)	Labour retraining	Do different active labour market policies differ in their effectiveness?	Data set of Händel and Akstat which includes labour market information of unemployed in Sweden.	Labour training programs are most effective in helping unemployed workers when they provide work experience and on-the-job training.

2.3.1 Motivation for Chapters 3 and 4

The policy focus of Chapters 3 and 4 is about student loans, provision of HE to mature students and labour retraining programs. The first income-contingent loans (ICL) for students were implemented by the Australian government in 1989 by the Higher Education Contribution Scheme (HECS). In England, the public universities charged no tuition fees before 1998 and offered maintenance loans. After 1998 upfront tuition fees of up to £1,000 were introduced. A mortgage-style repayment system was replaced by an income-contingent repayment system. Throughout the years many reforms were made to England HE finance with a shift towards higher tuition fees and an increase in the amount available for ICL. The most famous change was perhaps the tripling of the tuition fee to £9,000 per year in 2012. The history of the UK HE student finance can be found in many of the papers assessing the impact of UK HE reforms, for example Murphy, Scott-Clayton and Wyness (2018).

The UK government also implements policies to support mature students and vocational training programmes although in a more limited way (UK Government, 2022a). The government currently does not seem to offer a "second chance" at completing HE for students who desire and are capable of undertaking HE studies but who for some reason have failed. However, the government does provide policies that encourage the employees to give a "second chance" with apprenticeship training by paying up to 95% of the cost up to the funding band maximum to the training provider. The employment allowance also encourages small employers to hire more workers by reducing eligible employers' national insurance liability by up to £5,000, so students without qualifications will have a higher chance of being employed. The government also provides Advanced Learner Loans for courses of levels 3 to 6 qualifications at an approved college or training provider and it is income contingent (UK Government, 2022b). Despite the lower level of attention, the government is paying more attention to support for older students compared to the fall in spending by 35% between 2009-2010 and 2019-2020. A recent White Paper from the Department of Education includes a plan to increase and reform funding for programs taken by more mature students (see Sibieta, Tahir and Waltmann (2021) for a summary). One planned policy is to increase funding for mature students through a National Skills Fund which includes entitlements for three fully funded selected level 3 qualifications for adults who are earning below the National Living Wage.

HE has important consequences for the labour market outcomes throughout a worker's life. The benefits of HE on a worker's income are widely documented. The UK Department of Education (2020) reported that graduates have a higher employment rate (86.4%) compared to non-graduates (71.3%), as well as an extra £9,500 median salary premium over the non-graduates. Many papers estimate a positive labour market outcome for achieving a HE degree (Baum, Ma and Payea, 2010; Baum, Ma and Payea, 2013; Ma, Pender and Welch, 2016).

Interestingly, although many papers measure the benefits of HE, the benefits of obtaining HE for older students are not given much attention as reported by Bennett, Blundell and Salvanes (2020), Carnoy et al. (2012) and Caruth (2014). Donaldson and Townsend (2007) found that only around 1-2% of published papers in their seven selected HE journals published between 1990 and 2003 deal mainly with adult learners. Caruth (2014) argued that national policies are biased toward youth HE and universities are not designed for adult learners. Mason (2020) found the UK government's support for prospective adult learners over 25 years old to be weak and Desjardins and Lee (2016) found that adult learners are viewed as causing inefficiency in the HE systems.

In the empirical papers that do deal specifically with mature university students, they all find completing HE is beneficial for older graduates in the form of a higher probability of employment and wage level (Desjardins and Lee, 2016; Dorset, Lui and Weale, 2010; Kim and Baker, 2015). Completing high school late in life could also bring benefits. Bennett, Blundell and Salvanes (2020) measured the impact in Norway of lowering the opportunity cost of attending high school on previous high school dropouts. The late high school graduates' labour market prospects were improved. Given these benefits, it is not surprising to come across recommendations for easing access to education for adults, especially by offering financial support (Kim and Baker, 2015; Mason, 2020). Orr and Hovdhaugen (2014) compared the approaches taken by Sweden, Norway and Germany on widening access to HE by removing academic success at secondary school as the determining factor for entering HE. One of the aims of these policies is to allow older people to re-enter HE to update their skills as the population ages. Furthermore, these results indicate the potential labour market benefits of a government policy that gives another chance to workers to complete their HE program if they failed before.

Labour retraining aims to provide long-term help for displaced workers. There are different reasons for a worker in a developed country with a decent job to be displaced, e.g. technological changes including automation or competition from developing countries (Utar, 2018). These two factors often affect the manufacturing sector disproportionately. There have been many empirical papers that have estimated the scale of the displacement problem (Couch and Placzek, 2010). Autor, Dorn and Hansen (2013) assessed the impact of Chinese import competition on the US labour market and found it to result in not only lower wages and employment but also lower labour force participation and increased benefits usage.

However, the size of these negative effects is not without disagreement. Rothwell (2017) reassessed the impact and although agreed with the lower wage rate in the highly exposed manufacturing sector and its contribution to rising inequality, he didn't find the other negative effects measured in Autor, Dorn and Hansen (2013) which they attributed to being instead caused by a period of general poor macroeconomic performance. Acemoglu and Restrepo (2020) found the negative impact of implementing robots is concentrated on low and medium-skill occupations with one extra robot for 6 fewer workers. Nedelkoska and Quintini (2018) found 14% of jobs in the OECD to be highly automatable with the jobs in the manufacturing and agriculture sectors, characterised by a low level of education, being most at risk. These papers although all dealt with different topics, all similarly pointed out the fact that due to various factors, in developed countries workers that previously earned a decent level of income are facing a higher probability of unemployment and/or have lower income than before.

For the empirical papers that estimated the benefits of the various retraining programs in different countries. Bailey, Chapain and de Ruyter (2012) measured the effectiveness of a retraining policy response to a factory shutdown on workers' subsequent labour market experience. In combination with other factors, the retraining initiative was effective in bringing displaced workers back to employment although for lower wages. Kruppe and Lang (2018) analysed the impact of retraining in Germany and found a larger benefit for female participants and differences in effects between occupation choices. Jacobson, LaLonde and Sullivan (2005) found retraining to be a good investment as unemployment insurance cannot provide displaced workers with long-term help. For US studies, both Hotz, Imbens and Klermans (2006) and Dyke et al. (2006) have found that the more intensive retraining programs can have long-term benefits by increasing participant's human capital and these are more effective than the less intensive programs such as job search assistance. Sianesi (2008) evaluated the effectiveness of the various active labour market policies in Sweden in improving the employment probabilities of the unemployed. It found that a training program is more effective when it is more similar to actual employment and gives workplace experience to participants. For retraining workers, the results are more mixed in comparison to the results from completing HE degrees as adults. After retraining, the workers can return to the labour market but their earnings are not certain to be higher than their pre-displaced level and some retraining programmes are not as effective as others.

Many papers suggest either increasing spending on government retraining programmes or subsidising private firms' retraining. Vocational training has been suggested to deal with technological change such as automation that is making predominantly low-skill workers obsolete. (Gvaramadze, 2010; Manyika et al., 2017). Lerman, Loprest and Kuehn (2020) suggested ways to improve the labour retraining programme in the US by making it more straightforward and labour market focused to fix the skill mismatch between college graduates and employers. More recently because of Covid-19, the UK government intervened and used the Job Retention Scheme to avoid large-scale redundancies. Mayhew and Anand (2020) commented on the effectiveness and the problems associated with this policy and along with Li, Valero and Ventura (2020) suggested increasing funding for job-related training to deal with the Covid-19 labour market fallout which disproportionately affected young people.

The main difference between adult HE and labour retraining seems to be that labour retraining is specifically aimed to mitigate the problem of downward labour mobility while adult HE is just regular HE but specifically aimed at mature students. The UK currently uses income-contingent repayment for HE which offers important labour benefits but there is less support available for mature students who failed HE as well as for labour retraining programs. However, research has shown or at least indicated that the Second Chance policies are effective in improving the labour market outcomes of the participants. In many countries including the UK, HE is funded by student loans. Since the Second Chance policies are offered to older members of the economy who may or may not succeed in HE when they are younger, current students could reasonably expect the possibility of needing these policies when they are older. Given the benefits of these Second Chance policies, it is important to analyse the optimal design of current student loans that incorporate these Second Chance policies.

Furthermore, the various empirical papers naturally only estimated (or for the second chance at HE, only indicated) the impact of these Second Chance policies on those who directly experienced them. There is a gap in the literature on how these policies will affect others in the economy that doesn't need them for various reasons. Therefore Chapters 3 and 4 provide additional theoretical predictions on the effect of these optimally provided policies on the wider economy, which includes the workers that don't need them.

2.3.2 Related Theoretical Studies

In many countries, the HE is increasingly being funded by student loans. Many of these student loans differ from ordinary loans in that they are ICL. As a result, many papers that studied the method of funding HE at least include an analysis of the ICL. ICL is not a new idea in theory. Many papers have proposed ICL for funding HE and discussed its advantages and/or disadvantages. One of the reasons that the student loan market suffers from market failures is due to the riskiness of education investment as well as the nature of human capital itself. Friedman and Kuznets (1945) is perhaps the earliest paper that proposed ICL to solve these market failures. ICL is then restated and refined in Friedman (1955). ICL is proposed for vocational education instead of a mortgage-like loan because for lenders the students have no security but their future income unlike in capital investment with physical collateral and the high risk of failure led to underinvestment of human capital as lenders would provide insufficient capital for education. The risk is high since students may not successfully graduate and they could change their interest while the university may not provide the appropriate labour market skills. Friedman (1955) did not use ICL for income redistribution nor account for information asymmetry in the HE, but these aspects are introduced in later papers and importantly the idea of ICL will always be related to investment in one's human capital.

Shell et al. (1968) analysed the feasibility and desirability of an ICL proposal. The advantage of ICL is again recognized as a potential tool to improve the imperfect capital market for HE and repayment could be coordinated with a government income tax which in theory could make long-term collection feasible and relatively costless. Nerlove (1975) commented on the ICL proposal made by Friedman (1955) and discussed potential reasons for modifications. The concerns for the operation of an ICL at the time were the political and social resistance to such an idea and a lack of understanding of how individuals' behaviour would be affected by these loans makes planning for these loans difficult. It commented on the failure of the Yale plan which is a risk-pooling ICL which suffers from adverse selection problems by encouraging higher-ability students to enrol in other universities.

The above papers examined ICL before ICL was widely used in student loans. Chapman (2014) gave a general background on the conceptual and empirical basis for ICL and its adoption history around the world. It summarised the theoretical argument for using ICL to finance HE developed from past papers over time, they include lack of collateral in human capital investment as well as the inherent risks in education and the labour market. An ICL for HE compared to a typical bank loan that is insensitive to the borrower's financial situation, provides consumption smoothing and insurance against default. The borrower pays nothing when facing periods of low income and pays proportionally more in periods of high income. Chapman (2014) and Van Long (2019) also stated that the ICL should be provided by the government because the government has a superior ability to collect information about the income of its citizens through the income tax system. The use of ICL fits with the potential role of the government as a manager of risks and this concept has potential applications beyond student loans. Shireman (2017) also gave a history of ICL but is focused on the US. It pointed out that an ICL system is simply a way to tax the higher income resulting from HE in order to finance HE investment in the first place, so it is equivalent to a "graduate tax". If more and more people partake in HE, or if everyone goes to the university as in a model, the graduate tax would just be a tax on higher income.

Besides discussion on the potential benefits of introducing ICL to HE, many theoretical papers also dealt with this issue. These papers compare the different funding schemes for HE, to see which policy would achieve the largest welfare gain compared to the case without government intervention.

De Fraja (2002) found student support policy is justified and many other papers seek to find which policy instrument is the most justified without dealing with asymmetric information, for example, Migali (2012) used a calibrated model to examine how the degree of risk aversion and earning volatility would influence student's preference on the HE funding method between mortgage-based loan or ICL. It found that ICL is preferred especially by students from poorer families and with greater earning volatility because it offered insurance benefits over mortgage-based repayment. Del Rey and Racionero (2010) analysed the insurance role and effect on HE participation by several HE funding schemes. The model was built on Garcia-Penalosa and Wälde (2000) and solely focused on efficiency. In this paper, a policy is optimal if it maximizes the output and found that the optimal policy should fully insure the lowest ability individual who should enrol in HE which achieves optimal participation level. The policy to achieve this is an ICL that covers both the financial costs of education and forgone earnings. However, this is only acceptable if individuals have the same probability of succeeding in education. If this probability is positively dependent on ability then the higher-ability student wants to opt out of such a scheme. Garcia-Penalosa and Wälde (2000) and Hanushek, Leung and Yilmaz (2014) on the other hand using different methods, found ICL to not be the best policy when other options are introduced. However, all these papers lacked some important attributes related to student loans, for example, some of them assume the government have the same information on student ability as students themselves, so there is no adverse selection, and some of them did not account for individual study effort so there is no moral hazard. Moreover, some assume students have the same abilities.

Some student loan papers dealt explicitly with some aspects of asymmetric information. These papers examined a wide range of related issues using different methods. Cigno and

Luporini (2009) stated the problem of breaking even requires successful graduates to pay back more than a loan at a normal interest rate which causes students with the means to self-finance to not participate. It focused on the moral hazard aspect of the HE which is individual study effort. The way the paper assessed policy is like optimal tax literature, instead of starting with different policies and comparing them according to some criterion, they maximize a social welfare function and investigate how to implement it. However, there is no adverse selection as the government is assumed to have the same information as students on their abilities. The result is implementable by a "scholarship scheme financed by graduate tax" which is an ICL without restrictions on setting the size of loan/tuition fees and interest rate. If there are, then ICL would be suboptimal as it excludes more than the optimal amount of high-ability students from credit-constrained families. In the UK, the adjustment of ICL policy saw a massive increase in tuition fees and maximum student loans as well as the eligibility of who could borrow them over a relatively short period.

Del Rey (2012) focused on adverse selection to explore the scope for the government to produce an efficient scheme which guarantees student participation and does not rely on taxpayers or force students to participate in the programme. This question is also explored in Cigno and Luporini (2009) but theirs was from a moral hazard angle. The paper set up a set of risk-averse students with different innate abilities which affect their probabilities of success. The model's equilibrium depends on the proportion between high- and low-ability individuals. The results show that if there are too many high-ability individuals the equilibrium does not exist without government coercion for a programme like ICL, the high-ability students do not want this much insurance and would opt out if possible. This finding highlights the similarity between the ICL programme and traditional insurance which high-ability students find it beneficial to not be grouped with low-ability individuals. Consequently, ICL in real life is often provided by the government as a major or sole way to fund one's HE. This finding which paints government ICL in a negative light in that it restricts choices is contrasted to Stiglitz (2016) that laid out the many problems in the private loan market for funding education and the many potential advantages of government-provided ICL.

Chatterjee and Inonescu (2012) addressed the issue of the financial risk of attempting college and found it to be welfare-improving if loan forgiveness is offered to failed students. An ICL also offers insurance, but the loan is not immediately forgiven if students fail college. Both adverse selection and moral hazard are modelled but the paper differs from others by using quantitative analysis of US college statistics to model the optimal insurance to different ability groups given the modelled costs of moral hazard and adverse selection. In other papers, the student loan contract is designed to prevent moral hazard and/or adverse selection. Here loan forgiveness is found to be optimal despite it induces moral hazard and adverse selection. Matsuda and Mazur (2022) quantitatively evaluated the impact of ICL reform and found that ICL increases the welfare and the cost of moral hazard and adverse selection induced by the reform is mild.

Overall, despite the differences in questions and the way the problem is treated, the student loan literature all showed that the standard mortgage-style loan is not suitable for student loans, confirming the theoretical benefits in earlier papers on financing HE by replacing mortgage-style loans with ICL. The above literature generally does not derive the optimal policy but considers and compares specific policy interventions in the education market. Chapters 3 and 4 is related to the literature that analyses the optimal student loans using optimal taxation models with endogenous human capital. Optimal taxation is the study of how to design a tax system to maximize social welfare function subject to a set of constraints. Makris and Pavan (2021) studied the effect of learning by doing on the design of optimal taxation in which human capital accumulation is a side product of the labour supply process. In contrast, we focus on human capital accumulation through education. This design of the tax system could be translated into the design of student loans.

The optimal taxation literature starting from Mirrlees (1971) typically assumes exogenous abilities between agents and does not consider an endogenous human capital investment. Some papers included endogenous human capital and jointly considered optimal taxation policy with human capital policy. In one approach the education system is financed by education subsidies. For example, Bovenberg and Jacob (2005) used a static model that studies optimal education subsidy with endogenous human capital formation in terms of redistributive concerns and finds optimal subsidy alleviates the distortion in human capital investment induced by the redistributive policy. Unlike in other papers, human capital accumulation had no risks involved. Benabou (2002) examined the impact of progressive income tax and education subsidies on income levels and distribution in a dynamic setting. For optimal government policy in the dynamic setting with the government cannot observe private abilities, Bohacek and Kapicka (2008) found positive education subsidy is an optimal policy since labour supply is distorted by positive marginal income tax which causes downward distortion to the level of schooling. This distortion is reduced by a positive education subsidy. Kapicka and Neira (2019) modelled a two-period life-cycle economy with partially observable human capital investment and found the optimal education subsidy is substantial (between 86 to 100% of the cost of education).

The other approach to financing the education system is by analysing the optimal repayment schedule. With ex-ante identical agents but unobservable human capital investments and productivity shocks, Anderberg (2009) considered the consequences on optimal education and tax policy when individuals make a human capital investment while facing future wage uncertainty due to shocks. It finds optimal education policy and tax policy depend on whether education increases or decreases the future wage risk, if it increases wage risks optimal policy should ensure that there is less education in equilibrium and if it decreases wage risk then optimal policy should ensure there is more education in equilibrium. Grochulski and Piskorski (2010) extended this analysis to include multiple working periods and found that optimal policy requires "deferred taxation of capital". The marginal tax rate must be history-dependent with high capital tax for those with low labour income to prevent high-skilled agents from pretending to be low-skilled agents later in life. Deferred capital tax is efficient since more information is revealed about an individual's human capital investment. The optimal policy can be implemented by an income-contingent repayment scheme in which the repayment depends on shocks experienced throughout the working period. In both papers at optimum, the consumption is distorted towards current consumption.

Conceptually the following three papers are closer to this thesis, compared to Anderberg (2009) and Grochulski and Piskorski (2010). They additionally include heterogeneity between agents, individuals with different innate abilities will be affected differently by education. In Findeisen and Sachs (2016), the wage is determined endogenously by an individual's one-time binary education decision before entering the labour market. The result is that government-provided ICL can implement the second-best Pareto efficient optimum. Extension to the one-time education decision is to include human capital investment in every period, this is done in Stantcheva (2017) and ICL is still found to be an optimal policy. Koeniger and Prat (2018) extended the literature by focusing on financial and human capital transmission from parents to children and studied the optimal taxation of intergenerational transfers in a model with altruistic dynasties. Again, the social optimum is implemented by ICL and education finance alone can achieve this optimum. The timing of human capital investment is also different, in other papers like Stantcheva (2017) human capital investment affects productivity "today" while in Koeniger and Prat (2018) the investment benefits the children in the next period.

Gary-Bobo and Trannoy (2015) combined the theoretical student loan problem with an optimal income taxation problem that also included heterogeneity in innate abilities as well as uncertainty in the labour market. The student's innate type affects the probability of success in education. The labour market type is affected by the quality of education but is not 100% determined. The conclusion is still that optimal policy is based on ICL.

The one area that is mostly absent from other research papers is moral hazard constraints on the educational decision, the students are assumed to undertake their education with a high level of effort and no effort cost needs to be satisfied. Therefore, the key departure/extension in Gary-Bobo and Trannoy (2015) is the inclusion of genuine moral hazard for study effort and is combined with unobserved ex-ante types. The human capital investment is completely unobservable. There aren't many papers on student loans that include asymmetric information let alone consider both hidden action and hidden types. The ex-ante moral hazard is about the unobservable study effort which affects the study's success. Information asymmetry is an important aspect of the HE loan market and given that it is unrealistic to assume that study effort is perfectly observable or has no effect, I decided to follow and extend the model in Gary-Bobo and Trannoy (2015) which is less complicated than models with continuous types but capture more properties of the HE decisions. In Gary-Bobo and Trannoy (2015) a static model is used but the following aspects of HE investment are included: risky labour market outcomes, adverse selection, moral hazard and risk aversion. The three papers that were said to be conceptually close used more complicated dynamic models but didn't include genuine moral hazard problems for education. Table 2.2 presents a synopsis table to show the differences between the most relevant papers on ICL in HE.

Table 2.2: Papers of Optimal Taxation Model with Endogenous Human Capital

Paper	Number of Time Periods	Moral Hazard in Learning	Adverse Selection/Ex-ante Differences	Number of Individual Types	Main Findings
Anderberg (2009)	2	No	No	Singular	Optimal tax/education policy is dependent on the riskiness of education.
Grochulski and Piskorski (2010)	Dynamic	No	No	Singular	Deferred taxation is necessary when the human capital investment has a risky impact on wages and is private information which is only revealed later by observing labour income history. This deferred taxation is very similar to an income-contingent repayment scheme.
Findeisen and Sachs (2016)	Dynamic	No	Yes	Continuous	The optimal repayment schedule for education investment could be approximated by a linear schedule that depends on income.

Stantcheva (2017)	Dynamic	No	Yes	Continuous	Building on Findeisen and Sachs (2016), even when risky human capital investment occurs in every period along with other shocks, an ICL that is also conditioned on human capital investment can implement the optimum.
Koeniger and Prat (2018)	Dynamic with dynasties	No	Yes	Continuous	In a dynastic economy with endogenous human capital investment and transmission of financial and human capital from parents to children. The social optimum can be implemented using ICL alone.
Gary-Bobo and Trannoy (2015)	Static	Yes	Yes	2	The optimal way of financing HE is by ICL or a graduate tax even with moral hazards in education efforts. The paper found the second-best optimum to exhibit "equal treatment", the poststudy prework expected utility of different student types is equalized.

2.3.3 Contributions of Chapters 3 and 4

Studying the optimal policy for HE combined with another government programme has been done before in Paluszynski and Yu (2023) which examined the optimal design of student loans and retirement policies jointly. Despite the evidence of the benefits of HE for more mature students, to our knowledge, there is no paper from the optimal taxation perspective or other theoretical studies on financing HE that explicitly includes the second chance education for adults into the optimal income taxation model with human capital, that also includes moral hazard problem for the education and labour market risk. Chapter 3 addresses the following two questions: 1. How do we optimally provide a HE system that offers a second chance to all failed students? 2. What is the impact of such an optimally provided second chance on inequality and insurance?

In some of the dynamic models, agents can invest in human capital each period, but here the focus is solely on investment by those who failed HE initially. There is no paper explicitly exploring the effect on allocations if the government guarantees a second chance to those who have failed in their first HE opportunity, how that will affect an economy's equity and how that will affect the design of student loans and income tax. By directly forming these problems, we believe it is much clearer to see their effects. Intuitively, the economy could be more equitable given that there are more opportunities for a student to ultimately succeed in HE and earn a higher wage. Therefore, the first paper of this thesis is to extend the model used in Gary-Bobo and Trannoy (2015) by adding another period for a second chance education for those who have failed their youth education in the first period and explore any insights into optimal policy designs. The modelling of this second chance education is based on a political economy paper by Arawatari and Ono (2009) that examined what are the factors that determine a nation's voters' choice of taxation policy when the poor have a second chance of success at a later period. Therefore, this paper could also be seen as how the government should optimally provide this second chance.

The second paper of the thesis is another extension of Gary-Bobo and Trannoy's model by including a downward mobility risk for those who succeeded in their HE initially. Chapter 4's research questions mirror Chapter 3: 1. How do we optimally provide HE with a labour retraining program? 2. What is the impact of this optimally provided labour retraining program on inequality and insurance?

Similar to our first paper, there also haven't been papers that combined downward mobility and labour retraining to student loans within optimal taxation literature with endogenous human capital. There is no paper explicitly exploring the effect of an optimally provided labour retraining program on an economy's equity, we will fill in this literature gap by extending the model in Gary-Bobo and Trannoy (2015) and compare our results to it. As shown by stylised facts, this is another important part of the current economy which is about the people who felt their skills are made redundant by various factors that are outside their control and the benefits of government providing retraining opportunities for those who face this risk to update their skills. This downward mobility risk is modelled based on Arawatari and Ono (2015) and the retraining programme has its moral hazard constraints. The more talented agents are more likely to succeed initially but it also means they are subjected to more downward mobility risks. I want to examine the influence of this retraining programme on the model's results. Furthermore, both papers would be further extended by relaxing the assumption that the government is assumed to be able to commit to its promises of future policies and examine time-consistent policies.

2.4 Literature Review for Chapter 5

The previous surveyed literature so far is shedding light on what kind of education finance and tax policy the government should implement and not on what determined the level of support for HE and related policies by the government. Chapter 5 explores the potential difference in HE spending between presidential and parliamentary regimes. Section 2.4.1 gives some examples of the theories that have been applied in the research of HE funding and identifies the weakness within both economics literature and empirical papers concerning the political analysis of HE funding.

Data show presidential regimes are associated with a higher level of inequality than parliamentary regimes. Chapter 5 also examines whether the presidential regime itself is responsible for the higher level of inequality compared to the parliamentary regime. Section 2.4.2 first examines the difference between presidential and parliamentary regimes and how they are modelled in the literature. We also identify the gaps in the existing literature and explain how our third paper will fill those gaps. In Table 2.3 we summarised some of the theoretical and empirical papers within the positive constitutional economics and their findings. Table 2.3 highlights the lack of theoretical analysis of the effect of different political systems on the degree of inequality within the nation.

2.4.1 Literature Gap About the Political Determinants of the Level of HE Spending

Various theories have been used in the research related to the funding of HE (Elbasir and Siddiqui, 2018; Zhang, Kang and Barnes, 2016). There are many issues concerning HE funding. The principal-agent theory is concerned with the degree of control the government (the principal) has over the public universities (the agent) through policies and regulations (Gornitzka et al., 2004; Kivistö, 2007; Liefner, 2003; McLendon, 2003; Sanginmvibool and

Conglerttham, 2016). It is also used to analyse the adverse selection and moral hazard problem that exist between the government and the public university as the university may present false information to obtain funding or the public university pursue their private interest rather than accomplish the agreed goal set by the government (Kivistö, 2005; Kivistö, 2008). The Research Assessment Exercise (RAE) used in the UK and the current Research Excellence Framework (REF) is the way the government screens the universities to provide information on the quality of different universities and to inform the allocation of funding for research (Thomas, 2001; UKRI, 2022) (UKRI stands for UK Research and Innovation). The resource dependence theory assumes that the essential aim of any organisation, like a university, is to survive by obtaining an uninterrupted flow of resources from the environment (Pfeffer and Salancik, 2003). It predicts the more an organisation is reliant on a particular supplier, the greater the impact that supplier has on the said organisation. Some papers applied the resource dependence theory to HE funding (Pilbeam, 2012; Santos, 2007), for example, the resource dependence theory is used by empirical papers as a guide to examine the effect of greater reliance on tuition fees on the share of expenditure on education by the university (Fowles, 2013; Kholmuminov, Kholmuminov and Wright, 2019). Sanford and Hunter (2011) also used the resource dependence theory to explain the limited effect of performance-based funding on universities, arguing that the size of the performance-based funding is too small to meaningfully influence the university’s behaviour. Despite the applications of various theories, to our best knowledge, there hasn’t been any paper that applies positive constitutional economics to analyse the funding of HE.

This paper's main contribution is to examine whether political institutions have an impact on the level of support for HE in political economy. Existing papers have also analysed the reasons for individuals' preferences/voting behaviour towards different policy options to finance HE. The voters differ in their ability to benefit from HE and/or their inherited wealth. Fernandez and Rogerson (1995) examined the level of HE subsidy chosen through majority voting by voters who differ in income and cannot borrow. They showed that high- and middle-class voters vote for a low level of subsidy which excludes the poor from choosing HE and simultaneously extracts resources from them as the subsidy is only given to those who participate in HE. A similar regressive element is also found in Anderberg and Balestrino (2008) who showed in their model that subsidy to HE involves a transfer from the poor to the middle class. In De Fraja (2001), the voters did not have a choice between different financing options but they voted on the level of a university admission test and the level of university subsidy. In Del Rey and Racionero (2012), the voters were choosing between a tax subsidy and a (risk-sharing) ICL. In Del Rey and Racionero (2014), the choice was between 2 different kinds of ICL, risk-sharing and risk-pooling and it also considered the impact of giving student mobility after graduation. Risk-sharing meant defaulted student loans would be covered by general tax revenue and risk-pooling meant defaulted loans would be covered by the income of the successful students. In Borck and Wimbersky (2014) all funding schemes are considered alongside a normal loan and while the voters only differed by income, the paper extends by having the wage rates being determined endogenously in the model. A common finding in these papers is that the higher the degree of risk aversion, the more likely there will be majority support for ICL over other funding schemes. This literature in general is trying to find the factors that led to voters' support for different funding methods for HE and is ultimately implying the policy enacted by the government is what the voters want and that most politics involved in these papers are just majority voting. This is unrealistic because it assumes a government with no agency problem and that policies are chosen by direct democracy. In real life, it is rare that voters would vote on a method for funding a specific program and are especially unlikely to vote on specific details like risk-sharing versus risk-pooling ICL as in Del Rey and Racionero (2014). Related to the political economy of HE there is a relatively larger literature on the political economy of public education. They examined the level of government spending on public education with an option for households to opt out and pay for private education. The public education in question here refers to elementary and secondary education. Much of the literature also only used majority voting as the political mechanism to determine the equilibrium level of funding for public education (Glomm, Ravikumar and Schiopu, 2011). Glomm and Ravikumar (1998) and Epple and Romano (1996) are seminal papers that established the existence of a political equilibrium with voters who differ in their income. Bearse, Glomm and Janeba (2001) and Bearse, Glomm and Patterson (2005) extended the model by adding another policy-lump sum transfer and by allowing the household to supplant public education with private services, respectively. Besides income heterogeneity among the voters, other types of heterogeneity have also been examined, for example, heterogeneity in student ability (Epple and Romano, 1998; Epple and Romano, 2008), heterogeneity in the preferences over different types of public education (Alesina, Baqir and Easterly, 1999) and differences in preferences for public education between the young and the old (Levy, 2005).

Older studies that addressed the determinants of government support for HE have often focused on economic and demographic factors such as unemployment rate and enrolment levels (Clotfelter, 1976; Humphreys, 2000; Leslie and Ramey, 1986; Lowry, 2001; Peterson, 1976). More recently, there are some empirical papers focused on examining the potential impact of different political factors on levels of support for HE (Archibald and Feldman, 2006;

McLendon, Hearn and Mokher, 2009; Tandberg and Griffith, 2013). These different factors include the presence of term limits for the legislators, the political view of the governor and the number of HE interest groups. Tandberg (2010a) and Tandberg (2010b) developed and tested a fiscal policy framework that attempts to understand and explain the differences in state support for HE based on new institutionalism with different political factors, non-governmental special interest groups and socioeconomic factors which include the share of the population over 65 years old and the unemployment rate. They found that a stronger presence of the HE interest groups will result in higher state spending on HE. Ness, Tandberg and McLendon (2015) reviewed the literature on the impact of interest groups. Dar and Lee (2014) examined the influence of political parties and polarisation on the downward trend of HE funding among the state governments in the US. Tandberg, Fowles and McLendon (2017) extended these studies by including the potential influence of State HE executive officers who are in charge of the agency that oversees public post-secondary education. Overall, these works studied how the interplays between different institutions within the society determine the level of support for HE with empirical work to test out whether one of the included variables has a significant effect. Some papers (Ness and Tandberg, 2013; Tandberg, 2010b) measured the potential impact of the budgetary power of the governor on state education spending. A higher budgetary power is an indication of separation of powers and a lack of legislative cohesion in the government and this powerful governor could be likened to a powerful president who could directly control a nation’s policy. There seems to be no consensus on the effect of the budgetary power of the governor on education spending. Some papers such as Tandberg (2010b) did not find a significant impact on the budgetary power of the governor while some papers such as Ness and Tandberg (2013) found greater budgetary power meant higher funding for HE.

The main difference between the listed economics literature and empirical papers is that the first strand simply models voters directly choosing between HE-related policies while the second strand includes multiple factors, such as the presence of interest groups and the budgetary power of the governor but these papers did not use economic models to derive the government decisions on the HE spending based on the behaviours and interactions of utility maximizing agents including voters and politicians. The models used in the public choice literature based on Persson, Roland and Tabellini (2000) and Persson and Tabellini (1999) should be able to provide more realistic modelling on the political side of HE spending combined with utility maximizing agents. Acemoglou and Robinson (2013) argued for the importance of understanding the political economy to give sound economic policies. Withers (2013) also considered the political economy of HE to be an underexplored area in the literature when recommending possible future directions for research in ICL. One area that has not been properly explored is how constitutions between different political regimes (presidential regime and parliamentary regime) affect the levels of HE spending in a legislative bargaining model despite evidence that separation of powers could influence the level of HE spending. The policy outcome should be influenced by both the design of the political institutions and the different voter's utilities. This is one of the two contributions of our third paper and this paper could be seen as a first step into a potentially fruitful field of research.

2.4.2 Literature Review of the Key Papers and Literature Gap on the Effects of the Constitution on Inequality

The second contribution of this paper is to model the potential link between a political regime (presidential or parliamentary) and its level of inequality. The theoretical foundation is built directly on the following papers, especially Persson, Roland and Tabellini (2000). Diermeier and Feddersen (1998) showed that a government coalition should be more stable when political institutions remove the legislators' agenda-setting power after their policy proposals are rejected by a majority, this creates an incentive for politicians to vote together which changes the policy outcome. Persson, Roland and Tabellini (1998) and Persson, Roland and Tabellini (2000) as well as Persson and Tabellini (1999) utilized this insight and examined the differences in equilibrium outcome between the presidential regime and parliamentary regime in a legislative bargaining game. They built the model by explicitly combining three assumptions, (1) No benevolent actors, all agents motivated by self-interest. (2) No direct democracy. (3) Politicians cannot commit to policies. The parliamentary regimes have more legislative cohesion as members of the government in the parliamentary regime require continuous support from the legislature to avoid a government crisis. The politicians in government have an incentive to maintain a stable legislative majority that does not shift from issue to issue. As a result, the parliamentary regime is less competitive for voters and has a larger scope for colluding against them to obtain higher political rents. In the presidential regime, this cohesion does not exist as no stable congressional majority is needed to support the executive. The presidential regime is characterized by its separation of powers over taxation and expenditure as well as stronger competition among voters. The voters for the tax minister can exploit this separation of powers to limit government spending that does not benefit them. The papers predicted that the parliamentary regime has a higher level of government spending, a higher level of corruption and a higher level of transfers than the presidential regime. Interestingly, there is a lack of papers examining the link between the political regimes of a nation to inequality within the said nation. In the literature review, we will therefore also list down several related fields that have been done to investigate the gap that exists within the current literature.

Congleton (2018) gave a brief overview of the history of intellectual development over comparative constitutional research and demonstrated that the details of a democratic constitution matter for the final economic outcome. Voigt (2011) offered a very detailed review of the various theoretical and empirical papers that studied the economic effects of different constitutions. In Table 2 in Voigt (2011) various empirical papers that have constitutional rules as an explanatory variable are listed along with their results. There isn't a single paper there directly analysing the effect on the level of inequality. Various other fields are examined such as the size of government, composition of government spending and government corruption in Persson (2002) and Scartascini and Crain (2021). In Agnello and Sousa (2014), even a relatively niche topic, the volatility of discretionary fiscal policy between presidential and parliamentary regimes had been examined. I could only find two papers that explicitly estimated the potential impact of a presidential regime on a nation's inequality (Feld and Schnellenbach, 2014; McManus and Ozkan, 2018) and they did not have many relevant papers in their literature review. This paper is contributing to a relatively scarce field of positive constitutional economics and the Table 2.3 below provides a summary of a sample of the papers within this field.

Table 2.3: Summary of a Sample of the Theoretical and Empirical Papers on Positive Constitutional Economics

Paper	Constitutional Rule	Main Policy and Economic Outcomes	Does It Include Predictions from Its Theoretical Model(s)	Main Results

Persson and Tabellini (1999)	Proportional vs Majoritarian elections. Presidential vs Parliamentary government.	The size of the government.	Yes	Strong support for the prediction that the presidential regimes have smaller governments but weaker support for the prediction that majoritarian elections are associated with smaller governments.
Persson, Roland and Tabellini (2000)	Presidential vs Parliamentary government.	The level of transfers and size of the government.	Yes.	The parliamentary regimes are predicted to have more transfers and shown to have a larger government than the presidential regimes.
Agnello and Sousa (2014)	Presidential vs Parliamentary government.	The volatility of the discretionary component of fiscal policy	No.	Parliamentary governments are associated with less volatility in fiscal policy discretion.
Feld and Schnellenbach (2014)	Presidential vs Parliamentary government.	The degree of income inequality.	No.	The presidential regimes are associated with a less equal distribution of disposable income.
McManus and Ozkan (2018)	Presidential vs Parliamentary government.	The degree of income inequality.	No.	The presidential regimes are associated with Gini coefficients between 11% and 24% larger than the parliamentary regimes.
Iverson and Soskice (2006)	Proportional vs Majoritarian elections.	The level of redistribution.	Yes.	The electoral rules affect the party composition of the governing coalition, hence the level of redistribution.

Genicot, Bouton and Castanheira (2021)	Proportional vs Majoritarian elections.	The inequality in government interventions.	Yes.	The paper uncovered a novel relative electoral sensitivity effect only in majoritarian elections. This meant the majoritarian electoral rules may cause a more equal distribution of government intervention than proportional representation.
Kammas and Sarantides (2019)	Democracy vs Dictatorship.	The level of redistribution.	No.	Dictatorial regimes have a significantly greater fiscal redistribution as they rely more heavily on cash transfers. The democratic regimes spend more on public goods and services.

Within Voigt (2011), it could be seen that there are more papers devoted to examining the differences between majoritarian and proportional electoral rules. Iverson and Soskice (2006) used a model and found that the majoritarian system favours centre right coalition while the proportional system favours the centre-left coalition and this is a major determinant of the level of redistribution in a democracy as it changes the political position of the incumbent. However, more recent theoretical papers indicate that a majoritarian system can provide a more equal outcome as the government may have less incentive to intervene for specific voter groups (Genicot, Bouton and Castanheira, 2021). Another branch of the relevant studies compares the differences in economic effects and policy choices between democracies and autocracies. Between targeted direct transfer and public goods, the democracies favour higher levels of public goods provision as the government is controlled by a larger fraction of the population which would dilute the direct transfer. In contrast, a dictatorship would prefer targeted spending to powerful groups (Deacon, 2009). This finding is also found in Kammas and Sarantides (2019) but it suggested that because dictatorships rely more on cash transfers, they could end up having a larger impact on reducing the market Gini index than democracy which relies more on investment on various public goods and services. Link to Persson and Tabellini (1999) in terms of the mechanism by which voters pay rent to their politicians to ensure they do not take everything while in office, a similar method was used in Acemoglu, Ticchi and Vinbigni (2010) to outline a theory of why a nation's military would support a non-democratic regime or its democratic successor. The ruling oligarchs need to pay an efficient wage to the military so they do not turn on oligarchs.

Despite not much attention has been paid to examining the link between presidential regimes and inequality, the obvious fact that the USA is more unequal than Europe on average has not gone unnoticed (Atkinson, 1996; Blanchet, Chancel and Gethin, 2022; Piketty and Saez, 2014) and many papers have attempted to list down potential reasons for this. Alesina, Glaeser and Sacerdote (2001) attribute this reason to the lack of proportional representation and lack of major political upheaval in the US which caused the federalist structure that was strictly designed to protect property to be retained. Furthermore, it was suggested that racial heterogeneity and discord in the US led to a dislike towards redistribution as minorities are overrepresented among poor Americans. Bonica et al. (2013) listed 5 potential political reasons for the US to be more unequal than other advanced countries. It mentioned the presidential compared to the parliamentary regime citing Persson, Roland and Tabellini (2000) but its main point on the potential impact of political institution in the US is that filibuster and a greater degree of polarisation have led to the status quo policy being gridlocked. The US therefore could not react quickly to a rise in inequality by updating the social safety net and regulations. Nikoloski (2007) focused on examining the economic and political determinants of inequality. The degree of democracy measured by the Freedom House Index is the only political factor being considered and the type of constitution a regime has is not considered. Almås, Cappenlen and Tungodden (2020) directly surveyed the difference in social preferences between Americans and Scandinavians on inequality. It found when presenting the same situation to redistribute income, Americans accept a higher degree of inequality than Norwegians because more Americans consider inequality (no matter due to luck or difference in productivity) to be fair. The paper suggests this difference in preference corresponds to the different ways society is organized in these countries. Inequality in the US is not the only phenomenon that has been studied. The high level of inequality in Latin America has naturally led to questions about why the democratic governments in that region haven't implemented policies to reduce inequality. Ardanaz and Scartascini (2011) examined the question of why many developing countries (especially Latin American ones) have a low personal income tax despite economic development and democratization through a more in-depth look into the proportional versus majoritarian systems. The paper attributed "legislative malapportionment", a discrepancy between the share of legislative seats and the share of the population held by an electoral district, as one of the determinants. Higher legislative malapportionment enables the elite to ally with overrepresented regions to block progressive taxes and protect their interest. In addition to the above works, this paper therefore also serves as a contribution to the literature that tries to answer this difference in the degree of inequality between these countries using political factors.

The lack of literature formally examining how presidential regimes have a higher level of inequality is surprising. Figure 2.1 is a map of Gini coefficients from World Bank data indicating the differences in the degree of income inequality between different countries and Figure 2.2 shows the map of democratic countries in the world divided between majoritarian and proportional electoral rules and presidential and parliamentary forms of government. Figure 2.1 is extracted from Kohnert (2022) and Figure 2.2 is extracted from Persson and Tabellini (2003).

Figure 2.1: World Map of Gini Coefficients by Country Extracted from Kohnert (2022)

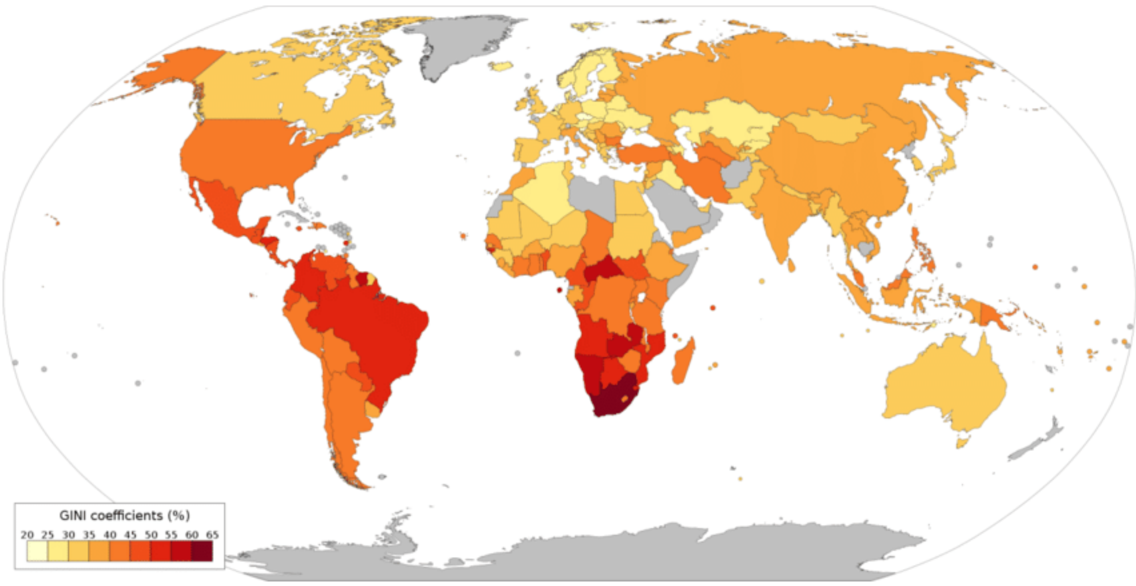
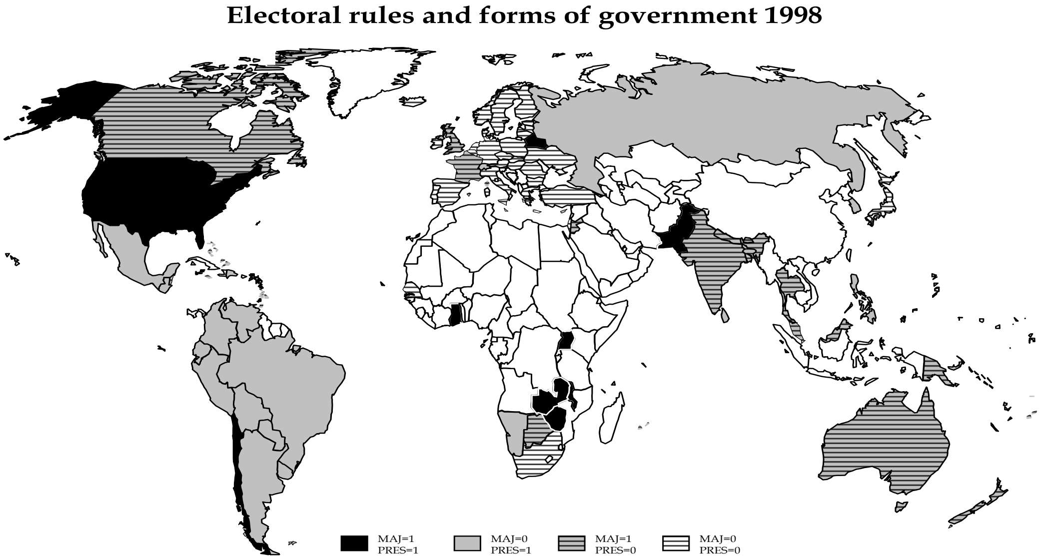


Figure 2.2: Electoral Rules and Forms of Government Extracted from Persson and Tabellini (2003)



Immediately, it could be seen that the map of presidential versus parliamentary regimes in Figure 2.2 roughly matches up with the degree of income inequality in Figure 2.1, especially when comparing between Americas and Europe.

Persson and Tabellini (1999) provided a link between voting rules and inequality. The mechanism behind majoritarian electoral rule caused stiffer electoral competition by concentrating it towards a key marginal district. This caused the district to receive more targeted redistribution from both politicians than under proportional electoral rules. Under this insight, majoritarian electoral rules could therefore cause higher inequality. However, for presidential versus parliamentary regimes, the reason that the parliamentary regime has a higher level of redistribution is due to the voters behind the parliamentary majority forming a bilateral monopoly and jointly exploiting the remaining minority who is out of power. The redistribution is not based on income and given to low-income groups but it is a transfer to the groups based on their political power. Parliamentary regimes are predicted to be more corrupt and more prone to exploit the minority to redistribute to the powerful majority. It is hard to conclude from this that parliamentary regimes are more equal just because they also have a larger provision of public goods. In this paper, the previous group transfers of one group have changed into a policy that directly augments the income of voters of said group, for example, education spending. This allows the tax minister in the presidential regime to use separation of powers to directly increase the income of his voters or limit the income of others to benefit his group. Under this model, it could be concluded more naturally that a parliamentary regime that relied more on cohesion between the members of the ruling majority would achieve a more equal outcome.

2.5 Literature Review for Chapter 6

Section 2.5.1 will first show the significance of the student loan crisis. It then shows how our fourth paper contributes to the analysis of the building up of student loan debt from a political economy perspective, embedding political uncertainty and polarisation.

Section 2.5.2 presents and discusses the relevant theoretical literature that models the effect of political uncertainty and polarisation on the incumbent government’s decisions in terms of public debt and public investment. Table 2.4 summarises various papers that examined the potential effect of political uncertainty and polarisation on public investment.

2.5.1 Motivation for and Contribution of Chapter 6

Closely linked to the government's decision on HE spending is its effect on the level of student debt. Historically many HE systems offer free HE to qualified students but there is a worldwide trend of shifting the cost of HE onto the students themselves (Johnstone, 2003; Johnstone, 2004; Marcucci and Johnstone, 2007). When a student loan scheme is the major way of funding HE, the government could increase the level of HE spending by increasing the cap on tuition fees and student loans. The total level of student debt in England has grown each year and has reached £182 billion at the end of 2021-22 while the total level of student loans in the US has been growing to $1.762 trillion in 2021 with 91.2% being federal loans (Bolton, 2022; Hanson, 2022a). For a more recent overview of the history of student loans in the UK look at Bolton (2019), which contains detailed statistics on the student loan in the UK and documents their increase over the years. Both the size of the student debt and its growth have not been unnoticed by many observers. Friedman (2018) a much-cited Forbes article on the US student loan statistics in 2018 called it a crisis. This growth of student loan debt in the UK and the US can be seen clearly in the figures below.

Figure 2.3: The Growth of Student Loan Debt in the UK. Source: Bolton (2022)

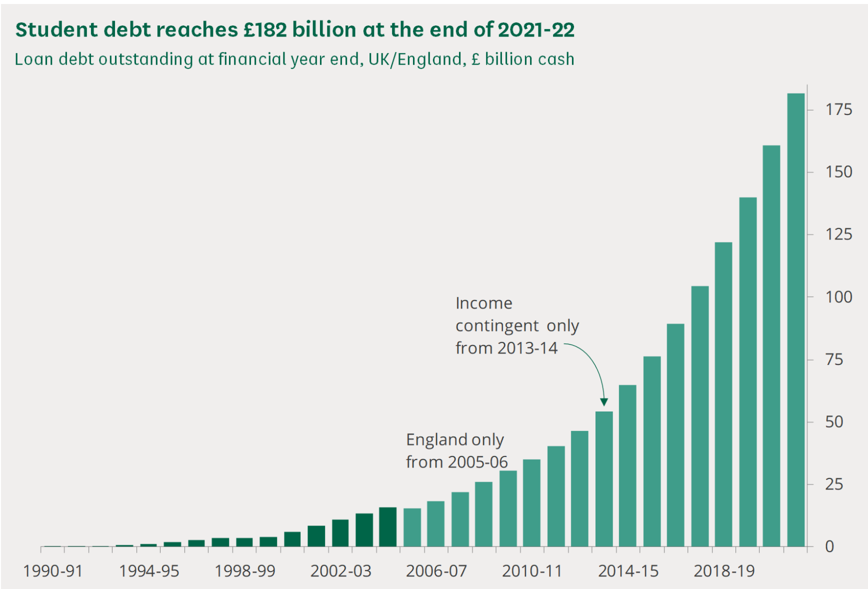
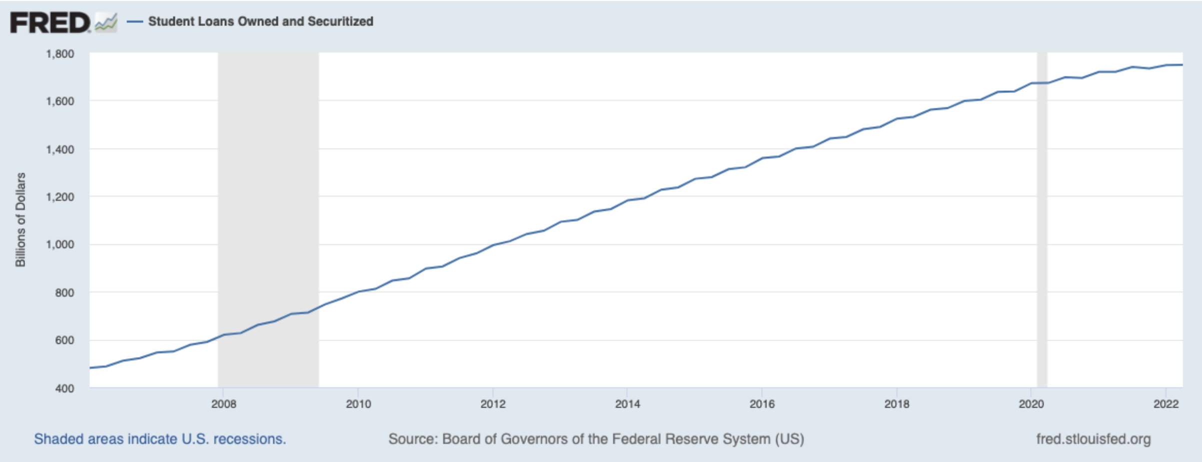


Figure 2.4: The Growth of Student Loan Debt in the US. Source: Board of Governors of the Federal Reserve System (US) (2022).



The build-up of student loans and their potential default by a substantial number of graduates is a motivating factor in searching for an optimal way of funding higher education (Hedlund, 2022; Radomska, 2019). Over the last three decades, the level of tuition fees has grown significantly in the US, although the average net tuition fees (difference between tuition fees and average grant aid received by first-time full-time students) has declined in recent years (Ma and Pender, 2022; Mitchell and Leachman, 2015). Despite the optimism in earlier student loan papers in the US which found that university students taking an excessive level of debt that would lead to default are a minority case (Hansen and Rhodes, 1988), this proved to be false as the rate of default of student debt is much higher currently with one out of every ten Americans having defaulted on a student loan (Hanson, 2022b) and many papers aim to find factors that lead to the higher average level of debt taken by students as well as a higher rate of default (Addo, Houle and Simon, 2016; Belfield, 2013; Dean Craig and Raisanen, 2014; Goodell, 2016; Hillman, 2014; Houle and Addo, 2019; Looney and Yannelis, 2015; Scott-Clayton, 2018), the most important and commonly found institutional factors that led to a higher level of student loan debt and higher rate of student loan default is attending for-profit colleges instead of non-profit or public colleges. For students' characteristics, having a poorer education performance, a lower income family background as well as being a minority are found to be associated with higher levels of student loan debt and default rates. Akers, Chingos and Henriques (2015) and Hershbein and Hollenbeck (2014) found that increases in tuition fees are an important factor in causing the rise in student loan debt and borrowing in the USA. In the US there are not only papers exploring the long-run impact of holding student loans on graduates (Gayardon et al., 2018; Houle and Berger, 2015; Mezza et al., 2020; Roots, 1999; Rothstein and Rouse, 2011) but also on how the student loans could negatively impact the community around the debt holder as the debt affects student's behaviour (Deller and Parr, 2021).

In the UK, many student loans motivated papers are focused on the impact of the important 2012-2013 reform. Crawford, Crawford and Jin (2014) aimed to estimate the long-run cost of this program to the government as a portion of the student loan would inevitably be written off and paid by higher future taxes or lower spending. There are many papers estimating the effect of the dramatic reform in 2012 on student enrolment rate, university funding and the distributional effect of this reform (Azmat and Simion, 2018; Chowdry et al., 2012; Murphy, Scott-Clayton and Wyness, 2019; Sá, 2019). Marginson (2018) found the 2012 tuition fee increase has increased the total funding for universities, reversed the trend in the 1990s and increased the funding per student. Some papers could indirectly point to the effect of the reform by estimating the degree of debt aversion among potential UK undergraduates (Callender and Mason, 2017).

As listed in the paragraphs above, many US papers were trying to determine which factors are causing the rising average level of student debt and rate of default while in the UK the relevant papers often outline the history of policies leading to the tripling of the tuition fee in 2012-2013 reform and linked it to the global trend of seeing HE as a private investment instead of a public good and shifting the cost of HE from the taxpayers onto the students themselves (Callender and Mason, 2017; Saunders, 2012). However, why doesn't the US government enforce much greater regulation on the ability of for-profit colleges to set tuition fees and recommend bad loans or limit the increase in tuition fees in general? Overall, I could not find many papers that formally question and examine "why do government allow tuition fees and student debt to increase?". In the UK case, the Conservative-led government actively tripled tuition fees and tuition loans which did not increase its popularity with young voters who already favoured the Labour Party. Could political uncertainty facing the incumbent government and polarisation between the parties influence its setting of the student loan levels in theory? Formally the question I am asking in Chapter 6 is: Why would an incumbent government increase spending on HE and student loan levels when its voters are not directly benefitting from HE?

2.5.2 Relevant Theoretical Studies

The question I am answering in Chapter 6 is related to the topic of the political economy of public debt and within the field of literature that argues incumbent government uses public debt as a strategic variable to influence future government. Mawejje and Odhiambo (2020) briefly discussed this topic and various other economic explanations of public debt. The review paper on the political economy of government debt, Alesina and Passalacqua (2016), offered a more detailed analysis of the literature in this field. The two most important papers in this field are Alesina and Tabellini (1990) and Persson and Svensson (1989). The models in the two papers are similar to each other. Two groups of voters are equivalent to each other, and each is represented by a party that acts fully in their own voters' interest. The voters only differ in their preference for public goods. The government must honour the debt it inherited from its predecessors. In Alesina and Tabellini (1990), the voters disagree on the composition of the public goods, they each want the government to spend on different public goods. When the incumbent government is unsure of his re-election, he would issue more debt to increase spending on current public goods his voters preferred optimally since the cost of lower public goods in the future due to higher debt is no longer internalized by him as the other government would not provide the public goods he wanted. In Persson and Svensson (1989), there is only one public good but the voters differ on the overall size of the spending. When a "stubborn" conservative government knows it will be replaced by a liberal government in the future, he would issue a higher level of debt to strategically tie the hands of his successor to force the liberal government to cut spending on its preferred level of the public good. The insights behind these papers are that debt is a state variable that allows the incumbent to control the future government by essentially taking tax revenue away from the future government. Given the similarity between the two papers and the same insights they offered, either could be chosen to extend upon. I choose to build upon Persson and Svensson (1989) because in 2012 it was a Conservative-led coalition that tripled the tuition fee and loans in the UK which can be compared to the "conservative government" discussed in Persson and Svensson (1989).

Various papers have extended the model based on the above two papers to explore various topics. Aghion and Bolton (1990) used a model in which voters have different incomes but identical preferences, the incumbent left-wing government voted by low-income voters knows it will be replaced by a conservative government uses higher debt to fund higher current spending while forcing higher tax revenue in the future on the higher income group. Aghion and Bolton (1990) also consider a situation with costless default and debt policies affecting incumbent re-election probability. Lizzeri (1999) linked the strategic use of debt to a redistribution policy in which rational voters reward the incumbent's myopic behaviour as the future government might not distribute resources to them. Alt and Lassen (2006) used the model to relate government transparency to its fiscal performance. The career-concerned but incompetent politicians have the incentive to use debt to appear competent to voters whose knowledge of debt depends on government transparency. Song, Storesletten and Zilibotti (2012) modelled intergenerational conflict over debt, taxes and public goods. They found that countries with stronger public good preferences have a stronger preference for fiscal discipline and have smaller debt.

Table 2.4: Public Investment under Political Uncertainty and Polarisation

Paper	Conflicts between Voter Groups	Is the Political Conflict between Two Partisan Parties	Probability of Re-election	Specific Government Policy/Policies Analysed	Effect of Uncertainty and Polarisation
Azzimonti (2011)	Preferences over the composition of public expenditure and preferences over policymakers.	Yes	Endogenous	Public spending and distortionary tax.	Underinvestment by voters due to higher levels of distortionary taxation to fund higher levels of public goods.

Azzimonti (2015)	Preferences over the composition of public expenditure.	Yes	Exogenous	Public investment.	Lower level of public investment and fluctuation in public investment.
Battaglini and Coate (2007)	Preferences over the composition of public expenditure.	No	Exogenous	Public investment and distortionary tax.	If public investment is not important to the voters, then public investment will be too low and the tax will be too high to fund district-specific transfers.
Besley and Coate (1998)	The level of redistribution.	No	Endogenous	A discrete public investment.	Public investment is not undertaken.
Devereux and Wen (1998)	Preferences over the composition of public expenditure.	Yes	Exogenous	Capital tax.	Capital tax is too high and reduces the long-run economic growth.
Peletier, Dur and Swank (1999)	Preferences over the composition of public expenditure.	No	Exogenous	Public investment and a deficit rule.	A balanced budget rule removed excessive public borrowing but resulted in too little public investment.
Svensson (1998)	The conflict is between the two potential governments.	Yes	Exogenous	Investment (reform) in legal infrastructure.	No reform in the legal system results in low private investment.
Bohn (2007)	Preferences over the composition of public expenditure.	Yes	Exogenous	Public investment.	Underinvestment with polarisation and uncertainty reinforce one another in their impact.
Elder and Wagner (2015)	Preferences over the composition	Yes	Exogenous	Level of pension funding.	Pension funding decreases.

	of public expenditure.				
Beetsma and Ploeg (2007)	Different ideological returns over different public investments .	Yes	Exogenous.	Public investment.	Increase public investment by issuing more debt.
Glazer (1989)	Different utilities over a government project.	No	Exogenous	Whether to undertake an investment project and the durability of said project.	A durable project is built even though under private decisions the decisive voter would prefer no project.

The tuition fees and loans set up and given by the UK government are a public investment into HE in the form of student debt which unpaid parts will be written off in a set future. Many other papers have extended from Alesina and Tabellini (1990) and Persson and Svensson (1989) to study the incumbent government's incentives for public investment in a politically unstable and polarised society. Table 2.4 summarises the models and the findings in these papers. Many papers' models found the following reasons for underinvestment. First, the incumbent fully internalizes the cost of public investment but does not fully benefit from the investment's future benefit if the future government has a different policy preference. Second, lowering current investment limits the future resources or tax base of a future government. This increase in the cost of tax collection reduces the tax revenue levied by the future government that would be spent on public goods that the incumbent does not want. This lack of investment is not only about physical investment but also about costly legal reforms in lowering the barrier to private investment or protecting property rights (Azzimonti, 2011; Azzimonti, 2015; Svensson, 1998). Third, in the citizen-candidate model, the incumbent also wouldn't invest if it either changed the identity of the future policymaker or changed the future policies in ways that are disadvantageous to the incumbent (Besley and Coate, 1998). Peletier, Dur and Swank (1999) added public investment and a rule that eliminates excessive borrowing to the model used in Tabellini and Alesina (1990) and found it caused underinvestment. The model in Devereux and Wen (1998) is related to the optimal capital tax of Chamley (1986) but the idea is similar to the first and second points as the incumbent optimally leaves the future government with smaller assets. More uniquely, Natvik (2009) and Fiva and Natvik (2013) emphasised the importance of the complementarity of invested public capital and other factors of production in the economy. If the successor could control labour supply, and that capital and labour are complements in production, then a lower re-election probability meant lower investment. Elder and Wagner (2015) used the same insight to create a model that explains polarisation and electoral uncertainty led incumbents to discount the future more heavily and underfund pensions to expand their current preferred public goods. Political uncertainty encourages policies that favour short-term payoffs over long-term payoffs. An obvious gap we are filling is that these papers are explaining underinvestment but the 2012-13 student loan reform in the UK was a rise in investment that further increases graduates' income. It is not obvious how this benefitted the Conservative coalition as this policy is unpopular with the student group that already prefers voting for Labour and from around 2010 to 2012 the Conservatives were expected to lose the next general election to Labour. Furthermore, assuming Conservative voters are older and do not participate in HE in general, they always only benefit from improvement in HE indirectly, so the first reason linking political uncertainty to public investment also cannot be applied here at face value. This seeming paradox cannot be explained directly by any of the above papers and I will explore these questions in Chapter 6.

Some papers linked political uncertainty and polarisation to excessive public investment. In Beetsma and Ploeg (2007), the reason for the incumbent to increase investment in a project they otherwise would not do is because the excessive public investment in their model is funded by excessive borrowing, so the investment in period 1 is limiting the choices available for future government. The future government needs to service the debt and would not be able to undertake activities that the incumbent doesn't want. The same insights are shown in Glazer (1989) as the decisive voter in period 1 chooses to invest in a durable project, despite preferring no projects privately, to limit the choices available to the future decisive voter. This explanation cannot answer the student loan question. Investment in HE funded by student loans not only increases the future income but also leads to future loan repayment, both of which increase the revenue and therefore choices available for future government. Therefore, a new model is needed to link political uncertainty and polarisation to higher public investment in this case.

Finally, completely unrelated to political uncertainty and polarisation, there are also papers using a political economy framework to investigate how a group of voters would vote on HE issues such as education subsidies. For example, Poutvaara (2011) used a median voter model to present a situation where a majority of voters agree on a generous subsidy of HE so that the median voter also undertakes HE and would not vote for a higher level of redistribution in the future. Haupt (2012) used an overlapping generation model to illustrate the evolution of the government subsidy for HE as a result of the increase in the number and voting power of skilled people. When the skilled people reach a majority, they will vote for an increase in the level of HE subsidy which increase the number of students and skilled people in the future.

However, this higher level of subsidy per student is politically unsustainable with the higher number of students which results in the fall of the level of subsidy per student. Our paper could be seen as another way of formulating a political economy explanation of HE policy.

Chapter 3: Optimal Provision of Higher Education with Second Chance

3.1 Introduction

The private returns of higher education (HE) have been widely documented in the economics literature (Baum, Ma and Payea, 2010; Baum, Ma and Payea, 2013; Ma, Pender and Welch, 2016). Nevertheless, there is relatively less attention paid to private returns to education for mature university students. Donaldson and Townsend (2007) found only 1-2% of the published papers in their seven selected HE journals published between 1990 and 2003 deal mainly with mature students. In empirical papers, a university student usually is considered as mature or old if he is older than 25 years old. The empirical papers that specifically measure the benefits of HE for mature university students all found that older graduates' probability of employment and wage level have increased after completing their HE (Carnoy et al., 2012; Desjardins and Lee, 2016; Dorset, Lui and Weale, 2010; Kim and Baker, 2015). Both young and mature students benefitted from HE. This benefit of completing education later in life is not exclusive to completing HE, Bennett, Blundell and Salvanes (2020) found high school dropouts in Norway that were over 25 years old obtain higher earnings and a higher probability of employment after completing high school. However, the current government policy does not consider education support for mature students to be a priority; Mason (2020) found that the UK education system has been heavily biased towards HE for 18-24 years old and Desjardins and Lee (2016) stated that in some countries education for mature students are discouraged because the mature students in HE system are often seen as an indication of inefficiency. The high returns to education accruing to mature students suggest that the government should pay more attention to the education needs of these older prospective students. These results also indicate the potential labour market and public finance benefits if the government gives a second chance to all failed students to complete their HE program. Despite the UK government not offering a second chance at HE, the government does provide policies that could improve the labour market performance for these failed students with apprenticeship funding and employment allowance to encourage apprenticeship training and small business employment. Although not policies that target the students who failed in HE specifically, the effect of these policies could fit into our general framework as these policies also improve the earnings of workers with the lowest wage. In this chapter, the second chance policy refers to the government providing all students who have failed their first try at HE another chance to succeed at HE. These students have all engaged with the labour market and this second chance is offered when they are older.

Given that student loans are an important source of funding for HE in many countries including the UK, many papers have analysed the optimal design of these student loans. This paper is also within the theoretical literature investigating the role of student loans in funding HE. In this field, many papers have recommended the use of income-contingent loans (ICL) to fund HE. The ICL is a type of loan for which the borrower's repayment is contingent on his income. The earliest papers that recommended ICL include Friedman and Kuznets (1945), Friedman (1955) and Shell et al. (1968). An ICL scheme was argued to be able to solve the student loan market failures resulting from the riskiness of education and the fact that human capital cannot be used as collateral. Nerlove (1975) commented on the ICL proposal made by Freidman (1955) and expressed concern for its feasibility due to the potential social and political resistance to such a policy and at the time a lack of understanding of how individuals' behaviour would be affected by these loans. These papers mainly discussed the potential benefits and feasibility of ICL before ICL were widely used in funding HE. Chapman (2014) summarised the conceptual and empirical basis for ICL and Shireman (2017) provided a focused history of the debate and applications of the ICL in the US.

There are also more recent papers that examined the potential benefits of ICL in HE. These papers compared different funding schemes for HE, to investigate which policy would achieve the largest welfare gain compared to the case without government intervention. Migali (2012) examined the factors influencing students' preference for HE funding methods between mortgage-styled loans and ICL. He found that ICL is preferred by students from poorer families and with greater earning volatility because ICL offers a greater degree of insurance. Del Rey and Racionero (2010) considered and compared the insurance role of several different HE funding schemes against uncertain education outcomes and their effects on HE participation. They proposed an ICL scheme that covers both the financial cost of education and foregone earnings to fully insure the lowest-ability individuals who should enrol in education and induce an optimal level of participation. Chatterjee and Ionescu (2012) considered student loans that offered insurance against failing education. They explicitly account for the costs of moral hazard in the form of low effort in education and adverse selection in the form of students who originally aimed to leave college but now choose to stay in college. They found that if loan forgiveness is offered to students who failed university, there would be a significant welfare gain without many adverse effects on graduation rates.

The model of this paper is also related to the literature that uses optimal taxation models with endogenous human capital to study the optimal design of student loans. Optimal taxation is the study of how to design the tax system to maximize a social welfare function subject to a set of constraints. The optimal labour taxation literature started with the static model of Mirrlees (1971) that assumes exogenous individual abilities and has subsequently been extended into a multiperiod model still with exogenous individual abilities (Farhi and Werning, 2013; Golosov, Tsyvinski and Werning, 2006; Kapicka, 2013). A few papers also considered endogenous human capital in the framework of the optimal taxation model. Anderberg (2009) and Grochulski and Piskorski (2010) analysed the optimal tax policy with ex-ante identical agents but unobservable human capital investments and productivity shocks. Realistically, different agents should have different private abilities. Findeisen and Sachs (2016), Stantcheva (2017) and Koeniger and Prat (2018) explicitly studied the feasibility of using ICL to implement the second-best allocations with heterogeneous (by abilities) individuals. Radomska (2019) gave an overview of the important literature in this field of study and summarised their results. In all these papers, they found that the second-best optimum can be implemented by an ICL.

Among the different optimal taxation papers with endogenous human capital, this paper's model is a direct extension of the model used in Gary-Bobo and Trannoy (2015). Gary-Bobo and Trannoy (2015) differed from other optimal taxation papers with endogenous human capital by considering the combination of private abilities with genuine moral hazard in education. In their model, individuals are born with an unobservable high- or low-ability type (ex-ante type) and need to choose their quality of education. After succeeding in HE, students will gain an unobservable labour market type (ex-post type) that can be interpreted as some job-market skills. Students could choose to exert an unobservable effort in HE to increase their chance of success. There is also an unobservable labour market effort which affects an individual's labour earnings. Consequently, there is an adverse selection problem in terms of students choosing the different qualities of education and also moral hazard problems in both HE and the labour market. The government wants to assign each student the quality of education that matches their ability and incentivise high study effort for all students but only high labour market effort for those of high ex-post type. Similarly to other papers, Gary-Bobo and Trannoy (2015) confirmed the need for ICL to implement the second-best allocations and that second-best loan repayments are always income contingent. Besides, in Gary-Bobo and Trannoy (2015), the second-best optimum has several implications for the degree of ex-ante inequality, ex-post inequality and insurance in the economy. The second-best optimum resulted in ex-ante inequality which meant that the high-ability students have a higher expected utility net of study effort cost than low-ability students before they know the outcome of education. It also resulted in ex-post inequality in the labour market as the students with a higher ex-post type have higher utilities as well as incomplete insurance against failing in HE because the government needs to give more utility to those who succeeded in HE to incentivise the students to exert a high study effort. The second-best optimum also exhibits a form of insurance called equal treatment: the students' expected utility after finishing education but before knowing their ex-post types are equalized between different ex-ante types conditional on academic success.

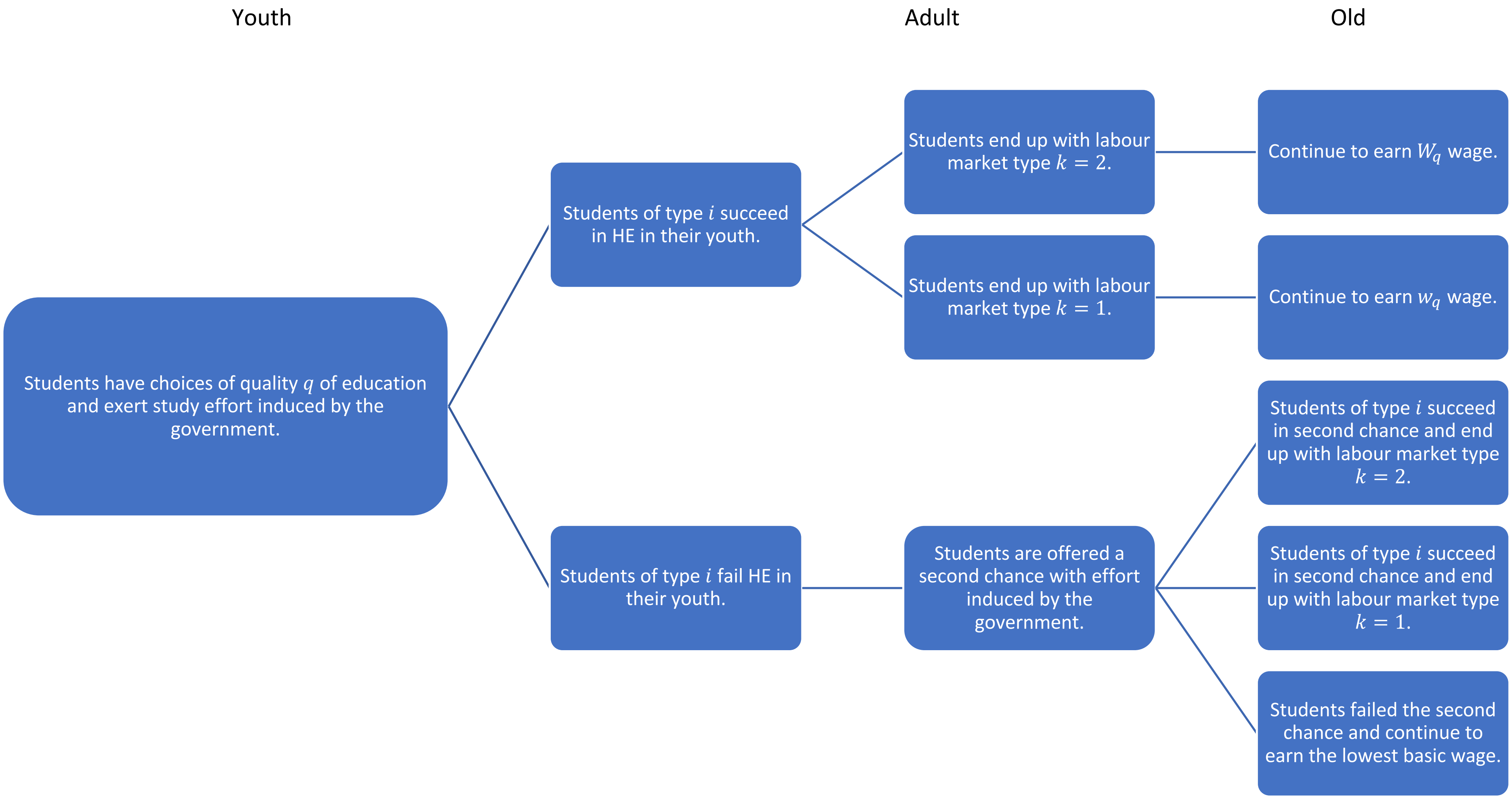
There is insufficient attention paid to the optimal design of student loans that explicitly includes a HE policy for older students. By extending the model in Gary-Bobo and Trannoy (2015) with an additional period, we include a second chance given to all failed students. The modelling of the second chance is based on the political economy paper-Arawatari and Ono (2009) in which the poor have a second chance of success through education at a later period.

This paper could inform governments on how to design the current student loan contracts in such a way as to provide incentives to induce the failed student to put effort into trying HE again later in life.

This paper explores two main questions: 1. How do we optimally provide a HE system that offers a second chance to all failed students? 2. What is the impact of such an optimally provided second chance on inequality and insurance? The second question fills in another literature gap. The empirical papers only examined the impact of HE on those mature students who enrolled in adult HE. This paper thus provides a theoretical framework for the impact of an optimally provided guaranteed second chance focusing on a) the utilities of the mature students who failed the second chance at HE and b) the expected utilities of students before finishing their first chance at HE. Because the model is a direct extension of Gary-Bobo and Trannoy (2015), the changes in inequality and insurance due to the introduction of the second chance will be compared to the results found in Gary-Bobo and Trannoy (2015) that did not include the second chance policy.

This paper is organized in the following way. Section 3.2 sets up the model and studies the first-best optima. Section 3.3 analyses different incentive constraints. Section 3.4 derives the common second-best optimality properties and answers the two questions. Section 3.5 extends the model by providing policy implications in the time-consistent optimum. Section 3.6 concludes.

Figure 3.1: Possible Education and Labour Market Outcomes Facing Type i Students



3.2 Model Setup

The main difference with Gary-Bobo and Trannoy's (GT's) model is that the students who failed in school when young have now a second chance in HE. Thus, when the failed students are earning the lowest wage $\underline{w}$ in their adult period, they are also simultaneously having a "second chance" where they are given the same quality q HE and choose to exert study effort.

If they have finally succeeded, they could earn a higher wage depending on their ex-post type k. The failed student does not have their ex-post type k revealed, to do that they had to succeed in education. Figure 3.1 above shows the different possible education and labour market outcomes facing students of ex-ante type i from when they are young till when they are old. To better understand the model and the analysis, below we will define several important terms that will be widely used in this chapter:

Ex-ante: Before taking the first chance at HE.

Interim expected utility: The expected utility of a student after taking the first chance at HE but before entering the labour market.

Ex-post utility: The actual utility a worker obtains when working in the labour market.

Equal treatment second-best optimum: The second-best optimum in which the students' interim expected utility is equalized between different ex-ante types conditional on academic success.

The government is assumed to have full commitment to the contract/repayment and cannot change the policies when old after learning what ex-ante type i each individual has. Section 3.2.1 will begin to set up the model.

3.2.1 Assumptions

There are three periods in this model: a youth period, an adult period and an old period. All students have the same intertemporal von Neumann-Morgenstern utility:

$$\delta u_a(.) + \delta^2 u_o(.),$$

where δ is the discount factor. The subscripts "a" and "o" denote "the adult period" and "the old period" respectively. $u_a(.)$ and $u_o(.)$ are the same function. The student's intertemporal utility function is additively separable over the two time periods. There is no storage technology in the economy. Each student uses all his net income for consumption within a period. We omit the youth period utility function to simplify the model's presentation. We could add a youth period utility function with an exogenous government transfer in the youth period so the students have positive utility in the youth period but its inclusion will not affect the analysis in this model.

The students have two-dimensional unobservable characteristics: ex-ante type i which represents the student's study/cognitive skill and ex-post type k which represents job market skills/opportunities.

The timing of information revelation is the following. Both the ex-ante type i and ex-post type k are private information. Ex-ante type i is known to students before they decide on the choice of HE quality while the ex-post type k is only revealed to students after they succeed in HE and enter the labour market. The unsuccessful students earning $\underline{w}$ could learn their type k if they succeeded in the second chance in their adult period.

An individual's skill is characterized by his pair of types (i, k). Assume the types can each take only one of the two values, $i, k = 1, 2$. Assume the students' ex-ante types are independently drawn from the same distribution. The frequency of ex-ante type i in the student population (the proportion of type i students in the population) is denoted by λ_i, and since there are only two possible types of i, $\lambda_1 + \lambda_2 = 1$.

The three building blocks of the model can be divided into Education, the Job Market and the Loan Contract. Each of them will be examined below.

