Application of Low Volume Statistical Process Control compared against the methodology proposed by BS ISO 7870-8:2017

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by

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Abstract

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The introduction of Statistical Process Control (SPC) by Shewhart [1] in the early 1920s demonstrated significant potential for improving quality parameters in high-volume production. However, since the original SPC concepts were primarily designed for mass production, numerous methods have been developed and adapted for low-volume scenarios starting from the 1970s. Statistical Process Control (SPC) methods have been developed and adapted to low-volume scenarios. These include short-run production with multiple products and parts and a just-in-time methodology to ensure quality control. However, current methods often generate numerous charts, leading to interpretation issues and significant time waste[2]. The BS ISO 7870-8 standard, issued in 2017, provides a framework for addressing deviations from the target in the application of statistical methods rather than focusing on individual values. However, it is essential to note that this standard is designed primarily for one-off quantities. It advises seeking specialized advice for other production volumes, which does not offer a comprehensive solution. This limitation highlights the need for a more inclusive approach. To address this gap, a case study will be conducted on Rotary Power, a UK-based company that manufactures engines and hydraulic pumps. The objective is to analyze how the company could manage the absence of a standard guideline for applying statistical methods to all short-run scenarios. Currently, no such comprehensive guideline exists, posing a significant challenge. The approach will prioritize deviation from the target as the primary value, rather than the measured value, to more accurately display process performance. This will enable data collection from various production stages while precisely monitoring process performance. In this study, the term "data transformation" will refer to values derived from deviations from the target, which are applied in the Statistical Process Control (SPC) methods presented in this work. Utilizing production data from Rotary Power, this research will evaluate the effectiveness of various control charts compared to the methodology proposed by the British Standard BS ISO 7870-8:2017. Specifically, the study will compare the effectiveness of the BS ISO 7870-8:2017 methods against the Q Chart, CUSUM, EWMA, and Moving Average methods. In conclusion, the methods successfully highlight potential risks and offer an efficient approach to identifying and managing these risks, promising an improvement in process quality for low-volume situations. Nevertheless, additional research is necessary to cover a wider variety of low-volume cases, since the BS ISO 7870-8:2017 standard is limited to scenarios where the size of each sample equals one and does not adequately address many real industry scenarios.

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List of Abbreviations

BS	British Standard
CUSUM	Cumulative Sum
EWMA	Exponentially Weighted Moving Average
ISO	International Organization for Standardization
SPC	Statistical Process Control

Nomenclature

CL	Centre Line
C_p	Process capability precision metric
δ	Deviation from target
H	T student distribution function
LCL	Lower Control Limit
m	Number of data points
σ	Standard Deviation
$\bar{x}$	Mean (Average) of a sample
$\tilde{x}$	Median of a sample
R	Range. Measure of any linear trend between two variables
T	Target Value
UCL	Upper Control Limit
Q	Value of Q in Q charts
K	One-half of the amount of change or shift of standard deviation
Φ	Normal Distribution function

Declaration

This thesis represents the culmination of my research on Statistical Process Control for low-volume industry scenarios, which was conducted at the Department of Engineering of Durham University in the United Kingdom. I would like to confirm that all of the material presented in this thesis is original and has not been submitted elsewhere for any other degree of qualification. Moreover, I attest that all of the work presented here is entirely my own unless otherwise referenced within the text.

I would like to express my gratitude to my family and tech tools, who have always stood by me despite the distance between us and have always believed in me. I am also grateful to my friends, who are an indispensable part of my life. I would like to thank my supervisor for supporting me throughout the development of this research. Finally, I would like to express my deep appreciation to my partner for their support in helping me achieve my goals in every project I undertake.

Chapter1

Introduction

In the past decades, the increasing and constant demand for customised products has led many companies and industries to transform their old processes to be more flexible [3]. To maintain such flexibility, some companies implement production models to manufacture a limited quantity of their products. Low Volume production is utilised in a wide variety and high specialisation industries, which may include the following arrangements: many different product types produced in relatively low quantities, the high variance within cycle times depending on the product type, high variance in demand of each separate part number, Just in Time techniques, and mass customisation [4]. In this regard, allowing businesses to increase their ability to customise their products may help them achieve the quality their consumers expect [5].

To respond to a wider variety of demands, in the late 1980s, the concept of Mass Customization (MC) was introduced [6]. Companies must consider several factors to succeed through Mass Customization, including customer demand, market, value chain, technology, a customised offer and knowledge [7].

While adopting new strategies to add flexibility to manufacturing processes may bring many benefits, it is essential to highlight the inherent challenges they could also produce. Achieving quality should be a necessary goal while delivering products or services to a customer; however, it sometimes represents an issue to maintain a high-efficiency level in an increased product variety situation [8]. Traditionally, product reliability and quality are some of the indicators of success, and quality is generally achieved through standardisation and process control [9]. Consequently, when a firm wishes to implement flexibility models with various products, ensuring quality must be one of their primary concerns.

Statistical Process Control (SPC) is a highly effective technique for ensuring process quality [10] and reducing variability [11]. It comprises a range of practical problem-solving tools that can stabilize processes and enhance their capability. However, SPC methods may not be as efficient in small mixed batches as they are in series manufacturing because the data collected from the processes is limited. It is generally recommended to have at least 25 samples to estimate population parameters [12], but this can be challenging in short-run operations where sufficient data may not be available. In such cases, there are differing recommendations in the literature, and ISO 7870-8:2017 offers charting techniques specifically for short runs and small mixed batches. However, in some scenarios, the best course of action may be to seek advice from a specialist as there is no one-size-fits-all solution.

1.1 Statistical Process Control for Low Volume Manufacturing

Statistical Process Control is a term extensively used to describe a compilation of techniques for auditing parts and processes to guarantee quality, prevent deterioration and improve operations. It aims to detect every time a process is not under correct performance so corrective actions can set the process into control. Traditionally, SPC is associated with Shewhart Control Charts since Walter Shewhart was the first to introduce this type of chart to detect special variation causes in processes back in the 1920s [13] and it has been used in a variety of industries ever since. Some of the most common tools of SPC (Statistical Process Control) are Shewhart Charts, Exponentially Weighted Moving Average (EWMA) charts, and Cumulative Sum (CUSUM) charts [14].

As mentioned above, the increasing and constant demand for complex products and high customisation have led many companies and industries to transform their production to low volume, mass customisation or Just in-Time schemes. This tendency generated a new focus towards SPC.

One of the most remarkable disadvantages of using traditional SPC methods in Low Volume production is that the conventional techniques usually require at least 25 samples of data [15] whilst, in versatile manufacturing processes, such an amount of data of one production run would not be attainable. Moreover, processes with a great variety of products would need to develop a chart for every quality characteristic, resulting in an increased number of charts of parts that might just be produced a few times a year, possibly reducing the level of accuracy of the tool if the data points are taken months apart from each other. Therefore, research is still needed to address the challenges associated with SPC for low-volume manufacturing.

1.2 Need of sufficiently low volume SPC research

In recent years diverse work has been presented in the field of Low Volume SPC to adapt to the continuous changes of Low Volume industry; however, most of these new methods fail to be applied in actual manufacturing situations for multiple reasons. Firstly, some of the most significant factors for these methods not to be used rely on the high complexity mathematical models that make them hard to apply in actual production situations and the difficulty of collecting reliable data. Secondly, most of the work related to low-volume SPC has been done using simulated data, Finally, as mentioned above, the paperwork increments as most techniques still need to build a control chart for each part, or characteristic quality [16].

In this regard, there has been a discussion over the years mentioned by Woodall [17] whether the use of SPC is still relevant in modern industry or not; however, as he pointed out, both (traditional and contemporary) statistical tools are significant for the current dynamic manufacturing environment and certainly for short-run high variety production. Thus, more research needs to be done regarding more sophisticated and flexible data collection and analysis methods to ensure process stability in Low volume manufacturing.

Considering the industrial needs described above and the few recent publications in the field, there is a lack of application of these tools as well. One notable exception is BS ISO 7870-8 "Charting techniques for short runs and small mixed batches" being the first major publication in 2017 using the concept of deviation from the target rather than the actual measurement value. Still, they only propose a methodology to process runs of size equal one, and for all other scenarios with different production quantities, the authors refer to 'seek specialist advice' when determining appropriate tools.

This research seeks to address the following question:

Does the method proposed by ISO 7870-8 in 2017 perform better in detecting deviations in the process than traditional SPC tools?

The main aim of this investigation is to expand the research on Low Volume Statistical Process Control, parting from the principles stated in the BS ISO 7870-8, compare it with more traditional techniques such as Q-charts, EWMA charts and CUSUM charts to design a weighting method afterwards which will provide more certainty when using historical data. It is expected to enhance the application of SPC techniques to make them more adaptable to industries with low volumes processes. These methods will be applied to data obtained from a real low-volume scenario.

It is beyond the scope of this study to examine other statistical methods not mentioned in this work, as well as any data from a different source not being the company used for this study.

1.3 Industry Case Description

To address the shortcomings of the research for low-volume statistical process control, this research work was developed in collaboration with a company part of the British Engines Group. This company designs and produces hydraulic pumps and motors based in the North East of England.

By the time of this study, the company in question was using internal inspection software. The data from this inspection was provided in different sets from different dates and parts. The values needed for this study work were extracted from the data sets provided by the company.

1.4 Thesis Structure

The thesis comprises five chapters, with the second chapter outlining the theoretical framework and investigates how the cases of low-volume production environments have been resolved over the years. Chapter three presents the data and applied methodology, while Chapter 4 analyses the results from all calculations applied to the data set in the three proposed approaches. Finally, Chapter five contains this study's conclusion.

Chapter2

Literature Review

This chapter presents the fundamental concepts to understand and apply statistical process control tools. Likewise, the characteristics of low-volume processes will be further explained.

2.1 Traditional Statistical Process Control

In the everlasting pursuit of quality, statistical process control (SPC) has been a fundamental tool for achieving this objective. It can be defined as a set of tools to reduce variability in production processes to achieve stability and improve capability [11]. The term Statistical Process Control was first introduced at the beginning of the Twentieth Century by Walter A. Shewhart to reduce variation and detect special-cause variations in a production process [13], which opposed to common-cause variations; these are not inherent to the process. To achieve this purpose, they utilised Control Charts [18].

As discussed before, variability is a significant concern of quality management. The continuous improvement of processes is crucial in the application of statistical process control [19]. By the time Shewhart introduced the new philosophy of process monitoring, the predominant culture in most western companies, where they separated the work of shop floor operators from the management team, treating operators as part of the manufacturing entity and the managers being the only ones able to make decisions about the process [20]. The change in management ideology included empowering shop floor employees; this way, they could identify changes in the process and find the appropriate solutions.

To summarize, SPC is a collection of statistical and graphical techniques which allow the representation of the status of a process to detect shifts in the process. They are oriented to detect and eliminate variability, and their primary tool is control charts [21].

The process capability indices depend directly on the specification limits. No unique specification limits exist since we use quality characteristics with dissimilar aims. In this regard, applying these process capability indices is impossible when working with low-volume processes.

2.2 Control Charts

Control charts are one of the simplest and most applied statistical process control techniques [11]. They are a graphical representation of the status of a process. The construction of traditional control charts assumes that there is existing data to estimate the parameters of the process before and/or during a production run [22]. Historically, the goal of SPC is to reduce variability considerably, being the control charts the most effective tool to reduce such variability [11]. Using control charts and appropriately interpreting what it displays can help promptly identify any particular cause of variation.

The essential components of a control chart are the Centre line (CL), which commonly represents an average value and indicates how "in-control" a quality characteristic is. Likewise, there are two other parallel lines to the CL, which are called Upper control limit (UCL) and Lower control limit (LCL); if the sample points fall between the control limits, it can be assumed that the process is somehow in-control [11]. In this regard, the control chart becomes a monitoring tool that utilises collected data from the process to compare it against the established control parameters. An example of a basic control chart can be observed in figure 2.1.

There are two predominant control chart types, which can be classified as memory-

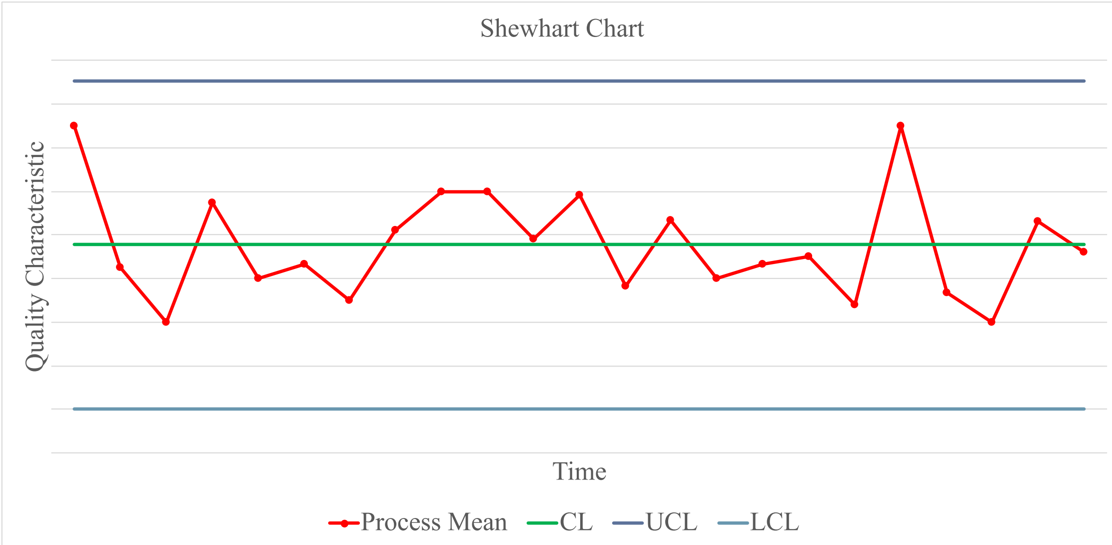

Figure 2.1: Example of a Shewhart Control Chart

less and memory-type control charts [23]. Depending on the literature, some memory-type charts can be classified as time-weighted control charts. The main difference between these two types of charts is that the *memory-less* type of charts only use current data from the process rather than the past and present information like the *memory-type* charts do [24]. Memory-type charts include the CUmulative SUM (CUSUM) and Exponentially Weighted Moving Average (EWMA) charts. On the other hand, the Shewhart chart is the clearest example of a memory-less chart with the disadvantage of being less sensitive to recognising small changes.

2.3 Shewhart Control Charts

Traditionally, a Shewhart chart will display m data points or subgroups from a quality characteristic [13]. These points display the average ($\bar{x}$) of a subgroup, and these subgroups are composed of n samples or observations. The CL can be calculated as the average of the plotted statistic or a derivation of the specification target [25]. Likewise, the upper and lower control limits are plotted at a distance of 3σ where σ refers to the standard deviation of the statistic in question [15]. The basic formulas to calculate the centre line and control limits in control charts are shown below.

$$UCL = \mu_{\bar{x}} + L\sigma_{\bar{x}} \tag{2.1}$$

$$CentreLine = \mu_{\bar{x}} \tag{2.2}$$

$$LCL = \mu_{\bar{x}} - L\sigma_{\bar{x}} \tag{2.3}$$

Moreover, it is generally assumed that the variables of a control chart follow a normal distribution. Kostyszyn [15] states that since normal distributions characterise data by their location and variation, control charts are usually presented as pairs: a) average ($\bar{x}$) chart and range (R) chart or standard deviation (s) chart; b) Individuals (x) chart and moving range (R_m); c) Median ($\tilde{x}$) and range (R) chart.

2.4 Time Weighted Control charts

Unlike memory-less control charts, memory-type control charts are better at detecting small shifts in the process. Memory-type charts are also time-weighted control charts because they incorporate information from previous observations [26]. These charts use recent and past information to determine if the process is in control or out-of-control [27]. Among these charts, the most popular are the Moving average chart, the CUmulative SUM (CUSUM) chart, and the Exponentially Weighted Moving Average (EWMA) chart.

2.4.1 CUSUM charts

Page first introduced CUSUM charts in 1954 [28]. It was developed as an alternative to Shewhart charts for its inefficacy in detecting small shifts in the process. ISO 7870-4 defines CUSUM charts as a ”running total of deviations from pre-selected reference value” [26]; this value is frequently represented as k. This chart cumulatively sums the deviations of the values of the samples from the target or reference value, such as the process mean. The general equation to obtain C value for CUSUM charts is shown in 2.4.

$$C_i = \sum_{j=1}^{i} (\bar{x} - \mu_0) \tag{2.4}$$

Observing the performance of the CUSUM function is essential to interpret the patterns of the point on the chart. Thus, if the process means is greater than the reference value, the plotted points might show an upward trend; otherwise, if the situation is the opposite, the graph should show a downward trend [25]. If the process is in-control, the data points should fluctuate around the horizontal line of zero. Generally, CUSUM charts can be used for three main purposes: for research, as a control tool, and for prediction of performance in the immediate future [26]

There are two accepted methods for making decisions from CUSUM charts. These methods are the V-mask chart proposed by Bernard in 1959 [29] and the Tabular (or algorithmic) CUSUM or numerical charts. Montgomery [11] states that the tabular CUSUM is preferable; however, the two methods will be briefly described as follows.

2.4.1.1 Tabular CUSUM

The tabular CUSUM is a decision-making tool that helps us monitor the process's means. This method can be used for individual observations and the averages of subgroups [11]. This type of CUSUM utilises the differences above or below the target μ_0 and accumulates these differences in new statistics C^+ and C^-, respectively. To calculate the statistics C^+ and C^- one can follow equations 2.5 and 2.6

$$C_i^+ = max[0, x_i - (\mu_0 + K) + C_{i-1}^+] \tag{2.5}$$

$$C_i^- = max[0, (\mu_0 + K) - x_i + C_{i-1}^-] \tag{2.6}$$

Where the starting values are $C_0^+ = C_0^- = 0$

As shown above, the tabular CUSUM accumulates and tabulates in a separate form the difference between the observed value and the sum of the target value and the K value (or reference value), which can be defined as one-half of the amount of change or shift of standard deviation. It can be calculated as follows in 2.7:

$$K = \frac{\delta}{2}\sigma = \frac{|\mu_1 - \mu_0|}{2} \tag{2.7}$$

Likewise, there is a decision interval H which must be met for the process to be considered to be in control. Choosing the value of H is very important, and an acceptable option is $H = 5\sigma$.

The appropriate choice of the parameters K and H is essential to get good results from the CUSUM chart [30].

Alternative techniques are also suggested in the literature to utilise CUSUM charts in different situations, such as CUSUM for rational subgroups, one-sided CUSUM, and CUSUM for monitoring variability, V-mask, among others [11].

2.4.2 Exponentially Weighted Moving Average (EWMA)

The Exponentially Weighted Moving Average was first introduced by Roberts (1959), and this technique was originally called *Geometric* moving average [31]. The method consists of assigning weights, decreasing as a geometric progression from the newest to the oldest value. The weights are assigned as fractions of λ, where the most recent value gets $0 < \lambda < 1$, and the rest of the data is assigned a weight of (1-λ). The general equation to compute the EWMA (z) values is shown below in 2.8.

$$z_i = \lambda x_i + (1-\lambda)z_{i-1} \tag{2.8}$$

EWMA control charts were intended to detect small shifts in the process more accurately than the classical Shewhart chart and even more sensitive than the standard Moving Average charts.

Generally, the starting point of this chart is $z_0 = \mu_0$, but when previous data is used as a starting point, it can be written as $z_0 = \bar{x}$.

Montgomery [11] illustrates how the EWMA z_i is actually a weighted average of all sample means with the following equations,

$$z_i = \lambda x_i + \lambda(1-\lambda)x_{i-1} + (1-\lambda)^2 z_{i-2} \tag{2.9}$$

Repeatedly substituting z_{i-j}, $j = 2, 3, ..., t$, we have

$$z_i = \lambda \sum_{j=0}^{i-1} (1-\lambda)^j x_{i-j} + (1-\lambda)^i z_0 \tag{2.10}$$

The construction of the EWMA chart involves the calculation of the centre line and control limits. These can be obtained with the following equations,

$$UCL = \mu_0 + L\sigma\sqrt{\frac{\lambda}{(2-\lambda}[1-(1-\lambda)^{2i}]} \tag{2.11}$$

$$CenterLine = \mu_0 \tag{2.12}$$

$$LCL = \mu_0 - L\sigma\sqrt{\frac{\lambda}{(2-\lambda}[1-(1-\lambda)^{2i}]} \tag{2.13}$$

Where L represents the width of the control limits.

Likewise, estimating λ is an important part of the EWMA process. Typically, *lambda* sure represents a smoothing parameter, a weighting tool that can help to give more or less value to the data depending on how much relevance it has according to the date [21]. This parameter can take any value from 0 to 1.

The values of L and λ can be chosen according to the performance desired in the EWMA Control Chart. Montgomery [11] proposed a table, which was at the same time adapted from [32] to guide the selection of these parameters.

Shift in Mean (multiple of σ)	L=3.054	2.998	2.962	2.814	2.615
	λ=0.40	0.25	0.20	0.10	0.05
0	500	500	500	500	500
0.25	224	170	150	106	84.1
0.50	71.2	48.2	41.8	31.3	28.8
0.75	28.4	20.1	18.2	15.9	16.4
1.00	14.3	11.1	10.5	10.3	11.4
1.50	5.9	5.5	5.5	6.1	7.1
2.00	3.5	3.6	3.7	4.4	5.2
2.50	2.5	2.7	2.9	3.4	4.2
3.00	2.0	2.3	2.4	2.9	3.5
4.00	1.4	1.7	1.9	2.2	2.7

Table 2.1: Average Run Lengths for Several EWMA Control Schemes

The application of this control chart is further explained in chapter 3.2.4.

2.4.3 Moving Average (MA)

The moving Average control chart is another type of time-weighted control chart; however, unlike the EWMA control chart, the MA chart weighs the current and previous values of the process equally to estimate the average. This control chart presents a graphical analysis of data collected continuously to provide an updated picture of the performance of the process [31].

The general equation to calculate the MA of period w at time i is displayed in 2.14

$$M_i = \frac{x_i + x_{i-1} + ... + x_{i-w+1}}{w} \tag{2.14}$$

The MA chart is preferred over the classical Shewhart chart because it is more effective in detecting small shifts in the process; nevertheless, CUSUM and EWMA charts are more sensitive [11].

The general equations to estimate Upper and Lower control limits in MA charts are described in 2.15 and 2.16, respectively.

$$UCL = \mu_0 + \frac{3\sigma}{\sqrt{w}} \tag{2.15}$$

$$LCL = \mu_0 - \frac{3\sigma}{\sqrt{w}} \tag{2.16}$$

The application of moving average charts will be expanded in Chapter 3.

2.5 Q-Charts

In 1992 Quesenberry [33] defined Q Statistics that could be used in different situations where either one or neither of the parameters (process mean or variance) are known.

One must follow the next equation to calculate the Q statistics:

$$Q_i = \phi^{-1}(G_{i-2}(T_i)), i = 3, 4, 5... \tag{2.17}$$

where $\phi^{-1}(.)$ is the inverse of the standard normal cumulative distribution function.

The principle of using Q-charts is that if $x_1, x_2, x_3...$ are normally distributed with common mean μ and variance σ^2, the Q_i values should be plotted within the control limits.

Some of the advantages [2] of using Q Charts are:

- They are easy to obtain in real-time, e.g. when the process parameters cannot be estimated before the beginning of the process run.

- They can be managed more easily by plotting different variables on the same chart.

- The rules to find patterns, such as the Western Electric rules can be used to improve the capability of the chart to detect special causes of variation.

There are different to obtain the Q statistics depending on the conditions of the data. These conditions can be, as stated by Quesenberry [25], regarding the arrangement of the data and whether the mean and the variance are known or unknown.

The first 4 cases presented by the author are derived from sample means and sub-grouped data.

