The Economics of Public Policy and Endogenous Health

WANG, JIUNN

How to cite:

Use policy
The full-text may be used and/or reproduced, and given to third parties in any format or medium, without prior permission or charge, for personal research or study, educational, or not-for-profit purposes provided that:

- a full bibliographic reference is made to the original source
- a link is made to the metadata record in Durham E-Theses
- the full-text is not changed in any way

The full-text must not be sold in any format or medium without the formal permission of the copyright holders.

Please consult the full Durham E-Theses policy for further details.
The Economics of Public Policy and Endogenous Health

Jiunn Wang

Department of Economics and Finance
Durham University
April 2019

A Thesis submitted for the degree of
Doctor of Philosophy
Dedicated to

My parents
The Economics of Public Policy and Endogenous Health

Jiunn Wang

Abstract

In this thesis, we explore the relationship between public policy and endogenous health in the economy from three aspects.

First, we explore the health effects underlying the implementation of taxes. Taxes on unhealthy commodities may fail in promoting health because the beneficial effects of reducing unhealthy consumption could be offset by the simultaneous decrease in health investment. However, when coupled with the revenue-neutral tax reforms where income taxes are adjusted, taxes on unhealthy commodities can improve both health and welfare more effectively.

Second, we take a non-paternalistic view to justify the role of sin taxes in terms of fiscal externalities. Although the Pigouvian element in optimal sin taxes decreases in the second-best setting, optimal sin taxes are not necessarily lower due to the presence of the efficiency element. Our calibration on the UK economy shows that the implementation of sin taxes have double-dividends which improve not only health but also economic performance as well as welfare.

Third, we explore the relationship between labor supply and public policy. Old-age labor supply increases with higher spending on health unless the additional spending is funded through taxes on old-age labor income. The economic impacts of changes in technologies are also examined. Furthermore, we find that the optimal tax scheme is determined by the coefficients of relative risk aversion.
Declaration

The work in this thesis is based on research carried out at the Department of Economics and Finance in Durham University, UK. The material contained in this thesis has not been submitted in support of an application for another degree or qualification in this or any other institution.

Copyright © 2019 by Jiunn Wang.

“The copyright of this thesis rests with the author. No quotations from it should be published without the author’s prior written consent and information derived from it should be acknowledged”.

iv
I would like to convey my deepest appreciation to my supervisors, Dr. Thomas Renström and Dr. Laura Marsiliani, for their valuable advice, patient guidance, and immense knowledge. I have greatly benefited from every supervision meeting with them, and could not have completed my thesis without them.

I want to express my sincere gratitude to Professor Luca Spataro and Professor Nigar Hashimzade for taking the time to review my thesis and for making constructive comments.

I appreciate the comments from academics at Durham University, Dr. Mauro Bambi, Dr. Xiaogang Che, Professor Tatiana Damjanovic, Dr. Vladislav Damjanovic, Dr. Leslie J Reinhorn, and Dr. Hong-Il Yoo; academics at Academia Sinica, Professor Been-Lon Chen, Dr. Ting Yuen Terry Cheung, and Dr. An-Chi Tung; and participants in the Scottish Economic Society Annual Conference 2019, Royal Economic Society PhD meeting 2018, 4th Taiwan Economics Research, 11th RGS Doctoral Conference in Economics, 18th meeting of the Association for Public Economic Theory, and the Tools for Macroeconomics workshop in LSE.

I am grateful to my friend, Mrs. Liz Smedley, for proofreading my thesis. Her continuous support and help throughout my postgraduate life means a lot to me.
I owe my profound gratitude to my parents, Mr. Yun-Lung Wang and Mrs. Li-Li Ma. Their support helps me overcome every obstacle in life. I will always be indebted to them. I want to thank my sister, Beryl Wang, my brother-in-law, Bruce Chen, and my little nephew, Pin-You Chen for giving me the warmth from family. I am lucky to have this supportive family and I will not take it for granted.

I am grateful to my friends, Chang-Mao Chao and Yuan-Yuan Tsai, who provided accommodation to me and helped me to resume my faith in God. I am equally grateful to my friend Xiaoxiao Ma, who encouraged me and shared so much reading material. I want to thank my friends from Ustinov College, Paula Rondon-Burgos, Marianna Iliadou, Connie Kwong, Jarno Välimäki, Yuqian (Linda) Wang, and Leah Rie Zou; my PhD colleagues, Adwoa Asantewaa, Helena Brennan, Tevy Chawwa, Jingyuan Di, Sara Gracey, Yunzi He, Bledar Hoda, Lu Li, Xiao Liang, Yoadong Liu, Johannes Schmalisch, Changhyun Park, Jeongseon Seo, Handling Sun, Matt Walker, and Narongchai Yaisawang; and my Taiwanese friends in Durham, Hung-Ying Chen, Pin-Chu Chen, Pen-Yuan Hsin, and Yo-Hsin Yang. I will always cherish our friendship.

Last but not least, I would like to thank God for accompanying me and giving me peace during challenging times. I am glad that I resumed my faith in God during my PhD journey. God reminds me of the reason why I want to pursue a PhD. I hope that I will never forget why I started, and will not be led astray by fame or fortune.
Contents

Abstract iii

Declaration iv

Acknowledgements v

1 Introduction 1

1.1 Health in the economy . . . . . . . . . . . . . . . . . . . . . . . . . . 2

1.2 Health and public policies . . . . . . . . . . . . . . . . . . . . . . . . 5

1.3 A road map for this thesis . . . . . . . . . . . . . . . . . . . . . . . . 8

2 Tax reform, unhealthy commodities and endogenous health 11

2.1 Introduction . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 12

2.2 The model . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 16

2.2.1 The decentralized economy . . . . . . . . . . . . . . . . . . . . . 18

2.2.2 The optimization problem . . . . . . . . . . . . . . . . . . . . . . 19

2.2.3 The equilibrium . . . . . . . . . . . . . . . . . . . . . . . . . . . . 21

2.2.4 Parameterization . . . . . . . . . . . . . . . . . . . . . . . . . . . 22

2.3 Policy analysis . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 25
<table>
<thead>
<tr>
<th>Contents</th>
<th>viii</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.3.1 Calibration</td>
<td>26</td>
</tr>
<tr>
<td>2.3.2 Comparative-static analysis of changing $\tau_x$ alone</td>
<td>29</td>
</tr>
<tr>
<td>2.3.3 Tax reform</td>
<td>32</td>
</tr>
<tr>
<td>2.3.4 Welfare analysis</td>
<td>38</td>
</tr>
<tr>
<td>2.4 Conclusion</td>
<td>40</td>
</tr>
<tr>
<td>2.A Appendix</td>
<td>41</td>
</tr>
<tr>
<td>2.A.1 Closed-form solutions</td>
<td>41</td>
</tr>
<tr>
<td>2.A.2 Comparative-static effects of income taxes</td>
<td>42</td>
</tr>
<tr>
<td>2.A.3 Health sector with DRTS technologies</td>
<td>43</td>
</tr>
<tr>
<td>3 Optimal sin taxes in the presence of income taxes and health care</td>
<td>47</td>
</tr>
<tr>
<td>3.1 Introduction</td>
<td>48</td>
</tr>
<tr>
<td>3.2 The economy</td>
<td>50</td>
</tr>
<tr>
<td>3.3 The Ramsey problem</td>
<td>53</td>
</tr>
<tr>
<td>3.4 Optimal sin taxes</td>
<td>56</td>
</tr>
<tr>
<td>3.5 Sin tax reform</td>
<td>60</td>
</tr>
<tr>
<td>3.6 Conclusion</td>
<td>61</td>
</tr>
<tr>
<td>3.A Appendix</td>
<td>62</td>
</tr>
<tr>
<td>3.A.1 Optimal conditions of the Ramsey problem</td>
<td>62</td>
</tr>
<tr>
<td>3.A.2 Calibration</td>
<td>63</td>
</tr>
<tr>
<td>3.A.3 Sensitivity analysis</td>
<td>65</td>
</tr>
<tr>
<td>4 Labor supply and endogenous lifetime</td>
<td>69</td>
</tr>
<tr>
<td>4.1 Introduction</td>
<td>70</td>
</tr>
<tr>
<td>4.2 The model</td>
<td>73</td>
</tr>
<tr>
<td>Section</td>
<td>Title</td>
</tr>
<tr>
<td>---------</td>
<td>--------------------------------------------</td>
</tr>
<tr>
<td>4.3</td>
<td>The equilibrium</td>
</tr>
<tr>
<td>4.3.1</td>
<td>The effects of the taxes</td>
</tr>
<tr>
<td>4.3.2</td>
<td>The effects of changes in technologies</td>
</tr>
<tr>
<td>4.4</td>
<td>Optimal taxation</td>
</tr>
<tr>
<td>4.4.1</td>
<td>The effects of changes in technologies with optimal taxation</td>
</tr>
<tr>
<td>4.5</td>
<td>Conclusion</td>
</tr>
<tr>
<td>4.A</td>
<td>Appendix</td>
</tr>
<tr>
<td>4.A.1</td>
<td>Analytical analysis</td>
</tr>
<tr>
<td>4.A.2</td>
<td>Calibration</td>
</tr>
<tr>
<td>4.A.3</td>
<td>The effects of taxes on utilities</td>
</tr>
<tr>
<td>5</td>
<td>Conclusions and future works</td>
</tr>
<tr>
<td>5.1</td>
<td>Conclusions</td>
</tr>
<tr>
<td>5.2</td>
<td>Future works</td>
</tr>
<tr>
<td>5.3</td>
<td>Concluding remarks</td>
</tr>
</tbody>
</table>