Education

Both the youth period education and the second chance will be analysed here with the first chance at HE described first and the second chance described next. The students are given their first chance at HE when they are young. In the beginning, the student learns her ex-ante type i, this ex-ante type i will not change and remain the same for both the first chance and the second chance if the student needs it. The student then chooses the quantity/quality of her education q which can either be $q = 1$ or $q = 2$. The cost of education (in terms of resources) for the first chance is denoted by y_q for choice q and assumes that $y_2 \geq y_1$.

The students' decision on choosing the education quality q for their first chance depends only on ex-ante type i since the ex-post type k is unknown when the education decision on q is made. The government is assumed to be able to observe education choice q.

Let q_i be the education choice of students with ex-ante type i, and the efficient choice of q for the first chance at HE for the ex-ante type i students will be $q_i = i$ (they choose the same level of education as their ex-ante type), this will be presented in Assumption 3.4 (b). The students either succeed or fail in their education and if they succeed when young they will not need to try again in the adult period. If they fail their first chance, they have a second chance to succeed in HE in the adult period. The variable determining the probability of success for the first chance at HE is explained below.

Each student will choose an ex-ante effort for study in the first chance. This effort has two possible values: 1 or 0. We denote the study effort for the first chance as:

e_i^y is the ex-ante effort for the first chance.

The study effort cost for type i's first chance is $c_i^y e_i^y$. The parameter c_i^y is >0. The government cannot observe study efforts for the first chance and this creates ex-ante moral hazard problems. Also assume that $c_1^y \geq c_2^y$.

The probability of success of student i exerting an ex-ante effort is denoted as $p_i(e_i^y)$ for the first chance, where

$$p_i\left(e_i^y\right) = \Pr(success \mid i \ and \ e_i^y).$$

Denote $P_i = p_i(1)$ as the probability of success in the first chance under high ex-ante effort and denote $p_i = p_i(0)$ as the probability of success in the first chance under low ex-ante effort.

If the students fail their first chance, they have a second chance to succeed in HE in the adult period. The cost of education (in terms of resources) for the second chance is denoted by $\eta_a y_q$ for choice q. The parameter η_a is for the second chance (for first chance $\eta_a = 1$), so if $\eta_a > 1$, then the second chance is more expensive than the first chance.

For the students engaged in the second chance, their choice of education quality q also depend only on ex-ante type i because they also don't know their ex-post type k since that failing school does not reveal students' ex-post type k.

The efficient choice of q for the second chance at HE for the ex-ante type i students will also be $q_i = i$ (they choose the same level of education as their ex-ante type), this will be presented in Assumption 3.4 (c). The variable determining the probability of success for the second chance at HE is explained below.

Each student will also choose an ex-ante effort for study in the second chance if needed. This effort will have the same two possible values: 1 or 0. We denote the second chance study effort as:

e_i^a is the ex-ante effort for the second chance.

The study effort cost for type i's second chance is $c_i^a e_i^a$. The parameter c_i^a is >0. Furthermore, assume that $c_i^a > c_i^y$, meaning to exert study effort is more costly for an older individual. The government also cannot observe the study effort for the second chance and this creates additional ex-ante moral hazard problems. Also, we assume that $c_1^a \geq c_2^a$.

The probability of success of student i exerting an ex-ante effort is denoted as $p_i^a(e\,_i^a)$ for the second chance, where

$$p_i^a(e\,_i^y) = \Pr(success \mid i\ and\ e\,_i^a).$$

Denote $P_i^a = p_i(1)$ as the probability of success in the second chance under high ex-ante effort and denote $p_i^a = p_i(0)$ as the probability of success in the second chance under low ex-ante effort. Besides the difference in the cost of education and study effort cost, we also allow the probability of success to differ between the first chance and the second chance. The government can observe whether an individual succeeds or fails his education.

Furthermore, assume the more talented student ($i = 2$) is more likely to succeed given the study effort. Formally, we make the assumption:

Assumption 3.1: $0 < \mathrm{p_i} < \mathrm{P_i} < 1, \mathrm{i} = 1, 2$ and $\mathrm{P_2} > \mathrm{P_1}$, $\mathrm{p_2} > \mathrm{p_1}$. $0 < \mathrm{p_i^a} < \mathrm{P_i^a} < 1, \mathrm{i} = 1, 2$ and $\mathrm{P_2^a} > \mathrm{P_1^a}$, $\mathrm{p_2^a} > \mathrm{p_1^a}$.

In summary, the student's ex-ante type and their effort are not observable by the government but the individual wage, the choice of education q and the success or failure of education are observed by the government.

Job Market

In this section, we will first describe how the wage is determined in this model and introduce the ex-post moral hazard problem. Then, we describe all the possible wage levels and our assumptions of their ranking. Next, we will list all the disutility of effort at work and the total effort costs in this model, and finally, we will state our assumption on the size of the disutility of effort at work.

Now the students work for two periods. In both the adult and the old period: Failure at school means earning the lowest/basic wage $\underline{w}$ and success at school means the wage would be dependent on the quality of education q and the ex-post type k. Once type k is determined, it will not change from period to period.

The student's ex-post type k is not determined with certainty. The probability of ex-post type k, given success in HE and education quality q, is defined as,

$$\Pr(k = q|q, success) = 1 - \pi,$$

and $$\Pr(k \neq q|q, success) = \pi,$$ assume that $\pi < \frac{1}{2}$.

Following the model of Gary-Bobo and Trannoy (2015) we assume that π is exogenous. If a student has an ex-post type $k = 2$, it means that he can occupy a top job and if a student has an ex-post type $k = 1$, it means that he will occupy a middle job. These are related to the later assumption of labour effort cost. Since we assume $\pi < \frac{1}{2}$, this is interpreted as there is a small probability that an individual with quality q HE will be endowed with ex-post characteristics $k \neq q$. For example, for students with ex-ante type $i = 2$ who chooses quality $q = 2$ HE, most of them can meet expectations and become a type $(i, k) = (2,2)$, with a high ex-ante and ex-post type, and occupy a top job. However, a small proportion of these students, type $(i, k) = (2,1)$, will lack the necessary skills/opportunities and only find a middle job. Similarly, the assumption meant that most of the ex-ante type $i = 1$ student who succeeded in quality $q = 1$ HE will obtain a middle job but a small part of said students possess the necessary labour skills/opportunities to obtain the top job. This interpretation fits findings in real life, in the UK, on average, students who obtained a first earn more than students who obtained a 2.1 who earn more than those with 2.2 while male Russell Group graduates earn over 40% more than those who attended the average post 1992 institution (35% for women) (Belfield, Britton and Erve, 2018; Social Mobility Commission, 2023). Both earning a higher university degree and being in Russel group could be interpreted as having a higher ex-ante type i in this model.

The ex-post type k creates an ex-post moral hazard problem. Assume that a failed student does not need to exert any ex-post labour market effort and the successful student needs to choose her level of ex-post labour market effort. Let ε_{ik} denotes the ex-post effort of a type (i, k) student. ε_{ik} could either be 0 or 1. For it to be a moral hazard problem related to the realization of the ex-post type k, assume that high effort $\varepsilon = 1$ is required for a top job and low effort $\varepsilon = 0$ is required for a middle job. The effect of type k is to affect the utility cost of ex-post effort. The moral hazard is for a $k = 2$ type to exert no effort and occupy a middle job.

The possible wage levels are assumed to be the same in both periods. The wage is determined by the quality of education q and the effort at work ε_{ik} only. The wage level is denoted by:

$$\omega(q, \varepsilon)$$

Let $\omega(q, 1) = W_q$ be the top job wage (top job needs $\varepsilon = 1$).

Let $\omega(q, 0) = w_q$ be the middle job wage (middle job needs $\varepsilon = 0$).

In total, in each period, there are five possible wage levels in the economy: $\underline{w}$, w_1, w_2, W_1 and W_2. We make the following assumption on the ranking between the wage levels.

Assumption 3.2.

(a). $\omega(q, \varepsilon) \geq \underline{w}$ for all q and ε.

(b). $\omega(2, 0) > \omega(1, 0)$, meaning that $w_2 > w_1$.

(c). $\omega(2, 1) - \omega(2, 0) \geq \omega(1, 1) - \omega(1, 0) > 0$, meaning that $W_2 - w_2 \geq W_1 - w_1 > 0$ and implies $W_q > w_q$.

(d). $\omega(1, 1) \geq \omega(2, 0)$, meaning that $W_1 \geq w_2$.

$\omega(q, \varepsilon)$ is the function measuring the wage level for a worker who succeeded in quality q HE and exert ε labour market effort. $\omega(1, 1)$ and $\omega(1, 0)$ are the functions for the wage levels, W_1 and w_1, for a worker who succeeded in quality $q = 1$ HE exerting high and low labour market effort respectively. $\omega(2, 1)$ and $\omega(2, 0)$ are the functions for the wage levels, W_2 and w_2, for a worker who succeeded in quality $q = 1$ HE exerting high and low labour market effort respectively. From Assumptions 3.2 (b) and (c) we can derive the following relation:

$$W_2 - W_1 \geq w_2 - w_1 > 0. \tag{3.1}$$

Write the two wage levels that a worker could face in the two periods as (adult period wage level, old period wage level), these are the following possible combinations of wage levels that an individual would earn in the two periods:

1. $(\underline{w}, \underline{w})$
2. $(\underline{w}, W_q)$
3. $(\underline{w}, w_q)$

4. (W_q, W_q)
5. (w_q, w_q)

The disutility of effort at work would be determined by the ex-post type k.

$\beta_{qk}\varepsilon_{ik}$ denotes the disutility of effort at work in state (i,k), the β_{qk} is assumed to not change across periods.

Assume that

$$\beta_{22} = \beta_{12} = b > 0,$$

$$\beta_{11} = \beta_{21} = B > 0.$$

The total cost of the effort includes all periods and both the study and work effort. They are denoted respectively:

For students who succeeded in the first chance: $c_i^y e_i^y + \delta\beta_{qk}\varepsilon_{ik} + \delta^2\beta_{qk}\varepsilon_{ik}$.

For students who succeeded in the second chance: $c_i^y e_i^y + \delta c_i^a e_i^a + \delta^2\beta_{qk}\varepsilon_{ik}$.

For students that failed twice: $c_i^y e_i^y + \delta c_i^a e_i^a$.

Let $B > b$. An ex-post type $k = 2$ incurs a disutility of b when doing the top job (as it requires ex-post effort $\varepsilon = 1$) while an ex-post type $k = 1$ incurs a disutility B. Same as in GT's model, assume the level of B is so large that no student of type $k = 1$ will ever exert high effort to obtain the top job. Only the $k = 2$ type will consider obtaining the top job and the type $k = 1$ will only have the middle job.

The government cannot observe the ex-post type k and the individual's labour effort ε_{ik} but can observe the wage rate.

Loans

For the government, the student loan repayments cover the full financial cost of HE for both the first chance and the second chance.

Given the assumptions on B and b as well as the assumption on the government's desired labour effort stated in section 3.2.2, the possible combination of repayment schedules for a student choosing a quality q level of education is as follows, the superscript "a" for the adult period and "o" for the old period:

Students fail twice:	$r_q^a, \quad r_q^o(\underline{w})$
Students succeed in second chance with ex-post type $k = 2$:	$r_q^a, \quad R_{q2}^o(\underline{w})$
Students succeed in second chance with ex-post type $k = 1$:	$r_q^a, \quad R_{q1}^o(\underline{w})$
Students succeed in the first chance with ex-post type $k = 2$:	$R_{q2}^a, \quad R_{q2}^o(W_q)$
Students succeed in the first chance with ex-post type $k = 1$:	$R_{q1}^a, \quad R_{q1}^o(w_q)$

The government is assumed to have full commitment and when they designed the repayment contract they do not know the individuals' ex-ante type i as students have not self-select themselves yet. In the beginning, the government announces to everyone all of the possible repayment schedules for both quality q HE for the adult and the old period that a student would face. As shown by the combinations of repayment schedules above, the repayment scheme when old is possibly dependent on the adult period repayment level, which is itself dependent on the adult period wage level. The government cannot change its promised policies when students become old. In the original GT's model, the assumption of the government having full commitment is unnecessary because there is only one period for policies. The addition of an old period meant that if the government cannot commit to its future policies it will change them when students are old. As a result, the youth, as well as the government, would take this into account when offering the contract to the youth.

The allocation in this model is an array $\{(e_i^y, e_i^a, q_i, \varepsilon_{ik}, r_i^a, r_i^o(\underline{w}), R_{ik}^a, R_{ik}^o(\overline{w}))\}$ i,k=1,2. $\overline{w}$ represents the wage level in the adult period. The menu of contracts is an array $\{(q^y, q^a, r_q^a, r_q^o(\underline{w}), R_{q1}^a, R_{q2}^a, R_{q1}^o(\underline{w}), R_{q1}^o(w_q), R_{q2}^o(\underline{w}), R_{q2}^o(W_q))\}$q=1,2.

We will now write the resource constraint when the above menu is chosen by students who chose education $q = i$ (the quality of education is the same as the ex-ante type). Note that for the second chance education, the government will provide the same quality of HE as the students' chosen quality when young and the students know this when choosing their quality q when young.

Resource Constraint

Assume the government discounts future revenue at the same rate as students discount their utility, at δ. Through assumptions on B and b as well as the assumption on the government's desired labour effort stated in section 3.2.2, the labour effort has the following numerical meaning $\varepsilon_{21} = \varepsilon_{11} = 0$, $\varepsilon_{22} = \varepsilon_{12} = 1$.

(I) For students who succeeded in education when young and with type $i = k$. This kind of student has a probability $p_i(e_i^y)(1-\pi)$ to happen. The government collects the following amount from these type i students:

$$X_i = \delta\varepsilon_{ii}R_{i2}^a + \delta(1-\varepsilon_{ii})R_{i1}^a + \delta^2[\varepsilon_{ii}R_{i2}^o(W_i) + (1-\varepsilon_{ii})R_{i1}^o(w_i)] = \delta X_i^a + \delta^2 X_i^o.$$

(II) For students who succeeded in education when young and with type $i \neq k$. This kind of student has a probability $p_i(e_i^y)\pi$ to happen. From these type i students, the government collects the amount:

$$Y_i = \delta\varepsilon_{ik}R_{i2}^a + \delta(1-\varepsilon_{ik})R_{i1}^a + \delta^2[\varepsilon_{ik}R_{i2}^o(W_i) + (1-\varepsilon_{ik})R_{i1}^o(w_i)]$$
$$= \delta Y_i^a + \delta^2 Y_i^o.$$

(III) For students who failed the first chance but then succeeded in the second chance and have type $i = k$. These students have a probability $(1-p_i(e_i^y))p_i^a(e_i^a)(1-\pi)$ to happen. From these type i students, the government collects the amount:

$$Z_i = \delta r_i^a + \delta^2\left[\varepsilon_{ii}R_{i2}^o(\underline{w}) + (1-\varepsilon_{ii})R_{i1}^o(\underline{w})\right] = \delta Z_i^a + \delta^2 Z_i^o.$$

(IV) For students who failed the first chance but then succeeded in the second chance and have type $i \neq k$. These students have a probability $(1-p_i(e_i^y))p_i^a(e_i^a)\pi$ to happen. From these type i students, the government collects the amount:

$$T_i = \delta r_i^a + \delta^2\left[\varepsilon_{ik}R_{i2}^o(\underline{w}) + (1-\varepsilon_{ik})R_{i1}^o(\underline{w})\right] = \delta T_i^a + \delta^2 T_i^o.$$

(V) For students who failed both chances and don't know their ex-post type k. This has a probability $(1-p_i(e_i^y))(1-p_i^a(e_i^a))$ to happen. From these ex-ante type i students, the government collects the amount:

$$A_i = \delta r_i^a + \delta^2 r_i^o(\underline{w}) = \delta A_i^a + \delta^2 A_i^o.$$

Note that $Z_i^a = T_i^a = A_i^a$ by definition.

Now with the different probabilities for each situation to happen, we can calculate the expected cost of education. The cost for first chance education is y_i and for the discounted second chance is $\delta\eta_a y_i$. The expected cost of education for type i students is:

$$y_i + \left(1 - p_i\left(e_i^y\right)\right)\delta\eta_a y_i.$$

Using the notations just defined, and given that there are two different values of ex-ante type i, the resource constraint can be written as,

$$(RC) \quad \sum_i \lambda_i \{ p_i\left(e_i^y\right)[(1-\pi)X_i + \pi Y_i] + \left(1 - p_i\left(e_i^y\right)\right) p_i^a(e_i^a)[(1-\pi)Z_i + \pi T_i]$$
$$+ \left(1 - p_i\left(e_i^y\right)\right)\left(1 - p_i^a(e_i^a)\right)A_i - y_i - \left(1 - p_i\left(e_i^y\right)\right)\delta\eta_a y_i\} \geq 0.$$

We also make a formal assumption about students being risk averse.

Assumption 3.3: $u_a(.)$ and $u_o(.)$ are strictly increasing, strictly concave and continuously differentiable for both adult and old age utility functions.

3.2.2 First-Best Optimality

In this model, there are five possibilities of ex-post utilities to be realized by a student of ex-ante type i. The ex-post utility is the actual utility that an individual would receive after he first finished his HE and started working in the labour market. First, for the students who succeed in the first chance, denote the following:

V_i: The ex-post utility of a successful student with education $q_i = i$, when the ex-post type is $k = i$.

v_i: The ex-post utility of a successful student with education $q_i = i$, when the ex-post type is $k \neq i$.

The interim expected utility of a successful student who chooses education $q_i = i$ is by definition

$$(3.2a) \qquad U_i^s = \pi(v_i - \delta\beta_{ik}\varepsilon_{ik} - \delta^2\beta_{ik}\varepsilon_{ik}) + (1-\pi)(V_i - \delta\beta_{ii}\varepsilon_{ii} - \delta^2\beta_{ii}\varepsilon_{ii}),$$

where $v_i = \delta u_a[\omega(q_i, \varepsilon_{ik}) - Y_i^a] + \delta^2 u_o[\omega(q_i, \varepsilon_{ik}) - Y_i^o]$,

$V_i = \delta u_a[\omega(q_i, \varepsilon_{ii}) - X_i^a] + \delta^2 u_o[\omega(q_i, \varepsilon_{ii}) - X_i^o]$, with $i \neq k$.

Second, for the ex-post utilities of students that failed the first time but succeeded in the second chance. "ex-post" also includes the adult period for them since the utility at the adult period for the failed students is the same whether they succeed in the second chance or not. Denote the following:

F_i: The ex-post utility of a student who succeeds in the second chance with education $q_i = i$, when the ex-post type is $k = i$.

f_i: The ex-post utility of a student who succeeds in the second chance with education $q_i = i$, when the ex-post type is $k \neq i$.

The interim expected utility of a student who succeeds in the second chance and chooses education $q_i = i$ is by definition

$$(3.2b) \qquad U_i^c = \pi(f_i - \delta^2\beta_{ik}\varepsilon_{ik}) + (1-\pi)(F_i - \delta^2\beta_{ii}\varepsilon_{ii}),$$

where, $f_i = \delta u_a[\underline{w} - T_i^a] + \delta^2 u_o[\omega(q_i, \varepsilon_{ik}) - T_i^o]$,

$F_i = \delta u_a[\underline{w} - Z_i^a] + \delta^2 u_o[\omega(q_i, \varepsilon_{ii}) - Z_i^o]$, with $i \neq k$.

Third, for the utility of a student of ex-ante type i that failed both of his education chances. Denote the following,

u_i: the utility of a type i student that failed twice,

where $u_i = \delta u_a[\underline{w} - A_i^a] + \delta^2 u_o[\underline{w} - A_i^o]$.

The ex-ante expected utility of a student with ex-ante type i, net of expected study effort cost, is defined as follows

(3.3) $p_i(e_i^y)U_i^s + (1 - p_i(e_i^y))p_i^a(e_i^a)U_i^c + (1 - p_i(e_i^y))(1 - p_i^a(e_i^a))u_i - c_i^y e_i^y - (1 - p_i(e_i^y))\delta c_i^a e_i^a$, where $i = 1, 2$.

Solved the first-best problem in the standard utilitarian case, in which the social welfare weight placed on ex-ante type i is equal to the proportion of ex-ante type i in the population. As the government wants to provide students of ex-ante type i with quality i HE (formal assumption in Assumption 3.4 (b) and (c)), the first-best utilitarian optimum can be obtained by solving the following:

(3.4) $$Max \sum_i \lambda_i \, [p_i(e_i^y)U_i^s + (1 - p_i(e_i^y))p_i^a(e_i^a)U_i^c + (1 - p_i(e_i^y))(1 - p_i^a(e_i^a))u_i - c_i^y e_i^y - (1 - p_i(e_i^y))\delta c_i^a e_i^a]$$

with respect to $\{(e_i^y, e_i^a, q_i, \varepsilon_{ik}, r_i^a, r_i^o(\underline{w}), R_{ik}^a, R_{q1}^o(\overline{w}))\}_{\text{i,k=1,2}}$, subject to RC, with each effort level chosen in the set $\{0, 1\}$.

Given the many possible effort level combinations in this model each with its level of welfare, the only case of the government's intended effort levels that will be analysed here is the case in which succeeding in education is valuable to the social welfare of both education chances and those with ex-post type $k = 2$ should not waste their labour market opportunity. In terms of effort vector, assume the government requires a high study effort for both types of students for both education chances, $\left(e_1^{y*}, e_2^{y*}, e_1^{a*}, e_2^{a*}\right) = (1, 1, 1, 1)$, and that the government requires the ex-post labour effort to be high if and only if the individuals' ex-post type k is high at 2, it meant that $(e_{i1}^*, e_{i2}^*) = (0, 1)$. We assume that the provision of HE benefits society in the case we studied but our model can study other cases as well. For example, our model can present the case in which HE is an inefficient policy if the study effort is very costly for a student type. In that scenario, the optimal student effort is zero for said type as the government does not aim to induce any study effort from said students. The objective of the government and the interests of said students are automatically aligned from the start.

Given the assumption on the socially efficient effort level, we can derive the ex-post utility of both ex-ante types, firstly for the students who succeeded in the first chance and then for students who succeeded in the second chance. The ex-post utilities are presented in the following equations:

(3.5)
$$V_2 = \delta V_2^a + \delta^2 V_2^o = \delta u_a(W_2 - R_{22}^a) + \delta^2 u_o\big(W_2 - R_{22}^o(W_2)\big),$$
$$v_2 = \delta v_2^a + \delta^2 v_2^o = \delta u_a(w_1 - R_{21}^a) + \delta^2 u_o(w_2 - R_{21}^o(w_2)),$$
$$V_1 = \delta V_1^a + \delta^2 V_1^o = \delta u_a(w_1 - R_{11}^a) + \delta^2 u_o(w_1 - R_{11}^o(w_1)),$$
$$v_1 = \delta v_1^a + \delta^2 v_1^o = \delta u_a(W_1 - R_{12}^a) + \delta^2 u_o(W_1 - R_{12}^o(W_1)),$$
$$F_2 = \delta F_2^a + \delta^2 F_2^o = \delta u_a\big(\underline{w} - r_2^a\big) + \delta^2 u_o\left(W_2 - R_{22}^o\big(\underline{w}\big)\right),$$
$$f_2 = \delta f_2^a + \delta^2 f_2^o = \delta u_a\big(\underline{w} - r_2^a\big) + \delta^2 u_o\left(w_2 - R_{21}^o\big(\underline{w}\big)\right),$$
$$F_1 = \delta F_1^a + \delta^2 F_1^o = \delta u_a\big(\underline{w} - r_1^a\big) + \delta^2 u_o\left(w_1 - R_{11}^o\big(\underline{w}\big)\right),$$
$$f_1 = \delta f_1^a + \delta^2 f_1^o = \delta u_a\big(\underline{w} - r_1^a\big) + \delta^2 u_o\left(W_1 - R_{12}^o\big(\underline{w}\big)\right).$$

note that $F_2^a = f_2^a = u_2^a$ and $F_1^a = f_1^a = u_1^a$, furthermore $u_1 = \delta u_1^a + \delta^2 u_1^o$ with $u_1^o = u_o\left(\underline{w} - r_1^o(\underline{w})\right)$ and $u_2 = \delta u_2^a + \delta^2 u_2^o$ with $u_2^o = u_o\left(\underline{w} - r_2^o(\underline{w})\right)$.

Given the assumption of the optimal choice of labour efforts, the expected wage E_{wq}^s of a student who succeeds in the first chance and with education quality q is

$$E_{w1}^s = (1-\pi)(\delta w_1 + \delta^2 w_1) + \pi(\delta W_1 + \delta^2 W_1),$$

$$E_{w2}^s = (1-\pi)(\delta W_2 + \delta^2 W_2) + \pi(\delta w_2 + \delta^2 w_2).$$

The expected wage E_{wq}^c of a student who succeeds in the second chance and with education quality q is

$$E_{w1}^c = \delta \underline{w} + (1-\pi)\delta^2 w_1 + \pi \delta^2 W_1,$$

$$E_{w2}^c = \delta \underline{w} + (1-\pi)\delta^2 W_2 + \pi \delta^2 w_2.$$

Denote the social surplus by S_i, it is the expected social benefit of education for a type i student choosing $q = i$ with the amount of ex-ante study effort and ex-post labour effort that is assumed to be socially efficient above

$$S_i = P_i E_{wi}^s + (1-P_i) P_i^a E_{wi}^c + (1-P_i)(1-P_i^a)\left(\delta \underline{w} + \delta^2 \underline{w}\right) - y_i - (1-P_i)\delta \eta_a y_i.$$

Similar to GT's model we make Assumption 3.4:

Assumption 3.4

(a) $S_i \geq \delta \underline{w} + \delta^2 \underline{w}$ for all i;

(b) $P_2[E_{w2}^s - E_{w1}^s] \geq y_2 - y_1 \geq P_1[E_{w2}^s - E_{w1}^s]$;

(c) $P_2^a[E_{w2}^c - E_{w1}^c] \geq \delta \eta_a (y_2 - y_1) \geq P_1^a[E_{w2}^c - E_{w1}^c]$.

$P_2[E^s_{w2} - E^s_{w1}]$ is the expected wage gain for an ex-ante type $i = 2$ student to choose quality $q = 2$ HE instead of quality $q = 1$ HE for the first chance at HE and $P_1[E^s_{w2} - E^s_{w1}]$ is the expected wage gain for an ex-ante type $i = 1$ student to choose quality $q = 2$ HE instead of quality $q = 1$ HE for the first chance at HE. $P^a_2[E^c_{w2} - E^c_{w1}]$ is the expected wage gain for an ex-ante type $i = 2$ student to choose quality $q = 2$ HE instead of quality $q = 1$ HE for the second chance at HE and $P^a_1[E^c_{w2} - E^c_{w1}]$ is the expected wage gain for an ex-ante type $i = 1$ student to choose quality $q = 2$ HE instead of quality $q = 1$ HE for the second chance at HE. Assumption 3.4 (b) meant that for an ex-ante type i student who succeeds in the first chance, choosing education quality $q = i$ generates more social surplus than choosing the alternative, and Assumption 3.4 (c) states that for an ex-ante type i student that succeeds in the second chance, choosing education quality $q = i$ also generates more social surplus than choosing the alternative, given the probability of success. Combined, it means a student of ex-ante type i should choose education $q = i$.

There need to be two inverse utility functions, one for each of the utility functions in the two periods,

(3.6) $$h_a(x) = u_a^{-1}(x), \qquad h_o(x) = u_o^{-1}(x).$$

By using the inverse functions on the student's utility function of each period, we can obtain the following function of repayments,

(3.7) $$\delta R^a_{22} + \delta^2 R^o_{22}(W_2) = \delta W_2 + \delta^2 W_2 - \delta h_a(V^a_2) - \delta^2 h_o(V^o_2),$$

$$\delta R^a_{21} + \delta^2 R^o_{21}(w_2) = \delta w_2 + \delta^2 w_2 - \delta h_a(v^a_2) - \delta^2 h_o(v^o_2),$$

$$\delta R^a_{11} + \delta^2 R^o_{11}(w_1) = \delta w_1 + \delta^2 w_1 - \delta h_a(V^a_1) - \delta^2 h_o(V^o_1),$$

$$\delta R^a_{12} + \delta^2 R^o_{12}(W_1) = \delta W_1 + \delta^2 W_1 - \delta h_a(v^a_1) - \delta^2 h_o(v^o_1),$$

$$\delta r_2^a + \delta^2 R_{22}^o(\underline{w}) = \delta\underline{w} + \delta^2 W_2 - \delta h_a(F_2^a) - \delta^2 h_o(F_2^o),$$

$$\delta r_2^a + \delta^2 R_{21}^o(\underline{w}) = \delta\underline{w} + \delta^2 w_2 - \delta h_a(f_2^a) - \delta^2 h_o(f_2^o),$$

$$\delta r_1^a + \delta^2 R_{11}^o(\underline{w}) = \delta\underline{w} + \delta^2 w_1 - \delta h_a(F_1^a) - \delta^2 h_o(F_1^o),$$

$$\delta r_1^a + \delta^2 R_{12}^o(\underline{w}) = \delta\underline{w} + \delta^2 W_1 - \delta h_a(f_1^a) - \delta^2 h_o(f_1^o),$$

$$\delta r_1^a + \delta^2 r_1^o(\underline{w}) = \delta\underline{w} + \delta^2 \underline{w} - \delta h_a(u_1^a) - \delta^2 h_o(u_1^o),$$

$$\delta r_2^a + \delta^2 r_2^o(\underline{w}) = \delta\underline{w} + \delta^2 \underline{w} - \delta h_a(u_2^a) - \delta^2 h_o(u_2^o).$$

Similarly, if this is done without writing down what the student's specific ex-ante type i is, we obtain

$$Y_i = \delta Y_i^a + \delta^2 Y_i^o = \delta\omega(q_i, \varepsilon_{ik}) + \delta^2\omega(q_i, \varepsilon_{ik}) - \delta h_a(v_i^a) - \delta^2 h_o(v_i^o),$$

$$X_i = \delta X_i^a + \delta^2 X_i^o = \delta\omega(q_i, \varepsilon_{ii}) + \delta^2\omega(q_i, \varepsilon_{ii}) - \delta h_a(V_i^a) - \delta^2 h_o(V_i^o),$$

$$T_i = \delta T_i^a + \delta^2 T_i^o = \delta\underline{w} + \delta^2\omega(q_i, \varepsilon_{ik}) - \delta h_a(f_i^a) - \delta^2 h_o(f_i^o),$$

$$Z_i = \delta Z_i^a + \delta^2 Z_i^o = \delta\underline{w} + \delta^2\omega(q_i, \varepsilon_{ii}) - \delta h_a(F_i^a) - \delta^2 h_o(F_i^o),$$

$$A_i = \delta A_i^a + \delta^2 A_i^o = \delta\underline{w} + \delta^2\underline{w} - \delta h_a(u_i^a) - \delta^2 h_o(u_i^o).$$

The expected amount of resources that are needed to provide the expected utility $P_i U_i^s + (1 - P_i)P_i^a U_i^c + (1 - P_i)(1 - P_i^a)u_i$, is denoted as

(3.8) $$E(h_i) = P_i\big[(1-\pi)\big(\delta h_a(V_i^a) + \delta^2 h_o(V_i^o)\big) + \pi\big(\delta h_a(v_i^a) + \delta^2 h_o(v_i^o)\big)\big] + (1 - P_i)P_i^a\big[(1-\pi)\big(\delta h_a(F_i^a) + \delta^2 h_o(F_i^o)\big) + \pi\big(\delta h_a(f_i^a) + \delta^2 h_o(f_i^o)\big)\big] + (1 - P_i)(1 - P_i^a)\big(\delta h_a(u_i^a) + \delta^2 h_o(u_i^o)\big).$$

The resource constraint RC can be simplified by substituting the terms for X_i, Y_i, Z_i, T_i and A_i and separating the terms into the expression for S_i and $E(h_i)$, the resource constraint is now,

$(\overline{RC})$ $$\sum_i \lambda_i \{S_i - E(h_i)\} \geq 0.$$

The repayment schemes are eliminated from the resource constraint and since $P_i = p_i(1)$ and $P_i^a = p_i^a(1)$, the first-best utilitarian problem is expressed as the following

(3.9) $$Max \sum_i \lambda_i \left[P_i U_i^s + (1 - P_i)P_i^a U_i^c + (1 - P_i)(1 - P_i^a)u_i - c_i^y e_i^y - (1 - P_i)\delta c_i^a e_i^a\right],$$

with respect to $(V_i^a, V_i^o, v_i^a, v_i^o, F_i^a, F_i^o, f_i^o, u_i^o)_{i=1,2}$, subject to $\overline{RC}$. Note that due to the model's construction, $F_i^a = f_i^a = u_i^a$ by definition.

Solving problem (3.9) and obtain the following proposition. The proposition is obtained in the Appendix A3.1.

Proposition 3.1:

Under Assumption 3.1-3.4, the first-best optimum implies full insurance for any student, that is for all i

$$V_i^a = V_i^o = v_i^a = v_i^o = F_i^a = F_i^o = f_i^o = u_i^o,$$

and in the standard utilitarian case, the first-best optimum exhibits full equality between ex-ante types, which means

$$V_1^a = V_2^a, V_1^o = V_2^o, v_1^a = v_2^a, v_1^o = v_2^o,$$

$$F_1^a = F_2^a, F_1^o = F_2^o, f_1^o = f_2^o, u_1^o = u_2^o.$$

The resource constraint $\overline{RC}$ must be binding at the first-best optimum.

The students are insured against education risks and labour market risks. Students are also provided with intertemporal insurance. This is a common first-best result in which the government offered the maximum possible insurance in the economy. This result would not be attainable with asymmetric information as students with ex-ante type $i = 2$ will pretend to be ex-ante type $i = 1$ students since if they tell the truth with quality $q = 2$ education they are more likely to obtain a high ex-post type and resulting in extra effort cost in the labour market. Also, no students would exert effort in either HE or the labour market since they would not be compensated for exerting effort in either HE or the labour market.

3.3 Asymmetric Information and Incentive Constraints

For the second-best case, the same assumptions on the effort levels are maintained, which are $\left(e_{1}^{y*}, e_{2}^{y*}, e_{1}^{a*}, e_{2}^{a*}\right) = (1, 1, 1, 1)$ for study effort and $(e_{i1}^{*}, e_{i2}^{*}) = (0, 1)$ for the ex-post effort. The second-best effort levels that the government wishes to induce are the same as the first-best effort levels for both study and ex-post effort for both of the periods that they respectively occur.

In the second-best case, the students' ex-ante types, ex-post types and effort levels are not observable by the government. The problem for the government is to satisfy the adverse selection and moral hazard incentive constraints. The students must self-select the contracts designed for them.

3.3.1 Incentives

The ex-post incentives (labour effort constraints) must be satisfied for both the students who succeeded in the first chance and the students who succeeded in the second chance. Due to the assumption of the large size of B, the only ex-post effort that needs to worry about are the students with ex-post type $k = 2$ pretending to be $k = 1$ to obtain a middle-level job and save on ex-post effort cost. The ex-post moral hazard constraints, which will be called labour effort constraints, should be separated for the two periods since the type $k = 2$ student needs to exert effort in both periods.

For the students who succeeded in the first chance. Those who chose education quality $q = 1$ and with ex-post type $k = 2$ will not choose a middle-level job in either period ex-post if and only if

(ICX_1^s) $\quad v_1^a - b \geq V_1^a \quad$ and $\quad v_1^o - b \geq V_1^o.$

Those students have education quality $q = 2$ and with ex-post type $k = 2,$ they will not choose a middle-level job in either period ex-post if and only if

(ICX_2^s) $\quad V_2^a - b \geq v_2^a \quad$ and $\quad V_2^o - b \geq v_2^o.$

For students who succeeded in the second chance. Those students have chosen education quality $q = 1$ and with ex-post type $k = 2$, they will not choose a middle-level job when old ex-post if and only if

(ICX_1^c) $\quad f_1^o - b \geq F_1^o.$

If the student has chosen education quality $q = 2$ and with ex-post type $k = 2$, he will not choose a middle-level job ex-post when old if and only if

(ICX_2^c) $\quad F_2^o - b \geq f_2^o.$

Note that different to the students who succeeded in the first chance, there are no labour effort constraints to satisfy in the adult period for the students who succeeded in the second chance.

The interim expected utilities from (3.2a) and (3.2b) of ex-ante type $i = 1, 2$, for both the students who succeeded in the first chance and the students who succeeded in the second chance, with their ex-post utilities can be written as

$(EU_1^s)\quad U_1^s = \pi(\delta v_1^a + \delta^2 v_1^o - \delta b - \delta^2 b) + (1-\pi)(\delta V_1^a + \delta^2 V_1^o),$

$(EU_2^s)\quad U_2^s = \pi(\delta v_2^a + \delta^2 v_2^o) + (1-\pi)(\delta V_2^a + \delta^2 V_2^o - \delta b - \delta^2 b),$

$(EU_1^c)\quad U_1^c = \pi(\delta f_1^a + \delta^2 f_1^o - \delta^2 b) + (1-\pi)(\delta F_1^a + \delta^2 F_1^o),$

$(EU_2^c)\quad U_2^c = \pi(\delta f_2^a + \delta^2 f_2^o) + (1-\pi)(\delta F_2^a + \delta^2 F_2^o - \delta^2 b).$

Given values of the ex-post utilities that satisfy all the labour effort constraints, the students need to simultaneously self-select the right contract that reveals their ex-ante type and exert the correct ex-ante (study) effort. Now we will present the ex-ante self-selection constraints and the study effort constraints for both the first chance at HE when young and the second chance.

The ex-ante self-selection constraint $\overline{IC_i}$ means that a student of ex-ante type i does not want to mimic to be ex-ante type j while exerting a high study effort at both periods (the expected intertemporal study effort costs cancelled each other out),

$(\overline{IC_i})\quad P_i U_i^s + (1-P_i)P_i^a U_i^c + (1-P_i)(1-P_i^a)u_i \geq P_i U_j^s + (1-P_i)P_i^a U_j^c + (1-P_i)(1-P_i^a)u_j,$

for all $i = 1, 2$ and $i \neq j$.

The study effort constraints are needed because the government wants a high study effort for both education opportunities and both types of students.

For a student with an ex-ante type i who experiences a second chance, the second chance effort constraint states that he will prefer to exert a high study effort level over a low study effort during the second chance,

$$(MH_i) \qquad P_i^a U_i^c + (1 - P_i^a)u_i - \delta c_i^a \geq p_i^a U_i^c + (1 - p_i^a)u_i.$$

For students in their youth who just learnt their ex-ante type i, and are determining whether to exert high study effort for the first chance or not, given the possibility of receiving a second chance in the future adult period, their first chance effort constraint meant they should prefer high study effort for the first chance at HE over low study effort, given a high effort at the second chance.

$$(\overline{MH_i}) \qquad P_i U_i^s + (1 - P_i)P_i^a U_i^c + (1 - P_i)(1 - P_i^a)u_i - c_i^y - (1 - P_i)\delta c_i^a \geq p_i U_i^s + (1 - p_i)P_i^a U_i^c + (1 - p_i)(1 - P_i^a)u_i - (1 - p_i)\delta c_i^a.$$

If both MH_i and $\overline{MH_i}$ are satisfied, then the combined moral hazard constraint of exerting high study effort on both the first chance and the second chance over low study effort for both is automatically satisfied.

In this model, it is not possible for an ex-ante type i student who self-selects and is revealed to be type i when young for the first chance at HE to mimic being an ex-ante type j for the second chance since no contract would be designed that way.

In addition, there are additional combined incentive constraints so that an ex-ante type i student would prefer to exert high study effort for the first chance at HE than to mimic type j and exert low study effort for the first chance at HE, given high study effort level in the second chance,

(IC_i) $P_i U_i^s + (1-P_i)P_i^a U_i^c + (1-P_i)(1-P_i^a)u_i - c_i^y - (1-P_i)\delta c_i^a \geq p_i U_j^s + (1-p_i)P_i^a U_j^c + (1-p_i)(1-P_i^a)u_j - (1-p_i)\delta c_i^a$.

Also, ex-ante type i students should prefer high study effort to low study effort at both education opportunities and to mimic ex-ante type j,

$(\underline{IC_i})$ $P_i U_i^s + (1-P_i)P_i^a U_i^c + (1-P_i)(1-P_i^a)u_i - c_i^y - (1-P_i)\delta c_i^a \geq p_i U_j^s + (1-p_i)p_i^a U_j^c + (1-p_i)(1-p_i^a)u_j$.

One additional combined incentive constraint is that ex-ante type i students should prefer to exert high study effort at both study opportunities compared to mimic ex-ante type j and exert high study effort at the first chance and low study effort at the second chance,

(ICA_i) $P_i U_i^s + (1-P_i)P_i^a U_i^c + (1-P_i)(1-P_i^a)u_i - c_i^y - (1-P_i)\delta c_i^a \geq P_i U_j^s + (1-P_i)p_i^a U_j^c + (1-P_i)(1-p_i^a)u_j - c_i^y$.

The second-best optimality problem is the following:

$$Max \sum_i l_i^{OB} \left[P_i U_i^s + (1-P_i)P_i^a U_i^c + (1-P_i)(1-P_i^a)u_i - c_i^y - (1-P_i)\delta c_i^a\right],$$

subject to $\overline{RC}$, $\overline{IC_i}$, MH_i, $\overline{MH}_i$, IC_i, $\underline{IC_i}$, ICA_i, ICX_i and EU_i along with $F_1^a = f_1^a = u_1^a$ and $F_2^a = f_2^a = u_2^a$, $i = 1, 2$. l_i^{OB} represents the social welfare weight given to the ex-ante type i

student by the government in the welfare function. Assume that $l_1^{OB} + l_2^{OB} = 1$, and $l_i^{OB} > 0$ for both i. In the second-best problem without any treatment, there are 20 variables to be differentiated and 16 incentive constraints. Preliminary treatment to simplify the problem should be done first. The second-best optimum will be characterised by the relations between the ex-post utilities obtained from solving the second-best optimality problem.

3.3.2 Labour Effort Constraints

In the second-best optimality problem, if any of the labour effort constraints are not binding, based on the first-order conditions the respective ex-post utilities in that labour effort constraint are equal to each other which violates that labour effort constraint. Therefore, in any second-best optimum, the ICX_i constraints need to be binding.

Lemma 3.1. The ICX_i constraints must be binding in any second-best optimum, which means,

(3.10) $$v_1^a - b = V_1^a, v_1^o - b = V_1^o, V_2^a - b = v_2^a, V_2^o - b = v_2^o,$$

$$f_1^o - b = F_1^o, F_2^o - b = f_2^o.$$

(3.11) $$U_1^s = \delta V_1^a + \delta^2 V_1^o, \qquad U_2^s = \delta v_2^a + \delta^2 v_2^o,$$

$$U_1^c = \delta F_1^a + \delta^2 F_1^o, \qquad U_2^c = \delta f_2^a + \delta^2 f_2^o.$$

We will replace the ex-ante expected utilities with the ex-post utilities in the study effort constraints using (3.11).

The relationships between the sums of the ex-post utilities are as follows,

(3.12) $$\delta v_1^a + \delta^2 v_1^o - \delta b - \delta^2 b = \delta V_1^a + \delta^2 V_1^o, \delta V_2^a + \delta^2 V_2^o - \delta b - \delta^2 b = \delta v_2^a + \delta^2 v_2^o,$$

$$\delta f_1^a + \delta^2 f_1^o - \delta^2 b = \delta F_1^a + \delta^2 F_1^o, \delta F_2^a + \delta^2 F_2^o - \delta^2 b = \delta f_2^a + \delta^2 f_2^o.$$

Using (3.10) and the fact that $F_1^a = f_1^a = u_1^a$ and $F_2^a = f_2^a = u_2^a$, the following ex-post utilities can be eliminated from the second-best problem, they are v_1^a, v_1^o, V_2^a, V_2^o, f_1^a, f_1^o, f_2^a, F_2^o, u_1^a and u_2^a. After these eliminations, there are ten remaining variables in the second-best problem.

3.3.3 Study Effort Constraints for the Second Chance

The second chance study effort constraint for the student with ex-ante type i (MH_i) can be rewritten as

$$(MH_i) \qquad U_i^c - u_i \geq \delta K_i^a, \qquad \text{where } K_i^a = \frac{c_i^a}{P_i^a - p_i^a}.$$

We assume that an ex-ante type $i = 2$ student is more efficient than an ex-ante type $i = 1$ student at exerting study effort, formally with the following assumption

Assumption 3.5: $K_1^a \geq K_2^a$.

In terms of ex-post utilities, for students of ex-ante type $i = 1$, we can derive that

$$(MH_1) \qquad \delta^2 F_1^o - \delta^2 u_1^o \geq \delta K_1^a.$$

For students of ex-ante type $i = 2$, we can derive that

$$(MH_2) \qquad \delta^2 f_2^o - \delta^2 u_2^o \geq \delta K_2^a.$$

3.3.4 Study Effort Constraints for the First Chance

The study effort constraints for the first chance at HE when young for a student of ex-ante type i ($\overline{MH_i}$) can be similarly rewritten as

$$(\overline{MH_i}) \qquad U_i^s - P_i^a U_i^c - (1 - P_i^a)u_i + \delta c_i^a \geq K_i^y, \qquad \text{where } K_i^y = \frac{c_i^y}{P_i - p_i}.$$

It is also reasonable to have a similar assumption about ex-ante type $i = 2$ students being more efficient at exerting study effort for the first chance as well, which formally we assume

Assumption 3.6: $K_1^y \geq K_2^y$.

In terms of ex-post utilities, for students of ex-ante type $i = 1$, we can derive that

$$(\overline{MH_1}) \qquad \delta V_1^a + \delta^2 V_1^o - P_1^a \delta^2 F_1^o - \delta F_1^a - (1 - P_1^a)\delta^2 u_1^o + \delta c_1^a \geq K_1^y.$$

For students of ex-ante type $i = 2$, we have

$$(\overline{MH_2}) \qquad \delta v_2^a + \delta^2 v_2^o - P_2^a \delta^2 f_2^o - \delta F_2^a - (1 - P_2^a)\delta^2 u_2^o + \delta c_2^a \geq K_2^y.$$

3.3.5 Adverse Selection

Rewrite the ex-ante self-selection constraints $\overline{IC_i}$ by putting u_i on one side and combining them to obtain the following string of inequality,

$$\frac{P_2}{(1 - P_2)(1 - P_2^a)}(U_2^s - U_1^s) + \frac{P_2^a}{(1 - P_2^a)}(U_2^c - U_1^c) \geq u_1 - u_2$$

$$\geq \frac{P_1}{(1 - P_1)(1 - P_1^a)}(U_2^s - U_1^s) + \frac{P_1^a}{(1 - P_1^a)}(U_2^c - U_1^c),$$

(3.13) $\frac{P_2}{(1-P_2)(1-P_2^a)}\left[U_2^s - U_1^s + \frac{P_2^a}{P_2}(1-P_2)(U_2^c - U_1^c)\right] \geq u_1 - u_2 \geq \frac{P_1}{(1-P_1)(1-P_1^a)}\left[U_2^s - U_1^s + \frac{P_1^a}{P_1}(1-P_1)(U_2^c - U_1^c)\right]$.

By assumption $P_2 > P_1$, $P_2^a > P_1^a$, therefore $\frac{P_2}{(1-P_2)(1-P_2^a)} > \frac{P_1}{(1-P_1)(1-P_1^a)}$, but in (3.13) there are also $\frac{P_2^a}{P_2}(1-P_2)$ and $\frac{P_1^a}{P_1}(1-P_1)$ inside the [.] as well. For (3.13), the various terms of interim expected utilities can take on different relationships to each other for example $U_2^c \leq U_1^c$, while keeping the inequality satisfied. Unlike the adverse selection constraint without a second chance, it does not require strict restrictions on what U_i and u_i need to be. If both $\overline{IC_i}$ are binding in the second-best optimum, the GT's model's equal treatment result $U_2^s = U_1^s$, $u_1 = u_2$ is not the only result that would satisfy the condition of both $\overline{IC_i}$ being binding.

3.3.6 Simplifying the Combined Incentive Constraints

Given the combination of adverse selection and moral hazard as well as the possibility of second chance education, there are six more additional combined incentive constraints in this model and two of them are automatically satisfied.

Lemma 3.2

(a). For the incentive constraint $\underline{IC_2}$, if IC_2 and MH_1 are satisfied, then by Assumption 3.5 the $\underline{IC_2}$ is satisfied.

(b). For the incentive constraint ICA_2, if $\overline{IC_2}$ and MH_1 are satisfied, then by Assumption 3.5 the ICA_2 is satisfied.

3.4 Properties of the Second-Best Optimums

In section 3.4.1, the common properties of the second-best optimums and the effect of introducing the second chance are discussed. In section 3.4.2, we examined the necessity of using ICL to implement the second-best optimum. In section 3.4.3, we analysed the effect of introducing the second chance on the degree of ex-ante inequality.

3.4.1 Common Second-Best Relationships and Implications on Inequality and Insurance

There are relationships between the ex-post utilities that are common across all possible cases of second-best optimum in this model and they are derived in Appendix A3.3.

Proposition 3.2: In all the second-best optimums the ex-post utilities have the following relations:

(a). $V_1^a = V_1^o, v_2^a = v_2^o$.

(b). $f_1^o > F_1^a > u_1^o, F_2^o > F_2^a > u_2^o$,

(c). $V_2^o = V_2^a > F_2^a$,

Additionally, in all possible second-best optimums, the resource constraint $\overline{RC}$ and the second chance study effort constraint MH_1 must be binding.

Corollary 3.1: The common properties of the second-best optimums meant the optimal repayment schedule must obey the following relations:

(a). $V_1^a = V_1^o$ implies $R_{11}^a = R_{11}^o(w_1)$,

$v_2^a = v_2^o$ implies $R_{21}^a = R_{21}^o(w_2)$.

(b). $f_1^o > F_1^a > u_1^o$ implies $W_1 - R_{12}^o(\underline{w}) > \underline{w} - r_1^a, r_1^o(\underline{w}) > r_1^a$,

$F_2^o > F_2^a > u_2^o$ implies $W_2 - R_{22}^o(\underline{w}) > \underline{w} - r_2^a, r_2^o(\underline{w}) > r_2^a$.

(c). $V_2^o = V_2^a > F_2^a$ implies $R_{22}^a = R_{22}^o(W_2), W_2 - R_{22}^a = W_2 - R_{22}^o(W_2) > \underline{w} - r_2^a$.

As shown in Corollary 3.1 (a), the government continue to provide intertemporal insurance for the students who succeed in their first chance of HE as their loan repayment level is the same across periods for each ex-ante type i student because it is optimal for the government to maintain the same level of ex-post utilities for those students who succeeded in their first chance of HE.

Proposition 3.2 (b) meant the introduction of the second chance removes intertemporal insurance for those who failed their first chance at HE. From Corollary 3.1 (b), for students who failed in their first chance but succeeded in the second chance and gained the ex-post type $k = 2$, the net wage at old age is higher than their net wage in the adult period and for students who failed twice, they will repay more or receive fewer transfers when old than in the adult period.

Corollary 3.1 (c) implies that there is a limit to the degree of redistribution in the adult period. The most successful student in terms of labour market performance who succeeds in their first chance needs to be made better off than students who have failed their first chance. Otherwise, it would be difficult to create incentives to exert study effort for HE when young.

Because of the additional incentive constraints, we cannot fully characterize the relations in the second-best optimums but a valid potential solution to the second-best problem is $U_2^s = U_1^s$, $U_2^c = U_1^c$ and $u_2 = u_1$ with both $\overline{IC_1}$ and $\overline{IC_2}$ being binding constraints. All the additional combined incentive constraints are automatically satisfied along with MH_2 in this new equal treatment outcome.

In the second-best optimum of GT's model without a second chance, their $\overline{MH_1}$ constraint is binding with $U_1^s - u_1 = K_1^y$, the insurance against failing HE is incomplete. With the addition of the second chance the second chance study effort constraint MH_1 must be binding which substitute into our $\overline{MH_1}$ implies $U_1^s - u_1 \geq K_1^y + p_1^a \delta K_1^a > K_1^y$. The difference in utilities between those who succeeded HE when young and those who failed HE in general and earned the lowest wage in both periods has widened. The second chance reduced the degree of insurance against failing HE for the type $i = 1$ (low ability) students compared with the corresponding insurance in GT's model when taking the incomplete insurance against failing second chance into account.

The final common properties in the model are derived through the satisfaction of the study effort constraints MH_i and $\overline{MH_i}$, which together with Proposition 3.2 (b) concludes that for each ex-ante type i student, u_i^o, the ex-post utility at old age when he failed his second chance is the lowest possible utility he would obtain.

$$\{V_i^a, V_i^o, v_i^a, v_i^o, F_i^a, f_i^a, u_i^a, F_i^o, f_i^o\} > u_i^o, i = 1, 2.$$

Policy-wise, this could be implemented if the student loan contract includes a clause that decreases the repayment threshold for an individual student if said student fails in HE again. Additionally, because the intertemporal insurance is maintained for the workers who succeeded in their first chance at HE while the workers earning the lowest income will receive a lower utility if they fail their second chance, the introduction of a second chance requires the optimal repayment schedule to widen the inequality between those who succeed in HE when young and those who failed in HE if they failed their second chance at HE. Furthermore, ex-post inequality in the labour market which is the difference in utility between type $k = 2$ and $k = 1$ workers after succeeding in the HE with the same quality and chance still exists between these workers of different ex-post types due to labour effort constraints. From this point of view, the second chance policy has also created more cases of ex-post inequality in the labour market as the workers that succeeded in second chance are previously treated the same because their ex-post type was unknown. Now they face inequality depending on their labour market outcomes because of the labour market effort constraints.

In conclusion, from these common properties, the optimal second-best policy offers intertemporal insurance only to students who succeed in youth HE. In the second-best optimums, the second chance policy improves the utility of those who succeed in the second chance. However, by removing the intertemporal insurance for those who failed youth HE as well as preventing moral hazard in both chances of HE, when old, the workers that have failed twice in HE would be worse off than they are in the adult period and any other workers of the same ex-ante type in the economy. The inequality is widened when old between those who succeeded in HE when young and those who earn the lowest wage.

The aim of a second chance policy in real life is to help out those who missed their first chance at succeeding in HE. However, because of the effects of the second chance policy listed above, a second chance policy should not be used as a redistributive policy.

3.4.2 The Necessity of ICL

For the question of whether a non-income-contingent loan (non-ICL) could implement the second-best optimum. Given that this model's possible repayment levels, agent types and assumptions on wage levels are not largely changed from GT's model, for the equal treatment case we could use the same method to show that it is only in rare conditions could a non-ICL be optimal, which is when education increases earning in a completely additive way for different types of jobs, $W_2 - W_1 - (w_2 - w_1) = 0$. Non-ICL means the optimal repayment schedule can be decomposed into an ordinary loan repayment that depends only on education quality and education history and an ordinary tax that does not depend on education factors. Here "does not depend on education factors" means the income tax depends only on the ex-post type and ignores the students' past education outcomes and the effects of education on income level between workers of the same ex-post type. The student loan should be income contingent, or the income tax needs to depend on the quality and history of education. Intuitively, in our model and real life, the wage rate depends on both education quality and labour effort, but in a non-specific way. This means that within a job type, a higher quality education has a different effect on wage rate than in other job types and for students with a level of quality of education increasing the labour effort will have a different effect on wage rate than students with other levels of quality of education. However, this first method did not consider the impact of the government providing a second chance and is only applicable to the equal treatment case.

The second way of showing non-ICL is not optimal applies to all second-best optimums. The intuition of this method is that we aim to show the government cannot use non-ICL to simultaneously implement all of the ex-post utility relations common in all the second-best optimums. We assume policies cannot be age dependent, meaning an ordinary tax and ordinary loan repayment cannot change across periods if the worker's income and education history are not changed. When old, students succeeded in their first chance and students who succeeded in second chance are engaged in their separate repayment schedules. For the students that have the same ex-post types and chose the same education quality, their only difference to the government is their education history (superscript "s" will indicate both education opportunities taken). We have the following eight equations for the levels of repayment for different old workers if tax schedule and loan repayment can be separated from each other,

$$R_{11}^{o}(w_1) = T_1 + L_1, \quad R_{11}^{o}(\underline{w}) = T_1 + L_1^{s},$$

$$R_{22}^{o}(W_2) = T_2 + L_2, \quad R_{22}^{o}(\underline{w}) = T_2 + L_2^{s},$$

$$R_{12}^{o}(W_1) = T_2 + L_1, \quad R_{12}^{o}(\underline{w}) = T_2 + L_1^{s},$$

$$R_{21}^{o}(w_2) = T_1 + L_2, \quad R_{21}^{o}(\underline{w}) = T_1 + L_2^{s}.$$

This implies that

1. $R_{11}^{o}(\underline{w}) - R_{11}^{o}(w_1) = L_1^{s} - L_1 = R_{12}^{o}(\underline{w}) - R_{12}^{o}(W_1),$
2. $R_{22}^{o}(\underline{w}) - R_{22}^{o}(W_2) = L_2^{s} - L_2 = R_{21}^{o}(\underline{w}) - R_{21}^{o}(w_2).$

Define

$$\Delta^1 = \left[R_{11}^{o}(\underline{w}) - R_{11}^{o}(w_1)\right] - \left[R_{12}^{o}(\underline{w}) - R_{12}^{o}(W_1)\right],$$

$$\Delta^2 = \left[R_{22}^{o}(\underline{w}) - R_{22}^{o}(W_2)\right] - \left[R_{21}^{o}(\underline{w}) - R_{21}^{o}(w_2)\right].$$

If $\Delta^1 = 0$, then it can be shown that an ICL and/or graduate tax is unnecessary. The only observable difference between students with the same ex-post type within equation Δ^1 is their education history as they are earning the same wage and chose the same quality HE and its loan when young. Utilizing equation (3.7) and substituting Δ^1 with the repayment schedules expressed by wage rate and the inverse utility function, we obtain

$$\Delta^1 = w_1 - h_o(F_1^o) - \left(w_1 - h_o(V_1^o)\right) - \left(W_1 - h_o(f_1^o)\right) + \left(W_1 - h_o(v_1^o)\right),$$

$$\Delta^1 = h_o(V_1^o) - h_o(F_1^o) - \left(h_o(v_1^o) - h_o(f_1^o)\right).$$

Although the differences between pairs of ex-post utilities are the same, $v_1^o - V_1^o = f_1^o - F_1^o = b$ due to labour effort constraints, because of Assumption 3.3, $h(.)$ is a strictly convex function, as a result $\Delta^1 \neq 0$ unless $v_1^o = f_1^o$ and $V_1^o = F_1^o$. Now consider the type $i = 1$ workers who are earning the lowest wage in both periods. Their repayment schedules with just an ordinary tax and ordinary loan would be: $r_1^a = t_1 + L_1$, $r_1^o(\underline{w}) = t_1 + L_1^s$, with t_1 denoting the tax charged to those earning the lowest wage. $u_1^a > u_1^o$ is a common property of the second-best optimum. This meant $L_1^s > L_1$ is needed to implement this second-best optimum property. However, this policy meant $v_1^o > f_1^o$ and $V_1^o > F_1^o$ would also be implemented regardless of their relationship. Therefore, even if $v_1^o = f_1^o$ and $V_1^o = F_1^o$ is the relation in the second-best optimum, non-ICL cannot be used to implement it without destroying other properties in the second-best optimum. The only non-ICL method to implement $v_1^o = f_1^o$ and $V_1^o = F_1^o$ is to enforce $L_1^s = L_1$ which would cause $u_1^a = u_1^o$, a violation of a common property of the second-best optimum.

The same transformation can be applied to Δ^2 and results in

$$\Delta^2 = h_o(V_2^o) - h_o(F_2^o) - \left(h_o(v_2^o) - h_o(f_2^o)\right).$$

The utility relationship to achieve $\Delta^2 = 0$ to conclude that ICL is unnecessary are $V_2^o = F_2^o$ and $v_2^o = f_2^o$. Using another common property of the second-best optimum, $u_2^a > u_2^o$. We use the same method and conclude that even if $V_2^o = F_2^o$ and $v_2^o = f_2^o$ is in the second-best optimum, enforcing $V_2^o = F_2^o$ and $v_2^o = f_2^o$ would violate the property of $u_2^a > u_2^o$ in the second-best optimum.

So far we have assumed the repayment for both the first chance and the government-provided second chance HE is in a standard loan repayment format. A further question would be "Could the second-best optimum be implemented by charging a separate ordinary loan for using the second chance at HE while using the first chance at HE is charged by ICL?". The answer is no, even if the student loan for HE of the youth is provided by an ICL, the government-provided second chance still cannot be provided by a non-ICL loan. To see this, the formulation is changed a bit with the other parts remaining the same, for example, let $R_{11}^o(\underline{w}) = R_{11}^o(w_1) + a_1$, $R_{12}^o(\underline{w}) = R_{12}^o(W_1) + a_1$ and $r_1^o(\underline{w}) = r_1^a + a_1$ and again unless $v_1^o = f_1^o$ and $V_1^o = F_1^o$ this non-income-contingent second chance loan cannot implement the second-best. "a_1" is interpreted as a separate loan repayment when old for the workers that used the second chance. The optimal relation $u_1^a > u_1^o$ meant a_1 must be positive. We have an corresponding argument as even if $v_1^o = f_1^o$ and $V_1^o = F_1^o$ is the relation in the second-best optimum, we cannot implement the second-best optimum with an ordinary loan for the second chance without destroying other properties of the second-best optimum. The same argument applies to the type $i = 2$ workers. In terms of policy implication, this meant that when the government promises the current youth a guaranteed second chance to all who fail in HE, the government cannot ignore this second chance policy in the design of the student loan repayment. In the student loan contract, the government need to specify the difference in the repayment levels if the student fails HE but later succeeds in the second chance and earns an equally high wage rate. This feature currently does not exist in UK student loans despite it being an ICL. The introduction of the second chance meant that the optimal ICL or equivalent graduate tax needed to take all the possible workers' education paths or their education history into account.

We still obtain the same results if we only allow the ordinary loan repayment to be age dependent. Because the utilities for those who succeeded in HE in the first chance are the same across periods in the second-best optimum, the loan repayment L_1 and L_2 still need to be the same across periods which allows us to still examine the utilities' relations within a period with utilities' relations across periods.

3.4.3 The Effect of the Second Chance on the Degree of Ex-ante Inequality

The degree of ex-ante inequality is defined by using equation (3.3) to compare the ex-ante expected utility of ex-ante type i student net of expected study effort cost. We can formally compare the difference in expected utility net of expected study effort cost between type $i = 2$ and type $i = 1$ students. In GT's model that doesn't have a second chance, the ex-ante expected utility of type $i = 2$ minus the ex-ante expected utility of type $i = 1$, net of study effort cost, in the equal treatment second-best optimum is:

$$(P_2 - P_1)K_1^y + c_1^y - c_2^y > 0$$

The $(P_2 - P_1)K_1^y$ exists because the ex-ante type $i = 2$ student has a higher probability of success in HE and the government needs to satisfy the effort constraints in HE. $c_1^y - c_2^y$ is the difference in study effort cost of HE between the two types of students. This term is positive because $P_2 > P_1$ and $c_1^y \geq c_2^y$. For ease of comparison, we assume the parameters of the model are such that in equal treatment with $U_1^s = U_2^s$, $U_1^c = U_2^c$ and $u_1 = u_2$, if $\overline{MH_1}$ is satisfied then $\overline{MH_2}$ is automatically satisfied. If instead the parameters meant $\overline{MH_1}$ is automatically satisfied in equal treatment, our analysis still applies since in that case the degree of ex-ante inequality is even larger. This new equal treatment outcome is a potential solution for our second-best optimality problem. With the addition of the second chance the ex-ante expected utility of type $i = 2$ minus the ex-ante expected utility of type $i = 1$ in the equal treatment second-best optimum with $\overline{MH_1}$ binding is now equal to:

$$(P_2 - P_1)K_1^y + (P_2 - P_1)p_1^a\delta K_1^a + [(1 - P_2)P_2^a - (1 - P_1)P_1^a]\delta K_1^a + c_1^y - c_2^y$$
$$+ (1 - P_1)\delta c_1^a - (1 - P_2)\delta c_2^a$$

The terms $(P_2 - P_1)K_1^y$ and $c_1^y - c_2^y$ are still present. There are additional terms introduced by the second chance. $p_1^a\delta K_1^a$ is the additional utility that the government needs to give to students to exert effort in their first chance at HE because the students who failed in their youth expect another chance to succeed in HE. From a student's perspective, this reduces the benefit of succeeding in HE when young relative to failing in them before a second chance is introduced. Therefore, the government need to give extra utility for students to put high effort into their first chance at HE. $(P_2 - P_1)p_1^a\delta K_1^a$ is strictly positive. Another additional term $(1 - P_1)\delta c_1^a - (1 - P_2)\delta c_2^a$ is also positive since by assumption, $(1 - P_1) > (1 - P_2)$ and $c_1^a \geq c_2^a$. The ex-ante type $i = 1$'s expected effort cost for a second chance is greater than ex-ante type $i = 2$ since they are more likely to fail in HE and their cost of effort in the second chance is not smaller than the ex-ante type $i = 2$ students. The term δK_1^a arises due to the government giving additional utility to those who succeeded in the second chance to induce study effort in the second chance. The sign of the term $[(1 - P_2)P_2^a - (1 -$

$P_1)P_1^a]\delta K_1^a$ depends on the difference in the size of the probabilities of a student's expected chance of succeeding in the second chance program before they know the education outcome of their first chance in HE which could be negative since low-ability students are more likely to need a second chance.

Simplify all the additional terms and obtain $(1 - P_2)[(P_2^a - p_1^a)\delta K_1^a - (P_2^a - p_2^a)\delta K_2^a] > 0$, the introduction of the second chance has increased the degree of ex-ante inequality in the equal treatment second-best case and reduced the insurance provided to youth against being born with low innate ability.

Is it possible for the second chance to lower the degree of ex-ante inequality found in the equal treatment case of GT's model in other possible second-best optimums?

Proposition 3.3: The introduction of the second chance increases the degree of the ex-ante inequality compared to the equal treatment outcome without the second chance.

The equal treatment case with the second chance saw an increase in the degree of ex-ante inequality, utilizing this insight, for the ex-ante inequality to fall, the government must offer a higher degree of insurance against failing first chance or second chance for type $i = 2$ students than their type $i = 1$ counterpart. However, both cannot be the case since it would violate the adverse selection constraint for ex-ante type $i = 2$ student $\overline{IC_2}$ as he is not compensated enough for his study effort. Can there be a combination of the difference between U_2^s to u_2, U_2^c to u_2 and U_1^s to u_1 such that the degree of ex-ante inequality has fallen? No, because it will still lead to a violation of the adverse selection constraint for type

$i = 2, \overline{IC_2}$. The introduction of a second chance increases the degree of ex-ante inequality. The proof of Proposition 3.3 is in Appendix A3.4.

3.5 Time-Consistent Optimum

The analysis so far has assumed that the government can commit to its promises of the old period policy when it announces its policy to the youth when they are choosing their quality of education. If the government cannot commit to its future policy it has an incentive to change its old period policy in the future while the students take this into account when making education decisions, the second-best results are no longer valid. The derivation of the properties in the time-consistent optimum will be shown in the Appendix A3.5.

The results derived will be summarised below:

3.5.1 The Degree of Insurance Against Failing the Second Chance

Given Assumption 3.5, if $K_1^a > K_2^a$ then in the time-consistent optimum $U_1^c - u_1 > U_2^c - u_2$ because in the time-consistent optimum not only does MH_1 needs to be binding but the constraint MH_2 also needs to bind. This relationship implies a lower degree of insurance for failing the second chance for the low-ability students in the time-consistent optimums. If $K_1^a = K_2^a$, the government will provide the same degree of insurance for failing the second chance for both types of students.

3.5.2 The Need for ICL

We obtain $F_1^o > V_1^o$, $f_2^o > v_2^o$ in the time-consistent optimum. Using the same argument as in section 3.4.2, this property of the old workers' ex-post utilities meant that an ICL is needed to implement the time-consistent optimum. The reason that the utility of those who succeeded in the second chance is greater than those who do not need a second chance is that the government still must satisfy the study effort constraint for the second chance for both ex-ante types of students. To induce effort for the second chance, the level of F_1^o and f_2^o are now certain to be higher than V_1^o and v_2^o. If an individual's past income is low (at basic wage) and only increased due to success at a second chance. Then he will face a smaller repayment level, policy-wise it might be some cancelling of the student debt based on whether he succeeds in the second chance.

3.5.3 Relationships of Ex-post Utilities

Proposition 3.4: In the time-consistent optimum with $l_1^{OB} \geq \lambda_1$ and $l_2^{OB} \leq \lambda_2$, $V_1^o > v_2^o$. Without a second chance, $u_1^o > V_1^o$ and $u_2^o > v_2^o$ in all time-consistent optimums. With the addition of a second chance, the relation between u_1^o and V_1^o and the relation between u_2^o and v_2^o are uncertain.

In terms of repayment level, $V_1^o > v_2^o$ means that $R_{21}^o(w_2) > R_{11}^o(w_1)$. The increase in the level of repayment is higher than the increase in wage rate from w_1 to w_2, the same relationship applies for the ex-post type $k = 2$ workers as well. Politically if such a policy is to be implemented, loan repayment is probably more feasible than a tax.

The time-consistent optimum implies a stronger level of insurance in old age because it is optimal for the government to ensure the utility relationships in old age are as close to the first-best outcomes as possible. For the students who succeed in their first chance at HE, the various first-order conditions derived with and without a utilitarian social welfare objective are equivalent to the first-best outcome only with the addition of labour effort constraints. This degree of insurance does not exist in the adult period. The time-consistent optimum also lacks intertemporal insurance. The second-best optimum always results in $R_{11}^{o}(w_1) = R_{11}^{a}$ and $R_{21}^{o}(w_2) = R_{21}^{a}$. However, when the government cannot commit, we lose the certainty of the intertemporal insurance provided to these workers no matter whether a second chance is introduced or not.

Now examine additional policy implications for the time-consistent policy without the second chance for the utility relationships of $u_1^o > V_1^o$ and $u_2^o > v_2^o$. When old, the students who succeeded in HE are worse off than the students who failed. However, for students to exert effort in HE when young, this would mean a much higher value of V_1^a and v_2^a compared to u_1^o and u_2^o relative to the second-best case. With the addition of the second chance, because of the need to satisfy the second chance study effort constraint, the utility relations between students who succeeded in HE and those who failed twice are no longer certain when old.

3.6 Conclusion

This paper addresses the following two questions: 1. How do we optimally provide a HE system that offers a second chance to all failed students? 2. What is the impact of such an optimally provided second chance on inequality and insurance?

For the first question, we assume that government policies cannot be age dependent. We demonstrate the necessity of using the ICL or a graduate tax to implement all the optimal relations in the second-best optimum. Even if the first chance of HE is implemented by ICL or a graduate tax, the government still cannot charge an ordinary loan for using the second chance.

For the second question, we benchmarked the results against the inequality and insurance properties found in Gary-Bobo and Trannoy (2015). The addition of a second chance meant the “equal treatment” second-best found in Gary-Bobo and Trannoy (2015) rarely occurred in our framework. Besides, in the “equal treatment” second-best optimum, we find that the second chance increases the degree of ex-ante inequality between those of high and low abilities compared to the results in Gary-Bobo and Trannoy (2015). Furthermore, utilising the insights from this result we conclude that the second chance increases the degree of ex-ante inequality in all possible second-best scenarios compared to the equal treatment second-best optimum without the second chance. We also found that it is only optimal to provide intertemporal insurance to university students who succeeded on their first try at HE. The introduction of a second chance policy removes intertemporal insurance for students who failed their first chance at HE and this combined with the requirement to induce study effort in both the first chance at HE and their second chance at HE leads to the students who failed twice being worse off than all other students when old. The second chance also reduced the degree of insurance against failing HE in general for the low-ability students when compared with the corresponding utilities in Gary-Bobo and Trannoy (2015). Moreover, the second chance will widen the degree of inequality between those who succeed in their first chance at HE and those earning the lowest wage when old.

The government will not provide a higher degree of insurance for failing second chances for the low-ability students in the time-consistent optimum. The time-consistent optimum also cannot be implemented by an ordinary loan.

This paper confirms the usefulness of ICL schemes found in the related literature on funding HE with student loans. The optimal student loan needs to be designed with the second chance in mind when the government promised to give a second chance to all failed students. The repayment for the second chance also needs to be income contingent. Furthermore, although the second chance policy seems to be redistributive, it has increased the degree of ex-ante inequality in the economy and worsened the individual ex-post utility when old for those who failed twice in HE. Therefore, if the government is considering expanding education to older workers, it should not view this second chance as a tool for redistribution.

Chapter 4: Optimal Provision of Higher Education with Labour Retraining

4.1 Introduction

Completing Higher Education (HE) brings significant labour market benefits to graduates including higher average wage rate and higher probability of employment and these benefits have been widely documented in the economics literature (Baum, Ma and Payea, 2010; Baum, Ma and Payea, 2013; Ma, Pender and Welch, 2016). However, it does not mean that all those workers will earn a high level of income throughout their lifetime and some of them will face downward mobility in the labour market. Many papers have found various factors that caused the workers in developed countries who previously earned a decent level of income to now face a lower level of income and/or a higher probability of unemployment. The factors that caused this labour displacement include technological changes such as improvement in ICT and automation (Acemoglu and Restrepo, 2020; Michaels, Natraj and Van Reenen, 2014; Nedelkoska and Quintini, 2018) and international competition from developing countries (Autor, Dorn and Hansen, 2013; Rothwell, 2017). Education is inherently risky, for example, what seems to be a good subject choice at the time could turn out to be a poor choice as the labour market is undergoing constant changes. The labour retraining program aims to provide long-term help to those workers, unlike unemployment insurance which does not change the skill set of the unemployed workers nor increase their human capital which means the displaced workers will be reemployed in low-paying jobs (Jacobson, LaLonde and Sullivan, 2005). Various empirical papers have found this policy to be effective in bringing those displaced workers back to employment (Bailey, Chapain and de Ruyter, 2012; Dyke et al., 2006;

Hotz, Imebns and Klermans, 2006; Kruppe and Lang, 2018). Consequently, there are papers recommending that the government should increase spending on labour retraining programs (Gvaramadze, 2010; Manyika et al., 2017). UK government also planned to provide three fully funded selected Level 3 qualifications for adults who are earning below the National Living Wage (Sibieta, Tahir and Waltmann, 2021). In this paper, in addition to providing HE, the government also provide a labour retraining program to all workers in danger of downward labour mobility.

The first income-contingent loans (ICL) for university students were implemented by the Australian government in 1989 through the Higher Education Contribution Scheme (HECS). Since then, HE is increasingly being funded by student loans and many papers have analysed the optimal design of these student loans. This paper is also within the theoretical literature investigating the role of student loans in funding HE. An ICL is a loan in which the repayment depends on the income of the borrower. Some papers recommended the use of ICL to solve student loan market failures before ICL became widely used in funding HE, these include Friedman and Kuznets (1945), Friedman (1955) and Shell et al. (1968). Chapman (2014) summarised the conceptual and empirical basis for ICL. The student loan market suffers market failure because the educational investments are risky since the students have a probability of failing HE and even after succeeding in HE the students face labour market risks. These risks are combined with the fact that if the future income of the student is lower than expected the student cannot sell a part of the HE investment and there is no collateral for the lenders. This meant the lenders would not provide student loans to students that are considered risky and students without any loan repayment guarantors. The ICL is recommended since it provides consumption smoothing to the students and insurance against default in periods of low income. With ICL, the graduates' human capital is used to finance HE.

There are also more recent papers that examined the potential benefits of using ICL to finance HE and compared different funding schemes for HE according to some criteria set in their model. Garcia-Penalosa and Wälde (2000) found using education subsidies funded by general taxation to finance the HE cannot simultaneously achieve the efficiency and equity targets. Del Rey and Racionero (2010) analysed the insurance role of several HE funding schemes with a model built on Garcia-Penalosa and Wälde (2000). In their model, the optimal education policy maximizes the output. They found the optimal policy to be an ICL scheme that covers both the financial cost of education and foregone earnings to fully insure the lowest ability individuals that should enrol in education. Hanushek, Leung and Yilmaz (2014) used a three-period live OLG model with intergenerational bequests and exogenous borrowing constraints to examine the welfare properties between different education policies. They found that the ICL is effective in providing insurance against poor labour market outcomes and reducing inequality, it is also possible for the ICL to subsidize low-ability students from rich parents at the cost of high-ability students from poor parents since high-ability students are subject to a higher level of repayment under ICL. Del Rey (2012) focused on adverse selection to explore the scope for the government to produce an efficient scheme which guarantees student participation and does not rely on forcing students to participate in any programme. It found that when a student's ability is private information and there are a low number of high-ability students in the economy, without government coercion, the low-ability student would choose ICL and the high-ability student would opt out and prefer to repay his loan as a percentage of his income. When the number of low-ability students is too low, no equilibrium would exist and all students would be better off if the government forced them to choose the ICL funding scheme.

The model of this paper is also related to the literature that uses optimal taxation models with endogenous human capital. These papers aim to design the tax system and the repayment schedule to maximize a social welfare function subject to a set of constraints. Radomska (2019) gave an overview of the important literature in this field of study and summarised their results. The optimal income taxation literature started with the static model of Mirrlees (1971) that assumes exogenous individual abilities and has subsequently been extended into a multiperiod model still with exogenous individual abilities (Farhi and Werning, 2013; Golosov, Tsyvinski and Werning, 2006; Kapicka, 2013). Some papers have included endogenous human capital and considered an education system financed by education subsidy as an instrument to alleviate the distortion in human capital investment by taxation (Benabou, 2002; Bovenberg and Jacob, 2005). The other approach to financing the HE system is by analysing the optimal repayment schedule. Anderberg (2009) and Grochulski and Piskorski (2010) both analysed the optimal tax policy with ex-ante identical agents but unobservable human capital investments and productivity shocks. Some papers included agents with private heterogeneous abilities. Findeisen and Sachs (2016) considered the optimal repayment schedule for an individual's one-time binary education investment and found an ICL could implement the second-best optimum, the optimal repayment schedule could be approximated by a schedule that linearly increases in income up to a threshold and constant afterwards. Stantcheva (2017) included risky human capital investment in every period and agreed on the usefulness of ICL to implement the second-best optimum. Koeniger and Prat (2018) examined the optimal taxation with endogenous human capital investment combined with the transmission of financial and human capital from parents to children in a dynastic economy. Again, the social optimum is implemented by ICL.

Among the different optimal taxation papers with endogenous human capital, this paper's model is a direct extension of the model used by Gary-Bobo and Trannoy (2015). Gary-Bobo and Trannoy (2015) differed from other optimal taxation papers with endogenous human capital by considering the combination of private abilities with genuine moral hazard in education. In their model, individuals are born with an unobservable high- or low-ability type (ex-ante type) and need to choose their quality of HE. After succeeding in HE, the individual will gain an unobservable labour market type (ex-post type) that represents job market skills and opportunities. Consequently, the model included an adverse selection problem in terms of students choosing the different qualities of education and also a moral hazard problem in both HE and the labour market. In other papers the moral hazard problem of education decisions is absent, the students are assumed to undertake their education with a high level of study effort. The government aims to assign each student the quality of HE that matches their ex-ante type and incentivises high study effort for all students but only high labour market effort for those of high ex-post type. Gary-Bobo and Trannoy (2015) also found the second-best optimum needs to be implemented by an ICL. Gary-Bobo and Trannoy (2015) also found ex-ante inequality in the second-best optimum meaning the high-ability students have a higher expected utility net of study effort cost than the low-ability students before the education outcome is known. The second-best optimum also resulted in ex-post inequality: after the workers gain their ex-post types, the workers with a higher ex-post type will have a higher level of utility because of the moral hazard problem in the labour market. The second-best optimum also exhibits equal treatment: the students' expected utility after finishing education but before knowing their ex-post types are equalized between different ex-ante types conditional on academic success.

