Essays on Sovereign Default with Unobservable Physical Capital and Debt Relief

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Abstract

This thesis provides three novel theoretical frameworks of sovereign default with unobservable physical capital and debt relief. The models are calibrated to the Argentina's economy and simulate the effect of productivity shocks between 1980 Q1 and 2017 Q4. We show results in terms of bond prices and a borrower's capital accumulation, debt, consumption, and debt relief, in addition to the default options at the steady-state and on the transition path with productivity shocks.

The first paper is concerned with a model of sovereign default with unobservable physical capital. To our knowledge, this is the first paper in the literature of sovereign default to model default probability with unobservable capital accumulation. It aims to solve the bond premium and perceived equilibrium default with incomplete information about the borrower's capital accumulation. The model applies to developing countries with undetectable capital accumulation. We derive the bond premium as well as the optimal level of foreign assets. In comparison with the existing literatures, the steady-state of capital and foreign assets from our model is lower and closer to the actual data. This is because the sovereign borrowers have less incentive to accumulate capital for influencing the future bond price. The bond premium is provided from the perceived information only.

In the second paper, we introduce a partial default option with the debt reduction and the probability of re-entry as exogenous variables. The novelty of this paper is the illustration of how sovereign borrowers decide on a partial default. Previous literature only includes absolute default that is sovereign borrowers are not required future repayments. In this model we include an additional option to

default that is partial repayment and a shorter stay in debt hangover. These two main variables are exogenously added into the default option. Thus, sovereign borrowers can decide among repayment, absolute default, and partial default. The outcome of the simulations provides the optimal level of debt relief and the length of stay in default for sovereign defaulters. The option of partial default leads to a higher utility for indebted countries. Under the calibration for the Argentina economy, the optimal debt relief in the simulation matches actual data. Results confirm the importance of using partial default as the utility of sovereign borrowers can be significantly higher than with absolute default. More importantly, the results show that sovereign defaulters who receive debt relief experience an increase in both consumption and investment. This outcome is in accordance with the statistics found by Fonchamnyo (2009) and Arslanalp and Henry (2005) that the countries receiving debt relief will spend more on both consumption and capital investment after re-entry into the credit market. On the other hand, our result contradicts the model of Romero-Barrutieta et al. (2015) where their results show only an increase in consumption but lower capital investment after receiving debt relief. Specifically, this is because Romero-Barrutieta et al. (2015) implements the fully observable capital accumulation and the exogenous bond price[1]. Therefore, our model can show an importance of unobservable physical capital which should be considered in the bond price schedule because our results from simulation are in accordance with the stylised facts of developing countries over the past decades; consumption and investment are both increased after receiving debt relief.

Finally, the debt relief and timing of debt restructuring become endogenous variables

[1]Romero-Barrutieta et al. (2015) models the bond premium as an independent and identically distributed random variable (exogenous variable).

in the third paper. The results show the dynamic choices of optimal debt relief under different levels of initial capital, foreign assets, and productivity. Countries facing a large negative productivity shock prefer an absolute default regardless of foreign assets. On the contrary, indebted countries with small or moderate productivity shocks optimise their own utilities over debt relief choices. The model simulations capture the behaviour of the Argentina's economy whose defaults happened for different levels of capital, foreign assets and productivity. The main novelty of this model is the dynamic choice of debt relief and its impact on the sovereign borrowers' decision. In comparison with the existing literature, our model provides a non-linear choice of debt relief over various level of initial capital, foreign assets, and productivity. Our theoretical framework is flexible enough to embody further development such as heterogenous agents, switching regime of default scheme, and partial default with mixture of bond maturities. This is left for future work.

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Chapter 1

Introduction

This thesis contributes to the existing literature by providing three main theoretical frameworks related to sovereign default. The first model represents a sovereign default with unobservable physical capital, whilst the second and third theoretical studies are about sovereign default with debt relief and its timing of financial exclusion. Debt relief is included as an exogenous variable in the second model, whilst in the third model, it is endogenous.

In these three studies of sovereign default, we calibrate our models to the Argentina's economy and simulate the effect of productivity shocks between 1980 Q1 and 2017 Q4. We show results in terms of bond prices and a borrower's capital accumulation, debt, consumption and ex-post debt relief, in addition to the default options at the steady-state and on the transition path with productivity shocks.

The thesis starts by addressing the issue of default probability of sovereign debtors under incomplete information of capital accumulation. The perceived equilibrium and bond price under incomplete bond contract are presented. This framework allows us to analyse whether it is optimal for a country to borrow for investment or consumption. The canonical position of lenders (e.g., IMF see Abbott et al. (2010)) is that foreign assets should be used for investments or for promoting long-run economic growth and, in turn, for generating returns to be used for debt repayment.

Nevertheless, the experience of several countries, whose borrowing was not followed by the agreed level of investment programmes, such as Argentina, Nigeria and, more recently, Greece, opens up the issue for debate.

Previous literature has either not modelled a borrower's physical capital (Aguiar and Gopinath, 2006; Arellano, 2008) or assumed that it is fully observable by lenders (Bai and Zhang, 2011; Gordon and Guerrón-Quintana, 2013). With fully-observable capital, it is possible to write a contract between borrowers and lenders and, typically, the resulting use of foreign assets would be for investment rather than for consumption. Furthermore, the existing literature does not provide a good fit with real world data in relation to equilibrium debt to GDP ratio, especially for Latin America and Africa (Reinhart and Rogoff, 2008).

In this thesis, the modelling of the borrower side follows the sovereign default literature in allowing the debtor to decide on the future level of capital, consumption and debt with an option to default. However, unlike the literature, a borrower's capital investment cannot be observed by the lender. The lender cannot observe physical capital; therefore, the pricing schedule does not depend on capital. The borrower can therefore choose to borrow for consumption rather than for investment in capital, without altering the price of borrowing. Our model is better suited than existing models in the literature to fit the default experience of several countries, especially in Latin America and Africa, by showing higher debt to GDP ratio at the steady-state. Therefore, it confirms the importance of imperfect information for understanding the behaviours of international borrowers and lenders and informing international debt policies.

We also find that below the steady-state level of physical capital, the borrower will find optimal ways to use foreign assets for investment; meanwhile, if capital is above the steady-state, the borrower will sell the physical capital for consumption. Furthermore, the level of capital will steadily move towards the steady-state. Notably, if the borrower holds a large amount of debt and low capital, she/he will not be able to borrow further and will stay below the steady-state level of capital. Furthermore, with a negative productivity shock, the borrower will be able to smooth consumption through sacrificing capital if the initial capital is at or above the steady-state level. On the other hand, if the initial capital is below the steady-state level, the borrower is unable to smooth consumption through borrowing more or selling physical capital. The simulation results therefore show that it is not optimal to force debtors to use foreign assets to invest only because, under certain conditions, consumption is the optimal decision. These results have not been captured by the previous literature.

The second study of sovereign default in this thesis addresses the option of partial default. In accordance with the previous theoretical framework, the partial default option is developed and included additionally into the seminal model. Absolute default is defined as a severe default where the defaulter is not obliged to any further repayment after default. The partial default option instead requires defaulters to partially repay some of the default bond. Cruces and Trebesch (2013) find that, over the past two decades, defaulters experienced a variety of debt relief and time of debt restructuring. Hence, the study of partial default in this thesis aims to find the optimal rates of debt relief and the timing of financial exclusion as exogenous variables. The rate of debt relief and timing of debt restructuring are given into the model in order to find the maximum utility of social welfare. We can find that at each level of debt relief and the timing, the model will converge to a different equilibrium as well as optimal level of capital accumulation and foreign assets. Therefore, the social planner can achieve a higher welfare level from the prior rate of debt relief and timing of debt hangover before default is realised.

To model partial default as an additional option for sovereign defaulters, a function of bond price is adjusted to respond to the probability of partial default. The option of partial default affects sovereign borrowers in the credit market; sovereign debtors must decide the optimal rate of debt relief and the appropriate length of stay in default in order to maximise their social welfare during default. Thus, the perceived probability of partial default's incentive requires an adjustment to the bond premium. Under the same parameters and calibration, we find a non-monotonic shift of bond prices for given debt relief and probability of re-entry. When including the partial default option for a country with small debt, the bond premium is higher when both debt relief and probability of re-entry are high. Conversely, when adding the partial default option for a highly indebted country, the reverse impact for partial default emerges. This indicates that absolute default is preferred for highly indebted states and a probability of partial repayment in the future reduces the current bond premium for large borrowing.

From the Monte Carlo simulation, we simultaneously find the optimal levels of debt relief and probability of re-entry. For Argentina, debt relief at 39% and probability of re-entry at 10% will maximise utility at the steady-state. This outcome is in accordance with 37% average haircuts of 180 sovereign bonds between 1980 and 2010 and 40% in Argentina[2] (Cruces and Trebesch, 2013). Under different scenarios

[2] The rate of haircuts can be varied due to the fluctuation of face value of bonds. At that time, there are 24% of participants who received nothing, and the average haircuts rate calculated by Cruces and Trebesch (2013) over years of default was around 40%.

of capital accumulation and foreign assets, the theoretical model provided in this thesis reveals the optimal rate of debt relief and its timing.

In the last paper, we augment the previous model by endogenising debt relief and timing of debt restructuring. We find the optimal choice of ex-post debt relief and its timing at the perceived equilibrium default. Specifically, the sovereign debtor can decide the ex-post debt relief as well as the time for re-entry into the international credit market. A defaulter with ex-post debt relief is required to make repayments in the re-entry period, but it has also the option of absolute default, that is of refusing future repayment with the penalty of a maximum stay in default[3]. Simultaneously, the creditors perceive the probability of default as well as the ex-post debt relief and, consequently, the bond premium is adjusted in equilibrium. In this framework, the indebted countries can obtain higher utility through a dynamic choice of optimal debt relief over the absolute default. Besides, there is a risk adjustment on sovereign bond contracts because there will be a probability that creditors will receive the partial repayment after default. We are also able to provide a function of debt relief in relation to the probability of re-entry.

At the same time, creditors also perceive the probability of default and the ex-post debt relief and its timing; thereby, the bond premium is adjusted. In this way, a sovereign debtor can obtain higher utility through a dynamic choice of optimal debt relief over the absolute default. We also provide the value of debt relief as a function of the probability of re-entry. The amount of debt relief includes compensation for the reduction of time spent in default.

[3]When the defaulter is penalised by the maximum length of debt hangover, the probability of re-entry into the credit market is set at 5%. On the other hand, if the defaulter choose to partially repay some debt, the probability of re-entry will be greater than 5% and as a result, the period of debt hangover becomes shorter.

Under the same parameters with respect to the previous literature and model calibration, we find that bond premium is higher because there is the additional probability of applying for debt relief after default. Computations illustrate the dynamic choice of optimal debt relief under different capital accumulation, foreign assets, and productivity. The choice of debt relief is significantly correlated to perceived productivity. We find that sovereign debtors are better off from being in absolute default when output is low regardless of the amount of foreign assets. Default with an expectation of an endogenous debt relief is preferred if output is not too low. In addition, the model indicates a different level of debt relief at borrowing from different productivity, initial capital and foreign assets. Generally, debt relief between 20% and 40% is optimal for sovereign debtors under negative shocks. The rate of debt relief can become higher during a good period and lower during a bad period. In accordance with the previous model of exogenous debt relief, bond price is not only monotonically influenced by the ex-post debt relief but also includes the time influence of the credit exclusion period. If the probability of re-entry is low (close to 5%), the default incentive will be less and close to the absolute default; hence, any gains from the partial default option result from a trade-off between the amount of debt relief and its timing.

The model also presents the steady-state of foreign assets and capital at the perceived equilibrium with the absolute and partial default options. Sovereign debtors can reach the steady-state when there is no productivity shock. If there is a temporary shock, the indebted countries will smooth consumption through capital accumulation and foreign assets. The results for this model are similar to our seminal models where the sovereign debtor will borrow during a good period and sell capital or default during a bad period. From the simulations of this model, we provide frequencies of absolute default and partial default under different rates of debt relief and timing. However, for the same parameters, the model with debt relief is more volatile than the seminal model with only absolute default. This is because of the higher probability of default in this model and, consequently, the fact that the bond premium is increased to compensate for an expected higher default.

This thesis is structured as follows: Chapter 1 provides the introduction to the thesis; Chapter 2 contains an overall review of previous theoretical and empirical literature on sovereign default; Chapter 3 includes the seminal theoretical model of sovereign default with unobservable capital, quantitative results and model simulations; Chapter 4 contains the theoretical model of sovereign default with debt relief and timing of debt restructuring as exogenous variables; Chapter 5 endogenise those variables. The final chapter provides a summary and conclusions.

Chapter 2

A Survey of the Literature

2.1 Introduction

Numerous studies have attempted to establish the reasons why debtor countries decide to default. There are several issues in this topic that remain unsolved. Martinez and Sandleris (2011) argue that estimating the costs and the benefits of sovereign default are difficult, because of the fluctuations in economic variables around the defaulting period and several unique problems. Presently, the global financial markets are more integrated compared with the past. The effects of default can spread over the international credit markets. For instance, if a country declares default, lenders who hold its sovereign bonds will be negatively effected. International lenders are aware of the future default. They will receive market information and decide on the risk premiums of the government's bonds. From this point, the government's decision on sovereign default is not totally isolated from the international credit markets. Hence, the study of sovereign default begins with the government's decision based on two main functions; repayment and default. The repayment function will take the international bonds into account, while the default function will be considered as autarky.

Eaton and Gersovitz (1981), began a study of sovereign default by approaching the endogenous relationship between the business cycle and probability of default. The researcher demonstrates that when the country has an output shock, the government will gather such information and decide on the new borrowing amount. The model is countercyclical; where the negative output shocks have a tendency to increase an amount of new debt. The authors explain that the country needs to borrow from international credit markets, in order to smooth domestic consumption. Expectedly, countries borrow more during bad times and repay during good times. However, if the overall debt is too high from serial output shocks, the government can choose to default. This assumption is consistent with empirical evidence from Tomz and Wright (2007), Yeyati and Panizza (2011) and Reinhart and Rogoff (2011) stating that sovereign defaults have occurred from high debt and high output shocks, since the early nineteenth century.

Aguiar and Gopinath (2006), provided a seminal study of bailout from the third part (IMF). They implemented the model with bailout before the default period. Benjamin and Wright (2009), Yue (2010) and Bai and Zhang (2012), extended a study into sovereign default with a debt renegotiation. They use a game theory of two agents; borrower and lender. The differences in in the defaulting duration is based on the bargaining powers between the two agents and levels of debt reduction. Alternatively, the model of sovereign default with political instability is approached by Cuadra and Sapriza (2008). They presented a model of a small open economy with two political parties. The outcome showed countries with higher degrees of conflict come higher rates of default. Besides, Lizarazo (2013), provided a study of risk-aversion for international investors. The authors found the importance of risk-aversion on the lender's side, because it has a significant impact on risk-premiums and bond prices. Chatterjee and Eyigungor (2012), studied the model with different maturity of debts; short-term and long-term. Finally, Cuadra et al. (2010), Arellano and Bai (2014), and Aguiar and Amador (2016), studied sovereign default with fiscal policies. They assumed that fiscal policy is pro-cyclical, where the sovereign debtor increases tax during bad times and decrease tax during good times.

On the empirical side, Schaltegger and Weder (2015), studied the relationship between fiscal adjustment and future default. The outcomes showed no significant support for a temporary decrease in the size of debt related with default probability. Surprisingly, a reduction in the composition of short-term to total debt significantly reduced default probability. The outcomes also showed that higher output growth, higher government revenue and higher international reserves can lead to lower rates of default the following year. More importantly, Martinez and Sandleris (2011), studied the punishments for sovereign default from creditor countries. The results showed no evidence to support for the sanctions, while defaulting countries significantly received a punishment in terms of financial exclusion. Lastly, Reinhart and Rogoff (2011), study sovereign default and crises. They divided crises into three main groups; banking, public debt and inflation. Interestingly, only banking crisis had a significant relationship with default probability.

From the above, this chapter is separated into two sections. The first and the second will be a review of theoretical and empirical literature respectively.

2.2 Theoretical Literature on Sovereign Default

Over the last decade, global financial sectors have become more integrated and share risk throughout the international credit markets. A government's decision to declare default has become more important to the world economy. There are several researchers have demonstrate the reasons for sovereign default with theoretical models. In 1981, the theoretical framework of Jonathan Eaton and Mark Gersovitz become well-known study of borrowing with default. They published the paper by assuming the decision on default is an optional strategy for debtor countries. Therefore, this section will provide a theoretical explanation of sovereign default from the previous literature.

2.2.1 Eaton and Gersovitz (1981) Model

Eaton and Gersovitz (1981) provided a seminal study with an endogenous relationship between default probability and output shocks. The authors demonstrated that government has a role to maximise consumption through allocating funds. When the economy receives a negative output shock, government can stimulate demand and increase consumption through spending. However, government budgets are limited in the autarky. Hence, the international credit markets will play an important role through lending money to government. From this view, the theoretical framework can be explained in that government will gather information about output shocks in the economy, and then react by borrowing at optimal proportion to smooth consumption. When a positive output shock occurs, government will repay their debt. This theoretical framework will follow the countercyclical policy. Countries will borrow more during bad times and repay during good times. Thus, the international debt is defined as an endogenous variable to the output.

From the above assumption, Eaton and Gersovitz (1981), demonstrated the government's decision on default by presenting two functions; repayment and default. The repayment function will include the amount of the zero-coupon bond as a proxy for international debt, while the default function will be assumed as autarky's state. They described that default is a possible strategy for borrowing agents, but they will be excluded from future borrowing, forever. Hence, the debtor countries will consider the expected outcomes from repayment and default. The sovereign default will become more favourable if the outcome from choosing default is larger than from choosing repayment. When the value of default is higher, the government will be more encouragement to destroy their obligations and be excluded from the international credit markets. Especially, some debtors with high domestic consumption and low borrowing intentions are more encouraged to default. Alternatively, lenders will gather the information and decide on the risk-premiums of loans based on the default probability. Countries with higher rates of future default will be charged higher risk-premiums. Under the symmetric information and incomplete bond markets, lenders will never lend to those who are certain to default in the following period. Hence, the model is designed to characterise borrower behaviour under the above assumptions.

The Model

In the model, output will be only consumed in a particular period. It is not storable over periods. Therefore, the model for a closed economy is given by:

$$c_t = y_t \tag{2.2.1a}$$

where consumption (c_t) is only determined by output (y_t) within a country. Besides, the output is stochastic variable with a production function [$g_t(y_t)$]:

$$\int_0^{\bar{y}} g_t(y_t)dy_t = 1. \tag{2.2.1b}$$

Remarkably, consumption is only equal to output in the country. The government is restricted to enhance consumption above output. In this way, consumption is relatively fluctuating because a decline in output immediately decreases level of consumption. Without fiscal policies and the government's support, the economy will enter recession during periods of negative output shocks. Hence, the government has a role to smooth the economy by increasing spending. In this model, the government will use the full amount of borrowing for sustaining consumption. With access to international credit markets, the government budget constraint can be significantly increased to an unlimited amount. However, higher debt will raise the burden (repayment) in the following period. From this assumption, the budget constraint for a small open economy is the following:

$$c_t = y_t + b_t - p_t \tag{2.2.1c}$$

where b_t is the borrowing contract and p_t is debt-service payments. Debtor countries have cash inflows from borrowing and cash outflows from repayment. Noticeably, the model in this paper is restricted to any investment, such that the set of available loan amounts in period t is $[0, \bar{b_t}]$. A high proportion of current sovereign debt means a large amount of repayment in the future. The government needs to consider the optimal level of borrowing to maximise the utility over periods. In this way, the borrower's utility, or objective function, is:

$$E[\sum_{t=0}^{\infty} \beta^{\prime} U(c_t - P_t)], \tag{2.2.1d}$$

$$U(X) = \begin{cases} \frac{X^{1-\gamma}}{1-\gamma} & \gamma > 0, \gamma \neq 1, \\ lnX & \gamma = 1 \end{cases} \tag{2.2.1e}$$

where P_t is a period of debt exclusion, β is a discount factor and γ is a rate of risk aversion. From this model, the benevolent government will borrow more during bad times and repay during good times. However, under incomplete markets, the government cannot roll over their debts because bonds are not available during very high default risk. Besides, the repayment function is defined by:

$$d_{t+1} = R(b_t). \tag{2.2.1f}$$

Debt service obligation is d_{t+1} in period $t+1$; usually $d_{t+1} = (1+r_t)b_t$ where r_t is the interest rate, and R is 1+ the market interest rate at the particular debt contract (b_t). From these above equations, the borrower decides the amount of debt contract (b_t) and payments (p_t) at the given market interest rate. If $p_t < d_t$, such that debt payments fall short of debt obligations, borrowers will obviously choose either repayment $(p_t = d_t)$ or non-repayment $(p_t = 0)$. The value function of remaining in credit or repayment can be written as:

$$V^R(y_t, d_t) = \sup_{b_t \in B_t} \left\{ U(y_t + b_t - d_t) + \beta E \max[V^R(y_{t+1}, d_{t+1}), V^D(y_{t+1}] \right\}. \quad (2.2.1g)$$

The value function of default or non-repayment is:

$$V^D(y_t) = E[\sum_{T=t}^{\infty} \beta^{T-t} U(y_t - P_t)]. \quad (2.2.1h)$$

In each period, the government has to decide whether default or not by comparing these two equations. Hence, the probability λ of default exists and can be determined as:

$$\lambda(d_t) = Pr[V_t^D > V_t^R]. \quad (2.2.1i)$$

Creditors will lend when it guarantees them a return of at least the market interest rate (R). It can be shown as:

$$[1 - \lambda^*(R^*(b_t))]R^*(b_t) = (1 + \bar{r})b_t. \quad (2.2.1j)$$

If the government has zero debt, the market interest rate will be determined by the risk-free only. In addition, a positive default probability will raise the market interest rate (risk-premium) of particular bonds. Therefore, the government is unable to borrow an infinite amount because the higher amount of loans will raise the default probability and burden in the next period. From the default function, an increase in debt will raise an expected outcome of non-repayment. The government will choose to default, if the value function of default is higher than that of repayment.

Model Calibration

Eaton and Gersovitz (1981), approach calibrated parameters based on a cross-sectional dataset of 45 developing countries in two years; 1970 and 1974. The crucial point in this model is that the amount of debts (b) is observed, while the joint estimation of parameters is estimated by maximum likelihood methods. Consequently, the authors initially explored a study of observed debts among endogenous variables and opened up new opportunities for other researchers to investigate more on this and related issues.

Model Result

This theoretical model is a good starting point to understand the behaviour of poor countries borrowing in the international credit markets. They provide a theoretical framework of interactions among consumption, output, loans and default probabilities, through an endogenous market interest rate. As a result, the credit ceiling, or safe threshold of debts, is obviously defined in the literature. A borrower can optimally choose an amount of loans and manage the default probability in order to maximise the utility function. However the model is proved, by recent economic circumstances, to have some assumptions that are not practicable in the modern economy. Essentially, the authors assumed that a borrower has no intention to store investment over time, and the financial exclusion is permanent after default. In contrast, recent empirical papers prove that consumers or governments usually save some money to invest for future returns, and the usual exclusion period is not long-lasting. Nonetheless, the seminal framework by Eaton and Gersovitz (1981), has been extended to approach the recent macroeconomic criteria, as well as literature that will be introduced in the next section.

2.2.2 Arellano (2008) Model

The theoretical model of Arellano (2008), became a seminal study of sovereign default in the past decade. The model was developed from the previous study of Eaton and Gersovitz (1981). It solved the previous issues by implementing the model that closes to the current bond market. The author provided the model with independent and identically distributed (i.i.d) shocks, based on a discrete-time stochastic process, or a Markov chain. For the novelty, the author assumes that the government always enters into a bond contract during bad and good times. The government will save through buying one-period discount bonds B' at price $q(B', y)$, whilst borrowing is the opposite. In this way, the model explains an endogenous relationship between the business cycle and the international bond markets. However, lenders notice that the sovereign debtor has an optional strategy to default, with unenforceable contract. Thereby, they will gather the market information and give the risk-premiums on the bond contracts if the sovereign borrowers have more encouragement to default. Furthermore, the author also added the probability to re-enter the default function as an exogenous variable. The sovereign debtors can re-enter the international credit markets after some periods of debt exclusion. The probability of re-entry can be calibrated from the previous debt exclusions. This assumption is consistent with recent empirical evidence that most defaulting duration is between three and six years (Reinhart and Rogoff, 2008; Sandleris et al., 2004).

The Model

From the stylised facts, sovereign debtors will choose to default when output is below trend (Krebs, 2006). The author assumes that lower output is entailed by a direct cost from choosing default. During default periods, budget constraints of a closed economy are slightly different from those of Eaton and Gersovitz (1981), as the following shows:

$$c = y^{def}, \tag{2.2.2a}$$

where $y^{def} = h(y) \leq y$. Output in autarky is assumed to be lower or equal to the output in an open economy depending on the direct cost of default $h(y)$. Furthermore, the government need to consider the utility function of consumption in order to maximise the country's welfare over time. Households utility function with risk averse is simply identified by:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \tag{2.2.2b}$$

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma} \tag{2.2.2c}$$

where $0 < \beta < 1$ is the discount factor, σ is the risk-aversion rate and c is consumption. Next, when the government enters into the international bond markets and chooses not to default, total amount of debt or investment, quantity of bonds and prices are taken into the model as endogenous factors. In this manner, the researcher characterised the key model of budget constraint for the small open economy as the following shows:

$$c = y + B - q(B', y)B', \tag{2.2.2d}$$

where B' is the quantity of one-period discounted bond that are decided by the government at time t. Bond price $q(B', y)$ is an endogenous variable which is decided by the market. The author annotated that a negative B' means borrowing to receive a cash inflow of $-q(B', y)B'$ from international lenders at time t, and promise to repay at B' the following period. To invest, the government chooses a positive B' with cash outflow $-q(B', y)B'$ and then, intending to receive cash inflow of B' the next period. Importantly, the bench-mark model of the pricing bond is given by:

$$q(B', y) = \frac{(1 - \delta(B', y))}{1 + r}, \tag{2.2.2e}$$

where δ is the default probability. As given schedules of output y and loan size B', the distribution probability of default and pricing schedules can be calculated. Then, the government will take the updated bench-mark of bonds into consideration and may choose new size of loan to optimise the objective function. Through the recursive process, loan size, default probability and price of bonds are recursively computed until reaching equilibrium, where the creditors and debtors can maximise their objectives. Furthermore, a function of the government's decision including the default option can be presented as follows:

$$v^0(B, y) = \max_{\{c,d\}} \left\{ v^c(B, y), v^d(y) \right\}, \tag{2.2.2f}$$

where $v^c(B, y)$ is the value of repayment and $v^d(y)$ is the value associated with default. These two main functions are given by the following:

$$v^c(B, y) = \max_{(B')} \left\{ u(y - q(B', y)B' + B) + \beta \int_{y'} v^0(B', y') f(y', y') dy' \right\}, \tag{2.2.2g}$$

$$v^d(y) = u(y^{def}) + \beta \int_{y'} [\theta v^0(0, y') + (1-\theta) v^d(y')] f(y', y) dy'. \tag{2.2.2h}$$

From the above equations, the government will decide to default if the value of default is greater than repayment. However the default option is permitted only when the government has negative assets. After default, the government will be excluded from international credit markets and regain access with the probability θ. The author assumed that the probability of reentry is an exogenous variable and constant over time.

Model Calibration

The model is calibrated from 74 quarterly observations in Argentina between 1983 and 2001; prior to the default of 2001. Parameters are simply estimated to match with the historical data and other literature. The stochastic structure of output is estimated from the Argentine GDP by a log-normal AR(1) process. Essentially, discount factor β, the probability of reentry θ and output costs $\hat{y}$ are calibrated to match the default probability of 3% and other empirical data. The probability of re-entry at 0.282 is consistent to the default in Argentina. They spent approximately three years of debt exclusion before re-entry.

Model Result

This model can explain an endogenous relationship between the business cycle and the international credit markets. It presents the theoretical framework of the countries to invest and borrow in bonds. As a result, the outcomes show that the borrowing constraint is countercyclical. The government will borrow and spend more during bad times with the budget restriction from the bond risk-premium. For example, when the government holds very large amounts of debt, the default risk will be increased as well as the bond risk-premium. The sovereign debtors cannot always roll-over their debt, because lenders will avoid lending money to countries with high default probability. When the debt is too large, they will decide to not repay and be excluded for some periods. The results are consistent with the past empirical data of Argentine's business cycle. The model can be used to predict the default in 2001. The clear explanations and results make the theoretical model of Arellano (2008), become widespread and seminal for a study of sovereign default.

2.2.3 Sovereign Default with Political Instability

Several literature studies the model of sovereign default with political instability. Citron and Nickelsburg (1987), provided a stylised fact of default pressure from political instability. An increase in the level of political instability will enhance the pressure and the probability to default. They also explained that during periods of political instability, the defaulting decision is an optional strategy to adjust the government's budget. If the new government chose to default, they will ignore repaying the principal and interest. This helps to reduce the burden from previous policies, when there is a change in political parties. If the government requires some funds to run policies, default is also less costly when compared with a sharp increase in tax. From this assumption, Cuadra and Sapriza (2008), provided a theoretical framework with two political parties under the internal conflict. The level of political instability will be increased during a period of changing sovereign agencies. There will be a trade-off between repayment and default during high and low rates of political instability. When the previous party is out of power, they will be replaced by the new party. Both political parties can have different ideas regarding repayment and default. Hence, the value functions have to include the situation in which they will make different decisions under the period of transition. For example, the country may deicide to repay during the control of the first party only. After they are replaced, the new government may want to default. Opposite, the first party may decide to default, while the second party wants to repay (or re-enter if they default).

The Model

The main novelty in this model is an internal conflict between two agents. From the above assumption, there are two political parties that take turns in office. The government still has a role to choose consumption and debt as in the previous literature. However, this is slightly different, because the household can be separated into two groups due to two political parties. An agent who is in power can allocate resources for both parties. For example, if party 1 is in office, it can choose to allocate resources to party 1 more than party 2. Each political party will aim to maximise its consumption, while it is still in office. At the end of each period, they can be replaced by another agent with probability π. The resources will be reallocated, after choosing the new government's party. If party 2 is chosen, the resources will be reallocated based on its consumption preferences. An agent who is not in office will always receive resources less than or equal to the government's party; based on the level of disagreement. The consumption preferences and the utility function of two political parties are the follows:

$$u(C_i) = \theta \left(\frac{C_i^{(1-\sigma)}}{1-\sigma} \right) + (1-\theta) \left(\frac{C_j^{(1-\sigma)}}{1-\sigma} \right) \tag{2.2.3a}$$

where θ is between 0.5 and 1. In the above equation, the government is an agent i, while j is another political party. When the party i has power, the party j will only receive resources based on what the party i leave. In this way, the allocation of consumptions for both parties depend on the disagreement level θ. If the disagreement rate between two political parties is very high, the resources will reallocate to C_i only. Thus, the budget constraint with two political parties can be written as:

$$C_1 + C_2 = y + B - q(B', y)B'. \tag{2.2.3b}$$

In this equation, C_1 and C_2 are consumption allocation for party 1 and party 2 respectively. Next, the government's decision function still follows the same path as with Eaton and Gersovitz (1981) and Arellano (2008). The decision is based on the value functions between repayment and default. The country will decide to default, if the expected outcome of the defaulting function is higher than repayment. Therefore, the government's decision function can be shown as:

$$V_0(B, y) = max \left\{ v^c(B, y), V^d(y) \right\} \tag{2.2.3c}$$

In this theoretical framework, the value functions of default and repayment can be varied, due to the decisions of two political parties in the periods of transition. Each decision will be made in order to maximise the utility of the government's party. When the government's party make a decision, it will have an effect on the following period. The allocation of resources and decisions on default and repayment will be decided again, after the new government is chosen.

Model Calibration

The model calibration is mainly based on 74 quarterly datasets of Argentina between 1983 and 2001. Parameters are calibrated to follow the characteristics of the Argentine economy before default at the end of 2001. In addition, the value of important parameters in the model are political risk (π) at 0.90 and level of disagreement (θ) at 0.62. At 90% of political risk, each party is relatively certain to be replaced by another party in the following period. Moreover, the disagreement rate at 0.62 means each party will allocate most of the resources to itself. The result is consistent with the historical characteristics of Argentina at that time.

Model Result

The outcome shows a negative relationship between political risk and credit spread. Countries with high political risk will be charged higher risk-premiums. The authors explained that if the government's party knows it will be replaced by another agent, it will try to consume more and save less today. For instance, Argentina with 90% of political risk frequently changed the government's party. The current party in office would think that the new government's party will not reallocate resources for its preferences. Hence, high political risk will lead to an impatient fiscal policy. They will borrow and consume regardless of the future. The impatient policies from high political risk will lead to an increase in the future default probability and risk- premiums. In the same manner, a stronger the level of disagreement between political agents also raises the credit spread. The current government will act to reallocate the resources to its preferences only. They will behave regardless of the future outcome.

2.2.4 Sovereign Default and the Firm's Decision Problem

Mendoza and Yue (2012), added an allocation of the firm's resources. The main novelty in this paper is adding labour supply, working capital, domestic inputs and foreign inputs into the productivity function and the sovereign default framework. They assumed that households will simultaneously consume and supply labour. Labour will work for firms to produce products. Firms need labour, working capital, domestic inputs and foreign inputs in order to produce the final products. If there is a significant change in interest rates, it will change the allocation of above resources. For example, an increase in interest rates decreases working capital required by firms. It can lead to a change in the amount of labour and inputs. Therefore, the key in this model is the default probability that significantly impacts on the factor allocation. Import inputs are sold in international markets and may be purchased by working capital outside the country only. During debt exclusion, firms still have to import inputs, but the financing cost of importing inputs is very costly when compared with domestic inputs. Thereby, domestic firms have to reallocate the resources. The country will lose firms efficiency and decrease the future output. In this way, it can imply that the cost of default tends to be higher than the previous literature, because the domestic firms will not be able to compete with foreign competitors. The model is designed to follow this theoretical framework as the follows.

The Model

Households in this model have a role to consume and supply labour to firms. Hence, consumption's equation and household's utility functions are different from the previous literature, as follows:

$$c_t = w_t L_t + \pi_t^f + \pi_t^m + T_t, \tag{2.2.4a}$$

$$u(c) = \frac{\left(c - \frac{L^\omega}{\omega}\right)^{1-\sigma} - 1}{1-\sigma}. \tag{2.2.4b}$$

From the above consumption's equation, households will receive profits (π) paid by firms from a sector f of final products and a sector m of intermediate products. They also receive the given wage rate w_t and government transfers T_t. In addition, the author used the Cobb-Douglas production function as follow:

$$y_t = \varepsilon_t \left(M\left(m_t^d, m_t^*\right)\right)^{\alpha_M} \left(L_t^f\right)^{\alpha_L} k^{\alpha_k}, \tag{2.2.4c}$$

where $0 < \alpha_L, \alpha_M, \alpha_k < 1$ and $\alpha_L + \alpha_M + \alpha_k = 1$. An output shock is denoted by ε. Furthermore, the overall budget constraints associated with debt can be shown as:

$$c = y + b_t - q_t(b_{t+1}, \varepsilon_t). \tag{2.2.4d}$$

At end of each period, the government will evaluate the output shock and choose a new amount of bonds that optimise the utility function through the budget constraint. The decision function will be based on the new amount of bonds chosen by the government and the output shock as follow:

$$V(b_t, \varepsilon) = max\left\{V^{nd}(b_t, \varepsilon_t), v^d(\varepsilon_t)\right\}. \tag{2.2.4e}$$

The government's decision function follows the previous literature. Default will be favourable, if the expected outcome from default is larger than those of repayment.

Model Calibration

This paper used quarterly data from Argentina between 1980 and 2005 with seasonal adjustment. However, with the limitation of observing data, some parameters are calibrated from annual data. For instance, the average ratio of intermediate goods is calibrated from annual data between 1993 and 2005.

Model Result

The result is consistent with the other literature. It can present a negative relationship between default risk and firm efficiency. In the theoretical framework, an increase in default risk will lead to a higher financing cost for imported inputs and significantly effect the factor allocation. Firms during periods of high default risk or debt exclusion will decrease their efficiency.

2.2.5 Sovereign Default and Investor's Risk-Aversion

Lizarazo (2013) studied an endogenous interaction between default risk and international investor's risk-aversion. The author explained that the future default probability is determined by the country's fundamentals and investors beliefs. In the international credit markets, investors will observe the market information and purchase the government bonds based on the expected default risk. Besides, different investor's preferences will demand different risk-premiums. An increase in risk-aversion will require an excess risk-premium and impact on the government's allocation of resources. From this assumption, the author provided a theoretical framework that include investor's preferences as in the following model:

The Model

The utility function and budget constraint follow the previous literature as follows:

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}, \tag{2.2.5a}$$

$$c = y - q(b', y, W)b' + b. \tag{2.2.5b}$$

From the above budget constraint, the bond's price q will depend on new amount of debt b', output y and observing wealth of international investors W. Besides, the government's decision function can be shown as:

$$V(b, y, W) = \max\left\{V^R(b, y, W), V^D(b, y, w)\right\}. \tag{2.2.5c}$$

The government's decision will follow the previous literature. The default is more preferable if the expected outcome of default is larger than repayment. Next, the utility function of international investors is presented as:

$$v(c^L) = \frac{\left(c^L\right)^{1-\gamma^L}}{1-\gamma^L}, \tag{2.2.5d}$$

where γ^L refers to the risk-aversion of international investors. The author assumes that investors will hold two types of assets in the portfolio. First are treasury bills which provide a return equal to a risk-free rate. Second are bonds of emerging countries that have a positive risk-premium. They will purchase these two assets based on the budget constraint and risk-aversion. Asset's allocation of investor's portfolio can be defined as:

$$W = \vartheta^{TB} + \vartheta, \tag{2.2.5e}$$

where ϑ^{TB} is asset position in T-Bills and ϑ is bonds of emerging economy. In addition, investor's consumption functions reflected from the sovereign repayment and default are as the follow:

$$c^{L,ndef} = X + W - q^f \vartheta^{TB'} - q\vartheta', \tag{2.2.5f}$$

$$c^{L,def} = X + W - q^f \vartheta^{TB'}, \tag{2.2.5g}$$

where X represents other income that is not from T-Bills and emerging bonds. From these above equations, investors' budget constraints will have an impact on the bonds of emerging countries. From solving these conditions, prices of bonds of emerging countries can be determined by two components as follow:

$$q = \zeta^{RA} + q^{RN}. \tag{2.2.5h}$$

First component ζ^{RA} is an excess risk-premium corresponding with investor's risk-aversion. Second component q^{RN} is default risk. Therefore, higher risk-aversion of international investors will induce risk-premium in bonds of emerging countries.

Model Calibration

The model calibration is based on quarterly data of Argentina between 1983 and 2001. This paper approached the effect of the risk-aversion from international investors on Argentine default in 2001. It presented the sensitivity analysis by setting the international investors' risk-aversion at 0, 0.5, 1, 2 and 5. In addition, investor's income from other sources (X) is estimated by using the U.S. stock index, as a proxy between 1983 and 2001.

Model Result

The results presented a framework with a positive relationship between risk-aversion rate and bond risk-premium. An increase in risk-aversion will induce risk-premium of bonds. Moreover, the model also novelly defined the investors' budget constraints; which is absent in the previous literature. It presented an optimal proportion of assets in the investors' portfolios. The result also matches with the Argentine default and U.S. stock index at the same time.

2.2.6 Sovereign Default and Fiscal Policy

Cuadra et al. (2010), provided a theoretical framework of sovereign default with tax rates and government spending. The author assumed that the government has a role to use fiscal policy, in order to optimise social welfare. They studied the endogenous relationship between fiscal policy and default risk. Notably, tax rates and government spending in this framework are pro-cyclical to the business cycle. The government is expected to decrease spending and increase tax rate during bad times, while raise spending and lower tax rates during good times. This pro-cyclical fiscal policy is consistent with theoretical frameworks provided by Bi (2012), Bi and Traum (2012) and Liu and Shen (2016). Arellano and Bai (2014), suggested austerity fiscal policy to relax the budget constraint. They explained that during periods of high default risk, the cost of international borrowing is costly compared to a rise in tax rates. Therefore, the framework of Cuadra et al. (2010) will novelly include two main variables; government spending and tax rates as in the following model.

The Model

In the model, production function will include a variable of labour as follow:

$$y_t = A_t F(l_t), \tag{2.2.6a}$$

where A_t is the productivity factor and l is labour. Next, household's utility function can be presented as the following:

$$U(C, G, 1-l) = \pi \left(\frac{G^{1-\sigma}}{1-\sigma} \right) + (1-\pi) \left(\frac{\left(C - \frac{l^{1+\psi}}{1+\psi} \right)^{1-\sigma}}{1-\sigma} \right), \tag{2.2.6b}$$

where labour elasticity is $1/\psi$ and the government's proportion in the utility function is π. The authors use the first order condition of households' functions with budget constraint, in order to solve the optimal function of labour supply and private consumption as the following shows:

$$l^* = \left(\frac{A}{1+T} \right)^{1/\psi}, \tag{2.2.6c}$$

$$C^* = \frac{Al^*}{1+T}, \tag{2.2.6d}$$

where tax rate is defined by T and private consumption by C. Furthermore, the government spending which associates with tax and international borrowing can be specified as follows:

$$G = TC^* + B - q(B', A)B', \tag{2.2.6e}$$

where G determines the government's budget constraint or public expenditure. From these above equations, the main utility function under optimal consumption and labour is as follows:

$$U(C^*, G, 1 - l^*) = \pi \frac{\left(C^* - \frac{(l^*)^{1+\psi}}{1+\psi}\right)^{1-\sigma}}{1-\sigma} + (1-\pi)\frac{G^{1-\sigma}}{1-\sigma}. \tag{2.2.6f}$$

The government will optimise the utility function by choosing new borrowing amount, public expenditure and tax rate as the follows:

$$V_o(B, A) = \max\{V^c(B, A), V^d(A)\} \tag{2.2.6g}$$

$$V^c(B, A) = \max_{T,G,B} \left\{ U(C^*, G, 1 - l^*) + \beta \sum_{A'} V_o(B', A') Q\left(\frac{A'}{A}\right) \right\}, \tag{2.2.6h}$$

$$V^d(A) = \max_{T_d,G_d} \left\{ U(C_d^*, G_d, 1 - l^*) + \beta \sum_{A'} \left(\mu V_o(0, A') + (1-\mu) V^d(A') Q\left(\frac{A'}{A}\right) \right) \right\}. \tag{2.2.6i}$$

From the above equations, bond is only available in the repayment's period. The government's decision is also based on expected utility between choosing default and repayment. In each function, the government will optimise through tax and government spending.

Model Calibration

The model is calibrated from quarterly data in Mexico between 1980 and 2007 with seasonal adjustment. Notably, the labour elasticity $\left(\frac{1}{\psi}\right)$ is exogenously calibrated at 2.22. The proportion of the government spending (pi) is 0.30. The probability of reentry (μ) is relatively lower than Argentina at 0.10.

Model Result

The result from Cuadra et al. (2010) is consist with the frameworks of Bi (2012) and

Liu and Shen (2016). They provided a theoretical model of sovereign default with the pro-cyclical behaviour of fiscal policy. The government is suggested to increase taxes and lower government spending during periods of high default risk. Changing taxes and government spending are less costly compared with international financing.

2.2.7 Sovereign Default and Debt Renegotiation

Benjamin and Wright (2009), Yue (2010) and Bai and Zhang (2012) presented theoretical frameworks for sovereign default and debt renegotiation. The main novelty of these literature is a game theory between a sovereign borrower and lenders during periods of high default risk and debt exclusion. Benjamin and Wright (2009), provided a stylised fact of default's duration and haircuts. The length of debt exclusion has a positive relationship with a ratio of debt to GDP. The average duration was around eight years in the sample of 73 countries between 1989 and 2006. Moreover, there is evidence to support a positive relationship between debt's duration and haircuts. If sovereign borrowers spend longer in debt renegotiation during default, the rate of debt reduction will be increased. Additionally, creditors had an average loss from sovereign default at 44% in the sample. From these stylised facts, Yue (2010), provided a model with an endogenous rate of debt recovery from periods of post-default to repayment's decision. On the other hand, Bai and Zhang (2012), studied debt renegotiation with a specification of bargaining power between debtors and creditors during debt renegotiation only. Hence, this section will be based on the model of Yue (2010), in order to give an overview of debt renegotiation from the government's decision function before default.

The Model

In the model, households preference follows the previous literature as follow:

$$u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma}. \tag{2.2.7a}$$

Next, the government's decision can be divided into two separate functions for a good credit record and a bad credit record as the following shows:

$$v(b,0,y) = \max v^r(b,0,y), v^d(b,0,y), \tag{2.2.7b}$$

$$v(b,1,y) = \max_{c,b' \in [b,0]: c + \frac{b'}{1+r} = (1-\lambda)y + b} u(c) + \beta \int_y v(b',1,y') d\mu(y'|y). \tag{2.2.7c}$$

where v(b,0,y) is the government's decision with a good credit record, whilst v(b,1,y) is a decision on a record of previous default. *lambda* is a direct cost in output from choosing default. When the debt is repaid, the sovereign borrower can reentry to international credit market with the value function as the follow:

$$v(0,1,y) = v(0,0,y). \tag{2.2.7d}$$

In the previous literature, the probability of default is exogenous without a rate of debt reduction. Oppositely, this paper will provide a bargaining power or surplus of debtors and creditors between accepting and rejecting on the agreement of debt recovery rate. The surplus of borrowers and lenders are as the followings:

$$\triangle^B (a;b,y) = [u(y) + \beta \int_y v(ab,1,y') d\mu(y'|y)] - v^{aut}(y), \tag{2.2.7e}$$

$$\triangle^{L}(a;b,y) = -\frac{ab}{1+r}. \tag{2.2.7f}$$

where $\triangle^{B}(a;b,y)$ is the value function of borrower's surplus and, $\triangle^{L}(a;b,y)$ is the value function of lender's surplus during debt renegotiation. The debt recovery rate can be defined due to the bargaining of the two agents as follows:

$$\alpha(b,y) = arg \max_{a\in[0,1]} \left[(\triangle^{B}(a;b,y))^{\theta}(\triangle^{L}(a;b,y))^{1-\theta}\right] \tag{2.2.7g}$$

where θ is a bargaining power of the borrower and, $1-\theta$ is the lender's bargaining power. From the bargaining power between two agents and the set of debt recovery rate, the optimal recovery rate in the following period can be presented as:

$$\gamma^{*}(b',y) = \frac{\int_{D^{*}(b')} \frac{\alpha^{*}(b',y')}{1+r} d\mu(y'|y)}{p^{*}(b',y)} \tag{2.2.7h}$$

From the above equation, the expected recovery rate is defined from the new amount of debt chosen by the government and the output shock. Importantly, the value of recovery rate can be plugged into the government's decision function. As a result, the government will consider the debt recovery rate before deciding to default or repay.

Model Calibration

The parameters were calibrated by using quarterly data from Argentina between 1980 and 2003. Notably, the bargaining power (θ) and time discount factor(β) are estimated as 0.72 and 0.72 respectively, for targeting default frequency and debt recovery rates at 2.78% and 27% consecutively. Moreover, the direct cost of default (λ) is 2%.

Model Result

The result shows a positive relationship between debt reduction and the initial amount of debt. The debtor countries with small amounts of debt will experience low debt reduction during renegotiation's periods. Debtor countries are more favourable to default, if the debt recovery is low. Moreover, comparing between good and bad states, default during bad states tend to have higher debt reductions. This framework helps to understand the debt renegotiation for the Argentine default in 2001.

2.2.8 Sovereign Default with Bailouts

Aguiar and Gopinath (2006), studied sovereign default and bailouts. Notably, they used an excess of assets instead of total international bonds. The authors demonstrated that if the amount of assets exceed the current debt the country will have a positive net wealth with zero default risk. On the opposite, if the net wealth is negative, the country will have a positive default risk from holding debts higher than assets. Through using the net wealth instead of total bonds, the model did not account the different characteristics between assets and debts. The authors assumed that there is no difference between the return on assets and debts. Besides, the depreciation from holding assets in this model is not taken into consideration. Furthermore, the researchers added bailouts from the third party (IMF). They assumed that the third party will inject money to rescue the debtor countries without any condition. In the model, bailouts will directly grant to lenders and decrease risk premium of bond from lower default risk. This framework can be a seminal study for using bailouts to rescue sovereign borrowers from default. The model will be designed under the above assumptions as the following section shows.

