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In the present thesis, dynamic risk management solutions are developed for the effective hedging of exchange rate risk with futures contracts in single and multiple currency positions. The theory of hedging and the motives for corporate hedging are presented, along with the existing evidence on foreign exchange risk management. Dynamic hedging models are derived, that account for the limitations of the related empirical studies. The portfolio effects on the hedging effectiveness of a model are studied empirically for a multicurrency hedging problem. A multivariate GARCH-X model is applied to the spot and futures exchange rate returns of four major currencies and dynamic hedge ratios are derived. The hedging efficiency of the dynamic portfolio hedge ratios, measured by both risk and utility performance, is supported in sample and out-of-sample. An additional empirical issue is the assumption of non-stochastic interest rates in the futures hedging problem, that is relaxed and, a dynamic model accounting for basis risk and the marking-to-market effect on the hedge ratio is estimated for the four major currencies. In-sample and out-of-sample comparisons of the basis risk dynamic model and the simple GARCH-X dynamic model reveal substantial gains in terms of utility and risk reduction for the hedger who accounts for basis risk in his combined spot-futures position. Finally, the cross hedging problem in the foreign exchange market is examined in a dynamic context for a spot portfolio of five EMS currencies with strong implications for the role of the German currency in the EMU and the hedging potential of spot positions in the Euro. In all cases, the evaluation of the proposed hedge ratios is supported by both ex post and ex ante measures of utility and risk.
Empirical Issues of Foreign Exchange Risk Management with Futures Contracts

by Sevasti D. Kaplanoglou

Submitted for a Doctor of Philosophy in Finance

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Department of Economics and Finance

UNIVERSITY OF DURHAM

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CHAPTER 1: Introduction

Exchange rate uncertainty plays an important role in the economics of capital markets since currency movements affect the returns on foreign investments and the monetary transactions of multinational corporations. Hedging currency risk with futures contracts results in substantial risk reduction in foreign portfolios and ensures that internal funding is available in the firm for value enhancing investments. However, the success of a risk management strategy depends on the correct specification of the optimal hedge ratio. The present study is motivated by the limitations of the existing empirical studies on hedging to account for the empirical regularities that are present in exchange rate and interest rate data, leading to uncovered risk and limited effectiveness of the proposed hedging solutions. The contribution of the present thesis to existing financial research lies in the development of dynamic foreign exchange risk management solutions under realistic scenarios adapted to the existing conditions in international financial markets. The empirical issues considered could help financial managers organize effective hedging programs that lead to higher utility for the investors and increasing value for a corporation.

The empirical questions answered in the present study are derived on the basis of their effect on the derivation of the optimal hedge ratio with currency futures. The bulk of evidence on currency risk management involves, with a few exceptions, the hedging problem of single spot currency positions. However, international investment creates exposures in more than one currency, thus leading to the problem of hedging multi-currency positions. This empirical issue is investigated in the present thesis in a dynamic portfolio context in order to detect differences in the size and the performance of the hedging demands derived when portfolio effects are accounted for in a hedging problem. The analysis extends the existing empirical studies by examining the effect of the transaction costs on the decision of rebalancing the hedged portfolio.
Second, the assumption of non-stochastic interest rates, followed by the majority of empirical studies on hedging, is not realistic in the presence of the high volatility of interest rate data. On the basis of the cost-of-carry futures pricing model, basis risk is due to uncertainty created by stochastic interest rates and the daily marking-to-market feature of futures contracts. A hedge ratio for foreign currency positions that ignores interest rate risk, may lead to unhedged risks and limited hedging effectiveness. In the present study, the effect of stochastic interest rates on the optimal currency hedge ratio is investigated for four major currencies, the British Pound, the German Mark, the Swiss Frank and the Japanese Yen in a dynamic context.

The increasing development of international trade and the international capital mobility create significant exchange rate risk not only for major but also for minor currencies, without active derivative markets. Extending the existing evidence into a dynamic portfolio framework, the present study examines the cross-hedging issue in the foreign exchange market from the perspective of an EMS currency portfolio. Additionally, the empirical regularities of the recently trading European exchange rate, the Euro, are studied for the development of the appropriate cross hedging model.

In Chapter 2, the hedging concept of the futures markets is analyzed both from an investor's as well as from a corporate standpoint. A brief review of the existing instruments for hedging is made as well as of the incentives for corporate risk management. The major theories of hedging are then presented including an analysis of their strengths and limitations. The optimal hedge ratio is defined and its relation to basis risk is examined. The traditional hedging theory is extended into a portfolio context and the assumption of non-stochastic interest rates is relaxed. The issue of cross hedging is also addressed for single and multiple currency positions. On the basis of this analysis, the empirical issues of the present thesis are raised and the review of the existing empirical evidence is presented.

The empirical issue of hedging multi-currency positions is addressed in Chapter 3,
where the limitations of the existing studies on portfolio hedging are considered in terms of the dynamic specification of foreign currency returns and the cointegration existing between spot and futures prices. A GARCH-X specification accounts for the effect of the disequilibrium in the previous period on both conditional moments of the exchange rate. *To the best of my knowledge, no previous study deals with this empirical issue in the hedging context.* The main point in this analysis is that different hedging demands are derived when portfolio effects are considered than when these effects are ignored. This finding has a further effect on the hedging performance of the two different strategies with the portfolio hedge ratios being more effective than their no-portfolio counterparts.

The effect of stochastic interest rates on the development of the optimal currency hedge ratio is the subject of *Chapter 4*. The intertemporal hedge ratio for basis risk and marking-to-market as derived by *Chang, Chang and Fang (1996)* is extended into a dynamic setting and estimated for the four major currencies. It is assumed that a default-free domestic discount bond is used as a hedge for the interest rate risk of the futures positions since the interest yield or paid on the margin account is expressed in terms of the domestic interest rate. *To the best of my knowledge, no previous effort has been made so far to model spot, futures exchange rates and interest rate series into a dynamic setting in order to jointly hedge exchange rate and interest rate risk.* In sample and out of sample comparisons of the basis risk and the traditional model are performed in terms of variance reduction and utility performance.

The cross hedging effectiveness of the BP, JY, DM and the SF futures contracts for a multi-currency EMS portfolio is the empirical issue investigated in *Chapter 5*. Additionally, the increasing power of the German Mark in the EMS countries is examined from the perspective of the hedging problem. *The model developed in this chapter is innovative both in terms of multi-currency hedging as well as a dynamic specification of minor currency returns for hedging purposes.* The conclusions to this analysis have strong implications for hedging European currency risk since the spot EMS portfolio can replicate the Euro as a basket of currencies. Additionally, a small dataset is collected for the recently trading Euro
and the hedging effectiveness of major futures contracts is compared with the Euro futures contract.

Finally, the thesis concludes with Chapter 6, where the empirical findings of the previous chapters are summarized with suggestions for further research.
CHAPTER 2: Foreign Exchange hedging with futures contracts: theoretical issues

Section A: Introduction

The increasing volatility of financial returns and the substantial growth of the derivatives markets are the major characteristics of international financial markets over the last two decades. Increased foreign investment motivated by the need for international diversification and combined with the extremely high volatility of exchange rates, has created significant foreign exchange risk for foreign currency denominated portfolios. The need of an international investor to hedge foreign currency risk is straightforward. Futures contracts are one of the most commonly used derivative instruments for hedging in the financial markets.

In Section 2.1 of the present chapter, the theoretical aspects of the futures markets are reported along with their use in the management of foreign exchange risk. In Section 2.2, the corporate incentives for hedging are analyzed in terms of the value of the corporation and the wealth of its shareholders. The existing empirical evidence on the corporate use of foreign exchange (FX) derivatives implies strong motivations for FX risk management. In Section 2.3, the traditional theories of hedging are presented, as introduced by Johnson (1960) and Ederington (1979), as well as more recent hedging theories based on the corporate definition of risk and the risk-return tradeoff. The limitations of the naïve method are reported as evidenced by existing empirical studies. The derivation of the optimal and the risk minimizing hedge ratio is presented as well as its relation to basis risk. However, it is shown that existing theories suffer from many limitations created by the time-varying distribution of exchange rates, the stochastic properties of interest rates as well as the degree of correlation between currencies and futures contracts. Empirical issues thus arise in the development of appropriate hedging models for the effective risk management of foreign currency positions.
First, hedging multi-currency positions creates portfolio effects arising from the covariance between spot currencies and futures returns, resulting in different demands for futures contracts relative to the case of a single currency position. The theoretical framework for the risk management of multi-currency portfolios as well as the existing empirical evidence are presented in Section 2.5 of the present chapter while the effectiveness of portfolio hedging is examined empirically in Chapter 3.

Second, in the presence of the substantial interest rate volatility, the assumption of non-stochastic interest rates is not realistic when hedging with futures contracts. In Section 2.4, the cost-of-carry futures pricing model is used in order to show that basis risk is due to uncertainty created by stochastic interest rates and the daily marking-to-market feature of futures contracts. The effect of interest rate risk on the optimal currency hedge ratio is discussed in the present chapter while the same issue is analyzed empirically in a dynamic framework in Chapter 4 of the present thesis.

Finally, the effectiveness of cross hedging minor currency risk with major currencies futures contracts depends primarily on the degree of correlation between major and minor currencies. In Section 2.6 of the present chapter, the review of the literature on cross hedging is presented as well as the basic limitations of existing empirical studies. On the basis of this analysis, a hedging model is developed in Chapter 5 for an EMS spot portfolio as well as a spot position in the single European currency.
Section B: Theoretical Analysis

(2.1) Foreign exchange risk management: definitions and instruments

*Currency risk* is defined as the variance in the value of future cash flows arising from currency fluctuations. In a *fixed exchange rate regime*, a *pegged currency* is one whose value is set by the government. In this case, currency risk is small since small deviations are allowed for the exchange rate from its stated par value. Alternatively, in a *floating exchange rate regime*, the equilibrium between supply and demand for currencies in the *foreign exchange market* determines the exchange rates as market clearing prices. Since the early 1970's, many exchange rates have been floating instead of fixed, exhibiting high volatility as most financial returns. The increasing exchange rate volatility, combined with substantial increases in the volume of international trade, has led to significant foreign currency risk for most international monetary transactions.

Most economic agents face exchange rate risk if they have foreign currency denominated cash flows. Large corporations are subject to foreign currency exposure arising from exports, imports of raw materials or capital equipment, or foreign borrowing or lending. Additionally, international investors face foreign exchange risk since the returns to their foreign investments depend not only on the risk associated with investing in the specific securities market but also on the risk of adverse price movements in the exchange rates. Currency movements affect returns on foreign denominated bonds and stocks both directly and indirectly. The *direct* influence is due to the fact that these returns must be translated from foreign to domestic currencies. The *indirect* effect is due to the reaction of asset prices to exchange rate adjustments. *Jorion (1994)* shows that significant under-performance in foreign portfolios is evident when their optimization is performed separately for the two effects in the form of “currency overlays”.

Foreign exchange *risk management* refers to actions that reduce the probability of bearing a monetary loss due to exchange rate changes. One way of reducing
currency risk is to create a well-diversified portfolio of currencies, so that the losses in one currency being offset by the gains in another currency. However, diversification eliminates only the unsystematic (diversifiable) risk of the portfolio while the market risk of the portfolio still exists and cannot be diversified away. On the contrary, hedging can reduce the systematic (undiversifiable) risk of the spot portfolio. Hedging means taking a position, such as acquiring a cash flow or an asset or a contract that will rise in value and offset a decrease in the value of the underlying position, therefore protecting the owner of the existing asset from loss. In this way, the maintenance of a well-diversified spot currency portfolio is ensured since any decreases in its value due to adverse exchange rate movements can be controlled with hedging and not with selling the spot currencies. However, a basic disadvantage of hedging is that it eliminates potential positive returns arising from a favourable change in the value of the spot position.

*Perold and Schulman (1988)* show that with a naïve currency hedge, U.S. investors can achieve a further risk reduction of similar magnitude as the reduction achieved by international diversification in the first place. The authors characterize currency hedging as a free lunch since, as it is on average a zero expected return strategy, it leads to risk reduction without cost. However, *Adler and Prasad (1992)* and *Jorion (1994)* show that a naïve hedge is optimal only when the foreign investment returns in domestic currency are uncorrelated with exchange rates and at the same time expected returns on foreign bonds are equal to zero. The former authors derive a universal currency hedge ratio that is defined as the demand by any investor of a proportion of a hedge portfolio, that contains positions in all foreign bonds and the domestic bond, per unit of his spot portfolio. This approach extends the theoretical work of *Black (1990)* in the sense that it relaxes the assumption of equal risk tolerances for all investors. Indeed, *Abken and Shrikhande (1997)* find that the degree of risk aversion of the individual investor plays a significant role in the development of the optimal hedge ratio. However, *Filatov and Rappoport (1992)* show that the optimal hedging policy is a function of the domestic currency of the investor and the composition of his portfolio. The results provided by *Glen and Jorion (1993)* and *Beltratti, Laurent and Zenios (1999)* support the latter
conclusion, ruling out the existence of a universal currency hedge ratio.

Additionally, it is worth noting that conflicting evidence is provided in the role of hedging in a currency crisis. According to Krueger (1996), the main source of capital outflows in a currency crisis is hedging and not speculation. Domestic investors with foreign debt and foreigners with local currency investments tend to close their open foreign exchange positions when confidence to the weak currency is reduced. As Garber and Spencer (1995, 1996) state, in a currency crisis, the sales of the weak currency are met by the central bank that defends the exchange rate regime with forward purchases of the currency. However, if its foreign exchange reserves are not sufficient for the defense of the domestic currency, the central bank raises the discount rate, thus preventing further sales by increasing the interest costs of the investors that are short in the domestic currency. However, the same authors recognize that dynamic hedging is a widely used strategy that can explain a large part of trading volume and price changes in the foreign exchange market. In this sense, being based on the option pricing formula of Garman and Kohlhagen (1983), they show that, under the assumption of constant exchange rate volatility, a rise in the interest rate can lead to further sales of the domestic currency through an increase in the dynamic hedge ratio. The final result would be the unsuccessful interest rate defense of a fixed exchange rate regime. However, as Krueger (1999) shows, in a currency crisis, volatility increases sharply, thus reducing the effect of the interest rate on the hedge ratio. Additionally, bid-ask spreads become extremely large making dynamic hedging strategies very costly. We can conclude that the usefulness of a foreign exchange risk management program is limited in the presence of a currency crisis. However, this is a minor result since a currency crisis is considered as an extreme event that should be accounted for separately. The substantial risk reduction, provided with hedging in periods of high exchange rate volatility, is evident in most empirical studies on currency risk management.

The derivative securities or contracts are used as hedging instruments in the sense that their payoff depends primarily on the price of the underlying asset. The foreign
exchange derivatives are contracts whose payoff and value is a function of the underlying exchange rate. An agent with a foreign currency exposure can hedge his spot position by taking an opposite position in the derivatives market. The payoff to the derivative is added to the cost or return of his spot position. The advantage of trading in the derivatives market lies in the difficulties existing in the cash market such as the low liquidity, high transaction costs, short-selling restrictions, execution problems, internal policy or government regulations and credit risk. High tradability and liquidity characterize derivative contracts, since large positions can be traded without affecting prices, as well as high leverage, since only a small cash down payment is sufficient for very large gains. They can easily substitute for a cash position, since the transaction costs are lower in the derivatives market and the orders can be executed more quickly. The most commonly used derivatives for hedging foreign currency risk are forwards, futures, options and swaps.

A foreign exchange forward contract is a private agreement made today to purchase or sell a pre-specified amount of foreign currency in exchange for domestic currency, on a future date and at a fixed exchange rate. A forward transaction has a unique time to maturity and it is used in order to hedge a cash flow in foreign currency for a short period of time. On the contrary, a foreign currency swap can be described as a series of forward contracts over multiple periods and it can be used in order to hedge foreign currency cash flows for longer horizons. Both contracts are traded in the over-the-counter (OTC) market in the sense that they are private agreements between banks and corporations and they are tailor-made to the specific needs of the client.

On the other hand, futures and options contracts are traded in organized derivatives exchanges that provide a central location where buyers and sellers make bids and offers for contracts with delivery in later months. A currency futures contract is a standardized agreement between a buyer and a seller, specifying a trade in the underlying spot currency for a given quantity at a specific time for an agreed exchange rate. Currency futures contracts are traded in the International Monetary Market (IMM) that is a division of the Chicago Mercantile Exchange and it was
founded in 1972. The currency futures traded on the IMM include Canadian Dollars, Australian Dollars, French Francs, Dutch Guilders, Mexican Pesos, British Pounds, Swiss Francs, German Marks and Japanese Yen. However, only the last four contracts are actively traded. Currency futures are also traded in London (London International Financial Futures Exchange) and Singapore. The delivery months of these contracts are March, June, September and December. The actual delivery of the underlying asset does not take place very often since, usually, market participants take offsetting positions before the expiry date of the contract. However, the possibility of delivery makes the spot and the futures price of the same asset move together with the futures price being equal to the spot price at the maturity of the futures contract. The equality of the two prices on the delivery date of the futures contract is due to arbitrage forces that exploit any profit opportunities by buying the cheap asset and selling the expensive one.

Although forwards and swaps are widely used in the foreign currency market, a rapid growth is observed in the futures market over the last decades. This is due to the fact that exchange traded contracts have standardized terms, concerning the underlying asset, the contract size, and the exact time and place of delivery. Except for standardization, futures contracts differ from forwards in the range of delivery dates they provide and the possibility of closing out a position before the maturity of the contract. Additionally, the moral hazard problem of OTC products is not present in the futures market due to the existence of the margin requirements. The buyer of the futures contract is required by the exchange to deposit cash, called initial margin, in a margin account as an insurance against adverse futures price movements. The daily marking-to-market refers to the adjustment of the margin account at the end of each trading day, for any gains or losses in the futures position. If the balance in the margin account falls below a certain amount, called maintenance margin, the owner of the futures contract receives a margin call i.e. he is required by the exchange to deposit additional funds in his margin account in order to cover the loss of his position. If he fails to do so, the broker closes out his position by selling the contract.
Futures contracts are zero-value investments since no up-front cost is required at their initiation. The margin requirements do not represent a real cost for the investor since, usually, he is allowed to earn interest on the balance of his margin account. The daily marking-to-market of futures contracts is, in effect, a daily settlement and it maintains their zero value at the end of the trading day. On the contrary, any gains or losses in a forward position are realized at the end of the life of the contract since forward contracts are settled at their maturity. A conclusion arising from the previous analysis is that the major difference between futures and forward contracts is the marking-to-market effect. Although no financing cost exists for the initial margin of futures contracts, any daily losses on the futures position must be paid or financed, resulting in an uncertain financing cost as a function of the variance of the futures price. In the following sections, it will be shown that the uncertainty arising from the marking-to-market effect of futures contracts depends on the uncertainty on the interest rate used to finance the futures margins. Additionally, any differences between futures and forward prices are explained by the correlation between interest rates and spot prices.

A currency option contract is a standardized agreement for the right but not the obligation to deliver (put option) or take delivery (call option) of an amount of foreign currency at a fixed exchange rate, the strike rate, in exchange for domestic currency. In this case, the owner of the option will buy (sell) the foreign currency only if the agreed exchange rate (strike price) is lower (higher) than the prevailing market rate. In other words, the currency option contract serves as an insurance against adverse currency movements while it still allows the option holder to benefit from favourable currency movements.

It is straightforward that the participant in the options market is subject to limited loss relative to a futures holder since the latter is obliged to realize the agreement before or at the expiration of the contract irrespective of the prevailing spot price of the underlying asset. However, the insurance provided by the option contract does not come at zero cost since an option premium must be paid in order to enter the option market. On the contrary, as it was stated in the previous paragraph, no up-
front cost is required at the initiation of a futures contract except for margin requirements. The choice between options and futures contracts depends on the needs of the specific investor and it is beyond the scope of the present study.
One of the major roles of management in financial and non-financial institutions is to maximize the aggregate net present value of the firm's cash flow. The firm's value increases with assets of positive net present value while it decreases with assets of negative net present value. Every firm is operating in two markets: the product market and the capital market. Product market decisions include decisions about prices, marketing, operating systems, labor costs, technology, quality, product distribution and strategic management. Capital market decisions provide the firm with the necessary financial instruments in order to increase the net present value of its aggregate cash flows. In both markets the firm faces currency risk i.e. the risk of exchange rate volatility that affects the firm's assets and liabilities denominated in foreign currencies. One way of coping with exchange rate risk is to avoid it by refusing risky transactions, like exports or foreign investments. However, bearing a certain amount of risk compensates the firm with significant revenues. Another way of facing risk is to try to hedge it with the use of derivative instruments. The substantial increase in the volume of trade in the derivatives markets indicates that corporations and financial institutions are the major participants. Since the issue of speculation, as a motive for the corporate use of derivatives, is ruled out by most empirical studies, it is worth examining the motives for corporate hedging.

According to the Modigliani and Miller theorem (1958), in perfect and complete capital markets, the overall firm value depends on the quality of the investments of the corporation that add value to the firm's operating assets, and is independent of the firm's capital structure - debt, equity or retained earnings. Assuming that exchange rate, interest rate or commodity price risk do not increase the unsystematic risk of a well-diversified portfolio, investors can themselves diversify away any unsystematic risks of the firm by holding a well-diversified portfolio. By hedging its diversifiable risks, the company incurs costs that decrease the value of the firm without creating any benefits for the shareholders. On the other hand, the

corporate hedging of the systematic risk just reallocates risk from the shareholders to the investors in the capital market for a risk premium. In this way, shareholders give up the expected return associated with their shares while the firm value is decreased by the transaction costs of hedging. Based on these assumptions, portfolio theory implies that corporate hedging is irrelevant in terms of the value of the company since the rate of return required by the investors is unaffected by any risk management strategies followed by the company.

According to Smith and Stulz (1985), Stulz (1996) and Hu (1996), a serious limitation of the Modigliani-Miller and the Capital Asset Pricing Model approach is that they ignore market imperfections such as costs of external funding, taxes, bankruptcy and agency costs that constitute the basic economic motives for corporate hedging. In order to undertake value-enhancing investments, the firms demand funds that are available through the equity and debt markets as well as by retained earnings. However, raising capital by issuing equity comes at a cost since it sometimes results in a stock price fall as investors may translate this action with the notion that the stock price is overvalued. On the other hand, debt financing is also costly since a high level of debt, rules out the possibility of raising funds later and increases the probability of financial distress. For these reasons, firms tend to cut down investment unless enough cash is generated internally by retained earnings. However, internal cash flows are sensitive to external risk factors such as exchange rates, commodity prices and interest rates. By bearing systematic and / or unsystematic risk, the company faces the probability of non-availability of internal funding needed for investments. Firms tend to lose strategic investment opportunities and bear underinvestment costs in the presence of foreign exchange or any other price risk (Bessembinder (1991)). According to Froot, Scharfstein and Stein (1994), the role of risk management is to protect the firm’s cash flows and earnings from the adverse price movements of external factors and allow for the availability of internal funds for value increasing projects.

The level of corporate hedging is a function of the sensitivity of the firm’s cash flows and investment decision to risk factors such as exchange rates. In the special
case of currency risk, the demand for derivatives is higher the more are the opportunities for foreign investment, due to the increased exposure of the firm to currency risk. According to Bartov and Bodnar (1994), the multinationality of a firm introduces higher foreign exchange exposure due to the foreign currency translation issue of its financial statements. A positive relationship between multinationality and the extent of currency hedging is evident in the survey-based study of Jesswein, Kwok and Folks (1995). The size of the firms examined does not seem to affect much the use of derivative products while the industry effect on currency hedging is more pronounced in the case of the finance, real estate and insurance industry. Goldberg et al (1998) conclude from firm disclosures that the use of foreign exchange derivatives is not limited to hedging foreign investments but also exports and all monetary transactions.

The significant effect of financial constraints on the decision of the firm to hedge its future capital expenditures is evident in the empirical study of Adam (1998). He reports that firms with lower liquidity and less diversification in sales tend to hedge more than firms without these characteristics do. The latter author derives optimal hedging portfolios in the context of matching cash inflows and outflows so that external funding is not necessary for future investments. A negative relationship between corporate hedging and liquidity is also evident in the empirical studies of Gay and Nam (1997), Geczy, Minton and Schrand (1997), Nance, Smith and Smithson (1993), Ahmed, Beatty and Takeda (1997), Mian (1996), Goldberg et. al (1998), Allayannis and Ofek (1998) and Howton and Perfect (1998). Similarly, a negative relationship between capitalization and hedging is found by Cummins, Phillips and Smith (1998), showing that firms with enough internal funds have no reason to hedge. Relevant implications can be derived by the positive relationship between the use of currency derivatives and growth opportunities, size, foreign exchange exposure and financial constraints evident in the previous empirical studies as well as in the articles of Goldberg et al (1998) and Graham and Rogers (1999). The former authors also report important economies of scale in currency hedging referring to the use of additional types of derivatives by the corporation. Haushalter (1999) supports this finding by showing that economies of scale in
hedging costs have a positive effect on the hedging decision.

The substantial value added to the firm by hedging is not limited to the expansion of investment opportunities. In incomplete and imperfect markets, taxes and contracting costs, as well as the costs of financial distress are present, thus affecting the availability of internal funding. According to Smith and Stulz (1985) and Stulz (1996), by reducing the variability of cash flows and earnings relative to external risk factors, a credible risk management strategy increases the availability of external borrowing (debt capacity), thus reducing the probability of financial distress and the expected costs of bankruptcy. Since expectations of bankruptcy reduce the market value of the firm, this cost reduction increases the expected value of the claims owned by the bondholders and the shareholders. Ross (1997), Leland (1998) and Graham and Rogers (1999) provide evidence that hedging increases debt capacity, leading to increased firm value. The latter authors also reveal a strong relationship between corporate hedging and financial distress costs as well as leverage. Similar results are obtained by Dolde (1996), Haushalter (1999), Berkman and Bradbury (1996), Gay and Nam (1999), Visvanathan (1995), Howton and Perfect (1998) and Graham and Rogers (1999). Likewise, Adam (1998) detects a positive relationship between a firm's debt ratio and the extent of corporate hedging. However, no significant results are obtained in the study of Allayannis and Ofek (1997) for the expected costs of bankruptcy. Similarly, no support for the general hypothesis of the relation between the firm's capital structure and risk management is provided by Mian (1996), Nance, Smith and Smithson (1993), Geczy, Minton and Schrand (1997) and Gunther and Siems (1995) with the exception of Sinkey and Carter (1994). Under the assumption that the expected costs of bankruptcy are the basic motive for hedging, Cooper and Mello (1999) examine the effect of credit risk on the decision of corporate hedging. In this context, firms with high default risk avoid hedging since the default spread of the hedging contract increases the liabilities of the firm thus keeping the probability of bankruptcy unaffected.

A significant motive for corporate hedging arises from the presence of contracting
costs that affect the value of the firm. These costs include the salaries and wages of
the management and the employees respectively, as well as payments made to
suppliers and trade creditors. All these non-investor groups demand higher
compensation and tighter contract terms the higher is the risk of the company. To
the extent that hedging can reduce the investment risk of these groups, it directly
enhances the firm value. Additionally, if the manager’s compensation depends on
changes in the firm’s value, risk-averse managers use risk management in order to
reduce the risk of unsuccessful results for the firm, thus reducing their employment
risk. Supportive evidence on the managerial risk aversion as a motive for corporate
hedging is provided by Tufano (1996) and Schrand and Unal (1998) who found a
negative relation between managerial options holdings and commodity hedging and
a positive relation between stock holdings and hedging. The former relation can be
explained by the fact that the probability of the exercise of an option is higher the
higher is the volatility of the underlying stock price. On the contrary, Geczy,
Minton and Schrand (1997), Haushalter (1997), Alayannis and Ofek (1997) and
Ahmed, Beatty and Takeda (1997) report lack of evidence on the effect of
managerial risk aversion on the hedging decision. Similarly, mixed evidence on the
effect of contracting costs on corporate hedging is provided by Mian (1996) and
Graham and Rogers (1999).

Smith and Stulz (1985) and Stulz (1996) also state that the effect of hedging on the
corporate tax liability is important if there is a positive relationship between the
marginal corporate tax rates and the firm’s pre-tax value. By reducing the
variability of the firm’s pre-tax value, hedging reduces the tax liability thus
increasing the expected post-tax value of the firm under the assumption of a
relatively low cost of hedging. In the case of tax shields of debt, by smoothing the
firm’s pretax income, hedging allows the levered firm to take full advantage of the
deductions of interest from its income. The importance of the tax benefits of
hedging is evident in the simulation study of Graham and Smith (1996). They show
that, for half the firms they examined, a one per cent decrease in the volatility of
taxable income, achieved with hedging, leads to a one per cent decrease in the
present value of taxes in the same year. Graham and Smith (1999) report extreme
cases of a reduction of forty per cent of the expected tax liabilities, achieved with corporate hedging. A significant relationship between the hedging decision and the tax function is also evident in the empirical studies of Nance, Smith and Smithson (1993), Howton and Perfect (1998), Ahmed, Beatty and Takeda (1997) and Cummins, Phillips and Smith (1998). Mixed evidence on the effect of taxes on corporate hedging is provided by Mian (1996) and Geczy, Minton and Schrand (1997). Finally, no direct support for the tax hypothesis is provided by Graham and Rogers (1999) although evidence in favour of the tax benefits from increased debt capacity through hedging is found.

DeMarzo and Duffie (1991) and Breeden and Viswanathan (1998) give an important incentive for corporate hedging in terms of the informational asymmetry existing in equity markets. The former authors developed a model in which managers have private information about a risk factor that cannot be detected by equity holders and introduces noise in their information set with respect to the firm’s expected payoffs. The corporate hedging of this exposure reduces the variability of the firm’s cash flows and the informational asymmetry of the shareholders, making them able to form an optimum portfolio. However, no significant results are obtained by Geczy, Minton and Schrand (1997) and Graham and Rogers (1999) in a study on the effect of hedging on informational asymmetries.

Finally, a recent study of Allayannis and Weston (1998) directly tests whether corporate hedging affects firm value, providing positive results for multinational firms that use currency derivatives. The implication of the previous studies for the present thesis is that the increased investment opportunities in foreign countries and the multinationality of large corporations, as well as several imperfections of the financial markets, introduce the necessity of foreign exchange risk management. By increasing the value of the firm, hedging provides an important strategy that must be followed by financial managers in both financial and non-financial firms.
One basic operation of the futures market is to enable market participants to transfer the risks they face from adverse spot price changes. Futures contracts serve as a hedge when a position is taken in the futures market that is opposite to the existing or anticipated cash position. A short hedge is the action of selling a futures contract when a long position is taken in the spot market while a long hedge involves buying futures contracts in order to reduce the risk of a short position in the spot market. Although hedging with futures can reduce or eliminate the risk of a cash position, it also reduces the possibility of a gain from a favourable move in the price of the spot position. The number of futures contracts that must be bought or sold in order to reduce the risk of a cash position is called hedge ratio. The choice of the optimal hedge ratio for single and multiple currency positions is the subject of the present empirical study.

In the present section, the major theories of hedging will be presented, involving the derivation of the optimal hedge ratio. There are three major theories of hedging using futures markets: (1) the traditional or pure risk-minimization approach, (2) Working's hypothesis or profit-motivated approach (1953) and (3) the portfolio approach (Johnson (1960), Ederington (1979)). However, there are also two more recent hedging theories, the a-t model, that is a corporate approach to the derivation of the optimal hedge ratio, and the Sharpe ratio model that is based on a risk-return framework.

(2.3.1) Traditional or naïve approach: definitions and limitations

The traditional hedging theory considers hedgers as pure risk-minimizers who take futures market positions of equal magnitude but opposite sign to their spot asset positions. This strategy is known as naïve or one-to-one hedging strategy. The principal value of the position in the futures market is equal to the principal value
of the position in the spot market. If there is perfect correlation between the spot and the futures price, the risk of the spot position can be totally eliminated and a *perfect hedge* can be accomplished. This is the case when the spot asset and the asset underlying the futures contract are identical and the gain (loss) on the futures position exactly offsets the loss (gain) on the cash position. *Hauser, Garcia and Tumblin (1982)* and *Brown (1985, 1986)* defend the naïve hedging strategy on the basis of its simplicity and effectiveness when the underlying asset of the contract is similar to the spot asset. However, if the two assets are different, the hedge is called a *cross hedge* and total elimination of the risk cannot be accomplished. In this case, the hedger is left with residual (or cross hedge) risk. The choice of the optimal hedging instrument depends on the correlation of the spot asset with the futures contract. The higher is the correlation, the more effective will be the futures contract.

By distinguishing futures market participants into hedgers and speculators, the *traditional hedging theory* has important implications for the derivation of futures prices. The *hedging pressure theory* is based on this distinction and its effect, through the supply and demand forces, on the price of futures contracts. The hedger is considered as an unsophisticated participant in the futures market who is willing to pay a risk premium in order to transfer the price risk inherent in his given cash position. On the contrary, a speculator is willing to enter the futures market only with the expectation to collect a premium.

We assume that hedgers take short positions in the futures market and speculators are long and net short hedging exceeds net long speculation (points A and B in Figure 2.1). The current futures price must fall below the expected futures price by the amount of a risk premium in order to compensate speculators for buying additional futures contracts. If the expected futures price is equal to the current spot price, the current futures price must fall below the current spot price by the amount of the risk premium. This is the *theory of normal backwardation* of *J.M Keynes (1923)* and *Hicks (1946)* according to which, the futures price sells always at a discount relative to the spot price.
On the contrary, an assumption is made that hedgers take, on average, long positions in the futures market and speculators are short and net long hedging exceeds net short speculation (points C and D in Figure 2.1). In this case, futures contracts must be overpriced relative to their expected value in order to compensate speculators for selling additional futures contracts. This concept of futures pricing is known as the contango theory and it causes the current futures prices to sell at a premium relative to the current spot price. Finally, the unbiased expectations hypothesis implies that the current futures price is an unbiased predictor of the future spot price. In this case, no risk premium is expected from holding futures contracts and futures prices follow martingales.

Figure 2.1: The effect of net open interest on the futures price.
(Source: Daigler, 1985, p.9).

A serious limitation of the naïve hedging strategy is that it ignores that since futures prices reflect market expectations, they would change proportionally but not equally to the spot price movement. The naïve method cannot lead to a perfect hedge even in the case of direct hedging i.e. when the spot asset and the underlying
asset of the futures contract are the same. This result is due to the failure of the naïve hedge ratio to account for changes in the basis of the spot-futures position, with the basis being equal to the difference between the spot and the futures price over the life of the futures contract.

Basis = Spot price – Futures price

If the underlying asset of the futures contract and the asset to be hedged are the same, the basis is equal to zero at the expiration of the futures contract since, due to arbitrage forces, the futures price converges to the spot price at delivery. Over the hedging period, the basis can be negative or positive, depending on the relation between spot and futures prices. The variance of the basis is called basis risk and it forms a significant part of uncertainty for hedged positions. When the spot price increases more than the futures price, the basis increases and the opposite happens when the futures price increases more than the spot price. This finding implies that a negative relationship exists between changes in the basis and futures price changes.

As the time to expiration of the futures contract increases, the variance of the basis becomes larger. An investor can totally eliminate basis risk if he sells futures contracts that expire at the end of his investment horizon. However, according to Netz (1996) the importance of basis risk is explained by the fact that less than the 3% of futures contracts are offset through delivery. The hedger can eliminate a portion of basis risk by using nearby contracts, although significant transaction costs are incurred since the hedge must be rolled over to the next contract when the current contract reaches expiration.

The following example shows that any unexpected changes in the basis affect the riskiness of the hedge.

An investor who intends to sell an asset at time 2, faces the risk of a decrease in the price $S$ from $t_1$ to $t_2$:
\[ \Delta S = S_{t_1} - S_{t_2} \]

We assume that the investor decides to hedge his spot position by taking a short position in the futures market at a price \( F_{t_1} \). At time 2, before the contract expires, he sells the asset for \( S_{t_2} \) and he closes his futures position with a gain \( F_{t_1} - F_{t_2} \). His total inflow from the combined spot and futures position is therefore

\[ S_{t_2} + F_{t_1} - F_{t_2} = F_{t_1} + B_{t_2} \]

where \( B_{t_2} \) is the basis at time 2.

If the hedger knew with certainty at time 1 what the basis \( B_{t_2} \) would be at time 2, he could have designed a perfect hedge. This is the assumption of the traditional hedging theory, leading to the naïve hedging strategy. However, \( B_{t_2} \) depends on the relation between the spot and the futures price at time 2 and forms the source of uncertainty for the hedger at time 1, when he undertakes the futures position. In other words, the hedger’s profit depends on the variance of the basis over the life of the hedge.

The variation in the basis can be explained by the new information arriving in the market between the initiation of the hedge and the time the hedge is lifted. Ederington (1979) shows that, in the case of Treasury bills, the hedged portfolio with the naïve strategy is more risky than the unhedged. Moser and Helms (1990) attribute basis risk to a nonstationary variance-covariance matrix of spot and futures returns that introduces the need of adjusting the hedge ratio. The conclusion from the present section is that, in the presence of basis risk, the naïve hedging strategy is unsuccessful in reducing the risk of a cash position. A hedge ratio that accounts for the relationship of spot and futures returns over the hedging period becomes necessary for successful risk management solutions.
(2.3.2) Working's Arbitrage Theory or profit-motivated approach

Working (1953) criticized the traditional hedging theory on the basis of the assumption of risk reduction as the basic motive for hedging and the limited hedging effectiveness in view of basis risk. In his theory, hedgers do not aim to transfer the risk of a given cash position, but they enter both spot and futures markets in order to make a profit from the relative price movements in the spot asset and the futures contract. In other words, a hedger's motive is to speculate on the basis. In this context, there is no distinction between a hedger and a speculator since they both aim to maximize their profit.

Castelino (1992) gives a good description of Working's hypothesis in terms of basis risk and under the assumption of unbiasedness in the futures market. He assumes that a hedger is taking a position in the spot market at time 1 at a price \( S_{t_1} \) and, at the same time, a position in the futures market at a price \( F_{t_1} \) for delivery at time 3. The hedging period is assumed to end at time 2 (i.e. before the maturity of the futures contract) and the prices \( S_{t_2} \) and \( F_{t_2} \) are assumed for the spot and futures positions respectively. If \( b \) is the portion of the spot position hedged, at time 2, the return on the hedged position, \( R_H \), will be:

\[
R_H = (S_{t_2} - S_{t_1}) - b(F_{t_2} - F_{t_1}) \tag{2.1}
\]

Adding and subtracting the terms \( F_1 \) and \( F_2 \), equation (2.1) becomes:

\[
R_H = (S_{t_2} - F_{t_2}) - (S_{t_1} - F_{t_1}) + (1 - b)(F_{t_2} - F_{t_1}) \tag{2.2}
\]

Rewriting the return, \( R_H \), in terms of the basis, \( B \), (2.2) becomes equal to:

\[
R_H = (B_{t_2} - B_{t_1}) + (1 - b)(F_{t_2} - F_{t_1}) \tag{2.3}
\]

Taking expectations, the expected return on the hedged position \( E(R_H) \) is equal to:
E(R_H) = [E(B_t) - B_{t_1}] + (1 - b)[E(F_t) - F_{t_1}] \quad (2.4)

and the variance of the return R_H is equal to:

\[ \text{var}(R_H) = \text{var}(B_t) + \{1 - b\}^2 \left[ \text{var}(F_t) \right] + \{2(1 - b) \text{cov}[B_t, F_t] \} \] \quad (2.5)

Assuming unbiasedness for the futures markets (F_1 = E(F_2)), as the theory of Working predicts, the second term in equation (2.4) is set equal to zero and the expected return on the hedged position is only a function of the expected movements in the basis:

\[ E(R_H) = [E(B_t) - B_{t_1}] \quad (2.6) \]

According to Working (1953), the result of this strategy would be either to hedge 100% of the asset position or not hedge at all, the positions in the spot and futures markets depending on expectations on the movements of the basis. The hedger is \textit{speculating on the basis} since the decision of hedging depends on the forecast a hedger can make for the basis at time 2. If the basis at time 2 represents an active profit opportunity compared with the basis at time 1, a full hedge will be applied. Otherwise, no hedging takes place. Under a full hedge (b=1), the variance of the return R_H in equation (2.5) is equal to the variance of the basis at time 2:

\[ \text{var}(R_H) = \text{var}(B_{t_2}) \quad (2.7) \]

In other words, the hedger faces basis risk, unless the hedge is lifted at the expiration of the futures contract, at time 3. In the latter case, the hedge is riskless since the basis is equal to zero at the expiration of the futures contract and the hedger can realize a return equal to the basis at time 1, i.e. at the initiation of the hedge.
Johnson (1960) criticizes this approach on the basis that, in contrast to a speculator, a hedger is defined as an investor with a given position in the spot market. Additionally, Brown (1985) reports the lack of practicality in estimating hedge ratios on the basis of price expectations. As Johnson (1960) states, the basic motive for hedging is the risk reduction of a given position in the spot market. However, the distinction between a hedger and a speculator is not very clear since usually hedgers adjust their cash positions according to their expectations of absolute spot price changes. The decision to hedge thus depends on their expectations of future spot price movements, casting doubt on the assertion of the traditional theory that hedgers are pure risk minimizers. The main conclusion is that both theories suffer from limitations and that a theory combining profit motivation and risk reduction should be derived.

(2.3.3) The portfolio approach: a combination of theories

On the basis of the limitations of the previous theories, a reformulation of the hedging theory is attempted by Johnson (1960) who defines hedging as taking a position in the futures market of a certain size so that the risk of the combined spot and futures market position is minimized. The main difference of the new theory from Working’s hypothesis is that a given spot market position is assumed before the hedge is undertaken. According to Johnson (1960), the primary spot market, in which a hedger is operating before taking a position in the futures market, is what distinguishes him from a speculator.

In order to combine the risk reduction approach of the traditional theory with the profit maximization hypothesis, the portfolio theory is applied for the derivation of the optimal hedge ratio. The optimal hedge ratio is derived as the one that maximizes the expected utility of the specific hedger and incorporates price change expectations. Suppose that there is an investor with a long position of one unit in the spot market and a short position of -b units in the futures market. If \( E(R_F) \) is the return on the futures position and \( E(R_S) \) is the return on the spot position, the expected return on the combined spot-futures position \( E(R_H) \) is equal to:
\[ E(R_H) = E(R_S) - b \cdot E(R_F) \quad (2.8) \]

Assuming that the investor has a mean-variance utility function,

\[ EU(R_H) = E(R_H) - \frac{1}{2} \gamma \cdot \text{var}(R_H) \quad (2.9) \]

with \( \gamma \) the degree of risk aversion (\( \gamma > 0 \)), his problem is to find the optimum number of futures contracts \( b_{\text{optimal}} \), that will maximize his expected utility, \( EU(R_H) \):

\[
\max_b EU(R_H) = \max_b \{ E(R_S) - bE(R_F) - \frac{1}{2} \gamma [\text{var}(R_S) + b^2 \text{var}(R_F) - 2b \text{cov}(R_S, R_F)] \}
\]

In order to solve the optimization problem, the first derivative of \( EU(R_H) \) is taken with respect to \( b \):

\[
\frac{\partial EU(R_H)}{\partial b} = -E(R_F) - \frac{1}{2} \gamma [2b \text{var}(R_F) - 2 \text{cov}(R_S, R_F)] \quad (2.10)
\]

By setting equation (2.10) equal to zero, we obtain the first solution for the maximization problem. Solving for \( b \), we obtain the optimal number of futures contracts, which is equal to

\[
b_{\text{optimal}} = \frac{-E(R_F)}{\gamma} + \frac{\text{cov}(R_S, R_F)}{\text{var}(R_F)} \quad (2.11)
\]

where \( \text{cov}(R_S, R_F) \) and \( \text{var}(R_F) \) are the covariance between spot and futures returns and the variance of futures returns respectively.

The optimal hedge ratio is the ratio of the futures position to the spot position that provides the best risk-return trade-off for the hedger. Since the hedge ratio depends on expectations about future price changes and the degree of risk aversion, it is
investor specific and requires the indifference curves of the investor with respect to risk and return. In figure 2.2, each indifference curve (I₁, I₂, and I₃) represents risk-return combinations for which the hedger is indifferent since they offer the same level of utility. The utility of the investor is maximized at the highest indifference curve (I₃) where the highest return is gained for the same level of risk. However, the efficient set of the hedger, described by the curve labelled AB, imposes a limitation on the maximum return the investor can achieve at several levels of risk. The optimal combination of the spot and futures position (b_{optimal}), that maximizes the utility of the investor subject to his efficient set, is represented by the tangent point of the highest indifference curve (I₂) with the efficient set (point O). If the investor wants to achieve the highest possible level of return, he can decide not to hedge at all (b = 0).

The equation (2.11) that represents the optimal hedge ratio can be decomposed into two components: a speculative component and a risk-minimizing component. This is shown by Anderson and Danthine (1981) and applied later by Briys and Solnik (1992) and Tong (1996). The speculative term depends on any bias in the futures price and it is investor-specific since it requires the degree of risk aversion of the investor as well as his expectation with respect to the return of the futures position. The second component depends on the covariance of spot and futures returns and the variance of futures returns and it is usually referred to as a pure hedge.
**Figure 2.2:** Risk-return trade-off in the hedge portfolio

(Source: Daigler (1994, p.188)
Briys, Crouhy and Schlesinger (1990) provide some interesting comments on the effect of different values of the speculative component on the size of the hedge ratio, assuming that perfect correlation exists between spot and futures returns. If the futures prices follow martingales, then \( \mathbb{E}(R_f) = 0 \), and the speculative component disappears from the optimal hedge ratio (see equation (2.11)). In this case, a perfect hedge can be applied to the unhedged position. In the case of normal backwardation, where a positive risk premium, \( \mathbb{E}(R_f) > 0 \), is expected by speculators in order to enter the futures market, less than full coverage is possible since the speculative component is now negative. However, the higher the risk aversion (\( \gamma \)) is, the closer to the full hedge is the optimal hedge ratio. In the case of contango, \( \mathbb{E}(R_f) < 0 \), more than full coverage is optimal since the speculative component is now positive. According to Gardner and Wuilloud (1995), the negative relationship between the expected return on the futures price and the size of the hedge ratio can be explained by the fact that the higher (lower) is the expected return, the higher (lower) will be the return from a high foreign exchange exposure. A lower (higher) hedge ratio thus seems more adequate in this case. In a later study Briys, Crouhy and Schlesinger (1993) prove that, independently of the evolution of the futures price, basis risk tends to eliminate the speculative part of the optimal hedge ratio, making it equal to the risk minimizing hedge ratio.

The optimal hedge ratio is difficult to be applied in practice since it requires the indifference curves of the individual hedger and reliable predictions of price changes over the life of the hedge. Gardner and Wuilloud (1995) show that for short hedging horizons there is a high probability of underperformance of the optimal hedge ratio versus a simple hedging strategy, for example a hedge ratio of 50% of the spot position. Additionally, there is a debate on the existence of significant bias in the futures market that is subject to the model and the methodology used. According to Gardner and Stone (1995), using the sample mean, as an estimate of the expected return, is a method of low precision, leading to useless risk management solutions.

Significant evidence of a risk premium in the currency futures market is provided
by many studies applying the Capital Asset Pricing Model (CAPM) in currency futures returns (Bessembinder (1992), Bessembinder and Chan (1992), McCurdy and Morgan (1992)). On the other hand, supportive evidence of the random walk hypothesis in the futures prices is provided by Carter, Rausser and Schmitz (1983), Raynaud and Tessier (1984), Beck (1987) and Pan, and Chan and Fok (1997). However, since developing the appropriate risk premium model for currency futures is beyond the scope of the present study, an assumption can be made that a large number of speculators operating in the futures market are risk neutral so that, in equilibrium, the profit in a futures position is zero. The assumption of risk neutrality implies that the hedging model developed in the present thesis cannot be tested in a risk-return framework but only from a risk reduction perspective.

Additionally, Castelino (1992) argues that the risk-return trade-off inherent in the portfolio theory of hedging focuses on price forecasting rather than basis risk, that seems irrelevant since the basic motivation for hedging is risk management. The latter author shows that in the case of unbiased futures markets, the choice of the hedge ratio does not affect the return on the hedged position (see equation (2.8)). For this reason, a risk-minimizing hedge seems more appropriate since the return is unaffected. The author also suggests a combination of the two theories of profit motivation and risk reduction, by first examining the basis for any profitable opportunities and only in the case of favourable results apply the risk minimizing hedge ratio. As Hartzmark (1987) states, even large commercial firms aim to minimize their risk rather than speculate on expected price changes. Additionally, Kahl (1986) suggests that the “full” portfolio model is not necessary in practice since hedgers either speculate on the basis or aim to minimize their risk in a zero return strategy.

(2.3.4) The risk-minimizing hedge ratio

Under the assumption of unbiasedness in the futures market, the optimal hedge ratio (2.11) becomes equal to the risk-minimizing hedge ratio:
\[ b^* = \frac{\text{cov}(R_s, R_F)}{\text{var}(R_F)} - \frac{\rho_{SF}\sigma_s}{\sigma_F} \]  \hspace{1cm} (2.12)  

Where \( \rho_{SF} \) is the correlation between spot and futures returns

\( \sigma_s \) is the standard deviation of spot returns

\( \sigma_F \) is the standard deviation of futures returns

It is obvious from equation (2.12) that the risk minimizing hedge ratio depends on the correlation between spot and futures prices as well as the standard deviation of the spot and futures price changes. The higher the correlation between the two assets, the more effective will be the risk minimizing hedge ratio in reducing the risk of the unhedged position. If there is perfect correlation between the spot asset and the futures contract, the hedge ratio is equal to one and a perfect hedge can be accomplished. In figure 2.2, the risk minimizing portfolio of spot and futures contracts is represented by the point \( b^* \) on the curve AB.