There has been insufficient attention paid to the optimal design of student loans that combine downward labour mobility and a labour retraining program. In this model, the government is not only financing the HE system but is also financing a labour retraining program that some students need after graduation. The modelling of downward mobility and a retraining program is based on a political economy paper-Arawatari and Ono (2015). After succeeding in HE, the workers with the low ex-post type are at risk of downward mobility. Those who suffer from downward labour mobility will earn the same wage as those who have failed HE in the next period. The government knows this risk of downward mobility to workers and offers a labour retraining program to all workers who need retraining to maintain their wage level in the next period. This paper is exploring two questions: 1. How do we optimally provide HE with a labour retraining program? 2. What is the impact of this optimally provided labour retraining program on inequality and insurance? The effect on inequality and insurance is examined by comparing the outcome to the results found in Gary-Bobo and Trannoy (2015). We will also present the different results of the two questions that resulted from whether the government could observe who needs retraining or not.

The subsequent sections of this paper are structured as follows. Section 4.2 sets up the model and studies the first-best optima. Section 4.3 analyses the incentive constraints. Section 4.4 presents the relations between the ex-post utilities in the second-best optimum and answers the two questions. Section 4.5 extends the model by presenting the policy implications in the time-consistent optimum. Section 4.6 concludes.

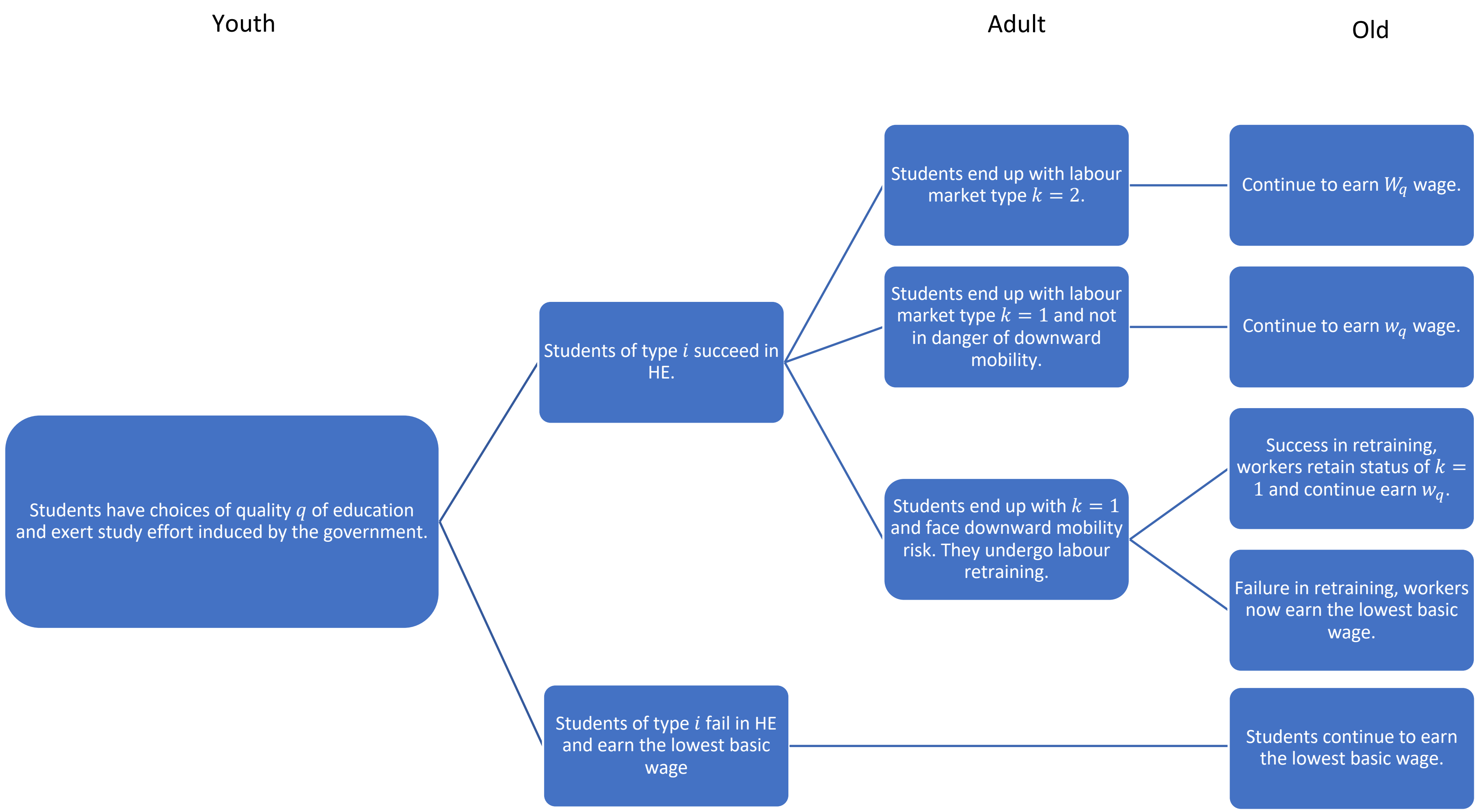


Figure 4.1: Possible Education and Labour Market Outcomes Facing Type i Students

4.2 Model Setup

The main extension of the model compared to the Gary-Bobo and Trannoy's (GT's) model is that downward labour mobility risk is included. In this model, downward labour mobility is formulated as an exogenous possibility that will happen to adults who succeeded in HE but only have ex-post type $k = 1$ (the lowest ex-post type). For these at-risk workers, the way to prevent earnings from falling to the lowest wage when old is to succeed in retraining. If the worker is successful then he will retain his status as an ex-post type $k = 1$ worker. If he fails then he will only earn the basic wage $\underline{w}$ in his old age.

Figure 4.1 above shows the different possible education and labour market outcomes facing students of ex-ante type i from when they are young till when they are old. To better understand the model and the analysis, below we will define several important terms that will be widely used in this chapter:

Ex-ante: Before taking HE.

Interim expected utility: The expected utility of a student after finishing his HE but before entering the labour market.

Ex-post utility: The actual utility a worker obtains when working in the labour market.

Equal treatment second-best optimum: The second-best optimum in which the students' interim expected utility is equalized between different ex-ante types conditional on academic success.

4.2.1 Assumptions

There are three periods in this model: a youth period, an adult period and an old period. All students have the same intertemporal von Neumann-Morgenstern utility:

$$\delta u_a(.) + \delta^2 u_o(.)$$

where δ is the discount factor. u_a and u_o are the same functions just for utility in the adult and old periods respectively. The student's intertemporal utility function is additively separable over the two time periods. There is no storage technology in the economy. Each student uses all of his net income for consumption within a period. We omit the youth period utility function to simplify the model's presentation. We could add a youth period utility function with an exogenous government transfer in the youth period so the students have positive utility in the youth period but its inclusion will not affect the analysis in this model.

The students have two-dimensional unobservable characteristics: ex-ante type i and ex-post type k. The ex-ante type could be interpreted as representing the individual's cognitive skills while the ex-post type could be interpreted as the individual's labour market skills and opportunities. The timing of information revelation is the following: Both the ex-ante type i and ex-post type k are private information. Students know their ex-ante type i before they decide on the choice of HE quality while the ex-post type k is only revealed to students after they succeed in HE and enter the labour market.

An individual is characterised by his pair of types (i, k). Each type can only take one of the two values, $i, k = 1, 2$. The students' ex-ante types are independently drawn from the same distribution. The proportion of ex-ante type i in the population is denoted by l_i, and since there are only two possible types $l_1 + l_2 = 1$.

The three building blocks of the model are: Education and Retraining, the Job Market and the Loan Contract. Each of them will be examined in the following sections.

Education and Retraining

The youth period education will be described first, followed by the retraining program. The students learn their ex-ante type i at the beginning and their ex-ante type remains the same across periods. The students then choose the quantity/quality of education q which is either low or high (1 or 2).

The cost of HE in terms of resources is denoted by y_q for education choice q and assumes that $y_2 \geq y_1$.

Let q_i be the education choice of students with ex-ante type i, and assume the efficient choice of education quality for ex-ante type i students is $q_i = i$, the same quality as their ex-ante type i.

The ex-ante effort for HE is denoted by e_i^y and it can be either 1 or 0 (high or low), with the study effort cost for the ex-ante type i agent being $c_i^y e_i^y$. The government cannot observe the study effort. The parameter c_i^y is >0. Assume that $c_1^y \geq c_2^y$.

For HE, the probability of success for students with ex-ante type i is denoted by $P_i = p_i(1)$ for high ex-ante study effort and $p_i = p_i(0)$ for low ex-ante study effort. The workers with the higher ex-ante type are more likely to succeed in HE, given the same effort.

The retraining program in the adult period is different from the HE in the youth period in that there is no difference in the retraining program offered between different ex-ante types as the government provides the same quality of retraining program to workers and the worker retains his ex-post type after succeeding in retraining. This is made to show that retraining as a policy is different to HE with a greater focus on labour market skills.

The cost of retraining in terms of resources is denoted by η_r. This parameter η_r can be higher or lower than the cost of HE, y_q. The cost of retraining η_r does not depend on the workers' ex-ante type.

The students either succeed or fail their HE. If they succeed and end up with high ex-post type $k = 2$ they will face no danger of downward mobility in the labour market. However, if they end up with $k = 1$ we include a probability they will be downgraded in the next period if the student is not retrained. The government will offer a labour retraining program in response to the risk of downward mobility. If the student succeeds in retraining he will maintain his ex-post type and continue earning the same wage.

The choice of effort for retraining in the adult period is denoted by e_i^r which also takes either 1 or 0 and the retraining effort cost for ex-ante type i agent is $c_i^r e_i^r$. The government cannot observe retraining efforts either. The parameter c_i^r is >0. c_i^r can be larger or smaller than c_i^y. Assume that $c_1^r \geq c_2^r$.

For the retraining, the probability of success of a student exerting ex-ante effort e_i^r, is denoted by $p_i^r(e_i^r)$. $P_i^r = p_i^r(1)$ is denoted as the probability of success with high retraining effort and $p_i^r = p_i^r(0)$ is for low retraining effort. The workers with the higher ex-ante type are also more likely to succeed in retraining, given the same effort. Formally, we make the following assumption on the probabilities of success for HE and retraining,

Assumption 4.1: $0 < p_i < P_i < 1$, $i = 1, 2$ and $P_2 > P_1$, $p_2 > p_1$. $0 < p_i^r < P_i^r < 1$, $i = 1, 2$ and $P_2^r > P_1^r$, $p_2^r > p_1^r$.

In terms of what the government can and cannot observe: the student's ex-ante type and their effort in HE and retraining are not observable by the government but the individual wage, the choice of education q and the success or failure in HE and retraining are observed by the government.

Job Market

In this section, we will first describe how the wage is determined in this model and how we model downward mobility. Then, we describe the possible wage levels facing the students and our assumption of their ranking. Next, we will list all the disutility of effort at work and the total effort costs in this model and state our assumption on the size of the disutility of effort at work. Finally, we state our assumption on the availability of information on who needs retraining and examine its implications.

The students work for two periods. In both periods: Failing HE means earning the lowest/basic wage $\underline{w}$ and success at school means the wage would be dependent on the quality of education q and the ex-post type k.

The student's ex-post type k is not determined with certainty after success in HE. The probability of ex-post type k, given success in HE and education quality q, is defined as,

$$\Pr(k = q|q, success) = 1 - \pi,$$

and $$\Pr(k \neq q|q, success) = \pi,$$ assume that $\pi < \frac{1}{2}$.

Following the model of Gary-Bobo and Trannoy (2015) we assume that π is exogenous. π is the probability for an ex-ante type i student to end up with a different ex-post type k in the labour market than his choice of HE quality q. The assumption that $\pi < \frac{1}{2}$ is interpreted as there is a small probability that an individual with quality q HE will be endowed with ex-post characteristics $k \neq q$. For example, for students with ex-ante type $i = 2$ who chooses quality $q = 2$ HE, most of them can meet expectations and become a type $(i, k) = (2,2)$, with a high ex-ante and ex-post type, and occupy a top job. However, a small proportion of these students, type $(i, k) = (2,1)$, will lack the necessary skills/opportunities and only find a middle job. Similarly, the assumption meant that most of the ex-ante type $i = 1$ student who succeeded in quality $q = 1$ HE will obtain a middle job but a small part of said students possess the necessary labour skills/opportunities to obtain the top job. This interpretation fits findings in real life, in the UK, on average, students who obtained a first earn more than students who obtained a 2.1 who earn more than those with 2.2 while male Russell Group graduates earn over 40% more than those who attended the average post 1992 institution (35% for women)

(Belfield, Britton and Erve, 2018; Social Mobility Commission, 2023). Both earning a higher university degree and being in Russel group could be interpreted as having a higher ex-ante type i in this model.

The probability of the need for retraining for a worker is added by the potential of downward labour market mobility for one who succeeded in HE but only has an ex-post type $k = 1$. Formally, for someone who only has ex-post type $k = 1$, there is a probability P_i^d that he will be risking a downward labour shift when old. If this person fails in retraining he will lose his ex-post type and only earn the lowest basic wage when old, the same wage as those who failed in HE. We assume the probability of facing downward mobility is purely dependent on exogenous factors that are not influenced by the worker's ex-ante type. This meant the probability P_i^d between ex-ante type $i = 1$ and $i = 2$ workers are equal, $P_1^d = P_2^d$. Formally,

Assumption 4.2: $P_1^d = P_2^d = P^d$.

The labour market effort of a type (i, k) individual is denoted by ε_{ik}. We assume that a high labour market effort $\varepsilon = 1$ is required for a top job and only a low effort $\varepsilon = 0$ is necessary for a middle job. The wage is determined by the quality of HE q and the labour effort ε_{ik}. The wage level is denoted by:

$$\omega(q, \varepsilon),$$

$\omega(q, 1) = W_q$ denotes the top job wage and $\omega(q, 0) = w_q$ denotes the middle job wage. We make the following assumption on the ranking between the wage levels.

Assumption 4.3.

(a). $\omega(q, \varepsilon) \geq \underline{w}$ for all q and ε.

(b). $\omega(2,0) > \omega(1,0)$,   meaning that $w_2 > w_1$.

(c). $\omega(2,1) - \omega(2,0) \geq \omega(1,1) - \omega(1,0) > 0$, meaning that $W_2 - w_2 \geq W_1 - w_1 > 0$ and implies $W_q > w_q$.

(d). $\omega(1,1) \geq \omega(2,0)$,   meaning that $W_1 \geq w_2$.

$\omega(q, \varepsilon)$ is the function measuring the wage level for a worker who succeeded in quality q HE and exert ε labour market effort. $\omega(1,1)$ and $\omega(1,0)$ are the functions for the wage levels, W_1 and w_1, for a worker who succeeded in quality $q = 1$ HE exerting high and low labour market effort respectively. $\omega(2,1)$ and $\omega(2,0)$ are the functions for the wage levels, W_2 and w_2, for a worker who succeeded in quality $q = 1$ HE exerting high and low labour market effort respectively. From Assumptions 4.3 (b) and (c) we can derive the following relation:

$$W_2 - W_1 \geq w_2 - w_1 > 0. \tag{4.1}$$

In the two periods of working, there are five possible wage levels: $\underline{w}$, w_1, w_2, W_1 and W_2. The two wage levels that the worker could face in the two periods are denoted as (the adult period wage level, the old period wage level). The possible combinations are:

1. $(\underline{w}, \underline{w})$
2. (W_q, W_q)
3. (w_q, w_q)
4. $(w_q, \underline{w})$

The disutility of effort at work would be determined by the ex-post type k. $\beta_{qk}\varepsilon_{ik}$ denotes the disutility of effort at work in state (i,k), the β_{qk} is assumed to not change across periods.

Assume that

$$\beta_{22} = \beta_{12} = b > 0,$$

$$\beta_{11} = \beta_{21} = B > 0.$$

The possible total cost of study and labour effort for both periods are denoted by:

Students succeed in HE and don't need retraining: $c_i^y e_i^y + \delta\beta_{qk}\varepsilon_{ik} + \delta^2\beta_{qk}\varepsilon_{ik}$.

Students who need retraining and succeeded: $c_i^y e_i^y + \delta c_i^r e_i^r + \delta\beta_{q1}\varepsilon_{i1} + \delta^2\beta_{q1}\varepsilon_{i1}$.

Students who need retraining and failed: $c_i^y e_i^y + \delta c_i^r e_i^r + \delta\beta_{q1}\varepsilon_{i1}$.

Students failed their HE: $c_i^y e_i^y$.

Let $B > b$. An ex-post type $k = 2$ incurs a disutility of b when doing the top job while an ex-post type $k = 1$ incurs a disutility B. Same as in GT's model, assume the level of B is so large that no student of type $k = 1$ will ever exert high effort to obtain the top job. Only the $k = 2$ type will consider obtaining the top job and the type $k = 1$ will only have the middle job. The labour market moral hazard is for a high ex-post type $k = 2$ to exert no labour effort and occupy a middle job.

There are two kinds of workers with ex-post type $k = 1$, one needs retraining and the other doesn't. We assume during the adult period, no workers are at risk of downward mobility and they do not know whether they will be at risk of downward mobility when old. The factors causing downward mobility come at the end of the adult period which influences the affected workers' labour market outcomes when old and it is only afterwards the individual worker knows whether they will face downward mobility when old. We will examine the case with the assumption that the government can observe which worker is at risk of downward mobility and the case in which the government cannot observe this. The utility relations in the second-best problem are solved under the assumption that the government knows who needs retraining, and then we analyse what is changed when the government cannot observe who needs retraining. When the government have information on which worker is subject to downward mobility, an individual with ex-post type $k = 1$ and is not at risk of downward mobility cannot pretend that his career is at risk and undergo the retraining process. This assumption may not be unrealistic. If a worker knows that his job will be made redundant in the future, the government could potentially also observe the factors that caused this and reach the same conclusion on said worker's future job security. Note that this assumption is different to knowing someone's innate ability, this is about whether the government could observe whether someone's job is at risk. It is still the case that the government cannot observe an individual's ex-ante type i. The government doesn't want to provide labour retraining to those workers who are not in danger of downward mobility since it would just be a waste of resources spent on extra labour retraining programs. We assume that after HE and in the labour market, the worker knows if he is facing a downward mobility risk. The alternative assumption that workers don't know themselves does not add anything interesting to the question. If they don't know but the government knows who needs retraining, then the results are the same as assuming both workers and the government know. If neither the workers nor the government knows, then the model would only change into one in which the government assigns all workers with a middle job into the retraining program which results in a waste of resources.

We assume the government can track an individual's learning record so someone who succeeds in HE but fails in retraining cannot pretend that he is someone who failed in HE.

Loans

For the government, the loan repayments will cover the full cost of HE and labour retraining. Given the assumptions on B and b as well as the assumption on the government's desired labour effort stated in section 4.2.2, all the possible combinations of repayments for students choosing quality q HE in the two periods are listed below:

For students who failed in HE: $r_q^a, r_q^o(\underline{w})$

For successful students with ex-post type $k = 2$: $R_{q2}^a, R_{q2}^o(W_q)$

For successful students with ex-post type $k = 1$ and don't need retraining: $R_{q1}^a, R_{q1}^o(w_q)$

For successful students with ex-post type $k = 1$ that succeeded in retraining: $R_{q1}^a, R_{q1}^{or}(w_q)$

For successful students with ex-post type $k = 1$ that failed in retraining: $R_{q1}^a, r_q^o(w_q)$

The government is assumed to have full commitment when they offer the contract to the youth and when they designed the repayment contract they do not know the individuals' ex-ante type i as students have not self-select themselves yet. The government cannot change its promised policies when students become old. The repayment level when old is dependent on the need for retraining and if retraining is undertaken then also whether the worker succeeded in retraining or not.

The allocation in this model can be represented by the array $\{\left(e_i^y, e_i^r, q_i, \varepsilon_{ik}, r_i^a, r_i^o(\overline{w}), R_{ik}^a, R_{ik}^o(\overline{w})\right)\}_{i,k=1,2}$. $\overline{w}$ represents the wage level in the adult period and R^{or} is included in the R_{ik}^o. The menu of contracts is an array $\{\left(q, r_q^a, r_q^o(\underline{w}), r_q^o(w_q), R_{q1}^a, R_{q2}^a, R_{q1}^o(w_q), R_{q1}^{or}(w_q), R_{q2}^o(W_q)\right)\}_{q=1,2}$.

Resource Constraints

We will now write the amount of revenue collected by the government from different students when the above menu is chosen by students who chose education $q = i$ (the quality of education is the same as the ex-ante type). Assume the government discounts future revenue at the same rate as students discount their utility, at δ. Through assumptions on B and b as well as the assumption on the government's desired labour effort stated in section 4.2.2, the labour effort has the following numerical meaning $\varepsilon_{21} = \varepsilon_{11} = 0$, $\varepsilon_{22} = \varepsilon_{12} = 1$.

(I) For students with type $i = k$ that succeeded in education in youth and don't need retraining. The probability for this to happen to an ex-ante type $i = 1$ student is $p_i(e_i^y)(1-\pi)(1-P^d)$ and the probability for this to happen to an ex-ante type $i = 2$ student is $p_i(e_i^y)(1-\pi)$. The government collects the following amount from these type i students:

$$X_i = \varepsilon_{ii}\delta R_{i2}^a + (1-\varepsilon_{ii})\delta R_{i1}^a + \delta^2[\varepsilon_{ii}R_{i2}^o(W_i) + (1-\varepsilon_{ii})R_{i1}^o(w_i)] = \delta X_i^a + \delta^2 X_i^o$$

(II) For students with type $i \neq k$ that succeeded in HE and don't need retraining. The probability for this to happen to an ex-ante type $i = 1$ student is $p_i(e_i^y)\pi$ and the probability for this to happen to an ex-ante type $i = 2$ student is $p_i(e_i^y)\pi(1-P^d)$. From these ex-ante type i students, the government collects the amount:

$$Y_i = \varepsilon_{ik}\delta R_{i2}^a + (1-\varepsilon_{ik})\delta R_{i1}^a + \delta^2[\varepsilon_{ik}R_{i2}^o(W_i) + (1-\varepsilon_{ik})R_{i1}^o(w_i)] = \delta Y_i^a + \delta^2 Y_i^o$$

(III) For students with type $i = k$ that succeeded in HE and succeeded in retraining. This is only possible for ex-ante type $i = 1$ students, the probability of this happening is $p_1(e_1^y)(1-\pi)P^d p_1^r(e_1^r)$. This is not possible for an ex-ante type $i = 2$ student. The government collects the following amount from these ex-ante type $i = 1$ students:

$$Q_1 = \delta R_{11}^{a} + \delta^2 R_{11}^{or}(w_1)$$

(IV) For students with type $i \neq k$ that succeeded in HE and succeeded in retraining. This is only possible for ex-ante type $i = 2$ students and the probability of this happening is $p_2(e_2^y)\pi P^d p_2^r(e_2^r)$. The government collects the following amount from these ex-ante type $i = 2$ students:

$$Q_2 = \delta R_{21}^{a} + \delta^2 R_{21}^{or}(w_2)$$

(V) For students with type $i = k$ that succeeded in HE but failed retraining. This is only possible for ex-ante type $i = 1$ students, the probability of this happening is $p_1(e_1^y)(1-\pi)P^d(1 - p_1^r(e_1^r))$. The government collects the following amount from these ex-ante type $i = 1$ students:

$$N_1 = \delta R_{11}^{a} + \delta^2 r_1^{o}(w_1)$$

(VI) For students with type $i \neq k$ that succeeded in HE but failed retraining. This is only possible to happen to ex-ante type $i = 2$ students and the probability of this happening is $p_2(e_2^y)\pi P^d(1 - p_2^r(e_2^r))$. The government collects the following amount from these ex-ante type $i = 2$ students

$$N_2 = \delta R_{21}^{a} + \delta^2 r_2^{o}(w_2)$$

(VII) For students that failed in HE and don't know their ex-post type k. This has a probability of $1 - p_i(e_i^y)$ to happen to ex-ante type i students. From these ex-ante type i students, the government collects the amount:

$$A_i = \delta r_i^{a} + \delta^2 r_i^{o}(\underline{w}) = \delta A_i^{a} + \delta^2 A_i^{o}$$

Now we formulate the expected cost of education and retraining. Due to the difference in the probability of facing downward mobility, the expected cost is different between ex-ante types in terms of formulation. We obtain the following:

The expected cost for ex-ante type $i = 1$ student is: $y_1 + p_1(e_1^y)(1-\pi)P^d\delta\eta_r$.

The expected cost for ex-ante type $i = 2$ student is: $y_2 + p_2(e_2^y)\pi P^d\delta\eta_r$.

The resource constraint RC is the summation between both ex-ante types i students of the proportion of population times the equation of the expected amount of revenue collection minus the expected cost of HE and retraining.

We also make a formal assumption about students being risk averse.

Assumption 4.4: $u_a(.)$ and $u_o(.)$ are strictly increasing, strictly concave and continuously differentiable.

4.2.2 First-Best Optimality

In this model, four possible routes of ex-post utilities could be realized by a student of ex-ante type i when he succeeds in HE. The ex-post utility is the actual utility that an individual would receive after he first finished his HE and started working in the labour market. First, for the students who succeeded in HE and don't need retraining, denote the following:

V_i: The ex-post utility of a student that succeeded in HE and does not need retraining, when the ex-post type is $k = i$.

v_i: The ex-post utility of a student that succeeded in HE and does not need retraining, when the ex-post type is $k \neq i$.

where $V_i = \delta u_a[\omega(q_i, \varepsilon_{ii}) - X_i^a] + \delta^2 u_o[\omega(q_i, \varepsilon_{ii}) - X_i^o]$,

$v_i = \delta u_a[\omega(q_i, \varepsilon_{ik}) - Y_i^a] + \delta^2 u_o[\omega(q_i, \varepsilon_{ik}) - Y_i^o]$, with $i \neq k$.

For students that need retraining but succeeded in them:

B_1: The ex-post utility of a student of ex-ante type $i = 1$ with ex-post type $k = 1$ that succeeded in HE and succeeded in the needed retraining.

b_2: The ex-post utility of a student of ex-ante type $i = 2$ with ex-post type $k = 1$ that succeeded in HE and succeeded in the needed retraining.

where $B_1 = \delta u_a[w_1 - R_{11}^a] + \delta^2 u_o[w_1 - R_{11}^{or}(w_1)]$,

$b_2 = \delta u_a[w_2 - R_{21}^a] + \delta^2 u_o[w_2 - R_{21}^{or}(w_2)]$.

For students that succeeded in HE but failed the retraining:

BF_1: The ex-post utility of a student of ex-ante type $i = 1$ with ex-post type $k = 1$ that succeeded in HE but failed the retraining.

bf_2: The ex-post utility of a student of ex-ante type $i = 2$ with ex-post type $k = 1$ that succeeded in HE but failed the retraining.

where $BF_1 = \delta u_a[w_1 - R_{11}^a] + \delta^2 u_o[\underline{w} - r_1^o(w_1)]$,

$bf_2 = \delta u_a[w_2 - R_{21}^a] + \delta^2 u_o[\underline{w} - r_2^o(w_2)]$,

The interim expected utility is the expected utility of a student poststudy but before they know their ex-post type if they are successful in HE. The interim expected utility of a student who succeeded in HE who chose education $q_i = i$ is by definition:

(4.2)$U_1^s = (1-\pi)\left[(1-P^d)V_1 + P^d p_1^r(e_1^r)B_1 + P^d\left(1 - p_1^r(e_1^r)\right)BF_1\right] + \pi[v_1 - \delta b - \delta^2 b]$,

$U_2^s = (1-\pi)[V_2 - \delta b - \delta^2 b] + \pi\left[(1-P^d)v_2 + P^d p_2^r(e_2^r)b_2 + P^d\left(1 - p_2^r(e_2^r)\right)bf_2\right\}$.

Lastly, for the utility of a student of ex-ante type i that failed in HE. Denote the following,

u_i: the utility of a type i student that failed in HE, where $u_i = \delta u_a[\underline{w} - A_i^a] + \delta^2 u_o[\underline{w} - A_i^o] = \delta u_i^a + \delta^2 u_i^o$.

The ex-ante expected utility of a type i student net of expected study effort cost and expected retraining effort cost is defined as follows:

(4.3) $p_1(e_1^y)U_1^s + \left(1 - p_1(e_1^y)\right)u_1 - c_1^y e_1^y - p_1(e_1^y)(1-\pi)P^d \delta c_1^r e_1^r$ for $i = 1$,

$$p_2(e_2^y)U_2^s + \left(1 - p_2(e_2^y)\right)u_2 - c_2^y e_2^y - p_2(e_2^y)\pi P^d \delta c_2^r e_2^r \text{ for } i = 2.$$

The first-best optimum is obtained by maximizing the sum of the ex-ante expected utility with the social welfare weight placed on each ex-ante type i student subject to the resource constraint. Given the many possible effort level combinations in this model each with its level of welfare, the only case of the government's intended effort levels that will be analysed here is the case in which succeeding in education and retraining is valuable to the social welfare and those with ex-post type $k = 2$ should not waste their labour market opportunity. In terms of effort vector, assume the government requires a high study and retraining effort for both types of students, $\left(e_1^{y*}, e_2^{y*}, e_1^{r*}, e_2^{r*}\right) = (1, 1, 1, 1)$, and that the government requires the ex-post labour effort to be high if and only if the individuals' ex-post type k is high at 2, it meant that $(e_{i1}^*, e_{i2}^*) = (0, 1)$. Furthermore, the government only wants to provide labour retraining to those at risk of downward mobility. We assume that the provision of HE benefits society in the case we studied but our model can study other cases as well. For example, our model can present the case in which HE is an inefficient policy if the study effort is very costly for a student type. In that scenario, the optimal student effort is zero for said type as the government does not aim to induce any study effort from said students. The objective of the government and the interests of said students are automatically aligned from the start.

Given the assumption on the socially efficient effort level, we can derive the ex-post utility of both ex-ante types i in the optimum. The ex-post utilities are presented in the following equations:

(4.4) $$V_2 = \delta V_2^a + \delta^2 V_2^o = \delta u_a(W_2 - R_{22}^a) + \delta^2 u_o\big(W_2 - R_{22}^o(W_2)\big),$$

$$v_2 = \delta v_2^a + \delta^2 v_2^o = \delta u_a(w_2 - R_{21}^a) + \delta^2 u_o\big(w_2 - R_{21}^o(w_2)\big),$$

$$V_1 = \delta V_1^a + \delta^2 V_1^o = \delta u_a(w_1 - R_{11}^a) + \delta^2 u_o\big(w_1 - R_{11}^o(w_1)\big),$$

$$v_1 = \delta v_1^a + \delta^2 v_1^o = \delta u_a(W_1 - R_{12}^a) + \delta^2 u_o\big(W_1 - R_{12}^o(W_1)\big),$$

$$b_2 = \delta b_2^a + \delta^2 b_2^o = \delta u_a(w_2 - R_{21}^a) + \delta^2 u_o\big(w_2 - R_{21}^{or}(w_2)\big),$$

$$B_1 = \delta B_1^a + \delta^2 B_1^o = \delta u_a(w_1 - R_{11}^a) + \delta^2 u_o\big(w_1 - R_{11}^{or}(w_1)\big),$$

$$bf_2 = \delta bf_2^a + \delta^2 bf_2^o = \delta u_a(w_2 - R_{21}^a) + \delta^2 u_o\left(\underline{w} - r_2^o(w_2)\right),$$

$$BF_1 = \delta BF_1^a + \delta^2 BF_1^o = \delta u_a(w_1 - R_{11}^a) + \delta^2 u_o\left(\underline{w} - r_1^o(w_1)\right),$$

$$u_1 = \delta u_1^a + \delta^2 u_1^o = \delta u_a\big(\underline{w} - r_1^a\big) + \delta^2 u_o\left(\underline{w} - r_1^o\big(\underline{w}\big)\right),$$

$$u_2 = \delta u_2^a + \delta^2 u_2^o = \delta u_a\big(\underline{w} - r_2^a\big) + \delta^2 u_o\left(\underline{w} - r_2^o\big(\underline{w}\big)\right).$$

Note that $V_1^a = B_1^a = BF_1^a$ and $v_2^a = b_2^a = bf_2^a$.

Given the assumption of the optimal choice of labour effort, the expected wage E_{wq}^s of an ex-ante type i student who succeeded at HE and with education $i = q$ is

$$E_{w1}^s = (1-\pi)\big[\delta w_1 + (1-P^d)\delta^2 w_1 + P^d P_1^r \delta^2 w_1 + P^d(1-P_1^r)\delta^2 \underline{w}\big] + \pi(\delta W_1 + \delta^2 W_1),$$

$$E_{w2}^s = (1-\pi)(\delta W_2 + \delta^2 W_2) + \pi\big[\delta w_2 + (1-P^d)\delta^2 w_2 + P^d P_2^r \delta^2 w_2 + P^d(1-P_2^r)\delta^2 \underline{w}\big].$$

The expected wage E_{wq}^{sa} of an ex-ante type i student who succeeded at HE and with education $i \neq q$ is

$$E_{w1}^{sa} = (1-\pi)\left[\delta w_1 + (1-P^d)\delta^2 w_1 + P^d P_2^r \delta^2 w_1 + P^d(1-P_2^r)\delta^2 \underline{w}\right] + \pi(\delta W_1 + \delta^2 W_1),$$

$$E_{w2}^{sa} = (1-\pi)(\delta W_2 + \delta^2 W_2) + \pi\left[\delta w_2 + (1-P^d)\delta^2 w_2 + P^d P_1^r \delta^2 w_2 + P^d(1-P_1^r)\delta^2 \underline{w}\right].$$

Denote the social surplus by S_i, it is the expected social benefit of HE for an ex-ante type i student choosing $q = i$ with the amount of ex-ante study effort, retraining effort and ex-post labour effort that is assumed to be socially efficient above. The S_i for each ex-ante type is formulated below

$$S_1 = P_1 E_{w1}^s + (1-P_1)\left(\delta \underline{w} + \delta^2 \underline{w}\right) - y_1 - P_1(1-\pi)P^d \delta \eta_r,$$

$$S_2 = P_2 E_{w2}^s + (1-P_2)\left(\delta \underline{w} + \delta^2 \underline{w}\right) - y_2 - P_2 \pi P^d \delta \eta_r.$$

Similar to GT's model we make Assumption 4.5:

Assumption 4.5.

(a) $S_i \geq \delta \underline{w} + \delta^2 \underline{w}$ for all i;

(b) $P_2[E_{w2}^s - E_{w1}^{sa}] \geq y_2 - y_1 + [2\pi - 1]P_2 P^d \delta \eta_r$;

(c) $P_1[E_{w2}^{sa} - E_{w1}^s] \leq y_2 - y_1 + [2\pi - 1]P_1 P^d \delta \eta_r$.

$P_2[E_{w2}^s - E_{w1}^{sa}]$ is the expected wage gain for an ex-ante type $i = 2$ student to choose quality $q = 2$ HE instead of quality $q = 1$ HE and $y_2 - y_1 + [2\pi - 1]P_2 P^d \delta \eta_r$ is the expected amount of additional resources needed for this choice of the ex-ante type $i = 2$ student. $P_1[E_{w2}^{sa} - E_{w1}^s]$ is the expected wage gain for an ex-ante type $i = 1$ student to choose quality $q = 2$ HE instead of quality $q = 1$ HE and $y_2 - y_1 + [2\pi - 1]P_1 P^d \delta \eta_r$ is the expected amount of additional resources needed for this choice of the ex-ante type $i = 1$ student. Assumptions 4.5 (b) and (c) meant that for an ex-ante type i student, choosing education quality $q = i$ generates more social surplus than choosing the alternative. Consequently, the government should design a policy that provides quality $q = i$ HE to ex-ante type i students.

There are two inverse utility functions, one for each period,

$$(4.5) \qquad h_a(x) = u_a^{-1}(x), \qquad h_o(x) = u_o^{-1}(x).$$

Apply the inverse functions on the ex-post utility functions, we will write down every pair of functions of repayments below,

$$(4.6) \qquad \delta R_{22}^a + \delta^2 R_{22}^o(W_2) = \delta W_2 + \delta^2 W_2 - \delta h_a(V_2^a) - \delta^2 h_o(V_2^o),$$

$$\delta R_{21}^a + \delta^2 R_{21}^o(w_2) = \delta w_2 + \delta^2 w_2 - \delta h_a(v_2^a) - \delta^2 h_o(v_2^o),$$

$$\delta R_{11}^a + \delta^2 R_{11}^o(w_1) = \delta w_1 + \delta^2 w_1 - \delta h_a(V_1^a) - \delta^2 h_o(V_1^o),$$

$$\delta R_{12}^a + \delta^2 R_{12}^o(W_1) = \delta W_1 + \delta^2 W_1 - \delta h_a(v_1^a) - \delta^2 h_o(v_1^o),$$

$$\delta R_{21}^a + \delta^2 R_{21}^{or}(w_2) = \delta w_2 + \delta^2 w_2 - \delta h_a(b_2^a) - \delta^2 h_o(b_2^o),$$

$$\delta R_{11}^a + \delta^2 R_{11}^{or}(w_1) = \delta w_1 + \delta^2 w_1 - \delta h_a(B_1^a) - \delta^2 h_o(B_1^o),$$

$$\delta R_{21}^a + \delta^2 r_2^o(w_2) = \delta w_2 + \delta^2 \underline{w} - \delta h_a(bf_2^a) - \delta^2 h_o(bf_2^o),$$

$$\delta R_{11}^a + \delta^2 r_1^o(w_1) = \delta w_1 + \delta^2 \underline{w} - \delta h_a(BF_1^a) - \delta^2 h_o(BF_1^o),$$

$$\delta r_1^a + \delta^2 r_1^o(\underline{w}) = \delta \underline{w} + \delta^2 \underline{w} - \delta h_a(u_1^a) - \delta^2 h_o(u_1^o),$$

$$\delta r_2^a + \delta^2 r_2^o(\underline{w}) = \delta \underline{w} + \delta^2 \underline{w} - \delta h_a(u_2^a) - \delta^2 h_o(u_2^o).$$

Define $E(h_i)$ as the expected amount of resources needed to provide the expected utility $P_i U_i^s + (1 - P_i) u_i$. It is formulated differently for different ex-ante types and is respectively

$$(4.7) \qquad E(h_1) = P_1\big[(1-\pi)[\delta h_a(V_1^a) + (1 - P^d)\delta^2 h_o(V_1^o) + P^d P_1^r \delta^2 h_o(B_1^o) + P^d(1 - P_1^r)\delta^2 h_o(BF_1^o)] + \pi\big(\delta h_a(v_1^a) + \delta^2 h_o(v_1^o)\big)\big] + (1 - P_1)\big(\delta h_a(u_1^a) + \delta^2 h_o(u_1^o)\big),$$

(4.8) $$E(h_2) = P_2\Big[(1-\pi)[\delta h_a(V_2^a) + \delta^2 h_o(V_2^o)] + \pi[\delta h_a(v_2^a) + (1-P^d)\delta^2 h_o(v_2^o) + P^d P_2^r \delta^2 h_o(b_2^o) + P^d(1-P_2^r)\delta^2 h_o(bf_2^o)]\Big] + (1-P_2)\big(\delta h_a(u_2^a) + \delta^2 h_o(u_2^o)\big).$$

The resource constraint can be written as

$(\overline{RC})$ $$\sum_i \lambda_i \{S_i - E(h_i)\} \geq 0.$$

If the government has a utilitarian social welfare function, it will attach the same weight to the utility of each ex-ante type as the proportion of this ex-ante type in the population. The first best utilitarian optimum is obtained by solving the following problem,

(4.9) $$Max \sum_i \lambda_i \left[P_i U_i^s + (1-P_i)u_i - c_i^y\right] - \lambda_1 P_1 (1-\pi) P^d \delta c_1^r - \lambda_2 P_2 \pi P^d \delta c_2^r,$$

with respect to $(V_i^a, V_i^o, v_i^a, v_i^o, b_2^o, B_1^o, bf_2^o, BF_1^o, u_i^a, u_i^o)$ i=1,2, subject to $\overline{RC}$. Note that by model's construction $V_1^a = B_1^a = BF_1^a$ and $v_2^a = b_2^a = bf_2^a$.

The following proposition for the first-best optimum is obtained in the Appendix A4.1.

Proposition 4.1:

Under Assumptions 4.1 to 4.5, the first-best Pareto optimality implies full insurance for any ex-ante type i student,

$$V_i^a = V_i^o = v_i^a = v_i^o = u_i^a = u_i^o = B_1^o = BF_1^o \text{ for } i = 1 \text{ and}$$

$$V_i^a = V_i^o = v_i^a = v_i^o = u_i^a = u_i^o = b_2^o = bf_2^o \text{ for } i = 2.$$

In the standard utilitarian case

$$V_1^a = V_2^a, V_1^o = V_2^o, v_1^a = v_2^a, v_1^o = v_2^o, u_1^a = u_2^a, u_1^o = u_2^o, B_1^o = b_2^o, BF_1^o = bf_2^o.$$

The resource constraint $\overline{RC}$ must be binding in the first-best optimum.

The students are insured against education risks and labour market risks. Students are also provided with intertemporal insurance. This is a common first-best result in which the government offered the maximum possible insurance in the economy. This result would not be attainable with asymmetric information as students with ex-ante type $i = 2$ will pretend to be ex-ante type $i = 1$ students since if they tell the truth with quality $q = 2$ education they are more likely to obtain a high ex-post type and resulting in extra effort cost required for the top job. Also, no students would exert effort in either HE, labour retraining or the top job since they would not be compensated for exerting any effort.

4.3 Asymmetric Information and Incentive Constraints

For the second-best case, the same assumptions on the required effort levels are maintained, which are $\left(e_1^{y*}, e_2^{y*}, e_1^{r*}, e_2^{r*}\right) = (1, 1, 1, 1)$ for study and retraining effort and $(e_{i1}^*, e_{i2}^*) = (0, 1)$ for the labour effort. The second-best effort levels that the government wishes to induce are the same as the first-best effort levels.

In the second-best problem, the government can still observe the choice of HE quality and its outcome, the outcome of the retraining and the wage level. We are also currently assuming that the government could observe who is at risk of downward mobility in the labour market. The government does not know the students' ex-ante types and ex-post types. It cannot observe their study effort, retraining effort and labour effort. The problem for the government is to satisfy the adverse selection and moral hazard incentive constraints. The students must self-select the contracts designed for them.

4.3.1 Incentives

The downward mobility does not affect labour effort constraints. In the construction of the model, the possibility of experiencing the downward labour mobility risk is tied to a worker's ex-post type k and not on occupying the "middle-level job". This is reasonable because the ex-post type k represents a worker's labour market skills and opportunities, which is a highly important factor in determining whether a worker could re-enter the labour market and maintain his previous level of earning if he is displaced. Consequently, if an ex-post type $k = 2$ worker pretends to be ex-post type $k = 1$ he does not need to worry about downward labour mobility. Furthermore, even though effort cost B is already assumed to be too large so ex-post type $k = 1$ worker does not want to mimic ex-post type $k = 2$, in this case even if he mimics he will still face the possibility of downward mobility risk. Now to list the labour effort constraints.

For the successful students who chose education $q = 1$ and with ex-post type $k = 2$, they will not choose a middle-level job if and only if,

(ICX_1^s) $\quad v_1^a - b \geq V_1^a \quad$ and $\quad v_1^o - b \geq V_1^o$.

For the successful students who chose education $q = 2$ and with ex-post type $k = 2$, they will not choose a middle-level job if and only if,

(ICX_2^s) $\quad V_2^a - b \geq v_2^a \quad$ and $\quad V_2^o - b \geq v_2^o$.

If the government does not offer the worker with ex-post type $k = 2$ a high utility when old to compensate for the labour effort, these workers will not reveal themselves to be high ex-post type at all since their total utility would be lower than pretending to be ex-post type $k = 1$. The interim expected utilities of ex-ante type $i = 1, 2$ for the students who succeeded in HE with high effort in retraining can be written in their ex-post utilities as

(EU_1^s) $U_1^s = (1-\pi)[\delta V_1^a + (1-P^d)\delta^2 V_1^o + P^d P_1^r \delta^2 B_1^o + P^d(1 - P_1^r)\delta^2 BF_1^o] + \pi[\delta v_1^a + \delta^2 v_1^o - \delta b - \delta^2 b]$,

(EU_2^s) $U_2^s = (1-\pi)[\delta V_2^a + \delta^2 V_2^o - \delta b - \delta^2 b] + \pi[\delta v_2^a + (1-P^d)\delta^2 v_2^o + P^d P_2^r \delta^2 b_2^o + P^d(1-P_2^r)\delta^2 bf_2^o]$.

We will introduce additional notations that will be used in other constraints. EU_1^s and EU_2^s are not the interim expected utility for ex-ante type $i = 1$ and $i = 2$ if they choose the contract designed for the other ex-ante type j. This is because the probability of success in retraining is dependent on the ex-ante type i.

The interim expected utility for an ex-ante type $i = 1$ student who mimics an ex-ante type $i = 2$, has succeeded in HE, and exerts high retraining effort is

(EU_2^{s1}) $U_2^{s1} = (1-\pi)[\delta V_2^a + \delta^2 V_2^o - \delta b - \delta^2 b] + \pi[\delta v_2^a + (1-P^d)\delta^2 v_2^o + P^d P_1^r \delta^2 b_2^o + P^d(1-P_1^r)\delta^2 bf_2^o]$.

If he exerts low retraining effort the interim expected utility would be

(EUL_2^{s1}) $U_2^{sr1} = (1-\pi)[\delta V_2^a + \delta^2 V_2^o - \delta b - \delta^2 b] + \pi[\delta v_2^a + (1-P^d)\delta^2 v_2^o + P^d p_1^r \delta^2 b_2^o + P^d(1-p_1^r)\delta^2 bf_2^o]$.

The interim expected utility for an ex-ante type $i = 2$ student who mimics ex-ante type $i = 1$, has succeeded in HE, and exerts high retraining effort is

(EU_1^{s2}) $U_1^{s2} = (1-\pi)[\delta V_1^a + (1-P^d)\delta^2 V_1^o + P^d P_2^r \delta^2 B_1^o + P^d(1-P_2^r)\delta^2 BF_1^o] + \pi[\delta v_1^a + \delta^2 v_1^o - \delta b - \delta^2 b]$.

If he exerts low retraining effort the interim expected utility will be

(EUL_1^{s2}) $U_1^{sr2} = (1-\pi)[\delta V_1^a + (1-P^d)\delta^2 V_1^o + P^d p_2^r \delta^2 B_1^o + P^d(1-p_2^r)\delta^2 BF_1^o] + \pi[\delta v_1^a + \delta^2 v_1^o - \delta b - \delta^2 b]$.

U_2^{s1}, U_2^{sr1}, U_1^{s2} and U_1^{sr2} will be denoted as U_j^{si} and U_j^{sri}.

The ex-ante self-selection constraint $\overline{IC_i}$ means that students of ex-ante type i do not want to mimic to be ex-ante type j by selecting the contract designed for ex-ante type j while exerting high education effort as well as retraining effort, for all $i = 1, 2$ and $i \neq j$. The self-selection constraint for the ex-ante type $i = 1$ is,

$(\overline{IC_1})$ $P_1 U_1^s + (1-P_1)u_1 - c_1^y - P_1(1-\pi)P^d \delta c_1^r \geq P_1 U_2^{s1} + (1-P_1)u_2 - c_1^y - P_1 \pi P^d \delta c_1^r$,

and for the ex-ante type $i = 2$ is,

$(\overline{IC_2})$ $P_2 U_2^s + (1-P_2)u_2 - c_2^y - P_2 \pi P^d \delta c_2^r \geq P_2 U_1^{s2} + (1-P_2)u_1 - c_2^y - P_2(1-\pi)P^d \delta c_2^r$.

The model includes a moral hazard constraint for retraining to those who face downward mobility and it means ex-ante type i students should prefer to exert high retraining effort to low retraining effort. The retraining effort constraint for ex-ante type $i = 1$ students is,

(MR_1) $P_1^r \delta^2 B_1^o + (1-P_1^r)\delta^2 BF_1^o - \delta c_1^r \geq p_1^r \delta^2 B_1^o + (1-p_1^r)\delta^2 BF_1^o$, and for the ex-ante type $i = 2$ students,

(MR_2) $$P_2^r \delta^2 b_2^o + (1 - P_2^r)\delta^2 bf_2^o - \delta c_2^r \geq p_2^r \delta^2 b_2^o + (1 - p_2^r)\delta^2 bf_2^o.$$

For students who just learnt their ex-ante type i, the study effort constraint meant they should prefer high study effort over low study effort, given a high effort at the retraining if needed. The study effort constraint for the ex-ante type $i = 1$ students is,

$(\overline{MH_1})$ $$P_1 U_1^s + (1 - P_1)u_1 - c_1^y - P_1(1 - \pi)P^d \delta c_1^r \geq p_1 U_1^s + (1 - p_1)u_1 - p_1(1 - \pi)P^d \delta c_1^r,$$

for the ex-ante type $i = 2$ student,

$(\overline{MH_2})$ $$P_2 U_2^s + (1 - P_2)u_2 - c_2^y - P_2 \pi P^d \delta c_2^r \geq p_2 U_2^s + (1 - p_2)u_2 - p_2 \pi P^d \delta c_2^r.$$

If both MH_i and $\overline{MH_i}$ are satisfied, then the combined moral hazard constraint of exerting high study effort and retraining effort over low effort for both is automatically satisfied.

There are additional incentive constraints that are a combination of adverse selection and moral hazard constraints. The ex-ante type $i = 1$ students should prefer to exert high education effort than to mimic ex-ante type $i = 2$ and exert low education effort, given high retraining effort

(IC_1) $$P_1 U_1^s + (1 - P_1)u_1 - c_1^y - P_1(1 - \pi)P^d \delta c_1^r \geq p_1 U_2^{s1} + (1 - p_1)u_2 - p_1 \pi P^d \delta c_1^r,$$

and for the ex-ante type $i = 2$ students

(IC_2) $$P_2 U_2^s + (1 - P_2)u_2 - c_2^y - P_2 \pi P^d \delta c_2^r \geq p_2 U_1^{s2} + (1 - p_2)u_1 - p_2(1 - \pi)P^d \delta c_2^r.$$

Ex-ante type $i = 1$ students should prefer to exert high retraining effort than to mimic ex-ante type $i = 2$ and exert low retraining effort, given high education effort,

(ICA_1) $P_1 U_1^s + (1 - P_1)u_1 - c_1^y - P_1(1 - \pi)P^d \delta c_1^r \geq P_1 U_2^{sr1} + (1 - P_1)u_2 - c_1^y$,

and for the ex-ante type $i = 2$ students

(ICA_2) $P_2 U_2^s + (1 - P_2)u_2 - c_2^y - P_2 \pi P^d \delta c_2^r \geq P_2 U_1^{sr2} + (1 - P_2)u_1 - c_2^y$.

Ex-ante type $i = 1$ students should prefer to exert high effort in both education and retraining than to mimic ex-ante type $i = 2$ and exert low effort in both,

(ICB_1) $P_1 U_1^s + (1 - P_1)u_1 - c_1^y - P_1(1 - \pi)P^d \delta c_1^r \geq p_1 U_2^{sr1} + (1 - p_1)u_2$,

and for the ex-ante type $i = 2$ students,

(ICB_2) $P_2 U_2^s + (1 - P_2)u_2 - c_2^y - P_2 \pi P^d \delta c_2^r \geq p_2 U_1^{sr2} + (1 - p_2)u_1$.

4.3.2 Labour Effort Constraints

The second-best optimum will be characterised by the relations between the ex-post utilities obtained from solving the second-best optimality problem. In the second-best optimality problem, if any of the labour effort constraints are not binding, based on the first-order conditions the respective ex-post utilities in that labour effort constraint are equal to each other which violates that labour effort constraint. Therefore, in any second-best optimum, the ICX_i^s constraints need to be binding.

Lemma 4.1: In the second-best optimum, the labour effort constraints, ICX_i^s, $i = 1, 2$ must be binding,

(4.10) $v_1^a - b = V_1^a = B_1^a = BF_1^a$, and $v_1^o - b = V_1^o$,

$V_2^a - b = v_2^a = b_2^a = bf_2^a$, and $V_2^o - b = v_2^o$.

The interim expected utility U_i^s can now be written as,

$$(4.11)\ \ U_1^s = \delta V_1^a + (1 - P^d + \pi P^d)\delta^2 V_1^o + (1-\pi)[P^d P_1^r \delta^2 B_1^o + P^d(1 - P_1^r)\delta^2 BF_1^o],$$

$$U_2^s = \delta v_2^a + (1 - \pi P^d)\delta^2 v_2^o + \pi[P^d P_2^r \delta^2 b_2^o + P^d(1 - P_2^r)\delta^2 bf_2^o].$$

U_j^{si} and U_j^{sri} are now written as

$$(4.12)\ U_1^{s2} = \delta V_1^a + (1 - P^d + \pi P^d)\delta^2 V_1^o + (1-\pi)[P^d P_2^r \delta^2 B_1^o + P^d(1 - P_2^r)\delta^2 BF_1^o],$$

$$U_2^{s1} = \delta v_2^a + (1 - \pi P^d)\delta^2 v_2^o + \pi[P^d P_1^r \delta^2 b_2^o + P^d(1 - P_1^r)\delta^2 bf_2^o],$$

$$(4.13)\ U_1^{sr2} = \delta V_1^a + (1 - P^d + \pi P^d)\delta^2 V_1^o + (1-\pi)[P^d p_2^r \delta^2 B_1^o + P^d(1 - p_2^r)\delta^2 BF_1^o],$$

$$U_2^{sr1} = \delta v_2^a + (1 - \pi P^d)\delta^2 v_2^o + \pi[P^d p_1^r \delta^2 b_2^o + P^d(1 - p_1^r)\delta^2 bf_2^o].$$

4.3.3 Retraining Effort Constraints

For the ex-ante type $i = 1$ students, the retraining effort constraint MR_1 can be rewritten as,

$$(MR_1) \qquad \delta^2 B_1^o - \delta^2 BF_1^o \geq \delta K_1^r, \quad \text{with } K_1^r = \frac{c_1^r}{P_1^r - p_1^r}.$$

For the ex-ante type $i = 2$ students, the retraining effort constraint MR_2 can be rewritten as

$$(MR_2) \qquad \delta^2 b_2^o - \delta^2 bf_2^o \geq \delta K_2^r, \quad \text{with } K_2^r = \frac{c_2^r}{P_2^r - p_2^r}.$$

We assume the ex-ante type $i = 2$ workers are more efficient than ex-ante type $i = 1$ workers in exerting retraining effort. Formally

Assumption 4.6: $K_1^r \geq K_2^r$.

4.3.4 Study Effort Constraints

For the ex-ante type $i = 1$ students, the study effort constraint $\overline{MH_1}$ can be rewritten as

$(\overline{MH_1})$ $$U_1^s - u_1 - (1-\pi)P^d\delta c_1^r \geq K_1^y, \quad \text{with } K_1^y = \frac{c_1^y}{P_1 - p_1}.$$

The possibility of needing to exert effort in retraining has made study effort constraints harder to satisfy compared to a model without retraining due to the additional effort cost in retraining.

For ex-ante type $i = 2$ student, the study effort constraint $\overline{MH_2}$ can be rewritten as

$(\overline{MH_2})$ $$U_2^s - u_2 - \pi P^d\delta c_2^r \geq K_2^y, \quad \text{with } K_2^y = \frac{c_2^y}{P_2 - p_2}.$$

We assume the ex-ante type $i = 2$ workers are more efficient than ex-ante type $i = 1$ workers in exerting study effort. Formally

Assumption 4.7: $K_1^y \geq K_2^y$.

4.3.5 Ex-ante Self-Selection Constraints

The ex-ante self-selection constraints have U_2^{s1} on the RHS of $\overline{IC_1}$ and U_1^{s2} on the RHS of $\overline{IC_2}$. They are not U_2^s and U_1^s since even if ex-ante type i students choose quality j education, they still have a probability P_i^r of succeeding in retraining.

Rewriting $\overline{IC_i}$ in the same style as GT's model, we obtain the following string of inequalities,

$$(4.14) \qquad \frac{P_2}{(1-P_2)}(U_2^s - U_1^{s2} + (1-2\pi)P^d\delta c_2^r) \geq u_1 - u_2$$

$$\geq \frac{P_1}{(1-P_1)}(U_2^{s1} - U_1^s + (1-2\pi)P^d\delta c_1^r).$$

4.3.6 Simplifying the Combined Incentive Constraints

In GT's model without downward mobility and the labour retraining program, both $\overline{IC_i}$ being binding in equilibrium gives strict conditions on the relationships between interim expected utilities and the ex-post utilities. Namely, it meant the equilibrium has an equal treatment condition in which $U_1^s = U_2^s$ and $u_1 = u_2$, the prework but poststudy expected utility is equal between different ex-ante types conditional on outcomes of HE. The ex-post utility relations are that $u_1 = u_2 < V_1 = v_2 < v_1 = V_2$. Although (4.14) looks similar, the $\overline{IC_i}$ being binding in equilibrium does not have any strict implications on the relations of the utilities because it also has U_1^{s2} and U_2^{s1} in the brackets of (4.14) instead of just U_1^s and U_2^s. The "equal treatment" becomes not possible to achieve for both $U_1^s = U_2^s$ and $u_1 = u_2$ by simple modification of the original model unless $b_2^o - bf_2^o$ has a specific value in the optimum based on the parameters. "Equal treatment" shouldn't be an official goal for the government when they are designing ICL offers to students with different innate abilities. Furthermore, because of the introduction of downward labour mobility, the relations between U_1^s and U_2^s no longer derive any relations between different ex-post utilities inside the interim expected utility. To answer the paper's questions, we will examine the common second-best relations between ex-post utilities that could be derived from the first-order conditions (FOC) in the maximization problem.

There are still combined incentive constraints that could be simplified and dropped out of the second-best maximization problem.

Lemma 4.2: In the second-best optimum

(a). If both $\overline{IC_2}$ and MR_1 are satisfied then due to Assumption 4.6 ICA_2 is automatically satisfied.

(b). If IC_2 and MR_1 are both satisfied then due to Assumption 4.6 ICB_2 is automatically satisfied.

The second-bet maximization problem is the following:

$$Max \sum_i l_i^{OB} \left[P_i U_i^s + (1 - P_i) u_i - c_i^y\right] - l_1^{OB} P_1 (1 - \pi) P^d \delta c_1^r - l_2^{OB} P_2 \pi P^d \delta c_2^r,$$

with respect to $(V_1^a, V_1^o, v_2^a, v_2^o, b_2^o, B_1^o, bf_2^o, BF_1^o, u_i^a, u_i^o)$, subject to $\overline{RC}$, $\overline{IC_i}$, MR_i, $\overline{MH_i}$, IC_i, ICA_1 and ICB_1, $i = 1, 2$. l_i^{OB} represents the social welfare weight given to the ex-ante type i student by the government in the welfare function. Assume that $l_1^{OB} + l_2^{OB} = 1$, and $l_i^{OB} > 0$ for both i.

4.4 Properties of the Second-Best Optimums

In section 4.4.1, we listed the certain relations between the ex-post utilities in the second-best optimum when the government is assumed to be able to identify who needs retraining and what happens if it cannot identify them. We also listed its implications on the levels of repayment. At the end, we discuss the potential solution to the second-best problem and the effect of labour retraining on the incomplete insurance against failing HE which are not affected by the assumption of the government's ability to identify who needs retraining. In section 4.4.2, we show in general, an ICL or a graduate tax is needed to implement the second-best optimum. In section 4.4.3 and section 4.4.4, we examine the effect of the impact of labour retraining program on the degree of ex-ante inequality compared to its level in the equal treatment second-best and on the degree of ex-post utility under the different assumptions on whether the government could identify who needs retraining.

4.4.1 The Common Relations between Ex-post Utilities and Implications on the Repayment Schedules

When the Government Knows Who Needs Retraining

Proposition 4.2: In the second-best optimums with the government knowing who needs to be retrained, the ex-post utilities have the following certain relations:

(a). $u_1^a = u_1^o, u_2^a = u_2^o$;

(b). $V_1^a > V_1^o, V_1^a + b > V_1^o + b$;

(c). $v_2^a > v_2^o, v_2^a + b > v_2^o + b$;

(d). $B_1^o > V_1^a$;

(e). $b_2^o > v_2^a$;

(f). $v_2^a + b > bf_2^o$ and $v_2^o + b > bf_2^o$.

Besides the relationship between the ex-post utilities, the resource constraint $\overline{RC}$ and the retraining effort constraint MR_1 must be binding in every second-best optimum.

To implement the second-best optimum, the optimal repayment schedule needs to obey the following relations

Corollary 4.1: When the government knows who needs retraining, the optimal repayment schedule has the following properties in the second-best optimum:

(1). $u_1^a = u_1^o$ implies $r_1^a = r_1^o(\underline{w})$,

$u_2^a = u_2^o$ implies $r_2^a = r_2^o(\underline{w})$.

(2). $V_1^a > V_1^o$ implies $R_{11}^o(w_1) > R_{11}^a$,

$V_1^a + b > V_1^o + b$ implies $R_{12}^o(W_1) > R_{12}^a$.

(3). $v_2^a > v_2^o$ implies $R_{21}^o(w_2) > R_{21}^a$,

$v_2^a + b > v_2^o + b$ implies $R_{22}^o(W_2) > R_{22}^a$.

(4). $B_1^o > V_1^a$ implies $R_{11}^a > R_{11}^{or}(w_1)$,

$B_1^o > V_1^o$ implies $R_{11}^o(w_1) > R_{11}^{or}(w_1)$.

(5). $b_2^o > v_2^a$ implies $R_{12}^a > R_{21}^{or}(w_2)$,

$b_2^o > v_2^o$ implies $R_{21}^o(w_2) > R_{21}^{or}(w_2)$.

(6). $v_2^a + b > bf_2^o$ implies $W_2 - \underline{w} > R_{22}^a - r_2^o(w_2)$,

$v_2^o + b > bf_2^o$ implies $W_2 - \underline{w} > R_{22}^o(W_2) - r_2^o(w_2)$.

Corollary 4.1 (1) meant that the optimal level of repayment/transfer for a student of ex-ante type i who has failed HE is the same across the two periods. The introduction of the retraining program does not mean the second-best optimum stop providing these failed students intertemporal insurance.

Corollary 4.1 (2) is about the ex-ante type $i = 1$ students who succeeded in HE and are at no risk of downward mobility. For these students, the effect of the introduction of downward mobility and the government retraining programme is to remove the insurance across periods enjoyed in the first best optimum and the level of repayment when old is higher than in the adult period.

The relationship presented in Corollary 4.1 (3) is for the ex-ante type $i = 2$ and is equivalent to the relationship presented in Corollary 4.1 (2). The policy implication is the same but for ex-ante type $i = 2$ students.

Corollary 4.1 (4) is for ex-ante type $i = 1$ students who are at risk of downward mobility. The repayment levels $R_{11}^{a} > R_{11}^{or}(w_1)$ and $R_{11}^{o}(w_1) > R_{11}^{or}(w_1)$ meant that for students with ex-ante type $i = 1$ and ex-post type $k = 1$, the students at risk of downward mobility but succeeded in the retraining faced the lowest level of repayment in the second-best optimum. In terms of policy design, this meant that the student loan contract should include a clause such that if the student's skills become obsolete and he encounters a labour market problem after successfully graduating from university, his loan repayment would be reduced if he signed up for government provided retraining program and succeeded in it. Note the outcome $B_1^o > V_1^o$ is not possible if the government is assumed to not know who needs retraining. Because if $B_1^o > V_1^o$ and the government does not know who needs retraining, those who were at no risk of downward mobility can pretend they need the retraining while never in any danger of failure even without any effort.

The relationship presented in Corollary 4.1 (5) is for the ex-ante type $i = 2$ and is equivalent to the relationship presented in Corollary 4.1 (4). The policy implication is the same but for ex-ante type $i = 2$ students.

In Corollary 4.1 (6) both $W_2 - \underline{w} > R_{22}^{a} - r_2^{o}(w_2)$ and $W_2 - \underline{w} > R_{22}^{o}(W_2) - r_2^{o}(w_2)$ simply mean that the students with ex-ante type $i = 2$ and ex-post $k = 2$ should always have a higher ex-post utility than students with ex-ante type $i = 2$ and ex-post type $k = 1$ who failed the retraining.

When the government knows who needs retraining and uses a graduate tax to implement the second-best optimum, succeeding in retraining means facing lower taxes compared to other students earning the same wage. Furthermore, the optimal policy should make sure the displaced workers who failed the retraining and remained in the industry threatened by foreign competition and/or technological change have a lower utility than the successfully retrained workers who went to the healthier industry. This tax policy may seem to be counterintuitive as it seems to be actively undermining the sectors that are already in danger but the reason is to incentivise those workers in actual danger of displacement to put the effort into the retraining programme and to move out of the ailing sectors.

New Utility Relations When the Government Does Not Know Who Needs Retraining

Proposition 4.2 is obtained based on the assumption that the government could observe who needs retraining. Proposition 4.2 points (a) and (f) do not depend on this assumption and still apply but Proposition 4.2 points (b) to (e) are dependent on this assumption. If the government does not know and still allocates the utilities such that $B_1^o > V_1^o$ and $b_2^o > v_2^o$, the workers that are not in danger of downward mobility will pretend to need retraining, exert no retraining effort and pretend to succeed to obtain the higher utility. Consequently, the government have additional constraints $B_1^o \leq V_1^o$ and $b_2^o \leq v_2^o$ for the second-best problem.

Proposition 4.3: In the second-best optimum with the government not knowing who needs to be retrained, it results in new relations between the ex-post utilities:

(a). $B_1^o = V_1^o > V_1^a$;

(b). $b_2^o = v_2^o > v_2^a$.

The relationships between the ex-post utilities in Propositions 4.2 and 4.3 are derived in Appendix A4.3.

To maintain the same level of utilities between B_1^o and V_1^o as well as between b_2^o and v_2^o, the level of repayment between these workers of the same wage must be the same regardless of their participation in the labour retraining program, although we still don't know the relation between BF_1^o and u_1^o as well as between bf_2^o and u_2^o who also are earning the same lowest basic wage. The government still does not provide intertemporal insurance to the students who succeeded in HE but opposite to the results in Proposition 4.2, when the government cannot identify who needs training, the level of repayment is lower when old. Also, Proposition 4.3 meant it is certain now in the second-best optimum $V_1^o > BF_1^o$ and $v_2^o > bf_2^o$.

Because of the additional incentive constraints, we cannot fully characterize the relations in the second-best optimum but a valid potential solution to the second-best problem is $U_2^s - U_1^{s2} + (1-2\pi)P^d\delta c_2^r = U_2^{s1} - U_1^s + (1-2\pi)P^d\delta c_1^r = 0$ and $u_1 = u_2$ with both $\overline{IC_1}$ and $\overline{IC_2}$ being binding constraints. All the combined adverse selection and moral hazard incentive constraints are automatically satisfied along with MR_2 and $\overline{MH_2}$. If we have $u_1 \leq u_2$ in the second-best optimum, it is certain that $\delta b_2^o - \delta bf_2^o \geq \frac{1-p}{p}(\delta B_1^o - \delta BF_1^o) +$

$\frac{1-2\pi}{\pi}\frac{1}{(P_2^r-P_1^r)}(\delta c_1^r - \delta c_2^r)$, the optimal policy offer less insurance against failing in retraining for the high ex-ante type students than for low ex-ante type students.

In the second-best optimum of GT's model without labour retraining their $\overline{MH_1}$ is binding with $U_1^s - u_1 = K_1^y$ which meant incomplete insurance against failing HE (adding downward mobility would not change this result). As discussed in section 4.3.4 the need to exert effort in retraining meant the $\overline{MH_1}$ now implies $U_1^s - u_1 \geq K_1^y + (1-\pi)P^d\delta c_1^r > K_1^y$. The labour retraining has reduced the degree of insurance against failing HE for the type $i = 1$ (low ability) students as the difference in utility between succeeding and failing HE has widened.

4.4.2 The Necessity of ICL or Graduate Tax

Could a non-graduate tax or a non-ICL be possible to implement the second-best outcome? Given that this model's possible repayment levels, agent types and assumptions on wage levels are not largely changed from GT's model, for the second-best optimum with $V_1^o = v_2^o < v_1^o = V_2^o$ and/or $V_1^a = v_2^a < v_1^a = V_2^a$ we could use the same method to show that an ICL is needed to implement the second-best optimum in general. GT's model specifically showed that in the equal treatment second-best optimum, unless HE increases earning in a completely additive way for different types of jobs, $W_2 - W_1 - (w_2 - w_1) = 0$, a non-ICL will not be optimal. Intuitively, in our model and real life, the wage rate depends on both education quality and labour effort, but in a non-specific way. This means that within a job type a higher quality education has a different effect on wages than in other job types and for students with a level of quality of education increasing the labour effort will have a different effect on wage rate than students with other levels of quality of education. However, this first method did not consider the impact of downward mobility and labour retraining and is only applicable to the case with $V_1^o = v_2^o < v_1^o = V_2^o$ and/or $V_1^a = v_2^a < v_1^a = V_2^a$.

The second way of showing non-ICL is not optimal includes labour retraining and applies to all second-best optimums. The intuition of this method is that we aim to show the government cannot use non-ICL to simultaneously implement all of the ex-post utility relations common in all the second-best optimums both when the government knows who needs retraining and when it does not. In this model with the addition of the labour retraining program, non-ICL or non-graduate tax means the optimal repayment schedule can be decomposed into an ordinary loan repayment that depends only on the education factors including the choice of HE quality and the use of the retraining program and an ordinary tax that only determined by the workers' ex-post type k. We assume the government policies cannot be age dependent meaning the ordinary loan repayment and the ordinary tax cannot change over time if the only thing changed for the worker is his age. We will examine the optimal repayment schedule both when the government knows who needs retraining and when the government doesn't know who needs retraining. Here are some of the non-ICL repayment schedules:

$$r_1^a = t_1 + L_1, r_1^o(\underline{w}) = t_1 + L_1,$$

$$r_2^a = t_1 + L_2, r_2^o(\underline{w}) = t_1 + L_2,$$

$$R_{11}^a = T_1 + L_1, R_{11}^o(w_1) = T_1 + L_1,$$

$$R_{21}^a = T_1 + L_2, R_{21}^o(w_2) = T_1 + L_2.$$

When the government knows who needs retraining, the second-best optimum requires $V_1^a > V_1^o$ and $v_2^a > v_2^o$ based on Proposition 4.2 points (b) and (c). Without age-dependent policies,

$R_{11}^a = R_{11}^o(w_1)$ and $R_{21}^a = R_{21}^o(w_2)$ cannot implement the second-best optimum with the non-ICL policies when the government knows who needs retaining. Note that if the government wants to lower the utility of V_1^o and v_2^o with a non-ICL, they can charge a fee on the workers that do not need labour retraining when the retraining is offered. However, because the workers that failed HE also do not need retraining by definition, this fee will also lower the utility of u_1^o and u_2^o. This will violate Proposition 4.2 (a) which requires $u_1^a = u_1^o$ and $u_2^a = u_2^o$. If the government only charge this fee to the workers who succeeded in HE, this policy then becomes an income-contingent repayment at old age as only the workers who earn above the lowest wage will pay (it could also be interpreted as a graduate tax as only those who succeeded in HE will pay).

When the government does not know who needs retraining, the second-best optimum requires $V_1^o > V_1^a$ and $v_2^o > v_2^a$ based on Proposition 4.3 points (a) and (b). Since the second-best optimum relation of Proposition 4.2 (a) is not changed while V_1^o and v_2^o is still different to V_1^a and v_2^a, the government still cannot use non-ICL to implement the second-best optimum. If the government states they will offer a loan repayment reduction to those who are offered labour retraining, this policy is also income contingent since only those with ex-post type $k = 1$ and not earning the lowest wage would be offered labour retraining.

For a further question, we ask "Could the second-best optimum be implemented by an ICL for HE while charging an ordinary loan or subsidy for using the labour retraining, both when the government knows who needs retraining and when the government does not know who needs retraining?" First we consider the case when the government knows who needs retraining. Let a to denote a charge given to those workers that don't need retraining and succeeded in HE and let b to denote a "reward" (flat payment) to those that need retraining. We have the following repayment schemes:

$$R_{11}^{or}(w_1) = ICL_1 - b, r_1^o(w_1) = ICL_0 - b,$$

$$R_{11}^a = ICL_1, r_1^a = ICL_0,$$

$$R_{11}^o(w_1) = ICL_1 + a, r_1^o(\underline{w}) = ICL_0.$$

a is already an income-contingent policy as it is only charged to those who don't need retraining while earning above the lowest wage. So even if the policy for using labour retraining is ordinary, we need additional income-contingent policies for those who don't use retraining. This repayment scheme implies $R_{11}^{or}(w_1) - r_1^o(w_1) = ICL_1 - ICL_0 = R_{11}^a - r_1^a$. The retraining effort constraint MR_1 must be binding in every second-best optimum, this meant the difference between V_1^a and u_1^a must be fixed at a level that is directly related to K_1^r under this loan repayment scheme. There is no reason this must be the case in the second-best optimum so giving an equal b to both B_1^o and BF_1^o is inappropriate. For example, if $c_1^r \longrightarrow 0$, $K_1^r \longrightarrow 0$ then $B_1^o \approx BF_1^o$. Under the above repayment scheme, this meant $V_1^a \approx u_1^a = u_1^o$ would be implemented despite $K_1^r \longrightarrow 0$ does not imply $V_1^a \approx u_1^a$ should be the case at the second-best optimum. If the amount b given to B_1^o and BF_1^o are different then b is no longer ordinary and is income contingent. In the second way to illustrate why b should not be ordinary we consider ICL_1 and ICL_0 are at the level so V_1^a and u_1^a are at their optimal value in the second-best optimum. Because $u(.)$ is strictly concave, if $V_1^a > u_1^a$ and $V_1^a - u_1^a < K_1^r$, then b must be negative to satisfy the binding MR_1 constraint. However, if b is negative the above repayment scheme results in $V_1^a > B_1^o$ which violates Proposition 4.2 (d).

Next, we consider the case when the government does not know who needs retraining. Proposition 4.3 (a) states $B_1^o = V_1^o > V_1^a$ will be the optimal relation. The retraining effort constraint MR_1 still binds. We will show that to implement the optimal relation $B_1^o = V_1^o > V_1^a$, the additional policy for using the retraining program "b" can no longer be ordinary. The optimal repayment scheme for V_1^o, B_1^o and BF_1^o can be written as follows:

$$R_{11}^{o}(w_1) = ICL_1 - a,$$

$$R_{11}^{or}(w_1) = ICL_1 - a - b + b,$$

$$r_1^{o}(w_1) = ICL_0 + b.$$

Now a is a "reward" that is given to every worker in old age who doesn't need retraining or succeeded in it, it is still income contingent as it is only given to those earning a high wage at w_1. Now b could be interpreted as a loan or a fee charged to all those who used labour retraining but is forgiven or waived if the worker succeeded in retraining. Since succeeding in retraining directly affects the income, b is now also income contingent. The second-best optimum also cannot be implemented by an ICL for HE while charging an ordinary loan for using the labour retraining program when the government does not know who needs retraining. The conclusion still applies even if age-dependent policies are allowed as we demonstrated the inadequacy of ordinary loans without using utilities across periods.

The government's objective of social welfare maximization is achieved by altering workers' disposable income based on their differences in HE, retraining and labour market outcomes. Ideally for a government to achieve this, the policy instrument should treat ex-ante types, ex-post types and the need for retraining together since a worker's wage is determined by the combination of these factors. Policies such as ICL or graduate tax are more flexible than an ordinary mortgage style loan that is insensitive to a borrower's income or income tax that does not consider an individual's education history since they can be designed to fit different workers based on an individual's choice of quality of HE, need of retraining and the student's labour market outcomes.

4.4.3 The Effect of the Labour Retraining Program on the Degree of Ex-ante Inequality

The degree of the ex-ante inequality is the difference in the expected utility net of study effort cost and expected retraining effort cost between ex-ante type i=2 and ex-ante type i=1 students before they know their outcomes in HE. In the GT's model, the only degree of ex-ante inequality that could be formulated is in the equal treatment second-best optimum. The "equal treatment" meant in the second-best optimum the interim expected utility is equal between the two ex-ante types conditional on HE outcome. In the following sections, we will show whether the introduction of labour retraining could result in an equilibrium that has a lower level of ex-ante inequality.

When the Government Knows Who Needs Retraining

Without labour retraining, this model is not meaningfully different from the GT's model. With only downward labour mobility and no labour retraining, the ex-post utilities B_1^o and b_2^o does not exist. We have equal treatment in which $u_1 = u_2$ and $U_1^s = U_2^s$ when both ex-ante self-selection constraints are binding in the second-best optimum. In this optimum, the study effort constraint for the ex-ante type $i = 1$ students binds, and for the ex-ante type $i = 2$ students, it is automatically satisfied,

$$U_2^s - u_2 = U_1^s - u_1 = K_1^y.$$

The degree of ex-ante inequality of the equal treatment case is the same as in GT's model and is equal to

$$P_2U_2^s + (1-P_2)u_2 - c_2^y - \left[P_1U_1^s + (1-P_1)u_1 - c_1^y\right] = (P_2-P_1)K_1^y + c_1^y - c_2^y > 0.$$

This term is positive since by assumption $P_2 > P_1$ and $c_1^y \geq c_2^y$. K_1^y is the utility difference between U^s and u because the government need to satisfy the study effort constraint in the second best. The students with the higher ex-ante type have a strictly higher expected utility than the students with lower ex-ante type before knowing the outcome of their HE.