Case KK: Where both mean $\mu = \mu_0$ and variance $\sigma = \sigma_0$ are known.

$$Q_i(\bar{X}_i) = \frac{\sqrt{n_i}(\bar{X}_i - \mu_0)}{\sigma_0} \tag{2.18}$$

where $i = 1, 2, ...$

Case UK: Where mean μ is unknown, and $\sigma = \sigma_0$ is known.

$$Q_i(\bar{X}_i) = \sqrt{\frac{n_i(n_1 + ... + n_{i-1})}{n_1 + ... + n_i}} \left(\frac{\bar{X}_i - \bar{\bar{X}}_{i-1}}{\sigma_0} \right) \tag{2.19}$$

where $i = 2, 3, ...$

Case KU: Where mean $\mu = \mu_0$ is known, and σ is unknown.

$$Q_i(\bar{X}_i) = \Phi^{-1} \left[H_{n_1 + ... + n_i - i} \left(\frac{\sqrt{n_i}(\bar{X}_i - \mu_0)}{S_{p,i}} \right) \right] \tag{2.20}$$

where $i = 1, 2, ...$

Case UU: Where μ and σ are unknown.

$$Q_i(\bar{X}_i) = \Phi^{-1} \left\{ H_{n_1 + ... + n_i - i} \left[\sqrt{\frac{n_i(n_1 + ... + n_{i-1})}{n_1 + ... + n_i}} \left(\frac{\bar{X}_i - \bar{\bar{X}}_{i-1}}{S_{p,i}} \right) \right] \right\} \tag{2.21}$$

where $i = 2, 3, ...$

Likewise, four other methods exist to obtain the Q statistics based on individual measurements for the process mean. The following equations were developed by Quesenberry

[25] for the same four cases.

Case KK: Where both mean $\mu = \mu_0$ and variance $\sigma = \sigma_0$ are known.

$$Q_r(X_r) = \frac{X_r - \mu_0}{\sigma_0} \tag{2.22}$$

where $r = 1, 2, ...$

Case UK: Where mean μ is unknown, and $\sigma = \sigma_0$ is known.

$$Q_r(X_r) = \sqrt{\frac{r-1}{r}} \frac{(X_r - \bar{X}_{r-1}}{\sigma_0}) \tag{2.23}$$

where $r = 2, 3, ...$

Case KU: Where mean $\mu = \mu_0$ is known, and σ is unknown.

$$Q_r(X_r) = \Phi^{-1}\left\{H_{r-2}\left(\frac{X_r - \mu_0}{S_{r-1}}\right)\right\} \tag{2.24}$$

where $r = 3, 4, ...$

Case UU: Where μ and σ are unknown.

$$Q_r(X_r) = \Phi^{-1}\left\{H_{r-2}\left[\sqrt{\frac{r-1}{r}}\left(\frac{X_r - \bar{X}_{r-1}}{S_{r-1}}\right)\right]\right\} \tag{2.25}$$

where $r = 3, 4, ...$

Where H stands for the t-student distribution, and Φ^{-1} represents the inverse of the Normal distribution.

For the purposes of this thesis, the approach to use will be the case "UU" for individual measurements described in 2.25.

2.6 Short-Run Small Mixed Batches

One of the first improvements for the application of SPC on low-volume processes resulted in a new generation of statistical control, known now as Short-run SPC. Bothe [34] promptly identified the primary needs of these smaller runs and proposed the deviation from the target as the reference value instead of the traditional method, which is to use the actual value obtained from an inspection. This approach intended to solve the problem of insufficient data to analyse and even allowed part numbers with different attributes to be plotted on the same chart. Other contributions are attributed to Montgomery [11], who, in the 1990s, included multivariate methods for monitoring several related variables simultaneously. Furthermore, Quesenberry [25] also proposed an approach to solve the problem of estimating mean and standard deviation in short-run production.

One cannot assume that traditional control charts can be applied to Low volume processes because the conditions are different from those in high-volume production, firstly because of the small amount of data generated and the diversity of the parts produced, and secondly of the natural irregularity of batch sizes and the difference in dates between these batches.

Nevertheless, many manufacturing systems in modern days rely on low-volume high-variety production; hence they do not meet the requirements for classical statistical tools [35]. Short-run processes can be found in multiple industries. Some of the most common situations where short runs can be applied are when the philosophy of Just in Time is adopted [34]; when there is little to null historical data available, for example, at the start-up stage of a process [36]; when different products are manufactured, and there is need of constant set-up changes or in workshops where small batches of many parts are produced [16].

One of the first researchers to propose solutions for SPC to be used in situations where there are only a few groups of data inspected (instead of 25, which is the standard) was Hillier [37], who developed a short-run theory of two stages for X-bar and R control charts. Hillier derived mathematical equations from reducing the probability of having a false alarm when charting a process with few inspections, which due to their nature, lacks available data. In the first stage (I), he proposed that for m, subgroups of size n are portrayed in the control charts to determine if the process s was stable or not. The second stage (II) begins only when the process is under control and the performance of the process is to be monitored. Subsequently, Yang joined Hillier [38] to work further on the matter of the two-stage argument to calculate the limits of control for the (X-bar, v) and (X-bar,$\sqrt{v}$) charts, using v as the variance of a subgroup. Elam et al. later upgraded the theory [39] and introduced computer programs to solve the problem of limited data. They worked by determining the factor of the charts more precisely.

One of the disadvantages of the two-stage technique is that it depends on previous data to calculate the process parameters [40]. This issue was identified and studied by Quesenberry [33], who proposed Q-Charts, of which one of the biggest advantages is displaying the performance in a single chart of many quality characteristics.

These charts are designed to detect changes in the process parameters at the time of the process initiation, and they do not require massive historical data. Q-charts' goal is to use a transformation to consider data as independent and identically distributed to build Q-statistics.

Q-charts are one of the most popular self-starting tools for observing short-run production processes. Self-starting methods allow the plotting process to be in real-time since they can start building charts with just two samples, whilst the parameters are estimated and updated as the production advances. Moreover, Q-charts permit the inspection of a process with various products when the variances of the different quality characteristics are known. If the variances are unknown, using a deviation chart is preferred [41]. Other self-staring approaches are CUSUM charts proposed by Hawkins [42], whose objective is to detect minor changes in dispersion and location parameters.

Data transformation is present in other SPC Short-run approaches; one of these methods is t-Charts. It utilises the T-Statistics, and preliminary estimation of the standard deviation of the in-control process is not required to build a t-chart, which makes it applicable for the start-up of a process. Some authors have made improvements to t-charts, but they were first proposed by Zhang et al. [43] when they presented the X-bar t-chart and EWMA t-chart. Gu et al. [35] also worked with t-charts and adjusted them for use in multi-variety and small-batch production runs.

Several methods have been proposed to improve Statistical Process Control for short-run productions. An extensive review of these techniques is presented by Marques et al. [44] in their Decision-Model proposal to select the most suitable SPC tool in short-run situations.

Despite all the progress and improvement made on the subject of Low volume SPC, some shortcomings have been identified, and they will be the subject of matter of this research. These shortcomings can be described as follows. The first issue is that the available data (e.g. from inspections of a quality characteristic of a part/product) are insufficient to generate a control chart with the standard methods, which involve using actual measurement values to develop the calculations. The second deficiency is that developing a control chart for each quality characteristic might result in an overload of administration work and the loss of worth from historical data as it is used to create present parameters. Moreover, the fact that most research work has been done with simulated data leaves a gap for real-world applications.

2.7 Standard Guidelines and Applications for Short Run Small Mixed Batches

The International Standardization Organization has evaluated and established normative documents to facilitate and guide the use of control charts with the ISO-7870, and specifically, the ISO-7870-8 *Control charts — Part 8: Charting techniques for short runs and small mixed batches* which the last version was updated and published in 2017. Despite this relatively new ISO regulation, it does not provide a solution to implement SPC in all low-volume situations, i.e. when the size is $n \neq 1$. An example of the flow diagram proposed by ISO is displayed in figure 2.2.

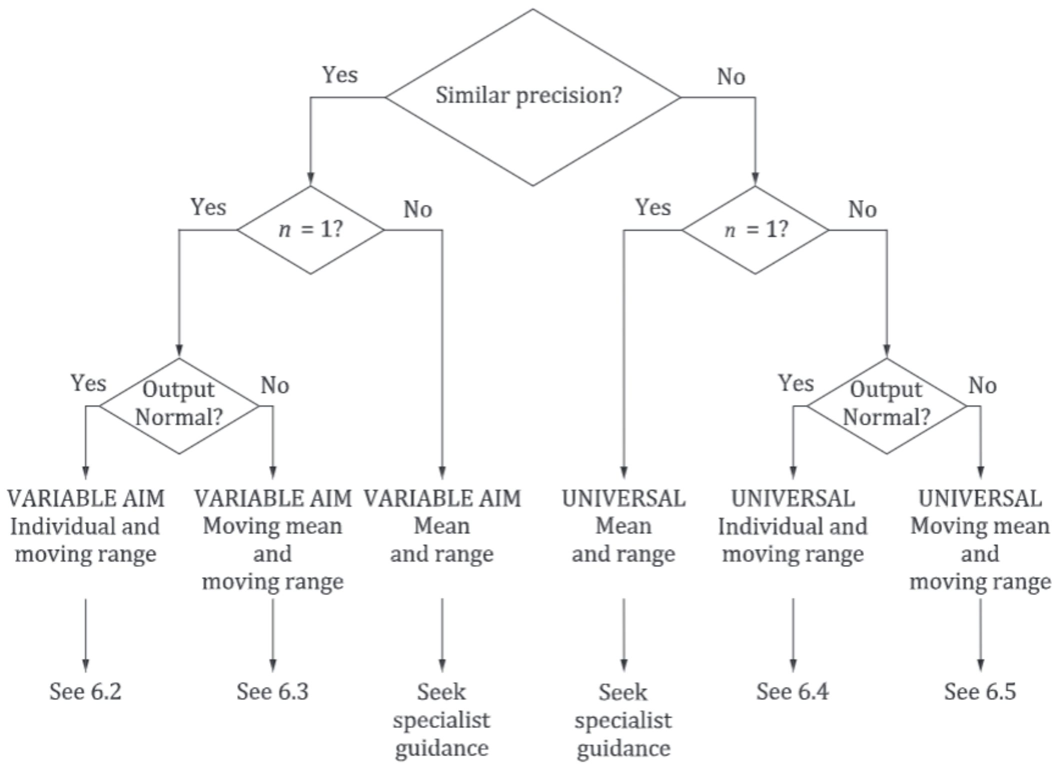

Figure 2.2: Control Chart Selection for short runs and small batches, taken from BSI ISO 7870-8:2017 [45]

The absence of a specific solution for industry and companies to refer to leaves researchers with an opportunity to research the subject in depth and find an alternative solution to this issue.

This British Standard proposes four scenarios for short runs and small batches where the subgroup size equals 1. The characteristics of these charts are presented in table 2.2.

The construction of the individual and moving range charts for individual measurements without constant aim should follow the procedure shown in 2.3 according to the BSI ISO 7870-8:2017 [45].

These values were obtained from control chart constants such as A_3s, d_2 and D_4.

Table 2.2: Chart Selection table for short runs and small batches (subgroup size, n=1) from ISO 7078-8:2017

Parameter or Characteristic	**Process Aim**	**Process Spread**	**Output**	**Chart Name**
Single	Dissimilar	Similar	Normal	Variable aim, individual and moving range
Single	Dissimilar	Similar	Approximately Normal	Variable aim, moving mean and moving range
Multiple	Dissimilar	Dissimilar	Approximately Normal	Universal, individual and moving range
Multiple	Dissimilar	Dissimilar	Non-normal	Universal, moving mean and moving range

Table 2.3: Data for constructing a not constant aim, individual and moving range chart from BS ISO 7870-8:2017

	Individual	**Range**
Plot point	X-T	Moving Range
CL	0	R_{exp}*
UCL	$+2.66\,R_{\text{exp}}$	$3.27\,R_{\text{exp}}$
LCL	$-2.66\,R_{\text{exp}}$	0
* $R_{\text{exp}} = 1.128\times$ expected standard deviation for a moving range of two.		

2.8 Chapter Summary

In this chapter, we have discussed the statistical process control techniques that are applicable to low-volume manufacturing processes. Additionally, we have provided an overview of the process control methodology recommended by the BS ISO 7870-8:2017, which is the primary standard documentation evaluated in this thesis work.

Chapter3

Proposed Methodology for Data Processing for Low-Volume Manufacturing

This chapter aims to apply and assess a selection of statistical methods to a set of low-volume data. Following the guidance provided by BS ISO 7870-8:2017, this study will focus on finding the best Statistical Methods to achieve process control for small mixed batches and short-run situations that are not covered by the ISO documentation.

The proposed initial step is to transform the data. This method transforms the primary variable to a 'deviation from the target' variable. Using the deviation from the target makes it possible to study the performance of multiple part numbers in the same chart, reducing the number of charts used over time.

The data will be analysed using three different grouping approaches. The first approach groups the data in groups of one, meaning all values are analysed individually. The second approach groups the data in small groups of equal or less than five values, with the condition for those values to be from the same date to be within the same subgroup. Moreover, the third approach groups the data in groups of five to maintain consistency within the subgroups.

3.1 Data set from low volume industry

The dataset employed in this study comprises measurements from three distinct grinding processes conducted on the Studer S145 machine at Rotary Power. These processes are designated as Operations 70, 80, and 140, each with specific target values. Due to the inherent variability in production, the number of pieces processed in each run differs. Detailed information regarding these processes is provided in Table 3.1 .

Machine	Part ID	Operation	Description	Target
Studer S145	C01C27157	70	Internal Grind	1.75
Studer S145	C01C27157	80	Internal Grind	1.97
Studer S145	C07C27145	140	Internal Grind	1.97

Table 3.1: Specification of parts used for analysis

The measurements forming the dataset were sourced from the company's Enterprise Resource Planning (ERP) system, which includes a tool called SUPA (Set-Up Process Algorithm). SUPA was developed by Stephen Cox during their PhD research at Durham University and employs probability theory to ensure, with 98% confidence, that a process meets a minimum level of process capability using as few as five samples. Despite its efficacy, one significant drawback of this system is the high complexity associated with the optimization calculations and the data-gathering methodology. Currently, operators manually input the measurements into the ERP software, which is characterized by a lack of flexibility and significant potential for improvement.

The data from March 2018 to August 2018 is suitable for this study due to several key factors. Firstly, it provides a record of the grinding process, capturing both the variability and specific target values necessary for the analysis. This detailed recording is essential because it gives an understanding of the grinding process's fluctuations and ensures that the analysis can account for a range of operational scenarios.

Secondly, using data from the ERP system ensures systematic collection, accurately reflecting actual production conditions. The ERP system's structured approach to data collection minimizes the risk of inconsistencies or gaps in the data, enhancing the reliability of the analysis. This systematic collection method ensures that the dataset is representative of real-world production scenarios, making the study's findings more applicable and credible. These characteristics contribute to a robust dataset, providing a solid foundation for the study and helping to ensure that the conclusions are based on accurate information. The complete dataset can be found in the Appendix A.2.

The measurement values from the inspection were transformed to be δ, or 'deviation from target'. All calculations will be performed using this δ value from here.

The data set found in appendix A.2 was organized as individual values, and it is presented in table 3.2

Table 3.2: Data Set arranged as individual values

Subgroup	Part	Op.	Date	Value	Target	X-T
1	C07C27145	140	3/23/2018	2	1.97	0.03
2	C07C27145	140	3/23/2018	2	1.97	0.03
3	C07C27145	140	3/26/2018	1.93	1.97	-0.04
4	C07C27145	140	3/26/2018	1.94	1.97	-0.03
5	C07C27145	140	3/27/2018	1.91	1.97	-0.06
6	C07C27145	140	3/27/2018	1.91	1.97	-0.06
7	C07C27145	140	4/22/2018	1.93	1.97	-0.04
8	C07C27145	140	4/22/2018	2	1.97	0.03
9	C07C27145	140	4/23/2018	1.93	1.97	-0.04
10	C07C27145	140	7/16/2018	1.93	1.97	-0.04
11	C07C27145	140	7/16/2018	2	1.97	0.03
12	C07C27145	140	7/16/2018	1.93	1.97	-0.04
13	C07C27145	140	7/16/2018	1.94	1.97	-0.03
14	C07C27145	140	7/16/2018	1.91	1.97	-0.06
15	C07C27145	140	7/16/2018	1.91	1.97	-0.06
16	C01C27157	70	8/8/2018	1.7	1.75	-0.05
17	C01C27157	70	8/8/2018	1.73	1.75	-0.02
18	C01C27157	70	8/8/2018	1.73	1.75	-0.02
19	C01C27157	70	8/8/2018	1.73	1.75	-0.02
20	C01C27157	70	8/8/2018	1.73	1.75	-0.02
21	C01C27157	70	8/8/2018	1.74	1.75	-0.01
22	C01C27157	70	8/10/2018	1.74	1.75	-0.01
23	C01C27157	70	8/10/2018	1.74	1.75	-0.01
24	C01C27157	70	8/10/2018	1.74	1.75	-0.01
25	C01C27157	70	8/10/2018	1.78	1.75	0.03
26	C01C27157	70	8/13/2018	1.75	1.75	0
27	C01C27157	70	8/14/2018	1.75	1.75	0
28	C01C27157	70	8/14/2018	1.77	1.75	0.02
29	C01C27157	70	8/14/2018	1.72	1.75	-0.03
30	C01C27157	70	8/14/2018	1.77	1.75	0.02
31	C01C27157	70	8/14/2018	1.74	1.75	-0.01
32	C01C27157	70	8/14/2018	1.78	1.75	0.03
33	C01C27157	70	8/14/2018	1.7	1.75	-0.05
34	C01C27157	70	8/14/2018	1.73	1.75	-0.02
35	C01C27157	70	8/14/2018	1.71	1.75	-0.04
36	C01C27157	70	8/14/2018	1.74	1.75	-0.01
37	C01C27157	70	8/14/2018	1.7	1.75	-0.05
38	C01C27157	70	8/14/2018	1.75	1.75	0
39	C01C27157	70	8/14/2018	1.6	1.75	-0.15
40	C01C27157	70	8/14/2018	1.71	1.75	-0.04
41	C01C27157	70	8/14/2018	1.79	1.75	0.04
42	C01C27157	70	8/14/2018	1.7	1.75	-0.05
43	C01C27157	70	8/14/2018	1.72	1.75	-0.03
44	C01C27157	70	8/14/2018	1.7	1.75	-0.05
45	C01C27157	70	8/14/2018	1.78	1.75	0.03
46	C01C27157	70	8/14/2018	1.7	1.75	-0.05
47	C01C27157	70	8/15/2018	1.7	1.75	-0.05
48	C01C27157	70	8/15/2018	1.72	1.75	-0.03
49	C01C27157	70	8/15/2018	1.77	1.75	0.02
50	C01C27157	70	8/15/2018	1.73	1.75	-0.02
51	C01C27157	70	8/15/2018	1.73	1.75	-0.02
52	C01C27157	70	8/15/2018	1.76	1.75	0.01
53	C01C27157	70	8/15/2018	1.77	1.75	0.02
54	C01C27157	70	8/15/2018	1.78	1.75	0.03
55	C01C27157	70	8/15/2018	1.78	1.75	0.03
56	C01C27157	70	8/15/2018	1.75	1.75	0
57	C01C27157	70	8/15/2018	1.73	1.75	-0.02
58	C01C27157	70	8/15/2018	1.75	1.75	0
59	C01C27157	70	8/15/2018	1.76	1.75	0.01
60	C01C27157	80	8/21/2018	1.95	1.97	-0.02
61	C01C27157	80	8/21/2018	1.93	1.97	-0.04
62	C01C27157	80	8/21/2018	1.9	1.97	-0.07
63	C01C27157	80	8/22/2018	1.9	1.97	-0.07
64	C01C27157	80	8/22/2018	1.94	1.97	-0.03
65	C01C27157	80	8/22/2018	2.03	1.97	0.06
66	C01C27157	80	8/23/2018	1.93	1.97	-0.04
67	C01C27157	80	8/24/2018	1.92	1.97	-0.05
68	C01C27157	80	8/24/2018	1.94	1.97	-0.03
69	C01C27157	80	8/24/2018	1.95	1.97	-0.02
70	C07C27145	140	8/25/2018	1.94	1.97	-0.03
71	C01C27157	80	8/25/2018	1.91	1.97	-0.06
72	C01C27157	80	8/25/2018	1.98	1.97	0.01
73	C01C27157	80	8/25/2018	1.9	1.97	-0.07
74	C01C27157	80	8/25/2018	1.9	1.97	-0.07
75	C01C27157	80	8/25/2018	1.9	1.97	-0.07
76	C07C27145	140	8/29/2018	2	1.97	0.03
77	C01C27157	80	8/30/2018	1.91	1.97	-0.06
78	C01C27157	80	8/30/2018	1.94	1.97	-0.03
79	C01C27157	80	8/30/2018	1.92	1.97	-0.05
80	C01C27157	80	8/30/2018	1.92	1.97	-0.05
81	C01C27157	80	8/30/2018	1.95	1.97	-0.02
82	C01C27157	80	8/30/2018	1.95	1.97	-0.02
83	C01C27157	80	8/30/2018	1.92	1.97	-0.05
84	C01C27157	80	8/30/2018	1.9	1.97	-0.07
85	C01C27157	80	8/30/2018	1.9	1.97	-0.07
86	C01C27157	80	8/30/2018	1.9	1.97	-0.07
87	C01C27157	80	8/30/2018	1.98	1.97	0.01
88	C01C27157	80	8/30/2018	1.93	1.97	-0.04
89	C01C27157	80	8/30/2018	1.91	1.97	-0.06
90	C01C27157	80	8/30/2018	1.91	1.97	-0.06
91	C01C27157	80	8/30/2018	1.91	1.97	-0.06
92	C07C27145	140	8/31/2018	1.91	1.97	-0.06
93	C01C27157	80	8/31/2018	1.95	1.97	-0.02
94	C01C27157	80	8/31/2018	1.93	1.97	-0.04
95	C01C27157	80	8/31/2018	1.94	1.97	-0.03
96	C01C27157	80	8/31/2018	1.91	1.97	-0.06
97	C01C27157	80	8/31/2018	1.99	1.97	0.02
98	C01C27157	80	8/31/2018	1.97	1.97	0
99	C01C27157	80	8/31/2018	1.97	1.97	0
100	C01C27157	80	8/31/2018	1.99	1.97	0.02

Data set from March 2018 to August 2018 arranged as individual values

3.2 SPC techniques to observations as Individual Values

This section evaluates the Statistical Process Control (SPC) methods using a data set organized as individual values rather than in subgroups or batches. The intent is to apply SPC techniques to monitor and analyze the process performance and quality over time, using data points representing single measurements taken at different time intervals.

3.2.1 Application of British Standard ISO 7870-8:2017

In this subsection, the methods applied were the control chart for individual values and the moving range proposed by BS ISO 7078-8 [45].

According to the British Standard mentioned in chapter 2 and the table 2.3, the operations to calculate the control limits of this control chart using this data set are as follows:

$$\delta Value = X - T \tag{3.1}$$

$$CL = 0 \tag{3.2}$$

$$UCL = 2.66 * R_{exp} \tag{3.3}$$

$R_{exp} = 1.28*$ Expected standard deviation for a moving range of two.

$$UCL = -2.66 * R_{exp} \tag{3.4}$$

$R_{exp} = 1.28*$ Expected standard deviation for a moving range of two.