The Model

The utility function follow the previous literature as:

$$u = \frac{c^{1-\gamma}}{1-\gamma}. \tag{2.2.8a}$$

The budget constraint and the government decision function are presented as the following shows:

$$c_t = y_t + a_t - q_t a_{t+1}, \tag{2.2.8b}$$

$$V(a_t, z_t, \Gamma_t) = \max(V_t^G, V_t^B), \tag{2.2.8c}$$

where a_t is assets, z_t is a transitory shock and T_t is a trend in output. A positive assets a_t means an excess of assets on top of international bonds. Besides, the government's decision will be based on the expected value of repayment (V_t^G) and default (V_t^B). The model can be used to calculate a change in the price of bonds from saving more assets in a region of positive net wealth as follows:

$$\hat{q}(\hat{a}_{t+1} + \triangle\, \hat{a}, z_t) = \frac{-1}{1+r^*} \sum_{\bar{z}(\hat{a}) \leq z_{t+1} \leq \bar{z}(\hat{a}+\triangle \hat{a})} \pi(z_{t+1}|z_t). \tag{2.2.8d}$$

Furthermore, the bailouts can be used to intervene in the price of bonds as follow:

$$\hat{q}_t = \frac{1}{1+r^*} \left\{ \min[1, \frac{\hat{a}^*}{\hat{a}}] + E_t\{1 - D_{t+1}\} \max[1 - \frac{\hat{a}^*}{\hat{a}}, 0] \right\}, \tag{2.2.8e}$$

where $\hat{a}^*$ represents bailouts from the third party. The bailouts will be granted to lenders with a risk-free rate. Debts will be repaid up to an amount of bailouts $\hat{a}^*$. An excess of debt above $\hat{a}^*$ will be accounted as a creditor's loss. Through this process, the bailouts will help to decrease risk-premium because the interest rate of bailouts is significantly below the international borrowing rate during periods of high default risk.

Model Calibration

The paper used quarterly observations of the Argentine economy between 1983 and 2000. The parameters were estimated to match the statistics of default in 2001. The probability of reentry is only 10%. It is relatively low compared with other literature. Importantly, the estimated average bailout limit is 18%.

Model Result

The results can present an interaction between bailouts and a bond's risk-premium. The bailouts from the third party (IMF) can rescue the debtor countries through decreasing the bond's risk-premium. In the model, bailouts are granted directly to lenders with risk-free rate. As a result, the amount of debt can be repaid up to the limit of bailouts without any bad condition. It will rescue sovereign countries and allow them to borrow more for smoothing consumption. This framework is consistent with the historical data during defaulting periods. For instance, Argentina in 2001 received a bailout from the IMF at 15% of its GDP. In addition, this model can be a seminal framework for the future study of bailouts during repayment. There can be a possibility that countries ask for the bailouts before the defaulting decision, if the value function from repayment with bailout is higher than choosing default.

2.2.9 Sovereign Default with Physical Capital

Bai and Zhang (2010), Bai and Zhang (2011), Gordon and Guerrón-Quintana (2013), Liu and Shen (2016) and Park (2017), provided theoretical frameworks of sovereign default by including physical capital in the model. In these papers, physical capital was assumed to be an optional assets as well as bonds. The government could choose to invest through two types of assets; bonds and physical capital. The total amount of bonds can be negative (borrowing), whilst the accumulated physical capital must be positive. If the government invests more in physical capital, it can enhance output through the production function. However, physical capital will be depreciated every year. Furthermore, the literature assumes that the borrowing rate is related to capital investment. In the creditor's view, debtor countries with high assets have more capability to payback their debts. Liu and Shen (2016), presented a positive relationship between capital investment and bond prices. Higher accumulated capital will decrease risk-premium and raise bond prices. However, Park (2017), indicated a problem of using two correlated assets in the model. For instance, if the government holds high physical capital, it allows them to borrow more from international credit markets with lower risk-premiums. As a result, the government invests more and borrows more. During the period of negative output shock, the government can sell physical capital for debt repayment. Nevertheless, without a variable of capital adjustment, the model is more volatile than the actual data and other literature. In the model, lenders are assumed to be rational, with perfect information. They will always give the risk-premium based on the borrower's optimal decision only. In this way, they leave an unsolved issue for the probability distribution of borrower's decisions. The models exclude the probability of the government's decision that can be diverted from the optimal level. This section will review the seminal model of Bai and Zhang (2011) for an overview of sovereign default with physical capital.

The Model

The utility function follows the standard CRRA form as the following shows:

$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}. \tag{2.2.9a}$$

Next, the author assumed the production function follows the standard Cobb-Douglas which can be written as follows:

$$Y = aK^{\alpha}L^{1-\alpha}, \tag{2.2.9b}$$

where a is the idiosyncratic shock and K is the capital input. Hence, the production function can be rearranged to present the output driven by capital per labour as:

$$f(k) = ak^{\alpha}. \tag{2.2.9c}$$

The key model of this paper is the function of budget constraint that is additionally driven by capital investment as follows:

$$c = ak^{\alpha} + b - q(a, k', b')b' - \tau|b'| - k' + (1 - \delta)k - \Phi(k', k), \tag{2.2.9d}$$

where τ is the real resource cost to enter international credit markets, Φ is the capital adjustment and δ is capital depreciation. In this way, the government has to choose an amount of bond and capital in order to maximise consumption. Consequently, the government's decision function is given by:

$$V(s, N) = \max \left\{W^R(s), W^D(a, k)\right\}, \tag{2.2.9e}$$

where s is a set of (a, k, b). From the above equation, the default will be more favourable, if the expected outcome of choosing default is larger than repayment. Next, the bond price and default probability are presented as follows:

$$q(a, k', b') = \frac{[1 - p(a, k', b')]}{R}, \tag{2.2.9f}$$

$$p(a, k', b') = \sum_{a'|a} \pi(a'|a) d(a', k', b'), \tag{2.2.9g}$$

where bond price is denoted by q and default risk denoted by p. In this way, default probability and bond risk-premium are driven by an amount of bond and capital. The government will optimise utility through choosing bond and capital. Hence, the default threshold is calculated based on the government's optimal decision.

Model Calibration

This research used data from 21 OECD countries and 22 emerging countries. The data was separated into two periods; 1970 - 1986 and 1987 - 2004. The parameters were separately estimated to match the data with two specific periods. The authors expected a significant difference in the level of global financial integration. In the extension, the model includes a risk-sharing coefficient; which is calibrated from those two periods.

Model Result

The result can present an interaction among capital investment, bond and default probability. The author demonstrated that an increase in capital investment will reduce the bond's risk-premium. It can enhance the capability of countries to borrow more. However, the model needs to include an accurate capital investment (Φ) and the real resource cost to enter international credit market (τ). Notably, without these two parameters, the result cannot fit with the actual data. In the extension, the model can be used to specify the international risk sharing between countries. The result shows a low rate of financial integration. In addition, there is only little evidence to support an increase in international risk-sharing over time.

2.3 Empirical Literature on Sovereign Default

This section draws a big picture of sovereign default through an extensive number of empirical literature and the stylised facts. The starting point for sovereign default is ambiguous. There are a lot of causes that can lead to the default decision. In the past, countries frequently experienced default and debt restructuring. After default, they would be temporarily excluded from international credit market. Some countries spent over 50 years of debt exclusion such as Russia in 1918, Greece in 1826 and Honduras in 1873. Surprisingly, countries in Latin America spend over 40% of their years in debt restructuring (Reinhart and Rogoff, 2008). The default countries can regain access to the international credit markets after completing debt renegotiation with lenders. From this view, there are several reasons and causes that make the government decide to default and spend a lot of time in debt restructuring. This section will explore empirical results from several literature on the government's decision to default.

2.3.1 Sovereign Default and Crises

Reinhart and Rogoff (2011), provided an empirical study on sovereign default and crises. From the stylised facts, sovereign default frequently occurred during periods of high inflation, high public debt and a large number of private bankruptcies in the financial sector. In order to investigate the relationship between default risk and crises the author separated the crises into three main types; inflation, public debt and banking crises. The results can show a significant relationship between banking crises and default probability. Hence, this section will review this empirical result and stylised facts from Reinhart and Rogoff (2008) and Reinhart and Rogoff (2011).

Reinhart and Rogoff (2008), provided an overall statistic of sovereign default from 70 countries across, six regions, over the last eight centuries. The GDP of these 70 countries can represent over 90 percent of world GDP in the specified periods. There is a stylised fact that suggest that all countries defaulted at least once during the emerging phase (pre-industrialised) of their economies. For instance, England in 1340 is the first country that officially declared sovereign default on its external debt. Besides, Spain, France, Portugal, Germany (Prussia) initially declared default on its external debt in 1557, 1558, 1560 and 1683 respectively. They frequently experienced sovereign default several times during the transition periods. Furthermore, the second stylised fact is that the average length of debt restructuring during the nineteenth century is six years, whilst it is reduced to three years after the World War II. The current development of international credit markets significantly reduces the sovereign default's duration. The author demonstrated that this change may be associated with an intervention by government and the third-party institutions; particularly by the World Bank and the International Monetary Fund (IMF). Bailouts from third parties can rescue debtor countries, but they frequently ask for the austerity fiscal policies and the acceptance of haircut rates. Moreover, the author presented a significant difference between Latin American and Asian countries. Countries in Latin America spent more than 40% of their years in default, whilst Asian countries spent only 10%. Countries in Latin America had an average frequency of sovereign default at seven times, whilst Asian countries faced few times. Next, the author found that an occurrence of sovereign default tended to be clustered around the periods of world crises. For example, countries representing 40% of the world GDP struggled with debt restructuring during the Great Depression. From this fact, the author provided a study of the relationship between sovereign default and crises.

Next, Reinhart and Rogoff (2011), separated crises into three main categories. The first type was defined by inflation. The starting point of financial collapses could have emerged from the hyperinflation and large changes in currency rates. A high fluctuation in inflation and exchange rates have a direct impact on both internal and external debt contracts. The price of bonds and their risk premiums are significantly determined by expected inflation and forward exchange rates between countries. From hyperinflation and a rapid depreciation in currency, debtor countries will face a difficulty to roll over and honour their obligations. Moreover, the bond prices and credit spreads are more complex and sensitive compared to the past. For instance, the sovereign default clustering in the 1930s and the 1980s have a high correlation with inflation and currency crises compared to the nineteenth century. The author explain that higher financial integration and risk-sharing behaviour in the modern economy tend to be one of explanations towards a difference in this correlation.

The second type of crisis is classified by banking problems. The authors provided an indicator (proxy) of the banking crises by the number of public acquisitions and amount of the government support in the financial sector. During the banking crises, there were many firms acquired or funded by the government. If this indicator starts to increase, it suggests the beginning of a banking crisis. Throughout history, banking problems generally arise from a sharp fall in asset quality. A destruction of property markets and bankruptcies of non-financial firms can increase the default risk of banks from holding mortgages or collaterals. A rapid increase in nonperforming loans and scandals in the financial markets can lead to banking panic. In the past, several banks collapsed from panics because of a dramatic change in asset (debt) composition. A collapse of firms in financial sector can spill over to non-financial firms, and a whole economy, through a sharp increase in financing costs. The financial sector is the core of the economy because it provides a care of capital to other sectors. A collapse of the financial sector frequently causes the economy to go into recession. Therefore, the government has to support the financial sector through funding. For instance, in the 1930s, the banking crisis caused sovereign defaults in many countries. Debts in the private sectors are transferred to the government's balance sheet. A destruction of the financial sector will significantly change the government's asset composition and increase default probability. The author found significant evidence to support a positive relationship between sovereign defaults and banking crises.

The third type is indicated by the overall level of public debt; both internal and external. Between 1800 and 2007, there were at least 250 events of sovereign default on external debts, and at least 70 events on domestic debts (Reinhart and Rogoff, 2008). Basically, sovereign default will not occur without a substantial level of debts. The overall level of debt can be an indicator to default risk. For example, 27 developing countries experienced sovereign defaults with 70.6% of debt to GNP between 1824 and 1999 (Reinhart et al., 2003). However, it may not be so straightforward. In the past, sovereign default occurred during high and low external debt ratios, depending on the government's decision. For example, Russia in 1991, Turkey in 1978, Chile in 1972 and The Dominican Republic in 1982 chose to default with the debt ratios of 12.5%, 21.0%, 31.1% and 31.8% respectively. On the contrary, Guyana in 1982, Jordan in 1989, Costa Rica in 1981 and Egypt in 1984 defaulted with external debt to GNP ratios of 214.3%, 179.5%, 136.9% and 112.0% respectively. Moreover, Reinhart and Rogoff (2011), presented an insignificant direct relationship between default probability and public debt crises. The author explained that there had been a change in a safe thresholds of external debt over the past century. It can cause a difficulty to investigate the linear relationship between default risk and public debt over centuries. For example, presently, many countries can hold higher debt ratios with lower bond risk-premiums than in the past.

2.3.2 Sovereign Default and Output

From the stylised facts, sovereign defaults frequently occurred during bad times (Yeyati and Panizza, 2011). There is a lot of literature investigating a change in economic variables around an event of default. Interestingly, output is one of the main variables that significantly drops in the defaulting period and quickly recovers in the following period. This section will provide a review of empirical studies for a relationship between sovereign default and output.

Tomz and Wright (2007), used the dataset of 175 countries between 1820 and 2004. In the sample, the total number of sovereign default was 250 times from 106 countries. Since the Napoleonic wars, new sovereign default happened every decade. The statistics are consistent with the dataset of Reinhart and Rogoff (2011), where sovereign default clustered around the financial crises. For instance, in the 1980s, 50 debtor countries failed to hold their debt obligations. Moreover, many sovereign defaults correlated with large drops in output. For example, the Argentine defaults of 1980, 1982 and 2001. However, there are a few countries that decided to default, even if output still increase such as, Chile in 1826 and 1880. Overall, the author found that debtor countries will experience an average drop of output at 1.6% below the trend[4] in the first year of default. In addition, this relationship becomes stronger (falls more than 7% below the trend), if the debtor countries default during the downtrend (severe crises). This outcome is consistent with the parameters of direct cost from sovereign default in the literature (in the theoretical section). Moreover, there were 61.5% of total countries that decided to default when the output is below the trend. This statistical evidence can support a significant relationship between default and output drop. Additionally, from these results, the author suggests that sovereign default and output are more likely to have a negative and non-linear relationships.

Furthermore, Yeyati and Panizza (2011), studied by using both annual and quarterly data of 39 countries between 1970 and 2005. Results from both annual and quarterly data provided a significantly negative relationship between sovereign default and output. In detail, the statistical evidence from annual data suggested that the

[4]The trend is defined by an average growth rate of the past GDP (Tomz and Wright, 2007)

output significantly drop one year before the default ($T-1$) and in the default's period (T) at 3% respectively. Then, it keeps decreasing in the following year of default ($T+1$) at 2%. Moreover, if the sovereign default occurred during the banking crisis, the output will drop more than 1% compared to defaults in other periods. On the other side, the result from using quarterly data also shows a significantly negative relationship between default and output. The output significantly decreases three quarters before default and in the default's period from 0% to 3%; the highest drop is a period before default. Surprisingly, the quarterly results suggested that output significantly increase after default for the following three quarters. From this result, it suggested that an official announcement of sovereign default can be a signal for the recovery in the following periods. The debtor countries can use default as an optional strategy to maximise their welfare.

In addition, Neumeyer and Perri (2005), provided a study among output, default risk and interest rates. The author used quarterly data from five emerging and five developed countries between 1983 and 2001. The author found evidence for an endogenous relationship between output and default risk. The output in emerging countries is significantly effected by a change in country risk. Unsurprisingly, output in developed countries is less volatile than emerging economies. Moreover, the output volatility can be reduced by 27%, if default risk is eliminated.

To conclude, the empirical results suggest a negative relationship between sovereign default and output. The government frequently decides to default during bad times. If the debtor countries default around the financial periods, the direct cost of default is always larger than in other periods. However, the authors suggest that its relationship is non-linear because default often occurs during good times.

2.3.3 Sovereign Default, Trade and Sanctions

This section will review the previous literature on the relationship between sovereign default and international trade. Rose (2005), argued that default countries can be punished by creditors through sanction of international trade. Sovereign default will negatively impact on bilateral trade between debtors and creditors. The literature also found that debt renegotiation is significantly related to international trade. On the opposite side, Martinez and Sandleris (2011), disagreed with the punishment of creditor's sanction in international trade by using the same sample of bilateral trades. Hence, this section is mainly based on the empirical evidence from these two studies.

From the article by Rose (2005), the author used the dataset from the International Monetary Fund (IMF) for the bilateral trades. All samples were under the Paris Club[5] for 217 trading entities[6] between 1948 and 1997. The empirical results showed a negative and significant relationship between sovereign default and international trade. Moreover, the author found that the international trade kept decreasing about 8% annually during periods of debt renegotiation. A fall in the bilateral trade (only creditor partners) can have a persistent effect over the following 15 years from default. In addition, debt renegotiation have a negative impact on exports higher than imports. Besides, there is no evidence to support the negative diversion from the bilateral trade to other trading partners (outside the Paris Club).

[5]The Paris Club is unofficially formed by a group of creditors for the purpose of debt renegotiation. It will be used to ensure the treatment of all creditors in the group. Details of the Paris Club is published by Sevigny (1990) and Eichengreen and Portes (1995)

[6]Some trading partners are not countries. For example, Guam, Bermuda and Guinea-Bissau are territory, colony and independent state respectively

On the opposite side, Martinez and Sandleris (2011), used the same dataset of the bilateral trades for 217 trading partners between 1948 and 1997. Other variables such as population and GDP were provided by the World Bank. The empirical results showed insignificant relationships between sovereign default and bilateral trade. The volume of bilateral trade between debtors and creditors is not significantly changed during debt renegotiation. This outcome goes against the the argument from Rose (2005). Besides, the results ensure a negative and significant relationship between sovereign default and general trade; including other trade partners. If debtor countries decide to default, the general trade volume will be negatively affected by 12.25% during that period (T). The author also found that the general trade volume for the following five years (after default) significantly decreased between 1% and 7% per year (on average 3.2%). The relationship between sovereign default and the general trade became weaker from six to fifteen years after default. This outcome is consistent with the empirical evidence from Borensztein and Panizza (2010) that sovereign default has a significant and negative effect on exports, but it is not persistent. In this way, there is no evidence to support the sanction from creditor countries for sovereign default. The author explained that the fall in the general trade may be caused by a reputation loss in the credit markets. A fall in economic fundamentals from default tends to reduce efficiency and output that are associated with the trade volume.

In conclusion, the above empirical literature ensures a significantly negative relationship between sovereign default and trade volume. It suggests that the government's decision should take a fall in international trade into consideration as an additional cost of default. The trade volume in the defaulting period will be negatively effected around 8% to 12%. The negative impact from default can be persistent up to the following fifteen years; comparing with normal phases.

2.3.4 The Cost of Sovereign Default on Bond Spread

From the above theories, bond price is mainly determined by the default risk and the real interest rate. Higher bond spreads will raise the government's incentive to default. Borrowers cannot always roll over their debt under the incomplete credit markets. The government will face difficulty to issue bonds during uncertain periods (high default risk). In this way, the risk-premium of bonds is mainly driven by the future default probability. Thus, debtor countries frequently default when the bond spread reaches its peak (Uribe and Yue, 2006). Alternatively, the past record of sovereign default is found to be another determinant of the current bond spread (Borensztein and Panizza, 2009). The credit spread of countries with default records is significantly higher than countries without default records. Thereby, this section will review the empirical literature on the sovereign bond spread after default.

From the above theoretical part, bond price is mainly determined by the default risk and the real interest rate. Higher bond spread will raise the government's incentive on default. Borrowers cannot always roll over their debt under the incomplete credit market. The government will face a difficulty to issue bonds during uncertain periods (high default risk). In this way, the risk-premium of bonds is mainly driven by the future default probability. Thus, debtor countries frequently default when the bond spread reaches its peak (Uribe and Yue, 2006). On the other side, the past record of sovereign default is found to be another determinant of the current bond spread (Borensztein and Panizza, 2009). The credit spread of countries with default record is significantly higher than countries without default record. Thereby, this section will review the empirical literature on the sovereign bond spread after default.

Ozler (1992), provided an empirical study on a dataset of 70 developing countries between 1820 and 1981. The author divided time periods into four groups; 1820 - 1929, 1930 - 1955, 1956 - 1968 and 1969 - 1981. The empirical evidence showed a stronger effect of the defaulting decision on the sovereign bond spread (after re-enter) in the more recent years. For instance, the impact from choosing default is insignificant between 1820 and 1929, whilst it becomes significant with higher correlation coefficient (approximately 10%) after 1930s. This results can imply that debtor countries with bad records will pay higher bond spreads than countries with good records. The author explained that if the debtor countries choose to default, they will be penalised by losing their future reputations in the international credit markets. The bond spreads will be charged about 20% more (from the normal rate), if the standard deviation in the cost of default is increased by 1 unit. This outcome is consistent with Martinez and Sandleris (2011), where the default countries tended to be penalised by lower reputations. However, some countries may not care about their future reputations because they expect more difficulties for the future repayments. For example, delaying default may cause borrowers to repay more in the following periods. The borrowers may have costs higher than benefits from repayment.

Cantor and Packer (1996), Borensztein and Panizza (2009) and Panizza et al. (2009), provided empirical evidence on the costs of sovereign default to bond spreads. Borensztein and Panizza (2009), used data from 68 countries between 1800 and 2002 to find the relationship between default and credit ratings. The results presented a negative and significant impact of default on credit ratings. When debtor countries defaulted, the credit rating decreased by notches. This outcome was close to the decrease of credit rating at 2.5 notches Cantor and Packer (1996). Moreover, Borensztein and Panizza (2009), used dummy variables from 1 to 25 years after default to estimate the effect on credit spread. The first and second years after default were statistically significant with positive coefficients. The sovereign default raised credit spreads for the following two years at 4% and 2.5% respectively. Panizza et al. (2009), provided an empirical study for 32 countries between 1994 and 2008. The results from controlling the credit rating suggested that, credit spreads during the first three years after default ($T-1$ to $T-3$) were insignificant. Alternatively, the credit rating during the first three years after default was significant with small coefficients. These results can imply that the cost of sovereign default on credit spreads is significant and temporary. The effect from default on sovereign bond spreads will disappear after a few years.

To conclude, debtor countries frequently default when experiencing a large drop in output and high credit spread. The bond spread is countercyclical and reaches its peak before sovereign default occurs (Uribe and Yue, 2006; Neumeyer and Perri, 2005). After default, countries with default records will be penalised by charging an additional bond spread. Sovereign default will cause borrowers to lose reputations and have lower credit rating. However, the cost of sovereign default on credit spread is not persistent.

2.3.5 Sovereign Default and Public Debt

From the previous theoretical sections, the future default probability can be determined by an amount of public debt. The theoretical models are mainly based on the debt level. Higher debt will raise the future default probability. The government will have more incentive to default when debt and interest payments are higher. Hence, this section will examine the positive relationship between default risk and public debt through empirical literature.

Reinhart et al. (2003), provided an empirical study of sovereign default and debt. The author used a dataset across 62 countries including both developing and developed countries from 1979 to 2002. The results showed developing countries between 1824 and 1999 default with an average debt to GNP ratio of 71%. The higher debt ratio seemed to have higher default risk. However, it is not certain to compare debt ratios across countries. The majority of this sample defaulted with debt ratios lower than 60%. Some countries did not default even though they had debt ratios above 100%. For example, Russia in 1991 and Turkey in 1978 defaulted with debt ratios at 12.5% and 21% respectively. Guyana in 1982 and Jordan in 1989 defaulted with debt ratios at 214.3% and 179.5% respectively. The author explained that different countries may decide differently due to their fundamentals. Nevertheless, some countries with the same fundamentals may still deicide differently. Comparing between developed and developing countries with controlling debt ratios, developing countries were more likely to default. The author defines these diverse decisions as debt intolerance levels. Countries with the same fundamentals will choose to default, if they have higher level of debt intolerance. From this view, countries with high levels of debt intolerance will have much lower safe debt ratio to default. The author used the indicators of debt intolerance and debt ratio in order to find the pattern of default. The prediction from these two variables in the model is significantly predictable. They can show empirical evidence for safe debt thresholds for each countries. The overall of the safe debt thresholds can be varied due to individuals' debt intolerance level and debt ratio.

Reinhart and Rogoff (2008) and Reinhart (2010), provide empirical results that are consistent with the above literature. They used data from 70 countries between 1800 and 2009. Several borrowers defaulted when the external debt was significantly increased in the prior periods of default. However, there was evidences to support the default probability from debt ratios across countries. Each country seemed to have their unique safe ratio of debt. Broner et al. (2014), showed a stylised fact of debt ratio from developed countries in the recent decade. If the debt ratio exceeds the individual safe level of debt, credit spread will suddenly increase. Moreover, debt ratios may not be the best parameter for comparing default risk across countries, because some countries may have hidden debt (Reinhart and Rogoff, 2011).

Furthermore, Schaltegger and Weder (2015), studied the relationship among fiscal adjustment, debt composition and default risk. They used a dataset of 58 emerging countries between 1980 and 2009. The statistical evidence suggested that a temporary change in size of debt prior to default has no significant effect on the future default probability. Alternatively, a change in the debt composition can significantly decrease default risk. For instance, if the government decreases the ratio of short-term to overall debt by 1%, the default probability in the following period will be reduced by approximately 1.4%. Besides, fiscal adjustments from the government revenue base has a highly negative and significant relationship with the future default risk. If the government receives a 1% increase in revenue base (ratio of revenue to GDP), it will reduce the default risk by 48% in the following period. For other empirical results, the international reserves, real GDP growth and political ratio have a significant and negative impact on default risk.

To conclude, the safe debt thresholds are varied due to the individual debt intolerance and public debt ratios. Some countries can default with higher or lower debt ratios with the same fundamentals. Overall, sovereign default frequently occurs from holding high external short-term debt. The outcomes show a significant effect on default probability from a change in debt composition, government revenue, and international reserves.

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Appendix

This appendix provides the summary of theoretical equations from the previous literature under the topic of sovereign default. As can be seen in Table 2.1, the main equations for sovereign default include the functions of closed economy, open economy, utility, sovereign decision, repayment and default. Therefore, Table 2.1 will show an overall comparison and differences among these literature as a previous review in this chapter.

Table 2.1: A Table of Theoretical Models

Model	Closed Economy	Open Economy	Utility Function	Government Decision Function	Repayment Function	Default Function
Aguiar and Gopinath (2006)	$c = (1-\delta)y_t$	$c_t = y_t + a_t - q_t a_{t+1}$	$u = \frac{c^{1-\gamma}}{1-\gamma}$	$V(a_t, z_t, \Gamma_t) = \max(V_t^G, V_t^B)$	$V^G(a_t, z_t, \Gamma_t) = max_{c_t}\{u(c_t) + \beta E_t V(a_{t+1}, z_{t+1}, \Gamma_{t+1})\}$	$V^B(z_t, \Gamma_t) = u((1-\delta)y_t) + \lambda\beta E_t V(0, z_{t+1}, \Gamma_{t+1}) + (1-\lambda)\beta E_t V^B(z_{t+1}, \Gamma_{t+1})$
Arellano (2008)	$c = y^{def}$	$c = y + B - q(B', y)B'$	$u(c) = \frac{c^{1-\sigma}}{1-\sigma}$	$v^0(B, y) = \max_{\{c,d\}} \left\{v^c(B, y), v^d(y)\right\}$	$v^c(B, y) = \max_{(B')}\{u(y - q(B', y)B' + B) + \beta \int_{y'} v^0(B', y')f(y', y')dy'\}$	$v^d(y) = u(y^{def}) + \beta \int_{y'}[\theta v^0(0, y') + (1-\theta)\, v^d(y')]f(y', y)dy'$
Arellano and Bai (2014)	$c + g \leq f(\tilde{z}, l)$	$c + g \leq f(\tilde{z}, l) + q(z, b')b' - b$	$u(c, g, l) = \frac{(c - \frac{l^{1+\frac{1}{v}}}{1+\frac{1}{v}})^{1-\sigma-1}}{1-\sigma} + \frac{g^{1-\sigma}}{1-\sigma}$	$v(z, b, h = 0) = \max_{\{d \in \{0,1\}\}}\{(1-d)v^r(z, b, h) + dv^d(z, b, h)\}$	$v^r(z, b, h) = \max_{\{c,g,l,b'\}} u(c, g, l) + \beta Ev(z', b', h' = 0)$	$v^d(z, b, h) = \max_{\{c,g,l\}} u(c, g, l) + \beta Ev(z', b' = b, h' = 1)$
Bai and Zhang (2010)	$c(a^\tau) + k(a^\tau) - (1-\delta)k \cdot (a^{\tau-1}) \leq (1-\lambda)\, a_\tau k(a^{\tau-1})^\alpha$	$c(a^t) + k(a^t) - (1-\delta)k(a^{t-1}) + b(a^t) \leq a_t k(a^{t-1})^\alpha + R_t b(a^{t-1})$	$u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$	$\max\{W(a', k', b'), V^{AUT}(a', k')\}$	$W(a, k, b) = \max_{\{c,k',b'\}} u(c) + \beta \sum_{a'\|a} \pi(a'\|a)W(a', k', b')$	$V^{AUT}(a^{t+1}, k(a^t)) = \max_{\{c(a^\tau),k(a^\tau)\}} \sum_{\tau=t+1}^{\infty} \sum_{a^\tau} \beta^{\tau-t-1}\pi(a^\tau\|a^{t+1})u(c(a^\tau))$
Bai and Zhang (2011)	$c = (1-\gamma)ak^\alpha - k' + (1-\alpha)\,k - \Phi(k', k)$	$c = ak^\alpha + b - q(a, k', b')b' - \tau\|b'\| - k' + (1-\delta)\,k - \Phi(k', k)$	$u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$	$V(s, N) = \max\{W^R(s), W^D(a, k)\}$	$W^R(s) = max_{\{c,k',b'\}}u(c) + \beta \sum_{\{a'\|a\}} \pi(a'\|a)V(s', N)$	$W^D(a, k) = \max_{c,k'} u(c) + \beta \sum_{\{a'\|a\}} \pi(a'\|a)V(a', k', 0, P)$
Benjamin and Wright (2009)	$c = e^{def}(s)$	$c = e(s) + b - q(b', s)b')$	$u = \frac{c^{1-\gamma}}{1-\gamma}$	$V(b, s) = \max V^R(b, s), \tilde{V}^D(b, s)$	$V^R(b, s) = \max_{c,b'} u(c) + \beta \sum \pi(s'\|s)V(b', s')$	$V^D(t^*, s^{t^*}, (\tau, b')) = \sum \beta^r u(e^{def}(s_r)) + \beta^{t^*} \cdot \{u(e^{def}(s_{t^*}) - \tau) + \beta E[V(b', s_{t^*+1})\|s_t^*]\}$
Bi (2012)	$c_t = A_t(1 - L_t) - g_t$	$c_t = A_t\,(1-\tau_t) \cdot (1 - L_t) + z_t - b_t q_t + (1-\Delta_t)\, b_{t-1}$	$u(c) = \frac{c^{1-\sigma}}{1-\sigma}$	$B * (A_t, g_t, rs_t^z) = E_t \sum \beta^j \cdot \left\{\frac{u_c^{max}(A_{t+j}, g_{t+j}}{u_c^{max}(A_t, g_t)}\right\} \cdot (T^{max}(A_{t+j}, g_{t+j}) - g_{t+j} - z(rs_{t+j}^z, A_{t+j}))$	-	-

Model	Closed Economy	Open Economy	Utility Function	Government Decision Function	Repayment Function	Default Function
Bi and Traum (2012)	$c_t = A_t n_t - g_t$	$c_t = A_t n_t (1-\tau_t) + z_t - b_t q_t + b_t^d$	$u(c) = \frac{c^{1-\sigma}}{1-\sigma}$	$B* = E_t \sum \beta^{t+1} \beta^{pol} \cdot \left\{ \frac{U_c^{max}(A_{t+j}, g_{t+1}}{U_c^{max}(A_0, g_0)} \right\} \cdot s_t^{max}$	-	-
Chatterjee and Eyigungor (2012)	$c = y - \phi(y) + m$	$c = y + m + [\lambda + (1-\lambda)z]\, b - q(y, b') \cdot [b' - (1-\lambda) b]$	$u(c) = \frac{c^{1-\gamma}}{1-\gamma}$	$W(y, m, b) = \max V(y, m, b), X(y, -\bar{m}$	$V(y, m, b) = \max_{b'} u(c) + \beta E_{(y' m' \vert y)} W(y', m', b')$	$X(y, m) = u(c) + \beta\{[1-\xi] \cdot E_{(y' m' \vert y)} X(y', m') + \xi E_{(y' m' \vert y)} W(y', m', 0)\}$
Cuadra and Sapriza (2008)	$y^d = C_1^d + c_2^d$	$C_1 + C_2 = y + B - q(B', y) B'$	$u(C_i) = \theta \left(\frac{C_i^{(1-\sigma)}}{1-\sigma} \right) + (1-\theta) \left(\frac{C_j^{(1-\sigma)}}{1-\sigma} \right)$	$V_0(B, y) = \max\{V^c(B, y), V^d(y)\}$	$V^c(B, y) = \max_{\{C_1, C_2, B'\}} \{\theta u(C_1) + (1-\theta)\, u(C_2) + \beta[\sum_{y'} \pi V_0(B', y') + (1-\pi)\, \bar{V_0}(B', y')] Q(\frac{y'}{y}]\}$	$V^d(B, y) = \max_{C_1^d, C_2^d} \{\theta u(c_1^d) + (1-\theta)\, u(C_2^d) + \beta\{\mu \sum_{y'} [\pi V_0(0, y') + (1-\pi)\, \bar{V_0}(0, y')] Q(\frac{y'}{y}) + (1-\mu) \sum_{y'} [\pi V^d(y') + (1-\pi)\, \bar{V}^d(y')] Q(\frac{y'}{y}\}\}$
Cuadra et al. (2010)	$G_d = T_d C_d^*$	$G = TC^* + B - q(B', A) B'$	$U(C, G, 1-l) = \pi \left(\frac{G^{1-\sigma}}{1-\sigma} \right) + (1-\pi) \cdot \left(\frac{\left(C - \frac{l^{1+\psi}}{1+\psi} \right)^{1-\sigma}}{1-\sigma} \right)$	$V_o(B, A) = \max\{V^c(B, A), V^d(A)\}$	$V^c(B, A) = \max_{T, G, B} \{U(C^*, G, 1-l^*) + \beta \sum_{A'} V_o(B', A') Q \left(\frac{A'}{A} \right)\}$	$V^d(A) = \max_{T_d, G_d} \{U(C_d^*, G_d, 1-l^*) + \beta \sum_{A'} (\mu V_o(0, A') + (1-\mu) V^d(A') Q \left(\frac{A'}{A} \right))\}$
Eaton and Gersovitz (1981)	$c = y^{def}$	$c_t = y_t + b_t - p_t$	$U(X) = \frac{X^{1-\gamma}}{1-\gamma}$	$V_t^O = \max[V_t^D, V_t^R]$	$V^R(y_t, d_t) = \sup_{b_t \in B_t} \{U(y_t + b_t - d_t) + \beta E \max[V^R(y_{t+1}, d_{t+1}), V^D(y_{t+1}]\}$	$V^D(y_t) = E[\sum_{T=t}^{\infty} \beta^{T-t} U(y_t - P_t)]$
Gordon and Guerrón-Quintana (2013)	$c_t \leq (1-\kappa) A_t k_t^{\alpha} l_t^{1-\alpha}$	$c_t + q_t(b_{t+1}, k_t, A_t) \cdot b_{t+1} \leq A_t K_t^{\alpha} l_t^{1-\alpha} + b_t$	$u(c) = \frac{c_t - \eta \frac{l_t^{\omega}}{\omega}^{1-\sigma}}{1-\sigma}$	$V(b_t, k_t, m_t, A_t) = \max\{V^{nd}(b_t, k_t, m_t, A_t), V^d(k_t, A_t)\}$	$V^{nd}(b_t, k_t, m_t, A_t) = \max_{\{c_t, b_{t+1}, i_t, k_{t+1}, l_t\}} u(c_t, l_t) + \beta E_t V(b_{t+1}, k_{t+1}, m_{t+1}, A_{t+1}$	$V^d(k_t, A_t) = \max_{c_t, l_t} u(c_t, l_t) + \beta\, (1-\phi)\, E_t V^d(k_{t+1}, A_{t+1}) + \beta \phi E_t V(0, k_{t+1}, A_{t+1})$

Model	Closed Economy	Open Economy	Utility Function	Government Decision Function	Repayment Function	Default Function
Liu and Shen (2016)	$\tilde{c}_2 + \tilde{k}_3 = y^*(h(a_2), k_2, \tilde{\tau})$	$c_1 + k_2 = q(k_2, b_2)b_2 + k_1$ $c_2 + k_3 = y*(a_2, k_2, \tau) - b_2$	$u^1(c_1) = \frac{c_1^{1-\sigma}}{1-\sigma}$ $u^2(c_2, l_2) = \frac{(c_2 - \chi \frac{l_2^{1+\psi}}{1+\psi})^{1-\sigma}}{1-\sigma}$	$V^1 = \max_{\{c_1, k_2, b_2\}} u^1(c_1) + E_{a_2} V^2(k_2, b_2, a_2)$ $V^2(k_2, b_2, a_2) = \max_{\{d \in \{0,1\}\}} \{(1 - d)V^{2,R}(k_2, b_2, a_2) + dV^{2,D}(k_2, a_2)\}$	$V^{2,R}(k_2, b_2, a_2) = \max_{\{\tau, c_2, k_3\}} u^2(c_2, l_2) + v(k_3)$	$V^{2,D}(k_2, a_2) = \max_{\{\tilde{\tau}, \tilde{c}_2, \tilde{k}_3} u^2(\tilde{c}_2, \tilde{l}_2) + \eta v(\tilde{k}_3)$
Lizarazo (2013)	$c = y^{def}$	$c = y - q(b', y, W)b' + b$	$u(c) = \frac{c^{1-\gamma}}{1-\gamma}$	$V^0(b, y, W) = \max_{R,D}\{V^R(b, y, W), V^D(b, y, W)\}$	$V^R(b, y, W) = \max_{b'}\{u(y - q(b', y, W)b' + b) + \beta \int_{y'} V^0(b', y', W')f(y', y)dy'\}$	$V^D(b, y, W) = u(y^{def}) + \beta \int_{y'} [\theta V^0(0, y', W') + (1 - \theta)\, V^D(0, y', W')]f(y, y')dy'$
Martinez and Sandleris (2011)	$Tr = T$	$Tr = T - xRb$	NA	$\max_{x \in [0,1]} W(\theta, a(p'(x)), G, Tr)$	$W\,(\theta, a(p'(1)), G, T - Rb)$	$W\,(\theta, a(p'(0)), G, T)$
Mendoza and Yue (2012)	$c_t = \varepsilon_t f(\tilde{M}, \tilde{L}^f, k) - m^*(\varepsilon_t)\tilde{P}^* + \kappa_{t-1}$	$c_t = f_y(M, L^f, b', \varepsilon_t) + b_t - q_t(b', \varepsilon_t)$	$u(c) = \frac{\left(c - \frac{L^\omega}{\omega}\right)^{1-\sigma} - 1}{1-\sigma}$	$V(b_t, \varepsilon) = max\left\{V^{nd}(b_t, \varepsilon_t), v^d(\varepsilon_t)\right\}$	$v^n d(b_t, \varepsilon_t) = \max_{c_t, b_{t+1}}\{u(c_t - g(L(q_t(b_{t+1}, \varepsilon_t))) + \beta E[V(b_t + 1, \kappa(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t), \varepsilon_{t+1})]\}$	$v^d(\kappa_{t-1}, \varepsilon_t) = \max_{c_t}\{u(c_t - g(\tilde{L}(\varepsilon_t))) + \beta(1 - \phi EV(0, 0, \varepsilon_{t+1})\}$
Park (2017)	$c + p_k^{aut}(K, K')I + \Phi(K, K') = exp(A)f(K, l)$	$c + p_k I + q(B', K', A)B' + \Phi(K, K') = exp(A)f(K, l) + B$	$U(c_t, l_t) = \frac{[c_t - \frac{l_t^\omega}{\omega}]^{1-\gamma} - 1}{1-\gamma}$	$V^o(B, K, A) = \max(V^{nd}(B, K, A), V^d(K, A))$	$V^{nd}(B, K, A) = \max_{\{c, l, B', i_d, i_f, K'\}} u(c, l) + \beta E[V^o(B', K', A')\|A]$	$V^d(K, A) = \max_{\{c, l, i^d, K'\}} u(c, l) + \theta\beta E[V^o(0, K', A')\|A] + (1 - \theta)\,\beta E[V^d(K', A')\|A']$
Yue (2010)	$c = (1 - \lambda)y$	$c = y + b - q(b', y)b'$	$u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma}$	$v(b, 0, y) = \max v^r(b, 0, y), v^d(b, 0, y)$ $v(b, 1, y) = \max_{\{c, b'\}} u(c) + \beta \int_y v(b', 1, y')d\mu(y'\|y)$	$v^r(b, 0, y) = \max_{\{c, b'\}} u(c) + \beta \int_Y v(b', 0, y')d\mu(y'\|y)$	$v^d(b, 0, y) = u(y) + \beta \int_Y v(\alpha(b, y)b, 1, y')d\mu(y'\|y)$

Chapter 3

Sovereign Default with Unobservable Physical Capital

This chapter contributes to the literature on sovereign debt default by modelling imperfect information about borrowers' physical capital. Borrowers will make decisions on consumption and investment, strategically, with an option to default. We calibrate our model to the Argentine economy and simulate the effect of productivity shocks between 1980 Q1 and 2017 Q4. We compute the dynamics of equilibrium bond prices, unobservable physical capital, debt and consumption in addition to equilibrium default. As capital is unobservable we show that borrowing for consumption can be optimal under certain circumstances. This is a novel result as previous literature models complete bond contracts, where borrowing for investment is always optimal. We also find that our model better fits Argentina's default experience than existing models in the literature. This chapter therefore highlights the importance of modelling imperfect information for understanding the behaviour of international borrowers and lenders and informing international debt policies.

3.1 Related Literature

Over the last decade, global financial sectors have become more integrated and have shared risk throughout international credit markets. A government's decision to declare default has become more important to the world's economy and several researchers have demonstrated the reasons for sovereign default by using theoretical models. In 1981, the theoretical framework of Jonathan Eaton and Mark Gersovitz became a well-known study of borrowing with default when they published their paper assuming that the decision to default is an optional strategy for debtor countries.

A seminal framework of an endogenous relationship between default probability and output shocks was initially provided by Eaton and Gersovitz (1981) and developed by Arellano (2008). In these models borrowers strive to maximise consumption by using foreign assets with imperfect information. Therefore, they find that the borrowers will borrow more during bad times and repay during good times (counter-cyclical policy). In this framework, international debt is an endogenous variable in the resource constraint depending on the shocks. In addition, the authors also add the probability of re-entry into the default function as an exogenous variable. Sovereign debtors can re-enter international credit markets after some periods of debt exclusion; this assumption is consistent with recent empirical evidence that most defaulting durations are between three and six years (Reinhart and Rogoff, 2008; Sandleris et al., 2004).

Bai and Zhang (2010), Bai and Zhang (2011), Gordon and Guerrón-Quintana (2013), Liu and Shen (2016) and Park (2017) include observable physical capital in their models. In these papers, physical capital is assumed as being optional assets that are fully observed, as well as bonds. The government could then choose to invest through two types of asset, bonds and physical capital with the total amount of bonds allowed to be negative (net borrowing). If the government invests more in physical capital, it can enhance output; however, physical capital will depreciate every year. The role of lenders is implicit in these models as the market can fully observe borrowers and always accept the requested amount of borrowing by adjusting the risk premium, providing borrowers do not default in the next period. Thereby, perfect information about physical capital in this standard of literature provides a direct incentive for borrowers to increase capital investment in order to reduce the risk premium.

Liu and Shen (2016), present a positive relationship between capital investment and bond prices, where higher accumulated capital decreases risk-premium and increases bond prices. Park (2017) raise an issue concerning the use of two correlated assets; foreign asset and domestic capital in the model. If borrowers hold high physical capital, they are allowed to borrow more from an international credit market and will enjoy a lower risk-premium. Consequently, borrowers invest more and borrow more. Therefore, countries will have accumulated physical capital above the optimal level.

This chapter expands upon this literature by assuming that the borrower's physical capital is not observed by the lender. This allows us to capture the effects of imperfect information on the the outcome of the negotiation between borrowers and lenders.

3.2 The Model Economy

In this section, we provide a new model of borrower's default decision when capital is unobservable. It will be used to derive the optimal values of physical capital and debt in response to positive and negative productivity shocks.

3.2.1 Utility Function

The representative individual has the following utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t), \tag{3.2.1}$$

$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}, \tag{3.2.2}$$

where c is consumption, σ is the risk aversion rate and β is the discount factor. As standard in the literature, utility function assumes the constant relative risk aversion (CRRA) specification.

3.2.2 Production Functions

Production is assumed to take the following Cobb-Douglas specification:

$$y = ak^{\alpha}, \tag{3.2.3}$$

where y is output, k is physical capital and a is total factor productivity (TFP) shock. An increase in the level of capital accumulation (k) positively influence output (y). In addition, the TFP shock for normal periods is assumed to follow AR(1) stochastic process with the gaussian distribution[1]:

$$a_t = \mu + \rho\, a_{t-1} + \epsilon_t \;, \tag{3.2.4}$$

where $\mu = 1$, $\epsilon_t \sim N(0, \sigma_\epsilon^2)$ and $\rho \leq 1$. If the country defaults, the borrower will be excluded from the international credit market and penalised. The Cobb-Douglas production function becomes:

$$y^{def} = a^{def} k^{\alpha}, \tag{3.2.5}$$

where a^{def} is TFP shock during the default period and y^{def} is output during default with $y^{def} < y$. Stylised fact from defaulting countries confirm that output tends to drop below the trend[2] during default periods (Reinhart and Rogoff, 2011; Tomz and Wright, 2007).

Hence, there is a default cost in terms of output as a penalty, if the government decides to default. The boundary for the TFP shock during repayment and default periods can be written as: $a \in (\underline{a}, \overline{a})$ and $a^{def} \in (\underline{a}, \overline{a}^{def})$ respectively; where $\overline{a}^{def} \leq \overline{a}$. Output during the penalty phase will drop by the default output cost (χ) below the trend. If $\chi = 0$, there will be no default cost and $a = a^{def}$.

3.2.3 The Resource Constraints of the Sovereign Borrower

When the country is excluded from the international credit market, it is modelled as a closed economy. The country is not then able to borrow from international lenders.

[1]We discretise a continuous stochastic process of TFP by using quadrature method (Tauchen, 1986; Tauchen and Hussey, 1991) (see Appendix 3B)

[2]The trend is defined by an average growth rate of the past GDP (Tomz and Wright, 2007)

Physical capital is the only asset type to smooth consumption. Hence, the country's resource constraint in terms of consumption is based on the level of output which also depends on the choice of capital investment (k'). If the government decides to invest more in capital, there will be less resources left to consume in the current period and higher capital depreciation in the following period. On the contrary, selling physical capital instantly increase consumption and will decrease capital depreciation in the following period.

3.2.3.1 Closed Economy

Based on the assumptions, the resource constraint for a closed economy is given as follows:

$$c = y + (1 - \delta)k - k' - \Phi(k', k), \tag{3.2.6}$$

where c is consumption, y is output, k is current physical capital and k' is the next period's physical capital. Besides, capital depreciation and capital adjustment cost are denoted by δ and $\Phi(k', k)$ respectively. Specifically, the cost of capital adjustment $\Phi(k', k)$ in Equation 3.2.6 is positive for a change in capital and an increasing function of the change in capital. Moreover, a change in capital accumulation has an endogenous relationship with output through the Cobb-Douglas production function.

