Essays on Conventional and Unconventional Monetary Policies

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A Thesis submitted in fulfilment of the requirements for the degree of

Doctor of Philosophy

Department of Economics and Finance

Durham University

September, 2022

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Abstract

On one hand, this thesis contributes to the emerging literature on monetary policy for oil exporting countries by identifying new transmission channels and assessing the welfare and macroeconomic impacts of monetary targeting objectives. On another hand, it contributes to the new literature on heterogeneous agent models for policy analysis. Therefore, this thesis is split into two; the first part develops a DSGE model to assess oil price shocks and to determine welfare optimising policy responses for low- and middle-income net exporters of oil. While the second part, utilises heterogeneous agent and two-agent new Keynesian (HANK and TANK) models to assess the impact Large Scale Asset Purchases (LSAPs) have on the economy and inequality. The results show that firstly, output gap targeting is welfare improving under declining oil prices, but inflation targeting is welfare improving under increasing oil prices. Secondly, foreign bias in the home consumption basket induces different transmission channels for net exporters of oil. Thirdly, oil price fluctuation transmits differently to the exchange rate and terms of trade, which plays a crucial role in the occurrence of the Dutch disease. Fourth, given financial frictions, the capital access channel dominates the propagation of shocks. While the borrowing constraints induce different input demand dynamics for oil firms which constricts the magnitude effects of negative oil price shocks but expands positive effects. Fifth, the results from HANK and TANK show a positive but short-lived impact of asset purchases on the economy. It reveals a different transmission channel to consumption – transfers and wages. While portfolio rebalancing occurs, aggregate consumption is driven by transfers and wages. This explains the modest impacts of LSAPs within the literature as transfers have smaller fiscal multipliers. Sixth, the transfers and wages transmission channel underpin the decrease in consumption inequality and growth in income inequality respectively, while labour supply decisions and portfolio reallocation underpin the increase in illiquid wealth inequality.

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Declaration

I, Henry Eshemokhai Aviomoh, declare that the material contained in this thesis is my own work. Where information has been derived from other sources, I confirm that this has been indicated in the thesis. The work in this thesis is based on research carried out at the Department of Economics and Finance, Durham University Business School, United Kingdom. No part of this thesis has been submitted elsewhere for any other degree or qualification.

Statement of Copy Right

Acknowledgements

I would like to express gratitude to my supervisors Tatiana Damjanovic and David Chivers for their guidance through out the writing of this Thesis. I would like to also thank Cesár Calderon, David McCausland, Keith Bender, Moussa Blimpo, Woubet Kassa, Tevy Chawwa, Oyinkansola Ojo-Aromokhudu and Lander Bosch for valuable feedback and comments provided. I would like to specially acknowledge my family most especially my parents Clement and Regina Aviomoh as well as Mr Phillip and Mrs. Ifeyinwa Chukwu, Mr. and Mrs Enyabine, and Dr. Adeyinka Okuwoga for their endless support throughout. I am immensely grateful to my siblings Odufa, Osikhena, Oshiobughie and most especially Aloaye.

I would like to acknowledge my friends especially Femi Adeniran, Bryan and Lisa Mache, Chukwuka and Chinenye Enyabine, Febechi (Banks) and Obianulika Chukwu, Chukwudebelu Okudo, Adaeze Egwagu, Amaka Ajaegbu, Ogechi Eze, Oselumese Tony-Eigbokhan, Fliss McDowall, Yasmin Ilkhani, Maleeka Rahma Metteden, Wuraola Fanimokun, Efunlola Hughes, Aliu and Maimounat Ibrahim, Kingsley Niteh, Simon Zubairu, Damilola Ojo, Nandom Fwangyil, Jude Dawute, Ernest Hodo, and Tolu.

Most importantly, I would like to extend gratitude to my wife Uwaye Tony-Eigbokhan for her endless love, courage and support.

Dedication

To

Victoria Aloaye Aviomoh

Chapter 1

Introductory Overview

In 2015 and 2020 two distinct oil supply shocks led to the catastrophic contraction of global oil prices, effectively changing its equilibrium level and growth path[1]. In addition, 73% of the top 30 net exporters of oil are low- and middle-income economies[2], which significantly suffered much of the negative macroeconomic implications from the decline in oil prices. The decline in oil prices has renewed academic interest in studying the effectiveness of monetary policies for insulating and limiting the effects of international oil price shocks through the innovative perspective of the oil exporter. Close attention must be paid to the overwhelming dominance of low- and middle-income countries to the group of oil exporting economies. With much of the literature on policy design in these economies still at its infancy, I develop two large scale Dynamic Stochastic General Equilibrium (DSGE) models for a net exporting oil country such that oil markets and non-oil sectors are jointly determined, with international oil prices endogenously determined. My models produce different transmission channels than established within the literature and most importantly prescribe better welfare-improving monetary policies for low- and middle-income net exporting oil countries. Additionally, I characterise the role commodity cycles play in driving credit cycles in these economies and how credit frictions tied to changes in international oil prices amplify and constrict the magnitude of oil price shock effects by changing the dynamics of input demand choices of oil producing firms. The major implication of this work is that it showcases how central banks in low- and middle-income net

[1]The first shock in 2015 occurred after the U.S. production of shale oil led to the U.S. being the largest oil producer, which subsequently led to a reaction from Saudi Arabia increasing oil production in an attempt to retain its position as the largest oil producer. The Second shock in 2020 also occurred in similar consequences as Russia increased oil production with Saudi Arabia retaliating. There was also a contraction in demand for oil following the COVID-19 pandemic, but I restrict attention to the supply side shock.

[2]Data is from the CIA World Factbook and from 2018

exporting oil countries can design targeting objectives of monetary policy to optimise welfare metrics. Furthermore, my results and models pave the way on how research should re-imagine the transmission channels for these types of economies. These form the main contributions of the proceeding two chapters of this thesis.

To this end, the second chapter of this thesis aims to assess the propagation of shocks from international economies to domestic economies by investigating the different transmission channels at play in low- and middle-income net exporting oil countries. Though within the literature the propagation of oil price shocks from international to domestic economies has been extensively studied, it has typically been studied from the perspectives of oil-importing countries, of which the U.S. dominates the literature[3]. Furthermore, in understanding the propagation channels for oil price shocks, one must first consider the important drivers of oil prices. Initially the literature was hinged on oil demand being the major driver of oil prices. This was anchored by two important studies: Killian (2012) and Killian and Murphy (2014). This meant that initial studies on monetary policies for oil price shocks were studied ignoring the supply side component of oil prices. However, considering measurement error with oil inventories data, Baumesiter and Hamilton (2019) show that oil prices are driven by oil supply shocks unlike previously established within the literature[4]. Demand or supply side-driven oil price shocks are important in characterising the changes in interest rates, as monetary policy should respond to the underlying reason oil prices changed. Additionally, international oil markets contemporaneously matter for not only determining oil prices but also for propagation channels. Depending on the size of the domestic countries non-oil export and import, trade will determine its ability in spreading the effects of the shock given intertemporal consumption smoothing behaviours. For example, an oil-importing country such as the United States will attempt to insulate itself from higher oil prices by increasing non-oil exports. However, oil exporting counties such as, Nigeria, Mexico or Angola, where oil makes up 87, 5.32, 95.2 percent of all merchandise exports[5] are unable to insulate national accounts from negative oil price shocks. This is because the negative effects of a decrease in oil prices are unable to be offset by any increases in non-oil exports.

[3]Until the recent discovery of shale oil, the U.S. was a dominant oil importer.

[4]There is still much debate on the true major driver for oil prices, as Killian and Zhou (2019) rehearse the point for oil demand being the major driver and Baumeister and Hamilton (2020) counters this. Here I follow oil supply shocks as the major driver for oil prices as oil supply shocks have underpinned the last two major innovations in oil prices.

[5]Data is from the World Bank's World development Indicators (WDI) for the year 2019.

The change in dynamics for monetary policy for oil-exporting countries was initially postulated by Ferrero and Seneca (2015) before Bergholt, Larsen and Seneca (2019) shed more light on the dynamics and showed the completely distinct propagation channel for oil-exporting countries. This sheds light on the different transmission mechanisms that could persist when assessing policies from the perspective of the oil-exporter. A new important transmission channel uncovered was from the international economy to domestic oil produces, which gives rise to possible channels that are hinged on how the domestic oil producers are connected to the domestic economy. In my framework where domestic oil producers produce alongside the non-oil sector, it allows for transmission of oil price shocks through wages paid in oil and non-oil sector as well as reallocation of workers between sectors. But most importantly the value added from oil producers becomes an important aspect to total GDP that creates a trade-off for monetary policy.

The main contribution of the second chapter is to characterise welfare improving monetary policies in oil-exporting countries in response to oil price shocks as well as establish the differences in transmission channels between oil-importing and oil-exporting countries. As oil companies' value added becomes an integral aspect of national accounts, it becomes welfare improving for monetary policy to stabilise the output gap over inflation when oil prices decline following oil supply surges. Attempts at stabilising inflation become ineffective as inflationary pressures are neither cost-push nor demand-pull, but rather from exchange rate pass through from abroad given depreciating exchange rates within the oil-exporting country. Understanding that the goal of monetary policy is ultimately to preserve monetary stability, central banks can support aggregate demand so long as it does not jeopardise its inflationary targets. With inflation expectations anchored, central banks can turn sight on to output stabilisation. But this comes with a caveat, as inflation targeting requires commitment in the form of credibility of the institution supporting the monetary policy. This need for credibility is larger in low- and middle-income countries which form the vast majority of net exporting oil economies, and with previous episodes of higher levels of inflation, inflation targeting becomes more difficult to implement in these economies. Furthermore, with sectoral production it allows for further prying into the *Dutch Disease* hypothesis, which constitutes another transmission channel for oil price shocks to oil-exporters. It follows from the exchange rate and terms of trade rebalancing channel with newer contributions including labour market channels. Under the hypothesis, oil price hikes that appreciate the exchange rate lead to a worsening in the terms of trade. This is not always case as the model showcases oil price hikes that create exchange rate appreciation but also an improvement in the terms of trade. The major policy implication is simply central bankers will have to trade-off stabilising domestic household welfare with prevalence of the Dutch Disease effects.

The literature[6] has emphasised conventional trade channels as the main transmission channel for propagation of oil price shocks. Additionally, inflationary pressures induced during oil price hikes are as a result of differences in real wages and marginal product of labour giving rise to higher marginal cost. These two channels are considerably different when considering oil-exporting countries. For instance, under Bergholt, Larsen and Seneca (2019), where the oil sector abstracts from labour input, its sectoral value added is through the sovereign wealth fund supply chain channels. Here the conventional trade channels that are particularly important for oil importers has less of a significant effect on the oil exporter.

An interesting occurrence when looking at consumption behaviour in the low- and middle-income net exporter is the absence of expenditure switching effects of exchange rate movements. Within open economies, consumption reallocates towards the cheaper consumption good, which is nested in the consumption preference structure of consumption goods within the home economy. In low- and middle-income net exporters the intermediate basket of goods is dominated by imported consumption goods (this is also at the heart of welfare maximising monetary policy target choice), so household preference is skewed significantly towards imported goods such that exchange rate movements are unable to reallocate consumption towards the cheaper good. These mutes the terms of trade transmission channel that drives the transmission mechanism in net importing countries. As a result, the trade balance does not respond to favourable terms of trades within the domestic economy and are simply driven by oil exports.

Secondly, in chapter 3, I extend the model to account for financial frictions within the domestic economy. The motivation behind this chapter is twofold. Firstly, the literature establishes the role financial frictions play in exacerbating the magnitude of effects of international shocks on domestic macroeconomic variables[7]. Secondly, there is increasing observation of the cor-

[6] Bodenstein, Guerreri and Killian (2012); Bodenstein and Guerreri (2011)
[7] See for example Gertler and Karadi (2011).

relation between the commodity cycles and the availability of credit within commodity driven economies. Dreschel and Tenreryo (2018) empirically provide evidence of the negative relationship between commodity prices and borrowing conditions in middle-income countries. Therefore, I account for this correlation by integrating a financial sector within the model developed in chapter 2 to account for the interaction between commodity cycles and financial constraints in low- and middle-income net exporters of oil. I utilize a simple modelling approach such that in equilibrium all financial intermediaries charge the same loan rate and further abstract from the risk of default by oil producing firms. Thus, this interaction between financial intermediaries and oil producing firms works via access to working capital of the domestic oil producing firms such that commodity booms create greater access to working capital via loosening of the borrowing constraint financial intermediaries enact. In addition to this transmission channel, the inclusion of financial intermediaries (banks) creates a second possible transmission channel – the financial accelerator channel. To outline this transmission channel, I further include an adjustment cost function for banks that imposes a minimum capital requirement such that they incur a cost for leverage ratio falling below the minimum capital requirement[8]. In addition to understanding the role financial frictions play in transmission of oil price shocks, I also identify the consequences for monetary policy.

I find that financial frictions induce smaller negative consequences of oil price declines but higher positive consequences for oil price hikes. This is embedded in the interaction between loan constraints and factor input demands. The interaction between banks and oil producing firms creates a relationship between oil producers input demands, such that optimality conditions for input demands is increasing in how firms respond to easing of the borrowing constraint as well as the tightness of borrowing constraint. This induces different transmission dynamics of factor input demands and thus output of domestic oil producing firms. As a consequence, during oil price declines oil producing firms demand for inputs does not contract instantly and immediately but gradually. This changes the transition dynamics for oil sector value added such that without an impact contraction in value added, the aggregate GDP contracts gradually and not on impact. During an oil price decline, credit frictions (financial frictions) constrict the oil sector effects on the domestic economy. Although, during oil price increases (from higher oil

[8]While I do not assess any macroprudential policies, I highlight the role macroprudential policies could play in creating monetary expansion via changes to the minimum capital requirements of banks in the model.

demand) financial frictions exacerbates the magnitude of shock effects in the same sequence of events through the oil sector. Furthermore, through my analysis, I find that the capital access channel plays a significant role in the transmission of international oil price shocks, with the financial accelerator creating little to no effects. The conclusions on the magnitude of the financial accelerators effects are based on the modelling approach I utilized in specification of the financial sector[9] as without loan defaults and availability of banks to fund loans through own equity, leverage ratio constraints do not have an extensive effect on the banks' balance sheet and as such the financial accelerator channel effects become diminished.

A common observation between both models (with and without financial frictions) is that nominal exchange rate targeting in central bank's Taylor Rule, which for lack of a better term acts as a *Dirty Target*[10] is unable to fully offset the nominal and real depreciations that accompany commodity busts, which ultimately create inflationary pressures. Within the data this is particularly evident from lasting periods of elevated levels of inflation in low and middle-income commodity economies. As such, I also discuss an alternative monetary policy approach to preserving exchange rates – exchange rate interventions. Here I provide thoughts on how (sterilised) exchange rate interventions can be jointly used with central bank targeting regimes to offset the depreciations of domestic currency induced by commodity price shocks.

Throughout both models presented in chapters 2 and 3, I restrict attention to monetary policy responses and abstract from fiscal policy responding to shocks arising within the model. Though I do understand and highlight the role fiscal policy could play in limiting exchange rate pass through from international economy to domestic economy, this presents opportunity for future research.

The final chapter of this thesis is dedicated a different research question – The role of Unconventional Monetary Policies. The last decade has also observed the prominence of unconventional monetary policies (Large Scale Asset Purchases usually referred to as Quantitative Easing) in the face of ineffective conventional monetary policies (interest rate changes). Following the financial crisis of 2008, nearly all advanced economies reached the zero-lower bound (ZLB)

[9] Brzoza-Brzezina, Kolasa and Makarski (2013) in an extensive analysis of the literature on financial frictions and financial accelerators in DSGE models, show that conclusions are model implied.

[10] Benes *et. al.* (2015)

on interest rates, with Japan implementing negative interest rates. In this region of ineffective interest rates, central banks turned to large scale asset purchases in an attempt to ease monetary policy stance and increase money supply within economies[11]. Similarly, in 2020 the COVID-19 pandemic induced both demand and supply side shocks and with interest rates hovering around the zero lower-bound central banks turned to LSAPs to stimulate economies. These two individual events created academic interests in understanding how LSAPs propagate and affect households and firms. I take this a bit further and assess how LSAPs affect wealth and consumption inequality. I present an alternative approach to assessing the impacts of LSAPs, by estimating and simulating an Asset Purchase Programme (APP) proxied through increases in government denoted bonds under a Heterogeneous Agent New Keynesian (HANK) model. By decomposing consumption, the results reveal that while the portfolio rebalancing channel holds, the main transmission channel for APPs to domestic consumption is through government direct transfers and wages. This transmission channel plays a huger role in decreasing consumption inequality while portfolio rebalancing channel drives wealth inequality. Ultimately, and in line with the literature I find that even under HANK, LSAP effects are still small and short lasting. However, unlike the literature I am able to further identify the reason behind the modest effects LSAPs have on aggregate economic activity. While the results on the assessment of LSAPs under HANK and the effect on inequality form the main contributions of the chapter, it also creates an additional question for future research, that is: how to maximise the effects of QE.

The literature documenting LSAPs is predominantly dominated by vector Autoregression (VAR) and Representative Agent New Keynesian (RANK – DSGE). Thus, the introduction of heterogeneity of households presents an innovative approach to understanding the true propagation of LSAPs. Furthermore, These two innovations present the two primary contributions of this chapter to the literature.

In the assessment of QE I also contrast the differences between Two-Agent New Keynesian Models (TANK) and HANK models. In the TANK case, I present a simple structure of constrained *Hand-to-Mouth* households that consume all of wage income together with transfers from the government, along with unconstrained households that have access to both equity

[11] This was also accompanied by changes in macroprudential policies to prevent commercial banks recreating the scenarios that led to the crisis.

capital holdings and bond holdings as in the HANK case. Both models are presented in a closed economy framework abstracting from international bond and currency markets. Under the HANK model I follow Kaplan, Moll and Violante (2018) and estimate the wage earnings process utilising male earnings data, before proceeding to simulate the model. However, since only the earnings data is estimated, both the RANK and TANK model are simulated. Similarly, the inclusion of differentiated assets in the form of liquid (bonds) and illiquid (stocks) allows for differentiated portfolio holdings by households, allowing choices of asset holdings across households to differ[12]. As not all households can invest in short-term government bonds under the HANK and TANK models, this bypasses Wallace (1981) irrelevance theorem for representative agent models where general equilibrium effects prevent open market operations from having out of equilibria effect on consumption and prices, as they are independent of the path of government portfolio[13].

The influential paper by Chen, Curdia and Ferrero (2012) on LSAPs[14] ascertained that the effects of LSAPs would have lower and moderate effects on macroeconomic variables. They reached this conclusion after utilising U.S data and Bayesian methods to estimate and simulate the Federal Reserve LSAP in a DSGE model. From their findings they show that LSAPs have a moderate effect on GDP, without inducing any inflationary pressures. Adding that effects are more likely to be uncertain and in comparison, to a 50bp cut to short-term interest rates. Previously noting that the theoretical criticisms are due to non-segmented asset markets where households view government short-term bonds as equivalent to reserves. They circumvent this by creating segmented asset markets, with investors (households) having different preferences for asset maturities. This form of asset market segmentation creates implications for long-term interest rates effects on aggregate demand, such that while short-term interest rates can be constrained by the zero lower-bound the LSAP will be effective by affecting current long-term rates. Although not wholly stated, this influential paper actually presents one of the first TANK mod-

[12]This holds only in the HANK and TANK models where heterogeneity across households exist with the HANK model appropriating richer heterogeneity. Under the RANK model, there is simply one representative household that chooses the different levels of asset holdings.

[13]Modigliani-Miller theorem is at the heart of this proof. See Wallace (1981).

[14]Other studies before these estimate the effects of LSAPs on long-term interest rates using VAR models. Hamilton and Wu (2010), D'Amico and King (2013) Neely (2010) all found the first round of Federal Reserve LSAP had positive impacts on long-term interest rates, increasing rates by 13 bp, 45bp and 107 bp respectively. Krishnamurthy and Vissing-Jorgensen (2011) estimate the second round of LSAPs to have a negative impact of 16bp on the U.S. ten-year yield.

els within the literature[15], of which I can ascribe as the working mechanism behind the reason LSAPs have effects on macroeconomic variables.

The main results from the empirical literature both from VAR and DSGE is that LSAPs can affect key macroeconomic variables and present a viable alternative when conventional monetary policy becomes effective. In essence unconventional monetary policies can be used to mitigate the economic costs of the ZLB by preventing liquidity traps. The main transmission channel corroborated within the literature is the portfolio rebalancing channel. Asset returns are particularly very important when deciding asset composition of portfolios. Hence any changes to asset returns will see reallocation of asset holding towards the asset with higher returns.

I identify a transmission channel not touted in the literature due to the deconstruction of consumption response following asset purchases by the government. I observe that LSAPs effect on consumption transmits through wages and transfers from the government. The intuition behind this originates from the consumption of *hand-to-mouth* individuals as well as portfolio choices. That is given a portfolio of liquid and illiquid assets such that consumption from liquid asset account is costless, with consumption from illiquid asset account incurring a cost, LSAPs change the allocation of portfolios and skews it towards illiquid asset holdings, which individuals typically do not consume from. Therefore, individual consumption choices are reallocated towards current wages, with *hand-to-mouth* (HTM) consumers dominating consumption through obtaining transfers from the government. This deconstruction of consumption follows through between both HANK and TANK.

LSAPs is shown to have a positive impact on other aggregate macroeconomic variables but like the literature the effects on aggregate macro variables is small and short lived under the HANK model, with a slightly longer effects under the TANK framework. This is particularly due to the transmission channel identified for consumption. That is, government transfers have been shown to have a smaller fiscal multiplier[16] than government consumption. Thus, with portfolio reallocation channel playing a backseat role and transfers and wages dominating the transmis-

[15] They present two distinct households, a restricted and an unrestricted household, while both households have access to the same assets the risk premium required by the restricted households changes the optimisation behaviour and decision that effectively transforms them to a different representative household, hence, the TANK structure.

[16] In the U.S. See Oh and Reis (2012).

sion channel the overall effect of asset purchases is a modest one as observed in my results and the literature. Similarly, I find that the TANK model does a good job at approximating HANK, such that the calibration of hand-to-mouth households in the TANK model plays a significant role in this approximation.

When considering inequality in macroeconomics, one of the most influential papers by Krusell and Smith (1998) assess how inequality affects the macroeconomy, but how do macroeconomic shocks and policy affect inequality within the macroeconomy? To this end, the second half of final chapter assesses the effects of LSAPs on Income, wealth and consumption inequality, utilising the HANK and TANK models. The introduction of heterogeneity allows me to understand another fundamental question in macroeconomics, that is – what the effects of economic policy on Wealth inequality are. Though the literature on wealth (and consumption) inequality has typically studied the reverse – the effects of inequality on policy. The heterogeneity within this model allows for a realistic interaction between the wealth and income inequality, and crucial macroeconomic variables. Understanding that though I observe aggregates, inequality in essence shapes aggregates, while monetary policies shapes inequalities. The share of wealth held by the 1% in the U.S declines following the onset of a recession but rebounds exponentially not necessarily following an economic expansion or LSAPs. This shows that changes in asset holdings could be a possible driver given differences in capital and income taxes. Hence, this innovation of heterogeneity allows for an assessment of policy effect on wealth inequality following LSAPs. Under HANK, heterogeneity in productivity means not all households are able to purchase short-term government bonds, and this creates differences in portfolio structure across households, hence, can create higher inequality in asset holdings and asset returns (capital income). Since household aggregate consumption depends on aggregate wealth, not only would the LSAP induce wealth inequality, but it would also induce consumption inequality.

The main finding for inequality is that LSAPs will induce more income and illiquid wealth inequality but decrease consumption and liquid wealth inequality. By utilising both HANK and TANK I can ascribe how this comes about. Firstly, from both models the dominant transmission channel is through wages and transfers but realising HTM receive the majority of transfers, which are consumed in current period induces the decrease in consumption inequality. The income inequality is driven by labour supply choices of both HTM and non-HTM households.

Given non-HTM (more productive under HANK specification) receive increase labour supply and HTM decrease labour supply (evident from TANK) the wages earned are skewed towards the non-HTM households generating more income inequality. Finally, in the dimension of illiquid wealth inequality, higher wages by non-HTM households allows for more purchase of capital, and the initial reallocation of portfolio choices away from liquid towards illiquid assets induces the increase in illiquid wealth inequality. Liquid wealth inequality decreases as increase in supply from LSAP allows distribution to lower income households in the HANK model, but non-HTM households reallocate away from bonds creating the decline in liquid wealth inequality.

These findings on the assessment LSAPs through HANK, the revelation of the dominance of transfers and wages for the transmission mechanism, and the implications these transmission channels have on inequality form the main contribution of my final chapter.

Chapter 2

Monetary Policy in Low- and Middle-Income Commodity Driven Economies

2.1 Introduction

Understanding the transmission of shocks in economies has been studied widely through the lenses of commodity importing countries. However, the literature surrounding monetary policy responses under the transmission of shocks in commodity driven economies is in its infancy[1]. The volatility of commodity prices creates challenges for stabilisation policies within commodity driven economies. Exchange rates and terms of trade movements are highly influenced by these volatile commodity prices, which are then transmitted to non-commodity sectors in the form of price adjustments and reallocation of labour and products along extensive margins. These forces drive business cycles in commodity driven economies. The crucial questions remain unchanged - what are the implications for optimal policy design given these challenges. The 2015 and subsequently 2020 crash in oil prices was a major concern and instigated renewed interest in these questions but from the perspective of the commodity exporting economy. Examining the literature on policy, one observes that researchers have abstracted from studying the role of commodity prices - for example the influential paper by Smets and Wouters (2003; 2007), Justiniano, Primceri and Tamblalotti (2011), Corsetti, Dedola and Leduc (2008). More

[1]Here oil commodity economies refer to commodity driven economies.

commonly, researchers study the role from the perspective of the commodity importing country – Bodenstein, Guerrieri and Killian (2012), Bodenstein, Erceg and Guerrieri (2011), Blanchard and Galí (2010), and Nakov and Pescatori (2010a,b)[2]. The research on policy from the perspective of commodity exporting country is minuscule, with papers dominated by Norges Bank (Norwegian Central Bank) researchers, including Bergholt, Larsen and Seneca (2019), Bergholt and Larsen (2016), Bergholt (2015). While Allegret and Benkhodja (2015), and Benkhodja (2014) present the only strand of literature encompassing nominal rigidities in wage and price setting when investigating monetary policy from a net-exporting middle income country[3].

In addition to this, the observation that low- and middle-income oil dependent economies depend heavily on intermediate goods imports in consumption and capital and are therefore subject to external shocks with little buffers in the way for absorbing these shocks. Secondly, these nations depend heavily on oil earnings, which are subject to volatile oil prices, and with oil earnings contributing heavily to domestic output creating larger business cycle volatility in contrast to other low- and middle-income countries. Therefore, a framework is needed that models these conditions in order to effectively assess how monetary authorities should conduct monetary policy in light of external shocks that also create fluctuations in oil prices. So, in this chapter I develop a large scale two-sector Dynamic Stochastic General Equilibrium (DSGE) Model for a commodity exporting country, such that both non-commodity activities as well as commodity activities are jointly determined. My motivation stems from the lack of such structural models for commodity exporters and the increased demand for such models following recent oil market changes. The main contribution of this chapter highlights the efficiency in policy design when reacting to oil price fluctuations within net-exporting oil countries. Particularly, it is aimed at low- and middle-income countries with highly underdeveloped non-oil sectors that rely heavily on imports, while relying heavily on oil income as a major driver of economic growth.

The core of the frame work is a Two-Country New Keynesian Model as in Corsetti, Dedola and Leduc (2008) [4], but with a more focused attention on the structure of commodity driven

[2]These papers focused on propagation of shocks in the U.S. economy, which until its boom in shale oil, was a major net commodity importer

[3]Bergholt, Larsen and Seneca (2019), Bergholt and Larsen (2016) study policy design from the Norwegian economy's perspective- a high income country. While Allegret and Benkhodja (2015), and Benkhodja (2014) look at Algeria, a middle-income country and only utilise exogenous oil price fluctuations.

[4]If required, the Two-Country Model can be adjusted to reflect a Small Open Economy framework by assuming the case in which the second country approximates a closed economy.

economies as in Bergholt, Larsen and Seneca (2019), which at its core is based on Smets and Wouters (2003; 2007). The model differs from the one developed by Bergholt *et. al.* (2019) in the following ways – Firstly, I model oil producing firms as part of the economy (not off-shore) such that wages and dividends are redistributed to the households. Secondly, I do not abstract from demand of oil goods by domestic households and firms so that the endogenously determined oil prices would be affected by these demands. This allows me to scale how much the home country and foreign block can affect the determination of international oil prices as in Bodenstein *et. al* (2012). Furthermore, I abstract from assuming international financial markets are complete, so, there are no cross-border trade in bonds, and I do not assume the presence of a sovereign wealth fund that governments can use to balance their budgets. Finally, a crucial underpinning within their model relies on the existence of this sovereign wealth fund that governments balance their budgets with. While the relevance of a wealth fund here seems intuitive, given the lower development by middle and low-income countries with lower levels of capital stock and a less than funded wealth fund, it would be redundant to include this within the framework. Similarly, low- and middle-income countries have far from complete capital markets and risk sharing with the international economy does not typically hold. A description of the full model is outlined in section 2.3.

Two different stabilisation policies are examined here: a strict inflation targeting and a strict output gap stabilisation. I choose to assess the policy rules for the following reasons: (i) under standard New Keynesian framework inflation targeting regimes are welfare improving over output gap targeting, (ii) Given domestic output now includes oil production technology, would an output gap targeting regime prove more effective than an inflation targeting regime contrary to standard NK framework; and (iii) what do strict targeting policies mean for the Dutch Disease. These policies are assessed by their ability to improve household welfare.

Though I understand the burden of adjustment of economies to volatile oil prices is not constrained to monetary policy. For example, commodity exporters have in the past insulated their economies from volatile commodity prices by utilising options contracts to hedge against short-term changes in commodity prices as Mexico has previously done. Similarly, commodity dependent economies can issue commodity linked bonds that insure against longer-term risks and establishes a counterpart in multinational corporations that require large commodities as input for production. Finally, another possible measure accrues to forecasting, by reducing optimism bias in commodity price prospects, commodity dependent economies are able to prevent fiscal procyclicality biases. While these measures exist, I restrict attention to monetary policy as a tool for adjusting to changes in oil prices.

My main findings follow below. Firstly, with the economy's dependence on oil sector activity the weight on the stabilisation of output increases in the importance of the oil sector. Therefore, on one hand during business cycle recessions induced by lower oil prices, an output gap targeting regime will perform best for household welfare. This contradicts the standard NK framework but follows in line with the works of Ferrero and Seneca (2015). The results follow from the reliance of the domestic economy on intermediate imported consumption goods such that Producer Price Inflation (PPI) is driven by import prices and thus the Exchange Rate pass through. Therefore, depreciation of the RER that occurs during oil price declines create inflationary pressures that domestic central banks cannot control, hence, attempts at stabilising higher inflation during from these external shocks will prove redundant. Additionally, with the inclusion of oil production technology in domestic output stabilising output will create faster recovery for GDP and higher domestic consumption. On the other hand, during oil revenue windfalls, inflation targeting regime will perform better for household welfare over strict output gap stabilisation. Secondly, in assessing the prevalence of the Dutch Disease, I obtain a trade-off between household welfare and the prevalence of the Dutch Disease. That is, policies that improve household welfare will exacerbate the effects of the Dutch Disease.

The rest of this chapter is structured as follows: Section 2.2 reviews the related literature, section 2.3 estimates a simple structural VAR for an example commodity driven middle income country and reports how they are affected by oil price shocks. Section 2.4 outlines the DSGE model developed for policy analysis. Proceeding, section 2.5 describes how the model is calibrated. In section 2.6 I consider three different pricing regimes for price setting behaviour and compare the difference in transmission channels through impulse response across them. Section 2.7 provides a counterfactual experiment to see how different monetary policy stabilising rules can impact business cycles in the home economy, and thus, their comparing welfare metrics. Section 2.8 concludes.

2.2 Relevant Literature

A key paper within the literature by Bodenstein, Guerreri and Killian (2012) argues that there is no general answer as to how monetary policy should react to oil price fluctuations. Rather the optimal policy to the oil price shock should depend on why the price of oil has changed. Their insight followed from Killian (2009) who first argued that the price of oil is a symptom rather than the cause. Therefore, policy makers should not respond directly to the price of oil, rather central banks must identify the intrinsic supply and demand shocks as well as macroeconomic variables driving the price of oil. This point is further argued by Nakov and Pescatori (2010) who demonstrate using an endogenous oil price DSGE model that it is optimal for a welfare maximising central banker to not respond directly to oil price shocks. Bodenstein *et. al.*(2012) develop a DSGE model with a heavy demand side inclination and show that the sign, pattern and magnitude of the monetary policy response will differ depending on the origin of the oil price fluctuation. A striking result obtained regarding monetary policies across countries mirrors Corsetti and Pesenti (2008), i.e. there are little gains to cooperation, however, larger than those obtained in Corsetti and Pesenti (2008) whose model abstracts from oil trade and also under dollar pricing[5]. However, Blanchard and Galí (2010) argue that the source of the oil price shock does not matter for the policy response, as long as the shock is exogenous (originates abroad). What matters they say is the price at which households and firms can purchase the oil.

While these aspect of the literature gives insight into how optimal monetary policy should respond, they are done purely from a net-importer perspective[6]. The most relevant paper by Bergholt (2014) attempts to characterise a Ramsey optimal policy rule in a net exporter of oil environment. One major improvement in his model is the role for fiscal regimes. Here the supply chain is modelled such that firms in the oil sector buy inputs from non-oil sectoral firms giving rise to spillover effects from oil to non-oil, which limits the scope for fiscal insulation policies[7]. He finds that the Ramsey optimal response to oil price fluctuation is an aggressive

[5]Corsetti and Pesenti (2008) considered what the optimal monetary policy and international policy cooperation response to exogenous productivity shocks when firms use Producer Currency Pricing (PCP), Dollar Pricing and Local Currency Pricing (LCP). They find that there are no welfare gains to cooperation under PCP and LCP, but little gains to cooperation under Dollar pricing.

[6]Bodenstein *et. al.*(2012), Bodenstein *et. al.*(2008), Nakov and Pescatori (2009), Plante (2009; 2014), Leduc and Sill (2004), Killian and Lewis (2011), Killian (2009), Blanchard and Galí (2010), and Bernarke *et. al.*(1997) all carried out research on the U.S. economy, which until recently was a net importer of oil.

[7]Norway practises the "Bird-in-Hand" rule, where funds from oil revenue are saved in a sovereign wealth fund, which are then used to balance the government budget deficit each period (van der Ploeg and Venables, 2011).

increase (decrease) in nominal interest rates when the marginal rate of substitution is above (below) the real wage. While the optimal monetary response under a Taylor rule assigns a high weight to nominal wage stability due to rigidities in wage setting. Under productivity shocks, the optimal policy rule (following a Taylor rule) targets CPI inflation stabilization by dropping nominal interest rates. Similarly, the optimal Ramsey policy rule (approximated by a Taylor rule with high interest rate inertia) under productivity shocks stabilises both CPI and PPI inflation, mirroring the optimal monetary policy. Additionally, and somewhat contradicting to the works of Cataõ and Chang (2013), he argues that from a welfare perspective, policies aimed at stabilising wages in the oil sector can have disastrous consequences on welfare.

Another interesting result that I also obtain complimentary to Ferrero and Seneca (2015). They show that with domestic output in oil driven economies depending on oil revenues, monetary policy should respond to output gap deviations ahead of inflation stabilisation. CPI inflation is driven primarily by exchange rate depreciations, hence, attempts at stabilising inflations during oil price declines will prove ineffective, as inflation is now being driven by import prices. Similarly, the weight on output gap stabilisation in Taylor rule dynamics would hinge on the dependence of domestic output on oil sector value added. However, they simply assume oil prices are exogenous and as such unable to truly prescribe policy given their model cannot account for why oil prices have changed. Likewise, Allegret and Benkhodja (2015) estimate a DSGE model for the Algerian economy and find that a strict inflation targeting regime is not optimal for an oil-exporting economy as it creates large volatility in macroeconomic variables. Here they find that a monetary rule targeting non-oil sector inflation together with headline inflation provides the best response for macro variables and household welfare. Although they assess the effectiveness, of other policies – including a fixed exchange rate and strict exchange rate stabilisation, they do not consider output gap in monetary authority response or a strict output gap rule.

The most relevant paper that closely relates to my research here is the work of Bergholt *et. al.* (2019) who develop a two-country open economy New Keynesian DSGE model to observe the effects on oil prices shocks on a net-exporter commodity driven economy (Norway). What is particular about their model is the presence of an oil supply chain network that permits the transmission of oil shocks from offshore oil companies to the mainland economy. Their model allows for oil revenues to be accumulated by offshore oil companies (and government in a sovereign fund) that rent capital from supply chain firms on the mainland economy. Therefore, factor input demand by supply chain firms drive the transmission mechanism for oil price movements from offshore oil companies to the domestic economy. They find that the oil supply chain is more important for transmission channel mechanism than conventional channels from Bodenstein *et. al.* (2011) and Bodenstein, Guerreri and Killian (2012). Although they do not concern themselves with the optimal monetary response to oil price shocks, they shed light on if the source of the oil price fluctuations matters as well as the effect of oil price fluctuations for business cycles in Norway. They also find that in line with Bodenstein *et. al.*(2012), oil price fluctuations with differing origins propagate differently, as a given oil price increase from non-oil shocks propagates far more than those originating from oil-specific fluctuations. Secondly, the presence of Norway's fiscal regime of a Bird-In-Hand rule for its oil revenue seems to provide an optimal fiscal response to oil price shocks. Currently Norway accumulates wealth in a sovereign wealth fund of which 4% every year is used to balance its government budget deficit. They find that this fiscal regime provides substantial protection against external shocks, allowing the Norwegian economy to smooth consumption over time[8].

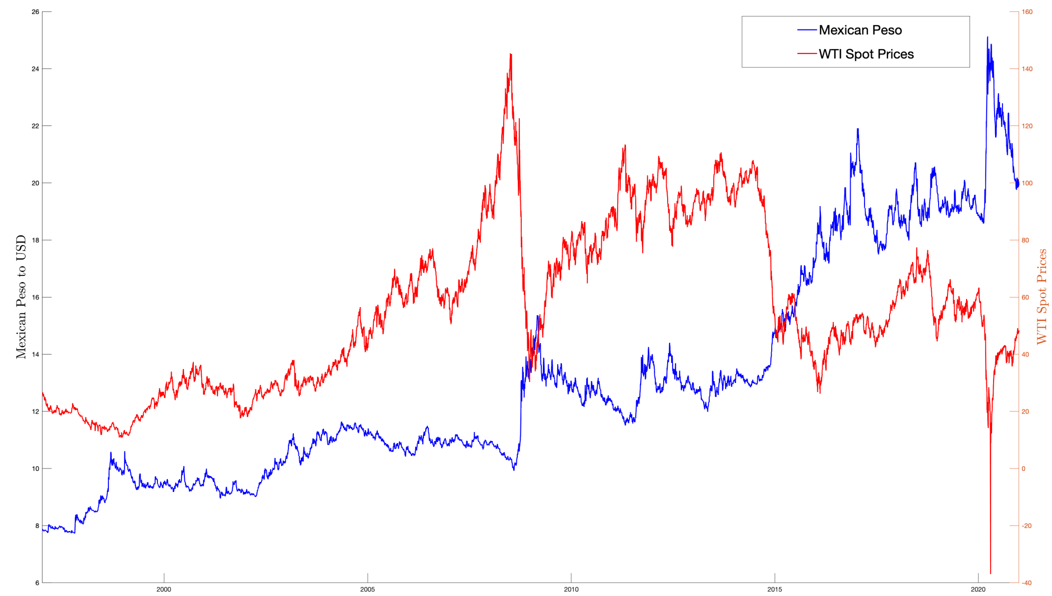
2.3 Oil Prices and Business Cycles In Oil Exporting Economies

To motivate the model presented here and an aspect of its contribution to the literature, consider Figure 2.1 where two key variables of interest for two middle income countries - Mexico and Nigeria respectively are plotted. Between 2011 and July 2020 oil prices have declined twice - firstly, by more than 50% in February of 2016 and again in February of 2020 respectively. Therefore, in order to interpret these fluctuations within oil prices, I need to account for oil activity within the DSGE model. Immediately observable is the how large exchange rate depreciations within the currencies of both countries proceed large declines in oil prices but importantly, the jumps in inflation. In any central bank regime that targets inflation, monetary authorities in these two economies should therefore raise interest rates due to higher inflationary pressures derived from the weakening of domestic currency as seen in Figure 2.2 and Figure 2.3. Ferrero and Seneca (2015) identify that this policy needs to be called to question, as falling oil prices creates lesser activity in oil sector (lower value added from oil sector to overall output), and with oil firms reacting by decreasing labour and capital input demand supplied by households,

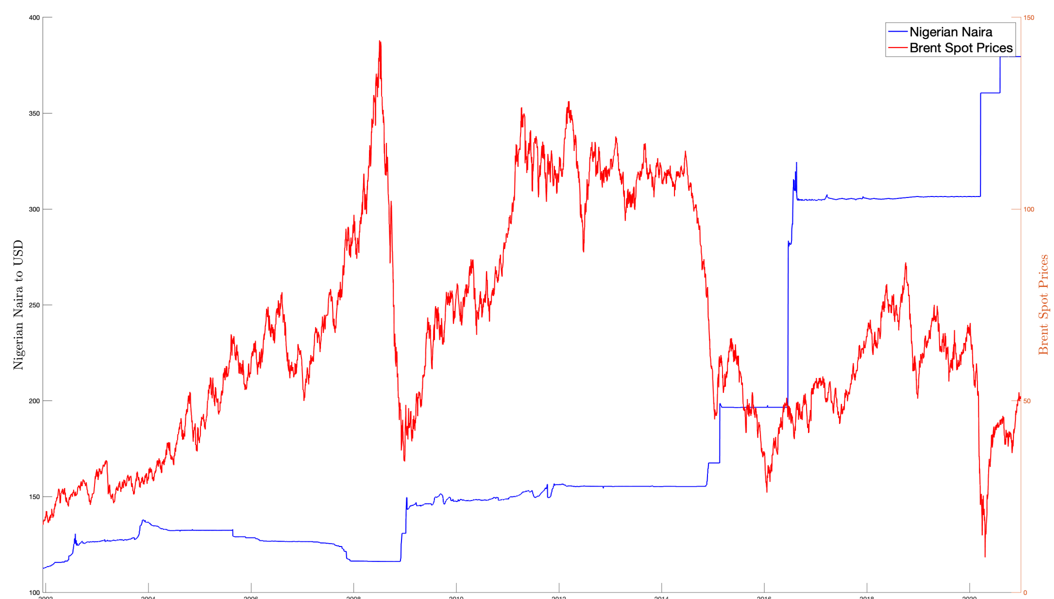
[8] A different strand of literature looks at the relationship between government spending and oil revenue, which generates procyclical fiscal policy. See e.g. El Anshasy and Bradley (2012).

this propagates further to the entire economy. Therefore, central bankers would face a trade-off between stabilising inflation and domestic non-oil activity.

Figure 2.1: Exchange Rate and Oil prices in Nigeria and Mexico



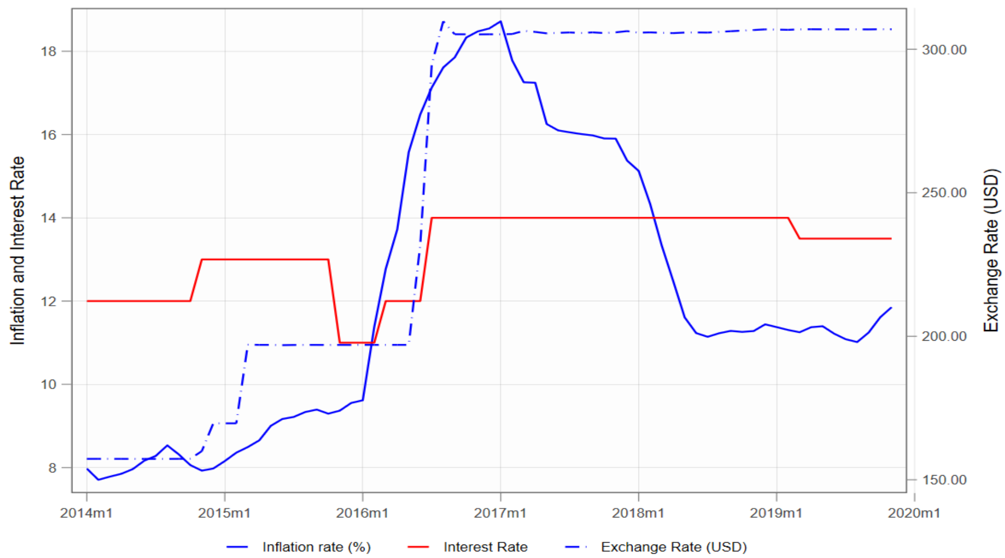
(a) Movement in Mexico’s Exchange Rate and Oil Prices



(b) Movement in Nigeria’s Exchagne Rate and Oil Prices

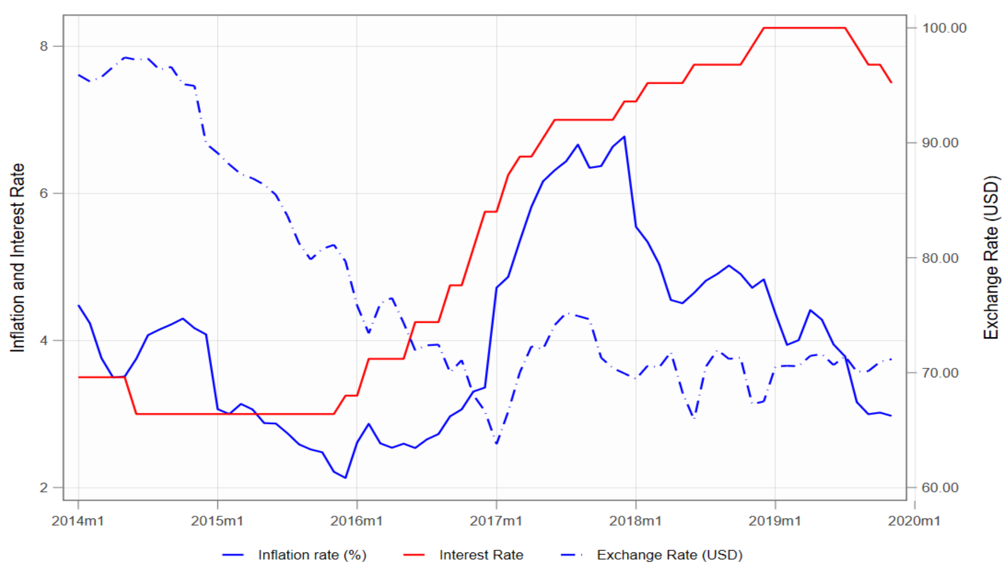
In (a) and (b) the red line depicts the spot price of West Texas Intermediate (WTI) and Brent prices respectively, while the blue line depicts the exchange rate of the Mexican Peso and Nigerian Naira to the dollar respectively.

Figure 2.2: Movement of Key Macroeconomic Variables in Nigeria



Note: Movement of Exchange Rate, Interest Rate and Inflation rate in Nigeria. Data covers the period 2014 (m1) to 2019(m12).

Figure 2.3: Movement of Key Macroeconomic Variables in Mexico



Note: Movement of Exchange Rate, Interest Rate and Inflation rate in Mexico. Data covers the period 2014 (m1) to 2019(m12).

To further comprehend the importance of commodity price shocks for commodity driven economies, one can also observe the relationship between commodity prices and the valuation of traded asset classes of those economies. The financialization of commodity markets has induced a debate on the linkages of trading commodity futures with increasing commodity price volatility. I do not make a stand here[9] but rather highlight the correlations between commodity price volatility and valuation of domestic issued assets (bonds).

Figure 2.4: Commodity Prices and Nigerian Government Bonds Yield

(a) Movement in GSCI and 10-Year Government Bond Index

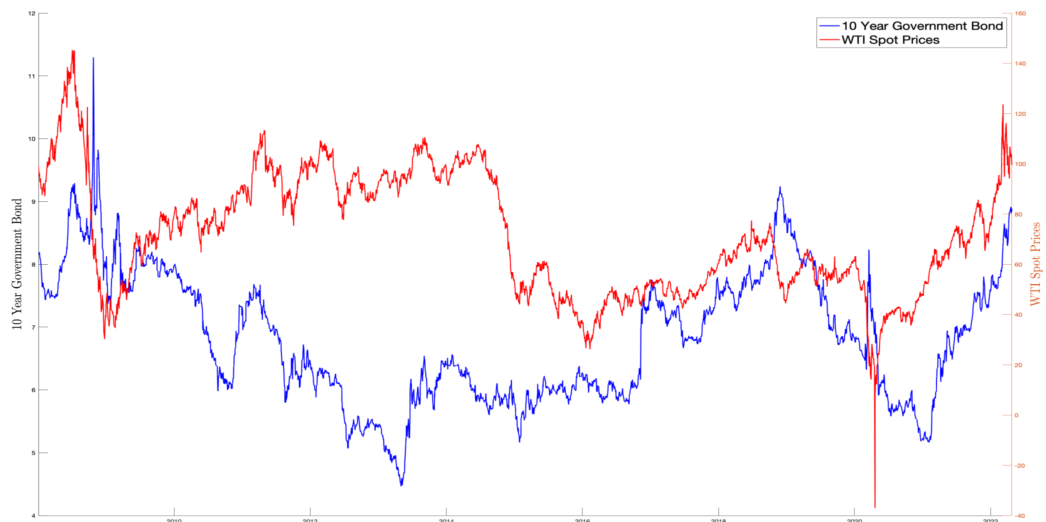
(b) Movement in Brent Prices and 10-Year Government Bond Yield

In (a) and (b) the red line depicts the Goldman Sachs Commodity Index (GSCI) and spot price of Brent Respectively. While the blue and blue line depicts the 10-Year Government Bond Yield. Data is daily and spans the period $1995 - 2021$.

[9] See Cheng and Xiong (2014) for a review of the literature.

Figure 2.5: Commodity Prices and Mexican Government Bonds Yield

(a) Movement in GSCI and 10-Year Government Bond Index



(b) Movement in Oil Prices and 10-Year Government Bond Yield

In (a) and (b) the red line depicts the Goldman Sachs Commodity Index (GSCI) and spot price of WTI Respectively. While the blue and blue line depicts the 10-Year Government Bond Yield. Data is daily and spans the period $1995-2021$.

Figure 2.4 and Figure 2.5 showcases some empirical patterns between 10-year issued government bond yields and commodity price index and oil prices. It shows that spot prices of both Brent and West Texas intermediate (WTI) in the last two decades $(2000-2020)$ have become increasingly correlated with the 10-year return government bonds. This has implications for domestic currency denominated bonds, which constricts domestic government's ability to increase revenue through issuing debt.

What is the Underlying Cause Behind Oil Price Movements?

A majority of the preceding literature has focused on demand side dynamics when assessing monetary policy responses to oil price fluctuations. This has primarily been due to early VAR evidence by Killian and Murphy (2014), Killian and Lewis (2013), and Killian (2009) who argued that what is important for innovations in oil prices are demand side drivers. As a result, earlier literature[10] focused primarily on the demand side and ignored the supply side aspect of oil activity in DSGE modelling. However, Baumeister and Hamilton (2019) show that supply disruptions are a bigger factor in historical oil price innovations, while inventory accumulations by countries plays a smaller role than earlier estimates as in Killian and Lewis (2013). While the data utilised in their estimation only accounts for the 2016 crash in oil prices, I re-estimate their model extending the data period to include the 2020 crash in oil prices. Following Baumeister and Hamilton (2019) I estimate the following structural equations[11]:

$$q_t = \alpha_{pq} p_t + \mathbf{b'}_1 x_{t-1} + u_{1t} \tag{2.3.1}$$

$$y_t = \alpha_{yp} p_t + \mathbf{b'}_2 x_{t-1} + u_{2t} \tag{2.3.2}$$

$$q_t = \beta_{qy} y_t + \beta_{qp} p_t + \chi^{-1} \Delta i_t + \mathbf{b'}_3 x_{t-1} + u_{3t} - \chi^{-1} e_t \tag{2.3.3}$$

$$\Delta i_t = \gamma_1 q_t + \gamma_2 y_t + \gamma_3 p_t + \mathbf{b'}_4 x_{t-1} + \chi u_{4t} + e_t \tag{2.3.4}$$

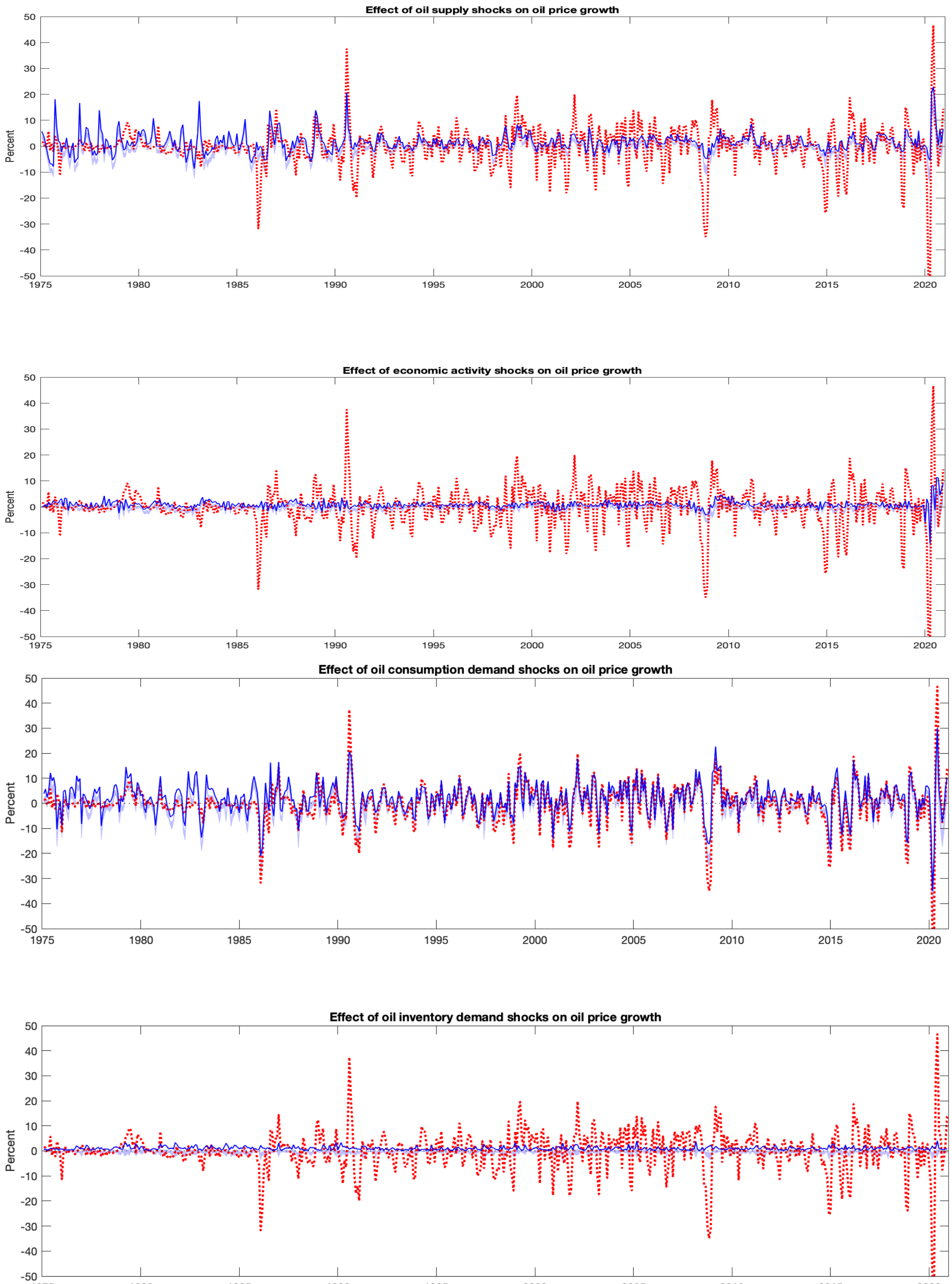
Equation $(2.3.1)$ describes the oil supply curve (world oil production), where α_{pq} and α_{qy} captures the possibility that world economic activity could have an effect on oil supply for other reasons other than oil prices. Equation $(2.3.2)$ is the world economic activity equation that allows oil supply and oil prices to affect it contemporaneously through α_{yq} and α_{yp} respectively. Equation $(2.3.3)$ is the inverse demand equation, but re-written in terms of oil supply. Equation $(2.3.4)$ captures global inventories demand[12]. I restrict attention to just the historical decompositions as it shows how much each shock contributes to displacing oil price growth from its unconditional mean. Therefore, the plots for historical decomposition of the shocks identify what truly matters for oil prices.

[10] Bodenstein *et. al.*(2011; 2012), Bodenstein *et. al.*(2008)

[11] See section 4 on the data use in the estimation process.

[12] See Baumeister and Hamilton (2019) for a full description and explanation of the Bayesian estimation method

Figure 2.6: Oil Price Determinants - Historical Decompositions of Oil Price Growth



Notes: Actual changes in oil prices (red dashed lines) and historical contribution of seperate structural shocks with 95% posterior credibility regions (blue and shaded) for the 4-variable model. Adapted from Baumeister and Hamilton (2019) then extended to 2020.

From Figure 2.6 it is immediately observable that oil supply shocks matter the most for oil prices, which accords to the observable shocks in 2016 and 2020. Therefore, from the perspective of a net-oil exporter the transmission channels require a richer framework than what earlier literature had required. This calls for more inclusive oil sectors that control oil supply in both the home and foreign country, in order to properly identify transmission channels and identify policy responses with higher welfare improvements. Therefore, from the perspective of a net-oil exporter the transmission channels requires a richer framework than what earlier literature had required. This calls for more inclusive oil sectors that control oil supply in both the home and foreign country, in order to properly identify transmission channels and identify policy responses with higher welfare improvements.

Stylised Facts for Nigeria and Mexico

Understanding how foreign variables, in this simple case oil prices affects net-exporters of oil presents an overview of how middle-income countries' business cycles react to international shocks and so, I begin my analysis with a simple numerical exercise. To proxy the lower spectrum of middle-income countries, I use data on Nigeria and and the upper spectrum Mexico[13]. The empirical exercise utilises a simple SVAR (Structural Vector Autoregression) model in identifying the impact international oil prices have on domestic variables. The identification strategy here is a Cholesky decomposition. So, consider the structural form of the economy given by:

$$B_0 Z_t = \Gamma_1 Z_{t-1} + \Gamma_2 Z_{t-2} + ... + \Gamma_k Z_{t-k} + \Upsilon \varepsilon_t \tag{2.3.5}$$

$$B_0 Z_t = \sum_{j=1}^{k} \Gamma_j Z_{t-j} + \Upsilon \varepsilon_t \tag{2.3.6}$$

$$Z_t = [P^*_{o,t} \; GDP^*_t \; \mathcal{S}_t \; \Pi_t \; GDP_t \; GDP_{n,t}]'.$$

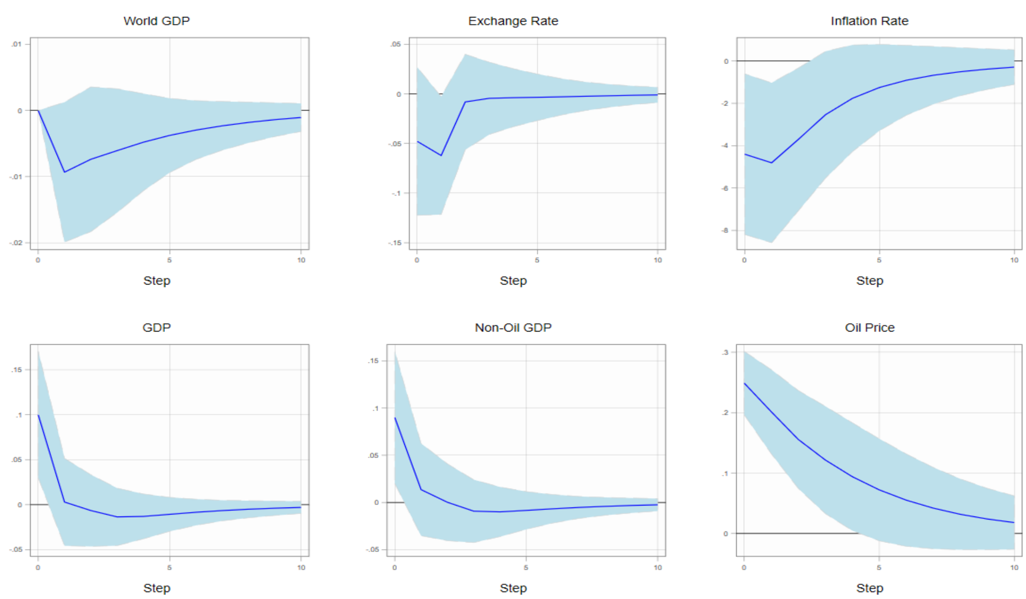
$$\varepsilon_t \sim N(0,1)$$

$P^*_{o,t}$ is international oil price, GDP^*_t is world GDP, GDP_t is domestic GDP, $GDP_{n,t}$ is non-oil GDP, $\mathcal{S}_t$ is the real exchange rate (measured to the U.S. dollar), Π_t is Consumer Price Index. Equation $(2.3.5)$ is simply the transition equation for impulse responses. Here, I consider a shock to the international oil price. I consider a lag length of 1 based on the Akaike

[13]The classification of economies is based on World Bank WDI meta data, see WorldBank (2019). Due to data limitations particularly for central bank policy rates, I utilise the deposit rate for the period $1976 - 2002$ for Mexico.

Information Criterion (AIC) and Swhartz Bayesian Information Criterion (SBIC)[14]. Under the Cholesky identification, the impact matrix B_0 only the first element the error term ε_t affects the world output level, GDP_t^*. Under the Cholesky identification the ordering of variables affects the impulse responses. Similarly, the ordering of other variables implies contemporaneous effects of proceeding variables. It is also worth noting that I do not pin down the driver of the oil price shock. In other words, the shock could be driven by either oil demand or supply disturbances, hence, it is simply interpreted as an oil price shock here. The SVAR is estimated using annual data from the World Bank World Development Index database for Nigeria and Mexico as well as data from the Central Bank of Nigeria and International Monetary Fund (IMF). World GDP (proxy for the international output) is also obtained from the WDI and oil prices are annual (averaged) for West Texas Intermediate (WTI) obtained from the U.S. energy information administration[15]. Finally, the data are log first differenced excluding inflation rate and oil prices to induce stationarity. I also perform a cointegration test to identify any cointegrating relationships between the variables and there is none.

Figure 2.7: Nigeria: SVAR Impulse Responses To an Oil Price Shock



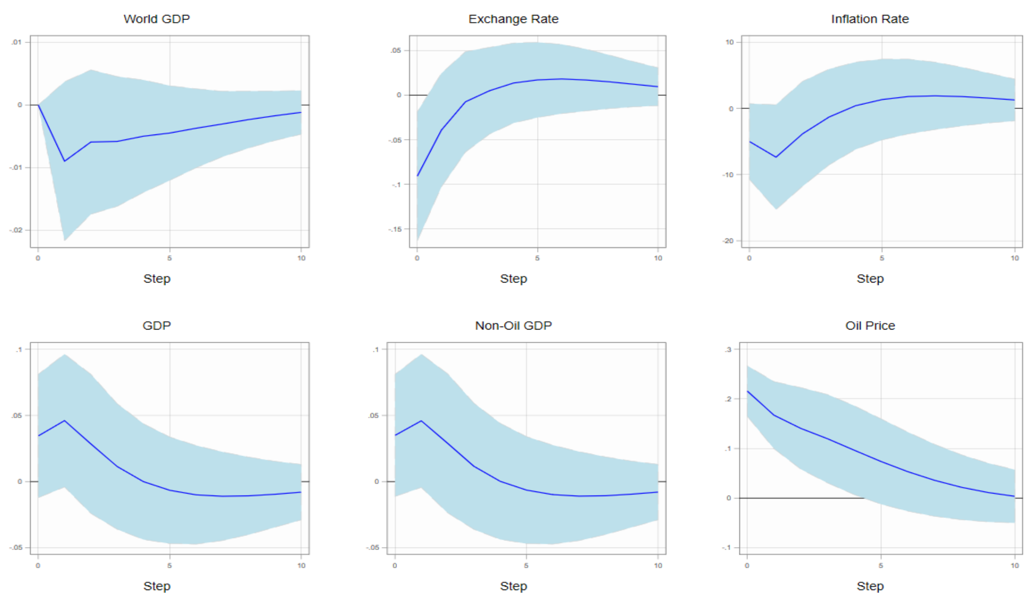
Note: Impulse Response to a unit standard deviation to yearly average of real crude oil price (deflated using U.S. Annual CPI). Based on Cholesky decomposition of first draws. Exchange rate is measure to the U.S. Dollar. GDP and non-oil Value added are also measured in real terms. Stability of the SVAR displayed in Figure 5.1 (b) in Appendix E.

[14]This is a limitation imposed by the annual data available.

[15]Section 5 and Appendix D has a complete description of the data used for estimation

The impulse responses of the domestic variables and world GDP is shown in Figure 2.8, the rise in oil prices leads to an appreciation of home currencies and a fall in inflation rate. Importantly the effects on domestic output are pronounced, and though not observable the more dependent the economy is on exporting oil as in the case of Nigeria, the greater the increase in domestic output in these economies. Furthermore, value added in non-oil sectors (Manufacturing and Services) responds positively to the higher oil prices. Although, based on the structural specification I am unable to account for the mechanism through which this occurs, I can speculate a general spillover effect from oil sector to financial services. Finally, looking at global economic activity, higher oil process does have a negative impact on global economic activity simply through increasing marginal cost of firms. Clearly, oil prices are driving business cycle fluctuations in the Nigerian economy presented here, but this can be extended to other low- or middle-income oil dependent economies.

Figure 2.8: Mexico: SVAR Impulse Responses To an Oil Price Shock



Note: Impulse Response to a unit standard deviation to yearly average of real crude oil price (deflated using U.S. Annual CPI). Based on Cholesky decomposition of first draws. Exchange rate is measure to the U.S. Dollar. GDP and non-oil Value added are also measured in real terms. Stability of the SVAR displayed in Figure 5.1 (a) in Appendix E.

Two conclusions can be drawn from the SVAR analysis. Firstly, somewhat in line with Dutch Disease hypothesis, oil price movements does have an effect on non-oil output[16] but unlike

[16] I refer to it as non-oil output as in the model developed, I only distinguish between non-oil and oil sectors

the prediction of the Dutch disease, the effect here is positive. Though the effects are small, it is worth noting that effects are bigger, the greater the dependence on the commodity. The SVAR is too basic to infer evidence of the Dutch Disease from these findings. Secondly, and perhaps the most important, oil price shocks induce spillover from the international economy to home economies. In this case, a rise in oil prices - the spillover is positive. Although, this exercise displays an overview, it doesn't present what the transmission channels between the international economy and domestic economy. As Fiscal policy would play a role in exchange rate pass through see Pieschahon (2012), monetary policy would play a role in transmission of oil price shocks to the domestic economy, see Bodenstein *et. al.* (2012), Killian (2009), Leduc and Sill (2004). Therefore, the development of the DSGE model is crucial in disentangling transmission channels.

2.4 The Model

While the neoclassical approach can produce interesting results for business cycle analysis, I choose to model the economy in a New Keynesian (NK) framework. Firstly, under the neoclassical framework we assume full price and wage flexibility in the economy. In this case one would not be able to observe the difference between monetary regimes, as the absence of nominal rigidities induce the *Divine Coincidence* for monetary policy. In this case both output-gap and inflation targeting will coincide. Secondly, for the model to capture macroeconomic data's empirical persistence it must embody both nominal and real frictions (rigidities)[17]. Though NK models are subject to the Lucas critique i.e., it will always disregard effects not common in data periods used to calibrate parameters of the model. This is the major disadvantage as it is why the financial crisis was unpredicted by NK models[18]. Ultimately, my choice of the New Keynesian model hinges on the research question, which ascertains central banks designs of monetary policy in low- and middle-income commodity driven economies. Given New Keynesian models have become the cornerstone for monetary policy analysis, it further motivates the choice of an NK model for my research into monetary policy conduct for low- and middle-income commodity economies.

without going into further depth.

[17]Under neoclassical models, firms acts as price takers and make choices on how much to produce. As opposed to price setting behaviour in the NK model.

[18]Another notable disadvantage lies in the assumption of rationality of households during business cycle booms. However, the neoclassical approaches also fails in this regard.

Following the works of Corsetti *et. al.* (2008), Challe and Giannitsarou (2014), and Bergholt *et. al.* (2019), I model a two-country two sector New Keynesian model, where the home country is a strict net-exporter of oil. The home country consists of a non-oil production sector and an oil production sector (subscript u). Both sectors provide consumption goods for home and foreign block, while only the non-oil production sector provides investment goods for home and foreign block. Oil consumption good produced in the oil sector, is consumed by household as direct fuel consumption and energy consumption. While oil consumption by non-oil sector firms is utilised in their production function in producing the final non-oil consumption good, and oil and non-oil investment goods. Households supply labour, rent out capital to both sector firms (rigs in the case of oil sector firms), consume domestically produced goods and imports from the foreign block, and save. I have abstracted from cross-border trade in assets as the no arbitrage condition arising from uncovered interest parity (UIP) would cause movements in nominal exchange rates such that difference in interest rate equalise relative exchange rate movements. To mitigate the Divine Coincidence as documented by Blanchard and Galí (2007), price and nominal wage rigidities are included in the model. Initially, price stickiness at home and abroad plays no role in exchange rate pass through, but the model is extended to account of imperfect exchange rate pass through by utilising Local Currency Pricing (LCP) in firms' pricing decisions at home and abroad. Finally, oil markets focuses on both the demand side and supply side as opposed to only the demand side popularised by Killian (2010), Bodenstein, Guerrieri and Killian (2012), Backus and Crucini (1998)[19].

Households

The home country[20] is populated by a continuum of identical monopolistically competitive households indexed $l \in [0, 1]$. Each household maximises lifetime utility by optimising its consumption $C_t(l)$ relative to habit formation, hours worked, $N_{j,t}(l)$[21], such that the preferences of

[19] Their works focussed on demands side based on empirical evidence that attributed oil price fluctuations to demand shocks see Bodenstein, Guerrieri and Killian (2012). However, the 2015 crash in oil prices was as a result of a supply side shock, with the U.S. boom in shale oil production

[20] Foreign block is an identical representation of the home country, and wherever differences exist would be noted

[21] Consumption is aggregate while hours worked is sectoral

the representative household are given by:

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left[\frac{[C_t(l) - hC_{t-1}(l)]^{1-\sigma_c}}{1-\sigma_c} - \frac{N_{j,t}(l)^{1+\sigma_n}}{1+\sigma_n} \right] \qquad \sigma_c, \sigma_n > 0 \tag{2.4.1}$$

Here β is discount factor, σ_n is the inverse of Frisch elasticity of labour supply, σ_c is the intertemporal elasticity of substitution for consumption, and h governs habit persistence in consumption. Standard optimisation behaviour gives rise to the standard optimality conditions that are derived in Appendix A. Household final consumption is aggregated, such that the consumption bundle for each household is aggregated into a consumption basket by a Dixit-Stiglitz aggregator, with constant elasticity of substitution (CES). So, Non-oil consumption goods are combined with oil consumption good to produce the final consumption good[22]:

$$C_t = \left[a_c^{\frac{1}{\vartheta}} C_{c,t}^{\frac{\vartheta-1}{\vartheta}} + a_o^{\frac{1}{\vartheta}} O_{c,t}^{u\,\frac{\vartheta-1}{\vartheta}} \right]^{\frac{\vartheta}{\vartheta-1}} \tag{2.4.2}$$

ϑ captures the intertemporal elasticity of substitution between consumption good $C_{c,t}$ and oil consumption good, $O^u_{c,t}$. Oil is directly consumed as fuel in daily activities. The shares of core consumption and oil consumption good in the basket are captured by a_c, a_o respectively. Similarly, $C_{c,t}$ is also aggregated by a Dixit-Stiglitz aggregator of domestically produced goods $C_{H,t}$, and imported goods $C_{F,t}$ with CES assumption[23]:

$$C_{c,t} = \left[a_H^{\frac{1}{\varpi}} C_{H,t}^{\frac{\varpi-1}{\varpi}} + a_F^{\frac{1}{\varpi}} C_{F,t}^{\frac{\varpi-1}{\varpi}} \right]^{\frac{\varpi}{\varpi-1}} \tag{2.4.3}$$

Where ϖ is the intertemporal elasticity of substitution between home produce and foreign produced goods. Analogously, a_H, $a_F = 1 - a_H$ are the shares of home and imported goods in the core consumption basket respectively. Cost minimisation gives rise to the following price

[22] International households aggregate consumption basket differs by the inclusion of a demand shock for oil consumption good as in Bodenstein *et. al.*(2010). $C_t = \left[a_c^{*\frac{1}{\vartheta^*}} C_{c,t}^{*\frac{\vartheta^*-1}{\vartheta^*}} + a_o^{*\frac{1}{\vartheta^*}} \left(\varepsilon^*_{d,t} O^{*u}_{c,t}\right)^{\frac{\vartheta^*-1}{\vartheta^*}} \right]^{\frac{\vartheta^*}{\vartheta^*-1}}$. Where $\varepsilon^*_{d,t}$ is a AR(1) exogenous shock to foreign households demand for oil, which is common to firms as well.

[23] Variables denoted with the F subscript are produced in the foreign block, while variables with * are foreign consumed variables.

indices:

$$P_{c,t} = \left[a_H P_{H,t}^{1-\varpi} + a_F P_{F,t}^{1-\varpi}\right]^{\frac{1}{1-\varpi}} \tag{2.4.4}$$

$$P_t = \left[a_o P_{o,t}^{*\,1-\vartheta} + a_c P_{c,t}^{1-\vartheta}\right]^{\frac{1}{1-\vartheta}} \tag{2.4.5}$$

P_t is the price of the aggregated consumption good price index and $P_{c,t}$[24] is the price of aggregated core consumption goods (consumption-based CPI). Within each of the indexes, I have $P_{o,t}^*$ is the real international oil prices and are priced in foreign currency, $P_{H,t}$ price of home produced goods and $P_{F,t}$, import prices.

2.4.1 Non-oil Production Sector

Final Goods

The final good is a composite commodity of both home produced goods and imports. It is aggregated by a Dixit-Stiglitz aggregator with CES as in the core consumption basket of goods. Thus, it is given by:

$$Y_t = \left[a_H^{\frac{1}{\varpi}} Y_{H,t}^{d\,\frac{\varpi-1}{\varpi}} + a_F^{\frac{1}{\varpi}} Y_{F,t}^{d\,\frac{\varpi-1}{\varpi}}\right]^{\frac{\varpi}{\varpi-1}} \tag{2.4.6}$$

Here $Y_{H,t}$ is home produced and $Y_{F,t}$ are imports. a_H is the degree of home bias and a_F is the degree of foreign bias. Though the initial model assumes Producer Currency Pricing (PCP) which induces the Law of one Price (LooP), Purchasing Power Parity (PPP) need not hold as long as $a_H, a_F^* > \frac{1}{2}$. Later on, I induce deviations from the LooP by assuming LCP. As in Kimball (1995) the final home and foreign goods are a composite made of a continuum of intermediate goods $Y_{H,t}(f)$ & $Y_{F,t}(f)$ such that:

$$Y_{H,t}^d = \left(\int_0^1 Y_{H,t}(f)^{\frac{\theta_p-1}{\theta_p}} df\right)^{\frac{\theta_p}{\theta_p-1}} \qquad Y_{F,t}^d = \left(\int_0^1 Y_{F,t}(f)^{\frac{\theta_p^*-1}{\theta_p^*}} df\right)^{\frac{\theta_p^*}{\theta_p^*-1}} , \quad \theta_p, \theta_p^* > 1$$

As with household labour supply, θ_p is the degree of substitutability between intermediate goods, such that as $\theta_p \to 1$ market structure becomes more of a monopoly and as $\theta_p \to \infty$ more of a

[24] The corresponding aggregated price for domestic investment, $P_{i,t}$ holds. The difference between the CES aggregated basket of investment goods I_t, and CES aggregated core consumption goods basket $C_{c,t}$, is the share of domestic and foreign goods. Under the investment basket, domestic share in investment is larger than foreign share (interpreted as FDI). See Table 2.1.

perfectly competitive structure. Cost minimisation gives rise to the following demands:

$$Y_{H,t}(f) = \left(\frac{P_{H,t}(f)}{P_{H,t}}\right)^{-\theta_p} Y_{H,t}^d \qquad Y_{H,t}(f) = \left(\frac{P_{F,t}(f)}{P_{F,t}}\right)^{-\theta_p^*} Y_{F,t}^d$$

Supply of intermediate goods therefore follows

$$Y_{H,t} = \int_0^1 Y_{H,t}(f)df = \int_0^1 \left(\frac{P_{H,t}(f)}{P_{H,t}}\right)^{-\theta_p} Y_{H,t}^d df = \Delta_{Hp,t} Y_{H,t}^d$$
$$Y_{F,t} = \int_0^1 Y_{F,t}(f)df = \int_0^1 \left(\frac{P_{F,t}(f)}{P_{F,t}}\right)^{-\theta_p^*} Y_{F,t}^d df = \Delta_{Fp,t} Y_{F,t}^d$$

Where $\Delta_{Hp,t}$ and $\Delta_{Fp,t}$ are measures of the cross-sectional price dispersion within the intermediate good sector. Furthermore, total production of output by the home firm must satisfy the following:

$$Y_t(f) = Y_{H,t}(f) + Y_{H,t}^*(f) \tag{2.4.7}$$

That is the domestic firms' total output $Y_t(f)$ must equal goods sold at home ($Y_{H,t}$) and those exported ($Y_{H,t}^*$). Therefore, the aggregate supply of home goods is given by

$$Y_t^d = \int_0^1 Y_t(f)df = \int_0^1 Y_{H,t}(f) + Y_{H,t}^*(f)df = \Delta_{Hp,t} Y_{H,t}^d + \Delta_{Hp,t}^* Y_{H,t}^{*d} \tag{2.4.8}$$

2.4.2 Intermediate Production Firms

Output in domestic firm f is given by the Cobb-Douglas production function:

$$Y_t(f) = \varepsilon_t^a K_t(f)^\alpha O_{uH,t}(f)^\mu N_{n,t}(f)^{1-\alpha-\mu} \tag{2.4.9}$$

This production function is adapted from Bergohlt (2014), though it is different as he includes materials as a production input for firms that generates sectoral spillovers[25]. However, I abstract from inter-sectoral spillovers as there is only one non-oil sector. Similarly, the data also supports this choice of abstracting from non-oil sector inter-sectoral inputs in intermediate goods firms. For example in Nigeria and Mexico, manufacturing only accounts for 14.61% and 17.95% of

[25] Given I only have one non-oil sector, there can be no inter-sectoral spillover in the non-oil sector.

GDP respectively, while imports of manufactures accounts for about 72.73% and 76.93% of imports respectively[26]. What this showcase is the lack of domestically utilised materials within these countries and their reliance on imports for inputs[27]. Given I have only one non-oil producing sector, it is modelled as oil products input for intermediate goods firms. It is also the first place where oil products input enters the firm's problem. $K_t(f)$ is the capital rented by firms f, $O_{uH,t}(f)$ is the oil products (energy) input used by firm f, and $N_{n,t}(f)$ is the final labour used by firm f, and ε_t^a is productivity shock (TFP) which follows an AR(1) process with high persistence. The representative intermediate goods firm takes factor prices as given and maximise expected discounted lifetime real profits $\mathbb{E}_t \sum_{t=0}^{\infty} M_{t,t} \mathcal{D}_t(f)$, where $M_{t,t}$ is the stochastic discount factor common across all households and $\mathcal{D}_t(f)$ is real profit given by:

$$\begin{aligned} \mathcal{D}_t(f) = \frac{P_{H,t}(f)}{P_{H,t}} \frac{P_{H,t}}{P_t} Y_{H,t}(f) + \mathcal{E}_t \frac{P^*_{H,t}(f)}{P^*_{H,t}} \frac{P^*_{H,t}}{P_t} Y^*_{H,t}(f) \\ - R_t^k K_t(f) - \mathcal{S}_t \Phi_{F,t} P^*_{o,t} O_{uH,t}(f) - \Omega_{n,t} N_{n,t}(f) \end{aligned} \quad (2.4.10)$$

Here, $\mathcal{E}_t$ and $\mathcal{S}_t$ are the nominal and real exchange rate respectively. I have used that $\frac{P^*_{o,t}}{P_t} = \mathcal{S}_t \Phi_{F,t} P^*_{o,t}$ where $\Phi_{F,t}$ governs exchange rate pass through as oil prices are denominated in foreign currency. Cost minimisation results in the following relative demands such that the marginal product of each factor equals its factor price:

$$\frac{K_t(f)}{N_{n,t}(f)} = \frac{\alpha}{1-\alpha-\mu} \frac{\Omega_{n,t}}{R_t^k} \qquad \frac{O_{uH,t}(f)}{N_{n,t}(f)} = \frac{\mu}{1-\alpha-\mu} \frac{\Omega_{n,t}}{\mathcal{S}_t \Phi_{F,t} P^*_{o,t}}$$

$\Omega_{n,t}$ is the real wage rate and R_t^k is the real rental rate of capital. Therefore, the real marginal cost Φ_t will be independent of firm type, implying that the real marginal cost will be the same for all firms, such that:

$$\Phi_t = \frac{1}{\varepsilon_t^a} \left(\frac{\Omega_{n,t}}{1-\alpha-\mu} \right)^{1-\alpha-\mu} \left(\frac{R_t^k}{\alpha} \right)^{\alpha} \left(\frac{\mathcal{S}_t \Phi_{F,t} P^*_{o,t}}{\mu} \right)^{\mu} \quad (2.4.11)$$

Price setting is subject to nominal rigidities as in wage setting, so that prices are set *á la* Calvo (1963).

[26]Data is obtained from the WDI and for the period 2019 to be consistent with data later on the aggregate economic ratios.

[27]Another reason for this abstraction, is in computation, as with the inclusion of a second non-oil sector will solving for the non-stochastic steady state becomes more computationally challenging, but with less analytic benefit.

Producer Currency Pricing

That is, export prices are pre-set in foreign currency. Every period, each firm is allowed to re-optimise its price with probability $1-\psi_p \in [0,1]$ or with probability ψ_p, it partially indexes to the past inflation $\Pi_{t-1} \equiv \frac{P_{H,t-1}}{P_{H,t-2}}$. So firm f chooses $P_{H,t}(f)$ at home to maximise expected profit. Defining the optimal reset price as $\bar{P}_{H,t}$, optimal prices satisfy:

$$\frac{\bar{P}_{H,t}}{P_{H,t}} = \frac{\theta_p}{\theta_p - 1} \frac{\mathbb{E}_i \sum_{i=0}^{\infty} \psi_p^i M_{t,t+i} Y_{H,t+i}^d \Phi_{t+i} \left(\frac{J_{t,t+i}^p P_{H,t}}{P_{H,t+i}} \right)^{-\theta_p}}{\mathbb{E}_i \sum_{i=0}^{\infty} \psi_p^i M_{t,t+i} Y_{Hj,t+i}^d \left(\frac{J_{t,t+i}^p P_{H,t}}{P_{H,t+i}} \right)^{1-\theta_p} \frac{P_{H,t+i}}{P_{t+i}}} \tag{2.4.12}$$

Then defining the optimal pre-set export price $\mathcal{E}_t P_{H,t}^*(f)$, optimal export prices satisfy:

$$\mathcal{E}_t \frac{\bar{P}_{H,t}^*}{P_{H,t}^*} = \frac{\theta_p}{\theta_p - 1} \frac{\mathbb{E}_i \sum_{i=0}^{\infty} \psi_p^i M_{t,t+i} \mathcal{E}_{t+i}^{\theta_p} Y_{H,t+i}^{*d} \Phi_{t+i} \left(\frac{J_{t,t+i}^p P_{H,t}}{P_{H,t+i}} \right)^{-\theta_p}}{\mathbb{E}_i \sum_{i=0}^{\infty} \psi_p^i M_{t,t+i} \mathcal{E}_{t+i}^{\theta_p} Y_{H,t+i}^{*d} \left(\frac{J_{t,t+i}^p P_{H,t}}{P_{H,t+i}} \right)^{1-\theta_p} \frac{P_{H,t+i}^*}{P_{t+i}}} \tag{2.4.13}$$

A direct result of PCP is the Law of one Price (LooP), i.e. $P_{H,t} = \mathcal{E}_t P_{H,t}^*$. So, firms let prices abroad move one-to-one with exchange rate movements. As a result I have perfect exchange rate pass through to import prices at the border and consumer price level. A direct implication is that nominal exchange rate depreciation leads to worsening of home terms of trade and appreciation of foreign terms of trade and vice versa. The relative differences in prices between home and foreign goods leads to expenditure switching effects of exchange rate movements, as demand will continually adjust in favour of the cheaper good. However, Purchasing Power Parity (PPP) need not hold as long as there is bias in the consumption basket for home and foreign produced goods[28] i.e. $a_H \neq \frac{1}{2}$. Under PCP I define the terms of trade in terms of import prices:

$$\mathcal{T}_t = \frac{P_{F,t}}{\mathcal{E}_t P_{H,t}^*} \Rightarrow \frac{P_{F,t}}{P_{H,t}} \tag{2.4.14}$$

Local Currency Pricing

Now I consider the case where firms set prices in domestic currency, the literature has termed this Local Currency Pricing (LCP). While I assume LCP pricing both at home and abroad I then extend to account for the dominant currency paradigm as in Corsetti, Kuester and Muller (2018). Here, I assume the dominant currency in this case is the foreign currency. Under the

[28] See e.g Corsetti and Pesenti (2001; 2005) and Obstfeld and Rogoff (1995).

