The Economics of Innovation, Investment, and Taxation

SUN, HAORAN

The Economics of Innovation, Investment, and Taxation

Haoran Sun

A thesis presented for the degree of

Doctor of Philosophy

Department of Economics and Finance

The University of Durham

United Kingdom

18th February 2023

Dedicated to

my family

This work was supported by

the UK Economics and Social Research Council (ESRC)

Award Number: ES/P000762/1

The Economics of Innovation, Investment, and Taxation

Haoran Sun

Abstract

The digitalization of economy has been creating profound difficulties in the tax policy and intensive debates among policy makers, economists and entrepreneurs. In this thesis, a series questions relating to innovation and tax policy are explored in three aspects.

Chapter 1 evaluates the role of public R&D support on labour productivity and productivity growth using non-parametric matching with up-to-date ONS data. We find that public support for R&D has a negative impact on labour productivity and such impact is insignificant for productivity growth. The significant negative effect on labour productivity and insignificant negative effect on productivity growth are mainly driven by high-tech firms and SMEs.

Chapter 2 presents a model of taxation of multinational businesses operating in a competitive international digital economy. The model includes important features of the digital economy, such as the network externality in consumption, significant market power of the providers, and the role of digital technology innovation. In particular, the focus is on the investment in innovation of two types: (i) innovation that reduces production cost, or process innovation, and (ii) innovation that improves the quality of the good and thus boosts the consumer demand, or product innovation.

Chapter 3 analyses a potential solution to taxing the digital economy based on the idea of division of the tax base. Firstly, we consider a two-country model in two approaches: a non-cooperative approach and a cooperative approach. The non-cooperative approach means each country decides what proportion of the firm's profit to tax. We consider a simple case when there is no profit split; next, we introduce profit split and analyse the situation where a firm earns profits from both countries. The cooperative solution is the case when two

countries jointly decide how to share the taxable profits. In this case, we first consider a simple case when a firm earns all profits from sales in the source country; next, we relax this assumption and analyse the situation where a firm also earns profits in the residence country. Secondly, we investigate the outcome when there are more than one source countries.

Supervisors: Nigar Hashimade, John Moffat, Laura Marsiliani and Thomas Renstrom

Acknowledgements

I would like to express my deepest gratitude to my supervisor Professor Nigar Hashimzade for her invaluable advice and immense knowledge. I could not have undertaken this journey without her patience and guidance. I learned a lot from her, not only as my supervisor but also as a dear friend who gave me constant encouragement to accept life changes and keep the research going. I am deeply indebted to Dr. John Moffat, who guided me on how to conduct empirical research. His Stata technique and salutary approach widened and deepened my knowledge of empirical research. I would also like to express my appreciation to Dr. Laura Marsiliani and Dr. Thomas Renstrom, for their support after Nigar's left. I have greatly benefited from every member of my supervisor team, and could not have completed my thesis without them.

Acknowledgement is made to the UK Economic and Social Research Council (ESRC) for funding support and UK Data Service for data support. I appreciate Durham University NINE DTP members, especially Mr. Stuart Gladwell, for their timely support on my off-campus academic visiting. I also appreciate support team of the UKDS SecureLab, who help to solve technique issues.

I would like to express my appreciation to my thesis committee, Professor Clive D. Fraser and Dr. Daniel Li for their time to review my thesis and make constructive comments.

Special thanks to Ms. Bekah Irwin, Ms. Abi McNally, and Ms. Joanna Quinlan for their support on my mental health.

I am grateful to my friend, Dr. Jingyuan Di, for his optimistic attitude toward life that helped me to feel positive. I appreciate Dr. Xiaoxiao Ma and Dr. Jiunn Wang for helping me to learn Matlab for calibration and simulation. I am also thankful to Ms. Helena Brennan for her advice on empirical research.

I would like to recognize the greatness of my wife's family on this journey, especially my parent-in-law, Mr. Yanqing Xu and Mrs. Li Peng, who are always supportive and treat me like their own child.

I owe profound gratitude to my parents, Mr. Jinlong Sun and Mrs. Shufang Wu, who set their wishes aside for the joy of mine. Their love and support help me to overcome every obstacle in life. I want to thank my dearest sister, Miss. Xiaoqi Sun, for giving me the warmth from family. I'm extremely fortunate to have such a loving sister to lean on and share my thoughts with.

Last but not least, I would like to thank my wife, Mrs. Yunfan Xu, who lights up my life and makes the last stage of my PhD journey meaningful. I cannot imagine what life would be like with anybody else as my wife than you, nor would I ever want to.

To all my family and friends, thank you and love you all.

Contents

Declaration

Chapter 1 is based on data from the UK Innovation Survey(UKIS) and Business Structure Database (BSD), produced by the Office of National Statistics (ONS) and provided by the UK Data Service SecureLab (UKDS Securelab). The data are Crown Copyright and reproduced with the permission of the controller of HMSO and King's Printer for Scotland. ONS or UKDS bears no responsibility for the analysis of the statistical data or the opinions presented in this chapter. The views expressed are the authors and do not necessarily represent the views of any organization. This chapter uses research data sets, which may not exactly reproduce National Statistics aggregations.

No part of this thesis has been submitted elsewhere for any other degree or qualification, and it is the sole work of the authors unless referenced to the contrary in the text.

List of Figures

List of Tables

Chapter 1

The Effect of Public Support for R&D on Productivity: An Empirical Study of UK Firms

1.1 Introduction

An economy's ability to grow is limited by the scarcity of resources. Endogenous growth theory suggests sustained growth can only be achieved by productivity enhancement (Romer, 1986, 1990; Lucas, 1988; Aghion and Howitt, 1992). Thus, technological change through innovation fundamentally determines a country's capacity for economic growth. In recognition of its importance, research and innovation were placed at the heart of the five foundations in the UK's 2017 industrial strategy while the more recent 'Plan for Growth' aims to boost UK investment in research and development (R&D) to 2.4% of GDP by 2027 (HM Treasury, 2021). Such increases in R&D expenditure require not only increased government spending but also the engagement of the private sector (House of Commons, 2021).

Crepon et al. (1998) propose a structural model (CDM model) to illustrate the relationship between firm level productivity, innovation output, and R&D investment.

The CDM model is based on Griliches (1979) knowledge production function which posits that R&D investment determines productivity indirectly through innovation output: productivity is driven by innovation output, and innovation output is generated from investment in R&D. However, it is likely that the private sector will provide insufficient R&D from a societal perspective due to market failures. Government intervention in the form of financial support is often used as a tool to correct this market failure. However, the logic that public support for R&D encourages technological improvements, which will boost productivity and economic growth, requires that such support is effective (Goldberg et al., 2015).

In developed countries, especially those at the technological frontier, policies to stimulate R&D in the private sector are actively implemented. Public support for R&D generally takes two forms: direct support such as subsidies or grants, and indirect support in the form of tax incentives. Subsidies and grants increase government expenditure. R&D tax incentives induce tax revenue loss unless they lead to a proportionally larger increase in the tax base. Understanding the effectiveness of different forms of public support is particularly urgent for the UK at a time of large budget deficits and low rates of economic growth.

The benefits of public support for R&D are hard to measure. Public support for R&D is supposed to boost investment in R&D in the private sector and thus productivity and economic growth. However, R&D support policies are used to fund medium- to long-term projects, the success of which will be influenced by factors other than the level of public support. Moreover, it is difficult to know what productivity and growth would be without public supports. As such, it is difficult to assess whether public support for R&D are successful in correcting the market failure of the sub-optimal level of investment in R&D in private sector.

This study will aim to analyse the effects of receiving UK regional, UK central, and EU supports on labour productivity and productivity growth. The determinants of participating in direct and indirect public support programmes for innovation will also be investigated. Using up-to-date data from UKIS and BSD, evidence shows public R&D support has a negative impact on UK firms' labour productivity using non-parametric matching.

This study is structured as follows. Section 2 presents a detailed review of the empirical literature. Section 3 explains the methodology, followed by section 4, which describes the data source and samples used for the analyses. Section 5 reports the main results. Section 6 concludes.

1.2 Literature Review

1.2.1 Justification for intervention

The topic of the impact of R&D investment on productivity was firstly studied by the seminal contribution by Griliches (1979) and sparked by the CDM model (Crepon et al., 1998). Many empirical studies estimate the effect of R&D investment on productivity (literature survey by Hall et al. 2010; meta-analyses by Ugur et al. 2020) and find that the effect of investment in R&D on productivity is expected to be positive. Therefore, stimulating investment in R&D through government support policies has always been an important part of industrial policy (Lazzarini, 2013).

The rationale for the existence of government support for innovation can be based on three paradigms: the market failure paradigm, the mission-oriented paradigm, and the cooperation paradigm (Bozeman and Dietz, 2001).

The market failure paradigm is rooted in three aspects of knowledge production: indivisibility, inappropriability, and uncertainty (Arrow, 1962). Indivisibility means that there exists a minimum investment in knowledge prior to the production of new knowledge (Chaminade and Esquist, 2010). Since the production of new knowledge heavily relies on both horizontal (i.e. complementarity or additionality with other advances) and vertical indivisibility (i.e. standing on the shoulder of giants), new knowledge production is turning into a more complex and demanding process (Antonelli, 2000). Thus, one cannot break knowledge production into simple steps such that the increasing barriers are tackled (Gopalakrishnan and Damanpour, 1994). For this reason, firms actively seek collaboration with other organizations and public fundings to reduce innovation barriers.

Inappropriability is an issue that arises because knowledge has public good characteristics which cause the social benefits from knowledge creation to exceed the benefits to the investor (Nelson, 1959; Arrow, 1962). Stiglitz (1999) explains that knowledge can be viewed as a public good because it is both non-rivalrous and non-excludable. Knowledge is non-rivalrous because the amount of knowledge is not reduced by consumption. Knowledge is also non-excludable. Raising barriers to knowledge or preventing others to learn is ineffective and unavailing in general since an idea can be easily copied and transferred. Knowledge creation from a private innovator benefits others but the innovator is not fully compensated for this benefit to others, which limits the return to the innovator because rivals will have access to the newly created knowledge. This lowers the incentives to innovation. Therefore, in a free market where goods and services are efficiently allocated, imperfect appropriability leads to less knowledge creation and private innovation than the socially desirable optimal. Czarnitzki et al. (2011) argue that even if the imperfect appropriation did not exist or was somehow remedied by government intervention, private innovation would still be insufficient due to the gap between the private return and cost of capital. Firms may be also discouraged from innovation because protection of own R&D from peer imitation would incur additional secrecy costs that reduce the private return from R&D further (Duguet, 2012). From the point of view of the investment theory, Hall and Lerner (2010, p. 661) argue *"... some innovations will fail to be provided purely because the cost of external capital is too high, even when they would pass the private-returns hurdle if funds were available at 'normal' interest rates"*. Therefore, government has incentives to encourage private innovations by compensating the gap between the private and social returns to R&D expenditure.

Uncertainty is an inherent characteristic in innovation, and it has two aspects: known uncertainty and unknown uncertainty (Jalonen, 2011). Known uncertainty, defined by Ellsberg (1961), means variables and outcomes are known but the actual values are missing (Chow and Sarin, 2002). For instance,the neo-classical model assumes economic agents hold complete and perfect information. However, in practice, innovators are likely to hold private information about R&D projects which may not be acknowledged by investors (Aboody and Lev, 2000). Such private information can consist of knowledge of the innovator's objectives, the potential outcomes from the innovation, or simply of the relevant field (Barbaroux, 2014). Imperfect information creates uncertainty and increases the difficulty of realising value from the commercialisation of innovations by investors and hence discourages innovation investment. However, uncertainty in innovation is beyond incomplete information alone because otherwise one could simply argue such innovation uncertainty can be reduced by increasing the amount of available information (Jalonen, 2011). Instead, innovation is a process where some missing information is unavailable to anyone, i.e. unknown uncertainty. Such unknown uncertainty includes, for example, complexity of innovation, non-deterministic innovation outcomes, whether markets for new products exists or not. Nevertheless, these three characteristics of innovation (indivisibility, inappropriability, and uncertainty) create under-investment in R&D by private sector and are the targets when justifying public support for R&D (Bloom et al., 2019). Government policies step in to correct market failures by reducing the cost of risky, uncertain R&D projects and introducing lawful appropriation mechanism to enforce intellectual property rights and thus increase private firms' expected returns.

The mission-oriented paradigm argues that government policies can be used to support the production of public goods relating to national defence, aerospace, public health etc (Bozeman and Dietz, 2001). Mission-oriented policies aim to achieve specific radical, high-end technological objectives which may lead to many smaller scale applications (Chiang, 1991). Such policies require large scale collaboration between the public and private sectors (Mazzucato, 2018). Private business becomes a contractor by signing government R&D contracts. Contractors could utilize government funds to rapidly accumulate experience, increase technical capabilities, produce new products and services, and gain private returns through commercial applications.

The cooperation paradigm emphasizes cooperation among industries, universities, and the government to facilitate knowledge flows and structural change in the national innovation system (Bozeman and Dietz, 2001). The rationale behind the cooperation paradigm is that the market is not always the most efficient place to conduct innovation. Instead, universities and laboratories at the knowledge frontier could innovate better than private businesses but lack the ability to commercialize innovation outcomes. Therefore, governments can carry out centralized plans for civilian technology development and manage technology flows among industries, universities, and rival firms. Even though Bozeman and Dietz (2001)argue that the cooperation paradigm has become less important, cooperation among different agents are still considered important aspects when evaluating public support applications (Huergo and Moreno, 2017).

Within these three paradigms, scholars generally agree that government should intervene to correct under-investment in innovation (Petrin, 2018). However, there are disagreements on how governments should intervene. On the one side, governments are viewed as the driving force to promote the national system of innovation and to prevent the private sector from taking on excessive risk (Mohnen, 2018). However, advocates of the free market believe that government should intervene as little as possible and leave most initiatives to the private sector (Hall and Reenen, 2000). The former leads to direct support, and the latter provides the rationale for indirect support.

1.2.2 Direct support and indirect support

Governments can directly provide R&D grants to private firms to address the gap between the private and social returns and reduce the cost of funding (Azoulay and Li, 2020). Direct supports generally require some form of application and decisions made by an awarding body on which firms will receive funding based on certain criteria in a competitive setting. However, decisions on which firms will be awarded grants by government agencies are often problematic and lead to government failures in practice. For instance, asymmetric information about innovation projects between government agencies and innovators cannot be eliminated and moral hazard on behalf of grant recipients may lead them to invest poorly. These problems increase the difficulties of distributing R&D grants in accordance with societal objectives.

In addition, Czarnitzki et al. (2011) argue that government agencies may be biased when administering R&D grants by factors such as political pressure, bureaucratic objectives, corruption, etc. Since government failure may lead to more severe consequences than the market failures that the grants are supposed to correct, indirect fiscal support such as R&D tax credits have gradually replaced or supplemented R&D grants (Dechezleprêtre et al., 2016). Unlike the application process required for direct support, firms do not compete to win R&D tax incentives. Instead, R&D tax incentives are automatically applied if minimum requirements are met.

The rationale for introducing R&D tax credits rests on user cost of capital which bridges taxes to investment decisions (Billings et al., 2001). Theoretically, firms will invest in R&D up to the point where the marginal cost of R&D equals the marginal benefit of R&D (David et al., 2000). Assuming other investment factors are fixed, a change in tax policy will influence cost of capital, and in turn affect marginal cost of investment. Thus, firm's investment behaviour will be altered.

Billings et al. (2001, p. 468) argues that user cost of capital can be used to explain how taxes affect firm investment behaviour. It can be calculated based on the neoclassical theory, Tobin's Q theory, or the effective tax rate theory: the *"present* *value of expected cash flow"* is used in neoclassical theory; Tobin's Q uses *"changes in firm's market value"*; and the effective tax rate approach uses a *"weighted average of the company-wide cost of capital, rate of interest, and the effective tax rate for specific classes of depreciable assets"*. Despite the differences in measuring the user cost of capital, these three methods give similar properties between taxes and investments. They also derive an equation for the user cost of capital and show that it has a negative correlation with the rate of R&D tax credits. R&D tax credit participants experience a reduction in their own costs of R&D and thus profits can be maximized through increasing outputs. Thus, they argue that higher R&D related output is expected because of a higher marginal product of R&D resulting from a higher rate of tax credit. Furthermore, in their derived user cost of capital equation, the depreciation rate of R&D is positively correlated with the user cost of capital. That is, firms are likely to be discouraged from investing in R&D as the user cost of capital increases because of an increase in depreciation rate. Holding all other parameters constant in the user cost equation, if the tax credit rate increases, firms are likely to increase investment in R&D. Therefore, they hypothesize that R&D tax credit recipients spend more on R&D than firms that do not receive the credit.

While the lack of prior evaluation may be considered an advantage, since it prevents governments from attempting to 'pick winners', it also increases the likelihood that government expenditure fails to stimulate additional R&D expenditure. For instance, Duguet (2012) argues that tax credits can be used for different purposes by private firms. Firms may respond to the reduction in the price of R&D by increasing investment so the tax credit has the desired effect of stimulating R&D. Alternatively, it is possible that firms who apply for R&D tax credits only do so due to the increased profits that can be obtained because of the reduction in costs. In the first case, one would argue that the tax credit meets its objective and encourages additional R&D expenditure, while it may not affect private R&D, or even introduce 'redundant' or 'useless' R&D expenditure in the second case.

Moreover, it has been argued that recipient firms are likely to invest in short-term projects with the highest private rate of return (Hall and Reenen, 2000; David et al., 2000). These short-term R&D projects are not necessarily the projects that are most deserving of public support, casting doubt on whether R&D tax credits can stimulate projects with high social and low private rate of return. From this point of view, R&D tax credits are not likely to be the most efficient policy tool to correct the market failure that arises due to the gap between the private and social rate of returns from innovation. Furthermore, researchers often believe that the incentive to innovate *"is driven by advances in basic science and perhaps by market demand, rather than by tax incentives"* (Bloom et al., 2019, p 169). Therefore, even if the R&D tax credit seems to have a significantly positive correlation with private R&D expenditure, the causal effect of R&D tax credits requires more investigation.

1.2.3 Determinants of receiving public R&D support

Evaluation of policy objectives is rather difficult in situations that one cannot distinguish a firm's true purpose of using public support. Nevertheless, the fundamental question in the literature of government support for R&D is how effective the programs are in stimulating private R&D, increasing innovation output, or boosting productivity. Since both direct and indirect support are not randomly awarded to firms, studying the determinants of participation may help to understand whether support decisions are consistent with policy goals, and if there are unexplained aspects affecting participation in R&D supports.

Moreover, understanding how public R&D supports are distributed may help to control potential selection bias when evaluating the effectiveness of such support policies (Aschhoff, 2010). Duguet (2004) explains selection bias can be controlled when variables that influence the outcome of public R&D policies also determine the participation. On the one hand, if a variable cannot affect firm's participation in public R&D policies, it creates no selection bias. On the other hand, if a variable does not influence the outcome but affects participation, it is not necessary to include such variable because it only creates 'noise' in the evaluation process. Therefore, based on previous literature, factors which potentially predict receiving public R&D support and at the same time can affect outcome variables (i.e. labour productivity or productivity growth) will be discussed below.

1.2.3.1 Size

Firm size (often measured as the natural logarithm of employment) can influence the probability of receiving support for R&D through different channels. Larger firms may have more efficient organization structures to conduct R&D (Carboni, 2011). The high cost of capital associated with innovation is likely to be overcome by larger firms which have relatively more internal funds available than SMEs (Clausen, 2009), whereas SMEs are likely to experience financial constraints (Hall, 2002). In addition, larger firms may experience internal spillover that could encourage R&D (Klette, 1996). Therefore, larger firms may have more potential and better ability to utilize public R&D support. From this perspective, governments might think larger firms will use public support better than the small firms and hence larger firms are more likely to receive public support for R&D. However, Barajas et al. (2021)argue that there is no solid empirical conclusion on how size would affect public support participation. This might be because, on the one hand, government policies tend to be more generous for SMEs because they are more likely to suffer from innovation constraints. On the other hand, large firms may have more opportunities to engage in R&D and capacity to meet government requirements and apply for financial support. Thus, if the policy is oriented towards firm size, then size will positively impact the probability of receiving public R&D support.

Most studies show that larger firms have a higher chance of receiving public support for R&D (Busom, 2000; Duguet, 2004; Kaiser, 2006; Hussinger, 2008; Aschhoff, 2010; Yang et al., 2012; Dumont, 2013; Kobayashi, 2013; Huang, 2014; Busom et al., 2016; Silva et al., 2017; Hottenrott et al., 2017; Huergo and Moreno, 2017; Albis et al., 2021); but others have found smaller firms are more likely to be awarded (González and Pazó, 2008; Czarnitzki and Lopes-Bento, 2014; Ugur et al., 2015; Barajas et al., 2021). Effect from size could also be irrelevant (Czarnitzki and Delanote, 2015).

Furthermore, the effect of size may differ across the economic cycle. For instance, Barajas et al. (2021) find that SMEs have a higher probability of participating in a subsidy programme before the 2008 financial crisis in Spain, but this probability fell in the post-crisis period. This might be because the survival rate of large firms is high, or the cost of participating in public R&D support increases for SMEs during the crisis period.

1.2.3.2 Age

A firm's experience in conducting R&D or effects from 'learning-by-doing' could be measured by its age. However, the effect of age in the empirical literature is ambiguous. Start-up firms may suffer more from financial constraints due to a lack of resources and limited access to financial markets. While established firms may have more experience and information about innovation and public support. Experienced firms are likely to receive public support (González and Pazó, 2008). Young firms may have higher probability of receiving public support (Reikowski et al., 2010; Yang et al., 2012; Czarnitzki and Lopes-Bento, 2014; Hottenrott et al., 2017; Barajas et al., 2021). Also, most literature has found that age is not a strong predictor of receiving public support for R&D (Heijs and Herrera Danny, 2004; Hussinger, 2008; Clausen, 2009; Herrera and Ibarra, 2010; Huang, 2014; Czarnitzki and Delanote, 2015; Beck et al., 2016; Silva et al., 2017; Hottenrott et al., 2017; Čadil et al., 2017). Huergo and Moreno (2017) argue that whether age is significant or not might be subject to the programme type. They found young firms are likely to have higher probability of receiving CDTI loan but lower probability of receiving EU support. Radicic and Pugh (2015) also find age is irrelevant for receiving UK national support for R&D but increase probability of receiving international support. Therefore, we hypothesize age will not affect the probability of receiving public support for R&D.

1.2.3.3 Share of qualified employees

The share of qualified employees or share of employees with a university degree is often used to measure a firm's capacity to acquire and generate knowledge for innovation (Aschhoff, 2010). Qualified support staff may help firm to be successful in funding stage (Czarnitzki and Fier, 2002). Literature generally find this variable to be positively affecting participation (Dumont, 2013; Czarnitzki and Fier, 2002; Aschhoff, 2010; Reikowski et al., 2010; Alecke et al., 2012; Beck et al., 2016; Busom et al., 2016; Albis et al., 2021).

1.2.3.4 Ownership

Ownership also affects the probability of participation. Access to a larger international market could leads to higher profit but also benefit from knowledge spillovers from overseas rivals (Love and Roper, 2015). Foreign owned firms are often able to access parent firm's internal funds and find financial opportunities in the international capital markets. Foreign ownership, whether total or partial, reduces the likelihood to be awarded a public support (Busom, 2000; Heijs and Herrera Danny, 2004; Hussinger, 2008; Herrera and Ibarra, 2010; Hottenrott et al., 2017; Barajas et al., 2021). Clausen (2009) explains this could be due to firms having no incentive to conduct R&D in foreign country since R&D has already been developed in parent company. This variable has also been found to be insignificant in some studies (Görg and Strobl, 2007; Clausen, 2009; Alecke et al., 2012; Beck et al., 2016; Silva et al., 2017).

1.2.3.5 Cooperation

Cooperation with other organization allows firm benefits from external knowledge spillover (Aschhoff, 2009) and reduces liquidity constraints and hence increase the probability of receiving public R&D supports (Czarnitzki and Hottenrott, 2012). Cooperation with other organisations is likely to increase the probability of receiving a subsidy (Kaiser, 2006; Aschhoff, 2009; Dumont, 2013; Beck et al., 2016; Huergo and Moreno, 2017; Hottenrott et al., 2017).

1.2.3.6 Constraints

R&D constraints including financial constraints, knowledge constraints, and market constraints are likely to limit the number and quality of R&D activities that a firm can engage. Firms are likely to be restricted to external financial opportunities when conducting R&D because of the uncertain nature of innovation (Ali-Yrkkö, 2005). Especially for SMEs and start-up firms, they have limited ability to finance their R&D activities due to less capital for business collateral (Becker, 2015). Accessing private funds is also risky for innovators who have un-patented ideas since it raises possibility for funders to steal an idea from innovators (Bloom et al., 2019). Financial constraints are expected to increase chances to receive public support (Duguet, 2004; Ali-Yrkkö, 2005; Kobayashi, 2013). However, Busom et al. (2016) and Carboni (2011) find no significant correlation between financial constraints and consistently receiving public supports and Busom et al. (2014) find negative relation between receiving tax credit and financial constraints for SMEs using Spanish firm level data.

In addition to all factors described above, innovation behaviour of a firm can differ in different phases of a business cycle, and can also depend on the type of industry and on the geographical location of the firm. In the empirical analysis, the potential effect of these additional factors can be captured using dummy variables.

1.2.3.7 Business cycle

Empirical analyses include time dummy to control for cyclical changes. It allows to determine whether the time-fixed effect on the dependent variable is different among different periods and thus test whether such effect is stable across time (Hardy, 1993).

1.2.3.8 Industry heterogeneity

Industry heterogeneity affects innovation behaviour across-industry. For instance, high-tech industries are more likely to conduct in R&D and thus actively participate in public support. Wu (2008) find high-tech industries are the primary recipients of R&D tax credit in the US because these industries are more R&D intensive. Ugur et al. (2015) include dummies for different Pavitt technology classes and find that Pavitt class 1-3 (sciences-based, specialized suppliers, and scale intensive firms) are more likely to receive EU and UK subsidies [1]. Furthermore, Fowkes et al. (2015) shows that R&D tax credit claims are different across industries. Thus, industry dummy is used to control heterogeneity across industries and is expected to affect the probability of receiving public support for R&D.

1.2.3.9 Region heterogeneity

Innovation behaviour and ability are significantly affected by a firm's location if technological environment, local policies, and funding availability are different across different localities (Yang et al., 2012)). Fowkes et al. (2015) also shows that claim rate of R&D tax credit is different across regions. Thus, regional dummy is used to control heterogeneity across regions and is expected to affect the probability of receiving public support for R&D.

1.2.4 Empirical Findings

For both direct and indirect support, the causal effects of public support are often estimated on variables measuring R&D inputs, R&D outputs, and broader measures of firm performance such as productivity (Petrin, 2018). Studies that evaluate effects on R&D inputs often try to answer whether government support encourages additional R&D expenditures. Studies on R&D outputs focus on evaluating the

[1] Pavitt technology classes, also known as Pavitt taxonomy, are a classification system used to categorize industries based on their level of technological intensity and innovation (Pavitt,1984)

impact and effectiveness of public support on product and/or process innovation, number of patents etc. Studies that consider firm performance more broadly evaluate effects on variables such as productivity, economic growth, or social welfare.

Even though knowledge of the effects on firm performance seems very important for evaluating government support policies, most of the empirical literature studies whether government support encourages or discourages private R&D expenditure and the magnitude of the impact. Most literature concludes that support leads to additionality but of different magnitudes. Studies of R&D output additionality and outcome variables are obstructed by the unavailability of relevant data and the difficulty of controlling for unobserved effects. For instance, it is hard to identify the pure effect of the policy. Naturally, and ideally, public support for R&D is used to achieve medium-term to long-term development projects, which will be influenced by factors other than policy itself. The effect can also be different in the short run and long run (Rao, 2016). Also, there may be unintended consequences, which are unknown at the time when a policy is implemented.

Empirical studies in this field should always consider the potential selection bias seriously, regardless of which type of support firms receive. The fundamental problem is that public support is not distributed randomly so recipients of public support are likely to be systematically different from non-recipients. For instance, recipients might be larger, belong to more technology-advanced industries, be located in wealthier regions, etc. These factors could indicate that recipients would have performed differently than the non-recipients, even if they had not received support (Czarnitzki et al., 2011). The use of public support thus should be considered as an endogenous variable.

David et al. (2000) investigate the pre-2000 literature that explores the sign and size of the impact of R&D subsidies on R&D expenditure. Studies are grouped and compared at four levels: laboratory, firm, industry, and country. They deliberately avoid drawing a definitive conclusion regarding the sign and size of R&D subsidy impact because most pre-2000 literature can be criticized on the grounds that the potential endogeneity of public funds is ignored. Consequently, estimated effects are likely to be inconsistent and thus they suggest that firm heterogeneity and selection bias should be seriously considered in both model construction and estimation phases. Therefore, this study will mainly focus on the literature before 2000.

There are generally four approaches to address the problem of selection bias in literature, namely, selection models, instrumental variable (IV), difference-in-difference (DID), and non-parametric matching. Regression discontinuity design is also used in recent studies. Petrin (2018) argues different estimation strategies, specification of equations, measurement of variables will significantly affect estimated results and makes comparing estimates across studies more difficult. Below, the discussion of the literature is organised by the estimation strategy employed.

1.2.4.1 Instrumental variables

Instrumental variables is an approach to control potential endogeneity (Arellano and Bover, 1995). In this context, a valid IV has a non-trivial effect upon the probability of receiving public support but no direct effect upon the outcome variable as well as being independent of unobserved determinants of the dependent variable. By utilizing IV to control the problem of endogeneity, Ali-Yrkkö (2005); Koga (2005); Clausen (2009); Harris et al. (2009); Mulkay and Mairesse (2013); Kasahara et al. (2014); Huang (2014); Fowkes et al. (2015); Chen and Yang (2019) find public R&D support has a positive effect on private R&D expenditure, and Lee (2011) finds mixed results.

Lee (2011) uses cross-sectional firm level World Bank data (including Canada, Japan, Korea, Taiwan, India, and China) for the year 1997 and concludes that public R&D support crowds in private R&D expenditure for firms with low technological competence in high-tech industries and intense market competition. However, crowd out effect is found for fast growing firms with high technological competence.

Harris et al. (2009) study the impact of R&D expenditure on output using Northern Ireland manufacturing sector data. The plant-level sample is constructed using BERD and ARD data for the period 1998-2003 [2]. GMM panel estimator is employed, and endogeneity is controlled by lagged variables. They find that output is positively and significantly affected by R&D stock. They also investigate effect from R&D tax credit. Using the same methodology and simulating a higher tax credit rate, they conclude a significant effect on productivity could be observed if tax credit become much generous.

Fowkes et al. (2015) evaluate the cost effectiveness of the tax credit in increasing the R&D expenditure through its impact on the user cost of R&D and re-estimating the price elasticity of R&D expenditure with 2002-2012 UK data. Cost effectiveness is defined as how much R&D investment changes by the private firms in response to the change in their user cost of capital for R&D investment, which is the price elasticity of R&D investment. An econometric equation, including R&D expenditure as the dependent variable and user cost and other firm characteristics as independent variables, is estimated using a dynamic GMM estimator. Potential endogeneity is controlled by lagged regressors. The estimated additional ratio shows that £1.53 and £2.35 R&D expenditure is stimulated by £1 of tax forgone for UK SMEs and large firms respectively. The elasticity of R&D expenditure with respect to the user cost of capital is -1.96. The negative sign confirms that private firms increase their R&D expenditure as cost of R&D falls. However, along with the potential flaws of the A-B GMM estimator that used in this paper, the additional ratios might be overestimated due to the following reasons. Firstly, according to the latest realise from ONS (2022, p 4), *"...HMRC R&D statistics have historically been higher than BERD statistics and have seen a larger rate of growth in recent years. This has resulted in the HMRC R&D statistics for the financial year 2020 to 2021 being £11.2 billion (42%) higher than the BERD estimate of £26.9 billion*

[2] The Business Enterprise Research and Development (BERD) is a database that includes information on R&D performed by UK businesses. Annual Respondents Database (ARD) contains information on firm characteristics, financial information, activities of businesses, and firm-level measure of gross value added in the UK.

for the calendar year 2020... ". This is because financial industries and overseas R&D expenditure are allowed to claim R&D tax credit but excluded from BERD, which only covers R&D activities that are carried in the UK. As a results, there are non-BERD R&D expenditures that used to claim tax credits and included in the HMRC R&D statistics. Secondly, businesses have incentive to 'relabel' their expenditure as R&D expenses for the maximum claim (Bloom et al., 2019). Indeed, HMRC (2022) reports the growing rate of R&D error and fraud that used to claim R&D tax credits [3].

1.2.4.2 Selection model

Since the allocation of public R&D support is not random, selection model based on the Heckit technique (i.e. Heckman correction, see Heckman, 1976, 1979) can be used to correct bias from non-randomly selected samples. The procedure first estimates a selection equation and then an outcome equation. In the context of estimating effect of public R&D support, a probit model is estimated firstly to explain the determinants of participation with a binary dependent variable of public R&D support. Then, an outcome equation is estimated by least squares with an additional explanatory variable called 'inverse Mills ratio (IMR)'. IMR is generated from first step's probit model to control selection bias. Hussinger (2008) and Dumont (2013) find public R&D subsidy has a crowd-in effect on private R&D using two-step selection model.

Ugur et al. (2015) also use selection model to study the effect of UK and EU subsidies on private R&D intensity. Their sample is constructed using Business Enterprise Research and Development (BERD), UK Innovation Survey (UKIS), and Annual Business Survey (ABS). The sampling frame contains more than 22,000 UK firms for the period 1998-2012. They find that both UK and EU funding agencies prefer growing firms and R&D intensive firms. Both funding agencies are

[3] According to the latest HMRC annual report, the estimate rate of error and fraud involved in R&D tax credit during 2021-2022 is 7.3% for SME, 1.1% for RDEC, and 4.9% for total R&D tax relief expenditure. These figures are 5.5%, 0.9%, and 3.6% correspondingly during 2020-2021

more likely to award firms at both ends in terms of size. This might be because funding agencies are likely to incubate small size of SMEs, whereas larger firms at the other end have more ability to successfully complete R&D projects. Start-up firms, measured by a dummy of new entrant, are more likely to receive UK subsidy. To evaluate the treatment effect from the UK and EU subsidies, Ugur et al. (2015) compare OLS and fixed-effect with no correction for selection bias to selection-corrected models and conclude no additionality effect on private R&D intensity from UK subsidy. However, such effect is positive and significant for EU subsidy.Even though the problem of endogeneity is be controlled in this study, their estimation could be inconsistent because both selection and outcome equations require correct specifications. This could be a result of limited information on funding regimes and data availability on firm characteristics as they argue *"... there is no commonly-agreed set of covariates that should enter the behavioural equation for the funding agency... "*(Ugur et al., 2015, p 12).

1.2.4.3 Difference-in-Difference

Difference-in-Differences (DID) is a popular approach used to evaluate the effect of public R&D support in international studies. DID estimator assumes common trends of variables on both treated and control group prior to the treatment. Studies using DID often reject crowd out effect on private R&D expenditure (Lach, 2002; Kaiser, 2006; Hægeland and Møen, 2007; Klette and Møen, 2012; Karhunen and Huovari, 2015; Guceri and Liu, 2019; Yang et al., 2020).

Karhunen and Huovari (2015) evaluate the effect of R&D subsidy on labour productivity using Finnish SME firms level data from 2000 to 2012. Using DID estimator, they find that R&D subsidies have a negative impact on labour productivity after controlling potential time-invariant effects. Such negative impact continues up to five years. They argue that negative impact on labour productivity is reasonable because benefit of R&D is lagged while recruiting new employees happened immediately.

Guceri and Liu (2019) study the effectiveness of the UK R&D tax incentives using firm level data in the period of 2002-2011. The DID estimator is used to control potential endogeneity to study the impact of the change in the UK R&D tax regime. A panel sample with 30,056 firm-year observations is constructed from HMRC UK corporation tax assessments, HMRC R&D spending, and FAME annual company accounts data. They find that R&D spending is increased 33% on average in the "medium-sized firm" due to 21% deduction in the R&D user cost, which is the results of the change definition of SME in 2008. The estimated price elasticity of R&D expenditure with respect to user cost of capital based on new definition of SME is about -1.53. This means that a one percent reduction in the price of R&D results in an increase of 1.53 percent R&D expenditure and about additional £1 R&D expenditure for every pound foregone in corporation tax revenue. Their estimates are in line with the findings in Fowkes et al. (2015). Therefore, Guceri and Liu (2019) confirm that more generous indirect public support for R&D strongly encourages private R&D expenditure, and even larger effects on consistent R&D investors and young firms after the policy reform.

1.2.4.4 Non-parametric matching

The common trend assumption is hard to meet and cannot be tested statistically, and evaluation using DID generally requires a policy change. Görg and Strobl (2007) argue that unobserved individual heterogeneity that affects participation decision cannot be controlled by DID and result in an overestimated effect of R&D policy. Non-parametric method of propensity score matching (PSM), on the other hand, restricts the difference between treated and control group only on public R&D support, if matching assumptions are met. Vast literature found positive effect on R&D inputs and outputs using different matching algorithms (Reikowski et al., 2010; Czarnitzki et al., 2011; Duguet, 2012; Czarnitzki and Lopes-Bento, 2014; Radicic and Pugh, 2015; Czarnitzki and Delanote, 2015; Becker, 2015; Ratinger et al., 2020; Barajas et al., 2021).

Becker (2015) analyses the effectiveness of public support using Eurostat's Community Innovation Survey for the period of 2006 to 2008. PSM estimator and post-regression adjustment are used to control potential endogeneity and to evaluate the impact on labour productivity, turnover, and employment. Labour productivity is measured by turnover per employee. The unbalanced panel sample is used as cross-sectional data. After controlling country heterogeneity with nearest neighbour PSM, the author finds ambiguous results. The effects on turnover and employment are negative and effect on labour productivity is positive, although none is statistically significant. Further investigating the ambiguous results, Becker (2015) uses weighted OLS and finds significant coefficients.

Ratinger et al. (2020)shows estimated treatment effects by R&D public support differs by firm size. They use a sample of 375 Czech firms covering period 2009 to 2016. Programme effects are measured by firm revenue, gross value added, labour productivity, and capital return. Generalized PSM is used because due to continues treatment variable. Unlike previous literature, a Dose-Response function is estimated to show how different level of R&D support affect outcome variables. They find impact on small firms' profit, value-added, and productivity are negative unless the program fund exceeds certain threshold. Impact of R&D support on medium and large firms are positive and negative, respectively. Different economic performance might be explained by distinct R&D strategy in each size category. For instance, projects with faster return on capital are more likely to be preferred by small firms due to their pressure to survive. Large firms might focus on long-term projects in their R&D strategy which require longer time to mature.

1.2.4.5 Combination of DID and PSM

Both DID and PSM on their own require strong assumptions. Combining the two methods could overcome such shortcomings (Görg and Strobl, 2007). Studies combining these methods also show positive effect on private R&D (Görg and Strobl, 2007; Albis et al., 2021).

Albis et al. (2021) study the impact of public R&D support on private R&D investment and firm productivity for Colombia manufacturing and services companies using firm level panel data for the period 2008 to 2014. PSM is used to construct a valid common support in the first stage. Then, DID is employed to evaluate the impact on productivity and R&D intensity. They find significant and positive effects on R&D intensity which confirms positive input additionality for firms in both manufacturing and services. Evidence also shows that productivity is significantly increased for the recipients in manufacturing sector.

1.2.4.6 Regression discontinuity

In more recent studies, regression discontinuity design is also used to determine the causal effects of receiving public R&D support. Dechezleprêtre et al. (2016) present a regression discontinuity approach to identify the causal effects of R&D tax incentives on R&D inputs and outputs using UK firm level data. In 2008, the UK R&D tax reform expanded the definition of SMEs which allowed some firms to be re-categorized as SMEs and hence qualify more generous deductions after the reform. Differences between innovation behaviour before and after the reform can be evaluated by regression discontinuity approach. Three data sources are used to construct their sample: patent data is extracted from PATSTAT; HMRC data provides information about R&D tax incentives and expenditures; and firm characteristics such as employment and total assets are taken from FAME. They find that R&D tax credit has a significant positive impact on patents, R&D, and spillovers. Further, financial constraint is also taken into consideration. Financial constraint is measured at industry level using average cash holding to capital ratio. Evidence shows that R&D tax credit has larger effects on innovation in more financially constrained firms.

Santoleri et al. (2022) also evaluate the effect of European R&D grants on SMEs using a regression discontinuity design. They construct a panel sample by linking data for all grant applicants to ORBIS firm-level data from 2014 to 2017. They find impacts on young SMEs are positive and significant on both R&D inputs and outputs including investment, patents, firm growth, revenue and employment, and survival chances. Regarding financial constraints, they obtain similar results to Dechezleprêtre et al. (2016): R&D grants are more beneficial for firms facing financial constraints. Dechezleprêtre et al. (2016) argue this is because financial constrained firms are more responsive to public support. Santoleri et al. (2022) explains such positive impact is from funding mechanism of public support, because extra funding reduces uncertainties in market and technological development, and increase the possibility of future external investments.

What emerges from post-2000 studies using firm level data from different countries with respect to different public R&D policies is the robust evidence of positive additionality after controlling for endogeneity of participation. However, there is no clear-cut evidence of the policy effect on the productivity in the firm level data. This study contributes to the existing literature by investigating the effect of public support for R&D on productivity for the UK firms using data from the UK Innovation Survey (UKIS). This dataset is unique in that it allows us to analyse and compare the effects of a range of different types of support. Moreover, we apply non-parametric matching to estimate the treatment effect, which has not previously been used in the UK context.

1.3 Methodology

1.3.1 Participation in public R&D support programmes

As discussed in the previous section, participation in public support for R&D depends on firm characteristics. The process through which firms are awarded public support depends on the programme under which the support is provided. For instance, Smart (previously known as Grant for R&D) is a form of direct support that is made available to micro-firms and SMEs on a competitive basis.

The decision that public agencies make on which firm wins depends on the extent to which applications fulfil certain criteria. This can therefore be modelled as a latent variable. On the contrary, some forms of support are automatic (e.g., R&D tax credits) if firms meet some criteria so no decision is required by a public agency. In this case, we may consider the profitability to the firm of applying for support as a latent variable.

However, in practice, there are two stages in firms' participation in R&D public supports. In the first stage, firms decide to apply supports. And, in the second stage, public agencies make decisions on firms' applications (Silva et al., 2017). Using a single probit model to study the determinants of public support participation assumes that firm's decision to participate is consistent with its application outcome. Due to data availability, researchers are unlikely to access to data which has the information about firm's decision. Instead, the outcome of public support application is generally observed. Therefore, the two-dimension participation collapse into a single dimension, i.e., firm received a public support for R&D or did not receive any support due to application being accepted or rejected.

Therefore, we assume that there will be a latent variable, y^*_{it} , that determines whether a firm receives R&D support. For firm i at time t, the latent variable can be expressed by:

$$y^*_{it} = \boldsymbol{X}'_{it}\beta + \varepsilon_{it} = \beta_0 + \beta_1 x_{1it} + \cdots + \beta_k x_{kit} + \varepsilon_{it} \tag{1.1}$$

where X'_{it} is a $(K \times 1)$ vector of explanatory variables (including time, region, industry indicators), β is a coefficient vector, and ε_{it} is an error term and ε_{it} follows normal distribution, i.e. $\varepsilon_{it} \sim N(0, \sigma^2)$.

However, the latent variable cannot be directly observed. A firm can only have one observed status: participation or non-participation in the programme. Therefore, we define participation status y_{it} as 1 if the firm participates in the programme and

0 otherwise with the following equation:

$$y_{it} = \begin{cases} 1 \text{ if } y_{it}^* > 0 \\ 0 \text{ if } y_{it}^* \leq 0 \end{cases}$$

Firm i will participate in the programme $y_{it} = 1$ if the criteria y_{it}^* meets the condition and will otherwise not participate. Therefore, we have a binary choice model. Here, a probit model is used and estimated by maximum likelihood. The estimated probability of participation is given by:

$$\Pr\left(y_{it} = 1 \mid \boldsymbol{X}_{it}\right) = \Phi\left(\boldsymbol{X}_{it}'\beta\right) \tag{1.2}$$

where $\Phi(\cdot)$ is the standard normal cumulative density function of the error term. Equation 1.2 can also be written as:

$$\Pr\left(y_{it} = 1 \mid \boldsymbol{X}_{it}\right) = \Phi\left(\beta_0 + \beta_1 x_{1it} + \cdots + \beta_k x_{kit} + \varepsilon_{it}\right) \tag{1.3}$$

That is, the probability that outcome variable y_{it} equals 1 is a linear function of of the explanatory variables. For the probit model, the estimated coefficients directly do not have an intuitive interpretation. To interpret the results of the probit model, marginal effects are more informative. These show how the probability that the outcome variable y_{it} equals 1 changes when the value of an explanatory variable changes, holding all other explanatory variables constant. The marginal effect for the *kth regressor* can be calculated by:

$$\frac{\partial \Pr\left(y_{it} = 1 \mid \boldsymbol{X}_{it}\right)}{\partial x_{kit}} = \beta_k \phi\left(\boldsymbol{X}_{it}'\beta\right) \tag{1.4}$$

Since the marginal effect depends on the covariates, the average marginal effect is used in this study for policy analysis. A consistent estimator of the average marginal effect when the independent variable is continuous is:

$$AME_{\text{continuous}} = \widehat{\beta_k}\left[N^{-1}\sum_{i=1}^{N}\frac{\partial \Pr\left(y_{it} = 1 \mid \boldsymbol{X}_{it}\right)}{\partial x_{kit}}\right] \tag{1.5}$$

When the independent variable is binary, the average marginal effect can be estimated as:

$$AME_{\text{binary}} = N^{-1}\sum_{i=1}^{N}\left[\Pr\left(y_{it} = 1 \mid x_{kit} = 1\right) - \Pr\left(y_{it} = 1 \mid x_{kit} = 0\right)\right] \tag{1.6}$$

Since UKIS is survey data that is stratified on exogenous variables, the sampling weight is applied when calculating average marginal effects. The sum of sample weights is:

$$W = \sum_{i=1}^{N} w_{it} \tag{1.7}$$

The estimated marginal effects can be rewritten as:

$$AME_{\text{continuous}} = \widehat{\beta_k} \left[W^{-1} \sum_{i=1}^{N} w_{it} \frac{\partial \Pr(y_{it} = 1 \mid \boldsymbol{X}_{it})}{\partial x_{kit}} \right] \tag{1.8}$$

$$AME_{\text{binary}} = W^{-1} \sum_{i=1}^{N} w_{it} \left[\Pr(y_{it} = 1 \mid x_{kit} = 1) - \Pr(y_{it} = 1 \mid x_{kit} = 0) \right] \tag{1.9}$$

The independent variables $\boldsymbol{X}_{it}$ are measures of receiving public support, which will be defined in the following section. Models for each form of public support are estimated using the full sample and four subsamples: high-tech, low-tech, large firms, and SMEs.

1.3.2 Effects of public support on productivity

Given the potential selection bias problem, we investigate how labour productivity and productivity growth is affected when a firm received public support using matching techniques. The basic principle of matching is to find an observation in the control group with the same or similar characteristics for each observation in the treatment group. Then, these observations are used to compute the counterfactual outcome without treatment for treated observations. The average treatment effect on the treated can then be estimated as the mean difference in the outcome variable between the treatment group and 'imputed' counterfactuals.

The main drawback of the matching estimator is that its ability to give unbiased estimates depends on the conditional independence assumption: it requires that there are no unobserved factors that affect participation in a programme and the outcome variable. Therefore, we have to assume that all determinants of participation and of the outcome variable are observed.

Using the matching estimator, we can answer the question *'what value would the outcome variable take for a treated firm if it had not been treated?'* In our study, a treatment is receipt of public support for R&D. The treatment effect for a given firm can be defined as:

$$t_{it} = Y_{it}\left(T_{it} = 1\right) - Y_{it}\left(T_{it} = 0\right) \tag{1.10}$$

Where t_{it} is the treatment effects for $i = 1, 2, \ldots, n$ firms at time t. T_{it} is the treatment variable that equals one when the firm receives public support and zero otherwise at time t. Y_{it} is the outcome variable, which in our case is labour productivity or productivity growth at time t. The average treatment effect is:

$$t^{ATE} = E\left(t_{it}\right) = E\left[Y_{it}\left(T_{it} = 1\right) - Y_{it}\left(T_{it} = 0\right)\right] \tag{1.11}$$

However, for any firm that receives support, $Y_{it}\left(T_{it} = 0\right)$ cannot be observed. Therefore, it is impossible to calculate a treatment effect for any firm or the average treatment effect Since ATE is the expected casual effect of the treatment across all observations in the population, it includes effect of the treatment on businesses who were never targeted by the public support. Thus, using ATE in this study is inappropriate as it captures effects on firms for which the program was never intended. We need to restrict the analysis to the firms for which the program is actually intended. Thus, we can use the average treatment effect on the treated (ATT):

$$t^{ATT} = E\left[Y_{it}(1) \mid T_{it} = 1\right] - E\left[Y_{it}(0) \mid T_{it} = 1\right] \tag{1.12}$$

Still, the counterfactual mean $E\left[Y_{it}(0) \mid T_{it} = 1\right]$ cannot be observed. What we can observe is the expected outcome of untreated firms, $\left[Y_{it}(0) \mid T_{it} = 0\right]$. However, using the expected outcome of the outcome variable for the untreated firms as a substitute for the expected value of the output variable for treated firms, had they not been treated, leads to selection bias. Thus, the ATT can be written as:

$$t^{ATT} = E\left[Y_{it}(1) \mid T_{it} = 1\right] - E\left[Y_{it}(0) \mid T_{it} = 0\right] - (\text{ Selection Bias }) \tag{1.13}$$

where

$$\text{Selection Bias } = E\left[Y_{it}(0) \mid T_{it} = 1\right] - E\left[Y_{it}(0) \mid T_{it} = 0\right] \tag{1.14}$$

t^{ATT} can be estimated accurately if the selection bias is zero. Rubin (1977) sets out the conditional independence assumption (CIA) to address this issue. CIA assume that the participation and outcome variables are independent for firms with identical values of X_i, which is defined in the previous section. If the CIA is satisfied, the following holds:

$$E[Y_{it}(0) \mid T_{it} = 1, X_{it}] = E[Y_{it}(0) \mid T_{it} = 0, X_{it}] \tag{1.15}$$

And thus

$$t^{ATT} = E[Y_{it}(1) \mid T_{it} = 1, X_{it}] - E[Y_{it}(0) \mid T_{it} = 0, X_{it}] \tag{1.16}$$

Equation 1.15 indicates that there is no systematic difference between participants and nonparticipants firms (i.e. there is covariate balance). Thus the counterfactual outcome can be substituted by the mean of the outcome variable for an appropriately constructed group of the untreated. Thus, the formula for the matching ATT estimator is:

$$t = \frac{1}{N^{T=1}} \sum_{i=1}^{N} T_{it} Y_{it}(1) - \frac{1}{N^{T=1}} \sum_{i=1}^{N} T_{it} Y_{it}(0) \tag{1.17}$$

Additionally, each unit must have a positive probability of receiving each treatment, i.e., the overlap assumption needs to be held:

$$0 < \Pr[T_{it} = 1 \mid X_{it}] < 1$$

The overlap assumption (or matching assumption) ensures that both treated and control cases exist for each value of X_{it}. In other words, the subsample of treated overlaps with the subsample of controls. If this assumption failed to be satisfied, there could be observations with X_{it} that are all controls and observations who are all treated with different value of X_{it}. If this assumption held, there must be a control observation with a similar X_{it} can be matched to the treated we are interested in (i.e. matching estimator exists).

One could think matching observations with the same characteristics might give a perfect counterfactual. However, such exact matching becomes impractical if there are several variables considered at once.

An approach to avoid such ‘dimensionality curse’ is to use a distance metric that measures the proximity between observations in treated and control group. Then, two observations are matched if the distance of similarity is small enough. Furthermore, given a comparison group with N^C observations for the treated observation i, the weight of the j th observation in the comparison group is defined as $w(i,j)$ when making the comparison to the treated observation i, such that:

$$\sum_j w(i,j) = 1 \tag{1.18}$$

Thus, the general formula for the ATT matching estimator can be rewritten as:

$$ATT = \frac{1}{N^{T=1}} \sum_{i \in \{T=1\}} \left[Y_{it}(1) - \sum_j w(i,j) Y_{it}(0) \right] \tag{1.19}$$

How the ATT is estimated differently depends on the choice of $w(i,j)$ which in turn depends on the choice of distance calculation.

In this study, we restrict a treated firm to be matched with a control firm within the same time period. Two matching approaches, multivariate-distance matching and propensity-score matching (MDM and PSM) are used. Both MDM and PSM use their own measures of distance between observations of pre-treatment covariates. MDM measures the distance between two observations (X_{it}, X_{jt}) using the Mahalanobis Distance:

$$M_{\text{Distance}} \ (X_{it}, X_{jt}) = \sqrt{(X_{it} - X_{jt})' \, S^{-1} \, (X_{it} - X_{jt})} \tag{1.20}$$

where S^{-1} is the covariance matrix of X in the sample. If X_i, X_j have a Mahalanobis distance of 0, they must have exact the same values of covariates. The larger the Mahalanobis distance, the more different are the two observations. Therefore, if a set of comparison units can be found to have small enough Mahalanobis distance to the treated units, then a valid comparison group is constructed for estimating ATT.

Unlike MDM, PSM collapses all information into a single probability index (the propensity score) so that the distance between observations is measured as a scalar. The propensity scores are calculated as the predicted probabilities from a probit (or logit) model in which the dependent variable is the indicator of public support and the independent variables consist of a set of characteristics and dummy variables. The predicted probability will be utilized in the second stage of matching.

In this study, the probability is estimated by the estimated propensity score from a probit model, $\Pr\left(T_{it}=1 \mid X_{it}\right)=\int_{-\infty}^{X\prime_{it}\beta} \phi(z) dz$. The distance between X_{it}, X_{jt} is the scalar difference between two probabilities:

$$P_{\text{Distance }}\left(X_{it}, X_{jt}\right)=\Pr\left(\widehat{T_{it}=1} \mid X_{it}\right)-\Pr\left(\widehat{T_{jt}=1} \mid X_{jt}\right) \tag{1.21}$$

There are several matching algorithms that can be used determine the matching weight $w(i, j)$ attached to each control group observation. One-to-one nearest neighbour matching without replacement (pair matching without replacement) matches each treated to the nearest control unit. Each control unit can only be matched once. However, Jann (2017) argues that pair-matching without replacement delete observations with the same propensity score which leads to biased estimate results.

One alternative to pair matching without replacement is to allow control units to be matched to several treated observations (i.e. with replacement). This strategy eliminates "random pruning" and includes more matched controls, which could create better balance and might lead to a better estimate (Jann, 2017).

Another matching algorithm, the kernel matching, uses pre-defined kernel functions to assign different weights to control observations within an agreed matching distance to the matched treated observation. More similar observations are given larger weights than those less similar. The agreed matching distance, or the bandwidth, determines the threshold dissimilarity. There are two methods to determine the bandwidth: a pair-matching algorithm (Huber et al., 2013, 2015) and cross-validation. The former determines the bandwidth as 1.5 times the 90% quantile of the non-zero distance in pair matching with replacement. The latter determines the bandwidth by using iterations on different portions of the data to test the model's ability to predict the means of propensity score in PSM or means of covariates in MDM. Jann (2017) argues that the pair-matching algorithm might lead to rather small bandwidths.

Following Jann (2017), regression adjustment can be used to remove remaining imbalance after matching. Regression adjustment estimates a regression model in which the treatment variable is interacted with all other variables in the model. For instance, assuming a simple regression with one regressor:

$$Y_{it} = \beta_0 + \beta_1 X_{it} + t_0 T_{it} + t_1 T_{it} X_{it} + u_{it}$$

The ATT can be calculated by:

$$t^{ATT}\left(X_{it}\right) = t_0 + E\left[t_1 X_{it} \mid T_{it} = 1\right] \tag{1.22}$$

Treatment effect is estimated by the results from that regression and thus depends on the means of covariates.

Moreover, control observations are restricted in a common support range for matchings. The common support range is defined as:

$$CS \in \{\max[\min(PS \mid \mathrm{T}), \min(PS \mid \mathrm{C})], \min[\max(PS \mid \mathrm{T}), \max(PS \mid \mathrm{C})]\}$$

where PS stands for propensity score, T and C stand for treated and control respectively. Control units outside the common support range will not be matched[4].

To assess the matching quality, several aspects are considered to evaluate the likelihood that equation 2 is satisfied. Firstly, the mean bias and median bias (absolute standardised difference) in the matched sample are calculated. The bias should be small between treated and controls in the matched sample for a good matching. Secondly, the covariates in the probit model that are used to predict the propensity score should be jointly insignificant in the matched sample. This means that the pseudo R2 should be close to zero and the likelihood ratios are not statistically significant after matching. Thirdly, Rubin's B and Rubin's R are used to summarize covariate balancing (Rubin, 2001). Rubin's B presents the absolute

[4] This Leads to different observation counts in different matchings

standardized difference of the means of the propensity score in the treated and untreated. Rubin's R is the ratio of the treated to control variables of the propensity scores. Rubin (2001) suggests that the value of B should be below 25 and the value of R should lie between 0.5 and 2 for overall balance to be sufficient. Finally, the standardised difference between treated and control for each variable should be small in the matched samples. The matching results with good covariate balance statistics will be preferred.

1.4 Data

1.4.1 Firm-level data on outcome variables and characteristics

The empirical analysis is based on an unbalanced panel constructed from the UK Innovation Survey (UKIS)[5] and Business Structure Database (BSD)[6] covering the period 2012-2020. The UKIS is conducted every two years and each wave covers a three-year period. It is drawn from the Inter-Departmental Business Register (IDBR)[7] to form a stratified random sample by industry, region, and size which represents the entire population of UK firms with more than 10 employees. The BSD is the research version of IDBR which includes data on firm characteristics such as year of birth, year of death, turnover, employment, and ownership (foreign or domestic) for the population of firms.

Four waves of UKIS were used in this analysis: 2012-2014 (UKIS 2015), 2014-2016 (UKIS 2017), 2016-2018 (UKIS 2019), and 2018-2020 (UKIS 2021). The survey sampled around 30,000 UK enterprise in each wave and received 15,091 responses in UKIS 2015, 13,194 responses in UKIS 2017, 14,040 responses in UKIS 2019 and 13,598 responses in UKIS 2021[8].

[5] Department for Business, Innovation and Skills et al. (2022)

[6] Office for National Statistics (2023)

[7] IDBR is a comprehensive list of a UK businesses used by government for statistical purposes.

[8] Response rates are 51% for UKIS 2015, 43% for UKIS 2017, 45% for UKIS2019, and 43% for UKIS 2021.

We matched the annual enterprise level BSD data to UKIS using the unique firm identifier available in both datasets (IDBR reporting unit reference number). The original enterprise level BSD data did not contain reporting unit reference numbers directly. Instead, the enterprise level BSD was firstly linked to a lookup file that can match the enterprise reference to the reporting unit reference. Then, the BSD was matched to UKIS using the reporting unit reference number.