3.2.3.2 Open Economy

A country with access to the international financial market has the additional option to borrow or invest in international financial assets. In this way, the government has two available type of assets; foreign assets and physical capital that can be used to maximises utility. Hence, the resource constraint for a small open economy can be expressed as following:

$$c = y + b - q(b', a)b' + (1 - \delta)k - k' - \Phi(k', k), \tag{3.2.7}$$

where $q(b', a)$ is bond price and, b and b' are current and next period's bond. Notably, b denotes the foreign asset of the country, so a negative value of b implies borrowing (debt). In an open economy, the government decides upon the levels of both k' and b' to maximise utility. However, bonds are only available during repayment periods as borrowers have to continue repayment in order to get future access to the international financial market. Nevertheless international lenders have no power to either force a sovereign defaulter to repay or to seize assets.

3.2.4 Decision Function of the Sovereign Borrower

Next, the sovereign borrower must decide whether to repayment or to default. If the sovereign borrower chooses to default, they will be forced to enter the financial autarky with a tendency for lower output. A sovereign defaulter will be unable to enter the international financial market for a certain period after a default decision. For all periods, the government has to decide between repayment and default by taking into account all possible outcomes after the decision. Hence, the government decision function is derived as follows:

$$v_g^0(b, k, a) = \max_{\{d,r\}} \{v_g^d(k, a), v_g^r(b, k, a)\}, \tag{3.2.8}$$

where v_g^0 is the present value of the government decision, v_g^d is the value of the default decision and v_g^r is the value of the repayment decision. The government will choose whether to default or repay based on the valuation of these two decisions' functions. For instance, the government will choose to repay their debt if the value of the repayment decision is greater than the value of default. By choosing repayment, a sovereign borrower can continue to enter the financial market; however, if the government avoids repayment, they will be punished by exclusion from the international financial market and tend to return after a period.

3.2.4.1 Value Function of Default

Therefore, the default function can be derived as the following:

$$v_g^d(k,a) = \max_{\{k'\}} u(c) + \beta\,\mathbb{E}[\theta v_g^0(0,k',a') + (1-\theta)v_g^d(k',a')], \tag{3.2.9a}$$

subject to:

$$c = y^{def} + (1-\delta)k - k' - \Phi(k',k), \tag{3.2.9b}$$

and

$$c, k' \geq 0, \tag{3.2.9c}$$

where y^{def} is output during default periods ($y^{def} \leq y$) and θ is an exogenous probability of re-entry into the international financial market.

3.2.4.2 Value Function of Repayment

Next, the repayment function is given by:

$$v_g^r(b,k,a) = \max_{\{b',k'\}} u(c) + \beta\,\mathbb{E}\,v_g^0(b',k',a'), \tag{3.2.10a}$$

subject to:

$$c = y + b - q(b',a)b' + (1-\delta)k - k' - \Phi(k',k), \tag{3.2.10b}$$

and

$$c, k', q(b',a) \geq 0. \tag{3.2.10c}$$

From the above government decision function, it is obvious a sovereign borrower will decide to default when the value of the default function is greater than repayment. They will therefore enter into the penalty phase with the probability to re-enter into the international market at θ.

3.2.5 Capital Adjustment Costs

In addition, the capital adjustment cost $\Phi(k',k)$ is assumed to be of the following quadratic form.

$$\Phi(k',k) = \frac{\Phi}{2}\left(\frac{k'-(1-\delta)k}{k}\right)^2 k, \tag{3.2.11}$$

where Φ is set at 3 or 5 in general real business cycles (RBC) literature to match the output moments (Bai and Zhang, 2012; Gordon and Guerrón-Quintana, 2013). this thesis assumes that physical capital can be traded anytime with a capital adjustment cost even in the default periods. On the other hand, a financial asset or bond is only available during the repayment periods, based on the borrower's default probability.

3.2.6 Bond Price Schedule

International lenders are assumed to be risk neutral. They will be indifferent in getting the return $1 + r$ for sure and getting the expected return $(1–\Psi)/q$ from lending to the sovereign borrower, where Ψ is the equilibrium default probability and q is the bond price. Consequently,

$$q(b', a) = \frac{1 - \Psi\,(b', a)}{1 + r}, \tag{3.2.12}$$

where Ψ is the default probability of the sovereign borrower, as perceived by international lenders. Since capital is not observed, the perceived default probability is only a function of borrowing and total factor productivity.

3.2.7 Perceived Equilibrium Default Probability

This section shows how the lender seeks to infer the default probability based on what is observed. Specifically, as lenders do not observe capital, we assume they act agnostically. Their best guess of the level of physical capital is its steady state level, k^*. Consequently, we look at the equilibrium default probability of a borrower endowed with k^*. The resource constraints of the borrower when defaulting is

$$c = y^{def} - \delta k^*, \tag{3.2.13}$$

and when not defaulting

$$c = y + b - q(b', a)b' - \delta k^*, \tag{3.2.14}$$

The borrower will choose to default if the value function of default is greater. From viewpoint of the lender, the borrower solves

$$v_l^0(b, a) = \max_{\{d,r\}}\{v_l^d(a), v_l^r(b, a)\}. \tag{3.2.15}$$

Next, the value function of the default decision can be expressed as follows:

$$v_l^d(a) = u(c) + \beta\,\mathbb{E}[\theta v_l^0(0, a') + (1-\theta)v_l^d(a')] \tag{3.2.16a}$$

subject to:

$$c = y^{def} - \delta k^*, \tag{3.2.16b}$$

and

$$c \geq 0, \tag{3.2.16c}$$

where y^{def} is obtained from observing a^{def} and guessing k^*.

The value function in case of repayment is

$$v_l^r(b, a) = \max_{\{b'\}} u(c) + \beta\,\mathbb{E}\,v_l^0(b', a'), \tag{3.2.17a}$$

subject to:

$$c = y + b - q(b', a)b' - \delta k^*, \tag{3.2.17b}$$

and

$$c, q(b', a) \geq 0. \tag{3.2.17c}$$

where y is also obtained from observing a and guessing k^*. Therefore, the sets of repayment and default decisions under the lender's viewpoint are given by:

$$R(b) = \{a \in A : v_l^r(b, a) \geq v_l^d(a^{def})\}, \tag{3.2.18}$$

$$D(b) = \{a \in A : v_l^r(b, a) < v_l^d(a^{def})\}, \tag{3.2.19}$$

where $R(b)$ is the repayment set and $D(b)$ is the default set under the lender's aspect(v_l^0). Next, the default set under the lender's view can be achieved as:

$$\Psi(b', a) = \int_{D(b')} f(a', a)\, da' \tag{3.2.20}$$

where $f(a', a)$ is the stochastic process provided in Equation (5.3.4). It is obvious that $0 \leq \Psi \leq 1$. When $D(b') = \emptyset$, the default probability is zero. From the above recursive equations, the sovereign borrower will choose the amount of assets b' and k' after recognising the current TFP shock (a). Simultaneously, the international lender will obtain b' and a from the sovereign borrower and return the bond price (q) based on the decision function (v_l^0). Finally, the set of all possible outcomes (v_g^0) with policy functions of physical capital, debt and default decisions with the corresponding TFP shock can be obtained.

3.3 Data and Calibration

The model is calibrated to target the Argentine economy by using quarterly data from 1980 Q1 to 2017 Q4 with some parameters set based on recent RBC literature. The main contribution is to explain the interactions among physical capital, debt, output, and risk premium and default probability. We use a discrete model with Monte Carlo simulation and the moment matching method in order to explore and fit the model to actual data. As a result, the model in this thesis can provide a closer fit to the average actual level of debt and consumption ratio as well as the moments and the correlation coefficients.

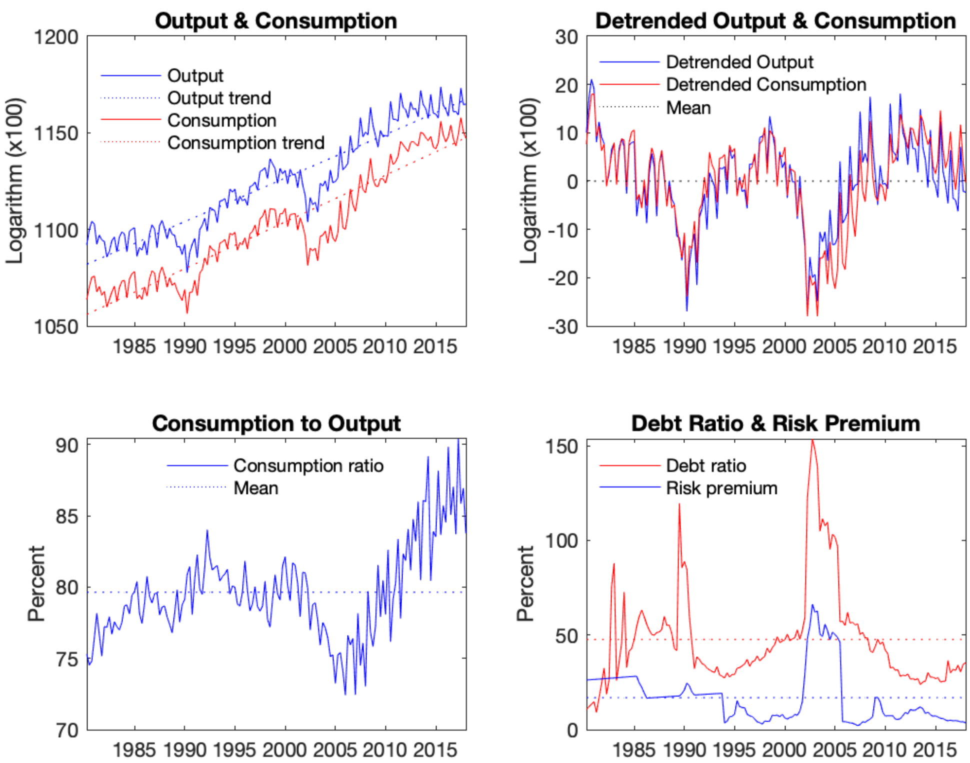


Figure 3.1: Output, Consumption, Trend and Debt

Data are from the Argentine Ministry of Economy (MECON). The data are available quarterly, non-seasonally adjusted and at constant 2010 U.S. dollars, starting from the first quarter of 1980 to the last quarter of 2017 over 152 periods. Figure 3.1 shows plots of output, consumption, debt, risk premium and trends. GDP and consumption are in log and linearly de-trended. Figure 3.1 clearly shows an upward trend for both output and consumption in Argentina with a standard deviation for consumption and output of 0.0969 (σ_c) and 0.0917 (σ_y) respectively.

Thus, consumption is 1.07 times more volatile than output (σ_c/σ_y). Moreover, there is a significant and positive correlation between output and consumption equal to 0.9223 (ρ_{cy}). In Figure 3.1, the consumption to output ratio of Argentina was 75% in 1980 Q1, and peaked at 90% in 2017 Q4. Besides, mean and standard deviation of the consumption ratio over 38 years are 79.63% ($\mu_{c/y}$) and 0.0336 ($\sigma_{c/y}$) respectively.

Table 3.1 below presents business cycle statistics for the Argentina's economy between 1980 Q1 and 2017 Q4. The first-order auto-regression model of output without a constant term has 0.6786 (ρ_y) auto-correlation. Next, we use the ratio of external debt to GDP to measure the external debt level in the model. Table 3.1 presents the debt ratio with a mean of 47.67% (μ_b) and standard deviation of 0.2719 (σ_b). before the late 1980s, the external debt to output ratio of Argentina had never exceeded 50%. It reached the bottom value at 9.07% in 1981 Q1 and increased to a peak of 153.63% in 2002 Q3. Spikes were clustering around the period of financial crisis and recession in 1990 and 2003. Noticeably, there is a negative correlation coefficient between output and debt ratio of -0.6552 (ρ_{by}). This negative relationship was obviously seen in 1990 and 2002 when the debt ratio increased to its peak at the same time as output experienced the biggest drop in Argentina.

Table 3.1 also shows that debt is 2.97 (σ_b/σ_y) times more volatile than output and

2.81 (σ_b/σ_c) times than consumption. The risk premium is obtained from Argentina's overall debt.

In Table 3.1, the mean and standard deviations of risk premium are 0.1689 (μ_ψ) and 0.1420 (σ_ψ) respectively. The correlation coefficient between output and risk premium is negative at -0.4713 ($\rho_{\psi y}$) whilst, debt ratio and risk premium have a positive correlation at 0.6953 ($\rho_{b\psi}$). Overall, the signs of correlation coefficients for Argentina are consistent with the RBC literature and stylised facts from developing countries (Aguiar and Gopinath, 2006; Yue, 2010).

Table 3.1: Argentina's business cycle statistics (1980 Q1 - 2017 Q4)

Parameters	Values	Parameters	Values
μ_y	0	σ_y	0.0917
μ_c	0	σ_c	0.0969
$\mu_{\frac{c}{y}}$	0.7963	$\sigma_{\frac{c}{y}}$	0.0336
μ_b	0.4767	σ_b	0.2719
μ_ψ	0.1689	σ_ψ	0.1420
$\frac{\sigma_c}{\sigma_y}$	1.0708	$\frac{\sigma_{c/y}}{\sigma_y}$	0.3664
$\frac{\sigma_b}{\sigma_y}$	2.9651	$\frac{\sigma_\psi}{\sigma_y}$	1.5485
ρ_y	0.6786	ρ_c	0.7860
ρ_b	0.8797	ρ_ψ	0.9440
ρ_{cy}	0.9223	ρ_{by}	-0.6552
$\rho_{\psi y}$	-0.4713	$\rho_{b\psi}$	0.6953
ρ_{cb}	-0.7616	$\rho_{c\psi}$	-0.542

Table 3.2 induces over model parameters for the Argentine economy. The discount factor (β) is borrowed from the sovereign default literature (Arellano, 2008) and it is equal to 0.35. Risk aversion (γ_g, γ_l), risk-free rate (r), capital share (α) and capital depreciation (δ) are 2, 1%, 0.35 and 0.05 respectively, based on the RBC literature for developing countries.

Table 3.2: Model specific parameter values

Parameters		Values	References
Discount factor	β	0.95	Arellano (2008)
Risk aversion of borrower	γ_g	2	Aguiar and Gopinath (2006)
Risk aversion of lender	γ_l	2	Lizarazo (2013)
Risk-free rate	r	0.01	Aguiar and Gopinath (2006)
Capital share	α	0.35	Park (2017)
Capital depreciation	δ	0.05	Romero-Barrutieta et al. (2015)
Output default cost	χ	7%	Tomz and Wright (2007)

We calibrate other parameters of interest as shown in Table 3.3. The stochastic structure of TFP shock, which follows the AR(1) process, is specified by using the moment matching method to target the Argentine economy (see Table 3.4). ρ_a and σ_ϵ are found to be 0.982 and 0.014 respectively. This stochastic structure fits the actual volatility of output (σ_y) and consumption (σ_c) and correlation coefficient between them (ρ_{cy}) at 0.0917, 0.0969 and 0.9223 respectively. The probability of re-entry (θ) and output default cost (χ) are fixed at 5% and 7% consistently in the literature. Following the standard quadratic form of capital movement (Bai and Zhang, 2011; Gordon and Guerrón-Quintana, 2013), capital adjustment cost is 2.4.

Table 3.3: Calibration parameters

Parameters		Values
Stochastic structure (TFP)	ρ_a, σ_ϵ	0.982 , 0.014
Probability of re-entry	θ	5%
Capital adjustment cost	Φ	2.4

Moreover, relative consumption and output volatility ($\frac{\sigma_c}{\sigma_y}$), average consumption ($\mu_{\frac{c}{y}}$) and debt ratio ($\mu_b$) are the targets of calibration at 1.0563 times, 79.63% and 47.67% respectively (see Table 3.4 below).

Table 3.4: Target statistics

Parameters		Values
Output volatility	σ_y	0.0917
Consumption volatility	σ_c	0.0969
Consumption output volatility	$\frac{\sigma_c}{\sigma_y}$	1.0563
Correlation coefficient (c,y)	ρ_{cy}	0.9223
Average consumption ratio	$\mu_{\frac{c}{y}}$	79.63 %
Average debt ratio	μ_b	47.67 %
Default periods		12.5 %

Furthermore, this thesis uses a discrete model and follows Tauchen's method[3] for obtaining a finite state Markov chain approximation of the nonlinear asset (Tauchen, 1986). Hence, the TFP shock with the above stochastic structure is discretised into a 31-state Markov chain.

The possible length of debt and physical capital in the grids are specified from the Argentine economy between 1980 Q1 and 2017 Q4 as $\{\underline{b}, \overline{b}\} = \{-4.5, 0.5\}$ and $\{\underline{k}, \overline{k}\} = \{2, 13\}$ respectively. From these lengths of the grids, debt ratio lies between $\frac{b}{\underline{y}}$ and $\frac{\overline{b}}{y}$, whilst capital ratio lies within $\frac{k}{\underline{y}}$ and $\frac{\overline{k}}{y}$. Hence, the upper and the lower bounds are set to cover the possible length of debt and capital with respect to the past experience of the Argentina's economy. Specifically, the total number of grids for debt and physical capital are 51 and 71. The 51-finite state of debt is equally distributed between the interval $\{\underline{b}, \overline{b}\}$, while the grid of capital is set to cluster around the expected steady state [4]. Finally, we simulate the model using the Monte Carlo simulation over 100,000 times.

[3]This method has been widely used by the previous literature on nonlinear model with a discrete-valued Markov chain see (Aguiar and Gopinath, 2006; Arellano, 2008; Gordon and Guerrón-Quintana, 2013; Liu and Shen, 2016; Schaltegger and Weder, 2015)

[4]The total number of grids for capital is 71. There are 7 and 5 state spaces that equivalently located with 0.5 distance for the value of capital at the intervals $\{2, 5\}$ and $\{11, 13\}$ respectively. Beside, other 59 state spaces are equivalently located within the interval $\{5, 11\}$. This setup helps to reduce the number of time spending in the computation and the distance between grid around the steady state of capital; the optimal choice of capital tends to cluster around the steady state.

3.4 Quantitative Results

This section will provide the result of the stimulation and their discussion. It includes three main parts. The first part will define the equilibrium bond price schedule, default probability and policy functions of a sovereign borrower. The second part will analyse impulse response functions and the Monte Carlo simulation. Finally, the third part will provide a comparison between actual data for the Argentina's economy from 1980 Q1 to 2017 Q4 and the model simulations.

The bond price schedule at the steady-state level of capital is given in Figure 3.2. We can derive an equilibrium a bond price (q) schedule that responds to the next period's bond (b') and the current level of output (y) in turn affected by current capital (k) and technology shock (a).The plot of bond price schedule indicates the negative relationship between next period's level of debt and bond price. An international lender will respond to a borrower's selection of debt amount and the current period's level of output. If a country intends to borrow large amount of debt, the risk premium increase in the international market and the borrower will be charged a high price. The opposite is true if the borrower selects a lower amount of debt.

Moreover, Figure 3.2 also shows differences in bond price schedules for several technology shocks where a_{higher}, a_{high}, a_{ss}, a_{low} and a_{lower} represent TFP percentage changes of 6%, 3%, 0%, -3% and -6% from the steady-state value (calculated without shocks).

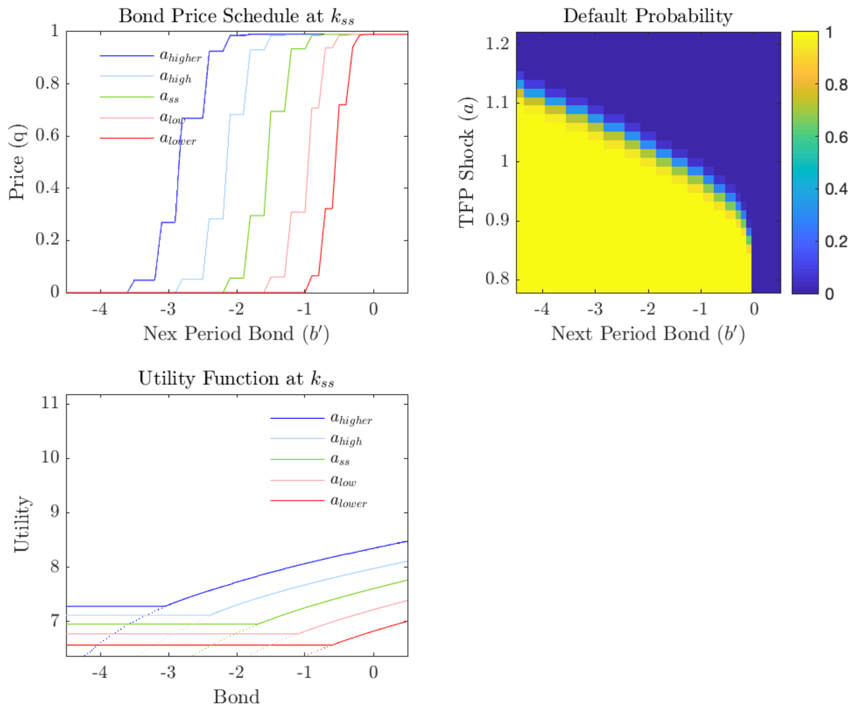


Figure 3.2: Bond Price Schedule, Default Probability and Utility Functions

The sovereign country can borrow a higher amounts of debt at a lower risk premium with a positive TFP shock. On the other hand, a negative TFP shock can lead to an instant increase in bond risk premium for the same amount of debt. Thereby, a large negative shock can instantly induce a sovereign country with large debt to default because of an expensive borrowing rate (higher risk premium) to be faced in the following period.

The future default probability is shown as a heat map chart in Figure 3.2. The figure is plotted from three dimensional data for bond selection (b'), TFP shock (a) and default probability (Ψ) where the default probability lies between zero (blue) and one (yellow). From the plot, if a country intends to save with a bond (b') above or equal to zero, there will be no default in the following period. With $\Psi = 0$, the country can borrow or save money at the risk-free rate (without risk premium; see the blue shade area in Figure 3.2). However, if the country intends to borrow, there will be a positive default probability (the yellow area) in the next period with respect to the size of the TFP shock. With a negative shock, the borrower will be charged at a higher risk premium in the following period because of a positive default probability. For a default probability of 100%, the borrower will be unable to borrow on the international market.

Furthermore, Figure 3.2 also shows the utility functions at the steady-state level of capital with various TFP shocks and debt. The solid lines in the figure are optimal utility values (depending on repayment and defaults). From the figure, the utility tends to be higher if there is a positive TFP shock in the current period. For the same value of TFP, the utility will be higher with more current bonds or foreign assets (b). In addition, the dotted lines in Figure 3.2 indicate the utility values when the sovereign borrower continues to repay debt instead of choosing to default. The utility values with a default decision will be flat as bond selection is unavailable for default countries. From a comparison between the solid and dotted lines in Figure 3.2 for large debt, we notice that default gives rise to higher utility. The sovereign borrower with large debt tends to default for a drop in current output or a lower value of TFP.

Figure 3.3 plots debt and capital functions at the steady-state level of capital (k_{ss}) and debt (b_{ss}) with positive and negative TFP shocks. In the figure, the red and blue lines indicate the policy function (choosing next period bond and capital) responding to a negative and positive TFP shocks respectively. The black line is defined when there is no shock to the economy, while the dotted line is the 45 degree line that helps to define an unchanged level of current and future period resources.

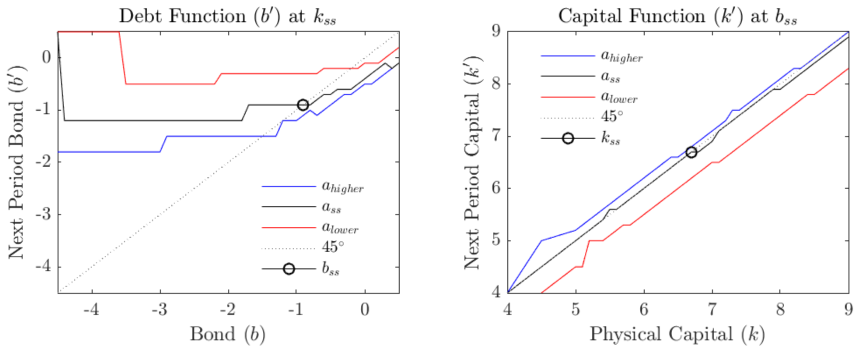


Figure 3.3: Policy Functions for Debt and Capital Investment

In the figure of debt function, there will be a shift from the black line to the blue line when the positive TFP shock occurs. It indicates that the sovereign borrower with current debt ($b < 0$) will choose to borrow more if a positive TFP shock occurs. The policy function will shift towards the red line when there is a negative TFP shock. It implies that the borrower will reduce debt to a certain level in response to a negative TFP shock. Moreover, the intersection of the policy line without any shock (black) and the 45° line (dotted line) indicates the steady state level of debt. If the economy starts with bond at a level above or below the steady state level of debt (i.e. the intersection), the bond function will converge to the steady stage.

The policy function for future capital with respect to its current level is also given in Figure 3.3. At the steady state level of debt, a sovereign borrower will increase/decrease capital investment from the positive/negative TFP shock. From a comparison of the policy line without shock (black solid line) and the 45° line (dotted line), we find the steady-state level of capital at 6.94. If the borrower start with capital above the steady-state level, the policy line will be slightly below the 45° line. This means that a sovereign borrower will decide to sell physical capital, if its current level is above the steady-state. The policy line is slightly above the 45° line when the current capital is below the steady-state level. This suggests that the country will choose to invest more until it reaches the steady state.

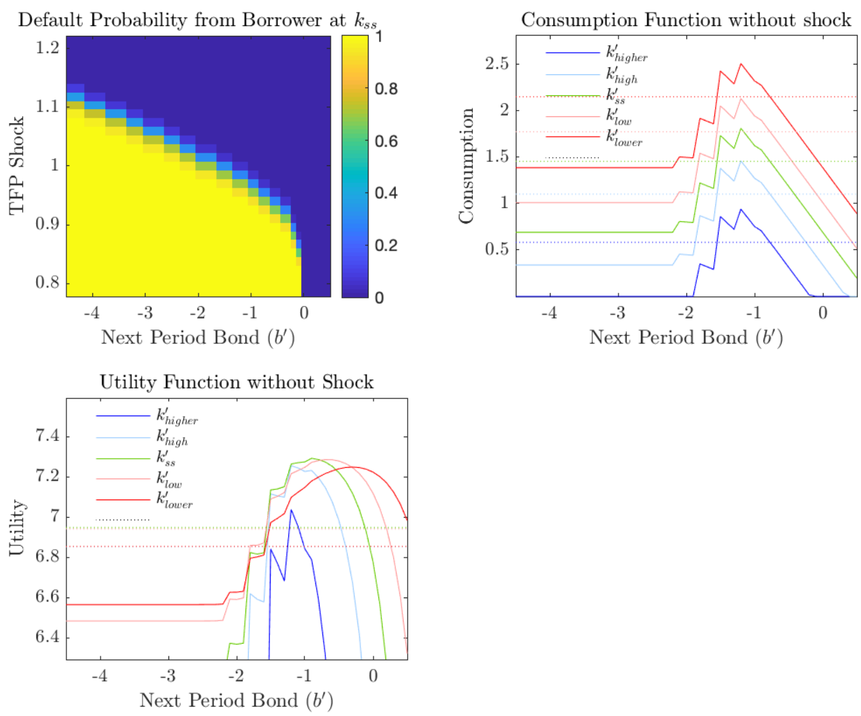


Figure 3.4: Default Probability, Consumption and Utility Functions of the Borrower

Figure 3.4 gives the heat map plot of the borrower's default probability the steady-state level of capital (k_{ss}). The future default intention of the sovereign borrower can be deduced from this plot as a function of the TFP shock (a) and the next period's bond (b'). As shown in the figure, a country with large debt has more intention to default after a drop in output (yellow area with TFP less than 1). For a TFP > 1 (blue area), debtors will remain at a high level of debt.

Figure 3.4 also shows plots of the consumption and utility functions with respect to the next period's bond and capital investment without shocks. The solid and dotted lines show consumption from repayment and default decisions respectively. From the consumption plot, selling physical capital can instantly relax the budget constraint, which in turn leads to an increase in current consumption. Comparing between the repayment (solid lines) and default (dotted lines) decisions, the debtor is better off staying around its peak with repayment decisions. At its peak, the country will be able to consume more from borrowing, if they continue to repay and borrow from the international bond market. However, the price of bonds is also based on the default risk which will make the bond price expensive with large amounts of debt. If instead the country saves money in the bond market, it will reduce the current budget constraint and consumption.

The utility function for the next period's bond and capital investment is also given in Figure 3.4. The plot shows an inverted u-shape relationship between utility values and the next period's bond under repayment. The utility curve of repayment peaks at the steady-state level of debt (b'_{ss}) at -0.9. When we compare repayment and default decisions plot we notice the utility values of repayment at the steady state is always higher than under default because the sovereign debtor has no intention to default at the steady state, if no shocks occur. More importantly, comparing utility curves for different capital investments, we observe that capital level at the steady state (k'_{ss}) generates the highest utility value.

Figure 3.5 shows the dimensional heat map charts of the default probability based on the borrower's selection of capital investment (k'). Areas of default intention (yellow) in each layer clearly define the probability of default in the next period. The top layer is generated from future default intention with high capital investment, whilst the middle and bottom layers are the default probability at the steady state and low capital investment respectively. From the sliced layers in the chart, we notice that the default probability in the following period changes due to the amount of the next period's capital (k'). An increase in physical capital will endogenously enhance productivity as well as raise the number of secured assets that can be used as a last resort during recession. Therefore, an increase in capital investment significantly reduces the default probability in the following period, as shown in Figure 3.5.

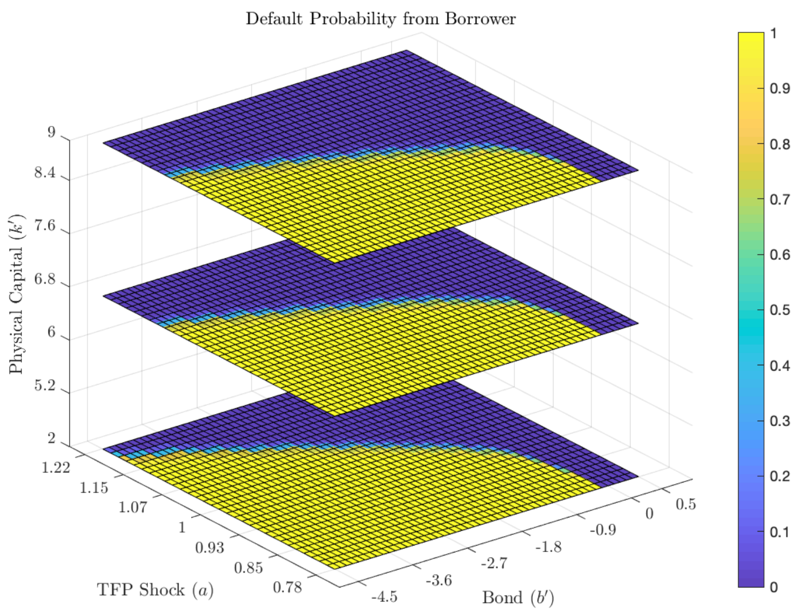


Figure 3.5: Default Probability from Borrower based on Physical Capital

On the other hand, a decrease in capital investment may raise current budget constraint and consumption instantly, but it tends to reduce future default probability. The debtors with lower physical capital obviously have fewer assets to sell during recession and, thereby, the future default probability is higher. More importantly, a sovereign borrower with little capital has higher probability to default with little debt compared to those with high physical capital. This characteristic of default matches the actual data according to which poor countries with low capital have a higher number of defaults (Reinhart and Rogoff, 2008).

The second part of this section will analyse the impulse response functions and the Monte Carlo simulation. At the steady state level of capital, the impulse-response functions following a positive and a negative TFP shock of 3% without a persistence are given in Figure 3.6. Notably, Figure 3.6 shows the response of foreign asset (b), net borrowing ($-b' + b$), output (y), physical capital (k), capital investment ($k' - k$), net asset ($k + b$), consumption (c) and net cashflow ($-\Delta b - \Delta k$).

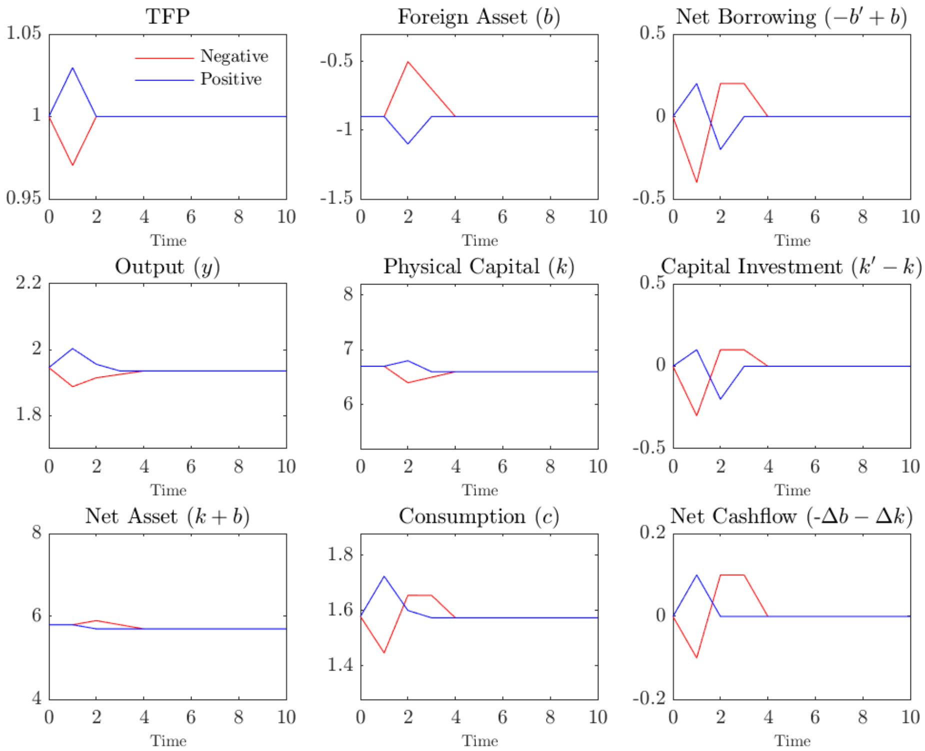


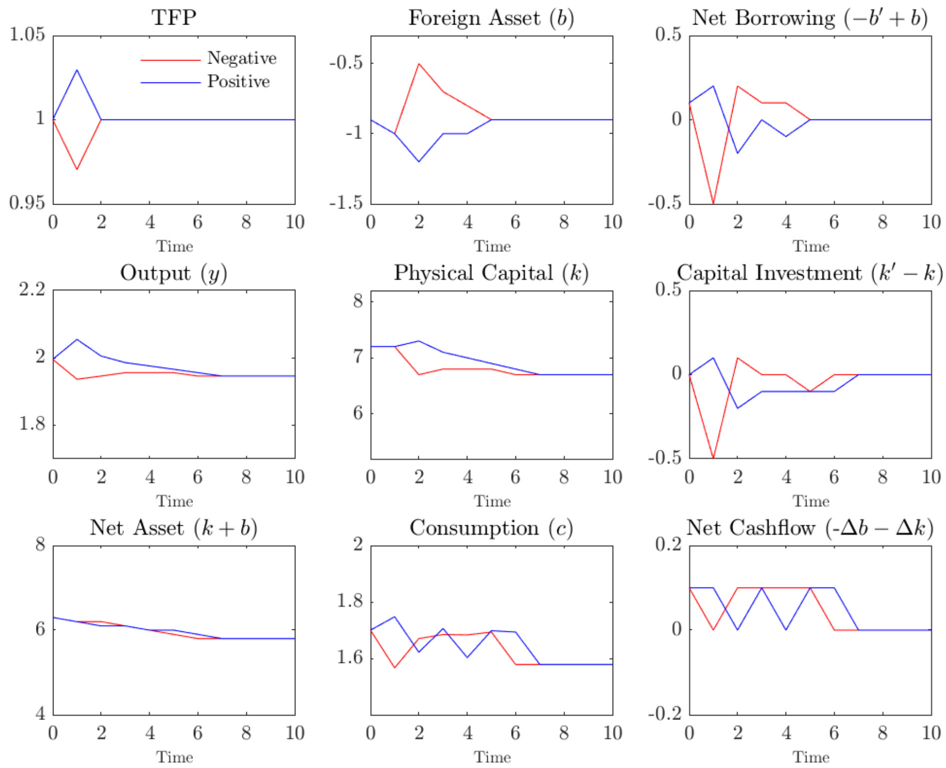
Figure 3.6: Impulse Response Functions at the steady state level of physical capital

At the steady state, when a positive shock occurs in period 1, the country starts to borrow more for both consumption and investment; net borrowing becomes negative, while capital investment and consumption increase. After that, the country slightly decreases capital, repays its debt and reduces consumption until the TFP shock fades. Hence, we can deduce that a country tends to borrow for both investment and consumption during a period of positive TFP shock.

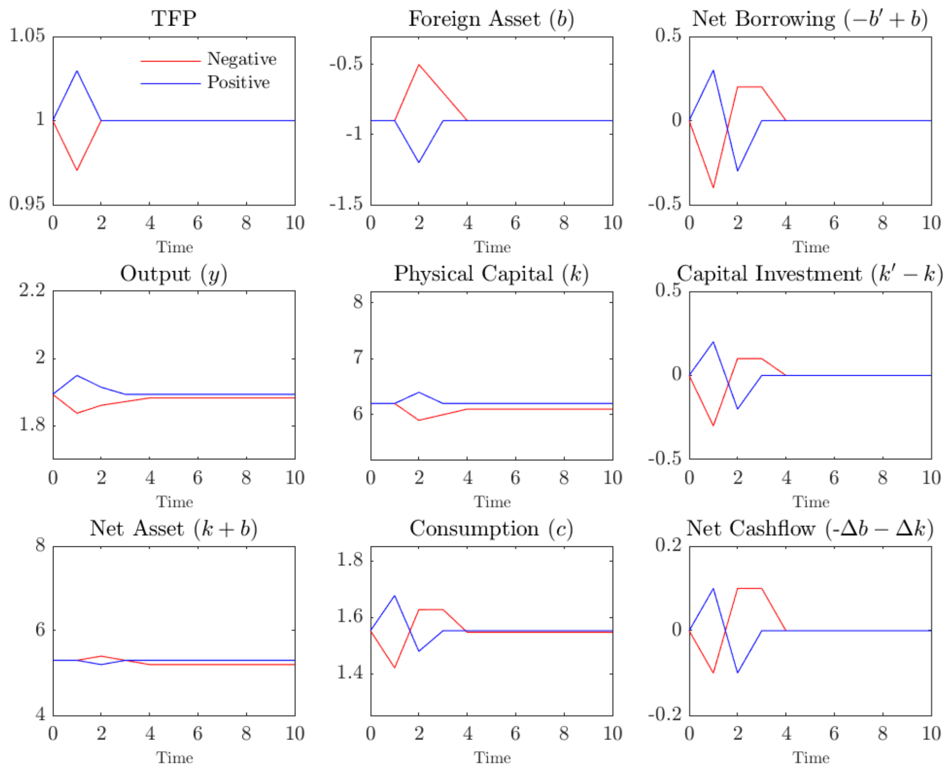
On the other hand, for a negative TFP shock in period 1, the country decides to repay with a sharp decrease in capital investment, while consumption is slightly reduced. Interestingly, Figure 3.6 shows consumption smoothing achieved by selling capital. This result can be explained by the fact that borrowing may not be better off because of the higher risk premium that will be face charged during a bad period (see Figure 3.2). As a result, the country can choose to smoothen consumption through selling physical capital.

Moreover, this section also provides the impulse response functions at above and below the steady state level of capital in Figure 3.7. As shown in Figure 3.7a, the country begins with an initial level of capital above the steady state. When a positive shock occurs in period 1, the country will choose to borrow more for both investment and consumption. The country can maintain a high level of debt with bond higher than the initial level, while the physical capital is above its steady state. However, after period 1, the country starts to sell physical capital and repay its debt. As can be seen in Figure 3.7a, net asset continuously declines until reaching its steady-state at period 7, as same as consumption. Comparing with the previous figure, consumption in Figure 3.7a can be maintained above the steady-state for 6 periods. Thus, a country with capital above the steady-state will sell assets for consumption.

Figure 3.7b shows opposite result for an initial capital below the steady state. With an increase in TFP in period 1, the country will start to borrow for both consumption and investment. However, as the initial capital is below the steady-state, the borrower has lower ability to borrow in comparison with the previous outcomes. Hence, physical capital slightly increases after the positive TFP shock, but it still remains below the actual steady state.



(a) Impulse Response Functions at the above steady state level of capital



(b) Impulse Response Functions at the below steady state level of capital

Figure 3.7: Impulse Response Functions at the above and below steady state of capital

Therefore, the impulse response functions analysis from 3.7 implies that a country will respond differently to TFP shocks, depending on initial the level of physical capital. If initial capital is above the steady state level, they can borrow more for both consumption and investment during a good period, and then sell capital to enjoy consumption above the steady-state. If initial capital is below the steady state, they will have a lower ability to borrow for investment. Physical capital is unable to return to its steady state.

Additionally, we also provide a simulation plot for a country starting with a positive foreign asset and various levels of physical capital (without shocks) in Figure 3.8.

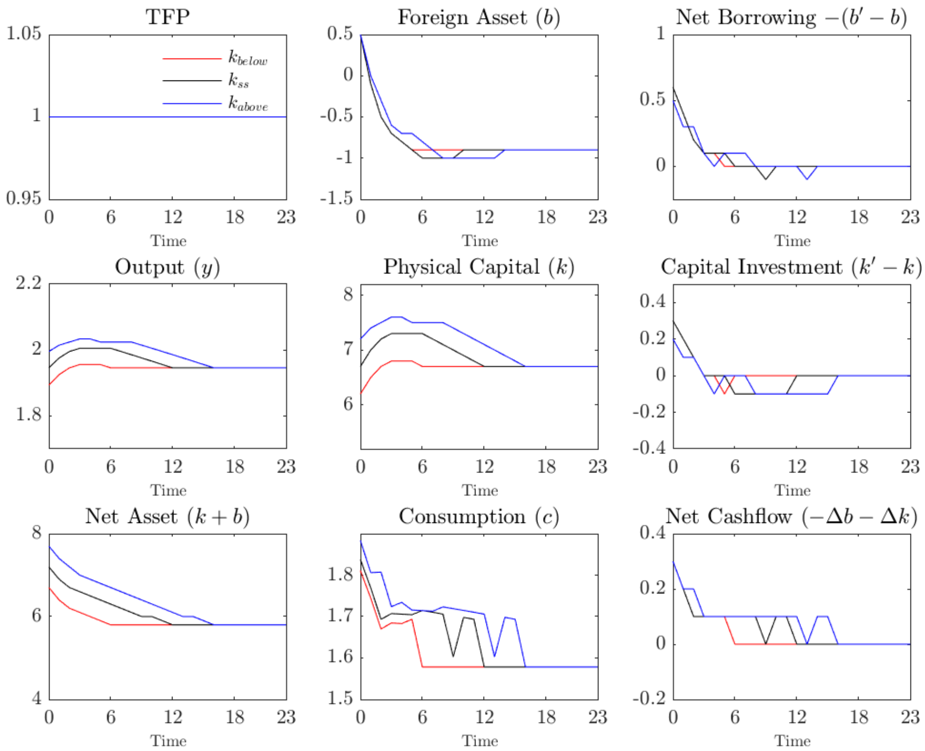


Figure 3.8: Impulse Response Functions at the steady state level of physical capital

As can be seen in the figure above, regardless of the initial levels of physical capital, a country will converge to the same steady-state of capital after 15 periods. A country with initial debt below the optimal level will choose to borrow more.From the plot, it is obvious that the majority of its debt is used for investment. During the first three periods, capital investment is significantly higher than a change in consumption. After that, the country stops borrowing and smoothen consumption through selling capital until it reaches its steady-state. This outcome confirms that there is only one steady-state level of capital and debt. However, the country will not be able to invest and reach the steady-state level of capital, if it starts with less capital and high debt.

The Monte Carlo simulation over 1,000 periods is shown in Figure 3.9. All variables are plotted in real term.

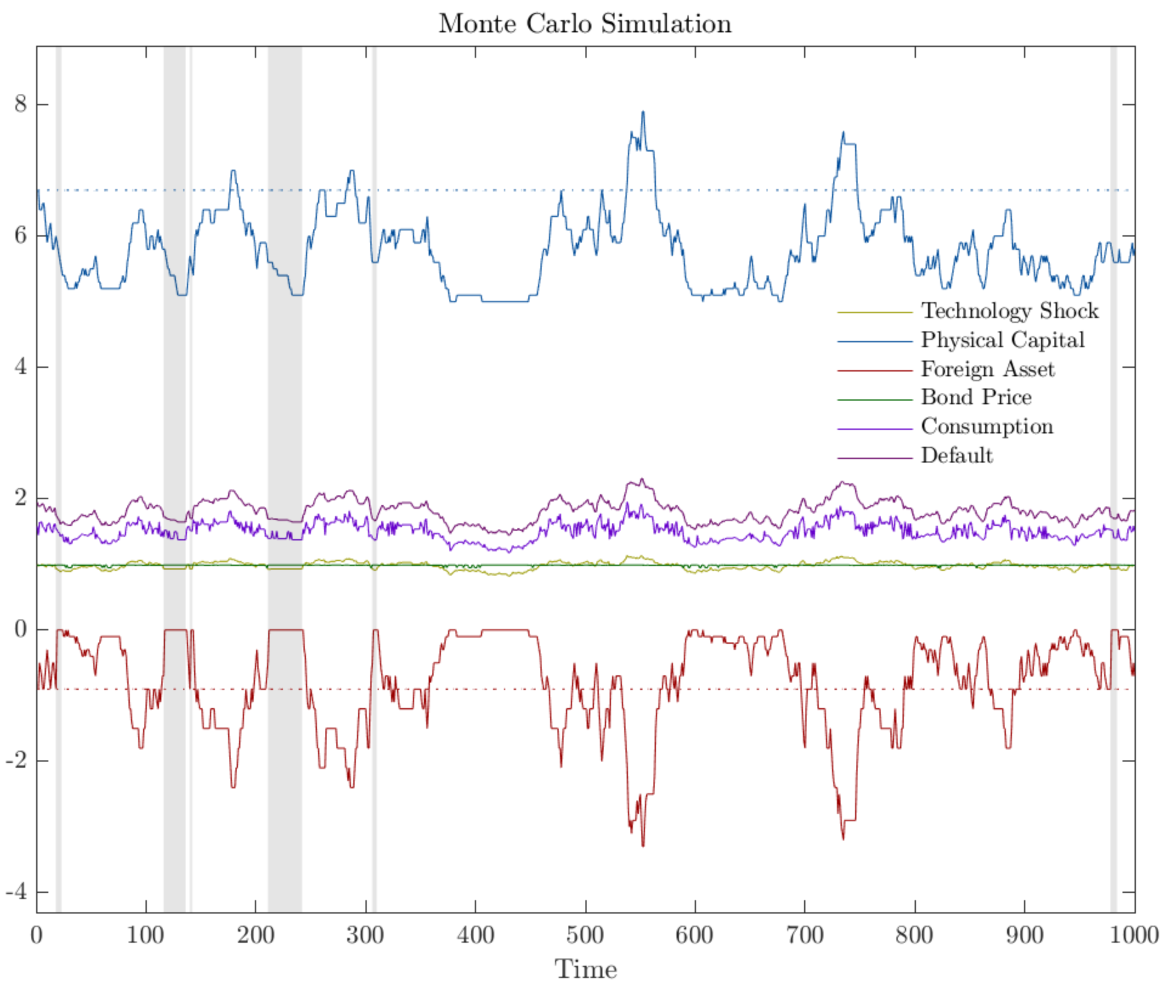


Figure 3.9: Monte Carlo Simulation over 1,000 periods

In the figure, the simulation starts at the economy steady-state and target the business cycle statistics of the Argentine economy, as shown in Table 3.4. The overlay default band corresponds to the sovereign default decision in the model; the dotted lines represent the steady-state level of capital and debt.

From Figure 3.9, we can see that the government tends to default after facing a series of negative shocks or a very large drop in output. After each default, the country will be excluded from the international bond market with a probability of re-entry (θ) equal to 5%. Output is capped at 7% below its mean during the exclusion periods. As can be seen in Figure 3.9, debt is always zero during defaults. Furthermore, after re-entering the bond market, the government will start borrowing and consuming more for a few periods until achieving that economy steady-state.