LCP regime I have that LooP does not hold both at home and abroad. Once again given nominal price rigidities, price setting is a lá Calvo (1963) and optimal price setting at homes satisfies:

$$\frac{\bar{P}_{H,t}}{P_{H,t}} = \frac{\theta_p}{\theta_p - 1} \frac{\mathbb{E}_i \sum_{i=0}^{\infty} \psi_p^i M_{t,t+i} Y_{H,t+i}^d \Phi_{J,t+i} \left(\frac{J_{t,t+i}^p P_{H,t}}{P_{H,t+i}} \right)^{-\theta_p}}{\mathbb{E}_i \sum_{i=0}^{\infty} \psi_p^i M_{t,t+i} Y_{Hj,t+i}^d \left(\frac{J_{t,t+i}^p P_{H,t}}{P_{H,t+i}} \right)^{1-\theta_p} \frac{P_{H,t+i}}{P_{t+i}}} \tag{2.4.15}$$

While export price setting satisfies:

$$\frac{\bar{P}_{H,t}^*}{P_{H,t}^*} = \frac{\theta_p}{\theta_p - 1} \frac{\mathbb{E}_t \sum_{i=0}^{\infty} \psi_t^i M_{t,t+i} \Phi_{t+i} Y_{H,t+i}^{*d} \left(\frac{J_{t,t+i}^{*p} P_{H,t}^*}{P_{H,t+i}^*} \right)^{-\theta_p}}{\mathbb{E}_t \sum_{i=0}^{\infty} \psi_t^i M_{t,t+1} Y_{H,t+i}^{*d} \mathcal{E}_{t+i} \left(\frac{J_{t,t+i}^{*p} P_{H,t}^*}{P_{H,t+i}^*} \right)^{1-\theta_p} \frac{P_{H,t+i}^*}{P_{t+i}}} \tag{2.4.16}$$

Both optimal prices are different – I have imperfect exchange rate pass through to prices at the border and consumer price level. Without full anticipation of exchange rate adjustments, the LooP fails. According to Corsetti and Pesenti (2005), unless a fixed exchange rate regime is in place or the exchange rate is perfectly forecastable the domestic and export prices will be different. Additionally, the effects of exchange rate movements on demands preclude expenditure switching effects as prices are preset in local currency. Following Monacelli (2015), the law of one price gap for exports and imports are defined respectively as:

$$\Phi_{H,t} = \frac{\mathcal{E}_t P_{H,t}^*}{P_{H,t}} \qquad \Phi_{F,t} = \frac{\mathcal{E}_t P_{F,t}^*}{P_{F,t}}$$

Under the LooP, $\Phi_{H,t} = \Phi_{F,t} = 1$, while under the dominant currency paradigm, $\Phi_{F,t} = 1$ and $\Phi_{H,t} = \frac{\mathcal{E}_t P_{H,t}^*}{P_{H,t}}$.

Dominant Currency Paradigm

Recent systematic empirical evidence prescribes the Dollar Currency Pricing (DCP) or Dominant Currency Paradigm for its implication for monetary policy stabilisation and international monetary policy cooperation. See Boz, Casas, Diez, Gopinath and Gourinchas (2020). Under DCP, I assume the dominant currency is foreign currency. Therefore, the LooP holds only for foreign currency, while there are deviations from the LooP for home export prices. As stated above, a direct implication is that there is perfect exchange rate pass through at the border and consumer price level for import prices at home and imperfect exchange rate pass through in the

ROW. So, equations (2.4.15) & (2.4.16) hold here.

2.4.3 Oil Production Sector

A Brief Overview of Oil and Gas Sector in Nigeria and Mexico

Oil production in Mexico is dominated by the state-owned oil company Pemex, which produces more than 90% of total oil barrels per day[29]. Pemex is currently fully state-owned without any private stakeholders. In the case of Nigeria, oil production is dominated by the state-owned oil-company NNPC (Nigerian National Petroleum Cooperation) along with a few international oil companies (IOCs)[30]. NNPC currently operates joint ventures with other upstream exploration companies in Nigeria for its oil production[31]. NNPC was until July 2022 a fully state-owned company, but currently is owned by the ministry of petroleum and finance in Nigeria. Furthermore, both Nigeria and Mexico implement policy enforcing utilisation of domestic sourced services in all aspects of the oil and gas industry including supply chains in oil and gas sectors. In Nigeria this is enforced by the Nigerian Content Development and Monitoring Board (NCDMB) and was established in 2010. In Mexico, the local content policy came into law in 2008 and initially required 25% of subcontractors of Pemex; from supply chain to other aspects of the industry to be locally sourced. This number has since grown to around 43% in 2019. Finally, in the calibration of oil sector variables, it is important to understand and distinguish between low- and middle-income net-exporters of oil such as Nigeria and Mexico, from high-income net-exporters such as Norway. This reflected in the calibration parameters that reflect lower capital utilisation, higher capital adjustment costs, reliance on FDI for investments in the sectors, more labour employed within the sector amongst others.

Oil Production

Following Nakov and Pescataori (2010a,b), Peersman and Stevens (2013), oil sector firms utilise labour from households and active oil rigs to produce the oil consumption good. Oil production

[29] At its inauguration Pemex had monopoly in refining, production and distribution of petroleum products in Mexico. However, after a series of energy reforms were passed by the Mexican congress, Pemex is allowed to contract upstream and downstream works to international oil companies (IOCs). In a similar trend in 2014 a series of reforms saw Pemex's monopoly end in exploration within Mexico. See Grunstein and Diaz-Wionczek (2017) for more detail on the series of reforms in Mexican Oil and Gas sector.

[30] These companies include Shell Petroleum Development Corporation of Nigeria, a subsidiary of Royal Dutch shell corporation; Chevron Nigeria, a subsidiary of the Chevron Corporation; ExxonMobil Nigeria, a subsidiary of Exxon; and TotalEnergies.

[31] Energy Information Administration.https://www.eia.gov/international/content/analysis/countries_long/Nigeria/

is by the Cobb-Douglas production function:

$$O_t^u = \varepsilon_{u,t}(\Psi_{u,t}^s)^{\alpha_u} N_{u,t}^{1-\alpha_u} \qquad \alpha_u \in [0,1) \tag{2.4.17}$$

O_t^u is oil output produced, $N_{u,t}$ is labour demand by oil sector firms. As the focus is on business cycle dynamics, I abstract from modelling oil reserves as without depletion or rate of extraction it would simply take the form of a constant, and thus, can be normalised to numeraire. $\varepsilon_{u,t}$ is the productivity shock for oil producing firms - when positive, it is interpreted as the oil supply shock in the model and vice versa. Its foreign counterpart $\varepsilon_{u,t}^*$ is the international foreign oil TFP shock, which is simulated later on. $\Psi_{u,t}^s$ are active oil rigs , where $\Psi_{u,t}^s = U_{u,t}\Psi_{u,t}$, with $U_{u,t}$ being utilisation rate of rigs. Short-run oil output is therefore governed by utilisation rate of active oil rigs. Utilisation function $a(U_{u,t})$ is adapted from Bergholt, Larsen and Seneca (2019), where $a(U_{u,t}) = z_1(U_{u,t}-1) + \frac{z_1 z_2}{2}(U_{u,t}-1)^2$ and satisfies $a(1) = 0$ and $a'(1) > 0$. The cost of changing rig utilisation is therefore $\tilde{P}_{i,t}a(U_{u,t})\Psi_{u,t}$. As in capital utilised by non-oil firms, rigs follow a law of motion that includes depreciation and an adjustment cost according to standard investment theory. Oil production firms at home and abroad are price takers and make decisions along two dimensions. Firstly, with oil prices moving freely, oil firms are free to own the rigs as they would never have suboptimal level of capital as in the case of Calvo pricing, which would break down if firms owned the capital and was unable to re-optimize its price. This creates a forward looking decision making as firms respond to oil price innovations immediately considering future productive capacity. Although in reality many oil firms seek to hedge their prices in times of uncertainty and as such do not immediately adjust their utilisation rate to match current demands. Secondly, Oil firms make inter-temporal decision for labour and utilisation as non-oil sector intermediate firms. So, the representative oil firm maximises expected lifetime discounted real profits $\mathbb{E}_t \sum_{t=0}^{\infty} M_{t,t}\mathcal{D}_{u,t}$ such that:

$$\mathbb{E}_t \sum_{s=0}^{\infty} M_{t,s}\left[P_{o,s}^* \mathcal{S}_s \Phi_{F,s} O_{u,s} - \tilde{P}_{i,s}a(U_s)\Psi_{u,s} - \Omega_{u,s}N_{u,s} - \tilde{P}_{i,s}I_{u,s}\right] \tag{2.4.18}$$

Here $\tilde{P}_{i,s} = \frac{P_{i,s}}{P_s}$ is the real price for investment goods. Optimisation is subject to the law of motion for active oil rigs:

$$\Psi_{u,t+1} = (1-\delta_u)\Psi_{u,t} + \varepsilon_{\psi,t}^i\left[1 - S_u\left(\frac{I_{u,t}}{I_{u,t-1}}\right)\right] I_{u,t}$$

Optimality conditions for utilisation rate, active oil rigs and level of investment are given by:

$$\mathbb{E}_t\left[M_{t,t+1}\left[\alpha_u\frac{\mathcal{S}_{t+1}\Phi_{F,t+1}P^*_{o,t+1}O_{u,t+1}}{\Psi_{t+1}}\right.\right.$$
$$\left.\left. - \tilde{P}_{i,t+1}a(U_{u,t+1}) + \mathcal{Q}_{u,t+1}(1-\delta_u)\right]\right] = \mathcal{Q}_{u,t} \quad (2.4.19)$$

$$\mathbb{E}_t\left[M_{t,t+1}\epsilon^i_{\psi,t+1}\mathcal{Q}_{u,t+1}S'\left(\frac{I_{u,t+1}}{I_{u,t}}\right)\left(\frac{I_{u,t+1}}{I_{u,t}}\right)^2\right]$$
$$+\epsilon^i_{\psi,t}\mathcal{Q}_{u,t}\left[1-S\left(\frac{I_{u,t}}{I_{u,t-1}}\right)-S'\left(\frac{I_{u,t}}{I_{u,t-1}}\right)\frac{I_{u,t}}{I_{u,t-1}}\right] = \tilde{P}_{i,t} \quad (2.4.20)$$

Here $\mathcal{Q}_{u,t}$ is simply Tobins-q for installed oil rigs, such that the optimisation solutions in equation (2.4.19) simply determines the discounted present marginal value for active oil rigs. The cost from increasing the number of rigs is given by $\tilde{P}_{i,t+1}a(U_{u,t+1})$ while $\frac{\mathcal{S}_{t+1}\Phi_{F,t+1}P^*_{o,t+1}O_{u,t+1}}{\Psi_{t+1}}$ denotes the revenues from increasing the number of rigs. Equation (2.4.20) simply equates the marginal cost increasing investment in active rigs today with the marginal gain from the increased number of rigs in the next period. Analogous to non-oil firms, the static optimisation problem results in:

$$\alpha_u\frac{\mathcal{S}_t\Phi_{F,t}P^*_{o,t}O_{u,t}}{U_t} = \tilde{P}_{i,t}a'(U_t)\Psi_t \quad (2.4.21)$$

$$(1-\alpha_u)\frac{\mathcal{S}_t\Phi_{F,t}P^*_{o,t}O_{u,t}}{N_{u,t}} = \Omega_{u,t} \quad (2.4.22)$$

$$\Rightarrow \frac{\Psi^s_{u,t}}{N_u,t} = \frac{\alpha_u}{(1-\alpha_u)}\frac{\Omega_{u,t}}{\tilde{P}_{c,t}a'(U_t)} \quad (2.4.23)$$

Equation (2.4.21) denotes the optimal rig utilisation for oil firms i.e. they increase utilisation up to the point where marginal revenues from higher utilisation equates marginal cost. This is the standard profit maximising condition. Equation (2.4.22) is simply the static labour demand optimality condition. The intertemporal and static conditions govern the operation of oil firms in the model. Business cycle dynamics in oil firms are dominated by utilisation rate, such that firms adjust their utilisation of active in order to change output. Long run dynamics are governed by investment decisions to increase the number of operational rigs. This creates forward-looking behaviour in oil firms, such that oil prices today are not the only drivers but the entire expected price path (Bergholt *et. al.,* 2019).

2.4.4 Oil Price Determination

Following Bodenstein *et. al.* (2012) and as in global markets, the price of oil is endogenously determined, therefore, the optimal response to monetary policy will be different depending on the source of the oil price shock. Market clearing in international oil markets requires that world supply of oil equals world demand for oil. Now assuming the Foreign Block has identical oil production technology, then global oil market clearing requires that demand for oil by firms and households in the Foreign block and at home equals optimal oil supply by oil firms at home and in the foreign block. The production technology of non-oil firms abroad has the following Cobb-Douglas structure:

$$Y_t(f)^* = \varepsilon_t^{*a} K_t(f)^{*\alpha^*} \left(O_{uH,t}(f)^{*\nu^*} O_{uF,t}(f)^{*(1-\nu^*)} \right)^{(\varepsilon_{d,t}^* \mu^*)} N_{n,t}^*(f)^{1-\alpha^*-\mu^*} \tag{2.4.24}$$

Here $O_{uH,t}^*$ is a oil goods produced in the home country and utilised in the foreign block by foreign non-oil sector firms. It guarantees that in the non-stochastic steady state, the home country is a net-exporter of oil. $\varepsilon_{d,t}^*$ is an oil intensity demand shock for oil input goods common to foreign households as well. Then global oil market clearing requires that the following market condition holds:

$$\frac{1}{\eta} O_t^u + O_t^{*u} = \frac{1}{\eta} \left(O_{c,t}^u + O_{uH,t} + O_{uH,t}^* \right) + O_{c,t}^{*u} + O_{uF,t}^* \tag{2.4.25}$$

Where $O_{u,t}^*$ is oil supply from the foreign block, $O_{c,t}^{*u}$ is foreign produced oil consumption good consumed by foreign households, and $O_{uF,t}^*$ is foreign produced oil consumption good utilised by foreign non-oil sector firms. As in Bondenstein *et. al.* (2012), with both oil supply and demands determined endogenously, the real price of oil $P_{o,t}^*$ is therefore endogenously determined and will adjust to clear international oil markets. Furthermore, it is possible to scale this international market clearing condition such that supply and/or demand by home firms would have a smaller effect on international oil prices than the foreign block. The parameter $\frac{1}{\eta}$ achieves this, such that as $\frac{1}{\eta} \to 0$ home production and demand has absolutely no effect on international oil prices. The model focuses on both the demand and supply side of the economies as opposed to those developed by Bodenstein, Guerrieri and Killian (2012), Killian (2009) that focuses on demand side. Similarly, it differs from those developed by Nakov and Pescatori (2010a,b), Nakov and Nũno (2011) and Peersman and Steven (2013) which focusses on imperfect competition in the supply block through modelling a third country (OPEC in their cases) that controls oil supply and prices. Somewhat like the discussion surrounding the true drivers of oil prices, the literature around competition in oil markets is also divided. Initial argument by Adelman (1993) emphasised an oligopolistic market structure based on the position that the long-term marginal cost of oil production was a small fraction of oil prices even after considering depletion of oil reserves. Several authors corroborate this oligopolistic market structure for oil markets, see e.g. Alhajji and Huettner, (2000); and Dees *et. al.* (2003). Though finding direct evidence for such imperfect competition models is difficult due to insufficiencies in the data. See e.g. Douglas and Herrera (2011), Bodenstein, Guerreri and Killian (2012). Further along this arguments, Colgan (2014) shows that the OPEC has little to no power on production capacity of member countries. Concluding that the cartel power of OPEC was nothing more than a rational myth. A similar conclusion is reached by Alhajji and Huettner (2000) where they empirically found that given the demand for oil remains considerably price inelastic, neither OPEC nor its largest producers could act as a dominant producer. On the other hand, Berk and Cam (2020) find that while the oligopolistic market structure fits the observed oil market structure, international oil markets have evolved and become closer to a competitive market structure. This they argue lies in the heart of the $2015 - 2016$ oil price crashes. They further posit that beginning form 2014 the market structure of oil markets began evolving closer to a competitive one with the market power of OPEC and other major suppliers dwindling. Similarly, Huppmann and Holz (2012) perform analysis of oil market structure within the $2005 - 2009$ period and find that during the oil price hike of 2005 and 2007, oil market structure mimicked an oligopolistic market structure and particularly Stackelberg competition with first mover advantage. However, following the 2008 price decline and global economic crisis, the market structure has become more competitive. Additionally, empirically, Baumeister and Killian (2016) show that post-2014, world oil markets have evolved into a more competitive market structure compared to prior to 2014. Analogously, Prest (2018) also advocates for the competitive market structure, arguing that both OPEC and Saudi Arabia have lost dominant market power status following the oil price crash of $2015 - 2016$.

Additionally, the assumption of international oil markets clearing has been argued to be insufficient and not be in the case in the real world. Smith (2009) provides initial arguments on oil market out of equilibrium dynamics. Similarly, arguments of the financialization of oil commodity markets posit that oil as an asset classes creates volatility of international oil prices rather than market clearing conditions[32]. In a similar relation to oil being related to other asset classes Peersman and Stevens (2013) identify the relationship between oil price evolution and the rate of return on other assets (bonds in this case) to be linked by deviations from the arbitrage condition for oil inventories[33]. Here, arbitrage evokes disturbances in oil markets by inducing trading in oil inventories. In other words, changes in oil prices invoke investors to readjust portfolio choices to more lucrative assets which invoke selling or buying oil inventories on the oil markets depending on the direction of oil price changes. Another significant strand of the literature that deviates from market equilibrium determining oil prices identifies oil price setting behaviour through modelling a dominant oil supplier that seeks to maximise welfare for its owners[34].

2.4.5 Monetary and Fiscal Policy

I distinguish between the government and central bank, such that the central bank conducts monetary policy while government conducts fiscal policy. I assume the central bank operates a flexible output, inflation and exchange rate targeting regime[35] that follows an interest rate smoothing Taylor rule as in Smets and Wouters (2007), but with exchange rate stabilisation included as in:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R} \left[\left(\frac{\Pi_t}{\Pi}\right)^{\rho_\pi} \left(\frac{GDP_t}{GDP}\right)^{\rho_y} \left(\frac{\mathcal{E}_t}{\mathcal{E}_{t-1}}\right)^{\rho_e}\right]^{1-\rho_R} \left(\frac{GDP_t}{GDP_{t-1}}\right)^{\rho_{\Delta y}} \varepsilon_t^R \qquad (2.4.26)$$

The inclusion of nominal exchange rates follows from Corsetti *et. al.* (2008) who argue that in small open economies, monetary policy should respond to exchange rate movements. Here Π_t is consumer price index, R is the steady state interest rate for R_t, ε_t^R is monetary policy shock which follows an AR(1) exogenous process. Π and GDP are the steady state values of gross CPI and the aggregate output respectively. Monetary policy in the foreign block approximates

[32] see Cheng and Xiong (2014)

[33] The arbitrage condition here equates the return on bonds to the expected future price of oil (Peersman and Stevens, 2013).

[34] See Nakov and Pescatori (2010a;b), as they model OPEC as a dominant oil supplier that sets oil prices through dynamic price setting.

[35] With nominal rigidities in wage and price setting, stabilising inflation does not coincide with stabilising output – the Divine Coincidence, see Blanchard and Galí (2007)

an identical Taylor rule given by:

$$\frac{R_t^*}{R^*} = \left(\frac{R_{t-1}^*}{R^*}\right)^{\rho_R^*} \left[\left(\frac{\Pi_t^*}{\Pi^*}\right)^{\rho^*{}_\pi} \left(\frac{GDP_t^*}{GDP^*}\right)^{\rho^*{}_y} \left(\frac{\mathcal{E}_t}{\mathcal{E}_{t-1}}\right)^{\rho_e^*}\right]^{1-\rho_R^*} \left(\frac{GDP_t^*}{GDP_{t-1}^*}\right)^{\rho_{\Delta y}^*} \varepsilon_t^{*R}$$

Though policy choices in the foreign block influence the impact of foreign shocks on foreign macro variables, they do not impact how foreign shocks affect the home economy (De Paoli, 2009). On the fiscal side, the government is assumed to always runs a balanced budget every period, with intertemporal budget constraint given by:

$$T_t + \frac{B_{H,t}(l)}{\epsilon_t^b P_t} = F_t + \frac{R_{t-1} B_{H,t-1}(l)}{\epsilon_t^b P_t} \tag{2.4.27}$$

$$\int_0^1 B_{H,t}(l) = B_{H,t}$$

The left-hand side represents government revenue and the right-hand side government expenditure. Here, F_t denotes government spending, which I assume to be a constant. At the moment fiscal policy plays no role in affecting the level of pass through. Though I can model the presence of a sovereign wealth fund as in Bergholt *et. al.* (2019), I currently abstract from this.

2.5 Data and Calibration

I calibrate the home economy parameters to those found in literature for net-exporting oil countries to match macro data for Nigeria and Mexico. As Nigeria forms lower-middle income and Mexico higher middle income net exporters economies. The foreign block is calibrated to match the U.S. economy based on estimated parameters from DSGE estimation literature.

2.5.1 Data

The data used in generating the simple SVAR is obtained from the world bank world development indicators. For Nigeria the data is obtained for the years 1981 – 2019. Logarithm of the data is taken after which they are first differenced to remove the time trend. Data on oil prices are taken from the U.S. Energy Information Administration for West Texas Intermediate (WTI). The data on real GDP and non-oil real GDP is obtained from the Central Bank of Nigeria. While data on Mexican GDP and non-oil GDP are extracted from the WDI. Inflation rate for Nigeria and Mexico are annual averages and also extracted from the WDI. All data on exchange rates (nominal) averages, and central bank policy rates are extracted from the IMF's International Financial Statistics dataset. The Data for replicating Baumeister and Hamilton (2019) is obtained from U.S. Energy Information Administration for WTI and is deflated using U.S. CPI from the U.S. Federal Reserve Economic Data. The data for global economic activity is constructed as OECD members industrial production plus 6 non-member countries[36] and downloaded from Hamilton's research page[37]. Data about net exporters of oil is obtained from the CIA World Factbook from 2018. Appendix D contains a richer description of the data sources.

2.5.2 Calibration

To solve the model, I first solve for the non-stochastic steady state, normalising rig utilisation rate to unity. Gross inflation rate is assumed to be unity in the steady state in accordance to New Keynesian literature. The calibrated values are described in Table 1 along with their sources. Based on these figures the model is solved recursively for the non-stochastic steady state. For the foreign block proxied through the U.S. economy, I set the discount rate $\beta = 0.99$ implying an annual interest rate of around 4%. The inverse Frisch elasticity of labour supply is set to unity, in the range of both micro and macro literature. The intertemporal elasticity of consumption is also set to unity implying log preferences in consumption. Capital share in production is set to $\frac{1}{3}$ in accordance to literature and based on estimate by Bodenstein *et. al.* (2012), with oil input in production function estimated at $\mu^* = 0.026$, implying labour input in production to be 0.64 in the rage estimated by Smets and Wouters (2007). Effective rigs share in oil production sector is set at 0.32 in the foreign block based on Bergholt *et. al.*(2019), while in the home block it is set at 0.42 based on Benkhodja (2014).The Kimball (1995) aggregators (wage and price markup) are set $\theta_p^* = 7.66$, $\theta_w^* = 6$ implying markups of 15% and 20% in prices and wages respectively. Calvo parameters are calibrated based on estimates derived in Bodenstein *et. al.* (2012) so, $\Psi_w^* = \Psi_{uw}^* = 0.88$, and $\Psi_p^* = 0.89$ implying wages and prices are set nearly every 5 quarters. Wages and price indexation parameters, $\iota_w \& \iota_p$ are set to the Smets and Wouters (2007) estimate of 0.24 and 0.58 respectively. Capital adjustment cost $\varrho_k^* = 5.45$ based on estimates from Smets and Wouters (2007). Parameters relating the CES aggregators are calibrated based on estimates obtained from Bodenstein, Guerreri and Killian (2012), the elasticity of substitution between

[36] Brazil, China, South Africa, India, Indonesia, and Russia

[37] https://econweb.ucsd.edu/~jhamilto/software.htm

exports and foreign produced goods is set to 0.569, and elasticity of consumption between oil and non-oil consumption is set to -1.73. The policy parameters are based on estimates from Smets and Wouters (2007), so interest rate smoothing parameter is set to 0.81, while monetary policy response to inflation and output is set to 2.04 and 0.08 respectively. Following Bergholt and Larsen (2016), fiscal policy parameter is set to 0.9, while fiscal policy response to GDP and inflation are 0.5 and 0.1 respectively. Finally, as bayesian estimation of DSGE models does poorly in identifying input shares in consumption basket, Bodenstein *et. al.* (2012) calibrate the share of imports in the U.S. consumption basket to be 0.068, this is also used in this model. The feedback of GDP growth $\rho^*_{\Delta y}$ is set to 0.22 based on the estimated parameter mode from Smets and Wouters (2007). Finally, since the U.S. does not respond to exchange rate fluctuation, the feedback parameter $\rho^*_{\mathcal{E}}$ is set at 0.001.

The Home country is calibrated to match an emerging economy, which includes literature values for Nigeria and Mexico and other oil dependent economies. Table 2 reports the parameters and their values. Based on Benkhodja (2014), the discount rate is set to 0.99, following Berg *et. al.* (2013)[38], inverse Frisch elasticity set to 10 and $\sigma_c = 2$ and degree of home bias in private consumption set to 0.4, so foreign bias in home consumption basket is 0.6. Capital share in production is set to 0.31 which is within the range specified in the empirical literature for low to middle income countries, see Cavalcanti, Mohaddes and Raissi (2011), with labour share in production around 0.67, this implies oil input in production to be 0.02. The parameterisation of capital adjustment cost within the literature varies, Benkhodja (2014), Allegret and Benkhodja (2015), Berg *et. al* (2013) and Devereux, Lane and Xu (2006) all set vastly different parameters, so I set capital adjustment cost to 7.5, which is in the range of the figures specified in the literature. This is also applied to the adjustment cost for rigs. The Kimball aggregator for firms and labour bundler is set to 8 implying price and wage markups of around 10%. Assuming slightly lower indexation to the foreign counterparts, price indexation parameter is set to 0.5 and wages to 0.2. The lower indexation parameters for low- and middle-income countries is driven by the observance that these types of economies are usually plagued by high and persistent levels of inflation. Following Medina and Soto (2005) I set the calvo probability for prices ψ_p at 0.75. The Calvo probability for wages ψ_w & ψ_{wu} is set at 0.75 as well for both oil and non-oil sec-

[38]Berg *et. al.* (2013) calibration is done to match CEMAC region in Africa

tors[39]. Following Ravenna and Natalucci (2008), the elasticity of substitution between imports and domestic goods is set to 0.42. The elasticity of substitution between core consumption and oil goods is set to -1.93 implying higher complementarity between oil consumed and consumption good. This is due to low- and middle-income countries relying heavily on fuels compared to the developed world, hence, core consumption is highly complemented by oil. Following Medina and Soto (2005), the interest rate smoothing parameter is set at 0.7 below the developed world but in line with emerging markets literature[40].

Table 2.1: Calibration - Foreign and Home Block

Foreign Country		Home Country			
Parameter	Value	Parameter	Value	Description	Source
β^*	0.99	β	0.99	Discount factor	Literature; Benkhodja (2014).
α^*	$\frac{1}{3}$	α	0.31	Capital share in production	Empirical literature (Author's calibration)
μ^*	0.026	μ	0.02	Oil share in production	Empirical Literature (Author's calibration)
ν^*	0.5	-	-	Imported oil share in foreign non-oil sector firms production	Author's calibration
α_u^*	0.32	α_u	0.42	Effective rigs share in oil production	Bergholt *et. al.* (2019); Benkhodja (2014)
σ_c^*	1	σ_c	2	Intertemporal consumption elasticity	Bodenstein *et. al.* (2012); Berg *et. al.* (2013)
σ_n^*	1	σ_n	10	Inverse Frisch elasticity	Bodenstein *et. al.* (2012); Berg *et. al.* (2013)
ψ_w^* ψ_{wu}^*	0.88	ψ_w ψ_{wu}	0.75	Calvo probability - Wages	Bodenstein *et. al.* (2012); Persistent Inflation
ψ_p^*	0.89	ψ_p	0.75	Calvo probability - Prices	Bodenstein *et. al.* (2012); Author's calibration
ι_w^*	0.24	ι_w	0.2	Degree of wage indexation	Smets and Wouters (2007); Author's calibration
ι_p^*	0.58	ι_p	0.5	Degree of price indexation	Smets and Wouters (2007); Author's calibration
a_O^*	0.021	a_O	0.054	Oil share in core consumption basket	Nakov and Pescatori (2010a,b); Author's calibration
θ_w^*	6	θ_w	8	Kimball aggregator - Wages	Author's Calibration; 10% Markup
θ_p^*	7.66	θ_p	8	Kimball gggregator - Prices	Nakov and Pescatori (2010a); 10% Markup
ϖ^* ϖ_i^* ϖ_u^*	0.569	ϖ ϖ_i ϖ_u	0.42	E.O.S domestic goods and imports	Bodenstein *et. al.* (2012); Ravenna and Natalucci (2008)
ϑ^*	-1.73	ϑ	-1.93	E.O.S between oil and core consumption	Bodenstein *et. al.* (2012); Author's calibration
$(1-\xi^*)$ $(1-\xi_u^*)$	0.9320	ξ ξ_u	0.8	Degree of home bias in investment goods	Bodenstein *et. al.* (2012); Berg *et. al.* (2013)
a_F^*	0.9320	a_H	0.4	Degree of home bias in core consumption basket	Bodenstein *et. al.* (2012); Berg *et. al.* (2013)
ϱ_k^*	5.74	ϱ_k	7.5	Capital adjustment cost	Smets and Wouters (2007); Author's calibration
ϱ_u^*	5.74	ϱ_u	7.5	Rigs adjustment cost	Similar Estimate as Capital; Author's calibration
δ^*	0.025	δ	0.035	Depreciation rate - Capital	Empirical Literature (Author's calibration)
δ_u^*	0.025	δ_u	0.035	Depreciation rate - Rigs	Bergholt *et. al.* (2019); Author's calibration
h^*	0.8	h	0.5	Habit formation parameter	Smets and Wouters (2007)
υ^*	0.02	υ	0.03	Elasticity of oil supply	Bergholt *et. al.* (2019)
-	-	$\frac{1}{\eta}$	0.01	Home share in global oil production	Data
ρ_R^*	0.81	ρ_R	0.7	Interest rate smoothing in Taylor Rule	Smets and Wouters (2007); Author's calibration
ρ_π^*	2.04	ρ_π	1.2	Inflation inertia in Taylor Rule	Smets and Wouters (2007); Author's calibration
ρ_y^*	0.08	ρ_y	0.16	Output gap inertia in Taylor Rule	Smets and Wouters (2007); Author's calibration
$\rho_{\mathcal{E}}^*$	0.001	$\rho_{\mathcal{E}}$	0.2	Nominal Exchange rate feedback in Taylor Rule	Author's calibration
$\rho_{\Delta y}^*$	0.22	$\rho_{\Delta y}$	0.2	Output difference feedback in Taylor Rule	Smets and Wouters (2007); Author's calibration

Notes: In the source column, the first item before the semicolon is the source for the foreign block value while the proceeding item after the semicolon is for the home block value.

[39]Medina and Soto (2005) estimate (posterior modes) the Calvo probabilities for prices and wages to be 0.74 and 0.82 respectively. Given the non-segmented labour market in their estimation, I chose to retain the same Calvo wage probability for wages and prices.

[40]Medina and Soto (2005) estimate the interest rate smoothing parameter for Chile to be around 0.73.

While monetary policy response to inflation, and GDP are set to 1.2 and 0.16 respectively. These calibration figures are based on estimates from literature on emerging economies[41]. The nominal exchange rate feedback parameter $\rho_{\mathcal{E}}$ is set at 0.2, which is higher than within the literature but is set this was as low- and middle-income oil countries typically have measures aimed at stabilising exchange rates[42]. Based on global oil production data from the U.S. Energy Information Administration, I set $\frac{1}{\eta}$ to 0.01, as Nigeria and Mexico currently contribute around 0.0098% and $0.009\% \approx 0.01$ respectively to global oil production. Oil share in the consumption basket is calibrated to 0.054 to be higher than Nakov and Pescatori (2010a,b) estimate for the U.S. as low- and middle-income country population rely heavily on fuel. The degree of habit formation in domestic country is set at 0.5 which is lower than specified in Araujo *et. al.* (2016) but in line with estimates from the empirical literature on emerging economies[43]. Depreciation rate for physical capital is set at 0.035 in range with empirical literature, while depreciation rate for rigs is also set at 0.035[44]. Finally, I define the parameters υ & υ^* that govern the elasticity of oil supply at home and abroad respectively. They are set to 0.03 & 0.02 respectively based on estimates from Bergholt *et. al.*(2019). While I understand that parameters for Norway are generally different from low- and middle-income countries, the elasticity of oil supply by oil producers will generally be similar independent of the country. This is why I set the elasticity figure to match Bergholt *et. al.*(2019). With that, I can recover the value for $z_2 \equiv \frac{a''(1)}{a'(1)} = \frac{\alpha_u}{\upsilon} + \alpha_u - 1$, a similar identity holds for the foreign counterpart.

Table 2.2: Benchmark - Data and Model Ratios

VARIABLES	Nigeria	Mexico	Model
Imports (% of GDP)	19.8	39.1	48.2
Exports (% of GDP)	14.2	38.8	24.19
Fuel Exports (% of Total Merchandise)	87.0	5.32	84.06
Final Consumption (% of GDP)	80.2	76.3	77.19
Gross Capital Formation (% of GDP)	25.4	21.2	22.25
GDP	\$502bn	\$1.2 tn	-
GDP Per Capita	\$2,502	\$9,819	-

Notes: Data is from the World Bank's WDI and for the period 2019. Model ratios represent the non-stochastic steady state ratios from the solved model.

[41]For Chile, Medina and Soto (2005) estimate the inflation feedback in Taylor rule to be 1.6, and GDP changes to 0.28. Carvalho *et. al* (2013) for Brazil, estimate the inflation inertia to be 1.9, with the output gap feedback at 0.1

[42]For example Banco de Mexico performs exchange rate interventions.

[43]Medina and Soto (2005) estimate the habit formation parameter at 0.32, Poghosyan and Beidas-Strom (2011) estimate it at 0.48. Araujo *et. al.* (2016) calibrate it to 0.7, which is closer to literature estimates of advanced economies.

[44]Berg *et. al.* (2013, Allegret and Benkhodja (2015), Agenor (2016) all set values in the range of 0.020–0.05.

Without prior estimates or sources for $\rho_{\Delta y}$ for low- or middle-income countries, I set it to 0.2 so, slightly lower than the estimated parameter for the U.S. from Smets and Wouters (2007) and Medina and Soto (2005). Table 2.1 displays all variable calibrations within the model.

The dynamics of the economy are driven by 6 shocks in the foreign economy - 2 sector specific TFP shocks, one in the oil sector and the other in the non-oil sector; 2 sector specific investment shocks; one oil intensity demand shock; and one monetary policy shock. Table 2.2 above represents the implied non-stochastic steady state ratio of the model, and compares them with actual observed data ratios from both Nigeria and Mexico.

2.6 Transmission of International Shocks

Empirical literature by Killian (2009), Bodenstein and Guerrieri (2011) have determined that foreign oil intensity shocks are key drivers of real oil prices. Similarly, Killian (2009) showed how monetary policy in the U.S. affected global oil prices through its impact on domestic productivity. Therefore, understanding how policy makers should respond to both surges and crashes in oil prices has always been a crucial question for oil importing countries. However, the literature investigating the converse is fairly in its infancy, of which the little that does exist is purely from high income resource rich countries such as Norway. See e.g. Bergholt (2014), Ferrero and Seneca (2015), Bergholt, Larsen and Seneca (2019). Thus, the model evaluated here sheds some light when the resource rich country is a low- or middle-income country.

Although not possible to evaluate each structural shock in the model, I provide comparisons between the monetary policy responses to different international structural shocks[45]. As stated in my introduction, the currency in which prices are set does have a role in international spillovers of shocks. Hence, each of the structural shocks is analysed under three different pricing regimes and one exchange rate regime (flexible exchange rate regime[46]). I show that depending on why the price of oil has changed generates different responses in both magnitude and sign of

[45]Though the model incorporates domestic shocks, they are of no use as the aim is to understand the spillover of international shocks to the domestic economy.

[46]Corsetti and Pesenti (2005) show that flexible exchange rate regimes are optimal. Though Frankel (2018) suggests that if the commodity driven economy pegs its exchange rate, it should consider including the export commodity to the basket of currencies to which they peg creating a Commodity-plus-Currency basket, such that the exchange rate targets would allow for better accommodation of terms of trade shocks. I do not delve into this discussion in the chapter, but see Frankel (2018) for further discussion.

monetary policy. To understand the implications of shocks as well as the transmission channels, the model is solved at the second order as in Kim, Kim, Schaumburg and Sims (2008), with impulse responses pruned to avoid explosive paths and then HP-filtered at 1600 draws. The results are analysed below.

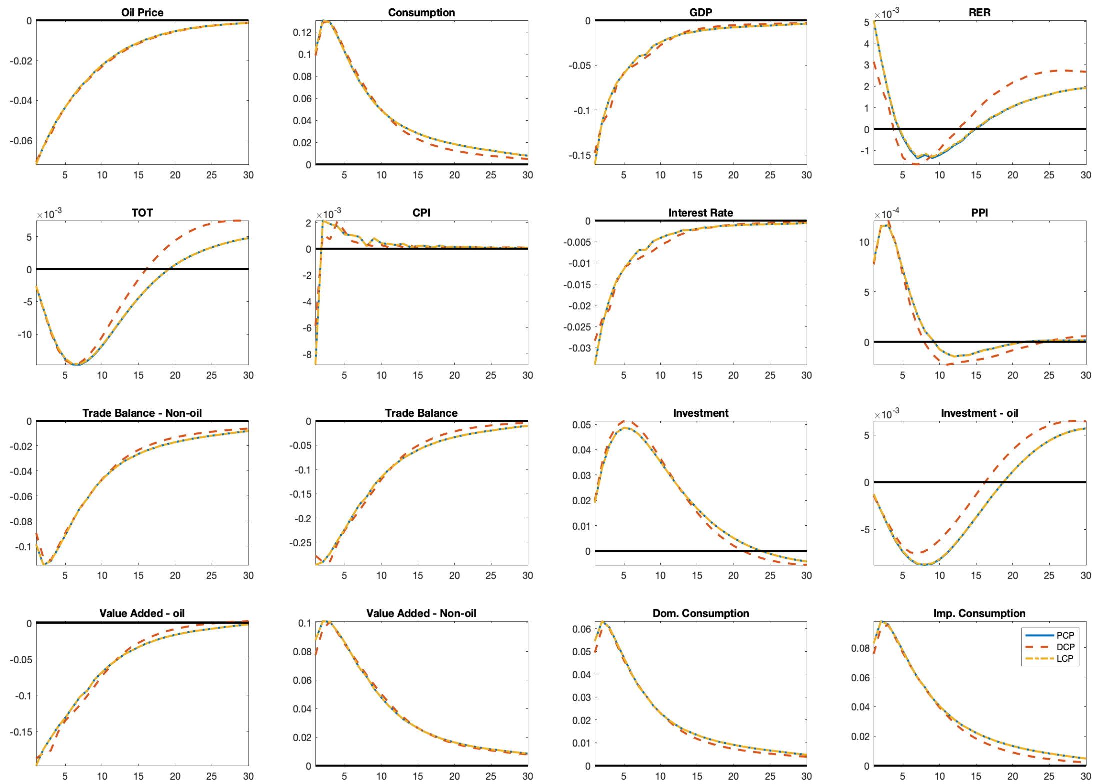
2.6.1 Oil Supply Innovations

One can think of oil supply innovations as productivity gains in the oil sector that increases efficiency in production. Figure 2.9 shows the simulation of an increase in international oil production activity, and the proceeding response by domestic variables. On the aggregate country level, there is a prolonged contraction to GDP plunging the economy into a recession. This recession is driven primarily by a contraction in oil sector and a depreciation in RER (real exchange rate). The transmission channels at play here are explained below. Firstly, oil production firms within oil sector react to the crash in oil prices by decreasing oil output through curtailing utilisation rates. Given the forward-looking optimisation behaviour of oil firms, the entire path of oil prices is determining their investment behaviour, thus, as oil prices crashed the present value of installed oil rigs falls below price of new rig installations[47]. Oil sector firms respond by decreasing the level of investment in rigs. As capital adjustment incurs a cost, the process is slower than the immediate changes in oil prices. Hence, while oil prices recovered slowly, investments in oil rigs continues to fall before rebounding following oil prices readjustment. This creates the contraction in oil sector (lower oil sector value added). Furthermore, as non-oil firms produce the investment goods in both oil and non-oil sector, the fall in oil sector investments leads to lower inter-sectoral demand. On the other hand, lower oil prices lead to lower marginal cost of production creating higher value added for domestic non-oil firms. These two channels show how oil sector activity creates spillover effects into non-oil sector within the model. However, due to lower substitution effect between oil input and capital and labour, the lower marginal cost is not substantial enough to generate order of magnitudes large enough to offset the negative value added from oil sector. Secondly, the fall in oil prices decreases the level of oil revenues and the domestic economy's asset become less valuable and a depreciation of domestic currency is the result. In addition to this, depreciation RER creates an improvement in the ToT (terms of Trade). Though the improvement in home ToT would have been beneficial if non-oil consumption dominated domestic exports, however, the converse is true and the ag-

[47] This is presented in equation (2.4.21)

gregate trade balances contract. These two channels lead to recessionary impacts of falling oil prices on domestic net oil exporters.

Figure 2.9: Home Variables IRFs to Increase Foreign Oil Producing Firms Productivity



Notes: Unit Standard deviation innovation in $\varepsilon^*_{u,t}$. Blue solid line represents PCP, red dashed line DCP and yellow dashed line LCP

Additionally, as with any aggregate supply shock, the fall in oil prices creates further adjustments in other key macro variables. Consumption and Investment in non-oil sector increase following the fall in oil prices. There are also inflationary pressure in producer price inflation (PPI)[48], while impact effect on CPI is negative, there is a sharp and exponential increase in CPI following the impact of the shock. The presence of imported intermediate goods in the consumption basket drives this wedge between domestic inflation and core inflation. Furthermore, with domestic consumption basket driven mainly by imported goods, core inflation and CPI are dependent on exchange rates. Therefore, these inflationary pressures are driven by RER depreciation and the composition of domestic consumption baskets. As the domestic economy prioritises imports above domestic produced goods, PPI is driven by import prices, thus, depre-

[48] I also refer to this as the consumption based CPI as it is based on $P_{H,t}$ and $P_{F,t}$

ciation in RER creates inflationary pressures through higher import prices[49]. As exchange rate depreciation passes through to PPI, the resulting cost push effect leads to the exponential jump in CPI. Monetary authorities react by adjusting the interest rate following the exponential rise in inflation. Although since interest rate also react to the output gap, changes in interest rates are not one-to-one with CPI[50].

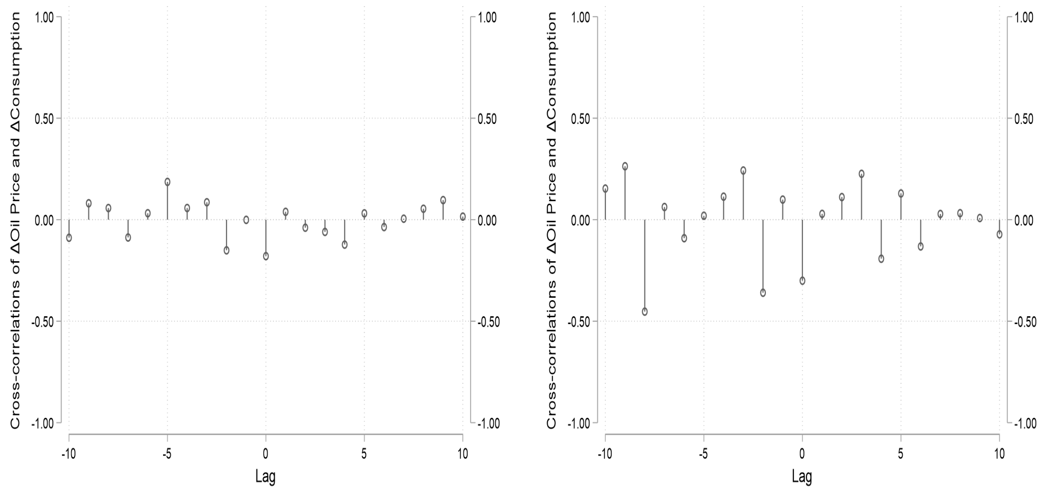
Understanding the channels through which consumption increases requires understanding the inflation dynamics explained above. With the immediate adjustment of the RER given the oil price shock, CPI inflation only occurs in the first period and gradually adjust to its steady state level. As already explained, RER pass through to PPI is high but subsequently not to CPI. From the Euler equation, understand that current consumption depends on the ex-ante real interest rate, and following the Taylor rule dynamics, monetary policy responds to contemporaneous CPI. Therefore, with lower CPI monetary authorities lower nominal interest rates and expected level of inflation rises, which leads the path of real interest rates to fall. This causes current consumption to rise as well. On the other hand, consumption is also driven by imported intermediate goods such that increased production abroad will lead to higher imported goods and higher level of consumption in the domestic economy. The RER pass through to PPI but not to CPI as well as the increase in international production, consumption increases even in the face of RER depreciation and contraction of the domestic economy.

While the IRFS showing the impact of higher oil prices on domestic consumption may seem counter intuitive for a net-exporting oil country, the data corroborates this response. Figure 2.10 above shows the responses of correlation between changes in oil prices and in consumption, and as the IRFS direct, impact relationship is indeed negative. In other words, impact effects of oil price growth is inversely related to consumption growth.

[49]In standard open economy models, the depreciation of RER creates expenditure switching effects of exchange rate movements under producer currency pricing and domestic households substitute imports for domestically produced goods. See Corsetti *et.al.* (2008). However, the domestic preference towards imports in consumption bundle in the model prevents this effect from coming into play.

[50]It is also worth observing that the difference in responses across all pricing regimes is only in the magnitude of the effect. This I attribute to the size of the shocks considered here, as the effect on the NER (nominal exchange rate) is not substantial enough such that it drives NER movements considerably different, thus, the resulting differences responses across the pricing regimes is not overly pronounced.

Figure 2.10: Correlelogram showing Relationship between Oil price movements and Consumption



(a) Mexico

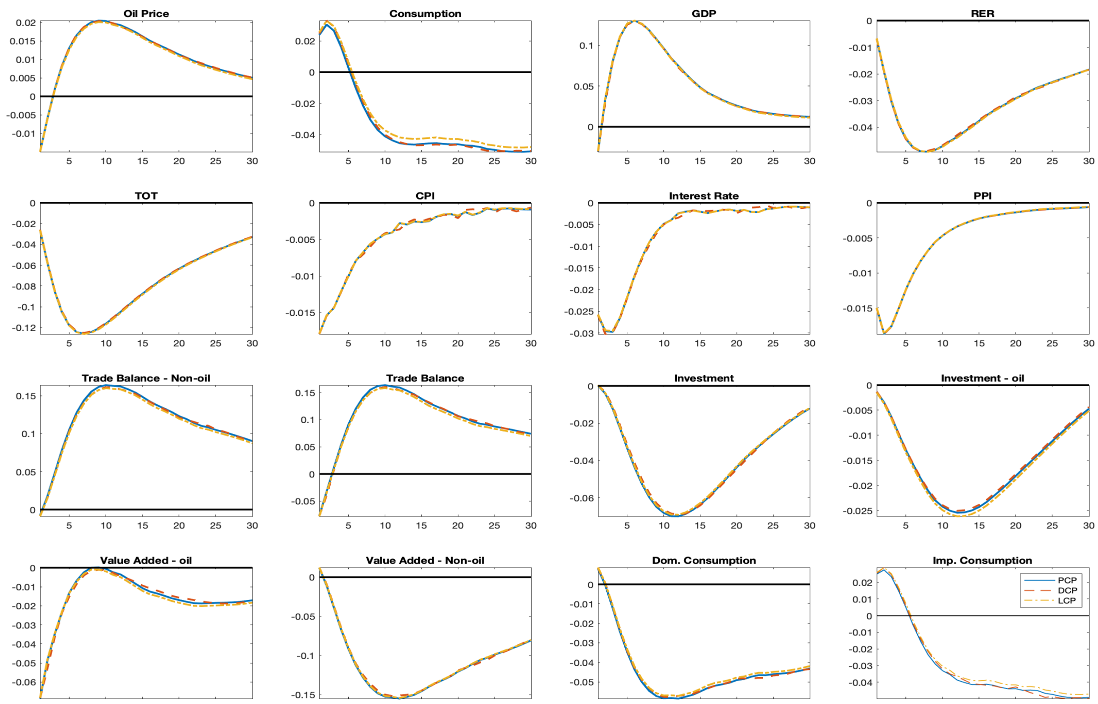
(b) Nigeria

Notes: Data on oil prices is WTI and Brent, which is reported annually and obtained from the U.S> Energy and Information Administration. Data on consumption is obtained from the World Bank's World Development Indicators (WDI). Data period is for 1971 − 2019

2.6.2 Oil Demand Shocks

Next, I look at demand side shocks originating from increase in non-oil sector firm productivity and increasing intensity of oil input usage. Figure 2.11 shows the domestic economy's response to an increase in productivity of foreign firms. The productivity shock induces an exponential and drawn-out period of higher economic activity. The economic boom persists despite the contraction consumption that occurs after about a year. Quite analogous but opposite to the supply shock, it is driven in part by an appreciation of RER, higher oil prices and a balance of trade surplus. Though, there is not an increase in value added within the oil sector. Looking at the transmission channels at play her: firstly, in line with the supply shock, value added in oil sector for the domestic economy falls. This is attributed to the fact that elasticity of substitution for oil input with non-oil sector firms production technologies is small, hence, increase in productivity generates less than substantial increase in oil demand. Additionally, given foreign economy also produces oil, their production capacity compensates for this increase in demand. As a result, domestic oil producers do not increase utilisation rates in an attempt to increase production capacity. This creates the fall in value added within the oil sector. Similarly, to the supply shock, this leads to lower investments in rigs in the domestic country. The higher oil prices generate higher oil revenues for domestic economy, which improves its current asset position. This creates an appreciation of the RER but does not lead to a worsening in the ToT. This has the following implications. Firstly, in contrast to Corden and Neary (1982), Corden (1984), and van Wijnbergen (1986) that hypothesise under the *Dutch Disease* that an appreciation of the RER creates worsening ToT is not observable here. This originates from higher domestic export prices given a higher marginal cost for domestic non-oil firms. Therefore, the higher domestic export prices and lower import prices creates an improvement in the home ToT.

Figure 2.11: Home Variables IRFs to Increase in Foreign Intermediate Goods Firm Productivity

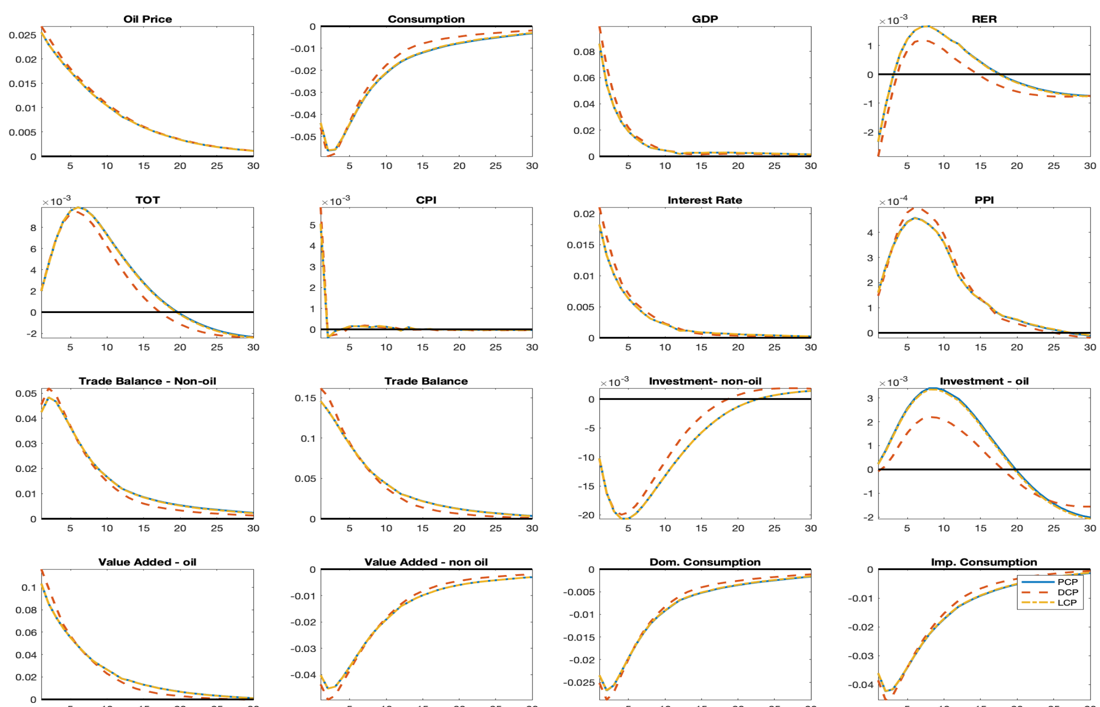


Notes: Unit Standard deviation innovation in ε_t^*. See Figure 2.9

On the other hand, Figure 2.12 displays responses increase intensity of oil usage, which creates a response that mirrors the converse of the oil supply shock. The expansionary effect on economic activity though not prolonged is driven by the same channels but with opposing effect. Here increasing in oil usage both by non-oil sector firms and households creates substantial increase in oil demand that induces domestic oil firms to increase their utilisation rates and increase production capacity to meet new demand. As present value of installed rigs rises above the current price of oil, oil producing firms respond by increasing their investment in new rigs.

Analogously, the adjustment costs of changing installed capital delays the response such that the incremental increases in investment for rigs lags behind the oil price adjustment. Higher oil prices induced by higher oil demand creates this increase in oil sector value added. Additionally, the country runs a balance of trade surplus as excess oil exports exacerbates the increase in positive trade balances. In line with the productivity shock induced higher oil prices, I observe little evidence for the *Dutch Disease* hypothesis, as the worsening of home terms of trade coincides with the appreciation of the RER. However, even with these conditions the non-oil sector trade balances are still positive. Additionally, the responses from producer price inflation (PPI) and CPI are different when compared to the productivity induced higher oil prices. Here, as it is only the intensity of oil usage that increase, the resulting oil price rise leads to an increase in marginal cost for foreign intermediate goods firms that then raises prices for foreign goods.

Figure 2.12: Home Variables IRFs to Increase in Foreign Oil Demand Intensity



Notes: Unit Standard deviation innovation in $\varepsilon^*_{d,t}$. See Figure 2.9

The effect of both demand shocks on consumption shows that transmission channel through import consumption outweighs the transmission channel through RER appreciation pass through to PPI and subsequently to CPI, as under the productivity shock, the path of nominal interest rates is kept lower due to appreciation of exchange rate passing through to PPI and subsequently to

CPI. However, rather than consumption increasing with lower nominal interest rates, consumption declines. This further reflects the greater impact imported goods have on final consumption.

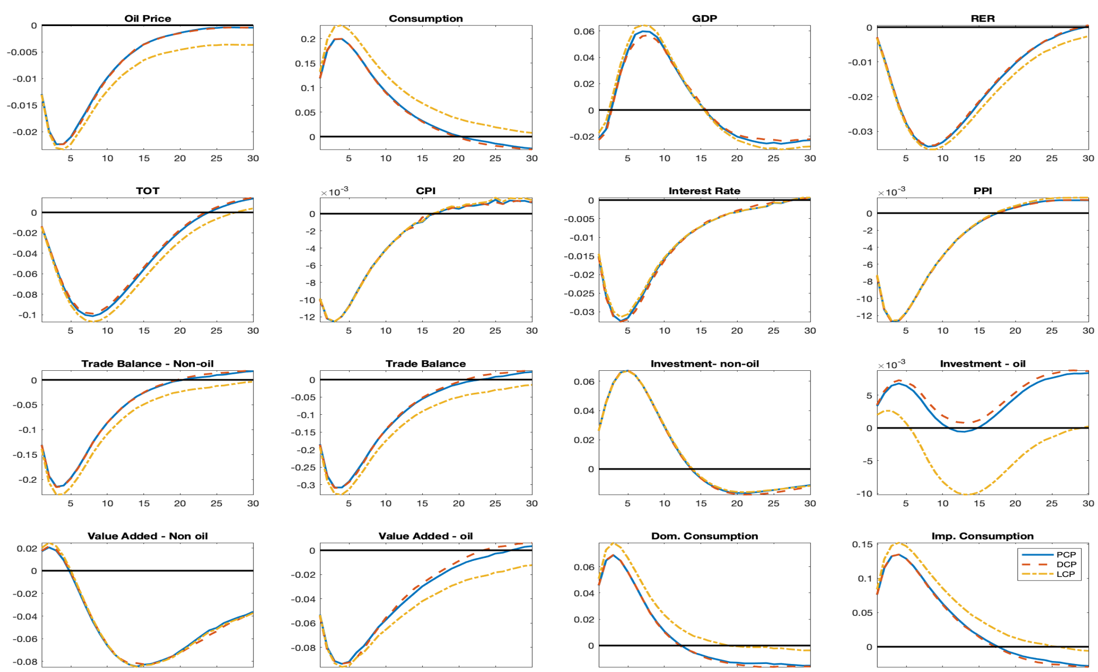
Perhaps these two shocks that are both demand side shocks corroborates one of the fundamental ideas behind this paper. While both demand shocks have induced higher oil prices, following the same transmission channels, they also differ within these channels as under higher productivity, the appreciation of the RER is not driven by the improvement in the home ToT, and the driver of the economic boom is not necessarily the oil sector in spite of higher oil prices. However, under a demand shock driven by increasing oil demand, oil sector motivates the economic boom, and the appreciation of RER is driven in parts by the improvement in home ToT. Hence, the as Bodenstein *et. al.* (2012) posited oil policy should respond to why oil prices have changed. I take this further in noting that the transmission channels for net oil exporters is considerably different for net oil importers and that differences within these channels also persist which creates a difficult environment for monetary authorities when setting interest rates.

2.6.3 Foreign Monetary Policy Shock

Killian (2009) argues that U.S. monetary policy plays a role in determining global oil prices. To understand the transmission channel, one needs to understand the way interest rates affects foreign firms. Figure 3.9 shows the response of domestic variables to an increase in foreign interest rates. The responses here tell a rich story on momentary transmission mechanisms. Firstly, it creates a demand side shock effect through lower oil demand (input materials in general). Higher interest rates increase the rental rate of capital for non-oil production firms, which decreases their level of investment. As investment goods are produced by these same firms, lower investment goods production leads to an overall lower productive output that causes the fall in demand for input materials. This leads to the lower input demand for oil. Note that the foreign economy generates more than 99% of all oil demand within the world economy, such that this lower oil demand leads to fall in oil prices.

Secondly, the rise in interest rates has expansionary effects in the domestic economy. Though the impact effect on domestic GDP is negative, there is an exponential rise and prolonged boom following the impact of the shock. Consumption increases for a prolonged period driven by both domestic and import goods consumption. This contrast the results of a *beggar thy neighbour* policy effect obtained in two-country open economy models. Here, the appreciation of the RER outweighs the contraction in oil sector from lower oil prices[51] and as such domestic consumers with preference towards imported goods are able to afford and consume more imported goods. Furthermore, the pass through of the exchange rate appreciation to PPI creates the prolonged deflationary periods, which when combined with lower oil prices creates lower CPI. This is further reflected in the trade balances, as non-oil trade balances remained negative as import consumption increased. Furthermore, I obtain that the appreciation of the RER is not coupled with a worsening in the home ToT but rather an improvement in the home ToT. To understand this, one needs to understand that the appreciation is not driven by higher revenues from oil prices (as oil prices fall), but rather from falling international prices.

Figure 2.13: Home Variables IRFS to International Monetary Policy Shock - Higher World Interest Rates



Notes: Unit Standard deviation innovation in $\varepsilon^*_{R,t}$. See Figure 2.9

Analogous to other demand side shocks, it generates effects on other key macroeconomic variables. Here, investment in the domestic economy rises, which is driven by both domestic investment and FDI given lower interest rates from lower CPI that resulted from RER appreciation.

[51] Although this may also reflect non cross-border trade in assets as the Uncovered interest parity condition would cause an immediate appreciation of foreign exchange rate, as return on foreign assets grow with higher nominal interest rates.

Whilst investment in the non-oil sector booms, investment in oil-sector booms, but only under when exchange rate pass through from the international economy to the domestic economy is perfect i.e. under PCP and DCP. With imperfect pass through, an increase in foreign interest rates generates larger nominal exchange rate (NER) appreciation than under perfect pass through, thus creating this divergence in investment within the oil sector. Notice that even though oil prices are lower, the investment in oil sector under perfect pass through is not negative, reflecting how domestic oil companies perceive the real price of oil (which is denoted in foreign currency), thus they keep investing despite the present value of installed rigs falling below the real price of oil.

2.6.4 Foreign Investments Shocks

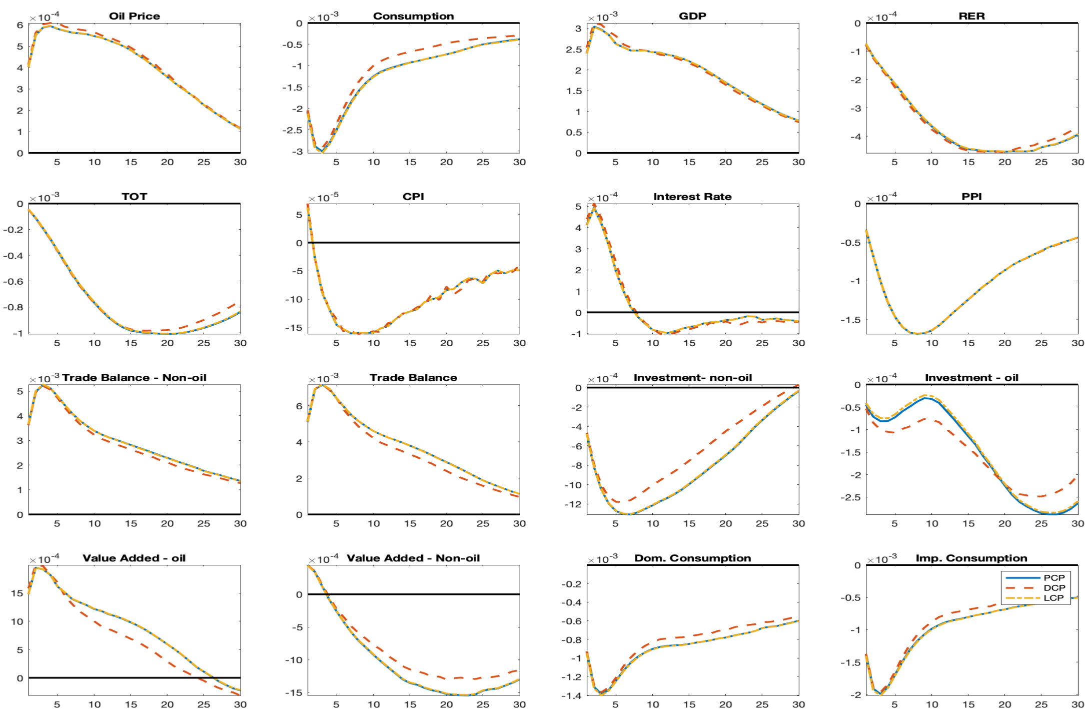
Here I look at domestic variables responses to an increase in international investment efficiency that creates higher investment demand than that obtained from investment prices and capital asset returns. Figure 2.14 plots domestic variables response to an investment shock and once again obtain an expansionary effect on domestic economic activity. GDP increases substantially and sustains this increase considerably. Here the expansionary effects arise through the increase in oil revenue from higher oil prices as well and the expansion within the world economy that generates demand for investment goods produced in the domestic economy and abroad. All observable from the higher value added generated by the oil sector and the positive trade balance held within the non-oil sector. The continual increase in oil prices improves the foreign position of the domestic economy within the world economy. In line with the previous demand side shocks, the exchange rate appreciation does not induce worsening ToT but rather an improvement with ToT. The exchange rate appreciation induces similar results as in previous shocks given the decline in domestic CPI as well as in PPI. This leads to the gradual decline in nominal interest rates as well, which responds slowly due to the impact effect of the shock on output and CPI.

On the other hand, if one looks at the shock in the oil sector in Figure 2.15, which I can interpret as an oil field discovery shock[52]. As expected, any increases in the number of oil fields creates a long-run supply-side shock that creates an increase in oil supply within global markets. This windfall shock decreases the return in oil investments. This result propagates similarly to the increase in foreign TFP in oil firms in foreign markets, but with a delayed response given the

[52] Bergholt, Larsen and Seneca (2019), Bergholt (2014).

adjustment cost and time required for new fields to begin to produce. Thus, a continual decrease in oil prices after the impact effect. This when considered with lower returns, lowers the present value of installed rigs below the price of oil and domestic oil firms respond by decreasing their own level of investment despite higher FDI in oil investment. The crucial difference between the windfall shock and the oil supply shock lies in the adjustment of exchange rates following the shock. Here there is an immediate appreciation following the impact depreciation of the RER, that begins to appreciate continually for about 3 years before beginning to depreciate again.

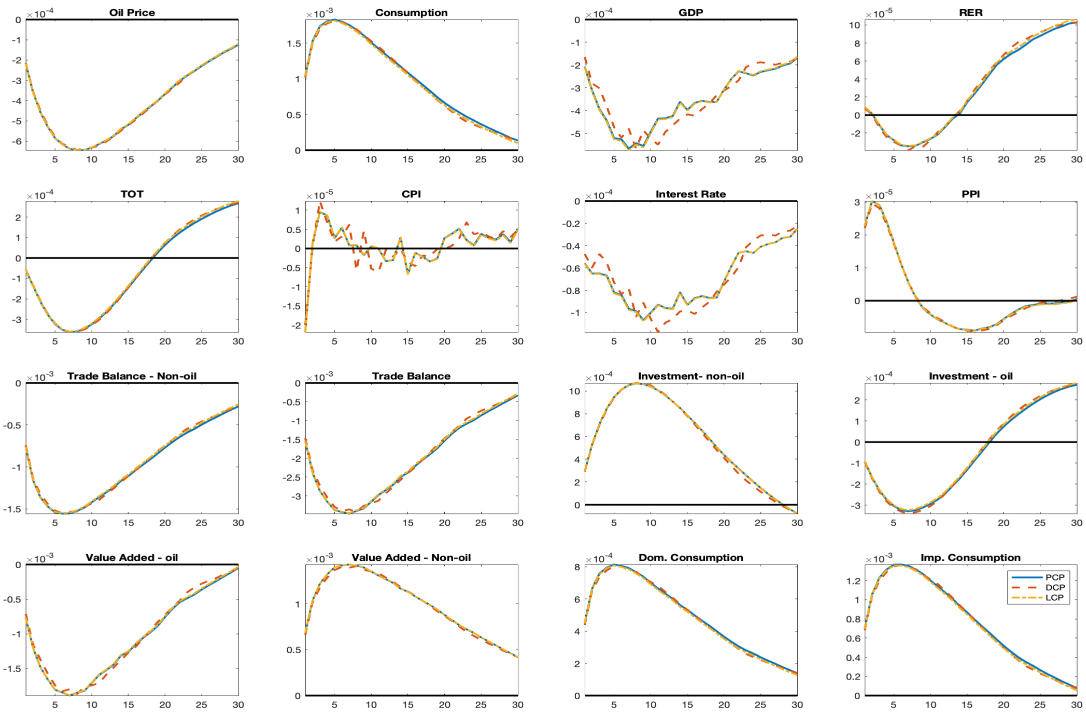
Figure 2.14: Home Variables IRFs to Increase in Foreign Non-Oil Sector Investment



Notes: Unit Standard deviation innovation in $\varepsilon^{*}_{i,t}$. See Figure 2.9

Additionally, the initial improvement in the domestic ToT also lasts for about 4 years, before gradually worsening. The resulting price responses from PPI reflects the exchange rate pass through and CPI leads to the lower path of nominal interest rates. The effect on economic activity is once again higher consumption following higher international production and lower nominal interest rates, while the contraction in the economy is driven by the value added from the oil sector. Given the initial impact effect, oil revenues decline as oil prices continues to fall, that creates the fall in sectoral value added and eventually economic output. Similarly, with a contraction in oil sector value added and non-oil sectoral trade balance, aggregate trade balance continues to decline.

Figure 2.15: Home Variables IRFs to Increase in Foreign Oil Sector Investment (Oil Discovery Shock)



Notes: Unit Standard deviation innovation in $\varepsilon^*_{iu,t}$ (Figure 2.15) . See Figure 2.9

2.7 Monetary Policy and Welfare

In standard new Keynesian framework, policy makers face the trade-off between stabilising the relevant output gap and inflation. However, within the model presented here, output gap depends on the oil technology such that the weight of output gap stabilisation is increasing in the domestic economy's dependence on oil sector. As already established, out economy is a net exporter of oil, with trade balances highly hinged on oil exports and the entire economy reliant on oil sector output. Furthermore, as detailed in the previous section, nominal interest rates play a crucial role in determining the path of real interest rates. Hence, monetary policy will play and import role in household welfare following any oil price shock. So, to observe the effect of monetary feedback parameters on welfare, I follow the works of Schmitt-Grohe and Uribe (2007), Bergholt (2014), and GalÍ and Monacelli (2016). Noting that the analytical expression of the welfare loss function is not easily derived, so I follow Galí, and Monacelli (2016) and evaluate it numerically. Specifically, I define lifetime utility under the monetary regime:

$$\mathcal{W} = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left[\frac{[C_t(l) - hC_{t-1}(l)]^{1-\sigma_c}}{1-\sigma_c} - \frac{\sum_{j=N,u} N_{j,t}^{1+\sigma_n} \Delta_{wj,t}}{1+\sigma_n} \right] \tag{2.7.1}$$

$$\mathbb{E}\left[U(C_t, N_{j,t}(l) : \varepsilon_t)\right] = U(C(1-h), N_j) \tag{2.7.2}$$

Here $\mathbb{E}$ is unconditional expectations operator, while $U(C(1-h), N_j)$ measures utility in the non-stochastic steady state. In line with Kim *et. al.* (2008), the conditional welfare criterion is chosen while I utilise the non-stochastic steady state as the initial condition. To efficiently obtain the welfare function, I simulate the model and evaluate it up to the second order as Schmitt-Grohe and Uribe (2007) have shown that up to the first order, the expected unconditional utility equals its value in the non-stochastic steady state. I simply compare the welfare responses across different policy regimes to identify what policy induces the best welfare responses.

2.7.1 Strict Targeting Regimes

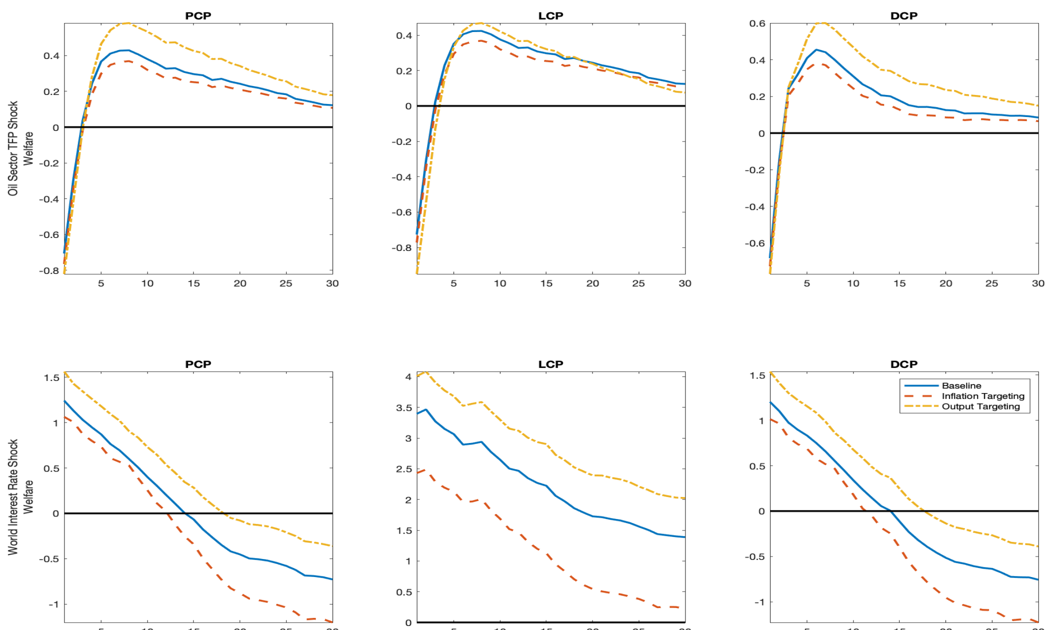
Though the Taylor rule design above has proven more efficient for policy makers in the net-exporting country to target output gap stabilisation over inflation stabilisation, it does not answer how these policies would affect welfare in the home economy. Given the transmission mechanisms of international shocks to domestic variables occurs through distinct channels, its final effect on domestic final output, consumption, inflation e.t.c. depends on some country specific factors such as monetary and fiscal policy. In this section, I inspect one of the transmission channels - monetary policy. Simply seeking to answer the question of how monetary policy can be used to insulate the home economy from prominent volatilities in commodity markets. To this end I conduct a series of counterfactual experiments over the three pricing regimes and display the results for specific variables of interest. In doing so, I consider two cases here: i) a strict output targeting regime with higher output feedback such that $\rho_y = 2$, $\rho_{\Delta_y} = 0.5$, $\rho_\pi = 0.1$ and $\rho_\mathcal{E} = 0$ and a strict inflation stabilisation regime i.e. $\rho_y = 0.1$ $\rho_\mathcal{E} = \rho_{\Delta_y} = 0$[53]. Finally, I will only consider four shocks - a positive foreign oil TFP shock, a foreign monetary policy shock, a positive foreign non-oil TFP shock and a demand intensity shock, i.e. a supply side shocks and a demand side shocks.

[53]While I refer to this policy regimes as strict, they are not completely strict as I cannot utilise strict output gap targeting by setting $\rho_\pi = 0$ as this will result in indeterminacy of Blanchard-Kahn conditions. So, to keep consistency across regimes I utilise the minimum inertia 0.1 for output gap when looking at strict inflation targeting and vice versa.

Oil Price Declines

To contrast the different strict targeting regimes I consider two shocks processes that led to a fall in oil prices[54]. Figure 2.16 contrasts the impulse responses under different strict Taylor rule targeting regimes given a fall in oil prices. I obtain an important result when comparing the different policy targeting regimes. Strict output gap targeting performs best in terms of household welfare compared to strict inflation targeting and the baseline Taylor rule. Output gap targeting performs better than inflation targeting due to the presence of oil sector output driving domestic output.

Figure 2.16: Home Welfare Responses to Oil Sector Productivity and Foreign Monetary Shocks



Notes: Impulse response to one standard deviation innovations in $\varepsilon^*_{u,t}$ (first row) and $\varepsilon^*_{R,t}$ (second row). Blue solid line represent the baseline Taylor rule, Red dashed line represents strict Inflation Targeting, and Yellow dashed line represents strict output gap targeting.

Additionally, the presence of intermediate imported consumption goods within household consumption basket implies that PPI is driven mainly by import prices rather than domestic prices and ultimately exchange rate pass through. In other words, the depreciation of real exchange rate, terms of trade and household preference towards imports induces inflationary pressures driven by import prices that are set in the foreign country. Hence any attempt to stabilise this

[54] A positive shock to oil sector productivity and a positive shock to world interest rates.

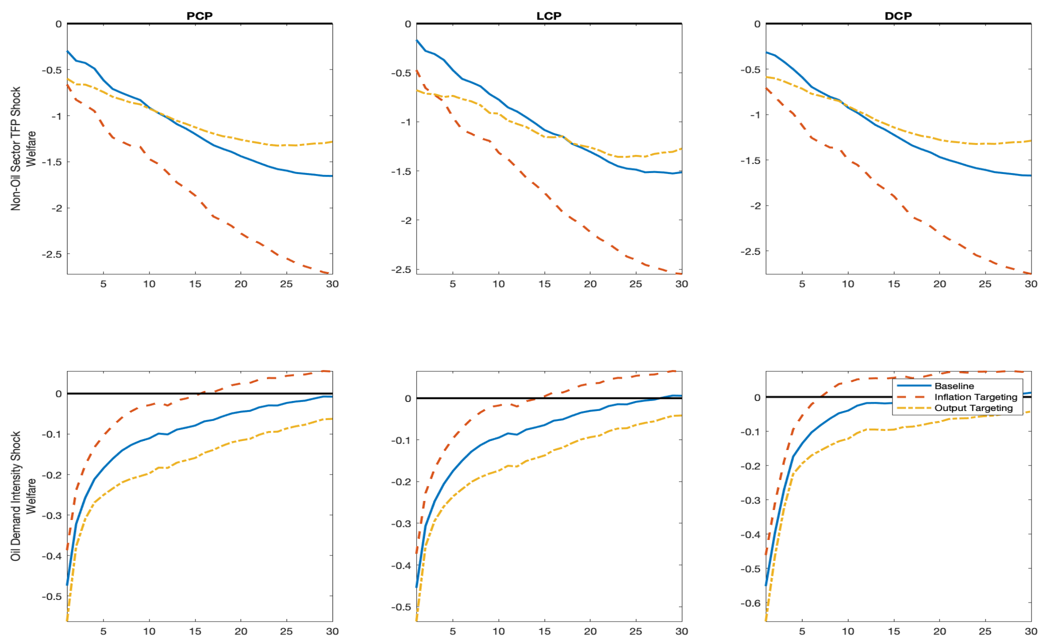
inflation by increasing interest rates as inflation exponentially rises will be mute. Therefore, keeping interest rates lower would not necessarily worsen inflation, but will have benefits towards GDP given oil technology contributes significantly to domestic output.

Understanding the macroeconomic inefficiencies arising from Taylor rule dynamics is crucial in understanding the short-run effects of monetary policy stabilising output, inflation, and the nominal exchange rate. As the Taylor rule would look to stabilise inflation over GDP, but given GDP now includes oil production technology, the impact effect of the shock on core CPI induces an impact fall in nominal interest rates, which exponentially rises following inflation. This also contrasts the result from Galí and Monacelli (2005) that show that in small open economies PPI targeting is optimal. In this case (though not shown here), PPI targeting will result in a stronger interest rate response that will worsen the recessionary impacts following oil prices fall. This inefficiency arises from increasing interest rates attempting to stabilise inflationary pressures from exchange rate depreciation, oil and non-oil sector outputs are unable to recover from shock impact as quickly as it can. Intuitively, the inflationary pressures from exchange rate depreciation would eventually stabilise as higher import prices would eventually generate expenditure switching effects. Therefore, keeping interest rates lower should induce firm investments and increase output thereby reducing the duration of the recession.

Increases in Oil Prices

Contrasting the supply side shocks, I look at demand shocks that increase oil prices in Figure 2.17. Immediately observe that indeed no one policy objective is optimal for the central banker. Under the supply side shocks, output gap targeting ranked top for household welfare. However, under higher oil demand intensity, inflation targeting becomes best. Intuitively, the pass through from RER to PPI does not create inflationary pressures as exchange rate actually appreciates. Rather, since CPI is driven by higher oil prices and PPI is driven by higher domestic price inflation from higher input prices. Hence, inflationary pressures are driven both domestic and abroad, therefore aims at stabilising inflation will prove efficient. In this case a better option than output gap targeting. To understand this, look at consumption, domestic output and CPI inflation responses. With higher domestic output, nominal interest rates are kept higher which shifts the path of real interest rates and decreases domestic consumption through the Euler equation. Similarly, the higher interest rates cause a faster contraction to domestic output. Since the effect on CPI occurs in the first period, the higher interest rates also target the higher level of inflation although not its actual aim. The result is lower consumption and a contraction in GDP that leads to lower household welfare.

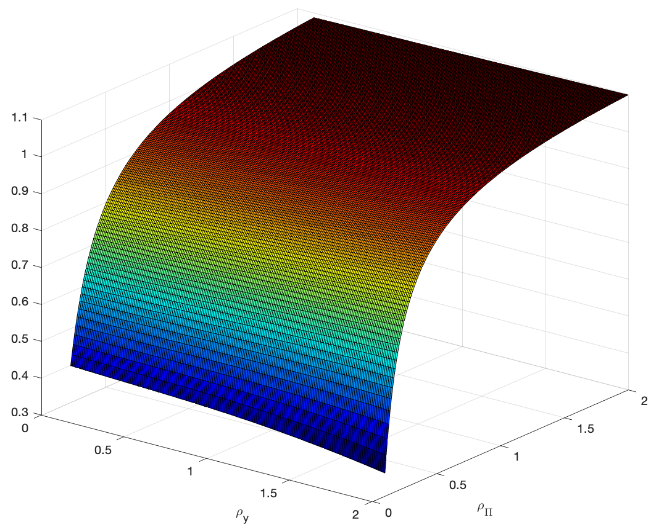
Figure 2.17: Home Welfare Responses to Oil Demand Side Shocks



Notes: Impulse response to one standard deviation innovations in ε_t^* (first row) and $\varepsilon_{d,t}^*$ (second row). Blue solid line represent the baseline Taylor rule, Red dashed line represents strict Inflation Targeting, and Yellow dashed line represents strict output gap targeting.

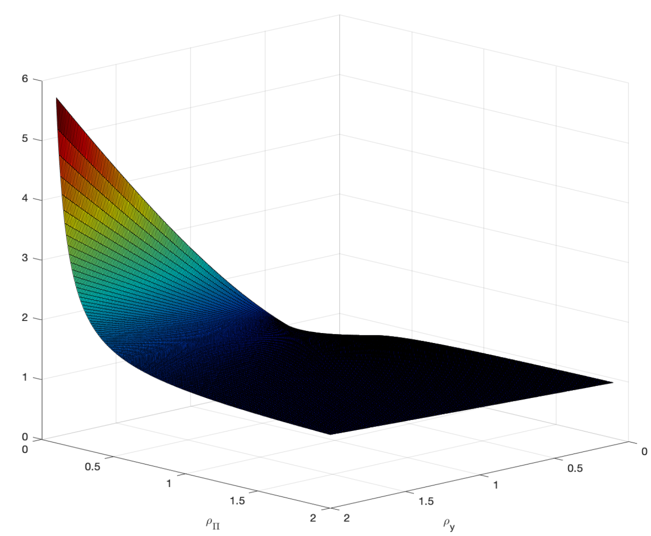
Though the result from non-oil sector productivity shock also showcases that even though oil prices have increased, the optimal targeting regime is not identical. Here I find that policy aimed at both inflation and output gap has optimal household welfare effects, as opposed to a stricter targeting regime that either stabilises inflation alone or output gap alone. While strict targeting regimes gives an idea of how central bankers should set policy, implementing them requires that monetary authorities have true knowledge about Taylor Rule feedback policy for private agents within the economy, which ultimately determines the macroeconomic dynamics. Furthermore, it requires monetary authorities have complete information about exogenous shock processes and about the true state of the economy.

