Monstrous Musical Moonshine: Explorations of Harmonic Spaces

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Abstract

This commentary provides context for a collection of compositions whose underlying structures are derived from and extend the harmonic theory based on rational intervals between pitches introduced by composer and music theorist James Tenney. The core component taken from Tenney is the measure of harmonic distance (distinct from pitch distance) between two notes. I have extended this to a measure of harmonic dispersion within any number of notes and applied this to developing chord sequences that are either gradually increasing or decreasing in harmonic dispersion. Furthermore, within such sequences the gradation in dispersion is as smooth as possible, both in terms of voicing (usually only one voice in the chord changes at a time) and dispersion level changing by a minimal amount at each stage. A further structural change, developed in later works, is what I term 'unfolding', by which a controlled level of repetition is introduced into the sequence, determined by both parametrised and aleatoric elements. The musical results are often microtonal but take into account the pragmatics of performance, and so are often quarter-tonal or based on the usual twelve-tone equal-tempered system. The resulting compositions are scored for live performers (soloists and ensembles), electronic synthesis, and hybrid scoring. The culminating work is a setting of texts from Zen Buddhist sources for vocal soloists, choir, chanters, an instrumental ensemble and electronic synthesisers. Aside from the structural technicalities, my music focuses on non-narrative, often long-duration expressions, influenced by composers such as Morton Feldman and Eliane Radigue. While technicalities are interesting, the key objective is always to produce music, not mathematics, with the hope that the work will be of interest to listeners, not mathematicians. Ultimately, this project has been experimental, researching how complex structures could be manifest in music.

Monstrous Musical Moonshine

Explorations of Harmonic Spaces

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Doctor of Philosophy

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List of Works Submitted

Properties of Matter: Expansion and Contraction for string quartet
Duration 25 minutes
Performed by Quatuor Bozzini, Berwick-upon-Tweed, 2019
Score at filename Worboys000736479-PoM expansion contraction.pdf
Recording at filename Worboys000736479-PoM expansion contraction.aif

Properties of Matter: Damped Harmonic Motion for instrumental group
Duration 15 minutes
Performed by the Ives Ensemble, Durham, 2019
Score at filename Worboys000736479-PoM damped HM.pdf
Recording at filename Worboys000736479-PoM damped HM.WAV

Properties of Matter: Controlled Descent for solo piano
Duration 7 minutes
Recorded by Ben Smith, London, 2021
Score at filename Worboys000736479-PoM controlled descent.pdf
Recording at filename Worboys000736479-PoM controlled descent.wav

Melum Scalenum for bass viol
Duration 5 minutes
Performed by Liam Byrne, Berlin, 2020
Score at filename Worboys000736479-melum scalenum.pdf
Video recording at filename Worboys000736479-melum scalenum.mp4

2-Spem-70 for 40 synthesizers
Duration 70 minutes
To be performed as 8-track version, 8-Spem-70, Berwick-upon-Tweed, 2022
Video recording at filename Worboys000736479-2-spem-70.mp4

Presence/Absence for eight solo voices, choir, instruments and electronics, to be performed 2022. Duration 60 minutes

Duration 60 minutes

Performed by Exaudi, choir, chanters and performers, Durham Cathedral, 2022

Score at filename Worboys000736479-Presence Absence.pdf

Recording at filename Worboys000736479-presence absence.wav

Supporting research data for this thesis has been published in the Durham University research data repository. Link: https://collections.durham.ac.uk/collections/r20k225b12z

Statement of Copyright

Acknowledgements

I would like to warmly acknowledge the help and support of my three supervisors, Dr James Weeks, Dr Eric Skytterholm Egan and Prof Nick Collins. I am particularly grateful to James for introducing me to rational intonation and setting me on the trail that led me to the work of James Tenney and thence to this project exploring and extending Tenney's theory of harmony. My relationship with the two James's has been at the heart of this work. All three supervisors have been very generous with their time. James and Eric have gone out of their way to provide and suggest opportunities for performance of my music. Nick has provided help with the SuperCollider audio synthesis platform.

To Moira and our family

Chapter 1

Introduction

1.1 Monstrous motivations

When I was younger I gained a PhD in pure mathematics by studying some very large collections of symmetries in many dimensions, called finite simple groups. One of these groups, actually the largest, was known as the Monster Group. In the course of my PhD studies, Donald Livingstone, my supervisor, was working to prove that this structure actually existed, and during this period two mathematicians, Conway and Norton (1979) noticed some remarkable properties of the Monster, if it existed. They titled their paper *Monstrous Moonshine*; 'Moonshine' because it was all predicated on the beast's existence. Eventually, the 'Monster' was proved to exist. Furthermore, in one of the greatest mathematical achievements of the 20th century, the Classification of Finite Simple Groups, the Monster was shown to complete the collection of such structures – there really were no more to be found!

How does this relate to this, my second PhD experience? In the spirit of Conway and Norton's work, this research is monstrous, in that some of the harmonic structures discussed later in the thesis are extremely large, maybe not as large as the Monster, but containing many thousands of harmonies. It is 'moonshine' in the sense that the work is experimental with uncertain outcomes; I want to try to show that such structures can indeed provide a framework for meaningful musical experiences.

1.2 Compositional objectives

As will be discussed in the next chapter, the structures underlying my compositions presented here derive from a harmonic theory based on rational intervals between pitches. This theory has a long history, going back at least as far as Pythagoras, and was developed into the notion of a harmonic space by James Tenney in a series of papers beginning in the late 1970s and now published in the edited collection Tenney (2015). Tenney did not have the computational resources to develop this concept beyond some relatively small-scale examples. I have taken advantage of my background as a mathematician and computer scientist to investigate large-scale harmonic spaces. Thus, a general goal of this research is to investigate approaches to composition using harmonic spaces that are process-based, algorithmic in nature, with some reliance on mathematical principles.

Mathematical structures, no matter how elegant and well-founded on fundamental harmonic principles, do not necessarily immediately translate into meaningful musical experiences. Throughout, I have sought to balance aesthetic and affective considerations with structural principles, with the goal of arriving at works where each reinforces the other. In addition, I have spent some years as a Zen practitioner, studying in the UK, USA and Japan, with a spell as head monk of a Zen centre in the USA. A third general objective is to bring some elements of this practice into the musical works. Areas where this has been developed include focus on non-narrative work, experiments with long duration, and work which facilitates mindfulness and attention to detail.

1.3 The portfolio

The works included in the portfolio are listed in the preamble to this commentary. I thought it important to show the range of my compositional activity, which includes a mix of solo and ensemble instrumental writing, a choral piece and work with electronic synthesisers. Because the choral and electronic works are long-duration, both around an hour, I have had to exceed the normal limit for total duration of music in a composition portfolio. The total duration of my portfolio is around three hours. This has been done with the approval of my supervisors and the Department.

1.4 Summary of the thesis

Having set out the principal objectives of this study, Chapter Two provides a description of the general compositional structures and processes used in my work. That chapter will review Tenney's notion of harmonic distance and use it to define my own notion of harmonic dispersion and other related properties. This chapter also provides some general details concerning the computations required to generate pitch material. Chapter Three will review the compositions submitted as part of the PhD in the context of related compositions by other composers that either influenced and motivated my works or are closely related to them. The final chapter will critically review the overall project, provide a general research context for the work, assess strengths and weaknesses of the overall research, and look forward to future works.

Chapter 2

Compositional approach

2.1 Introduction

This chapter describes the structures and processes involved in my compositions. The framework described below relates mainly to the harmonic structures underlying my compositions, along with some other strategies, including repetition and use of randomness and uncertainty. The rhythmic, timbral and gestural components are of course equally important, but are more piece-specific and so will be discussed in Chapter Three when describing individual works. These latter aspects are combined with the process-based framework below to bring out the musicality of the works.

2.2 General compositional strategy

In his introduction to 'Patterns of Intuition', Nierhaus (2015) presents the steps in a strategy that process-based composition can take place. They include presentation and formalisation of the compositional principles, implementation and computer generation of musical material, and cycles of generation and evaluation based upon the composer's aesthetic preferences. While this strategy was geared to algorithmic composition, there is sufficient overlap with my own methods to be a useful framework. The compositional principle for my work is founded on Tenney's concept of harmonic distance (Tenney, 2015), discussed in more detail in the next section, enhanced by specific factors related to the individual piece being composed. Factors that I have further developed include harmonic dispersion, repetition, voice-leading, and randomness. The implementations necessary to generate the basic musical material are performed using the programming language Python (Rossum and Drake, 2003) to generate the initial pitch collection sequences. SuperCollider (McCartney, 1996; Wilson et al., 2011) was used for generating synthesisers of electronic sounds when the piece was wholly or in part electro-acoustic. This process as a whole is discussed in more detail later in this chapter, while aspects of the process that apply to specific compositions are described in the next chapter, when considering the pieces themselves.

2.3 Harmonic structures and relationships

2.3.1 Harmonic spaces

This thesis and my works composed for the PhD are based on microtonal music, where pitches are combined based upon rational relationships within harmonic spectra. Thus, this undertaking can be seen as an extension of works based on just intonation practice. Each pitch has an associated sequence of harmonics, beginning with itself, its octave, the third harmonic, and so on. The guiding principle of this framework is that two or more pitches are more or less related together as far as their individual harmonic sequences are related. So, two pitches in unison have identical harmonic sequences and therefore are maximally related; two pitches an octave apart are slightly less related but more related than two pitches distant by a third harmonic. This notion can be precisely quantified using a formula developed originally by James Tenney, in his 1979 work 'The Structure of Harmonic Aggregates' and expanded on in his 1983 paper 'John Cage and the Theory of Harmony', both published in the collection (Polansky et al., 2015).

We begin with a pitch taken as the fundamental for the framework. For this exposition, let us assume this to be the pitch A1(55Hz)[1] . Then any other pitch, assuming it has some harmonic relationship to the fundamental, can be represented by a sequence of integers $[r_1, r_2, r_3, \ldots]$ to indicate by how many iterations of second, third, ... harmonics it is related to the fundamental. For example, A2(110Hz) is the 2nd harmonic of A1(55Hz), and so would be represented by sequence $[1,0,0,\ldots]$. A3(220Hz) is the 2nd harmonic of the second harmonic, and so represented as $[2,0,0,\ldots]$. The 3rd harmonic pitch 165Hz is represented approximately by the E3 that is an interval of an octave and a perfect fifth above A1(55Hz) and has sequence $[0,1,0,\ldots]$. A0(27.5Hz), being the octave below A1(55Hz) and so the 2nd undertone harmonic (to borrow a word from Harry Partch (Partch, 1974)), has representation $[-1,0,0,\ldots]$. The position of an integer in a sequence relates to the prime number for that harmonic and the integer itself provides information on the number of iterations (overtones or undertones) of that harmonic. The entire collection of all these sequences determines the

[1] Scientific Pitch Notation is used throughout this commentary, usually combined with frequency measured in Hz, so as to avoid any ambiguity.)

structure of an infinite dimensional vector space, which we term the *harmonic space based on A1(55Hz)*. It is this space that is the basis for all the work to follow.

In the compositional works below, limits are placed on the number of prime number components used. For example, if only primes up to and including 11 are used, extending usual terminology, the space is referred to the 11-limit space and is 5-dimensional (being based upon the five primes 2,3,5,7,11). For brevity of notation, we abbreviate the harmonic sequences by eliminating all the trailing zeroes, for example $[2,1,0,-1,3]$ rather than $[2,1,0,-1,3,0,0,\ldots]$.

2.3.2 Harmonic distance HD

When two pitches in a harmonic space are combined, we need to consider their harmonic relationship according to the above scheme. Traditional theory quantifies pitch relationship according to the pitch interval between them. In this work we use a different quantification based on their relationship in harmonic space, termed by Tenney the *harmonic distance* or HD between them. Our formulation is essentially Tenney's construction in (Tenney, 2015). Suppose we have pitches p_1 and p_2, with sequences $[r_1, r_2, r_3, \ldots]$ and $[s_1, s_2, s_3, \ldots]$, then the harmonic distance $\delta(p_1, p_2)$ is defined as follows:

$$\mathrm{HD}(p_1, p_2) = |r_1 - s_1| \log_2 2 + |r_2 - s_2| \log_2 3 + |r_3 - s_3| \log_2 5 + \ldots$$

where $2, 3, 5, \ldots$ is the sequence of prime numbers.

It is worth noting that the harmonic distance between two pitches is independent of which fundamental is chosen, provided that the pitches can be related in that harmonic space. Also, it is of mathematical interest that HD has the properties of a metric, and enhances the structure of the harmonic space to a metric space.

James Tenney, in the previously cited paper 'John Cage and the Theory of Harmony' (Tenney, 2015) provides an example of what he calls a '2,3 plane of harmonic space'. In the notation above, this is an example of a 2-dimensional harmonic space, based on the first two primes, 2 and 3. Harry Partch's '11-limit' scale, in (Partch, 1974), implies a harmonic space of five dimensions, using the primes 2,3,5,7 and 11.

It is also interesting to apply our notion of harmonic distance to traditional just 7-limit intervals in a chromatic scale. Table 2.1 provides a table of these rationals and their harmonic distances from the tonic A. We may note that these distances accord very well with traditional notions of consonance, distances ranging in increasing value from unison to octave to perfect fifth, perfect fourth, major sixth, major third, through to minor second. There is not an exact match; for example, the augmented fourth is quite low and the minor seventh quite high.

Table 2.1 Harmonic distances of some just intervals within an octave

Note	Interval	Rational	Prime powers 2	3	5	7	HD
A	Unison	$\frac{1}{1}$	0	0	0	0	0.00
A♯	Just Minor 2nd	$\frac{16}{15}$	4	-1	-1	0	7.91
B	Pythagorean Major 2nd	$\frac{9}{8}$	-3	2	0	0	6.17
C	Just Minor 3rd	$\frac{6}{5}$	1	1	-1	0	4.91
C♯	Just Major 3rd	$\frac{5}{4}$	-2	0	1	0	4.32
D	Pythagorean Perfect 4th	$\frac{4}{3}$	2	-1	0	0	3.58
D♯	Septimal Diminished 5th	$\frac{7}{5}$	0	0	-1	1	5.13
E	Pythagorean Perfect 5th	$\frac{3}{2}$	-1	1	0	0	2.58
F	Just Minor 6th	$\frac{8}{5}$	3	0	-1	0	5.32
F♯	Just Major 6th	$\frac{5}{3}$	0	-1	1	0	3.91
G	Pythagorean Minor 7th	$\frac{16}{9}$	4	-2	0	0	7.17
G♯	Just Major 7th	$\frac{15}{8}$	-3	1	1	0	6.91
A	Octave	$\frac{2}{1}$	1	0	0	0	1.00

However, the table does provide some evidence of a relationship between harmonic distance and traditional harmonic practice.

2.3.3 Harmonic dispersion

The next step, and this takes us beyond Tenney's theory, is to extend the definition of harmonic distance to clusters of more than two pitches. Suppose we have a collection S of n pitches $p_1, p_2, \ldots, p_n$. The *harmonic dispersion* of S, $\Delta(S)$, is defined as the mean of the harmonic distances between all pairs of distinct pitches. For example, consider a root just major tonic triad, (C,E,G), with rationals (1, 5/4, 3/2). The harmonic dispersion of this triad is computed as the mean of harmonic distances between C and E, E and G, and C and G, which is the mean of 4.32, 4.91, and 2.58, which computes to 3.93. On the other hand, the dispersion of the diminished triad (C,E♭,G♭) computes to around 5. While by no means conclusive, these examples and others suggest that lower dispersion values correlate with harmonic stability in the traditional sense[2] .

[2]It would be interesting to further investigate the relationship between dispersion and harmonic stability for a wider set of chords and for higher values of n. However, this is beyond the scope of this research.