In addition, the government will choose to stay in the bond market with a repayment decision, if there is insufficient TFP shock or enough physical capital. During a slight drop in output, the sovereign debtor tends to sell physical capital or reduce consumption in order to repay its debt. Bond price fluctuates after a TFP shock with higher debt ratio. Interestingly, in some periods with a series of small negative shocks, the borrower sells physical capital to slowly repay its debt. With small debt and sufficient capital, the country can remain in the financial market during recession and wait for a positive shock.

The final and third part of this section will present the quantitative results of the Monte Carlo simulations over 100,000 periods for the Argentina's economy between 1980 Q1 and 2017 Q4.

Table 3.5: Model and Target Statistics

Parameters		Model	Target
Output volatility	σ_y	0.0940	0.0917
Consumption volatility	σ_c	0.0924	0.0969
Consumption output volatility	$\frac{\sigma_c}{\sigma_y}$	0.9830	1.0563
Correlation coefficient (c,y)	ρ_{cy}	0.9314	0.9223
Average consumption ratio	$\mu_{\frac{c}{y}}$	82.73 %	79.63 %
Debt ratio at steady state	μ_b	46.25 %	47.67 %
Length of default periods		7.40%	12.50%

As shown in Table 3.5, output volatility (σ_y), consumption volatility (σ_y) and consumption output volatility ($\frac{\sigma_c}{\sigma_y}$) are 0.0940, 0.0924 and 0.9830 respectively. The correlation coefficient between consumption and output (ρ_{cy}) is 0.9314. The results are consistent with the Argentine data. More importantly, the model can target an average consumption and debt ratio[5] of 82.73% and 46.25% respectively. However, the length of the default period from the model simulation is 7.40%, while the actual data is 12.50 %.

Table 3.6: Other Business Cycle Statistics from the Model and the Actual Data

Business Cycle Statistics		Model	Actual
Debt Volatility	σ_b	0.3472	0.2719
Risk Premium Volatility	σ_ψ	0.0083	0.1420
Autocorrelation of Output	ρ_y	0.9838	0.6786
Autocorrelation of Consumption	ρ_c	0.8569	0.7860
Correlation coefficient (y,ψ)	$\rho_{y\psi}$	-0.18806	-0.4713
Correlation coefficient (c,ψ)	$\rho_{c\psi}$	-0.2138	-0.5402
Minimum Consumption Ratio	$\frac{c}{y}_{min}$	71.28 %	72.42 %
Maximum Consumption Ratio	$\frac{c}{y}_{max}$	95.21 %	90.39 %
Minimum Debt Ratio	b_{min}	0 %	9.07 %
Maximum Debt Ratio	b_{max}	184.73 %	153.63 %

Finally, the other statistic from the theoretical model and the actual data is shown in Table 3.6. Volatilities of debt and risk premium are 0.3472 and 0.0083 respectively.

[5]The model uses debt ratio at the steady state in order to target the average debt ratio of Argentina between 1980 Q1 and 2017 Q4.

The autocorrelation of output and consumption are 0.9838 and 0.8569. Besides, the model shows a negative relationship between output and risk premium of -0.18806, whilst the relationship between consumption and risk premium is -0.2138. Minimum and maximum values of the of consumption and debt ratios are 71.28%, 95.21%, 0% and 184.73% respectively. The correlation coefficient that ensure the policy function among output and risk premium matches actual data. However, risk premium in the model is less volatile, because the sovereign debtor will choose to default when borrowing is expensive. As a result, in comparison of the previous literature, the statistical results of this thesis can provide a closer fit to the actual data.

3.5 Conclusion

This thesis contributes to the literature on sovereign debt default by modelling incomplete information about borrower's physical capital. We find that incomplete information about a borrower's physical capital affects the borrower's default decision and their optimal capital, consumption and debt.

In our model, the default decision by a borrower will be punished with their financial exclusion and by them bearing the cost of default. These two punishments will result in lower utility in future periods. Equilibrium bond price will be higher than predicted by previous literature as the lender cannot observe the borrower's capital and therefore in equilibrium a higher risk premium emerges. Without productivity shocks the steady-state equilibrium will be characterised by no default occurring. On the other hand, productivity shocks will affect capital, consumption and debt and may lead to the default decision.

Furthermore, we find that borrowers choose to borrow more when it is possible. In comparison with previous literature, our model simulations indicate a higher level of debt (in and out of the steady state) which fits to the actual data for Argentina.

Unobservable capital implies a higher risk premium in equilibrium. In equilibrium, bond price will not be directly affected by a change in future capital investment. The risk premium and bond price emerge when the actual output is revealed. The borrowers cannot change the bond price schedule by changing their future capital investment.

Another novel result is that the steady-state level of physical capital is typically less than in the previous literature because borrowers have no incentive to accumulate capital for influencing the bond price in the current period.

This framework also suggests a smoothing of physical capital when shocks occur, which is not captured by previous literature, as changes in future capital investment have no direct impact on the current bond price schedule.

The results of impulse response functions suggest that it is not optimal to force debtors to use foreign assets to invest only. The borrower will find optimal to consume rather than invest, if physical capital is above the steady-state level. On the other hand, at or below the steady-state level of capital, the country will borrow more for both consumption and investment in the presence of a positive shock.

Our simulations with physical capital at or above steady-state level also show consumption smoothing behaviour through sacrificing physical capital during bad periods. This, consistently with previous literature, is because consumers will take decisions on the amount of capital investment and debt in order to keep consumption at an optimal level. However, when large negative productivity shock occurs, we find that borrowers are better off if they reduce consumption and choose to repay their debt in order to avoid default.

The simulation results show that our model is better suited to fitting Argentina's default experience than existing models in the literature and it confirms the importance of limited information about a borrower's physical capital for understanding the behaviour of international borrowers. Therefore, this thesis provides a seminal framework for further studies on sovereign default. An interesting extension would be to model borrowers and lenders with different features; for example, different risk aversion rates and discount rates and the possibility of partial default.

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Appendices

This appendix will be separated into two main parts. The first part will describe the data sources. The second part will describe the computational algorithm on the model of sovereign default and productivity shock with the finite state Markov-Chain approximation (Tauchen, 1986; Tauchen and Hussey, 1991).

3.A Data Description

The data series are taken from the Ministry of Economy (MECON) of Argentina and complied by Datastream. These include real GDP, household consumption, government spending that present in real term and external debt as a percentage to GDP. Besides, The overall risk premium on US denominated debt is from JP Morgan and compiled by Oxford Economics. All data are quarterly series with non-seasonal adjustment between 1980 Q1 and 2017 Q4.

3.B Computational Algorithm

In this thesis, we discretise a continuous stochastic process of TFP by using Tauchen's method (1986). The TFP level is defined as follows:

$$a_t = \rho\, a_{t-1} + \epsilon_t \,, \tag{3.B.1}$$

where $|\rho| < 1$ and $\epsilon_t \sim N(0, \sigma_\epsilon^2)$. From this continuous stochastic process, we will assume $\tilde{a}$ as the discrete value from the approximation of a with the finite set of possible realisation $\{a_1, a_2, ...a_N\}$. Tauchen (1986) also suggests that a maximum value of $\tilde{a}$ (a_N) is given by:

$$a_N = m\left(\frac{\sigma_\epsilon^2}{1-\rho^2}\right)^{\frac{1}{2}}, \tag{3.B.2}$$

where m is a multiple of the unconditional standard deviation. In general case, m is equal to 3. From the symmetric assumption of the distribution, the minimum value of $\tilde{a}$ (a_1) is $-a_N$. Besides, $\{a2, a3, ..., a_{N-1}\}$ will be equivalently located between the interval $[a_1, a_N]$ with the distance (d). Therefore, the finite state of possible realisation of $\tilde{a}$ can be obtained as the state space form.

Next, Tauchen (1986) provides a solution to compute the transition probabilities as follows:

$$\pi_{jk} = P\left\{\tilde{a}_t = a_k | \tilde{a}_{t-1} = a_j\right\} = P\left\{a_k - \frac{d}{2} - \rho a_j < \epsilon_t \leq a_k + \frac{d}{2} - \rho a_j\right\}. \tag{3.B.3}$$

If $1 < k < N-1$, the transition probability π_{jk} will be given by:

$$\pi_{jk} = F\left(\frac{a_k + \frac{d}{2} - \rho a_j}{\sigma_\epsilon}\right) - F\left(\frac{a_k - \frac{d}{2} - \rho a_j}{\sigma_\epsilon}\right), \tag{3.B.4}$$

where F is the standardisation process. When $k = 1$ and $k = N$ the transition probability at these boundaries are given by:

$$\pi_{j1} = F\left(\frac{a_1 + \frac{d}{2} - \rho a_j}{\sigma_\epsilon}\right), \tag{3.B.5}$$

$$\pi_{jN} = 1 - F\left(\frac{a_N - \frac{d}{2} - \rho a_j}{\sigma_\epsilon}\right). \tag{3.B.6}$$

From the above method, the possible set of $\tilde{a}$ with its transition probability will converge to a_t when the distance (d) is approaching 0 and N is closed to ∞. Therefore, a is defined by the finite vector of possible realisation of TFP with the interval $[\underline{a}, \overline{a}]$, whilst the continuous stochastic process of TFP is approximated by the given transition probability matrix.

3.C The 31 Finite State Markov-Chain

From the previous section, it derives the quadrature-based method for obtaining approximations of continuous stochastic process. Hence, this section will provide a graphical plot of 31 finite state Markov-Chain of the TFP.

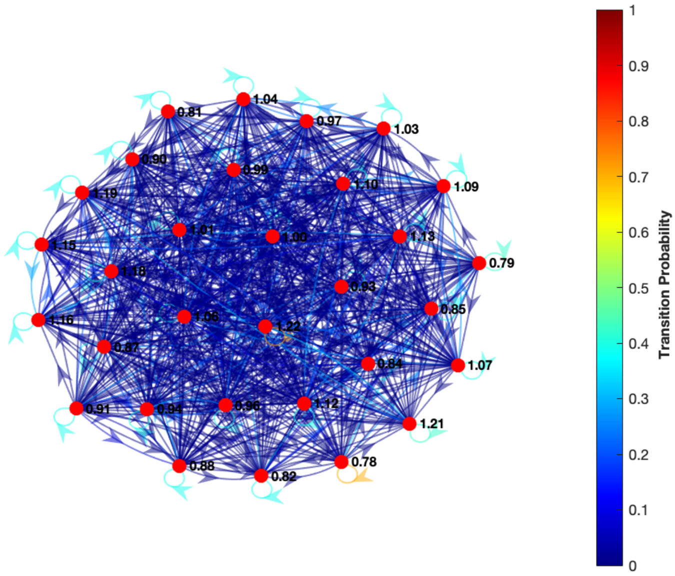


Figure 3.10: 31 Finite State Markov-Chain Approximations

Figure 3.10 gives the transition probabilities of TFP. The cool tone indicates a low transition probability, whilst the hot tone is a high chance of transition from one to either the current or another state. The number on each node denotes the state value of TFP. The maximum and minimum are 1.22 and 0.78 respectively.

Chapter 4

Sovereign Default, Debt Relief and Its Timing

4.1 Introduction

This chapter provides extension of partial default from the previous seminal framework of sovereign default and unobservable physical capital. Sovereign debt relief has been widely discussed in the international financial market since 1920 following the debt rescheduling of many European countries, including the UK. Numerous countries had experienced a sovereign default and had asked for bailout or debt relief. Remarkably, average debt relief for the middle-high income countries in recent decades is approximately 36.1% of the external public debt, which is crucial (Reinhart and Trebesch, 2016). Several literatures have attempted to provide an optimal amount for debt relief and its timing (Panizza et al., 2009) and modelling for its implications (Romero-Barrutieta et al., 2015). However, their results are still unclear, having several restrictions in the models and leaving a puzzle to be solved for the latter discussion.

From the stylised facts, many countries have made agreements with international bondholders and re-entered from default within a short period of time in order to avoid debt hangover. Some countries, such as Argentina, Brazil and Mexico, spent several years in debt restructuring before re-entering the international financial market. The amount of debt relief for defaulters and the participation rate of bondholders also varies. The creditors want to minimise the loss of principal and interest as well as avoiding the non-performing periods, whilst debtors need to maximise the amount of debt reduction and shorten debt hangover. Consequently, the optimal amount of debt relief and its timing are still a puzzle for sovereign defaulters and international bondholders. Therefore, this chapter will aim to provide a theoretical model for sovereign default and debt relief. The optimal amount of debt reduction and the period for debt hangover will be thoroughly computed.

In this chapter, we will add the option of partial default (debt relief) into the seminal framework of the sovereign default model from the previous chapter. We also borrow the parameter calibration from the previous chapter that targets the quarterly data of Argentina's economy between 1980 Q1 and 2017 Q4. The continuous structure of the stochastic process will be approximately transformed into the discrete state of the Markov chain with discrete variability of the state variables, physical capital and foreign assets. Therefore, the probability of default and partial default, including the optimal amount of debt relief and its timing, can be solved with respect to the current total factor productivity (TFP) shock, foreign assets and capital accumulation.

Moreover, our results also suggest that debt relief can encourage both consumption and capital investment, which contradicts the previous literature of Romero-Barrutieta et al. (2015); in which debt relief leads to higher consumption and lower capital investment. Notably, our results are sustained with the stylised facts from Fonchamnyo (2009) and Arslanalp and Henry (2005) that show statistical evidence of an increase in capital investment after the period of debt relief. Moreover, the optimal percentage for debt relief is also around the suggestion from an evidence of actual statistics for the overall debt relief in several countries (Miller and Thomas, 2007).

This chapter will be separated into five main sections. First is the introduction, second is a review of related literature, while data and the methodology is presented in the third part; fourth is the quantitative results and its simulation and the final part contains the conclusion.

4.2 Related Literature

This section provides the stylised facts of sovereign default and debt relief from the related literature. It will give the general idea based on past experience and the theoretical incentive of bailout to sovereign defaulters. Hence, this section will be separated into two main parts for empirical and theoretical studies, respectively.

4.2.1 Empirical Studies

To begin with, Reinhart et al. (2003) and Porzecanski (2004) provide overall statistics pertaining to sovereign indebtedness. From the 16^{th} to 19^{th} centuries,many developed countries, such as France, Germany and Spain, experienced sovereign default several times. At that time, borrowing by these countries often occurred because of wars and economic transition from agriculture to the industrial era. After that, in the late 20^{th} century, many countries experiencing sovereign default were from emerging markets, namely low-income to middle-income countries. Porzecanski (2004) suggests that sovereign default can be inferred from a portion of debt to government's revenue and the debt to GDP[6] ratio. Cantor et al. (2008)shows the statistical evidence of credit ratings for emerging countries that is significantly correlated with the debt ratio to sovereign revenue. Sovereign bonds with lower credit rating will face a higher bond premium, requiring higher debt service and being more difficult to roll-over. Moreover, in 1995, the ratio of external debt to GDP for Africa, Developing Asia, Middle East, Latin America and Emerging Europe was 72%, 33%, 58.5%, 37% and 38% respectively (IMF, 2004). As a result, countries in Africa and Latin America often faced sovereign default and debt hangover rather than other regions. In addition, after the sovereign default, many countries asked for debt relief or bailout from external debtors. Numerous proposals were made from highly indebted poor countries (HIPC) for debt relief, as stated in the Brady Plan in 1990. Thus, several literatures discuss incentives for debt relief to both debtors and creditors. Why should debtors decide on the debt relief?

Reinhart and Trebesch (2016) provide the overall statistics for sovereign debt relief from 1920 to 2010. Debt relief over the past century can be divided into two main eras or four subsections. The first era is the period between 1920s and 1930s, in which the US and UK provided war debt relief to several European countries, such as Austria, Belgium, France, Greece, Italy, Germany and Portugal, at totals of 17.6% and 28.6% to GDP1 (of debtor), respectively. In this era, the resolution was often in soft and hard forms through debt restructuring; replacing the non- performing debt by a new bond with a reduced interest rate and a longer length of maturity.

[6]Reinhart and Trebesch (2016) provide the percentage of debt relief to GDP of the US and the UK respectively.

The second era is the period between 1978 and 2010, in which sovereign defaulters were from the middle-high income emerging market, especially in Latin America and Africa. The average debt relief to these countries, expressed as the percentage of total external debt and external public debt, are 25% and 36%, respectively. In this period, Reinhart and Trebesch (2016) found resolutions were in a softer form with many debt reschedulings through a temporary freeze of payments or bridging loans. Hence, countries struggled with debt restructuring until being alleviated by debt relief.

Furthermore, Reinhart and Trebesch (2016) state that the sovereign crisis can be divided into four subsections, in which the first is the debt rescheduling during the 1920s and the second is the Hoover Moratorium in 1931. Third is the Baker Plan in 1986 and the fourth is the Brady plan in 1990. The last two events were widely acknowledged as the main debt relief for emerging markets over the last few decades. During the first two events, the literature shows a significant increase of real GDP per capita after the last five years of debt relief. On the other hand, there is no significant effect of debt relief on GDP for the last two events but a significant increase in the debt to GDP ratio is found with a lower amount of debt service. The rating of sovereign defaulters after re-entry was significantly higher and they could borrow more at the lower bond premium. Hence, this stylised fact highlights debt relief significantly shortens the period of debt overhang for countries that implemented the Brady plan, agreeing to the conditionality of debt restructuring. Remarkably, the literature also points to the advantage for bondholders where the debt relief reduces the non-performing period (debt overhang) and repays them some principal and interest. However, creditors between 1978 and 2010 had to sacrifice haircuts for the middle-high income defaulters, at approximately 27% (Cruces and Trebesch, 2013).

Furthermore, as widely discussed in the literature and the above stylised facts, debt relief seems to be an important issue for both sovereign borrowers and international lenders. Notably, in corporate finance terms, it may not be as important because the lender can seize collateral assets from defaulters. However, Hornbeck (2004), Gelpern (2005) and Miller and Thomas (2007), state that trade intervention and seizures were not authorised by a court in the case of Argentina and several cases in other countries, especially as trade intervention against defaulters is illegal under WTO and international law. Seizures of land and state-owned enterprises are not always possible, depending on the law of individual countries; most sovereign bonds in the market do not require collateralising with sovereign assets. Motivation for international creditors' litigation against sovereign borrowers is very limited (Alfaro, 2015). Debt relief with conditions of an appropriate restructuring seems to be the last resort of defaulters from debt hangover and creditors being partially repaid. In the 1990s, 16 countries signed on for the Brady initiative plan of debt relief (Reinhart and Trebesch, 2016). 22 HIPCs signed on in 2002 for conditional agreement of debt relief (Edwards, 2003).

Furthermore, under the debt relief for highly indebted poor countries (HIPCs), Fonchamnyo (2009) found empirical results of an increase in capital investment, health care, education, and growth of GDP per capita. The study shows a significant growth of GDP and physical capital for those countries with institutional reforms and either partial or full relief. Specifically, countries that completed the reform and obtained debt forgiveness had significant growth in capital, at 13.8% in 2005. The author suggests that the reformed condition of debt relief to HIPCs tends to improve the behaviour of these sovereign defaulters.

However, post-periods of debt forgiveness can be difficult as they are dependent on the characteristics and initial components of countries. Edwards (2003) suggests three main aspects for considering whether the sovereign defaulter will successfully grow after the debt relief, which are the domestic cost of capital, availability of future loans and future support from outsiders. Cohen (2001) shows statistical evidence that the amount of debt forgiveness was less than the initial promises; however, the successful defaulters who completed on conditions and required agreements sustainably grew.

Next, Nguyen et al. (2005) provided a study of debt relief in 55 low-income countries between 1970 and 1999. The study found that these countries tend to increase output from more borrowing at the threshold between 30 and 37 of debt to GDP ratio and that exceeding debt above this threshold tends to reduce the marginal rate of return on capital as well as decrease investors' confidence. The high debt to GDP ratio of the countries shows the lower capability to repay their debt and a higher probability of default. The author also studied the period after default and debt relief. The reduction of external debt for countries signed on to the HIPC initiative significantly induces growth of GDP per capita from 0.8% to 1.1% and public investment increased from 0.75% to 1% from a 6% decrease in debt service. Nevertheless, it is apparent that external debt tends to rise following an increase in public investment. Defaulters who receive debt relief should therefore spend more on public investment in order to sustain long-term economic growth. For instance, according to the IMF's reform program, countries that require financial support through the program have to strictly follow the advice of a 0.5% annual increase in public investment to GDP ratio (Nguyen et al., 2005).

On the other hand, a study by Rajan and Subramanian (2005) indicates an adverse relationship between debt relief and industrial competitiveness. The relative growth of labour intensity on exporting products is adversely affected by an inflow from debt relief through the exchange rate. Statistical evidence suggests a tendancy for overvaluation of the exchange rate during the period of debt relief. Moreover, the debt relief will induce the labour-intensive sector rather than manufacturing.

Furthermore, Krugman (1988a) provides a discussion on creditors' incentive of debt relief. The author defines debt overhang as the situation when a country has debt larger than the expected present value of future steams of resource transfer. Even at the maximum value of resource transfer with an extreme willingness to repay its debt, the creditor will be unable to get full repayment because the debt is higher. However, the author also indicates that debtors may have an incentive to lend, although there will be an expectation of loss from avoiding an immediate default. In this way, the interest rate will be charged at a higher level in order to cover the probability of a debtor's loss. Nevertheless, the interest rate needs not be too high because the debtor has maximum repayment capability through the present value of resource transfer. Even if creditors charge higher interest rates for more borrowing, they will not get full repayment that exceeds the debtor's resource transfer. Moreover, if there is a chance of sovereign default and debt overhang, debt forgiveness needs to be considered. The repurchase of sovereign bonds is suggested to be unnecessary because, at the maximum level of a debtor's ability, they will not be able to repay. The debt relief can be an incentive to debtors for considering partial repayment; therefore, debt relief can be an incentive for creditors to increase chances of regaining the remaining resource transfer from debtors.

Next, the optimal amount of debt relief and its timing are still important questions. Cruces and Trebesch (2013) provide an empirical study on the debt's haircuts. The main highlight in the literature is the average haircut of 180 sovereign bonds between 1980 and 2010 at 37%. However, there is significantly high variation in these losses with more than half of the sample having haircuts above 53% or below 23%. In addition, the authors give statistical evidence of the relationship between the debt restructuring and subsequent borrowing of sovereign debtors. The results contradict Lindert and Morton (1989) and Miller and Thomas (2007) where they indicate a small punishment for sovereign default with a sharp decrease in bond spread for subsequent borrowing. Conversely, Cruces and Trebesch (2013) show a significant correlation between them with higher loss of creditors from debtors' default resulting in an increase of the spread after debt restructuring. Under the sample of 47 emerging countries, the size of debt relief had a significant impact on the subsequent bond spread of up to 7 years. The increase of subsequent bond spread is highlighted when the debt relief is above 40%. An increase of 1 standard deviation in the current debt relief will increase the bond spread at the 120 basis point between 4 and 7 years after re-entry.

Notably, Cruces and Trebesch (2013) also highlight that a higher amount of debt relief also relates to an additional period of debt hangover. An increase in 1 standard deviation of debt relief will reduce the probability of re-entry approximately 50% to any period after re-entry. Based on this statistic, it can be interpreted that many cases with very short periods of debt hangover would experience very small debt relief. For example, Pakistan in 1999, Ukraine in 1998 and Uruguay in 2003 re-entered the market within 1, 3 and 9 months, respectively. These countries with very short lengths of debt hangover had to fully repay creditors without any debt forgiveness. On the opposite side, Argentina in 2005, Ecuador in 2000 and Russia in 1998 returned to the market within 38, 10 and 18 months with debt reductions of 66.3%, 40% and 37.5%[7], respectively. This result can be interpreted as the amount of debt relief tending to be higher if the period of debt negotiation between the two parties is higher. Porzecanski (2012) states that debtors will be forced to choose between some partial repayment or nothing. Some debtors will choose to sacrifice haircuts, whilst there is still the free rider who waits and expects to regain the full amount of their principal after the creditor's restructuring. Therefore, a longer period of debt hangover means loss of opportunities for bondholders to recover the principal and its interest.

In addition, the stylised facts of default duration and the number of haircuts are sustained with the results from Benjamin and Wright (2009). The empirical study shows that the length of debt overhang has a positive relationship with debt to GDP ratio. The authors study a sample of 73 countries between 1989 and 2006, showing there is statistically significant evidence of a positive coefficient between debt restructuring and debt relief. If sovereign borrowers spend longer in debt renegotiation during default, the rate of debt reduction will be increased. With an average of eight years in debt restructuring, an average loss for creditors from sovereign default will be around 44% in the sample.

More specifically, in the case of Argentina, Reinhart and Trebesch (2016) provide details of Argentina's decision on sovereign default in 2001. In 2001, the debt to

[7] Porzecanski (2005) and Porzecanski (2004) provide a statistical evidence of haircut in discount bond (%).

GDP ratio of the country was only 62.5%, whilst it jumped to 150.6% in 2002 with a decision on default and hyperinflation. The Argentine sovereign debt which would be restructuringd included performing debt and non-performing debt to be restructuringd at 43.3% and 53.3%, respectively. These two main parts of the public debt were required to be restructuringd and reduced with participation rates at 66.3%[8] and 76%, respectively. In this case, the participation rate can be interpreted as only 76%[9] of bond holders accepting the haircuts, whilst the remaining 24% would receive nothing. Miller and Miller and Thomas (2007) show that after debt relief was confirmed, the Argentine bond spread sharply reduced to be close to the spread of emerging markets bond index (EMBI). Debt relief can increase an investor's confidence in the Argentine economy; however, sustainability was not long term as Argentina experienced technical default in 2014 and 2020.

To conclude, review of these empirical literatures can provide an overview of past experience for sovereign defaulters as well as stylised facts over past decades. The statistical evidences are useful for the latter theoretical model provided in this chapter. Our model of sovereign default and debt relief will attempt to determine the optimal rate of haircut and length of debt hangover.

4.2.2 Theoretical Studies

There have been several theoretical studies on sovereign default and debt relief. Regarding discussion in the previous chapter, Eaton and Eaton and Gersovitz (1981)

[8]The ratio of haircut can be varied depending on the present value of bond. In 2005, Cruces and Trebesch (2013) shows the full debt relief with bond swap at only 40% of the principle.

[9]In other cases, the participation rate is almost 100% with the debt relief below 40% (Miller and Thomas, 2007).

and Arellano (2008) give a seminal framework on a government's default decision with respect to size of the output shock. Following these two literatures, several papers have provided improved versions of the model, in addition to ours. In the previous chapter, we developed the theoretical model of sovereign default with an unobservable physical capital[10]. Therefore, this section will provide a review of recent theoretical studies after sovereign default; such as a model with debt relief incentive, bargaining between debtors and creditors, debt renegotiation and the debt Laffer curve.

As previously discussed, under a series of negative shocks, the sovereign bond premium will sharply rise along with the borrower's intention to default. The present value of future resource transfer will be reduced, whilst the debt value is increasing. In order to avoid sovereign default under a serious shock, there will be two options. First, the debtor can choose to sell the physical capital in order to earn some revenue for covering the bond repayment. Second, the debtor may ask the creditor to lower the bond premium or bailout on the debt. However, if the shock is sufficiently large, the default decision is the only option left for sovereign debtors. After sovereign default, the country will fall into a financial autarky or financial exclusion with no ability to borrow in the international debt market. During debt overhang, Krugman (1988a) provides a theoretical framework of incentives for debtors and creditors on shortening the exclusion period. On the creditor's side, there will be a cost of default during any period of financial restructuring. The longer they stay in debt overhang, the longer they will enjoy

[10]Our sovereign model of debt and physical capital can thoroughly estimate the probability of full default; without debt relief, as well as solving the optimal level of physical capital and foreign asset. We provide the convenient ability to the current DSGE framework where the physical capital needs not to be observed dedicatedly.

utility in the lower state; therefore, defaulters have an incentive to re-enter the international financial market to borrow for higher consumption and investment again. On the other hand, creditors have an incentive to regain the principal and interest on their debt and they need to negotiate with debtors in order to minimise their loss. Debt relief is the only solution as this will be an incentive for debtors to partially repay their debt; if the debt value is less than the future steam of resource transfer.

Yue (2010) provides a model with an endogenous rate of debt recovery from periods post-default to the repayment decision. Besides, Bai and Zhang (2012), studied debt renegotiation with a specification of bargaining power between debtors and creditors during debt renegotiation only. The main novelty of these literatures is a game theory between a sovereign debtor and a creditor during periods of high default risk and debt overhang. From these literatures, the model is consistent with the stylised fact where a positive relationship between debt reduction and the initial amount of debt occurs. Debtor countries with small amounts of debt will experience low debt reduction during renegotiation periods; moreover, debtor countries reach higher utility towards default if debt recovery is low. Moreover, a comparison between good and bad states shows that sovereign defaults in bad states occasionally get more debt relief.

Romero-Barrutieta et al. (2015)provide a model of dynamic stochastic general equilibrium (DSGE) with debt relief. The model is computed in a discrete form and the continuous TFP shock is exogenous and approximately transformed into the discrete state space. Besides, default probability and debt relief are given[11]

[11] The default probability and debt relief just randomly occur based on the structure controlled by the logistic regression of the actual data.

based on the stochastic structure of empirical data of Uganda between 1982 and 2006. The theoretical model shows that positive debt relief will increase consumption but decrease capital investment. In this way, the result can be interpreted as defaulters using the reduction on their debt to consume rather than make investments.

Next, Corden (1988) illustrates a significant difference between exogenous and endogenous relief with a minimum consumption level. The theoretical diagram of proportions among output, consumption and investment is illustrated to explain future capability. The author highlights that sovereign default tends to occur when debtors have low capability to repay due to a lack of previous investment. Too much consumption and too little investment in the current period will reduce a capacity to pay their debt. On the other hand, the author also argues that creditors have an incentive to sacrifice additional exogenous relief to regain the initial principal. Hence, the amount of debt relief that should be granted is suggested at the minimum amount required to avert future default.

Furthermore, Sachs (1989) provides a theoretical framework of interactions between debtors and creditors for debt relief with conditionality. In a scenario of extreme indebtedness, the author illustrates the decision tree and argues that creditors have an incentive for partial debt forgiveness, whilst defaulters have a greater motivation to accept the conditions, such as economic reform and trade liberalisation. Besides, Krugman (1988b) presents a theoretical discussion on debt-equity swap, debt buy-back and securitisation as options from debt overhang. However, the author finds that the only option is debt-equity swap, which is mutually attractive to both creditors and debtors.

Besides, Dooley (1989) defines the incentive of debt relief through the leverage buy-out of the existing debt. External investors with higher collective ability than existing creditors will purpose to buy the existing debt at the rate where its return is still above the world interest rate. Consequently, the rate of debt relief can be evaluated based on the purchase intention of external investors as well as the sovereign debtors. Next, Claessens (1990) shows an inference of the debt Laffer curve in which the present value of future debt is changed due to the amount of debt and the probability of partial repayment. If the amount of current debt is high, there will be more probability or less incentive for debtors to fully pay their debt; therefore, the present value of future repayment will be less. Accordingly, as the debt grows high, the market value of debt will be less and the curve is lower than the 45-degree line of full repayment.

Interestingly, as can be seen in the previous discussion, existing literatures have not yet approached the theoretical model for sovereign partial default or estimating debt relief before the debt hangover actually happens. Moreover, the optimal level of debt relief and its timing are still unclear. Only a literature by Romero-Barrutieta et al. (2015) provided the DSGE model with a random debt relief and default probability; however, the relationship between capital investment and debt relief is still against the stylised facts from Fonchamnyo (2009). Therefore, the remainder of this chapter will provide the theoretical model for the sovereign borrower to decide on either full or partial default; where the debtor is preparing to enter and exit from partial repayment. Simultaneously, we also provide an adjustment of bond premium by a foreseen probability of full and partial default options from a debtor's incentive. In addition, we also find the optimality of debt relief and length of financial exclusion that maximise the utility of sovereign borrowers.

4.3 The Model of Sovereign Partial Default

This section provides extension to the seminal model from the previous chapter in order to approach the optimal decision of full and partial defaults with respect to the given amount of debt relief and the financial exclusion (default) period.

4.3.1 The Resource Constraint and Decision Functions of the Sovereign Borrower

Mainly, there are three available choices for a sovereign debtor under a negative foreign asset. First, the debtor can choose to follow the debt agreement through full repayment of their debt. In this way, the debtor will remain in the open economy with an option for subsequent borrowing from the credit market. Second, the debtor can fully default on their debt and there will be no enforcement for seizures of collateral assets. However, the defaulter will be excluded from the credit market with no foreign assets and they will experience a significant drop of output (below the trend). A period of debt overhang will be extremely large because the defaulter will no longer be obligated for any repayment when making re-entry. Third, the debtor can choose to default with a guarantee for future partial repayment. This paper defines it as 'partial default' on the debt and the amount of debt relief and length of debt restructuring will be given prior to the debtor falling into debt hangover. In this way, the length of exclusion is reasonably less than for the full default option; therefore, we can derive repayment, full default and partial default with conditionality as in the following:

A closed economy (when excluded from the international credit market):

$$c = y + (1-\delta)k - k' - \Phi(k', k). \tag{4.3.1}$$

An open economy :

$$c = y + b - q(b', a)b' + (1-\delta)k - k' - \Phi(k', k). \tag{4.3.2}$$

Sovereign Decision function between default and repayment:

$$v_g^0(b, k, a) = \max_{\{d,r\}}\{v_g^d(k, a), v_g^r(b, k, a)\}. \tag{4.3.3}$$

Functions of default from a decision between full and partial default:

$$v_g^d(k, a) = \max_{\{k'\}}\{u(c) + \max(v_g^{fd}, v_g^{pd})\}, \tag{4.3.4}$$

subject to:

$$v_g^{fd} = \beta\mathbb{E}[\theta_1 v_g^0(0, k', a')] + (1-\theta_1)v_g^{fd}(k', a')], \tag{4.3.5}$$

$$v_g^{pd} = \beta\mathbb{E}[\theta_2 v_g^0(b'_{pd}, k', a')] + (1-\theta_2)v_g^{pd}(k', a')], \tag{4.3.6}$$

$$c = y^{def} + (1-\delta)k - k' - \Phi(k', k), \tag{4.3.7}$$

$$c, k' \geq 0, \tag{4.3.8}$$

$$b < b'_{pd} < 0, \tag{4.3.9}$$

where v_g^d is the value function of default based on the decision between full default (v_g^{fd}) and partial default (v_g^{pd}). Notably, with full default decision, the sovereign borrower will receive the probability of re-entry at θ_1 or an exclusion period at $(1-\theta_1)$ and will re-enter with zero debt ($b'=0$), as given in Equation 4.3.5.

On the other hand, the debtor can negotiate with the creditor and promise to partially repay their debt when re-entering the credit market. Hence, with the partial default decision in Equation 4.3.6, the sovereign debtor will receive the probability of re-entry at θ_2; where $\theta_2 > \theta_1$, and the debtor will guarantee to repay a partial debt at b'_{pd}; where $b < b'_{pd} < 0$ (see Appendix).

Next, if the borrower has not yet chosen partial default in the previous period and has no guarantee for partial repayment, the value function of repayment is defined as the following:

$$v_g^r(b,k,a) = \max_{\{b',k'\}} u(c) + \beta\,\mathbb{E}\,v_g^0(b',k',a'), \tag{4.3.10a}$$

subject to:

$$c = y + b - q(b',a)b' + (1-\delta)k - k' - \Phi(k',k), \tag{4.3.10b}$$

and

$$c, k', q(b',a) \geq 0. \tag{4.3.10c}$$

On the other way, if the borrower has chosen the partial default and being obligated for the partial repayment when re-entry, the repayment function with the obligation of partial repayment can be derived as follows:

$$v_g^r(b_{pd},k,a) = \max_{\{b'\geq 0,k'\}} u(c) + \beta\,\mathbb{E}\,v_g^0(b',k',a'), \tag{4.3.11a}$$

subject to:

$$c = y + b_{pd} - q(b', a)b' + (1 - \delta)k - k' - \Phi(k', k), \tag{4.3.11b}$$

and

$$c, k', q(b', a), b' \geq 0, \tag{4.3.11c}$$

where b_{pd} is the partial amount of debt the sovereign borrower is obligated to repay when making re-entry into the market. Notably, from the above constraint, the borrower has no ability to borrow or roll-over the debt during the period for partial repayment; which is expected to be paid within one period. After the debt has been paid, the sovereign debtor will be allowed to borrow from the creditor again.

4.3.2 Perceived equilibrium of full default and partial default probability

From the previous chapter, the international creditors are passive risk neutral lenders and will supply $-b'$ as long as the expected return is equal to the risk-free rate of interest. This will define the equilibrium (country specific) bond price schedule. Notably, the physical capital cannot be observed by the international lender. The outsider can only guess the optimal physical capital in the steady-state at the very beginning of a computation. The change of output (y) or a transition matrix will be significantly based on the discrete transformation of the stochastic process for TFP (a). Hence, the lender can still infer the bond premium or a borrower's intention to default regarding the TFP shock or the current output. From this condition, the model for sovereign partial default intention can be provided as follows:

A closed economy :

$$c = y^{def} - \delta k^*. \tag{4.3.12}$$

An open economy :

$$c = y + b - q(b', a)b' - \delta k^*. \tag{4.3.13}$$

An expected borrower's intention on default and repayment:

$$v_l^0(b, a) = \max_{\{d,r\}}\{v_l^d(a), v_l^r(b, a)\}. \tag{4.3.14}$$

Value functions on full default and partial default:

$$v_l^d(a) = u(c) + \max[v_l^{fd}, v_l^{pd}], \tag{4.3.15}$$

subject to:

$$v_l^{fd} = \beta\mathbb{E}[\theta_1 v_l^0(0, a') + (1 - \theta_1)v_l^{fd}(a')], \tag{4.3.16}$$

$$v_l^{pd} = \beta\mathbb{E}[\theta_2 v_l^0(b'_{pd}, a') + (1 - \theta_2)v_l^{pd}(a')], \tag{4.3.17}$$

$$c = y^{def} - \delta k^*, \tag{4.3.18}$$

$$c \geq 0, \tag{4.3.19}$$

$$b > b'_{pd} > 0, \tag{4.3.20}$$

where v_l^d is the expected value function of a borrower's default intention without observing physical capital for a decision between full default (v_l^{fd}) and partial default (v_l^{pd}).

Next, the repayment function without observing physical capital can be expressed as follows:

$$v_l^r(b,a) = \max_{\{b'\}} u(c) + \beta\,\mathbb{E}\,v_l^0(b',a'), \tag{4.3.21a}$$

subject to:

$$c = y + b - q(b',a)b' - \delta k^*, \tag{4.3.21b}$$

and

$$c, q(b',a) \geq 0. \tag{4.3.21c}$$

Notably, under the partial default decision, the probability of re-entry from partial default (θ_2) and the partial repayment of debt (b'_{pd}) must be given by the lender. In this way, the lender can also foresee a borrower's intention for full default and partial default before the event actually happens. Hence, the probability of full default and partial default can be calculated and derived to adjust the bond premium in the bond price schedule, which is shown in the following section.

Thereby, the bond price schedule from the probability of full default and partial default with bailout can be determined. From the previous chapter, the bond price schedule with an option of full default only can be characterised as the following:

$$q(1+r) = \Psi^{fd} \cdot \mathbb{E}[R^{fd}] + (1 - \Psi^{fd}) \cdot \mathbb{E}[R^{fr}], \tag{4.3.22}$$

where probability of full default is Ψ^{fd}, and expected return on full default and full repayment are $\mathbb{E}[R^{fd}] = 0$ and $\mathbb{E}[R^{fr}] = 1$, respectively. Hence, the bond price of an option with full default only is given as:

$$q = \frac{1 - \Psi^{fd}}{1+r}. \tag{4.3.23}$$

Next, the bond price schedule with an extension of partial default can be derived using the following:

$$q(1+r) = \Psi^{fd} \cdot \mathbb{E}[R^{fd}] + \Psi^{pd} \cdot \mathbb{E}[R^{pr}] + (1 - \Psi^{fd} - \Psi^{pd}) \cdot \mathbb{E}[R^{fr}], \tag{4.3.24}$$

where $\mathbb{E}[R^{fd}] = 0$, $\mathbb{E}[R^{fr}] = 1$. Hence, the equation can be re-written as below:

$$q(1+r) = \Psi^{pd} \cdot E[R^{pr}] + (1 - \Psi^{fd} - \Psi^{pd}). \tag{4.3.25}$$

Moreover, the expected return on partial default $\mathbb{E}[R^{pr}]$ will be derived from the partial repayment with bailout and the probability of re-entry (θ_2) as follows:

$$\mathbb{E}[R^{pr}] = \frac{1}{1+r} \left\{ \theta_2\lambda + \left(\frac{1-\theta_2}{1+r}\right) \theta_2\lambda + \left(\frac{1-\theta_2}{1+r}\right)^2 \theta_2\lambda + ... + \left(\frac{1-\theta_2}{1+r}\right)^\infty \theta_2\lambda \right\}, \tag{4.3.26}$$

$$\mathbb{E}[R^{pr}] = \frac{\theta_2\lambda}{1+r} \left\{ 1 + \left(\frac{1-\theta_2}{1+r}\right) + \left(\frac{1-\theta_2}{1+r}\right)^2 + ... + \left(\frac{1-\theta_2}{1+r}\right)^\infty \right\}, \tag{4.3.27}$$

$$\mathbb{E}[R^{pr}] = \frac{\theta_2\lambda}{1+r} \cdot \frac{1+r}{r+\theta_2}, \tag{4.3.28}$$

$$\mathbb{E}[R^{pr}] = \frac{\theta_2\lambda}{r+\theta_2}. \tag{4.3.29}$$

Therefore, the expected return on partial repayment in Equation 4.3.25 can be substituted by Equation 5.3.32 as follows:

$$\begin{aligned} q(1+r) &= \Psi^{pd} \cdot \frac{\theta_2\lambda}{r+\theta_2} + (1 - \Psi^{fd} - \Psi^{pd}) \\ &= 1 - \Psi^{fd} - \Psi^{pd} \left(1 - \frac{\theta_2\lambda}{r+\theta_2}\right), \end{aligned} \tag{4.3.30}$$

$$q(b', a) = \left[1 - \Psi^{fd}(b', a) - \Psi^{pd}(b', a) \left(\frac{r + \theta_2(1-\lambda)}{r+\theta_2}\right)\right] \cdot \frac{1}{1+r}, \tag{4.3.31}$$

where the probability of full default and repayment are denoted by Ψ^{fd} and Ψ^{pd}, the probability of re-entry from partial default is θ_2 and partial repayment is λ; where $0 \leq \lambda \leq 1$. Notably, with partial default, the international lender has to sacrifice some amount of lending or bailout at $1 - \lambda$.

Moreover, the percentage of debt relief to the initial debt $(1 - \lambda)$ can be derived as follows:

$$\Theta = 1 - \lambda = 1 - \frac{b_{pd}}{b}, \tag{4.3.32}$$

where the rate of debt relief is denoted by Θ and b_{pd} is the chosen amount of the partial debt approximation (see appendix) at the subsequent borrowing.

4.4 Data and Methodology

This section provides detail of data and methodology in the theoretical model for sovereign debt with full and partial defaults options (debt relief). Parameters in this theoretical model are calibrated to target the quarterly data for Argentina's economy from 1980 Q1 to 2017 Q4; details of the calibration can be seen in the previous chapter (see Table 3.2 in Chapter 3)[12]. The value of parameters are shown in the following table.

[12] In the previous chapter, the sovereign model of full default is solely used for the calibration to target Argentine economy. For a comparison between the model of full and partial defaults, all parameters will be obtained from the previous chapter (see Table 3.2 and Table 3.3).

Table 4.1: Model specific parameter values

Parameters		Values
Stochastic structure (TFP)	ρ_a, σ_ϵ	0.982 , 0.014
Discount factor	β	0.95
Risk aversion of borrower	γ_g	2
Risk aversion of lender	γ_l	2
Risk-free rate	r	0.01
Capital share	α	0.35
Capital depreciation	δ	0.05
Capital adjustment cost	Φ	2.4
Output default cost	χ	7%
Probability of re-entry from full default	θ_1	5%
Probability of re-entry from partial default	θ_2	$\{0, 100\%\}$
Partial debt repayment ratio	λ	$\{0, 100\%\}$

As can be seen in Table 5.1, the probability of re-entry from full default is set at 0.05, whilst the probability of re-entry θ_2 and debt repayment ratio λ from partial default will be computed from 0 to 1 with a distance of intervals at 0.01. Notably, this chapter aims to provide a full computation in order to target the optimal value of lender's bailout and the period of financial exclusion and therefore, the range of θ_2 and λ will be implemented. Notably, the availability of partial default's option is controlled with the possible range of maximum and minimum allowance of foreign asset at $\{-4.5, 0.5\}$ (see Appendix).

Next, this chapter uses a discrete model following the Tauchen's method (Tauchen, 1986) to approximately transform the continuously stochastic structure into the discrete-state Markov chain (Tauchen, 1986). Utilising the AR(1) process, the stochastic structure of the specific parameters in Table 5.1 is discretised into a 31-state Markov chain.

Furthermore, the discrete model requires specifying the possible length of state variables; physical capital and foreign asset. According to Argentina's economy from 1980 Q1 and 2017 Q4, physical capital $\{\underline{k}, \overline{k}\}$ are set between $\{2, 13\}$ and foreign assets $\{\underline{b}, \bar{b}\}$ are set between $\{-4.5, 0.5\}$, respectively. The upper and the lower bounds of these two variables are arranged to cover the possible value of capital and foreign assets from the past. Specifically, the number of grids for TFP, foreign assets and physical capital are arranged at 31, 51 and 71 (see Appendix).

To conclude, the option of partial default with debt relief will be added into the sovereign model with a specific range of parameters for probability of re-entry (θ_2) and partial debt repayment (λ). In order to find the optimal value of these two variables, the Monte Carlo simulation over 100,000 periods for each set of parameters will be presented.

4.5 Quantitative Results

This section will provide the quantitative results of the computation and simulations of the theoretical model of sovereign decisions on full default and partial default, as well as the optimal debt relief. It is separated into three main parts. First, it will give an overall result to approaching optimal debt relief and its timing. Second, the interactions of sovereign debtors from debt relief will be provided. Third, it will present bond price schedules with various level of perceived debt relief. Finally, the overall results for the Monte Carlo simulation among different parameters will be shown.

4.5.1 Debt Relief and Probability of Re-entry

Based on the theoretical model in this chapter, the sovereign value function is based on the decisions from the return on continuing repayment, full default and partial default. If the value function of repayment is the highest, a sovereign borrower will obviously continue to repay their debt and strictly follow the debt agreement. Besides, if the value of full default is highest, the borrower will decide to default and enter into the financial autarky, resulting in the country being excluded from the international market. Markedly, the value of the default decision is significantly based on the probability of re-entry (θ_1), which will control how the defaulter takes decisions for re-entering the international financial market. In the full default decision, the lender will lose all initial loans and the ability to seize a defaulter's assets. On the other hand, there is an option for the debtor to shorten the exclusion period of a full default decision through negotiation with the lender and reaching agreement for partial repayment. Therefore, the value function of the sovereign partial default decision is significantly based on the rate of debt relief (Θ) and the probability of re-entry (θ_2). Notably, the rate of debt relief is equal to $1 - \lambda$ (1 - partial repayment). Thus, this section will show the quantitative outcomes from the theoretical model among various levels of debt relief and their timing.

To begin with, Figure 4.1 provides the scatter plot of value functions at the steady state level of capital (k) and debt (b) among various level of debt relief and their timing. Notably, the probability of re-entry from a full default (θ_1) is set at 0.05 for all computations[13], whilst the probability of re-entry from a partial default (θ_2) lies between 0 and 1 with an interval of 0.01 as well as the debt relief (Θ). Hence, from these given values of debt relief and their timing, we can fully obtain computations

[13] The probability of re-entry from a full default is 5%. It follows the calibration and the stylised fact that the full default without debt relief will take a longer period in debt restructuring. Thereby, the probability of re-entry from full default is expected to be extremely low.

from all ranges of the two parameters and present the scatter plot of value functions for each outcome among different sets of parameters.