References 109
## List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Comparative-static effects of implementing ( \tau_x ) alone</td>
<td>29</td>
</tr>
<tr>
<td>2.2</td>
<td>The effects of ( \tau_x ) on ( m ) when other taxes are held constant</td>
<td>31</td>
</tr>
<tr>
<td>2.3</td>
<td>The replacement of ( \tau_l ) and ( \tau_k ) with ( \tau_x ) under revenue-neutral schemes</td>
<td>33</td>
</tr>
<tr>
<td>2.4</td>
<td>Tax reform of replacing ( \tau_l ) with ( \tau_x )</td>
<td>34</td>
</tr>
<tr>
<td>2.5</td>
<td>Tax reform of replacing ( \tau_k ) with ( \tau_x )</td>
<td>34</td>
</tr>
<tr>
<td>2.6</td>
<td>Changes in welfare in response to the implementation of the tax reforms</td>
<td>39</td>
</tr>
<tr>
<td>2.A.1</td>
<td>Comparative-static effects of ( \tau_l )</td>
<td>42</td>
</tr>
<tr>
<td>2.A.2</td>
<td>Comparative-static effects of ( \tau_k )</td>
<td>42</td>
</tr>
<tr>
<td>2.A.3</td>
<td>The comparative-static effects of ( \tau_x ) when ( m ) has DRTS</td>
<td>44</td>
</tr>
<tr>
<td>2.A.4</td>
<td>The comparative-static effects of ( \tau_x ) on ( m ) when ( m ) has DRTS</td>
<td>45</td>
</tr>
<tr>
<td>2.A.5</td>
<td>Tax reform of replacing ( \tau_l ) with ( \tau_x ) when ( m ) has DRTS</td>
<td>45</td>
</tr>
<tr>
<td>2.A.6</td>
<td>Tax reform of replacing ( \tau_k ) with ( \tau_x ) when ( m ) has DRTS</td>
<td>46</td>
</tr>
<tr>
<td>3.1</td>
<td>The first- and second-best ( \tau_x^p ) when ( \epsilon_x = \epsilon_h = 0 ) with different levels of ( \bar{m} )</td>
<td>58</td>
</tr>
<tr>
<td>3.2</td>
<td>The first- and second-best ( \tau_x ) when ( \epsilon_x = \epsilon_h = 0 ) with different levels of ( \bar{m} )</td>
<td>59</td>
</tr>
</tbody>
</table>
List of Figures

3.3 The economic impacts of a sin tax reform ........................................... 60
3.A.1 The first- and second-best $\tau^p_x$ with different $\epsilon_x$ and $\epsilon_h$ .......... 66
3.A.2 The first- and second-best $\tau_x$ with different $\epsilon_x$ and $\epsilon_h$ .......... 66
3.A.3 The first- and second-best $\tau^p_x$ with different $\epsilon_x$ and $\epsilon_h$ when $\kappa_h < 0$ . 67
3.A.4 The first- and second-best $\tau_x$ with different $\epsilon_x$ and $\epsilon_h$ when $\kappa_h < 0$ . 68
4.1 Number of male survivors in England and Wales across all ages .......... 70
4.2 Number of male deaths by age in England and Wales across all ages 71
4.3 The impacts of adjusting $\tau_l$ ............................................................. 77
4.4 The impacts of adjusting $\tau_z$ ............................................................. 78
4.5 The impacts of adjusting $\tau_k$ ............................................................. 79
4.6 The impacts of improving production technologies ............................. 80
4.7 The impacts of improving medical technologies ................................. 81
4.8 Variations in optimal taxes in response to the changes in the coef-
cients of relative risk aversion ................................................................. 84
4.9 Variations in optimal taxes in response to the changes in production

technologies and medical technologies ................................................... 85
4.10 The variations in $\Omega$ in response to the changes in production and

medical technologies ............................................................................. 86
4.11 The impacts of improving production technologies under optimal
taxation ................................................................................................... 87
4.12 Utilities changes under optimal taxation in response to the changes

in production technologies .................................................................... 87
4.13 The impacts of improving medical technologies under optimal taxation 88
4.14 Utilities changes under optimal taxation in response to the changes in medical technologies 89
4.A.1 Changes in utilities in response to the adjustments in taxes 103
# List of Tables

<table>
<thead>
<tr>
<th>Table</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Benchmark parameters and variables</td>
<td>27</td>
</tr>
<tr>
<td>2.2</td>
<td>Benchmark calibration</td>
<td>28</td>
</tr>
<tr>
<td>2.3</td>
<td>Calibrated parameters</td>
<td>29</td>
</tr>
<tr>
<td>2.4</td>
<td>The comparative-static effects of $\tau_x$ with lump-sum transfers</td>
<td>30</td>
</tr>
<tr>
<td>2.A.1</td>
<td>Changes in tax rates with lump-sum transfers</td>
<td>43</td>
</tr>
<tr>
<td>2.A.2</td>
<td>Calibrated parameters with DRTS production in the health sector</td>
<td>43</td>
</tr>
<tr>
<td>3.1</td>
<td>Benchmark parameters</td>
<td>57</td>
</tr>
<tr>
<td>3.2</td>
<td>Calibration</td>
<td>58</td>
</tr>
<tr>
<td>4.1</td>
<td>Benchmark parameters and calibration</td>
<td>76</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

In recent decades, the leading risk factors of the global disease burden has changed from communicable diseases in children to non-communicable diseases in adults (Lim et al., 2013).\(^1\) According to the WHO report in 2018, non-communicable diseases kill 41 million people each year, which is equivalent to 71% of all deaths globally.\(^2\) In addition to their impacts on individual life span, these diseases also reduce individual quality of life dramatically. As these diseases are usually related to lifestyles, policy makers have recently focused more on individual lifestyle choices. For example, in 2011, the Hungarian government levied the “public health product tax” on food products containing high levels of salt, sugar or other ingredients; the collected tax revenue is earmarked for public health spending. The public health product tax is generally believed to be effective in improving the dietary habits of the population.

\(^1\)Risk factors regarding communicable diseases in children include micro-nutrient deficiencies and poor sanitation. Risk factors regarding non-communicable diseases include the consumption of tobacco, alcohol, and ultra-processed food.

\(^2\)https://www.who.int/en/news-room/fact-sheets/detail/noncommunicable-diseases
in Hungary (Bíró, 2015). In 2012, the Philippine government passed a “sin tax reform” with higher excise taxes on tobacco which are earmarked for public health services. This sin tax reform not only creates more revenue for the government but also reduces the prevalence of smoking (World Health Organization, 2015).

Although fiscal policies which target health in the economy have been widely implemented in different countries, the mechanisms behind the health effects of these policies have seldom been examined. The essential purpose of this thesis is to find out exactly how public policies contribute to individual health and welfare in the economy. We investigate variations in the economy regarding the changes in policies when individual health is endogenized. Furthermore, we explore the optimal policies for a government to maximize welfare in the economy.

In the following sections, we will illustrate the main literature discussed in this thesis. In Section 1.1, we will introduce the concept of health in economics. In Section 1.2, we will present the usual application of endogenous health in policy analysis. In Section 1.3, we will provide a road map for this thesis.

1.1 Health in the economy

In the seminal work of Grossman (1972), the level of health is endogenous in that it depends on the resources allocated to its production. The accumulated resources in health can be viewed as a form of human capital which follows the law of motion as below

\[ h_{t+1} - h_t = M_t - \delta_t h_t, \]  

(1.1.1)
where $h_t$ is the health capital in period $t$, $M$ is the gross investment in health, and $\delta$ is the rate of depreciation.

The difference between health capital and other forms of human capital is that health capital does not directly increase labor productivity but enhances the amount of time that one can spend on creating monetary earnings and commodities. Grossman (1972) distinguishes individual endowment time into “sick time” and “healthy time”. Individuals can only work in their healthy time but not sick time. Therefore, decisions about the investment in health are affected by not only the mental benefits from higher levels of health but also the monetary benefits from more healthy time. Health capital contributes to healthy time through a concave function (e.g. Galama et al., 2012; Grossman, 2000; Kelly, 2017).

$$H_t = H(h_t), \quad (1.1.2)$$

where

$$H_h > 0, \quad H_{hh} \leq 0. \quad (1.1.3)$$

In the above equation, $H_t$ denotes individual healthy time in period $t$.

The concept of endogenous health is widely applied in macroeconomics. Ehrlich and Chuma (1990) specify a demand function for longevity to explain the trends in life expectancy and variations in exposure to risk factors across different populations. In their model, individual lifetime utility ($LU$) is set to be separable over time within a finite time framework:

$$LU = \int_0^T e^{-\rho t} U(c(t), H(h(t))) \, dt, \quad (1.1.4)$$

where $c$ denotes consumption and $\rho$ denotes the rate of time preference. An important feature of this model is that individuals have to choose the length of their life
span ($T$) along with consumption and health investment. Hence, this study implies that individuals have to make the trade-off between quality of life – consumption and healthy time – and quantity of life – longevity. The results of this study provide the link between longevity and other economic variables, such as initial wealth, education, medical costs, and age.

In line with Ehrlich and Chuma (1990), Hall and Jones (2007) also explore the trade-off between quality and quantity of life by solving the optimal allocation between consumption and health investment. They construct a dynamic model to rationalize the steady growth in health spending in the US. In their model, they provide a direct link between life expectancy and health investment so as to cover the discussion about mortality in the economy. The basic setting of their model follows:

$$\max f(h)u(c), \quad (1.1.5)$$

$$\text{s.t. } c + h = y, \quad (1.1.6)$$

where $y$ denotes income and $f$ denotes life expectancy which is an increasing function of health investment $h$. Moreover, they employ a utility function which allows for the elasticity of marginal utility to be well above one.$^3$ The marginal utility of consumption thus falls rapidly as the level of consumption increases. A rise in health spending in the US is therefore the rational response to its steady growth of income per person.

$^3$In Hall and Jones (2007), the utility function takes the form of $u(c) = b + c^{1-\gamma}/(1-\gamma)$, where $b$ and $\gamma$ are both constants. In this case, the consumption elasticity $u'(c)c/u$ becomes a decreasing function in the level of consumption.
To investigate the relationship between health and portfolio choices after retirement, Yogo (2016) develops a life-cycle model where retirees face stochastic depreciation in health. This stochastic depreciation affects not only the marginal utility of consumption but also the life expectancy of retirees. The role of health is endogenized in that retirees make decisions on the amount of spending on health investment which determines the levels their health. This study indicates a positive relationship between health and the portfolio share in stocks. The rationale is that healthier retirees expect longer life time and are thus more willing to invest in risky assets. Moreover, this study points out that retirees with poor health generally have higher levels of out-of-pocket health expenditure. The reason is that the marginal product of health is higher for individuals with poor health because of the decreasing returns in health investment.

The concept of endogenous health is also employed in other topics in economics. For example, Baird et al. (2016) estimate long-run impacts of school-based deworming programs and find that these programs bring more future government revenue than costs. Deschênes et al. (2017) develop a measure of willingness to pay for air quality improvement based on Grossman’s idea of demand for health.

### 1.2 Health and public policies

The inclusion of health in economic models provides the room to analyze health related policies. Fiscal policies targeting certain health goals are one type of policy which have been explored most extensively in the literature. However, we find that these studies generally take a paternalistic view which assumes that the government
1.2. Health and public policies

knows better than individuals in terms of their own health and decisions.