2.4 Generation of musical material

Generating the draft musical material for all the pieces submitted as part of this thesis follow some or all of the following stages:

1. Choice and construction of pitch palette;
2. Generation of primitive chord sequences;
3. Generation of final chord sequences.

I now describe each of these stages in general terms. In the section following this, I provide some details of the computational processes used to realise these constructions.

2.4.1 Choice and construction of pitch palette

Each musical work in my collection is formed from a finite collection of pitches, called here the *pitch palette* of that work. There are three factors that determine the pitch palette. The first is the desired ranges of the instruments for which the work is composed. I include here the desired range of the synthesisers in an electro-acoustic piece; for example, it would not be appropriate to use pitches outside the range of human hearing. The second is limited by the number of harmonics used. For example, a particular work might only use pitches in an 11-limit harmonic space. The third factor is related to harmonic range rather than pitch range. All the pieces here use pitches that are within a pre-determined harmonic distance from a central pitch. This central pitch depends on the particular work, but might for example be A1(55 Hz). So, in summary, a pitch palette for a particular work is determined by tessitura and harmonic constraints.

2.4.2 Generation of primitive chord sequences

With the exception of *A-Orbital-40*[3], an early piece, not included in the portfolio and which will be discussed in Chapter 3, each work presented here is based on a piece-specific sequence of chords, called its *primitive chord sequence*. Each piece will have a fixed number of voices that determines the size of the chords. For example, the string quartet has a primitive tetrad sequence. The procedure for generating the primitive sequence is a recursive process, as follows.

Initialisation: The first chord is a unison, and therefore has zero harmonic dispersion.

[3] *A-Orbital-40* can be found at https://www.youtube.com/watch?v=Mqb7VxOQ3YE as the soundtrack to the movie *Crossing*, that was shown as part of the 2018 Berwick Film Festival Fringe.

Iteration: If the previous chord X has harmonic dispersion δ, then go through all possible chords that are different from X in just one voice and choose the chord from this collection that results in a minimal non-zero increase to δ.

Conclusion: Continue the iteration step until it is no longer possible to choose a new chord that satisfies the two recursion constraints.

It is noteworthy that this process must always stop as the pitch palette provides only a finite number of choices of pitch and so the harmonic dispersion cannot grow indefinitely.

2.4.3 Generation of extended chord sequences

Generating the primitive chord sequence may immediately provide the chord sequence for the work. However, experience with earlier works showed the benefits of repetition. An *extended chord sequence* is derived from a primitive chord sequence using a folding process that allows repetitions in a controlled manner that might in some cases involve elements of randomness.

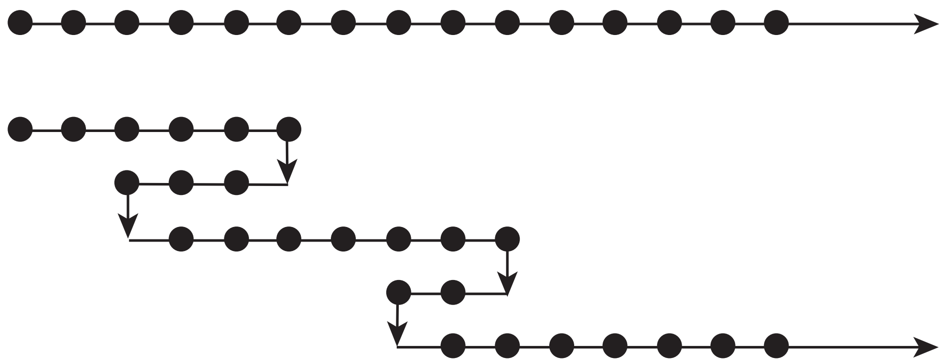

Fig. 2.1 Folding process to produce controlled repetition under randomness

Figure 2.1 shows an example of the folding process in action. The top row is the primitive chord sequence, from leftmost unison upwards. The extended sequence below has been formed by proceeding along a controlled random number of chords up the sequence (the *up-subsequence*), then backtracking through a second random number of chords (the *down-subsequence*), and continuing in this manner until the end of the primitive sequence is reached. The choice of random numbers for the up-subsequences and down-subsequences is controlled by parameters that are determined by aesthetic considerations related to the particular piece, and ensure that an endpoint is finally reached. The randomisation process is described in more detail in subsection 2.5.3 below.

2.4.4 Notational considerations

The Helmholtz-Ellis JI Pitch Notation (HEJI), devised by Sabat and von Schweinitz (2004) explicitly notates microtonal pitches based on rational intervals. Initially I considered using this notation for all microtonal pieces designed for performance on acoustic instruments. However, in the end, I decided against this, using instead microtonal adjustments to the equal-tempered scale measured in cents, for the following reasons. Firstly, many of the harmonic relationships in my works are extremely complex, usually involving combinations of powers of several primes, and using HEJI would be very cumbersome, requiring much new extra symbolic notation that would need learning for each piece. This would not necessarily provide an easily performable score. Secondly, two ensembles who performed two of my pieces expressed a preference against HEJI. HEJI is very expressive in many ways, and has the advantage that it exposes the precise harmonic relationship between pitches. I do not rule out using HEJI in future work, particularly in cases where the harmonic relationships are simple enough for this notation to be useful.

2.5 Computational considerations

As already stated, the platforms used for programming and audio synthesis are respectively the programming language Python (Rossum and Drake, 2003) and the audio server, programming language and editor, bundled together as SuperCollider (McCartney, 1996; Wilson et al., 2011). While SuperCollider does have the capability for algorithmic development, almost all of this component of the work was programmed in Python, using the Integrated Development Environment (IDE) PyCharm (Nguyen, 2019). All computations were executed on a MacBook Pro (2016). The stages of the computations are now discussed in more detail.

2.5.1 Generating the pitch palette

The pitch palette provides the set of pitches to be used in the work in question. To generate these pitches, the following parameters need to be defined:

1. The prime limit of the harmonic space used in the piece.
2. The lower and upper bounds to the range of allowable frequencies. This will be dependent on the voice ranges, instrument ranges, limits of human hearing, and other considerations.
3. The fundamental or central pitch.

4. The allowable harmonic distance from the fundamental pitch that all pitches must be within.

There may be further constraints. For example, it might be necessary to limit the pitches to be close in frequency (see later discussions on precision) to pitches in the equal-tempered semitone (12-tone) or quarter-tone (24-tone) scales.

After all constraints have been specified, an iteration within the harmonic space is performed and all pitches that satisfy the constraints above are saved to a file for input to the next stage of the process.

2.5.2 Computation of primitive chord sequences

The general idea of the process for generating a primitive chord sequence is described in subsection 2.4.2. This subsection provides more details on the algorithm. The following input needs to be provided and parameters defined:

1. The file containing the pitch palette, coming from the computation in subsection 2.5.1.
2. The number of voices in each chord.
3. Any further data about the instrument line up in each chord, and the limitations of frequencies based on the *tessitura* of each instrument, or other factors.
4. The initial pitch from which an initial unison chord is constructed.
5. The upper bound on the length of the sequence (optional).[4]

The generation of chords proceeds as described in subsection 2.4.2. Once the process is complete, or the upper bound has been reached, a file containing the sequence of chords is output. Each chord is itself a sequence of pitches, in an order determined by the specified instrument line-up.

2.5.3 Computation of extended chord sequences

In those compositions where repetition is a feature, a further stage in the computational process is required. An extended chord sequence is generated from a primitive chord sequence by a process of randomised repetition, described briefly in subsection 2.4.3 and here described in further detail.

[4] As stated earlier, generation of a primitive chord sequence will always terminate. However, it may be desirable in a particular case to specify an earlier termination point.

Figure 2.1 shows the general pattern of repetition, requiring a series of forward motions through up-subsequences followed by retrograde motions through down-subsequences, until the process terminates. Randomisation is achieved by the introduction of four parameters, two for the up-sequences and two for the down-sequences.

For the up-subsequences, an integer parameter is given, called *up-mean*, that specifies the mean of their lengths, and a second integer parameter, *up-var*, specifies the variance around the mean. In a similar way, *down-mean* and *down-var* are specified for the down-subsequences. The higher the variance, the more variability there is in each subsequence. The higher the mean, the longer each subsequence will be. The difference between the up-mean and the down-mean is the critical factor that determines the length of the entire extended sequence. In general, the down-mean should be less than the up-mean, and the greater the difference between the two means, the shorter the extended sequence will be, and therefore also the composition. The setting of these parameters is a key musical decision for each piece using repetition.

The output from this computational process will be a sequence of chords, where each chord is a sequence of pitches, in an order determined by the specified instrument line-up. The repetition of the sequence is randomised in a controlled fashion as detailed above.

2.5.4 Output of results

There are two forms in which the results from the above processes are output. In the case where a pitch from a chord in the sequence is to be subjected to audio synthesis, then its frequency in Hz is all that is required. In the case where a pitch from a chord in the sequence is to be played by a human performer, the output is given in the form of a pitch specified in Scientific Pitch Notation, so that it can be transcribed to traditional musical notation in scores. The algorithm for transforming a frequency to Scientific Pitch Notation is well-known[5] and will not be described further here.

2.5.5 Audio synthesis

All pitches input to the audio synthesis process are specified as frequencies, measured in Hz. The audio synthesis process itself takes place within the SuperCollider platform, as described above. The chord sequence is input as a file of pitches that, for each pitch, contains information about its frequency, its position in which chord, and the time in the piece at which it was to be sounded. Based on this data, the envelope of each note (attack, decay, sustain and release) is computed. Supercollider is then programmed to sequence the synthesis

[5] See, for example, Puckette (2007), page 7

of each pitch as a sine wave with parameters determined as above, taking account of placing the note in the multichannel (usually stereo) soundstage.

In all pieces in my portfolio, the pitches are synthesised into relatively simple combinations of sine waves, with some granulation. The purpose of the granulation is to provide some added character to the pure sine wave timbre. As this applies to several of my works discussed in this commentary[6], this is described in a general form here.

Granulation, in the context of electronic audio processing, is the segmentation of a sound wave into small portions, called *grains*. Grains may be so small in duration that characteristics such as frequency and amplitude may be at the limit of human perception (Winckel, 1967). These grains may then be further manipulated and recombined to alter the character of the original signal. Typical results of such granulation techniques include pitch shifting, temporal stretching, adding drone-like effects, and freezing. For a comprehensive exposition of granulation, the reader is referred to Roads (2001).

2.5.6 Further comments on the overall computational process

Not all pieces in this portfolio require all the stages described above. Figure 2.2 provides a schematic of the flow through the modules and shows routes through the network of modules for the works covered by this commentary.

Works discussed in the commentary that require data preparation provided by the human performance module are:

- *Properties of Matter: Expansion and Contraction*
- *Properties of Matter: Damped Harmonic Motion* (repetition done manually)
- *Properties of Matter: Controlled Descent* (with the extended sequence module)
- *Melum Scalenum* (with the extended sequence module)
- *Presence/Absence* (with the extended sequence module)

Works discussed in the commentary that require the data preparation provided by the synthesis module are:[7]

- *2-Spem-70* (with the extended sequence module)

[6] *A-Orbital-40*, *2-Spem-70*, and *Presence/Absence*

[7] The work *Presence/Absence* is scored for both human and synthesiser performance and so is included in both lists.

- *A-Orbital-40*
- *Presence/Absence* (with the extended sequence module)

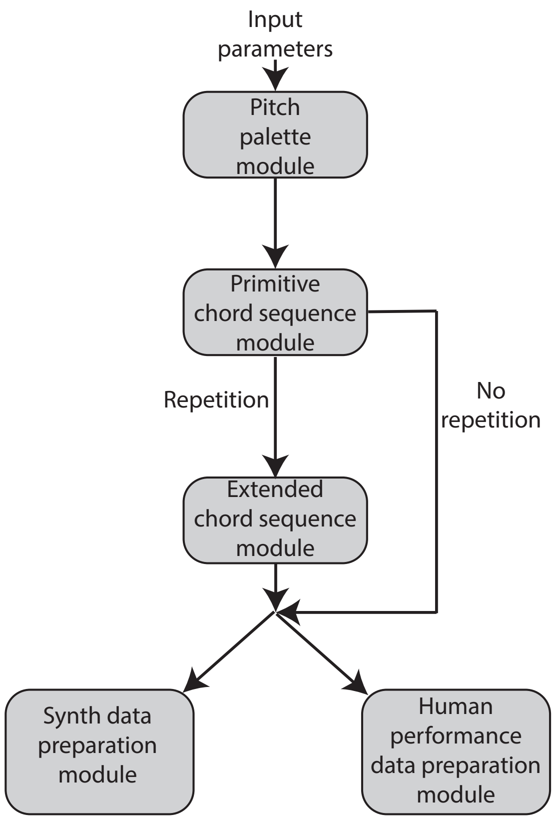

Fig. 2.2 Flow diagram of the computational process

2.6 Summary

This chapter has described the harmonic distance, harmonic space, harmonic dispersion concepts as well as the principles by which primitive and extended chord sequences are produced. I have discussed, firstly in a general sense and then in some detail, how these constructions have been used to produce the pitch material for my compositions. In the next chapter I will discuss the specifics of how they are applied to each work.

It is important to say that the generation of sound material therefrom is but a part of the composition process. Such material provides an armature for the construction of the musical material that proceeds, dependent on piece-specific aesthetic considerations, as discussed in the next chapters.

Chapter 3

My selected works

3.1 Introduction

This chapter introduces the works selected for the portfolio and places them in the context of my own thinking and listening experiences. The work can be broadly divided into three groups: purely acoustic, purely electronic, and hybrid. A further classification is that some of the works form part of a sequence of compositions under the heading, ‘Properties of Matter’, each relating to some fundamental physical phenomenon. All works use the underlying framework described in the previous chapter.

The chapter opens with a commentary on *Properties of Matter: Expansion and Contraction*, for string quartet, which is the first substantial work that I composed as part of my doctoral studies and illustrates the processes described in Chapter 2 in their ‘purest’ form, that is, without the addition of processes related to repetition and forms of diminution that are important for my later works in this study. The string quartet is discussed alongside a discussion of the string quartet *Arbor Vitae* composed by James Tenney.

I then proceed to discuss the other works in the portfolio. My work *Presence/Absence* is the most ambitious work, embodying most of the processes described in Chapter 2. Hence, I give this work substantially more attention, using it as an exemplar of many aspects of my approach to composition.

3.2 Harmonic Space and Two String Quartets: *Properties of Matter: Expansion and Contraction*

I cannot remember how I was introduced to the work of James Tenney. I do remember a visit from the violinist Mira Benjamin in the first term of studies at Durham, who demonstrated rational harmonies on her instrument. Some time in that first year I made a study of the score of Tenney's string quartet *Arbor Vitae*, with a recording by Quatuor Bozzini, who are world-leading exponents in this area. The details of the construction of this work are set out in detail in a paper by Michael Winter (2008). *Arbor Vitae* was Tenney's last work, completed shortly before his death, with the assistance of Michael Winter. On writing this now, I realise how influential this work has been for me and my own string quartet, although once immersed in the composition I was not consciously remembering or even aware of Tenney's compositional processes.

Properties of Matter: Expansion and Contraction is a microtonal work for string quartet, 25 minutes in duration, composed over the course of about a year in 2018-2019, and given its first performance in November 2019 at the Watchtower Gallery, Berwick-upon-Tweed. *Arbor Vitae*, is also a microtonal composition for string quartet. It is James Tenney's last work, completed in 2006, and is thirteen minutes in duration. Both works were given their first performances by Quatuor Bozzini.