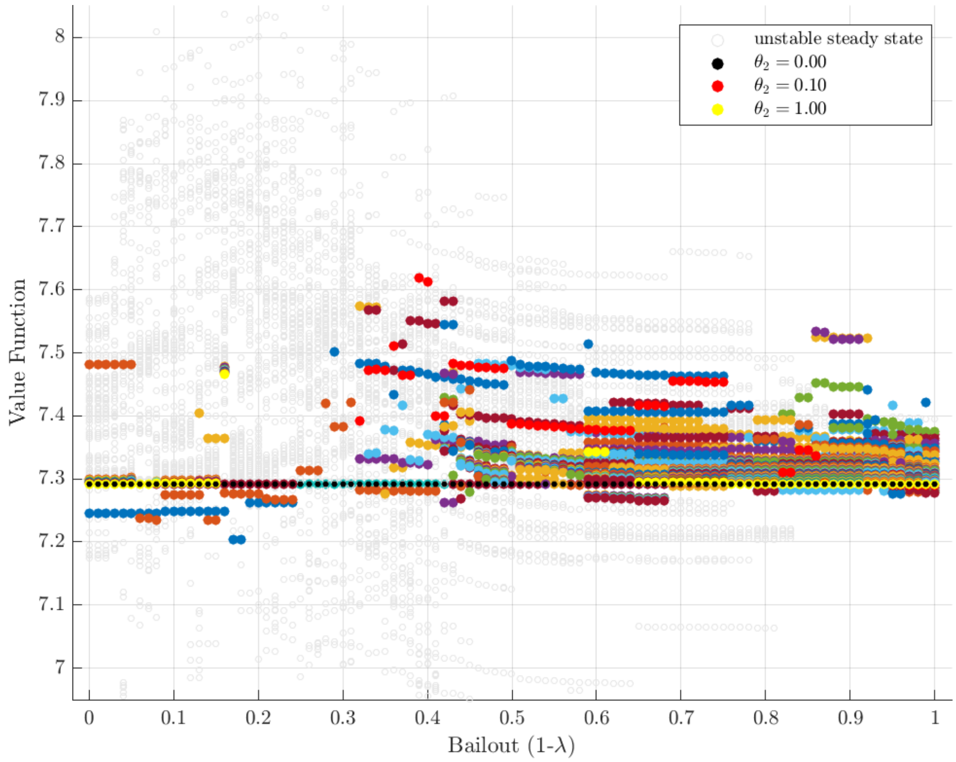


Figure 4.1: Value functions with partial default at the steady state for the given θ_2 and $1 - \lambda$

Figure 4.1 shows the scatter plot of value functions with different colours based on different values for debt relief timing (θ_2). As can be seen, the black dot in the plot illustrates the value functions from the given θ_2 at 0, which means there will be no exit for debt restructuring. The value function in this given parameter is the baseline scenario where the sovereign debtor will clearly choose either repayment or full default instead of partial default in all situations. The value functions with θ_2 between 0 and 0.05 is constant at 7.29 for all value of debt relief (Θ or $1 - \lambda$). This result suggests there will be no difference in the model with an additional option of partial default to the full default model, if $\theta_2 \leq \theta_1$. This is because the full default decision will be more beneficial in all situations.

Next, Figure 4.1 also provides the plot of value functions for all ranges of θ_2 from

0 to 1 with a 0.01 interval. As can be seen in the figure, if θ_2 is greater than 0.05 or θ_1, various colour dots are plotted above the baseline scenario at 7.29. This can imply that an additional option of partial default or debt relief can increase the value functions of a sovereign borrower when compared with the model for full default in the steady-state. However, some given values of θ_2 lead to the weak steady-state level of capital and debt. For instance, some economies may choose to maximise the utility function through always choosing the partial default with an expectation of receiving debt relief in either the steady-state or an occurrence of a very small TFP shock.

Additionally, the option of partial default will increase the utility for a borrower with a big debt to gain more from a lender's sacrifice and, thereby, some values for the debt relief and their timing can possibly lead to a borrower always preferring partial default in all situations. It can be said that it will be expected to pay debt relief in the steady-state of a defaulter; hence, these outcomes will be omitted with a grey colour in Figure 4.1. Remarkably, the cluster of grey scatters in the figure shows the density of the unstable state that occurs from the given debt relief (Θ or $1-\lambda$) between 0 and 0.30. This outcome suggests that too low a value of debt relief ($\leq$ 0.30) will create an unstable route for sovereign borrowers that always choose partial default, even under the smallest TFP shock. Notably, from observing value functions of the outcomes over the unstable steady-state, the model with too low a value for debt reduction will create uncertainty of choices from the utilities that are too closed from choosing among repayment, full default and partial default in the steady-state and under low TFP shocks. This outcome can be interpreted as too low a level of debt relief is unable to sustain the economy of defaulters in the long-term and there will therefore be a chance that the sovereign borrower will default again in the near future. On the other hand, a higher value for debt reduction distinctively improves the capability of a defaulter and makes it easier to decide whether to opt for repayment, full default or partial default in each state of shocks.

Next, Figure 4.1 also illustrates the maximum value function under the steady-state at 7.59 from the given probability of re-entry (θ_2) and debt relief (Θ) at 0.10 and 0.39, respectively. The value functions from θ_2 at 0.10 are plotted with red dots. As can be seen in the scatter plot, the value functions are above the baseline at 7.29 when the debt reduction is above 0.33. Moreover, the value function at the steady-state will reach its peak at 7.59 when debt relief is 0.39. After that, the value functions are steadily decreased as the relief is increased. From these results, it can be shown that the theoretical model with an option of partial default is important to the sovereign borrower from increasing the utility over the full default model. Moreover, Figure 4.1 can show the optimal decisions from choosing the amount of debt reduction (Θ) and the agreed period of financial exclusion (θ_2). Furthermore, the results can be clearly seen from the three-dimensional heat map in Figure 4.2.

Next, Figure 4.2 shows the three-dimensional data with a heat map over the value functions by giving debt relief (Θ) from 0 to 1 and probability of re-entry (θ_2) between 0.05 and 0.12. The above three-dimensional heat map plot clearly shows the range of value functions with colour. The blue area in Figure 4.1 indicates a low value of utility; most of the blue area is approximately 7.29 as the baseline from a full default model. On the opposite side, the red area indicates the value functions at its peak.

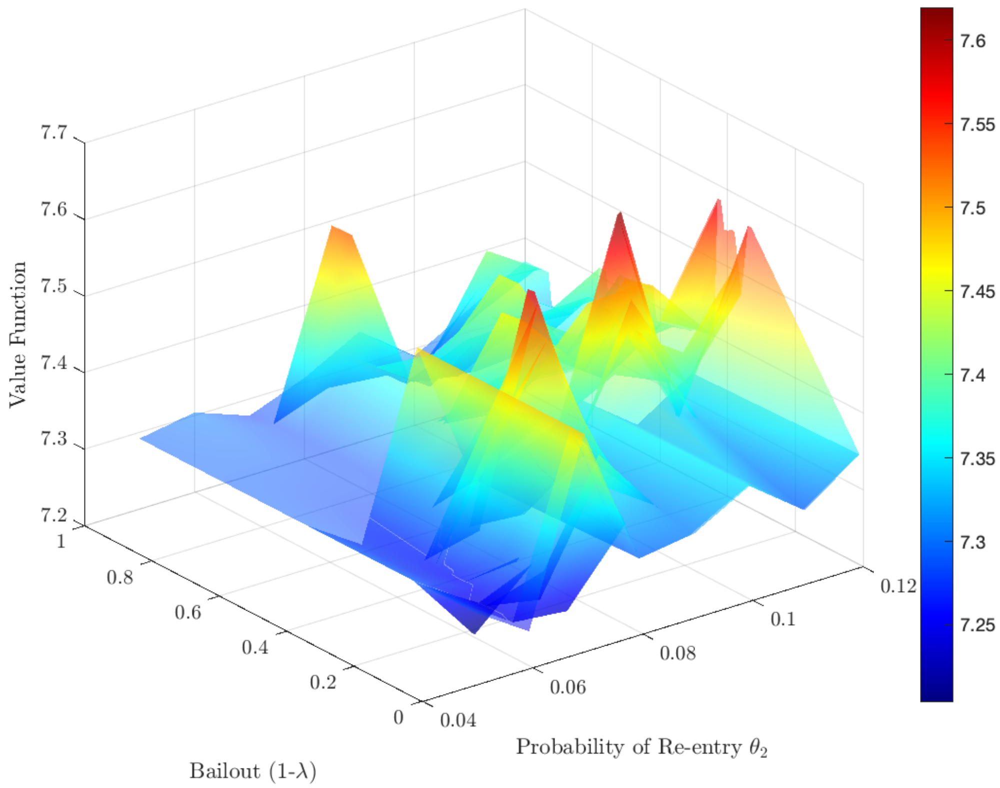


Figure 4.2: 3D heat map of value functions with partial default at the steady-state for the given θ_2 and $1 - \lambda$

Remarkably, the figure obviously shows the peak of value functions for each level of θ_2. In the figure, when θ_2 is 0.05, there is only a blue flat area at 7.29 because of unchanged value functions regardless of debt relief. In addition, from a given θ_2 between 0.06 and 0.12, the optimal debt reliefs (Θ) for each timing (θ_2) are 1, 0.05, 0.32, 0.51, 0.39, 0.46 and 0.43, respectively. In this way, it can be interpreted that a small adjustment of financial exclusion ($\theta_1 = 0.05$, $\theta_2 = 0.06$) does not provide sufficient incentive for a sovereign debtor to shift from full default. On the other hand, higher adjustment of financial exclusion tends to increase the utility with a subsequent debt relief.

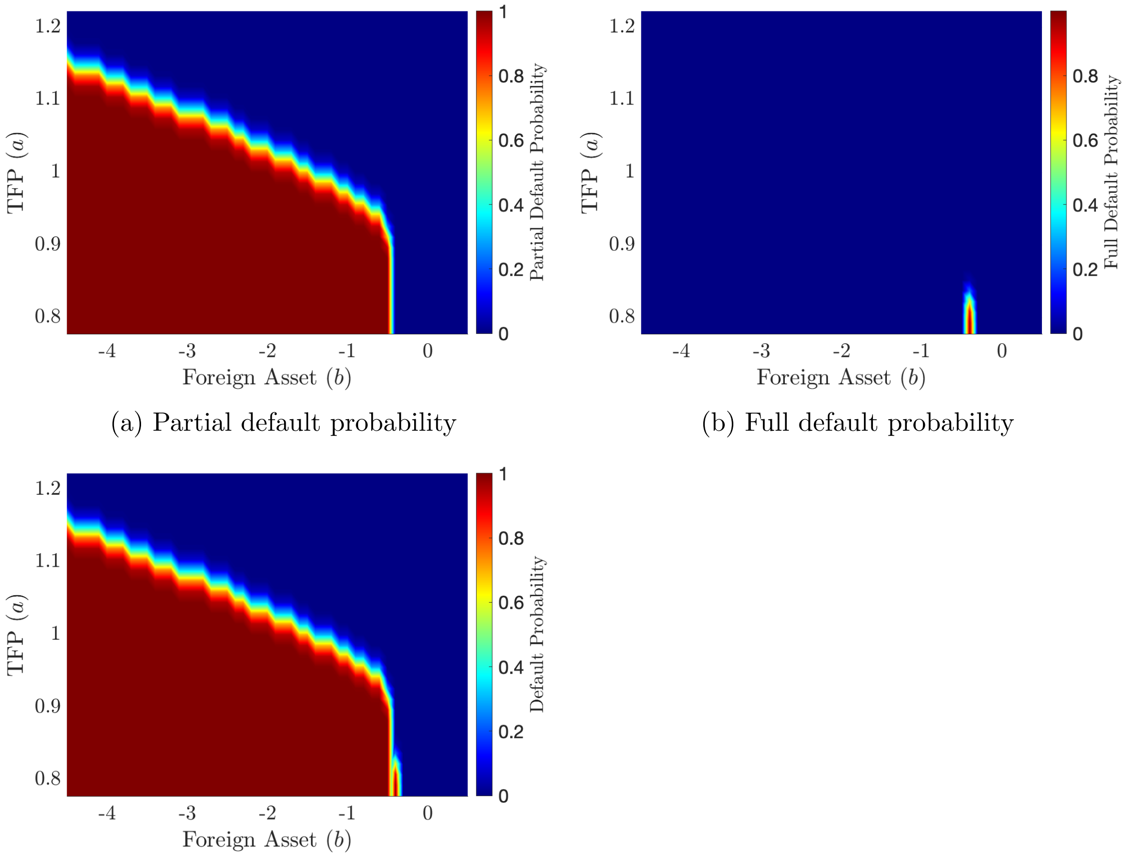
Moreover, the peak values at the optimal bailout for each level of θ_2 are 7.03, 7.48, 7.58, 7.47, 7.62, 7.48 and 7.58, respectively. Interestingly, the optimal bailout for each level of θ_2 is not monotonic. Each level of debt relief timing has its own optimal value for debt relief. However, the highest peak of value functions in Figure 4.2 is

7.62 from the given debt relief and its timing at 0.39 and 0.10, respectively. These results are sustained with an average haircut of 180 sovereign bonds between 1980 and 2010 at 37% (Cruces and Trebesch, 2013).

Therefore, the result can imply that the theoretical model with an option of partial default will increase the utility of a sovereign borrower greater than the model with only the option of full default. On the other hand, it also provides an incentive to creditors on debt relief as they can increase their expectation of obtaining the debt remaining on default. Moreover, the international lender who is risk-neutral will rationally foresee the probability of both full and partial default and, therefore, the bond premium will be adjusted in order to cover the risks.

4.5.2 Economic Interactions and Debt Relief

Next, we explore more deeply the outcomes at a specific parameter in order to find the interactions among TFP, capital accumulation, foreign assets and bond price on debt relief and its timing. Under the given probability of re-entry (θ_2) at 0.08 and debt relief (Θ) at 0.86, we can provide three figures to illustrate future probability of default options. Figure 4.3 illustrates the decisions for partial default and full default options at the steady-state level of capital with respect to the total factors of productivity (a) and the foreign assets (b). Figure 4.3a shows the probability of future partial default. As can be seen in the colour bar, the blue shade in the figure illustrates low probability of partial default in the next period, whilst the red area is where the sovereign debtor will choose to default with the guarantee of partial repayment. At the steady-state level of capital, the sovereign debtor tends to partially default in the next period if a severe TFP shock with high borrowing occurs. From the figure, the borrower is required to maintain foreign assets above -0.80 in order to secure future partial default. A high possibility of future partial default will lead to a higher bond premium on the current bond price (see Equation 4.3.31). If the country excessively borrows more to respond to the negative shock, creditors will secure the partial default risk through charging more bond premium at the current bond price. In the opposite side, a positive TFP shock at the steady-state level of capital will provide an opportunity to a sovereign debtor for more borrowing because there will be less incentive for future default and the bond premium will be steady.



(a) Partial default probability

(b) Full default probability

(c) Partial and full default probability

Figure 4.3: Default probability with partial and full default at the steady state level of capital (k) and foreign asset (b)

Besides, Figure 4.3b illustrates the probability of the full defaults option with respect to levels of foreign assets (b) and TFP (a). As can be seen in the figure, the possibility of the full default option is significantly lower than the partial default in Figure 4.3a. At the steady-state level of capital (k), the borrower will decide to fully default without any guarantee for future repayment when an extremely severe shock occurs at foreign assets between -0.50 and -0.30. From comparison between the two figures for partial default, it can be concluded that the partial default option leads to higher utility.

Furthermore, in order to fully illustrate the default probability under the two options, the possibility of partial and full defaults can be combined in Figure 4.3c. The figure shows the probability of chosen choices among these two options with respect to the foreign assets (b) and TFP (a). From this figure, it can obviously be shown that an incentive from sovereign default starts to increase when the foreign asset is below -0.30. Remarkably, without a severe (negative) TFP shock, a borrower at the steady-state level of capital at 7.10 and foreign asset at -0.80 will pay a very small bond premium. From the figure, it can be seen that it will require around 10% negative shock of TFP to induce a possibility of partial default in the next period. Obviously, from this figure, the sovereign borrower at the steady-state will choose the partial defaults option when the debt is large and TFP is low.

In addition, we can also provide full illustrations of partial and full default options at the different levels of physical capital. Figure 4.4 gives the three-dimensional plot of a heat map of default probabilities with respect to physical capital (z-axis), TFP (y-axis) and foreign assets (x-axis).

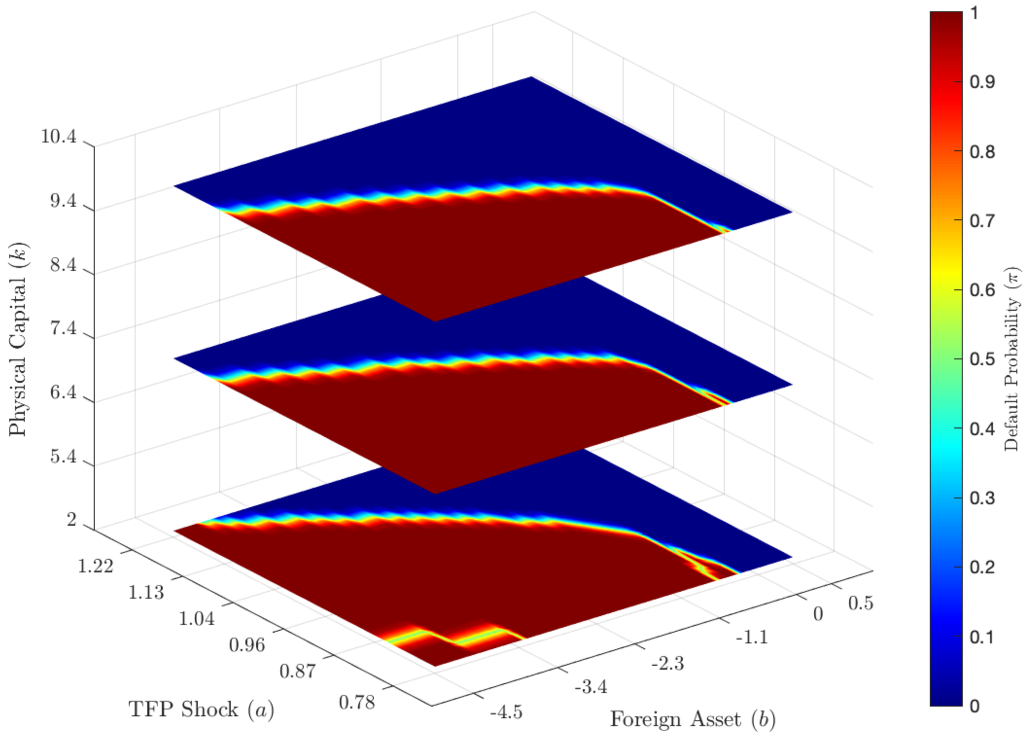


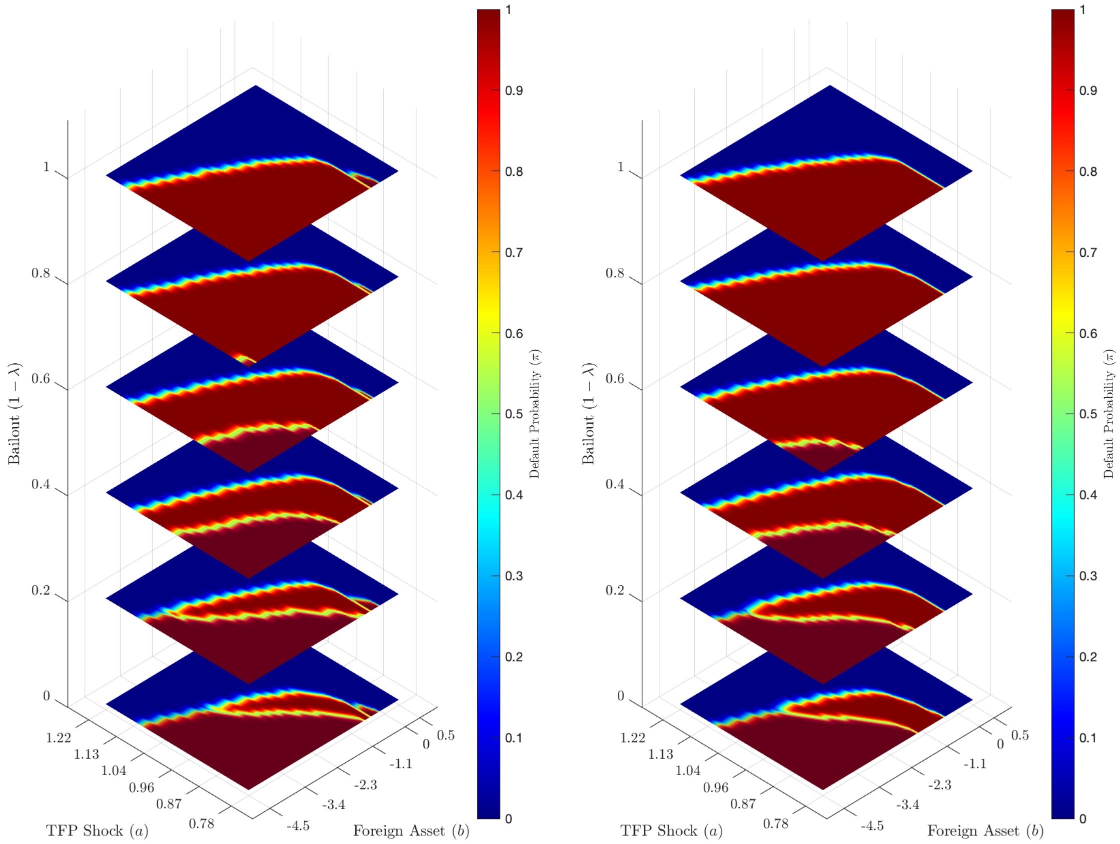
Figure 4.4: Partial and full default probability at different levels of physical capital (k)

In Figure 4.4, the red area indicates where the probability of either full default or partial default is high, whilst the blue area shows the zero-default probability. When the level of physical capital is 7.10 at the steady-state, the figure will show a similar plot to Figure 4.3c. Interestingly, partial and full default decisions will be varied based on the level of physical capital, as is shown in the figure. In Figure 4.4, at the below level of physical capital (first layer from the bottom), the full default probability occurs when the economy with a high level of debt receives a severe shock of TFP, as can be seen in the red and yellow areas at the bottom left of the first layer. Besides, the full default option provides higher utility when there is a large negative TFP shock with a low amount of debt, as can be seen in the figure with a larger area of full default probability. Additionally, the probability of partial default at the low level of capital is larger. With the similar negative TFP shock at the high level of debt, the economy with a low level of physical capital will face a high probability of default, including both the partial and full default options.

In Figure 4.4, the red area indicates where the probability of either full default or partial default is high, whilst the blue area shows the zero-default probability. When the level of physical capital is 7.10 at the steady-state, the figure will show a similar plot to Figure 4.3c. Interestingly, partial and full default decisions will be varied based on the level of physical capital, as is shown in the figure. In Figure 4.4, at the below level of physical capital (first layer from the bottom), the full default probability occurs when the economy with a high level of debt receives a severe shock of TFP, as can be seen in the red and yellow areas at the bottom left of the first layer. Besides, the full default option is more favourable when there is a large negative TFP shock with a low amount of debt, as can be seen in the figure with a larger area of full default probability. Additionally, the probability of partial default at the low level of capital is larger. With the similar negative TFP shock at the high level of debt, the economy with a low level of physical capital will face a high probability of default, including both the partial and full default options.

Next, Figure 4.4 can also indicate the probability of default at the above steady-state level of capital (the top layer in the figure). When the economy has a high level of physical capital, the default probability is significantly lower than at the lower state of capital. The figure shows a smaller area for partial default probability when comparing layers between the economy with high and low levels of capital. In this way, it can be explained that the economy with a high amount of previous capital accumulation will have sufficient assets in order to sustain optimal consumption from appearance of TFP shock. This result is sustained, according to the literature and the previous chapter with the model for only the full default option.

However, with the additional option for partial default, the result obviously shows that a partial default decision will provide an incentive of default where sovereign defaulters can shorten financial exclusion with partial repayments. Notably, the full default option will not be preferred if partial default is available at a high level of capital accumulation. Furthermore, not only does accumulation of physical capital have an effect on future default probability but also the amount of debt relief (Θ) and its timing (θ_2), which have a direct influence on the default decision.



(a) Default Probability at $\theta_2 = 0.08$

(b) Default Probability at $\theta_2 = 0.10$

Figure 4.5: A comparison of default probability (both partial and full default) with different rates of re-entry (θ_2)

Figure 4.5 provides a three-dimensional heat map of default probability from two different levels of θ_2 with foreign assets (x-axis), TFP shock (y-axis) and the amount of bailout (z-axis). Similar to the previous figures, the red area represents a high probability of default, whilst the blue area defines non-default probability. Notably, the darker shade of red also represents the favourability of full default over partial default.

For both figures, the results are plotted at the same level of physical capital (k) and the probability of full default (θ_1) is set at 0.05. Figure 4.5a and Figure 4.5b are given with different rates of re-entry from partial default (θ_2) at 0.08 and 0.10, respectively. Interestingly, a comparison between these two plots shows a significant difference of default scheme. In Figure 4.5a, a decision for full default provides higher utility than partial default if the rate of bailout is relatively low. At zero bailout, the area of full default (dark red) covers most of the total area of default decisions. Moreover, the partial default leads to higher utility when the rate of bailout is increased. This result can be clearly seen in the figure where the area of full default is slightly replaced by partial default when the rate of bailout is higher. The top of the diagram shows that a sovereign borrower with negative TFP shocks will have high probability of choosing partial default, whilst the option of full default is very small.

On the other hand, Figure 4.5b illustrates the default probability with the higher rate of re-entry (θ_2) at 0.10. The result of this diagram indicates a higher decay of full default favourability at each rate of debt relief. Comparison between Figures 4.5a and Figures 4.5b shows that the overall area of choosing default is relatively similar but the full default option has lower utility with θ_2 at 0.10. Additionally, from these two plots, the full default probability in Figure 4.5b is reduced faster than Figure 4.5a when the rate of debt relief is increased.

Therefore, two significant points can be concluded from our comparison of these two figures. First, the partial default option provides higher utility than full default when the probability of re-entry from partial default (θ_2) is high. Second, the probability of full default will be lower and partial default will be higher if the rate of debt relief (Θ) is increased. Partial default leads to higher utility and provides an additional option to a sovereign debtor. Remarkably, the partial default option for a sovereign debtor will not negatively affect the international lender because the bond premium is adjusted to consistent with the probability of partial default. As a result, the outcome is sustained with the stylised fact that several countries often choose to receive bailout or some amount of debt reduction instead of a long period of debt hangover (Edwards, 2003; Reinhart and Trebesch, 2016).

4.5.3 Bond Price Schedules with Perceived Debt Relief

Next, from the probability of full and partial default options, the bond price schedule can be obtained through the Equation at 4.3.31. Figure 4.6 illustrates the multiple lines of bond price at the steady-state level of capital with respect to the future amount of foreign assets (b') and the current bond price (q). In this figure, it is clearly seen that the bond price schedules for all levels of TFP have a positive slope. The bond premium will be low when the future amount of borrowing is less, whilst a high volume of debt will lead to a high bond premium.

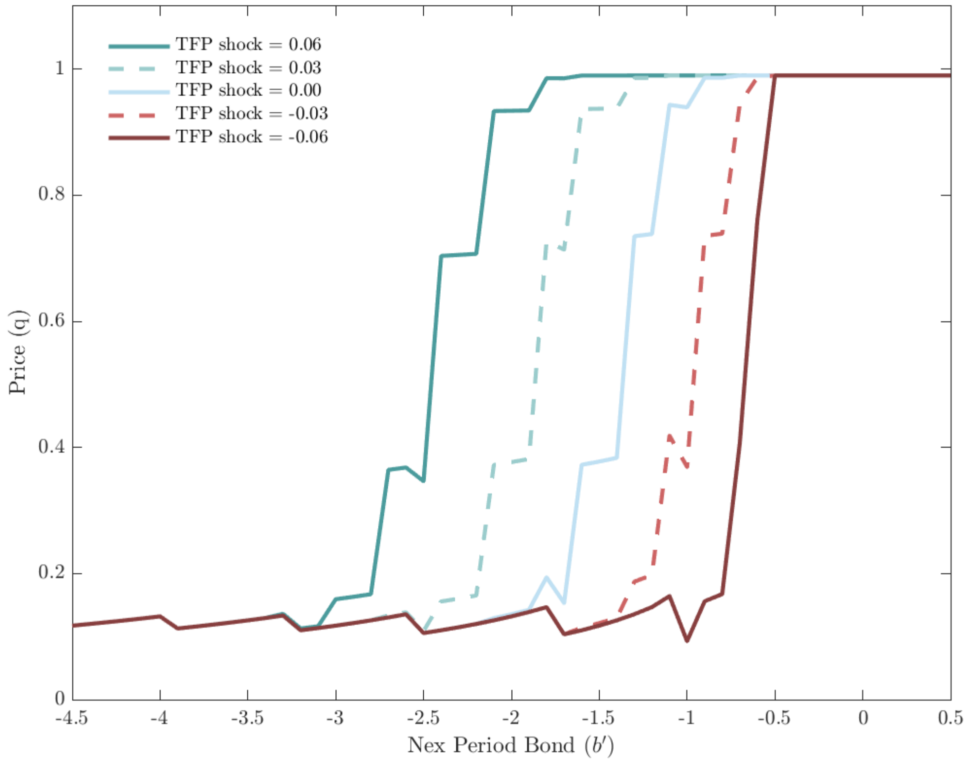
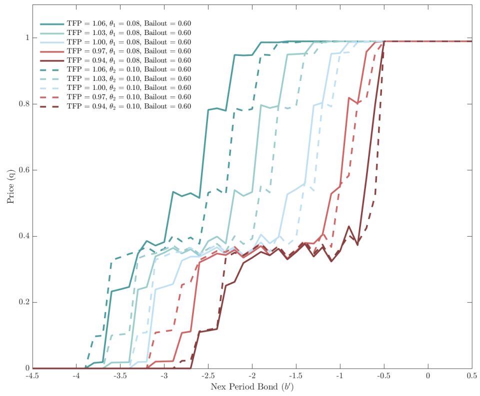
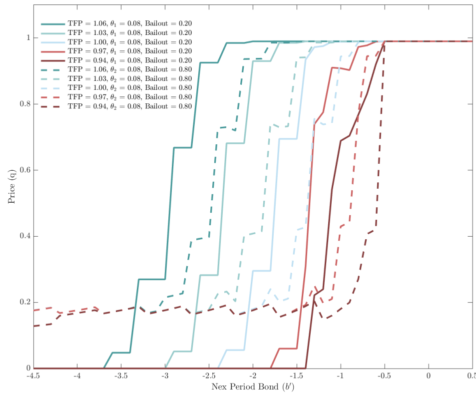


Figure 4.6: Bond price schedule with partial default option at different levels of TFP shock (a)

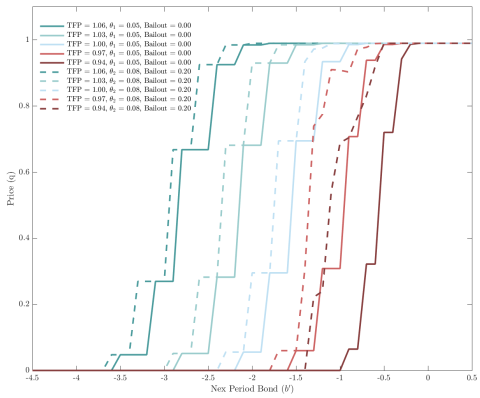
In addition, Figure 4.6 indicates a shift of bond price schedule when there is a TFP shock. A positive TFP shock will shift the bond price schedule to the left, meaning a sovereign debtor can borrow more because the bond premium is lower at the same level of debt. This result is certain with the previous figures where a sovereign debtor has more ability to borrow under higher levels of TFP. On the other hand, the bond price schedule will shift to the right when there is a negative TFP shock. The economy will have less capability to borrow because there will be a higher default incentive in the next period. This result is sustained with the previous results that capture a higher default incentive when a serious negative TFP shock appears.



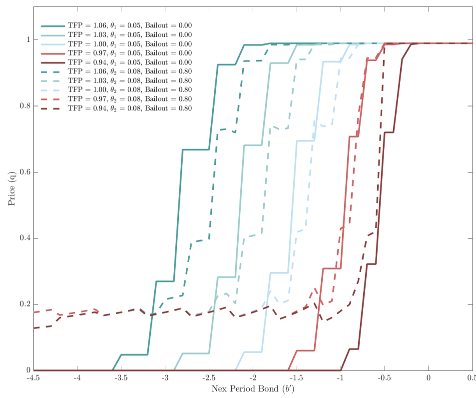
(a) Bond price at different level of θ_2



(b) Bond price at different level of bailout



(c) Bond price with bailout at 0.20



(d) Bond price with bailout at 0.80

Figure 4.7: Bond price schedules with partial default and different sets of parameters

Figure 4.7 provides four diagrams of bond price schedules with different sets of parameters. Identical to the previous figure showing a bond price schedule (Figure 4.6), the diagram provides multiple lines of bond price with respect to various TFP shocks and the amount of borrowing. The bond price schedule is computed at the steady-state level of both physical capital (k) and foreign assets (b). From the figure, it is clear that the bond premium will be higher if a negative shock occurs or there is a higher amount of borrowing (lower foreign assets).

Figure 4.7a shows a comparison of bond price from two different levels of θ_2. The linear lines are plotted with θ_2 at 0.08, whilst the dash lines are plotted with θ_2 at

0.10. Consequently, the bond premium is increased from an increase of θ_2 when the debt is relatively small. At the point where the debt is still small, the bond price with a higher θ_2 will be below the bond price of a lower θ_2, which suggests a lower bond premium at a lower θ_2. In this case, a sovereign borrower with a small amount of debt will be better off if θ_2 is smaller.

However, the opposite is true if the country expects to borrow more in the future. As can be seen in Figure 4.7, the bond price with a high θ_2 is above the bond price with a low θ_2. At the same amount of debt relief at 0.60, the sovereign borrower will have less default probability (less bond premium) with an extremely high amount of future debt under positive TFP shocks or small negative TFP shocks. Interestingly, we find a non-monotonic shift of the bond price schedule from a change in the rate of re-entry following partial default. An increase in θ_2 will also result in a lower bond premium, if the future amount of borrowing is extremely high. The outcome is sustained with the previous figures for partial and full default probability where the sovereign borrower with an extreme amount of debt tends to plan full default. For example, if sovereign borrowers with an extremely large debt choose partial default, they will be obligated to partially repay their debt and the partial amount of debt is still large because of the initial amount. In this way, it leads to a situation where full default presents higher utility than the partial default option. Therefore, an increase of θ_2 which favours the option of partial default can have an inverse impact on the bond premium at an extreme amount of borrowing. It can be concluded that the increased probability of re-entry following partial default (θ_2) will increase the bond premium when debt is low and decrease the bond premium when debt is extremely high (lower utility from partial default).

Next, Figure 4.7b shows a comparison of bond price schedules at different levels of debt relief. The dash and solid lines consecutively define bond price schedules at 0.20 and 0.80 debt relief with the probability of re-entry from partial default at 0.08. From this diagram, it is clearly shown that a higher rate of debt relief at the same level of θ_2 will cause an increased bond premium in general. It is obvious that if lenders sacrifice a higher rate of debt relief in order to encourage borrowers to make partial repayment, it will influence the probability of choosing partial default and increase the bond premium. On the other hand, the bond premium tends to reduce at an extreme amount of borrowing if the rate of debt relief is higher. In this way, it can be explained that a higher rate of debt relief will replace the possibility of choosing the full default option and, thereby, the bond premium can be decreased because the lender can expect to obtain some amount of repayment.

Furthermore, Figure 4.7c and Figure 4.7d compare bond price schedules between the model with and without partial default probability. To present the bond price at the baseline scenario, the solid lines of these two figures are computed with θ_2 and Θ at zero. On the other hand, the dash lines are plotted with θ_2 at 0.08 and Θ at 0.20 in Figure 4.7c and Θ at 0.80 in Figure 4.7d.

As can be seen in the figure, the bond price schedule with a low rate of debt relief leads to a lower bond premium than the baseline for both positive and negative TFP shocks. In contrast, the bond price schedule with a high rate of perceived debt relief provides a higher premium for a bond from the baseline without partial default. These results are sustained with the previous diagrams where the bond premiums will be increased if the partial default option is more interesting.

In conclusion, increased debt reduction and a probability of re-entry will clearly encourage a sovereign borrower to decide more on the partial default and increase the possibility of future default at low and moderate amounts of borrowing.

4.5.4 Overall Result from Monte Carlo Simulations

We use the Monte Carlo simulation to illustrate the Argentine economy, following the theoretical model with full and partial default options over 100,000 periods for each of the given sets of debt relief and their timing. The results can show how the economy reacts in response to the TFP shocks and frequencies of the three decisions; namely, repayment, full default and partial default. Figure 4.8 and Figure 4.9 show the percentage of economies staying in partial and full default over 100,000 periods, respectively. These three-dimensional heat maps provide the plot of θ_2 (x-axis) and bailout (y-axis) against the default density (z-axis). The yellow area in the figure indicates a high frequency of staying in default over 100,000 periods, whilst the blue area defines the low density of staying in default. Notably, Figure 4.8 has the maximum percentage for staying in partial default (yellow area) at 55%, whilst the maximum density of full default in Figure 4.9 is only 6.23%.

As can be seen in Figure 4.9, the yellow area is widely spread over the figure when θ_2 is 0.05 and sharply reduced until reaching zero when θ_2 is 0.07. Hence, this can imply that an option of full default give higher utility to a sovereign borrower when the probability of re-entry from partial default (θ_2) is less than 0.07 regardless of bailout. In addition, Figure 4.8 shows that partial default will be zero when θ_2 is 0.05. Besides, the partial default option will give higher utility when θ_2 is above 0.05. The higher probability of re-entry from partial default (θ_2) increases the chance of a sovereign borrower being better off from choosing partial default. Moreover, the results for Figure 4.8 relate with the previous figure where the optimal level of bailout will lead to higher utility from the partial default option; the default density in the plot tends to be higher than other levels of bailout.

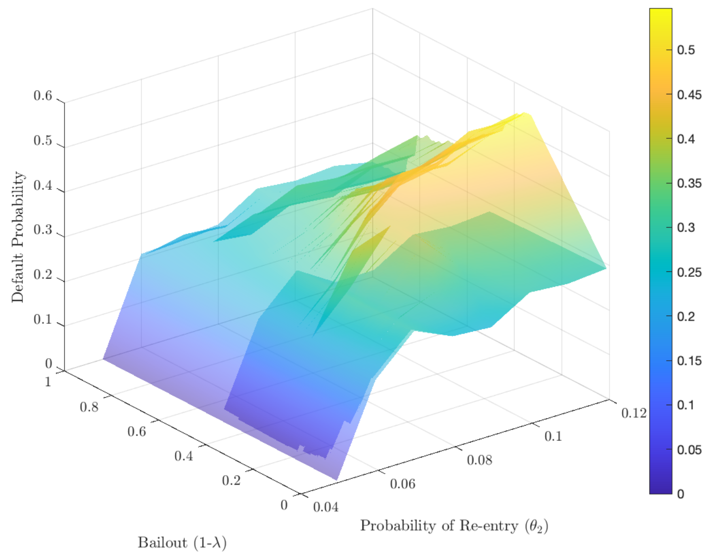


Figure 4.8: Partial default's experiences over the simulations for different parameters

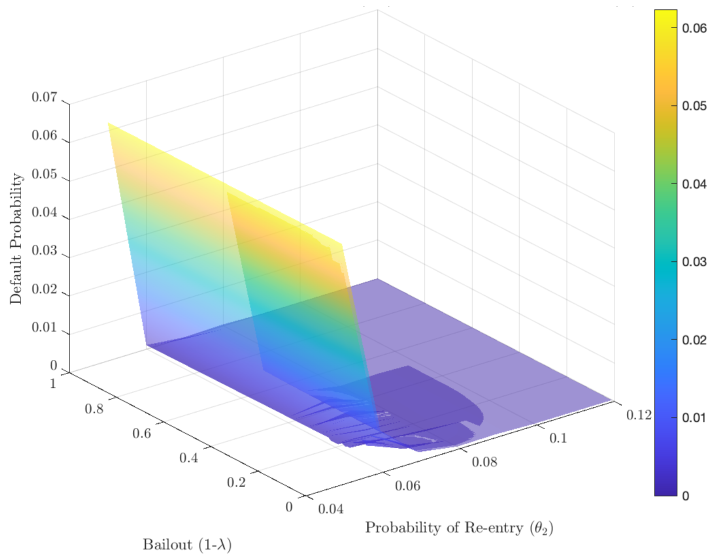


Figure 4.9: Full default's experiences over the simulations for different parameters

4.5.5 Conclusion

This chapter provides a contribution to existing literature regarding optimal debt relief and its timing. In the existing literature for debt relief, a puzzle remains where the debt relief is unclear at the prior period. Thus, we developed the theoretical model from the previous seminal framework for sovereign default with an unobservable physical capital. An option of debt relief with a subsequent length of exclusion are added into the model. The default probability and bond risk-premium are adjusted and perceived in the equilibrium. Thereby, the optimal debt relief, its timing and economic interactions of sovereign debtors can be thoroughly observed, as well as the novelties. Mainly, optimal debt relief and its timing are sustained with the stylised fact of overall defaulters who received debt relief (Miller and Thomas, 2007). We contribute to the previous model for bond price schedules by inducing an adjustment from the seminal framework to cover the partial default incentive. There is a non-monotonic change of bond price scheme due to the relief. Bond premium around the steady-state debt level is increased when debt relief and probability of re-entry are higher, whilst an adverse impact is found at an extreme amount of debt. At a low level of debt relief, defaulters will be unable to sustain the economy in the long-term and frequently decide on default.

Thoroughly, under negative foreign assets, a sovereign debtor is fully obligated to future repayment of its debt. However, there are options for default in which a social planner can choose maximising their own welfare and devalue international creditors. According to the stylised fact, international creditors are ineffective at collecting sovereign borrowing; moreover, trade intervention and seizures are not authorised (Hornbeck, 2004). Defaulters will be forced into a financial autarky and receive serial output shocks during debt hangover. There are two ways to re-enter the credit market. The first is to wait until full exclusion expires and to re-enter without any repayment, whilst the second way is by financial renegotiation with creditors for partial repayment. In this way, our theoretical model provides perceived probabilities for these two options on the bond price schedule with a subsequent borrowing amount before default is unavoidable. The rate of debt relief and its timing are given. With these options, the results show a higher utility of sovereign debtors in the steady-state compared with the seminal framework. This outcome is sustained with the stylised fact where most defaulters accepted debt relief with conditions rather than full default (Edwards, 2003; Reinhart and Trebesch, 2016). An option of partial default allows a social planner to obtain higher utility under serious negative shock from planning on entering and exiting from default with a reduced debt. In addition, too low a level of debt relief is found to be an unstable route for defaulters. Our results show that the model with too low a rate of debt relief cannot be sustained in the long-term.

Next, the probability of both full and partial default is significantly lower when physical capital is high. Interestingly, the two options of full and partial default are found to be chosen differently due to an initial level of capital accumulation. At the steady-state and above, partial default is preferable under negative TFP shocks. Moreover, partial default gives higher utility when the perceived debt relief is increased along with the probability of re-entry. The incentive for sovereign debtors adopting partial default will be higher when future reduction of the debt is high. On the opposite side, at below steady-state, full default is preferred under an extreme amount of debt and negative TFP shocks.

The full default option becomes more interesting if the perceived debt relief is decreased. This outcome can imply that countries with poor economic conditions; low productivity, low capital accumulation and high debt level will choose to default without a guarantee for future repayment. Defaulters with low capital accumulation will prefer to stay in debt hangover rather than repay high amounts of their debt.

Furthermore, under debt relief and a shorter debt hangover, we found that sovereign defaulters have both higher consumption and capital investment compared with the model without debt relief. This can imply that the amount of debt relief is spent on both consumption and physical capital. Interestingly, this outcome is confirmed with statistical evidence of defaulters in recent decades from Fonchamnyo (2009) and Arslanalp and Henry (2005), showing an increase in capital investment after the period of debt relief. On the other hand, our result contradicts the theoretical model of Romero-Barrutieta et al. (2015), in which debt relief leads to higher consumption and lower capital investment.

Moreover, we found a non-monotonic shift of bond price schedules from a change in debt relief and its timing. At the small debt level, the bond premium is higher from quicker and bigger debt relief. Sovereign defaulters have more incentive from partial default if the period of debt hangover is short and the relief is high. They can be better off re-entering quickly and receiving some reduction under a guarantee for partial repayment. On the other hand, we found that the bond premium will be adverse at an extreme amount of debt whereas it is lower when there is a shorter length of debt hangover and higher debt relief. In this way, two reasons are behind this adverse effect. First, a full default option become more interesting; second, creditors can expect some amount of repayment. Under the probability of partial default above zero in the model, the adjustment on the bond price schedule will lower the bond premium compared with bonds for the full default model. Creditors can expect some partial repayment on the principal and interest in the future. The present value of future resource transfer will, at least, be positive instead of a zero return. Therefore, increased probability of re-entry from partial default will increase the bond premium when debt is low and decrease the bond premium when debt is extremely high. Increased debt reduction and the probability of re-entry will also encourage a sovereign borrower to decide on partial repayment and increase the chance of creditors' claims succeeding.

Next, the model can show the optimal level of debt relief for different timings. Each level of debt relief has its own timing for maximising the utility functions of a sovereign defaulter. Additionally, using the Monte Carlo simulation for each set of the given parameters, the overall results show more favourability of the partial default option at the optimal level of debt relief and its timing. The percentage for staying in partial default is increased when the rate of debt relief and probability of re-entry are moved towards optimality.

To conclude, the incentive from partial default is obvious when we discuss the debtors' side as sovereign defaulters will have higher utility from a shorter period of debt hangover and also obtain some amount of debt reduction. Besides, a small length of debt restructuring will lead to a smaller amount of output cost during a default period. Our theoretical model can show a higher present value for the future stream of resource transfer under optimal debt relief and its timing, which is sustained with the statistical evidence of actual data.

Moreover, our model provides an adjustment to the bond premium to consistent with an incentive of partial default. Given the perceived amount of debt relief and its timing, the bond premium can be adjusted based on the perceived probability of future default with respect to various levels of foreign assets and physical capital. The probability of partial default with respect to the size of TFP and a subsequent debt relief can be inferred and covered from an adjustment of bond premium in the model.

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Appendices

This appendix will be separated into two main parts. The first part will describe the data sources. The second part will describe the computational algorithm on the model of sovereign default and productivity shock with the finite state Markov-Chain approximation (Tauchen, 1986; Tauchen and Hussey, 1991).

4.A Data Description

The data series are taken from the Ministry of Economy (MECON) of Argentina and complied by Datastream. These include real GDP, household consumption, government spending that present in real term and external debt as a percentage to GDP. Besides, The overall risk premium on US denominated debt is from JP Morgan and compiled by Oxford Economics. All data are quarterly series with non-seasonal adjustment between 1980 Q1 and 2017 Q4.

4.B Computational Algorithm

In this thesis, we discretise a continuous stochastic process of TFP by using Tauchen's method (1986). The TFP level is defined as follows:

$$a_t = \rho\, a_{t-1} + \epsilon_t \,, \tag{4.B.1}$$

where $|\rho| < 1$ and $\epsilon_t \sim N(0, \sigma_\epsilon^2)$. From this continuous stochastic process, we will assume $\tilde{a}$ as the discrete value from the approximation of a with the finite set of possible realisation $\{a_1, a_2, ...a_N\}$. Tauchen (1986) also suggests that a maximum value of $\tilde{a}$ (a_N) is given by:

$$a_N = m\left(\frac{\sigma_\epsilon^2}{1-\rho^2}\right)^{\frac{1}{2}}, \tag{4.B.2}$$

where m is a multiple of the unconditional standard deviation. In general case, m is equal to 3. From the symmetric assumption of the distribution, the minimum value of $\tilde{a}$ (a_1) is $-a_N$. Besides, $\{a2, a3, ..., a_{N-1}\}$ will be equivalently located between the interval $[a_1, a_N]$ with the distance (d). Therefore, the finite state of possible realisation of $\tilde{a}$ can be obtained as the state space form.