The risk minimizing hedge ratio is used by the majority of empirical studies on hedging due to its computational simplicity and the fact that it represents the basic motive for hedging: risk reduction. It is more practical since it can be estimated as the slope coefficient in the regression of the futures price change on the spot price change.

\[ R_s = a + b^* R_F + u_t \]  \hspace{1cm} (2.13)

Ederington (1979) introduced the following measure of hedging effectiveness as the percentage reduction of the variance of the spot position.

\[ HE_{\text{risk min}} = 1 - \frac{\text{var}(R_H)}{\text{var}(R_S)} \]  \hspace{1cm} (2.14)

where
HE_{risk \min} \text{ is the measure of hedging effectiveness based on risk reduction only}

\text{Var} \ (R_H) \text{ is the variance of the return on the hedged portfolio and,}

\text{Var} \ (R_S) \text{ is the variance of the return on the unhedged position}

The hedging effectiveness of the risk minimizing hedge ratio can easily be computed as the coefficient of determination \( R^2 \) of regression (2.13) that describes the good fit of the model i.e. the portion of the spot price change that can be explained by the change in the futures price. The residuals, ut, from this regression correspond to basis risk that cannot be hedged.

The empirical application of the risk minimizing hedge ratio to the GNMA and T-bills futures markets by Ederington (1979) indicates that, in most cases, the estimated hedge ratio is less than one. This finding supports the initial criticism of the traditional approach that the variation in the basis leads to a risk-minimizing hedge different than 100%. Additionally, hedge ratios are different for different assets because of basis risk. For example, Ederington (1979) estimates a risk minimizing hedge ratio of 31% for 90-day T-bills and a 92% for corn. As basis risk increases, i.e. as the correlation between spot and futures prices falls, the hedge ratio falls as well. Similarly, in the case of currency futures, Hill and Schneeweis (1981) estimate risk minimizing hedge ratios significantly different from one. On the contrary, higher hedge ratios and hedging effectiveness are estimated by Dale (1981) for the same dataset. However, his results are criticized by the former authors on the basis of the use of price levels instead of returns that introduces biases in the estimated hedge ratios and it overstates hedging effectiveness. The use of price levels is ruled out in studies of hedging since, in general, financial prices are nonstationary processes and they have to be first differenced for the removal of any stochastic trends. This empirical issue is examined in detail in the third chapter of the present thesis.

Significant deviations of the risk minimizing hedge ratios from the full hedge are also reported by Kamara and Siegel (1987), Cecchetti, Cumby and Figlewski (1988), Castelino, Francis and Wolf (1991), Hsin, Kuo and Lee (1994) and
Parhizgari and Chunhachinda (1996). DeJong, DeRoon and Veld (1997) perform tests of the hypothesis that $b = 1$ for hedge ratios derived from three different utility models for currency futures and they reject the null hypothesis for all three models, concluding that a hedge ratio lower than one should be applied in all cases.

Hill and Schneeweiss (1982) report substantial risk reduction from the application of the naïve and the risk minimizing hedge ratio to British Pound and German Mark currency futures. However, the latter method is found to be more effective in terms of variance reduction of the unhedged position. Similar results are provided by Naidu and Shin (1980) who show that a risk reduction of 70% - 90% can be achieved with the risk minimizing hedge ratio for two out of the three currencies examined. Significant variance reduction from the application of the risk minimizing hedge ratio in an out-of-sample analysis is reported by Malliaris and Urrutia (1991), Benet (1992) and Geppert (1995). Kroner and Sultan (1991) report that the naïve hedge is the least effective hedging strategy for the Japanese Yen in terms of variance reduction both in the in sample as well as in the out-of-sample comparisons. In a later empirical study (1993) including five currency spot positions, the latter authors obtain the same results. In an empirical study on the hedging performance of stock index futures, Figlewski (1984) and Holmes (1996) find that the beta hedge ratio, i.e. a naïve hedge ratio equal to the stock index portfolio beta, results in higher variance than the OLS hedge.

The substantial performance of the risk minimizing hedge ratio over the naïve hedge ratio is also evident in cross hedging studies. Using the Sharpe Performance Index (SPI) as a measure of hedging effectiveness, Aggarwal and DeMaskey (1997) report significant performance of the risk minimizing hedge ratio over a full hedge ratio in improving the SPIs of Asian currencies positions. In a similar study of hedging Asian currency risk, Mun and Morgan (1997) show that the naïve hedge performs the worst of all hedging strategies both as a single as well as a portfolio hedge leading to increased risk for the spot position for three out of five countries examined.
The apparent success of the risk minimizing hedge ratio over the naïve hedge ratio lies within its connection with the basis behaviour. Shafer (1993) argues that the risk minimizing hedge ratio, estimated from the regression between spot and futures price levels or changes, reflects the systematic basis behaviour and thus gives a better estimate of the effective price of the hedge than the naïve hedge ratio. However, the change in the basis cannot be forecasted and the hedger is still subject to residual basis risk. The effect of basis risk on the minimum-variance hedge ratio is well described by the theoretical model of Castelino (1992). The risk-minimizing hedge ratio \( b^* \), described by equation (2.12) is the price that the hedge ratio \( b \) in equation (2.5) takes when the variance of the return in the hedged portfolio is minimized. The latter author derives the risk-minimizing hedge ratio \( b^* \) in terms of basis risk by calculating the first derivative of the variance of the hedged return (equation (2.5)) with respect to \( b^* \), equating it to zero and solving for \( b^* \).

\[
\frac{\partial \text{var}(R_{H})}{\partial b^*} = 0
\]

\[
-2(1-b^*) \text{var}(F_{t_t}) - 2 \text{cov}(B_{t_t}, F_{t_t}) = 0
\]  
(2.15)

\[
b^* = 1 + \frac{\text{cov}(B_{t_t}, F_{t_t})}{\text{var}(F_{t_t})}
\]

\[
b^* = 1 + \frac{\rho_{BF} \sigma_{B_{t_t}}}{\sigma_{F_{t_t}}}
\]

It is obvious from equation (2.15) that basis risk is directly related to the risk minimizing hedge ratio. If basis risk is zero \( (\sigma_{B_{t_t}}=0) \), the second term of the hedge ratio disappears and it is possible to full hedge the spot position and achieve a perfect hedge. In other words, basis risk is introducing the difference between the naïve and the risk minimizing hedge ratio. Under the assumption of a negative relationship between the basis and the futures price \( (\rho_{BF} < 0) \), an increase in basis risk always results in decreases in the risk minimizing hedge ratio. Moser and Helms (1990) support this result by showing that the demand of futures contracts
decreases as basis risk increases. Similarly, *Haushalter (1999)* shows that a negative relationship exists between basis risk and corporate hedging.

*Lioui (1997)* proves that, under basis risk, the minimum variance hedge ratio should be less than one even in cases with perfect correlation between the asset to be hedged and the underlying asset of the futures contract. A factor is necessary in order to adjust the hedge ratio for the difference in instantaneous volatilities between the spot and the futures price. This finding is supported by the results of the earlier studies of *Figlewski (1984)* and *Park and Bera (1987)* that basis risk is present even when there is perfect correlation between spot and futures prices.

Substituting expression (2.15) into the variance equation (2.5), we have the residual risk that cannot be hedged with the risk minimizing hedge ratio and it is a function of the variance of the basis and the correlation between the basis and the futures price.

\[
\text{Residual} = \sigma_{Bt}^2 (1 - \rho_{BF}^2) \quad (2.16)
\]

Without basis risk \((\sigma_{Bt}^2 = 0)\), there is no residual risk for the hedger who can completely eliminate the risk of his cash position.

We can conclude that hedging in the futures markets, substitutes basis risk for price risk. The naïve hedge ratio does not account for intertemporal variations in the spot-futures relationship over the hedging period. On the other hand, the risk minimizing hedge ratio of *Johnson (1960)* and *Ederington (1979)* accounts for basis risk since it can be derived by the regression of the spot price change on the futures price change over the hedging period. However, only the anticipated variation in the basis can be accounted for by the risk minimizing hedge ratio. The unanticipated variance in the basis is represented by the residuals of the OLS regression and forms the residual risk of the hedge. According to *Garcia, Leuthold and Sarhan (1984)*, unexpected changes in the basis impose limitations on the ability of the futures market to transfer risk from hedgers to speculators and reduce the income levels of the former. Analyzing the magnitude and variance of the basis
is thus necessary in order to develop the appropriate risk management solutions. This issue is discussed in detail in Section 2.4.

(2.3.5) The a-t model: a corporate approach

As it is reported in the previous section, the risk minimizing hedge ratio is derived by the portfolio approach of hedging theory, where the risk is computed as the variance of returns. However, there is another perception of risk, usually followed by financial managers in corporate policy. According to Crum, Laughhunn and Payne (1983), risk can be defined as the failure to achieve a specific level of return. In this sense, Fishburn (1977) derived the measure of risk as the expected utility of a loss that is a function of a target return, \( t \) and the degree of risk aversion, \( \alpha \). In this framework, a hedge ratio \( b \) is applied in the derivatives market to prevent a loss for the firm that is due to an unexpected adverse outcome.

\[
G_a(t) = \int_{-\infty}^{t} (t - Y(b))^\alpha dF(Y(b))
\]  \hspace{1cm} (2.17)

Where

\( b \) = the hedge ratio

\( G_a(t) \) = the expected utility of a loss

\( t \) = the target return

\( \alpha \) = the degree of risk aversion for returns lower than \( t \)

\( Y(b) \) = the return below the target return and,

\( F(Y(b)) \) = the probability distribution of \( Y(b) \).

The optimal hedge ratio \( b^* \) of the a-t model can be derived, by minimizing the utility function (2.17) with respect to \( b \).

\[
b^* = \min_b G_{Na}(t) = \frac{1}{N} \sum_{i=1}^{N} (t - Y_i(b))^\alpha I(Y_i(b) \leq t)
\]  \hspace{1cm} (2.18)
where

\[ G_{Na}(t) = \text{the estimate of the expected utility of a loss} \]
\[ I = \text{the indicator function and,} \]
\[ N = \text{the number of periods} \]

As in the case of the risk minimizing hedge ratio, the measure of hedging effectiveness of the a-t model is defined as the percentage variance reduction achieved with the hedging model relative to the spot position.

\[ HE_{a-t} = 1 - \frac{G_{Na,h}(t)}{G_{Na,s}(t)} \quad (2.19) \]

Johnson and Walther (1984) and Ahmadi, Sharp and Walther (1986) use the a-t model in order to estimate the hedging effectiveness of forwards, currency futures and options. However, DeJong et al (1997) criticize these studies since they use the naïve and risk minimizing methods for the estimation of the hedge ratio while they apply the a-t measure of hedging effectiveness. Additionally, the same authors criticize the a-t model for deriving a risk minimizing measure of hedging effectiveness although it is based on utility objectives. In general, the a-t model can only be applied in cases of hedging downside risk and it is not applicable in the present thesis that is initially developed by portfolio analysis and utility maximizing objectives.

(2.3.6) The Sharpe-Ratio Model

In order to derive an optimal hedge ratio and a measure of hedging effectiveness, that account for both risk and return, Howard and D'Antonio (1984) concentrate the investor's problem into maximizing the Sharpe ratio \(^2\) of the hedged portfolio \(\delta_{H}\). If \(E(R_{H})\) is the expected return on the combined spot and futures position.

\(^2\) The Sharpe ratio is equal to the excess return per unit of risk.
\( \text{var}(R_H) \) is the variance of this position and \( i \) is the risk-free rate of return, the investor’s problem can be expressed as:

\[
\max_b \delta_H = \frac{E(R_H) - i}{\text{var}(R_H)}
\]

(2.20)

The optimal hedge ratio \( b^* \), derived from the first order condition of the relation (2.20) is the following:

\[
b^* = \frac{-\text{var}(R_S)(\rho_{SF} - \lambda)}{\text{var}(R_F)(1 - \lambda \rho_{SF})}
\]

(2.21)

where

\( \text{var}(R_S), \text{var}(R_F) \) are the variances of the spot and futures returns respectively,
\( \rho_{SF} \) is the correlation coefficient between spot and futures returns,
\( \lambda = \frac{\delta_F}{\delta_S} \) is the relative return of the futures position versus the spot position per unit of risk,
\( \delta_S = \frac{E(R_S) - i}{\text{var}(R_S)} \) is the expected excess return per unit of risk of the spot position and,
\( \delta_F = \frac{E(R_F)}{\text{var}(R_F)} \) is the expected excess return per unit of risk of the futures position.

The measure of hedging effectiveness is defined as the ratio of the Sharpe index of the hedged position to the Sharpe index of the unhedged position and it shows the relative return per unit of risk of the hedged portfolio to the unhedged position:

\[
\text{HE}_\text{Sharpe} = \frac{\delta_H}{\delta_S} = \sqrt{\frac{(1 - 2\lambda \rho_{SF} + \lambda^2)}{1 - \rho_{SF}^2}}
\]

(2.22)
In contrast with the other measures of hedging effectiveness reported so far (minimum risk (Ederington) and a-t model), this measure encompasses both risk and return. It depends not only on the correlation between spot and futures returns, $\rho_{SF}$, but also on the relative payoff of the futures versus the spot position per unit of risk, $\lambda$. The effectiveness of the hedge is high if $H_{E_{\text{Sharpe}}}$ is greater than one.

However, a negative excess return of the spot position, $(E(R_S) - i < 0)$, implies that $\delta_s < 0$. In this case, conflicting results are derived as to the effectiveness of the hedge. In order to account for this effect, Chang and Shanker (1986, 1987) propose an alternative measure of hedging effectiveness that they use in a comparison of the hedging effectiveness between currency futures and currency options.

$$H_{E_{\text{Sharpe}_2}} = \frac{\delta_H - \delta_S}{|\delta_S|} \quad (2.23)$$

Howard and D’Antonio (1987) criticize the measure (2.23) of being undefined if $\delta_s = 0$ and propose the following measure of hedging effectiveness:

$$H_{E_{\text{Sharpe}_3}} = \delta_H - \delta_S \quad (2.24)$$

However, Chang and Fang (1990) criticize (2.24) as being of limited use since it is based on differences, thus not permitting comparisons across commodities or over time. Additionally, Hsin, Kuo and Lee (1994) criticize all three measures as being inconsistent due to the use of the Sharpe index $\delta$, that is a proper measure of portfolio performance only when excess returns are positive. However, in the foreign exchange market, negative excess returns are quite common (Hammer (1988)), casting doubt on the applicability of the Sharpe ratio in currency hedging.

Hsin, Kuo and Lee (1994) following the work of Anderson and Danthine (1980), Gjerde (1987) and Cecchetti, Cumby and Figlewski (1988), propose a new measure of hedging effectiveness that accounts for both risk and return and it is derived
from the maximization of the mean-variance utility function.

\[ HE_{\text{mean-var}} = R_{\text{ceH}} - R_{\text{ceS}} \]  

(2.25)

Where

- \( R_{\text{ceH}} \) is the certainty equivalent return to the hedged position and,
- \( R_{\text{ceS}} \) is the certainty equivalent return to the unhedged spot position.

The certainty equivalent returns \( R_{\text{ceH}} \) and \( R_{\text{ceS}} \) are defined as the returns realized when the variance of the hedged portfolio and the spot position respectively are zero. The advantage of this measure over the Sharpe ratio is that it provides consistent results despite the sign of the expected excess return on the spot currency. A positive HE, indicates that the hedge is effective since it increases the certainty equivalent returns of the spot position, while a negative HE indicates the opposite. The application of measure (2.25) to currency futures and options by Hsin, Kuo and Lee (1994) reveals that the former contracts are more efficient in terms of improving the returns on the hedged portfolio. On the other hand, in terms of risk reduction, the Ederington measure (2.14) shows that both contracts are equally effective in minimizing the risk of a spot position. Shanker (1992) obtains similar results in a risk-return framework when the effect of margin requirements is included in the model while the two currency contracts have a similar hedging effectiveness in the absence of margin requirements. However, a comparison between futures and options in terms of hedging effectiveness is beyond the scope of the present study and it is left for further research.

DeJong et. al (1997) perform an out of sample analysis on the hedging effectiveness of the three utility models (risk minimizing, a-t, and Sharpe ratio model) in the currency futures market. Their comparisons reveal that the models based on the risk minimization criterion are effective in terms of risk reduction while the Sharpe ratio has decreased the utility of the hedger in most cases examined. The analysis of the different hedging theories shows that hedge ratios and measures of hedging effectiveness that are based on the risk-return framework
suffer from several limitations. Additionally, as it was mentioned previously, a basic assumption of the present thesis is the risk neutrality in the futures market that rules out the use of utility-based measures. For this reason, in the present study, the risk minimizing hedge ratio is used and the objective of risk reduction is analyzed for all currencies.
(2.4) Sources of basis risk and their effect on the optimal hedge ratio

In Section 2.3.4, it is concluded that the risk minimizing hedge ratio is successful in eliminating anticipated changes in the basis while any unexpected changes are left unhedged in the form of residual risk. However, an appropriate hedging strategy should be able to reduce the risk of unexpected changes in the relationship of spot and futures returns. In order to detect the sources of basis risk, the determinants of the relation between spot and futures prices must be defined. The latter relation can be derived by the cost-of-carry futures pricing model that is based on a no-arbitrage condition. It consists of the interest rate paid in order to finance the spot asset less the income earned on the asset. In the case of exchange rates, the cost-of-carry pricing model can be derived by the Covered Interest Rate Parity Condition.

We assume that \( S_t \) is the spot exchange rate at time \( t \), \( f_{t-k} \) is the price, at time \( t-k \), of a forward contract that expires at time \( t \), and \( B_{t-k}^f \) and \( B_{t-k}^d \) are the prices of a foreign and a domestic bond respectively, that pay one unit of domestic or foreign currency at time \( t \). In order to prevent arbitrage in the foreign exchange market, international investors must be indifferent between a) investing in domestic bonds or b) exchanging domestic currency for foreign currency at the current spot exchange rate \( S_{t-k} \), investing these funds in foreign bonds and converting them into domestic currency at time \( t \) at the agreed forward rate \( f_{t-k} \). This no-arbitrage relation is called Covered Interest Rate Parity and it is expressed by the following equation.

\[
f_{t-k} = S_{t-k} B_{t-k}^f / B_{t-k}^d \quad (2.26)
\]

or

\[
f_{t-k} = S_{t-k} e^{(i_{t-k}^d - i_{t-k}^f)}
\]

where \( i_{t-k}^d \) and \( i_{t-k}^f \) are the domestic and foreign interest rates respectively over the life of the forward contract.
As it is shown by the CIRP condition, in the case of forward contracts, the basis is equal to the difference between the domestic and foreign interest rates. If futures prices were equal to forward prices, the CIRP condition could be directly applied to the pricing of currency futures. However, although forward and futures contracts have many similarities, they have some important differences.

(2.4.1) The futures-forwards hedging differential

In the case of the forward contract, no cash exchange takes place over the life of the contract until the delivery date, while participants in the futures market realize gains or losses on their position on a daily basis due to the marking-to-market feature of futures contracts. The funds or losses occurring from the marking-to-market can be invested or financed respectively at the current domestic interest rate.

Cox, Ingersoll and Ross (CIR) (1981) derived a general theoretical relationship between forward and futures prices, by being based on a strategy that replicates a forward contract with a number of futures contracts and daily borrowing or lending. In this way, CIR show, that the difference at time $t$ between forward and futures prices of the same asset, is equal to the value at time $t$ of a cash flow to be received at time $m$:

$$ F(t) - f(t) = \sum_{i=t}^{m-1} [F(i + 1) - F(i)] \frac{B^d(i)}{B^d(i + 1)} - 1 \bigg/ B^d(t) $$

(2.27)

where $t =$ current time

$m =$ maturity date for forward and futures contracts

$F(t) =$ futures price at time $t$

$f(t) =$ forward price at time $t$

$B^d(t) =$ price at time $t$ of a domestic discount bond paying one dollar at time $m$.

(1981) and French (1982), Levy (1989) have argued that forward and futures prices are equal only if interest rates are non-stochastic. From equation (2.27), it is clear that if the discount bond prices B, on which the loss/profit from the daily resettlement of the futures contract is financed/invested, are constant over the life of the futures contract, then the value at time t of the funds earned or lost should be zero. The futures and forward prices would thus be equal over the life of the contracts of the same maturity. However, if interest rates are stochastic, then the bond price of the following period is not known at the initiation of the futures contract. There is risk inherent in the futures position related to the reinvestment of cash flows under random interest rates. This risk introduces the main difference between forward and futures prices.

Cornell and Reinganum (1981), Chang and Chang (1990) and Benninga and Protopapadakis (1994) are unable to find any statistical significance on the forward-futures differential. However, their methodology is criticized by Polakoff and Grier (1991) and Dezhbakhsh (1994) who use more sophisticated econometric techniques and provide evidence supporting the initial theoretical proposition of CIR, that under stochastic interest rates, there is significant difference between forward and futures prices. The latter author emphasizes the role of the marking-to-market as the main reason for the divergence between the two derivative prices. The significance of the marking-to-market effect is also acknowledged by Berck (1981), Chang and Loo (1987) and Kenyon and Klay (1987).

Levy (1989) proposes a strategy in order to minimize the risk emerging from daily marking-to-market futures contracts under stochastic interest rates. He argues that since the futures contracts are marked to market daily, the forecast error of the interest rate process is very small if forecasting is limited to one-day horizons. However, according to Jabbour and Sachlis (1993) this strategy requires that the forecasting procedure be repeated (n-1) times over the contract’s life, where n is the number of days of the life of the contract. The latter authors prove in a theoretical model that, under stochastic interest rates, the variance of the final futures cash-flow increases with the number of days remaining until maturity.
The effects of the marking-to-market feature of futures markets on the hedger have been usually ignored in the development of hedging models as being trivial (Peterson and Leuthold (1987)). The statistical significance of the forward-futures price differential, found in recent empirical studies, leads to the idea that there must be a futures-forward hedging differential due to the marking-to-market effect. Blank (1990) argues that not accounting for the capital requirements of margin calls means that the unrealistic assumption of a frictionless market is made, where no limitations or costs are imposed on raising capital for hedging purposes. Figlewski et al (1991) state that even under non-stochastic interest rates, the forward hedging strategy is different than the futures hedging strategy since in the futures market the hedger needs to tail his hedge with an adjustment factor until the hedge is lifted. According to Lioui and Eldor (1998), the investor must hold less futures contracts than in the corresponding forward contract positions due to the marking-to-market procedure of the futures positions.

The effects of margin requirements on the hedge ratios and the hedging effectiveness of currency futures and options are examined by Shanker (1992). The results show that an increase in the margin requirements leads to a reduction in the demand of futures contracts by the hedgers and the hedging effectiveness of futures contracts. The existence of margins leads to the construction of a suboptimal hedge portfolio. This result can be explained by the fact that although the hedger can earn interest in his margin account if he deposits T-Bills, (Anderson (1981)), access to these funds is restricted, thus creating a liquidity cost to the investor (Telser (1981), Telser and Yamey (1965) and Hartzmark (1986)). The significant effect of margin requirements on the risk and return of a hedged portfolio is evident in the empirical study of Meyer (1999) for interest rate and exchange rate futures and option hedges.

Finding the determinants of the hedging differential is crucial for the development of an appropriate hedging strategy that accounts for the marking-to-market effect. In the special case of currency futures, the difference between forward and futures
prices is derived by *Amin and Jarrow (1991)* and it is used in the present study as reference for the forward-futures hedging differential. The latter authors apply the term structure model of *Heath, Jarrow and Morton (1992)* in the pricing of currency futures and derive the following relation between foreign exchange forward and futures prices:

\[
F(0, T) = f(0, T) \exp\left[\sum_{i=1}^{2} \int_0^T a_d(v, T) [a_d(v, T) - a_f(v, T) - \delta_d(v, T)] dv\right] \tag{2.28}
\]

where \(F(0, T)\) is the price of a futures contract maturing at time \(T\)

\(f(0, T)\) is the price of a forward contract maturing at time \(T\)

\(a_d(v, T)\) is the standard deviation of the domestic bond price

\(a_f(v, T)\) is the standard deviation of the foreign bond price and,

\(\delta_d(v, T)\) is the standard deviation of the spot exchange rate.

According to *Amin and Jarrow (1991)*, the exponential term in equation (2.28) represents the covariance between the exchange rate and the long-term bond prices. This result becomes more obvious by the following representation given by Lioui (1998) based on the *Amin and Jarrow (1991)* framework.

\[
F(t, t_1) = f(t, t_1) \exp\left\{\left(\nu_{d1}^2 + \nu_{d2}^2\right) \frac{(t_1 - t)^3}{3} + \left(\nu_{d1} \nu_{s1} + \nu_{d2} \nu_{s2}\right) \frac{(t_1 - t)^2}{2} - \nu_{d2} \nu_{s2} \frac{(t_1 - t)^3}{3}\right\} \tag{2.29}
\]

where \(F(t, t_1)\) is the price of a futures contract maturing at time \(t_1\)

\(f(t, t_1)\) is the price of a forward contract maturing at time \(t_1\) and,

\(\nu_{d1}, \nu_{d2}, \nu_{f1}, \nu_{s1}, \nu_{s2}\) are strictly positive constant parameters of the domestic interest rate, foreign interest rate and spot exchange rate instantaneous volatilities respectively.

In both studies the authors observe that a sufficient condition for the equality of forward and futures prices in the foreign exchange market is that the domestic interest rate is a deterministic function of time. In this case, the variance of the
domestic interest rate would be equal to zero, making $v_{d1}$, $v_{d2}$ equal to zero. The exponential term or the forward-futures differential in equation (2.29) would thus become equal to one and forward and futures prices would be equal.

Lioui (1998) attributes this result to the fact that the margin account gains or losses that represent the main difference between forward and futures prices are expressed in terms of the domestic interest rate. This observation can be explained if we consider the foreign currency as a stock paying a stochastic dividend yield equal to the foreign interest rate. Jarrow and Oldfield (1981) state that the forward-futures differential for a stock is due to the stochastic interest rate and not to the stochastic dividend yield. The futures-forward hedging differential is thus independent of the foreign interest rate volatility. The same conclusion is made by Figlewski, Landskroner and Silber (1991) who state that the tailing factor of the futures contract, accounting for the marking-to-market effect, is a function of interest rates only and is independent of the underlying asset and the decision of the hedger.

It is evident from the previous analysis that the single source of uncertainty in the forward-futures differential is the domestic interest rate on which the loss/profit of the futures position is financed/reinvested. Accounting for the domestic interest rate risk is thus necessary for the development of the appropriate hedging strategy in the futures market.

(2.4.2) The cost-of-carry futures pricing model and the sources of basis risk

As it is mentioned in Section 2.3.4, basis risk creates uncertainty in the combined spot-futures position, making traditional hedging techniques ineffective. Identifying the sources of basis risk is crucial in order to derive an appropriate hedge ratio in the futures market. The study of Brenner and Kroner (1995) is based on the Covered Interest Rate Parity condition and the diffusion process for the spot price and derives the cost-of-carry futures pricing model. The price, at time $t-k$, of a futures contract maturing at time $t$ can be given by the following equation:
\[ F_{t/t-k} = S_{t-k} \frac{B^f_{t/t-k}}{B^d_{t/t-k}} \exp\{Q_{t/t-k}\} = S_{t-k} D_{t/t-k} \exp\{Q_{t/t-k}\} \]  

(2.30)

where

\( Q_{t/t-k} \) is equal to the exponential term of equation (2.29), as a function of the spot exchange rate and the domestic and foreign interest rates and it represents the adjustment term for the marking-to-market effect of the futures contracts and,

\( D_{t/t-k} \) is the differential between the domestic and foreign interest rates.

Taking the natural logarithm of equation (2.30), we have the following expression for the futures price:

\[ \ln F_{t/t-k} = \ln S_{t-k} + \ln D_{t/t-k} + Q_{t/t-k} \]  

(2.31)

It is clear from equation (2.31) that the spot-futures basis is equal to the difference between the domestic and the foreign interest rate plus a stochastic term that represents the marking-to-market effect and also depends on the domestic interest rate. We can easily observe that any movement in the basis is due to the uncertainty in the movement of interest rates.

An analogous relation holds for stock index futures with the foreign interest rate being replaced by the dividend yield on the index. This happens because the foreign currency is considered as a security paying a known dividend yield. Since the yield earned on the foreign bond is denominated in foreign currency, its value for the international investor depends on the exchange rate when the yield is converted into domestic currency. In general, basis risk for investment assets arises from a stochastic risk-free interest rate and uncertainty as to the asset’s yield in the future. An exception to this rule is the case where the investment horizon of the hedger does not correspond with the maturity of the futures contract, creating basis risk even when interest rates are non-stochastic.
Figlewski (1984) has made an attempt to identify the sources of basis risk in the stock index futures market. He examined basis risk with respect to the non-market risk arising from cross hedging, dividend risk, hedging duration and time to expiration. Dividends are not found to play an important role as a source of basis risk while the hedging duration and the time to expiration are more significant. The implication of these results for our study is that it is not the asset's yield that creates basis risk in the case of investment assets. Additionally, even in the case of hedging the S&P 500-index portfolio with the relevant futures contract, basis risk remains significant, suggesting that a different than the non-market risk source of basis risk may be present. However, Figlewski (1984) does not examine the domestic risk-free interest rate as a possible source of basis risk.

A different approach is followed by Park and Bera (1987) who account for basis risk through a Box-Cox transformation on the risk-minimizing hedge ratio in order to capture any nonlinearities present in the spot-futures relationship. The application of the Box-Cox hedge ratio on the GNMA (mortgages) market results in superior performance of the latter relative to the OLS hedge ratio both in the direct (GNMA futures) and the cross hedging (T-Bills futures) case. The implication of this finding is that the non-market risk, arising through cross hedging, is not the main source of basis risk.

The significance of basis risk for the present study comes from its effect on the derivation of the optimal hedge ratio. Beltratti, Laurent and Zenios (1999) show that an investor with a low domestic interest rate should completely hedge his spot portfolio since the cost of hedging (spot-futures basis) is mainly negative. Moser and Helms (1990), argue that any unpredictable changes in the variance-covariance matrix of spot and futures prices increase basis risk and lead to a lower risk minimizing hedge ratio. The latter authors attribute this result to the decrease in the demand of futures by hedgers implying that basis risk is partly due to futures pricing specifications like the marking-to-market effect, delivery or quality issues. The relation between basis risk and margin requirements is also acknowledged by
Blank (1990), who states that when the basis risk decreases, the variance of the return on the hedged portfolio is lower, resulting in lower reserve requirements for the futures position.

According to Lioui (1997, 1998), under non-stochastic interest rates, a hedger can construct a perfect hedge when there is perfect negative correlation between the futures contract and the underlying asset. However stochastic interest rates create basis risk making the derivative an imperfect substitute of the asset to be hedged and a perfect hedge cannot be accomplished. The latter author argues that, in this case, the investor is able to realize only a risk-minimizing hedge. A factor that depends on the stochastic interest rates is necessary in order to adjust the hedge ratio for the difference in instantaneous volatilities between the spot and the futures price. The factor accounting for basis risk is the same for forward and futures contracts and it adds a time dimension in the hedging strategy that introduces the need to continuously rebalance the hedged portfolio. However, futures contracts need a further adjustment due to the marking-to-market effect that depends on the difference between spot and futures prices over the hedging period. The interesting result is that, for interest rate sensitive securities, the single source of uncertainty of the futures position is the domestic interest rate.

We can conclude that the basis risk for currency futures is related to the marking-to-market effect and it is due to the same source of uncertainty, which is the randomness of the domestic risk-free interest rate. Existing hedging strategies that use only the correlation between spot and futures prices as an optimal hedge ratio ignore the effect of basis risk, arising from stochastic interest rates, on the hedging effectiveness of a hedging policy. A hedge ratio that accounts for the correlation between spot prices and domestic interest rates and futures prices and domestic interest rates would eliminate the risk emerging from changes in both the basis and the cash resettlement of the futures contract.

This empirical issue is the subject of Chapter 4 “Basis risk, marking-to-market and the optimal currency hedge ratio”. In this chapter, a dynamic risk-minimizing
hedge ratio, accounting for basis risk and stochastic interest rates, will be applied to four major currencies and it will be compared with the traditional risk minimizing hedge ratio.
(2.5) Hedging a multi-currency position: a portfolio approach

The hedging problem, examined so far, involves the exposure of a single currency position. However, according to Stein (1986), the bulk of trading in the futures markets is made by banks and financial institutions that act as intermediaries and they hedge multiple spot positions. Additionally, international trade and diversification through foreign investments create multiple currency risks for multinational firms and international investors. Selective hedging is useful only if foreign exchange risk exists for only one currency and the remaining currencies can be left unhedged. However, hedging each currency in isolation is suboptimal not only because of administrative difficulties, but also due to portfolio effects arising from the correlation existing between currencies.

The advantage of creating a multi-currency portfolio lies in the fact that the residual risk of each currency is partly diversified away, due to the negative correlation existing between some currencies. In this way, the depreciation in one currency can be offset by the appreciation of another so that the portfolio return is left unaffected. Hedging all the components of the portfolio is thus not necessary since a part of currency risk is already diversified away. This result leads to a decrease in the demand for futures contracts for hedging the multi-currency portfolio relative to the demand for hedging each currency in isolation. Since a futures contract can be used as a direct as well as a cross-hedging instrument, an investor can hedge his spot currency position with multiple futures contracts. In this way, portfolio effects arise not only from the covariance between currencies but also from the covariance between futures returns. A joint estimation of the hedge ratios is thus required in order to derive adequate risk management solutions.

Anderson and Danthine (1980, 1981) and Levy (1987) expand the single futures hedging theory in order to account for the possibility of multiple futures hedges for a spot position. The proportion of the spot portfolio that is hedged by each futures contract (minimum variance hedge ratio) is derived as the coefficient in a multiple regression of the spot portfolio return on the multiple futures returns.
\[ D \ln s_t = c + \beta_1 d \ln f_{it} + \beta_2 d \ln f_{it} + \ldots + \beta_n d \ln f_{nt} + \epsilon_t \] (2.32)

Where \( D \ln s_t \) is the return on the spot portfolio.

\( D \ln f_{it} \) is the return on each futures position with \( i = 1, \ldots, n \) the number of different futures contracts.

\( \beta_i \) is the hedge ratio for each futures position with \( i = 1, \ldots, n \).

Miller (1985) and Grant and Eaker (1989) use the proposed methodology but they report no significant advantage from using multiple futures contracts in agricultural commodities. Peterson and Leuthold (1987) find that the optimal hedge for multiple spot positions in a commodity is a partial hedge due to the diversification benefits provided by the correlations between different combinations of spot and futures positions. For a similar market, Noussinov and Leuthold (1998) report significant differences between the single hedges and the multiproduct hedges with the latter being consistently lower than the former. In the currency markets, Eaker and Grant (1987) compare the effectiveness of single and multiple hedges in reducing the risk of single and portfolio spot positions respectively, in the British Pound, Canadian Dollar and Japanese Yen. Multivariate OLS regressions of the spot portfolio return on the components futures returns are used for the derivation of the risk-minimizing hedge ratios. Comparisons of the size of the futures positions required by each strategy and the hedging effectiveness achieved, reveal no significant differences between single and complex hedges. Similarly, Lypny (1988) reports no significant portfolio effects for the currencies he examines. However, a serious disadvantage of the latter studies is that they did not use a dynamic hedging strategy required by the significant time variation evident in financial returns.

Portfolio effects can also arise in the case of cross hedging a single currency position with multiple futures contracts. Eaker and Grant (1987) also examine the effectiveness of cross-hedging the Italian Lira, Greek Drachma, Spanish Peseta and
the South African Rand with single and multiple hedges of major currencies futures contracts. Significant differences in the estimated hedge ratios are found between the single and the multiple case, with the latter strategy providing smaller futures positions for each contract examined relative to the single case. The multiple hedge ratios are found to be superior to the single hedges in terms of hedging effectiveness and intertemporal stability. The superiority of the portfolio strategy in cross hedging currency risk is also evident in the empirical study of Benet (1990), who derives single and multiple commodity cross hedges by OLS univariate and multivariate regressions of minor currency returns on export commodity futures returns. Although temporal instability is observed in the estimated hedge ratios, in sample and out-of-sample comparisons reveal the superiority of the commodity portfolio hedges to their single counterparts in terms of hedging effectiveness.

However, in a similar empirical study, Benet (1992) reports that the temporal instability of the hedge ratios estimated from multivariate OLS regressions creates serious problems in the performance of portfolio cross hedges. Although significant risk reduction is achieved ex post, the ex ante results show a decrease in hedging effectiveness, with some commodity and currency futures portfolios increasing the risk of the minor currency spot positions. The loss in hedging performance is more evident in the case of futures portfolios, due to the simultaneous determination of the hedge ratios and it cancels the diversification advantage achieved by the portfolio hedge. The implication of the previous studies for the present thesis is that although portfolio effects are important in the development of multicurrency hedging models, accounting for these effects with a dynamic model is an essential condition for deriving adequate risk management solutions.

Glen and Jorion (1993) also acknowledge the importance of accounting for portfolio effects in hedging arising from the correlation between assets. The authors attribute the fact that no significant improvement is observed when cross hedging passive indices of stocks and bonds with currency forwards, to the limitation of the model to account for the correlation between stocks and bonds with the exchange rates. Additionally, the usefulness of introducing time-variation in the hedge ratio is
evident both in sample and out-of-sample, by the significant improvement in the risk-return characteristics of the hedged portfolio observed when the hedging strategy is conditioned on the forward premium.

*Meyer (1994)* uses the standard regression methodology in a different application of hedging London spot metal positions with single hedges of U.S. metal futures versus complex hedges consisting of British Pound futures and metal futures. However, no significant improvement in the performance of complex hedges is reported, a finding that is attributed to the low exchange rate risk in international metal markets. *Mun and Morgan (1997)* examine the effectiveness of major currency futures portfolios for cross hedging the risk of net foreign currency positions of banks in five Asian emerging markets. Three alternative methods are used for the construction of the weighting vectors of the futures portfolios: equally weighted, naïve and weights based on the variance covariance matrix of futures returns. The risk-return comparisons between the naïve, single and portfolio hedges reveal that the minimum variance portfolio cross hedge is the superior strategy for three out of the five countries examined, with the naïve single and portfolio method being the worse. These results are supported by mean-variance efficiency tests of each hedging strategy with the Generalized Method of Moments.

Although the studies reported in the previous paragraphs account for portfolio effects in the multicurrency hedging problem, they suffer from the ignorance of the time-variation in the empirical distribution of foreign exchange returns that demands a dynamic hedging strategy for foreign currency positions. *Gagnon, Lypny and McCurdy (1998)* criticize the previous studies for ignoring the time variation in hedge ratios and they attribute the lack of evidence on portfolio effects to the constant portfolio setting assumed and the hedging effectiveness criterion used. The latter authors use a multivariate GARCH (1,1) model in order to estimate the optimal hedge ratios for two alternative currency portfolios. Their in-sample tests on the hedging performance of the static and dynamic portfolio and no-portfolio hedging strategies reveal that, even after accounting for transaction costs, the dynamic portfolio hedge outperforms the no-portfolio hedges from both the
variance reduction and the utility standpoint. The latter authors provide the theoretical framework for hedging a multicurrency position on the basis of the well-known portfolio theory of hedging.

We assume a portfolio consisting of $N$ currencies with $S = (S_1, S_2, \ldots, S_n)'$ be the vector of spot exchange rates and $w = (w_1, w_2, \ldots, w_n)$ be the vector of weights in the spot currency portfolio. If $F = (F_1, F_2, \ldots, F_n)'$ is the vector of the prices of the respective futures contracts and $b = (b_1, b_2, \ldots, b_n)$ is the vector of hedge ratios for each currency futures, the return on the hedged portfolio is equal to:

$$ R_{H} = R_{P} - R_{F}'b $$

Where $R_{P} = $ the return on the spot portfolio with $P = S'w$

$R_{F} = $ the return on the futures portfolio

If the hedger has a mean-variance utility function, the hedging problem reduces to the choice of the optimal hedge ratio $b$ that will maximize the following utility function:

$$ \max_{b} E(R_{H}) - \frac{1}{2} \gamma \text{var}(R_{H}) $$

Where $\gamma$ is the degree of risk aversion ($\gamma > 0$) and,

$$ E(R_{H}) = E(R_{P}) - E(R_{F})'b $$

It is worth noting at this point that, the main difference of the present analysis from the derivation of the optimal hedge ratio for a single spot position in Section 2.3.3, is that the returns and variances are now expressed as vectors and matrices respectively, since they involve more than one spot and futures positions.
Before proceeding to the solution of the optimization problem, we denote the variance-covariance matrix of the spot and futures positions with $\Sigma$.

$$
\Sigma = \begin{bmatrix}
\sigma_p^2 & \Sigma_{pf} \\
\Sigma_{pf}' & \Sigma_{ff}
\end{bmatrix}
$$

(2.36)

Where $\sigma_p^2$ is the variance of the spot currency portfolio

$\Sigma_{ff}$ is the $n \times n$ variance matrix of futures returns and,

$\Sigma_{pf}$ is the $1 \times n$ covariance matrix of spot and futures returns.

The first order condition for maximizing the utility function (2.34) is to estimate the first derivative with respect to $b$ and set it equal to zero.

$$
-E(R_f) - \frac{1}{2} \gamma [2b\Sigma_{ff} - 2\Sigma_{pf}] = 0
$$

(2.37)

Solving for $b$, we obtain the optimal number of futures contracts that is equal to

$$
b_{optimal} = \frac{-E(R_f)}{\gamma \Sigma_{ff}} + \frac{\Sigma_{pf}}{\Sigma_{ff}}
$$

(2.38)

Expression (2.38) generalizes the optimal hedge ratio to the case where a multi-currency portfolio is hedged with multiple futures contracts. Assuming unbiasedness in the futures market, the optimal hedge ratio in (2.38) is set equal to the risk minimizing hedge ratio $b^*$:

$$
b^* = \frac{\Sigma_{pf}}{\Sigma_{ff}}
$$

(2.39)

To the extent that the returns of different futures positions are correlated, portfolio effects will be reflected on the estimated risk-minimizing hedge ratio. The portfolio risk-minimizing hedge ratio should be more effective in terms of risk reduction.
than the single hedge ratio. In Chapter 3, with title "Dynamic portfolio effects in a complex currency hedging problem", this empirical issue is investigated for two spot portfolios, the first consisting of British Pounds and Japanese Yen and the second including Swiss Francs and German Marks.

*Lien (1990)* argues that the marginal contribution of a futures contract, as a component in a futures portfolio, to the risk reduction of the spot position is different from its hedging effectiveness when it is used in isolation. This difference is due to portfolio effects arising from the composition of the spot and the futures portfolio. The latter author emphasizes that the specific futures contract may improve little or worsen the spot portfolio in terms of risk reduction, although it may be a very effective hedging instrument for individual spot positions. For example, *Saunders and Sienkiewicz (1988)* find that a portfolio of BP and DM futures is a more effective hedge than the ECU futures for a spot portfolio of European currencies. Based on these findings, *Lien (1990)* proposes that, in order to include a futures contract in the original futures portfolio, its contribution to the effectiveness of the hedge should be examined. However, this action should not be made without accounting for the transaction costs incurred by the additional contract. The role of transaction costs in the hedging effectiveness of a dynamic strategy is evident in the empirical study of *Meyer (1999)* who shows that the introduction of transaction costs in the hedging model results in substantial differences in terms of risk and return of the hedged portfolio. Although daily portfolio rebalancing produces returns with the lowest variance, accounting for transaction costs results in higher returns for lower frequencies of portfolio rebalancing. Similarly, *Beltratti, Laurent and Zenios (1999)* report the relevance of transaction costs in the choice of the best dynamic strategy under changing market conditions.

In this context, portfolio effects are examined in the empirical Chapters 3 and 5, for both direct and cross hedging cases in the presence of transaction costs. In Chapter 3, the problem of direct multicurrency hedging is analyzed for four major currencies. Following the methodology introduced by *Gagnon, Lypny and*
McCurdy (1998), two passive indices are constructed and two dynamic multivariate GARCH-X models are estimated for the spot portfolios and their respective futures contracts in order to account for the two limitations of the conventional (OLS) hedging model. A basic contribution of the chapter is that it accounts for the effect of the cointegrating relationship not only on the levels of spot and futures exchange rate returns, but also on their conditional variances. A conditional hedging strategy is also applied in the estimated models accounting for transaction costs and allowing the investor to selectively rebalance his futures positions. The main conclusion emerging from Chapter 3 is that, accounting for portfolio effects in a dynamic setting when hedging multicurrency positions, results in substantial risk reduction and utility gains, relative to single hedges, both in the in sample and the out of sample analysis.

In Chapter 5, dynamic portfolio effects are accounted for in cross hedging a European currency portfolio with major currency futures contracts. A portfolio of five European currencies is constructed consisting of positions in the Netherlands Guilder, French Franc, Danish Crone, Spanish Peseta and the Italian Lira. A bivariate GARCH system is estimated for the spot portfolio and each one of the most actively traded currency futures contracts (the British Pound, the Swiss Franc, the Deutsche Mark and the Japanese Yen) in order to derive dynamic risk minimizing cross hedge ratios. The hedging performance of each contract is investigated in an ex post and ex ante analysis. Additionally, a futures portfolio hedge is constructed with a structure depending on the hedging effectiveness of the individual futures contracts and the transaction costs. The model developed in Chapter 5 is innovative both in terms of cross hedging a European currency portfolio as well as a dynamic specification of minor currency returns for hedging purposes.
International investment and foreign currency transactions often take place in countries where no forward or futures contracts are actively traded. In this case, an investor with an exposure to a minor currency can hedge his spot position with a futures contract or multiple futures contracts denominated in the major currencies. This action is called a *cross hedge* and the resulting hedge portfolio of spot and futures positions will have a lower risk than the spot position alone under the assumption that the spot currency and the underlying currency of the futures contract are highly correlated.

The importance of cross hedging is extremely high in the foreign exchange market since only major currencies are actively traded in the international futures exchanges. Kogut (1983) shows that the availability of cross hedging instruments for exchange rate risk, affects the foreign investment decision of managers who take advantage of strategic opportunities in foreign countries achieving optimal resource allocation. The lack of a hedging vehicle for foreign exchange risk can also lead to a decrease in the exports of an international firm. However, if a derivative instrument exists, that is denominated in a currency highly correlated with the currency under question, exchange rate risk can be eliminated significantly. This is the case of cross hedging that, as Broll and Wahl (1998) show, leads to an increase in the output and economic welfare of an exporting firm and it can further improve national exports and international trade. In general, cross hedging leads to the expansion of hedging opportunities for assets without actively traded futures contracts.

However, in contrast with the case of direct hedging, an investor cannot fully hedge his foreign currency position since there is no perfect correlation between the spot asset and the hedging instrument (Broll, Wahl and Zilcha (1995), Broll and Eckwert (1996)). This is due to the fact that, in the case of cross hedging, the characteristics of the cash asset underlying the futures contract differ from those of the cash instrument being hedged, affecting the performance of the cross hedge.
The efficiency of hedging a spot currency position with a futures contract of another currency is limited when different economic conditions exist in the two countries.

The identification of the adequate futures contract to cross hedge a spot currency position is based on the correlation existing between the return on the spot position and the return on the futures position. The higher the correlation coefficient between the two positions, the more effective is the cross hedge. On the contrary, the lower is the correlation the less appropriate is the specific hedging instrument for the spot position. This is due to the large basis risk created since spot and futures prices do not move together. Basis risk is crucial in the development of cross-hedging solutions since, sometimes, it may introduce even higher risk than the unhedged case.

The importance of the correlation existing between spot and futures returns is evident in most empirical studies on cross hedging. Eaker and Grant (1987) examine the potential of cross hedging the Italian Lira, the Greek Drachma, the Spanish Peseta and the African Rand with major currency futures contracts (British Pound, German Mark, Japanese Yen, Canadian Dollar and Swiss Frank) and gold futures. Significant effects of the European market integration and the trade relations between countries on the hedging effectiveness of each contract are revealed, with European currency futures being the most effective cross hedges of European currencies. Braga, Martin and Meilke (1989) provide additional evidence that the DM futures contract is an efficient hedging instrument for spot positions in the Italian Lira with strong implications for the interdependence of EMS currencies. Park et. al (1987) investigate the cross-hedging performance of the DM futures hedge for several EMS currencies.

Under the hypothesis of a positive relationship between exchange rate returns and primary export commodities returns, Benet (1990) derives commodity futures risk minimizing hedge ratios for several minor currencies. In the out-of-sample analysis, the commodity futures are as successful as the benchmark currency
futures portfolio in reducing the risk of the spot positions while the commodity futures portfolio outperforms the currency portfolio. Ghosh (1996) uses an error correction model in order to derive risk minimizing hedge ratios for the Belgian Franc, the Italian Lira and the Dutch Guilder using the U.S. Dollar Index futures contract. Out-of-sample forecasts support the superiority of the error correction model over the traditional OLS model. Eaker, Grant and Woodard (1993) use major currency futures contracts in order to cross hedge the equity and bond exposure of foreign assets denominated in major currencies. Although currency hedges are very effective in reducing exchange rate risk, they reduce only a small portion of the securities risk. Similarly, Meyer (1994) reports no significant advantages from using complex U.S. metal / British pound futures hedges over single U.S. metal futures positions for London spot metal positions.

Mun and Morgan (1997) examine the effectiveness of major currency futures for cross hedging the risk of net foreign currency positions of banks in five Asian emerging markets. For the single futures positions, the results are quite mixed and no general conclusions can be made for the best futures contract to cross hedge Asian currency risk. While the JY futures contract outperforms the remaining contracts for Indonesia and Thailand, Korea is better off by hedging with DM futures, Malaysia with Canadian futures and Singapore with BP futures. However, mean-variance efficiency tests of each contract with the Generalized Method of Moments, show that the JY futures contract can provide a successful cross hedge for all countries examined. The implications for cross hedging of the presence of a Japanese Yen bloc in Asian emerging markets is more evident in the empirical studies of Aggarwal and DeMaskey (1997). Futures and options contracts in the five major currencies are used in order to hedge the currency risk in seven Asian emerging markets. Using the Sharpe Performance Index as a measure of hedging effectiveness, the Japanese Yen futures and options contracts are found to be the most effective for the majority of the countries examined. This empirical finding casts doubt on the conclusion of Hauser, Marcus and Yaari (1994) that it is not beneficial to hedge emerging market currency risk.
Additionally, the stability of the spot-futures relationship over time is a major determinant of the success of a cross hedge. Broil, Wahl and Zilcha (1995) show that an international firm can benefit from cross-hedging exchange rate risk under the assumption of a stable correlation between the hedging instrument and the exchange rate. If the derived hedge ratios in one period are no longer valid in the following period, the effectiveness of the cross hedge is limited or totally eliminated. Intertemporal instability is more evident in the cross hedging case than in direct hedging since different assets are affected by different economic factors over time. The substantial volatility of financial returns introduces dramatical shifts in the structural relationship of different assets, making constant hedging strategies inappropriate in most cases.