O’Donoghue and Rabin (2006) justify the role of sin taxes on unhealthy commodities when individuals have self-control problems. The objective function in their model follows the form as below.

\[ u(x, c) \equiv v(x; \rho) - \beta f(x; \gamma) + c, \]  

(1.2.7)

where \( v(\cdot) \) represents the immediate utility benefits from consuming unhealthy commodities \( x \), \( f(\cdot) \) represents the negative health consequences from consuming \( x \), \( c \) represents the composite good, \( \beta \) is the parameter which captures the time-inconsistent preference for immediate gratification, and \( \rho \) and \( \gamma \) are the parameters which capture population characteristics. The cases with \( \beta < 1 \) implies that individuals have short-term desires for \( x \) and that they may consume too many unhealthy commodities. To restore the Pareto optimum, the government should implement sin taxes to repress the extra consumption of \( x \). Moreover, O’Donoghue and Rabin (2006) believe that sin taxes are beneficial also to individuals who do not have self-control problems as they receive the subsidies transferred from sin taxes.

Cremer et al. (2012) investigate the effects of sin taxes when individuals do not fully recognize the role of health. They develop a two-period model with the objective function as below:

\[ U_i = u(c_i) + \psi(x_i) + u(d_i) + \alpha_i h(x_i, e_i), \]  

(1.2.8)

where \( c_i \) and \( d_i \) are the first- and second-period consumption of individual \( i \), \( x_i \) is the consumption of sin goods, \( e_i \) is the investment in health. Both \( u \) and \( \psi \) are concave functions, and \( h(\cdot) \) is the health function which is decreasing in \( x \) but increasing in
e. The parameter $\alpha$ denotes the level of recognition toward the role of health in the second period. This parameter captures the issue of myopia when individuals underestimate the detrimental effects of unhealthy commodities $x$ on health. With myopic individuals in the economy, the government should implement sin taxes along with subsidies to restore the social optimum. Cremer et al. (2012) further analyze the second-best case where the revenue of sin taxes is earmarked for health care spending. Their results show that this policy combination is also welfare improving.

To identify the impacts of certain unhealthy commodities on health, Goulao and Pérez-Barahona (2014) extend Grossman’s model by including the detrimental effects of unhealthy commodities into the health accumulation function:

$$
 h_{t+1} = (1 - \delta)h_t + \sigma m_t - \alpha x_t,
$$

(1.2.9)

where $h$ is the stock of health, $m$ is the health investment, $\delta$ is the rate of depreciation, $x$ is the consumption of unhealthy commodities, and both $\sigma$ and $\alpha$ are parameters. They further consider the case where individual have limited information about health by forming the following law of motion perceived by individuals:

$$
 h_{t+1} = (1 - \delta)h_t + \sigma m_t - \epsilon \alpha x_t,
$$

(1.2.10)

where $0 < \epsilon < 1$ represents individual level of recognition toward the detrimental effects of unhealthy commodities. With $\epsilon$ lower than unity, individuals tend to reach a lower level of health than what is socially optimal. Nevertheless, the government can restore the social optimum by taxing unhealthy commodities and subsidizing health investment.
1.3 A road map for this thesis

In this thesis, we explore the interrelationship between health and policies from three aspects: the mechanism underlying the health effects of taxes, the structure of optimal sin taxes, and the role of taxes in the economy when health affects the duration of individual lifetime.

In Chapter 2, we explore the mechanism underlying the health effects of taxes on unhealthy commodities by developing a general equilibrium model with endogenous health. This model is characterized by a goods sector which produces commodities and a health sector which generates the stock of health. In line with empirical findings in the epidemiological literature, our analytical results suggest that the implementation of unhealthy commodities taxes alone may be ineffective in improving health in the long run. The explanation toward this well-documented fact is that individuals would decrease the investment in health in response to higher unhealthy commodity taxes. We further find that, when coupled with a reduction in income tax rate through a revenue-neutral reform, the implementation of unhealthy commodity taxes can effectively improve health through income effects. Moreover, the level of welfare can be raised by the reforms. These findings are backed with simulation analysis on the US economy.

In Chapter 3, we explore the structure of optimal sin taxes in the presence of income taxes by extending the model developed in Chapter 2. In this chapter, we take a non-paternalistic view to justify the role of sin taxes based on the short- and the long-term externalities on public health care. We find that the additive property between the Pigouvian element and the efficiency element proposed by Sandmo
A road map for this thesis

(1975) is retained in our model. Although the Pigouvian element is distorted downward by income taxes, the second-best optimal sin taxes are not necessarily lower due to the presence of the efficiency element. This analytical result is supported by our calibration on the UK economy. Moreover, to further explore the property of sin taxes, we construct a “sin tax reform” with reductions in labor income tax rate. The simulation regarding this reform shows that sin taxes have the double-dividends which improve not only individual health but also economic performance as well as welfare in the UK.

In Chapter 4, we examine the role of taxes in an economy with endogenous lifetime. We extend the basic model of Fletcher et al. (2010) and Leroux and Ponthiere (2018) by endogenizing the formation of health and including the role of tax policies in the economy. We divide an individual life cycle into two periods: the period when individuals are young and the period when individuals are old. In our model, the government collects taxes to fund public health care. With higher provision of health care, individuals enjoy higher probabilities of survival into old age. The calibration on the UK economy indicates that a higher level of spending on health care can generally encourage labor supply during old age. However, this statement would be reversed if the additional spending is mainly funded through heavier taxes on old-age labor income. Moreover, we examine the economic impacts of variations in production and medical technologies in this framework. In response to improvements in production technologies, individuals increase consumption and labor supply in both young and old age. Our calibration results suggest that these increases translate into higher survival probabilities in equilibrium. In response to
improvements in medical technologies, individuals offer more labor supply during old age but less when they are young. These changes have to be applied so that the intertemporal marginal rate of substitution toward labor supply can be maintained. The resulting survival probabilities are lower because the fiscal impacts of decreasing the young-age labor supply outweigh those of increasing the old-age labor supply. Furthermore, we derive an optimal tax scheme for welfare maximization. We find that optimal labor income taxes depend on the coefficients of risk aversion toward consumption and labor supply. The implementation of the optimal tax scheme contributes to a smoother consumption path with higher levels of consumption over lifetime. Moreover, individuals also obtain a smoother labor choice over time under the optimal tax scheme with labor supply lower during young age but higher during old age.

Finally, summaries of each chapters and future works are provided in Chapter 5 to conclude this thesis.
Chapter 2

Tax reform, unhealthy commodities and endogenous health

In this chapter, we examine the impacts of taxes on unhealthy commodities on consumer behavior and welfare by developing a dynamic general equilibrium model with endogenous health. Analytical results suggest that taxes on unhealthy commodities may fail to improve health because the beneficial impacts of reducing unhealthy consumption could be offset by a simultaneous decrease in health investment. The simulation results in this chapter show that, when coupled with tax reforms where income taxes are adjusted, taxes on unhealthy commodities can improve health and welfare more effectively. This analysis may inform policy making decisions on taxation of unhealthy commodities when a government can adjust pre-existing taxes.
2.1 Introduction

The rising global burden of non-communicable diseases such as diabetes, cardiovascular and coronary heart diseases, and certain types of cancer has driven policymakers to explore approaches to improve population health (Lim et al., 2013). Since many major health problems are due to individual behaviors such as over-consumption of foods and beverages with high fat, sugar and salt content, it is possible to use fiscal policy to target these unhealthy commodities (e.g. Chokshi and Farley, 2014; Lustig et al., 2012). Changing the relative prices of these commodities via taxation is one of the policies which has been proposed and explored the most in the public arena. Examples include the public health product tax in Hungary, several taxes on saturated fat in Denmark, and the Soft Drink Industry Levy (also known as the “sugar tax”) in the UK.

Taxes on unhealthy commodities discourage consumption and therefore should contribute to a higher level of population health. However, existing studies do not always support this intuition. Fletcher et al. (2010) find that soft drink taxes in the US induce significant changes in consumer behavior, but the impacts on body mass index (BMI) are small in magnitude. Mytton et al. (2007) even show that taxes on saturated fat in the UK would not reduce the incidence of cardiovascular diseases because the beneficial effects from decreases in saturated fat would be offset by increases in salt intake. Schroeter et al. (2008) warn that people could consume other, untaxed, unhealthy commodities in response, and thus increase their BMI. Yaniv et al. (2009) explain that, even if a fat tax reduces the consumption of junk foods, obesity could still rise because individuals might spend less time exercising.
One interesting finding in the literature is that, when coupled with other fiscal instruments such as subsidies, taxes on unhealthy commodities are more likely to be beneficial to health (e.g. Cornelsen and Carreido, 2015; Franck et al., 2013; Nnoaham et al., 2009). This finding suggests that the government should consider more comprehensive policies to ensure a positive impact on health.

One mechanism which may help explain the results is the health investment, such as exercise and health care (e.g. Goulao and Pérez-Barahona, 2014; Grossman, 1972).\(^1\) Indeed, if taxes on unhealthy commodities induce individuals to invest less in health, the taxes might fail to improve population health. In the canonical model of Grossman (1972), health is considered as a capital stock. Individuals invest in health not only because it provides utility, but also because it increases the amount of healthy time available for labor supply. Goulao and Pérez-Barahona (2014) include the detrimental effects of unhealthy commodities into Grossman’s health function, so that they can identify the impacts of the consumption on health. Their results show that taxes on unhealthy commodities, when coupled with subsidies on health investment, can restore the optimal level of health when individuals are myopic.\(^2\)

The mechanism of health investment is also used in this chapter. Different from Grossman (1972) and Goulao and Pérez-Barahona (2014), we fully endogenize

---

\(^1\)The epidemiological literature provides some evidence that exercise and health care could improve the level of health both in the short and long run (e.g. Lucas et al., 2003; Nemet et al., 2005; Oja et al., 2016).

\(^2\)O’Donoghue and Rabin (2006) and Cremer et al. (2012) also study the role of taxes on unhealthy commodities in an economy with myopic households, but they do not specify the equations to illustrate the law of motion of health.
2.1. Introduction

health investment by specifying the health production function. We construct a
dynamic general equilibrium model with two sectors: the goods sector, which pro-
duces consumption commodities, and the health sector, which provides individuals
with health. The economy is populated with infinitely-lived dynastic representative
individuals.\(^3\) The concept of health in our model is in line with Grossman (1972)
in that health provides both utility and income benefits, so that we can examine
how individual levels of health are determined clearly. This endogenous health de-
cision is important in this chapter because unhealthy commodity taxes are usually
employed to target health problems related to consumption choices. Different from
Goulao and Pérez-Barahona (2014), we endogenize individual income by addressing
both labor supply and capital in the economy. By doing so, we are able to highlight
the roles of different fiscal instruments (taxes on labor income, capital income, and
consumption), and thus the efficacy of the tax reforms. One novelty of our model
is found in how it embeds individual preference for health in that of leisure. To be
more specific, we model how the stock of health affects individual healthy time.\(^4\)
Individuals can allocate healthy time into either leisure or labor supply. This novelty
helps us to clarify the trade-off between leisure and labor supply when it comes to

\(^3\) The dynastic model is also used by Tobing (2011) and Kelly (2017) to discuss the role of health
on growth. This chapter is different in that (1) we focus on the role of healthy time on labor-leisure
choices; and (2) we examine the impacts of taxes on health and welfare instead of growth.