While linking the BSD to UKIS, we need to consider firm characteristics before and after the treatment since covariates (control variables) should be determined prior to treatment. This is to identify pre-treatment variables that are not affected by the treatment (i.e. public support). Since treatment variables refer to three years in the UKIS, control variables in BSD should be matched at the beginning of that three-year period or even before. Relative to the pre-treatment period, three year leads of the BSD are used to capture post-treatment effects. For instance, we took 2015 BSD data and linked it with UKIS 2012-2014, and so on[9]. This was due to the way in which the original BSD was constructed. The original BSD might be problematic because the data was actually reflecting what was happening in the previous year (or even earlier) due to the lags in updating the IDBR. In addition, the BSD did not contain any precise time stamp which reflects the date of the observations. However, it is possible that for some firms the BSD records have even longer lags, so that the value of the outcome variables which we assume to be post-treatment correspond to the times before treatment. Nevertheless, the one year lag BSD was suggested to be the best practice assumption (BEIS, 2017).

Thus, 2015, 2017, 2019, and 2021 BSD with data on turnover and employment were matched to the corresponding UKIS wave. For the pre-treatment, the BSD of each initial years of UKIS wave: 2012, 2014, 2016, and 2018 were matched to the corresponding UKIS wave. Given the lag in updating the BSD, matching data corresponding to the UKIS initial years may not affect the pre- and post- treatment

[9]For the convenience, 2015, 2017, 2019 and 2021 BSD are called end year's BSD. 2012, 2014, 2016, and 2018 BSD will be initial year's BSD.

analysis because variables would still be determined prior to the treatment. Each pre-treatment BSD included data on turnover, employment, ownership, year of birth, and year of death.

Particularly, current and previous year data of turnover and employment were used to calculate growth rate variables instead of corresponding end year of data between current wave and previous wave. For instance, the growth rate of productivity of 2019 was calculated using data from 2018 and 2019 turnover and employment instead of using data from 2017 (end year of UKIS 2017) and 2019 (end year of UKIS 2019). This is because the latter would lead to the loss of all the observations of UKIS 2015.

1.4.2 Data cleaning and subsampling

After matching BSD to UKIS, a total sample of 34,228 firms with 55,923 observations pooled over four wave was available. Micro firms with less than 10 employees and the firms with zero turnovers were dropped which reduces the observations to 50,714. Micro firms are excluded because UKIS is constructed using non-micro firms. A zero turnover could mean that this business is inactive.

Pellegrino and Savona (2017) suggest potential selection bias can be corrected by identifying the relevant sample. Firms who are not willing to innovate and those who do not engage in innovation for reasons other than obstacles described in UKIS are excluded. Businesses that have no incentives to innovate or are not engaged in any R&D activities are unlikely to receive public support. This is important because it allows us to limit our analysis to the relevant sample. We therefore include firms who answered yes to *"whether engaged in any innovation related activities"*[10]. This

[10]This variable was derived by UKIS. The value of one indicates the business engaged in at least one of the following activities: internal R&D, external R&D, acquisition of advanced machinery, acquisition of computer hardware, acquisition of computer software, acquisition of external knowledge, training for innovative activities, all forms of design, changes to product or service design, market research, changes to marketing methods, launch advertising.

step led to the loss of 26,343 observations and therefore a final sample of 24,371 observations[11].

We also construct four subsamples for small-medium and large firms, and high-tech and low-tech firms. SMEs are defined as firms with less than 250 employees; the rest are large. high-tech firms consist of medium-high tech manufacturing and high-tech knowledge intensive services. This aggregation is based on 2-digit SIC2007 (ONS, 2018). High-tech knowledge intensive services include SIC 59, 60, 61, 62, 63 and 72. Medium-high tech manufacturing includes SIC 19, 20, 21, 26, 27, 28, 29 and 30. The remaining industries will be categorized as low-tech[12].

1.4.3 Outcome variables

Two outcome variables are of interest for the post treatment. One is the labour productivity calculated by the natural log of turnover over employment. The other is the growth rate of labour productivity capturing the change in the actual level after the treatment. Both variables were calculated using end year's BSD data. Labour productivity is used because of the absence of data on the capital stock and value added.

1.4.4 Treatment variables: public supports

The treatment variables in this study are defined as public supports. Six public support variables are constructed from UKIS: four of them measuring support from

[11]There are firms who received public support without engaged in any innovation activities in our sample. In detail, 320 firms received the UK regional support; 194 firms received UK central support; 63 firms received EU support.

[12]High-tech knowledge intensive services: 59(Motion picture, video and television programme production, sound recording and music publishing activities); 60(Programming and broadcasting activities); 61(Telecommunications); 62(Computer programming, consultancy and related activities); 63(Information service activities); 72(Scientific research and development). Medium-high tech manufacturing: 19(Manufacture of coke and refined petroleum products); 20(Manufacture of chemicals and chemical products); 21(Manufacture of basic pharmaceutical products and pharmaceutical preparations); 26(Manufacture of computer, electronic and optical products); 27(Manufacture of electrical equipment); 28(Manufacture of machinery and equipment n.e.c); 29(Manufacture of motor vehicles, trailers and semi-tralers); 30(Manufacture of other transport equipment).

different funding sources (EU support, UK regional support, UK central support, and any public support) and two measure different funding types (direct support and indirect support). We define the regional public support treatment variable as equal to one if the firm received UK regional support and zero otherwise. That is, UK regional support is defined as one if the firm ticked *"UK local and regional authorities"* in the UKIS question *"During three years, which of the following levels of government did this business receive public financial support for innovation activities"*, and zero otherwise. The same procedure applies to all the treatment variables.

1.4.5 Independent variables

All the pre-treatment firm characteristics are calculated in the initial years.

The size of the firm is measured by BSD employment in natural logarithm. Age is calculated by birth year and death year from BSD. If the firm is still in business, then the age is calculated by the difference between year 2022 and its birth year. Both size and age are transformed in natural logarithms. The skill structure of workforce is measured by two variables: the share of science and engineer with degree, and share of workers with degree in other subjects.

Two binary variables are also included: an indicator identifying those who cooperated on innovation activities (Cooperate); an indicator measuring whether the business is foreign owned (based on the ultimate Foreign Ownership code).

In addition, a set of dummy variables measuring the constraints that the business faced in innovation were included. The different constraints are: economic risk, direct cost, cost of finance; availability of finance; lack of qualified personnel; lack of information on the market; lack of information on the technology; marketed dominated by established firms; uncertain demand; UK regulations; and EU regulation. Each constraint is coded to one if the firm rated it as highly important in the constraint question, and zero otherwise. For responses who does not answer this question but treated as a valid response by UKIS, missing indicator method (MIM) is used. The reason that missing data is not discarded and only estimate on the fully observed data is because such data could produce biased results as discarded observations may still have important information.

To implement MIM, a corresponding indicator is added for each constraint variables to indicate whether the observation is missing or not. If the observation is missing, the corresponding indicator equals 1, otherwise equals 0.

The summary statistics are shown in Table 1.1, Table A.2, Table A.3, and Table A.4. A list of variables and their definitions is shown in Table A.5.

Table 1.1: Summary Statistics of full Sample, 2012-2020

Category	Variable	mean	sd	min	max
Public Support	Any Public Support	0.176	0.38	0	1
	EU Support	0.03	0.171	0	1
	UK Central Support	0.114	0.318	0	1
	UK Regional Support	0.074	0.262	0	1
	Indirect Support	0.094	0.292	0	1
	Direct Support	0.037	0.189	0	1
	All Source Supports	0.008	0.087	0	1
	EU Support Only	0.009	0.093	0	1
	UK Central Support Only	0.081	0.272	0	1
	UK Regional Support Only	0.048	0.214	0	1
	EU and UK Central Support	0.009	0.096	0	1
	EU and UK Regional Support	0.004	0.067	0	1
	UK Central and Regional Support	0.014	0.117	0	1
	Indirect Support Only	0.072	0.259	0	1
	Direct Support Only	0.015	0.121	0	1
	Both Indirect and Direct Supports	0.022	0.147	0	1
Firm Characteristics	Time	2.403	1.146	1	4
	ONS 12 regions. UKIS	6.521	3.119	1	12
	Industry	7.069	3.836	1	13
	Log BSD Employment	4.414	1.528	2.303	12.03
	Log Age	3.326	0.484	1.609	4.043
	Cooperate	0.55	0.497	0	1
	Foreign Ultimate Ownership	0.153	0.36	0	1
	% Qualified Scientists and Engineers	9.16	19.644	0	100
	% Qualified Other Staff	11.181	19.855	0	100
Constraints	Economic Risk	0.124	0.33	0	1
	Direct Cost	0.137	0.343	0	1
	Financial Cost	0.121	0.326	0	1
	Finance Availability	0.126	0.332	0	1
	Qualified Personnel	0.1	0.3	0	1
	Technology Information	0.044	0.206	0	1
	Market Information	0.039	0.193	0	1
	Dominated Market	0.074	0.261	0	1
	Uncertain Demand	0.086	0.28	0	1

Table 1.1 continued from previous page

Category	Variable	mean	sd	min	max
	UK Regulation	0.092	0.289	0	1
	EU regulation	0.076	0.265	0	1
Missing Indicator	% Qualified Scientists and Engineers	0.316	0.465	0	1
	% Qualified Other Staff	0.284	0.451	0	1
	Economic Risk	0.193	0.395	0	1
	Direct Cost	0.195	0.396	0	1
	Financial Cost	0.194	0.396	0	1
	Finance Availability	0.195	0.396	0	1
	Qualified Personnel	0.193	0.395	0	1
	Technology Information	0.197	0.398	0	1
	Market Information	0.197	0.398	0	1
	Dominated Market	0.195	0.396	0	1
	Uncertain Demand	0.197	0.398	0	1
	UK Regulation	0.195	0.396	0	1
	EU regulation	0.196	0.397	0	1
Count	IDBR Unique Reporting Unit	16899			
	IDBR Unique Enterprise	16632			
	Total Observations	24371			

1.5 Results

1.5.1 Participation in public R&D support programmes

In this section, evidence on the effect of firm characteristics including obstacles to innovate on a firm's probability of receiving public support is presented. All specifications in Table A6 - A12 are estimated using a pooled probit model. We first examine the determinants of different funding sources using the full sample (Table 1.2. Complete results can be found in Table A.6). A further investigation with the same specifications using four subsamples (large firms, SMEs, high-tech, and low-tech, Table A7 - A10) will be carried out next. This allows us to explore more details and heterogeneity between different groups. Finally, the determinants of different funding types will also be investigated using the full sample and all subsamples (Table A11 - A12). Since the UKIS is a stratified sample drawn from the IDBR, weights provided by UKIS are used in all specifications in order to show effects on the whole population.

Table 1.2 shows the average marginal effect of each explanatory variable on different funding sources using the full sample of 24,371 observations.

Firms are more likely to receive UK supports in more recent waves. This might reflect increased government contributions to meet the R&D/GDP goal. The probability of receiving EU support is 1.5 percentage points less in the most recent wave, which might be the result of UK withdrawal from European Union.

Compared to the London area, businesses in all other areas have significantly higher probabilities of receiving at least one kind of public support except for the South East region. The North East (1.9 percentage points), Yorkshire and The Humber (3 percentage points), West Midlands (3.4 percentage points), South West (2.3 percentage points), Scotland (2 percentage points), and Northern Ireland (1.8 percentage points) have a higher probability of receiving EU support. This might be the result of EU support being targeted towards less developed regions.

Table 1.2: Estimated Average Marginal Effect from Probit Models for Different Sources of Public Support using Full Sample, 2012-2020

	Any Public Support	EU Support	UK Regional Support	UK Central Support
2014	-0.003	-0.007	0.006	0.001
2016	-0.016	-0.001	-0.031***	-0.007
2018	0.036**	-0.015**	0.028**	0.021**
North East	0.068***	0.019**	0.066***	0.013
North West	0.054**	0.01	0.052***	0.011
Yorkshire and The Humber	0.084***	0.030**	0.053***	0.019
East Midlands	0.109**	0.012	0.095**	0.014
West Midlands	0.069***	0.034***	0.057***	0.027
Eastern	0.034*	0.003	0.029**	0.014
South East	0.02	-0.001	0.015	0.009
South West	0.057***	0.023**	0.040***	0.016
Wales	0.129***	0.009	0.137***	-0.013
Scotland	0.102***	0.020*	0.110***	0.002
Northern Ireland	0.115***	0.018*	0.146***	-0.009
Manufacturing (C)	0.097***	0.026***	0.036	0.067**
Electricity Gas Steam and A/C (D)	0.022	0.039	-0.002	-0.009
Water Supply and Waste Management (F)	0.01	0.016	0.001	0.009
Construction (F)	0.002	0.008	0.002	-0.001
Wholesale Retail and Motor Trade (G)	-0.009	0.001	0.012	-0.021
Transportation and Storage (H)	-0.022	0.009	-0.001	-0.02
Accommodation and Catering (I)	0.008	0.016	0.033	-0.059*
Information and Communication (J)	0.098**	0.009	0.015	0.078**
Financial and Insurance (K)	-0.031	0.002	-0.041*	-0.018
Real Estate Activities (L)	-0.057	0	-0.003	-0.055*
Professional Science and Tech. (M)	0.009	0.025**	-0.008	0.004
Administrative and Support Services (N)	-0.038	0.009	-0.01	-0.039
Log BSD Employment	0.001	-0.003	-0.003	0.010***
Log Age	-0.028**	-0.006	-0.023***	-0.014*
Cooperate	0.117***	0.033***	0.057***	0.071***
Foreign Ultimate Ownership	-0.040***	-0.011*	-0.037***	-0.022*
% Qualified Scientists and Engineers	0.002***	0.000***	0.000*	0.001***
% Qualified Other Staff	0	0	0	0
Economic Risk	0.034*	0.014*	0.018	0.001
Direct Cost	0.005	-0.009	0.006	0.017*
Financial Cost	-0.018	0.001	0.006	-0.034***
Finance Availability	0.056**	0.011	0.030*	0.022*
Qualified Personnel	0.038**	0.006	0.027***	0.015
Technology Information	-0.007	0.002	0.014	-0.034*
Market Information	0.001	-0.004	0.007	0.014
Dominated Market	-0.007	-0.011	-0.004	0
Uncertain Demand	0.015	0.016	-0.026**	0.01
UK Regulation	0.005	-0.008	0.019	0.007
EU regulation	0.004	0.008	-0.004	-0.005
Chi_sq (Overall)	668.820***	187.520***	467.540***	520.300***
Chi_sq (Time)	13.830**	11.080*	70.610***	17.800***
Chi_sq (Region)	86.600***	43.420***	172.790***	17.94*
Chi_sq (Industry)	105.74***	40.540***	56.520***	169.46***
Total Observations	24371	24371	24371	24371

Note: Complete results are shown in Table A.6. Chi squared statistics refer to the joint significance of the overall model, time dummy, region dummy, and industry dummy respectively. * p<0.05; ** p<0.01;*** p<0.001.

Unsurprisingly, the coefficients on the region dummies are all significant except for South East when the dependent variable is UK regional support, suggesting that this type of support is also targeted towards deprived regions. By contrast, the coefficients are not significant when the dependent variable is UK central support.

In terms of industry effects, the Manufacturing (C) sector is likely to receive more EU, UK central, and at least one kind of support. Firms in the Information and Communication (J) sector are on average 9.8 percentage points more likely to receive at least one kind of support and 7.8 percentage points more likely to receive UK central support. EU support is more likely to be received by firms in the Professional Science and Tech (M) industry comparing with other supports. In addition, industry is not a significant predictor of UK regional support other than for Financial and Insurance (K), which implies that there might not be policy preference across different industries.

After controlling for time, region, and industry effects, the estimated effect of employment is insignificant except for the UK central support: an increase of 1% of employment significantly increases the probability of receiving UK central support by approximately 1 percentage point.

Age, as a proxy for experience, is insignificant in the EU support model. However, age has negative and significant effects on the likelihood of receiving UK support. This could mean that UK support focuses more on young and start-up firms who have less experience in R&D.

Cooperation shows consistent significant and positive effects across the four specifications. Cooperation results in a much higher propensity to receive UK regional support (5.7 percentage points) and UK central support (7.1 percentage points) compared to EU support (3.3 percentage points). Also, the probability of receiving at least one type of support is about 12 percentage points higher when a firm cooperates with external partners. This implies that public agencies have specifically favoured R&D projects which involve cooperation when awarding financial supports.

Foreign ownership also shows consistent results across the four specifications. The negative and significant results implies that foreign owned firms are less likely to receive public support. Foreign ownership is associated with a 3.7 percentage points lower probability of receiving UK regional support while only a 1.1 percentage points lower probability of receiving EU support. This might be because support policies prefer to target domestic firms than multinational firms which have already reached a competitive position. It also could be a result that foreign-owned firms conduct most of their R&D outside of the UK so have less need of support.

The share of qualified scientists and engineers show a positive and significant effect on the probability of receiving public support. However, the magnitudes of such effects among all funding source specifications are rather small. The share of other qualified support staff does not have a significant impact at all.

Regarding the constraints to innovation, cost factors affect the probability of receiving government support but in different ways. The probability of receiving at least one support is increased by about 3.4 percentage points or 5.6 percentage points if a firm suffers from high economics risk or limited availability of finance respectively. Similarly, UK central support is more likely to be awarded to firms who experience high direct innovation costs and limited finance availability. However, high financial costs leads to a 3.4 percentage points lower probability of receiving UK central support. EU support is likely to take economic risks of innovation in consideration. The probability of receiving UK regional support is affected by financial availability (3 percentage points higher).

Knowledge factors seem less relevant in determining receipt of EU support. Firms who lack qualified personnel are 2.7 percentage points more likely to receive UK regional support or 3.8 percentage points more likely to receive at least one type of support. However, if a firm has difficulty in innovating due to a lack of information on technology, it is 3.4 percentage point less likely to receive UK central support.

Market factors and regulation factors generally have insignificant effects on the likelihood of receiving public support. Firms who face uncertain demand have a 2.6 percentage points lower probability of receiving UK regional support.

Table A.7 and Table A.8 show the results from large firm subsample and SME subsample. Both large firms and SMEs are less likely to receive EU support in recent years, which is consistent with the full sample results.

Regional effects seem less significant for large firms. The results for SMEs are very similar to the full sample results. All industry indicators are insignificant in the large firm subsample. Interestingly, SMEs in Manufacturing (C), Construction (F), Accommodation and Catering (I), Information and Communication (J), Professional Science and Technology (M), and Administrative and Support Services (N) have a higher probability of receiving EU support.

The size of large firms is relevant for receipt of EU support but is still insignificant for SMEs. For both large firms and SMEs, a 1% increase in employment at the start year of UKIS wave increases by approximately 1.1-1.2 percentage points the likelihood of receiving the UK central support, which is consistent with the full sample result.

The results suggest that age is not affecting public agency's decision making for large firms. But negative and significant results for SMEs (except EU support) suggest that young and start-up SMEs are more likely to apply for UK support.

Cooperation status, foreign ownership, and qualified scientists and engineers for both large firms and SMEs are significant and similar to the full sample results.

Most constraints for innovation show similar results to those obtained from the SME sample and the full sample. Unsurprisingly, high direct costs of innovation does not have a significant impact on the likelihood of receiving public support for large firms. This might be because large firms tend to have the ability to initiate costly R&D projects. Factors that do contribute to the probability of receiving public support for large firms are financial availability for UK regional support and uncertain demand for UK central support. However, public agencies may have different criteria for awarding supports to SME. SMEs who face high economic risk are 3.5 percentage points or 1.5 percentage points more likely to receive at least one support or EU support, respectively. Financial availability show significant positive impacts on receiving at least one support (5.8 percentage points), UK regional support (3.1 percentage points), and UK central support(2.2 percentage points). In addition, SMEs who lacks qualified personnel are 3.9 percentage points and 2.9 percentage points more likely to receive at least one support or UK regional support respectively. However, high financial cost and uncertain demand for innovation goods reduces the chance that SMEs receive UK central support and UK regional support respectively.

Table A.9 and Table A.10 show the estimated results from the probit models for the high-tech sample and low-tech sample.

Interestingly, the size and age of a high-tech firm does not have a significant effect on the probability of receiving public support except for the case of EU support where size has a positive and significant effect, though the magnitude of the effect is small: larger companies may have an approximately 1 percentage point higher probability of receiving EU support. For low-tech firms, size has a positive and significant effect on the probability of receiving UK central support and age has a negative effect on receipt of UK regional support and at least one support.

Cooperation is the most important factor for both high-tech and low-tech firms in determining whether a firm receives support. Cooperating with others significantly increases the probability of receiving UK central support by 17.5 percentage points for high-tech firms and 5.6 percentage points for low-tech firms. The effects on EU and regional support are smaller but still significant. Overall, cooperating firms have a 20.5 percentage points and 10.3 percentage points higher chance of receiving at least one support for high-tech and low-tech respectively.

Foreign ownership and the share of qualified scientist and engineer have the same effects in the full sample for high-tech firms. However, foreign ownership is not a significant determinant for low-tech firms of participating in EU and UK central support.

Regarding the factors that constrain innovation, cost obstacles significantly affect the probability of receiving UK central support for firms in both high-tech and low-tech sectors. High direct costs and finance availability positively and significantly increases the probability of receiving at least one kind of public support for high-tech firms. Besides, high direct costs increase the probability of receiving UK central support significantly by 7.8 percentage points while finance availability increases the likelihood of receiving EU support by 3 percentage points. Finance availability significantly increases the probability of receiving public supports in low-tech sectors.

Knowledge obstacles have an insignificant effect on receiving public support for high-tech firms. This might be explained by high-tech new entrants generally having a sufficient knowledge base for innovation. However, knowledge obstacles constraints innovation activities for low-tech firms: public support is likely to be awarded to low-tech firms who face lack of qualified personnel.

Table A.11 and Table A.12 show results for direct and indirect UK central supports respectively.

The estimated time effects show that the probability of receiving direct support among all samples significantly decreases since wave 2014 as the magnitude of negative effects increases through waves in most cases. Indirect support shows positive results for the most recent wave. However, the coefficient on the 2018 dummy is only significant in the large firms sample, which could mean that public agencies prefer large companies for awarding indirect support or simply that large firms tend to innovate more in the most recent years.

After controlling for additional regional and industry heterogeneity, size and age show significant effects on receiving direct support in most cases but is less important in the indirect specifications. Cooperation is positive and significant in all specifications but the magnitude of the effect on indirect support is larger than that for direct support. The share of qualified scientists and engineers increases the likelihood of receiving both direct and indirect support significantly.

Innovation constraints are less important in direct support specifications compared with indirect support. The presence of high direct cost and finance availability significantly increases the firms' probability of receiving indirect support except in the large firm sample. Having difficulty covering financial costs significantly reduce the chances of being awarded indirect support. Being restricted by lacking qualified personnel is associated with a higher probability of receiving both direct and indirect support. Operating in a market that is dominated by establish firms no longer makes a significant difference, but uncertain demand increases the likelihood of receiving indirect support.

1.5.2 Effects of public support on productivity

The matching results for labour productivity and productivity growth using the full sample are shown in Table 1.3.

Table 1.3: PSM Kernel Matching with Regression Adjustment for Labour Productivity and Productivity Growth using Full Sample, 2012-2020

	Any Public Support	EU Support	Central Support	Regional Support	Indirect Support	Direct Support
Labour Productivity	-0.060**	-0.210***	-0.064**	-0.068*	-0.057*	-0.207***
Observations	24359	20817	22870	21887	22330	20947
Productivity Growth	-0.01	-0.039	-0.015	-0.031	-0.031	-0.037
Observations	24354	20813	22866	21883	22326	20944

Note: Complete results are shown in Table A.13. Observation refers to the number of observations in the full (i.e., unmatched) sample. * p<0.05; ** p<0.01;*** p<0.001.

Table A.14 and Table A.15 show the covariate balance statistics when labour productivity and productivity growth, respectively, are used as outcome variables. There is a slight difference between the number of observations used when estimating ATT on labour productivity and productivity growth using the same sample. The total number of observations in the labour productivity sample is 24,359 for any public support but the number of observations is 24,354 for labour productivity. This is because 5 observations were dropped when matching on productivity growth due to matching on common support. The mean and median bias in the matched sample are significantly lower for all forms of public support than in the full sample. The pseudo R^2 approaches zero after matching. Likelihood ratios are not statistically significant after matching. Rubin's B and Rubin's R are smaller in the matched sample. Table A.16 and Table A.17 show the standardised difference between the treated and untreated groups in the full and matched samples for each treatment variable. The significant reductions in the standardised differences in the matched sample suggest that using propensity score kernel matching with regression adjustments improves the comparability between the treated and untreated. In conclusion, the covariate balance statistics strongly show that PSM kernel with regression adjustment gives a good quality of matching for both labour productivity and productivity growth. Therefore, the results using propensity score kernel matching with regression adjustment are preferred.

The estimated ATTs in Table 1.3 suggest that public support generally has a significant negative effect on labour productivity, regardless of the funding source and type. The relationships between public support and productivity growth are negative though insignificant. Receiving at least one kind of support has a significant and negative effect on labour productivity. The estimated effect of receiving at least one type of support is to reduce labour productivity by 6.2%. The negative impact on labour productivity from EU support is greater than UK support, which is to reduce productivity by 21%. Labour productivity is estimated to fall about 6.5% as a result of receiving UK central and regional support. In terms of funding types, UK direct support shows a much larger effect than UK indirect support: the estimated decline in labour productivity due to UK direct support is about 21% whereas the reduction is 5.7% for UK indirect support. This might be because of the nature of these two type funding: firms can directly utilize grants and loans quicker than tax credits. The negative estimated impacts from public supports differs from the positive effects obtained in most literature[13].

[13] See Literature section for more details

To further investigate the negative relationship between the treatment variables and the dependent variables, the methodology is applied to four subsamples: high-tech; low-tech; large firms; and SMEs. Table 1.4 shows the summary of matching results on labour productivity for each subsample. The high-tech and SME samples show consistent negative and significant relationships between public supports and labour productivity. These estimated negative impacts are much larger than the full sample results. For instance, high-tech firms who receive UK direct support experience the largest decline in labour productivity of 49.2%. However, low-tech and large firms show different results. UK central support and indirect support have significant and positive impacts on low-tech firms' labour productivity, of 8% and 9.8% respectively. Labour productivity is negatively affected by EU and UK regional support but such effects are not statistically significant. The effect from direct support is positive but not significant. For large firms, all public support variables show statistically insignificant impacts on labour productivity. Effects from EU support and direct support are negative and the other support variables show positive influence on labour productivity. The inconsistent matching results for low-tech and large firms suggest that the effects are heterogeneous across technology and firm size.

Table 1.4: PSM Kernel Matching with Regression Adjustment for Labour Productivity (Subsamples), 2012-2020

	Any Public Support	EU Support	Central Support	Regional Support	Indirect Support	Direct Support
High-tech	-0.207***	-0.416***	-0.230***	-0.190**	-0.250***	-0.492***
Low-tech	0.016	-0.036	0.080**	-0.031	0.098**	0.023
Large Firm	0.001	0.028	-0.004	0.052	0.029	-0.159
SME	-0.098***	-0.337***	-0.153***	-0.140***	-0.102**	-0.221***

Note: Complete results are show in Table A.18. * p<0.05; ** p<0.01;*** p<0.001.

Table 1.5 presents the matching results on productivity growth for each subsample. Though effects on productivity growth remain insignificant in almost all subsamples, the high-tech subsample shows different results from the full sample. Receiving at least one public support or EU support significantly decrease productivity growth for high-tech firms. For low-tech firms, receiving at least one kind of public support seems to have a positive effect on productivity growth. EU and UK regional support has negative impacts on productivity growth as in the full sample, but UK central support has a positive relation to productivity growth. Moreover, direct support has a positive effect on productivity growth.

Table 1.5: PSM Kernel Matching with Regression Adjustment for Productivity Growth (Subsamples), 2012-2020

	Any Public Support	EU Support	Central Support	Regional Support	Indirect Support	Direct Support
High-tech	-0.065*	-0.128*	-0.051	-0.008	-0.048	-0.039
Low-tech	0.015	-0.045	0.002	-0.001	-0.006	0.033
Large Firm	0.007	-0.034	0.004	-0.02	0.009	-0.07
SME	-0.013	-0.066	-0.028	0.008	-0.009	-0.008

Note: Complete results are show in Table A.19. * p<0.05; ** p<0.01;*** p<0.001.

Matching results for large firms show positive but not significant impacts from receiving at least one public support, UK central support, or indirect support.

In conclusion, the significant negative effect on labour productivity and insignificant negative effect are mainly driven by high-tech firms and SMEs. Low-tech and large firms show different results which suggest that the effects are heterogeneous across technology and firm size.

1.5.3 Robustness checks

In this section, robustness checks of the matching results in the previous section are presented.

Firstly, we show matching results using other matching techniques as a further check of the robustness of the main results. If results obtained from using other matching algorithms show the same results obtained using PSM Kernel matching with regression adjustment, this confirms that the results in Table 1.3 are robust. Table A.20 and Table A.21 show matching results on labour productivity and productivity growth respectively using MDM and PSM with different matching algorithms. Results in Table A.20 shows consistent negative and significant effects on labour productivity regardless of funding sources and funding types among all specifications. The insignificant negative impacts on productivity growth are also consistent among all specifications, presented in Table A.21. Matching results using different algorithms are similar and support the findings in the previous section.

Secondly, we consider the potential effect of the policy mix following Czarnitzki and Lopes-Bento (2014). Firms with no support at all are used to construct the control group. We then separate the remaining firms into 10 groups by funding source: 1) firms which received UK regional support only, 2) firms which received UK central support only, 3) firms which received EU support only, 4) firms which received both UK regional and central support, 5) firms which received both UK regional and EU support, 6) firms which received UK central and EU support, and 7) firms which received all three public supports. In addition, we also separate firms into 3 groups by funding type: 8) firms which received direct support only, 9) firms which received indirect support only, 10) firms which received both indirect and direct support.

With such group separation, when matching the 1-10 groups to the control group, we can evaluate the single/mix effect of public support funding source(s) and funding type(s) on labour productivity and productivity growth so that one could investigate which support or support combination leads to the negative results. In this case, the treatment variable equals one for each group 1-10.

Analysing policy mix could distinguish and simultaneously presents effects on outcome variables by different policy settings. For instance, there are firms which only received the UK central support and firms which received both UK central support and EU support. Results from the former group show the pure effect from the UK central support whereas the latter shows the effect not only from the UK central support, but also EU support. In addition, such analysis might improve matching quality because unobserved characteristics of firms which only received one support and firms who received several supports are eliminated.

Table 1.6 presents the matching results for the funding source policy mix using PSM kernel matching with regression adjustment. Firms who received all three supports (UK central, UK regional, and EU) show statistically significant negative impacts on labour productivity, but such negative impact turns insignificant for productivity growth. Firms which received EU support only or UK regional support show negative but insignificant effects on both labour productivity and productivity growth. Firms which received both EU support and UK regional support show consistent results as receiving a single policy: the effects on both outcome variables are negative but not significant. Including UK central support into the policy mix makes results more interesting. Firms which received UK central support only show positive but insignificant effects on labour productivity and productivity growth. Once the UK central support is mixed with EU support or UK regional support, effects on labour productivity turns into negative and significant. However, such negative effects are not significant on productivity growth. Table 1.7 effects from any type are negative on both outcome variables. Firms which received direct supports show significant impact on labour productivity but not on productivity growth. Effect from indirect support is positive on labour productivity but not statistically significant.

Table 1.6: PSM Kernel Matching with Regression Adjustment for Founding Source Policy Mix using Full Sample, 2012-2020

	All Public Support	EU Support Only	Central Support Only	Regional Support Only	EU and Central	EU and Region	Central and Region
Labour Productivity	-0.261**	-0.042	0.01	-0.064*	-0.426***	-0.196	-0.163**
Productivity Growth	-0.115	-0.015	0	0.006	-0.03	-0.16	0.013

Note: Complete results are shown in Table A.22. * p<0.05; ** p<0.01;*** p<0.001.

Table 1.7: PSM Kernel Matching with Regression Adjustment for Funding Type Policy Mix using Full Sample, 2012-2020

	Indirect Support Only	Direct Support Only	Direct and Indirect
Labour Productivity	0.003	-0.143*	-0.242***
Productivity Growth	-0.016	-0.012	-0.054

Note: Complete results are shown in Table A.22. * p<0.05; ** p<0.01;*** p<0.001.

Thirdly, a sample is constructed that only includes firms that exist in the UKIS for at least two consecutive waves. The four subsamples (i.e. high-tech, low-tech, large firm, and SMEs) are also created based on the sample that only has consecutive observations. This allows to identify long term effect from public support on next period outcome variables.

Table 1.8 presents the matching results for the consecutive full sample using PSM kernel match with regression adjustment. Public supports have similar effects on labour productivity and productivity growth. Firms that receive at least one kind of public support show negative but insignificant impacts on both outcome variables. EU and UK central support are negatively associated with both outcome variables. Regional support has an insignificant positive effect on labour, and the magnitude of the effect is small. In terms of funding types, both indirect and direct support negatively affect the outcome variables but the effect from direct support is significant on labour productivity.

Table 1.8: PSM Kernel with Regression Adjustment using Consecutive Full Sample, 2012-2020

	Any Public Support	EU Support	Central Support	Regional Support	Indirect Support	Direct Support
Labour Productivity	-0.008	-0.179*	-0.023	-0.03	-0.025	-0.221***
Productivity Growth	-0.019	-0.035	-0.016	0.005	-0.005	-0.044

Note: Complete results are shown in Table A.23. * p<0.05; ** p<0.01;*** p<0.001.

Table 1.9 shows the PSM Kernel with regression adjustment matching results for labour productivity using consecutive subsamples. Public supports have negative and significant effects on labour productivity for high-tech firms and SMEs. This is consistent with the results in the previous section. For low-tech firms, the effect from public support is positive and significant except for UK regional support and direct support. This might suggest that public support generally could boost labour productivity for low-tech firms. For large firms, though all supports have insignificant impacts, public supports has a positive relationship with labour productivity except for direct support.

Table 1.10 shows matching results for productivity growth using consecutive subsamples. Public support has a consistent negative relationship with high-tech firms' labour productivity. Productivity growth of low-tech firm has positive relation to EU support and UK regional support but negative to the UK central support. This result is the opposite of the results from the low-tech non-consecutive sample in the previous section (i.e. Table 1.5). This might suggest that the longer-term effect on productivity growth from EU support and UK regional support turns positive. Effects on large firms' productivity growth are very small. For SMEs, public supports are negatively associated with productivity growth except for UK regional support.

Table 1.9: PSM Kernel with Regression Adjustments for Labour Productivity (Consecutive Subsamples), 2012-2020

	Any Public Support	EU Support	Central Support	Regional Support	Indirect Support	Direct Support
High-tech	-0.217***	-0.441***	-0.237***	-0.194*	-0.198**	-0.466***
Low-tech	0.133***	0.167*	0.134***	0.057	0.111**	0.047
Large firm	0.021	0.105	0.057	0.028	0.065	-0.144
SME	-0.061	-0.226**	-0.072	-0.064	-0.123**	-0.210**

Note: Complete results are shown in Table A.24. * $p<0.05$; ** $p<0.01$;*** $p<0.001$.

Table 1.10: PSM Kernel with Regression Adjustments for Productivity Growth (Consecutive Subsamples), 2012-2020

	Any Public Support	EU Support	Central Support	Regional Support	Indirect Support	Direct Support
High-tech	-0.073	-0.166*	-0.03	-0.006	-0.043	-0.171*
Low-tech	0	0.006	-0.004	0.023	-0.008	-0.045
Large firm	-0.023	-0.042	0.004	0.004	0.019	0.049
SME	-0.003	-0.058	-0.008	0.028	-0.02	-0.077

Note: Complete results are shown in Table A.25. * $p<0.05$; ** $p<0.01$;*** $p<0.001$.

Finally, a long term effect is estimated by matching treatment variables in the current wave to dependent variables in the next wave using the consecutive samples. Table 1.11 shows the matching results using consecutive next wave full sample. In general, firms that get at least one kind of public support experience positive but not significant impacts on next wave's labour productivity. Effects from any funding source are negative but not significant impact persist to next wave on labour productivity. Interestingly, the impact from direct support is strong enough to significantly lower labour productivity in long run.

The long term effect on productivity growth remains negative for all founding sources and funding types except for UK regional support. However, the insignificant results question long run effectiveness.

Table 1.11: PSM Kernel with regression adjustment on next wave outcomes using consecutive sample, 2012-2020

	Any Public Support	EU Support	Central Support	Regional Support	Indirect Support	Direct Support
Labour Productivity	0.002	-0.129	-0.04	-0.07	-0.028	-0.178*
Productivity Growth	-0.009	-0.013	-0.036	0.005	-0.026	-0.04

Note: Complete results are shown in Table A.26. * p<0.05; ** p<0.01;*** p<0.001.

1.6 Conclusion

The effect of public support for R&D on UK firm productivity is investigated using a sample constructed from UKIS and BSD in this study. The sample includes more than 16,000 R&D active enterprises from 2012-2020. The results are obtained using non-parametric matching to control potential endogeneity. We find that public support for R&D has a negative impact on labour productivity and the impact is insignificant for productivity growth. The significant negative effect on labour productivity and insignificant negative effect on productivity growth are mainly driven by high-tech firms and SMEs. Low-tech and large firms show different results which suggest that the effects are heterogeneous across technology and firm size.

The effectiveness of industrial policies to boost productivity should be seriously reconsidered, especially to meet the UK's 2.4% GDP R&D spending goal. Evidence in this study shows that high-tech firms and SMEs are targets of the UK R&D supports, and effects on productivity in both groups are negative and significant. It is therefore unlikely that current policy framework will boost productivity in the UK in the short run, especially since the UK economy is currently in recession. Public support for R&D might be effective if it targets certain regions and industries.

The negative impact on productivity should not come as a surprise, as similar findings have been concluded in other studies. For instance, Karhunen and Huovari (2015) and Ratinger et al. (2020) also report negative effects on productivity. Although such negative impacts on productivity may appear paradoxical, considering that governments continue to implement such policies to enhance productivity, caution is necessary when interpreting them. Firstly, it is important to note that this study does not encompass a comprehensive cost-benefit analysis. Thus, potential positive externalities like spillover effects are not captured. The productivity of other firms may increase due to these spillover effects, which are not fully accounted for. Secondly, it is crucial to consider that the impact of public support can vary depending on the short-term or long-term perspective (Rao, 2016). The benefits of research and development may have a lagged effect, while the recruitment of new employees can happen immediately (Karhunen and Huovari, 2015). Therefore, it is essential to approach these results with caution, considering the limitations of the study and the potential for different effects in various time frames.

Even though negative impact on productivity is found in this study, whether public support has an additional effect on private R&D expenditure is unknown due to lack of relevant data. Potential violation of the PSM assumptions presents a threat to the validity of the analysis.Instead of gross output, gross value added can be used in the future study.

Chapter 2

Two Country Model of R&D Investment

2.1 Introduction

This study is motivated by the growing interest and critical issues in the expansion of the digital economy.

One critical issue that tax administrators across the world face is who has taxing rights. Avi-Yonah (1997) presents this issue with proliferation of international e-commerce. He argues that communication between different parties is the fundamental requirement to facilitate e-commerce. And indeed, the ability of communication has been growing exponentially through the creation of telegraph, telephone, and Internet. Advancements in communication technology have not only enabled the rapid and accurate exchange of information, but have also created electronic payment systems. It gives the possibility that buyers and sellers can locate in different jurisdiction, which creates the issue - who has taxing rights.

Therefore, governments believe that the current international tax system is inadequate in the digital age. The digitalization of the economy presents challenges to governments and tax authorities globally. Digital technologies allow firms to provide services across the borders remotely. Such economic activities are provided without relying on the permanent establishment (PE) of the seller, (i.e. the country where the seller has significant "brick and mortar" presence). Consequently, international taxation under current tax law systems, being built upon PE, often fails to address this challenge.

OECD (2014*a*, p. 13) proposed redefining PE because "*the digital economy and its business models present some key features which are potentially relevant from a tax perspective. These features include mobility... reliance on data... network effects... the spread of multi-sided business models... a tendency toward monopoly or oligopoly... and volatility...*". Digital businesses that conclude contracts with consumers under the current tax system can take full advantage of infrastructure and rule of law institutions while they are not considered to present for tax purposes. If governments keep current tax systems and tax laws unchanged, there would be a tax revenue loss from the inability to effectively levy taxes on the digital economy.

The long-term solution is of course to seek a radical change, which is to reform the international tax system. The reform could or should include, for example, tax multinationals as single and unified firms, and working towards an equitable distribution of taxing rights. The task of achieving global equitable distribution of taxation rights is complex and full of challenges. This could be due to a multitude of reasons such as inequal economic power, imbalance of economic development, and differing, often conflicting, national interests, etc. Countries are different in economic structures, development stages, governance capabilities, which might view "equitable distribution of taxation" diversely. For instance, residence countries might perceive such distribution of taxation right as a potential threat to their economic "attractiveness", while source countries might view it as an opportunity to enhance their fiscal rights. Despite these complexities and conflicts, it is reasonable to suggest that the pursuit of equity in taxation rights can provide a strong foundation for sustainable global economic growth and development.

The reform must also be 'fit for purpose' considering increased globalisation, and the growth of the digital economy. On the one hand, these radical changes, take time to implement, while some remain under consideration.

To address such a problem in the short term, the European Commission (2018) proposed a “digital service tax” (DST) of 3% of revenues earned from services that were created through “user participation”. Cross-jurisdiction transactions with user participation can be referred to as "digital sales". Initially, these transactions primarily involved the sale of physical goods, but now it is sales of both goods and services. The use of the term "digital services tax" is due to the particular challenge in determining the tax base for services, although the same issue applies to digital sales of physical or digital goods.

The UK government also announced that a 2% tax will be levied on “search engine, social media services, and online marketplaces which derive value from UK users” from April 2020. The creation of DST primarily motivates this study.

Secondly, DST is levied on the revenue from sales, which could discourage firms to invest in R&D. The growth of the world economy has been largely driven by international trade. This creates more challenges for firms to compete with rivals overseas. The key factor for firms to be competitive in the international market, and which is the result of investing in R&D, is to be productive (Long et al., 2011). From a government’s perspective, innovation plays an important role to boost productivity, economic growth, employment, and social welfare (Siebert, 2019). There will be underinvestment in innovation if the society relies on the market process alone. The creation of DST might make this situation worse because it reduces the firm’s profitability. Thus, the DST is sometimes opposed as detrimental to innovation in digital technologies. Therefore, we need to understand how, in terms of production and innovation decisions, DST will shape a firm’s behaviour .

Thirdly, to support the digital firms facing DST, and to encourage investment in innovation, governments have R&D policies to subsidise the cost of R&D. These policies aim at making domestic businesses more competitive internationally and improving home welfare. Governments, while introducing DST, also subsidize firms to invest in R&D. There are several reasons that the government positions R&D policies as the centre of industry strategy. Firstly, R&D can benefit both firms and consumers by improving product quality or reducing the cost of production. It could increase the consumer surplus and firm's profit margin, and further improve social welfare. Secondly, Haaland and Kind (2006) argue that the typical characteristics of R&D, such as positive externalities and public goods aspect, can reduce market efficiency and discourage firms to invest in R&D. Thirdly, R&D policies are one of the few tools that a government can use to influence industrial activities. For instance, trade and industrial policies, such as export subsidies, are often precluded in the international agreement for fair trade, but R&D policies are not. R&D policies can be used as a strategic tool to advance a domestic industry to maintain competitiveness in the international market. Therefore, we need also to understand how the DST will interact with R&D policies, and the overall effect on welfare.

An important issue is a potential trade-off between tax revenue and the subsidies that encourage firms to invest in R&D. It is unclear that a comprehensive international agreement would improve both domestic and international welfare. The analysis of the economic foundations of the transformation of international tax law in the areas potentially adversely affected by the development of digital technologies, such as transnational cloud computing and data storage services will form part of my doctoral research. We aim to develop a theoretical framework that allows quantitative investigation of various international tax law reforms proposed by legal scholars.

Current trading and R&D literature overlooks the importance the digital economy, and thus ignore its unique characteristics. Most literature focuses either on trade liberalization or interactions between different types of R&D in the traditional economy. (Lin and Saggi, 2002; Symeonidis, 2003; Haaland and Kind, 2006, 2008; Leahy and Neary, 2001; Long et al., 2011; Ishii, 2013; Pires, 2015; Yang, 2018) In this paper, we aim to develop a model of taxation for multinational businesses operating in a competitive international digital economy. This work will contribute to the understanding of how to design an international tax law system that would deliver efficient and fair outcomes in the modern global economy increasingly ruled by digital technologies. The model includes important features of the digital economy, such as the network externality in consumption, the significant market power of the providers, and the role of digital technology innovation. In particular, the focus is on the investment in innovation of two types: (i) innovation that reduces production cost, or process innovation, and (ii) innovation that improves the quality of the good and thus boosts the consumer demand, or product innovation.

The model is built upon several strands of literature. The idea of modelling process R&D is from Haaland and Kind (2008), who argue that marginal production cost is reduced by the effort that the firm invests in process R&D. However, they focus on the effect of trade liberalization on a firm's behaviour in output and R&D. For example, we do not consider the effect of lowering trade costs in this paper. Instead, we use sales tax on the imported good to study the effect of DST. We also study the effect on product R&D. To model network externalities in consumption, we follow the concepts introduced by Leibenstein (1950) and Grilo et al. (2001) . However, we do not use the Hotelling model of spatial competition that was used in Grilo et al. (2001). The function of network externalities in consumption is completely new with similar assumptions to Grilo et al. (2001). The new feature of the model in this paper allows calculating the optimal tax and subsidy policies in the presence of network externalities in consumption, and the focus on tax revenue.

The model will be used to answer the following research questions.

- How does the DST affect the decision of the firms' behaviour, the price and quantity of the output, and investment in innovation?

- What is the welfare effect of the strategic interaction between governments in setting the R&D policies and DST in the presence of network externalities in the markets?

2.2 Literature Review

Firms' R&D investment has been an important objective of industrial strategy. There is a large literature that has highlighted the welfare consequences of process and product R&D investment.

The focus of the earlier literature has been primarily on the process of R&D,i.e. investment in technology that helps to reduce production costs. More recently, the literature on R&D has started to centre on product R&D, i.e. investment in the improvement of product quality, and its link with the process of R&D.

2.2.1 Process R&D

Brander and Spencer (1983) presented a pioneer theoretical model for process R&D and subsidy policies in the setting of an imperfect competitive international market. They argue that subsidize R&D is an important instrument for the government to keep domestic firms competitive in the international market. The incentive for the government to subsidize private firms' R&D is not because of positive externalities, but the large domestic share of international profit the firm can obtain.

In their model, two firms, domestic and foreign, located in two countries compete in a two-stage setting. It is a duopoly game with a Stackelberg-Cournot setting. Brander and Spencer (1983) assume that all outputs are for export to other countries. Domestic consumption is not considered in this model, even though it would encourage the government to subsidize private firms since output might increase with decreasing prices. The objective for the domestic government is to maximize the domestic firm's profit net the cost of R&D subsidy/taxation.

In the first stage, the government subsidizes private firms with the assumption that government can credibly commit itself to subsidizing the R&D process before R&D decisions are made by private firms. The equilibrium outcome of the multi-stage game is built upon this assumption. Brander and Spencer (1983) argue that such government behaviour that acts as a leadership role is natural in the real world owing to two reasons. One is bureaucratic sluggishness. This simply means that government is inflexible to changing a policy once it has been set. They argue that this characteristic of government is tied closely to R&D subsidy policies because the R&D phase is presumably relatively short. Thus, the subsidy period might not have to be maintained for very long. The second reason is that government needs to maintain its reputation for future policies.

In the second stage, firms choose the quantity of output. By taking the R&D decision as given, the output function can be written as a function of the R&D decision. Thus, the firm's profit is a function of the R&D decision, and the optimal solution is described by a Nash equilibrium. Thus, the solution to the two-stage game is a sub-game perfect equilibrium. This setting can also be extended into a three-stage game where there is a leading government makes R&D decisions first, with other governments taking this decision as given to make subsequent decision about R&D. The model is solved by backward induction.

They conclude that under the assumptions made, the domestic government can subsidize domestic private firms to maintain competitiveness in the international market, and thus increase domestic welfare. This result is contrary to findings in the previous international trade literature, where government intervention is socially suboptimal. The result also suggests that the optimal policy would be, if the government can tax or subsidize both R&D and export, to tax R&D to counteract the overused R&D investments by firms. In this case, if the export subsidies are allowed, the optimal policy is to subsidize R&D.

Leahy and Neary (2001) study international policy coordination in a model with process R&D and spill-overs between firms. The objective for government is to maximize social welfare. The result is ambiguous as government can either over-subsidize or under-subsidize investments, which depends on the degree of spill overs between firms. They conclude that R&D might be a more robust tool than export subsidies. In addition, the prisoner dilemma solution suggests that both countries might be worse off by over-subsidizing their domestic firm.

Haaland and Kind (2006) study the implications of cooperation and non-cooperation of government R&D policies. Building upon what they find, Haaland and Kind(2008) study international competition between investments in process R&D and import tariffs. They develop a two-firm two-county model with trade costs. Two firms can invest in process R&D to reduce the marginal production costs of horizontally differentiated goods. Having trade cost in this model is motivated by the assumption that firms would like to increase the profit margin. Thus, they tend to invest more in cost-reducing R&D with the expansion in the market size. Consequently, the domestic price will fall, and consumer surplus increases. The government, therefore, has incentives to subsidize R&D. Haaland and Kind (2008) defines such an incentive as a consumer surplus motive. Both Brander and Spencer (1983) and Leahy and Neary (2001) have the third pure import country in their model to study the consumer surplus effect. Both conclude that excessive R&D is expected if there is policy competition.

In addition, there is a strategy motive for the government to subsidize the domestic firm when goods in the international market are close substitutes. Haaland and Kind (2008) argue the strategy motive has a business stealing effect which may lead to policy competition between governments. The term 'business stealing' refers to a strategic economic manoeuvre where the government influences market competitiveness indirectly through subsidizing R&D. Unlike pure export subsidies which might contravene international trade regulations, R&D subsidies can reduce the risks and costs of investing in cost-efficient technologies and higher-quality products. These innovations enable domestic firms to be more competitive and thus 'steal' market share from international competitors without contravening any regulations[1].The trend of studying the business stealing motive to subsidize R&D begins with Brander and Spencer (1983). The main reason is that export subsidy is

[1]Export subsidies are not allowed under the WTO rules, although some very few exceptions can apply. Please see `https://www.wto.org/english/docs_e/legal_e/24-scm.pdf`

regulated by international trade agreements.

The model Haaland and Kind (2008) assumed there are two firms located in two countries with the same market size (normalised to 1). The positive trade costs occurred when the domestic country exported goods to the foreign country. Such trade costs do not include any forms of the tariff, but barriers such as transport costs and different product standards . Haaland and Kind (2008) develop a two-stage game where the government chooses optimal R&D pol icy at the first stage, and firms make R&D investment decisions and output decisions in the second stage. Similar to Brander and Spencer (1983) model, the outcome of this two-stage game is sub-optimal. They conclude that trade liberalization in terms of lowering trade costs might encourage more R&D and may increase the firm's competitiveness in both the domestic and international markets. The policy competition between countries significantly depends on market competitiveness. However, unlike previous findings, policy competition does not always encourage R&D policies. The key determining factor is the degree of product differentiation.

Long et al. (2011) also study the relationships between trade liberalization and R&D incentives of private firms. They further link them with industry productivity. The novel contribution this paper has made is that it considers both the short run, where there is no entry for other firms, and the long run, where there is free entry. Even though Long et al. (2011) show that the outcome has no significant difference between the short run and long run, their results are the opposite of models that consider homogeneous firms in the innovation and trade literature.

They develop a two-firm reciprocal dumping model located in two markets. The trade barrier is defined the same as Haaland and Kind (2008). Tariffs are not considered in this model. Firstly, they consider a benchmark model that two firms are homogeneous and compete in the Cournot fashion. Long et al. (2011) emphasise that the assumption of homogeneity refers to all firms having the same marginal cost function and the market having complete information. The game has two stages. Firms need to decide on entry or exit in the first stage. Then investment in R&D

and quantity of outputs will be solved in the second stage. The results are consistent with other literature in this case. Trade liberalization encourages firms to invest in process R&D and profit margin increases, in both the short run and long run. Specifically, trade liberalization raises the overall market competitiveness. Though increase in competitiveness and increase in profit margin seems contradictory, Long et al. (2011) explain such effects can coexist due to the expansion of firm output when it participates in the international market. The expansion of firm output and rise in process R&D expenditure both allow higher profits, which induce more entries. The investment in R&D introduces higher productivity by lowing marginal cost of production, both at the firm and industry level. Long et al. (2011) refer such complexity as the direct effect of trade liberalization.

Secondly, the homogeneous firms are replaced by two heterogeneous firms and two cases are considered: no entry in the short run; and free entry in the long run. Long et al. (2011) conclude that if trade cost is low, trade liberalization in the short run raises investments in process R&D, but if trade cost is high they discourage investment in process R&D. This is the opposite of the homogeneous case where the relationship between trade liberalization and investment in process R&D is monotonic. Furthermore, they argue there is a selection effect at the industry level. Since firms are heterogeneous, firms with the least efficiency and highest marginal cost are likely to choose to exit the market. Thus, the overall industry productivity might rise. In the long run, a highly competitive export market requires the firm to increase output to keep zero profit. The number of firms operating in the market might fall due to the selection effect. They explain this is because the risk/cost of facing import competition is higher than the benefit from better access to the export market.

Chang et al. (2013) include technology licensing in the competitive model to study its effect on incentives of R&D. In addition to the common conclusion from other literature about process R&D, and how it can increase the firm's competitiveness, they also argue that firms who invest in process R&D could benefit from being a licensor.

The model setting is different from other literature. There is one competitive market that has only one firm that invests in process R&D and could be a licensor. The rest of the n firms are assumed to be homogeneous, which means all firms have the same production function, marginal cost function, and produce identical goods. Chang et al. (2013) assume that the licensor firm can make benefit through a two-part tariff contract. This includes a fixed one-time payment, and a royalty rate. The total number of n + 1 firms compete in Cournot fashion and the game is three-stage. In the first stage, the licensor firm makes the process R&D investment decision. In the second stage, licensee firms sign the two-part tariff to gain access to the technology the licensor invented. In the third stage, all firms including the licensor compete in Cournot fashion.

Chang et al. (2013) conclude that in contrast to the traditional view that a firm licensing out its technology might have a positive effect on R&D. Firms may invest less in R&D as a licensor if R&D efficiency is high enough. Consequently, social welfare is lower if R&D efficiency is high. Thus, they suggest that *"blanket encouragement of licensing may not be socially desirable"* [2].

More recently, Baik and Kim (2019) studied whether complete information in the market would affect the process of R&D. The reason they extend the traditional duopoly model in such a way is that, on the one hand, some countries such as the UK and the USA require firms to disclosure their R&D effort. Conversely, countries such as France and Germany do not. In addition, even if firms are required to provide their R&D information, they might provide incomplete or incorrect R&D efforts to the public. Thus, understanding whether such asymmetric information would affect R&D investment, firm profits, and social welfare are important.

The models are built upon the Brander-Spencer model. Firms are assumed to be homogeneous, and make their own process R&D investment decision. They analyse

[2] See Chang et al. (2013) page 339

two cases: the observable-investment model; and the unobservable-investment model. In the first case, they assume that the market has complete information on R&D efforts for both firms. In the second case, R&D investment is private information.

In the complete information case, the game is two-stage. In the first stage, firms optimise their R&D investment and announce it to the public. In the second stage, firms compete in the Cournot fashion. In the private information case, firms solve their process R&D problem, but keep such information private. In the second stage, firms choose the quantity of out put simultaneously. They conclude that firms in the complete information market invest more in process R&D than in the market with no R&D disclosure. Baik and Kim (2019) explain this is because firms announce “large” R&D investments to gain competitive advantage. Additionally, they find that equilibrium profits, consumer surplus, and social welfare are smaller in the complete information case. Based on this result, they argue that governments should not require firms to disclose their R&D efforts.

Naskar and Pal (2020) also study the differences between Bertrand and Cournot competition when firms invest in process R&D to produce differentiated network goods. Two profit-maximizing firms compete either in the Bertrand or Cournot competition. Two networks are formed around two products produced by each firm. This network externality component is added to the consumer’s linear quadratic utility function by assuming that the individual willingness to pay for the good is an increasing function of the number of other buyers of the good. Naskar and Pal (2020) argue that because they cannot credibly commit to their outputs/prices, both firms are “network-size taking” while deciding outputs/prices. Thus, before making competition decisions, the network size is regarded as an exogenous variable.

This is a two-stage game. In the first stage, firm 1 and firm 2 decide their own R&D process decision simultaneously. In the second stage, firms compete either in Bertrand or Cournot fashion. Naskar and Pal (2020) found that firms’ incentive to invest in process R&D, with the existence of network externalities, is higher in both the Bertrand and Cournot competition . Network externality affects the

Bertrand competition more significantly, which leads to Bertrand firms investing more in process R&D than Cournot firms. Unlike previous literature Lin and Saggi (2002) conclude Cournot firms are likely to invest more in process R&D than Bertrand firms, Naskar and Pal (2020) argue that such order is reversed because regardless degree of product differentiation, the effect from network externalities in the Bertrand competition is larger.

2.2.2 Product R&D

Since Scherer and Ross (1990) showed that approximately three-fourths of R&D in US firms is product R&D, literature on R&D has begun to centre on product R&D, and its link to process R&D.

Lin and Saggi (2002) present a pioneer model that studies the relationship between process and product R&D in both Bertrand and Cournot competition . The purpose of their study is to provide a theoretical model to study the link between two kinds of R&D.The model they present has two firms producing differentiated goods. Two firms compete in a three-stage game. In the first stage, firms choose the level of product R&D. This determines the degree of product differentiation. In the second stage, firms decide the investment in process R&D. In the third stage, two firms compete in either the quantity of output or price. Such a sequential move between product and process R&D shows the intrinsic link between these two types of R&D. Lin and Saggi (2002) argue that such a sequential move is in line with the reality, where firms invest in product R&D first in order to have a high-quality or differentiated product. Firms will then tend to invest in process R&D to reduce the cost of production. Both product and process R&D could raise the firm's profit margin but differently. Product R&D increases the consumer's willingness to pay, whereas process R&D lowers the cost of production. This is defined as the output effect by Lin and Saggi (2002). Furthermore, product and process R&D are two-way complementary. On the one hand, product R&D shifts the demand curve outwards so that output increases. Such an increase in output makes process R&D more attractive. On the other hand, process R&D reduces the cost of production. This also increases the output level and further encourages firms to invest in product R&D.

They conclude that product R&D and process R&D have a positive relationship. Therefore, firms might invest more in process R&D if goods are differentiated. Also, firms that can reduce the cost of production tend to invest more in product R&D than those that cannot. In addition, firms who compete in price in the market are likely to invest more in product R&D than those firms who compete in output. However, firms have a strong incentive to invest in process R&D in output competition than price competition.

As one of the early works that study product R&D, Symeonidis (2003) argues that product R&D is less studied in theoretical research, even though it is empirically more important. In line with empirical relevance, Symeonidis (2003) studies the Bertrand and Cournot duopoly competition with the product R&D involved. The model has two firms with two differentiated goods produced by each firm. Firms have had their own investment decision in product R&D, and such a decision is made independently. The competition between two firms is a two-stage game. In the first stage, firms choose how much to invest in product R&D that can increase product variety. In the second stage, firms compete either in Bertrand or Cournot fashion.