Eaker and Grant (1987) report significant inter-temporal instability in the hedging effectiveness of some cross hedges, introducing more risk in hedged positions than unhedged ones and they propose a rolling regression technique in order to incorporate the most recent information in the estimated hedge ratio. Benet (1990) attributes the ex ante superiority of the commodity futures portfolio over the currency futures portfolio to the reported temporal instability of currency hedge ratios. In a later study (1992) on minor currency cross hedging, the same author observes significant variability of the estimated hedge ratios over time, leading to a decrease in performance of the cross hedges from ex post to ex ante. Aggarwal and DeMaskey (1997) also perform tests for intertemporal stability in the estimated cross hedge ratios for the seven Asian markets, by dividing the sample period into smaller subperiods. However, instability is found in very few cases indicating a good ex ante performance of the cross hedge ratios.

A conclusion emerging from the present section is that considering the correlation existing between assets is important in order to choose the appropriate cross hedging instrument for a spot position. While a European currency future is the most successful candidate in a cross hedging problem of minor European currencies, the Japanese Yen future should be more efficient in cross hedging Asian exchange rate risk. However, an important limitation of the previous studies is that
they fail to account for the time variation present in financial returns. This is the reason why significant temporal instability is observed in the estimated hedge ratios with serious effects on the ex ante hedging performance. In order to account for this effect, in the present study, a dynamic hedging strategy will be applied to the currencies examined.

In Chapter 5 of the present thesis, “Cross-hedging European currency portfolios with major futures contracts: implications for the single European currency risk”, the efficiency of the four major currency futures in hedging a spot portfolio consisting of five EMS exchange rates will be examined. Dynamic portfolio effects will be considered so that the instability of the cross hedge ratios is accounted for. Strong emphasis will be given in the degree of correlation between major and minor currencies and its effect on the efficiency of cross hedge ratios. Implications for the risk management of the Euro as the single European currency are also drawn on the basis of the analysis of the EMS portfolio as well as of a small sample on the prices of the Euro spot and futures contract.
Section C: Conclusions

In the present chapter, a discussion of the theoretical issues of hedging in the futures markets is performed. The definitions and the basic concepts of the derivatives markets as the principal hedging instruments are first stated as well as the contribution of risk management in maximizing the value of a multinational firm. Additionally, the traditional theories of hedging are reported, with strong emphasis to the effect of basis risk on the development of the optimal hedge ratio. The measures of hedging effectiveness, as derived by the traditional and more recent theories, are presented and their strengths and weaknesses are analyzed on the basis of existing empirical evidence.

The effect of stochastic interest rates on the optimal hedge ratio is studied through the futures-forward hedging differential and the cost-of-carry futures pricing model. Significant empirical issues for foreign exchange hedging arise from the relation between basis risk and the domestic interest rate. Additionally, the limitations of the relevant empirical studies on portfolio hedging lead to the development of a multivariate hedging model that must be estimated in a dynamic portfolio context. Finally, a literature review on foreign currency cross hedging is presented and empirical questions are derived as to the stability of hedge ratios and their ex ante performance.

In the following chapters, the empirical issues developed in the previous sections will be investigated with data analysis and empirical model estimation. Significant results will be derived that are of interest to financial managers as well as international investors who face foreign exchange exposures.
CHAPTER 3: Dynamic portfolio effects in a complex currency hedging problem.

Section A: Introduction

The problem of foreign exchange hedging has been the subject of many empirical studies, following the theoretical work of Johnson (1960), Stein (1961) and Ederington (1979). The latter studies are based on the portfolio theory introduced by Markowitz (1952) and derive the risk minimizing hedge ratio by dividing the unconditional covariance between spot and futures returns by the unconditional variance of futures returns. According to this theory, the hedge ratio can be estimated as the slope coefficient in the regression of the spot price change on the futures price change. This approach is followed by many studies on hedging due to its simplicity and ease of calculation. It is applied to commodities by Johnson (1960) and Stein (1961), to interest rates by Ederington (1979), to exchange rates by Dale (1981) and Hill and Schneeweiss (1981) and stock prices by Figlewski (1984).

However, the reported instability in the hedging effectiveness of the OLS model (Grammatikos and Saunders (1983), Figlewski (1984)), and the substantial evidence on the presence of conditional heteroscedasticity in financial returns, has led researchers to the development of dynamic risk management solutions. As it is reported in the following section, the success of the GARCH model in capturing the empirical regularities of financial returns is extended in the hedging literature. Time-varying risk minimizing hedge ratios have been derived by the application of ARCH models to interest rates (Cecchetti et. al (1988)), commodities (Myers (1991), Baillie and Myers (1991), Sephton (1993)), exchange rates (Kroner and Claessens (1991), Kroner and Sultan (1991), Kroner and Sultan (1993), Lien and Luo (1994), Tong (1996), Lin, Najand and Yung (1994)), and stock index futures (Holmes (1996) and Park and Switzer (1995)).
A basic limitation of the studies mentioned above is that they ignore the problem of hedging multicurrency positions. When multiple futures contracts are used to hedge a portfolio consisting of their underlying currencies, portfolio effects are created by the covariances between the futures contracts as well as the components of the spot portfolio. The hedge ratio should depend not only on the covariance of the futures contract with the underlying currency, but also, on its covariance with the other currency futures. Ignoring portfolio effects by hedging each currency in isolation may result in substantial losses for the hedger. In order to account for portfolio effects in a complex hedging problem, the joint estimation of spot and futures prices is necessary.

In the present study, the problem of direct multicurrency hedging is analyzed for four major currencies. Following the methodology introduced by Gagnon, Lypny and McCurdy (1998), two passive indices are constructed, one including positions in the Swiss Franc and the Deutsche Mark and a second consisting of British Pounds and Japanese Yen. Two multivariate GARCH-X models are estimated for the spot portfolios and their respective futures contracts in order to account for the two basic limitations of the conventional (OLS) hedging model. A basic contribution of the present study is that it accounts for the effect of the cointegrating relationship not only on the levels of spot and futures exchange rate returns, but also on their conditional variances. A squared error correction term is included in the conditional variance equation of spot and futures returns as in the GARCH-X model of Lee (1994) for forward contracts. Restrictions for the significance of the error correction term and the GARCH effects are imposed and tested. Static and dynamic portfolio risk minimizing hedge ratios are then derived by the restricted and unrestricted multivariate models. Additionally, four bivariate GARCH-X models are estimated for each spot currency return and the return on its relevant futures contract and no portfolio risk minimizing hedge ratios are also derived.

The comparison between the risk minimizing hedge ratios generated by the portfolio and the no-portfolio model shows that hedging each currency in isolation
tends to overestimate the optimal number of futures contracts. The estimated hedge ratios from the dynamic portfolio model are half in size and more variable than their no-portfolio counterparts. The portfolio effects on the performance of the hedge are also found to be significant as it is shown by the variance reduction and utility tests. The comparison between the static and dynamic strategies shows that the estimated dynamic hedge ratios outperform the constant ones both in terms of variance reduction as well as utility maximization.

However, this analysis proceeds one step further from the empirical study of Gagnon et al (1998). A conditional hedging strategy is applied in the estimated models accounting for transaction costs and allowing the investor to selectively rebalance his futures positions. In the presence of transaction costs, selective rebalancing leads to higher gains in utility for the dynamic portfolio strategy. Additionally, in contrast with the previous study, an out of sample analysis is also performed in order to examine the true efficiency gains resulting from the dynamic portfolio strategy. In terms of the ex-ante performance, the dynamic portfolio hedge is the dominant strategy with increased utility and variance improvement over the static and single hedges, showing the superior forecasting performance of the GARCH models in short investment horizons. The main conclusion emerging from the present study is that, accounting for portfolio effects in a dynamic setting when hedging multicurrency positions, results in substantial risk reduction and utility gains, relative to single hedges, both in the in sample and the out of sample analysis.

Section B contains a review of the empirical issues of exchange rate data that lead to the building of the econometric specification used in the present study. Section C describes the data and preliminary tests performed. The empirical results from the estimation of the multivariate and bivariate GARCH-X models are reported and the risk minimizing hedge ratios are derived, as implied by each strategy, with their descriptive statistics. In Section D, the results of the tests on the hedging performance of each strategy are reported for the in sample and the out of sample analysis. Finally, Section E contains concluding comments on the empirical issues.
addressed in the present chapter.
Section B: Theoretical Issues and Empirical Design

(3.1) Cointegration of spot and futures exchange rates: implications for the development of a hedging model

Existing evidence on spot and forward/futures exchange rates shows that these two variables are found to be cointegrated independently of the data frequencies used in each study. This finding has some implications for the development of hedging models for currency risk since it casts doubt on the widely used OLS methodology for the estimation of the optimal hedge ratio.

It is well recognized in the empirical literature that spot and forward/futures exchange rates are best represented as non-stationary series\(^3\). An important empirical question that follows and has been extensively examined, is whether cointegration\(^4\) exists between foreign currency spot and futures rates. According to Engle and Granger\(^5\) (1987), a cointegrated system can be estimated in a two-step procedure. In the first step, the following OLS regression is estimated:

\[
S_t = \alpha F_t + e_t
\]

In order for cointegration to exist, the residuals, \(e_t\), of the OLS regression must be stationary. The second step involves the introduction of the residuals from the cointegrating regression in a general error correction model. According to the Granger Representation Theorem (1987) any change in one of the two cointegrated

---


\(^4\)Cointegration between two or more time series means that the series move towards a long-run equilibrium relationship and that information from previous periods has explanatory power in the model of the two variables. If two series, for example the spot and futures exchange rates, \(S_t\) and \(F_t\), are integrated of the same order \(d\), \(I(d)\) and there exists a value \(\alpha\) so that \((S_t - \alpha F_t)\) is stationary, then \(S_t\) and \(F_t\) are said to be cointegrated of order \(d\), \(\alpha \approx [S_t, F_t]_{CI(d, \alpha)}\), where \(\alpha\) is called cointegrating vector.

\(^5\)In the present empirical study, the Engle and Granger test is preferred among many other cointegration tests, due to its bivariate design and computational tractability that make it appropriate in order to test for cointegration between spot and futures exchange rates.
variables in this period is related to last period's deviations from equilibrium through an error correction term, and to past changes in both variables. The latter authors proposed the following error correction representation for modelling spot and futures returns in the presence of cointegration:

\[
\begin{align*}
\text{dln}s_t &= c_s + b_{e}\text{ect}_{t-1} + \sum_{i=1}^{n} \alpha_i \text{dln}s_{t-i} + \sum_{j=1}^{m} \beta_j \text{dln}f_{t-j} + \varepsilon_{1t} \\
\text{dln}f_t &= c_f - b_{e}\text{ect}_{t-1} + \sum_{i=1}^{n} \gamma_i \text{dln}s_{t-i} + \sum_{j=1}^{s} \delta_j \text{dln}f_{t-j} + \varepsilon_{2t}
\end{align*}
\]  

(3.1a, 3.1b)

where \(\text{dln}s_t\) is the spot exchange rate first difference
\n\(\text{dln}f_t\) is the futures exchange rate first difference
\n\(\text{dln}s_{t-i}, \text{dln}f_{t-j}\) are the lagged differences of spot and futures prices respectively,
\n\(c_s, c_f\) are constant terms and,
\n\(\text{ect}_{t-1}\) is the error correction term, i.e. the residuals, \(\varepsilon_{t-1}\), of the cointegrating regression that represent the short-run adjustment to deviations from equilibrium.

In view of the significant evidence of cointegration \(^6\) between spot and futures or between spot and forward exchange rates, the use of an error correction model is dictated in the decision making of currency risk management. This empirical issue invalidates the risk minimizing hedge ratio calculated as the slope coefficient from an OLS regression of the spot price change on the futures price change. The omission of dynamics from the OLS equation and the non-stationarity of the data introduce serial correlation in the error term and noise in the asymptotic distribution of the estimates (Engle and Granger (1987)). The standard errors of the static regression are thus biased, leading to a misspecified risk minimizing hedge ratio. As Brenner and Kroner (1995) state, the exclusion of the error correction term from the hedging model, gives a downward biased estimate of the hedge ratio. This finding is supported by Ghosh (1993) who shows empirically that failure to
account for cointegration in a hedging model leads the hedger to a smaller than optimal futures position. Supportive to these results is the theoretical analysis of Lien (1996) who proves that the cost of this suboptimal position is greater the more the spot and futures prices adjust to past disequilibrium. The latter author explains the effect of the cointegrating relationship on the derivation of the risk minimizing hedge ratio on the basis of the error correction model in (3.1).

Using the notation of the system (3.1), the risk minimizing hedge ratio for a given position in the spot market is equal to the covariance of the spot and futures return divided by the variance of the futures return:

\[ h^* = \frac{\text{Cov}(d \ln s_t, d \ln f_t / \Omega_{t-1})}{\text{var}(d \ln f_t / \Omega_{t-1})} \]  \hspace{1cm} (3.2)

where \( \Omega_{t-1} \) is the information set of the previous period, containing a number of conditioning variables.

A hedger who is aware of the cointegrating relationship between spot and futures prices includes in his information set the following variables:

\( \Omega_{t-1} = (\text{ect}_{t-1}, \text{dln}_{s_{t-1}}, \text{dln}_{f_{t-j}}) \)

The hedging model in this case is well specified and the risk minimizing hedge ratio can be computed as:

\[ h_{\text{ECM}}^* = \frac{\text{Cov}(d \ln s_t, d \ln f_t, d \ln s_{t-j}, d \ln f_{t-j}, \text{ect}_{t-1})}{\text{var}(d \ln f_t / d \ln s_{t-j}, d \ln f_{t-j}, \text{ect}_{t-1})} = \frac{\text{Cov}(\varepsilon_{11}, \varepsilon_{21})}{\text{Var}(\varepsilon_{21})} = \frac{\rho \sigma_1}{\sigma_2} \]  \hspace{1cm} (3.3)

---

Where

\( h_{\text{ECM}}^* \) is the optimal demand for futures contracts in the presence of cointegration between spot and futures prices, 
\( \sigma_1, \sigma_2 \) is the standard deviation of the error term \( \varepsilon_1, \varepsilon_2 \) respectively and, 
\( \rho \) is the correlation between \( \varepsilon_1 \) and \( \varepsilon_2 \).

On the other hand, a hedger, who ignores the cointegrating relationship, does not condition his hedging strategy on the error correction term, and his information set is the following:

\[ \Omega_{t-1} = (\text{dln} s_{t-i}, \text{dln} f_{t-j}) \]

The hedging model in this case is misspecified and the risk minimizing hedge ratio is derived as:

\[
h_{\text{OLS}}^* = \frac{\text{Cov}(\text{dln} s_t, \text{dln} f_t / \text{dln} s_{t-i}, \text{dln} f_{t-j})}{\text{var}(\text{dln} f_t / \text{dln} s_{t-i}, \text{dln} f_{t-j})}
\]

\[
= \frac{\text{Cov}(b_s \text{ect}_{t-i} + \varepsilon_{1t}, -b_f \text{ect}_{t-i} + \varepsilon_{2t} / \text{dln} s_{t-i}, \text{dln} f_{t-j})}{\text{var}(\text{ect}_{t-i} + \varepsilon_{2t} / \text{dln} s_{t-i}, \text{dln} f_{t-j})}
\]

\[
= \frac{(b_s^2 \text{var}(\text{ect}_{t-i} / \text{dln} s_{t-i}, \text{dln} f_{t-j}) + \rho \sigma_1 \sigma_2)}{(b_f^2 \text{var}(\text{ect}_{t-i} / \text{dln} s_{t-i}, \text{dln} f_{t-j}) + \sigma_2^2)} \quad (3.4)
\]

A comparison between the two hedge ratios, \( h_{\text{OLS}}^* \) and \( h_{\text{ECM}}^* \), shows that they become equal only when \( b_f = 0 \). However, for \( b_s, b_f > 0, \) or \( b_s = 0, h_{\text{OLS}} < h_{\text{ECM}}, \) implying that the conventional OLS hedging strategy leads to a suboptimal futures position. It is obvious that the higher the \( b_s, b_f \) i.e. the more responsive spot and futures prices are to deviations from equilibrium, the higher is the difference between the optimal and the suboptimal hedge ratios, leading to higher costs for the hedger who ignores the cointegrating relationship. Lien (1996) concludes that the
presence of conditional heteroscedasticity in the series considered does not affect
the previous results and that even when there is time variation in the hedge ratios,
the hedger who ignores cointegration takes a smaller than optimal futures position.

Another important implication of the cointegrating relationship between spot and
futures prices for the usefulness of hedging is given by Lim (1996). He shows that
the hedged portfolio i.e. the combined spot and futures position has a constant
variance over time if spot and futures prices are cointegrated. On the contrary, the
lack of cointegration between these two prices means that the hedged portfolio
value behaves like a random walk with time-varying variance. In this case, the
hedge is no longer a protection against adverse spot price movements, but it leads
the hedger to increased price risk. Testing for cointegration becomes thus crucial
for the development of the appropriate hedging model. As Lien and Luo (1993),
Geppert (1995) and Brenner and Kroner (1995) show, the presence or not of the
cointegrating relationship of spot and futures prices is directly related to the cost-
of-carry futures pricing model. Lien (1992) shows that the satisfaction of the no
arbitrage conditions of the model leads to an improvement of the correlation of the
spot and futures prices, thus increasing the effectiveness of the hedging model.

The importance of cointegration tests and error correction models is acknowledged
by most empirical studies on currency hedging. Ghosh and Chew (1994) apply the
cointegration methodology in order to estimate risk minimizing hedge ratios for the
U.S. Dollar Index. The Error Correction Model is found to be superior to the OLS
model both in terms of statistical specification and forecasting performance,
leading to more effective hedge ratios for multicurrency portfolios. Kroner and
Sultan (1993) find that, although the error correction model of spot and futures
price changes is statistically superior to the OLS model, both models provide
similar hedge ratios for the British Pound, the Canadian Dollar, the German Mark,
estimate error correction models for the same currencies in order to derive
multiperiod optimal hedge ratios, reaching the same conclusions. Additionally,
although evidence of cointegration between spot and forward prices of the JY is
found by Tong (1996) no improvement in the hedging effectiveness of the dynamic strategy is reported after adding an error correction term in the hedging model. The implication of the previous studies for the present thesis is that, although an error correction model does not always provide an improvement in hedging effectiveness, the existence of cointegration between spot and futures rates of the same currency is itself a necessary condition for the success of the futures contract as a hedging vehicle of the underlying currency.

A limitation of the previous studies is that they ignore the effect of cointegration on the conditional second moments of the spot and futures exchange rates that define the risk minimizing hedge ratio. According to Lee (1994), the importance of the error correction models is not limited to the increase in the forecasting power of the conditional mean of the cointegrated series (Engle and Yoo (1987)) but it can also be used for forecasting the conditional variance of the series of interest. In an attempt to model the conditional heteroscedasticity of the forecast error of seven currencies, Lee (1994) included the squared error correction term in a new dynamic model, called GARCH-X, finding that there is a positive relationship between the spot-forward spread and the volatility of the forecast error. The implication of this finding is that error correction models lead to better risk management solutions through the improvement in volatility forecasting. In the present study, two multicurrency spot portfolios are constructed and their components' futures contracts are used for hedging purposes. In the presence of cointegration and conditional heteroscedasticity between spot and futures returns of the same currency, a GARCH-X model will be used for the derivation of dynamic hedge ratios, accounting for the effect of the cointegrating relationship on both the level and variance of spot and futures returns.
(3.2) Conditional heteroscedasticity and the development of a dynamic hedging model

(3.2.1) Hedge ratio instability and the conditional distribution of exchange rates

The correct specification of the levels of spot and futures exchange rates is important in the derivation of the optimal hedge ratios, since misspecifications of the mean equations lead to suboptimal positions. However, according to Hopper (1997), the development of risk management solutions is based on the volatility of currency returns as a measure of currency risk. It is straightforward that the correct modeling of the volatility of exchange rate returns is crucial for the development of hedging models in the foreign exchange market.

Hedging decisions involve the minimization of the risk of systematic price changes in an exchange rate position. A major drawback of the optimal hedge ratio (2.11) derived in Chapter 2, is that it uses, as a measure of risk, unconditional variances and covariances that include unsystematic and predictable elements since they are not conditioned on the existing information set. On the other hand, conditional second moments of the exchange rate changes distribution, involve purely systematic risk components, since they are conditioned on the existing information in the market. According to Kroner and Sultan (1993), the conditional hedge ratio is equal to:

\[ b^* = \frac{E_t(d \ln f_{t+1}) + 2\gamma \text{cov}_t(d \ln s_{t+1}, d \ln f_{t+1})}{2\gamma \text{var}_t(d \ln f_{t+1})} \]  

(3.5)

The \( t \) subscripts in expression (3.5) show that all the expectations and measures of risk are conditioned on the information set available at time \( t \). In this case, the optimal hedge ratio depends on the conditional covariances and variances of spot and futures returns and it is thus time varying. The only exception to this case exists when the joint distribution of spot and futures returns is constant over time,
setting equal the conditional and the unconditional hedge ratios. However, evidence of leptokurtosis and high autocorrelation of squared returns violates the hypothesis of normal and identically and independently distributed spot and futures exchange rate returns. The most popular reason found in the literature for the observed leptokurtosis in financial time series is the time variation in the parameters of the normal distribution.

The time dependence in the joint distribution of spot and futures exchange rates creates many limitations for constant hedging strategies. The imposition of the restriction that the regression coefficients are stable over time may significantly bias the estimation of the optimal hedge ratio and the measure of hedging effectiveness, leading to costly and suboptimal hedging decisions. Indeed, the effect of the misspecifications of the OLS methodology on the derived hedge ratio and its hedging effectiveness is reported by many studies in the hedging literature. Large discrepancies between the in sample and out of sample performance are attributed to the reported instability of estimated hedge ratios.

Lypny (1988) attributes the poor performance of his constant hedge ratio to the time variation of the spot and futures returns distribution. The same factor can explain the results of the empirical studies of Marmer (1986), Malliaris and Urrutia (1991) and Grammatikos and Saunders (1983). The latter authors report upward trends in the estimated hedge ratios and the hedging effectiveness for two of the five currencies examined. Eaker and Grant (1987) and Benet (1990, 1992) report important differences between the ex post and the ex-ante measures of the hedging performance of cross hedging strategies, showing that actual hedgers can incur huge losses in their hedged portfolios when they implement misspecified hedge ratios ex-ante. The time-varying relationship between spot and futures exchange rates is also considered to be the source of statistical problems in the empirical study of Sercu and Wu (1999) for the estimation of currency hedge ratios. The use of a dynamic strategy accounting for the time variation in currency returns is thus required.

\[\text{Friedman and Vandersteel (1982), Hsieh (1989a,b), Fujihara and Park (1990), Diebold and Nason (1990).}\]
According to Bollerslev (1990) and Kroner and Sultan (1991), as new information enters the foreign exchange market, the conditional moments of exchange rate series are changing, leading to time variation in the risk of foreign exchange positions. Hedge ratios must be adapted to this variation in risk through time, in order to minimize the risk of the net-hedged position. Hsieh (1993) and Fujihara and Mougoue (1997) acknowledge the importance of the correct modeling of nonlinearities in the distribution of spot and futures returns, as improving the forecasting performance of hedging models. They state that the probability distribution of futures price changes determines the hedge ratios and the amount of capital needed to cover losses in the margin account. Specifically, Hsieh (1993), uses the estimated conditional densities of the predictable and unpredictable parts of currency futures returns, in order to compute the minimum capital required to cover losses in short and long futures positions.

Holmes (1996) gives another motivation for a dynamic hedging strategy, in terms of the basis risk existing in combined spot and futures positions. According to hedging theory, the risk minimizing hedge ratio is different than the naïve hedge ratio, due to basis risk. Changes in the basis, i.e. the difference between spot and futures prices of the same asset, depend on the behaviour of arbitrageurs in the spot and futures market. Since the behaviour of arbitrageurs changes continuously, the risk minimizing hedge ratio must also change over time.

An additional limitation of the conventional OLS and cointegration hedging models is that they are developed in an one-period framework and can be extended to a multiperiod hedging strategy only under the assumption that the joint distribution of the spot and futures returns is constant over time. A more appropriate model for the estimation of a dynamic hedge ratio should account for the conditional heteroscedasticity of the exchange rate changes. In this case, in order to move from a single period framework to a multiperiod setting, we apply the rebalancing decision in a conditional hedging strategy.

(3.2.2) *Symmetric and asymmetric GARCH models and their use in financial risk management*

Volatility clustering and leptokurtosis can be jointly explained through a time varying model for the series of exchange rate returns. An appropriate model for the conditional second moments of financial returns is the *Generalized Autoregressive Conditional Heteroscedasticity model (GARCH (p,q))* proposed by Bollerslev (1986). In this model, the temporal dependence in the error term is expressed by parameterizing the current conditional variance of the series as a linear autoregressive function of q lagged squared residuals and p lagged conditional variances. Assuming that the error term \( \epsilon_t \) of a stochastic process follows a discrete time stochastic process of the form:

\[
\epsilon_t = z_t \sigma_t,
\]

where \( z_t \) i.i.d., \( E(z_t) = 0, \text{var}(z_t) = 1 \)

and that, by definition, \( \epsilon_t \) is serially uncorrelated with zero mean and conditional variance \( \sigma_t^2 \) that changes through time, the GARCH model is defined as:

\[
\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2
\]

where \( p, q \) is the number of lags.

In order for this model to be well specified, the parameters \( \alpha_i \) and \( \beta_i \) must be nonnegative. Moreover, the sum of \( \alpha_i \)'s and \( \beta_i \)'s must be less than unity so that the variance \( \sigma_t^2 \) is finite. The coefficient of kurtosis for GARCH (1,1) is greater than zero by assumption, making the unconditional error distribution of the GARCH (1,1) process leptokurtic by construction. This explains why the GARCH model has
been found successful in capturing the nonlinearity of financial returns in many econometric applications.

There exists substantial empirical evidence on the presence of conditional heteroscedasticity in models of foreign exchange data. The majority of documented studies conclude that the GARCH (1,1) model is the most successful in capturing the empirical regularities of spot and futures exchange rates. It is straightforward that the conditional variance of foreign exchange spot and futures returns can be modeled with a GARCH specification that allows for time variation and persistence in the information arrival process. From the general structure of the GARCH model, several models of this type can be constructed, depending on the behaviour of the speculative prices under research. For example, there is the exponential GARCH model (EGARCH), introduced by Nelson (1990), that accounts for asymmetric reactions of volatility to different signs and sizes of information shocks.

\[
\log \sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \left( \delta z_{t-i} + \gamma \left[ |z_{t-i}| - E|z_{t-i}| \right] \right) + \sum_{i=1}^{p} \beta_i \log \sigma_{t-i}^2 \quad (3.7)
\]

As it is shown by the expression (3.7), the EGARCH model allows the conditional variance to respond differently to a positive versus a negative shock of the underlying error term. This asymmetry is introduced through the second term in the right-hand side of the model. If \( \alpha_i \delta < 0 \), a negative (positive) shock in the previous period will increase (decrease) the conditional variance in the present period. This effect is known as the “leverage effect” observed in equities markets; bad news, in the form of lower expected returns, tend to increase volatility more than good news, due to the increase of the debt to equity ratio that increases corporate leverage and the risk of holding stocks. The parameter \( \gamma \) measures the size effect of past shocks.

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a positive $\gamma$ means that, a deviation of the absolute value of a price shock from its expected value will produce large unexpected price changes.

Although many empirical studies exist on the development of conditional volatility models for exchange rates, only a few have attempted to investigate the existence of asymmetric behaviour of exchange rates. Most of them conclude that an asymmetric GARCH model is superior to the standard GARCH model in terms of statistical validity but they fail to find any statistical significance of the asymmetry parameter\textsuperscript{10}. According to Gallant, Hsieh and Tauchen (1991), the lack of asymmetric behaviour in the conditional variance of exchange rate returns is due to the two-sided nature of the foreign exchange market. Additionally, Ederington and Guan (1999) find no evidence of substantial superiority of the EGARCH model over the GARCH (1,1) model for exchange rates.

Two exceptions to the previous results are the empirical studies of Koutmos (1993) and Laopodis (1997) that provide evidence of asymmetric behaviour of EMS and non-EMS daily exchange rates with respect to appreciations and depreciations of the U.S. dollar. The asymmetry parameter is negative and statistically significant for five out of six currencies examined, with the exception of the JY. These results imply that U.S. dollar depreciations lead to greater exchange rate volatility than U.S. dollar appreciations. According to Koutmos (1993), this finding can be explained by the fact that the dollar is used globally as a reserve currency and any depreciation can lead to greater uncertainty. However, when weekly data are used, asymmetries remain significant only for EMS currencies. This finding implies that the evidence of asymmetric behaviour in exchange rate data depends on the sample period and the data frequency used. In the present study, both a symmetric and an exponential GARCH specification will be estimated for the variables of interest and statistical comparisons will be performed in order to choose the most appropriate representation for the derivation of the dynamic hedge ratios.

The advantages of GARCH hedging models versus constant hedging models in terms of statistical performance are cited in all relevant studies. Kroner and

Claessens (1991) derive the optimal debt portfolio for Indonesia against foreign exchange risk, by estimating the conditional variances and covariances of the rupiah with respect to eight currencies and a price variable in a GARCH (1,1) model. Kroner and Sultan (1991) estimate the optimal hedge ratio of currency futures for major currencies with a constant correlation bivariate GARCH (1,1) specification. The model is found statistically valid by several diagnostic tests, outperforming all constant hedging models in terms of in sample variance reduction. Lin, Najand and Yung (1994) estimate OLS and GARCH models for five major currencies and they report significant variance reductions for the GARCH model for the 2-week hedges, although no advantage is found for the 4-week hedges. Similar conclusions are made for the same currencies from Lien and Luo (1994) who derive multiperiod hedge ratios from the OLS, the ECM and the GARCH model. They report significant differences in the estimated optimal hedge ratios with the GARCH model outperforming ex-ante the constant hedging strategies in terms of variance reduction. Tong (1996) reports similar results after applying a GARCH (1,1) model to JY spot and forward returns that reduces the variance of the hedged portfolio by 6% ex post and by 2% ex-ante compared to the constant hedging strategies.

A basic characteristic of the previous studies is that they use as a measure of hedging performance only the variance reduction performed by each hedging model. Kroner and Sultan (1993) also use the utility performance of each model as a measure of the economic significance of each strategy, taking into account the transaction costs. The dynamic hedge ratio, derived by the Error Correction GARCH (1,1) model, is found to outperform the constant hedge ratios both in terms of risk reduction and utility increases in the in sample and out of sample tests. The dynamic strategy also provides the hedger with the possibility of rebalancing only when the utility from revising the hedged position is greater than the transaction cost. Gagnon, Lypny and McCurdy (1998) use a trivariate GARCH (1,1) model in order to estimate the optimal hedge ratio for two two-currency portfolios. Their in sample tests reveal that, even with daily rebalancing and in the presence of transaction costs, the dynamic hedge outperforms the static hedges in
terms of risk reduction and utility.

The main conclusion from the empirical evidence presented above is that, the success of GARCH models over static models in estimating effective currency hedge ratios dictates the use of a dynamic model in order to estimate time varying hedge ratios for the two currency portfolios examined. The previous studies are extended by examining the effectiveness of the spot-futures spread in explaining the volatility of spot and futures exchange rate returns in a GARCH-X model. To the best of our knowledge, no empirical study has attempted so far to capture this effect in the case of hedging with futures contracts.
(3.3) The GARCH-X hedging model

Concluding, the following GARCH-X (1,1) representation will be used for the estimation of the time-varying risk minimizing hedge ratio for the two alternative currency portfolios. This model accounts for the misspecifications of the traditional OLS hedging model that provides the constant risk minimizing hedge ratio as derived by Ederington (1979).

\[
\begin{align*}
\text{d} \ln p_t &= C_{\text{spot}} + (\text{Beta}_{\text{spot}} \times \text{ect}_{\text{spot}, t-1}) + \sum_{i=1}^{\alpha} (\gamma_{in} \times \text{d} \ln p_{t-i}) + \sum_{j=1}^{\beta} (\gamma_{jf} \times \text{d} \ln F_{p,t-j}) + \epsilon_{pt} \\
\text{d} \ln f_{1,t} &= C_1 + (\text{Beta}_1 \times \text{ect}_{f1,t-1}) + \sum_{h=1}^{p} (\phi_{hf} \times \text{d} \ln f_{1,t-h}) + \sum_{k=1}^{q} (\phi_{ks} \times \text{d} \ln s_{1,t-k}) + \epsilon_{f1,t} \\
\text{d} \ln f_{2,t} &= C_2 + (\text{Beta}_2 \times \text{ect}_{f2,t-1}) + \sum_{l=1}^{p} (\theta_{lf} \times \text{d} \ln f_{2,t-l}) + \sum_{r=1}^{q} (\theta_{rs} \times \text{d} \ln s_{2,t-r}) + \epsilon_{f2,t} \\
\h^2_{st} &= D_0 + D_1 h^2_{st(t-1)} + D_2 \epsilon^2_{st(t-1)} + D_3 \text{ect}^2_{\text{spot},t-1} \\
\h^2_{n1} &= E_0 + E_1 h^2_{n1(t-1)} + E_2 \epsilon^2_{n1(t-1)} + E_3 \text{ect}^2_{n1,t-1} \\
\h^2_{n2} &= F_0 + F_1 h^2_{n2(t-1)} + F_2 \epsilon^2_{n2(t-1)} + F_3 \text{ect}^2_{n2,t-1} \\
h_{\text{spot},f1} &= P_{\text{spot},F1} \times (h_{sf} \times h_{fn}) \\
h_{\text{spot},f2} &= P_{\text{spot},F2} \times (h_{sf} \times h_{f2}) \\
h_{fn},h_{f2} &= P_{F1,F2} \times (h_{fn} \times h_{f2}) \\
\end{align*}
\]

where \( \text{dlnp}_t \) is the spot currency portfolio return,
\( \text{dlnf}_{1,t} \) is the return of the first futures contract,
\( \text{dlnf}_{2,t} \) is the return of the second futures contract,
\( c_{\text{spot}}, c_1, c_2 \) are constant terms,
\( \text{ect}_{\text{spot},t-1} \) is the error correction term for the spot portfolio,
estimated by the lagged residuals of the regression of \( \ln p_t \) on \( f_{p,t} \)
with \( f_{p,t} = \ln (w_1 \times F_{1,t} + w_2 \times F_{2,t}) \) (\( w_1, w_2 \) are the weights of the
currencies used in each spot portfolio and $F_{1,t}, F_{2,t}$ are the levels of the prices of the respective futures contracts).

dlns_{1,t-k}, dlnf_{1,t-h}, dlns_{2,t-r}, dlnf_{2,t-l}$ are the lagged differences of spot and futures prices respectively,
dlnp_{t,i}, dlnf_{p,t-j}$ are the lagged returns on the spot and futures portfolio respectively and,

$e_{ct1,t-1}$ and $e_{ct2,t-1}$ are the error correction terms for the two futures contracts of each multicurrency portfolio, estimated by the lagged residuals of the regression of $lns_{1,t}$ on $lnf_{1,t}$ and $lns_{2,t}$ on $lnf_{2,t}$ respectively.

$h_{st}^2, h_{ft}^2$ and $h_{fr}^2$ are the conditional variances of the spot and futures returns respectively

$h_{spot,1}, h_{spot,2}$ are the conditional covariances between spot portfolio and futures returns,

$p_{spot,1}, p_{spot,2}$ are the conditional correlations between spot portfolio and futures returns

$h_{1,2}$ is the conditional covariance of futures returns,
p_{1,2} is the conditional correlation of futures returns and,

e_{pt}, e_{f1}, e_{f2,t}$ are the spot and futures returns innovations at time $t$.

In the presence of cointegration between spot and futures exchange rates an error correction model will be used for the mean equations of the spot portfolio and futures returns, as described in equations (a), (b) and (c) of the system (3.8). A GARCH-X (1,1) model is used for the conditional variances of spot portfolio and futures returns as described in equations (d), (e) and (f). As it is reported in the previous paragraphs, the lag order of one for the variances and squared errors is supported by many empirical studies on foreign exchange rates. The squared error correction term, as estimated by the residuals of the cointegrating regression, is also included in the conditional variance equations. Equations (g) and (h) describe the conditional covariances of spot and futures returns as a proportion of the product of their conditional standard deviations, while the conditional correlation is assumed to be constant over time. Although Gagnon et al (1998) used the Baba, Engle and Kroner (BEKK) (1991) representation allowing for time varying conditional
correlations in their multivariate GARCH model, in the present study this specification is not used due to the reported "internal inconsistency problem". According to Ding (1994), the BEKK model is not parsimonious and the estimated unconditional covariance matrices may not be positive definite during estimation. Additionally, the full model requires the estimation of a large number of parameters and convergence is difficult to obtain. On the other hand, the constant conditional correlation model leads to a major reduction in the computational complexity, and its validity can be examined with tests of remaining cross correlation in the standardized residuals of the GARCH-X model. Finally, portfolio effects are accounted for in the present study by estimating the conditional covariance of the futures returns (equation (i)).

An additional advantage of model (3.8) is that it specifies spot and futures returns as a system of simultaneous equations. According to Chatrath and Song (1998), the linear combination of the two series, may lead to less persistence in their conditional variances than their univariate representations since the persistence may be common across the series.

(3.4) Distributional issues

As it is reported in Section 3.2, GARCH models are used in order to capture the observed nonlinearity in the conditional variance of asset returns. However, excess kurtosis is detected on the standardized residuals from these models, after the assumption of conditional normality for the distribution of FX returns. This finding has led many authors to the assumption of fat-tailed distributions such as the Student-t, with degrees of freedom estimated by the model (see Baillie and Bollerslev (1989a)) or other non-normal distributions (Hsieh (1989)). According to Huisman et al (1998), this distributional assumption may lead to a misspecified model and significant estimation errors because the data-generating processes of alternative distributions are non-nested. Additionally, the same authors state that for foreign currencies, a symmetric Student-t distribution is inappropriate due to the inequality of left and right tails reported for many currencies.
As Bollerslev and Woolridge (1989) and Weiss (1986) state, under the assumption that the conditional mean and variance equations are correctly specified, the quasi-maximum likelihood estimates are still consistent and asymptotically normal, even when the assumption of conditional normality is violated. However, the usual standard errors need to be modified. According to Lumsdaine (1995), the Quasi-MLE is a generalization of MLE when the true underlying distribution is unknown. In the present study, the method of Quasi-Maximum Likelihood estimation is used with robust standard errors, since it provides consistent estimates of the dynamic multivariate models.
Section C: Data and Empirical Results

(3.5) Data

Daily exchange rates vis-à-vis the U.S. Dollar are obtained for the British Pound ($/BP), German Mark ($/DM), Swiss Frank ($/SF) and Japanese Yen ($/JY) as traded in the New York Foreign Exchange Market. The specific currencies are chosen on the basis of their active trading in the New York and London foreign exchange markets. Futures closing prices for the four major currencies are obtained from the International Monetary Market (IMM). The nearby contract is used since both the open interest and the volume of trade, are higher than for more distant contracts, introducing high liquidity in the futures market. However, according to Castelino (1992) both measures of trading drop sharply near the expiration of the futures contract, introducing distortions in the futures prices. Continuous series of the futures prices are constructed by replacing contracts ten days before maturity by the next nearest contracts. The sample covered the period from January 7, 1988 to October 2, 1997 (2541 observations).

The marking-to-market feature of futures contracts causes daily changes to their value, making the use of daily data necessary in order to derive financial risk management solutions. According to Frankle (1980), the use of daily observations for the estimation of the optimal futures hedge, leads to increased covariance of spot and futures prices and an important reduction in variance, relative to the use of weekly data. The use of daily data is also dictated by the fact that more precise inference is possible when a long span of high frequency data is used in tests for cointegration. Additionally, as evidenced by Baillie and Bollerslev (1989) and other authors, the degree and persistence of the time-dependent heteroscedasticity and the level of leptokurtosis which are characteristics of many financial time series, are more evident in daily rather than weekly exchange rate data. McCurdy and Morgan (1987), attribute this result to the highly organized nature and activity of foreign exchange markets that allow only short lived persistence and shocks to the mean and volatility.
(3.6) Seasonality tests

Daily return series of the spot and futures exchange rates are constructed as the differences between the natural logs of the present and the previous period.

\[ D\ln S_t = \ln S_t - \ln S_{t-1} \]
\[ D\ln F_t = \ln F_t - \ln F_{t-1} \]

Where

\( \ln S_t \) is the natural logarithm of the spot exchange rate in the present period
\( \ln S_{t-1} \) is the natural logarithm of the spot exchange rate in the previous period
\( \ln F_t \) is the natural logarithm of the futures exchange rate in the present period
\( \ln F_{t-1} \) is the natural logarithm of the futures exchange rate in the previous period

However, daily economic series exhibit seasonal patterns in the form of day-of-the-week and holiday effects that lead to increased variance and low forecasting performance of the estimated models. Before proceeding to the analysis of spot and futures exchange rates, it is thus necessary to remove any seasonalties that may be present in the dataset (see McFarland (1982) and Baillie and Bollerslev (1989)). According to Birchenall et al (1987), not accounting for these effects, causes serious problems in dynamic models. For this reason, spot and futures exchange rate returns are filtered by day-of-the-week effect adjustment regressions. The adjustments consist of regressions of the series under investigation on dummy variables, representing each day of the week, \( D_{\text{Mon}} \), \( D_{\text{Tu}} \), \( D_{\text{W}} \) and \( D_{\text{Th}} \), and a dummy variable that accounts for holidays over the sample period, \( D_{\text{Hol}} \). From table 3.1, it is obvious that for most return series examined, there is a significant Monday and end-of-the-week effect. Holidays do not seem to affect foreign exchange returns for the specific sample period.

Additionally, since our sample period includes the date of the ERM crisis of the British Pound, which is the 16th of September 1992 (obs.1225), a dummy variable
accounting for this effect is introduced in the filtering regressions of the BP spot and futures returns ($D_{ERM}$). As it can be seen, in Table 3.1, the coefficient $C_{ERM}$ is highly significant in both equations of the BP, indicating that the inclusion of this observation in our dataset would affect the estimation results. A new dataset is constructed by the residuals of the filtering regressions, which is free of any seasonalities. For the remaining of the chapter, all preliminary tests and estimations will be performed on the new series of spot and futures returns.
Table 3.1: Day-of-the-week effect adjustment regressions of foreign exchange returns

<table>
<thead>
<tr>
<th></th>
<th>DLSSF</th>
<th>DLFSF</th>
<th>DLSDM</th>
<th>DLFDM</th>
<th>DLSBP</th>
<th>DLFBP</th>
<th>DLSJY</th>
<th>DLFJY</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>-0.0605</td>
<td>-0.0514</td>
<td>-0.0624</td>
<td>-0.0511</td>
<td>-0.0005</td>
<td>-0.0007</td>
<td>-0.0397</td>
<td>-0.0480</td>
</tr>
<tr>
<td></td>
<td>(0.0341)</td>
<td>(0.0345)</td>
<td>(0.0309)*</td>
<td>(0.0311)*</td>
<td>(0.0003)</td>
<td>(0.0003)*</td>
<td>(0.0306)</td>
<td>(0.0311)</td>
</tr>
<tr>
<td>C&lt;sub&gt;Mon&lt;/sub&gt;</td>
<td>0.1038</td>
<td>0.1031</td>
<td>0.0841</td>
<td>0.0758</td>
<td>0.0005</td>
<td>0.0009</td>
<td>0.0536</td>
<td>0.0916</td>
</tr>
<tr>
<td></td>
<td>(0.0483)*</td>
<td>(0.0488)*</td>
<td>(0.0437)*</td>
<td>(0.044)*</td>
<td>(0.0004)</td>
<td>(0.00044)*</td>
<td>(0.0434)</td>
<td>(0.0441)*</td>
</tr>
<tr>
<td>C&lt;sub&gt;Tu&lt;/sub&gt;</td>
<td>0.0601</td>
<td>0.0365</td>
<td>0.0771</td>
<td>0.0399</td>
<td>0.0008</td>
<td>0.0009</td>
<td>0.0575</td>
<td>0.0585</td>
</tr>
<tr>
<td></td>
<td>(0.0481)</td>
<td>(0.0487)</td>
<td>(0.0436)*</td>
<td>(0.0439)</td>
<td>(0.0004)*</td>
<td>(0.00044)*</td>
<td>(0.0432)</td>
<td>(0.0439)</td>
</tr>
<tr>
<td>C&lt;sub&gt;W&lt;/sub&gt;</td>
<td>0.0311</td>
<td>0.0252</td>
<td>0.0509</td>
<td>0.0493</td>
<td>0.0003</td>
<td>0.0007</td>
<td>-0.0031</td>
<td>-0.0035</td>
</tr>
<tr>
<td></td>
<td>(0.0481)</td>
<td>(0.0486)</td>
<td>(0.0436)</td>
<td>(0.0439)</td>
<td>(0.0004)</td>
<td>(0.00044)</td>
<td>(0.0432)</td>
<td>(0.0439)</td>
</tr>
<tr>
<td>C&lt;sub&gt;Th&lt;/sub&gt;</td>
<td>0.0977</td>
<td>0.08144</td>
<td>0.0943</td>
<td>0.0799</td>
<td>0.0005</td>
<td>0.0011</td>
<td>0.1067</td>
<td>0.1133</td>
</tr>
<tr>
<td></td>
<td>(0.0480)*</td>
<td>(0.0486)*</td>
<td>(0.0435)*</td>
<td>(0.0438)*</td>
<td>(0.0004)</td>
<td>(0.00044)*</td>
<td>(0.0432)*</td>
<td>(0.0439)*</td>
</tr>
<tr>
<td>C&lt;sub&gt;Hol&lt;/sub&gt;</td>
<td>-0.0396</td>
<td>-0.0368</td>
<td>-0.0568</td>
<td>-0.0295</td>
<td>-0.0002</td>
<td>-0.0001</td>
<td>-0.0267</td>
<td>-0.047</td>
</tr>
<tr>
<td></td>
<td>(0.0903)</td>
<td>(0.0914)</td>
<td>(0.0818)</td>
<td>(0.0824)</td>
<td>(0.0008)</td>
<td>(0.00083)</td>
<td>(0.0812)</td>
<td>(0.0826)</td>
</tr>
<tr>
<td>C&lt;sub&gt;ERM&lt;/sub&gt;</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-0.0426</td>
<td>-0.0447</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.0066)*</td>
<td>(0.0070)*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: The following regression is estimated for the spot and futures returns of all currencies examined:

\[ D_{\text{Line}} = C + C_{\text{Mon}} + D_{\text{Mon}} + C_{\text{Tu}} + D_{\text{Tu}} + C_{\text{W}} + D_{\text{W}} + C_{\text{Th}} + D_{\text{Th}} + C_{\text{Hol}} + D_{\text{Hol}} + C_{\text{ERM}} + D_{\text{ERM}} + u \]

where \( D_{\text{Line}} \) is the first difference of the natural logarithm of the spot (or future) exchange rate, \( D_{\text{Mon}}, D_{\text{Tu}}, D_{\text{W}}, D_{\text{Th}}, D_{\text{Hol}} \) are dummy variables accounting for each day of the week and holidays and, \( D_{\text{ERM}} \) is a dummy variable accounting for the ERM crisis of September 1992. Asymptotic standard errors are given in parentheses and asterisks are used to indicate the significance of the estimated coefficients at the 95% level.
(3.7) Preliminary data analysis of spot and futures exchange rates

In the present section, the procedure followed for the construction of the two spot portfolios is described and preliminary statistical tests are performed on their returns and the returns of their respective futures contracts. However, before proceeding to the analysis of spot and futures returns, it is necessary to perform cointegration tests between the natural logs of spot and futures exchange rates in order to test whether an error correction term should be included in the hedging model.

As most financial series, exchange rates exhibit non-stationarity making their econometric analysis complex and the use of returns instead of levels necessary. Before testing for cointegration, the Dickey-Fuller (DF) (1981) and Phillips-Perron (PP) (1988) unit root test statistics are applied on the natural logs of the spot and futures prices of the four currencies under investigation. These tests examine the null of non-stationarity against the alternative hypothesis of stationarity. The DF test is based on the assumption that the error terms of the test regressions are statistically independent and they have constant variance. The PP test is generally considered to be more robust to serially correlated and heteroscedastic errors than the DF test.

In the DF unit root tests performed on the natural logs of spot and futures exchange rates, the procedure proposed by Dolado, Jenkinson and Sosvilla Rivero (1990) is followed in order to test for the significance of the deterministic regressors. The most general model is tested first, i.e. the one including both the drift and the trend. Since the null of a unit root cannot be rejected, we proceed to testing for the significance of the trend using the $\phi_3^{11}$ and $\tau_{\beta T}^{12}$ statistics provided by Dickey and Fuller (1981). In table 3.2, both statistics show that the trend is not significant for any of the series examined. The second step of the proposed procedure involves an

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$^{11}$The $\phi_3$ statistic is an F-test for the joint hypothesis of a unit root and an insignificant trend term ($\gamma = \alpha_2 = 0$) in the model: $\Delta y_t = \alpha_0 + \gamma y_{t-1} + \alpha_2 \tau_t + \Sigma \beta_k \Delta y_{t+k} + \epsilon_t$

$^{12}$The $\tau_{\beta T}$ statistic is a t-test for the significance of the trend ($\alpha_2 = 0$) under the null of a unit root ($\gamma = 0$) in the model: $\Delta y_t = \alpha_0 + \gamma y_{t-1} + \alpha_2 \tau_t + \Sigma \beta_k \Delta y_{t+k} + \epsilon_t$
ADF test with a constant but without a trend term. The null hypothesis is not rejected and we proceed to the DF tests for the significance of the constant term with the $\tau_{nm}$ and the $\phi_1$ statistics. It is clear from table 3.2 that the constant term is insignificant for all series examined. The results of the tests performed so far indicate that the unit root tests on the natural logs of spot and futures exchange rates should be made without a constant or a trend. In table 3.2, columns (5) and (6) contain the results of the ADF ($\tau$) and Phillips-Perron ($Z(t_{a*})$) tests performed on the natural logs of spot and futures prices of the four currencies. The hypothesis of non-stationarity cannot be rejected for any of the currencies examined. The data are first differenced and the same tests are performed on the first differences (not reported for reasons of limited space), rejecting the null hypothesis of non-stationarity. We can conclude that all the series of spot and futures prices are first order integrated (I (1)).