\(^4\) Another way of modeling health is to embed it in the concept of longevity. These studies
usually employ a finite horizon (e.g. Ehrlich and Chuma, 1990; Kuhn et al., 2015) or allow health
to affect mortality (or survival probability) at each point of time (e.g. Hall and Jones, 2007; Jones,
2016). However, longevity is not the focus of this chapter.
the changes in taxes on labor income. Unhealthy commodities, which also provide individuals with utility, pose detrimental effects on the accumulation of health. Individuals have to find the balance between the utility and detrimental effects from these commodities. In addition to the utility function, we also specify the production function of health with labor supply and capital. This specification allows us to carefully examine the changes in individual investment decision between the two sectors in response to the taxes. Our results indicate that the implementation of taxes on unhealthy commodities does not improve the level of health directly in the steady state. As documented in the literature, one way to improve both the levels of health and welfare through unhealthy commodity taxes is to earmark the tax revenue for health investment subsidies (e.g. Aronsson and Thunström, 2008; Cremer et al., 2016). In this chapter, we propose alternative revenue-neutral tax reforms which raise taxes on unhealthy commodities but lower those on income. These alternative reforms lead to similar desired results in that the steady state levels of health and welfare are improved more efficiently.

The chapter proceeds as follows. A two-sector model with endogenous health and its optimal conditions will be discussed in Section 2.2. In Section 2.3, we will calibrate the model on the US economy. A tax analysis will be performed based on the steady state solutions. Two revenue-neutral tax reforms will be proposed: the reform which levies unhealthy commodity taxes while adjusting labor income taxes, and the reform which levies unhealthy commodity taxes while adjusting capital income taxes. A welfare analysis of the reforms will be performed as well. Finally, conclusions will be offered in Section 2.4.
2.2 The model

In our model, individuals can freely allocate healthy time into leisure or labor supply. Healthy time can be obtained through the accumulation of the stock of health, $h$, with decreasing marginal returns (Grossman, 1972). $h$ is between zero and a maximum value which is assumed to be above its own steady state level. When $h$ is zero, no healthy time is produced. Individuals determine $l$ fraction of healthy time spent on labor supply and leaves $(1 - l)$ for leisure. The individual lifetime utility is set as follows:

$$U = \int_0^\infty u(c, x, L)e^{-\rho t}dt, \quad (2.2.1)$$

where

$$L \equiv (1 - l)h^\mu.$$

In the above equations, $c$ denotes the numeraire commodities, $x$ denotes the unhealthy commodities, $L$ denotes the leisure when individuals are healthy, $\rho$ denotes the rate of time preference, and $\mu$ denotes the efficiency of the stock of health in generating healthy time. Following Grossman (1972), $h$ generates healthy time through a concave function. Therefore, we assume that $0 < \mu < 1$.

The economy is constituted by the goods sector and the health sector, and both sectors require inputs of capital and labor supply.

$$y = f(sk, vlh^\mu), \quad (2.2.2)$$

---

As in Grossman (1972), total individual time is given by healthy time plus sick time. In this chapter, individuals can enjoy leisure or offer labor supply during their healthy time but not during sick time.
2.2. The model

\[ m = m((1 - s)k; (1 - v)lh^\mu), \]  

(2.2.3)

where \( y \) is the output of the goods sector, \( m \) is the flow of health services generated from the health sector, \( k \) is the physical capital, and \( s \) and \( v \) are the fractions of capital and labor supply devoted into the goods sector. Equation (2.2.2) and (3.2.6) are both assumed to have homogeneity of degree one. Hence, both sectors can be described by the representative firms.

The goods sector produces \( c \) and \( x \) which are distinguishable in the market demand.\(^6\) The prices of the two goods are standardized into unity for simplicity. Consequently, the law of motion for \( k \) is set as follows:

\[ \dot{k} = y - c - x, \]  

(2.2.4)

where the variables with a dot on the top represents the growth of that variable hereafter.

As in Goulao and Pérez-Barahona (2014), \( x \) enters the law of motion for \( h \) in the following form:

\[ \dot{h} = m - \eta x - \delta h, \]  

(2.2.5)

where \( \eta \geq 0 \) and \( \delta \geq 0 \). In this expression, \( \eta \) is the measure of the detrimental effects of \( x \) on \( h \). The extreme case of \( \eta = 0 \) refers to the situation where the detrimental effects of \( x \) are negligible.

---

\(^6\)Some studies investigate role of unhealthy commodities from the supply side (e.g. Bonita et al., 2013). This chapter however focuses on the consumer behaviors from a macroeconomic perspective. Hence, without the loss of generality, the goods sector in our model is represented by the aggregate production of \( c \) and \( x \).
2.2. The model

Notice that, with $\mu < 1$, our model cannot produce endogenous growth. For endogenous growth to occur, $k$ and $h$ have to grow at the same rate (followed by the inspection of equations (2.2.4) and (2.2.5)). Also, output divided by $k$ must be constant. Since the production function (2.2.2) is homogeneous of degree one, $lh^\mu/k$ has to be constant. This ratio would not be constant if $h$ and $k$ grow at the same rate. Also note that, individuals own the representative firms and receive their profits accordingly. Nevertheless, the profits are zero in equilibrium because we assume homogeneity of degree one in productions.

2.2.1 The decentralized economy

In a decentralized economy, the firms in both sectors seek to maximize their own profits. The rental prices of capital and labor supply are thus

\begin{align*}
    r_y &= f_1, \quad (2.2.6a) \\
    w_y &= f_2, \quad (2.2.6b) \\
    r_m &= p_mm_1, \quad (2.2.6c) \\
    w_m &= p_mm_2. \quad (2.2.6d)
\end{align*}

In this expression, $r_i$ and $w_i$ are the rental prices of capital ($k$) and labor ($lh^\mu$) in the $i$ sector, $f_1$ and $f_2$ are the marginal product of capital and that of labor supply in the goods sector ($y$), $m_1$ and $m_2$ are the marginal product of capital and that of labor supply in the health sector ($m$). $p_m$ is the relative price of health services.

The government receives tax revenue from taxes on capital income, labor income, and commodities. Assuming the government balances its budget by financing a
lump-sum transfer, $G$, the government budget can be presented as below

$$G = \tau_k (sr_y + (1 - s) r_m) k + \tau_l (vw_y + (1 - v) w_m) lh^\mu + \tau_c c + \tau_x x,$$

(2.2.7)

where $\tau_k, \tau_l, \tau_c$, and $\tau_x$ are the taxes on capital income, labor income, numeraire commodities, and unhealthy commodities respectively.

Individuals purchase $c$, $x$, and $m$ and pay taxes. The taxes they pay are transferred back to them in the form of $G$. Consequently, we transform equation (2.2.4) into

$$\dot{k} = (1 - \tau_k)(r_y sk + r_m(1 - s)k) + (1 - \tau_l)(w_y vlh^\mu + w_m(1 - v)lh^\mu)$$

$$- (1 + \tau_c)c - (1 + \tau_x)x - p_m m + G + R.$$

(2.2.8)

where $R$ denotes profits of the representative firm received by individuals. It should be noted that $R = 0$ when the firm has constant returns to scale (CRST) and that $R > 0$ when the firm has decreasing returns to scale (DRTS). In line with our assumption on the production function, $R = 0$. When any two equations of (2.2.4), (2.2.7), and (2.2.8) hold, the third one also holds.

### 2.2.2 The optimization problem

Individuals maximize the utility (2.2.1) with the constraints of (2.2.5) and (2.2.8). The Hamiltonian function is

$$H = u(c, x, L) + \lambda[(1 - \tau_k)(r_y sk + r_m(1 - s)k) + (1 - \tau_l)(w_y vlh^\mu + w_m(1 - v)lh^\mu)$$

$$- (1 + \tau_c)c - (1 + \tau_x)x - p_m m + G + R] + q[m - \eta x - \delta h],$$

where $\lambda$ is the shadow price of capital, and $q$ is the shadow price of health. The first-order conditions for this optimization problem are:

$$u_c = \lambda (1 + \tau_c),$$

(2.2.9a)
2.2. The model

\begin{align*}
  u_x &= \lambda(1 + \tau_x) + q\eta, \quad \text{(2.2.9b)} \\
  u_L &= \lambda(1 - \tau_l)(w_y v + w_m(1 - v)), \quad \text{(2.2.9c)} \\
  r_y &= r_m, \quad \text{(2.2.9d)} \\
  w_y &= w_m, \quad \text{(2.2.9e)} \\
  \lambda p_m &= q, \quad \text{(2.2.9f)} \\
  \lambda(1 - \tau_k)(r_y s + r_m (1 - s)) &= \lambda \rho - \dot{\lambda}, \quad \text{(2.2.9g)} \\
  \mu u_L(1 - l)h^{\mu - 1} + \lambda \mu(1 - \tau_l)(w_y v l h^{\mu - 1} + w_m(1 - v) l h^{\mu - 1}) - q\delta &= q\rho - \dot{q}, \quad \text{(2.2.9h)}
\end{align*}

along with the transversality conditions,

\begin{align*}
  \lim_{t \to \infty} e^{-\rho t} \lambda(t) k(t) &= 0, \quad \text{(2.2.9i)} \\
  \lim_{t \to \infty} e^{-\rho t} q(t) h(t) &= 0. \quad \text{(2.2.9j)}
\end{align*}

Equation (2.2.9a) equates the optimal consumption of numeraire goods to the product of the shadow price of \( k \) and the after-tax price of the goods; equation (2.2.9b) shows that the optimal consumption of unhealthy commodities is related to both the shadow price of \( k \) and that of \( h \); equation (2.2.9c) equates the marginal utility of leisure to the marginal costs of labor supply; equations (2.2.9d) and (2.2.9e) describe the optimal allocation of inputs between the two sectors; equation (2.2.9f) implies that the relative value of the two shadow prices depends on \( p_m \); equations (2.2.9g) and (2.2.9h) are the Euler equations; and equations (2.2.9i) and (2.2.9j) are the conditions to exclude Ponzi games in the economy.