Symeonidis (2003) finds that Cournot competition leads to higher investment in product R&D than Bertrand competition. Since product R&D increases the consumer willingness to pay, the price in the Cournot competition thus is higher than in the Bertrand competition. Consequently, Cournot's net profit are higher. However, Symeonidis (2003) argues that the comparison of output for both types of competition is debatable. The output level in the Cournot competition can be either higher or lower than in the Bertrand competition. The determinant variables are R&D spillovers, and the degree of product differentiation. If two goods are highly differentiated, and R&D spillovers are strong, output in Bertrand the competition is likely to be higher, and vice versa. For any given level of quality, Bertrand's competition certainly produces more outputs. However, since product R&D is higher in the Cournot competition and such R&D pushes the demand curve outwards , the conclusion is ambiguous. Since the consumer surplus is a function of output, the ranking of the two types competition is also ambiguous, and so is social welfare.

Symeonidis (2003) also discusses process R&D. He illustrates that cost reduction R&D can increase the consumer surplus, but indirectly through reduction in marginal cost and an increase in output. Furthermore, he argues that the relationship between product and process R&D is unclear, as already discussed by Lin and Saggi (2002). These two types of R&D are complementary. This is proved to be correct later by (Braun, 2008).

Haaland and Kind (2006) believe R&D policies should be like other international agreements that need to be regulated. They develop a simple duopoly model to investigate the implications of cooperative and non-cooperative R&D policies across the country. The model is like Brander-Spencer's model, holding two country settings with the absence of spill over and no export policies available. Two firms located in two countries produce both horizontally and vertically differentiated goods, and can invest in product R&D to improve the quality, and hence the demand. Social welfare is affected by two aspects. Firstly, product R&D improves the quality of the product that domestic firms produced, and thus consumer surplus is increased through the increasing willingness to pay. Secondly, the domestic firms may make a higher profit due to a better competitive position in both domestic and international markets. They argue that the former can encourage cooperation in R&D policies, but the latter might end up with policy competition between governments because it is "profit-shifting". Unlike Brander-Spencer's model, Haaland and Kind (2006) include consumption and consumer surplus and allow for active policies in both countries in their model.

The model is a two-stage game. In the first stage, governments need to solve the R&D subsidy problem. Firms will make an R&D investment decision, and compete quantity of output (Cournot game) in the second stage. The model is solved by backward induction. They conclude that R&D subsidies are affected by the degree of product differentiation. The subsidy would be small if two goods are highly differentiated and are high for goods that are close substitutes. Additionally, economic unions, such as the EU, should have centralized R&D policies or perfect coordination of R&D policy among all countries to achieve optimum. Furthermore, if countries are symmetric , coordinated R&D policies would increase joint social welfare. In the asymmetric case, the optimal coordination of R&D policy depends on the degree of product differentiation. This means that countries might prefer policy competition when the joint welfare is maximized. With a similar focus as Leahy and Neary (2001)'s paper, the key variable in Haaland and Kind (2006) model is the degree of product differentiation, which is the core reason why firms, from a government standpoint, invest in product R&D and policy cooperation. In addition, the model that Leahy and Neary (2001) use only considered symmetric case, whereas Haaland and Kind (2006) study an asymmetric case.

Ishii (2013) presents a novel model of a competition between firms in developing countries and firms in developed countries. The intuition behind such a game setting is that firms from BRIC countries entered and survived in the highly competitive international market that is dominated by incumbent firms from developed countries. One common strategy of the newly entered firm is to set lower prices. More importantly, newly entered firms cannot use price strategy forever to keep the competition position. Ishii (2013) argues that product quality from these firms tends to be inferior because firms from developing countries are generally later starters. Therefore, they must invest in product R&D to improve the quality of their product to the level that is acceptable in the international market. However, firms in developing countries with less usable profit to invest in R&D re- quires subsidies from the government. This might lead to policy competition between countries.

The model has two firms located in one developing country and one developed country. Each firm produces its good with different quality. There is no domestic consumption in both countries. All outputs from the two firms are exported to a third country. Since the firm from a developed country is an incumbent, it has already established a good reputation in the third country due to high-quality products. The firm from a developing country has newly entered a third country with a low-quality product. Both firms can invest in product R&D at constant cost. The cost of doing product R&D firm from developing countries is assumed to be higher.

From the above settings, the competitive game is three-stage. In the first stage, governments announce R&D subsidy policies. In the second stage, two firms simultaneously determine the level of product R&D. In the third stage, two firms compete in Bertrand fashion. The game is solved by backward induction to have subgame perfect equilibrium.

The objective for both firms is to maximize their profit and for countries to maximize their welfare. By solving these maximize problems, Ishii (2013) makes the following conclusions. Firstly, product R&D has a positive effect on its own price, but a negative effect on rival's price. This is consistent with other literature that product R&D increases the consumers' willingness to pay. Product R&D shifts its demand curve outwards so that demand and price both increased. Also, improving their product quality, (i.e. investing more in product R&D), reduces the demand for the product of rival's price. In a general Bertrand competition without product R&D, firms compete in price so that they are unwilling to raise the price. This is because demand will fall. However, the price can be increased if firms improve their product quality through R&D. Secondly, a larger price difference leads to less intense price competition. Consequently, firms from a developing country that invest in product R&D make the price competition more intense. Ishii (2013) argues one implication of this finding is that firms from developing countries could intentionally produce a low-quality good to avoid price competition.

Thirdly, R&D subsidy encourages firms to invest more in R&D for both countries, which makes both quality and price competition more intense. However, Ishii (2013) ignores the degree of product differentiation in his model. This is often regarded as an important variable to determine the behaviour of governments and firms.

Hoefele (2016) studies the optimal product R&D policy for firms that operate in the international market. R&D policies have been becoming an important tool for policy makers. In addition, countries (including the UK) give tax breaks for R&D expenditure in private sectors, such as R&D credits, to further increase the support from governments.

Hoefele (2016) defines product innovation R&D as which changes the characteristics of products and reducing the substitutability among them. Thus, investment in product R&D determines the degree of differentiation between products. This definition is in line with Lin and Saggi (2002) argument. Furthermore, the author argues that there are two forms of product innovation. One is a definite change in quality, which is called vertical differentiation, and the other is called horizontal differentiation, where firms change the characteristics of their product.

Yang (2018) developed a model based on Lin and Saggi (2002)'s paper. Yang studies the relationship between product R&D and trade costs, the effect of R&D competition/cooperation in the international market, and how competition/cooperation affects social welfare. The market consists of two homogeneous firms located in two symmetric countries with the same size population.

Unlike Lin and Saggi (2002)'s model, the competitive game Yang (2018) developed is two-stage. In the first stage, firms choose product R&D. The degree of differentiation between products thus is defined. In the second stage, firms compete in the Bertrand fashion. Product R&D has a direct effect on firms' profit margins. It shifts the demand curve outwards so that firms can charge higher prices. Yang (2018) finds that trade liberalization might reduce domestic output, but increase export, regardless of the level of product differentiation. Lowering trade costs also leads to an increase in product R&D. This is consistent with other literature finding for an increase in competitiveness. In addition, Yang (2018) analyses the three different interactions between two firms. If two firms agree to set the same level of product R&D, the equilibrium level of product differentiation is higher than product R&D competition. Thus, the aggregate welfare is larger than product R&D competition. If firms have an asymmetric investment in product R&D, the product differentiation is lower than in the previous case, but larger than R&D competition, so as welfare.

Unlike other trade literature that considers the duopoly model with simultaneous move Pires (2015) presented a leader-follower model to study the strategic trade policies. The model has two homogeneous firms located in two countries, domestic and foreign. A third country is assumed to be a pure importer where two firms compete in Cournot fashion. In the first stage, the government decides whether or not to subsidize R&D. In the second stage, the magnitude of the R&D subsidy is determined and awarded to a local firm. In the third stage, the foreign firm chooses process R&D before making an output decision. In the fourth stage, the foreign firm decides output and the domestic firm chooses process R&D and output simultaneously. The game is solved by backward induction.

Pires (2015) concludes that the foreign firm, who is the R&D leader, can achieve higher competitiveness than the domestic firm, the R&D fol- lower. However, a subsidy for the R&D follower from the domestic government can eliminate such a leader advantage. In addition, the domestic country is richer than the domestic firm that received an R&D subsidy, even if the foreign country also provides a subsidy to the foreign firm. More interestingly, Pires (2015) points out that the R&D subsidy works differently from the export subsidy because export subsidy only has a profit-shifting effect, while R&D subsidy can trigger competitiveness-shifting effects. Thus, an R&D policy is better than an export subsidy for catch-up countries.

2.2.3 Network Externality

Network externality has been the focus of digital market literature. Such characteristics can often be observed in two-sided digital platforms such as Airbnb, Amazon, or even Xbox. A common definition is that two-sided markets are those in which network externality is present. Such a network externality is defined as one consumer's demand would affect or be affected by other consumers' demands.

Leibenstein (1950) presents a pioneering paper that introduces three types of network externalities in consumption, namely the Bandwagon effect, Snob effect, and the Veblen effect. The bandwagon effect is a positive externality in that an individual demand for a good increases when more of other consumers are also demanding the same good. Consumers exhibiting the bandwagon effect are likely to be in the same "style" or "tastes" as the people they would like to be associated with.

The snob effect is the opposite of the Bandwagon effect. It is a negative externality in that an individual demand for a good decreases when more of others consume the same good. This presents those people who want to be "exclusive" or "different" from others. The Veblen effect refers to the phenomenon the demand for an individual increasing due to its high price (This effect is not discussed in this paper).

Liebenstein (1950) suggests to consider separately the effect of price and the effect of other consumers' demand on the individual demand. A decrease in price leads to an increase in the individual quantity demanded, keeping everything else (including the other consumers' demand) constant. On the individual demand diagram, this corresponds to a downward movement along the demand curve. An increase in the other consumers' demand leads (in the presence of bandwagon effect) to an increase in the individual demand, keeping everything else (including the price) constant. On the individual demand diagram, this corresponds to an outward shift in the demand curve. Thus, an individual demand can be written as $q = q(p; Q)$, where p is the price and Q , the 'shifting parameter', is the demand of other consumers

Even though Leibenstein (1950) introduces different types of consumption externalities, the microeconomic foundations of these effects remain unclear.

Grilo et al. (2001) present a model that consists of the concepts Leibenstein (1950) introduced, and a spatial competition. The model has a basic Hotelling setting with two stores selling a homogeneous good at two locations. Consumers are uniformly distributed, and each consumer is assumed to buy one unit of goods. Therefore, the externality is a function of the number of other consumers buy from a store, as shown in the following.

$$E(n_i) = \alpha n_i - \beta n_i^2 \tag{2.1}$$

where n_i is the number of consumers buying from the store i. α determines the type of externality and β is the degree of concavity of this function. If α is positive/negative, the bandwagon/snob effect is taken place.

Economides (1996) models the inverse demand function as the highest willingness to pay, plus the network effect $f(S)$, which is a function of total sales. If the coefficient of the network effect is positive, there is a positive network externality in consumption.

$$P(Q, S) = P(Q, 0) + f(S), f'(S) > 0 \tag{2.2}$$

Similar to Economides (1996)'s model, we define willingness to pay as a function of individual demand plus the network effect $f(Q)$, which is a function of the market aggregate demand.

$$P(q, Q) = P(q, 0) + f(Q), f'(Q) > 0 \tag{2.3}$$

Economides (1996) models supply-side issue where firms solve profit-maximization problem. q is the individual supply, Q is the actual market-wide sales, S is the expected sales, and in the equilirium the expectations are fufilled, i.e. $S^* = Q(S^*)$. Using the same approach for consumers, we would assume that q is individual demand, Q is the market-wide demand, and S is the expected market demand. Then, as in Economides's model, in equilibrium $S^* = Q(S^*)$.

Equation 2.2.3 means that each consumer's willingness to pay is shaped by an individual's preference and the network effect that is influenced by the market demand.

If the network effect is positive, there is a positive effect from market demand, so an increase in market aggregate demand would positively affect an individual's demand. In equilibrium, the total demand is the aggregated individual demand. In our model, since there is only one representative consumer, in equilibrium $Q = q$.

2.3 Model Summary

In this model, there are two countries, home and foreign, two monopolistic firms, one in each country, and three consumption goods, labelled 0, 1, and 2. All goods are produced using labour as the only input. Good 0 is produced by perfectly competitive firms using one unit of labour for each unit produced, and is traded freely and costlessly. Good 1 is produced by the foreign monopoly, and good 2 is produced by the home monopoly. For simplicity, we consider only consumers located in the home country. The consumers in the home country derive utility from consumption of the three goods, supply inelastically L units of labour, own equal shares in the home firms, and receive a lump-sum transfer from the government. Both home goods and foreign goods exhibit network externalities in consumption, and they are imperfect substitutes. Both monopolies can invest in process and product innovation. Both countries subsidize innovation costs for their own firms. The home country imposes DST in the form of the sales tax on imported foreign goods.

2.3.1 Consumer

The representative home country consumer maximize quasi-linear utility function as in Singh and Vives (1984) subject to the budget constraint:

$$\max : \; U(q_0, q_1, q_2) = (A_1)q_1 + (A_2)q_2 - \frac{1}{2}(q_1^2 + q_2^2 + 2mq_1q_2) + q_0 \tag{2.4}$$

$$\text{subject to} : \; p_1q_1 + p_2q_2 + q_0 = M = L + \pi_2 + T \tag{2.5}$$

Here, good 0 is chosen as a numeraire, and the assumption of its production ensures that the home wage is equal to one. Thus, on the right side of the budget constraint, L is the labour income, π_2 is the profit of the home monopoly and T is the lump-sum transfer. Parameter $m \in [-1, 1]$ measures the degree of substitutability between goods 1 and 2. The goods are substitutes in consumption when m is positive, and complements when m is negative; the goods are identical when $m = 1$. Solving the above problem gives the demand functions:

$$p_1 = A_1 - q_1 - mq_2$$
$$p_2 = A_2 - q_2 - mq_1$$

where A_1 and A_2 are the vertical intercepts in the demand functions. It is the price at and above with the quantity demanded is zero, which is the choke price. Therefore, A_1 and A_2 are the consumers' highest willingness to pay.

$$A_i = a_i + r_i$$

Where $i = 1, 2$, a_i is the network externality for good 1 and 2, and r_i is the quality increased by investing in product R&D. The reason A_1 and A_2 are the functions above is that both network externality and product quality will improve the highest willingness to pay, but these two effects are assumed to be completely independent.

Product R&D increases product quality. This makes the product more attractive to the consumers so that their highest willingness to pay rises. Therefore, the demand curve parallelly shifts upwards, which raises the vertical intercept by r_1. In the presence of a consumption network externality, the willingness to pay depends on consumption by others. If the externality is positive, the stronger the network effect, the higher the willingness to pay.

Modelling the network externality in consumption follows the concepts introduced by Leibenstein (1950) and Grilo et al. (2001)'s model. In this paper, we only focus on the positive network externality, the bandwagon effect. Since positive network externality is one reason that the digital economy grows so fast, the logic behind this argument is that as more consumer demands one product, the stronger the positive network effect will be. This further encourages consumers to buy more, and leads to a virtuous circle. We model the willingness to pay as an increasing function of the aggregate consumption[3]:

$$a_i = a + b_i Q_i + d_i Q_i^2 \tag{2.6}$$

where Q_i is the perceived market demand and $a > 0, 0 < b_i < 1$. Parameter $d_i < 0$ introduces concavity/diminishing effect into the function. That is, the effect of each next unit's increase in aggregate consumption is smaller than the effect of the previous unit. We assume $d_i = 0$ for simplicity. Each one-unit increase in aggregate consumption has the same constant positive effect on the willingness to pay. In equilibrium, the market demand is perceived correctly by the representative consumer, i.e. $q_i = Q_i$.

Our formal description of the externality in consumption follows Grilo et al. (2001). In their model, individual utility of consumption, and, therefore, individual demand, depends on the total number of consumers buying the same good. We depart from Grilo et al. (2001) by assuming that the individual demand depends on the quantity demanded by all other consumers, rather than the number of consumers. Formally, we assume that an individual consumer's maximal willingness to pay for the good is a function of the market demand. For analytical tractability, we assume that this function is linear. Thus a positive network externality in consumption is present when this function is increasing in the market-wide quantity demanded.

Therefore, demand functions:

$$p_1 = a + r_1 - (1 - b_1)q_1 - mq_2 \tag{2.7}$$

$$p_2 = a + r_2 - (1 - b_2)q_2 - mq_1 \tag{2.8}$$

[3]Naskar and Pal (2020) also assume that the individual willingness to pay for the good is an increasing function of the number of other buyers of the good. But they use a completely different function.

2.3.2 Producers

Both firm 1 and firm 2 are profit-maximizing firms:

Foreign producer:

$$\pi_1 = (1-t)p_1q_1 - (c_1 - k_1)q_1 - (1-s_1)\frac{\varphi_1 k_1^2}{2} - (1-\sigma_1)\frac{\theta_1 r_1^2}{2} \tag{2.9}$$

Home producer:

$$\pi_2 = p_2q_2 - (c_2 - k_2)q_2 - (1-s_2)\frac{\varphi_2 k_2^2}{2} - (1-\sigma_2)\frac{\theta_2 r_2^2}{2} \tag{2.10}$$

where $t \in (0,1)$ is the sales tax rate. The reason we model DST as sales tax is because DST is levied on the revenue of the multinational firm from sales of digital goods to home consumers. k_i is the marginal production cost reduced by investing in process R&D, φ_i and θ_i are processes and product R&D efficiency respectively. Specifically, firm i can reduce its marginal product costs to $(c_i - k_i)$ by investing $\frac{\varphi_i k_i^2}{2}$. Cost of investing in product is $\frac{\theta_i r_i^2}{2}$. Both firms receive R&D subsidies from their government by $s_i\frac{\varphi_i k_i^2}{2} + \sigma_i\frac{\theta_i r_i^2}{2}$, where s_i and σ_i are subsidy rates for process and product R&D respectively.

2.3.3 Governments

The foreign government maximises the profit of its firm net of subsidies:

$$W_1 = \pi_1 - s_1\frac{\varphi_1 k_1^2}{2} - \sigma_1\frac{\theta_1 r_1^2}{2} \tag{2.11}$$

The home government maximises the utility of the representative consumer:

$$W_2 = U + \pi_2 - s2\frac{\varphi_2 k_2^2}{2} - \sigma_2\frac{\theta_2 r_2^2}{2} + tp_1q_1 \tag{2.12}$$

This expression takes into account that the tax revenues net of subsidies is returned to the consumers as a lump-sum transfer.

2.4 Dynamic Game

Stage 1 The governments announce tax and subsidy policies.

Stage 2 The firms choose the number of outputs and investments in both types of R&D.

To obtain sub-game perfect Nash Equilibrium, this game is solved by backward induction.

2.4.1 Stage 2: Profit maximization in Cournot game

Substitute demand functions 2.7 and 2.8 into foreign producer profit function 2.9:

$$\pi_1 = (1-t)[a+r_1-(1-b_1)q_1-mq_2]q_1-(c_1-k_1)q_1-(1-s_1)\frac{\varphi_1 k_1^2}{2}-(1-\sigma_1)\frac{\theta_1 r_1^2}{2} \quad (2.13)$$

First-order conditions for an interior optimum gives:

$$q_1 = \frac{1}{2(1-b_1)}\left[a + r_1 - mq_2 - \frac{c_1 - k_1}{1-t}\right] \quad (2.14)$$

$$k_1 = \frac{q_1}{[1-s_1]\,\varphi_1} \quad (2.15)$$

$$r_1 = \frac{[1-t]\,q_1}{[1-\sigma_1]\,\theta_1} \quad (2.16)$$

$$(2.17)$$

Similarly, substitute demand functions 2.7 and 2.8 into home producer profit function 2.10, the first order conditions for an interior optimum gives:

$$q_2 = \frac{1}{2(1-b_2)}\left[a + r_2 - mq_1 - [c_2 - k_2]\right] \quad (2.18)$$

$$k_2 = \frac{q_2}{[1-s_2]\,\varphi_2} \quad (2.19)$$

$$r_2 = \frac{q_2}{[1-\sigma_2]\,\theta_2} \quad (2.20)$$

$$(2.21)$$

For simplicity, process R&D and product R&D will be discussed separately.

2.5 Case 1: Process R&D

Set $r_1 = r_2 = 0$. The second-order conditions for interior maximum requires:

$$\frac{\partial^2 \pi_i}{\partial q_i^2} < 0, \frac{\partial^2 \pi_i}{\partial k_i^2} < 0, \Delta_i^{k*} > 0$$

So, for $\Delta_{1,2}^{k*}$ we have:

$$\Delta_1^{k*} = 2(1-b_1)[1-t][1-s_1]\varphi_1 \left[1 - \frac{1}{2(1-b_1)[1-t][1-s_1]\varphi_1}\right] \quad (2.22)$$

$$\Delta_2^{k*} = 2(1-b_2)[1-s_2]\varphi_2 \left[1 - \frac{1}{2(1-b_2)[1-s_2]\varphi_2}\right] \quad (2.23)$$

Thus, for the second-order conditions to hold, it must be the case that:

$$1 - \frac{1}{2(1-b_1)[1-t][1-s_1]\varphi_1} > 0 \quad (2.24)$$

$$1 - \frac{1}{2(1-b_2)[1-s_2]\varphi_2} > 0 \quad (2.25)$$

The second order conditions are satisfied for $0 < b_1 < 1$ and $0 < s_1 < 1$, $0 < \sigma_1 < 1$. Hence the profit has a unique interior maximum, and the equilibrium is described by the linear system of equations:

$$q_1 = \frac{1}{2(1-b_1)}\left[a + r_1 - mq_2 - \frac{c_1 - k_1}{1-t}\right] \quad (2.26)$$

$$k_1 = \frac{q_1}{[1-s_1]\varphi_1} \quad (2.27)$$

$$q_2 = \frac{1}{2(1-b_2)}[a + r_2 - mq_1 - [c_2 - k_2]] \quad (2.28)$$

$$k_2 = \frac{q_2}{[1-s_2]\varphi_2} \quad (2.29)$$

By rewrite the above linear system equations into matrix form, q_1 and q_2 can be solved explicitly by Cramer's rule:

$$q_1 = \frac{1}{2(1-b_1)} \frac{\left[a - \frac{c_1}{1-t}\right]\left[1 - \frac{1}{2(1-b_2)[1-s_2]\varphi_2}\right] - \frac{m[a-c_2]}{2(1-b_2)}}{\left[1 - \frac{1}{2(1-b_1)[1-t][1-s_1]\varphi_1}\right]\left[1 - \frac{1}{2(1-b_2)[1-s_2]\varphi_2}\right] - \frac{m^2}{4(1-b_1)(1-b_2)}}$$

$$q_2 = \frac{1}{2(1-b_2)} \frac{[a - c_2]\left[1 - \frac{1}{2(1-b_1)[1-t][1-s_1]\varphi_1}\right] - \frac{m\left[a - \frac{c_1}{1-t}\right]}{2(1-b_1)}}{\left[1 - \frac{1}{2(1-b_1)[1-t][1-s_1]\varphi_1}\right]\left[1 - \frac{1}{2(1-b_2)[1-s_2]\varphi_2}\right] - \frac{m^2}{4(1-b_1)(1-b_2)}}$$

Since we focus on the interior solution, our analysis is restricted to the case where:

$$\left[1-\frac{1}{2\left(1-b_1\right)\left[1-t\right]\left[1-s_1\right]\varphi_1}\right]\left[1-\frac{1}{2\left(1-b_2\right)\left[1-s_2\right]\varphi_2}\right] -\frac{m^2}{4\left(1-b_1\right)\left(1-b_2\right)} > 0 \quad (2.30)$$

$$\left[a-\frac{c_1}{1-t}\right]\left[1-\frac{1}{2\left(1-b_2\right)\left[1-s_2\right]\varphi_2}\right]-\frac{m\left[a-c_2\right]}{2\left(1-b_2\right)} > 0 \quad (2.31)$$

$$\left[a-c_2\right]\left[1-\frac{1}{2\left(1-b_1\right)\left[1-t\right]\left[1-s_1\right]\varphi_1}\right]-\frac{m\left[a-\frac{c_1}{1-t}\right]}{2\left(1-b_1\right)} > 0 \quad (2.32)$$

2.5.1 Policy effect on Process R&D

How equilibrium quantities and investment in R&D depend on tax and subsidies can be calculated by taking the total differentials of the system of linear equations:

$$dq_1-\frac{1}{2\left(1-b_1\right)\left[1-t\right]}dk_1+\frac{m}{2\left(1-b_1\right)}dq_2 = -\frac{c_1-k_1}{2\left(1-b_1\right)\left[1-t\right]^2}dt \quad (2.33)$$

$$-\frac{1}{\left[1-s_1\right]\varphi_1}dq_1+dk_1 = \frac{q_1}{\left[1-s_1\right]^2\varphi_1}ds_1 \quad (2.34)$$

$$\frac{m}{2\left(1-b_2\right)}dq_1+dq_2-\frac{1}{2\left(1-b_2\right)}dk_2 = 0 \quad (2.35)$$

$$-\frac{1}{\left[1-s_2\right]\varphi_2}dq_2+dk_2 = \frac{q_2}{\left[1-s_2\right]^2\varphi_2}ds_2 \quad (2.36)$$

In the matrix form:

$$\begin{bmatrix} 1 & -\frac{1}{2(1-b_1)[1-t]} & \frac{m}{2(1-b_1)} & 0 \\ -\frac{1}{[1-s_1]\varphi_1} & 1 & 0 & 0 \\ \frac{m}{2(1-b_2)} & 0 & 1 & -\frac{1}{2(1-b_2)} \\ 0 & 0 & -\frac{1}{[1-s_2]\varphi_2} & 1 \end{bmatrix}\begin{bmatrix} dq_1 \\ dk_1 \\ dq_2 \\ dk_2 \end{bmatrix} = \begin{bmatrix} -\frac{c_1-k_1}{2(1-b_1)[1-t]^2}dt \\ \frac{q_1}{[1-s_1]^2\varphi_1}ds_1 \\ 0 \\ \frac{q_2}{[1-s_2]^2\varphi_2}ds_2 \end{bmatrix}$$

This gives:

$$dq_1 = \frac{\begin{vmatrix} -\frac{c_1-k_1}{2(1-b_1)[1-t]^2}dt & -\frac{1}{2(1-b_1)[1-t]} & \frac{m}{2(1-b_1)} & 0 \\ \frac{q_1}{[1-s_1]^2\varphi_1}ds_1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -\frac{1}{2(1-b_2)} \\ \frac{q_2}{[1-s_2]^2\varphi_2}ds_2 & 0 & -\frac{1}{[1-s_2]\varphi_2} & 1 \end{vmatrix}}{\begin{vmatrix} 1 & -\frac{1}{2(1-b_1)[1-t]} & \frac{m}{2(1-b_1)} & 0 \\ -\frac{1}{[1-s_1]\varphi_1} & 1 & 0 & 0 \\ \frac{m}{2(1-b_2)} & 0 & 1 & -\frac{1}{2(1-b_2)} \\ 0 & 0 & -\frac{1}{[1-s_2]\varphi_2} & 1 \end{vmatrix}} = \frac{\Delta_{q_1}}{\Delta}$$

For an interior solution, $\Delta > 0$ must hold. In that case:

$$\frac{\partial q_1}{\partial t} < 0 \tag{2.37}$$

$$\frac{\partial q_1}{\partial s_1} > 0 \tag{2.38}$$

$$\frac{\partial q_1}{\partial s_2} \lesseqqgtr 0 \text{ for } m \gtreqqless 0 \tag{2.39}$$

Similarly, we have:

$$\frac{\partial k_1}{\partial t} < 0 \tag{2.40}$$

$$\frac{\partial k_1}{\partial s_1} > 0 \tag{2.41}$$

$$\frac{\partial k_1}{\partial s_2} \lesseqqgtr 0 \text{ for } m \gtreqqless 0 \tag{2.42}$$

Derivatives 2.37 and 2.40 describe the effect of DST on foreign firm's output and investment in process R&D. The effect of DST on imported foreign good has a negative effect on foreign firm's output and investment. DST reduces the demand of the foreign good and decreases the profit-maximising quantity produced by the foreign firm. This weakens the incentive for the foreign firm to reduce the marginal cost of production, i.e. investment in process R&D.

Derivatives 2.38-2.39 and 2.41-2.42 show how subsidy policies from both countries affect the foreign firm's output and investment decisions. According to 2.38 and

2.41, foreign country's subsidy always has a positive effect on the foreign firm's output and investment. Foreign country's subsidy lowers the cost of process R&D and thus increases the foreign firm's incentive to invest in process R&D. As a result, marginal cost of production is reduced, and hence, the foreign firm can expand it's production, i.e. output increased.

According to 2.39 and 2.42, the effect of home country's subsidy policy on the foreign firm's output and investment depends on whether the goods/services produced by two firms are substitutes or complements. If two goods are substitutes ($m > 0$), the effect is negative, and if two goods are complements ($m < 0$), the effect is positive. Home country's subsidy has no efffect on the foreign firm's output and investment decisions if two goods are independent.

For home firm:

$$\begin{aligned} \frac{\partial q_2}{\partial t} &\lesseqqgtr 0 \text{ for } m \lesseqqgtr 0 && (2.43) \\ \frac{\partial q_2}{\partial s_1} &\gtreqqless 0 \text{ for } m \lesseqqgtr 0 && (2.44) \\ \frac{\partial q_2}{\partial s_2} &> 0 && (2.45) \end{aligned}$$

$$\begin{aligned} \frac{\partial k_2}{\partial t} &\gtreqqless 0 \text{ for } m \lesseqqgtr 0 && (2.46) \\ \frac{\partial k_2}{\partial s_1} &\gtreqqless 0 \text{ for } m \lesseqqgtr 0 && (2.47) \\ \frac{\partial k_2}{\partial s_2} &> 0 && (2.48) \end{aligned}$$

Derivatives 2.43 and 2.46 describe the effect of DST on home firm's output and investment in process R&D, which depend on the substitutability/complementarity between two goods/services produced by both home and foreign firms. If two goods are substitutes ($m < 0$), DST increases the demand for the home goods/services. Home firm is incentivized to produce more output and invest more in process R&D to reduce marginal cost of production. The situation is the opposite in the case of complements.

Derivatives 2.44-2.45 and 2.47-2.48 present the effect of subsidy policies from both countries on home firm's output and investment in process R&D. According to 2.45

and 2.48, home subsidy has positive effects on home firm's output and investment in process R&D. The effect of foreign country's subsidy on home firm's output and investment depends on whether two goods/ services are substitutes or complements. If two goods are substitutes ($m > 0$), DST encourage home firm produce more because demand for the foreign product/service falls (2.39). Home firm has more incentives to invest more in process R&D to reduce marginal cost of production. The situation is the opposite in the case of complements ($m < 0$).

Therefore, we can conclude that investment in process R&D can always be boosted by subsidies from own government. Foreign firm's decisions on outputs and investment in process R&D are, unless two goods are complements, always negatively affected by home country's tax policy. Home firm's outputs and investment in process R&D will be hurt if two goods are complementary. If two goods are independent, home government's subsidy policy will not affect the foreign firm's behaviour.

2.5.2 The role of network externalities in consumption

We can also calculate how the decisions of the firms depend on network externalities in consumption by taking the total derivatives of the linear system equations:

$$\begin{aligned} dq_1 - \frac{1}{2\left(1-b_1\right)\left[1-t\right]}dk_1 + \frac{m}{2\left(1-b_1\right)}dq_2 + 0 \times dk_2 &= \frac{a - \frac{c_1-k_1}{1-t} - mq_2}{2\left(1-b_1\right)^2}db_1 \\ -\frac{1}{\left[1-s_1\right]\varphi_1}dq_1 + dk_1 + 0 \times dq_2 + 0 \times dk_2 &= 0 \\ \frac{m}{2\left(1-b_2\right)}dq_1 + 0 \times dk_1 + dq_2 - \frac{1}{2\left(1-b_2\right)}dk_2 &= \frac{a-\left(c_2-k_2\right)-mq_1}{2\left(1-b_2\right)^2}db_2 \\ 0 \times q_1 + 0 \times k_1 - \frac{1}{\left[1-s_2\right]\varphi_2}q_2 + k_2 &= 0 \end{aligned}$$

By rewriting the above equations in matrix form, we can obtain the signs for an interior solution:

$$\frac{\partial q_1}{\partial b_1} > 0 \tag{2.49}$$

$$\frac{\partial q_1}{\partial b_2} \gtreqqless 0 \text{ for } m \lesseqqgtr 0 \tag{2.50}$$

$$\frac{\partial k_1}{\partial b_1} > 0 \tag{2.51}$$

$$\frac{\partial k_1}{\partial b_2} \gtreqqless 0 \text{ for } m \lesseqqgtr 0 \tag{2.52}$$

Similarly:

$$\frac{\partial q_2}{\partial b_2} > 0 \tag{2.53}$$

$$\frac{\partial q_2}{\partial b_1} \gtreqqless 0 \text{ for } m \lesseqqgtr 0 \tag{2.54}$$

$$\frac{\partial k_2}{\partial b_2} > 0 \tag{2.55}$$

$$\frac{\partial k_2}{\partial b_1} \gtreqqless 0 \text{ for } m \lesseqqgtr 0 \tag{2.56}$$

Derivatives 2.51-2.52 and 2.55-2.56 describe the effect of externality in consumption of a good on the investment in the process R&D by the firm producing this good and by the other firm. According to 2.51 and 2.55, a network externality in consumption of the good produced by the firm always has a positive effect on that firm's investment in process R&D. Positive consumption externality increases the demand and makes it more price-elastic. This strengthens the incentives for the producer to reduce the marginal cost of production.

The effect of externality on the other firm's investment depends on whether the goods produced by two firms are substitutes or complements. According to 2.52 and 2.56, if two goods are substitutes ($m > 0$) , the effect is negative, and if the two goods are complements ($m < 0$), the effect is positive. The externality has no effect on the other firm's investment when the demands for two goods are independent.

Clearly, in the case of independent demands there is no strategic interdependence between the two firms, and so the investment decision of on firms does not depend on the properties of the demand for the good produced by other firm. Strategic interdependence generated by substitutability or complementarity between two goods creates the externality effect on investment.

Consider the case of substitutes. Positive externallity in consumption of good 1 makes the demand more elastic and raises the profit-maximising quantity produced by firm 1. The best response of firm 2 under Cournot competition is to reduce its output, i.e. the quantity of good 2. This weakens the incentive for firm 2 to reduce the marginal cost of production. Thus, its investment in the process R&D is lower in the presence of externality in consumption of good 1. This situation is the opposite in the case of complements.

2.5.3 Stage 1: Optimal tax and subsidy policies

Set$r_1 = r_2 = 0$, governments' welfare functions are:

$$W_1 = \pi_1 - s_1 \frac{\varphi_1 k_1^2}{2} \tag{2.57}$$

$$W_2 = U + \pi_2 - s_2 \frac{\varphi_2 k_2^2}{2} + tp_1 q_1 \tag{2.58}$$

2.5.4 Foreign country optimizes subsidy policy

Solving the first order condition for foreign country welfare function 2.57, we have the following equation:

$$s_1 = \frac{\frac{m^2}{4(1-b_1)(1-b_2)}}{1 - \frac{1}{2(1-b_2)[1-s_2]\varphi_2}} \tag{2.59}$$

which is the best response (reaction function) of the foreign country to the subsidy policy of the home country. Surprisingly, the subsidy of the foreign country does not depend on the home country's tax policy. When two goods are independent, $m = 0$, the foreign country will not respond to the home country's subsidy policy. Otherwise, a foreign country will always subsidize its firm, even when the home country does not have a subsidy policy, irrespective of whether two goods are substitutes or complements:

$$\frac{ds_1}{ds_2} = \frac{1}{\left[1-\frac{1}{2(1-b_2)[1-s_2]\varphi_2}\right]^2}\frac{m^2}{8\left(1-b_1\right)\left(1-b_2\right)^2\left[1-s_2\right]^2\varphi_2} > 0 \text{ for } m \neq 0$$

2.5.5 Home country optimizes tax and subsidy policy

Solving the first order condition for home country welfare function 2.58, when $m = 0$, we can obtain:

$$s_2 = \frac{1}{1+2(1-b_2)} \tag{2.60}$$

In contrast to the foreign country, the home country will always subsidize its firm, even if two goods are independent, since social welfare can be increased by raising the home consumers' utility. $\frac{ds_2}{db_2} > 0$ implies that the stronger the network externality of home goods is, the higher the optimal the subsidy will be.

For home country, the model structure of the first order condition with respect to tax does not allow for the closed form solutions for the general case, $m \neq 0$, of tax and subsidy policies. This equation will be solved numerically in the final section.

2.6 Case 2: Product R&D

Set $k_1 = k_2 = 0$. The second-order conditions for interior maximum requires:

$$\frac{\partial^2\pi_i}{\partial q_i^2} < 0, \frac{\partial^2\pi_i}{\partial r_i^2} < 0, \Delta_i^{r*} > 0$$

So, for $\Delta_{1,2}^{r*}$ we have :

$$\Delta_1^{r*} = 2\left(1-b_1\right)\left[1-t\right]\left[1-\sigma_1\right]\theta_1\left[1-\frac{1-t}{2\left(1-b_1\right)\left[1-\sigma_1\right]\theta_1}\right] > 0 \tag{2.61}$$

$$\Delta_2^{r*} = 2\left(1-b_2\right)\left[1-\sigma_2\right]\theta_2\left[1-\frac{1}{2\left(1-b_2\right)\left[1-\sigma_2\right]\theta_2}\right] > 0 \tag{2.62}$$

Thus, for the second-order conditions to hold, it must be the case that:

$$1-\frac{1-t}{2\left(1-b_1\right)\left[1-\sigma_1\right]\theta_1} > 0 \tag{2.63}$$

$$1-\frac{1}{2\left(1-b_2\right)\left[1-\sigma_2\right]\theta_2} > 0 \tag{2.64}$$

Therefore, the equilibrium is described by:

$$q_1 = \frac{1}{2(1-b_1)}\left[a + r_1 - mq_2 - \frac{c_1}{1-t}\right] \quad (2.65)$$

$$r_1 = \frac{[1-t]\,q_1}{[1-\sigma_1]\,\theta_1} \quad (2.66)$$

$$q_2 = \frac{1}{2(1-b_2)}\left[a + r_2 - mq_1 - c_2\right] \quad (2.67)$$

$$r_2 = \frac{q_2}{[1-\sigma_2]\,\theta_2} \quad (2.68)$$

The equilibrium $\{q_1, q_2\}$ can be solved as:

$$q_1 = \frac{1}{2(1-b_1)} \frac{\left[a - \frac{c_1}{1-t}\right]\left[1 - \frac{1}{2(1-b_2)[1-\sigma_2]\theta_2}\right] - \frac{m}{2(1-b_2)}\left[a - c_2\right]}{\left[1 - \frac{1-t}{2(1-b_1)[1-\sigma_1]\theta_1}\right]\left[1 - \frac{1}{2(1-b_2)[1-\sigma_2]\theta_2}\right] - \frac{m^2}{4(1-b_1)(1-b_2)}}$$

$$q_2 = \frac{1}{2(1-b_2)} \frac{\left[a - c_2\right]\left[1 - \frac{1-t}{2(1-b_1)[1-\sigma_1]\theta_1}\right] - \frac{m}{2(1-b_1)}\left[a - \frac{c_1}{1-t}\right]}{\left[1 - \frac{1-t}{2(1-b_1)[1-\sigma_1]\theta_1}\right]\left[1 - \frac{1}{2(1-b_2)[1-\sigma_2]\theta_2}\right] - \frac{m^2}{4(1-b_1)(1-b_2)}}$$

2.6.1 Policy effect on Product R&D

Similar to the case of process R&D, the linear systems equations of product R&D will be differentiated to investigate how equilibrium outputs and investments in product R&D depend on policy variables.

Total differential:

$$dq_1 - \frac{1}{2(1-b_1)}dr_1 + \frac{m}{2(1-b_1)}dq_2 + 0 \times dr_2 = -\frac{c_1}{2(1-b_1)[1-t]^2}dt$$

$$-\frac{[1-t]}{[1-\sigma_1]\,\theta_1}dq_1 + dr_1 + 0 \times dq_2 + 0 \times dr_2 = -\frac{q_1}{[1-\sigma_1]\,\theta_1}dt + \frac{[1-t]\,q_1}{[1-\sigma_1]^2\,\theta_1}d\sigma_1$$

$$\frac{m}{2(1-b_2)}dq_1 + 0 \times dr_1 + dq_2 - \frac{1}{2(1-b_2)}dr_2 = 0$$

$$0 \times dq_1 + 0 \times dr_1 - \frac{1}{[1-\sigma_2]\,\theta_2}dq_2 + dr_2 = \frac{q_2}{[1-\sigma_2]^2\,\theta_2}d\sigma_2$$

In matrix form:

$$\begin{bmatrix} 1 & -\frac{1}{2(1-b_1)} & \frac{m}{2(1-b_1)} & 0 \\ -\frac{[1-t]}{[1-\sigma_1]\theta_1} & 1 & 0 & 0 \\ \frac{m}{2(1-b_2)} & 0 & 1 & -\frac{1}{2(1-b_2)} \\ 0 & 0 & -\frac{1}{[1-\sigma_2]\theta_2} & 1 \end{bmatrix} \begin{bmatrix} dq_1 \\ dr_1 \\ dq_2 \\ dr_1 \end{bmatrix} = \begin{bmatrix} -\frac{c_1}{2(1-b_1)[1-t]^2}dt \\ -\frac{q_1}{[1-\sigma_1]\theta_1}dt + \frac{[1-t]q_1}{[1-\sigma_1]^2\theta_1}d\sigma_1 \\ 0 \\ \frac{q_2}{[1-\sigma_2]^2\theta_2}d\sigma_2 \end{bmatrix}$$

The denominator:

$$\Delta^r \equiv \left[1 - \frac{1}{2(1-b_2)(1-\sigma_2)\theta_2}\right]\left[1 - \frac{1-t}{2(1-b_1)(1-\sigma_1)\theta_1}\right] - \frac{m^2}{4(1-b_1)(1-b_2)} > 0 \tag{2.69}$$

Thus, the effect of policies on q_1 is:

$$\frac{dq_1}{dt} < 0 \tag{2.70}$$
$$\frac{dq_1}{d\sigma_1} > 0 \tag{2.71}$$
$$\frac{dq_1}{d\sigma_2} \lesseqqgtr 0 \text{ for } m \gtreqqless 0 \tag{2.72}$$

Policy effect on product R&D:

$$\frac{dr_1}{dt} < 0 \tag{2.73}$$
$$\frac{dr_1}{d\sigma_1} > 0 \tag{2.74}$$
$$\frac{dr_1}{d\sigma_2} \lesseqqgtr 0 \text{ for } m \gtreqqless 0 \tag{2.75}$$

Similarly, for the home firm, we have:

$$\frac{dq_2}{dt} \gtreqqless 0 \text{ for } m \gtreqqless 0 \tag{2.76}$$
$$\frac{dq_2}{d\sigma_1} \lesseqqgtr 0 \text{ for } m \gtreqqless 0 \tag{2.77}$$
$$\frac{dq_2}{d\sigma_2} > 0 \tag{2.78}$$

and:

$$\frac{dr_2}{dt} \gtreqqless 0 \text{ for } m \gtreqqless 0 \tag{2.79}$$
$$\frac{dr_2}{d\sigma_1} \lesseqqgtr 0 \text{ for } m \gtreqqless 0 \tag{2.80}$$
$$\frac{dr_2}{d\sigma_2} > 0 \tag{2.81}$$

Therefore, the home country DST always hurts the foreign firm's output and investment in product R&D (2.70 and 2.73) and hurts home firm's investment in product R&D and output if two goods are complements ($m < 0$, 2.76 and 2.79). The quantity of outputs and Investment in product R&D are always increasing with own country subsidy, irrespective of product substitutability, m (2.71, 2.74, 2.78, and 2.81). However, firms' output decision and investment in product R&D decrease (increase) in the other country's subsidy if two goods are substitutes(complements), and if two goods are independent, one country's subsidy policy does not affect the other country firm's behaviour (2.72, 2.75, 2.77, and 2.80).

2.6.2 The role of network externalities in consumption

Total differentiation gives

$$\begin{aligned}
dq_1 - \frac{1}{2(1-b_1)}dr_1 + \frac{m}{2(1-b_1)}dq_2 + 0 \times dr_2 &= \frac{1}{2(1-b_1)^2}\left[a + r_1 - \frac{c_1}{1-t} - mq_2\right] db_1 \\
-\frac{[1-t]}{[1-\sigma_1]\theta_1}dq_1 + dr_1 + 0 \times dq_2 + 0 \times dr_2 &= 0 \\
\frac{m}{2(1-b_2)}dq_1 + 0 \times dr_1 + dq_2 - \frac{1}{2(1-b_2)}r_2 &= \frac{1}{2(1-b_2)^2}\left[a + r_2 - c_2 - mq_1\right] db_2 \\
0 \times dq_1 + 0 \times dr_1 - \frac{1}{[1-\sigma_2]\theta_2}dq_2 + dr_2 &= 0
\end{aligned}$$

Solving in the matrix form, we have:

$$\frac{dq_1}{db_1} > 0 \tag{2.82}$$
$$\frac{dq_1}{db_2} \lesseqqgtr 0 \text{ for } m \gtreqqless 0 \tag{2.83}$$
$$\frac{dr_1}{db_1} > 0 \tag{2.84}$$
$$\frac{dr_1}{db_2} \lesseqqgtr 0 \text{ for } m \gtreqqless 0 \tag{2.85}$$

similarly:

$$\frac{dq_2}{db_1} \lesseqqgtr 0 \text{ for } m \gtreqqless 0 \tag{2.86}$$
$$\frac{dq_2}{db_2} > 0 \tag{2.87}$$
$$\frac{dr_2}{db_1} \lesseqqgtr 0 \text{ for } m \gtreqqless 0 \tag{2.88}$$
$$\frac{dr_2}{db_2} > 0 \tag{2.89}$$

Derivatives 2.84-2.85 and 2.88-2.89 present the effect of externality in consumption go a good/service on the investment in the product R&D by the firm producing this good and by the other firm. According to 2.84 and 2.89, a network externality in consumption of the good produced by the firm always has a positive effect on that firm's investment in the product R&D. A new or improved quality product/service attract consumers to buy more product and thus demand increases (2.82 and 2.87). This encourages the producer to invest more in new and/or higher quality goods and services.

The effect of network externality on the other firm's investment depends on whether the goods are substitutes or complements. According to 2.85 and 2.88, if two goods are subsititutes ($m > 0$), the network externality has negative effect on the other firm's investment in product R&D. Lower quality products have lower demand. Thus, reduced demand for the other firm's goods and services discourage its production, i.e. output decreases (2.83 and 2.86). The situation is the opposite in the case of complements.

In the case that two goods/services are independent, there is no strategic interdependence between the two firms, which means investment in product R&D of one firm is not affected by the demand of the good/service producded by the other firm. Similar to process R&D cases, strategic interdependence generated by substitutability or complementarity between the two good creates the externality effect on investment.

2.6.3 Stage 1: Optimal tax and subsidy policies

Set $k_1 = k_2 = 0$, welfare functions are:

$$W_1 = \pi_1 - \sigma_1 \frac{\theta_1 r_1^2}{2} \tag{2.90}$$

$$W_2 = U + \pi_2 - \sigma_2 \frac{\theta_2 r_2^2}{2} + t p_1 q_1 \tag{2.91}$$

2.6.3.1 Foreign country optimizes subsidy policy

We can simplify the first order condition $\frac{dW_1}{d\sigma_1}$ to obtain:

$$\sigma_1 = \frac{\frac{m^2}{4(1-b_1)(1-b_2)}}{1 - \frac{1}{2(1-b_2)} \frac{1}{[1-\sigma_2]\theta_2}} \tag{2.92}$$

This best response function is similar to the case of process R&D. If two goods are independent, the optimal subsidy policy for the foreign government is zero. Otherwise, the foreign government will always subsidize its firm, even if there is no subsidy policy of the home government. Also, this function indicates that the home country's tax policy will not affect foreign governments' subsidy policy. The only factor that has an effect is the degree of substitutability between two goods, m.

2.6.3.2 Home country optimizes tax and subsidy policies

Home country solves the following two problems:

$$\text{optimal subsidy:} \quad \frac{dW_2}{d\sigma_2} = 0 \tag{2.93}$$

$$\text{optimal tax:} \quad \frac{dW_2}{dt} = 0 \tag{2.94}$$

When $m = 0$, we can simplify equation 2.93:

$$\sigma_2 = \frac{1}{1 + 2(1 - b_2)} \tag{2.95}$$

This is similar to the equation 2.60 for s_2 in the case of process R&D. That is, when demands are independent, the home country chooses to subsidize the product R&D to the home firm.

The model structure does not allow for the closed form solutions for the general case, $m \neq 0$, of tax and subsidy policies. These equations will be solved numerically in the final section.

2.7 Policy Analysis

In this section, we will discuss how optimal tax and subsidy rates depend on the degree of substitutability m, and degree of network externalities,b1,b2. The theoretical models for optimal tax and subsidy for the home country for both cases cannot be solved in the closed form when $m \neq 0$. By assigning certain numerical values to coefficients, we could solve the theoretical models numerically for optimal tax and subsidies. By changing m, b_1,and b_2 in certain ranges, we can simulate the model to see how tax and subsidies respond.

2.7.1 Calibration and simulation

2.7.1.1 Calibration the degree of substitutability

To our best knowledge, there is little empirical literature that would allow us to calibrate the model parameters. It is difficult to find a relevant empirical observation for comparison. Instead, we calibrate the parameters by following assumptions for the existence of an interior solution, and varify that the solutions are economically meaningful. For instance, $a > 0$ is a positive component of the willingness to pay. In the absence of product R&D, $A_i = a = p_i$ means the highest willingness to pay with zero demand is the same as network externality which equals price. We set the value of a as 1. Since we assume network effect is an increasing function of market aggregate demand, $0 < b_i < 1$, we set $b_1 = 0.15$ and $b_2 = 0.1$. Marginal costs are set to $c_1 = 0.7$ and $c_2 = 0.9$. Coefficients of R&D efficiency are scalars which are set to 2 for both product R&D and process R&D.

The determination of m is crucial and more or like guesswork. Ultimately, any chosen value of m must give the positive quantity of output as $q_1 > 0, q_2 > 0$. We also restrict corresponding numerical results of price(p_1, p_2), quality improved by product R&D (r_1, r_2), marginal cost reduced by process R&D (k_1, k_2), utility and welfare (U, W_1, W_2) must also be positive. We also require conditions (2.30 ,2.31, and 2.32) must be held with each value of m. Furthermore, we exclude cases that satisfy all previous conditions, but which generates a home tax that is unrealistically high. For example, a tax rate set to 50% is highly unlikely in practice. With all previous restrictions, in the case that two goods are substitutes, we limit the value of m lies in the range $[0.25, 0.42]$ for process R&D, and $[0.2, 0.4]$ for product R&D. The range of m if two goods are complementary is $[-0.6, 0)$ and $[-0.7, 0)$. We exclude $m = 0$ when defining ranges for complementary goods. However, $m = 0$ is used in the simulation

2.7.1.2 Calibration the degree of network externality

To calibrate the degree of network externality for foreign and home goods, $b1$ and b_2 respectively, we have fixed m in the range from the pre-calibrated values. Regarding process R&D, we set $b1$ and b_2 in the range $[0, 0.4]$ for both substitute and complementary goods. For product R&D, we choose $[0, 0.45]$ as the range for $b1$ and b_2. Details can be found in table 2.1.

Table 2.1: Calibration for m,b_1,b_2

R&D	Good Type	m	b_1	b_2
Process R&D	Substitution	[0.25,0.42]	[0,0.4]	[0,0.4]
	Complementary	[-0.6,0)		
Product R&D	Substitution	[0.25,0.44]	[0,0.45]	[0,0.45]
	Conplementary	[-0.7,0)		

2.7.1.3 Numerical simulation

To analyse how optimal tax and subsidies depend on the degree of the network externality, we consider strong and weak substitutes/complementary to analyse changes in $b1$ and b_2 for both process and product R&D. That is,taking process R&D as an example, we choose one upper value and one lower value of m from the pre-calibrated range for substitution good $[0.25, 0.42]$. For each fixed m, we simulate how optimal tax and subsidies change in the $b_1 \in [0, 0.4]$ or $b_2 \in [0, 0.4]$. For complementary goods, we also fix two values of m first and simulate optimal tax and subsidies in $b_1 \in [0, 0.4]$ or $b_2 \in [0, 0.4]$ after. For the case of product R&D, the simulation procedure for both $b1$ and b_2 are the same. In total, there are 16 simulations. Details of each set of parameters are given in the Appendix B Figure A11-A26.

2.7.2 Process R&D

In this section, how optimal tax and subsidies respond to changes in model parameters m, b_1, and b_2 with the existence of process R&D will be presented.

2.7.2.1 Degree of substitution

Figure C.1 shows how optimal tax and subsidies change in the degree of substitution when home goods and foreign goods are substituted. The optimal subsidy for foreign goods, s_1, shows an inverted-U shape relation to m. When the goods are weak substitutes (small m), s_1 is positive, meaning that the foreign government subsidies its firm to increase its competitiveness. As the goods become closer substitutes (m increases), the home country decreases the tax on foreign good and, at the same time, decreases the subsidy to its firm. For sufficiently large m, home country's subsidy becomes negative, i.e. home country starts taxing its firm when two goods are sufficiently strong substitutes. As the home country lowers subsidies to its firm, the competitive pressure on foreign goods decreases which gives the foreign government incentive to lower the subsidy rate. This helps explain the inverted-U shape of s_1 as a function of m: the foreign subsidy rate first increases with the degree of substitutability and then decreases.

Figure C.2 presents changes in profit and investment responding to m. When consumers view two goods as closer substitutes, this leads to a monotonically increase in foreign firm profit, but a dramatic decrease in home firm profit. Foreign firm investment in process R&D shows the same pattern as foreign subsidy s_1 when the degree of substitute is higher. Strong substitution leads to a reduce in the investment of home firms.

Welfare and utility are shown in Figure C.3. The welfare of the foreign country shows a monotonic increase in m. This means profit gained exceeds subsidy costs and such surplus expands as the degree of substitution increasing. For the home country, the profit of the home firm, home consumer's utility, and tax revenue from foreign goods contribute to the increase in-home welfare. Figure C.3(b) shows that home welfare is slightly increasing, while m increases and s_2 is positive. This means welfare gain is greater than the cost of the subsidy. However, home welfare dramatically decreases after reaching the highest point. This is because firm profit, consumer utility, and tax revenue are reducing as the substitution between home goods and foreign good is stronger. When s_2 turns into negative, the additional tax applied on home firms, combining a drop in process R&D investment, the competitiveness of home goods reduces further. The significantly reduction in-home profit and consumer utility dominates tax revenue gained. Therefore, the aggregate welfare of the two countries desreases in m because the reduction of home welfare dominates the increase in foreign welfare, as shown in Figure C.3(c).

When two goods are complementary, how optimal tax and subsidies change in m is illustrated in Figure C.4. Both foreign and home subsidies decrease while the two goods are becoming weaker complements Foreign government stops subsidising their firm when two goods are independent, i,e $m = 0$. Once can notice s_2 is always positive even when two goods are independent. This means the home country will always subsidize its firm as long as two goods are not substitutes. Tax levied on the foreign product reduces first and then increases in m.

Profits of foreign and home countries decrease in m, as shown in C.5(a) and Figure C.5(b). Investment in process R&D also decreases for both countries while the two goods become weaker complements. One can notice foreign firms still invest in process R&D with the absence of subsidy when two goods are independent.

As a result of decreasing profits and decreasing consumer utility, which is shown in Figure C.6(d), both foreign and home country welfare and the aggregate welfare decrease in m.

2.7.2.2 Degree of network externality

Figure C.11 , Figure C.12, Figure C.13 , and Figure C.14 show the simulated results regarding the change in $b1$ and b_2, considering weak and strong substitution. These Figures clearly show that utility and welfare increase when the network externalises are greater, given two goods are substitutes. With higher network externality, demand for foreign goods, foreign investment, and foreign profit are higher while these Figures are smaller for the home country. Higher network externality boosts the competitiveness of foreign goods. However, the subsidies show different results at different levels of b_1 when $m = 0.4$ and $m = 0.25$. While the home government may set a lower optimal subsidy as a higher degree of the network externality, this effect is stronger when the goods are stronger substitutes. In addition, the foreign subsidy rate stops increasing, and starts to fall when the network externalities are high enough. This might be because due to an increase in the degree of externality, the competitiveness of foreign goods is strong enough to compete with home goods. Thus, it might not be necessary for a foreign government to subsidise its firm for extra international competitiveness.

Figure C.15, Figure C.16, Figure C.17, and Figure C.18 show how how endogenous variables respond to a different level of network externality when two goods are complementary. The patterns are very similar between weak complementarity and strong complementarity except for the home country's tax policy. That is, tax has inverted-U shape relation with b_1 and b_2 if two goods are strongly complementary, but it is monotonically decreasing in b_1 and b_2 if two goods are weak complementary. This might be because, while two goods are strongly complementary, demand increases due to a high degree of network externality for foreign goods, and will not be hurt much by applying a tax on the foreign product. The home country will be better off from the increased tax revenue. If two goods are weak substitutes, the home country will be better off by encouraging more demand for foreign goods.

2.7.3 Product R&D

This section presents how optimal tax and subsidies respond to changes in model parameters m, b_1, and b_2 with the existence of product R&D.

2.7.3.1 Degree of substitution

Figure C.7 presents the simulation results for optimal tax and subsidies regarding changing in $m \in [0.2, 0.4]$. The optimal subsidies have the same pattern as in the case of process R&D. Home government set a lower subsidy rate when two goods are strong substitutes. Optimal tax decreases in the degree of substitution when two goods are strong substitutes.

Figure C.8 shows the changes in optimal profit and investment in product R&D in m. Foreign firm profit increases steadily when two goods are weak substitutes till such substitute relation becomes stronger. Profit increasing is the result of the positive net benefit of investing in product R&D. As investment in product R&D by the foreign firm rises, the highest willingness to pay $a + r_1$ is increased when two goods are strong substitutes. Together with the decreasing q_2 in Figure C.9(d), one can notice that p_1 is higher if two goods are more similar, Figure C.9(d). However, a constant increase in price will discourage demanding more foreign goods when two goods are strongly substitutable, as shown by the trajectory at the end of the curve in C.9(c). Accordingly, the profit of the foreign firm drops when the degree of substitution is large enough. This is an interesting result because investment in product R&D could lead to a higher willingness to pay and a strong preference for the product, which not only increases the market share but also increase the firm's profitability, at least when m is small enough. For a home firm, both price and output fall when two goods become strong substitutes, which leads to a monotonic decreasing profit.

Welfare and utility when two goods are substitutes are shown in Figure C.10. These four graphs present the same pattern. Foreign country welfare starts decreasing when profits decrease. Therefore, because the cost of subsidy exceeds the benefit gained from the firm, which is shown in Figure C.7(a), when the degree of substitution is large enough, the foreign subsidy starts to fall. Home welfare is mainly driven by the consumer's utility. Thus, even when the home firm's profit is smaller when two goods are strong substitutes, the home country is still better off.