Arbor Vitae and *Expansion and Contraction* both use pitches from an 11-limit harmonic space. Tenney's pitches are drawn from those represented in a tree diagram (Figure 1 in Winter (2008)) and are roughly speaking simple combinations of prime harmonics up to 11. Formally, each pitch is some integer multiple of B♭1 (approx. 58.27Hz), where the integer is of form $3^a * 5^b * 7^c * 11^d$, where $a+b+c+d \leq 3$. There is also some flexibility about moving up and down octaves. This then constrains Tenney's pitch palette to around 35 pitches. My work constrains the 11-limit ball of all pitches within a fixed harmonic distance 20 from A3 (220Hz). A further constraint is that all pitches are in frequency range 65-1568Hz [1] and results in a pitch palette of around 600 pitches.

On a macro-level, there are some similarities. Polanski (1983) used the term 'swell' to denote an enlargement followed by diminution in one or more musical parameters. *Arbor Vitae* has swells in dynamics and pitch range; *Expansion and Contraction* has a swell in harmonic dispersion, as defined in the previous chapter. The piece begins on a unison A3 (220Hz), stated immediately, and then the primitive chord sequence is exposed one note at a time until, roughly two-thirds through the piece, maximum harmonic dispersion is achieved, and then the process is reversed.

Once initial parameters have been set by the composer, *Arbor Vitae* is then completely determined by the processes set in motion, each note following the next according to strict deterministic and probabilistic rules, and all instruments sounding pitches at all times. By

[1] This range corresponds to the low C2 on the cello up to G6. There are of course higher frequencies available to be played by the instruments and Tenney's work has considerable higher pitches. However, in order to play my piece pitch-accurately, meters are required; Bozzini player inform ed me that the meters cease to be accurate above the range given.

contrast, although the pitch palette and primitive chord sequence are process-driven in *Expansion and Contraction*, the disposition of notes on the page is much more intuitive and dependent on the composer's inner ear, taking into account rhythmic, timbral and gestural considerations.

As mentioned above, all instruments play all the time in *Arbor Vitae*, and natural and artificial harmonics provide timbral variety. In *Expansion and Contraction* no harmonics are used, but an additional element that can provide textural change is silence.. Figure 3.1 shows examples of this from the score. Example (a) shows all instruments playing; (b) shows a staggered texture with instruments entering and exiting at differing times; (c) shows a block texture with all instruments entering together with a short silence between; (d) shows a longer silence; and (e) shows a short solo passage for first violin. The principle gestural component is the widespread use of fairly localised increases and decreases in dynamics. Other than these factors, the piece is relatively featureless, with prominence given to the global feature of harmonic expansion and contraction.

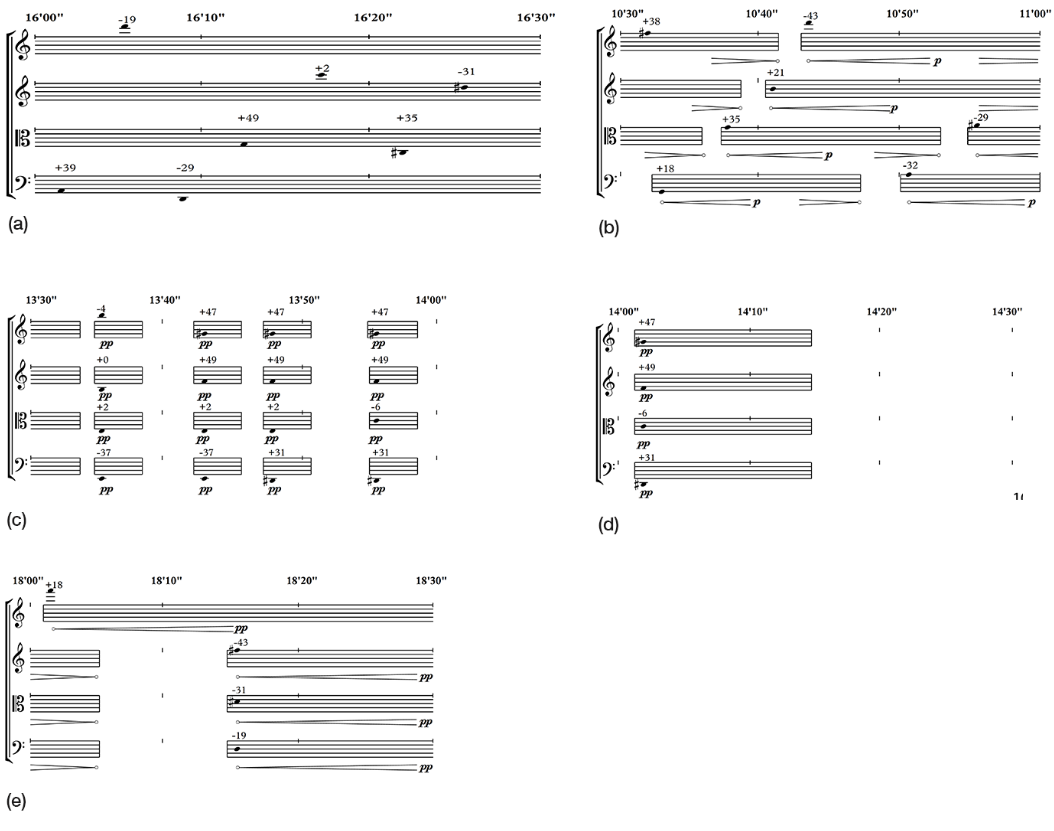

Fig. 3.1 Examples of textures in *Expansion and Contraction*

3.3 Music or mathematics?

Chapter 2 leaned heavily towards the technical side, setting up the rather complex framework provided by harmonic spaces as a basis for my own music. As a mathematician, these aspects are endlessly fascinating, and I believe ground the harmonic material in real physical properties of sound in ways that perhaps not all earlier theories do[2]. However, music is not mathematics, and in this section I would like to explore some musical works that move me and have influenced my work as a composer. I should say at this point that as a child I was a singer in a choir committed to the Catholic Tridentine Mass. (This was before the Second Vatican Council, after which such practices became quite rare). Later in life I spent some years immersed in Zen, acting for a while as head monk in a U.S. Zen centre and staying for a short visit at an important Zen monastery in Japan. These experiences have definitely had an impact on the kinds of music to which I am drawn and try to create. It is difficult to sum up in a a few words, but 'contemplative', 'non-narrative', 'going nowhere but where we are right now' and 'mindful of the sound rather than what it symbolises' all point in the right direction. In this section I introduce some musical works that speak this language and have influenced the work in this PhD in ways that I hope to demonstrate.

Trilogie de la Mort was composed by Éliane Radigue between 1985 and 1993 and was premiered in November 1993 at the monastery at Cimiez in Nice. The work is purely electronic, the sounds being generated by an ARP 2500 synthesizer. *Trilogie de la Mort* comprises three sections, *Kyema*, *Kailasha*, and *Koumé*, each being roughly an hour in length. Radigue has been a practicing Buddhist in the Tibetan tradition since the mid-1970s and this work is inspired by Tibetan Buddhist concepts and practice. *Kyema*'s structure mirrors the six bardos set out in the Tibetan Book Of The Dead; *Kailasha* is named after Mount Kailash, the most sacred of Tibet's mountains; and *Koumé* refers to the deaths of Éliane's spiritual teacher Tsuglak Mawe Wangchuk and of her son Yves Arman. All three sections concern the passage from one state to another, whether this be a physical or spiritual transition.

The sound world of *Trilogie de la Mort* is 'relatively featureless', to reuse a term from the previous section describing *Expansion and Contraction*. It comprises long-duration, synthesised, imperceptibly changing drones. It repays a special kind of attention, not unlike the 'bare attention' technique used in a meditation, and in aural terms can be termed 'immersive listening' (see Glover and Harrison (2013)) or 'deep listening' (see Oliveros (2005)). However, once engaged, the seemingly grey sound-world begins to disclose a shining array of colours and experiences, at times not far short of ecstatic. I vividly remember

[2] I believe that processes such as serialism have more to do with pattern recognition than the way that sound waves actually work together. Of course, it is not a requirement of music that it is grounded in physical properties of sound, nor is this a statement about the value of such music.

my first experience of *Kyema*, being profoundly moved to tears about 25 minutes into the piece.

The second work that I want to bring forward as influencing not a single work but all the works in this study is *For Bunita Marcus*, a work for solo piano composed by Morton Feldman, published in 1985, and one of a trilogy of late works for the piano that also included *Triadic Memories* (1981) and *Palais de Mari* (1986). I could have chosen several long-duration works by Feldman, but I selected this because, as well as having an overall influence, it also has specific aspects — use of repetition, written for solo piano — that has a bearing on specific works in the portfolio, and will be discussed later with regard to those connections. For the moment, I would like to provide some initial thoughts. I am drawn to *For Bunita Marcus* for many of the same reasons as I am to *Trilogie de la Mort*. The work is of long duration, unfolding over the course of around 90 minutes, is muted in its gestures, contemplative and non-narrative. Unlike *Trilogie de la Mort* it has no spiritual subtext. In Radigue's piece, changes occur continuously over long periods of time; in Feldman's work changes are more discrete, with long periods of stasis broken by the introduction of new material. For example, the work opens with an almost obsessive focus on the three pitch classes played D♭, D and E♭ in different octave combinations, and only after around 80 bars are new pitch classes (E♮ and F) introduced. However, there is a feel that these local changes are evolving through longer durations.

For Bunita Marcus (1985) was written in conventional notation, with a metronome marking ♩= 63-66 and each bar with a different time signature, with a large preponderance of signatures $\frac{3}{8}$ and $\frac{5}{16}$, thus giving the listener a feel of free-floating pitches with no discernible rhythmic pattern. The sustaining pedal is down throughout, apart from two instances. Feldman, describing his compositional process, is quoted as saying,"You know the expression 'playing by ear'? I compose by ear, and there you have it.[3] From this and many other quotes, it is clear that Feldman classified himself as an 'intuitive' and not a 'process-based' composer. In *For Bunita Marcus*, as with Feldman's other late works, I feel an underlying framework of empty bars, similar to the warp foundation of Persian carpets (an interest of Feldman's), that provides the basis for him to 'weave' his musical material, using repetition and variation. This is the extent of the process in Feldman's compositional style.

I will return to these pieces when looking at some further works from my portfolio.

[3] Extracted from a 1967 interview with Jean-Yves Bosseur, *Ecrits et Paroles*, 1998, accessed from https://www.musicandliterature.org/features/2015/9/19/feldman.

3.4 Time's arrow: Swings and roundabouts, *Properties of Matter: Damped Harmonic Motion*

As a scientist, I am often struck by two aspects of the world that are extremely simple to state and yet quite astonishing in their universal effect.[4] These are the phenomena of cause and effect and the irreversibility of time, often called 'time's arrow'. Focusing for the moment on the latter, it is also interesting that for the most part and for most natural laws time is reversible; if you ran a movie showing the movement of balls on a snooker table backwards and observed the results, you would be unable to see any natural law being broken, *with one important exception*: balls that had been slowing down after being struck or after collision with other balls would now be speeding up. This is an example of the only classical law that requires the arrow of time to point in one direction only. The action of friction between a snooker ball and the surface upon which it moves generates heat energy and because total energy is conserved (another fundamental law), the ball's kinetic energy is reduced - its slows down and eventually comes to a halt. The irreversibility of the generation of heat by friction is an example of the action of the Second Law of Thermodynamics, and it is this unique law that ensures the irreversibility of time.

It is this fascinating property of the material world that provides a focus for several of my works, two of which I will now discuss in the context of *Momento* for string quartet by Aldo Clementi. *Momento* was published in 2005 and recorded by Quatuor Bozzini. The piece lasts just over 17 minutes and is a threefold repetition of 36 bars of music. Each repetition has its own quality, the first being played *normale*, the second more slowly with mutes and *poco vibrato*, the third even slower, quieter still, with mutes, *sul ponticello* and *non vibrato*. The 36 bars are themselves further divided into three subgroups of 12 bars each. The second and third subgroups repeat the material of the first with some rhythmic and pitch variation. The material itself is rather sparse and contains many symmetries; for example, the first violin enters with a seven-note palindrome G-E-A-D-A-E-G in canon with the second violin playing its inversion D-F-C-G-C-F-D, starting an octave and a perfect fourth below. The viola and cello enter with their own five-note palindromes. The work is notated diatonically with no accidentals, but the viola and cello are tuned a semitone down, thus providing some appealingly crunchy dissonances complementing the melodic consonances.

Time's arrow is pointing definitely in one direction here; we go three times around on the carousel, each circular view itself divided into three, and each time the roundabout quietens, becomes more ethereal, slows and eventually halts. What we see as we revolve around is a somewhat monochromatic, featureless but quietly beautiful landscape, full of interesting

[4] Except perhaps at quantum levels.

sonic details. Over the course of the 17 minutes, the listener is aware of material that is constantly under change, but nevertheless familiar to the memory, leaving me with a slight feeling of *déjà vu*. Although on a smaller scale, the affect (and effect) of this piece has much in common with *Trilogie de la Mort* and *For Bunita Marcus*.

Apart from the non-narrative and focus on sound aspects of *Momento*, the gradual slowing down and decrease in dynamics of the work link directly to the Second Law of Thermodynamics, although there is no evidence that this law was explicitly in the composer's mind. Both this focus on temporal directionality and the importance of repetition[5] provide a link to the motivations and inspirations behind my work *Properties of Matter: Damped Harmonic Motion*, now the subject of discussion.

There are many ways in which the Second Law of Thermodynamics manifests itself as an observable property of matter. An example that led to the next work of mine that I will discuss is the motion of a pendulum. We are all familiar with the swinging motion of a pendulum, known technically as *simple harmonic motion*. A so-called 'ideal' pendulum is one that once released will continue to swing for ever. Real pendulums of course, if left to swing under their own steam, do not swing for ever but eventually come to a halt. This is because of friction due to rubbing on the pivot and to air resistance. Their swinging motion is known technically as *damped harmonic motion*, and this physical property is the inspiration for my work *Properties of Matter: Damped Harmonic Motion*. This work is scored for alto flute, two clarinets in B♭, bass clarinet, piano, violin, viola and cello. It was commissioned by the Ives Ensemble[6] and premiered by them in November 2019 as part of the Durham University Musicon Series. It lasts 15 minutes.

The pitch palette is formed by a collection of pitches in 11-limit harmonic space based on A3 (220Hz) within the ranges of the instruments. I was immediately presented with a problem because the pitch palette contains many pitches that do not fit precisely with the equal-tempered scale provided by the piano. It was impractical to retune the piano, and even that would still result in only a limited range of pitches available. This problem is part of the more general issue of precision in the manner in which music is notated, performed and perceived, and this is discussed further in Section 4.3. Based on those later considerations regarding pitch precision, any pitch from the pitch palette within $\pm 5 - 8$ cents of an equal-tempered pitch was admitted as an equal-tempered pitch. In *Properties of Matter: Damped Harmonic Motion*, the stringed instruments were treated as microtonal instruments. All other

[5]It is worth pointing out that macro-repetition in *Momento* and *Properties of Matter: Damped Harmonic Motion* are quite different from the unfolding described in Chapter 2 and evinced in works like *Controlled Descent*.

[6]The Ives Ensemble comprises a pool of around 14 musicians dedicated to performance of contemporary chamber music. https://ives-ensemble.nl

instruments were provided with pitches within range of equal-tempered pitches as set out by the bounds given above.

The work is structured as a number of 'swings', in each of which the harmonic dispersions and tempi are correlated with the displacement of the pendulum from its rest position and its speed, respectively. Each swing is smaller than the previous one, and the work gradually slows down to almost stasis, and a harmonic unison. Unlike *Momento*, where the descent to stasis is in three discrete stages, *Properties of Matter: Damped Harmonic Motion* provides a nearly continuous descent. Figure 3.2 provides an overall schematic of the work. The left of the figure shows a schematic of the pendulum and the first movements of the bob from the left starting position to its lowest point (fastest speed), then to A and to B, and so on. The figure on the right shows the movement from the start until the bob eventually comes to rest as a wave form. The letters A to F are the letters marked in the score and indicate the places, left or right, where the swing meets its maximum.