However, the ERM crisis that took place over the sample period examined (16th of September 1992 (obs. 1225)), may have created structural breaks in the series of spot and futures BP prices. In this case, the ADF and PP tests are biased toward the non-rejection of a unit root. For the case of the United Kingdom, the effect of Black Wednesday must be considered. One method of testing for a unit root in the presence of a structural break is splitting the sample into two subsets and performing unit root tests in each subset. However, this procedure leads to less degrees of freedom for each regression of the test, so, it is preferable to perform the unit root test on the full sample. Perron (1989) developed a formal procedure to test for unit roots in the presence of a structural break. If $\tau$ is the date of the structural break, the most general model involves estimating the following regression:

$$y_t = a_0 + a_1 y_{t-1} + a_2 t + \mu_2 D_L + \mu_3 D_T + \sum_{i=1}^{k} \beta_i \Delta y_{t-i} + \epsilon_t$$

13The $\tau_{nm}$ statistic is a t-test for the significance of the constant term ($a_0=0$) under the null of a unit root ($\gamma=0$) in the model: $\Delta y_t = a_0 + \gamma y_{t-1} + \sum \phi_k \Delta y_{t-k} + \epsilon_t$

14The $\phi_1$ statistic is an F-test of the joint hypothesis of a unit root and an insignificant constant term ($a_0=\gamma=0$) in the model: $\Delta y_t = a_0 + \gamma y_{t-1} + \sum \phi_k \Delta y_{t-k} + \epsilon_t$

15 Different numbers of truncation lags were used in the PP tests without any significant differences in the results. The reported results are those with eight lags which is considered as a compromise between the large size distortions under the null hypothesis for $l=4$ and the low power of the test for $l=12$. 99
where \( D_L = 0 \) for \( n = 1, \ldots, \tau \)
\[
D_L = 1 \quad \text{for} \quad n = \tau + 1, \ldots, T
\]
and \( D_T = 0 \) for \( n = 1, \ldots, \tau \)
\[
D_T = t \quad \text{for} \quad n = \tau + 1, \ldots, T
\]

The dummy variables \( D_L \) and \( D_T \) represent a change in the intercept and the slope of the trend \( t \) respectively. After estimating the regression above, the t-statistic for the null \( a_1 = 1 \) can be compared to the appropriate critical value calculated by Perron for the value of \( \lambda \) corresponding to the specific date of the structural break. The value of \( \lambda \) for the UK is 0.5. The dummy variable \( D_L \) is found statistically significant for both spot and futures exchange rate series examined, while the dummy \( D_T \) is insignificant in both cases. The hypothesis \( a_1=1 \) is accepted\(^6\) supporting the previous results thus implying that the UK spot and futures exchange rates are non-stationary.

We can now proceed to the two step methodology\(^7\) of Engle and Granger in order to find out whether spot and futures prices of the same currency share a common stochastic trend. In columns (7) and (8) of Table 3.2, both the ADF (\( \tau_{Res} \)) and Phillips-Perron (\( Z(t_{a_i}) \)) cointegration tests suggest that the null hypothesis of no cointegration between the spot and the futures price of the same currency is rejected for all currencies examined. This result is supported by many empirical studies (Kroner and Sultan (1993), Baillie and Bollerslev (1989) and Hakkio and Rush (1989)). The cointegrating vector \( b \) is estimated to be close to 1, suggesting that the spot-futures basis is stationary. We can conclude that spot and futures exchange rates are cointegrated with cointegrating vector \( (1, -1) \). An error correction term, equal to the lagged residuals of the cointegrating regression, should be included in the mean equations of the hedging models to ensure that the long run relationship between spot and futures prices is maintained. Additionally, following the example of Lee (1994), in the presence of conditional heteroscedasticity of spot and futures returns, a GARCH-X specification will be

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\(^6\) The estimated t-statistics for the BP spot and futures exchange rates are -2.48 and -1.674 respectively while the PP critical value for \( \lambda=0.5 \) is -3.76.

\(^7\) This methodology is preferred in the present chapter on the basis of simplicity and efficiency in the bivariate case. However, in the multivariate case, the Engle and Granger methodology has important defects such as dealing with the existence of more than one cointegrating vectors (see chapter 5).
applied to the conditional variance equations in order to account for the disequilibrium in higher moments of exchange rate returns.

Before proceeding to further statistical tests, two spot currency portfolios are constructed on the basis of the unconditional correlation matrix of the returns of the currencies examined. One portfolio consisting of two highly correlated currencies will be compared with another consisting of two currencies of low correlation, so that the importance of the level of correlation between currency returns is examined for the realization of portfolio effects.

Based on Table 3.3.1, the first portfolio constructed includes spot positions in the SF and the DM, two highly correlated currencies, while the second portfolio consists of positions in the BP and the JY that are shown to have the lowest unconditional correlation of 0.46338. Following the methodology of Gagnon et. al (1998), each portfolio is a passive index of the two currencies, with value-based weights that are set on the first day of the sample and remain constant for the whole sample period. In this way, the decision of rebalancing the futures portfolio depends solely on spot price risk while quantity risk is not an issue for the hedger. The SF-DM spot portfolio consists of \( w_{SF} = 1.3444 \) Swiss Francs and \( w_{DM} = 1.649077 \) Deutsche Marks, while the BP-JY portfolio consists of \( w_{BP} = 0.55371 \) British Pounds and \( w_{JY} = 1.293996 \) Japanese Yen. Table 3.3.2 represents the unconditional correlation matrices of the two spot portfolio returns with their component currency futures returns.

In Table 3.4, the insignificant t-statistics for the means, reported in parentheses, show that the hypothesis of zero-mean returns cannot be rejected for any of the futures contracts. This finding is consistent with the assumption of zero futures returns (\( E(R_f) = 0 \) in (2.11)) in the present study and that the optimal hedge ratio (2.11) reduces to the risk minimizing objective (2.12). Substantial departures from normality for the unconditional distributions of the spot portfolio and futures returns are indicated by the highly significant skewness and kurtosis measures. This result is supported by the significance of the Bera-Jarque statistic that rejects the
null hypothesis of normal distribution for all series examined. As it is reported in section B of this chapter, evidence of non-normality is mainly due to the temporal dependence of the higher moments of a series. In order to test for linear and non-linear dependence, the Ljung-Box (1978) portmanteau test for serial correlation is applied to the returns and squared returns of spot and futures exchange rates. As the LB (20) statistic shows, no evidence of linear dependence is provided, since the first moments of the series are not found to be autocorrelated, with the exception of the JY futures return. However, when the same statistic is applied on squared data (LBsQ (20)), in order to test for nonlinear dependence, the results are highly significant. Since time-variation in the variances is the main source of nonlinear dependence, the introduction of a GARCH error structure in the hedging model becomes necessary.

The main conclusion from the preliminary analysis of spot and futures exchange rate returns, is that they can be described as linearly unpredictable, leptokurtic and conditionally heteroscedastic. In the following section, a multivariate GARCH-X (1,1) model that accounts for these empirical regularities will be estimated for the two spot portfolios and their respective currency futures components.
Table 3.2: Unit Root and Cointegration tests of spot and futures exchange rates

<table>
<thead>
<tr>
<th>Variables</th>
<th>$\tau_{pr}$</th>
<th>$\varphi_3$</th>
<th>$\tau_{qu}$</th>
<th>$\phi_1$</th>
<th>$\tau^{18}$</th>
<th>$Z(t_a)^{19}$</th>
<th>$\tau_{Res}^{20}$</th>
<th>$Z(t_a)^{21}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SF spot</td>
<td>0.0213</td>
<td>5.0063</td>
<td>-0.2319</td>
<td>3.4916</td>
<td>-0.31</td>
<td>-0.34</td>
<td>-4.37</td>
<td>-8.71</td>
</tr>
<tr>
<td>SF futures</td>
<td>0.0853</td>
<td>5.2412</td>
<td>-0.2374</td>
<td>3.5063</td>
<td>-0.31</td>
<td>-0.33</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DM spot</td>
<td>-0.4352</td>
<td>4.0137</td>
<td>-0.2283</td>
<td>3.6239</td>
<td>-0.10</td>
<td>-0.11</td>
<td>-4.18</td>
<td>-7.80</td>
</tr>
<tr>
<td>DM futures</td>
<td>-0.3598</td>
<td>4.3709</td>
<td>-0.2390</td>
<td>3.7937</td>
<td>-0.09</td>
<td>-0.10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BP spot</td>
<td>0.3544</td>
<td>6.2651</td>
<td>-0.3286</td>
<td>5.3754</td>
<td>-0.68</td>
<td>-0.68</td>
<td>-7.30</td>
<td>-13.51</td>
</tr>
<tr>
<td>BP futures</td>
<td>0.3764</td>
<td>6.4861</td>
<td>-0.3260</td>
<td>6.0923</td>
<td>-0.68</td>
<td>-0.68</td>
<td></td>
<td></td>
</tr>
<tr>
<td>JY spot</td>
<td>-0.4193</td>
<td>1.7301</td>
<td>0.1421</td>
<td>1.4283</td>
<td>-0.86</td>
<td>-0.91</td>
<td>-5.87</td>
<td>-13.40</td>
</tr>
<tr>
<td>JY futures</td>
<td>-0.3844</td>
<td>1.7833</td>
<td>0.1565</td>
<td>1.4264</td>
<td>-0.87</td>
<td>-0.92</td>
<td></td>
<td></td>
</tr>
<tr>
<td>95% cr.</td>
<td>2.79</td>
<td>6.49</td>
<td>2.54</td>
<td>4.71</td>
<td>-1.95</td>
<td>-1.94</td>
<td>-3.34</td>
<td>-3.37</td>
</tr>
<tr>
<td>99% cr.</td>
<td>3.53</td>
<td>8.73</td>
<td>3.22</td>
<td>6.70</td>
<td>-2.60</td>
<td>-2.58</td>
<td></td>
<td>-4.07</td>
</tr>
</tbody>
</table>

$\tau$ is the Augmented Dickey Fuller test statistic for the hypothesis $\gamma = 0$ in the model

$$\Delta Y_t = \gamma Y_{t-1} + \sum_k \delta Y_{t-k} + \epsilon_t$$

where $k$ is the number of lags of the differenced series added to the model in order to remove any serial correlation from the residuals. Three lags were added in the case of the JY and the SF, four lags in the case of DM and no lags in the case of the BP.

$Z(t_a)$ is the Phillips and Perron (1988) test statistic for a unit root, with a truncation lag of 8.

$\tau_{Res}$ is the Augmented Dickey Fuller test statistic for the hypothesis $\delta = 0$ in the model

$$\Delta e_t = \delta e_{t-1} + \sum_k \Delta e_{t-k} + \epsilon_t$$

where $\epsilon_t$ are the residuals from the cointegrating regression.

$Z(t_a)$ is the Engle and Granger (1987) test for cointegration.
Table 3.3.1: Unconditional Correlation Matrix of major currency returns

<table>
<thead>
<tr>
<th></th>
<th>DLSSF</th>
<th>DLSH</th>
<th>DLSBP</th>
<th>DLSY</th>
</tr>
</thead>
<tbody>
<tr>
<td>DLSSF</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DLSH</td>
<td>0.92003</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DLSBP</td>
<td>0.70325</td>
<td>0.73506</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>DLSY</td>
<td>0.58997</td>
<td>0.59245</td>
<td>0.46338</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 3.3.2: Unconditional correlation matrices for the BP-JY portfolio and the SF-DM portfolio

<table>
<thead>
<tr>
<th>BP-JY portfolio</th>
<th>SF-DM portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Portfolio</td>
<td>BP</td>
</tr>
<tr>
<td>BP</td>
<td>0.78248</td>
</tr>
<tr>
<td>JY</td>
<td>0.83353</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 3.4: Descriptive Statistics on the spot portfolios and futures returns multiplied by 100\(^{22}\)

<table>
<thead>
<tr>
<th>Statistics</th>
<th>SF-DM port</th>
<th>BP-JY port</th>
<th>SF futures</th>
<th>DM futures</th>
<th>BP futures</th>
<th>JY futures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>-0.00298</td>
<td>-0.00082</td>
<td>-0.00327</td>
<td>-0.00303</td>
<td>-0.00442</td>
<td>0.00253</td>
</tr>
<tr>
<td></td>
<td>(-0.21025)</td>
<td>(-0.06949)</td>
<td>(-0.21261)</td>
<td>(-0.21865)</td>
<td>(-0.31483)</td>
<td>(0.18189)</td>
</tr>
<tr>
<td>Variance</td>
<td>0.50939</td>
<td>0.35124</td>
<td>0.60168</td>
<td>0.48925</td>
<td>0.49972</td>
<td>0.49222</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>2.14507</td>
<td>3.54062</td>
<td>2.08456</td>
<td>2.29594</td>
<td>4.06195</td>
<td>4.28838</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.01672</td>
<td>0.18811</td>
<td>0.09433</td>
<td>-0.00796</td>
<td>-0.34254</td>
<td>0.30080</td>
</tr>
<tr>
<td>Bera-Jarque</td>
<td>84.47000*</td>
<td>110.56000*</td>
<td>128.62500*</td>
<td>49.09200*</td>
<td>-25.65000*</td>
<td>303.01400*</td>
</tr>
<tr>
<td>LB(20)</td>
<td>14.64000</td>
<td>31.12000</td>
<td>15.21000</td>
<td>16.29000</td>
<td>30.65000</td>
<td>40.87000*</td>
</tr>
<tr>
<td>LB_{SQ}(20)</td>
<td>169.65000*</td>
<td>98.00000*</td>
<td>153.44000*</td>
<td>145.63000*</td>
<td>216.70000*</td>
<td>96.19000*</td>
</tr>
</tbody>
</table>

\(^{22}\)Bera-Jarque is the test for normality. BJ = n[skewness\(^2\)/6 + (kurtosis-3)/24] \sim \chi^2(2) (95% c.v 5.99).
In the present section, the results of the quasi-maximum likelihood estimation of the system (3.8) are reported for the two spot currency portfolios and their respective futures contracts. First, the estimates of the multivariate GARCH-X (1,1) are presented accounting for portfolio effects in the two-currency hedging problem. Second, for comparison purposes, a bivariate GARCH-X (1,1) model is estimated for the spot and futures returns of each currency in isolation. Diagnostic tests are performed in order to examine the statistical validity of each econometric specification and static and dynamic hedge ratios are derived by each restricted and unrestricted model. The portfolio dynamic model is expected to outperform in terms of econometric specification as well as economic significance since it accounts for the factor of dynamic covariance between currencies that is crucial in the development of portfolio construction.

(3.8.1) Quasi-maximum Likelihood Estimation of the multivariate Error Correction GARCH models

In Table 3.5, the Quasi-Maximum Likelihood estimates of the GARCH-X (1,1) hedging models for the SF-DM and the BP-JY portfolios respectively are provided with their robust standard errors. The statistical significance of the coefficients shows the econometric validity of the model and its ability to describe adequately the means and volatility of spot and futures exchange rate returns and any statistical relations existing between the two series.

As expected by the preliminary data analysis, the intercepts in the spot (\(C_p\)) and futures (\(C_{SF}, C_{DM}\) and \(C_{JY}\)) equations are found to be statistically insignificant, with the exception of the British Pound (\(C_{BP}\)), a finding that implies the absence of a linear trend from the data generation process. The hypothesis of cointegration between spot and futures returns is again examined through the statistical significance of the error correction terms. As expected, the coefficients of the error
correction term (β's) in the spot and futures equations are, with the exception of the
spot equation of the BP-JY portfolio, statistically significant, showing that both
spot and futures prices continuously adjust to the market conditions in order to
eliminate arbitrage opportunities. The implication of this finding for hedging
models is that the two futures contracts used in each portfolio are a valid protection
against adverse spot price movements and they can provide a hedged portfolio with
constant variance (Lim (1996)). The number of lags of spot and futures returns
included in spot and futures equations are chosen so that the Akaike Information
Criterion is minimized. As it can be seen in table 3.5, all lagged values included in
both models (γ's) are statistically significant at the 5% level. This finding supports
the theory of cointegration that, any change in one of the two cointegrated variables
in this period is related to past changes in both variables.

The relevance of the GARCH-X specification in explaining a part of the exchange
rate volatility is tested through the statistical significance of the GARCH-X
coefficients. In the variance equations of both portfolios, the constant terms (D₀, E₀
and F₀), that represent the unconditional variances of spot and futures returns, are
all statistically significant. The same result applies to the estimated coefficients of
the lagged conditional variances (D₁, E₁ and F₁) and squared error terms (D₂, E₂
and F₂), indicating strong GARCH effects for spot and futures returns and time
variation in the estimated hedge ratios. The presence of IGARCH effects is not an
issue in the present empirical analysis, as it is shown by the low persistence in
variance evident in all cases. In all variance equations, the coefficients of the
squared error correction term (D₃, E₃ and F₃) are highly significant, supporting the
findings of Lee (1994), that past disequilibrium can explain a portion of the
conditional heteroscedasticity of financial returns. This finding is important on the
basis of the provided ability to forecast a larger part of exchange rate volatility that
is due to deviations from equilibrium. An economic implication of this result is that
a further risk reduction can be achieved with a hedging model that accounts for the
effect of cointegration on exchange rate volatility.

The most important aspect of the GARCH-X dynamic portfolio model is its ability
to capture the dynamic effects of the covariance between different currency futures returns. The estimates of the conditional correlations between spot and futures returns and between futures returns are all positive and highly significant in both portfolios examined. This finding implies substantial efficiency gains from the construction of a hedging model that captures portfolio effects. Significant interaction exists between currency futures returns thus leading to a different hedging solution when a currency portfolio is constructed. However, as it is expected by the unconditional correlation matrices in Table 3.3.2, the conditional correlations between spot and futures returns in the SF-DM portfolio ($P_{SF}, P_{DM}$) are also much higher than those in the BP-JY portfolio ($P_{BP}, P_{JY}$). On the basis of these empirical findings, the BP and JY futures contracts are expected to be less effective hedging instruments for the BP-JY spot portfolio than the SF and DM futures are for the SF-DM portfolio. This result should be mainly due to the low conditional correlation between the former currency futures returns that is expected since UK and Japan belong in two different geographic and economic blocs. On the contrary, the high correlation between the DM and SF futures returns can be explained by the fact that, although Switzerland is not an EEC member, it is highly integrated with the European Union in terms of its balance of trade and capital account (Aguirre and Saidi (1998)).

The econometric validity of the dynamic GARCH-X specification is tested in Table 3.6, where diagnostic tests on the standardized residuals of the two multivariate GARCH-X (1,1) models are reported, in the form of remaining serial correlation and ARCH effects, revealing no significant misspecifications. No additional lags of the dependent variables need to be introduced in the models since, as the Ljung-Box (20) (LB) statistic shows, there is no serial correlation present in the standardized residuals of the estimated models. Ljung-Box (20) portmanteau statistics on the squared residuals of the two models show no remaining heteroscedasticity in the GARCH-X error structure. The hypothesis of constant conditional correlations is supported by the LB (20) test on the cross product of the standardized residuals. No neglected structure is found in the GARCH-X (1,1) conditional variance-covariance specification.
The main conclusion of the present section is that the GARCH-X portfolio model is an adequate representation of the mean and volatility of spot and futures currency returns\footnote{Following the studies reporting evidence of asymmetries in foreign exchange data and the success of the EGARCH model in terms of statistical validity, an ECM with an exponential variance specification is also estimated for each portfolio. However, since the estimated model is misspecified in terms of remaining heteroscedasticity and likelihood ratio tests, these results are not reported.} as well as of any dynamic interaction existing between spot and futures and currency futures returns. However, apart from the statistical validity of a hedging model, the hedger is mostly interested in its hedging performance, both in an in sample as well as in an out of sample analysis. The economic implication of this finding is that improved hedging results are expected from the application of the dynamic portfolio model to the risk management of multi-currency portfolios. A dynamic hedging strategy is derived by the GARCH-X (1,1) model, in the form of time varying hedge ratio series. The dynamic portfolio hedge ratios are estimated for each currency by dividing the conditional covariance of the spot and the futures price with the conditional variance of the futures price. Statistical tests on the estimated hedge ratios are reported in Section (3.8.3). The hedging performance of the model is further examined with variance reduction and utility tests in Section D of the present chapter in a comparison with the no-portfolio dynamic GARCH-X model, that is estimated in the following paragraphs.
Table 3.5: Quasi-maximum Likelihood Estimation of the multivariate GARCH-X (1,1) model for the SF-DM and the BP-JY hedging portfolios.

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>SF-DM portfolio</th>
<th>Robust Coefficients</th>
<th>BP-JY portfolio</th>
<th>Robust Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimates</td>
<td>Stand. Errors</td>
<td>Estimates</td>
<td>Stand. Errors</td>
</tr>
<tr>
<td>$C_{p1}$</td>
<td>0.01950</td>
<td>0.065400</td>
<td>$C_{p2}$</td>
<td>0.01029</td>
</tr>
<tr>
<td>$C_{SF}$</td>
<td>0.02131</td>
<td>0.072900</td>
<td>$C_{BP}$</td>
<td>0.02447</td>
</tr>
<tr>
<td>$C_{DM}$</td>
<td>0.02150</td>
<td>0.064200</td>
<td>$C_{JY}$</td>
<td>0.00582</td>
</tr>
<tr>
<td>$\beta_{p1}$</td>
<td>0.05389</td>
<td>0.01901*</td>
<td>$\beta_{p2}$</td>
<td>0.01822</td>
</tr>
<tr>
<td>$\gamma_{1, p1}$</td>
<td>-0.42782</td>
<td>0.03453*</td>
<td>$\gamma_{1, p2}$</td>
<td>-0.31697</td>
</tr>
<tr>
<td>$\gamma_{1, p1}$</td>
<td>0.42779</td>
<td>0.03429*</td>
<td>$\gamma_{1, p2}$</td>
<td>0.31738</td>
</tr>
<tr>
<td>$\gamma_{2, p1}$</td>
<td>-0.17080</td>
<td>0.02806*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_{2, p1}$</td>
<td>0.17660</td>
<td>0.02713*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_{SF}$</td>
<td>0.10266</td>
<td>0.02214*</td>
<td>$\beta_{BP}$</td>
<td>0.09366</td>
</tr>
<tr>
<td>$\beta_{DM}$</td>
<td>0.08207</td>
<td>0.02014*</td>
<td>$\beta_{JY}$</td>
<td>0.12293</td>
</tr>
<tr>
<td>$D_{0, p1}$</td>
<td>0.01371</td>
<td>0.00672*</td>
<td>$D_{0, p2}$</td>
<td>0.02068</td>
</tr>
<tr>
<td>$D_{1, p1}$</td>
<td>0.93974</td>
<td>0.02030*</td>
<td>$D_{1, p2}$</td>
<td>0.88964</td>
</tr>
<tr>
<td>$D_{2, p1}$</td>
<td>0.03873</td>
<td>0.00821*</td>
<td>$D_{2, p2}$</td>
<td>0.04405</td>
</tr>
<tr>
<td>$D_{3, p1}$</td>
<td>0.05052</td>
<td>0.00279*</td>
<td>$D_{3, p2}$</td>
<td>0.09420</td>
</tr>
<tr>
<td>$E_{0, SF}$</td>
<td>0.01483</td>
<td>0.009140</td>
<td>$E_{0, BP}$</td>
<td>0.01959</td>
</tr>
<tr>
<td>$E_{1, SF}$</td>
<td>0.94794</td>
<td>0.02265*</td>
<td>$E_{1, BP}$</td>
<td>0.91202</td>
</tr>
<tr>
<td>$E_{2, SF}$</td>
<td>0.02465</td>
<td>0.00872*</td>
<td>$E_{2, BP}$</td>
<td>0.04455</td>
</tr>
<tr>
<td>$E_{3, SF}$</td>
<td>0.03460</td>
<td>0.00323*</td>
<td>$E_{3, BP}$</td>
<td>0.09000</td>
</tr>
<tr>
<td>$F_{0, DM}$</td>
<td>0.00985</td>
<td>0.00558*</td>
<td>$F_{0, JY}$</td>
<td>0.03397</td>
</tr>
<tr>
<td>$F_{1, DM}$</td>
<td>0.94787</td>
<td>0.01833*</td>
<td>$F_{1, JY}$</td>
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</tr>
<tr>
<td>$F_{2, DM}$</td>
<td>0.02876</td>
<td>0.00805*</td>
<td>$F_{2, JY}$</td>
<td>0.05643</td>
</tr>
<tr>
<td>$F_{3, DM}$</td>
<td>0.03370</td>
<td>0.00277*</td>
<td>$F_{3, JY}$</td>
<td>0.03590</td>
</tr>
<tr>
<td>$P_{0, SF}$</td>
<td>0.94858</td>
<td>0.00169*</td>
<td>$P_{0, BP}$</td>
<td>0.78309</td>
</tr>
<tr>
<td>$P_{0, DM}$</td>
<td>0.91897</td>
<td>0.00293*</td>
<td>$P_{0, BP}$</td>
<td>0.46996</td>
</tr>
<tr>
<td>$P_{P, DM}$</td>
<td>0.94615</td>
<td>0.00178*</td>
<td>$P_{P, JY}$</td>
<td>0.84443</td>
</tr>
</tbody>
</table>

Table 3.5 contains the estimated coefficients of the following GARCH-X (1,1) model:

\[
\begin{align*}
\ln p_t &= C_p + (\beta_p \ast \text{ect}_{p,t-1}) + \sum_{i=1}^{n} (\gamma_{p.i} \ast \ln p_{t-i}) + \sum_{j=1}^{m} (\gamma_{j.p} \ast \ln F_{p,t-j}) + \epsilon_{p,t} \\
\ln f_{1,t} &= C_1 + (\beta_1 \ast \text{ect}_{f1,t-1}) + \epsilon_{1,t} \\
\ln f_{2,t} &= C_2 + (\beta_2 \ast \text{ect}_{f2,t-1}) + \epsilon_{2,t} \\
h_{p,t-1} &= D_0 + D_1 h_{p,t-1}^2 + D_2 e_{p,t-1}^2 + D_3 \text{ect}_{p,t-1} \\
h_{\Omega_{t-1}} &= E_0 + E_1 h_{\Omega_{t-1}} + E_2 e_{\Omega,t-1}^2 + E_3 \text{ect}_{\Omega,t-1} \\
h_{\Omega,t} &= F_0 + F_1 h_{\Omega,t-1}^2 + F_2 e_{\Omega,t-1}^2 + F_3 \text{ect}_{\Omega,t-1} \\
h_{p,t} &= P_{p} h_{p,t} \ast (h_{p,t} \ast h_{\Omega,t}) \\
h_{p,\Omega} &= P_{p} h_{p,t} \ast (h_{p,t} \ast h_{\Omega,t}) \\
h_{\Omega,t} &= P_{\Omega} h_{\Omega,t} \ast (h_{\Omega,t} \ast h_{\Omega,t})
\end{align*}
\]
Table 3.6: Misspecification tests\(^{24}\) on the standardized residuals of the GARCH-X (1,1) model for the SF-DM and the BP-JY portfolio.

<table>
<thead>
<tr>
<th>Tests for remaining serial correlation</th>
<th>Statistics</th>
<th>Tests for remaining ARCH effects</th>
<th>Statistics</th>
<th>Tests for remaining cross correlation</th>
<th>Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>LB(_{p1})</td>
<td>14.55</td>
<td>LB(_{p1}^2)</td>
<td>19.53</td>
<td>LB(_{p1, SF})</td>
<td>24.56</td>
</tr>
<tr>
<td>LB(_{SF})</td>
<td>15.54</td>
<td>LB(_{SF}^2)</td>
<td>26.29</td>
<td>LB(_{p1, DM})</td>
<td>19.07</td>
</tr>
<tr>
<td>LB(_{DM})</td>
<td>15.89</td>
<td>LB(_{DM}^2)</td>
<td>15.62</td>
<td>LB(_{SF, DM})</td>
<td>21.24</td>
</tr>
<tr>
<td>LB(_{p2})</td>
<td>27.39</td>
<td>LB(_{p2}^2)</td>
<td>18.73</td>
<td>LB(_{p2, BP})</td>
<td>31.72</td>
</tr>
<tr>
<td>LB(_{BP})</td>
<td>22.57</td>
<td>LB(_{BP}^2)</td>
<td>19.81</td>
<td>LB(_{p2, JY})</td>
<td>15.21</td>
</tr>
<tr>
<td>LB(_{JY})</td>
<td>30.68</td>
<td>LB(_{JY}^2)</td>
<td>11.28</td>
<td>LB(_{BP, JY})</td>
<td>31.36</td>
</tr>
</tbody>
</table>

(3.8.2) Quasi-maximum Likelihood Estimation of the bivariate GARCH-X models

In order to make a comparison between the efficiency of hedging a multicurrency portfolio with this of hedging each spot currency position in isolation, four separate bivariate systems are estimated. A GARCH-X (1,1) model with constant conditional correlations is estimated for each of the four currencies and its corresponding futures contract. This approach ignores the significant covariances between currencies and between futures contracts and it does not thus consider any portfolio effects.

In Table 3.7.1, the Quasi-Maximum Likelihood estimates of the bivariate GARCH-X (1,1) model are provided with robust standard errors in parentheses. The estimated coefficients of the mean and variance equations resemble those of the respective multivariate systems in Table 3.5 in terms of statistical significance. The coefficients of the error correction terms in the spot (\(b_s\)) and futures (\(b_f\)) equations are all highly significant at the 5% level of significance, as well as the lagged differences of the dependent variables (\(\alpha\)’s and \(\delta\)’s) included in the level equations. Similarly, in the variance equations of all spot and futures returns examined, the coefficients are statistically significant, indicating strong GARCH effects for spot

\(^{24}\) LB\(_{p1}\), LB\(_{p2}\), LB\(_{SF}\), LB\(_{DM}\), LB\(_{BP}\) and LB\(_{JY}\) are the Ljung-Box (20) statistics for 20th order serial correlation on the residuals of the portfolios and futures equations respectively. It is \(\chi^2\) distributed and has 95% critical value 31.41 and 99% c.v. 37.57. LB\(_{p1}^2\), LB\(_{p2}^2\), LB\(_{BP}^2\), LB\(_{SF}^2\), LB\(_{DM}^2\) and LB\(_{BP, JY}\) are the Ljung-Box (20) statistics for 20th order serial correlation on the squared residuals of the spot portfolios and futures equations respectively. LB\(_{p1, SF}\), LB\(_{p1, DM}\), LB\(_{p2, BP}\), LB\(_{p2, JY}\), LB\(_{SF, DM}\) and LB\(_{BP, JY}\) are the Ljung-Box (20) statistic for 20th order serial correlation on the cross products of the standardized residuals of the spot portfolios and futures equations.
and futures returns and time variation in the estimated hedge ratios. The significance of the coefficients of the squared error correction terms, $b_3$ and $g_3$, implies that a part of exchange rate risk can be attributed to deviations from the equilibrium relationship between spot and futures prices. The estimate of the conditional correlation, $p_{sf}$, between spot and futures returns is positive and highly significant with a value close to one for most currencies examined. This finding implies strong interaction between spot and futures exchange rates and indicates the benefits resulting from modeling spot and futures price changes jointly.

A more detailed analysis of the BP variance equations reveals that there is a significant difference between the multivariate system in Table 3.5 and the bivariate system in Table 3.7.1. Modelling the variance of the BP spot and futures returns in isolation leads to a high persistence in variance (Table 3.7.1: $g_3 = 0.98412$). On the contrary, modeling currencies in portfolios results in a substantial reduction of IGARCH effects (Table 3.5: $E_{1,BP} = 0.91202$) and an improvement in the forecasted volatilities. However, the presence of IGARCH in the bivariate case is not an estimation problem since, according to Lumsdaine (1991) and Lee and Hansen (1994), the log likelihood of an IGARCH process is well behaved asymptotically and the MLE estimators are asymptotically normal.

In Table 3.7.2, diagnostic tests on the standardized residuals of the bivariate ECM-GARCH (1,1) models, in the form of remaining serial correlation and ARCH effects, reveal no significant misspecifications. The hypothesis of constant conditional correlation is supported by the insignificance of the $LB_{SF} (20)$ test on the cross product of the standardized residuals. The analysis of the no-portfolio GARCH-X model reveals that it captures well the empirical regularities of spot and futures exchange rate returns. However, the limitations of the no portfolio model relative to a multi-currency specification as well as the advantages of the dynamic specification can be examined with an econometric comparison between the no-portfolio GARCH-X model and the portfolio GARCH-X model estimated in the previous section.
The estimates of the log-likelihood functions of each static and dynamic model for the portfolio and the no-portfolio case are reported in Table 3.8. First, the comparison between bivariate and multivariate GARCH-X specifications shows that, in all cases examined, the multivariate GARCH-X model has a higher log likelihood function than the respective bivariate models, a result that implies strong portfolio effects and an advantage from modeling and hedging jointly multi-currency positions.

Second, the statistical significance of the squared error correction term in the conditional variance equations is tested by imposing the restriction \( D_3 = E_3 = F_3 \) in each multivariate GARCH-X (1,1) model. In the bivariate cases the same hypothesis is tested by imposing the restriction \( b_3 = g_3 = 0 \), thus leading to the simple GARCH model. It is straightforward that the omission of the squared error correction term from the variance equations leads to a lower log-likelihood function and a decrease in the explanatory power of all models. This finding supports the previous results of the present chapter on the ability of the deviations from equilibrium to explain a significant part of the heteroscedasticity of spot and futures exchange rates.

Third, the significance of the dynamic GARCH specification is tested by imposing on the latter model the restriction of a constant conditional covariance matrix, thus leading to the Error Correction Model. In the no-portfolio case, the ECM is derived from the bivariate GARCH-X (1,1) model by imposing the restriction \( b_0 = b_1 = b_2 = g_0 = g_1 = g_2 = 0 \) while in the multivariate model the restriction \( D_0 = D_1 = D_2 = E_0 = E_1 = E_2 = F_0 = F_1 = F_2 \) is imposed. In all cases, the Error Correction Model has a lower log likelihood function than the GARCH model, showing the substantial statistical gains resulting from the dynamic specification of spot and futures exchange rate returns.

Finally, the statistical significance of the error correction model is tested by imposing the restriction of zero coefficients in the error correction terms of the ECM, thus leading to the simple OLS model. In the SF-DM model, the restriction
\( \beta_{p1} = \beta_{SF} = \beta_{DM} = 0 \) is imposed while for the BP-JY model, the relevant restriction is \( \beta_{p2} = \beta_{BP} = \beta_{JY} = 0 \). In the no-portfolio case, the OLS model can be derived with the additional restriction \( b_s = b_f = 0 \). From Table 3.8, it is straightforward that the omission of an error correction term from a hedging model results in a lower likelihood function and a loss in the explanatory power of the model, giving support to the statistical significance of the error correction term.

In general, in terms of statistical performance, the unrestricted GARCH-X (1,1) model is the best specification for all currencies examined in both the bivariate and the multivariate case. Comparing the results from the estimated portfolio and no-portfolio models, we can conclude that the development of a hedging solution that ignores portfolio effects will lead in losses in terms of efficiency since it omits the important factor of the covariance between currency returns. The economic significance of this factor can be evaluated with comparisons among the estimated hedge ratios as well as in terms of risk reduction after their application.

In the following section, the risk minimizing hedge ratios estimated by the static and dynamic bivariate and multivariate models will be compared in terms of size and variance. The hedged returns that result from the bivariate cases will be aggregated for each two-currency portfolio with the weights used in the construction of the spot portfolios. Comparisons of the hedging performance between the models can then be made from both a risk minimizing as well as from a utility standpoint.
### Table 3.7.1: Quasi-maximum Likelihood Estimation of the bivariate GARCH-X (1,1) model

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>BP</th>
<th>JY</th>
<th>SF</th>
<th>DM</th>
</tr>
</thead>
<tbody>
<tr>
<td>C_s</td>
<td>0.01881</td>
<td>-0.00351</td>
<td>0.00794</td>
<td>0.01410</td>
</tr>
<tr>
<td></td>
<td>(0.00373)*</td>
<td>(0.00430)</td>
<td>(0.01390)</td>
<td>(0.01397)</td>
</tr>
<tr>
<td>C_f</td>
<td>0.03375</td>
<td>-0.00473</td>
<td>0.00965</td>
<td>0.01855</td>
</tr>
<tr>
<td></td>
<td>(0.00363)*</td>
<td>(0.00428)</td>
<td>(0.01434)</td>
<td>(0.01393)</td>
</tr>
<tr>
<td>B_s</td>
<td>0.00906</td>
<td>0.03661</td>
<td>0.0322</td>
<td>0.0680</td>
</tr>
<tr>
<td></td>
<td>(0.00481)*</td>
<td>(0.00935)*</td>
<td>(0.00242)*</td>
<td>(0.00245)*</td>
</tr>
<tr>
<td>B_f</td>
<td>0.10581</td>
<td>0.11034</td>
<td>0.05650</td>
<td>0.05402</td>
</tr>
<tr>
<td></td>
<td>(0.00522)*</td>
<td>(0.00942)*</td>
<td>(0.02426)*</td>
<td>(0.02526)*</td>
</tr>
<tr>
<td>α_{1s}</td>
<td>-0.21554</td>
<td>-0.33020</td>
<td>-0.34531</td>
<td>-0.38970</td>
</tr>
<tr>
<td></td>
<td>(0.00914)*</td>
<td>(0.00714)*</td>
<td>(0.01239)*</td>
<td>(0.01498)*</td>
</tr>
<tr>
<td>α_{1f}</td>
<td>0.21386</td>
<td>0.33216</td>
<td>0.35020</td>
<td>0.38838</td>
</tr>
<tr>
<td></td>
<td>(0.00397)*</td>
<td>(0.00646)*</td>
<td>(0.00526)*</td>
<td>(0.01456)*</td>
</tr>
<tr>
<td>δ_{1s}</td>
<td>0.11057</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.00505)*</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>δ_{3s}</td>
<td>-</td>
<td>-0.01554</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>(0.00688)*</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>δ_{1f}</td>
<td>-0.11323</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.00398)*</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>δ_{3f}</td>
<td>-</td>
<td>-0.02115</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>(0.00630)*</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>B_0</td>
<td>0.000069</td>
<td>0.21931</td>
<td>0.00919</td>
<td>0.00346</td>
</tr>
<tr>
<td></td>
<td>(0.00006)*</td>
<td>(0.00292)*</td>
<td>(0.00418)*</td>
<td>(0.00113)*</td>
</tr>
<tr>
<td>B_1</td>
<td>0.98265</td>
<td>0.27983</td>
<td>0.96111</td>
<td>0.96335</td>
</tr>
<tr>
<td></td>
<td>(0.00019)*</td>
<td>(0.00676)*</td>
<td>(0.01239)*</td>
<td>(0.01010)*</td>
</tr>
<tr>
<td>B_2</td>
<td>0.011385</td>
<td>0.13676</td>
<td>0.01740</td>
<td>0.02017</td>
</tr>
<tr>
<td></td>
<td>(0.00021)*</td>
<td>(0.00615)*</td>
<td>(0.00531)*</td>
<td>(0.00214)*</td>
</tr>
<tr>
<td>B_3</td>
<td>0.01210</td>
<td>0.28954</td>
<td>0.00986</td>
<td>0.01514</td>
</tr>
<tr>
<td></td>
<td>(0.00039)*</td>
<td>(0.01172)*</td>
<td>(0.00053)*</td>
<td>(0.00086)*</td>
</tr>
<tr>
<td>G_0</td>
<td>-0.00081</td>
<td>0.21231</td>
<td>0.00718</td>
<td>0.00261</td>
</tr>
<tr>
<td></td>
<td>(0.00002)*</td>
<td>(0.00234)*</td>
<td>(0.00316)*</td>
<td>(0.00082)*</td>
</tr>
<tr>
<td>G_1</td>
<td>0.98412</td>
<td>0.32310</td>
<td>0.96779</td>
<td>0.96828</td>
</tr>
<tr>
<td></td>
<td>(0.00006)*</td>
<td>(0.00530)*</td>
<td>(0.01008)*</td>
<td>(0.00053)*</td>
</tr>
<tr>
<td>G_2</td>
<td>0.01294</td>
<td>0.14076</td>
<td>0.01584</td>
<td>0.01867</td>
</tr>
<tr>
<td></td>
<td>(0.00007)*</td>
<td>(0.00538)*</td>
<td>(0.00520)*</td>
<td>(0.00227)*</td>
</tr>
<tr>
<td>G_3</td>
<td>0.01259</td>
<td>0.27318</td>
<td>0.00778</td>
<td>0.01309</td>
</tr>
<tr>
<td></td>
<td>(0.00017)*</td>
<td>(0.01250)*</td>
<td>(0.00076)*</td>
<td>(0.00073)*</td>
</tr>
<tr>
<td>P_{sf}</td>
<td>0.94845</td>
<td>0.94801</td>
<td>0.96198</td>
<td>0.96176</td>
</tr>
<tr>
<td></td>
<td>(0.00024)*</td>
<td>(0.00041)*</td>
<td>(0.00203)*</td>
<td>(0.00067)*</td>
</tr>
</tbody>
</table>

---

25 Table 3.7.1 reports the estimates of the following model:

\[
\begin{align*}
\text{dlns}_t &= C_s + B_s \text{ect}_{t-1} + \alpha_s \Sigma \text{dlns}_{t-1} + \gamma_s \Sigma \text{dlnf}_{t-1} + \epsilon_t \\
\text{dlnf}_t &= C_f + B_f \text{ect}_{t-1} + \delta_s \Sigma \text{dlns}_{t-1} + \delta_f \Sigma \text{dlnf}_{t-1} + \epsilon_t \\
\text{h}_{ss}^{tt} &= B_{ss} + B_{ss} \text{ect}_{t-1} + B_2 \epsilon^2_{t-1} + B_3 \text{ect}^2_{t-1} \\
\text{h}_{sf}^{tt} &= G_0 + G_1 \text{h}_{ss}^{tt-1} + G_2 \epsilon^2_{t-1} + G_3 \text{ect}^2_{t-1}
\end{align*}
\]

Asymptotic standard errors are given in parentheses and asterisks are used for the significant coefficients at the 95% level of significance.
Table 3.7.2: Misspecification tests on the standardized residuals of the bivariate GARCH-X (1,1) model

<table>
<thead>
<tr>
<th></th>
<th>BP</th>
<th>JY</th>
<th>SF</th>
<th>DM</th>
</tr>
</thead>
<tbody>
<tr>
<td>LB_S</td>
<td>24.65</td>
<td>31.28</td>
<td>14.82</td>
<td>17.21</td>
</tr>
<tr>
<td>LB_F</td>
<td>23.95</td>
<td>31.47</td>
<td>15.93</td>
<td>16.80</td>
</tr>
<tr>
<td>LB_SS</td>
<td>31.49</td>
<td>30.95</td>
<td>30.08</td>
<td>31.00</td>
</tr>
<tr>
<td>LB_FF</td>
<td>30.17</td>
<td>31.29</td>
<td>31.10</td>
<td>31.46</td>
</tr>
<tr>
<td>LB_SF</td>
<td>31.62</td>
<td>31.70</td>
<td>31.80</td>
<td>31.13</td>
</tr>
</tbody>
</table>

Table 3.8: Log-likelihood estimates of the multivariate and bivariate models.

<table>
<thead>
<tr>
<th>Models</th>
<th>SF-DM multivariate</th>
<th>SF bivariate</th>
<th>DM bivariate</th>
</tr>
</thead>
<tbody>
<tr>
<td>OLS</td>
<td>2254.200</td>
<td>2090.680</td>
<td>2127.190</td>
</tr>
<tr>
<td>ECM</td>
<td>2256.700</td>
<td>2091.810</td>
<td>2128.310</td>
</tr>
<tr>
<td>GARCH</td>
<td>2661.620</td>
<td>2559.370</td>
<td>2601.370</td>
</tr>
<tr>
<td>GARCH-X</td>
<td>2769.788</td>
<td>2629.700</td>
<td>2708.154</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Models</th>
<th>BP-JY multivariate</th>
<th>BP bivariate</th>
<th>JY bivariate</th>
</tr>
</thead>
<tbody>
<tr>
<td>OLS</td>
<td>2259.700</td>
<td>2094.470</td>
<td>2081.800</td>
</tr>
<tr>
<td>ECM</td>
<td>2261.600</td>
<td>2097.550</td>
<td>2083.980</td>
</tr>
<tr>
<td>GARCH</td>
<td>2991.810</td>
<td>2588.580</td>
<td>2557.950</td>
</tr>
<tr>
<td>GARCH-X</td>
<td>2994.990</td>
<td>2594.148</td>
<td>2611.380</td>
</tr>
</tbody>
</table>

26 LB_S and LB_F are the Ljung-Box (20) statistics for 20th order serial correlation on the residuals of the spot and futures equation respectively. It is $\chi^2_{20}$ distributed and has 95% critical value 31.41 and 99% c.v. 37.57.

LB_SS and LB_FF are the Ljung-Box (20) statistics for 20th order serial correlation on the squared residuals of the spot and futures equation respectively. LB_SF is the Ljung-Box (20) statistic for 20th order serial correlation on the cross products of the standardized residuals of the spot and futures equation.
(3.8.3) Estimated hedge ratios

In the present section, a statistical analysis is performed on the static and dynamic hedge ratios estimated by the restricted and unrestricted multivariate and bivariate models. The risk minimizing hedge ratios are computed for each currency and each portfolio by dividing the estimated covariance of the spot and the futures return with the variance of the futures return. Six alternative hedging strategies occur for each currency and comparisons are made in terms of the size and variance of the futures position as well as unit root and serial correlation tests on the dynamic hedge ratios, with implications on the hedging performance of each strategy.

The conditional second moments used for the estimation of the dynamic portfolio hedging strategy are those estimated by the multivariate GARCH-X (1,1) model of section 3.8.1. The dynamic no-portfolio strategy is based on risk minimizing hedge ratios estimated for each currency in isolation by using the conditional variances and covariances of the bivariate GARCH-X (1,1) model of section 3.8.2. Two static portfolio strategies occur for each currency from the estimation of the OLS and ECM models for each spot portfolio and the futures returns of its components. The estimated covariances and variances give two constant risk-minimizing hedge ratios for each currency examined. Additionally, two static no-portfolio strategies can be derived by estimating single OLS and ECM regressions of each spot currency return on its corresponding futures return.

In Table 3.9, a comparison between the hedge ratios (means) derived by the OLS model and the ones derived by the Error Correction Model reveals that, for both portfolio and no-portfolio cases and for all currencies examined, the hedge ratios of the OLS model are smaller. This finding implies that the OLS hedge ratios will underhedge the spot position, giving support to the empirical finding of Ghosh (1993) that the omission of the error correction term from the hedging model leads the hedger to a suboptimal position. The degree of inefficiency of the OLS method will be examined in the following section in terms of variance reduction and utility maximization.
The importance of hedging jointly multi-currency positions can be understood by comparisons of the means between the portfolio and no-portfolio case for the same currencies for both static and dynamic hedge ratios. As it is evident in Table 3.9, in all cases, the hedge ratios accounting for portfolio effects are almost half in size than their no-portfolio counterparts. The difference in the size of the dynamic futures demand derived by each model is shown quite clearly in the figures 3.1 - 3.4. This finding implies that ignoring portfolio effects leads the hedger to overhedge his spot currency position. Comparing the portfolio and no-portfolio dynamic hedging strategies, the portfolio hedge ratios exhibit more variance than their no-portfolio counterparts, a result probably due to the additional time variation introduced by the covariance between futures contracts of the same portfolio.

In figures 3.1 – 3.4, it is shown that the conditional hedge ratios estimated by both the multivariate (portfolio) and the bivariate (no-portfolio) GARCH-X (1,1) models are clearly time-varying. However, the Phillips-Perron unit root tests (PP in Table 3.9) on the estimated hedge ratios show that after a shock, they all tend to revert to their long-run means. This finding is supported by the empirical findings of Kroner and Sultan (1993) for the same currencies. Additionally, the estimated hedge ratios are serially positively correlated as it is indicated by the first order correlation coefficient \( \rho \). A large hedge ratio in one day tends to be followed by a large hedge ratio in the following day. In the case of the DM, our findings are different from those of Kroner and Sultan (1993), who report negative autocorrelation for the hedge ratio of the DM, a result attributed to the estimation period used.

In the following section, the hedging effectiveness of each hedging strategy is estimated in terms of variance reduction and utility performance. The utility comparisons for the dynamic cases are based on a strategy of selective rebalancing the futures positions, taking into account the transaction costs.
Table 3.9: Descriptive statistics of the risk minimizing hedge ratios estimated for the portfolio and the no-portfolio case for static and dynamic models.