2.7.3.2 Degree of network externality

Figure C.20 , Figure C.19, Figure C.24 , and Figure C.19 show the simulated results regarding the network externality when two goods are substitutes. The Figures show that a higher degree of network externality leads to both countries being better off in terms of welfare. While the network externality in consumption of good 1 is greater, home firm profit, investment in product R&D, output, and price are lower. This means the effect of an increase in the network externality for foreign goods is significant on the home product. However, the home country's welfare is increasing in $b1$ means that welfare gained from consumer's utility compensates for the falls in firm profit.

Figure C.21 , Figure C.22, Figure C.25 , and Figure C.26 present the simulated results regarding the change in $b1$ and b_2 when two goods are complementary. All results show positive relation to the network externality except for the optimal tax. This is because the home firm and consumer benefit more from a lower tax rate when the network externality is greater.

2.8 Conclusion

This chapter presents a theoretical model of taxation for multinational businesses strategically competing in an international digital economy with network externality in consumption, significant market power of the producers, and investment in process and product R&D. We contribute to the literature of understanding of how to design an international tax law system with the following findings. First, investments in both process R&D and product R&D can always be boosted by subsidies from own government. Firm's decisions on outputs and investments in R&D are negatively affected by home country's tax policy unless two goods are complementary. If two goods are independent, home government's subsidy policies will not influence foreign firm's output and investment decisions. Second, effect of network externality always has a positive effect on own firm's output and investments in R&D. The cross effect of network externality is negative if two goods are substitutes. Such effect turns into positive when two goods are complementary. If two goods are independent, one good's network externality will not affect the other firm's output and investment decisions. Third, foreign subsidy policy does not depend on home country's tax. When two goods are independent, optimal subsidy policy for the foreign government is zero. Otherwise, foreign country will always subsidize even when there is no subsidy policy on home country, no matter two goods are substitutes or complements. Home country will always subsidize own firm even if two goods are independent. Finally, aggregate welfare in the case of process R&D increase when two goods are weak substitutes but dramatically decreases when two goods are strong substitutes. Foreign country's high welfare cannot compensate the loss of the home country in this case. However, welfare in the case of product R&D decreases when two goods are weak substitutes but increases when two goods are strong substitutes. If two goods are complementary, with the presence of tax policy, welfare in both countries decreases when two goods are weak complementary. In all simulations, network externality presents substantial positive effect on welfare.

Subsidy rate can be lower when the network externality is high enough. The strong network externality suggests that home tax should be low so that both countries are better off.

Chapter 3

Taxing the Clouds: An Economic Analysis

3.1 Introduction

The advent of a digital economy under the definition of permanent establishment (PE) challenges governments' ability to tax business income from operating in the international market. Businesses in the digital economy can relocate their physical facilities to tax heaven where they can benefit from the low effective tax rates while earn profits from customers based in the countries where businesses may have little or no physical presence. For example, a business that has a digital platform operating in search engines can install computer servers and data storage facilities in a low-tax jurisdiction and derive a substantial part of its profits from selling cloud-computing services (such as advertising) to customers in high-tax jurisdictions where it will not be liable for profit tax. This is because it does not have a physical presence, for the PE threshold purpose.

The concept of the permanent establishment (PE) was introduced in the late 19th century when there was an expeditious change due to the second industrial revolution (Hoffart, 2007). The purpose of PE was to avoid double taxation in the international market, and the concept persisted mainly through the OECD

Model Tax Convention (OECD, 2014*b*). The flourishing of the digital economy urges redefining PE or changing international tax rules for digital businesses as the tax laws for traditional economy are becoming obsolete.

Cockfield (2002) suggests that a sensible solution to the current challenge of digitialization is to shift PE that uses the physical presence of production to a new rule using economic presence at the location of consumption. Indeed, suggestions for the tax reform embrace this idea (Hoffart, 2007). However, the implementation of the idea around economic presence still lacks a definite plan. A Two-Pillar solution was introduced, which was accepted by 136 tax jurisdictions in 2021 (OECD, 2021). In short, Pillar One pledges equitable distribution of profits and taxing rights among countries where large multinational enterprises(MNEs) generate profits. Pillar Two proposes a global minimum tax. Specifically, implementing Pillar One ceases current Digital Services Taxes(DST) which arguably has a negative impact on international trade. However, there are also problems with Pillar One. For example, tax assessment to eliminate double taxation is time-consuming and costly. And, Pillar one only considers large MNEs.

In this paper, we propose a tax reform based on the idea of division of the tax base. The residence country and source country mutually agree on the shares of profits of a company in digital business through a Multilateral Convention. While the details of such an agreement can be complex and can depend on the exact nature of the business and the assessed shares of revenues and costs attributable to each state involved, useful insights can be gained from a stylized framework developed in this paper. An agreement on taxing the shares of profits is modeled as an outcome of strategic interaction between the resident and the source countries. Firstly, we consider a two-country model with two approaches: a non-cooperative approach and a cooperative approach. The non-cooperative approach means each country decides the proportion of the firm's profits earned in the source country to tax. We consider a simple case when there is no profit split. Next, we introduce a profit split and analyse the situation where a firm earns profits from both countries. The cooperative solution is the case when two countries jointly decide how to share the taxable profits earned in the source country. In this case, we first consider a simple case when a firm earns all profits from sales in the source country. Next, we relax this assumption and analyse the situation where a firm also earns profits in the residence country. Secondly, we investigate the effect of more than one source country.

The strategic interaction is modelled as a two-stage game. In the first stage, countries choose their own shares of profits earned in the source country to be taxed by both. In the second stage, the firm decides on the allocation of its productive resource, taking the tax policies as given. The game is solved by backward induction. We analyse the first stage using the concept of coupled-constrained Nash Equilibrium in the non-cooperative approach, and cooperative Nash bargaining in the cooperative approach.

3.2 A two-country model

There are two countries in the model. Assuming a firm operates in a digital business that is registered in one country and sells digital goods and/or services to both countries, we define the country in which the firm is registered for a legal purpose as the residence country and the other one as the source country. The gross profit earned in the two countries is:

$$\pi = \pi_R + \pi_S$$

where π_R is the profit generated from the residence country and π_S is the profit gained from the source country. Productions are identical in the two countries. We assume that productions require two inputs: productive resources and public infrastructure (k). Production functions have a Cobb-Douglas form. Profits are assumed to be linear in outputs in both countries.

The firm divides its fixed one-unit productive resource between the residence and the source, in proportions $1-\lambda$ and λ respectively, with $\lambda \in (0,1)$. With productivity being A_R, the profit function in the residence country is:

$$\pi_R = A_R(1-\lambda)^{1-\eta}k_R^{\eta}$$

Similarly, we have the following profit equation for the source country:

$$\pi_S = A_S\lambda^{1-\eta}k_S^{\eta}$$

where A_S is the productivity in the source country. The firm also needs to pay taxes by on profits generated in both countries. The total tax, T, the firm pays includes tax paid to the residence country T_R, and tax paid to the source country T_S :

$$T = T_R + T_S$$

3.3 Non-Cooperative Approach

There are two cases in a non-cooperative situation: no profit splitting and profit splitting.

3.3.1 Non-splitting

In the absence of profit splitting, the source country gains zero tax revenue $T_S = 0$. Total tax revenue is the tax revenue earned by the residence country government with tax rate t_R, which is:

$$T = T_R = t_R\pi = t_R(\pi_R + \pi_S)$$

3.3.2 Profit Splitting

In the case of profit splitting, profits earned in the source country are taxed by both the residence and the source country. Under the profit-split agreement, share μ_R is taxed by the residence country and share μ_S is taxed by the source country . Profits earned in the residence country are subject to tax only in the residence country. Tax revenues in the residence and the source country can be written as:

$$\begin{aligned} T_R &= t_R\pi_R + t_R(\mu_R\pi_S) \\ T_S &= t_s(\mu_S\pi_S) \end{aligned}$$

Therefore, the net profit after tax is the following:

$$\begin{aligned} \hat{\pi} = \pi - T = (\pi_R + \pi_S) - (T_R - T_S) \\ &= (1 - t_R)\pi_R + (1 - t_R\mu_R - t_S\mu_S)\pi_S \\ &= (1 - t_R)A_R[(1-\lambda)]^{1-\eta}k_R^\eta + (1 - t_R\mu_R - t_S\mu_S)A_S(\lambda)^{1-\eta}k_s^\eta \end{aligned} \quad (3.1)$$

The firm's objective is to maximise the net profit after tax by choosing λ, the allocation of its productive resource.

The profit-splitting can be described as a sequential move game. There are two players in the first stage: the residence country; and the source country. They choose their own strategy (μ_R and μ_S) to maximise their payoff function, (for example, the welfare). One country's payoff function depends not only on its own strategy, but also on the other's. We also assume information in this game is perfect. Countries know how the firm decision will be affected by μ_R and μ_S. Countries simultaneously decide μ_R and μ_S. In the second stage, the firm observes μ_R and μ_S with given tax rates and chooses the allocation of productive resources, λ. Thus, λ is a function of μ_R and μ_S.

The firm is not allowed to decide which country is the residence country before making the allocation decision of its productive resource. If choosing a location is allowed, this means that the firm could potentially de-register from the previous residence country, and register in the new residence country for every project. This is not allowed by company law in many countries. For example, according to the international manual under HMRC internal manual, "*Under company law relating to the United Kingdom a company cannot move its place of incorporation or registered office out of the country in which it is domiciled whilst at the same time retaining its identity ... In English law an English company which uses those procedures and is struck off the English register ceases to exist. The company, now foreign incorporated, is considered to be a different company*". This means the firm loses its identity. Furthermore, changing its residence country can be challenged by the tax authority precisely on the grounds of there being no business purpose, and tax avoidance being the only reason for this change. Thus, assuming the firm can choose a residence country, it might make sense economically, but in reality it is against company law in many countries.

This game is solved by backward induction.

3.3.2.1 Firm Decision

By taking profit-splitting shares and tax rates as given, the firm maximises the net tax profit described by equation 3.1. The interior solution has the following form:

$$\lambda_n = \frac{\epsilon}{1+\epsilon} \tag{3.2}$$

where λ_n is the optimal allocation under net profit after tax. And:

$$\epsilon = \left[\frac{b_S}{b_R}\right]^{\frac{1}{\eta}}$$
$$b_S = (1 - \mu_R t_R - \mu_S t_S) A_S k_S^{\eta}$$
$$b_R = (1 - t_R) A_R k_R^{\eta}$$

3.3.2.2 Government Decision

The residence country's welfare is defined as the sum of its own tax revenues and the net profit of the firm, and the source country's welfare is defined as its tax revenue. That is:

$$W_R = \hat{\pi} + T_R = \pi_R + (1 - \mu_S t_S)\pi_S \tag{3.3}$$
$$W_S = \mu_S t_S \pi_S \tag{3.4}$$

Both countries anticipate the firm's production decision when choosing the profit-splitting shares. The objective for each country is to maximise its own welfare taking the other country's choice as given, the outcome, if it exists, is the Nash equilibrium. However, it is reasonable to assume that the choices of the residence and the source country must satisfy the constraint $\mu_R + \mu_S = 1$. This condition avoids double taxation in that corporate income tax is paid twice on the same source of income, and also emphasizes that no income should go un-taxed (Avi-Yonah, 1997). Since the two countries' strategies are joint, the solution is the coupled-constraint equilibrium, and there will be typically infinitely many equilibria. The objectives of the two countries are the following:

$$\max_{\mu_R}\{W_R : \mu_R + \mu_S - 1 \geq 0\}$$
$$\max_{\mu_S}\{W_S : \mu_R + \mu_S - 1 \geq 0\}$$

The constraint is binding with strict equality at equilibrium, which is shown below.

For country i, $i \in [R, S]$, the Lagrangian of the constrained optimisation problem with a Kuhn-Tucker multiplier u_i, is defined as:

$$\mathcal{L}_i = W_i + u_i(1 - \mu_R - \mu_S) \tag{3.5}$$

In the equilirium, the following conditions hold:

$$\begin{aligned} \frac{\partial \mathcal{L}_R}{\partial \mu_R} &= 0 \\ \frac{\partial \mathcal{L}_S}{\partial \mu_S} &= 0 \\ u_i &\geq 0 \\ u_i(1 - \mu_R - \mu_S) &= 0 \\ 1 - \mu_R - \mu_S &\geq 0 \end{aligned}$$

Following Rosen (1965) approach, the equilibrium is normalized if the multiplier u_i, $i \in [R, S]$, is co-linear with a common u_0, such that the normalized equilibrium point is:

$$u_i = \frac{u_0}{r_i} \tag{3.6}$$

where r_i is a weight assigned to country i. Then, the Lagrangian is:

$$\mathcal{L}_0(\mu_i, u_o) = r_i W_i + u_0(1 - \mu_R - \mu_S)$$

The Kuhn-Tucker conditions thus can be written as:

$$\begin{aligned}
\frac{\partial \mathcal{L}_0(\mu_R, u_o)}{\partial \mu_R} &= r_R \frac{\partial W_R}{\partial \mu_R} - u_0 = 0 \\
\frac{\partial \mathcal{L}_0(\mu_S, u_o)}{\partial \mu_S} &= r_S \frac{\partial W_S}{\partial \mu_S} - u_0 = 0 \\
u_0 &\geq 0 \\
u_0(1 - \mu_R - \mu_S) &= 0 \\
1 - \mu_R - \mu_S &\geq 0
\end{aligned}$$

And therefore, we have:

$$r_R \frac{\partial W_R}{\partial \mu_R} = r_S \frac{\partial W_S}{\partial \mu_S}$$

which shows that the weighted shadow price of the constraints should be the same for the residence country and the source country.

Let $\rho \equiv \frac{r_S}{r_R} > 0$. Thus, for a given $\rho \in (0, \infty)$ the profit split in an interior equilibrium satisfies:

$$\begin{aligned}
\frac{\partial W_R}{\partial \mu_R} - \rho \frac{\partial W_S}{\partial \mu_S} &= 0 \\
1 - \mu_R - \mu_S &= 0
\end{aligned}$$

Note that, in general, the equilibria will be in the interior only for a subset of the values of ρ, and, depending on the model parameters, this subset can be empty. By construction, ρ is the ratio of the resident country's shadow price of the constraint to the source country's shadow price of the constraint:

$$\rho = \left(\frac{\partial W_R}{\partial \mu_R}\right) / \left(\frac{\partial W_S}{\partial \mu_S}\right) \tag{3.7}$$

where

$$\begin{aligned}
\frac{\partial W_R}{\partial \mu_R} &= \frac{1-\eta}{\eta}\frac{t_R \pi_R \lambda}{1 - t_s\mu_S - t_R\mu_R} + \frac{1-\eta}{\eta}(1 - t_S\mu_S)\frac{(1-\lambda)\pi_S t_R}{1 - t_s\mu_S - t_R\mu_R} \\
\frac{\partial W_S}{\partial \mu_S} &= -\frac{1-\eta}{\eta} t_S\mu_S \frac{(1-\lambda)\pi_S t_S}{1 - t_s\mu_S - t_R\mu_R} + t_S\pi_S
\end{aligned}$$

Therefore

$$\begin{aligned}
\rho &= \left(\frac{\partial W_R}{\partial \mu_R}\right) / \left(\frac{\partial W_S}{\partial \mu_S}\right) \\
&= \frac{1-\eta}{\eta}\frac{t_R^2}{t_s^2}\frac{1}{1-t_R}\frac{1 - t_s\mu_S - \mu_R}{\frac{1 - t_s\mu_S - t_R\mu_R}{t_S(1-\lambda)} - \mu_S\frac{1-\eta}{\eta}}
\end{aligned}$$

Defining $\mu = \mu_S = 1 - \mu_R$, the equilibrium ρ can be written as:

$$\rho = \frac{1-\eta}{\eta}\frac{1-t_S}{1-t_R}\left(\frac{t_R}{t_S}\right)^2 \frac{\mu}{\frac{1-t_R}{t_S}\left[1 + \frac{A_R k_R^\eta}{A_S k_S^\eta}\cdot \epsilon^{1+1/\eta}\right] - \mu\left[\frac{1}{\eta} - \frac{t_R}{t_S}\right]}$$

This can be rearranged as:

$$\mu = \frac{\rho\delta}{1 + \rho\delta\left(\frac{1}{\eta} - \frac{t_R}{t_S}\right)}\frac{1-t_R}{t_S}\left[1 + \left(\frac{A_S}{A_R}\right)^{1/\eta}\frac{k_S}{k_R}\left(1 - \mu\frac{t_S - t_R}{1 - t_R}\right)^{1+1/\eta}\right] \tag{3.8}$$

where

$$\delta = \frac{1-\eta}{\eta}\frac{1-t_S}{1-t_R}\left(\frac{t_R}{t_S}\right)^2 > 0$$

Let:

$$h(\mu) = \frac{\rho\delta}{1+\rho\delta\left(\frac{1}{\eta}-\frac{t_R}{t_S}\right)}\frac{1-t_R}{t_S}\left[1+\left(\frac{A_S}{A_R}\right)^{1/\eta}\frac{k_S}{k_R}\left(1-\mu\frac{t_S-t_R}{1-t_R}\right)^{1+1/\eta}\right] \tag{3.9}$$

Solving equation $\mu = h(\mu)$ gives the equilibrium profit split μ^*. While equation 3.9 does not have a closed-form solution, we can describe the conditions on the model parameters under which the equilibrium exists for some ρ. Assuming the firm locates in a residence country because of the low tax rate, i.e. $t_R < t_S$, we have the following derivative:

$$\frac{\partial h(\mu)}{\partial \mu} = -\frac{t_S-t_R}{t_S}\frac{\rho\delta}{1+\rho\delta\left(\frac{1}{\eta}-\frac{t_R}{t_S}\right)}\left[1+\left(\frac{A_S}{A_R}\right)^{1/\eta}\frac{k_S}{k_R}\left(1-\mu\frac{t_S-t_R}{1-t_R}\right)^{1/\eta}\right] < 0$$

Note that for $\mu = 0$

$$h(\mu) = \frac{1-t_R}{t_S}\frac{\rho\delta}{1+\rho\delta\left(\frac{1}{\eta}-\frac{t_R}{t_S}\right)}\left[1+\left(\frac{A_S}{A_R}\right)^{1/\eta}\frac{k_S}{k_R}\right] > 0$$

and, for $\mu = 1$

$$h(\mu) = \frac{1-t_R}{t_S}\frac{\rho\delta}{1+\rho\delta\left(\frac{1}{\eta}-\frac{t_R}{t_S}\right)}\left[1+\left(\frac{A_S}{A_R}\right)^{1/\eta}\frac{k_S}{k_R}\left(\frac{1-t_S}{1-t_R}\right)^{1+1/\eta}\right] > 0$$

That is, the right-hand side of equation 3.9 is a strictly decreasing convex function, strictly positive for $0 \le \mu \le 1$. μ^* thus can be found at the intersection of $h(\mu)$ and a 45-degree line, i.e. $\mu = \mu$. Deine m as:

$$m \equiv h(1) = \frac{\rho\delta\psi}{1+\rho\delta\left(\frac{1}{\eta}-\frac{t_R}{t_S}\right)} \tag{3.10}$$

where

$$\psi = \frac{1-t_R}{t_S}\left[1+\left(\frac{A_S}{A_R}\right)^{1/\eta}\frac{k_S}{k_R}\left(\frac{1-t_S}{1-t_R}\right)^{1+1/\eta}\right] \tag{3.11}$$

Now we can give the full characterisation of the coupled-constraint equilibrium.

For $m < 1$, there is a uniue $\mu = \mu^* \in (0,1)$ for a given value of ρ:

$$m \equiv \frac{\rho\delta\psi}{1+\rho\delta\left(\frac{1}{\eta}-\frac{t_R}{t_S}\right)} < 1 \Longleftrightarrow \rho\delta\phi(\eta) < 1 \tag{3.12}$$

where

$$\phi(\eta) = -\frac{1}{\eta}+\frac{1}{t_S}\left[(1-t_S)\left(\frac{A_S}{A_R}\frac{1-t_S}{1-t_R}\right)^{1/\eta}\frac{k_S}{k_R}+1\right]$$

To see how condition 3.12 holds, each component need to be considered. Since $\rho = r_S/r_R > 0$ and $\delta > 0$, given equation $\phi(\eta)$, condition 3.12 holds (i) for any $\rho > 0$ if $\phi(\eta) \leq 0$ and (ii) for any $0 < \rho < 1/\delta\phi(\eta)$ if $\phi(\eta) > 0$.

To fully describe $\phi(\eta)$, we will need the properties of its derivative:

$$\frac{\partial\phi(\eta)}{\partial\eta} = \frac{1}{\eta^2}\left[1-\frac{1-t_S}{t_S}\left(\frac{A_S}{A_R}\frac{1-t_S}{1-t_R}\right)^{1/\eta}\frac{k_S}{k_R}\ln\left(\frac{A_S}{A_R}\frac{1-t_S}{1-t_R}\right)\right]$$

We need consider two cases:

a) $\ln\left(\frac{A_S}{A_R}\frac{1-t_S}{1-t_R}\right) \leq 0$ if $\frac{A_S}{A_R}\frac{1-t_S}{1-t_R} \leq 1$. In this case, $\frac{\partial\phi(\eta)}{\partial\eta} > 0$ for all $\eta \in (0,1)$. Therefore, $\phi(\eta)$ is an increasing function of η for all $\eta \in (0,1)$ if $\frac{A_S}{A_R}\frac{1-t_S}{1-t_R} \leq 1$. Also,

$$\lim_{\eta\to 0}\phi(\eta) = -\infty$$
$$\lim_{\eta\to 1}\phi(\eta) = -1+\frac{1}{t_S}\left[1+(1-t_S)\left(\frac{A_S}{A_R}\frac{1-t_S}{1-t_R}\frac{k_S}{k_R}\right)\right] > 0$$

Thus, $\exists! \eta^* \in (0,1)$ so that $\phi(\eta) \lesseqqgtr 0$ for each $\eta \lesseqqgtr \eta^*$.

b) $\frac{A_S}{A_R}\frac{1-t_S}{1-t_R} > 1$. In this case

$$\lim_{\eta \to 0} \phi(\eta) = \infty$$
$$\lim_{\eta \to 1} \phi(\eta) = -1 + \frac{1}{t_S}\left[1 + (1-t_S)\left(\frac{A_S}{A_R}\frac{1-t_S}{1-t_R}\frac{k_S}{k_R}\right)\right] > 0$$

which indicates $\phi(\eta)$ is a decreasing function in $\eta \in (0,1)$. Furthermore, we need to verify the sign of the $\phi(\eta)$. Let η_0 be such that $d\phi/d\eta = 0$ at $\eta = \eta_0$. Then

$$\phi(\eta_0) = -\frac{1}{\eta_0} + \frac{1}{t_S}\left[(1-t_S)\left(\frac{A_S}{A_R}\frac{1-t_S}{1-t_R}\right)^{1/\eta_0}\frac{k_S}{k_R} + 1\right]$$

such that

$$0 = \left.\frac{d\phi}{d\eta}\right|_{\eta=\eta_0} = \frac{1}{\eta_0^2}\left[1 - \frac{1-t_S}{t_S}\left(\frac{A_S}{A_R}\frac{1-t_S}{1-t_R}\right)^{1/\eta_0}\frac{k_S}{k_R}\ln\left(\frac{A_S}{A_R}\frac{1-t_S}{1-t_R}\right)\right]$$
$$\Rightarrow \frac{1}{\ln\left(\frac{A_S}{A_R}\frac{1-t_S}{1-t_R}\right)} = \frac{1-t_S}{t_S}\left(\frac{A_S}{A_R}\frac{1-t_S}{1-t_R}\right)^{1/\eta_0}\frac{k_S}{k_R} > 0$$

and

$$\phi(\eta_0) = -\frac{1}{\eta_0} + \frac{1}{t_S}\left[(1-t_S)\left(\frac{A_S}{A_R}\frac{1-t_S}{1-t_R}\right)^{1/\eta_0}\frac{k_S}{k_R} + 1\right]$$
$$= -\frac{1}{\eta_0} + \frac{1}{t_S} + \frac{1}{\ln\left(\frac{A_S}{A_R}\frac{1-t_S}{1-t_R}\right)} > 0$$

Therefore, we can conclude that $\phi(\eta)$ is decreasing and strictly positive for $\eta \in (0,1)$ when $\frac{A_S}{A_R}\frac{1-t_S}{1-t_R} > 1$

And, for $m = h(1) \geq 1$, we have a corner solution $\mu = \mu^* = 1$ for a given value of ρ:

$$m \equiv \frac{\rho\delta\psi}{1 + \rho\delta\left(\frac{1}{\eta} - \frac{t_R}{t_S}\right)} \geq 1 \iff \rho\delta\phi(\eta) \geq 1 \tag{3.13}$$

Condition 3.13 holds for $\rho \geq 1/\delta\phi(\eta)$ if $\phi(\eta)$ is positive. Let $\bar{\rho} \equiv 1/\delta\phi(\eta)$

Proposition 1 *In the coupled-constraint equilibrium, given ρ, the equilibrium profit split μ^* exists and is unique for each given value of ρ, so that*
(a) if $\phi(\eta) > 0$ then (i) $\mu^* \in (0,1)$ is the unique solution of equation 3.8 for $0 < \rho < \bar{\rho}$ and (ii) $\mu^* = 1$ for $\rho \geq \bar{\rho}$;
(b) if $\phi(\eta) \leq 0$ then $\mu^* \in (0,1)$ is the unique solution of equation 3.8 for any $\rho > 0$.

3.3.2.3 Aggregate Welfare

The aggregate welfare is the sum of two countries' welfares, and it is equal to the pre-tax profit of the firm.

$$
\begin{aligned}
W = W_R + W_S &= \pi_R + (1 - \mu_S t_S)\pi_S + \mu_S t_S \pi_S = \pi_R + \pi_S \\
&= A_R[(1-\lambda)]^{1-\eta} k_R^\eta + A_S[\lambda]^{1-\eta} k_S^\eta
\end{aligned}
$$

The social optimum $\lambda = \lambda_a$ can be solved from the following derivative:

$$
\begin{aligned}
\left.\frac{dW}{d\lambda}\right|_{\lambda=\lambda_a} &= 0 \\
\Rightarrow \lambda_a &= \frac{1}{1 + \left(\frac{A_R k_R}{A_S k_S}\right)^{1/\eta}}
\end{aligned}
$$

Under profit split, if the proportion of productive resources allocated to the source country, λ_n (equation 3.2), both maximized aggregate welfare W, and net profit after tax $\hat{\pi}$, then $\lambda_n = \lambda_a$, which must be true for the following condition:

$$
\begin{aligned}
\frac{1}{1 + \left[\frac{A_R k_R}{A_S k_S}\right]^{1/\eta}} &= \frac{1}{1 + \left[\frac{(1-t_R)A_R k_R^\eta}{(1-\mu_R t_R - \mu_S t_S)A_S k_S^\eta}\right]^{1/\eta}} \\
\Rightarrow 1 &= \frac{1 - t_R}{1 - t_R + t_R \mu_R - t_S \mu_S}
\end{aligned}
$$

which can be simplified as:

$$\mu(t_R - t_S) = 0 \tag{3.14}$$

To maximise aggregate welfare, condition 3.14 must hold. Therefore, (i) $\mu = 0$ if $t_R \neq t_s$. This means the source country should not tax the firm's profit if tax rates in the two countries are different. Or (ii) $t_R = t_S$ for any $\mu \in (0,1)$. That is if the two countries have the same tax rates any profit split is optimal.

3.4 Cooperative Approach

In the cooperative approach we define the equilibrium as the cooperative Nash bargaining solution. The equilibrium profit split will depend on the relative bargaining power of the two countries.

3.4.1 Case 1. Sales only in the source country

3.4.1.1 Firm Decision

In this case, we assume that there are no sales in the residence country. Instead, the firm is registered in the residence country exports its all outputs to the source country. Production in the residence country requires productive resources, and public infrastructure, k. The production function is Cobb-Douglas, and we assume profit is linear in output. The pre-tax profit is:

$$\pi = Ak^{\eta}$$

The residence country and the source country bargain over the split of profits subject to tax. Let $\mu \in [0,1]$ be the profit share taxed by the source government.

The net profit after tax is:

$$\hat{\pi} = \pi - t_S(\mu\pi) - t_R(1-\mu)\pi$$

where t_S and t_R are tax rates set by source country and residence country, respectively.

3.4.1.2 Government Decision

The residence and the source country want to divide the profit to gain tax revenue. If there is no agreement between the two countries, firms do not produce, and welfare in both countries is zero, i.e. $W_R^D = W_S^D = 0$.

If two countries agree to profit split, the objectives for the source country and residence country are to maximise their own welfare and can be written as follows:

$$\begin{aligned} W_S &= t_S\mu\pi \\ W_R &= \hat{\pi} + t_R(1-\mu\pi) = \pi - t_S\mu\pi \end{aligned}$$

The optimal split share can be solved by maximising the weighted Nash product, which is:

$$\mu^* = \arg\max_{[0,1]} \left(N = W_S^{\beta} W_R^{1-\beta} \right)$$

where $\beta \in (0,1)$ is the bargaining power of the source country. The first order condition gives:

$$\begin{aligned} \frac{dN}{d\mu} &= 0 \\ &\Rightarrow \frac{\beta}{\mu} - \frac{(1-\beta)t_S}{1-t_S\mu} = 0 \\ &\Rightarrow \mu = \frac{\beta}{t_S} \end{aligned}$$

If $\beta > t_S$, the source country's bargaining power exceeds its profit tax rate, $\mu = \frac{\beta}{t_S} > 1$, which means the proportion of the taxed profit by the source country will be 1, the source country will tax all and the residence country will tax nothing. Therefore, in equilibrium:

$$\mu = \min\left\{\frac{\beta}{t_S}, 1\right\} \tag{3.15}$$

3.4.2 Case 2. Sales in both residence and source countries

3.4.2.1 Firm Decision

In this case, the firm's decision is the same as the case of profit splitting under a non-cooperative approach. The optimal allocation decision of a firm's productive resource is shown by equation 3.2.

3.4.2.2 Government Decision

Similar to Case 1, two situations will be considered if there is no agreement between two countries: (i) No production at the disagreement point; (ii) the firm produces in the residence country and pays all tax to the residence. In both situations, the objective of each country is to maximise its own welfare function.

We consider (i) first. The social welfare for the residence and source country are the following:

$$W_R = \hat{\pi} + T_R = \pi_R + (1 - \mu t_S)\pi_S \tag{3.16}$$

$$W_S = \mu t_S \pi_S \tag{3.17}$$

The optimal split maximises the weighted Nash product:

$$\mu^* = \arg\max_{[0,1]} \left(N = W_S^{\beta} W_R^{1-\beta}\right)$$

which can be solved by:

$$\left.\frac{dN}{d\mu}\right|_{\mu=\mu^i} = 0$$

$$\Rightarrow \frac{\beta}{1-\beta} = -\frac{W_S}{W_R}\frac{\frac{dW_R}{d\mu}}{\frac{dW_S}{d\mu}} \tag{3.18}$$

$$\Rightarrow \frac{\beta}{1-\beta} = \frac{\mu t_S}{1-\mu t_S + \frac{\pi_R}{\pi_S}} \frac{1+\frac{1}{t_S\mu}\left(1-t_S-\frac{\lambda}{1-\lambda}\frac{\pi_R}{\pi_S}\right)\phi}{1-\phi}$$

$$\Rightarrow \frac{\beta}{1-\beta} = \frac{\mu t_S}{1-\mu t_S + \frac{\pi_R}{\pi_S}} \frac{1-\frac{t_R}{t_S}\frac{1-t_S}{1-t_R}\phi}{1-\phi} \tag{3.19}$$

where

$$\phi = -\mu\frac{1-\lambda}{\lambda}\frac{d\lambda}{d\mu}$$

By taking the derivative of λ with respect to μ, we have:

$$\frac{d\lambda}{d\mu} = -\frac{1}{\eta}\frac{\lambda(1-\lambda)(t_S-t_R)}{1-t_R-\mu(t_S-t_R)}$$

so that:

$$\phi = \frac{1-\eta}{\eta}(1-\lambda)\frac{\mu(t_S-t_R)}{1-t_R-\mu(t_s-t_R)}$$

$$= \frac{1-\eta}{\eta}\left[1+\left(1-\frac{\mu(t_S-t_R)}{1-t_R}\right)^{1/\eta}\frac{A_S^{1/\eta}k_S}{A_R^{1/\eta}k_R}\right]^{-1}\frac{\mu(t_S-t_R)}{1-t_R-\mu(t_S-t_R)}$$

In the symmetric case, equation 3.19 can be simplified as the following:

$$\frac{\beta}{1-\beta} = \frac{\mu t_S}{2-\mu t_S} \Rightarrow \hat{\mu} = \frac{2\beta}{t}$$

An interior solution exists if $\beta < t/2$. This is more likely to hold for small β (i.e. low bargaining power of the source country). Conversely, $\hat{\mu} = 1$ if $\hat{\mu} > \frac{2\beta}{t}$, which means that the source country will tax all profit earned in that country if the source country's bargaining power is high enough.

If two countries are not symmetric, equation 3.19 also has a unique solution. The right-hand side of equation 3.19 is a strictly positive, monotone increasing function of μ, say $g(\mu)$, as long as $\phi < 1$ and $\mu < \frac{1-t_R}{t_S-t_R}$. The latter condition is always true if $t_S \in (0,1)$ and $t_S > t_R$. If these conditions hold, $\phi < 1$ is always true. Therefore, equation 3.19 has a unique solution, μ^i, and since $g(0) = 0$, the solution is strictly positive for any β. Moreover, $d\mu/d\beta > 0$. The equilibrium profit split is then given by:

$$\hat{\mu} = min\{\mu^i, 1\}$$

We now consider the situation (ii): production only in the residence country at the disagreement point. When there is no agreement on profit split and the firm only produce in the residence country, the welfare of the residence country equals the pre-tax profit of the firm, and the welfare of the source country is zero.

$$W_R^D = \pi^D = A_R k_R^\eta$$

The optimal profit split maximises the weighted Nash product

$$\mu^* = \arg\max_{[0,1]} \left(N = W_S^\beta [W_R - W_R^D]^{1-\beta} \right)$$

and can be solved by

$$\begin{aligned} \left.\frac{dN}{d\mu}\right|_{\mu=\mu^{ii}} &= 0 \\ \Rightarrow \frac{\beta}{1-\beta} &= -\frac{W_S}{W_R - W_R^D}\frac{\frac{dW_R}{d\mu}}{\frac{dW_S}{d\mu}} \\ \Rightarrow \frac{\beta}{1-\beta} &= \frac{\mu t_S}{1-\mu t_S + \frac{\pi_R}{\pi_S} - \frac{\pi^D}{\pi_S}} \frac{1 - \frac{t_R}{t_S}\frac{1-t_S}{1-t_R}\phi}{1-\phi} \end{aligned} \tag{3.20}$$

where

$$\phi = \frac{1-\eta}{\eta}\left[1+\left(1-\frac{\mu(t_S-t_R)}{1-t_R}\right)^{1/\eta}\frac{A_S^{1/\eta}k_S}{A_R^{1/\eta}k_R}\right]^{-1}\frac{\mu(t_S-t_R)}{1-t_R-\mu(t_S-t_R)}$$

Since $\beta \in (0,1)$, the right-hand of equation 3.20 is strictly positive and monotonically increase in μ if $t_S \in (0,1)$ and $t_S > t_R$. There exists a unique solution, μ^{ii} and since $g(0) = 0$, the solution is strictly positive for any β. Moreover, $d\mu/d\beta > 0$. The equilibrium profit split is then given by

$$\hat{\mu} = min\{\mu^{ii}, 1\}$$

3.4.3 The equivalence of Non-cooperative and Cooperative

By comparing the cooperative and non-cooperative solutions, we can establish the equivalence between the ratio of bargaining powers of the two countries in the cooperative equilibria and the ratio of shadow prices of the constraint for two countries in the non-cooperative equilibria.

From the Lagrangian of the constrained optimisation, u_i in equation 3.5 is the shadow price. For example, if the source country gained an extra unit of profit split share μ_S at the optimal where the marginal welfare per unit share of profit split is equal to u_S, then the change in the maximal welfare per unit of an additional share of profit split will be equal to u_S.

And, using equation 3.6, ρ can be written as follows:

$$\rho = \frac{r_S}{r_R} = \frac{u_R}{u_S}$$

By definition from Rosen's (1965), the above equation shows that ρ is the ratio of the shadow price of constraint, which is the inverse ratio of weights of two countries, as r_i is the weight assigned to country i.

Furthermore, welfare functions 3.3 and 3.4 under the non-cooperative approach are equivalent to welfare functions 3.16 and 3.17 at the optimum where $\mu = \mu_S = 1 - \mu_R$. And, equation 3.7 ρ can be written as:

$$\rho = \left(\frac{\partial W_R}{\partial \mu_R}\right) / \left(\frac{\partial W_S}{\partial \mu_S}\right) = -\left(\frac{\partial W_R}{\partial \mu}\right) / \left(\frac{\partial W_S}{\partial \mu}\right)$$

therefore, equation 3.18 can be written as:

$$\frac{\beta}{1-\beta} = \frac{W_S}{W_R}\rho \equiv \frac{W_S}{W_R}\frac{r_S}{r_R} \tag{3.21}$$

where β and $1 - \beta$ are the bargaining powers of the source country and the residence country respectively. The above equation shows the relationship between the cooperative and non-cooperative approaches. In both formulations, the welfares of the residence and source are interlinked. In the cooperative formulation, they are linked via the joint surplus function. In the non-cooperative formulation, they are interlinked via joint constraint.

To interpret equation 3.21, we can reformulate the coupled-constraint maximization as a dual problem.

In the original coupled-constraint maximization, by taking $\{\sigma, \mu_j\}$ as given, country i solves:

$$\max_{\mu_i \in [0,1]} W_i(\mu_i) \quad \text{s.t. } \mu_i \leq \sigma - \mu_j$$

In particular, σ can be equal to 1. This constrained maximization problem is equivalent to the following unconstrained maximization problem using Lagrange function:

$$\max_{\mu_i \in [0,1]} \mathcal{L}_i(\mu_i, u_i | \sigma, \mu_j) \equiv W_i(\mu_i) + u_i(\sigma - \mu_i - \mu_j)$$

where $u_i \geq 0$. First-order conditions for an interior solution $\{\mu_i^*, u_i^*\}$ at $\mu_i = \mu_i^*, u_i = u_i^*$ are:

$$\begin{aligned} \frac{\partial \mathcal{L}_i}{\partial \mu_i} &= 0 \\ \frac{\partial \mathcal{L}_i}{\partial u_i} &= 0 \\ \frac{\partial W_i}{\partial \mu_i} &= u_i \\ \sigma - \mu_i - \mu_j &= 0 \end{aligned}$$

where $u_i \geq 0$. The Lagrangean equals the constrained maximum of the welfare at optimum. Let $\mathcal{L}_i^*(\sigma, \mu_j) = \mathcal{L}_i(\mu_i^*, u_i^* | \sigma, \mu_j) = W_i(\mu_i^* | \sigma, \mu_j) \equiv W_i^*(\sigma, \mu_j)$. Using comparative static, we have:

$$\frac{\mathcal{L}_i^*(\sigma, \mu_j)}{\partial \sigma} = \frac{\partial W_i^*(\sigma, \mu_j)}{\partial \sigma} = u_i^* \tag{3.22}$$

In the optimum, the value of the Lagrange multiplier u_i^* shows how much the objective function increases when the constraint level increases by one unit. Note that both μ_i^* and u_i^* depend on σ, μ_j.

We can reformulate the original coupled constrained maximization problem as the following minimization problem: country i solves:

$$\min \ \ \sigma_i(\mu_i) = \mu_i + \mu_j \quad \text{s.t. } W_i(\mu_i) \geq \widehat{W}_i$$

taking $\{W_i^*, \mu_j\}$ as given. This constrained minimization problem is equivalent to the following unconstrained minimization problem:

$$\min \ \ \mathcal{L}_i^*(\mu_i, r_i|\widehat{W}_i, \mu_j) = \mu_i + \mu_j - r_i[W_i(\mu_i) - \widehat{W}_i]$$

where $r_i \geq 0$. F.O.C for an interior solution $\{\mu_i^*, r_i^*\}$ at $\mu_i = \mu_i^*, r_i = r_i^*$ are:

$$\begin{aligned} \frac{\partial \mathcal{L}_i}{\partial \mu_i} &= 0 \\ \frac{\partial \mathcal{L}_i}{\partial r_i} &= 0 \\ \frac{\partial W_i}{\partial \mu_i} &= u_i \\ W_i(\mu_i) - \widehat{W}_i &= 0 \end{aligned}$$

Let $\mathcal{L}_i^*(\widehat{W}_i, \mu_j) = \mathcal{L}_i(\mu_i^*, r_i^*|\widehat{W}_i, \mu_j) = \sigma_i(\mu_i^*|\widehat{W}_i, \mu_j) \equiv \sigma_i^*(\widehat{W}_i, \mu_j)$. Using envelope function theorem:

$$\frac{\mathcal{L}_i^*(\widehat{W}_i, \mu_j)}{\partial \widehat{W}_i} = \frac{\partial \sigma_i^*(\widehat{W}_i, \mu_j)}{\partial \widehat{W}_i} = r_i^* \tag{3.23}$$

In the optimum, the value of Lagrange multiplier r_i^* shows by how much the objective function decreases when the constraint level decreases by one unit. In other words, if the welfare of country i decreases by one unit, the sum $\mu_i^* + \mu_j$ will decrease by r_i^*.

Moreover, the quantity:

$$\zeta_i \equiv \frac{\widehat{W_i}}{\sigma_i^*(\widehat{W_i}, \mu_j)} \frac{\partial \sigma_i^*(\widehat{W_i}, \mu_j)}{\partial \widehat{W_i}} = \frac{r_i^* \widehat{W_i}}{\sigma_i^*(\widehat{W_i}, \mu_j)}$$

measures the sensitivity (elasticity) of $\sigma_i^*(\widehat{W_i}, \mu_j)$ to a change in $\widehat{W_i}$. The higher this ratio, the more sensitive the joint constraint to the welfare of country i. In other words, increasing the country i's welfare by a unit would require relaxing the constraint by more than it would be required if instead we wanted to increase the country j's welfare. It can also be interpreted as the opportunity cost of the constraint $\widehat{W_i}$ imposed on W_i: if we want to increase this target level $r_i\widehat{W_i}$, the minimal(optimal) level of σ_i will rise.

Suppose, we solve this dual problem for both residence(R) and source(S) countries and there exist optimal $\sigma_S(\widehat{W_S}, \mu_R) = \mu_S^* + \mu_R$ corresponding to the solution $\{\mu_S = \mu_S^*, r_S = r_S^*\}$ and $\sigma_R(\widehat{W_R}, \mu_S) = \mu_R^* + \mu_S$ corresponding to the solution $\{\mu_R = \mu_R^*, r_R = r_R^*\}$. Then, setting $\mu_R = \mu_R^*$ in σ_S^* and $\mu_S = \mu_S^*$ in σ_R^*, we get:

$$\sigma_S^*(\widehat{W_S}, \mu_R^*) = \mu_S^* + \mu_R^* = \sigma_R^*(\widehat{W_R}, \mu_S^*)*$$

which is a Nash equilibrium, because each country does its best(minimize its objective function by choosing $\mu_i = \mu_i*$) given the choice of the other country. By the analogy of the "coupled-constraint"equilibrium, we can call it the "coupled-objective"equilibrium, because two countries have the same objective, which is to minimise the sum of shares $\mu_S + \mu_R$. In the equilirium, we have:

$$\sigma^*(\widehat{W_S}, \widehat{W_R}) = \mu_S^* + \mu_R^*$$

We can match it to the equilibrium in the original problem if $\widehat{W_S} = W_S^*$ and $\widehat{W_R} = W_R^*$. Then the optimal μ_S and μ_R in the dual problem will be the same as the optimal μ_S and μ_R in the original problem. So, if in the original problem their sum was equal to 1, it has to be the case that $\sigma^*(\widehat{W_S}, \widehat{W_R}) = 1$.

Therefore, the ratio $\frac{r_S W_S}{r_R W_R}$ in equation 3.21 is equivalent to the ratio $\frac{r_S^* \widehat{W_S}}{r_R^* \widehat{W_R}}$ in the “coupled objective ”equilibrium at optimum. The ratio measure the relative sensitivities of the common objective to the changes in the target welfare levels of the two countries, which, as argued above, can also be interpreted as the ratio of the opportunity costs of the countries’ target welfare levels.

Finally, equations 3.22 and 3.23 imply that in the equilibrium the Lagrange multipliers of the original and the dual problem are related as the derivatives of the inverse functions:

$$u_i* = \frac{u_0}{r_i*}$$

which is defined in equation 3.6. Now, we can fully interpret equation 3.21. In the cooperative formulation, the bargaining power of country i is the elasticity of the joint welfare concerning the individual welfare of country i. If $\beta_i > \beta_j$, it means the joint welfare is more sensitive to the changes in the country i’s welfare. In other words, it is more beneficial for joint welfare to increase the country i’s welfare. Conversely, it is more harmful to decrease the welfare of the country i. Therefore, equation 3.21 shows that the cooperative and non-cooperative solutions are equivalent when the bargaining powers of the countr in the cooperative solution (sensitivities of the joint welfare to each country’s welfare) are proportional to the opportunity costs of the country’s welfare in the non-cooperative solution (sensitives of the joint constraint to each country’s welfare.)

3.4.4 Welfare Possibility Frontier

In this section, we simulate a welfare possibility frontier (WPF) under a non-cooperative solution. The welfare possibility frontier captures the different possible combinations of the welfare of two countries that can be achieved by a given level of ρ. Equation 3.8 shows that optimal μ is a function of ρ. Since ρ links the non-cooperative and cooperative solutions, simulation WPF for a range of ρ could give more meaningful insights when considering both cases.

Simulation of WPF requires assigning numerical values for the model parameters. Multi-factor productivity in both countries is set to be 1, i.e. $A_S = A_R = 1$. The stock of public infrastructure in production $k_S = k_R = 0.2$. Corporate tax rates in Ireland and the UK are set to be the residence country and source country's tax rates. That is $t_R = 0.125$ and $t_S = 0.19$. The share of public infrastructure used in production $\eta = 0.14$. We set the range of ρ gives a meaningful optimal profit split. That is, the range of ρ gives $\mu \in [0, 1]$ with predefined parameter values.

Aggregate welfare is defined as a weighted sum of the country's welfare:

$$W_{WPF} = W_S + \rho W_R$$

We can show the simulation results in Figure D.1.When the opportunity costs of the countries' welfare are higher (or the joint constraint is more sensitive to the source country's welfare), the source country can have a larger share of the profit split to gain tax revenue. This leads to greater welfare in the source country, less welfare in the residence, and greater aggregate welfare.

3.5 A three-country model

Now we introduce one more source country into the model. In this game, governments and the firm move in sequence. In the first stage, the residence country simultaneously negotiates the profit split with each source country. In the second stage, the firm takes profit splits as given, and chooses an allocation decision of its productive resources. Assuming the information is perfect, this game is solved by backward induction.

3.5.1 Firm Decision

Assuming the firm divides its one unit productive resource between two source countries by proportion λ and $1-\lambda$. The firm's production follows Cobb-Douglas form that requires productive resources, and public infrastructure k to produce digital good. The profit is linear in the output of the consumption good. Profits for source country 1 and source country 2 are the following:

$$\pi_1 = A_1 \lambda^{1-\eta} k_1^{\eta}$$
$$\pi_2 = A_2 (1-\lambda)^{1-\eta} k_2^{\eta}$$

Source country $i = 1, 2$ charges tax $t_i \in (0,1)$ on the share of profit equal to $\mu_i \in [0,1]$. Total tax is:

$$T = T_1 + T_2 + T_R$$

where

$$T_1 = t_1 \mu_1 \pi_1$$
$$T_2 = t_2 \mu_2 \pi_2$$
$$T_R = t(1-\mu_1)\pi_1 + t(1-\mu_2)\pi_2$$

$(1-\mu_i)\pi_i$ is the proportion of profit left in source country i that can be taxed by the residence country. The net profit after tax is:

$$\begin{aligned}
\hat{\pi} &= \pi_1 + \pi_2 - T \\
&= [1-\mu_1 t_1 - t(1-\mu_1)]\pi_1 + [1-\mu_2 t_2 - t(1-\mu_2)]\pi_2 \\
&= [1-\mu_1 t_1 - t(1-\mu_1)]A_1[\lambda]^{1-\eta}k_1^{\eta} + [1-\mu_2 t_2 - t(1-\mu_2)]A_2[(1-\lambda)]^{1-\eta}k_2^{\eta} \\
&= b_1[\lambda]^{1-\eta} + b_2[(1-\lambda)]^{1-\eta} \qquad (3.24)
\end{aligned}$$

where

$$b_1 = [1-\mu_1 t_1 - t(1-\mu_1)]A_1 k_1^{\eta}$$
$$b_2 = [1-\mu_2 t_2 - t(1-\mu_2)]A_2 k_2^{\eta}$$

The firm maximises net profit after tax by choosing λ between two source countries, taking tax rates and profit split as given:

$$\lambda^* = \arg\max_{\lambda\in[0,1]}(\hat{\pi})$$

At optimal,

$$\lambda^* = \frac{1}{1+\left[\frac{b2}{b1}\right]^{1/\eta}}$$

Upon substitution of the optimal λ^* into after-tax net profit function 3.24, the maximised profit can be expressed as:

$$\hat{\pi} = \left(b_1^{1/\eta} + b_2^{1/\eta}\right)^{\eta} \qquad (3.25)$$

Equation 3.25 is a constant elasticity of the substitute function of profits earned in each country per unit of investment. We can also show how optimal net profit responds to the profit split by:

$$\frac{\partial\hat{\pi}}{\partial\mu_i} = -\left(b_1^{1/\eta} + b_2^{1/\eta}\right)^{\eta-1} b_i^{1/\eta-1}A_i k_i^{\eta}(t_i - t)$$

The above equation shows that optimal net profit is a decreasing function in profit split μ. This means, if the source country's tax rate is higher than the residence's, i.e.$t_i - t > 0$, the net profit falls as the share taxed by the source increases.

3.5.2 Governments Decision

In this stage, the objectives for governments in this three-country model is to maximise their own welfare. The welfare functions for the residence country W and source country W_i are

$$W = \hat{\pi} + T_R = (1-\mu_1 t)\pi_1 + (1-\mu_2 t)\pi_2 = \sum_{i=1,2}(1-\mu_i t_i)\pi_i$$
$$W_i = T_i = t_i \mu_i \pi_i$$

The profit splits are negotiated simultaneously in bilateral agreements between the residence and each source country. If there is no agreement between the residence and only one source country, the 3-country model is collapsed as a two-country model as discussed in the previous section. If the residence country has no agreement with both source counties, the firm produces nothing and welfare in all countries is zero, i.e. $W_1^D = W_2^D = W^D$. Therefore, for optimal profit split μ_i^* where $i = 1, 2$, it maximises the weighted Nash

$$\mu^* = \arg\max_{\mu\in[0,1]} \left(N_i = W_i^{\beta_i} W^{1-\beta_i}\right)$$

Where $\beta_i \in (0, 1)$ represents the strength of the bargaining power of the source country i. The interior solution satisfies the following equation:

$$\frac{dN_i}{d\mu_i} = 0$$
$$\Rightarrow \frac{\beta_i}{1-\beta_i} = -\frac{W_i}{W}\frac{\frac{dW}{d\mu_i}}{\frac{dW_i}{d\mu_i}}$$

For source country 1:

$$\frac{\beta_1}{1-\beta_1} = -\frac{W_1}{W}\frac{\frac{dW}{d\mu_1}}{\frac{dW_1}{d\mu_1}}$$
$$\Rightarrow \frac{\beta_1}{1-\beta_1} = -\frac{1}{1+(1-\lambda)\phi_1}\left[\frac{(1-\mu_1 t_1)\phi - \mu_1 t_1}{(1-\mu_1 t_1)+(1-\mu_2 t_2)\frac{A_2 k_2}{A_1 k_1}\left(\frac{b_1}{b_2}\right)^{1-1/\eta}} - \lambda\phi_1\right] \tag{3.26}$$

where

$$\phi_1 = \frac{1-\eta}{\eta}\frac{(t-t_1)\mu_1}{1-\mu_1 t_1 - t(1-\mu_1)}$$

Similarly, in the interior solution the optimal condition for source country 2 is given by the following equation:

$$\frac{\beta_2}{1-\beta_2} = -\frac{W_2}{W}\frac{\frac{dW}{d\mu_2}}{\frac{dW_2}{d\mu_2}}$$
$$\Rightarrow \frac{\beta_2}{1-\beta_2} = -\frac{1}{1-\lambda\phi_2}\left[\frac{\mu_2 t_2 + (1-\mu_1 t_1)\phi_2\frac{A_1 k_2^{\eta}}{A_2 k_2^{\eta}}(\frac{b1}{b2})^{1/\eta-1}}{(1-\mu_2 t_2)+(1-\mu_1 t_1)\frac{A_1 k_2^{\eta}}{A_2 k_2^{\eta}}(\frac{b1}{b2})^{1/\eta-1}} - \lambda\phi_2\right] \tag{3.27}$$

where

$$\phi_2 = \frac{1-\eta}{\eta}\frac{(t-t_2)\mu_2}{1-\mu_2 t_2 - t(1-\mu_2)}$$

There is no closed-form solution for the system of two equations 3.26 and 3.27. It can be solved numerically to find μ_1 and μ_2.

3.5.3 Calibration and Simulation

Model parameters are calibrated in the first step. The multi-factor productivity in both source countries ($A_i, i = 1, 2$) is set to be 1. This means that one unit of production input generates one unit of output. Production requires the use of public infrastructure k_i. We set $k_1 = 0.2$ and $k_2 = 0.5$. The percentage of public infrastructure used in the production process is set to be 0.2, i.e. $\eta = 0.2$. The tax rate in the residence country is assumed to be lower than in the source countries. Instead of assigning value to tax rates, we use corporate tax from reality, the residence country's tax rate t is set to 0.125, which is the corporate tax rate in Ireland. France and the UK corporate tax rates are used for source countries 1 and 2 respectively, i.e. $t_1 = 0.32$ and $t_2 = 0.19$.

3.5.3.1 Bargaining Power

Bargaining power β_i in source countries will be controlled separately. For instance, bargaining power of source country 2 is fixed at $\beta_2 = 0.1$ or $\beta_2 = 0.7$ when calculating how variables responds to change in β_1, and vice versa. We set the range of β_1 is $[0, 0.5]$ and the range of $beta_2$ is $[0, 0.3]$ to obtain economically meaningful solutions.

Figure D.2 and D.3 show how optimal shares of profit split change in the source country 1's bargaining power. Shares of profit split in both cases are larger when the source 1 country has more bargaining power. Interestingly, source country 2's share of profit split is larger than 1 when source 2 country has a high enough bargaining power irrespective of how bargaining power differs in source country 1. The residence country's welfare is lower but higher in source countries' when source country 1 has more bargaining power. Profit splitting allows the redistribution of welfare among three countries. Both Figures show consistent patterns for each variable except the aggregate welfare W_a. When source country 2 has low bargaining power, there is a maximum in the aggregate welfare when $\beta_1 \approx 0.07$. However, aggregate welfare shows monotonic relation in $beta_1$ when source country 2 has high bargaining power.

Figures D.4 and D.5 show how optimal shares of profit split change in the source country 1's bargaining power. When source country 1 has low bargaining power, profit split μ_1 decreases but μ_2 is larger when source country 2 has more bargaining power. Source country 2 can tax all profits if it has larger enough bargaining power over the residence country. Notably, source country 1's welfare starts to rise when β_2 is large enough though μ_1 is small. This is because the profit in source country 1 is large enough to compensate for the small taxable share.

In conclusion, a profit split will lower the residence country's welfare but greater aggregate welfare exists.

3.5.3.2 Tax rates

Firstly, we simulate changes in the tax rate of source country 1. Source country 2's tax rate is set to be $t_2 = 0.19$. Bargaining power β_1 and β_2 are set to be 0.1. The range of t_1 is $[0.125, 0.32]$.

Figure D.6 shows the simulated results regarding the change in the source country 1's tax rate. Source country 1's taxable share $\mu 1$ and profits are lower when t_1 is greater. As a consequence, welfare in source country 1 is smaller. This means high tax rate leads to low social welfare, giving the bargaining power unchanged. μ_2 shows a U-shape relation to t_1. The aggregate welfare presents the highest value when t_1 is around source country 2's tax rate 0.19.

Secondly, we simulate changes in the tax rate of source country 2. Source country 1's tax rate is set to be $t_1 = 0.32$. Bargaining power β_1 and β_2 are set to be 0.1. The range of t_2 is $[0.125, 0.32]$. Simulated results are shown in Figure D.7. When the tax rate in source country 2 is higher, the profit split share is greater in source country 1 but lower in source country 2. Welfare curves show similar pattern as in the case of changing in t_1.

Thirdly, we consider how optimal taxable shares changes in the residence country's tax rate. The results are presented in Figure D.8. The high tax rate in the residence country leads to larger taxable shares by source countries. The highest profit split share of the source country 2 is achieved when $t \approx 0.125$, at which the aggregate welfare also be the greatest.

3.6 Conclusion

We develop a two-country model and extend it to a three-country model to analyze how the division of the tax base could be achieved. In our model, the division of tax base is modelled as a share of the profit generated in source countries. By constructing a two-stage game, we conclude the following:

First, in a two-country model, the equilibrium profit split share exists, and is unique in both non-cooperative and cooperative situations. If two countries cooperatively split the taxable profits, the source country can tax all profits if its bargaining power is high enough.

Second, the non-cooperative equilibria are equivalent to the cooperative equilibria when the ratio of the opportunity costs of the countries' welfare equals the ratio of countries' bargaining powers.

Third, we extend the two-country model to a three-country model. By numerically simulating the model, we conclude that a country can have more share of the profit split if it has high bargaining power. Profit split reduces the residence country's welfare, but greater aggregate welfare exists by doing so.