Next, Tauchen (1986) provides a solution to compute the transition probabilities as follows:

$$\pi_{jk} = P\left\{\tilde{a}_t = a_k|\tilde{a}_{t-1} = a_j\right\} = P\left\{a_k - \frac{d}{2} - \rho a_j < \epsilon_t \leq a_k + \frac{d}{2} - \rho a_j\right\}. \tag{4.B.3}$$

If $1 < k < N-1$, the transition probability π_{jk} will be given by:

$$\pi_{jk} = F\left(\frac{a_k + \frac{d}{2} - \rho a_j}{\sigma_\epsilon}\right) - F\left(\frac{a_k - \frac{d}{2} - \rho a_j}{\sigma_\epsilon}\right), \tag{4.B.4}$$

where F is the standardisation process. When $k = 1$ and $k = N$ the transition probability at these boundaries are given by:

$$\pi_{j1} = F\left(\frac{a_1 + \frac{d}{2} - \rho a_j}{\sigma_\epsilon}\right), \tag{4.B.5}$$

$$\pi_{jN} = 1 - F\left(\frac{a_N - \frac{d}{2} - \rho a_j}{\sigma_\epsilon}\right). \tag{4.B.6}$$

From the above method, the possible set of $\tilde{a}$ with its transition probability will converge to a_t when the distance (d) is approaching 0 and N is closed to ∞. Therefore, a is defined by the finite vector of possible realisation of TFP with the interval $[\underline{a}, \overline{a}]$, whilst the continuous stochastic process of TFP is approximated by the given transition probability matrix.

4.C Approximation of Partial Debt Repayment

In the model of partial default, when the sovereign borrower chooses to negotiate with an international lender for the partial repayment, the borrower is obligated to partially repay the debt as the percentage of the initial amount. The function of partial repayment or debt with bailout can be derived as the following:

$$b_{pd} = \lambda \cdot b \,, \tag{4.C.1}$$

where b is the amount of initial debt, b_{pd} is the amount of debt after re-entry from partial default and λ is a percentage of debt repayment ($0 \leq \lambda \leq 1$). Hence, if the partial default is decided, the lender has to sacrifice a percentage of lending amount at $1 - \lambda$. However, with the discrete model, the amount of partial repayment has to be approximated in order to locate equivalently in the given grid. The amount of partial repayment (b_{pd}) in the grid form is approximately located to the nearest grid within $\pm 3\%$ of the required λ. In addition, the minimum amount of initial debt ($\underline{b}$) for partial default is set at -0.50. There will be only an option of full default, if the initial debt is smaller than the minimum amount ($-0.50 < b$). The discrete grid of the initial debt (b) and the partial debt approximation (b_{pd}) is illustrated as below:

$$
\begin{array}{ccccc}
b & & b_{pd}(1-\lambda \approxeq 0.50) & & 1-\lambda \\
\begin{bmatrix} -4.5 \\ -4.4 \\ -4.3 \\ -4.2 \\ -4.1 \\ -4.0 \\ -3.9 \\ -3.8 \\ -3.7 \\ -3.6 \\ . \\ . \\ . \\ -0.50 \\ -0.40 \\ -0.30 \\ -0.20 \\ -0.10 \\ 0 \\ 0.10 \\ 0.20 \\ 0.30 \\ 0.40 \\ 0.50 \end{bmatrix}
& \Rightarrow &
\begin{bmatrix} -2.3 \\ -2.2 \\ -2.1 \\ -2.1 \\ -2.0 \\ -2.0 \\ -2.0 \\ -1.9 \\ -1.8 \\ -1.8 \\ . \\ . \\ . \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0.10 \\ 0.20 \\ 0.30 \\ 0.40 \\ 0.50 \end{bmatrix}
& \Rightarrow &
\begin{bmatrix} 0.4889 \\ 0.5000 \\ 0.5116 \\ 0.5000 \\ 0.5122 \\ 0.5000 \\ 0.4872 \\ 0.5000 \\ 0.4865 \\ 0.5000 \\ . \\ . \\ . \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
\end{array}
$$

As can be seen in the above vectors, it can illustrate the value of each element in the grid of the initial debt (b), debt after re-entry from partial default (b_{pd}) and amount of bailout or debt reduction ($1 - \lambda$). Notably, to comfort the computation with the discrete form, the value of b_{pd} is required to locate within the finite set of possible debt with interval $[\underline{b}, \bar{b}]$ or $[-4.5, 0.50]$ and the distance for each element at 0.10. Therefore, with the given debt relief at 50% of the initial debt, the amount of debt repayment with debt relief will be approximated to the nearest grid as can be seen in the vector of b_{pd}. Moreover, the percentage of debt reduction to the initial debt ($1 - \lambda$) can be derived as follows:

$$1 - \lambda = 1 - \frac{b_{pd}}{b}, \tag{4.C.2}$$

$$\Theta = 1 - \lambda, \tag{4.C.3}$$

where debt relief is denoted by Θ or $1 - \lambda$. As the above vector of debt relief, with the given Θ at 50% of the initial debt, the ratio of debt reduction is shown at an approximate equivalence to 0.50. In addition, when the foreign asset (b) is equal or greater than -0.50, there will be no option of partial default and as a result, the lender has to sacrifice all of the lending amount with full default decision of a sovereign borrower. Remarkably, the borrower is unable to default with a positive total amount of foreign asset.

Chapter 5

Sovereign Default and Endogenous Debt Relief

5.1 Introduction

In accordance to the previous chapter, the following contains a thorough discussion on the theoretical model for sovereign default to illustrate the optimal level of debt relief and the re-entry period. The quantitative results and stylised facts from the late of 20^{th} century provide evidence to support differences of optimal selections for debt relief in various scenarios; different level of capital, foreign assets, and productivty. Some countries, such as Pakistan in 1999 and Ukraine in 1998, decided to re-enter after a period of default with absolute repayment, whilst some countries, such as Argentina in 2004, Ecuador in 2000 and Russia in 1998, spent several periods in debt restructuring and received some relief (Cruces and Trebesch, 2013). Therefore, this chapter aims to provide a theoretical model for endogenous debt relief enabling a sovereign debtor to optimise social welfare through a selection of ex-post debt relief.

In this chapter, we calibrate parameters by targeting the real business cycle from quarterly data of Argentina's economy between 1980 Q1 and 2017 Q4. For simplicity and comparability of the models, some parameters are borrowed from the related literature and the seminal model for an absolute default, which allows only a full default option. We follow the quadrature-based method to approximately discretise the continuous structure of a stochastic process (Tauchen, 1986). The Monte Carlo simulation with statistical evidence is provided.

The quantitative result can contribute to existing literature on sovereign default and debt relief. We find a dynamic choice for optimal debt relief over various levels of foreign assets and outputs. The bond price schedule with ex-post debt relief is more risk-averse than a bond contract with only an absolute default. The model framework can illustrate the reasons for sovereign default as well as a motive for debt relief from international creditors. Interestingly, the bond price schedule is not monotonic under a change of debt relief only. An option of partial default is preferable if the probability of re-entry is significantly low.

Moreover, we find that partial default gives higher utility if output is high. A full default option is preferable when output is significantly low, regardless of foreign assets. A relief rate between 20% and 40% has high density of occurrences in the simulation, which is sustained to the actual evidence of default and debt relief. The volatility and correlations of bond premium over variables follow actual data. The theoretical model can capture the default density of Argentina's economy; however, we find that the model with endogenous debt relief is more volatile than the seminal model of an absolute default under the same parameters.

Hence, this chapter will be separated into six main sections. First is this introduction and the second is a review of related literature, whilst the data and methodology will be given in the third part. Fourth is the theoretical model of endogenous debt relief, the fifth contains the quantitative results and the model simulations and the final part provides a conclusion for this chapter.

5.2 Related Literature

This section gives a big picture of sovereign default and debt relief from the existing literature and includes stylised facts from the past to the present century, as well as the theoretical models from the seminal framework through to the advanced one. It will explain and give reasons why countries have to prepare for partial default as much as the international creditors. This section is divided into two sub-sections comprising empirical and theoretical reviews.

5.2.1 Empirical Studies

This part illustrates an in-depth overview of sovereign default through numerous studies of empirical literature and stylised facts. There are several reasons for sovereign debtors deciding to default in past centuries, as well as external causes for default, which defaulters used to excuse their failures for debt repayment. In the 19^{th} and 20^{th} century, Reinhart and Rogoff (2008) give statistical evidence of countries in Latin America that spent over 40% of their time in the debt restructuring process. In addition, many European countries experienced default several times with a shorter period of debt restructuring between the 16^{th} and 19^{th} centuries (Porzecanski, 2004; Reinhart et al., 2003). Even in a modern economy, some countries spent less than a quarter in default, whilst some countries faced a long period of debt hangover (Cruces and Trebesch, 2013). In addition, various rates of debt relief were applied after the first implementation of the war relief from the US and UK in 1920 to other European countries (Reinhart and Trebesch, 2016). From this empirical study, there seems to be a complicated puzzle among sovereign default, debt relief and the timing of re-entry; thus, we will study the existing literature in order to draw any interactions among them.

Firstly, sovereign default is found to occur frequently in developing countries rather than in developed countries (Neumeyer and Perri, 2005). In the past, several European countries, such as France, Spain, Germany and the United Kingdom, defaulted on their debts; however, these default experiences appeared in war and pre-industrialised periods. The economies of these European countries are not sufficiently developed. Reinhart (2010) shows an empirical result for developing countries between 1800 and 2009 and that the number of defaulters in developing countries is significantly larger than an empirical result for developed countries. Reinhart (2010) also indicates that developing countries in Latin America defaulted seven times, whilst Asian and European countries default less. From the empirical statistics, it is obvious that sovereign debtors with less output default more. Poor countries with less capital accumulation and lower productivity have fewer choices compared to rich countries with more capital accumulation and higher productivity.

Yeyati and Panizza (2011) studied the density of sovereign default and output, finding significant evidence of default often occurred during bad times rather than good times. Reinhart and Rogoff (2011) provide an empirical study on a default cluster with financial crises. The authors show many sovereign defaults correlated with a large drop of output; for instance, the Argentine defaults of 1980, 1982 and 2001 experienced a significant drop in productivity and output[14]. From these results, the authors suggested that sovereign default and output are more likely to have a negative and non-linear relationship. Moreover, Neumeyer and Perri (2005) studied the volatility of output with a risk of default. In rich countries, output is significantly less volatile than in poor countries. Unsurprisingly, the empirical evidence of Neumeyer and Perri (2005) illustrates volatility of output is also affected by country risk, showing a higher premium of bond appears in a country with more volatility of output.

Furthermore, debtor countries frequently default when the bond spread is significantly large (Uribe and Yue, 2006). Borensztein and Panizza (2009) give the historical record of sovereign default that is found to be another determinant of current bond spread. In addition, Ozler (1992) shows an empirical result for a sovereign debtor with a bad record of default has approximately a 10% increase in bond spread than a good debtor had between 1930 and 1981. This evidence is sustained with Martinez and Sandleris (2011) who asserted defaulters are penalised with a lower reputation. If countries become familiar with default, the future possibility of default tends to be higher. In addition, Panizza et al. (2009) studied 32 countries between 1994 and 2008 and find that the cost of sovereign default on credit spread is significant but temporary. The impact of a default record is not persistent and it will disappear after a few years.

Next, there is evidence to support a negative relationship between default and subsequent credit ratings (Borensztein and Panizza, 2009). A default decision significantly decreases the country's credit rating by 2.5 notches (Cantor and

[14]The large drop of output was also related with a hyperinflation and a significant change in the currency depreciation

Packer, 1996). Cantor et al. (2008) also find that the credit rating of an emerging economy significantly correlates to a proportion of debt and revenue. The composition of sovereign revenue, total debt and debt service has a negative correlation to the country bond premium (Porzecanski, 2004).

Furthermore, Reinhart et al. (2003) provide an empirical study of sovereign default and borrowing amount, finding that a higher debt ratio is a trigger for an increase in default risk. Interestingly, some countries did not default with a debt ratio above 100%; for instance, Guyana in 1982 and Jordan in 1989 defaulted with debt ratios at 214.3% and 179.5% respectively (Reinhart et al., 2003). Conversely, there was evidence of defaulters who decided to default with a small debt ratio; for example, Russia in 1991 and Turkey in 1978 defaulted with debt ratios at 12.5% and 21% respectively (Reinhart et al., 2003). These outcomes suggest debt intolerance levels among countries vary and the bond premium is less volatile in a country with a good record of default. Additionally, the bond premium can be more volatile and very sensitive to the borrowing amount in a country with low intolerance or a bad record of default.

Specifically, the external debt to output ratio in 1995 for Africa, Developing Asia, Middle East, Latin America and Emerging Europe are 72%, 33%, 58.5%, 37% and 38% respectively (IMF, 2004). Comparisons among small economies show countries with a high external debt ratio have high volatile bond premium as well as default experience.

Broner et al. (2014) indicate that the sensitivity of a bond premium is due to individual tolerance levels. If the borrowing amount is still below the debt tolerance, the bond premium will be small. However, if the borrowing amount exceeds the debt tolerance, the bond premium will be very sensitive to the additional amount of borrowing. This explanation is consistent with Reinhart and Rogoff (2008) and Reinhart (2010), in which the authors find a significant increase of debt in the period prior to default.

However, debt ratio may not be the only parameter for default risk because a sovereign debtor may have hidden debt (Reinhart and Rogoff, 2011). Additionally, Schaltegger and Weder (2015) define that a change in debt composition and fiscal adjustment can deviate the default risk. For instance, an increase of 1% in a government revenue base will subsequently decrease risk of default by 48% in the following period. However, sovereign debtors find it very difficult to increase their revenue base or restructuring the composition of sovereign revenue during a bad period and default is therefore inevitable.

Next, the role of debt relief became more essential after the war relief in 1920, the Hoover Moratorium in 1931, the Baker Plan in 1986 and the Brady plan in 1990 respectively (Reinhart and Trebesch, 2016). Especially, the last two events were globally acknowledged as the seminal framework of debt relief for several emerging countries. As a result, it has been inevitable to not include debt relief as a tool in negotiation between defaulters and creditors. The sovereign defaulter is not the only one gaining advantage from debt relief, as international creditors also have motives for giving some relief. In the sovereign bond market, international creditors outside the country have difficulty in forcing sovereign debtors to obey the bond agreement.

Notably, in the case of Argentina, seizures of collateral assets and trade intervention were not authorised by the court (Hornbeck, 2004; Miller and Thomas, 2007). Trade intervention to defaulters is illegal under the WTO and international law (Gelpern, 2005). The motives for international creditors' litigation against sovereign borrowers are very limited (Alfaro, 2015); therefore, debt relief is the only tool for negotiating with a sovereign defaulter. International creditors can request partial repayment on the initial debt with conditions. In 2002, 22 highly indebted poor countries (HIPCs) agreed to a creditor's conditions for an exchange of debt relief (Edwards, 2003).

Moreover, debt relief is necessary for countries with a maximum value for future streams of resource transfer below their debt (Krugman, 1988a). There is no possibility that the defaulter can fully repay if they have insufficient funds and the creditor is certain to lose all of the loan and subsequent interest. Thereby, some relief on the initial debt allows the defaulter to have an ability for a partial repayment as well as the motive to repay.

Additionally, Cruces and Trebesch (2013) provide a statistical report of debt relief for the middle-high income defaulters between 1978 and 2010 showing approximately 27%. From this period, sovereign defaulters struggling with default trap or debt restructuring had been assisted through debt relief granted by their creditors (Reinhart and Trebesch, 2016). There is significant evidence of improved credit rating after a debt restructuring process with conditions of debt relief (Reinhart and Trebesch, 2016). The composition of borrowing amount and debt service is more straightforward and conservative.

Next, Fonchamnyo (2009) indicates a significant increase in capital investment, healthcare, education and growth of GDP per capita in HIP countries. Specifically, countries that completed reform and obtained debt forgiveness had significant growth in capital at 13.8% in the following period. Conditional reform with debt relief is found to be a significant indicator for HIP countries in an improved debt structure. However, the success of debt forgiveness is based on three main dimensions, which are the domestic cost of capital, the availability of future loans and future support from outsiders (Edwards, 2003). If these three dimensions are high, the periods post-default tend to be successful. Cohen (2001) shows statistically evidence that the amount of debt forgiveness was less than the initial promises; however, successful defaulters who completed on conditions and required agreements grow sustainably.

Debt forgiveness can be another source for increasing the GDP per capita through public investment for low-income countries (Nguyen et al., 2005). Under the IMF's reforming program for HIP countries, sovereign debtors who received debt relief were required to strictly follow the guarantee for a 0.5% annual increase in a proportion of public investment and GDP. Nguyen et al. (2005) provide statistical evidence of a study of HIP countries showing there will be a 1% increase in public investment for every 6% decrease in debt service. However, the effects of debt relief can vary due to the production factor. Rajan and Subramanian (2005)find that debt relief is more effective in a labour-intensive sector rather than in manufacturing.

Furthermore, an empirical seminal study on the optimal rate of debt relief by Cruces and Trebesch (2013) shows the proportion of debt haircuts. The main highlight of this study is an average haircut of 180 sovereign bonds from 1980 to 2010 at 37%. However, more than half of the sample had relief below 23% and above 53%. The authors also find statistical evidence of subsequent punishment and default. A higher loss by creditors from a debtors' default will result in an increase in the bond spread after debt restructuring. Under the sample of 47 emerging countries, the size of debt relief has a significant impact on the subsequent bond spread up to 7 years. The increase of subsequent bond spread is highlighted when the debt relief is above 40%. An increase of 1 standard deviation in the current debt relief will increase the bond spread at the 120 basis point between 4 and 7 years after re-entry. These results oppose Lindert and Morton (1989) and Miller and Thomas (2007) who indicate small punishment of sovereign default through subsequent bond spread.

Moreover, a seminal study on the probability of staying in default by Cruces and Trebesch (2013)) shows that an increase in 1 standard deviation of debt relief will reduce the probability of re-entry by 50%. This outcome can demonstrate that countries with small debt relief will suffer shorter debt hangover. For example, Pakistan in 1999, Ukraine in 1998 and Uruguay in 2003 stayed in default only 1, 3 and 9 months respectively. Unsurprisingly, these defaulters made full repayment in exchange for re-entering the credit market. Conversely, Argentina in 2005, Ecuador in 2000 and Russia in 1998 returned to the market within 38, 10 and 18 months with debt reductions of 66.3%, 40% and 37.5%[15]. Hence, the amount of debt relief is higher if the sovereign debtor takes longer in the restructuring process. Porzecanski (2012) states creditors will be forced to decide on either a partial return or nothing. Some creditors will choose to sacrifice some relief, whilst there are plenty of free riders who wait and expect to regain the full amount of their principal after the debtor restructuring process. Therefore, a longer period of debt hangover means loss of opportunities for bondholders to recover the principal and interest.

Next, Benjamin and Wright (2009) provide statistical evidence of a positive relationship between the timing of debt hangover and the debt to GDP ratio. From 73 samples between 1989 and 2006, the length of debt structure and the

[15] Porzecanski (2005) and Porzecanski (2004) provide a statistical evidence of haircut in discount bond (%).

amount of relief have a positive correlation. For every eight years in sovereign default or debt restructuring, the creditors will lose 44% of their loans; hence, a longer time spent in the debt restructuring process will increase ex-post debt relief.

Specifically, Reinhart and Trebesch (2016) provide details of Argentina's decision for sovereign default in 2001. In that year, the debt to GDP ratio of the country was only 62.5%, whilst it jumped to 150.6% in 2002 with a decision to default and hyperinflation. The Argentine sovereign debt to be restructuringd, including performing debt and non-performing debt to be restructuringd, was 43.3% and 53.3% respectively. These two main parts of the public debt were required to be restructuringd and reduced with participation rates at 66.3% and 76% respectively. In this case, the participation rate can be interpreted as only 76% of bond holders accepting the haircuts, whilst the remainder would receive nothing. Miller and Thomas (2007) show that after the debt relief was confirmed, the Argentine bond spread sharply reduced to come close to the spread of the emerging market bond index (EMBI). Debt relief can increase an investor's confidence in Argentina's economy; however, sustainability did not last that long as Argentina experienced technical default in 2014 and 2020.

5.2.2 Theoretical Studies

From the last two decades, there have been many sovereign defaults during financial crises when a government has failed to make debt repayments. Hence, there are several theories concerning future probability of sovereign default. One seminal framework began by Jonathan Eaton and Mark Gersovitz in 1981 who approached the probability of sovereign default using a real business cycle model.

In the seminal model, the sovereign debtor will decide on default based upon the size of output. Unsurprisingly, countries will choose to default if the negative shock is large. After that, Arellano (2008) developed a model to correlate the borrowing amount and timing of the default period, in which default probability is perceived through the borrowing amount and output. Defaulters will be punished by a drop of output and time spent in financial exclusion. Following these seminal frameworks, several papers have provided more advanced and complicated models in the field of sovereign default. Moreover, after the Baker Plan in 1986 and the Brady plan in 1990, a plan for issuing debt relief became widely acknowledged by sovereign debtors and international creditors (Reinhart and Trebesch, 2016). Therefore, this section focuses on theoretical literature in the related fields of sovereign default, debt relief and the timing of debt restructuring.

Firstly, there are several frameworks for sovereign default models. Eaton and Gersovitz (1981) and Arellano (2008) provide seminal models for sovereign default with an absolute default option. A sovereign debtor has the two choices of repayment or default. The sovereign debtor who continues to repay will be in the general equilibrium, whilst the defaulter will be excluded from the credit market as well as penalised by a drop of output during default. In the model, the future probability of choosing default can be inferred from the current level of foreign assets and output. Mendoza and Yue (2012) developed the model by adding micro variables into the firm and household function, in which sovereign default is affected from the allocation of resources factor.

Furthermore, the default decision is significantly related with fiscal policies, including government spending, revenue and taxation. As in the previous discussion, the composition of sovereign revenue and debt service can divert the country risk (Porzecanski, 2004). Cuadra et al. (2010), Liu and Shen (2016), Bi and Traum (2012), Arellano and Bai (2014), Bi (2012), and Aguiar and Amador (2016) obtain a theoretical model for sovereign default with fiscal policies. They assumed that fiscal policy is pro-cyclical, where the sovereign debtor increases tax during bad times and decreases tax during good times. A change in composition of tax and debt service will affect the future probability of sovereign default.

Next, Lizarazo (2013) provides a study of risk-aversion for international investors. The author found the importance of risk-aversion is on the lender's side because it has a significant impact on bond premiums and bond prices. Alternatively, Chatterjee and Eyigungor (2012) give a study of short-term and long-term maturity of bond can have diverse effects on country risk. Besides, the model for sovereign default with political instability is approached by Cuadra and Sapriza (2008) and Citron and Nickelsburg (1987), who presented a model of a small open economy with two political parties. The outcome showed countries with higher degrees of conflict lead to higher rates of default.

Additionally, several literatures give a theoretical structure including sovereign default, debt forgiveness and the timing of restructuring. Aguiar and Gopinath (2006) provide a seminal study of bailout from a third party (IMF) and implement the model with bailout before the default period. Benjamin and Wright (2009) and Bai and Zhang (2012) extended into a study of sovereign default with debt renegotiation. They use a game theory of two agents; borrower and lender. The differences in the defaulting duration are based on the bargaining powers between the two agents and the levels of debt reduction. Yue (2010) additionally provides a model with an endogenous rate of debt recovery from periods of post-default to the repayment decision, finding that defaulters are better off staying in default if the debt relief is not big enough. Moreover, a comparison between good and bad states shows that sovereign default in bad states occasionally receives more debt relief than in the good states. Besides, Corden (1988) gives a theoretical diagram of proportions among output, consumption and investment under exogenous and endogenous relief. The amount of debt relief is suggested at the minimum amount required to avert future default.

Next, Krugman (1988a) provides a theoretical framework in the default time where the two parties have the incentive to shorten the default period. The defaulter has an incentive to return to the credit market, whilst creditors want to recover the loan. In the same way, Dooley (1989) defines the incentive of debt relief through the leverage to buy-out the existing debt. Claessens (1990) shows an inference of debt using the Laffer curve where the present value of future debt is changed due to the amount of debt and probability of partial repayment. If the amount of current debt is high, there will be more probability of less incentive for debtors to fully repay their debt; thereby, the present value of future repayment will be less. From this literature, creditors also have significant motivation to sacrifice some debt in order to gain some return.

In addition, Sachs (1989) give a theoretical framework for interactions between debtors and creditors for debt relief with conditionality with various assumptions provided for the illustration under general and extreme scenarios. Besides, Krugman (1988b) gives a theoretical discussion on debt-equity swap, debt buy-back and securitisation for being options from debt overhang. The author finds that the only option is debt-equity swap, which is mutually attractive to both creditors and debtors.

Finally, Romero-Barrutieta et al. (2015) give a dynamic stochastic general equilibrium (DSGE) with debt relief. Default probability and debt relief are randomly selected under the controlled structure of logistic regression, which finds that positive debt relief will increase consumption but decrease capital investment. Defaulters will consume more from debt relief rather than from capital investment.

To conclude, the existing literature has not yet approached the theoretical model for sovereign default and the posterior function of debt relief. In the literature, debt relief is either fixed or random; moreover, the optimal level of debt relief and its timing are still unclear. Only a literature by Romero-Barrutieta et al. (2015) provided a DSGE model with random debt relief and default probability; however, the relationship between capital investment and debt relief is still in conflict with the stylised facts from Fonchamnyo (2009). Therefore, the remainder of this chapter will provide a theoretical model for sovereign default and the dynamic choice of ex-post debt relief. The sovereign debtor is offered three options, which are absolute default, partial default and repayment. The posterior function of debt relief and its timing of re-entry to the credit market will be given.

5.3 The Model of Endogenous Debt Relief

This section provides a theoretical model for sovereign default and endogenous debt relief. The model approaches a sovereign decision for future default and interactions among capital accumulation, debt amount, bond price and debt relief. The sovereign debtor has a right to prepare for entering default with the posterior function of debt relief and its timing. Simultaneously, international creditors also adjust the availability of a bond and bond price schedule over various choices of debt relief and its timing. Therefore, the model economy of this framework can be derived as in the following.

5.3.1 Utility Function

As stated in the related literature and previous chapters, the representative agent generally follows a constant relative risk-aversion (CRRA) form of utility function as given in:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t), \tag{5.3.1}$$

$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}, \tag{5.3.2}$$

where c is consumption, σ is the risk aversion rate and β is the discount factor. Notably, creditors also perceive the consumption preference of a sovereign debtor following the CRRA function.

5.3.2 Production Functions

As presented in the previous chapter, there are two main production functions for the economy during repayment and default. During a repayment or normal period, the economy will stochastically produce output with the Cobb-Douglas production function, as shown below:

$$y = ak^{\alpha}, \tag{5.3.3}$$

where y is output (GDP), k is physical capital, a is total factor productivity (TFP) and α is a share of capital. In this small economy, a change in domestic capital accumulation has an endogenous relationship with output through the Cobb-Douglas production function. An increase in capital accumulation (k) can positively influence output (y) at the level of capital share (α). In addition, the TFP for normal periods is assumed to follow the stochastic process of AR(1) with a gaussian distribution derived as in the following:

$$a_t = \mu + \rho\, a_{t-1} + \epsilon_t\ , \tag{5.3.4}$$

where $\mu = 1$, $\epsilon_t \sim N(0, \sigma_\epsilon^2)$ and $\rho \leq 1$. Notably, the stochastic process of TFP is an approximated transition probability matrix of continuous distribution of possible realisation[16]. On the other side, the economy during exclusion (penalty) periods from the default decision will receive a cost of default in production process. Thereby, output during default ($\tilde{y}$) will be penalised to be less or equal to output during repayment (y). Hence, the production of this framework can be derived from the Cobb-Douglas as follows:

$$\tilde{y} = \tilde{a}k^\alpha, \tag{5.3.5}$$

where $\tilde{a}$ is TFP during the default period. From the stylised fact, output tends to drop below the trend[17] during default periods (Reinhart and Rogoff, 2011; Tomz and Wright, 2007). Hence, there is a default cost to output as a penalty, if a sovereign

[16]We discretise a continuous stochastic process of TFP by using quadrature method (Tauchen, 1986; Tauchen and Hussey, 1991) (see Appendix)

[17]The trend is defined by an average growth rate of the past GDP (Tomz and Wright, 2007).

debtor decides to enter the financial autarky with a default decision[18]. The upper and lower bounds of TFP during repayment and default can be written as: $a \in (\underline{a}, \overline{a})$ and $\tilde{a} \in (\underline{a}, \tilde{\overline{a}})$ respectively; where $\tilde{\overline{a}} \leq \overline{a}$. Output during penalty periods will be dropped by the penalised cost of output (χ) below the trend. If $\chi = 0$, there will be no default cost and $a = \tilde{a}$ and $y = \tilde{y}$.

5.3.3 The Resource Constraint and Decision Functions of the Sovereign Borrower with Choices of Debt Relief

Firstly, sovereign debtors under a negative foreign asset are generally obligated to the bond contract on future repayment. If debtors fully repay their debt, countries will remain in an open economy with an option for subsequent borrowing from the international credit market. In addition, sovereign debtors also have an option to default on their debt, whilst international creditors cannot enforce seizure of collateral assets. However, defaulters will be penalised in the production function (see Equation 5.3.5) and excluded from the credit market during debt hangover. Thus, the resource constraints over open and closed economies are given as follows:

A closed economy (when excluded from the international credit market):

$$c = \tilde{y} + (1-\delta)k - k' - \Phi(k', k). \tag{5.3.6}$$

An open economy:

$$c = y + b - q(b', a, \theta')b' + (1-\delta)k - k' - \Phi(k', k), \tag{5.3.7}$$

[18]Entering default with full or partial decision will receive the same penalty to the production.

where b is foreign asset, k is physical capital, δ is a depreciation rate, Φ is a cost adjustment of capital and q is bond price.

From Equation 5.3.6 and Equation 5.3.7, the main difference is an absence of foreign asset in a closed economy. Remarkably, the bond price schedule (q) is a function of foreign assets, TFP and the perceived debt relief, which obligates defaulters when seeking re-entry.

Furthermore, sovereign debtors with a negative foreign asset have two main choices for consideration, which are repayment and default. The decision function of these options can be derived from the following value functions.

Sovereign Decision function between default and repayment:

$$v_0(b,k,a,\theta) = \max_{\{v_d,v_r\}} \{v_d(k,a,\theta), v_r(b,k,a,\theta)\}, \tag{5.3.8}$$

where v_0 is the current value of future stream of social welfare from a decision of default (v_d) and repayment (v_r). A sovereign debtor will choose to continue repayment if the value function of a repayment decision is greater than for default. On the contrary, if staying in default is better, the sovereign debtor will refuse to repay and stay in a closed economy under the agreed period as a function of the perceived debt relief. The sovereign default function can be given as follows:

Sovereign default function:

$$v_d = \max\left[v_{d1}(k,a,\underline{\theta}), v_{d2}(k,a,\theta)\right], \tag{5.3.9a}$$

where v_{d1} is the value function of an absolute default ($\underline{\theta}$ and v_{d2} is the value function of default with optimal debt relief (θ). Notably, absolute default is always an option for a sovereign borrower to default. The probability of re-entry $\underline{\theta}$ is given at the global minimum. Besides, the debt relief and its probability of re-entry is optimised and obligated from previous debt borrowing. The current utility of a sovereign borrower at default is obtained from maximising these two options. In addition, the value functions of absolute default and default at optimal debt relief are given as the following:

$$v_{d1}(k, a, \underline{\theta}) = \max_{\{k'\}}\{u(c) + \beta\mathbb{E}[\underline{\theta} \cdot v_0(\tilde{b}, k', a', \underline{\theta})] + (1 - \underline{\theta}) \cdot v_{d1}(k', a', \underline{\theta})]\}, \quad (5.3.10)$$

subject to:

$$c = \tilde{y} + (1 - \delta)k - k' - \Phi(k', k), \quad (5.3.11a)$$

$$c, k' \geq 0, \quad (5.3.11b)$$

$$\tilde{b} = 0. \quad (5.3.11c)$$

and

$$v_{d2}(k, a, \theta) = \max_{\{k'\}}\{u(c) + \beta\mathbb{E}[\theta \cdot v_0(\tilde{b}, k', a', \theta)] + (1 - \theta) \cdot v_{d2}(k', a', \theta)]\}, \quad (5.3.12a)$$

subject to:

$$c = \tilde{y} + (1 - \delta)k - k' - \Phi(k', k), \quad (5.3.12b)$$

$$c, k' \geq 0, \quad (5.3.12c)$$

$$b < \tilde{b} < 0, \quad (5.3.12d)$$

where $\tilde{b}$ is the amount of foreign asset at re-entry, β is a discount factor, $\underline{\theta}$ is the perceived probability of re-entry from absolute default and θ is the perceived probability of re-entry at optimal debt relief. The default function is a sum of current default and future probabilities of staying in default $(1 - \theta)$ and re-entry (θ) to the credit market. Notably, the posterior function of debt relief and timing of financial exclusion are given from the previous period when the sovereign debtor borrows from creditors; debtors are promised to the possible ex-post debt relief and the timing of re-entry into the credit market. Alternatively, the sovereign borrower can also choose absolute default and suffer at the minimum probability of re-entry $(\underline{\theta})$ without relief. In this case, if $\hat{v}_{d1} > \hat{v}_{d2}$, the probability of re-entry will be minimised $\theta = \underline{\theta}$.

Next, if the borrower has not yet chosen default in the previous period and has no guarantee for partial repayment with any debt relief, the value function of repayment is defined as the following:

$$v_r(b, k, a) = \max_{\{b', k', \theta'\}} u(c) \ + \beta \, \mathbb{E} \left[v_0(b', k', a', \theta') \right], \tag{5.3.13a}$$

subject to:

$$c = y + b - q(b', a, \theta')b' + (1 - \delta)k - k' - \Phi(k', k), \tag{5.3.13b}$$

and

$$c, k', q(b', a, \theta') \geq 0. \tag{5.3.13c}$$

The repayment function is a current value of the future stream of a repayment decision. A sovereign debtor maximises social welfare through choosing the amount of future foreign assets (b'), physical capital (k') and ex-post debt relief and the probability of re-entry (θ').

On the other hand, if the borrower has chosen default and being obligated for partial repayment (receiving debt relief), the repayment function with a guarantee for partial repayment can be derived as follows:

$$v_r(\tilde{b}, k, a) = \max_{\{b' \geq 0, k', \theta'\}} u(c) + \beta\, \mathbb{E}\left[v_0(b', k', a', \theta')\right], \tag{5.3.14a}$$

subject to:

$$c = y + \tilde{b} - q(b', a)b' + (1 - \delta)k - k' - \Phi(k', k), \tag{5.3.14b}$$

and

$$c, k', q(b', a, \theta'), b' \geq 0. \tag{5.3.14c}$$

If the sovereign debtor is obligated to partial repayment $(\tilde{b})$, the debtor will re-enter the financial market when the previous debt agreement has been fulfilled. In other words, the sovereign debtor who had been in default must spend one period repaying their debt without the ability for roll-over or subsequent default. After the debt has been paid, the sovereign debtor is allowed to borrow a new amount from international creditors. Thus, the future foreign asset is required not to be negative for one period ($b' \geq 0$).

5.3.4 Perceived Equilibrium of Default Probability

As in the discussion in previous chapters, international creditors are passive risk neutral lenders, and will supply -b' as long as the expected return is equal to the risk free rate of interest. This will define the equilibrium (country specific) bond price schedule. The perceived equilibrium of default probability is derived from an expectation of steady-state capital accumulation. Therefore, the perceived equilibrium of sovereign intention can be provided as in the following:

A closed economy :

$$c = \tilde{y} - \delta k^*. \tag{5.3.15}$$

An open economy :

$$c = y + b - q(b', a, \theta')b' - \delta k^*, \tag{5.3.16}$$

A perceived intention for default and repayment:

$$\hat{v}_0(b, a, \theta) = \max_{\{\hat{v}_d, \hat{v}_r\}} \{\hat{v}_d(a, \theta), \hat{v}_r(b, a, \theta)\}. \tag{5.3.17}$$

where k^* is the steady-state physical capital and $\hat{v}_0$ is the perceived value function of a sovereign debtor's intention for the future repayment and default without observing physical capital. The perceived value functions of sovereign default and repayment are denoted by $\hat{v}_d$ and $\hat{v}_r$, respectively.

The perceived value function of default is given as:

$$\hat{v}_d(a, \theta) = \max\left[\hat{v}_{d1}(a, \underline{\theta}), \hat{v}_{d2}(a, \theta)\right], \tag{5.3.18}$$

where $\hat{v}_{d1}(a, \underline{\theta})$ and $\hat{v}_{d2}(a, \theta)$ are the perceived value functions of absolute default and default at optimal relief respectively. The details of these two functions are shown below:

$$\hat{v}_{d1}(a, \underline{\theta}) = u(c) + \beta \mathbb{E}[\underline{\theta}\, \hat{v}_0(\tilde{b}, a', \underline{\theta}') + (1 - \underline{\theta})\, \hat{v}_{d1}(a', \underline{\theta})] \tag{5.3.19a}$$

subject to:

$$c = \tilde{y} - \delta k^*, \tag{5.3.19b}$$

$$c \geq 0, \tag{5.3.19c}$$

$$\tilde{b} = 0. \tag{5.3.19d}$$

and

$$\hat{v}_{d2}(a, \theta) = u(c) + \beta \mathbb{E}[\theta \, \hat{v}_0(\tilde{b}, a', \theta') + (1 - \theta) \, \hat{v}_{d2}(a', \theta)] \tag{5.3.20a}$$

subject to:

$$c = \tilde{y} - \delta k^*, \tag{5.3.20b}$$

$$c \geq 0, \tag{5.3.20c}$$

$$b < \tilde{b} < 0. \tag{5.3.20d}$$

The perceived value of debt relief and timing for financial exclusion are given from optimisation in the previous period. The debtor is promised to the possible ex-post debt relief with an option for absolute default. If the absolute default is chosen, the probability of re-entry will be minimised $\theta = \underline{\theta}$.

Next, the perceived repayment function without observing physical capital can be expressed as follows:

$$\hat{v}_r(b, a, \theta) = \max_{\{b', \theta'\}} u(c) + \beta \, \mathbb{E} \, \hat{v}_0(b', a', \theta'), \tag{5.3.21a}$$

subject to:

$$c = y + b - q(b', a, \theta')b' - \delta k^*, \tag{5.3.21b}$$

and

$$c, q(b', a) \geq 0. \tag{5.3.21c}$$

Notably, Equation 5.3.21a shows the perceived value of repayment function with a condition that the sovereign debtor has not been obligated to receive debt relief or future partial repayment. If the sovereign debtor has been obligated for partial repayment, the value function of repayment with a guarantee for debt relief can be derived as shown below:

$$\hat{v}_r(\tilde{b}, a, \theta) = \max_{\{b' \geq 0, \theta'\}} u(c) + \beta\, \mathbb{E}\, \hat{v}_0(b', a', \theta'), \tag{5.3.22a}$$

subject to:

$$c = y + \tilde{b} - q(b', a, \theta')b' - \delta k^*, \tag{5.3.22b}$$

and

$$c, q(b', a, \theta'), b' \geq 0. \tag{5.3.22c}$$

From Equation 5.3.22a, the sovereign debtor must spend one period making the partial repayment ($\tilde{b}$).

Furthermore, as can be seen in Equation 5.3.17, the decision function can be derived from a comparison between a value function of repayment ($\hat{v}_r$) and a value function of default ($\hat{v}_d$) as follows:

$$\mathbb{Z}(b, a, \theta) = \begin{cases} z(b, a, \theta) = 1; \quad \hat{v}_d(b, a, \theta) > \hat{v}_r(b, a, \theta), \\ z(b, a, \theta) = 0; \quad \hat{v}_d(b, a, \theta) \leq \hat{v}_r(b, a, \theta), \end{cases} \tag{5.3.23a}$$

$$z \in \mathbb{Z}(b, a, \theta), \tag{5.3.23b}$$

where $\mathbb{Z}$ is a set of perceived default intention, and z is an element of the total set. If the perceived value of default is greater than repayment, z is equal to one. Conversely, z is equal to zero when the perceived value of default is less than or equal to repayment; therefore, default probability in the following period can be given as the following:

$$\Psi(b', a, \theta') = \mathbb{Z}(b', a, \theta') \cdot \mathbb{P}\left\{\hat{y}_t = y | \hat{y}_{t-1} = y_t\right\}, \tag{5.3.24}$$

where Ψ is the default probability in the future based on the given amount of borrowing, TFP and the obligation of posterior debt relief. $\mathbb{P}\left\{\hat{y}_t = y | \hat{y}_{t-1} = y_t\right\}$ is a transition probability matrix of Markov Chain (see Appendix).

Furthermore, the bond price schedule with only a full default intention from the default probability (Ψ) of a subsequent borrowing as follows:

$$q(b', a, \theta) = \frac{1 - \Psi(b', a, \theta')}{1 + r}, \tag{5.3.25}$$

where q is the current bond price and r is the interest rate. Notably, the international creditor is risk-neutral; thereby, there will not be additional cost of borrowing to the interest rate if there is no default intention.

However, if a sovereign defaulter is obligated to any further repayment from the debt relief condition, creditors can expect future repayment. Therefore, the bond price schedule should be adjusted for possible realisation of further repayment. From Equation 5.3.25, the bond price schedule can be written with the future repayment as follows:

$$q(1 + r) = \Psi \cdot \mathbb{E}[\tilde{R}] + (1 - \Psi) \cdot \mathbb{E}[R], \tag{5.3.26}$$

where $\mathbb{E}[\tilde{R}]$ is the current value of possible realisation of future repayment from debt relief condition and $\mathbb{E}[R]$ is the current value of a certain repayment if there is no default. Under a condition of the risk-neutral creditors, $\mathbb{E}[R] = 1$. The bond price schedule can be re-written as below:

$$q(1+r) = \Psi \cdot \mathbb{E}[\tilde{R}] + (1-\Psi). \tag{5.3.27}$$

From Equation 5.3.27, the bond price contains two possible realisations for future repayment. The first is a probability of not defaulting, which will return the full repayment, whilst the second is a probability of default, which will return partial repayment with respect to the perceived agreement for debt relief. Thereby, an expected function of repayment with debt relief can be derived from the future stream of possible repayments as follows:

$$\mathbb{E}[\tilde{R}] = \frac{1}{1+r} \cdot \sum_{t=0}^{\infty} \left\{ \left(\frac{1-\theta}{1+r} \right)^{t} \theta\lambda \right\}, \tag{5.3.28}$$

$$\mathbb{E}[\tilde{R}] = \frac{1}{1+r} \left\{ \theta\lambda + \left(\frac{1-\theta}{1+r} \right) \theta\lambda + \left(\frac{1-\theta}{1+r} \right)^{2} \theta\lambda + \ldots + \left(\frac{1-\theta}{1+r} \right)^{\infty} \theta\lambda \right\}, \tag{5.3.29}$$

$$\mathbb{E}[\tilde{R}] = \frac{\theta\lambda}{1+r} \left\{ 1 + \left(\frac{1-\theta}{1+r} \right) + \left(\frac{1-\theta}{1+r} \right)^{2} + \ldots + \left(\frac{1-\theta}{1+r} \right)^{\infty} \right\}, \tag{5.3.30}$$

$$\mathbb{E}[\tilde{R}] = \frac{\theta\lambda}{1+r} \cdot \frac{1+r}{r+\theta}, \tag{5.3.31}$$

$$\mathbb{E}[\tilde{R}] = \frac{\theta\lambda}{r+\theta}, \tag{5.3.32}$$

where θ is a probability of re-entry into the credit market and λ is a ratio of repayment to the initial amount of debt; $0 \leq \lambda \leq 1$. The expected repayment by a sovereign defaulter with a guarantee for debt relief in Equation 5.3.32 can be substituted into the bond price schedule in Equation 5.3.27 as below:

$$\begin{aligned} q(1+r) &= \Psi \cdot \frac{\theta\lambda}{r+\theta} + (1-\Psi) \\ &= 1 - \Psi\left(1 - \frac{\theta\lambda}{r+\theta}\right), \end{aligned} \tag{5.3.33}$$

$$q = \left[1 - \Psi\left(\frac{r+\theta\,(1-\lambda)}{r+\theta}\right)\right] \cdot \frac{1}{1+r}, \tag{5.3.34}$$

$$q(b', a, \theta') = \left[1 - \Psi(b', a, \theta')\left(\frac{r+\theta'\,(1-\lambda)}{r+\theta'}\right)\right] \cdot \frac{1}{1+r}. \tag{5.3.35}$$

In Equation 5.3.35, the bond price schedule will be adjusted to possible realisation of future repayment if there is a guarantee perceived debt relief.

5.3.5 Debt Relief and Financial Exclusion

When a sovereign debtor decides on borrowing in the credit market, the perceived debt relief and its timing has an endogenous relationship with the bond price schedule. The ratio for debt relief or sacrifice of creditors can be given as follows:

$$\omega = 1 - \lambda = 1 - \frac{\tilde{b}}{b}, \tag{5.3.36}$$

where the ratio of debt relief is denoted by ω and $\tilde{b}$ is the foreign asset after discount (see Appendix); if there is no debt relief ($\lambda = 1$), $\tilde{b} = b$. Moreover, from the stylised fact, a long period of debt hangover significantly relates to a high amount of debt relief, whilst a short period of debt hangover leads to a smaller discount. Thus, a function of debt relief (ω)and its timing (θ) can be simply given as follows:

$$\omega = \left[\frac{-1}{\overline{\theta} - \underline{\theta}} \cdot \theta \;+\; \frac{1}{\overline{\theta} - \underline{\theta}}\right]^{\tau}, \tag{5.3.37a}$$

subject to:

$$\theta \in [\underline{\theta}, \overline{\theta}], \tag{5.3.37b}$$

where τ is a parameter in the debt relief function. The boundaries of debt relief are determined by $\underline{\omega}$ and $\overline{\omega}$[19]. Besides, the boundaries of probability of re-entry are $\underline{\theta}$ and $\overline{\theta}$[20]. This function captures the possibility of full default option where the minimum probability of re-entry ($\underline{\theta} = 0.05$) is given for the case of no repayment (see footnote 20). Notably, if $\tau = 1$, it will determine a linear structure of debt relief and financial exclusion (see Appendix).

5.4 Data and Methodology

This section gives the data description and calibration of the economy under a theoretical model for sovereign default with endogenous debt relief. The parameters in this theoretical model are calibrated to target the quarterly data of Argentina's economy[21] from 1980 Q1 to 2017 Q4. Notably, some parameters in this chapter is borrowed from the existing literature under the real business cycle (RBC) model (see Table 3.2 in Chapter 3) , whilst some parameters are obtained from the previous chapters[22]. For a simplicity and a comparability of the models, this chapter focuses more on the data of Argentina and the overall stylised facts of debt relief and periods of financial exclusion over the recent decades. The value of parameters are shown in the following table.

[19]Under the possible amount of debt relief, $[\underline{\omega}, \overline{\omega}] = [0, 1]$.

[20]From the previous chapter, the full default requires $\theta = 0.05$ and hence, $[\underline{\theta}, \overline{\theta}] = [0.05, 1]$.

[21]The data is updated from the previous chapters which targets Argentina between 1980 Q1 and 2017 Q4.

[22]The detail of parameters can be seen in the previous chapter (see Table 3.2 and Table 3.3.

Table 5.1: Model specific parameter values

Parameters		Values
Stochastic structure (TFP)	ρ_a, σ_ϵ	0.982 , 0.014
Constant term of Stochastic process	μ_a	1
Discount factor	β	0.95
Risk aversion	γ	2
Perceived risk aversion	$\hat{\gamma}$	2
Risk-free rate	r	0.01
Capital share	α	0.35
Capital depreciation	δ	0.05
Capital adjustment cost	Φ	2.4
Output default cost	χ	0.07
linear structure of debt relief	τ	1
Boundaries of re-entry probability	$[\underline{\theta}, \overline{\theta}]$	$[0.05, 1]$
Boundaries of debt relief	$[\underline{\omega}, \overline{\omega}]$	$[0, 1]$
Boundaries of foreign asset	$[\underline{b}, \overline{b}]$	$[-4.5, 0.5]$
Boundaries of physical capital	$[\underline{k}, \overline{k}]$	$[2, 13]$

From Table 5.1, the stochastic structure of TFP $(\rho_a, \sigma_\epsilon, \mu_a)$ are 0.982, 0.014 and 1. A risk-free rate (β) and risk-aversion $(\gamma, \hat{\gamma})$ are 0.95 and 2. Besides, capital share (α), depreciation (δ) and cost adjustment (Φ) of capital are 0.35, 0.05, 2.4. A penalty rate or the output cost of default[23] (χ) is 0.07. The quadratic structure of debt relief is 1 which simply implies the linear structure of debt relief function (see Appendix).