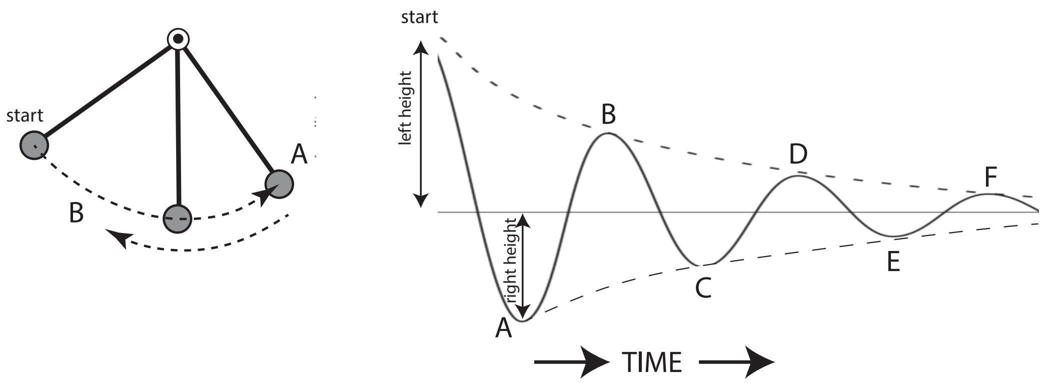

Fig. 3.2 The overall structure of *Damped Harmonic Motion*

When the bob of the pendulum is released (the start of the work), its height is at the maximum, represented by maximum harmonic dispersion. The ball starts to pick up speed, represented by a gradual acceleration of tempo and increase in dynamics, reaching maximum speed (tempo) and minimum height (unison) at the bottom of the swing, and the process is reversed as the bob rises to a new height (music loses tempo, quietens and becomes more harmonically diverse). The process repeats through maxima A to F, finally coming to rest in very quiet unison at the end of the work. The continuous changes in tempo are handled by frequently changing metronome markings. For practical performance reasons, a click track is provided to keep the players in time throughout.

The harmonic sequence of chords is determined by the generation of a primitive triadic chord sequence, as described in Chapter 2, and as the piece progresses and the swings become less pronounced, successively smaller subsequences are used. The resulting sequence of 41 pitches is given in Appendix 1, along with details on harmonic dispersion of triads, vector of harmonic components, frequencies, and number of cents deviation from equal temperament. In the first swing to the bottom, pitches are introduced in order 1 to 42, using the numbering in Appendix 1. As the bob rises to its maximum, pitches are introduced in reverse order 42 – 12, then down again to pitch 1, and so the subsequences narrow and become harmonically closer to A3 (220Hz).

The above description makes the process sound quite mechanistic, but in fact there is much scope for the application of a listening ear. While this system provides an underlying framework, the work is intended as a musical experience for the listener, and care has been taken over instrumentation and variation of timbre. For a single example, on the first ascent, pitch 25 (A3 220Hz) is introduced in bar 57 by the second clarinet and piano over a G2 (98 Hz), the A3 (220Hz) reinforced in the next bar by bass clarinet and viola, all at *mezzoforte* and ♩ = 120. On the next ascent, these pitches are sounded nearly at the top of the swing at bar 118, and pitch 25 (A3 220Hz) is sounded *pianissimo* initially by the bass clarinet and piano with later reinforcement by alto flute and first clarinet at a tempo of ♩ = 60. These kinds of changes in instrumentation result in gradations of timbre corresponding to the gradual lowering of energy levels as the pendulum slows down.

Pendulums can have hypnotic effects, and the general mood of this piece is contemplative. As a curiosity, I calculated how large a real pendulum would have to be to swing at the rate required for the piece. Altogether there are just under three full swings back and forth before the pendulum comes to rest, each taking about six minutes. So an actual pendulum for this piece would be about 35km in length!

3.5 Unfolding material: *Properties of Matter: Controlled Descent*

Properties of Matter: Expansion and Contraction had limited repetition in the form of repeated block chords, and the overall arch structure indicating a return to the beginning, although returning through a different pitch set. In *Properties of Matter: Damped Harmonic Motion*, repetition played an important role, as the increasingly truncated chord sequences were repeated through each swing of the bob. While composing this work, listening to its performance, and listening to the work of other composers, I decided to develop repetition from an essentially intuitive process to something more systematised, providing an algorithmic basis and allowing controlled randomness.

I was also influenced at the time by the work *three bodies (moving)* by Catherine Lamb. Lamb was born in the USA in 1982.[7] Many of her compositions employ harmonies based on rational harmonic relationships. She is a joint founder, with Rebecca Lane and Marc Sabat, of the Harmonic Space Orchestra, which is “a Berlin-based experimental research and performance collective with rational intonation at the core of its practice” (HSO, 2021).

Catherine Lamb’s work *three bodies (moving)* was composed in 2010 and lasts around 45 minutes. It is scored for violin, cello and B♭ clarinet. The work is in three movements and is quiet throughout, with dynamic indications between ***ppp*** and ***mp***. Harmonically, the work uses notes that are the complete range of harmonics from 3 to 40 (plus 48 and 80) with fundamental B♭ of frequency approximately 29.135. In order to accommodate that range of harmonics, primes from 2 to 37 are needed, so we could say that the work is in a 37-limit harmonic space. This is a space with many (12) dimensions; however, apart from primes 2 and 3, no primes go beyond the second power.

To facilitate playing, the cello is tuned with string III as B♭ and the other strings simple rational multiples of this; the violin is tuned with strings IV and II as B♭ and octave, respectively, with the other strings simple rational multiples.

In each movement, the ‘principal’ instrument for that movement (first cello, second violin, third clarinet) oscillates around one or more fixed notes, with the other instruments oscillating more slowly. The general progression is from high to low harmonics, with the first movement mostly focused on harmonics 25 upwards, the second mostly 12-25, and the final movement 10 downwards for the clarinet, with support from the other instruments playing notes harmonically close to the fundamental B♭. Figure 3.3 shows an examples from the opening and closing of the work. The number above each note indicates the number of the harmonic. The first section opens with a preoccupation with harmonics around the 32nd, while the third section opens with a focus on the 8th harmonic. Repetition and melodic material are clearly foreground features of *three bodies (moving)*. While the repetition is not systematised in the sense of having an algorithmic basis, there is clearly a thought-through use of it as the work descends through the harmonic series and pitch register. Catherine Lamb describes[8] the repetition and exploration of pitches and their relationship to what has already been exposed as ‘unfolding’. This unfolding process is reminiscent of the slow opening sections of Hindustani ragas, where the performer ruminates on the exposure of each new piece of material as it becomes exposed.

The ‘blandness’ (to use one of Lamb’s words in (Reynell, 2011) of the music results in a wonderfully calming experience for the patient listener (at least in my listening experience).

[7] Interestingly, one of Lamb’s teachers was James Tenney, thus providing a satisfying link to the main theme of this project.

[8] Private communication, May 3, 2021

Fig. 3.3 Opening of the first and third sections of *three bodies (moving)* (reproduced with the composer's permission)

To use a metaphor, the work evokes a sense of gentle ripples on harmonic waters. As Michael Pisaro put it, "three bodies (moving)? [*sic*] is a wonderful kind of 'becoming song' - beginning in a narrow, intense, winding manner and progressing, step by step towards melody. As it moves between the instruments it begins to widen, in a gently curved space. Just at the verge of tunefulness, the piece stops, in one of the most beautiful endings I have heard in recent years." (Extracted from (Reynell, 2011))

The next work in my portfolio, *Properties of Matter: Controlled Descent*, albeit in a widely different context, unfolds and looks back on material as it is being exposed in a similar way to Lamb's work. *Properties of Matter: Controlled Descent* is a further addition to the collection of works under the heading *Properties of Matter* and is also concerned with a simple physical phenomenon, in this case descent under gravity controlled by some braking force. A serendipitous recent example of this is the descent of the Perseverance Rover to the surface of Mars on February 18, 2021.

Controlled Descent is a work for solo piano of duration just under seven minutes. The harmonic structure of the piece is based on James Tenney's concept of harmonic space, using eight-voice chords with constituent pitches selected from 11-limit pitches harmonically relatively close to A3 (220Hz) and in close proximity to equal-tempered pitches. A primitive 8-voice chord sequence was generated using the process described in subsection 2.4.2. Pitches from this chord sequence, starting with pitches from the most harmonically diverse chord and working downwards to the unison chord pitch A (220Hz), then provide material for the extended chord sequence, as described in subsection 2.4.3. I view repetition as being an important foreground feature for this composition, so the parameters were set in the process of generating the extended chord sequence in such a way that there was a high degree of folding.

The next process was a ‘controlled descent’ in terms of the duration of pitches in the work. Starting with a sequence of pitches all of equal duration, at ♩= 150, the total duration was around 400 seconds. I wanted to construct a temporal distortion process that would apply a multiplying factor to each pitch that would bring earlier pitches forward in time, thus shortening their duration, and as the piece progressed the shortening would be less and less. This should all result in no change to the work’s total duration. After some experimentation, a cubic curve was chosen. The equation of the curve is:

$$t_{new} = 0.75\frac{t_{old}^3}{T^2} + 0.25t_{old}$$

where T is the total duration of the work, in our case around 400 seconds. The shape of the curve in the region of interest is shown in figure 3.4. So, for example, a pitch that occurred at time 100 seconds in the original would occur at time roughly 30 seconds, shortening the duration by a ratio of 10 : 3, while a pitch that occurred at time 350 seconds in the original would occur at time roughly 300 seconds, shortening the duration by a ratio of 7 : 6.

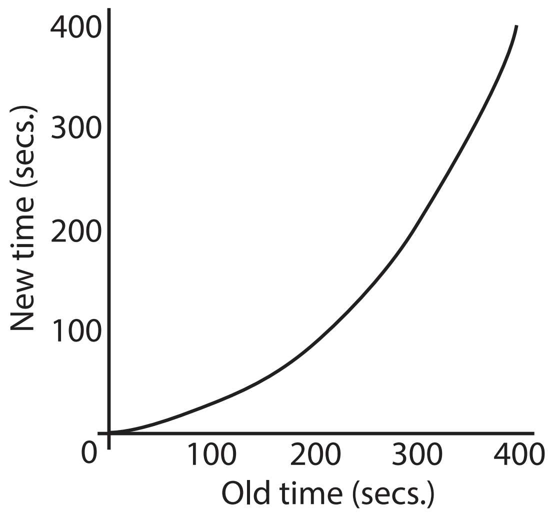

Fig. 3.4 The shape of the function that provides for a gradual slowing down of tempo.

In summary, at the beginning of the piece each chord is harmonically highly dispersed and over the course of time reduces to the most compact harmony: a unison. A combination of continual repetition and gradual reduction in tempo and dynamics represents the controlling braking force being applied.

I wanted to include in my portfolio a work that was contrasting in style to the other pieces. *Properties of Matter: Controlled Descent* is relatively short, dynamic and percussive, while still conforming to the global harmonic processes of the other works. Each pitch was introduced in sequence within a temporal precision of one quaver at ♩= 150. Any pitches that came one after another within a time shorter than this were played together. This results in at most three notes sounded together, therefore so as to keep some impression of 8-part harmony, half-pedalling was introduced throughout the piece. I also needed to make some adjustments so that the piece was playable, in particular that intervals in chords or successive notes could be reached with the hands; this necessitated a few changes of octave and some temporal displacements so that a note was sounded a short time before or after that specified by the algorithmic process.

I was very fortunate that the pianist Ben Smith[9] agreed to rehearse and record *Properties of Matter: Controlled Descent* . It was he that suggested half-pedalling as an appropriate sustain that would carry the 8-part harmony, as described in the previous paragraph.

3.6 Introduction of some melody: *Melum Scalenum*

Melum Scalenum is a composition for bass viol that I wrote for a Durham University workshop in 2020 and was performed by Liam Byrne as part of an online concert on Facebook. The work lasts around three minutes. I reused a pitch sequence from the initial draft electronic track of *Absence / Presence*, discussed later in this commentary. The sequence chosen was one that was not finally used in *Absence / Presence*. The sequence was further filtered so that it contained only pitches playable by the bass viol, after it had been modified. The bass viol has the property that its frets are adjustable, being made of gut. Figure 3.5 shows the modifications that were made. The bass viol was tuned in the usual way with equal-tempered pitches A1, D2, G2, C3, E3, A3, D4. The first fret was then moved so that pitches playable on that fret were tuned down 14 cents. Similarly, the sixth and seventh frets were moved resulting in retuning of pitches down 50 cents and 30 cents, respectively. This enabled all harmonics up to the eleventh harmonic to be played according to rational tuning based on fundamental A 27.5Hz. The pitches that were used for the piece are shown in figure 3.5.

Unlike many of the other works in the portfolio, this work has some melodic elements in the foreground. The title, *Melum Scalenum*[10], suggests a melody in three unequal parts. The action is predominantly in the lower strings, with the top two strings activated only in the

[9] https://www.bensmithmusic.co.uk

[10] *scalenum* is the obsolete form of *scalene* and applies to triangles with sides of unequal lengths.

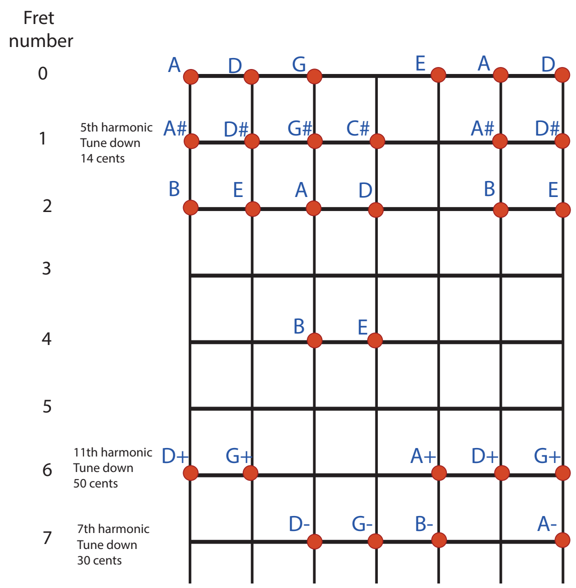

Fig. 3.5 Pitches provided by adjusting frets on the bass viol.

middle and end sections. The mood is lyrical with a focus on the glorious gritty sounds that the bass viol can produce.

All the works in the *Properties of Matter* series take a highly structured approach to the generation of their musical materials. But the focus on sound and mathematically organised process in my work does not preclude the possibility of basing the music on other types of material, including found materials. As I was composing this work I remembered a work that influenced me as a teenager; Symphony No. 4 in A minor, Op. 63 (1910-11), composed by Jean Sibelius, opens with a slow oscillation in the bassoon and lower strings (see Figure 3.6) and I referenced this in bars 18-21, 31-33 and 51-54 of *Melum Scalenum* (see Figure 3.7).