Appendix A

Tables for Chapter 1

Table A.1: Summary Statistics of full Sample, 2012-2020

Category	Variable	mean	sd	min	max
Public Support	Any Public Support	0.176	0.38	0	1
	EU Support	0.03	0.171	0	1
	UK Central Support	0.114	0.318	0	1
	UK Regional Support	0.074	0.262	0	1
	Indirect Support	0.094	0.292	0	1
	Direct Support	0.037	0.189	0	1
	All Source Supports	0.008	0.087	0	1
	EU Support Only	0.009	0.093	0	1
	UK Central Support Only	0.081	0.272	0	1
	UK Regional Support Only	0.048	0.214	0	1
	EU and UK Central Support	0.009	0.096	0	1
	EU and UK Regional Support	0.004	0.067	0	1
	UK Central and Regional Support	0.014	0.117	0	1
	Indirect Support Only	0.072	0.259	0	1
	Direct Support Only	0.015	0.121	0	1
	Both Indirect and Direct Supports	0.022	0.147	0	1
Firm Characteristics	Time	2.403	1.146	1	4
	ONS 12 regions. UKIS	6.521	3.119	1	12
	Industry	7.069	3.836	1	13
	Log BSD Employment	4.414	1.528	2.303	12.03
	Log Age	3.326	0.484	1.609	4.043
	Cooperate	0.55	0.497	0	1
	Foreign Ultimate Ownership	0.153	0.36	0	1
	% Qualified Scientists and Engineers	9.16	19.644	0	100
	% Qualified Other Staff	11.181	19.855	0	100
Constraints	Economic Risk	0.124	0.33	0	1
	Direct Cost	0.137	0.343	0	1
	Financial Cost	0.121	0.326	0	1
	Finance Availability	0.126	0.332	0	1
	Qualified Personnel	0.1	0.3	0	1
	Technology Information	0.044	0.206	0	1
	Market Information	0.039	0.193	0	1
	Dominated Market	0.074	0.261	0	1
	Uncertain Demand	0.086	0.28	0	1

Table A.1 continued from previous page

Category	Variable	mean	sd	min	max
	UK Regulation	0.092	0.289	0	1
	EU regulation	0.076	0.265	0	1
Missing Indicator	% Qualified Scientists and Engineers	0.316	0.465	0	1
	% Qualified Other Staff	0.284	0.451	0	1
	Economic Risk	0.193	0.395	0	1
	Direct Cost	0.195	0.396	0	1
	Financial Cost	0.194	0.396	0	1
	Finance Availability	0.195	0.396	0	1
	Qualified Personnel	0.193	0.395	0	1
	Technology Information	0.197	0.398	0	1
	Market Information	0.197	0.398	0	1
	Dominated Market	0.195	0.396	0	1
	Uncertain Demand	0.197	0.398	0	1
	UK Regulation	0.195	0.396	0	1
	EU regulation	0.196	0.397	0	1
Count	IDBR Unique Reporting Unit	16899			
	IDBR Unique Enterprise	16632			
	Total Observations	24371			

Table A.2: Observations of Time Dummy per Wave

Time	Observations	Unique Reporting Units	Unique Enterprise
2012	7185	7185	7105
2014	6066	6066	6004
2016	5244	5244	5213
2018	5876	5876	5829
Total	24371	24371	24151

Table A.3: Observations of Region Dummy, 2012-2020

Region	Observations	Unique Reporting Units	Unique Enterprise
North East	1057	730	726
North West	2407	1637	1630
Yorkshire and The Humber	1880	1326	1316
East Midlands	1796	1240	1228
West Midlands	2021	1417	1402
Eastern	2156	1515	1510
London	3338	2398	2393
South East	2968	2125	2116
South West	1910	1329	1318
Wales	1403	980	978
Scotland	2357	1625	1613
Northern Ireland	1078	781	777
Total	24371	17103	17007

Table A.4: Observations of Industry Dummy, 2012-2020

Industry	Observations	Unique Reporting Units	Unique Enterprise
Mining and Quarrying (B)	213	146	143
Manufacturing (C)	5787	3628	3504
Electricity Gas Steam and A/C (D)	90	64	61
Water Supply and Waste Management (F)	447	301	300
Construction (F)	1295	936	920
Wholesale Retail and Motor Trade (G)	4812	3773	3736
Transportation and Storage (H)	1192	850	843
Accommodation and Catering (I)	1332	995	993
Information and Communication (J)	1809	1219	1211
Financial and Insurance (K)	1012	741	737
Real Estate Activities (L)	751	518	517
Professional Science and Tech. (M)	3769	2590	2585
Administrative and Support Services (N)	1862	1293	1290
Total	24371	17054	16840

Table A.5: List of Variables

Variable Name	Description
Any Public Support	Dummy variable equals to 1 if the firm received at least any one of public support from EU, UK central, and UK regional
EU Support	Dummy variable equals to 1 if the firm received EU support
UK Central Support	Dummy variable equals to 1 if the firm received UK central support
UK Regional Support	Dummy variable equals to 1 if the firm received UK regional support
Indirect Support	Dummy variable equals to 1 if the firm received UK indirect support
Direct Support	Dummy variable equals to 1 if the firm received UK direct support
All Source Supports	Dummy variables equals to 1 if the firm received all of EU, UK central, and UK regional supports
EU Support Only	Dummy variable equals to 1 if the firm received EU support only
UK Central Support Only	Dummy variable equals to 1 if the firm received UK central support only
UK Regional Support Only	Dummy variable equals to 1 if the firm received UK regional support only
EU and UK Central Support	Dummy variable equals to 1 if the firm received EU and UK central supports only
EU and UK Regional Support	Dummy variable equals to 1 if the firm received EU and UK regional supports only
UK Central and Regional Support	Dummy variable equals to 1 if the firm received UK regional and UK central supports only
Indirect Support Only	Dummy variable equals to 1 if the firm received UK indirect support only
Direct Support Only	Dummy variable equals to 1 if the firm received UK direct support only
Both Indirect and Direct Supports	Dummy variable equals to 1 if the firm received UK direct and indirect supports
Time	Time dummy. 1 = wave 2012; 2 = wave 2014; 3 = wave 2016; 4 = wave 2018.
ONS 12 regions. UKIS	ONS region dummy. 1 = North East; 2 = North West; 3 = Yorkshire and the Humber; 4 = East Midlands; 5 = West Midlands; 6 = Eastern; 7 = London; 8 = South East; 9 = South West; 10 = Wales; 11 = Scotland; 12 = Northern Ireland.
Industry	Industry Category by sic2007. 1 = Mining and Quarrying (B); 2 = Manufacturing (C); 3 = Electricity Gas Steam and A/C (D); 4 = Water Supply and Waste Management (F); 5 = Construction (F); 6 = Wholesales Retail and Motor Trade (G); 7 = Transportation and Storage (H); 8 = Accommodation and Catering (I); 9 = Information and Communication (J); 10 = Financial and Insurance (K); 11 = Real Estate Activities (L); 12 = Professional Science and Tech. (M); 13 = Administrative and Support Service (N).
Log BSD Employment	BSD Employment in Log value
Log Age	Age in Log value
Cooperate	Whether business co-operated on any innovation activities. 1= Yes; 0 = No.
Foreign Ultimate Ownership	BSD Ultimate foreign Ownership. 1 = foreign owned; 0 = UK firm.
% Qualified Scientists and Engineers	Share of qualified Scientist and Engineer
% Qualified Other Staff	Share of other qualified staff
Economic Risk	Dummy variable equals to 1 if the firm rated highly important in excessive perceived economic risks in constraining innovation activities

Table A.5 continued from previous page

Variable Name	Description
Direct Cost	Dummy variable equals to 1 if the firm rated highly important in high direct innovation cost in constraining innovation activities
Financial Cost	Dummy variable equals to 1 if the firm rated highly important in cost of finance in constraining innovation activities
Finance Availability	Dummy variable equals to 1 if the firm rated highly important in availability of finance in constraining innovation activities
Qualified Personnel	Dummy variable equals to 1 if the firm rated highly important in lack of qualified personnel in constraining innovation activities
Technology Information	Dummy variable equals to 1 if the firm rated highly important in lack of information on technology in constraining innovation activities
Market Information	Dummy variable equals to 1 if the firm rated highly important in lack of information on markets in constraining innovation activities
Dominated Market	Dummy variable equals to 1 if the firm rated highly important in market dominated by established businesses in constraining innovation activities
Uncertain Demand	Dummy variable equals to 1 if the firm rated highly important in uncertain demand for innovative goods or services in constraining innovation activities
UK Regulation	Dummy variable equals to 1 if the firm rated highly important in the UK government regulation in constraining innovation activities
EU regulation	Dummy variable equals to 1 if the firm rated highly important in the EU regulations in constraining innovation activities
% Qualified Scientists and Engineers	Missing indicator of qualified scientist and engineer
% Qualified Other Staff	Missing indicator of other staff
Economic Risk	Missing indicator of economic risk
Direct Cost	Missing indicator of high direct cost
Financial Cost	Missing indicator of financial cost
Finance Availability	Missing indicator of availability of finance
Qualified Personnel	Missing indicator of lack of qualified personnel
Technology Information	Missing indicator of lack of technology information
Market Information	Missing indicator of lack of markets information
Dominated Market	Missing indicator of market dominated by established firms
Uncertain Demand	Missing indicator of uncertain demand
UK Regulation	Missing indicator of UK regulation
EU regulation	Missing indicator of EU regulation

Table A.6: Estimated Average Marginal Effect from Probit Models for Different Sources of Public Support using Full Sample, 2012-2020

	Any Public Support	EU Support	UK Regional Support	UK Central Support
2014	-0.003	-0.007	0.006	0.001
	(0.019)	*(0.005)*	*(0.014)*	*(0.008)*
2016	-0.016	-0.001	-0.031***	-0.007
	(0.013)	*(0.007)*	*(0.007)*	*(0.007)*
2018	0.036**	-0.015**	0.028**	0.021**
	(0.012)	*(0.005)*	*(0.009)*	*(0.008)*
North East	0.068***	0.019**	0.066***	0.013
	(0.02)	*(0.007)*	*(0.014)*	*(0.014)*
North West	0.054**	0.01	0.052***	0.011
	(0.017)	*(0.006)*	*(0.011)*	*(0.012)*
Yorkshire and The Humber	0.084***	0.030**	0.053***	0.019
	(0.017)	*(0.011)*	*(0.012)*	*(0.013)*
East Midlands	0.109**	0.012	0.095**	0.014
	(0.036)	*(0.006)*	*(0.029)*	*(0.015)*
West Midlands	0.069***	0.034***	0.057***	0.027
	(0.018)	*(0.008)*	*(0.013)*	*(0.015)*
Eastern	0.034*	0.003	0.029**	0.014
	(0.015)	*(0.005)*	*(0.009)*	*(0.011)*
South East	0.02	-0.001	0.015	0.009
	(0.014)	*(0.004)*	*(0.008)*	*(0.011)*
South West	0.057***	0.023**	0.040***	0.016
	(0.017)	*(0.008)*	*(0.011)*	*(0.012)*
Wales	0.129***	0.009	0.137***	-0.013
	(0.031)	*(0.005)*	*(0.025)*	*(0.011)*
Scotland	0.102***	0.020*	0.110***	0.002
	(0.019)	*(0.009)*	*(0.015)*	*(0.01)*
Northern Ireland	0.115***	0.018*	0.146***	-0.009
	(0.022)	*(0.008)*	*(0.019)*	*(0.014)*
Manufacturing (C)	0.097***	0.026***	0.036	0.067**
	(0.029)	*(0.008)*	*(0.02)*	*(0.025)*
Electricity Gas Steam and A/C (D)	0.022	0.039	-0.002	-0.009
	(0.054)	*(0.039)*	*(0.038)*	*(0.039)*
Water Supply and Waste Management (F)	0.01	0.016	0.001	0.009
	(0.039)	*(0.014)*	*(0.028)*	*(0.032)*
Construction (F)	0.002	0.008	0.002	-0.001
	(0.031)	*(0.009)*	*(0.022)*	*(0.026)*
Wholesale Retail and Motor Trade (G)	-0.009	0.001	0.012	-0.021
	(0.029)	*(0.007)*	*(0.021)*	*(0.025)*
Transportation and Storage (H)	-0.022	0.009	-0.001	-0.02
	(0.031)	*(0.01)*	*(0.022)*	*(0.026)*
Accommodation and Catering (I)	0.008	0.016	0.033	-0.059*
	(0.037)	*(0.012)*	*(0.027)*	*(0.024)*
Information and Communication (J)	0.098**	0.009	0.015	0.078**
	(0.032)	*(0.008)*	*(0.022)*	*(0.028)*
Financial and Insurance (K)	-0.031	0.002	-0.041*	-0.018
	(0.045)	*(0.012)*	*(0.02)*	*(0.036)*
Real Estate Activities (L)	-0.057	0	-0.003	-0.055*
	(0.03)	*(0.009)*	*(0.022)*	*(0.025)*
Professional Science and Tech. (M)	0.009	0.025**	-0.008	0.004
	(0.029)	*(0.008)*	*(0.02)*	*(0.025)*

Table A.6 continued from previous page

	Any Public Support	EU Support	UK Regional Support	UK Central Support
Administrative and Support Services (N)	-0.038	0.009	-0.01	-0.039
	(0.03)	*(0.009)*	*(0.021)*	*(0.025)*
Log BSD Employment	0.001	-0.003	-0.003	0.010***
	(0.003)	*(0.002)*	*(0.002)*	*(0.002)*
Log Age	-0.028**	-0.006	-0.023***	-0.014*
	(0.01)	*(0.004)*	*(0.007)*	*(0.006)*
Cooperate	0.117***	0.033***	0.057***	0.071***
	(0.011)	*(0.006)*	*(0.007)*	*(0.007)*
Foreign Ultimate Ownership	-0.040***	-0.011*	-0.037***	-0.022*
	(0.012)	*(0.005)*	*(0.008)*	*(0.009)*
% Qualified Scientists and Engineers	0.002***	0.000***	0.000*	0.001***
	(0.)	*(0.)*	*(0.)*	*(0.)*
% Qualified Other Staff	0	0	0	0
	(0.)	*(0.)*	*(0.)*	*(0.)*
Economic Risk	0.034*	0.014*	0.018	0.001
	(0.014)	*(0.006)*	*(0.01)*	*(0.007)*
Direct Cost	0.005	-0.009	0.006	0.017*
	(0.018)	*(0.008)*	*(0.009)*	*(0.007)*
Financial Cost	-0.018	0.001	0.006	-0.034***
	(0.02)	*(0.009)*	*(0.011)*	*(0.009)*
Finance Availability	0.056**	0.011	0.030*	0.022*
	(0.017)	*(0.007)*	*(0.012)*	*(0.009)*
Qualified Personnel	0.038**	0.006	0.027***	0.015
	(0.012)	*(0.004)*	*(0.008)*	*(0.008)*
Technology Information	-0.007	0.002	0.014	-0.034*
	(0.02)	*(0.007)*	*(0.014)*	*(0.014)*
Market Information	0.001	-0.004	0.007	0.014
	(0.023)	*(0.007)*	*(0.014)*	*(0.016)*
Dominated Market	-0.007	-0.011	-0.004	0
	(0.015)	*(0.006)*	*(0.011)*	*(0.009)*
Uncertain Demand	0.015	0.016	-0.026**	0.01
	(0.018)	*(0.008)*	*(0.01)*	*(0.008)*
UK Regulation	0.005	-0.008	0.019	0.007
	(0.021)	*(0.006)*	*(0.014)*	*(0.014)*
EU regulation	0.004	0.008	-0.004	-0.005
	(0.021)	*(0.005)*	*(0.015)*	*(0.014)*
Chi_sq (Overall)	668.820***	187.520***	467.540***	520.300***
Chi_sq (Time)	13.830**	11.080*	70.610***	17.800***
Chi_sq (Region)	86.600***	43.420***	172.790***	17.94*
Chi_sq (Industry)	105.74***	40.540***	56.520***	169.46***
Total Observations	24371	24371	24371	24371
Total Unique Reporting Units	16899	16899	16899	16899
Total Unique Enterprises	16632	16632	16632	16632

Note: Standard errors in parentheses. Chi squared statistics refer to the joint significance of the overall model, time dummy, region dummy, and industry dummy respectively. * p<0.05; ** p<0.01;*** p<0.001.

Table A.7: Estimated average marginal effects from Probit models for different sources of public support using large firm sample, 2012-2020

	Any Public Support	EU Support	UK Regional Support	UK Central Support
2014	-0.011	-0.003	0.008	-0.012
	(0.011)	*(0.005)*	*(0.008)*	*(0.01)*
2016	-0.021	-0.005	-0.01	-0.020*
	(0.011)	*(0.005)*	*(0.007)*	*(0.01)*
2018	0.015	-0.011*	-0.001	0.013
	(0.012)	*(0.005)*	*(0.008)*	*(0.011)*
North East	0.090**	0.034	0.007	0.061*
	(0.033)	*(0.017)*	*(0.017)*	*(0.029)*
North West	0.017	0.017*	0.004	0.012
	(0.017)	*(0.009)*	*(0.01)*	*(0.014)*
Yorkshire and The Humber	0.034	0.006	0.001	0.025
	(0.02)	*(0.007)*	*(0.011)*	*(0.018)*
East Midlands	0.03	0.008	0.004	0.022
	(0.02)	*(0.008)*	*(0.012)*	*(0.019)*
West Midlands	0.039*	0.004	0.022	0.029
	(0.019)	*(0.007)*	*(0.012)*	*(0.018)*
Eastern	0.016	0.014	-0.002	0.02
	(0.02)	*(0.008)*	*(0.011)*	*(0.018)*
South East	0.019	0.009	-0.001	0.017
	(0.016)	*(0.006)*	*(0.01)*	*(0.014)*
South West	0.054*	0.003	0.001	0.048*
	(0.023)	*(0.007)*	*(0.011)*	*(0.019)*
Wales	0.074*	0.025	0.056*	0.035
	(0.032)	*(0.014)*	*(0.023)*	*(0.028)*
Scotland	0.087***	0.016	0.076***	0.024
	(0.022)	*(0.009)*	*(0.016)*	*(0.017)*
Northern Ireland	0.114***	0.034	0.135***	0.008
	(0.032)	*(0.02)*	*(0.028)*	*(0.024)*
Manufacturing (C)	0.077	0	0.026	0.057
	(0.056)	*(0.027)*	*(0.034)*	*(0.052)*
Electricity Gas Steam and A/C (D)	0.085	-0.021	-0.009	0.039
	(0.083)	*(0.032)*	*(0.042)*	*(0.078)*
Water Supply and Waste Management (F)	0.065	0.026	0.081	0.033
	(0.072)	*(0.037)*	*(0.05)*	*(0.066)*
Construction (F)	-0.028	0	-0.025	-0.007
	(0.059)	*(.)*	*(0.034)*	*(0.055)*
Wholesale Retail and Motor Trade (G)	-0.074	-0.033	-0.024	-0.061
	(0.056)	*(0.026)*	*(0.033)*	*(0.052)*
Transportation and Storage (H)	0.011	-0.025	0.042	-0.03
	(0.059)	*(0.027)*	*(0.036)*	*(0.054)*
Accommodation and Catering (I)	-0.092	-0.034	-0.014	-0.092
	(0.056)	*(0.027)*	*(0.034)*	*(0.052)*
Information and Communication (J)	0.036	-0.021	-0.004	0.038
	(0.059)	*(0.027)*	*(0.035)*	*(0.055)*
Financial and Insurance (K)	-0.042	-0.025	-0.008	-0.038
	(0.059)	*(0.028)*	*(0.035)*	*(0.054)*
Real Estate Activities (L)	-0.012	-0.021	0.044	-0.049
	(0.061)	*(0.028)*	*(0.04)*	*(0.055)*
Professional Science and Tech. (M)	-0.012	-0.005	-0.007	-0.013
	(0.057)	*(0.027)*	*(0.034)*	*(0.053)*

Table A.7 continued from previous page

	Any Public Support	EU Support	UK Regional Support	UK Central Support
Administrative and Support Services (N)	-0.102	-0.031	-0.028	-0.092
	(0.056)	*(0.026)*	*(0.033)*	*(0.051)*
Log BSD Employment	0.008	0.008***	0.002	0.011**
	(0.004)	*(0.002)*	*(0.003)*	*(0.004)*
Log Age	-0.015	-0.008	-0.009	-0.012
	(0.012)	*(0.005)*	*(0.007)*	*(0.01)*
Cooperate	0.124***	0.035***	0.051***	0.098***
	(0.012)	*(0.008)*	*(0.008)*	*(0.011)*
Foreign Ultimate Ownership	-0.030**	-0.005	-0.016*	-0.022*
	(0.011)	*(0.004)*	*(0.007)*	*(0.009)*
% Qualified Scientists and Engineers	0.002***	0.000***	0.001***	0.001***
	(0.)	*(0.)*	*(0.)*	*(0.)*
% Qualified Other Staff	0	0	0	0
	(0.)	*(0.)*	*(0.)*	*(0.)*
Economic Risk	0.001	0.007	0.001	-0.002
	(0.015)	*(0.005)*	*(0.009)*	*(0.013)*
Direct Cost	0.002	0.006	0.003	0.002
	(0.014)	*(0.005)*	*(0.009)*	*(0.013)*
Financial Cost	-0.024	0.002	-0.003	-0.02
	(0.017)	*(0.006)*	*(0.01)*	*(0.016)*
Finance Availability	0.029	0.004	0.022*	0.009
	(0.016)	*(0.006)*	*(0.009)*	*(0.015)*
Qualified Personnel	0.011	0.006	-0.004	0.01
	(0.017)	*(0.006)*	*(0.01)*	*(0.015)*
Technology Information	0.002	-0.016*	0.008	-0.017
	(0.022)	*(0.008)*	*(0.012)*	*(0.021)*
Market Information	0.008	0.007	-0.005	0.017
	(0.025)	*(0.009)*	*(0.013)*	*(0.022)*
Dominated Market	-0.019	-0.002	0.003	-0.021
	(0.019)	*(0.007)*	*(0.011)*	*(0.017)*
Uncertain Demand	0.027	0.004	0	0.038**
	(0.015)	*(0.006)*	*(0.01)*	*(0.013)*
UK Regulation	-0.004	0.015*	0.016	-0.011
	(0.019)	*(0.006)*	*(0.012)*	*(0.02)*
EU regulation	0.004	-0.012	0.007	0
	(0.021)	*(0.006)*	*(0.011)*	*(0.02)*
Chi_sq (Overall)	324.03***	49.54	248.520***	283.500***
Chi_sq (Time)	10.81*	4.38	6.11	11.330*
Chi_sq (Region)	36.16***	8.65	19.090***	11.45
Chi_sq (Industry)	131.30***	13.68	64.840***	120.110***
Total Observations	6445	6137	6445	6445
Total Unique Reporting Units	4262	4051	4262	4262
Total Unique Enterprises	4111	3912	4111	4111

Note: Standard errors in parentheses. Chi squared statistics refer to the joint significance of the overall model, time dummy, region dummy, and industry dummy respectively. * p<0.05; ** p<0.01;*** p<0.001.

Table A.8: Estimated average marginal effects from Probit models for different sources of public support using SME sample, 2012-2020

	Any Public Support	EU Support	UK Regional Support	UK Central Support
2014	-0.002	-0.007	0.006	0.003
	(0.019)	*(0.005)*	*(0.014)*	*(0.008)*
2016	-0.015	0	-0.032***	-0.005
	(0.013)	*(0.007)*	*(0.008)*	*(0.007)*
2018	0.038**	-0.014**	0.030**	0.021**
	(0.012)	*(0.005)*	*(0.009)*	*(0.008)*
North East	0.066**	0.018*	0.069***	0.01
	(0.021)	*(0.008)*	*(0.015)*	*(0.014)*
North West	0.055**	0.009	0.054***	0.011
	(0.018)	*(0.006)*	*(0.012)*	*(0.013)*
Yorkshire and The Humber	0.086***	0.031**	0.056***	0.019
	(0.018)	*(0.011)*	*(0.013)*	*(0.013)*
East Midlands	0.111**	0.012	0.099**	0.013
	(0.037)	*(0.007)*	*(0.03)*	*(0.015)*
West Midlands	0.070***	0.035***	0.059***	0.027
	(0.019)	*(0.009)*	*(0.014)*	*(0.015)*
Eastern	0.034*	0.003	0.031**	0.014
	(0.016)	*(0.005)*	*(0.01)*	*(0.012)*
South East	0.019	-0.002	0.016*	0.009
	(0.015)	*(0.005)*	*(0.008)*	*(0.011)*
South West	0.056**	0.024**	0.042***	0.014
	(0.018)	*(0.008)*	*(0.012)*	*(0.013)*
Wales	0.130***	0.008	0.140***	-0.015
	(0.032)	*(0.006)*	*(0.026)*	*(0.011)*
Scotland	0.101***	0.019*	0.113***	0
	(0.02)	*(0.009)*	*(0.015)*	*(0.011)*
Northern Ireland	0.114***	0.018*	0.146***	-0.01
	(0.023)	*(0.009)*	*(0.02)*	*(0.014)*
Manufacturing (C)	0.103**	0.034***	0.041	0.070*
	(0.032)	*(0.006)*	*(0.022)*	*(0.027)*
Electricity Gas Steam and A/C (D)	0.013	0.05	0.004	-0.01
	(0.06)	*(0.045)*	*(0.043)*	*(0.044)*
Water Supply and Waste Management (F)	0.01	0.02	0	0.01
	(0.042)	*(0.014)*	*(0.03)*	*(0.034)*
Construction (F)	0.008	0.016*	0.006	0.003
	(0.034)	*(0.008)*	*(0.024)*	*(0.028)*
Wholesale Retail and Motor Trade (G)	-0.001	0.009	0.018	-0.016
	(0.032)	*(0.005)*	*(0.023)*	*(0.027)*
Transportation and Storage (H)	-0.021	0.017	0	-0.017
	(0.034)	*(0.009)*	*(0.024)*	*(0.028)*
Accommodation and Catering (I)	0.015	0.024*	0.037	-0.054*
	(0.04)	*(0.011)*	*(0.028)*	*(0.027)*
Information and Communication (J)	0.106**	0.017*	0.019	0.083**
	(0.035)	*(0.007)*	*(0.024)*	*(0.03)*
Financial and Insurance (K)	-0.028	0.008	-0.041	-0.013
	(0.05)	*(0.011)*	*(0.022)*	*(0.041)*
Real Estate Activities (L)	-0.057	0.007	-0.002	-0.053
	(0.033)	*(0.007)*	*(0.024)*	*(0.027)*
Professional Science and Tech. (M)	0.016	0.033***	-0.004	0.009
	(0.032)	*(0.007)*	*(0.022)*	*(0.027)*

Table A.8 continued from previous page

	Any Public Support	EU Support	UK Regional Support	UK Central Support
Administrative and Support Services (N)	-0.027	0.017*	-0.004	-0.031
	(0.033)	*(0.008)*	*(0.023)*	*(0.028)*
Log BSD Employment	0	-0.006	-0.004	0.012***
	(0.005)	*(0.003)*	*(0.004)*	*(0.003)*
Log Age	-0.028**	-0.006	-0.023***	-0.015*
	(0.01)	*(0.004)*	*(0.007)*	*(0.007)*
Cooperate	0.116***	0.033***	0.057***	0.070***
	(0.012)	*(0.007)*	*(0.008)*	*(0.007)*
Foreign Ultimate Ownership	-0.044**	-0.014*	-0.043***	-0.022*
	(0.015)	*(0.006)*	*(0.011)*	*(0.01)*
% Qualified Scientists and Engineers	0.002***	0.000***	0.000*	0.001***
	(0.)	*(0.)*	*(0.)*	*(0.)*
% Qualified Other Staff	0	0	0	0
	(0.)	*(0.)*	*(0.)*	*(0.)*
Economic Risk	0.035*	0.015*	0.018	0.001
	(0.014)	*(0.006)*	*(0.01)*	*(0.008)*
Direct Cost	0.006	-0.009	0.006	0.019*
	(0.019)	*(0.008)*	*(0.009)*	*(0.007)*
Financial Cost	-0.018	0.001	0.007	-0.035***
	(0.021)	*(0.01)*	*(0.012)*	*(0.009)*
Finance Availability	0.058**	0.012	0.031*	0.022*
	(0.018)	*(0.008)*	*(0.012)*	*(0.009)*
Qualified Personnel	0.039**	0.006	0.029***	0.015
	(0.012)	*(0.004)*	*(0.009)*	*(0.008)*
Technology Information	-0.007	0.003	0.014	-0.035*
	(0.021)	*(0.008)*	*(0.014)*	*(0.014)*
Market Information	0.001	-0.005	0.008	0.014
	(0.024)	*(0.007)*	*(0.015)*	*(0.016)*
Dominated Market	-0.006	-0.011	-0.004	0.001
	(0.015)	*(0.007)*	*(0.012)*	*(0.01)*
Uncertain Demand	0.014	0.016	-0.027**	0.009
	(0.019)	*(0.009)*	*(0.01)*	*(0.008)*
UK Regulation	0.005	-0.009	0.019	0.008
	(0.021)	*(0.006)*	*(0.014)*	*(0.014)*
EU regulation	0.004	0.009	-0.005	-0.005
	(0.022)	*(0.006)*	*(0.016)*	*(0.014)*
Chi_sq (Overall)	603.170***	173.610***	429.390***	450.460***
Chi_sq (Time)	13.360**	10.030**	67.900***	15.520**
Chi_sq (Region)	79.570***	42.020***	159.920***	17.530***
Chi_sq (Industry)	95.550***	38.620***	53.610***	145.820***
Total Observations	17926	17926	17926	17926
Total Unique Reporting Units	12879	12879	12879	12879
Total Unique Enterprises	12831	12831	12831	12831

Note: Standard errors in parentheses. Chi squared statistics refer to the joint significance of the overall model, time dummy, region dummy, and industry dummy respectively. * p<0.05; ** p<0.01;*** p<0.001.

Table A.9: Estimated average marginal effects from Probit models for different sources of public support using High-tech sample, 2012-2020

	Any Public Support	EU Support	UK Regional Support	UK Central Support
2014	-0.049	-0.005	0.002	-0.066**
	(0.025)	*(0.015)*	*(0.016)*	*(0.024)*
2016	-0.026	-0.019	-0.025	-0.042
	(0.028)	*(0.016)*	*(0.017)*	*(0.027)*
2018	0.007	-0.025	-0.002	-0.013
	(0.028)	*(0.015)*	*(0.017)*	*(0.027)*
North East	0.02	0.032	0.054	0.017
	(0.056)	*(0.026)*	*(0.032)*	*(0.053)*
North West	-0.044	-0.001	0.016	-0.033
	(0.048)	*(0.02)*	*(0.021)*	*(0.045)*
Yorkshire and The Humber	0.038	0.012	0.081**	0.014
	(0.051)	*(0.019)*	*(0.03)*	*(0.049)*
East Midlands	0.042	0.024	0.028	0.017
	(0.055)	*(0.022)*	*(0.026)*	*(0.055)*
West Midlands	0.145**	0.112***	0.096**	0.041
	(0.053)	*(0.032)*	*(0.034)*	*(0.053)*
Eastern	-0.05	0.025	-0.002	-0.02
	(0.046)	*(0.018)*	*(0.019)*	*(0.043)*
South East	0.026	0.016	0.007	0.031
	(0.045)	*(0.018)*	*(0.019)*	*(0.043)*
South West	0.051	0.075*	0.036	0.028
	(0.052)	*(0.03)*	*(0.023)*	*(0.051)*
Wales	0.045	0.024	0.146***	-0.029
	(0.051)	*(0.021)*	*(0.03)*	*(0.046)*
Scotland	0.096*	0.011	0.205***	-0.026
	(0.048)	*(0.018)*	*(0.034)*	*(0.044)*
Northern Ireland	0.212***	0.090*	0.375***	-0.056
	(0.056)	*(0.037)*	*(0.049)*	*(0.049)*
Log BSD Employment	0.003	0.010**	0.007	0.006
	(0.009)	*(0.004)*	*(0.004)*	*(0.009)*
Log Age	-0.008	-0.025	-0.025	-0.004
	(0.024)	*(0.013)*	*(0.013)*	*(0.025)*
Cooperate	0.205***	0.058***	0.075***	0.175***
	(0.025)	*(0.016)*	*(0.017)*	*(0.025)*
Foreign Ultimate Ownership	-0.087**	-0.035**	-0.033*	-0.061*
	(0.031)	*(0.012)*	*(0.016)*	*(0.029)*
% Qualified Scientists and Engineers	0.003***	0.001***	0	0.003***
	(0.)	*(0.)*	*(0.)*	*(0.)*
% Qualified Other Staff	-0.001	0	0	0
	(0.001)	*(0.)*	*(0.)*	*(0.001)*
Economic Risk	-0.013	0.007	0.031	-0.055*
	(0.027)	*(0.013)*	*(0.017)*	*(0.025)*
Direct Cost	0.083**	0.001	0.015	0.078**
	(0.028)	*(0.013)*	*(0.015)*	*(0.026)*
Financial Cost	-0.012	0.021	0.041	0
	(0.037)	*(0.014)*	*(0.021)*	*(0.034)*
Finance Availability	0.068*	0.030*	-0.003	0.03
	(0.034)	*(0.014)*	*(0.019)*	*(0.032)*
Qualified Personnel	0.026	-0.011	0.014	0.033
	(0.029)	*(0.015)*	*(0.015)*	*(0.027)*

Table A.9 continued from previous page

	Any Public Support	EU Support	UK Regional Support	UK Central Support
Technology Information	-0.025	-0.021	0.021	-0.08
	(0.047)	*(0.023)*	*(0.028)*	*(0.042)*
Market Information	-0.045	-0.007	-0.026	-0.008
	(0.05)	*(0.022)*	*(0.034)*	*(0.039)*
Dominated Market	-0.001	0.009	-0.027	0.001
	(0.035)	*(0.018)*	*(0.02)*	*(0.032)*
Uncertain Demand	0.034	-0.001	0.003	0.022
	(0.03)	*(0.014)*	*(0.019)*	*(0.028)*
UK Regulation	-0.004	0.001	0.001	0.003
	(0.052)	*(0.018)*	*(0.019)*	*(0.048)*
EU regulation	0.01	0.03	0.014	-0.008
	(0.046)	*(0.017)*	*(0.02)*	*(0.045)*
Chi_sq (Overall)	198.540***	96.850***	142.24***	183.760***
Chi_sq (Time)	5.88	3.4	3.82	9.240*
Chi_sq (Region)	38.210***	28.560**	97.950***	8.71
Total Observations	4699	4699	4699	4699
Total Unique Reporting Units	2996	2996	2996	2996
Total Unique Enterprises	2941	2941	2941	2941

Note: Standard errors in parentheses. Chi squared statistics refer to the joint significance of the overall model, time dummy, region dummy, and industry dummy respectively. * p<0.05; ** p<0.01;*** p<0.001.

Table A.10: Estimated average marginal effects from Probit models for different sources of public support using Low-tech sample, 2012-2020

	Any Public Support	EU Support	UK Regional Support	UK Central Support
2014	0.001	-0.008	0.007	0.007
	(0.02)	*(0.005)*	*(0.016)*	*(0.008)*
2016	-0.017	0	-0.032***	-0.005
	(0.013)	*(0.007)*	*(0.008)*	*(0.007)*
2018	0.040**	-0.013*	0.033***	0.023**
	(0.013)	*(0.005)*	*(0.01)*	*(0.008)*
North East	0.079***	0.018*	0.067***	0.018
	(0.021)	*(0.007)*	*(0.015)*	*(0.013)*
North West	0.070***	0.012*	0.057***	0.02
	(0.017)	*(0.006)*	*(0.013)*	*(0.013)*
Yorkshire and The Humber	0.099***	0.033**	0.054***	0.025
	(0.018)	*(0.011)*	*(0.013)*	*(0.013)*
East Midlands	0.126***	0.011	0.110**	0.02
	(0.038)	*(0.006)*	*(0.037)*	*(0.016)*
West Midlands	0.065***	0.024**	0.054***	0.030*
	(0.018)	*(0.008)*	*(0.014)*	*(0.014)*
Eastern	0.047**	0.001	0.033***	0.02
	(0.015)	*(0.004)*	*(0.01)*	*(0.012)*
South East	0.018	-0.004	0.015*	0.004
	(0.014)	*(0.003)*	*(0.008)*	*(0.011)*
South West	0.062***	0.016*	0.041***	0.016
	(0.017)	*(0.007)*	*(0.012)*	*(0.012)*
Wales	0.141***	0.008	0.139***	-0.005
	(0.033)	*(0.005)*	*(0.029)*	*(0.01)*
Scotland	0.108***	0.022*	0.103***	0.008
	(0.019)	*(0.01)*	*(0.015)*	*(0.01)*
Northern Ireland	0.116***	0.012	0.124***	0.003
	(0.022)	*(0.007)*	*(0.019)*	*(0.012)*
Log BSD Employment	0.002	-0.004	-0.004	0.010***
	(0.003)	*(0.002)*	*(0.003)*	*(0.002)*
Log Age	-0.020*	-0.003	-0.020**	-0.003
	(0.009)	*(0.003)*	*(0.007)*	*(0.006)*
Cooperate	0.103***	0.028***	0.055***	0.056***
	(0.012)	*(0.006)*	*(0.008)*	*(0.007)*
Foreign Ultimate Ownership	-0.029*	-0.01	-0.040***	-0.01
	(0.012)	*(0.006)*	*(0.01)*	*(0.009)*
% Qualified Scientists and Engineers	0.001***	0.000**	0	0.001***
	(0.)	*(0.)*	*(0.)*	*(0.)*
% Qualified Other Staff	0	0	0	0
	(0.)	*(0.)*	*(0.)*	*(0.)*
Economic Risk	0.042**	0.014*	0.019	0.006
	(0.015)	*(0.006)*	*(0.012)*	*(0.008)*
Direct Cost	-0.007	-0.008	0.006	0.01
	(0.02)	*(0.009)*	*(0.011)*	*(0.008)*
Financial Cost	-0.014	-0.001	-0.001	-0.039***
	(0.022)	*(0.01)*	*(0.013)*	*(0.009)*
Finance Availability	0.052**	0.007	0.037**	0.018*
	(0.019)	*(0.008)*	*(0.014)*	*(0.009)*
Qualified Personnel	0.039**	0.010*	0.029**	0.013
	(0.013)	*(0.004)*	*(0.009)*	*(0.008)*

Table A.10 continued from previous page

	Any Public Support	EU Support	UK Regional Support	UK Central Support
Technology Information	-0.006	0.006	0.008	-0.027
	(0.021)	*(0.007)*	*(0.015)*	*(0.014)*
Market Information	0.011	-0.002	0.019	0.018
	(0.025)	*(0.007)*	*(0.015)*	*(0.016)*
Dominated Market	-0.011	-0.018**	0	0.001
	(0.016)	*(0.007)*	*(0.012)*	*(0.01)*
Uncertain Demand	0.011	0.016	-0.031**	0.01
	(0.02)	*(0.008)*	*(0.011)*	*(0.008)*
UK Regulation	0.003	-0.008	0.02	0.002
	(0.021)	*(0.006)*	*(0.015)*	*(0.014)*
EU regulation	0.002	0.004	-0.007	-0.001
	(0.022)	*(0.005)*	*(0.016)*	*(0.014)*
Chi_sq (Overall)	379.200***	112.35***	376.510***	252.270***
Chi_sq (Time)	14.090**	8.000*	65.55***	18.190***
Chi_sq (Region)	99.610***	34.440***	146.280***	17.07
Total Observations	19672	19672	19672	19672
Total Unique Reporting Units	13965	13965	13965	13965
Total Unique Enterprises	13782	13782	13782	13782

Note: Standard errors in parentheses. Chi squared statistics refer to the joint significance of the overall model, time dummy, region dummy, and industry dummy respectively. * p<0.05; ** p<0.01;*** p<0.001.

Table A.11: Estimated average marginal effects on Direct Support from Probit models using all samples, 2012-2020

	Full Sample	High-Tech	Low-Tech	Large	SME
2014	-0.023**	-0.046**	-0.020**	-0.009	-0.023**
	(0.007)	*(0.015)*	*(0.007)*	*(0.006)*	*(0.007)*
2016	-0.025***	-0.038*	-0.024***	-0.006	-0.026***
	(0.006)	*(0.016)*	*(0.006)*	*(0.006)*	*(0.006)*
2018	-0.026***	-0.043**	-0.023***	-0.008	-0.026***
	(0.006)	*(0.014)*	*(0.006)*	*(0.006)*	*(0.006)*
North East	0.01	0.036	0.008	0.029	0.008
	(0.005)	*(0.026)*	*(0.004)*	*(0.02)*	*(0.005)*
North West	0.009	-0.013	0.013**	0.002	0.01
	(0.005)	*(0.016)*	*(0.005)*	*(0.008)*	*(0.005)*
Yorkshire and The Humber	0.017**	0.059*	0.014**	-0.002	0.018**
	(0.006)	*(0.028)*	*(0.005)*	*(0.008)*	*(0.006)*
East Midlands	0.006	0.023	0.006	0.01	0.005
	(0.004)	*(0.022)*	*(0.004)*	*(0.012)*	*(0.005)*
West Midlands	0.040***	0.090**	0.036***	0.002	0.043***
	(0.011)	*(0.033)*	*(0.011)*	*(0.009)*	*(0.011)*
Eastern	0.014**	0.048*	0.012**	0.014	0.014**
	(0.005)	*(0.023)*	*(0.005)*	*(0.01)*	*(0.005)*
South East	0.012*	0.040*	0.006	0.004	0.012*
	(0.005)	*(0.019)*	*(0.004)*	*(0.007)*	*(0.005)*
South West	0.018*	0.03	0.018**	0.018	0.018*
	(0.007)	*(0.02)*	*(0.007)*	*(0.011)*	*(0.007)*
Wales	0.035*	0.077*	0.029	-0.002	0.037*
	(0.017)	*(0.03)*	*(0.016)*	*(0.01)*	*(0.018)*
Scotland	0.037***	0.058*	0.033**	0.016	0.037***
	(0.011)	*(0.023)*	*(0.011)*	*(0.011)*	*(0.011)*
Northern Ireland	0.023**	0.071	0.018**	0.003	0.025**
	(0.008)	*(0.039)*	*(0.006)*	*(0.014)*	*(0.008)*
Manufacturing (C)	0.033***			0.027	0.037***
	(0.008)			*(0.028)*	*(0.008)*
Electricity Gas Steam and A/C (D)	0.003			0.031	-0.003
	(0.012)			*(0.047)*	*(0.008)*
Water Supply and Waste Management (F)	0.01			0.014	0.012
	(0.014)			*(0.035)*	*(0.015)*
Construction (F)	0.002			-0.019	0.006
	(0.008)			*(0.029)*	*(0.008)*
Wholesale Retail and Motor Trade (G)	0.005			-0.035	0.009
	(0.008)			*(0.027)*	*(0.008)*
Transportation and Storage (H)	0.013			0.013	0.016
	(0.01)			*(0.03)*	*(0.01)*
Accommodation and Catering (I)	0.006			-0.035	0.011
	(0.01)			*(0.028)*	*(0.01)*
Information and Communication (J)	0.011			-0.025	0.016
	(0.008)			*(0.028)*	*(0.008)*
Financial and Insurance (K)	-0.009			-0.023	-0.007
	(0.007)			*(0.028)*	*(0.007)*
Real Estate Activities (L)	-0.002			-0.014	0.001
	(0.008)			*(0.029)*	*(0.008)*
Professional Science and Tech. (M)	0.020*			-0.008	0.025**
	(0.008)			*(0.028)*	*(0.008)*
Administrative and Support Services (N)	0			-0.036	0.005

Table A.11 continued from previous page

	Full Sample	High-Tech	Low-Tech	Large	SME
	(0.008)			*(0.027)*	*(0.008)*
Log BSD Employment	0.003*	0.008	0.003**	0.007**	0.001
	(0.001)	*(0.004)*	*(0.001)*	*(0.002)*	*(0.002)*
Log Age	-0.011**	-0.046**	-0.003	-0.018**	-0.010*
	(0.004)	*(0.014)*	*(0.004)*	*(0.006)*	*(0.004)*
Cooperate	0.021***	0.066***	0.016***	0.029***	0.021***
	(0.004)	*(0.015)*	*(0.004)*	*(0.008)*	*(0.004)*
Foreign Ultimate Ownership	-0.011**	-0.02	-0.011*	-0.006	-0.013*
	(0.004)	*(0.013)*	*(0.004)*	*(0.005)*	*(0.005)*
% Qualified Scientists and Engineers	0.001***	0.001***	0.000***	0.001***	0.001***
	(0.)	*(0.)*	*(0.)*	*(0.)*	*(0.)*
% Qualified Other Staff	0	-0.001	0	0	0
	(0.)	*(0.)*	*(0.)*	*(0.)*	*(0.)*
Economic Risk	0.004	-0.009	0.004	0.006	0.004
	(0.005)	*(0.013)*	*(0.005)*	*(0.007)*	*(0.005)*
Direct Cost	0.010*	0.015	0.009	-0.002	0.011*
	(0.005)	*(0.015)*	*(0.005)*	*(0.007)*	*(0.005)*
Financial Cost	-0.004	-0.001	-0.004	-0.005	-0.005
	(0.005)	*(0.015)*	*(0.005)*	*(0.008)*	*(0.005)*
Finance Availability	0.010*	0.040**	0.005	0.007	0.010*
	(0.004)	*(0.015)*	*(0.004)*	*(0.007)*	*(0.005)*
Qualified Personnel	0.013**	0.018	0.012**	0.001	0.013**
	(0.004)	*(0.012)*	*(0.004)*	*(0.009)*	*(0.004)*
Technology Information	0.003	-0.017	0.003	0.002	0.003
	(0.009)	*(0.025)*	*(0.008)*	*(0.012)*	*(0.01)*
Market Information	0	-0.013	0.004	0.02	-0.001
	(0.007)	*(0.021)*	*(0.007)*	*(0.011)*	*(0.007)*
Dominated Market	-0.003	-0.014	-0.003	-0.016	-0.003
	(0.006)	*(0.019)*	*(0.006)*	*(0.009)*	*(0.006)*
Uncertain Demand	-0.006	-0.002	-0.006	0.015*	-0.007
	(0.005)	*(0.013)*	*(0.005)*	*(0.007)*	*(0.005)*
UK Regulation	0.014	0.025	0.01	0.014	0.014
	(0.009)	*(0.023)*	*(0.009)*	*(0.009)*	*(0.009)*
EU regulation	-0.021**	-0.049*	-0.017*	-0.015	-0.022**
	(0.008)	*(0.022)*	*(0.008)*	*(0.01)*	*(0.008)*
Chi_sq (Overall)	316.170***	80.140***	174.760***	115.810***	277.220***
Chi_sq (Time)	26.610***	11.010*	23.290***	2.51	24.960***
Chi_sq (Region)	50.940***	19.160*	50.590***	10.08	50.520***
Chi_sq (Industry)	60.970***			61.610***	53.750***
Total Observations	24371	4699	19672	6445	17926
Total Unique Reporting Units	16899	2996	13965	4262	12879
Total Unique Enterprises	16632	2941	13782	4111	12831

Note: Standard errors in parentheses. Chi squared statistics refer to the joint significance of the overall model, time dummy, region dummy, and industry dummy respectively. * p<0.05; ** p<0.01;*** p<0.001.

Table A.12: Estimated average marginal effects on Indirect Support from Probit models using all samples, 2012-2020

	Full Sample	High-Tech	Low-Tech	Large	SME
2014	-0.013*	-0.060*	-0.008	-0.007	-0.013*
	(0.006)	*(0.023)*	*(0.005)*	*(0.009)*	*(0.006)*
2016	-0.006	-0.042	-0.004	-0.006	-0.007
	(0.006)	*(0.026)*	*(0.006)*	*(0.009)*	*(0.006)*
2018	0.012	0.005	0.011	0.029**	0.011
	(0.006)	*(0.026)*	*(0.006)*	*(0.01)*	*(0.006)*
North East	0.021	0.024	0.023*	0.035	0.02
	(0.012)	*(0.053)*	*(0.011)*	*(0.024)*	*(0.013)*
North West	0.013	-0.005	0.018	0.004	0.014
	(0.01)	*(0.047)*	*(0.01)*	*(0.013)*	*(0.011)*
Yorkshire and The Humber	0.026*	0.034	0.029*	0.025	0.027*
	(0.011)	*(0.049)*	*(0.011)*	*(0.017)*	*(0.012)*
East Midlands	0.011	0.023	0.014	0.016	0.011
	(0.011)	*(0.055)*	*(0.011)*	*(0.017)*	*(0.011)*
West Midlands	0.008	0.041	0.008	0.026	0.007
	(0.01)	*(0.052)*	*(0.009)*	*(0.016)*	*(0.01)*
Eastern	0.013	-0.023	0.018	0.012	0.013
	(0.01)	*(0.042)*	*(0.01)*	*(0.016)*	*(0.01)*
South East	0.009	0.04	0.001	0.01	0.009
	(0.009)	*(0.043)*	*(0.008)*	*(0.013)*	*(0.009)*
South West	0.022	0.023	0.021	0.035	0.022
	(0.013)	*(0.05)*	*(0.013)*	*(0.018)*	*(0.014)*
Wales	-0.001	-0.011	0.004	0.023	-0.002
	(0.009)	*(0.045)*	*(0.008)*	*(0.025)*	*(0.01)*
Scotland	-0.005	-0.034	0.001	0.014	-0.006
	(0.008)	*(0.043)*	*(0.007)*	*(0.016)*	*(0.009)*
Northern Ireland	-0.01	-0.07	0.001	0.001	-0.011
	(0.01)	*(0.049)*	*(0.009)*	*(0.022)*	*(0.011)*
Manufacturing (C)	0.048*			0.055	0.049
	(0.024)			*(0.05)*	*(0.027)*
Electricity Gas Steam and A/C (D)	-0.017			0.003	-0.013
	(0.037)			*(0.072)*	*(0.042)*
Water Supply and Waste Management (F)	0.004			0.008	0.005
	(0.033)			*(0.06)*	*(0.036)*
Construction (F)	-0.021			-0.015	-0.019
	(0.025)			*(0.051)*	*(0.027)*
Wholesale Retail and Motor Trade (G)	-0.041			-0.054	-0.039
	(0.024)			*(0.049)*	*(0.026)*
Transportation and Storage (H)	-0.045			-0.071	-0.042
	(0.024)			*(0.05)*	*(0.027)*
Accommodation and Catering (I)	-0.071**			-0.083	-0.068**
	(0.023)			*(0.049)*	*(0.026)*
Information and Communication (J)	0.072**			0.039	0.075*
	(0.027)			*(0.053)*	*(0.03)*
Financial and Insurance (K)	-0.009			-0.024	-0.007
	(0.036)			*(0.052)*	*(0.04)*
Real Estate Activities (L)	-0.062**			-0.085	-0.059*
	(0.024)			*(0.05)*	*(0.027)*
Professional Science and Tech. (M)	-0.006			-0.016	-0.004
	(0.024)			*(0.05)*	*(0.027)*
Administrative and Support Services (N)	-0.046			-0.083	-0.041

Table A.12 continued from previous page

	Full Sample	High-Tech	Low-Tech	Large	SME
	(0.024)			*(0.049)*	*(0.027)*
Log BSD Employment	0.009***	0.003	0.010***	0.004	0.012***
	(0.002)	*(0.009)*	*(0.001)*	*(0.004)*	*(0.003)*
Log Age	-0.012*	0.019	-0.004	0.001	-0.013*
	(0.005)	*(0.025)*	*(0.005)*	*(0.01)*	*(0.006)*
Cooperate	0.058***	0.167***	0.042***	0.077***	0.057***
	(0.006)	*(0.025)*	*(0.006)*	*(0.011)*	*(0.006)*
Foreign Ultimate Ownership	-0.013	-0.062*	0	-0.016	-0.013
	(0.007)	*(0.028)*	*(0.007)*	*(0.008)*	*(0.009)*
% Qualified Scientists and Engineers	0.001***	0.002***	0.001***	0.001***	0.001***
	(0.)	*(0.)*	*(0.)*	*(0.)*	*(0.)*
% Qualified Other Staff	0	0	0	0	0
	(0.)	*(0.001)*	*(0.)*	*(0.)*	*(0.)*
Economic Risk	-0.004	-0.045	-0.001	-0.006	-0.004
	(0.006)	*(0.025)*	*(0.006)*	*(0.012)*	*(0.006)*
Direct Cost	0.013*	0.056*	0.009	0.01	0.013*
	(0.006)	*(0.025)*	*(0.007)*	*(0.012)*	*(0.007)*
Financial Cost	-0.019*	-0.011	-0.021*	-0.03	-0.018*
	(0.008)	*(0.033)*	*(0.008)*	*(0.016)*	*(0.009)*
Finance Availability	0.022**	0.033	0.019**	0.009	0.023**
	(0.008)	*(0.031)*	*(0.007)*	*(0.015)*	*(0.008)*
Qualified Personnel	0.011	0.027	0.01	0.013	0.01
	(0.007)	*(0.026)*	*(0.006)*	*(0.013)*	*(0.007)*
Technology Information	-0.037***	-0.095*	-0.029**	-0.016	-0.037**
	(0.011)	*(0.043)*	*(0.011)*	*(0.019)*	*(0.012)*
Market Information	0.003	0.016	0.004	-0.004	0.004
	(0.009)	*(0.037)*	*(0.009)*	*(0.02)*	*(0.01)*
Dominated Market	0.005	0.014	0.003	-0.01	0.005
	(0.007)	*(0.031)*	*(0.007)*	*(0.015)*	*(0.008)*
Uncertain Demand	0.025***	0.037	0.023***	0.036**	0.025***
	(0.007)	*(0.027)*	*(0.006)*	*(0.012)*	*(0.007)*
UK Regulation	-0.014	0.007	-0.018*	-0.021	-0.013
	(0.009)	*(0.048)*	*(0.008)*	*(0.02)*	*(0.009)*
EU regulation	0.006	0.002	0.008	-0.002	0.008
	(0.009)	*(0.045)*	*(0.008)*	*(0.02)*	*(0.009)*
Chi_sq (Overall)	437.260***	161.710***	196.280***	258.02***	367.42***
Chi_sq (Time)	21.380***	11.520**	13.920**	16.420***	17.930***
Chi_sq (Region)	21.730*	10.02	22.060*	8.38	20.870*
Chi_sq (Industry)	174.220***			125.310***	147.160***
Total Observations	24371	4699	19672	6445	17926
Total Unique Reporting Units	16899	2996	13965	4262	12879
Total Unique Enterprises	16632	2941	13782	4111	12831

Note: Standard errors in parentheses. Chi squared statistics refer to the joint significance of the overall model, time dummy, region dummy, and industry dummy respectively. * p<0.05; ** p<0.01;*** p<0.001.

Table A.13: PSM Kernel Matching with Regression Adjustment for Labour Productivity and Productivity Growth using Full Sample, 2012-2020

	Any Public Support	EU Support	Central Support	Regional Support	Indirect Support	Direct Support
Labour Productivity	-0.060**	-0.210***	-0.064**	-0.068*	-0.057*	-0.207***
	(0.02)	*(0.054)*	*(0.024)*	*(0.028)*	*(0.027)*	*(0.045)*
Total Observations	24359	20817	22870	21887	22330	20947
Total Unique Reporting Units	16891	15232	16171	15810	15930	15309
Total Unique Enterprises	16626	14998	15917	15572	15680	15074
Productivity Growth	-0.01	-0.039	-0.015	-0.031	-0.031	-0.037
	(0.012)	*(0.031)*	*(0.016)*	*(0.017)*	*(0.02)*	*(0.029)*
Total Observations	24354	20813	22866	21883	22326	20944
Total Unique Reporting Units	16888	15228	16168	15807	15927	15306
Total Unique Enterprises	16623	14994	15914	15569	15677	15071

Note: Standard errors in parentheses.* p<0.05; ** p<0.01;*** p<0.001.

Table A.14: PSM Kernel with Regression Adjustment Matching Statistics for Labour Productivity using Full Sample, 2012-2020

	Any Public Support	EU Support	Central Support	Regional Support	Indirect Support	Direct Support
Mean bias Unmatched	0.274	0.313	0.279	0.298	0.293	0.302
Mean bias matched	0.009	0.028	0.01	0.013	0.01	0.018
Median bias Unmatched	0.164	0.213	0.159	0.202	0.203	0.238
Median bias Matched	0.008	0.028	0.006	0.011	0.006	0.011
Ps R^2 Unmatched	0.193	0.23	0.215	0.192	0.241	0.253
LR Unmatched	4362.910***	1462.400***	3648.670***	2394.18***	3547.16***	1873.870***
B Unmatched	120.164	150.321	132.996	127.715	145.168	157.715
R Unmatched	0.604	1.022	0.662	0.586	0.585	1.013
Ps R^2 Matched	0.002	0.008	0.002	0.003	0.003	0.005
LR Matched	20.56	14.21	15.97	12.55	14.8	11.6
B Matched	10.68	57.709	19.075	27.846	22.121	50.983
R Matched	1.091	0.545	0.841	0.811	0.576	0.742
Unmatched Control	20080	20080	20080	20080	20052	20052
Unmatched Treated	4279	737	2790	1807	2278	895
Unmatched Total Observations	24359	20817	22870	21887	22330	20947
Unmatched Total Unique Reporting Units	16891	15232	16171	15810	15930	15309
Unmatched Total Unique Enterprises	16626	14998	15917	15572	15680	15074
Matched Control	3977	670	2593	1666	2118	827
Matched Treated	17700	19593	17879	17845	17858	17493
Matched Total Observations	21677	20263	20472	19511	19976	18320
Matched Total Unique Reporting Units	15456	14929	14808	14533	14592	13565
Matched Total Unique Enterprises	15232	14706	14575	14335	14363	13359

Note: Ps R^2 stands for Pseudo R^2. LR stands for likelihood ratio. B and R refer to Rubin's B and Rubin's R respectively. * p<0.05; ** p<0.01;*** p<0.001.

Table A.15: PSM Kernel with Regression Adjustment Matching Statistics for Productivity Growth using Full Sample, 2012-2020

	Any Public Support	EU Support	Central Support	Regional Support	Indirect Support	Direct Support
Mean bias Unmatched	0.274	0.313	0.279	0.298	0.293	0.302
Mean bias matched	0.007	0.029	0.01	0.013	0.011	0.018
Median bias Unmatched	0.164	0.213	0.159	0.201	0.203	0.238
Median bias Matched	0.004	0.026	0.005	0.011	0.007	0.011
Ps R^2 Unmatched	0.193	0.23	0.215	0.192	0.241	0.253
LR Unmatched	4362.910***	1462.400***	3648.670***	2394.180***	3547.160***	1873.870***
B Unmatched	120.164	150.321	132.996	127.715	145.168	157.715
R Unmatched	0.604	1.022	0.662	0.856	0.585	1.013
Ps R^2 Matched	0.002	0.008	0.002	0.003	0.003	0.005
LR Matched	16.06	13.96	14.71	13.03	15.67	11.6
B Matched	7.302	49.895	21.922	21.235	21.525	50.981
R Matched	1.03	0.638	0.877	1.054	0.603	0.742
Unmatched Control	20077	20077	20077	20077	20049	20049
Unmatched Treated	4277	736	2789	1806	2277	895
Unmatched Total Observations	24354	20813	22866	21883	22326	20944
Unmatched Total Unique Reporting Units	16888	15228	16168	15807	15927	15306
Unmatched Total Unique Enterprises	16623	14994	15914	15569	15677	15071
Matched Control	17474	19565	17874	17823	17915	17491
Matched Treated	3956	677	2586	1668	2113	827
Matched Total Observations	21430	20242	20460	19491	20028	18318
Matched Total Unique Reporting Units	15313	14912	14800	14535	14606	13563
Matched Total Unique Enterprises	15089	14690	14566	14337	14377	13357

Note: Ps R^2 stands for Pseudo R^2. LR stands for likelihood ratio. B and R refer to Rubin's B and Rubin's R respectively. * $p<0.05$; ** $p<0.01$;*** $p<0.001$.