With equation (2.2.9d), we can reform the evolution of \( \lambda \) from equation (2.2.9g)
2.2. The model

as

\[ \frac{\dot{\lambda}}{\lambda} = \rho - (1 - \tau_k)r, \]  

(2.2.10)

where \( r = r_y = r_m \). With equations (2.2.9c), (2.2.9e), and (2.2.9f), the evolution of \( q \) in equation (2.2.9h) can be written as

\[ \frac{\dot{q}}{q} = \rho + \delta - \frac{\mu}{p_m}(1 - \tau_l)wh^{\mu - 1}, \]  

(2.2.11)

where \( w = w_y = w_m \).

2.2.3 The equilibrium

With equations (2.2.6a), (2.2.6b), (2.2.9g), and (2.2.9h), we obtain the steady state level of \( h \) as follows

\[ h^* = \left( \frac{w\mu(1 - \tau_l)\rho}{r_p m(\rho + \delta)(1 - \tau_k)} \right)^{\frac{1}{1-\mu}}. \]  

(2.2.12)

The asterisk indicates steady states hereafter. Providing \( m \) has CRTS, the labor-capital ratio in the \( m \) sector would be fixed in equilibrium. Accordingly, the ratio of \( r_m \) to \( w_m \) and thus \( p_m \) would also be pinned down. Referring to equations (2.2.9d) and (2.2.9e), the rental prices of capital and wages are identical across both sectors. Therefore, the ratio of \( r_y \) to \( w_y \) is also fixed in equilibrium. Moreover, with the linearity of \( x \) in equation (2.2.5), reductions in \( x \) due higher \( \tau_x \) cannot alter this fixed ratio. This inference implies the following proposition:

**Proposition 2.1** With a constant labor-capital ratio in the health sector, taxes on unhealthy commodities do not affect the level of health in equilibrium.

The reason why \( \tau_x \) may fail in improving \( h^* \) is that \( m \) has to decrease in response to the decrease in \( x \), so that the steady state condition for equation (2.2.5) can be
held. The story behind this proposition is that individuals find it beneficial to decrease the investment in health when the detrimental effects from $x$ diminish. Therefore, the effects of reducing health investment offset the beneficial effects of reducing $x$. The finding that the simultaneous decrease in $m$ weakens the beneficial impacts of $\tau_x$ on $h^*$ also holds in the case where $m$ does not have CRTS.\(^7\)

### 2.2.4 Parameterization

To examine the economic impacts of tax policies through simulation, we adopt specific functions for equation (2.2.1)-(3.2.6) in this subsection. The utility function (2.2.1) is set to be the following form:

$$u(c, x, L) = \ln c + \theta \ln x + \psi \ln((1 - l)h^\mu),\tag{2.2.13}$$

where $\theta$ is the preference to $x$, and $\psi$ is the preference to $L$. The production functions follow the Cobb-Douglas forms:

$$y = A (sk)^\alpha (vlh^\mu)^{1-\alpha},\tag{2.2.14}$$

$$m = B ((1 - s)k)^\beta ((1 - v)lh^\mu)^{1-\beta},\tag{2.2.15}$$

where $\alpha$ and $\beta$ are the shares of capital and $A$ and $B$ are production efficiency factors in the two sectors. In line with Grossman (1972), we take the CRTS $m$ as the benchmark. Our inference that the beneficial effects of $\tau_x$ would be weakened by the decreases in $m$ holds even without the restriction of CRTS. As suggested in Galama et al. (2012) and Halliday et al. (2017), $m$ could have DRTS. The analysis with the DRTS $m$ is presented in Appendix 2.A.3.

\(^7\)More detailed analysis can be found in Appendix 2.A.3.
With the specified production functions, we are able to clarify the relationship between \( s \) and \( v \) by using equations (2.2.9d) and (2.2.9e):

\[
v = \frac{\beta (1 - \alpha) s}{\alpha (1 - \beta) - (\alpha - \beta) s},
\]

(2.2.16)

which implies that \( v \) is increasing in \( s \) when \( \alpha \geq \beta \).

With \( \dot{\lambda} = 0 \) in the steady state, we derive the following condition from equations (2.2.6a), (2.2.10), and (2.2.14).

\[
lh^\mu k = \frac{s}{v} \left( \frac{\rho}{\alpha A (1 - \tau_k)} \right)^{1/\beta}.
\]

(2.2.17)

The left-hand side of the equation is the labor-capital ratio in the economy. With equations (2.2.6b), (3.2.9), (2.2.14), and the condition \( \dot{q} = 0 \), we find that

\[
lh^\mu k = \frac{s}{v} \left( \frac{\mu A (1 - \tau_l) (1 - \alpha)}{p_m (\delta + \rho)} \right)^{1/\alpha} h^{-(1-\mu)}.
\]

(2.2.18)

By combining equations (2.2.17) and (2.2.18), we can characterize \( h^* \) as

\[
h^* = \left( \frac{\alpha A (1 - \tau_k)}{\rho} \right)^{1/\alpha} \left( \frac{\mu A (1 - \tau_l) (1 - \alpha)}{p_m (\delta + \rho)} \right)^{1/\alpha}.
\]

(2.2.19)

The specified function \( p_m \) can be obtained from equations (2.2.6c), (2.2.10), and (2.2.17).

\[
p_m = \frac{\alpha}{\beta} \left( \frac{1 - \alpha}{1 - \beta} \right)^{1/\beta} \left( \frac{A^{1-\alpha}}{B} \right)^{\frac{\alpha - \beta}{1-\alpha}} \left( \frac{1 - \tau_k}{\rho} \right)^{\frac{\alpha - \beta}{1-\alpha}}.
\]

(2.2.20)

This equation indicates that the relative price of \( m \) is affected by the production efficiency factors in both sectors. Given inputs in both sectors, a higher \( A \) contributes to a more efficient production in the goods sector compared to that in the health sector, and thus decreases the relative prices of the products in the goods sector. On the other hand, \( p_m \) decreases as \( B \) raises the relative productivity in the health sector. Equation (2.2.20) also implies a negative relationship between \( \tau_k \) and \( p_m \).
The reason behind this negative relationship is that higher capital income taxes \((\tau_k)\) reduce the after-tax marginal product of \(k\) (as in equation (2.2.9g)) and thus decrease the relative shadow price of \(q\) to \(\lambda\). Referring to equation (2.2.9f), \(p_m\) is determined by the ratio of \(q\) to \(\lambda\). A relatively lower level of \(q\) thus implies a lower level of \(p_m\).

The steady state level of \(k\) can be obtained through equations (2.2.17) and (2.2.19):

\[
k^* = \frac{v^*}{s^*} \left( \frac{\alpha A(1 - \tau_k)}{\rho} \right) \frac{1}{\alpha} l^*(h^*)^\mu, \tag{2.2.21}
\]

where \(k^*\) has to increase as \(l^*(h^*)^\mu\) increases, so that the labor-capital ratio is fixed in equilibrium.

By using equation (2.2.9b), we have

\[
x^* = \frac{\theta(1 + \tau_c)}{1 + \tau_x + p_m\eta} c^* \tag{2.2.22}
\]

We find that \(\partial x^*/\partial \tau_x < 0\) given \(c^*\), indicating that an increase in \(\tau_x\) does deter the consumption of \(x\). This finding is in line with the prevailing hypothesis of the supporters of taxes on unhealthy commodities.

Next, we rewrite equation (2.2.9c) into:

\[
c^* = \frac{p_m(\rho + \delta)}{\mu \psi(1 + \tau_c)} (1 - l^*)h^*. \tag{2.2.23}
\]

With equations (2.2.15), (2.2.18), and (2.2.22), equation (2.2.5) in the steady state can be rewritten as

\[
\frac{\rho + \delta}{\mu(1 - \beta)(1 - \gamma)} (1 - v)l^*h^* = \frac{\eta \theta(1 + \tau_c)}{1 + \tau_x + p_m\eta} c^* + \delta h^*. \tag{2.2.24}
\]

With equations (2.2.17) and (2.2.22), the market clearing condition in the goods
sector, \( y = x + c \), can be transformed into

\[
\frac{p_m(\rho + \delta)v^*l^*h^*}{\mu(1 - \tau_l)(1 - \alpha)} = \frac{(\pi + \theta(1 + \tau_c))c^*}{\pi},
\]

(2.2.25)

where

\[
\pi = 1 + \tau_x + p_m\eta.
\]

Equations (2.2.19), (2.2.23), (2.2.24), and (2.2.25) form a system which could be used to solve for the solutions of \( c^* \), \( l^* \), and \( v^* \):

\[
c^* = \frac{\omega(1 - \tau_l)(1 - l^*)h^*}{\psi(1 + \tau_c)},
\]

(2.2.26)

\[
l^* = \frac{p_m(1 - \beta)(\delta \pi h^* + \eta \theta(1 + \tau_c)c^*)}{\omega(1 - v^*)\pi h^*},
\]

(2.2.27)

\[
v^* = \frac{(1 - \alpha)(\pi + \theta(1 + \tau_c))c^*}{\pi \omega l^* h^*},
\]

(2.2.28)

where

\[
\omega = \frac{p_m(\rho + \delta)}{\mu(1 - \tau_l)}.
\]

With equation (2.2.16), we can further obtain the solution of \( s^* \).

\[
s^* = \frac{\alpha(1 - \beta)(\pi + \theta(1 + \tau_c))c^*}{(\alpha - \beta)(\pi + \theta(1 + \tau_c))c^* + \beta \pi \omega l^* h^*}
\]

(2.2.29)

With the specified functions, we can rewrite equations (2.2.26)-(2.2.29) into closed-form solutions. The closed-form solutions are presented in Appendix 2.A.1.

### 2.3 Policy analysis

In this section, we calibrate the model on the US economy, and then simulate the economic impacts of implementing \( \tau_x \) alone and those of our proposed revenue-neutral tax reforms: the reform of implementing \( \tau_x \) with adjustments in \( \tau_l \), and the reform of implementing \( \tau_x \) with adjustments in \( \tau_k \).
2.3. Policy analysis

2.3.1 Calibration

Based on the estimation of Mitnitski et al. (2002), we calibrate the value of $\delta$ as 0.043.\(^8\) The initial tax rates are set to be $\tau_l = 0.20$, $\tau_k = 0.27$ and $\tau_c = \tau_x = 0.08$ in accordance with the average tax rates in the US from 1970 to 2013.\(^9\) $\rho$ is selected as 0.04 following Azariadis et al. (2013). $\alpha$ is set to be 0.3 as in Chen and Lu (2013) and $\beta$ is set to be 0.22 as in Acemoglu and Guerrieri (2008). It is worth noting that, referring to equation (2.2.16), the observation of $\alpha = 0.3$ and $\beta = 0.22$ implies $v'(s) \geq 0$. The initial level of $l^*$ is selected as 0.25 in accordance with the observation of Prescott (2006). We normalize the initial level of $y$ to unity for simplicity.