<table>
<thead>
<tr>
<th></th>
<th>Swiss Franc and German Mark</th>
<th>British Pound and Japanese Yen</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Portfolio case</td>
<td></td>
</tr>
<tr>
<td>Currencies</td>
<td>Models</td>
<td>Mean</td>
</tr>
<tr>
<td>SF</td>
<td>Dynamic</td>
<td>0.43309</td>
</tr>
<tr>
<td></td>
<td>OLS</td>
<td>0.43366</td>
</tr>
<tr>
<td></td>
<td>ECM</td>
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</tr>
<tr>
<td>DM</td>
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</tr>
<tr>
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</tr>
<tr>
<td></td>
<td>ECM</td>
<td>0.48009</td>
</tr>
<tr>
<td></td>
<td>British Pound and Japanese Yen</td>
<td>Portfolio case</td>
</tr>
<tr>
<td>Currencies</td>
<td>Models</td>
<td>Mean</td>
</tr>
<tr>
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<td>Dynamic</td>
<td>0.33962</td>
</tr>
<tr>
<td></td>
<td>OLS</td>
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</tr>
<tr>
<td></td>
<td>ECM</td>
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</tr>
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<td>DM</td>
<td>Dynamic</td>
<td>0.35904</td>
</tr>
<tr>
<td></td>
<td>OLS</td>
<td>0.35251</td>
</tr>
<tr>
<td></td>
<td>ECM</td>
<td>0.35464</td>
</tr>
</tbody>
</table>

|              | No portfolio case          |                               |
| Currencies   | models                     | Mean | Variance | PP       | p          |
| SF           | Dynamic                    | 0.94851 | 0.00040 | -7.64026 | 0.94920 |
|              | OLS                        | 0.94752 |
|              | ECM                        | 0.94831 |
| DM           | Dynamic                    | 0.94590 | 0.00123 | -6.99537 | 0.96356 |
|              | OLS                        | 0.94596 |
|              | ECM                        | 0.94647 |
|              | British Pound and Japanese Yen | No portfolio case |               |
| Currencies   | models                     | Mean | Variance | PP       | p          |
| SF           | Dynamic                    | 0.90068 | 0.00085 | -4.3992  | 0.98520 |
|              | OLS                        | 0.89619 |
|              | ECM                        | 0.90000 |
| DM           | Dynamic                    | 0.92836 | 0.00086 | -16.844  | 0.79420 |
|              | OLS                        | 0.91605 |
|              | ECM                        | 0.91959 |

Notes: PP is the Phillips-Perron unit root test on the dynamic hedge ratio and ρ is the first order autocorrelation coefficient of the dynamic hedge ratio.
Figure 3.1: Time variation and size differences between the portfolio and no-portfolio hedge ratios for the Swiss Franc.
Figure 3.3: Time variation and size differences between the portfolio and no-portfolio hedge ratios for the British Pound.
Hedge ratio for the DM

Figure 3.4: Time variation and size differences between the portfolio and no-portfolio hedge ratios for the Deutsche Mark.
Section D: Hedging Efficiency Tests

(3.9) In sample comparisons of hedging performance

In the present section, the evaluation of the hedging efficiency of each static and dynamic strategy for both portfolio and no-portfolio cases is performed from a risk minimizing and utility standpoint. In order to be able to make comparisons between the no-portfolio and the portfolio case, the hedged return series estimated by the hedge ratios of the bivariate models are aggregated for the SF-DM and the BP-JY portfolios by the weights used for the construction of the two-currency portfolios.

Specifically, following the methodology used by many studies on hedging, (Kroner and Sultan (1993) and others) hedged portfolios are constructed as implied by the hedge ratios of each strategy (static and dynamic). Series of returns on the portfolios hedged by multiple futures contracts are computed for the existing sample by the following equation:

\[ X_{t+1} = D\ln S_{t+1} - (b_1^* \ D\ln f_{1,t+1}) - (b_2^* \ D\ln f_{2,t+1}) \]  

(3.9)

where \( D\ln S_{t+1} \) is the return on the spot portfolio, 
\( D\ln f_{1,t+1}, D\ln f_{2,t+1} \) are the returns on the futures contracts and, 
\( b_1^*, b_2^* \) are the hedge ratios implied by each model

In the bivariate case, where a single futures contract is used to hedge the underlying spot currency, the return on the hedged portfolio is computed by the following equation:

\[ X_{t+1} = D\ln S_{t+1} - b^* D\ln f_{t+1} \]  

(3.10)

where \( D\ln S_{t+1} \) is the return on the spot exchange rate, 
\( D\ln f_{t+1} \) is the return on the futures contract and, 
\( b \) is the hedge ratio implied by each model
The returns derived by the hedge ratios of the bivariate models are aggregated for each currency portfolio by using the value weights \( w \) of each currency \( S \) in the spot portfolio \( P \). Specifically, each return series is multiplied by \((S_{t-1}w_i/P_{t-1})\) \((I = SF, DM, BP, JY)\) before it is added to the second return series of the same portfolio. Comparisons between the static and dynamic models and the no-portfolio and portfolio cases are then made in terms of the variance reduction performed by each model and the utility gains implied for the investor.

The variances of the hedged portfolios are computed for each case and they are reported in table 3.10.1. Moreover, the absolute and the percentage variance improvement of the dynamic over the constant hedging techniques, as well as, the improvement of the portfolio over the no-portfolio case are reported in the same table. An interesting result is the additional risk reduction achieved with the Error Correction model relative to the traditional model, showing the significance of including the error correction term in the hedging model. In Table 3.10.1, a comparison between static and dynamic models shows that the dynamic strategy outperforms slightly the constant hedging strategies in terms of variance reduction. Although most of the variance reduction is achieved by the naïve and static strategies, an additional risk reduction of a minimum of 0.03% per day (BP-JY no portfolio case) and a maximum of 3.20% per day (SF-DM portfolio case) is possible with the use of a dynamic hedge. This finding contradicts the empirical findings of Lypny (1988) who reports a poor performance of his portfolio hedge ratios relative to the no-portfolio case. However, the latter author attributes his results to the constant variance assumption used in his model, which is most probably the case since in the present study a dynamic specification for the conditional variances was used.

The comparison between the portfolio and the no-portfolio case for the two alternative spot portfolios shows that, in general, hedging currencies in isolation is not efficient in terms of risk. The efficiency gain resulting from modeling and hedging jointly spot currencies is shown by comparing similar strategies between
the portfolio and the no-portfolio case. For every strategy examined the portfolio case has a lower variance than the no-portfolio case. For example, a risk reduction of 4% is achieved by the dynamic SF-DM portfolio case relative to the dynamic no-portfolio case. However, a similar comparison for the BP-JY portfolio shows an additional risk reduction of only 0.76% for the dynamic portfolio case relative to the dynamic no-portfolio case. The lower variance reduction achieved in the BP-JY portfolio is explained by the low correlation between the component currencies. Accounting for the covariance of their futures returns in a portfolio context is thus not as effective in terms of risk as in the case of two futures contracts with almost perfect conditional correlation.

Nevertheless, a general conclusion that emerges from the comparison among different hedging strategies is that, accounting for portfolio effects in a multi-currency hedging problem, is important when the risk reduction criterion is examined. Additionally, a dynamic strategy is required in order to achieve the risk-minimizing objective. Although, in the present study, the assumption of risk neutrality rules out the use of a utility approach, the measure of utility can be applied in combination with the effect of the transaction costs, in order to form a selective rebalancing hedging strategy.

We assume that \( x \) is the series of returns from the hedged portfolios and that the mean-variance utility function of the investor, is given by the following equation:

\[
EU(x_{t+1}) = E(x_{t+1}) - \gamma \text{var}(x_{t+1})
\]  

(3.11)

Where \( \gamma \) is the degree of risk aversion (\( \gamma > 0 \))

If the expected return on the hedged portfolio is equal to zero and the degree of risk aversion is equal to 4, (3.11) becomes:

\[
EU(x_{t+1}) = -4 \times \text{var}(x_{t+1})
\]  

(3.12)
and for a period of 2537 days:

\[ EU(x_{t+1}) = -4 \times 2537 \times \text{var}(x_{t+1}) \]  

(3.13)

Where \( \text{var}(x_{t+1}) \) is the variance of the hedged portfolio implied by the different strategies.

Table 3.10.2 presents utility comparisons among the three constant hedging strategies and the dynamic hedge for the 2537-days period for both the portfolio and the no-portfolio case. As it is expected, the dynamic hedge outperforms the static hedges in both cases even with daily rebalancing of the futures position. It is also quite clear that, accounting for portfolio effects in multicurrency hedges, results in additional utility gains for the hedger.

Nevertheless, in the real world, investors cannot rebalance their portfolios on a daily basis since transaction costs make continuous rebalancing extremely costly. The basic advantage of the dynamic strategy is not only the variance reduction but also the possibility offered to the investor to selectively rebalance his hedged position, taking the transaction costs into consideration. Every day, the hedger can choose whether or not to change his/her futures position, only when the expected utility gains from rebalancing offset the transaction costs incurred when a new position is taken in the futures market. In this case, the utility from rebalancing is estimated by subtracting an amount \( y \), equal to the transaction costs per cent, from the estimated expected utility. Assuming again a zero expected return to the hedged portfolio, in the case of continuous rebalancing, equation (3.12) becomes:

\[ EU(x_{t+1}) = -y - \gamma \text{var}(x_{t+1}) \]  

(3.14)

Extending the methodology of *Kroner and Sultan (1993)* to the case of multiple currency positions, an investor with a mean-variance expected utility function rebalances his futures positions at time \( t \), if and only if the expected utility from rebalancing is greater from the expected utility from no rebalancing:
\[-y - \gamma \left( h_{s,t+1}^2 - 2b_{fl,t}^* h_{sf,t+1} - 2b_{f2,t}^* h_{sf2,t+1} + b_{fl,t}^{*2} h_{f1,t+1} - 2b_{fl,t}^* b_{f2,t}^* h_{f1,f2,t+1} + b_{f2,t}^{*2} h_{f2,t+1}^2 \right) >
\]
\[-y \left( h_{s,t+1}^2 - 2b_{fl,t}^* h_{sf,t+1} - 2b_{f2,t}^* h_{sf2,t+1} + b_{fl,t}^{*2} h_{f1,t+1} - 2b_{fl,t}^* b_{f2,t}^* h_{f1,f2,t+1} + b_{f2,t}^{*2} h_{f2,t+1}^2 \right) \right]

where $h_{s,t+1}^2$ is the conditional variance of the spot portfolio return,

$hs_{fl,t+1}, hs_{f2,t+1}$ is the conditional covariance between the spot portfolio return and each futures return,

$h_{fl,t+1}^2, h_{f2,t+1}^2$ are the conditional variances of the futures returns,

$h_{fl,f2,t+1}$ is the conditional covariance of the futures returns,

$b_{fl,t}^{*}, b_{f2,t}^{*}$ are the dynamic hedge ratios for continuous rebalancing,

$b_{fl,t}^{*}, b_{f2,t}^{*}$ are the hedge ratios from the most recent rebalancing,

$\gamma$ is the degree of relative risk aversion.

The relations in the parentheses of (3.15) express the variance of the hedged portfolio in the case of rebalancing and in the case of no rebalancing respectively.

The conditional variances and covariances estimated by the bivariate and multivariate dynamic models are substituted in the conditional hedging strategy presented above. In the bivariate case, the term representing the conditional covariance of the futures returns ($h_{fl,f2,t+1}$) is set equal to zero since the latter model ignores portfolio effects. Although a round trip in the futures market, i.e. one buy and one sell, usually costs around $10-$15, we assume that $\gamma$ can take values from 0.001 - 0.003, in order to show the magnitude of the effect of the transaction costs on the effectiveness of the dynamic hedging strategy.

The column named “rebalancing” in Table 3.10.2 shows how many times an investor rebalances his position over the 2537-days period under six different scenarios for the amount of transaction costs. For example, the investor with the BP-JY portfolio and for $\gamma = 0.001$, rebalances his futures positions 1279 times over
the 2537-days period and he has an expected utility of −363.162. On the contrary, if he uses the best constant strategy (ECM hedge), avoiding transaction costs, his utility function is equal to −364.250. It is quite clear that even though the percentage risk reduction achieved by the dynamic hedge over the ECM is quite small (0.094%), the dynamic strategy outperforms the constant hedging strategy on the basis of utility comparisons.

In the case of selective rebalancing, the dynamic portfolio strategy still dominates, as it is evident by its superior utility performance versus the no-portfolio case. However, the effect of the transaction costs on the utility of the dynamic hedging strategy is very significant. In Table 3.10.2, it is obvious that a round trip transaction cost over $20 will decrease the number of possible rebalancings of the hedged position leading to a decrease in the estimated utility function. This finding supports the results of Meyer (1999) and Laurent and Zenios (1999) on the effect of the transaction costs on the hedging model. It is thus very important to account for transaction costs in the decision making of a dynamic strategy.

The results of the in sample analysis are based on the assumption that the risk minimizing hedge ratio for the following period is known a priori at the time the hedge is initiated. However, since this is not a realistic assumption, the investor in the real world is concerned about the forecasting ability and the hedging performance of the model ex-ante. In the following section, the same comparisons on the basis of variance reduction and utility maximization will be performed in an out of sample analysis.
### Table 3.10.1: Variance comparisons of the in sample Hedging Effectiveness of hedging models

<table>
<thead>
<tr>
<th>Hedging model</th>
<th>Portfolio case</th>
<th>No portfolio case</th>
<th>Hedging model</th>
<th>Portfolio case</th>
<th>No portfolio case</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Variances</td>
<td></td>
<td></td>
<td>Variances</td>
<td></td>
</tr>
<tr>
<td>Unheded</td>
<td>0.50939</td>
<td>0.50939</td>
<td>Unheded</td>
<td>0.35124</td>
<td>0.35124</td>
</tr>
<tr>
<td>Naïve</td>
<td>0.04143</td>
<td>0.04144</td>
<td>Naïve</td>
<td>0.03758</td>
<td>0.03861</td>
</tr>
<tr>
<td>Traditional</td>
<td>0.03973</td>
<td>0.04108</td>
<td>Traditional</td>
<td>0.03596</td>
<td>0.03617</td>
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<tr>
<td>ECM</td>
<td>0.03969</td>
<td>0.04107</td>
<td>ECM</td>
<td>0.03589</td>
<td>0.03615</td>
</tr>
<tr>
<td>Dynamic</td>
<td>0.03841</td>
<td>0.04002</td>
<td>Dynamic</td>
<td>0.03586</td>
<td>0.03614</td>
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</tbody>
</table>

Absolute and percentage variance reduction of the dynamic model:

<table>
<thead>
<tr>
<th>Hedging model</th>
<th>Portfolio case</th>
<th>No portfolio case</th>
<th>Hedging model</th>
<th>Portfolio case</th>
<th>No portfolio case</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Variances</td>
<td></td>
<td></td>
<td>Variances</td>
<td></td>
</tr>
<tr>
<td>Unheded</td>
<td>0.47098</td>
<td>92.46%</td>
<td>Unheded</td>
<td>0.31538</td>
<td>89.79%</td>
</tr>
<tr>
<td>Naïve</td>
<td>0.00302</td>
<td>7.29%</td>
<td>Naïve</td>
<td>0.00172</td>
<td>4.57%</td>
</tr>
<tr>
<td>Traditional</td>
<td>0.00132</td>
<td>3.30%</td>
<td>Traditional</td>
<td>0.00010</td>
<td>0.27%</td>
</tr>
<tr>
<td>ECM</td>
<td>0.00128</td>
<td>3.20%</td>
<td>ECM</td>
<td>0.00003</td>
<td>0.09%</td>
</tr>
<tr>
<td>Dynamic</td>
<td>0.00161</td>
<td>4.006%</td>
<td>Dynamic</td>
<td>0.00028</td>
<td>0.76%</td>
</tr>
</tbody>
</table>

Absolute and percentage variance reduction of the portfolio model:

<table>
<thead>
<tr>
<th>Hedging model</th>
<th>Portfolio case</th>
<th>No portfolio case</th>
<th>Hedging model</th>
<th>Portfolio case</th>
<th>No portfolio case</th>
</tr>
</thead>
<tbody>
<tr>
<td>Naïve</td>
<td>0.00001</td>
<td>0.008%</td>
<td>Naïve</td>
<td>0.00103</td>
<td>2.68%</td>
</tr>
<tr>
<td>Traditional</td>
<td>0.00135</td>
<td>3.286%</td>
<td>Traditional</td>
<td>0.00021</td>
<td>0.59%</td>
</tr>
<tr>
<td>ECM</td>
<td>0.00138</td>
<td>3.375%</td>
<td>ECM</td>
<td>0.00026</td>
<td>0.70%</td>
</tr>
<tr>
<td>Dynamic</td>
<td>0.00161</td>
<td>4.006%</td>
<td>Dynamic</td>
<td>0.00028</td>
<td>0.76%</td>
</tr>
</tbody>
</table>

27 The formula used for the computation of the % variance reduction is:

\[
\text{Var}_{\text{Old model}} - \text{Var}_{\text{Dynamic model}} \over \text{Var}_{\text{Old model}}
\]

where old model: unhedged, naïve, traditional and ECM.
Table 3.10.2: Utility comparisons of the in sample Hedging Effectiveness of hedging models.

<table>
<thead>
<tr>
<th>Model</th>
<th>Transaction Costs</th>
<th>Portfolio case</th>
<th>No portfolio case</th>
<th>Model</th>
<th>Transaction Costs</th>
<th>Portfolio case</th>
<th>No portfolio case</th>
</tr>
</thead>
<tbody>
<tr>
<td>Swiss Franc and German Mark</td>
<td></td>
<td></td>
<td></td>
<td>British Pound and Japanese Yen</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unhedged</td>
<td>0.00000</td>
<td>-5171.339</td>
<td>0</td>
<td>Unhedged</td>
<td>0.00000</td>
<td>-3565.803</td>
<td>0</td>
</tr>
<tr>
<td>Naive</td>
<td>0.00000</td>
<td>-420.643</td>
<td>0</td>
<td>Naive</td>
<td>0.00000</td>
<td>-381.321</td>
<td>0</td>
</tr>
<tr>
<td>Traditional</td>
<td>0.00000</td>
<td>-403.295</td>
<td>0</td>
<td>Traditional</td>
<td>0.00000</td>
<td>-364.908</td>
<td>0</td>
</tr>
<tr>
<td>ECM</td>
<td>0.00000</td>
<td>-402.892</td>
<td>0</td>
<td>ECM</td>
<td>0.00000</td>
<td>-364.250</td>
<td>0</td>
</tr>
<tr>
<td>Dynamic</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td>0.00125</td>
<td>-389.411</td>
<td>1939</td>
<td>0.00125</td>
<td>-363.156</td>
<td>1130</td>
<td>-366.583</td>
</tr>
<tr>
<td></td>
<td>0.00150</td>
<td>-389.400</td>
<td>1844</td>
<td>0.00150</td>
<td>-363.147</td>
<td>977</td>
<td>-366.557</td>
</tr>
<tr>
<td></td>
<td>0.00200</td>
<td>-389.577</td>
<td>1789</td>
<td>0.00200</td>
<td>-363.265</td>
<td>963</td>
<td>-366.619</td>
</tr>
<tr>
<td></td>
<td>0.00250</td>
<td>-389.767</td>
<td>1777</td>
<td>0.00250</td>
<td>-363.345</td>
<td>959</td>
<td>-366.641</td>
</tr>
<tr>
<td></td>
<td>0.00300</td>
<td>-389.816</td>
<td>1679</td>
<td>0.00300</td>
<td>-363.422</td>
<td>947</td>
<td>-366.659</td>
</tr>
</tbody>
</table>
In the present section, the ex-ante performance of the proposed hedging strategies is examined with the use of the out of sample analysis. It is expected that the hedging effectiveness ex ante is lower than ex post since the future price changes of financial assets cannot be forecasted with certainty. In order to perform an out of sample comparison of the hedging effectiveness of each model, the last 100 observations are withheld and the bivariate and multivariate models are estimated with the first 2441 observations. One step forecasts are made for the daily spot and futures return of each currency and portfolio, and forecasted variances and covariances are computed. This analysis is repeated 100 times by adding one observation at a time in the dataset, until all the observations of the initial sample are used. The forecasted hedge ratios are estimated by dividing the forecasted covariances by the forecasted variances of the futures returns. In this way, 100 forecasted hedged portfolios are constructed for each model, and the variances of their returns are computed and given in Table 3.11.1. Moreover, the absolute and percentage variance improvement of the dynamic strategy over the traditional hedging techniques as well as the improvement of the portfolio over the no portfolio method are reported in the same table.

Although the magnitude of the efficiency gains is limited in the out of sample analysis relative to the in sample case, the dynamic strategy slightly outperforms the constant hedging strategies, giving support to the forecasting performance of GARCH models in short-term investment horizons (Christoffersen, Diebold and Schuermann (1998)). It is worth noting that the ECM again outperforms the OLS model in all cases examined. The implication of this finding for an international investor is that a hedging model that does not include an error correction term will lead to unhedged risks and losses in the combined spot and futures portfolio returns.

The comparison between the no-portfolio and the portfolio case, reveals that for all models examined, portfolio effects are important in the hedging decision. For
example, even though in the case of the BP-JY portfolio, the improvement of the dynamic portfolio hedge over the no-portfolio hedge is quite small, (0.120 %), in the SF-DM portfolio this reduction is of 2.50%. The implication of this finding for the present study is that hedging currencies in a dynamic portfolio context improves volatility forecasting and hedging effectiveness.

The superiority of the dynamic strategy is more meaningful when it is applied to the decision of rebalancing the combined spot–futures position. The investor can rebalance his portfolio only when the gains in the expected utility offset the transaction costs. In the out of sample analysis, selective rebalancing can also be applied to the traditional and ECM hedges since the method of rolling regressions has generated a series of time-varying forecasted hedge ratios for each model. Table 3.11.2 presents utility comparisons among the three traditional hedging strategies and the dynamic hedge for the portfolio and the no-portfolio case over the 100-days period. The column named “rebal.” shows how many times the hedger would rebalance his position over the 100-days period, under six different scenarios for the amount of transaction costs.

The results of Table 3.11.2 reveal that the same pattern applies to the utility performance of the proposed hedging models as in the in sample analysis, with the dynamic portfolio strategy being the most successful. An interesting observation in the dynamic cases is that for both currency portfolios, the investor is able to increase his expected utility by rebalancing his futures positions more times if he follows the portfolio method than if he hedges his currency holdings separately. However, it is worth noticing that for an amount of transaction costs greater than $20, a decrease in the utility and a limitation in the possibility of rebalancing is observed, showing the importance of the transaction costs in dynamic hedging.

We can conclude that, in the out of sample analysis, the dynamic portfolio strategy again outperforms the constant and dynamic no portfolio hedging strategies both on the basis of variance reduction as well as of utility maximization in the presence of transaction costs. Although the results of the out of sample analysis show that the in
sample measures overstated the real hedging effectiveness of hedging strategies, they still give empirical support to the ability of the dynamic portfolio strategy to provide an adequate risk management solution in the foreign exchange market.
Table 3.11.1: Variance comparisons of the out of sample hedging effectiveness of hedging models.

<table>
<thead>
<tr>
<th>Hedging model</th>
<th>Swiss Franc and German Mark</th>
<th></th>
<th>British Pound and Japanese Yen</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Portfolio case</td>
<td>No portfolio case</td>
<td>Hedging model</td>
<td>Portfolio case</td>
</tr>
<tr>
<td>----------------</td>
<td>-----------------</td>
<td>------------------</td>
<td>----------------</td>
<td>----------------</td>
</tr>
<tr>
<td>Unheded</td>
<td>0.43273</td>
<td>0.43273</td>
<td>Unheded</td>
<td>0.32541</td>
</tr>
<tr>
<td>Naïve</td>
<td>0.02930</td>
<td>0.02928</td>
<td>Naïve</td>
<td>0.01665</td>
</tr>
<tr>
<td>Traditional</td>
<td>0.02834</td>
<td>0.02898</td>
<td>Traditional</td>
<td>0.01658</td>
</tr>
<tr>
<td>ECM</td>
<td>0.02825</td>
<td>0.02893</td>
<td>ECM</td>
<td>0.01655</td>
</tr>
<tr>
<td>Dynamic</td>
<td>0.02817</td>
<td>0.0289</td>
<td>Dynamic</td>
<td>0.01653</td>
</tr>
</tbody>
</table>

**Absolute and percentage variance reduction of the dynamic model**

<table>
<thead>
<tr>
<th>Hedging model</th>
<th>Variance Reduction</th>
<th>Portfolio case</th>
<th>No portfolio case</th>
<th>Hedging model</th>
<th>Variance Reduction</th>
<th>Portfolio case</th>
<th>No portfolio case</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unheded</td>
<td>93.32%</td>
<td>0.40456</td>
<td>93.49%</td>
<td>0.40383</td>
<td>93.2%</td>
<td>0.30888</td>
<td>94.92%</td>
</tr>
<tr>
<td>Naïve</td>
<td>3.85%</td>
<td>0.00113</td>
<td>0.00038</td>
<td>1.32%</td>
<td>0.00012</td>
<td>0.00073</td>
<td>4.21%</td>
</tr>
<tr>
<td>Traditional</td>
<td>0.61%</td>
<td>0.00017</td>
<td>0.00008</td>
<td>0.31%</td>
<td>0.00005</td>
<td>0.00013</td>
<td>0.78%</td>
</tr>
<tr>
<td>ECM</td>
<td>0.26%</td>
<td>0.00008</td>
<td>0.00003</td>
<td>0.12%</td>
<td>0.00002</td>
<td>0.00003</td>
<td>0.18%</td>
</tr>
</tbody>
</table>

**Absolute and percentage variance reduction of the portfolio model**

<table>
<thead>
<tr>
<th>Hedging model</th>
<th>Variance Reduction</th>
<th>Portfolio case</th>
<th>No portfolio case</th>
<th>Hedging model</th>
<th>Variance Reduction</th>
<th>Portfolio case</th>
<th>No portfolio case</th>
</tr>
</thead>
<tbody>
<tr>
<td>Naïve</td>
<td>-0.07%</td>
<td>-0.00002</td>
<td>-0.00002</td>
<td>Naïve</td>
<td>3.62%</td>
<td>0.00063</td>
<td>3.62%</td>
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<tr>
<td>Traditional</td>
<td>2.21%</td>
<td>0.00064</td>
<td>0.00064</td>
<td>Traditional</td>
<td>0.60%</td>
<td>0.00010</td>
<td>0.60%</td>
</tr>
<tr>
<td>ECM</td>
<td>2.37%</td>
<td>0.00068</td>
<td>0.00068</td>
<td>ECM</td>
<td>0.18%</td>
<td>0.00003</td>
<td>0.18%</td>
</tr>
<tr>
<td>Dynamic</td>
<td>2.50%</td>
<td>0.00073</td>
<td>0.00073</td>
<td>Dynamic</td>
<td>0.12%</td>
<td>0.00002</td>
<td>0.12%</td>
</tr>
</tbody>
</table>
Table 3.11.2: Utility comparisons of the out-of-sample hedging effectiveness of hedging models.

<table>
<thead>
<tr>
<th>Swiss Franc and German Mark</th>
<th>British Pound and Japanese Yen</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hedging Model</td>
<td>Trans. Costs</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Unhedged</td>
<td>0.00000</td>
</tr>
<tr>
<td>Naïve</td>
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</tr>
<tr>
<td></td>
<td>0.00000</td>
</tr>
<tr>
<td></td>
<td>0.00000</td>
</tr>
<tr>
<td>Traditional</td>
<td>0.00000</td>
</tr>
<tr>
<td></td>
<td>0.00020</td>
</tr>
<tr>
<td></td>
<td>0.00025</td>
</tr>
<tr>
<td></td>
<td>0.00030</td>
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<tr>
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<td>0.00000</td>
</tr>
<tr>
<td></td>
<td>0.00100</td>
</tr>
<tr>
<td></td>
<td>0.00125</td>
</tr>
<tr>
<td></td>
<td>0.00150</td>
</tr>
<tr>
<td></td>
<td>0.00200</td>
</tr>
<tr>
<td></td>
<td>0.00250</td>
</tr>
<tr>
<td></td>
<td>0.00300</td>
</tr>
<tr>
<td></td>
<td>0.00000</td>
</tr>
<tr>
<td></td>
<td>0.00100</td>
</tr>
<tr>
<td></td>
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</tr>
<tr>
<td></td>
<td>0.00150</td>
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<tr>
<td></td>
<td>0.00200</td>
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<tr>
<td></td>
<td>0.00250</td>
</tr>
<tr>
<td></td>
<td>0.00300</td>
</tr>
</tbody>
</table>
Section E: Conclusions

The bulk of empirical evidence on foreign exchange hedging is based on direct and cross-hedges of a single currency position. However, hedging multiple currency positions is a usual investment problem that has to be accounted for with the adequate methodology. In the present chapter, the problem of direct multicurrency hedging was analyzed for four major currencies. Following the methodology introduced by Gagnon, Lypny and McCurdy (1998), two passive indices were constructed, one including positions in the Swiss Frank and the Deutsche Mark and a second consisting of British Pounds and Japanese Yen.

The cointegration between spot and futures exchange rates and the volatility clustering of financial returns have given rise to the development of new risk management solutions for foreign exchange risk. In the present study, the effect of cointegration was accounted for in a dynamic context on both the level and the variance of spot and futures exchange rate returns, providing significant results for the derivation of dynamic hedge ratios. To the best of our knowledge, no previous effort has been made so far to incorporate the effect of this equilibrium relationship on both conditional moments of spot and futures returns in an application to risk management. Two multivariate GARCH-X models were estimated for the spot currency portfolios and their respective futures contracts in order to account for the basic limitations of the conventional (OLS) hedging model. The estimated dynamic models were found statistically superior to the constant methods, giving statistically significant coefficients for all additional terms. Additionally, four bivariate GARCH-X models were estimated for each spot currency return and the return on its relevant futures contract, giving similar results.

A comparison between the risk minimizing hedge ratios generated by the bivariate systems with those of the multivariate systems revealed that single hedges tend to overhedge the spot position. The estimated hedge ratios from the portfolio model were half in size and more variable than their no-portfolio counterparts. It is also
worth mentioning that the estimated error correction models have given risk minimizing hedge ratios that were higher than those derived by the OLS model, supporting the critique to the conventional method for underhedging the spot position.

In order to examine the portfolio effects on the performance of a hedge, the single hedged return series were aggregated with the weights used in the construction of the spot portfolios. The hedging performance of the hedge ratios derived by the multivariate models was compared to this of the bivariate case. For all strategies examined, the portfolio case outperformed the no-portfolio case both in terms of risk reduction as well as in terms of utility maximization. The comparison between the static and dynamic strategies showed that the estimated dynamic hedge ratios have outperformed the constant ones in terms of variance reduction. Utility comparisons revealed that even with the daily rebalancing of his position, a hedger could achieve a higher utility with the dynamic strategy than with the constant hedging strategies.

However, this analysis has extended the empirical study of Gagnon et. al (1998) in two ways. A conditional hedging strategy was applied in the estimated dynamic models, accounting for transaction costs and, allowing to the investor to selectively rebalance his futures positions only when the gains in expected utility were higher than the transaction costs. In the presence of transaction costs, selective rebalancing has led to higher gains in utility for the dynamic portfolio model compared to the OLS and ECM hedging models. Additionally, in contrast with the analysis of the previous study, an out of sample analysis was also performed in order to examine the true efficiency gains resulting from the dynamic portfolio strategy. Although the out of sample performance of the models was limited relative to their in sample performance, the dynamic portfolio hedge was again the dominant strategy.

*The main conclusion emerging from the present study is that, accounting for portfolio effects when hedging multicurrency positions is crucial for the construction of a successful hedge. Additionally, the close arbitrage relationship*
existing between spot and futures exchange rates and the time-varying nature of most financial and high-frequency returns, make the use of a dynamic hedging strategy necessary in order to hedge efficiently a spot currency position. The forecasting performance of GARCH models in short horizons enhances their success in the development of dynamic hedge ratios.
CHAPTER 4: Basis Risk, Marking-to-market and the Optimal Currency Hedge Ratio

Section A: Introduction

The dramatic volatility of interest rates over the past years, makes the assumption of non-stochastic interest rates unrealistic when hedging with futures contracts. On the basis of the cost-of-carry futures pricing model, under stochastic interest rates, interest rate risk exists not only for interest rate sensitive portfolios, but also for all investment assets. Traditional theories of exchange rate determination relate exchange rates to interest rates. According to Covered Interest Rate Parity, in foreign exchange markets, the forward basis is equal to the interest rate differential between the two currencies involved. In other words, basis risk is due to uncertainty created by stochastic interest rates. When hedging with futures contracts, additional risk is transferred to the hedger through the daily marking-to-market feature of futures contracts. A hedge ratio for a foreign currency position, that ignores interest rate risk, may lead to unhedged risks and limited hedging effectiveness. In other words, basis risk and the marking-to-market effect can reduce the attractiveness of the futures contract as a risk management tool.

In the present chapter, the assumption of non-stochastic interest rates is relaxed. The estimation of the risk minimizing hedge ratio is based on the inter-temporal hedge ratio for basis risk and marking-to-market as derived by Chang and Fang (1990) and Chang, Chang and Fang (1996). It is assumed that a default-free domestic discount bond is used as a hedge for the interest rate risk of the futures positions since the interest yield or paid on the margin account is expressed in terms of the domestic interest rate. However, an extension to the Chang et. al model into a dynamic context is performed, by assuming time-varying variances and covariances for the spot and futures exchange rates and Eurocurrency interest rates. The domestic (US) interest rate is assumed to follow a mean-reverting square root process (Cox, Ingersoll and Ross, 1985) with a time-varying volatility parameter (TVP-Levels model, Brenner et. al, 1996). The spot and futures
exchange rate returns are modeled in a GARCH-X (1,1) process in order to account for the cointegration and time-variation in their conditional distribution, as evidenced in the previous chapter. A multivariate system is thus formed which is estimated jointly in order to provide time-varying variances and covariances of the variables of interest. *To the best of our knowledge, no previous effort has been made so far to model all three series into a dynamic setting in order to jointly hedge exchange rate and interest rate risk.* The dynamic risk minimizing hedge ratio under stochastic interest rates is then estimated as derived by Chang *et al* and compared with the dynamic hedge ratio under non-stochastic interest rates, in terms of hedging performance.

Section B of the present chapter presents the theoretical model of Chang and Fang (1990) and the derivation of the optimal hedge ratio accounting for basis risk. A brief review of the literature on relevant empirical studies is also included. The remainder of the section involves all empirical issues with respect to the term structure of interest rates, leading to the building of the final model to be estimated for the domestic interest rate. In Section C, the preliminary data analysis is performed and the multivariate GARCH system is estimated for four major currencies, the British Pound, German Mark, Swiss Franc and the Japanese Yen. Dynamic basis risk minimizing hedge ratios are estimated and compared in terms of size with the dynamic risk minimizing hedge ratios estimated in the previous chapter, under the assumption of non-stochastic interest rates. In Section D, in sample and out-of-sample hedging effectiveness comparisons in terms of variance reduction and utility performance are performed between constant and dynamic strategies and between strategies ignoring and accounting for stochastic interest rates. Finally, in Section E the conclusions of the present empirical chapter are discussed.
Section B: Theoretical Issues and Empirical Design

(4.1) Interest Rate Risk and the optimal currency hedge ratio

Although there is a vast empirical literature on the development of hedge ratios for exchange rate risk, only a few empirical studies focus on the effect of interest rate risk on the effectiveness of a derivative contract to hedge currency risk. The first attempt to incorporate the influence of the forward basis on the optimal currency hedge ratio is made by Solnik (1990), based on the Covered Interest Rate Parity Condition. The derived hedge ratio is decomposed into two terms: a macroeconomic term that depends on the covariance between exchange rates and the interest rate differential and an asset-specific term that is a function of the covariance of the asset return with the exchange rate and the interest rate differential. Briys and Solnik (1992) make a similar approach to currency hedging by emphasizing on the currency hedging decision of predetermined foreign stock and bond portfolios. However, a speculative term is also included in the derived five-term hedge ratio, accounting for the presence of any bias in the forward market. The remaining terms are related to the interest rate risk and are sensitive to the utility function of the investor. None of these studies made any tests on the performance of the proposed hedging policy, focusing on the relative importance of each hedge component across countries.

Tong (1996) extends the previous studies to a dynamic framework by allowing for time-varying variances and covariances of the spot and forward exchange rates and stock returns. He focuses on the minimum-variance components of the hedge ratio that are represented by the macroeconomic and the asset specific component. However, he concludes that, for the Japanese market, GARCH hedging methods do not lead to a large improvement in performance relative to constant methods. Glen and Jorion (1993) follow a different approach by focusing on the investment decision. They account for the currency risk of foreign investments by simultaneously constructing optimal portfolios consisting of foreign assets and forward contracts. They use the forward premium, as defined by the interest rate
differential, for a conditional hedging strategy that results in higher portfolio performance both in an in-sample and out-of-sample analysis.

It is worth noting that the above mentioned hedge ratios cannot be applied in the futures market since the marking-to-market effect introduces more complexity in the futures pricing model. Chang and Fang (1990) provide a general derivation of the optimal inter-temporal futures hedge ratio, under the assumption of stochastic interest rates. The main innovation of this study is that the hedge ratio depends on the bond price variance and its covariance with the spot and the futures price, and it includes the traditional hedge ratios as special cases. Comparisons between the new and the traditional hedge ratios reveal that the effect of the cash resettlement and basis risk is significant. The empirical application of the inter-temporal hedge ratio to stock index futures from Chang, Chang and Fang (1996) reveals that traditional hedge ratios tend to over-hedge the cash positions, especially in shorter investment horizons and for hedge ratios that use risk-return optimization.

The last result is expected on the basis of the theoretical explanation given by Lioui (1998). The latter author states that the volatility (risk) of the hedged portfolio does not change if futures contracts are used instead of forwards since both contracts have equal instantaneous volatility. For this reason, hedging interest rate risk stemming from the futures-forward hedging differential is not expected to affect significantly the risk minimizing hedge ratio. On the contrary, the effectiveness of the hedge is affected when the risk-return trade-off of the hedged portfolio is considered. This happens because the drifts of the hedged portfolios are different, depending on the derivative contract used. Lioui (1998) proves that the return on the hedged portfolio is a function of the interest accrued in the margin account and the covariance between the futures price and the domestic interest rate. Hedging interest rate risk would thus affect more the risk-return hedge ratio than the risk minimizing hedge ratio, as it is shown by Chang et al (1996). The significance of the effect of interest rate risk on the risk minimizing hedge ratio is an empirical issue of the present chapter.

A serious limitation of the studies mentioned above, with the exception of Tong
(1996), is the assumption that the variances and the covariances of spot, forward/futures prices and interest rates are stationary, which is invalid in the presence of the high volatility of financial returns. The estimated hedge ratios are static, so, they do not capture the dynamics of exchange rates and interest rates, leading probably to unhedged risks. In the present study, the methodology of Chang et. al (1990, 1996) is extended into a dynamic setting. Spot and futures exchange rate returns are modeled in a GARCH-X (1,1) model, while the interest rate is assumed to follow a Time Varying Parameter-Level process28. In this way, the variances and the covariances of the variables of interest are allowed to vary over time. The inter-temporal hedge ratio is then estimated for four foreign currency positions under the assumption of stochastic interest rates.

28 The Time Varying Parameter LEVEL model is presented in the following section.
(4.1.1) The inter-temporal futures hedge ratio for basis risk under stochastic interest rates

In order to derive the optimal hedge ratio under stochastic interest rates, Chang et. al (1990, 1996) were based on the following assumptions. The interest rate $i$ is assumed to follow the CIR (1985) stochastic, square root, mean-reverting process:

$$\text{di} = \kappa(\mu - i)dt + \sigma_i \sqrt{dW_i} \quad (4.1)$$

where $\mu$ is the long run mean of the interest rate.

$\kappa$ is the speed of adjustment to the long run mean

$\sigma_i$ is the instantaneous standard deviation of the interest rate change

$dW_i$ is the increment of a Wiener process

The spot and futures prices $S$ and $F$ respectively can be represented by the following stochastic differential equations:

$$\text{dS} = \mu_s(S,i,t)dt + \sigma_s(S,i,t)dW_s \quad (4.2)$$

$$\text{dF} = \mu_F(F,i,t)dt + \sigma_F(F,i,t)dW_F \quad (4.3)$$

where $\mu_S$ ($\mu_F$): expected change on the spot (futures) exchange rate

$\sigma_S$ ($\sigma_F$): instantaneous standard deviation of the spot (futures) exchange rate

$dW_S$ ($dW_F$): increments of standard Wiener processes

However, since interest rates are not tradable assets, an interest rate based instrument is needed in order to hedge the interest rate risk of the combined spot and futures currency position. The specification followed by Chang et. al (1990, 1996) for the default-free pure discount bond price corresponds to the CIR (1985) term structure model and it is given as follows:
\[ dB = \mu_B \, dt + \sigma_B \, dZ_i \]  

(4.4)

where \( \mu_B = Bi(1 - \pi G) \) is the expected change on the bond price.

\[ \sigma_B = -BG\sqrt{i} \sigma \] is the instantaneous standard deviation of the bond price.

\[ B(i, t, T) = D(t, T)e^{-\tau G(t, T)} \]

\[ D(t, T) = \left[ \frac{2te^{(\kappa + \tau + \pi)(T-t)/2}}{(\kappa + \tau + \pi)(e^\tau(T-t) - 1) + 2\tau} \right]^{2\kappa / \sigma_i^2} \]

\[ G(t, T) = \frac{2(e^\tau(T-t) - 1)}{(\tau + \kappa + \pi)(e^\tau(T-t) - 1) + 2\tau} \]

\[ \tau = [(k + \pi)^2 + 2\sigma_i^2]^{1/2} \]

and,

\[ \pi = \text{the market risk parameter of interest rates} \]

Chang, Chang and Fang (1996) were based on the previous relations and, using dynamic programming techniques, they derived the optimal hedging strategy under stochastic interest rates. Their model allows the hedger to take a long position in a domestic bond in order to hedge the interest rate risk of his futures position. The optimal hedge ratios for futures contracts and domestic default-free bonds respectively can be described by the following formulas:

\[ \frac{h_{str}}{\sigma_i R_1} = -\frac{\sigma_s R_2}{\sigma_i R_1} \]  

(4.5.1)

\[ \frac{h_{Bond}}{\sigma_i R_1} = \frac{\sigma_s R_2}{\sigma_B R_1} \]  

(4.5.2)

where

\[ R_1 = \mu_s[(1-\rho_2^2)\theta_1(\rho_1\rho_2\rho_3) + \theta_2(\rho_1\rho_2-\rho_3)] \]

\[ R_2 = \mu_s[(\rho_1\rho_2\rho_3 - \theta_1(1-\rho_3^2) + \theta_2(\rho_2 - \rho_1\rho_3)) \]

\[ \theta_1 = \mu_F \sigma_S / \sigma_F \mu_S \] is the relative price change of the futures versus the cash position per unit of risk, and

\[ \theta_2 = -i(1-\pi G)\sigma_S / G\sqrt{i} \mu_S \] is the relative price change of the futures.
The inter-temporal risk minimizing hedge ratios for futures contracts and domestic bonds can be derived from (4.5.1), assuming that \( \mu_f = 0 \), i.e. that the expected return on the futures position is equal to zero. This assumption sets \( \theta_1 = 0 \) and the hedge ratios (4.5.1) and (4.5.2) become respectively:

\[
\begin{align*}
    h_{\text{min}} &= \frac{-\sigma_s[(\rho_1 - \rho_2 \rho_3) + \theta_2 (\rho_2 - \rho_1 \rho_3)]}{\sigma_f[(1 - \rho_f^2) + \theta_2 (\rho_1 \rho_2 - \rho_3)]} \\
    h_{\text{Bond}} &= \frac{\sigma_s[(\rho \rho_3 - \rho_s) + \theta_2 (1 - \rho_s^2)]}{\sigma_f[(1 - \rho_f^2) + \theta_2 (\rho_1 \rho_2 - \rho_3)]}
\end{align*}
\]

The new risk minimizing hedge ratio for futures contracts \( h_{\text{min}} \) does not depend only on the variances of spot and futures prices, \( \sigma_s, \sigma_f \), and their correlation, \( \rho_1 \). It is also a function of the correlation of the futures prices with interest rates, \( \rho_2 \), the correlation between interest rates and spot prices, \( \rho_3 \), and the relative payoff of the default-free bond versus the cash position per unit of risk, \( \theta_2 \). An interesting aspect of this formula is that it nests the one-period risk minimizing hedge ratio formula as derived by Ederington (1979). By assuming that the interest rates are not correlated with the spot and the futures prices, i.e. setting \( \rho_2 = \rho_3 = 0 \), (4.6.1) becomes equal to the traditional risk minimizing hedge ratio:

\[
h_{\text{min}}' = \frac{\rho_1 \sigma_s}{\sigma_f}
\]

In the following sections the model used for the estimation of the inter-temporal
hedge ratio will be presented and applied on four currency futures positions.

(4.2) Dynamic hedge ratios and the appropriate model for the term structure of interest rates

As it is shown in the previous section, the derivation of the optimal currency hedge ratio under the assumption of stochastic interest rates is based on the CIR term structure model. The ability of a term-structure model to capture interest rate volatility is very important when hedging interest rate risk. Dynamic hedge ratios are sensitive to the current level of volatility and its stochastic properties. An incorrect model of the term structure of interest rates may lead to mishedged or unhedged risks. In the present section, different aspects of the term structure model are analyzed and the model used for the estimation of the interest rate process is presented.

(4.2.1) Evidence of unit roots in the term structure of interest rates

The existence of a stochastic trend in interest rate series is very important for the choice of the appropriate term structure model. Most theoretical models on the term structure of interest rates are based on the assumption that interest rate series exhibit mean reversion. However, there exists controversial empirical evidence on the stationarity of international interest rates, depending on the data and methodology used, as well as, the time period covered. Not being able to reject the hypothesis that interest rates are integrated time series, many studies proceed to testing for cointegration between interest rates of different maturities or international interest rates of the same maturity. However, evidence of cointegration in the term structure, is not consistent with the theoretical model of the term structure of Cox, Ingersoll and Ross (1985) in which the instantaneous rate is the only common factor to all yields.

Treasury bills rates for U.S. and other countries have been extensively used in order to test for unit roots and cointegration in the term structure of interest rates. Engle and Granger (1987) and Stock and Watson (1988) examine U.S. short-term and
long-term interest rates and they accept the unit root and the cointegration hypothesis. The same hypothesis is accepted in the case of U.S. Treasury securities of seven different maturities examined by Bradley and Lumpkin (1992) and between short-term (1-month U.S. T-bill rate) and long term interest rates (20-year U.S. Treasury bond yields) examined by Campbell and Shiller (1987). The latter authors use two samples in their empirical tests; a full sample and a shorter, covering a period corresponding to a single regime. An important finding of this study is that, the unit root hypothesis for the short and long rates cannot be rejected in the short sample while it can be rejected for the short rate in the full sample when a trend is included in the test regression. Although the authors rule out the existence of a trend, we can interpret this finding as evidence of trend stationarity of the short rate around a broken trend line, which is due to a regime change.

Shea (1992) replicates the empirical study of Campbell and Shiller (1987) with a new term structure dataset of zero-coupon yields with several maturities. The Stock and Watson (1988) and Sims's (1988) test statistics reject the unit root hypothesis only under the assumption of a time trend. Additionally, Perron's (1989) unit root test with a structural break rejects the null of a unit root only for the model with a changing trend level and slope and only for short-term and three year yields. However, Shea (1992) criticizes trend stationary models on the basis of their finite sample behaviour and concludes that nonstationary models describes best the term structure. Hall, Anderson and Granger (1992), use a sample of U.S. T-bills of several maturities that covers three monetary regimes that are shown by Huizinga and Mishkin (1986) and Hardouvelis (1988) to have caused structural breaks in term structure data. They cannot reject the unit root hypothesis for the full sample and the three subsamples that account for each regime. An interesting observation in the previous studies is that the role of regime changes is very important and should be considered adequately in unit root tests.

In the Eurocurrency interest rate market, Mougoue (1992) shows evidence of unit roots and one cointegrating vector between four different maturities of the same interest rate for all six countries examined. Daily Eurocurrency deposit rates on eight major currencies are used by Arshanapalli and Doukas (1994) in unit root and
cointegration tests between five different maturities of the same deposit rate. The hypothesis of the unit root can be rejected only in the case of the French franc with the standard ADF and Phillips and Perron tests. Patel and Akella (1996) cannot reject the hypothesis of a stochastic trend for the Eurocurrency deposit rates of U.S., Germany and Canada. The same result is given by Bremnes, Gjerde and Saetten (1997) for five major three-month Eurocurrency interest rates (German, U.S., Japanese, U.K. and French) and De Gennaro, Kunkel and Lee (1994) for five international long-term interest rates. However, none of these studies examined the interest rate series for a structural break that could be due to a policy regime change over the sample period.


An important limitation of the previous empirical studies supporting the presence of unit roots in the term structure, with the exception of Shea (1992), is that they ignore that classical unit root tests have low power against trend stationarity in time series (see DeJong et al (1992)). Additionally, financial crises such as the oil price
shock and the great crash, as well as interventions from monetary authorities, introduce breaks in macroeconomic time series, like interest rates, that should be taken into account in unit root tests. The traditional techniques of splitting the sample into subsamples corresponding to different regimes lead to the loss of degrees of freedom and to lower power of the unit root tests.

(4.2.2) Evidence of mean reversion in the term structure of interest rates

Recognizing the limitations of the traditional unit root tests, Perron (1989) developed a new methodology in tests of the unit root hypothesis in the presence of structural breaks in time series. This method is followed and further developed by many empirical studies (Perron and Vogelsang (1992), Christiano (1992), Zivot and Andrews (1992), Banerjee, Lumsdaine and Stock (1992)) and it is followed by the present study as well. According to Banerjee et al (1992), invalid inferences are made when stationary time series around a broken trend line are incorrectly classified as integrated. This finding casts doubt on the validity of unit roots and cointegration theory for many time series. The proponents of stationary models for interest rates are based on the idea that interest rates cannot be nonstationary since they are bounded below by zero. However, according to Shea (1992), stationary models give bounded variances and not strictly bounded levels although positive interest rates can be more reliably simulated in stationary models.

Mean reversion of interest rates is present in many theoretical models of the term structure as for example in Lutz (1940), Vasicek (1977), Courtadon (1982) and Cox, Ingersoll and Ross (1985). McCulloch (1990) shows that the returns-to-maturity hypothesis and the local expectations hypothesis, that are two expressions of the expectations hypothesis of the term structure, are mutually consistent under the assumption of a strictly trend-stationary model for the term structure.

29 Long-term yields are arithmetic averages of expected future short-term yields.
30 Short-term spot yields equal expected short-term holding period yields of long term bonds.
The literature on the evidence of mean reversion in interest rate series is innovative in that it is does not use any classical unit root tests. It is based either on new econometric techniques or theoretical term-structure models. It does not ignore the presence of structural breaks in interest rate data due to regime shifts. Sanders and Unal (1988), use Vasicek’s mean reverting model in order to test the inter-temporal behaviour of one-month T-bill yields. They support the mean reverting hypothesis, through the stability of the estimated coefficients, for the whole sample period and for only one of the four subperiods, corresponding to different regimes. They thus emphasize on the importance of accounting for a regime change in interest rate studies. Applying the methodology of unit root tests with a structural break on the ex-post real U.S. interest rates, Perron (1990) rejects the null of a unit root against the hypothesis of stationarity with a changing constant. Accounting for the shifts in the inflation rates with a Markov switching model, Evans and Lewis (1995) cannot reject the hypothesis of stationarity of real U.S. interest rates.