With the introduction of the labour retraining program, we have the interim expected utilities U_1^s, U_1^{s2}, U_2^s and U_2^{s1} written in (4.11) and (4.12). Given that $P_2^r > P_1^r$, it is certain $U_1^{s2} > U_1^s$ and $U_2^s > U_2^{s1}$. We could examine whether the labour retraining program has increased the degree of ex-ante inequality in the optimum with double binding self-selection constraint and $u_1 = u_2$, which has equal treatment for students who failed in HE. Now in the equal treatment second-best optimum with double binding self-selection constraint and $u_1 = u_2$, the following can be derived from (4.14): $U_2^s - U_1^{s2} + (1-2\pi)P^d\delta c_2^r = U_2^{s1} - U_1^s + (1-2\pi)P^d\delta c_1^r = 0$. This new equal treatment outcome is a potential solution for our second-best optimality problem. The difference between $U_2^s - u_2$ and $U_1^s - u_1$ is $(1-\pi)P^d[P_2^r - P_1^r]\delta K_1^r - (1-2\pi)P^d\delta c_2^r$. Therefore, the minimum degree of ex-ante inequality in the equal treatment case with the addition of the labour retraining program is

$$(P_2-P_1)K_1^y + c_1^y - c_2^y + (P_2-P_1)(1-\pi)P^d\delta c_1^r + P_2(1-\pi)P^d(P_2^r - P_1^r)\delta K_1^r - P_2(1-2\pi)P^d\delta c_2^r + P_1(1-\pi)P^d\delta c_1^r - P_2\pi P^d\delta c_2^r.$$

$(P_2 - P_1)K_1^y + c_1^y - c_2^y$ is the original term for ex-ante inequality without labour retraining. For the additional terms, $(1-\pi)P^d\delta c_1^r$ is the additional utility needed to satisfy the study effort constraint due to the introduction of labour retraining. The promised labour retraining program increased the expected effort cost of succeeding in HE for the students since they expect a probability of facing downward mobility in the labour market after succeeding and are required to exert effort in the retraining program. This additional utility increased the degree of ex-ante inequality because the ex-ante type $i = 2$ students are more likely to succeed in HE so they are more likely to benefit from the additional utility. The second additional term, $(1-\pi)P^d(P_2^r - P_1^r)\delta K_1^r - (1-2\pi)P^d\delta c_2^r$, is the additional utility over $U_1^s - u_1$ that is needed to satisfy the utility difference between U_2^s and u_2 in the equal treatment second-best optimum. For the final additional term, $P_1(1-\pi)P^d\delta c_1^r - P_2\pi P^d\delta c_2^r$, it is the difference in the expected effort cost for the labour retraining between ex-ante type $i = 1$ and ex-ante type $i = 2$ students before knowing the outcome of HE. Its sign is uncertain and depends on the size of the parameters, the ex-ante type $i = 1$ is less likely to succeed in HE but after succeeding in HE they are more likely to obtain an ex-post type $k = 1$ and be at risk of downward mobility. Since $(P_2 - P_1)(1-\pi)P^d\delta c_1^r$ plus $P_1(1-\pi)P^d\delta c_1^r - P_2\pi P^d\delta c_2^r - P_2(1-2\pi)P^d\delta c_2^r$ is not negative, overall, the degree of ex-ante inequality is increased compared to the equal treatment optimum in GT's model. Moreover, this meant even if $U_2^s - u_2$ is equal to $U_1^s - u_1$ in equal treatment, the degree of ex-ante inequality is still increased.

Is it possible for the labour retraining program to reduce the degree of ex-ante inequality in any second-best optimum below the level found in the equal treatment case in GT's model? It is shown in the Appendix A4.5 that it is not possible because of any combination of utility differences of $U_2^s - u_2$ and $U_1^s - u_1$ that reduces the ex-ante inequality below the GT's level would violate ex-ante self-selection constraint $\overline{IC_2}$. The government labour retraining program can only increase the degree of ex-ante inequality.

When the Government Does Not Know Who Needs Retraining

The above analysis is done assuming the government knows which worker is at risk of downward labour mobility. Is it possible for labour retraining to reduce the degree of ex-ante inequality below the equal treatment level when the government does not know who needs retraining?

This answer is not changed because the method of proving the rise in the degree of ex-ante inequality is the same. The labour retraining program always increases the degree of ex-ante inequality.

Proposition 4.4: The introduction of labour retraining increases the degree of ex-ante inequality compared to the equal treatment outcome independent of whether the government could identify who needs retraining or not.

In conclusion, although the labour retraining program seems like a policy capable of reducing the degree of ex-ante inequality. No matter the assumption on the government's ability to identify who needs retraining between workers of the same ex-post types, the program will lead to an increase in the degree of ex-ante inequality above the level in the equal treatment case without the labour retraining program.

4.4.4 The Effect of the Labour Retraining Program on the Degree of Ex-post Inequality

Ex-post inequality is the difference in utility between type $k = 2$ and $k = 1$ workers after succeeding in HE of the same quality. In the original GT's model without labour retraining, among the successful students, those with the higher ex-post type $k = 2$ will always have a higher utility than those with the lower ex-post type $k = 1$ due to the labour effort constraints, specifically, $v_1 - b = V_1$ and $V_2 - b = v_2$. Could the introduction of labour retraining change the difference in ex-post utility between workers with ex-post type $k = 2$ and workers with ex-post type $k = 1$?

When the Government Knows Who Needs Retraining

For the ex-ante type $i = 1$ workers when old, we have

$v_1^o - b = V_1^o$ but since $B_1^o > V_1^o$, the relationship between v_1^o and B_1^o is uncertain.

This is the effect of labour retraining. For all students with ex-ante type $i = 1$ and succeed in HE, a portion of "$(1 - \pi)P^d P_1^r$" will end up with B_1^o. For successful ex-ante type $i = 1$ students, the labour retraining program has at least reduced the difference in ex-post utility between those workers with ex-post type $k = 2$ and those workers with ex-post type $k = 1$ and succeeded in retraining below b. For all the students born with ex-ante type $i = 1$, being successful in the labour market and gaining ex-post type $k = 2$ does not guarantee obtaining the highest possible utility for them when old. Even if $B_1^o > v_1^o$, the ex-post type $k = 2$ workers will not pretend to be ex-post type $k = 1$ and need retraining since we assume the government knows who needs retraining here. The ex-post type $k = 2$ does not need retraining by assumption and when they pretend to be ex-post type $k = 1$, they will be assigned the lower utility of V_1^o.

For the ex-ante type $i = 2$ students, the effect of the labour retraining program on the ex-post inequality is the same since it is certain that $b_2^o > v_2^o$ in the second-best optimum. For successful ex-ante type $i = 2$ students, the labour retraining program has also at least reduced the difference in ex-post utility between those workers with ex-post type $k = 2$ and those workers with ex-post type $k = 1$ and succeeded in retraining below b.

When the Government Does Not Know Who Needs Retraining

When the government does not know who needs retraining, the labour effort constraints ICX_i^s are still binding in the second-best optimum.

For the ex-ante type $i = 1$ workers when old, the government ensures that in the second-best optimum

$$v_1^o - b = V_1^o = B_1^o.$$

The government needs to make sure $V_1^o = B_1^o$ since if $B_1^o > V_1^o$, the workers that don't need retraining will pretend to need retraining and pretend to succeed in them. As a result of the government's lack of information, the labour retraining program does not reduce the difference between the utility of those with ex-post type $k = 2$ and ex-post type $k = 1$, which is still b.

For the ex-ante type $i = 2$ workers when old, we reach the same conclusion. Since $b_2^o > v_2^o$ is the second-best outcome when the government knows who needs retraining and now the government ensures that $V_2^o - b = v_2^o = b_2^o$ when the government does not know who needs retraining. The difference between the utility of those with ex-post type $k = 2$ and ex-post type $k = 1$ is still b.

In conclusion, with the introduction of the labour retraining program, the workers with ex-post type $k = 1$ who succeeded in retraining will obtain a higher ex-post utility than the workers earning the same wage who did not participate in retraining only if the government could identify who needs retraining. If the government lacks information on who needs retraining, the labour retraining program can only maintain the same degree of ex-post inequality compared to when the labour retraining is not offered by the government.

4.5 Time-Consistent Optimum

The second-best optimum assumes that the government can commit to its promises of future policies when old when it announces its policy to the youth when they are choosing their quality of education. If the government cannot commit to its future policy it has an incentive to change its policies when old while the students take this into account when making education decisions, the second-best results are no longer valid. The government time-consistent maximization problem in the old period is equivalent to the first-best problem but with the addition of labour effort constraints and retraining effort constraints. The derivation of the properties in the time-consistent optimum is shown in the Appendix A4.6. The following properties are found.

4.5.1 The Necessity of ICL and the Low Utility for Those Who Failed the Retraining

Proposition 4.5: In all the time-consistent optimum without labour retraining, $u_1^o = BF_1^o$, $u_2^o = bf_2^o$. With labour retraining, $u_1^o > BF_1^o$, $u_2^o > bf_2^o$. If the government knows who needs retraining, $B_1^o > V_1^o$, $b_2^o > v_2^o$ and if the government does not know who needs retraining, $B_1^o = V_1^o$, $b_2^o = v_2^o$.

We will demonstrate the necessity of the ICL to implement the time-consistent optimum using the non-ICL repayment schedules of V_1^o, B_1^o, u_1^o and BF_1^o, these workers can be arranged into pairs that earn the same wage and take the same quality of HE and only differ in that one of them also took the labour retraining:

$$R_{11}^o(w_1) = T_1 + L_1, R_{11}^{or}(w_1) = T_1 + L_1^r,$$

$$r_1^o(\underline{w}) = t_1 + L_1, r_1^o(w_1) = t_1 + L_1^r.$$

We obtain

$$R_{11}^o(w_1) - R_{11}^{or}(w_1) = L_1 - L_1^r = r_1^o(\underline{w}) - r_1^o(w_1),$$

$$\begin{aligned}\Delta_1 &= R_{11}^o(w_1) - R_{11}^{or}(w_1) - [r_1^o(\underline{w}) - r_1^o(w_1)] \\ &= w_1 - h_o(V_1^o) - [w_1 - h_o(B_1^o)] - [\underline{w} - h_o(u_1^o) - \left(\underline{w} - h_o(BF_1^o)\right)] \\ &= h_o(B_1^o) - h_o(V_1^o) + h_o(u_1^o) - h_o(BF_1^o).\end{aligned}$$

If Δ_1 is equal to zero, then an ICL or a graduate tax is unnecessary. If $u_1^o > BF_1^o$, B_1^o must be smaller than V_1^o for Δ_1 to be zero. From Proposition 4.5, $B_1^o \geq V_1^o$, $u_1^o > BF_1^o$ in the time-consistent optimum, Δ_1 is strictly positive, so the time-consistent optimum must be implemented using an ICL. The same argument can be applied to the ex-ante type $i = 2$

students to demonstrate the necessity of ICL. Using the same method, we conclude that the time-consistent optimum also cannot be implemented by an ICL for HE while charging an ordinary loan for using the labour retraining.

For students who are earning the lowest basic wage $\underline{w}$, those who failed retraining become worse off than those who failed HE. If the current government cannot commit, when the need to retrain workers who are facing downward mobility arises, the optimum policy entails a harsh term for those who failed the retraining program. If labour retraining is not offered, then workers earning the basic wage with the same quality of HE would obtain the same level of utility when old.

For the relationship between u_1^o and V_1^o and the relationship between u_2^o and v_2^o, it is unclear when the government is providing labour retraining but does not know who needs it, in all other time-consistent optimums, $u_1^o > V_1^o$ and $u_2^o > v_2^o$.

4.5.2 The Degree of Insurance Against Failing the Retraining

Both retraining effort constraints MR_1 and MR_2 are binding in the time-consistent optimum, this meant that $B_1^o - BF_1^o > b_2^o - bf_2^o$ would be the case when $K_1^r > K_2^r$. The ex-ante type $i = 1$ workers who are at risk of downward mobility receive less insurance against failing the retraining than the ex-ante type $i = 2$ workers. Only when $K_1^r = K_2^r$ would the two different ex-ante types i workers receive the same level of insurance against failing labour retraining.

4.6 Conclusion

This paper addresses the two following questions: 1. How do we optimally provide HE with a labour retraining program? 2. What is the impact of this optimally provided labour retraining program on inequality and insurance?

For the first question. The ICL is a more flexible instrument compared to an ordinary mortgage-style loan. We assume the government policies are not allowed to be age dependent and show the ICL is needed to implement the second-best optimum with HE and labour retraining. Moreover, the second-best optimum cannot be optimally provided by charging a separate ordinary loan to students who used the labour retraining program that is independent of their income while charging an ICL for using the HE.

For the second question, we benchmarked the results against the inequality and insurance properties found in Gary-Bobo and Trannoy (2015). The labour retraining has made the "equal treatment" second-best optimum found in Gary-Bobo and Trannoy (2015) into an unlikely outcome. The introduction of labour retraining has increased the degree of ex-ante inequality in all possible second-best optimums compared to the level of ex-ante inequality found in the equal treatment second-best optimum without labour retraining both when the government could identify who needs retraining and when the government could not identify who needs retraining. The effect of labour retraining on the degree of ex-post inequality for workers with the same ex-ante types who succeeded in HE is dependent on whether the government can identify who needs retraining. If the government can, then the labour retraining program will reduce the degree of ex-post inequality for those workers when old. If the government cannot, the same level of ex-post inequality is maintained. It is still optimal to provide intertemporal insurance to students who failed HE. However, the introduction of downward labour mobility and labour retraining has removed intertemporal insurance for the students who succeeded in HE. Labour retraining also has reduced the degree of insurance against failing HE for the low-ability students because of the possibility of the need to exert effort in retraining after succeeding in HE.

The time-consistent optimum also needs to be implemented by an ICL. The government will not offer the high ex-ante type less insurance against failing labour retraining than to low ex-ante type. To satisfy the retraining effort constraints, the utility of the workers who failed retraining will be lower than those who failed HE.

This paper confirms the usefulness of ICL schemes found in the related literature on funding HE with student loans. When the government plans to introduce labour retraining programs because it is expected that a part of the graduates with lower labour market skills and opportunities will face downward mobility, our findings suggest that the government could implement the second-best optimum by including the need for retraining in the ICL for HE to differ the loan repayment between those who need retraining and those who don't. The labour retraining has increased the degree of ex-ante inequality resulting in a larger difference in the net expected utility before knowing HE outcomes between the high- and low-ability students. In terms of the effect on ex-post inequality, the effect is dependent on whether the government can identify which worker needs to be retrained. We show that the ex-post inequality can only be reduced if the government can identify who needs retraining. This meant if the government wished to use the labour retraining program to achieve a redistributive effect and reduce the level of inequality, it needed to improve its information on factors causing labour displacement in the nation such as which sector of the economy is facing pressure from international competition and what skills are potentially made obsolete by technological changes to better identify which worker needs to be retrained.

Chapter 5: Political Equilibrium and Higher Education Spending in Presidential and Parliamentary Regimes

5.1 Introduction

There are economics papers examining the provision of higher education (HE) from a political angle and we contribute to this literature by examining whether a nation's constitution affects the level of its HE spending. Some of these papers aimed to determine which factors affect the voters' support for different methods of funding HE (Borck and Wimbersky, 2014; Del Rey and Racionero, 2012; Del Rey and Racionero, 2014) and different levels of university subsidy (Anderberg and Balestrino, 2008; De Fraja, 2001; Fernandez and Rogerson, 1995). Haupt (2012) illustrated that the level of HE subsidy would first increase when the number of skilled people reaches a majority but then falls as this high level of subsidy per student is politically unsustainable with a large number of students. However, the modelling of these papers involves unrealistic assumptions on politics, they assume the government would enact the exact policy demanded by the voters and the policy is chosen by direct democracy. They ignore any agency problem and the most politics involved in these papers is just majority voting. The question of whether the government constitution could affect the level of HE spending has not been properly explored in the literature despite the evidence that the form of the government being a presidential or parliamentary regime directly affects many aspects of the economy.

Some empirical papers examined the potential impact of different political factors, such as the number of HE interest groups and the budgetary power of the governor, on the level of state spending on HE (Ness and Tandberg, 2013; Tandberg, 2010a; Tandberg, 2010b). A higher budgetary power for the governor indicates a stronger separation of powers and there is a lack of consensus on its effect. Tandberg (2010b) did not find a greater budgetary power would have a significant effect on HE spending while Ness and Tandberg (2013) found that greater budgetary power increased the spending on HE. McLendon, Hearn and Mokher (2009) found more powerful governors decreased the spending on HE. However, these papers did not use economic models to derive the effect of different political factors on the government's decision on HE spending with utility maximizing voters and politicians. The constitution of the presidential regime gives it more separation of powers and less legislative cohesion when compared to the parliamentary regimes. This paper fills in a literature gap between the economic literature and relevant empirical papers on the government's decision on HE spending by combining more realistic assumptions of politics with utility-maximizing agents using a model based on Persson, Roland and Tabellini (2000). We provide a tailored modelling framework demonstrating the different decision-making processes for HE in the two regimes. The first question in this paper is: Do the different political systems such as presidential and parliamentary regimes differ in the level of HE spending? Different theories such as the principal-agent theory and the resource dependence theory have been applied to analyse the funding of HE (Elbasir and Siddiqui, 2018; Fowles, 2013; Kivistö, 2008; Zhang, Kang and Barnes, 2016). To answer our question, we will use the methods from positive constitutional economics.

There are a few papers that linked HE spending to regressive policies that would lead to a higher level of inequality. Fernandez and Rogerson (1995) showed that high- and middle-class voters would vote for a low level of HE subsidy which excludes the poor from HE. This subsidy also extracts resources from the poor because it is only given to those who participate in HE. Anderberg and Balestrino (2008) showed that subsidy to HE involves a transfer from the poor to the middle class in their model. Poutvaara (2011) showed how voters would vote for a large subsidy of HE so the median voter would also participate in HE and not vote for a high level of redistribution in the future and choose lower taxes instead. However, Anderberg (2013) constructed a quantitative political economy model of public spending on post-compulsory education, calibrated to the UK economy. It found the UK level of public spending on post-compulsory education corresponded to a majority voting equilibrium that is supported primarily by low- and middle-income groups which suggest the HE policy need not be as regressive. Di Gioacchino and Sabini (2009) developed a model that explains why a more unequal society tends to spend proportionately more on higher levels of education than on basic education. In their model lower ability individuals benefit more from basic education, so public investment in basic education would reduce future labour income inequality. Since labour income inequality is reduced, the future government would redistribute resources using capital income taxation. Di Gioacchino and Sabini (2009) found the greater the wealth inequality, the more likely that public investment in basic education would be politically unfeasible and public investment would be biased towards HE. Because we directly contrast the outcome of the HE spending between presidential and parliamentary regimes, we also answer the following question: Does a presidential regime lead to a higher degree of inequality compared to a parliamentary regime? The review paper-Voigt (2011) has a detailed description and analysis of the various papers that studied the economic effects of different constitutions and there isn’t any paper in Voigt (2011) directly analysed the effect on the level of inequality. Various other topics are examined including the difference in the size of the government and corruption between presidential and parliamentary regimes (Blume, Müller and Voigt, 2009; Gerring and Thacker, 2004; Lederman, Loayza and Soares, 2005; Persson and Tabellini, 1999; Persson and Tabellini, 2003). Fumagalli and Narciso (2012) argued the reason why parliamentary regimes have a larger government spending than presidential regimes is because parliamentary regimes are associated with higher voter participation. It is higher voter participation that increases government spending. Some papers examined the difference in policy outcomes between majoritarian and proportional electoral rules (Lizzeri and Persico, 2001; Milesi-Ferretti, Perotti and Rostagno, 2002; Persson and Tabellini, 2004) as well as between democracy and dictatorships (Deacon, 2009; Kammas and Sarantides, 2019). The only empirical papers we could find that estimate the potential effect of the presidential system on the countries' inequality compared to the parliamentary system are Feld and Schnellenbach (2014) and MacManus and Ozkan (2018). To our best knowledge, there also aren't any theoretical papers after 2011 that analysed the effect of presidential and parliamentary regimes on inequality. This paper aims to contribute to this relatively scarce field of positive constitutional economics.

Despite the lack of attention paid to examining the potential link between being a presidential regime and higher inequality, the fact that the USA is more unequal than Europe on average has not gone unnoticed. We also contribute to the papers examining the differences in the level of inequality between the USA and Europe (Alesina, Glaeser and Sacerdote, 2001; Almås, Cappenlen and Tungodden, 2020; Bonica et al, 2013). These papers attributed various factors including lack of proportional representation, a greater degree of polarisation in politics and differences in social preferences towards inequality as potential factors causing the US to have a higher degree of inequality than Europe on average. We present another explanation of how the US, a presidential regime is more unequal than Europe, a continent dominated by parliamentary regimes by illustrating how the differences in the design of the constitution would lead to different policies that result in different levels of inequality.

The model of this paper is based on Persson, Roland and Tabellini (2000). They included three assumptions to create more realistic politics: (1) No benevolent actors: All agents, including politicians, are motivated by self-interests. (2) No direct democracy: The voters elect their representatives to participate in the government's decision-making process. (3) No outside enforcement: Politicians cannot commit to any policies before they are elected. The three groups of voters are identical in all aspects except each group desires a specific transfer to their group only. The political process determines the level of the common tax rate and allocates the tax revenue to public goods, transfers and rents for the politicians. This combined with the three assumptions creates three conflicts of interest: politicians and voters disagree over the size and the use of tax revenue; the different groups of voters disagree over the distribution of the transfers; the politicians disagree over the distribution of the rents. The model focuses on two important features of the presidential and parliamentary regimes-separation of powers and legislative cohesion. The difference in the constitution between the two regimes meant the presidential regime has more separation of powers but less legislative cohesion than the parliamentary regime. A strong legislative cohesion means the government coalition is stable and does not shift from issue to issue. Legislative cohesion arises when it is costly for the coalition to break up (Diermeier and Feddersen, 1998). The presidential regime is modelled by sequential voting over taxation and spending by separate agenda setters. In the parliamentary regime, the majority coalition government in power is subject to a vote of confidence which could trigger a government crisis. The taxation and spending decisions are voted together and require the support of both members of the coalition. They found parliamentary regimes would result in a higher level of redistribution than presidential regimes. However, this is because the voters behind the parliamentary majority coalition can form a bilateral monopoly and jointly exploit the minority group that is out of power. This redistribution is not given to groups with low income but is transferred to groups with political power. We cannot conclude that this is the mechanism that causes the parliamentary regime to be able to produce a lower level of inequality than the presidential regime.

To answer the two questions, we did not make any major changes to the political aspect of the model from Persson, Roland and Tabellini (2000). The definition of an equilibrium of a political regime is also the same. To include the HE spending into the model, not every group of voters has a fixed level of income as in the original model and one requires HE spending to gain a positive level of income. This HE spending replaces the transfer for this group. Completing HE will improve the wage rate of the graduates on average is a well-documented finding (Baum, Ma and Payea, 2010; Baum, Ma and Payea, 2013; Ma, Pender and Welch, 2016). A higher level of HE spending will result in a higher level of income for the recipient group. The common tax rate assumption is unrealistic but completely removing it in the original model will simply result in the groups in power charging a 100% tax rate on others while a 0% tax rate on themselves. We allow the tax rates to differ between different groups of voters but the government is constrained by the fact that it must obtain an equal level of tax revenue from each group. In the original model with only fixed income, this new assumption is equivalent to the common tax rate assumption.

The paper is organized in the following ways: Section 5.2 presents the equilibriums of the infinite horizon model based on Persson, Roland and Tabellini (2000) with one group needing the HE spending. Section 5.3 answers the two questions based on the equilibriums of the model. Section 5.4 concludes.

5.2 Infinite Horizon Model

We will present the model in section 5.2.1 and the equilibriums of the simple legislature, the presidential and parliamentary regimes in sections 5.2.2, 5.2.3 and 5.2.4.

5.2.1 The New Model of Public Finance

Assume three groups of voters, $i = 1, 2, 3$, each group has a continuum of voters with unit mass. Time is measured discretely: a typical period is denoted by t and we consider an infinite horizon model.

The preferences of a member of group i in an arbitrary starting period j are given by

$$u_j^i = \sum_{t=j}^{\infty} \beta^{t-j} \, U^i(q_t),$$

where $\beta < 1$ is the discount factor for the future utilities, q_t is the vector of policies at period t and U^i is the utility function of a member of group i per period. We assume that in each period, there will only be one group of voters needing education spending and each group has an equal chance to be the group needing the education, the other two groups will be the traditional groups which each have a fixed income equal to one and bargain for the group-specific transfers. The group's needs or nature in that period deciding whether a group needs HE or not is random and cannot be known beforehand. For clarity in the notations, we assume in the period that we study for all different regimes, group 1 ends up being the group needing the HE spending while the other groups are traditional groups. If there is no HE spending in the period, then group 1's income in the period is equal to zero. The group 1 utility function U^1 in period $t = j$ is written as

$$U^1(q_j) = c_j^1 + H(g_j) = P_1 f_j^e - \tau_j + H(g_j).$$

If the other groups are chosen to be the ones needing HE, we assume every group's ability to benefit from HE spending is the same, $P_1 = P_2 = P_3 > \frac{1}{2}$. For group 2 and group 3, their utility functions in period $t = j$ are written as

$$U^i(q_j) = c_j^i + H(g_j) = 1 - \tau_j + f_j^i + H(g_j), i = 2, 3.$$

We assume the tax rates could differ between different groups of voters but it is restricted by the constraint that each group of voters need to contribute the same amount of tax revenue, τ_j. f_j^e is the income increasing HE spending, f_j^i is a group-specific transfer payment to group i that does not need HE spending (groups 2 and 3 in the case we study) and g_j is the public goods. The function $H(.)$ is concave and monotonically increasing and we assume that the public goods are valuable to the citizens, meaning $H'(0) > \max\,[1, P_1]$.

Define the public policy vector q as

$$q_t = [\tau_t, g_t, \{f_t^i\}, \{r_t^l\}].$$

$\{r_t^l\}$ denotes the political rents taken by the legislators at period t with r_t^l being the political rents taken by legislator l, but not other legislators.

The public policy vector in period t, q_t, must satisfy the government budget constraint:

$$(5.1) \qquad 3\tau_t = \sum_i f_t^i + \sum_i r_t^l + g_t \equiv f_t + r_t + g_t,$$

where f_t is the aggregate spending on HE and transfers at period t and r_t is the aggregate rent taken by the legislators.

5.2.2 A Simple Legislature

We set up a simple legislature which lacks separation of powers and legislative cohesion. The equilibrium outcome of the simple legislature is used as a comparison to the presidential regime and parliamentary regime. In the simple legislature and other political regimes, each group i is represented by one legislator so that $i = l = 1, 2, 3$. Separate elections under plurality rule take place in each of these groups for electing the legislator to represent them in parliament.

In period j, the incumbent legislator has preferences over outcomes, given by

$$v_j^l = \sum_{t=j}^{\infty} \mathrm{b}^{t-j} V^l(q_t) D_t^l,$$

where the utility per period is

$$V^l(q_t) = r_t^l.$$

The legislator's utility in a period is the rent he takes in that period and D_t^l is a dummy variable that is equal to one if legislator l holds office in period t and zero otherwise.

At the end of each period, each group holds an election for re-electing the legislator and the candidate with the largest number of votes wins. The incumbent runs against a randomly drawn opponent that is identical in preferences and is not different in any attitudes that affect their competence. An incumbent who is not re-elected can never return to being a legislator.

In period t, the incumbent legislators elected to the simple legislature at the end of period $t-1$ decide on public policy in a legislative bargaining game with the following sequence of events:

1. Nature randomly selects an agenda setter a among the three legislators. The voters' groups' nature/needs are also revealed with one of them needing HE spending and this becomes public information.
2. Voters set their re-election strategy and reservation utilities, which become publicly visible.
3. The agenda setter proposes a public policy q_t.
4. The legislature vote on the proposal. If a majority (≥2) supports the proposal, it is implemented. If not a default policy is implemented, with $P_1 f^e = \tau = \overline{r} > 0$, $r^l = \frac{3-\frac{1}{P_1}}{3}\overline{r}$ and $g = f^2 = f^3 = 0$.
5. Elections are held.

Throughout this chapter, any information stated to be public meant that every voter and legislator would be able to observe it. In every regime studied, the political positions of the legislators and the groups' nature/needs will be publicly known after they are determined by nature. The voters' reservation utilities also become public information immediately after they are set. The legislators cannot commit, the voters can observe the outcomes in the policy vectors after the policy is implemented and before elections. We restrict the attention to the equilibria in which voters of the same group coordinate their strategy but the voters across different groups do not cooperate. For every possible type of regime, we assume both voters and legislators are restricted to using strategies that condition their actions in period t on observable payoff relevant information in period t only and not on outcomes in any earlier periods. This essentially makes the equilibrium outcome stationary and the time subscript will be dropped for the equilibrium outcomes.

The voters within the same group coordinate their voting rule and they re-elect their incumbent if their reservation utility $\overline{B_t^i}$ is satisfied,

$$D_{t+1}^l = 1 \text{ if } U^i(q_t) \geq \overline{B_t^i}, i = l,$$

for $i = a$ and $i \neq a$ at t. All voters simultaneously set their reservation utilities $\overline{B_t^i}$ in a utility-maximizing fashion.

Define W as the expected equilibrium continuation value for each legislator at the start of each period, before nature has selected the agenda setter and revealed the needs of their voters being HE spending or a traditional transfer. W should be interpreted as how much a legislator at the beginning expects to gain for all future periods given he will always be re-elected.

The equilibrium of the simple legislature game is defined below with $\overline{B_t}$ denotes the vector of the reservation utilities $\overline{B_t^i}$ (The L superscript stands for the simple legislature).

An equilibrium of the simple legislature is a vector of policies $q_t^L(\overline{B_t})$ and a vector of reservation utilities $\overline{B_t^L}$, such that, in any period t, when all players take as given the equilibrium outcomes of periods $t+k, k \geq 1$:

1. for any given $\overline{B_t}$, at least one legislator $i \neq a$ weakly prefers $q_t^L(\overline{B_t})$ to the default outcome;
2. for any given $\overline{B_t}$, the agenda setter a prefers $q_t^L(\overline{B_t})$ to any other policy satisfying the first condition;
3. the reservation utilities $\overline{B_t^{iL}}$ are optimal for the voters in each group i, when one takes into account that policies in the current period are set according to $q_t^L(\overline{B_t})$ and taken as given the reservation utilities of other groups and the identity of the agenda setter.

Two cases of simple legislature with different agenda setters will be examined. One is the legislator from group 1, a_1, whose voters need HE spending as the agenda setter and the other one is the legislator from group 3, a_3, being the agenda setter. The equilibrium outcomes of the two cases are summarised in Propositions 5.1 and 5.2. The steps used to prove the Propositions are written in the Appendix A5.1.

Proposition 5.1: In the equilibrium of the simple legislature with the legislator of the group needing HE spending being the agenda setter,

$$\tau = 1,$$

$$r = 3 - 2\beta \frac{1 - \frac{1}{3}\frac{1}{P_1}}{1 - \frac{1}{3}\beta} - \frac{1}{P_1},$$

$$g = \min\left[H'^{-1}(P_1), 2\beta\frac{1-\frac{1}{3}\frac{1}{P_1}}{1-\frac{1}{3}\beta}\right],$$

$$f^e = \frac{1}{P_1} + 2\beta\frac{1-\frac{1}{3}\frac{1}{P_1}}{1-\frac{1}{3}\beta} - g, f^2 = f^3 = 0.$$

All legislators are re-elected.

Proposition 5.2: In the equilibrium of the simple legislature with the legislator of the traditional group being the agenda setter,

$$\tau = 1,$$

$$r = 3 - 2\beta\frac{1-\frac{1}{3}\frac{1}{P_1}}{1-\frac{1}{3}\beta} - \frac{1}{P_1},$$

$$g = \min\left[H'^{-1}(1), 2\beta\frac{1-\frac{1}{3}\frac{1}{P_1}}{1-\frac{1}{3}\beta}\right],$$

$$f^3 = 2\beta\frac{1-\frac{1}{3}\frac{1}{P_1}}{1-\frac{1}{3}\beta} - g, f^2 = 0, f^e = \frac{1}{P_1}.$$

All legislators are re-elected.

In the equilibrium, the only way for there to be any additional HE spending above $\frac{1}{P_1}$ is for the legislator of the group needing HE to be the agenda setter. Otherwise, f^e would be kept at the minimum level ($\frac{1}{P_1}$) that is needed to maximize the tax revenue. The agenda setter acts as a "residual claimant of tax revenue" by minimizing the rent to the coalition partner and voters can only obtain public goods and transfers by appealing to the future benefits of re-election with the rest of the tax revenue being claimed as rents.

5.2.3 A Presidential Regime

In the presidential regime, the voters' voting rule for their representatives remains the same as in the simple legislature. They condition their reservation utilities on whether their agenda representative is the agenda setter for the allocation of spending ($i = a_g$), for taxes ($i = a_t$) or neither ($i = n$):

$$D_{t+1}^{l} = 1 \text{ if } U^{i}(q_t) \geq \overline{B_t^{i}}, i = l \text{ at } t.$$

The timing of the game in a typical period follows this sequence of events and is based closely on Persson, Roland and Tabellini (2000):

1. Nature randomly selects two different agenda setters among the incumbent legislators, one for taxes and one for the allocation of public spending, a_t and a_g, respectively. The voters' groups' nature/needs are also revealed with one of them needing HE spending and this becomes public information.
2. Voters set reservation utilities for their voting rule, $\{\overline{B^{i}}\}$. The voters could choose to set their reservation utilities to be dependent on the tax level/rates they face.
3. The agenda setter a_t proposes different tax rates for the different groups and τ which is the common tax level.
4. Congress vote. If at least two legislators are in favour of the proposal, the policy is implemented. Otherwise, a default tax level $\tau = \overline{r} < 1$ is implemented with the tax

rates for the group needing HE spending not being decided. Note if the tax default occurs, the value of f^e must obey $P_1 f^e \geq \overline{r}$ in the final equilibrium outcome.

5. Agenda setter a_g proposes $[g, f^e, f^2, f^3, \{r^i\}]$ subject to the budget constraint: $f + r + g \leq 3\tau$. The agenda setter chooses a legislator as the coalition partner to support his policies.
6. Congress votes. If at least two legislators are in favour, the policy is implemented. If the coalition partner votes no to the package, the spending agenda setter will choose the other legislator as the replacement for the coalition partner and vote on another spending proposal with the same level of rent to the coalition partner. If failed again, a default policy of $P_1 f^e = \tau, r^l = \frac{3-\frac{1}{P_1}}{3}\tau$ and $g = f^2 = f^3 = 0$ is implemented. If the default occurred at stage (4) and happened again at this stage, $P_1 f^e = \tau = \overline{r}$ will be implemented in default.
7. Elections are held.

We assume the voters can set their reservation utilities to be dependent on the tax level they face. This removes a complication in the first presidential case in which the voters of the legislator not in any government position cause a disequilibrium to reduce the tax level in which the tax agenda setter is not re-elected, but both his legislator and the spending agenda setter are re-elected. The voters of the spending agenda setter will set their reservation utility to be dependent on the tax level. The other voters, especially the voters of the tax agenda setter will not set their utility to be dependent on the tax level.

At stage (5), legislator a_g attempts to form a coalition that maximizes his utility. Since a_g is indifferent between the other two legislators, we assume they have the same probability of being included in the winning coalition. This is important because legislators are forward-looking, their behaviour at stages (3) and (4) depends on their expectation of being a part of the coalition later.

The equilibrium of the presidential regime is defined below (The superscript P stands for the presidential regime).

An equilibrium of the presidential regime is a vector of policies $q_t^C(\overline{B_t})$ and a vector of reservation utilities $\overline{B_t^C}$, such that, in any period t, with all players taking as given the expected equilibrium outcomes of periods $t+k$, $k \geq 1$:

1. for any given $\overline{B_t}$, at stage 4, at least one legislator $i \neq a_\tau$, weakly prefers accepting rather than rejecting the tax policy proposal, given the expected equilibrium proposals and decisions at stages 5 and 6;
2. for any given $\overline{B_t}$, a_τ prefers proposing the equilibrium tax policy to any other tax policies that satisfies condition 1, given the expected equilibrium proposals and decisions at stages 5 and 6;
3. for any given $\overline{B_t}$ and tax policy, at stage 6, at least one legislator $i \neq a_g$ weakly prefers accepting rather than rejecting the spending policy proposal;
4. for any given $\overline{B_t}$ and tax policy, at stage 5, a_g prefers the equilibrium spending proposal to any other proposal satisfying condition 3 and the budget constraint;

5. the reservation utilities $\overline{B_t^{iC}}$ are optimal for the voters of each group i when one considers that policies of the current period will be set according to $q_t^C(\overline{B_t})$ and take as given the other group's reservation utilities as well as the identity of the agenda setters a_g and a_τ.

We will analyse three different cases of presidential regimes with the legislator of the group 1 voters who are assumed to turn out needing HE spending being the tax agenda setter, being the spending agenda setter and not being an agenda setter:

1. The tax agenda setter is from group 1, a_{t1} and the spending agenda setter is from group 3, a_{g3}.
2. The tax agenda setter is from group 3, a_{t3} and the spending agenda setter is from group 1, a_{g1}.
3. The tax agenda setter is from group 3, a_{t3} and the spending agenda setter is from group 2, a_{g2}.

We only consider the presidential regime's equilibriums in which all legislators are re-elected. The voters of the tax agenda setter will not receive a higher utility from manipulating the public goods set by the spending agenda setter to deviate from the level preferred by the voters of the spending agenda setter. In the infinite horizon model, this meant assuming for groups 2 and 3, $H'(g) > 1$ when $g < \beta W$, look in the Appendix A5.2 which explains why this is the case. The assumption that when the spending proposal fails, the spending agenda setter could choose the other legislator as the replacement for the coalition partner removes a possible disequilibrium in the third presidential case in which the tax agenda setter could be incentivised to cause legislator 1 to not be re-elected. We also assume the maximum f^e level to be $\tau + 2\frac{\frac{1}{P_1}}{3}\tau$ because if f^e has no upper limit the tax agenda setter can and will force the default to occur in which he receives a higher expected utility.

The steps used to prove the equilibriums of all three cases of presidential regimes are shown in the Appendix A5.3. For the presidential regimes, r^m denote the rents given to the spending agenda setter's coalition partner.

Proposition 5.3: In the equilibrium of the presidential regime with the tax agenda setter a_{t1}, who represents the group needing HE spending, and the spending agenda setter a_{g3}, who represents a traditional group,

$$\tau = 1,$$

$$r^{a_{g3}} = r^m = 1 - \frac{1}{3}\frac{1}{P_1} - \frac{2}{3}\frac{9}{9+\beta}\left(1 - \frac{1}{3}\frac{1}{P_1}\right)\beta,$$

$$g = \min\left[H'^{-1}(1), \frac{2}{3}\frac{9}{9+\beta}\left(1 - \frac{1}{3}\frac{1}{P_1}\right)2\beta\right],$$

$$f^3 = \frac{2}{3}\frac{9}{9+\beta}\left(1 - \frac{1}{3}\frac{1}{P_1}\right)2\beta - g,$$

$$f^e = 1 + 2\frac{\frac{1}{P_1}}{3},$$

$$f^2 = 0.$$

All legislators are re-elected.

The separation of powers interacting with the restrictions on the tax system gives power to the voters behind the tax agenda setter who use the position to manipulate the spending on

HE f^e which changes the relative income between different groups to maximize their income. The W is also smaller than in the simple legislature case, which means more revenue for spending on HE f^e and less spending on transfers f^3.

A confusion that could be had here is: Isn't it in the spending agenda setter's interest to charge the maximum tax at $\tau = 1$ and take all additional revenue as his rent? What is to stop the spending agenda setter to not spending the massive tax revenue on f^e after the tax agenda setter announces the tax plan? This problem can be explained by differentiating the tax rates from the tax level. The taxation plan made by the tax agenda setter could be made to force the spending agenda setter to propose such high spending on f^e by setting a low tax rate on group 1 voters, for example, 20% and the tax rate on group 2 and group 3 voters to be 100%. Group 1's voters' income only exists after the spending on f^e has occurred so the spending agenda setter cannot take all the tax revenue as his political rents without spending the required amount on f^e constraint by the taxation set by the tax agenda setter.

If the group 2 voters are unsatisfied with the equilibrium outcome and want to cause the default tax $\tau = \overline{r} < 1$ to happen, the only possible action they could take is to set their reservation utility above $H(g)$ so their legislator will perhaps not support a_{t1}'s proposal to earn his re-election. If no other legislators support a_{t1}'s proposal then a_{t1} will not be re-elected since a_{t1} failed to satisfy his own voters' reservation utility, foreseeing this when the different groups of voters set up their reservation utilities at stage 2, a_{t1} will propose the tax rates such that $\tau = 1, f^e = \frac{1}{P_1}$ for the disequilibrium in which no legislator is re-elected. a_{g3} will always support this tax proposal and plan for no re-election because he will receive a higher utility than to reject the proposal (and plan for either re-election or no re-election) or support the proposal but plan for re-election. When facing $\tau = 1$, $f^e = \frac{1}{P_1}$ proposal, a_{g3} should always take everything and leave $g = f^2 = f^3 = 0$. Therefore no group 2 nor 3 voters will set their reservation utility purposefully high so that this disequilibrium happens.

Proposition 5.4: In the equilibrium of the presidential regime with the tax agenda setter a_{t3}, who represents the traditional group, and the spending agenda setter a_{g1}, who represents the group needing HE,

$$\tau = 1 - \frac{3\beta}{\left(3 - \frac{1}{P_1}\right)} \frac{2}{3} \frac{9}{9+\beta}\left(1 - \frac{1}{3}\frac{1}{P_1}\right) < 1,$$

$$r^{a_{g1}} = r^m = \left[\frac{3 - \frac{1}{P_1}}{3}\right]\left[1 - \frac{3\beta}{\left(3 - \frac{1}{P_1}\right)} \frac{2}{3} \frac{9}{9+\beta}\left(1 - \frac{1}{3}\frac{1}{P_1}\right)\right] - \beta \frac{2}{3}\frac{9}{9+\beta}\left(1 - \frac{1}{3}\frac{1}{P_1}\right),$$

$$g = \frac{2}{3}\frac{9}{9+\beta}\left(1 - \frac{1}{3}\frac{1}{P_1}\right) 2\beta,$$

$$f^e = \left(1 + 2\frac{\frac{1}{P_1}}{3}\right)\left[1 - \frac{3\beta}{\left(3 - \frac{1}{P_1}\right)} \frac{2}{3} \frac{9}{9+\beta}\left(1 - \frac{1}{3}\frac{1}{P_1}\right)\right],$$

$$f^2 = f^3 = 0.$$

All legislators are re-elected.

Proposition 5.5: In the equilibrium of the presidential regime with the tax agenda setter a_{t3} and the spending agenda setter a_{g2}, both represent their respective traditional groups,

$$\tau = 1 - \frac{3\beta}{\left(3 - \frac{1}{P_1}\right)} \frac{2}{3} \frac{9}{9+\beta}\left(1 - \frac{1}{3}\frac{1}{P_1}\right) < 1,$$

$$r^{a_{g2}} = r^m = \left[\frac{3 - \frac{1}{P_1}}{3}\right]\left[1 - \frac{3\beta}{\left(3 - \frac{1}{P_1}\right)} \frac{2}{3} \frac{9}{9+\beta}\left(1 - \frac{1}{3}\frac{1}{P_1}\right)\right] - \beta \frac{2}{3}\frac{9}{9+\mathrm{b}}\left(1 - \frac{1}{3}\frac{1}{P_1}\right),$$

$$g = \min\left[H'^{-1}(1), \frac{2}{3}\frac{9}{9+\beta}\left(1-\frac{1}{3}\frac{1}{P_1}\right)2\beta\right],$$

$$f^2 = \frac{2}{3}\frac{9}{9+\beta}\left(1-\frac{1}{3}\frac{1}{P_1}\right)2\beta - g,$$

$$f^e = \left(1+2\frac{\frac{1}{P_1}}{3}\right)\left[1-\frac{3\beta}{\left(3-\frac{1}{P_1}\right)}\frac{2}{3}\frac{9}{9+\beta}\left(1-\frac{1}{3}\frac{1}{P_1}\right)\right],$$

$$f^3 = 0.$$

All legislators are re-elected.

For the three cases of presidential regimes, the important mechanism of the interaction of the separation of powers, the tax revenue requirement and the HE spending causes a high level of spending on f^e and it implies a high degree of inequality. The position of the tax agenda setter is the most politically important position due to how the tax rates can determine spending on f^e. In the first case, group 1's voters behind the tax agenda setter exploit his power to maximize their utility at the expense of the other groups. In the other two cases, the traditional groups are indifferent to the level of HE spending since an increase in the level of f^e does not mean an increase in their tax burden. Using this indifference, the level of f^e could be maximized at the tax level even if group 1's legislator is not any agenda setter. Because there is no collusion, the tax agenda setter is indifferent to the rents given to the spending agenda setter, the level of f^e is therefore maximized by reducing the rents given to the spending agenda setter and the spending agenda setter receives the same level of rents as rents given to the coalition partner. This outcome in the presidential regime also isn't caused solely by allowing the tax rates to differ between different groups because if there is no spending that alters relative income between different groups our results would not be generated.

We can modify and extend the constitutional rules of the presidential regimes following Persson, Roland and Tabellini (2000) by adding a nationally elected president to the model. One way of doing it is to add a president with a block veto after Congress votes on the spending proposal. The president would be given rents to not exercise his veto power in the equilibrium. We can also give the power of the tax agenda setter to the president instead of one of the legislators. Same as in Persson, Roland and Tabellini (2000), this creates a stronger separation of powers and reduces the equilibrium tax level because the president would never be a part of the spending agenda setter's coalition. The president only wishes to be re-elected and satisfy the low tax demand from the majority of the voters. Moreover, because the legislator of the group 1 voter can never be the tax agenda setter, the tax level is minimized in all cases of the presidential regime. However, our insight that the voters that need HE maximize the level of f^e at a given tax level by minimizing the rents to the spending agenda setter is not changed in either modification of the presidential regime. Similar to the tax agenda setter, the president is indifferent to the rents given to the spending agenda setter. The addition of a president did not remove any of the mechanisms that created the high level of f^e.

Adding a Large Exogenous Threat of No Re-election

In the preceding analysis, the voters cannot stop the legislators from taking away everything by appealing to any future exogenous utility the legislators obtain if they are continuously re-elected because we assume there are no exogenous rents from office. We could modify the model for the presidential regime by adding an exogenous utility R for the legislator for being in office. The utility of the legislator per period is

$$V^l(q_t) = r_t^l + R.$$

The incumbent legislator's preferences over the outcome in period j in equilibrium with all legislators re-elected is

$$v_j^l = \sum_{t=j}^{\infty} \mathrm{b}^{t-j}\, r_t^l D_t^l + \frac{R}{1-\beta}.$$

We assume $\frac{\beta}{1-\beta} R > \frac{3-\frac{1}{P_1}}{3}$, the voters have a strong exogenous threat of no re-election against their legislators. Under this assumption, the spending agenda setter will not give his coalition partner any rent if he plans for re-election and the spending agenda setter prefers to be re-elected than default even if he receives zero rents in re-election. For the tax agenda setter, the minimum tax level in equilibrium is determined by the inequality below

$$\frac{r^m}{2} + \beta W + \frac{R}{1-\beta} \geq \frac{1}{2} - \frac{\frac{1}{P_1}}{6} + R,$$

substitute $r^m = \frac{3-\frac{1}{P_1}}{3}\tau - \beta W - \frac{R}{1-\beta}$, we obtain

$$\tau \geq 1 - \frac{3}{3-\frac{1}{P_1}} \frac{\beta}{1-\beta} R - \frac{3}{3-\frac{1}{P_1}} \beta W.$$

Given the assumption on R, the minimum tax level the tax agenda setter is willing to set in equilibrium is zero. The political rents are eliminated in all equilibriums and the spending agenda setter is indifferent between re-election and taking away everything. If the tax agenda setter's voters have fixed income they want to minimize both the tax level and the f^e. The level of f^e will only increase group 1 voters' income to the tax level to generate the tax revenue needed to spend on other policies. In the second case of the presidential regime, $P_1 f^e = \tau = \frac{f^e+g}{3}$ (note that the group 1 voters also want the maximum possible level of public goods in this case) and in the third case $P_1 f^e = \tau = \frac{f^e+g+f^2}{3}$. In the first presidential case the tax agenda setter's voters do benefit from higher f^e, they still want to maximize the tax level to maximize the level of f^e. Group 1 voters remain as the residual claimant of tax revenue, after spending on the transfers to group 3 and the public goods, all the tax revenue is spent on f^e.

The tax level and f^e in the new equilibriums corresponding to each presidential case analysed above are: 1. $\tau = 1$, $f^e = 2 + \frac{1}{3P_1} - \frac{\beta}{1-\beta}R$. 2. $\tau = \frac{P_1}{3P_1-1}\frac{3}{2}\frac{\beta}{1-\beta}R$, $f^e = \frac{1}{3P_1-1}\frac{3}{2}\frac{\beta}{1-\beta}R$. 3. $\tau = \frac{P_1}{3P_1-1}\frac{3}{2}\frac{\beta}{1-\beta}R$, $f^e = \frac{1}{3P_1-1}\frac{3}{2}\frac{\beta}{1-\beta}R$.

5.2.4 A Parliamentary Regime

At the beginning of each period, nature picks two legislators as members of a majority coalition which makes up the government. One of the legislators prepares a budget proposal which is then voted on in the parliament. In this vote, each coalition partner has a veto right and a veto is essentially a vote of confidence on the government. When the veto is exercised by either partner, a government crisis ensues. A new agenda setter is picked at random and the decision-making becomes the same as the simple legislature analysed in section 5.2.2.

The sequence of events in a parliamentary regime game consists of the following stages:

1. Nature randomly selects two coalition partners among the incumbent legislators; one becomes the agenda setter for all the public finance decisions (tax and spending), a, and the other his junior partner, m.
2. The voters' groups' nature/needs are revealed. One of the groups would be the one needing f^e (assume this group is group 1). All groups have an equal chance of being the group needs f^e spending.
3. The voters set their reservation utilities for their voting rule, $\{\overline{B^i}\}$.
4. The agenda setter a proposes $[\tau_a, g_a, \{f_a^i\}, \{r_a^l\}]$ subject to $3\tau_a \geq f_a + r_a + g_a$.
5. The junior partner can veto the joint proposal from stage 4. If approved, the proposal is implemented and the game goes to stage 10. If not the government falls and the game goes to stage 6'.

6'. Nature randomly selects a new agenda setter a' among the three legislators.

7'. Voters reformulate their re-election strategies, conditional on the status of their representatives after the government crisis.

8'. The agenda setter a' proposes an entire allocation $q_{a'}$.

9'. The parliament votes on this proposal. If approved by at least two legislators, $q_{a'}$ is implemented. If not, the legislature bargaining ends and a default outcome with $P_1 f^e = \tau = \overline{r} > 0, r^l = \frac{3-\frac{1}{P_1}}{3}\overline{r}$ and $g = f^2 = f^3 = 0$ is implemented.

10. Elections are held.

Compared to the simple legislature, the current junior partner has pre-assigned veto rights in the parliamentary regime. These veto rights give the junior coalition partner and the voters she represents more bargaining power and induce legislative cohesion. The voters' re-election strategy in each group is the same as in the simple legislature and the presidential regime. They set their reservation utilities based on the political position of their legislator.

An alternative formulation of the parliamentary regime is to add sequential proposals within the government. It is stated in Persson, Roland and Tabellini (2000) that in a previous version of their paper with sequential proposals, they obtained identical results to their current paper without sequential proposals. This statement is incorrect as the tax level will be reduced from the maximum level due to sequential proposals in their model. The main insight of sequential proposals is that the legislative cohesion induced by the veto did limit the separation of powers induced by the sequential proposals compared to the presidential regime due to the simple legislature subgame the agenda setters could trigger if either vetoed. f^e cannot be maximized by minimizing the level of rent and making the spending agenda setter indifferent to the default outcome. Sequential proposals did not change the results that different to the presidential regime, additional spending on f^e does not exist unless the group 1's legislator is within the governing coalition and different to the simple legislature, additional spending on f^e would exist if group 1's legislator is the coalition partner to the spending agenda setter. To focus on the effect of legislative cohesion in the parliamentary regime, we choose to model the constitutional rules of our parliamentary regime as closely to Persson, Roland and Tabellini (2000) as possible.

The equilibrium of the parliamentary regime is defined below. We use $\overline{B_t'}$ to denote the vector of the reservation utilities set by the voters if the government falls at stage 5 (The superscript R stands for parliamentary regime).

An equilibrium of the parliamentary regime is defined by a vector of policies $q_t^R(\overline{B_t})$ and vectors of reservation utilities $\overline{B_t^R}$ and $\overline{B_t^{R'}}$, such that, in any period t, given the expected equilibrium outcomes of periods $t+k, k \geq 1$:

1. for any given vector $\overline{B_t}$ and given the proposal made at stage 4, at stage 5, the junior partner of the coalition optimally chooses whether to accept or veto the proposal, given the expected reservation utilities and the expected policy outcomes in stages 6' to 9';
2. the reservation utilities $\overline{B_t^{iR'}}$ set at stage 6' after the fall of the government are optimal for the voters of each group when taking into account that the policies will be set as in the simple legislature equilibrium and take as given the reservation utilities set by other groups;
3. for any given vectors of reservation utilities $\overline{B_t}$ and $\overline{B_t'}$, the agenda setter in the coalition prefers to pass their proposal $q_t^R(\overline{B_t})$ over the government crisis given conditions 1 and 2 and the government budget constraint;
4. the reservation utilities $\overline{B_t^{iR}}$ are optimal for the voters of each group i when one takes into account that the policies in the current period set by the agenda setter follow condition 3, take as given the expected reservation utilities set by other groups if the government falls, and the fact that policies will be set as in the simple legislature equilibrium after a government crisis at stage 5, and also take as given the reservation utilities set by other groups at stage 7'.

We examine the case of group 1 and group 3's representatives being in the coalition government. Which is the agenda setter and which is the junior partner is not important for deriving the equilibrium outcome and the mechanism of policy formation in the parliamentary regime is the same, we assume the agenda setter will be from group 1, a_1 and the junior partner is from group 3, m_3. The alternative case with groups 2 and 3 being inside the coalition government will not be studied since its equilibrium outcome will be nearly identical to Persson, Roland and Tabellini (2000) with the addition of $f^e = \frac{1}{P_1}$.

The equilibrium outcome is summarised in the following proposition and the steps used to prove the equilibrium are in the Appendix A5.4. r^P is the total rent taken by the legislators in the parliamentary regime. A denotes the additional spending on HE that pushes group 1 voter's income above the tax level. Similar to the presidential regime, for the parliamentary regime, we use r^m to denote the rents given to the agenda setter's junior partner.

Proposition 5.6: In the parliamentary regime with the agenda setter from group 1 and the junior partner from group 3, there is a continuum of equilibria such that

$$\tau = 1,$$

$$r^a = \frac{2}{3}r^P, r^m = \frac{1}{3}r^P, r^P = 3 - \frac{1}{P_1} - 2\beta\frac{1-\frac{1}{3}\frac{1}{P_1}}{1-\frac{1}{3}\beta},$$

$$f^e = \frac{1}{P_1} + A,$$

$$A + f^3 = 2\beta\frac{1-\frac{1}{3}\frac{1}{P_1}}{1-\frac{1}{3}\beta} - g,$$

$$f^2 = r^2 = 0 \text{ as } n = 2.$$

If $A > 0$, $f^3 > 0$ then

$$H'(g) = \frac{P_1}{1 + P_1}.$$

If $A = 0, f^3 = 0$ then

$$g = 2\beta \frac{1 - \frac{1}{3}\frac{1}{P_1}}{1 - \frac{1}{3}\beta}.$$

In other possible equilibria, if $P_1 < 1$, in the equilibrium with $A = 0$ but $f^3 > 0$ or $A > 0$ but $f^3 = 0$, then the level of public goods obey the following condition

$$H(g) \geq \frac{1}{3}H(g'_e) + \frac{2}{3}H(g'_T).$$

Since $P_1 < 1$, $g'_e > g'_T$, it is possible for $g < g'_e$ in equilibrium.

If $P_1 > 1$, in the equilibrium with $A > 0$ but $f^3 = 0$, then the public goods also obey the same condition

$$H(g) \geq \frac{1}{3}H(g'_e) + \frac{2}{3}H(g'_T).$$

Since $P_1 > 1$, $g'_e < g'_T$, then it is possible for $g < g'_T$ in equilibrium, with $g'_e = \min\,[H'^{-1}(P_1), 2\beta \frac{1-\frac{1}{3}\frac{1}{P_1}}{1-\frac{1}{3}\beta}]$ and $g'_T = \min\,[H'^{-1}(1), 2\beta \frac{1-\frac{1}{3}\frac{1}{P_1}}{1-\frac{1}{3}\beta}]$.

All legislators are re-elected.

Different to the simple legislature, even if group 1's legislator is not the agenda setter, they can still obtain additional spending on f^e if their legislator is the junior partner within the government coalition. For the level of public goods, the level of public goods in the parliamentary regime can be lower than in the presidential regime. The level of public goods in the presidential regime is either the same as the simple legislature counterpart in the interior solutions or in the case when the spending agenda setter represents the group that needs HE, potentially higher than the simple legislature counterpart. The reason for the level of public goods in the parliamentary regime to be possible to be smaller than in the presidential regime case is because in the subsequent simple legislature subgame if the government crisis occurred, the agenda setter's level of public goods is uncertain as different groups being chosen as a' will result in different levels of public goods. This affected the reservation utilities and the possible public goods level in the equilibrium.

Following the focus of Becher (2019) and Becher and Christiansen (2015) on the ability of the prime minister to dissolve the parliament to call an early election, we could extend the parliamentary regime by giving the agenda setter the power to dissolve the parliament. The agenda setter could choose to directly move to the election stage instead of the simple legislature subgame when he triggers a government crisis. This threat of dissolution would not affect the equilibrium outcome. It is an empty threat because the agenda setter is always worse off using the dissolution than not using it.

5.3 Analysis of the Outcomes

In this section, we will use our findings to answer the two questions. In section 5.3.1, we examine the effect of the constitutions in different regimes on the level of HE spending. In section 5.3.2, we examine its implications on the degree of income inequality within a nation. In section 5.3.3, we discuss the implications on the public goods level and political rents level.

5.3.1 The Difference in HE Spending between Presidential and Parliamentary Regimes

The key finding of our model is to present the different ways that HE spending is determined in the two regimes. In our model, the separation of powers property in the presidential regimes allows the voters needing HE spending to become the residual claimant of tax revenue instead of the spending agenda setter because of the tax revenue requirement and the income-increasing property of the HE spending. The group needing HE requires HE spending to generate any income. Because we allow tax rates to differ between different groups and the requirement of each group to generate an equal amount of tax revenue, the spending agenda setter is restricted by the tax policy set at the previous period and cannot freely choose the level of HE spending. Instead, he must spend the required amount of f^e to meet the tax revenue requirement based on the tax rates. When the threat of no re-election is endogenously determined the political rents cannot be eliminated. There is a minimum required tax level that is needed for the expected utility of the tax agenda setter of being re-elected to be equal to his highest expected utility of proposing a tax plan that led to no re-election for all legislators. Within the same tax level, even though the legislator of the group of voters that needs HE is not the tax agenda setter, they can increase their spending on HE by minimizing the rents to the spending agenda setter (the spending agenda setter is indifferent between being re-elected and the default outcome). The tax burden on the voters with fixed income is not increased. Since there is no collusion because the spending agenda setter cannot make credible promises to other legislators, the tax agenda setter is indifferent to the utility of the spending agenda setter and would propose the tax plan that results in the desired HE spending to ensure his re-election. The highest level of spending of HE occurs when the legislators of the voters needing HE is the tax agenda setter because the maximum tax level would be charged.

In the parliamentary regime, the coalition government remains the residual claimant of tax revenue. Without separation of powers and with legislative cohesion in the parliamentary regime, the voters needing HE cannot minimize the rents to the agenda setter. It is always optimal for the government coalition to charge the maximum tax revenue to maximize the political rents and it is also optimal for their voters as it allows them to obtain as many transfers as possible. Any additional spending on HE reflects the voters' legislator's position of power. If the group needing HE is not a part of the coalition government, they will not receive any additional spending on HE. Because of the different ways the HE spending is determined under the two regimes, comparing the highest level of HE spending between the two regimes is uncertain. However, we do know that HE spending is always set at the maximum level that is possible in the equilibriums of the presidential regime whereas in the parliamentary regime additional spending on HE is shared with the available tax revenue on public goods and transfers that the voters obtained from appealing to re-election.

Additionally, for the presidential regime, we present an extreme case in which the voters possessed a strong threat of no re-election over their legislator represented by a large exogenous utility for being in office. The utility of re-election becomes exogenously determined and the utility the non-spending agenda setters gain in default and no re-election is smaller than the exogenous utility of being re-elected even if they receive no rents. This meant in equilibrium with the separation of powers, the voters behind the tax agenda setter were capable of letting the tax agenda setter propose a tax plan that eliminates political rents through the threat of no re-election with the tax revenue only spent on policies that benefit the voters. Here the voters can truly minimize the tax level in equilibrium, any increase in the HE spending will increase the tax level. If the voters have fixed incomes, they are no longer indifferent to HE spending, they want to minimize the tax level and therefore the HE spending. The level of HE spending would either be maximized or minimized depending on the identity of the tax agenda setter. It will be maximized if the tax agenda setter represents the group that benefited directly from HE spending with the tax revenue being solely used to fund the relatively low level of public goods and transfers with the rest being used to spend on HE. It will be minimized if the tax agenda setter represent a fixed income group as the tax agenda setter charges the minimum tax revenue possible combined with 100% tax rates on the group needing HE to be just enough to fund the various spending.

We also predict the HE spending between different parliamentary regimes is less varied on average than the HE spending between different presidential regimes meaning the extremely high and extremely low levels of HE spending should be more likely to be found in the different presidential regimes.

There is some evidence supporting the prediction of a greater variation in HE spending between different presidential nations. Figure 5.1 is based on the latest OECD data on tertiary education spending between different countries measured in US dollars per student. The presidential regimes are indicated by red dots and the parliamentary regimes are indicated by blue dots. Except for Luxembourg, the USA had the highest spending at $35,347 per student above the OECD average of $17,599 per student. For the countries with the lowest tertiary education spending per student, except Greece and Lithuania, all these nations are presidential regimes. The only presidential regime with a spending level close to the OECD average is Costa Rica which spends $15,786 per student on tertiary education.

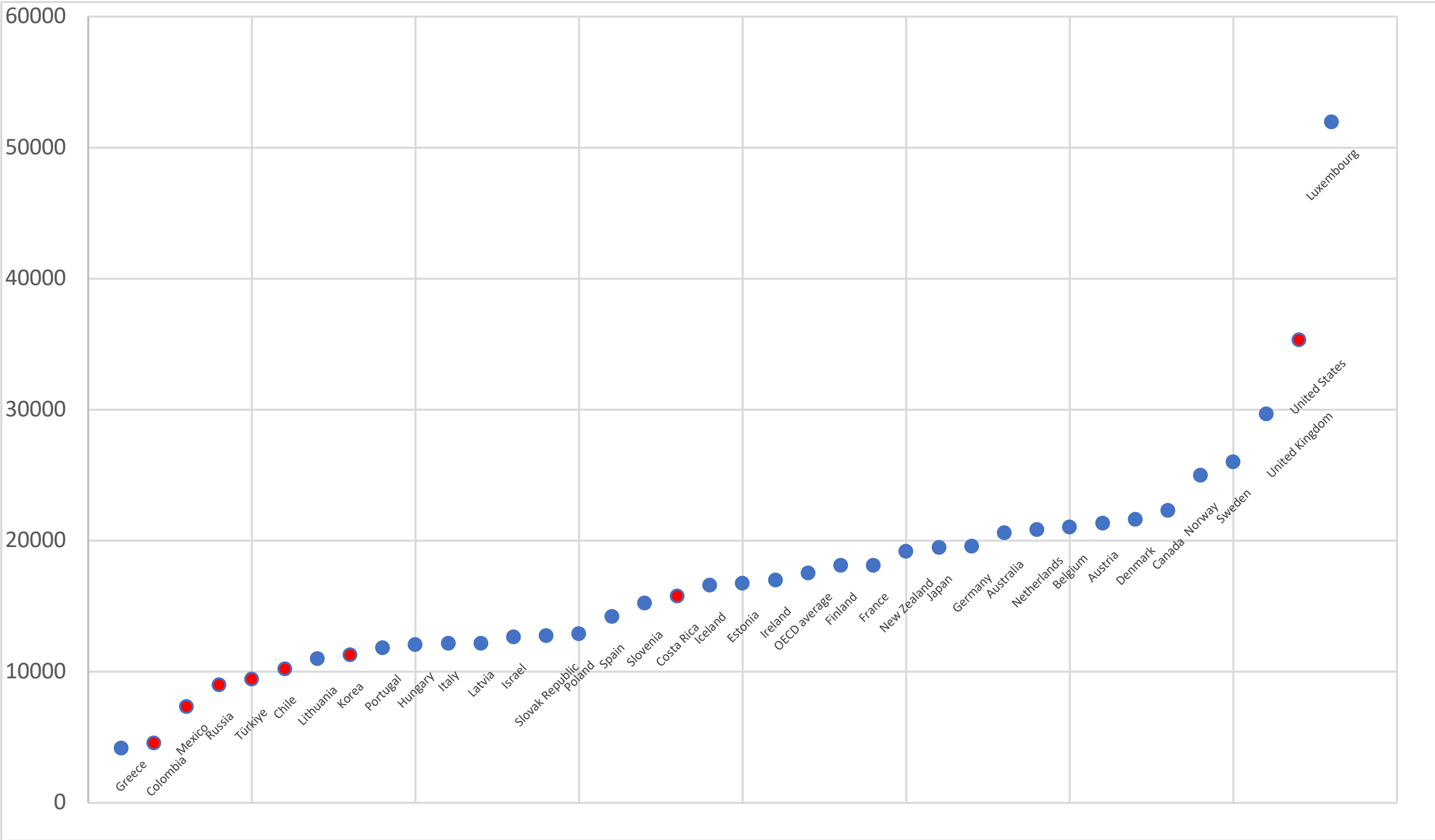


Figure 5.1: Tertiary Education Spending, US Dollars/Student. Source: OCED (2022)

5.3.2 The Effect of the Political Institution on Inequality within a Nation

Because the HE spending directly affects the relative income between different voters, the different ways the HE spending is determined also explain why the presidential regime has a higher degree of inequality on average compared to the parliamentary regime. In the presidential regime, the HE spending is always either maximized or minimized which maximizes the difference between group 1's income and the fixed income. This greater degree of income inequality in the presidential regime is a direct result of the constitution's design. Although generating inequality is not the goal of any voters or legislators, due to the separation of powers property and the tax revenue requirement, the voters would generate a higher level of income inequality through utility maximization.

This high degree of inequality is not found in the presidential regime of Persson, Roland and Tabellini (2000) that does not include the HE spending changing the relative income between different groups. In their model, even if inequality is artificially introduced by changing the fixed income in the different groups, this inequality is not affected by whether the richer or poorer groups are in positions of power. When the poor group's legislator is the tax agenda setter they cannot charge low tax on themselves and high tax on the rich group even if the tax rates are allowed to be different between groups since they are constraint by the need to raise an equal amount of revenue from every group. If the voters behind the tax agenda setter are a relatively poor group, it is unrealistic that they would choose to perpetuate the current inequality that does not benefit them when they become a powerful group in the government and are in charge of government taxation.

The f^e in this model could have other interpretations than HE spending. For example, f^e could be a public investment or represent the ease of private investment into a specific region or in a specific economic sector. Our model illustrates that the legislator could exploit the separation of powers by announcing plans for tax breaks on certain regions or certain economic sectors that his voters are more heavily involved with while requiring a roughly equal amount of tax revenue to be generated from different parts of the economy. After passing the tax proposal, the government is under pressure to generate enough tax revenue from different parts of the country or different sectors of the economy to fund the supply of various public goods and spending while restricted by the tax policy set previously. As a result, it will enact the level of public investment desired by the voters who benefit from them.

It is a well-established fact that on average, presidential regimes have a higher level of income inequality than parliamentary regimes. It is useful to examine the differences in the level of income inequality within each country between parliamentary regimes and presidential regimes. Table 5.1 reports the available data for the average Gini index (Gini) (World Bank Open Data, 2021) and the Top 20% pre-tax income share (Top20) (WID.world, 2022) (WID stands for World Inequality Database) from 2015 to 2019 for all the available parliamentary and presidential countries. Appendix A5.5 details the definitions of Gini and Top20, along with their sources. The classification of a country as being a presidential regime or a parliamentary regime is based on the classification in Persson and Tabellini (2003). Based on their classification, if the country's government's survival is not subject to a vote of confidence, that country would be classified as a presidential regime and if its survival is subject to a vote of confidence, the country would be classified as a parliamentary regime. In countries with an elected president, its classification depends on whether the president or the legislative assembly has more control over the appointment and/or dismissal of the executive. If such control rests primarily with the legislative assembly, the country is classified as parliamentary. Therefore, many Eastern European countries, like Ukraine, are classified as parliamentary regimes despite having a president.

Table 5.1: Degrees of Income Inequality between Different Countries

Parliamentary Regime			Presidential Regime		
Country	Gini	Top20	Country	Gini	Top20
United Kingdom	34.37	0.506	Cyprus	32.44	0.5198
Austria	30.4	0.4828	United States	41.28	0.6076
Belgium	27.42	0.481	Switzerland	32.78	0.4709
Denmark	28.2	0.4782	Argentina	41.825	0.5776
France	32.15	0.4728	Bolivia	44.16	0.6635

Germany	31.53	0.5277	Brazil	53.18	0.7261
Italy	35.43	0.479	Chile	44.4	0.7496
Luxembourg	33.74	0.4904	Colombia	50.6	0.6899
Netherlands	28.44	0.4528	Dominican Republic	43.74	0.6635
Norway	27.66	0.4496	Ecuador	45.36	0.5777
Sweden	29.38	0.4412	El Salvador	39.2	0.6211
Canada	33.23	0.5608	Guatemala		0.6635
Japan	33.4	0.5892	Mexico	47.2	0.7491
Finland	27.32	0.489	Nicaragua		0.6635
Greece	34.28	0.4916	Paraguay	47.14	0.6635
Iceland	26.7	0.4374	Peru	42.86	0.7081
Ireland	31.65	0.5044	Uruguay	39.74	0.572
Malta	29.48	0.4918	Venezuela		0.6635
Portugal	34.16	0.529	Egypt	31.65	0.6371
Spain	35.14	0.4942	Sri Lanka	39.3	0.6296
Australia	34	0.5078	Korean Republic	31.4	0.5986
New Zealand		0.479	Nepal		0.5693
Trinidad and Tobago		0.66346	Philippines	43.45	0.6166
Israel	38.88	0.64	Gambia	35.9	0.6024
India		0.6788	Russia	36.05	0.5853
Malaysia	41.1	0.5543	Costa Rica	48.32	0.685
Botswana	53.3	0.7335	Ghana	43.5	0.6401
Papa New Guinea		0.6222	Honduras	49.1	0.6635
Estonia	31.08	0.5164	Malawi	41.6	0.6857
Latvia	34.74	0.5153	Namibia	59.1	0.7746
Poland	30.73	0.5185	Zambia	57.1	0.7551
Ukraine	25.84	0.4785			
Czech Republic	25.3				
Slovak Republic	24.62	0.4311			
Hungary	30.18	0.4699			
Romania	35.38	0.5565			
Bulgaria	40.24	0.5643			
Pakistan	30.45	0.5763			
Bangladesh	32.4	0.5719			
Fiji	30.7				
Mauritius	36.8	0.6171			
Singapore		0.5931			
Average:	32.43	0.5261	Average:	43.05	0.645

Note: The missing area indicates there is no available data on either the Gini index or the Top 20% income share for that country between 2015 to 2019 in World Bank Open Data (2021) or WID.world (2022).

A clear picture emerged suggesting the presidential regime has a higher degree of income inequality both before and after taxes and transfers, agreeing with our model's prediction. The presidential regimes' Gini index on average is about 33% higher than parliamentary regimes and the top 20% income share on average is about 12 percentage points higher than parliamentary regimes. However, there is a wide range of results within different regimes with the Korean Republic and Switzerland having a low level of inequality despite both being presidential regimes. Furthermore, the nation with the highest Gini index is a parliamentary regime, Botswana, signifying that there are other factors involved in determining the level of inequality inside a country.

5.3.3 Public Goods Level and Level of Political Rents

In general, our equilibrium results agree with the prediction of the level of public goods between the presidential and parliamentary regimes in Persson, Roland and Tabellini (2000). Unless the equilibrium in the parliamentary regime is a corner solution, the level of public goods is greater than the presidential regime in Propositions 5.3 and 5.5. However, we do provide a different and new prediction for the level of public goods in the presidential regime with the HE needing group's legislator being the spending agenda setter and the traditional group's voter being the tax agenda setter. When the level of HE spending is capped by assumption or minimized by the tax agenda setter, the voters needing HE optimally set their reservation utility to obtain the maximum level of public goods that is possible in the presidential regime.

In terms of the total level of rents in equilibrium, Persson, Roland and Tabellini (2000) predicted the parliamentary regime also results in a higher level of rents than the presidential regime. In our model with the value of re-election being determined endogenously, the voters desiring HE spending exploit the separation of powers to minimize the rents to the spending agenda setter that made him indifferent between re-election and default. However, this means the equilibrium continuation value for the legislators is lower in the presidential regime than in the parliamentary regime. The threat of no re-election is weaker in the presidential regime. Although the total level of rents in the parliamentary regime in Proposition 5.6 is greater than the presidential regime with the minimized tax level in Propositions 5.4 and 5.5. Comparing the level of total rents between Proposition 5.3 and Proposition 5.6, if the discount factor β is large enough ($\beta > 0.739$), the total rents in the presidential regime with the maximum tax level is larger than the level of rents in the parliamentary regime in Proposition 5.6.

The original model does not offer any suggestions on which political institution is strictly better than the other but instead finds a trade-off: the parliamentary regime solves the under-provision of public goods g better while the presidential regime is strictly better in terms of solving the political agency problem. Political institution strictly dominates in one aspect of the outcome. The importance of education spending f^e for both voters and the legislators has offered more possibilities in the final economic outcome than the strict "parliamentary regime provides more public good while presidential regime limits taxation and waste". It is not simply that one government must be better at one aspect than the other government, now the composition of the government and the underlying economic nature of the voters will affect the outcome. It is still not possible to give any normative implications on constitution reforms as the outcome becomes more varied but it does offer a richer prediction of the outcome between different political regimes than the original model.

5.4 Conclusions

In this paper we address the two following questions: 1. Do the different political systems such as presidential and parliamentary regimes differ in the level of HE spending? 2. Does a presidential regime lead to a higher degree of inequality compared to a parliamentary regime?

The constitutional difference between the presidential and parliamentary regimes we focused on is that the presidential regimes have more separation of powers but less legislative cohesion than parliamentary regimes. For the first question, the levels of HE spending differ because of the different ways the level of HE spending is determined in the two regimes. In the presidential regime, the voters needing the HE spending can maximize the spending on HE by becoming the residual claimant of tax revenue no matter their legislator's political position. In the parliamentary regime, the coalition government remains the residual claimant of tax revenue due to the threat of veto power and the subsequent simple legislature game. There will only be additional spending on HE if the group is included in the coalition and they need to share the tax revenue with their coalition partner's transfers. Additionally in the presidential regime with a large exogenous utility for being in office, we show the level of HE spending is minimized when the voters of the tax agenda setter have fixed income. We predict the presidential regimes have a greater variation in HE spending than the parliamentary regimes. For the second question, our model predicts due to the separation of powers the presidential regimes will generate a higher level of income inequality than the parliamentary regimes. Our prediction is supported by previous research in Feld and Schnellenbach (2014) and McManus and Ozkan (2018). They showed that presidential regimes have a statistically significant higher post-tax income inequality compared to the parliamentary regime.

There is no collusion between the tax agenda setter and the spending agenda setter in the presidential regime. The tax agenda setter effectively controls the HE spending decision of the spending agenda setter despite not officially controlling it, even if the spending agenda setter tries to take away everything. The tax agenda setter would propose a tax plan that maximizes the spending on HE at a given tax level by minimizing the rents to the spending agenda setter. The spending agenda setter is indifferent between re-election and the default outcome. With a high exogenous utility from office, the voters behind the tax agenda setters could eliminate all rents to both agenda setters. Here any increase in HE spending above the minimum meant an increase in the tax level. If the voters behind the tax agenda setter have fixed income their utility would decrease from a higher tax level, to maximize their utility they would require the minimum tax burden and minimum spending on HE as well. In the parliamentary regime with greater legislative cohesion, the level of HE spending will be relatively moderate because the tax revenue will be shared with spending on the transfers to other groups. The greater legislative cohesion also meant the voters could no longer exploit the indifference between the different agenda setters to use the HE spending to minimize the rents.

In terms of additional findings in our infinite horizon model, we generally agree with the conclusions of Persson, Roland and Tabellini (2000) that the parliamentary regimes provide a higher level of public goods and result in a higher level of rents but we find in corner solutions the parliamentary regime can produce a lower level of public goods. Moreover, with a high discount factor, the total level of rents in the parliamentary regime is lower than the total level of rents in the presidential regimes when the tax agenda setter's voters need HE spending because of the lower equilibrium continuation value in the presidential regime and when the tax agenda setter's voters need HE, the tax level is maximized instead of minimized.

We contribute to the economics literature on the political decision on HE with a more realistic political environment including self-interested agents, representative democracy and politicians being incapable of commitment which the previous papers lack. We answer a previously unexplored question on the effect of the design of the constitution on the nation's HE spending. This analysis could also be applied in future research to analyse the effect of the electoral rule, i.e., between the proportional and majoritarian electoral systems, on government decisions on HE spending. Furthermore, we provide a formal mechanism on how the nature of the presidential regime's constitution with its separation of powers and lack of legislative cohesion results in a higher level of income inequality. This adds to the literature examining the potential reasons for the difference in the within-nation income inequality between different countries, especially between the USA, Canada and various European nations.

Chapter 6: Government's Student Loans and HE Spending Policy under Uncertainty and Polarisation

6.1 Introduction

Historically many countries offered free higher education (HE) to qualified students but there is a worldwide trend of shifting the cost of HE onto the students themselves in the form of tuition fees (Johnstone, 2003; Johnstone, 2004; Marcucci and Johnstone, 2007). In the UK and the USA, the total level of student loan debt has been increasing over the years as shown in the two figures below. A large concern for the build-up of student loan debt is the risk of default and Glater (2016) even compared this to the financial mortgage crisis.

Figure 6.1: The Growth of Student Loan Debt in the UK. Source: Bolton (2022)

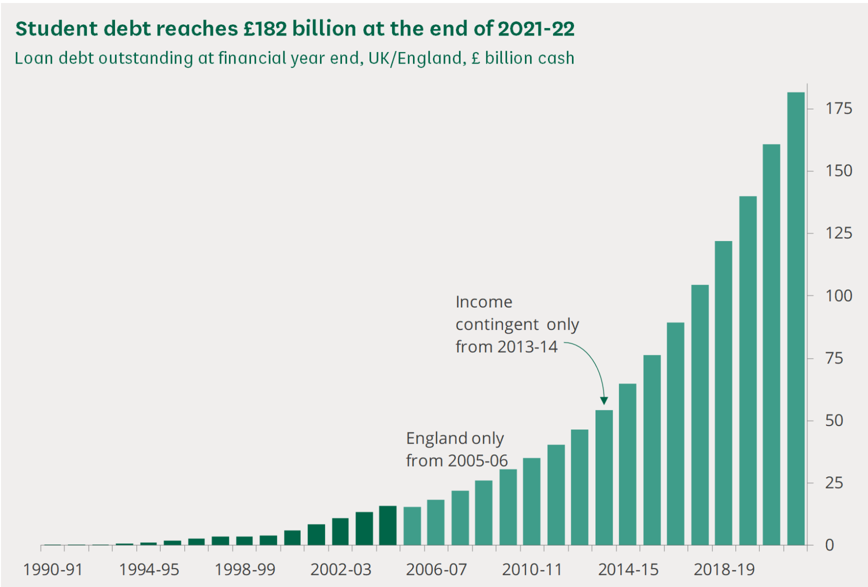
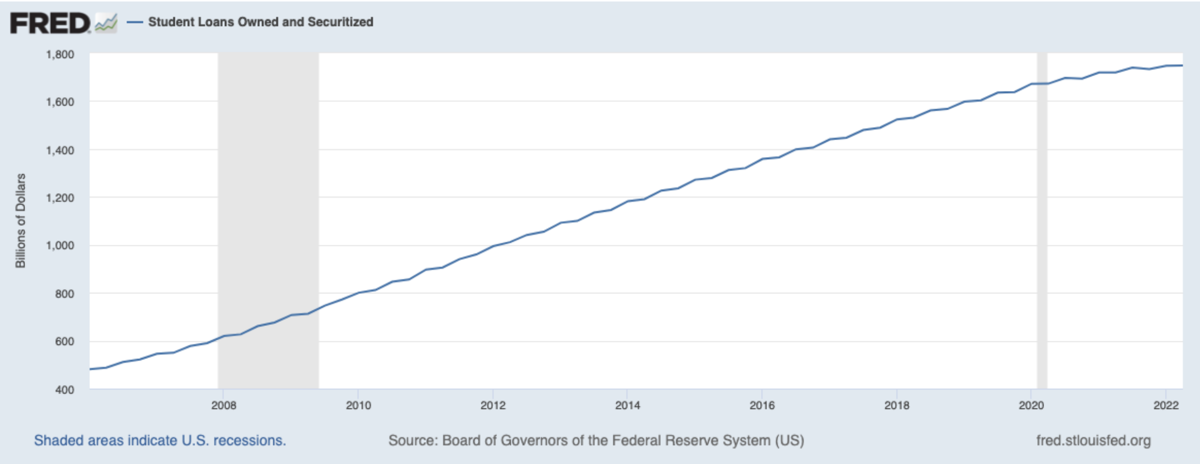


Figure 6.2: the Growth of Student Loan Debt in the US. Source: Board of Governors of the Federal Reserve System (US) (2022).