Fig. 3.6 Opening bars of Sibelius Symphony 4

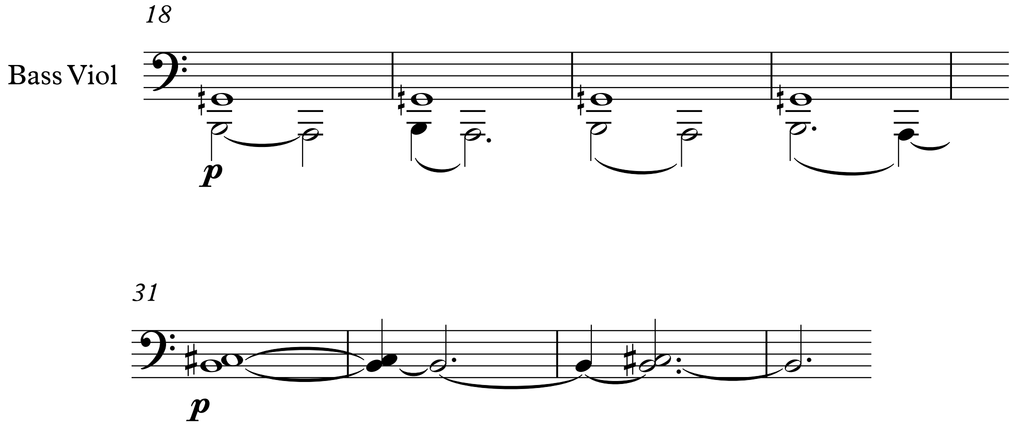

Fig. 3.7 Extracted bars 18-21 and 31-34 from *Melum Scalenum*

3.7 Electronic works and *2-Spem-70*

In the course of my PhD studies I have composed several works for electronic synthesisers using the software package SuperCollider (McCartney, 1996; Wilson et al., 2011). An early piece, not included in the portfolio, was *A-Orbital-40*, based on a harmonic space of 10 dimensions using the sequence of primes from 2 to 29. This piece was in essence an experiment with harmonic spaces. The idea was to construct a ring of pitches, all around 40 harmonic units from A4 (440Hz) such that the harmonic distance between any neighbouring pair of pitches is no more than 20HD. These pitches were realised by sine wave oscillator with some perturbation using granulation techniques, as discussed in subsection 2.5.5 of Chapter 2. . The piece was constructed so that we move around the pitch ring, arriving back at the starting point. The final work was used as the soundtrack for a ‘slow’[11] movie titled ‘Crossing’, recording the progress of a 60 minute walk along the Pilgrim’s Way from the Northumberland mainland to the Island of Lindisfarne. It was screened as part of the Berwick Film Festival Fringe in October 2018.[12]

The principal electronic work that I would like to discuss and include in my portfolio is the electronic composition *2-Spem-70*, which is one of a family of works *c-Spem-d*, where *c* and *d* are a pair of parameters that specify the number of output channels and total duration in minutes, respectively. The specific manifestation included in the portfolio is a stereo work of duration 70 minutes. *2-Spem-70* is based on an 11-limit harmonic space and uses 40 sine wave oscillators. The structure of the piece is based on *Spem in Alium*, a 40-part motet by

[11]The adjective ‘slow’ has been used in many contexts, such as ‘slow food’ and ‘slow travel’. It is used here to indicate an approach to cinema that is characterised by long takes with minimal narrative content. A detailed investigation and discussion of slow cinema can be found in (de Luca and Jorge, 2015).

[12]*Crossing* can be accessed at https://www.youtube.com/watch?v=Mqb7VxOQ3YE.

Thomas Tallis, composed around 1570 for eight choirs each of five voices. The choir entries of *Spem in Alium* are shown in black in figure 3.8 and this structure is mirrored precisely, except for total duration, in *c-Spem-d*. Although the total duration *d* of *c-Spem-d* may vary, the choir entries are in exact proportion to the entries in *Spem in Alium*.

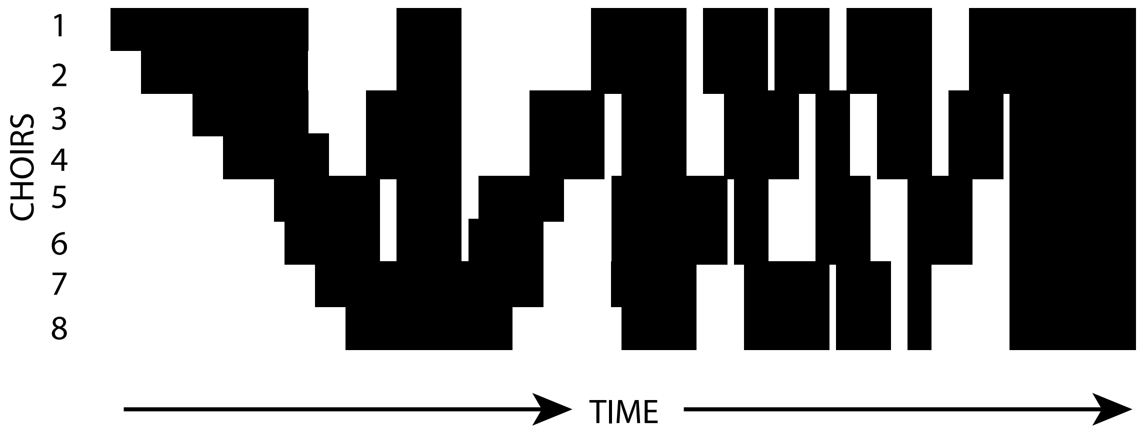

Fig. 3.8 Schematic showing the choir entries in *c-Spem-d*

The compositional process that provides the framework for *c-Spem-d* has all the elements discussed in Chapter 2, including generation of a pitch palette, primitive 40-voice chord sequence that descends to unison and an extended chord sequence using controlled but randomised repetition. In this case, each pitch was synthesised using a sinusoidal wave oscillator realised by SuperCollider. Each individual pitch was prolonged up to when the next pitch in that voice is sounded, and was given the envelope of a rapid attack phase, followed by decay, sustain and release phases in proportion to the duration of the note. (Longer notes will decay more slowly than shorter notes). The sine wave timbre of each pitch was modified (coloured) by the addition of a small amount of granulation, as discussed in subsection 2.5.5 of Chapter 2. The choir entry/exit structure shown in figure 3.8 provides a filter. For example, at the beginning of the work all voices are muted except voices 1-5 from the first choir.

The effects in *Spem in Alium* that I was interested in mirroring were the spatiality of the sound and the surges of sound as different combinations of choirs entered and exited. I focused on two versions of my work, namely *2-Spem-70*, included in this portfolio, and *8-Spem-70* .

The two-channel version *2-Spem-70* is mainly intended for accessing from a digital device. It was composed as a purely sonic work. However, my long-time friend and collaborator, John Stell listened to the work and proposed that we work together to develop an accompanying animation that could be viewed on platforms such as YouTube. John Stell[13] is a visual artist

[13]http://johnstell.com

and computer scientist. The collaboration began by John firstly becoming familiar with *2-Spem-70* by repeated listenings. It then proceeded iteratively by us jointly developing an animation that we both felt worked well for the piece. The animation was developed using the *Processing* software development system.[14]

The eight choirs are arranged uniformly from left to right on a linear soundstage. software platform. Each choir is visually represented by a rotating regular pentagon, the vertices of which point to the five voices in the choir. The coordination of entrances and exits of the pentagons directly correspond to the choir entries and exits in the sound piece. Each pentagon traces out a *Lissajous curve*[15], derived from the harmonic motions of two pendulums (thus linking with my work *Properties of Matter: Damped Harmonic Motion*). However, there is no audio reference to the curve in *2-Spem-70*.

The eight-channel version *8-Spem-70* is intended for performance in a physical space, and will be performed in 2022 at the Watchtower Gallery, Berwick-upon-Tweed. In this case the eight choirs are arranged uniformly on a circular soundstage and each choir is assigned its own channel.

3.8 Bring it all together: *Presence/Absence*

3.8.1 Overview

Presence/Absence is a work for eight solo singers (SSAATTBB), choir (SATB), chanters, two bass trombones, percussion and electronic synthesisers. The work's duration is around an hour. *Presence/Absence* is in a sense a culmination of my PhD studies and a synthesis of several musical and non-musical strands in my life. Harmonically, it is an example of my extensions to Tenney's harmonic space structure realised for synthesiser, instruments and voices. It also draws on my experience as a Zen practitioner in its use of (a) chants of the type that we did daily in the monastery and (b) texts that express ideas and practices that are fundamental to the Zen experience. In terms of the general ambience of the work, it sums up my general appreciation of works in the non-narrative tradition, as described earlier in the chapter. Initially it was conceived as a requiem, but as I got more into the texts, I came to the view that the work could deal with loss in a more general way, focusing on the absence of an enduring identity within the presence of the phenomenal world, and this led me to the title of the work.

[14] *Processing* (https://processing.org) is a software environment and programming language developed for coding in the context of the visual arts.

[15] A Lissajous curve, or Bowditch curve, results from the joint motion of two pendulums with axes at right angles to each other. They were first considered by the mathematician Nathaniel Bowditch in 1815.

Zen has its own musical tradition, mostly (at least in my experience) a collection of near monotone chants. The chant that is sung everyday in Zen centres and monasteries is the *Hanya Shingyo*, known in English as the Heart Sutra. [16] The *Hanya Shingyo* expresses in its opening sentences a fundamental precept, common to many forms of Buddhism, that no phenomenon has any ultimate, unchanging essence. In Japanese, the chant is less than 300 syllables in length. In *Presence/Absence*, the chant is used in three of the sections, along with another chant, *om ture tutare tura*, which is an ancient Tibetan mantra. Both chants are vocalised on a single tone that each chanter can choose freely.

The texts of the work are from the Wu-Men Kuan in the translation from the original Chinese by David Hinton (2018) and entitled *No-Gate Gateway*.[17] Composed in 1228 by *Wu-Men Hui-Kai*, *Wu-Men Kuan* is a collection of 48 chapters, each containing (a) a 'case', (b) a commentary on the case and (c) a short verse, called a *gatha*, that provides a reflection and encapsulation of the case and commentary. Each case may be an interchange between ancient Zen masters, a scene from the Buddha's life or a record of some other historical event. They are usually paradoxical, defying rational interpretation, and contain the essence of Zen teaching. I have chosen nine extracts as texts for *Presence/Absence*, from cases or *gathas*, sometimes making very small modifications to the text in order to work better with the music.

3.8.2 Structure

Presence/Absence is in eleven movements, each movement being the setting of a chant, one or more extracts from *Wu-Men Kuan*, or both. Excepting the chanting of the *Hanya Shingyo*, *om ture tutare tura*, and one other passage in an early movement, all the texts are in English.

Presence/Absence is in ten parts as shown below.

1. Ritual
2. Bright lightning flash
3. I always remember
4. Chorus 1
5. Chorus 2
6. Before your mother

[16] More details on the Heart Sutra, and indeed on Buddhism generally may be found in (Harvey, 2012)
[17] David Hinton has kindly given me permission to use his translation in this work.

7. Hearing the name
8. Interlude
9. Sword-blade's edge
10. Koan
11. Finale

Ritual is one of three movements without electronics and is a chant of the *Hanya Shingyo* for chanters, chorus and solo basses. This introductory movement announces a ritualistic element to the work and proceeds *attacca* to movements two to six, comprising a continuous electronic track with interventions from voices and instruments. These movements include two choruses (very distant echoes of Feldman's *Rothko Chapel*, both being wordless, having overlapping voices to create a continuous sound, and both evoking a sense of mystery), settings of extracts from *No-Gate Gateway* and a second chanting of the *Hanya Shingyo*. The next movement is an *a cappella* trio for two sopranos and alto solo voices. The work continues with an interlude for electronics and bass drum that is followed by another setting from *No-Gate Gateway*, underpinned by a chanting of the mantra, *om ture tutare tura*. Then follows movement ten, *Koan*, described below in the discussion about *noh* theatre. This movement is very different in character from the rest of the work, I hope throwing into relief the prevailing sound world of the piece. In a sense it contains the crux of the work, with the interchange:

> Is there an ultimate truth that no-one has spoken?
> There is.
> What is this truth?
> It is not mind, it is not God, it is not any thing.

The final eleventh movement of the work returns to the previous sound world, with settings from *No-Gate Gateway* and a last chant of the *Hanya Shingyo*.

The musical nature of the work derives from several sources of thought, as well as musical and other influences and experiences:

- Instrumentation and contextual influences
- Harmonic space
- Zen chanting practice as a foundation for *Presence/Absence*
- Japanese *noh* theatre in the *koan* movement

These aspects are discussed hereafter.

3.8.3 Instrumentation and contextual influences

Presence/Absence was conceived with the professional vocal ensemble EXAUDI[18] at its centre. This core group of singers is extended by a choir and a group of chanters. The ritual nature of the work and its connection with Tibetan and Zen Buddhist practices suggested instrumental support by two bass trombones and a range of percussion instruments. The intention behind the two bass trombones was that they should both provide a bass pedal to sections of the work and evoke the sound of the Tibetan horn (*dungchen*), used in Tibetan Buddhist and Mongolian Buddhist ceremonies (see figure 3.9[19]). A work that influenced me musically, ever since I heard a recording[20] in 2009, is *Body Mandala* for orchestra, composed by Jonathan Harvey. The work opens with a unison low A♭ sounded by French horn, tenor trombone and bass trombone accompanied by a single beat of the bass drum with wooden stick. The details of the form of the trombone part is shown are figure 3.10, where the tied notes with accents are not separately tongued and individual notes demarcated by strong lip *vibrato*. This trombone technique is used in several places in *Presence/Absence* as an *homage* to Harvey, whose approach to conveying a sense of the spiritual in music I have been drawn to over the years.

I should also mention the influence that Éliane Radigue's *Trilogie de la Mort* had on the development of *Presence/Absence*. Radigue became a student of Tibetan Buddhism in 1975, and several of her compositions, in particular *Trilogie de la Mort* (1988-1993), *Songs of Milarepa* (1984), *Jetsun Mila* (1986), are influenced by Tibetan Buddhist concepts and practices (Waschbüsch, 2015). I have discussed *Trilogie de la Mort* earlier in the chapter. The long duration and gradual unfolding of material in that piece influenced the development of the electronic component (composed first) of *Presence/Absence* and from there the expansion to the entire work. While my work is not a requiem, an examination of mortality is certainly an element of it.

3.8.4 Overall architecture and harmonic space

The harmonic structure of *Presence/Absence* is based on a sequence of 12-voice chords with pitches situated in 11-limit harmonic space. The pitch palette for the piece is designed in the first instance for synthesisers and could therefore include pitches in a wide frequency range, from 55Hz to 2093Hz. All pitches were constrained to be within 12 harmonic units from pitch A1 (55Hz). A 12-voice primitive chord sequence was generated by the method

[18] http://www.exaudi.org.uk

[19] Image licensed under the Creative Commons Attribution-Share Alike 3.0 Germany

[20] *Harvey: Body Mandala, Timepieces, Tranquil Abiding, White As Jasmine ...Towards A Pure Land*, Ilan Volkov, BBC Scottish Symphony Orchestra, NMC, 2008

Fig. 3.9 Tibetan horns (*dungchens*) in action

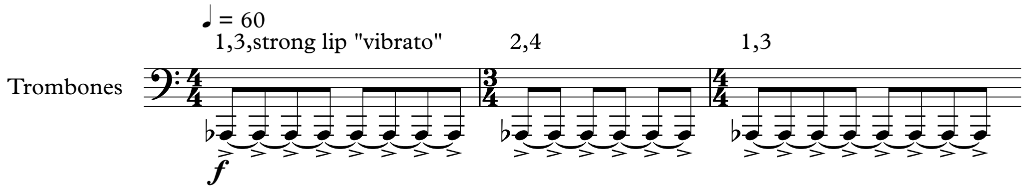

Fig. 3.10 The opening bars for the trombones in *Body Mandala*

outlined in section 2.4.2. The total number of pitches in the sequence is 1882. From this primitive chord sequence an extended chord sequence was constructed as in section 2.4.3, using a controlled but randomised amount of folding. The end result was a sequence of 5969 pitches that at any point in their playing will form a 12-voice chord, and the sequence of such chords moves from maximal to zero dispersion. As with my work *Controlled Descent*, a scaling function was applied to the uniform duration of the pitches so that there was a gradual slowing down. The total duration of the final pitch sequence is approximately one hour.