Table A.16: PSM Kernel with Regression Adjustment Balance Statistics for Labour Productivity using Full Sample, 2012-2020

	Any Public Support		EU Support		UK Central Support		UK Regional Support		Indirect Support		Direct Support	
	Raw	Matched	Raw	Matched	Raw	Matched	Raw	Matched	Raw	Matched	Raw	Matched
North East	0.062	-0.002	0.123	-0.028	0.048	0.007	0.064	0.02	0.066	-0.009	0.039	0.005
North West	-0.056	-0.004	-0.06	0.035	-0.045	-0.005	-0.064	0.009	-0.041	-0.004	-0.099	0.009
Yorkshire and The Humber	0.016	-0.009	0.033	0.009	0.026	-0.008	-0.011	0.01	0.04	0.01	0.035	-0.008
East Midlands	-0.01	-0.006	-0.041	-0.007	0	-0.031	-0.051	0.005	-0.011	-0.032	-0.079	0
West Midlands	0.015	-0.004	0.059	0.001	0.019	-0.008	0.023	0.031	0.021	0.014	0.029	-0.011
Eastern	-0.013	0.013	0.043	-0.001	0.034	-0.001	-0.126	-0.013	0.032	0.007	0.054	0.021
South East	-0.048	-0.008	-0.108	0.009	0.033	0.021	-0.225	0.002	0.049	0	0.026	-0.022
South West	0.017	0.033	0.043	-0.016	0.05	0.026	-0.066	0	0.048	-0.004	0.05	0.038
Wales	0.05	-0.009	0.041	0.007	-0.017	0	0.158	-0.05	-0.01	0.006	0.035	0.021
Scotland	0.117	0.005	0.124	-0.014	0.025	-0.009	0.306	0.003	0	0.022	0.112	-0.001
Northern Ireland	0.123	-0.01	0.077	0.029	-0.001	-0.003	0.287	-0.011	-0.011	0.01	0.073	-0.003
Manufacturing (C)	0.328	0.027	0.291	0.084	0.352	0.003	0.304	0.025	0.412	0.002	0.349	0.054
Electricity Gas Steam and A/C (D)	0.014	-0.01	0.009	-0.006	0.018	-0.014	-0.014	0.005	0.007	-0.014	0.03	0.018
Water Supply and Waste Management (F)	0.011	0.013	0.026	0.021	0.017	0.016	0.033	0	0	0.014	0.015	0.028
Construction (F)	-0.072	-0.001	-0.169	-0.001	-0.072	0.011	-0.084	-0.014	-0.081	0.003	-0.174	0.004
Wholesale Retail and Motor Trade (G)	-0.279	-0.015	-0.383	-0.029	-0.353	-0.002	-0.219	-0.01	-0.402	-0.002	-0.419	-0.031
Transportation and Storage (H)	-0.104	-0.001	-0.155	0.003	-0.121	0.001	-0.061	-0.002	-0.203	0.001	-0.07	0.01
Accommodation and Catering (I)	-0.196	-0.01	-0.254	0.002	-0.267	-0.01	-0.088	-0.019	-0.317	-0.012	-0.263	0
Information and Communication (J)	0.145	-0.012	0.024	-0.055	0.182	-0.024	0.094	0.014	0.219	-0.004	0.009	0.025
Financial and Insurance (K)	-0.146	-0.01	-0.213	-0.011	-0.151	-0.002	-0.158	-0.011	-0.136	-0.006	-0.238	-0.005
Real Estate Activities (L)	-0.092	0	-0.131	0.002	-0.149	-0.005	0.014	0.004	-0.21	-0.007	-0.124	0.005
Professional Science and Tech. (M)	0.207	0.01	0.51	-0.024	0.276	0.027	0.105	0.008	0.288	0.012	0.498	-0.067
Adminitrative and Support Services (N)	-0.201	-0.019	-0.203	-0.015	-0.247	-0.019	-0.174	-0.02	-0.265	-0.002	-0.307	0.005
Log BSD Employment	-0.087	-0.001	-0.183	0.028	0.01	-0.025	-0.202	0.013	0.007	-0.042	-0.087	0.034
Log Age	-0.132	-0.006	-0.215	0.004	-0.095	0.011	-0.198	0.017	-0.095	0	-0.225	-0.035
Cooperate	0.843	0.028	0.993	0.068	0.906	0.03	0.842	0.031	0.918	0.027	1.024	0.05
Foreign Ultimate Ownership	-0.004	-0.004	-0.103	0.019	0.057	-0.027	-0.115	0.034	0.074	0.005	0.015	0.026
% Qualified Scientists and Engineers	0.586	-0.026	0.863	-0.112	0.733	-0.016	0.409	-0.012	0.798	-0.029	0.93	-0.077
% Qualified Other Staff	0.133	0.004	0.116	0.042	0.139	0.003	0.131	-0.009	0.135	-0.02	0.069	0.026

Economic Risk	0.239	0.017	0.321	0.033	0.211	-0.013	0.324	-0.021	0.197	-0.018	0.298	-0.002
Direct Cost	0.273	0.017	0.377	0.03	0.259	0.028	0.342	-0.006	0.263	0.015	0.362	0.003
Financial Cost	0.194	0.023	0.299	0.041	0.121	0.015	0.326	-0.007	0.104	-0.001	0.246	0.037
Finance Availability	0.29	0.01	0.457	0.037	0.256	0.003	0.378	0.001	0.25	0.008	0.407	0.04
Qualified Personnel	0.257	-0.006	0.288	0.035	0.242	0	0.297	0.025	0.256	-0.012	0.301	0.011
Technology Information	0.114	0.004	0.119	0.044	0.056	-0.003	0.176	-0.001	0.033	-0.003	0.06	-0.002
Market Information	0.164	-0.009	0.208	0.03	0.159	-0.001	0.201	0.029	0.166	-0.017	0.194	0.023
Dominated Market	0.141	-0.014	0.143	0.018	0.126	0.001	0.165	0.026	0.13	-0.016	0.119	0.003
Uncertain Demand	0.233	-0.007	0.277	0.063	0.262	-0.004	0.184	0	0.292	-0.011	0.302	0.008
UK Regulation	0.136	-0.015	0.184	0.045	0.094	-0.029	0.234	-0.003	0.059	-0.033	0.12	0.024
EU regulation	0.127	-0.004	0.194	0.038	0.097	-0.023	0.172	-0.007	0.084	-0.019	0.099	0.036
Total Observations	24359	21677	20817	20263	22870	20472	21887	19511	22330	19976	20947	18320
Total Unique Reporting Units	16891	15456	15232	14929	16171	14808	15810	14533	15930	14592	15309	13565
Total Unique Enterprises	16626	15232	14998	14706	15917	14575	15572	14335	15680	14363	15074	13359

Table A.17: PSM Kernel with Regression Adjustment Balance Statistics for Productivity Growth using Full Sample, 2012-2020

	Any Public Support		EU Support		UK Central Support		UK Regional Support		Indirect Support		Direct Support	
	Raw	Matched	Raw	Matched	Raw	Matched	Raw	Matched	Raw	Matched	Raw	Matched
North East	0.062	-0.004	0.124	-0.02	0.049	-0.001	0.064	0.011	0.066	-0.002	0.039	0.004
North West	-0.056	-0.001	-0.06	0.046	-0.045	-0.002	-0.064	0.015	-0.041	-0.01	-0.099	0.009
Yorkshire and The Humber	0.016	-0.002	0.033	0.01	0.026	0	-0.011	0.011	0.04	0.007	0.035	-0.008
East Midlands	-0.01	0.001	-0.04	-0.002	0	-0.021	-0.05	0.011	-0.011	-0.036	-0.079	0.001
West Midlands	0.016	0.001	0.059	-0.009	0.019	-0.008	0.023	0.03	0.022	0.003	0.029	-0.011
Eastern	-0.013	0.006	0.044	0.016	0.034	-0.002	-0.126	-0.012	0.032	0.008	0.054	0.021
South East	-0.048	0.004	-0.113	-0.006	0.032	0.017	-0.225	0.002	0.048	-0.002	0.026	-0.022
South West	0.018	0.023	0.044	0.007	0.05	0.023	-0.066	-0.006	0.048	-0.006	0.05	0.038
Wales	0.051	-0.008	0.041	-0.022	-0.017	0	0.159	-0.02	-0.01	0.012	0.035	0.021
Scotland	0.117	0.002	0.124	-0.025	0.025	-0.004	0.306	-0.002	0	0.031	0.112	-0.001
Northern Ireland	0.122	-0.024	0.077	0.034	-0.001	0.001	0.285	-0.034	-0.011	0.008	0.072	-0.003
Manufacturing (C)	0.328	0.026	0.292	0.085	0.352	0.013	0.304	0.018	0.412	-0.001	0.349	0.054
Electricity Gas Steam and A/C (D)	0.014	-0.004	0.009	0.018	0.018	-0.015	-0.014	0.007	0.007	-0.026	0.03	0.018
Water Supply and Waste Management (F)	0.011	0.012	0.026	0.033	0.017	0.017	0.033	-0.004	0	0.014	0.015	0.028
Construction (F)	-0.072	-0.004	-0.169	0	-0.072	0.005	-0.084	-0.01	-0.081	0.005	-0.174	0.004
Wholesale Retail and Motor Trade (G)	-0.279	-0.018	-0.383	-0.028	-0.353	-0.004	-0.219	-0.013	-0.401	-0.004	-0.419	-0.031
Transportation and Storage (H)	-0.103	-0.003	-0.155	0.003	-0.121	0	-0.06	-0.011	-0.203	0.001	-0.07	0.01
Accommodation and Catering (I)	-0.196	-0.01	-0.254	0.001	-0.267	-0.011	-0.088	-0.027	-0.317	-0.013	-0.263	0
Information and Communication (J)	0.144	-0.001	0.025	-0.04	0.182	-0.029	0.092	0.023	0.219	-0.007	0.009	0.025
Financial and Insurance (K)	-0.145	-0.011	-0.213	-0.014	-0.15	0	-0.158	-0.012	-0.136	-0.005	-0.238	-0.005
Real Estate Activities (L)	-0.092	0.002	-0.131	0.003	-0.149	-0.005	0.014	0.006	-0.21	-0.007	-0.124	0.005
Professional Science and Tech. (M)	0.207	0.009	0.508	-0.037	0.275	0.024	0.105	0.015	0.288	0.019	0.498	-0.067
Adminitrative and Support Services (N)	-0.201	-0.022	-0.203	-0.024	-0.247	-0.02	-0.174	-0.012	-0.265	-0.001	-0.307	0.005
Log BSD Employment	-0.086	-0.002	-0.182	0.044	0.011	-0.025	-0.201	0.006	0.008	-0.047	-0.087	0.034
Log Age	-0.131	-0.013	-0.214	0.03	-0.094	0.016	-0.197	0.004	-0.094	-0.004	-0.225	-0.035
Cooperate	0.843	0.027	0.993	0.06	0.906	0.031	0.841	0.035	0.917	0.026	1.024	0.05
Foreign Ultimate Ownership	-0.004	0.006	-0.102	0.025	0.057	-0.024	-0.117	0.014	0.074	0.006	0.015	0.026
% Qualified Scientists and Engineers	0.585	-0.017	0.862	-0.099	0.733	-0.015	0.41	-0.007	0.797	-0.018	0.93	-0.077
% Qualified Other Staff	0.133	0.005	0.115	0.023	0.139	0.001	0.132	0.002	0.134	-0.024	0.069	0.026

Economic Risk	0.239	0.007	0.322	0.028	0.211	-0.012	0.324	-0.031	0.197	-0.009	0.297	-0.002
Direct Cost	0.274	0.003	0.378	0.047	0.26	0.03	0.343	-0.006	0.263	0.02	0.362	0.003
Financial Cost	0.194	0.012	0.3	0.043	0.121	0.01	0.325	-0.002	0.104	0.012	0.246	0.037
Finance Availability	0.29	0.004	0.455	0.043	0.256	-0.004	0.378	0.003	0.249	0.018	0.407	0.04
Qualified Personnel	0.257	-0.004	0.289	0.007	0.242	0.01	0.298	0.023	0.256	-0.015	0.301	0.011
Technology Information	0.114	0.003	0.119	0.053	0.056	-0.004	0.176	-0.019	0.033	-0.005	0.06	-0.002
Market Information	0.164	0.001	0.208	0.015	0.159	0.002	0.201	0.034	0.166	-0.024	0.194	0.023
Dominated Market	0.141	-0.012	0.144	0.008	0.126	-0.002	0.165	0.023	0.13	-0.023	0.119	0.003
Uncertain Demand	0.233	-0.011	0.277	0.071	0.262	-0.012	0.184	-0.004	0.292	-0.013	0.302	0.008
UK Regulation	0.136	-0.019	0.184	0.04	0.094	-0.033	0.234	-0.011	0.059	-0.033	0.12	0.024
EU regulation	0.127	-0.007	0.194	0.045	0.097	-0.027	0.172	-0.007	0.084	-0.02	0.099	0.036
Total Observations	24354	21430	20813	20242	22866	20460	21883	19491	22326	20028	20944	18318
Total Unique Reporting Units	16888	15313	15228	14912	16168	14800	15807	14535	15927	14606	15306	13563
Total Unique Enterprises	16623	15089	14994	14690	15914	14566	15569	14337	15677	14377	15071	13357

Table A.18: PSM Kernel Matching with Regression Adjustment for Labour Productivity (Subsamples), 2012-2020

		Any Public Support	EU Support	Central Support	Regional Support	Indirect Support	Direct Support
High-tech	Labour Productivity	-0.207***	-0.416***	-0.230***	-0.190**	-0.250***	-0.492***
		(0.041)	*(0.087)*	*(0.045)*	*(0.061)*	*(0.048)*	*(0.081)*
	Total Observations	4699	3476	4321	3638	4175	3551
	Total Unique Reporting Units	2996	2441	2819	2531	2755	2489
	Total Unique Enterprises	2941	2398	2768	2486	2706	2444
Low-tech	Labour Productivity	0.016	-0.036	0.080**	-0.031	0.098**	0.023
		(0.023)	*(0.061)*	*(0.028)*	*(0.031)*	*(0.032)*	*(0.05)*
	Total Observations	19660	17341	18549	18249	18155	17396
	Total Unique Reporting Units	13957	12824	13402	13325	13222	12856
	Total Unique Enterprises	13776	12659	13226	13159	13046	12693
Large Firm	Labour Productivity	0.001	0.028	-0.004	0.052	0.029	-0.159
		(0.043)	*(0.11)*	*(0.047)*	*(0.073)*	*(0.051)*	*(0.103)*
	Total Observations	6439	5633	6198	5836	6060	5692
	Total Unique Reporting Units	4258	3927	4162	4022	4112	3956
	Total Unique Enterprises	4108	3797	4018	3888	3971	3825
SME	Labour Productivity	-0.098***	-0.337***	-0.153***	-0.140***	-0.102**	-0.221***
		(0.023)	*(0.061)*	*(0.029)*	*(0.029)*	*(0.031)*	*(0.054)*
	Total Observations	17920	15184	16672	16051	16270	15255
	Total Unique Reporting Units	12875	11491	12224	11993	12022	11539
	Total Unique Enterprises	12827	11444	12178	11948	11976	11494

Note: Standard errors in parentheses. * $p<0.05$; ** $p<0.01$;*** $p<0.001$.

Table A.19: PSM Kernel Matching with Regression Adjustment for Productivity Growth (Subsamples), 2012-2020

		Any Public Support	EU Support	Central Support	Regional Support	Indirect Support	Direct Support
High-tech	Productivity Growth	-0.065*	-0.128*	-0.051	-0.008	-0.048	-0.039
		(0.028)	*(0.058)*	*(0.029)*	*(0.047)*	*(0.03)*	*(0.059)*
	Total Observations	4697	3475	4320	3637	4174	3551
	Total Unique Reporting Units	2996	2440	2819	2531	2755	2489
	Total Unique Enterprises	2941	2397	2768	2486	2706	2444
Low-tech	Productivity Growth	0.015	-0.045	0.002	-0.001	-0.006	0.033
		(0.013)	*(0.039)*	*(0.017)*	*(0.019)*	*(0.017)*	*(0.027)*
	Total Observations	19657	17338	18546	18246	18152	17393
	Total Unique Reporting Units	13954	12821	13399	13322	13219	12853
	Total Unique Enterprises	13773	12656	13223	13156	13043	12690
Large Firm	Productivity Growth	0.007	-0.034	0.004	-0.02	0.009	-0.07
		(0.024)	*(0.043)*	*(0.025)*	*(0.044)*	*(0.03)*	*(0.054)*
	Total Observations	6437	5631	6196	5834	6058	5690
	Total Unique Reporting Units	4256	3925	4160	4020	4110	3954
	Total Unique Enterprises	4106	3795	4016	3886	3969	3823
SME	Productivity Growth	-0.013	-0.066	-0.028	0.008	-0.009	-0.008
		(0.013)	*(0.043)*	*(0.021)*	*(0.019)*	*(0.019)*	*(0.038)*
	Total Observations	17917	15182	16670	16049	16268	15254
	Total Unique Reporting Units	12873	11489	12223	11991	12021	11538
	Total Unique Enterprises	12825	11442	12177	11946	11975	11493

Note: Standard errors in parentheses. * $p<0.05$; ** $p<0.01$;*** $p<0.001$.

Table A.20: All Matchings for Labour Productivity using Full Sample, 2012-2020

	Any Public Support		EU Support		Central Support		Regional Support		Indirect Support		Direct Support	
MDM NN	-0.138***	*(0.023)*	-0.321***	*(0.062)*	-0.160***	*(0.029)*	-0.147***	*(0.035)*	-0.160***	*(0.031)*	-0.340***	*(0.053)*
MDM NN. PS	-0.092**	*(0.028)*	-0.289***	*(0.065)*	-0.101**	*(0.034)*	-0.100*	*(0.041)*	-0.136***	*(0.04)*	-0.212***	*(0.058)*
MDM Kernel	-0.130***	*(0.02)*	-0.397***	*(0.053)*	-0.107***	*(0.025)*	-0.213***	*(0.028)*	-0.088**	*(0.027)*	-0.310***	*(0.047)*
MDM Kernel. RA	-0.117***	*(0.02)*	-0.320***	*(0.052)*	-0.134***	*(0.025)*	-0.131***	*(0.027)*	-0.126***	*(0.027)*	-0.287***	*(0.045)*
MDM Kernel. CV	-0.073***	*(0.02)*	-0.351***	*(0.053)*	-0.038	*(0.024)*	-0.179***	*(0.028)*	-0.110***	*(0.031)*	-0.283***	*(0.056)*
MDM Kernel. CV+RA	-0.095***	*(0.019)*	-0.299***	*(0.051)*	-0.111***	*(0.024)*	-0.116***	*(0.026)*	-0.108***	*(0.031)*	-0.273***	*(0.054)*
MDM Kernel. PS	-0.064**	*(0.022)*	-0.206***	*(0.058)*	-0.083**	*(0.027)*	-0.057	*(0.032)*	-0.059*	*(0.029)*	-0.196***	*(0.05)*
MDM Kernel. PS+CV	-0.084***	*(0.022)*	-0.255***	*(0.057)*	-0.100***	*(0.027)*	-0.105***	*(0.03)*	-0.099**	*(0.032)*	-0.233***	*(0.05)*
PSN NN	-0.086**	*(0.028)*	-0.284***	*(0.067)*	-0.094**	*(0.036)*	-0.083*	*(0.04)*	-0.131***	*(0.037)*	-0.250***	*(0.059)*
PSM NN. RA	-0.094***	*(0.025)*	-0.262***	*(0.062)*	-0.084**	*(0.031)*	-0.080*	*(0.036)*	-0.122***	*(0.033)*	-0.266***	*(0.052)*
PSM Kernel	-0.057*	*(0.022)*	-0.199***	*(0.059)*	-0.066*	*(0.027)*	-0.054	*(0.032)*	-0.062*	*(0.029)*	-0.198***	*(0.05)*
PSM Kernel. RA	-0.060**	*(0.02)*	-0.210***	*(0.054)*	-0.064**	*(0.024)*	-0.068*	*(0.028)*	-0.057*	*(0.027)*	-0.207***	*(0.045)*
PSM Kernel. CV	-0.084***	*(0.022)*	-0.250***	*(0.057)*	-0.100***	*(0.027)*	-0.104***	*(0.03)*	-0.101**	*(0.032)*	-0.233***	*(0.05)*
PSM Kernel. CV+RA	-0.084***	*(0.02)*	-0.264***	*(0.052)*	-0.091***	*(0.024)*	-0.108***	*(0.027)*	-0.083**	*(0.028)*	-0.234***	*(0.044)*
Total Reporting Units	24359		20817		22870		21887		22330		20947	
Total Unique Reporting Units	16891		15232		16171		15810		15930		15309	
Total Unique Enterprises	16626		14998		15917		15572		15680		15074	

Note: Standard errors in brackets. * p<0.05; ** p<0.01;*** p<0.001. MDM stands for multivariate distance matching. PSM stands for propensity score matching. MDM NN refers to MDM with one to one nearest neighbour matching with replacement. MDM NN PS refer to propensity score as the only dependent variable in MDM NN. MDM Kernel refers to MDM using Kernel function to determine weights for matched controls. MDM Kernel RA refers to regression adjustment for MDM Kernel matching. MDM Kernel CV refers to the bandwidth of the MDM Kernel matching is calculated by cross-validation method. MDM Kernel CV+RA refers to regression adjustment for MDM Kernel matching that bandwidth is cross-validated. MDM Kernel PS refers to the only dependent variable in MDM Kernel matching is propensity score. MDM Kernel PS+CV refers to bandwidth in MDM Kernel PS is calculated by cross-validation method. PSM NN refers to using one-to-one nearest neighbour with replacement in PSM. PSM NN RA stands for regression adjustment for PSM NN. PSM Kernel refers to PSM using kernel function to determine weights for matched controls. PSM Kernel RA refers to regression adjustment for PSM Kernel matching. PSM Kernel CV refers to bandwidth of PSM Kernel matching is calculated by cross-validation method. PSM Kernel CV+RA means PSM Kernel Matching using cross-validated bandwidth and regression adjustment.

Table A.21: All Matchings for Productivity Growth using Full Sample, 2012-2020

	Any Public Support		EU Support		Central Support		Regional Support		Indirect Support		Direct Support	
MDM NN	-0.003	*(0.014)*	-0.061	*(0.036)*	-0.016	*(0.018)*	0.028	*(0.02)*	-0.026	*(0.018)*	-0.034	*(0.034)*
MDM NN. PS	-0.007	*(0.016)*	-0.051	*(0.037)*	0.002	*(0.02)*	-0.026	*(0.026)*	-0.055	*(0.03)*	-0.03	*(0.036)*
MDM Kernel	-0.002	*(0.011)*	-0.037	*(0.032)*	0.001	*(0.014)*	-0.009	*(0.015)*	-0.003	*(0.015)*	0	*(0.028)*
MDM Kernel. RA	-0.005	*(0.012)*	-0.049	*(0.032)*	-0.008	*(0.015)*	-0.003	*(0.016)*	-0.014	*(0.015)*	-0.009	*(0.029)*
MDM Kernel. CV	-0.003	*(0.011)*	-0.042	*(0.032)*	-0.002	*(0.014)*	-0.01	*(0.016)*	-0.008	*(0.017)*	-0.03	*(0.032)*
MDM Kernel. CV+RA	-0.01	*(0.012)*	-0.057	*(0.032)*	-0.012	*(0.015)*	-0.011	*(0.017)*	-0.016	*(0.018)*	-0.045	*(0.035)*
MDM Kernel. PS	-0.012	*(0.012)*	-0.06	*(0.032)*	-0.011	*(0.015)*	-0.022	*(0.019)*	-0.044	*(0.028)*	-0.031	*(0.029)*
MDM Kernel. PS+CV	-0.01	*(0.013)*	-0.068*	*(0.033)*	-0.016	*(0.016)*	-0.014	*(0.019)*	-0.028	*(0.019)*	-0.019	*(0.03)*
PSN NN	-0.008	*(0.015)*	-0.049	*(0.036)*	-0.012	*(0.021)*	-0.017	*(0.021)*	-0.042	*(0.028)*	-0.035	*(0.034)*
PSM NN. RA	-0.004	*(0.014)*	-0.051	*(0.036)*	-0.01	*(0.02)*	-0.017	*(0.022)*	-0.035	*(0.024)*	-0.03	*(0.034)*
PSM Kernel	-0.01	*(0.012)*	-0.043	*(0.031)*	-0.018	*(0.018)*	-0.033	*(0.018)*	-0.035	*(0.022)*	-0.038	*(0.029)*
PSM Kernel. RA	-0.01	*(0.012)*	-0.039	*(0.031)*	-0.015	*(0.016)*	-0.031	*(0.017)*	-0.031	*(0.02)*	-0.037	*(0.029)*
PSM Kernel. CV	-0.01	*(0.013)*	-0.068*	*(0.033)*	-0.016	*(0.016)*	-0.014	*(0.019)*	-0.027	*(0.019)*	-0.018	*(0.03)*
PSM Kernel. CV+RA	-0.009	*(0.012)*	-0.068*	*(0.033)*	-0.015	*(0.016)*	-0.012	*(0.019)*	-0.024	*(0.018)*	-0.02	*(0.031)*
Total Reporting Units	24354		20813		22866		21883		22326		20944	
Total Unique Reporting Units	16888		15228		16168		15807		15927		15306	
Total Unique Enterprises	16623		14994		15914		15569		15677		15071	

Note: Standard errors in brackets. * p<0.05; ** p<0.01;*** p<0.001. MDM stands for multivariate distance matching. PSM stands for propensity score matching. MDM NN refers to MDM with one to one nearest neighbour matching with replacement. MDM NN PS refer to propensity score as the only dependent variable in MDM NN. MDM Kernel refers to MDM using Kernel function to determine weights for matched controls. MDM Kernel RA refers to regression adjustment for MDM Kernel matching. MDM Kernel CV refers to the bandwidth of the MDM Kernel matching is calculated by cross-validation method. MDM Kernel CV+RA refers to regression adjustment for MDM Kernel matching that bandwidth is cross-validated. MDM Kernel PS refers to the only dependent variable in MDM Kernel matching is propensity score. MDM Kernel PS+CV refers to bandwidth in MDM Kernel PS is calculated by cross-validation method. PSM NN refers to using one-to-one nearest neighbour with replacement in PSM. PSM NN RA stands for regression adjustment for PSM NN. PSM Kernel refers to PSM using kernel function to determine weights for matched controls. PSM Kernel RA refers to regression adjustment for PSM Kernel matching. PSM Kernel CV refers to bandwidth of PSM Kernel matching is calculated by cross-validation method. PSM Kernel CV+RA means PSM Kernel Matching using cross-validated bandwidth and regression adjustment.

Table A.22: PSM Kernel Matching with Regression Adjustment for Policy Mix using Full Sample, 2012-2020

	All Public Support	EU Support Only	Central Support Only	Regional Support Only	EU and Central	EU and Region	Central and Region	Indirect Support Only	Direct Support Only	Direct and Indirect
Labour Productivity	-0.261**	-0.042	0.01	-0.064*	-0.426***	-0.196	-0.163**	0.003	-0.143*	-0.242***
	(0.101)	*(0.094)*	*(0.026)*	*(0.03)*	*(0.104)*	*(0.132)*	*(0.061)*	*(0.029)*	*(0.061)*	*(0.065)*
Total Observations	20268	20291	22045	21249	20309	20189	20421	21791	20408	20591
Total Unique Reporting Units	14928	14950	15776	15460	14950	14893	15036	15668	15018	15115
Total Unique Enterprises	14703	14722	15530	15227	14722	14669	14808	15424	14790	14884
Productivity Growth	-0.115	-0.015	0	0.006	-0.03	-0.16	0.013	-0.016	-0.012	-0.054
	(0.072)	*(0.036)*	*(0.017)*	*(0.017)*	*(0.066)*	*(0.107)*	*(0.041)*	*(0.016)*	*(0.036)*	*(0.045)*
Total Observations	20265	20288	22042	21245	20305	20186	20418	21787	20405	20588
Total Unique Reporting Units	14925	14947	15773	15457	14946	14890	15033	15665	15015	15112
Total Unique Enterprises	14700	14719	15527	15224	14718	14666	14805	15421	14787	14881

Note: Standard errors in brackets. * p<0.05; ** p<0.01;*** p<0.001.

Table A.23: PSM Kernel with Regression Adjustment using Consecutive Full Sample, 2012-2020

	Any Public Support	EU Support	Central Support	Regional Support	Indirect Support	Direct Support
Labour Productivity	-0.008	-0.179*	-0.023	-0.03	-0.025	-0.221***
	(0.027)	*(0.071)*	*(0.033)*	*(0.04)*	*(0.038)*	*(0.062)*
Productivity Growth	-0.019	-0.035	-0.016	0.005	-0.005	-0.044
	(0.016)	*(0.04)*	*(0.021)*	*(0.02)*	*(0.019)*	*(0.034)*
Total Observations	9904	8170	9250	8623	8981	8267
Total Unique Reporting Units	4234	3965	4159	4066	4122	4011
Total Unique Enterprises	4196	3934	4121	4031	4085	3976

Note: Standard errors in brackets. * p<0.05; ** p<0.01;*** p<0.001.

Table A.24: PSM Kernel with Regression Adjustments for Labour Productivity (Consecutive Subsamples), 2012-2020

		Any Public Support	EU Support	Central Support	Regional Support	Indirect Support	Direct Support
High-tech	Labour Productivity	-0.217***	-0.441***	-0.237***	-0.194*	-0.198**	-0.466***
		(0.052)	*(0.12)*	*(0.061)*	*(0.085)*	*(0.061)*	*(0.104)*
	Total Observations	2355	1683	2172	1764	2090	1736
	Total Unique Reporting Units	999	874	971	905	958	898
	Total Unique Enterprises	985	863	957	892	944	886
Low-tech	Labour Productivity	0.133***	0.167*	0.134***	0.057	0.111**	0.047
		(0.031)	*(0.068)*	*(0.036)*	*(0.045)*	*(0.041)*	*(0.06)*
	Total Observations	7549	6487	7078	6859	6891	6531
	Total Unique Reporting Units	3274	3110	3219	3187	3192	3135
	Total Unique Enterprises	3254	3093	3199	3168	3172	3117
Large firm	Labour Productivity	0.021	0.105	0.057	0.028	0.065	-0.144
		(0.05)	*(0.186)*	*(0.059)*	*(0.092)*	*(0.063)*	*(0.134)*
	Total Observations	3037	2569	2908	2695	2822	2619
	Total Unique Reporting Units	1366	1279	1349	1314	1336	1302
	Total Unique Enterprises	1344	1262	1327	1294	1315	1282
SME	Labour Productivity	-0.061	-0.226**	-0.072	-0.064	-0.123**	-0.210**
		(0.033)	*(0.081)*	*(0.04)*	*(0.05)*	*(0.046)*	*(0.071)*
	Total Observations	6867	5601	6342	5928	6159	5648
	Total Unique Reporting Units	3010	2797	2934	2874	2907	2821
	Total Unique Enterprises	3006	2793	2930	2870	2903	2817

Note: Standard errors in parentheses. * p<0.05; ** p<0.01;*** p<0.001.

Table A.25: PSM Kernel with Regression Adjustments for Productivity Growth (Consecutive Subsamples), 2012-2020

		Any Public Support	EU Support	Central Support	Regional Support	Indirect Support	Direct Support
High-tech	Productivity Growth	-0.073	-0.166*	-0.03	-0.006	-0.043	-0.171*
		(0.045)	*(0.081)*	*(0.034)*	*(0.061)*	*(0.036)*	*(0.083)*
	Total Observations	2355	1683	2172	1764	2090	1736
	Total Unique Reporting Units	999	874	971	905	958	898
	Total Unique Enterprises	985	863	957	892	944	886
Low-tech	Productivity Growth	0	0.006	-0.004	0.023	-0.008	-0.045
		(0.016)	*(0.035)*	*(0.019)*	*(0.022)*	*(0.021)*	*(0.03)*
	Total Observations	7549	6487	7078	6859	6891	6531
	Total Unique Reporting Units	3274	3110	3219	3187	3192	3135
	Total Unique Enterprises	3254	3093	3199	3168	3172	3117
Large firm	Productivity Growth	-0.023	-0.042	0.004	0.004	0.019	0.049
		(0.024)	*(0.056)*	*(0.027)*	*(0.037)*	*(0.032)*	*(0.061)*
	Total Observations	3037	2569	2908	2695	2822	2619
	Total Unique Reporting Units	1366	1279	1349	1314	1336	1302
	Total Unique Enterprises	1344	1262	1327	1294	1315	1282
SME	Productivity Growth	-0.003	-0.058	-0.008	0.028	-0.02	-0.077
		(0.018)	*(0.053)*	*(0.023)*	*(0.031)*	*(0.029)*	*(0.051)*
	Total Observations	6867	5601	6342	5928	6159	5648
	Total Unique Reporting Units	3010	2797	2934	2874	2907	2821
	Total Unique Enterprises	3006	2793	2930	2870	2903	2817

Note: Standard errors in parentheses. * $p<0.05$; ** $p<0.01$;*** $p<0.001$.

Table A.26: PSM Kernel with regression adjustment on next wave outcomes using consecutive sample, 2012-2020

	Any Public Support	EU Support	Central Support	Regional Support	Indirect Support	Direct Support
Labour Productivity	0.002	-0.129	-0.04	-0.07	-0.028	-0.178*
	(0.035)	*(0.075)*	*(0.046)*	*(0.053)*	*(0.048)*	*(0.074)*
Productivity Growth	-0.009	-0.013	-0.036	0.005	-0.026	-0.04
	(0.02)	*(0.037)*	*(0.027)*	*(0.03)*	*(0.028)*	*(0.043)*
Total Observations	5670	4719	5288	4945	5136	4762
Total Unique Reporting Units	4132	3588	3912	3744	3820	3623
Total Unique Enterprises	4097	3565	3881	3716	3792	3597

Note: Standard errors in brackets. * p<0.05; ** p<0.01;*** p<0.001.

Appendix **B**

Calculations for Chapter 2

B.1 Simultaneous Move Game

*Stage*1 The governments announce tax and subsidy policies.

*Stage*2 The firms choose the number of outputs and investments in both types of R&D.

To obtain sub-game perfect Nash Equilibrium, this game is solved by backward induction.

B.1.1 Stage 2: Profit maximization in Cournot game

Given:

$$p_1 = a + r_1 - (1 - b_1)q_1 - mq_2$$
$$p_2 = a + r_2 - (1 - b_2)q_2 - mq_1$$

Foreign firm:

$$\pi_1 = (1 - t)p_1q_1 - (c_1 - k_1)q_1 - (1 - s_1)\frac{\varphi_1 k_1^2}{2} - (1 - \sigma_1)\frac{\theta_1 r_1^2}{2}$$
$$= (1 - t)[a + r_1 - (1 - b_1)q_1 - mq_2]q_1 - (c_1 - k_1)q_1 - (1 - s_1)\frac{\varphi_1 k_1^2}{2} - (1 - \sigma_1)\frac{\theta_1 r_1^2}{2}$$

First-order conditions for an interior optimum:

$$
\begin{aligned}
0 &= \frac{\partial \pi_1}{\partial q_1} = [1-t]\left[a + r_1 - 2\left(1-b_1\right) q_1 - m q_2\right] - [c_1 - k_1] \\
0 &= \frac{\partial \pi_1}{\partial k_1} = q_1 - [1-s_1]\,\varphi_1 k_1 \\
0 &= \frac{\partial \pi_1}{\partial r_1} = [1-t]\,q_1 - [1-\sigma_1]\,\theta_1 r_1
\end{aligned}
$$

which gives:

$$
\begin{aligned}
q_1 &= \frac{1}{2\left(1-b_1\right)}\left[a + r_1 - m q_2 - \frac{c_1 - k_1}{1-t}\right] \\
k_1 &= \frac{q_1}{[1-s_1]\,\varphi_1} \\
r_1 &= \frac{[1-t]\,q_1}{[1-\sigma_1]\,\theta_1}
\end{aligned}
$$

Similarly, for home firm:

$$
\begin{aligned}
q_2 &= \frac{1}{2\left(1-b_2\right)}\left[a + r_2 - m q_1 - [c_2 - k_2]\right] \\
k_2 &= \frac{q_2}{[1-s_2]\,\varphi_2} \\
r_2 &= \frac{q_2}{[1-\sigma_2]\,\theta_2}
\end{aligned}
$$

For simplicity, process R&D and product R&D will be discussed seperately.

B.2 Case 1: Process R&D

Set $r_1 = r_2 = 0$. The second-order conditions for interior maximum requires:

$$
\begin{aligned}
\frac{\partial^2 \pi_i}{\partial q_i^2} &< 0, \frac{\partial^2 \pi_i}{\partial k_i^2} < 0, \\
\Delta_i^{k*} &\equiv \begin{vmatrix} \frac{\partial^2 \pi_i}{\partial q_i^2} & \frac{\partial^2 \pi_i}{\partial q_i \partial k_i} \\ \frac{\partial^2 \pi_1}{\partial q_i \partial k_i} & \frac{\partial^2 \pi_i}{\partial k_i^2} \end{vmatrix} > 0
\end{aligned}
$$

So, for $\Delta^{k*}_{1,2}$ we have:

$$\begin{aligned}
\Delta^{k*}_1 &\equiv \begin{vmatrix} -2\left(1-b_1\right)\left[1-t\right] & 1 \\ 1 & -\left[1-s_1\right]\varphi_1 \end{vmatrix} \\
&= 2\left(1-b_1\right)\left[1-t\right]\left[1-s_1\right]\varphi_1\left[1-\frac{1}{2\left(1-b_1\right)\left[1-t\right]\left[1-s_1\right]\varphi_1}\right] \\
\Delta^{k*}_2 &= 2\left(1-b_2\right)\left[1-s_2\right]\varphi_2\left[1-\frac{1}{2\left(1-b_2\right)\left[1-s_2\right]\varphi_2}\right]
\end{aligned}$$

Thus, for the second-order conditions to hold, it must be the case that:

$$\begin{aligned}
1-\frac{1}{2\left(1-b_1\right)\left[1-t\right]\left[1-s_1\right]\varphi_1} &> 0 \\
1-\frac{1}{2\left(1-b_2\right)\left[1-s_2\right]\varphi_2} &> 0
\end{aligned}$$

The second order conditions are satisfied for $0 < b_1 < 1$ and $0 < s_1 < 1$, $0 < \sigma_1 < 1$. Hence the profit has a unique interior maximum, and the equilibrium is described by the linear system of equations:

$$\begin{aligned}
q_1 &= \frac{1}{2\left(1-b_1\right)}\left[a+r_1-mq_2-\frac{c_1-k_1}{1-t}\right] \\
k_1 &= \frac{q_1}{\left[1-s_1\right]\varphi_1} \\
q_2 &= \frac{1}{2\left(1-b_2\right)}\left[a+r_2-mq_1-\left[c_2-k_2\right]\right] \\
k_2 &= \frac{q_2}{\left[1-s_2\right]\varphi_2}
\end{aligned}$$

In matrix form:

$$\begin{bmatrix} 1 & -\frac{1}{2(1-b_1)[1-t]} & \frac{m}{2(1-b_1)} & 0 \\ -\frac{1}{[1-s_1]\varphi_1} & 1 & 0 & 0 \\ \frac{m}{2(1-b_2)} & 0 & 1 & -\frac{1}{2(1-b_2)}k_2 \\ 0 & 0 & -\frac{1}{[1-s_2]\varphi_2} & 1 \end{bmatrix}\begin{bmatrix} q_1 \\ k_1 \\ q_2 \\ k_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2(1-b_1)}\left[a-\frac{c_1}{1-t}\right] \\ 0 \\ \frac{1}{2(1-b_2)}\left[a-c_2\right] \\ 0 \end{bmatrix}$$

q_1 and q_2 can be solved explicitly by Cramer's rule:

$$\begin{aligned}
q_1 &= \frac{1}{2\left(1-b_1\right)}\frac{\left[a-\frac{c_1}{1-t}\right]\left[1-\frac{1}{2(1-b_2)[1-s_2]\varphi_2}\right]-\frac{m[a-c_2]}{2(1-b_2)}}{\left[1-\frac{1}{2(1-b_1)[1-t][1-s_1]\varphi_1}\right]\left[1-\frac{1}{2(1-b_2)[1-s_2]\varphi_2}\right]-\frac{m^2}{4(1-b_1)(1-b_2)}} \\
q_2 &= \frac{1}{2\left(1-b_2\right)}\frac{\left[a-c_2\right]\left[1-\frac{1}{2(1-b_1)[1-t][1-s_1]\varphi_1}\right]-\frac{m\left[a-\frac{c_1}{1-t}\right]}{2(1-b_1)}}{\left[1-\frac{1}{2(1-b_1)[1-t][1-s_1]\varphi_1}\right]\left[1-\frac{1}{2(1-b_2)[1-s_2]\varphi_2}\right]-\frac{m^2}{4(1-b_1)(1-b_2)}}
\end{aligned}$$

Since we focus on the interior solution, our analysis is restricted to the case where:

$$\begin{aligned}
\left[1-\frac{1}{2\left(1-b_1\right)\left[1-t\right]\left[1-s_1\right]\varphi_1}\right]\left[1-\frac{1}{2\left(1-b_2\right)\left[1-s_2\right]\varphi_2}\right]-\frac{m^2}{4\left(1-b_1\right)\left(1-b_2\right)} &> 0 \\
\left[a-\frac{c_1}{1-t}\right]\left[1-\frac{1}{2\left(1-b_2\right)\left[1-s_2\right]\varphi_2}\right]-\frac{m\left[a-c_2\right]}{2\left(1-b_2\right)} &> 0 \\
\left[a-c_2\right]\left[1-\frac{1}{2\left(1-b_1\right)\left[1-t\right]\left[1-s_1\right]\varphi_1}\right]-\frac{m\left[a-\frac{c_1}{1-t}\right]}{2\left(1-b_1\right)} &> 0
\end{aligned}$$

B.2.1 Policy effect on Process R&D

How equilibrium quantities and investment in R&D depend on tax and subsidies can be calculated by taking the total derivatives of the ~~above linear system equations~~ following system of linear equations:

$$\begin{aligned}
dq_1-\frac{1}{2\left(1-b_1\right)\left[1-t\right]}dk_1+\frac{m}{2\left(1-b_1\right)}dq_2 &= -\frac{c_1-k_1}{2\left(1-b_1\right)\left[1-t\right]^2}dt \\
-\frac{1}{\left[1-s_1\right]\varphi_1}dq_1+dk_1 &= \frac{q_1}{\left[1-s_1\right]^2\varphi_1}ds_1 \\
\frac{m}{2\left(1-b_2\right)}dq_1+dq_2-\frac{1}{2\left(1-b_2\right)}dk_2 &= 0 \\
-\frac{1}{\left[1-s_2\right]\varphi_2}dq_2+dk_2 &= \frac{q_2}{\left[1-s_2\right]^2\varphi_2}ds_2
\end{aligned}$$

In the matrix form:

$$\begin{bmatrix} 1 & -\frac{1}{2(1-b_1)[1-t]} & \frac{m}{2(1-b_1)} & 0 \\ -\frac{1}{[1-s_1]\varphi_1} & 1 & 0 & 0 \\ \frac{m}{2(1-b_2)} & 0 & 1 & -\frac{1}{2(1-b_2)} \\ 0 & 0 & -\frac{1}{[1-s_2]\varphi_2} & 1 \end{bmatrix}\begin{bmatrix} dq_1 \\ dk_1 \\ dq_2 \\ dk_2 \end{bmatrix}=\begin{bmatrix} -\frac{c_1-k_1}{2(1-b_1)[1-t]^2}dt \\ \frac{q_1}{[1-s_1]^2\varphi_1}ds_1 \\ 0 \\ \frac{q_2}{[1-s_2]^2\varphi_2}ds_2 \end{bmatrix}$$

let:

$$dq_1 = \frac{\begin{vmatrix} -\frac{c_1-k_1}{2(1-b_1)[1-t]^2}dt & -\frac{1}{2(1-b_1)[1-t]} & \frac{m}{2(1-b_1)} & 0 \\ \frac{q_1}{[1-s_1]^2\varphi_1}ds_1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -\frac{1}{2(1-b_2)} \\ \frac{q_2}{[1-s_2]^2\varphi_2}ds_2 & 0 & -\frac{1}{[1-s_2]\varphi_2} & 1 \end{vmatrix}}{\begin{vmatrix} 1 & -\frac{1}{2(1-b_1)[1-t]} & \frac{m}{2(1-b_1)} & 0 \\ -\frac{1}{[1-s_1]\varphi_1} & 1 & 0 & 0 \\ \frac{m}{2(1-b_2)} & 0 & 1 & -\frac{1}{2(1-b_2)} \\ 0 & 0 & -\frac{1}{[1-s_2]\varphi_2} & 1 \end{vmatrix}} = \frac{\Delta_{q_1}}{\Delta}$$

Numerator:

$$\begin{aligned} \Delta_{q_1} &\equiv \begin{vmatrix} -\frac{c_1-k_1}{2(1-b_1)[1-t]^2}dt & -\frac{1}{2(1-b_1)[1-t]} & \frac{m}{2(1-b_1)} & 0 \\ \frac{q_1}{[1-s_1]^2\varphi_1}ds_1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -\frac{1}{2(1-b_2)} \\ \frac{q_2}{[1-s_2]^2\varphi_2}ds_2 & 0 & -\frac{1}{[1-s_2]\varphi_2} & 1 \end{vmatrix} \\ &= -\frac{c_1-k_1}{2(1-b_1)[1-t]^2}\left[1-\frac{1}{2(1-b_2)[1-s_2]\varphi_2}\right]dt \\ &\quad +\frac{q_1}{2(1-b_1)[1-t][1-s_1]^2\varphi_1}\left[1-\frac{1}{2(1-b_2)[1-s_2]\varphi_2}\right]ds_1 \\ &\quad -\frac{mq_2}{4(1-b_1)(1-b_2)[1-s_2]^2\varphi_2}ds_2 \end{aligned}$$

Denominator:

$$\begin{aligned} \Delta &\equiv \begin{vmatrix} 1 & -\frac{1}{2(1-b_1)[1-t]} & \frac{m}{2(1-b_1)} & 0 \\ -\frac{1}{[1-s_1]\varphi_1} & 1 & 0 & 0 \\ \frac{m}{2(1-b_2)} & 0 & 1 & -\frac{1}{2(1-b_2)} \\ 0 & 0 & -\frac{1}{[1-s_2]\varphi_2} & 1 \end{vmatrix} \\ &= \left[1-\frac{1}{2(1-b_1)[1-t][1-s_1]\varphi_1}\right]\left[1-\frac{1}{2(1-b_2)[1-s_2]\varphi_2}\right]-\frac{m^2}{4(1-b_1)(1-b_2)} \end{aligned}$$

For an interior solution, $\Delta > 0$ must hold. In that case:

$$\begin{aligned}
\frac{\partial q_1}{\partial t} &= -\frac{1}{\Delta}\frac{c_1 - k_1}{2(1-b_1)[1-t]^2}\left[1 - \frac{1}{2(1-b_2)[1-s_2]\varphi_2}\right] \\
\frac{\partial q_1}{\partial s_1} &= \frac{1}{\Delta}\frac{q_1}{2(1-b_1)[1-t][1-s_1]^2\varphi_1}\left[1 - \frac{1}{2(1-b_2)[1-s_2]\varphi_2}\right] \\
\frac{\partial q_1}{\partial s_2} &= -\frac{1}{\Delta}\frac{mq_2}{4(1-b_1)(1-b_2)[1-s_2]^2\varphi_2}
\end{aligned}$$

Therefore:

$$\frac{\partial q_1}{\partial t} < 0,\ \frac{\partial q_1}{\partial s_1} > 0,\ \text{and}\ \frac{\partial q_1}{\partial s_2} \lesseqqgtr 0 \text{ for } m \gtreqqless 0.$$

Similarly, we can have:

$$\begin{aligned}
\frac{\partial k_1}{\partial t} &= -\frac{1}{\Delta}\frac{c_1 - k_1}{2(1-b_1)[1-t]^2[1-s_1]\varphi_1}\left[1 - \frac{1}{2(1-b_2)[1-s_2]\varphi_2}\right] < 0 \\
\frac{\partial k_1}{\partial s_1} &= \frac{1}{\Delta}\frac{q_1}{[1-s_1]^2\varphi_1}\left[1 - \frac{1}{2(1-b_2)[1-s_2]\varphi_2} - \frac{m^2}{4(1-b_1)(1-b_2)}\right] > 0 \\
\frac{\partial k_1}{\partial s_2} &= -\frac{1}{\Delta}\frac{mq_2}{4(1-b_1)(1-b_2)[1-s_1][1-s_2]^2\varphi_1\varphi_2} \lesseqqgtr 0 \text{ for } m \gtreqqless 0
\end{aligned}$$

For home firm:

$$\begin{aligned}
\frac{\partial q_2}{\partial t} &= \frac{1}{\Delta}\frac{m[c_1 - k_1]}{4(1-b_1)(1-b_2)[1-t]^2} \lesseqqgtr 0 \text{ for } m \lesseqqgtr 0 \\
\frac{\partial q_2}{\partial s_1} &= -\frac{1}{\Delta}\frac{mq_1}{4(1-b_1)(1-b_2)[1-t][1-s_1]^2\varphi_1} \gtreqqless 0 \text{ for } m \lesseqqgtr 0 \\
\frac{\partial q_2}{\partial s_2} &= \frac{1}{\Delta}\frac{q_2}{2(1-b_2)[1-s_2]^2\varphi_2}\left[1 - \frac{1}{2(1-b_1)[1-t][1-s_1]\varphi_1}\right] > 0
\end{aligned}$$

$$\begin{aligned}
\frac{\partial k_2}{\partial t} &= \frac{1}{\Delta}\frac{m}{4(1-b_1)(1-b_2)}\frac{c_1 - k_1}{[1-t]^2}\frac{1}{[1-s_2]\varphi_2} \gtreqqless 0 \text{ for } m \lesseqqgtr 0 \\
\frac{\partial k_2}{\partial s_1} &= -\frac{1}{\Delta}\frac{mq_1}{4(1-b_1)(1-b_2)[1-t][1-s_1]^2\varphi_1}\frac{1}{[1-s_2]\varphi_2} \gtreqqless 0 \text{ for } m \lesseqqgtr 0 \\
\frac{\partial k_2}{\partial s_2} &= \frac{1}{\Delta}\frac{q_2}{[1-s_2]^2\varphi_2}\left[1 - \frac{1}{2(1-b_1)[1-t][1-s_1]\varphi_1} - \frac{m^2}{4(1-b_1)(1-b_2)}\right] > 0
\end{aligned}$$

B.2.2 The role of network externalities in consumption

We can also calculate how the decisions of the firms depend on network externalities in consumption by taking the total derivatives of the linear system equations:

$$\begin{aligned}
dq_1 - \frac{1}{2(1-b_1)[1-t]}dk_1 + \frac{m}{2(1-b_1)}dq_2 + 0 \times dk_2 &= \frac{a - \frac{c_1-k_1}{1-t} - mq_2}{2(1-b_1)^2}db_1 \\
-\frac{1}{[1-s_1]\varphi_1}dq_1 + dk_1 + 0 \times dq_2 + 0 \times dk_2 &= 0 \\
\frac{m}{2(1-b_2)}dq_1 + 0 \times dk_1 + dq_2 - \frac{1}{2(1-b_2)}dk_2 &= \frac{a-(c_2-k_2)-mq_1}{2(1-b_2)^2}db_2 \\
0 \times q_1 + 0 \times k_1 - \frac{1}{[1-s_2]\varphi_2}q_2 + k_2 &= 0
\end{aligned}$$

In the matrix form,

$$\begin{bmatrix} 1 & -\frac{1}{2(1-b_1)[1-t]} & \frac{m}{2(1-b_1)} & 0 \\ -\frac{1}{[1-s_1]\varphi_1} & 1 & 0 & 0 \\ \frac{m}{2(1-b_2)} & 0 & 1 & -\frac{1}{2(1-b_2)} \\ 0 & 0 & -\frac{1}{[1-s_2]\varphi_2} & 1 \end{bmatrix} \begin{bmatrix} dq_1 \\ dk_1 \\ dq_2 \\ dk_2 \end{bmatrix} = \begin{bmatrix} \frac{a-\frac{c_1-k_1}{1-t}-mq_2}{2(1-b_1)^2}db_1 \\ 0 \\ \frac{a-(c_2-k_2)-mq_1}{2(1-b_2)^2}db_2 \\ 0 \end{bmatrix}$$

Therefore:

$$\begin{aligned}
\frac{\partial q_1}{\partial b_1} &= \frac{1}{\Delta}\frac{a - \frac{c_1-k_1}{1-t} - mq_2}{2(1-b_1)^2}\left[1 - \frac{1}{2(1-b_2)}\frac{1}{[1-s_2]\varphi_2}\right] > 0 \\
\frac{\partial q_1}{\partial b_2} &= -\frac{m}{\Delta}\frac{a-(c_2-k_2)-mq_1}{4(1-b_1)(1-b_2)^2} \gtreqless 0 \text{ for } m \lesseqgtr 0
\end{aligned}$$

$$\begin{aligned}
\frac{\partial k_1}{\partial b_1} &= \frac{1}{\Delta}\frac{a - \frac{c_1-k_1}{1-t} - mq_2}{2(1-b_1)^2}\frac{1}{[1-s_1]\varphi_1}\left[1 - \frac{1}{2(1-b_2)}\frac{1}{[1-s_2]\varphi_2}\right] > 0 \\
\frac{\partial k_1}{\partial b_2} &= -\frac{m}{\Delta}\frac{a-(c_2-k_2)-mq_1}{4(1-b_1)(1-b_2)^2[1-s_1]\varphi_1} \gtreqless 0 \text{ for } m \lesseqgtr 0
\end{aligned}$$

Similarly:

$$\begin{aligned}
\frac{\partial q_2}{\partial b_2} &= \frac{1}{\Delta}\frac{a-(c_2-k_2)-mq_1}{2(1-b_2)^2}\left[1 - \frac{1}{2(1-b_1)}\frac{1}{[1-s_1]\varphi_1}\right] > 0 \\
\frac{\partial q_2}{\partial b_1} &= -\frac{m}{\Delta}\frac{a - \frac{c_1-k_1}{1-t} - mq_2}{4(1-b_1)^2(1-b_2)} \gtreqless 0 \text{ for } m \lesseqgtr 0
\end{aligned}$$

$$\begin{aligned}
\frac{\partial k_2}{\partial b_2} &= \frac{1}{\Delta}\frac{a-(c_2-k_2)-mq_1}{2(1-b_2)^2}\frac{1}{[1-s_2]\varphi_2}\left[1 - \frac{1}{2(1-b_1)}\frac{1}{[1-s_1]\varphi_1}\right] > 0 \\
\frac{\partial k_2}{\partial b_1} &= -\frac{m}{\Delta}\frac{a - \frac{c_1-k_1}{1-t} - mq_2}{4(1-b_1)^2(1-b_2)[1-s_2]\varphi_2} \gtreqless 0 \text{ for } m \lesseqgtr 0
\end{aligned}$$

B.2.3 Stage 1: Optimal tax and subsidy policies

Set$r_1 = r_2 = 0$, governments' welfare functions are:

$$
\begin{aligned}
W_1 &= \pi_1 - s_1 \frac{\varphi_1 k_1^2}{2} \\
W_2 &= U + \pi_2 - s_2 \frac{\varphi_2 k_2^2}{2} + t p_1 q_1
\end{aligned}
$$

B.2.4 Foreign country optimizes subsidy policy

$$
\begin{aligned}
0 &= \frac{dW_1}{ds_1} = \frac{d\pi_1}{ds_1} - \frac{\varphi_1 k_1^2}{2} - s_1 \varphi_1 k_1 \frac{dk_1}{ds_1} \\
&= [1-t]\, q_1 \frac{dp_1}{dq_2} \frac{dq_2}{ds_1} + \frac{\varphi_1 k_1^2}{2} - \frac{\varphi_1 k_1^2}{2} - s_1 \varphi_1 k_1 \frac{dk_1}{ds_1} \\
&= -[1-t]\, q_1 m \frac{dq_2}{ds_1} - s_1 \varphi_1 k_1 \frac{dk_1}{ds_1}
\end{aligned}
$$

where:

$$
\begin{aligned}
\frac{\partial q_2}{\partial s_1} &= -\frac{1}{\Delta} \frac{m q_1}{4(1-b_1)(1-b_2)[1-t][1-s_1]^2 \varphi_1} \\
\frac{\partial k_1}{\partial s_1} &= \frac{1}{\Delta} \frac{q_1}{[1-s_1]^2 \varphi_1} \left[1 - \frac{1}{2(1-b_2)[1-s_2]\varphi_2} - \frac{m^2}{4(1-b_1)(1-b_2)} \right]
\end{aligned}
$$

Subsituting $\frac{\partial q_2}{\partial s_1}$ and $\frac{\partial k_1}{\partial s_1}$ into $\frac{dW_1}{ds_1}$, we can have the following equation:

$$
s_1 = \frac{\frac{m^2}{4(1-b_1)(1-b_2)}}{1 - \frac{1}{2(1-b_2)[1-s_2]\varphi_2}}
$$

And:

$$
\frac{ds_1}{ds_2} = \frac{1}{\left[1 - \frac{1}{2(1-b_2)[1-s_2]\varphi_2}\right]^2} \frac{m^2}{8(1-b_1)(1-b_2)^2[1-s_2]^2\varphi_2} > 0 \text{ for } m \neq 0
$$

B.2.5 Home country optimizes tax and subsidy policy

For subsidy:

$$
\begin{aligned}
\frac{dW_2}{ds_2} &= \left[q_1 + t \frac{c_1 - k_1}{1-t} \right] \frac{dq_1}{ds_2} - \frac{s_2 q_2}{1-s_2} \frac{dk_2}{ds_2} \\
&\quad + [m[1-t]\, q_1 + q_2] \frac{dq_2}{ds_2}
\end{aligned}
$$

Substitutes $\frac{dq_1}{ds_2}, \frac{dk_2}{ds_2}, and \frac{dq_2}{ds_2}$ into the above equation, when $m = 0$, we can obtain:

$$s_2 = \frac{1}{1 + 2(1 - b_2)}$$

For tax policy:

$$\begin{aligned} 0 &= \frac{dW_2}{ds_2} = q_1 \left[\frac{dq_1}{ds_2} + m\frac{dq_2}{ds_2}\right] + q_2 \left[\frac{dq_2}{ds_2} + m\frac{dq_1}{ds_2}\right] \\ & -mq_2\frac{dq_1}{ds_2} - s_2\varphi_2 k_2 \frac{dk_2}{ds_2} + t\left[p_1\frac{dq_1}{ds_2} + q_1\left[-\left(1 - b_1\right)\frac{dq_1}{ds_2} - m\frac{dq_2}{ds_2}\right]\right] \end{aligned}$$

The model structure does not allow for the closed form solutions for the general case, $m \neq 0$, of tax and subsidy policies. This equation will be solved numerically in the final section.