The equations 3.2, 3.3 and 3.4 were applied to the values of column "$X - T$" from table

3.2. These are represented as follows:

Upper control limit:

$$\begin{aligned} UCL &= 2.66 \times R_{exp} \\ &= 2.66 \times (1.128 \times (\frac{\bar{R}}{d_2})) \\ &= 2.66 \times (1.128 \times (\frac{0.03141}{1.128})) \\ UCL &= 0.0836 \end{aligned} \tag{3.5}$$

And for the lower control limit:

$$\begin{aligned} UCL &= -2.66 \times R_{exp} \\ &= -2.66 \times (1.128 \times (\frac{\bar{R}}{d_2})) \\ &= -2.66 \times (1.128 \times (\frac{0.03141}{1.128})) \\ UCL &= -0.0836 \end{aligned} \tag{3.6}$$

The control chart result of these operations is displayed in figure 3.1

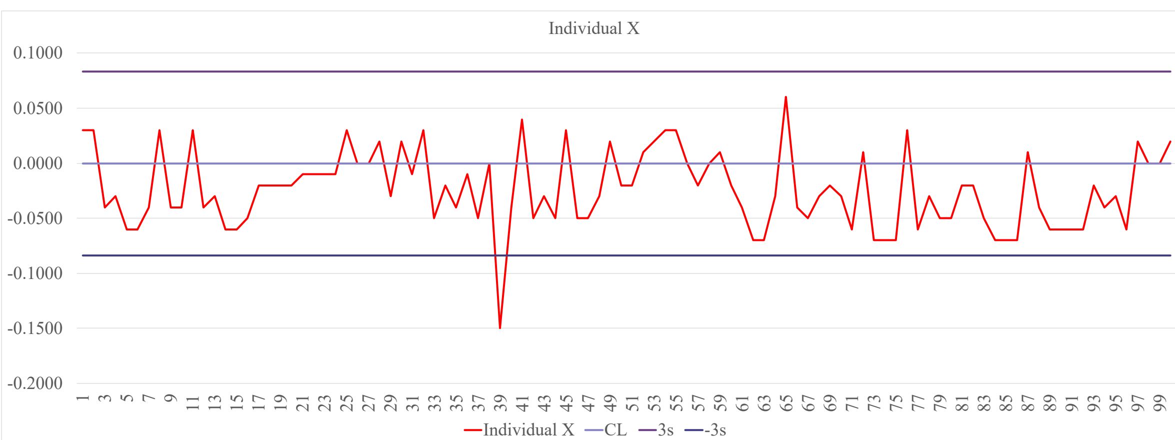

Figure 3.1: Shewhart chart for individual values

Likewise, the moving range chart for this data set was calculated from the equations proposed by the British Standard Organization and are listed below in 3.7, 3.8 and 3.9:

$$CL = R_{exp} \tag{3.7}$$

$$UCL = 3.27 * R_{exp} \tag{3.8}$$

$$LCL = 0 \tag{3.9}$$

For this case, the control limits are calculated as:

$$\begin{aligned} CL &= R_{exp} \\ &= 1.128 \times (\frac{\bar{R}}{d_2}) \\ &= 1.128 \times (\frac{0.03141}{1.128}) \\ CL &= 0.03141 \end{aligned} \tag{3.10}$$

$$\begin{aligned} UCL &= 3.27 \times R_{exp} \\ &= 3.27 \times (1.128 \times (\frac{\bar{R}}{d_2})) \\ &= 3.27 \times (1.128 \times (\frac{0.03141}{1.128})) \\ UCL &= 0.1027 \end{aligned} \tag{3.11}$$

The moving range chart for this data set is presented in figure 3.2.

Partial Results

In charts 3.1 and 3.2, a variation in the process can be observed; however, most of the values seem to fall within the upper and lower control limits except for value no. 39, this value corresponds to the inspection performed on the 18th of August 2018.

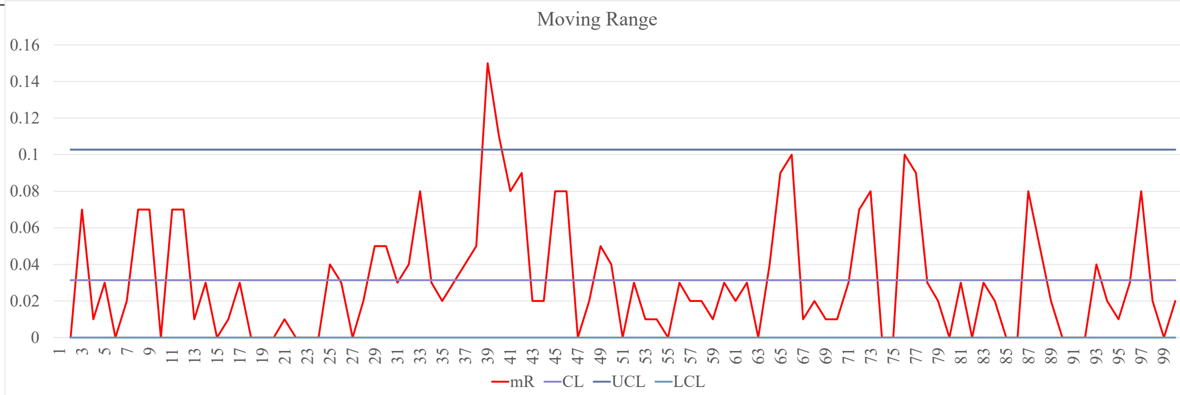

Figure 3.2: Moving range chart for individual values of figure 3.1

3.2.2 Application of Q-Charts to subgroups n=1

This section will analyse the Q charts in the proposed arrangement mentioned before. The method selection was based on those proposed by Charles Quesenberry [25] according to the theory presented in Chapter 2. For individual values, we will use the case of "UU", where the mean and variance of the process are unknown.

According to the literature, the equations to calculate the Q values for this control chart are shown in 2.25 as:

$$Q_r(X_r) = \Phi^{-1}\left\{H_{r-2}\left[\sqrt{\frac{r-1}{r}}\left(\frac{X_r - \bar{X}_{r-1}}{S_{r-1}}\right)\right]\right\}$$

where $r = 3, 4, ...,$ and Φ^{-1} represents the Inverse Normal Distribution of the data. The operations performed in Excel to calculate each Q value are as follows:
$= NORM.S.INV(T.DIST((SQRT((COUNT(X_i : X_{n-1})) - 1/(COUNT(X_i : X_{n-1})))) * ((X_n - AVERAGE(X_i : X_{n-1})/STDEV.P(X_i : X_{n-1}))), COUNT(X_i : X_{n-1}) - 1, TRUE))$
After performing this calculation through the complete data set, the Q values are displayed in table 3.3.

The upper and lower control limits were calculated using the 3σ deviation from the central line, which in this case is zero (0).

Table 3.3: Q Values for individual values

Subgroup	Individual X	Q Statistic	CL	UCL	LCL
1	0.03				
2	0.03				
3	-0.04		0	4.16	-4.16
4	-0.03	-0.33	0	4.16	-4.16
5	-0.06	0.03	0	4.16	-4.16
6	-0.06	0.63	0	4.16	-4.16
7	-0.04	1.13	0	4.16	-4.16
8	0.03	1.58	0	4.16	-4.16
9	-0.04	1.09	0	4.16	-4.16
10	-0.04	1.38	0	4.16	-4.16
11	0.03	1.82	0	4.16	-4.16
12	-0.04	1.32	0	4.16	-4.16
13	-0.03	1.61	0	4.16	-4.16
14	-0.06	1.72	0	4.16	-4.16
15	-0.06	2.02	0	4.16	-4.16
16	-0.05	2.32	0	4.16	-4.16
17	-0.02	2.65	0	4.16	-4.16
18	-0.02	2.77	0	4.16	-4.16
19	-0.02	2.88	0	4.16	-4.16
20	-0.02	3.00	0	4.16	-4.16
21	-0.01	3.14	0	4.16	-4.16
22	-0.01	3.20	0	4.16	-4.16
23	-0.01	3.25	0	4.16	-4.16
24	-0.01	3.30	0	4.16	-4.16
25	0.03	3.49	0	4.16	-4.16
26	0	3.09	0	4.16	-4.16
27	0	3.08	0	4.16	-4.16
28	0.02	3.15	0	4.16	-4.16
29	-0.03	2.78	0	4.16	-4.16
30	0.02	3.14	0	4.16	-4.16
31	-0.01	2.85	0	4.16	-4.16
32	0.03	3.08	0	4.16	-4.16
33	-0.05	2.47	0	4.16	-4.16
34	-0.02	2.81	0	4.16	-4.16
35	-0.04	2.82	0	4.16	-4.16
36	-0.01	3.13	0	4.16	-4.16
37	-0.05	2.99	0	4.16	-4.16
38	0	3.41	0	4.16	-4.16
39	-0.15	2.64	0	4.16	-4.16
40	-0.04	3.17	0	4.16	-4.16
41	0.04	3.71	0	4.16	-4.16
42	-0.05	3.01	0	4.16	-4.16
43	-0.03	3.27	0	4.16	-4.16
44	-0.05	3.27	0	4.16	-4.16
45	0.03	3.84	0	4.16	-4.16
46	-0.05	3.25	0	4.16	-4.16
47	-0.05	3.40	0	4.16	-4.16
48	-0.03	3.65	0	4.16	-4.16
49	0.02	4.02	0	4.16	-4.16
50	-0.02	3.70	0	4.16	-4.16

Subgroup	Individual X	Q Statistic	CL	UCL	LCL
51	-0.02	3.77	0	4.16	-4.16
52	0.01	4.01	0	4.16	-4.16
53	0.02	4.01	0	4.16	-4.16
54	0.03	3.97	0	4.16	-4.16
55	0.03	3.83	0	4.16	-4.16
56	0	3.50	0	4.16	-4.16
57	-0.02	3.37	0	4.16	-4.16
58	0	3.56	0	4.16	-4.16
59	0.01	3.62	0	4.16	-4.16
60	-0.02	3.38	0	4.16	-4.16
61	-0.04	3.32	0	4.16	-4.16
62	-0.07	3.23	0	4.16	-4.16
63	-0.07	3.39	0	4.16	-4.16
64	-0.03	3.81	0	4.16	-4.16
65	0.06	4.49	0	4.16	-4.16
66	-0.04	3.52	0	4.16	-4.16
67	-0.05	3.56	0	4.16	-4.16
68	-0.03	3.82	0	4.16	-4.16
69	-0.02	3.98	0	4.16	-4.16
70	-0.03	3.97	0	4.16	-4.16
71	-0.06	3.84	0	4.16	-4.16
72	0.01	4.47	0	4.16	-4.16
73	-0.07	3.86	0	4.16	-4.16
74	-0.07	4.00	0	4.16	-4.16
75	-0.07	4.14	0	4.16	-4.16
76	0.03	4.97	0	4.16	-4.16
77	-0.06	4.21	0	4.16	-4.16
78	-0.03	4.55	0	4.16	-4.16
79	-0.05	4.49	0	4.16	-4.16
80	-0.05	4.60	0	4.16	-4.16
81	-0.02	4.93	0	4.16	-4.16
82	-0.02	4.98	0	4.16	-4.16
83	-0.05	4.82	0	4.16	-4.16
84	-0.07	4.78	0	4.16	-4.16
85	-0.07	4.91	0	4.16	-4.16
86	-0.07	5.03	0	4.16	-4.16
87	0.01	5.73	0	4.16	-4.16
88	-0.04	5.33	0	4.16	-4.16
89	-0.06	5.27	0	4.16	-4.16
90	-0.06	5.39	0	4.16	-4.16
91	-0.06	5.50	0	4.16	-4.16
92	-0.06	5.61	0	4.16	-4.16
93	-0.02	6.01	0	4.16	-4.16
94	-0.04	5.92	0	4.16	-4.16
95	-0.03	6.08	0	4.16	-4.16
96	-0.06	5.93	0	4.16	-4.16
97	0.02	6.61	0	4.16	-4.16
98	0	6.39	0	4.16	-4.16
99	0	6.38	0	4.16	-4.16
100	0.02	6.52	0	4.16	-4.16

The Q values obtained from table 3.3 are plotted in the chart shown in figure 3.3.

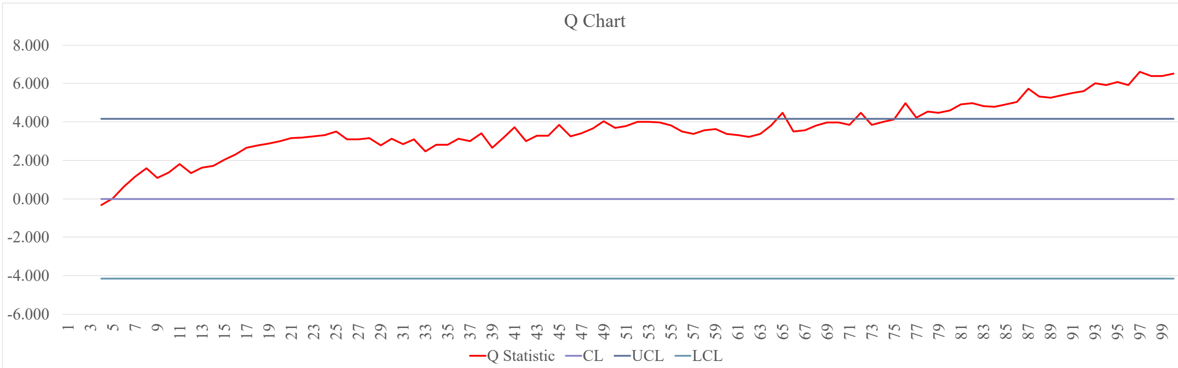

Figure 3.3: Q chart for individual values

Partial Results

What can be clearly seen in this chart is a marked increase in the value of Q. This behaviour suggests a drastic change in the process mean.

3.2.3 Application of CUSUM charts to subgroups n=1

This section will analyse the CUSUM chart approach for individual values according to the literature in chapter 2.4.1. Secondly, the Tabular CUSUM method will be applied to each CUSUM chart to evaluate its efficiency.

In conformity with the aforementioned methods, the calculations for the CUSUM values should be as follows:

In equation 2.4, the general equation is displayed to calculate the C value. Using the data from the table 3.2, the values are displayed in 3.12:

$$
\begin{aligned}
&C_1 = 0.030 - 0 = 0.030 \\
&C_2 = (0.030 - 0) + C_1 \\
&C_2 = (0.030 - 0) + 0.030 = 0.060 \\
&C_3 = (-0.040 - 0) + C_2 \\
&C_3 = (-0.040 - 0) + 0.060 = 0.020 \\
&\quad . \\
&\quad . \\
&\quad . \\
&C_i = (x_i - \mu_0) + C_{i-1}
\end{aligned}
\tag{3.12}
$$

Correspondingly, to calculate the Tabular CUSUM, the values C^+ and C^- are required to calculate the accumulation of deviations above and below the target, respectively.

As indicated in equations 2.5 and 2.6, to estimate these values, it is essential to select a K value, which is basically half of the magnitude of the shift in deviation.

In this case the equation 2.7 was applied to the data set in table 3.2 to estimate K as follows:

$$K = \frac{|\mu_1 - \mu_0|}{2}$$
$$K = \frac{|\mu_1 - 0|}{2}$$
$$K = \frac{|-0.024 - 0|}{2}$$
$$K = 0.012$$

With this value of K, the operations to calculate the one-sided upper and lower CUSUMs are displayed next.

$$C_i^+ = max[0, x_i - (\mu_0 + K) + C_{i-1}^+]$$
$$C_i^- = max[0, (\mu_0 + K) - x_i + C_{i-1}^-]$$

Operations in Excel are performed as follows:

$$C_i^+ = MAX(0, (x_i - (0 + 0.012) + x_{i-1}))$$
$$C_i^- = MAX((((0 - 0.012) - x_i) + x_{i-1}), 0)$$

The control limits were calculated using three standard deviations from the central line, which is 0.

Upper control limit:

$$
\begin{aligned}
UCL &= 0 + (3 \times 0.183) \\
UCL &= 0.549
\end{aligned} \tag{3.13}
$$

And for the lower control limit:

$$
\begin{aligned}
LCL &= 0 - (3 \times 0.025) \\
LCL &= -0.549
\end{aligned} \tag{3.14}
$$

The values plotted on the CUSUM chart are shown in the table 3.4.

The CUSUM values from table 3.4, are plotted in the chart displayed as figure 3.4.

Partial Results From the information provided in the CUSUM chart from figure 3.4, it can be concluded that there exists an evident change in mean from observation 21 in the general C value and observation 73 in the C^- chart. In this case, both graphs go in a direction opposite to what is expected, and the main reason for that is the nature of the values taken as "individual observations", which are actually deviations from the target in the first place. The results of this control chart will be further discussed and compared in chapter 4.

Table 3.4: CUSUM Values

Subgroup	X-T	C value	C+	C-	Subgroup	X-T	C value	C+	C-
1	0.03	0.030	0.018	0.000	51	-0.02	-1.060	0.000	0.443
2	0.03	0.060	0.048	0.000	52	0.01	-1.050	0.000	0.421
3	-0.04	0.020	0.008	0.000	53	0.02	-1.030	0.000	0.389
4	-0.03	-0.010	0.000	0.000	54	0.03	-1.000	0.000	0.347
5	-0.06	-0.070	0.000	0.010	55	0.03	-0.970	0.000	0.305
6	-0.06	-0.130	0.000	0.057	56	0	-0.970	0.000	0.292
7	-0.04	-0.170	0.000	0.085	57	-0.02	-0.990	0.000	0.300
8	0.03	-0.140	0.000	0.043	58	0	-0.990	0.000	0.288
9	-0.04	-0.180	0.000	0.071	59	0.01	-0.980	0.000	0.266
10	-0.04	-0.220	0.000	0.099	60	-0.02	-1.000	0.000	0.274
11	0.03	-0.190	0.000	0.057	61	-0.04	-1.040	0.000	0.302
12	-0.04	-0.230	0.000	0.085	62	-0.07	-1.110	0.000	0.360
13	-0.03	-0.260	0.000	0.103	63	-0.07	-1.180	0.000	0.418
14	-0.06	-0.320	0.000	0.151	64	-0.03	-1.210	0.000	0.436
15	-0.06	-0.380	0.000	0.199	65	0.06	-1.150	0.000	0.364
16	-0.05	-0.430	0.000	0.236	66	-0.04	-1.190	0.000	0.391
17	-0.02	-0.450	0.000	0.244	67	-0.05	-1.240	0.000	0.429
18	-0.02	-0.470	0.000	0.252	68	-0.03	-1.270	0.000	0.447
19	-0.02	-0.490	0.000	0.260	69	-0.02	-1.290	0.000	0.455
20	-0.02	-0.510	0.000	0.268	70	-0.03	-1.320	0.000	0.473
21	-0.01	-0.520	0.000	0.266	71	-0.06	-1.380	0.000	0.521
22	-0.01	-0.530	0.000	0.264	72	0.01	-1.370	0.000	0.499
23	-0.01	-0.540	0.000	0.262	73	-0.07	-1.440	0.000	0.557
24	-0.01	-0.550	0.000	0.260	74	-0.07	-1.510	0.000	0.615
25	0.03	-0.520	0.000	0.218	75	-0.07	-1.580	0.000	0.673
26	0	-0.520	0.000	0.205	76	0.03	-1.550	0.000	0.630
27	0	-0.520	0.000	0.193	77	-0.06	-1.610	0.000	0.678
28	0.02	-0.500	0.000	0.161	78	-0.03	-1.640	0.000	0.696
29	-0.03	-0.530	0.000	0.179	79	-0.05	-1.690	0.000	0.734
30	0.02	-0.510	0.000	0.147	80	-0.05	-1.740	0.000	0.772
31	-0.01	-0.520	0.000	0.145	81	-0.02	-1.760	0.000	0.780
32	0.03	-0.490	0.000	0.103	82	-0.02	-1.780	0.000	0.788
33	-0.05	-0.540	0.000	0.141	83	-0.05	-1.830	0.000	0.826
34	-0.02	-0.560	0.000	0.149	84	-0.07	-1.900	0.000	0.884
35	-0.04	-0.600	0.000	0.177	85	-0.07	-1.970	0.000	0.942
36	-0.01	-0.610	0.000	0.174	86	-0.07	-2.040	0.000	0.999
37	-0.05	-0.660	0.000	0.212	87	0.01	-2.030	0.000	0.977
38	0	-0.660	0.000	0.200	88	-0.04	-2.070	0.000	1.005
39	-0.15	-0.810	0.000	0.338	89	-0.06	-2.130	0.000	1.053
40	-0.04	-0.850	0.000	0.366	90	-0.06	-2.190	0.000	1.101
41	0.04	-0.810	0.000	0.314	91	-0.06	-2.250	0.000	1.149
42	-0.05	-0.860	0.000	0.352	92	-0.06	-2.310	0.000	1.197
43	-0.03	-0.890	0.000	0.370	93	-0.02	-2.330	0.000	1.205
44	-0.05	-0.940	0.000	0.408	94	-0.04	-2.370	0.000	1.233
45	0.03	-0.910	0.000	0.366	95	-0.03	-2.400	0.000	1.251
46	-0.05	-0.960	0.000	0.403	96	-0.06	-2.460	0.000	1.298
47	-0.05	-1.010	0.000	0.441	97	0.02	-2.440	0.000	1.266
48	-0.03	-1.040	0.000	0.459	98	0	-2.440	0.000	1.254
49	0.02	-1.020	0.000	0.427	99	0	-2.440	0.000	1.242
50	-0.02	-1.040	0.000	0.435	100	0.02	-2.420	0.000	1.210

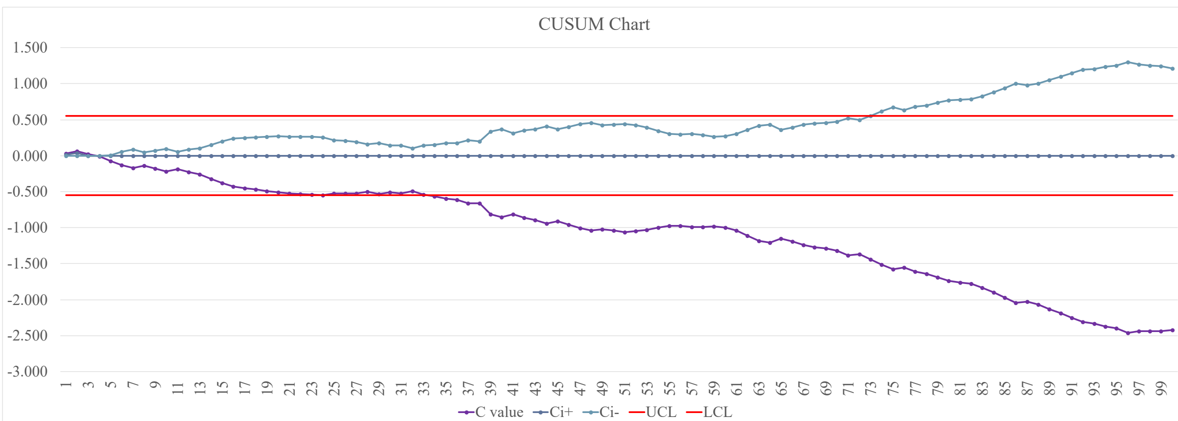

Figure 3.4: CUSUM chart for individual values

3.2.4 Application of EWMA charts to subgroup size n=1

This section will apply the EWMA chart technique to the data set presented in table 3.2. This technique will be developed from the EWMA methods described in chapter 2.4.2.

According to the literature review, the principal value of the EWMA chart is z, and it should be calculated with the following equation:

$$z_i = \lambda x_i + (1 - \lambda) z_{i-1}$$

As it can be observed, it is also necessary to estimate the value of λ. In this case, this parameter was obtained as indicated in chapter 2.4.2.

In order to conduct the research, I have carefully selected a lambda value of 0.050. The decision was based on the values suggested by Montgomery, which are outlined in Table 2.1. The operations are executed as follows:

$$z_i = \lambda x_i + (1 - \lambda) z_{i-1}$$

$$z_1 = \mu_1 = -0.024$$

$$z_2 = (0.050 \times 0.030) + ((1 - 0.050) \times -0.024) = -0.021$$

$$z_3 = (0.050 \times -0.040) + ((1 - 0.050) \times -0.021) = -0.022$$

All the z values are calculated with the same procedure until value No. 100.