Figure 3.11 shows the overall harmonic architecture of *Presence/Absence* mapped with time on the horizontal axis, and harmonic dispersion on the vertical axis. The eleven movements are indicated, and above them can be seen a measure of harmonic dispersion at that time. The electronics tracks are shown as Electronics I, II and III. After the chant in

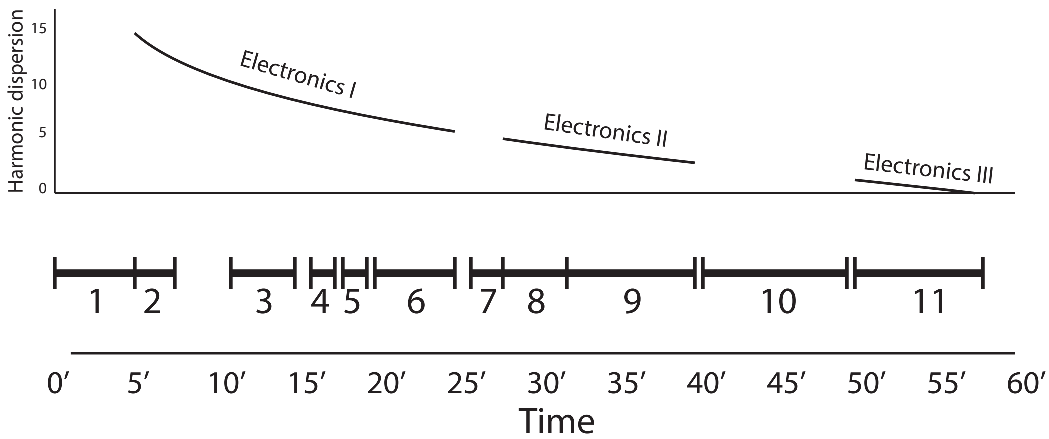

Fig. 3.11 Overall structure of *Presence/Absence*

movement 1, this harmonic structure was followed throughout the piece by all voices and instruments, with the exception that movement 10 had no specific pitches.

I decided that only the synthesisers were to be unconstrained in the pitches that they could sound. A subsequence of 1215 pitches that were within six cents of notes in the quarter-tone equal-tempered scale was extracted from the full extended sequence of 5969 pitches. This sequence was for use by the soloists. A smaller subsequence containing pitches that were within six cents of notes in the usual chromatic equal-tempered scale was extracted for use by the amateur choir and bass trombones. The pitches then became available for composition at the times when they appeared in the overall extended sequence.

3.8.5 Zen chanting practice as a foundation for *Presence/Absence*

I spent some years as a Zen practitioner, including a period as head monk in an American Zen centre and a short time as a visiting monk in a Japanese Zen monastery. Zen is divided into two main schools: *Soto* and *Rinzai*, and I was a member of the Soto Zen school. Chanting in both traditions takes the form of recitation of texts, and the musical forms used in the chant vary between and within the schools. Greene et al. (2002) provide a survey of the variety of musical forms in Zen and other forms of Buddhism.

The musical practices in my own experience consisted of chants rendered at a uniform rhythm, accompanied by a drum, small bell (*kinhin*) and singing bowl. The ‘cantor’ leading the chant would usually precede the chant with a short solo introduction, essentially monotone in nature but with some embellishment of the melodic line and a more variable approach to the rhythm. I have employed this structure in modified form in *Presence/Absence* with an extended passage at the beginning of the work for finger cymbals, gong, and drum. This is

Fig. 3.12 Bass cantors introduce chant in *Ritual*.

followed by the intoning of the preamble to the chant by the two solo bass singers. Figure 3.12 shows an example of the intonation in *Ritual*. This declamation is used to dramatic effect in the final movement, when the solo bass singers interrupt the chorus to once again intone the preamble "maka hannya haramita shingyo", this being shown in figure 3.13

In the monastic setting, there is no melodic variation in the general chant; each chanter begins on a pitch of choice and then proceeds with a monotonic line, which may vary a little when influenced by the pitches of other chanters. The duration of a chant is highly variable, from a few minutes to an hour or more. During the chant, dynamics might change gradually in response to the feeling in the group. In a more formal ritualist setting, the dynamic would remain relatively fixed at *mezzoforte*. The chant *Hanya Shingyo* is sung every day in Zen monasteries, including the one in which I practiced, and we chanted the *om ture tutare tura* regularly during intensive retreats. In my Zen practice, chants are usually prefaced by sequences of bell rings, and beatings or ringings of a singing bowl and claps using wood clappers. The chant itself is supported rhythmically by the regular beating of a drum and punctuated at designated moments with the ringing of a bell.

I have followed more or less closely the above practices in my writing for voices in *Presence/Absence*. The chanters follow the prescription for chanting in the previous paragraph, except I have specified dynamics throughout. In *Presence/Absence* the bass and tenor drums are used as the main workhorses for chant accompaniment with support from other percussion

Fig. 3.13 Bass cantors declaim the chant preamble in *Finale*.

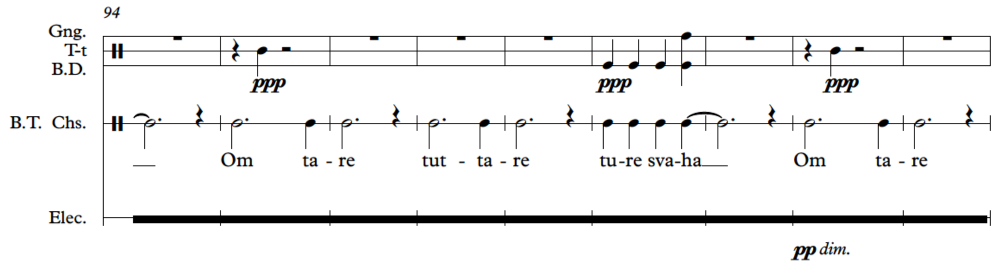

Fig. 3.14 Excerpt from the chant *Om tare* in *Sword-blade's edge*

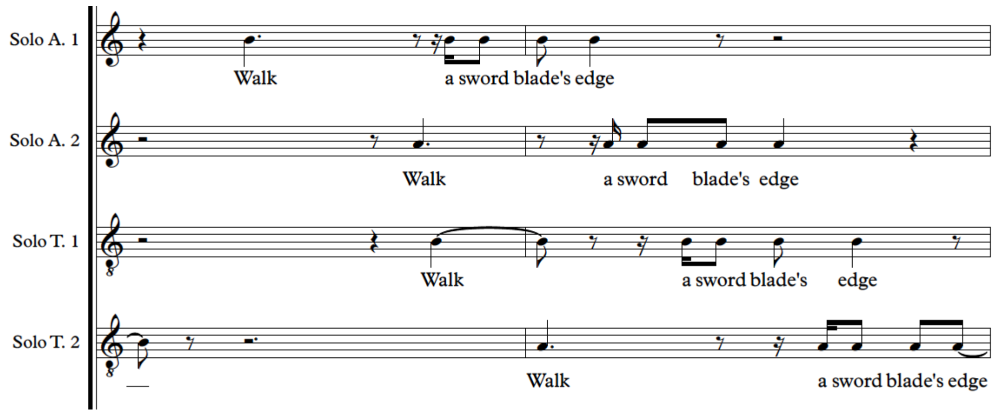

Fig. 3.15 Excerpt from *Sword-blade's edge*.

instruments, particularly a medium gong and tam-tam. Figure 3.14 gives an example of the chanting of the *om ture* chant, accompanied by bass drum, gong and electronics.

The choir and soloists follow a similar practice: pitches are provided from the electronic pitch sequence and are sounded for as long as the pitch is sounded in the electronic sequence, with some variation to take account of breathing and for musical effect (synchronisation of parts, adaptation to text, etc.). The trombones are usually provided with pedal pitches related to the electronic pitch sequence, and are sometimes in advance of choral parts to aid singers in finding initial pitches. Whilst maintaining the same principles of pitch choice for the soloists, I have introduced considerable rhythmic variation, for musical interest and variety, and also to enhance the text. Figure 3.15 shows an excerpt from *Sword-blade's edge*, marked 'With some trepidation', to indicate the perilous nature of the activity indicated in the text.

In just a few places, some minimal melodic activity has been developed by slight deviation from a single pitch line. Two examples are provided here. The first is the opening of *Hearing the name*, shown in figure 3.16, where the two sopranos change pitches by major and minor thirds on specific, single syllables. The second is from the *Finale*, where a small degree of

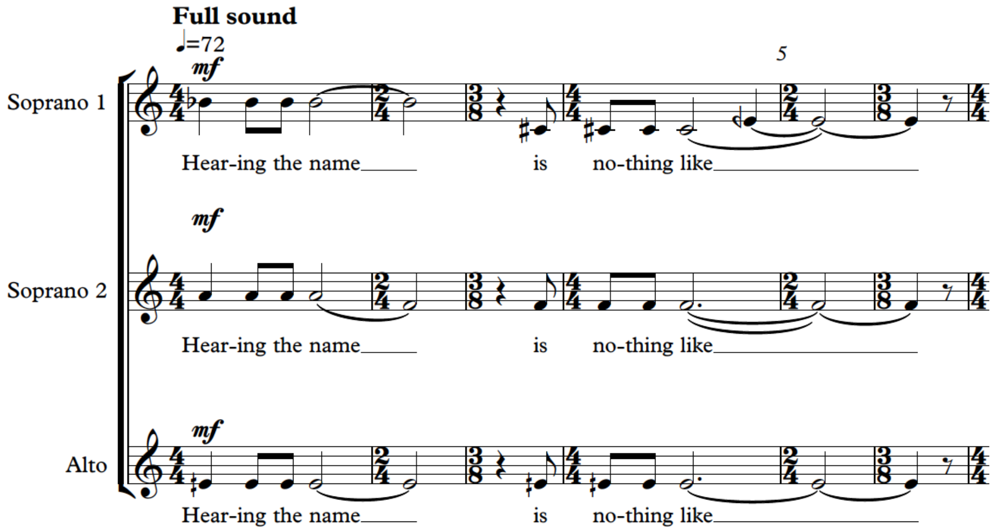

Fig. 3.16 Opening of the trio in *Hearing the name*.

embroidery to the final text of the piece is developed by deviations of small intervals in the melodic lines, shown in figure 3.17.

3.8.6 Japanese *noh* theatre in the *koan* movement of *Presence/Absence*

When in Japan, I attended some *noh* theatre performances and retain an interest in the form. The music of a *noh* play is performed by an ensemble *hayashi*, comprising transverse bamboo flute, two hand drums (small and large) and a barrel-type taiko drum hit with wooden sticks. Melody is secondary to rhythm, and the function of the music is not for its own sake but to

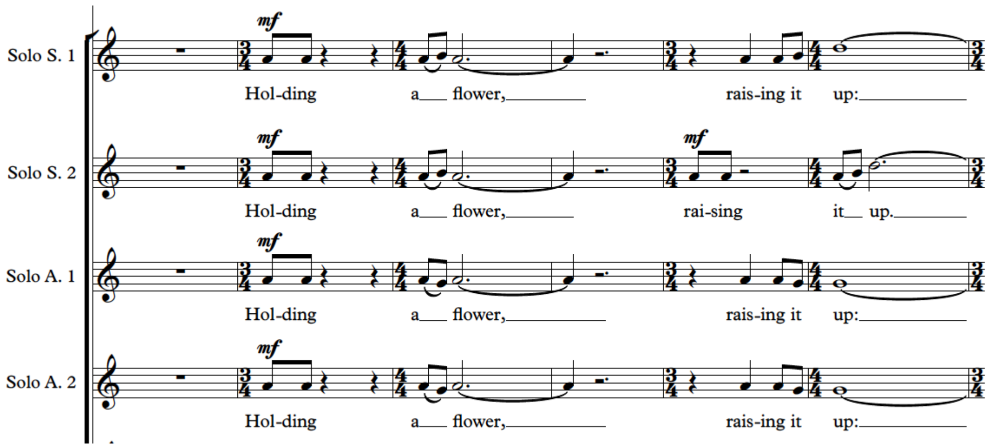

Fig. 3.17 Excerpt from *Finale*.

ornament and support the voices. Voices may either be solo or in a group. The voices chant or speak their lines. In speaking each sentence, the performer employs a gradually rising pitch followed by a falling off. In chanting, performers produce definite pitches, characterised as low, medium and high, and a pronounced amount of vibrato is used. Rhythmically, a chant may be performed in a regular rhythm or in a varied recitative manner. For more information about *noh* music, see Malm (2001) and Minagawa (1957).[21] Several composers have either been influenced or have incorporated *noh* music and *noh* drama into their work.[22]

The *Koan* movement of *Presence/Absence* is not meant to imitate or follow precisely any of the formal or performative aspects of *noh*. Rather, I have introduced some elements which are reminiscent of my experience of attending *noh* theatre performances. In particular, I have not specified particular pitches but indicated ranges, and added glissandi notated by curved lines, again of unspecified pitch. The two bass trombones, muted throughout are used to support the singers, as well as to indirectly suggest the bamboo flute. Percussion in the form of high and low wood blocks, tenor drum played with sticks, and tam-tam evoke in a generalised manner the sounds of *noh* percussion.

Figure 3.18 shows bars 10 – 14 of *Koan*. It shows a range of techniques including air sounds and glissandi within specified ranges for the muted bass trombones; speech and pitch glissandi for the voices; and coin slides and tremolos for the tam-tam.

[21] I was lucky to be able to consult David Hughes, an Emeritus Research Associate of the Japan Research Centre, SOAS, and an expert in *noh*. David suggested several sources of information on *noh* practice.

[22] A notable example is Curlew River, composed by Benjamin Britten, which is based on the noh play *Sumidagawa* by Juro Motomasa] (1395-1431). Harrison Birtwistle uses a 'Noh harp' in *The Mask of Orpheus*, comprising five metal bars over a timpani, but I can find no evidence that such an instrument was or is used in *noh* theatre.

Fig. 3.18 Excerpt from *Koan*.

Chapter 4

Conclusions

I have always been concerned with the tangible experiencing of sound, and I have failed more often than I have succeeded in realizing this or manifesting it in my music.

James Tenney[1]

4.1 Introduction

One begins a PhD programme in one place and, particularly for a part-time student, over a number of years and experiences ends up at another, maybe quite different place. My original idea and proposal to the admissions committee was, in a nutshell, to investigate the application of some advanced structures in mathematics, hitherto unresearched by composers in the compositional process. The investigation would survey possible structures, engage with them in the compositional process, and evaluate them in musical terms. In the first period of the work, I did consider several structures with which I was familiar as a mathematician, including finite algebraic structures with rich sets of symmetries and extensions to neo-Riemannian structures that went beyond traditional harmonic ideas. (Cohn (1998) provides an exposition of neo-Riemannian theory). Indeed the classical *Tonnetz*, which plays an important role in the latter does lead on to my eventual area of work, as Tenney's harmonic-distance pitch maps can be seen as quite closely related to *Tonnetz* structures.

Looking back now, patterns emerge and general areas of influence become apparent. In this final chapter I take the opportunity to provide some more context of works that relate to the general area of my own research, under the headings: process-based composition, the harmonic spectrum, and works that are motivated by spirituality. I then draw together some threads of thought that continue the discussion on music and mathematics, and add some

[1]James Tenney, opening lines of *Valentine Manifesto*, 8 January 1973, Music Scores files, 1978-018/003 (1), Clara Thomas Archives and Special Collections, York University Libraries.

general conclusions about musical precision. Finally, I summarise and evaluate the research, and look towards the future.

4.2 Research context

In this section I will discuss works by others that relate closely to the project as a whole, rather than to any individual work in the portfolio, thus providing a context in which the project takes place. I have divided the discussion into three areas: process-based composition, the harmonic spectrum, and spiritual themes.