Fama and Bliss (1987), based on the evidence of the forecasting power of forward rates for changes in long-term interest rates, conclude that mean reversion is present in long-term interest rates. However, Shea (1992) criticizes their methodology as an informal test for mean reversion without any specific alternative. Supportive evidence to the results of Fama and Bliss (1987) and doubt on the current literature on unit roots and cointegration of interest rates is provided by Wu and Zhang (1996). The latter authors develop a new multivariate testing procedure in order to test for unit roots in international interest rates. They form a system of autoregressive processes with monthly data on various interest rates from twelve OECD countries. In recognizing the drawbacks of the traditional univariate unit root tests, they use the cross-correlations of the international interest rates as additional information in their tests. By restricting the first order autoregressive coefficients to be equal across countries, they reject the unit root hypothesis for all series examined.

Although there is a vast literature on the presence of unit roots in interest rate data compared to this of mean reversion, the former case ignores the effect of structural breaks that makes a series look non-stationary even when it is not. The standard
unit root tests, used in most empirical studies, are no longer valid in the presence of structural breaks. In the present study, the data will be searched for regime shifts or other events that may create a structural break and the methodology of Perron (1989) will be followed. This approach is necessary before the application of the theoretical term-structure model that imposes mean reversion on the interest rate series.

(4.2.3) The LEVELS model

The term structure model on which Chang et al. (1990, 1996) were based for the derivation of the optimal hedge ratio, is the one-factor square-root mean reverting model of Cox, Ingersoll and Ross (1985)

\[ di = \kappa (\mu - i) dt + \sigma i^{\gamma} dw_i \]  \hspace{1cm} (4.1)

where \( \gamma = 0.5 \)

This model incorporates mean reversion for the short rate \( i \) which is pulled to a level \( \mu \) at a rate \( \kappa \). The higher \( \kappa \) is, the faster the interest rate \( i \) responds to deviations from its long-run mean \( \mu \). The standard deviation of the short rate is proportional to \( \sqrt{i} \), which implies that high volatility in interest rates is associated with high interest rate levels. In this model, heteroscedasticity in interest rates is accounted for through the squared level of the interest rate. The CIR model has some advantages and disadvantages compared to other term structure models. We will refer to them briefly since the development of a model for the term structure of interest rates is a separate issue and is not the main subject of the present study.

According to Subrahmanyam (1996) the CIR model is completely internally consistent since it is based on a general equilibrium framework. The functional form and the dynamics of the bond prices are implicitly derived and the market price of risk is determined endogenously. The square root type process imposes stationarity on the interest rate which can never be negative or zero. Additionally, the well-documented dependence of the variance of the interest rate on its level is introduced through the parameter \( \gamma \). Reported high correlations between the
level and its variance, imply that periods with high short rates are associated with periods of high interest rate volatility. This idea is supported by the empirical findings of Ball and Torous (1995), who found that estimated volatilities for the periods 1974-1975 and post-1985, corresponding to periods where interest rates were much lower, were lower than those of 1979-1983.

However, a basic disadvantage of the CIR model is the assumption that investors' preferences are logarithmic, implying that their decisions are independent of their wealth and are thus myopic. Additionally, the fact that the CIR model does not consider the current term structure that is observed in the market, invalidates the model in terms of pricing contingent claims and forecasting performance. Brown and Dybvig (1986), Marsh and Rosenfeld (1983) and Pearson and Sun (1994) reject the CIR model in terms of stability tests of the estimated parameters and nested tests respectively. Chan, Karolyi, Longstaff and Sanders (CKLS) (1992) also reject the CIR model since the estimate of $\gamma$ is found to be equal to 1.5.

However, as Subrahmanyam (1996) states, the general version of the CIR model is not necessarily misspecified and an extension to the model may show important improvements in the pricing of interest-rate sensitive assets. As Eom (1995) shows, the results of the previous studies can be attributed to inadequate methods of estimation. Indeed, the estimation of term structure models usually requires the following restriction for stationarity: $0 \leq \gamma \leq 1$. A value of $\gamma$ greater than 1, would introduce non-stationarity in the model, something that is not consistent with the theory of mean reverting interest rate processes. According to Hamilton (1990), the non-stationarity can be explained by the presence of regime shifts; interest rates may exhibit mean reversion in each regime but, taken on average, they may exhibit non-stationarity.

According to Ball and Torous (1995), the omission of the restriction: $0 \leq \gamma \leq 1$ is the main reason why CKLS cannot find any evidence for a structural break in October 1979, when the Federal Reserve announced a change in its operating policy. By definition, a structural break makes a stationary time series look non-stationary and CKLS's estimate of $\gamma = 1.5$ captures the non-stationarity created.
by the regime shift. However, when *Ball and Torous (1995)* test the CKLS model with respect to a structural break, under the restriction: \(0 \leq \gamma \leq 1\), evidence for a structural break in October 1979 is found. More specifically, the *Cox, Ingersoll and Ross (1985)* hypothesis of \(\gamma = 0.5\) cannot be rejected by the same authors even when they apply a model of Markov regime shifts and a stochastic volatility model. *Ball and Torous (1999)* accept the same hypothesis after applying a stochastic volatility model to a sample of Euro-currency interest rates. The *CIR square-root mean reverting model is also supported by Brenner, Harjes and Kroner (1996)* for capturing time variation in the interest rate volatility better than other one-factor and two-factor models when the investment horizon is short. The value of \(\gamma\) is not significantly different from 0.5 in the estimation of a two-factor Level EGARCH model by *Andersen and Lund (1997)*.

(4.2.4) The TVP-LEVELS model

The empirical evidence provided in the previous subsection shows that the CIR model that describes the interest rate as a mean reverting square root type process with volatility depending on its level, is a valid model for the representation of the domestic interest rate. However, while the sensitivity of the volatility of the interest rate to its level should not be ignored, the interest rate volatility process should also depend on unexpected interest rate shocks. According to *Brenner, Harjes and Kroner (1996)*, the dynamic hedge ratios of interest rate risk are affected not only by the current level of volatility but also by the stochastic properties of volatility. It becomes thus important to allow the volatility parameter to be also a function of the news arrival process with a GARCH model. There have been several applications of the GARCH model to interest rate data, like those of *Engle, Lilien and Robins (1987), Park and Bera (1997), Longstaff and Schwartz (1992), Gagnon and Lypny (1995), Koutmos (1996)*.

However, modeling interest rates in a GARCH process results in high persistence in the volatility function and allows for negative interest rates. Additionally, it ignores a well- evidenced theoretical implication of the term structure models: the dependence of the volatility to the interest rate level. Accounting for these weaknesses of the GARCH model, *Koedijk, Nissen, Schotman and Wolff*...
(1997) and Brenner, Harjes and Kroner (1996) nest the GARCH and LEVELS models in a more general discrete-time approximation of the interest rate process. Brenner, Harjes and Kroner (1996) propose the following model for the interest rate process that allows the variance to be a function of the level of the interest rate as well as of unexpected shocks to the interest rate market.

\[ i_t - i_{t-1} = \alpha + \beta i_{t-1} + \epsilon_t \]
\[ E(\epsilon_t / \Omega_{t-1}) = 0 \]
\[ E(\epsilon_t^2 / \Omega_{t-1}) = \sigma_t^2 = h_t^2 i_{t-1}^2 \]
\[ h_t^2 = c_0 + c_1 \epsilon_{t-1}^2 + c_2 h_{t-1}^2 \] (4.7)

The specification proposed above nests both the LEVEL (for \(c_1 = c_2 = 0\)) and the GARCH model (for \(\gamma = 0\)). It is called Time Varying Parameter LEVELS model (TVP-LEVELS) since it can be interpreted as a version of the LEVELS model with time-varying parameters of the variance process. In this model, the sensitivity of the volatility to the level of the interest rate depends on the size of the information shocks. Large shocks (\(\epsilon_t\)) lead to a higher dependence of volatility to levels than small shocks. In this way, the well-evidenced sensitivity of the volatility to levels is maintained and the information impact on the volatility is also accounted for. Andersen and Lund (1997) extend this analysis to a stochastic volatility framework by estimating a two-factor continuous-time model for the U.S. interest rates. They support the use of a Levels-EGARCH specification instead of Levels-GARCH for the interest rate volatility since the former is found to capture better the conditional heteroscedasticity of the interest rate process.

In the present study, the TVP-Levels model, proposed by BHK, will be used for the estimation of the variance of the interest rate. The main difference of the estimated model from the BHK model is the drift used for the interest rate process. The specific drift is necessary for the estimation of the inter-temporal hedge ratio as implied by the theoretical model of CIR. It is worth noting here that the TVP-LEVELS model is a time-varying parameter version of the LEVELS model. The theoretical bond prices of the CIR model are assumed to be unaffected by the time-
varying volatility parameter. As Longstaff and Schwartz (1992) state, the use of the GARCH process in the estimation procedure is independent of the functional form of prices implied by the theoretical model. The latter authors use a GARCH estimate for the conditional volatility of the interest rate process which is introduced in their term structure model as the second state variable.

In the present study, the model to be estimated for the term structure of interest rates is the following:

\[ i_t - i_{t-1} = \kappa (\mu - i_{t-1}) + \varepsilon_t \]
\[ \mathbb{E}(\varepsilon_t / \Omega_{t-1}) = 0 \]
\[ \mathbb{E}(\varepsilon_t^2 / \Omega_{t-1}) \equiv \sigma^2_t = \omega_1^2 \sigma_{t-1}^2 \]
\[ \omega_1^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \omega_{t-1}^2 \]  

(4.8)
Section C: Data and Empirical Results

(4.3) Data

The data on the spot and futures exchange rates, used in the present chapter, are the same as those used in Chapter 3. Daily spot and futures prices on the four major currencies, the British Pound / $U.S., Deutsche Mark / $U.S., Swiss Frank / $U.S. and Japanese Yen / $U.S. were collected\(^{31}\), covering the period January 7, 1988 to October 2, 1997 (2541 observations). An additional reason for the use of daily data in empirical studies on the behaviour of the spot-futures basis is the fact that the cash flows of the futures position are generated daily due to the marking-to-market effect under a stochastic overnight interest rate. According to McCurdy and Morgan (1988), the use of an interval of observation longer than one day for the futures price change would result in a loss of information that may reduce the power of the empirical tests.

For the estimation of the interest rate model, daily closing prices of Eurocurrency interest rates with 3-month maturity were obtained for the same sample period, as traded in the London money market for the UK, Germany, Switzerland, Japan and the U.S. The Eurocurrency market is preferred since it is more frictionless and informationally more efficient than the domestic interest rate markets as it is not subject to the same bank regulations (Aliber (1972)). As in Chapter 3, all the data series were filtered by day-of-the week adjustment regressions. The residuals of these regressions are used as the dataset of the present empirical chapter.

\(^{31}\) Source: DATASTREAM International.
(4.4) Preliminary data analysis of Eurocurrency interest rates

In the present section, the stochastic properties of the five Eurocurrency interest rates will be examined. Unit root tests will be performed in order to test the hypothesis of mean reversion. The interest rate series will also be examined for serial correlation and heteroscedasticity before estimating the dynamic model for the derivation of the optimal hedge ratio.

The CIR model, on which Chang et. al (1990, 1996) were based for the development of the optimal hedge ratio, makes the assumption that interest rate series are mean reverting processes. Before the imposition of mean reversion to the domestic interest rate, unit root tests are performed on the natural logs of each interest rate series. Although the time series properties of the domestic (US) interest rate series are of interest in the present study, the preliminary analysis is applied to all interest rate series in order to be able to draw general conclusions for the stochastic properties of interest rates.

The Phillips-Perron (PP) unit root test is used in the present study since it is more robust to heteroscedasticity and serial correlation. In Table 4.1, the PP statistic shows that the Eurocurrency interest rates of Germany and Switzerland, are trend stationary processes. Since the null of a unit root can be rejected by the most general model (with a constant and a trend), this result is accepted, given that unit root tests have low power in rejecting the unit root hypothesis. According to Shea (1992), trend stationarity can be accepted for periods with a run-up in interest rates. However, for the remaining countries, tests become more complicated. An ERM crisis, as well as several regime changes, took place over the sample period between 7 January 1988 and 2 October 1997. These events tend to create structural breaks in time series, and the ADF and PP tests are biased toward the non-rejection of a unit root. For the case of the United Kingdom, the effect of Black Wednesday on the 16th of September 1992 (obs.1225), must be considered. In the case of Japan, a regime shift has taken place in August 1995 (obs.1996), when the Bank of Japan, set interest rates at a low level. In the United States, the Federal Reserve
Board raised the target for the federal funds rate in February 1994 (obs.1585).

One method of testing for a unit root in the presence of a structural break is splitting the sample into two subsets and performing unit root tests in each subset. However, this procedure leads to less degrees of freedom for each regression of the test, so, it is preferable to perform the unit root test on the full sample. Perron (1989) developed a formal procedure to test for unit roots in the presence of a structural break. If τ is the date of the structural break, the most general model involves estimating the following regression:

\[ y_t = a_0 + a_1 y_{t-1} + a_2 t + \mu_2 D_L + \mu_3 D_T + \sum_{i=1}^{\lambda} \beta_i \Delta y_{t-i} + \epsilon_t \]  

where:

- \( D_L = 0 \) for \( n = 1, \ldots, \tau \)
- \( D_L = 1 \) for \( n = \tau + 1, \ldots, T \)

and:

- \( D_T = 0 \) for \( n = 1, \ldots, \tau \)
- \( D_T = t \) for \( n = \tau + 1, \ldots, T \)

The dummy variables \( D_L \) and \( D_T \) represent a change in the intercept and the slope of the trend \( t \) respectively. After estimating the regression above, the t-statistic for the null \( a_1 = 1 \) can be compared to the appropriate critical value calculated by Perron for the value of \( \lambda \) corresponding to the specific date of the structural break. The value of \( \lambda \) for the US and UK is 0.5, while for Japan it is equal to 0.2. Table 4.1 below represents the results of the unit root tests for the levels of interest rates.

The dummy variable \( D_L \) is found statistically significant for all three interest rate series examined, while the dummy \( D_T \) is significant only in the case of Japan and UK. From table 4.1, it is clear that the interest rates of Switzerland and Germany are trend stationary processes. The interest rates of Japan and UK are stationary processes around a changing constant and slope trend line. The US interest rate is found to be stationary around a changing constant due to a structural break caused by a policy regime change of the Federal Reserve Bank. This finding means that the two regimes had different mean parameters. Assuming that regimes with high mean parameters are regimes with high volatility parameters, a mean-reverting model of U.S interest rates with volatility sensitive to the level, seems appropriate in
The conclusion from the unit root tests on the interest rate levels is that, in general, interest rate series are stationary processes. This finding casts doubt on the vast existing empirical literature on the non-stationarity and common stochastic trends in systems of interest rates. However, it is supported by most term structure models (Cox, Ingersoll and Ross, Vasicek (1977) etc.) that describe interest rates as mean-reverting processes. An interest rate model that incorporates mean-reversion in the interest rate series is thus adequate in the present case.

Before proceeding to any other preliminary tests on the interest rate series, the interest rate levels must be detrended. The removal of the deterministic trend is performed by the regression of each interest rate series on a constant and a polynomial time trend. The appropriate degree of the polynomial is determined by the significance of the coefficients of the trend terms as indicated by the standard t-test. This procedure of detrending results in two trend terms for all interest rate series except for the case of the Japanese interest rate where the degree of the polynomial is three. Additionally, the series with a structural break are also regressed on the appropriate level and trend dummy variables. The residuals from these regressions are considered to be the new detrended interest rate series that will be used in the remaining empirical tests and for the estimation of the term structure model.

In Table 4.1, tests for serial correlation and conditional heteroscedasticity are performed on the detrended series. The first moments of the series are found to be autocorrelated, as the Ljung-Box test (LB(20)) shows. The high degree of persistence in the interest rate series will be accounted for by including an autoregressive term in the conditional mean equation. When the Ljung-Box test is applied on squared data (LB^2 (20)) to test for ARCH effects, the results are highly significant supporting the hypothesis of time-variation in the variances and the need for the introduction of a GARCH error structure in the term-structure model.
Table 4.1: Tests for unit roots, serial correlation and conditional heteroscedasticity

<table>
<thead>
<tr>
<th></th>
<th>Unit Root tests on levels</th>
<th>PP critical values</th>
<th>LB(20)(4)</th>
<th>LB^2(20)(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Switzerland</td>
<td>-3.15(1)</td>
<td>-3.13 (10%)</td>
<td>62.58</td>
<td>395.37</td>
</tr>
<tr>
<td>Germany</td>
<td>-3.21(1)</td>
<td>-3.13 (10%)</td>
<td>88.48</td>
<td>460.33</td>
</tr>
<tr>
<td>US</td>
<td>-4.14(2)</td>
<td>-3.76 (5%)</td>
<td>52.06</td>
<td>173.57</td>
</tr>
<tr>
<td>Japan</td>
<td>-5.06(3)</td>
<td>-3.66 (10%)</td>
<td>65.99</td>
<td>845.83</td>
</tr>
<tr>
<td>UK</td>
<td>-4.45(3)</td>
<td>-4.24 (5%)</td>
<td>42.47</td>
<td>71.15</td>
</tr>
</tbody>
</table>

Notes:

(1) This statistic is the t-ratio for the hypothesis $a_1 = 1$ in the following model:

$$y_t = a_0 + a_1 y_{t-1} + a_2 (t - T/2) + \varepsilon_t$$

(2) This statistic is the t-ratio for the hypothesis $a_1 = 1$ in the following model (model a):

$$y_t = a_0 + a_1 y_{t-1} + a_2 t + \mu_2 D_t + \sum_{i=1}^{k} \beta_i \Delta y_{t-i} + \varepsilon_t$$

(3) This statistic is the t-ratio for the hypothesis $a_1 = 1$ in the following model (model c):

$$y_t = a_0 + a_1 y_{t-1} + a_2 t + \mu_2 D_t + \mu_3 D_T + \sum_{i=1}^{k} \beta_i \Delta y_{t-i} + \varepsilon_t$$

(4) This is the Ljung-Box statistic for serial correlation of 20th order on the interest rate levels.

(5) This is the Ljung-Box statistic for serial correlation of 20th order on the interest rate squared levels.
(4.5) Quasi-maximum Likelihood Estimation of the multivariate GARCH model

(4.5.1) The Model

In the present section, the following multivariate GARCH (1,1) model will be estimated in order to derive dynamic basis risk minimizing hedge ratios for the BP, DM, SF and the JY.

\[ d\ln s_t = c_s + b_s e_{ct-1} + \alpha_{ss} \sum_{j=1}^{\infty} d\ln s_{t-j-1} + \alpha_{sf} \sum_{j=1}^{\infty} d\ln f_{t-j-1} + \epsilon_{st} \]  \hspace{1cm} (4.10a)

\[ d\ln f_t = c_f + b_f e_{ct-1} + \delta_{ss} \sum_{j=1}^{\infty} d\ln s_{t-j} + \delta_{sf} \sum_{j=1}^{\infty} d\ln f_{t-j} + \epsilon_{ft} \]  \hspace{1cm} (4.10b)

\[ i_t - i_{t-1} = \kappa (\mu - i_{t-1}) + \Delta \delta_{DFR} + \epsilon_t \]  \hspace{1cm} (4.10c)

\[ h_{ss}^2 = B_0 + B_1 h_{ss(t-1)}^2 + B_2 e_{st-1}^2 + B_3 e_{ct-1}^2 \]  \hspace{1cm} (4.10d)

\[ h_{sf}^2 = G_0 + G_1 h_{sf(t-1)}^2 + G_2 e_{st-1}^2 + G_3 e_{ct-1}^2 \]  \hspace{1cm} (4.10e)

\[ E(e_t^2 / \Omega_{t-1}) = h_t^2 = \omega_t^2 i_{t-1}^2 \]  \hspace{1cm} (4.10f)

\[ \omega_t^2 = C_0 + C_1 \epsilon_{t-1}^2 + C_2 \omega_{t-1}^2 \]  \hspace{1cm} (4.10g)

\[ h_{sf} = p_{sf}(h_{ss} h_{sf}) \]  \hspace{1cm} (4.10h)

\[ h_{ss} = p_{ss}(h_{ss} h_{ss}) \]  \hspace{1cm} (4.10i)

\[ h_{sf} = p_{sf}(h_{sf} h_{sf}) \]  \hspace{1cm} (4.10j)

where

- \( d\ln s_t \) is the spot return of each currency
- \( d\ln f_t \) is the futures returns of each currency
- \( e_{ct-1} \) is the error correction term
- \( i_t \) is the domestic (US) interest rate
- \( \Omega_{t-1} \) is the information set in the previous period
- \( \mu \) is the mean of the interest rate process
- \( \kappa \) is the speed of adjustment to deviations from the mean
- \( h_{ss}^2 \) is the conditional variance of the spot return
$h^2_{ft}$ is the conditional variance of the futures return

$h^2_{ii}$ is the conditional variance of the US interest rate

$h_{sf}$ is the conditional covariance between spot and futures returns

$h_{is}$ is the conditional covariance of interest rates and spot returns

$h_{if}$ is the conditional covariance of interest rates and futures returns

$p_{sf}$ is the conditional correlation between spot and futures returns

$p_{is}$ is the conditional correlation of interest rates and spot returns

$p_{if}$ is the conditional correlation of interest rates and futures returns and

$\epsilon_{st}, \epsilon_{sf}$ and $\epsilon_{ii}$ are the spot, futures and interest rates returns innovations at time $t$.

$\omega$ is a time-varying parameter

$\gamma$ is a constant

delta is the coefficient of the dummy variable

$DFRB$ is a dummy variable that takes the value 1 for observations 1584-1603.

In equation (4.10a) and (4.10b), the introduction of the error correction term is necessary due to the well-evidenced cointegration between spot and futures prices of all currencies examined. For the same reason, the squared error correction term is included in the conditional variance equations of the spot and futures returns (4.10d) and (4.10e) respectively, since it was found to explain a significant portion of exchange rate volatility in Chapter 3. With equation (4.10c), the interest rate model incorporates mean reversion for the short rate $i$ which is pulled to a level $\mu$ at a rate $\kappa$. Following the paradigm of Brenner et. al (1996), the dummy variable $DFRB$ is introduced in the model accounting for the structural break caused by the regime change of the Federal Reserve Board.

The variance of the short rate $h^2_{ii}$ is proportional to its level, as it is described in equation (4.10f), which implies that high volatility in interest rates is associated with high interest rate levels. The restriction $0 \leq \gamma \leq 1$ is imposed during estimation so that the hypothesis of stationarity is maintained. Additionally, with equation (4.10g), the sensitivity of the volatility to the level of the interest rate is modelled with a GARCH (1,1) process. The significance of the estimated coefficients $C_0, C_1$ and $C_2$ will be tested in order to comment on whether large shocks ($\epsilon_i$) lead to a
higher dependence of volatility to levels than small shocks. The GARCH-X (1,1) model is used for the conditional variances of spot and futures returns as described by equations (4.10d) and (4.10e) since it was found to be an adequate representation of the series of interest in the previous empirical chapter.

Equation (4.10h) describes the conditional covariance of spot and futures returns as a proportion of the product of the conditional standard deviations of spot and futures returns, while the conditional correlation is assumed to be constant over time. This restriction leads to a major reduction in the computational complexity and it was supported in the previous chapter by empirical tests on the cross-correlation of standardized residuals. The same representation is chosen for the conditional covariances of spot and interest rates and futures and interest rates in the equations (4.10i) and (4.10j) respectively. The estimated conditional covariances $h_{is}^2, h_{if}^2$ are time varying although the conditional correlations $p_{is}, p_{if}$ are assumed to be constant.

It is worth noting at this point that the model developed in the present section is innovative in the sense that the variances and covariances of interest rates and spot and futures returns are allowed to vary with time. This specification extends the empirical study of Chang, et al (1996), where the inter-temporal hedge ratio for basis risk was estimated with constant variances and covariances.
(4.5.2) Estimation results

In the present study, the method of Quasi-Maximum Likelihood estimation will be used since it is known\(^\text{32}\) to provide consistent estimates of dynamic multivariate models. In Table 4.2.1, the Quasi-Maximum Likelihood estimates of the multivariate GARCH (1,1) model (4.10) are provided with robust standard errors in parentheses. The significance of the estimated coefficients is examined in the following paragraphs in order to find out whether the econometric specification of (4.10) explains adequately the behaviour of spot and futures exchange rates and interest rates.

As it was found in the previous empirical chapter, with the exception of the constant terms, \(C_s\) and \(C_f\), the coefficients of the mean equations of spot and futures returns are statistically significant for all currencies examined. The hypothesis of cointegration between spot and futures exchange rates is supported as it is shown by the significance of the error correction terms, \(b_s\) and \(b_f\). The negative sign of the coefficient of the error correction term in the SF spot equation implies that, with everything else being equal and a positive error correction term, the spot exchange rate should fall and the futures exchange rate should rise in order to adjust to deviations from equilibrium. However, this is a more complicated empirical issue depending on whether futures prices are at a discount or a premium over the life of the futures contract. The coefficients of the lagged returns of spot and futures exchange rates (\(\alpha\)'s and \(\delta\)'s) are all statistically significant, showing the power of past changes in both variables to explain the behaviour of currency returns. In the mean equation of the U.S interest rate level, the hypothesis of mean reversion, evidenced in the unit root tests, is supported by the statistical significance of the mean \(\mu\) and the speed of adjustment \(\kappa\) in all cases examined. The coefficient of the dummy variable Delta was also found highly significant for all cases, indicating the importance of accounting for structural breaks in studies of interest rate data.

\(^{32}\) For a detailed analysis of the Quasi-Maximum Likelihood estimation, refer to the previous empirical chapter.
In spot and futures variance equations, the constant terms, $B_0$ and $G_0$, representing the unconditional variances of spot and futures returns respectively, are both statistically significant for all cases examined. The estimated coefficients of the lagged conditional variances and squared error terms, $B_1$, $B_2$, $G_1$ and $G_2$ are also highly significant, indicating strong GARCH effects for spot and futures returns and time variation in the estimated hedge ratios. As in Chapter 3, the deviation from the long-run equilibrium, measured by the squared error correction term explains a significant portion of the conditional volatility as it is shown by the statistical significance of $B_3$ and $G_3$. The presence of IGARCH effects is implied by the high persistence in variance evident in all cases with the exception of the JY. However, according to Lumsdaine (1991) and Lee and Hansen (1994), the log likelihood of an IGARCH process is well behaved asymptotically and the MLE estimators will be asymptotically normal. The estimate of the conditional correlation, $p_{sf}$, between spot and futures returns is positive and highly significant with a value close to one for all currencies examined. This finding implies strong interaction between spot and futures exchange rates and indicates the substantial efficiency gains resulting from modeling spot and futures price changes jointly.

The CIR definition of the interest rate process as a “square-root type” specification is supported in the present thesis as it is shown by the estimated coefficient $\gamma$. The latter coefficient is close to 0.5 and significant for all cases examined, indicating that the variance is an increasing function of the interest rate level. As Brenner et. al (1996) argue, the statistical significance of the parameter $\gamma$ implies that, for high data frequencies, the interest rate in the TVP-Levels model can never be negative. Additionally, the value of $\gamma$ casts doubt on the findings of CKLS (1992) who failed to incorporate the restriction $0 < \gamma < 1$ in their model, thus leading to a non-stationary interest rate specification. On the contrary, the estimated value of $\gamma$ is consistent with the presence of structural breaks in the interest rate series evidenced by the significance of the dummy variable Delta as well as the findings of Ball and Torous (1995, 1999) and Andersen and Lund (1997).

However, the level effect is not the only determinant of the interest rate volatility. The hypothesis of a time-varying interest rate volatility parameter, $\omega^2_t$, is
supported by the statistical significance of all coefficients in the interest rate GARCH equation. The GARCH specification captures a part of the reported heteroscedasticity in interest rates, as the TVP-Levels model suggests. This finding implies that failure to incorporate dependence on past shocks in the interest rate volatility would lead to a misspecified model. The presence of IGARCH cannot be tested directly ($\alpha_1 + \alpha_2$) in the interest rate case, since the volatility persistence is now a function of the persistence in both the volatility parameter $\omega^2$, and the interest rate level (see Brenner et.al (1996)).

The conditional correlation between spot returns and domestic (U.S.) interest rates $p_{is}$ is negative and statistically significant in all cases. This finding is consistent with the theory of “leaning-against-the-wind” or exchange rate “policy-reaction” of governments who raise the domestic interest rate when the domestic currency depreciates (see Branson (1984)). Briys and Solnik (1992) state that in this case investors in the forward market would underhedge their spot positions since basis risk would move to the same direction as the exchange rate risk. The same result applies in the conditional correlation of futures returns and domestic (U.S.) interest rates $p_{if}$. According to Chang et. al (1996) when $p_{is}$ and $p_{if}$ are negative and $p_{is}$ is higher than $\theta_2$, the payoff of the domestic bond relative to the cash position, the hedger is short in the futures market. This is the case with all four currencies examined in the present chapter and it is expected since, as Chang and Fang (1990) state, in the case of financial futures, $p_{if}$ is always negative. The futures contract can be used as a direct hedge of exchange rate risk and as a cross hedge of interest rate risk. The size of the short futures position will increase the higher the correlation between futures prices and interest rates and the higher the difference between $\theta_2$ and $p_{is}$. The latter authors state that the biases in the hedging demands, introduced by the stochastic interest rates, are lower in the case of financial futures than in the case of commodity futures where $p_{if}$ is positive. However, this is an empirical issue that is left for further research.

In Table 4.2.2, diagnostic tests on the standardized residuals of the multivariate system, in the form of remaining serial correlation, reveal, with a few exceptions, no significant misspecifications. As the Ljung-Box (20) statistic shows, there is no
serial correlation present in the standardized residuals of the spot (LB₅) and futures (LB₇) equations. However, significance of the same statistic (LB₁) reveals that the mean equation of the interest rate process is misspecified. Following the results of Brenner et. al (1996) and Andersen and Lund (1997), this result is attributed to the fact that the interest rate drift may not be correct, something quite difficult to achieve.

Tests for remaining ARCH effects (LB₅S and LB₇F) and cross-correlation (CORRₛₕ) on the residuals of the model reveal that the GARCH-X error structure with constant conditional correlation captures well the conditional heteroscedasticity present in foreign exchange data. The LB₅₁ statistic for remaining heteroscedasticity in the interest rate process is insignificant in all cases, showing that the TVP-Levels model is an adequate representation of the U.S interest rate process since it captures the dependence of volatility both to past shocks and the interest rate level. The hypothesis of constant conditional correlations, πₛ and πₛₖ, is supported by the Ljung-Box (20) test on the cross product of the standardized residuals. In all cases the CORRₛₛ and CORRₛₖ tests are insignificant. We can conclude that the GARCH-X (1,1) model for spot and futures exchange rate returns jointly estimated with a TVP-Levels model for the domestic interest rate, is an adequate representation of the series of interest.

In order to estimate the dynamic currency hedge ratios for basis risk, the equation (4.4) is used as derived by Chang et. al. The market price of risk, π, is estimated by the non-linear least squares estimation of the following equation, after incorporating the estimated values of κ, μ and hᵢᵢ² from the multivariate system (4.10).

\[ i = B(i, t, T) + u_i \]  \hspace{1cm} (4.11)

Where

\[ B(i, t, T) = D(t, T)e^{-\theta(t, T)} \]  \hspace{1cm} (4.4)
\[ D(t, T) = \left[ \frac{2\tau e^{(\kappa + \tau + \pi)(T-t)/2}}{2\kappa \tau + \pi (e^{(T-t)} - 1) + 2\tau} \right]^{2\kappa \tau + \pi} \]

\[ G(t, T) = \frac{2(e^{\tau(T-t)} - 1)}{(\tau + \kappa + \pi)(e^{\tau(T-t)} - 1) + 2\tau} \]

\[ \tau = [(\kappa + \pi)^2 + 2h^2]^{1/2} \]

and \( i \) is the 3-month Eurodollar interest rate.

Equation 4.4 is also used for the estimation of the price of the domestic bond \( B \) and its variance \( \sigma_B \), so that the risk minimizing hedge ratio of the domestic bond, \( h_{\text{BOND}} \) (see equation 4.5.2) is computed. In Table 4.2.2, the means of the two dynamic hedge ratios for basis risk are presented. The hedge ratio for the domestic bond takes its highest value for the Japanese Yen case. This result is expected since the JY futures contract has a low correlation with the U.S. interest rate. It cannot thus form a very effective cross hedge for the domestic interest rate risk and a large long position in the domestic bond is thus necessary.

As it can be seen in the figures 4.1 to 4.4, the dynamic risk minimizing hedge ratios for the futures contracts (\( h_{\text{basis}} \)) estimated by the multivariate GARCH (1,1) model are clearly time-varying, following the same pattern with the hedge ratios estimated by the simple GARCH-X (1,1) model (\( h_{\text{dynamic}} \)). Additionally, as expected, the PP test in Table 4.2.2 shows that they are stationary, implying that the persistence of any shock to the hedge ratios is very low. The estimated hedge ratios are serially positively correlated as is indicated by the first order correlation coefficient \( \phi \). A large hedge ratio in one day tends to be followed by a large hedge ratio in the following day.

In order to compare the traditional dynamic hedge ratios estimated in the previous chapter and the dynamic futures hedge ratios for basis risk, the averages of their absolute (\( h_{\text{basis}} - h_{\text{dyn}} \)) and relative [(\( h_{\text{basis}} - h_{\text{dyn}} \))/\( h_{\text{basis}} \)] differences are calculated in Table 4.2.2. Both measures show that the risk minimizing hedge ratio that ignores basis risk tends to overhedge the currency position in all cases examined.
The theory of “leaning-against-the-wind” is again supported by this result since, as we argued above, investors who hedge not only currency risk but interest rate risk as well, tend to underhedge their spot positions when there is negative correlation between exchange rates and interest rates. This finding can be also observed in the figures 4.1 - 4.4. Although both hedge ratios follow the same pattern, those accounting for basis risk are generally lower than the simple dynamic hedge ratios. This finding is supported by the results of Chang et. al (1996) where the risk minimizing hedge ratio tended to overhedge the stock index position. Assuming that the basis risk hedge ratio is more effective in terms of hedging performance than the simple dynamic hedge ratio, a strategy of lower cost can be produced since a smaller number of futures contracts is needed in order to hedge the spot position.

The main conclusion of the present section is that, as long as the spot currency return is correlated with the domestic interest rate, and, in the presence of stochastic interest rates, the hedge ratio for basis risk has to be applied. Additionally, a long position in a domestic bond is necessary as a direct hedge against adverse interest rate movements. The size of the latter position also depends on the conditional correlation of the spot and futures currency returns with the domestic interest rate. However, apart from the statistical validity of a hedging model, the hedger is mostly interested to its hedging performance, both in an in sample as well as in an out-of-sample analysis. In the following section, the dynamic hedging strategy accounting for basis risk is compared with the dynamic hedge ratio that ignores basis risk in terms of hedging performance. Additionally, comparisons are made with the constant hedging strategies based on OLS and ECM as well as the constant interest rate model. In sample and out-of-sample tests of variance reduction and utility performance are made in the presence of transaction costs, in order to choose the appropriate hedging model.
### Table 4.2.1: Maximum Likelihood Estimation of the multivariate GARCH Hedging Model

<table>
<thead>
<tr>
<th></th>
<th>BP</th>
<th>JY</th>
<th>SF</th>
<th>DM</th>
<th>BP</th>
<th>JY</th>
<th>SF</th>
<th>DM</th>
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</thead>
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<td>$C_s$</td>
<td>0.019110</td>
<td>-0.002348</td>
<td>0.008814</td>
<td>0.015266</td>
<td>$B_0$</td>
<td>0.009000</td>
<td>0.221283</td>
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Table 4.2.2: Hedge Ratio Statistics and Misspecification Tests of the Basis Risk GARCH Model

Descriptive Statistics on the basis risk dynamic hedge ratios for the futures contracts and the domestic bond.

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<th>JY</th>
<th>SF</th>
<th>DM</th>
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<tbody>
<tr>
<td>Mean(hrbasis)</td>
<td>0.858862</td>
<td>0.912320</td>
<td>0.930861</td>
<td>0.930998</td>
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<tr>
<td>Mean(hrBOND2)</td>
<td>0.186972</td>
<td>0.832910</td>
<td>0.367320</td>
<td>0.196770</td>
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<tr>
<td>PP</td>
<td>-4.299700</td>
<td>-7.647500</td>
<td>-8.240100</td>
<td>-7.197800</td>
</tr>
<tr>
<td>φ</td>
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<td>0.957850</td>
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</tr>
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<td>Hrbasis-hrdyn</td>
<td>-0.04140</td>
<td>-0.016670</td>
<td>-0.017710</td>
<td>-0.014930</td>
</tr>
<tr>
<td>(hrbasis-hrdyn)/hbasis</td>
<td>-5.218%</td>
<td>-2.2486%</td>
<td>-1.9960%</td>
<td>-1.8840%</td>
</tr>
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Misspecification Tests of the multivariate GARCH model.

<table>
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<th>SF</th>
<th>DM</th>
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<td>14.47</td>
<td>17.08</td>
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<td>LBf(20)</td>
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<td>31.19</td>
<td>15.79</td>
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<td>LBd(20)</td>
<td>39.59*</td>
<td>39.55*</td>
<td>39.59*</td>
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<tr>
<td>LBSS(20)</td>
<td>30.21</td>
<td>30.57</td>
<td>30.92</td>
<td>30.60</td>
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<td>30.02</td>
<td>27.06</td>
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<td>LBFF(20)</td>
<td>28.09</td>
<td>29.58</td>
<td>25.20</td>
<td>26.95</td>
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<tr>
<td>CORRSF(20)</td>
<td>27.60</td>
<td>25.76</td>
<td>26.69</td>
<td>25.75</td>
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<tr>
<td>CORRIS(20)</td>
<td>24.98</td>
<td>18.48</td>
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<td>CORRIF(20)</td>
<td>22.20</td>
<td>12.53</td>
<td>19.72</td>
<td>28.60</td>
</tr>
</tbody>
</table>

33 LBs, LBf and LBd are the Ljung-Box (20) statistics for 20th order serial correlation on the standardized residuals of the estimated GARCH model, while LBSS , LBff and LBFF are the Ljung-Box (20) statistics for remaining conditional heteroscedasticity on the squared standardized residuals of the model. CORRSF(20), CORRIS(20) and CORRIF(20) are the Ljung-Box (20) statistics for 20th order serial correlation on the cross products of the standardized residuals of the estimated GARCH model. These tests are $\chi^2$ distributed and they have 95% critical value 31.41 and 99% c.v. 37.57. The values (hrbasis-hrdyn) and (hrbasis-hrdyn)/|hrbasis| measure respectively the average and relative degree of mishedging of the dynamic hedge ratio compared to the basis risk hedge. PP is the unit root test statistic of the estimated hedge ratios with critical value -2.86, and $\phi$ is the first-order autocorrelation coefficient.
Figure 4.1: Size differences between the dynamic basis risk and traditional hedge ratios for the DM futures contract. The traditional hedge ratio HRDM1 tends to overhedge the spot position.\textsuperscript{34}

\textsuperscript{34} HRDM1 represents the dynamic risk minimizing hedge ratio for the DM, estimated by the bivariate GARCH-X model in the previous chapter, under the assumption of non-stochastic interest rates. HRDM2 represents the dynamic basis risk hedge ratio for the DM, estimated by the multivariate GARCH model in the previous section under the assumption of stochastic interest rates.
Figure 4.2: Size differences between the dynamic basis risk and traditional hedge ratios for the BP futures contract. The traditional hedge ratio HRBP1 tends to overhedge the spot position.\textsuperscript{35}

\textsuperscript{35}HRBP1 represents the dynamic risk minimizing hedge ratio for the BP, estimated by the bivariate GARCH-X model in the previous chapter, under the assumption of non-stochastic interest rates. HRBP2 represents the dynamic basis risk hedge ratio for the BP, estimated by the multivariate GARCH model in the previous section under the assumption of stochastic interest rates.
Figure 4.3: Size differences between the dynamic basis risk and traditional hedge ratios for the SF futures contract. The traditional hedge ratio HRSF1 tends to overhedge the spot position.\textsuperscript{36}

\textsuperscript{36}HRSF1 represents the dynamic risk minimizing hedge ratio for the SF, estimated by the bivariate GARCH-X model in the previous chapter, under the assumption of non-stochastic interest rates. HRSF2 represents the dynamic basis risk hedge ratio for the SF, estimated by the multivariate GARCH model in the previous section under the assumption of stochastic interest rates.
Figure 4.4: Size differences between the dynamic basis risk and traditional hedge ratios for the JY futures contract. The traditional hedge ratio HRJY1 tends to overhedge the spot position.\textsuperscript{37}

\textsuperscript{37}HRJY1 represents the dynamic risk minimizing hedge ratio for the JY, estimated by the bivariate GARCH-X model in the previous chapter, under the assumption of non-stochastic interest rates. HRJY2 represents the dynamic basis risk hedge ratio for the JY, estimated by the multivariate GARCH model in the previous section under the assumption of stochastic interest rates.
Section D: Hedging Efficiency Tests

(4.6) In-sample hedging performance of basis risk hedging strategies

In order to examine the effectiveness of the proposed basis risk dynamic hedging strategy (Basis Dynamic) relative to the simple constant (OLS and ECM) and dynamic (GARCH-X) models that ignore basis risk, the hedging performance of each hedge ratio has to be estimated. Additionally, for comparison purposes, the hedging performance of a constant model that accounts for basis risk (Basis Constant) is also examined. In the latter model, the assumption of stationary variances and covariances is followed for interest rates and spot and futures exchange rates as in Chang, Chang and Fang (1996). Following the methodology of Kroner and Sultan (1993), hedged portfolios are constructed as implied by the hedge ratios of each strategy. As it is discussed in the previous sections, a short position of $b_{12}$ futures contracts and a long position of $b_{\text{bond}}$ domestic bonds are assumed in order to hedge the basis risk of a combined spot-futures position. On the contrary, no bonds are used for a hedging strategy that ignores basis risk and a short position of $b_{\text{f}}$ futures is assumed. Series of returns on the hedged portfolios are computed for the existing sample by the following equations:

\[ X_{\text{no bonds}} = D\ln S_t - b_{\text{f}} \cdot D\ln f_t \]  \hspace{1cm} (4.12.1)

\[ X_{\text{bonds}} = D\ln S_t - b_{\text{f}} \cdot D\ln f_t + b_{\text{bond}} \cdot D\ln B_t \]  \hspace{1cm} (4.12.2)

Where $D\ln S_t$ is the spot price change,

$D\ln f_t$ is the futures price change

$D\ln B_t$ is the bond price change as estimated by the model (4.4)

$b_{\text{f}}$ is the hedge ratio (OLS, ECM and GARCH-X) for futures contracts that ignores basis risk

$b_{\text{f}}$ is the hedge ratio (constant and dynamic) for futures contracts that accounts for basis risk and,

$b_{\text{bond}}$ is the hedge ratio (constant and dynamic) for the domestic bond.
The variances of the constant and dynamic hedged portfolios are computed for each currency and they are reported in table 4.3.1. Moreover, the absolute and percentage variance improvement of the basis risk dynamic strategy over the remaining constant and traditional dynamic hedging techniques is reported in the second panel of the same table. A comparison among the constant hedging strategies reveals that the model accounting for stochastic interest rates (Basis Constant) has the lowest variance, supporting the results of Chang et. al. Accounting for the interest rate risk of a currency futures position results in a further risk reduction of 0.5% in the case of the DM relative to the ECM hedging strategy. However, for the remaining currencies, the effect of the basis risk hedge ratio is minimal. The superiority of the basis risk hedge ratio is more significant in the dynamic case as compared with the GARCH-X hedging model. A further risk reduction is possible with the basis risk dynamic hedge ratio that outperforms the constant and the traditional dynamic techniques in all cases. It is obvious that accounting for basis risk in a dynamic context results in better risk management solutions even from the pure risk minimizing perspective. An active hedger who would hedge not only the exchange rate risk of his position but the domestic interest rate risk as well, would have achieved an additional risk reduction of a minimum of 0.856 % for the BP reaching the maximum of 4.605 % for the DM.

However, an investor is not only concerned with the variance reduction in his spot portfolio but also for the economic usefulness of the specific hedging strategy. The economic significance of the variance reduction, provided by each hedging model, is also examined with respect to the utility of the investor. Although in the present study the assumption of risk neutrality rules out the use of a utility approach, the measure of utility can be applied in combination with the effect of the transaction costs, in order to form a selective rebalancing hedging strategy. As in the previous chapter, we assume that \( x \) is the series of returns from the hedged portfolios and that the utility function of the investor, is given by the following equation:

\[
EU(x) = -4 \times \text{var}(x) \quad (4.13)
\]
and for a period of 2537 days:

\[ \text{EU} (x) = -4 \times 2537 \times \text{var} (x) \]

Where var \((x)\) is the variance of the hedged portfolio implied by the different strategies and reported in Table 4.3.1.

Table 4.3.2 presents utility comparisons among the constant and dynamic hedging strategies for the 2537-days period. The basis constant model is again the best constant hedging strategy. In the dynamic cases, it is shown that even with daily rebalancing, the basis risk dynamic hedge ratio provides the highest utility to the investor, giving support to the economic rationale for a hedging model that incorporates stochastic interest rates. However, in the real world, investors cannot rebalance their portfolios on an every day basis since transaction costs make continuous rebalancing extremely costly. As in Chapter 3, the method of selective rebalancing under six different scenarios for the amount of the transaction costs is also applied in the present chapter. Extending the methodology of Kroner and Sultan (1993) to the case of hedging jointly exchange rate and interest rate risk, the investor can rebalance his futures and domestic bond positions only when the expected utility from rebalancing is higher than the expected utility from no-rebalancing:

\[
-y - \gamma \left( h_{s,t+1}^2 - 2b_{f_{2,t}} h_{sf,t+1} + b_{f_{2,t}}^2 h_{f,t+1}^2 + b_{bond,t}^2 h_{b,t+1}^2 + 2b_{bond,t} h_{ab,t+1} - 2b_{f_{2,t}} b_{bond,t} h_{fb,t+1} \right) > -\gamma \left( h_{s,t+1}^2 - 2b_{f_{2,t}} h_{sf,t+1} + b_{f_{2,t}}^2 h_{f,t+1}^2 + b_{bond,t}^2 h_{b,t+1}^2 + 2b_{bond,t} h_{ab,t+1} - 2b_{f_{2,t}} b_{bond,t} h_{fb,t+1} \right)
\]

Where \(h_{s,t+1}^2\) is the conditional variance of the spot portfolio return, \(h_{sf,t+1}\) is the conditional covariance between spot and futures returns, \(h_{f,t+1}^2\) is the conditional variance of the futures return, \(h_{b,t+1}^2\) is the conditional variance of the domestic bond return, \(h_{ab,t+1}\) is the conditional covariance of the spot and bond returns, \(h_{fb,t+1}\) is the conditional covariance of the futures and bond returns,
$b_{\Omega,t}$ is the dynamic futures hedge ratio for continuous rebalancing,
$b_{\Omega,t-1}$ is the futures hedge ratio from the most recent rebalancing,
$b_{\text{bond},t}$ is the dynamic bond hedge ratio for continuous rebalancing,
$b_{\text{bond},t-1}$ is the bond hedge ratio from the most recent rebalancing,
y is the amount of transaction costs and,
y is the degree of relative risk aversion.

The number of possible times of rebalancing for each currency over the 2537-days period is shown in the columns “Rebal.” of Table 4.3.2. An interesting result in the utility comparisons of the dynamic strategies is that, over the 2537-days period and under the dynamic strategy for basis risk, the hedger would be able to rebalance his position more times than in the simple dynamic strategy (GARCH-X). The higher frequency of rebalancing can be explained by the fact that the dynamic basis risk hedge ratio accounts for both the exchange rate risk of the spot position and the interest rate risk of the futures position. The need for the continuous adjustment of the futures position is thus increased. Substantial differences in the estimated utilities are also evident, with the basis risk dynamic strategy being superior in all cases. We can conclude that, even in the presence of transaction costs, a dynamic hedge ratio that accounts for stochastic interest rates allows for more frequent rebalancing of the hedged portfolio, leading to higher utility for the international investor. However, a round trip transaction cost over $20 will decrease the number of possible rebalancings of the hedged position leading to a decrease in the estimated utility function. It is thus very important to account for the transaction costs in a dynamic hedging strategy.

It is quite clear that the basis risk dynamic strategy outperforms the other constant and dynamic hedging strategies both on the basis of variance and utility comparisons. However, an investor is more concerned with the hedging performance of a model in the future. In the following section, the same comparisons on the basis of variance reduction and utility maximization will be performed in an out-of-sample analysis.
Table 4.3.1: Variance Comparisons of the Within-Sample Hedging Effectiveness of static and dynamic hedging models.