There is a large number of US studies examined the factors that led to the rising student loan debt and rising rates of default (Addo, Houle and Simon; 2016; Belfield, 2013; Dean Craig and Raisanen, 2014; Goodell, 2016; Hillman, 2014; Scott-Clayton, 2018). A common institutional factor found in these papers is that attending for-profit colleges will lead to a significantly higher average level of debt after graduation and a higher risk of default in the future. Individual factors such as a low-income background or belonging to a minority group are also found to be associated with higher rates of default. Moreover, Akers, Chingos and Henriques (2015) and Hershbein and Hollenbeck (2014) found the rise in tuition fees to be an important factor in causing the rise of student loan debt and borrowing in the USA. However, these papers did not question why the government allowed this rise in tuition and student loans. Similarly, although many UK papers examined the impact of the various UK HE funding reforms (Azmat and Simion, 2018; Murphy, Scott-Clayton and Wyness, 2019; Sá, 2019), they did not formally examine the potential reasons for the government to increase the tuition fee cap. When the HE spending of a country is mainly funded by student loans like in the UK and the USA, an increase in the tuition fee represents an increase in HE spending. Marginson (2018) found the UK's tripling of tuition fees from £3,000 to £9,000 in 2012 increased both the total funding for universities and funding per student. This reform of HE finance is unpopular and caused student protests against the Conservative-led government in 2010. Could the government be motivated to increase student loans and tuition fees when facing political uncertainty and polarisation between the voters in theory? Formally, we examine the following question in this paper: Why would an incumbent government increase spending on HE and student loan levels when its voters are not directly benefitting from HE?

The model used in this paper is based on the model in Persson and Svensson (1989), one of the most important papers on examining the role of public debt as a strategic variable for the incumbent government and the effect of political uncertainty on the government debt policy (Alesina and Passalacqua, 2016; Mawejje and Odhiambo, 2020; Yared, 2019). Persson and Svensson (1989) examined the incentives facing a conservative government's borrowing level when the conservative government knows it will be replaced by a liberal successor next period. Alesina and Tabellini (1990) used a similar model but the different governments prefer different kinds of public goods (e.g., pork-barrel spending) which results in conflict over the composition of spending. In Persson and Svensson (1989) the different governments disagreed over the levels of government spending with the conservative government favouring a lower level of government spending. There are two groups of voters each represented by a partisan politician. The voters are identical except for their preferences towards public goods. There are two periods in the model. In the first period, the incumbent government sets up the level of tax rates and could choose to issue government debt for borrowing. In the second period, the successor government sets up a new tax rate and the level of public goods and will pay back all of the government debt it inherited. The political uncertainty meant the incumbent government was unsure of its re-election next period and polarisation refers to the different preferences for the level of public goods between the two groups of voters in this model. When the incumbent government knows it will be succeeded by a liberal government that prefers a larger level of public goods, a stubborn conservative government that puts a relatively large weight on public goods distortion will borrow more than it would had it remained in power in the future. This is because public debt acts as a strategic variable that allows the incumbent government to influence the policy of the future government. By increasing the level of public debt, the incumbent government ties the hands of the liberal successor as the successor must honour the inherited debt by using tax revenue and make it optimal for this future government to spend less on public goods. Various other papers have extended this model to explore different topics including redistribution policy, government transparency and intergenerational conflicts over debt, taxes and public goods (Aghion and Bolton, 1990; Alt and Lassen, 2006; Lizzeri, 1999; Song, Storesletten and Zilibotti, 2012).

The tuition loans set up and given by the UK government using the Student Loans Company are a public investment into HE in the form of private debt which the unrepaid parts will be written off after 30 years. Other papers have extended from Alesina and Tabellini (1990) and Persson and Svensson (1989) to study the reasons for government public underinvestment in a politically unstable and polarised society (Azzimonti, 2011; Azzimonti, 2015; Battaglini and Coate, 2007; Besley and Coate, 1998; Devereux and Wen, 1998; Peletier, Dur and Swank, 1999; Svensson, 1998). These papers have found the following reasons for the government's underinvestment: (1). The incumbent government fully internalizes the cost of public investment but does not fully benefit from it if the future government has a different preference. (2). Reducing investment reduces the future resources available for the successor government so it cannot spend as much on the public goods the incumbent does not want. (3). The incumbent would not invest if the investment changes the identity of the future policymakers in a way that is not beneficial to the incumbent. Bohn (2007) and Elder and Wagner (2015) used their model to show that when facing political uncertainty and polarisation the incumbent will choose to favour short-term payoffs instead of longer-term payoffs resulting from public investment. Besides explaining underinvestment, a smaller number of papers examined how uncertainty and polarisation could lead to an excessive level of public investment (Beetsma and Ploeg, 2007; Glazer, 1989).

Given the subject matter, the papers explaining underinvestment cannot be readily applied to explain the student loan question. The rise in the UK's tuition fees and loans led to greater investment in HE and around 2010 to 2012 the Conservative party were expected to lose the 2015 general election to the Labour party. For the papers that deal with over-investment, excessive public investment is used to limit the choices available for the future government. This explanation is similar to explaining how a conservative government could choose an excessive level of borrowing. However, increasing investment in HE funded by student loans should not only increase the recipients' future income but also increase the future loan repayment, both of which increase the revenue and therefore choices available for the future government. Consequently, this explanation cannot answer the student loan question. Since the current literature cannot offer suitable explanations, this question will be answered using our model.

We modify the model in Persson and Svensson (1989) by adding a student loan level decision for the incumbent government. We assume the HE is completely funded by student loans and a higher level of student loans means a higher level of investment into HE and results in a higher level of income for the graduates. In the UK, university students who are predominately young people have a low level of support for the Conservative party. To answer the question, in our model only one group of voters' future income will be affected by the student loan levels while the other group's future income is not dependent on the student loan level. Although the other group is not directly benefitting from HE investment, they are indirectly benefiting from them in the form of higher tax revenue from the other group in the future. We present the results both with government borrowing and without government borrowing in period 1. In addition to the theoretical analysis, we will also present the computational results with our general model that includes government borrowing substituted with specific functions.

The paper is organized in the following way: Section 6.2 presents the model and examines the different policy preferences of the voters. Section 6.3 presents the result of the government borrowing decision in Persson and Svensson (1989) and examines the incentives facing the incumbent government on its student loan decision when its probability of re-election changes. Section 6.4 presents the computational model and results. Section 6.5 concludes.

6.2 Model of Student Loans with Time-Consistent Preferences

We assume a small open economy. There are two periods in the model, $t = 1, 2$. The government can borrow and lend at a world interest rate of zero which is also the interest rate on individual savings. The individual preferences over the private economic outcomes (private consumption c_t^n, c_t^e and leisure) are additively separable and concave which is increasing in consumption and decreasing in labour supply. There are two groups in the economy. They are group n whose income does not depend on the level of student loans and group e whose income depends on the level of student loans. A higher level of student loans is interpreted as paying for higher tuition and results in higher spending on HE. The representative consumer in both groups is endowed with one unit of time in each period which they choose between labour denoted by l_t^n, l_t^e and leisure denoted by x_t^n, x_t^e.

For group n, their utility function is

(6.1) $$u^n(c_1^n, c_2^n, l_1^n, l_2^n) = F(c_1^n) + c_2^n + V_1^n(1 - l_1^n) + V_2^n(1 - l_2^n).$$

Group e is assumed to not work during period 1 and focus on education, assuming they are working in period 1 does not change the result, their utility function is

(6.2) $$u^e(c_1^e, c_2^e, l_2^e) = F(c_1^e) + c_2^e + V_1^e(1) + V_2^e(1 - l_2^e).$$

All $V(.)$ functions are concave functions. $V_t^n(.)$ and $V_t^e(.)$ are the same function with "n" and "e" denoting the group that the individual belongs.

For the private budget constraint, the group n's budget constraints for each of the two periods are

$$c_1^n + s^n = (1 - \tau_1)l_1^n,$$

$$c_2^n = (1 - \tau_2)l_2^n + s^n.$$

For group e, their private budget constraint in each period is

$$c_1^e + s^e = w,$$

$$c_2^e = P(f^e)(1-\tau_2)l_2^e + s^e - (1+r)mf^e.$$

To explain each term, τ_1 and τ_2 are the labour tax rates in periods 1 and 2. f^e is the government spending on HE in the form of student loans and $P(f^e)$ is a concave function that is increasing in f^e. $P(f^e)$ is the group e's wage and it represents the quality of HE. Group e's wage is increasing in f^e. Even if group e members have not received any HE, they should still be able to obtain the same jobs as group n members and receive the same wage which is equal to one. Formally, we assume

Assumption 6.1: $P(f^e) = 1$ when $f^e = 0$.

s^n and s^e are the private saving decisions made by consumers in group n and group e respectively. m is an exogenous parameter that signifies the proportion of student loan repayment the group e will be responsible for in the second period. $1-m$ could represent either the proportion of students who defaulted on their student loan in mortgage-style student loans or the proportion of loans not repaid due to having too low of an income to fully repay their income-contingent student loan (ICL). r is the interest rate on a student loan that is above the world interest rate of zero. The parameter w represents an exogenous endowment for group e in period 1.

Combine the two budget constraints into an intertemporal budget constraint and for group n it will be

$$c_1^n + c_2^n = (1-\tau_1)l_1^n + (1-\tau_2)l_2^n.$$

The intertemporal budget constraint for group e will be

$$c_1^e + c_2^e = P(f^e)(1-\tau_2)l_2^e + w - (1+r)mf^e.$$

The representative consumer in each group will maximize their utility function choosing consumption and labour supply subject to their respective private budget constraint, taking the government policies as given. Substitute the intertemporal budget constraint into the utility function, both group n and group e will optimally consume at period 1 to the point where the marginal utility of period 1 consumption is equal to one. The labour supply for group n members at the two periods are $L_1^n(\tau_1)$ and $L_2^n(\tau_2)$, both are decreasing in the tax rates in the same period and are not affected by other policies. The labour supply for group e members at period 2 is $L_2^e(f^e, \tau_2)$ which is decreasing in τ_2 and increasing in f^e. Substitute the labour supply functions and optimal consumption, c_t^{n*}, c_t^{e*} creates the indirect utility functions for each group's representative consumer. We assume the labour supply functions are concave.

For the government behaviour, following Persson and Tabellini (1989) we only include government consumption g in period 2. Each group is represented by a partisan party, denoted by party n and party e. The government for a period would be controlled by either party n or party e. The two groups could have different preferences for this public good and they are expressed by a concave utility function added to the private indirect utility, $\alpha^n H(g)$ and $\alpha^e H(g)$. If $\alpha^n < \alpha^e$, it means that group n has a lower preference for public goods g than group e. We define the conservative group as having a lower preference for public goods and we assume that group n is the conservative group. Formally

Assumption 6.2: $\alpha^n < \alpha^e$.

The government could decide to borrow in period 1, denoted by b, if b is a negative it means the government is saving/accumulating tax revenue. In period 2, the government is assumed to always honour the debt it inherited. The government budget constraint in each period is written as

Period 1: $$\tau_1 L_1^n(\tau_1) = -b + f^e,$$

Period 2: $$(1+r)mf^e + \tau_2 L_2^n(\tau_2) + \tau_2 L_2^e(f^e, \tau_2)P(f^e) = g + b.$$

The incumbent government set the student loan level f^e in period 1. The nation's HE is funded by government-sanctioned student loans with the loans paying for the group e's tuition fee. A larger loan is needed for a higher tuition fee. The graduates then pay off the debt themselves in the future period. Since HE and the labour market have risks, it is possible that not all the student loans would be repaid, the proportion of student loans that are not repaid in period 2, $(1-m)f^e$, would be the losses covered by the general taxpayer's revenue.

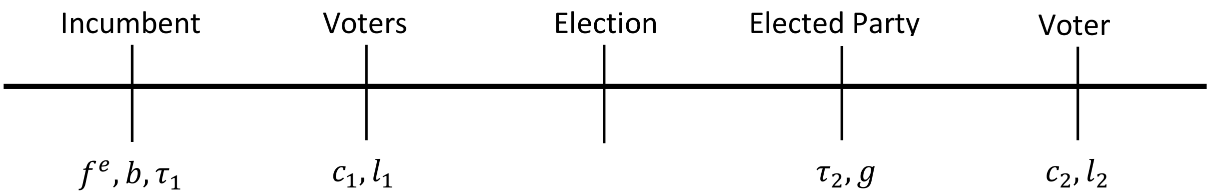
In terms of the information available to each group and their partisan party, we assume the individuals know which group they belong to and they know the policies they face when they make their private economic decisions. The incumbent government knows its probability of re-election when making its policy decisions in period 1 and both parties know how the voters change their private economic decisions given the policies. Below I will define the equilibrium of the model with the * superscript to signify a variable at the equilibrium.

An equilibrium of the model is defined by a vector of policies $[f^{e*}, b^*, \tau_1^*, \tau_2^*, g^*]$ and a vector of private economic decisions $[c_1^{n*}, c_2^{n*}, l_1^{n*}, l_2^{n*}, c_1^{e*}, c_2^{e*}, l_2^{e*}]$ such that:

1. the $[c_1^{n*}, c_2^{n*}, l_1^{n*}, l_2^{n*}, c_1^{e*}, c_2^{e*}, l_2^{e*}]$ are optimal for the consumers in the respective group in each period given the policies, $[f^{e*}, b^*, \tau_1^*, \tau_2^*, g^*]$, they face and their private budget constraints;
2. In period 2, given the period 1 policies, the elected party prefers τ_2^* and g^* to any other τ_2 and g that satisfy the period 2 government budget constraint, taking the consumers' decisions into account;
3. In period 1, given the probability of re-election, the incumbent government prefers policies f^{e*}, b^* and τ_1^* to any other f^e, b and τ_1 that satisfy the period 1 government budget constraint, taking the potential future government and the consumers' decisions into account.

The events' timing is as follows: 1. One of the parties, party n or party e holds office in period 1; the party sets various period 1 public policies f^e, b and τ_1. Then the private economic decisions in period 1 are made. 2. Election is held. 3. The elected party took office and sets public policies for period 2, the period 2 public policies are τ_2 and g; after which private economic decisions in period 2 are made. This sequence of events is illustrated in Figure 6.3. Note in section 6.3.3 we examine the results without government borrowing. The only change in the model is the removal of b.

Figure 6.3: Sequence of Events



In period 2 either party could be in office and they will make decisions based on the policies of the government made in period 1 and subject to period 2's government budget constraint. Given that the policies are made sequentially (potentially by different governments) and the period 2 policies are made after the election, it is reasonable to assume that the incumbent government cannot commit to future policies in period 1. Following Persson and Svensson (1989), for the period 1 government, the assumption of having commitment and no commitment would not have any impact on the result for optimal f^e level when the period 1 government knows it will be re-elected for sure in the next period because the preferences are time-consistent, the first-order conditions are the same. The time-consistent preferences also meant the first-order conditions for the optimal choice of τ_2 and g are the same for the period 1 government ex-ante (in period 1) and ex-post (in period 2). When the period 2 party in charge is different to the party in period 1, the assumption of commitment and no commitment again results in the same outcome since even if the period 1 government could commit to period 2 policies we assume they are not binding for a different government.

Polarisation between Group n and Group e

Polarisation in our model refers to the difference in the preferred levels of the optimal period 2 policies between group n and group e. In Appendix A6.1 and A6.2, we have derived the first-order condition that determined the optimal period 2 tax rate chosen by each party and their reaction functions. The reaction function of τ_2 as a function of period 1 HE investment level f^e and government borrowing b for party n is $T^n(f^e, b)$, and for party e is $T^e(f^e, b)$. The optimal level of public goods g is determined by the period 2 government budget constraint with the optimal period 2 tax rate. The marginal tax revenue (MTR) is positive in all optimums for both party n and party e, so the optimal level of public goods is increasing in τ_2.

The size of period 2 policies of government spending g and tax rate τ_2 between party n and party e in charge when they face the same period 1 government policies is compared in Appendix A6.3. For the level of government spending, the party that set the higher tax rate will have a higher level of spending on g. For the period 2 tax rate τ_2 and government spending g, let T^n and G^n denote the period 2 tax rate and public goods set by party n and T^e and G^e denote the period 2 tax rate and public goods set up by party e, we obtain the following relations based on the first-order conditions

- If $\frac{1}{\alpha^n} L_2^n(T) > \frac{1}{\alpha^e} P(f^e) L_2^e(f^e, T)$, then $T^n < T^e$ and $G^n < G^e$.
- If $\frac{1}{\alpha^n} L_2^n(T) < \frac{1}{\alpha^e} P(f^e) L_2^e(f^e, T)$, then $T^n > T^e$ and $G^n > G^e$.
- If $\frac{1}{\alpha^n} L_2^n(T) = \frac{1}{\alpha^e} P(f^e) L_2^e(f^e, T)$, then $T^n = T^e$ and $G^n = G^e$.

$T = T^n, T^e$.

With both HE investment and Assumption 6.2, the optimal level of policies and their comparison depends on both the preference for public goods and $P(f^e)$, which represents the quality of HE. These two factors jointly determine the degree of polarisation in our model. In the model in Persson and Svensson (1989) without HE investment group n and group e have the same preferences for private economic outcomes and only differ in their preference for public goods, the party with the higher preference for public goods will always set a higher tax rate and result in higher public goods level. With Assumption 6.2, this meant $T^n < T^e$ and $G^n < G^e$ is always the outcome in Persson and Svensson (1989).

6.3 The Government Decision on Student Loans

In section 6.3.1, we examine the conflicting incentives facing the incumbent conservative on government borrowing based on the same model as in Persson and Svensson (1989) and the modified model with student loans as an exogenous variable. In section 6.3.2, we present the conditions that determine how the period 2 government will change the period 2 tax rate when facing a higher level of student loans. In section 6.3.3, we examine the incentives facing the incumbent on the level of student loans without government borrowing. The incentives examined in sections 6.3.1 and 6.3.3 are both presented by functions derived by applying the implicit function theorem when the incumbent's re-election probability falls. In section 6.3.4, using the results in sections 6.3.1 and 6.3.3, we present the government's decision on the level of student loans with the ability to borrow.

6.3.1 Results of the Model in Persson and Svensson (1989)

Here we will present the results of Persson and Svensson (1989) in function (6.3) below. Because our model is built on Persson and Svensson (1989), our results of a government decision on student loan level without government borrowing are presented in a similar function to (6.3). In Persson and Svensson (1989), only the inventive facing government borrowing b under uncertainty and polarisation is analysed and the HE investment does not exist. Both groups' income and labour supply in period 2 are only affected by the period 2 tax rate and the two group's income and labour supply are the same when facing the same tax rate. The only difference between the two groups is that group n has a lower preference for public goods represented by Assumption 6.2. When the incumbent government n faces a lower probability of re-election, whether it will increase or decrease the level of government debt is determined by the sign of the following term:

(6.3) $$\alpha^n[H'(G^n) - H'(G^e)] + \left(\frac{\alpha^n}{\alpha^e} - 1\right) L_2(T^e)T_b^e(b).$$

In Persson and Svensson (1989), whether the incumbent will increase or decrease the level of borrowing is uncertain and the above function (6.3) represents the two incentives for changing the level of borrowing facing the government n when its re-election probability falls. As discussed in section 6.2, group n's preferred level of period 2 tax rate and public goods is always lower than group e when facing the same level of b in Persson and Svensson (1989). The first term $\alpha^n[H'(G^n) - H'(G^e)] > 0$ represents the incentive to reduce the spending on public goods by issuing more debt. As shown by the period 2 government budget constraint, an increase in b (with others being equal) will reduce the available tax revenue to spend on public goods and constrain party e in period 2. For the second term $\left(\frac{\alpha^n}{\alpha^e} - 1\right) L_2(T^e)T_b^e(b) < 0$, note that $T_b^e(b) > 0$, a rise in debt will result in an increase in the period 2 tax rate by party e. This term represents the expected future tax distortion as higher debt causes the period 2 government e to increase the tax rates. Since group n preferred a lower tax rate than group e in period 2, government n has an incentive to reduce the level of debt so party e would lower the period 2 tax rate.

Results with an Exogenous Level of HE Investment

Now the group e's period 2 labour supply depends on the level of $P(f^e)$. When $P(f^e) > 1$, the group e's labour supply is greater than group n under the same period 2 tax rate and when $P(f^e) = 1$, the labour supply are equal. The incumbent government n still cannot choose the level of HE investment. Assume here the two groups have the same preference for public goods, $\alpha^n = \alpha^e$. When the incumbent government n faces a lower probability of re-election, whether it will increase or decrease the level of borrowing is determined by the following similar function:

$$\alpha^n[H'(G^n) - H'(G^e)] + [P(f^e)L_2^e(f^e, T^e) - L_2^n(T^e)]T_b^e(b).$$

The two incentives facing the incumbent government n on changing the public goods and the period 2 tax rate of party e are represented by the two terms respectively but the key determinant of the different preferences on the period 2 tax rates and the level of public goods between group n and group e is represented by the quality of HE, $P(f^e)$. When $P(f^e) > 1$, the signs of the two terms are reversed compared to their counterpart in the original model in Persson and Svensson (1989). When $P(f^e) = 1$, the incumbent has no incentive to change the level of borrowing since the party e's preferred policy is the same as party n (meaning no polarisation).

If $\alpha^n < \alpha^e$, then whether the incumbent n will increase or decrease the level of borrowing is determined by the following function:

$$(6.4) \qquad \alpha^n[H'(G^n) - H'(G^e)] + \left[\frac{\alpha^n}{\alpha^e}P(f^e)L_2^e(f^e, T^e) - L_2^n(T^e)\right]T_b^e(b).$$

The two terms of (6.4) still represent the same two incentives facing the incumbent government n. Note that if $T^e = T^n$, (6.4) is equal to zero and the incumbent will not change the level of b when its probability of re-election falls. In section 6.3.3 with f^e but no government borrowing decisions, we will compare the similarities and differences of the incentives facing the incumbent on f^e by comparing the function to (6.4). In section 6.3.4 the incumbent chooses both f^e and b, condition (6.4) still exists and is part of the indirect effect of the change in probability of re-election on the incumbent decision on f^e.

6.3.2 The Decision on Period 2 Tax Rates by Party e and Party n

Now the incumbent government is choosing the level of HE investment f^e in period 1, we need to know whether the period 2 party would increase or decrease the tax rate when facing a higher level of f^e. For the condition that determined the sign of the party e's reaction functions of τ_2 to changes in f^e:

(6.5) $T_f^e(f^e) > 0$ if $-\left[P_f(f^e)L_2^e(f^e,T^e) + P(f^e)L_{2f}^e(f^e,T^e)\right] +$

$\frac{H''(G^e)}{H'(G^e)}P(f^e)L_2^e(f^e,T^e)\left[(1+r)m + T^e\left[P_f(f^e)L_2^e(f^e,T^e) + P(f^e)L_{2f}^e(f^e,T^e)\right]\right] +$

$\alpha^e H'(G^e)\left[P_f(f^e)L_2^e(f^e,T^e) + P(f^e)L_{2f}^e(f^e,T^e) + T^e\left[P_f(f^e)L_{2\tau_2}^e(f^e,T^e) +\right.\right.$

$\left.\left.P(f^e)L_{2\tau_2 f}^e(f^e,T^e)\right]\right] > 0,$

$T_f^e(f^e) < 0$ if $-\left[P_f(f^e)L_2^e(f^e,T^e) + P(f^e)L_{2f}^e(f^e,T^e)\right] +$

$\frac{H''(G^e)}{H'(G^e)}P(f^e)L_2^e(f^e,T^e)\left[(1+r)m + T^e\left[P_f(f^e)L_2^e(f^e,T^e) + P(f^e)L_{2f}^e(f^e,T^e)\right]\right] +$

$\alpha^e H'(G^e)\left[P_f(f^e)L_2^e(f^e,T^e) + P(f^e)L_{2f}^e(f^e,T^e) + T^e\left[P_f(f^e)L_{2\tau_2}^e(f^e,T^e) +\right.\right.$

$\left.\left.P(f^e)L_{2\tau_2 f}^e(f^e,T^e)\right]\right] < 0,$

with $G^e = (1+r)mf^e - b + T^eL_2^n(T^e) + T^eL_2^e(f^e,T^e)P(f^e)$.

If T_f^e is negative, a higher f^e will lead to a reduction in tax rate by party e in period 2 and if T_f^e is positive, a higher f^e will lead to an increase in tax rate by party e in period 2.

For the condition that determined the sign of party n's reaction functions of τ_2 to changes in f^e:

(6.6) $T_f^n(f^e) > 0$ if $\frac{H''(G^n)}{H'(G^n)} L_2^n(T^n) \Big[(1+r)m + T^n\big[P_f(f^e)L_2^e(f^e, T^n) + P(f^e)L_{2f}^e(f^e, T^n)\big]\Big] + \alpha^n H'(G^n) \Big[P_f(f^e)L_2^e(f^e, T^n) + P(f^e)L_{2f}^e(f^e, T^n) + T^n\big[P_f(f^e)L_{2\tau_2}^e(f^e, T^n) + P(f^e)L_{2\tau_2 f}^e(f^e, T^n)\big]\Big] > 0$,

$T_f^n(f^e) < 0$ if $\frac{H''(G^n)}{H'(G^n)} L_2^n(T^n) \Big[(1+r)m + T^n\big[P_f(f^e)L_2^e(f^e, T^n) + P(f^e)L_{2f}^e(f^e, T^n)\big]\Big] + \alpha^n H'(G^n) \Big[P_f(f^e)L_2^e(f^e, T^n) + P(f^e)L_{2f}^e(f^e, T^n) + T^n\big[P_f(f^e)L_{2\tau_2}^e(f^e, T^n) + P(f^e)L_{2\tau_2 f}^e(f^e, T^n)\big]\Big] < 0$,

with $G^n = (1+r)mf^e - b + T^n L_2^n(T^n) + T^n L_2^e(f^e, T^n)P(f^e)$.

If T_f^n is negative, a higher f^e will lead to a reduction in tax rate by party n in period 2 and if T_f^n is positive, a higher f^e will lead to an increase in tax rate by party n in period 2.

The condition that determines the sign of the period 2 tax rate's reaction function of party e and party n is the change in the net marginal benefit of the tax rate of party e and party n respectively when f^e rises. If an increase in the level of HE investment increases the net marginal benefit of tax, then the period 2 government should increase the tax rate to maximize utility and if it decreases the net marginal benefit of tax, then the period 2 government should decrease the tax rate. The conditions in (6.5) and (6.6) are derived in Appendix A6.4.

6.3.3 The Period 1 Incumbent's Incentive on the Level of f^e without Government Borrowing

To focus on the incentives facing the incumbent's decision on the student loans level f^e, first, we consider the case in which the incumbent cannot issue debt b in period 1. The model is the same as the one in section 6.2 except there is no government debt b. Assume party n is in charge in period 1. The optimal period 1 tax rate set by incumbent n will always be an interior solution, meaning it is below the rate that maximizes the period 1 tax revenue. Let p^n denote the probability that party n will be re-elected next period and $p^e = 1 - p^n$ be the probability that party e will be elected next period.

$$(6.7)\ \ \alpha^n(1+r)m[H'(G^e) - H'(G^n)]$$
$$+ \alpha^n \Big[H'(G^e)T^e\big[P_f(f_n^e)L_2^e(f_n^e, T^e) + P(f_n^e)L_{2f}^e(f_n^e, T^e)\big]$$
$$- H'(G^n)T^n\big[P_f(f_n^e)L_2^e(f_n^e, T^n) + P(f_n^e)L_{2f}^e(f_n^e, T^n)\big]\Big]$$
$$+ T_f^e(f_n^e)\left[\frac{\alpha^n}{\alpha^e}P(f_n^e)L_2^e(f_n^e, T^e) - L_2^n(T^e)\right].$$

Proposition 6.1: Whether party n would increase or decrease its optimal choice of f^e, f_n^e, when it realizes its re-election chances are decreased depends on the sign of (6.7):

- If $(6.7) > 0$, then party n increases f_n^e.
- If $(6.7) < 0$, then party n decreases f_n^e.
- If $(6.7) = 0$, then party n does not change f_n^e.

When $V'(1-l) = \phi l$ with ϕ being a positive constant, as $H''(g) \longrightarrow 0$, incumbent n will increase f^e when its probability of re-election falls if $T^e > T^n$ and decrease f^e when its probability of re-election falls if $T^e < T^n$.

$V'(1-l)$ is the marginal utility of leisure. Condition (6.7) and the incumbent n's decision under specific leisure functions that result in $V'(1-l) = \phi l$ are derived in Appendix A6.5. We will use (6.7) and (6.4) to compare the incentives of changing the level of f^e and changing the level of b. Moreover, the sign of condition (6.7) allows us to compare the optimal level of f^e when the re-election is certain to not happen and when the re-election is certain for the incumbent. If (6.7) is positive, then incumbent n will set f^e at a higher level when re-election does not happen compared to when it is certain to happen. If (6.7) is negative, then incumbent n will set f^e at a lower level when re-election does not happen compared to when it is certain to happen.

Note that if $T^e = T^n$, then $(6.7) = 0$ and party n will not change its optimal choice of f^e when its re-election probability falls. This is because if the incumbent and its potential successor preferred the same level of period 2 policies, the partisan incumbent is indifferent between being re-elected and not re-elected. Changing its re-election prospect will not change its optimal policy. If there is no polarisation, increased political uncertainty will not affect the incumbent's policies.

The second term $\alpha^n[H'(G^e)T^e[P_f(f_n^e)L_2^e(f_n^e,T^e) + P(f_n^e)L_{2f}^e(f_n^e,T^e)] - H'(G^n)T^n[P_f(f_n^e)L_2^e(f_n^e,T^n) + P(f_n^e)L_{2f}^e(f_n^e,T^n)]]$ is new compared to (6.4) and can be treated as the "pure investment" incentive facing incumbent n under increased political uncertainty. This term exists in this case with HE investment because of an increase in f^e results in an increase in the labour supply from group e and their labour productivity which increases the tax revenue available in period 2 and the size of this increase is different under different tax rates. This term is the difference between group e and group n of the product of the marginal utility of public goods and the marginal revenue of investment (MRI). If $T^n < T^e$ and the MRI is increasing in period 2 tax rate, the incumbent n has the incentive to increase f^e when it is less likely to be re-elected as it expects a higher tax rate in period 2. Here, this higher tax rate increases the return on investment for the incumbent n. However, the MRI is weighted by the marginal utility of public goods as g is what the tax revenue is spent on. When $G^n < G^e$, the marginal utility of public goods is lower when party e is in charge and this creates an incentive to decrease f^e. Overall, the sign of this second term is unknown.

The remaining terms have their counterpart in condition (6.4) with the modified model based on Persson and Svensson (1989) with only government borrowing and are focused on the differences in the preferred level of period 2 policies between the two groups. In the first term $\alpha^n(1+r)m[H'(G^e) - H'(G^n)]$, the position of G^e and G^n is switched compared to (6.4) and represents the opposite incentives for the decision on f^e compared to the decision on b. This term comes from the student loan repayment from group e in period 2, the higher the student loan level in period 1 the higher the level of loan repayment available in period 2 to be spent on public goods. Assuming we are in the case with $G^n < G^e$, the conservative group n prefers a lower public goods level than group e. The term $\alpha^n(1+r)m[H'(G^e) - H'(G^n)]$ is negative, the incumbent n has the incentive to reduce the level of f^e when its probability of re-election falls to lower the level of loan repayment in period 2 so that party e has less revenue available and will reduce spending on g, ceteris paribus.

The final term in (6.7), $T_f^e(f_n^e)[\frac{\alpha^n}{\alpha^e}P(f_n^e)L_2^e(f_n^e,T^e) - L_2^n(T^e)]$, is the expected future tax distortion and can find a similar function/incentive represented in (6.4). The bracket $[\frac{\alpha^n}{\alpha^e}P(f_n^e)L_2^e(f_n^e,T^e) - L_2^n(T^e)]$ is the difference in the marginal cost of taxation weighted by the inverse of public goods preferences at the tax rate set by party e in period 2. The group with the higher weighted marginal cost of taxation prefers a lower tax rate, e.g. if $\frac{\alpha^n}{\alpha^e}P(f_n^e)L_2^e(f_n^e,T^e) - L_2^n(T^e) < 0$, then $T^n < T^e$. The term outside the bracket, $T_f^e(f_n^e)$, represents how would party e change the tax rate when facing a higher level of f^e. The incumbent has an incentive to change f^e so its potential successor's period 2 tax rate will be set closer to its preferred level.

When party e is in charge in period 1, the incentives facing party e on whether it will increase or decrease the level of student loans under increased political uncertainty are evaluated by a similar function to (6.7). Because group e does not supply labour in period 1, a rise in period 1 tax rate does not reduce the group e's utility. Incumbent e will set the period 1 tax rate to extract tax revenue from group n to match the optimal level of f^e. The period 1 budget constraint for incumbent e could be thought of as an inequality constraint with an upper limit for f^e. We assume this constraint does not bind, meaning the optimal level of f^e is below the maximum period 1 tax revenue.

$$(6.8)\ P_f(f_e^e)[(1-T^n)L_2^e(f_e^e,T^n)-(1-T^e)L_2^e(f_e^e,T^e)]$$
$$+\alpha^e(1+r)m[H'(G^n)-H'(G^e)]$$
$$+\alpha^e\Big[H'(G^n)T^n\big[P_f(f_e^e)L_2^e(f_e^e,T^n)+P(f_e^e)L_{2f}^e(f_e^e,T^n)\big]$$
$$-H'(G^e)T^e\big[P_f(f_e^e)L_2^e(f_e^e,T^e)+P(f_e^e)L_{2f}^e(f_e^e,T^e)\big]\Big]$$
$$+T_f^n(f_e^e)\left[\frac{\alpha^e}{\alpha^n}L_2^n(T^n)-P(f_e^e)L_2^e(f_e^e,T^n)\right].$$

Proposition 6.2: Whether party e would increase or decrease its optimal level of f^e, f_e^e, when it realizes it is less likely to be re-elected depends on the sign of (6.8):

- If $(6.8) > 0$, then party e increases f_e^e,
- If $(6.8) < 0$, then party e decreases f_e^e,
- If $(6.8) = 0$, then party e does not change f_e^e.

When $V'(1-l) = \phi l$ with ϕ being a positive constant, as $H''(g) \longrightarrow 0$, incumbent e will increase f^e when its probability of re-election falls if $T^e > T^n$ and decrease f^e when its probability of re-election falls if $T^e < T^n$.

Condition (6.8) and the incumbent e's decision under specific leisure functions are derived in Appendix A6.6. Similar to condition (6.7), if (6.8) is positive, incumbent e will set f^e at a higher level when its re-election would not happen compared to when its re-election is certain. If (6.8) is negative, incumbent e will set f^e at a lower level when re-election would not occur compared to when it is certain.

For the same reason, incumbent e will also not change the level of f_e^e when its probability of re-election falls if $T^e = T^n$.

The last 3 terms of condition (6.8) $\alpha^e(1+r)m[H'(G^n)-H'(G^e)]$, $\alpha^e[H'(G^n)T^n[P_f(f_e^e)L_2^e(f_e^e,T^n)+P(f_e^e)L_{2f}^e(f_e^e,T^n)]-H'(G^e)T^e[P_f(f_e^e)L_2^e(f_e^e,T^e)+P(f_e^e)L_{2f}^e(f_e^e,T^e)]]$ and $T_f^n(f_e^e)[\frac{\alpha^e}{\alpha^n}L_2^n(T^n)-P(f_e^e)L_2^e(f_e^e,T^n)]$ all have its symmetric counterpart in condition (6.7) but with $T_f^n(f_e^e)$ since now it is incumbent e trying to influence the period 2 tax rate set by party n as the incumbent e expect a higher probability of party n being elected in period 2.

The additional term $P_f(f_e^e)[(1-T^n)L_2^e(f_e^e,T^n)-(1-T^e)L_2^e(f_e^e,T^e)]$ exists because group e's wage directly benefits from investment in HE. $P_f(f_e^e)$ is the increase in group e's wage because of higher f^e. This is not affected by the identity of the period 2 government. The bracket $[(1-T^n)L_2^e(f_e^e,T^n)-(1-T^e)L_2^e(f_e^e,T^e)]$ is the difference in the net income of group e (divided by $P(f^e)$) between when party n is in charge in period 2 and when party e is in charge in period 2. Combined, this new term is the difference in the benefit of a higher f^e through the increase in wage on the group e's net income between the party n's period 2 tax rate and the party e's period 2 tax rate and could be interpreted as the private economic incentive facing incumbent e when its re-election chances fall. It represents the difference in the benefit of improving the quality of HE on the group e's net income between the two period 2 governments. This term is positive if $T^n < T^e$ and is negative if $T^n > T^e$. Therefore, if group n prefers a lower period 2 tax rate than group e, this term represents an incentive for incumbent e to increase the level of f^e because the private benefit of HE investment is greater the lower the period 2 tax rate is expected.

Using the insights from our specific example, we conclude that when facing the same period 1 policies, if the two parties' response to a change in the level of f^e is always consistent to itself and opposite to each other, then we can always make a certain prediction on the incumbent's decision on f^e when its probability of re-election changes by comparing the preferred levels of period 2 policies.

As conditions (6.7) and (6.8) have shown, the direction that the incumbent government in period 1 chooses to respond to a change in its re-election probability by changing the optimal issuing of f^e depends on several factors:

1. The function $P(.)$ which represents the quality of HE.
2. The difference between public goods preferences, α^n and α^e, between the two groups of voters.
3. The direction and magnitude of the period 2 party changing their period 2 tax rate in response to the increase in student loan debt f^e, which depends on conditions in (6.5) and (6.6).

The first and second factor determines the difference in the preferred levels of period 2 tax rate and public goods between group n and group e. The third factor determines everything else in (6.7) and (6.8) given the preferred tax rates due to the equivalence of cross derivatives. Using conditions (6.7) and (6.8), we investigate how the incentives facing the incumbent on the student loan level decision under political uncertainty relate to and also differ from the decision on government borrowing in Persson and Svensson (1989), these different kinds of incentives are summarised in Table 6.1.

Table 5.1: Summary of the Incentives Facing the Incumbent Presented in Sections 6.3.1 and 6.3.3

	b	f^e with Incumbent n	f^e with Incumbent e
Public Goods Distortion	Exist	Exist	Exist
Period 2 Tax Rate Distortion	Exist	Exist	Exist
Investment Incentive for Public Goods	Does not exist	Exist	Exist
Private Economic Incentive	Does not exist	Does not exist	Exist

Same as Persson and Svensson (1989), it is uncertain whether the incumbent will increase or decrease the student loan level in general even though we provide a specific example in which the incumbent decision is certain given the difference in preferred period 2 tax rates. Even if the incumbent conservative government's voters' income does not increase from higher investment in HE, it is still possible for the conservative government to increase the student loan level when it expects to be replaced by a different government in the future.

6.3.4 The Period 1 Incumbent's Incentive on the Level of f^e with Government Borrowing

In sections 6.3.1 and 6.3.3, we have separately considered the case with only government borrowing b and only student loans f^e and analysed the incentives facing the incumbent when its probability of re-election changed. In this section, we will consider the incumbent decision on f^e when the incumbent is deciding both the level of f^e and b in period 1. The incentives examined in sections 6.3.1 and 6.3.3 still apply. The period 2 tax rate T^n and T^e is now a function of both f^e and b set by the incumbent.

In Appendix A6.7, we derive the condition that decides whether the incumbent would increase or decrease the level of student loans using the implicit function theorem for a system of equations. Conditions (6.7) and (6.8) are still important determinants of the incumbent's decision on f^e but we also need to take the government decision on debt b into account since the government is also incentivised to change the level of b when its re-election probability falls.

When the incumbent government is party n the incumbent is maximizing the expected utility of group n (EU^n) with respect to f^e and b. The optimal level of f^e and b are determined by two first-order conditions:

$$(FOC6.1)\quad \frac{\partial EU^n}{\partial f^e}$$

$$= -L_1^n\big(T_1^n(f^e - b)\big)T_{1f}^n(f^e - b)$$

$$+ p^n\left[\alpha^n H'(G^n)\left[(1+r)m + T^n\left[P_f(f^e)L_2^e(f^e, T^n) + P(f^e)L_{2f}^e(f^e, T^n)\right]\right]\right]$$

$$+ (1 - p^n)\left[-T_f^e(f^e, b)L_2^n(T^e) + T_f^e(f^e, b)\frac{\alpha^n}{\alpha^e}P(f^e)L_2^e(f^e, T^e)\right.$$

$$\left. + \alpha^n H'(G^e)\left[(1+r)m + T^e\left[P_f(f^e)L_2^e(f^e, T^e) + P(f^e)L_{2f}^e(f^e, T^e)\right]\right]\right] = 0.$$

$$(FOC6.2)\quad \frac{\partial EU^n}{\partial b}$$

$$= -L_1^n\big(T_1^n(f^e - b)\big)T_{1b}^n(f^e - b) + p^n[-\alpha^n H'(G^n)]$$

$$+ (1 - p^n)\left[T_b^e(f^e, b)\left[\frac{\alpha^n}{\alpha^e}P(f^e)L_2^e(f^e, T^e) - L_2^n(T^e)\right] - \alpha^n H'(G^e)\right] = 0.$$

As shown in Appendix A6.7, whether incumbent n will increase or decrease the level of f^e when its probability of re-election falls is the same as the sign of the following term (with $\frac{\partial FOC6.2}{\partial b}$ denoted by $(-)$):

(6.9) $$-\left[\frac{\partial FOC6.1}{\partial p^e}(-) - \frac{\partial FOC6.2}{\partial p^e}\frac{\partial FOC6.1}{\partial b}\right].$$

The first part of the condition (6.9), $\frac{\partial FOC6.1}{\partial p^e}$, is the condition (6.7) with the period 2 tax rate being dependent on both f^e and b. The economic interpretations of the various terms in $\frac{\partial FOC6.1}{\partial p^e}$ are the same as in the case without government borrowing. This first part of (6.9) represents the direct effect of an increase in p^e. For the second part of the condition (6.9), the term $\frac{\partial FOC6.1}{\partial b}$ is the change in the net marginal benefit of HE investment f^e after an increase in the government borrowing b. If $\frac{\partial FOC6.1}{\partial b}$ is positive, it means the government should increase the level of HE investment if the government decides to increase the level of government borrowing, ceteris paribus. The term $\frac{\partial FOC6.2}{\partial p^e}$ is the same as condition (6.4) on the incumbent n's decision on the level of b with exogenous HE investment and the terms have the same economic interpretation. Multiply the two terms, and if both terms are positive, the economic interpretation of the product is that under lower re-election chances, the incumbent n is incentivised to increase the level of borrowing and this creates an incentive for the incumbent to increase the level of f^e because the net marginal benefit of HE investment increases under a higher level of b. The second part of (6.9) is the indirect influence of p^e on the government decision of f^e through affecting the borrowing decision. In this example $\frac{\partial FOC6.2}{\partial p^e}$ and $\frac{\partial FOC6.1}{\partial b}$ having the same sign, the incumbent is incentivised to increase f^e from the indirect effect of p^e through b.

If the incumbent government is party e and the period 1 tax rate is an interior solution, the sign of $\frac{\partial f^e}{\partial p^n}$ is determined by a similar condition to (6.9) with the same interpretation but for party e. In Appendix A6.7, we derived this condition using the same method.

$$-\left[\frac{\partial FOC6.3}{\partial p^n}(-) - \frac{\partial FOC6.4}{\partial p^n}\frac{\partial FOC6.3}{\partial b}\right],$$

with $\frac{\partial FOC6.4}{\partial b}$ denoted by $(-)$ and

$$(FOC6.3)\ \frac{\partial EU^e}{\partial f^e}$$

$$= -(1+r)m$$

$$+ p^n \Bigg[P_f(f^e)(1-T^n)L_2^e(f^e,T^n) - P(f^e)T_f^n(f^e,b)L_2^e(f^e,T^n)$$

$$+ \frac{\alpha^e}{\alpha^n}T_f^n(f^e,b)L_2^n(T^n)$$

$$+ \alpha^e H'(G^n)\left[(1+r)m + T^n\left[P_f(f^e)L_2^e(f^e,T^n) + P(f^e)L_{2f}^e(f^e,T^n)\right]\right]\Bigg]$$

$$+ (1-p^n)\Bigg[P_f(f^e)(1-T^e)L_2^e(f^e,T^e)$$

$$+ \alpha^e H'(G^e)\left[(1+r)m + T^e\left[P_f(f^e)L_2^e(f^e,T^e) + P(f^e)L_{2f}^e(f^e,T^e)\right]\right]\Bigg] = 0.$$

$$(FOC6.4)\ \frac{\partial EU^e}{\partial b}$$

$$= p^n\left[T_b^n(f^e,b)\left[\frac{\alpha^e}{\alpha^n}L_2^n(T^n) - P(f^e)L_2^e(f^e,T^n)\right] - \alpha^e H'(G^n)\right]$$

$$+ (1-p^n)[-\alpha^e H'(G^e)] = 0.$$

When the period 1 tax rate is at the corner solution, the incumbent e is effectively only choosing the optimal level of f^e with b written as a function of f^e based on the period 1 budget constraint with $\frac{\partial b}{\partial f^e} = 1$. An additional unit of HE investment f^e will be funded by an additional unit of debt. Whether incumbent e will increase or decrease the level of f^e when its probability of re-election falls depends on the sign of the following term

$$(6.10)\ \ P_f(f^e)[(1-T^n)L_2^e(f^e,T^n) - (1-T^e)L_2^e(f^e,T^e)] + \alpha^e(1+r)m[H'(G^n) - H'(G^e)] + \alpha^e\Big[H'(G^n)T^n\big[P_f(f^e)L_2^e(f^e,T^n) + P(f^e)L_{2f}^e(f^e,T^n)\big] - H'(G^e)T^e\big[P_f(f^e)L_2^e(f^e,T^e) + P(f^e)L_{2f}^e(f^e,T^e)\big]\Big] + \big[T_f^n(f^e,b) + T_b^n(f^e,b)\big]\Big[\frac{\alpha^e}{\alpha^n}L_2^n(T^n) - P(f^e)L_2^e(f^e,T^n)\Big] + \alpha^e[H'(G^e) - H'(G^n)].$$

The condition (6.10) is similar to condition (6.8). If (6.10) is positive, incumbent e will increase the optimal level of f^e and b when its probability of re-election falls. If (6.10) is negative, incumbent e will instead lower the optimal level of f^e and b when the probability of re-election falls. The condition (6.10) is $\frac{\partial FOC6.3}{\partial p^n} + \frac{\partial FOC6.4}{\partial p^n}$ because $\frac{\partial b}{\partial f^e} = 1$. When the incumbent e's probability of re-election falls, its decision to change the optimal level of f^e is jointly determined by the terms representing the incentives of only changing f^e plus the terms representing the incentives of only changing b since when incumbent e changes f^e, he will also change b by the same amount.

Note that in all the cases we examined above, if $T^e = T^n$, the incumbent will not change the level of f^e and b when its probability of re-election changes.

Similar to Propositions 6.1 and 6.2, under specific leisure functions and with $H(.)$ being as linear as possible, we will obtain certain predictions for the incumbent decision on f^e.

As $H''(g) \longrightarrow 0$, $(\frac{\partial FOC6.2}{\partial p^e}, \frac{\partial FOC6.4}{\partial p^n}) \longrightarrow 0$. The incumbent n's decision on f^e is determined by the sign of $\frac{\partial FOC6.1}{\partial p^e}$ and Proposition 6.1 applies to this case with both f^e and b. The incumbent e's decision on f^e is determined by the sign of $\frac{\partial FOC6.3}{\partial p^n}$ both when the period 1 tax rate is an interior solution and when it is a corner solution, so Proposition 6.2 applies here.

Proposition 6.3: When the incumbent controls both f^e and b and the voters' leisure function results in $V'(1-l) = \phi l$ with ϕ being a positive constant, as $H''(g) \longrightarrow 0$, both incumbent n and e will increase f^e when its probability of re-election falls if $T^e > T^n$ and decrease f^e when its probability of re-election falls if $T^e < T^n$.

Under these assumptions, as voters' utility from public goods approaches a linear function, the incumbent's decision on f^e under increasing political uncertainty is determined by the preferred levels of period 2 policies.

6.4 Computational Model and Results

In this section, we will present the computational results of the model generated using MATLAB. We will first introduce the specific functions used in our computation. Then we will present the values of the parameters. Finally, we will present and discuss the quantitative results.

6.4.1 The Computational Model

Table 6.2 matches the specific functions used to the general functions presented in Section 6.2.

Table 6.2: Specific Functions Used for Computation

General Function	Specific Function
$F(c_1)$	$\ln(c_1)$
$V(1-l)$	$1-\varphi l^2$
$P(f^e)$	$1+(f^e)^\gamma$
$H(g)$	$\ln(g)$

For computing our results, we removed the assumption that there is one unit of time in each period, and we added an assumption that the incumbent is making sure all period 2 consumptions are not negative when maximizing its group's utility. For our theoretical results based on the general model, we assumed that all period 2 consumptions are positive even without the constraints which is a reasonable assumption that is also implicitly assumed in Persson and Svensson (1989). For our specific functions which created a stylized model, we take the possibility that some of the period 2 consumption could be negative without taking these constraints into account. The code for incumbent n's optimization problem is in Appendix 6.8 and the code for incumbent e's optimization problem is in Appendix 6.9.

6.4.2 Parameters

Table 6.3 presents the values of our parameters.

Table 6.3: Parameters' Values

Parameters	Values
γ	0.13
α^n	0.677
α^e	1.323
r	0
m	1
w	1
φ	0.6

The value of γ is estimated using two relations: 1. $\frac{dP(f^e)}{df^e}\frac{f^e}{P(f^e)} = elasticity\ of\ wage\ with\ respect\ to\ HE\ spending.$ 2. $\frac{P(f^e)}{1} = graduate\ wage\ premium.$ For the wage elasticity, we used Tremblay (1986) which estimated the elasticity for the US southern and non-southern men aged 24-34 in 1976. We weighted the elasticity based on the number of southern and non-southern men used in Tremblay (1986) and obtained 0.0399. Our wage premium is 1.444 which is the wage premium for men of all experience from 1975 to 1980 estimated by Murphy and Welch (1989). Based on these data, we obtained $\gamma = 0.13$.

Using data from the International Monetary Fund (IMF) website for the US government expenditure as a percentage of GDP from 1973 to 2020, we calculated that when the president is a Republican this percentage is 35.67% and when the president is a Democrat this percentage is 36.37%. For parameter α^n, its value is chosen so that incumbent n's government expenditure over GDP ratio is close to 35.76% when $p^n = 1$. For parameter α^e, its value is chosen so that incumbent e's government expenditure over GDP ratio is close to 36.37% when $p^n = 0$.

We assume $r = 0$, meaning the student loans' interest rate is equal to the world interest rate and $m = 0$, meaning all the student loans will be repaid. The assumption $w = 1$ means group e does not need to borrow anything to cover their consumption in period 1. We also assume $\varphi = 0.6$.

Given our specific functions and parameters' values, unless the optimal f^e is around 0.000835, there will be polarisation in the model. If f^e is greater than 0.000835, group n will prefer a higher period 2 tax rate and government spending than group e.

6.4.3 Results and Discussion

Figure 6.4 shows the values of the incumbent n's optimal period 1 policies and the expected utility of group n at every level of p^n. Figure 6.5 shows the values of the incumbent e's optimal period 1 policies and the expected utility of group e at every level of p^n. Both incumbents' groups' expected utility is higher when the incumbent's re-election is more certain.

Figure 6.4: Incumbent n's Optimal Period 1 Policies and Group n's Expected Utility

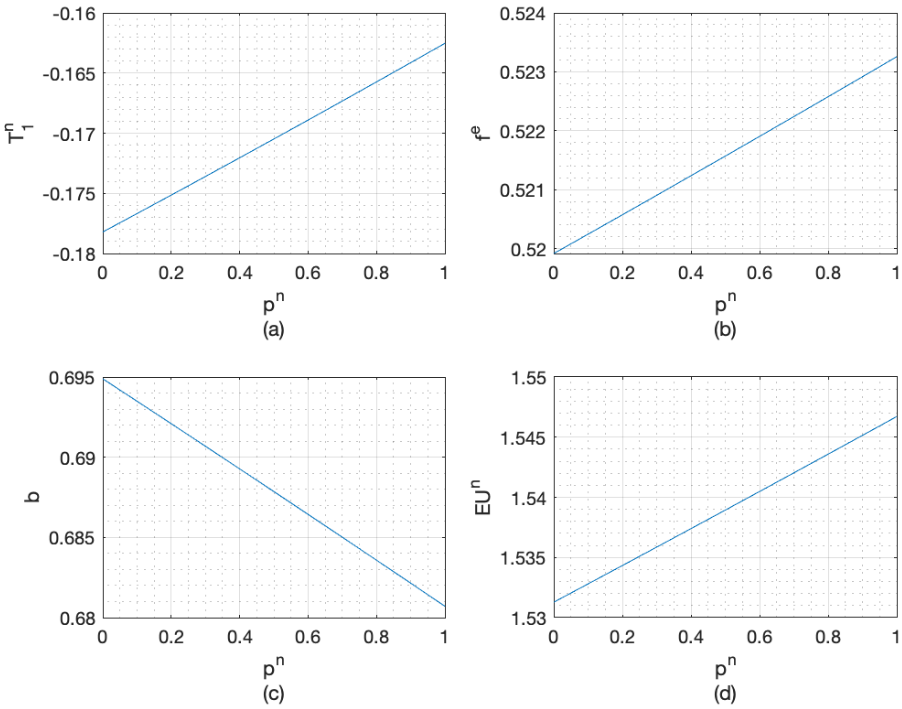
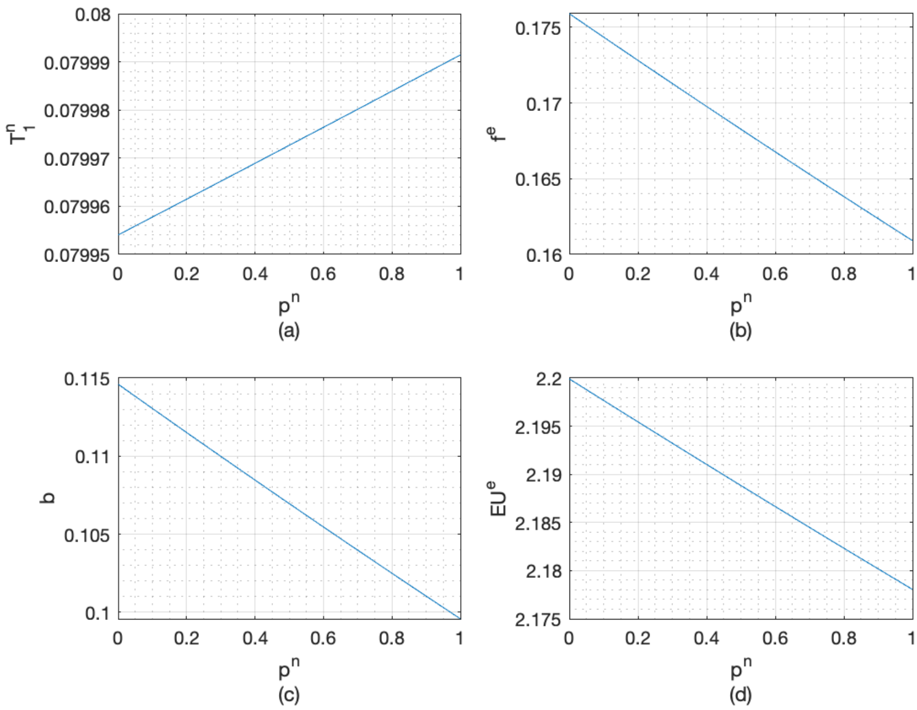


Figure 6.5: Incumbent e's Optimal Period 1 Policies and Group e's Expected Utility



Our computational results support the argument that changes in the incumbent's re-election probability will affect public investment (f^e) and public debt (b). This is in line with our theoretical results that the incumbent government is incentivised to change the level of f^e when its probability of re-election changes. In both Figure 6.4 and Figure 6.5, when the incumbent's re-election becomes more uncertain, the incumbent will reduce the level of investment into HE, this result agrees with the papers that found political uncertainty and polarisation lower the level of public investment. However, this observation needs to be interpreted with caution as our computational model is based on specific functions which limits the generalizability of this finding. Table 6.4 presents the percentage change in f^e when the incumbent switches from a certain re-election to its re-election being certain to not happen. In terms of percentage, incumbent e decreases f^e by 8.525% which is larger than incumbent n which only decreases f^e by 0.639%.

Table 6.4: Percentage Change in f^e

Incumbent	f^e when $p^n = 0$	f^e when $p^n = 1$	Percentage Change
n	0.5199	0.5223	-0.639
e	0.1759	0.1609	-8.525

Figure 6.6 and Figure 6.7 show the incumbent's group's actual utility when the incumbent is re-elected and when the incumbent is not re-elected. The incumbent sets the period 1 policies taking into account the effects on period 2 policies so that the actual utility when re-elected increases when re-election becomes more certain and the actual utility when not re-elected

increases when re-election becomes more uncertain as p^n is the weight the incumbent attaches to the utilities in the two possible scenarios.

Figure 6.6: Group n's Actual Utility at the Two Election Possibilities with Incumbent n

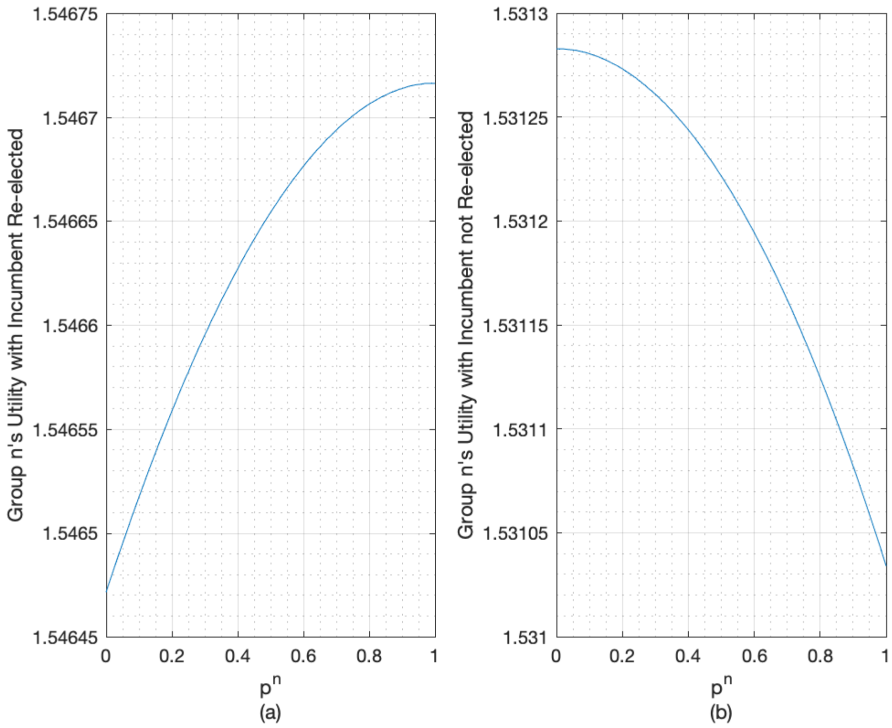
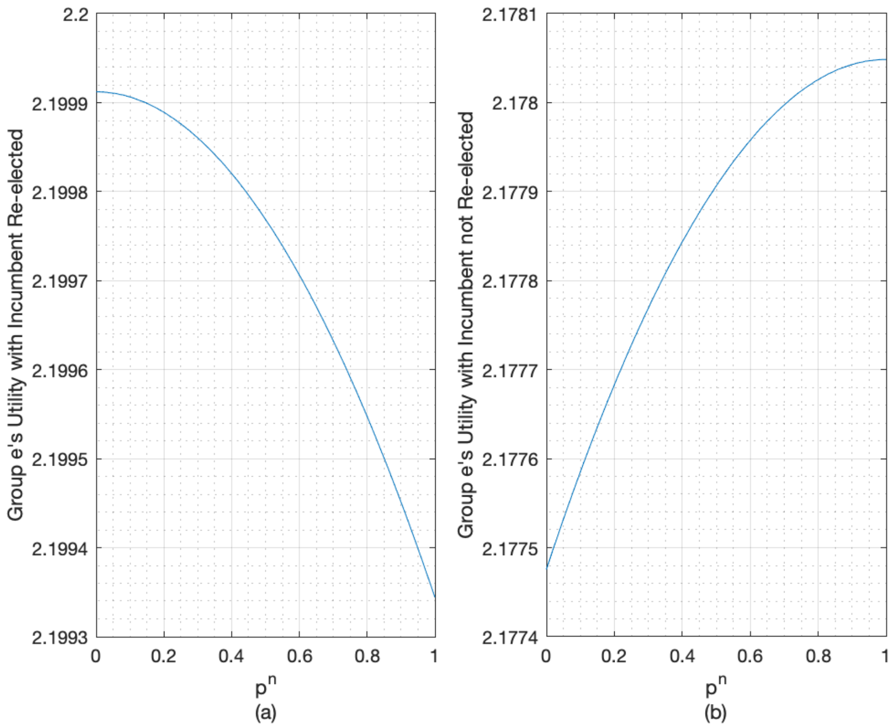


Figure 6.7: Group e's Actual Utility at the Two Election Possibilities with Incumbent e



The two incumbents set different period 1 policies at every re-election probability. Incumbent n subsidize the period 1 labour supply with a negative tax rate with the period 1 government budget entirely funded by debt. It makes sense for incumbent n to offer a lower period 1 tax rate than incumbent e because only group n supplies labour and earns income in period 1 and would benefit from such a policy. Perhaps surprisingly, incumbent n invests more into HE than incumbent e despite group n's wage is not dependent on f^e. This is likely a result of the fact that this HE investment is in the form of a loan that group e needs to pay back which reduces group e's period 2 consumption. Because incumbent n does not maximize group e's expected utility, it does not take this into account, unlike incumbent e.

Figure 6.8: Period 2 Consumptions under Incumbent e

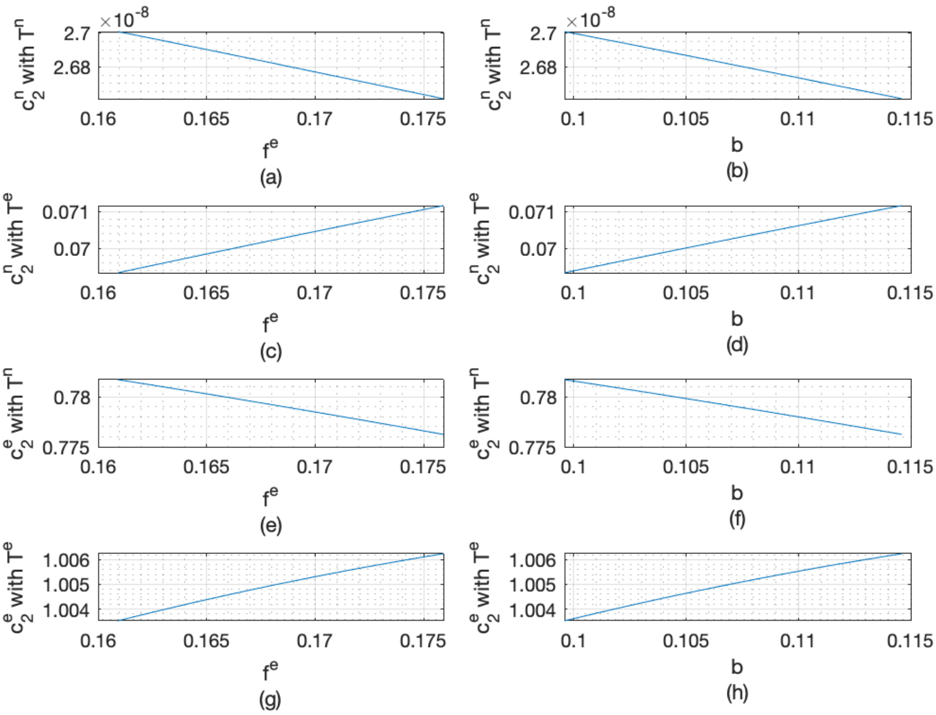


Figure 6.9: Incumbent e's Optimal Period 1 Policies and Group e's Expected Utility with φ Reduced from 0.6 to 0.3

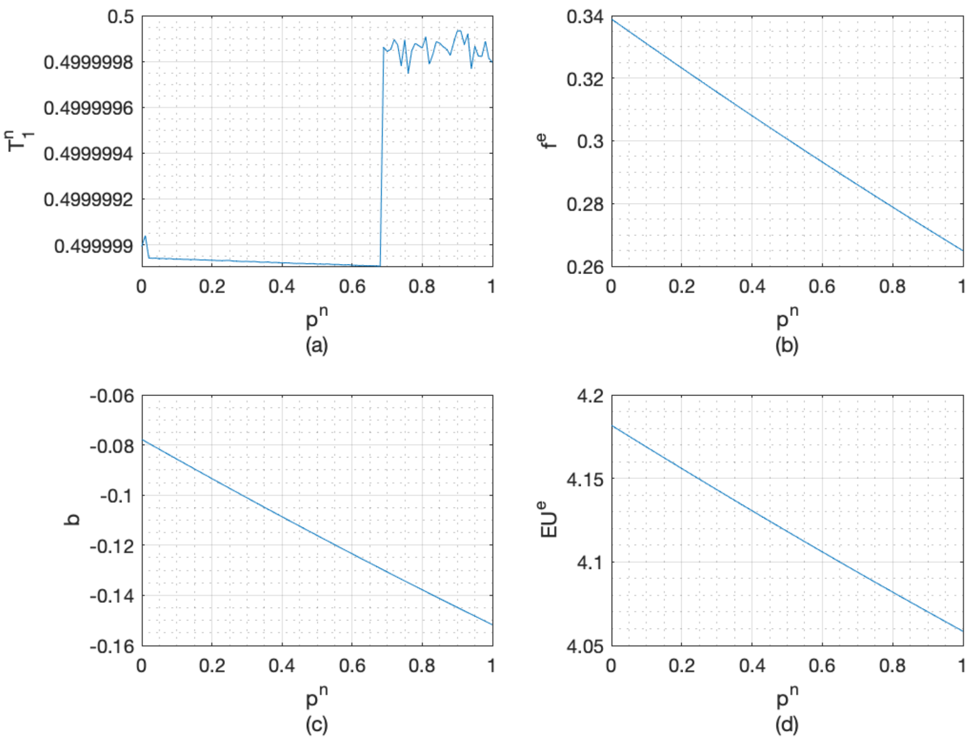
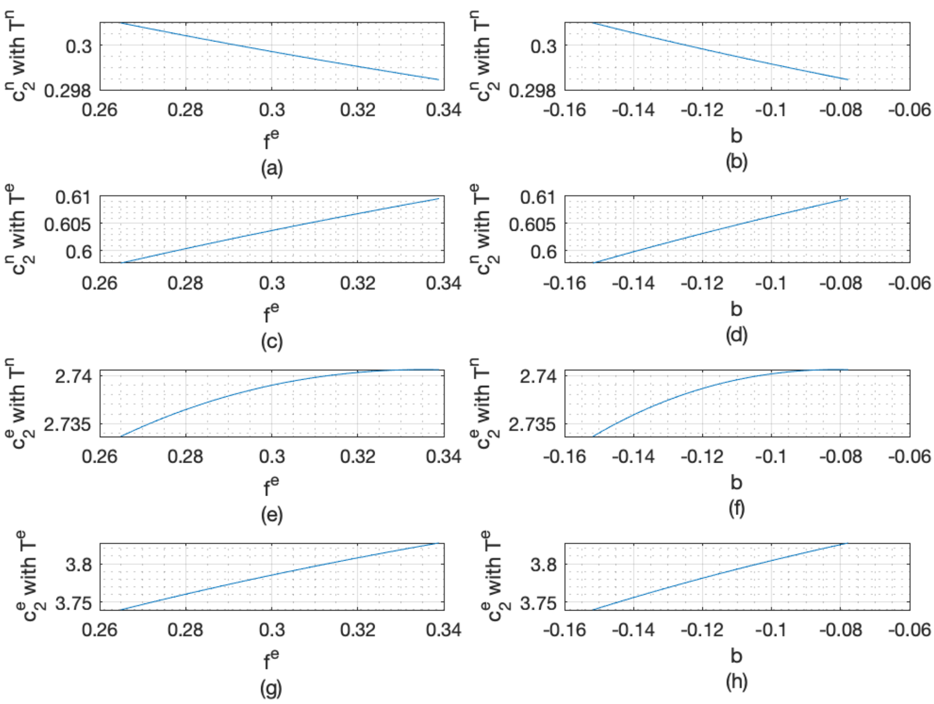


Figure 6.10: Period 2 Consumptions under Incumbent e with φ Reduced from 0.6 to 0.3



A limitation of our model is that some of the results are dependent on the assumptions of parameter values. As shown in Figure 6.8, group n's period 2 consumptions are at a corner solution with incumbent e. The reason incumbent e set a low period 1 tax rate is to make sure group n's period 2 consumption will not be negative. Figure 6.9 and Figure 6.10 show that if we change φ to 0.3 (reduce the marginal disutility of labour) and keep all other parameters constant, the group n's period 2 consumption will be strictly positive and the period 1 tax rate will be close to 0.5 which maximize the period 1 tax revenue which are either saved or spend on f^e. Despite incumbent e effectively no longer changing its period 1 tax rate when its probability of re-election changes, he still changes f^e and b. Unless the different groups in the economy prefer the same period 2 policies (no polarisation), we should expect the partisan incumbent to change its policies when its probability of re-election changes.

6.5 Conclusion

In this paper, we examine the question: Why would an incumbent government increase spending on HE and student loan levels when its voters are not directly benefitting from HE?

Using our model without government borrowing, we show in a polarised society, when the incumbent conservative government knows its re-election probability in the next period falls, there are several incentives determining whether it will increase or decrease the tuition fees and student loan level. These incentives are presented in condition (6.7) of Proposition 6.1. The first is a "pure investment" incentive and it is the difference in the product of the marginal utility of public goods and the MRI between the two different parties being in charge in period 2. The other two terms can be interpreted as the incentives behind the decision of the incumbent to optimally distort the level of student loans to influence the period 2 government's decision on tax rates and public goods and they have their corresponding counterpart in the model of Persson and Svensson (1989) on government borrowing. For example, when the potential liberal successor government prefers a higher level of tax rates and public goods than the incumbent conservative government, if the liberal government's preferred level of tax rates falls when their voters face a higher level of student loan debt, then the incumbent conservative government has conflicting incentives, it wants to increase the level of student loans to reduce the period 2 tax rates but it also wants to reduce the level of loans to reduce the level of student loans repayment which lowers the level of public goods set by the liberal successor. In our model, the conservative group have a lower preference for public goods but if the quality of HE is high enough, the liberal group can prefer lower tax rates and public goods due to the high cost of taxation. If the incumbent government is liberal, it faces an additional incentive to change the student loan level when its probability of re-election falls because its own voters' wage is increasing in f^e and the benefit of this wage increase on the net income is larger the lower the period 2 tax rate. Consequently, if the conservative successor prefers a lower period 2 tax rate, the liberal incumbent is incentivised to increase f^e. All the incentives discussed above are summarised in Table 6.1.

In addition to the other papers within the literature that include only government borrowing or only some types of public investment, we present the incentives facing the conservative incumbent on the student loan decision under political uncertainty in which the incumbent is simultaneously deciding the level of government borrowing. Condition (6.7) in Proposition 6.1 derived from the model without government borrowing still exists and represents the direct effect of a change in the re-election probability. Since the marginal benefit of f^e is different under different levels of debt, the change in re-election probability also indirectly affects the decision on f^e by affecting the borrowing decision.

We also show that under specific leisure functions, as the utility function for public goods becomes more and more linear, whether the incumbent will increase or decrease f^e when its probability of re-election falls is dependent on the difference of the preferred period 2 tax rate between the two possible period 2 governments under the same period 1 policies. The incumbent will increase f^e if $T^e > T^n$ and will decrease f^e if $T^e < T^n$.

Finally, our computational results based on a stylized model also showed that the incumbent government will set different levels of student loans at different probabilities of re-election. It provides support to our theoretical results that the incumbent is incentivised to change the level of student loans when its probability of re-election changes.

We applied the literature on the theories of public investment under uncertainty and polarisation to the government's decision on setting tuition fees and student loan levels. Unlike other papers in the literature, our results did not strictly predict the government must reduce investment or increase investment when the probability of re-election falls. Instead, the government's decision depends on several incentives which could act in opposite directions to each other. Like government debt and public investment, in theory, the HE policy is another way for the government to exert some influence over future policies. We present an additional potential factor to explain the rise in student loan debt and tuition fees that has not been studied before.

Chapter 7: Conclusions

This chapter will conclude the thesis by summarising the key research findings of each of the four papers, corresponding to Chapters 3 to 6, and discussing their contributions. It will also review the limitations of each of the four papers and propose directions for future research.

7.1 Overall Findings in Each Paper

Our first paper which is in Chapter 3 aims to investigate the optimal way to provide a higher education (HE) system with the government providing a second chance to all failed students and analyse the effect of this optimally provided second chance on inequality. By not allowing the policies to be age dependent, our results show that the optimal repayments must be income contingent. Moreover, the optimal repayments in the second-best optimum cannot be provided by charging a separate ordinary loan for using the second chance while the first chance at HE is charged by an income-contingent loan (ICL). Compared to the benchmark "equal treatment" second-best optimum in Gary-Bobo and Trannoy (2015), the second chance policy increases the degree of ex-ante inequality between the low- and high-ability type students. Furthermore, the second chance makes the students who failed twice in HE to be worse off than everyone when old. Additionally, we found that the second chance has reduced the degree of insurance against failing HE for the low-ability students when compared to the corresponding utilities in Gary-Bobo and Trannoy (2015).

Our second paper which is in Chapter 4 aims to investigate the optimal way to provide a HE system with a labour retraining program given to those at risk of downward labour mobility and analyse the effect of this retraining program on inequality. Similar to our first paper we do not allow the policies to be age dependent, our results show that the optimal repayments must be income contingent and the second-best optimum cannot be implemented by charging a separate ordinary loan to those who used the retraining program that is independent of their income while the repayments for enrolling in the HE is income contingent. The labour retraining program has increased the degree of ex-ante inequality compared to the benchmark "equal treatment" second-best optimum in Gary-Bobo and Trannoy (2015). Our results indicate the effect of the labour retraining program on the degree of ex-post inequality depends on whether the government can identify which workers are at risk of downward mobility. The labour retraining program can only reduce the degree of ex-post inequality if the government can identify who needs retraining and the degree of ex-post inequality is maintained if the government cannot identify who needs retraining. Labour retraining has also reduced the degree of insurance against failing HE for the low-ability students.

In both papers, we also extend the model by removing the assumption that the government can commit to its future policies and analyse the time-consistent optimum. Our results indicate that the optimal repayments to implement the time-consistent optimum must be income contingent.

Our third paper which is in Chapter 5 aims to investigate the effect of the design of the constitution on the level of HE spending and income inequality between the presidential regime and the parliamentary regime. We show the differences in the constitution results in different ways the HE spending is determined in the presidential and the parliamentary regimes. In the parliamentary regime, the coalition government remained as the residual claimant of tax revenue and any revenue available for the additional HE spending is shared with the transfers desired by the voters of the coalition partner. In the presidential regime, the group of voters needing HE exploits the separation of powers property and can maximize the HE spending by minimizing the rent to the spending agenda setter and becomes the residual claimant of tax revenue. Moreover, we show when the exogenous threat of re-election is high the spending on HE is minimized in the presidential regime when the tax agenda setter's voters do not benefit from additional HE spending and these voters only want to minimize the tax burden. Since HE spending affects the relative income between different groups of voters, both the maximized case and minimized case explain how the constitution of the presidential regime would lead to a higher level of income inequality compared to the parliamentary regime. Our results also indicate the parliamentary regime should produce more public goods and result in more political rents than the presidential regime but in certain cases the opposite is true.

Our fourth paper which is in Chapter 6 aims to investigate the incentives facing the incumbent government on tuition fees and student loan levels when facing political uncertainty and polarisation and examine whether the incumbent government could be motivated to increase their levels in theory. Our research reveals a range of strategic incentives for the incumbent government. One key finding indicates that when the incumbent government's re-election prospect decreases, the incumbent government is incentivised to distort the level of student loans to change the next period's government's policies on tax rates and public goods level to be closer to its preferred level, for example, if the successor government prefer a higher level of public goods and tax rates than the incumbent and the successor government would reduce the level of tax rates when the student loans rise, the incumbent government is incentivised to reduce the level of student loan to reduce the successor's spending on public goods but is also incentivised to increase the level of student loans so the successor would lower the tax rate in the next period. Additionally, our study identifies several other incentives, including a pure investment incentive for public goods and a private economic incentive, demonstrating the complex interplay between the incumbent's re-election prospects and its student loans and HE spending policy in a polarised society. We also show that under specific leisure functions, as the utility function for public goods becomes more and more linear, whether the incumbent will increase or decrease f^e when its probability of re-election falls is dependent on the difference of the preferred period 2 tax rate between the two possible period 2 governments under the same period 1 policies. Incumbent will increase f^e if the group needing HE prefers a higher level of period 2 tax rate and public goods and will decrease f^e if they prefer a lower level instead. Our computational results also indicate that the incumbent government will change the level of student loans when its re-election prospects change.