The OECD statistics shows that the ratio of health expenditure to GDP in the US between 1970 and 2013 is around 11\%.\(^10\) Note that GDP in our model is $y + p_m m$. Therefore, with the normalization of $y$, $p_m m$ is calculated as 0.1236. Moreover, Data retrieved from the Bureau of Labor Statistics Consumer Survey shows that the household expenditure on food-away-from-home 2013-2015 is around 5\% as a ratio of average annual expenditure.\(^11\) Consequently, we calibrate the initial level of $x^*$ as 0.0562. With the goods market clearing condition $y = c + x$, the initial $c^*$ is

---

\(^8\)Considering the similarity in natural forces of health depreciation, the data of Canadian population can be a good approximation for the US population (e.g. Dalgaard and Strulik, 2014; Rockwood and Mitnitski, 2007; Strulik, 2015).

\(^9\)Data source: http://www.caramcdaniel.com/researchpapers. We obtain the updated data in February 2017. These average tax rates are calculated by using the methods provided in McDaniel (2007).

\(^10\)Data Source: http://stats.oecd.org/.

\(^11\)Data source: https://www.bls.gov/cex/.
calibrated as 0.9438.

Referring to equations (2.2.9d) and (2.2.9e), we find that the initial $s$ and $v$ are 0.9169 and 0.8790 with the specified parameters. Accordingly, the value of $\psi$ can be calculated as 1.8751 by using equations (2.2.6b), (2.2.6d), and (2.2.9e). The initial $k^*$ is calibrated as 5.9712 by using equations (2.2.9g), (2.2.6a), (2.2.6c), and (2.2.9d).

The determination of the initial $p_m$ is relatively flexible, because a different $p_m$ could be the result of $m$ being calculated in different units. For simplicity, we set the initial $p_m$ to be unity.

To provide a clearer view, we summarize the benchmark parameters and variables in Table 2.1 and the calibration Table 2.2.

### Table 2.1: Benchmark parameters and variables

<table>
<thead>
<tr>
<th>Benchmark parameters and variables</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Share of physical capital in the goods sector</td>
<td>$\alpha$</td>
</tr>
<tr>
<td>Share of physical capital in the health sector</td>
<td>$\beta$</td>
</tr>
<tr>
<td>Rate of time preference</td>
<td>$\rho$</td>
</tr>
<tr>
<td>Natural depreciation of health</td>
<td>$\delta$</td>
</tr>
<tr>
<td>Capital income taxes</td>
<td>$\tau_k$</td>
</tr>
<tr>
<td>Labor income taxes</td>
<td>$\tau_l$</td>
</tr>
<tr>
<td>Commodity taxes</td>
<td>$\tau_c$ &amp; $\tau_x$</td>
</tr>
<tr>
<td>Initial fraction of time allocated to labor supply</td>
<td>$l_0$</td>
</tr>
<tr>
<td>Initial production in the goods sector</td>
<td>$y_0$</td>
</tr>
<tr>
<td>Ratio of health expenditure to GDP</td>
<td>$p_m m/GDP$</td>
</tr>
<tr>
<td>Ratio of unhealthy commodities to GDP</td>
<td>$x/GDP$</td>
</tr>
</tbody>
</table>
Table 2.2: Benchmark calibration

<table>
<thead>
<tr>
<th>Benchmark calibration</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative preference to leisure</td>
<td>$\psi$</td>
</tr>
<tr>
<td>Ratio of numeraire commodities to output production</td>
<td>$c_0/y_0$</td>
</tr>
<tr>
<td>Ratio of unhealthy commodities to output production</td>
<td>$x_0/y_0$</td>
</tr>
<tr>
<td>Ratio of physical capital to output production</td>
<td>$k_0/y_0$</td>
</tr>
<tr>
<td>Fraction of physical capital invested in the goods sector</td>
<td>$s_0$</td>
</tr>
<tr>
<td>Fraction of labor supply invested in the goods sector</td>
<td>$\theta_0$</td>
</tr>
</tbody>
</table>

With equations (2.2.9c) and (2.2.9h), $h^*$ can be presented as

$$h^* = \frac{\mu \psi (1 + \tau_c) c^*}{(1 - l^*) (\rho + \delta)}.$$  \hspace{1cm} (2.3.30)

With equation (2.3.30) and the steady state condition $\dot{h} = 0$, we rewrite equation (2.2.5) as

$$\eta = \frac{y^*}{x^*} \left( \frac{m^*}{y^*} - \frac{\mu \psi (1 + \tau_c) c^*}{(1 - l^*) (\rho + \delta) y^*} \right).$$  \hspace{1cm} (2.3.31)

This equation indicates that the determination of $\mu$ affects the value of $\eta$. For $\eta$ to be non-negative, the term in the brackets on the right-hand side has to be greater than or equal to zero. Therefore,

$$\mu \leq \bar{\mu} \equiv \frac{(1 - l^*) (\rho + \delta) m^*}{\delta \psi (1 + \tau_c) c^*}.$$  \hspace{1cm} (2.3.32)

In our calibration, the upper limit $\bar{\mu} = 0.0936$. In this paper, we select $\mu = 0.08$ as the benchmark. Consequently, the benchmark $\eta$ is 0.3199 (from equation (2.3.31)), and the initial $h^*$ is 2.4564 (from equation (2.3.30)). $\theta$ is calibrated as 0.0772 from equation (2.2.9b). Furthermore, by using equations (2.2.2) and (3.2.6), we find that $A = 1.6493$ and $B = 2.0869$. We further include the other two parameter sets:
Table 2.3: Calibrated parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Parameter set 1 (benchmark)</th>
<th>Parameter set 2</th>
<th>Parameter set 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>0.0800</td>
<td>0.0936</td>
<td>0.0400</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.0772</td>
<td>0.0595</td>
<td>0.1290</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.3199</td>
<td>0.0000</td>
<td>1.2599</td>
</tr>
<tr>
<td>$A$</td>
<td>1.6493</td>
<td>1.6185</td>
<td>1.7245</td>
</tr>
<tr>
<td>$B$</td>
<td>2.0869</td>
<td>2.0435</td>
<td>2.1931</td>
</tr>
</tbody>
</table>

It should be noted that the parameter set 2 is the extreme case where unhealthy commodities do not exert any detrimental effect on health. The reason why we include the parameter set 3 is to show the case where $\eta$ is above 1.

2.3.2 Comparative-static analysis of changing $\tau_x$ alone

Figure 2.1 depicts the effects of implementing $\tau_x$ alone.

![Diagram](image)

Figure 2.1: Comparative-static effects of implementing $\tau_x$ alone
This figure presents the comparative-static effects of $\tau_x$ with the three parameter sets listed in Table 2.3: (1) The solid curves denote the simulation results with the benchmark parameter set; (2) the dashed curves denote the simulation results with parameter set 2; and (3) the dash-dotted curves denote the simulation results with parameter set 3.

We summarize the effects of implementing $\tau_x$ with lump-sum transfers in the benchmark case as below:

Table 2.4: The comparative-static effects of $\tau_x$ with lump-sum transfers

<table>
<thead>
<tr>
<th></th>
<th>$c^*$</th>
<th>$x^*$</th>
<th>$l^*$</th>
<th>$s^*$</th>
<th>$k^*$</th>
<th>$h^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_x$</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>0</td>
</tr>
</tbody>
</table>

The comprehensive comparative-static effects of the taxes are shown in Appendix 2.A.2.

Taking the derivative of equation (2.2.22) with respect to $\tau_x$, we find that increases in $\tau_x$ reduce $x^*$. This result is in line with the intuition that higher prices deter the consumption of $x$. Intuitively, since $\tau_x$ reduces $x^*$, higher $\tau_x$ should be beneficial to $h^*$. However, referring to equation (2.2.19), we find that increases in $\tau_x$ do not improve $h^*$. Although $\tau_x$ reduces the consumption of $x$, individuals have to reduce the investment in the health sector in order to hold the steady state condition in equation (2.2.5). The decreased investment in the health sector offsets the positive force from the reduced $x^*$.

Figure 2.2 shows the effects of $\tau_x$ on $m$ when other taxes are held constant.
Figure 2.2 shows that, in the special case where $\mu = \bar{\mu}$ (parameter set 2), health investment is basically fixed regardless of the changes in $\tau_x$. However, when $\mu < \bar{\mu}$ (parameter sets 1 and 3), individuals decrease $m$ in response to the increases in $\tau_x$. To illustrate this reduction in $m$, we first examine the impacts of implementing $\tau_x$ on labor supply $lh^\mu$ and physical capital $k$. As shown in Table 2.4, increases in $\tau_x$ result in the decreases in both the steady state labor supply and physical capital. Moreover, with the benchmark parameter set (parameter set 1), the fractions of inputs allocated to the health sector, $(1 - s^*)$ and $(1 - v^*)$, decrease in response to the implementation of $\tau_x$. Consequently, $m$ decreases without ambiguity. This simulation result accords to our Proposition 2.1.

Although the implementation of $\tau_x$ alone does not improve $h^*$, we find that $h^*$
can be affected by changes in $\tau_l$ and $\tau_k$:

$$\frac{dh^*}{d\tau_l} = \frac{\partial h^*}{\partial \tau_l} < 0, \quad (2.3.33)$$

$$\frac{dh^*}{d\tau_k} = \frac{\partial h^*}{\partial \tau_k} + \frac{\partial h^*}{\partial p_m} \frac{\partial p_m}{\partial \tau_k} < 0, \quad (2.3.34)$$

where

$$\frac{\partial h^*}{\partial \tau_l} = \frac{-h^*}{(1 - \mu)(1 - \tau_l)} < 0,$$

$$\frac{\partial h^*}{\partial \tau_k} = \frac{-\alpha h^*}{(1 - \alpha)(1 - \mu)(1 - \tau_k)} < 0,$$

$$\frac{\partial h^*}{\partial p_m} = \frac{-h^*}{(1 - \mu)p_m} < 0.$$

The direct effects of $\tau_l$ and $\tau_k$ on $h^*$ are both negative. The reason is that increases in $\tau_l$ or $\tau_k$ crowd out the resources available for the health sector directly. In addition, $\tau_k$ affect $h^*$ through the channel of $p_m$. $h^*$ is decreasing in $p_m$ because higher prices on health services discourage individuals from investing in health. Note that $\frac{\partial p_m}{\partial \tau_k} < 0$ (see equation (2.2.20)). Consequently, $\tau_k$ have positive impacts on $h^*$ through this indirect channel. Nevertheless, the direct effect of $\tau_k$ outweighs the indirect effect, so increases in $\tau_k$ reduce $h^*$. Therefore, $h^*$ could be improved more effectively if the government can combine the implementation of $\tau_x$ with reductions in income taxes.