B.3 Case 2: Product R&D

Set $k_1 = k_2 = 0$. The second-order conditions for interior maximum requires:

$$\begin{aligned} \frac{\partial^2 \pi_i}{\partial q_i^2} &< 0, \frac{\partial^2 \pi_i}{\partial r_i^2} < 0, \\ \Delta_i^{r*} &\equiv \begin{vmatrix} \frac{\partial^2 \pi_i}{\partial q_i^2} & \frac{\partial^2 \pi_i}{\partial q_i \partial r_i} \\ \frac{\partial^2 \pi_1}{\partial q_i \partial r_i} & \frac{\partial^2 \pi_i}{\partial r_i^2} \end{vmatrix} > 0 \end{aligned}$$

So, for $\Delta_{1,2}^{r*}$ we have :

$$\begin{aligned} \Delta_1^{r*} &= 2\left(1 - b_1\right)\left[1 - t\right]\left[1 - \sigma_1\right]\theta_1 \left[1 - \frac{1 - t}{2\left(1 - b_1\right)\left[1 - \sigma_1\right]\theta_1}\right] > 0 \\ \Delta_2^{r*} &= 2\left(1 - b_2\right)\left[1 - \sigma_2\right]\theta_2 \left[1 - \frac{1}{2\left(1 - b_2\right)\left[1 - \sigma_2\right]\theta_2}\right] > 0 \end{aligned}$$

Thus, for the second-order conditions to hold, it must be the case that:

$$\begin{aligned} 1 - \frac{1 - t}{2\left(1 - b_1\right)\left[1 - \sigma_1\right]\theta_1} &> 0 \\ 1 - \frac{1}{2\left(1 - b_2\right)\left[1 - \sigma_2\right]\theta_2} &> 0 \end{aligned}$$

Therefore, the quilibrium is described by:

$$\begin{aligned}
q_1 &= \frac{1}{2(1-b_1)}\left[a + r_1 - mq_2 - \frac{c_1}{1-t}\right] \\
r_1 &= \frac{[1-t]\,q_1}{[1-\sigma_1]\,\theta_1} \\
q_2 &= \frac{1}{2(1-b_2)}\left[a + r_2 - mq_1 - c_2\right] \\
r_2 &= \frac{q_2}{[1-\sigma_2]\,\theta_2}
\end{aligned}$$

The equilibrium $\{q_1, q_2\}$ can be solved as:

$$\begin{aligned}
q_1 &= \frac{1}{2(1-b_1)} \frac{\left[a - \frac{c_1}{1-t}\right]\left[1 - \frac{1}{2(1-b_2)[1-\sigma_2]\theta_2}\right] - \frac{m}{2(1-b_2)}\left[a - c_2\right]}{\left[1 - \frac{1-t}{2(1-b_1)[1-\sigma_1]\theta_1}\right]\left[1 - \frac{1}{2(1-b_2)[1-\sigma_2]\theta_2}\right] - \frac{m^2}{4(1-b_1)(1-b_2)}} \\
q_2 &= \frac{1}{2(1-b_2)} \frac{\left[a - c_2\right]\left[1 - \frac{1-t}{2(1-b_1)[1-\sigma_1]\theta_1}\right] - \frac{m}{2(1-b_1)}\left[a - \frac{c_1}{1-t}\right]}{\left[1 - \frac{1-t}{2(1-b_1)[1-\sigma_1]\theta_1}\right]\left[1 - \frac{1}{2(1-b_2)[1-\sigma_2]\theta_2}\right] - \frac{m^2}{4(1-b_1)(1-b_2)}}
\end{aligned}$$

B.3.1 Policy effect on Product R&D

Similar to the case of process R&D, the linear systems equations of product R&D will be differentiated to investigate how equilibrium outputs and investments in product R&D depend on policy variables.

Total differentiation:

$$\begin{aligned}
dq_1 - \frac{1}{2(1-b_1)}dr_1 + \frac{m}{2(1-b_1)}dq_2 + 0 \times dr_2 &= -\frac{c_1}{2(1-b_1)[1-t]^2}dt \\
-\frac{[1-t]}{[1-\sigma_1]\,\theta_1}dq_1 + dr_1 + 0 \times dq_2 + 0 \times dr_2 &= -\frac{q_1}{[1-\sigma_1]\,\theta_1}dt + \frac{[1-t]\,q_1}{[1-\sigma_1]^2\,\theta_1}d\sigma_1 \\
\frac{m}{2(1-b_2)}dq_1 + 0 \times dr_1 + dq_2 - \frac{1}{2(1-b_2)}dr_2 &= 0 \\
0 \times dq_1 + 0 \times dr_1 - \frac{1}{[1-\sigma_2]\,\theta_2}dq_2 + dr_2 &= \frac{q_2}{[1-\sigma_2]^2\,\theta_2}d\sigma_2
\end{aligned}$$

In matrix form:

$$\begin{bmatrix}
1 & -\frac{1}{2(1-b_1)} & \frac{m}{2(1-b_1)} & 0 \\
-\frac{[1-t]}{[1-\sigma_1]\theta_1} & 1 & 0 & 0 \\
\frac{m}{2(1-b_2)} & 0 & 1 & -\frac{1}{2(1-b_2)} \\
0 & 0 & -\frac{1}{[1-\sigma_2]\theta_2} & 1
\end{bmatrix}
\begin{bmatrix} dq_1 \\ dr_1 \\ dq_2 \\ dr_1 \end{bmatrix}
=
\begin{bmatrix}
-\frac{c_1}{2(1-b_1)[1-t]^2}dt \\
-\frac{q_1}{[1-\sigma_1]\theta_1}dt + \frac{[1-t]q_1}{[1-\sigma_1]^2\theta_1}d\sigma_1 \\
0 \\
\frac{q_2}{[1-\sigma_2]^2\theta_2}d\sigma_2
\end{bmatrix}$$

The denominator:

$$\begin{aligned}
\Delta^r \equiv & \begin{vmatrix} 1 & -\frac{1}{2(1-b_1)} & \frac{m}{2(1-b_1)} & 0 \\ -\frac{[1-t]}{[1-\sigma_1]\theta_1} & 1 & 0 & 0 \\ \frac{m}{2(1-b_2)} & 0 & 1 & -\frac{1}{2(1-b_2)} \\ 0 & 0 & -\frac{1}{[1-\sigma_2]\theta_2} & 1 \end{vmatrix} \\
= & \left[1 - \frac{1}{2(1-b_2)(1-\sigma_2)\theta_2}\right]\left[1 - \frac{1-t}{2(1-b_1)(1-\sigma_1)\theta_1}\right] - \frac{m^2}{4(1-b_1)(1-b_2)} > 0
\end{aligned}$$

Thus, for the effect of policies on q_1:

$$\begin{aligned}
\frac{dq_1}{dt} &= -\frac{1}{\Delta^r}\frac{1}{2(1-b_1)}\left[\frac{q_1}{[1-\sigma_1]\theta_1} + \frac{c_1}{[1-t]^2}\right]\left[1 - \frac{1}{2(1-b_2)[1-\sigma_2]\theta_2}\right] < 0 \\
\frac{dq_1}{d\sigma_1} &= \frac{1}{\Delta^r}\frac{[1-t]q_1}{2(1-b_1)[1-\sigma_1]^2\theta_1}\left[1 - \frac{1}{2(1-b_2)[1-\sigma_2]\theta_2}\right] > 0 \\
\frac{dq_1}{d\sigma_2} &= -\frac{1}{\Delta^r}\frac{q_2}{[1-\sigma_2]^2\theta_2}\frac{m}{4(1-b_1)(1-b_2)} \lesseqqgtr 0 \text{ for } m \gtreqqless 0
\end{aligned}$$

Policy effect on product R&D:

$$\begin{aligned}
\frac{dr_1}{dt} &= -\frac{1}{[1-\sigma_1]\theta_1}\left[q_1 + \frac{1}{\Delta^r}\frac{1-t}{2(1-b_1)}\left[\frac{q_1}{[1-\sigma_1]\theta_1} + \frac{c_1}{[1-t]^2}\right]\left[1 - \frac{1}{2(1-b_2)[1-\sigma_2]\theta_2}\right]\right] < 0 \\
\frac{dr_1}{d\sigma_1} &= \frac{[1-t]q_1}{[1-\sigma_1]^2\theta_1}\left[1 + \frac{1}{\Delta^r}\frac{[1-t]}{2(1-b_1)[1-\sigma_1]\theta_1}\left[1 - \frac{1}{2(1-b_2)[1-\sigma_2]\theta_2}\right]\right] > 0 \\
\frac{dr_1}{d\sigma_2} &= -\frac{1}{\Delta^r}\frac{[1-t]q_2}{[1-\sigma_1][1-\sigma_2]^2\theta_1\theta_2}\frac{m}{4(1-b_1)(1-b_2)} \lesseqqgtr 0 \text{ for } m \gtreqqless 0
\end{aligned}$$

Similarly, for the home firm, we have:

$$\begin{aligned}
\frac{dq_2}{dt} &= \frac{m}{\Delta^r}\frac{1}{2(1-b_2)}\left[\frac{q_1}{2(1-b_1)(1-\sigma_1)\theta_1} + \frac{c_1}{2(1-t)^2(1-b_1)}\right] \gtreqqless 0 \text{ for } m \gtreqqless 0 \\
\frac{dq_2}{d\sigma_1} &= -\frac{m}{\Delta^r}\frac{(1-t)q_1}{4(1-b_1)(1-b_2)(1-\sigma_1)^2\theta_1} \lesseqqgtr 0 \text{ for } m \gtreqqless 0 \\
\frac{dq_2}{d\sigma_2} &= \frac{1}{\Delta^r}\frac{q_2}{\theta_2(1-b_2)(1-\sigma_2)^2}\left[1 - \frac{1-t}{2(1-b_1)(1-\sigma_1)\theta_1}\right] > 0
\end{aligned}$$

and:

$$\begin{aligned}
\frac{dr_2}{dt} &= \frac{m}{\Delta^r}\frac{1}{2(1-b_2)(1-\sigma_2)\theta_2}\left[\frac{q_1}{2(1-b_1)(1-\sigma_1)\theta_1} + \frac{c_1}{2(1-b_1)(1-t)^2}\right] \gtreqqless 0 \text{ for } m \gtreqqless 0 \\
\frac{dr_2}{d\sigma_1} &= -\frac{m}{\Delta^r}\frac{q_1(1-t)}{4(1-b_1)(1-b_2)(1-\sigma_1)^2(1-\sigma_2)\theta_1\theta_2} \lesseqqgtr 0 \text{ for } m \gtreqqless 0 \\
\frac{dr_2}{d\sigma_2} &= \frac{1}{\Delta^r}\frac{q_2}{(1-\sigma_2)^2\theta_2}\left[1 - \frac{1-t}{2(1-b_1)(1-\sigma_1)\theta_1} - \frac{m^2}{4(1-b_1)(1-b_2)}\right] > 0
\end{aligned}$$

B.3.2 The role of network externalities in consumption

Total differentiation gives

$$\begin{aligned}
dq_1 - \frac{1}{2(1-b_1)}dr_1 + \frac{m}{2(1-b_1)}dq_2 + 0\times dr_2 &= \frac{1}{2(1-b_1)^2}\left[a + r_1 - \frac{c_1}{1-t} - mq_2\right]db_1 \\
-\frac{[1-t]}{[1-\sigma_1]\theta_1}dq_1 + dr_1 + 0\times dq_2 + 0\times dr_2 &= 0 \\
\frac{m}{2(1-b_2)}dq_1 + 0\times dr_1 + dq_2 - \frac{1}{2(1-b_2)}r_2 &= \frac{1}{2(1-b_2)^2}\left[a + r_2 - c_2 - mq_1\right]db_2 \\
0\times dq_1 + 0\times dr_1 - \frac{1}{[1-\sigma_2]\theta_2}dq_2 + dr_2 &= 0
\end{aligned}$$

In the matrix form

$$\begin{bmatrix} 1 & -\frac{1}{2(1-b_1)} & \frac{m}{2(1-b_1)} & 0 \\ -\frac{[1-t]}{[1-\sigma_1]\theta_1} & 1 & 0 & 0 \\ \frac{m}{2(1-b_2)} & 0 & 1 & -\frac{1}{2(1-b_2)} \\ 0 & 0 & -\frac{1}{[1-\sigma_2]\theta_2} & 1 \end{bmatrix}\begin{bmatrix} dq_1 \\ dr_1 \\ dq_2 \\ dr_1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2(1-b_1)^2}\left[a + r_1 - \frac{c_1}{1-t} - mq_2\right]db_1 \\ 0 \\ \frac{1}{2(1-b_2)^2}\left[a + r_2 - c_2 - mq_1\right]db_2 \\ 0 \end{bmatrix}$$

Thus:

$$\begin{aligned}
\frac{dq_1}{db_1} &= \frac{1}{\Delta^r}\frac{1}{2(1-b_1)^2}\left[a + r_1 - \frac{c_1}{1-t} - mq_2\right]\left[1 - \frac{1}{(1-\sigma_2)\theta_2}\frac{1}{2(1-b_2)}\right] > 0 \\
\frac{dq_1}{db_2} &= -\frac{1}{\Delta^r}\frac{m}{4(1-b_1)(1-b_2)^2}\left[a + r_2 - c_2 - mq_1\right] \lesseqqgtr 0 \text{ for } m \gtreqqless 0
\end{aligned}$$

And:

$$\begin{aligned}
\frac{dr_1}{db_1} &= \frac{1}{\Delta^r}\frac{1-t}{2(1-b_1)^2[1-\sigma_1]\theta_1}\left[a + r_1 - \frac{c_1}{1-t} - mq_2\right]\left[1 - \frac{1}{2(1-b_2)[1-\sigma_2]\theta_2}\right] > 0 \\
\frac{dr_1}{db_2} &= -\frac{1}{\Delta^r}\frac{[1-t]m}{4(1-b_1)(1-b_2)^2[1-\sigma_1]\theta_1}\left[a + r_2 - c_2 - mq_1\right] \lesseqqgtr 0 \text{ for } m \gtreqqless 0
\end{aligned}$$

similarly:

$$\begin{aligned}
\frac{dq_2}{db_1} &= -\frac{1}{\Delta^r}\frac{m}{4(1-b_1)(1-b_2)^2}\left[a + r_1 - \frac{c_1}{1-t} - mq_2\right] \lesseqqgtr 0 \text{ for } m \gtreqqless 0 \\
\frac{dq_2}{db_2} &= \frac{1}{\Delta^r}\frac{1}{2(1-b_2)^2}\left[a + r_2 - c_2 - mq_1\right]\left[1 - \frac{1-t}{2(1-b_1)(1-\sigma_1)\theta_2}\right] > 0
\end{aligned}$$

$$\begin{aligned}
\frac{dr_2}{db_1} &= -\frac{1}{\Delta^r}\frac{m}{4(1-b_1)^2(1-b_2)[1-\sigma_1]\theta_1}\left[a + r_1 - \frac{c_1}{1-t} - mq_2\right] \lesseqqgtr 0 \text{ for } m \gtreqqless 0 \\
\frac{dr_2}{db_2} &= \frac{1}{\Delta^r}\frac{1}{2(1-b_2)^2[1-\sigma_1]\theta_1}\left[a + r_2 - c_2 - mq_1\right]\left[1 - \frac{1-t}{2(1-b_1)[1-\sigma_1]\theta_1}\right] > 0
\end{aligned}$$

B.3.3 Stage 1: Optimal tax and subsidy policies

Set $k_1 = k_2 = 0$, welfare functions are:

$$\begin{aligned} W_1 &= \pi_1 - \sigma_1 \frac{\theta_1 r_1^2}{2} \\ W_2 &= U + \pi_2 - \sigma_2 \frac{\theta_2 r_2^2}{2} + t p_1 q_1 \end{aligned}$$

B.3.3.1 Foreign country optimizes subsidy policy

$$0 = \frac{dW_1}{d\sigma_1} = -m\left[1-t\right] q_1 \frac{dq_2}{d\sigma_1} - \sigma_1 \theta_1 r_1 \frac{dr_1}{d\sigma_1}$$

where:

$$\begin{aligned} \frac{\partial r_1}{\partial \sigma_1} &= \frac{1}{\Delta^r} \frac{\left[1-t\right] q_1}{\left[1-\sigma_1\right]^2 \theta_1} \left[1 - \frac{1}{2\left(1-b_2\right)} \frac{1}{\left[1-\sigma_2\right]\theta_2} - \frac{m^2}{4\left(1-b_1\right)\left(1-b_2\right)}\right] > 0 \\ -m\frac{dq_2}{d\sigma_1} &= \frac{1}{\Delta^r} \frac{m^2\left[1-t\right] q_1}{4\left(1-b_1\right)\left(1-b_2\right)\left[1-\sigma_1\right]^2 \theta_1} > 0 \end{aligned}$$

Thus, we can simplify $\frac{dW_1}{d\sigma_1}$ to obtain:

$$\sigma_1 = \frac{\frac{m^2}{4(1-b_1)(1-b_2)}}{1 - \frac{1}{2(1-b_2)} \frac{1}{[1-\sigma_2]\theta_2}}$$

B.3.3.2 Home country optimizes tax and subsidy policies

Home country solves the following two problems:

$$\begin{aligned} \text{optimal subsidy:} \quad \frac{dW_2}{d\sigma_2} &= 0 \\ \text{optimal tax:} \quad \frac{dW_2}{dt} &= 0 \end{aligned}$$

Given:

$$W_2 = U + \pi_2 - \sigma_2 \frac{\theta_2 r_2^2}{2} + t p_1 q_1$$

$$U = \left[a_1 + r_1\right] q_1 + \left[a_2 + r_2\right] q_2 - \frac{1}{2}(q_1^2 + q_2^2 + 2m q_1 q_2) + M_0 - p_1 q_1 - p_2 q_2$$

$$\begin{aligned} p_1 &= a + r_1 - \left(1-b_1\right) q_1 - m q_2 \\ p_2 &= a + r_2 - \left(1-b_2\right) q_2 - m q_1 \end{aligned}$$

$$\pi_2 = p_2 q_2 - c_2 q_2 - [1-\sigma_2] \frac{\theta_2 r_2^2}{2}$$

For optimal subsidy:

$$\begin{aligned} 0 \quad = \quad & \frac{dW_2}{d\sigma_2} = q_1 \frac{d}{d\sigma_2}(a_1 + r_1 - p_1) + q_2 \frac{d}{d\sigma_2}(a_2 + r_2 - p_2) \text{ (from } U) \\ & + q_2 \frac{dp_2}{dq_1} \frac{dq_1}{d\sigma_2} + \frac{\theta_2 r_2^2}{2} \text{ (from } \pi_2) \\ & - \frac{\theta_2 r_2^2}{2} - \sigma_2 \theta_2 r_2 \frac{dr_2}{d\sigma_2} + t \left[p_1 \frac{dq_1}{d\sigma_2} + q_1 \frac{dp_1}{d\sigma_2} \right] \text{ (last two terms)} \end{aligned}$$

Simplify the equation so that:

$$\begin{aligned} 0 \quad = \quad & \frac{dW_2}{d\sigma_2} = \left[q_1 + \frac{t}{1-t} c_1 \right] \frac{dq_1}{d\sigma_2} \\ & + [q_2 + m[1-t] q_1] \frac{dq_2}{d\sigma_2} + t q_1 \frac{dr_1}{d\sigma_2} - \frac{\sigma_2}{1-\sigma_2} q_2 \frac{dr_2}{d\sigma_2} \end{aligned}$$

Substitute $\frac{dq_1}{d\sigma_2}$,$\frac{dq_2}{d\sigma_2}$,$\frac{dr_1}{d\sigma_2}$,and $\frac{dr_2}{d\sigma_2}$ into the above equation, when $m = 0$, we can obtain:

$$\sigma_2 \quad = \quad \frac{1}{1 + 2(1 - b_2)}$$

This is similar to the equation for s_2 in the case of process R&D. That is, when demands are independent, the home country chooses to subsidize the product R&D to the home firm.

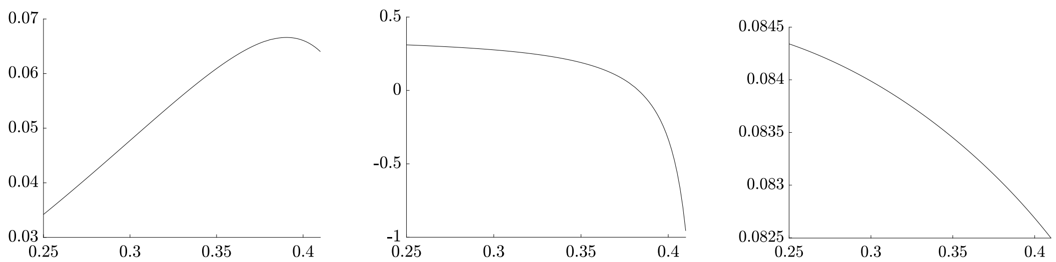
For optimal tax:

$$\begin{aligned} 0 \quad = \quad & \frac{dW_2}{dt} = q_1 \frac{d}{dt}(a_1 + r_1 - p_1) + q_2 \frac{d}{dt}(a_2 + r_2 - p_2) \text{ (from } U) \\ & + q_2 \frac{dp_2}{dq_1} \frac{dq_1}{dt} \text{ (from } \pi_2) \\ & - \sigma_2 \theta_2 r_2 \frac{dr_2}{dt} + p_1 q_1 + t \left[p_1 \frac{dq_1}{dt} + q_1 \frac{dp_1}{dt} \right] \text{ (last two terms)} \end{aligned}$$

The model structure does not allow for the closed form solutions for the general case, $m \neq 0$, of tax and subsidy policies. This equation will be solved numerically in the final section.

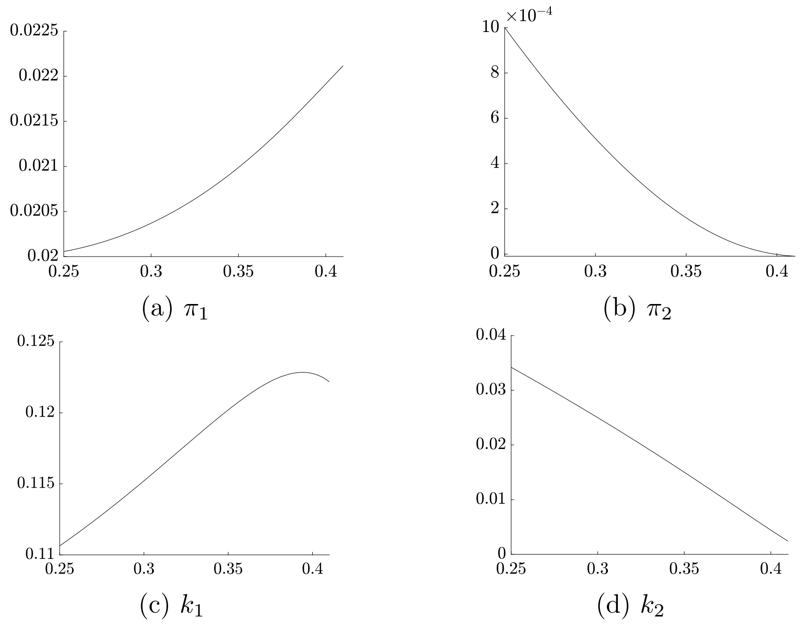
Appendix **C**

Figures for Chapter 2



(a) s_1 (b) s_2 (c) t

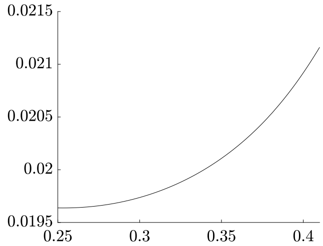
Figure C.1: Process R&D: Optimal tax and subsidies, $m > 0$



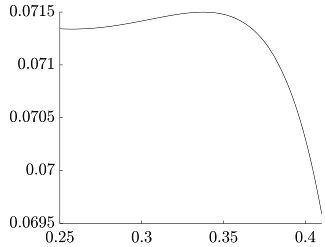
(a) π_1 (b) π_2

(c) k_1 (d) k_2

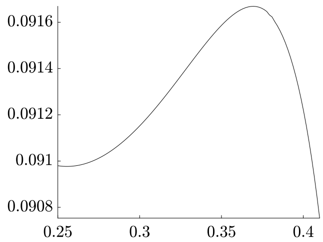
Figure C.2: Process R&D: Optimal profit and investment, $m > 0$



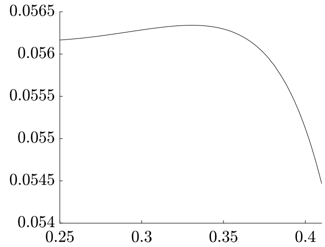
(a) W_1



(b) W_2

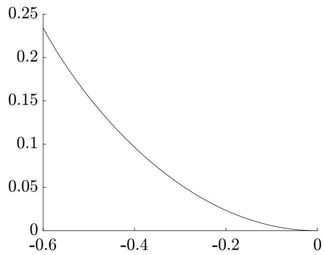


(c) W

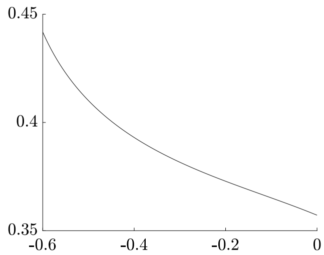


(d) U

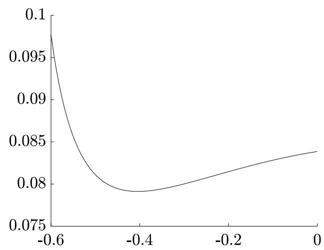
Figure C.3: Process R&D: Optimal profit and investment, $m > 0$



(a) s_1

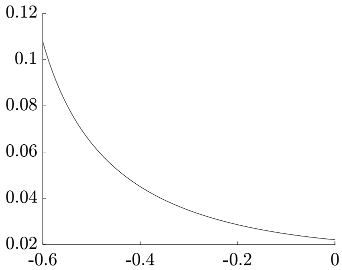


(b) s_2

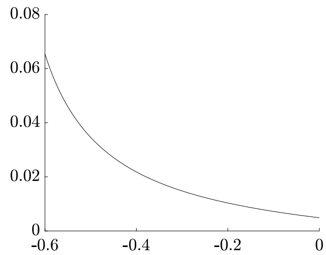


(c) t

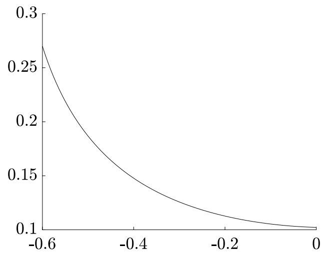
Figure C.4: Process R&D: Optimal tax and subsidies, $m < 0$



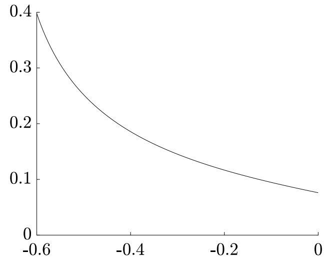
(a) π_1



(b) π_2

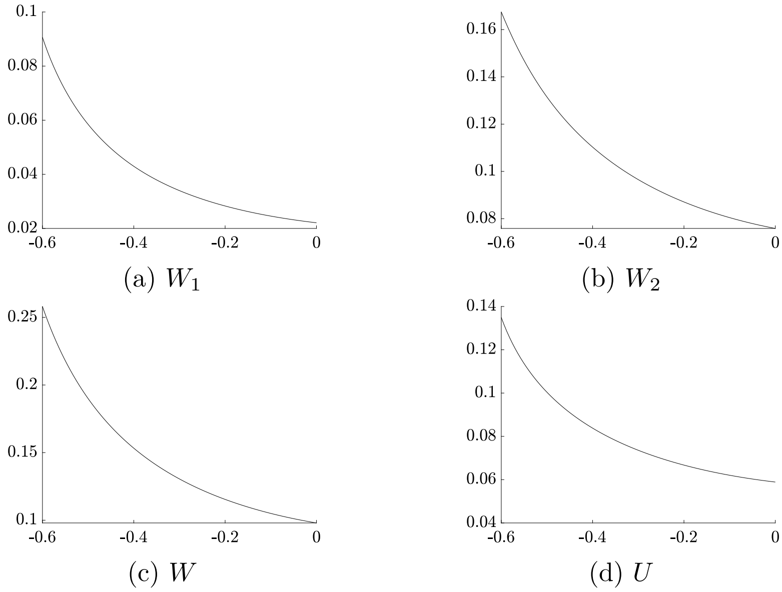


(c) k_1



(d) k_2

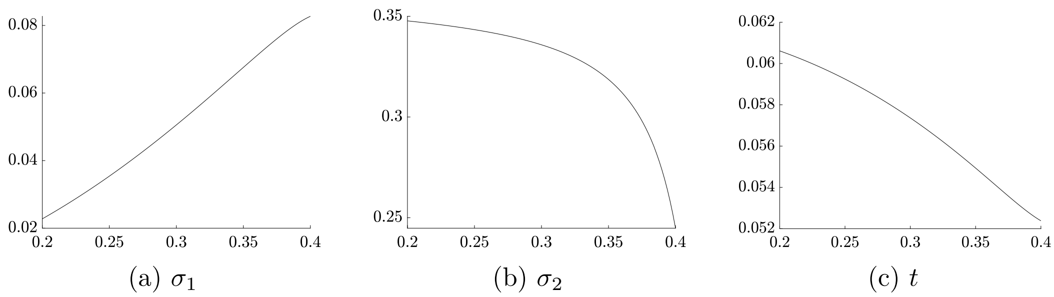
Figure C.5: Process R&D: Optimal profit and investment, $m < 0$



(a) W_1 (b) W_2

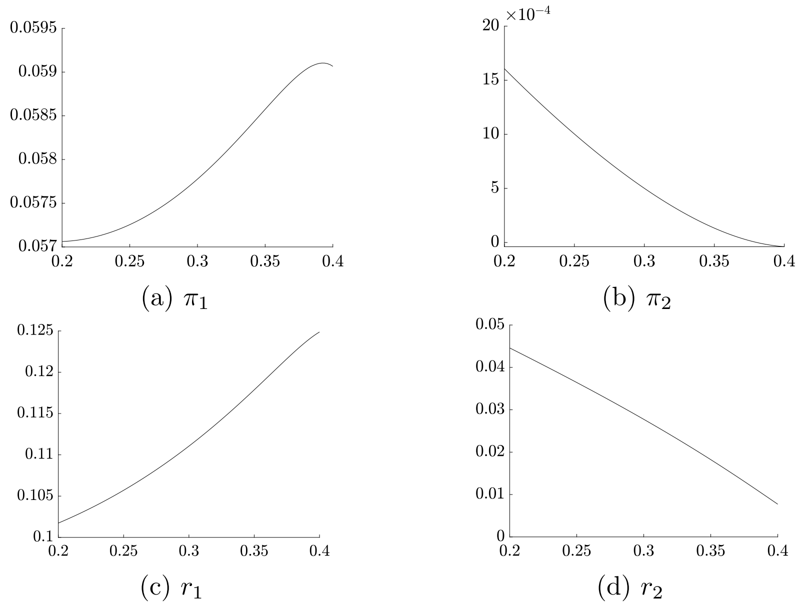
(c) W (d) U

Figure C.6: Process R&D: Optimal welfare and utility, $m < 0$



(a) σ_1 (b) σ_2 (c) t

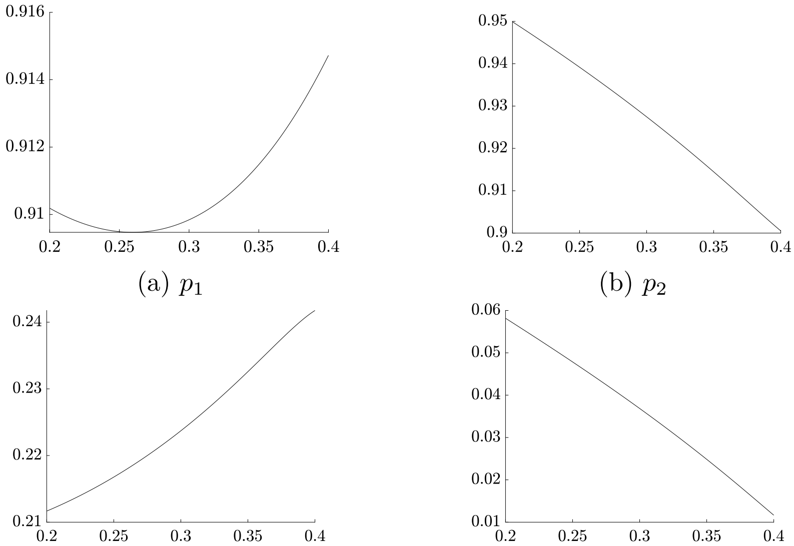
Figure C.7: Product R&D: Optimal tax and subsidies, $m > 0$



(a) π_1 (b) π_2

(c) r_1 (d) r_2

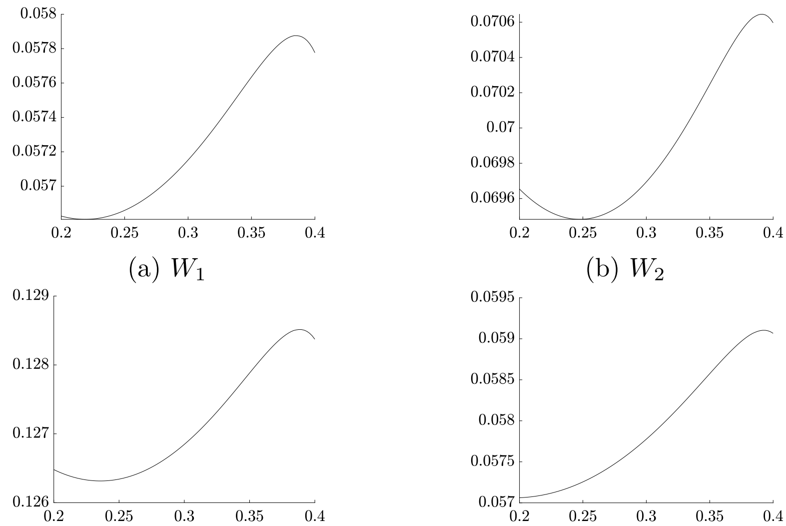
Figure C.8: Product R&D: Optimal profit and investment, $m > 0$