In chapter 2, formulas 2.11 and 2.11 were used to calculate control limits. Table 2.1 shows values of L associated with λ, and the value $L = 2.615$ was used to calculate control limits for this chart because it corresponds to $\lambda = 0.050$.

The calculation of the Control Limits is performed in Excel with the following equations:

Upper Control Limit:

$UCL_n = \bar{X} + (L * \sigma) * (SQRT((\lambda/(2-\lambda)) * (1-((1-\lambda)^n))))$

Lower Control Limit:

$UCL_n = \bar{X} - (L * \sigma) * (SQRT((\lambda/(2-\lambda)) * (1-((1-\lambda)^n))))$

All z values were obtained with these operations and presented in table 3.5.

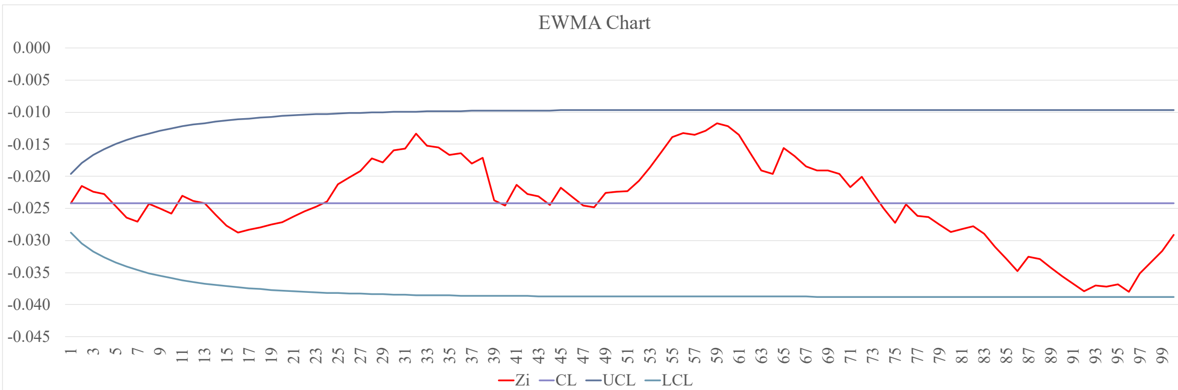

Figure 3.5: EWMA chart for individual values

Partial Results

The EWMA chart for individual values displayed a high variability, with the values going around the central line, almost crossing the upper and lower control limits.

Table 3.5: EWMA Values

Subgroup	X-T	z value	UCL	LCL
1	0.03	-0.024	-0.020	-0.029
2	0.03	-0.021	-0.018	-0.030
3	-0.04	-0.022	-0.017	-0.032
4	-0.03	-0.023	-0.016	-0.033
5	-0.06	-0.025	-0.015	-0.033
6	-0.06	-0.026	-0.014	-0.034
7	-0.04	-0.027	-0.014	-0.035
8	0.03	-0.024	-0.013	-0.035
9	-0.04	-0.025	-0.013	-0.036
10	-0.04	-0.026	-0.013	-0.036
11	0.03	-0.023	-0.012	-0.036
12	-0.04	-0.024	-0.012	-0.036
13	-0.03	-0.024	-0.012	-0.037
14	-0.06	-0.026	-0.011	-0.037
15	-0.06	-0.028	-0.011	-0.037
16	-0.05	-0.029	-0.011	-0.037
17	-0.02	-0.028	-0.011	-0.037
18	-0.02	-0.028	-0.011	-0.038
19	-0.02	-0.028	-0.011	-0.038
20	-0.02	-0.027	-0.011	-0.038
21	-0.01	-0.026	-0.011	-0.038
22	-0.01	-0.025	-0.010	-0.038
23	-0.01	-0.025	-0.010	-0.038
24	-0.01	-0.024	-0.010	-0.038
25	0.03	-0.021	-0.010	-0.038
26	0	-0.020	-0.010	-0.038
27	0	-0.019	-0.010	-0.038
28	0.02	-0.017	-0.010	-0.038
29	-0.03	-0.018	-0.010	-0.038
30	0.02	-0.016	-0.010	-0.038
31	-0.01	-0.016	-0.010	-0.038
32	0.03	-0.013	-0.010	-0.038
33	-0.05	-0.015	-0.010	-0.039
34	-0.02	-0.015	-0.010	-0.039
35	-0.04	-0.017	-0.010	-0.039
36	-0.01	-0.016	-0.010	-0.039
37	-0.05	-0.018	-0.010	-0.039
38	0	-0.017	-0.010	-0.039
39	-0.15	-0.024	-0.010	-0.039
40	-0.04	-0.025	-0.010	-0.039
41	0.04	-0.021	-0.010	-0.039
42	-0.05	-0.023	-0.010	-0.039
43	-0.03	-0.023	-0.010	-0.039
44	-0.05	-0.024	-0.010	-0.039
45	0.03	-0.022	-0.010	-0.039
46	-0.05	-0.023	-0.010	-0.039
47	-0.05	-0.025	-0.010	-0.039
48	-0.03	-0.025	-0.010	-0.039
49	0.02	-0.023	-0.010	-0.039
50	-0.02	-0.022	-0.010	-0.039

Subgroup	X-T	z value	UCL	LCL
51	-0.02	-0.022	-0.010	-0.039
52	0.01	-0.021	-0.010	-0.039
53	0.02	-0.019	-0.010	-0.039
54	0.03	-0.016	-0.010	-0.039
55	0.03	-0.014	-0.010	-0.039
56	0	-0.013	-0.010	-0.039
57	-0.02	-0.014	-0.010	-0.039
58	0	-0.013	-0.010	-0.039
59	0.01	-0.012	-0.010	-0.039
60	-0.02	-0.012	-0.010	-0.039
61	-0.04	-0.014	-0.010	-0.039
62	-0.07	-0.016	-0.010	-0.039
63	-0.07	-0.019	-0.010	-0.039
64	-0.03	-0.020	-0.010	-0.039
65	0.06	-0.016	-0.010	-0.039
66	-0.04	-0.017	-0.010	-0.039
67	-0.05	-0.018	-0.010	-0.039
68	-0.03	-0.019	-0.010	-0.039
69	-0.02	-0.019	-0.010	-0.039
70	-0.03	-0.020	-0.010	-0.039
71	-0.06	-0.022	-0.010	-0.039
72	0.01	-0.020	-0.010	-0.039
73	-0.07	-0.023	-0.010	-0.039
74	-0.07	-0.025	-0.010	-0.039
75	-0.07	-0.027	-0.010	-0.039
76	0.03	-0.024	-0.010	-0.039
77	-0.06	-0.026	-0.010	-0.039
78	-0.03	-0.026	-0.010	-0.039
79	-0.05	-0.028	-0.010	-0.039
80	-0.05	-0.029	-0.010	-0.039
81	-0.02	-0.028	-0.010	-0.039
82	-0.02	-0.028	-0.010	-0.039
83	-0.05	-0.029	-0.010	-0.039
84	-0.07	-0.031	-0.010	-0.039
85	-0.07	-0.033	-0.010	-0.039
86	-0.07	-0.035	-0.010	-0.039
87	0.01	-0.033	-0.010	-0.039
88	-0.04	-0.033	-0.010	-0.039
89	-0.06	-0.034	-0.010	-0.039
90	-0.06	-0.036	-0.010	-0.039
91	-0.06	-0.037	-0.010	-0.039
92	-0.06	-0.038	-0.010	-0.039
93	-0.02	-0.037	-0.010	-0.039
94	-0.04	-0.037	-0.010	-0.039
95	-0.03	-0.037	-0.010	-0.039
96	-0.06	-0.038	-0.010	-0.039
97	0.02	-0.035	-0.010	-0.039
98	0	-0.033	-0.010	-0.039
99	0	-0.032	-0.010	-0.039
100	0.02	-0.029	-0.010	-0.039

3.2.5 Application of Low Volume Moving Average charts

This section will analyse the Moving average chart in the three proposed arrangements. Moreover, the data was analysed with the Moving Average chart method mentioned by Quesenberry [25], and Montgomery [11], and it was decided to evaluate three different moving period lengths. The first-period length involves the total data, the second length is 2, and the third is 5.

3.2.5.1 Moving average with n=1

The utilization of Moving Average charts can facilitate the swift identification of minor deviations in process mean values within a range of 0.5 to 2.0 sigma. The initial methodology entails the employment of the complete dataset as subgroups with a sample size of n=1, as stated in previous sections.

Chapter 2.4.3 outlines the methodologies for computing control chart values based on the Moving Average approach. The performance of the chart is defined by the length of the moving period chosen for the analysis.

In the initial phase, the Moving Average was calculated by incorporating all the available data points, resulting in the development of a graph that displayed diminishing fluctuations over time.

The values for this control chart were calculated using the equations in chapter 2, equation 2.14.

The application of this method looks as follows for data points 1, 2 and 3:

$$
\begin{aligned}
M_1 &= 0.030 \\
M_2 &= \frac{0.030 + 0.030}{2} = 0.030 \\
M_3 &= \frac{0.030 + 0.030 + (-0.040)}{3} = 0.007
\end{aligned}
$$

According to equation 2.15, the UCL for the first 3 data points in this arrangement are:

$$
\begin{aligned}
UCL_1 &= -0.024 + \frac{0.035}{\sqrt{1}} = 0.080 \\
UCL_2 &= -0.024 + \frac{0.035}{\sqrt{2}} = 0.050 \\
UCL_2 &= -0.024 + \frac{0.035}{\sqrt{3}} = 0.036
\end{aligned}
$$

According to equation 2.16, the LCL for the first 3 data points in this arrangement are:

$$
\begin{aligned}
LCL_1 &= -0.024 - \frac{0.035}{\sqrt{1}} = -0.128 \\
LCL_2 &= -0.024 - \frac{0.035}{\sqrt{2}} = -0.098 \\
LCL_2 &= -0.024 - \frac{0.035}{\sqrt{3}} = -0.084
\end{aligned}
$$

All the calculated values for this chart can be found in table 3.6.

Furthermore, the control chart with these data is displayed in figure 3.6.

Employing the same methodology, we have generated the Moving Average control chart,

Table 3.6: Moving Average Values

Subgroup	X-T	MA from top	CL	UCL	LCL
1	0.03	0.030	-0.024	0.080	-0.128
2	0.03	0.030	-0.024	0.050	-0.098
3	-0.04	0.007	-0.024	0.036	-0.084
4	-0.03	-0.003	-0.024	0.028	-0.076
5	-0.06	-0.014	-0.024	0.022	-0.071
6	-0.06	-0.022	-0.024	0.018	-0.067
7	-0.04	-0.024	-0.024	0.015	-0.064
8	0.03	-0.018	-0.024	0.013	-0.061
9	-0.04	-0.020	-0.024	0.011	-0.059
10	-0.04	-0.022	-0.024	0.009	-0.057
11	0.03	-0.017	-0.024	0.007	-0.056
12	-0.04	-0.019	-0.024	0.006	-0.054
13	-0.03	-0.020	-0.024	0.005	-0.053
14	-0.06	-0.023	-0.024	0.004	-0.052
15	-0.06	-0.025	-0.024	0.003	-0.051
16	-0.05	-0.027	-0.024	0.002	-0.050
17	-0.02	-0.026	-0.024	0.001	-0.049
18	-0.02	-0.026	-0.024	0.000	-0.049
19	-0.02	-0.026	-0.024	0.000	-0.048
20	-0.02	-0.026	-0.024	-0.001	-0.048
21	-0.01	-0.025	-0.024	-0.001	-0.048
22	-0.01	-0.024	-0.024	-0.001	-0.048
23	-0.01	-0.023	-0.024	-0.001	-0.048
24	-0.01	-0.023	-0.024	-0.001	-0.048
25	0.03	-0.021	-0.024	-0.001	-0.048
26	0	-0.020	-0.024	-0.001	-0.048
27	0	-0.019	-0.024	-0.001	-0.048
28	0.02	-0.018	-0.024	-0.001	-0.048
29	-0.03	-0.018	-0.024	-0.001	-0.048
30	0.02	-0.017	-0.024	-0.001	-0.048
31	-0.01	-0.017	-0.024	-0.001	-0.048
32	0.03	-0.015	-0.024	-0.001	-0.048
33	-0.05	-0.016	-0.024	-0.001	-0.048
34	-0.02	-0.016	-0.024	-0.001	-0.048
35	-0.04	-0.017	-0.024	-0.001	-0.048
36	-0.01	-0.017	-0.024	-0.001	-0.048
37	-0.05	-0.018	-0.024	-0.001	-0.048
38	0	-0.017	-0.024	-0.001	-0.048
39	-0.15	-0.021	-0.024	-0.001	-0.048
40	-0.04	-0.021	-0.024	-0.001	-0.048
41	0.04	-0.020	-0.024	-0.001	-0.048
42	-0.05	-0.020	-0.024	-0.001	-0.048
43	-0.03	-0.021	-0.024	-0.001	-0.048
44	-0.05	-0.021	-0.024	-0.001	-0.048
45	0.03	-0.020	-0.024	-0.001	-0.048
46	-0.05	-0.021	-0.024	-0.001	-0.048
47	-0.05	-0.021	-0.024	-0.001	-0.048
48	-0.03	-0.022	-0.024	-0.001	-0.048
49	0.02	-0.021	-0.024	-0.001	-0.048
50	-0.02	-0.021	-0.024	-0.001	-0.048
51	-0.02	-0.021	-0.024	-0.001	-0.048
52	0.01	-0.020	-0.024	-0.001	-0.048
53	0.02	-0.019	-0.024	-0.001	-0.048
54	0.03	-0.019	-0.024	-0.001	-0.048
55	0.03	-0.018	-0.024	-0.001	-0.048
56	0	-0.017	-0.024	-0.001	-0.048
57	-0.02	-0.017	-0.024	-0.001	-0.048
58	0	-0.017	-0.024	-0.001	-0.048
59	0.01	-0.017	-0.024	-0.001	-0.048
60	-0.02	-0.017	-0.024	-0.001	-0.048
61	-0.04	-0.017	-0.024	-0.001	-0.048
62	-0.07	-0.018	-0.024	-0.001	-0.048
63	-0.07	-0.019	-0.024	-0.001	-0.048
64	-0.03	-0.019	-0.024	-0.001	-0.048
65	0.06	-0.018	-0.024	-0.001	-0.048
66	-0.04	-0.018	-0.024	-0.001	-0.048
67	-0.05	-0.019	-0.024	-0.001	-0.048
68	-0.03	-0.019	-0.024	-0.001	-0.048
69	-0.02	-0.019	-0.024	-0.001	-0.048
70	-0.03	-0.019	-0.024	-0.001	-0.048
71	-0.06	-0.019	-0.024	-0.001	-0.048
72	0.01	-0.019	-0.024	-0.001	-0.048
73	-0.07	-0.020	-0.024	-0.001	-0.048
74	-0.07	-0.020	-0.024	-0.001	-0.048
75	-0.07	-0.021	-0.024	-0.001	-0.048
76	0.03	-0.020	-0.024	-0.001	-0.048
77	-0.06	-0.021	-0.024	-0.001	-0.048
78	-0.03	-0.021	-0.024	-0.001	-0.048
79	-0.05	-0.021	-0.024	-0.001	-0.048
80	-0.05	-0.022	-0.024	-0.001	-0.048
81	-0.02	-0.022	-0.024	-0.001	-0.048
82	-0.02	-0.022	-0.024	-0.001	-0.048
83	-0.05	-0.022	-0.024	-0.001	-0.048
84	-0.07	-0.023	-0.024	-0.001	-0.048
85	-0.07	-0.023	-0.024	-0.001	-0.048
86	-0.07	-0.024	-0.024	-0.001	-0.048
87	0.01	-0.023	-0.024	-0.001	-0.048
88	-0.04	-0.024	-0.024	-0.001	-0.048
89	-0.06	-0.024	-0.024	-0.001	-0.048
90	-0.06	-0.024	-0.024	-0.001	-0.048
91	-0.06	-0.025	-0.024	-0.001	-0.048
92	-0.06	-0.025	-0.024	-0.001	-0.048
93	-0.02	-0.025	-0.024	-0.001	-0.048
94	-0.04	-0.025	-0.024	-0.001	-0.048
95	-0.03	-0.025	-0.024	-0.001	-0.048
96	-0.06	-0.026	-0.024	-0.001	-0.048
97	0.02	-0.025	-0.024	-0.001	-0.048
98	0	-0.025	-0.024	-0.001	-0.048
99	0	-0.025	-0.024	-0.001	-0.048
100	0.02	-0.024	-0.024	-0.001	-0.048

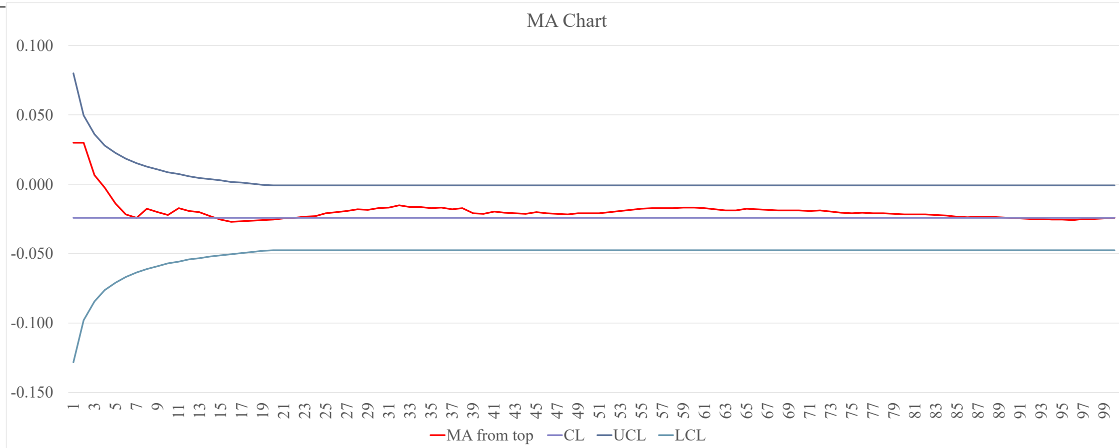

Figure 3.6: Moving Average chart for individual values

utilizing moving periods of 2 and 5. The data tables for the values with a moving period of 2 can be found in table A.3, while those for a moving period of 5 are contained in table A.4, as detailed in Appendix A.

The Moving Average control charts are shown in figures 3.7 and 3.8.

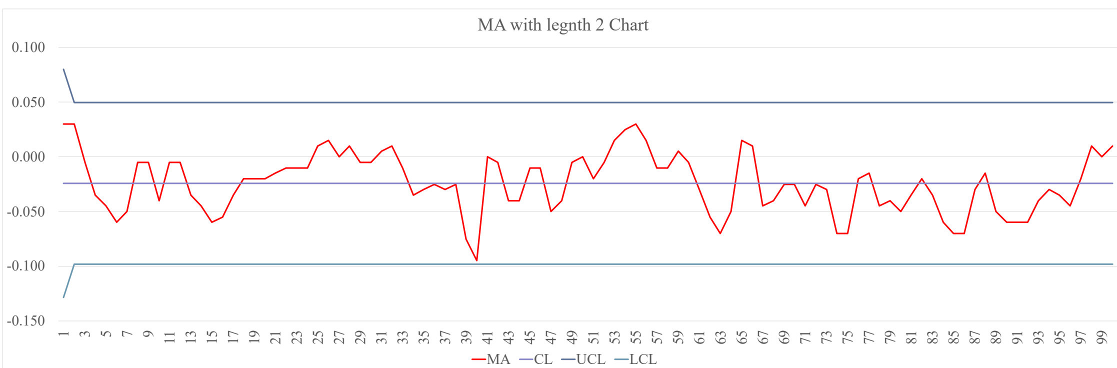

Figure 3.7: Moving Average chart of length 2 for subgroups of size n=1

Partial Results

The three different lengths were applied to the Moving average for individual values. None of the charts displayed values outside the control limits; however, there are two points to remark. Firstly, the total moving average chart is performed as a fitting tool, where the values tend to the average $\bar{X}$. Secondly, in the MA chart with length 5, a more precise cyclic trend can be observed moving around the centre line.

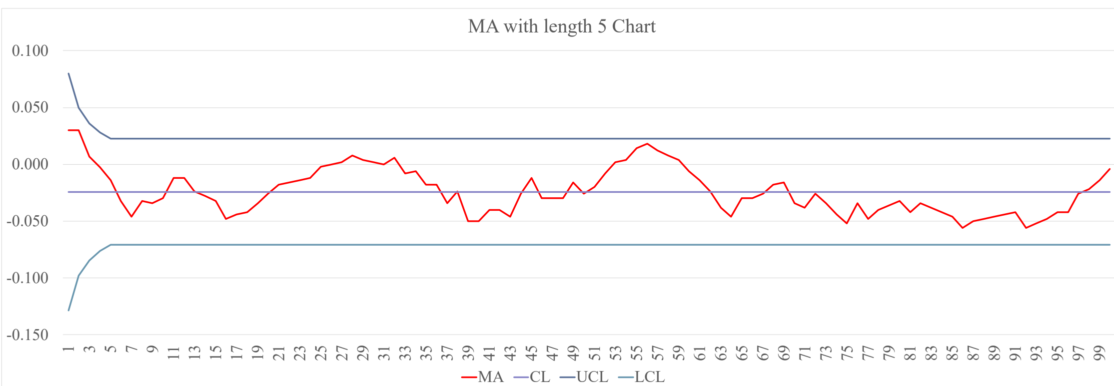

Figure 3.8: Moving Average chart of length 5 for subgroups of size n=1

3.3 SPC Techniques applied to Data organised in subgroups by date and of different sizes

In this section, the methods applied in section 3.2 will be replicated and adapted to the data organised in subgroups. These subgroups share the characteristic of being produced on the same date, resulting in different n sizes in each subgroup.

According to this logic, the data from table A.2, the original inspection data, will be organised into 22 subgroups as displayed in table 3.7.

Table 3.7: Data set with subgroups of different size

Subgroup	Part/OP	Date	Average Measurement from inspection
1	C07C27145-140	23/03/2018	2.00
2	C07C27145-140	26/03/2018	1.94
3	C07C27145-140	27/03/2018	1.91
4	C07C27145-140	22/04/2018	1.97
5	C07C27145-140	23/04/2018	1.93
6	C07C27145-140	16/07/2018	1.95
7	C07C27145-140	16/07/2018	1.92
8	C07C27157-70	08/08/2018	1.70
9	C07C27157-70	08/08/2018	1.73
10	C07C27157-70	10/08/2018	1.75
11	C07C27157-70	13/08/2018	1.75
12	C07C27157-70	14/08/2018	1.75
13	C07C27157-70	14/08/2018	1.73
14	C07C27157-70	14/08/2018	1.71
15	C07C27157-70	14/08/2018	1.72
16	C07C27157-70	15/08/2018	1.73
17	C07C27157-70	15/08/2018	1.77
18	C07C27157-70	15/08/2018	1.75
19	C07C27157-80	21/08/2018	1.93
20	C07C27157-80	22/08/2018	1.96
21	C07C27157-80	23/08/2018	1.93
22	C07C27157-80	24/08/2018	1.94
23	C07C27145-140	25/08/2018	1.94
24	C07C27157-80	25/08/2018	1.92
25	C07C27145-140	29/08/2018	2.00
26	C07C27157-80	30/08/2018	1.93
27	C07C27157-80	30/08/2018	1.91
28	C07C27157-80	30/08/2018	1.93
29	C07C27145-140	31/08/2018	1.91
30	C07C27157-80	31/08/2018	1.94
31	C07C27157-80	31/08/2018	1.98

The main characteristics of this data arrangement are listed below:

- The target value of the subgroups is different from each other.
- The size of n is determined by the size of the original production/inspection batch; therefore, the subgroup size is not constant.