Figure 2.18: Taylor Rules and Home Welfare Under Increased Foreign Oil Firms Productivity



Notes: Impulse response to one standard deviation innovations in $\varepsilon^*_{u,t}$ looped over the the policy feedback parameters ρ_y, $\rho_\pi \in (0.1, 2]$ in the Taylor Rule.

Figure 2.19: Taylor Rules and Home Welfare under Increased Foreign Intermediate Good Firms Productivity



Notes: Impulse response to one standard deviation innovations in ε^*_t looped over the the policy feedback parameters ρ_y, $\rho_\pi \in (0.1, 2]$ in the Taylor Rule.

This makes the strict targeting regimes discussed above difficult to implement as this information is not available to monetary authorities. Therefore, though it is possible to explicitly observe the best responses, in reality a response that puts significant weight on both inflation stabilisation and output gap will prove more effective.

In summary, looking at Figure 2.18 & Figure 2.19, under the oil sector TFP shock the best outcome arises when output gap has highest feedback to the Taylor rule, with the worst outcome arising from inflation stabilisation. Similarly, under a non-oil sector TFP shock, the best outcome arises when inflation stabilisation is prioritised as opposed to output gap, and the worst outcome occurring if policy maker chooses to not do anything.

2.7.2 Monetary Policy, Oil Prices, Exchange Rates and Inflation

While I have argued based on simulation exercises the role oil prices play in driving exchange rates, and ultimately inflation that renders interest rates movement almost moot in affecting inflation, I turn to the data once more to further understand the relationships. Here I look at Historical decompositions, as historical decompositions outline individual structural shocks contributions in displacing the endogenous variables (Exchange Rates, Inflation, and Interest rates) within the VAR from their unconditional means. To obtain the historical decompositions, I first estimate a 4-Variable SVAR with Cholesky identification to decompose the contributions of oil price movements to movements in exchange rates and ultimately in inflation. I estimate the following[55]:

$$Z_t = A_1 Z_{t-1} + A_2 Z_{t-2} + ... + A_k Z_{t-k} + B\varepsilon_t \tag{2.7.3}$$

$$Z_t = \sum_{j=1}^{k} A_j Z_{t-j} + B\varepsilon_t \tag{2.7.4}$$

To obtain the Historical decomposition, represent the SVAR model in equation (2.7.4) with the

[55]The Shawrz Bayesian Information Criterion (SBIC) and Hanna-Quinn Information Criterion (HQIC) specify an optimal lag length of 2 for the Model, this what is chosen in the estimations.

Wold representation[56] given by:

$$Z_t = A^t Z_0 + \sum_{j=0}^{t-1} A^j B \varepsilon_{t-j} \tag{2.7.5}$$

$$Z_t = [P^*_{o,t}\ \mathcal{S}_t\ \Pi_t\ i_t]'. \qquad \varepsilon_t \sim N(0,1)$$

Under the Wold representation of the VAR, I can write Z_i as a function of the present and past structural shocks ε_t^{oil}, ε_t^{ner}, and ε_t^{i} plus initial conditions given by Z_0. Here $P^*_{o,t}$ represents international spot prices for Brent or WTI, $\mathcal{S}_t$ is nominal exchange rate, Π_t is inflation (CPI), and i_t is the lending rate[57]. The price of Nigerian oil is closely related to Brent prices, hence, I use Brent oil spot prices in the estimation with Nigerian sample, while WTI is used in the estimation of Mexican oil[58].

The results of the Historical decompositions of identified structural shocks for Exchange rates and Inflation are plotted in figures below[59]. From the Figure 2.20 and Figure 2.21 below, the increasing contribution of Brent/WTI price shocks to displacing CPI and the nominal exchange rates from their unconditional means is clear. What is particularly evident is inflation's unresponsiveness to changes in interest rates, as interest rates shocks contribute the least to the displacement of CPI from its unconditional mean. In fact, looking at nominal exchange rates decompositions, shocks to interest rates play little to no role in displacement from its unconditional mean. Here is the main catch, while nominal exchange rates and CPI are unresponsive to change in the interest rates, the biggest displacement of interest rates is due to changes in CPI (which is expected given the inflation targeting regimes the central banks utilise). Though this exercise was simplistic, its aim was to showcase that exchange rates and inflation in oil economies are particularly driven by oil price shocks, with interest rates playing a second-hand role. This further accentuates how inflation targeting regimes can become redundant in these economies and less welfare improving in the face of oil price declines.

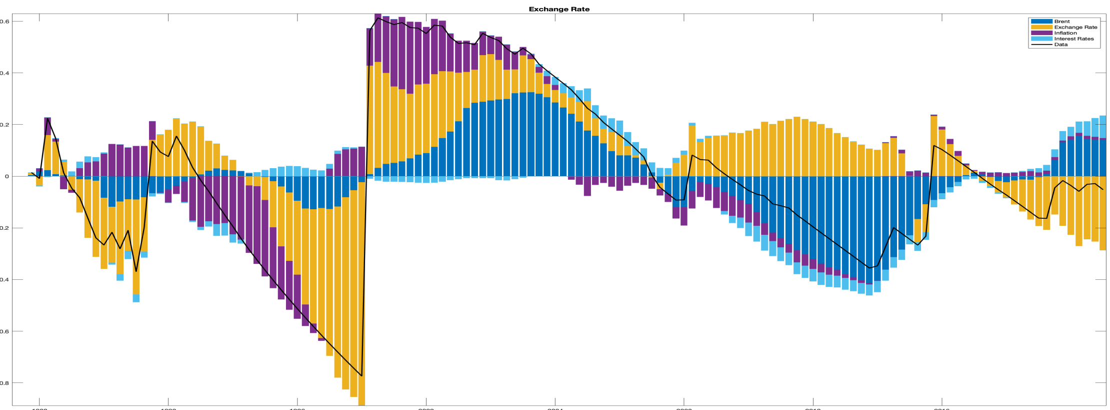
[56] Wold representation re-expresses the combination of each observation within the SVAR model, in other words it shows that each observation in the SVAR can be expressed as a combination of both a stochastic and deterministic series. Here the stochastic series is the sum of previous and present structural shocks, while the deterministic series is the initial condition. See Wold (1938).

[57] Central Bank Policy rate is unavailable in both countries until late 2000s, which decreases the sample size considerably. So, I utilise the lending rate given the lending rate is the policy rate with an additional "mark-up" to be crude.

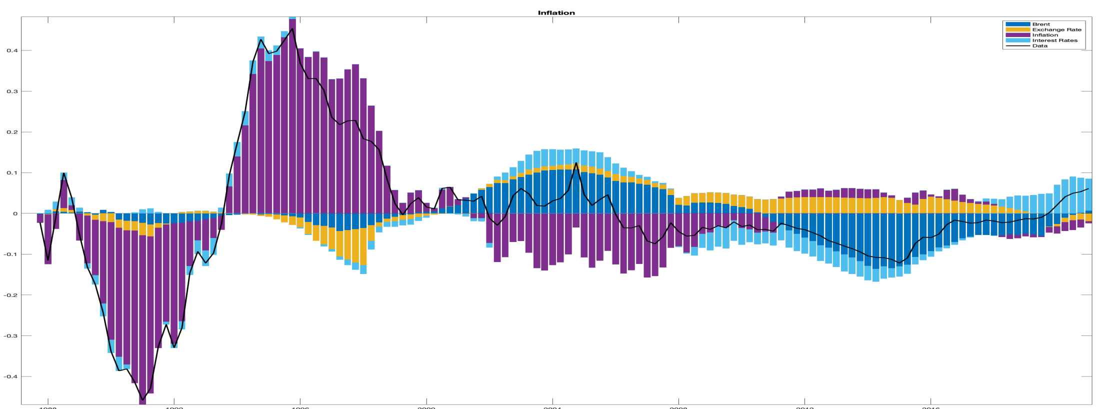
[58] For robustness, the estimation is redone utilising WTI prices in Nigeria's case and Brent prices in Mexico's case. The results are similar and are shown in Figure 5.11 for Nigeria and Figure 5.12 for Mexico in Appendix E

[59] The corresponding IRFs are not of imminent interest and can be found in Figures 5.4 & 5.6 in Appendix E.

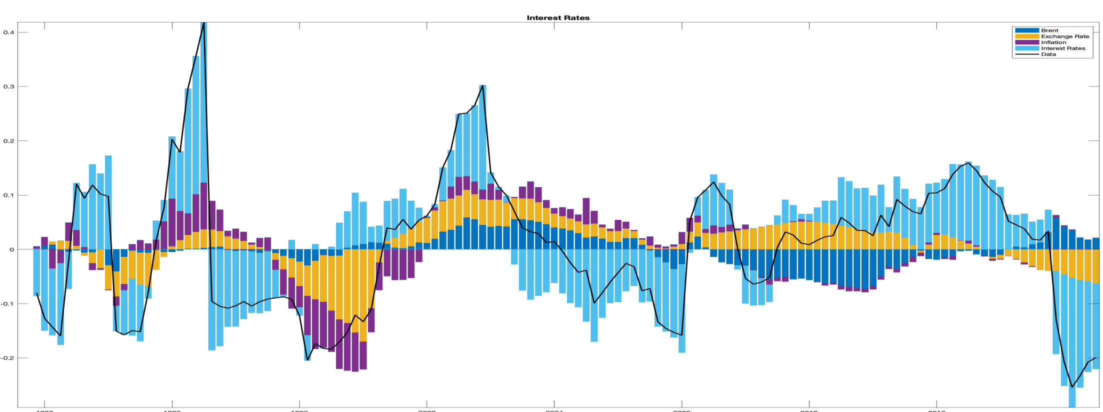
Figure 2.20: Historical Decompositions of Exchange Rates, Inflation and Interest Rates for Nigeria



(a) Historical Decomposition of Nominal Exchange Rates



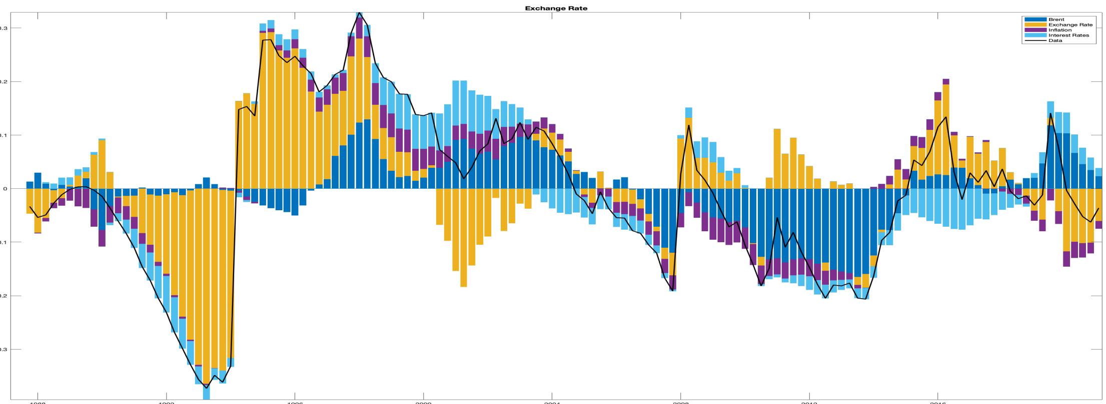
(b) Historical Decomposition of Inflation



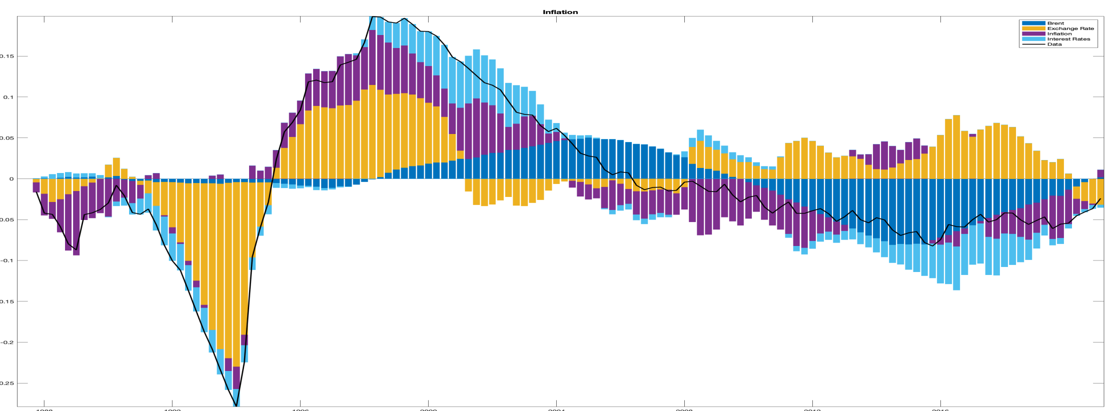
(c) Historical Decomposition of Interest Rates

In (a), (b) and (c) the solid black line depicts the data for log of Nominal Exchange rates, Consumer Price Index and Lending rates, respectively. Blue bars represent historical contribution of Brent oil prices, Yellow, Nominal Exchange rates, Purple; Inflation, and light blue; Lending Rates. Data period is between 1988Q1: 2021Q4. Stability of the SVAR displayed in Figure 5.2 (b) in Appendix E.

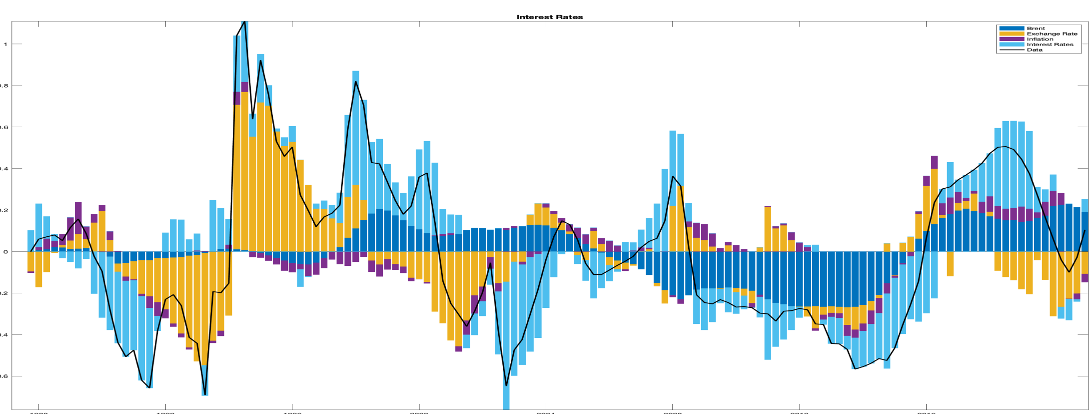
Figure 2.21: Historical Decompositions of Exchange Rates, Inflation and Interest Rates for Mexico



(a) Historical Decomposition of Nominal Exchange Rates



(b) Historical Decomposition of Inflation



(c) Historical Decomposition of Interest Rates

In (a), (b) and (c) the solid black line depicts the data for log of Nominal Exchange rates, Consumer Price Index and Lending rates, respectively. Blue bars represent historical contribution of WTI oil prices, Yellow; Nominal Exchange rates, Purple; Inflation, and light blue; Lending Rates. Data period is between 1988Q1: 2021Q4. Stability of the SVAR displayed in Figure 5.2 (a) in Appendix E.

2.7.3 Inspecting The Dutch Disease

Corden and Neary (1982), Corden (1984), Neary and van Wijnbergen (1986), theoretically argued that surges in oil prices create exchange rate appreciations that crowd out non-oil exports through expenditure switching effects arising from improving terms of trade[60] of the resource rich country. The model here allows me to put this hypothesis to the test and inspect transmission channels through which the Dutch Disease could emanate. In accordance, I simulate two shocks that lead to both oil price increases and oil price falls and look at the pass through to the RER, Terms of Trade and aggregate trade balances.

Under the demand shocks, I observe differing paths of the RER and ToT with higher oil prices. The depreciation of the RER after the oil intensity demand shock and the worsening of domestic ToT contradicts the prediction of the Dutch Disease hypothesis. The worsening of domestic ToT here implies home exports prices are cheaper than relative import prices, which accords from the depreciation of the RER, which under the hypothesis should be the reverse. However, the appreciation of the RER and improvement of home following the rise in oil prices is in line with the prediction of the hypothesis. Though aggregate trade balances have differing impact effects, it eventually becomes positive, with TFP shock starting out negative following the path of oil prices.

On the other hand, with lower oil prices, the pass through to exchange rate is not identical. The exchange rate appreciates and continues to appreciate under higher interest rates induced oil price declines, while under oil supply induced oil price declines the RER depreciates on impact but appreciates only for about 1 year before further depreciation. The distinction between both transmission to RER occurs due to lack of cross border trade in assets. Without the UPI condition holding, higher interest rate in the foreign economy does not create capital outflow from the domestic economy to the foreign economy with higher asset returns and thus does not lead to a domestic currency depreciation. Therefore, holding debt in the domestic economy outweighs asset returns in the foreign economy. Similarly, the ToT under both shocks is improving, with aggregate trade balances declining following lower oil revenues.

[60] Improving terms of Trade reflects higher domestic export prices relative to import prices. In Corden and Neary (1982), Corden (1984), Neary and van Wijnbergen (1986), this is viewed as a worsening in the terms of Trade as manufactured goods within the domestic country become relatively more expensive, thus, making it less competitive.

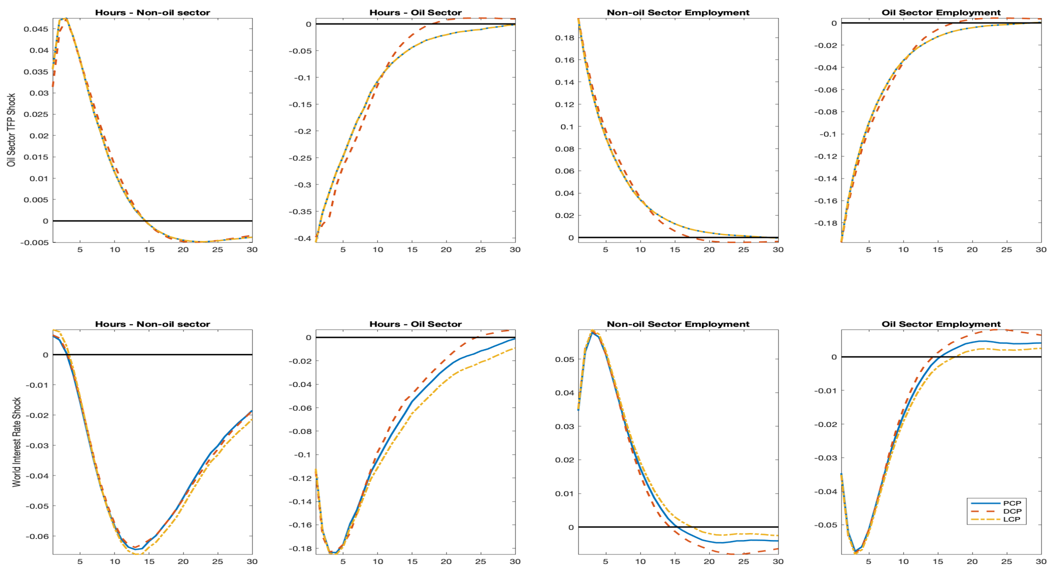
Additionally, Caselli and Cunnigham (2009) argue that oil windfalls create reallocation of productive labour from service sector to resource sectors, which are inefficient. With sectoral labour supply I am also able to observe reallocation effects of labour following oil price changes. Labour demand is bundled by:

$$N_{j,t}^{d} = \left(\left(\frac{1}{\eta_j} \right)^{\frac{1}{\theta_w}} \int_0^1 N_{j,t}(l)^{\frac{\theta_w - 1}{\theta_w}} dl \right)^{\frac{\theta_w}{\theta_w - 1}} , \quad \theta_w > 1$$

$N_{j,t}(l)$ is the sectoral supply of type l homogenous labour and $N_{j,t}^{d}$ is the final sectoral demand of labour by intermediate goods firm, and η_j measure the portion of the workforce employed in each individual sector such that $\sum_{j=N,O} \eta_j = 1$. Conversely to Bergholt, Larsen and Seneca (2019), I assume free mobility of workers across sectors. This allows me to investigate the impact of oil price fluctuations on level of workforce employed in each sector. The model is simulated with the baseline calibration in section 4.

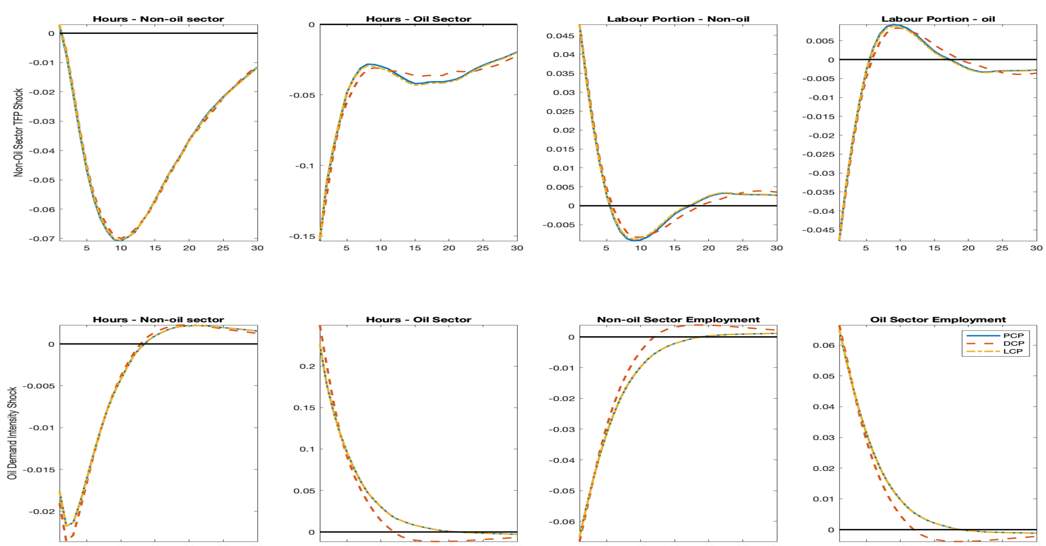
Figure 2.22 showcases the impulse responses of hours worked following a decrease in oil prices from higher oil supply and higher foreign interest rates. In line with the Dutch Disease, when oil revenues fall following a crash in oil prices, hours across oil sectors contracts, with reallocation of labour from oil-sector to the non-oil sector. Although under higher interest rates induced oil price decline, hours in both sectors fall. So, the fall in oil sector hours worked is due to lower oil revenues and lower wages within the oil sector as real wages in oil sector are increasing in oil prices. Under demand driven higher oil price hikes shown in Figure 2.23, the consequence is higher hours worked and reallocation of labour towards oil sector under the oil demand intensity shock. This simply follows from the fact that it is only under oil demand intensity shock that domestic oil firms benefit from the most as shown by the value added in oil sector GDP. Hence, the higher revenues create more demand for labour as well as higher wages for workers. Similarly, the response of sectoral GDP mimic the responses of labour and hours worked. From Figure 2.24a I obtain the same sectoral shifts in sectoral GDP (value added). Here given oil price hikes from oil specific demand intensity, the sectoral value added for GDP shifts towards the oil sector as oil prices increase marginal cost for non-sector oil firms and decrease their level of production. Although, under the oil price hike from increase in foreign non-oil sector firms' productivity the sectoral shifts are not as significant as the oil intensity demand specific price hike.

Figure 2.22: Home Labour Market Responses to Foreign Oil Sector TFP and Monetary Policy Shocks



Notes: Impulse response to one standard deviation innovations in $\varepsilon^*_{u,t}$ (top row) and $\varepsilon^*_{R,t}$ (bottom row). Blue solid line represents PCP, Red dashed line DCP, and yellow dashed line LCP.

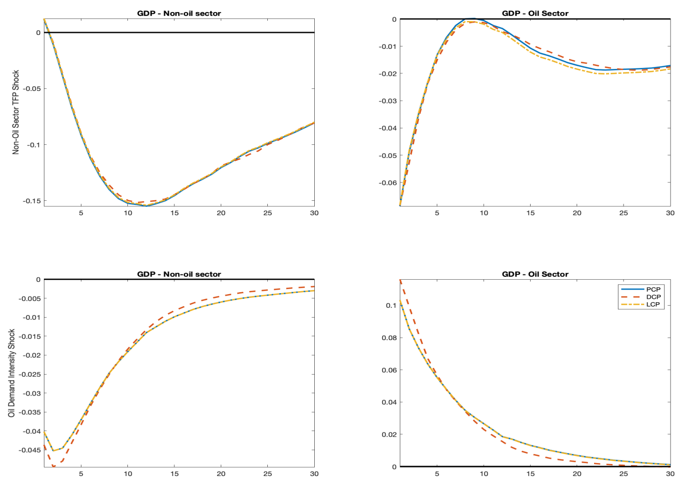
Figure 2.23: Home Labour Market Responses to Foreign Oil Demand Side Shocks



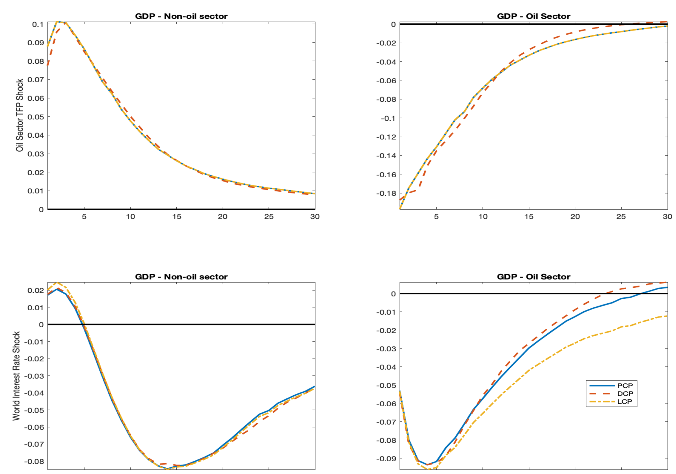
Notes: Impulse response to one standard deviation innovations in ε^*_t (top row) and $\varepsilon^*_{d,t}$ (bottom row). Blue solid line represents PCP, Red dashed line DCP, and yellow dashed line LCP.

This is owing to the foreign oil production capacity being able to account for the increase in oil demand from higher productivity levels, such that the domestic oil production is unable to benefit from the higher oil prices[61]. Analogously, from Figure 2.24b during oil price declines there is a sectoral shift from oil sector towards non-oil sector as GDP from non-oil sector booms while oil sector GDP declines following lower oil prices. This dynamic adjustment is more pronounced under the oil supply shock (oil sector TFP shock) than from higher world interest rates.

Figure 2.24: Home Sectoral GDP Responses to Foreign Oil Demand and Supply Side Shocks



(a) Demand Side Shocks



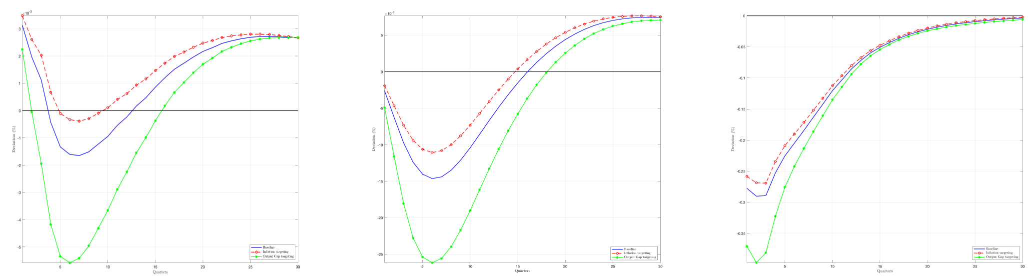
(b) Supply Side Shocks

Notes: In panel (a) impulse response to one standard deviation innovations in ε_t^* (top row) and $\varepsilon_{d,t}^*$ (bottom row). While panel (b), impulse response to one standard deviation innovations in ε_t^* (top row) and $\varepsilon_{d,t}^*$ (bottom row). Blue solid line represents PCP, Red dashed line DCP, and yellow dashed line LCP.

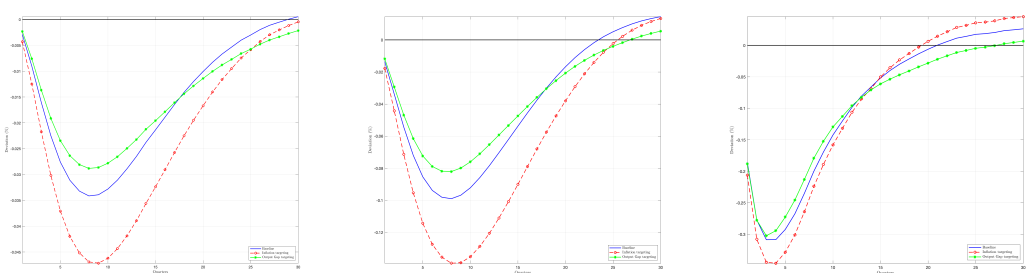
[61] This was already explained in more detail in section 2.6.2.

Furthermore, Caselli and Cunningham (2009) argue that central banks can play a role in helping resource rich developing countries avoid the Dutch disease through policies that stabilise the exchange rate. Similarly, Agenõr (2016) shows how governments are capable of impacting the pass through of international shocks to the RER. So, to this end, I want to observe how the counterfactual specified Taylor rules perform in avoiding the Dutch disease. Here I simulate the two TFP shocks, one in oil sector and one in non-oil sector, which from above create an economic outlook that is in line with the Dutch Disease[62]. The result is displayed in Figure 2.25 Under

Figure 2.25: Exchange Rate, Terms of Trade Fluctuations and Trade Balances Under Strict Targeting



(a) Real Exchange Rate (b) Terms of Trade (c) Trade Balance



(d) Real Exchange Rate (e) Terms of Trade (f) Trade Balance

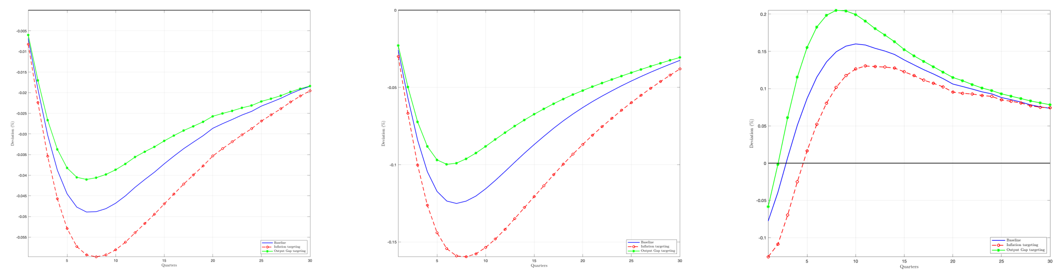
Note: One Standard Deviation Innovation in $\varepsilon^{*}_{u,t}$ (first row) and $\varepsilon^{*}_{R,t}$ (second row). Blue solid line is baseline Taylor Rule, Red is inflation targeting and Green is Output gap Targeting.

an oil supply shock monetary authorities face another trade-off between the RER and aggregate trade balances. Output stabilising policies will appreciate the and prolong the appreciation of the RER, which also prolongs the improvement in ToT. However, with lower domestic import prices and higher domestic export prices, the aggregate trade balances decline even more, but consumption increases with domestic output recovering faster. On the contrary when examin-

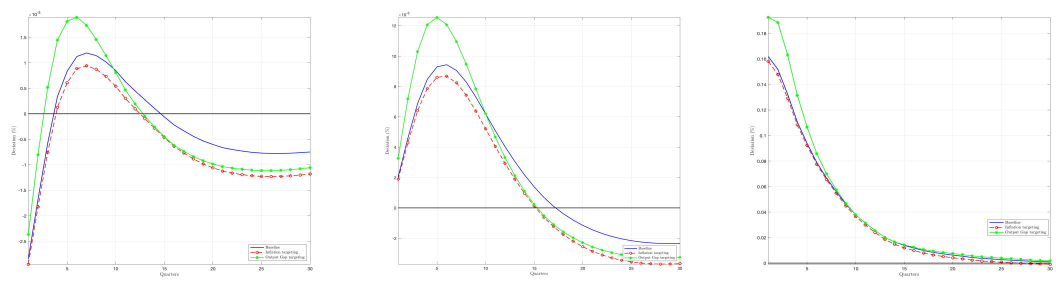
[62] As the size of the shocks do not create large significant differences between policy regimes, I only report the result for impulse responses under DCP pricing regime

ing the impulse responses for a monetary shock, the results are reversed for policy objectives. Here output stabilising policies will limit the magnitude of appreciation and improvement of ToT,which ultimately limits the benefits to aggregate trade balances. . Here output gap target-

Figure 2.26: Exchange Rate, Terms of Trade Fluctuations and Trade Balances Under Strict Targeting



(a) Real Exchange Rate (b) Terms of Trade (c) Trade Balance



(d) Real Exchange Rate (e) Terms of Trade (f) Trade Balance

Note: One Standard Deviation Innovation in ε_t^* (first row) and $\varepsilon_{d,t}^*$ (second row). Blue solid line is baseline Taylor Rule, Red is inflation targeting and Green is Output gap Targeting.

ing decreases the level of appreciation and improvement in domestic ToT and decreases the level of trade deficit. This is why under the welfare effects, output gap targeting under the oil TFP shock increase welfare substantially compared to inflation targeting. Thus, monetary authorities can trade domestic output recovery and higher consumption with higher trade deficits. On the other hand, looking at oil demand shocks in Figure 2.26, with higher foreign productivity monetary authorities also face this trade-off between the RER and trade balances. Here, output targeting decreases the level of appreciation of the RER as well as the level of improvement in the ToT but leads to a faster recovery of trade balances and even higher peak, with prolonged duration. Analogously, even though the policy rule targets output gap, domestic GDP falls the most and never becomes positive, with consumption falling on impact and declining onwards.

Thus, inflation targeting proves more useful here as it appreciates the RER, improves ToT, recovers domestic output faster and even leads to higher peak consumption and a prolonged positive duration of consumption. Looking at an oil intensity shock, I get the same trade-off story and welfare effects. I have arrived at a commonality - policies that limit the effects of the Dutch Disease perform poorly for household welfare.

While I have obtained some results that are in line with the Dutch Disease, I have to make the same addendum as with policy response. That is, given oil prices are simply symptoms of underlying shocks - not all oil price changes transmit in the same way and are thus not going to create a Dutch Disease scenario for central banks and governments to deal with. Hence it is possible for an oil price hike or crash to not appreciate or depreciate the net-exporters real exchange rate as in the case with foreign monetary shock that crashes oil prices but appreciates real exchange rate. Therefore, the effect of oil price volatility on non-oil exports in the net-oil exporter will hinge on its transmission to real exchange rate and terms of trade. Therefore, in line Caselli and Cunningham (2009) monetary authorities through exchange rate stabilising policies reduce the impact of the Dutch Disease but require information of the transmission channel through which oil price volatility is acting[63].

2.8 Conclusion

IMF (2015) shows that a substantial fraction of global economic activity can be accounted for by commodity driven economies. Nonetheless, the existing literature on policy conduct considering oil price fluctuations are primarily done from the aspect of the commodity importer, with a simplistic view on the supply side of global oil markets. In regard to policy conduct in commodity exporters, the literature is still at its infancy and prescribed through the lenses of a high-income country, while ignoring the fact that most commodity driven economies are low or middle income. In my analysis, I relax these assumptions about the supply and demand side of global oil markets and analyse how policy should be conducted given the economy is a low or middle-income commodity net-exporter.

[63] It is also worth noting that the volatility of ToT is greater in developing economies than it is in already established industrial economies. Edwards and Levy-Yeyati (2005) find that countries with greater exchange rate flexibility are able to better absorb ToT shocks through exchange rate adjustments than output adjustments.

I develop on a two-country New Keynesian DSGE model that encompasses a supply-side oil block within each country and trade in oil and non-oil goods, with the home country being a strict net-exporter of oil. The model displays how monetary and fiscal policy conduct is influenced by a substantial number of international shocks that drive fluctuations in real oil prices, and crucial domestic variables such as output, inflation, and consumption. In line with Bodenstein *et. al.* (2012) the model shows that it is necessary to distinguish between structural shocks and oil demand shocks, as structural shocks themselves create fluctuations in the real price of oil and further transmits to fluctuations in crucial macroeconomic variables at home.

The model here does provide insight on transmission channels in commodity driven economies. Here, international shocks that create fluctuations in oil prices transit to the home country through consumption basket and through domestic oil producing firms demand value added to the domestic economy. Firstly, with foreign bias in home country's domestic consumption basket import prices and exchange rate pass through will drive PPI and subsequently CPI, thus, exchange rate depreciation will induce inflationary pressures within the domestic country. Therefore, focussing on monetary policy design, with aggregate output including oil producing technology, output gap inertia in the Taylor rule is increasing with the importance of the oil sector. So, optimal policy design during oil price decline prioritises stabilising output gap over inflation as this decreases length of recessions, and is welfare improving above inflation targeting. On the other hand, during periods of higher oil prices monetary policy should prioritise stabilising inflation over output gap as exchange rate appreciation creates lower inflationary pressures and performs better for household welfare over output gap stabilisation. Furthermore, I observe a trade-off between welfare optimising policies and Dutch Disease prevalence policies.

These results obtained here are in line with those obtained by Ferrero and Seneca (2015), who argued that in net-exporting oil countries, policy makers shouldn't raise interest rates due to inflationary pressures induced by exchange rate deprecations as a result of falling oil prices, as aggregate output now encompasses oil production technology. Additionally, Frankel (2018) reaches a similar conclusion to ours, he argues that middle income commodity exporting economies should also abandon inflation targeting in favour of output targeting. Finally, though Bodenstein *et. al.* (2012) and Killian (2010) are correct in how monetary policy should react to the underlying reason oil prices have changed, it has to be amended for the net-exporter especially in the case of low- or middle-income countries due to different transmission channels arising from foreign bias in home consumption basket. Furthermore, even if the net-exporter is a high-income country, Bergholt, Larsen and Seneca (2019) also show that with offshore oil producing technology, the transmission channels of oil price fluctuations to a high-income net-exporting country are different to that of a net-importing country.

Though the model presented here I feel provides a step in the right direction in improving our understanding of the interaction between oil exporting countries and monetary policy conduct, I do have ideas for future research. Firstly, it would be useful to extend the model to focus on optimal monetary policy design under a Ramsey welfare system. This extension would provide optimal weights on monetary policy rule when stabilising output differences and inflation gap. It would also provide comparisons for the other two monetary policy alternatives studied here. Similarly, low- and middle-income commodity economies are typically plagued with inefficient institutions that lead to more than optimal government consumption that move cyclically with higher oil revenues, which ends up creating procyclical fiscal policy. Hence, including a fiscal framework that accounts for these inefficient government consumption habits will prove useful. Despite these limitations of the model, this analysis constitutes one of the first formal monetary policy responses to international structural shocks given the economy is a middle-income country.

Analogously, one must make the following observations about assumptions in the model. Firstly, the assumption of complete pass through of international oil prices to the domestic economy does not always hold in low- and middle-income countries. Here it is important to understand that low and middle-income countries governments enact subsidies for refined oil products due to a high percentage of population being low-income earners. As such the international price of oil (and refined oil products) does not pass through completely to domestic households[64]. In other words, according Amin *et. al.* (2021) since governments purchase the oil (and refined oil products) from producers before reselling at a discounted price to households, oil markets need not necessarily clear since the subsidised (discounted) price of oil differs from market prices. Furthermore, Plante (2014) also shows that in the presence of substantial fuel subsidies,

[64] The International Energy Agency (IEA) produces data on fuel subsidies for oil as well as gas, coal and electricity. See `https://www.iea.org/data-and-statistics/data-product/fossil-fuel-subsidies-database` for data on fuel subsidies by countries.

international oil markets do not necessarily clear and create price distortions, further decreasing welfare for the net-exporting oil country as well as the net-importing oil country. The inclusion of subsidies within the model will pose consequences for monetary transmission channels through the New Keynesian Phillips Curve, as higher oil prices under subsidised regimes will not induce inflationary pressures from higher real marginal cost to firms as well as to households.

Finally, I abstracted from oil depletion given my focus was on short-run business cycle dynamics. This forms a limitation of the model as the inclusion of depletion of natural resources would affect the utilisation rate for active oil rigs both home and foreign country. The consequences on supply outcomes should depletion be considered will inevitably create supply pressures that will affect market prices as well as create different dynamics regarding the importance of new oil discoveries. This further creates more dynamics within the model as oil in the ground will follow a unique law of motion that interacts with active oil rigs. While not considered, this presents an avenue for future research that could further understand how limited oil in the ground affects oil markets outcomes and ultimately economies that rely heavily on them.

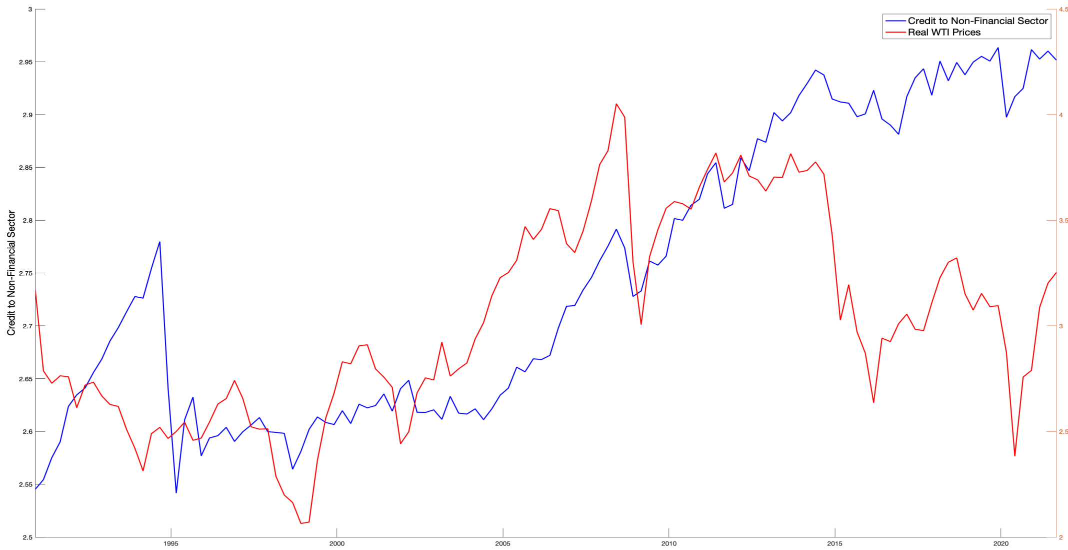
Chapter 3

Commodity Cycles, Credit Cycles and Monetary Policy in Low- and Middle-Income Oil Countries

3.1 Introduction

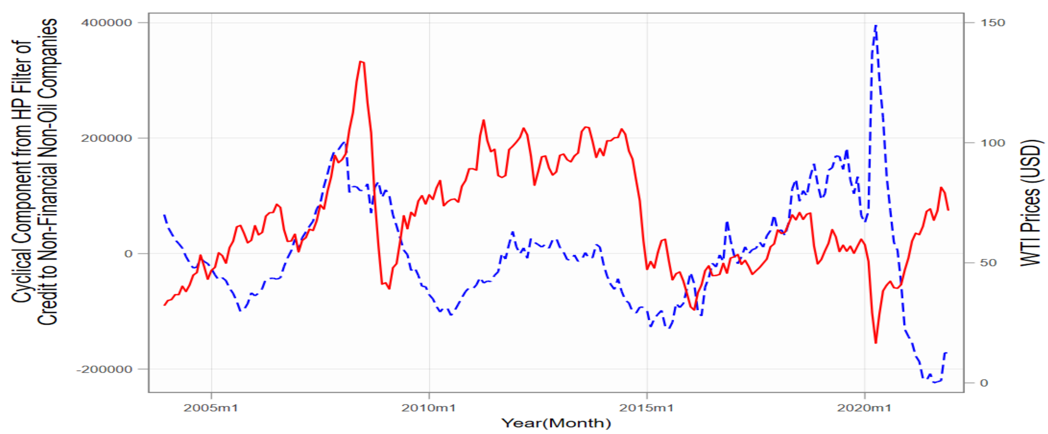
Financial frictions have been established as a source of business cycle fluctuations and importantly as a shock propagator. Based on the established role of financial frictions as shock propagators, in this chapter I extend my original contribution with the integration of financial frictions into a net exporting oil economy in order to identify the relationship between the financial frictions, commodity cycles, domestic credit availability and the transmission of international shocks. While I have established how business cycles in commodity driven economies are driven by commodity price volatility, and most importantly how to conduct monetary policy during commodity booms and busts, these financial frictions interacting with international shocks could create implications for monetary policy. Since, financial frictions have been shown to play a significant role in amplifying the transmission of international shocks for net exporting oil countries as capital access constraints are highly correlated with the price of oil for net exporting oil countries. To be precise, there is increasing evidence on the relationship between borrowing constraints (borrowing conditions) and commodity price volatility in commodity driven economies. That is, higher commodity prices favour looser borrowing conditions thereby increasing credit availability and vice versa[1] (see Figure 3.1, Figure 3.2 and Figure 3.3).

Figure 3.1: Oil Prices and Credit to Non-Financial Sector in Mexico



The blue line depicts the spot price of West Texas Intermediate (WTI), while the red line depicts the aggregate credit to non-financial sector in Mexico. Data on Credit to non-financial sector is obtained from the Bank of International Settlement (BIS) and WTI prices are obtained from the U.S. Energy Information Administration.

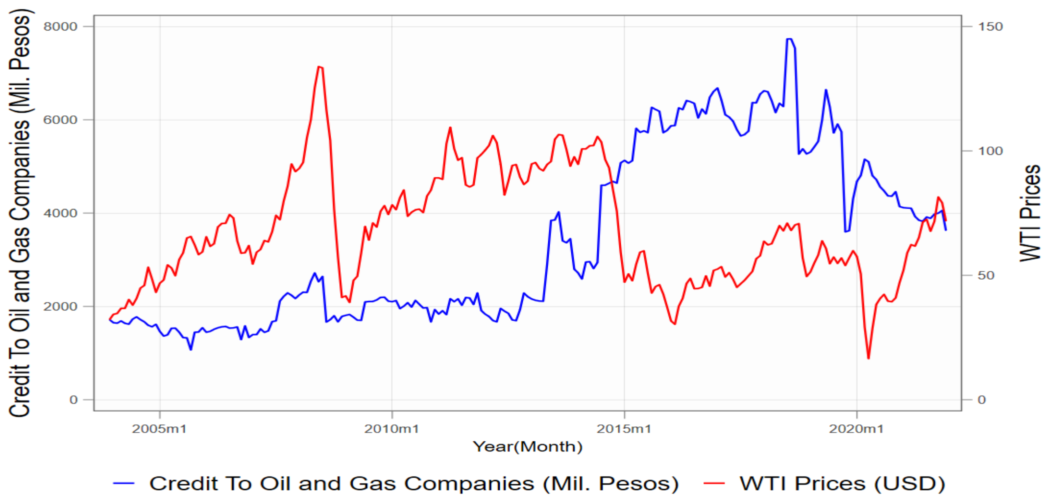
Figure 3.2: Oil Prices and Credit to Non-Financial Non-Oil Companies in Mexico



The blue line depicts the Cyclical Component from HP filtering Credit to Non-Financial companies excluding those who categorise their activity as oil and gas extraction or petroleum products (including coal), while the red line depicts the aggregate WTI Prices. Data on Credit is obtained from Haver Analytics and WTI prices are obtained from the U.S. Energy Information Administration. Data is monthly and covers the period December 2003− December 2021. I choose $\lambda = 129600$ for the smoothing parameter.

[1] See Shousha (2016), Fernandez *et. al. (2018)* and Drechsel and Tenreyro (2018) for more on the relationship between commodity cycles and lending conditions

Figure 3.3: Oil Prices and Credit to Non-Financial Oil and Gas Companies in Mexico



Data on Credit to non-financial oil and gas companies is calculated as credit to companies whose activities are described as oil and gas extraction as well as petroleum products (and coal). The data is obtained from the Haver Analytics, while WTI prices are obtained from the U.S. Energy Information Administration. Data is monthly and covers the period December 2003– December 2021.

Figure 3.1, Figure 3.2 and Figure 3.3 highlight the growing procyclical relationship between credit availability in Mexico and international oil prices. This relationship even exist beyond credit to non-oil and gas companies as in Figure 3.2 further emphasizing the role oil prices play in creating and driving financial frictions through the entire economy of Mexico[2]. To understand the importance of financial frictions for transmission of shocks, firstly consider the importance of financial frictions on the monetary transmission mechanism. Here credit channel theory emphasizes the level of financial frictions strengthens the monetary transmission mechanism as loans are highly sensitive to banks financial leverage. This occurs through two additional channels, the balance sheet channel and the capital access channel (or bank lending channel)[3]. While the monetary transmission mechanism is one way financial frictions affect the transmission of shocks, the literature has also obtained other results. For instance, García-Cicco, Kirchner and Justel (2014) find that domestic financial frictions increase the effects of foreign shocks on crucial macroeconomic variables (excluding output and investment) through foreign exchange

[2]While Mexico here is used as an example simply because of data availability, this highly procyclical relationship between credit availability and oil prices holds in highly oil dependent economies that are low- and middle-income.

[3]See Bernanke and Gertler (1995) for a review on the credit channel. See Bernanke, Gertler, and Gilchrist (1998) for more on the role financial frictions play in the monetary transmission mechanism. See Bean, Larsen and Nikolov (2002) for a review on the literature of financial frictions and the monetary transmission mechanism.

rates interactions with these financial frictions. Gertler, Gilchrist and Natalucci (2007) show in a replication exercise of the Asian financial crisis, that financial frictions particularly the financial accelerator mechanism accounted for about half the decline in economic activity. [4] [5]

Based on this increasing evidence, I extend the model in chapter 2 to include a banking sector that constrains the acquisition of capital by oil firms and study the dynamics of the role financial frictions play in transmission of international oil price shocks as well as the role monetary policy plays in insulating the economy from international oil price shocks. With the addition of the banking sector, I can therefore assess the role two additional channels play in transmission of international oil price shocks to the domestic economy. The first being changes in interest rate that change the cost of acquiring new capital by increasing the loan rate charged by banking firms – the capital access channel. This channel is also affected by the banking sector borrowing conditions that are tied to changes in oil prices. The second channel is presented in the sensitivity analysis and is based on subjective leverage constraints faced by banks as capital constraints imposed by minimum capital requirements create credit supply constraints for banks trying to lend to oil sector firms – the financial accelerator channel. The rationale behind assessing these transmission channels is to better understand what transmission channels play significant roles in shock propagation as well as monetary transmission. This further has implications for monetary policies, which are a secondary objective of this chapter.

I present a richer two-country open economy framework that fully endogenizes international oil markets, with domestic financial frictions constricting domestic oil sector activity. The financial sector includes an elasticity property that allows the domestic oil producing firm and representative bank to respond accordingly to changes in international oil prices. Within this framework,

[4] Other transmission channels highlighted in the literature involve credit spreads as in Dedola and Lombardo (2012) that emphasize the exposure financially integrated countries to one another's credit spreads such that unexpected changes in credit spread across one country induces similar changes in credit spreads across the other financially integrated country.

[5] Considering financial frictions as constraints that restrict firms from funding capital acquisition and investment decisions through external sources as in the model provided, then in the context of low- and middle-income countries this varies widely. For example, in Pakistan, Rashid and Jabeen (2018) show that the relationship between external financing availability of firms and their internal funds availability are negatively related. Similarly, when looking at interest rate spreads, Banerjee (2003) finds that in developing countries, interest rate spreads are about $30 - 60$ percent. For comparison, the International Financial Statistics published by the IMF estimates the average interest rate spread to be approximately 5% in Italy and 3% in the United States. Calvacanti *et. al.* (2021) using Brazilian data find that the standard deviation on lending rates for formal loans was about 35 percentage points even after controlling past defaults. See Buera *et. al.* (2016) for a review on the micro-financial evidence of financial frictions in developing economies.

an increase (decrease) in commodity prices eases (constrains) tightness of the borrowing constraint and increases (decreases) the elasticity of capital acquisition and utilisation to oil prices for the oil producing firm. This induces both the amplification and constrained responses of domestic macroeconomic variables to changes in international oil prices as the oil sector drives the domestic economy. While I am not the first to study the dimensions of banking in an open economy framework, I present one of the first papers to illustrate the relationship between commodity cycles, banking conditions and business cycles in net-exporting oil countries. The paper closest to mine is Drechsel, McLeay and Tenreyro (2019) who also present a commodity exporting country with banking constraints. The model presented here draws extensively on the banking sector dynamics[6] from Karmelavicius and Ramanauskas (2019) as well as Drechsel *et. al.* (2019). However, the model deviates extensively from Drechsel *et. al.* (2019) as I present an endogenous oil price determination, domestic sectoral capital and abstract from international risk sharing in the form of non-cross border trade in bonds. Similarly, the incorporation of a supply chain from Bergholt, Larsen and Seneca (2019) presents richer dynamics as it creates another linkage between oil industry and domestic households (not only through labour income now, but through capital rents to supply chain firms[7]), the sectoral employment of labour allows for reallocation of across different sectors and captures labour market dynamics. Finally, production is only constrained domestically, so foreign oil sector firms are not subject to the working capital constraint and are in the original model presented in chapter 2.

As with the preceding chapter, the aim of this chapter is to examine the crucial role monetary policy can play in stabilising commodity driven economies during commodity busts. The caveat here is the financial sectors restrictions on capital within oil sectors can have broader consequences for stabilisation policies through the capital access channel, as lending rates are intrinsically tied to nominal interest rates. Here I once again analyse the decisions of the central bank on whether to target inflation or target output gap when concerned with household welfare. As before I assume a floating exchange rate regime as Drechsel *et. al.* (2019) and others[8] have shown that exchange rate pegs perform poorly in open economy settings even in the case of net

[6]In the sensitivity analysis, I consider a banking sector with capital constraints that creates room for possible macroprudential policies as a stabilisation tool.

[7]Ferrero and Seneca (2015) in their analysis identify that supply chains as important for developed commodity exporters. However, as I am focussing on underdeveloped exporters, the supply chain serves as oil sector capital producers that allows efficient integration of the banking sector.

[8]See. Corsetti, Dedola and Leduc (2008), Corsetti and Pesenti (2005).

exporting commodity economies.

Given the modelling framework, a decline in oil prices (from increasing oil supply) creates unfavourable borrowing conditions for oil producing firms, which limits their access to capital. The borrowing constraint not only limits their access to capital but optimality conditions for labour and utilisation of rigs demand are increasing in the loan binding constraint as well as the tightness of the binding constraint. Hence, input demand (labour and utilisation) does not immediately contract on impact but undergoes a steady decline. This effect is transmitted to oil output such that it does not immediately decline on impact but rather steadily declines forcing the contraction of oil sector value added to be slower. Under these conditions, financial frictions do not amplify the magnitude of the effects from international oil shocks but rather constrains the magnitude of the effects by limiting the propagation of the oil sector shock spillovers to the rest of the domestic economy. After conducting sensitivity analysis, I find that the main transmission channel from the financial sector to the oil sector is through the capital access channel, with the financial accelerator not playing a significant role in the constricting the representative bank's lending capacity to the oil producing firm. While this result could be due to the modelling simplicity of the financial sector, there is no consensus on the magnitude and significance of the financial accelerator channel. For instance, Liu, Wang and Zha (2013), Cordoba and Ripoli (2004) show that the financial accelerator amplification can be close to zero depending on the magnitude of impact the shock has on domestic prices, such that only shocks that affect prices directly induce strong amplification effects of the financial accelerator channel. Although Brunnermeier and Sannikov (2014) accentuate the relevance of asymmetries and non-linearities in creating strong financial accelerator channel amplifications.

I also find that inflation targeting still performs best during higher oil price booms from increased foreign productivity with the added caveat of preventing any negative impact on domestic welfare. This follows from an interesting finding as I find that financial frictions in the model improve the welfare of households beyond the baseline model that excluded financial frictions. This is due to the change in input demand dynamics by oil producing firms. Furthermore, I provide some thoughts on how sterilised exchange rate interventions could become a crucial monetary tool for central banks in oil countries in order to stabilise volatile exchange rates. In other words, I argue that in order to achieve a greater offset to exchange rate depreciations induced by oil price declines, central banks in commodity driven economies should practise sterilised exchange rate interventions in conjunction with their targeting objective.

The rest of this chapter is structured as follows. Section 3.2 presents the literature relevant to the chapter here. The proceeding section presents an empirical analysis of the role commodity cycles play in driving credit availability before looking at the model, which is an augmentation of the model presented in the preceding chapter. In section 3.4 I present the results of international shock consequences for the domestic economy. Section 3.5 presents counterfactual analysis on different monetary policy regimes, with an extension to include adjustment costs and leverage ratio for banks to characterise the financial accelerator channel. Section 3.6 outlines sterilised exchange rate intervention and the role they could play in offsetting the inflationary pressures induced by exchange rate depreciation, and finally section 3.7 concludes.

3.2 Relevant Literature

As with the preceding chapter this paper is related to a strand of literature that characterises open economy monetary policy responses to international shocks[9]. Secondly my work contributes to the line of literature that assess monetary policy in the presence of commodity sector dynamics[10]. Finally, my work also contributes to an upcoming strand of the literature that looks at monetary policy responses for oil exporting countries [11].

The strand of literature on the financialization of commodity markets that motivates the inclusion of a banking sector within the model, provides evidence in both directions on the argument of the financialization of commodity markets. Fernandez, Rodirguez and Gonzalez (2018) find that commodity prices induce a countercyclical effect on the spread of sovereign bonds especially in emerging market economies. That is, they find that commodity prices produce negative effects on the risk premium of sovereign bonds of these economies. That is, lower commodity prices increases the risk premium and thus the interest rates payable. Also noting that these emerging market economies are most likely to be commodity exporters. Shousha (2016) finds

[9]See Gali and Monacelli (2005). Other noteworth contributions to the open economy monetary policy include but not limited to Benigno and Benigno (2003), Corsetti and Pesenti (2005, 2008).

[10]See for example Bodenstein, Erceg and Guerreri (2011), Bodenstein and Guerrieri (2011), Bodenstein, Killian and Guerrieri (2012), Killian and Murphy (2014), Killian and Lewis (2013).

[11]See for example Bergholt (2014) and Bergolt, Larsen and Seneca (2019)

that one of the major channels between transmission of commodity price shocks to commodity exporting economy was through the differences in capital constraints faced by firms within the economy[12] [13].

When looking at financial frictions[14], Gertler *et. al.* (2007) show that in the context of a small open economy, the financial accelerator channel can account for a substantial portion of economic activity. In their exercise, the financial accelerator channel accounted for half of the decline in economic activity following the Asian financial crisis. Cespedes and Velasco (2012) empirically show that economies with less developed financial sectors significantly experience worse effects on output and investment from commodity price volatility compared to economies with better developed financial sectors. Earlier works by Neumeyer and Perri (2005) and Uribe and Yue (2006) find that these financial frictions provide the necessary mechanisms for their developed models to account for observed business cycle dynamics. Similarly, Fuentes-Albero (2012) shows through the lens of the financial accelerator that financial frictions are significant contributors to business cycle dynamics in developing economies[15] [16].

Drechsel *et. al.* (2019) positive commodity price booms prevent efficient stabilisation of the economy by monetary policy. The inefficiency arises from the inability of households to benefit from commodity price driven expansions, as households are unable to benefit from higher production in the commodity sector. The relaxation of borrowing constraint[17] amplifies the inefficiency further and the increase in demand for domestically produced goods leads to domestic inflationary pressures and real exchange rate appreciations that eventually lead to expenditure switching effects of exchange rate movements as households eventually switch to foreign cheaper goods. They find that the optimal monetary policy response is to allow exchange rates appreciate and increase policy rates such that int hike in policy rates increases in proportion

[12]Though Shousha (2016) notes that imbalances in balance sheet of banks and leverage constraints did not create any further amplification of international shocks for emerging market economies.

[13]See Cheng and Xiong (2014) for a review of the literature.

[14]Bernanke, Gertler and Gilchrist (1996, 1998) pioneer the literature and though their initial work is not related to commodity exporting economies, it provided insight on the effects of financial accelerators in propagation of shocks. See others Kiyotaki and Moore (1997), Christiano *et. al.*(2015), Negro *et. al.*(2014).

[15]Her model looks at financial shocks to be specific and its importance in driving aggregate volatility

[16]Though I will still be abstracting from cross—border trade in bonds, these papers assessing risk premia provide another channel through which financial frictions amplify both domestic and international shocks.

[17]In their model and analogously in ours, borrowing constraints are dynamic and follow changes in international oil prices.

to the strength of the financial channel. Though in their characterisation of welfare improving policies they only look at differing inflation targets (either CPI or domestic price inflation) in contrast to an exchange rate peg regime and show that optimal policy is welfare improving when it targets either of these inflation targets as both gave rise to similar welfare losses.

3.3 The Model

What The Data Says about Credit and oil Prices:

Before proceeding to the model setup, I would like to conduct a preliminary exercise on the how oil prices change the availability of credit in oil dependent economies. To do so, I simply estimate a SVAR for the Mexican economy. Because the goal here is a crude overview of what the data shows, I restrict identification strategy to Short-run Cholesky identification. In other words, I estimate the following equation:

$$Y_t = A^t Y_0 + \sum_{j=0}^{t-1} A^j B \varepsilon_{t-j} \tag{3.3.1}$$

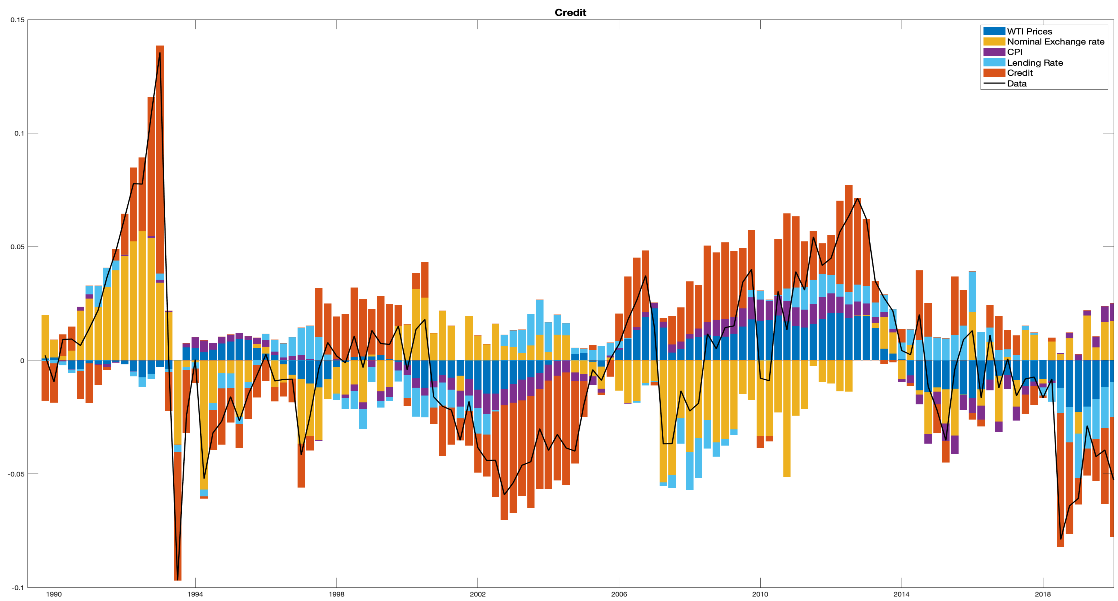
$$Y_t = [P^*_{o,t}\ \mathcal{E}_t\ \Pi_t\ L_t\ Cr_t]'. \qquad \varepsilon_t \sim N(0,1)$$

Where $P^*_{o,t}$ is international oil prices, measured using West Texas intermediate (WTI) prices, $\mathcal{E}_t$ is the nominal exchange rate, Π_t is the consumer price index, L_t is the lending rate, and Cr_t is credit measured as aggregate credit to non-financial sector[18]. The inclusion of the CPI captures the fact that the nominal interest rates move with changes to inflation and therefore since the nominal rates represents the base from which the lending rate to be crude represents a markup over. To identify the contribution of oil prices to changes in both credit and the lending rate, I compute the historical decomposition following the estimation of the SVAR. Figure 3.4 display the historical decompositions of credit and lending rate respectively[19].

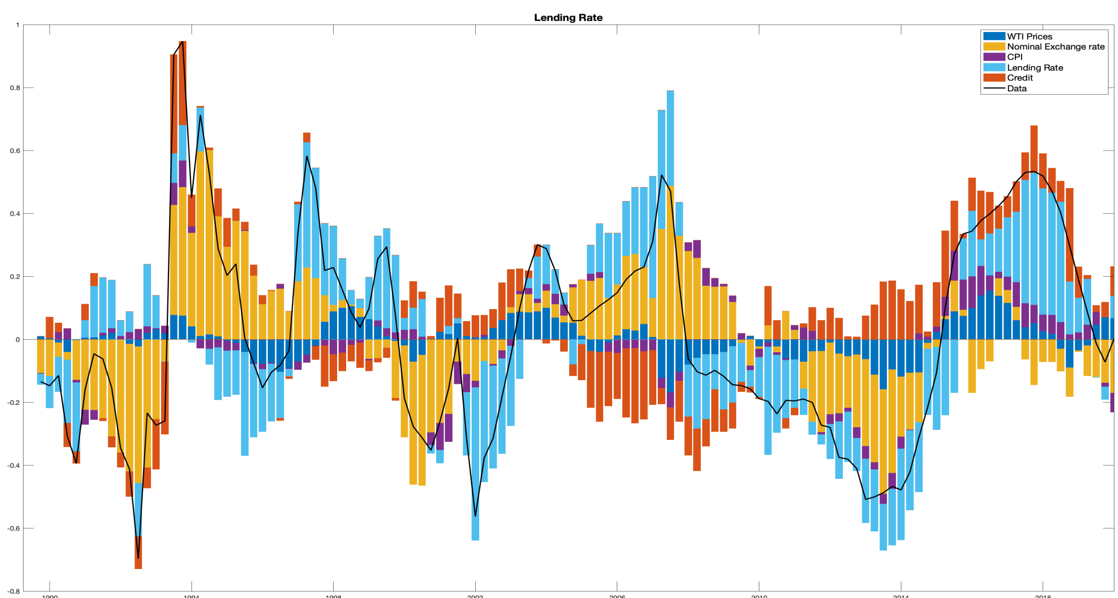
[18]Data on credit is obtained from the Bank of International Settlement (BIS) credit database `https://www.bis.org/statistics/totcredit.htm`. All other data sources have been previously listed.

[19]In consistency with the previous data exercises, the optimal lag length for the SVAR is chosen based on Schwarz's Bayesian information criterion (SBIC), and Hannan and Quinn information criterion (HQIC). The optimal lag length based SBIC and HQIC is 2.

Figure 3.4: Historical Decomposition of Credit and Lending Rate from Structural VAR



(a) Historical Decomposition of Credit to Non-Financial Sector



(b) Historical Decomposition of Lending Rate

The first pane depicts the Historical Decomposition for Credit to non-financial sector and the second pane the lending rate. Key to contribution of each variable is depicted in each pane. The solid black line depicts the observed data. All variables are log-transformed. Stability of the SVAR displayed in Figure 5.3 in Appendix E.

From the impulse responses (see Figure 5.8 in Appendix E), an increase in oil prices leads to an increase in the availability of credit and increased lending rate[20]. The historical decompositions depict a financialization of the commodity market. Over the past two decades, oil prices have been considerably gaining share in driving the credit cycles. That is higher credit availability has been associated with positive shocks to oil prices and lower credit associated with negative oil price shocks. Somewhat similar, the historical decomposition for lending rates reveals the growing importance of oil price shocks in driving lending rates, particularly in the latter half of the 2010s. While this exercise presented a simple understanding of the data, it enables inference on the relationship between international oil prices and the domestic availability of credit in an oil driven economy (Mexico).

3.3.1 Setup - Oil Sector

The model here is as presented in chapter 2, only I now abstract from the foreign investment in the oil sector such that $I_{u,t}$ is no longer a composite good consisting of both home and foreign investment but simply produced by a domestic capital producer with price determined by the market clearing conditions. Analogously, households now earn dividend payment from the banking sector with the introduction of banks. Finally, I will only consider one pricing regime – Dominant Currency Paradigm (Dollar Currency Pricing). The main changes from chapter 2 accrue to the oil sector which is explained below.

Oil Sector Capital Producers - Rig Producers

There are now two firms within the oil sector. Following Bregholt (2014) and Bergholt, Larsen and Seneca (2019) I include a supply chain firm that produces the investment and capital goods required by oil sector firms. Therefore, representative supply chain firm's production function is given by[21]:

$$Y_{\psi,t} = K_{\psi,t}^{\alpha_k} N_{\psi,t}^{1-\alpha_k} \tag{3.3.2}$$

[20]The impulse responses for CPI and the nominal exchange rate are consistent with the findings from the data exercise in chapter 2, section 2.3 that was conducted utilising both Nigerian and Mexican data. Figure 5.9 also shows the historical decomposition for CPI, which is also consistent with the findings from chapter 2, section 2.7.2 on the data exercise assessing the role exchange rates play for in creating inflationary pressures.

[21]The specified production function differs slightly from Bregholt (2014) and Bergholt, Larsen and Seneca (2019) as they include inter-sectoral inputs, which creates the linkages between non-oil sector firms and the oil sector. I abstract from this and utilise this simpler specification for two reasons: 1-, low- and middle-income developing countries typically required imported inputs in their production function and as such are not significantly dependent on inter-sectoral inputs. 2. The simpler specification makes it computationally easing.

Here $Y_{\psi,t}$ is the output good from supply chain firms, $N_{\psi,t}$ and $K_{\psi,t}$ are respectively labour and capital inputs utilised by supply chain firms. Notice that both inputs are subject to household conditions, providing another linkage between oil sector and non-oil sector. The supply chain firm maximises intertemporal profits, taking all prices as given. So it solves:

$$\mathcal{D}_{\psi,t} = \tilde{P}_{\psi,t} Y_{\psi,t} - R_t^k K_{\psi,t} - \Omega_{\psi,t} N_{\psi,t}^{1-\alpha_k} \tag{3.3.3}$$

All variables are expressed in real terms, with $\tilde{P}_{\psi,t}$ being the real price of capital in the oil sector[22]. The problem is a static one and the first order conditions are generic[23]. Market clearing for supply chain firms is thus given by

$$Y_{\psi,t} = I_{u,t}^{agg} + a(U_t)\Psi_{u,t} \tag{3.3.4}$$

Here I define $I_{u,t}^{agg}$ as the aggregate investment good produced. With capital constrained oil producing firms, not every single investment good or capital good produced is bought by oil producing firm[24].

Oil Producers

As shown in the previous chapter, oil production subject to the following production function:

$$O_t^u = \varepsilon_{u,t} (\Psi_{u,t}^s)^{\alpha_u} N_{u,t}^{1-\alpha_u} \qquad \alpha_u \in [0,1) \tag{3.3.5}$$

Where $(\Psi_{u,t}^s = U_t \Psi_{u,t})$ is the effective utilisation of active oil rigs, with $\Psi_{u,t}$ being active oil rigs and $U_{u,t}$ being the utilisation rate of active oil rigs, and $N_{u,t}$ is labour employed by oil producing firms. Oil producing firms are subject to the law of motion for active oil rigs:

$$\Psi_{u,t+1} = (1-\delta_u)\Psi_{u,t} + \varepsilon_{\psi,t}^i \left[1 - S_u\left(\frac{I_{u,t}}{I_{u,t-1}}\right)\right] I_{u,t}$$

[22]Note that in chapter 2 when investment is aggregated by a Dixit-Stiglitz aggregator, the price of investment is the same for oil and non-oil sector investment (given as $\tilde{P}_{i,t}$). However, in this model the price of oil sector investment $\tilde{P}_{\psi,t}$ is now determined by market clearing and this price is different. So, $\tilde{P}_{\psi,t} \neq \tilde{P}_{i,t}$ even in the steady state.

[23]See Appendix B for the first order conditions to the intertemporal optimisation problem.

[24]In the deterministic steady state, I compute the residual investment good as the aggregate investment good produced net of the investment good purchased by oil producing firms.

Now I will deviate from the previous chapter as oil producing firms are now subject to an additional constraint. They must finance their rig purchases utilising loans secured from the financial sector, such that:

$$\tilde{P}_{\psi,t}\Psi_{u,t} = L_t^B \tag{3.3.6}$$

$$L_t^B \leq \amalg_t \mathcal{S}_t P_{o,t}^* O_t^u \tag{3.3.7}$$

Where L_t^B represents the loan secured by the oil producing firm from the bank, and $\amalg_t$ reflects how tight borrowing conditions are. Domestic capital access channels in low- and middle-income countries contribute to financial frictions experienced within these economies. For example, in Nigeria and Mexico, domestic banks credit to the private sector accounts around 11% and 29% of GDP respectively[25]. Analogously, as conveyed in Figure 3.1, Figure 3.2 and Figure 3.3 credit availability within low- and middle-income countries are particularly tied to the oil sector such that whenever the oil sector experiences windfalls, the level of credit within these countries increases[26]. As already stated in the previous chapter, the local content legislations in these countries[27] force the State-Owned Enterprises (SOEs) to utilise domestic supply chains[28]. While FDIs are highly important in the oil sector and particularly in capital supply chains, for the sake of this exercise I will restrict attention to only domestic capital access as well as supply chain. The motivation behind this lies in the modelling objective of understanding how domestic capital access channels within domestically embedded financial frictions affect the propagation of international oil price shocks. Additionally, this is important as it allows me to isolate the role domestic capital access channel alone plays in the propagation of international shocks. As already depicted, Figure 3.1 and Figure 3.3 show the domestic credit availability as well as domestic credit to non-financial oil sector companies in the example country Mexico. Without access to firm level data, it is impossible to tell the level of credit allocated to oil sector supply chain firms, so, in the model presented I abstract from oil supply chain firms relying on loans and only the representative oil producer relying on loans from domestic banks.

Though the assumptions here allow me to assess and isolate the role domestic capital access

[25]Data is from the World Bank's WDI for the year 2020.

[26]Another consideration is that fact that the representative oil company mimics the state-owned oil company that finances operations utilising partial domestic funding as in the case of Pemex of Mexico or NNPC of Nigeria.

[27]Example being Nigeria and Mexico

[28]See Grunstein and Díaz-Wionczek (2017)

channel within financial friction plays in propagation of shocks, the assumption restricting supply chain and investment to the domestic country is not fully realistic. With this consideration in mind, future research will relax this assumption and integrate high FDI in the domestic supply chain and most importantly integrating external capital access channels as observed in the data[29].

As in Drechsel *et. al.* (2019) I also allow the tightness constraint to vary with commodity prices, that is I model $\amalg_t$ as[30]:

$$\amalg_t = \amalg_1 \left(\frac{P^*_{o,t} \mathcal{S}_t}{P^*_{o,t-1} \mathcal{S}_{t-1}} \right)^{\amalg_2} \tag{3.3.8}$$

This positive correlation between the tightness constraint and commodity prices captures banks' incentive to ease and increase lending to oil producing firms under favourable conditions[31]. The inclusion of these constraints is an indication of real-world commodity sectors. As Drechsel and Tenreryo (2018) put it, the introduction of financial frictions indicates that borrowing constraints in commodity economies are typically eased when commodity cycles are booming. In optimisation I assume the borrowing constraint always binds[32], such that $L^B_t = \amalg_t P^*_{o,t} O_{u,t} \mathcal{S}_t$. I can then derive what the constrained capital allocation for oil producing firms will be as:

$$\Psi_{u,t} = \left[\frac{\amalg_t \mathcal{S}_t P^*_{o,t} \varepsilon_{u,t} U_t^{\alpha_u} N_{u,t}^{1-\alpha_u}}{\tilde{P}_{\psi,t}} \right]^{\frac{1}{1-\alpha_u}} \tag{3.3.9}$$

Substituting the borrowing condition (tightness constraint) $\amalg_t$, one obtains the following:

$$\Psi_{u,t} = \left[\frac{\left(\mathcal{S}_t P^*_{o,t} \right)^{1+\amalg_2} \varepsilon_{u,t} U_t^{\alpha_u} N_{u,t}^{1-\alpha_u}}{\left(\mathcal{S}_{t-1} P^*_{o,t-1} \right)^{\amalg_2} \tilde{P}_{\psi,t}} \right]^{\frac{1}{1-\alpha_u}}$$

It has the following deduction: without the financial sector, the model returns to the baseline

[29]This will integrate two additional channels for propagation of international shocks, as movements in the world interest rates will either ease or constrain external capital access independent of oil price movements.

[30]The specification in Drechsel *et. al.* (2019) is slightly different, they model it taking into account only current oil prices, while I take into account previous oil prices in addition to current oil prices, which is a more realistic approach as it takes into account forward looking behaviours of banks' positions on their loans.

[31]$\amalg_1$ is set to 1.22 so the constraint always binds in the steady state. This is also so i can attain steady state ratios close to what is observed in the data. Similar to standard demand functions, $\amalg_2$ captures the elasticity of borrowing conditions to changes in international oil prices. The higher the elasticity of borrowing conditions to international oil price changes the greater the transmission from commodity prices to oil producing firms and thus the rest of the economy.

[32]This makes it computationally easy, as it does not account for occasionally binding constraints in the simulations.

model presented in chapter 2 such that the elasticity of rigs, $\Psi_{u,t}$ and utilisation U_t, to international oil prices $P^*_{o,t}$ is $\frac{1}{1-\alpha_u}$. While with the inclusion of financial frictions, the elasticity becomes $\frac{1+\text{II}_2}{1-\alpha_u}$ such that for $\text{II}_2 > 0$ then $\frac{1+\text{II}_2}{1-\alpha_u} > \frac{1}{1-\alpha_u}$ with the transmission of international oil prices to oil producing firms hinging on II_2 (see footnote 22). Taking the combined borrowing constraints into one equation. The optimisation problem of firms is analogous to that in chapter 2 only with the inclusion of the combined capital constraint. The first order conditions now include the borrowing constraint and are given by:

$$\begin{aligned}\mathbb{E}_t\Bigg[M_{t,t+1}\Bigg[\alpha_u\frac{\mathcal{S}_{t+1}P^*_{o,t+1}O_{u,t+1}}{\Psi_{t+1}} - \tilde{P}_{\psi,t+1}a(U_{u,t+1})\\ + \mathcal{Q}_{u,t+1}(1-\delta_u) + \mathbb{Z}_{t+1}\left(\alpha_u\frac{\text{II}_{t+1}\mathcal{S}_{t+1}P^*_{o,t+1}O_{u,t+1}}{\Psi_{t+1}} - \tilde{P}_{\psi,t+1}\right)\Bigg]\Bigg] = \mathcal{Q}_{u,t} \quad (3.3.10)\end{aligned}$$

$$\begin{aligned}\mathbb{E}_t\left[M_{t,t+1}\epsilon^i_{\psi,t+1}\mathcal{Q}_{u,t+1}S'\left(\frac{I_{u,t+1}}{I_{u,t}}\right)\left(\frac{I_{u,t+1}}{I_{u,t}}\right)^2\right]\\ + \epsilon^i_{\psi,t}\mathcal{Q}_{u,t}\left[1 - S\left(\frac{I_{u,t}}{I_{u,t-1}}\right) - S'\left(\frac{I_{u,t}}{I_{u,t-1}}\right)\frac{I_{u,t}}{I_{u,t-1}}\right] = \tilde{P}_{\psi,t} \quad (3.3.11)\end{aligned}$$

$$\begin{aligned}\alpha_u\frac{\mathcal{S}_tP^*_{o,t}O_{u,t}}{U_t} &= \tilde{P}_{\psi,t}a'(U_t)\Psi_t - \mathbb{Z}_t\alpha_u\frac{\text{II}_t\mathcal{S}_tP^*_{o,t}O_{u,t}}{U_t}\\ \Rightarrow \alpha_u(1+\mathbb{Z}_t\text{II}_t)\frac{\mathcal{S}_tP^*_{o,t}O_{u,t}}{U_t} &= \tilde{P}_{\psi,t}a'(U_t)\Psi_t \quad (3.3.12)\\ (1-\alpha_u)\frac{\mathcal{S}_tP^*_{o,t}O_{u,t}}{N_{u,t}} &= \Omega_{u,t} - \mathbb{Z}_t(1-\alpha_u)\frac{\text{II}_t\mathcal{S}_tP^*_{o,t}O_{u,t}}{N_{u,t}}\\ \Rightarrow (1-\alpha_u)(1+\mathbb{Z}_t\text{II}_t)\frac{\mathcal{S}_tP^*_{o,t}O_{u,t}}{N_{u,t}} &= \Omega_{u,t} \quad (3.3.13)\\ \Rightarrow \frac{\bar{\Psi}_{u,t}}{N_u,t} &= \frac{\alpha_u}{(1-\alpha_u)}\frac{\Omega_{u,t}}{\tilde{P}_{\psi,t}a'(U_t)} \quad (3.3.14)\end{aligned}$$

The only new variable here is $\mathbb{Z}_t$, where $\mathbb{Z}_t = \frac{Z_t}{\lambda_t}$ with λ_t being the marginal utility for household consumption and Z_t being the Lagrange multiplier on the combined borrowing constraint i.e., the shadow value of increasing loans by one unit. Therefore, $\mathbb{Z}_t$ indicates oil firms valuation of relaxing the borrowing constraint. An addendum, the first order conditions now show that oil sector firms will demand higher inputs in production function whenever the prices for oil are favourable. Here $\mathcal{Q}_{u,t}$ is simply Tobins-q for installed oil rigs, such that the optimisation solutions in equation (3.10) simply determines the discounted marginal value for oil rigs. Though, with the added addendum that the marginal value is now increasing in the tightness of the borrowing constraint and the firms valuation of relaxing the borrowing constraint.

$\tilde{P}_{\psi,t+1}(a(U_{u,t+1}) + \mathbb{Z}_t)$ now denotes the cost from increasing the number of rigs which is increasing in how much slack banks allow when lending, while $\frac{(1+\amalg_{t+1}\mathbb{Z}_{t+1})\mathcal{S}_{t+1}P^*_{o,t+1}O_{u,t+1}}{\Psi_{t+1}}$ denotes the revenues from increasing the number of rigs, which is increasing in oil prices but the rate of increase is constrained by the valuation of relaxing future borrowing constraint $\mathbb{Z}_{t+1}$ and the tightness of the future borrowing constraint itself $\amalg_{t+1}$. Equation (3.3.11) simply equates the marginal cost of increasing investment in active rigs today with the marginal gain from the increased number of rigs in the next period. Equation (3.3.12) denotes the optimal rig utilisation for oil firms i.e. they increase utilisation up to the point where marginal revenues from higher utilisation equates marginal cost. Though the increase in utilisation is now constrained by the path of oil prices, such that higher oil prices will loosen the tightness of borrowing constraint and increase the level of utilisation and vice versa. Equation (3.3.13) is simply the static labour demand optimality condition that also depend on the path of oil prices. Business cycle dynamics in oil firms are thus now dominated by the path of oil prices and utilisation rate, such that firms adjust their utilisation of active in order to change output and adjust the levels of input demand based on the entire path of oil prices as this determines their ability to borrow and finance capital and investment decisions. Long run dynamics are governed by investment decisions to increase the number of operational rigs, which creates forward-looking behaviour in oil firms, such that oil prices today are not the only drivers but the entire expected price path (Bergholt *et. al., 2019*).

Furthermore, I can derive the equation characterising $\mathbb{Z}_t$ by taking the first order conditions for rig utilisation as well as the constrained capital allocation, in other words combining equations (3.3.9) and (3.3.12), one can derive the following:

$$\amalg_t = \alpha_u \frac{(1 + \mathbb{Z}_t \amalg_t)}{U_t a'(U_t)}$$
$$\Leftrightarrow \mathbb{Z}_t = \frac{\amalg_t U_t a'(U_t) - \alpha_u}{\alpha_u \amalg_t}$$

if $\mathbb{Z}_t \ > \ 0$ then

$$\frac{\amalg_t U_t a'(U_t) - \alpha_u}{\alpha_u \amalg_t} > 0$$
$$\Leftrightarrow \amalg_t > \frac{\alpha_u}{U_t a'(U_t)} \quad \& \quad \alpha_u > 0 \quad \& \quad \amalg_t > 0 \tag{3.3.15}$$

To understand how the tightness of the borrowing constraint affects oil firms valuation of easing the borrowing constraint, take the partial derivative of $\mathbb{Z}_t$ with respect to $\amalg_t$. This gives:

$$\frac{\partial \mathbb{Z}_t}{\partial \amalg_t} = \frac{1}{\amalg_t^2} \quad > 0$$

Therefore, tighter borrowing constraints will always increase how much oil firms value an easing of the borrowing conditions. So, easing the tightness of borrowing constraint will always be beneficial to oil producing firms. Furthermore, given the assumption that $\mathbb{Z}_t$ always binds then it must be that the following holds[33]. Equations in (3.3.15) therefore determine the conditions required for the borrowing constraint in equation (3.3.7) to always bind[34]. These new conditions now characterise the new dynamic of the oil sector.