7.2 Contributions of the Thesis

Although using student loans to fund HE is not new in both theory and practice, the issue of the optimal design of the student loan system is not resolved yet. Given the evidence of improvement in the labour market outcomes after completing HE for mature students and after completing the labour retraining program, we extend the literature on the optimal way to finance HE by adding additional policies with a second chance at HE in our first paper and a labour retraining program in our second paper. We agree with other papers in the literature on the necessity of using ICL to implement the second-best optimum because of the flexibility of ICL as a policy instrument compared to separating policies into an ordinary loan that only depends on education and an income tax that only depends on income.

In terms of additional policy recommendations, our results suggest if the government is planning to provide future policies targeted at the current prospective HE students, these policies cannot be separately funded by an ordinary loan. The government also should not use the second chance at HE as a tool for redistribution. If the government want to use the optimally provided labour retraining program to reduce the difference in utility between high- and middle-earning workers, the government needs to improve their ability to identify which workers need to be retrained and understand what factors are causing downward mobility in the economy.

Our third paper contributes to the literature focusing on the political economy dimension of the HE and compares it to the previous economic literature on the provision of HE (De Fraja, 2001) we include more realistic assumptions on politics. To our best knowledge, there are no theoretical papers using methods from political economy to examine the impact of the design of the constitution on the level of HE spending despite empirical evidence that separation of powers could influence the level of state HE spending. Additionally, we contribute to the literature that examines the potential reasons for the differences in the within nation inequality between different nations. There is a lack of papers examining the potential link between a nation's constitution and its level of inequality. By adding an income-increasing HE spending and relaxing the equal tax rates assumption in Persson, Roland and Tabellini (2000), we demonstrate that the way the level of HE spending is determined in the presidential and parliamentary regimes are fundamentally different and results in a higher degree of income inequality in the presidential regime. Our prediction on inequality is supported by empirical papers such as Feld and Schnellenbach (2014) and McManus and Ozkan (2018) which both found that presidential regimes are associated with greater income inequality.

In the UK and the US, rising tuition fees are hotly debated topics and the building up of student loan debt has raised concerns. To our best knowledge, there are no theoretical papers using methods from political economy to analyse the effect of political uncertainty and polarisation on the decisions on the level of tuition fees and student loans as in our fourth paper. We confirm the insights in other literature on public investment under uncertainty and polarisation that the incumbent government is incentivised to use available policies to influence the decisions made by the successor government. We contribute to the literature that studies the factors leading to rising student loans and rising rates of default (Belfield, 2013; Goodell, 2016; Scott-Clayton, 2018) by presenting an additional potential factor to explain the rise in student loan debt and tuition fees that have not been studied before.

7.3 Limitations of the Thesis

Our first two papers only included two ability types based on the model of Gary-Bobo and Trannoy (2015) instead of a continuum of types and it could be argued that this is too simplistic. However, like Gary-Bobo and Trannoy (2015), the essential ideas and intuitions on the usefulness of ICL are conveyed. Furthermore, we use this model with two ability types to assess the effect of the second chance and labour retraining program by directly contrasting it to the benchmark “equal treatment” second-best optimum results in Gary-Bobo and Trannoy (2015) with two ability types.

For our third paper, the policies are determined in the legislative bargaining game and it can be criticized that the results are sensitive to the particular extensive form of the game. In the presidential regime of the infinite horizon model, because of the combination of the addition of the income-increasing HE spending and the relaxation of the equal tax rates assumption, we make assumptions on preferences and policies as well as additions to the sequence of events of the legislative bargaining game to limit the possible equilibriums and only consider the one in which all legislators are re-elected. However, the key feature of the presidential regime-the separation of powers and the key feature of the parliamentary regime-the legislative cohesion are fully captured in the game and are both key determinants of the equilibrium outcome.

In our fourth paper, we analyse the case in which the HE sector is completely funded by government-sanctioned student loans and any increase in tuition fees/student loan level results in an increase in investment in the HE sector. Our model cannot be used to meaningfully analyse the decision between choosing different methods of funding the HE system by the government because of the assumption of partisan politicians. With a partisan government that only takes its own group’s voters’ welfare into account, if the incumbent government’s supporters are voters not engaged in HE, the HE sector will be chosen to be completely funded by student loans. We cannot use the model to explain why historically the HE system is funded by the government using tax revenue in the UK and other countries. Furthermore, our computational results are derived from a stylized model with specific functions and precise parameter values which inherently constrained the generalizability of our findings. For example, while the computational results support our theoretical results that the incumbent government is incentivised to change the tuition fees/student loan level in response to diminishing re-election prospects by showing a reduction, it cannot be conclusively determined that the response must be a reduction in tuition fees/student loan level.

7.4 Recommendations for Future Research

Students' migration after graduation poses a well-known problem for collecting loan repayments as it is more difficult for the government to pursue debts of overseas workers in the absence of bilateral or multilateral agreements between countries for coordinating the collection of student loan repayment. Chapman and Higgins (2013) estimated that the amount of uncollected overseas debt from the ICL of the Higher Education Contribution Scheme (HECS) between 1989-2011 from the graduates working overseas is between \$400-\$800 million. We could extend the models of our first two papers by including both the emigrations of domestic students and the immigration of workers to examine the impact of migration on the second-best allocations and the optimal design of the student loans system that combines both adverse selection and moral hazard. We could also examine any potential change in the effect of these Second Chance policies when migration is introduced since now fewer domestic graduates are using these policies while the immigrant workers may now engage in these policies if the government allows it.

Our third and fourth papers studied and applied the HE and student loans in two specific fields within political economy, one is the positive analysis of the effect of the constitution design and one is the effect of political uncertainty and polarisation. There are multiple directions to extend the analysis of the political economy of HE without the benevolent government and direct democracy. Close to our third paper, we could analyse the effect of different electoral rules on the level of HE spending. We could examine any potential difference in the level of HE spending between countries with proportional elections, like Belgium and Netherlands, and countries with majoritarian elections (first-past-the-post), like the UK. Furthermore, as the HE sector is increasingly being funded by student loans, future research in the political economy could also model and examine any potential factors causing the government to switch from a taxpayer-funded HE system to one being funded by students themselves.

Appendices

Appendix for Chapter 3

A3.1: Appendix for Proposition 3.1

Let θ be the Lagrange multiplier for the resource constraint in Proposition 3.1. Use λ_1^{OB} and λ_2^{OB} to indicate the social welfare weight attached to the ex-ante type i's utility and in the standard utilitarian case it will be $\lambda_1^{OB} = \lambda_1$ and $\lambda_2^{OB} = \lambda_2$. The Lagrange function for the first-best optimization problem can be written as the following,

$$L = \sum_i \lambda_i^{OB} \left[P_i U_i^s + (1 - P_i)P_i^a U_i^c + (1 - P_i)(1 - P_i^a)u_i - c_i^y e_i^y - (1 - P_i)\delta c_i^a e_i^a\right]$$

$$+ \theta \sum_i \lambda_i \{S_i - E(h_i)\}$$

Differentiate w.r.t the ex-post utility levels and obtain the following equality for $i = 1, 2$.

$$\frac{\lambda_i^{OB}}{\lambda_i} = \theta h_a'(V_i^a) = \theta h_o'(V_i^o) = \theta h_a'(v_i^a) = \theta h_o'(v_i^o) = \theta h_a'(F_i^a) = \theta h_o'(F_i^o) = \theta h_o'(f_i^o)$$

$$= \theta h_o'(u_i^o).$$

First, the above equalities yield $\theta > 0$ since $h(.)$ is a strictly convex function and that $h_a'(.)$ and $h_o'(.)$ are both positive, so θ must be positive for the above equalities to be satisfied. The resource constraint is binding at the first-best optimum.

Second, in the standard utilitarian case with $\lambda_i^{OB} = \lambda_i$, divide the above term of equalities between $i = 1$ and $i = 2$, obtain

$$1=\frac{h_a'(V_1^a)}{h_a'(V_2^a)}=\frac{h_o'(V_1^o)}{h_o'(V_2^o)}=\frac{h_a'(v_1^a)}{h_a'(v_2^a)}=\frac{h_o'(v_1^o)}{h_o'(v_2^o)}=\frac{h_a'(F_1^a)}{h_a'(F_2^a)}=\frac{h_o'(F_1^o)}{h_o'(F_2^o)}=\frac{h_o'(f_1^o)}{h_o'(f_2^o)}=\frac{h_o'(u_1^o)}{h_o'(u_2^o)}.$$

The two terms of equalities meant that

$$V_i^a=V_i^o=v_i^a=v_i^o=F_i^a=F_i^o=f_i^o=u_i^o$$

in the first-best optimum and in the standard utilitarian case,

$$V_1^a=V_2^a, V_1^o=V_2^o, v_1^a=v_2^a, v_1^o=v_2^o,$$

$$F_1^a=F_2^a, F_1^o=F_2^o, f_1^o=f_2^o, u_1^o=u_2^o.$$

A3.2: Appendix for Lemma 3.2

(a). The LHS of IC_2 and $\underline{IC_2}$ are the same, so if IC_2 is satisfied and the RHS of IC_2 is larger than $\underline{IC_2}$, then $\underline{IC_2}$ is satisfied as well. The RHS of IC_2 compare to the RHS of $\underline{IC_2}$ leads to the inequality below,

$$p_2U_1^s+(1-p_2)P_2^aU_1^c+(1-p_2)(1-P_2^a)u_1-(1-p_2)\delta c_2^a$$
$$\geq p_2U_1^s+(1-p_2)p_2^aU_1^c+(1-p_2)(1-p_2^a)u_1.$$

Through simplification and dividing both sides first by $(1-p_2)$ and then by $(P_2^a-p_2^a)$, we obtain

$$U_1^c-u_1\geq\delta K_2^a.$$

By Assumption 3.5 $K_1^a\geq K_2^a$, and if MH_1 holds $U_1^c-u_1\geq\delta K_1^a$, then the above inequality automatically holds.

In summary, if MH_1 and IC_2 are satisfied, $\underline{IC_2}$ is satisfied.

(b). For the incentive constraint ICA_2, if the RHS of $\overline{IC_2}$ is greater than or equal to the RHS of ICA_2, we will obtain the following inequality,

$$P_2U_1^s + (1-P_2)P_2^aU_1^c + (1-P_2)(1-P_2^a)u_1 - c_2^y - (1-P_2)\delta c_2^a$$
$$\geq P_2U_1^s + (1-P_2)p_2^aU_1^c + (1-P_2)(1-p_2^a)u_1 - c_2^y.$$

Through the same simplification process, the following can be obtained

$$U_1^c - u_1 \geq \delta K_2^a.$$

By Assumption 3.5 $K_1^a \geq K_2^a$, this meant that if $\overline{IC_2}$ and MH_1 are satisfied, then ICA_2 is automatically satisfied.

A3.3: Appendix for the Common Properties of the Second-Best Optima

The interim expected utilities in the incentive constraints are substituted by Lemma 3.1's ex-post utility equivalent. Let κ, μ_1, μ_2, ψ_1, ψ_2, σ_1, σ_2, ξ_1, ζ_1, ξ_2 and φ_1 be the non-negative Lagrange multipliers of $\overline{RC}$, $\overline{IC_1}$, $\overline{IC_2}$, MH_1, MH_2, $\overline{MH_1}$, $\overline{MH_2}$, IC_1, $\underline{IC_1}$, IC_2 and ICA_1. Let the objective function OB be defined as

$$\sum_i \lambda_i^{OB}\left[P_iU_i^s + (1-P_i)P_i^aU_i^c + (1-P_i)(1-P_i^a)u_i - c_i^ye_i^y - (1-P_i)\delta c_i^ae_i^a\right].$$

The Lagrange function will be of the form

$$L = OB + \kappa\overline{RC} + \mu_1\overline{IC_1} + \mu_2\overline{IC_2} + \psi_1MH_1 + \psi_2MH_2 + \sigma_1\overline{MH_1} + \sigma_2\overline{MH_2} + \xi_1IC_1 + \xi_2IC_2$$
$$+ \zeta_1\underline{IC_1} + \varphi_1ICA_1.$$

The first-order conditions are

$$(FOC3.1a)\ \frac{\partial L}{\partial V_1^a}$$

$$= \lambda_1^{OB} P_1 \delta - \lambda_1 \kappa P_1 [(1-\pi)\delta h_a'(V_1^a) + \pi \delta h_a'(V_1^a + b)] + \mu_1 P_1 \delta - \mu_2 P_2 \delta$$

$$+ \sigma_1 \delta + \xi_1 P_1 \delta - \xi_2 p_2 \delta + \zeta_1 P_1 \delta + \varphi_1 P_1 \delta = 0,$$

$$(FOC3.2a)\ \frac{\partial L}{\partial V_1^o}$$

$$= \lambda_1^{OB} P_1 \delta^2 - \lambda_1 \kappa P_1 [(1-\pi)\delta^2 h_o'(V_1^o) + \pi \delta^2 h_o'(V_1^o + b)] + \mu_1 P_1 \delta^2$$

$$- \mu_2 P_2 \delta^2 + \sigma_1 \delta^2 + \xi_1 P_1 \delta^2 - \xi_2 p_2 \delta^2 + \zeta_1 P_1 \delta^2 + \varphi_1 P_1 \delta^2 = 0,$$

$$(FOC3.3a)\ \frac{\partial L}{\partial v_2^a}$$

$$= \lambda_2^{OB} P_2 \delta - \lambda_2 \kappa P_2 [(1-\pi)\delta h_a'(v_2^a + b) + \pi \delta h_a'(v_2^a)] - \mu_1 P_1 \delta + \mu_2 P_2 \delta$$

$$+ \sigma_2 \delta - \xi_1 p_1 \delta + \xi_2 P_2 \delta - \zeta_1 p_1 \delta - \varphi_1 P_1 \delta = 0,$$

$$(FOC3.4a)\ \frac{\partial L}{\partial v_2^o}$$

$$= \lambda_2^{OB} P_2 \delta^2 - \lambda_2 \kappa P_2 [(1-\pi)\delta^2 h_o'(v_2^o + b) + \pi \delta^2 h_o'(v_2^o)] - \mu_1 \delta^2 P_1$$

$$+ \mu_2 P_2 \delta^2 + \sigma_2 \delta^2 - \xi_1 p_1 \delta^2 + \xi_2 P_2 \delta^2 - \zeta_1 p_1 \delta^2 - \varphi_1 P_1 \delta^2 = 0,$$

$$(FOC3.5a)\ \frac{\partial L}{\partial F_1^a}$$

$$= \lambda_1^{OB} (1 - P_1) \delta$$

$$- \lambda_1 \kappa [(1 - P_1) P_1^a [(1-\pi)\delta h_a'(F_1^a) + \pi \delta h_a'(F_1^a)]$$

$$+ (1 - P_1)(1 - P_1^a) \delta h_a'(F_1^a)] + \mu_1 (1 - P_1) \delta - \mu_2 (1 - P_2) \delta - \sigma_1 \delta$$

$$+ \xi_1 (1 - P_1) \delta - \xi_2 (1 - p_2) \delta + \zeta_1 (1 - P_1) \delta + \varphi_1 (1 - P_1) \delta = 0,$$

$$(FOC3.6a)\ \frac{\partial L}{\partial F_2^a}$$

$$= \lambda_2^{OB}(1-P_2)\delta$$

$$- \lambda_2\kappa[(1-P_2)P_2^a[(1-\pi)\delta h_a'(F_2^a) + \pi\delta h_a'(F_2^a)]$$

$$+ (1-P_2)(1-P_2^a)\delta h_a'(F_2^a)] - \mu_1(1-P_1)\delta + \mu_2(1-P_2)\delta - \sigma_2\delta$$

$$- \xi_1(1-p_1)\delta + \xi_2(1-P_2)\delta - \zeta_1(1-p_1)\delta - \varphi_1(1-P_1)\delta = 0,$$

$$(FOC3.7a)\ \frac{\partial L}{\partial F_1^o}$$

$$= \lambda_1^{OB}(1-P_1)P_1^a\delta^2 - \lambda_1\kappa(1-P_1)P_1^a[(1-\pi)\delta^2 h_o'(F_1^o) + \pi\delta^2 h_o'(F_1^o + b)]$$

$$+ \psi_1\delta^2 + \mu_1(1-P_1)P_1^a\delta^2 - \mu_2(1-P_2)P_2^a\delta^2 - \sigma_1 P_1^a\delta^2 + \xi_1(1-P_1)P_1^a\delta^2$$

$$- \xi_2(1-p_2)P_2^a\delta^2 + \zeta_1(1-P_1)P_1^a\delta^2 + \varphi_1(1-P_1)P_1^a\delta^2 = 0,$$

$$(FOC3.8a)\ \frac{\partial L}{\partial f_2^o}$$

$$= \lambda_2^{OB}(1-P_2)P_2^a\delta^2 - \lambda_2\kappa(1-P_2)P_2^a[(1-\pi)\delta^2 h_o'(f_2^o + b) + \pi\delta^2 h_o'(f_2^o)]$$

$$+ \psi_2\delta^2 - \mu_1(1-P_1)P_1^a\delta^2 + \mu_2(1-P_2)P_2^a\delta^2 - \sigma_2 P_2^a\delta^2 - \xi_1(1-p_1)P_1^a\delta^2$$

$$+ \xi_2(1-P_2)P_2^a\delta^2 - \zeta_1(1-p_1)p_1^a\delta^2 - \varphi_1(1-P_1)p_1^a\delta^2 = 0,$$

$$(FOC3.9a)\ \frac{\partial L}{\partial u_1^o}$$

$$= \lambda_1^{OB}(1-P_1)(1-P_1^a)\delta^2 - \lambda_1\kappa(1-P_1)(1-P_1^a)\delta^2 h_o'(u_1^o) - \psi_1\delta^2$$

$$+ \mu_1(1-P_1)(1-P_1^a)\delta^2 - \mu_2(1-P_2)(1-P_2^a)\delta^2 - \sigma_1(1-P_1^a)\delta^2$$

$$+ \xi_1(1-P_1)(1-P_1^a)\delta^2 - \xi_2(1-p_2)(1-P_2^a)\delta^2 + \zeta_1(1-P_1)(1-P_1^a)\delta^2$$

$$+ \varphi_1(1-P_1)(1-P_1^a)\delta^2 = 0,$$

$(FOC10a)\ \dfrac{\partial L}{\partial u_2^o}$

$$= \lambda_2^{OB}(1-P_2)(1-P_2^a)\delta^2 - \lambda_2\kappa(1-P_2)(1-P_2^a)\delta^2 h_o'(u_2^o) - \psi_2\delta^2$$

$$- \mu_1(1-P_1)(1-P_1^a)\delta^2 + \mu_2(1-P_2)(1-P_2^a)\delta^2 - \sigma_2(1-P_2^a)\delta^2$$

$$- \xi_1(1-p_1)(1-P_1^a)\delta^2 + \xi_2(1-P_2)(1-P_2^a)\delta^2 - \zeta_1(1-p_1)(1-p_1^a)\delta^2$$

$$- \varphi_1(1-P_1)(1-p_1^a)\delta^2 = 0.$$

Simplify the 10 FOCs, we obtain

$(FOC3.1b)$ $\lambda_1^{OB}P_1 + \mu_1P_1 - \mu_2P_2 + \sigma_1 + \xi_1P_1 - \xi_2p_2 + \zeta_1P_1 + \varphi_1P_1 = \lambda_1\kappa P_1Eh'(V_1^a)$,

$(FOC3.2b)$ $\lambda_1^{OB}P_1 + \mu_1P_1 - \mu_2P_2 + \sigma_1 + \xi_1P_1 - \xi_2p_2 + \zeta_1P_1 + \varphi_1P_1 = \lambda_1\kappa P_1Eh'(V_1^o)$,

$(FOC3.3b)$ $\lambda_2^{OB}P_2 - \mu_1P_1 + \mu_2P_2 + \sigma_2 - \xi_1p_1 + \xi_2P_2 - \zeta_1p_1 - \varphi_1P_1 = \lambda_2\kappa P_2Eh'(v_2^a)$,

$(FOC3.4b)$ $\lambda_2^{OB}P_2 - \mu_1P_1 + \mu_2P_2 + \sigma_2 - \xi_1p_1 + \xi_2P_2 - \zeta_1p_1 - \varphi_1P_1 = \lambda_2\kappa P_2Eh'(v_2^o)$,

$(FOC3.5b)$ $\lambda_1^{OB}(1-P_1) + \mu_1(1-P_1) - \mu_2(1-P_2) - \sigma_1 + \xi_1(1-P_1) - \xi_2(1-p_2)$

$$+ \zeta_1(1-P_1) + \varphi_1(1-P_1) = \lambda_1\kappa(1-P_1)h_a'(F_1^a),$$

$(FOC3.6b)$ $\lambda_2^{OB}(1-P_2) - \mu_1(1-P_1) + \mu_2(1-P_2) - \sigma_2 - \xi_1(1-p_1) + \xi_2(1-P_2)$

$$- \zeta_1(1-p_1) - \varphi_1(1-P_1) = \lambda_2\kappa(1-P_2)h_a'(F_2^a),$$

$(FOC3.7b)$ $\lambda_1^{OB}(1-P_1)P_1^a + \psi_1 + \mu_1(1-P_1)P_1^a - \mu_2(1-P_2)P_2^a - \sigma_1P_1^a + \xi_1(1-P_1)P_1^a$

$$- \xi_2(1-p_2)P_2^a + \zeta_1(1-P_1)P_1^a + \varphi_1(1-P_1)P_1^a = \lambda_1\kappa(1-P_1)P_1^aEh'(F_1^o),$$

$(FOC3.8b)$ $\lambda_2^{OB}(1-P_2)P_2^a + \psi_2 - \mu_1(1-P_1)P_1^a + \mu_2(1-P_2)P_2^a - \sigma_2P_2^a - \xi_1(1-p_1)P_1^a$

$$+ \xi_2(1-P_2)P_2^a - \zeta_1(1-p_1)p_1^a - \varphi_1(1-P_1)p_1^a = \lambda_2\kappa(1-P_2)P_2^aEh'(f_2^o),$$

$(FOC3.9b)$ $\lambda_1^{OB}(1-P_1)(1-P_1^a) - \psi_1 + \mu_1(1-P_1)(1-P_1^a) - \mu_2(1-P_2)(1-P_2^a)$

$$- \sigma_1(1-P_1^a) + \xi_1(1-P_1)(1-P_1^a) - \xi_2(1-p_2)(1-P_2^a)$$

$$+ \zeta_1(1-P_1)(1-P_1^a) + \varphi_1(1-P_1)(1-P_1^a)$$

$$= \lambda_1\kappa(1-P_1)(1-P_1^a)h_o'(u_1^o),$$

$$(FOC3.10b)\ \lambda_2^{OB}(1-P_2)(1-P_2^a)-\psi_2-\mu_1(1-P_1)(1-P_1^a)+\mu_2(1-P_2)(1-P_2^a)$$

$$-\sigma_2(1-P_2^a)-\xi_1(1-p_1)(1-P_1^a)+\xi_2(1-P_2)(1-P_2^a)$$

$$-\zeta_1(1-p_1)(1-p_1^a)-\varphi_1(1-P_1)(1-p_1^a)$$

$$=\lambda_2\kappa(1-P_2)(1-P_2^a)h_o'(u_2^o),$$

where

$Eh'(V_1^a)$ denote $(1-\pi)h_a'(V_1^a)+\pi h_a'(V_1^a+b)$,

$Eh'(V_1^o)$ denote $(1-\pi)h_o'(V_1^o)+\pi h_o'(V_1^o+b)$,

$Eh'(v_2^a)$ denote $(1-\pi)h_a'(v_2^a+b)+\pi h_a'(v_2^a)$,

$Eh'(v_2^o)$ denote $(1-\pi)h_o'(v_2^o+b)+\pi h_o'(v_2^o)$,

$Eh'(F_1^o)$ denote $(1-\pi)h_o'(F_1^o)+\pi h_o'(F_1^o+b)$,

$Eh'(f_2^o)$ denote $(1-\pi)h_o'(f_2^o+b)+\pi h_o'(f_2^o)$.

Lemma 3.5: In the second-best optimum:

(a) The $\overline{RC}$ is a binding constraint.

(b) MH_1 needs to be binding in every possible second-best optimum.

Proof of Lemma 3.5(a)

Adding up the ten $FOC3b$s, for the LHS we obtain two and for the RHS the following term is obtained,

$$\kappa\sum_i \lambda_i\,[P_i[Eh'(V_i^a)+Eh'(V_i^o)]+(1-P_i)h_a'(F_i^a)+(1-P_i)P_i^aEh'(F_i^o)$$
$$+(1-P_i)(1-P_i^a)h_o'(u_i^o)].$$

Consequently

$$(\sum FOC)\ \ \frac{2}{\kappa} = \sum_i \lambda_i \left[P_i[Eh'(V_i^a) + Eh'(V_i^o)] + (1-P_i)h_a'(F_i^a) + (1-P_i)P_i^a Eh'(F_i^o)\right.$$

$$\left. + (1-P_i)(1-P_i^a)h_o'(u_i^o)\right] > 0.$$

$\kappa > 0$ as the different $h'(.)$ are positive. By the complementary slackness condition $\kappa\big[\lambda_1[S_1 - E(h_1)] + \lambda_2[S_2 - E(h_2)]\big] = 0$, we conclude that $\overline{RC}$ is a binding constraint.

Proof of Lemma 3.5(b)

Using $FOC3.7b$ and $FOC3.9b$, if MH_1 does not bind ($\psi_1 = 0$), then $u_1^o > F_1^o$ will be the result based on the first-order conditions which violate the study effort constraint MH_1.

Analysing the Common Properties in the Model

Common properties meant common relationships between the ex-post utilities that we can always derive from the model no matter what constraints are binding or not.

Lemma 3.6: In all the second-best optimums the ex-post utilities have the following relations:

(a). $V_1^a = V_1^o, v_2^a = v_2^o$.

(b). $f_1^o > F_1^a > u_1^o, F_2^o > F_2^a > u_2^o$.

(c). $V_2^o = V_2^a > F_2^a$.

Intertemporal Insurance

Firstly, $FOC3.1b$ equals $FOC3.2b$ and $FOC3.3b$ equals $FOC3.4b$, which means in terms of ex-post utilities,

$$V_1^a = V_1^o \text{ and } v_2^a = v_2^o.$$

Students who succeeded in HE in their youth will receive constant utility across periods.

Secondly, $FOC3.7b + FOC3.9b = FOC3.5b$, which means,

$$h_a'(F_1^a) = P_1^a Eh'(F_1^o) + (1 - P_1^a)h_o'(u_1^o).$$

By definition $Eh'(F_1^o) > h_o'(F_1^o)$, and constraint MH_1 implies that $h_o'(F_1^o) > h_o'(u_1^o)$. Based on these relations, the above equation implies $f_1^o > F_1^a > u_1^o$ in the second-best optimum.

Equally, $FOC3.8b + FOC3.10b = FOC3.6b$, which is,

$$h_a'(F_2^a) = P_2^a Eh'(f_2^o) + (1 - P_2^a)h_o'(u_2^o).$$

By definition $Eh'(f_2^o) > h_o'(f_2^o)$, and constraint MH_2 implies that $h_o'(f_2^o) > h_o'(u_2^o)$. Based on these relations, the above equation implies $F_2^o > F_2^a > u_2^o$ in the second-best optimum.

$V_2^o = V_2^a > F_2^a$

There is one more additional common property that could be obtained by comparing $FOC3.3b$ and $FOC3.6b$. It is certain that $v_2^a + b > F_2^a$ because based on $FOC3.3b$ and $FOC3.6b$, $Eh'(v_2^a)$ will not be smaller than $h_a'(F_2^a)$ in any second-best optimum. Since $h_a'(v_2^a + b) > Eh'(v_2^a)$, this meant that $v_2^a + b > F_2^a$. With $v_2^a = v_2^o$, it is also the case that $v_2^o + b > F_2^a$.

A3.4: Appendix for Proposition 3.3

In the equal treatment second-best optimum of $U_2^s - u_2 = U_1^s - u_1$ and $U_2^c - u_2 = U_1^c - u_1$ with both MH_1 and $\overline{MH_1}$ being binding constraints. The degree of ex-ante inequality has increased due to the second chance. Utilizing this insight, the only possible way to decrease the degree of inequality is to let $U_1^s - u_1 > U_2^s - u_2$ or $U_1^c - u_1 > U_2^c - u_2$ be the case in the second-best optimum but both cannot be the case because $\overline{IC_2}$ would be violated.

In every possible second-best optimum, the second chance effort constraint MH_1 binds. Is there any arrangement of the other study effort constraints MH_2, $\overline{MH_1}$ and $\overline{MH_2}$ in any second-best optimum that led to a lower level of ex-ante inequality compared to the result in GT's model's equal treatment optimum? Write down the study effort constraints with the potential deviation from the equal treatment second-best,

$$U_2^s = K_1^y + p_1^a \delta K_1^a + u_2 - \varepsilon_2^s, U_2^c = \delta K_1^a + u_2 - \varepsilon_2^c,$$

$$U_1^s = K_1^y + p_1^a \delta K_1^a + u_1 + \varepsilon_1^s, U_1^c = \delta K_1^a + u_1.$$

The difference in the degree of ex-ante inequality is written below

$$\begin{aligned} &P_2 U_2^s + (1 - P_2) P_2^a U_2^c + (1 - P_2)(1 - P_2^a) u_2 - c_2^y - (1 - P_2) \delta c_2^a \\ &\quad - \left[P_1 U_1^s + (1 - P_1) P_1^a U_1^c + (1 - P_1)(1 - P_1^a) u_1 - c_1^y - (1 - P_1) \delta c_1^a\right] \\ &\quad - (P_2 - P_1) K_1^y - c_1^y + c_2^y. \end{aligned}$$

Substituting U_2^s, U_2^c, U_1^s and U_1^c into the above function and obtain

$$\begin{aligned} &u_2 + (1 - P_2)(P_2^a - p_1^a) \delta K_1^a - (1 - P_2)(P_2^a - p_2^a) \delta K_2^a - u_1 - P_2 \varepsilon_2^s - (1 - P_2) P_2^a \varepsilon_2^c \\ &\quad - P_1 \varepsilon_1^s. \end{aligned}$$

For the degree of ex-ante inequality to not be increased by the second chance, this term must be not positive, which meant $P_2 \varepsilon_2^s + (1 - P_2) P_2^a \varepsilon_2^c + P_1 \varepsilon_1^s$ have a minimum value needed to not increase the inequality

$$P_2\varepsilon_2^s + (1-P_2)P_2^a\varepsilon_2^c + P_1\varepsilon_1^s \geq u_2 - u_1 + (1-P_2)[(P_2^a - p_1^a)\delta K_1^a - (P_2^a - p_2^a)\delta K_2^a].$$

Substituting U_2^s, U_2^c, U_1^s and U_1^c into the ex-ante self-selection constraint for ex-ante type $i = 2$ students $\overline{IC_2}$, we obtain

$$P_2\left[K_1^y + p_1\delta K_1^a + u_2 - \varepsilon_2^s\right] + (1-P_2)P_2^a[\delta K_1^a + u_2 - \varepsilon_2^c] + (1-P_2)(1-P_2^a)u_2$$
$$\geq P_2\left[K_1^y + p_1\delta K_1^a + u_1 + \varepsilon_1^s\right] + (1-P_2)P_2^a[\delta K_1^a + u_1]$$
$$+ (1-P_2)(1-P_2^a)u_1,$$
$$u_2 - u_1 \geq P_2\varepsilon_2^s + (1-P_2)P_2^a\varepsilon_2^c + P_2\varepsilon_1^s.$$

Substituting the minimum required value of $P_2\varepsilon_2^s + (1-P_2)P_2^a\varepsilon_2^c + P_1\varepsilon_1^s$ needed to not increase inequality into

$$u_2 - u_1 \geq P_2\varepsilon_2^s + (1-P_2)P_2^a\varepsilon_2^c + P_1\varepsilon_1^s$$

and obtain the following

$$0 \geq (1-P_2)[(P_2^a - p_1^a)\delta K_1^a - (P_2^a - p_2^a)\delta K_2^a]$$

which is not possible as by Assumptions 3.1 and 3.5 on the value of the parameters the term on the RHS is positive. Given that $P_2\varepsilon_2^s + (1-P_2)P_2^a\varepsilon_2^c + P_2\varepsilon_1^s \geq P_2\varepsilon_2^s + (1-P_2)P_2^a\varepsilon_2^c + P_1\varepsilon_1^s$ as $P_2 > P_1$ and $\varepsilon_1^s \geq 0$, $u_2 - u_1 \geq P_2\varepsilon_2^s + (1-P_2)P_2^a\varepsilon_2^c + P_2\varepsilon_1^s$ is also not possible as the RHS is larger than the LHS. Any potential arrangement of the study effort constraint that would not lead to an increase in the degree of ex-ante inequality led to the violation of $\overline{IC_2}$. The second chance can only increase the degree of ex-ante inequality in any valid second-best optimum.

A3.5: The Properties of the Time-Consistent Optimum

The properties of the time-consistent optimum are derived from the relations of the FOCs in the utility maximization problem for the last period. Consequently, only the utility maximization problem for students when old will be written here. The labour effort constraints are still binding in the time-consistent optimum. The social welfare when old for the government to maximize is

$$Max\ W_g = \lambda_1^{OB}\left\{P_1V_1^o + (1-P_1)\left[P_1^aF_1^o + (1-P_1^a)u_1^o - \frac{1}{\delta}c_1^a\right]\right\}$$
$$+\lambda_2^{OB}\left\{P_2v_2^o + (1-P_2)\left[P_2^af_2^o + (1-P_2^a)u_2^o - \frac{1}{\delta}c_2^a\right]\right\}.$$

This will be maximized subject to the relevant resource constraint when old: $Rev^o - Cost^o - (1+r)T \geq 0$, the term T is introduced to account for inter-period transfers that could be done by the government. If T is positive, it means the government is repaying the borrowing from the last period along with the interest rate. The part of the old period resource constraint that mattered in the maximization is the part with the inverse utility functions, this part is

$$\ldots - [\lambda_1\{P_1[(1-\pi)h_o(V_1^o) + \pi h_o(V_1^o + b)] + (1-P_1)P_1^a[(1-\pi)h_o(F_1^o) + \pi h_o(F_1^o + b)]$$
$$+ (1-P_1)(1-P_1^a)h_o(u_1^o)\}$$
$$+ \lambda_2\{P_2[(1-\pi)h_o(v_2^o + b) + \pi h_o(v_2^o)]$$
$$+ (1-P_2)P_2^a[(1-\pi)h_o(f_2^o + b) + \pi h_o(f_2^o)] + (1-P_2)(1-P_2^a)h_o(u_2^o)\}]$$
$$- (1+r)T \geq 0,$$

The government also need to satisfy the second chance study effort constraints, MH_1 and MH_2,

MH_1: $F_1^o - u_1^o \geq \frac{1}{\delta} K_1^a$,

MH_2: $f_2^o - u_2^o \geq \frac{1}{\delta} K_2^a$.

The students have already made their education quality choices in their youth and they can no longer change them. The government is assumed to have a record of students' past education and therefore the government is not concerned with education choice incentive constraints in the second period.

The Lagrange for the maximization problem is

$$L = W_g + \kappa^o[Rev^o - Cost^o - (1+r)T] + \psi_1\left[F_1^o - u_1^o - \frac{1}{\delta}K_1^a\right] + \psi_2\left[f_2^o - u_2^o - \frac{1}{\delta}K_2^a\right].$$

The Lagrange function is maximized with respect to the utilities V_1^o, F_1^o, u_1^o, v_2^o, f_2^o and u_2^o.

$$(FOC3.1c)\ \frac{\partial L}{\partial V_1^o} = \lambda_1^{OB}P_1 - \lambda_1\kappa^o P_1[(1-\pi)h_o'(V_1^o) + \pi h_o'(V_1^o + b)] = 0,$$

$$(FOC3.2c)\ \frac{\partial L}{\partial F_1^o}$$

$$= \lambda_1^{OB}(1-P_1)P_1^a - \lambda_1\kappa^o(1-P_1)P_1^a[(1-\pi)h_o'(F_1^o) + \pi h_o'(F_1^o + b)] + \psi_1$$

$$= 0,$$

$$(FOC3.3c)\ \frac{\partial L}{\partial u_1^o} = \lambda_1^{OB}(1-P_1)(1-P_1^a) - \lambda_1\kappa^o(1-P_1)(1-P_1^a)h_o'(u_1^o) - \psi_1 = 0,$$

$$(FOC3.4c)\ \frac{\partial L}{\partial v_2^o} = \lambda_2^{OB}P_2 - \lambda_2\kappa^o P_2[(1-\pi)h_o'(v_2^o + b) + \pi h_o'(v_2^o)] = 0,$$

$$(FOC3.5c)\ \frac{\partial L}{\partial f_2^o}$$

$$= \lambda_2^{OB}(1-P_2)P_2^a - \lambda_2\kappa^o(1-P_2)P_2^a[(1-\pi)h_o'(f_2^o + b) + \pi h_o'(f_2^o)] + \psi_2$$

$$= 0,$$

$$(FOC3.6c)\ \frac{\partial L}{\partial u_2^o} = \lambda_2^{OB}(1-P_2)(1-P_2^a) - \lambda_2\kappa^o(1-P_2)(1-P_2^a)h_o'(u_2^o) - \psi_2 = 0.$$

Simplifying the FOCs and obtain the following,

$$(FOC3.1d)\ \frac{\lambda_1^{OB}}{\lambda_1} = \kappa^o Eh'(V_1^o),$$

$$(FOC3.2d)\ \frac{\lambda_1^{OB}}{\lambda_1} + \psi_1 \frac{1}{(1-P_1)P_1^a\lambda_1} = \kappa^o Eh'(F_1^o),$$

$$(FOC3.3d)\ \frac{\lambda_1^{OB}}{\lambda_1} - \psi_1 \frac{1}{(1-P_1)(1-P_1^a)\lambda_1} = \kappa^o h_o'(u_1^o),$$

$$(FOC3.4d)\ \frac{\lambda_2^{OB}}{\lambda_2} = \kappa^o Eh'(v_2^o),$$

$$(FOC3.5d)\ \frac{\lambda_2^{OB}}{\lambda_2} + \psi_2 \frac{1}{(1-P_2)P_2^a\lambda_2} = \kappa^o Eh'(f_2^o),$$

$$(FOC3.6d)\ \frac{\lambda_2^{OB}}{\lambda_2} - \psi_2 \frac{1}{(1-P_2)(1-P_2^a)\lambda_2} = \kappa^o h_o'(u_2^o).$$

Both MH_1 and MH_2 are Binding Constraints

If the MH_2 is slack in the optimum, which meant $\psi_2 = 0$, a comparison between $FOC3.5d$ and $FOC3.6d$ suggests that $f_2^o < u_2^o$ which would violate the constraint MH_2. Consequently, it meant that MH_2 must be a binding constraint. The same comparison between $FOC3.2d$ and $FOC3.3d$ leads to the conclusion that MH_1 must be binding in the time-consistent optimum as well.

The Ex-post Utility Relationships $F_1^o > V_1^o$, $f_2^o > v_2^o$

The two relationships are derived from the comparison between $FOC3.1d$ and $FOC3.2d$ and between $FOC3.4d$ and $FOC3.5d$, the fact that both MH_1 and MH_2 are binding results in

$$Eh'(F_1^o) > Eh'(V_1^o) \text{ and } Eh'(f_2^o) > Eh'(v_2^o).$$

These two inequalities meant $F_1^o > V_1^o$ and $f_2^o > v_2^o$.

Additional Properties in the Time-Consistent Optimum

If $l_1^{OB} \geq \lambda_1$ and $l_2^{OB} \leq \lambda_2$, then the following relation could be discerned by comparing the $FOC3.1d$ with $FOC3.4d$,

$$V_1^o > v_2^o.$$

If $l_1^{OB} < \lambda_1$ and $l_2^{OB} > \lambda_2$, then $Eh'(V_1^o) < Eh'(v_2^o)$ and the relation between V_1^o and v_2^o is uncertain.

When the government is offering a second chance, the relations of u_1^o, u_2^o to V_1^o, v_2^o is uncertain. If there is no second chance, there are no second chance effort constraints. If the model is simply extended by one period, then compare $FOC3.1d$ with $FOC3.3d$, we obtain $u_1^o > V_1^o$ and compare $FOC3.4d$ with $FOC3.6d$ we obtain $u_2^o > v_2^o$.

Appendix for Chapter 4

A4.1: Appendix for Proposition 4.1

For the first-best utilitarian problem (4.9), use κ as the Lagrange multiplier for the resource constraint. For the objective function we use λ_1^{OB} and λ_2^{OB} to indicate the social welfare weight attached to the ex-ante type i's utility. In the standard utilitarian case, it will be $\lambda_1^{OB} = \lambda_1$ and $\lambda_2^{OB} = \lambda_2$. The Lagrange function for solving the first-best problem is

$$L = \sum_i \lambda_i^{OB}\left[P_i U_i^s + (1-P_i)u_i - c_i^y\right] - \lambda_1^{OB} P_1(1-\pi)P^d \delta c_1^r - \lambda_2^{OB} P_2 \pi P^d \delta c_2^r$$

$$+ \kappa \sum_i \lambda_i \{S_i - E(h_i)\}.$$

Differentiating the Lagrange function with respect to the ex-post utility levels in (4.9). From the first-order conditions, the following is derived:

$$\frac{\lambda_i^{OB}}{\kappa\lambda_i} = h'_a(V_i^a) = h'_o(V_i^o) = h'_a(v_i^a) = h'_o(v_i^o) = h'_o(BF_i^o) = h'_o(B_i^o) = h'_a(u_i^a) = h'_o(u_i^o).$$

Here BF_i^o denotes either BF_1^o or bf_2^o and B_i^o denotes either B_1^o or b_2^o depends on i. The resource constraint $\overline{RC}$ is binding as $\kappa > 0$ since $h_a(.)$ and $h_o(.)$ are strictly convex and h'_a and h'_o are positive. The above string of equalities meant that $\kappa > 0$. The above equalities create the result in Proposition 4.1 in which

$$V_i^a = V_i^o = v_i^a = v_i^o = u_i^a = u_i^o = (B_1^o = BF_1^o \text{ or } b_2^o = bf_2^o \text{ depend on } i).$$

Furthermore, in the standard utilitarian case with $\lambda_i^{OB} = \lambda_i$, we obtain

$$1 = \frac{h'_a(V_1^a)}{h'_a(V_2^a)} = \frac{h'_o(V_1^o)}{h'_o(V_2^o)} = \frac{h'_a(v_1^a)}{h'_a(v_2^a)} = \frac{h'_o(v_1^o)}{h'_o(v_2^o)} = \frac{h'_a(u_1^a)}{h'_a(u_2^a)} = \frac{h'_o(u_1^o)}{h'_o(u_2^o)} = \frac{h'_o(B_1^o)}{h'_o(b_2^o)} = \frac{h'_o(BF_1^o)}{h'_o(bf_2^o)}.$$

This meant in the standard utilitarian case,

$$V_1^a = V_2^a, V_1^o = V_2^o, v_1^a = v_2^a, v_1^o = v_2^o, u_1^a = u_2^a, u_1^o = u_2^o, B_1^o = b_2^o, BF_1^o = bf_2^o.$$

A4.2: Appendix for Lemma 4.2

(a). The LHS of $\overline{IC_2}$ and ICA_2 are the same, so if $\overline{IC_2}$'s RHS is greater than ICA_2's RHS we know ICA_2 would be satisfied. Let's compare the RHS of $\overline{IC_2}$ to the RHS of ICA_2 and obtains,

$$P_2 U_1^{s2} + (1 - P_2)u_1 - c_2^y - P_2 \pi P^d \delta c_2^r \geq P_2 U_1^{sr2} + (1 - P_2)u_1 - c_2^y,$$

$$\delta^2 B_1^o - \delta^2 BF_1^o \geq \frac{\pi}{1 - \pi} \delta K_2^r.$$

Since $\pi < \frac{1}{2}$ and due to Assumption 4.6, the above inequality is satisfied if MR_1 is satisfied. Consequently, if $\overline{IC_2}$ and MR_1 are satisfied, then due to Assumption 4.6, ICA_2 is automatically satisfied.

(b). The LHS of IC_2 and ICB_2 is the same, so if IC_2's RHS is greater than ICB_2's RHS we know ICB_2 would be satisfied. Compare the RHS of IC_2 to the RHS of ICB_2 and obtains,

$$p_2 U_1^{s2} + (1 - p_2)u_1 - p_2 \pi P^d \delta c_2^r \geq p_2 U_1^{sr2} + (1 - p_2)u_1,$$

$$\delta^2 B_1^o - \delta^2 BF_1^o \geq \frac{\pi}{1 - \pi} \delta K_2^r.$$

If IC_2 and MR_1 are satisfied then due to Assumption 4.6, ICB_2 is automatically satisfied.

A4.3: Appendix for Common Properties in the Second-Best Retraining Case

Let κ, μ_1, μ_2, ψ_1, ψ_2, σ_1, σ_2, ξ_1, ξ_2, ζ_1 and φ_1 be the non-negative Lagrange multipliers of the following incentive constraints $\overline{RC}$, $\overline{IC_1}$, $\overline{IC_2}$, MR_1, MR_2, $\overline{MH_1}$, $\overline{MH_2}$, IC_1, IC_2, ICA_1 and ICB_1.

The objective function is written as follows:

$$\sum_i \lambda_i^{OB}\left[P_i U_i^s + (1-P_i)u_i - c_i^y\right] - \lambda_1^{OB} P_1(1-\pi)P^d\delta c_1^r - \lambda_2^{OB} P_2 \pi P^d \delta c_2^r.$$

This is the objective function that will be maximized with respect to $[V_1^a, V_1^o, v_2^a, v_2^o, B_1^o, BF_1^o, b_2^o, bf_2^o, u_i^a, u_i^o]$ with $i = 1, 2$.

The Lagrange will be of the form,

$$L = OB + \kappa\overline{RC} + \mu_1\overline{IC_1} + \mu_2\overline{IC_2} + \psi_1 MR_1 + \psi_2 MR_2 + \sigma_1\overline{MH_1} + \sigma_2\overline{MH_2} + \xi_1 IC_1 + \xi_2 IC_2 + \zeta_1 ICA_1 + \varphi_1 ICB_1.$$

The first-order conditions are

$$(FOC4.1a)\ \frac{\partial L}{\partial V_1^a} = \lambda_1^{OB} P_1\delta - \lambda_1\kappa P_1[(1-\pi)\delta h_a'(V_1^a) + \pi\delta h_a'(V_1^a + b)] + \mu_1 P_1\delta - \mu_2 P_2\delta + \sigma_1\delta + \xi_1 P_1\delta - \xi_2 p_2\delta + \zeta_1 P_1\delta + \varphi_1 P_1\delta = 0,$$

$$(FOC4.2a)\ \frac{\partial L}{\partial V_1^o}$$

$$= \lambda_1^{OB} P_1(1-(1-\pi)P^d)\delta^2$$

$$-\lambda_1 \kappa P_1[(1-\pi)(1-P^d)\delta^2 h_o'(V_1^o) + \pi\delta^2 h_o'(V_1^o + b)]$$

$$+\mu_1 P_1(1-(1-\pi)P^d)\delta^2 - \mu_2 P_2(1-(1-\pi)P^d)\delta^2$$

$$+\sigma_1\delta^2(1-(1-\pi)P^d) + \xi_1 P_1(1-(1-\pi)P^d)\delta^2$$

$$-\xi_2 p_2(1-(1-\pi)P^d)\delta^2 + \zeta_1 P_1(1-(1-\pi)P^d)\delta^2$$

$$+\varphi_1 P_1(1-(1-\pi)P^d)\delta^2 = 0,$$

$$(FOC4.3a)\ \frac{\partial L}{\partial v_2^a}$$

$$= \lambda_2^{OB} P_2\delta - \lambda_2\kappa P_2[(1-\pi)\delta h_a'(v_2^a + b) + \pi\delta h_a'(v_2^a)] - \mu_1 P_1\delta + \mu_2 P_2\delta$$

$$+\sigma_2\delta - \xi_1 p_1\delta + \xi_2 P_2\delta - \zeta_1 P_1\delta - \varphi_1 p_1\delta = 0,$$

$$(FOC4.4a)\ \frac{\partial L}{\partial v_2^o}$$

$$= \lambda_2^{OB} P_2(1-\pi P^d)\delta^2$$

$$-\lambda_2\kappa P_2[(1-\pi)\delta^2 h_o'(v_2^o + b) + \pi(1-P^d)\delta^2 h_o'(v_2^o)] - \mu_1\delta^2 P_1(1-\pi P^d)$$

$$+\mu_2 P_2(1-\pi P^d)\delta^2 + \sigma_2(1-\pi P^d)\delta^2 - \xi_1 p_1(1-\pi P^d)\delta^2$$

$$+\xi_2 P_2(1-\pi P^d)\delta^2 - \zeta_1 P_1(1-\pi P^d)\delta^2 - \varphi_1 p_1(1-\pi P^d)\delta^2 = 0,$$

$$(FOC4.5a)\ \frac{\partial L}{\partial B_1^o}$$

$$= \lambda_1^{OB} P_1(1-\pi)P^d P_1^r\delta^2 - \lambda_1\kappa P_1(1-\pi)P^d P_1^r\delta^2 h_o'(B_1^o)$$

$$+\mu_1 P_1(1-\pi)P^d P_1^r\delta^2 - \mu_2 P_2(1-\pi)P^d P_2^r\delta^2 + \psi_1\delta^2 + \sigma_1(1-\pi)P^d P_1^r\delta^2$$

$$+\xi_1 P_1(1-\pi)P^d P_1^r\delta^2 - \xi_2 p_2(1-\pi)P^d P_2^r\delta^2 + \zeta_1 P_1(1-\pi)P^d P_1^r\delta^2$$

$$+\varphi_1 P_1(1-\pi)P^d P_1^r\delta^2 = 0,$$

$(FOC4.6a)\ \frac{\partial L}{\partial BF_1^o}$

$$= \lambda_1^{OB} P_1(1-\pi)P^d(1-P_1^r)\delta^2 - \lambda_1 \kappa P_1(1-\pi)P^d(1-P_1^r)\delta^2 h_o'(BF_1^o)$$

$$+ \mu_1 P_1(1-\pi)P^d(1-P_1^r)\delta^2 - \mu_2 P_2(1-\pi)P^d(1-P_2^r)\delta^2 - \psi_1\delta^2 + \sigma_1(1$$

$$- \pi)P^d(1-P_1^r)\delta^2 + \xi_1 P_1(1-\pi)P^d(1-P_1^r)\delta^2 - \xi_2 p_2(1$$

$$- \pi)P^d(1-P_2^r)\delta^2 + \zeta_1 P_1(1-\pi)P^d(1-P_1^r)\delta^2 + \varphi_1 P_1(1$$

$$- \pi)P^d(1-P_1^r)\delta^2 = 0,$$

$(FOC4.7a)\ \frac{\partial L}{\partial b_2^o}$

$$= \lambda_2^{OB} P_2 \pi P^d P_2^r \delta^2 - \lambda_2 \kappa P_2 \pi P^d P_2^r \delta^2 h_o'(b_2^o) - \mu_1 \pi P_1 P^d P_1^r \delta^2$$

$$+ \mu_2 P_2 \pi P^d P_2^r \delta^2 + \psi_2 \delta^2 + \sigma_2 \pi P^d P_2^r \delta^2 - \xi_1 p_1 \pi P^d P_1^r \delta^2 + \xi_2 P_2 \pi P^d P_2^r \delta^2$$

$$- \zeta_1 P_1 \pi P^d p_1^r \delta^2 - \varphi_1 p_1 \pi P^d p_1^r \delta^2 = 0,$$

$(FOC4.8a)\ \frac{\partial L}{\partial bf_2^o}$

$$= \lambda_2^{OB} P_2 \pi P^d(1-P_2^r)\delta^2 - \lambda_2 \kappa P_2 \pi P^d(1-P_2^r)\delta^2 h_o'(bf_2^o)$$

$$- \mu_1 P_1 \pi P^d(1-P_1^r)\delta^2 + \mu_2 P_2 \pi P^d(1-P_2^r)\delta^2 - \psi_2\delta^2 + \sigma_2 \pi P^d(1-P_2^r)\delta^2$$

$$- \xi_1 p_1 \pi P^d(1-P_1^r)\delta^2 + \xi_2 P_2 \pi P^d(1-P_2^r)\delta^2 - \zeta_1 P_1 \pi P^d(1-p_1^r)\delta^2$$

$$- \varphi_1 p_1 \pi P^d(1-p_1^r)\delta^2 = 0,$$

$(FOC4.9a)\ \frac{\partial L}{\partial u_1^a}$

$$= \lambda_1^{OB}(1-P_1)\delta - \lambda_1\kappa(1-P_1)h_a'(u_1^a)\delta + \mu_1(1-P_1)\delta - \mu_2(1-P_2)\delta$$

$$- \sigma_1\delta + \xi_1(1-P_1)\delta - \xi_2(1-p_2)\delta + \zeta_1(1-P_1)\delta + \varphi_1(1-P_1)\delta = 0,$$

$(FOC4.10a)\ \frac{\partial L}{\partial u_1^o}$

$$= \lambda_1^{OB}(1-P_1)\delta^2 - \lambda_1\kappa(1-P_1)\delta^2 h_o'(u_1^o) + \mu_1(1-P_1)\delta^2 - \mu_2(1-P_2)\delta^2$$

$$- \sigma_1\delta^2 + \xi_1(1-P_1)\delta^2 - \xi_2(1-p_2)\delta^2 + \zeta_1(1-P_1)\delta^2 + \varphi_1(1-P_1)\delta^2$$

$$= 0,$$

$$(FOC4.11a)\ \frac{\partial L}{\partial u_2^a}$$

$$= \lambda_2^{OB}(1-P_2)\delta - \lambda_2\kappa(1-P_2)\delta h_a'(u_2^a) - \mu_1(1-P_1)\delta + \mu_2(1-P_2)\delta$$

$$- \sigma_2\delta - \xi_1(1-p_1)\delta + \xi_2(1-P_2)\delta - \zeta_1(1-P_1)\delta - \varphi_1(1-p_1)\delta = 0,$$

$$(FOC4.12a)\ \frac{\partial L}{\partial u_2^o}$$

$$= \lambda_2^{OB}(1-P_2)\delta^2 - \lambda_2\kappa(1-P_2)\delta^2 h_o'(u_2^o) - \mu_1(1-P_1)\delta^2 + \mu_2(1-P_2)\delta^2$$

$$- \sigma_2\delta^2 - \xi_1(1-p_1)\delta^2 + \xi_2(1-P_2)\delta^2 - \zeta_1(1-P_1)\delta^2 - \varphi_1(1-p_1)\delta^2$$

$$= 0,$$

Simplify the 12 FOCs, we obtain

$$(FOC4.1b)\ \lambda_1^{OB} + \mu_1 - \mu_2\frac{P_2}{P_1} + \sigma_1\frac{1}{P_1} + \xi_1 - \xi_2\frac{p_2}{P_1} + \zeta_1 + \varphi_1 = \lambda_1\kappa Eh'(V_1^a),$$

$$(FOC4.2b)\ \lambda_1^{OB}(1-(1-\pi)P^d) + \mu_1(1-(1-\pi)P^d) - \mu_2\frac{P_2}{P_1}(1-(1-\pi)P^d)$$

$$+ \sigma_1(1-(1-\pi)P^d)\frac{1}{P_1} + \xi_1(1-(1-\pi)P^d) - \xi_2\frac{p_2}{P_1}(1-(1-\pi)P^d)$$

$$+ \zeta_1(1-(1-\pi)P^d) + \varphi_1(1-(1-\pi)P^d) = \lambda_1\kappa Eh'(V_1^o),$$

$$(FOC4.3b)\ \lambda_2^{OB} - \mu_1\frac{P_1}{P_2} + \mu_2 + \sigma_2\frac{1}{P_2} - \xi_1\frac{p_1}{P_2} + \xi_2 - \zeta_1\frac{P_1}{P_2} - \varphi_1\frac{p_1}{P_2} = \lambda_2\kappa Eh'(v_2^a),$$

$$(FOC4.4b)\ \lambda_2^{OB}(1-\pi P^d) - \mu_1\frac{P_1}{P_2}(1-\pi P^d) + \mu_2(1-\pi P^d) + \sigma_2(1-\pi P^d)\frac{1}{P_2}$$

$$- \xi_1\frac{p_1}{P_2}(1-\pi P^d) + \xi_2(1-\pi P^d) - \zeta_1\frac{P_1}{P_2}(1-\pi P^d) - \varphi_1\frac{p_1}{P_2}(1-\pi P^d)$$

$$= \lambda_2\kappa Eh'(v_2^o),$$

$$(FOC4.5b)\ \lambda_1^{OB} + \mu_1 - \mu_2\frac{P_2P_2^r}{P_1P_1^r} + \psi_1\frac{1}{P_1(1-\pi)P^dP_1^r} + \sigma_1\frac{1}{P_1} + \xi_1 - \xi_2\frac{p_2P_2^r}{P_1P_1^r} + \zeta_1 + \varphi_1$$

$$= \lambda_1\kappa h_o'(B_1^o),$$

$$(FOC4.6b)\ \lambda_1^{OB} + \mu_1 - \mu_2 \frac{P_2(1-P_2^r)}{P_1(1-P_1^r)} - \psi_1 \frac{1}{P_1(1-\pi)P^d(1-P_1^r)} + \sigma_1 \frac{1}{P_1} + \xi_1$$

$$- \xi_2 \frac{p_2(1-P_2^r)}{P_1(1-P_1^r)} + \zeta_1 + \varphi_1 = \lambda_1 \kappa h_o'(BF_1^o),$$

$$(FOC4.7b)\ \lambda_2^{OB} - \mu_1 \frac{P_1 P_1^r}{P_2 P_2^r} + \mu_2 + \psi_2 \frac{1}{P_2 \pi P^d P_2^r} + \sigma_2 \frac{1}{P_2} - \xi_1 \frac{p_1 P_1^r}{P_2 P_2^r} + \xi_2 - \zeta_1 \frac{P_1 p_1^r}{P_2 P_2^r}$$

$$- \varphi_1 \frac{p_1 p_1^r}{P_2 P_2^r} = \lambda_2 \kappa h_o'(b_2^o),$$

$$(FOC4.8b)\ l_2^{OB} - \mu_1 \frac{P_1(1-P_1^r)}{P_2(1-P_2^r)} + \mu_2 - \psi_2 \frac{1}{P_2 \pi P^d (1-P_2^r)} + \sigma_2 \frac{1}{P_2} - \xi_1 \frac{p_1(1-P_1^r)}{P_2(1-P_2^r)} + \xi_2$$

$$- \zeta_1 \frac{P_1(1-p_1^r)}{P_2(1-P_2^r)} - \varphi_1 \frac{p_1(1-p_1^r)}{P_2(1-P_2^r)} = \lambda_2 \kappa h_o'(bf_2^o),$$

$$(FOC4.9b)\ \lambda_1^{OB} + \mu_1 - \mu_2 \frac{(1-P_2)}{(1-P_1)} - \sigma_1 \frac{1}{(1-P_1)} + \xi_1 - \xi_2 \frac{(1-p_2)}{(1-P_1)} + \zeta_1 + \varphi_1$$

$$= \lambda_1 \kappa h_a'(u_1^a),$$

$$(FOC4.10b)\ \lambda_1^{OB} + \mu_1 - \mu_2 \frac{(1-P_2)}{(1-P_1)} - \sigma_1 \frac{1}{(1-P_1)} + \xi_1 - \xi_2 \frac{(1-p_2)}{(1-P_1)} + \zeta_1 + \varphi_1$$

$$= \lambda_1 \kappa h_o'(u_1^o),$$

$$(FOC4.11b)\ \lambda_2^{OB} - \mu_1 \frac{(1-P_1)}{(1-P_2)} + \mu_2 - \sigma_2 \frac{1}{(1-P_2)} - \xi_1 \frac{(1-p_1)}{(1-P_2)} + \xi_2 - \zeta_1 \frac{(1-P_1)}{(1-P_2)}$$

$$- \varphi_1 \frac{(1-p_1)}{(1-P_2)} = \lambda_2 \kappa h_a'(u_2^a),$$

$$(FOC4.12b)\ \lambda_2^{OB} - \mu_1 \frac{(1-P_1)}{(1-P_2)} + \mu_2 - \sigma_2 \frac{1}{(1-P_2)} - \xi_1 \frac{(1-p_1)}{(1-P_2)} + \xi_2 - \zeta_1 \frac{(1-P_1)}{(1-P_2)}$$

$$- \varphi_1 \frac{(1-p_1)}{(1-P_2)} = \lambda_2 \kappa h_o'(u_2^o),$$

The notations used in the FOCs have the following meaning:

$Eh'(V_1^a)$ denote $(1-\pi)h_a'(V_1^a) + \pi h_a'(V_1^a + b)$,

$Eh'(V_1^o)$ denote $(1-\pi)(1-P^d)h_o'(V_1^o) + \pi h_o'(V_1^o + b)$,

$$Eh'(v_2^a) \text{ denote } (1-\pi)h_a'(v_2^a+b)+\pi h_a'(v_2^a),$$

$$Eh'(v_2^o) \text{ denote } (1-\pi)h_o'(v_2^o+b)+\pi(1-P^d)h_o'(v_2^o).$$

Lemma 4.3: In any second-best optimum in the retraining case:

(a). $\overline{RC}$ is a binding constraint.

(b). MR_1 must be a binding constraint.

(c). If $\overline{IC_1}$, IC_1, ICA_1 and ICB_1 are all slack then MR_2 must be binding in the optimum.

Proof of Lemma 4.3 (a)

Adding up all the 12 FOCs and all the terms involving the following Lagrange multipliers are cancelled out: $\mu_1, \mu_2, \psi_1, \psi_2, \sigma_1, \sigma_2, \xi_1, \xi_2, \zeta_1$ and φ_1.

When adding all of the FOCs for the terms with λ_1^{OB} and λ_2^{OB}, we obtain $2\lambda_1^{OB}$ and $2\lambda_2^{OB}$. Since $\lambda_1^{OB}+\lambda_2^{OB}=1$ by definition, $2\lambda_1^{OB}+2\lambda_2^{OB}$ adds up to two. Adding together the resource constraint in all the FOCs, the summation results in κ times the various $h_a'(.)$ and $h_o'(.)$ which are all positive. Putting κ to the side of λ_i^{OB} terms we can derive the following inequality:

$$\frac{2}{\kappa}>0.$$

For this to be valid, κ must be positive and it means $\overline{RC}$ is a binding constraint.

Proof of Lemma 4.3 (b)

Let $FOC4.5b$ minus $FOC4.6b$ and let $\psi_1=0$ (MR_1 is slack), and doesn't matter if any other multipliers are zero or positive, we obtain

$$\mu_2 \frac{P_2}{P_1} \frac{(P_1^r - P_2^r)}{P_1^r(1 - P_1^r)} + \xi_2 \frac{p_2}{P_1} \frac{(P_1^r - P_2^r)}{P_1^r(1 - P_1^r)} = \lambda_1 \kappa [h_o'(B_1^o) - h_o'(BF_1^o)].$$

Since $P_1^r < P_2^r$, the LHS is either negative or equal to zero which is a violation of MR_1. This meant in equilibrium MR_1 must be binding.

Proof of Lemma 4.3 (c)

Assume $\psi_2 = 0$. Let μ_2 be expressed in terms of μ_1 using $FOC4.7b$ and then substituting the μ_2 into $FOC4.8b$ and gain the following expression for μ_1,

$$\mu_1 = \lambda_2 \kappa \frac{P_2 P_2^r (1 - P_2^r)}{P_1 (P_1^r - P_2^r)} [h_o'(bf_2^o) - h_o'(b_2^o)] - \xi_1 \frac{p_1}{P_1} - \zeta_1 \frac{p_1^r - P_2^r}{P_1^r - P_2^r} - \varphi_1 \frac{p_1 (p_1^r - P_2^r)}{P_1 (P_1^r - P_2^r)}.$$

Rearrange the equation,

$$\mu_1 \frac{P_1}{P_2} \frac{(P_2^r - P_1^r)}{P_2^r(1 - P_2^r)} + \xi_1 \frac{p_1}{P_2} \frac{(P_2^r - P_1^r)}{P_2^r(1 - P_2^r)} + \zeta_1 \frac{P_1}{P_2} \frac{(P_2^r - p_1^r)}{P_2^r(1 - P_2^r)} + \varphi_1 \frac{p_1}{P_2} \frac{(P_2^r - p_1^r)}{P_2^r(1 - P_2^r)}$$
$$= \lambda_2 \kappa [h_o'(b_2^o) - h_o'(bf_2^o)].$$

Based on this equation, if none of the constraints $\overline{IC_1}$, IC_1, ICA_1 or ICB_1 binds in the second-best optimum, then it meant that MR_2 must be binding instead.

Analysing the First-Order Conditions

Lemma 4.4 contains the relations between ex-post utilities that are certain in all second-best optimums regardless of what incentive constraints are binding or not.

Lemma 4.4: At every second-best optimum when the government could identify who needs retraining:

(a). $u_1^a = u_1^o$ and $u_2^a = u_2^o$.

(b). $V_1^a > V_1^o$ and $V_1^a + b > V_1^o + b$.

(c). $v_2^a > v_2^o$ and $v_2^a + b > v_2^o + b$.

(d). $B_1^o > V_1^a$.

(e). $b_2^o > v_2^a$.

(f). $v_2^a + b > bf_2^o$ and $v_2^o + b > bf_2^o$.

In all second-best optimums, $FOC4.9b = FOC4.10b$, the expected increase in resources for an additional unit of ex-post utilities for an ex-ante type $i = 1$ student who failed youth education is equal between the two periods.

$$(a4.1) \qquad h_a'(u_1^a) = h_o'(u_1^o).$$

Since $h_a(.)$ and $h_o(.)$ are the same equation but for different periods utilities, (a4.1) meant that in the second-best optimum $u_1^a = u_1^o$.

For the ex-ante type $i = 2$ students, in all second-best optimums, $FOC4.11b = FOC4.12b$.

$$(a4.2) \qquad h_a'(u_2^a) = h_o'(u_2^o).$$

(a4.2) means that in the second-best optimum, $u_2^a = u_2^o$. These two relations are Lemma 4.4 (a).

The following relationships will be less obvious. Let's first compare $FOC4.1b$ and $FOC4.2b$, divide $FOC4.2b$ by $(1-(1-\pi)P^d)$. We have the following equality,

$$(a4.3) \qquad Eh'(V_1^a) = \frac{Eh'(V_1^o)}{(1-(1-\pi)P^d)}.$$

From this equality, we derive that $V_1^a > V_1^o$ in the second-best optimum. This is Lemma 4.4 (b).

Compare $FOC4.3b$ and $FOC4.4b$. We have the following equality,

$$(a4.4) \qquad Eh'(v_2^a) = \frac{Eh'(v_2^o)}{(1-\pi P^d)}.$$

From this equality, we drive that in the second-best optimum $v_2^a > v_2^o$. This is Lemma 4.4 (c).

$FOC4.1a$ is equal to the sum of $FOC4.2a$, $FOC4.5a$ and $FOC4.6a$ and form the equation

$$Eh'(V_1^a) = Eh'(V_1^o) + (1-\pi)P^d P_1^r h_o'(B_1^o) + (1-\pi)P^d(1-P_1^r)h_o'(BF_1^o).$$

Relations can be derived by splitting the $Eh'(V_1^o)$ term and reorganizing the equation into

$$(a4.5) \quad Eh'(V_1^a) = (1-\pi)[(1-P^d)h_o'(V_1^o) + P^d P_1^r h_o'(B_1^o) + P^d(1-P_1^r)h_o'(BF_1^o)] + \pi h_o'(V_1^o + b).$$

Given that $V_1^a > V_1^o$, the $\pi h_a'(V_1^a + b)$ is greater than $\pi h_o'(V_1^o + b)$, this meant $h_a'(V_1^a) < (1-P^d)h_o'(V_1^o) + P^d P_1^r h_o'(B_1^o) + P^d(1-P_1^r)h_o'(BF_1^o)$. Given that $V_1^a > V_1^o$, and due to

MR_1, $B_1^o > BF_1^o$, it can be deduced that $B_1^o > V_1^a$ in all cases of second-best optimum. This is Lemma 4.4 (d).

$FOC4.3a$ is equal to the sum of $FOC4.4a$, $FOC4.7a$ and $FOC4.8a$. This summation together with (a4.5) meant the optimum allocation ensures the increase in resources for a unit of extra utility at the adult period is equal to the expected increase in resources for an extra unit of utility when old for both ex-ante types of students. The summation results in

$$Eh'(v_2^a) = Eh'(v_2^o) + \pi P^d P_2^r h_o'(b_2^o) + \pi P^d (1 - P_2^r) h_o'(bf_2^o).$$

Rearrange the equation and obtain

$$(a4.6)\ \ Eh'(v_2^a)$$
$$= (1-\pi)h_o'(v_2^o + b) + \pi[(1 - P^d)h_o'(v_2^o) + P^d P_2^r h_o'(b_2^o)$$
$$+ P^d(1 - P_2^r)h_o'(bf_2^o)].$$

From Lemma 4.4 (c) we know $v_2^a > v_2^o$ in the second-best optimum. Equivalent to the ex-ante type $i = 1$ case, we also obtain $h_a'(v_2^a) < (1 - P^d)h_o'(v_2^o) + P^d P_2^r h_o'(b_2^o) + P^d(1 - P_2^r)h_o'(bf_2^o)$. Since $b_2^o > bf_2^o$ due to MR_2, for this inequality to be viable it must be the case that $b_2^o > v_2^a$. This is Lemma 4.4 (e).

Compare between $FOC4.3b$ and $FOC4.8b$, if $\mu_1 P_1 \pi P^d [P_2^r - P_1^r] + \xi_1 p_1 \pi P^d [P_2^r - P_1^r] + \zeta_1 P_1 \pi P^d [P_2^r - p_1^r] + \varphi_1 p_1 \pi P^d [P_2^r - p_1^r] > -\psi_2$, then $v_2^a + b > bf_2^o$ is certain in the second best. Since the fact that by assumption $p_1^r < P_2^r$ and $P_1^r < P_2^r$, the LHS cannot be negative. $v_2^a + b > bf_2^o$ is a certain result due to Lemma 4.3 (c), if all of the incentive constraints that are associated with μ_1, ξ_1, ζ_1, φ_1 were slack in the second-best optimum, then for MR_2 to not be violated ψ_2 needs to be positive, so it is certain $v_2^a + b > bf_2^o$ is in the second-best optimum.

The same logic applies to $FOC4.4b$ and $FOC4.8b$, divide both sides of $FOC4.4b$ by $(1 - \pi P^d)$, we obtain that if $\mu_1 P_1 \pi P^d [P_2^r - P_1^r] + \xi_1 p_1 \pi P^d [P_2^r - P_1^r] + \zeta_1 P_1 \pi P^d [P_2^r - p_1^r] + \varphi_1 p_1 \pi P^d [P_2^r - p_1^r] > -\psi_2$, then $v_2^o + b > bf_2^o$ is certain in the second-best optimum. Applying Lemma 4.3 (c), it is certain that $v_2^o + b > bf_2^o$. These two relations are Lemma 4.4 (f).

What Happens When the Government Cannot Identify Who Needs Retraining

If the government cannot identify who needs training, we have two additional constraints added to the maximization problem:

$$V_1^o \geq B_1^o \text{ and } v_2^o \geq b_2^o.$$

Both constraints will be binding in the second-best optimum since if they do not bind the constraint will be violated. This meant in the second-best optimum when the government cannot identify who needs retraining, we have:

$$V_1^o = B_1^o \text{ and } v_2^o = b_2^o.$$

Both (a4.5) and (a4.6) still apply and we can derive the relations of the optimum utilities across periods from their equalities. We found that for these equalities to be viable, we must have:

$$V_1^o = B_1^o > V_1^a \text{ and } v_2^o = b_2^o > v_2^a.$$

Furthermore, the retraining effort constraint MR_1 must be binding in the second-best optimum. All of Lemmas 4.3 still applies in the second-best optimum. Lemma 4.4 (a) and 4.4 (f) are also unchanged in the second-best optimum when the government does not know who needs retraining.

A4.4: Appendix for Proposition 4.4

Is it possible for the retraining program to reduce the degree of ex-ante inequality in any second-best optimum below the equal treatment optimum of $U_1^s = U_2^s$ and $u_1 = u_2$ found in GT's model when the government knows who needs retraining? To formally examine this, we set up the following equation based on $\overline{MH_1}$,

$$U_1^s = K_1^y + (1-\pi)P^d\delta c_1^r + u_1 + \varepsilon_1^s, U_2^s = K_1^y + (1-\pi)P^d\delta c_1^r + u_2 - \varepsilon_2^s.$$

The difference in the degree of ex-ante inequality is written below

$$P_2U_2^s + (1-P_2)u_2 - c_2^y - P_2\pi P^d\delta c_2^r - \left[P_1U_1^s + (1-P_1)u_1 - c_1^y - P_1(1-\pi)P^d\delta c_1^r\right] - (P_2 - P_1)K_1^y - c_1^y + c_2^y.$$

Substituting U_1^s and U_2^s into the difference and obtain

$$u_2 - u_1 + P_2(1-\pi)P^d\delta c_1^r - P_2\pi P^d\delta c_2^r - P_2\varepsilon_2^s - P_1\varepsilon_1^s.$$

For the degree of ex-ante inequality to not increase above the degree in the equal treatment optimum of GT's model, the above term needs to be not positive and we obtain

$$P_2\varepsilon_2^s + P_1\varepsilon_1^s \geq u_2 - u_1 + P_2(1-\pi)P^d\delta c_1^r - P_2\pi P^d\delta c_2^r.$$

Given that MR_1 is always binding in the second-best optimum, U_1^{s2} can be written as: $U_1^{s2} = U_1^s + (1-\pi)P^d(P_2^r - P_1^r)\delta K_1^r$. Substituting U_2^s and U_1^{s2} into the self-selection constraint $\overline{IC_2}$ and obtains

$$u_2 - u_1 \geq P_2\varepsilon_2^s + P_2\varepsilon_1^s + P_2(1-\pi)P^d(P_2^r - P_1^r)\delta K_1^r - P_2(1-2\pi)P^d\delta c_2^r.$$

Since in any valid second-best optimum $\varepsilon_1^s \geq 0$, it meant $P_2\varepsilon_2^s + P_2\varepsilon_1^s \geq P_2\varepsilon_2^s + P_1\varepsilon_1^s$. Substituting in the minimum value of $P_2\varepsilon_2^s + P_1\varepsilon_1^s$ needed for not increasing the degree of ex-ante inequality in place of $P_2\varepsilon_2^s + P_2\varepsilon_1^s$, we obtain

$$0 \geq P_2(1-\pi)P^d(P_2^r - p_1^r)\delta K_1^r - P_2(1-\pi)P^d(P_2^r - p_2^r)\delta K_2^r.$$

Given $P_2^r - p_1^r > P_2^r - p_2^r$ and $K_1^r \geq K_2^r$, we conclude that

$$P_2(1-\pi)P^d(P_2^r - p_1^r)\delta K_1^r - P_2(1-\pi)P^d(P_2^r - p_2^r)\delta K_2^r > 0.$$

In the original inequality "$u_2 - u_1 \geq P_2\varepsilon_2^s + P_2\varepsilon_1^s + P_2(1-\pi)P^d(P_2^r - P_1^r)\delta K_1^r - P_2(1-2\pi)P^d\delta c_2^r$" would also be violated by any set of values of ε_2^s and ε_1^s that does not increase the degree of ex-ante inequality. The introduction of the labour retraining program can only increase the degree of ex-ante inequality because any combination of ε_2^s and ε_1^s in the study effort constraints that does not increase the degree of ex-ante inequality would violate the self-selection constraint $\overline{IC_2}$.

This answer is not changed when the government don't know who needs retraining because this method of proving the degree of ex-ante inequality has increased is not changed.

A4.5: The Properties of the Time-Consistent Optimum

The properties of the time-consistent optimum are derived from the relations of the FOCs in the utility maximization problem for the last period. Consequently, only the utility maximization problem for students when old will be written here. The labour effort constraints are still binding in the time-consistent optimum.

The objective function when old is written as

$$W_g = \lambda_1^{OB}\left\{P_1\left[(1-(1-\pi)P^d)V_1^o + (1-\pi)[P^dP_1^rB_1^o + P^d(1-P_1^r)BF_1^o]\right] + (1-P_1)u_1^o\right\}$$

$$+\lambda_2^{OB}\left\{P_2\left[(1-\pi P^d)v_2^o + \pi[P^dP_2^rb_2^o + P^d(1-P_2^r)bf_2^o]\right] + (1-P_2)u_2^o\right\}$$

$$-\lambda_1^{OB}P_1(1-\pi)P^d\frac{1}{\delta}c_1^r - \lambda_2^{OB}P_2\pi P^d\frac{1}{\delta}c_2^r.$$

For the resource constraint when old $Rev^o - Cost^o - (1+r)T \geq 0$, the part with the inverse utility when old is written as

$$\ldots - \Big[\lambda_1[P_1[(1-\pi)[(1-P^d)h_o(V_1^o) + P^dP_1^rh_o(B_1^o) + P^d(1-P_1^r)h_o(BF_1^o)]$$

$$+\pi h_o(V_1^o + b)] + (1-P_1)h_o(u_1^o)]$$

$$+\lambda_2\Big[P_2\Big[(1-\pi)h_o(v_2^o + b)$$

$$+\pi[(1-P^d)h_o(v_2^o) + P^dP_2^rh_o(b_2^o) + P^d(1-P_2^r)h_o(bf_2^o)]\Big]$$

$$+(1-P_2)h_o(u_2^o)\Big]\Big] - (1+r)T \geq 0.$$

Although the government is no longer concerned with the constraint relevant to the HE, in the last period, the government still wants the workers to exert effort in retraining. The government must satisfy retraining effort constraints for both ex-ante type workers, MR_1 and MR_2 when old.