- The date range between one subgroup and another is not constant.

According to Sefik [46], the most critical factor to consider when creating subgroups is that all the samples within the subgroups should have been produced under the same fundamental conditions.However, in the absence of enough data to apply standard techniques, choosing the same date appears to be a sensible option. For this reason, this arrangement is one with a less precise construction.

The following subsections will evaluate the application of different Control charts to the data presented in table 3.7.

3.3.1 Application of Shewhart Charts to data in subgroups of different size

As exposed in previous chapters, this section aims to apply and evaluate the performance of Shewhart Charts to the data arranged in the subgroups described above. The data were analysed using $\bar{X}$ and R charts in this case.

The first step transforms the data values from table 3.7 to 'deviation from target' or 'δ' values.

The elements of this chart are Centre Line (CL), Upper Control Limit (UCL) and Lower Control Limit (LCL). The control limits were calculated using $\pm 3\sigma$.

In this case, the control limits were calculated as follows:

$CL = 0$ because it represents the target value.

Upper control limit:

$$\begin{aligned} UCL &= 0 + (3 \times 0.025) \\ UCL &= 0.0758 \end{aligned} \tag{3.15}$$

And for the lower control limit:

$$\begin{aligned} LCL &= 0 - (3 \times 0.025) \\ LCL &= -0.0758 \end{aligned} \tag{3.16}$$

The calculated values are displayed in table 3.8 below.

Likewise,the chart for the data from table 3.8 is displayed in figure 3.9, and the corresponding moving range chart is displayed in figure 3.10

Partial Results

The Shewhart chart for values grouped by date in charts 3.9 and 3.10 showed no data

Table 3.8: Shewhart Chart values for subgroups of different size

Subgroup	X-T	CL	UCL	LCL
1	0.03	0.00	0.0758	-0.0758
2	-0.03	0.00	0.0758	-0.0758
3	-0.06	0.00	0.0758	-0.0758
4	-0.01	0.00	0.0758	-0.0758
5	-0.04	0.00	0.0758	-0.0758
6	-0.02	0.00	0.0758	-0.0758
7	-0.05	0.00	0.0758	-0.0758
8	-0.05	0.00	0.0758	-0.0758
9	-0.02	0.00	0.0758	-0.0758
10	0.00	0.00	0.0758	-0.0758
11	0.00	0.00	0.0758	-0.0758
12	0.00	0.00	0.0758	-0.0758
13	-0.02	0.00	0.0758	-0.0758
14	-0.04	0.00	0.0758	-0.0758
15	-0.03	0.00	0.0758	-0.0758
16	-0.02	0.00	0.0758	-0.0758
17	0.02	0.00	0.0758	-0.0758
18	0.00	0.00	0.0758	-0.0758
19	-0.04	0.00	0.0758	-0.0758
20	-0.01	0.00	0.0758	-0.0758
21	-0.04	0.00	0.0758	-0.0758
22	-0.03	0.00	0.0758	-0.0758
23	-0.03	0.00	0.0758	-0.0758
24	-0.05	0.00	0.0758	-0.0758
25	0.03	0.00	0.0758	-0.0758
26	-0.04	0.00	0.0758	-0.0758
27	-0.06	0.00	0.0758	-0.0758
28	-0.04	0.00	0.0758	-0.0758
29	-0.06	0.00	0.0758	-0.0758
30	-0.03	0.00	0.0758	-0.0758
31	0.01	0.00	0.0758	-0.0758

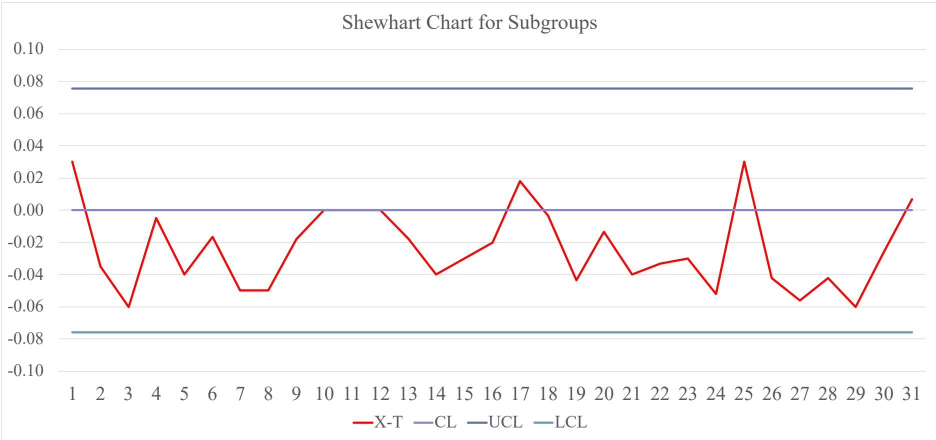

Figure 3.9: Shewhart chart for subgroups divided by date

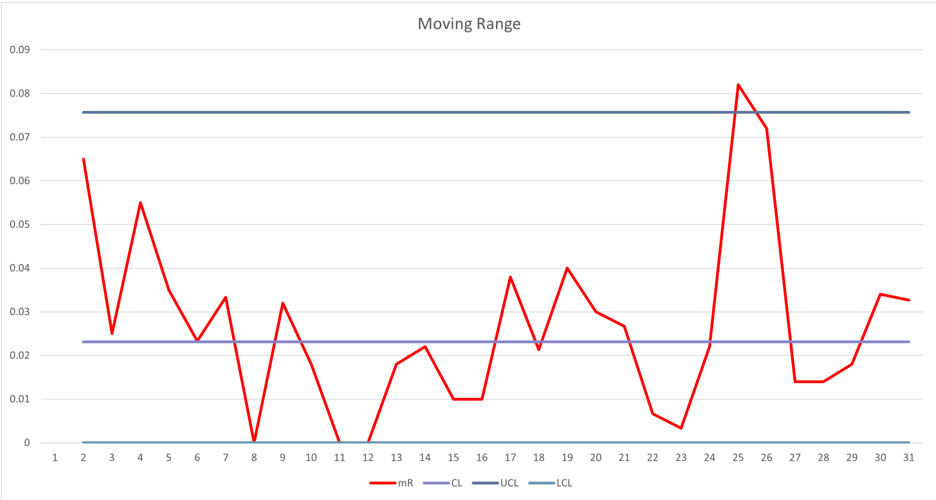

Figure 3.10: Moving range chart for subgroups divided by the date of figure 3.9

outside the control limits computed at 3σ. Nevertheless, a high level of variation can be observed in the individual and moving range chart. If the charts are analysed with the 1σ and 2σ criteria, it can be found that values 1 and 19 fall outside of the 2σ limits. Point 1 belongs to the observations from the 23rd of March 2018; on the other hand, point 19 represents the observations taken on the 29th of August 2018.

3.3.2 Application of Q charts to data in subgroups of different size

The Q chart for individual values of the first group of data showed that only value no. 4 was out of the control limits; however, a trend can be observed from point 16 to point 28, where the values consistently appear above the Centre Line.

For this situation, the Q Chart was calculated using the methods for variables in subgroups; specifically, the Q Statistic for sample means proposed by Quesenberry [25].

The equation is presented in equation 2.21 and is displayed below:

$$Q_i(\bar{X}_i) = \Phi^{-1}\left\{H_{n_1+\ldots+n_i-i}\left[\sqrt{\frac{n_i(n_1+\ldots+n_{i-1})}{n_1+\ldots+n_i}}\left(\frac{\bar{X}_i-\bar{\bar{X}}_{i-1}}{S_{p,i}}\right)\right]\right\}$$

Where $i = 2, 3, \ldots$, and H represents the t-Student distribution.

Using this method, the calculations in Excel were executed as described in the following paragraph.

$= NORM.S.INV(T.DIST(((n_i*(SUM(n_1:n_{i-1}))/(SUM(n_1:n_i)))*((\bar{X}_i-(AVERAGE(n_1:n_{i-1})))/(STDEV(n_1:n_i))),(COUNT(n_1:n_i),FALSE))$

The data from table 3.7 was used to compute the Q values for this data set, and they are presented in table 3.9. The upper and lower control limits were set using $\pm 3\sigma$.

The Q chart displaying these values is shown in figure 3.11.

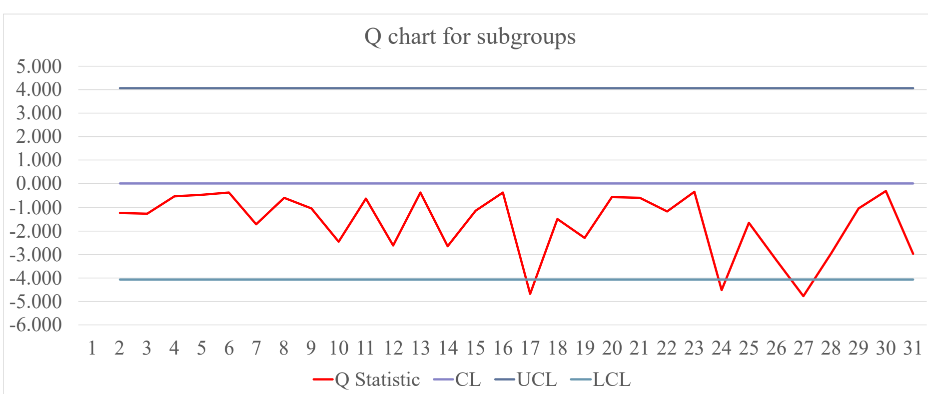

Figure 3.11: Q Chart for subgroups of different size

Table 3.9: Q values for Data arranged in subgroups of different size

Subgroup	X-T	Q Statistic	UCL	LCL
1	0.030	0.000	4.06	-4.06
2	-0.035	-1.248	4.06	-4.06
3	-0.060	-1.276	4.06	-4.06
4	-0.005	-0.544	4.06	-4.06
5	-0.040	-0.484	4.06	-4.06
6	-0.017	-0.380	4.06	-4.06
7	-0.050	-1.714	4.06	-4.06
8	-0.050	-0.588	4.06	-4.06
9	-0.018	-1.061	4.06	-4.06
10	0.000	-2.459	4.06	-4.06
11	0.000	-0.630	4.06	-4.06
12	0.000	-2.628	4.06	-4.06
13	-0.018	-0.360	4.06	-4.06
14	-0.040	-2.650	4.06	-4.06
15	-0.030	-1.156	4.06	-4.06
16	-0.020	-0.362	4.06	-4.06
17	0.018	-4.677	4.06	-4.06
18	-0.003	-1.485	4.06	-4.06
19	-0.043	-2.300	4.06	-4.06
20	-0.013	-0.580	4.06	-4.06
21	-0.040	-0.604	4.06	-4.06
22	-0.033	-1.186	4.06	-4.06
23	-0.030	-0.341	4.06	-4.06
24	-0.052	-4.524	4.06	-4.06
25	0.030	-1.657	4.06	-4.06
26	-0.042	-3.240	4.06	-4.06
27	-0.056	-4.772	4.06	-4.06
28	-0.042	-2.958	4.06	-4.06
29	-0.060	-1.056	4.06	-4.06
30	-0.026	-0.291	4.06	-4.06
31	0.007	-2.992	0.00	0.00

Partial Results

Through the application of Q charts, it can be observed that all recorded values are situated below the Lower Control Limit. Additionally, there are three data points, specifically 17, 24, and 27, which have exceeded the established control limits.

3.3.3 Application of CUSUM Charts to Data in subgroups of different size

The work in this section explores the application of CUSUM charts in a context where the data is organised in subgroups of different sizes. All the calculations were performed in Excel to compute the values, and the equations utilised to do so are displayed below.

To calculate the C Values, one can use the following general formula:

$C_i = (x_i - \mu_0) + C_{i-1}$

Furthermore, for this data set, the K value used in calculating C^- and C^+ is also 0.0017, as in the previous section.

The control limits were calculated using three standard deviations from the central line which in this case is 0.

Upper control limit:

$$\begin{aligned} UCL &= 0 + (3 \times 0.216) \\ UCL &= 0.649 \end{aligned} \tag{3.17}$$

And for the lower control limit:

$$\begin{aligned} LCL &= 0 - (3 \times 0.216) \\ LCL &= -0.649 \end{aligned} \tag{3.18}$$

The values obtained from these operations are displayed below in table 3.10

The CUSUM chart for these values is shown in figure 3.12.

Partial Results

Two points need to be remarked on in this chart. The first thing to highlight is that the central values with which this study is developed are the differences from the target of the

Table 3.10: CUSUM Values for Subgroups with different sizes

Subgroup	X-T	C value	Ci+	Ci-	UCL	LCL
1	0.030	0.030	0.028	0.000	0.649	-0.649
2	-0.035	-0.005	0.000	0.002	0.649	-0.649
3	-0.060	-0.065	0.000	0.060	0.649	-0.649
4	-0.005	-0.070	0.000	0.063	0.649	-0.649
5	-0.040	-0.110	0.000	0.102	0.649	-0.649
6	-0.017	-0.127	0.000	0.117	0.649	-0.649
7	-0.050	-0.177	0.000	0.165	0.649	-0.649
8	-0.050	-0.227	0.000	0.213	0.649	-0.649
9	-0.018	-0.245	0.000	0.230	0.649	-0.649
10	0.000	-0.245	0.000	0.228	0.649	-0.649
11	0.00	-0.245	0.000	0.226	0.649	-0.649
12	0.00	-0.245	0.000	0.225	0.649	-0.649
13	-0.02	-0.263	0.000	0.241	0.649	-0.649
14	-0.04	-0.303	0.000	0.280	0.649	-0.649
15	-0.03	-0.333	0.000	0.308	0.649	-0.649
16	-0.02	-0.353	0.000	0.326	0.649	-0.649
17	0.02	-0.335	0.000	0.307	0.649	-0.649
18	0.00	-0.338	0.000	0.308	0.649	-0.649
19	-0.04	-0.381	0.000	0.350	0.649	-0.649
20	-0.01	-0.395	0.000	0.362	0.649	-0.649
21	-0.04	-0.435	0.000	0.400	0.649	-0.649
22	-0.03	-0.468	0.000	0.432	0.649	-0.649
23	-0.03	-0.498	0.000	0.460	0.649	-0.649
24	-0.05	-0.550	0.000	0.510	0.649	-0.649
25	0.03	-0.520	0.000	0.479	0.649	-0.649
26	-0.04	-0.562	0.000	0.519	0.649	-0.649
27	-0.06	-0.618	0.000	0.573	0.649	-0.649
28	-0.04	-0.660	0.000	0.614	0.649	-0.649
29	-0.06	-0.720	0.000	0.672	0.649	-0.649
30	-0.03	-0.746	0.000	0.696	0.649	-0.649
31	0.01	-0.739	0.000	0.688	0.649	-0.649

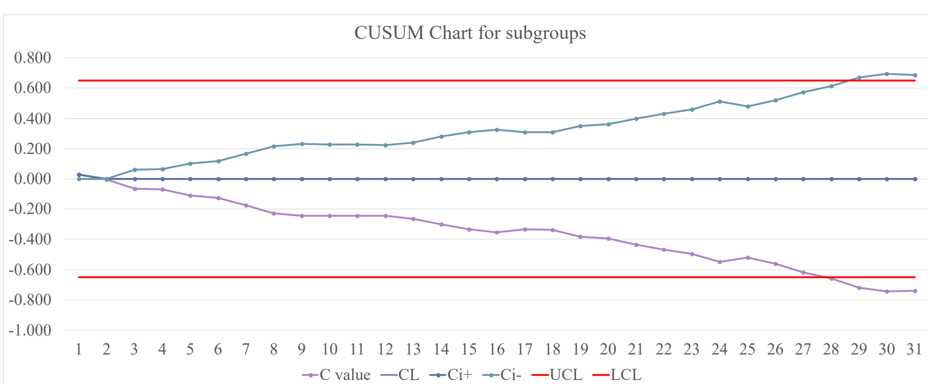

Figure 3.12: CUSUM Chart for subgroups of different size

individual observations; some are negative (-) values. This fact makes the C Value behave in a downward direction. The second point is that two subgroups fall beyond the Lower Control Limit: points 21 and 22.

3.3.4 Application of EWMA charts to data in subgroups of different size

Similar to other sections, this section will explore the performance of the EWMA charts with low-volume data grouped according to the production date.

In this occasion, a total of 31 subgroups on size $0 \leq n \leq 5$ will be analysed with the method described in section 2.4.2, with the general equation for EWMA Charts which is as follows:

$$z_i = \lambda x_i + (1 - \lambda) z_{i-1}$$

Using the values from table 3.7, the calculations to get the Z-Values (EWMA Values) are as follows:

$$z_1 = \mu_1 = -0.024$$

$$z_2 = (0.400 \times (-0.035)) + ((1 - 0.400) \times -0.024) = -0.028$$

$$z_3 = (0.400 \times (-0.060)) + ((1 - 0.400) \times -0.028) = -0.041$$

Following this method, all the values obtained for this chart are displayed in table 3.11

The EWMA values from table 3.11 are plotted in figure 3.13

Partial Result

The data displayed in chart 3.13 shows 6 points that appear out of control, which in this case are points 3, 8, 24, 28, 29 and 30.

Table 3.11: EWMA Values for Subgroups with different sizes

Subgroup	X-T	Zi	CL	UCL	LCL
1	0.030	-0.024	0	0.0309	-0.0309
2	-0.035	-0.028	0	0.0360	-0.0360
3	-0.060	-0.041	0	0.0377	-0.0377
4	-0.005	-0.027	0	0.0382	-0.0382
5	-0.040	-0.032	0	0.0384	-0.0384
6	-0.017	-0.026	0	0.0385	-0.0385
7	-0.050	-0.036	0	0.0385	-0.0385
8	-0.050	-0.041	0	0.0386	-0.0386
9	-0.018	-0.032	0	0.0386	-0.0386
10	0.000	-0.019	0	0.0386	-0.0386
11	0.000	-0.012	0	0.0386	-0.0386
12	0.000	-0.007	0	0.0386	-0.0386
13	-0.018	-0.011	0	0.0386	-0.0386
14	-0.040	-0.023	0	0.0386	-0.0386
15	-0.030	-0.026	0	0.0386	-0.0386
16	-0.020	-0.023	0	0.0386	-0.0386
17	0.018	-0.007	0	0.0386	-0.0386
18	-0.003	-0.005	0	0.0386	-0.0386
19	-0.043	-0.021	0	0.0386	-0.0386
20	-0.013	-0.018	0	0.0386	-0.0386
21	-0.040	-0.027	0	0.0386	-0.0386
22	-0.033	-0.029	0	0.0386	-0.0386
23	-0.030	-0.030	0	0.0386	-0.0386
24	-0.052	-0.039	0	0.0386	-0.0386
25	0.030	-0.011	0	0.0386	-0.0386
26	-0.042	-0.023	0	0.0386	-0.0386
27	-0.056	-0.036	0	0.0386	-0.0386
28	-0.042	-0.039	0	0.0386	-0.0386
29	-0.060	-0.047	0	0.0386	-0.0386
30	-0.026	-0.039	0	0.0386	-0.0386
31	0.007	-0.021	0	0.0386	-0.0386

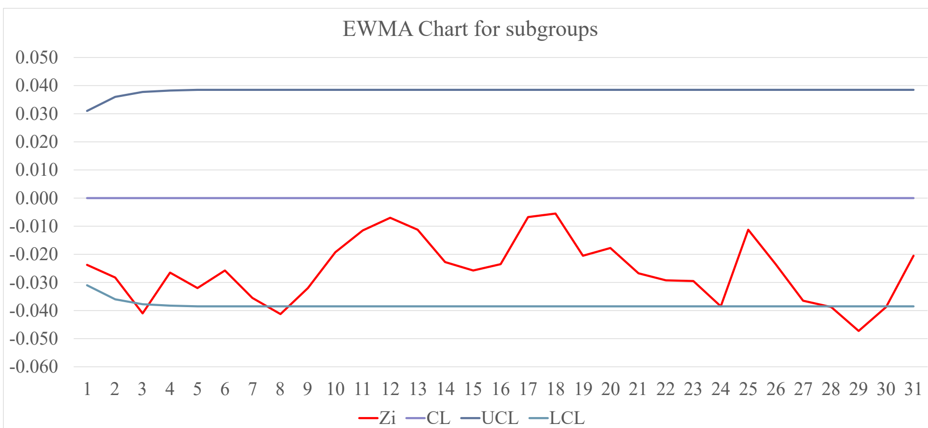

Figure 3.13: EWMA chart of data in subgroups of different sizes

3.3.5 Application of Moving Average Charts to Data in subgroups of different size

Moving Average control charts are applied to the data grouped in subgroups shown in table 3.7. The test was performed under three different scenarios to analyse the data: in the first approach, the Moving Average formula was run throughout all the data points. This approach helps fit the totality of the historical information. The second approach uses a two-period moving average, and the third approach analyses a five-period moving average.

3.3.5.1 Approach One: Using an n-period moving average

The application of the method for the first approach utilises Excel operations, and each point is given by the following formula: $= +AVERAGE(n_1 : n_i)$

Correspondingly, the Upper and Lower Control Limits are also fitted in this graph. To achieve that, in equations 2.15 and 2.16, the value of w was always replaced by n in each data point.

Using this method, the data points for this control chart were generated and displayed in the table 3.12.

The Control chart for this approach is presented in figure 3.14.

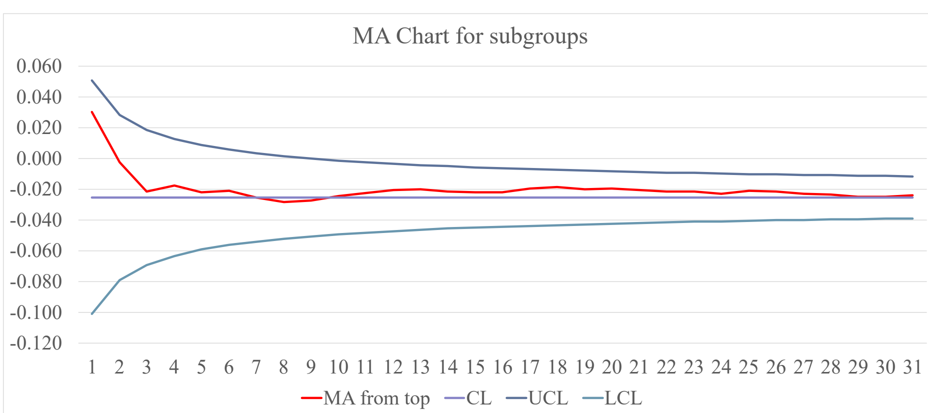

Figure 3.14: Moving Average chart using subgroups of different sizes

Table 3.12: MA Values for Subgroups with different sizes

Subgroup	X-T	MA from top	CL	UCL	LCL
1	0.030	0.030	-0.025	0.050	-0.101
2	-0.035	-0.002	-0.025	0.028	-0.079
3	-0.060	-0.022	-0.025	0.018	-0.069
4	-0.005	-0.018	-0.025	0.013	-0.063
5	-0.040	-0.022	-0.025	0.009	-0.059
6	-0.017	-0.021	-0.025	0.006	-0.056
7	-0.050	-0.025	-0.025	0.003	-0.054
8	-0.050	-0.028	-0.025	0.001	-0.052
9	-0.018	-0.027	-0.025	0.000	-0.051
10	0.000	-0.024	-0.025	-0.001	-0.049
11	0.000	-0.022	-0.025	-0.002	-0.048
12	0.000	-0.020	-0.025	-0.003	-0.047
13	-0.018	-0.020	-0.025	-0.004	-0.046
14	-0.040	-0.022	-0.025	-0.005	-0.046
15	-0.030	-0.022	-0.025	-0.006	-0.045
16	-0.020	-0.022	-0.025	-0.006	-0.044
17	0.018	-0.020	-0.025	-0.007	-0.044
18	-0.003	-0.019	-0.025	-0.007	-0.043
19	-0.043	-0.020	-0.025	-0.008	-0.043
20	-0.013	-0.020	-0.025	-0.008	-0.042
21	-0.040	-0.021	-0.025	-0.009	-0.042
22	-0.033	-0.021	-0.025	-0.009	-0.041
23	-0.030	-0.022	-0.025	-0.010	-0.041
24	-0.052	-0.023	-0.025	-0.010	-0.041
25	0.030	-0.021	-0.025	-0.010	-0.040
26	-0.042	-0.022	-0.025	-0.010	-0.040
27	-0.056	-0.023	-0.025	-0.011	-0.040
28	-0.042	-0.024	-0.025	-0.011	-0.040
29	-0.060	-0.025	-0.025	-0.011	-0.039
30	-0.026	-0.025	-0.025	-0.011	-0.039
31	0.007	-0.024	-0.025	-0.012	-0.039

3.3.5.2 Approach Two: Using a 2-period moving average

In this case, the analysis was performed using a 2-period moving average. Equally, the operations for this approach were done using Excel.