3.3.2 The Banking Sector

The financial sector within the model consists of a continuum of domestically owned banks h, in a perfectly competitive environment and as such have do not set prices. In line with Crucinello and Signoretti (2015), I assume all bank profits are redistributed as dividends to the households that own them such that $\sum_h \mathcal{D}_t^B(h) = \mathcal{D}_t^B$. I also assume that loans to oil sector firms acts as banks' sole asset and deposits from households acting as its sole liability. For simplicity, I assume that loans must equal deposits such that the banks' balance sheet is given by:

$$L_t^B(h) = B_{H,t}(h) \tag{3.3.16}$$

Where L_t^B represents loans to oil sector firms and $B_{H,t}$ represents deposits from households. With the simplified banking sector, I abstract from credit risk and loan default by oil producing firms[35]. Banks flow budget constraint consist of loans granted to oil producing firms $L_t^B(h)$, gross interest payments received on previous loans $\frac{R_{L,t-1}}{\pi_t} L_{t-1}^B$, deposits from households $B_{H,t}$ and interest payments to households for deposits $\frac{R_{t-1}}{\pi_t} B_{H,t-1}$. I can therefore express the flow budget constraint for banks as:

$$\mathcal{D}_t^B(h) + L_t^B(h) + \frac{R_{t-1}}{\pi_t} B_{H,t-1}(h) = B_{H,t} + \frac{R_{L,t-1}}{\pi_t} L_{t-1}^B(h) \tag{3.3.17}$$

[33] In the deterministic steady state I obtain $\mathbb{Z}_t = 0.6464$. Analogously every other constraint holds.

[34] Given I assume the constraint always binds in solving the model, these conditions characterise the overall parametrisation of financial sector variables

[35] In the sensitivity analysis I impose a financial adjustment cost function in addition to a minimum capital requirement by banks á lá Basel-style

Substituting equation (3.3.16) into the flow budget constraint of banks, it is possible to express the budget constraint in terms of only loans:

$$\mathcal{D}_t^B(h) = \frac{1}{\pi_t}(R_{L,t-1} - R_{t-1})L_{t-1}^B \tag{3.3.18}$$

Here $R_{L,t}$ represents the nominal lending rate, and R_t the nominal interest rate set by the central bank and the rate at which households earn returns on their deposits. Note that even though past decisions determine profit today, the absence of credit risk neutralises the timing choice of optimising bank behaviour[36]. As with intermediate good firms each bank then chooses the loans $L_t^B(h)$ to maximise discounted lifetime profits:

$$\mathbb{E}_t \sum_{s=o}^{\infty} M_{t,t+s}\mathcal{D}_t^B \tag{3.3.19}$$

The first order condition yields

$$\mathbb{E}_t \left[\frac{M_{t,t+1}}{\pi_{t+1}}(R_{L,t} - R_t) \right] = 0 \tag{3.3.20}$$

From the first order condition since optimal loans are independent of bank (i.e. all banks choose the same loans), a consequence of the perfect banking competition is that the lending rate will always equal the nominal interest rate. This holds as long as the household discount factor $M_{t,t+1}$ and the inflation rate π_t are none-negative[37].

3.3.3 Monetary Policy

I specify the same Taylor Rule as in the previous chapter here such that interest rates responds to both movement in the aggregate inflation Π_t, output gap and changes in the nominal exchange rate as a exchange rate stabilisation policy for the central bank. The Taylor rule is given by:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R} \left[\left(\frac{\Pi_t}{\Pi}\right)^{\rho_\pi} \left(\frac{GDP_t}{GDP}\right)^{\rho_y} \left(\frac{\mathcal{E}_t}{\mathcal{E}_{t-1}}\right)^{\rho_e} \right]^{1-\rho_R} \left(\frac{GDP_t}{GDP_{t-1}}\right)^{\rho_{\Delta_y}} \varepsilon_t^R \tag{3.3.21}$$

[36]within this literature authors impose different timing conditions dependent on research objective. See for example Lacoviello (2015), and Gerali, Neri, Sessa, and Signoretti (2010).

[37]The introduction of adjustment costs for banks attempts to remedy this occurrence. This is done later on in the sensitivity analysis section.

Monetary policy in the foreign block is identical to the domestic block. The Entire model is a mirror of the model presented chapter 2 but with the inclusion of a financial sector and supply chain firms in the oil sector. That is, the model includes both wage and price rigidities through Calvo wage and price setting processes. See Appendix A and B for all equations in the model.

3.3.4 Calibration

Table 3.1: Calibration - Foreign and Home Block

Foreign Country		Home Country			
Parameter	Value	Parameter	Value	Description	Source
β^*	0.99	β	0.99	Discount factor	Literature; Benkhodja (2014).
α^*	$\frac{1}{3}$	α	0.31	Capital share in production	Empirical literature (Author's calibration)
μ^*	0.026	μ	0.02	Oil share in production	Empirical Literature (Author's calibration)
ν^*	0.5	-	-	Imported oil share in foreign non-oil sector firms production	Author's calibration
α_u^*	0.32	α_u	0.42	Effective rigs share in oil production	Bergholt *et. al.* (2019); Benkhodja (2014)
σ_c^*	1	σ_c	2	Intertemporal consumption elasticity	Bodenstein *et. al.* (2012); Berg *et. al.* (2013)
σ_n^*	1	σ_n	10	Inverse Frisch elasticity	Bodenstein *et. al.* (2012); Berg *et. al.* (2013)
ψ_w^* ψ_{wu}^*	0.88	ψ_w ψ_{wu}	0.75	Calvo probability - Wages	Bodenstein *et. al.* (2012); Persistent Inflation
ψ_p^*	0.89	ψ_p	0.75	Calvo probability - Prices	Bodenstein *et. al.* (2012); Author's calibration
ι_w^*	0.24	ι_w	0.2	Degree of wage indexation	Smets and Wouters (2007); Author's calibration
ι_p^*	0.58	ι_p	0.5	Degree of price indexation	Smets and Wouters (2007); Author's calibration
a_O^*	0.021	a_O	0.054	Oil share in core consumption basket	Nakov and Pescataori (2010a,b); Author's calibration
θ_w^*	6	θ_w	8	Kimball aggregator - Wages	Author's calibration; 10% Markup
θ_p^*	7.66	θ_p	8	Kimball gggregator - Prices	Nakov and Pescataori (2010a); 10% Markup
ϖ^* ϖ_i^* ϖ_u^*	0.569	ϖ ϖ_i ϖ_u	0.42	E.O.S domestic goods and imports	Bodenstein *et. al.* (2012); Ravenna and Natalucci (2008)
ϑ^*	-1.73	ϑ	-1.93	E.O.S between oil and core consumption	Bodenstein *et. al.* (2012); Author's calibration
$(1-\xi^*)$ $(1-\xi_u^*)$	0.9320	ξ ξ_u	0.8	Degree of home bias in investment goods	Bodenstein *et. al.* (2012); Berg *et. al.* (2013)
a_F^*	0.9320	a_H	0.45	Degree of home bias in core consumption basket	Bodenstein *et. al.* (2012); Berg *et. al.* (2013)
ϱ_k^*	5.74	ϱ_k	7.5	Capital adjustment cost	Smets and Wouters (2007); Author's calibration
ϱ_u^*	5.74	ϱ_u	7.5	Rigs adjustment cost	Similar Estimate as Capital; Author's calibration
δ^*	0.025	δ	0.035	Depreciation rate - Capital	Empirical Literature (Author's calibration)
δ_u^*	0.025	δ_u	0.035	Depreciation rate - Rigs	Bergholt *et. al.* (2019); Author's calibration
h^*	0.8	h	0.5	Habit formation parameter	Smets and Wouters (2007)
υ^*	0.02	υ	0.03	Elasticity of oil supply	Bergholt *et. al.* (2019)
-	-	$\frac{1}{\eta}$	0.01	Home share in global oil production	Data
ρ_R^*	0.81	ρ_R	0.7	Interest rate smoothing in Taylor Rule	Smets and Wouters (2007); Author's calibration
ρ_π^*	2.04	ρ_π	1.2	Inflation inertia in Taylor Rule	Smets and Wouters (2007); Author's calibration
ρ_y^*	0.08	ρ_y	0.16	Output gap inertia in Taylor Rule	Smets and Wouters (2007); Author's calibration
$\rho_{\mathcal{E}}^*$	0.001	$\rho_{\mathcal{E}}$	0.2	Nominal Exchange rate feedback in Taylor Rule	Author's calibration
$\rho_{\Delta y}^*$	0.22	$\rho_{\Delta y}$	0.2	Output difference feedback in Taylor Rule	Smets and Wouters (2007); Author's calibration
		Financial Frictions			
α_k^*	0.32	α_k	0.2	Capital share in supply chains production	Bergholt *et. al.* (2019)
-	-	Π_1	1.22	Loan tightness constraint adjustment parameter	Drechsel *et. al.* (2019)
-	-	Π_2	0.5	Elasticity of loan tightness to changes in oil Prices	Drechsel *et. al.* (2019)

Notes: In the source column, the first item before the semicolon is the source for the foreign block value while the proceeding item after the semicolon is for the home block value.

The model is calibrated exactly as in the previous chapter with two changes. I decrease the value of import consumption in the domestic basket of goods to 0.55, so that the consumption of domestic goods is 0.45. While the value has slightly changed, I still maintain foreign bias in home consumption basket. These changes were made to closely match the steady state ratios of the model without financial frictions. In the Oil sector, the rig producer's capital input $\alpha_k = 0.2$ is calibrated to match Bergholt, Larsen and Seneca (2019). In the financial sector, I set the elasticity of the binding constraint $\text{II}_2 = 0.5$ and the adjustment parameter $\text{II}_1 = 1.22$. Drechsel *et. al.* (2019) calibrate these figures to be between $[0,\ 2]$. Table 3.2 displays the steady state ratios of the model compared to the previous chapter's and to two middle-income oil driven economies.

Table 3.2: Benchmark - Data and Model Ratios

VARIABLES	Nigeria	Mexico	Model (Without F.F.)	Model (with F.F.)
Imports (% of GDP)	19.8	39.1	48.2	48.1
Exports (% of GDP)	14.2	38.8	24.19	15.1
Fuel Exports (% of Total Merchandise)	87.0	5.32	84.06	70.1
Final Consumption (% of GDP)	80.2	76.3	77.19	83.4
Gross Capital Formation (% of GDP)	25.4	21.2	22.25	20.1
GDP	\$502bn	\$1.2 tn	-	-
GDP Per Capita	\$2,502	\$9,819	-	-

Notes: Data is from the World Bank's WDI and for the period 2019. Model ratios represent the non-stochastic steady state ratios from the solved model.

While the model has several shocks, in what follows I only assess two shocks that are the two foreign sector TFP shocks; one in the oil sector to mimic oil supply shocks and the other in non-oil sector to mimic oil demand shocks.

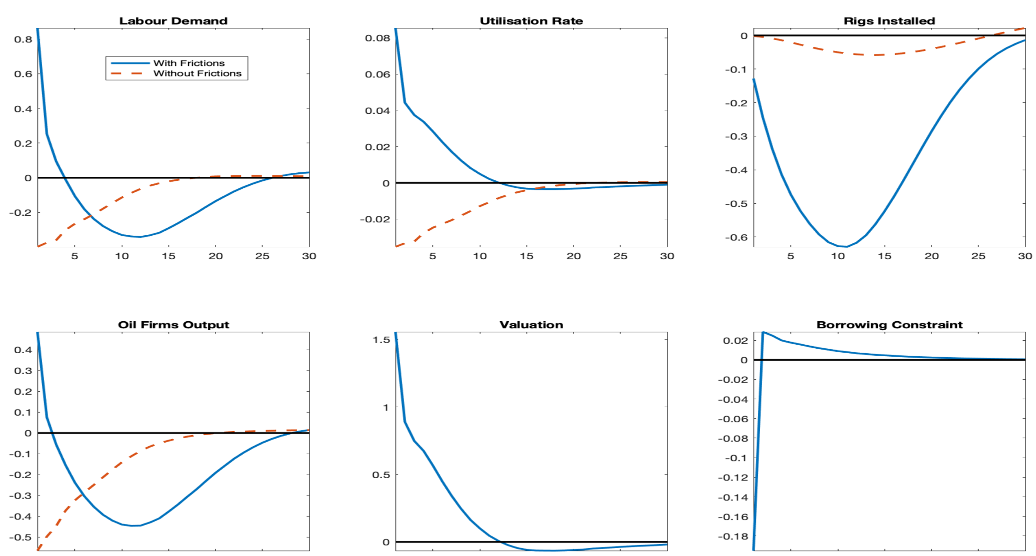
3.4 Transmission Channels under Financial Frictions

3.4.1 Oil Innovations

Figure 3.6 plots the impulse responses of the aggregate macroeconomic variables to an increase in foreign oil sector productivity (to mimic an increase in foreign oil supply). As immediately observed and analogous to the previous chapter, the increase in foreign oil supply causes a contraction in international oil prices, which also leads to a contraction in the domestic economy.

What is immediately observable is the contraction in the economy is considerably less when compared to the same model without frictions. To understand this, observe that the magnitude of the decline in oil prices is significantly smaller simply because the domestic oil producers are unable to increase oil production due to the constraint in lending becoming tighter from lower oil prices preventing any additional demand in inputs to be realised. This is evident from the impulse responses for investment in the oil sector, which contracts considerably and significantly more under the model with financial frictions than in the absence of these frictions. Secondly, the lower decline in oil prices creates lower contraction in oil sector value added and smaller expansion in non-oil sector value added. To understand why contraction in the oil sector is lower, recall that factor inputs by oil producing firms are now increasing in the valuation of easing binding constraint as well as the tightness of binding constraint, Π_t. Therefore during the fall in oil prices, the factor input demands do not contract on impact as the case without financial frictions, but rather steadily decline over time. This induces similar transition dynamics for oil output by domestic oil firms such that with steadily declining factor inputs, oil output steadily declines rather than an immediate and impact contraction (see Figure 3.5).

Figure 3.5: Home Oil Sector Impulse Responses to Increase in Foreign Oil Sector Productivity



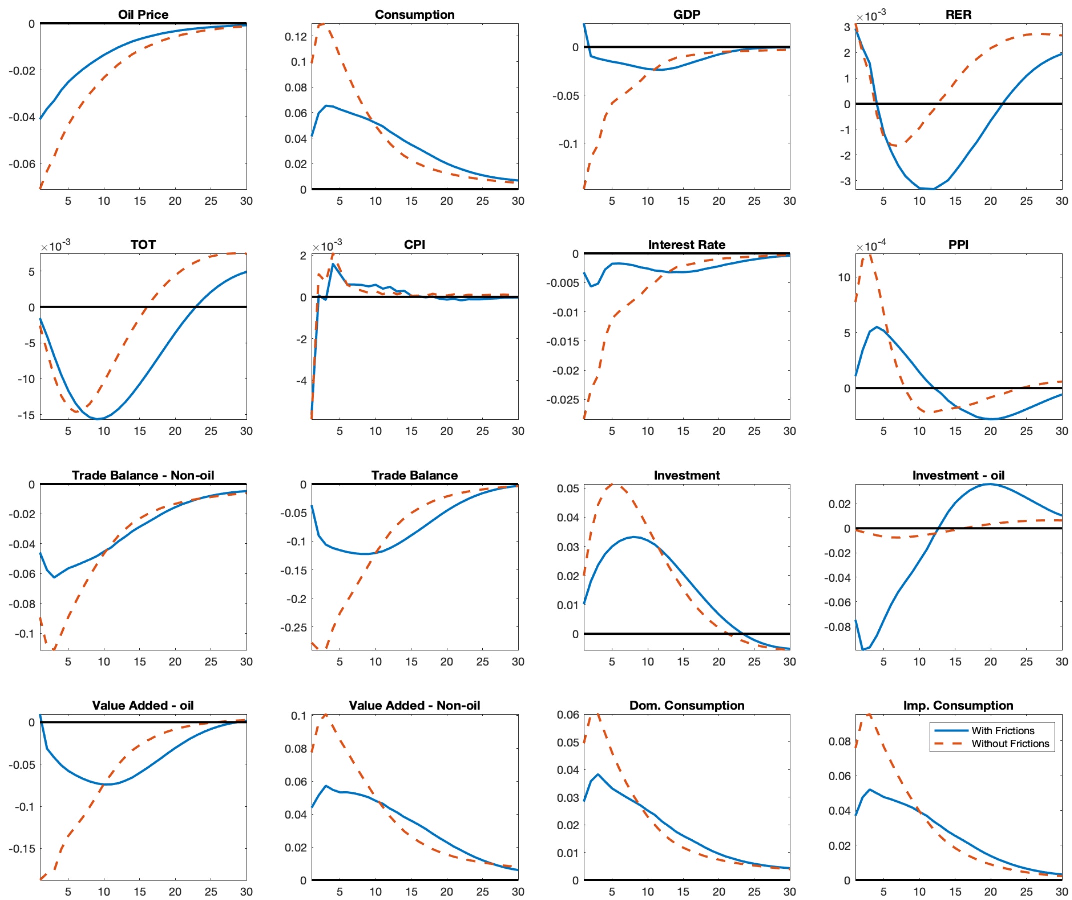
Notes: Unit Standard deviation innovation in $\varepsilon^*_{u,t}$. Blue solid line depicts Impulse responses from the Model with Financial frictions embedded, while the red dashed line depicts the impulse response from the model presented in Chapter 2. Valuation here refers to $\mathbb{Z}_t$ from the optimality conditions and borrowing constraint is the tightness of the borrowing constraint Π_t.

The intuition behind this different dynamics imposed on input demand by the financial frictions is from how oil firms value the relaxation of the tightness of borrowing constraint as well as forward looking behaviour of oil firms who make decisions based on entire oil price path and not only current oil prices. With the valuation parameter $\mathbb{Z}_t$ binding then the working capital constraint induces stronger responses of factor inputs to oil price movements. Emphasis on the movement because every period after the initial impact will always have $P^*_{o,t} > P^*_{o,t-1}$ and given the initial impact on oil prices is negative then this tightens the borrowing constraint on impact but leads to the jump and sudden easing of the constraint. This is also evident in the input demand responses as the sudden declining (jump) of input demands that relaxes following the easing of the borrowing constraint. Additionally, because of the inverse relationship between the tightness of the borrowing constraint and oil firms valuation of easing this constraint, the negative impact effect on tightness of the borrowing constraint is therefore mirrored with a positive impact effect on their valuation. That is if borrowing conditions become extremely unfavourable, oil firms will valuation of easing this borrowing constraint will become larger. Since input demands are also responding strongly to this valuation then it generates the positive impact effect on input demands. This works in tandem with the forward looking behaviour of oil price firms as anticipating tighter borrowing conditions, decrease the number of installed rigs. The differences in effects on capital (rigs) and input demand is simply because input demand decisions are intertemporal but capital decisions are forward looking.

The exchange rate depreciates on impact but appreciates more under financial frictions. This is very straightforward as the decline in oil prices is considerably less, and movement in oil prices induces these appreciations and deprecations in the Real exchange rate. In addition, the lower decline in interest rates retains pressure on future consumption such that consumption today expands by less than the case without financial frictions where interest rates declined by more.

To summarise, have two channels at play here: the financial access channel through working capital constraint is constricting capital acquisition, but preventing the immediate impact contraction of factor input demands. While on the other hand the lower contraction in oil prices is creating less severe economic environment for domestic oil firms, which contribute more to domestic GDP. Therefore, output in oil sector which dominates aggregate GDP contracts less than without financial frictions (while output in non-oil sector expands less, its contribution to aggregate GDP is less), leading to the lower contraction in aggregate GDP.

Figure 3.6: Home Variables Impulse Responses to Increase in Foreign Oil Sector Productivity



Notes: Unit Standard deviation innovation in $\varepsilon^*_{u,t}$. Blue solid line depicts Impulse responses from the Model with Financial frictions embedded, while the red dashed line depicts the impulse response from the model presented in Chapter 2.

In this case the presence of financial frictions and the financial access channel is not necessarily exacerbating the effects of foreign oil shocks. Instead it is constricting the oil sector's volatility through banking constrains, and thus constrains the overall effects the oil sector has on the domestic economy. Hence, contrarily the financial sector is not amplifying the effects of international shocks from the oil sector[38].

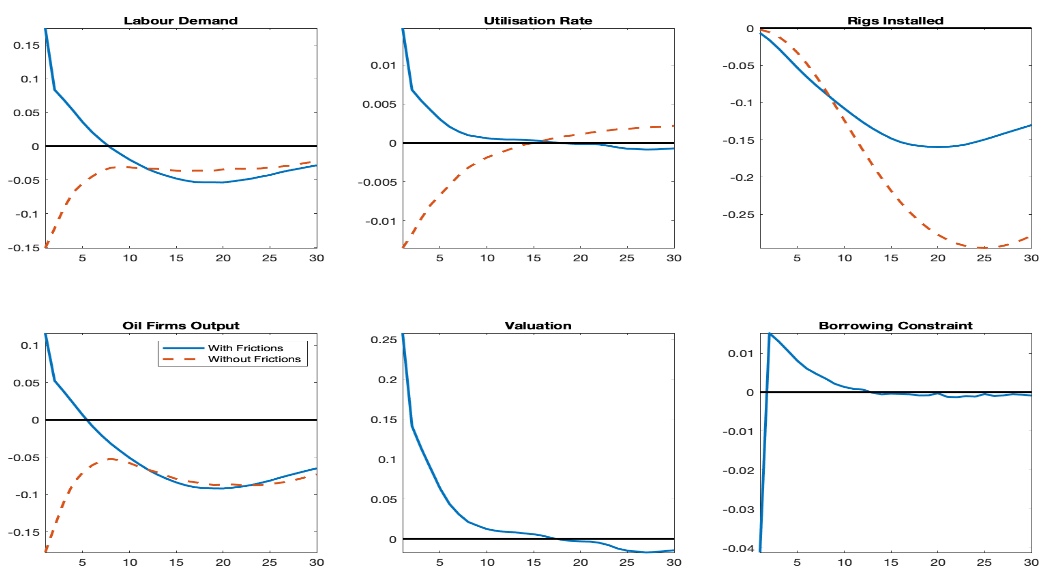
3.4.2 Increase in Foreign Productivity

Figure 3.8 depicts the domestic impulse responses given an increase in foreign non-oil sector firms productivity. The results here show the financial accelerator channel at work with the cap-

[38] However, this result could be due to the arrangement of the financial sector only impacting the domestic oil sector.

ital access channel. Higher oil prices loosen the borrowing constraint on oil producing firms' capital acquisition along with lower nominal interest rates. As a result, domestic oil producing firms can take advantage of the increasing oil prices to increase their output, from higher input demands of factor inputs (which are increasing in $\amalg_t$) evident in the responses from oil sector value added. To grasp this recall the elasticity of rigs and utilisation to oil prices is given by $\frac{1+\amalg_2}{1-\alpha_u} > \frac{1}{1-\alpha_u}$ and $\frac{1+\amalg_2}{\alpha_u} > \frac{1}{\alpha_u}$ respectively, the presence of the financial frictions increases the responsiveness of oil producing firms to higher oil prices. Similarly, from the optimisation behaviour, under financial frictions oil producing firms increase their input demands under favourable oil prices. However, because the immediate impact of the productivity shock is not a positive impact effect on oil prices, the transition dynamics for input demands follows that of the supply shock but since the negative impact effect is significantly smaller and the jump to easing of borrowing constraint somewhat identical the magnitudes of the impact effects and subsequent transition is different. These two conditions characterise the rise in oil sector value added beyond the baseline model without financial frictions.

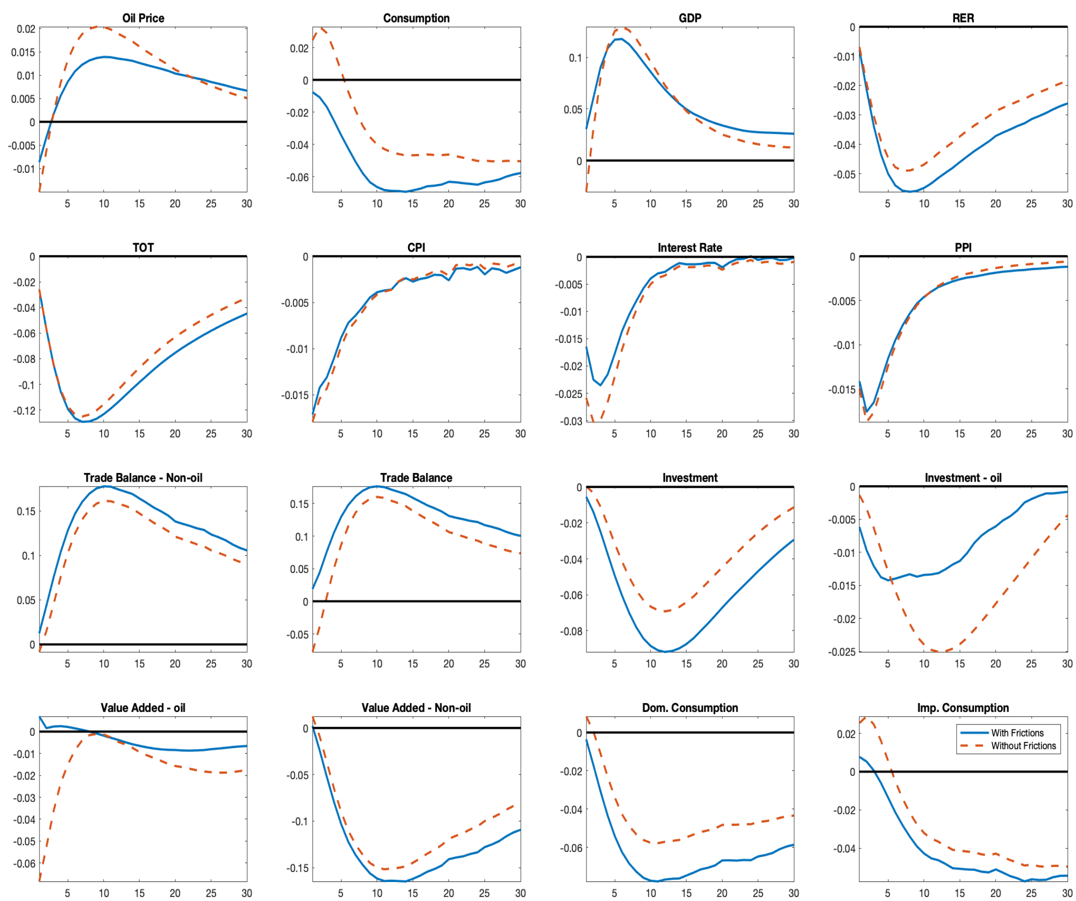
Figure 3.7: Home Oil Sector Impulse Responses to Increase in Foreign Intermediate Good Firms Productivity



Notes: Unit Standard deviation innovation in $\varepsilon^*_{f,t}$. See Figure 3.6

The consequence is an amplifying effect here for domestic aggregate macroeconomic variables. Consumption (aggregate and both domestic and imports) is contracting at a greater magnitude, the real exchange rate is appreciating even more, trade balances are expanding even more, value added in oil sector and non-oil sector are expanding and contracting by more respectively. One could say the lower contraction in oil investments presents an anomaly but given the conditions behind optimal investment choice of firms being constrained by forward looking oil prices, this should be the case. This is contrary to the oil supply shock, and one can ascertain that the financial frictions are exacerbating shock effects when compared to the baseline model without frictions. This is intuitive because, under the productivity shock the higher oil prices ease the financial constraints on oil producing firms. These firms that drive aggregate economic activity by internalising this ease of constraints and produce more leading to higher oil sector value added.

Figure 3.8: Home Variables IRFs to Increase in Foreign Productivity of Intermediate Good Firms



Notes: Unit Standard deviation innovation in $\varepsilon^*_{f,t}$. See Figure 3.6

3.5 Sensitivity Analysis

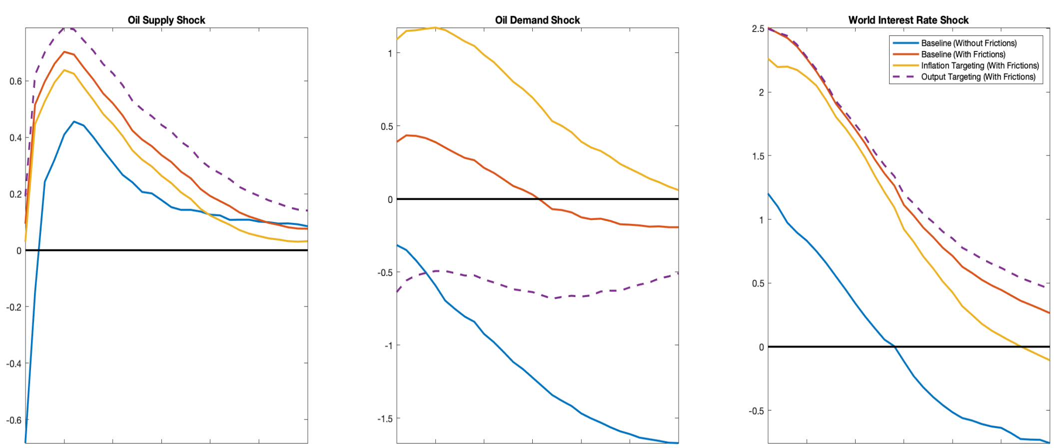
The inclusion of financial sector within the model induced two additional transmission channels from which international shocks transmit to the domestic economy. In this section I investigate how the financial frictions included affect the level of welfare households are subject to. I once again define the welfare in the economy by:

$$\mathcal{W} = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left[\frac{[C_t(l) - hC_{t-1}(l)]^{1-\sigma_c}}{1-\sigma_c} - \frac{\sum_{j=N,k,u} N_{j,t}^{1+\sigma_n} \Delta_{wj,t}}{1+\sigma_n} \right] \tag{3.5.1}$$

In line with Kim *et. al.* (2008), the conditional welfare criterion is chosen while I utilise the non-stochastic steady state as the initial condition. To efficiently obtain the welfare function, the model is simulated and evaluated up to the second order as Schmitt-Grohe and Uribe (2007) have shown that up to the first order, the expected unconditional utility equals its value in the non-stochastic state. I simply compare the welfare responses across different policy regimes to identify what policy induces the best welfare responses.

3.5.1 Monetary Policy Regimes in the Presence of Financial Frictions

Figure 3.9: Home Welfare Impulse Response to Monetary Policy Targeting Regimes



Notes: The three figures represent shocks to international oil sector productivity, non-oil sector productivity and interest rates respectively. The blue dashed line represents the model without financial frictions presented in chapter 2, the red solid line is the baseline model with financial frictions, the yellow solid depicts inflation targeting and the dashed line represents output gap targeting all with financial frictions embedded. Oil demand shock is a shock to ε_t^*. Oil supply shock refers to shock to $\varepsilon_{u,t}^*$ and World interest rate shock is a shock to $\varepsilon_{R,t}^*$

In the previous chapter it was shown that output targeting regimes are a better choice for the central banker in the case of oil price declines induced by increases in foreign oil supply. While inflation targeting was a better alternative when oil prices rose from higher demand shocks inducing higher oil prices. I want to conduct the same experiment here, but within the framework that included financial frictions. The interest rate channel through which monetary policy affected firms (oil producing firms are of interest here) can now be accelerated by the presence of banks that charge a lending rate that restricts access to capital – the capital access channel.

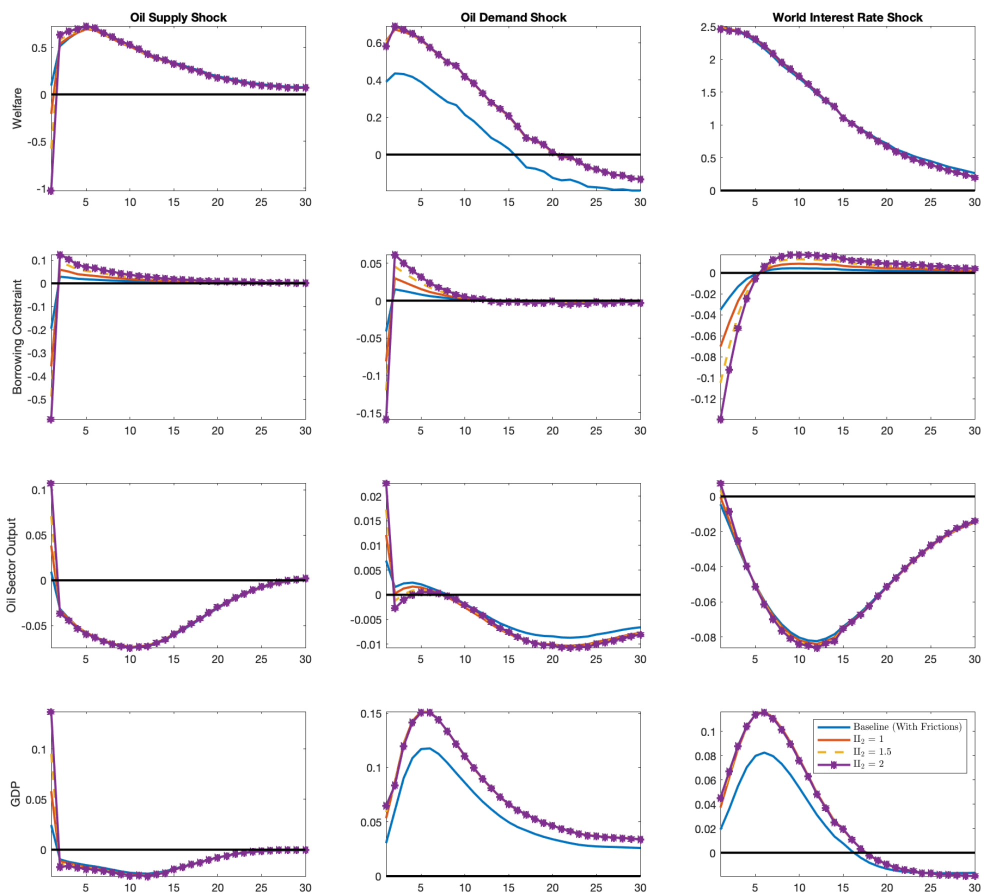
What is immediately evident is that the model with financial frictions induces better welfare metrics that without these frictions. When looking at the oil supply shock the intuition behind this is follows from the constraint placed on oil producing firms. Since the oil sector drives the aggregate economy, volatility in their output transmits to volatility in aggregate domestic output. Therefore, any mechanism that constrains these firms from uncontrollably adjusting capital and investment (basically an ad hoc adjustment cost) prevents them from creating higher peaks and lower busts. In other words, the financial frictions prevent the loss in welfare as with the other standard macroeconomic variables in section 5.1. Similarly, when looking at the productivity shocks and interest rate shocks, under financial frictions, the amplification of effects passes through to domestic welfare and financial frictions induce better welfare metrics for domestic households. Note that this runs in both directions, that is, the amplification of positive welfare also becomes and amplification of negative welfare effects if the shocks run in the other direction.

Like the previous chapter, I observe that inflation targeting regimes perform worst when oil prices decline whether as a result of increased foreign oil supply or increased foreign interest rates driving down demand - the output gap targeting will tend to perform better. Analogously, inflation targeting performs better when oil price increases as a result of foreign demand from higher foreign productivity.

3.5.2 Elasticity of The Binding Constraints

In order to assess the impact oil prices have on credit availability, I simulate the model with different parameter values for the elasticity of the loan binding constraint to oil prices. Here I compare domestic household welfare and tightness of borrowing constraint as well as other key aggregate macroeconomic variables based on changes to the elasticity of the binding constraint[39]. That is, I choose $\Pi_2 \in [0.5,\ 1,\ 1.5,\ 2]$. Figure 3.10 and compares the results of the differing choices of Π_2. The results confirm that while the elasticity of the binding constraint to oil prices plays a significant role in the amplification of shock effects, the differences are not always pronounced and sometimes insignificant. Though the responses are quite similar, the presence of the financial sector intensifies the propagation of shocks through the elasticity of the binding constraint, as for every elasticity greater than the baseline 0.5 the response is either identical or greater.

Figure 3.10: Sensitivity To the Elasticity of The Binding Constraint



Notes: Oil demand shock is a shock to ε_t^*. Oil supply shock refers to shock to $\varepsilon_{u,t}^*$ and World interest rate shock is a shock to $\varepsilon_{R,t}^*$. See Image Key.

In summary, the elasticity of the borrowing constraint directly affects the sensitivity of oil pro-

[39] Note that I keep the same Taylor rule parameters as the baseline with only the elasticity of the binding constraint changing.

ducing firms to changes in international oil prices such that the greater the sensitivity (i.e. the greater the elasticity) the greater the effect on oil producing firms demand. Recall that input demands from optimisation are increasing in $\amalg_t$, therefore increasing the elasticity $\amalg_2$ amplifies the response of $\amalg_t$. However, the quantitative differences are minuscule and present themselves in certain aggregate macroeconomic variables and are shock specific. Furthermore, higher elasticity is associated with better household welfare when oil prices increase from demand side shocks. This is simply transmitting through the capital access channel that is increasing oil sector activity, which in turn increases the expansion of domestic consumption and labour supply and thus, welfare. It is worth noting that this sensitivity runs in both directions, in other words higher elasticity will always create either bigger peaks or bigger troughs in the borrowing constraint evident through the responses in the productivity and interest rate shocks.

3.5.3 The Financial Accelerator: Leverage Constraints and The Elasticity of The Binding Constraints

So far banks are unable to enact a spread between their lending rate and the nominal interest rates. To remedy this, I consider the case where banks face an adjustment cost in their lending decision, such that the adjustment cost function introduces a persistent minimum capital requirement. Additionally, the adjustment cost induces leverage constraints in the form of Debt-to-Equity ratio for firms. Banks now finance loan operations utilising both deposits as well as bank equity[40]. So equation (3.3.16) then becomes:

$$L_t^B(h) = B_{H,t}(h) + E_t(h) \tag{3.5.2}$$

Bank equity now evolves according to:

$$E_t = E_{t-1} + \mathcal{D}_t^B(h) \tag{3.5.3}$$

This imposes a strict condition on bank's equity - it can only accumulate equity based on profit earned[41]. I can now express the bank's new profit function with adjustment costs and equity

[40] Equity will act as the banks capital.

[41] In the steady state the bank's profit will equal zero.

financing of loans. That is the banks profit function is now given by:

$$\mathcal{D}_t^B(h) = \frac{1}{\pi_t}(R_{L,t-1} - R_{t-1})L_{t-1}^B + \frac{1}{\pi_t}R_tE_t - S_t^B(L_t^B)$$

Where $S^B(L_t^B) = \frac{\chi^B}{2}\left(\frac{E_t}{\omega_b L_t^B} - \mu_t^B\right)^2 L_t^B$ is the adjustment cost for banks[42]. I choose this specification based on the assumption of no loan default by oil producing firms, thus creates no credit risks for banks. Here $\frac{E_t}{\omega_b L_t^B}$ is the inverse of the Debt-to-Equity ratio and represents the leverage constraints. μ_t^B is the minimum capital requirement, which I assume to be a very persistent auto regressive process (I set the auto regressive coefficient $\rho_B = 0.99$)[43]. ω_b is the risk weighting factor for loans and χ_b is the adjustment cost parameter, which I assume to be non-negative.The optimal loans and equity for banks is therefore given by:

$$\mathbb{E}_t\left[M_{t,t+1}\left[\frac{1}{\pi_{t+1}}(R_{L,t} - R_t) + \frac{\chi_b}{2}\left(\frac{E_t}{\omega_b L_t^B} - \mu_t^B\right)^2 - \frac{\chi_b E_t}{\omega_b L_t^B}\left(\frac{E_t}{\omega_b L_t^B} - \mu_t^B\right)\right]\right] = 0 \tag{3.5.4}$$

$$\mathbb{E}_t\left[M_{t,t+1}\left[\frac{1}{\pi_{t+1}}R_t + \frac{\chi_b}{\omega_b}\left(\frac{E_t}{\omega_b L_t^B} - \mu_t^B\right) - \mathcal{Q}_{t+1}^B\right]\right] = \mathcal{Q}_t^B \tag{3.5.5}$$

Where $\mathcal{Q}_t^B$ can be interpreted as the bank's Tobin's Q. Equation (3.5.4) now implies that the bank's capital buffers are increasing along with an increasing nominal interest rate margins. In other words higher interst rates generates more cpaital buffers for banks. Equation (3.5.5) pays resemblance to households capital Euler. It states that the bank increases equity up to the point where the marginal cost of accumulating equity equates the price. I calibrate $\omega_b = 0.9$ and $\chi_b = 0.00384$ according to posterior modes parameter estimates from Carvalho *et. al.* (2013) and Karmelaviciusa and Ramanauskas (2019) respectively. Furthermore, I also simulate the model with adjustment costs based on different elasticities of the binding constraint as in the previous subsection.

I obtain very interesting results. Figure 3.11 & Figure 3.12 show the results with the inclusion of the adjustment cost as well as under different elasticities of the loan binding constraint[44].

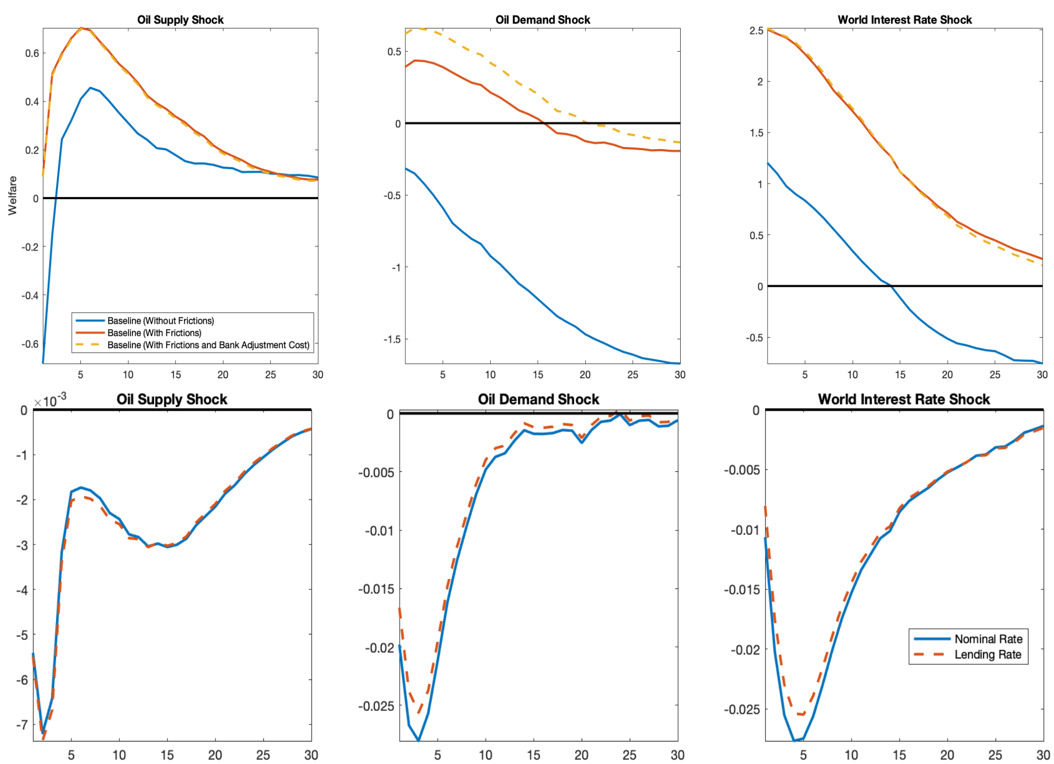
[42] Several authors specify varying adjustment costs for banks depending on their desired goal.

[43] While I do not go into macroprudential policies, the presence of the minimum capital requirement allows the central bank to loosen capital constraints placed on oil producing firms by changing the minimum capital requirements for banks.

[44] Note that I select the same elasticities for the binding constraint as in the previous analysis.

Even though a spread exists between the lending rate and the nominal interest rate, there is no difference on the effect of including the adjustment cost to the banks optimisation problem. The financial accelerator does not create any significant amplification of shocks[45]. Even when considering the model with adjustment costs and higher elasticity of binding constraint, the model still produces no significant differences in the amplification of shocks. I offer the following explanation. The simple balance sheet representation of banks does not consider net worth of the banks or net worth of the oil producing firms. Therefore, the leverage constraints do not create any differences in how banks would change their borrowing conditions Π_t in the presence of changing oil prices. In addition to this, without the risk of default in the model banks' lending operation is considerably simple.

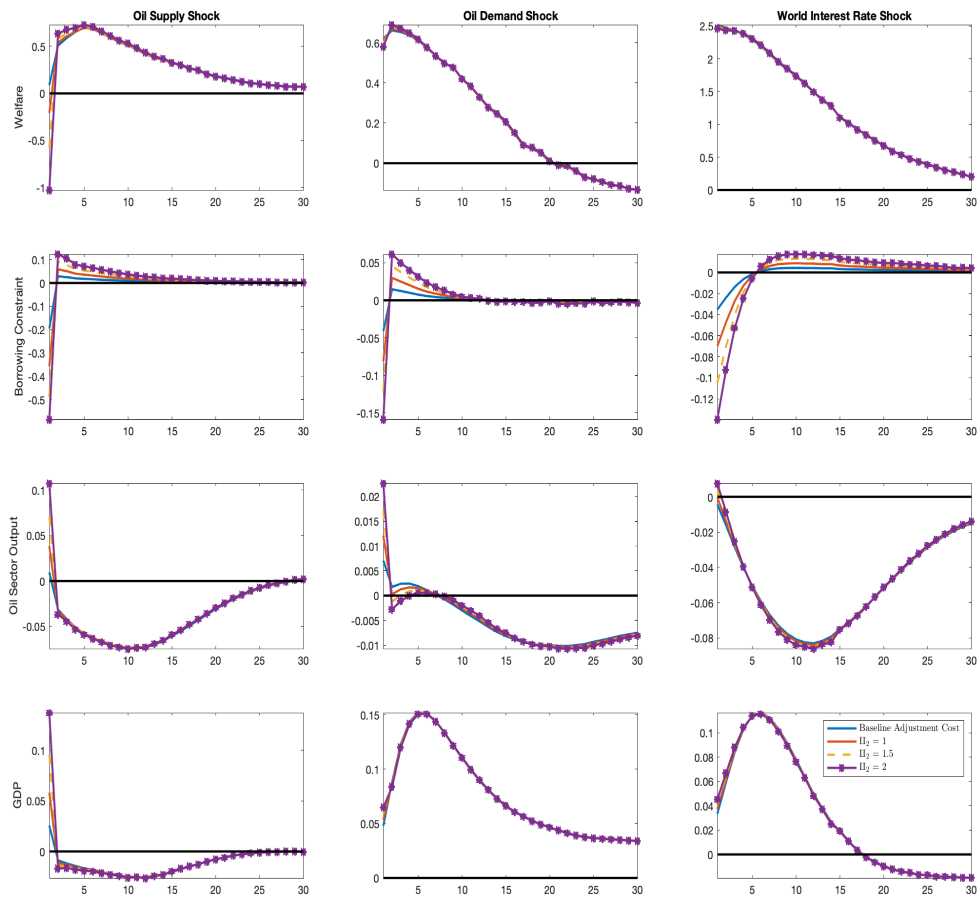
Figure 3.11: Sensitivity to Inclusion of Adjustment Costs



Notes: Oil demand shock is a shock to ε_t^*. Oil supply shock refers to shock to $\varepsilon_{u,t}^*$ and World interest rate shock is a shock to $\varepsilon_{R,t}^*$. The first row represents welfare, with the blue solid line representing the model without financial frictions as presented in chapter 2, while the red solid and yellow dashed line represent the model with financial frictions under the baseline and with adjustment costs respectively. The second wow represent responses of interest rates and lending rates, where the blue solid line represents the nominal rate and the red dashed line represents the lending rate.

[45]Shousha (2016) obtains a similar result where the inclusion of leverage constraint or the imbalances between bank balance sheets do not have any significant differences in the amplification of international shocks.

Figure 3.12: Sensitivity to Adjustment Costs and Changes in Elasticity of The Binding Constraint.



Notes: Oil demand shock is a shock to ε_t^*. Oil supply shock refers to shock to $\varepsilon_{u,t}^*$ and World interest rate shock is a shock to $\varepsilon_{R,t}^*$. See Image Key.

Though I note that this does present an opportunity for central bankers to utilise macroprudential policies (altering the minimum capital requirement) in easing lending constraints of banks during bust periods. In theory, by changing the reserve requirement, it allows the bank to change its lending operation and possible increase capital access to oil producing firms as it decreases the spread between the nominal interest rate and the lending rate[46]. Given the capital access channel dominates the propagation of shocks, increasing capital access to the oil sector will improve oil sector's value added, which ultimately drives the aggregate economy[47].

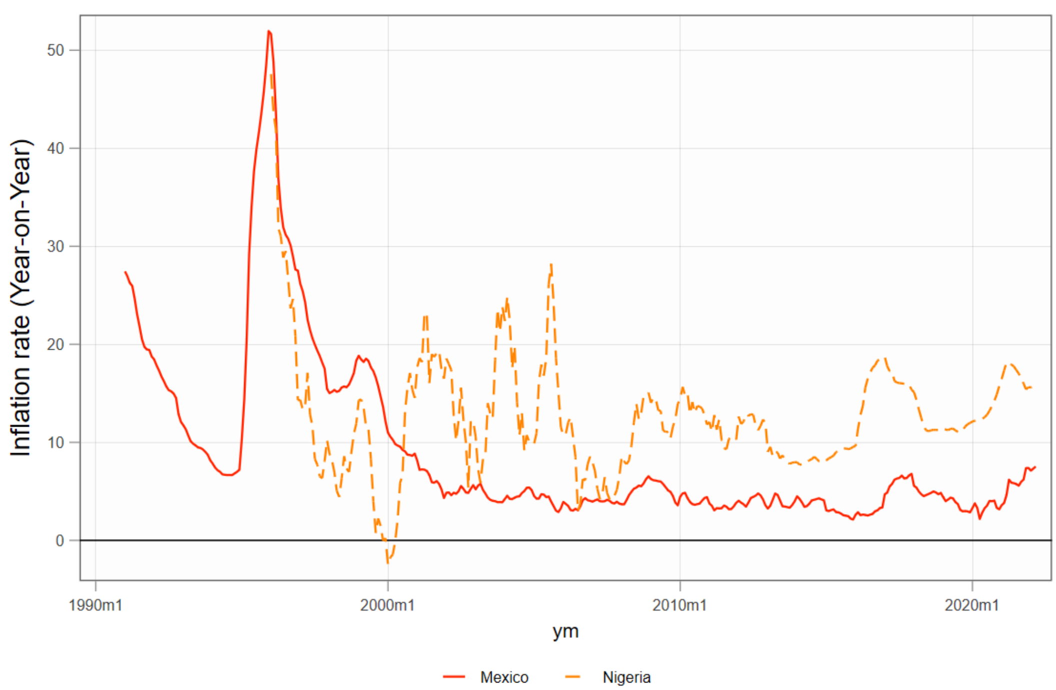
[46]The literature on the effects of the reserve requirement is vast and finds evidence though modest, for the effects of shocks to the reserve requirements on the economy. See e.g. Glocker and Towbin (2012), Tovar, Garcia-Escribano and Martin (2012), Areosa and Coelho (2013).

[47]I do not delve into macroprudential policies as it present opportunity for future research that can be explored.

3.6 A Different Kind of Monetary Policy – Exchange Rate Interventions

Since interest rate adjustments cannot completely offset the inefficient fluctuations of aggregate macroeconomic variables and volatility in exchange rate induced by international spillovers, what more can commodity driven economies utilise to their advantage[48]. Lower and middle-income commodity driven economies have over the years suffered periods of elevated levels of inflation[49] (see Figure 3.13) accompanied by depreciations in their exchange rates, then perhaps an additional monetary policy should be exchange rate interventions. In this section I explain how exchange rate interventions, which are not present in the model could be adopted in these types of economies to increase the efficacy of interest rate adjustments in the face of international spillovers.

Figure 3.13: Year-on-Year Inflation Rates in Mexico and Nigeria



Notes: Data is from Haver Analytics, CPI are seasonally adjusted before Year on Year differences are taken. Data is reported monthly and covers the period 1992 − 2021.

[48] I restrict attention to monetary policy and do not consider Fiscal policy as that warrants a completely new research on its own.

[49] Sargent (1982) backed by historical data argues that the periods of high inflation are due to behaviours of both monetary and fiscal policy.

I have so far shown how inflationary pressures induced by nominal exchange rate depreciations render any inflation targeting rule by the central banker to be ineffective, with output gap targeting proving to be the better choice of target under specific shocks. However, given the important role exchange rates play, one should understand more in-depth how to control and manage these depreciations to prevent periods of extraordinarily high inflation periods that plagues low and middle-income commodity economies and emerging market economies. In the chapters presented I introduced a '*dirty*' nominal exchange rate target that central banker respond to as they do output gap and inflation. Several authors[50] have studied the effects of the inclusion of nominal or real exchange rates into the Taylor rule and find limited evidence that the inclusion of the exchange rates plays a significant role. Roger *et. al.* (2009) show that inclusion of the exchange rate into the Taylor rule comes at a trade-off between reducing the volatility of exchange rates and increasing the volatility of inflation and output.

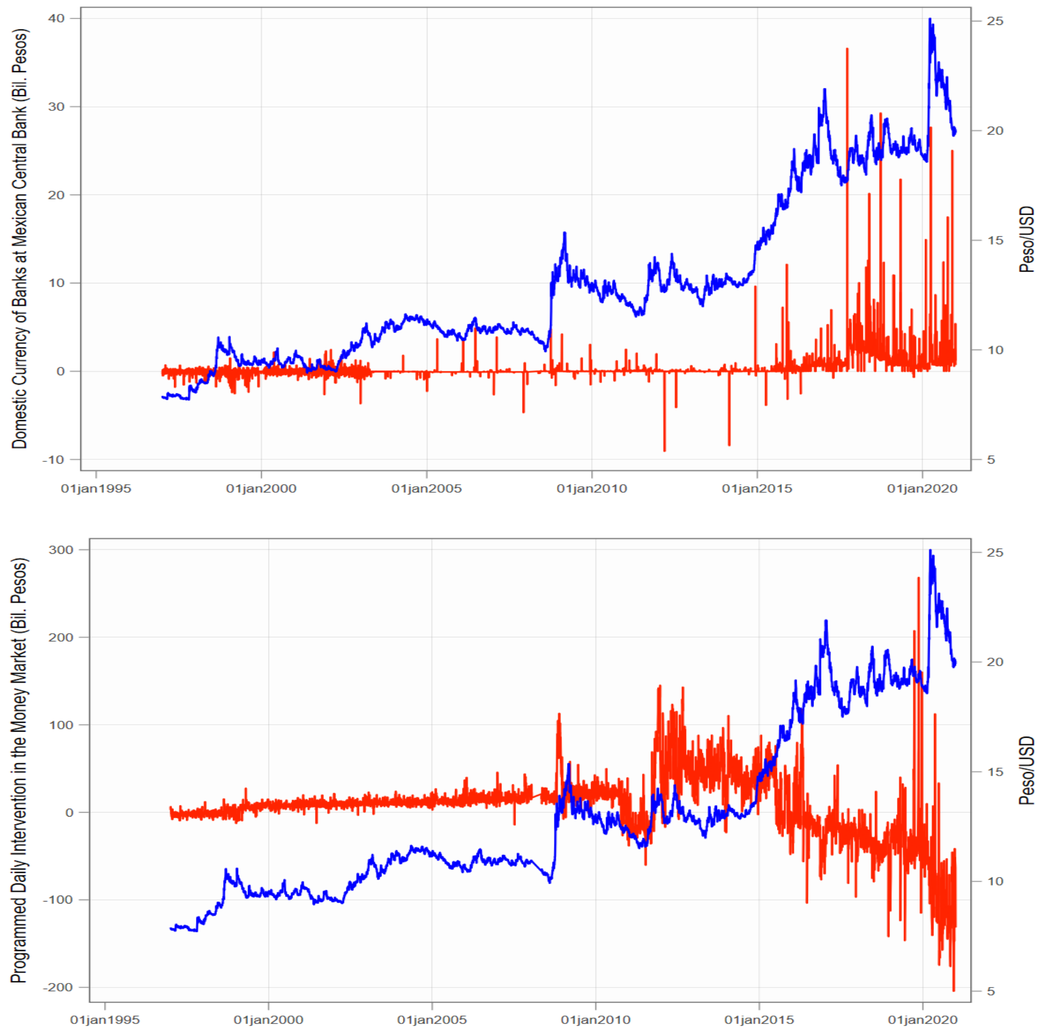
Given that exchange rate interventions are the primary instrument EMDEs use in managing exchange rates (Benes *et. al.*, 2015), I must make the distinction between exchange rate targeting and exchange rate intervention. Exchange rate targeting simply put is movement of interest rates in response to changes in the exchange rate, while exchange rate intervention is central bank directly attempting to affect the behaviour of exchange rates through foreign exchange market operations, such that the exchange rate becomes an operational target (see Figure 3.14). Additionally, when referring to exchange rate interventions I refer to *sterilised exchange rate interventions*[51] that affect exchange rates through portfolio balancing with foreign and domestic bonds being imperfect asset substitutes such that sterilised interventions alter the relative supply of domestic bonds that in turn alter the composition of portfolios for investors. The changes in portfolio composition challenge the views of investors, here investors will require either higher or lower returns based on the risk premium associated with the bonds to absorb the increasing or decreasing supply of the bond. This combined with the increase in the demand for foreign domiciled bonds creates either and appreciation or depreciation of the exchange rate[52].

[50]See e.g. Morn and Winkelried, (2005), Roger *et. al.* (2009). These authors study the concept in terms of emerging markets and developing economies (EMDEs) but their results can be applicable to low and middle-income commodity driven economies given a significant number or EMDEs are commodity dependent economies.

[51]*Non-sterilised exchange rate interventions* work through the liquidity channel as proportional central bank dollar purchases may depreciate the underlying domestic currency through increases in liquidity of money market. See Domac and Mendoza (2004), Dominguez and Frankel (1993).

[52]A second channel for sterilised exchange rates exist in the form of signalling based on central bank credibility and effectiveness in utilisation of exchange rate interventions. I do not go into this as studies have shown that the signalling channel is a weaker transmission channel in emerging markets due to the failure of institutional policies

Figure 3.14: Exchange Rates and Programmed Intervention by *Banco de* Mexico



Notes: Data is from Haver Analytics and is reported Daily. Data period is from 1997 – 2021. In both plots, the blue line depicts the nominal exchange rate while the red line depicts the left y-axis variable.

Early strand of exchange rate intervention such as Branson and Henderson (1985) showed that by allowing different composition of assets in portfolio, one could influence the risk premium. Though Dornbusch (1976) positioned the research on exchange rate through perfect asset substitutability, with Backus and Kehoe (1989) showing that constant time paths for monetary policy had no impact on asset markets as private sector decisions remained unchanged due to perfect substitutability, thus, having no impact on the risk premium. However, as done in the model, I abstract from complete markets (hence perfect asset substitutability) because in general risk

to back up their commitments. In other words, failure of central bank credibility in emerging markets weakens transmission through signalling channel. See Canales-Kriljenko *et. al.* (2003), Domac and Mendoza (2004) sharing between low- and middle-income countries (whether commodity driven or not) and the rest of the world does not hold. Furthermore, Kumhof (2010) shows that through government spending shocks that induce nominal exchange rate adjustment, one can create imperfect substitutability between public assets. Thus, given private sector exposure to exchange rate risk, exchange rate interventions work. Therefore, thinking about low- and middle-income commodity driven economies, with individual country default risk and incomplete asset markets the idea of imperfect asset substitutability is not farfetched[53].

Earlier literature on FX interventions focused on advanced economies such as Dominguez and Frankel (1993) that looked at the Swiss franc, Fatum and Hutchison (2003) that looked at the German deutsche mark, and Fatum and Hutchinson (2010) that looked at the Japanese Yen. However, since the concern is with low- and middle-income commodity economies which also make a significant portion of the EMDEs. For example, Domac and Mendoza (2004) utilising data on Mexico (and turkey) in a GARCH framework show that the daily exchange rate interventions by the Banco De Mexico considerably decreased the volatility of its exchange rate. They also conclude that the use of exchange rate intervention in line with an active (inflation) targeting regime could be used in offsetting the effects of temporary exchange rate shocks that induce inflationary pressures. Montoro and Ortiz (2013) construct a DSGE model with foreign exchange dealers that drive deviations from the UIP condition. They find that exchange rate interventions have important applications for monetary policy transmission mechanism as it is capable of disrupting transmission channels and decreasing the impact of shocks on inflation. Similarly, they find that interventions have stronger stabilisation power than conventional targeting as interventions exploit the expectations channel. Vargas *et. al.* (2013) also construct a DSGE model with financial frictions and find that given imperfect asset substitutability, interventions can have a positive impact on credit supply through bank balance sheet alterations. Ostry, Ghosh and Chamon (2012) argue that abstracting from perfect capital mobility, then interventions in conjunction with inflation targeting can be useful in managing exchange rate fluctuations. The BIS (2005) shows that interventions could be utilised in offsetting short-run undesired volatility in exchange rates if the shocks it stems from are temporary.

In contrast to this, Sarno and Taylor (2001) argue that industrialised countries with highly in-

[53] Benes *et. al.* (2015).

tegrated capital markets exchange rates interventions are ineffective. Similarly, Watabe and Harada (2001) applying a GARCH framework to Japan's programmed exchange rate intervention shows that interventions had a short-term effect but no long-term effects.

While the literature also provides some evidence that exchange rate interventions remain ineffective, the arguments behind these conclusions are based on advanced capital markets with improved asset substitutability. As already stated, and within the literature for low- and middle-income countries, capital markets are far from complete, risk sharing with the world does not hold and assets do not hold as perfect substitutes. Therefore, it is indicatively plausible that given the thin nature of markets changes in supplies will induce significant changes in relative prices. I argue that given the right monetary responsiveness in conjunction with efficient exchange rate interventions, low- and middle-income commodity economies can limit the depreciations associated with temporary declines associated with commodity busts and prevent inflationary pressures that accompany them. This allows for faster economic recovery without aggressive inflationary pressures.

3.7 Conclusion

Commodity price cycles have become ever so important for business cycle dynamics of low- and middle-income commodity driven economies. Yet very little can be said about the conduct of monetary policies in these countries and the role it should play in insulating their economies from commodity price volatility. Additionally, the interrelationship between the financial sector and the commodity cycles is highly procyclical, with commodity price booms driving financial sectors output. Thus, in my assessment I introduce a financial sector to an already rich two-country commodity DSGE model in order to understand the role commodity price relationship with financial constraints play in driving business cycle dynamics in low-and middle-income commodity economies. Furthermore, I assess the effects of monetary policy regimes from a welfare perspective in order to characterise the targeting objective central banks should pay attention to when reacting to international commodity price shocks.

I am able to show that commodity driven booms and busts can be amplified or constrained by the presence of the financial sector owing to the tightness of the borrowing constraint enacted by banks within the financial sector as well as how much oil firms value the easing of this tightness. With the elasticity of oil rig purchases and utilisation of oil active oil rigs is increasing the borrowing constraint parameter, during commodity booms the relaxation of the borrowing constrain increases the responsiveness of oil producing firms to higher oil prices, which in turn increases the capital acquisition and utilisation by domestic oil producers, which allows the increase in domestic oil sector value added that drives the aggregate economy.

In continuing with the goal of offsetting the effects commodity prices have on the exchange rate of low- and middle-income country, I also provide thoughts on how exchange rate interventions can be a useful tool in managing the depreciations of domestic exchange rates. This in conjunction with an appropriate targeting regime which is dependent on the underlying reason oil prices changed, can be the most effective in limiting periods of high inflation. Analogously, it could work in the other direction in preventing realisations of the Dutch disease by limiting the appreciations of exchange rates.

Finally, while I focus on monetary policy, there is growing argument on the importance of fiscal policy in managing exchange rate pass-through to domestic inflation[54]. This presents an opportunity for future research by extending the model to capture the full dynamics of both fiscal and monetary policy. Once again I present one of the most complete dynamic models for assessing monetary policy in low- and middle-income commodity driven economies that not only takes into account the relationship between oil markets and the domestic economy, but as well as the relationship between oil prices and credit availability.

[54] See Agenor (2016).

Chapter 4

Unconventional Monetary Policies: Large Scale Asset Purchases in HANK

4.1 Introduction

Unconventional monetary policies in the form of large scale asset purchases (typically referred to as quantitative easing) have become more instrumental since the financial crisis, and as central banks reached the zero lower bound. Once again in response to COVID-19 induced recessions and central banks attaining the zero lower bound on interest rates, many governments and central banks have engaged in more large scale asset purchase (LSAP) programs. In March 2020 the Federal Reserve System in the U.S. announced the reinstatement of its asset purchase programs and since July of 2020 has purchased $80 Billion Treasury Securities and $40 Billions of agencies backed mortgage securities monthly. In the European Union (EU) the European Central Bank (ECB) initiated the Pandemic Emergency Purchase Programme in March 2020 that was subsequently up-scaled at later dates to reach a total of €1850 Billion in asset purchases. Similarly, the Bank of England initiated its own asset purchase programmes in response to the pandemic.

The Objective of these LSAPs is to support the aggregate economic activity as countries dealt with this pandemic. This once again calls into question the effectiveness of LSAPs in increasing aggregate economic activity and its transmission channels within the economy. Unconventional policies target the cost and availability of external finance to households, banks, other financial and non-financial operations by operating on different aspects of the yield curve through a reduction of maturities (Chen *e. al.*, 2012; Curdia *et. al.*, 2010; Woddford, 2010). Though the effect of asset purchases could diminish as economies recover given the transmission channel of portfolio rebalancing could be more effective under strained economic conditions and thus, weakens as macroeconomic variables normalise within the economies. In other words, the financial market impacts of asset purchases are hinged on the economic conditions especially given the level of interest rates. Similarly, Borio and Hofmann (2017) posit that with extended periods of low interest rates, the macroeconomic effect asset purchases have on financial markets will diminish as the economy recovers.

I augment the HANK model created by Kaplan, Moll and Violante (2018) to include a bond shock *a lá* Chen *et. al.* (2012). Following from Kaplan, Moll and Violante (2018), first, I estimate the earnings distribution of heterogenous agents using SCF data in order to generate the distribution of income earnings processes, then proceed to simulating the model with calibrations matching estimated DSGE parameter estimates for the U.S. Although I do not explicitly model the imperfect substitutability of liquid and illiquid assets, the presence of heterogeneity in income earnings creates a financial adjustment cost to substituting between liquid and illiquid assets. In addition to this, households face an adjustment costs for transferring wealth across liquid and illiquid accounts[1]. Similarly, to Chen *et. al.* (2012) I do not explicitly model LSAPs through a central bank, rather governments control the supply of liquid assets such that LSAPs are a straightforward increase in government debt by increasing bond supply. As the HANK model is presented in continuous time, I do not impose a strict government bond supply rule, rather a straightforward bond innovation that increases the number of government bonds within the economy. As the model is developed in a closed economy setting, LSAPs implemented by the government translate directly to higher household bond holding without foreign intervention.

Though there is a large extensive literature on LSAPs, to my knowledge I present one of the first economic analysis of the effects of LSAPs under a heterogeneous agent new Keynesian (HANK) model. In this chapter, I estimate and simulate a closed economy HANK model to assess the impacts LSAPs have on crucial macroeconomic and financial variables. Given the economy is not at the zero lower bound, which was the case of several economies at the start of their asset

[1] I interpret this as a proxy that induces some form of imperfect asset substitutability.

purchase programmes. I also present a comparison with a relevant Two-Agent new Keynesian (TANK) model that mimics the structure of the HANK model. In both models here, households hold both government bonds and firm equity such that government asset purchases transmit to the economy by creating a redistribution of portfolio adjustment choices of households such that they rebalance their portfolio mix of equity and government bonds. A reallocation towards firm equity increases the price of equity, transmits to productivity of firms by increasing investment and capital accumulation given the structure of equity within the model. Similarly, the rise in stock market from higher equity prices creates the larger returns for holding equity and drives the spread[2].

To ascertain the source of heterogeneity in both models, I follow the original authors of both model Kaplan *et. al.* (2018), also related to Werning (2015) and Derbotoli and Galí (2017) there are two dimensions to first consider that create the divergence of both HANK and TANK from their RANK counterpart. The first dimension of heterogeneity is given by time varying differences in average consumption between households facing the binding borrowing constraint (constrained households - *Hand-to-mouth* households) and those that do not (unconstrained households - saving households). This dimension of heterogeneity is embedded in both TANK and HANK models and forms the underlying basis of heterogeneity in TANK. The second dimension of heterogeneity is given by time varying consumption dispersion (or differences) within households that are unconstrained by the binding borrowing constraint. Because of the income and wealth distribution of the HANK model, only HANK satisfies this second dimension of heterogeneity[3].

Another dimension of policy implication I assess is with regards for inequality. While inequality in general affects policy, I ask the question on how policy affects inequality. With heterogeneity of agents, the difference in bond and equity holdings across distributed household groups creates wealth inequality. With the estimated earning process such that income depends on productivity, which induces income inequality. The amalgamation of both income and wealth inequality will further create consumption inequality and I'm able to assess how LSAPs drive inequality across households.

[2]The spread is the difference between the return on holding equity and the return on government bonds

[3]In Derbotoli and Galí (2017) this dimensions of heterogeneity materialises in the form of wedges in the aggregate consumption's Euler equation.

Taking into account heterogeneity of agents is vital in understanding the redistributive effects (across income groups) of both conventional and unconventional monetary policy[4]. The structural construct of agents' heterogeneity will have consequences on how inequality manifests in both HANK and TANK models. Here the distributional effects will impinge on how asset purchases affect income and wealth distribution of agents with different marginal propensities to consume. Since under TANK, *Hand-to-mouth* households do not hold any savings whether in the form of liquid or illiquid wealth, and consume their entire income, their marginal propensity to consume is ultimately always 100%. Additionally, because income in TANK is determined by only market clearing as opposed to the additional earnings process in HANK, all agents earn the same labour income. Therefore, based on the redistribution channels in Auclert (2018) an unequal income gains channel will not suffice here but rather inequality particularly wealth inequality will be driven by the fisher channel and interest rate exposure channel, which respectively arise from unexpected inflation and changes in the path of the real interest rates. While under HANK, the earnings process creates heterogeneity in labour income earnings. Thus, in combination with the fisher and interest rate exposure channel, the heterogeneity income channel from unequal labour income gains will drive both income inequality and wealth inequality. Finally, this changes in both wealth and income inequality will drive changes in consumption inequality given agents consumption path is determined by both income and wealth[5].

Based on the simulation analysis, I find that asset purchases increases GDP by about 1.4% and 0.25% under HANK and TANK respectively with the highest magnitude effect corresponding to the impact effects.. Additionally the increases for inflation is quite modest, with inflation increasing by 0.7 and 1 percentage points annually under HANK and TANK respectively. With the derivation of return on bonds in the HANK model based on the Fisherain equation, the model does not generate countervailing bond return response to asset purchases. In other words, the increase in bond supply does not create the negative impact effect on bond return as in the TANK model and established DSGE literature. So, while bond markets clear based on a market rate return, this return is directly linked to policy rates that respond to inflation. This deviation of bond return response forms the only result difference I obtain that is not in line with the literature.

[4] Auclert (2018) highlights the channels for the redistributive effects of monetary policy.

[5] I do not explicitly produce wealth inequality transitions for the TANK model as given the composition of only two agents where only one agent holds wealth this will be redundant.

When deconstructing consumptions response I provide true insight into how asset purchases induces modest responses for output. On breaking down the transmission channels for consumption, I find that what matters the most for consumption is not the portfolio rebalancing channel but rather income and direct transfers. This is owing towards the distribution of *Hand-to-Mouth* (HTM) households within the economy in combination with the nature of portfolio composition and its relationship with current period consumption. Under LSAPs, HTM consumers drive the aggregate level of consumption (observable through TANK disaggregation) of which their primary source of consumption is through income and transfers. Additionally, portfolio readjustments towards illiquid assets, which cannot be utilised for immediate consumption shocks further reduces the impacts of the portfolio rebalancing transmission channel for consumption. How this translates to modest output returns is one needs to understand that the size of fiscal multipliers for fiscal instruments are different. Given transfers form the major transmission mechanism here, noting that government transfers produce the smallest multipliers compared to government spending[6]. Furthermore, investment is driven by increases in capital stick from portfolio readjustment towards illiquid assets but this is dominated by non-HTM households of which they are lower, combined with investments impact on GDP being smaller than consumption. Hence, with the major GDP driver being consumption and with consumption being driven by transfers, the small multiplier effect of transfers creates the modest impact of QE on aggregate output.

What I'm also particularly interested in, are the results for wealth, consumption and income inequality. I find that asset purchases decrease the consumption inequality gap significantly due to the transmission channel through which asset purchases affect consumption. However, with portfolio reallocation towards illiquid assets asset purchases increases illiquid wealth inequality, while decreasing liquid wealth inequality. To understand this, one needs to understand that asset purchases also increases income inequality such that non-HTM (more productive) households are able to save more through illiquid assets (from portfolio rebalancing channel) and with closed economy framework, the higher increases in domestic bonds leads to less productive households holding more bonds creating the decrease in liquid asset inequality. The counterfactual analysis cements this point further as asset purchases lasting for longer periods inducing more wealth in-

[6]see Oh and Reis (2012).

equality along illiquid assets dimension while decreasing wealth inequality along liquid assets dimension, with higher income inequality.

The rest of this chapter is structured as follows: section 3.2 reviews the related literature for asset purchases, section 3.3 presents the HANK model utilised here, section 3.4 outlines the data and calibration parameters, section 3.5 presents the results from the simulations exercise. Section 3.6 presents an analysis of the effects LSAPs have on all dimensions of inequality in the models, and section 3.7 provides concluding remarks.

4.2 Related Literature

Within the literature, the assessment of unconventional monetary policies impacts on the economy have been approached from a DSGE and Vector Autoregressions (VAR) frameworks, with each study corroborating the portfolio rebalancing transmission channel or confidence and signalling channels. Papers with the portfolio rebalancing channel highlight imperfect substitutability between assets originating from adjustment costs as the mechanism that underpins the transmission of Asset Purchase Programs (APPs) to the macroeconomy. On the other hand, the signalling channel operates through expectations regarding future short-term rates paths from bond yields arising from the APPs. As to my knowledge I present the first to assessment of how APPs operate in a HANK framework, there is no literature to outline but instead I will present relevant literature from both VAR analysis and NK DSGE models.

To begin, Tobin (1956) provided the contributions of imperfect asset substitutability through portfolio choices for holding liquid cash and interest-bearing assets. Though Tobin's initially contribution looks at cash in hand and interest-bearing assets the imperfect substitutability channel can be applied to other assets as the literature has done. Empirically, several studies have quantitatively derived the impacts APPs have on yield structure. For example, in the Euro Area (EA) Andrade *et. al.* (2016) show that following the ECBs purchase programmes, inflation expectations and higher equity prices were accompanied by a 27-64 basis point (bp) decline in EA government bond yield. Similarly, Altavilla *et. al.* (2015) shows higher equity prices and higher euro depreciations were accompanied by lower corporate bond yield and a decline of 30 – 60 bp of 10-year EA government bonds after the ECB APPs from the financial crisis. In the context of the U.S., Gagnon *et. al.* (2011) estimates the effect of the first round of APPs in the U.S. and finds that it lowered the U.S. 10-year treasury yield by 58 bp [7].

When looking at the model-based literature that has identified portfolio rebalancing channel as the transmission mechanism, this is usually hinged on imperfect asset substitutability derived from the different composition of asset maturities as they are subject to adjustment costs. Here Chen *et. al.* (2012) in their analysis of the effects of QE in the U.S. extend the standard NK DSGE model to include long- and short-term bonds with market segmentation and frictionless financial markets. They show that the effect of APPs in the U.S. has a small impact on GDP growth but a persistent impact on level GDP, with very little effect on inflation. Furthermore, they show that even if nominal short-term rates are at the zero lower bound (ZLB) the effects of the APP on GDP and inflation is significantly larger and increases with commitment to keeping nominal short-term rates at the ZLB[8]. Falagiarda (2013) follows the setup of Chen *et. al.* (2012) and extends it to account for portfolio frictions as a proxy for household preference towards liquidity risk and preferences over bond maturities. He finds that APPs have a negative effect on long-term bond yields and creates higher output and inflation. Similarly, he finds that the effects of the APPs are sensitive to the exit strategy of central banks i.e., if they keep interest rates at the ZLB. Curdia and Woodford (2011) show that in a New Keynesian framework with credit frictions, as long as households perceive the asset purchases in the form of government bonds as being equivalent to reserves, then the APP will have no effect on the economy[9]. Though the effectiveness of the programmes can be realised if the government buys securities that are not equivalent to reserves.[10].

These papers outlined above have all assessed the unconventional policies under closed economy frameworks. However, Hohberger *et. al.* (2019) remedies this and extends the DSGE model to an open economy framework where government bond issuances can be taken up by foreign households. The presence of the open economy in their model allows for reallocation

[7] Greenwood and Vayanos (2014); Joyce *et. al.* (2011) also arrive at similar estimates for the U.S.

[8] Chen *et. al.* (2012) also empirically estimate a 13 bp decline in bond premium following a $600 Billion APP

[9] Given the heterogeneity of household income processes in the model, this creates inequality in asset market investment that acts as financial frictions to households and prevents this outcome to occur.

[10] While the imperfect substitutability channels allow APPs to have non-neutral effects on the economy, it is not unique in its regard, as other studies have analysed bank lending channels as alternative transmission mechanisms. Though in the model I present here, this channel is unobservable as I do not include a direct financial system to observe bank lending channels.

of assets into foreign currency assets as well as devaluation of domestic currency. Similar to Falagiarda (2013) investors here have preferences over asset maturities, along with transaction costs from adjusting portfolio choices allows them to isolate the portfolio rebalancing channel and captures the movement of relative asset prices depending on relative asset supply. When imposing an endogenously binding ZLB they find that the ECB's QE increased annual GDP by around 0.3 percentage points and increased CPI inflation on average by 0.5 percentage points.

Burlon *et. al.* (2019) also present an open economy model and provides an alternative approach to assessing APPs. Here as opposed to a committed end date programme where the central bank announces duration of asset purchases allowing investors to have perfect foresight, they allow for the central bank to not announce (basically an open ended asset purchase) the duration of asset purchases forcing investors decisions to be based on expectations. Following Chen *et. al.* (2012) they also assume imperfect substitutability of assets such that given the open economy framework, these financial assets become imperfect substitutes for the international sovereign bonds bought by the central bank, thus, creating real effects within their model[11]. They show that open-ended APPs are more effective in stimulating the economy. Similarly, Burlon, Gerali, Notarpietro, and Pisani (2017) show that the farther the end-date of an APP, the greater the effect it has on simulating the economy. Their result highlights the channels through which central banks can manage the agents' expectations through announcements of APP dates.

On the other hand, when looking at VAR empirical literature, authors obtain similar outcomes to model based literature, sitting both bank-lending and portfolio rebalancing transmission channels[12]. However, unlike model-based literature, they are unable to identify specific transmission channels but rather identify there are heterogeneous transmission channels at play given unconventional monetary policies. Altavilla *et. al.* (2016) shows that the heterogeneous effects of interest rates on bank lending rates hinge on the level of non-performing loans, capital ratio and sovereign debt exposure. Similarly, Boeckx *et. al.* (2017) also finds that lower capitalised banks create lower output effects in response to the ECB's APPs. They also find that the ECB's APPs had a significant and positive effect on output in the Eurozone. In the context of the U.S.

[11] Given this structure the *Wallace Irrelevance Proposition* will not hold, therefore, APPs will have effects on price levels and allocations within the economy

[12] However, some empirical literature have highlighted that these transmission channels have changed over time see for example Huber *et. al.* (2019); Breitfub *et. al.* (2017)

Feldkircher and Huber (2018) utilise a Time varying parameter stochastic volatility VAR and identify a wealth transmission channel for APPs[13], with no evidence of the bank-lending transmission channel. Engen *et. al.* (2015) provides empirical evidence for the signalling channel. Here they emphasised how the effectiveness of the signalling channel depends on the level of uncertainty of economic conditions as well as the strength of financial markets.

4.3 The Models

4.3.1 A Two Asset Two-Agent New Keynesian Model

I utilise the two asset TANK model from Kaplan, Moll and Violante (2018), which is based on the HANK model presented later on here. As in standard DSGE literature there are optimising households but without sticky wage setting. Households are divided into *Hand-to-mouth* households that consume only labour income, redistributed firm profits and transfers from the government, and non-HTM (saving) households that choose the level of bond holdings and equity holdings, which is a combination of capital and stocks. There is a final goods producer that aggregates intermediate goods produced by optimising firms that face price rigidities. The government is the sole issuer of bonds within the model and the central bank sets monetary policy that follows a Taylor rule. The TANK framework diverges from the RANK model as conversely to the later, at any point in time, the optimisation behaviour of a fraction of agents in the economy is subject to a borrowing constraint[14]. To be precise, this fraction of agents (*Hand-to-mouth* agents) behave as if they face a borrowing constraint and thus only change consumption in response to current income without considering any changes to the nominal or real interest rates. This condition generates the heterogeneity in the TANK model[15].

Households

Deviating from the standard DSGE literature, saving households can save using bonds and equity, though, the return on bonds is determined by the Fisherian channel as opposed to being equivalent to the nominal interest rates. There is a continuum of households that populate the

[13] The wealth here is measured as in household consumption wealth as well as assets of banks and non-profit organisations.

[14] This is also true in the HANK model as well

[15] This is alteration underlies the heterogeneity in the TANK model in Derbotoli and Gal(́2017) and the Kaplan *e. al.* (2018) amongst others.

economy, consume, C_t, that supply labour ℓ_t. Household lifetime utility is given by:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\ln C_t - \varrho \frac{\ell_t^{1+\sigma_n}}{1+\sigma_n} \right] \qquad \varrho, \sigma_n > 0 \tag{4.3.1}$$

Were β is the household discount factor, σ_n is the inverse elasticity of labour supply, and ϱ is the disutility from labour supply.

Whilst most of the RANK models in the literature have focused on two types of bonds short- and long-term bonds[16] the model here utilises two different assets short-term bonds and equity to better assess the portfolio rebalancing channel and understand the changes in spread. Non-HTM households can save in short-term bonds that are one-period securities that pay nominal return r_t^b in period $t+1$. They also have access to equity assets a_t that are illiquid such that they face a cost (adjustment cost) for depositing or withdrawing from equity accounts, that pays out a return r_t^a in period $t+1$. The choice of depositing into equity accounts is d_t and the adjustment cost for depositing is $\chi(d_t)$. Additionally, non-HTM Households supply labour and face a fixed income tax τ and earn dividend from firms' profit Γ_t, where these dividends are also subject to the income tax. Therefore, the non-HTM household's problem is given by:

$$\max_{\{c_t \ \ell_t \ d_t \ a_{t+1} \ b_{t+1}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\ln C_t - \varrho \frac{\ell_t^{1+\sigma_n}}{1+\sigma_n} \right] \tag{4.3.2}$$

With the flow budget constraint for liquid and illiquid assets:

$$C_t + B_{t+1} + d_t + \chi(d_t) = (1-\tau)(w_t \ell_t + \Gamma_t) + T_t + (1 + r_t^b) b_t \tag{4.3.3}$$

$$a_{t+1} = (1 + r_t^a) a_t + d_t \tag{4.3.4}$$

$$\chi(d_t) = \chi_1 |d_t|^{\chi_2} \tag{4.3.5}$$

On the other hand, HTM households solve the following problem:

$$\max_{\{c_t \ \ell_t\}} \ln C_t^{htm} - \varrho \frac{\ell_t^{htm\ (1+\sigma_n)}}{1+\sigma_n} \tag{4.3.6}$$

$$\text{S.T.} \quad C_t^{htm} = (1-\tau)(w_t \ell_t^{htm} + \Gamma_t^{htm}) + T_t^{htm} \tag{4.3.7}$$

[16] See e.g. Chen *et. al.* (2012).