(a) p_1 (b) p_2

(c) q_1 (d) q_2

Figure C.9: Product R&D: Optimal price and output, $m > 0$



(a) W_1 (b) W_2

(c) W (d) U

Figure C.10: Product R&D: Optimal welfare and utility, $m < 0$

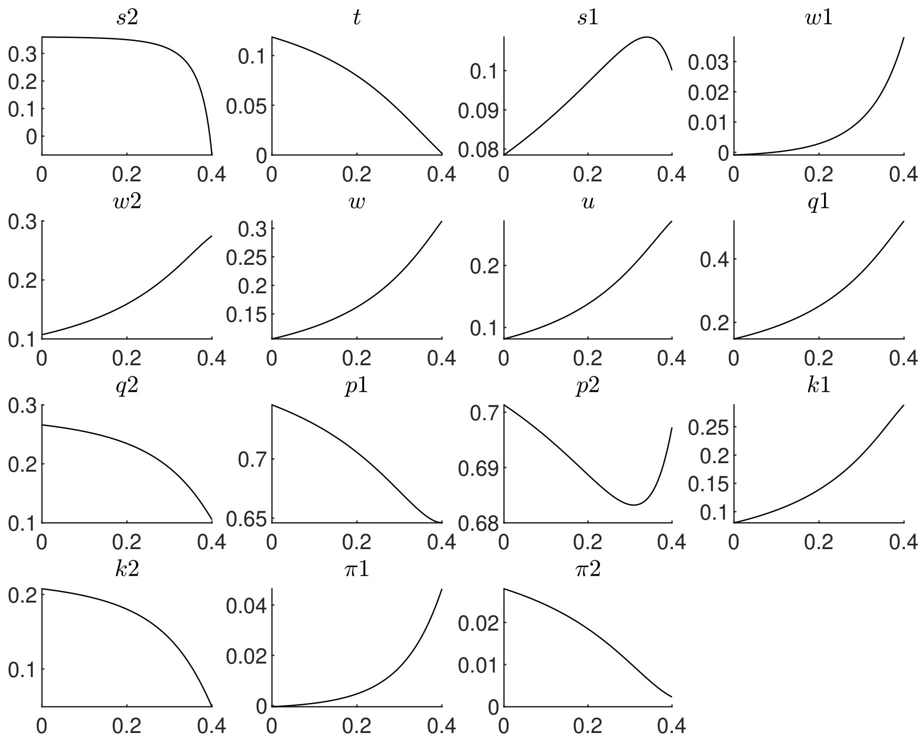


Figure C.11: Process b1 m=0.4

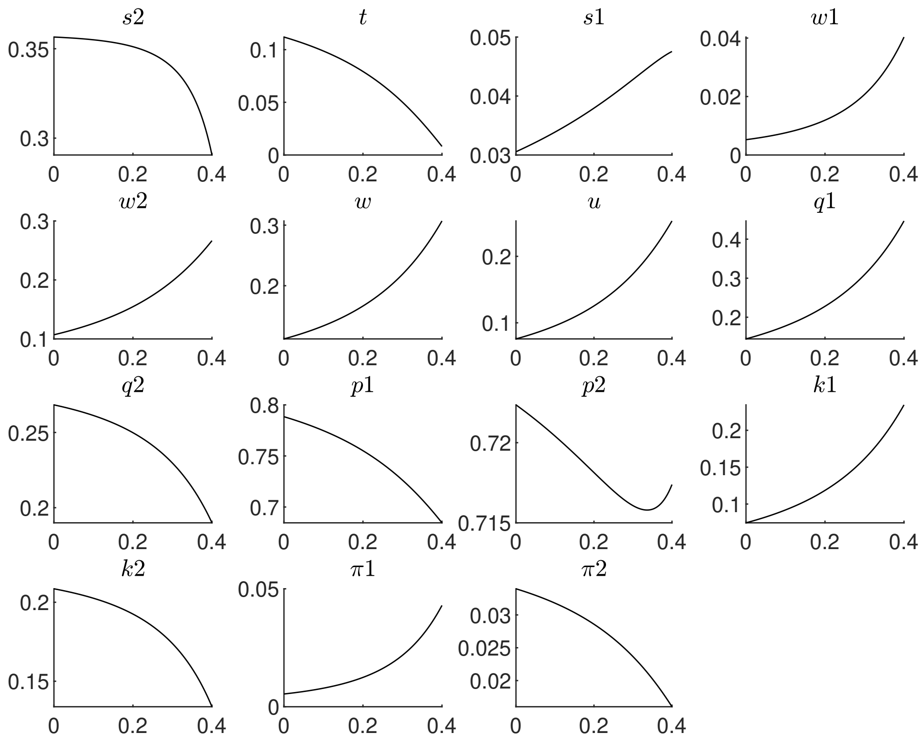


Figure C.12: Process b1 m=0.25

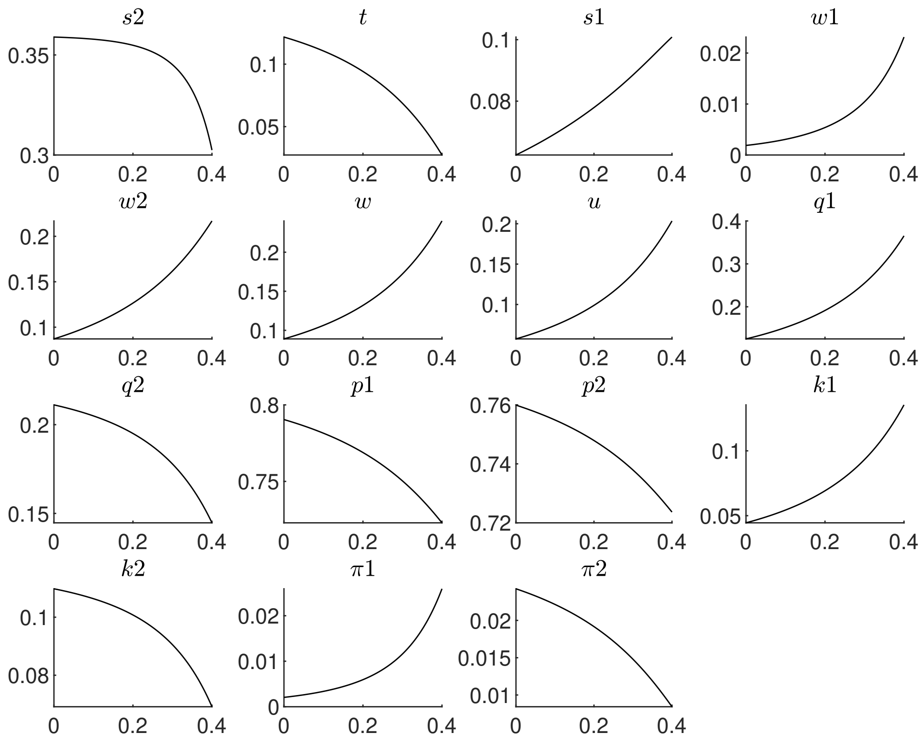


Figure C.13: Process b2 m=0.4

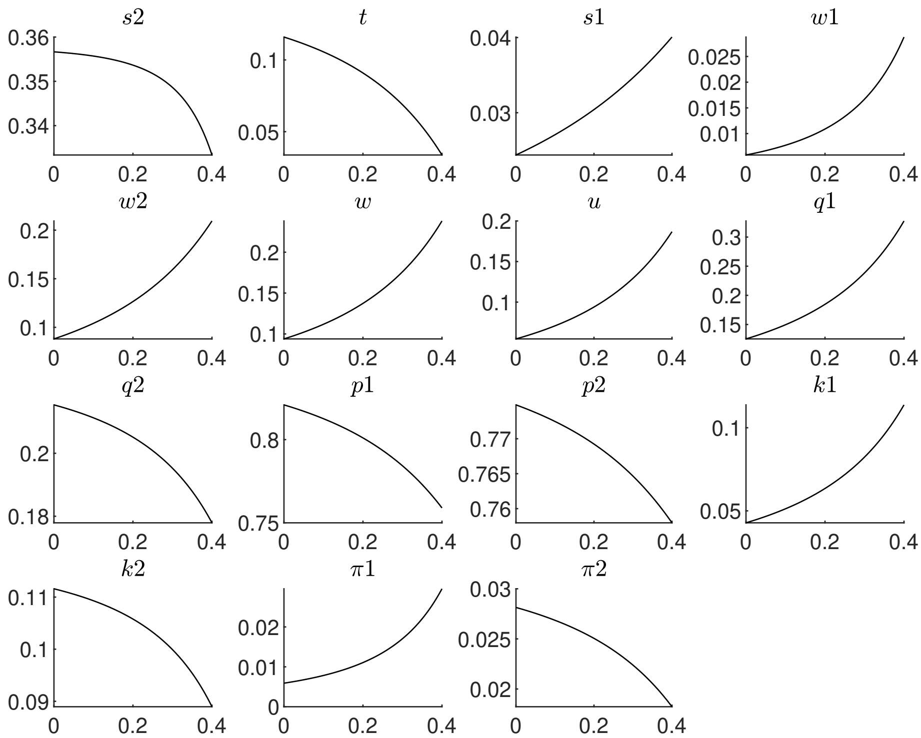


Figure C.14: Process b2 m=0.25

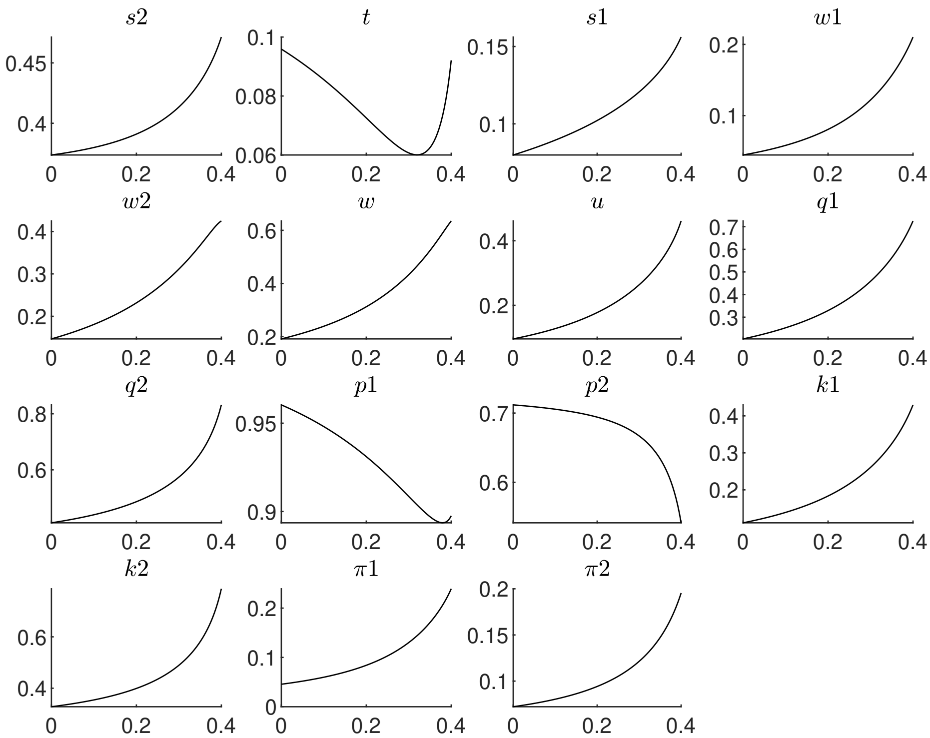


Figure C.15: Process b1 m=-0.4

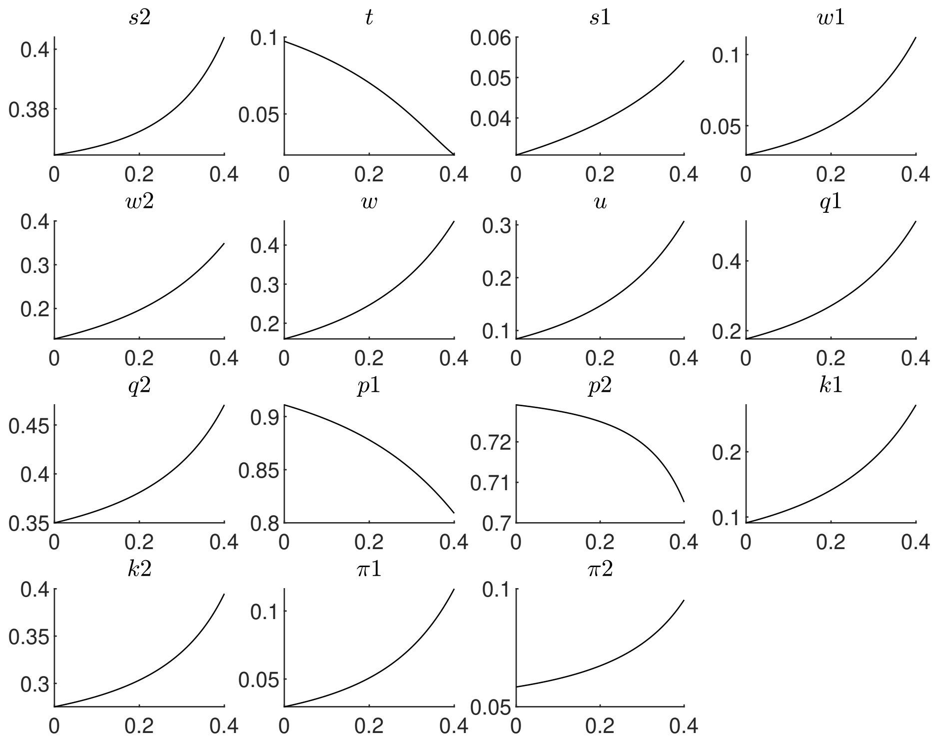


Figure C.16: Process b1 m=-0.25

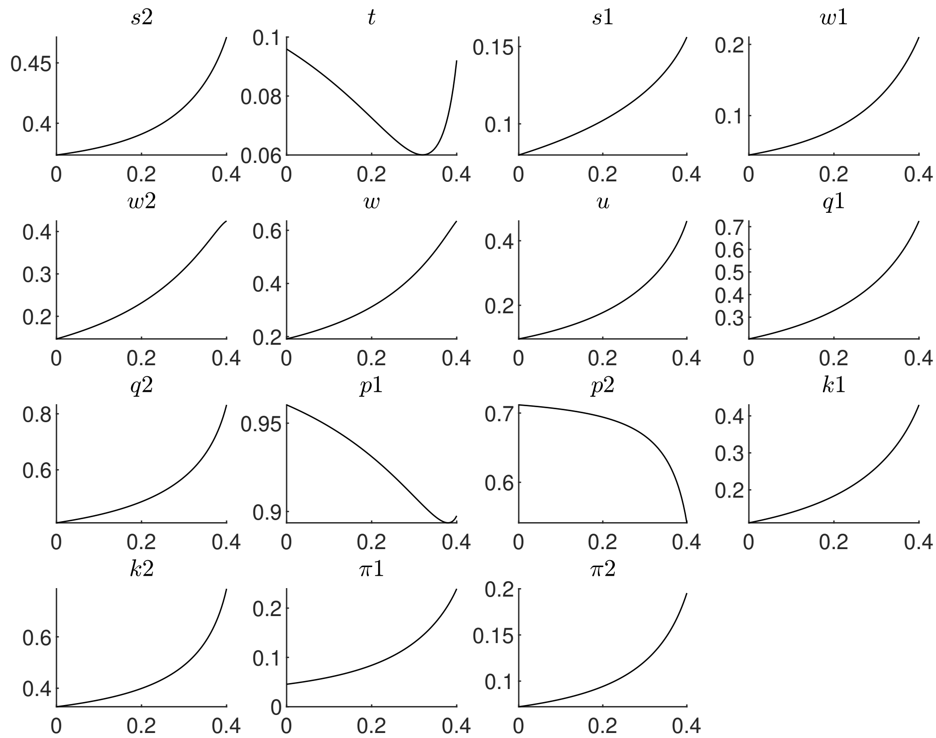


Figure C.17: Process b2 m=-0.4

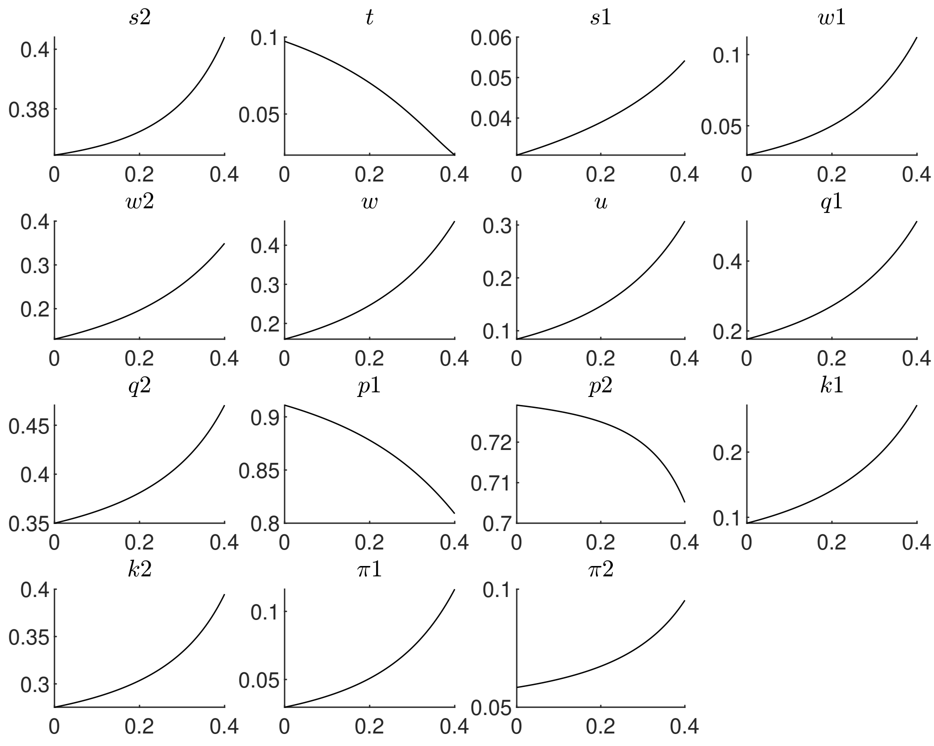


Figure C.18: Process b2 m=-0.25

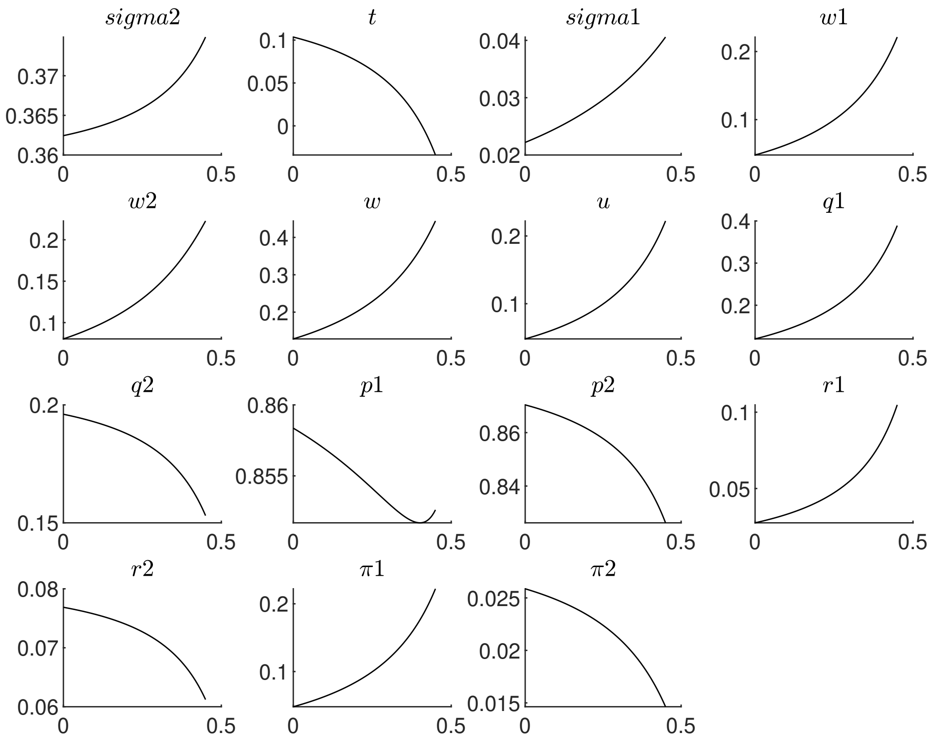


Figure C.19: Product b1 m=0.25

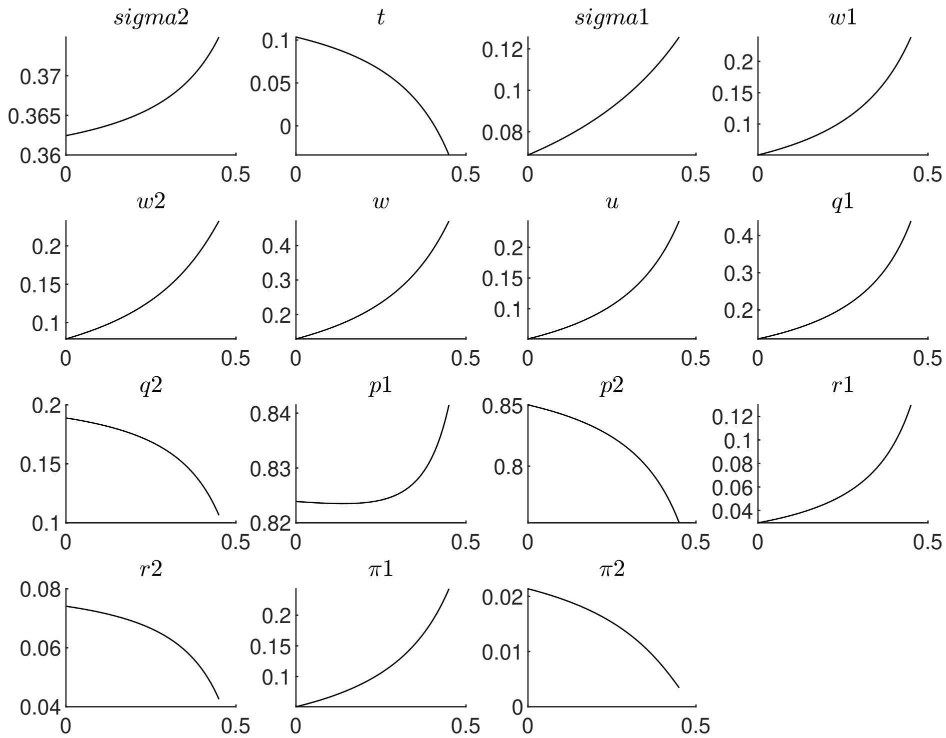


Figure C.20: Product b1 m=0.44

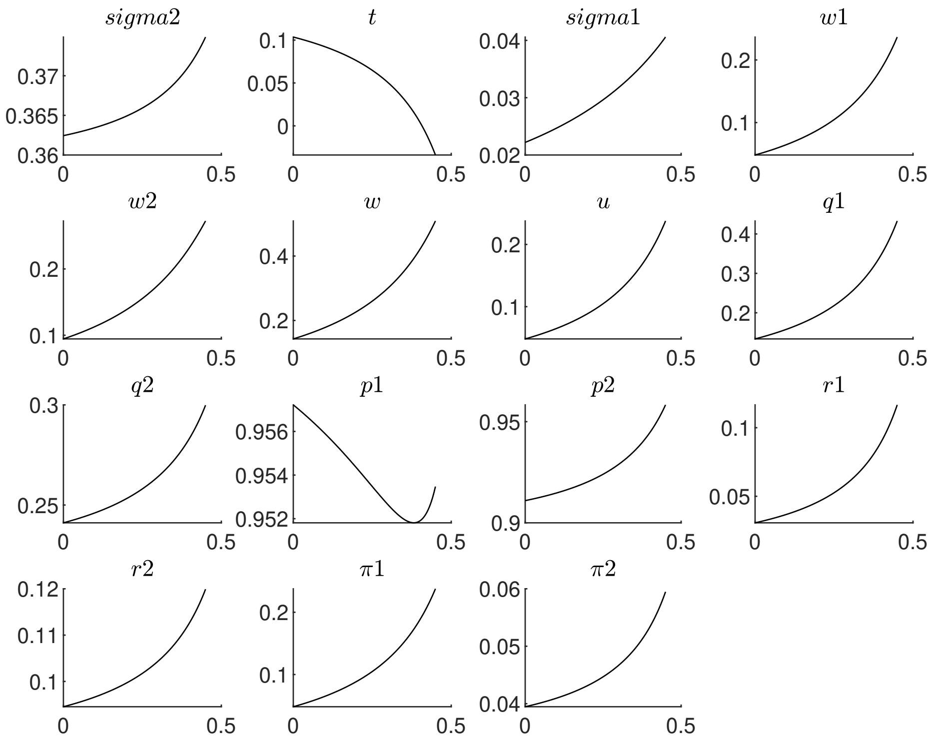


Figure C.21: Product b1 m=-0.25

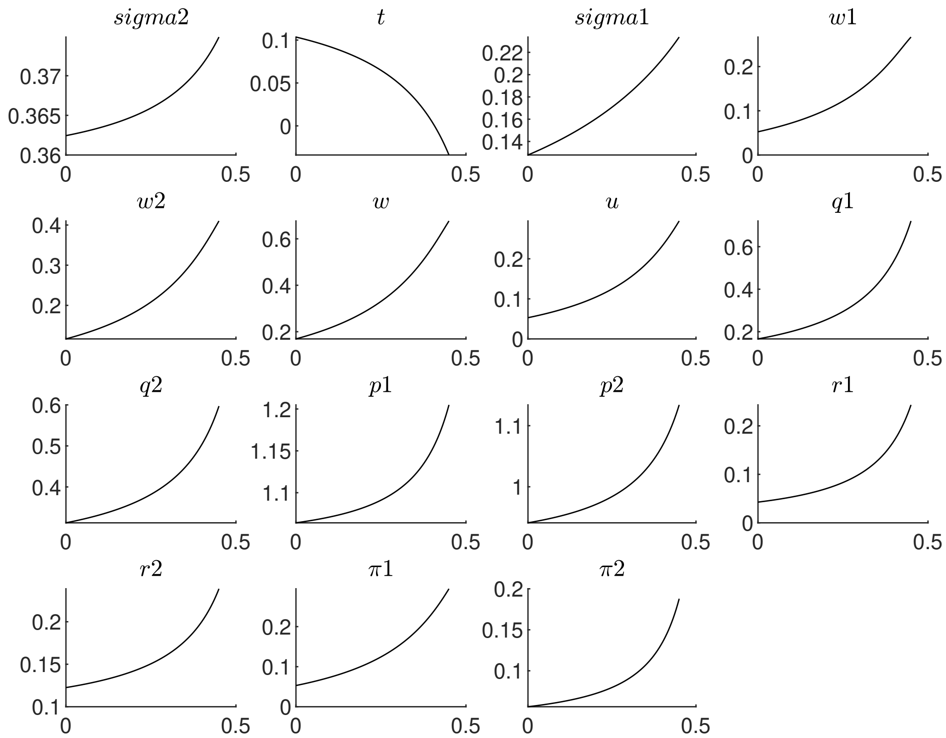


Figure C.22: Product b1 m=-0.6

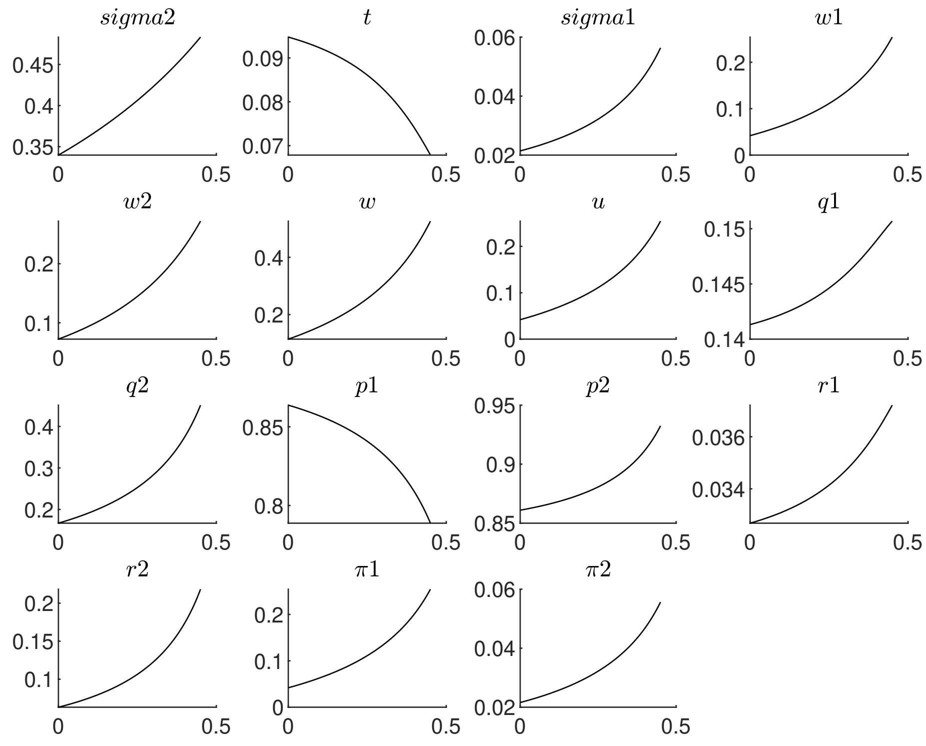


Figure C.23: Product b2 m=0.25

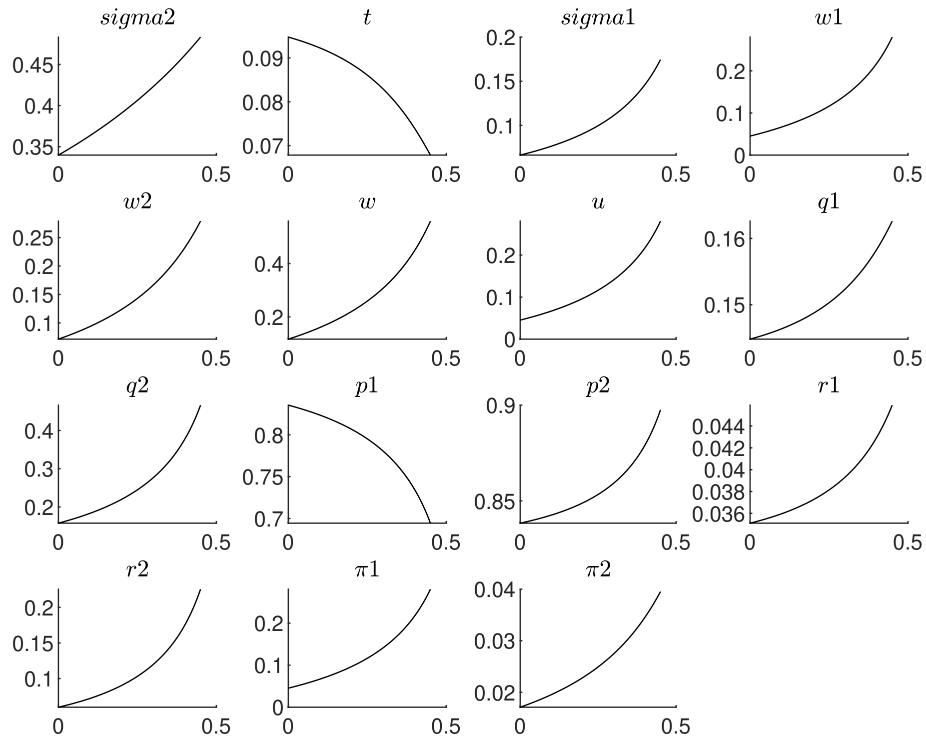


Figure C.24: Product b2 m=0.44

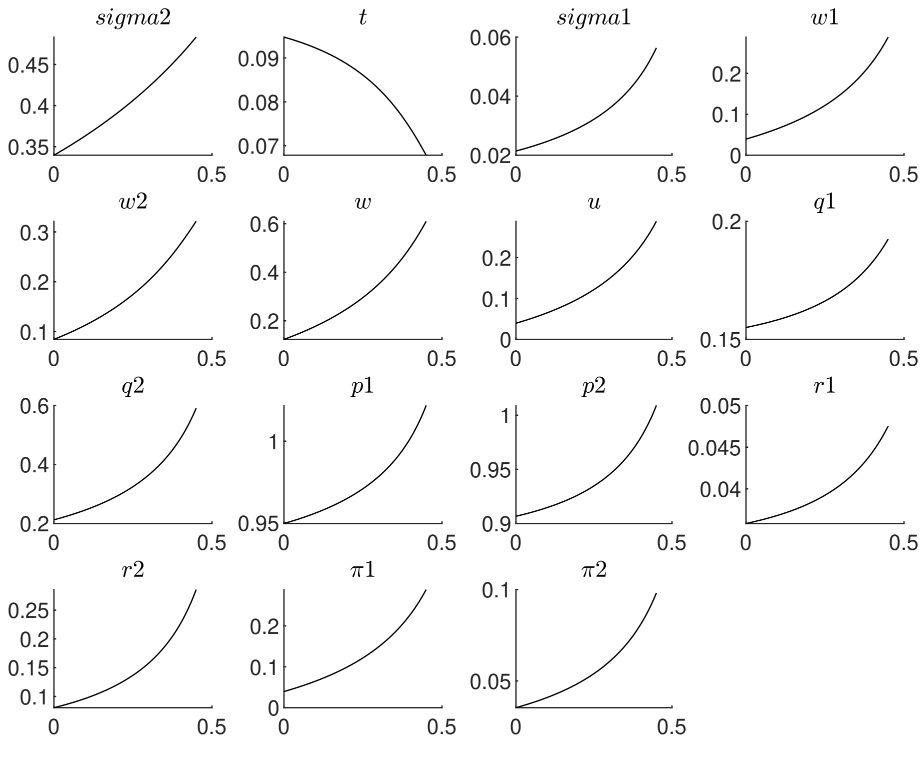


Figure C.25: Product b2 m=-0.25

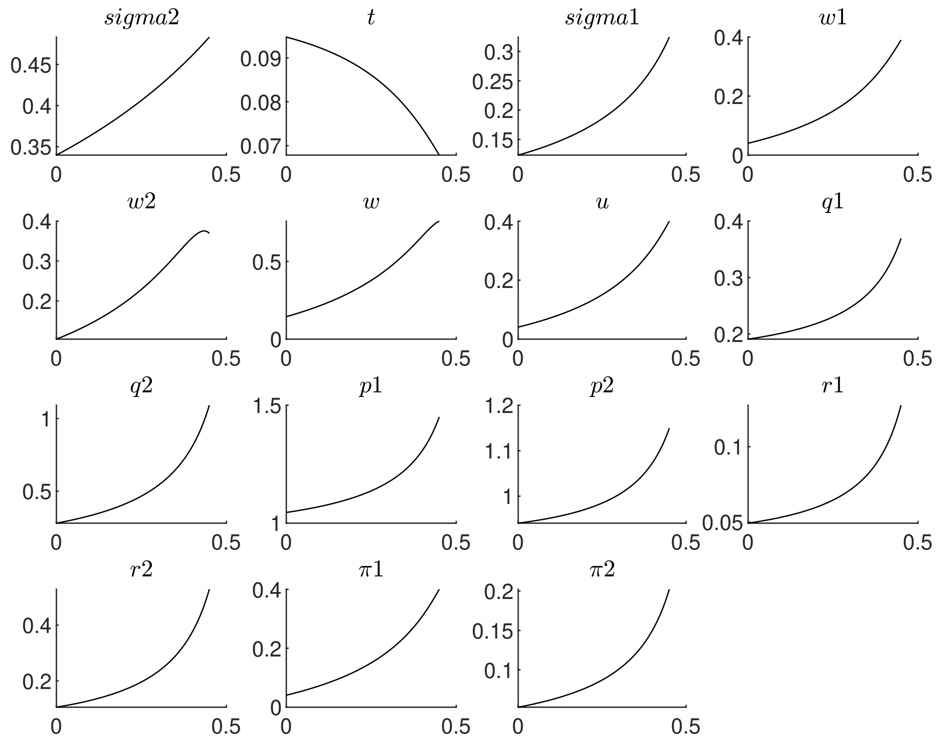


Figure C.26: Product b2 m=-0.6

Appendix D

Figures for Chapter 3

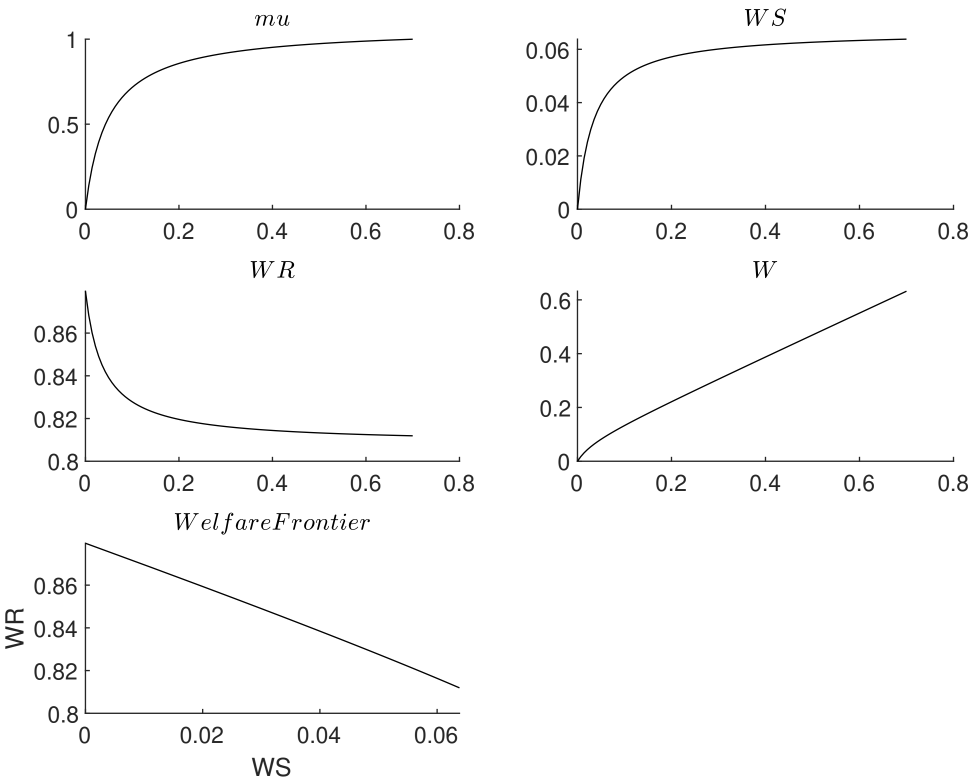


Figure D.1: Welfare Possibility Frontier

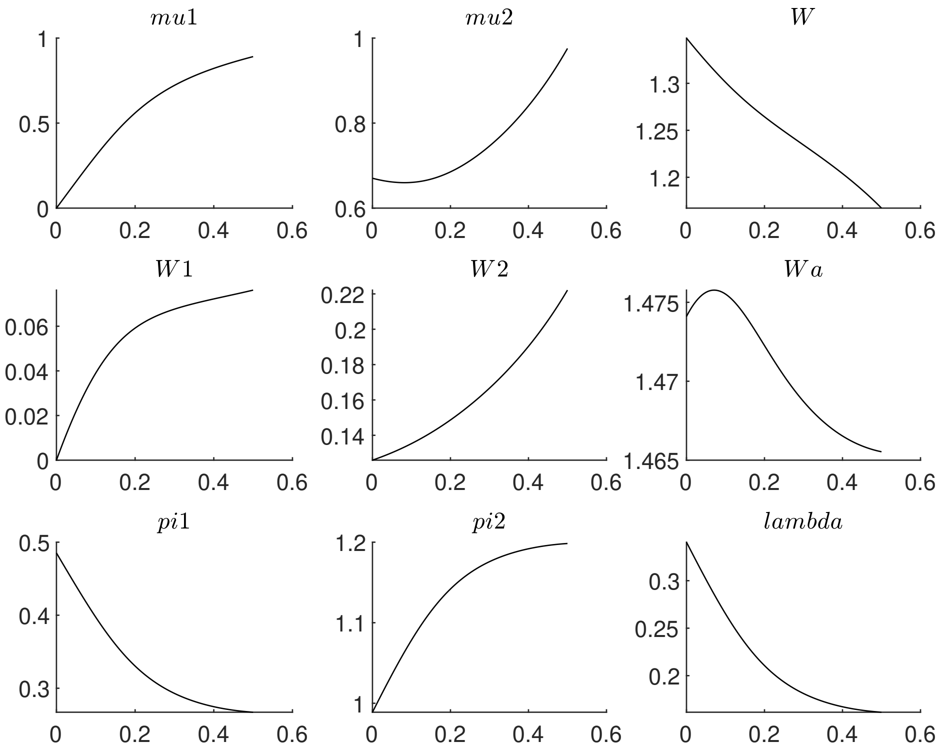


Figure D.2: Nash-bargaining equilibrium outcomes as a function of β_1 when $\beta_2 = 0.1$

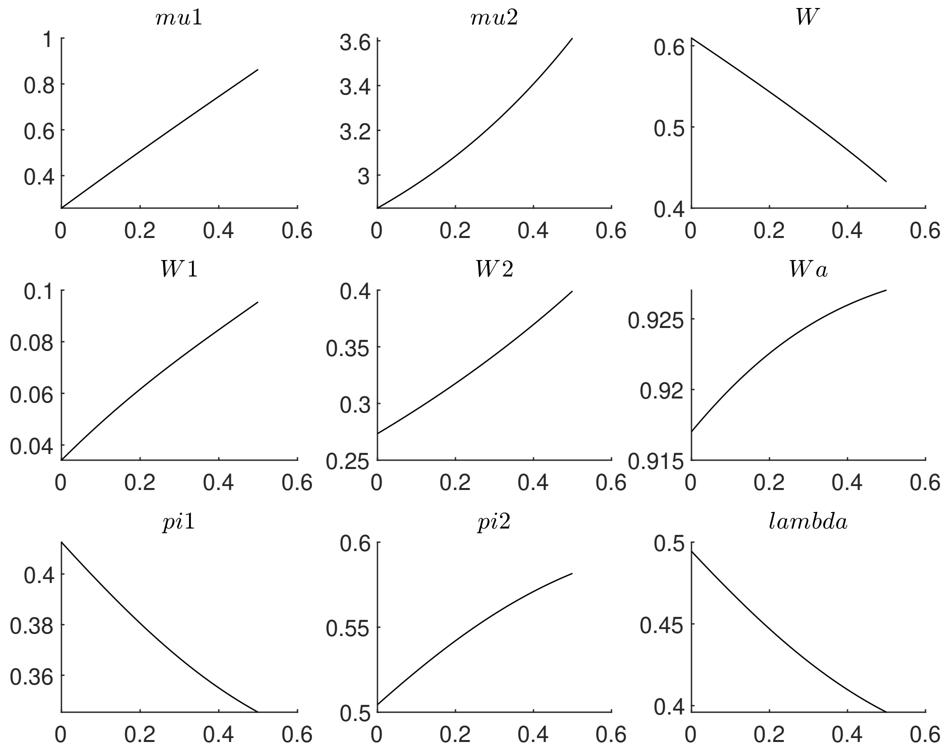


Figure D.3: Nash-bargaining equilibrium outcomes as a function of β_1 when $\beta_2 = 0.7$

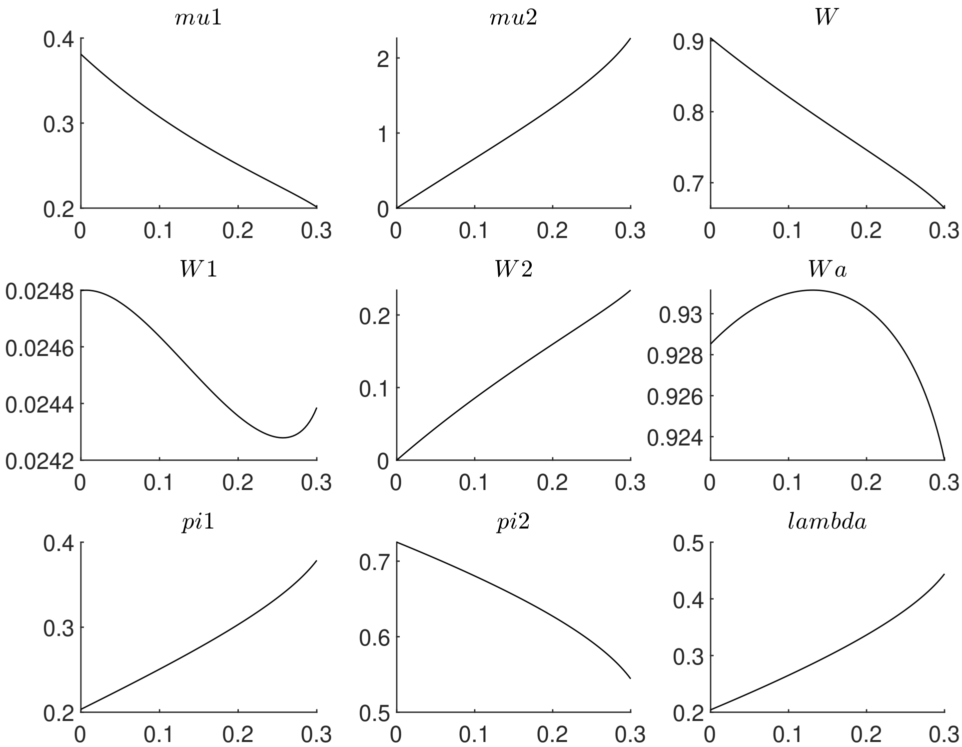


Figure D.4: Nash-bargaining equilibrium outcomes as a function of β_2 when $\beta_1 = 0.1$

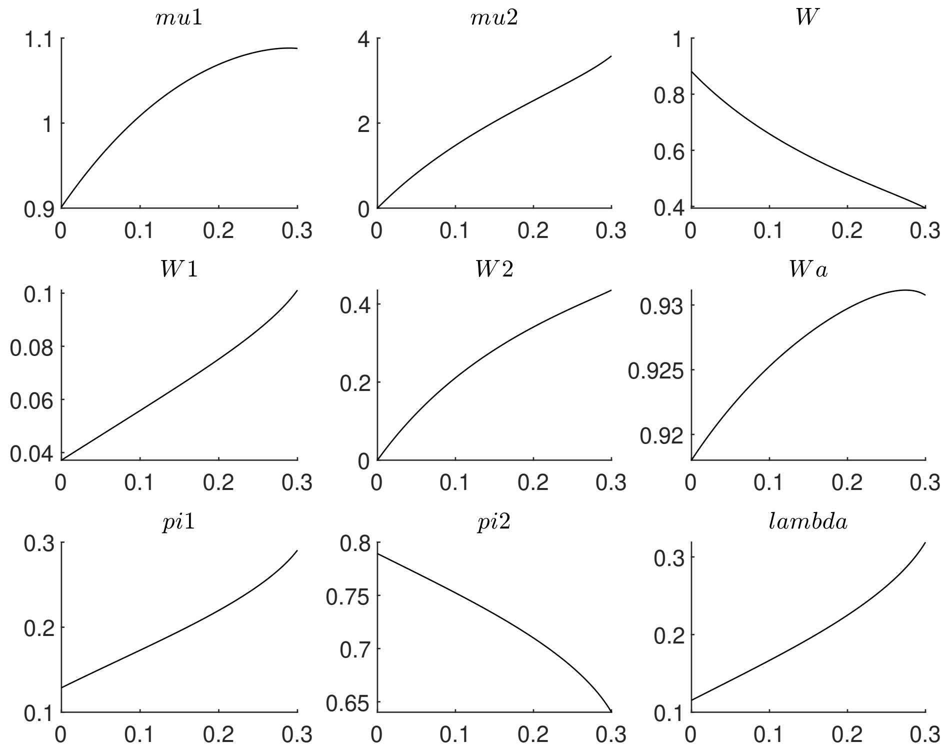


Figure D.5: Nash-bargaining equilibrium outcomes as a function of β_2 when $\beta_1 = 0.7$

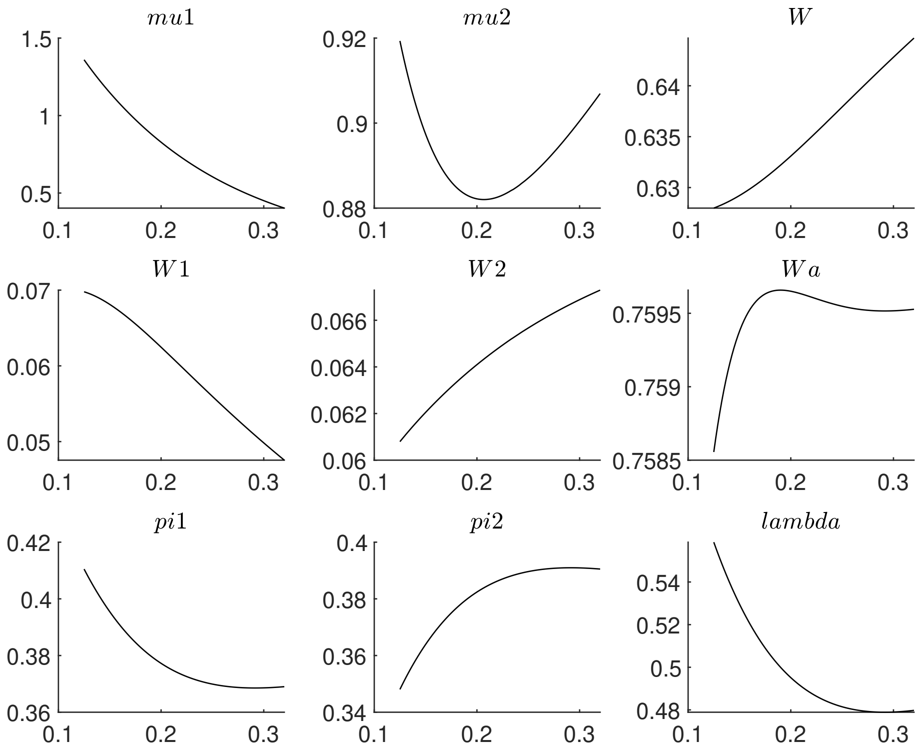


Figure D.6: Nash-bargaining equilibrium outcomes as a function of t_1 when $t_2 = 0.19$

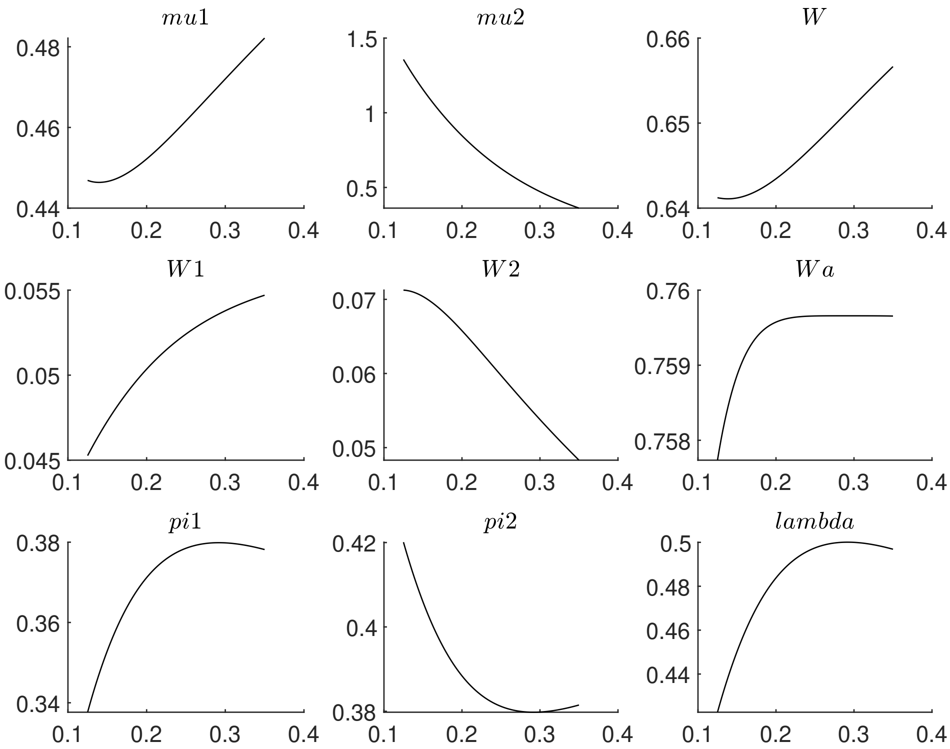


Figure D.7: Nash-bargaining equilibrium outcomes as a function of t_2 when $t_1 = 0.32$

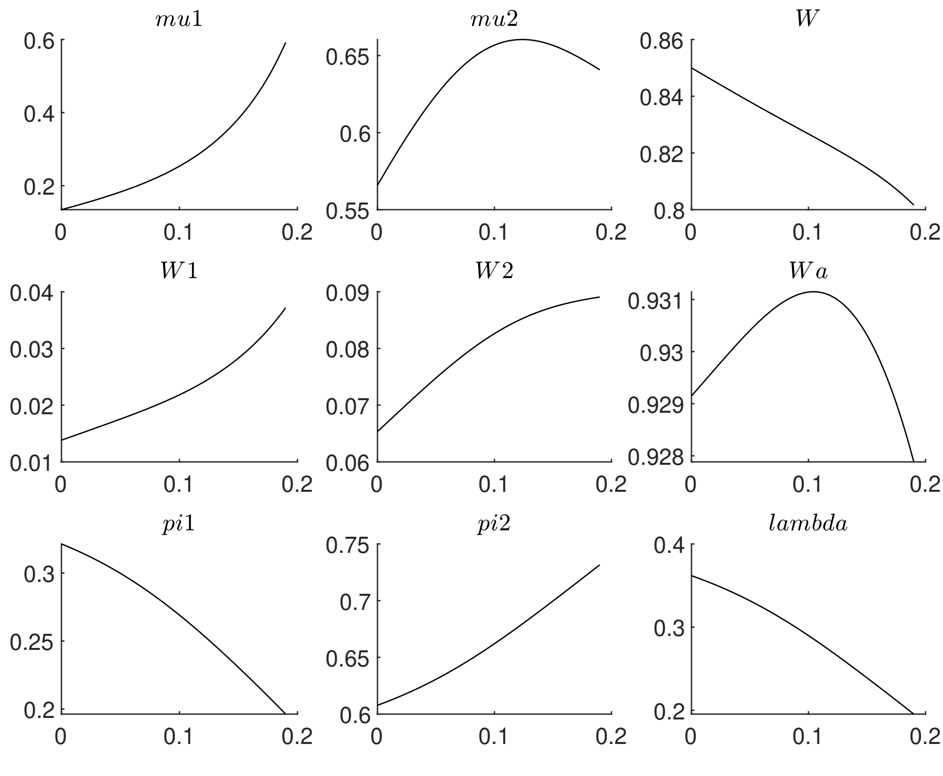


Figure D.8: Nash-bargaining equilibrium outcomes as a function of t when $t_1 = 0.32$ $t_2 = 0.19$

Bibliography

Aboody, D. and Lev, B. (2000), 'Information asymmetry, R&D, and insider gains', *The Journal of Finance* **55**(6), 2747–2766.
URL: *https://doi.org/10.1111%2F0022-1082.00305*

Aghion, P. and Howitt, P. (1992), 'A model of growth through creative destruction', *Econometrica* **60**(2), 323–51.
URL: *https://EconPapers.repec.org/RePEc:ecm:emetrp:v:60:y:1992:i:2:p:323-51*

Albis, N., Marín, R., Sánchez, E., Bayona-Rodríguez, H. and García, J. M. (2021), 'The impacts of public support for innovation on firm productivity and on private investment in R&D in manufacturing and services in colombia', *Innovation and Development* pp. 1–20.
URL: *https://doi.org/10.1080%2F2157930x.2021.1996312*

Alecke, B., Reinkowski, J., Mitze, T. and Untiedt, G. (2012), 'Does firm size make a difference? analysing the effectiveness of R&D subsidies in east germany', *German Economic Review* **13**(2), 174–195.
URL: *https://doi.org/10.1111%2Fj.1468-0475.2011.00546.x*

Ali-Yrkkö, J. (2005), Impact of public R&D financing on private R&D: Does financial constraint matter?, Technical Report 030, European Network of Economic Policy

Research Institutes.
URL: *https://EconPapers.repec.org/RePEc:epr:enepwp:030*

Antonelli, C. (2000), 'Collective knowledge communication and innovation: The evidence of technological districts', *Regional Studies* **34**(6), 535–547.
URL: *https://doi.org/10.1080%2F00343400050085657*

Arellano, M. and Bover, O. (1995), 'Another look at the instrumental variable estimation of error-components models', *Journal of econometrics* **68**(1), 29–51.

Arrow, K. (1962), Economic welfare and the allocation of resources for invention, *in* 'The Rate and Direction of Inventive Activity: Economic and Social Factors', National Bureau of Economic Research, Inc, pp. 609–626.
URL: *https://EconPapers.repec.org/RePEc:nbr:nberch:2144*

Aschhoff, B. (2009), 'The effect of subsidies on R&D investment and success – do subsidy history and size matter?', *SSRN Electronic Journal* .
URL: *https://doi.org/10.2139%2Fssrn.1433275*

Aschhoff, B. (2010), 'Who gets the money? the dynamics of r&d project subsidies in germany', *Jahrbücher für Nationalökonomie und Statistik* **230**(5), 522–546.
URL: *https://doi.org/10.1515%2Fjbnst-2010-0503*

Avi-Yonah, R. S. (1997), 'International taxation of electronic commerce'.
URL: *https://repository.law.umich.edu/articles/2328/*

Azoulay, P. and Li, D. (2020), Scientific grant funding, Technical report.
URL: *https://doi.org/10.3386%2Fw26889*

Baik, K. H. and Kim, S.-K. (2019), 'Observable versus unobservable R&D investments in duopolies', *Journal of Economics* **130**(1), 37–66.
URL: *https://doi.org/10.1007%2Fs00712-019-00679-3*

Barajas, A., Huergo, E. and Moreno, L. (2021), 'The role of public loans in financing business R&D through the economic cycle', *Economia Politica* **38**(2), 505–538.
URL: *https://doi.org/10.1007%2Fs40888-021-00225-9*

Barbaroux, P. (2014), 'From market failures to market opportunities: managing innovation under asymmetric information', *Journal of Innovation and Entrepreneurship* **3**(1), 5.
URL: *https://doi.org/10.1186%2F2192-5372-3-5*

Beck, M., Lopes-Bento, C. and Schenker-Wicki, A. (2016), 'Radical or incremental: Where does R&D policy hit?', *Research Policy* **45**(4), 869–883.
URL: *https://doi.org/10.1016%2Fj.respol.2016.01.010*

Becker, L. (2015), 'Effectiveness of public innovation support in europe. does public support foster turnover, employment and labour productivity?', *SSRN Electronic Journal* .
URL: *https://doi.org/10.2139%2Fssrn.2585442*

BEIS (2017), 'The impact of public support for innovation on firm outcomes'.
URL: *https://www.bl.uk/collection-items/beisc-the-impact-of-public-support-for-innovation-on-firm-outcomes-2017*

Billings, A., Glazunov, S. and Houston, M. (2001), 'The role of taxes in corporate research and development spending', *R&D Management* **31**(4), 465–477.
URL: *https://doi.org/10.1111%2F1467-9310.00232*

Bloom, N., Reenen, J. V. and Williams, H. (2019), 'A toolkit of policies to promote innovation', *Journal of Economic Perspectives* **33**(3), 163–184.
URL: *https://doi.org/10.1257%2Fjep.33.3.163*

Bozeman, B. and Dietz, J. S. (2001), 'Research policy trends in the united states: Civilian technology programs, defense technology and teh deployment of the national laboratories', *Research and innovation policies in the new global economy: An international comparative analysis* pp. 47–78.

Brander, J. A. and Spencer, B. J. (1983), 'Strategic commitment with R&D: The symmetric case', *The Bell Journal of Economics* **14**(1), 225.
URL: *https://doi.org/10.2307/3003549*

Braun, S. (2008), 'Economic integration, process and product innovation, and relative skill demand', *Review of International Economics* **16**(5), 864–873.
URL: *https://doi.org/10.1111/j.1467-9396.2008.00754.x*

Busom, I. (2000), 'An empirical evaluation of the effects of R&D subsidies', *Economics of Innovation and New Technology* **9**(2), 111–148.
URL: *https://doi.org/10.1080%2F10438590000000006*

Busom, I., Corchuelo, B. and Martínez-Ros, E. (2014), 'Tax incentives or subsidies for business r&d?', *Small Business Economics* **43**(3), 571–596.
URL: *https://doi.org/10.1007%2Fs11187-014-9569-1*

Busom, I., Corchuelo, B. and Martínez-Ros, E. (2016), 'Participation inertia in R&D tax incentive and subsidy programs', *Small Business Economics* **48**(1), 153–177.
URL: *https://doi.org/10.1007%2Fs11187-016-9770-5*

Čadil, J., Mirošník, K. and Rehák, J. (2017), 'The lack of short-term impact of cohesion policy on the competitiveness of SMEs', *International Small Business Journal: Researching Entrepreneurship* **35**(8), 991–1009.
URL: *https://doi.org/10.1177%2F0266242617695382*

Carboni, O. A. (2011), 'R&D subsidies and private R&D expenditures: evidence from italian manufacturing data', *International Review of Applied Economics* **25**(4), 419–439.
URL: *https://doi.org/10.1080%2F02692171.2010.529427*

Chaminade, C. and Esquist, C. (2010), Rationales for public policy intervention in the innovation process: Systems of innovation approach, *in* 'The theory and practice of innovation policy', Edward Elgar Publishing.

Chang, R.-Y., Hwang, H. and Peng, C.-H. (2013), 'Technology licensing, R&D and welfare', *Economics Letters* **118**(2), 396–399.
URL: *https://doi.org/10.1016%2Fj.econlet.2012.11.016*

Chen, L. and Yang, W. (2019), 'R&D tax credits and firm innovation: Evidence from china', *Technological Forecasting and Social Change* **146**, 233–241.
URL: *https://doi.org/10.1016%2Fj.techfore.2019.05.018*

Chiang, J.-T. (1991), 'From mission-oriented to diffusion-oriented paradigm: the new trend of u.s. industrial technology policy', *Technovation* **11**(6), 339–356.
URL: *https://doi.org/10.1016%2F0166-4972%2891%2990017-x*

Chow, C. C. and Sarin, R. (2002), 'Known, Unknown, and Unknowable Uncertainties', *Theory and Decision* **52**(2), 127–138.
URL: *https://doi.org/10.1023/A:1015544715608*

Clausen, T. H. (2009), 'Do subsidies have positive impacts on R&D and innovation activities at the firm level?', *Structural Change and Economic Dynamics* **20**(4), 239–253.
URL: *https://doi.org/10.1016%2Fj.strueco.2009.09.004*

Cockfield, A. J. (2002), 'The law and economics of digital taxation: Challenges to traditional tax laws and principles', *Bulletin for International Fiscal Documentation* **56**, 606.

Crepon, B., Duguet, E. and Mairessec, J. (1998), 'Research, innovation and productivity: An econometric analysis at the firm level', *Economics of Innovation and New Technology* **7**(2), 115–158.
URL: *https://doi.org/10.1080%2F10438599800000031*

Czarnitzki, D. and Delanote, J. (2015), 'R&D policies for young SMEs: input and output effects', *Small Business Economics* **45**(3), 465–485.
URL: *https://doi.org/10.1007%2Fs11187-015-9661-1*

Czarnitzki, D. and Fier, A. (2002), Do innovation subsidies crowd out private investment? evidence from the german service sector, Technical Report 02-04, ZEW - Leibniz Centre for European Economic Research.
URL: *https://EconPapers.repec.org/RePEc:zbw:zewdip:893*

Czarnitzki, D., Hanel, P. and Rosa, J. M. (2011), 'Evaluating the impact of R&D tax credits on innovation: A microeconometric study on canadian firms', *Research Policy* **40**(2), 217–229.
URL: *https://doi.org/10.1016%2Fj.respol.2010.09.017*

Czarnitzki, D. and Hottenrott, H. (2012), 'Collaborative R&D as a strategy to attenuate financing constraints', *ZEW-Centre for European Economic Research Discussion Paper* (12-049).

Czarnitzki, D. and Lopes-Bento, C. (2014), 'Innovation subsidies: Does the funding source matter for innovation intensity and performance? empirical evidence from germany', *Industry and Innovation* **21**(5), 380–409.
URL: *https://doi.org/10.1080%2F13662716.2014.973246*

David, P. A., Hall, B. H. and Toole, A. A. (2000), 'Is public R&D a complement or substitute for private R&D? a review of the econometric evidence', *Research Policy* **29**(4-5), 497–529.
URL: *https://doi.org/10.1016%2Fs0048-7333%2899%2900087-6*

Dechezleprêtre, A., Einiö, E., Martin, R., Nguyen, K.-T. and Van Reenen, J. (2016), Do tax incentives for research increase firm innovation? an RD design for R&D, Technical report, National Bureau of Economic Research.

Department for Business, Innovation and Skills, Northern Ireland. Department of Enterprise, Trade and Investment and Office for National Statistics (2022), *UK Innovation Survey, 1994-2020: Secure access*, 8th edn, UK Data Service. SN 6699.
URL: *http://doi.org/10.5255/UKDA-SN-6699-8*

Duguet, E. (2004), 'Are r&d subsidies a substitute or a complement to privately funded R&D?', *Revue d'économie politique* **Vol. 114**(2), 245–274.
URL: *https://doi.org/10.3917%2Fredp.142.0245*

Duguet, E. (2012), 'The effect of the incremental R&D tax credit on the private funding of R&D an econometric evaluation on french firm level data', *Revue d'économie politique* **Vol. 122**(3), 405–435.
URL: *https://doi.org/10.3917%2Fredp.223.0405*

Dumont, M. (2013), 'The impact of subsidies and fiscal incentives on corporate R&D expenditures in belgium (2001-2009)', *Reflets et perspectives de la vie économique* **Tome LII**(1), 69–91.
URL: *https://doi.org/10.3917%2Frpve.521.0069*

Economides, N. (1996), 'Network externalities, complementarities, and invitations to enter', *European Journal of Political Economy* **12**(2), 211–233.

Ellsberg, D. (1961), 'Risk, ambiguity, and the savage axioms', *The Quarterly Journal of Economics* **75**(4), 643–669.
URL: *https://doi.org/10.2307/1884324*

European Commission (2018), 'Fair Taxation of the Digital Economy'.
URL: *https://taxation-customs.ec.europa.eu/fair-taxation-digital-economy_en*

Fowkes, R. K., Sousa, J. and Duncan, N. (2015), 'Evaluation of research and development tax credit', *HM Revenue and Customs HMRC Working Paper* **17**.

Goldberg, I., Berg-Andersson, B., Rouvinen, P., Jensen, C., Kox, H., Straathof, B., Paterson, I., Kaseva, H., Lindholm, P., Schönpflug, K., Valkonen, T., Gaillard-Ladinska, E., Sobolewski, M., Sellner, R. and Mocking, R. (2015), *A study on R&D tax incentives : final report*, Publications Office.

González, X. and Pazó, C. (2008), 'Do public subsidies stimulate private R&D spending?', *Research Policy* **37**(3), 371–389.
URL: *https://doi.org/10.1016%2Fj.respol.2007.10.009*

Gopalakrishnan, S. and Damanpour, F. (1994), 'Patterns of generation and adoption of innovation in organizations: Contingency models of innovation attributes', *Journal of Engineering and Technology Management* **11**(2), 95–116.
URL: *https://doi.org/10.1016%2F0923-4748%2894%2990001-9*

Görg, H. and Strobl, E. (2007), 'The effect of R&D subsidies on private R&D', *Economica* **74**(294), 215–234.
URL: *https://doi.org/10.1111%2Fj.1468-0335.2006.00547.x*

Griliches, Z. (1979), 'Issues in assessing the contribution of research and development to productivity growth', *The bell journal of economics* pp. 92–116.

Grilo, I., Shy, O. and Thisse, J.-F. (2001), 'Price competition when consumer behavior is characterized by conformity or vanity', *Journal of Public Economics* **80**(3), 385–408.
URL: *https://doi.org/10.1016%2Fs0047-2727%2800%2900115-8*

Guceri, I. and Liu, L. (2019), 'Effectiveness of fiscal incentives for R&D: Quasi-experimental evidence', *American Economic Journal: Economic Policy* **11**(1), 266–291.
URL: *https://doi.org/10.1257%2Fpol.20170403*

Haaland, J. I. and Kind, H. J. (2008), 'R&D policies, trade and process innovation', *Journal of International Economics* **74**(1), 170–187.
URL: *https://doi.org/10.1016%2Fj.jinteco.2007.04.001*

Haaland, J. and Kind, H. J. (2006), 'Cooperative and non-cooperative R&D policy in an economic union', *Review of World Economics (Weltwirtschaftliches Archiv)* **142**(4), 720–745.
URL: *https://ideas.repec.org/a/spr/weltar/v142y2006i4p720-745.html*

Hægeland, T. and Møen, J. (2007), 'The relationship between the norwegian R&D tax credit scheme and other innovation policy instruments', *Statistics Norway, Oslo-Kongsvinger* .

Hall, B. (2002), The financing of research and development, Technical report.
URL: *https://doi.org/10.3386%2Fw8773*

Hall, B. H. and Lerner, J. (2010), The financing of R&D and innovation, *in* 'Handbook of The Economics of Innovation, Vol. 1', Elsevier, pp. 609–639.
URL: *https://doi.org/10.1016%2Fs0169-7218%2810%2901014-2*

Hall, B. H., Mairesse, J. and Mohnen, P. (2010), Measuring the returns to R&D, *in* 'Handbook of the Economics of Innovation, Volume 2', Elsevier, pp. 1033–1082.
URL: *https://doi.org/10.1016%2Fs0169-7218%2810%2902008-3*

Hall, B. and Reenen, J. V. (2000), 'How effective are fiscal incentives for R&D? a review of the evidence', *Research Policy* **29**(4-5), 449–469.
URL: *https://doi.org/10.1016%2Fs0048-7333%2899%2900085-2*

Hardy, M. A. (1993), *Regression with dummy variables*, Vol. 93, Sage.

Harris, R., Li, Q. C. and Trainor, M. (2009), 'Is a higher rate of R&D tax credit a panacea for low levels of R&D in disadvantaged regions?', *Research Policy* **38**(1), 192–205.
URL: *https://doi.org/10.1016%2Fj.respol.2008.10.016*

Heckman, J. J. (1976), The common structure of statistical models of truncation, sample selection and limited dependent variables and a simple estimator for such models, *in* 'Annals of economic and social measurement, volume 5, number 4', NBER, pp. 475–492.

Heckman, J. J. (1979), 'Sample selection bias as a specification error', *Econometrica: Journal of the econometric society* pp. 153–161.

Heijs, J. and Herrera Danny, L. (2004), 'The distribution of R&D subsidies and its effect on the final outcome of innovation policy'.

Herrera, L. and Ibarra, E. R. B. (2010), 'Distribution and effect of R&D subsidies: A comparative analysis according to firm size', *Intangible Capital* **6**(2).
URL: *https://doi.org/10.3926%2Fic.2010.v6n2.p272-299*

HM Treasury (2021), 'Build back better: our plan for growth'.
URL: *https://www.gov.uk/government/publications/build-back-better-our-plan-for-growth*

HMRC (2022), 'Hmrc annual report and accounts 2021 to 2022 - gov.uk'.
URL: *https://www.gov.uk/government/publications/hmrc-annual-report-and-accounts-2021-to-2022/hmrc-annual-report-and-accounts-2021-to-2022–2*

Hoefele, A. (2016), 'Endogenous product differentiation and international R&D policy', *International Review of Economics & Finance* **41**, 335–346.
URL: *https://doi.org/10.1016%2Fj.iref.2015.07.005*

Hoffart, B. (2007), 'Permanent establishment in the digital age: Improving and stimulating debate through and access to markets proxy approach', *Nw. J. Tech. & Intell. Prop.* **6**, 106.

Hottenrott, H., Lopes-Bento, C. and Veugelers, R. (2017), 'Direct and cross scheme effects in a research and development subsidy program', *Research Policy* **46**(6), 1118–1132.
URL: *https://doi.org/10.1016%2Fj.respol.2017.04.004*

House of Commons (2021), 'Post-pandemic economic growth: Industrial policy in the uk'.
URL: *https://committees.parliament.uk/publications/6452/documents/70401/default/*

Huang, C.-H. (2014), 'Tax credits and total factor productivity: firm-level evidence from taiwan', *The Journal of Technology Transfer* **40**(6), 932–947.
URL: *https://doi.org/10.1007%2Fs10961-014-9358-7*

Huber, M., Lechner, M. and Steinmayr, A. (2015), 'Radius matching on the propensity score with bias adjustment: tuning parameters and finite sample

behaviour', *Empirical Economics* **49**(1), 1–31.
URL: *https://ideas.repec.org/a/spr/empeco/v49y2015i1p1-31.html*

Huber, M., Lechner, M. and Wunsch, C. (2013), 'The performance of estimators based on the propensity score', *Journal of Econometrics* **175**(1), 1–21.

Huergo, E. and Moreno, L. (2017), 'Subsidies or loans? evaluating the impact of R&D support programmes', *Research Policy* **46**(7), 1198–1214.
URL: *https://doi.org/10.1016%2Fj.respol.2017.05.006*

Hussinger, K. (2008), 'R&D and subsidies at the firm level: an application of parametric and semiparametric two-step selection models', *Journal of Applied Econometrics* **23**(6), 729–747.
URL: *https://doi.org/10.1002%2Fjae.1016*

Ishii, Y. (2013), 'Quality-price competition and product R&D investment policies in developing and developed countries', *Economic Record* **90**(289), 197–206.
URL: *https://doi.org/10.1111%2F1475-4932.12076*

Jalonen, H. (2011), 'The uncertainty of innovation: a systematic review of the literature', *Journal of Management Research* **4**(1).
URL: *https://doi.org/10.5296%2Fjmr.v4i1.1039*

Jann, B. (2017), 'KMATCH: Stata module module for multivariate-distance and propensity-score matching, including entropy balancing, inverse probability weighting, (coarsened) exact matching, and regression adjustment', Statistical Software Components, Boston College Department of Economics.
URL: *https://ideas.repec.org/c/boc/bocode/s458346.html*

Kaiser, U. (2006), 'Private R&D and public R&D subsidies: Microeconometric evidence for denmark', *Nationaløkonomisk tidsskrift* **2006**(1), 1–17.
URL: *https://ideas.repec.org/a/hhs/jdaecn/0138.html*

Karhunen, H. and Huovari, J. (2015), 'R&D subsidies and productivity in SMEs', *Small Business Economics* **45**(4), 805–823.
URL: *https://doi.org/10.1007%2Fs11187-015-9658-9*

Kasahara, H., Shimotsu, K. and Suzuki, M. (2014), 'Does an R&D tax credit affect R&D expenditure? the japanese R&D tax credit reform in 2003', *Journal of the Japanese and International Economies* **31**, 72–97.
URL: *https://doi.org/10.1016%2Fj.jjie.2013.10.005*

Klette, T. J. (1996), 'R&d, scope economies, and plant performance', *The RAND Journal of Economics* pp. 502–522.

Klette, T. J. and Møen, J. (2012), 'R&D investment responses to R&D subsidies: a theoretical analysis and a microeconometric study', *World Review of Science, Technology and Sustainable Development* **9**(2/3/4), 169.
URL: *https://doi.org/10.1504%2Fwrstsd.2012.047687*

Kobayashi, Y. (2013), 'Effect of R&D tax credits for SMEs in japan: a microeconometric analysis focused on liquidity constraints', *Small Business Economics* **42**(2), 311–327.
URL: *https://doi.org/10.1007%2Fs11187-013-9477-9*

Koga, T. (2005), 'R&D subsidy and self-financed R&D: The case of japanese high-technology start-ups', *Small Business Economics* **24**(1), 53–62.
URL: *https://doi.org/10.1007%2Fs11187-005-3096-z*

Lach, S. (2002), 'Do R&D subsidies stimulate or displace private R&D? evidence from israel', *Journal of Industrial Economics* **50**(4), 369–390.
URL: *https://doi.org/10.1111%2F1467-6451.00182*

Lazzarini, S. G. (2013), 'Strategizing by the government: Can industrial policy create firm-level competitive advantage?', *Strategic Management Journal* **36**(1), 97–112.
URL: *https://doi.org/10.1002%2Fsmj.2204*

Leahy, D. and Neary, J. P. (2001), 'Robust rules for industrial policy open economies', *The Journal of International Trade & Economic Development* **10**(4), 393–409.
URL: *https://doi.org/10.1080%2F09638190110073778*

Lee, C.-Y. (2011), 'The differential effects of public R&D support on firm R&D: Theory and evidence from multi-country data', *Technovation* **31**(5-6), 256–269.
URL: *https://doi.org/10.1016%2Fj.technovation.2011.01.006*

Leibenstein, H. (1950), 'Bandwagon, snob, and veblen effects in the theory of consumers' demand', *The Quarterly Journal of Economics* **64**(2), 183.
URL: *https://doi.org/10.2307%2F1882692*

Lin, P. and Saggi, K. (2002), 'Product differentiation, process R&D, and the nature of market competition', *European Economic Review* **46**(1), 201–211.
URL: *https://doi.org/10.1016%2Fs0014-2921%2800%2900090-8*

Long, N. V., Raff, H. and Stähler, F. (2011), 'Innovation and trade with heterogeneous firms', *Journal of International Economics* **84**(2), 149–159.
URL: *https://doi.org/10.1016%2Fj.jinteco.2011.03.008*

Love, J. H. and Roper, S. (2015), 'SME innovation, exporting and growth: A review of existing evidence', *International Small Business Journal: Researching Entrepreneurship* **33**(1), 28–48.
URL: *https://doi.org/10.1177%2F0266242614550190*

Lucas, R. E. (1988), 'On the mechanics of economic development', *Journal of Monetary Economics* **22**(1), 3–42.
URL: *https://doi.org/10.1016%2F0304-3932%2888%2990168-7*

Mazzucato, M. (2018), 'Mission-oriented innovation policies: challenges and opportunities', *Industrial and Corporate Change* **27**(5), 803–815.
URL: *https://doi.org/10.1093%2Ficc%2Fdty034*

Mohnen, P. (2018), Effectiveness of direct and indirect R&D support, *in* 'Innovation Systems, Policy and Management', Cambridge University Press, pp. 53–78.
URL: *https://doi.org/10.1017%2F9781108529525.004*

Mulkay, B. and Mairesse, J. (2013), The R&D tax credit in france: Assessment and ex-ante evaluation of the 2008 reform, Technical report.
URL: *https://doi.org/10.3386%2Fw19073*

Naskar, M. and Pal, R. (2020), 'Network externalities and process R&D: A cournot–bertrand comparison', *Mathematical Social Sciences* **103**, 51–58.
URL: *https://doi.org/10.1016/j.mathsocsci.2019.11.006*

Nelson, R. R. (1959), 'The simple economics of basic scientific research', *Journal of Political Economy* **67**(3), 297–306.
URL: *https://doi.org/10.1086%2F258177*

OECD (2014*a*), *Addressing the Tax Challenges of the Digital Economy*, OECD.
URL: *https://doi.org/10.1787%2F9789264218789-en*

OECD (2014*b*), 'Update to the oecd model tax convention - oecd'.
URL: *https://www.oecd.org/tax/2014-update-model-tax-convention.htm*

OECD (2021), 'Statement on a two-pillar solution to address the tax challenges arising from the digitalisation of the economy'.
URL: *https://www.oecd.org/tax/beps/statement-on-a-two-pillar-solution-to-address-the-tax-challenges-arising-from-the-digitalisation-of-the-economy-october-2021.htm*

Office for National Statistics (2023), *Business Structure Database, 1997-2022: Secure access*, 15th edn, UK Data Service. SN 6697.
URL: *http://doi.org/10.5255/UKDA-SN-6697-15*

ONS (2018), 'Examining regional gross value added growth in the uk: 1998 to 2016 - office for national statistics'.
URL: *https://www.ons.gov.uk/economy/nationalaccounts/uksectoraccounts/compendium/economicr*

ONS (2022), 'Comparison of ons business enterprise research and development statistics with hmrc research and development tax credit statistics - office for national statistics'.

URL: *https://www.ons.gov.uk/economy/governmentpublicsectorandtaxes/researchanddevelopmentex 09-29*

Pellegrino, G. and Savona, M. (2017), 'No money, no honey? financial versus knowledge and demand constraints on innovation', *Research Policy* **46**(2), 510–521.

URL: *https://doi.org/10.1016/j.respol.2017.01.001*

Petrin, T. (2018), *A literature review on the impact and effectiveness of government support for R&D and innovation*, Vol. 5, ISIGrowth Pisa, Italy.

Pires, A. J. G. (2015), 'Competitiveness-shifting effects and the prisoner's dilemma in international R&D subsidy wars', *International Economics* **142**, 32–49.

URL: *https://doi.org/10.1016%2Fj.inteco.2015.01.002*

Radicic, D. and Pugh, G. (2015), Input and output additionality of R&D programmes in european smes, *in* 'European Conference on Innovation and Entrepreneurship', Academic Conferences International Limited, p. 567.

Rao, N. (2016), 'Do tax credits stimulate R&D spending? the effect of the R&D tax credit in its first decade', *Journal of Public Economics* **140**, 1–12.

Ratinger, T., Čadil, V. and Agyemang, S. A. (2020), 'Are there any economic impacts of business R&D support? the case of the czech republic', *Central European Business Review* **9**(5), 45–62.

URL: *https://doi.org/10.18267%2Fj.cebr.251*

Reikowski, J., Alecke, B., Mitze, T. and Untiedt, G. (2010), 'Effectiveness of public R&D subsidies in east germany – is it a matter of firm size?', *SSRN Electronic Journal* .

URL: *https://doi.org/10.2139%2Fssrn.1707992*

Romer, P. M. (1986), 'Increasing returns and long-run growth', *Journal of Political Economy* **94**(5), 1002–1037.
URL: *https://doi.org/10.1086%2F261420*

Romer, P. M. (1990), 'Endogenous technological change', *Journal of Political Economy* **98**(5, Part 2), S71–S102.
URL: *https://doi.org/10.1086%2F261725*

Rosen, J. B. (1965), 'Existence and uniqueness of equilibrium points for concave n-person games', *Econometrica: Journal of the Econometric Society* pp. 520–534.
URL: *https://doi.org/10.2307%2F1911749*

Rubin, D. B. (1977), 'Assignment to treatment group on the basis of a covariate', *Journal of educational Statistics* **2**(1), 1–26.

Rubin, D. B. (2001), 'Using propensity scores to help design observational studies: Application to the tobacco litigation', *Health Services and Outcomes Research Methodology* **2**(3/4), 169–188.
URL: *https://doi.org/10.1023%2Fa%3A1020363010465*

Santoleri, P., Mina, A., Minin, A. D. and Martelli, I. (2022), 'The causal effects of R&D grants: Evidence from a regression discontinuity', *The Review of Economics and Statistics* pp. 1–42.
URL: *https://doi.org/10.1162%2Frest_a_01233*

Scherer, F. M. and Ross, D. R. (1990), *Industrial Market Structure and Economic Performance*, Rand McNally.

Siebert, R. B. (2019), 'The differential impact of subsidies on product improvements', *Journal of Institutional and Theoretical Economics* **175**(4), 583.
URL: *https://doi.org/10.1628%2Fjite-2019-0038*

Silva, A. M., Silva, S. T. and Carneiro, A. (2017), 'Determinants of grant decisions in R&D subsidy programmes: Evidence from firms and S&T organisations in

portugal', *Science and Public Policy* **44**(5), 683–697.
URL: *https://doi.org/10.1093%2Fscipol%2Fscx002*

Singh, N. and Vives, X. (1984), 'Price and quantity competition in a differentiated duopoly', *The Rand journal of economics* pp. 546–554.

Stiglitz, J. E. (1999), Knowledge as a global public good, *in* 'Global Public Goods', Oxford University PressNew York, pp. 308–325.
URL: *https://doi.org/10.1093%2F0195130529.003.0015*

Symeonidis, G. (2003), 'Comparing cournot and bertrand equilibria in a differentiated duopoly with product R&D', *International Journal of Industrial Organization* **21**(1), 39–55.
URL: *https://doi.org/10.1016/s0167-7187(02)00052-8*

Ugur, M., Churchill, S. A. and Luong, H. M. (2020), 'What do we know about R&D spillovers and productivity? meta-analysis evidence on heterogeneity and statistical power', *Research Policy* **49**(1), 103866.
URL: *https://doi.org/10.1016%2Fj.respol.2019.103866*

Ugur, M., Trushin, E. and Solomon, E. (2015), 'UK and EU subsidies and private R&D investment: Is there input additionality?', *SSRN Electronic Journal* .
URL: *https://doi.org/10.2139%2Fssrn.2693944*

Wu, Y. (2008), 'State R&D tax credits and high-technology establishments', *Economic Development Quarterly* **22**(2), 136–148.
URL: *https://doi.org/10.1177%2F0891242408316728*

Yang, C.-H., Huang, C.-H. and Chang, W.-H. (2020), 'Does reduction in the tax credit rate retrad R&D activity? evidence from taiwan R&D tax credit reform in 2010', *Contemporary Economic Policy* **39**(2), 398–415.
URL: *https://doi.org/10.1111%2Fcoep.12506*

Yang, C.-H., Huang, C.-H. and Hou, T. C.-T. (2012), 'Tax incentives and R&D activity: Firm-level evidence from taiwan', *Research Policy* **41**(9), 1578–1588.
URL: *https://doi.org/10.1016%2Fj.respol.2012.04.006*

Yang, I. (2018), 'Product R&D and international trade under bertrand competition', *Korea International Trade Research Institute* **14**(2), 1–20.
URL: *https://doi.org/10.16980%2Fjitc.14.2.201804.1*

Colophon

This thesis is based on a template developed by Matthew Townson and Andrew Reeves. It was typeset with LaTeX 2_ε. It was created using the *memoir* package, maintained by Lars Madsen, with the *madsen* chapter style. The font used is Latin Modern, derived from fonts designed by Donald E. Kunith.