Table 3.13: 2-Period MA Values for Subgroups with different sizes

Subgroup	X-T	MA from top	CL	UCL	LCL
1	0.030	0.030	-0.025	0.050	-0.101
2	-0.035	-0.002	-0.025	0.028	-0.079
3	-0.060	-0.022	-0.025	0.018	-0.069
4	-0.005	-0.018	-0.025	0.013	-0.063
5	-0.040	-0.022	-0.025	0.009	-0.059
6	-0.017	-0.021	-0.025	0.006	-0.056
7	-0.050	-0.025	-0.025	0.003	-0.054
8	-0.050	-0.028	-0.025	0.001	-0.052
9	-0.018	-0.027	-0.025	0.000	-0.051
10	0.000	-0.024	-0.025	-0.001	-0.049
11	0.000	-0.022	-0.025	-0.002	-0.048
12	0.000	-0.020	-0.025	-0.003	-0.047
13	-0.018	-0.020	-0.025	-0.004	-0.046
14	-0.040	-0.022	-0.025	-0.005	-0.046
15	-0.030	-0.022	-0.025	-0.006	-0.045
16	-0.020	-0.022	-0.025	-0.006	-0.044
17	0.018	-0.020	-0.025	-0.007	-0.044
18	-0.003	-0.019	-0.025	-0.007	-0.043
19	-0.043	-0.020	-0.025	-0.008	-0.043
20	-0.013	-0.020	-0.025	-0.008	-0.042
21	-0.040	-0.021	-0.025	-0.009	-0.042
22	-0.033	-0.021	-0.025	-0.009	-0.041
23	-0.030	-0.022	-0.025	-0.010	-0.041
24	-0.052	-0.023	-0.025	-0.010	-0.041
25	0.030	-0.021	-0.025	-0.010	-0.040
26	-0.042	-0.022	-0.025	-0.010	-0.040
27	-0.056	-0.023	-0.025	-0.011	-0.040
28	-0.042	-0.024	-0.025	-0.011	-0.040
29	-0.060	-0.025	-0.025	-0.011	-0.039
30	-0.026	-0.025	-0.025	-0.011	-0.039
31	0.007	-0.024	-0.025	-0.012	-0.039

3.3.5.3 Approach Two: Using a 5-period moving average

Similar to the two previous approaches, the Moving Average method was applied to this arrangement, but in this case using a 5-period set.

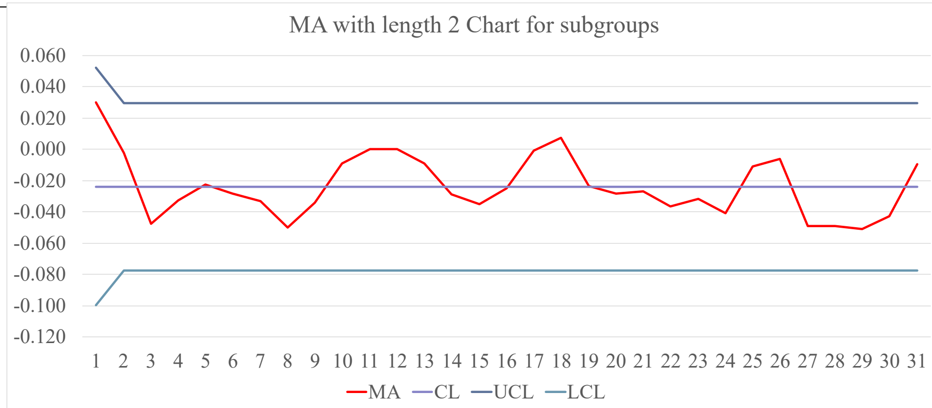

Figure 3.15: 2-Period Moving Average chart for subgroups of different sizes

The calculations were performed in Excel and the obtained values are presented in table 3.14.

Subsequently, the values were plotted in the moving average chart that can be observed in figure 3.14.

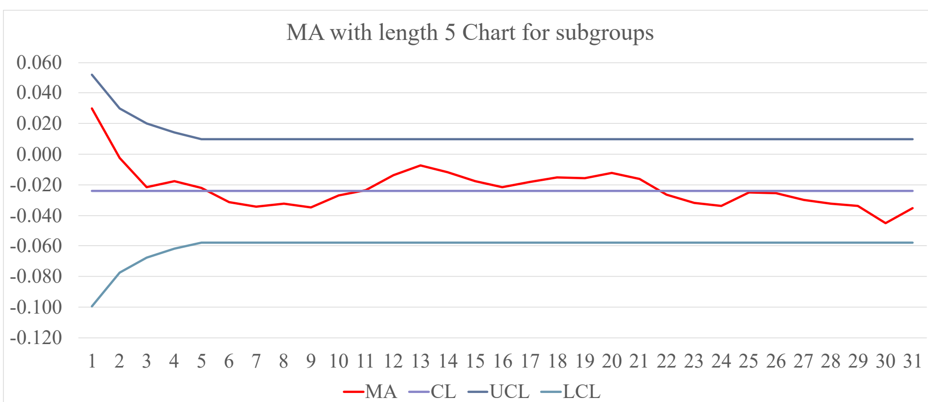

Figure 3.16: 5-Period Moving Average chart for subgroups of different sizes

Partial Results

The performance of charts with Moving Average using all the data as in 3.14, 2-period 3.15, and 5-period 3.16 was found to be similar to those for individual values. However, the length of 5 was observed to provide a fitted trend line, without fitting all the values near to the central line as when using all the values. Notably, none of the charts exhibited any outliers.

Table 3.14: 5-Period MA Values for Subgroups with different sizes

Subgroup	X-T	MA	CL	UCL	LCL
1	0.030	0.030	-0.024	0.052	-0.100
2	-0.035	-0.002	-0.024	0.030	-0.077
3	-0.060	-0.022	-0.024	0.020	-0.068
4	-0.005	-0.018	-0.024	0.014	-0.062
5	-0.040	-0.022	-0.024	0.010	-0.058
6	-0.017	-0.031	-0.024	0.010	-0.058
7	-0.050	-0.034	-0.024	0.010	-0.058
8	-0.050	-0.032	-0.024	0.010	-0.058
9	-0.018	-0.035	-0.024	0.010	-0.058
10	0.000	-0.027	-0.024	0.010	-0.058
11	0.000	-0.024	-0.024	0.010	-0.058
12	0.000	-0.014	-0.024	0.010	-0.058
13	-0.018	-0.007	-0.024	0.010	-0.058
14	-0.040	-0.012	-0.024	0.010	-0.058
15	-0.030	-0.018	-0.024	0.010	-0.058
16	-0.020	-0.022	-0.024	0.010	-0.058
17	0.018	-0.018	-0.024	0.010	-0.058
18	-0.003	-0.015	-0.024	0.010	-0.058
19	-0.043	-0.016	-0.024	0.010	-0.058
20	-0.013	-0.012	-0.024	0.010	-0.058
21	-0.040	-0.016	-0.024	0.010	-0.058
22	-0.033	-0.027	-0.024	0.010	-0.058
23	-0.030	-0.032	-0.024	0.010	-0.058
24	-0.052	-0.034	-0.024	0.010	-0.058
25	0.030	-0.025	-0.024	0.010	-0.058
26	-0.042	-0.025	-0.024	0.010	-0.058
27	-0.056	-0.030	-0.024	0.010	-0.058
28	-0.042	-0.032	-0.024	0.010	-0.058
29	-0.060	-0.034	-0.024	0.010	-0.058
30	-0.026	-0.045	-0.024	0.010	-0.058
31	0.007	-0.035	-0.024	0.010	-0.058

3.4 SPC Techniques applied to Data organised in subgroups of size n=5

This section aims to apply and assess the performance of statistical tools used in this study, but on this occasion, under an approach of consistent subgroups of size five. According to this logic, the data from table **??** will be organised into 21 subgroups of five data points each. Table 3.15 displays the data set arrangement.

Table 3.15: Data Set from March 2018 to August 2018 arranged in subgroups of size 5

Subgroup	X-T Average Value
1	-0.014
2	-0.03
3	-0.032
4	-0.026
5	-0.002
6	0.002
7	-0.018
8	-0.05
9	-0.012
10	-0.026
11	0.014
12	-0.006
13	-0.03
14	-0.034
15	-0.052
16	-0.032
17	-0.046
18	-0.044
19	-0.042
20	-0.004
21	-0.028

The main characteristics of this arrangement are:

- Uniformity throughout subgroups.
- Subgroups are conformed by data points from different dates.

- Subgroups contain data from the inspection of different parts. The target values within the subgroup are not constant.

The next sections will describe the utilization of the method for this arrangement.

3.4.1 Application of Shewhart Charts to data organised in subgroups of size n=5

In this subsection, Shewhart chart methods were applied to the data inspection with a dissimilar aim and subgroup size of $n = 5$. The characteristic considered for arranging the subgroups was the uniformity of subgroup size, regardless of the inspection date.

Similar to the sections above, the values obtained from the Shewhart calculations are presented in table 3.16.

In this case, the control limits were calculated as follows:

$CL = 0$ because it represents the target value.

Upper control limit:

$$\begin{aligned} UCL &= 0 + (3 \times 0.0181) \\ UCL &= 0.0545 \end{aligned} \tag{3.19}$$

And for the lower control limit:

$$\begin{aligned} LCL &= 0 - (3 \times 0.0181) \\ LCL &= -0.0545 \end{aligned} \tag{3.20}$$

The Shewhart values from table 3.16 are plotted and displayed in figure 3.17.

Table 3.16: Shewhart Chart Values for data set organised in subgroups of 5

Subgroup	X-T	CL	UCL	LCL
1	-0.014	0.00	0.0545	-0.0545
2	-0.030	0.00	0.0545	-0.0545
3	-0.032	0.00	0.0545	-0.0545
4	-0.026	0.00	0.0545	-0.0545
5	-0.002	0.00	0.0545	-0.0545
6	0.002	0.00	0.0545	-0.0545
7	-0.018	0.00	0.0545	-0.0545
8	-0.050	0.00	0.0545	-0.0545
9	-0.012	0.00	0.0545	-0.0545
10	-0.026	0.00	0.0545	-0.0545
11	0.014	0.00	0.0545	-0.0545
12	-0.006	0.00	0.0545	-0.0545
13	-0.030	0.00	0.0545	-0.0545
14	-0.034	0.00	0.0545	-0.0545
15	-0.052	0.00	0.0545	-0.0545
16	-0.032	0.00	0.0545	-0.0545
17	-0.046	0.00	0.0545	-0.0545
18	-0.044	0.00	0.0545	-0.0545
19	-0.042	0.00	0.0545	-0.0545
20	-0.004	0.00	0.0545	-0.0545
21	-0.028	0.00	0.0545	-0.0545

The moving range chart is presented in figure 3.18 along with the Shewhart chart for subgroups.

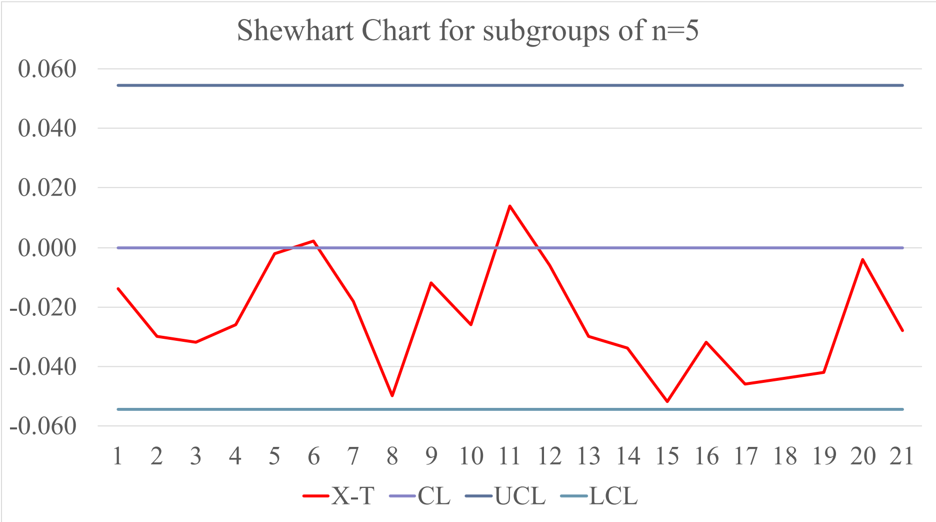

Figure 3.17: Shewhart chart for subgroups of $n = 5$

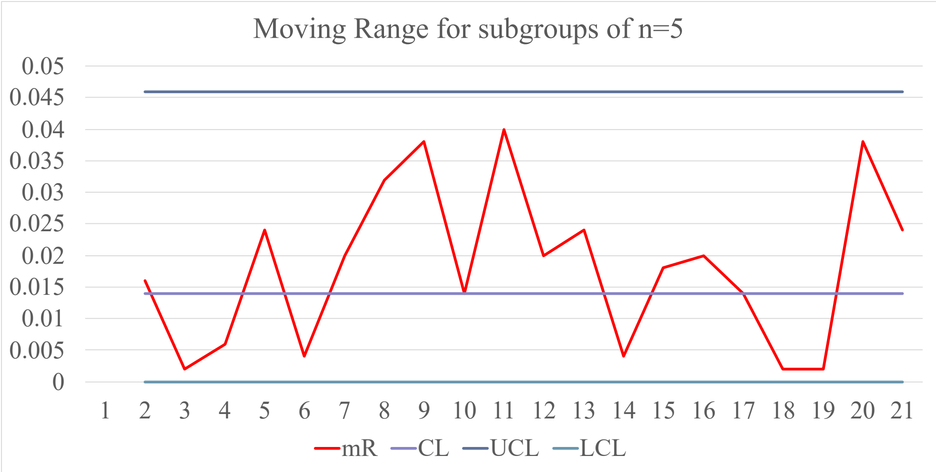

Figure 3.18: Moving range chart for subgroups of $n = 5$ of figure 3.17

Partial Results

The chosen arrangement for this control chart satisfies the essential sampling specification, even in cases where values for subgroups differ significantly. The chart's purpose is to evaluate its performance compared to those that organize data in a manner more akin to production scenarios. In this case, none of the values in charts 3.17, and chart 3.18 showed values out of control limits. A more comprehensive analysis will be presented in Chapter 4.

3.4.2 Application of Q Charts to data organised in subgroups of size n=5

The Q chart method was applied to data inspection in this subsection. This is data with dissimilar aims and divided into subgroup sizes of $n = 5$. Just as in the previous case of subgroups, the analysis for this arrangement used the method of Q charts for variables in subgroups.

$$Q_i(\bar{X}_i) = \Phi^{-1}\left\{ H_{n_1+...+n_i-i}\left[\sqrt{\frac{n_i(n_1+...+n_{i-1})}{n_1+...+n_i}}\left(\frac{\bar{X}_i - \bar{\bar{X}}_{i-1}}{S_{p,i}}\right)\right]\right\}$$

The upper and lower control limits were set using $\pm 3\sigma$.

Upon completion of the calculation process, the resulting Q values have been presented in table 3.17.

Table 3.17: Q Values for Data Set with subgroups of size n=5

Subgroup	X-T	Q Statistic	CL	UCL	LCL
1	-0.014				
2	-0.030				
3	-0.032	0.992	1.140	1.836	0.443
4	-0.026	1.460	1.140	1.836	0.443
5	-0.002	1.870	1.140	1.836	0.443
6	0.002	1.284	1.140	1.836	0.443
7	-0.018	0.972	1.140	1.836	0.443
8	-0.050	1.065	1.140	1.836	0.443
9	-0.012	1.088	1.140	1.836	0.443
10	-0.026	1.090	1.140	1.836	0.443
11	0.014	1.220	1.140	1.836	0.443
12	-0.006	0.897	1.140	1.836	0.443
13	-0.030	0.858	1.140	1.836	0.443
14	-0.034	0.931	1.140	1.836	0.443
15	-0.052	0.987	1.140	1.836	0.443
16	-0.032	1.036	1.140	1.836	0.443
17	-0.046	1.086	1.140	1.836	0.443
18	-0.044	1.139	1.140	1.836	0.443
19	-0.042	1.194	1.140	1.836	0.443
20	-0.004	1.281	1.140	1.836	0.443
21	-0.028	1.204	1.140	1.836	0.443

The corresponding Q chart for the table above is presented in figure 3.19.

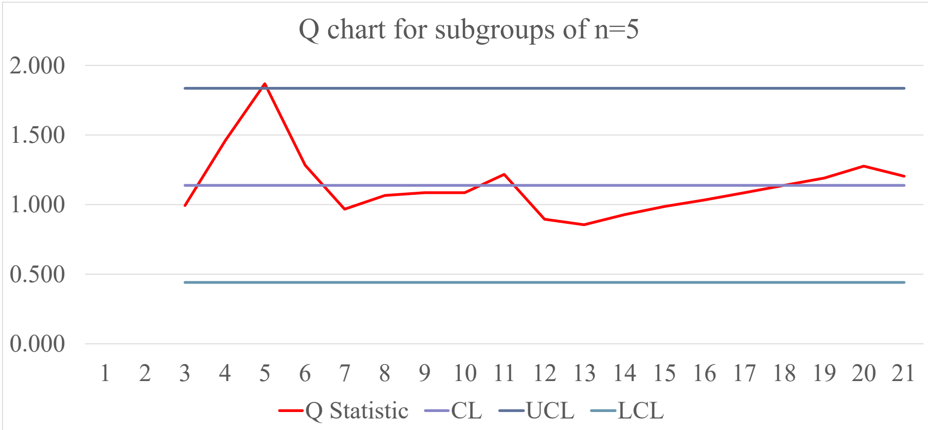

Figure 3.19: Q chart for data in subgroups of $n = 5$

Partial Results

This arrangement displayed a different trend. In this case, the values were almost all around the Centre Line, except for point 5, which appeared above the upper control limit.

3.4.3 Application of CUSUM Charts to data organised in subgroups of size n=5

In this subsection, the methods applied were the CUSUM chart to data inspection with a different aim subgroup size of $n = 5$. CUSUM values were calculated and are presented in table 3.18.

Table 3.18: CUSUM Values for Data Set with subgroups of size n=5

Subgroup	X-T	C value	Ci+	Ci+	UCL	LCL
1	-0.014	-0.014	0.000	0.002	0.461	-0.461
2	-0.030	-0.044	0.000	0.020	0.461	-0.461
3	-0.032	-0.076	0.000	0.039	0.461	-0.461
4	-0.026	-0.102	0.000	0.053	0.461	-0.461
5	-0.002	-0.104	0.000	0.043	0.461	-0.461
6	0.002	-0.102	0.000	0.029	0.461	-0.461
7	-0.018	-0.120	0.000	0.035	0.461	-0.461
8	-0.050	-0.170	0.000	0.072	0.461	-0.461
9	-0.012	-0.182	0.000	0.072	0.461	-0.461
10	-0.026	-0.208	0.000	0.086	0.461	-0.461
11	0.014	-0.194	0.000	0.060	0.461	-0.461
12	-0.006	-0.200	0.000	0.054	0.461	-0.461
13	-0.030	-0.230	0.000	0.072	0.461	-0.461
14	-0.034	-0.264	0.000	0.093	0.461	-0.461
15	-0.052	-0.316	0.000	0.133	0.461	-0.461
16	-0.032	-0.348	0.000	0.153	0.461	-0.461
17	-0.046	-0.394	0.000	0.187	0.461	-0.461
18	-0.044	-0.438	0.000	0.219	0.461	-0.461
19	-0.042	-0.480	0.000	0.248	0.461	-0.461
20	-0.004	-0.484	0.000	0.240	0.461	-0.461
21	-0.028	-0.512	0.000	0.256	0.461	-0.461

The chart corresponding to table 3.18 can be found in figure 3.20.

Partial Results

Regularly, the CUSUM chart generates an upwards graph; in this case, the chart accumulated downwards because the studies values are the deviation from target δ, many of which are negative.

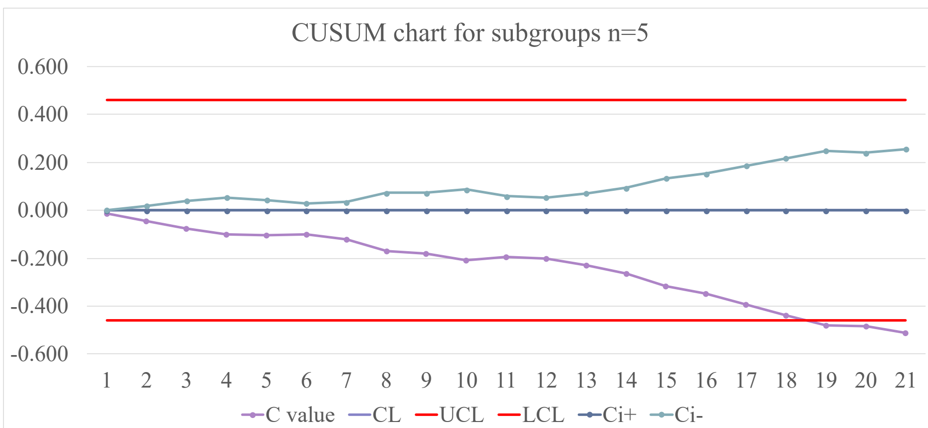

Figure 3.20: CUSUM chart for subgroups of $n = 5$

3.4.4 Application of EWMA Charts to data organised in subgroups of size n=5

In this subsection, the methods applied were the EWMA chart with data inspection with a dissimilar aim subgroup size of $n = 5$. The characteristic considered for arranging the subgroups was the uniformity of subgroup size, regardless of the inspection date.

Calculations were performed as follows:

For the z values:

$$
\begin{aligned}
z_1 &= \mu_1 = -0.027 \\
z_2 &= (0.040 \times -0.035) + ((1 - 0.040) \times -0.027) = -0.030 \\
z_3 &= (0.040 \times -0.060) + ((1 - 0.040) \times -0.030) = -0.042 \\
&\ldots \\
z_{21} &= (0.040 \times -0.028) + ((1 - 0.40) \times -0.027) = -0.027
\end{aligned}
$$

For the Upper and Lower Control Limits:

$$
UCL_n = \bar{X} + (L * \sigma) * (SQRT((\lambda/(2 - \lambda)) * (1 - ((1 - \lambda)^{2n}))))
$$

$$
UCL_n = \bar{X} - (L * \sigma) * (SQRT((\lambda/(2 - \lambda)) * (1 - ((1 - \lambda)^{2n}))))
$$

Substituting the formulas above, the EWMA chart values were obtained and presented in table 3.19.

The corresponding chart for the values from table 3.19 is displayed in figure 3.21.