Next, the lower bound of re-entry probability $(\underline{\theta})$ is set at 0.05 as well as the full default from the previous chapter[24] The upper bound of re-entry probability $(\overline{\theta})$ is 1 from the stylised fact; that some defaulters have fallen into debt hangover only a period. The boundaries of debt relief $([\underline{\omega}, \overline{\omega}])$ is $[0, 1]$. Thus, from the boundaries of debt relief and its timing, it can be assumed that a choice of θ at 0.05 and ω at 0 means an application of full default. On the contrary, the upper bounds of θ and ω at 1 suggest a full repayment with a certain return to the credit market.

[23]The boundaries of stochastic structure under default (χ) from $[\underline{a}, \overline{a}]$ is $[\underline{a}, \overline{a} \cdot (1 - \chi)]$.

[24]The calibration of re-entry probability with full default is given in the previous chapter.

Remarkably, an availability of partial default is adjusted by the function of debt relief as well as boundaries and minimum allowance of foreign asset (see Appendex).

Furthermore, this chapter uses a discrete model following the Tauchen's method (Tauchen, 1986) to approximate the transition probability matrix from the continuous stochastic process (see Appendix). As can be seen in Equation 5.3.24, the transition probability matrix of output $(\mathbb{P}\left\{\hat{y}_t = y | \hat{y}_{t-1} = y_t\right\})$ follows the AR(1) process with a stochastic structure in Table 5.1 and being discretised into the 31 finite-state Markov Chain. Besides, the discrete modelling of the dynamic stochastic general equilibrium (DSGE) requires a specification of the possible realisation of state variables; physical capital and foreign asset. Based on the Argentina's economy from 1980 Q1 and 2017 Q4, the boundaries of capital accumulation $([\underline{k}, \overline{k}])$ is $\{2, 13\}$ and the boundaries of foreign asset $([\underline{b}, \overline{b}])$ is $[-4.5, 0.5]$. The upper and the lower bounds of these two state variables are arranged to cover the possible realisation of capital and foreign asset over the statistical evidence of Argentina's economy. Specifically, the number of grid (state space form) for TFP, foreign asset, physical capital and debt relief are arranged at 31, 51, 71 and 96 (see Appendix).

To conclude, the function of endogenous debt relief and its timing has been added into the theoretical model for sovereign default. This can provide a study of an optimal choice for sovereign debtors in choosing ex-post debt relief, as well as for creditors in adjustment of the bond price schedule from perceived default and debt relief probability. Finally, the model in this chapter will be calibrated to target recent data for the Argentine economy and simulated by the Monte Carlo over 100,000 periods to give an optimal choice of debt relief over various economic condition of capital, foreign assets and productivity.

5.5 Quantitative Results

This section provides quantitative results for the model economy with sovereign default and the endogenous decision of debt relief. The theoretical model aims to illustrate an optimal choice of debt relief and its timing with endogenous interactions among capital, foreign assets and productivity. The sovereign debtors can maximise their welfare from default options (both absolute and partial default) as well as foreign assets and capital accumulation. Thus, this section is separated into five subsections, the first of which relates to bond price schedule and optimal debt relief and the second concerns value functions at different debt reliefs. The third concerns default probability, the fourth discusses Monte Carlo simulations and the final part shows a comparison between the theoretical model and actual data.

5.5.1 Bond Price and Debt Relief

Firstly, the bond price schedule is given from the perceived probability of future default through the amount of foreign assets and a choice of debt relief. As can be seen in Equation 5.3.35, the bond price schedule is related to the prior choice of debt relief and its timing[25]. The sovereign defaulter is obligated to the subsequent repayment after the amount of possible realisation of debt relief chosen during the borrowing period. Thus, sovereign debtors who borrow under different options will

[25] Equation 5.3.37 defines a debt relief as a function of probability of re-entry. Thus, each choice of debt relief is obligated to the subsequent periods of debt hangover (θ).

experience a variety of bond prices depending on the specific option. A sovereign debtor can maximise social welfare from choosing the optimal option of prior realisation of debt relief and its timing under different values of state-variables. Hence, a plot of two bond price schedules from different options of default is given in Figure 5.1.

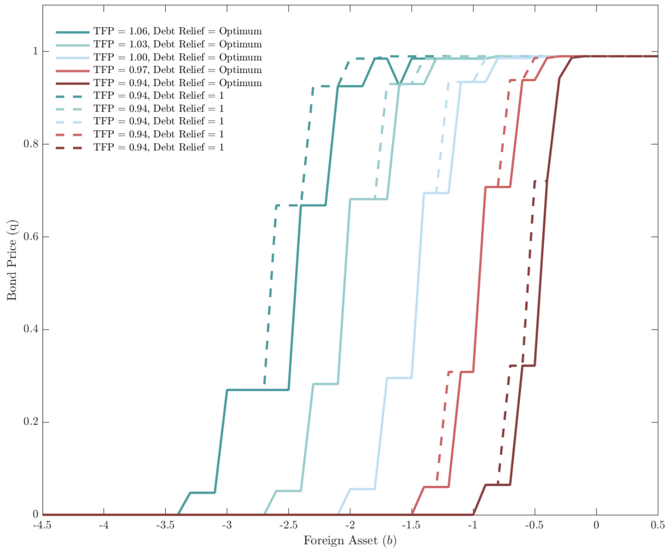


Figure 5.1: A comparison of two bond price schedules (partial and full default)

Figure 5.1 illustrates the differences of two bond price schedules; bond contract with an option of debt relief ($\omega = optimal$) and bond contract with a full default option with an absolute debt relief ($\omega = 1$). In the figure, the solid lines represent the bond price schedule at the dynamic choice of optimal debt relief under different scenarios of TFP. It is obvious that the bond price schedule from a positive shock of TFP is on the left, whilst the bond price schedule from a negative shock is on the right of the diagram. This gives an overview of a sovereign debtor under a positive shock of productivity borrowing at a lower bond premium compared with the economy under a negative shock.

Besides, the dot lines represent the bond price schedule with an absolute debt relief ($\omega = 1$)[26]. From a comparison of these two bond price schedules in Figure 5.1, the bond premium with a dynamic option of optimal debt relief is slightly higher than the bond premium without the option. Unsurprisingly, an option of debt relief is an addition to favour a sovereign debtor; therefore, adjustment to the bond contract over this option is added as an increased bond premium. Optimal debt relief tends to increase the utility of the sovereign defaulter, thereby increasing the default intention. When the future possibility of calling this option is perceived by creditors, the bond premium is subsequently adjusted and shown as a higher bond premium bond in Figure 5.1.

In addition, there are various bond price schedules under two different options of debt relief. Figure 5.2 illustrates two bond price schedules with options of debt relief (ω) 20% and 80%, respectively. The solid lines show the bond price with 20% debt relief and the dot lines show the bond price with 80% debt relief. From the diagram, a positive TFP shock will stimulate the economy and lead to a lower bond premium bond, whilst a sovereign borrower under a negative shock has to borrow at a higher bond premium. Moreover, from a comparison of different options of debt relief, the bond price with a small proportion of debt relief is offered at a lower bond premium. Obviously, a higher rate of debt relief increases the default intention of sovereign debtors as well as increasing the probability of future default; consequently, a higher bond premium will be added to the bond price.

[26]Full default is always an option when a debtor choose not to repay and hence, the defaulter will experience a maximum period of debt hangover at $\underline{\theta}$ without any subsequent repayment.

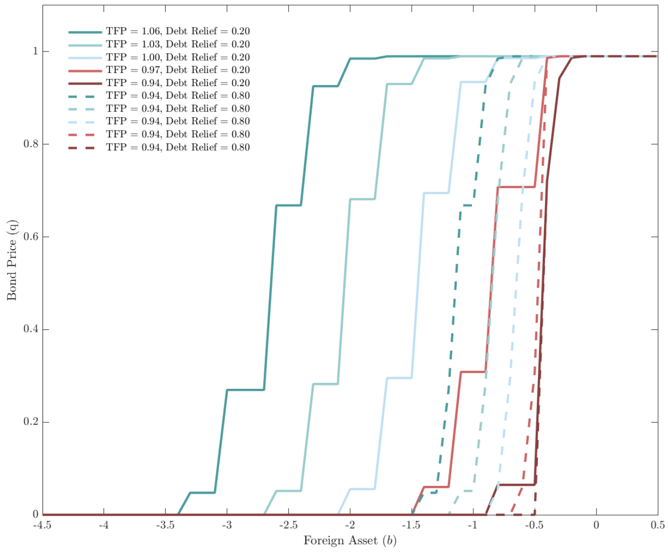
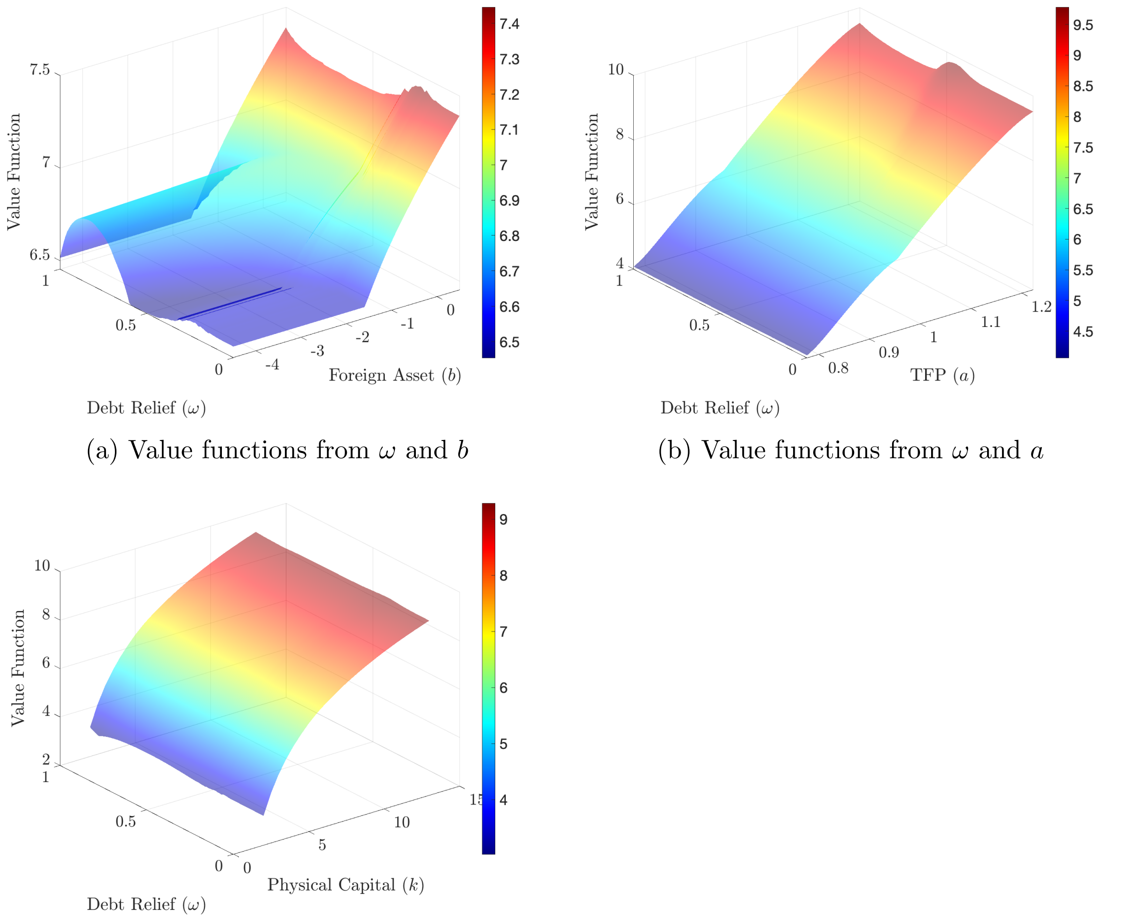


Figure 5.2: Bond price schedule with partial default and debt relief ω at 0.20 and 0.80

Nevertheless, Figure 5.1 and 5.2 can indicate that the bond price schedule is not always monotonic from a change in a rate of debt relief among several options of debt relief and absolute debt relief under different scenarios of capital accumulation, foreign assets and productivity. Without a debt relief option or absolute debt relief ($\omega = 1$), the bond price has a lower bond premium compared with an option for 80% debt relief. This result is based on the utility of the defaulter, in which absolute debt relief or full default causes the defaulter to experience a maximum period of debt hangover ($\theta = \underline{\theta}$), whilst partial default can lead to a higher probability of re-entry ($\theta > \underline{\theta}$) into the market. If the probability of re-entry (θ) is too low, it will decrease the utility of default as well as lower the bond premium.

5.5.2 Value Functions for Different Values of Debt Relief

Regarding to Equation 5.3.8, a sovereign debtor has to maximise welfare through choosing a bond contract at an optimal level of debt relief. Illustrations of value functions from debt relief (ω), foreign asset (b), TFP (a) and physical capital (k) are given in Figure 5.3.



(a) Value functions from ω and b

(b) Value functions from ω and a

(c) Value functions from ω and k

Figure 5.3: Three dimensional heatmaps of value functions with both full and partial default among variables

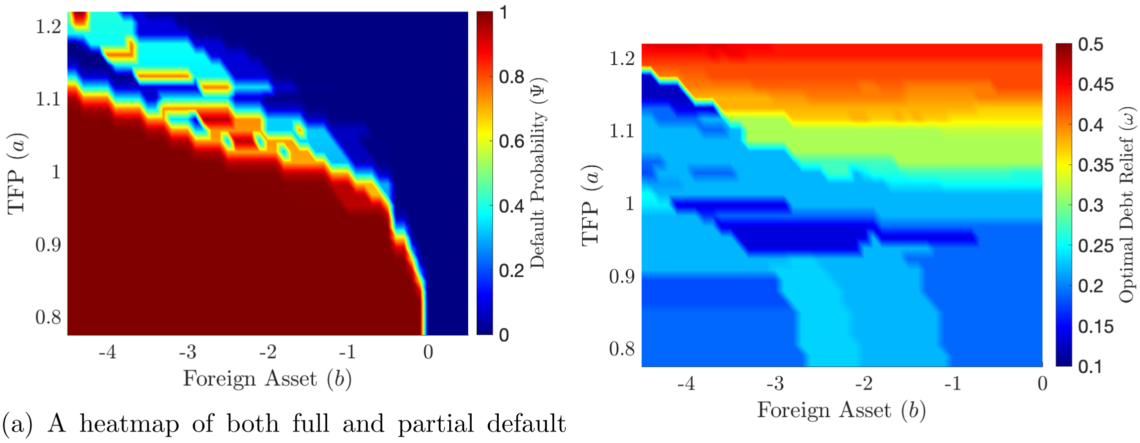
Figure 5.3a is three dimensional heatmap of value functions at various choices of debt relief and current level of foreign assets. Notably, Figure 5.3a is plotted at steady-state capital and productivity. In the diagram, there is a slight difference in the value function when a foreign asset is between 0 and -1. When the borrowing amount and debt relief are small, a positive curve on the surface can be seen, which indicates the optimal choice of debt relief at 0.20. A bond contract with an option of low debt relief is suitable and increase the overall welfare when debt is small ($b > -1$). Conversely, Figure 5.3a also shows a positive curve on the surface when the borrowing amount and debt relief are both large. If the foreign asset is between -4.5 and -1, the value function can be maximised with optimal debt relief at approximately 0.80. Therefore, a theoretical result for the model in Figure 5.3a can be summarised as debt relief at low and high amounts of borrowing are maximised by debt relief being at 0.20 and 0.80 respectively.

Next, Figure 5.3b is a three-dimensional plot of the value function for TFP and debt relief at the steady-state level of capital and foreign asset. From the figure, a positive curve can be seen on the surface when the TFP is larger than 1 and debt relief is approximately 0.20. This theoretical result indicates that the value function can be maximised if the sovereign debtor chooses to borrow with a possible realisation of 20% future debt relief. Besides, there is no significant difference when there is a negative TFP shock ($a < 1$).

In addition, Figure 5.3b illustrates the value function for various choices of debt relief and a current level of physical capital at steady-state foreign asset and productivity. It is obvious that there is no difference among various ratios of debt relief at each level of capital accumulation.

5.5.3 Default Probability and Optimal Debt Relief

The future possibility of sovereign default can be obtained through Equations 5.3.23 and Equation 5.3.24. With respect to current productivity and borrowing intention, the default probability at the steady-state capital is shown in Figure 5.4a.



(a) A heatmap of both full and partial default probability (Ψ)

(b) A heatmap of optimal debt relief (ω)

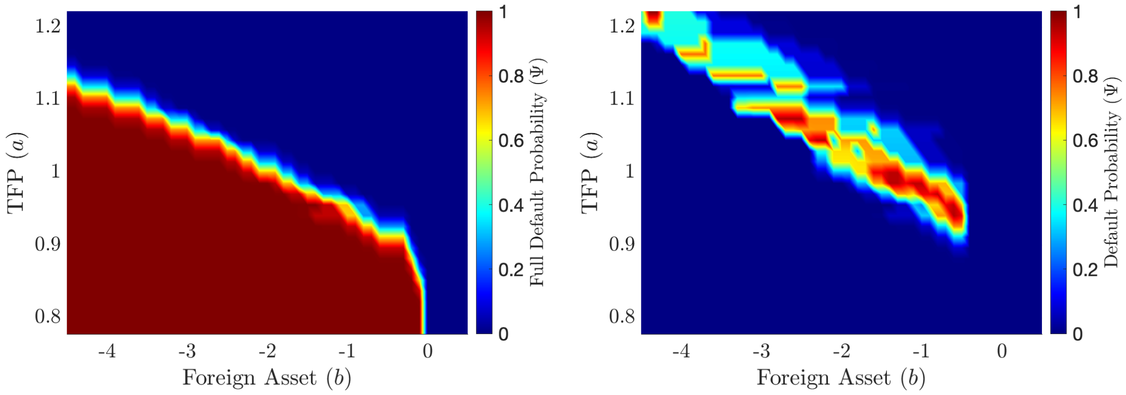
Figure 5.4: Three dimensional heatmaps of default probability (Ψ) and debt relief (ω)

Figure 5.4a is a three-dimensional heatmap of default probability where the hot colour is defined as a high possibility of default, whilst the cold colour represents the opposite. As can be seen in the plot, the default probability is high when both productivity and foreign assets are negative. If the economy experiences a positive shock, a sovereign debtor can borrow more at a lower bond premium. However, there can be default at a positive shock if the foreign asset is too high. Notably, each of the state-spaces is dynamically specified by the optimal debt relief in Figure 5.4b.

Figure 5.4b illustrates the dynamic optimisation of debt relief for various levels of productivity and foreign asset under steady-state capital. Remarkably, cold and hot colours specify the optimal levels of debt relief from low to high proportions. In this figure, if a productivity is below one, optimal debt relief is certainly below 0.30. Conversely, if there is a positive shock, a sovereign debtor can borrow with a guarantee for debt relief from 0.30 to 0.45. Both figures indicate that optimal debt relief for the state-space of high default possibility is between 0.15 and 0.30, whilst the lower default state is between 0.30 and 0.45. Obviously, optimal debt relief tends to be low when the economy is at risk in order to borrow at a lower bond premium.

Oppositely, under high productivity, a sovereign debtor will have less intention to default; therefore, borrowing at a high rate for the possible realisation of debt relief is beneficial.

Moreover, Figure 5.5 illustrates the probability of full default and partial default at the steady-state of capital accumulation under the given optimal of debt relief.



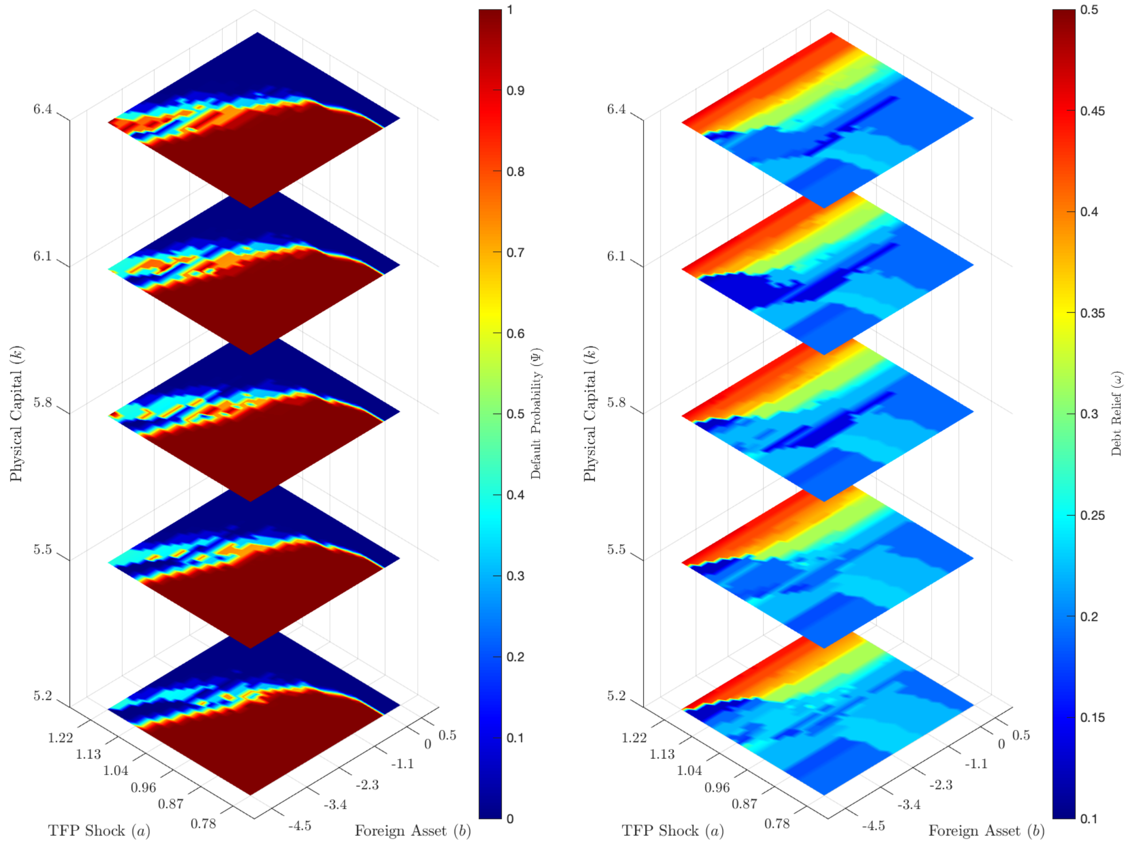
(a) A heatmap of full default

(b) A heatmap of partial default

Figure 5.5: Three dimensional heatmaps of full default and partial default

Figure 5.5a shows probability of choosing a full default over a partial default at the given debt relief, whilst Figure 5.5a shows default probability at the optimal debt relief. From these two figures, it is obvious that full default appears when productivity is low and the borrowing amount is high. Oppositely, the partial default with optimal debt relief is preferred under a positive shock and high borrowing.

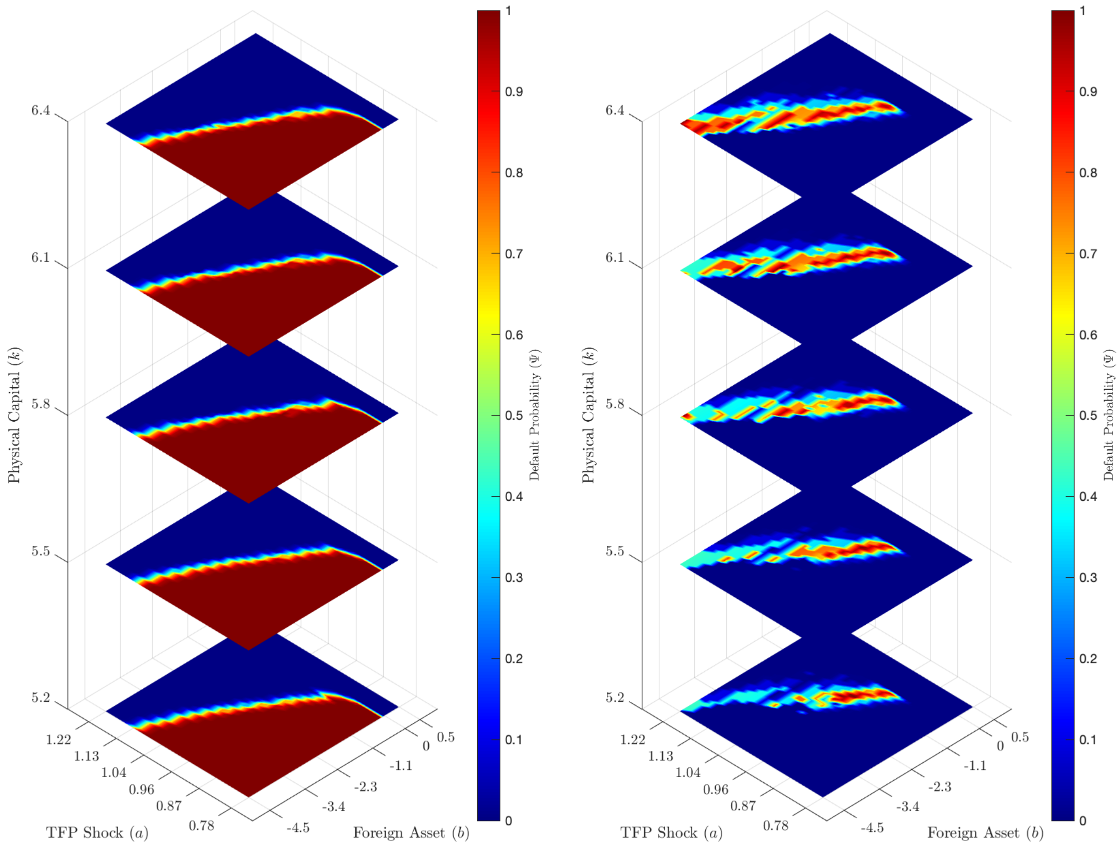
In addition, Figure 5.6 illustrates default probability and optimal debt relief at each level of given capital accumulation. Interestingly, Figure 5.6a shows an increase in the number of high default states when capital accumulation is increased. The economy under a positive shock is more likely to default with subsequent debt relief when capital accumulation is increased.



(a) Default Probability at each level of capital (b) Optimal debt relief at each level of capital

Figure 5.6: Default probability with full and partial default at the optimal debt relief and given levels of capital

Figure 5.6b shows a three-dimensional heatmap of the dynamics of optimal debt relief at the simultaneous state-space of Figure 5.6a. This diagram clearly shows a dynamic result for debt relief under different scenarios of productivity and foreign assets at each level of capital accumulation. From both diagrams, partial default intention appears more when low debt relief is shown when capital accumulation is higher. This explains that when capital accumulation is too high, there is a possible call for technical default with a fast-restructuring process for debt. A technical default with debt relief between 0 and 0.20 is obligated with a re-entry probability from 100% to 80% respectively ($\tau = 1$). The distinction between an absolute default and partial default under different states of capital is plotted in Figure 5.7.



(a) Full default probability under various capital (b) Partial default under various capital

Figure 5.7: Full and partial default probability at various capital accumulation

Figure 5.7 gives a comparison of full default and partial default for different levels of given capital. In Figure 5.7a, the full default probability over capital accumulation is slightly decreased. If a sovereign debtor has a high amount of initial capital, the full default option is less preferable compared with a lower state of capital.

On the contrary, Figure 5.7b shows an increase in partial default intention under a high state of capital accumulation. From this diagram, it is obvious that a sovereign debtor tends to default more with a possible realisation of debt relief under a good economy. Under poor productivity, the partial default probability is dramatically less, whilst full default leads to higher utility.

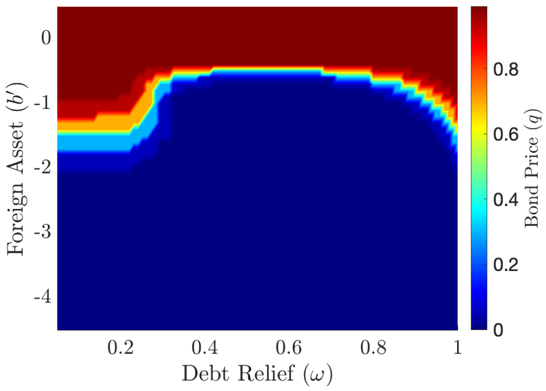
Therefore, this outcome can be interpreted as a technical default for a rich state of capital because a partial default with debt relief provides an option of a fast-restructuring process. A defaulter may call for a partial default under high productivity because there will be less cost of default compared with the amount of debt relief. On the other hand, the full default option is obviously beneficial when output is relatively low.

Furthermore, the default probability is a premium for the bond price and, hence, the price of a bond can be varied depending on the amount of foreign assets, TFP and a possible realisation of subsequent debt relief, as is shown in Figure 5.8. Figure 5.8a illustrates the heatmap of bond price for the borrowing amount and debt relief at steady-state capital. From the diagram, cold and hot colours define the price of a zero-coupon bond from 0 to $\left(\frac{1}{1+r}\right)$ respectively[27]. As can be seen in the figure, debt relief between 0 and 0.2 can lead to a lowest bond premium bond when the borrowing amount is between -1 and -2. Remarkably, under a similar bond premium, one with a guarantee for higher debt relief is beneficial. Thereby, Figure 5.8a suggests an optimal rate of debt relief at approximately 0.20 where a debtor can borrow more with a low bond premium. Furthermore, Figure 5.8a also indicates that a bond without debt relief ($\omega = 1$) can provide a low risk-premium, although the defaulter has to stay longer in a period of debt hangover.

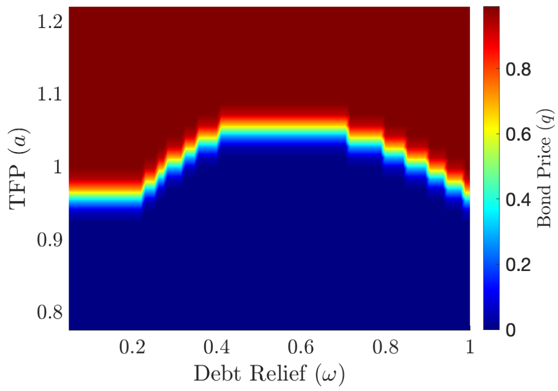
Moreover, Figure 5.8b shows a heatmap of the bond price with respect to productivity and debt relief at the steady-state of both capital and foreign assets. From the diagram, an economy under a positive TFP shock ($a > 1$) can borrow at the steady-state level of debt with high debt relief. Besides, an economy under a

[27] The price of a zero-coupon bond at $\left(\frac{1}{1+r}\right)$ indicates a zero bond premium.

small negative shock can sustain the amount of debt and capital at the steady-state only when debt relief is low.



(a) Bond Price, Foreign Asset and Debt Relief

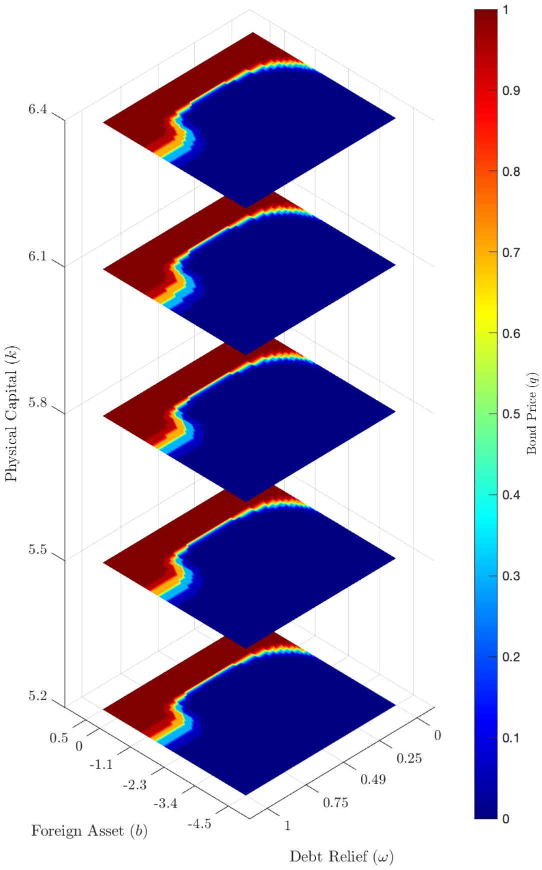


(b) Bond Price, TFP and Debt Relief

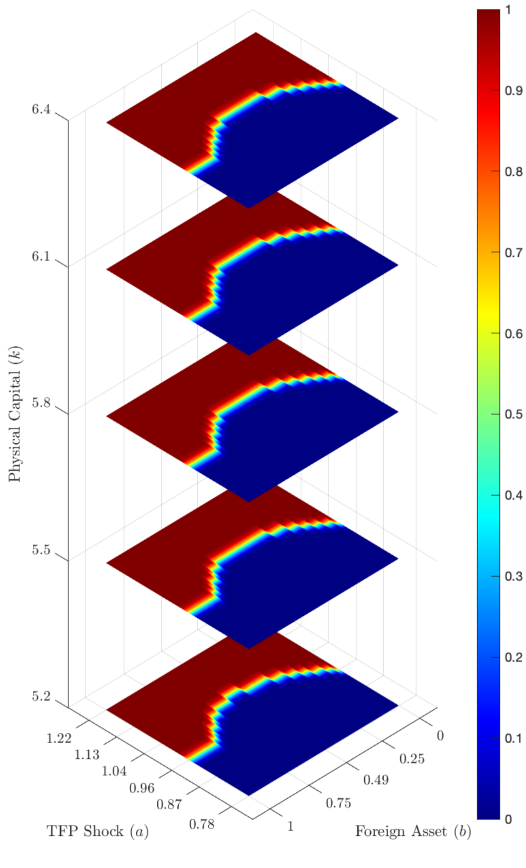
Figure 5.8: Heatmaps of bond price with full and partial default at the given foreign asset, TFP and debt relief

Additionally, Figure 5.8b can be interpreted as the economy under a large negative shock is unable to sustain the steady-state. The sovereign debtor has to sacrifice capital accumulation or reduce their borrowing amount. Otherwise, the bond premium will either be high or unavailable because of increased future default intention.

Next, Figure 5.9 shows a comparison of bond price, debt relief and foreign assets over different levels of given capital accumulation. From the diagram, no significant difference is shown for a change in the steady-state level of capital to the interaction between bond price and debt relief.



(a) Bond Price, Foreign Asset and Capital



(b) Bond Price, TFP and Capital

Figure 5.9: Heatmaps of bond price with full and partial default at the given debt relief and various levels of capital

From Equation 5.3.35, there is no capital in the function of bond price. Unsurprisingly, a change in steady-state capital has an insignificant effect on bond price regarding a change in debt relief.

5.5.4 Model Simulation

This section implements the Monte Carlo simulation of the theoretical model with optimal debt relief from the parameters in Table 5.1. Figure 5.10 illustrates the economy and its interactions through the simulation over 1,500 periods.

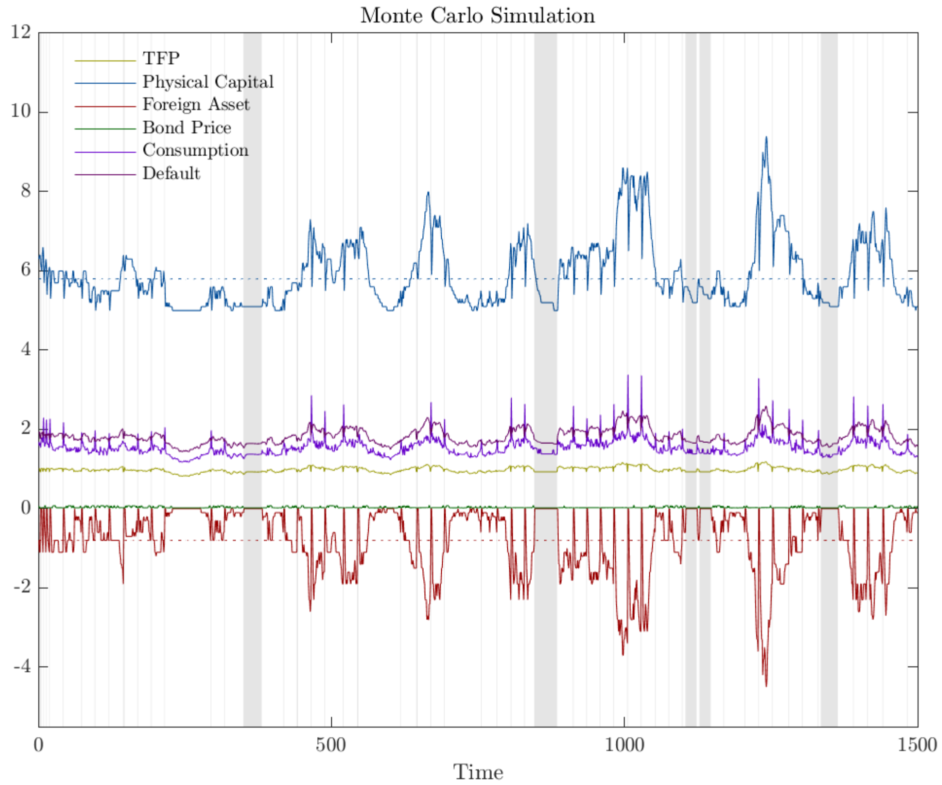


Figure 5.10: Monte Carlo simulation of the mode over 1,500 periods

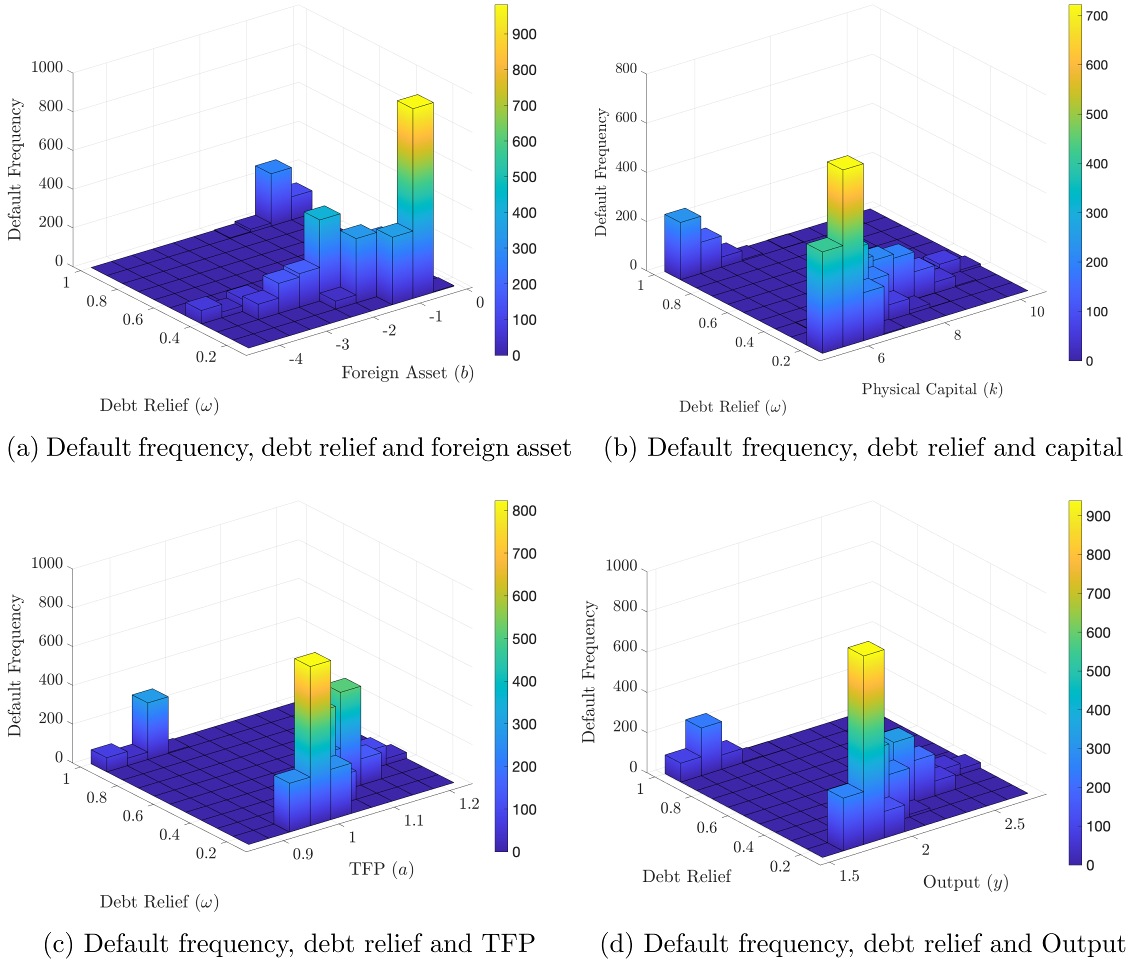
The economic simulation begins with the steady-state level of capital and foreign assets, where the solid lines are current values and the dot lines are the steady-state of subsequent variables. Physical capital, foreign assets, consumption and productivity are in real terms, whilst bond premium is a percentage of bond price. Remarkably, the dark shade indicates the period for the debt restructuring process or default. As can be seen in the figure, there are two main types of default. A short period for the dark shade is defined as a partial default with debt relief, whilst a long period for the dark shade indicates a full default decision under a maximum period for the debt restructuring process. From the simulation in Figure 5.10, the economy can default several times under the option of partial default.

As can be seen in the figure, consumption is varied, depending on a change in productivity. During the repayment periods, consumption is smoothed through changing capital and debt. When a sovereign debtor enters a debt restructuring period, consumption after default will slightly decrease because the defaulter has to sell capital accumulation for smoother consumption in substitution of borrowing exclusion. Conversely, in the initial period of borrowing, consumption temporarily jumps from initial borrowing after re-entry. Hence, a big fall and increase in consumption only appears when the sovereign debtor enters and exits debt restructuring respectively.

Besides, capital accumulation dramatically falls prior and during default periods. The economy can accumulate capital from a positive TFP shock and gain large borrowing at a low bond premium. On the other hand, if there is a negative shock, the physical capital and borrowing amount will be lower because the economy cannot afford to stay in the steady-state, as shown in Figure 5.8, when the bond premium is increased. During repayment under a negative shock, physical capital is another source for a consumption smoothing pattern through a reduction of capital accumulation.

Next, Figure 5.10 indicates that a sovereign debtor with the ability to borrow will borrow more during a positive shock and less during a negative shock. The interaction among foreign assets and other variables also shows that the debtor borrows more for both consumption and investment.

Interestingly, a sovereign debtor can default with either low or high volumes of debt under a debt relief condition due to the size of productivity shock. The full default option is only called upon when the size of a negative shock is large (greater than 10%). In addition, the model can be simulated over 100,000 periods for a dedicated illustration of results. The detail for default frequency over a big period is plotted into Figure 5.11.



(a) Default frequency, debt relief and foreign asset (b) Default frequency, debt relief and capital

(c) Default frequency, debt relief and TFP (d) Default frequency, debt relief and Output

Figure 5.11: Histograms of default frequency with both full and partial default and economic interactions

Figure 5.11a is a three-dimensional histogram of default frequency with respect to debt relief and foreign assets when a sovereign debtor chooses to default. From this histogram, most of the default realise with 20% debt relief at low debt. Only a few partial defaults with 40% debt relief realise at large borrowing and some periods of full default appear with low and moderate debt periods.

Furthermore, Figure 5.11b shows a histogram of default frequency, debt relief and physical capital during a prior period of default. From this plot, it is obvious that there is high default frequency when physical capital is low; both partial and full default appear when capital accumulation is lower than 8. Moreover, Figure 5.11b indicates that full default only appears when productivity is dramatically low, whilst partial default can realise during positive and negative shocks.

Interestingly, Figure 5.11d illustrates default frequency and debt relief with respect to output. From this plot, it is more obvious that a sovereign debtor will decide to default based on output rather than capital or productivity. The debtor will choose to default with absolute relief (full default) if current output is significantly low (30% lower than the steady-state output). Moreover, the debtor will choose partial default if output is moderately or slightly low. Notably, only a few defaults appear when output is high because the debtor has a large amount of debt. Next, Figure 5.12 gives a histogram of default frequency with a ratio for debt to output.

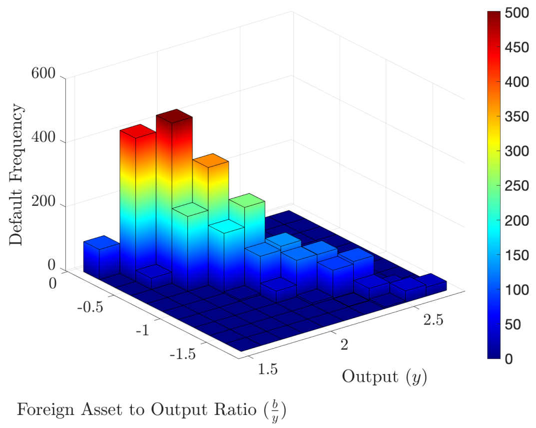
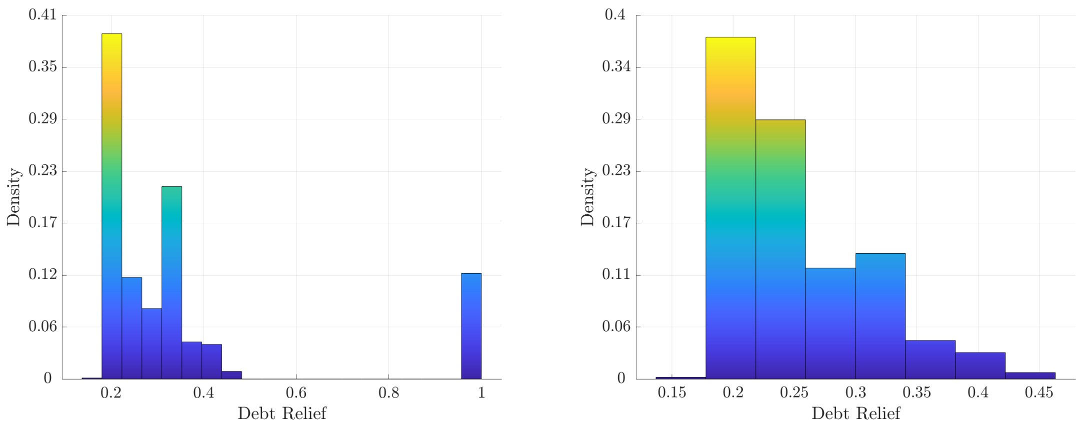


Figure 5.12: Histogram of default (full and partial) frequency, output and debt ratio

From Figure 5.12, it is obvious that default often appears when the debt ratio is less than 100% and, simultaneously, default is realised more during a recession when output is significantly low. During high productivity, the debtor will choose to default only when the debt ratio is between 100% and 200% and less frequently compared with periods of low output.

In addition, the histogram for a chosen debt relief during default and repayment over 100,000 periods of Monte Carlo simulation is given in Figure 5.13.