Because households face no wage rigidities, there is no monopolistic bundler of labour, and optimal labour supply is determined by individual household optimisation behaviour. Wages are therefore determined by firms optimising behaviour. Illiquid assets a_t is the sum of the fraction of non-HTM household's capital stock K_t and equity stock s_t claims at price q_t to the fraction η of firms' profit Π_t. Finally, Γ_t and Γ_t^{htm} are the direct transfers non-HTM and HTM households respectively receive from the redistributed profits of intermediate goods firms. Under the assumption of no aggregate uncertainty then the no arbitrage condition requires that the return on holding capital and return from holding stocks must be equivalent. I describe this further when looking at the HANK model. Therefore, under any unanticipated shock, the return on holding capital stock and equity can differ on impact, however, under no arbitrage the price of equity stock q_t will jump so as to equalise the capital returns and equity returns from period $t+1$ onwards.

Firms

Final Good Producers: Final good producers combine the heterogeneous goods produced by intermediate good producers into a homogeneous consumption good with the following technology:

$$Y_t^d = \left(\int Y_t(f)^{\frac{(\theta_p - 1)}{\theta_p}} \right)^{\frac{\theta_p}{(\theta_p - 1)}} \tag{4.3.8}$$

The degree of substitutability between heterogeneous intermediate goods is governed by θ_p, such that as $\theta_p \to 1$market structure tends towards a monopoly and as $\theta_p \to \infty$ towards a perfectly competitive structure. Cost minimisation gives rise to the demand for the jth firm intermediate good:

$$Y_t(f) = \left(\frac{P_t(f)}{P_t} \right)^{-\theta_p} Y_t^d \tag{4.3.9}$$

Intermediate Goods Producers: There is a continuum of intermediate good producers that produce a heterogeneous selection of goods by employing both labour and capital in the following Cobb-Douglas structure:

$$Y_t(f) = K_t(f)^{\alpha} N_t(f)^{1-\alpha} \tag{4.3.10}$$

The problem for capital and labour is an inter-temporal problem. Hence, cost minimisation gives rise to the standard optimality conditions given by:

$$w_t = (1-\alpha)\Phi_t(f)K_t(f)^{\alpha}N_t(f)^{-\alpha} \tag{4.3.11}$$

$$r_t^k = \alpha\Phi_t(f)K_t(f)^{\alpha}N_t(f)^{-\alpha} \tag{4.3.12}$$

$$\Rightarrow \frac{K_t}{N_t} = \frac{\alpha}{1-\alpha}\frac{w_t}{r_t^k} \tag{4.3.13}$$

One can show that the Real marginal cost $\Phi_t(f)$ is independent of intermediate inputs such that:

$$\Phi_t(f) = \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{r_t^k}{\alpha}\right)^{\alpha} = \Phi_t \tag{4.3.14}$$

Where r_t^k is the real rental rate of capital, Ω_t is real wage paid to labour and Φ_t is the real marginal cost. Conversely to chapter 2, price setting is based on Rotemberg (1982), with Rotemberg adjustment cost function given by[17]:

$$\Theta\left(\frac{P_t(f)}{P_{t-1}(f)}\right) = \frac{\theta}{2}\left(\frac{P_t(f)}{P_{t-1}(f)} - 1\right)^2 Y_t^d \tag{4.3.15}$$

θ is the adjustment cost constant, and each firm chooses prices $P_t(f)$ to maximise lifetime discounted profits in:

$$\max_{\{P_t(f)\}_0^\infty} \sum_{t=0}^{\infty} \lambda_t Q_t \left[\frac{P_t(f)}{P_t}Y_t(f) - w_t N_t(f) - r_t^k K_t(f) - \Theta\left(\frac{P_t(f)}{P_{t-1}(f)}\right)\right]$$

$$\max_{\{P_t(f)\}_0^\infty} \sum_{t=0}^{\infty} \lambda_t Q_t \left[\left(\frac{P_t(f)}{P_t} - \Phi_t\right)Y_t(f) - \Theta\left(\frac{P_t(f)}{P_{t-1}(f)}\right)\right] \tag{4.3.16}$$

Subject to:

$$Y_t(f) = \left(\frac{P_t(f)}{P_t}\right)^{-\theta_p} Y_t^d$$

Price optimisation gives rise to the New Keynesian Phillips Curve (NKPC):

$$1 - \theta\pi_t(1+\pi_t) + \frac{\lambda_{t+1}\mathcal{Q}_{t+1}}{\lambda_t Q_t}\left[\theta\pi_{t+1}\frac{Y_{t+1}^d}{Y_t^d}(\pi_{t+1}+1)\right] = (1-\Phi_t)\theta_p \tag{4.3.17}$$

[17] With the Rotemberg price setting and continuous price setting, there is not need to log-linearise around the deterministic steady state to obtain the NKPC

Under the flexible price equilibrium, all firms set the same prices and one obtains $\Phi_t = \frac{\theta_p - 1}{\theta_p}$. Therefore, in the sticky price setting, firms will raise prices when the markup $\frac{\theta_p - 1}{\theta_p}$ is above the flexible price equilibrium.

Illiquid asset Holdings: Illiquid asset is a constitute of both capital holdings and claims on equity as a fraction of firms profit. The no arbitrage condition dictates that the return on capital must equal the return on equity such that:

$$r_t^a \equiv \frac{\eta \Pi_t + (q_t - q_{t-1})}{q_{t-1}} = r_t^k - \delta \tag{4.3.18}$$

$$\Leftrightarrow q_t = (1 + r_t^a) q_{t-1} - \eta \Pi_t \Rightarrow q_{t+1} = (1 + r_{t+1}^a) q_t - \eta \Pi_{t+1}$$

$$q_t = \frac{1}{(1 + r_{t+1}^a)} (q_{t+1} + \eta \Pi_t) \tag{4.3.19}$$

Also I can express $(1 + r_t^a) q_{t-1} = \eta \Pi_t + q_t$. Then the law of motion for illiquid wealth can be expressed as:

$$a_{t+1} \equiv K_{t+1} + S_{t+1} q_t \Rightarrow a_t = K_t + S_t q_{t-1}$$

Recall from household budget constraint $a_{t+1} = (1 + r_t^a) a_t + d_t$, then:

$$\begin{aligned} a_{t+1} &= (1 + r_t^a) a_t + d_t = (1 + r_t^a)(K_t + S_t q_{t-1}) + d_t \\ &= (1 + r_t^k - \delta) K_t + (1 + r_t^a) q_{t-1} S_t + d_t \\ &\Rightarrow (1 + r_t^k - \delta) K_t + (\eta \Pi_t + q_t) S_t + d_t \end{aligned}$$

$$a_{t+1} = (1 + r_t^k - \delta) K_t + (\eta \Pi_t + q_t) S_t + d_t \tag{4.3.20}$$

The remaining fraction of profits are then redistributed to households as direct transfers, such that:

$$\Gamma_t^{agg} = (1 - \eta) \Pi_t \quad \Gamma_t^{agg} = \Gamma_t = \Gamma_t^{htm}$$

Where Γ_t^{agg} is the aggregate redistributed fraction of profits, while Γ_t and Γ_t^{htm} are the fraction directly transferred to non-HTM and HTM households respectively. The fraction of hand to mouth households is calibrated to form 30% of aggregate households.

Monetary Authority and Government

Government and Fiscal Policy: Government issues Bond denoted B_t^g such that a negative value indicates government debt. Therefore a higher negative corresponds to higher government debt, which is the focus of this paper. The Flow budget constraint of the government is given by:

$$B_{t+1}^g = (1 + r_t^b)B_t^g + \tau(wtN_t + \Gamma_t^{agg}) - T_t^{agg} - G_t \tag{4.3.21}$$

Where:

$$B_t^g = B_t g + \varepsilon_t$$

Where ε_t is a Bond shock that follows an AR(1) process:

$$\varepsilon_t = \rho\varepsilon_{t-1} + e_t$$

Government transfers are then distributed across households such that:

$$T_t^{agg} = \Lambda T_t^{htm} + (1 - \Lambda)T_t \tag{4.3.22}$$

The share of HTM fraction of transfers is Λ^{htm} such that:

$$\Lambda^{htm} T_t^{agg} = \Lambda T_t^{htm} \tag{4.3.23}$$

That is. the HTM households share of government transfers is equal to the fraction of the population that re HTM households.

Monetary Policy:The central bank follows a simple Taylor rule that seeks to stabilise only inflation:

$$i_t = \bar{r}_t^b + \rho_\pi \pi_t \tag{4.3.24}$$

Given inflation and the nominal interest rate, the real return on liquid assets is therefore given

by:

$$1 + r_t^b = \frac{1 + i_{t-1}}{1 + \pi_t} \tag{4.3.25}$$

Equation (4.3.25) is the Fisherian channel that says the return on liquid assets (bonds, b_t) is equal to the nominal interest rate adjusted for inflation. Everything else follows as in the representative agent model[18]. Derbotoli and Galí (2017) conclude that when TANK and HANK models are calibrated such that the shares of *Hand-to-mouth* households are comparable, the TANK model provides a good approximation of the implications of HANK models. This is another motivation for providing TANK as a comparison to HANK in the results.

4.3.2 Two Asset Heterogeneous Agent New Keynesian Model

The HANK model presented here is as in Kaplan, Moll and Violante (2018). The model is presented in continuous time; households have a calibrated death rate and an exogenous productivity process which are different from the RANK/TANK model but otherwise all other things are still identical.

Households

There is a continuum of households that consume and supply labour, however, now labour productivity follows an exogenous Markov process z_t. Households hold both bonds (liquid assets) b_t and illiquid assets a_t, and this combined with their exogenous productivity process gives rise to the joint distribution $\mu_t(da\ db\ dz)$. With exogenous death rate η households die and produce offspring with zero wealth allocation. To match the data, death rate is stochastic such that there is a significant number of households without illiquid wealth relative to the data. Annuity markets are assumed to be perfect to encompass proportion redistribution of assets to other individuals with asset holdings. Household utility function is therefore given as:

$$\max_{\{c_t \ell_t d_t\}} \quad \mathbb{E}_0 \int_0^{\infty} e^{(-\beta+\eta)} U(c_t, \ell_t)\ \mathrm{d}t \tag{4.3.26}$$

Here ℓ_t are number of hours worked, with households discounting the future with β which is conditional on them surviving. Households once again can borrow up to the limit in $\underline{\text{b}}$ and also

[18] See Appendix C for a detailed version of the TANK model

save in b_t with return[19] $r_t^b(b_t)$. Illiquid assets a_t incur a transaction cost $\chi(d_t, a_t)$ for depositing and withdrawing from illiquid accounts. Here d_t represents the deposits from flow budget constraint to illiquid account, with $d_t < 0$ representing a withdrawal from illiquid account. The transaction cost has two separable components that play significant role in controlling deposits and withdrawals from illiquid accounts. It also differs slightly from that specified in the RANK/TANK models. Here it is given as:

$$\chi(d_t, a_t) = \chi_0|d| + \frac{\chi_1}{2}\left(\frac{d}{a}\right)^2 a \tag{4.3.27}$$

Here the linear component $\chi_0|d_t|$, creates an inaction region pertaining to optimal deposits due to decreasing marginal gains from withdrawing or depositing. The convex component $\chi_1|d_t|^{\chi_2}$ holds when $\chi_1 > 0$ and $\chi_2 > 0$ and acts as a transversality condition that ensures deposits and withdrawals are finite i.e. $|d_t| < \infty$. Finally, scaling deposit rates by illiquid assets a_t ensures homogeneity of degree zero in deposit rate $\frac{d_t}{a_t}$ for marginal cost of depositing or withdrawing $\chi'_d(d_t, a_t)$. This further ensures that the marginal cost of depositing or withdrawing hinges solely on the fraction of illiquid assets being transacted, with the size of the transaction being insignificant. This transactional cost creates disparities between steady state returns from liquid and illiquid asset[20] such that $r_t^a > r_t^b$. Households flow budget constraints are then given by:

$$\dot{b_t} = (1-\tau)w_t e^{y_t}\ell_t + r^b(b_t)b_t + T_t - d_t - \chi(d_t, a_t) - c_t \tag{4.3.28}$$

$$\dot{a_t} = r_t^a a_t + d_t \tag{4.3.29}$$

$$b_t \geq -b, \quad a_t \geq 0 \tag{4.3.30}$$

Here $\dot{b_t}$ represents households continuous savings, which equals households income stream. τ is income tax payment, with z_t representing individual productivity and follows the Ornstein-Uhlenbeck process[21]. $\dot{a_t}$ is households continuous net-savings in illiquid assets, which accumulates returns according to r_t^a. The household problem is as in standard NK literature, and they maximise their lifetime utility in equation (4.3.26) subject to their different budget constraints in equations (4.3.28) – (4.3.30). The household problem is represented as a Hamiltonian-Jacobi-

[19]Given the return on bond is linked to real interest rates using the Fisherian channel, the true rate of return on bond can be expressed as $\tilde{r}_t^b = r_t^b + wedge$, where the wedge is the difference between lending rates (that encompass a risk premium) and borrowing rates.

[20]In the TANK and RANK models, this does not hold true such that in the steady state $r_t^a = r_t^b$

[21]where dWt is the innovation to a standard Brownian motion, σ is the rate of mean reversion, and σ captures the size of innovations

Bellman equation:

$$(\beta+\eta)V(a_t,b_t,y_t)=\max_{\{c_t\ell_t d_t\}}\ U(c_t,\ell_t)+V_b'(a_t,b_t,y_t)\big[(1-\tau)w_t e^{y_t}\ell_t+r^b(b_t)b_t \\ +T_t-d_t-\chi(d_t,a_t)-c_t\big]V_a'(a_t,b_t,y_t)\left[r_t^a a_t+d_t\right]+V_y'(a_t,b_t,y_t)(-\rho y) \\ +\lambda\int_{-\infty}^{\infty}\big(V(a_t,b_t,y_t')-V(a_t,b_t,y_t)\big)\,\phi(y_t')\mathrm{d}y_t' \quad (4.3.31)$$

Here $V(a_t,b_t,y_t)$ is the value function of the bellman equation, $V_i'(a_t,b_t,y_t),\ i=a,\ b,\ y$ is the derivative of the value function with respect to either $a,\ b,\ y$. W here λ is the Poisson arrival rate for jumps[22], and ρ is the rate at which earnings process drifts towards zero after each new jump is realised[23]. Then the stationary version of the households HJB equation is therefore given as:

$$(\beta+\eta)V(a,b,y)=\max_{\{c,\ell,d\}}\ U(c,\ell)+V_b'(a,b,y)\Big[(1-\tau)we^{y}\ell+r^b(b)b+T \\ -d-\chi(d,a)-c\Big]V_a'(a,b,y)\left[r^a a+d\right]+V_y'(a,b,y)(-\rho y) \\ +\lambda\int_{-\infty}^{\infty}\big(V(a,b,y')-V(a,b,y)\big)\,\phi(y')\mathrm{d}y'$$

The evolution of the joint distribution of liquid and illiquid wealth can be described by the Fockker-Plankt equation (Kolmogorov Forward Equation). Now one can define $g(a,b,y,t)$ as the density function that corresponds to the distribution of $\mu_t(a,b,z)$, where I have used that $y=logz$. Finally, I can define the optimal drifts in in the stationary HJB equation above for liquid and illiquid state space as $s^b(a,b,y)$ and $s^a(a,b,y)$ respectively. These functions define the optimal savings policy for liquid and illiquid assets respectively. Then I have that the stationary density satisfies the Fokker-Plankt equation:

$$0=\partial_a\left(s^a(a,b,y)g(a,b,y)\right)-\partial_b\left(s^b(a,b,y)g(a,b,y)\right)-\partial_y\left(-\rho y g(a,b,y)\right)-\lambda g(a,b,y) \\ +\lambda\phi(y)\int_{-\infty}^{+\infty}g(a,b,y')dy'-\eta g(a,b,y)+\eta\nu(a-a_0)\nu(b-b_0)g^*(y)$$

Here, ν denotes the dirac delta function and a_0, b_0 are initial assets for new born agents, and $g^*(y)$ is the stationary distribution for y. I show in the numerical solution section how to solve

[22]The wage earning process (explained later on) utilises a jump-drift process, and λ here is the rate at which these jumps arrive

[23]As shown in section 4.4.2, the earnings process is given by the jump-drift process $dy_{i,t}=-\rho y_{i,t}dt+dJ_{i,t}$.

both the HJB and Fokker-Plankt equations using a finite difference approach as in Achdou *et. al.* (2014; 2017) and Kaplan *et. al.* (2018).

Drawing attention to the first order conditions for deposits as this is crucial in determining the accumulation of illiquid wealth. The fist order condition for deposits is given as:

$$\text{if } d > 0;\ \chi_d(d,a) = \chi_0 + \chi_1 \frac{d}{a^2} a \Rightarrow d = \left(\frac{V'_a}{V'_b} - 1 - \chi_0 \right) \frac{a}{\chi_1} \tag{4.3.32}$$

$$\text{if } d < 0;\ \chi_d(d,a) = -\chi_0 + \chi_1 \frac{d}{a^2} a \Rightarrow d = \left(\frac{V'_a}{V'_b} - 1 + \chi_0 \right) \frac{a}{\chi_1} \tag{4.3.33}$$

Optimal deposits are therefore conditional on households having paid the fixed costs associated with the transaction, thus, optimal deposits are:

$$d = \left(\frac{V'_a}{V'_b} - 1 + \chi_0 \right)^{-} \frac{a}{\chi_1} + \left(\frac{V'_a}{V'_b} - 1 - \chi_0 \right)^{+} \frac{a}{\chi_1} \tag{4.3.34}$$

It holds that for $d = 0, -\chi_0 < \frac{V'_a}{V'_b} - 1 < \chi_0$. This implies that it is optimal for households to withdraw from illiquid accounts when the marginal value of liquid wealth is relatively higher than the marginal value of illiquid wealth. The converse holds; that is, households deposit into their liquid wealth accounts when the marginal value of illiquid wealth is relatively larger than the marginal value of liquid wealth[24].

As stated, households can choose to invest in liquid or illiquid wealth. Illiquid wealth here comprises of savings in the form of capital K_t (which is rented to firms), and stocks s_t of intermediate goods firms. This stocks guarantees a fraction of intermediate goods firms profit net of fixed cost[25]. Now define q_t as the price of a stock in the intermediate goods firm and as such the value of a given household's illiquid wealth is therefore given by:

$$a_t = k_t + q_t(s_t)s_t \tag{4.3.35}$$

An important assumption here is the absence of frictions between capital and stocks, such that

[24]Here V'_a & V'_b represent the first order conditions with respect to illiquid wealth a and liquid wealth b respectively. Furthermore, the optimality condition for deposits shows that households will cease to withdraw or deposit when the marginal values of both liquid and illiquid wealth are similar or identical

[25]Fixed cost here simply refer to the adjustment cost which is introduced later when solving the intermediate goods firm optimisation problem

individuals can immediately adjust their composition of illiquid wealth without incurring any cost or time. This implies a no-arbitrage condition most hold between stocks and capital. Therefore, the return from holding capital must equal the return from holding intermediate good firms stocks and this condition is given by:

$$\frac{\Pi_t + \dot{q}_t(s_t)}{q_t(s_t)} = R_t^k - \delta =: r_t^a \tag{4.3.36}$$

Where Π_t is the profit of intermediate good firms. This makes it computationally easy when simulating the model as it consists of three state space dimensions rather than four given the one dimension for illiquid assets as opposed to two dimensions.

The competitive equilibrium for households is then defined as a set of prices and transfers $\{w_t \; r_t^b \; r_t^a \; \tau_t \; T_t\}_{t\geq 0}$ such that households maximise their utility in equation (4.3.26) subject to budget constraints in equations (4.3.28) – (4.3.30), and all markets clear. In the model, the markets are for bonds, capital and labour, such that, w_t determines labour market clearing, r_t^b determines bond market clearing, where r_t^b is determined through the Taylor rule and Fisherian channel; finally, r_t^a clears the capital markets[26]. The deterministic steady state solution is a recursive set of policy rules for consumption, $c(a,\ b,\ z;\ \Gamma)$, deposits, $d(a,\ b,\ z;\ \Gamma)$, and labour supply $\ell(a,\ b,\ z;\ \Gamma)$ such that these policies imply optimal drifts for both assets as well as a stochastic process for individual productivity and induce a stationary joint distribution[27] $\mu(da,\ db,\ dz;\ \Gamma)$

Firms

As in standard NK literature, there is a final goods producer that combines intermediate inputs to produce the final good. This is exactly as in the TANK model description, but is presented here in continuous time as opposed to discrete time. The intermediate firms problem in continuous time is given as:

$$\underset{K_t(f),N_t(f),P_t(f)}{\text{Max :}} \quad \mathbb{E}_t \int_0^\infty e^{-\int_0^t r_s^a ds} \left[\mathcal{D}_t(f) - \Psi\left(\frac{\dot{P}_t}{P_t}\right)\right]$$
$$s.t. \qquad Y_t(f) = K_t(f)^\alpha N_t(f)^{1-\alpha-\mu}$$

[26]This follows from standard Heterogeneous models, see e.g., Aiyagari (1994)
[27]See Appendix C for how the model is solved following Achdou *et. al.* (2017) and Kaplan *et. al.* (2018)

All other results hold, but the continuous time NKPC can be expressed as:

$$\left(r_t^a - \frac{\dot{Y}_t}{Y_t}\right)\pi_t = \frac{\theta}{\psi}\left(\Phi_t - \frac{\theta-1}{\theta}\right) + \dot{\pi}_t \tag{4.3.37}$$

Monetary and Fiscal Policy

Monetary Policy: Central bank sets interest rates according to a Taylor rule given by:

$$i_t = \bar{r}^b + \rho_\pi \pi_t \tag{4.3.38}$$

Here the central bank practises a strict inflation targeting regime without any interest rate smoothing parameters[28].

Fiscal Policy: Finally, I assume government expenditure to be exogenous, with a progressive taxation on household income, and a lump-sum transfer T_t, with a proportional tax rate τ_t. The government in this model is the sole issuer of bonds (liquid) assets, B_t^g, such that negativity refers to government debt. The government intertemporal budget constraint is therefore given as:

$$\dot{B}_t^g + G_t + T_t = \tau_t \int \omega_t z \ell_t(a,b,z)d\mu_t + r_t^b B_t^g \tag{4.3.39}$$

Here B_t^g is government debt in the form of government bonds. In the application of LSAPs, I simply assume the government issues more debt (more bonds) i.e. such that:

$$B_t^g = B^g + \varepsilon_t^b \tag{4.3.40}$$

Where ε_t^b is a mean reverting autoregressive process, with mean reverting rate ρ_t such that $\varepsilon_t^b = e^{-\rho^c t}\ \varepsilon_0$. Given the inflation and nominal interest rate, I can derive the real return on liquid assets pinned down by the Fisher equation $r_t^b = i_t - \pi_t$. The liquid asset return r_t^b pinned down by the fisher equation is also consistent with the bond market equilibrium above.

[28] I found that under the LSAP simulation, the introduction of interest rate smoothing violated the borrowing constraint for households and as such the model was not solvable.

In equilibrium, liquid markets clear and thus:

$$B_t^h = B_t^g$$
$$B_t^h = \int b \; d\mu_t$$

$B_t^h = \int b \; d\mu_t$ are total household holdings of liquid (bonds) assets. This procedure for debt issuance differs from Chen *et. al.* (2012) where they impose a government bond rule that depends on previous level of government bonds on which the shock is induced. As I follow the original authors of the HANK model, the model is presented in continuous time and as such discrete time transitional processes do not translate[29].

Therefore a complete competitive equilibrium is a set of prices $\{w_t,\ r_t^a,\ w_t,\ r_t^b,\ q_t,\ \pi_t,\}_{t \geq 0}$, a path for household and firm variables $\{c_t,\ \ell_t,\ n_t,\ b_t,\ a_t,\ k_t,\ d_t\}_{t \geq 0}$, a set of fiscal policy variables $\{T_t,\ B_t,\ G_t\}_{t \geq 0}$, and measures $\{\mu_t\}_{t \geq 0}$ such that for every t: (i) Households and intermediate goods firm solve their optimisation problems taken prices, and transfers as given; (ii) government budget constraint binds; (iii)bond market clears, illiquid asset market clears, labour markets clear, and goods markets clear.

4.4 Data and Calibration

4.4.1 Data

I collect data from two sources of household wealth for the United States (U.S.), the Flow of Funds Data for household balance sheet and the Survey of Consumer Finances data. Both data periods are for the year 2019[30]. These two data sets allows accurate structuring of the liquid-illiquid asset dynamics within the model. Note that the balance sheets are aggregated thus they do not provide a source of heterogeneity and inequality. However, more importantly the data allows for observed distributions of liquid and illiquid wealth to be mapped to the model's distribution of liquid and illiquid wealth. Accurate distribution of both liquid and illiquid wealth holdings bears direct importance when looking at distributional impacts changes in interest rates

[29] Under discrete time, t and $t-1$ are perfectly distinguishable. However, with the continuous time approach this is not possible. Thus, I simply present a simple one time shock to government bonds.

[30] In the original paper Kaplan *et. al.* (2018) utilise data from 2004, but given the overwhelming changes in balance sheet of households and federal government from the financial crisis of 2008, I decided on updating the data period to 2019 to account for the increases in bond issuances over the quantitative easing periods.

(that directly affects changes in the returns on bonds) have on consumption.

4.4.2 Calibration and Estimation

On categorising assets into liquid and illiquid, I follow the same type of assets as specifications in Kaplan *et. al.* (2018). In the Flow of Funds dataset variables are accurately defined. However, under the SCF data set, to classify an asset as liquid it should bear little transactional costs, while illiquid assets bear high transactional costs. The 2004 SCF dataset utilised by Kaplan *et. al.* (2018) bears different distinctions between the 2019 SCF dataset. Therefore, I describe the specification of assets as defined in the calibrations. Classification of liquid assets includes all money market accounts along with certificate of deposits, government bonds, corporate bonds that includes foreign bonds, and all net of revolving consumer credit balances (which includes credit card balances and instalment loans). On the other hand, the specification of illiquid wealth includes direct stocks held (defined as corporate equity), equity held in non-corporate businesses (defined as all equity held net of direct stocks), real estate wealth (defined as primary residential houses, plus residential property excluding primary residences, equity in non-residential real estate.) net of total mortgage debt held, and consumer durables (defined as all other non-financial assets plus vehicles that includes cars, planes etc.) net of non-revolving consumer credit[31]. When aggregating the SCF data I choose to report aggregates over means at it matches closer to the aggregates reported in the FoF. When generating the final ratios, I utilise the SCF for bonds and deposits, while utilising the FoF for illiquid asset specifications [32].

Table 4.1: HANK - Ratios of Liquid and Illiquid Asset to GDP Disaggregation

Liquid Assets (Bonds B_t^h)		Illiquid Assets (Equity; $a_t = K_t + s_t$)	
Government Bonds ($\frac{B_t^g}{GDP_t}$)	0.024	Net housing	0.911
Corporate Bonds	0.051	Net Consumer Durables	0.187
Deposits ($\frac{d_t}{GDP_t}$)	0.332	Corporate Equity ($\frac{s_t}{GDP_t}$)	1.351
Consumer Debt ($\frac{-b}{GDP_t}$)	-0.051	Private Equity ($\frac{K_t}{GDP_t}$)	0.556
Total	0.356		3.085

Notes: Data is from 2019 Survey of Consumer Finances and Flow of Funds. All variables are estimated as a fraction of U.S. GDP from 2019. U.S. GDP for 2019 is $\$21,400$B, and is obtained from Federal Reserve Economic Data (FRED).

[31] See net worth flow chart of SCF data for these definitions `https://www.federalreserve.gov/econres/files/networth%20flowchart.pdf`

[32] Kaplan *et. al.* (2018) provide detail comparisons between the two datasets before settling on this specifications. See Appendix C in Kaplan *et. al.* (2018) for this detail comparison.

As in standard new Keynesian literature and in Kaplan *et. al.* (2018), I calibrate the rest of the model parameters to match literature estimates. Household discount rate β is set to 0.9876 implying an annual discount rate of around 5.1 percent. Coefficient of risk aversion in consumption, $\sigma_c = 1$ implying log utility. The inverse Frisch elasticity of labour supply σ_n is set to 1 as well. ϱ is set to 2.2 such that the steady state average hours worked is equal to 1.5. Kaplan *et. al.* (2018) set η to $\frac{1}{180}$ to attain an average lifespan of 45 years for households.

Table 4.2: Model Calibrations

Parameter	Value	Description
β	0.988	Annual Discount Rate
ζ	$\frac{1}{180}$	Household death rate
σ_c	1	Intertemporal elasticity of substitution
σ_n	1	Inverse Frisch elasticity of substitution
ϱ	2.2	Disutility of Labour Supply
τ	0.3	Proportional tax rate
α	$\frac{1}{3}$	Capital share in production function
ω	$\frac{1}{3}$	Profit redistribution parameter
δ	0.17	depreciation rate
θ_p	10	Elasticity of Substitution
Θ	100	Rotemberg Adjustment cost
$\frac{T}{Y}$	0.06	Lump-sum transfers
ρ_π	1.25	Inflation feedback in Taylor Rule
χ_0	0.0438	Linear component of Deposit adjustment cost function
χ_1	0.956	Convex component of Deposit adjustment cost function
χ_2	1.402	Convex component of Deposit adjustment cost function
$\underline{\text{b}}$	16,500	Borrowing limit
r^{borr}	0.08	Borrowing rate
$\underline{\text{a}}$	1000	denominator for illiquid asset
		TANK Parameters Only
Λ	0.3	Fraction of population that are HTM
Λ^{htm}	0.3	HTM share of transfers from government.
		Shock Parameters
ε^b	0.025	LSAP shock size
ρ	0.81	Shock Persistence

Notes: Parameters are calibrated as in Kaplan *et. al.* (2018) excluding the shock parameters. These are calibrated to match estimated LSAP programme parameters from Chen *et. al.* (2012).

When considering household borrowing, I follow the original authors calibration set the natural borrowing limit $\underline{\text{b}}$ to $\$16,500$, with the borrowing rate set at 8% annually. The borrowing rate is determined by the borrowing wedge κ, which is set at ≈ 0.015 implying an annual borrowing wedge of approximately 6 percent. Elasticity of capital input in production function is set to $\frac{1}{3}$, which is standard in the literature for developed countries. Depreciation rate δ is set to 0.017, the elasticity of substitution for final goods producer θ_p is set to 10 such that there is about 11 percent markup for intermediate goods producers when price setting. The Rotemberg constant θ is set at 100 implying a New Keynesian Phillips Curve (NKPC) slope of 0.1. To neutralise countercyclicality of markups Kaplan *et. al.* (2018) set the redistributive share of profits (share of profits reinvested into illiquid wealth) ω to $\frac{1}{3}$, same as the share of capital in production function. The proportional tax rate τ is set to 0.3 with steady state Transfer to GDP ratio $\frac{T}{Y}$, set to 0.06. In the baseline model of the simulations, I utilise a strict inflation targeting regime without interest rate smoothing. The inflation feedback in Taylor rule ρ_π is set to 1.25. See Table 4.2 for complete parameter values. For the deposit adjustment cost function parameters one needs to take into account the steady state return on liquid and illiquid assets[33]. As in Kaplan *et. al.* (2018) χ_0, χ_1, χ_2 are set to 0.0438, 0.956, and 1.402 respectively. This combined with the annual discount rate β, and the borrowing wedge κ, matches the 5 key moments when considering the distribution of wealth and illiquid wealth. The steady state return on liquid assets r^b is calibrated at 0.05 implying an annual return of 2 percent. With the difference in balance sheet from 2004 and 2019, the target for illiquid assets is now $3.1\ \times$ GDP from 2019, while the liquid asset holdings in the economy B^h is 31 percent of annual GDP reaching $\$6,600$B

Estimation

Only income process within the model is estimated. This follows from Guvenen, Karahan, Ozkan and Song (2015), who utilise Social Security Administration Data on male earnings. As this data is not made public, I am unable to update the male earnings within the model and follow the estimation procedure of Kaplan *et. al.* (2018). Here they estimate the male earnings process utilising simulated methods of moments, which attempts to match eight crucial moments from Guvenen *et. al.* (2015). The eight moments required in matching are higher order moments and thus, they allow one to infer high frequency earning dynamics from these annual earnings data of SSA. Additionally, understanding that households that face small but frequent earnings shocks, in other words households that consume more of their disposable income save with liquid assets as it allows for immediate responses to changes in consumption. On the other hand households

[33]in the RANK and TANK model, in the steady state I obtain $d = -r^a a$. Though, in the HANK model it is calibrated to match the data.

that face infrequent but large shocks, will consume less of disposable income and will more likely hold illiquid assets to generate higher returns. Taking this into consideration, the income generating process is modelled as a sum of two independent components that closely relates to an AR(1) processes in discrete time. The logarithm of income $y_{it} = \ln z_{it}$ where $\ln z_{it}$ follows:

$$lnz_{it} = lnz_{1,it} + lnz_{2,it} \tag{4.4.1}$$

Here each $z_{j,it}$ follows a jump drift process, such that jumps arrive at a Poisson rate λ_j. Once a new jump is achieved, a new earnings state $z'_{j,it}$ is drawn, where $z'_{j,it} \sim N(0,\ \sigma_i^2)$. Finally, ξ_i is the rate at which the earnings process drifts towards zero after each new jump. Therefore, defining the earnings process $z_{j,it}$:

$$\mathrm{d}y_{it} = -\rho y_{it}\mathrm{d}t + dJ_{i,t} \tag{4.4.2}$$

Table 4.3 reports the model and data fittings. The parameter estimates are consistent with the Permanent Income Hypothesis (PIH), as it shows both a transitory and persistent component of earnings. The persistent component of earnings is interpreted as a permanent component when you consider the arrival rate of the earnings component combined with the lifespan of each household in the model. That is, each household can expect to achieve this permanent component of income a maximum of twice every lifetime. The transitory component for earnings is $j = 1$, with the persistent component $j = 2$. The arrival rate of each component of earnings here is 3 and 38 years respectively[34]. This earnings process embodies the heterogeneity in the model. To understand this recall that earnings process flows according to $y_{it} := w_{it} z_{it} \ell_{it}$ such that earnings will be determined by the choice of labour supply ℓ_{it} and most importantly the realisation of productivity shocks z_{it}. The estimated earnings process features a left skew such that a large right tail inequality is evident. In the steady state this results in the top 0.1%, 1%, and 10% of households accounting for respectively 2%, 7% and 32% of labour earnings[35]. Since z_{it} accrues to the productivity shocks of households combined with he labour supply choices from household optimisation, this results in a fully heterogeneous earnings process that allows households to independently invest in different asset holdings creating the rich heterogeneity in

[34]From the lens of infinitely lived households, this can be interpreted as a large and persistent career shock with perturbations of smaller temporary shocks (Kaplan *et. al.*, 2018)

[35]This is the assumption behind the model as built by Kaplan, Moll and Violante (2018). Ahn *et. al.* (2017) and Alves *et. al.* (2020) utilize the same underpinnings in their source of heterogeneity. Similarly, Achdou *et. al.* (2017) utilises a similar approach but with a lower number of agents.

the model. Note that it is this skewed labour earnings process that allows the model to generate skewed distributions along both liquid and illiquid wealth.

Table 4.3: HANK - Estimated Earnings Fit

Moment	Data	Model
Variance: annual log earns	0.70	0.70
Variance: 1-year change	0.23	0.23
Variance: 5-year change	0.46	0.46
Kurtosis: 1-year change	17.8	16.5
Kurtosis: 5-year change	11.6	12.1
Frac. 1-year change $< 10\%$	0.54	0.56
Frac. 1-year change $< 20\%$	0.71	0.67
Frac. 1-year change $< 50\%$	0.86	0.85

	Parameter	Component $j = 1$	Component $j = 2$
Arrival rate	λ_j	0.080	0.007
Mean reversion	ρ_j	0.761	0.009
Standard deviation of innovations	σ_j	1.74	1.53

Source: Kaplan, Moll, and Violante (2018)

Profit Redistribution:

In standard NK models, both price and wage rigidities play a huge role in the transmission of movements in interest rates to households. As already discussed in Chapter 2, the *Divine Coincidence* occurs in the absence of wage rigidities due to the countercyclicality of markups. In the Kaplan *et. al.* (2018) model, the absence of wage rigidities causes the HANK model to also suffer from the same issues as in standard NK models without wage rigidities. Since changes in interest rates have an inverse relationship with markups such that increasing interest rates decreases markups and vice versa, this has a similar effect on firms' profit and therefore on the price of stocks within the two-asset framework. This induces a countercyclical behaviour of firms profits. There, one needs to understand how monopoly profits within the model are distributed to households. In other words, are firm profits captured in the form of dividend payments paid into liquid or illiquid accounts? Due to transaction costs associated with withdrawing from illiquid accounts, the Marginal Propensity to Consume (MPC) for illiquid assets is significantly different (lower) than the MPC to consume from liquid asset holdings. Kaplan *et. al.* (2018) provide further assumptions about the distribution of profits within the model[36].

They fractionalise the redistribution of profits such that a given fraction of profits is paid into illiquid accounts. Here $\omega \in [0,\ 1]$ controls the fraction of profits paid into illiquid accounts. Thus, by appropriately choosing the parameter ω they implement a profit redistribution scheme to avoid the implication of negating rigid wages with included rigid prices. The choice of $\omega = \alpha$ renders the distributional consequences of movements in markups moot by transforming aggregate income flowing to illiquid accounts to be independent of marginal cost.

Residual share of profit $(1 - \omega)$ is then paid as dividends in the form of lump-sum transfers to individual households as in standard NK models. However, to further boost the heterogeneity in incomes (in this case liquid asset income) the portion of profit redistributed to the household is dependent on the productivity level of households such that more productive households receive greater share of redistributed profits.

4.5 Simulating LSAPs - HANK vs. TANK

Simulating the baseline LSAP, government issues more debt by increasing the short-term bond supply in the economy. I calibrate the model to match Chen *et al.* (2012), which in turn is calibrated to match the U.S. second LSAP programme following the financial crisis. The same parameters are chosen to match in the TANK model as well. Based on these estimations for earnings process and calibrated variables within the model I report the Impulse Response Functions for corresponding crucial macroeconomic variables. A key transmission for shocks within the model lies in the government budget constraint in equation (4.3.39). With increases in government debt, under the HANK model I allow for transfers to adjust in response to any shock process. Therefore, any changes in $\dot{B}$ require movements in the government budget constraint

[36] In the RANK or TANK case profits are redistributed as a lump-sum to households, even then, the assumptions behind the distributions of profit matters.

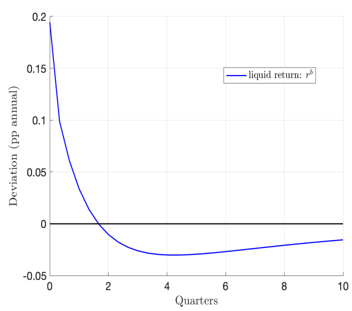
and transfers. [37]. The shock processes for HANK and TANK are respectively given as:

$$\varepsilon_t^B = e^{-\rho^c t}$$
$$\varepsilon_t^B = \rho \varepsilon_{t-1}^B + u_t$$

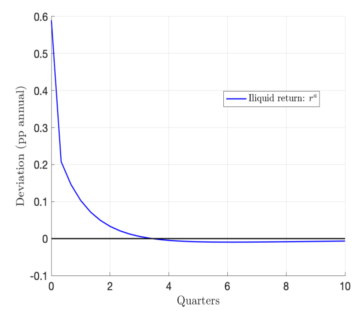
To equalise the persistence of shock processes one needs to consider how the deviation in continuous and discrete time works. Taking the cumulative deviation of liquid assets from $t = 0$ then one obtains that in continuous time $\int_0^\infty exp(-\rho^c s)ds = \frac{1}{\rho^c}$ while in discrete time $\sum_{t=0}^\infty \rho^t = \frac{1}{1-\rho}$. Using this formulation I choose persistence parameters such that $\rho^c \approx \rho$.

4.5.1 Aggregate Responses to LSAPs

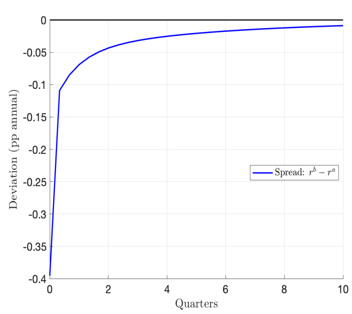
Figure 4.1: HANK: IRFs of The Share Price, Liquid, Illiquid Return, and the Resulting Spread



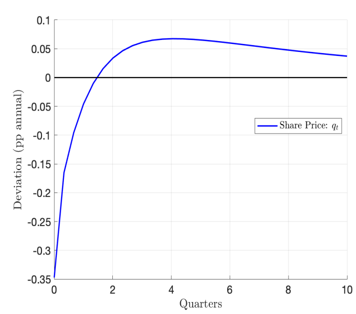
(a) IRF Liquid Return: r^b



(b) IRF Illiquid return: r^a



(c) IRF Spread ($r^b - r^a$)



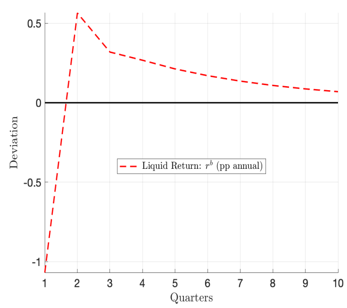
(d) IRF Share Price

Figure 4.1 and Figure 4.2 represent the responses of returns on bond holdings and illiquid asset as well as the share price. The issuance of more short-term bonds by the government leads to an initial positive impact effect on the returns on bonds. However, this is immediately proceeded

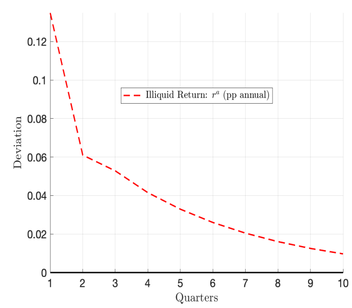
[37] An immediate observation under the TANK model lies in the shape of IRFs, this is due to the nature of the Taylor rule. Without interest rate smoothing in the Taylor rule, the nominal interest rate variable R_t is a jump variable that attains a new level following impact of shocks and changes to inflation.

by a sharp contraction on the return on bonds. This holds true under both models, with HANK having a steeper decline effect. Similarly, the effect on the return on illiquid asset is positive with a sharp decline, but with an exponentially smaller magnitude of decline in the return compared to the return on bonds. Given the preference structure of households holding a mix in their portfolio, there is observable substitution away from liquid assets towards illiquid assets. The higher demand for illiquid assets causes an increase in private investment (through capital) but recall from equation (4.3.35) that stock prices (q_t) are decreasing in capital. Thus, the decline in intermediate goods firm stock prices following asset purchases. Additionally, with reallocation towards equity assets, the result is an increase in equity returns from [38].

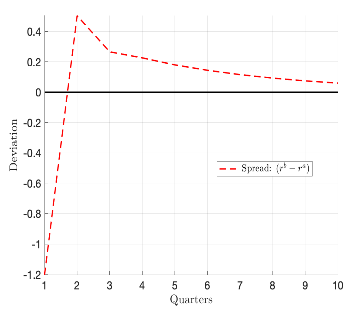
Figure 4.2: TANK: IRFs of The Share Price, Liquid, Illiquid Return, and the Resulting Spread



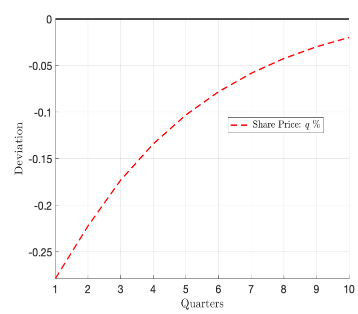
(a) IRF Liquid Return: r^b



(b) IRF Illiquid return: r^a



(c) IRF Spread ($r^b - r^a$)



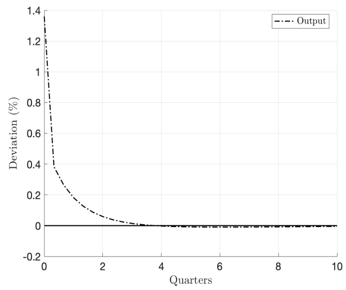
(d) IRF Share Price

The corresponding spread, defined as the difference between the return on bonds and the return on equity, is negative following the impact of higher bond issuance and remains negative before gradually returning to the steady state. These outcomes mimic real world outcomes (see Figure 4.1), while the literature on quantitative easing has primarily focussed on long-term vs

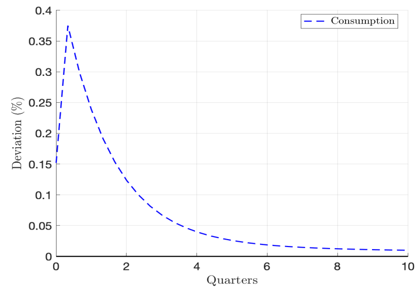
[38] As the economy is closed there is no role for currency in the model. What one would typically expect is the decline in the demand to hold domestic government bonds will induce a depreciation of domestic currency. This is likely to be explored in future research

short-term bonds, corporate finance literature on asset pricing and literature attempting to decompose the equity premium puzzle obtains that increases in bonds supply has a negative impact on the spread[39].

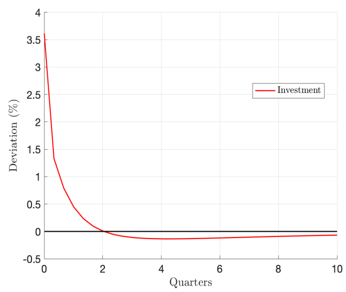
Figure 4.3: HANK: IRFs of Investment, Consumption, Output and Policy Variables



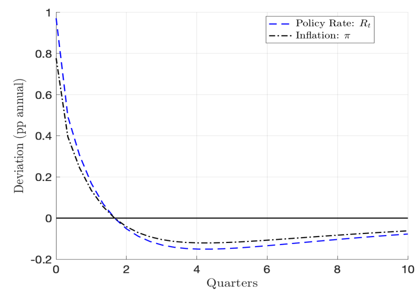
(a) Output



(b) Consumption

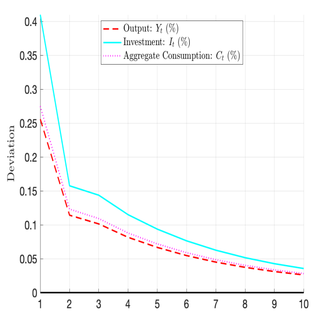


(c) Investment

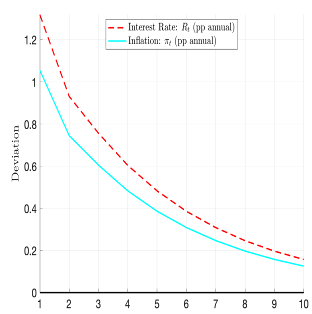


(d) Inflation and Policy Rate

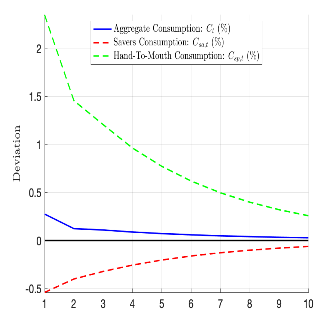
Figure 4.4: TANK: IRFs of Investment, Consumption, Output and Policy Variables



(a) Macro Variables



(b) Policy Variables



(c) Disaggregated Consumption

[39]see for example Chen, Collin-Dufrense and Goldstein (2009). It is worth noting that the literature focusses on corporate bonds. In the model both corporate and treasury bonds are aggregated to obtain the aggregate bond supply. Hence, one can generalise the results from this aspect of the literature. However, one must pay attention as a host of the literature looks at spreads between corporate and treasury bonds.

Figure 4.3 and Figure 4.4 report the aggregate responses under HANK and TANK respectively. In response to the bond issuance, the level of inflation on impact increases but exponentially decreases immediately after the first period. Analogously, so does nominal interest rates as it responds only to inflationary changes. The lower policy rates stimulate consumption to increase, as well as a positive impact on investment and output. The second transmission channel occurs through the portfolio rebalancing channel. With households readjusting portfolio wealth towards equity, this higher demand for equity decreases the premium for equity and decreasing financial costs for corporations. The higher increases in holding equity and lower corporations financial costs translates to higher investment from household capital accumulation. If you recall the illiquid asset $a_t = K_t + q_t(s_t)s_t$ so, with the no-arbitrage condition higher equity leads to more capital accumulation (see Figure 4.5 for capital's response) that increases investment. On the other hand, the TANK model exposes a similar pattern of responses, but with the added benefit of being able to disaggregate consumptions response. This further disaggregation of consumptions response shows that *hand-to-mouth* benefit from the bond issuance, while saving consumers suffer more than proportionately, ultimately biasing the aggregate level of consumption upward.

To understand this, the bond Euler equation in the model no longer depends on the nominal interest rate as in standard NK literature but rather on the returns to bonds, which in turn depends on the lagged nominal interest rate as well as lagged inflation rate. With returns on bonds decreasing following the increase in issuance, non-HTM consumption income decreases and they further substitute future consumption for current consumption following lower liquid returns[40]. To understand the reasoning behind smaller effects of LSAP on output, the transmission channels at play here need to be fully understood. By decomposing consumption's response I can investigate the driving transmission mechanisms (see Figure 4.6). While the portfolio rebalancing channel comes into effect, its effects on consumption are outweighed by the effects from transfers on consumption and as Oh and Reis (2012) show that (targeted) transfers have smaller fiscal multipliers for the economy than government consumption.

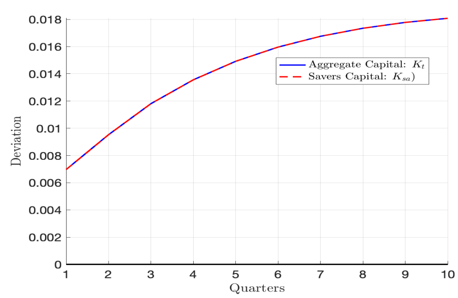
The higher increases in holding equity assets increases the capital stock and induces an increase in the productivity of workers. This holds in both HANK and TANK models, and arises as the

[40] This follows from the household Euler equation for bonds.

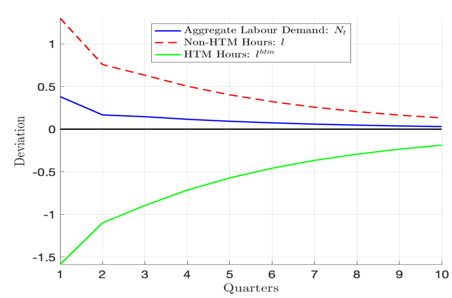
effects on the level of investment within both models is similar and modest. Under HANK, the impact effect on bond return of the increases in bond is positive, that exponentially dissipates and becomes negative, while there is a negative impact effect that exponentially increases before returning to the steady state level under the TANK model.

The aggregate labour supply (demand[41]) increases as non-HTM households desire more wage income for current period consumption. This is assessment is based on observing the deconstructed response from the TANK model as the HANK model does not explicitly model the labour supply choices of each household. Intuitively, with greater access to liquid assets (savings from bonds), which increases exposure to return volatility, savers (non-HTM[42]) have more incentive to provide labour hours to compensate for loss in income from bond savings, while *hand-to-mouth* consumers need not compensate by increasing the number of hours worked, given no exposure to the return on savings but most importantly the increase in transfers from the government.

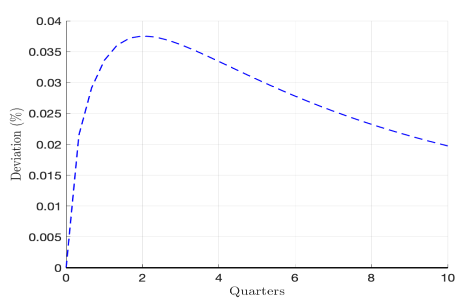
Figure 4.5: Capital and Labour Responses



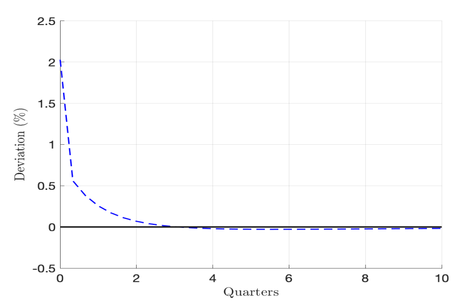
(a) TANK: Aggregate and Saver Capital



(b) TANK: Labour Supply



(c) HANK: Aggregate Capital



(d) HANK: Aggregate Labour Supply

Hence, the response of labour supply is driven by the choices of savers, such that increasing

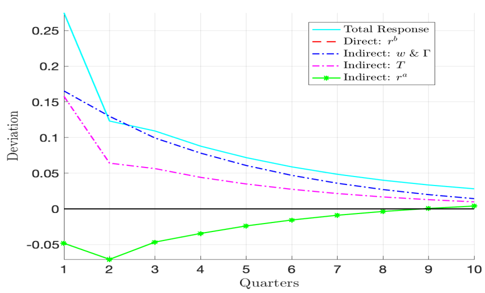
[41] In equilibrium all markets clear so supply equals demand.

[42] I use savers and non-HTM interchangeably.

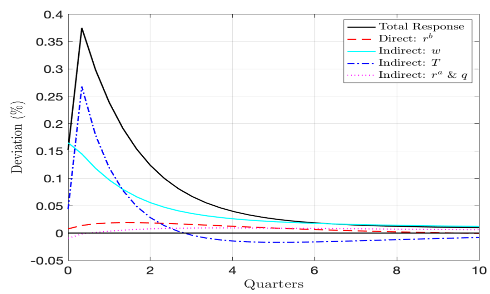
the number of *hand-to-mouth* consumers will alter aggregate variable movements (deviations from their steady state levels). This further plays into the consequences for income inequality which I will explore later on, as non-HTM households increasing labour supply leads to higher wages earned compared to HTM households, thus, driving income inequality. Understanding the HANK model has no arbitrary constraints to holding bonds (like TANK) such that the aggregate choices do not reflect the choice of a single group given the in-existence of a strict single group. This richness of the HANK model displays its superiority in matching the real world.

4.5.2 Breaking Down Consumption's Response

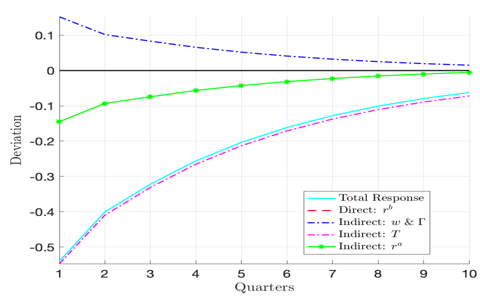
Figure 4.6: Consumption Decomposition



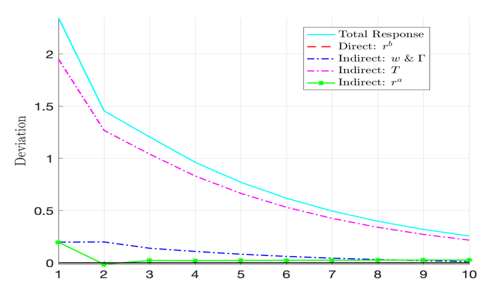
(a) TANK: Consumption Decomposition



(b) HANK: Consumption Decomposition



(c) TANK: Non-HTM Consumption Decomposition



(d) TANK: HTM Consumption Decomposition

Here I decompose the direct and indirect effects of prices (wages, return on liquid and illiquid assets, transfers) on consumption. Following Kaplan *et. al.* (2018) counterfactual experiments are used in computing the direct and indirect effects on consumption. Here to obtain the direct effect any one price has on consumption, I allow the price in question, for example wages to move as in the baseline specification, while holding all other prices constant[43].

[43] For the TANK model, I compute it in a similar way. However, there is indeterminacy in the TANK model when return on bonds does not move due to the Euler relationship, so, I compute the responses by setting variables equal

Conversely to Kaplan *et. al.* (2018) and as already explained, the direct transmission channel accounts for a far smaller portion of consumption compared to the indirect channels and particularly the transfers, wages and redistributed profit channels. The intuitive explanation behind this is simple - The issuance of more bonds creates the rate drop in return for bonds, which in turn has a profound impact on the consumption level for non hand-to-mouth households (higher productivity households), to which the response of consumption in their case is dominated by the direct transmission mechanism. Hence, a decrease in consumption for these non-HTM households.

Therefore, aggregate consumption's response is driven by *hand-to-mouth* households where consumption channels in their case is driven by wages earned and profit redistributed to them, along with direct transfers from the government, in other words the indirect transmission channel. To better understand this wage channel, observe the impact effect that arises in the form of higher level of aggregate output within the economy following the bond issuances. Higher output requires higher inputs in capital and labour which pushes the demand for employment and thus the income expansion. This is the wage transmission mechanism in action.

A consequence of the two transmission channels highlighted here is that households that gain from wages, higher equity prices and return on illiquid assets are completely different from households that gain from the transfers from governments. Additionally, the employment (wage) gains are skewed towards higher productive households, such that higher productive households gain more income from higher labour supply following current consumption requirements[44].

4.5.3 Sensitivity Analysis

This section evaluates two sensitivity analysis of both models. Firstly, inflationary pressures are subject to the rate of adjustment of prices, such that inflations response will be induced by how quickly firms can adjust their prices, and so will the nominal interest rate as well as the return on liquid assets. Therefore, I consider alternative price adjustment costs. Secondly, I want to understand the role the length of the asset purchase program plays following Burlon *et.*

to their steady state and simulating the model then subtracting the responses from when only bond return is allowed to move.

[44] I note that this is a consequence of the income generating process of the model.

al. (2019) comments on the length of APPs.

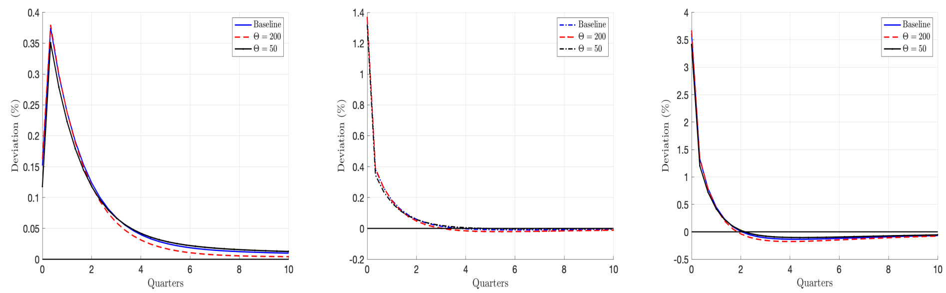
What Happens When Price Rigidity Changes?

The response for inflation and the slope of the NKPC is hinged on the Rotemberg price adjustment cost function parameter Θ_t. Setting $\Theta = 0$ implies a fully flexible price equilibrium and higher values of Θ_t implies more price rigidity. Both the financial crisis and subsequently COVID-19 could have led to significant structural changes in the way firms set their prices. In DSGE literature, there is evidence for both lower and higher price rigidity, and so, I want to quantify the sensitivity of the results to both lower and higher price rigidities. On the one hand, I consider $\Theta = 200$ for higher price rigidity such that the slope of the NKPC $\frac{\theta_p}{\Theta} = 0.05$. On the other hand, for lower price rigidity I consider $\Theta = 50$, implying an NKPC slope of 0.2.

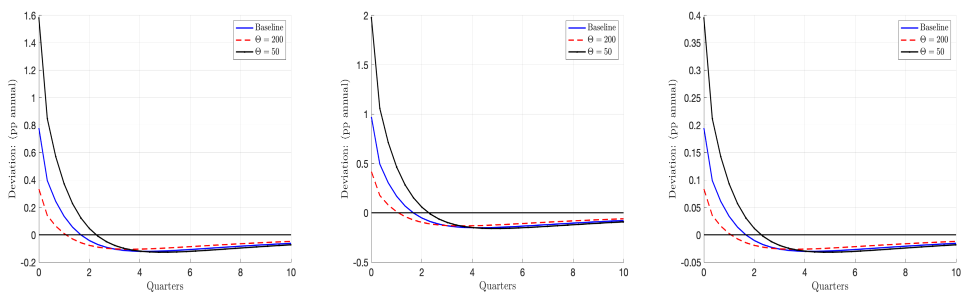
As expected, Figure 4.7 & Figure 4.8 show that under both models, the degree of price rigidity plays an import role in inflations response during the APP. Though, the effect on inflation is mostly relegated to the impact effects under HANK. With higher price rigidity, inflation increases by almost 0.4 percentage points annually compared to about 0.06 percentage points annually under less sticky prices. However, the effects on aggregate macro variables are insignificant, especially with aggregate output where difference do not occur on impact or in transition. I offer the following explanation: under HANK the degree of market segmentation is considerably lower such that adjustment processes cannot be transferred from inflation (as well as interest rates and bond returns) to core aggregate macroeconomic variables.

With the TANK model, the results are similar with regards to inflation, such that more rigidity decreases the increase of inflation as firms do not adjust prices as quickly as they can. However, this transmission channel permeates differently to other aggregate variables with the model such that higher price rigidity increases the APPs effect on output, consumption and investment (unlike the HANK case). The degree of price rigidity not only determines the adjustment of prices but also the adjustment of responses to the LSAP. Here, less price rigidity decreases the response of output, consumption and investment with the converse holding. The strictness of segmented markets in the TANK model is the driving force behind this result, as it is something common with Chen *et.al.* (2012) results. However, under the HANK model there are no significant impacts on the aggregate variables and price adjustment only changes the rate of adjustment of inflation.

Figure 4.7: HANK: Sensitivity of Aggregate Variables to Price Rigidity

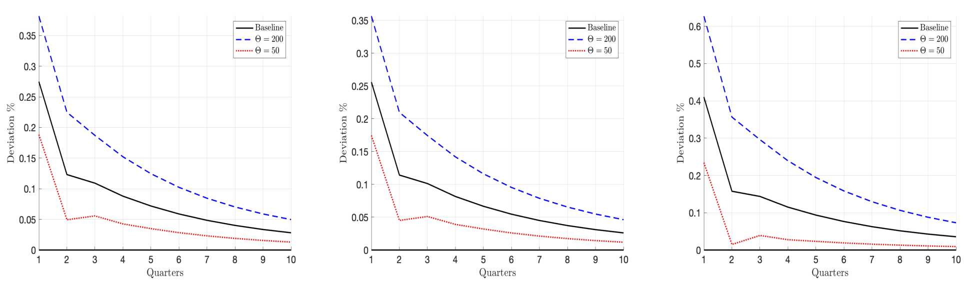


(a) Aggregate Consumption (b) Aggregate Output (c) Aggregate Investment

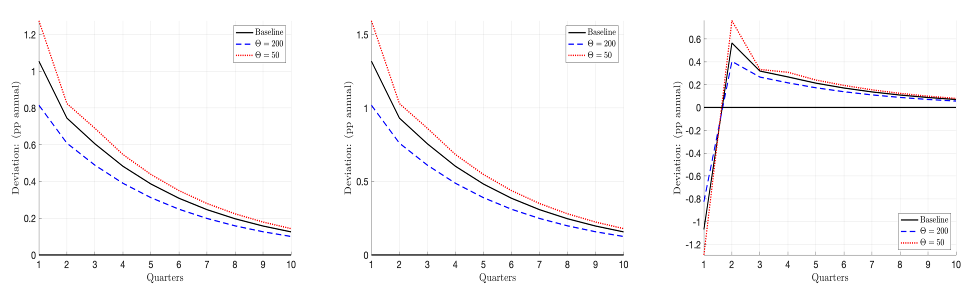


(d) Inflation (e) Interest Rates (f) Bond Return

Figure 4.8: TANK: Sensitivity of Aggregate Variables to Price Rigidity



(a) Aggregate Consumption (b) Aggregate Output (c) Aggregate Investment



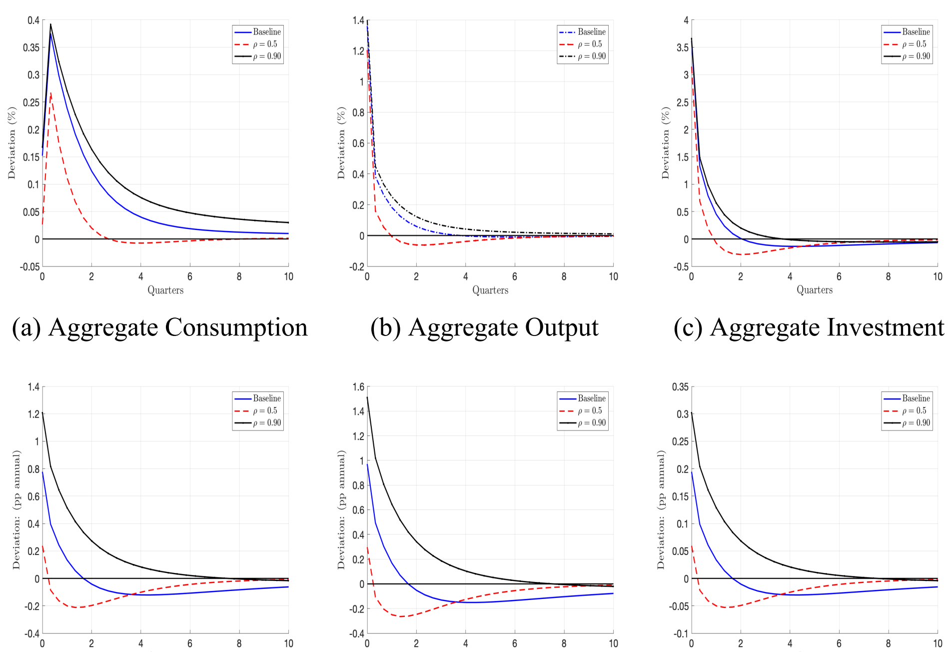
(d) Inflation (e) Interest Rates (f) Bond Return

To summarise, under the HANK model where market segmentation is to a lower degree, changes in firm behaviour towards price setting only has consequences for inflation interest rates and bond return that are all linked through the Fisherian channel. While under the TANK model with highly segmented markets, higher (lower) degree of price rigidity shifts macro variable responses from inflation (GDP) to GDP (inflation) by increasing (decreasing) the response of interest rates and liquid return.

What Happens When the Length of Asset Purchases Changes?

In the baseline scenario for the LSAP program, the government accumulates more debt for about 4 quarters before winding down the purchases. While this is plausible, the length of the LSAP will depend on the economic conditions, for example the APPs during the COVID-19 pandemic and the APPs during the financial crisis have different purchase lengths. To better understand the effects LSAPs have, I follow Chen *et. al.* (2012) and observe the effects the duration of LSAPs have on the economy. So, I evaluate this at two different persistence levels: $\rho = 0.5$ and $\rho = 0.1$. This corresponds to quarterly autocorrelation levels of $e^{-0.5} = 0.61$ and $e^{-0.1} = 0.91$. Under the TANK model the corresponding values for are set at $\rho = 0.5$ and $\rho = 0.9$ respectively.

Figure 4.9: HANK: Sensitivity of Aggregate Variables to Persistence of Purchases



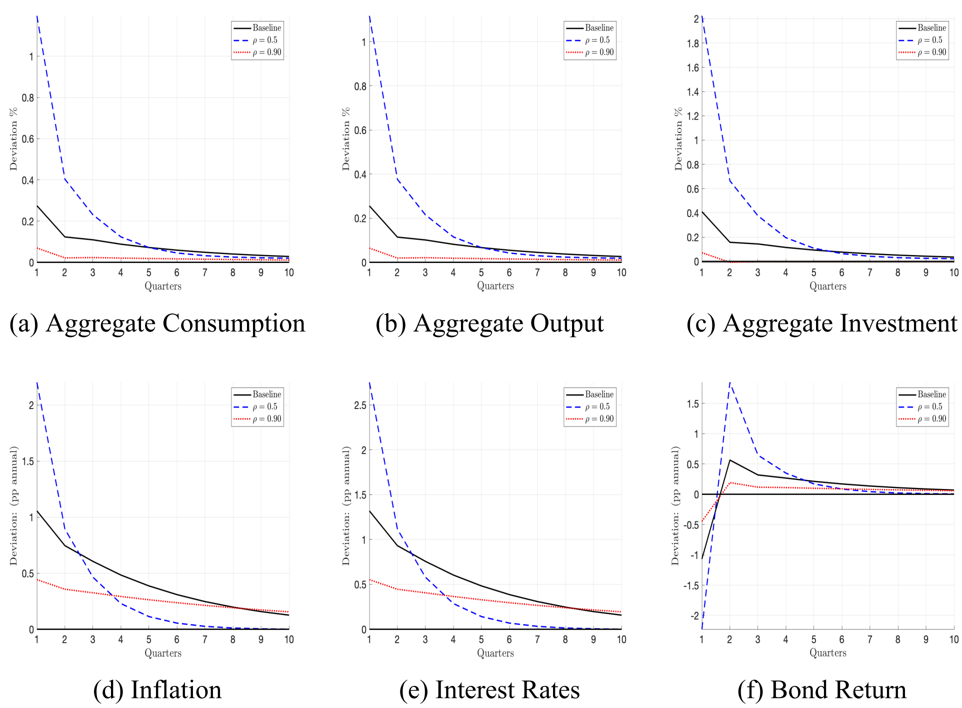
(a) Aggregate Consumption (b) Aggregate Output (c) Aggregate Investment

(d) Inflation (e) Interest Rates (f) Bond Return

Unsurprisingly, and in line with Chen *et. al.* (2012) and Burlon *et. al.* (2019) changes in the time profile for asset purchases induces greater responses from inflation, interest rates and bond return. Their responses are almost doubled under the longer purchase program, along with impact magnitudes for consumption. Additionally, the boost to consumption becomes more persistent lasting longer the baseline and a shorter windowed purchase program.

Although, the response for output under the longer windowed asset purchases is only slightly larger and corresponds to the magnitude of the impact effects. On the other hand, the TANK model produces considerably different results with longer lasting asset purchases inducing less than significant changes to aggregate variables, while shorter lasting windows for asset purchases induce higher magnitude for impact effects but with somewhat identical persistence to the baseline.

Figure 4.10: TANK: Sensitivity of Aggregate Variables to Persistence of Purchases



(a) Aggregate Consumption (b) Aggregate Output (c) Aggregate Investment

(d) Inflation (e) Interest Rates (f) Bond Return

In summary, under HANK, if governments increased the window period for asset purchases, it increases the impact effect on crucial macro economic variables with more persistent effects on inflation, and consumption. The effects on inflation are particularly more pronounced than the effects on output due to price setting behaviour. With sticky prices, price re-optimisation by firms is not immediate. Therefore, longer lasting asset purchases leads to more firms optimising their prices in response to the shocks but they do so more aggressively, which induces greater inflationary pressures. This converse holds true when looking at shorter windowed asset purchases.

4.6 Wealth Inequality Effects of LSAPs

Income inequality can serve as a source heterogeneity as individuals have considerable different marginal propensity to save and consume. Similarly, distributional sources of income also serve to create heterogeneity as individuals possess different sources of income and different levels of labour productivity. This creates a channel for interest rates through returns on savings and repayments on loans (savers vs. borrowers), so, monetary policy can act as a redistributive instrument and have distributional consequences for income and capital. In this case, the return on bonds is determined through the Fisherian channel and hence return on bonds is determined by the nominal interest rate. Additionally, fiscal policy can also become a redistributive instrument in the form of transfers to lower productive (lower income) households. Therefore, since LSAPs induce changes to government transfers as well as changes to the return on bonds, it is instinctive to seek understanding about the redistributive consequences all these changes can have on all forms of household inequality.

When considering the effects macroeconomic shocks have on inequality representative agent models become ineffective as all households are in essence represented by a single agent and with no forms no digression. Here, all transfers will become neutral, and the Ricardian equivalence neutralises all transfers across time. However, given the heterogeneous framework from the TANK and richer HANK model, disparities exist between household asset holdings and consumption. It is also worth noting that under the TANK model I do not measure any wealth inequality as there are only have two agents – HTM agents and non-HTM, thus, *hand-to-mouth* households hold no bonds or sticks and any attempt at characterising the differences in asset holdings as wealth inequality will generate exponential differences that will be empirically redundant. Though the effects on consumption inequality can be observed.

Within the literature the work that closely relates to mine is Ahn, Kaplan, Moll, Wineberry and

Wolf (2017) who utilising the HANK model from Kaplan *et. al.* (2018) and Kaplan and Violante (2014) augmented with mortgage debt payments and a fraction of *Hand-to-mouth* households show that under productivity shocks consumption responses are larger with richer heterogeneity, while representative agent models have lower responses given the lesser transitory component of the shock. Here the portion of *Hand-to-mouth* households drives these responses given their inability to increase consumption immediately in response to income growth, and therefore, their consumption response is considerably weaker. This result of inequality mattering for macroeconomic variables is similar to the influential paper by Krusell and Smith (1998) who also ascertained that inequality mattered for macroeconomic shocks[45].

When observing how macro shocks affect inequality Ahn *et. al.* (2017) show that negative productivity shocks restricted to lower-skilled individuals can be recessionary but generating more than disproportionate negative effects on the lower skilled workers and increasing the level of income and consumption inequality. Analogously, a positive shock to capital productivity has expansionary effects on the economy but the boom from expansion more than proportionately benefits higher skilled workers, which leads to higher income and consumption inequality. A broad aspect of the literature has specifically focused on the distributional effects of monetary policy. The most related paper is by Gornemann, Kuester and Nakajima (2021) that present a rich HANK model where heterogeneity is introduced in two distinct ways: retired households who do not earn labour income; and working households that face labour market frictions through search and matching, such that there are employed and unemployed agents within the economy. In order to capture the true consumption risks unemployed households face, they allow for persistent earnings proceeding redundancy. They show that what is particularly important for the distributional effects of monetary policy is the wage setting process. With this, retired individuals who do not face any labour market risk but face capital market gains, gain proportionally from strict inflation targeting rules. On the other hand, lower income working household benefit from employment targeting monetary policy rules. One important result from their model shows that the HANK model can instigate distributional concerns beyond household productive efficiency resulting from after-tax transfers[46]. This trade-off hinges importantly on the wage

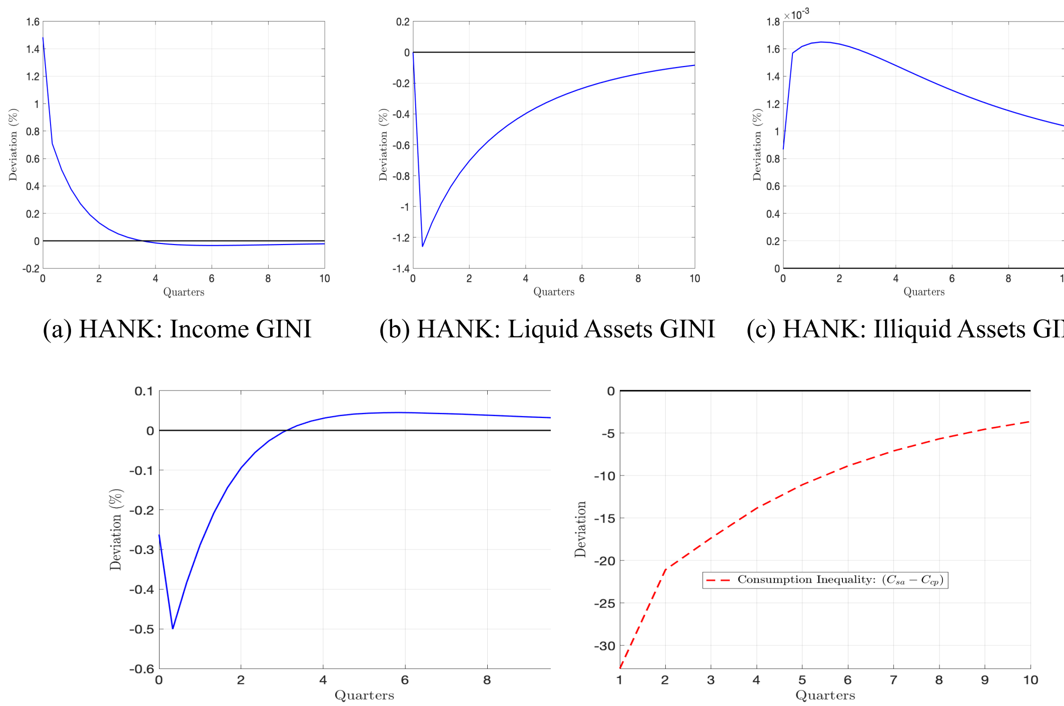
[45] I do not assess how inequality in the model affects the transmission of shocks, for further reading see Ahn, Kaplan, Moll, Wineberry and Wolf (2017); Krusell and Smith (1998), Campbell and Mankiw (1989)

[46] Oh and Reis (2012) look at targeted transfers during the financial crisis and conclude that while the transfer multiplier is considerably smaller than government consumption multiplier, lump-sum direct transfers can be expansionary and create negative wealth effects for some households. This happens through the Neoclassical labour

setting behaviour within the model. In the model, conversely to their search and matching wage setting, there is a simple household labour supply decision that in equilibrium must match firms labour demand[47].

So far, to my knowledge I am also amongst the first to present the distributional consequences of LSAPs on consumption, income and wealth inequality. The two channels at play here are those identified in the main body of this chapter. The first being the portfolio readjustment channel that induces accumulation of equity and bonds by higher income households capable of purchasing the new issued government bonds as well as the higher priced equity, and the second channel being direct transfers from the government. Though not an explicit transmission channel, an indirect transmission exists and acts through nominal interest rates movements in Taylor-Rule.

Figure 4.11: Wealth, Income and Consumption Inequality Responses to LSAP



(a) HANK: Income GINI (b) HANK: Liquid Assets GINI (c) HANK: Illiquid Assets GINI

(d) HANK: Consumption GINI (e) TANK: Consumption GINI

As any interest rate movements can become contractionary or expansionary following the re-

supply channel; with transfers increasing labour supply for some being more than proportionate than the positive wealth effects of transfers decreasing labour supply for others

[47] A significant and growing aspect of the literature looks at distributional effects of optimal monetary policy under incomplete markets. See e.g., Challe (2020); Berger, Dew-Becker, Schimdt and Takahashi (2019). Another strand of monetary policy literature looks at simple monetary policy shocks via Taylor-rule innovations in policy rate. See e.g., Bilbiie (2020); Acharya and Dogra (2020)

sponse to inflationary movements to increasing bond issuance. These then proportionally favour distinct income groups, that is non-HTM households will benefit from higher and more persistent interest rates. However, unlike Gornemann *et. al.* (2021) the model does not incorporate unemployment, and hence unable to observe the unemployment channel for nominal interest rate movements to affect households.

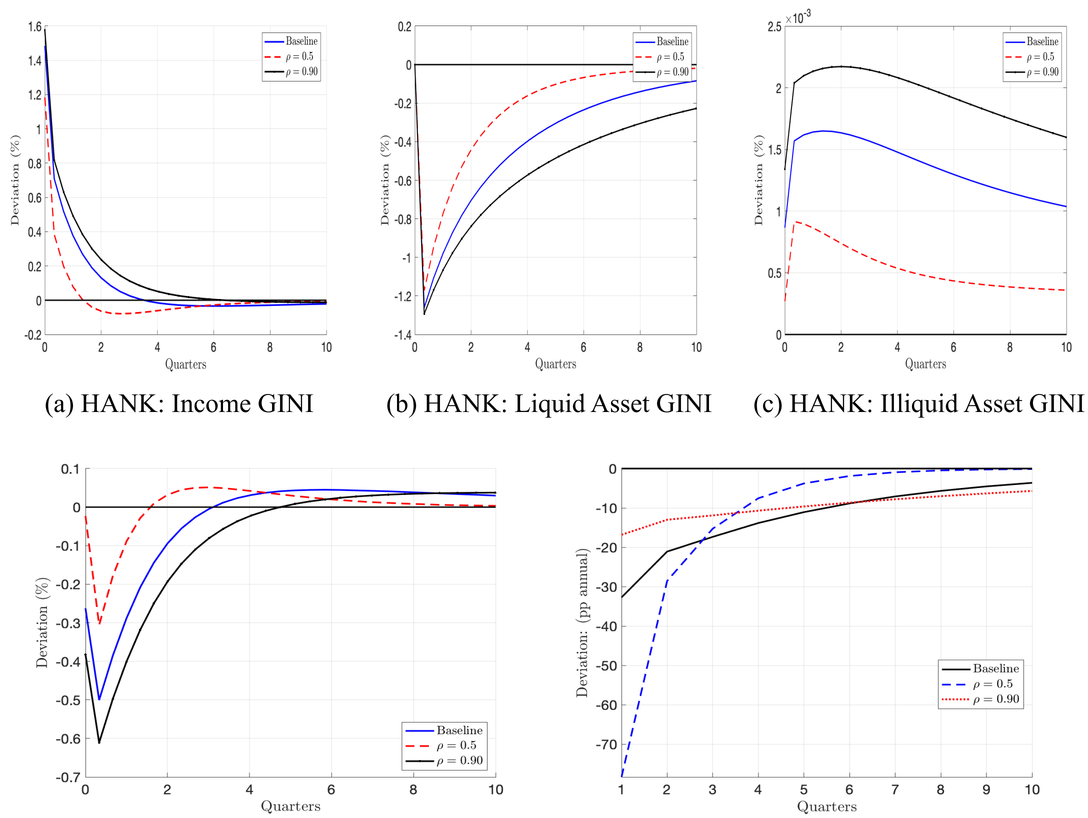
Figure 4.11 shows the effects of LSAPs on liquid, illiquid wealth, capital holdings and consumption inequality. Consistent between both models is that of consumption inequality, as the impact effect is negative in both cases before gradually dissipating. Under the TANK model I have already disaggregated the responses of consumption from both consumer groups (see Figure 4.4 (c)). The HANK responses derive from a similar the intuitive explanation: with the portfolio reallocation channel, higher income households rebalance asset holdings towards illiquid assets evident from the increase in inequality in illiquid asset holdings, to which illiquid assets cannot be utilised in response to current period consumption needs. Therefore, with the decreased returns from bonds, consumption today is highly hinged on wage income today as well as net transfers from the government but targeted transfers are aimed towards lower income households (HTM households) and not higher income/more productive households. In fact under TANK, the decomposition of consumption's response for non-HTM households reveals that non-HTM have negatively impacted by transfers (see Figure 4.6 (c)). That is, their wealth is redistributed towards HTM households by government. Once one understands that the indirect transmission channel i.e., wages and transfers, dominates consumptions response within the model, then it is clear why the consumption GINI is negative in both models.

Recall the explanation in section (4.5.2) on households that gain from illiquid wealth differs considerably from households that gain from transfers. In line with Oh and Reis (2012), the labour supply channel constitutes one of the major transmission channels, even in the simplified wage determination environment. The higher illiquid inequality effects are a consequence of the wage transmission channel and the resulting increase in equity prices. While consumption is shifted towards lower income households through increased transfers, the higher wages are skewed towards higher productivity households, which induces the higher income inequality. Thus, while higher income households benefit from increased wages, present period consumption is not priority and as such income is directed towards illiquid asset holdings that generate greater return creating higher illiquid asset inequality.

Finally, the last indirect transmission mechanism for inequality works through the nominal interest rates. While it is not my intention to explicitly observe monetary transmission mechanisms, it cannot be ignored given LSAPs induce inflationary pressures (though small) that induce nominal interest rate changes. Here, recall the findings of Gornemann *et. al.* (2021) on households that gain from capital markets are proportionately favoured by strict inflation targeting regimes, which the Taylor rule in the model follows. This transmission mechanism is what is partly responsible for the higher illiquid asset inequality, as strict targeting regimes creates capital gains. With illiquid assets consisting of capital, this translates to more illiquid asset gains and therefore, higher illiquid asset inequality.

4.6.1 Effects of the Length of Asset Purchase on Inequality

Figure 4.12: Policy Effects on Inequality



(a) HANK: Income GINI (b) HANK: Liquid Asset GINI (c) HANK: Illiquid Asset GINI

(d) HANK: Consumption GINI (e) TANK: Consumption GINI

In order to observe the effects of how longer lasting asset purchases would have on inequality, I extend the persistence of the shock variable, such that government issuance lasts for a longer period. This is contrasted to shorter lasting asset purchase programmes. When asset purchases are extended for a longer duration, the expectation would be a longer lasting effect. This is exactly what is observable in addition to a greater magnitude of the impact effect. What is particularly interesting is that shorter lasting asset purchases decrease the impact effect on all the measures of inequality under HANK. While longer lasting asset purchase programmes accomplishes the opposite. This creates somewhat of a trade-off as policy makers would have a choice between decreasing illiquid wealth and income inequality with current period consumption inequality. Though, the decrease in liquid wealth inequality also occurs, the return from liquid wealth in the short run decreases due to the increasing level of asset purchases. A direct implication of this result lays in addressing aggregate wealth inequality. In other words, increasing income and illiquid wealth inequality today will drive aggregate wealth inequality in the future, as growth in return from illiquid wealth increase based on market performance and consumption inequality will change depending on the of aggregate future wealth such that after asset purchases end, consumption will once again be driven by wealth and income, with current period consumption dominated by wages and liquid wealth, future consumption will be driven by aggregate wealth that includes illiquid wealth.