<table>
<thead>
<tr>
<th>Models</th>
<th>DM</th>
<th>JY</th>
<th>BP</th>
<th>SF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unhedged</td>
<td>0.48317</td>
<td>0.47606</td>
<td>0.45152</td>
<td>0.58820</td>
</tr>
<tr>
<td>Naïve</td>
<td>0.04946</td>
<td>0.07000</td>
<td>0.06253</td>
<td>0.05286</td>
</tr>
<tr>
<td>OLS</td>
<td>0.04786</td>
<td>0.06633</td>
<td>0.05637</td>
<td>0.05125</td>
</tr>
<tr>
<td>ECM</td>
<td>0.04787</td>
<td>0.06622</td>
<td>0.05641</td>
<td>0.05101</td>
</tr>
<tr>
<td>Basis Constant</td>
<td>0.04765</td>
<td>0.06622</td>
<td>0.05638</td>
<td>0.05102</td>
</tr>
<tr>
<td>GARCH-X</td>
<td>0.04758</td>
<td>0.06620</td>
<td>0.05624</td>
<td>0.05101</td>
</tr>
<tr>
<td>Basis Dyn.</td>
<td>0.04539</td>
<td>0.06536</td>
<td>0.05576</td>
<td>0.05012</td>
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Variance reduction

<table>
<thead>
<tr>
<th>Models</th>
<th>% absolute</th>
<th>% absolute</th>
<th>% absolute</th>
<th>% absolute</th>
<th>% absolute</th>
<th>% absolute</th>
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</thead>
<tbody>
<tr>
<td>Unhedged</td>
<td>90.607</td>
<td>86.270</td>
<td>87.651</td>
<td>91.480</td>
<td>0.43779</td>
<td>0.41070</td>
</tr>
<tr>
<td>Naïve</td>
<td>8.240</td>
<td>6.620</td>
<td>10.828</td>
<td>5.186</td>
<td>0.00408</td>
<td>0.00463</td>
</tr>
<tr>
<td>OLS</td>
<td>5.175</td>
<td>1.458</td>
<td>0.00097</td>
<td>1.755</td>
<td>0.00248</td>
<td>0.00036</td>
</tr>
<tr>
<td>ECM</td>
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<td>1.298</td>
<td>1.153</td>
<td>1.755</td>
<td>0.00248</td>
<td>0.00086</td>
</tr>
<tr>
<td>Basis Const.</td>
<td>4.744</td>
<td>1.292</td>
<td>1.108</td>
<td>1.755</td>
<td>0.00226</td>
<td>0.00085</td>
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<tr>
<td>GARCH-X</td>
<td>4.605</td>
<td>1.269</td>
<td>0.856</td>
<td>1.752</td>
<td>0.00219</td>
<td>0.00084</td>
</tr>
</tbody>
</table>
Table 4.3.2: Utility Comparisons of the Within-Sample Hedging Effectiveness of dynamic hedging models.

<table>
<thead>
<tr>
<th>Models</th>
<th>Transaction Costs</th>
<th>DM Rebalancing</th>
<th>JY Rebalancing</th>
<th>BP Rebalancing</th>
<th>SF Rebalancing</th>
<th>Utilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unheded</td>
<td>0.00000</td>
<td>0</td>
<td>-4903.252</td>
<td>0</td>
<td>-4831.0927</td>
<td>0</td>
</tr>
<tr>
<td>naive</td>
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<td>0</td>
<td>-501.932</td>
<td>0</td>
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<tr>
<td>OLS</td>
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<td>0</td>
<td>-673.1123</td>
<td>0</td>
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<tr>
<td>ECM</td>
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<td>0</td>
<td>-485.733</td>
<td>0</td>
<td>-672.0274</td>
<td>0</td>
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<tr>
<td>BASIS CONS</td>
<td>0.00000</td>
<td>0</td>
<td>-483.509</td>
<td>0</td>
<td>-671.9846</td>
<td>0</td>
</tr>
<tr>
<td>GARCH-X</td>
<td>0.00000</td>
<td>2537</td>
<td>-482.808</td>
<td>2536</td>
<td>-671.8265</td>
<td>2537</td>
</tr>
<tr>
<td>GARCH-X</td>
<td>0.00100</td>
<td>1767</td>
<td>-480.852</td>
<td>1667</td>
<td>-669.2516</td>
<td>1097</td>
</tr>
<tr>
<td>GARCH-X</td>
<td>0.00125</td>
<td>1657</td>
<td>-476.193</td>
<td>1482</td>
<td>-669.0229</td>
<td>951</td>
</tr>
<tr>
<td>GARCH-X</td>
<td>0.00150</td>
<td>1538</td>
<td>-475.647</td>
<td>1278</td>
<td>-665.7437</td>
<td>812</td>
</tr>
<tr>
<td>GARCH-X</td>
<td>0.00200</td>
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<td>-475.658</td>
<td>1254</td>
<td>-665.7582</td>
<td>809</td>
</tr>
<tr>
<td>GARCH-X</td>
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<td>801</td>
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<tr>
<td>GARCH-X</td>
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<td>1501</td>
<td>-475.683</td>
<td>1238</td>
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<tr>
<td>BASIS DYN</td>
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<td>2537</td>
<td>-460.573</td>
<td>2536</td>
<td>-663.3016</td>
<td>2537</td>
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<tr>
<td>BASIS DYN</td>
<td>0.00100</td>
<td>1840</td>
<td>-457.293</td>
<td>2416</td>
<td>-651.2493</td>
<td>1382</td>
</tr>
<tr>
<td>BASIS DYN</td>
<td>0.00125</td>
<td>1640</td>
<td>-455.579</td>
<td>2413</td>
<td>-651.2435</td>
<td>1382</td>
</tr>
<tr>
<td>BASIS DYN</td>
<td>0.00150</td>
<td>1420</td>
<td>-450.725</td>
<td>2412</td>
<td>-651.0053</td>
<td>1382</td>
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<tr>
<td>BASIS DYN</td>
<td>0.00200</td>
<td>1419</td>
<td>-450.728</td>
<td>2409</td>
<td>-651.0147</td>
<td>1365</td>
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<td>BASIS DYN</td>
<td>0.00250</td>
<td>1411</td>
<td>-450.734</td>
<td>2401</td>
<td>-651.0258</td>
<td>1357</td>
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<tr>
<td>BASIS DYN</td>
<td>0.00300</td>
<td>1408</td>
<td>-450.781</td>
<td>2397</td>
<td>-651.0971</td>
<td>1324</td>
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</tbody>
</table>

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(4.7) Out-of-sample hedging performance of the basis risk hedging strategies

In order to perform an out-of-sample comparison of the hedging effectiveness of the estimated models, one step forecasts are generated for the daily U.S. interest rate, the spot and the futures returns of each currency by using the last 100 observations of the sample. Forecasted variances and covariances are computed and a series of 100 forecasted hedge ratios is created. In this way, 100 forecasted hedged portfolios are constructed for each model, and the variances of their returns are computed and given in Table 4.4.1. Moreover, the absolute and percentage variance improvement of the basis risk dynamic strategy over the constant and dynamic hedging techniques is reported in the second panel of the same table.

As it can be seen in Table 4.4.1, the same pattern is followed by each hedging model in terms of hedging efficiency as in the in sample analysis. However, a drop in the performance of the hedging models is evident, a result that is expected since the hedge ratios are estimated ex ante. The basis risk dynamic strategy is still the superior strategy in terms of variance reduction, giving support to the use of a time varying hedge ratio for interest rate risk in the risk management of foreign currency positions. The implication of this finding for an international investor is that a hedging model that does not account for basis risk will lead to unhedged risks and losses in the combined spot and futures portfolio returns.

The superiority of the new strategy is more meaningful when it is applied to the decision of rebalancing the combined spot–futures position. As it is mentioned above, the main advantage of a conditional strategy is not the variance reduction but the choice given to the investor to selectively rebalance his portfolio only when the gains in the expected utility offset the transaction costs. Table 4.4.2 presents utility comparisons among the different hedges for the 100-days period. The decision of selective rebalancing is also applied to the constant models since the method of rolling regressions used in the forecasting procedure generated time-varying hedge ratios for the OLS, ECM and the constant basis model. In the present analysis, as it is shown in the relevant columns of Table 4.4.2, under three different
scenarios\textsuperscript{38} for the amount of the transaction costs, the hedger would rebalance his position more often with the basis risk dynamic strategy over the 100-days period, achieving higher levels of utility. The new dynamic strategy outperforms the simple hedging strategies both on the basis of variance reduction and utility maximization in the out-of-sample analysis.

We can conclude that the proposed model that incorporates time variation in the conditional second moments of spot and futures exchange rates and stochastic interest rates produces valid forecasts of the financial variables of interest. It is shown to provide successful ex ante risk management solutions for both exchange rate and interest rate risk. On the basis of the reported size differences between the traditional dynamic hedge ratios and the basis risk hedge ratio as well as the risk reduction and utility performance provided, a hedging strategy of lower cost and higher hedging effectiveness can be produced in the presence of stochastic interest rates.

\textsuperscript{38}No significant differences were found in the estimated utilities when higher transaction costs were introduced in the model. For reasons of limited space, only the results for the three different scenarios for the transaction costs are reported.
Table 4.4.1: Out-of-sample hedging effectiveness: variance comparisons.

<table>
<thead>
<tr>
<th>Models</th>
<th>Variance Comparisons</th>
<th>Variance reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DM</td>
<td>JY</td>
</tr>
<tr>
<td>Unhedged</td>
<td>0.462492</td>
<td>0.709337</td>
</tr>
<tr>
<td>Naïve</td>
<td>0.026466</td>
<td>0.062903</td>
</tr>
<tr>
<td>OLS</td>
<td>0.026383</td>
<td>0.062834</td>
</tr>
<tr>
<td>ECM</td>
<td>0.026322</td>
<td>0.062637</td>
</tr>
<tr>
<td>Basis Const.</td>
<td>0.026309</td>
<td>0.062629</td>
</tr>
<tr>
<td>GARCH-X</td>
<td>0.026275</td>
<td>0.062570</td>
</tr>
<tr>
<td>Basis Dyn.</td>
<td>0.025589</td>
<td>0.062456</td>
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</tbody>
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<table>
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<th>% absolute</th>
<th>% absolute</th>
<th>% absolute</th>
</tr>
</thead>
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<tr>
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<td>91.195</td>
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<tr>
<td>Naïve</td>
<td>3.314</td>
<td>0.000877</td>
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<td>3.008</td>
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<table>
<thead>
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<td>Unhedged Naïve</td>
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<td>-5.263</td>
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<td>100</td>
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<tr>
<td>GARCH-X</td>
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<td>-5.123</td>
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<td>GARCH-X</td>
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<td>-10.515</td>
<td>4</td>
<td>-25.049</td>
<td>5</td>
<td>-5.123</td>
<td>2</td>
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<td>-17.132</td>
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<td>-10.401</td>
<td>20</td>
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<td>-5.098</td>
<td>40</td>
<td>-17.087</td>
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<td>-10.236</td>
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<td>100</td>
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<td>Basis Dyn.</td>
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<td>-10.066</td>
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<td>-23.879</td>
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<td>42</td>
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</tbody>
</table>
Section E: Conclusions

In the present chapter, the assumption of non-stochastic interest rates was relaxed since it is not realistic when hedging currency risk with futures contracts. As the cost-of-carry futures pricing model predicts, basis risk can be attributed to the uncertainty created by the stochastic interest rates and the daily marking-to-market feature of futures contracts. It was shown that a hedge ratio for foreign currency positions that ignores interest rate risk leads to unhedged risks and limited hedging effectiveness.

In the present study, the inter-temporal hedge ratio for basis risk and marking-to-market, as derived by Chang et. al (1996), was estimated for four major currencies (DM, BP, JY, SF). It was assumed that a default-free domestic discount bond is used as a hedge for the interest rate risk of the futures positions since the interest yield or paid on the margin account is expressed in terms of the domestic interest rate. An extension to the Chang et. al model was performed by assuming time-varying variances and covariances for the spot and futures exchange rates. A GARCH-X error structure was assumed for these variables in order to account for the time-variation in their conditional distribution, as evidenced in the previous chapter. The domestic (U.S.) interest rate was assumed to follow a mean-reverting square root process (Cox, Ingersoll and Ross, 1985) with a time-varying volatility parameter (GARCH, Brenner et. al, 1996). The reported ability of the model to perform well under structural breaks is important for our study since the Eurocurrency interest rates in our sample were found to be stationary around a broken trend line due to structural changes. To the best of our knowledge, no previous effort has been made so far to model all three series into a dynamic setting in order to jointly hedge exchange rate and interest rate risk.

Following the methodological approach of the previous chapter, the multivariate GARCH model was estimated and a time varying basis risk minimizing hedge ratio was computed for each futures contract. The multivariate GARCH model was found to be a valid representation for the exchange rates and the interest rates.
examined. The estimated coefficients of the interest rate process were used for the computation of the domestic bond price and its variance, resulting to a series of dynamic hedge ratios for the domestic bond. The combined futures and domestic bond positions formed the dynamic basis risk hedging strategy that was applied to the four spot exchange rates. The in sample and out of sample analysis of the constant and dynamic models revealed that the currency portfolios hedged with the dynamic basis risk hedge ratios had the lowest variance and provided the hedger with the highest utility.

It can be concluded that basis risk can be accounted for in hedging decisions, resulting in substantial benefits for an international investor. The implication of these findings for treasurers and portfolio managers is that the risk management of foreign currencies can be improved by the dynamic hedging of the interest rate risk resulting from the daily marking to market of futures contracts. To the extent that a derivative contract is correlated with the domestic interest rate, the basis risk of the hedged portfolios can be significantly reduced, leading to increased performance of the initial hedge.

Section A: Introduction

The main objective for the foundation of the European Monetary System in 1979 is the monetary stability in Europe. Indeed, relevant empirical studies conclude that the bilateral EMS exchange rate volatility has decreased substantially over the post-EMS period. This is mainly due to the fact that the bilateral EMS exchange rates of the countries participating in the Exchange Rate Mechanism are allowed small fluctuations from the par value. According to the Maastricht Treaty, at the starting date of Stage Three of the European Monetary Union, the national currencies of the Member Countries will be linked by irrevocably fixed conversion rates and the new European currency (Euro) will substitute for them in the Member States.

The adoption of a single European currency implies the elimination of exchange rate risk in monetary transactions between the participating countries. However, U.S. dollar exchange rates are free floating, exhibiting extremely high volatility as most financial time series. Single or multiple positions in European currencies are subject to high U.S. dollar exchange rate risk, leading to an increased demand for derivative instruments for currency risk hedging in the countries where well developed risk sharing markets do not exist. Indeed, DeSantis, Gerard and Hillion (1997) provide evidence that the EMU component of currency risk in equity and Eurodeposit markets is smaller than the non-EMU component measured by U.S. Dollar, British Pound and Japanese Yen returns. A small positive risk premium is required for EMU risk while a negative risk premium is expected for non-EMU risk that indicates the willingness of the investors to forgo a part of expected returns in order to hedge a portion of non-EMU exchange rate risk.

In the present chapter, a portfolio of five European currencies is constructed since, usually, international firms take positions in more than one currency. The spot
portfolio consists of positions in the Netherlands Guilder, French Franc, Danish Crone, Spanish Peseta and the Italian Lira. A bivariate GARCH system is estimated for the spot portfolio and each one of the most actively traded currency futures contracts (the British Pound, the Swiss Franc, the Deutsche Mark and the Japanese Yen) in order to derive dynamic risk minimizing cross hedge ratios. The hedging performance of each contract is investigated in an ex post and ex ante analysis. Additionally, the increasing power of the Deutsche Mark in the EMS countries is examined with multivariate Johansen cointegration tests and an error correction term is included when appropriate in the hedged portfolios. The model developed in this chapter is innovative both in terms of cross hedging a European currency portfolio as well as a dynamic specification of minor currency returns for hedging purposes.

The conclusions to this analysis have strong implications for hedging the single European currency risk since the spot EMS portfolio is assumed to replicate the single European currency. Since the Euro started trading very recently (January 1999), limited observations are available for the construction of hedging models for Euro spot positions. The economic usefulness of estimating cross hedging models for a portfolio of representative EMS exchange rates lies in the formation of the single European currency as a basket of national currencies of the countries that belong to the European Monetary Union (EMU). The conclusions from the EMS spot portfolio estimations would thus be very helpful for portfolio managers wishing to hedge their reserves in the single European currency against fluctuations in the U.S. Dollar. As Dermine (1998) argues, the Euro is expected to replace the Deutsche Mark and, in view of the economic role of the European Economic Union, be a strong international currency competing with the Dollar. Additionally, we find it useful, for comparison purposes, to estimate a hedging model for the Euro as well, since the specific currency will substitute for all European currencies introduced in the European Monetary Union. One direct and four cross hedging models are estimated for the Euro futures and the four major currencies futures contracts for the period 4th January 1999 until the 22nd June 1999. To the best of our knowledge, no previous effort has been made to examine the hedging potential of the single European currency.
In Section B of the present chapter, a brief review of the literature is presented on the dominance of the DM in the European currency area with strong implications for the hedging effectiveness of the DM futures contract in cross hedging EMS currency risk. Additionally, the development of the econometric specification for spot and futures returns is included, based on the empirical evidence of cointegration and conditional heteroscedasticity in systems of European exchange rates. In Section C, the data and the empirical results from the estimation of the dynamic hedging models are reported. Section D describes the tests of hedging efficiency of the estimated models both for the five-currency spot portfolio as well as the Euro spot position. In Section E, the conclusions of the present study are analyzed and proposals for further research are given.
Section B: Theoretical Issues and Empirical Design

(5.1) The German Dominance in the EMS: implications for cross hedging European currency risk

The effectiveness of a derivative instrument in minimizing the risk of a foreign currency exposure depends on the degree of its correlation with the asset or the portfolio of assets that are being hedged. The higher the correlation, the more effective is the futures contract. In the case of direct hedging, almost perfect correlation exists between the currency futures contract and the underlying currency, since the two prices are moving closely, due to the cost-of-carry pricing relationship. However, since in the case of cross hedging perfect correlation cannot be achieved, a choice must be made among various futures contracts and the contract with the highest correlation with the spot portfolio should be chosen.

In the present study, the hedging problem is concentrated on a spot portfolio of EMS currencies for which there are no actively traded futures contracts. Consequently, a choice must be made among the four currency futures that are traded for the four major currencies: the British Pound, the Deutsche Mark, the Swiss Franc and the Japanese Yen. It is expected that the three contracts on European currencies will be more effective than the JY, since the latter involves a different geographic and economic bloc. However, the choice among the three European futures contracts is a more complicated issue that depends on the participation of the European countries examined in the European Monetary System.

The country that is considered to play a dominant role in the EMS, in terms of monetary policy, is Germany. In practice, the DM bilateral rates have been the most important exchange rates in the ERM. One view of the German dominance is the strict form of the "German Leadership Hypothesis" where Germany sets the monetary policy and the other EMS countries follow by adjusting their inflation and interest rates (Giavazzi and Pagano (1988)). According to the weak form of the
same hypothesis, there is an asymmetry in the EMS where a country sets the policy to be followed in the long run but the other members have autonomy in adjusting their policies in the short-run. However, the German monetary policy is the leading one in both cases.

The concept of cointegration is used by empirical studies on the German dominance in the EMS because monetary convergence is implied by the co-movement of interest rates or monetary bases. However, as it is reported in Chapter 4, the presence of unit roots in interest rate series is a separate empirical issue that should be examined with caution. Karfakis and Moschos (1990) examine the role of Germany in the EMS. Bivariate cointegration tests between the German interest rate and those of Belgium, France, Ireland, Italy and Netherlands cannot reject the hypothesis of no cointegration, revealing no long run co-movement between these rates. This result can be explained by differences in the monetary policy of the countries examined. On the other hand, Granger causality tests show that German interest rates have a forecasting power for the interest rate movement in the other EMS countries with the exception of Ireland. However, the contrary does not hold, supporting the "German Leadership Hypothesis" in the EMS.

Hafer and Kutan (1994) apply a multivariate cointegration analysis on the systems of overnight interest rates and monetary bases of Belgium, France, Germany, Italy and Netherlands in order to examine whether monetary policy convergence has occurred over the 1979-1990 period. The cointegration tests show that more than one common stochastic trend exists in both multivariate systems, implying that full policy convergence did not take place. Granger causality tests reveal that a strong but equal interaction exists among country policies in the short-run, with a significant influence of Germany when EMS countries are tested as a group. Similar conclusions are provided by Henry and Weidmann (1994) who observe that the asymmetry in the EMS is increased after the German unification. MacDonald and Taylor (1991) also use the monetary base as a proxy for the monetary policy and provide significant evidence for the German dominance in the group of Germany, France and Italy.
The German Leadership in the EMS is also examined by Caporale et al (1996) by performing unit root and cointegration tests on the differentials between German and EMS interest rates. The decrease in the mean and variance of the differentials in the second EMS period is a first indication of interest rate convergence. The presence of one common stochastic trend in the system of EMS interest rate differentials over the second EMS period is a more powerful indication of interest rate convergence. The authors conclude that the evidence provided supports the “weak” version of the German Leadership Hypothesis in the form of interest rate convergence to the Bundesbank low-inflation monetary policy. However, it is shown that EMS countries have kept some autonomy in the form of capital controls.

Bredin and Fountas (1998) extend the analysis of Hafer and Kutan (1994) with a larger sample period and additional countries examined: Denmark and Ireland. By splitting their sample period into two and using Granger causality tests, they reject the German dominance hypothesis for most countries examined. Tests on both interest rates and monetary bases reveal strong interaction of monetary policies in the EMS. However, the effect of the German monetary policy on the other countries’ policies is stronger than the effect of the other countries’ policies on Germany.

The previous studies on EMS countries conclude that Germany plays an important role in the EMS either by dominating in terms of monetary policy or by being the major determinant of convergence in the long run. The implication of these findings for the present study is that the DM futures contract is expected to be the most effective hedging vehicle for the basket of EMS currencies examined. Indeed, Eaker and Grant (1987) show that the effectiveness of hedging spot positions in European currencies is higher when European currency futures are used, especially for EMS countries. Specifically, the DM is found to be the most effective hedging instrument for the Italian Lira, the Greek Drachma and the Spanish Peseta. The success of the German exchange rate is attributed to the fact that Germany is the leading trade partner of these countries. However, a limitation of the latter study is that it ignores time variation and portfolio effects in the estimated hedge ratios.
The development of a hedging model must be based on sound theoretical reasoning in order to provide optimal hedge ratios for a multi-currency spot position. Any statistical relationships and empirical regularities of the exchange rates examined have to be accounted for in their econometric specification since they may improve the forecasting performance of the hedging model and the estimated hedge ratios. These important issues are described in the following sections, leading to the construction of the cross hedging model in Section 5.3.

(5.2) Evidence of cointegration in systems of exchange rates and implications for the construction of the multi-currency spot portfolio.

In Chapter 3 of the present thesis, we examined the effect of cointegration between spot and futures exchange rates on the development of a hedging model. However, there also exists substantial empirical evidence on the cointegration in systems of spot exchange rates with strong implications for the construction of a multi-currency portfolio. The effect of the cointegrating relationship on the problem of hedging the exposure of a spot currency portfolio is twofold. First, evidence of cointegration in the system of the EMS currencies implies that an error correction term should be included in the empirical modeling of the spot portfolio return. Second, the evidence of cointegration in a system of major and minor exchange rates means that the series move together over time. The implication of this finding for a hedging policy is that major currencies futures contracts can be very effective cross hedging instruments for the currency risk of European currencies without a well-developed futures market.

The evidence on the cointegration of spot exchange rates provided so far is mixed although significant improvement is observed in terms of the data and methodology used by recent empirical studies. It is worth noting that the existence of one common stochastic trend in two or more exchange rates, implies the possibility of forecasting one on the basis of the others, violating the martingale assumption that is necessary for the foreign exchange market efficiency. *Dwyer and Wallace (1992)* give a useful interpretation for the existence of cointegration between exchange
rates without the violation of foreign exchange market efficiency. They state that if two countries have fixed exchange rates relative to each other, their cross exchange rates will be stationary. The floating rates of these countries to a third country are expected to be cointegrated although the foreign exchange market efficiency is still maintained. Testing for cointegration in the context of a system of exchange rates is thus more realistic in the case of the EMS currencies since these rates are linked through the Exchange Rate Mechanism.

*McDonald and Taylor (1989)* cannot reject the unit root hypothesis for any of the ten currencies examined and they proceed to bivariate cointegration tests. The null hypothesis of no cointegration can be rejected only in one case out of the 45 cases examined and only at the ten per cent significance level. Using the same methodology, *Hakkio and Rush (1989)* reach the same conclusion for the BP and the DM. *Copeland (1991)* tests the DM, BP, FF, SF and the JY for cointegration in pairs but he cannot reject the hypothesis of no cointegration using the Johansen test. On the contrary, *Sephton and Larsen (1991)* reject the null of no cointegration after testing the BP, DM, CD and the JY as a system. However, the Johansen multivariate cointegration test is found to suffer from temporal instability for the sample period examined.

The same criticism is applied to the empirical study of *Baillie and Bollerslev (1989)* who provide evidence of cointegration in the systems of seven spot and forward U.S. Dollar rates. *Diebold et. al. (1994)* use the same data and they estimate a martingale, a vector autoregressive and an error correction model. The error correction model performs worse in terms of out-of-sample forecasting performance. Additionally, the hypothesis of no cointegration cannot be rejected when a drift is included in the tests. *Baillie and Bollerslev (1994)* repeat the cointegration tests on the same dataset by including an intercept and they support the findings of *Diebold et al (1994)*. They attribute this result to the long memory in the exchange rate relationship and a fractionally integrated error correction term. However, standard cointegration tests cannot capture this effect, leading to the above mentioned mixed results. An implication of this finding for multi-currency models is that the inclusion of an error correction term cannot improve the
forecasting performance of the martingale model.

Aggarwal and Mougoue (1993, 1996) perform multivariate cointegration tests on systems of several Asian currencies and the Japanese Yen. Both studies reject the hypothesis of no cointegration with the Japanese Yen, showing the increasing influence of the Japanese Yen in Asia and the formation of a Japanese bloc. The implication of these studies for hedging decisions is that derivative instruments denominated in the Japanese Yen can be very effective for cross hedging the exchange rate risk of portfolio investments in emerging Asian markets. Indeed, Aggarwal and Demaskey (1997) report that the JY futures and options contracts outperform other major currency contracts in cross hedging Asian exchange rates.

In the present study, the hypothesis of no cointegration will be tested for the system of the five EMS currencies. Since the Engle and Granger (1987) test is designed for the bivariate case, the Johansen and Juselius (1990) test will be used, as it provides a more valid test for cointegration in the multivariate setting. A constant term will be included in the Vector Autoregressive model since relevant empirical studies show that it has a definitive effect on the results of cointegration tests. If the hypothesis of no cointegration is rejected, an error correction term will be included in the mean equation of the spot portfolio return. Otherwise, a martingale model will be applied. Additionally, the cointegration hypothesis will be tested for a system including the five EMS exchange rates and each of the major exchange rates (DM, BP, SF and JY). Evidence of cointegration in this case would imply a substantial efficiency gain from using major currencies futures contracts to hedge minor currencies spot positions.
(5.3) *Time-varying conditional heteroscedasticity in EMS exchange rates and the GARCH (1,1) cross hedging model.*

As it is stated in the previous chapters of the present thesis, the correct econometric specification of the second moments of spot and futures currency returns is a necessary condition for the derivation of the appropriate risk minimizing hedge ratio. Failure to account for the time variation in the volatility of currency returns can lead to huge losses for the combined spot-futures position. This is evident in most empirical studies on cross hedging that apply constant methods for the derivation of the risk minimizing hedge ratios. Significant instability in the estimated coefficients is observed, leading to reduced hedging effectiveness when the constant hedge ratio is applied ex ante\(^{39}\).

Although most empirical studies report lower volatility estimates for bilateral EMS exchange rates relative to dollar EMS exchange rates, significant ARCH effects are evident in both cases, showing that GARCH models can be successful in forecasting short-run fluctuations in EMS exchange rates. *Bollerslev (1990)* examines the co-movement in European currencies with respect to the U.S. Dollar, by modeling them jointly in a multivariate GARCH model. Significant ARCH effects are also detected by *Koutmos (1994)* in six bilateral EMS exchange rates. *Engle and Gau (1997)* find significant ARCH effects in both DM and U.S.$ EMS exchange rates. An important observation of this study is that the multi-band model, that includes positions in all EMS rates, has the lowest persistence in variance, showing the gains of jointly modeling the EMS exchange rates. *Aguirre and Saidi (1998)* also use a GARCH model for nine European currencies vis-à-vis the Deutsche Mark and the U.S. Dollar, that is found to capture well the conditional variance of the European rates.

The implication of the previous studies for the present thesis is that a GARCH model is an adequate specification for the conditional variance of EMS exchange rate returns. Time variation in the conditional heteroscedasticity of the series

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examined implies the need for a conditional hedging strategy and the derivation of a dynamic risk minimizing hedge ratio. Under the hypothesis of no cointegration between the EMS exchange rates and in the presence of significant ARCH effects in the series examined, the following bivariate GARCH (1,1) model will be estimated in the following section, in order to derive optimal hedge ratios for the spot currency portfolio:

\[
\begin{align*}
\ln s_t &= c_s + e_{st} \tag{5.1.a} \\
\ln f_{i,t} &= c_{fi} + e_{fi,t} \tag{5.1.b} \\
h_{ss}^2 &= b_0 + b_1 h_{ss}^2(t-1) + b_2 e_{st}^2(t-1) \tag{5.1.c} \\
h_{ff, i}^2 &= g_0 + g_1 h_{ff, i}(t-1) + g_2 e_{fi}^2(t-1) \tag{5.1.d} \\
h_{sf, i} &= p_{sf, i}(h_{ss} h_{ff, i}) \tag{5.1.e}
\end{align*}
\]

where

dlns\_t is the return of the multi-currency portfolio
dlnf\_i,t is the futures return of each major currency with i = SF, DM, BP, JY.
h\_ss is the conditional variance of the portfolio return
h\_ff, i is the conditional variance of the futures return
h\_sf, i is the conditional covariance between spot and futures returns
p\_sf, i is the conditional correlation between spot and futures returns and,
e\_st and e\_fi,t are the spot and futures returns innovations at time t.

The mean equations (5.1.a) and (5.1.b) represent the spot and futures returns as random walks, a representation that is supported by many empirical studies on the series of interest. However, in the presence of cointegration in the system of the five EMS exchange rates, an error correction model will replace the above equations. The GARCH (1,1) model is used for the conditional variances of spot and futures returns as described by equations (5.1.c) and (5.1.d) since it is found to be an adequate representation of exchange rate return series by relevant empirical studies. Additionally, an exponential specification of the variance equations will also be applied in the form of an EGARCH (1,1) model\textsuperscript{40}, in order to test the European exchange rates for asymmetric behaviour. Equation (5.1.e) describes the
conditional covariance of spot and futures returns as a proportion of the product of the conditional standard deviations of spot and futures returns, while the conditional correlation is assumed to be constant over time. This restriction leads to a major reduction in the computational complexity and it can be investigated empirically with tests for remaining serial correlation on the cross products of the standardized residuals of the model. Time variation is allowed for the estimated conditional covariance $h_{sf}^2$ although the conditional correlation $p_{sf}$ is assumed to be constant. In the following section, the data and the estimation results of the system (5.1) are presented and the dynamic risk minimizing cross hedge ratios are computed for the spot portfolio.

\[\text{Refer to Chapter 3 for a detailed description of the EGARCH specification.}\]

\[201\]
Section C: Data and Empirical results

(5.4) Data

Daily futures prices on four major currencies, the British Pound, the Deutsche Mark, the Swiss Frank and the Japanese Yen vis-à-vis the U.S. Dollar are collected\(^{41}\), covering the period January 2, 1990 to October 2, 1997 (1952 observations). As in the previous chapters, the same reasons apply for the choice of daily data. Spot prices on five European exchange rates vis-à-vis the U.S. Dollar, the Spanish Peseta (SP), the Danish Crone (DEN), the French Franc (FF), the Italian Lira (IT) and the Netherlands Guilder (NETH) are collected for the same time period. The five currencies are chosen on the basis of their membership in the EEC and their participation in the ERM (Exchange Rate Mechanism) from the beginning of the period examined, with the exception of the Italian Lira that left on the 17\(^{th}\) of September 1992. Additionally, daily data on the spot and futures prices of the European currency (US$/Euro) and the futures prices of the SF (US$/SF), DM (US$/DM), BP (US$/BP) and the JY (US$/JY) are collected for the period 4\(^{th}\) January 1999 until the 22\(^{nd}\) June 1999. As in the previous empirical chapters, the data series of all spot and futures exchange rates are filtered by day-of-the-week adjustment regressions before the estimations of the econometric models.

(5.5) Preliminary data analysis

In the present section, the stochastic properties of the spot and futures exchange rates are analyzed. All series are examined for serial correlation and heteroscedasticity before the application of the dynamic model for the derivation of the optimal hedge ratio. Unit root tests are performed on each exchange rate series in order to test the null hypothesis of non-stationarity. The system of the five EMS exchange rates is investigated for the existence of a common stochastic trend in order to decide whether an error correction term should be included in the mean equation of the spot portfolio. Additionally, cointegration tests are performed on a system consisting of the five EMS exchange rate series and each major exchange

\(^{41}\) Source: DATASTREAM International.
rate series. The presence of a common stochastic trend in the second case would imply an increased hedging effectiveness of the respective futures contract.

Before proceeding to cointegration tests, each series is examined for the presence of unit roots. The Phillips-Perron (PP) test is used in the present chapter as in the previous ones since it is more robust to heteroscedasticity and serial correlation. In Table 5.1.1, the PP statistics show that all EMS exchange rates are first-order integrated processes, since the null hypothesis of a unit root cannot be rejected for the natural logs of the series (PP_L), but it is easily rejected for the differenced series (PP_D).

In order to test for cointegration in the system of the five EMS exchange rate series, the Johansen methodology is preferred to the Engle and Granger methodology as it is designed for multivariate systems in the absence of normality and homoscedasticity. An intercept is included in the testing model since it is reported to have a significant effect on the results of cointegration tests. (See Diebold et. al (1994), Baillie and Bollerslev (1994)). The null hypothesis of no cointegration is examined with respect to the maximal eigenvalue as well as the trace of the stochastic matrix. The test based on the trace statistic tests the null of zero cointegrating vectors (r = 0) against the general alternative of one or more cointegrating vectors (r > 0). The test based on the maximal eigenvalue tests the null of no cointegrating vector (r = 0) for the system of exchange rates against the alternative of one cointegrating vector (r = 1). As it can be seen on the column "EMS" in the second panel of Table 5.1.1, no test can reject the null of no cointegration between the five EMS exchange rates. The implication of this finding is that an error correction term should not be included in the spot portfolio. On the contrary, when the spot price of the DM is included in the system, (column "EMS & DM) both tests reject the null of no cointegration. The implication of this finding for our study is a potential for increased hedging effectiveness of the DM futures contract for cross hedging the risk of the spot currency portfolio.

The results of the cointegration tests with the remaining futures prices are mixed. While the test based on eigenvalues rejects the null of no cointegration at the 95%
critical level for the SF and the BP, and at the 90% critical level for the JY, the trace test can reject the null only for the JY. However, according to Johansen and Juselius (1990), the trace test lacks power relative to the maximum eigenvalue test, so, we can reject the hypothesis of no cointegration for all European currency futures at the 95% critical level. It is worth noting that the test statistics of the latter cases are much lower than those of the DM are where the evidence of cointegration is clearer. Additionally, the high log likelihood function of the system including the DM relative to the other systems implies that the DM futures contract will give a well-specified hedging model.

Since the purpose of the present study is to examine the effectiveness of major futures contracts for cross hedging the risk of multi-currency positions, a portfolio consisting of the five European currencies is constructed as a passive index, following the specification of Gagnon, Lypny and McCurdy (1998). The weight for each currency is equal to the value of the currency on the first day of the sample and it is held fixed all over the sample period. If \( \theta_i \) is the weight of each currency in the five-currency portfolio, with \( i = \text{SP, DEN, IT, NETH, FF} \), then \( \theta_{\text{SP}} = 110.3 \), \( \theta_{\text{DEN}} = 6.635 \), \( \theta_{\text{FF}} = 5.836 \), \( \theta_{\text{IT}} = 1279 \), \( \theta_{\text{NETH}} = 1.93 \). The main reason for keeping the weights of the spot currencies fixed all over the hedging period is to leave the portfolio rebalancing decision independent of changes in the quantity of the spot currencies, depending only on changes in the price of spot and futures exchange rates. As it is discussed in the first chapter of the present thesis, a hedger is an investor with a “given” position in the spot market.

In Table 5.1.2, tests for serial correlation and conditional heteroscedasticity are performed on the spot portfolio and futures returns. The first moments of the series are not found to be autocorrelated, as the Ljung-Box test (LB (20)) shows. As Chinn and Frankel (1994) show, the best model for major and EMS exchange rates is the random walk model. When the Ljung-Box test is applied on squared data (LB_{SQ} (20)) to test for ARCH effects, the results are highly significant supporting the hypothesis of time-variation in the variances and the need for the introduction of a GARCH error structure in the hedging model. The significant skewness and kurtosis measures for all spot and futures returns, imply significant non-linearities
in the exchange rate returns possibly due to the time variation in their conditional variances.

Table 5.1.3 represents the unconditional correlation matrix of the spot portfolio and futures returns with important implications for cross hedging. It is clear that the correlation of the spot portfolio return with the DM futures return is the highest of all, supporting the results of the cointegration tests. The correlation with the SF is also significant supporting the argument of Aguirre and Saidi (1998) that although Switzerland is not an EEC member, it is highly integrated with the European Union, in terms of its balance of trade and capital account. The correlation with the BP is lower although the United Kingdom is an EEC member. However, the BP is no longer in the EMS since September 1992. Not surprisingly, the correlation with the JY is the lowest since Japan belongs to a different geographic and economic bloc. An interesting observation in Table 5.1.3 is the high correlation existing between the SF and the DM futures contract. An empirical issue that seems worth investigating is whether a combination of the two contracts in a futures portfolio would provide an effective cross hedge for the spot EMS portfolio. A multivariate GARCH model is estimated in the following sections for the spot EMS portfolio return and the SF and DM futures returns. In this way, portfolio effects, arising from the correlation between futures returns, are captured in a cross-hedging model of European currency risk.

Table 5.1.4 contains diagnostic tests on the spot and futures returns of the Euro that started trading on the 4th of January 1999, and the futures returns of the major currencies for the same sample period. It is worth noting that general conclusions cannot be drawn easily from the results of the preliminary analysis and any empirical estimation due to the short sample period used (122 daily observations). In contrast with the currencies examined so far, the means of the spot and futures returns on the Euro are significant implying positive returns for positions in the Euro spot and futures markets. Additionally, a significant mean return is observed for the DM futures contract for the same sample period. Although the measures of skewness and kurtosis are insignificant for the majority of the series examined (with the exception of the JY futures return), substantial deviations from normality
are evident by the significance of the Bera-Jarque test. The tests for serial correlation (LB(20)) and conditional heteroscedasticity (LB_{SQ}(20)) are insignificant at the 95% level for most series examined (with the exception of the futures return of the DM and the Euro), implying that no serial correlation and ARCH effects are present in the series examined for the specific sample period. For this reason, in this case, a GARCH model is not considered to be very helpful for the estimation of a risk minimizing hedge ratio. However, lagged values of the spot and futures returns will be included in the mean equations of the hedging model on the basis of the Akaike Information Criterion in order to capture the serial correlation evident in the preliminary tests. Simple naïve and OLS hedging models are estimated for the Euro spot and futures returns and the returns on the major futures contracts, and hedging effectiveness comparisons are made among the different futures contracts.
**Table 5.1.1**: Unit Root and Cointegration tests on systems of exchange rates (unrestricted intercepts and no trends)\(^{42}\)

<table>
<thead>
<tr>
<th>Unit Root Tests</th>
<th>SP</th>
<th>DEN</th>
<th>FF</th>
<th>IT</th>
<th>NETH</th>
</tr>
</thead>
<tbody>
<tr>
<td>PP(_1)(8)</td>
<td>-0.631</td>
<td>-2.103</td>
<td>-2.194</td>
<td>-0.861</td>
<td>-2.009</td>
</tr>
<tr>
<td>PP(_0)(8)</td>
<td>-43.091</td>
<td>-43.940</td>
<td>-42.159</td>
<td>-42.794</td>
<td>-43.083</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cointegration Tests</th>
<th>Null</th>
<th>Alternative</th>
<th>EMS</th>
<th>EMS &amp; DM</th>
<th>EMS &amp; SF</th>
<th>EMS &amp; JY</th>
<th>EMS &amp; BP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max eigenvalue</td>
<td>R=0</td>
<td>R =1</td>
<td>31.645</td>
<td>177.895</td>
<td>40.068</td>
<td>39.480</td>
<td>40.541</td>
</tr>
<tr>
<td>Critical values</td>
<td>95%</td>
<td></td>
<td>33.640</td>
<td>39.830</td>
<td>39.830</td>
<td>39.830</td>
<td>39.830</td>
</tr>
<tr>
<td>trace</td>
<td>R=0</td>
<td>R &gt; = 1</td>
<td>31.020</td>
<td>36.840</td>
<td>36.840</td>
<td>36.840</td>
<td>36.840</td>
</tr>
<tr>
<td>Critical values</td>
<td>95%</td>
<td></td>
<td>62.268</td>
<td>239.744</td>
<td>85.793</td>
<td>99.488</td>
<td>84.388</td>
</tr>
<tr>
<td></td>
<td>90%</td>
<td></td>
<td>70.490</td>
<td>95.870</td>
<td>95.870</td>
<td>95.870</td>
<td>95.870</td>
</tr>
<tr>
<td>Log Likelihood</td>
<td>42161.400</td>
<td>50895.400</td>
<td>49882.100</td>
<td>49274.500</td>
<td>49755.300</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(^{42}\) The reported unit root statistics are performed on the natural logs of the exchange rates (PP\(_1\)) and on their differences (PP\(_0\)). The EMS system represents the following currencies: Italian Lira, Spanish Peseta, Netherlands Guilder, Danish Crone and French Franc.
Table 5.1.2: Preliminary data analysis on the series of the EEC portfolio and major futures returns

<table>
<thead>
<tr>
<th>Statistic</th>
<th>EMS Portfolio</th>
<th>SF futures</th>
<th>DM futures</th>
<th>BP futures</th>
<th>JY futures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>-0.61327</td>
<td>0.47302</td>
<td>-0.12252</td>
<td>0.07052</td>
<td>0.95703</td>
</tr>
<tr>
<td>Variance</td>
<td>0.39927</td>
<td>0.63162</td>
<td>0.51516</td>
<td>0.50265</td>
<td>0.51017</td>
</tr>
<tr>
<td>LB(20)</td>
<td>25.39000</td>
<td>28.65000</td>
<td>25.21000</td>
<td>24.89000</td>
<td>20.38000</td>
</tr>
<tr>
<td>LB_{sq}(20)</td>
<td>459.6800*</td>
<td>131.8600*</td>
<td>137.3800*</td>
<td>242.8200*</td>
<td>56.8700*</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>2.1270*</td>
<td>2.0051*</td>
<td>2.1999*</td>
<td>4.3615*</td>
<td>4.4195*</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.1360*</td>
<td>0.1156*</td>
<td>-0.01021</td>
<td>-0.3887*</td>
<td>0.4296*</td>
</tr>
</tbody>
</table>

43 The reported tests are performed on the log-differenced series multiplied by 10000 of the spot portfolio and the futures contracts prices. The spot portfolio consists of 110.3 Spanish Pesetas, 6.635 Danish Crones, 5.836 French Franks, 1279 Italian Liras, and 1.93 Netherlands Guilders.
Table 5.1.3: Unconditional Correlation Matrix of the spot portfolio and futures returns

<table>
<thead>
<tr>
<th></th>
<th>DLPORT</th>
<th>DLFM</th>
<th>DLFDM</th>
<th>DLFBP</th>
<th>DLFY</th>
</tr>
</thead>
<tbody>
<tr>
<td>DLPORT</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DLFM</td>
<td>0.73379</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DLFDM</td>
<td>0.80526</td>
<td>0.91171</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DLFBP</td>
<td>0.62445</td>
<td>0.69044</td>
<td>0.72447</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>DLFY</td>
<td>0.40740</td>
<td>0.54308</td>
<td>0.54382</td>
<td>0.40631</td>
<td>1</td>
</tr>
</tbody>
</table>
Table 5.1.4: Preliminary data analysis on the series of the single European currency (Euro) spot and futures returns and the major futures returns for the period 4th January 1999 until 22nd June 1999.

<table>
<thead>
<tr>
<th>Statistic</th>
<th>DLSEURO</th>
<th>DLFEURO</th>
<th>DLFSF</th>
<th>DLFDM</th>
<th>DLFBP</th>
<th>DLFJY</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>-0.11225</td>
<td>-0.11034</td>
<td>-0.09981</td>
<td>-0.11057</td>
<td>-0.03211</td>
<td>-0.06624</td>
</tr>
<tr>
<td>Variance</td>
<td>(-2.21562)*</td>
<td>(-2.12412)*</td>
<td>(-1.74317)*</td>
<td>(-2.13425)*</td>
<td>(-0.83824)*</td>
<td>(-0.77142)*</td>
</tr>
<tr>
<td>LB(20)</td>
<td>28.45510</td>
<td>33.33960</td>
<td>29.75250</td>
<td>32.50810</td>
<td>26.76950</td>
<td>21.62180</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.05745</td>
<td>0.09416</td>
<td>0.02065</td>
<td>0.09584</td>
<td>0.08093</td>
<td>-0.58752</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>0.48488</td>
<td>0.11410</td>
<td>0.42803</td>
<td>0.14189</td>
<td>0.15244</td>
<td>2.69297</td>
</tr>
<tr>
<td>Bera-Jarque</td>
<td>31.95928</td>
<td>42.16791</td>
<td>33.35937</td>
<td>41.36957</td>
<td>41.01293</td>
<td>7.43639</td>
</tr>
</tbody>
</table>

44 Bera - Jarque is the test for normality $BJ = n \frac{[\text{skewness}^2/6 + (\text{kurtosis}-3)^2/24]}{\chi^2(2)}$. The critical value at the 95% level of significance is 5.99. LB(20) is the test for serial correlation and $LB_{sq}(20)$ is the test for conditional heteroscedasticity on the squared returns. It is $\chi^{20}$ distributed and has 95% critical value 31.41 and 99% c.v. 37.57.
(5.6) Estimation results

(5.6.1) Quasi-maximum Likelihood Estimation of the bivariate GARCH model: no portfolio effects.

In the present section, a bivariate GARCH (1,1) model is estimated for the spot portfolio return and the return on each major futures contract. The method of Quasi-Maximum Likelihood estimation is used since it is known\textsuperscript{45} to provide consistent estimates of dynamic multivariate models. Single futures dynamic cross hedges are derived from the estimated conditional covariance of the spot and futures return and the conditional variance of the futures return.

In Table 5.2, the Quasi-Maximum Likelihood estimates of the bivariate GARCH (1,1) model are provided with robust standard errors in parentheses. The statistical significance of the estimated coefficients is examined in order to find out whether the GARCH specification adequately explains the behavior of EMS exchange rates. In the mean equations of spot and futures returns, the intercepts $C_S$, $C_F$ are found to be statistically insignificant implying the absence of a linear trend in exchange rate series. This finding shows the superior forecasting performance of the random walk model for the EMS exchange rates to any linear model. In the spot and futures variance equations, the constant terms, $b_0$ and $g_0$, representing the unconditional variances of spot and futures returns respectively, are both statistically significant. The estimated coefficients of the lagged conditional variances and squared error terms, $b_1$, $b_2$, $g_1$, $g_2$ are also highly significant, indicating strong GARCH effects for spot and futures returns and time variation in the estimated hedge ratios.

The estimate of the conditional correlation, $p_{sf}$, between spot and futures returns is positive and highly significant, taking its highest value for the DM and the SF and supporting the results of the previous section. The statistical significance of this coefficient shows that a strong interaction exists between the spot portfolio and the DM and SF futures returns, indicating the substantial gain resulting from cross hedging European multi-currency portfolios with DM and SF futures contracts. The
higher estimate for the DM futures return is expected on the basis of the present position of Germany in the Euro-zone in terms of the monetary policy followed. On the contrary, a low conditional correlation is shown to exist between the spot portfolio and the JY and BP futures return implying a low hedging effectiveness of the two futures contracts.

In order to test the validity of the GARCH model for the representation of the spot portfolio and futures returns, the restrictions \( g_0 = g_1 = g_2 = b_0 = b_1 = b_2 = 0 \) are imposed on the dynamic model, leading to the constant OLS strategy. The likelihood ratio tests (LR) in table 5.2 are highly significant for all currencies examined, rejecting the restrictions imposed. The implication of this finding is that the OLS model may lead to unhedged risks and losses in the combined spot-futures position, especially in the case of cross hedging where there is high instability in the relation between the spot and the futures price of the two assets. However, the constant hedge ratio is also estimated by dividing the covariance of spot and futures returns by the variance of futures returns so that it is compared with the dynamic hedging technique in terms of hedging efficiency.

Diagnostic tests on the standardized residuals of the bivariate GARCH system, in the form of remaining serial correlation and ARCH effects, reveal no significant misspecifications. In Table 5.2, the Ljung-Box (20) statistics (LBs, LBf) show that there is no serial correlation present in the standardized residuals of the spot and futures equations. Tests for remaining ARCH effects on the squared residuals of the model (LBsS, LBfF) reveal that the GARCH error structure captures well the conditional heteroscedasticity present in foreign exchange data. The hypothesis of constant conditional correlations \( p_{sf} \) is supported by the LB test on the cross product of the standardized residuals (LBsf (20)) which is insignificant for all cases examined. We can conclude that the GARCH (1,1) model with constant conditional correlations is an adequate representation of spot and futures exchange rate returns\(^{46}\).

\(^{45}\) For a detailed analysis of the Quasi-Maximum Likelihood estimation, refer to chapter 3.
\(^{46}\) An exponential-GARCH (1,1) model with constant conditional correlations is also estimated for the spot portfolio and the four currency futures returns. The estimation results are not reported in the present thesis since the EGARCH model is found to be seriously misspecified, providing dynamic hedge ratios with limited hedging performance relative to the GARCH (1,1) model.
The time varying hedge ratios for cross hedging the currency risk of the European portfolio are computed by dividing the conditional covariances of spot and futures returns by the conditional variances of futures returns, estimated by the GARCH model. In Section 5.6.3, descriptive statistics and graphs of the estimated hedge ratios are presented for all currencies examined in order to make comparisons between the different hedging models.

The main conclusion of the present section is that, as long as the spot currency portfolio return is highly correlated with the futures return of a major currency, the latter contract can provide an effective cross-hedge against minor currency risk. Additionally, accounting for the time variation in the conditional covariance between the spot and the futures return is crucial for the derivation of a successful cross-hedge, due to the lack of stability in the relations of spot and futures prices of different assets. However, apart from the statistical validity of a hedging model, the hedger is mostly interested to its hedging performance, both in an in sample as well as in an out-of-sample analysis. In section D, the efficiency gains from using highly correlated instruments in hedging a multi-currency portfolio are reported. The static and dynamic hedge ratios for the four futures contracts are compared in terms of hedging performance both in an in sample and out of sample analysis. Tests of variance reduction and utility performance are made in the presence of transaction costs, in order to choose the appropriate hedging vehicle for the specific spot portfolio.
Table 5.2: Quasi-maximum likelihood estimation of the GARCH (1,1) cross hedging model for the four major currency futures contracts\textsuperscript{47}.