### 2.3.3 Tax reform

We define the government budget $F$ by transforming equation (2.2.7) into

$$F \equiv \tau_k r_k + \tau_l w l h^* + \tau_c c + \tau_x x - G = 0. \quad (2.3.35)$$
In our calibration, \( dF/d\tau_k > 0, dF/d\tau_l > 0, dF/d\tau_x > 0, \) and \( dF/d\tau_c > 0. \) According to the implicit function theorem, we find \( d\tau_k/d\tau_x < 0 \) and \( d\tau_l/d\tau_x < 0 \) in our model. These negative relationships suggest that the government can keep the revenue constant by raising \( \tau_x \) while reducing \( \tau_l \) or \( \tau_k \). We propose two potential tax reforms: first, the reform which raises \( \tau_x \) while reducing \( \tau_l \); second, the reform which raises \( \tau_x \) while reducing \( \tau_k \). To calculate appropriate income tax rates, we endogenize \( \tau_l \) and \( \tau_k \) in the computation. The changes in the two income taxes are plotted in Figure 2.3.

![Figure 2.3: The replacement of \( \tau_l \) and \( \tau_k \) with \( \tau_x \) under revenue-neutral schemes](image_URL)

We then present the simulation results of the tax reforms in Figures 2.4 and 2.5.
2.3. Policy analysis

Figures 2.4 and 2.5 show that $x^*$ decreases in $\tau_x$ in both reforms. Decomposing
the effects of $\tau_x$ on $x^*$ in the tax reforms from equation (2.2.22), we find that

$$\frac{dx^*}{d\tau_x} = \frac{\partial x^*}{\partial \tau_x} + \frac{\partial x^*}{\partial \tau_i} d\tau_i,$$

where $i = l, k$. (2.3.36)

where $\partial x^*/\partial \tau_x < 0$, $\partial x^*/\partial \tau_l < 0$, and $\partial x^*/\partial \tau_k < 0$. The above full derivative shows that the negative relationship between $\tau_x$ and $x^*$ is composed of two opposite effects: the negative effect of $\tau_x$ and the positive effect through the channel of decreasing $\tau_l$ or $\tau_k$. The negative effect of $\tau_x$ can be examined from Table 2.4. To understand the positive effects through the channels of decreasing income taxes, we note that any changes in $\tau_l$ or $\tau_k$ which affects $c^*$ would require adjustments in $x^*$ so as to restore the marginal rate of substitution (MRS). Therefore, an increase in $c^*$ induced by smaller $\tau_l$ would prompt individuals to raise $x^*$; likewise, an increase in $c^*$ due to smaller $\tau_k$ would also make individuals increase $x^*$. However, with our calibration, the positive effect always dominates the negative effects through income taxes in both reforms.

It should be noted that $h^*$ increases in $\tau_x$ with both reforms. To understand the mechanisms, we take full derivations of $h^*$ as below:

$$\frac{dh^*}{d\tau_x} = \frac{\partial h^*}{\partial \tau_x} + \frac{\partial h^*}{\partial \tau_i} d\tau_i,$$

where $\partial h^*/\partial \tau_l < 0$ and $\partial h^*/\partial \tau_k < 0$ as shown in Appendix 2.A.2. The overall impacts of the tax reforms on $h^*$ are thus positive. It should be noted that these increases in $h^*$ are not due to the reduced detrimental effects of decreased $x^*$, but the indirect effects from the decreased $\tau_l$ or $\tau_k$.

The effects of the tax reforms of replacing $\tau_l$ or $\tau_k$ with $\tau_x$ on $c^*$ can be examined through

$$\frac{dc^*}{d\tau_x} = \frac{\partial c^*}{\partial \tau_x} + \frac{\partial c^*}{\partial \tau_i} d\tau_i,$$

(2.3.38)
where \( \frac{\partial c^*}{\partial \tau_x} > 0 \), \( \frac{\partial c^*}{\partial \tau_l} < 0 \), and \( \frac{\partial c^*}{\partial \tau_k} < 0 \). Referring to equation (2.2.26), higher \( \tau_x \) contribute to a higher level of \( c^* \) through the channel of \( l^* \). In addition to this effect, increases in \( \tau_x \) also affect \( c^* \) through the channel of decreasing \( \tau_l \) or that of \( \tau_k \) with the reforms. To illustrate the channel of decreasing \( \tau_l \): a decrease in \( \tau_l \) raises the marginal cost of leisure (as shown in equation (2.2.9c)). Individuals are thus encouraged to provide more labor supply, resulting in two opposing effects. First, more labor supply contributes to a higher output in the goods sector. To clear the goods market, the consumption of \( c^* \) has to increase in the long run. Second, more labor supply may lead to less leisure, so that individuals have to decrease \( c^* \) to hold the MRS constant. As shown in Figure 2.4, the negative effects are overshadowed by the positive effects, resulting in the case where \( c^* \) is increasing in \( \tau_x \). The channel of decreasing \( \tau_k \) can be examined from equation (2.2.9g): a decrease in \( \tau_k \) raises the after-tax marginal product of capital, so an increase in \( k^* \) is needed to reduce the pre-tax marginal product of capital. As a result, \( c^* \) has to increase so that the steady state condition for the resource constraint can be held.

To explain the impacts of the reforms on \( l^* \), we take the full derivatives of \( l^* \) with respect to \( \tau_x \)

\[
\frac{dl^*}{d\tau_x} = \frac{\partial l^*}{\partial \tau_x} + \frac{\partial l^*}{\partial \tau_l} \frac{d\tau_l}{d\tau_x} + \frac{\partial l^*}{\partial \tau_k} \frac{d\tau_k}{d\tau_x},
\]

(2.3.39)

where \( \frac{\partial l^*}{\partial \tau_x} < 0 \), \( \frac{\partial l^*}{\partial \tau_l} < 0 \), and \( \frac{\partial l^*}{\partial \tau_k} < 0 \). The impacts of the tax reforms on \( l^* \) can be separated into: the negative direct effect of \( \tau_x \) (see Figure 2.1), and the positive indirect effect through decreased \( \tau_l \) or \( \tau_k \). To understand the positive indirect effect through \( \tau_l \): a decrease in \( \tau_l \) raises the marginal cost of leisure (see equation (2.2.9c)), so individuals would find it optimal to reduce leisure by increasing
2.3. Policy analysis

Our simulation result shows that, in the reform of replacing \( \tau_l \), the positive indirect effect through decreasing \( \tau_l \) dominates the negative direct effect of \( \tau_x \) on \( l^* \). This combined effect overshadows the direct effect of \( \tau_x \), so \( l^* \) increases as \( \tau_x \) increases under this reform. To understand the positive indirect effect through \( \tau_k \):, the reduced \( \tau_k \) encourages the accumulation of \( k^* \) and thus results in two opposing effects. The first effect can be examined from equations (2.2.8) and (2.2.9c): a higher level of \( k^* \) results in a higher level of \( c^* \). individuals have to decrease \( l^* \) to maintain the MRS between consumption and leisure. The second effect can be viewed from equation (2.2.17): smaller \( \tau_k \) result in a lower labor-capital ratio. To restore the labor-capital ratio, \( l^* \) is required to increase. The positive indirect effect is overshadowed by the combined negative effect, so \( l^* \) decreases in the reform which adjusts \( \tau_k \).

As shown in Figures 2.4 and 2.5, \( s^* \) increases in both reforms. The full derivatives of \( s^* \) with respect to \( \tau_x \) in the two tax reforms yield

\[
\frac{ds^*}{d\tau_x} = \frac{\partial s^*}{\partial \tau_x} + \frac{\partial s^*}{\partial \tau_l} \frac{d\tau_l}{d\tau_x},
\]  

(2.3.40)

where \( \partial s^*/\partial \tau_x > 0 \), \( \partial s^*/\partial \tau_l > 0 \), and \( \partial s^*/\partial \tau_k > 0 \). To understand the mechanisms behind the changes in \( s^* \), we decompose the effects of the reforms into two: the direct effect of increasing \( \tau_x \) and the indirect effect of decreasing \( \tau_l \) or \( \tau_k \). We know from Table 2.4 that the direct effect of increasing \( \tau_x \) is positive on \( s^* \). To explain the indirect effects from \( \tau_l \) and \( \tau_k \), it is worth noting that decreases in either income taxes encourage the accumulation of \( k \) as in equation (2.2.8). To maintain \( \dot{k} = 0 \) in the steady state, individuals must shift the investment from the goods sector to the health sector in response to both tax reforms. However, our simulation results show
that the positive effect of \( \tau_x \) dominates. Consequently, \( s^* \) (and hence \( v^* \)) increases with the implementation of either reforms.

The comparative-static effects on \( k^* \) with the two types of tax reforms can be disentangled into two parts as below

\[
\frac{dk^*}{d\tau_x} = \frac{\partial k^*}{\partial \tau_x} + \frac{\partial k^*}{\partial (h^*)^\mu} \frac{dl(h^*)}{d\tau_i} \frac{d\tau_i}{d\tau_x},
\]

(2.3.41)

where \( \partial k^*/\partial \tau_x < 0, \partial k^*/\partial l(h^*)^{\mu} > 0, \partial l(h^*)^{\mu}/\partial \tau_l < 0, \) and \( \partial l(h^*)^{\mu}/\partial \tau_k < 0. \)

The two proposed tax reforms affect \( k^* \) through two channels. The first channel is through the crowding-out effect in the goods sector: increases in \( \tau_x \) directly crowd out the resource available for the accumulation of \( k \) (as in equation (2.2.8)). The second channel is through the changes in labor supply: to hold the labor-capital ratio constant in the steady state, \( k^* \) has to increase (decrease) as \( l^*(h^*)^{\mu} \) increases (decreases). Since the variations in \( h^* \) were discussed with equations (2.3.33) and (2.3.34) earlier, we focus on the analysis of the impacts of changes in \( l^* \) on \( k^* \). In the tax reform where \( \tau_l \) is replaced by \( \tau_x \), individuals find it optimal to raise \( l^* \). Following this increase in \( l^* \), \( k^* \) has to increase in order to fix the labor-capital ratio. In the reform with adjustments in \( \tau_k \), decreased \( \tau_k \) encourages individuals to accumulate more \( k \). Accordingly, the effects through the channel of decreasing income taxes are positive under both reforms. These positive effects dominate, so \( k^* \) increases in \( \tau_x \) with either reform.