To summarise, longer asset purchases create a higher decrease and in current period consumption inequality as well as liquid wealth inequality but generate higher increases in income and illiquid wealth inequality. Thus, future consumption which is driven by both forms of wealth and income and not transfers given the end of asset purchases will inevitably become skewed towards the higher income level due to higher accumulation of illiquid wealth and higher income that can be converted to both types of wealth. These results hold under HANK as under TANK only consumption inequality is observable and here TANK contrasts the results of HANK in this dimension of longer lasting asset purchases creating longer lasting effect on the decrease in consumption inequality but shorter asset purchases inducing a larger magnitude impact effect but short lived.

4.7 Conclusion

The chapter here presents the HANK and TANK models developed by Kaplan, Moll and Violante (2018) but utilises it in answering a different economic question – what the effects of LSAPs are and how do LSAPs affect inequality in the economy. The model allows households to purchase two different asset classes and allows portfolio readjustment to take place but with frictions modelled through adjustment costs. As such it allows the observation of the role the portfolio rebalancing transmission channel plays in assess the impacts of LSAPs. Additionally, it also incorporates government direct transfers providing another alternative transmission channel.

My results are in line with the literature obtaining modest magnitude impacts of LSAPs on aggregate macroeconomic variable even under the HANK approach. Overall, my findings show a positive but short-lived impact on aggregate macroeconomic variables with small inflationary pressures induced. It also does well in its realistic replication of asset markets as the LSAPs create increase in share prices along with an increase in the return on stocks, which ultimately induces the higher spread [48]. The model presented provides more insight int the transmission channels for LSAPs. Most importantly it establishes the importance of lower income households or *Hand-to-mouth* in driving aggregate consumption of the economy in response to LSAPs. Since government direct transfers and wages dominate the effect on consumption, these households are more likely to receive direct transfers and to consume their disposable wages. Additionally, though the portfolio rebalancing channel holds it plays more of a significant role in driving aggregate wealth inequality by driving up illiquid wealth inequality. While the transfers and wages drive down consumption inequality, the increase in labour supply (productivity) of higher income households drives the income inequality.

After conducting several robustness checks I find that aggregate variables under the HANK framework show very little response to changes in the level of price rigidity, with only inflation showing significant changes to how often firms change their prices. However, I find that the length of the asset purchases induces significant differences. In other words, I find that longer lasting asset purchase programmes induce longer lasting and greater magnitude of effects on

[48] Though the increase in share price is not replicated in the TANK model

aggregate macroeconomic variable under HANK, but the converse holds under TANK, with longer lasting programmes having little to no effects.

A limitation with this chapter is the interaction with monetary policy and particularly the ZLB. Understanding the effects of LSAPs when interest rates are at the ZLB along with the commitment of the central banker to keep interest rates at this point will provide more insight on the effects of LSAPs. This also forms a path for future research in this area, as well as in an open economy setting. Additionally, I have utilised a simple Taylor rule here that only accounts for inflation targeting and without interest rate smoothing. Future research will relax this assumption and look at as well as compare Taylor rules that account for the output gap and interest rate smoothing to identify welfare improving policies.

Chapter 5

General Conclusion

Oil prices have experienced two distinct supply side shocks in the last decade that have translated to negative macroeconomic implications for low- and middle-income net exporting oil countries. While these countries contribute significantly to global oil production little is still understood about transmission of oil price shocks to their economies and how monetary policies should be conducted in these economies in response to international oil price shocks. Considering this, in the second two chapters of this thesis I sought out to understand the propagation of international oil price shocks to low- and middle-income oil producing countries but most importantly how monetary policy should respond in order to mitigate the negative consequences accompanied by these shocks. To do this I develop a large scale DSGE model that jointly determines both international oil markets and domestic non-oil economy. With the joint determination I can fully endogenize oil prices and infer the transmission channels between international and domestic.

My results confirm that transmission channels to these net exporting oil countries is considerably different from the transmission channels for net importing oil countries. These channels in turn have consequences for the conduct of monetary policy. Firstly, the major transmission channels occur through consumption and final good production reliance on imports. As such oil price declines will induce exchange rate depreciations in the home economy and without a corresponding response from non-oil exports from the home economy national accounts contract considerably. Additionally, domestic aggregate activity is also driven by sectoral value added from the oil sector such that decreasing oil prices decreases oil sector value added. Since oil production is an integral part of their economies, traditional inflation targeting rules by central banks are not welfare improving under declining oil prices. While central banks seek to decrease inflation from through higher interest rates, inflationary pressures during oil price induced economic downturns are driven by depreciations of the real exchange rate. Hence any attempt at stabilising inflation will prove moot. Furthermore, strict inflation targeting will have disastrous consequences for domestic GDP and domestic consumption because all producers including oil producers (that drive the economy) suffer from higher interest rates. In such a case of declining oil prices, output gap targeting will be the most welfare improving monetary policy as interest rates' reaction will be expansionary even with inflationary pressures.

The linkages between exchange rates and oil prices in these low- and middle-income net exporters of oil also has different roots in the literature when considering oil windfalls. In other words the *Dutch Disease* hypothesis has ascribed poor performance of non-oil sector exports to worsening terms of trade that accompanies exchange rate appreciations. What I find is that the prevalence of the Dutch disease is also linked to the underlying reason oil prices have changed. Not all terms of trade worsening are accompanied by exchange rate appreciations and inf cat monetary policy can be utilised as mitigation too for the prevalence of the Dutch disease. However, central bankers face a trade-off between stabilising non-oil sector value added and non-oil exports.

Analogously, the increasing correlation between commodity price cycles and credit availability in commodity driven economies is becoming ever so prevalent in the literature[1]. In a similar sense, the propagation of shocks under financial frictions has been cemented in the literature to have an intensifying effect[2]. Considering these implications for low- and middle-income oil exporting countries, I extend the model in chapter 2 to incorporate credit frictions for the oil sector such that the direct relationship between oil prices and easing of borrowing is positive. I find that financial frictions induce different optimal input demand dynamics by oil sector firms and as a result either exacerbate or constrict the effects of rising or falling oil prices respectively. Additionally, in the sensitivity analysis, the integration of the leverage constraints to generate the financial accelerator channel shows that what matters for the propagation of shocks is the capital access channel. Here the Financial accelerator induces little to insignificant changes

[1] see Drechsel *et. al.* (2019)
[2] see Gerali *et. al* (2010).

to magnitude effects, even when considering changes to how responsive borrowing constraints are to changes in oil prices. Finally, I provide thoughts through evidence based literature that given conventional exchange rate stabilisation policy by central banks (exchange rate targeting) is unable to offset the depreciations of exchange rates from falling oil prices, then sterilised exchange rate interventions in conjunction with a policy targeting objective tailored to the underlying reason oil prices changed will perform best for both limiting exchange rate depreciation and inflationary pressures induced by the depreciation.

While the monetary policy studied in chapter 2 and 3 emphasises commitment of central bankers, the issue of credibility of central banks especially in low- and middle-income countries becomes a problem. Since Inflation targeting regimes typically rely on the credibility of the central banks supporting monetary policy. In this context, the need for credibility is bigger in developing countries (LICs or MICs) due to past high inflation episodes. Analogously understanding the time inconsistency of monetary policy is crucial to fully designing monetary policy[3]. As time inconsistent monetary policy alters transition pathways for economies in response to international shocks, it is crucial in economies with low levels of inflation and even more important in economies that experience elevated inflationary periods. In economies such as the low-and middle-income commodity driven ones, where periods of high inflation are a common occurrence, time-inconsistency of monetary policy could lead to longer periods of elevated inflation levels through discretionary inflation bias. In this case the central banker reneges against any announced target level of inflation in favour of an inflation surprise while attempting to stabilise output. Though this leads to lower credibility of the central bank and higher inflation expectation of firms and workers that creates higher levels of inflation. Over time becomes more costly for the central bank to renege on its inflation target as firms adapt to the central bank's behaviour, with policy announcement becoming time inconsistent. Though discretionary inflation bias can create elongated periods of inflation, the literature has focussed on time-inconsistency affecting the central bankers ability to stabilise inflation - *Stabilisation bias*. Since households and firms form inflation expectations, then any trade-off the central banker makes against inflation stabilisation will cause higher future inflation expectations of households and firms, prompting households to bargain higher wages and firms to increase prices today to compensate. However,

[3]The influential papers by Kydland and Prescott (1977) and Barro and Gordon (1983) outline the time-inconsistency theory of monetary policy.

it is worth noting that discretionary inflation and stabilisation biases induced by commitment and time-inconsistent policies are characteristics of central banks that commit to inflation targeting, and not output targeting. This issue of time inconsistency of policies and credibility of the central bank will create difficulties in monetary policy attempting to insulate the economy from international shocks.

Finally, given the increased reliance of central banks on unconventional monetary policy tools I assess the transmission channels as well as impacts of LSAPs on the aggregate economy and inequality from a heterogeneous agent framework. I utilise the Heterogeneous Agent New Keynesian (HANK) and Two-Agent New Keynesian (TANK) models developed by Kaplan, Moll and Violante (2018) to simulate increased government bond issuance. The models incorporate both liquid (bond) and illiquid assets (equity) for saving decisions of households, which allows readjustment of asset holdings following changes in return. I find that the portfolio reallocation channel plays a lower role in transmission mechanism. Here transfers and wages dominate the transmission mechanism for LSAPs impact. This transmission mechanisms also underpins the effect of LSAPs on wealth, consumption and income inequality, as reallocation towards equity drives illiquid asset inequality ad at the same time decreases consumption inequality due to the role illiquid assets affect current period consumption. While my findings are in line with the literature, I do obtain one differing result and it is the impact of LSAPs on bond returns in HANK. Due to the determination of bond returns from the Fisher equation I obtain countervailing bond return response to asset purchases. Furthermore, the deconstruction of consumptions response to LSAPs helps reveal transmission channels in HANK and I find an interesting reason behind the observed magnitude effect of asset purchases on output. The small size of transfers multiplier underpins the modest effect LSAPs have on output. Most importantly, the comparison with a TANK framework reveals the possibility of approximating heterogeneity through two agents, when considering the context of unconventional monetary policies.

Through the chapters presented here, I have advanced the understanding of monetary policy conduct as well as the transmission of oil price shocks in low- and middle-income oil exporters. Similarly, I have unveiled a different transmission mechanism for LSAPs through HANK that underpin the modest effects LSAPs have on output and inflation, which is observed in the literature. I also present a platform for future research on how we can tailor policies to decrease income and wealth inequality. In addition, I also show how we should re-imagine monetary policies for oil exporters and how the dynamics differ from oil importers especially if the countries in question are low- or middle-income.

Appendix A

The Model - Two Country Open Economy with Oil Sector

Households

The household utility function is therefore:

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left[\frac{[C_t(l) - hC_{t-1}(l)]^{1-\sigma_c}}{1-\sigma_c} - \frac{N_t(l)^{1+\sigma_n}}{1+\sigma_n} \right], \qquad \sigma_c, \sigma_n > 0$$

Here m_t denotes real money holdings. The household budget constraint is therefore:

$$C_t(l) + \frac{P_{i,t}}{P_t}\left(I_t(l)\right) + \frac{B_{H,t}(l)}{\epsilon_t^b P_t} = \Omega_t N_t(l) + R_t^k K_t(l) + \frac{R_{t-1} B_{H,t-1}(l)}{\epsilon_t^b P_t} - T_t(l) + \frac{M_{t-1}}{P_t} + \mathcal{D}_t$$

Households can invest in physical capital (non-oil sector) and in capital used in oil sector - they have law of motion:

$$K_t(l) = (1-\delta)K_{t-1}(l) + \epsilon_t^i \left[1 - S\left(\frac{I_t(l)}{I_{t-1}(l)} \right) \right] I_t(l)$$

Where $S\left(\frac{I_t(l)}{I_{t-1}(l)}\right) \equiv \frac{\varrho_k}{2}\left(\frac{I_t(l)}{I_{t-1}(l)} - 1\right)^2$ Is the capital adjustment cost function which satisfies $S(1) = 0$, $S'(1) = 0$ and $S''(.) > 0$. The household problem is to choose consumption, hours worked, bonds, real money holdings, energy and investment in physical capital and physical capital stock to maximise the objective function. The lagarnagian for the problem is:

$$
\begin{aligned}
\mathcal{L} = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left[\frac{[C_t(l) - hC_{t-1}(l)]^{1-\sigma_c}}{1-\sigma_c} - \frac{N_t(l)^{1+\sigma_n}}{1+\sigma_n} \right] \\
+\Lambda_t \Bigg(\Omega_t N_t(l) + R_t^k K_t(l) + \frac{R_{t-1} B_{H,t-1}(l)}{\epsilon_t^b P_t} \\
-T_t(l) + \mathcal{D}_t - C_t(l) - \frac{P_{i,t}}{P_t} \left(I_t(l) \right) - \frac{B_{H,t}(l)}{\epsilon_t^b P_t} \Bigg) \\
+Q_t \left[(1-\delta) K_{t-1} + \epsilon_t^i \left[1 - S\left(\frac{I_t}{I_{t-1}} \right) \right] I_t - K_t \right]
\end{aligned}
$$

First order conditions are as follows:

$$
C_t: \qquad [C_t - hC_{t-1}]^{-\sigma_c} - \Lambda_t(l) = 0 \Leftrightarrow [C_t - hC_{t-1}]^{-\sigma_c} = \Lambda_t
$$

This only holds when there is external habit formation in the model. Since consumption and habit formation are identical across households, it must be that the lagrange multiplier for households Λ_t is also identical across households. Therefore, I define:

$$
M_{t,t+i} = \beta \frac{\Lambda_{t+i}}{\Lambda_t}
$$

This is the stochastic discount factor that is common across households and firms, as households own the firms:

$$
\begin{aligned}
B_{H,t}: \qquad \beta\, \mathbb{E}_t\, V'(B_{H,t}) - \frac{\Lambda_t}{\epsilon_t^b P_t} \\
\text{where} \quad V'(B_{H,t-1}) = \Lambda_t \frac{R_{t-1}}{P_t} \Rightarrow V'(B_{H,t}) = \Lambda_{t+1} \frac{R_t}{P_{t+1}} \\
\epsilon_t^b\, \mathbb{E}_t \left[M_{t,t+1} \frac{R_t}{\Pi_{t+1}} \right] = 1
\end{aligned}
$$

$$
\begin{aligned}
K_t: \qquad \beta\, \mathbb{E}_t\, V'(K_{t-1}) - Q_t = 0 \\
V'(K_{t-1}) = \Lambda_t(l)[R_t^k] + Q_t[(1-\delta)] \Rightarrow V'(K_t) = \Lambda_{t+1}[R_{t+1}^k] + Q_{t+1}[(1-\delta)] \\
\beta\, \mathbb{E}_t \left[\Lambda_{t+1}[R_{t+1}^k] + Q_{t+1}[(1-\delta)] \right] = Q_t
\end{aligned}
$$

Now define Tobin's Q $\mathcal{Q} = \frac{Q_t}{\Lambda_t}$. Then dividing through by Λ_t yields:

$$\mathbb{E}_t\left[M_{t,t+1}[R^k_{t+1} + \mathcal{Q}_{t+1}(1-\delta)]\right] = \mathcal{Q}_t$$

$$I_t: \qquad \beta\,\mathbb{E}_t\,V'(I_t) - \Lambda_t + \epsilon^i_t Q_t\left[1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right]$$

$$V'(I_{t-1}) = \epsilon^i_t Q_t\left[S'\left(\frac{I_t}{I_{t-1}}\right)\left(\frac{I_t}{I_{t-1}}\right)^2\right]$$

$$\Rightarrow V'(I_t) = \epsilon^i_{t+1} Q_{t+1}\left[S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right]$$

$$\beta\,\mathbb{E}_t\left[\epsilon^i_{t+1} Q_{t+1}\left[S'\left(\frac{I_{t+1}(l)}{I_t}\right)\left(\frac{I_{t+1}(l)}{I_t}\right)^2\right]\right] + \epsilon^i_t Q_t\left[1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right] = \Lambda_t$$

$$\mathbb{E}_t\left[M_{t,t+1}\epsilon^i_{t+1}\mathcal{Q}_{t+1}S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right] + \epsilon^i_t\mathcal{Q}_t\left[1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right] = \frac{P_{i,t}}{P_t}$$

The solution for rig capital is analogous to that of physical capital. These FOCs for consumption and investment are aggregate , with sectoral consumption and investment, I assume households consume from both the manufacturing, service sectors goods produced both at home and abroad as well as energy provided from downstream oil firms. Non-oil goods and services are combined with energy demand to produce the final consumption good. The minimisation problem is given as:

$$min_{C_{c,t},O^u_{c,t}} : P_{c,t}C_{c,t} + P_{H,t}C_{H,t} + P_{F,t}C_{F,t} + P^o_{d,t}O^d_{c,t} \qquad S.T:$$

$$C_t = \left[a_c^{\frac{1}{\vartheta}} C_{c,t}^{\frac{\vartheta-1}{\vartheta}} + a_o^{\frac{1}{\vartheta}} O^{d\,\frac{\vartheta-1}{\vartheta}}_{c,t}\right]^{\frac{\vartheta}{\vartheta-1}}$$

$$C_{c,t} = \left[a_H^{\frac{1}{\varpi}} C_{H,t}^{\frac{\varpi-1}{\varpi}} + a_F^{\frac{1}{\varpi}} C_{F,t}^{\frac{\varpi-1}{\varpi}}\right]^{\frac{\varpi}{\varpi-1}}$$

$$\mathcal{L} = P_{c,t}C_{c,t} + P_{H,t}C_{H,t} + P_{F,t}C_{F,t} + P^o_{d,t}O^d_{c,t} + \lambda_1\left(\left[a_c^{\frac{1}{\vartheta}} C_{c,t}^{\frac{\vartheta-1}{\vartheta}} + a_o^{\frac{1}{\vartheta}} O^{d\,\frac{\vartheta-1}{\vartheta}}_{c,t}\right]^{\frac{\vartheta}{\vartheta-1}} - C_t\right)$$

$$+\lambda_2\left(C_{c,t} - \left[a_H^{\frac{1}{\varpi}} C_{H,t}^{\frac{\varpi-1}{\varpi}} + a_F^{\frac{1}{\varpi}} C_{F,t}^{\frac{\varpi-1}{\varpi}}\right]^{\frac{\varpi}{\varpi-1}}\right)$$

FOCs

$$P_{H,t} = \lambda_2 \left[a_H^{\frac{1}{\varpi}} C_{H,t}^{\frac{\varpi-1}{\varpi}} + a_F^{\frac{1}{\varpi}} C_{F,t}^{\frac{\varpi-1}{\varpi}} \right]^{\frac{1}{\varpi-1}} a_H^{\frac{1}{\varpi}} C_{H,t}^{-\frac{1}{\varpi}}$$
$$P_{F,t} = \lambda_2 \left[a_H^{\frac{1}{\varpi}} C_{H,t}^{\frac{\varpi-1}{\varpi}} + a_F^{\frac{1}{\varpi}} C_{F,t}^{\frac{\varpi-1}{\varpi}} \right]^{\frac{1}{\varpi-1}} a_F^{\frac{1}{\varpi}} C_{F,t}^{-\frac{1}{\varpi}}$$
$$P_{o,t}^{*} = \lambda_1 \left[a_c^{\frac{1}{\vartheta}} C_{c,t}^{\frac{\vartheta-1}{\vartheta}} + a_o^{\frac{1}{\vartheta}} O_{c,t}^{d\frac{\vartheta-1}{\vartheta}} \right]^{\frac{1}{\vartheta-1}} a_o^{\frac{1}{\vartheta}} O_{c,t}^{u-\frac{1}{\vartheta}}$$
$$\lambda_2 = \lambda_1 \left[a_c^{\frac{1}{\vartheta}} C_{c,t}^{\frac{\vartheta-1}{\vartheta}} + a_o^{\frac{1}{\vartheta}} O_{c,t}^{u\frac{\vartheta-1}{\vartheta}} \right]^{\frac{1}{\vartheta-1}} a_c^{\frac{1}{\vartheta}} C_{c,t}^{-\frac{1}{\vartheta}}$$

Taking ratio of the first two FOCs and obtain the ratio of demand for consumption of goods from non-oil sector:

$$\frac{P_{H,t}}{P_{F,t}} = \left(\frac{a_H}{a_F}\right)^{\frac{1}{\varpi}} \left(\frac{C_{H,t}}{C_{F,t}}\right)^{-\frac{1}{\varpi}}$$
$$\frac{C_{H,t}}{C_{F,t}} = \frac{a_H}{a_F} \left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\varpi}$$

This is the Dixit-Stiglitz relative demand for consumption of goods from nonoil sector. Now from the first two FOCs:

$$C_{H,t}^{-\frac{1}{\varpi}} = a_H^{-\frac{1}{\varpi}} P_{H,t} \lambda_2^{-1} \left[a_H^{\frac{1}{\varpi}} C_{H,t}^{\frac{\varpi-1}{\varpi}} + a_F^{\frac{1}{\varpi}} C_{F,t}^{\frac{\varpi-1}{\varpi}} \right]^{-\frac{1}{\varpi-1}}$$
$$C_{F,t}^{-\frac{1}{\varpi}} = a_F^{-\frac{1}{\varpi}} P_{F,t} \lambda_2^{-1} \left[a_H^{\frac{1}{\varpi}} C_{H,t}^{\frac{\varpi-1}{\varpi}} + a_F^{\frac{1}{\varpi}} C_{F,t}^{\frac{\varpi-1}{\varpi}} \right]^{-\frac{1}{\varpi-1}}$$
$$C_{H,t} = a_H \left(\frac{P_{H,t}}{\lambda_2}\right)^{-\varpi} C_{c,t} \qquad C_{F,t} = a_F \left(\frac{P_{F,t}}{\lambda_2}\right)^{-\varpi} C_{c,t}$$

Plugging these demands into the Dixit-Stiglitz aggregator budget constraint and obtain:

$$C_{c,t} = \left[a_H^{\frac{1}{\varpi}} \left(a_H \left(\frac{P_{H,t}}{\lambda_2}\right)^{-\varpi} C_{c,t} \right)^{\frac{\varpi-1}{\varpi}} + a_F^{\frac{1}{\varpi}} \left(a_F \left(\frac{P_{F,t}}{\lambda_2}\right)^{-\varpi} C_{c,t} \right)^{\frac{\varpi-1}{\varpi}} \right]^{\frac{\varpi}{\varpi-1}}$$
$$C_{c,t} = C_{c,t} \left[a_H \left(\frac{P_{H,t}}{\lambda_2}\right)^{1-\varpi} + a_F \left(\frac{P_{F,t}}{\lambda_2}\right)^{1-\varpi} \right]^{\frac{\varpi}{\varpi-1}}$$
$$\lambda_2 = \left[a_H P_{H,t}^{1-\varpi} + a_F P_{F,t}^{1-\varpi} \right]^{\frac{1}{1-\varpi}} \equiv P_{c,t}$$
$$C_{H,t} = a_H \left(\frac{P_{H,t}}{P_{c,t}}\right)^{-\varpi} C_{c,t} \qquad C_{F,t} = a_F \left(\frac{P_{F,t}}{P_{c,t}}\right)^{-\varpi} C_{c,t}$$

Analogously, I obtain the demand and aggregate price indexes for the other consumption baskets:

$$P_t = \left[a_o P_{o,t}^{*1-\vartheta} + a_c P_{c,t}^{1-\vartheta}\right]^{\frac{1}{1-\vartheta}}$$
$$O_{c,t}^d = a_o \left(\frac{P_{o,t}^*}{P_t}\right)^{-\vartheta} C_t \qquad C_{c,t} = a_c \left(\frac{P_{c,t}}{P_t}\right)^{-\vartheta} C_t$$

Furthermore, with the aggregate price index depending on the price of energy which ultimately depends on the international oil prices, volatility of exchange rate in oil dependent countries is driven by international oil prices. Therefore an increase in $P_{o,t}^*$ (international oil prices) will lead to an appreciation of the home currency and vice versa. Analogously, for sectoral investment decisions, I obtain:

$$I_t = \left[\xi^{\frac{1}{\varpi_i}} I_{H,t}^{\frac{\varpi_i - 1}{\varpi_i}} + (1-\xi)^{\frac{1}{\varpi_i}} I_{F,t}^{\frac{\varpi_i - 1}{\varpi_i}}\right]^{\frac{\varpi_i}{\varpi_i - 1}}$$
$$\frac{I_{H,t}}{I_{F,t}} = \frac{\xi}{1-\xi}\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\varpi_i}$$

Intermediate Labour Unions, Labour Packers and Dynamic Wage Setting:

Households have monopolistic power over their homogenous labour types which they supply. With different sectors, there is free mobility across sectors within the business cycle[4]. Now denote $\eta_j \in (0,1)$ as measure of household working in sector j, then $\sum_{j=P,O} \eta_j = 1$. Specifically, Households homogenous labour types are packed into homogenous final labour by a competitive labour producer with production function:

$$N_{j,t}^d = \left(\left(\frac{1}{\eta_j}\right)^{\frac{1}{\theta_w}} \int_0^1 N_{j,t}(l)^{\frac{\theta_w - 1}{\theta_w}} dl\right)^{\frac{\theta_w}{\theta_w - 1}}, \quad \theta_w > 1$$

Here $N_{j,t}(l)$ is the sectoral supply of type l homogenous labour and $N_{j,t}^d$ is the final sectoral demand of labour by intermediate goods firm (such that in the competitive equilibrium, it is equal to $\int_0^1 N_{j,t}(j)dj$, where $N_{j,t}(j)$ is the intermediate goods firm labour demand). The parameter θ_w captures the elasticity of substitution between different forms of homogenous labour, such that as $\theta_w \to 1$, a monopoly and as $\theta_w \to \infty$ perfectly competitive outcome. The final labour packers thus solve the following problem taking nominal sectoral wages $W_{j,t}(l)$ and the nominal

[4]The free mobility across sectors contrasts that of Bergholat and Larsen (2016)

aggregate wage index $W_{j,t}$ as given:

$$Max_{N_{j,t}(l)} : W_{j,t}N^d_{j,t} - \int_0^1 W_{j,t}(l)N_{j,t}(l)dl$$

$$s.t. \quad N^d_{j,t} = \left(\left(\frac{1}{\eta_j}\right)^{\frac{1}{\theta_w}} \int_0^1 N_{j,t}(l)^{\frac{\theta_w-1}{\theta_w}} dl \right)^{\frac{\theta_w}{\theta_w-1}}$$

$$\Rightarrow W_{j,t} \left(\left(\frac{1}{\eta_j}\right)^{\frac{1}{\theta_w}} \int_0^1 N_{j,t}(l)^{\frac{\theta_w-1}{\theta_w}} dl \right)^{\frac{\theta_w}{\theta_w-1}} - \int_0^1 W_{j,t}(l)N_{j,t}(l)dl$$

Then the FOC for this problem is:

$$W_{j,t}\frac{\theta_w - 1}{\theta_w} \left(\left(\frac{1}{\eta_j}\right)^{\frac{1}{\theta_w}} \int_0^1 N_{j,t}(l)^{\frac{\theta_w-1}{\theta_w}} dl \right)^{\frac{1}{\theta_w-1}} \frac{\theta_w}{\theta_w - 1} \left(\frac{1}{\eta_j}\right)^{\frac{1}{\theta_w}} N_{j,t}(l)^{\frac{-1}{\theta_w}} - W_{j,t}(l) = 0$$

$$W_{j,t} \left(\left(\frac{1}{\eta_j}\right)^{\frac{1}{\theta_w}} \int_0^1 N_{j,t}(l)^{\frac{\theta_w-1}{\theta_w}} dl \right)^{\frac{1}{\theta_w-1}} \left(\frac{1}{\eta_j}\right)^{\frac{1}{\theta_w}} N_{j,t}(l)^{\frac{-1}{\theta_w}} = W_{j,t}(l)$$

This FOC is true for all $l \in [0, 1]$, therefore it holds for any household l', then dividing through by FOC for household l':

$$\frac{W_{j,t}(l)}{W_{j,t}(l')} = \left(\frac{N_{j,t}(l)}{N_{j,t}(l')}\right)^{\frac{-1}{\theta_w}} \Rightarrow W_{j,t}(l') = W_{j},t(l) \left(\frac{N_{j,t}(l)}{N_{j,t}(l')}\right)^{\frac{1}{\theta_w}}$$

$$W_{j,t}(l')N_{j,t}(l') = W_{j,t}(l)N_{j,t}(l)^{\frac{1}{\theta_w}} N_{j,t}(l')^{\frac{\theta_w-1}{\theta_w}}$$

Integrating over all $l \in [0, 1]$:

$$\int_0^1 W_{j,t}(l')N_{jt}(l')dl' = W_{j,t}(l)N_{j,t}(l)^{\frac{1}{\theta_w}} \int_0^1 N_{j,t}(l')^{\frac{\theta_w-1}{\theta_w}} dl'$$

$$W_{j,t}N^d_{j,t} = W_{j,t}(l)N_{j,t}(l)^{\frac{1}{\theta_w}} (N^d_{j,t})^{\frac{\theta_w}{\theta_w-1}} \left(\frac{1}{\eta_j}\right)^{-\frac{1}{\theta_w}}$$

$$N_{j,t}(l) = \left(\frac{W_{j,t}(l)}{W_{j,t}}\right)^{-\theta_w} \frac{N^d_{j,t}}{\eta_j} = \left(\frac{W_{j,t}(l)}{W_{j,t}}\right)^{-\theta_w} \tilde{N}^d_{j,t}$$

$\tilde{N}^d_{j,t} \equiv \frac{N^d_{j,t}}{\eta_j}$ is defined as average effective labor hours per worker in the sector. Then the aggregate labour supply is:

$$N_{j,t} = \int_0^1 N_{j,t}(l)dl = \int_0^1 \left(\frac{W_{j,t}(l)}{W_{j,t}}\right)^{-\theta_w} \tilde{N}^d_{j,t} = \Delta_{wj,t}\tilde{N}^d_{j,t}$$

Here $\Delta_{wj,t}$ is a measure of sectoral wage dispersion. Next, the labour unions are an intermediary between households and final labour packers. Under staggered wage settings, households sets wages a la Calvo. That is periodically, households are allowed to reset its nominal wage optimally with probability $1 - \psi_{wj} \in (0, 1)$, allowing previous period wages $W_{j,t-1}(l)$ grow at a rate partially indexed to previous periods wage inflation $\Pi_{wj,t-1} \equiv \frac{W_{j,t-1}}{W_{j,t-2}}$ with probability ψ_{wj}. For ease, I introduce the variable:

$$J^w_{jt,t+i} = \begin{cases} \Pi^{i-1}_{j,k=0}\pi^{\iota_w}_{jw,t+k} & if \quad i > 0 \\ 1 & if \quad i = 0 \end{cases}$$

Where $\iota_w \in [0, 1]$ declares the degree of indexation to past wage inflation, such that there is full indexation if $\iota_w = 1$ and no indexation if $\iota_w = 0$. Now a household that is allowed to optimally reset its wage at date t and never again has wage at date $t + i$:

$$\tilde{W}_{j,t+i}(l) = W_{j,t}(l)J^w_{jt,t+i}$$
$$\frac{\tilde{W}_{j,t+i}(l)}{W_{t+i}} = \frac{W_{j,t}(l)}{W_{j,t+i}}J^w_{jt,t+i}$$

Then it must be that a household allowed to optimally reset its wage at date t and never again has wage at date $t + i$ has from the aggregate demand function for labour supply given by:

$$\tilde{N}_{j,t+i}(l) = \left(\frac{\tilde{W}_{j,t+i}(l)}{W_{j,t+i}}\right)^{-\theta_w} \tilde{N}^d_{j,t+i} \Rightarrow \tilde{N}_{j,t+i}(l) = \left(\frac{W_{j,t}(l)J^w_{jt,t+i}}{W_{j,t+i}}\right)^{-\theta_w} \frac{N_{j,t+i}}{\Delta_{w,t+i}}$$

Similarly, the corruption penalty for being caught in forgone wages for a household that is allowed to optimally reset its wage at date t and never again has the following penalty function:

$$\frac{\tilde{W}_{t+i}(l)}{P_{t+i}}\tilde{N}_{j,t+i}(l) = \frac{\tilde{W}_{j,t}(l)J^w_{jt,t+i}}{P_{t+i}} \left(\frac{W_{j,t}(l)J^w_{jt,t+i}}{W_{j,t+i}}\right)^{-\theta_w} \frac{N_{j,t+i}}{\Delta_{wj,t+i}}$$

So the relevant aspects of the lagrangian are:

$$\mathbb{E}_t \sum_{i=0}^{\infty} \psi_w^i \beta^i \left[\Lambda_{t+i} \left(\tilde{N}_{j,t+i}(l) \frac{W_{j,t}(l)}{P_{t+i}} J^w_{t,t+i} \right) - \frac{\tilde{N}_{j,t+i}(l)^{1+\sigma_n}}{1+\sigma_n} \right]$$

$$\mathbb{E}_t \sum_{i=0}^{\infty} \psi_w^i \beta^i \Bigg[\Lambda_{t+i} \Bigg(\frac{W_{j,t}(l) J^w_{t,t+i}}{P_{t+i}} \left(\frac{W_{j,t}(l) J^w_{t,t+i}}{W_{j,t+i}} \right)^{-\theta_w} \frac{N_{j,t+i}}{\Delta_{w,t+i}} -$$

$$\frac{\tilde{W}_{j,t}(l) J^w_{t,t+i}}{P_{t+i}} \left(\frac{W_{j,t}(l) J^w_{t,t+i}}{W_{j,t+i}} \right)^{-\theta_w} \frac{N_{j,t+i}}{\Delta_{w,t+i}} \Bigg) - \frac{1}{1+\sigma_n} \left(\left(\frac{W_{j,t}(l)}{W_{j,t+i}} J^w_{t,t+i} \right)^{-\theta_w} \frac{N_{j,t+i}}{\Delta_{w,t+i}} \right)^{1+\sigma_n} \Bigg]$$

Now define the equilibrium reset wage $W^*_{j,t}$, which is symmetric for all households, then the FOC w.r.t. $W_{j,t}(l)$:

$$\mathbb{E}_t \sum_{i=0}^{\infty} \psi_w^i \beta^i \Bigg[(1-\theta_w) W_{j,t}(l)^{-\theta_w} \frac{\Lambda_{t+i}}{P_{t+i}} \frac{J^{w(1-\theta_w}_{t,t+i}}{W^{-\theta_w}_{j,t+i}} \frac{N_{j,t+i}}{\Delta_{w,t+i}} - (1-\theta_w) W_{j,t}(l)^{-\theta_w} \frac{\Lambda_{t+i}}{P_{t+i}} \frac{J^{w(1-\theta_w}_{t,t+i}}{W^{-\theta_w}_{j,t+i}} \frac{N_{j,t+i}}{\Delta_{w,t+i}} \Bigg]$$

$$+\theta_w W_{j,t}(l)^{-\theta_w - 1} \left(\left(\frac{W_{j,t}(l) J^w_{t,t+i}}{W_{j,t+i}} \right)^{-\theta_w} \frac{N_{j,t+i}}{\Delta_{w,t+i}} \right)^{\sigma_n} \left(\frac{J^w_{t,t+i}}{W_{j,t+i}} \right)^{-\theta_w} \frac{N_{j,t+i}}{\Delta_{w,t+i}} = 0$$

$$\mathbb{E}_t \sum_{i=0}^{\infty} \psi_w^i \beta^i \Bigg[(1-\theta_w) W^{*-\theta_w}_{j,t} \frac{\Lambda_{t+i}}{P_{t+i}} \frac{J^{w(1-\theta_w}_{t,t+i}}{W^{-\theta_w}_{j,t+i}} \frac{N_{j,t+i}}{\Delta_{w,t+i}} \Bigg]$$

$$+\theta_w W^{*-\theta_w - 1}_{j,t} \left(\left(\frac{W_{j,t}(l) J^w_{t,t+i}}{W_{j,t+i}} \right)^{-\theta_w} \frac{N_{j,t+i}}{\Delta_{w,t+i}} \right)^{\sigma_n} \left(\frac{J^w_{t,t+i}}{W_{j,t+i}} \right)^{-\theta_w} \frac{N_{j,t+i}}{\Delta_{w,t+i}} = 0$$

$$\mathbb{E}_t \sum_{i=0}^{\infty} \psi_w^i \beta^i \Bigg[(1-\theta_w) W^{*-\theta_w}_{j,t} \frac{\Lambda_{t+i}}{P_{t+i}} \frac{J^{w(1-\theta_w}_{t,t+i}}{W^{-\theta_w}_{j,t+i}} \frac{N_{j,t+i}}{\Delta_{w,t+i}} \Bigg]$$

$$+\theta_w W^{*-\theta_w - 1}_{j,t} \left(\left(\frac{W_{j,t}(l) J^w_{t,t+i}}{W_{j,t+i}} \right)^{-\theta_w} \frac{N_{j,t+i}}{\Delta_{w,t+i}} \right)^{\sigma_n} \left(\frac{J^w_{t,t+i}}{W_{j,t+i}} \right)^{-\theta_w} \frac{N_{j,t+i}}{\Delta_{w,t+i}} = 0$$

$$\mathbb{E}_t \sum_{i=0}^{\infty} \psi_w^i \beta^i \Lambda_{t+i} \Bigg[(1-\theta_w) W^{*-\theta_w}_{j,t} \frac{1}{P_{t+i}} \frac{J^{w(1-\theta_w}_{t,t+i}}{W^{-\theta_w}_{j,t+i}} \frac{N_{j,t+i}}{\Delta_{w,t+i}}$$

$$+\theta_w W^{*-\theta_w - 1}_{j,t} \left(\left(\frac{W_{j,t}(l) J^w_{t,t+i}}{W_{j,t+i}} \right)^{-\theta_w} \frac{N_{j,t+i}}{\Delta_{w,t+i}} \right)^{\sigma_n} \left(\frac{J^w_{t,t+i}}{W_{j,t+i}} \right)^{-\theta_w} \frac{N_{j,t+i}}{\Delta_{w,t+i} \Lambda_{t+i}} \Bigg] = 0$$

$$\mathbb{E}_t \sum_{i=0}^{\infty} \psi_w^i \beta^i \Lambda_{t+i} \Bigg[W_{j,t}^* \frac{1}{P_{t+i}} \frac{J_{t,t+i}^{w(1-\theta_w}}{W_{j,t+i}^{-\theta_w}} \frac{N_{j,t+i}}{\Delta_{w,t+i}}$$

$$-\frac{\theta_w}{\theta_w - 1} \left(\left(\frac{W_{j,t}(l) J_{t,t+i}^w}{W_{j,t+i}} \right)^{-\theta_w} \frac{N_{j,t+i}}{\Delta_{w,t+i}} \right)^{\sigma_n} \left(\frac{J_{t,t+i}^w}{W_{j,t+i}} \right)^{-\theta_w} \frac{N_{j,t+i}}{\Delta_{w,t+i} \Lambda_{t+i}} \Bigg] = 0$$

$$\mathbb{E}_t \sum_{i=0}^{\infty} \psi_w^i \beta^i \frac{\Lambda_{t+i}}{\Lambda_t} \Bigg[W_{j,t}^* \frac{1}{P_{t+i}} \frac{J_{t,t+i}^{w(1-\theta_w}}{W_{j,t+i}^{-\theta_w}} \frac{N_{j,t+i}}{\Delta_{w,t+i}}$$

$$-\frac{\theta_w}{\theta_w - 1} \left(\left(\frac{W_{j,t}(l) J_{t,t+i}^w}{W_{j,t+i}} \right)^{-\theta_w} \frac{N_{j,t+i}}{\Delta_{w,t+i}} \right)^{\sigma_n} \left(\frac{J_{t,t+i}^w}{W_{j,t+i}} \right)^{-\theta_w} \frac{N_{j,t+i}}{\Delta_{w,t+i} \Lambda_{t+i}} \Bigg] = 0$$

$$\mathbb{E}_t \sum_{i=0}^{\infty} \psi_w^i M_{t,t+i} \Bigg[W_{j,t}^* \frac{J_{t,t+i}^w}{P_{t+i}} - \frac{\theta_w}{\theta_w - 1} \left(\frac{W_{j,t}(l) J_{t,t+i}^w}{W_{j,t+i}} \right)^{-\theta_w \sigma_n} \frac{N_{j,t+i}^{\sigma_n}}{\Delta_{w,t+i}^{\sigma_n} \Lambda_{t+i}} \Bigg]$$

$$\left(\frac{W_{j,t+i}}{W_{j,t}} \right)^{\theta_w} \frac{N_{j,t+i} J_{t,t+i}^{w-\theta_w}}{\Delta_{w,t+i}} = 0$$

One can break this down into two iterative equations:

$$\zeta_t \equiv \mathbb{E}_t \sum_{i=0}^{\infty} \psi_w^i M_{t,t+i} \frac{W_{j,t}^* J_{jt,t+i}^{w,(1-\theta_w)}}{P_{t+i}} \left(\frac{W_{j,t}}{W_{j,t+i}} \right)^{-\theta_w} \frac{N_{j,t+i}}{\Delta_{wj,t+i}}$$

$$\Upsilon_t \equiv \frac{\theta_w}{\theta_w - 1} \mathbb{E}_t \sum_{i=0}^{\infty} \psi_w^i M_{t,t+i} \left[\left(\frac{W_{j,t}^*}{W_{j,t+i}} \right)^{-\theta_w \sigma_n} \left(\frac{W_{j,t}}{W_{j,t+i}} \right)^{-\theta_w} \frac{N_{j,t+i}^{1+\sigma_n}}{\Delta_{wj,t+i}^{1+\sigma_n} \Lambda_{t+i}} J_{jt,t+i}^{w,-\theta_w(1+\sigma_n)} \right]$$

Now defining $G_{j,t} \equiv \frac{W_{j,t}^*}{W_{j,t}}$ the sectoral relative wage for optimising households, can then define the recursive form of both iterative equations above:

$$\zeta_{j,t} = G_{j,t} \left[\frac{\Omega_{j,t} N_{j,t}}{\Delta_{w,t}} + \psi_{wj} \, \mathbb{E}_t \left[\frac{M_{t,t+1} \zeta_{t+1}}{G_{j,t+1}} \left(\frac{\Pi_{wj,t+1}}{\Pi_{wj,t}^{\iota_w}} \right)^{\theta_w - 1} \right] \right]$$

$$\Upsilon_{j,t} = G_{j,t}^{-\sigma_n \theta_w} \left[\frac{\theta_w}{\theta_w - 1} \frac{N_{j,t}^{1+\sigma_n}}{\Delta_{wj,t}^{1+\sigma_n} \Lambda_t} + \psi_{wj} \, \mathbb{E}_t \left[\frac{M_{t,t+1} \Upsilon_{t+1}}{G_{j,t+1}^{-\sigma_n \theta_w}} \left(\frac{\Pi_{wj,t+1}}{\Pi_{wj,t}^{\iota_w}} \right)^{(1+\sigma_n)\theta_w} \right] \right]$$

Finally, one can derive the law of motion for sectoral wages. From the wage dynamic equation, define once again $W_{j,t}^*$ as the optimal sectoral wage which is common across households that are allowed to reset their wage, then the wage dynamics for W_t can be expressed as:

$$W_{j,t}^{1-\theta_w} = (1-\psi_{wj})W_{j,t}^{*1-\theta_w} + \int\limits_{\mathcal{I}_{w,t}} W_{j,t}^{1-\theta_w}(l)dl$$

Where $\mathcal{I}_{w,t}$ are households who failed to re-optimise their wages at time t, this has measure ψ_{wj}. Of these households who failed to re-optimise wage, a share $(1-\psi_{wj})$ of these households would adjust their wage at date $t-1$ but not at date t, thus this share is $(1-\psi_{wj})\psi_{wj}$ and by the indexation rule specified, their wage today is $\Pi_{w,t-1}^{\iota_w} W_{j,t-1}^{*}$. Therefore, it follows that:

$$W_{j,t}^{1-\theta_w} = (1-\psi_{wj})W_{j,t}^{*1-\theta_w} + (1-\psi_{wj})\psi_{wj}\Pi_{w,t-1}^{(1-\theta_w)\iota_w} W_{j,t-1}^{*} + \int\limits_{\mathcal{I}_{w,t}\cap Ia_{w,t-1}} W_{j,t-1}^{1-\theta_w}(l)dl$$

A measure $1-(1-\psi_{wj})-(1-\psi_{wj})\psi_{wj}$ of households would have failed to adjust their wages at date $t-1$, of these households, a share $(1-\psi_{wj})$ would adjust their wages at date $t-2$ but would have failed to adjust their wages at date $t-1$ and date t. Applying the indexation rule, their wage is therefore $(\Pi_{wj,t-1}\Pi_{wj,t-2})^{\iota_w} W_{j,t-2}^{*}$. Therefore, it follows that:

$$\begin{aligned}
W_{j,t}^{1-\theta_w} &= (1-\psi_{wj})W_{j,t}^{*1-\theta_w} + (1-\psi-wj)\psi_{wj}\Pi_{w,t-1}^{(1-\theta_w)\iota_w} W_{j,t-1}^{*} \\
&+\psi_{wj}^2(1-\psi_{wj})[(\Pi_{w,t-1}\Pi_{w,t-2})^{\iota_w} W_{j,t-2}^{*}]^{(1-\theta_w)} + \int\limits_{\mathcal{I}_{w,t}\cap Ia_{w,t-1}\cap Ia_{w,t-2}} W_{j,t-2}^{1-\theta_w}(l)dl \\
W_{j,t}^{1-\theta_w} &= (1-\psi_{wj})W_{j,t}^{*1-\theta_w} + \psi_{wj}\Pi_{w,t-1}^{(1-\theta_w)\iota_w}\Big[(1-\psi_{wj})W_{j,t-1}^{*1-\theta_w} \\
&+\psi_{wj}(1-\psi_{wj})[\Pi_{w,t-2}^{\iota_w} W_{j,t-2}^{*}]^{(1-\theta_w)}\Big] + \int\limits_{\mathcal{I}_{w,t}\cap Ia_{w,t-1}\cap Ia_{w,t-2}} W_{j,t-2}^{1-\theta_w}(l)dl
\end{aligned}$$

Iterating the expression forward and noting that as time approaches infinity, every household will eventually re-optimise their wage, so it follows that $\lim_{n\to\infty} Pr(l \in \cap_{i=0}^{h}\mathcal{I}_{w,t-i}) = 0$ $\forall h \in [0,1]$, then one obtains:

$$\begin{aligned}
W_{j,t}^{1-\theta_w} &= (1-\psi_{wj})W_{j,t}^{*1-\theta_w} + \psi_{wj}\Pi_{w,t-1}^{(1-\theta_w)\iota_w}[(1-\psi_{wj})W_{j,t-1}^{*1-\theta_w} \\
&+\psi_{wj}(1-\psi_{wj})[\Pi_{w,t-2}^{\iota_w} W_{j,t-2}^{*}]^{(1-\theta_w)} + \psi_{wj}^2(1-\psi_{wj})[(\Pi_{w,t-2}\Pi_{w,t-3})^{\iota_w} W_{j,t-3}^{*}]^{(1-\theta_w)} + \dots] \\
W_{j,t}^{1-\theta_w} &= (1-\psi_{wj})W_{j,t}^{*1-\theta_w} + \psi_{wj}\Pi_{w,t-1}^{(1-\theta_w)\iota_w} W_{j,t-1}^{1-\theta_w} \qquad (5.0.1)
\end{aligned}$$

Similarly, one can obtain the law of motion for the dispersion of wages. From the equation for wage dispersion obtained above:

$$\Delta_{wj,t} = \int_0^1 \left(\frac{W_{j,t}(l)}{W_{j,t}}\right)^{-\theta_w} dl = W_{j,t}^{\theta_w} \int_0^1 W_{j,t}(l)^{-\theta_w} dl$$

$$W_{j,t}^{-\theta_w} \Delta_{wj,t} = \int_0^1 W_{j,t}(l)^{-\theta_w} dl$$

There is now an identical expression to the one for wage dynamics, then applying the same technique to obtain the law of motion for wage dispersion:

$$W_{j,t}^{-\theta_w} \Delta_{w,t} = (1-\psi_{wj}) W_{j,t}^{*1-\theta_w} + \psi_{wj} \left(\Pi_{w,t-1}^{\iota_w} W_{j,t-1}\right)^{-\theta_w} \Delta_{wj,t-1}$$

$$\Delta_{wj,t} = (1-\psi_{wj}) G_{j,t}^{-\theta_w} + \psi_{wj} \left(\frac{\Pi_{w,t-1}^{\iota_w}}{\Pi_{w,t}}\right)^{-\theta_w} \Delta_{w,t-1}$$

Non-Oil Sector - Final Good Firms

Final goods are bundled with a Dixit-Stiglitz aggregator subject to a nested structure. So:

$$Y_t = \left[a_H^{\frac{1}{\varpi}} Y_{H,t}^{d\,\frac{\varpi-1}{\varpi}} + a_F^{\frac{1}{\varpi}} Y_{F,t}^{d\,\frac{\varpi-1}{\varpi}}\right]^{\frac{\varpi}{\varpi-1}}$$

Here a_H is the degree of home bias and a_F is the degree of foreign bias. If one assumes $a_H = a_F = \frac{1}{2}$ then there will be efficient risk sharing under complete markets[5]. However, I will not assume this as developing countries tend to have a bias towards foreign goods and services, therefore I assign bigger weights to a_F when calibrating the model. Simple cost minimisation problem delivers the demand for home and foreign output (I will also only solve this once and apply the results to further problems):

$$min_{Y_{H,t}^d, Y_{F,t}^d} : P_{H,t} Y_{H,t}^d + P_{F,t} Y_{F,t}^d \quad s.t \quad Y_t = \left[a_H^{\frac{1}{\varpi}} Y_{H,t}^{d\,\frac{\varpi-1}{\varpi}} + a_F^{\frac{1}{\varpi}} Y_{F,t}^{d\,\frac{\varpi-1}{\varpi}}\right]^{\frac{\varpi}{\varpi-1}}$$

$$\mathcal{L} = P_{H,t} Y_{H,t}^d + P_{F,t} Y_{F,t}^d + \lambda_t \left(Y_t - \left[a_H^{\frac{1}{\varpi}} Y_{H,t}^{d\,\frac{\varpi-1}{\varpi}} + a_F^{\frac{1}{\varpi}} Y_{F,t}^{d\,\frac{\varpi-1}{\varpi}}\right]^{\frac{\varpi}{\varpi-1}}\right)$$

[5] Without complete markets i.e. Financial Autarky, this breaks down and there is no longer efficient risk sharing

λ_t is the lagrange multiplier and the shadow price. FOCs are:

$$P_{H,t} = \lambda_t a_H^{\frac{1}{\varpi}} Y_{H,t}^{d-\frac{1}{\varpi}} \left[a_H^{\frac{1}{\varpi}} Y_{H,t}^{d\frac{\varpi-1}{\varpi}} + a_F^{\frac{1}{\varpi}} Y_{F,t}^{d\frac{\varpi-1}{\varpi}} \right]^{\frac{1}{\varpi-1}}$$

$$P_{F,t} = \lambda_t a_F^{\frac{1}{\varpi}} Y_{F,t}^{d-\frac{1}{\varpi}} \left[a_H^{\frac{1}{\varpi}} Y_{H,t}^{d\frac{\varpi-1}{\varpi}} + a_F^{\frac{1}{\varpi}} Y_{F,t}^{d\frac{\varpi-1}{\varpi}} \right]^{\frac{1}{\varpi-1}}$$

$$\Rightarrow Y_{H,t}^{d-\frac{1}{\varpi}} = P_{H,t} \lambda_t^{-1} a_H^{-\frac{1}{\varpi}} \left[a_H^{\frac{1}{\varpi}} Y_{H,t}^{d\frac{\varpi-1}{\varpi}} + a_F^{\frac{1}{\varpi}} Y_{F,t}^{d\frac{\varpi-1}{\varpi}} \right]^{-\frac{1}{\varpi-1}}$$

$$Y_{H,t}^{d} = P_{H,t}^{-\varpi} \lambda_t^{\varpi} a_H \left[a_H^{\frac{1}{\varpi}} Y_{H,t}^{d\frac{\varpi-1}{\varpi}} + a_F^{\frac{1}{\varpi}} Y_{F,t}^{d\frac{\varpi-1}{\varpi}} \right]^{\frac{\varpi}{\varpi-1}}$$

$$Y_{H,t}^{d} = \left(\frac{P_{H,t}}{\lambda_t} \right)^{-\varpi} a_H Y_t \qquad Y_{F,t}^{d} = \left(\frac{P_{F,t}}{\lambda_t} \right)^{-\varpi} a_F Y_t$$

From FOCs one can derive the relative demand for home and foreign goods (take ratio of both FOCs):

$$\frac{Y_{H,t}^{d}}{Y_{F,t}^{d}} = \frac{a_H}{a_F} \left(\frac{P_{H,t}}{P_{F,t}} \right)^{-\varpi}$$

Now plugging the demands into the Dixit-Stiglitz aggregator, one obtains:

$$Y_t = \left[a_H^{\frac{1}{\varpi}} \left(\left(\frac{P_{H,t}}{\lambda_t} \right)^{-\varpi} a_H Y_t^d \right)^{\frac{\varpi-1}{\varpi}} + a_F^{\frac{1}{\varpi}} \left(\left(\frac{P_{F,t}}{\lambda_t} \right)^{-\varpi} a_F Y_t^d \right)^{\frac{\varpi-1}{\varpi}} \right]^{\frac{\varpi}{\varpi-1}}$$

$$Y_t = \left[a_H \left(\frac{P_{H,t}}{\lambda_t} \right)^{1-\varpi} + a_F \left(\frac{P_{F,t}}{\lambda_t} \right)^{1-\varpi} \right]^{\frac{\varpi}{\varpi-1}} Y_t^d$$

$$1 = a_H \left(\frac{P_{H,t}}{\lambda_t} \right)^{1-\varpi} + a_F \left(\frac{P_{F,t}}{\lambda_t} \right)^{1-\varpi}$$

$$\lambda_t^{1-\varpi} = a_H P_{H,t}^{1-\varpi} + a_F P_{F,t}^{1-\varpi} \equiv P_{j,t}^{1-\varpi}$$

$$P_{c,t} = \left(a_H P_{H,t}^{1-\varpi} + a_F P_{F,t}^{1-\varpi} \right)^{\frac{1}{1-\varpi}}$$

Here $P_{c,t}$ is the aggregate price index[6]. Plugging this into the derived demand, one gets the cost minimising demand for home and foreign goods yields:

$$Y_{H,t}^{d} = \left(\frac{P_{H,t}}{P_{j,t}} \right)^{-\varpi} a_H Y_t \qquad Y_{F,t}^{d} = \left(\frac{P_{F,t}}{P_{j,t}} \right)^{-\varpi} a_F Y_t$$

[6] Identity holds as λ_t is the shadow price.

As in the closed economy case, following Kimball(1995) the final home and foreign sectoral goods are a composite made of a continuum of intermediate goods $Y_{H,t}(f)$ & $Y_{F,t}(f)$:

$$Y_{H,t}^{d}=\left(\int_{0}^{1}Y_{H,t}(f)^{\frac{\theta_p-1}{\theta_p}}df\right)^{\frac{\theta_p}{\theta_p-1}}\qquad Y_{F,t}^{d}=\left(\int_{0}^{1}Y_{F,t}(f)^{\frac{\theta_p-1}{\theta_p}}df\right)^{\frac{\theta_p}{\theta_p-1}}\quad,\quad\theta_p,\theta_p>1$$

As with household labour supply, θ_p is the degree of substitutability between intermediate goods, such that as $\theta_p\to 1$ a monopoly and as $\theta_p\to\infty$ a perfectly competitive outcome. The problem and solutions are analogous to the household labour supply problem and obtain:

$$Y_{H,t}(f)=\left(\frac{P_{H,t}(f)}{P_{H,t}}\right)^{-\theta_p}Y_{H,t}^{d}\qquad Y_{H,t}(f)=\left(\frac{P_{F,t}(f)}{P_{F,t}}\right)^{-\theta_p^*}Y_{F,t}^{d}$$

$$P_{H,t}=\left(\int_{0}^{1}P_{H,t}(f)^{1-\theta_p}dj\right)^{\frac{1}{1-\theta_p}}\qquad P_{F,t}=\left(\int_{0}^{1}P_{F,t}(f)^{1-\theta_p^*}dj\right)^{\frac{1}{1-\theta_p^*}}$$

Supply of intermediate goods therefore follows:

$$Y_{H,t}=\int_{0}^{1}Y_{H,t}(f)df=\int_{0}^{1}\left(\frac{P_{H,t}(f)}{P_{H,t}}\right)^{-\theta_p}Y_{H,t}^{d}df=\Delta_{Hp,t}Y_{H,t}^{d}$$

$$Y_{F,t}=\int_{0}^{1}Y_{F,t}(f)df=\int_{0}^{1}\left(\frac{P_{F,t}(f)}{P_{F,t}}\right)^{-\theta_p^*}Y_{F,t}^{d}df=\Delta_{Fp,t}Y_{F,t}^{d}$$

Where $\Delta_{Hp,t}$ and $\Delta_{Fp,t}$ are measures of the cross-sectional price dispersion within the intermediate good sector. Furthermore, total production of output by the home firm must satisfy:

$$Y_t(f)=Y_{H,t}(f)+Y_{H,t}^{*}(f)$$

That is the domestic firms goods sold at home and those exported. Therefore, the aggregate supply of home goods both those sold at home and abroad is:

$$Y_t^{d}=\int_{0}^{1}Y_t(f)df=\int_{0}^{1}Y_{H,t}(f)+Y_{H,t}^{*}(f)df=\Delta_{Hp,t}Y_{H,t}^{d}+\Delta_{Hp,t}^{*}Y_{H,t}^{*d}$$

Intermediate Good Firms:

I have that output in domestic firm f is given by the same cobb-douglas production function as in the closed economy case. Sectoral output is therefore:

$$Y_t(f) = \varepsilon_t^a K_t(f)^\alpha O_{uH,t}(f)^\mu N_{n,t}(f)^{1-\alpha-\mu}$$

This production function is adapted from Bergohlt (2014), where he includes materials as a production input for firms. Though here it is modelled as oil products input for intermediate goods firms. It is also the first place where oil products input enters the firm's problem, here $K_t(f)$ is the capital rented out by firm f, $O_{uH,t}(f)$ is the oil products (energy) input used by firm f, and $N_{n,t}(f)$ is the final labour used by firm f, and ε_t^a is aggregate sectoral productivity shock (TFP) which follows the following exogenous process:

$$\ln \varepsilon_t^a = (1-\rho_a)\ln \varepsilon^a + \rho_a \ln \varepsilon_{t-1}^a + e_t^a \quad e_{j,t}^a \sim N(0, \sigma_a)$$

Such that ρ_a is the persistence of the productivity shock.
Intermediate goods firm maximise expected discounted lifetime real profits, where the profits of firm f is:

$$\mathcal{D}_t(f) = \frac{P_{H,t}(f)}{P_{H,t}} Y_{H,t}(f) + \mathcal{E}_t \frac{P_{H,t}^*(f)}{P_{H,t}^*} Y_{H,t}^*(f) - R_t^k K_t(f) - P_{o,t}^* O_{uH,t}(f) - \Omega_{n,t} N_{n,t}(f)$$

So the firms problem is to:

$$Max_{K_t(f), O_{uH,t}(f), N_{n,t}(f) and P_{H,t}(f) P_{H,t}^*(f)} : \mathbb{E}_t \sum_{t=0}^{\infty} M_{t,t} \mathcal{D}_{j,t}(f)$$
$$s.t. \qquad Y_t(f) = \varepsilon_t^a K_t(f)^\alpha O_{uH,t}(f)^\mu N_{n,t}(f)^{1-\alpha-\mu}$$

The lagrangian reads:

$$\mathcal{L}_t = \mathbb{E}_t \sum_{t=0}^{\infty} M_{t,t} \Bigg[\frac{P_{H,t}(f)}{P_{H,t}} Y_{H,t}(f) + \mathcal{E}_t \frac{P_{H,t}^*(f)}{P_{H,t}^*} Y_{H,t}^*(f)$$
$$-R_t^k K_t(f) - P_{o,t}^* O_{uH,t}(f) - \Omega_{n,t} N_{n,t}(f) \Bigg] - \Phi_t(f) \left[Y_t(f) - \varepsilon_t^a K_t(f)^\alpha O_t(f)^\mu N_{n,t}(f)^{1-\alpha-\mu} \right]$$

$\Phi_t(j)$ is the shadow price, also known as the Real Marginal cost faced by firm f. The static problem is non different from the closed economy case and I obtain the following optimality conditions:

$$
\begin{aligned}
R_t^k &= \alpha \varepsilon_t^a K_t(f)^{\alpha-1} O_{H,t}(f)^{\mu} N_{n,t}(f)^{1-\alpha-\mu} \\
\mathcal{S}_t \Phi_{F,t} P_{o,t}^* &= \mu \varepsilon_t^a K_t(f)^{\alpha} O_{H,t}(f)^{\mu-1} N_{n,t}(f)^{1-\alpha-\mu} \\
\Omega_{n,t} &= (1-\alpha-\mu) \varepsilon_t^a K_t(f)^{\alpha} O_{uH,t}(f)^{\mu} N_{n,t}(f)^{-\alpha-\mu}
\end{aligned}
$$

Taking ratios of the equations above, it is straightforward to find the Capital-labour Ratio and Oil-Labour Ratio:

$$
\begin{aligned}
\mathcal{K}_t \equiv \frac{K_t(f)}{N_{n,t}(f)} &= \frac{\alpha}{1-\alpha-\mu} \frac{\Omega_{n,t}}{R_t^k} \\
\mathcal{O}_{uH,t} \equiv \frac{O_{uH,t}(f)}{N_{n,t}(f)} &= \frac{\mu}{1-\alpha-\mu} \frac{\Omega_{n,t}}{P_{o,t}^*}
\end{aligned}
$$

Observe that the Capital-Labour ratio and the Oil-labour ratio is independent of firm type f. From these, one can infer the input demand functions. Taking the production function, then:

$$
\begin{aligned}
\frac{Y_t(f)}{\varepsilon_t^a K_t(f)^{\alpha} O_{uH,t}(f)^{\mu}} &= N_{n,t}(f)^{1-\alpha-\mu} \\
N_{n,t}(f) &= \frac{Y_t(f)}{\varepsilon_t^a} \left(\frac{K_t(f)}{N_{n,t}(f)} \right)^{-\alpha} \left(\frac{O_{uH,t}(f)}{N_{n,t}(f)} \right)^{-\mu} \\
N_{n,t}(f) &= \frac{Y_t(f)}{\varepsilon_t^a} \left(\frac{1-\alpha-\mu}{\Omega_{n,t}} \right)^{\alpha+\mu} \left(\frac{R_t^k}{\alpha} \right)^{\alpha} \left(\frac{P_{o,t}^* \mathcal{S}_t \Phi_{F,t}}{\mu} \right)^{\mu}
\end{aligned}
$$

Similarly, from FOC for energy demand, the input demand for energy is:

$$
\begin{aligned}
P_{o,t}^* &= \mu \Phi_{j,t}(f) \varepsilon_t^a K_t(f)^{\alpha} O_{uH,t}(f)^{\mu-1} N_{n,t}(f)^{1-\alpha-\mu} \\
P_{o,t}^* O_{uH,t}(f) &= \mu \Phi_t(f) \varepsilon_t^a K_t(f)^{\alpha} O_{uH,t}(f)^{\mu} N_{n,t}(f)^{1-\alpha-\mu} \\
O_{uH,t}(f) &= \mu \left(\frac{\Phi_t}{P_{o,t}^* \mathcal{S}_t \Phi_{F,t}} \right) Y_t(f)
\end{aligned}
$$

These expressions imply that the total cost of producing the final good $Y_t(j)$ is:

$$
\begin{aligned}
R_t^k K_t(f) + P_{o,t}^* O_{uH,t}(f) + \Omega_{n,t} N_{n,t}(f) &= \Omega_{n,t} N_{n,t}(f) \\
&+ \frac{\alpha}{1-\alpha-\mu} \Omega_{n,t} N_{n,t}(f) + \frac{\mu}{1-\alpha-\mu} \Omega_{n,t} N_{n,t}(f) = \frac{\Omega_{n,t} N_{n,t}(f)}{1-\alpha-\mu}
\end{aligned}
$$

Combining these equations the real marginal cost $\Phi_{j,t}(f)$ is independent of firm type j:

$$\Phi_t(f) = \frac{\Omega_{n,t}}{(1-\alpha-\mu)\varepsilon_t^a K_t(f)^\alpha O_{uH,t}(f)^\mu N_{n,t}(f)^{-\alpha-\mu}}$$
$$\Phi_t(f) = \frac{\Omega_t N_{n,t}(f)}{(1-\alpha-\mu)\varepsilon_t^a K_t(f)^\alpha O_{uH,t}(f)^\mu N_{n,t}(f)^{1-\alpha-\mu}}$$
$$= \frac{\Omega_{n,t} N_{n,t}(f)}{(1-\alpha-\mu) Y_t(f)}$$

where

$$\frac{N_{n,t}(f)}{Y_t(f)} = \frac{1}{\varepsilon_t^a} \frac{1}{\left(\frac{K_t(f)}{N_{n,t}(f)}\right)^\alpha \left(\frac{O_{uH,t}(f)}{N_{n,t}(f)}\right)^\mu}$$

Therefore, it follows that:

$$\Phi_t(f) = \frac{1}{\varepsilon_t^a} \frac{\Omega_t}{(1-\alpha-\mu)} \left(\frac{\alpha}{1-\alpha-\mu} \frac{\Omega_{n,t}}{R_t^k}\right)^{-\alpha} \left(\frac{\mu}{1-\alpha-\mu} \frac{\Omega_{n,t}}{P_{o,t}^*}\right)^{-\mu}$$
$$\Phi_t(f) = \frac{1}{\varepsilon_t^a} \left(\frac{\Omega_{n,t}}{1-\alpha-\mu}\right)^{1-\alpha-\mu} \left(\frac{R_t^k}{\alpha}\right)^\alpha \left(\frac{P_{o,t}^*}{\mu}\right)^\mu \equiv \Phi_t$$

Then re-writing the profit of firm f at time t as:

$$\mathcal{D}_t(f) = \frac{P_{H,t}(f)}{P_{H,t}} \frac{P_{H,t}}{P_t} Y_{H,t}(f) + \mathcal{E}_t \frac{P_{H,t}^*(f)}{P_{H,t}^*} \frac{P_{H,t}^*}{P_t} Y_{H,t}^*(f) - R_t^k K_t(f) - P_{o,t}^* O_{uH,t}(f) - \Omega_{n,t} N_{n,t}(f)$$
$$\mathcal{D}_t(f) = \frac{P_{H,t}(f)}{P_{H,t}} \frac{P_{H,t}}{P_t} Y_{H,t}(f) + \mathcal{E}_t \frac{P_{H,t}^*(f)}{P_{H,t}^*} \frac{P_{H,t}^*}{P_t} Y_{H,t}^*(f) - \frac{\Omega_{n,t} N_{n,t}(f)}{1-\alpha-\mu}$$
$$\mathcal{D}_t(f) = \frac{P_{H,t}(f)}{P_{H,t}} \frac{P_{H,t}}{P_t} Y_{H,t}(f) + \mathcal{E}_t \frac{P_{H,t}^*(f)}{P_{H,t}^*} \frac{P_{H,t}^*}{P_t} Y_{H,t}^*(f) - \Phi_t Y_t(f)$$
$$\mathcal{D}_t(f) = \frac{P_{H,t}(f)}{P_{H,t}} \frac{P_{H,t}}{P_t} Y_{H,t}(f) + \mathcal{E}_t \frac{P_{H,t}^*(f)}{P_{H,t}^*} \frac{P_{H,t}^*}{P_t} Y_{H,t}^*(f) - \Phi_t \left(Y_{H,t}(f) + Y_{H,t}^*(f)\right)$$

Then expressing the profit function in real terms by dividing through with the aggregate price index P_t:

$$\mathcal{D}_t(f) = \frac{P_{H,t}(f)}{P_t} Y_{H,t}(f) + \mathcal{E}_t \frac{P_{H,t}^*(f)}{P_t} Y_{H,t}^*(f) - R_t^k K_t(f) - \mathcal{S}_t \Phi_{F,t} P_{o,t}^* O_{uH,t}(f) - \Omega_{n,t} N_{n,t}(f)$$

Finally, from firms FOC, total cost is given as:

$$R_t^k K_t(f) + \mathcal{S}_t \Phi_{F,t} P_{o,t}^* O_{uH,t}(f) + \Omega_{n,t} N_{n,t}(f) = \Omega_{n,t} N_{n,t}(f)$$
$$+ \frac{\alpha}{1-\alpha-\mu} \Omega_{n,t} N_{n,t}(f) + \frac{\mu}{1-\alpha-\mu} \Omega_{n,t} N_{n,t}(f) = \frac{\Omega_{n,t} N_{n,t}(f)}{1-\alpha-\mu} = \Phi_t Y_t(f)$$

Recall market clearing for final output:

$$Y_t(f) = Y_{H,t}(f) + Y_{H,t}^*(f)$$
$$\Rightarrow \Phi_t Y_t(f) = \Phi_t \left(Y_{H,t}(f) + Y_{H,t}^*(f) \right)$$

Recall the following demand equations hold:

$$Y_{H,t}(f) = \left(\frac{P_{H,t}(f)}{P_{H,t}} \right)^{-\theta_p} Y_{H,t}^d \qquad Y_{H,t}^*(f) = \left(\frac{P_{H,t}^*(f)}{P_{H,t}^*} \right)^{-\theta_p^*} Y_{H,t}^{*d}$$

I will assume Producer Currency Pricing (PCP) and extend to Local Currency Pricing (LCP) in optimisation[7]. That is, export prices are preset in foreign currency and in the case of PCP, prices preset in domestic (producer's) currency. As with households, every period, each firm is allowed to re-optimise its price with probability $1 - \psi_p \in [0, 1]$ or with probability ψ_p, it partially indexes to the past inflation $\Pi_{t-1} \equiv \frac{P_{Hj,t-1}}{P_{Hj,t-2}}$. As with households, define $\iota_p \in [0, 1]$ as the degree of indexation to past inflation. Similarly, I define the new term:

$$J_{t,t+i}^p = \begin{cases} \Pi_{H,k=0}^{i-1} \pi_{H,t+k}^{\iota_p} & if \quad i > 0 \\ 1 & if \quad i = 0 \end{cases}$$

$$J_{t,t+i}^{*p} = \begin{cases} \Pi_{H,k=0}^{*i-1} \pi_{H,t+k}^{*\iota_p} & if \quad i > 0 \\ 1 & if \quad i = 0 \end{cases}$$

Note that under PCP and LCP $J_{t,t+i}^{*p}$ doesn't change. Here $J_{t,t+i}^{*p} = \left(\frac{P_{H,t-1+i}^*}{P_{H,t-1}^*} \right)^{\chi_p}$. Finally, price of any firm that optimally resets its price at time t and never again, has price at time $t + i$

[7]The demand functions are the same under LCP, I can also write $\left(\frac{P_{H,t}^*(f)}{P_{H,t}^*} \right)^{-\theta_p} = \left(\frac{\mathcal{E}_t P_{H,t}^*(f)}{\mathcal{E}_t P_{H,t}^*} \right)^{-\theta_p}$, this will be useful when considering indexation of prices in the firm optimisation problem.

as:

$$P_{H,t+i}(f) = P_{H,t}(f)J^{p}_{t,t+i} \Rightarrow \frac{P_{H,t+i}(f)}{P_{H,t+i}} = \frac{P_{H,t}(f)}{P_{H,t+i}}J^{p}_{t,t+i}$$

Analogously for export prices, since producers can set prices based on either LCP or PCP, under LCP any firm that optimally resets its price at time t and never again, has price at time $t+i$ as:

$$P^{*}_{H,t+i}(f) = P^{*}_{H,t}(f)J^{*p}_{t,t+i} \Rightarrow \frac{P^{*}_{H,t+i}(f)}{P^{*}_{H,t+i}} = \frac{P^{*}_{H,t}(f)}{P^{*}_{H,t+i}}J^{*p}_{t,t+i}$$

While under PCP, any firm that optimally resets its price at time t and never again, has price at time $t+i$ as:

$$\mathcal{E}_{t+i}P^{*}_{H,t+i}(f) = \mathcal{E}_{t}P^{*}_{H,t}(f)J^{*p}_{t,t+i} \Rightarrow \frac{\mathcal{E}_{t+i}P^{*}_{H,t+i}(f)}{\mathcal{E}_{t+i}P^{*}_{H,t+i}} = \frac{\mathcal{E}_{t}P^{*}_{H,t}(f)}{\mathcal{E}_{t+i}P^{*}_{H,t+i}}J^{*p}_{t,t+i}$$

For ease, split the profit into two parts, the first aspect is the price setting at home and this holds whether it is PCP and LCP. That is:

$$\mathcal{D}_t(f) = \frac{P_{H,t}(f)}{P_{H,t}}\frac{P_{H,t}}{P_t}Y_{H,t}(f) - \Phi_t Y_{H,t}(f)$$
$$\Rightarrow \mathcal{D}_t(f) = \left(\frac{P_{H,t}(f)}{P_{H,t}}\right)^{(1-\theta_p)} Y^{d}_{H,t}\frac{P_{H,t}}{P_t} - \Phi_t \left(\frac{P_{H,t}(f)}{P_{H,t}}\right)^{-\theta_p} Y^{d}_{H,t}$$

Firms maximise price with respect to present discounted life time profits so that:

$$\mathbb{E}_i \sum_{i=0}^{\infty} \psi_p^i M_{t,t+i} \left(\frac{P_{H,t+i}(f)}{P_{H,t+i}}\right)^{(1-\theta_p)} Y^{d}_{H,t+i}\frac{P_{H,t+i}}{P_{t+i}} - \Phi_{t+i} \left(\frac{P_{H,t+i}(f)}{P_{H,t+i}}\right)^{-\theta_p} Y^{d}_{H,t+i}$$
$$\Rightarrow \mathbb{E}_i \sum_{i=0}^{\infty} \psi_p^i M_{t,t+i} \left(\frac{P_{H,t}(f)J^{p}_{t,t+i}}{P_{H,t+i}}\right)^{(1-\theta_p)} Y^{d}_{H,t+i}\frac{P_{H,t+i}}{P_{t+i}} - \Phi_{t+i} \left(\frac{P_{H,t}(f)J^{p}_{t,t+i}}{P_{H,t+i}}\right)^{-\theta_p} Y^{d}_{H,t+i}$$

Now define the optimally set price at home as $\bar{P}_{H,t}$, then FOC w.r.t. $P_{H,t}(f)$ is:

$$\mathbb{E}_i \sum_{i=0}^{\infty} \psi_p^i M_{t,t+i} \Bigg[\bar{P}_{H,t}^{-\theta_p}(1-\theta_p)\left(\frac{J^{p}_{t,t+i}}{P_{H,t+i}}\right)^{1-\theta_p} \frac{P_{H,t+i}}{P_{t+i}} + \theta_p \bar{P}_{H,t}^{-\theta_p-1}\Phi_{t+i}\left(\frac{J^{p}_{t,t+i}}{P_{H,t+i}}\right)^{-\theta_p}\Bigg] Y^{d}_{H,t+i} = 0$$
$$\mathbb{E}_i \sum_{i=0}^{\infty} \psi_p^i M_{t,t+i} \left[\bar{P}_{H,t}(1-\theta_p)\left(\frac{J^{p}_{t,t+i}}{P_{H,t+i}}\right)^{1-\theta_p} \frac{P_{H,t+i}}{P_{t+i}} + \theta_p \Phi_{t+i}\left(\frac{J^{p}_{t,t+i}}{P_{H,t+i}}\right)^{-\theta_p}\right] Y^{d}_{H,t+i} = 0$$

$$\bar{P}_{H,t}\,\mathbb{E}_i \sum_{i=0}^{\infty} \psi_p^i M_{t,t+i} Y_{H,t+i}^d \left[(1-\theta_p) \left(\frac{J_{t,t+i}^p}{P_{H,t+i}} \right)^{1-\theta_p} \frac{P_{H,t+i}}{P_{t+i}} \right] = \\ - \theta_p\,\mathbb{E}_i \sum_{i=0}^{\infty} \psi_p^i M_{t,t+i} \left[\Phi_{t+i} \left(\frac{J_{t,t+i}^p}{P_{H,t+i}} \right)^{-\theta_p} \right] Y_{H,t+i}^d$$

Defining the relative price $\frac{\bar{P}_{H,t}}{P_{H,t}}$ optimally set by the firm, obtain[8]:

$$\frac{\bar{P}_{H,t}}{P_{H,t}} = \frac{\theta_p}{\theta_p - 1} \frac{\mathbb{E}_i \sum_{i=0}^{\infty} \psi_p^i M_{t,t+i} Y_{H,t+i}^d \Phi_{t+i} \left(\frac{J_{t,t+i}^p P_{H,t}}{P_{H,t+i}} \right)^{-\theta_p}}{\mathbb{E}_i \sum_{i=0}^{\infty} \psi_p^i M_{t,t+i} Y_{Hj,t+i}^d \left(\frac{J_{t,t+i}^p P_{H,t}}{P_{H,t+i}} \right)^{1-\theta_p} \frac{P_{H,t+i}}{P_{t+i}}}$$

Recursively, the above can be expressed as:

$$L_{H,t} = \frac{\bar{P}_{H,t}}{P_{H,t}} = \frac{\Xi_{H,t}}{\Sigma_{H,t}}$$

Where $\Xi_{H,t}$ and $\Sigma_{H,t}$ are recursive auxiliary variables given as:

$$\Xi_{H,t} = \frac{\theta_p}{\theta_p - 1} \Phi_t Y_{H,t}^d + \psi_p\,\mathbb{E}_t \left[M_{t,t+1} \Xi_{H,t+1} \left(\frac{\Pi_{H,t+1}}{\Pi_{H,t}^{\iota_p}} \right)^{\theta_p} \right]$$

$$\Sigma_{H,t} = Y_{H,t}^d \frac{P_{H,t}}{P_t} + \psi_p\,\mathbb{E}_t \left[M_{t,t+1} \Sigma_{H,t+1} \left(\frac{\Pi_{H,t+1}}{\Pi_{H,t}^{\iota_p}} \right)^{\theta_p - 1} \right]$$

For export price setting, there are two scenarios: Producer currency pricing (PCP) and Local Currency Pricing (LCP). Under PCP, export prices are preset in foreign currency, so firms choose $\mathcal{E}_t P_{H,t}^*(f)$. The aspect of the lagrangian concerned with that is:

$$\mathcal{D}_t(f) = \mathcal{E}_t \frac{P_{H,t}^*(f)}{P_{H,t}^*} \frac{P_{H,t}^*}{P_t} Y_{H,t}^*(f) - \Phi_t Y_{H,t}^*(f)$$

[8] Simply dividing divide through by $P_{H,t}$

One can express the demand functions for exports with a trick:

$$Y^*_{H,t}(f) = \left(\frac{P^*_{H,t}(f)}{P^*_{H,t}}\right)^{-\theta^*_p} Y^{*d}_{H,t} \Rightarrow Y^*_{H,t}(f) = \left(\frac{\mathcal{E}_t P^*_{H,t}(f)}{\mathcal{E}_t P^*_{H,t}}\right)^{-\theta^*_p} Y^{*d}_{H,t}$$

$$\Rightarrow \mathcal{D}_t(f) = \left(\frac{\mathcal{E}_t P^*_{H,t}(f)}{P^*_{H,t}}\right)^{(1-\theta_p)} \mathcal{E}_t^{\theta_p} Y^{*d}_{H,t} \frac{P^*_{H,t}}{P_t} - \Phi_t \left(\frac{\mathcal{E}_t P^*_{H,t}(f)}{\mathcal{E}_t P^*_{H,t}}\right)^{-\theta_p} Y^{*d}_{H,t}$$

Once again using that:

$$\mathcal{E}_{t+i} P^*_{H,t+i}(f) = \mathcal{E}_t P^*_{H,t}(f) J^{*p}_{t,t+i} \Rightarrow \frac{\mathcal{E}_{t+i} P^*_{H,t+i}(f)}{\mathcal{E}_{t+i} P^*_{H,t+i}} = \frac{\mathcal{E}_t P^*_{H,t}(f)}{\mathcal{E}_{t+i} P^*_{H,t+i}} J^{*p}_{t,t+i}$$

$$\Rightarrow \mathbb{E}_i \sum_{i=0}^{\infty} \psi_p^i M_{t,t+i} \left(\frac{\mathcal{E}_t P^*_{H,t}(f) J^{*p}_{t,t+i}}{P^*_{H,t+i}}\right)^{(1-\theta_p)} \mathcal{E}_{t+i}^{\theta_p} Y^{*d}_{H,t+i} \frac{P^*_{H,t+i}}{P_{t+i}} - \Phi_{t+i} \left(\frac{\mathcal{E}_t P^*_{H,t}(f) J^{*p}_{t,t+i}}{\mathcal{E}_{t+i} P^*_{H,t+i}}\right)^{-\theta_p} Y^{*d}_{H,t+i}$$

Once again define the optimal set export price $\mathcal{E}_t \bar{P}^*_{H,t}$ The the FOC results in:

$$\mathbb{E}_i \sum_{i=0}^{\infty} \psi_p^i M_{t,t+i} \Bigg[(\mathcal{E}_t \bar{P}^*_{H,t})^{-\theta_p} (1-\theta_p) \left(\frac{J^{*p}_{t,t+i}}{P^*_{H,t+i}}\right)^{1-\theta_p} \mathcal{E}_{t+i}^{\theta_p} \frac{P^*_{H,t+i}}{P_{t+i}} + \theta_p (\mathcal{E}_t \bar{P}^*_{H,t})^{-\theta_p - 1} \Phi_{t+i} \left(\frac{J^{p}_{t,t+i}}{\mathcal{E}_{t+i} P_{H,t+i}}\right)^{-\theta_p} \Bigg] Y^{*d}_{H,t+i} = 0$$

$$\mathbb{E}_i \sum_{i=0}^{\infty} \psi_p^i M_{t,t+i} \Bigg[\mathcal{E}_t \bar{P}^*_{H,t} (1-\theta_p) \left(\frac{J^{*p}_{t,t+i}}{P^*_{H,t+i}}\right)^{1-\theta_p} \mathcal{E}_{t+i}^{\theta_p} \frac{P^*_{H,t+i}}{P_{t+i}} + \theta_p \Phi_{t+i} \left(\frac{J^{*p}_{t,t+i}}{\mathcal{E}_{t+i} P^*_{H,t+i}}\right)^{-\theta_p} \Bigg] Y^{*d}_{H,t+i} = 0$$

$$\mathcal{E}_t \frac{\bar{P}^*_{H,t}}{P^*_{H,t}} = \frac{\theta_p}{\theta_p - 1} \frac{\mathbb{E}_i \sum_{i=0}^{\infty} \psi_p^i M_{t,t+i} \mathcal{E}_{t+i}^{\theta_p} Y^{*d}_{H,t+i} \Phi_{t+i} \left(\frac{J^{*p}_{t,t+i} P^*_{H,t}}{P^*_{H,t+i}}\right)^{-\theta_p}}{\mathbb{E}_i \sum_{i=0}^{\infty} \psi_p^i M_{t,t+i} \mathcal{E}_{t+i}^{\theta_p} Y^{*d}_{H,t+i} \left(\frac{J^{p}_{t,t+i} P^*_{H,t}}{P^*_{H,t+i}}\right)^{1-\theta_p} \frac{P^*_{H,t+i}}{P_{t+i}}}$$

Without defining the recursive nature, one can already see that for $i = 0$, this breaks down to a markup over the marginal cost which is identical to the price at home for $i = 0$. Thus, this is how PCP induces the law of one price. Now one can analogously define the recursive formulation for this problem as:

$$L^*_{H,t} = \frac{\mathcal{E}_t \bar{P}^*_{H,t}}{P^*_{H,t}} = \frac{\Xi^*_{H,t}}{\Sigma^*_{H,t}}$$

Where $\Xi_{H,t}$ and $\Sigma_{H,t}$ are recursive auxiliary variables given as:

$$\Xi^*_{H,t} = \frac{\theta_p}{\theta_p - 1}\mathcal{E}_t^{\theta_p}\Phi_t Y^{*d}_{H,t} + \psi_p \,\mathbb{E}_t \left[M_{t,t+1}\Xi^*_{H,t+1} \left(\frac{\Pi^*_{H,t+1}}{\Pi^{*\iota_p}_{H,t}} \right)^{\theta_p} \right]$$
$$\Sigma^*_{H,t} = \mathcal{E}_t^{\theta_p} Y^{*d}_{H,t}\frac{P^*_{H,t}}{P_t} + \psi_p \,\mathbb{E}_t \left[M_{t,t+1}\Sigma^*_{H,t+1} \left(\frac{\Pi^*_{H,t+1}}{\Pi^{*\iota_p}_{H,t}} \right)^{\theta_p - 1} \right]$$

Under Local Currency Pricing, export prices are preset in local currency, that is producers choose $P^*_{H,t}(f)$, **NOT** $\mathcal{E}_t P^*_{H,t}(f)$. Here I utilise the demand equation without the trick and apply the reset condition. That is:

$$P^*_{H,t+i}(f) = P^*_{H,t}(f) J^{*p}_{t,t+i} \Rightarrow \frac{P^*_{H,t+i}(f)}{P^*_{H,t+i}} = \frac{P^*_{H,t}(f)}{P^*_{H,t+i}} J^{*p}_{t,t+i}$$
$$Y^*_{H,t}(f) = \left(\frac{P^*_{H,t}(f)}{P^*_{H,t}} \right)^{-\theta^*_p} Y^{*d}_{H,t}$$

Plugging these into the life time discounted profit condition:

$$\mathbb{E}_i \sum_{i=0}^{\infty} \psi_p^i M_{t,t+i} \left(\mathcal{E}_{t+i} \frac{P^*_{H,t}(f) J^{*p}_{t,t+i}}{P^*_{H,t+i}} \right)^{(1-\theta_p)} Y^{*d}_{H,t+i} \frac{P^*_{H,t+i}}{P_{t+i}} - \Phi_{t+i} \left(\frac{P^*_{H,t}(f) J^{*p}_{t,t+i}}{P^*_{H,t+i}} \right)^{-\theta_p} Y^{*d}_{H,t+i}$$

FOC reads

$$\mathbb{E}_i \sum_{i=0}^{\infty} \psi_p^i M_{t,t+i} \Bigg[\bar{P}^{*-\theta_p}_{H,t} (1-\theta_p) \mathcal{E}_{t+i} \left(\frac{J^{*p}_{t,t+i}}{P^*_{H,t+i}} \right)^{1-\theta_p} \frac{P^*_{H,t+i}}{P_{t+i}}$$
$$+ \theta_p \bar{P}^{*-\theta_p - 1}_{H,t} \Phi_{t+i} \left(\frac{J^{*p}_{t,t+i}}{P^*_{H,t+i}} \right)^{-\theta_p} \Bigg] Y^{*d}_{H,t+i} = 0$$

$$\mathbb{E}_i \sum_{i=0}^{\infty} \psi_p^i M_{t,t+i} \left[\bar{P}^*_{H,t}(1-\theta_p)\mathcal{E}_{t+i} \left(\frac{J^{*p}_{t,t+i}}{P_{H,t+i}} \right)^{1-\theta_p} \frac{P^*_{H,t+i}}{P_{t+i}} + \theta_p \Phi_{t+i} \left(\frac{J^{*p}_{t,t+i}}{P^*_{H,t+i}} \right)^{-\theta_p} \right] Y^{*d}_{H,t+i} = 0$$

$$\frac{\bar{P}^*_{H,t}}{P^*_{H,t}} = \frac{\theta_p}{\theta_p - 1} \frac{\mathbb{E}_t \sum_{i=0}^{\infty} \psi_t^i M_{t,t+i} \Phi_{t+i} Y^{*d}_{H,t+i} \left(\frac{J^{*p}_{t,t+i} P^*_{H,t}}{P^*_{H,t+i}} \right)^{-\theta_p}}{\mathbb{E}_t \sum_{i=0}^{\infty} \psi_t^i M_{t,t+i} Y^{*d}_{H,t+i} \mathcal{E}_{t+i} \left(\frac{J^{*p}_{t,t+i} P^*_{H,t}}{P^*_{H,t+i}} \right)^{1-\theta_p} \frac{P^*_{H,t+i}}{P_{t+i}}}$$

Without defining the recursive nature, one can already see that for $i = 0$, this reduces to a markup over the marginal cost divided by the nominal exchange rate which is different from the price at home for $i = 0$. This implies that $\bar{P}^*_{H,t} \neq \frac{\bar{P}_{H,t}}{\mathcal{E}_t}$. Thus, this is how LCP induces deviations from the law of one price. Now one can analogously define the recursive formulation for this problem as:

$$L^*_{H,t} = \frac{\bar{P}^*_{H,t}}{P^*_{H,t}} = \frac{\Xi^*_{H,t}}{\Sigma^*_{H,t}}$$

Where $\Xi^*_{H,t}$ and $\Sigma^*_{H,t}$ are recursive auxiliary variables given as:

$$\Xi^*_{H,t} = \frac{\theta_p}{\theta_p - 1} \Phi_t Y^{*d}_{H,t} + \psi_p \, \mathbb{E}_t \left[M_{t,t+1} \Xi^*_{H,t+1} \left(\frac{\Pi^*_{H,t+1}}{\Pi^{*\iota_p}_{H,t}} \right)^{\theta_p} \right]$$

$$\Sigma^*_{H,t} = Y^{*d}_{H,t} \mathcal{E}_t \frac{P^*_{H,t}}{P_t} + \psi_p \, \mathbb{E}_t \left[M_{t,t+1} \Sigma^*_{H,t+1} \left(\frac{\Pi^*_{H,t+1}}{\Pi^{*\iota_p}_{H,t}} \right)^{\theta_p - 1} \right]$$

There is a clear difference in both optimal prices, which has the following implications. Exchange rate movements will induce deviations from the law of one price[9] as home export prices expressed in foreign currency do not move when the nominal exchange rate changes. There is zero exchange rate pass through, implying Purchasing Power Parity (PPP) will not hold. Analogously to the wage setting condition, one can determine the law of motion for prices. For domestic prices:

$$P_{H,t}^{1-\theta_p} = (1-\psi_p) \bar{P}_{H,t}^{1-\theta_p} + \int_{\mathcal{I}_{pH,t}} P_{H,t}^{1-\theta_p}(f) dl$$

[9]the law of one Price holds if $\bar{P}_{H,t} = \mathcal{E}_t \bar{P}^*_{H,t}$

Analogously to the wage setting this gives:

$$P_{H,t}^{(1-\theta_p)} = (1-\psi_p)\bar{P}_{H,t}^{(1-\theta_p)} + \psi_p \left(\Pi_{H,t-1}^{\chi_p} P_{H,t-1}\right)^{(1-\theta_p)}$$

$$1 = (1-\psi_p)L_{H,t}^{(1-\theta_p)} + \psi_p \left(\frac{\Pi_{H,t-1}^{\chi_p}}{\Pi_{H,t}}\right)^{(1-\theta_p)}$$

As with the households, one can then derive the law of motion for prices and cross-sectional price dispersion. The Calvo pricing and partial indexation results in the following pricing dynamics (problem is analogous to households wage dispersion), one can derive the price dispersion law of motion, obtained for home prices:

$$\Delta_{pH,t} = (1-\psi_p)L_{H,t} + \psi_p \left(\frac{\Pi_{H,t-1}^{\chi_p}}{\Pi_{H,t}}\right)^{-\theta_p}$$

Under LCP, and PCP, the following law of motion for export prices and respective price dispersion terms respectively:

$$1 = (1-\psi_p){L^*}_{H,t}^{(1-\theta_p)} + \psi_p \left(\frac{\Pi_{H,t-1}^{*\chi_p}}{\Pi_{H,t}^*}\right)^{(1-\theta_p)}$$

$$\Delta_{pH,t}^* = (1-\psi_p)L_{H,t}^* + \psi_p \left(\frac{\Pi_{H,t-1}^{*\chi_p}}{\Pi_{H,t}^*}\right)^{-\theta_p}$$

Now define the Terms of Trade, $\mathcal{T}_t$ - expressing both prices in term of home currency, this is the relative price of imports to the price of exports:

$$\mathcal{T}_t = \frac{P_{F,t}}{\mathcal{E}_t P_{H,t}^*}$$

Under PCP, the law of one price holds and this becomes[10]:

$$\mathcal{T}_t = \frac{P_{F,t}}{P_{H,t}}$$

[10]Under DCP, since the law of one price does not hold for exports, $\mathcal{T}_t \equiv \frac{P_{F,t}}{\mathcal{E}_t P_{H,t}^*} = \mathcal{T}_t = \frac{P_{F,t}}{\Phi_{H,t} P_{H,t}}$. This definition is used in dynare for home terms of trade under DCP and LCP.