Table 3.19: EWMA Values for Data Set with subgroups of size n=5

Subgroup	X-T	Zi	CL	UCL	LCL
1	0.030	-0.027	-0.027	0.001	-0.054
2	-0.035	-0.030	-0.027	0.006	-0.059
3	-0.060	-0.042	-0.027	0.007	-0.060
4	-0.005	-0.027	-0.027	0.008	-0.061
5	-0.040	-0.032	-0.027	0.008	-0.061
6	-0.033	-0.033	-0.027	0.008	-0.061
7	-0.050	-0.040	-0.027	0.008	-0.061
8	-0.018	-0.031	-0.027	0.008	-0.061
9	0.000	-0.019	-0.027	0.008	-0.061
10	0.000	-0.011	-0.027	0.008	-0.061
11	-0.022	-0.015	-0.027	0.008	-0.061
12	-0.002	-0.010	-0.027	0.008	-0.061
13	-0.043	-0.023	-0.027	0.008	-0.061
14	-0.034	-0.028	-0.027	0.008	-0.061
15	-0.052	-0.037	-0.027	0.008	-0.061
16	-0.032	-0.035	-0.027	0.008	-0.061
17	-0.046	-0.040	-0.027	0.008	-0.061
18	-0.044	-0.041	-0.027	0.008	-0.061
19	-0.042	-0.042	-0.027	0.008	-0.061
20	-0.004	-0.027	-0.027	0.008	-0.061
21	-0.028	-0.027	-0.027	0.008	-0.061

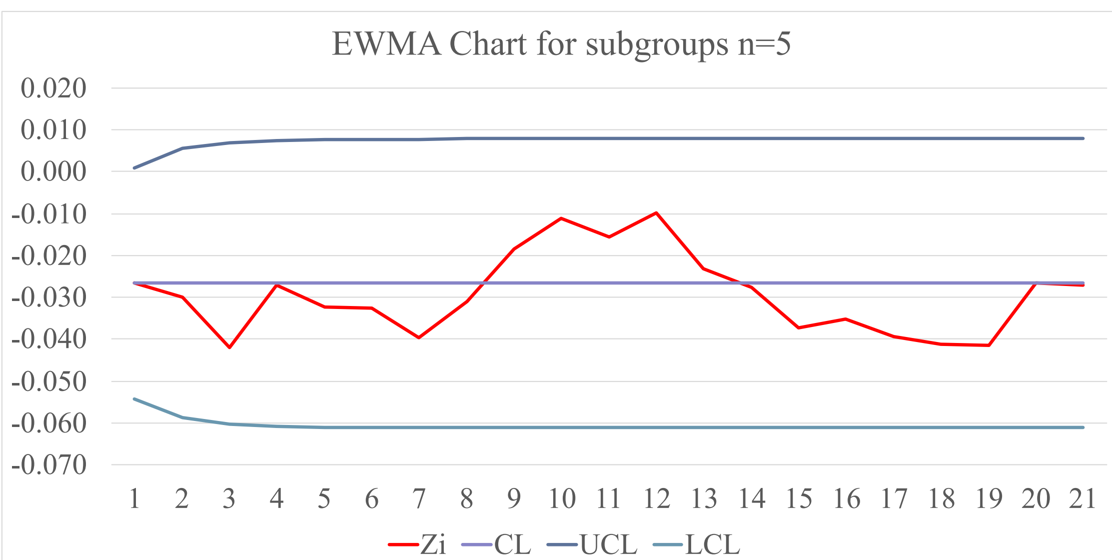

Figure 3.21: EWMA chart for subgroups of $n = 5$

Partial Results

The performance of this chart is visibly within control limits; however, a significant variation is observable in the data floating around the centre line.

3.5 Application of Moving Average Charts to data organised in subgroups of size n=5

This section proposes to apply three approaches of moving average charts to the data organised in subgroups of 5.

Calculations were performed in Excel, and the moving average values are displayed in table 3.20. The corresponding chart is shown in figure 3.22.

Firstly, the data set was analysed with an infinite moving average. The first moving average values are displayed in table 3.20, and the chart is in figure 3.22.

Table 3.20: Moving Average Values for Data Set with subgroups of size n=5

Subgroup	X-T	MA	CL	UCL	LCL
1	-0.014	-0.014	-0.024	0.030	-0.079
2	-0.030	-0.022	-0.024	0.014	-0.063
3	-0.032	-0.030	-0.024	0.007	-0.056
4	-0.026	-0.031	-0.024	0.003	-0.052
5	-0.002	-0.029	-0.0244	0.000	-0.049
6	0.002	-0.023	-0.0244	-0.002	-0.047
7	-0.018	-0.018	-0.0244	-0.004	-0.045
8	-0.050	-0.018	-0.0244	-0.005	-0.044
9	-0.012	-0.022	-0.0244	-0.006	-0.043
10	-0.026	-0.021	-0.0244	-0.007	-0.042
11	0.014	-0.022	-0.0244	-0.008	-0.041
12	-0.006	-0.018	-0.0244	-0.009	-0.040
13	-0.030	-0.017	-0.0244	-0.009	-0.039
14	-0.034	-0.018	-0.0244	-0.010	-0.039
15	-0.052	-0.019	-0.0244	-0.010	-0.038
16	-0.032	-0.022	-0.0244	-0.011	-0.038
17	-0.046	-0.022	-0.0244	-0.011	-0.038
18	-0.044	-0.024	-0.0244	-0.012	-0.037
19	-0.042	-0.025	-0.0244	-0.012	-0.037
20	-0.004	-0.026	-0.0244	-0.012	-0.037
21	-0.028	-0.025	-0.0244	-0.012	-0.036

Subsequently, the data were analysed with a two-period moving average approach. The data points are displayed in table 3.21 and the corresponding chart is shown in figure 3.23.

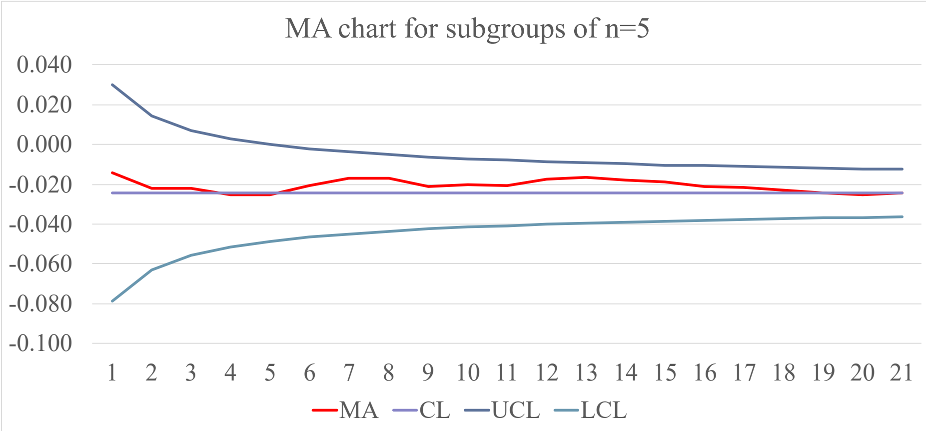

Figure 3.22: Moving Average chart with subgroups of n=5

Table 3.21: Values for a 2-Period Moving Average Chart for Data Set with subgroups of size n=5

Subgroup	X-T	MA	CL	UCL	LCL
1	-0.014	-0.014	-0.024	0.019	-0.068
2	-0.030	-0.022	-0.024	0.007	-0.055
3	-0.032	-0.031	-0.024	0.007	-0.055
4	-0.026	-0.029	-0.024	0.007	-0.055
5	-0.002	-0.014	-0.024	0.007	-0.055
6	0.002	0.000	-0.024	0.007	-0.055
7	-0.018	-0.008	-0.024	0.007	-0.055
8	-0.050	-0.034	-0.024	0.007	-0.055
9	-0.012	-0.031	-0.024	0.007	-0.055
10	-0.026	-0.019	-0.024	0.007	-0.055
11	0.014	-0.006	-0.024	0.007	-0.055
12	-0.006	0.004	-0.024	0.007	-0.055
13	-0.030	-0.018	-0.024	0.007	-0.055
14	-0.034	-0.032	-0.024	0.007	-0.055
15	-0.052	-0.043	-0.024	0.007	-0.055
16	-0.032	-0.042	-0.024	0.007	-0.055
17	-0.046	-0.039	-0.024	0.007	-0.055
18	-0.044	-0.045	-0.024	0.007	-0.055
19	-0.042	-0.043	-0.024	0.007	-0.055
20	-0.004	-0.023	-0.024	0.007	-0.055
21	-0.028	-0.016	-0.024	0.007	-0.055

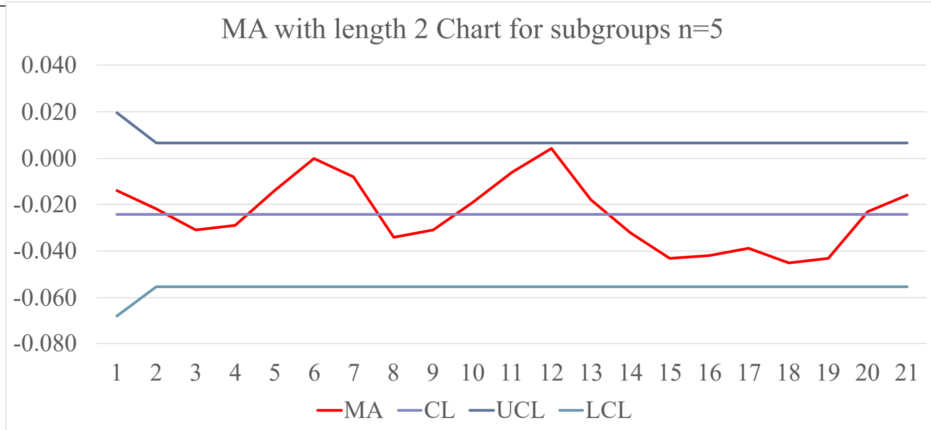

Figure 3.23: Moving Average chart of length 2 with subgroups of n=5

Finally, a five-period moving average was used to analyse the data set arranged in subgroups of 5. The values obtained from this analysis are shown in table 3.22, and the chart where these values are plotted can be found in figure 3.24.

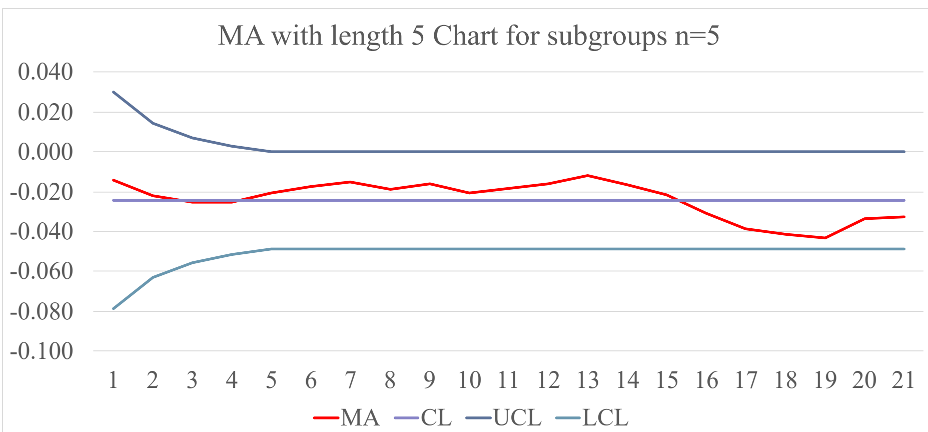

Figure 3.24: Moving Average chart of length 5 with subgroups of n=5

Partial Results

Analysing charts 3.22, 3.23 and 3.24, it can be observed that the moving average chart with a length of 2 performed better at detecting changes, even when the 3 charts maintained the data under the control limits.

Table 3.22: Values for a 5-Period Moving Average Chart for Data Set with subgroups of size n=5

Subgroup	X-T	MA	CL	UCL	LCL
1	-0.014	-0.014	-0.024	0.030	-0.079
2	-0.030	-0.022	-0.024	0.014	-0.063
3	-0.032	-0.025	-0.024	0.007	-0.056
4	-0.026	-0.026	-0.024	0.003	-0.052
5	-0.002	-0.021	-0.024	0.000	-0.049
6	0.002	-0.018	-0.024	0.000	-0.049
7	-0.018	-0.015	-0.024	0.000	-0.049
8	-0.050	-0.019	-0.024	0.000	-0.049
9	-0.012	-0.016	-0.024	0.000	-0.049
10	-0.026	-0.021	-0.024	0.000	-0.049
11	0.014	-0.018	-0.024	0.000	-0.049
12	-0.006	-0.016	-0.024	0.000	-0.049
13	-0.030	-0.012	-0.024	0.000	-0.049
14	-0.034	-0.016	-0.024	0.000	-0.049
15	-0.052	-0.022	-0.024	0.000	-0.049
16	-0.032	-0.031	-0.024	0.000	-0.049
17	-0.046	-0.039	-0.024	0.000	-0.049
18	-0.044	-0.042	-0.024	0.000	-0.049
19	-0.042	-0.043	-0.024	0.000	-0.049
20	-0.004	-0.034	-0.024	0.000	-0.049
21	-0.028	-0.033	-0.024	0.000	-0.049

Chapter4

Results and Discussion

In this section, the findings from the data analysis conducted in Chapter 3 will be presented. The statistical control methods employed in this study will be compared with the approach suggested by BS ISO 7078-8:2017. To better illustrate the results, a table that compares each method with each approach has been included.

The following paragraphs will comprehensively examine the outcomes achieved by applying diverse statistical techniques to identical data. Specifically, the Shewhart Chart, Moving Range, Q Chart, CUSUM Chart, EWMA Chart, and Moving Average were employed. The dataset was organized in three distinct ways to evaluate the efficacy of each approach and facilitate comparisons between them.

4.1 Research Limitations

The research indicates that traditional SPC methods struggle to detect outlier values in a process. A primary challenge in applying any of these techniques is gathering the minimum data required for analysis, which is often complex. However, the approach suggested by BS ISO 7078-8:2017, which combines individual values with a classic Shewhart approach, has proven effective in identifying inconsistent values.

This study was conducted under specific data availability constraints. Due to the low-volume nature of the processes evaluated, it was necessary to transform the dataset to achieve a larger joint dataset. The data transformation is displayed in Table 3.2, which facilitated the application of the four proposed methods.

4.2 SPC Methods Results

Various statistical techniques were utilized to analyze the low-volume data, including the Shewhart Chart, Moving Range, Q charts, CUSUM Charts, EWMA Charts, and Moving Average charts. The data was organized in three distinct ways: individual values, grouped data arranged by date, and data grouped into subgroups of five data points. This section provides an overview of the outcomes of applying these methods to the three arrangements.

As illustrated in Table 4.1 through the results comparison, only one out of the six settings demonstrated a consistent process, while the remaining five were deemed ineffective. Furthermore, approximately 80% of the results were identified as outliers. The next subsection will display in detail the outcomes of the control charts for each approach.

Table 4.1: SPC Method Comparison

Method	**Individual Values**	**Data in Subgroups of different size**	**Data in Subgroups of size** $n = 5$
Shewhart Charts	This chart showed an unstable process, with the values floating around the Central Line. Nevertheless, the only outlier value is value no. 39.	This chart shows a similar performance to the individual values. Generally, the values constantly go upper and lower in the central line. No values were plotted outside the control limits.	The Shewhart chart in the approach of subgroups of equal size did not show any specific trend. Several of the plotted values are out of control. These outlier values are found along the process. Values out of control are 6, 8, 11, 15, 17, 18 and 19.

Continued on next page

Table 4.1: SPC Method Comparison (Continued)

Method	**Individual Values**	**Data in Subgroups of different size**	**Data in Subgroups of size** $n = 5$
Moving Range	Like the Shewhart Chart, the process is displayed in this chart without any specific trend. In this chart, value 39 was again the only outlier going past the upper limit.	In this chart, it is observable that the 15 middle values fall below the central line. In this example, there are 4 outlier values, three below the control line and reaching the lower control limit. The fourth outlier is value 25, which stands out from the rest by creating a peak above the upper control limit.	Similar to the Shewhart chart for this approach, the moving range chart did not show any specific trend, particularly in this chart; almost all values fall consecutively in opposite sites of the central line. This behaviour created a spiked line, but no data points were plotted above the upper control limit.

Continued on next page

Table 4.1: SPC Method Comparison (Continued)

Method	**Individual Values**	**Data in Subgroups of different size**	**Data in Subgroups of size** $n = 5$
Q Charts	In this chart, it can be observed that the values started following an upwards trend commencing in value no. 4.	In contrast to the effects of Q Charts, in this arrangement, the Q chart did not behave following an upwards trend. However, in this case, the chart displayed three outlier values: values no. 17, 25 and 27, which fall below the lower control limit.	Using this approach, Q charts drew a steady chart just after spiking an outlier in subgroup 5, with this subgroup falling above the upper control limit.

Continued on next page

Table 4.1: SPC Method Comparison (Continued)

Method	**Individual Values**	**Data in Subgroups of different size**	**Data in Subgroups of size** $n = 5$
CUSUM Charts	This chart was arranged to show two different C Values. On one side, the CUSUM chart showed a shift upwards at around value 39; however, the data points started to fall out of the upper control limit after point 73.	This chart was organised to plot the standard C value and the tabular C values together. The results show that values of subgroups 28, 29, 30 and 31 fall out of the control limits, with a remarkable downward trend in the C value line.	As in the 'individual' and the 'data in subgroups of different sizes' approaches, for this example, the traditional CUSUM chart and tabular CUSUM chart were merged into a single graph. The result of this chart showed that subgroups 19, 20 and 21 fall below the lower control limit.

Continued on next page

Table 4.1: SPC Method Comparison (Continued)

Method	Individual Values	Data in Subgroups of different size	Data in Subgroups of size $n = 5$
EWMA Charts	The data points of this chart show a fluctuation around the central line with two peaks almost reaching the upper control limit. However, the last quarter of the data series shows a remarkable decline, creating a lower peak and almost reaching the lower control limit.	The behaviour of the EWMA chart with this approach did not show a steady performance; instead, the values formed waves that fluctuated between the central line and the lower control limit. Nonetheless, six values can be considered outliers. These values are positions 3, 8, 24, 28, 29 and 30. At that point, CUSUM and EWMA charts have spotted the same outlier values.	The EWMA chart for this approach did not display any outliers; however, it did not show a steady trend either.

Continued on next page

Table 4.1: SPC Method Comparison (Continued)

Method	**Individual Values**	**Data in Subgroups of different size**	**Data in Subgroups of size** $n = 5$
Moving Average; Moving Average 2-Period; Moving Average 5-Period	From these three moving average charts, the first approach was the best to adjust all the values around the central line; however, it does not give a realistic image of the process performance. The 2-period and 5-period MA, on the other hand, show a more defined process line. The 5-period moving average presents the best way to smooth the graph.	The moving average charts for this approach perform similarly to the moving average charts applied to individual values.	The three moving average charts performed similarly to the performance of these charts on the other approaches. None of the Moving Average charts applied to this approach showed outlier values.

As shown in table 4.1 through the Results Matrix, only one of the six settings demonstrated a consistent process, while the remaining five were unhelpful. Additionally, approximately 80% of the results were identified as outliers. In the following subsection, we will assess the effectiveness of the control charts for each approach.

4.2.1 Values arranged as individuals

Following the methodology outlined in BS ISO 7078-8:2017, the individual values approach utilizes Shewhart and Moving Range Charts. These charts are commonly employed to assess process stability and identify outliers. In this study, we utilized the same data arrangement to evaluate the effectiveness of alternative statistical methods.

The analysis revealed that the Shewhart, Moving Range, and CUSUM charts consistently identified outlier value 39 (see figures 3.1, 3.2, and 3.4). This consistent detection of process deviations suggests a reliable methodology for identifying and addressing potential system issues. Moreover, the ease of computation for these statistics makes this approach practical for real-world applications.

4.2.2 Values Arranged in Subgroups of Dissimilar Size

In contrast to the individual values approach, the Q, CUSUM, and EWMA charts effectively detected deviations from the centre of the process (refer to figures 3.11, 3.12, and 3.13). Specifically, subgroups falling between values 24 and 30 were identified as out of process control. The consistency in outcomes between the EWMA, CUSUM, and Q charts can be attributed to their time-weighted nature, which allows for better detection of subtle process shifts.

The consistency in outcomes between the EWMA, CUSUM, and Q charts can be attributed to their time-weighted nature. In contrast, the Shewhart chart is memory-less and fails to identify outlier values despite showcasing process instability. Variations in subgroup sizes may introduce biases or obscure underlying patterns in the data.

4.2.3 Values arranged in subgroups of size 5

This approach exhibited considerable instability in the process charts of the three trialled methods. Both the Shewhart chart (refer to 3.17) and the Q chart (refer to 3.19), as well as the CUSUM chart (refer to 3.20), revealed deviations that were out of control. The central values that fell outside of the control limits were situated between Subgroup 5 and Subgroup 19.

In contrast to the other two methods, this particular approach demonstrated distinct behaviour. The data points gathered to form the subgroups did not possess any shared attributes, such as the production date or the manufacturing process. As a result, it is not surprising that values arranged in equal-size subgroups as the sole parameter tend to be less dependable for this data set. Therefore, relying solely on equal size as the parameter for subgroup formation can be inadequate and may not accurately reflect the true state of the process.

4.3 Discussion and Evaluation of Results

This study evaluates different statistical methods for monitoring process stability and detecting deviations in low-volume production scenarios where standard Statistical Process Control (SPC) methods are not suitable. The effectiveness of these methods is confirmed under varying conditions described in Chapter 3. Each method of data grouping presented in this chapter provides unique conditions for applying statistical techniques to the study's data.

The first grouping method evaluates each observation individually as shown in section 3.2, making it ideal for small-batch production. While this approach accelerates the creation of control charts, it may be challenging to identify minor shifts or trends due to the limited data points available.

The second and third methods involve aggregating multiple observations into each data point. The second method, as done in section 3.3, groups data by production date, while the third method, as seen in section 3.4, arranges data into fixed-size groups. In industrial settings, the second method handles cases where subgroups may vary in size, potentially requiring more time to accumulate sufficient data for charting. The third method is suitable for scenarios involving small batches of processes performed in brief, continuous periods. Grouping data in this way reduces the impact of individual data point variability and provides a clearer picture of the process's central tendency.

The findings of this study highlight the importance of choosing appropriate statistical tools and subgroup arrangements designed to the specific characteristics of the dataset. Different methods offer varying levels of sensitivity and reliability depending on contextual factors. For example, while Shewhart and Moving Range Charts excel with individual values, time-weighted charts such as Q, CUSUM, and EWMA are better equipped to detect subtle process shifts, particularly in cases with irregular subgroup sizes. Conversely, relying solely on equal-size subgroups without considering other attributes can yield unreliable outcomes, underscoring the necessity of deliberate subgroup selection.

This research also provides insight into the effectiveness of various statistical methods for process monitoring but emphasises the need for further research to improve the reliability and applicability of these findings.

Firstly, it would be essential to involve a wider variety of datasets from diverse industries and contexts. This will help verify the methods' applicability across different types of processes and data characteristics. Moreover, longitudinal studies that track the performance of Shewhart, Moving Range, Q, CUSUM, and EWMA charts over extended periods would provide deeper insights into their reliability and potential drift over time and help accurately assess these statistical methods' long-term stability and effectiveness.

Secondly, the impact of different subgroup characteristics beyond size and date should be investigated. Factors such as environmental conditions and operator influence can significantly affect the process. Understanding how these attributes interact with statistical methods will enhance the precision of process monitoring.

Furthermore, integrating traditional statistical methods with modern data analysis techniques can offer more advanced and adaptive process control solutions. Research in this area could lead to combining models that target both approaches. While the ease of computation is an advantage worth seeking in this kind of project, further research into the user-friendliness and practicality of implementing these methods in real-world settings is crucial. Studies that investigate the implementation, training needs, and usability of these statistical tools in different industrial settings will help promote their wider adoption.

In conclusion, additional research is crucial to confirm and build upon the discoveries of this study. The low-volume statistical process control field can make significant progress by examining more extensive and diverse datasets and considering long-term stability, subgroup characteristics, integration with modern analytics, and user-friendliness. These efforts will lead to more precise and practical solutions for maintaining process stability and quality control in low-volume industries.