The Lagrange Function is

$$L = W_g + \kappa^o[Rev^o - Cost^o - (1+r)T] + \psi_1\left[B_1^o - BF_1^o - \frac{1}{\delta}K_1^r\right] + \psi_2[b_2^o - bf_2^o - \frac{1}{\delta}K_2^r]$$

and is maximized with respect to V_1^o, B_1^o, BF_1^o, u_1^o, v_2^o, b_2^o, bf_2^o and u_2^o.

$$\frac{\partial L}{\partial V_1^o} = \lambda_1^{OB}P_1(1-(1-\pi)P^d) - \lambda_1\kappa^o P_1[(1-\pi)(1-P^d)h_o'(V_1^o) + \pi h_o'(V_1^o + b)] = 0,$$

$$\frac{\partial L}{\partial B_1^o} = \lambda_1^{OB}P_1(1-\pi)P^d P_1^r - \lambda_1\kappa^o P_1(1-\pi)P^d P_1^r h_o'(B_1^o) + \psi_1 = 0,$$

$$\frac{\partial L}{\partial BF_1^o} = \lambda_1^{OB}P_1(1-\pi)P^d(1-P_1^r) - \lambda_1\kappa^o P_1(1-\pi)P^d(1-P_1^r)h_o'(BF_1^o) - \psi_1 = 0,$$

$$\frac{\partial L}{\partial u_1^o} = \lambda_1^{OB}(1-P_1) - \lambda_1\kappa^o(1-P_1)h_o'(u_1^o) = 0,$$

$$\frac{\partial L}{\partial v_2^o} = \lambda_2^{OB}P_2(1-\pi P^d) - \lambda_2\kappa^o P_2[(1-\pi)h_o'(v_2^o + b) + \pi(1-P^d)h_o'(v_2^o)] = 0,$$

$$\frac{\partial L}{\partial b_2^o} = \lambda_2^{OB}P_2\pi P^d P_2^r - \lambda_2\kappa^o P_2\pi P^d P_2^r h_o'(b_2^o) + \psi_2 = 0,$$

$$\frac{\partial L}{\partial bf_2^o} = \lambda_2^{OB}P_2\pi P^d(1-P_2^r) - \lambda_2\kappa^o P_2\pi P^d(1-P_2^r)h_o'(bf_2^o) - \psi_2 = 0,$$

$$\frac{\partial L}{\partial u_2^o} = \lambda_2^{OB}(1-P_2) - \lambda_2\kappa^o(1-P_2)h_o'(u_2^o) = 0.$$

Simplify the FOCs and obtain the following,

$$\frac{\lambda_1^{OB}}{\lambda_1}(1-(1-\pi)P^d) = \kappa^o Eh'(V_1^o),$$

$$\frac{\lambda_1^{OB}}{\lambda_1} + \psi_1\frac{1}{\lambda_1 P_1(1-\pi)P^d P_1^r} = \kappa^o h_o'(B_1^o),$$

$$\frac{\lambda_1^{OB}}{\lambda_1} - \psi_1 \frac{1}{\lambda_1 P_1 (1-\pi) P^d (1-P_1^r)} = \kappa^o h_o'(BF_1^o),$$

$$\frac{\lambda_1^{OB}}{\lambda_1} = \kappa^o h_o'(u_1^o),$$

$$\frac{\lambda_2^{OB}}{\lambda_2}(1-\pi P^d) = \kappa^o E h'(v_2^o),$$

$$\frac{\lambda_2^{OB}}{\lambda_2} + \psi_2 \frac{1}{\lambda_2 P_2 \pi P^d P_2^r} = \kappa^o h_o'(b_2^o),$$

$$\frac{\lambda_2^{OB}}{\lambda_2} - \psi_2 \frac{1}{\lambda_2 P_2 \pi P^d (1-P_2^r)} = \kappa^o h_o'(bf_2^o),$$

$$\frac{\lambda_2^{OB}}{\lambda_2} = \kappa^o h_o'(u_2^o).$$

In terms of the retraining effort constraints, if ψ_1 and ψ_2 are equal to zero, then based on the FOCs, we obtain $B_1^o = BF_1^o$ and $b_2^o = bf_2^o$ which violates the retraining effort constraints. In the time-consistent optimum, both MR_1 and MR_2 need to bind for $B_1^o > BF_1^o$ and $b_2^o > bf_2^o$.

If the government does not offer labour retraining, then in all time-consistent optimum: $u_1^o = BF_1^o > V_1^o$, $u_2^o = bf_2^o > v_2^o$.

When the government offers labour retraining and knows who needs it, the relations of $u_1^o > V_1^o$ and $u_2^o > v_2^o$ remains because their FOCs are not changed. Comparing the other FOCs we found the following relations in the time-consistent optimum,

- $h_o'(B_1^o) > h_o'(u_1^o) > h_o'(BF_1^o)$ means that $B_1^o > u_1^o > BF_1^o$,
- $h_o'(b_2^o) > h_o'(u_2^o) > h_o'(bf_2^o)$ means that $b_2^o > u_2^o > bf_2^o$,
- $h_o'(B_1^o) > \frac{Eh'(V_1^o)}{1-(1-\pi)P^d}$ meant that $B_1^o > V_1^o$,

- $h'_o(b_2^o) > \frac{Eh'(v_2^o)}{1-\pi P^d}$ meant that $b_2^o > v_2^o$.

When the government cannot identify who needs retraining: $B_1^o = V_1^o$, $b_2^o = v_2^o$, $u_1^o > BF_1^o$ and $u_2^o > bf_2^o$. The relation of B_1^o and V_1^o to u_1^o and relation of b_2^o and v_2^o to u_2^o are now uncertain.

Appendix for Chapter 5

A5.1: Proofs of Propositions 5.1 and 5.2 of the Simple Legislature in the Infinite Horizon Model

We will write down the steps used to prove Propositions 5.1 and 5.2.

The Transfers when Group 1's Legislator a_1 is the Agenda Setter

First, examine the transfers f^2 and f^3 when group 1's legislator is the agenda setter. In any equilibrium, the agenda setter will only seek the support of one other legislator since this is enough to implement the proposed policies and receiving the support of the third legislator would require additional resources that could be taken as rent for the agenda setter. The voters of groups 2 and 3 want to be included in the legislator a_1's coalition to obtain any redistribution to their group. Given that the legislators are the same in the eyes of the agenda setter as potential partners because all legislators receive the same payoffs in the event of a default, the voters engage in a Bertrand competition to undercut each other to the point which $f^2 = f^3 = 0$ to be more attractive to the agenda setter.

The Transfers when Group 3's Legislator a_3 is the Agenda Setter

The same incentive exists between voters of groups 1 and 2. As a result of underbidding again $f^2 = 0$. For the HE spending f^e, denote $f^e = f + A$ which $P_1 f = \tau$ and A is the additional spending on HE that pushes voter 1's income above the tax level. Due to the Bertrand competition against voters from group 2, $A = 0$ in equilibrium.

The Legislators' Rents when Group 1's Legislator is the Agenda Setter

For the agenda setter, he wants to give his coalition partner enough transfer so that the partner is indifferent between voting yes and being re-elected and voting no, receiving the default payoff $\frac{3-\frac{1}{P_1}}{3}\bar{r}$ and losing the election, which means the agenda setter will never offer more than

$$(a5.1) \qquad r^m = \frac{3 - \frac{1}{P_1}}{3}\bar{r} - \beta W$$

to his coalition partner.

If the agenda setter does not seek reappointment, first he has to offer $\frac{3-\frac{1}{P_1}}{3}\bar{r}$ to the coalition partner for him to agree on this plan. Since the agenda setter does not care about re-election and pleasing his voters here, the agenda setter will appropriate all resources, meaning $g = f^2 = f^3 = 0$, $P_1 f^e = \tau = 1$. Consequently, the agenda setter seeks re-election if and only if

$$(a5.2) \qquad r^a + \beta W \geq 3 - \frac{1}{P_1} - \frac{3 - \frac{1}{P_1}}{3}\bar{r}.$$

By (a5.1) and (a5.2), legislators a and m will implement a policy leading to their reappointment if and only if

$$(a5.3) \qquad r = r^a + r^m \geq 3 - 2\beta W - \frac{1}{P_1}.$$

The optimal voting rule must satisfy (a5.3) for the legislators to want to be re-elected. This is an incentive constraint that the voters need to consider when setting their voting rule/reservation utility.

The Legislators' Rents when Group 3's Legislator is the Agenda Setter

The level of aggregate rents still must satisfy (a5.3). This is not changed when the legislator is from a traditional group.

The Equilibrium Outcome when Group 1's Legislator is the Agenda Setter

As $f^2 = f^3 = 0$, the policy maximizing the utility of voters in group 1 is the solution to

$$Max \;\; P_1 f^e - \tau + H(g),$$

subject to the constraint for the maximum level of tax, $\tau \leq 1$, the government budget constraint, $3\tau = f^e + r + g$, and the incentive constraint (a5.3), $r \geq 3 - 2\beta W - \frac{1}{P_1}$.

Combine the budget and the incentive constraints and obtain: $3\tau - f^e - g \geq 3 - 2\beta W - \frac{1}{P_1}$, substitute in $f^e = \frac{t}{P_1} + A$, we have

$$3\tau - \frac{t}{P_1} - 3 + \frac{1}{P_1} + 2\beta W \geq A + g.$$

The solution to this optimization problem implies $\tau = 1$, $g = \min\,[H'^{-1}(P_1), 2\beta W]$, $A = 2\beta W - g$ and $r = 3 - 2\beta W - \frac{1}{P_1}$. Since the voters already know their legislator's political position when they set their reservation utilities at stage (2), in equilibrium, the reservation utility would be set in which all legislators are re-elected. We can obtain

$$(a5.4) \qquad W = \frac{r}{3} + \beta W.$$

Substitute in the value of r, $r = 3 - 2\beta W - \frac{1}{P_1}$ and obtain

$$W = \frac{1 - \frac{1}{3}\frac{1}{P_1}}{1 - \frac{1}{3}\beta}.$$

Substitute W into the equilibrium results we obtain Proposition 5.1.

The Equilibrium Outcome when Group 3's Legislator is the Agenda Setter

In equilibrium $A = f^2 = 0$ and the policy is maximizing the utility of voters from group 3,

$$Max\ 1 - \tau + f^3 + H(g),$$

subject to the constraint for the maximum level of tax, $\tau \leq 1$, the government budget constraint, $3\tau = f^3 + \frac{t}{P_1} + r + g$ and the same incentive constraint (a5.3), $r \geq 3 - 2\beta W - \frac{1}{P_1}$. Combine the incentive constraint and the budget constraint we have

$$3(\tau - 1) + \frac{1}{P_1} - \frac{t}{P_1} + 2\beta W \geq f^3 + g.$$

The solution is $\tau = 1$, $g = \min\,[H'^{-1}(1), 2\beta W]$, $f^3 = 2\beta W - g$ and $r = 3 - 2\beta W - \frac{1}{P_1}$. The value of W remains the same as before. Substitute W and obtain Proposition 5.2.

A5.2: The Assumption for the Incentives Facing the Voters of the Tax Agenda Setter in the Presidential Regime

Let's consider the case without the restriction on the maximum level of f^e first. If voters of group 1 of the tax agenda setter want to manipulate the spending agenda setter from group 3 into enacting a policy to change the level of public goods into the level of public goods preferred by voters of group 1, it will always result in the spending agenda setter not being re-elected by group 3's voters. The level of public goods that voters of group 1 want the spending agenda setter to implement might not be their optimal level of $H'^{-1}(P_1)$. There are three different scenarios for a policy combination of g and f^e here that results in the spending agenda setter enacting the policy instead of wanting the default outcome or taking everything. Note the policy proposal by the tax agenda setter would be approved by the legislator of group 2, since in this disequilibrium if voters of group 2 set their reservation utility differs from the policy desired by the voters of group 1, the tax agenda setter will enact the policy which causes everyone not re-elected and everything is taken. We consider here the case in which the voters of group 2 set its reservation utility equal to the policy desired by group 1 and jointly manipulate the spending agenda setter. If the spending agenda setter enacts the policy desired by group 1, he would pay the lower level rent to the coalition partner since the coalition partner is re-elected.

1. First, the spending agenda setter is indifferent between enacting the policy desired by group 1 and the default outcome but strictly prefers both to take away everything. In terms of utility, for the indifference

$$3\tau - g - f^e - \left(\frac{3 - \frac{1}{P_1}}{3} \tau - \beta W \right) = \frac{3 - \frac{1}{P_1}}{3} \tau,$$

$$g + f^e = \tau + 2 \frac{\frac{1}{P_1}}{3} \tau + \beta W.$$

Since the spending agenda setter strictly preferred enacting the desired policy to taking away everything, we obtain

$$3\tau - g - f^e - \left(\frac{3 - \frac{1}{P_1}}{3} \tau - \beta W \right) > 3\tau - f^e - \left(\frac{3 - \frac{1}{P_1}}{3} \tau \right),$$

$$g < \beta W.$$

Since the spending agenda setter strictly preferred the default outcome to take away everything, we obtain

$$\frac{3 - \frac{1}{P_1}}{3} \tau > 3\tau - f^e - \left(\frac{3 - \frac{1}{P_1}}{3} \tau \right),$$

$$f^e > \tau + 2 \frac{\frac{1}{P_1}}{3} \tau.$$

2. Second, the spending agenda setter is indifferent between enacting the policy desired by group 1 and taking away everything but strictly prefers both to the default outcome. In terms of the utilities, for the indifference relation we obtain

$$3\tau - g - f^e - \left(\frac{3 - \frac{1}{P_1}}{3} \tau - \beta W \right) = 3\tau - f^e - \left(\frac{3 - \frac{1}{P_1}}{3} \tau \right),$$

$$g = \beta W.$$

Since the spending agenda setter strictly preferred enacting the desired policy to the default outcome, we obtain

$$3\tau - g - f^e - \left(\frac{3 - \frac{1}{P_1}}{3} \tau - \beta W \right) > \frac{3 - \frac{1}{P_1}}{3} \tau,$$

$$g + f^e < \tau + 2 \frac{\frac{1}{P_1}}{3} \tau + \beta W.$$

Since the spending agenda setter strictly preferred taking away everything to the default outcome, we obtain

$$3\tau - f^e - \left(\frac{3 - \frac{1}{P_1}}{3} \tau \right) > \frac{3 - \frac{1}{P_1}}{3} \tau,$$

$$f^e < \tau + 2 \frac{\frac{1}{P_1}}{3} \tau.$$

3. Third, the spending agenda setter is indifferent between all three options, enacting the desired policy, taking away everything and the default outcome. The first indifference utility relation is

$$3\tau - g - f^e - \left(\frac{3 - \frac{1}{P_1}}{3} \tau - \beta W \right) = 3\tau - f^e - \left(\frac{3 - \frac{1}{P_1}}{3} \tau \right),$$

$$g = \beta W.$$

For the second indifference utility relations, we obtain

$$3\tau - g - f^{e} - \left(\frac{3 - \frac{1}{P_1}}{3}\tau - \beta W \right) = \frac{3 - \frac{1}{P_1}}{3}\tau,$$

$$g + f^{e} = \tau + 2\frac{\frac{1}{P_1}}{3}\tau + \beta W.$$

For the third indifference utility relations, we obtain

$$3\tau - f^{e} - \left(\frac{3 - \frac{1}{P_1}}{3}\tau \right) = \frac{3 - \frac{1}{P_1}}{3}\tau,$$

$$f^{e} = \tau + 2\frac{\frac{1}{P_1}}{3}\tau.$$

Let's consider the assumption that the optimal level of public goods preferred by group 3's voters, g^{*}, is greater than or equal to βW, $g^{*} \geq \beta W$. This meant $H'(g) > 1$ when $g < \beta W$. Would this remove the incentive for group 1's voters to manipulate the level of public goods? This would cover the second and the third scenario but it would not cover the first. Now in addition to the assumption of $g^{*} \geq \beta W$, we are also assuming the optimal level of policy preferred by voters of group 1, denoted by g^{1}, is also greater than or equal to βW, $g^{1} \geq \beta W$ which meant $H'(g) > \max\,[P_1, 1]$ when $g < \beta W$. Consequently, if he is trying to manipulate the level of public goods he would not be able to reach his preferred level g^{1} and that his optimal level of utility cannot be achieved in the first scenario. With our assumption, it meant that if the group 1's voters could act as a dictator that only pays $\frac{3-\frac{1}{P_1}}{3}\tau$ to spending agenda setter and $\frac{3-\frac{1}{P_1}}{3}\tau - \beta W$ to his tax agenda setter to obtain his preferred outcome, he will set $g \geq \beta W$ and $f^e = \tau + 2\frac{\frac{1}{P_1}}{3}\tau + \beta W - g \leq \tau + 2\frac{\frac{1}{P_1}}{3}\tau$.

Whether $P_1 > 1$ or $P_1 < 1$, the level of utility the voters of group 1 obtain in the equilibrium with all legislators re-elected ($f^e = \tau + 2\frac{\frac{1}{P_1}}{3}\tau$ and $g = g^*$) is greater than the level of utility obtain in the first scenario with ($f^e = \tau + 2\frac{\frac{1}{P_1}}{3}\tau + \beta W - g$ and $g < \beta W$). With all of the assumptions, it provides better utility to the voters of group 1 when they go along with the equilibrium, even if they preferred a different level of public goods than group 3, than trying to manipulate the level of g.

The same argument applies to the case when the group 1 voter's legislator is the spending agenda setter and the voters behind the tax agenda setter are a traditional group. The voters of the traditional group also have no incentive to change the level of the public good to be different from the preferred level set by the group 1 voters' spending agenda setter.

With the assumption on the maximum level of f^e in the presidential regime ($\max f^e = \tau + \frac{\frac{1}{P_1}}{3}\tau$), only the assumption of the optimal level of public goods preferred by group 3's voters, g^*, is greater than or equal to βW, $g^* \geq \beta W$ (or equivalently the assumption $H'(g) > 1$ when $g < \beta W$) is needed since the first case cannot occur as the level of f^e cannot exceed $\tau + 2\frac{\frac{1}{P_1}}{3}\tau$. Furthermore, when the group 1 voter's legislator is the spending agenda setter and voters behind the tax agenda setter are a traditional group, the level of public goods is maximized due to the restriction on the maximum level of f^e. The voters behind the tax agenda setter (the traditional group) have no intention of changing the level of public goods since it can only lead to a lower utility.

A5.3: Proofs of Propositions 5.3, 5.4 and 5.5 of the Presidential Regime in the Infinite Horizon Model

We will write down the steps used to prove Propositions 5.3, 5.4 and 5.5.

Stage (5) and (6) of the Game when a_{t1} a_{g3} is the Case

We begin at stages (5) and (6) of the game. The spending agenda setter a_{g3} take τ as given. In terms of the incentive compatibility constraint, the a_{g3} will offer the default payment $\frac{3-\frac{1}{P_1}}{3}\tau$ to the coalition partner if he plans no re-election. a_{g3} will seek reappointment if and only if

$$r^{a_{g3}} + \beta W \geq 3\tau - f^e - \frac{3-\frac{1}{P_1}}{3}\tau,$$

$$r^{a_{g3}} \geq 2\tau - f^e - \beta W + \frac{\frac{1}{P_1}}{3}\tau, \quad (a5.5)$$

when stage (4) default did not occur. Note that it is "f^e" written in (a5.5) as f^e is not certain to be equal to $\frac{t}{P_1}$ even when the agenda setter decides to not seek reappointment because of the restrictions/conditions on the tax rates. Since τ represents different tax rates for different groups conditioned on these different groups providing the same amount of tax revenue, with the tax agenda setter controlling the tax rates, he essentially has a large influence over f^e as well since it directly affects the relative income between different groups, e.g if a_{t1} set a low tax rate on voters of group 1 and high tax rates on other groups, a_{g3} must spend a high level of f^e to take τ from group 1 because otherwise, group 1 doesn't have enough income. Even when a_{g3} goes for default, he is still constrained by the taxation policy set previously. A higher level of f^e directly reduce the spending agenda setter's rents. In the simple legislature case without separation of powers, when the agenda setter can control both the spending and taxation, this constraint does not exist. For group 3, they are indifferent to the level of f^e and $r^{a_{g3}}$ in equilibrium. The incentive compatibility constraint for spending agenda setter's rents can be rewritten as

$$r^{a_{g3}} + f^e \geq 2\tau - \beta W + \frac{\frac{1}{P_1}}{3}\tau.$$

When group 1's legislator is the tax agenda setter, there is a minimum level of rent that needs to be given to the spending agenda setter for him to plan for re-election. This level of rent is "$\frac{3-\frac{1}{P_1}}{3}\tau - \beta W$" which gives spending agenda setter utility equal to $\frac{3-\frac{1}{P_1}}{3}\tau$, indifferent to the default outcome. We can calculate the maximum level of f^e in equilibrium:

$$2\tau - f^e + \frac{\frac{1}{P_1}}{3}\tau = \frac{3-\frac{1}{P_1}}{3}\tau,$$

$$f^e = \tau + 2\frac{\frac{1}{P_1}}{3}\tau.$$

At the maximum level of f^e, the spending agenda setter is indifferent between 1. Enact the desired policy and be re-elected. 2. The default outcome and no re-election. 3. Take everything and plan for no re-election.

When a_{g3} seek reappointment, he will offer

$$(a5.6) \qquad r^m = \frac{3 - \frac{1}{P_1}}{3}\tau - \beta W$$

to his junior coalition partner to win the approval of the spending plan. Adding up (a5.5) and (a5.6), the aggregate rents' incentive compatibility constraint for the legislators to want to be re-elected is

$$r = r^{a_{g3}} + r^m \geq 3\tau - f^e - 2\beta W,$$

$$f^e + r^{a_{g3}} + r^m \geq 3\tau - 2\beta W.$$

Together with the government budget constraint (5.1), we obtain

$$(a5.7) \qquad 2\beta W \geq f^2 + f^3 + g.$$

This is the maximum amount of public goods and transfers to groups 2 and 3 that the model can have while satisfying the incentive constraint of the agenda setter.

For the voters of group 1, because they are behind the tax agenda setter a_{t1}, they in effect have a large influence on the setting of f^e through their voting rule and they will not underbid anything against group 2 since even if a_{g3} does not take the tax agenda setter as the coalition partner, a_{g3} is still constrained by τ and f^e. Despite the assumption that the spending agenda setter could choose a replacement coalition partner the "Bertrand competition" still happens for the traditional groups who are not spending agenda setter's voters. If the legislator of group i is chosen first by the spending agenda setter and votes no, the spending proposal being voted on by the new coalition partner will not give any transfers to group i. When the spending proposal would result in re-elections for the legislators, the legislator chosen first as the coalition partner would always vote yes to the proposal, so only the group chosen as the initial partner will receive any possible transfers and the group chosen first is the group demanding the smaller transfers. For the voters of group 2 whose legislator is not any agenda setter, for them to gain any chance of being included in the coalition, they will set their reservation utility such that $f^2 = 0$ since if they set $f^2 > 0$, a_{g3} will not choose their legislator as the partner at all as it entails extra spending if a_{g3} wants to be re-elected.

For the equilibrium level of g and f^3, the voters in group 3 maximize their utility by taking τ as given and maximising $1 - \tau + f^3 + H(g)$ subject to (a5.7) with $f^2 = 0$. This gives $g = \min\,[H'^{-1}(1), 2\beta W]$, $f^3 = 2\beta W - g$, $f^e + r = 3\tau - 2\beta W$.

Under the assumptions that $H'(g) > 1$ when $g < \beta W$ and the maximum level of f^e is $\tau + 2\frac{\frac{1}{P_1}}{3}\tau$ for any given level of τ, the voters of group 1 will not obtain a higher utility by manipulating the level of public goods and transfers to differ from the level chosen by group 3.

Stage (3) and (4) of the Game when a_{t1} a_{g3} is the Case

Now we derive the tax level τ and the HE spending f^e that will be set in this model. Given that the voters behind the tax agenda setter are group 1, it is in group 1's voters' interest to set up the tax levels that maximize spending on f^e in equilibrium.

In the disequilibrium in which no legislators are re-elected, the agenda setter a_{t1} proposes a tax plan and f^e that is inconsistent with the reservation utility required by the voters, a_{t1} will set $\tau = 1$ and $f^e = \frac{1}{P_1}$. The agenda setter a_{g3} will agree with this tax plan and in the next stage the agenda setter a_{g3} will also take away everything by spending nothing on public goods and transfers f^2 and f^3 and leave his coalition junior partner with

$$1-\frac{\frac{1}{P_1}}{3}.$$

Since the probability of a_{t1} being chosen as the spending agenda setter's junior partner is $\frac{1}{2}$. The v^d, the expected utility of a_{t1} to go through with this tax plan and choose no re-election is $\frac{1}{2}-\frac{\frac{1}{P_1}}{6}$. If a_{t1} propose a tax plan that will be consistent with the reservation utility set up by group 1 voters, a_{t1} still only has a 50% chance to be chosen as the junior partner to a_{g3} and be offered r^m. For a_{t1} to make a proposal that satisfied the reservation utility of voter 1, a_{t1} must receive a payoff:

$$\frac{r^m}{2}+\beta W \geq \frac{1}{2}-\frac{\frac{1}{P_1}}{6}.$$

Substitute r^m with (a5.6), we obtain the condition for the minimum tax level τ

$$\tau \geq 1-\frac{3}{3-\frac{1}{P_1}}\beta W.$$

We assume the maximum level of f^e the tax agenda setter could set is $f^e=\tau+2\frac{\frac{1}{P_1}}{3}\tau$ for any given level of τ. This in effect is a restriction for the minimum tax rates that could be given to group 1 given the tax levels on other groups. The reason we added this assumption is that if the tax agenda setter can charge as high f^e as possible, he will propose a tax plan with $\tau=1$ that leads to the default outcome for the spending agenda setter by charging a high level of f^e because the expected utility is greater in default than taking away everything since it is certain he will receive $\frac{3-\frac{1}{P_1}}{3}$ in default instead of the 50% chance of being given $\frac{3-\frac{1}{P_1}}{3}$. This would lead to results in which the tax agenda setter does not want to be re-elected. Now with the assumption on the maximum level of f^e, the tax agenda setter cannot propose a tax plan that leads to default spending policies. At most, the spending agenda setter would be indifferent between taking away everything and default and in that case, we assume the spending agenda setter would choose to take away everything.

Both the voters of group 1 and a_{t1} wants to set τ at the maximum level at $\tau = 1$ but a_{t1} does not have a preference on the level of f^e since it does not affect r^m.

The voters of group 1 will set their reservation utility so a_{t1} will set the tax rates to make sure the level of f^e is maximized:

$$(a5.8) \qquad f^e = 1 + 2\frac{\frac{1}{P_1}}{3}.$$

In terms of tax rates, this would be a 100% tax rate on the income of voters of groups 2 and 3 while a low tax rate on the income of the group 1 voters themselves. This f^e in (a5.8) is the maximum amount of spending on the HE. It is equal to the maximum possible tax revenue three minus the spending on the public goods and group 3 transfer ($2\beta W$), the level of rents that needs to be paid to the junior partner by a_{g3} so the spending plan of a_{g3} will be approved and the minimum amount of rent that needs to be paid to the spending agenda setter a_{g3}. The level of rents to the spending agenda setter is equal to the rents to the junior partner with $r^{a_{g3}} = \frac{3-\frac{1}{P_1}}{3} - \beta W$. The voters of group 1 chose their reservation utility to set the level of f^e as the claimant of the residual tax revenue similar to the agenda setter in the simple legislature when he chose his rents.

Even if a_{g3} does not want to approve this plan, the other legislator of group 2 always will approve this since the amount r^m is maximized with $\tau = 1$ and is provided in the budget which is what the other legislator will receive if he is chosen to be the junior partner in stage (5). For the other legislator, it is self-defeating to vote no since a low tax level $\tau = \overline{r} < 1$ which means lower r^m would be the outcome.

Stage (5) and (6) of the Game when a_{t3} a_{g1} is the Case

Again the spending agenda setter a_{g1} takes τ as given. The incentive compatibility constraint, which is the required rent for a_{g1} to seek re-election is:

$$r^{a_{g1}} + \beta W \geq 3\tau - f^e - \frac{3-\frac{1}{P_1}}{3}\tau,$$

$(a5.9)$

$$r^{a_{g1}} \geq 2\tau - f^e - \beta W + \frac{\frac{1}{P_1}}{3}\tau.$$

When the a_{g1} is seeking reappointment, he offers

$$r^m = \frac{3-\frac{1}{P_1}}{3}\tau - \beta W$$

to his junior partner to win his approval. Thus the aggregate rents in the equilibrium must at least be

$(a5.10)$

$$r = r^{a_{g1}} + r^m \geq 3\tau - f^e - 2\beta W.$$

Again in (a5.9) and (a5.10) f^e is included, the reason is the same as in the case with $a_{\tau 1}$ a_{g3}.

Together with the government budget constraint, (a5.10) implies the voters cannot obtain more public goods and transfers than

$$2\beta W \geq f^2 + f^3 + g.$$

For the transfers to the voters not in the spending agenda setter a_{g1}'s group, the answer is the same as in the original model in Persson, Roland and Tabellini (2000). Both voters in groups 2 and 3 will engage in Bertrand competition which they underbid each other on the levels of f^2 and f^3 to be chosen first as the coalition partner with a_{g1}. This results in $f^2 = f^3 = 0$ in the equilibrium, group 2 and 3's voters demand no transfers from their representative.

The optimal level of public goods from the point of view of group 1's voters given $2\beta W \geq g$ should be

$$g = 2\beta W$$

and the level of HE spending f^e is

$$f^e = \tau + 2\frac{\frac{1}{P_1}}{3}\tau.$$

We have assumed the maximum level of f^e the tax agenda setter could charge at a given level of τ. Group 1's voters maximize their utility by asking for the highest level of g and the highest possible level of f^e given τ. The spending agenda setter is indifferent between the equilibrium outcome with re-election, taking away everything and the default outcome. If the group 1's voters' optimal level of g is above $2\beta W$ and they try to let the spending agenda setter charge a higher level of g by lowering f^e, it will always result in the spending agenda setter plan for taking everything and not be re-elected since it offers them more utility. The level of f^e is controlled by the tax agenda setter, if voters of group 1 demand a greater level of f^e than the maximum amount given τ, it will simply lead the tax agenda setter to propose the no re-election tax plan with $\tau = 1$ and $f^e = \frac{1}{P_1}$ since the tax agenda setter knows he will not be re-elected if he tries to match the reservation utility of group 1.

Although the level of f^e is now controlled by the tax agenda setter a_{t3}, the group 3's voters only want to minimize the tax level τ and are indifferent to the level of f^e as long as τ is minimized. Higher spending on f^e does not increase the tax level because the higher f^e comes from reducing the rents given to the spending agenda setter. Therefore, because group 1's voters want the level of f^e to be maximized given τ:

$$f^e = \tau + 2\frac{\frac{1}{P_1}}{3}\tau,$$

the rents to the spending agenda setter a_{g1} is minimized at

$$r^{a_{g1}} = \frac{3 - \frac{1}{P_1}}{3}\tau - \beta W.$$

Stage (3) and (4) of the Game when a_{t3} a_{g1} is the Case

Now we examine the incentive facing the voters of a_{t3}. Since in equilibrium these voters will receive zero transfers, they would want the level of the tax burden to be as low as possible. The optimal voting rule of group 3's voters requires a_{t3} to set the taxes as low as possible without a_{t3} planning for no re-election. Using the same method as in the case with $a_{\tau 1}$ a_{g3}, we reach the same condition for the minimum tax level τ

$$\tau \geq 1 - \frac{3}{3 - \frac{1}{P_1}} \beta W$$

that is needed for agenda setter a_{t3} to propose a plan that led to his re-election. We assume the tax level $\tau = 1 - \frac{3}{3-\frac{1}{P_1}} \beta W$ is enough to finance the maximum amount of public goods, $1 - \frac{3}{3-\frac{1}{P_1}} \beta W > \frac{2}{3} \beta W$. The optimal voting rule for the voters of a_{t3} is to let a_{t3} propose tax rates such that

$$\tau = 1 - \frac{3}{3 - \frac{1}{P_1}} \beta W.$$

This tax proposal would always be approved by the third legislator, the legislator from group 2. If the legislator of group 2 votes no he will cause the default tax level $\tau = \overline{r} < 1$ to occur. If $\overline{r} < 1 - \frac{3}{3-\frac{1}{P_1}} \beta W$, then this is self-defeating since it meant a lower level of rent r^m the legislator would obtain if he became the a_{g1}'s coalition partner next period. If $\overline{r} > 1 - \frac{3}{3-\frac{1}{P_1}} \beta W$, then if legislator 2 votes no and causes the default τ to occur, none of the legislators would be re-elected. Since $\frac{r^m}{2} + \beta W \geq v^d$ and $\overline{r} < 1$, the legislator receives a lower expected utility in this default outcome with no re-election than to accept the proposal made by a_{t3}. The tax agenda setter is indifferent to the level of f^e because the utility of the tax agenda setter is not directly dependent on the level of f^e. Since the tax agenda setter needs to make sure the spending agenda setter can meet the reservation utility of group 1 to be re-elected himself, the tax agenda setter will announce a tax proposal that provides the maximum level of f^e given the minimized tax level.

For the voters of group 1, they want the tax level τ to be maximized but they are not the tax agenda setter's group in this case. The tax agenda setter's re-election is dependent on the reservation utility of group 3's voters who want the τ to be minimized.

Stage (5) and (6) of the Game when $a_{t3}\ a_{g2}$ is the Case

Now both the voters of the spending agenda setter and the tax agenda setter are a part of the traditional group of voters. Group 1's legislator is not an agenda setter.

The level of public goods and transfer to group 2 is the same as in the case with $a_{\tau 1}\ a_{g3}$. In equilibrium we obtain

$$g = \min[H'^{-1}(1), 2\beta W]\,, f^2 = 2\beta W - g, f^e + r = 3\tau - 2\beta W.$$

Stage (3) and (4) of the Game when $a_{t3}\ a_{g2}$ is the Case

The voters of the tax agenda setter want to minimize the tax level τ but they are indifferent to the level of f^e given τ. The minimum level of τ is the same as before with $\tau = 1 - \frac{3}{3-\frac{1}{P_1}}\beta W$.

The tax agenda setter is also indifferent to the level of f^e as it does not affect the level of rent he would receive if he is chosen as the coalition partner. Therefore, although the group 1's voters' legislator is not in any position of power, they should set their reservation utility with the maximum level of f^e given τ to maximize their utility.

A question here is that is there any incentive for the tax agenda setter to propose $\tau = 1 - \frac{3}{3-\frac{1}{P_1}}\beta W$ but the level of HE spending being minimized at $f^e = \frac{\tau}{P_1}$ so that he and the spending agenda setter would be re-elected as groups 2 and 3's reservation utilities are met while legislator 1 will not be re-elected. The aim of the tax agenda setter is that the spending agenda setter will choose him as the coalition partner for certain. Under our assumption of the spending agenda setter's coalition formation at stage 6, the fact that the spending agenda setter could choose the other legislator as the replacement coalition partner to vote again on the same proposal removed this probability as even if the tax agenda setter's proposal of $\tau = 1 - \frac{3}{3-\frac{1}{P_1}}\beta W$ and $f^e = \frac{\tau}{P_1}$ is approved, both the tax agenda setter and the legislator of group 3 remain identical to the spending agenda setter despite the legislator 3 would not be re-elected under this proposal.

The reason is that when the spending agenda setter aims for re-election, the coalition partner can see his intention of re-election from the proposal he made. The legislator of group 1 knows he will not be re-elected due to the taxation constraint from the tax agenda setter and knows that the tax agenda setter will be re-elected. Then the legislator of group 1 will accept any positive level of rents even below the default outcome (as long as its level is $\geq \frac{3-\frac{1}{P_1}}{3}\tau - \beta W$) because he knows if he refuses, the tax agenda setter will be the next coalition partner and he is willing to accept $\frac{3-\frac{1}{P_1}}{3}\tau - \beta W$. The legislator of group 1 will vote yes despite knowing he will not be re-elected because he knows the tax agenda setter will vote yes and the default would not occur.

Therefore, in the equilibrium, the voters of group 3 maximize their utility by minimising the tax level τ and the voters of group 1 maximize their utility by maximising the level of f^e at the given level of τ. Since there is no incentive for the tax agenda setter to set a tax proposal that does not accommodate the level of f^e demanded by group 1, the tax agenda setter will satisfy the f^e required by voters of group 1 and result in the re-election of all legislators. The reason tax agenda setter will satisfy the required f^e is because that is what's needed for legislator 1 to accept the tax proposal and result in the tax agenda setter's re-election. In equilibrium we obtain:

$$\tau = 1 - \frac{3}{3 - \frac{1}{P_1}} \beta W,$$

$$f^e = \tau + 2 \frac{\frac{1}{P_1}}{3} \tau$$

and the level of rents are

$$r^{a_{g2}} = \frac{3 - \frac{1}{P_1}}{3} \left(1 - \frac{3}{3 - \frac{1}{P_1}} \beta W \right) - \beta W,$$

$$r^m = \frac{3 - \frac{1}{P_1}}{3} \left(1 - \frac{3}{3 - \frac{1}{P_1}} \beta W \right) - \beta W.$$

By the same argument, the voters of group 3 also cannot use the fact that their legislator controls the level of f^e to demand a positive level of f^3 by letting their tax agenda setter propose a tax plan that results in legislator 1's not being re-elected. Because if $f^3 > 0$, even if legislator 1 will not be re-elected and the spending agenda setter wishes for re-election, the spending agenda setter will approach legislator 1 first with a proposal that leads to the spending agenda setter's re-election with $f^3 = 0$. Legislator 1 will accept this proposal because he knows the spending agenda setter will offer a proposal to group 3 next that leads to re-election and default will not occur.

The Value of W in the Presidential Regimes

The value of W is obtained by the same equation as (a5.4), but the value of r is now different. In all three cases, the rents to both the spending agenda setter and the coalition partner are minimized given the level of τ. We have studied all three possible equilibria in the presidential regime with the group needing HE spending's legislator being at all three positions. Each case has an equal chance of appearing, and the equilibrium continuation value W can be calculated using the following equation:

$$W = \frac{1}{3}\left[\frac{1}{3}\left(2 - \frac{\frac{2}{P_1}}{3} - 2\beta W\right) + \frac{2}{3}\left(\left(2 - \frac{\frac{2}{P_1}}{3}\right)\left(1 - \frac{3}{3 - \frac{1}{P_1}}\beta W\right) - 2\beta W\right)\right] + \beta W,$$

$$(a5.11) \qquad W = \frac{2}{3}\frac{9}{9+\beta}\left(1 - \frac{1}{3}\frac{1}{P_1}\right)$$

which is smaller than the level of W in the simple legislature $\frac{1-\frac{1}{3}\frac{1}{P_1}}{1-\frac{1}{3}\beta}$. Substitute the value of W to obtain the equilibrium policy outcome in the three cases of presidential regimes.

A5.4: Proof of Proposition 5.6 of the Parliamentary Regime in the Infinite Horizon Model

We will write down the steps used to prove Proposition 5.6.

The Sum of the Legislator's Rents r^P in the Equilibrium of the Parliamentary Regime.

The equilibrium is solved by backward induction. We assume the government crisis occurs and analyse the outcomes in the subgame consisting of stages 6'-9'. There is a one-third probability the agenda setter a' would be the legislator representing the group needing HE spending and the equilibrium outcome would be:

$$g'_e = \min[H'^{-1}(P_1), 2\beta W], \tau' = 1, A' = 2\beta W - g'_e, f' = 0, r' = 3 - 2\beta W - \frac{1}{P_1}.$$

There is a two-thirds probability the agenda setter a' would be a legislator representing a traditional group and the equilibrium outcome of the simple legislature would be

$$g'_T = \min[H'^{-1}(1), 2\beta W], \tau' = 1, A' = 0, f' = 2\beta W - g'_T, r' = 3 - 2\beta W - \frac{1}{P_1}.$$

These equilibrium results are based on our findings of the equilibrium outcomes in the simple legislature section. W is common between the two possible equilibriums of the subgame and it is the equilibrium continuation value of holding office in the parliamentary regime.

The expected continuation value of reaching this subgame (where all legislators are re-elected) for all legislators is

$$(a5.12) \qquad E(v') = \frac{1}{3}r' + \beta W$$

since all legislator has a one-third chance of being the agenda setter a'. The expected utility for group 1 voters is,

$$(a5.13) \qquad E(u'_e) = \frac{1}{3}H(g'_e) + \frac{2}{3}H(g'_T) + \frac{1}{3}P_1 A', A' = 2\beta W - g'_e$$

and the expected utility for groups 2 and 3's voters is

$$E(u'_T) = \frac{1}{3}H(g'_e) + \frac{2}{3}H(g'_T) + \frac{1}{3}f', f' = 2\beta W - g'_T.$$

At stage 5, m will veto any proposal that does not give her the same utility as after the fall of the government coalition. m will accept the proposal that results in re-election for all legislators if $r^m + \beta W \geq E(v')$. The group outside the government which is group 2 in this case will have $f^2 = r^2 = 0$. As the agenda setter will not pay more than necessary for m's support, by (a5.12), $r^m = \frac{1}{3}r'$.

In the parliamentary regime, the agenda setter has the power to set both taxes and spending, which means if the agenda setter plans for no re-election he can simply take everything with $\tau = 1$, $f^e = \frac{1}{P_1}$, $g = f^2 = f^3 = 0$ and make m agree by giving him $\frac{1}{3}r' + \beta W$ for rents. Consequently, if the rent for the agenda setter is less than $\frac{2}{3}r'$, it would result in zero utilities for all voters, this value of the minimum rent level is obtained from $r^a + \beta W \geq 3 - \frac{1}{P_1} - \frac{1}{3}\left[3 - 2\beta W - \frac{1}{P_1}\right] - \beta W$ which led to $r^a \geq \frac{2}{3}r'$. The voters will not be able to reduce the equilibrium rents for the agenda setter below what he obtains after choosing no re-election, which implies the following incentive constraint

$$r^a \geq \frac{2}{3}r'.$$

In equilibrium, the voters will not leave excess rents to the legislators, this means the total rents is equal to $r^P = 3 - 2\beta W - \frac{1}{P_1}$.

Solve for W using the same equation as (a5.4) and obtain,

$$W = \frac{1}{3}r' + \beta W,$$

$$W = \frac{1 - \frac{1}{3}\frac{1}{P_1}}{1 - \frac{1}{3}\beta}.$$

This value of W is greater than the value of W in the presidential regime in (a5.11). Substitute W into the total level of political rents r^P in the parliamentary regime, we obtain

$$r^P = 3 - \frac{1}{P_1} - 2\beta \frac{1 - \frac{1}{3}\frac{1}{P_1}}{1 - \frac{1}{3}\beta}.$$

The Equilibrium Reservation Utilities Required by the Voters in the Governing Coalition

We substitute the equilibrium rents into the government budget constraint with $f^e = \frac{t}{P_1} + A$ and obtain

$$(a5.14) \qquad g + A + f^3 = 3(\tau - 1) - \frac{1}{P_1}(\tau - 1) + 2\beta \frac{1 - \frac{1}{3}\frac{1}{P_1}}{1 - \frac{1}{3}\beta}.$$

This is the only incentive constraint the voters need to be concerned with since the threat of the coalition to take away everything and plan for no re-election is already included in r^P.

The reservation utilities set by voters from group 1, $\overline{B^{a1}}$ and group 3, $\overline{B^{m3}}$ must be mutually consistent under the relevant constraints. The reservation utility $\overline{B^{a1}}$ must be optimal for the voters in group 1 in the equilibrium, giving the equilibrium value of $\overline{B^{m3}}$, and vice versa. However, this requirement is satisfied by many pairs of $(\overline{B^{a1}}, \overline{B^{m3}})$ because the voters choose their reservation utilities simultaneously. Hence there are multiple equilibria and the only policies that could be ruled out are policies dominated by the outcome/expected utility in the simple legislature from the point of view of the voters in the government coalition.

The voters know their expected utility when the government coalition falls due to the veto being triggered. Hence the equilibrium reservation utility for group 1 who needs HE spending f^e is

$$\overline{B^{i1}} \equiv P_1 f^e - \tau + H(g) \geq E(u'_e), i = a, m$$

and for group 3 which does not need f^e,

$$\overline{B^{i3}} \equiv 1 - \tau + H(g) + f^{i3} \geq E(u'_T), i = a, m.$$

"$i = a, m$" is used because the reservation utility condition is the same when the agenda setter represents group 3 and the junior partner represents group 1.

The Equilibrium Policy in the Parliamentary Regime with Agenda Setter a_1 and the Junior Partner m_3

Group 1 voters optimize their utility by choosing g, A, f^{m3} and τ, given any $\overline{B^{m3}}$ with their reservation utility also being satisfied:

$$Max \;\; P_1 A + H(g)$$

subject to

$$(a5.15) \qquad 1 - \tau + H(g) + f^{m3} \geq \frac{1}{3}H(g'_e) + \frac{2}{3}H(g'_T) + \frac{1}{3}\left[2\beta\frac{1 - \frac{1}{3}\frac{1}{P_1}}{1 - \frac{1}{3}\beta} - g'_T\right],$$

$$(a5.16) \qquad \tau \leq 1,$$

$(a5.17) \qquad\qquad A \geq 0,$

$(a5.18) \qquad\qquad f^{m3} \geq 0.$

Use the government budget constraint (a5.14) to replace the tax level

$$\tau = 1 - 2\beta\frac{\left(1-\frac{1}{3}\frac{1}{P_1}\right)}{\left(1-\frac{1}{3}\beta\right)}\frac{1}{3-\frac{1}{P_1}} + \frac{1}{3-\frac{1}{P_1}}g + \frac{1}{3-\frac{1}{P_1}}A + \frac{1}{3-\frac{1}{P_1}}f^{m3}$$

and denote, respectively, λ, μ, v^A, v^{m3} as the Lagrange multipliers of the constraints (a5.15)-(a5.18), results in the Lagrange function

$$L = P_1A + H(g)$$

$$+\lambda\left[1 - 1 + 2\beta\frac{\left(1-\frac{1}{3}\frac{1}{P_1}\right)}{\left(1-\frac{1}{3}\beta\right)}\frac{1}{3-\frac{1}{P_1}} - \frac{1}{3-\frac{1}{P_1}}g - \frac{1}{3-\frac{1}{P_1}}A - \frac{1}{3-\frac{1}{P_1}}f^{m3}\right.$$

$$\left. + H(g) + f^{m3} - \frac{1}{3}H(g'_e) - \frac{2}{3}H(g'_T) - \frac{1}{3}\left[2\beta\frac{1-\frac{1}{3}\frac{1}{P_1}}{1-\frac{1}{3}\beta} - g'_T\right]\right]$$

$$+\mu\left[1 - 1 + 2\beta\frac{\left(1-\frac{1}{3}\frac{1}{P_1}\right)}{\left(1-\frac{1}{3}\beta\right)}\frac{1}{3-\frac{1}{P_1}} - \frac{1}{3-\frac{1}{P_1}}g - \frac{1}{3-\frac{1}{P_1}}A - \frac{1}{3-\frac{1}{P_1}}f^{m3}\right]$$

$$+ v^AA + v^{m3}f^{m3}.$$

Obtain the following three FOCs,

$$\frac{\partial L}{\partial g} = H'(g) - \lambda\frac{1}{3-\frac{1}{P_1}} + \lambda H'(g) - \mu\frac{1}{3-\frac{1}{P_1}} = 0,$$

$$\frac{\partial L}{\partial A} = P_1 - \lambda\frac{1}{3-\frac{1}{P_1}} - \mu\frac{1}{3-\frac{1}{P_1}} + v^A = 0,$$

$$\frac{\partial L}{\partial f^{m3}} = -\lambda \frac{1}{3-\frac{1}{P_1}} + \lambda - \mu \frac{1}{3-\frac{1}{P_1}} + v^{m3} = 0.$$

We obtain

$$(a5.19) \qquad \lambda = P_1 + v^A - v^{m3},$$

$$(a5.20) \qquad H'(g) = \frac{P_1 + v^A}{1 + P_1 + v^A - v^{m3}}.$$

If $\tau < 1$, then the multiplier $\mu = 0$ and $\frac{\partial L}{\partial g}$ results in

$$H'(g) = \frac{P_1 + v^A - v^{m3}}{1 + P_1 + v^A - v^{m3}} \frac{1}{3-\frac{1}{P_1}} = \frac{P_1 + v^A}{1 + P_1 + v^A - v^{m3}}.$$

From the last two equalities, we obtain

$$(a5.21) \qquad -v^{m3} = \left[2 - \frac{1}{P_1}\right][P_1 + v^A].$$

Given the assumption of $P_1 > \frac{1}{2}$, (a5.21) is impossible even if $v^{m3} = v^A = 0$. Therefore we conclude that in the equilibrium of the parliamentary regime

$$\tau = 1.$$

For the level of public goods, g. If both $A > 0$ and $f^{m3} > 0$ in equilibrium which meant $v^{m3} = v^A = 0$, (a5.19) and (a5.20) imply $\lambda = P_1$ and $H'(g) = \frac{P_1}{1+P_1}$. If both $A = 0$ and $f^{m3} = 0$ in equilibrium we obtain $g = 2\beta \frac{1-\frac{1}{3}\frac{1}{P_1}}{1-\frac{1}{3}\beta}$ based on the government budget constraint.

Now consider the scenario in which $A = 0$ but $f^{m3} > 0$ which means for the multipliers $v^A > 0$ but $v^{m3} = 0$. The public goods g in (a5.20) is written as

$$(a5.22) \qquad H'(g) = \frac{P_1 + v^A}{1 + P_1 + v^A}.$$

We obtain the following relations

$$1 > \frac{P_1 + v^A}{1 + P_1 + v^A} > \frac{P_1}{1 + P_1}.$$

If $P_1 > 1$, then $P_1 > \frac{P_1+v^A}{1+P_1+v^A}$, otherwise the relation between P_1 and $\frac{P_1+v^A}{1+P_1+v^A}$ is uncertain.

Now consider the scenario in which $A > 0$ but $f^{m3} = 0$, for the multipliers this meant $v^A = 0$ but $v^{m3} > 0$, so (a5.20) is written as

$$(a5.23) \qquad H'(g) = \frac{P_1}{1 + P_1 - v^{m3}}.$$

We obtain the following relation

$$\frac{P_1}{1 + P_1 - v^{m3}} > \frac{P_1}{1 + P_1}.$$

The relations of 1 and P_1 to $\frac{P_1}{1+P_1-v^{m3}}$ is uncertain.

Now we compare the level of the public good in the two different cases of (a5.22) and (a5.23) to the level in the simple legislature subgame. For (a5.22), we focus on the utility of group 1 voters which in the case with $A = 0$ is only $H(g)$. Their reservation utility $\overline{B^{a1}}$ which must be

$\geq E(u_e')$. $E(u_e')$ is written at (a5.13). Note this analysis is only for the case in which $P_1 < 1$ since if $P_1 > 1$, the level of public goods is larger than the level in the simple legislature. To satisfy the reservation utility $\overline{B^{a1}}$, the equilibrium level of public goods need to satisfy the following condition:

$$H(g) \geq \frac{1}{3}H(g_e') + \frac{2}{3}H(g_T').$$

g_e' and g_T' are the level of public goods in the subsequent simple legislature game if the government falls. Since $P_1 < 1$, $g_e' > g_T'$. This condition states the level of public goods in the equilibrium can be lower than the level of public goods when group 1's legislator is chosen to be the agenda setter, g_e'.

For (a5.23), focus on the reservation utility of group 3's voters. In this equilibrium $A > 0$ but $f^{m3} = 0$, which means the utility of the voters from group 3 is only $H(g)$. To satisfy the reservation utility $\overline{B^{m3}}$, the equilibrium level of public goods needs to satisfy the following condition:

$$H(g) \geq \frac{1}{3}H(g_e') + \frac{2}{3}H(g_T').$$

If $P_1 < 1$, we have $g_e' > g_T'$, so it is possible in the equilibrium for $g < g_e'$. If $P_1 > 1$, then $g_e' < g_T'$ and it is possible in the equilibrium for $g < g_T'$.

A5.5: The Definitions of Gini and Top20, and Their Sources

Gini: The average of the Gini coefficient after tax and transfers that is based on available data from 2015 to 2019 in each country. Source: World Bank Open Data (2021) *Gini Index*. Available at: https://data.worldbank.org/indicator/SI.POV.GINI (Accessed: 1 June 2022).

Top20: The average of the top quintile pre-tax national income share of a country from 2015 to 2019. Source: WID.world (2022) *World Inequality Database*. Available at: https://wid.world/ (Accessed: 1 June 2022).

Appendix for Chapter 6

A6.1: Party n's Period 2 Tax Rate and the Public Goods

If party n is elected, the party n in period 2 is optimizing group n's utility by choosing τ_2 with g being substituted by the period 2 government budget constraint.

$$Max\ F(c_1^{n*}) + (1-\tau_2)L_2^n(\tau_2) + (1-\tau_1)L_1^n(\tau_1) - c_1^{n*} + V_1^n\big(1 - L_1^n(\tau_1)\big)$$
$$+ V_2^n\big(1 - L_2^n(\tau_2)\big) + \alpha^n H(g),$$

with g substituted by $g = (1+r)mf^e - b + \tau_2 L_2^n(\tau_2) + \tau_2 L_2^e(f^e, \tau_2)P(f^e)$.

Apply the first-order condition relationship derived from period 2 group n's labour supply, $1 - \tau_2 - V_{2x}^n\big(1 - L_2^n(\tau_2)\big) = 0$, we obtain

$$(FOC_n) \quad -L_2^n(\tau_2)$$
$$+ \alpha^n H'(g)\big[L_2^n(\tau_2) + \tau_2 L_{2\tau_2}^n(\tau_2) + L_2^e(f^e, \tau_2)P(f^e) + \tau_2 L_{2\tau_2}^e(f^e, \tau_2)P(f^e)\big]$$
$$= 0,$$

with $g = (1+r)mf^e - b + \tau_2 L_2^n(\tau_2) + \tau_2 L_2^e(f^e, \tau_2)P(f^e)$.

The FOC_n would determine the reaction function of the party n's period 2 tax rate. The reaction function of party n shows the period 2 party n's choices of τ_2 as a function of f^e and b, it is written as:

$$T^n = T^n(f^e, b).$$

The relationship of the party n's optimal level of τ_2 to changes in f^e and b could be obtained by applying the implicit function theorem to FOC_n. The level of government spending g in optimum is determined by the period 2 budget constraint with the optimum period 2 tax rate.

The term $[L_2^n(\tau_2) + \tau_2 L_{2\tau_2}^n(\tau_2) + L_2^e(f^e, \tau_2)P(f^e) + \tau_2 L_{2\tau_2}^e(f^e, \tau_2)P(f^e)]$ is the MTR. Based on FOC_n, the MTR must be positive at any optimum meaning the optimal level of g is increasing in τ_2.

A6.2: Party e's Period 2 Tax Rate and the Public Goods

If party e is elected, the party e in period 2 optimizes the utility of group e with respect to τ_2 with g being substituted by the period 2 government budget constraint.

$$Max\ F(c_1^{e*}) + P(f^e)(1 - \tau_2)L_2^e(f^e, \tau_2) + w - c_1^{e*} - (1 + r)mf^e + V_1^e(1) + V_2^e\big(1 - L_2^e(f^e, \tau_2)\big) + \alpha^e H(g),$$

with g substituted by $g = (1 + r)mf^e - b + \tau_2 L_2^n(\tau_2) + \tau_2 L_2^e(f^e, \tau_2)P(f^e)$.

Using the relationship obtained from the first-order condition of the group e's labour supply in period 2, $P(f^e)(1 - \tau_2) - V_{2x}^e\big(1 - L_2^e(f^e, \tau_2)\big) = 0$, we obtain

$$(FOC_e)\ \ -P(f^e)L_2^e(f^e, \tau_2) + \alpha^e H'(g)\big[L_2^n(\tau_2) + \tau_2 L_{2\tau_2}^n(\tau_2) + L_2^e(f^e, \tau_2)P(f^e) + \tau_2 L_{2\tau_2}^e(f^e, \tau_2)P(f^e)\big] = 0,$$

with $g = (1 + r)mf^e - b + \tau_2 L_2^n(\tau_2) + \tau_2 L_2^e(f^e, \tau_2)P(f^e)$.

The FOC_e would determine the reaction function of the party e's period 2 tax rate as a function of period 1 f^e and b

$$T^e = T^e(f^e, b).$$

The relationship of the party e's optimal level of τ_2 to changes in f^e and b could be obtained by applying the implicit function theorem to FOC_e. Same as Appendix A6.1, the optimal level of g chosen by party e is increasing in τ_2.

A6.3: Compare the Values of Period 2 Policies between the Two Parties

Compare the period 2 policy choices between party n and party e when facing the same level of f^e and b chosen by the period 1 party. The level of the period 2 tax rate determined by party n and party e is determined by FOC_n and FOC_e respectively. Since the level of g is determined by the period 2 government budget constraint and is increasing in the tax rate, the party with the higher tax rate will spend a higher level of g. The two FOCs can be simplified in the following way:

(FOC_n) $$\alpha^n H'(G^n)[MTR^n] = L_2^n(T^n),$$

(FOC_e) $$\alpha^e H'(G^e)[MTR^e] = P(f^e)L_2^e(f^e, T^e).$$

Both $H'(.)$ and [MTR] are positive and decreasing in tax rate τ_2. Substitute T^n from FOC_n into FOC_e for T^e. Here $H'(.)$ [MTR] are equalized between FOC_n and FOC_e. The only potential difference between the two terms are $\frac{1}{\alpha^n}L_2^n(T^n)$ and $\frac{1}{\alpha^e}P(f^e)L_2^e(f^e, T^n)$. Because the FOC is decreasing in τ_2, the relation between the optimum tax rate set by party n and party e in period 2 based on FOC_n and FOC_e are as follows:

- If $\frac{1}{\alpha^n}L_2^n(T^n) > \frac{1}{\alpha^e}P(f^e)L_2^e(f^e, T^n)$, then $T^n < T^e$ and $G^n < G^e$.
- If $\frac{1}{\alpha^n}L_2^n(T^n) < \frac{1}{\alpha^e}P(f^e)L_2^e(f^e, T^n)$, then $T^n > T^e$ and $G^n > G^e$.
- If $\frac{1}{\alpha^n}L_2^n(T^n) = \frac{1}{\alpha^e}P(f^e)L_2^e(f^e, T^n)$, then $T^n = T^e$ and $G^n = G^e$.

We obtain the same results if we substitute T^e for T^n.

A6.4: How Period 2 Policies are Affected by Period 1 Policy f^e based on the Reaction Functions

To understand how would party n optimally change the level of τ_2 due to a change in f^e, we used the implicit function theorem on equation FOC_n (derived in Appendix A6.1) and obtain

$$\frac{\partial \tau_2}{\partial f^e} = -\frac{\frac{\partial FOC_n}{\partial f^e}}{\frac{\partial FOC_n}{\partial \tau_2}}.$$

At the optimum value of T^n that maximize the utility of group n, $\frac{\partial FOC_n}{\partial \tau_2}$ must be negative. The sign of $\frac{\partial \tau_2}{\partial f^e}$ depends on the sign of $\frac{\partial FOC_n}{\partial f^e}$.

$$\frac{\partial FOC_n}{\partial f^e} = \frac{H''(G^n)}{H'(G^n)} L_2^n(T^n)\left[(1+r)m + T^n\left[P_f(f^e)L_2^e(f^e,T^n) + P(f^e)L_{2f}^e(f^e,T^n)\right]\right]$$

$$+ \alpha^n H'(G^n)\Big[P_f(f^e)L_2^e(f^e,T^n) + P(f^e)L_{2f}^e(f^e,T^n)$$

$$+ T^n\left[P_f(f^e)L_{2\tau_2}^e(f^e,T^n) + P(f^e)L_{2\tau_2 f}^e(f^e,T^n)\right]\Big],$$

with $G^n = (1+r)mf^e - b + T^n L_2^n(T^n) + T^n L_2^e(f^e,T^n)P(f^e)$.

$$\frac{\partial \tau_2}{\partial f^e} > 0 \text{ if } \frac{\partial FOC_n}{\partial f^e} > 0,$$

$$\frac{\partial \tau_2}{\partial f^e} < 0 \text{ if } \frac{\partial FOC_n}{\partial f^e} < 0.$$

When $V'(1-l)=\phi l$ with ϕ being a positive constant, we obtain $l_2^e=\frac{1}{\phi}P(f^e)(1-\tau_2)$, the term $\alpha^n H'(G^n)\left[P_f(f^e)L_2^e(f^e,T^n)+P(f^e)L_{2f}^e(f^e,T^n)+T^n\left[P_f(f^e)L_{2\tau_2}^e(f^e,T^n)+P(f^e)L_{2\tau_2 f}^e(f^e,T^n)\right]\right]$ is strictly positive. As $H''(g)\longrightarrow 0$, $\frac{\partial FOC_n}{\partial f^e}\longrightarrow R_{>0}$.

The same method of implicit function theorem is used to determine how party e optimally change the level of τ_2 due to a change in f^e but now FOC_e from Appendix A6.2 is used instead of FOC_n. We obtain the following,

$$\frac{\partial \tau_2}{\partial f^e}=-\frac{\frac{\partial FOC_e}{\partial f^e}}{\frac{\partial FOC_e}{\partial \tau_2}}.$$

The sign of T_f^e is determined by the sign of $\frac{\partial FOC_e}{\partial f^e}$.

$$\begin{aligned}\frac{\partial FOC_e}{\partial f^e}&=-\left[P_f(f^e)L_2^e(f^e,T^e)+P(f^e)L_{2f}^e(f^e,T^e)\right]\\&+\frac{H''(G^e)}{H'(G^e)}P(f^e)L_2^e(f^e,T^e)\Big[(1+r)m\\&+T^e\left[P_f(f^e)L_2^e(f^e,T^e)+P(f^e)L_{2f}^e(f^e,T^e)\right]\Big]\\&+\alpha^e H'(G^e)\Big[P_f(f^e)L_2^e(f^e,T^e)+P(f^e)L_{2f}^e(f^e,T^e)\\&+T^e\left[P_f(f^e)L_{2\tau_2}^e(f^e,T^e)+P(f^e)L_{2\tau_2 f}^e(f^e,T^e)\right]\Big],\end{aligned}$$

with $G^e=(1+r)mf^e-b+T^eL_2^n(T^e)+T^eL_2^e(f^e,T^e)P(f^e)$.

$$\frac{\partial \tau_2}{\partial f^e}>0 \text{ if } \frac{\partial FOC_e}{\partial f^e}>0,$$

$$\frac{\partial \tau_2}{\partial f^e}<0 \text{ if } \frac{\partial FOC_e}{\partial f^e}<0.$$

When $V'(1-l)=\phi l$ with ϕ being a positive constant, we obtain $l_2^e=\frac{1}{\phi}P(f^e)(1-\tau_2)$, the term $-\left[P_f(f^e)L_2^e(f^e,T^e)+P(f^e)L_{2f}^e(f^e,T^e)\right]+\alpha^e H'(G^e)\left[P_f(f^e)L_2^e(f^e,T^e)+P(f^e)L_{2f}^e(f^e,T^e)+T^e\left[P_f(f^e)L_{2\tau_2}^e(f^e,T^e)+P(f^e)L_{2\tau_2 f}^e(f^e,T^e)\right]\right]$ is strictly negative. Given that the function $H''(g)$ is negative, we conclude $\frac{\partial FOC_e}{\partial f^e}$ is also negative.

A6.5: Period 1 Party n's Decision on f^e without Government Debt

Assume party n is the incumbent government. At the start of period 1, party n knows that he will be re-elected next period with probability $p^n=1-p^e$. Take both party n and party e's reaction functions in period 2 into account, write the period 1 tax rate as a function of f^e ($T_1^n(f^e)$) based on the period 1 budget constraint and substitute the public goods with the period 2 budget constraint, the party n is maximizing the expected utility of group n by choosing f^e,

$$Max\ p^n\Big[F(c_1^{n*})+\big(1-T^n(f^e)\big)L_2^n\big(T^n(f^e)\big)+\big(1-T_1^n(f^e)\big)L_1^n\big(T_1^n(f^e)\big)-c_1^{n*}$$

$$+V_1^n\Big(1-L_1^n\big(T_1^n(f^e)\big)\Big)+V_2^n\Big(1-L_2^n\big(T^n(f^e)\big)\Big)+\alpha^n H(G^n)\Big]$$

$$+(1-p^n)\Big[F(c_1^{n*})+\big(1-T^e(f^e)\big)L_2^n\big(T^e(f^e)\big)+\big(1-T_1^n(f^e)\big)L_1^n\big(T_1^n(f^e)\big)$$

$$-c_1^{n*}+V_1^n\Big(1-L_1^n\big(T_1^n(f^e)\big)\Big)+V_2^n\Big(1-L_2^n\big(T^e(f^e)\big)\Big)+\alpha^n H(G^e)\Big],$$

with $T_1^n(f^e)L_1^n\big(T_1^n(f^e)\big)=f^e$,

$G^n=(1+r)mf^e+T^n(f^e)L_2^n\big(T^n(f^e)\big)+T^n(f^e)L_2^e(f^e,T^n(f^e))P(f^e)$,

$G^e=(1+r)mf^e+T^e(f^e)L_2^n\big(T^e(f^e)\big)+T^e(f^e)L_2^e(f^e,T^e(f^e))P(f^e)$.

$$\frac{\partial}{\partial f^e} = p^n \Bigg[-T_f^n(f^e)L_2^n(T^n) - L_1^n\big(T_1^n(f^e)\big)T_{1f}^n(f^e)$$

$$+ \alpha^n H'(G^n)\Bigg[(1+r)m + T_f^n(f^e)L_2^n(T^n) + T^n L_{2\tau_2}^n(T^n)T_f^n(f^e)$$

$$+ T_f^n(f^e)L_2^e(f^e, T^n)P(f^e)$$

$$+ T^n\left[\frac{\partial L_2^e\big(f^e,\ T^n(f^e)\big)}{\partial f^e}P(f^e) + L_2^e(f^e, T^n)P_f(f^e)\right]\Bigg]\Bigg]$$

$$+ (1-p^n)\Bigg[-T_f^e(f^e)L_2^n(T^e) - L_1^n\big(T_1^n(f^e)\big)T_{1f}^n(f^e)$$

$$+ \alpha^n H'(G^e)\Bigg[(1+r)m + T_f^e(f^e)L_2^n(T^e) + T^e L_{2\tau_2}^n(T^e)T_f^e(f^e)$$

$$+ T_f^e(f^e)L_2^e(f^e, T^e)P(f^e)$$

$$+ T^e\left[\frac{\partial L_2^e\big(f^e,\ T^e(f^e)\big)}{\partial f^e}P(f^e) + L_2^e(f^e, T^e)P_f(f^e)\right]\Bigg]\Bigg] = 0.$$

From FOC_n, the following relation is obtained,

$$T_f^n(f^e)\alpha^n H'(G^n)\big[L_2^n(T^n) + T^n L_{2\tau_2}^n(T^n) + L_2^e(f^e, T^n)P(f^e) + T^n L_{2\tau_2}^e(f^e, T^n)P(f^e)\big]$$

$$= T_f^n(f^e)L_2^n(T^n).$$

From FOC_e the following relation is obtained,

$$T_f^e(f^e)H'(G^e)\big[L_2^n(T^e) + T^e L_{2\tau_2}^n(T^e) + L_2^e(f^e, T^e)P(f^e) + T^e L_{2\tau_2}^e(f^e, T^e)P(f^e)\big]$$

$$= T_f^e(f^e)\frac{1}{\alpha^e}P(f^e)L_2^e(f^e, T^e).$$

$\frac{\partial}{\partial f^e}$ can be simplified into

$$(a6.1)\ \frac{\partial}{\partial f^e} = -L_1^n\big(T_1^n(f^e)\big)T_{1f}^n(f^e)$$

$$+ p^n\left[\alpha^n H'(G^n)\left[(1+r)m + T^n\big[P_f(f^e)L_2^e(f^e,T^n) + P(f^e)L_{2f}^e(f^e,T^n)\big]\right]\right]$$

$$+ (1-p^n)\Bigg[-T_f^e(f^e)L_2^n(T^e) + T_f^e(f^e)\frac{\alpha^n}{\alpha^e}P(f^e)L_2^e(f^e,T^e)$$

$$+ \alpha^n H'(G^e)\left[(1+r)m + T^e\big[P_f(f^e)L_2^e(f^e,T^e) + P(f^e)L_{2f}^e(f^e,T^e)\big]\right]\Bigg] = 0.$$

Using the implicit function theorem, the relationship between the optimal level of f^e and p^e is

$$\frac{\partial f^e}{\partial p^e} = -\frac{\frac{\partial(a6.1)}{\partial p^e}}{\frac{\partial(a6.1)}{\partial f^e}}.$$

Let f_n^e denote the f^e level chosen by the party n government in period 1. In any valid equilibrium that maximizes the expected utility of group n, the net marginal benefit of f^e needs to be decreasing with f^e since if the net marginal benefit of f^e is increasing in f^e, the optimal level of f^e would just be to set it as high as possible. Consequently, $\frac{\partial(a6.1)}{\partial f^e} < 0$ and the sign of $\frac{\partial f^e}{\partial p^e}$ is dependent on the sign of $\frac{\partial(a6.1)}{\partial p^e}$ which is

$$\frac{\partial(a6.1)}{\partial p^e} = \alpha^n(1+r)m[H'(G^e) - H'(G^n)]$$

$$+ \alpha^n \Big[H'(G^e)T^e\big[P_f(f_n^e)L_2^e(f_n^e, T^e) + P(f_n^e)L_{2f}^e(f_n^e, T^e)\big]$$

$$- H'(G^n)T^n\big[P_f(f_n^e)L_2^e(f_n^e, T^n) + P(f_n^e)L_{2f}^e(f_n^e, T^n)\big]\Big]$$

$$+ T_f^e(f_n^e)\left[\frac{\alpha^n}{\alpha^e}P(f_n^e)L_2^e(f_n^e, T^e) - L_2^n(T^e)\right].$$

This is the condition (6.7) in section 6.3.3. If $T^e = T^n$, then $\frac{\partial(a6.1)}{\partial p^e} = 0$.

If we differentiate $\alpha^n H'(g)\Big[(1+r)m + \tau_2\big[P_f(f^e)L_2^e(f^e, \tau_2) + P(f^e)L_{2f}^e(f^e, \tau_2)\big]\Big]$ with respect to the period 2 tax rate, then by Young's theorem the equivalence of cross derivative the resulting derivative is $\frac{\partial FOC_n}{\partial f^e}$. Under our assumption $V'(1-l) = \phi l$, as $H''(g) \longrightarrow 0$, we conclude that $\frac{\partial FOC_n}{\partial f^e}$ is strictly positive. Consequently, $\alpha^n H'(g)\Big[(1+r)m + \tau_2\big[P_f(f^e)L_2^e(f^e, \tau_2) + P(f^e)L_{2f}^e(f^e, \tau_2)\big]\Big]$ is increasing in period 2 tax rate. If $T^e > T^n$, $\alpha^n H'(G^e)\Big[(1+r)m + T^e\big[P_f(f^e)L_2^e(f^e, T^e) + P(f^e)L_{2f}^e(f^e, T^e)\big]\Big] - \alpha^n H'(G^n)\Big[(1+r)m + T^n\big[P_f(f^e)L_2^e(f^e, T^n) + P(f^e)L_{2f}^e(f^e, T^n)\big]\Big]$ is positive and if $T^e < T^n$, $\alpha^n H'(G^e)\Big[(1+r)m + T^e\big[P_f(f^e)L_2^e(f^e, T^e) + P(f^e)L_{2f}^e(f^e, T^e)\big]\Big] - \alpha^n H'(G^n)\Big[(1+r)m + T^n\big[P_f(f^e)L_2^e(f^e, T^n) + P(f^e)L_{2f}^e(f^e, T^n)\big]\Big]$ is negative.

In Appendix A6.4 under the same assumption, T_f^e is strictly negative. If $T^e > T^n$, $T_f^e(f_n^e)\left[\frac{\alpha^n}{\alpha^e}P(f_n^e)L_2^e(f_n^e, T^e) - L_2^n(T^e)\right]$ is positive and condition (6.7) is positive. If $T^e < T^n$, $T_f^e(f_n^e)\left[\frac{\alpha^n}{\alpha^e}P(f_n^e)L_2^e(f_n^e, T^e) - L_2^n(T^e)\right]$ is negative and condition (6.7) is negative.

From this specific example, we can conclude that in general if $\frac{\partial FOC_n}{\partial f^e}$ and $\frac{\partial FOC_e}{\partial f^e}$ have constant and opposite signs under the same period 1 policies, then the sign of condition (6.7) is certain.

A6.6: Period 1 Party e's Decision on f^e without Government Debt

If party e is the incumbent government, the method to solve the party e's optimal level of f^e is the same as in Appendix A6.5 but party e is maximizing the expected utility of group e. Since group e does not supply labour in period 1, party e will set the period 1 tax rate to raise enough revenue for f^e. We assume the optimal level of f^e for party e is below the maximum period 1 tax revenue.

$$Max\ p^n \left[F(c_1^{e*}) + P(f^e)\big(1 - T^n(f^e)\big)L_2^e\big(f^e, T^n(f^e)\big) + w - c_1^{e*} - (1+r)mf^e + V_1^e(1) + V_2^e\left(1 - L_2^e\big(f^e, T^n(f^e)\big)\right) + \alpha^e H(G^n)\right]$$
$$+ (1-p^n)\left[F(c_1^{e*}) + P(f^e)\big(1 - T^e(f^e)\big)L_2^e\big(f^e, T^e(f^e)\big) + w - c_1^{e*} - (1+r)mf^e + V_1^e(1) + V_2^e\left(1 - L_2^e\big(f^e, T^e(f^e)\big)\right) + \alpha^e H(G^e)\right],$$

with $G^n = (1+r)mf^e + T^n(f^e)L_2^n\big(T^n(f^e)\big) + T^n(f^e)L_2^e(f^e, T^n(f^e))P(f^e)$,

$$G^e = (1+r)mf^e + T^e(f^e)L_2^n\big(T^e(f^e)\big) + T^e(f^e)L_2^e(f^e, T^e(f^e))P(f^e).$$

$$\frac{\partial}{\partial f^e} = -(1+r)m$$

$$+ p^n \Bigg[P_f(f^e)(1-T^n)L_2^e(f^e,T^n) - P(f^e)T_f^n(f^e)L_2^e(f^e,T^n)$$

$$+ \alpha^e H'(G^n) \Bigg[(1+r)m + T_f^n(f^e)L_2^n(T^n) + T^n L_{2\tau_2}^n(T^n)T_f^n(f^e)$$

$$+ T_f^n(f^e)L_2^e(f^e,T^n)P(f^e)$$

$$+ T^n \left[\frac{\partial L_2^e\big(f^e,\ T^n(f^e)\big)}{\partial f^e} P(f^e) + L_2^e(f^e,T^n)P_f(f^e) \right] \Bigg] \Bigg]$$

$$+ (1-p^n) \Bigg[P_f(f^e)(1-T^e)L_2^e(f^e,T^e) - P(f^e)T_f^e(f^e)L_2^e(f^e,T^e)$$

$$+ \alpha^e H'(G^e) \Bigg[(1+r)m + T_f^e(f^e)L_2^n(T^e) + T^e L_{2\tau_2}^n(T^e)T_f^e(f^e)$$

$$+ T_f^e(f^e)L_2^e(f^e,T^e)P(f^e)$$

$$+ T^e \left[\frac{\partial L_2^e\big(f^e,\ T^e(f^e)\big)}{\partial f^e} P(f^e) + L_2^e(f^e,T^e)P_f(f^e) \right] \Bigg] \Bigg] = 0.$$

From FOC_e the following relation is obtained

$$T_f^e(f^e)\alpha^e H'(G^e)\left[L_2^n(T^e) + T^e L_{2\tau_2}^n(T^e) + L_2^e(f^e,T^e)P(f^e) + T^e L_{2\tau_2}^e(f^e,T^e)P(f^e)\right]$$
$$= T_f^e(f^e)L_2^e(f^e,T^e)P(f^e).$$

From FOC_n the following relation is obtained

$$T_f^n(f^e)H'(G^n)\left[L_2^n(T^n)+T^nL_{2\tau_2}^n(T^n)+L_2^e(f^e,T^n)P(f^e)+T^nL_{2\tau_2}^e(f^e,T^n)P(f^e)\right]$$

$$=T_f^n(f^e)\frac{1}{\alpha^n}L_2^n(T^n).$$

Same as the process in Appendix A6.5, $\frac{\partial}{\partial f^e}$ is simplified into

$$(a6.2)\ \frac{\partial}{\partial f^e}=-(1+r)m$$

$$+p^n\Bigg[P_f(f^e)(1-T^n)L_2^e(f^e,T^n)-P(f^e)T_f^n(f^e)L_2^e(f^e,T^n)$$

$$+\frac{\alpha^e}{\alpha^n}T_f^n(f^e)L_2^n(T^n)$$

$$+\alpha^eH'(G^n)\left[(1+r)m+T^n\left[P_f(f^e)L_2^e(f^e,T^n)+P(f^e)L_{2f}^e(f^e,T^n)\right]\right]\Bigg]$$

$$+(1-p^n)\Big[P_f(f^e)(1-T^e)L_2^e(f^e,T^e)$$

$$+\alpha^eH'(G^e)\left[(1+r)m+T^e\left[P_f(f^e)L_2^e(f^e,T^e)+P(f^e)L_{2f}^e(f^e,T^e)\right]\right]\Big]=0.$$

Use the implicit function theorem to determine whether party e would increase or decrease the level of f^e when its probability of re-election falls.

$$\frac{\partial f^e}{\partial p^n}=-\frac{\frac{\partial(a6.2)}{\partial p^n}}{\frac{\partial(a6.2)}{\partial f^e}}.$$

The same reasons as in Appendix A6.5, for any equilibrium that maximizes the expected utility of group e, the net marginal benefit of f^e is decreasing in f^e, the sign of $\frac{\partial f^e}{\partial p^n}$ is dependent on the sign of $\frac{\partial(a6.2)}{\partial p^n}$. Let f_e^e denote the f^e level chosen by the party e's government in period 1, $\frac{\partial(a6.2)}{\partial p^n}$ is written as

$$\frac{\partial(a6.2)}{\partial p^n} = P_f(f_e^e)[(1-T^n)L_2^e(f_e^e,T^n) - (1-T^e)L_2^e(f_e^e,T^e)]$$
$$+ \alpha^e(1+r)m[H'(G^n) - H'(G^e)]$$
$$+ \alpha^e \Big[H'(G^n)T^n\big[P_f(f_e^e)L_2^e(f_e^e,T^n) + P(f_e^e)L_{2f}^e(f_e^e,T^n)\big]$$
$$- H'(G^e)T^e\big[P_f(f_e^e)L_2^e(f_e^e,T^e) + P(f_e^e)L_{2f}^e(f_e^e,T^e)\big]\Big]$$
$$+ T_f^n(f_e^e)\left[\frac{\alpha^e}{\alpha^n}L_2^n(T^n) - P(f_e^e)L_2^e(f_e^e,T^n)\right].$$

This is the condition (6.8) in section 6.3.3. If $T^e = T^n$, then $\frac{\partial(a6.2)}{\partial p^n} = 0$.

If we differentiate $P_f(f^e)(1-\tau_2)L_2^e(f^e,\tau_2) + \alpha^e H'(g)\Big[(1+r)m + \tau_2\big[P_f(f^e)L_2^e(f^e,\tau_2) + P(f^e)L_{2f}^e(f^e,\tau_2)\big]\Big]$ with respect to the period 2 tax rate. Same as in Appendix A6.5 using Young's theorem the equivalence of cross derivative the resulting derivative is $\frac{\partial FOC_e}{\partial f^e}$. Under our assumption $V'(1-l) = \phi l$, $\frac{\partial FOC_e}{\partial f^e}$ is strictly negative. Consequently, $P_f(f^e)(1-\tau_2)L_2^e(f^e,\tau_2) + \alpha^e H'(g)\Big[(1+r)m + \tau_2\big[P_f(f^e)L_2^e(f^e,\tau_2) + P(f^e)L_{2f}^e(f^e,\tau_2)\big]\Big]$ is decreasing in period 2 tax rate. If $T^e > T^n$, $P_f(f^e)(1-T^n)L_2^e(f^e,T^n) + \alpha^e H'(G^n)\Big[(1+r)m + T^n\big[P_f(f^e)L_2^e(f^e,T^n) + P(f^e)L_{2f}^e(f^e,T^n)\big]\Big] - \Big[P_f(f^e)(1-T^e)L_2^e(f^e,T^e) + \alpha^e H'(G^e)\Big[(1+r)m + T^e\big[P_f(f^e)L_2^e(f^e,T^e) + P(f^e)L_{2f}^e(f^e,T^e)\big]\Big]\Big]$ is positive and if $T^e < T^n$, $P_f(f^e)(1-T^n)L_2^e(f^e,T^n) + \alpha^e H'(G^n)\Big[(1+r)m + T^n\big[P_f(f^e)L_2^e(f^e,T^n) +$

$P(f^e)L_{2f}^e(f^e,T^n)\big]\Big] - \Big[P_f(f^e)(1-T^e)L_2^e(f^e,T^e) + \alpha^e H'(G^e)\Big[(1+r)m +$

$T^e\big[P_f(f^e)L_2^e(f^e,T^e) + P(f^e)L_{2f}^e(f^e,T^e)\big]\Big]\Big]$ is negative.