Such that increases in $\mathcal{T}_t$ reflects worsening in the home country's Terms of Trade. Following Galí (2015) I can define the law of one price gap for imports and exports:

$$\Phi_{F,t} = \frac{\mathcal{E}_t P^*_{F,t}}{P_{F,t}} \qquad \Phi_{H,t} = \frac{\mathcal{E}_t P^*_{H,t}}{P_{H,t}}$$

Under LCP, the terms of trade for the foreign country are different and is given as $\mathcal{T}^*_t = \frac{P^*_{H,t}}{P^*_{F,t}}$. This can be expressed as a linear transformation in terms of the home country's terms of trade and law of one price gaps, such that $\mathcal{T}^*_t = \frac{\Phi_{H,t}}{\Phi_{F,t}\mathcal{T}_t}$. Then I can express the evolution of the law of one price gap for imports and exports is:

$$\frac{\Phi_{F,t}}{\Phi_{F,t-1}} = \frac{\mathcal{E}_t}{\mathcal{E}_{t-1}} \frac{\Pi^*_{F,t}}{\Pi_{F,t}} \qquad \frac{\Phi_{H,t}}{\Phi_{H,t-1}} = \frac{\mathcal{E}_t}{\mathcal{E}_{t-1}} \frac{\Pi^*_{H,t}}{\Pi_{H,t}}$$

Oil Sector

Upstream oil firms that produce the oil consumption good have the following production function:

$$O_{u,t} = \varepsilon_{u,t} \Psi^{s\alpha_u}_t N^{(1-\alpha_u)}_{u,t}$$

Here $\Psi^s_t = U_t \Psi_t$ represents effective rig utilisation, such that $\alpha_u \in [0, 1)$ induces decreasing returns to scale. The representative oil company seeks to maximise expected stream of life time cash flows:

$$\mathbb{E}_t \sum_{s=0}^{\infty} M_{t,s} \left[P^*_{o,s} \mathcal{S}_s \Phi_{F,t} O_{u,s} - \tilde{P}_{i,s} a(U_s) \Psi_{u,s} - \Omega_{u,s} N_{u,s} - \tilde{P}_{i,s} I_{u,s} \right]$$

Taking oil prices as given the representative oil company chooses the level of utilisation, Investment, labour demand and the future production capacity. As prices are not sticky, the firms decision on future productive capacity does not leave it with inefficient levels of capital when prices change. The maximisation problem is subject to law of motion of active rigs:

$$\Psi_{u,t+1} = (1 - \delta_u)\Psi_{u,t} + \varepsilon^i_{\psi,t} \left[1 - S_u \left(\frac{I_{u,t}}{I_{u,t-1}} \right) \right] I_{u,t}$$

I adapt Bergholt, Larsen and Seneca (2018), which is standard in investment theory literature specification for utilisation of capital to utilisation of rigs:

$$a(U_t) = z_1(U_t - 1) + \frac{z_1 z_2}{2}(U_t - 1)^2$$

This function satisfies $a(1) = 0$ and $a'(1) > 0$. Here z_1 is governs the utilisation rate and equals $\frac{1}{\beta} + \delta_u - 1$ in the steady state. I also have that $z_2 = \frac{a''(1)}{a'(1)}$. I calibrate z_2 such that the cost of changing utilisation rate s linear. The first order conditions for Ψ_{t+1} $I_{u,t}$ U_t $N_{u,t}$ respectively:

$$\alpha_u \frac{\mathcal{S}_t P^*_{o,t} O_{u,t}}{U_t} = \tilde{P}_{i,t} a'(U_t) \Psi_t$$

$$(1 - \alpha_u) \frac{\mathcal{S}_t P^*_{o,t} O_{u,t}}{N_{u,t}} = \Omega_{u,t}$$

$$\mathbb{E}_t \left[M_{t,t+1} [\alpha_u \frac{\mathcal{S}_{t+1} P^*_{o,t+1} O_{u,t+1}}{\Psi_{t+1}} - \tilde{P}_{i,t+1} a(U_{u,t+1}) + \mathcal{Q}_{u,t+1}(1 - \delta)] \right] = \mathcal{Q}_{u,t}$$

$$\mathbb{E}_t \left[M_{t,t+1} \epsilon^i_{\psi,t+1} \mathcal{Q}_{u,t+1} S' \left(\frac{I_{u,t+1}}{I_{u,t}} \right) \left(\frac{I_{u,t+1}}{I_{u,t}} \right)^2 \right] + \epsilon^i_{\psi,t} \mathcal{Q}_{u,t} \left[1 - S\left(\frac{I_{u,t}}{I_{u,t-1}} \right) - S'\left(\frac{I_{u,t}}{I_{u,t-1}} \right) \frac{I_{u,t}}{I_{u,t-1}} \right] = \frac{P_{i,t}}{P_t}$$

Combining the last two equations, I can get the effective rig utilisation - labour ratio[11]:

$$\frac{\Psi^s_{u,t}}{N_{u,t}} = \frac{\alpha_u}{(1 - \alpha_u)} \frac{\Omega_{u,t}}{\tilde{P}_{i,t} a'(U_t)}$$

The level of investment in the first order condition for the firm is aggregate. Given that firms in the home country are both IOCs (International oil companies) as well as home based companies, I have that the aggregate investment is aggregated by a dixit-stiglitz aggregator as in the household investment problem. So, cost minimisation gives rise to the following:

$$I_{u,t} = \left[\xi_u^{\frac{1}{\varpi_i}} I_{uH,t}^{\frac{\varpi_u - 1}{\varpi_i}} + (1 - \xi_u)^{\frac{1}{\varpi_u}} I_{uF,t}^{\frac{\varpi_i - 1}{\varpi_i}} \right]^{\frac{\varpi_i}{\varpi_i - 1}}$$

$$\frac{I_{uH,t}}{I_{uF,t}} = \frac{\xi_u}{1 - \xi_u} \left(\frac{P_{H,t}}{P_{F,t}} \right)^{-\varpi_u}$$

[11] Here $a'(U_t) = z_1 + z_1 z_2 (U_t - 1)$

Then oil exports are then given by:

$$Ex_t = O_{u,t} - O_{uH,t} - O^*_{uH,t} - O^u_{c,t}$$

Aggregation

As with the rest of the model, I only show the process for the Home country, the ROW block is determined in the same way. Non-oil sectoral market clearing requires:

$$Y_t(f) = Y_{H,t}(f) + Y^*_{H,t}(f)$$
$$\Rightarrow Y^d_t = \int_0^1 Y_{H,t}(f) + Y^*_{H,t}(f)df = \Delta_{Hp,t}Y^d_{H,t} + \Delta^*_{Hp,t}Y^{*d}_{H,t}$$
$$Y^d_t = \int_0^1 Y_t(f) = \int_0^1 \varepsilon^a_t K_t(f)^\alpha {O^d}_{f,t}(f)^\mu N_{n,t}(f)^{1-\alpha-\mu}df = \varepsilon^a_t K^\alpha_t {O^{d\mu}}_{f,t} N^{1-\alpha-\mu}_{n,t}$$

These represent total factor utilised by non-oil firms. Note that in the case of PCP, $\Delta^*_{Hp,t} = \Delta_{Hp,t}$. Total labour demand by intermediate firms both in oil and non-oil sector is:

$$\int_0^1 N_t(f)df = \tilde{N}^d_{j,t}$$
$$\Rightarrow \tilde{N}^d_{j,t} = \frac{N_t}{\Delta_{wj,t}}$$

This implies that the production function of oil and and non-oil firms becomes:

$$\Delta^{1-\alpha-\mu}_{w,t} Y^d_t = \varepsilon^a_t K^\alpha_t {O^{d\mu}}_{f,t} N^{1-\alpha-\mu}_{n,t}$$
$$\Delta^{1-\alpha_u}_{wu,t} O^u_t = \varepsilon_{u,t}(\Psi^s_{u,t})^{\alpha_u} N^{1-\alpha_u}_{u,t}$$

Finally, aggregate hours worked in the economy is:

$$N_t = \eta_u \tilde{N}^d_{u,t} + \eta N_t$$

With this, the share of workers in each sector is:

$$\eta_u = \frac{N_{u,t}}{N_t} \qquad \eta = \frac{N_{n,t}}{N_t}$$

Market clearing for home produced goods and for exported goods requires that:

$$Y^d_{H,t} = C_{H,t} + I_{H,t} + I_{uH,t} + a(U_{u,t})\Psi_{u,t-1} + F_t$$
$$Y^{*d}_{H,t} = C^*_{H,t} + I^*_{H,t} + I^*_{uH,t}$$

Then one obtains aggregate market clearing condition:

$$Y^d_t = \Delta_{Hp,t}\left(C_{H,t} + I_{H,t} + I_{uH,t} + a(U_{u,t})\Psi_{u,t-1}\right) + \Delta^*_{Hp,t}\left(C^*_{H,t} + I^*_{H,t} + I^*_{uH,t}\right)$$

With the final consumption good in the economy satisfying:

$$Y_t = \left[a_H^{\frac{1}{\varpi}} Y^{d\,\frac{\varpi-1}{\varpi}}_{H,t} + a_F^{\frac{1}{\varpi}} Y^{d\,\frac{\varpi-1}{\varpi}}_{F,t}\right]^{\frac{\varpi}{\varpi-1}}$$

Next I define trade balance in the economy:

$$TB_t = \Phi_{H,t}\frac{P^*_{H,t}}{P_t}Y^*_{H,t} + \mathcal{S}_t\Phi_{F,t}P^*_{o,t}Ex_t - \frac{P_{F,t}}{P_t}Y_{F,t}\Phi_{F,t}$$

Finally, total output in the economy, GDP_t:

$$GDP_t = C_t + \frac{P_{i,t}}{P_t}\left(I_t + I_{u,t} + a(U_{u,t})\Psi_{u,t}\right) + \frac{P_{H,t}}{P_t}\left(F_t\right) + TB_t$$

The Foreign Block is just an identical economy but with slight differences in non-oil sector production:

Solving the Model

Initialising

As I simulate the model with dynare, I need to transform all the pricing variables to ratios and inflation. This is what this section does. I begin with the Home country:

$$P_t = \left[a_o P_{o,t}^{*1-\vartheta} + a_c P_{j,t}^{1-\vartheta}\right]^{\frac{1}{1-\vartheta}}$$
$$P_{c,t} = \left[a_H P_{H,t}^{1-\varpi} + a_F P_{F,t}^{1-\varpi}\right]^{\frac{1}{1-\varpi}}$$
$$\frac{P_{c,t}}{P_{H,t}} = \left[a_H + a_F \left(\frac{P_{F,t}}{P_{H,t}}\right)^{1-\varpi}\right]^{\frac{1}{1-\varpi}}$$

Recalling the definition for Terms of trade:

$$\mathcal{T}_t = \frac{P_{F,t}}{P_{H,t}}$$
$$\frac{P_{c,t}}{P_{H,t}} = \left[a_H + a_F \mathcal{T}_t^{1-\varpi}\right]^{\frac{1}{1-\varpi}} \equiv g(\mathcal{T}_t)$$

This definition is used under PCP. However, under DCP and LCP, where the terms of trade are defined differently:

$$\mathcal{T}_t = \frac{P_{F,t}}{\mathcal{E}_t P_{H,t}^*}$$

Then the Consumption CPI and evolution of the terms of trade are given as:

$$\Pi_{c,t} = \Pi_{H,t}\left(\frac{g(\mathcal{T}_t)}{g(\mathcal{T}_{t-1})}\right)$$
$$\frac{\mathcal{T}_t}{\mathcal{T}_{t-1}} = \frac{\Pi_{F,t}}{\Pi_{H,t}} \qquad \frac{\mathcal{T}_t}{\mathcal{T}_{t-1}} = \frac{\Pi_{F,t}}{\Pi_{H,t}\frac{\Phi_{H,t}}{\Phi_{H,t-1}}}$$

Analogously, I can do the same for the actual CPI price index:

$$P_t = \left[a_o P_{o,t}^{*1-\vartheta} + a_c P_{c,t}^{1-\vartheta}\right]^{\frac{1}{1-\vartheta}}$$
$$\frac{P_t}{P_{c,t}} = \left[a_o \left(\frac{P_{o,t}^*}{P_{c,t}}\right)^{1-\vartheta} + a_c\right]^{\frac{1}{1-\vartheta}} \equiv g(\mathcal{O}_t)$$
$$\Pi_t = \Pi_{c,t}\frac{g(\mathcal{O}_t)}{g(\mathcal{O}_{t-1})}$$
$$\mathcal{O}_t = \frac{\Pi_{o,t}}{\Pi_{c,t}}\mathcal{O}_{t-1}$$

Analogously, for the foreign country, the the price indexes and their respective gross inflation rates differ depending on whether I have PCP, DCP or LCP:

$$\frac{P^*_{c,t}}{P^*_{F,t}} = \left[a^*_H \left(\frac{P^*_{H,t}}{P^*_{F,t}} \right)^{1-\varpi^*} + a_F \right]^{\frac{1}{1-\varpi}} \equiv g(\mathcal{T}^*_t)$$

$$\textbf{PCP:} \quad \frac{P^*_{H,t}}{P^*_{F,t}} = \frac{P_{H,t}}{P_{F,t}} = \frac{1}{\mathcal{T}_t} \quad \textbf{DCP:} \quad \frac{P^*_{H,t}}{P^*_{F,t}} = \frac{1}{\mathcal{T}_t} \quad \textbf{DCP:} \quad \frac{P^*_{H,t}}{P^*_{F,t}} = \mathcal{T}^*_t = \frac{\Phi_{H,t}}{\Phi_{F,t}\mathcal{T}_t}$$

$$\Pi^*_{c,t} = \Pi^*_{F,t} \left(\frac{g(\mathcal{T}^*_t)}{g(\mathcal{T}^*_{t-1})} \right)$$

$$\frac{P^*_t}{P^*_{c,t}} = \left[a^*_o \left(\frac{P^*_{o,t}}{P^*_{c,t}} \right)^{1-\vartheta^*} + a^*_c \right]^{\frac{1}{1-\vartheta^*}} \equiv g(\mathcal{O}^*_t)$$

$$\Pi^*_t = \Pi^*_{c,t} \frac{g(\mathcal{O}^*_t)}{g(\mathcal{O}^*_{t-1})}$$

The ratio of real oil prices to consumption good prices $\frac{P^*_{o,t}}{P^*_{c,t}}$ is different under PCP, DCP and LCP given the definition of the terms of trade. analogously, under DCP, one can simply set $\Phi_{H,t} = 1$ to get the PCP case, and under DCP, setting $\Phi_{H,t} = \Phi_{F,t} = 1$ to get PCP case:

$$\frac{P^*_{o,t}}{P^*_{c,t}} = \frac{P^*_{o,t}}{P_{c,t}} \frac{P_{c,t}}{P_{H,t}} \frac{P_{H,t}}{P^*_{H,t}} \frac{P^*_{H,t}}{P^*_{F,t}} \frac{P^*_{F,t}}{P^*_{c,t}}$$

From here, under PCP where the law of one price holds, $\frac{P_{H,t}}{P^*_{H,t}} = \mathcal{E}_t$ and then:

$$\frac{P^*_{o,t}}{P^*_{c,t}} = \frac{\mathcal{O}_t \mathcal{E}_t g(\mathcal{T}_t)}{\mathcal{T}_t g(\mathcal{T}^*_t)}$$

Under DCP, $\frac{P_{H,t}}{P^*_{H,t}} = \frac{\mathcal{E}_t P_{H,t}}{\mathcal{E}_t P^*_{H,t}} = \frac{\mathcal{E}_t}{\Phi^*_{H,t}}$, then:

$$\frac{P^*_{o,t}}{P^*_{c,t}} = \frac{\mathcal{O}_t \mathcal{E}_t g(\mathcal{T}_t)}{\mathcal{T}_t \Phi_{H,t} g(\mathcal{T}^*_t)}$$

Finally, under LCP, where $\mathcal{T}^*_t = \frac{P^*_{H,t}}{P^*_{F,t}} = \frac{\Phi_{H,t}}{\Phi_{F,t}\mathcal{T}_t}$, and $\frac{P_{H,t}}{P^*_{H,t}} = \frac{\mathcal{E}_t}{\Phi^*_{H,t}}$, then:

$$\frac{P^*_{o,t}}{P^*_{c,t}} = \frac{\mathcal{O}_t \mathcal{E}_t g(\mathcal{T}_t) \mathcal{T}^*_t}{\Phi_{H,t} g(\mathcal{T}^*_t)} = \frac{\mathcal{O}_t \mathcal{E}_t g(\mathcal{T}_t)}{\Phi_{F,t} \mathcal{T}_t g(\mathcal{T}^*_t)}$$

Analogously, the definition of the real exchange rate $\mathcal{S}_t$, differs depending whether it is PCP, LCP and DCP:

$$\mathcal{S}_t = \frac{\mathcal{E}_t P_t^*}{P_t} = \frac{P_{F,t}^* P_{F,t}}{P_{F,t}^* P_{F,t}} \frac{\mathcal{E}_t P_t^*}{P_t}$$

Under PCP, from the law of one price $\frac{\mathcal{E}_t P_{F,t}^*}{P_{F,t}} = 1$ and I have that:

$$\begin{aligned}
\frac{P_t^*}{P_{F,t}^*} &= \frac{P_t^*}{P_{c,t}^*} \frac{P_{c,t}^*}{P_{F,t}^*} = g(\mathcal{O}_t^*) g(\mathcal{T}_t^*) \\
\frac{P_{F,t}}{P_t} &= \frac{P_{F,t}}{P_{H,t}} \frac{P_{H,t}}{P_{c,t}} \frac{P_{c,t}}{P_t} = \frac{\mathcal{T}_t}{g(\mathcal{O}_t) g(\mathcal{T}_t)} \\
\Rightarrow \mathcal{S}_t &= \frac{\mathcal{T}_t g(\mathcal{O}_t^*) g(\mathcal{T}_t^*)}{g(\mathcal{O}_t) g(\mathcal{T}_t)}
\end{aligned}$$

Under DCP, the law of one price holds for imports $\frac{\mathcal{E}_t P_{F,t}^*}{P_{F,t}} = 1$, however, $\frac{P_{F,t}}{P_{H,t}} = \mathcal{T}_t \Phi_{H,t}$. Then:

$$\begin{aligned}
\frac{P_t^*}{P_{F,t}^*} &= \frac{P_t^*}{P_{c,t}^*} \frac{P_{c,t}^*}{P_{F,t}^*} = g(\mathcal{O}_t^*) g(\mathcal{T}_t^*) \\
\frac{P_{F,t}}{P_t} &= \frac{P_{F,t}}{P_{H,t}} \frac{P_{H,t}}{P_{c,t}} \frac{P_{c,t}}{P_t} = \frac{\mathcal{T}_t \Phi_{H,t}}{g(\mathcal{O}_t) g(\mathcal{T}_t)} \\
\Rightarrow \mathcal{S}_t &= \frac{\mathcal{T}_t \Phi_{H,t} g(\mathcal{O}_t^*) g(\mathcal{T}_t^*)}{g(\mathcal{O}_t) g(\mathcal{T}_t)}
\end{aligned}$$

Under LCP, I have deviations from the law of one price in both markets so, $\frac{\mathcal{E}_t P_{F,t}^*}{P_{F,t}} = \Phi_{F,t}$, and $\frac{P_{F,t}}{P_{H,t}} = \mathcal{T}_t \Phi_{H,t}$, then the real exchange rate:

$$\begin{aligned}
\frac{P_t^*}{P_{F,t}^*} &= \frac{P_t^*}{P_{c,t}^*} \frac{P_{c,t}^*}{P_{F,t}^*} = g(\mathcal{O}_t^*) g(\mathcal{T}_t^*) \\
\frac{P_{F,t}}{P_t} &= \frac{P_{F,t}}{P_{H,t}} \frac{P_{H,t}}{P_{c,t}} \frac{P_{c,t}}{P_t} = \frac{\mathcal{T}_t \Phi_{H,t}}{g(\mathcal{O}_t) g(\mathcal{T}_t)} \\
\Rightarrow \mathcal{S}_t &= \frac{\mathcal{T}_t \Phi_{H,t} \Phi_{F,t} g(\mathcal{O}_t^*) g(\mathcal{T}_t^*)}{g(\mathcal{O}_t) g(\mathcal{T}_t)}
\end{aligned}$$

Finally, I can define the evolution of nominal exchange rate as:

$$\frac{\mathcal{E}_t}{\mathcal{E}_{t-1}} = \frac{\Pi_t}{\Pi_t^*} \frac{\mathcal{S}_t}{\mathcal{S}_{t-1}}$$

The Non-Stochastic Steady State

In the Stochastic steady state, all shocks equal 1, and I make the following standard assumption as in New Keynesian framework, i.e. inflation rate is zero, therefore, the gross inflation rate is one. $\Phi_{F,t} = \Phi_{H,t} = \Pi = \Pi_c = \Pi_H = \Pi_H^* = 1$. These conditions have direct implications as I would have sectoral wage inflation equal to one i.e. $\Pi_{wj} = 1$. Finally I normalise steady state utilisation of rigs to 1 $(U = 1)$. From the specification for the utilisation function, I have that $a(1) = 0$. The same condition holds in the foreign block. I will derive the conditions for the home country, the same technique is used to derive conditions for the foreign block. These restrictions directly give $M\ G_j\ L_H\ L_H^*\ \Delta_{wj}\ \Delta_H\ \Delta_H^*$. From these restrictions, I have that:

$$M = \beta$$
$$G_j = L_H = \Delta_{wj} = \Delta_H = \Delta_H^* = L_H^* = 1$$

Derive the rental rate for capital:

$$R^k = \beta^{-1} - 1 + \delta$$

From law of motion for rigs, it follows that:

$$z_1 = \beta^{-1} - 1 + \delta_u$$

Which conveniently leads to $\tilde{P}_i = 1$. From the auxiliary variables from optimal price setting, it follows that:

$$\Xi_H = \Sigma_H \Rightarrow \frac{\theta_p}{(\theta_p - 1)(1 - \psi_p \beta)} \Phi Y_H^d = \frac{Y_{H,t}^d}{1 - \psi_p \beta}$$
$$\Phi = \frac{\theta_p - 1}{\theta_p}$$

Then from the equations for $\Phi \theta_p$ I obtain:

$$\frac{\theta_p - 1}{\theta_p} = \left(\frac{\Omega_n}{1 - \alpha - \mu}\right)^{1-\alpha-\mu} \left(\frac{R_j^k}{\alpha}\right)^{\alpha} \left(\frac{P_o^*}{\mu}\right)^{\mu}$$
$$\Rightarrow \Omega_n = (1 - \alpha - \mu) \left[\frac{\theta_p - 1}{\theta_p} \left(\frac{R^k}{\alpha}\right)^{-\alpha} \left(\frac{P_o^*}{\mu}\right)^{-\mu}\right]^{\frac{1}{1-\alpha-\mu}}$$

This only holds in the non-oil sector. In the oil sector since they are price takers, their marginal cost equals the price, so real wages in the oil sector is:

$$\Omega_u = (1-\alpha_u)\left[P_o^*\left(\frac{\tilde{P}_{i,t}a(U)}{\alpha_u}\right)^{-\alpha_u}\right]^{\frac{1}{1-\alpha_u}}$$

Similarly, in the steady state markup for a price taker is equal to 1 so real marginal cost for upstream firms which is equal to its price is equal to 1. From Wage dynamics steady state, I get:

$$\zeta_j = \Upsilon_j \Rightarrow \frac{\Omega_j N_j}{1-\psi_{wj}\beta} = \frac{\theta_{wj}}{(1-\theta_{wj})(1-\psi_{wj}\beta)}\frac{N_j^{1+\sigma_n}}{\Lambda}$$

$$\text{Recall:} \quad \Lambda = [1-h]^{-\sigma_c}C^{-\sigma_c}$$

$$\frac{\Omega_j N_j}{1-\psi_{wj}\beta} = \frac{\theta_{wj}}{(1-\theta_{wj})(1-\psi_{wj}\beta)}N_j^{1+\sigma_n}[1-h]^{\sigma_c}C^{\sigma_c}$$

Where $N_j^{\sigma_n}[1-h]^{\sigma_c}C^{\sigma_c}$ is the consumption-leisure Marginal rate of substitution, then I obtain:

$$N_j^{\sigma_n}[1-h]^{\sigma_c}C^{\sigma_c} = \frac{\theta_{wj}-1}{\theta_{wj}}\Omega_j$$

Substituting yields:

$$\Upsilon_j = \frac{\theta_{wj}}{(1-\theta_{wj})(1-\psi_{wj}\beta)}N_j^{1+\sigma_n}[1-h]^{\sigma_c}C^{\sigma_c} = \frac{\Omega_j N_j}{1-\psi_{wj}\beta} = \zeta_j$$

From the consumption-leisure Marginal rate of substitution I find the labour supply:

$$N_j = \left[\frac{(\theta_{wj}-1)\Omega_j[1-h]^{-\sigma_c}C^{-\sigma_c}}{\theta_{wj}}\right]^{\frac{1}{\sigma_n}}$$

Per capita variables from FOCs from intermediate firm problems:

$$\frac{K}{N_n} = \frac{\alpha_j}{1-\alpha-\mu}\frac{\Omega_n}{R^k}$$
$$\frac{O_{uHj}}{N_n} = \frac{\mu_j}{1-\alpha-\mu}\frac{\Omega_n}{P_o^*}$$
$$\frac{\Psi_u^s}{N_u} = \frac{\alpha_u}{(1-\alpha_u)}\frac{\Omega_u}{\tilde{P}_i a'(U)}$$
$$\frac{Y^d}{N_n} = \left(\frac{K}{N_n}\right)^{\alpha}\left(\frac{O_{uH}}{N_n}\right)^{\mu}$$
$$\frac{O_u}{N_u^*} = \left(\frac{\Psi_u^s}{N_u}\right)^{\alpha_u}$$

Finally I need to express the demands for individual goods. The following demands can be expressed as:

$$C_H = a_H\left(\frac{P_H}{P_c}\right)^{-\varpi} C_c = a_H\left(\frac{P_c}{P_H}\right)^{\varpi} C_c = a_H g(\mathcal{T})^{\varpi} C_c$$
$$C_c = a_c\left(\frac{P_c}{P}\right)^{-\vartheta} C = a_c g(\mathcal{O})^{\vartheta} C$$
$$O_{cu} = a_O\left(\frac{P_o^*}{P}\right)^{-\vartheta} C = a_O\left(\frac{P}{P_o^*}\right)^{\vartheta} C = a_O\left(\frac{P}{P_c}\frac{P_c}{P_o^*}\right)^{\vartheta} C = a_O\left(\frac{g(\mathcal{O})}{\mathcal{O}}\right)^{\vartheta} C$$
$$C_H^* = a_H^*\left(\frac{P_H^*}{P_c^*}\right)^{-\varpi^*} C_c^* = a_H^*\left(\frac{g(\mathcal{T}^*)}{\mathcal{T}^*}\right)^{\varpi^*} C_c^*$$
$$C_c^* = a_c^*\left(\frac{P_c^*}{P^*}\right)^{-\vartheta^*} C^* = a_c^* g(\mathcal{O}^*)^{\vartheta^*} C^*$$
$$I_H = \xi g(\mathcal{T})^{\varpi_i} I \qquad I_{Hu} = \xi_u g(\mathcal{T})^{\varpi_u} I_u$$
$$I_H^* = \xi^*\left(\frac{g(\mathcal{T}^*)}{\mathcal{T}^*}\right)^{\varpi_i^*} I^* \qquad I_{Hu}^* = \xi_u^*\left(\frac{g(\mathcal{T}^*)}{\mathcal{T}^*}\right)^{\varpi_i^*} I_u^*$$

The same can be applied for foreign produced goods and investment goods:

$$C_F^* = a_F^* g(\mathcal{T}^*)^{\varpi^*} C_c^* \qquad C_F = a_F \left(\frac{g(\mathcal{T})}{\mathcal{T}}\right)^{\varpi} C_C$$

$$C_c^* = a_C^* g(\mathcal{T}^*)^{\varpi^*} C^*$$

$$O_{cu} = a_O \left(\frac{g(\mathcal{O})}{\mathcal{O}}\right)^{\vartheta} C$$

$$I_F = (1-\xi)\left(\frac{g(\mathcal{T})}{\mathcal{T}}\right)^{\varpi_i} I \qquad I_F^* = (1-\xi^*) g(\mathcal{T}^*)^{\varpi_i^*} I^*$$

$$I_{Fu} = (1-\xi_u)\left(\frac{g(\mathcal{T})}{\mathcal{T}}\right)^{\varpi_u} I_u \qquad I_{Fu}^* = (1-\xi_u^*) g(\mathcal{T}^*)^{\varpi_u^*} I_u^*$$

Next I define the following variables for ease in matlab:

$$A_C \equiv a_H a_C g(\mathcal{T})^{\varpi} g(\mathcal{O})^{\vartheta}$$
$$A_{C^*} \equiv a_H^* a_C^* \left(\frac{g(\mathcal{T}^*)}{\mathcal{T}^*}\right)^{\varpi^*} g(\mathcal{O}^*)^{\vartheta^*}$$
$$A_I \equiv \xi g(\mathcal{T})^{\varpi_i}$$
$$A_{I_u} \equiv \xi_u g(\mathcal{T})^{\varpi_u}$$
$$A_{I^*} \equiv \xi^* \left(\frac{g(\mathcal{T}^*)}{\mathcal{T}^*}\right)^{\varpi_i^*}$$
$$A_{I_u^*} \equiv \xi_u^* \left(\frac{g(\mathcal{T}^*)}{\mathcal{T}^*}\right)^{\varpi_u^*}$$

From the foreign country perspective :

$$B_{C^*} \equiv a_F^* a_C^* g(\mathcal{T}^*)^{\varpi^*} g(\mathcal{O}^*)^{\vartheta^*}$$
$$B_C \equiv a_F a_C \left(\frac{g(\mathcal{T})}{\mathcal{T}}\right)^{\varpi} g(\mathcal{O})^{\vartheta}$$
$$B_I \equiv (1-\xi) \left(\frac{g(\mathcal{T})}{\mathcal{T}}\right)^{\varpi_i}$$
$$B_{I_u} \equiv (1-\xi_u) \left(\frac{g(\mathcal{T})}{\mathcal{T}}\right)^{\varpi_u}$$
$$B_{I^*} \equiv (1-\xi^*) g(\mathcal{T}^*)^{\varpi_i^*}$$
$$B_{I^*} \equiv (1-\xi_u^*) g(\mathcal{T}^*)^{\varpi_u^*}$$

Then from the market clearing condition for manufacturing, I obtain that:

$$Y^d = A_c C + A_{c^*} C^* + A_i I + A_{i^*} I^* + A_u I_u + A_{u^*} I_u^*$$

Similarly, I can write the market clearing condition for foreign manufacturing markets as:

$$Y_m^{*d} = B_c C + B_{c^*} C^* + B_i I + B_{i^*} I^* + B_u I_u + B_{u^*} I_u^*$$

With this, I require two more transformations, I get that in the steady state:

$$I = \delta K$$
$$I_u = \delta_u \Psi_u$$

Similar conditions hold in the foreign block as well. Then from the market clearing conditions, substituting these in, one can find the aggregate consumption:

$$C = A_c^{-1}\left[Y^d - A_{c^*}C^* - A_i\delta\frac{K}{N_n}N_n - B_{i^*}\delta^*\frac{K_n^*}{N_n^*}N_n^* - A_u\delta_u\frac{\Psi_u}{N_u}N_u + A_{u^*}\delta_u^*\frac{\Psi_u^*}{N_u^*}^*\right]$$
$$C^* = B_{c^*}^{-1}\left[Y^{*d} - B_{c*}C - B_i^*\delta\frac{K}{N_n}N_n - B_{i^*}\delta^*\frac{K}{N_n^*}N_n^* - B_u\delta_u\frac{\Psi_u}{N_u}N_u + B_{u^*}\delta_u^*\frac{\Psi_u^*}{N_u^*}^*\right]$$

Solving the model is straight forward from here. I utilise the FSOLVE solver in matlab to derive the steady state values recursively. The equations solved recursively are as follows:

$$C = A_C^{-1}\left[\frac{Y^d}{N}N - A_{C^*}C^* - A_I\delta\frac{K}{N_n}N_n - A_{I^*}\delta^*\frac{K^*}{N_n^*}N_n^* - A_{I_u}\delta_u\frac{\Psi_u}{N_u}N_u - A_{I_u^*}\delta_u^*\frac{\Psi_u^*}{N_u^*}N_u^*\right]$$

$$C^* = B_{C^*}^{-1}\left[\frac{Y^{*d}}{N^*}N^* - B_C C - B_I\delta\frac{K}{N_n}N_n - B_{I^*}\delta^*\frac{K^*}{N_n^*}N_n^* - B_{I_u}\delta_u\frac{\Psi_u}{N_u}N_u - B_{I_u^*}\delta_u^*\frac{\Psi_u^*}{N_u^*}N_u^*\right]$$

$$Y^d = \left(\frac{K}{N_n}\right)^{\alpha}\left(\frac{O_{Hu}}{N_n}\right)^{\mu}N_n$$

$$Y^{*d} = \left(\frac{K^*}{N_n^*}\right)^{\alpha^*}\left(\left(\frac{O_{Hu}^*}{N_n^*}\right)^{\nu}\left(\frac{O_{Fu}^*}{N_n^*}\right)^{1-\nu}\right)^{\mu^*}N^*$$

$$O_u = \left(\frac{\Psi_u}{N_u}\right)^{\alpha_u}N_u$$

$$O_u^* = \left(\frac{\Psi_u^*}{N_u^*}\right)^{\alpha_u^*}N_u^*$$

$$N_n = \left[\frac{(\theta_w - 1)\Omega_n[1-h]^{-\sigma_c}C^{-\sigma_c}}{\theta_w}\right]^{\frac{1}{\sigma_n}}$$

$$N_u = \left[\frac{(\theta_w - 1)\Omega_u[1-h]^{-\sigma_c}C^{-\sigma_c}}{\theta_w}\right]^{\frac{1}{\sigma_n}}$$

$$N_n^* = \left[\frac{(\theta_w^* - 1)\Omega_n^*[1-h]^{-\sigma_c^*}C^{*-\sigma_c^*}}{\theta_w^*}\right]^{\frac{1}{\sigma_n^*}}$$

$$N_u^* = \left[\frac{(\theta_w^* - 1)\Omega_u^*[1-h]^{-\sigma_c^*}C^{*-\sigma_c^*}}{\theta_w^*}\right]^{\frac{1}{\sigma_n^*}}$$

$$I = A_I^{-1}\left[Y^d - A_{C^*}C^* - A_C C - A_{I^*}\delta^*\frac{K^*}{N^*}N^* - A_{I_u}\delta_u\frac{\Psi_u}{N_u}N_u - A_{I_u^*}\delta_u^*\frac{\Psi_u^*}{N_u^*}N_u^*\right]$$

$$I^* = B_{I^*}^{-1}\left[Y^{*d} - B_{C^*}C^* - B_C C - A_I I - B_{I_u}\delta_u\frac{\Psi_u}{N_u}N_u - B_{I_u^*}\delta_u^*\frac{\Psi_u^*}{N_u^*}N_u^*\right]$$

$$I_u = A_{I_u}^{-1}\left[Y^d - A_{C^*}C^* - A_C C - -A_{I^*}I - A_I I - A_{I_u^*}\delta_u^*\frac{\Psi_u^*}{N_u^*}N_u^*\right]$$

$$I_u^* = B_{I_u^*}^{-1}\left[Y^{*d} - B_{C^*}C^* - B_C C - B_{I^*}I^* - B_{I_u}I_u - B_I I\right]$$

$$O_{Hu} = \frac{\mu}{1-\alpha-\mu}\frac{\Omega_n}{P_o^*}N_n$$
$$O_{Hu}^* = \frac{\mu^*\nu}{1-\alpha^*-\mu^*}\frac{\Omega_n^*}{P_o^*}N_n^*$$
$$O_{Fu}^* = \frac{\mu^*(1-\nu)}{1-\alpha^*-\mu^*}\frac{\Omega_n^*}{P_o^*}N_n^*$$
$$O_{cu} = a_O\left(\frac{g(\mathcal{O})}{\mathcal{O}}\right)^{\vartheta}C$$
$$O_{cu}^* = a_O^*\left(\frac{g(\mathcal{O}^*)g(\mathcal{T}^*)\mathcal{T}}{g(\mathcal{O})\mathcal{O}}\right)^{\vartheta^*}C^*$$
$$O_{uw} = O_u + O_u^*$$
$$O_{dw} = O_{Hu} + O_{cu} + O_{Hu}^* + O_{Fu}^* + O_{cu}^*$$
$$O_{uw} = O_{dw}$$

Appendix B

The rest of the model is exactly as presented in Appendix A. Only the extension is presented here.

Financial Frictions Model

Oil Sector

Supply Chain Firms

The profit function for supply chain firms is given by (noting supply chain firms take prices as given):

$$\mathcal{D}_{\psi,t} = \tilde{P}_{\psi,t} Y_{\psi,t} - R_t^k K_{\psi,t} - \Omega_{\psi,t} N_{\psi,t}^{1-\alpha_k}$$

First order conditions for the static problem results in capital-labour ratio:

$$\frac{K_{\psi,t}}{N_{\psi,t}} = \frac{\alpha_k}{(1-\alpha_k)} \frac{\Omega_{\psi,t}}{R_t}$$

Market clearing requires that

$$Y_{\psi,t} = I_{u,t}^{agg} + a(U_t)\Psi_{u,t}$$

Since not all investment goods are purchased by the oil producing firms due to capital constraints, the residual investment good is given as:

$$I_{u,t}^R = I_{u,t}^{agg} - I_{u,t}$$

Furthermore, under the financial frictions model, oil sector investment is constrained to domestic investments alone such that $I_{u,t}$ is not an aggregated good of home and foreign goods.

Oil Producers

$$O_t^u = \varepsilon_{u,t}(\Psi_{u,t}^s)^{\alpha_u} N_{u,t}^{1-\alpha_u} \qquad \alpha_u \in [0,1)$$

$$\mathbb{E}_t \sum_{s=0}^{\infty} M_{t,s}\left[P_{o,s}^* \mathcal{S}_s \Phi_{F,s} O_{u,s} - \tilde{P}_{i,s} a(U_s)\Psi_{u,s} - \Omega_{u,s} N_{u,s} - \tilde{P}_{i,s} I_{u,s}\right]$$

Oil firms now face an additional capital constraint:

$$\tilde{P}_{\psi,t}\Psi_{u,t} = L_t^B$$

$$L_t^B \leq \amalg_t \mathcal{S}_t \Phi_{F,t} P_{o,t}^* O_t^u$$

Assuming the constraint binds, I obtain:

$$\tilde{P}_{\psi,t}\Psi_{u,t} = \amalg_t \mathcal{S}_t P_{o,t}^* \Phi_{F,t} O_t^u$$

$$\Psi_{u,t} = \left[\frac{\amalg_t \mathcal{S}_t P_{o,t}^* \Phi_{F,t} \varepsilon_{u,t} U_t^{\alpha_u} N_{u,t}^{1-\alpha_u}}{\tilde{P}_{\psi,t}}\right]^{\frac{1}{1-\alpha_u}}$$

So the firms optimality conditions are now subject to two capital constraints:

$$\Psi_{u,t+1} = (1-\delta_u)\Psi_{u,t} + \varepsilon_{\psi,t}^i\left[1 - S_u\left(\frac{I_{u,t}}{I_{u,t-1}}\right)\right] I_{u,t}$$

$$\tilde{P}_{\psi,t}\Psi_{u,t} = \amalg_t \mathcal{S}_t P_{o,t}^* \Phi_{F,t} O_t^u$$

Optimality conditions for utilisation rate, active oil rigs and level of investment are now given

by:

$$\mathbb{E}_t\Bigg[M_{t,t+1}\Bigg[\alpha_u\frac{\mathcal{S}_{t+1}\Phi_{F,t+1}P^*_{o,t+1}O_{u,t+1}}{\Psi_{t+1}}-\tilde{P}_{\psi,t+1}a(U_{u,t+1})$$

$$+\mathcal{Q}_{u,t+1}(1-\delta_u)-\mathbb{Z}_{t+1}\left(\alpha_u\frac{\amalg_{t+1}\mathcal{S}_{t+1}\Phi_{F,t+1}P^*_{o,t+1}O_{u,t+1}}{\Psi_{t+1}}-\tilde{P}_{\psi,t+1}\right)\Bigg]\Bigg]=\mathcal{Q}_{u,t}$$

$$\mathbb{E}_t\left[M_{t,t+1}\epsilon^i_{\psi,t+1}\mathcal{Q}_{u,t+1}S'\left(\frac{I_{u,t+1}}{I_{u,t}}\right)\left(\frac{I_{u,t+1}}{I_{u,t}}\right)^2\right]$$

$$+\epsilon^i_{\psi,t}\mathcal{Q}_{u,t}\left[1-S\left(\frac{I_{u,t}}{I_{u,t-1}}\right)-S'\left(\frac{I_{u,t}}{I_{u,t-1}}\right)\frac{I_{u,t}}{I_{u,t-1}}\right]=\tilde{P}_{\psi,t}$$

The static FOCs are thus given by:

$$\alpha_u\frac{\mathcal{S}_t\Phi_{F,t}P^*_{o,t}O_{u,t}}{U_t}=\tilde{P}_{\psi,t}a'(U_t)\Psi_t-\mathbb{Z}_t\alpha_u\frac{\amalg_t\mathcal{S}_t\Phi_{F,t}P^*_{o,t}O_{u,t}}{U_t}$$

$$\Rightarrow \alpha_u(1+\mathbb{Z}_t\amalg_t)\frac{\mathcal{S}_t\Phi_{F,t}P^*_{o,t}O_{u,t}}{U_t}=\tilde{P}_{\psi,t}a'(U_t)\Psi_t$$

$$(1-\alpha_u)\frac{\mathcal{S}_t\Phi_{F,t}P^*_{o,t}O_{u,t}}{N_{u,t}}=\Omega_{u,t}-\mathbb{Z}_t(1-\alpha_u)\frac{\amalg_t\mathcal{S}_t\Phi_{F,t}P^*_{o,t}O_{u,t}}{N_{u,t}}$$

$$\Rightarrow (1-\alpha_u)(1+\mathbb{Z}_t\amalg_t)\frac{\mathcal{S}_t\Phi_{F,t}P^*_{o,t}O_{u,t}}{N_{u,t}}=\Omega_{u,t}$$

$$\Rightarrow \frac{\bar{\Psi}_{u,t}}{N_u,t}=\frac{\alpha_u}{(1-\alpha_u)}\frac{\Omega_{u,t}}{\tilde{P}_{c,t}a'(U_t)}$$

From combined constraints, solve for $\Psi_{u,t}$ and obtain:

$$\Psi_{u,t}=\left[\alpha_u(1+\mathbb{Z}_t\amalg_t)\frac{\mathcal{S}_t\Phi_{F,t}P^*_{o,t}\varepsilon_{u,t}U_t^{\alpha_u}N_{u,t}^{1-\alpha_u}}{\tilde{P}_{\psi,t}a'(U_t)U_t}\right]^{\frac{1}{1-\alpha_u}}$$

combining both equations yields:

$$\amalg_t=\alpha_u\frac{(1+\mathbb{Z}_t\amalg_t)}{U_ta'(U_t)}$$

$$\Leftrightarrow \mathbb{Z}_t=\frac{\amalg_tU_ta'(U_t)-\alpha_u}{\alpha_u\amalg_t}$$

if $\mathbb{Z}_t\ >\ 0$ then:

$$\frac{\amalg_tU_ta'(U_t)-\alpha_u}{\alpha_u\amalg_t}>0$$

$$\Leftrightarrow \amalg_t>\frac{\alpha_u}{U_ta'(U_t)}\quad \&\quad \alpha_u>0\quad \&\quad \amalg_t>0$$

I choose the loan parameter II_t as a function of international real oil prices:

$$\text{II}_t = \text{II}_1 \left(\frac{\mathcal{S}_t P^*_{o,t}}{\mathcal{S}_{t-1} P^*_{o,t-1}} \right)^{\text{II}_2}$$

Here, II_1 is used to determine the decision on binding constraint. I calibrate it accordingly to match the binding constraint.

Appendix C

Two Asset RANK Economy

Households

This is derived from the Kaplan *et. al.* (2018) model, a TANK model with two assets. The non-HTM households problem is:

$$\max_{\{c_t\ \ell_t\ d_t\ a_{t+1}\ b_{t+1}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\ln C_t - \varrho \frac{\ell_t^{1+\sigma_n}}{1+\sigma_n} \right]$$

Subject to the flow budget constraint:

$$C_t + B_{t+1} + d_t + \chi(d_t) = (1-\tau)(w_t \ell_t + \Gamma_t) + T_t + (1+r_t^b) b_t$$
$$a_{t+1} = (1+r_t^a) a_t + d_t$$
$$\chi(d_t) = \chi_1 |d_t|^{\chi_2}$$

The lagrangian reads:

$$\begin{aligned}
\mathcal{L}_t &= \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\ln C_t - \varrho \frac{\ell_t^{1+\sigma_n}}{1+\sigma_n} \right] + \lambda_t \Big[(1-\tau)(w_t \ell_t + \Gamma_t) + T_t + (1+r_t^b) b_t - C_t - b_{t+1} \\
&\qquad - d_t - \chi(d_t) \Big] + \mathcal{Q}_t \left[(1+r_t^a) a_t + d_t - a_{t+1} \right] \\
&= \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \lambda_t \Bigg(\Big[\ln C_t + \varrho \frac{\ell_t^{1+\sigma_n}}{1+\sigma_n} + (1-\tau)(w_t \ell_t + \Gamma_t) + T_t + (1+r_t^b) b_t - C_t - b_{t+1} \\
&\qquad - d_t - \chi(d_t) \Big] + \mathcal{Q}_t \left[(1+r_t^a) a_t + d_t - a_{t+1} \right] \Bigg)
\end{aligned}$$

First order conditions read:

$$\lambda_t = \frac{1}{C_t}$$

$$\varrho \ell_t^{\sigma_n} = (1-\tau) w_t \lambda_t$$

$$\Rightarrow \frac{\varrho \ell_t^{\sigma_n}}{\lambda_t} = (1-\tau) w_t \Rightarrow \varrho \ell_t^{\sigma_n} C_t = (1-\tau) w_t$$

$$\beta \, \mathbb{E}_t \, V'(b_{t+1}) = \lambda_t$$

$$V'(b_t) = \lambda_t (1 + r_t^b) \Rightarrow v'(b_{t+1}) = \lambda_{t+1}(1 + r_{t+1}^b)$$

$$\mathbb{E}_t \left[\beta \lambda_{t+1} (1 + r_{t+1}^b) \right] = \lambda_t \Rightarrow E_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} (1 + r_{t+1}^b) \right] = 1$$

Now I define the household stochastic discount factor, that is common to both households and firms:

$$M_{t,t+1} = \beta \frac{\lambda_{t+1}}{\lambda_t} = \beta \frac{C_t}{C_{t+1}}$$

Proceeding , the first order conditions for illiquid assets and deposits are therefore:

$$\beta \, \mathbb{E}_t \, V'(a_{t+1}) = \lambda_t \mathcal{Q}_t$$

$$\Rightarrow \mathcal{Q}_t = M_{t,t+1} \left[\mathcal{Q}_{t+1} (1 + r_{t+1}^a) \right]$$

$$\mathcal{Q}_t = 1 + sign(d_t) \times \chi_1 \chi_2 2 |d_t|^{\chi_2 - 1}$$

HTM households consume all their labour income and transfers for government but hold no liquid or illiquid assets. The HTM household's problem is:

$$\max_{\{c_t \; \ell_t\}} \ln C_t^{htm} - \varrho \frac{\ell_t^{htm \; (1+\sigma_n)}}{1+\sigma_n}$$

$$\text{S.T.} \quad C_t^{htm} = (1-\tau)(w_t \ell_t^{htm} + \Gamma_t^{htm}) + T_t^{htm}$$

The FOCs to the problem are:

$$C_t^{htm} = (1-\tau)(w_t \ell_t^{htm} + \Gamma_t^{htm}) + T_t^{htm}$$

$$\varrho \ell_t^{htm \sigma_n} C_t^{htm} = (1-\tau) w_t$$

Firms Problem

The Final Good Firm The final producer combines all the intermediate producer goods into an index of final consumption to met aggregate demand:

$$Y_t^d = \left(\int Y_t(f)^{\frac{(\theta_p - 1)}{\theta_p}} \right)^{\frac{\theta_p}{(\theta_p - 1)}}$$

The final producer problem is quite straight forward as it tries to solve:

$$\max_{Y_t(f)\}} P_t Y_t^d - \int_0^1 P_t(f) Y_t(f)$$

Optimisation problem results in the following demand:

$$Y_t(f) = \left(\frac{P_t(f)}{P_t} \right)^{-\theta_p} Y_t^d$$

The Intermediate Goo Firms As in standard DSGE literature, there is a continuum of intermediate good firms f with production function:

$$Y_t(f) = K_t(f)^{\alpha} N_t(f)^{1-\alpha}$$

Each intermediate good firm maximises profit:

$$\frac{P_t(f)}{P_t} Y_t(f) - w_t N_t(f) - r_t^k K_t(f) - \Theta\left(\frac{P_t(f)}{P_{t-1}(f)} \right)$$

price setting is based on Rotemberg pricing, with $\Theta\left(\frac{P_t(f)}{P_{t-1}(f)}\right)$ being the adjustment cost for changing prices each period. Following Kaplan *et. al.*(2018) this is a quadratic adjustment cost function such that:

$$\Theta\left(\frac{P_t(f)}{P_{t-1}(f)} \right) = \frac{\theta}{2} \left(\frac{P_t(f)}{P_{t-1}(f)} - 1 \right)^2 Y_t^d$$

The choice for $K_t(f)$ and $N_t(f)$ s a static one and first order conditions are straight forward to write as:

$$w_t = (1-\alpha)\Phi_t(f)K_t(f)^{\alpha}N_t(f)^{-\alpha}$$
$$r_t^k = \alpha\Phi_t(f)K_t(f)^{\alpha}N_t(f)^{-\alpha}$$
$$\Rightarrow \frac{K_t}{N_t} = \frac{\alpha}{1-\alpha}\frac{w_t}{r_t^k}$$

one can show that the real marginal cost $\Phi_t(f)$ is independent of intermediate inputs such that:

$$\Phi_t(f) = \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{r_t^k l}{\alpha}\right)^{\alpha} = \Phi_t$$

Each intermediate producer then choose the path for prices $\{P_t(f)\}_0^{\infty}$ to maximise the present value of discounted profits:

$$\max_{\{P_t(f)\}_0^{\infty}} \sum_{t=0}^{\infty} \lambda_t Q_t \left[\frac{P_t(f)}{P_t}Y_t(f) - w_t N_t(f) - r_t^k K_t(f) - \Theta\left(\frac{P_t(f)}{P_{t-1}(f)}\right)\right]$$
$$\max_{\{P_t(f)\}_0^{\infty}} \sum_{t=0}^{\infty} \lambda_t Q_t \left[\left(\frac{P_t(f)}{P_t} - \Phi_t\right)Y_t(f) - \Theta\left(\frac{P_t(f)}{P_{t-1}(f)}\right)\right]$$

Subject to

$$Y_t(f) = \left(\frac{P_t(f)}{P_t}\right)^{-\theta_p} Y_t^d$$

The firms problem in period $t+1$ is then given as:

$$\left(\frac{P_t(f)}{P_t} - \Phi_t\right)Y_t(f) - \Theta\left(\frac{P_t(f)}{P_{t-1}(f)}\right) + \left(\frac{P_{t+1}(f)}{P_{t+1}} - \Phi_{t+1}\right)Y_{t+1}(f) - \Theta\left(\frac{P_{t+1}(f)}{P_t(f)}\right)$$

So the firms problem become:

$$
\begin{aligned}
&\max_{\{P_t(f)\}_0^\infty} \sum_{t=0}^{\infty} \lambda_t Q_t \left[\left(\frac{P_t(f)}{P_t} - \Phi_t \right) Y_t(f) - \Theta\left(\frac{P_t(f)}{P_{t-1}(f)} \right) \right] \\
&+\lambda_{t+1} \mathcal{Q}_{t+1} \left(\frac{P_{t+1}(f)}{P_{t+1}} - \Phi_{t+1} \right) Y_{t+1}(f) - \Theta\left(\frac{P_{t+1}(f)}{P_t(f)} \right) \\
&\max_{\{P_t(f)\}_0^\infty} \sum_{t=0}^{\infty} \lambda_t Q_t \left[\left(\frac{P_t(f)}{P_t} - \Phi_t \right) \left(\frac{P_t(f)}{P_t} \right)^{-\theta_p} Y_t^d - \frac{\theta}{2} \left(\frac{P_t(f)}{P_{t-1}(f)} - 1 \right)^2 Y_t^d \right] \\
&+\lambda_{t+1} \mathcal{Q}_{t+1} \left[\left(\frac{P_{t+1}(f)}{P_{t+1}} - \Phi_{t+1} \right) \left(\frac{P_{t+1}(f)}{P_{t+1}} \right)^{-\theta_p} Y_{t+1}^d - \frac{\theta}{2} \left(\frac{P_{t+1}(f)}{P_t(f)} - 1 \right)^2 Y_{t+1}^d \right]
\end{aligned}
$$

The first order conditions are therefore:

$$
\begin{aligned}
&\lambda_t Q_t \Bigg[(1-\theta_p) P_t(f)^{-\theta_p} \left(\frac{1}{P_t} \right)^{1-\theta_p} Y_t^d + \theta_p \Phi_t P_t(f)^{-\theta_p - 1} \left(\frac{1}{p_t} \right) \\
&- \theta \left(\frac{P_t(f)}{P_{t-1}(f)} - 1 \right) \frac{1}{P_{t-1}(f)} Y_t^d \Bigg] + \lambda_{t+1} \mathcal{Q}_{t+1} \left[\theta \left(\frac{P_{t+1}(f)}{P_t(f)} - 1 \right) \frac{Y_{t+1}^d}{P_t(f)} \frac{P_{t+1}(f)}{P_t(f)} \right] = 0
\end{aligned}
$$

In a symmetric equilibrium all firms will choose the same price i.e. $P_t(f) = P_t$. So Imposing a symmetric equilibrium, one obtains:

$$
\begin{aligned}
&\lambda_t \mathcal{Q}_t \left[(1-\theta_p) \frac{Y_t^d}{P_t} + \theta_p \Phi_t \frac{Y_t^d}{P_t} - \theta \left(\frac{P_t(f)}{P_{t-1}(f)} - 1 \right) \frac{1}{P_{t-1}(f)} Y_t^d \right] \\
&\qquad + \lambda_{t+1} \mathcal{Q}_{t+1} \left[\theta \left(\frac{P_{t+1}}{P_t} - 1 \right) \frac{Y_{t+1}^d}{P_t} \frac{P_{t+1}}{P_t} \right] = 0
\end{aligned}
$$

Now using that $\pi_t = \frac{P_t}{P_{t-1}} - 1$ I can reduce the equation above to:

$$
\begin{aligned}
\lambda_t \mathcal{Q}_t \left[(1-\theta_p) \frac{Y_t^d}{P_t} + \theta_p \Phi_t \frac{Y_t^d}{P_t} - \theta \pi_t \frac{1}{P_{t-1}(f)} Y_t^d \right] + \lambda_{t+1} \mathcal{Q}_{t+1} \left[\theta \pi_{t+1} \frac{Y_{t+1}^d}{P_t} (\pi_{t+1} + 1) \right] &= 0 \\
(1-\theta_p) \frac{Y_t^d}{P_t} + \theta_p \Phi_t \frac{Y_t^d}{P_t} - \theta \pi_t \frac{1}{P_{t-1}(f)} Y_t^d + \frac{\lambda_{t+1} \mathcal{Q}_{t+1}}{\lambda_t Q_t} \left[\theta \pi_{t+1} \frac{Y_{t+1}^d}{P_t} (\pi_{t+1} + 1) \right] &= 0 \\
(1-\theta_p) + \theta_p \Phi_t - \theta \pi_t \frac{P_t}{P_{t-1}} + \frac{\lambda_{t+1} \mathcal{Q}_{t+1}}{\lambda_t Q_t} \left[\theta \pi_{t+1} \frac{Y_{t+1}^d}{Y_t^d} (\pi_{t+1} + 1) \right] &= 0 \\
1 - \theta \pi_t (1 + \pi_t) + \frac{\lambda_{t+1} \mathcal{Q}_{t+1}}{\lambda_t Q_t} \left[\theta \pi_{t+1} \frac{Y_{t+1}^d}{Y_t^d} (\pi_{t+1} + 1) \right] &= (1 - \Phi_t) \theta_p
\end{aligned}
$$

This last equation is the new Keynesian Phillips curve that relates inflation to the real marginal cost and demand. Finally I can now aggregate over ll intermediate firms such that:

$$Y_t(f) = K_t(f)^{\alpha} N_t(f)^{1-\alpha} \Rightarrow Y_t^d = K_t^{\alpha} N_t^{1-\alpha}$$

Then imposing the symmetric equilibrium prices and aggregating over all intermediate firms, profits are then:

$$\Pi_t = Y_t^d \left((1 - \Phi_t) - \frac{\theta}{2}\pi_t^2 \right)$$

Illiquid asset Holdings Illiquid asset is a constitute of both capital holding. and claims on equity as a fraction of firms profit. The no arbitrage condition dictates that the return on capital must equal the return on equity such that:

$$\begin{aligned} r_t^a \equiv \frac{\eta\Pi_t + (q_t - q_{t-1})}{q_{t-1}} &= r_t^k - \delta \\ \Leftrightarrow q_t = (1 + r_t^a)q_{t-1} - \eta\Pi_t &\Rightarrow q_{t+1} = (1 + r_{t+1}^a)q_t - \eta\Pi_{t+1} \\ q_t = \frac{1}{(1 + r_{t+1}^a)} (q_{t+1} + \eta\Pi_t) & \end{aligned} \tag{5.0.2}$$

Also I can express $(1 + r_t^a)q_{t-1} = \eta\Pi_t + q_t$. Then the law of motion for illiquid wealth can be expressed as:

$$a_{t+1} \equiv K_{t+1} + S_{t+1}q_t \Rightarrow a_t = K_t + S_t q_{t-1}$$

Recall from household budget constraint $a_{t+1} = (1 + r_t^a)a_t + d_t$, then:

$$\begin{aligned} a_{t+1} = (1 + r_t^a)a_t + d_t &= (1 + r_t^a)(K_t + S_t q_{t-1}) + d_t \\ &= (1 + r_t^k - \delta)K_t + (1 + r_t^a)q_{t-1}S_t + d_t \\ &\Rightarrow (1 + r_t^k - \delta)K_t + (\eta\Pi_t + q_t)S_t + d_t \end{aligned}$$

The remaining fraction of profits are then redistributed to households as direct transfers, such that:

$$\Gamma_t^{agg} = (1 - \eta)\Pi_t \quad \Gamma_t^{agg} = \Gamma_t = \Gamma_t^{htm}$$

Kaplan *et. al.* (2018) set $\eta = \alpha$ so as to neutralise the role of countercyclical profits.

Monetary Authority and Government

Government and LSAP: Government issues Bond denoted B_t^g such that a negative value indicates government debt. Therefore a higher negative corresponds to higher government debt, which is the focus of this paper. The Flow budget constraint of the government is given by:

$$B_{t+1}^g = (1 + r_t^b)B_t^g + \tau(wtN_t + \Gamma_t^{agg}) - T_t^{agg} - G_t$$

Where:

$$B_t^g = BTg + \varepsilon_t$$

Where ε_t is a Bond shock that follows an AR(1) process:

$$\varepsilon_t = \rho\varepsilon_{t-1} + e_t$$

Government transfers are then distributed across households such that:

$$T_t^{agg} = \Lambda T_t^{htm} + (1 - \Lambda)T_t$$

The share of HTM fraction of transfers is Λ^{htm} such that:

$$\Lambda^{htm} T_t^{agg} = \Lambda T_t^{htm}$$

That is. the HTM households share of government transfers is equal to the fraction of the population that re HTM households.

Central Bank The central bank follows a simple Taylor rule that seeks to stabilise only inflation:

$$i_t = \bar{r}_t^b + \rho_\pi \pi_t$$

Given inflation and the nominal interest rate, the real return on liquid assets is therefore given

by:

$$1 + r_t^b = \frac{1 + i_{t-1}}{1 + \pi_t}$$

Equilibrium Conditions Two Asset TANK

- Household Conditions

$$M_{t,t+1} = \beta \frac{\lambda_{t+1}}{\lambda_t} \tag{R1}$$

$$\lambda_t = \frac{1}{C_t} \tag{R2}$$

$$(1-\tau)w_t = \varrho \ell_t^{\sigma_n} C_t \tag{R3}$$

$$1 = E_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} (1 + r_{t+1}^b) \right] \tag{R4}$$

$$\mathcal{Q}_t = M_{t,t+1} \left[\mathcal{Q}_{t+1} (1 + r_{t+1}^a) \right] \tag{R5}$$

$$\mathcal{Q}_t = 1 + sign(d_t) \times \chi_1 \chi_2 2 |d_t|^{\chi_2 - 1} \tag{R6}$$

- Firm Conditions

$$\frac{K_t}{N_t} = \frac{\alpha}{1-\alpha} \frac{w_t}{r_t^k} \tag{R7}$$

$$\Phi_t = \left(\frac{w_t}{1-\alpha} \right)^{1-\alpha} \left(\frac{r_t^k}{\alpha} \right)^{\alpha} \tag{R8}$$

$$Y_t^d = K_t^\alpha N_t^{1-\alpha} \tag{R9}$$

$$\Theta = \frac{\theta}{2} \pi_t^2 Y_t^d \tag{R10}$$

$$(1-\Phi_t)\theta_p = 1 - \theta \pi_t (1 + \pi_t) + \beta \frac{\lambda_{t+1} \mathcal{Q}_{t+1}}{\lambda_t Q_t} \left[\theta \pi_{t+1} \frac{Y_{t+1}^d}{Y_t^d} (\pi_{t+1} + 1) \right] \tag{R11}$$

$$\Gamma_t = (1-\eta)\Pi_t \tag{R12}$$

$$\Pi_t = Y_t^d \left((1-\Phi_t) - \frac{\theta}{2} \pi_t^2 \right) \tag{R13}$$

- Illiquid Asset Market

$$a_{t+1} = K_{t+1} + s_{t+1}q \tag{R14}$$

$$a_{t+1} = (1 + r_t^k - \delta)K_t + (\eta\Pi_t + q_t)S_t + d_t \tag{R15}$$

$$q_t = \frac{1}{(1 + r_{t+1}^a)}\left(q_{t+1} + \eta\Pi_t\right) \tag{R16}$$

$$K_{t+1} = (1 - \delta)K_t + I_t \tag{R17}$$

$$r_t^k = r_t^a + \delta \tag{R18}$$

- Monetary and Fiscal Policy

$$i_t = \bar{r}_t^b + \rho_\pi \pi_t + \varepsilon_t \tag{R19}$$

$$1 + r_t^b = \frac{1 + i_{t-1}}{1 + \pi_t} \tag{R20}$$

- *Hand-to-Mouth* Household Optimality

$$C_t^{htm} = (1 - \tau)(w_t \ell_t^{htm} + \Gamma_t^{htm}) + T_t^{htm} \tag{R21}$$

$$\varrho \ell_t^{htm\sigma_n} C_t^{htm} = (1 - \tau)w_t \tag{R22}$$

- Market Clearing

$$Y_t^d = C_t + I_t + G_t + \Theta_t \tag{R23}$$

$$K_{t+1}^{agg} = (1 - \Lambda)K_{t+1} \tag{R24}$$

$$B_{t+1}^g = (1 + r_t^b)B_t^g + \tau(wtN_t + \Gamma_t) - T_t - G_t \tag{R25}$$

$$s_{t+1}(1 - \Lambda) = 1 \tag{R26}$$

$$\ell_t + ell_t^{htm} = N_t \tag{R27}$$

$$b_t(1 - \Lambda) = -B_t^g \tag{R28}$$

$$B_t^G = B^G + \varepsilon_t \tag{R29}$$

$$G_t = G \tag{R30}$$

- Transfers and Profit Redistribution

$$T_t^{agg} = \Lambda T_t^{htm} + (1-\Lambda)T_t \tag{R31}$$

$$\Lambda^{htm} T_t^{agg} = \Lambda T_t^{htm} \tag{R32}$$

$$\Gamma_t^{agg} = (1-\eta)\Pi_t \tag{R33}$$

$$\Gamma_t^{agg} = \Gamma_t = \Gamma_t^{htm} \tag{R34}$$

- Exogenous Shock Process

$$\varepsilon_t = \rho\varepsilon_{t-1} + e_t \tag{R35}$$

Two Asset HANK Closed Economy

Households

The Households problem is to choose the level of consumption, c_t, labour hours, ℓ_t, and depositis, d_t:

$$\begin{gathered}
\max_{\{c_t \ell_t d_t\}} \quad \mathbb{E}_0 \int_0^{\infty} e^{(-\beta+\eta)} U(c_t, \ell_t)\ \mathrm{d}t \\
\text{S.t.} \quad \dot{b}_t = (1-\tau)w_t e^{y_t}\ell_t + (r^b(b_t) + \zeta_t)b_t + T_t - d_t - \chi(d_t, a_t) - c_t \\
\dot{a}_t = r_t^a a_t + d_t \\
b_t \geq -b, \quad a_t \geq 0
\end{gathered}$$

Here, b_t are liquid assets (bonds), a_t are illiquid assets (investment in capital), y_t is income, w_t is wages from labour and χ is the transaction cost. r_t^a is return on capital assets, r_t^b is return on bonds. The budget constraint for liquid (bond) wealth differs from Kaplan *et. al.* (2018), to derive this, I follow the Posch (2018) and let the q_t^b denote the price of bonds at time t, and let $\tilde{b}_t$ be the number of bonds each individual has. Then the budget constraint for each individual is given as:

$$\dot{\tilde{b}}_t q_t^b = (1-\tau)w_t e^{y_t}\ell_t + r^b(b_t)b_t + T_t - d_t - \chi(d_t, a_t) - c_t$$

Similalry, let bond prices follow the process:

$$\dot{q_t}^b = \zeta_t q_t^b$$

Here ζ captures the price changes in the economy. Following Kaplan *et. al.* (2018) I will abuse the notation slightly and write:

$$b_t = \tilde{b_t} q_t^b$$
$$\dot{b_t} = (1-\tau) w_t e^{y_t} \ell_t + r^b(b_t) b_t + T_t - d_t - \chi(d_t, a_t) - c_t + \zeta_t b_t$$

Finally, the transaction costs function is given by:

$$\chi(d_t, a_t) = \chi_0 |d| + \frac{\chi_1}{2} \left(\frac{d}{a}\right)^2 a$$

With $\chi_0, \chi_1 > 0$, the adjustment cost function has a kink at $d = 0$, therefore one would have that at $d = 0$ house holds become inactive in deposits, while at the convex component of the adjustment cost function, households have a finite deposit rate. Finally, as in Kaplan *et. al.* (2018), assume the logarithm of income $y_{it} = \ln z_{it}$ follows a jump drift process:

$$\mathrm{d}y_{it} = -\rho y_{it} \mathrm{d}t + dJ_{i,t}$$

Jumps arrive at a Poisson arrival rate λ. Conditional on a jump, a new log-earnings state is drawn y'_{it} such that $y'_{it} \sim N(0, \sigma^2)$. Household composition of illiquid wealth is characterised by illiquid savings which is invested into Capital, k_t and equity shares, s_t. Here, equity shares represents dividend claims in standard New Keynesian literature, that is, it represents a claim on intermediate goods firms future stream of profits, i.e. $\Pi_t = \mathcal{D}_t - \Psi(\pi_t)$. Now define the price of a share $q_t(s_t)$, then household illiquid assets satisfy:

$$a_t = k_t + q_t(s_t) s_t$$

Then the dynamics of Capital and equity satisfy:

$$\dot{k_t} + q_t(s_t) \dot{s_t} = (R_t^k - \delta) k_t + \Pi_t s_t + d_t$$

Under the assumption of zero adjustment cost in illiquid account, illiquid savings can be redistributed between capital and equity without incurring any costs. Thus, no-arbitrage holds and the return from holding equity share must equal the return from holding capital:

$$\frac{\Pi_t + \dot{q}_t(s_t)}{q_t(s_t)} = R_t^k - \delta =: r_t^a$$

Given the reult above, the dimensionality of illiquid asset space can be reduced to the combined illiquid asset a_t with return on asset holding r_t^a and law of motion $\dot{a}_t = r_t^a + d_t$ This condition essentially reduces the illiquid asset to a single state variable and only holds in the absence of jumps in price of equity shares $q_t(s_t)$. Then express the present value Hamiltonian for the households problem as:

$$H(a,b,y) = U(c_t,\ell_t) + \xi_1 \left[(1-\tau)we^y\ell + (r^b(b)+\zeta)b + T - d - \chi(d,a) - c\right] \\ +\xi_2\left[r^a a + d\right] + \xi_3\left[-\rho y\right] \qquad \text{(H1)}$$

Now assuming there is no death in the model $\eta = 0$ and considering the discount factor $\beta(\Delta) = e^{-\rho\Delta}$, note that $\lim_{\Delta\to 0}\beta(\Delta) = 1$ and $\lim_{\Delta\to\infty}\beta(\Delta) = 0$. Now consider the bellman equation for the household problem in discrete time:

$$V(a_t,b_t,y_t) = \max_{\{c_t\ell_t d_t\}} \quad U(c_t,\ell_t) + e^{-\rho}V(a_{t+1},b_{t+1},y_{t+1})$$

$$\text{S.t.} \quad b_{t+1} = (1-\tau)w_t e^{y_t}\ell_t + (r^b(b_t)+\zeta_t)b_t + T_t - d_t - \chi(d_t,a_t) - c_t$$

$$a_{t+1} = r_t^a a_t + d_t$$

Then applying the following approximation:

$$b_{t+1} \approx \frac{b_{t+\Delta} - b_t}{\Delta t} \quad a_{t+1} \approx \frac{a_{t+\Delta} - a_t}{\Delta t}$$

The Bellman equation and household problem becomes:

$$V(a_t,b_t,y_t) = \max_{\{c_t\ell_t d_t\}} \quad \Delta U(c_t,\ell_t) + e^{-\rho}V(a_{t+\Delta},b_{t+\Delta},y_{t+\Delta})$$

$$\text{S.t.} \quad b_{t+\Delta} = \Delta\left((1-\tau)w_t e^{y_t}\ell_t + (r^b(b_t)+\zeta_t)b_t + T_t - d_t - \chi(d_t,a_t) - c_t\right) + b_t$$

$$a_{t+\Delta} = \Delta\left(r_t^a + d_t\right) + a_t$$

For small Δ, take $\Delta \to 0$, such that $e^{-\rho\Delta} = 1 - \rho\Delta$ then have that:

$$
\begin{aligned}
V(a_t, b_t, y_t) &= \Delta U(c_t, \ell_t) + (1 - \rho\Delta)V(a_{t+\Delta}, b_{t+\Delta}, y_{t+\Delta}) \\
V(a_t, b_t, y_t) - (1 - \rho\Delta)V(a_t, b_t, y_t) &= \Delta U(c_t, \ell_t) + (1 - \rho\Delta)\Big[V(a_{t+\Delta}, b_{t+\Delta}, y_{t+\Delta}) \\
&\quad - V(a_t, b_t, y_t)\Big] \\
\rho\Delta V(a_t, b_t, y_t) &= \Delta U(c_t, \ell_t) + (1 - \rho\Delta)\left[V(a_{t+\Delta}, b_{t+\Delta}, y_{t+\Delta}) - V(a_t, b_t, y_t)\right] \\
\rho V(a_t, b_t, y_t) &= U(c_t, \ell_t) + \frac{(1 - \rho\Delta)}{\Delta}\left[V(a_{t+\Delta}, b_{t+\Delta}, y_{t+\Delta}) - V(a_t, b_t, y_t)\right] \\
\rho V(a_t, b_t, y_t) &= U(c_t, \ell_t) + (1 - \rho\Delta)\Big[\frac{V(a_{t+\Delta}) - V(a_t)}{a_{t+\Delta} - a_t}\frac{a_{t+\Delta} - a_t}{\Delta} \\
&\quad + \frac{V(b_{t+\Delta}) - V(b_t)}{b_{t+\Delta} - b_t}\frac{b_{t+\Delta} - b_t}{\Delta} + \frac{V(y_{t+\Delta}) - V(y_t)}{y_{t+\Delta} - y_t}\frac{y_{t+\Delta} - y_t}{\Delta}\Big]
\end{aligned}
$$

Now taking $\Delta \to 0$

$$
\rho V(a_t, b_t, y_t) = U(c_t, \ell_t) + V'(a_t)\dot{a}_t + V'(b_t)\dot{b}_t + V'(y_t)\dot{y}_t \tag{B1}
$$

Now comparing equation $H1$ and equation $B1$, I get that $\xi_1 = V'(b_t)$, $\xi_2 = V'(a_t)$, and $\xi_3 = V'(y_t)$. That is the co-state variable from the Bellman equation equals the shadow value from the Present Value Hamiltonian. Therefore, the first order conditions along with the envelope conditions are necessary and sufficient for an optimum. Therefore I can write the household problem as a Hamiltonain-Jacobi-Bellman equation given by:

$$
\begin{aligned}
(\beta + \eta)V(a_t, b_t, y_t) = \max_{\{c_t \ell_t d_t\}} \; & U(c_t, \ell_t) + V_b'(a_t, b_t, y_t)\Big[(1 - \tau)w_t e^{y_t}\ell_t + (r^b(b_t) + \zeta)b_t \\
& + T_t - d_t - \chi(d_t, a_t) - c_t\Big] V_a'(a_t, b_t, y_t)\left[r_t^a a_t + d_t\right] + V_y'(a_t, b_t, y_t)(-\rho y) \\
& + \lambda \int_{-\infty}^{\infty} \big(V(a_t, b_t, y_t') - V(a_t, b_t, y_t)\big)\,\phi(y_t')\mathrm{d}y_t'
\end{aligned}
$$

Then the stationary version of the households HJB equation is given as:

$$\begin{aligned}(\beta+\eta)V(a,b,y) = \max_{\{c,\ell,d\}} \; & U(c,\ell) + V_b'(a,b,y)\Big[(1-\tau)we^y\ell + (r^b(b)+\zeta)b + T \\ & - d - \chi(d,a) - c\Big] V_a'(a,b,y)\left[r^a a + d\right] + V_y'(a,b,y)(-\rho y) \\ & + \lambda \int_{-\infty}^{\infty} \big(V(a,b,y') - V(a,b,y)\big)\,\phi(y')\mathrm{d}y'\end{aligned}$$

The evolution of the joint distribution of liquid and illiquid wealth can be described by the Fockker-Plankt equation (Kolmogorov Forward Equation). Now define $g(a,b,y,t)$ as the density function that corresponds to the distribution of $\mu_t(a,b,z)$, where I have used that $y = logz$. Finally, I can define the optimal drifts in in the stationary HJB equation above for liquid and illiquid state space as $s^b(a,b,y)$ and $s^a(a,b,y)$ respectively. These functions define the optimal savings policy for liquid and illiquid assets respectively. Then I have that the stationary density satisfies the Fokker-Plankt equation:

$$\begin{aligned}0 = \partial_a\left(s^a(a,b,y)g(a,b,y)\right) - \partial_b\left(s^b(a,b,y)g(a,b,y)\right) - \partial_y\left(-\rho y g(a,b,y)\right) - \lambda g(a,b,y) \\ +\lambda\phi(y)\int_{-\infty}^{+\infty} g(a,b,y')dy' - \eta g(a,b,y) + \eta\nu(a-a_0)\nu(b-b_0)g^*(y)\end{aligned}$$

Here, ν denotes the dirac delta function and a_0, b_0 are initial assets for new borns, and $g^*(y)$ is the stationary distribution for y. I show in the numerical solution section how to solve both the HJB and Fokker-Plankt equations using a finite differenc eapproach as in Achdou *et.al*(2014; 2017) and Kaplan *et. al.* (2018).