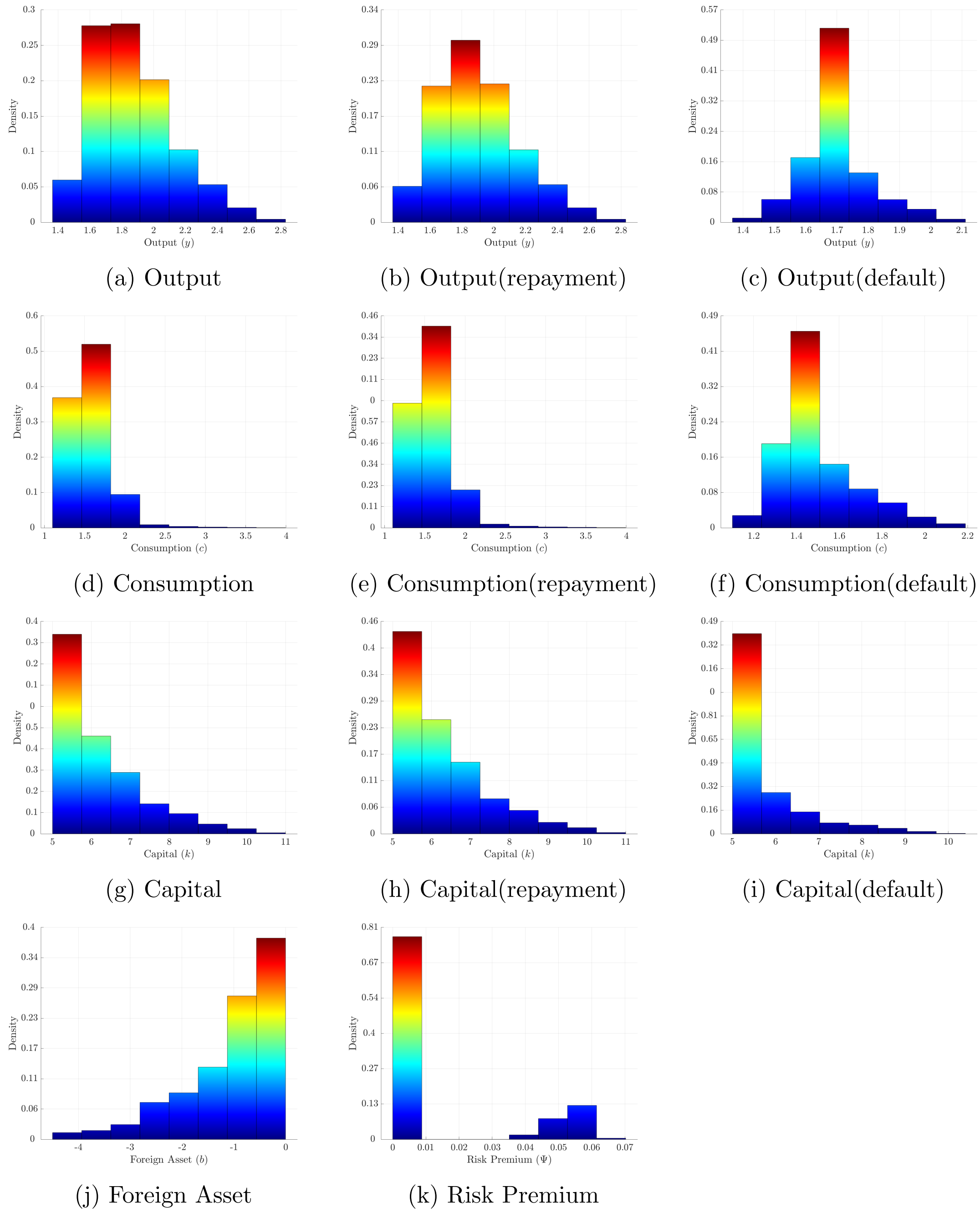
(a) A histogram of debt relief at default (b) A histogram of debt relief at repayment

Figure 5.13: Histogram of debt relief during partial default and repayment

Figure 5.13a shows a probability of choosing possible realisation of debt relief in a period prior to default. From the figure, it is obvious that debt relief at 20% is most frequent with 40% probability. Then, 30% and 100% debt relief have probability for being chosen at 20% and 12% respectively. Under the calibration parameters of Argentina's economy, the optimal debt reliefs chosen the most are 20%, 30% and 100% respectively.

Furthermore, Figure 5.13a illustrates the probability of debt relief chosen during the repayment periods. As can be seen in the diagram, a sovereign debtor tends to choose debt relief between 20% and 40%. The most frequently chosen at 36% probability is debt relief at 20%. From these two figures, it is obvious that a sovereign debtor under the parameters of Argentina's economy prefers possible realisation of debt relief at 20%, as well as a short period of debt hangover. Compared with the full default option, there is only a slight chance that absolute relief will be beneficial.

Finally, there is an outcome showing a smoothing pattern of consumption through capital accumulation. The density of output, consumption, capital, foreign assets and bond premium from the Monte Carlo simulation are plotted in Figure 5.14.



(a) Output (b) Output(repayment) (c) Output(default)
(d) Consumption (e) Consumption(repayment) (f) Consumption(default)
(g) Capital (h) Capital(repayment) (i) Capital(default)
(j) Foreign Asset (k) Risk Premium

Figure 5.14: Histogram of output, consumption, capital foreign asset and risk-premium during default (both full and partial) and repayment

In the figure, the density of output for overall, repayment and default periods are shown in Figure 5.14a, Figure 5.14b and Figure 5.14c respectively. This indicates that the density of output during repayment is left-skewed, whilst output during default is normal. The mean for output during repayment is higher than in the default period whereas output during default is obviously less fluctuated.

Moreover, the density of consumption is given in Figure 5.14d, Figure 5.14e and Figure 5.14f respectively. Consumption during default is close to normal with the lower mean, whilst consumption during repayment is left-skewed with the higher mean. Besides, capital accumulation is presented in Figure 5.14g, Figure 5.14h and Figure 5.14i. It shows left-skewed distributions for both repayment and default. Figure 5.14i shows a heavy tail and lower mean of capital during default.

These outcomes for output, consumption and capital indicate that a sovereign defaulter accumulates less capital during default. The defaulter cannot use physical capital for smoothing consumption. Therefore, the density of output and consumption is shown as normal following the stochastic structure of TFP with independent and identical distribution. Furthermore, a sovereign debtor during repayment accumulates more capital and, hence, they can use capital to smooth consumption. According to a change of capital, output during repayment fluctuates more, whilst consumption is more stable.

Finally, the density of foreign assets and bond premium are plotted in Figures 5.14j and Figure 5.14k respectively. The outcome shows a right-skewed distribution for foreign assets. A sovereign debtor chooses the amount of debt to borrow based on the output; besides, the density of bond premium below 1% is significantly high. Hence, these two figures can illustrate that a sovereign debtor often chooses to borrow at a low bond premium. There is only a 20% chance that a bond premium is deviates between 3% and 7%, whilst the other period shows borrowing below 1%.

5.5.5 Model and Target Statistics

This section gives statistical evidence of actual data and Monte Carlo simulations of the two theoretical models over 100,000 periods. Specifically, Model 1 is for an economy with only an option of full default whereas Model 2 is for an economy with dynamic choices of endogenous debt relief. The two models are simulated to target Argentina's economy under the given set of parameters[28] in Table 5.1. The quantitative results with targeting evidence are shown in Table 5.2.

Table 5.2: Model and Target Statistics

Parameters		Model 1	Model 2	Target
Output volatility	σ_y	0.0940	0.1286	0.0917
Consumption volatility	σ_c	0.0924	0.1422	0.0969
Consumption output volatility	$\frac{\sigma_c}{\sigma_y}$	0.9830	1.1057	1.0563
Correlation coefficient (c,y)	ρ_{cy}	0.9314	0.8496	0.9223
Average consumption ratio	$\mu_{\frac{c}{y}}$	82.73 %	83.78 %	79.63 %
Debt ratio	μ_b	46.25 %	45.24 %	47.67 %
Length of default periods		7.40%	12.34%	12.50%

From Table 5.2, output, consumption and a proportion of consumption to output volatility of the model 1 are 0.0940, 0.0924 and 0.983, whilst the model 2 is more volatile at 0.1286, 0.1422 and 1.1057 respectively. The correlation coefficient of consumption and output for model 1 and model 2 are 0.9314 and 0.8496. The debt ratio and default period of model 1 are 46.45% and 7.4%, whilst the model 2 is at 45.24% and 12.35% respectively. From the outcome, it can be seen that the model with an endogenous debt relief can target the actual statistic of Argentina for the proportion of default periods. The economy under a dynamic choice of debt relief is more frequent in experiencing default and thereby, the economy is more volatile.

[28]The detail of parameters' calibration can be seen in the previous chapter.

Furthermore, there are other statistical evidences of Argentina economy and model simulations given in Table 5.3.

Table 5.3: Other Business Cycle Statistics from the Models and Actual Data

Business Cycle Statistics		Model 1	Model 2	Actual
Debt Volatility	σ_b	0.3472	0.4262	0.2719
Risk Premium Volatility	σ_ψ	0.0083	0.0207	0.1420
Autocorrelation of Output	ρ_y	0.9838	0.9665	0.6786
Autocorrelation of Consumption	ρ_c	0.8569	0.8021	0.7860
Correlation coefficient (y,ψ)	$\rho_{y\psi}$	-0.18806	-0.1909	-0.4713
Correlation coefficient (c,ψ)	$\rho_{c\psi}$	-0.2138	-0.2244	-0.5402
Minimum Consumption Ratio	$\frac{c}{y}_{min}$	71.28 %	67.05 %	72.42 %
Maximum Consumption Ratio	$\frac{c}{y}_{max}$	95.21 %	155.88 %	90.39 %
Minimum Debt Ratio	b_{min}	0 %	0 %	9.07 %
Maximum Debt Ratio	b_{max}	184.73 %	226.03 %	153.63 %

Table 5.3 shows the volatility of debt and bond premium for model 1 at 0.3472 and 0.0083, whilst model is at 0.4262 and 0.0207 respectively. The autocorrelation of output and consumption for model 1 are 0.9838 and 0.8569 and for model 2 are 0.9665 and 0.8021. These results show the debt volatility of model simulations is more volatile than the actual statistics, whilst the bond premium is less. This outcome relates with the Monte Carlo simulation in Figure 5.10 as the model suggests a sovereign default when the bond premium is increased under a negative shock of productivity, whilst the actual data shows suffocating periods of high bond premiums. The discrete decision of model simulations leads to high volatility of debt, whilst the bond premium is less because the sovereign debtor chooses to default rather than suffer a high premium on borrowing.

Next, the correlation coefficient between output and bond premium of models 1 and 2 are -0.18806 and -0.1909 respectively, which shows a negative relationship between default risk and output. This outcome is sustained with Argentina where the bond premium is higher during a recession and lower during expansion. Besides, the correlation coefficient of consumption and output of models 1 and 2 are -0.2138 and -0.2244. The minimum and maximum consumption ratios for model 1 are 71.28% and 95.21%, whilst model 2 indicates 67.05% and 155.88%. The minimum and maximum ratios for debt for model 1 are 0% and 184.73% and for model 2 they are 0% and 226.03% respectively.

To conclude, the quantitative results from Table 5.2 and Table 5.3 show that the economic model for sovereign default with optimal choice of ex-post debt relief can target the default periods of Argentina. The sovereign debtor can choose to default with a short period for the debt restructuring process with small debt relief under a negative productivity shock, whilst the debtor can choose full default with a long period for debt restructuring under a large negative shock. This outcome shows the non-linear default probability and debt relief over the amount of foreign assets and output. The model simulation for model 2 can provide a better illustration of the optimal choice of debt relief rather than model 1. However, the Monte Carlo simulation for model 2 is more volatile because the economy frequently experiences a short period of debt restructuring.

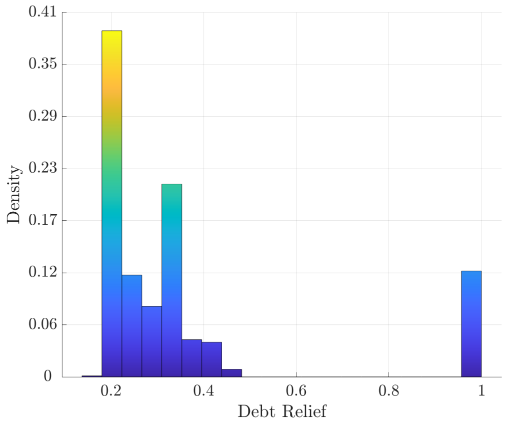
5.5.6 Sensitivity Analysis

This section provides the sensitivity analysis of a parameter in the debt relief function. From Equation 5.3.37, the structure of debt relief and timing of re-entry is controlled by τ. If $\tau = 1$, it will determine a linear structure of debt relief and financial exclusion. If τ is greater than 1, it represents a quadratic structure of debt relief and its timing. Hence, we provide the results of Monte Carlo simulation over 100,000 periods with different level of this parameter (τ) as follows:

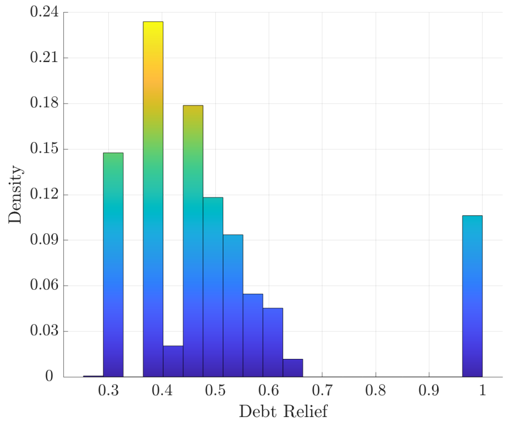
Table 5.4: Sensitivity

Parameters		$\tau = 1$	$\tau = 2$
Output volatility	σ_y	0.1286	0.1265
Consumption volatility	σ_c	0.1422	0.1378
Consumption output volatility	$\frac{\sigma_c}{\sigma_y}$	1.1057	1.0893
Correlation coefficient (c,y)	ρ_{cy}	0.8496	0.8466
Average consumption ratio	$\mu_{\frac{c}{y}}$	83.78 %	83.47 %
Debt ratio	μ_b	45.24 %	46.01 %
Length of default periods		12.34%	13.68%
Debt Volatility	σ_b	0.4262	0.4424
Risk Premium Volatility	σ_ψ	0.0207	0.0256
Autocorrelation of Output	ρ_y	0.9665	0.9659
Autocorrelation of Consumption	ρ_c	0.8021	0.8002
Correlation coefficient (y,ψ)	$\rho_{y\psi}$	-0.1909	-0.0710
Correlation coefficient (c,ψ)	$\rho_{c\psi}$	-0.2244	-0.1321
Minimum Consumption Ratio	$\frac{c}{y}_{min}$	67.05 %	66.02 %
Maximum Consumption Ratio	$\frac{c}{y}_{max}$	155.88 %	161.56 %
Minimum Debt Ratio	b_{min}	0 %	0 %
Maximum Debt Ratio	b_{max}	226.03 %	226.03 %

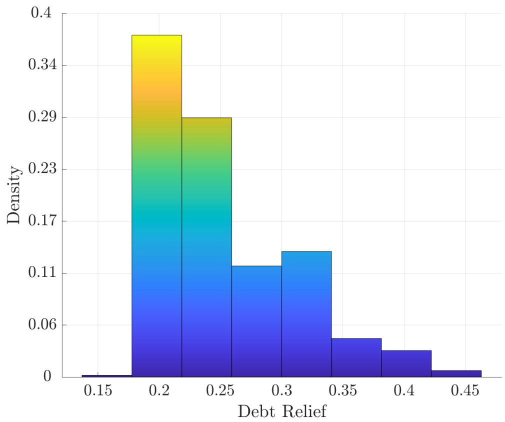
Table 5.4 presents the overall statistics of the simulation when $\tau = 1$ and $\tau = 2$. As can be seen in the table, the volatility of output and consumption is less with a quadratic structure of debt relief ($\tau = 2$). Output and consumption volatility are reduced to 0.1265 and 0.1378 from 0.1286 and 0.1422 respectively. Besides, there is a slight increase of debt volatility from 0.4262 to 0.4424 when $\tau = 2$. The overall statistics of these two simulations are relatively similar. However, there is an interesting outcome from a comparison between these two simulations. The length of default periods is increased from 12.34% to 13.68% when $\tau = 2$. The outcome of an increase in debt hangover period will be obvious from the following histogram of optimal debt relief over different level of this parameter (τ).



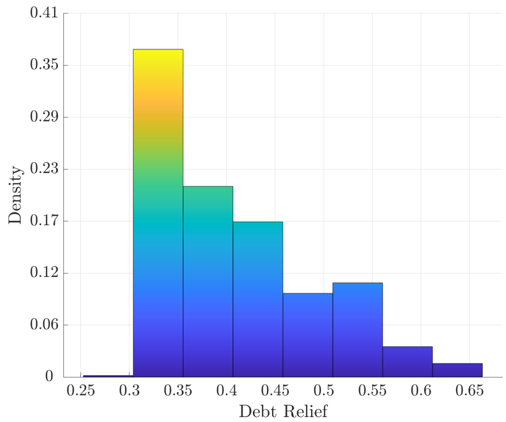
(a) Debt relief at default ($\tau = 1$)



(b) Debt relief at default ($\tau = 2$)



(c) Debt relief at repayment ($\tau = 1$)



(d) Debt relief at repayment ($\tau = 2$)

Figure 5.15: Results from different structure of debt relief function

Figure 5.15 presents the density of debt relief at default and repayment when $\tau = 1$ and $\tau = 2$. Obviously, the quadratic structure of debt relief (τ) has a different optimal point comparing with a linear structure. Figure 5.15b illustrates that optimal debt relief at default is between 0.3 and 0.6, whilst Figure 5.15a shows a lower rate of debt relief between 0.2 and 0.4. From this outcome, it can be concluded that the optimal debt relief is significantly higher when τ is increased and as a result, the probability of re-entry from default is lower. The simulation of debt relief function at $\tau = 2$ has a higher chance to stay longer in default for receiving higher relief rate. Additionally, Figure 5.15d with $\tau = 2$ also shows a higher density of debt relief between 0.3 and 0.6 comparing with Figure 5.15c.

5.6 Conclusion

This research contributes to the related literature on sovereign default and debt relief, providing a theoretical model for default with an optimal choice of ex-post debt relief. It finds that the bond price with a probability of default on the debt relief is more risk-averse. The model framework can illustrate the reason for sovereign default as well as a motive for debt relief by international creditors. The optimal level of debt relief can be found under various levels of foreign asset and output. The probability of sovereign decisions on repayment, full default and partial default with possible debt relief becomes obvious as well as the interactions among state-variables.

In the general model, a sovereign debtor has a guarantee to repay prior borrowing; however, the sovereign debtor can also choose to destroy the guarantee by default and suffer being excluded from the financial market. A defaulter also suffers from a direct reduction of productivity during the debt restructuring period. After a long period of debt hangover, the sovereign debtor can re-enter the international credit market.

In this research, we implement an additional option in the debt restructuring process where the timing of debt restructuring is negotiable as a function of the repayment amount. For example, if the sovereign debtor fully repays the debt, the defaulter will experience only a short period of default. On the other hand, if the sovereign debtor chooses not to repay at all, the defaulter has to experience a long period of debt hangover. In this way, it can be interpreted that there are incentives from creditors for debt relief in order to receive some return on the default loan as well as shortening the period of default.

The ex-post debt relief can be perceived in the model through the amount of foreign assets and output. A sovereign debtor can choose the optimal level of ex-post debt relief during repayment periods. The future probability of default on the optimal debt relief depends on the current level of state-variables and the future possibility of productivity shocks.

From the stylised facts for sovereign default, many defaulters have various experiences of debt restructuring. Some countries received numerous reliefs from creditors, whilst other countries received nothing. There are several causes behind sovereign default and subsequent debt relief, which this research attempts to identify and solve.

In order to provide a sophisticated framework for the model, we calibrate parameters by targeting a real business cycle from the quarterly data for Argentina's economy between 1980 Q1 and 2017 Q4. For simplicity and comparability of the models, the parameters are borrowed from existing literature and the previous chapter on a sovereign model for the full default option. The stylised facts for Argentina and other countries concerning the proportion of debt relief and its timing is also considered. Moreover, this research implements the discrete model following Tauchen's method (Tauchen, 1986) for an approximation of a transitional probability matrix on productivity. The Monte Carlo simulation over 100,000 periods and statistical evidence are also given.

The main novelty is a dynamic choice of optimal debt relief prior to default. The quantitative results show the policy function of debt relief under different levels of foreign asset and output. Sovereign defaulters choose a bond contract with possible ex-post debt relief differently according to economic conditions; level of capital, foreign assets and productivity. We find that the sovereign debtor under a negative productivity shock will borrow with a ex-post debt relief at 20%, if default occurs. With the expectation of small debt relief, the sovereign borrower is expected to partially repay some debt to compensate for shortening the financial exclusion period. International creditors have to sacrifice 20% of debt relief to receive returns instead of nothing. On the other hand, a sovereign debtor will choose to borrow with a higher rate of debt relief under a positive shock. From these outcomes, the sovereign debtor who intends to borrow is pressured more during a bad period. The debt relief positively influences the default intention through an increase in the utility from less repayment; therefore, debt relief is higher during a good period and lower during a bad period.

Interestingly, the bond price is not monotonically influenced by the possible amount of ex-post debt relief but includes the time influence of the exclusion period. The bond price schedule is related with a posterior function of debt relief and we find an increase of bond premium if the ex-post debt relief is higher. The bond price is a function of future default intention and, thereby, an increase in the possible amount of debt relief has an effect on the subsequent price of the bond. However, the bond price schedule is not a linear function of debt relief only. The default intention also includes timing of debt hangover. From the results, if the probability of re-entering is too low, the default intention will be less. For example, the bond contract with only absolute relief and a maximum period of debt hangover suggests a lower intention to default and, therefore, the bond premium becomes lower than a bond contract with a lower rate of possible ex-post debt relief.

Moreover, the amount of foreign asset is significantly related with output and bond premium. A debtor is pressured more during a bad period because the default intention is higher. Consequently, at the same amount of borrowing, the bond premium is higher from a decrease in output and becomes lower from an increase in output. In order to borrow at a small bond premium, debtors will borrow less during a bad period and borrow more during a good period.

Furthermore, a comparison of full and partial default is considered in the theoretical model and we find that the full default option is beneficial if output is low (approximately 30% lower than the steady-state output) regardless of foreign assets. From the policy function and Monte Carlo simulation, the defaulter experiences full default at high and low amounts of borrowing, whilst the defaulter only calls upon full default when output is significantly low. On the contrary, if output is not too low, default with some relief provides higher utility. Remarkably, the model also captures technical default with small debt relief. In the previous model for full default, there is no default intention during a positive shock; however, this research shows a default intention during a good period with an expectation of small exclusions and debt relief.

Next, the steady-state levels of capital accumulation and foreign asset are found with optimal debt relief. A debtor can sustain in the steady-state if there is no productivity shock. From the Monte Carlo simulation, sovereign debtors will borrow more for both consumption and capital investment during a good period. Conversely, debtors will not be able to sustain capital and foreign assets from a negative shock. We find that debtors sell their physical capital in order to smooth consumption as well as borrow less during a bad period. From this result, consumption is smoothed during repayment from a change in capital accumulation and foreign assets to compensate for loss of output.

However, we also find that there is a fluctuation in consumption during periods of default and re-entry. The consumption is lower when a defaulter has less output and small capital accumulation. At some point, the debtors will stop selling physical capital and suffer from negative productivity through lower consumption. Oppositely, consumption is sharply increased when a defaulter re-enters the credit market. At an initial period of borrowing, the debtor will borrow more for both consumption and investment; thereby, consumption temporarily increases for one period of re-entry.

Additionally, the theoretical model shows a difference in default intention under different levels of perceived capital accumulation. From the given perceived amount of capital accumulation, the probability of default is affected. We find a positive relationship of intention for partial default with perceived capital accumulation. If the given amount of perceived capital is higher, the probability of partial default will be increased. Besides, the full default intention slightly decreased if the perceived level of capital accumulation is higher. This result is sustained with a change in output that impacts on the default choice during a good period. A high volume of output will lead to an incentive of the sovereign defaulter for a technical default with debt relief. Nevertheless, we find insignificant impact on perceived capital accumulation and bond premium. Unsurprisingly, a change in future capital accumulation in this model is not a function of bond price schedule. The perceived capital accumulation is given from model equilibrium; therefore, the behaviour of a sovereign debtor is not directly related with perceived capital accumulation.

Finally, the statistical evidence from the model simulation shows that an economy with default options can capture the periods of Argentina's default. The model simulation can illustrate both short and long periods for debt restructuring. Amounts of debt relief ranging from 10% to 40% have been granted to shorten the period of debt hangover. The volatility of bond premium is closer to the actual bond; however, the economy is more volatile under the debt relief option.

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Appendices

This appendix will be separated into two main parts. The first part will describe the data sources. The second part will describe the computational algorithm on the model of sovereign default and productivity shock with the finite state Markov-Chain approximation (Tauchen, 1986; Tauchen and Hussey, 1991).

5.A Data Description

The data series are taken from the Ministry of Economy (MECON) of Argentina and complied by Datastream. These include real GDP, household consumption, government spending that present in real term and external debt as a percentage to the real GDP. Besides, The overall risk premium on US denominated debt is from JP Morgan and compiled by Oxford Economics. All data are quarterly series with non-seasonal adjustment between 1980 Q1 and 2017 Q4.

5.B Computational Algorithm

In this thesis, we discretise a continuous stochastic process of TFP by using Tauchen's method (1986). The TFP level is defined as follows:

$$a_t = \rho\, a_{t-1} + \epsilon_t \,, \tag{5.B.1}$$

where $|\rho| < 1$ and $\epsilon_t \sim N(0, \sigma_\epsilon^2)$. From this continuous stochastic process, we will assume $\hat{a}$ as the discrete value from the approximation of a with the finite set of possible realisation $\{a_1, a_2, ...a_N\}$. Thus, if a is a vector of possible realisation of TFP, $\hat{a}$ is defined as an approximated vector of continuous structure dominated by a. To discretise from the continuous structure, Tauchen (1986) suggests that a maximum value of $\hat{a}(a_N)$ is given by:

$$a_N = m \left(\frac{\sigma_\epsilon^2}{1-\rho^2} \right)^{\frac{1}{2}} , \tag{5.B.2}$$

where m is a multiple of the unconditional standard deviation. In general case, m is equal to 3. From the symmetric assumption of the distribution, the minimum value of $\hat{a}(a_1)$ is $-a_N$. Besides, $\{a2, a3, ..., a_{N-1}\}$ will be equivalently located between the interval $[a_1, a_N]$ with the distance (d). Therefore, the finite state of possible realisation of $\hat{a}$ can be obtained as the state space form.

Next, Tauchen (1986) provides a solution to compute the transition probabilities as follows:

$$\pi_{jk} = P\left\{\hat{a}_t = a_k | \hat{a}_{t-1} = a_j\right\} = P\left\{a_k - \frac{d}{2} - \rho a_j < \epsilon_t \leq a_k + \frac{d}{2} - \rho a_j\right\}. \tag{5.B.3}$$

If $1 < k < N-1$, the transition probability π_{jk} will be given by:

$$\pi_{jk} = F\left(\frac{a_k + \frac{d}{2} - \rho a_j}{\sigma_\epsilon}\right) - F\left(\frac{a_k - \frac{d}{2} - \rho a_j}{\sigma_\epsilon}\right) , \tag{5.B.4}$$

where F is the standardisation process. When $k = 1$ and $k = N$ the transition probability at these boundaries are given by:

$$\pi_{j1} = F\left(\frac{a_1 + \frac{d}{2} - \rho a_j}{\sigma_\epsilon}\right) , \tag{5.B.5}$$

$$\pi_{jN} = 1 - F\left(\frac{a_N - \frac{d}{2} - \rho a_j}{\sigma_\epsilon}\right). \tag{5.B.6}$$

From the above method, the possible set of $\hat{a}$ with its transition probability will converge to a_t when the distance (d) is approaching 0 and N is closed to ∞. Therefore, a is defined by the finite vector of possible realisation of TFP with the interval $[\underline{a}, \overline{a}]$[29], whilst the continuous stochastic process of TFP is approximated by the given transition probability matrix.

5.C Approximation of Partial Debt Repayment

In the model, the sovereign defaulter is required for repayment at the re-entry period. Under the assumption in this chapter, the defaulter is obligated for the partial repayment as a percentage of the initial amount. There is only absolute defaulter who has the repayment at zero. Thereby the function of repayment is given by:

$$\tilde{b} = \lambda \cdot b\,, \tag{5.C.1}$$

where b is the amount of initial debt, $\tilde{b}$ is the amount of debt at re-entry and λ is a percentage of debt repayment ($0 \leq \lambda \leq 1$). Specifically, the repayment at re-entry is zero ($\tilde{b} = 0$), if the absolute default is chosen.

Under the discrete model, the amount of partial repayment has to be allocated in the given grid of borrowing amount (b). Thereby, the repayment at re-entry if the debt relief is applied requires the approximation to reallocate the repayment to the

[29] $\underline{a}$ is the minimum state of TFP within the finite vector and $\overline{a}$ is a maximum state of TFP where the minimum and maximum are symmetrically defined by Equation 5.B.2

$$
\begin{array}{ccccc}
b & & \tilde{b}\,(\omega \approxeq 0.50) & & \omega \\
\begin{bmatrix} -4.5 \\ -4.4 \\ -4.3 \\ -4.2 \\ -4.1 \\ -4.0 \\ -3.9 \\ -3.8 \\ -3.7 \\ -3.6 \\ . \\ . \\ . \\ -0.50 \\ -0.40 \\ -0.30 \\ -0.20 \\ -0.10 \\ 0 \\ 0.10 \\ 0.20 \\ 0.30 \\ 0.40 \\ 0.50 \end{bmatrix}
& \Rightarrow &
\begin{bmatrix} -2.3 \\ -2.2 \\ -2.1 \\ -2.1 \\ -2.0 \\ -2.0 \\ -2.0 \\ -1.9 \\ -1.8 \\ -1.8 \\ . \\ . \\ . \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0.10 \\ 0.20 \\ 0.30 \\ 0.40 \\ 0.50 \end{bmatrix}
& \Rightarrow &
\begin{bmatrix} 0.4889 \\ 0.5000 \\ 0.5116 \\ 0.5000 \\ 0.5122 \\ 0.5000 \\ 0.4872 \\ 0.5000 \\ 0.4865 \\ 0.5000 \\ . \\ . \\ . \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
\end{array}
$$

Figure 5.16: Vectors of borrowing (b), repayment approximation ($\tilde{b}$) and debt relief (ω)

nearest neighbour within the given grid. To reduce the loss of approximation, we control the loss of reallocation grid of repayment to the nearest neighbour within ±3%. In order to achieve that, the state variable (number of grid) for borrowing amount is set at 71 and the minimum application for debt relief is at -0.5. Thus, the sovereign defaulter will not be able to apply for debt relief, if the real amount of foreign asset is equal or greater than -0.5. The given vector of the borrowing amount and repayment approximation are illustrated as follows:

From Figure 5.16, ω is the ratio of debt relief $1-\lambda$. It can illustrate the value of each element in the grid of the initial debt (b), debt at re-entry from default ($\tilde{b}$) and debt relief ($1-\lambda$). Specifically, to comfort the computation with the discrete form, the value of b_{pd} is required to locate within the finite set of possible debt with interval $[\underline{b}, \overline{b}]$ or $[-4.5, 0.50]$ and the distance for each element is equivalently allocated at 0.10. Hence, with the example of given debt relief at 50% of the initial debt, the amount of debt repayment with debt relief will be approximated and reallocated to the nearest grid as can be seen in the vector of $\tilde{b}$. In addition, when the foreign asset (b) is equal or greater than -0.50, the option of debt relief is unavailable. Remarkably, the borrower is unable to default with a positive total amount of foreign asset.

5.D Structure of Debt Relief Function

In this chapter, the amount of debt relief is a function of financial exclusion as given by Equation 5.3.37. The boundaries of debt and financial exclusion (probability of re-entry) are denoted by $[\underline{\omega}, \overline{\omega}] = [0, 1]$ and $[\underline{\theta}, \overline{\theta}] = [0.05, 1]$ respectively. From the setup, it can be interpreted that the ratio of debt relief can be varied between 0 and 100%, whilst the probability of re-entry is from 5% to 100%[30]. Moreover, the structure of debt relief function is controlled by a parameter τ. The structure of debt relief function is setup at $\tau = 1$ for the simplicity of debt relief structure. When τ is equal to 1, it indicates the linear structure between debt relief and probability of re-entry. Its structure can be transformed from the linear to the quadratic function when τ is greater than 1. The linear and quadratic structures of debt relief from

[30]The probability of re-entry cannot be 0, otherwise the defaulter cannot reenter to the credit market and will stay in default forever. Hence, we follow the previous chapter of absolute default with 5% probability of re-entry

various level of τ is illustrated as the following:

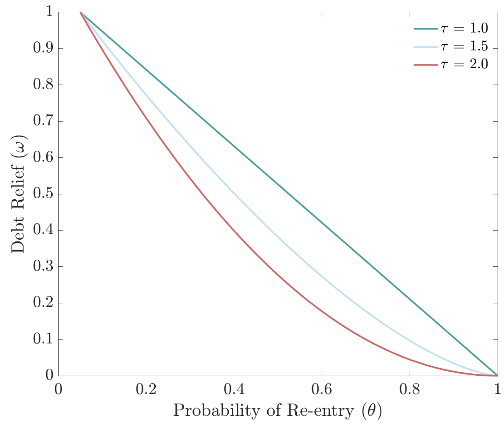

Figure 5.17: A structure of debt relief (ω) from different level of τ

Figure 5.17 illustrates plots of debt relief (ω) and financial exclusion (θ) through different levels of parameter τ. It is obvious that $\tau = 1$ represents a linearly negative relationship between debt relief and probability of re-entry. An increase in probability of re-entry leads to an equivalent decrease in the amount of debt relief, if τ is equal to 1. If τ is greater than 1, it will represent the quadratic structure of debt relief function. From the figure, if τ is 2, a decrease in probability of re-entry will lead to an exponential increase in debt relief. Therefore, it represents the rate of change in debt relief when a sovereign debtor stay in default. The longer stay in default relates to an increase of debt relief; which is in accordance with Cruces and Trebesch (2013).

Chapter 6

Conclusion

In this thesis we have presented three novel models of foreign default in order to improve our understanding of defaults and offer guidelines to policy makers. In this chapter we firstly provide an overall summary of the thesis, then discuss the policy implications of our analysis and finally suggest future research developments

6.1 Summary

This thesis contributes to the existing literature on sovereign default by providing three main theoretical models. The first one deals with sovereign default with unobservable physical capital and it finds the default probability of sovereign borrowers under the assumption that borrower's capital accumulation is unobservable. It presents equilibrium results for the optimal level of capital accumulation, foreign assets, and bond price under an incomplete bond contract in response to productivity shocks. The second model is concerned with sovereign default with debt relief and with the length of stay in default. To avoid a long period of debt hangover, we introduce debt relief and the exclusion rate as exogenous variables. We find that partial default emerges in equilibrium and depends on the rate of debt relief and the probability of re-entry into the credit market. Hence borrowers can obtain higher utility from choosing the partial default option instead of the absolute default. The third model captures sovereign default with endogenous debt relief. It allows us to find the steady state values and dynamics of debt relief and its timing for different values of productivity, capital accumulation, and foreign assets. Below we summarise the main features and results for each paper, highlighting the respective novelties.

In the first paper we develop a theoretical model of sovereign default with imperfect information about a debtor's physical capital. A sovereign debtor will make decisions on consumption and investment with an option for absolute default. We follow Tauchen's method of approximation of continuous stochastic process of productivity shocks and compute it in the form of discrete matrix (Tauchen, 1986). The model is calibrated to target Argentina's economy between 1980 Q1 and 2017 Q4. Monte Carlo simulation is applied with the moment matching method for the calibration. We mainly compute the dynamics of equilibrium bond prices, unobservable physical capital, foreign assets and consumption, in addition to the perceived equilibrium default. As capital is unobservable, we show that borrowing for consumption can be optimal for positive productivity shocks. Our model fits Argentina's default experience better than the existing literature. It also highlights the importance of modelling imperfect information to understand the behaviour of both sovereign debtors and international creditors.

We find that incomplete information about physical capital affects the bond premium, as well as the optimal decisions on capital, consumption, and foreign assets. We assume that international creditors cannot thoroughly observe a borrower's assets or guarantee a bond with domestic collateral. This assumption follows the experience of Argentina where seizures of collateral assets and trade intervention were not authorised by the court (Hornbeck, 2004; Miller and

Thomas, 2007). International creditors infer steady-state capital stock from realised output and productivity. The bond price schedule is computed under the perceived equilibrium default with unobservable physical capital and the bond price is not affected by a change in future capital accumulation. We show the different grid points of perceived equilibrium and optimal grid points of capital, consumption, and foreign assets.

We find that borrowers choose to borrow more whenever possible. In comparison with the previous literature, our model simulations indicate a higher level of debt. Unobservable capital also implies a higher premium in the perceived equilibrium because future capital investments are not be included in the current adjustment of bond prices. Moreover, the steady-state level of capital accumulation is typically lower than in the literature because sovereign debtors have less incentive to accumulate capital for influencing the bond price schedule in the current period. The impulse response functions also indicate that sovereign debtors find it optimal to consume rather than to invest if capital accumulation is currently above the stead-state. The model simulation also presents consumption smoothing behaviour through adjustment of capital accumulation during bad times. To avoid default from a large negative productivity shock, sovereign debtors are better off by reducing consumption and paying off their debt in the current period. Finally, computation experiments show that our model matches Argentina's experience; however, we find that each debt restructuring period is relatively long. The model can capture only a long length of debt hangover because only the absolute default option is included; that leads to a constant value of re-entry probability. Therefore, the next paper in the thesis adds to the framework the partial default option with various levels of probability of re-entry into the credit market as well as the rate of debt relief. In this way, we capture both short and long periods of debt hangover and make the sovereign borrowers reach higher utility than from the partial default option.

In the second model, we include the option of partial default to our seminal framework with unobservable physical capital. In the previous model, a sovereign debtor only has an option of absolute default; where the defaulter does not need to make any further repayments on the default bond. With an option of partial default, the sovereign defaulter is due a partial repayment at re-entry with a shorter period of debt restructuring. The option of partial default is constructed by observing that in real world defaulters are faced with various relief rates and different lengths of debt hangover (Cruces and Trebesch, 2013). Thus, the model for partial default includes two main exogenous variables, debt relief and length of stay in default; in order to find the optimal rates for repayment and timing of re-entry that maximise the borrower's utility.

Besides, another novelty provided by this model is the adjustment of bond premium from the partial default. The partial default option increase the utility of sovereign debtors; they are better off because it provides a fast track through which they can re-enter the credit market. In this way, if there is an incentive for partial default, it requires bond prices to adjust for responding with probability of default that is consistent to the debt relief. We find a non-monotonic shift of the bond price schedule due to debt relief and its timing. For small debt levels, the bond premium is higher when the amount of debt relief and the chance of re-entry is high. Interestingly, the reverse occurs for large debts. In particular we find that firstly, full default makes the sovereign borrowers with large debt have higher utility. Secondly, the probability of partial repayment (instead of absolute default) at large amount of borrowing makes the bond premium slightly lower because of the expectation of future partial repayment. Hence, an option of partial default increases the bond premium at low debt level and decreases the bond premium at high debt level[31].

Moreover, the model also presents the optimal level of debt relief for given timing. We find that utility is maximised for a rate of debt relief of 39% and probability of re-entry of 10% This result is in accordance with the average haircuts of 180 sovereign bonds between 1980 and 2010 at 37% (Cruces and Trebesch, 2013). Thus, this theoretical model can provide an illustration of debt relief and its timing as well as the adjustment of bond prices as a function of debt relief. However, the rate of debt relief and its timing are exogenous in the model. Therefore, the next study in the thesis aims to model debt relief and its timing as endogenous variables that can be chosen dynamically for different levels of state-variables, such as choosing high debt relief during a positive productivity shock and choosing low debt relief during a negative productivity shock.

Our third model includes a sovereign default and debt relief function. We model the optimal choice of possible ex-post debt relief and its timing. At the borrowing stage, sovereign debtors will decide the debt relief as well as the time of re-enter in the sovereign bond contract. If default occurs, sovereign debtors must make the subsequent repayment according to the debt-relief function. At the same time, creditors also infer the probability of default and the possible realisation of debt relief and its timing; thereby, the bond premium is adjusted accordingly. In this way, a

[31] From the simulation, the option of partial default will decrease the bond premium when the debt to output ratio is more than twice, whilst the option of partial default will increase the bond premium during positive productivity shocks and the moderate debt ratio.

sovereign debtor can obtain higher utility through a dynamic choice of optimal debt relief over absolute default.

We also model debt relief as a function of the probability of re-entry. The amount of debt relief can compensate for the reduction of time spent in default. We find that a sovereign defaulter chooses a bond contract with a rate of debt relief differently from the rate of productivity shocks. The optimal level of debt relief can be varied based on the different level of capital accumulation, foreign assets, and productivity.

For a negative productivity shock, the sovereign debtor will borrow with an expectation to receive a debt relief at 20% if default occurs. Additionally, the rate of the debt relief is higher during a good period and lower during a bad period. In accordance with the previous model of exogenous debt relief, the bond price is not monotonically influenced by the possible amount of debt relief, but it includes the time influence of an exclusion period. The bond price is not a linear function of debt relief only. The default intention also includes timing of the restructuring process. If the probability of re-entry is extremely low, the default intention will be less and close to the absolute default. Therefore, the gain from applying partial default is a trade-off between the amount of debt relief and its timing for re-entry.

Moreover, we find that the absolute default is preferred by the borrower if output is very low, regardless of foreign assets; conversely, if output is not too low, default with some relief becomes optimal. Our model also includes default with a small debt relief for a positive productivity shock. We show that the optimal decision is to plan for a default during good time. Subsequently, the model presents steady-state values for foreign assets and capital accumulation at the perceived equilibrium with optimal debt relief; when the default probability is still low. The sovereign debtor can reach the steady-state debt level if there is no productivity shock. If there is a shock, the country can smooth consumption through capital accumulation and exercise foreign assets with an optimal rate of relief. From the Monte Carlo simulation, we notice instances of absolute default and partial default under various rates of debt relief and length of stay in default. However, for the same parameters, the model with debt relief is more volatile because there are more available choices of default that offer to sovereign debtors; for maximising their utility. On the other hand, the bond-premium is higher during positive and negative productivity shocks for countries with moderate level of debt, in order to compensate for a higher default intention (higher probability of applying partial default).

In conclusion, the three theoretical models provided in this thesis illustrate reasons for sovereign default; where the sovereign borrower can achieve higher utility from being in default or choosing a partial default option during a large productivity shock. We find that the bond premium should be adjusted to be consistent with the probability of both absolute and partial default with a dynamic rate of debt relief. Hence, we confirm the importance of modelling limited information about capital accumulation as well as debt relief in the bond price schedule. The next section discusses the policy implications of our results.

6.2 Policy Implications

Several policy implications emerge from our theoretical analysis. The models can be used to plan the time to default. From the Monte Carlo simulations, default is optimal for a large productivity shock or a series of negative productivity shocks.

Bond premium is high during bad periods and relatively low during good periods. If the size of the shock is not too large, debtors can still be able to repay the initial debt and interest. However, it can be seen in the simulation that if the premium is high greater than 10%), there is a greater chance that discontinuing repayment is a better option. In this case, if the debtor decides to repay and continue in the credit market, there is a high probability that the bond premium of subsequent borrowing will continue to increase because of negative shocks. According to the structure of autoregressive function in the model, the persistency of productivity shocks is high and as a result, the total factor of productivity will fall for several periods from a large negative productivity shock.

Over the past twenty years, there have been several cases, including Argentina, of defaulters re-entering the credit market within four quarters. Under the default option, developing countries can avoid repaying a huge premium by escaping through a default route before returning when the economy is back to normal.

However, it is not always the case that default is inevitable. Sovereign debtors can avoid default by selling capital accumulation and borrowing less during bad times. The bond premium is high due to a huge amount of borrowing during bad times. Interestingly, we find that the default incentive under a negative shock becomes lower as the borrowing amount decreases. Thus, sovereign debtors have to borrow less in order to enjoy lower premiums. In this way, countries can avoid default by paying back some loans to prevent the bond premium from surging. Nevertheless, borrowing less can lead to a lack of consumption, therefore, sovereign debtors have to sell physical capital to smooth consumption. Our models can show the optimal level of capital and borrowing for sovereign debtors in order to avoid default. If the debtor has high capital stock, default can be avoided by selling capital. On the other hand, selling capital for consumption is not a good option if the country's capital stock is below the steady-state.

Furthermore, social planners are advised to borrow more for both consumption and investment during good times. From the Monte Carlo simulation, the bond premium is relatively low for a positive productivity shock. If capital stock is below the steady-state level, when there is a series of positive shocks, sovereign debtors can borrow at low cost. During a good period, countries that borrow for investment tend to grow more and achieve output at above the steady-state level. However, it is not optimal to borrow for investment only. The results of our models indicates that an increase in consumption is also a good choice if capital accumulation is higher than the steady-state level. Theoretically, countries cannot sustain capital accumulation at above steady-state level in the long term. Capital level will converge to the steady-state when productivity returns to normal. The marginal utility of an increase in capital accumulation is lower when capital is above the steady-state; therefore, borrowing for consumption is better than accumulating capital if the current physical capital is too high. From the Monte Carlo simulation, capital stock can be 50% higher than the steady-state level, and consumption can be sustained at above average level during a series of positive shocks. However, the level of capital accumulation will definitely converge to the steady-state when productivity is normal.

Our models also suggest the importance of information about capital accumulation. In developed countries, the information about capital accumulation is typically complete; in the sovereign bond market, physical capital is not used as collateral in the bond contract. International creditors cannot force sovereign debtors to sell their assets or seize their physical capital in the domestic market. Hence, our model can be useful for creditors in inferring the default probability. Our model with unobservable physical capital provides no incentive from accumulating capital in terms of a reduction in the bond premium. The bond premium is computed from the current information and for steady-state capital accumulation only.

The theoretical models with debt relief indicate better outcomes for sovereign debtors, as well as higher incentive for debt relief from international creditors. The result shows that if default is inevitable, partial default is preferable in most scenarios. Absolute default with a long stay in debt restructuring is only optimal, if the productivity shock is large. Moreover, a long stay in default is not beneficial for debtors or creditors. Debtors will be forced to the lower-state of output, whilst creditors will lose repayments during default. Debtors will not be able to repay the loans if their income stream is lower than the total debt. To avoid total loss, creditors need to grant some relief on the initial loan repayment in order to reduce the total amount of debt and, as a result, debtors will be able to pay back some of the debt. Additionally, the time spent in default also becomes shorter when debt relief is applied. In this way, there is an incentive for creditors to offer debt relief when default is inevitable. Moreover, from the Monte Carlo simulations for Argentina, we find that the optimal rate of debt relief ranges between 20% and 40%. These findings can inform both creditors and debtors about possible relief during default.

We also find that the bond premium adjusts with equilibrium debt relief. The probability of default in the presence of debt relief is higher than in the model without debt relief, which indicates that the bond price with only absolute default probability is too low. A sovereign debtor may be willing to default because there is a chance of receiving debt relief as well as a short period of debt hangover. The bond premium captures the risk of default with an expectation of future debt relief. To conclude, the theoretical models in this thesis indicate when the sovereign debtor should default on their debt as well as the policies available to avoid default. They highlight how the bond price adjusts in the presence of incomplete information about a debtor's capital accumulation and the expectation of debt relief. The models therefore capture the behaviour of sovereign defaulters, under both short and long periods of debt restructuring. However, there are limitations in our framework, and these will be discussed in the following section together with suggestions for further developments

6.3 Further Developments

From the framework of sovereign default with unobservable physical capital and debt relief, our model is flexible enough to be extended to capture some important features, such as default to different bond types, borrowing to specific creditors, bond availability for different maturities and premium, and foreign exchange rates.

Our model can be a seminal framework to study partial default to specific types of bond. In the credit market, sovereign bonds have different maturities and bond yields. Our framework of partial default can be further developed to specify an optimal option to default for some specific bond types in order to provide higher utility for sovereign borrowers.

The stochastic process in this thesis is structured with a discrete approximation (grid form). The accuracy is based on the number of state-space specified at the computation. Thus, the future development can be achieved by transforming the discrete computational model into a continuous form. However, it will be typically difficult to obtain the equilibrium with options to default through the continuous format because the equilibrium will always be at a corner, either always or never in default.

Our framework also allows to capture the effects of borrower government taxes on default and debt relief decisions. During a default period, there can be a requirement from international creditors such as the IMF for the borrower to implement a tax before receiving debt relief, in order to increase the probability of future repayment. Difference in tax rates can affect the probability of future default.

Finally, the impact of sanction and tariff between countries at default can be considered in this seminal framework. The international creditors who are a big lender to sovereign borrowers can consider forcing countries to repay within an international trade agreement and hence, the tariff between countries should be added into the model. Trade agreement and foreign exchange between countries are also important when sovereign borrowers decide on default.

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