<table>
<thead>
<tr>
<th></th>
<th>BP</th>
<th>JY</th>
<th>SF</th>
<th>DM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_s$</td>
<td>-0.00119</td>
<td>-0.00114</td>
<td>0.00011</td>
<td>-0.00232</td>
</tr>
<tr>
<td></td>
<td>(0.01345)</td>
<td>(0.01125)</td>
<td>(0.01116)</td>
<td>(0.00512)</td>
</tr>
<tr>
<td>$c_f$</td>
<td>0.01506</td>
<td>0.00670</td>
<td>0.00438</td>
<td>-0.00244</td>
</tr>
<tr>
<td></td>
<td>(0.01515)</td>
<td>(0.01354)</td>
<td>(0.01547)</td>
<td>(0.00533)</td>
</tr>
<tr>
<td>$b_0$</td>
<td>0.00862</td>
<td>0.00908</td>
<td>0.01142</td>
<td>0.00762</td>
</tr>
<tr>
<td></td>
<td>(0.00035)</td>
<td>(0.0038)*</td>
<td>(0.00429)</td>
<td>(0.00218)</td>
</tr>
<tr>
<td>$b_1$</td>
<td>0.93755</td>
<td>0.94754</td>
<td>0.95048</td>
<td>0.95608</td>
</tr>
<tr>
<td></td>
<td>(0.00093)</td>
<td>(0.0086)*</td>
<td>(0.01175)</td>
<td>(0.00717)</td>
</tr>
<tr>
<td>$b_2$</td>
<td>0.04362</td>
<td>0.03511</td>
<td>0.03067</td>
<td>0.02810</td>
</tr>
<tr>
<td></td>
<td>(0.00111)</td>
<td>(0.0097)*</td>
<td>(0.00672)</td>
<td>(0.00528)</td>
</tr>
<tr>
<td>$g_0$</td>
<td>0.00971</td>
<td>0.00516</td>
<td>0.00739</td>
<td>0.00694</td>
</tr>
<tr>
<td></td>
<td>(0.00050)</td>
<td>(0.0038)*</td>
<td>(0.00133)</td>
<td>(0.00129)</td>
</tr>
<tr>
<td>$g_1$</td>
<td>0.92061</td>
<td>0.93835</td>
<td>0.92227</td>
<td>0.93540</td>
</tr>
<tr>
<td></td>
<td>(0.00177)</td>
<td>(0.00151)</td>
<td>(0.00328)</td>
<td>(0.00240)</td>
</tr>
<tr>
<td>$g_2$</td>
<td>0.05240</td>
<td>0.05285</td>
<td>0.05812</td>
<td>0.04537</td>
</tr>
<tr>
<td></td>
<td>(0.00204)</td>
<td>(0.00178)</td>
<td>(0.00534)</td>
<td>(0.00490)</td>
</tr>
<tr>
<td>$p_{sf}$</td>
<td>0.59878</td>
<td>0.41487</td>
<td>0.73653</td>
<td>0.79672</td>
</tr>
<tr>
<td></td>
<td>(0.00908)</td>
<td>(0.01425)</td>
<td>(0.00858)</td>
<td>(0.01085)</td>
</tr>
</tbody>
</table>

\begin{itemize}
\item $LL_{GARCH}$: 336.676
\item $LL_{OL}$: 144.82500
\item LR: 383.702*
\item LB$_S$: 23.85
\item LB$_F$: 20.91
\item LB$_SS$: 18.16
\item LB$_FF$: 9.80
\item LB$_{SF}$: 47.04*
\end{itemize}

\textsuperscript{47}The reported coefficients are those estimated by the following model:

$$d \ln f_t = c_f + e_{ft}$$

$$d \ln s_t = c_s + e_{st}$$

\begin{align*}
\hat{h}_{ss} &= b_0 + b_1 \hat{h}_{ss}(t-1) + b_2 e_{st-1}^2 \\
\hat{h}_{ss} &= g_0 + g_1 \hat{h}_{ss}(t-1) + g_2 e_{st-1}^2
\end{align*}

Asymptotic standard errors are given in parentheses and asterisks are used for the significant coefficients. LL is the system log-likelihood estimated by each model. LB$_S$ and LB$_F$ are the Ljung-Box (20) statistics for 20th order serial correlation on the standardized residuals of the estimated GARCH model, while LB$_SS$ and LB$_FF$ are the Ljung-Box (20) statistics for 20th order serial correlation on the squared standardized residuals of the model. It is $\chi^2_{20}$ distributed and has 95% critical value 31.41 and 99% c.v. 37.57. PP (hr) is the unit root test statistic of the estimated hedge ratios with critical value \textminus2.86, and $\phi$ is the first-order autocorrelation coefficient.
(5.6.2) Quasi-maximum Likelihood Estimation of the multivariate GARCH model: portfolio effects.

Based on the high correlation between the SF and the DM futures contracts, estimated in section 5.5, a futures portfolio of the two major currencies is used in the present section in order to cross-hedge the EMS spot portfolio. This approach tends to account for the potential high covariance between the two futures contracts in generating portfolio effects in a cross-hedging model. A multivariate GARCH (1,1) model is estimated for the returns on the spot portfolio and the SF and DM futures contracts, and multiple dynamic risk minimizing hedge ratios are derived from the estimated conditional variances and covariances.

In Table 5.3, the Quasi-Maximum Likelihood estimates of the multivariate GARCH (1,1) model are provided with robust standard errors in parentheses. The results of the previous section apply in the portfolio case for all mean and variance equations in terms of the statistical significance of the estimated coefficients. The main benefit of the present section is implied by the high conditional correlation estimated between the two major futures contracts returns (pf_{DM, SF} = 0.90972). Substantial efficiency gains are thus expected from the use of the SF-DM portfolio as a cross-hedge of the EMS spot portfolio. Diagnostic tests on the standardized residuals of the multivariate GARCH system, in the form of remaining serial correlation and ARCH effects, reveal no significant misspecifications.

The DM and SF futures demands for cross hedging the currency risk of the European portfolio are computed by the conditional second moments estimated by the GARCH model. A detailed analysis of the derived hedge ratios is provided in Section 5.6.3. The superiority of the futures portfolio hedge to the single hedges in terms of risk reduction and utility performance is examined in the following section both in an in sample as well as in an out-of-sample analysis.
Table 5.3: Quasi-maximum likelihood estimation of the GARCH (1,1) cross hedging model for the DM and SF futures portfolio.

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Estimates</th>
<th>Coefficients</th>
<th>Estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{\text{spot}}$</td>
<td>0.00199 (0.01189)</td>
<td>$A_2$</td>
<td>0.030093 (0.006273)*</td>
</tr>
<tr>
<td>$C_{\text{FDM}}$</td>
<td>0.00362 (0.01390)</td>
<td>$G_0$</td>
<td>0.017802 (0.006906)*</td>
</tr>
<tr>
<td>$C_{\text{FSF}}$</td>
<td>0.00897 (0.01555)</td>
<td>$G_1$</td>
<td>0.94255 (0.01577)*</td>
</tr>
<tr>
<td>$B_0$</td>
<td>0.00894 (0.00347)*</td>
<td>$G_2$</td>
<td>0.02766 (0.00617)*</td>
</tr>
<tr>
<td>$B_1$</td>
<td>0.92594 (0.01892)*</td>
<td>$P_{\text{sfm}}$</td>
<td>0.79793 (0.00983)*</td>
</tr>
<tr>
<td>$B_2$</td>
<td>0.04897 (0.01102)*</td>
<td>$P_{\text{sfm}}$</td>
<td>0.73722 (0.01086)*</td>
</tr>
<tr>
<td>$A_0$</td>
<td>0.01212 (0.004914)*</td>
<td>$P_{\text{sfm}}$</td>
<td>0.90972 (0.00497)*</td>
</tr>
<tr>
<td>$A_1$</td>
<td>0.94454 (0.01468)*</td>
<td>LL</td>
<td>2033.158</td>
</tr>
</tbody>
</table>

Tests for remaining serial correlation:
- $LBP_2$ | 23.71
- $LB_{SF}$ | 26.20
- $LB_{DM}$ | 22.91

Tests for remaining ARCH effects:
- $LB_{p}^2$ | 19.03
- $LB_{SF}^2$ | 21.88
- $LB_{DM}^2$ | 14.45

Tests for remaining cross correlation:
- $LB_{PSF}$ | 22.29
- $LB_{DM}$ | 26.52
- $LB_{SFDM}$ | 19.25

The following model is estimated:

\[
\begin{align*}
\text{d} \ln s_t &= c_s + e_{st} \\
\text{d} \ln f_{DM,t} &= c_{\text{FDM}} + e_{\text{FDM},t} \\
\text{d} \ln f_{SF,t} &= c_{\text{FSF}} + e_{\text{FSF},t} \\
\text{h}^{2}_{s_t} &= bo + b1 \text{h}^{2}_{s_{t-1}} + b2 \text{e}^{2}_{s_{t-1}} \\
\text{h}^{2}_{\text{FDM}} &= a0 + a1 \text{h}^{2}_{\text{FDM}(t-1)} + a2 \text{e}^{2}_{\text{FDM}(t-1)} \\
\text{h}^{2}_{\text{FSF}} &= g0 + g1 \text{h}^{2}_{\text{FSF}(t-1)} + g2 \text{e}^{2}_{\text{FSF}(t-1)}
\end{align*}
\]

Asymptotic standard errors are given in parentheses and asterisks are used for the significant coefficients. LL is the system log-likelihood estimated by the model. LB is the Ljung-Box (20) statistics for 20th order serial correlation on the residuals of the portfolio and futures equations respectively. It is $\chi^2_{20}$ distributed and has 95% critical value 31.41 and 99% c.v. 37.57.
In the present section, an analysis is performed on the static and dynamic cross hedge ratios estimated by the futures portfolio and the single futures models. The risk-minimizing cross hedge ratios are computed for the EMS spot portfolio and each futures contract or the SF-DM futures portfolio by dividing the estimated covariance of the spot portfolio return and the futures return with the variance of the futures return. Comparisons can be made among the alternative hedging strategies and the four futures contracts in terms of the size of the futures position and the hedging performance.

The conditional second moments used for the estimation of the dynamic no-portfolio hedging strategy are those estimated by the bivariate GARCH (1,1) model of section 5.6.1. The dynamic SF-DM portfolio strategy is based on risk-minimizing hedge ratios estimated with the conditional variances and covariances of the multivariate GARCH (1,1) model of section 5.6.2. One static portfolio strategy occurs from the estimation of the OLS model for the EMS spot portfolio and the SF-DM futures portfolio. Additionally, four static no-portfolio strategies can be derived for the EMS portfolio by estimating single OLS regressions of the spot portfolio return on each major currency futures return. Descriptive statistics on the constant and dynamic risk minimizing hedge ratios are reported in Table 5.4.

As it can be seen in the figures 5.1 to 5.4, the dynamic hedge ratios estimated by the portfolio and the no-portfolio GARCH (1,1) models are clearly time-varying. However, the Phillips Perron (PP) test in Table 5.4 shows that they are stationary, implying that the persistence of any shock to the hedge ratios is very low. The estimated hedge ratios are serially positively correlated as it is indicated by the first order correlation coefficient ρ. The means of the estimated hedge ratios reported in Table 5.4 reveal significant differences between portfolio and no-portfolio futures demands for the SF and DM cross hedges. The hedge ratios of the SF-DM futures portfolio are almost half in size than their no-portfolio counterparts. The difference in the size of the dynamic futures demand derived by each model is shown quite clearly in the figures 5.1 and 5.4. This finding implies that ignoring portfolio
effects may lead the hedger to overhedge his spot currency portfolio.

In section D, the hedging effectiveness of each hedging strategy and futures contract/portfolio is estimated in terms of variance reduction and utility performance. The utility comparisons for the dynamic cases are based on a strategy of selective rebalancing the futures positions, taking into account the transaction costs.
Table 5.4: Descriptive Statistics of the constant and dynamic hedge ratios estimated by the portfolio and no portfolio hedging models

<table>
<thead>
<tr>
<th>Currencies</th>
<th>No portfolio case</th>
<th></th>
<th></th>
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<tbody>
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<td></td>
<td>Models</td>
<td>mean</td>
<td>variance</td>
<td>PP</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>BP</td>
<td>Dynamic</td>
<td>0.54446</td>
<td>0.006424</td>
<td>-5.9962</td>
<td>0.95959</td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td>Constant</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>JY</td>
<td>Dynamic</td>
<td>0.36571</td>
<td>0.011453</td>
<td>-4.0158</td>
<td>0.98524</td>
<td></td>
<td></td>
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<td></td>
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<td>variance</td>
<td>PP</td>
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</tr>
<tr>
<td>SF</td>
<td>Dynamic</td>
<td>0.284320</td>
<td>0.001865</td>
<td>-4.4891</td>
<td>0.9772</td>
<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td>Constant</td>
<td>0.291814</td>
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<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DM</td>
<td>Dynamic</td>
<td>0.342420</td>
<td>0.001290</td>
<td>-5.1379</td>
<td>0.96517</td>
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<td></td>
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</tr>
<tr>
<td></td>
<td>Constant</td>
<td>0.354567</td>
<td></td>
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<tr>
<th>Currencies</th>
<th>No portfolio case</th>
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<td>Models</td>
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<td>Variance</td>
<td>PP</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>SF</td>
<td>Dynamic</td>
<td>0.56897</td>
<td>0.008766</td>
<td>-4.7685</td>
<td>0.97399</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td>Constant</td>
<td>0.58372</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DM</td>
<td>Dynamic</td>
<td>0.68448</td>
<td>0.004728</td>
<td>-4.9570</td>
<td>0.96703</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Constant</td>
<td>0.70913</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

49 PP is the unit root test statistic of the estimated hedge ratios with critical value -2.86, and $\rho$ is the first-order autocorrelation coefficient.
**Figure 5.1:** Time-variation and size differences between single and portfolio cross-hedge ratios of Swiss Franc futures contracts for the spot EMS portfolio.
Figure 5.2: Time varying single cross-hedge ratio of Japanese Yen futures contracts for the spot EMS portfolio.
Single hedge ratio for the British Pound

Figure 5.3: Time varying single cross-hedge ratio of British Pound futures contracts for the spot EMS portfolio.
Figure 5.4: Time variation and size differences between single and portfolio cross-hedge ratios of Deutsche Mark futures for the spot EMS portfolio.
(5.6.4) SURE estimation of the OLS direct and cross hedging models for the Euro spot position

As it is stated in the introduction, the economic significance of estimating a cross hedging model for a multi-currency portfolio of EMS countries lies in the future changeover of the European currency area to the adoption of the Euro as the single currency. Since the Euro started trading just recently and limited observations exist so far, in the present study, the possibilities of hedging the exposure of the single currency are explored by replicating the Euro as a basket of currencies with a passive index of some representative EMS exchange rates. However, for comparison purposes, it is worth examining the direct and cross hedging possibilities existing for the European currency itself as it has been trading from January 1999. Although the availability of data is limited due to the short period of trading, we collected 122 daily observations on the spot and futures Euro rates. In the present section, an OLS model is estimated with the method of Seemingly Unrelated Regressions, for the Euro spot return and the Euro futures return as well as the major currencies futures returns.

In Table 5.5, the results of the OLS estimations are reported for each futures contract and hedge ratios are derived by the division of the estimated covariance between spot and futures returns to the estimated variance of the futures return. The constant terms in all spot equations (c_s) are statistically significant, implying the presence of a linear trend in the spot return series of the Euro. Similar conclusions apply in the case of the Euro, the SF and the DM futures returns (c_f). The lagged returns included in the spot and futures equations are chosen on the basis of the Akaike Information Criterion and tests of the statistical significance of the estimated coefficients. The lagged returns of the spot (b_{sm}) and futures (b_{fn}) prices are all significant with the exception of the JY, implying that information from the previous period can be useful in forecasting futures returns. However, it is worth noting that general conclusions cannot be drawn from the present model due to the very small sample used.
Diagnostic tests on the residuals of the estimated model reveal no significant remaining serial correlation and heteroscedasticity. This result is certainly due to the small sample period used since all the empirical evidence provided in the present thesis has shown that the OLS model is not an adequate description of major futures returns.

The constant direct and cross-hedge ratios are estimated by the unconditional covariances and variances of futures returns and they are provided in Table 5.5. An interesting observation is that the estimated correlation coefficient (\(p_{sd}\)) of the DM futures contract with the Eurocurrency spot return is of similar magnitude as the correlation of the spot and futures Euro returns. This result makes the direct hedge ratio of the EURO futures contract almost equal to the cross hedge ratio estimated for the DM. A comparison of the spot-DM correlation estimate of the present section (\(p_{sdm} = 0.9709\)) with the respective estimate of the conditional correlation provided in Section 5.6.1 (\(p_{sdm}=0.79672\)), reveals that higher correlation of the DM exists with the Euro than with the European spot portfolio examined before. This finding is expected since the European spot portfolio includes only a number of the currencies that participate in the construction of the Euro. However, it is obvious that the Euro tends to inherit the statistical properties of the DM. In the hedging decision of spot positions in the Euro, the DM futures contract can provide a cross hedging vehicle of equal or even higher efficiency than the direct hedge of the respective futures contract.

The estimated correlations and the hedge ratios for the remaining futures contracts also provide significant results. The SF futures contract is highly correlated with the Euro while the BP futures shows a much lower correlation. This result is expected on the basis of the present position of the BP with respect to the EMS and the high integration of Switzerland with the European community. The lowest correlation is again provided for the JY that leads to a hedge ratio of only 13% of the spot currency position with JY futures contracts. However, the true efficiency gains from each futures contract are explored in the following section.
Table 5.5: SURE estimation of the OLS direct and cross hedging models of the spot Euro return with the Euro futures contract and the four major currency futures contracts.

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>EURO</th>
<th>SF</th>
<th>DM</th>
<th>BP</th>
<th>JY</th>
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<tr>
<td>$C_s$</td>
<td>-0.15585</td>
<td>-0.12946</td>
<td>-0.15415</td>
<td>-0.13030</td>
<td>-0.12607</td>
</tr>
<tr>
<td></td>
<td>(0.05035)*</td>
<td>(0.05054)*</td>
<td>(-0.05033)*</td>
<td>(0.04991)*</td>
<td>(0.05092)*</td>
</tr>
<tr>
<td>$B_{s1}$</td>
<td>-0.35388</td>
<td>-0.17837</td>
<td>-0.34227</td>
<td>-0.14336</td>
<td>-0.14783</td>
</tr>
<tr>
<td></td>
<td>(0.06499)*</td>
<td>(0.06720)*</td>
<td>(0.06485)*</td>
<td>(0.07565)*</td>
<td>(0.08763)*</td>
</tr>
<tr>
<td>$B_{s2}$</td>
<td>-0.16311</td>
<td>-0.15901</td>
<td>-</td>
<td>(0.06544)*</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(-0.06565)*</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$B_{s4}$</td>
<td>-0.14843</td>
<td>-0.11712</td>
<td>-0.14650</td>
<td>-0.03569</td>
<td>-0.07470</td>
</tr>
<tr>
<td></td>
<td>(0.05089)*</td>
<td>(0.05637)*</td>
<td>(-0.05072)*</td>
<td>(-0.03823)</td>
<td>(-0.08609)</td>
</tr>
<tr>
<td>$C_{fi}$</td>
<td>-0.34352</td>
<td>-0.19005</td>
<td>-0.33329</td>
<td>-</td>
<td>-0.02541</td>
</tr>
<tr>
<td></td>
<td>(0.06485)*</td>
<td>(0.06659)*</td>
<td>(0.06462)*</td>
<td>-</td>
<td>(-0.08899)</td>
</tr>
<tr>
<td>$B_{fi1}$</td>
<td>-0.14534</td>
<td>-0.13846</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.06510)*</td>
<td>-</td>
<td>(0.06487)*</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$B_{fi2}$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-0.25043</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>(0.07880)*</td>
<td>-</td>
</tr>
<tr>
<td>$COV_{sf}$</td>
<td>0.28626</td>
<td>0.30067</td>
<td>0.28481</td>
<td>0.11623</td>
<td>0.11743</td>
</tr>
<tr>
<td>$VAR_s$</td>
<td>0.29336</td>
<td>0.29980</td>
<td>0.29233</td>
<td>0.27661</td>
<td>0.30010</td>
</tr>
<tr>
<td>$VAR_f$</td>
<td>0.29697</td>
<td>0.37663</td>
<td>0.29436</td>
<td>0.17016</td>
<td>0.88499</td>
</tr>
<tr>
<td>$P_{sf}$</td>
<td>0.96984</td>
<td>0.89478</td>
<td>0.97090</td>
<td>0.53573</td>
<td>0.22787</td>
</tr>
<tr>
<td>$HR$</td>
<td>0.96393</td>
<td>0.79832</td>
<td>0.96754</td>
<td>0.68306</td>
<td>0.13269</td>
</tr>
<tr>
<td>$LB_s$</td>
<td>7.76950</td>
<td>4.84540</td>
<td>7.21080</td>
<td>2.73070</td>
<td>5.17380</td>
</tr>
<tr>
<td>$LB_f$</td>
<td>8.46000</td>
<td>6.32750</td>
<td>7.98740</td>
<td>3.40830</td>
<td>1.83300</td>
</tr>
<tr>
<td>$LB_{ss}$</td>
<td>3.85550</td>
<td>0.90700</td>
<td>3.65940</td>
<td>2.40240</td>
<td>0.96600</td>
</tr>
<tr>
<td>$LB_{ff}$</td>
<td>2.37770</td>
<td>16.09790</td>
<td>2.46600</td>
<td>2.80860</td>
<td>2.88510</td>
</tr>
</tbody>
</table>

The reported coefficients are those estimated by the following model:

$$d\ln f_{i,t} = c_{fi} + b_{fi} \sum_{t=1}^{4} d\ln f_{i,t-1} + e_{fi}$$

$$d\ln s_{t} = c_{s} + b_{sm} \sum_{m=1}^{4} d\ln s_{t-1} + e_{st}$$

where $d\ln f_{i,t}$ is the return on the futures contract $i$, with $i =$ EURO, SF, DM, BP, JY.

d$\ln s_t$ is the return on the spot position in the Euro-currency exchange rate.

$COV_{sf}$, $VAR_s$, $VAR_f$ are the estimated covariance between the spot and futures returns, the variance of the spot return and the variance of the futures return respectively. $HR$ is the hedge ratio estimated by the ratio: $COV_{sf}/VAR_f$. $LB_{ss}$ and $LB_{ff}$ are the Ljung-Box (6) statistics for 6th order serial correlation on the standardized residuals of the estimated GARCH model, while $LB_{s}$ and $LB_{f}$ are the Ljung-Box (6) statistics for 6th order serial correlation on the squared standardized residuals of the model. It is $\chi^2_{0.05}$ distributed and has 95% critical value 12.59 and 99% c.v. 16.81.
Section D: Hedging Efficiency Tests

(5.7) Comparisons of the in-sample hedging performance of constant and time-varying hedge ratios with portfolio and no-portfolio effects.

In order to make comparisons among the four futures contracts or the futures portfolio and the different hedging strategies, the cross hedging performance of each hedge ratio has to be estimated. Following the methodology of Kroner and Sultan (1993), in the present study, hedged portfolios are constructed as implied by the hedge ratios of each model. Series of returns on the hedged portfolios are computed for the existing sample by the following equation:

\[ x = d\ln S_t - b_i^* d\ln f_{it} \]  

(5.2)

where \( d\ln S_t \) is the spot portfolio return,

\( d\ln f_{it} \) is the return on the futures contract \( i \), with \( i = SF, DM, BP, JY \).

and \( b_i^* \) is the hedge ratio implied by each futures contract and hedging strategy.

In the case of multiple futures contracts (SF and DM), the return on the hedged portfolio is equal to:

\[ x = d\ln S_t - b_{SF}^* d\ln f_{SF,t} - b_{DM}^* d\ln f_{DM,t} \]  

(5.3)

Where \( d\ln S_t \) is the spot portfolio return,

\( d\ln f_{SF,t} \) is the return on the SF futures contract

\( d\ln f_{DM,t} \) is the return on the DM futures contract

and \( b_{SF}^* , b_{DM}^* \) are the hedge ratios implied by each futures contract

The variances of the hedged portfolios are computed for each futures contract or portfolio and each hedging strategy and they are reported in table 5.6.1. Moreover, the absolute and percentage variance improvement of the dynamic risk minimizing hedge ratio over the constant hedge ratios is reported in the last rows of the same table.
A comparison of the hedging effectiveness of the four futures contracts shows that, as expected by the previous analysis, the DM futures contract is the most effective cross-hedging instrument for the currency risk of the spot portfolio. The dynamic cross-hedge of the EEC portfolio with a DM futures contract results in a reduction of 65.49% of the variance of the unhedged position. This finding implies that significant risk reductions can be achieved in the exchange rate exposure of European portfolios with the use of the DM futures contract in a dynamic hedging strategy. Additionally, the dynamic strategy is the best strategy in terms of variance reduction for all contracts examined. This finding shows the significant gains in efficiency from the use of a GARCH model in estimating time-varying hedge ratios. However, an interesting observation is that the naïve and OLS hedging strategy have achieved most of the variance reduction and only an additional reduction of 1.834% has been possible with the dynamic DM cross-hedge.

The dynamic SF futures hedge is also a successful hedge for the multi-currency portfolio leading to a reduction of 55.32% of the spot portfolio risk. As in the case of the DM, only an additional reduction of 3.05% is achieved with the dynamic hedge. The BP futures hedge follows the same pattern as the two previous cases but with lower hedging effectiveness in terms of risk improvement. The results for the JY futures hedge are striking: the naïve hedging strategy leads to increased risk relative to the unhedged case. This finding is an indication of the irrelevance of the one-to-one hedge in cross hedging cases where basis risk is highly significant. The OLS and dynamic cross-hedges are more successful, with the latter giving a variance reduction of 18.24% of the unhedged case. The low improvement of the latter contract relative to European currency futures is expected since Japan belongs to a different geographic and economic bloc.

Based on the estimated high conditional correlations between the spot EMS portfolio and the SF and DM futures returns, and the success of the two futures contracts in cross hedging the spot position, a futures portfolio of the two contracts is also used to cross hedge the EMS portfolio. As it can be seen in Table 5.6.1, the results are rather disappointing since the SF-DM futures portfolio cannot improve
the hedging performance of the single DM futures hedge. The dynamic portfolio hedge achieves a 63.11% variance reduction while the single DM futures succeeds in reducing 65.49% of the variance of the unhedged portfolio. No other combinations of major currency futures are used to form multiple hedges due to their lower hedging effectiveness relative to the DM case. The implication of the failure of multiple cross-hedges with contrast to the case of direct hedging (see Chapter 3), is that the use of a futures portfolio is not always beneficial in the cross-hedging problem. This finding is consistent with the empirical findings of Lien (1990) who emphasizes that a futures contract may improve little or worsen the spot portfolio in terms of risk reduction, although it may be a very effective hedging instrument for individual spot positions. The differences in terms of hedging effectiveness are due to portfolio effects arising from the composition of the spot and futures portfolio.

The assumption of risk neutrality rules out the use of utility in the present study, as a measure of the economic usefulness of a hedging strategy. However, a conditional rebalancing strategy can be formed under six different scenarios for the amount of the transaction costs and evaluated with respect to the estimated utilities. As in the previous chapter, we assume that \( x \) is the series of returns from the hedged portfolios and that the mean-variance utility function of the investor, is given by the following equation:

\[
EU(x) = E(x) - \gamma \text{var}(x) \tag{5.4}
\]

Where \( \gamma \) is the degree of risk aversion (\( \gamma > 0 \))

If the expected return on the hedged portfolio is equal to zero and the degree of risk aversion is equal to 4, the expected utility of the investor each day, can be computed by the following relation:

\[
EU(x) = -4* \text{var}(x) \tag{5.5}
\]

and for a period of 1952 days:
EU (x) = -4 * 1952 * var (x)

Where var (x) is the variance of the hedged portfolio implied by the different futures contracts or the futures portfolio and the constant and dynamic techniques.

In Table 5.6.2, utility comparisons are made among the constant and dynamic hedging strategies as implied by the four contracts and the SF-DM futures portfolio for the 1952-days period. It is clear that the dynamic DM futures contract outperforms the other contracts and the SF-DM futures portfolio even with daily rebalancing of the hedged portfolio. The computed utilities of the remaining futures contracts reveal the same pattern of hedging effectiveness as implied by the risk comparisons.

However, since transaction costs make daily trading very costly, the investor must rebalance his hedged position only when the gains in his expected utility from rebalancing offset the transaction costs incurred when a new position is taken in the futures market. Assuming six different prices for the amount of the transaction costs, the estimated utilities and the number of rebalancings are shown in Table 5.6.2. It is worth noting that, in the presence of transaction costs, an investor with a futures portfolio hedge would be able to rebalance approximately only half the times than with a single hedge. This finding is consistent with the empirical findings of Lien (1990) who argues that including a futures contract in a hedge portfolio should not ignore the additional transaction costs incurred. This argument is also supported for an amount of transaction costs over $20, that leads to a lower number of the possible times of rebalancing and a lower utility for the investor.

It is quite clear that the DM dynamic cross-hedge outperforms the other single futures cross hedges and the SF-DM futures portfolio both on the basis of variance and utility comparisons. However, in sample measures tend to overstate the true hedging effectiveness of a hedging model. In Section 5.9, out-of-sample tests of hedging performance are performed in the presence of transaction costs.
Table 5.6.1: Variance Comparisons of constant and dynamic single and multiple cross-hedges for the spot EMS portfolio.

<table>
<thead>
<tr>
<th>Model</th>
<th>BP</th>
<th>DM</th>
<th>JY</th>
<th>SF</th>
<th>SF-DM portfolio</th>
</tr>
</thead>
<tbody>
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<td>variances</td>
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<tr>
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<td>0.39927</td>
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<tr>
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<td>0.54105</td>
<td>0.29342</td>
<td>0.2121</td>
</tr>
<tr>
<td>OLS</td>
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<td>0.14036</td>
<td>0.33277</td>
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<td>0.1513</td>
</tr>
<tr>
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<td>0.13779</td>
<td>0.32643</td>
<td>0.17840</td>
<td>0.1473</td>
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Variance improvement of the dynamic model

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<tbody>
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<td>18.24%</td>
<td>0.0728</td>
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</tr>
<tr>
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<td>0.1010</td>
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<td>0.0462</td>
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<td>0.0024</td>
<td>1.83%</td>
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<td>3.05%</td>
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Table 5.6.2: Utility Comparisons of constant and dynamic single and multiple cross-hedges for the spot EMS portfolio.

<table>
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<th>Model</th>
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<th>BP Rebalancing</th>
<th>JY Utilities</th>
<th>JY Rebalancing</th>
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<td>-3114.34</td>
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<td>-1164.01</td>
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<td>-1066.30</td>
<td>1214</td>
<td>-1164.14</td>
<td>531</td>
</tr>
</tbody>
</table>
(5.8) *Comparisons of the in-sample hedging performance of constant direct and cross-hedge ratios for the Euro.*

As it is stated in the previous paragraphs, the main purpose of deriving single and multiple hedge solutions for a basket of representative EMS currencies, is for examining the hedging potential of major currencies futures contracts for the economic exposure in the Euro. However, since the Euro started trading in January 1999, a small dataset can be constructed from Euro spot and futures prices and the hedging effectiveness of the naïve and the OLS model can be studied for direct and cross-hedges.

In Table 5.7, the variance comparisons between the naïve and OLS hedging strategies show that the latter outperforms the former in all cases examined. In the case of the JY, the naïve strategy is shown to have increased the variance of the Euro spot position, indicating again the irrelevance of the full hedge for cross hedging problems. The comparison among the different futures contracts shows that the most successful is the DM cross hedge that outperforms the direct Euro futures hedge by 0.464%. Although this difference is not very large, it has important implications on the ineffectiveness of the EURO futures contract in hedging the underlying currency and the significant role of the Deutsche Mark in the European currency area.

The utility comparisons support the previous results with the DM futures contract outperforming slightly the Euro futures contract and the SF, BP and the JY futures contracts. However, it must be recognized that all the results of the present section are based on a very small sample period and they cannot thus lead to general conclusions on the hedging problem of the Euro. This empirical issue is left for further research.
Table 5.7: Comparisons of the in sample hedging effectiveness of constant direct and cross-hedges for the Euro.

<table>
<thead>
<tr>
<th>Model</th>
<th>EURO</th>
<th>SF</th>
<th>DM</th>
<th>BP</th>
<th>JY</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unheded</td>
<td>0.31055</td>
<td>0.31055</td>
<td>0.31055</td>
<td>0.31055</td>
<td>0.31055</td>
</tr>
<tr>
<td>Naïve</td>
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<td>0.07813</td>
<td>0.02126</td>
<td>0.25314</td>
<td>0.94571</td>
</tr>
<tr>
<td>OLS</td>
<td>0.02159</td>
<td>0.06113</td>
<td>0.02045</td>
<td>0.23290</td>
<td>0.29217</td>
</tr>
</tbody>
</table>

**Variance reduction of the OLS model**

<table>
<thead>
<tr>
<th></th>
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<th>% absolute</th>
<th>% absolute</th>
<th>% absolute</th>
<th>% absolute</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unheded</td>
<td>93.048%</td>
<td>0.289</td>
<td>80.316%</td>
<td>0.250</td>
<td>93.415%</td>
</tr>
<tr>
<td>Naïve</td>
<td>4.272%</td>
<td>0.001</td>
<td>21.758%</td>
<td>0.017</td>
<td>3.808%</td>
</tr>
</tbody>
</table>

**Utilities**

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Unheded</td>
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<td>-150.306</td>
<td>-150.306</td>
<td>-150.306</td>
<td>-150.306</td>
</tr>
<tr>
<td>Naïve</td>
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<td>-457.725</td>
</tr>
<tr>
<td>OLS</td>
<td>-10.4489</td>
<td>-29.5865</td>
<td>-9.89711</td>
<td>-112.726</td>
<td>-141.411</td>
</tr>
</tbody>
</table>
(5.9) Out-of-sample hedging performance of dynamic hedging strategies

An investor is not concerned with the risk reduction and utility performance he could achieve in the past but with the performance of a strategy in the future. The true efficiency gains from a dynamic strategy are evident only by the ex ante performance of the hedging model. In order to perform an out-of-sample comparison of the hedging effectiveness of each strategy, the last 100 observations are withheld from the original sample and the OLS and GARCH models are estimated with the first 1852 observations. One-day forecasts are made for the spot portfolio and futures returns and their forecasted conditional moments are computed. This analysis is repeated 100 times by adding one observation at a time in the dataset, until all observations of the initial sample are used. The forecasted hedge ratios are then estimated and 100 forecasted portfolios of combined spot and futures positions are constructed for each model. The variances of their returns and the percentage variance improvement of the dynamic strategy over the others are computed and reported in Table 5.8.1.

As it can be seen in Table 5.8.1, the ineffectiveness of the naïve JY futures strategy also applies in the case of the BP futures contract when the ex ante performance is studied. The use of the BP futures in a naïve hedging strategy would increase the portfolio risk by almost 14 % while the JY futures could lead to a 53 % variance increase, resulting in a huge loss for the hedger. These findings rule out the use of the two futures contracts as naïve hedges of the European currency portfolio risk. However, there is a gain in terms of risk reduction from the use of the BP and the JY futures contract respectively when the OLS cross-hedging strategy is used. Another interesting result from the out-of-sample analysis is that the dynamic strategy no longer outperforms the OLS strategy in the case of the BP futures contract. This finding is attributed to a possible misspecification of the GARCH model for the BP, which might be due to the ERM crisis of September 1992 that is included in the sample. Although this event was accounted for with a dummy variable, the limitations of hedging in a currency crisis still remain.

A striking result of the out-of-sample analysis is that, in the case of the DM, the
OLS strategy can no longer outperform the simple one-to-one hedge ratio. This finding supports the empirical results of Lypny (1988) who finds that a naïve hedge of a spot portfolio of Canadian Dollars and Deutsche Marks is more effective in terms of risk reduction than the risk minimizing hedge ratio. According to Aggarwal and DeMaskey (1997), this result is attributed to the limitations of the minimum-variance hedge ratio that rules out the possibility of equal price changes for the spot and the futures and it assumes stability of the estimated OLS coefficients. The success of the naïve hedge in this case can also be explained by the relatively stable relationship existing among the five European currencies and the DM, which is probably due to a weak or strong German monetary policy dominance in the EEC. In the case of Netherlands, there is a clear monetary union with Germany and over the 1980's the guilder was pegged to the DM. The high performance of the dynamic DM futures hedge shows the validity of GARCH models in producing one-day forecasts of exchange rate returns. The SF dynamic hedge is also a very effective hedging strategy when used out-of-sample although its performance is lower than in the in-sample analysis. The SF-DM dynamic portfolio shows a remarkable performance relative to the in-sample case with a risk reduction of 69.114 % relative to the unhedged case. However, it cannot outperform again the DM single dynamic futures hedge that is superior by offering an additional 10 % risk reduction to the spot portfolio.

The superiority of the dynamic strategy is also evident when it is applied to the decision of rebalancing the combined spot–futures position. Tables 5.8.2 and 5.8.3 present utility comparisons among the constant and dynamic hedges for the 100-days period. It is shown that, under six different scenarios for the amount of transaction costs, the DM dynamic strategy is the best hedge for the spot portfolio on the basis of utility maximization in the out-of-sample analysis. The same limitations in terms of the estimated utilities apply in the out-of-sample case as in the in-sample analysis after increasing the amount of the transaction costs over $20.

We can conclude that the problem of managing the risk of European currencies without actively traded derivative instruments can be met successfully with the use of the DM futures contract. A dynamic hedging program is necessary in order to
reduce remarkably the initial variance of the spot position and adapt selectively the currency portfolio. This conclusion also applies in the case of the Euro since the latter currency is considered to possess most of the empirical regularities of the German exchange rate.
Table 5.8.1: Out-of-sample variance comparisons of constant and dynamic cross-hedges.

<table>
<thead>
<tr>
<th>Model</th>
<th>BP</th>
<th>DM</th>
<th>JY</th>
<th>SF</th>
<th>SF-DM portfolio</th>
</tr>
</thead>
<tbody>
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<td></td>
<td>variances</td>
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<td></td>
<td></td>
<td></td>
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<td>0.40328</td>
<td>0.40328</td>
<td>0.40328</td>
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Table 5.8.2: Utility comparisons of constant and dynamic cross hedges with the BP and the JY futures contracts.

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<th>Rebal.</th>
<th>DM utilities</th>
<th>Rebal.</th>
<th>SF-DM port utilities</th>
<th>Rebal.</th>
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<tr>
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Section E: Conclusions

The adoption of a single European currency implies the elimination of exchange rate risk in monetary transactions between the participating countries of the European Monetary Union. However, the fluctuation of the Euro with respect to third countries currencies will be free floating, making spot positions in the Euro very risky, under the assumption of the high volatility of free floating exchange rates.

Since the Euro started trading very recently (January 1999), limited observations are available for the construction of hedging models for Euro spot positions. For this reason, in the present study, a portfolio of five EMS currencies is constructed in an effort to replicate the European currency. Since the specific currencies are not traded in the futures markets, four major currency futures contracts are used in order to construct single cross hedges for the spot portfolio. Additionally a portfolio of futures contracts is also used in order to account for portfolio effects in a multi-currency problem. However, we found it useful, for comparison purposes, to estimate a hedging model for the Euro as well, since the specific currency will substitute for all European currencies introduced in the EMU. One direct and four cross hedging models are estimated for the Euro futures and the four major currencies futures contracts for the period 4th January 1999 until 22nd June 1999.

A significant limitation of the empirical studies on cross hedging is the use of constant methods to derive risk minimizing hedge ratios. Significant temporal instability in the estimated coefficients introduces a drop in the hedging performance of the hedging models when they are applied ex ante. The present chapter accounts for the specific problem by applying a dynamic hedging strategy to the spot EMS currency portfolio and the major currency futures returns. Following the approach of the previous chapters, a multivariate GARCH model is estimated and the dynamic risk minimizing hedge ratio is computed for the four futures contracts and the futures portfolio. The multivariate GARCH model is found to perform well in all cases examined. Additionally, an in-sample and out-of-
sample comparison is made among the five dynamic hedges and constant strategies in terms of hedging performance.

The conclusions from the EMS spot portfolio and the Euro spot return estimations are very helpful for portfolio managers wishing to hedge their reserves in the single European currency against fluctuations in the U.S. Dollar. It is shown that the EMS currency portfolio hedged with the DM dynamic hedge ratio has the lowest variance and the best utility performance with strong implications for the dominance of the German monetary policy in the European currency area. This result is enhanced by the analysis of the Euro spot position that reveals a superior hedging performance for the DM futures contract even when it is compared to a direct hedge of the Euro by its respective futures contract. We can conclude with a proposal for further research on the hedging potential of currency futures for spot positions in the single European currency. Further examination is needed with a larger sample period and a more adequate econometric specification, in order to be able to draw general conclusions.
CHAPTER 6: Conclusions and Suggestions for Further Research

Foreign exchange risk forms a major source of uncertainty faced by investors and multinational firms. Motivated by the increasing volatility of foreign exchange rates and the substantial volume of international investment, the present thesis developed dynamic hedging models that account for special empirical issues. The derivation of effective strategies for currency risk management eliminates a significant portion of losses, leading to a well-maintained foreign equity or currency portfolio or, enough internal funds for corporate investment. The continuous growth of derivatives markets offers a large number of hedging products to the interested investors, reducing the adverse effects of financial risk in portfolio management.

In Chapter 1, the motivation and the objectives of the present thesis were briefly reported and an overview of the main points addressed in the remaining chapters was included. Chapter 2 aimed to provide a contemporary review of the main theoretical issues of foreign exchange risk management. An introduction to the theory and instruments of currency hedging was made with emphasis on the corporate benefits of risk management. The effect of the basis risk and the futures-forward hedging differential on the optimal hedge ratio was analyzed in the presence of stochastic interest rates. The theoretical framework of multi-currency hedging was presented along with the relevant empirical literature. Additionally, the issue of cross hedging was discussed and a review of the related studies was reported. The principal empirical issues addressed in the following chapters were derived by the limitations in the existing empirical evidence on hedging.

The complex hedging problem of the exposure in more than one currencies is faced by the majority of international investors and multinational corporations. The portfolio effects on the development of the optimal currency hedge ratio were investigated in Chapter 3. As in the empirical study of Gagnon et al (1998), two value-weighted spot portfolios were examined, consisting of British Pounds and Japanese Yen and, Swiss Francs and German Marks. Two multivariate dynamic
models were estimated for the spot currency portfolios and their respective futures contracts and dynamic portfolio hedge ratios were calculated. The estimated dynamic models were found statistically superior to the constant methods in all cases examined. Additionally, four bivariate dynamic models were estimated for each spot currency return and the return on its relevant futures contract and dynamic no-portfolio hedge ratios were also derived.

However, the present thesis extended the existing evidence in the following ways. First, the existence of a common stochastic trend in spot and futures prices was considered in a dynamic framework for both conditional moments of their distribution. The deviation from the general equilibrium relationship was found to explain a significant portion of the conditional variance, giving support to the use of the GARCH-X model for the estimation of the time-varying risk minimizing hedge ratio. Second, a selective rebalancing strategy was applied to the combined spot-futures portfolio accounting for the hedger's utility in the presence of transaction costs. Finally, an out-of-sample analysis was also performed in order to examine the true efficiency gains resulting from the dynamic portfolio strategy.

The comparison between portfolio and no-portfolio hedge ratios revealed significant differences in terms of size and variance as well as hedging effectiveness. The hedge ratios estimated from the portfolio model were half in size and more variable than their no-portfolio counterparts. The notion in the paper of Gagnon et al (1998) that hedging models accounting for portfolio effects are more effective than those ignoring them was supported in the present thesis with respect to both risk and utility comparisons. Additionally, dynamic hedge ratios outperformed the static ones stressing the need for a dynamic hedging strategy in the foreign exchange market. Although the out-of-sample effectiveness of the models was limited relative to their in-sample performance, the dynamic portfolio hedge was again the dominant strategy. The implication of this finding for portfolio management is that, accounting for portfolio effects in a dynamic hedging program, is a more efficient investment policy.

Following the discussion in Chapter 2, in the foreign exchange market, the
forward-futures hedging differential and the basis risk depend on the same source of uncertainty, that is the domestic interest rate. In Chapter 4, the assumption of non-stochastic interest rates was relaxed as being unrealistic in view of the high volatility of interest rates. The inter-temporal hedge ratio for basis risk and the marking-to-market effect was applied to four major currencies as derived by Chang et al (1996). It was assumed that a default-free domestic discount bond is used as a hedge for the interest rate risk of the futures positions since the interest, on which the margin account is financed, is expressed in terms of the domestic interest rate.

However, in the present thesis, the methodology of Chang et. al (1996)) was extended into a dynamic setting. A GARCH-X error structure was assumed for the spot and futures exchange rates in order to account for the time-variation in their conditional distribution, as evidenced in Chapter 3. The domestic (U.S.) interest rate was assumed to follow a mean-reverting square root process (Cox, Ingersoll and Ross, 1985) with a time-varying volatility parameter (TVP-Levels Model, Brenner et al, 1996). The reported ability of the model to perform well under structural breaks was important for our study since the Eurocurrency interest rates in our sample were found to be stationary around a broken trend line due to monetary regime changes. The model developed is innovative on the basis of the joint estimation of spot and futures exchange rates and the domestic interest rate in a dynamic context in order to hedge basis risk.

The dynamic basis risk model was found to be a valid representation for the exchange rates and the interest rates examined. The futures and domestic bond hedging demands were derived by the coefficients of the estimated model. The in-sample and out-of-sample comparisons of the constant and dynamic strategies revealed that the currency portfolios hedged with the dynamic basis risk hedge ratios were the most efficient in terms of variance reduction and utility performance. The implication of these findings for treasurers and portfolio managers is that the risk management of foreign currencies can be improved by the dynamic hedging of the interest rate risk resulting from the daily marking to market of futures contracts. To the extent that a derivative contract is correlated to the domestic interest rate, the basis risk of the hedged portfolios can be significantly
reduced, leading to increased performance of the initial hedge.

The introduction of the Euro as the single European currency in 2001 and the possibility of replicating it by a basket of representative EMS currencies, raised the empirical issue examined in Chapter 5. The cross-hedging potential of major currencies futures contracts was investigated for a portfolio consisting of five EMS currencies. Accounting for a significant shortcoming of the empirical studies on cross hedging, a dynamic hedging strategy was applied to the spot EMS currency portfolio and the major currency futures. Spot portfolio and futures returns were jointly estimated in a GARCH model and dynamic hedge ratios were derived for each futures contract. The multivariate GARCH model was found to capture the non-linearities of the data in all cases examined. As the preliminary cointegration tests had implied, the German Mark was the most effective hedging instrument for the spot portfolio. Additionally, a portfolio of the SF and DM futures contracts was also used in order to account for portfolio effects in the multi-currency problem. However, the futures portfolio could not outperform the DM futures contract in terms of variance reduction and utility maximization.

A basic limitation of the last empirical chapter is the fact that the results of the hedging efficiency tests are specific to the EMS currencies used for the construction of the spot portfolio. Different conclusions could be drawn from the use of a portfolio consisting of more than five EMS currencies. However, the success of a currency portfolio in replicating the Euro is a separate empirical issue that is left for further research.

Nevertheless, the main innovation of the present chapter lies in the exploration of the single European currency that was introduced in 1999. One direct and four cross hedging models were estimated for the Euro futures and the four major currencies futures contracts for the period 4th January 1999 until 22nd June 1999. The analysis of the Euro spot position revealed a superior hedging performance for the DM futures contract even relative to a direct hedge with the Euro futures contract. This finding has strong implications for the statistical properties of the new currency and the German Dominance in the EMU. However, no general
conclusions could be drawn from the latter model since the Euro started trading very recently and a limited sample was available. The conclusions from the EMS spot portfolio and the Euro spot return estimations are very helpful for investors wishing to hedge their reserves in the single European currency against fluctuations in the U.S. Dollar.

In all empirical chapters of the thesis, the use of the risk reduction as the basic measure of hedging effectiveness was explained by the fact that the primal objective of risk management is the minimization of the risk of a given spot position. However, the economic usefulness of the rebalancing strategy derived by the dynamic model could be examined only in terms of the implied utility of the investor. Although some empirical studies account for the risk-return tradeoff in hedging, they suffer from the limitation that this approach depends on market expectations and the utility functions of the specific investor. Using ex post returns on futures prices is not very useful in hedging decisions since market expectations change continuously as new information enters the financial markets. A suggestion for further research is to develop a conditional risk premium model for futures prices and estimate the expected return in the currency futures position. This estimate can then be introduced in the optimal currency hedge ratio and the issue of currency hedging can be studied in a risk-return framework.

Additionally, an implication for research arises from the application of the basis risk dynamic model to the hedging problem of the single European currency for a larger sample period. The hedging potential of the Euro and the major currencies' futures contracts can thus be studied under the realistic assumption of stochastic interest rates. In this way, a substantial part of the additional risk, arising from the marking-to-market effect and the basis variation, can be reduced, leading to a more adequate currency risk management strategy for the Euro. The conclusions from these estimations are very important to financial managers facing European currency risk on the basis of the significant fluctuation of the Euro versus the U.S. Dollar.

Finally, in view of the recent currency crises in countries of Eastern Europe, Latin
America and Asia, a risk analysis of the emerging markets' exchange rates can also be proposed. However, for the derivation of effective risk management solutions, an econometric specification that can capture extreme events in the distribution of exchange rates is required.

The scope of the present thesis was the derivation of effective risk management solutions for foreign currency portfolios of major and minor exchange rates. Its contribution to existing evidence lies in the development of dynamic hedging strategies under more realistic assumptions and the exploration of the single European currency. The empirical findings form hedging proposals for treasurers and investors participating in the foreign exchange market.


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