4.2.1 Process-based composition

A decision that needed to be made as I proceeded through the research was to what extent I wanted my compositions to be 'ruled' by the underlying processes upon which they were based and how much I wanted the 'intuitive' aspects of composition to be present. As the work evolved I determined that the balance I sought should be structured by the processual element, however when desirable allowing my 'inner ear' to overrule mathematical concepts in the interest of aesthetic considerations. I use the term *tightly-coupled* process-based compositions to denote those works that allow the process to 'rule' and *loosely-coupled* process-based compositions where intuition plays a prominent role. The previous chapter has provided examples of works by other composers who are 'process-based' to a greater or lesser extent. For example, the loosely-coupled approach in *three bodies (moving)* for violin, cello and B♭ clarinet, by Catherine Lamb, and the more tightly-coupled approach in Tenney's *Arbor Vitae* for string quartet. In this section I provide further context to this area of activity by citing some examples of works by prominent composers in this field.

Tom Johnson is often cited (see, for example, (Girard and Lucia, 2020)) as the prime example of a tightly-coupled process-based composer. In the course of his life as a composer, Johnson has used a range of mathematical structures, including counting processes, tessellations, and combinatorial designs. I think it is telling that in his interview with Bernard Girard (2020), he states, "There's a quote attributed to Charles Ives that I really like: 'What the hell does sound have to do with music?'". I think this indicates the importance he attaches to the structures underlying his music, and the primacy he gives to these structures as opposed to sonic issues such as timbre. For example, the first movement of his *Movements for Wind Quintet* is entirely structured according to the Fibonacci numbers $1, 2, 3, 5, 8, \ldots$. Figure 4.1 shows Johnson's plan of the opening of the first movement. This structure is followed rigorously, and it is clear that the primary purpose of the music is to reveal as transparently as possible the patterns arising from the structure as it unfolds.

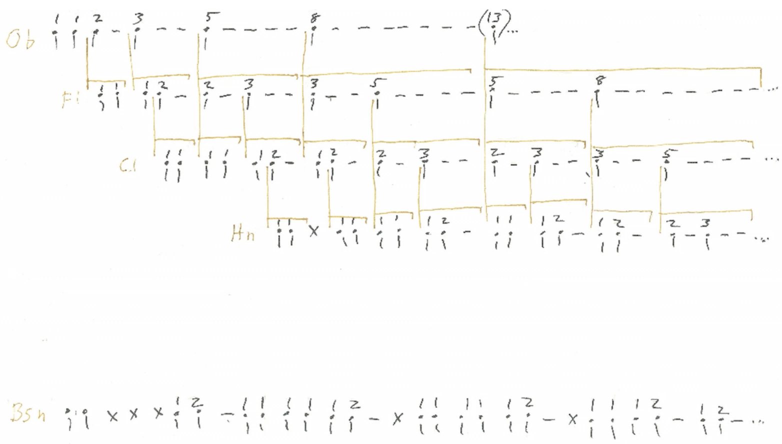

Fig. 4.1 Tom Johnson's diagram of the opening of his *Movements for Wind Quintet*

Richard Glover's composition *Logical Harmonies 2* for solo piano is structured in a simple but rigorous pattern and provides a good second example of the tightly-coupled end of the spectrum. Figure 4.2 shows the first three systems of the six-minute piece. Each letter represents the root pitch of a tonic triad, and the upper and lower letters indicate triads played by the right and left hand, respectively. The triads are arranged in a descending sequence of fifths, and in each system the lower row is shifted one place to the left. The only flexibility left to the performer is choice and positioning of the inversions, subject to further constraints imposed by the composer that the hands should not be too far apart. My own impression on listening to the piece is that I was initially interested in the harmonic relations between the shifting chords as they diverged and converged. However, my interest waned when the pattern became clear.

I decided not to take the tightly-coupled approach exemplified by the two preceding examples, as I wanted to have more control at the surface level of how my music actually sounded to the listener. An example of the loosely-coupled approach is *gradients of detail* for string quartet, composed by Chiyoko Szlavnics in 2005. In this work, and in others, Szlavinics uses drawing as the structural basis for composition. As she herself stated (Szlavnics (2006)):

> ... I do not create drawings in order to produce unique forms for the parametric control of material; rather, I create works of art to produce unique forms which almost literally represent frequency and duration.

Figure 4.3 shows one of the drawings that Szlavnics uses to structure *gradients of detail*. The vertical relationships between the line segments in the drawing represent rational harmonic

C	F	Bb	Eb	Ab	Db	Gb	B	E	A	D	G
C	F	Bb	Eb	Ab	Db	Gb	B	E	A	D	G

C	F	Bb	Eb	Ab	Db	Gb	B	E	A	D	G
F	Bb	Eb	Ab	Db	Gb	B	E	A	D	G	C

C	F	Bb	Eb	Ab	Db	Gb	B	E	A	D	G
Bb	Eb	Ab	Db	Gb	B	E	A	D	G	C	F

Fig. 4.2 Opening systems of *Logical Harmonies 2* (excerpted from Richard Glover's original score, available at https://richardglover.wordpress.com).

relationships and the pitches derived from them, which to use Szlavnics own words (Szlavnics, 2006), "produce a smooth aural surface". The composer disturbs this surface in a variety of ways, chief amongst them being long glissandi between stable harmonies and the use of extended silences. Partial silences can arise from the dropping out of a single instrument, or there can be total silences where all instruments are silent together. This is a technique that I used myself, particularly in *Properties of Matter: Expansion and Contraction* for string quartet.

Fig. 4.3 Drawing by Chiyoko Szlavnics shown in the introduction to *gradients of detail*

I will come to another example of the loosely-coupled, process-based approach when discussing a work of Marc Sabat. However, I have situated that discussion under the later heading of rational intonation as Sabat is a leading exponent of this method.

4.2.2 Focus on the harmonic spectrum

Works involving the harmonic spectrum and rational intonation have been considered in previous chapters of this commentary. Here, I will summarise some main themes and also make some comments about other uses of the harmonic spectrum in composition. Of course, all music is physical sound, and the physical properties of sound waves have to some degree a bearing on all musical styles. However, I want to distinguish here music for which the harmonic series is a prime element.

Spectralism

Grisey (translated by Fineberg) states that "spectral music offered a formal organization and sonic material that came directly from the physics of sound ... " (Grisey and Fineberg, 2000). Furthermore, in the same article, Grisey listed some of spectralism's characteristics as "[a] more 'ecological' approach to timbres, noises and intervals"; "Integration of harmony and timbre within a single entity", and "Integration of all sounds (from white noise to sinusoidal sounds)." Composers labelled by themselves or others as spectralists include Kaija Saariaho, Jonathan Harvey, Tristan Murail, and Gerard Grisey himself. This focus on timbre, while important in my music, has more of a surface-level rather than deep structural importance. Also, there is not the focus on harmonic relationships that my own work values. Hence, spectralism as defined by the work of the above composers has limited relevance to my work and will not be discussed further here.

Rational intonation and harmonic spaces

A constellation of composers, partly influenced by the teaching and work of James Tenney, used the themes of tuning, rational intonation and harmonic spaces as crucial structural frameworks for many of their works. I count myself lucky to now be within this fold. I have already discussed Catherine Lamb and Chiyoko Szlavnics in this and earlier chapters. Other composers currently working in the field of rational intonation include Marc Sabat, James Weeks and Wolfgang von Schweinitz. Ben Johnston, who died on 2019, is also an important figure in this field; Johnston's writings on rational intonation may be found in (Johnston, 2006).

As this area has been well discussed earlier, I would like to add just one more composer to this contextual analysis, namely Marc Sabat. Canadian composer Marc Sabat has written music for a variety of ensembles using rational intonation. His 2011 work, *Euler Lattice Spirals Scenery* for string quartet is in five movements and uses harmonic spaces, conceived by Sabat as extended Euler Lattices[2]. The heart of the work is the fourth movement, *Harmonium for Ben Johnston*. Marc Sabat (2019) writes of this:

[2]The *tonnetz* of Leonard Euler (1739) is a graphical representation of pitches and their rational harmonic relationships using prime numbers 3 and 5

> Harmonium follows a spiralling path of major and minor triads, wandering through an extended 5-limit space, resembling some of the lattices Ben employed in his early works.

Figure 4.4 shows the opening bars of this, and figure 4.5 shows the clear harmonic movement through the *tonnetz* from a G minor triad in bar 1, followed by a E♭ major triad in bar 2, to a C minor triad in bar 3.

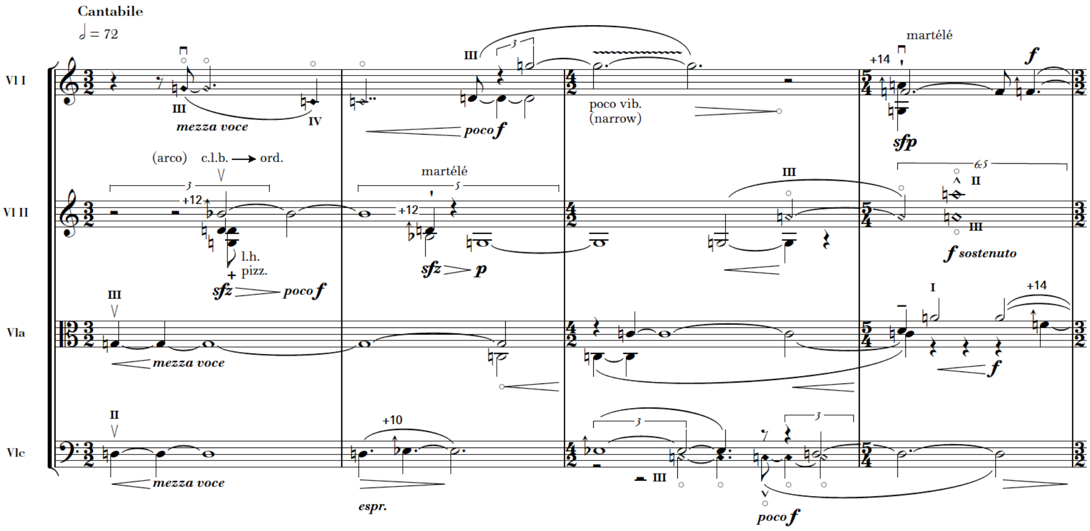

Fig. 4.4 Opening bars of *Harmonium for Ben Johnston* from *Euler Lattice Spirals Scenery* by Marc Sabat

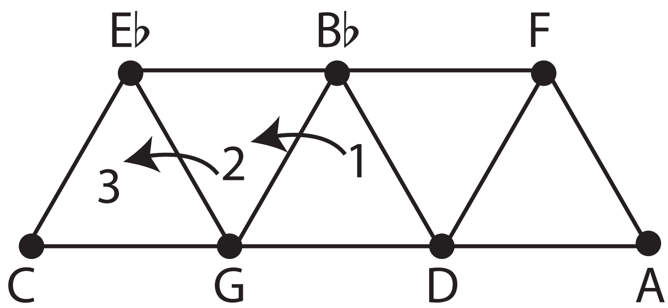

Fig. 4.5 Path through the *tonnetz* in opening bars of *Harmonium for Ben Johnston*

It is clear that Sabat is pursuing a rigorous harmonic process in composing this work. However, it is also clear by perusing the opening bars, shown in figure 4.4, that there is also much attention to the surface detail of the work. Examples include variable dynamics, careful pitch placement, harmonics, and bowing technique. As Sabat writes (Sabat, 2019):

> Although the piece is built on a systematic mapping of all possible triadic common-tone progressions and follows a symmetric form away from and back to a central point, as I wrote I found my mathematical constructions dissolving into an organic, musical surface.

Therefore, *Euler Lattice Spirals Scenery* provides a further example of a loosely-coupled process-based composition. I believe that Marc Sabat and I have much in common with regard to compositional process, being a combination of rigorous structure and musical elaboration. With the possible exception of my purely electronic work, all pieces follow this approach.

4.2.3 Spiritual themes

As already stated, I have been a Zen practitioner, and Zen ideas and practice have informed several of the works in my portfolio, either indirectly through a focus on non-narrative forms and meditative affects or more directly through the setting of Zen-related texts. I have also been interested in other composers who have Buddhism as a central part of their practice as composers. John Cage, in his paradoxical utterances, such as “What silence requires is that I go on talking”, is influenced by Zen thought (Shultis, 1995).

The inspiration that Éliane Radigue and Jonathan Harvey have taken from Tibetan Buddhism is well known. Radigue practised Tibetan Buddhism following a retreat that she attended in 1975. From 1979 onwards, she composed a series of works for synthesiser inspired by Buddhist themes. A later work, *Trilogie de la Mort* was discussed in the previous chapter. In an interview with Jenkins and Harvey (2006), Jonathan Harvey stated that, after encountering Christian mysticism around 1959, he began to practise in the Tibetan Buddhist tradition from about 1986, composing several works on the theme of emptiness. Harvey’s *Body Mandala* for large orchestra has been discussed in the previous chapter.

In this section, I would like to introduce one further work inspired by spiritual or religious themes. *Cinerum* (liturgy for Ash Wednesday) was composed by Horatiu Radulescu (2004) for four solo bass voices and instrumental ensemble, comprising string quartet, two trombones, theorbo, organ and percussion. The piece is a 70-minute setting of the Latin Mass. I only became aware of this work after *Presence/Absence* had been substantially composed. However, it is clear that it shares many characteristics with my work. It is monumental in duration, scale and affect. The vocal writing often has the character of chant and the text is based upon a religious tradition – in this case, Christianity. The harmonies are microtonal and reminiscent of the spectralist tradition. The percussion is extensive, as in *Presence/Absence*, with a focus on gongs. The brass, as in *Presence/Absence*, often features low pedal notes for trombones. An excellent review of its first performance by the Hilliard Ensemble can be found in (Gilmore, 2005).

4.3 Precision of pitch, tempo and dynamics

Issues of precision arise in several of the works discussed in the previous chapter, and I believe it is important to make some general remarks applicable to all works in the portfolio. Amongst the musical dimensions in which precision is important are tempo, dynamic level, and pitch. The granularity to which any of these parameters are specified needs careful consideration by the composer in terms of notation, and will impact the performer and ultimately the listener.

Regarding dynamic level, the traditional approach is to specify by using a sequence of direction, ranging from *pianissimo* to *fortissimo*, with extremes indicated above and below that range. These specifications may be supplemented by additional verbal comments. Although amplitude of a sound wave may be objectively and precisely measured using decibels, in practice there are many factors which introduce subjective and relative aspects. These include the acoustic properties of the performance venue, and the instrument(s) for which the instructions may be given. (An indication of *fortissimo* can mean something quite different when applied to a flute or to a trombone.) The psychoacoustic aspects of dynamic levels have been much researched (see, for example, Zwicker and Fasti (1999)).

In the works in this portfolio, pitches in any pitch palette are specified either as frequencies in Hz, or, when to be read by an instrumentalist, indicated with normal staff notation supplemented by deviations from equal temperament indicated by cents, each cent being a hundredth of a semitone. As with time values, dynamics, and other musical parameters, nothing can be absolutely precise. In the case of pitch, if the performer is using a meter, that again introduces imprecision. As discussed in Chapter 3, the question becomes: what degree of imprecision are we willing to accept? Based on the advice of Clemens Merkel, first violin of *Quatuor Bozzini*, I generally worked with a tolerance level of approximately $\pm 5 - 8$ cents. So, for example, if writing a piece in which microtonaI harmonies are employed, then a pitch would be permitted to be played on a well-tempered instrument if it was within $\pm 5 - 8$ cents of a well-tempered note.

There has been experimental work on precision as it applies to pitch. The term, *just noticeable difference (JND)*, is used to denote the amount something must be changed in order for a difference to be noticeable. In the case of pitch, Kollmeier et al. (2008) reported that the JND is approximately 3Hz for sine waves below 500Hz and about 1Hz (roughly 10 cents) for those above 1000Hz. These experimental findings accord well with Clemens

Merkel's advice. It should be said that these results apply to consecutive tones; when notes are played together the listener can use beats and difference tones to make much more accurate assessments.