Using the same method as in Appendix A6.5, we conclude if $T^e > T^n$, $T_f^n(f_e^e)\left[\frac{\alpha^e}{\alpha^n}L_2^n(T^n) - P(f_e^e)L_2^e(f_e^e,T^n)\right]$ and condition (6.8) is positive. If $T^e < T^n$, $T_f^n(f_e^e)\left[\frac{\alpha^e}{\alpha^n}L_2^n(T^n) - P(f_e^e)L_2^e(f_e^e,T^n)\right]$ and condition (6.8) is negative.

Similar to Appendix A6.5, we can also conclude that if $\frac{\partial FOC_n}{\partial f^e}$ and $\frac{\partial FOC_e}{\partial f^e}$ have constant and opposite signs under the same period 1 policies, then the sign of condition (6.8) is certain.

A6.7: Period 1 Government Decision on f^e with Government Debt

When party n is the incumbent and is choosing the optimal level of f^e and b to maximize the group n's expected utility (EU^n), it is solving the following problem with the period 1 tax rate written as $T_1^n(f^e - b)$ based on the period 1 government budget constraint:

$$\begin{aligned}
Max\ p^n \Big[&F(c_1^{n*}) + \big(1 - T^n(f^e,b)\big)L_2^n\big(T^n(f^e,b)\big) + \big(1 - T_1^n(f^e-b)\big)L_1^n\big(T_1^n(f^e-b)\big) \\
&- c_1^{n*} + V_1^n\Big(1 - L_1^n\big(T_1^n(f^e-b)\big)\Big) + V_2^n\Big(1 - L_2^n\big(T^n(f^e,b)\big)\Big) + \alpha^n H(G^n)\Big] \\
&+ (1-p^n)\Big[F(c_1^{n*}) + \big(1 - T^e(f^e,b)\big)L_2^n\big(T^e(f^e,b)\big) \\
&+ \big(1 - T_1^n(f^e-b)\big)L_1^n\big(T_1^n(f^e-b)\big) - c_1^{n*} + V_1^n\Big(1 - L_1^n\big(T_1^n(f^e-b)\big)\Big) \\
&+ V_2^n\Big(1 - L_2^n\big(T^e(f^e,b)\big)\Big) + \alpha^n H(G^e)\Big],
\end{aligned}$$

with $T_1^n(f^e-b)L_1^n\big(T_1^n(f^e-b)\big) = f^e - b,$

$$G^n = (1+r)mf^e - b + T^n(f^e,b)L_2^n\big(T^n(f^e,b)\big) + T^n(f^e,b)L_2^e\big(f^e,T^n(f^e,b)\big)P(f^e),$$

$$G^e = (1+r)mf^e - b + T^e(f^e,b)L_2^n\big(T^e(f^e,b)\big) + T^e(f^e,b)L_2^e(f^e,T^e(f^e,b))P(f^e).$$

We obtain the two following first-order conditions for the optimal level of f^e and b from differentiating with respect to f^e and b:

$$(FOC6.1)\ \frac{\partial EU^n}{\partial f^e}$$

$$= -L_1^n\big(T_1^n(f^e-b)\big)T_{1f}^n(f^e-b)$$

$$+ p^n\left[\alpha^n H'(G^n)\left[(1+r)m + T^n\left[P_f(f^e)L_2^e(f^e,T^n) + P(f^e)L_{2f}^e(f^e,T^n)\right]\right]\right]$$

$$+ (1-p^n)\Bigg[-T_f^e(f^e,b)L_2^n(T^e) + T_f^e(f^e,b)\frac{\alpha^n}{\alpha^e}P(f^e)L_2^e(f^e,T^e)$$

$$+ \alpha^n H'(G^e)\left[(1+r)m + T^e\left[P_f(f^e)L_2^e(f^e,T^e) + P(f^e)L_{2f}^e(f^e,T^e)\right]\right]\Bigg] = 0.$$

$$(FOC6.2)\ \frac{\partial EU^n}{\partial b}$$

$$= -L_1^n\big(T_1^n(f^e-b)\big)T_{1b}^n(f^e-b) + p^n[-\alpha^n H'(G^n)]$$

$$+ (1-p^n)\left[T_b^e(f^e,b)\left[\frac{\alpha^n}{\alpha^e}P(f^e)L_2^e(f^e,T^e) - L_2^n(T^e)\right] - \alpha^n H'(G^e)\right] = 0.$$

When the incumbent n's probability of re-election falls (p^e rises), whether the incumbent will increase or decrease the level of HE investment can be determined by applying Cramer's rule to $FOC6.1$ and $FOC6.2$:

$$\frac{\partial f^e}{\partial p^e} = -\frac{\det\begin{pmatrix}\frac{\partial FOC6.1}{\partial p^e} & \frac{\partial FOC6.1}{\partial b}\\ \frac{\partial FOC6.2}{\partial p^e} & \frac{\partial FOC6.2}{\partial b}\end{pmatrix}}{\det\begin{pmatrix}\frac{\partial FOC6.1}{\partial f^e} & \frac{\partial FOC6.1}{\partial b}\\ \frac{\partial FOC6.2}{\partial f^e} & \frac{\partial FOC6.2}{\partial b}\end{pmatrix}}.$$

For the determinant of the denominator matrix, it can be written as this:

$$\det\begin{pmatrix} \frac{\partial FOC6.1}{\partial f^e} & \frac{\partial FOC6.1}{\partial b} \\ \frac{\partial FOC6.2}{\partial f^e} & \frac{\partial FOC6.2}{\partial b} \end{pmatrix} = \det\begin{pmatrix} \frac{\partial^2(EU^n)}{\partial f^{e2}} & \frac{\partial^2(EU^n)}{\partial f^e \partial b} \\ \frac{\partial^2(EU^n)}{\partial b \partial f^e} & \frac{\partial^2(EU^n)}{\partial b^2} \end{pmatrix} > 0.$$

The denominator matrix is the Hessian matrix and for the optimum f^e and b to maximize the expected utility of group n, the Hessian matrix must be a negative definite symmetric matrix. For a matrix to be negative definite, the second-order leading principal minor is positive. The sign of $\frac{\partial f^e}{\partial p^e}$ is dependent on the sign of the determinant of the numerator matrix. Furthermore, the sign of $\frac{\partial FOC6.2}{\partial b} = \frac{\partial^2(EU^n)}{\partial b^2}$ must also be negative. For the other terms in the numerator matrix, $\frac{\partial FOC6.1}{\partial p^e}$ and $\frac{\partial FOC6.2}{\partial p^e}$ have already been derived, they are conditions (6.7) and (6.4) respectively.

$$\begin{aligned} \frac{\partial FOC6.1}{\partial p^e} = {} & \alpha^n(1+r)m[H'(G^e) - H'(G^n)] \\ & + \alpha^n \Big[H'(G^e)T^e\big[P_f(f^e)L_2^e(f^e,T^e) + P(f^e)L_{2f}^e(f^e,T^e)\big] \\ & - H'(G^n)T^n\big[P_f(f^e)L_2^e(f^e,T^n) + P(f^e)L_{2f}^e(f^e,T^n)\big]\Big] \\ & + T_f^e(f^e,b)\left[\frac{\alpha^n}{\alpha^e}P(f^e)L_2^e(f^e,T^e) - L_2^n(T^e)\right]. \end{aligned}$$

$$\frac{\partial FOC6.2}{\partial p^e} = \alpha^n[H'(G^n) - H'(G^e)] + \left[\frac{\alpha^n}{\alpha^e}P(f^e)L_2^e(f^e,T^e) - L_2^n(T^e)\right]T_b^e(f^e,b).$$

Based on the sign of the denominator matrix and the terms in the numerator matrix, the sign of $\frac{\partial f^e}{\partial p^e}$ is determined by the sign of the following term (with $\frac{\partial FOC6.2}{\partial b}$ denoted by $(-)$):

$$-\left[\frac{\partial FOC6.1}{\partial p^e}(-)-\frac{\partial FOC6.2}{\partial p^e}\frac{\partial FOC6.1}{\partial b}\right].$$

If $\frac{\partial FOC6.1}{\partial p^e}$ is positive and $\frac{\partial FOC6.2}{\partial p^e}$ and $\frac{\partial FOC6.1}{\partial b}$ have the same sign, then party n is certain to increase the level of f^e when its probability of re-election falls. If $\frac{\partial FOC6.1}{\partial p^e}$ is positive and $\frac{\partial FOC6.2}{\partial p^e}$ and $\frac{\partial FOC6.1}{\partial b}$ maintain having the same sign at all values of p^e, b and f^e, then party n is certain to set a higher level of f^e when it knows it will not be re-elected compared to when its re-election is certain. If $\frac{\partial FOC6.1}{\partial p^e}$ is negative and $\frac{\partial FOC6.2}{\partial p^e}$ and $\frac{\partial FOC6.1}{\partial b}$ have opposite signs, then party n is certain to decrease the level of f^e when its probability of re-election falls. If $\frac{\partial FOC6.1}{\partial p^e}$ is negative and $\frac{\partial FOC6.2}{\partial p^e}$ and $\frac{\partial FOC6.1}{\partial b}$ maintain having opposite signs for all values of p^e, b and f^e, then party n is certain to set a lower level of f^e when it knows it will not be re-elected compared to when its re-election is certain.

As $H''(g) \longrightarrow 0$, $(\frac{\partial FOC_n}{\partial b}, \frac{\partial FOC_e}{\partial b}) \longrightarrow 0$. If we differentiate $-\alpha^n H'(g)$ with respect to the period 2 tax rate, we obtain $\frac{\partial FOC_n}{\partial b}$. We conclude $-\alpha^n H'(g)$ is not changing with respect to the period 2 tax rate and $\frac{\partial FOC6.2}{\partial p^e}$ is zero as $T_b^e(f^e, b) \longrightarrow 0$. The sign of $\frac{\partial f^e}{\partial p^e}$ is solely determined by the sign of $\frac{\partial FOC6.1}{\partial p^e}$, the same as condition (6.7) and Proposition 6.1 applies.

When party e is the incumbent it is maximizing the expected utility of group e (EU^e) with respect to f^e and b. We assume in this case that the optimal period 1 tax rate is an interior solution meaning the incumbent e will not use the maximum possible period 1 tax revenue to supply its period 1 policies of f^e and b. The optimal level of f^e and b is determined by the two following first-order conditions:

$$(FOC6.3)\ \frac{\partial EU^e}{\partial f^e}$$

$$= -(1+r)m$$

$$+ p^n \Bigg[P_f(f^e)(1-T^n)L_2^e(f^e,T^n) - P(f^e)T_f^n(f^e,b)L_2^e(f^e,T^n)$$

$$+ \frac{\alpha^e}{\alpha^n} T_f^n(f^e,b)L_2^n(T^n)$$

$$+ \alpha^e H'(G^n)\left[(1+r)m + T^n\left[P_f(f^e)L_2^e(f^e,T^n) + P(f^e)L_{2f}^e(f^e,T^n)\right]\right]\Bigg]$$

$$+ (1-p^n)\Bigg[P_f(f^e)(1-T^e)L_2^e(f^e,T^e)$$

$$+ \alpha^e H'(G^e)\left[(1+r)m + T^e\left[P_f(f^e)L_2^e(f^e,T^e) + P(f^e)L_{2f}^e(f^e,T^e)\right]\right]\Bigg] = 0.$$

$$(FOC6.4)\ \frac{\partial EU^e}{\partial b}$$

$$= p^n \left[T_b^n(f^e,b)\left[\frac{\alpha^e}{\alpha^n}L_2^n(T^n) - P(f^e)L_2^e(f^e,T^n)\right] - \alpha^e H'(G^n)\right]$$

$$+ (1-p^n)[-\alpha^e H'(G^e)] = 0.$$

Whether the incumbent government e increases or decreases the level of HE investment when the probability of the party n's election chances increases can be determined by applying Cramer's rule:

$$\frac{\partial f^e}{\partial p^n} = -\frac{\det\begin{pmatrix} \frac{\partial FOC6.3}{\partial p^n} & \frac{\partial FOC6.3}{\partial b} \\ \frac{\partial FOC6.4}{\partial p^n} & \frac{\partial FOC6.4}{\partial b} \end{pmatrix}}{\det\begin{pmatrix} \frac{\partial FOC6.3}{\partial f^e} & \frac{\partial FOC6.3}{\partial b} \\ \frac{\partial FOC6.4}{\partial f^e} & \frac{\partial FOC6.4}{\partial b} \end{pmatrix}}.$$

Similar to its incumbent n's counterpart, the sign of $\frac{\partial f^e}{\partial p^n}$ is determined by the sign of the following condition (with $\frac{\partial FOC6.4}{\partial b}$ denoted by $(-)$):

$$-\left[\frac{\partial FOC6.3}{\partial p^n}(-) - \frac{\partial FOC6.4}{\partial p^n}\frac{\partial FOC6.3}{\partial b}\right].$$

The relations of $\frac{\partial FOC6.3}{\partial p^n}$ and $\frac{\partial FOC6.4}{\partial p^n}\frac{\partial FOC6.3}{\partial b}$ needed to make certain predictions of incumbent e's decision on f^e is the same as their counterpart for incumbent n. However, the sign of $\frac{\partial f^e}{\partial p^n}$ is only determined by the sign of "$-\left[\frac{\partial FOC6.3}{\partial p^n}(-) - \frac{\partial FOC6.4}{\partial p^n}\frac{\partial FOC6.3}{\partial b}\right]$" for the interior solutions of incumbent e, so we cannot compare the level of f^e between $p^n = 0$ and $p^n = 1$.

The above case for incumbent e is for an interior solution of the period 1 tax rate. If the incumbent e sets the period 1 tax rate to maximize the period 1 tax revenue, we could solve the optimal level of f^e for incumbent e with a single FOC like in Appendix 6.6 by writing b as a function of f^e based on the period 1 government budget constraint.

The same as the case when the incumbent is n, with $H''(g) \longrightarrow 0$, we conclude $\frac{\partial FOC6.4}{\partial p^n}$ is zero.

For both the interior solution and the corner solution for period 1 tax rate, the sign of $\frac{\partial f^e}{\partial p^n}$ is solely determined by the sign of $\frac{\partial FOC6.3}{\partial p^n}$, the same as condition (6.8) and Proposition 6.2 applies.

A6.8: MATLAB Code for Incumbent n

```
clc, clearvars

syms fe b t2n t1n t2e pn
an = 0.677;
tie = 0.13;
r = 0;
m = 1;
phi = 0.6;
ae = 1.323;
w = 1;
pfe = 1+fe^tie;

% Period 2 tax rates
afn = -1/(2*phi)*(1+pfe^2);
bfn = 1/(2*phi)*(2+2*pfe^2);
cfn = -((2*an+1/(2*phi))*(1+pfe^2)-(1+r)*m*fe+b);
dfn = an*(1+pfe^2)-(1+r)*m*fe+b;
focn = afn*t2n^3 + bfn*t2n^2 + cfn*t2n + dfn;
t2n = solve(focn, t2n, "MaxDegree",3);

afe = -1/(2*phi)*pfe^2*(1+pfe^2);
bfe = 1/(2*phi)*pfe^2*(2+2*pfe^2);
cfe = -((2*ae+1/(2*phi)*pfe^2)*(1+pfe^2)-(1+r)*m*fe*pfe^2+b*pfe^2);
dfe = ae*(1+pfe^2)-pfe^2*(1+r)*m*fe+pfe^2*b;
foce = afe*t2e^3 + bfe*t2e^2 + cfe*t2e + dfe;
t2e = solve(foce, t2e, "MaxDegree", 3);

i = 1;

% Incumbent n optimization problem
ln1 = 1/(2*phi)*(1-t1n);
ln2n = 1/(2*phi)*(1-t2n(i));
ln2e = 1/(2*phi)*(1-t2e(i));
le2n = 1/(2*phi)*pfe*(1-t2n(i));
le2e = 1/(2*phi)*pfe*(1-t2e(i));
cn2n = (1-t1n)*ln1+(1-t2n(i))*ln2n-1;
cn2e = (1-t1n)*ln1+(1-t2e(i))*ln2e-1;
ce2n = pfe*(1-t2n(i))*le2n+w-(1+r)*m*fe-1;
ce2e = pfe*(1-t2e(i))*le2e+w-(1+r)*m*fe-1;
gn = (1+r)*m*fe-b+t2n(i)*ln2n+t2n(i)*le2n*pfe;
ge = (1+r)*m*fe-b+t2e(i)*ln2e+t2e(i)*le2e*pfe;

upn = cn2n+1-phi*(ln1)^2+1-phi*(ln2n)^2;
hgn = log(gn);
upe = cn2e+1-phi*(ln1)^2+1-phi*(ln2e)^2;
hge = log(ge);
```

```
nonlcon = @confun;
x0 = [0.2, 0.2, 0.12];
A = [];
B = [];
Aeq = [];
Beq = [];
lb = [-inf, 0, -inf];
ub = [];


% Storing optimal values
optimal_t1n = zeros(1,101);
optimal_fe = zeros(1,101);
optimal_b = zeros(1,101);
optimal_t2n = zeros(1,101);
optimal_t2e = zeros(1,101);
optimal_un = zeros(1,101);
optimal_ln1 = zeros(1,101);
optimal_ln2n = zeros(1,101);
optimal_ln2e = zeros(1,101);
optimal_le2n = zeros(1,101);
optimal_le2e = zeros(1,101);
optimal_cn2n = zeros(1,101);
optimal_cn2e = zeros(1,101);
optimal_ce2n = zeros(1,101);
optimal_ce2e = zeros(1,101);
optimal_gn = zeros(1,101);
optimal_ge = zeros(1,101);
utiln_el = zeros(1,101);
utiln_unel = zeros(1,101);

pn_values = 0:0.01:1;
for re=1:length(pn_values)
    groupnutil = subs(-pn*(upn+an*hgn)-(1-pn)*(upe+an*hge), pn, pn_values(re));
    nobjective = @(x)double(subs(groupnutil, [t1n, fe, b], [x(1), x(2), x(3)]));
    [x, fval] = fmincon(nobjective, x0, A, B, Aeq, Beq, lb, ub, nonlcon);
    optimal_t1n(re) = x(1);
    optimal_fe(re) = x(2);
    optimal_b(re) = x(3);
    optimal_t2n(re) = double(subs(t2n(i), [fe, b], [optimal_fe(re),
optimal_b(re)]));
    optimal_t2e(re) = double(subs(t2e(i), [fe, b], [optimal_fe(re),
optimal_b(re)]));
    optimal_un(re) = -fval;
    optimal_ln1(re) = double(subs(ln1, t1n, optimal_t1n(re)));
    optimal_ln2n(re) = double(subs(ln2n, [fe, b], [optimal_fe(re),
optimal_b(re)]));
    optimal_ln2e(re) = double(subs(ln2e, [fe, b], [optimal_fe(re),
optimal_b(re)]));
    optimal_le2n(re) = double(subs(le2n, [fe, b], [optimal_fe(re),
optimal_b(re)]));
    optimal_le2e(re) = double(subs(le2e, [fe, b], [optimal_fe(re),
optimal_b(re)]));
    optimal_cn2n(re) = double(subs(cn2n, [t1n, fe, b], [optimal_t1n(re),
optimal_fe(re), optimal_b(re)]));
    optimal_cn2e(re) = double(subs(cn2e, [t1n, fe, b], [optimal_t1n(re),
optimal_fe(re), optimal_b(re)]));
    optimal_ce2n(re) = double(subs(ce2n, [fe, b], [optimal_fe(re),
optimal_b(re)]));
    optimal_ce2e(re) = double(subs(ce2e, [fe, b], [optimal_fe(re),
optimal_b(re)]));
    optimal_gn(re) = double(subs(gn, [fe, b], [optimal_fe(re), optimal_b(re)]));
    optimal_ge(re) = double(subs(ge, [fe, b], [optimal_fe(re), optimal_b(re)]));
    utiln_el(re) = double(subs(upn+an*hgn, [t1n, fe, b], [optimal_t1n(re),
optimal_fe(re), optimal_b(re)]));
    utiln_unel(re) = double(subs(upe+an*hge, [t1n, fe, b], [optimal_t1n(re),
optimal_fe(re), optimal_b(re)]));
```

```
end

% Plot the period 1 policies and group n’s expected utility against pn
figure(1);
subplot(2, 2, 1)
plot (pn_values, optimal_t1n)
xlabel({'p^n','(a)'})
ylabel('T^n_1')
grid on;
grid minor;
subplot (2, 2, 2);
plot (pn_values, optimal_fe)
xlabel({'p^n','(b)'})
ylabel('f^e')
grid on;
grid minor;
subplot(2, 2, 3);
plot (pn_values, optimal_b)
xlabel({'p^n','(c)'})
ylabel('b')
grid on;
grid minor;
subplot(2, 2, 4);
plot (pn_values, optimal_un)
xlabel({'p^n','(d)'})
ylabel('EU^n')
grid on;
grid minor;
% Plot the period 2 consumptions against fe and b
figure (2);
subplot (4 ,2, 1);
plot (optimal_fe, optimal_cn2n)
xlabel({'f^e','(a)'})
ylabel('c^n_2 with T^n')
grid on;
grid minor;
subplot (4, 2, 2);
plot (optimal_b, optimal_cn2n)
xlabel({'b','(b)'})
ylabel('c^n_2 with T^n')
grid on;
grid minor;
subplot (4, 2, 3);
plot (optimal_fe, optimal_cn2e)
xlabel({'f^e','(c)'})
ylabel('c^n_2 with T^e')
grid on;
grid minor;
subplot (4, 2, 4);
plot (optimal_b, optimal_cn2e)
xlabel({'b','(d)'})
ylabel('c^n_2 with T^e')
grid on;
grid minor;
subplot (4, 2, 5);
plot (optimal_fe, optimal_ce2n)
xlabel({'f^e','(e)'})
ylabel('c^e_2 with T^n')
grid on;
grid minor;
subplot (4, 2, 6);
plot (optimal_b, optimal_ce2n)
xlabel({'b','(f)'})
ylabel('c^e_2 with T^n')
grid on;
grid minor;
subplot (4, 2, 7);
plot (optimal_fe, optimal_ce2e)
```

```
xlabel({'f^e','(g)'})
ylabel('c^e_2 with T^e')
grid on;
grid minor;
subplot (4, 2, 8);
plot (optimal_b, optimal_ce2e)
xlabel({'b','(h)'})
ylabel('c^e_2 with T^e')
grid on;
grid minor;
% Plot the actual utility of the incumbent’s group against pn
figure (3);
subplot (1, 2, 1);
plot (pn_values, utiln_el)
xlabel({'p^n','(a)'})
ylabel('Group n''s Utility with Incumbent Re-elected')
grid on;
grid minor;
subplot (1, 2, 2);
plot (pn_values, utiln_unel)
xlabel({'p^n','(b)'})
ylabel('Group n''s Utility with Incumbent not Re-elected')
grid on;
grid minor;


% The constraints function
function [c, ceq] = confun(x)
syms t2n t2e
i = 1;
t1n = x(1);
fe = x(2);
b = x(3);
an = 0.677;
tie = 0.13;
r = 0;
m = 1;
phi = 0.6;
ae = 1.323;
w = 1;
pfe = 1+fe^tie;
afn = -1/(2*phi)*(1+pfe^2);
bfn = 1/(2*phi)*(2+2*pfe^2);
cfn = -((2*an+1/(2*phi))*(1+pfe^2)-(1+r)*m*fe+b);
dfn = an*(1+pfe^2)-(1+r)*m*fe+b;
focn = afn*t2n^3 + bfn*t2n^2 + cfn*t2n + dfn;
t2n = solve(focn, t2n, "MaxDegree",3);
afe = -1/(2*phi)*pfe^2*(1+pfe^2);
bfe = 1/(2*phi)*pfe^2*(2+2*pfe^2);
cfe = -((2*ae+1/(2*phi)*pfe^2)*(1+pfe^2)-(1+r)*m*fe*pfe^2+b*pfe^2);
dfe = ae*(1+pfe^2)-pfe^2*(1+r)*m*fe+pfe^2*b;
foce = afe*t2e^3 + bfe*t2e^2 + cfe*t2e + dfe;
t2e = solve(foce, t2e, "MaxDegree", 3);
c(1) = -((1-double(t2n(i)))^2*1/(2*phi)+(1-t1n)^2*1/(2*phi)-1);
c(2) = -((1-double(t2e(i)))^2*1/(2*phi)+(1-t1n)^2*1/(2*phi)-1);
c(3) = -(pfe*1/(2*phi)*(1-double(t2n(i)))^2+w-1-(1+r)*m*x(2));
c(4) = -(pfe*1/(2*phi)*(1-double(t2e(i)))^2+w-1-(1+r)*m*x(2));
ceq = t1n*1/(2*phi)*(1-t1n) - fe + b;
end
```

A6.9: MATLAB Code for Incumbent *e*

```
clc, clearvars
```

```
syms fe b t2n t1n t2e pn
an = 0.677;
tie = 0.13;
r = 0;
m = 1;
phi = 0.6;
ae = 1.323;
w = 1;
pfe = 1+fe^tie;

% Period 2 tax rates
afn = -1/(2*phi)*(1+pfe^2);
bfn = 1/(2*phi)*(2+2*pfe^2);
cfn = -((2*an+1/(2*phi))*(1+pfe^2)-(1+r)*m*fe+b);
dfn = an*(1+pfe^2)-(1+r)*m*fe+b;
focn = afn*t2n^3 + bfn*t2n^2 + cfn*t2n + dfn;
t2n = solve(focn, t2n, "MaxDegree",3);

afe = -1/(2*phi)*pfe^2*(1+pfe^2);
bfe = 1/(2*phi)*pfe^2*(2+2*pfe^2);
cfe = -((2*ae+1/(2*phi)*pfe^2)*(1+pfe^2)-(1+r)*m*fe*pfe^2+b*pfe^2);
dfe = ae*(1+pfe^2)-pfe^2*(1+r)*m*fe+pfe^2*b;
foce = afe*t2e^3 + bfe*t2e^2 + cfe*t2e + dfe;
t2e = solve(foce, t2e, "MaxDegree", 3);

i = 1;

% Incumbent e optimization problem
ln1 = 1/(2*phi)*(1-t1n);
ln2n = 1/(2*phi)*(1-t2n(i));
ln2e = 1/(2*phi)*(1-t2e(i));
le2n = 1/(2*phi)*pfe*(1-t2n(i));
le2e = 1/(2*phi)*pfe*(1-t2e(i));
cn2n = (1-t1n)*ln1+(1-t2n(i))*ln2n-1;
cn2e = (1-t1n)*ln1+(1-t2e(i))*ln2e-1;
ce2n = pfe*(1-t2n(i))*le2n+w-(1+r)*m*fe-1;
ce2e = pfe*(1-t2e(i))*le2e+w-(1+r)*m*fe-1;
gn = (1+r)*m*fe-b+t2n(i)*ln2n+t2n(i)*le2n*pfe;
ge = (1+r)*m*fe-b+t2e(i)*ln2e+t2e(i)*le2e*pfe;

upn = ce2n+1+1-phi*(le2n)^2;
hgn = log(gn);
upe = ce2e+1+1-phi*(le2e)^2;
hge = log(ge);


nonlcon = @confun;
x0 = [0.2, 0.2, 0.12];
A = [];
B = [];
Aeq = [];
Beq = [];
lb = [-inf, 0, -inf];
ub = [];


% Storing optimal values
optimal_t1n = zeros(1,101);
optimal_fe = zeros(1,101);
optimal_b = zeros(1,101);
optimal_t2n = zeros(1,101);
optimal_t2e = zeros(1,101);
optimal_ue = zeros(1,101);
optimal_ln1 = zeros(1,101);
optimal_ln2n = zeros(1,101);
optimal_ln2e = zeros(1,101);
optimal_le2n = zeros(1,101);
optimal_le2e = zeros(1,101);
```

```
optimal_cn2n = zeros(1,101);
optimal_cn2e = zeros(1,101);
optimal_ce2n = zeros(1,101);
optimal_ce2e = zeros(1,101);
optimal_gn = zeros(1,101);
optimal_ge = zeros(1,101);
utile_el = zeros(1,101);
utile_unel = zeros(1,101);

pn_values = 0:0.01:1;
for re=1:length(pn_values)
    groupeutil = subs(-pn*(upn+ae*hgn)-(1-pn)*(upe+ae*hge), pn, pn_values(re));
    nobjective = @(x)double(subs(groupeutil, [t1n, fe, b], [x(1), x(2), x(3)]));
    [x, fval] = fmincon(nobjective, x0, A, B, Aeq, Beq, lb, ub, nonlcon);
    optimal_t1n(re) = x(1);
    optimal_fe(re) = x(2);
    optimal_b(re) = x(3);
    optimal_t2n(re) = double(subs(t2n(i), [fe, b], [optimal_fe(re),
optimal_b(re)]));
    optimal_t2e(re) = double(subs(t2e(i), [fe, b], [optimal_fe(re),
optimal_b(re)]));
    optimal_ue(re) = -fval;
    optimal_ln1(re) = double(subs(ln1, t1n, optimal_t1n(re)));
    optimal_ln2n(re) = double(subs(ln2n, [fe, b], [optimal_fe(re),
optimal_b(re)]));
    optimal_ln2e(re) = double(subs(ln2e, [fe, b], [optimal_fe(re),
optimal_b(re)]));
    optimal_le2n(re) = double(subs(le2n, [fe, b], [optimal_fe(re),
optimal_b(re)]));
    optimal_le2e(re) = double(subs(le2e, [fe, b], [optimal_fe(re),
optimal_b(re)]));
    optimal_cn2n(re) = double(subs(cn2n, [t1n, fe, b], [optimal_t1n(re),
optimal_fe(re), optimal_b(re)]));
    optimal_cn2e(re) = double(subs(cn2e, [t1n, fe, b], [optimal_t1n(re),
optimal_fe(re), optimal_b(re)]));
    optimal_ce2n(re) = double(subs(ce2n, [fe, b], [optimal_fe(re),
optimal_b(re)]));
    optimal_ce2e(re) = double(subs(ce2e, [fe, b], [optimal_fe(re),
optimal_b(re)]));
    optimal_gn(re) = double(subs(gn, [fe, b], [optimal_fe(re), optimal_b(re)]));
    optimal_ge(re) = double(subs(ge, [fe, b], [optimal_fe(re), optimal_b(re)]));
    utile_el(re) = double(subs(upe+ae*hge, [fe, b], [optimal_fe(re),
optimal_b(re)]));
    utile_unel(re) = double(subs(upn+ae*hgn, [fe, b], [optimal_fe(re),
optimal_b(re)]));
end

% Plot the period 1 policies and group e’s expected utility against pn
figure(1);
subplot (2, 2, 1)
plot (pn_values, optimal_t1n)
xlabel({'p^n','(a)'})
ylabel('T^n_1')
grid on;
grid minor;
subplot (2, 2, 2);
plot (pn_values, optimal_fe)
xlabel({'p^n','(b)'})
ylabel('f^e')
grid on;
grid minor;
subplot(2, 2, 3);
plot (pn_values, optimal_b)
xlabel({'p^n','(c)'})
ylabel('b')
grid on;
grid minor;
subplot (2, 2, 4);
```

```
plot (pn_values, optimal_ue)
xlabel({'p^n','(d)'})
ylabel('EU^e')
grid on;
grid minor;
% Plot the period 2 consumptions against fe and b
figure (4);
subplot (4 ,2, 1);
plot (optimal_fe, optimal_cn2n)
xlabel({'f^e','(a)'})
ylabel('c^n_2 with T^n')
grid on;
grid minor;
subplot (4, 2, 2);
plot (optimal_b, optimal_cn2n)
xlabel({'b','(b)'})
ylabel('c^n_2 with T^n')
grid on;
grid minor;
subplot (4, 2, 3);
plot (optimal_fe, optimal_cn2e)
xlabel({'f^e','(c)'})
ylabel('c^n_2 with T^e')
grid on;
grid minor;
subplot (4, 2, 4);
plot (optimal_b, optimal_cn2e)
xlabel({'b','(d)'})
ylabel('c^n_2 with T^e')
grid on;
grid minor;
subplot (4, 2, 5);
plot (optimal_fe, optimal_ce2n)
xlabel({'f^e','(e)'})
ylabel('c^e_2 with T^n')
grid on;
grid minor;
subplot (4, 2, 6);
plot (optimal_b, optimal_ce2n)
xlabel({'b','(f)'})
ylabel('c^e_2 with T^n')
grid on;
grid minor;
subplot (4, 2, 7);
plot (optimal_fe, optimal_ce2e)
xlabel({'f^e','(g)'})
ylabel('c^e_2 with T^e')
grid on;
grid minor;
subplot (4, 2, 8);
plot (optimal_b, optimal_ce2e)
xlabel({'b','(h)'})
ylabel('c^e_2 with T^e')
grid on;
grid minor;
% Plot the actual utility of the incumbent’s group against pn
figure (5);
subplot (1, 2, 1);
plot (pn_values, utile_el)
xlabel({'p^n','(a)'})
ylabel('Group e''s Utility with Incumbent Re-elected')
grid on;
grid minor;
subplot (1, 2, 2);
plot (pn_values, utile_unel)
xlabel({'p^n','(b)'})
ylabel('Group e''s Utility with Incumbent not Re-elected')
grid on;
```

```
grid minor;


% The constraints function
function [c, ceq] = confun(x)
syms t2n t2e
i = 1;
t1n = x(1);
fe = x(2);
b = x(3);
an = 0.677;
tie = 0.13;
r = 0;
m = 1;
phi = 0.6;
ae = 1.323;
w = 1;
pfe = 1+fe^tie;
afn = -1/(2*phi)*(1+pfe^2);
bfn = 1/(2*phi)*(2+2*pfe^2);
cfn = -((2*an+1/(2*phi))*(1+pfe^2)-(1+r)*m*fe+b);
dfn = an*(1+pfe^2)-(1+r)*m*fe+b;
focn = afn*t2n^3 + bfn*t2n^2 + cfn*t2n + dfn;
t2n = solve(focn, t2n, "MaxDegree",3);
afe = -1/(2*phi)*pfe^2*(1+pfe^2);
bfe = 1/(2*phi)*pfe^2*(2+2*pfe^2);
cfe = -((2*ae+1/(2*phi)*pfe^2)*(1+pfe^2)-(1+r)*m*fe*pfe^2+b*pfe^2);
dfe = ae*(1+pfe^2)-pfe^2*(1+r)*m*fe+pfe^2*b;
foce = afe*t2e^3 + bfe*t2e^2 + cfe*t2e + dfe;
t2e = solve(foce, t2e, "MaxDegree", 3);
c(1) = -((1-double(t2n(i)))^2*1/(2*phi)+(1-t1n)^2*1/(2*phi)-1);
c(2) = -((1-double(t2e(i)))^2*1/(2*phi)+(1-t1n)^2*1/(2*phi)-1);
c(3) = -(pfe*1/(2*phi)*(1-double(t2n(i)))^2+w-1-(1+r)*m*x(2));
c(4) = -(pfe*1/(2*phi)*(1-double(t2e(i)))^2+w-1-(1+r)*m*x(2));
ceq = t1n*1/(2*phi)*(1-t1n) - fe + b;
end
```

Bibliography

Acemoglu, D. and Restrepo, P. (2020) 'Robots and jobs: Evidence from US labor markets', *Journal of Political Economy*, 128(6), pp. 2188-2244.

Acemoglu, D. and Robinson, J.A. (2013) 'Economics versus politics: Pitfalls of policy advice', *Journal of Economic Perspectives*, 27(2), pp. 173-92.

Acemoglu, D., Ticchi, D. and Vindigni, A. (2010) 'A theory of military dictatorships', *American Economic Journal: Macroeconomics*, 2(1), pp. 1-42.

Addo, F.R., Houle, J.N. and Simon, D. (2016) 'Young, black, and (still) in the red: Parental wealth, race, and student loan debt', *Race and Social Problems*, 8(1), pp. 64-76.

Aghion, P. and Bolton, P. (1990) 'Government domestic debt and the risk of default: a political-economic model of the strategic role of debt', in R. Dornbusch and M. Draghi (eds.) *Public debt management: theory and history*. Cambridge: Cambridge University Press, pp. 315-349.

Agnello, L. and Sousa, R.M. (2014) 'The determinants of the volatility of fiscal policy discretion', *Fiscal Studies*, 35(1), pp. 91-115.

Akers, E., Chingos, M., & Henriques, A. (2015) 'Understanding changes in the distribution of student loan debt over time', in B. Hershbein, and K. M. Hollenbeck (eds.) *Student loans and the dynamics of debt*. Kalamazoo, MI: W. E. Upjohn Institute for Employment Research, pp. 117-135.

Alesina, A., Baqir, R. and Easterly, W. (1999) 'Public goods and ethnic divisions', *The Quarterly Journal of Economics*, 114(4), pp. 1243-1284.

Alesina, A. and Passalacqua, A. (2016) 'The political economy of government debt', *Handbook of Macroeconomics*, 2, pp. 2599-2651.

Alesina, A. and Tabellini, G. (1990) 'A positive theory of fiscal deficits and government debt', *The Review of Economic Studies*, 57(3), pp. 403-414.

Alesina, A., Glaeser, E. and Sacerdote, B. (2001) 'Why Doesn't the United States Have a European-Style Welfare State?', *Brookings Paper on Economics Activity*, 2001(2), pp. 187-278.

Almås, I., Cappelen, A.W. and Tungodden, B. (2020) 'Cutthroat capitalism versus cuddly socialism: Are Americans more meritocratic and efficiency-seeking than Scandinavians?', *Journal of Political Economy*, 128(5), pp. 1753-1788.

Alt, J.E. and Lassen, D.D. (2006) 'Fiscal transparency, political parties, and debt in OECD countries', *European Economic Review*, 50(6), pp. 1403-1439.

Anderberg, D. (2009) 'Optimal policy and the risk properties of human capital reconsidered', *Journal of Public Economics*, 93(9-10), pp. 1017-1026.

Anderberg, D. (2013) 'Post-compulsory education: Participation and politics', *European Journal of Political Economy*, 29, pp. 134-150.

Anderberg, D. and Balestrino, A. (2008) *The political economy of post-compulsory education policy with endogenous credit constraints*. CESifo Working Paper Series No. 2304. Munich: Center for Economic Studies and ifo Institute (CESifo). Available at: https://www.cesifo.org/DocDL/cesifo1_wp2304.pdf (Accessed: 24 March 2023).

Arawatari, R. and Ono, T. (2009) 'A second chance at success: a political economy perspective', *Journal of Economic Theory*, 144(3), pp. 1249-1277.

Arawatari, R. and Ono, T. (2015) 'A Political Economy Model of Earnings Mobility and Redistribution Policy', *Journal of Public Economic Theory*, 17(3), pp. 346-382.

Archibald, R.B. and Feldman, D.H. (2006) 'State higher education spending and the tax revolt', *The Journal of Higher Education*, 77(4), pp. 618-644. Available at: https://doi.org/10.1080/00221546.2006.11772309

Ardanaz, M. and Scartascini, C. (2011) *Why don't we tax the rich? Inequality, legislative malapportionment, and personal income taxation around the world*. IDB Working Paper No. IDB-WP-282. Available at: http://dx.doi.org/10.2139/ssrn.1972115 (Accessed: 23 March 2023).

Atkinson, A.B. (1996) 'Income distribution in Europe and the United States', *Oxford Review of Economic Policy*, 12(1), pp. 15-28.

Autor, D., Dorn, D., and Hanson, G. (2013) 'The China syndrome: Local labor market effects of import competition in the United States', *American Economic Review*, 103(6), pp. 2121-2168.

Azmat, G. and Simion, S. (2018) *Higher education funding reforms: A comprehensive analysis of educational and labor market outcomes in England*. CEPR Discussion Paper No. DP12389. London: Centre for Economic Performance. Available at SSRN: https://ssrn.com/abstract=3057323 (Accessed: 16 July 2022).

Azzimonti, M. (2011) 'Barriers to investment in polarized societies', *American Economic Review*, 101(5), pp. 2182-2204.

Azzimonti, M. (2015) 'The dynamics of public investment under persistent electoral advantage', *Review of Economic Dynamics*, 18(3), pp. 653-678.

Battaglini, M. and Coate, S. (2007) 'Inefficiency in legislative policymaking: a dynamic analysis', *American Economic Review*, 97(1), pp. 118-149.

Bailey, D., Chapain, C. and de Ruyter, A. (2012) 'Employment outcomes and plant closure in a post-industrial city: an analysis of the labour market status of MG Rover workers three years on', *Urban Studies*, 49(7), pp. 1595-1612.

Baum, S., Ma, J. and Payea, K. (2010) 'Education Pays, 2010: The Benefits of Higher Education for Individuals and Society. Trends in Higher Education Series', *College Board Advocacy & Policy Center*.

Baum, S., Ma, J. and Payea, K. (2013) 'Education Pays, 2013: The Benefits of Higher Education for Individuals and Society. Trends in Higher Education Series', *College Board*.

Bearse, P., Glomm, G. and Janeba, E. (2001) 'Composition of Government Budget, Non-Single Peakedness, and Majority Voting', *Journal of Public Economic Theory*, 3(4), pp. 471-481.

Bearse, P., Glomm, G. and Patterson, D.M. (2005) 'Endogenous public expenditures on education', *Journal of Public Economic Theory*, 7(4), pp. 561-577.

Becher, M. (2019) 'Dissolution power, confidence votes, and policymaking in parliamentary democracies', *Journal of Theoretical Politics*, 31(2), pp. 183-208.

Becher, M. and Christiansen, F.J. (2015) 'Dissolution threats and legislative bargaining', *American Journal of Political Science*, 59(3), pp. 641-655.

Beetsma, R.M. and van der Ploeg, R. (2007) *Partisan public investment and debt: The case for fiscal restrictions*. EUI Working Paper No. 37. Italy: European University Institute. Available at: https://cadmus.eui.eu/handle/1814/7338 (Accessed: 24 March 2023).

Belfield, C.R. (2013) 'Student loans and repayment rates: The role of for-profit colleges', *Research in Higher Education*, 54(1), pp. 1-29.

Belfield, C. Britton, J. and Erve, L.V.D. (2018) *Family background has an important impact on graduates' future earnings, but subject and institution choice can be even more important*. Available at: https://ifs.org.uk/news/family-background-has-important-impact-graduates-future-earnings-subject-and-institution#:~:text=For%20example%2C%20five%20years%20after,(35%25%20for%20women). (Accessed: 20 October 2024).

Benabou, R. (2002) 'Tax and education policy in a heterogeneous-agent economy: What levels of redistribution maximize growth and efficiency?', *Econometrica*, 70(2), pp. 481-517.

Bennett, P., Blundell, R.W. and Salvanes, K.G. (2020) *A second chance? Labor market returns to adult education using school reforms*. IZA Discussion Paper No. 13592. Available at: https://docs.iza.org/dp13592.pdf (Accessed: 24 Mar 2023).

Besley, T. and Coate, S. (1998) 'Sources of inefficiency in a representative democracy: a dynamic analysis', *American Economic Review*, 88(1), pp. 139-156.

Blanchet, T., Chancel, L. and Gethin, A. (2022) 'Why is Europe more equal than the United States?', *American Economic Journal: Applied Economics*, 14(4), pp. 480-518.

Blume, L., Müller, J. and Voigt, S. (2009) 'The economic effects of direct democracy—a first global assessment', *Public Choice*, 140(3), pp. 431-461.

Board of Governors of the Federal Reserve System (US) (2022) *Student Loans Owned and Securitized [SLOAS]*. Available at: https://fred.stlouisfed.org/series/SLOAS# (Accessed: August 28, 2022).

Bohacek, R. and Kapicka, M. (2008) 'Optimal human capital policies', *Journal of Monetary Economics*, 55(1), pp. 1-16.

Bohn, F. (2007) ‘Polarisation, uncertainty and public investment failure’, *European Journal of Political Economy*, 23(4), pp. 1077-1087.

Bolton, P. (2019) ‘Student loan statistics’, *House of Commons Library*, Briefing paper number 1079.

Bolton, P. (2022) *Student loan statistics*. Available at: https://commonslibrary.parliament.uk/research-briefings/sn01079/#:~:text=Scale%20of%20student%20loans%20in,)%20by%20the%20mid%2D2040s. (Accessed: 16 July 2022).

Bonica, A., McCarty, N., Poole, K.T. and Rosenthal, H. (2013) ‘Why hasn't democracy slowed rising inequality?’, *Journal of Economic Perspectives*, 27(3), pp. 103-24.

Borck, R. and Wimbersky, M. (2014) ‘Political economics of higher education finance’, *Oxford Economic Papers*, 66(1), pp. 115-139.

Bovenberg, A.L. and Jacobs, B. (2005) ‘Redistribution and education subsidies are Siamese twins’, *Journal of Public Economics*, 89(11-12), pp. 2005-2035.

Callender, C. and Mason, G. (2017) ‘Does student loan debt deter higher education participation? New evidence from England’, *The ANNALS of the American Academy of Political and Social Science*, 671(1), pp. 20-48.

Carnoy, M., Rabling, B.J., Castaño-Muñoz, J., Montoliu, J.M.D. and Sancho-Vinuesa, T. (2012) ‘Does on-line distance higher education pay off for adult learners? The case of the Open University of Catalonia’, *Higher Education Quarterly*, 66(3), pp. 248-271.

Caruth, G.D. (2014) ‘Meeting the needs of older students in higher education’, *Participatory Educational Research*, 1(2), pp. 21-35.

Chamley, C. (1986) ‘Optimal taxation of capital income in general equilibrium with infinite lives’, *Econometrica*, 54(3), pp. 607-622.

Chapman, B. (2014) ‘Income contingent loans: Background’, in B. Chapman, T. Higgins, & J. Stiglitz (eds.) *Income contingent loans: Theory, practice and prospects*. London: Palgrave Macmillan, pp. 12–28.

Chapman, B. and Higgins, T. (2013) ‘The costs of unpaid higher education contribution scheme debts of graduates working abroad’, *Australian Economic Review*, 46(3), pp. 286-299.

Chatterjee, S. and Ionescu, F. (2012) ‘Insuring student loans against the financial risk of failing to complete college’, *Quantitative Economics*, 3(3), pp. 393-420.

Chinn, M.D. and Ito, H. (2006) ‘What matters for financial development? Capital controls, institutions, and interactions’, *Journal of Development Economics*, 81(1), pp. 163-192.

Chowdry, H., Dearden, L., Goodman, A., and Jin, W. (2012) 'The distributional impact of the 2012–13 higher education funding reforms in England', *Fiscal Studies*, 33(2), pp. 211-236.

Cigno, A. and Luporini, A. (2009) 'Scholarships or student loans? Subsidizing higher education in the presence of moral hazard', *Journal of Public Economic Theory*, 11(1), pp. 55-87.

Clotfelter, C.T. (1976) 'Public spending for higher education: An empirical test of two hypotheses', *Public Finance*, 31(2), pp. 177-195.

Congleton, R.D. (2018) 'Intellectual foundations of public choice, the forest from the trees', *Public Choice*, 175(3), pp. 229-244.

Couch, K.A. and Placzek, D.W. (2010) 'Earnings losses of displaced workers revisited', *American Economic Review*, 100(1), pp. 572-89.

Crawford, C., Crawford, R. and Jin, W. (2014) 'Estimating the public cost of student loans', *Institute for Fiscal Studies*, IFS Report R94.

Dar, L. and Lee, D.W., (2014) 'Partisanship, political polarization, and state higher education budget outcomes', *The Journal of Higher Education*, *85*(4), pp. 469-498. Available at: https://doi.org/10.1080/00221546.2014.11777337

David, H., Dorn, D. and Hanson, G.H. (2013) 'The China syndrome: Local labor market effects of import competition in the United States', *American Economic Review*, 103(6), pp. 2121-2168.

De Fraja, G. (2001) 'Education policies: Equity, efficiency and voting equilibrium', *The Economic Journal*, 111(471), pp. 104-119.

De Fraja, G. (2002) 'The design of optimal education policies', *The Review of Economic Studies*, 69(2), pp. 437-466.

De Gayardon, A., Callender, C., Deane, K.C. and DeJardines, S. (2018) *Graduate indebtedness: its perceived effects on behaviour and life choices–a literature review*. Working Paper no. 38. UK: Centre for Global Higher Education. Available at: https://www.researchcghe.org/publications/working-paper/graduate-indebtedness-its-perceived-effects-on-behaviour-and-life-choices-a-literature-review/ (Accessed: 24 March 2023).

Deacon, R.T. (2009) 'Public good provision under dictatorship and democracy', *Public Choice*, 139(1), pp. 241-262.

Dean Craig, J. and Raisanen, S.R. (2014) 'Institutional determinants of American undergraduate student debt', *Journal of Higher Education Policy and Management*, 36(6), pp. 661-673.

Dearden, L., Fitzsimons, E. and Wyness, G. (2014) 'Money for nothing: Estimating the impact of student aid on participation in higher education', *Economics of Education Review*, 43, pp. 66-78.

Del Rey, E. (2012) 'Deferring higher education fees without relying on contributions from non-students', *Education Economics*, 20(5), pp. 510-521.

Del Rey, E. and Racionero, M. (2010) 'Financing schemes for higher education', *European Journal of Political Economy*, 26(1), pp. 104-113.

Del Rey, E. and Racionero, M. (2012) 'Voting On Income-Contingent Loans For Higher Education', *Economic Record*, 88, pp. 38-50.

Del Rey, E. and Racionero, M. (2014) *Choosing the type of income-contingent loan: risk-sharing versus risk-pooling*. IEB Working Paper N. 2014/7. Available at: http://dx.doi.org/10.2139/ssrn.2410910 (Accessed: 24 March 2023).

Deller, S. and Parr, J. (2021) 'Does Student Loan Debt Hinder Community Well-Being?', *International Journal of Community Well-Being*, 4(2), pp. 263-285.

Department of Education (2021) *Graduate Labour Market Statistics.* Available at: https://explore-education-statistics.service.gov.uk/find-statistics/graduate-labour-markets/2020 (Accessed: 23 Feb 2022).

Desjardins, R. and Lee, J. (2016) ‘Earnings and employment benefits of adult higher education in comparative perspective: Evidence based on the OECD Survey of Adult Skills (PIAAC)’, Available at: https://escholarship.org/content/qt0jz0k1pp/qt0jz0k1pp.pdf (Accessed: 23 Feb 2022).

Devereux, M.B. and Wen, J.F. (1998) ‘Political instability, capital taxation, and growth’, *European Economic Review*, 42(9), pp. 1635-1651.

Di Gioacchino, D. and Sabani, L. (2009) ‘Education policy and inequality: a political economy approach’, *European Journal of Political Economy*, 25(4), pp. 463-478.

Diermeier, D. and Feddersen, T.J. (1998) ‘Cohesion in legislatures and the vote of confidence procedure’, *American Political Science Review*, 92(3), pp. 611-621.

Donaldson, J.F. and Townsend, B.K. (2007) ‘Higher education journals' discourse about adult undergraduate students’, *The Journal of Higher Education*, 78(1), pp. 27-50.

Dorsett, R., Lui, S. and Weale, M. (2010) ‘Economic benefits of lifelong learning’, *Centre for Learning and Life Chances in Knowledge Economies and Societies*, LLAKES Research Paper 13.

Dyke, A., Heinrich, C.J., Mueser, P.R., Troske, K.R. and Jeon, K.S. (2006) ‘The effects of welfare-to-work program activities on labor market outcomes’, *Journal of Labor Economics*, 24(3), pp. 567-607.

Elbasir, A. and Siddiqui, K. (2018) 'Higher education, funding, polices and politics: A critical review', *Journal of Social and Administrative Sciences*, 5(2), pp. 152-167.

Elder, E.M. and Wagner, G.A. (2015) 'Political effects on pension underfunding', *Economics & Politics*, 27(1), pp. 1-27.

Epple, D. and Romano, R.E. (1996) 'Ends against the middle: Determining public service provision when there are private alternatives', *Journal of Public Economics*, 62(3), pp. 297-325.

Epple, D. and Romano, R.E. (1998) 'Competition between private and public schools, vouchers, and peer-group effects', *American Economic Review*, 88(1), pp. 33-62.

Epple, D. and Romano, R. (2008) 'Educational vouchers and cream skimming', *International Economic Review*, 49(4), pp. 1395-1435.

Farhi, E. and Werning, I. (2013) 'Insurance and taxation over the life cycle', *Review of Economic Studies*, 80(2), pp. 596-635.

Feld, L.P. and Schnellenbach, J. (2014) 'Political institutions and income (re-) distribution: evidence from developed economies', *Public Choice*, 159(3), pp. 435-455.

Fernandez, R. and Rogerson, R. (1995) 'On the political economy of education subsidies', *The Review of Economic Studies*, 62(2), pp. 249-262.

Findeisen, S. and Sachs, D. (2016) 'Education and optimal dynamic taxation: The role of income-contingent student loans', *Journal of Public Economics*, 138, pp. 1-21.

Fiva, J.H. and Natvik, G.J. (2013) 'Do re-election probabilities influence public investment?', *Public Choice*, 157(1), pp. 305-331.

Fowles, J. (2014) 'Funding and focus: Resource dependence in public higher education', *Research in Higher Education*, 55, pp. 272-287. Available at: https://doi.org/10.1007/s11162-013-9311-x

Friedman, M. (1955) 'The role of government in education', Available at: https://tedsf.org/wp-content/uploads/2019/01/The-Role-of-Government-in-Education.pdf (Accessed: 10 Nov 2021).

Friedman, M. and Kuznets, S. (1945) *Income from Independent Professional Practice*. New York: National Bureau of Economic Research.

Friedman, Z. (2018) 'Student loan debt statistics in 2018: A $1.5 trillion crisis', *Forbes*, 13 June. Available at: https://www.paisboa.org/assets/aggie-blog/2018/09.21.18/StudentLoanDebt_$1.5TrillionCrisis.pdf (Accessed: 16 July 2022).

Fumagalli, E. and Narciso, G. (2012) 'Political institutions, voter turnout, and policy outcomes', *European Journal of Political Economy*, 28(2), pp. 162-173.

Garcia-Penalosa, C. and Wälde, K. (2000) 'Efficiency and equity effects of subsidies to higher education', *Oxford Economic Papers*, 52(4), pp. 702-722.

Gary-Bobo, R.J. and Trannoy, A. (2015) 'Optimal student loans and graduate tax under moral hazard and adverse selection', *The RAND Journal of Economics*, 46(3), pp. 546-576.

Genicot, G., Bouton, L. and Castanheira, M. (2021) 'Electoral systems and inequalities in government interventions', *Journal of the European Economic Association*, 19(6), pp. 3154-3206.

Gerring, J. and Thacker, S.C. (2004) 'Political institutions and corruption: The role of unitarism and parliamentarism', *British Journal of Political Science*, 34(2), pp. 295-330.

Glater, J.D. (2016) 'Student debt and the siren song of systemic risk', *Harvard Journal on Legislation*, 53(1), pp. 99-146.

Glazer, A. (1989) 'Politics and the Choice of Durability', *The American Economic Review*, 79(5), pp. 1207-1213.

Glomm, G. and Ravikumar, B. (1998) 'Opting out of publicly provided services: A majority voting result', *Social Choice and Welfare*, 15(2), pp. 187-199.

Glomm, G., Ravikumar, B. and Schiopu, I.C. (2011) ‘The political economy of education funding’, in E.A. Hanushek, S. Machin and L. Woessmann (ed.) *Handbook of the Economics of Education*. Amsterdam: North Holland, pp. 615-680.

Golosov, M., Tsyvinski, A. and Werning, I. (2006) ‘New Dynamic Public Finance: A User's Guide’, *NBER Macroeconomics Annual*, 21, pp. 317-363.

Goodell, J.W. (2016) ‘Do for-profit universities induce bad student loans?’, *The Quarterly Review of Economics and Finance*, 61, pp. 173-184.

Gornitzka, Å., Stensaker, B., Smeby, J.C. and De Boer, H. (2004) ‘Contract arrangements in the Nordic countries—solving the efficiency/effectiveness dilemma?’, *Higher Education in Europe*, 29(1), pp. 87-101.

Grochulski, B. and Piskorski, T. (2010) ‘Risky human capital and deferred capital income taxation’, *Journal of Economic Theory*, 145(3), pp. 908-943.

Gvaramadze, I. (2010) ‘Low-skilled workers and adult vocational skills-upgrading strategies in Denmark and South Korea’, *Journal of Vocational Education & Training*, 62(1), pp. 51-61.

Hall, R.E. and Jones, C.I., (1999) ‘Why do some countries produce so much more output per worker than others?’, *The Quarterly Journal of Economics*, 114(1), pp. 83-116.

Hansen, W.L. and Rhodes, M.S. (1988) 'Student debt crisis: Are students incurring excessive debt?', *Economics of Education Review*, 7(1), pp. 101-112.

Hanson, M. (2022a) *Student Loan Debt Statistics*. Available at: https://educationdata.org/student-loan-debt-statistics (Accessed: 16 July 2022).

Hanson, M. (2022b) *Student Loan Default Rate*. Available at: https://educationdata.org/student-loan-default-rate (Accessed: 25 Jan 2023).

Hanushek, E.A., Leung, C.K.Y. and Yilmaz, K. (2014) 'Borrowing constraints, college aid, and intergenerational mobility', *Journal of Human Capital*, 8(1), pp. 1-41.

Haupt, A. (2012) 'The evolution of public spending on higher education in a democracy', *European Journal of Political Economy*, 28(4), pp. 557-573.

Hedlund, A. (2022) 'What Can Be Done to Address Rising Student Debt?', *The Center for Growth and Opportunity*.

Hershbein, B. and Hollenbeck, K. (2014) 'The distribution of college graduate debt, 1990 to 2008: A decomposition approach', in B. Hershbein, and K. M. Hollenbeck (eds.) *Student loans and the dynamics of debt*. Kalamazoo, MI: W. E. Upjohn Institute for Employment Research, pp. 53-116.

Heyes, J. (2013) 'Vocational training, employability and the post-2008 jobs crisis: Responses in the European Union', *Economic and Industrial Democracy*, 34(2), pp. 291-311.

Hillman, N.W. (2014) 'College on credit: A multilevel analysis of student loan default', *The Review of Higher Education*, 37(2), pp. 169-195.

Hotz, V.J., Imbens, G.W. and Klerman, J.A. (2006) 'Evaluating the differential effects of alternative welfare-to-work training components: A reanalysis of the California GAIN program', *Journal of Labor Economics*, 24(3), pp. 521-566.

Houle, J.N. and Addo, F.R. (2019) 'Racial disparities in student debt and the reproduction of the fragile black middle class', *Sociology of Race and Ethnicity*, 5(4), pp. 562-577.

Houle, J.N. and Berger, L. (2015) 'Is student loan debt discouraging homeownership among young adults?', *Social Service Review*, 89(4), pp. 589-621.

Humphreys, B.R. (2000) 'Do business cycles affect state appropriations to higher education?', *Southern Economic Journal*, 67(2), pp. 398-413.

International Monetary Fund (2022) *Government expenditure, percent of GDP*. Available at: https://www.imf.org/external/datamapper/exp@FPP/USA/FRA/JPN/GBR/SWE/ESP/ITA/ZAF/IND (Accessed: 7 December 2023).

Iversen, T. and Soskice, D. (2006) 'Electoral institutions and the politics of coalitions: Why some democracies redistribute more than others', *American Political Science Review*, 100(2), pp. 165-181.

Jacobson, L.S., LaLonde, R. and Sullivan, D. (2005) 'Is retraining displaced workers a good investment?', *Economic Perspectives*, 29(2), pp. 47-66.

Johnstone, D.B. (2003) 'Cost sharing in higher education: Tuition, financial assistance, and accessibility in a comparative perspective', *Czech Sociological Review*, 39(3), pp. 351-374.

Johnstone, D.B. (2004) 'The economics and politics of cost sharing in higher education: comparative perspectives', *Economics of Education Review*, 23(4), pp. 403-410.

Kammas, P. and Sarantides, V. (2019) 'Do dictatorships redistribute more?', *Journal of Comparative Economics*, 47(1), pp. 176-195.

Kapička, M. (2013) 'Efficient allocations in dynamic private information economies with persistent shocks: A first-order approach', *Review of Economic Studies*, 80(3), pp. 1027-1054.

Kapička, M. and Neira, J. (2019) 'Optimal taxation with risky human capital', *American Economic Journal: Macroeconomics*, 11(4), pp. 271-309.

Kholmuminov, S., Kholmuminov, S. and Wright, R.E. (2019) 'Resource dependence theory analysis of higher education institutions in Uzbekistan', *Higher Education*, 77, pp. 59-79.

Kim, K.N. and Baker, R.M. (2015) ‘The assumed benefits and hidden costs of adult learners’ college enrollment’, *Research in Higher Education*, 56(5), pp. 510-533.

Kivistö, J. (2005) ‘The government-higher education institution relationship: Theoretical considerations from the perspective of agency theory’, *Tertiary Education & Management*, 11(1), pp. 1-17.

Kivistö, J. (2007) *Agency theory as a framework for the government-university relationship*. Tampere: Tampere University Press.

Kivistö, J. (2008) ‘An assessment of agency theory as a framework for the government–university relationship’, *Journal of Higher Education Policy and Management*, 30(4), pp. 339-350.

Koeniger, W. and Prat, J. (2018) ‘Human capital and optimal redistribution’, *Review of Economic Dynamics*, 27, pp. 1-26.

Kohnert, D. (2022) ‘Are Africans Happy? ‘Return to Laughter’ in Times of War, Famine and Misery’, Available at: http://dx.doi.org/10.2139/ssrn.4098094

Kruppe, T. and Lang, J. (2018) ‘Labour market effects of retraining for the unemployed: the role of occupations’, *Applied Economics*, 50(14), pp. 1578-1600.

Lakner, C. and Milanovic, B. (2015) ‘Global income distribution from the fall of the Berlin Wall to the great recession’, *Revista de Economía Institucional*, 17(32), pp. 71-128.

Lederman, D., Loayza, N.V. and Soares, R.R. (2005) ‘Accountability and corruption: Political institutions matter’, *Economics & Politics*, 17(1), pp. 1-35.

Lerman, R.I., Loprest, P.J. and Kuehn, D. (2020) ‘Training for jobs of the future: Improving access, certifying skills, and expanding apprenticeship’, Institute of Labour Economics (IZA) Policy Paper No. 166.

Leslie, L.L. and Ramey, G. (1986) ‘State appropriations and enrollments: Does enrollment growth still pay?’, *The Journal of Higher Education*, 57(1), pp. 1-19.

Levy, G. (2005) ‘The politics of public provision of education’, *The Quarterly Journal of Economics*, 120(4), pp. 1507-1534.

Li, J., Valero, A. and Ventura, G. (2020) *Trends in job-related training and policies for building future skills into the recovery*. Centre for Vocational Educational Research Discussion Paper 33. London: LSE. Available at: https://cver.lse.ac.uk/textonly/cver/pubs/cverdp033.pdf (Accessed: 24 March 2023).

Liefner, I. (2003) ‘Funding, resource allocation, and performance in higher education systems’, *Higher Education*, 46, pp. 469-489.

Lizzeri, A. (1999) ‘Budget deficits and redistributive politics’, *The Review of Economic Studies*, 66(4), pp. 909-928.

Lizzeri, A. and Persico, N. (2001) ‘The provision of public goods under alternative electoral incentives’, *American Economic Review*, 91(1), pp. 225-239.

Looney, A. and Yannelis, C. (2015) ‘A crisis in student loans?: How changes in the characteristics of borrowers and in the institutions they attended contributed to rising loan defaults’, *Brookings Papers on Economic Activity*, Fall 2015, pp. 1-89.

Lowry, R.C. (2001) ‘Governmental structure, trustee selection, and public university prices and spending: Multiple means to similar ends’, *American Journal of Political Science*, 45(4), pp. 845-861.

Ma, J and Pender, M. (2022) *Trends in College Pricing and Student Aid 2022*, New York: College Board.

Ma, J., Pender, M. and Welch, M. (2016) ‘Education Pays 2016: The Benefits of Higher Education for Individuals and Society. Trends in Higher Education Series’, *College Board*, Available at: https://eric.ed.gov/?id=ED572548 (Accessed: 11 Jan 2022).

Makris, M. and Pavan, A. (2021) ‘Taxation under learning by doing’, *Journal of Political Economy*, 129(6), pp. 1878-1944.

Manyika, J., Lund, S., Chui, M., Bughin, J., Woetzel, J., Batra, P., Ko, R. and Sanghvi, S. (2017) 'What the future of work will mean for jobs, skills, and wages', *McKinsey Global Institute*, Available at: https://www.mckinsey.com/featured-insights/future-of-work/jobs-lost-jobs-gained-what-the-future-of-work-will-mean-for-jobs-skills-and-wages (Accessed: 10 Jan 2022).

Marginson, S. (2018) 'Global trends in higher education financing: The United Kingdom', *International Journal of Educational Development*, 58, pp. 26-36.

Marcucci, P.N. and Johnstone, D.B. (2007) 'Tuition fee policies in a comparative perspective: Theoretical and political rationales', *Journal of Higher Education Policy and Management*, 29(1), pp. 25-40. Available at: https://doi.org/10.1080/13600800600980015

Mason, G. (2020) 'Higher education, initial vocational education and training and continuing education and training: where should the balance lie?', *Journal of Education and Work*, 33(7-8), pp. 468-490.

Matsuda, K. and Mazur, K. (2022) 'College education and income contingent loans in equilibrium', *Journal of Monetary Economics*, 132, pp. 100-117.

Mawejje, J. and Odhiambo, N.M. (2020) 'The determinants of fiscal deficits: a survey of literature', *International Review of Economics*, 67(3), pp. 403-417.

Mayhew, K. and Anand, P. (2020) 'COVID-19 and the UK labour market', *Oxford Review of Economic Policy*, 36(S1), pp. S215-S224.

McLendon, M.K. (2003) 'The politics of higher education: Toward an expanded research agenda', *Educational Policy*, 17(1), pp. 165-191.

McLendon, M.K., Hearn, J.C., and Mokher, C.G. (2009) 'Partisans, Professionals, and Power: The Role of Political Factors in State Higher Education Funding', *The Journal of Higher Education*, 80(6), pp. 686-713. Available at: https://doi.org/10.1080/00221546.2009.11779040

McManus, R. and Ozkan, F.G. (2018) 'Who does better for the economy? Presidents versus parliamentary democracies', *Public Choice*, 176(3), pp. 361-387.

Mezza, A., Ringo, D., Sherlund, S. and Sommer, K. (2020) 'Student loans and homeownership', *Journal of Labor Economics*, 38(1), pp. 215-260.

Michaels, G., Natraj, A. and Van Reenen, J. (2014) 'Has ICT polarized skill demand? Evidence from eleven countries over twenty-five years', *Review of Economics and Statistics*, 96(1), pp. 60-77.

Migali, G. (2012) 'Funding higher education and wage uncertainty: Income contingent loan versus mortgage loan', *Economics of Education Review*, 31(6), pp. 871-889.

Milesi-Ferretti, G.M., Perotti, R. and Rostagno, M. (2002) 'Electoral systems and public spending', *The Quarterly Journal of Economics*, 117(2), pp. 609-657.

Mirrlees, J.A. (1971) ‘An exploration in the theory of optimum income taxation’, *The Review of Economic Studies*, 38(2), pp. 175-208.

Mitchell, M. and Leachman, M. (2015) *Years of cuts threaten to put college out of reach for more students*. Available at: https://www.cbpp.org/research/state-budget-and-tax/years-of-cuts-threaten-to-put-college-out-of-reach-for-more-students (Assessed: 21 Jan 2023).

Murphy, R., Scott-Clayton, J. and Wyness, G. (2019) ‘The end of free college in England: Implications for enrolments, equity, and quality’, *Economics of Education Review*, 71, pp. 7-22.

Murphy, K. and Welch, F. (1989) ‘Wage premiums for college graduates: Recent growth and possible explanations’, *Educational researcher*, 18(4), pp. 17-26.

Natvik, G.J. (2013) ‘The political economy of fiscal deficits and government production’, *European Economic Review*, 58, pp. 81-94.

Nedelkoska, L. and Quintini, G. (2018) *Automation, skills use and training*. OECD Social, Employment and Migration Working Papers No.202. Paris: OECD Publishing. Available at: https://doi.org/10.1787/2e2f4eea-en (Accessed: 24 March 2023).

Nerlove, M. (1975) ‘Some problems in the use of income-contingent loans for the finance of higher education’, *Journal of Political Economy*, 83(1), pp. 157-183.

Ness, E.C. and Tandberg, D.A. (2013) 'The determinants of state spending on higher education: How capital project funding differs from general fund appropriations', *The Journal of Higher Education*, 84(3), pp. 329-362.

Ness, E.C., Tandberg, D.A., McLendon, M.K. (2015) 'Interest Groups and State Policy for Higher Education: New Conceptual Understandings and Future Research', *Higher Education: Handbook of Theory and Research*, 30, pp. 151-186. Available at: https://doi.org/10.1007/978-3-319-12835-1_4

Nikoloski, Z., (2007) *Economic and Political Determinants of Income Inequality*. PhD thesis. University College London. Available at: http://www.stat.unipg.it/aissec2009/Documents/papers/87_Nikoloski.pdf (Accessed: 23 March 2023).

OECD (2022) *Education Spending*. Available at: https://data.oecd.org/eduresource/education-spending.htm#indicator-chart (Accessed: 26 Oct 2022).

Orr, D. and Hovdhaugen, E. (2014) ''Second chance' routes into higher education: Sweden, Norway and Germany compared', *International Journal of Lifelong Education*, 33(1), pp. 45-61.

Paluszynski, R. and Yu, P.C. (2023) 'Efficient Consolidation of Incentives for Education and Retirement Savings', *American Economic Journal: Macroeconomics*, 15(3), pp. 153-190.

Peletier, B.D., Dur, R.A. and Swank, O.H. (1999) 'Voting on the budget deficit: comment', *American Economic Review*, 89(5), pp. 1377-1381.

Persson, T. (2002) 'Do political institutions shape economic policy?', *Econometrica*, *70*(3), pp. 883-905.

Persson, T., Roland, G. and Tabellini, G. (1998) 'Towards micropolitical foundations of public finance', *European Economic Review*, 42(3-5), pp. 685-694.

Persson, T., Roland, G. and Tabellini, G. (2000) 'Comparative politics and public finance', *Journal of Political Economy*, 108(6), pp. 1121-1161.

Persson, T. and Svensson, L.E. (1989) 'Why a stubborn conservative would run a deficit: Policy with time-inconsistent preferences', *The Quarterly Journal of Economics*, 104(2), pp. 325-345.

Persson, T. and Tabellini, G. (1999) 'The size and scope of government:: Comparative politics with rational politicians', *European Economic Review*, 43(4-6), pp. 699-735.

Persson, T. and Tabellini, G. (2003) *The economic effects of constitutions: what do the data say?*. Cambridge: MIT press.

Persson, T. and Tabellini, G. (2004) ‘Constitutions and economic policy’, *Journal of Economic Perspectives*, 18(1), pp. 75-98.

Peterson, R.G. (1976) ‘Environmental and political determinants of state higher education appropriations policies’, *The Journal of Higher Education*, 47(5), pp. 523-542.

Pfeffer, J. and Salancik, G.R. (2003) *The external control of organizations: A resource dependence perspective*. Stanford: Stanford University Press.

Piketty, T. and Saez, E. (2014) ‘Inequality in the long run’, *Science*, 344(6186), pp. 838-843.

Pilbeam, C. (2012) ‘Pursuing financial stability: a resource dependence perspective on interactions between pro-vice chancellors in a network of universities’, *Studies in Higher Education*, 37(4), pp. 415-429. Available at: https://doi.org/10.1080/03075079.2010.520696

Poutvaara, P. (2011) ‘The expansion of higher education and time-consistent taxation’, *European Journal of Political Economy*, 27(2), pp. 257-267.

Radomska, S. (2019) ‘Optimal Policy for Investment in Human Capital in the Light of Optimal Tax Theory’, *Studia i Materiały*, 30(1), pp. 34-42.

Roots, R. (1999) ‘The student loan debt crisis: A lesson in unintended consequences’, *Southwestern University Law Review*, 29(3), pp. 501-528.

Rothwell, J.T. (2017) 'Cutting the losses: Reassessing the costs of import competition to workers and communities', Available at SSRN: https://ssrn.com/abstract=2920188 (Accessed at: 13 Jan 2022).

Rothstein, J. and Rouse, C.E. (2011) 'Constrained after college: Student loans and early-career occupational choices', *Journal of Public Economics*, 95(1-2), pp. 149-163.

Sá, F. (2019) 'The effect of university fees on applications, attendance and course choice: Evidence from a natural experiment in the UK', *Economica*, 86(343), pp. 607-634.

Sanford, T. and Hunter, J.M. (2011) 'Impact of performance-funding on retention and graduation rates', *Education Policy Analysis Archives*, 19(33), pp. 1-30.

Sangiumvibool, P. and Chonglerttham, S. (2017) 'Performance-based budgeting for continuing and lifelong education services: the Thai higher education perspective', *Journal of Higher Education Policy and Management*, 39(1), pp. 58-74. Available at: https://doi.org/10.1080/1360080X.2016.1211977

Santos, J.L. (2007) 'Resource allocation in public research universities', *The Review of Higher Education*, 30(2), pp. 125-144. Available at: https://doi.org/10.1353/rhe.2006.0077

Saunders, M. (2012) 'A political economy of university funding: The English case', *Journal of Higher Education Policy and Management*, 34(4), pp. 389-399. Available at: https://doi.org/10.1080/1360080X.2012.689196

Scartascini, C.G. and Crain, W.M. (2021) 'The size and composition of government spending in multi-party systems', in J. Hall and B. Khoo (eds.) *Essays on Government Growth*. Cham: Springer, pp. 97-127.

Scott-Clayton, J.E. (2018) 'The looming student loan crisis is worse than we thought', *Evidence Speaks*, 2(34). Available at: https://doi.org/10.7916/D8WT05QV

Shell, K., Fisher, F.M., Foley, D.K., Friedlaender, A.F., Behr, Jr, J.J., Fischer, S. and Mosenson, R.D. (1968) 'The Educational Opportunity Bank: an economic analysis of a contingent repayment loan program for higher education', *National Tax Journal*, 21(1), pp. 2-45.

Shireman, R. (2017) 'Learn now, pay later: A history of income-contingent student loans in the United States', *The ANNALS of the American Academy of Political and Social Science*, 671(1), pp. 184-201.

Sianesi, B. (2008) 'Differential effects of active labour market programs for the unemployed', *Labour Economics*, 15(3), pp. 370-399.

Sibieta, L., Tahir, I. and Waltmann, B. (2021*) Big changes ahead for adult education funding? Definitely maybe*. Briefing Note 325, London: Institute for Fiscal Studies (IFS). Available at: https://doi.org/10.1920/BN.IFS.2021.BN0325

Social Mobility Commission (2023) *Labour market value of higher and further education qualifications: a summary report*. Available at: https://www.gov.uk/government/publications/labour-market-value-of-higher-and-further-education-qualifications-a-summary-report/labour-market-value-of-higher-and-further-education-qualifications-a-summary-report#executive-summary (Accessed: 20 October 2024).

Song, Z., Storesletten, K. and Zilibotti, F. (2012) 'Rotten parents and disciplined children: A politico-economic theory of public expenditure and debt', *Econometrica*, 80(6), pp. 2785-2803.

Stantcheva, S. (2017) 'Optimal taxation and human capital policies over the life cycle', *Journal of Political Economy*, 125(6), pp. 1931-1990.

Stiglitz, J.E. (2016) 'Income-contingent loans: some general theoretical considerations, with applications', in J.E. Stiglitz and M. Guzman (ed.) *Contemporary Issues in Microeconomics*. London: Palgrave Macmillan, pp. 129-136. Available at: https://doi.org/10.1057/9781137529718_7

Svensson, J. (1998) 'Investment, property rights and political instability: Theory and evidence', *European Economic Review*, 42(7), pp. 1317-1341.

Tabellini, G.E. and Alesina, A. (1990) 'Voting on the budget deficit', *The American Economic Review*, 80(1), pp. 37-49.

Tandberg, D. (2010a) ‘Interest groups and governmental institutions: The politics of state funding of public higher education’, *Educational Policy*, 24(5), pp. 735-778.

Tandberg, D. (2010b) ‘Politics, interest groups and state funding of public higher education’, *Research in Higher Education*, 51(5), pp. 416-450.

Tandberg, D.A., Fowles, J.T. and McLendon, M.K. (2017) ‘The governor and the state higher education executive officer: How the relationship shapes state financial support for higher education’, *The Journal of Higher Education*, 88(1), pp. 110-134.

Tandberg, D.A. and Griffith, C. (2013) ‘State support of higher education: Data, measures, findings, and directions for future research’, In Paulsen. M (ed.) *Higher Education: Handbook of Theory and Research: Volume 28*. Dordrecht: Springer Netherlands, pp. 613-685.

Thomas, H.G. (2001) ‘Funding mechanism or quality assessment: Responses to the research assessment exercise in English institutions’, *Journal of Higher Education Policy and Management*, 23(2), pp. 171-179. Available at: https://doi.org/10.1080/13600800120088601

Tremblay, C.H. (1986) ‘The impact of school and college expenditures on the wages of southern and non-southern workers’, *Journal of Labor Research*, 7, pp. 201-211.

UK Government (2022a) *Student Finance Eligibility*. Available at: https://www.gov.uk/student-finance/who-qualifies (Accessed: 23 Feb 2022).

UK Government (2022b) *Advanced Learner Loan*. Available at: https://www.gov.uk/advanced-learner-loan (Accessed: 23 Feb 2022).

UKRI (2022) *What is the REF?*. Available at: https://www.ref.ac.uk/about-the-ref/what-is-the-ref/ (Accessed: 17 Jan 2023).

Utar, H. (2018) 'Workers beneath the floodgates: Low-wage import competition and workers' adjustment', *Review of Economics and Statistics*, 100(4), pp. 631-647.

Van Long, N. (2019) 'Financing higher education in an imperfect world', *Economics of Education Review*, 71, pp. 23-31.

Voigt, S. (2011) 'Positive constitutional economics II—a survey of recent developments', *Public Choice*, 146(1), pp. 205-256.

WID.world (2022) *World Inequality Database*. Available at: https://wid.world/ (Accessed: 1 June 2022).

World Bank Open Data (2021) *Gini Index*. Available at: https://data.worldbank.org/indicator/SI.POV.GINI (Accessed: 1 June 2022).

Withers, G. (2014) 'Future Directions for ICL Theory', in *Income Contingent Loans: Theory, Practice and Prospects*. London: Palgrave Macmillan, pp. 241-247.

Yared, P. (2019) ‘Rising government debt: Causes and solutions for a decades-old trend’, *Journal of Economic Perspectives*, 33(2), pp. 115-40.

Zhang, Q., Ning, K. and Barnes, R. (2016) ‘A systematic literature review of funding for higher education institutions in developed countries’, *Frontiers of Education in China*, 11(4), pp. 519-542.