Chapter5

Conclusion and Future Research

This thesis evaluates the effectiveness of the BS ISO 7870-8:2017 method in low-volume scenarios compared to traditional SPC methods. Despite its utility, the standard lacks guidance for sample sizes other than 1. Four control charts were tested on datasets arranged in different formats, including Individual X, Shewhart, Moving Range, Q, CUSUM, EWMA, and Moving Average charts, as detailed in Chapter Three. The results of these applications are presented in Chapter Four, with the conclusion in Chapter Five.

The findings highlight the importance of tailoring the choice of the right statistical tools and subgroup arrangements based on data characteristics. While Shewhart and Moving Range Charts work well for individual values, time-weighted charts like Q, CUSUM, and EWMA better detect subtle shifts in processes with varying subgroup sizes. Proper subgroup composition is crucial to avoid unreliable results.

This study provides valuable insights for quality control professionals and process engineers. It highlights the need for customised approaches in low-volume scenarios. Recognising the strengths and weaknesses of each method enhances process monitoring and ensures stability, allowing for better process monitoring and stability.

Future research should involve diverse industry datasets to verify the methods' applicability. Longitudinal studies tracking the performance of different charts over time will provide deeper insights into their reliability. Investigating the impact of subgroup characteristics beyond size, such as production process and environmental conditions, will enhance precision. Integrating traditional statistical methods with modern data analysis techniques can offer more adaptive solutions, which are solutions that can adjust to changing data and conditions. Additionally, research should focus on the practical implementation and usability of these methods in real-world settings to promote wider adoption.

In conclusion, further research would be beneficial to build on this study's findings. More precise and practical solutions for process stability and quality control in low-volume production environments can be developed by examining diverse datasets, considering long-term stability, and integrating modern analytics.

AppendixA

Data Sets

Dataset No. 1

No.	Part/OP	Date	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12	X13	X14	X15	X16	X17	X18	X19	X20
1	C07C27145-140	3/23/2018	2	2																		
2	C07C27145-140	3/26/2018	1.93	1.94																		
3	C07C27145-140	3/27/2018	1.91	1.91																		
4	C07C27145-140	4/22/2018	1.93	2																		
5	C07C27145-140	4/23/2018	1.93																			
6	C07C27145-140	7/16/2018	1.93	2	1.93	1.94	1.91	1.91														
7	C07C27157-70	8/8/2018	1.7																			
8	C07C27157-70	8/8/2018	1.73	1.73	1.73	1.73	1.74															
9	C07C27157-70	8/10/2018	1.74	1.74	1.74	1.78																
10	C07C27157-70	8/13/2018	1.75																			
11	C07C27157-70	8/14/2018	1.75	1.77	1.72	1.77	1.74	1.78	1.7	1.73	1.71	1.74	1.7	1.75	1.6	1.71	1.79	1.7	1.72	1.7	1.78	1.7
12	C07C27157-70	8/15/2018	1.7	1.72	1.77	1.73	1.73	1.76	1.77	1.78	1.78	1.75	1.73	1.75	1.76							
13	C07C27157-80	8/21/2018	1.95	1.93	1.9																	
14	C07C27157-80	8/22/2018	1.9	1.94	2.03																	
15	C07C27157-80	8/23/2018	1.93																			
16	C07C27157-80	8/24/2018	1.92	1.94	1.95																	
17	C07C27145-140	8/25/2018	1.94																			
18	C07C27157-80	8/25/2018	1.91	1.98	1.9	1.9	1.9															
19	C07C27145-140	8/29/2018	2																			
20	C07C27157-80	8/30/2018	1.91	1.94	1.92	1.92	1.95	1.95	1.92	1.9	1.9	1.9	1.98	1.93	1.91	1.91	1.91					
21	C07C27145-140	8/31/2018	1.91																			
22	C07C27157-80	8/31/2018	1.95	1.93	1.94	1.91	1.99	1.97	1.97	1.99												

Table A.1: Data Set No. 1

Table A.2: General Data Set

Subgroup	Part	Op.	Date	Value	Target	X-T
1	C07C27145	140	23/03/2018	2	1.97	0.03
2	C07C27145	140	23/03/2018	2	1.97	0.03
3	C07C27145	140	26/03/2018	1.93	1.97	-0.04
4	C07C27145	140	26/03/2018	1.94	1.97	-0.03
5	C07C27145	140	27/03/2018	1.91	1.97	-0.06
6	C07C27145	140	27/03/2018	1.91	1.97	-0.06
7	C07C27145	140	22/04/2018	1.93	1.97	-0.04
8	C07C27145	140	22/04/2018	2	1.97	0.03
9	C07C27145	140	23/04/2018	1.93	1.97	-0.04
10	C07C27145	140	16/07/2018	1.93	1.97	-0.04
11	C07C27145	140	16/07/2018	2	1.97	0.03
12	C07C27145	140	16/07/2018	1.93	1.97	-0.04
13	C07C27145	140	16/07/2018	1.94	1.97	-0.03
14	C07C27145	140	16/07/2018	1.91	1.97	-0.06
15	C07C27145	140	16/07/2018	1.91	1.97	-0.06
16	C01C27157	70	08/08/2018	1.7	1.75	-0.05
17	C01C27157	70	08/08/2018	1.73	1.75	-0.02
18	C01C27157	70	08/08/2018	1.73	1.75	-0.02
19	C01C27157	70	08/08/2018	1.73	1.75	-0.02
20	C01C27157	70	08/08/2018	1.73	1.75	-0.02
21	C01C27157	70	08/08/2018	1.74	1.75	-0.01
22	C01C27157	70	10/08/2018	1.74	1.75	-0.01
23	C01C27157	70	10/08/2018	1.74	1.75	-0.01
24	C01C27157	70	10/08/2018	1.74	1.75	-0.01
25	C01C27157	70	10/08/2018	1.78	1.75	0.03
26	C01C27157	70	13/08/2018	1.75	1.75	0
27	C01C27157	70	14/08/2018	1.75	1.75	0
28	C01C27157	70	14/08/2018	1.77	1.75	0.02
29	C01C27157	70	14/08/2018	1.72	1.75	-0.03
30	C01C27157	70	14/08/2018	1.77	1.75	0.02
31	C01C27157	70	14/08/2018	1.74	1.75	-0.01
32	C01C27157	70	14/08/2018	1.78	1.75	0.03
33	C01C27157	70	14/08/2018	1.7	1.75	-0.05
34	C01C27157	70	14/08/2018	1.73	1.75	-0.02
35	C01C27157	70	14/08/2018	1.71	1.75	-0.04
36	C01C27157	70	14/08/2018	1.74	1.75	-0.01
37	C01C27157	70	14/08/2018	1.7	1.75	-0.05
38	C01C27157	70	14/08/2018	1.75	1.75	0
39	C01C27157	70	14/08/2018	1.6	1.75	-0.15
40	C01C27157	70	14/08/2018	1.71	1.75	-0.04
41	C01C27157	70	14/08/2018	1.79	1.75	0.04
42	C01C27157	70	14/08/2018	1.7	1.75	-0.05
43	C01C27157	70	14/08/2018	1.72	1.75	-0.03
44	C01C27157	70	14/08/2018	1.7	1.75	-0.05
45	C01C27157	70	14/08/2018	1.78	1.75	0.03
46	C01C27157	70	14/08/2018	1.7	1.75	-0.05
47	C01C27157	70	15/08/2018	1.7	1.75	-0.05
48	C01C27157	70	15/08/2018	1.72	1.75	-0.03
49	C01C27157	70	15/08/2018	1.77	1.75	0.02
50	C01C27157	70	15/08/2018	1.73	1.75	-0.02
51	C01C27157	70	15/08/2018	1.73	1.75	-0.02
52	C01C27157	70	15/08/2018	1.76	1.75	0.01
53	C01C27157	70	15/08/2018	1.77	1.75	0.02
54	C01C27157	70	15/08/2018	1.78	1.75	0.03
55	C01C27157	70	15/08/2018	1.78	1.75	0.03
56	C01C27157	70	15/08/2018	1.75	1.75	0
57	C01C27157	70	15/08/2018	1.73	1.75	-0.02
58	C01C27157	70	15/08/2018	1.75	1.75	0
59	C01C27157	70	15/08/2018	1.76	1.75	0.01
60	C01C27157	80	21/08/2018	1.95	1.97	-0.02
61	C01C27157	80	21/08/2018	1.93	1.97	-0.04
62	C01C27157	80	21/08/2018	1.9	1.97	-0.07
63	C01C27157	80	22/08/2018	1.9	1.97	-0.07
64	C01C27157	80	22/08/2018	1.94	1.97	-0.03
65	C01C27157	80	22/08/2018	2.03	1.97	0.06
66	C01C27157	80	23/08/2018	1.93	1.97	-0.04
67	C01C27157	80	24/08/2018	1.92	1.97	-0.05
68	C01C27157	80	24/08/2018	1.94	1.97	-0.03
69	C01C27157	80	24/08/2018	1.95	1.97	-0.02
70	C07C27145	140	25/08/2018	1.94	1.97	-0.03
71	C01C27157	80	25/08/2018	1.91	1.97	-0.06
72	C01C27157	80	25/08/2018	1.98	1.97	0.01
73	C01C27157	80	25/08/2018	1.9	1.97	-0.07
74	C01C27157	80	25/08/2018	1.9	1.97	-0.07
75	C01C27157	80	25/08/2018	1.9	1.97	-0.07
76	C07C27145	140	29/08/2018	2	1.97	0.03
77	C01C27157	80	30/08/2018	1.91	1.97	-0.06
78	C01C27157	80	30/08/2018	1.94	1.97	-0.03
79	C01C27157	80	30/08/2018	1.92	1.97	-0.05
80	C01C27157	80	30/08/2018	1.92	1.97	-0.05
81	C01C27157	80	30/08/2018	1.95	1.97	-0.02
82	C01C27157	80	30/08/2018	1.95	1.97	-0.02
83	C01C27157	80	30/08/2018	1.92	1.97	-0.05
84	C01C27157	80	30/08/2018	1.9	1.97	-0.07
85	C01C27157	80	30/08/2018	1.9	1.97	-0.07
86	C01C27157	80	30/08/2018	1.9	1.97	-0.07
87	C01C27157	80	30/08/2018	1.98	1.97	0.01
88	C01C27157	80	30/08/2018	1.93	1.97	-0.04
89	C01C27157	80	30/08/2018	1.91	1.97	-0.06
90	C01C27157	80	30/08/2018	1.91	1.97	-0.06
91	C01C27157	80	30/08/2018	1.91	1.97	-0.06
92	C07C27145	140	31/08/2018	1.91	1.97	-0.06
93	C01C27157	80	31/08/2018	1.95	1.97	-0.02
94	C01C27157	80	31/08/2018	1.93	1.97	-0.04
95	C01C27157	80	31/08/2018	1.94	1.97	-0.03
96	C01C27157	80	31/08/2018	1.91	1.97	-0.06
97	C01C27157	80	31/08/2018	1.99	1.97	0.02
98	C01C27157	80	31/08/2018	1.97	1.97	0
99	C01C27157	80	31/08/2018	1.97	1.97	0
100	C01C27157	80	31/08/2018	1.99	1.97	0.02

Table A.3: Moving Average Values for a Moving Period of 2

Subgroup	X-T	MA of 2	CL	UCL	LCL	Subgroup	X-T	MA of 2	CL	UCL	LCL
1	0.03	0.030	-0.024	0.080	-0.128	51	-0.02	-0.020	-0.024	0.050	-0.098
2	0.03	0.030	-0.024	0.050	-0.098	52	0.01	-0.005	-0.024	0.050	-0.098
3	-0.04	-0.005	-0.024	0.050	-0.098	53	0.02	0.015	-0.024	0.050	-0.098
4	-0.03	-0.035	-0.024	0.050	-0.098	54	0.03	0.025	-0.024	0.050	-0.098
5	-0.06	-0.045	-0.024	0.050	-0.098	55	0.03	0.030	-0.024	0.050	-0.098
6	-0.06	-0.060	-0.024	0.050	-0.098	56	0	0.015	-0.024	0.050	-0.098
7	-0.04	-0.050	-0.024	0.050	-0.098	57	-0.02	-0.010	-0.024	0.050	-0.098
8	0.03	-0.005	-0.024	0.050	-0.098	58	0	-0.010	-0.024	0.050	-0.098
9	-0.04	-0.005	-0.024	0.050	-0.098	59	0.01	0.005	-0.024	0.050	-0.098
10	-0.04	-0.040	-0.024	0.050	-0.098	60	-0.02	-0.005	-0.024	0.050	-0.098
11	0.03	-0.005	-0.024	0.050	-0.098	61	-0.04	-0.030	-0.024	0.050	-0.098
12	-0.04	-0.005	-0.024	0.050	-0.098	62	-0.07	-0.055	-0.024	0.050	-0.098
13	-0.03	-0.035	-0.024	0.050	-0.098	63	-0.07	-0.070	-0.024	0.050	-0.098
14	-0.06	-0.045	-0.024	0.050	-0.098	64	-0.03	-0.050	-0.024	0.050	-0.098
15	-0.06	-0.060	-0.024	0.050	-0.098	65	0.06	0.015	-0.024	0.050	-0.098
16	-0.05	-0.055	-0.024	0.050	-0.098	66	-0.04	0.010	-0.024	0.050	-0.098
17	-0.02	-0.035	-0.024	0.050	-0.098	67	-0.05	-0.045	-0.024	0.050	-0.098
18	-0.02	-0.020	-0.024	0.050	-0.098	68	-0.03	-0.040	-0.024	0.050	-0.098
19	-0.02	-0.020	-0.024	0.050	-0.098	69	-0.02	-0.025	-0.024	0.050	-0.098
20	-0.02	-0.020	-0.024	0.050	-0.098	70	-0.03	-0.025	-0.024	0.050	-0.098
21	-0.01	-0.015	-0.024	0.050	-0.098	71	-0.06	-0.045	-0.024	0.050	-0.098
22	-0.01	-0.010	-0.024	0.050	-0.098	72	0.01	-0.025	-0.024	0.050	-0.098
23	-0.01	-0.010	-0.024	0.050	-0.098	73	-0.07	-0.030	-0.024	0.050	-0.098
24	-0.01	-0.010	-0.024	0.050	-0.098	74	-0.07	-0.070	-0.024	0.050	-0.098
25	0.03	0.010	-0.024	0.050	-0.098	75	-0.07	-0.070	-0.024	0.050	-0.098
26	0	0.015	-0.024	0.050	-0.098	76	0.03	-0.020	-0.024	0.050	-0.098
27	0	0.000	-0.024	0.050	-0.098	77	-0.06	-0.015	-0.024	0.050	-0.098
28	0.02	0.010	-0.024	0.050	-0.098	78	-0.03	-0.045	-0.024	0.050	-0.098
29	-0.03	-0.005	-0.024	0.050	-0.098	79	-0.05	-0.040	-0.024	0.050	-0.098
30	0.02	-0.005	-0.024	0.050	-0.098	80	-0.05	-0.050	-0.024	0.050	-0.098
31	-0.01	0.005	-0.024	0.050	-0.098	81	-0.02	-0.035	-0.024	0.050	-0.098
32	0.03	0.010	-0.024	0.050	-0.098	82	-0.02	-0.020	-0.024	0.050	-0.098
33	-0.05	-0.010	-0.024	0.050	-0.098	83	-0.05	-0.035	-0.024	0.050	-0.098
34	-0.02	-0.035	-0.024	0.050	-0.098	84	-0.07	-0.060	-0.024	0.050	-0.098
35	-0.04	-0.030	-0.024	0.050	-0.098	85	-0.07	-0.070	-0.024	0.050	-0.098
36	-0.01	-0.025	-0.024	0.050	-0.098	86	-0.07	-0.070	-0.024	0.050	-0.098
37	-0.05	-0.030	-0.024	0.050	-0.098	87	0.01	-0.030	-0.024	0.050	-0.098
38	0	-0.025	-0.024	0.050	-0.098	88	-0.04	-0.015	-0.024	0.050	-0.098
39	-0.15	-0.075	-0.024	0.050	-0.098	89	-0.06	-0.050	-0.024	0.050	-0.098
40	-0.04	-0.095	-0.024	0.050	-0.098	90	-0.06	-0.060	-0.024	0.050	-0.098
41	0.04	0.000	-0.024	0.050	-0.098	91	-0.06	-0.060	-0.024	0.050	-0.098
42	-0.05	-0.005	-0.024	0.050	-0.098	92	-0.06	-0.060	-0.024	0.050	-0.098
43	-0.03	-0.040	-0.024	0.050	-0.098	93	-0.02	-0.040	-0.024	0.050	-0.098
44	-0.05	-0.040	-0.024	0.050	-0.098	94	-0.04	-0.030	-0.024	0.050	-0.098
45	0.03	-0.010	-0.024	0.050	-0.098	95	-0.03	-0.035	-0.024	0.050	-0.098
46	-0.05	-0.010	-0.024	0.050	-0.098	96	-0.06	-0.045	-0.024	0.050	-0.098
47	-0.05	-0.050	-0.024	0.050	-0.098	97	0.02	-0.020	-0.024	0.050	-0.098
48	-0.03	-0.040	-0.024	0.050	-0.098	98	0	0.010	-0.024	0.050	-0.098
49	0.02	-0.005	-0.024	0.050	-0.098	99	0	0.000	-0.024	0.050	-0.098
50	-0.02	0.000	-0.024	0.050	-0.098	100	0.02	0.010	-0.024	0.050	-0.098

Table A.4: Moving Average Values for a Moving Period of 5

Subgroup	X-T	MA of 5	CL	UCL	LCL
1	0.03	0.030	-0.024	0.080	-0.128
2	0.03	0.030	-0.024	0.050	-0.098
3	-0.04	0.007	-0.024	0.036	-0.084
4	-0.03	-0.003	-0.024	0.028	-0.076
5	-0.06	-0.014	-0.024	0.022	-0.071
6	-0.06	-0.032	-0.024	0.022	-0.071
7	-0.04	-0.046	-0.024	0.022	-0.071
8	0.03	-0.032	-0.024	0.022	-0.071
9	-0.04	-0.034	-0.024	0.022	-0.071
10	-0.04	-0.030	-0.024	0.022	-0.071
11	0.03	-0.012	-0.024	0.022	-0.071
12	-0.04	-0.012	-0.024	0.022	-0.071
13	-0.03	-0.024	-0.024	0.022	-0.071
14	-0.06	-0.028	-0.024	0.022	-0.071
15	-0.06	-0.032	-0.024	0.022	-0.071
16	-0.05	-0.048	-0.024	0.022	-0.071
17	-0.02	-0.044	-0.024	0.022	-0.071
18	-0.02	-0.042	-0.024	0.022	-0.071
19	-0.02	-0.034	-0.024	0.022	-0.071
20	-0.02	-0.026	-0.024	0.022	-0.071
21	-0.01	-0.018	-0.024	0.022	-0.071
22	-0.01	-0.016	-0.024	0.022	-0.071
23	-0.01	-0.014	-0.024	0.022	-0.071
24	-0.01	-0.012	-0.024	0.022	-0.071
25	0.03	-0.002	-0.024	0.022	-0.071
26	0	0.000	-0.024	0.022	-0.071
27	0	0.002	-0.024	0.022	-0.071
28	0.02	0.008	-0.024	0.022	-0.071
29	-0.03	0.004	-0.024	0.022	-0.071
30	0.02	0.002	-0.024	0.022	-0.071
31	-0.01	0.000	-0.024	0.022	-0.071
32	0.03	0.006	-0.024	0.022	-0.071
33	-0.05	-0.008	-0.024	0.022	-0.071
34	-0.02	-0.006	-0.024	0.022	-0.071
35	-0.04	-0.018	-0.024	0.022	-0.071
36	-0.01	-0.018	-0.024	0.022	-0.071
37	-0.05	-0.034	-0.024	0.022	-0.071
38	0	-0.024	-0.024	0.022	-0.071
39	-0.15	-0.050	-0.024	0.022	-0.071
40	-0.04	-0.050	-0.024	0.022	-0.071
41	0.04	-0.040	-0.024	0.022	-0.071
42	-0.05	-0.040	-0.024	0.022	-0.071
43	-0.03	-0.046	-0.024	0.022	-0.071
44	-0.05	-0.026	-0.024	0.022	-0.071
45	0.03	-0.012	-0.024	0.022	-0.071
46	-0.05	-0.030	-0.024	0.022	-0.071
47	-0.05	-0.030	-0.024	0.022	-0.071
48	-0.03	-0.030	-0.024	0.022	-0.071
49	0.02	-0.016	-0.024	0.022	-0.071
50	-0.02	-0.026	-0.024	0.022	-0.071

Subgroup	X-T	MA of 5	CL	UCL	LCL
51	-0.02	-0.020	-0.024	0.022	-0.071
52	0.01	-0.008	-0.024	0.022	-0.071
53	0.02	0.002	-0.024	0.022	-0.071
54	0.03	0.004	-0.024	0.022	-0.071
55	0.03	0.014	-0.024	0.022	-0.071
56	0	0.018	-0.024	0.022	-0.071
57	-0.02	0.012	-0.024	0.022	-0.071
58	0	0.008	-0.024	0.022	-0.071
59	0.01	0.004	-0.024	0.022	-0.071
60	-0.02	-0.006	-0.024	0.022	-0.071
61	-0.04	-0.014	-0.024	0.022	-0.071
62	-0.07	-0.024	-0.024	0.022	-0.071
63	-0.07	-0.038	-0.024	0.022	-0.071
64	-0.03	-0.046	-0.024	0.022	-0.071
65	0.06	-0.030	-0.024	0.022	-0.071
66	-0.04	-0.030	-0.024	0.022	-0.071
67	-0.05	-0.026	-0.024	0.022	-0.071
68	-0.03	-0.018	-0.024	0.022	-0.071
69	-0.02	-0.016	-0.024	0.022	-0.071
70	-0.03	-0.034	-0.024	0.022	-0.071
71	-0.06	-0.038	-0.024	0.022	-0.071
72	0.01	-0.026	-0.024	0.022	-0.071
73	-0.07	-0.034	-0.024	0.022	-0.071
74	-0.07	-0.044	-0.024	0.022	-0.071
75	-0.07	-0.052	-0.024	0.022	-0.071
76	0.03	-0.034	-0.024	0.022	-0.071
77	-0.06	-0.048	-0.024	0.022	-0.071
78	-0.03	-0.040	-0.024	0.022	-0.071
79	-0.05	-0.036	-0.024	0.022	-0.071
80	-0.05	-0.032	-0.024	0.022	-0.071
81	-0.02	-0.042	-0.024	0.022	-0.071
82	-0.02	-0.034	-0.024	0.022	-0.071
83	-0.05	-0.038	-0.024	0.022	-0.071
84	-0.07	-0.042	-0.024	0.022	-0.071
85	-0.07	-0.046	-0.024	0.022	-0.071
86	-0.07	-0.056	-0.024	0.022	-0.071
87	0.01	-0.050	-0.024	0.022	-0.071
88	-0.04	-0.048	-0.024	0.022	-0.071
89	-0.06	-0.046	-0.024	0.022	-0.071
90	-0.06	-0.044	-0.024	0.022	-0.071
91	-0.06	-0.042	-0.024	0.022	-0.071
92	-0.06	-0.056	-0.024	0.022	-0.071
93	-0.02	-0.052	-0.024	0.022	-0.071
94	-0.04	-0.048	-0.024	0.022	-0.071
95	-0.03	-0.042	-0.024	0.022	-0.071
96	-0.06	-0.042	-0.024	0.022	-0.071
97	0.02	-0.026	-0.024	0.022	-0.071
98	0	-0.022	-0.024	0.022	-0.071
99	0	-0.014	-0.024	0.022	-0.071
100	0.02	-0.004	-0.024	0.022	-0.071

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