As with dynamics and pitch, precision and accuracy also play an important role in tempo. Quantitative measures of tempo are usually specified by metronome markings indicating beats per minute. Such values originate from those measured by mechanical metronomes which provide measures in intervals of between two and eight units. Digital metronomes allow a finer precision, to at least one unit. However, metronome markings in my work are provided using the traditional values unless there is a cogent reason to not do so. An example where this happens is in *Properties of Matter: Damped Harmonic Motion* because the timings are linked directly to the motion of a pendulum.

4.4 Music or mathematics? (continued)

This section continues the discussion in section 3.3. In the earlier section, the focus was on the general point that while musical construction may have some mathematical foundation[3], ultimately it is a physical medium which is intended for listening. This section will dwell more on the way that mathematical structures deriving from harmonic spaces are transformed in my work into musical thought. I will draw some common threads together from the manner in which this has worked for me in the pieces in the portfolio. Sometimes, this aspect of composition is called 'intuitive', and I would add to that processes related to the 'inner ear' – hearing what is in my imagination related to the work currently being composed but also drawing on previous listening experience.

4.4.1 Structural and harmonic considerations

By the term, 'structural', I mean decisions about the overall shape of the music. Key input parameters here are the overall duration of the work along with its instrumentation. In terms of instrumentation, my work ranges from solo instrument (human performer or electronics), through small ensembles (e.g., string quartet), to larger scale works involving choirs, instruments and electronics, such as in *Presence/Absence*. Durations have spanned works of a few minutes (e.g., *Melum Scalenum*) to more than an hour (*Presence/Absence* and *2-Spem-70*).

In almost all of the longer works in the portfolio, I have employed one of the following with regard to harmony:

[3] After all, mathematics is nothing more than the study of abstract patterns and structures.

- A single arch structure, moving from minimum harmonic dispersion to maximal dispersion and back, (*Properties of Matter: Expansion and Contraction*).

- A series of such arches, diminishing in size of dispersion, (*Properties of Matter: Damped Harmonic Motion*).

- A single downward slope from maximal to minimum dispersion, (*Presence/Absence*). *Properties of Matter: Controlled Descent* can be included here even though not of excessively long duration.

For the shorter works, often a fragment of an extended chord sequence was chosen as the backbone of the piece.

The other important structural consideration that should be mentioned here is the introduction of repetition in some of the later works, and discussed elsewhere in this commentary.

On the issue of duration, some of my work is of long-duration, and as with each temporal scale, long-duration raises specific concerns. La Monte Young (1960) famously stated "[W]hen the sounds are very long ... it can be easier to get inside of them". This comment could relate to individual long sounds or to whole collections of sounds in the form of pieces. This states the most important motivation for me, in that with patience a listener can connect with the sounds themselves, not any extraneous meaning, and so access a contemplative approach to the listening experience. Morton Feldman (2000) expressed a similar thought when writing about his *String Quartet 2* (1983): "... on the third hour, I start to take away material rather than bring it in, make it more interesting, and for about an hour I have a very placid world". This makes explicit the idea in La Monte Young's statement, that for a long-duration piece (and *String Quartet 2* can run anything from four to six hours), one can resist the urge to continue to add new 'interesting' material. This advice was something that I implemented in *2-Spem-70*, where the only variation permitted is the harmonic variation resulting from the extended chord sequence and the spatial variation as the choirs come in and out of the performance.

4.4.2 Melodic aspects

Melody has been de-emphasised in earlier works in the research, where the focus was concentratedly on harmonic development and movement. This is particularly true of *Properties of Matter: Expansion and Contraction*, and *Properties of Matter: Damped Harmonic Motion*, but also of the later work, *Properties of Matter: Controlled Descent*, where melody is secondary to harmony. In these works, each 'voice' is provided with a sequence of pitches, each being sounded for an allotted period of time, and the interest is not in how one pitch in the sequence relates to the next, but how each contributes to the harmonic development.

I have begun to introduce melodic lines and melodic development in some of the later pieces. For example, in *Melum Scalenum*, the passage in the opening bars (1–4), with upward and downward leaps, is repeated in bars 11–14 with emphasis on upward leaps, and the upward leap aspect is considerably expanded in bars 22–27.

There are some melodic lines in *Presence/Absence*, usually arising serendipitously from the harmonic development in the electronic tracks. In the vocal writing, the emphasis is on chant-like lines, often on a single pitch with a small amount of variation. The use of chant in *Presence/Absence* was discussed in detail in section 3.8, when considering this work in detail.

4.4.3 Some contributions to affect

As expressed by Cespedes-Guevara and Eerola (2018), “One of music’s most pervasive features is its power to represent or express meanings. ... Among this variety of meanings, the ability of music to represent or express emotions stands out as one of the main reasons why music is omnipresent ...”. Each work in my research expresses a range of meanings, from deeply meditative in *2-Spem-70* to energy and excitement in *Properties of Matter: Controlled Descent*. Each work has its own techniques and properties, as discussed in Chapter 3, but I can draw together some general conclusions here.

Duration The duration of the work imposes a strong constraint on meanings expressed. The longer duration works, with their often inexorable harmonic logic, provide the possibility of conveying a sense of monumentality. The fact that they demand of the listener investment in time, concentration, and patience, along with their often underplayed gestures (not the case with *Presence/Absence*) can provide the listener with the opportunity, if so wished, to enter a state of ‘deep listening’.[4]

Energy A sense of energy is conveyed by *Properties of Matter: Damped Harmonic Motion*, and *Properties of Matter: Controlled Descent* in very different, but in each case, controlled ways. In *Properties of Matter: Damped Harmonic Motion*, the ever diminishing oscillations between low and high energy are conveyed by transitions between harmonic instability (high potential energy but low kinetic energy) and stability (low potential energy but high kinetic energy). This is reinforced by changes in tempo, dynamics and timbre, as discussed in Chapter 3. Similar techniques are used in the single

[4] This term is used in the sense of Oliveros (2005).

descent from high to low kinetic energy in *Properties of Matter: Controlled Descent* with an additional underlying constant quaver foundation providing a mechanistic affect.

Narrative Most of the works in this research are non-narrative in quality. They do not draw the listener to any goal or climax, but instead provide opportunities for the listener to focus on the sounds as they arise and cease, moment by moment, as the work unfolds. The exception to this is the composition *Presence/Absence* that sets texts and also does in a sense lead the listener through a series of risings and fallings of tensions to an eventual sense of release.[5] This is achieved by a collection of devices, as detailed in section 3.8.

4.5 Summary of the work

Where I have ended up, after several excursions in areas that did not quite fit my musical style, is the rich area of rational harmony in which I have extended some of the ideas of James Tenney, particular in areas relating to harmonic space. I have cast a 'mathematician's eye' over Tenney's harmonic space structures, particularly as were set out in Chapters 11, 12, 17 and 18 of the edited collection of his work (Tenney, 2015) and which formed the basis for his string quartet *Arbor Vitae*. This approach has resulted in a foundationally unified collection of works presented here, using harmonic space, harmonic distance, harmonic dispersion, chord sequencing, and unfolding to provide a unified underlying framework for a series of compositional processes. Even though the method has a common basis, it does lead to a wide diversity of forms when put into practice musically, from a piece for solo bass viol to a work for choir, chanters, solo singers, electronics, brass and percussion. There are two quite distinct parts to the compositional process for all these works; the first being constructing the programs and setting parameters for the generation of the pitch material, and the second configuring this material in a musical way, taking account of context (function, instrumentation, etc.).

4.6 Evaluation

All good research projects should include an evaluation component. In this case, this is multifaceted and includes self-evaluation, but also evaluation from my supervisors, examiners,

[5] It was very gratifying to receive an email from a member of the audience of the premiere of *Presence/Absence* that contained the sentence, " I felt a profound sense of peace by the end.".

listeners, and performers. I will leave it to others to appraise my work, if they wish to do so, but from my perspective this project has been a huge success. Of course, the whole project is experimental, investigating how musical meaning can be developed from quite complex mathematical structures that are extensions of Tenney's notion of harmonic space. As the quote from Tenney that heads this chapter indicates, even a master composer and theorist such as Tenney cannot be successful all the time. However, having said that, I do feel that I have been able to use my mathematical knowledge and programming skills to make a meaningful contribution to developing Tenney's theory, and to demonstrate, at least to myself, that this 'monstrous moonshine' really can lead to interesting musical matter. It has also been very interesting in my latter years on this project to create a work, *Presence/Absence*, that integrates so many strands of my work and life as a mathematician, musician, boy chorister of Gregorian chant and Zen practitioner.

A criticism I might level against myself would be that all the underlying structures rely on a single metric within harmonic space, namely that of dispersion. Even more, that the structures are narrowed further by looking at sequences of minimally increasing or decreasing dispersion. My response is to plead guilty as charged, but to offer mitigation that even within these strict limits it has been possible to construct a wide variety of musical experiences, with more to come.

I believe that apart from coming up with the Tenney extension itself, the factor that made the biggest difference to later compositions was a much more controlled use of the unfolding-repetition process. This process reached its zenith in *Controlled Descent*, in which I was able to control the amount of variation in unfolding as the piece progressed using an aleatoric algorithm.

4.7 Next steps

Returning to Tenney, there is still a whole world to explore using these extensions to harmonic spaces. An element somewhat neglected in my research is melody, and the idea of expanding the expressive potential of these structures by using my algorithms to build dynamic, microtonal scales; dynamic in the sense of the gradual emergence and disappearance of pitches within them, leading to a continuous transformation of the scale itself. These dynamic, microtonal scales would then be used as the bases for melodic as well as harmonic development. In Hindustani classical music, the long first stage (*alaap*) of a *raga* is the unfolding of a static, prescribed scale (*that*) associated with the *raga*. Chaudhuri (2021) describes the singer Govind Prasad Jaipurwale's approach to melodic evolution based on a fixed *that* as, "His ... scale wasn't 'made up' – it wasn't the sum – of seven notes; it was the unfolding, up and down, of an unbroken connection: the drawing out of a long moment." These melodic elements are nascent in my existing work, but I aim to develop them more systematically in works to come.

I also have the intention to return to the natural environment as a space in which to make music. I have for some years been a field recordist, and have sometimes incorporated these sounds into my music. But I do now understand that this approach is me (subject) making representations of the environment as object, and hence objectifying whatever is being represented. I am seeking a more involved, participative approach, somewhat along the lines of the work of Bennett Hogg, who recently gave a talk entitled "But what do we say to the birds?" [6] I aim to work on integrating this new approach with the harmonic (and melodic) developments stemming from harmonic spaces.

There is no straightforward way to end this account. There has been a good dose of the 'monstrous', in that some of the structures that I have developed have been very large, making their understanding and utilisation often beyond the powers of my brain and needing computational support. There has also been a good dash of 'moonshine' in the speculative nature of the project. However, I hope that the substantial presence of the resulting body of work has brought some of these ideas out into the light of the sun. Finally, what I can say is that sound and music have always been at the foundation of my life and I will continue to explore and create.

[6] "But what do we say to the birds?", Bennett Hogg, research seminar at Guildhall School of Music and Drama, London, May 24, 2021.

Appendix 1: Pitch Sequence for *Properties of Matter: Damped Harmonic Motion*

	HD of triad	Note vector	Note frequency	Note name	MIDI octave	Cents divergence
1	13.29	[0 2 0 0 -1]	180	F#	3	-47
2	12.85	[-4 0 1 0 0]	68.75	C#	2	-14
3	12.28	[1 -2 0 1 0]	342.22	F	4	-35
4	12.15	[-1 2 0 -1 0]	141.43	C#	3	35
5	11.58	[2 0 0 0 -1]	80	D#	2	49
6	11.54	[-1 0 -1 0 1]	242	B	3	-35
7	11.34	[0 0 1 -1 0]	157.14	D#	3	17
8	11.29	[-2 -1 0 1 0]	128.33	C	3	-33
9	11.05	[0 2 -1 0 0]	396	G	4	18
10	10.99	[0 -2 0 0 1]	268.89	C	4	47
11	10.87	[1 1 0 0 -1]	120	B	2	-49
12	10.78	[2 1 0 -1 0]	377.14	F#	4	33
13	10.74	[-3 0 0 0 1]	302.5	D#	4	-49
14	10.54	[-4 0 1 0 0]	68.75	C#	2	-14
15	10.44	[4 0 -1 0 0]	704	F	5	14
16	10.1	[-3 2 0 0 0]	247.5	B	3	4
17	9.97	[3 -2 0 0 0]	195.56	G	3	-4
18	9.89	[1 2 0 -1 0]	565.71	C#	5	35
19	9.84	[0 -1 0 1 0]	513.33	C	5	-33
20	9.45	[0 0 0 1 -1]	140	C#	3	18
21	9.26	[3 1 -1 0 0]	1056	C	6	16
22	9.26	[-1 1 0 0 0]	330	E	4	2
23	8.2	[-1 -2 0 1 0]	85.56	F	2	-35
24	8.09	[3 0 -1 0 0]	352	F	4	14
25	7.9	[0 0 0 0 0]	220	A	3	0
26	7.23	[-4 0 0 1 0]	96.25	G	2	-31
27	6.57	[-1 -1 0 0 1]	403.33	G	4	49
28	6.48	[-3 0 0 1 0]	192.5	G	3	-31
29	6.33	[-1 0 0 0 0]	110	A	2	0
30	6.15	[0 -1 1 0 0]	366.67	F#	4	-16
31	6.05	[3 -2 0 0 0]	195.56	G	3	-4
32	5.84	[3 -1 -1 0 0]	117.33	A#	2	12
33	5.81	[1 1 -1 0 0]	264	C	4	16
34	5.14	[-1 2 -1 0 0]	198	G	3	18
35	4.93	[-1 2 0 -1 0]	141.43	C#	3	35
36	4.6	[0 1 0 0 0]	660	E	5	2
37	3.94	[-3 2 0 0 0]	247.5	B	3	4
38	3.27	[-1 1 -1 0 0]	66	C	2	16
39	2.78	[-2 2 0 0 0]	495	B	4	4
40	2.39	[-2 1 0 0 0]	165	E	3	2
41	0.67	[-1 0 0 0 0]	110	A	2	0
42	0	[0 0 0 0 0]	220	A	3	0

Appendix 2: List of works by other composers

Britten, Benjamin (1964). *Curlew River - A Parable for Church Performance*. Score published by Faber Music, ISBN: 0571500021.

Clementi, Aldo (2005). *Momento* for string quartet. Score published by *Edizioni Suvini Zerboni*, Milan, Cat. No. S. 12619 Z. Recording by Quatuor Bozzini, label Collection QB, CQB 1615.

Feldman, Morton (1985). *For Bunita Marcus* for solo piano. Score published by Universal Edition, UE3525.

Frey, Jürg (2010-2014). *String Quartet (No. 3)*. Score and recording published by Edition Wandelweiser Records (https://www.wandelweiser.de)

Glover, Richard (2013). *Logical Harmonies 2*. Score available at https://richardglover.wordpress.com. Recording from www.anothertimbre.com.

Harvey, Jonathan (2006). *Body Mandala* for large orchestra. Score published by Faber Music, published 2011, ISBN 0571518494.

Johnson, Yom (1980), *Movements* for wind quartet. Score available at Editions 75, http://editions75.com/catalogue-ensemble.html.

Lamb, Catherine (2010). *three bodies (moving)* for violin, cello and B♭ clarinet. Score available from www.sacredrealism.org.

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Radigue, Éliane (1985-1993) *Trilogie de la Mort* for synthesisers. Recording published by the Phill Niblock's XI label (https://xirecords.org)

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