Since the first order conditions along with the envelope conditions are necessary and sufficient for an optimum, one can characterise the first order conditions for the HJB equation, which are:

$$\begin{aligned}U_c'(c,\ell) &= V_b'(a,b,y) \\ U_\ell'(c,\ell) &= V_b'(a,b,y)(1-\tau)we^y \\ V_b'(a,b,y)(1+\chi_d(d,a)) &= V_a'(a,b,y)\end{aligned}$$

The adjustment cost function takes two forms, depending on the value of d:

$$\text{if } d > 0;\ \chi_d(d,a) = \chi_0 + \chi_1 \frac{d}{a^2} a \Rightarrow d = \left(\frac{V_a'}{V_b'} - 1 - \chi_0\right) \frac{a}{\chi_1}$$
$$\text{if } d < 0;\ \chi_d(d,a) = -\chi_0 + \chi_1 \frac{d}{a^2} a \Rightarrow d = \left(\frac{V_a'}{V_b'} - 1 + \chi_0\right) \frac{a}{\chi_1}$$

Then conditional on paying the fixed cost, the optimal deposit are given by:

$$d = \left(\frac{V_a'}{V_b'} - 1 + \chi_0\right)^{-} \frac{a}{\chi_1} + \left(\frac{V_a'}{V_b'} - 1 - \chi_0\right)^{+} \frac{a}{\chi_1}$$

From here on out, the following notation will hold, for any variable $x^+ = max\{x, 0\}$ and $x^- = min\{x, 0\}$.

Firms

As with standard New Keynesian Literature, and from Chapter 1, Final goods are bundled with a Dixit-Stiglitz aggregator subject to a nested structure. So:

$$Y_t = \left(\int_0^1 Y_t(f)^{\frac{\theta-1}{\theta}} df\right)^{\frac{\theta}{\theta-1}}$$

θ is the elasticity of substitution between intermediate goods such that as $\theta \to 0$ there is perfect competition, and as $\theta \to \infty$, there is a monopoly. Cost minimisation results in the following demand quantities[12]:

$$Y_t(f) = \left(\frac{P_t(f)}{P_t}\right)^{-\theta} Y_t \quad \text{With} \quad P_t = \left(\int_0^1 P_t(f)^{1-\theta} df\right)^{\frac{1}{1-\theta}}$$

I have that output in domestic firm f is given by the same cobb-douglas production function as in the closed economy case. Sectoral output is therefore:

$$Y_t(f) = \varepsilon_t^a K_t(f)^{\alpha} N_t(f)^{1-\alpha}$$

This production function here, $K_t(f)$ is the capital rented out by firm and $N_t(f)$ is the final labour used by firm f, and ε_t is aggregate productivity shock (TFP) which follows the following

[12] A detailed derivation of the demand quantities is shown in Appendix A.

exogenous process:

$$\ln \varepsilon_t^a = (1-\rho_a)\ln \varepsilon^a + \rho_a \ln \varepsilon_{t-1}^a + e_t^a \quad e_t^a \sim N(0,\sigma_a)$$

Such that ρ_a is the persistence of the productivity shock. Intermediate goods firm maximise expected discounted lifetime real profits, where the profits of firm f is:

$$\mathcal{D}_t(f) = \frac{P_t(f)}{P_t}Y_t(f) - R_t^k K_t(f) - \Omega_t N_t(f)$$

So each intermediate goods firms solves the maximisation problem below. Conversely, to chapter 1 and following Kaplan *et. al.* (2018), Rotemberg (1982) pricing is utilised for the sticky price environment, where the price adjustment cost is: given by:

$$\underset{K_t(f),N_t(f),P_t(f)}{\text{Max :}} \quad \mathbb{E}_t \int_0^\infty e^{-\int_0^t r_s^a ds}\left[\mathcal{D}_t(f) - \Psi\left(\frac{\dot{P}_t}{P_t}\right)\right]$$
$$s.t. \qquad Y_t(f) = \varepsilon_t^a K_t(f)^\alpha N_t(f)^{1-\alpha-\mu}$$

As with standard New Keynesian literature with pricing *á la* Rotemberg, the adjustment costs are quadratic in the rate at which prices change $\frac{\dot{P}_t}{P_t}$, and expressed as a fraction of aggregate output Y_t so:

$$\Psi\left(\frac{\dot{P}_t}{P_t}\right) = \frac{\psi}{2}\left(\frac{\dot{P}_t}{P_t}\right)^2 Y_t$$

The lagrangian reads:

$$\mathcal{L}_t = \mathbb{E}_t \int_0^\infty e^{-\int_0^t r_s^a ds}\left[\mathcal{D}_t(f) - \Psi\left(\frac{\dot{P}_t}{P_t}\right)\right] - \Phi_t(f)\left[Y_t(f) - \varepsilon_t^a K_t(f)^\alpha N_t(f)^{1-\alpha}\right]$$

$\Phi_t(f)$ is the shadow price, also known as the real marginal cost faced by firm f. The static problem is non different from the from appendix A:

$$R_t^k = \alpha\varepsilon_t^a K_t(f)^{\alpha-1}N_t(f)^{1-\alpha}$$
$$\Omega_t = (1-\alpha)\varepsilon_t^a K_t(f)^\alpha N_t(f)^{-\alpha}$$

Taking ratios of the equations above, the Capital-labour Ratio is:

$$\mathcal{K}_t \equiv \frac{K_t(f)}{N_t(f)} = \frac{\alpha}{1-\alpha}\frac{\Omega_t}{R_t^k}$$

Observe that the capital-labour ratio is independent of firm type f. From these, I can infer the input demand functions. Taking the production function;

$$\frac{Y_t(f)}{\varepsilon_t^a K_t(f)^\alpha} = N_t(f)^{1-\alpha}$$
$$N_t(f) = \frac{Y_t(f)}{\varepsilon_t^a}\left(\frac{K_t(f)}{N_t(f)}\right)^{-\alpha}$$
$$N_t(f) = \frac{Y_t(f)}{\varepsilon_t^a}\left(\frac{1-\alpha}{\Omega_t}\right)^{\alpha}\left(\frac{R_t^k}{\alpha}\right)^{\alpha}$$

These expressions imply that the total cost of producing the final good $Y_t(j)$ is:

$$R_t^k K_t(f) + \Omega_t N_t(f) = \Omega_t N_t(f) + \frac{\alpha}{1-\alpha}\Omega_t N_t(f) = \frac{\Omega_t N_t(f)}{1-\alpha}$$

Combining these equations one can show that the Real marginal Cost $\Phi_{j,t}(f)$ is independent of firm type f:

$$\Phi_t(f) = \frac{\Omega_t}{(1-\alpha)\varepsilon_t^a K_t(f)^\alpha N_t(f)^{-\alpha}}$$
$$\Phi_t(f) = \frac{\Omega_t N_t(f)}{(1-\alpha)\varepsilon_t^a K_t(f)^\alpha N_t(f)^{1-\alpha}} = \frac{\Omega_t N_t(f)}{(1-\alpha)Y_t(f)}$$

where:

$$\frac{N_t(f)}{Y_t(f)} = \frac{1}{\varepsilon_t^a}\frac{1}{\left(\frac{K_t(f)}{N_t(f)}\right)^\alpha}$$

Therefore:

$$\Phi_t(f) = \frac{1}{\varepsilon_t^a}\frac{\Omega_t}{(1-\alpha)}\left(\frac{\alpha}{1-\alpha}\frac{\Omega_t}{R_t^k}\right)^{-\alpha}$$
$$\Phi_t(f) = \frac{1}{\varepsilon_t^a}\left(\frac{\Omega_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{R_t^k}{\alpha}\right)^{\alpha} \equiv \Phi_t$$

Using that total cost $\frac{\Omega_t N_t(f)}{1-\alpha} = Y_t(f)\Phi_t$, and substituting out the demand for intermediate goods

$Y_t(f)$, the profit function is given as:

$$\mathcal{D}_t = \left(\frac{P_t(f)}{P_t} - \Phi_t\right)\left(\frac{P_t(f)}{P_t}\right)^{-\theta} Y_t$$

Then the Pricing problem for each intermediate goods firm is:

$$\int_0^\infty e^{-\int_0^t r_s^a ds}\left[\left(\frac{P_t(f)}{P_t} - \Phi_t\right)\left(\frac{P_t(f)}{P_t}\right)^{-\theta} Y_t - \frac{\psi}{2}\left(\frac{\dot{P}_t}{P_t}\right)^2 Y_t\right]$$

Under the definition that change in prices equals inflation, $\frac{\dot{P}_t}{P_t} = \pi$, I can express the function above as:

$$\int_0^\infty e^{-\int_0^t r_s^a ds}\left[\left(\frac{P_t(f)}{P_t} - \Phi_t\right)\left(\frac{P_t(f)}{P_t}\right)^{-\theta} Y_t - \frac{\psi}{2}\pi^2 Y_t\right]$$

Such that the Hamiltonian-Bellman equation for this recursive continuous time problem is[13]:

$$r_t^a V_p(P(f),t) = max_\pi \left(\frac{P_t(f)}{P_t} - \Phi_t\right)\left(\frac{P_t(f)}{P_t}\right)^{-\theta} Y_t - \frac{\psi}{2}\pi^2 Y_t + V_p'(p(f),t)\dot{P}_t + V_t(P(f),t)$$
$$\frac{\dot{P}_t}{P_t} = \pi \Leftrightarrow \dot{P}_t = \pi P_t$$
$$r_t^a V_p(P(f),t) = max_\pi \left(\frac{P_t(f)}{P_t} - \Phi_t\right)\left(\frac{P_t(f)}{P_t}\right)^{-\theta} Y_t - \frac{\psi}{2}\pi^2 Y_t + V_p'(P(f),t)\pi P_t + V_t(p,t)$$

FOCs

$$[\pi] \qquad 0 = V_p'(p,t)p - \psi\pi Y_t \Leftrightarrow V_p'(p,t)p = \psi\pi Y_t$$
$$[P_t(f)] \qquad r_t^a V_p''(P(f),t) = -\theta\left(\frac{P_t(f)}{P_t} - \Phi_t\right)\frac{Y_t}{P_t}\left(\frac{P_t(f)}{P_t}\right)^{-\theta-1} + \frac{Y_t}{P_t}\left(\frac{P_t(f)}{P_t}\right)^{-\theta}$$
$$+V_p''(P(f),t)P_t(f)\pi + V_p'(P_t(f),t)\pi + V_{tp}''(P_t(f),t)$$
$$(r_t^a - \pi)V_p'(P_t(f),t) = -\theta\left(\frac{P_t(f)}{P_t} - \Phi_t\right)\frac{Y_t}{P_t}\left(\frac{P_t(f)}{P_t}\right)^{-\theta-1} + \frac{Y_t}{P_t}\left(\frac{P_t(f)}{P_t}\right)^{-\theta}$$
$$+V_p''(P(f),t)P_t(f)\pi + V_{tp}''(P_t(f),t)$$

[13] This is derived in the same way I derived the household recursive problem

Using that in a symmetric equilibrium, $P_t(f) = P_t$, both FOCs become:

$$V_p'(p,t) = \frac{\psi \pi Y_t}{P_t} \tag{B2}$$
$$(r_t^a - \pi)V_p'(P_t,t) = -\theta\left(1-\Phi_t\right)\frac{Y_t}{P_t} + \frac{Y_t}{P_t} + V_p''(P_t,t)P_t\pi_t + V_{tp}''(P_t,t) \tag{B3}$$

Now differentiating equation (B2) w.r.t to time t, yields:

$$V_p''(P_t,t)\dot{P}_t + V_{pt}''(P_t,t) = \frac{\psi\dot{\pi}_t Y_t}{P_t} + \frac{\psi\pi_t\dot{Y}_t}{P_t} - \frac{\psi\pi_t Y_t}{P_t^2}\dot{P}_t = \frac{\psi\dot{\pi}_t Y_t}{P_t} + \frac{\psi\pi_t\dot{Y}_t}{P_t} - \frac{\psi\pi_t^2 Y_t}{P_t}$$

Substituting into (B3) and also substituting (B2) into (B3) yields:

$$(r_t^a - \pi)\frac{\psi\pi Y_t}{P_t} = -\theta\left(1-\Phi_t\right)\frac{Y_t}{P_t} + \frac{Y_t}{P_t} + \frac{\psi\dot{\pi}_t Y_t}{P_t} + \frac{\psi\pi_t\dot{Y}_t}{P_t} - \frac{\psi\pi_t^2 Y_t}{P_t}$$

Multiplying through by $\frac{P_t}{\psi Y_t}$ yields:

$$\begin{aligned}
\left(r_t^a - \pi_t\right)\pi_t &= -\frac{1}{\psi}(1-\Phi)\theta + \frac{1}{\psi} + \dot{\pi}_t - \pi_t^2 \\
\left(r_t^a - \frac{\dot{Y}_t}{Y_t}\right)\pi_t &= \frac{1}{\psi}\left(1-(1-\Phi_t)\theta\right) + \dot{\pi}_t \\
\left(r_t^a - \frac{\dot{Y}_t}{Y_t}\right)\pi_t &= \frac{\theta}{\psi}\left(\Phi_t - \frac{\theta-1}{\theta}\right) + \dot{\pi}_t
\end{aligned}$$

Monetary Authority and Government

Monetary Policy is set utilising a Taylor Rule, In Kaplan *et. al* (2018) they assume a strict inflation targeting regime, such that:

$$i_t = \bar{r}^b + \rho_\pi \pi_t + \varepsilon_t^i$$

Where $\rho_\pi > 1$, and $\varepsilon_t^i = 0$ in the deterministic steady state. Finally, I assume government expenditure to be exogenous, with a progressive taxation on household labour income, and a lump-sum transfer T_t, with a proportional tax rate τ_t. The government in this model is the sole issuer of bonds (liquid) assets, B_t^g, such that negativity refers to government debt. The government intertemporal budget constraint is therefore given as:

$$\dot{B_t}^g + G_t + T_t = \tau_t \int \omega_t z \ell_t(a,b,z) d\mu_t + r_t^b B_t^g$$

Given the government budget constraint, I can consider various forms of fiscal policy, i.e. the government can choose to adjust either income tax, τ_t, lump-sum transfers, T_t or government expenditure, G_t. The unconventional aspect of the monetary policy design looks as QE through LSAPs, in this case, I follow Chen, Cùrida and Ferrero (2012) but with a simplification[14]. So, governments can control the supply of (bonds) illiquid assets using a simple autoregressive rule:

$$B_t^g = B^g + \varepsilon_t^b$$

Where ε_t^b is an i.i.d zero mean with disturbance term σ_B. Given the inflation and nominal interest rate, I can derive the real return on liquid assets pinned down by the Fisher equation $r_t^b = i_t - \pi_t$. The liquid asset return r_t^b pinned down by the fisher equation is also consistent with the bond market equilibrium above. In equilibrium, liquid markets clear and thus:

$$B_t^h = B_t^g$$
$$B_t^h = \int b \; d\mu_t$$

$B_t^h = \int b \; d\mu_t$ are total household holdings of liquid (bonds) assets.

Numerical Solution

Hamiton-Jacobi-Bellman Equation

To solve the HJB equation from equation, I follow Achdou *et. al.* (2017) who use an implicit method upwind finite difference scheme. The upwind method employed here splits the drift of liquid assets (bonds) into three different parts and upwinds them differently. These parts are

[14] Chen, Cùrida and Ferrero (2012) present a DSGE model in discrete time framework, due to the continuous framework of HANK model, in continuous time one cannot differentiate between t and $t-1$

given by:

$$s^c = r^b(b) + \zeta - c$$
$$s^d = -d - \chi(d, a)$$

To proceed, I need to discretise the continuous state space. So I denote the grid points for illiquid assets by, a_i, $i = 1, ...I$, liquid assets, b_j, $j = 1, ...J$ and income process, y_k, $k = 1, ...K$. So the value function is thus given by:

$$V^n_{ijk} = V(a_i, b_j, y_k)$$

Under the upwind scheme, the direction of winding is given by $-$ when it's backward and $+$ when it is forward. Finally, in the same way as Kaplan *et. al.* (2018), I use non-equispaced grids such that $\Delta a^+_i = a_{i+1} - a_i$ and $\Delta a^-_i = a_i - a_{i-1}$. In the same way I can denote the grid points for bonds as $\Delta b^+_i = b_{j+1} - b_j$ and $\Delta b^-_j = b_j - b_{j-1}$ and so on. Therefore, I can approximate the derivatives of the value function for liquid, illiquid assets and income process as:

$$V'_a(a_i, b_j, y_k) \approx V'^F_{(a:i,j,k)} = \frac{V'_{a:i+1,j,k} - V'_{a:i,j,k}}{\Delta a^+_i}$$
$$V'_a(a_i, b_j, y_k) \approx V'^B_{(a:i,j,k)} = \frac{V'_{a:i,j,k} - V'_{a:i-1,j,k}}{\Delta a^-_i}$$
$$V'_b(a_i, b_j, y_k) \approx V'^F_{(b:i,j,k)} = \frac{V'_{b:i,j+1,k} - V'_{b:i,j,k}}{\Delta b^+_j}$$
$$V'_b(a_i, b_j, y_k) \approx V'^B_{(b:i,j,k)} = \frac{V'_{b:i,j,k} - V'_{b:i,j-1,k}}{\Delta b^-_j}$$
$$V'_y(a_i, b_j, y_k) \approx V'^F_{(y:i,j,k)} = \frac{V'_{y:i,j,k+1} - V'_{y:i,j,k}}{\Delta y^+_k}$$
$$V'_y(a_i, b_j, y_k) \approx V'^B_{(y:i,j,k)} = \frac{V'_{y:i,j,k} - V'_{y:i,j,k-1}}{\Delta y^-_k}$$

Then the discretised version of the HJB equation can be expressed as:

$$\frac{V^{n+1}_{i,j,k} - V^n_{i,j,k}}{\Delta} + (\beta + \eta)V^{n+1}_{i,j,k} = U(c^n_{i,j,k}, \ell^n_{i,j,k}) + V'^{n+1}_{b:i,j,k} s^{b,n}_{i,j,k} + V'^{n+1}_{a:i,j,k}\left(r^a(a_i)a_i + d^n_{i,j,k}\right)$$
$$+ V'^{n+1}_{y:i,j,k}(-\rho y_k) + \lambda \int_{-\infty}^{\infty} V'^{n+1}_{y:i,j,k'} - V'^{n+1}_{y:i,j,k}\phi(k')dk'$$

The constraints and first order conditions can be expressed as:

$$U_c'(C_{i,j,k}^n, \ell_{i,j,k}^n) = V_{b:i,j,k}'^n$$
$$U_\ell'(C_{i,j,k}^n, \ell_{i,j,k}^n) = V_{b:i,j,k}'^n \left((1-\tau)we^{y_k}\right)$$
$$s_{i,j,k}^{b,n} = (1-\tau)we^{y_k}\ell + (r^b(b_j) + \zeta_j)b_j + T - \chi\left(d_{i,j,k}^n, a_i\right) - C_{i,j,k}^n$$
$$V_{b:i,j,k}'^n \left(1 + \chi(d_{i,j,k}^n, a_j)\right) = V_{a:i,j,k}'^n$$

Here it follows that $V_{a:i,j,k}'^{n+1}$, $V_{b:i,j,k}'^{n+1}$, and $V_{y:i,j,k}'^{n+1}$ are either the forward difference or backward difference approximations defined earlier, such that given an initial guess for the value function $V_{i,j,k}^n$ along with the direction of the upwind scheme, I can implicitly define the optimal levels for consumption, labour and deposits.

Under the upwind scheme, I'm able to choose the direction of the finite difference approximation. When upwinding the illiquid asset, a_j, I choose a forward difference approximation whenever the drift of the asset is positive, and a backward difference approximation whenever the drift is negative. Given the split of the drifts described above, I can define $C_{i,j,k}^B$ and $C_{i,j,k}^F$ as the optimal consumption whenever backward or forward difference approximation is used in the value function $V_{b:i,j,k}'^{B,n+1}$, and $V_{b:i,j,k}'^{F,n+1}$ respectively. Similarly, I define $\ell_{i,j,k}^B$ and $\ell_{i,j,k}^F$ as the optimal labour supplied whenever backward or forward difference approximation is used in the value function $V_{b:i,j,k}'^{B,n+1}$, and $V_{b:i,j,k}'^{F,n+1}$ respectively. The case of deposits is more complicated and will be explained in detail below. Therefore the partitioned drift components can be expressed as:

$$\begin{aligned} s_{ijk}^{c,B} &= (1-\tau)we^{y_k}\ell_{i,j,k}^n + r^b(b_j)b_j - C_{i,j,k}^{B,n} \\ &= (1-\tau)we^{y_k}\left(U_\ell'\right)^{-1}(1-\tau)we^{y_k}V_{b:i,j,k}'^{B,n} + (r^b(b_j)+\zeta_j)b_j - \left(U_C'\right)^{-1}V_{b:i,j,k}'^{B,n} \\ &= V_{b:i,j,k}'^{B,n}\left[(1-\tau)we^{y_k}\left(U_\ell'\right)^{-1}(1-\tau)we^{y_k} + (r^b(b_j)+\zeta_j)b_j - \left(U_C'\right)^{-1}\right] \\ s_{ijk}^{c,F} &= V_{b:i,j,k}'^{F,n}\left[(1-\tau)we^{y_k}\left(U_\ell'\right)^{-1}(1-\tau)we^{y_k} + (r^b(b_j)+\zeta_j)b_j - \left(U_C'\right)^{-1}\right] \end{aligned}$$

Then I approximate:

$$V_{i,j,k}s_{i,j,k}^c \approx V_{b:i,j,k}'^{F,n+1}\left(s_{i,j,k}^{c,F}\right)^+ + V_{b:i,j,k}'^{B,n+1}\left(s_{i,j,k}^{c,B}\right)^-$$

In the not so complicated case of deposit, because it's first order condition depends on both the value functions of illiquid and liquid wealth. therefore the upwind scheme will depend on four different cases of direction of the finite difference approach each of the value function takes. So, now I define the following $d_{i,j,k}^{BB}$ $d_{i,j,k}^{BF}$ $d_{i,j,k}^{FB}$ $d_{i,j,k}^{BB}$, such that udder $d_{i,j,k}^{BB}$, the optimal deposit is calculated using a backward difference approach with illiquid asses, a_i, and a backward difference approach with liquid asset b_j, i.e. I calculate optimal deposits using $V_{a:i,j,k}^{\prime F,n}$ and $V_{b:i,j,k}^{\prime F,n}$ respectively. Additionally, for clarity, $d_{i,j,k}^{BF}$ is the case when I calculate optimal deposit using backward difference wit illiquid asset a_i and forward difference with liquid asset b_j, i.e. it is obtained using $V_{a:i,j,k}^{\prime B,n}$ and $V_{b:i,j,k}^{\prime F,n}$ respectively. Therefore, $d_{i,j,k}^{BB}$ $d_{i,j,k}^{BF}$, satisfy the following equations respectively:

$$V_{b:i,j,k}^{\prime B,n}\left(1+\chi_d\left(d_{i,j,k}^{BB},a_j\right)\right)=V_{a:i,j,k}^{\prime B,n}$$
$$V_{b:i,j,k}^{\prime F,n}\left(1+\chi_d\left(d_{i,j,k}^{BF},a_j\right)\right)=V_{a:i,j,k}^{\prime B,n}$$

Now I can define $d_{i,j,k}^{B}$ and $d_{i,j,k}^{F}$ as deposit paths such that a backward difference or forward difference approximation is always utilised in calculating optimal deposits for one of the cases. So, they satisfy:

$$d_{i,j,k}^{B}=\left(d_{i,j,k}^{BB}\right)^{-}+\left(d_{i,j,k}^{BF}\right)^{+}$$
$$d_{i,j,k}^{F}=\left(d_{i,j,k}^{FF}\right)^{+}+\left(d_{i,j,k}^{FB}\right)^{-}$$

Then the drift component in the constraint for deposits are given by $s_{i,j,k}^{F}$ and $s_{i,j,k}^{B}$, which satisfy the following two equations respectively:

$$s_{i,j,k}^{F}=-d_{i,j,k}^{F,n}+\chi\left(d_{i,j,k}^{F,n},a_i\right)$$
$$s_{i,j,k}^{B}=-d_{i,j,k}^{B,n}+\chi\left(d_{i,j,k}^{B,n},a_i\right)$$

Then I can use the following approximation:

$$d_{i,j,k}=d_{i,j,k}^{B,n}\mathbb{1}_{s_{i,j,k}^{d,B}<0}+d_{i,j,k}^{F,n}\mathbb{1}_{s_{i,j,k}^{d,F}>0}+\bar{d}_{i,j,k}^{n}\mathbb{1}_{s_{i,j,k}^{d,B}\leq 0\leq s_{i,j,k}^{d,F}}$$

Then the upwind finite difference approximation for the HJB equation is given as:

$$\frac{V_{i,j,k}^{n+1}-V_{i,j,k}^{n}}{\Delta}+(\beta+\eta)V_{i,j,k}^{n+1}=U(c_{i,j,k}^{n},\ell_{i,j,k}^{n})+V_{b:i,j,k}^{\prime F,n+1}\left(s_{i,j,k}^{c,F}\right)^{+}+V_{b:i,j,k}^{\prime B,n+1}\left(s_{i,j,k}^{c,B}\right)^{-}$$
$$+V_{b:i,j,k}^{\prime B,n+1}\left(s_{i,j,k}^{d,B}\right)^{-}+V_{b:i,j,k}^{\prime B,n+1}\left(s_{i,j,k}^{d,F}\right)^{+}+V_{a:i,j,k}^{\prime B,n+1}d_{i,j,k}^{-}+V_{a:i,j,k}^{\prime B,n+1}\left(d_{i,j,k}^{+}+r^{a}a_{i}\right)$$
$$+\sum_{k\neq k'}^{K}\lambda_{k,k'}\left(V_{i,j,k'}^{n+1}-V_{i,j,k}^{n+1}\right) \tag{B8}$$

I can write this in matrix notation as:

$$\frac{1}{\Delta}\left(V^{n+1}-V^{n}\right)+(\beta+\eta)V^{n+1}=U^{n}+(\mathbb{A}^{n}+\Lambda)V^{n+1}$$

Here, you find that the length of V^n, V^{n+1} and U^n are the same length as the g-matrix $g(a,b,y)$ at $I\times J\times K$. The matrix Λ summarises the stochastic nature of the income process and has length of $(I\times J\times K)\times(I\times J\times K)$. This is the same length as the $\mathbb{A}$ matrix[15]. The system can then be rewritten as;

$$\mathbb{B}^{n}V^{n+1}=b^{n}$$
$$\mathbb{B}^{n}=\left(\frac{1}{\Delta}+(\beta+\eta)\right)I-(\mathbb{A}^{n}+\Lambda)\qquad b^{n}=U^{n}+V^{n}\left(\frac{1}{\Delta}\right)$$

When Calibrating solution parameters, Kaplan *et. al.* (2018) choose the numerical grid points for income to be 33, i.e. $k=33$. With such a large grid point, the sparse library suite in matlab becomes slow, so to remedy this, I consider an alternative application where I use an implicit method finite difference approach in the $a-$dimension and $b-$dimension, but an explicit method finite difference approach in the $z-$dimension[16]. I can thus rewrite equation (B8) as:

$$\frac{V_{i,j,k}^{n+1}-V_{i,j,k}^{n}}{\Delta}+(\beta+\eta)V_{i,j,k}^{n+1}=U(c_{i,j,k}^{n},\ell_{i,j,k}^{n})+V_{b:i,j,k}^{\prime F,n+1}\left(s_{i,j,k}^{c,F}\right)^{+}+V_{b:i,j,k}^{\prime B,n+1}\left(s_{i,j,k}^{c,B}\right)^{-}$$
$$+V_{b:i,j,k}^{\prime B,n+1}\left(s_{i,j,k}^{d,B}\right)^{-}+V_{b:i,j,k}^{\prime B,n+1}\left(s_{i,j,k}^{d,F}\right)^{+}+V_{a:i,j,k}^{\prime B,n+1}d_{i,j,k}^{-}+V_{a:i,j,k}^{\prime B,n+1}\left(d_{i,j,k}^{+}+r^{a}a_{i}\right)$$
$$+\sum_{k\neq k'}^{K}\lambda_{k,k'}\left(V_{i,j,k'}^{n}-V_{i,j,k}^{n}\right) \tag{B9}$$

[15] Achdou *et.al* (2017) explain in detail how the transition matrix $\mathbb{A}$ encodes the the stochastic processes evolution. Note, in their explanation, they compute the numerical solution for the HJB under a single asset case, with a Poisson income process i.e. $k=2$.

[16] In the matlab file I do not use this method, only written out for pure alternative purposes if one chooses a substantially large enough k such that the algorithm becomes incredibly slow

To grasp the method, I can express this discretised equation in matrix notation as:

$$\frac{1}{\Delta}\left(V^{n+1}-V\right)+(\beta+\eta)V^{n+1}=U^{n}+\mathbb{A}^{n}V^{n+1}+\Lambda V^{n}$$

$\mathbb{A}^{n}$ is a diagonal matrix such that all of its off-diagonal elements are zero and all its diagonal elements are non-zero. I can then rewrite the matrix notation as:

$$\mathbb{B}^{n}V^{n+1}=b^{n}$$
$$\mathbb{B}^{n}=\left(\frac{1}{\Delta}+(\beta+\eta)\right)I-\mathbb{A}^{n} \qquad b^{n}=U^{n}+V^{n}\left(\frac{1}{\Delta}+\Lambda\right)$$

The idea behind the implicit upwind scheme in i and j dimensions, with explicit upwind scheme in k dimension, means I can split the problem into k smaller problems. So one obtains:

$$\frac{1}{\Delta}\left(V_{k}^{n+1}-V_{k}^{n}\right)+(\beta+\eta)V_{k}^{n+1}=U_{k}^{n}+\mathbb{A}_{k}^{n}V_{k}^{n+1}+\sum_{k\neq k'}^{K}\lambda_{k,k'}\left(V_{k'}^{n}-V_{k}^{n}\right)$$

This effectively reduces the size of the matrices V_{k}^{n+1}, V_{k}^{n} and U_{k}^{n} such that they are K vectors with length $I\times J$. Finally one would get that $\mathbb{A}$ essentially reduces size as well, such that $\mathbb{A}_{k}^{n}$ are K matrices with sizes $(I\times J)\times(I\times J)$.

Fokker-Plankt Equation (Kolmogorov Forward Equation)

Finally, I can solve the Fokker-Plankt equation (Kolmogorov Forward equation) in the same way using a finite difference approach, with upwinding schemes. The Fokker-Plankt equation is given by[17];

$$\begin{aligned}0=&\,\partial_{a}\left(s^{a}(a,b,y)g(a,b,y)\right)-\partial_{b}\left(s^{b}(a,b,y)g(a,b,y)\right)-\partial_{y}\left(-\rho y g(a,b,y)\right)-\lambda g(a,b,y)\\&+\lambda\phi(y)\int_{-\infty}^{+\infty}g(a,b,y')dy'-\eta g(a,b,y)+\eta\nu(a-a_{0})\nu(b-b_{0})g^{*}(y)\end{aligned}$$

[17]Note that both the HJB and Fokker-Plankt equations are stationary i.e. they are time independent. I show how to solve the time dependent variations for these equations on the Transition Dynamics aspect of this Appendix.

Firstly, I need to discretise the continuous time HJB equation to:

$$-[s^a_{i,j,k}g_{i,j,k}]' - [s^b_{i,j,k}g_{i,j,k}]' - [-\rho y_k g_{i,j,k}]' - \lambda_k g_{i,j,k} + \lambda_{k'}\phi(y_k)\sum_{k'=1}^{K} g_{i,j,k'}\Delta y_{k'} - \eta g_{i,j,k}$$
$$+\eta\nu(a_i - a_0)\nu(b_j - b_0)g^*_k$$

To obtain the conditional expectation at boundaries, the Trapezoidal rule is utilised. See ConditionalExpectation.m matlab file. Once again I need to determine whether or not to use a forward or backward approach when approximating the derivative of the $[s^a_{i,j,k}g_{i,j,k}]'$, $[s^b_{i,j,k}g_{i,j,k}]'$ and $[-\rho y_k g_{i,j,k}]'$. Achdou *et.al.*(2017) show that the most convenient/correct approximation follows from:

$$\begin{aligned}
&-\frac{(s^{n:F}_{i,j,k})^+ g_{i,j,k} - g_{i-1,j,k}(s^{n;F}_{i-1,j,k})^+}{\Delta a} - \frac{(s^{n:B}_{i+1,j,k})^+ g_{i+1,j,k} - g_{i,j,k}(s^{n;B}_{i,j,k})^+}{\Delta a} \\
&\quad -\frac{(s^{n:F}_{i,j,k})^+ g_{i,j,k} - g_{i,j-1,k}(s^{n;F}_{i,j-1,k})^+}{\Delta b} - \frac{(s^{n:B}_{i,j+1,k})^+ g_{i,j+1,k} - g_{i,j,k}(s^{n;B}_{i,j,k})^+}{\Delta b} \\
&\quad -\frac{\rho y_k g_{i,j,k} - \rho y_{k-1} g_{i,j,k-1}}{\Delta y} - \frac{\rho y_{k+1} g_{i,j,k+1} - \rho y_k g_{i,j,k}}{\Delta y} \\
&\quad -\lambda_k g_{i,j,k} + \lambda_{k'}\phi(y_k)\sum_{k'=1}^{K} g_{i,j,k'}\Delta y_{k'} - \eta g_{i,j,k} + \eta\nu(a_i - a_0)\nu(b_j - b_0)g^*_k
\end{aligned}$$

Looking at the approximation, it resembles the approximation approach used in solving the HJB equation. Analogously, with this approximation approach, I can specify it's matrix formation as:

$$\mathbb{A}'g = 0$$

Here, $\mathbb{A}'$ as in standard matrix algebra, is the transpose of $\mathbb{A}$ that solves the HJB equation. Since the stochastic process' evolution is within the matrix $\mathbb{A}$, the eigenvalue problem denoted by $\mathbb{A}'g = 0$ can be solved to obtain the stationary distribution of the g matrix[18]. With the matrix $\mathbb{A}$ already constructed when solving the HJB equation in the first part, one can simply apply it's transpose when solving the Fokker-Plankt equation in this second part.

[18]For interested readers and more reading on this problem, see Achdou *et.al.*(2017) online appendix.

Transition Dynamics

In order to study the economy's transitional dynamics given an unanticipated shock, I need to solve the time dependent HJB and Fokker-Plankt equation. The same method is used to solve these two equations. The time dependent HJB and Fokker-Plankt equations are given as:

$$(\beta+\eta)V(a,b,y,t) = \max_{\{c\ell d\}}\ U(c,\ell) + V_b'(a,b,y,t)\Big[(1-\tau)we^y\ell + r^b(b)b + T$$
$$- d - \chi(d,a) - c\Big]V_a'(a,b,y,t)\left[r^a+d\right] + V_y'(a,b,y,t)(-\rho y)$$
$$+ \lambda\int_{-\infty}^{\infty}\big(V(a,b,y',t) - V(a,b,y,t)\big)\,\phi(y')\mathrm{d}y'$$

$$0 = \partial_a\left(s^a(a,b,y,t)g(a,b,y,t)\right) - \partial_b\left(s^b(a,b,y,t)g(a,b,y,t)\right) - \partial_y\left(-\rho y g(a,b,y,t)\right)$$
$$- \lambda g(a,b,y,t) + \lambda\phi(y)\int_{-\infty}^{+\infty} g(a,b,y',t)dy' - \eta g(a,b,y,t)$$
$$+ \eta\nu(a-a_0)\nu(b-b_0)g^*(y)$$

The time varying FOCs that caharcterise the optimum of the HJB are thus:

$$U_c'(c,\ell) = V_b'(a,b,y,t)$$
$$U_\ell'(c,\ell) = V_b'(a,b,y,t)(1-\tau)we^y$$
$$V_b'(a,b,y,t)(1+\chi_d(d,a)) = V_a'(a,b,y,t)$$

These are the system of equations that need to be solved along every step. To solve the time varying HJB equation, I need to approximate the value function at I, J and K discrete points along the wealth dimension and N discrete point in the time dimension[19]. I can redefine the notation above, such that the value function is now time dependent. So:

$$V_{ijk}^n = V(a_i,b_j,y_k,t^n)$$

[19] In the Matlab code, I choose the same N as Kaplan *et. al.* (2018), which is $N = 200$

Then the discretised version of the time dependent HJB is given as[20]:

$$
\begin{aligned}
(\beta+\eta)V^n_{i,j,k} &= U(c^{n+1}_{i,j,k}, \ell^{n+1}_{i,j,k}) + V'^n_{b:i,j,k} s^{b,n+1}_{i,j,k} + V'^n_{a:i,j,k}\left(r^{a,n+1}(a_i)a_i + d^n_{i,j,k}\right) \\
&+ V'^n_{y:i,j,k}(-\rho y_k) + \lambda \int_{-\infty}^{\infty} V'^n_{y:i,j,k'} - V'^n_{y:i,j,k}\phi(k')dk' + \frac{V^{n+1}_{i,j,k} - V^n_{i,j,k}}{\Delta t}
\end{aligned}
$$

where the constraints are now given as:

$$
\begin{aligned}
U'_c(C^{n+1}_{i,j,k}, \ell^{n+1}_{i,j,k}) &= V'^n_{b:i,j,k} \\
U'_\ell(C^{n+1}_{i,j,k}, \ell^{n+1}_{i,j,k}) &= V'^n_{b:i,j,k}\left((1-\tau)we^{y_k}\right) \\
s^{b,n+1}_{i,j,k} &= (1-\tau)we^{y_k}\ell + r^{b,n+1}(b_j)b_j + T - \chi\left(d^{n+1}_{i,j,k}, a_i\right) - C^{n+1}_{i,j,k} \\
V'^n_{b:i,j,k}\left(1 + \chi(d^{n+1}_{i,j,k}, a_j)\right) &= V'^n_{a:i,j,k}
\end{aligned}
$$

The upwind finite difference approach for this time varying HJB is then:

$$
\begin{aligned}
(\beta+\eta)V^n_{i,j,k} &= U(c^{n+1}_{i,j,k}, \ell^{n+1}_{i,j,k}) + V'^{F,n}_{b:i,j,k}\left(s^{c,F}_{i,j,k}\right)^+ + V'^{B,n}_{b:i,j,k}\left(s^{c,B}_{i,j,k}\right)^- \\
&+ V'^{B,n}_{b:i,j,k}\left(s^{d,B}_{i,j,k}\right)^- + V'^{B,n}_{b:i,j,k}\left(s^{d,F}_{i,j,k}\right)^+ + V'^{B,}_{a:i,j,k} d^-_{i,j,k} + V'^{B,}_{a:i,j,k}\left(d^+_{i,j,k} + r^a a_i\right) \\
&+ \sum_{k\neq k'}^{K} \lambda_{k,k'}\left(V^n_{i,j,k'} - V^n_{i,j,k}\right) + \frac{V^{n+1}_{i,j,k} - V^n_{i,j,k}}{\Delta t}
\end{aligned}
$$

$$
\begin{aligned}
(\beta+\eta)V^n &= U^{n+1} + (\mathbb{A}^{n+1} + \Lambda)V^n + \frac{1}{\Delta t}\left(V^{n+1} - V^n\right) \\
\mathbb{B}^n V^{n+1} &= b^n \\
\mathbb{B}^n = \left(\frac{1}{\Delta t} + (\beta+\eta)\right) I - (\mathbb{A}^{n+1} + \Lambda) \qquad & b^n = U^n + V^n\left(\frac{1}{\Delta t}\right)
\end{aligned}
$$

Analogously, $\mathbb{A}^{n+1}$ is defined as in $\mathbb{A}^n$ in the previous section where I solve the non time dependent HJB equation. Therefore, it can be explained in the same way in regards to discretised stochastic process is as (a_t, b_t, z_t). Achdou *et.al.*(2017) explain that the interpretation of each n becomes a time step rather than an iteration on the stationary value function. As so, by solving the time dependent problem backwards towards $t \to -\infty$ the stationary value function can thus be found.

[20]To derive the discretised version of the time dependent HJB, one simply follows the same steps as deriving the discretised version of the stationary HJB

Similarly to the time-dependent HJB equation, I can represent the time-dependent Fokker-Plankt equation as:

$$
\begin{aligned}
0 = &\partial_a \left(s^a(a,b,y,t)g(a,b,y,t)\right) - \partial_b \left(s^b(a,b,y,t)g(a,b,y,t)\right) - \partial_y \left(-\rho y g(a,b,y,t)\right) \\
&- \lambda g(a,b,y,t) + \lambda\phi(y)\int_{-\infty}^{+\infty} g(a,b,y',t)dy' - \eta g(a,b,y,t) + \eta\nu(a-a_0)\nu(b-b_0)g^*(y)
\end{aligned}
$$

The density function can then be approximated on K discrete points along the wealth dimension, while along the time dimension, I approximate on N discrete points. once again, redefining the notation above, so that the density function of the distribution corresponding to of $\mu_t(a,b,z)$ is now:

$$
g^n_{i,j,k} = g(a_i, b_j, y_k, t^n)
$$

Then the discretised version of the continuous time-dependent Fokker-Plankt equation above is:

$$
\begin{gathered}
-[s^a_{i,j,k}g^n_{i,j,k}]' - [s^b_{i,j,k}g^n_{i,j,k}]' - [-\rho y_k g^n_{i,j,k}]' - \lambda_k g^n_{i,j,k} + \lambda_{k'}\phi(y_k)\sum_{k'=1}^{K} g^n_{i,j,k'}\Delta y_{k'} - \eta g^n_{i,j,k} \\
+\eta\nu(a_i - a_0)\nu(b_j - b_0)g^*_k
\end{gathered}
$$

Given the solution method for solving the Fokker-Plankt equation without time-dependence, I can apply the same solution method by utilising the the transition matrix $\mathbb{A}^n$ obtained from solving the time-dependent HJB equation above. Therefore, with an initial condition (initial starting point) $g^0_{i,j,k} = g(i,j,k,0)$, the Fokker-Plankt equation can easily be solved by applying the implicit method[21]:

$$
\frac{g^{n+1} - g^n}{\Delta t} = (\mathbb{A}^n)' \, g^{n+1}
$$

[21] I do not go into detail of the explicit method for finite difference approach, but it can also be used to solve both the stationary problem of the HJB and Fokker-Plankt equation as well as the time-dependent versions of both equations. See Achodu *et. al.*(2017; 2014) for an exercise on how to use these methods when solving Ayagari-Bewley-Hugget models

Appendix D

Data Sources and Description

The annual data of domestic and world GDP, interest rates, inflation rates, for Nigeria is obtained from the World Bank's world development indicators (WDI) which can be accessed here https://databank.worldbank.org/source/world-development-indicators. GDP is estimated in constant U.S. dollars for the year 2015 and deflated utilising the country deflator.

Data on real GDP and real non-oil GDP for Nigeria is obtained from the Central Bank of Nigeria and can be accessed here https://www.cbn.gov.ng/rates/RealGDP.asp

Monthly data on CPI, lending rates and exchange rates are obtained from the IMF data archives as well as all IMF data quoted as well can be accessed fom the url provided. https://data.imf.org/?sk=388DFA60-1D26-4ADE-B505-A05A558D9A42.

Oil prices Brent and WTI are obtained from the U.S. Energy information Administration, which can be accessed here https://www.eia.gov/dnav/pet/hist/rbrteD.htm and https://www.eia.gov/dnav/pet/hist/rwtcD.htm respectively. Brent and WTI prices are deflated using U.S. CPI City average for All Urban Consumers: All Items in U.S. City, which can be accessed here https://fred.stlouisfed.org/series/CPIAUCSL.

Data for world industrial production to replicate Baumeister and Hamilotn (2019) can be found here https://econweb.ucsd.edu/~jhamilto/software.htm#book. See Baumeister and

Hamilton (2019) for further data sources on replicating their paper.

BIS data on credit to non-financial sector organisations for Mexico can be accessed here https://www.bis.org/statistics/totcredit.htm.

All data from Haver Analytics requires a subscription to the platform and is not publicly available. Historical data on the Goldman Sachs Commodity Index is publicly available on several sources but was collected from both Haver Analytics as well as Yahoo Finance accessed here https://finance.yahoo.com/quote/GD$%$3DF/history?p=GD$%$3DF. I utilise the trading day closing value in the rolling correlations and standard plots.

Data from the CIA world Factbook on oil exporters can be accessed form here https://www.cia.gov/the-world-factbook/.

The 2019 SCF and FoF Data

Following Kaplan *et. al.* (2018) I depict the disaggregation of Survey of Consumer Finances (SCF) and Flow of Funds (FoF) data. However, I report the aggregates from the SCF data as opposed to justify better comparisons with the FoF dataset which is strictly an aggregated data for U.S. households.

Table 5.1: HANK - Assets and Liabilities

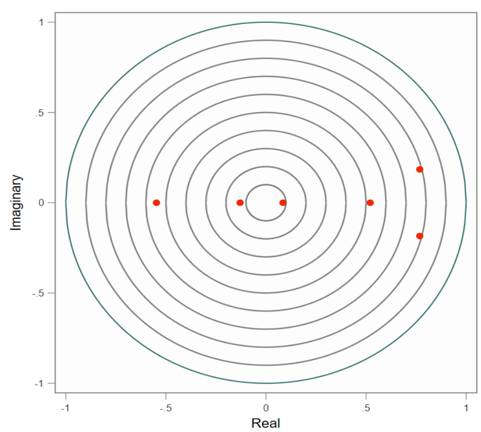
Assets			Liabilities		
Variables	FoF	SCF	Variables	FoF	SCF
Real Estate	30,000	44,300	Mortgage Debt	10,500	5,600
Consumer Durables	5,700	8,100	Non-revolving Consumer Credit	1,700	1,900
Deposits	12,700	7,100	Revolving Consumer Credit	4,200	1,100
Government Bonds	4,300	520			
Corporate Bonds	170	1,100			
Corporate Equity	28,900	34,400			
Non-Corporate Equity	11,900	15,200			
Total					

Notes: Data under the column SCF is from the SCF for 2019 and can be accessed here https://www.federalreserve.gov/econres/scfindex.htm and Data under the FoF is from the Flow of Funds from 2019 and can be accessed here https://fred.stlouisfed.org/release/tables?rid=52&eid=810090#snid=810130

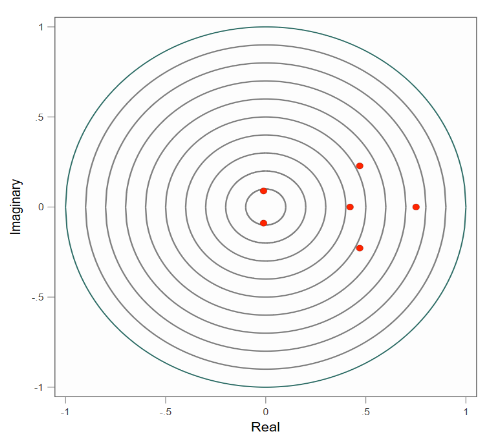
Appendix E

Additional Figures

Figure 5.1: Stability of SVARs (Roots of the Companion Matrix) in Figure 2.7 and 2.8

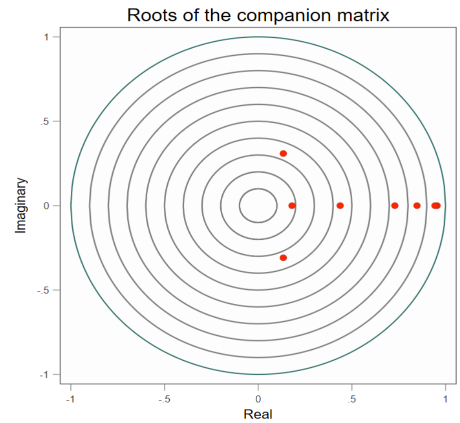


(a) SVAR with Mexico Data (Figure 2.8)

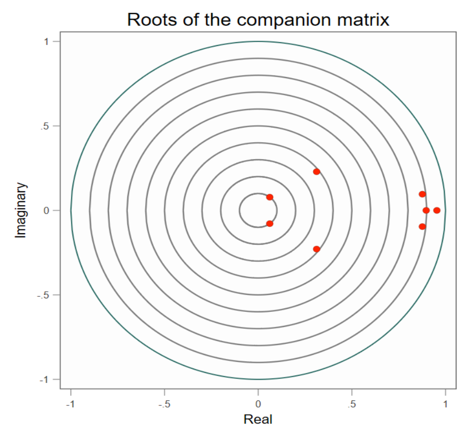


(b) SVAR with Nigeria Data (Figure 2.7)

Figure 5.2: Stability of SVARs in Figure 2.20 and 2.21



(a) SVAR with Mexico Data (Figure 2.21)



(b) SVAR with Nigeria Data (Figure 2.20)

Figure 5.3: Stability of SVAR in Figure 3.4

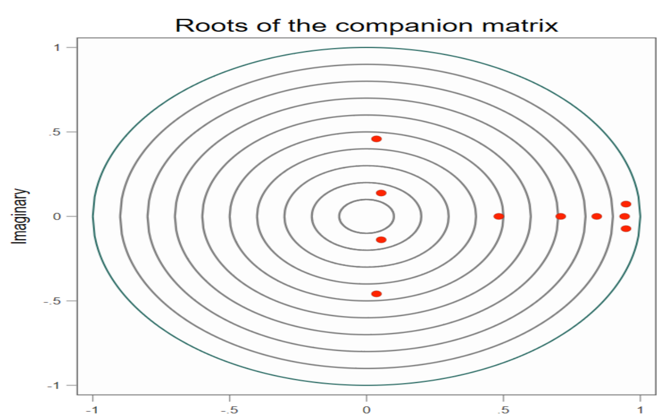
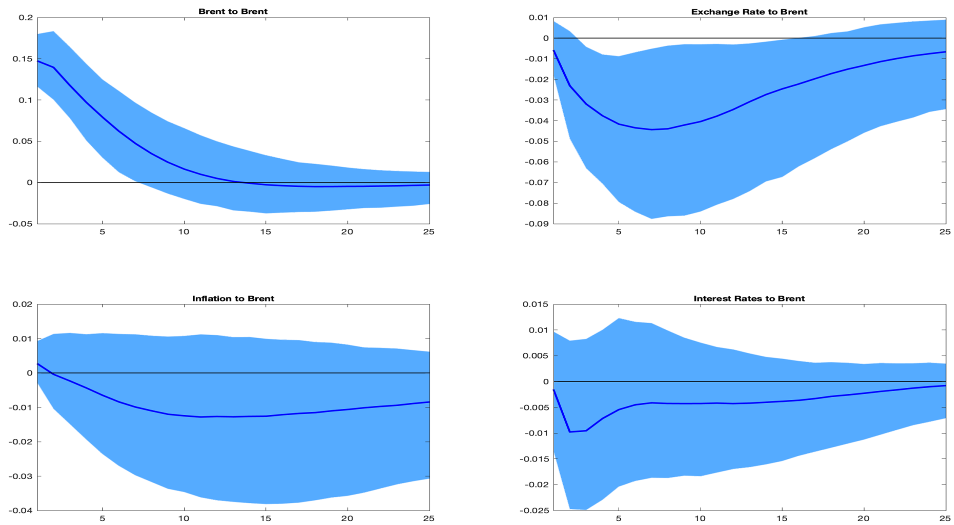
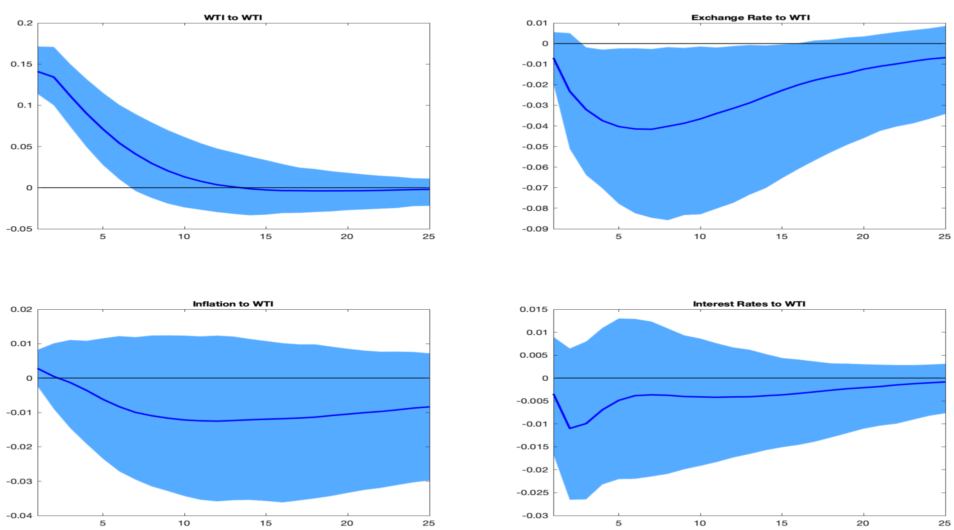


Figure 5.4: SVAR: IRFs of Brent oil Prices - Nigeria



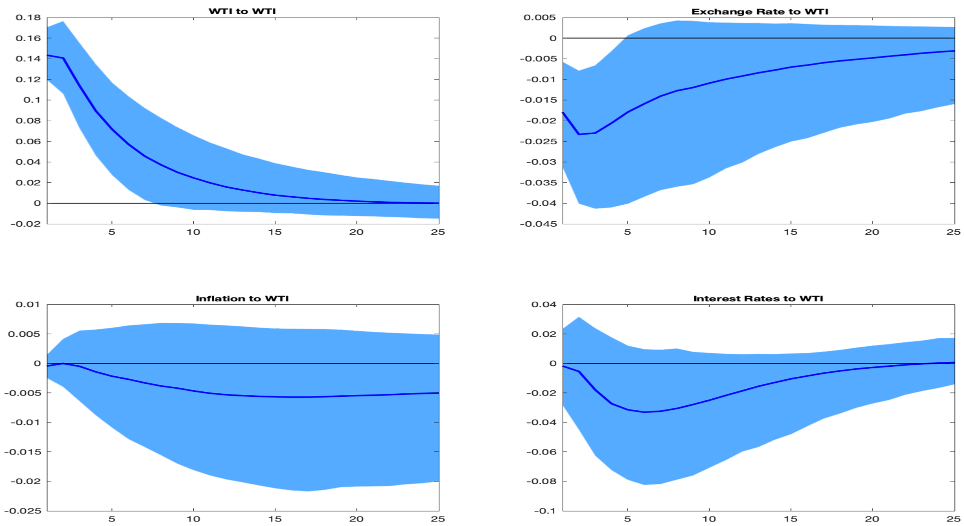
Notes: Impulse response from SVAR utilising Brent prices displayed in Figure 2.20 for historical decomposition for macroeconomic variables for Nigeria

Figure 5.5: SVAR: IRFs of WTI oil Prices - Nigeria



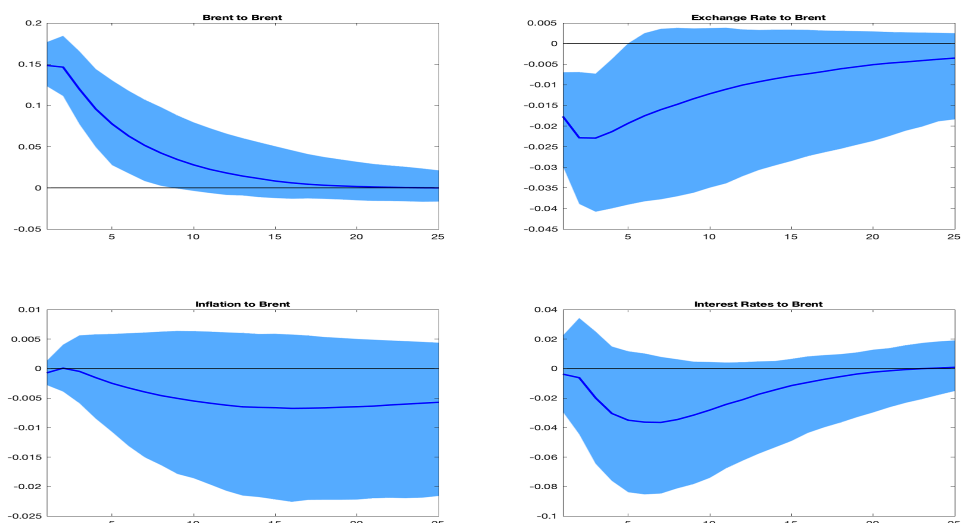
Notes: Impulse response from SVAR utilising WTI prices displayed in Figure 5.11 for historical decomposition for macroeconomic variables for Nigeria

Figure 5.6: SVAR: IRFs of WTI Oil Prices - Mexico



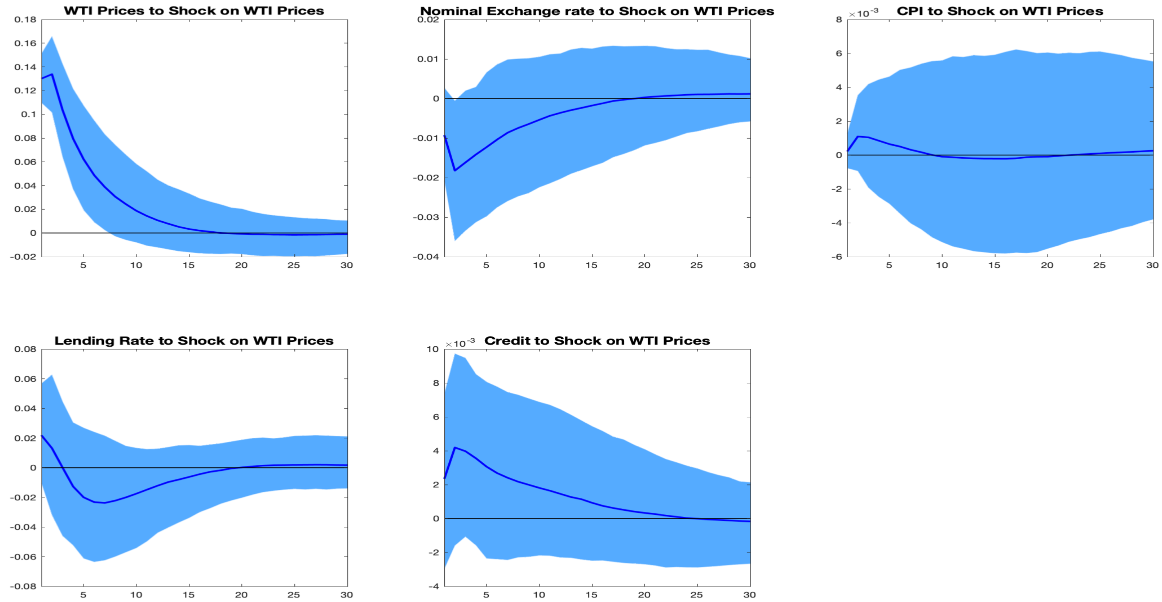
Notes: Impulse response from SVAR utilising WTI prices displayed in Figure 2.21 for historical decomposition for macroeconomic variables for Mexico

Figure 5.7: SVAR: IRFs of Brent Oil Prices - Mexico



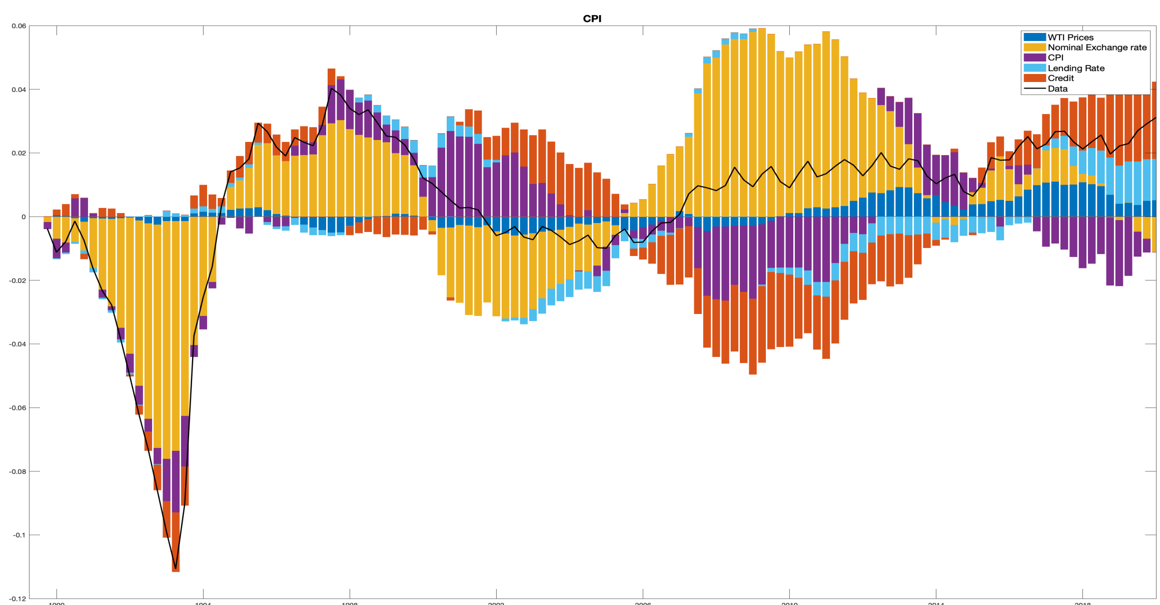
Notes: Impulse response from SVAR utilising Brent prices displayed in Figure 5.12 for historical decomposition for macroeconomic variables for Mexico

Figure 5.8: SVAR: IRFs for Credit, Lending Rate, NER, and CPi from shock on WTI Oil Prices (Figure 3.4)



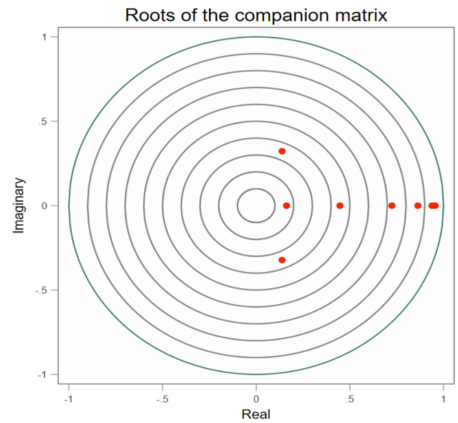
Notes: Impulse responses from SVAR assessing the impact of oil price shock on credit market in Mexico.

Figure 5.9: SVAR: Historical Decomposition for CPI (additional figure for Figure 3.4)

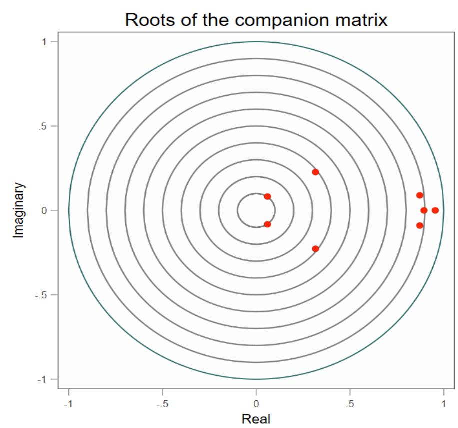


Notes: See Figure 3.4.

Figure 5.10: Stability of SVARs in Figure 5.11 and 5.12

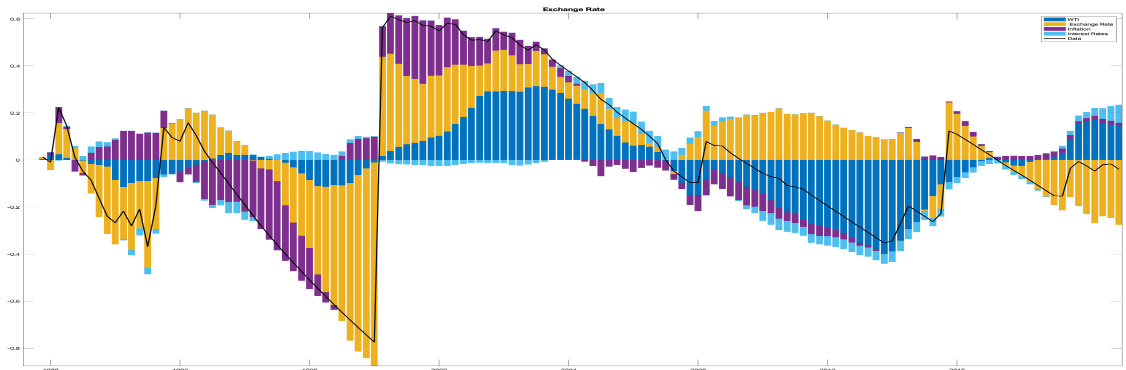


(a) SVAR with Mexico Data (Figure 5.12)

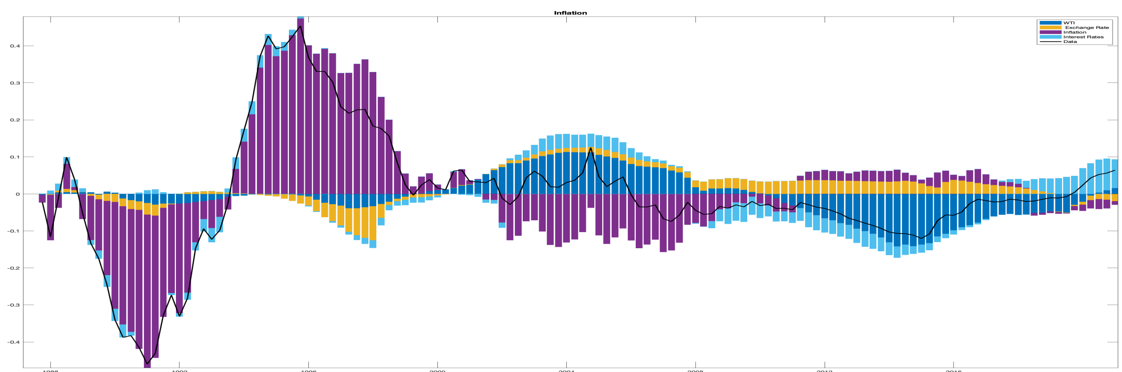


(b) SVAR with Nigeria Data (Figure 5.11)

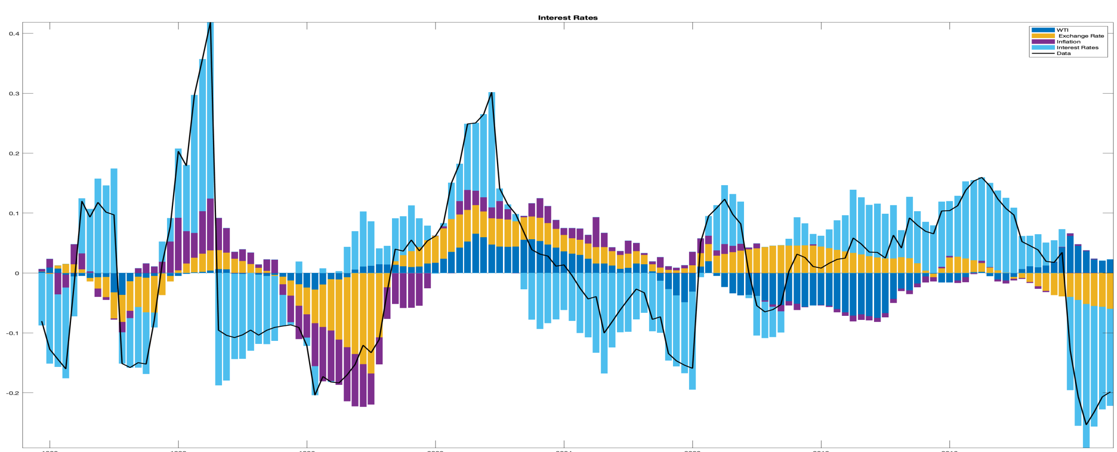
Figure 5.11: Historical Decompositions of Exchange Rates, Inflation and Interest Rates for Nigeria (WTI)



(a) Historical Decomposition of Nominal Exchange Rates



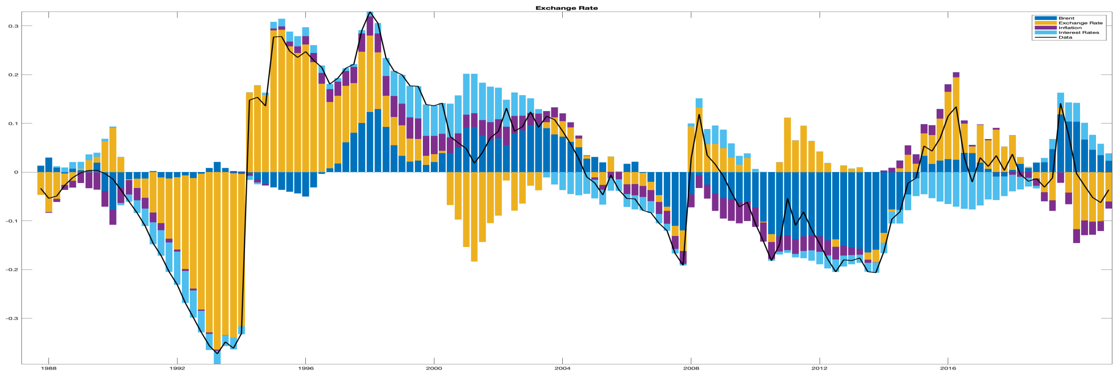
(b) Historical Decomposition of Inflation



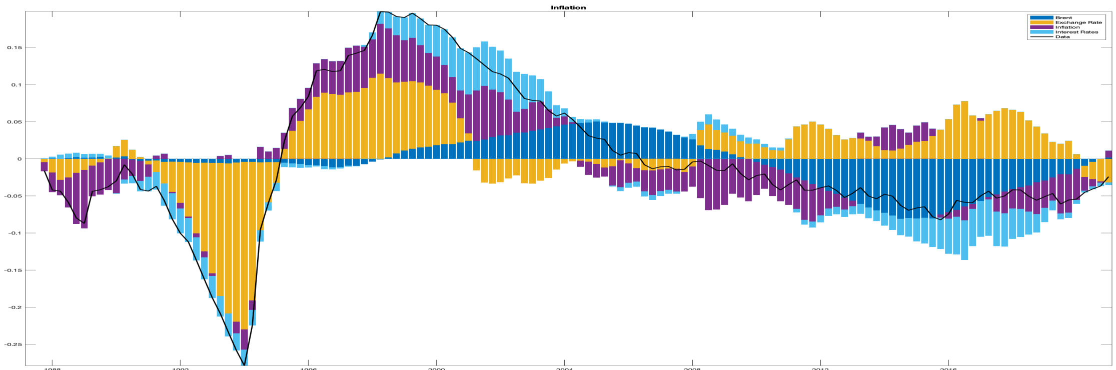
(c) Historical Decomposition of Interest Rates

In (a), (b) and (c) the solid black line depicts the data for log of Nominal Exchange rates, Consumer Price Index and Lending rates, respectively. Blue bars represent historical contribution of WTI oil prices, Yellow, Nominal Exchange rates, Purple; Inflation, and light blue; Lending Rates. Data period is between 1988Q1: 2021Q4.

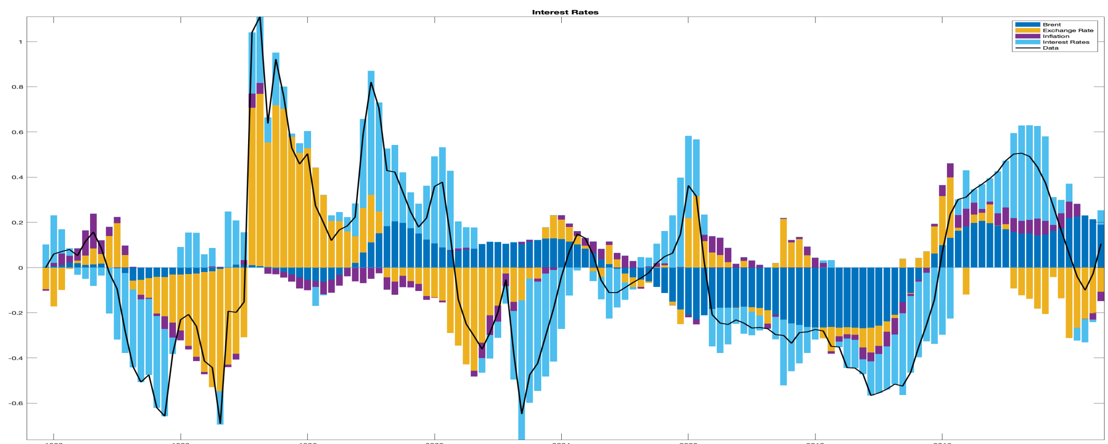
Figure 5.12: Historical Decompositions of Exchange Rates, Inflation and Interest Rates for Mexico (Brent)



(a) Historical Decomposition of Nominal Exchange Rates



(b) Historical Decomposition of Inflation



(c) Historical Decomposition of Interest Rates

In (a), (b) and (c) the solid black line depicts the data for log of Nominal Exchange rates, Consumer Price Index and Lending rates, respectively. Blue bars represent historical contribution of Brent oil prices, Yellow; Nominal Exchange rates, Purple; Inflation, and light blue; Lending Rates. Data period is between 1988Q1: 2021Q4.

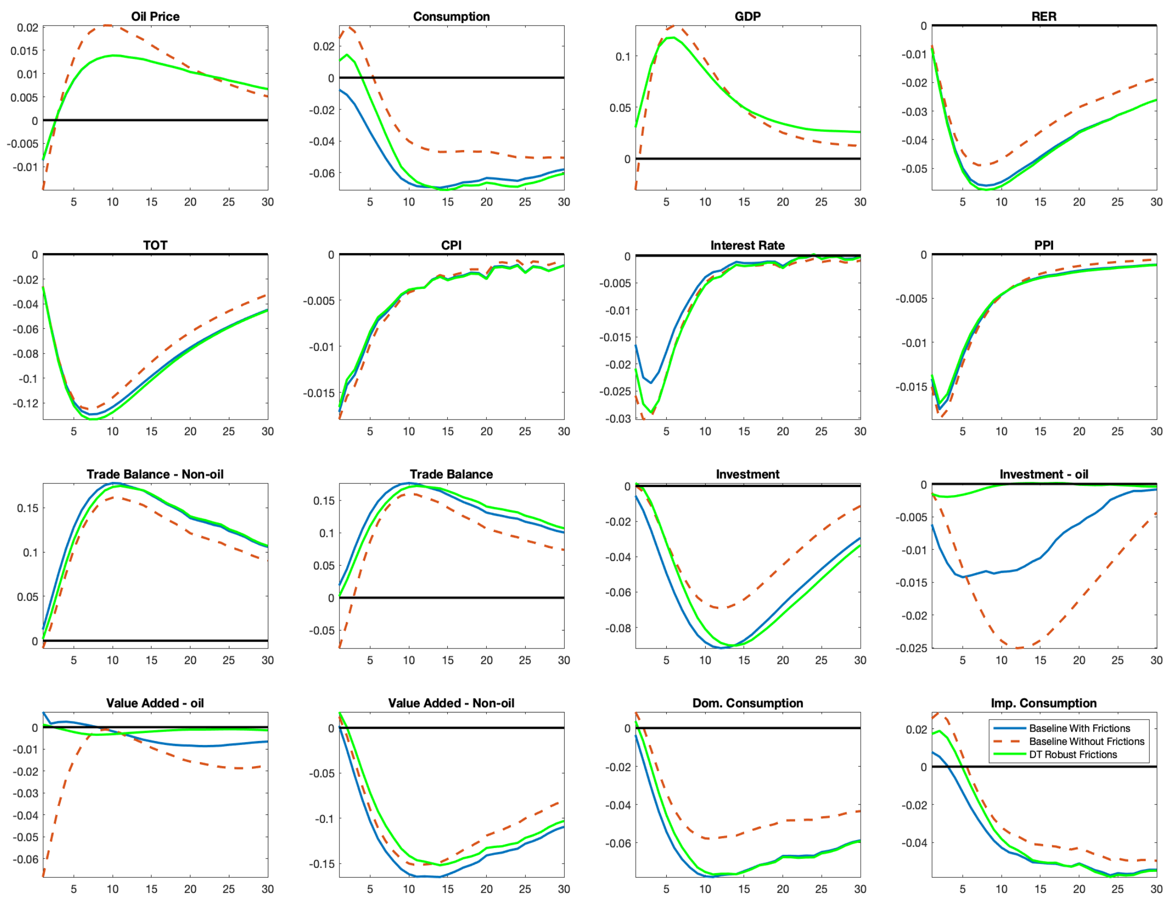
Further Sensitivity Analysis for Chapter 3

Here I specify the tightness of borrowing constraint according to Dreschel *et. al.* (2019). That is I set:

$$\mathrm{II}_t = \mathrm{II}_1(\mathcal{S}_t Po_t^*)^{\mathrm{II}_2}$$

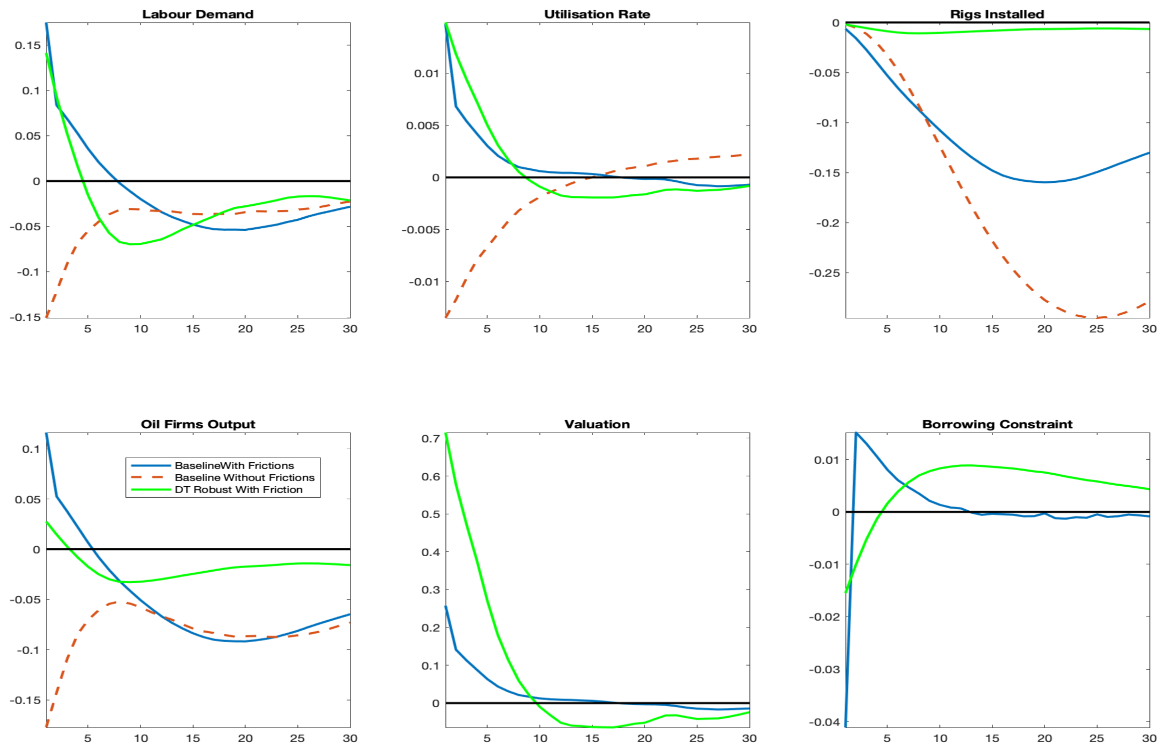
So the tightness of the borrowing constraint changes in response to current period prices alone as opposed to the evolution of oil prices.

Figure 5.13: Home Variable IRFs to Increase in Foreign Intermediate Goods Firms Productivity



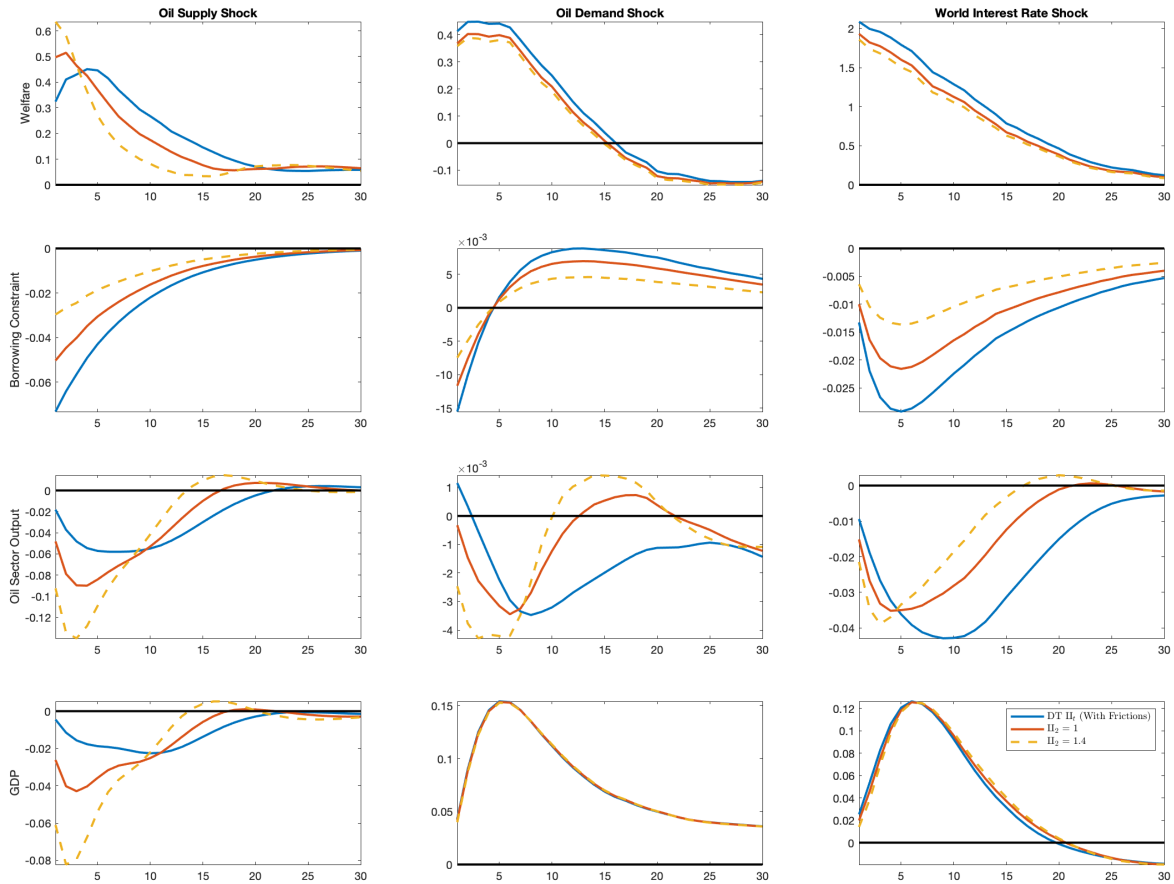
Notes: The blue line depicts the baseline of the model presented in chapter 3. The red dashed line represents the baseline of the model without financial frictions as presented in chapter 2. The green line depicts the model as the baseline in chapter 3 but with the tightness of the borrowing constraint specified according to Dreschel *et. al.* (2019) Shock is $\varepsilon^*_{t,f}$.

Figure 5.14: Home Oil Sector Variable IRFs to Increase in Foreign Intermediate Goods Firms Productivity



Notes: The blue line depicts the baseline of the model presented in chapter 3. The red dashed line represents the baseline of the model without financial frictions as presented in chapter 2. The green line depicts the model as the baseline in chapter 3 but with the tightness of the borrowing constraint specified according to Dreschel *et. al.* (2019) Shock is $\varepsilon^{*}_{t,f}$.

Figure 5.15: Selected Macroeconomic Variable IRFs to Changes in the Elasticity of the Binding Constraint



Notes: The blue line depicts the baseline of the model presented in chapter 3 but with the tightness of the borrowing constraint specified according to Dreschel *et. al.* (2019). The other lines are changes to the elasticity of the binding constrain to changes in oil prices. That is, for $\Pi_2 \in [1\ 1.4]$. Oil demand shock is a shock to ε^*_t. Oil supply shock refers to shock to $\varepsilon^*_{u,t}$ and World interest rate shock is a shock to $\varepsilon^*_{R,t}$

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