### 2.3.4 Welfare analysis

In this subsection, we simulate the effects on welfare in the economy by taking the quantitative results of the tax reforms into the utility function (2.2.1). Moreover,
we include the simulated effects of implementing $\tau_x$ alone as a comparison. We scale up the utility levels in order to attain positive values. The changes in welfare are plotted in Figure 2.6.

![Figure 2.6: Changes in welfare in response to the implementation of the tax reforms](image)

To better present the variations in welfare after implementing different policies, we simulate the results from the initial calibration where $\tau_x = 0.08$ in Figure 2.6. The tax reform where $\tau_l$ is replaced by $\tau_x$ results in the decreases of both leisure and $x^*$; nevertheless, due to its contribution to the increase in $c^*$, this tax reform still contributes to better welfare in the long run. Compared to the former reform, the replacement of $\tau_k$ with $\tau_x$ increase not only $c^*$ but also leisure in the long run. It should be noted that the implementation of $\tau_x$ alone reduces welfare in our calibration. This finding implies that, compared to the taxing $x$ alone, the reform with
2.4. Conclusion

This paper provides a rigorous theoretical framework to explain the findings from the epidemiological literature on population health: first, why taxes on unhealthy commodities alone might fail to improve population health, and second, why these taxes are more likely to be beneficial to health when they are coupled with other fiscal instruments. In addition, we offer insights on how taxation of unhealthy commodities affects the economy and overall welfare. For this purpose, we construct a dynamic general equilibrium two-sector model with endogenous health. The two sectors employed in the model are the goods sector, which produces consumption commodities, and the health sector, which provides individuals with health. Health not only raises individual labor supply, but also increases the level of utility by enhancing leisure time. Although unhealthy commodities provide individuals with utility, they pose detrimental effects on health. Intuitively, taxes on unhealthy commodities should directly improve health as long as the taxes are effective in reducing their consumption. However, the steady state solutions show that, even though taxes on unhealthy commodities decrease their consumption, they hardly improve the stock of health in the long run. The reason is that, as detrimental effects decrease, individuals would find it beneficial to reduce the investment in the health sector. Nevertheless, with revenue-neutral adjustments of taxes on labor income or on capital income, the implementation of taxes on unhealthy commodities can largely improve the level of health through income effects. In addition, both tax reforms contribute to higher

reduction in income taxes not only raise \( h \) but also welfare in the long run.
levels of welfare in the long run. The results offer important guidelines to policy makers: the introduction of a tax on unhealthy commodities, for example a “sugar tax”, should always be coupled with a reduction in other tax burdens in order to improve the level of population health and increase overall welfare effectively.

2.A Appendix

2.A.1 Closed-form solutions

We obtain closed-form solutions of the parameterized model by rearranging equations (2.2.12), (2.2.16), (2.2.18), (2.2.26), (2.2.27), and (2.2.28):

\[
c^* = \frac{(\omega - (1 - \beta)\delta p_m)(1 - \tau_l)\pi h^*}{(1 - \alpha)\pi(1 - \tau_l) + \left[((1 - \beta)\delta p_m + \theta(1 - \alpha))(1 - \tau_l) + \pi \psi\right](1 - \tau_c)},
\]

\[
x^* = \frac{\theta(1 + \tau_c)}{1 + \tau_x + p_m\eta}c^*,
\]

\[
l^* = \frac{(1 - \beta)p_m(\delta \pi \psi + \eta \theta \omega(1 - \tau_l))(1 + \tau_c) + (1 - \alpha)\omega(1 - \tau_l)(\pi + \theta(1 + \tau_c))}{\omega \left\{(\theta(1 - \alpha)(1 - \tau_l) + \pi \psi + (1 - \beta)\delta p_m(1 - \tau_l))(1 + \tau_c) + \pi(1 - \alpha)(1 - \tau_l)\right\}},
\]

\[
v^* = \frac{(1 - \alpha)(\omega - (1 - \beta)\delta p_m)(1 - \tau_l)(\pi + \theta(1 + \tau_c))}{(1 - \beta)p_m(\delta \pi \psi + \eta \theta \omega(1 - \tau_l))(1 + \tau_c) + (1 - \alpha)\omega(1 - \tau_l)(\pi + \theta(1 + \tau_c))},
\]

\[
s^* = \frac{\alpha(1 - \beta)v^*}{\beta(1 - \alpha) + (\alpha - \beta)v^*},
\]

\[
h^* = \left(\frac{\alpha A(1 - \tau_k)}{\rho}\right)^{\frac{\gamma}{\gamma - \rho}} \left(\frac{\mu A(1 - \alpha)(1 - \tau_l)}{p_m(\rho + \delta)}\right)^{\frac{1}{1 - \rho}},
\]

\[
p_m = \frac{\alpha^{\frac{\gamma(\gamma - \rho)}{\gamma - \rho}}}{\beta^\beta} \left(\frac{1 - \alpha}{1 - \beta}\right)^{1 - \beta} \left(\frac{A^{1 - \beta} \rho^\beta}{B}\right) \left(\frac{1 - \tau_k}{\rho}\right)^{\frac{\alpha - \beta}{1 - \rho}}.
\]
2.A.2 Comparative-static effects of income taxes

Figure 2.A.1: Comparative-static effects of $\tau_l$

Figure 2.A.2: Comparative-static effects of $\tau_k$
2.A. Appendix

Table 2.A.1: Changes in tax rates with lump-sum transfers

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( c^* )</th>
<th>( x^* )</th>
<th>( l^* )</th>
<th>( s^* )</th>
<th>( k^* )</th>
<th>( h^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau_l )</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( \tau_k )</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( \tau_x )</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>0</td>
</tr>
</tbody>
</table>

2.A.3 Health sector with DRTS technologies

In addition to the benchmark case where \( m \) has CRTS, we analyze the case where \( m \) has DRTS in this appendix. First of all, we transform equation (2.2.15) into the following form:

\[
m = B \left[ ((1 - s)k)^\beta ((1 - v)lh^\mu)^{(1-\beta)} \right]^\gamma,
\]

(2.A.1)

where \( \gamma \) denotes the degree of returns to scale.

In addition to the empirical data used in Section 2.3, we choose \( \gamma \) as 0.8 in accordance with Halliday et al. (2017). The new parameter sets for the DRTS case can then be obtained. The calibrated parameters are presented in Table 2.A.2.

Table 2.A.2: Calibrated parameters with DRTS production in the health sector

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Parameter set 1</th>
<th>Parameter set 2</th>
<th>Parameter set 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu )</td>
<td>0.0800</td>
<td>0.0959</td>
<td>0.0400</td>
</tr>
<tr>
<td>( \theta )</td>
<td>0.0797</td>
<td>0.0595</td>
<td>0.1302</td>
</tr>
<tr>
<td>( \eta )</td>
<td>0.3654</td>
<td>0.0000</td>
<td>1.2827</td>
</tr>
<tr>
<td>( A )</td>
<td>1.6235</td>
<td>1.5882</td>
<td>1.6963</td>
</tr>
<tr>
<td>( B )</td>
<td>1.3977</td>
<td>1.3706</td>
<td>1.4535</td>
</tr>
</tbody>
</table>
It is worth noting that parameter set 2 is the extreme case where the consumption of $x$ does not pose any detrimental effect on health. With the calibrated parameters listed in Table 2.A.2, we are able to analyze the impacts of implementing $\tau_x$ when $m$ has DRTS. The comparative-static effects of $\tau_x$ and those of the two tax reforms (one adjusts $\tau_l$ in response to the changes in $\tau_x$ and the other one adjusts $\tau_k$ in response to the changes in $\tau_x$) are shown in the following figures. In terms of the simulated results of the two tax reforms, we only present those with parameter set 1 for concision.

Figure 2.A.3 shows the changes in economic variables in response to the changes in $\tau_x$ in the benchmark case with DRTS in the health sector. Figure 2.A.4 further shows the impacts of $\tau_x$ on $m$. The effects of the two tax reforms are shown in Figures 2.A.5 and 2.A.6.

Figure 2.A.3: The comparative-static effects of $\tau_x$ when $m$ has DRTS
Figure 2.A.4: The comparative-static effects of $\tau_x$ on $m$ when $m$ has DRTS

Figure 2.A.5: Tax reform of replacing $\tau_l$ with $\tau_x$ when $m$ has DRTS
In the cases where $m$ has DRTS, the impacts of implementing $\tau_x$ alone on $h^*$ are still limited (see Figure 2.A.3), because the beneficial effects of the taxes on $h^*$ could be offset by the simultaneous reduction in $m$ (see Figure 2.A.4). Therefore, our inference in the previous section hold even when the health sector has DRTS. Moreover, the implementation of the policies present similar impacts on endogenous variables. Therefore, our analysis in the previous sections hold even with the DRTS.

Figure 2.A.6: Tax reform of replacing $\tau_k$ with $\tau_x$ when $m$ has DRTS
Chapter 3

Optimal sin taxes in the presence of income taxes and health care

In this chapter, we take a non-paternalistic view to justify the role of sin taxes (taxes on unhealthy commodities) in terms of externalities on public funds. The analytical results in this chapter show the additive property between the Pigouvian and the efficiency elements in optimal sin taxes. Although the Pigouvian element decreases in the presence of income taxes, optimal sin taxes are not necessarily lower due to the presence of the efficiency element in the second-best setting. The calibration results in this chapter show that the implementation of sin taxes have double-dividends which improve not only health but also economic performance as well as welfare in the UK economy.
3.1 Introduction

In this chapter, we take a non-paternalistic view to justify the role of sin taxes (taxes on unhealthy commodities such as cigarettes and alcohol) based on the fiscal externalities on public funds. Moreover, we explore the structure of optimal sin taxes in the presence of income taxes and the provision of health care. The consumption of sin goods poses detrimental effects on individual health. Although individuals can replenish health through health care services, they may incur undesirable externalities on the fiscal budget if they do not fully internalize (1) the crowding-out effect on health care resources while tackling short-term health problems (the short-term externalities), and (2) the relationship between individuals’ long-term health and the effectiveness of health care services (the long-term externalities).

In Grossman (1972), health not only enhances utility but also individual “healthy time” available for work. However, studies on sin taxes and health care generally treat labor supply independent of health (e.g. Cremer et al., 2012). In this chapter, we broaden the analysis to include the relationship between health and labor supply by combining the preferences toward health and leisure. By doing so, we can include

---

1Studies of sin taxes usually apply the paternalistic view which focuses on the corrective property of the taxes toward individual self-control problems or ignorance toward the detrimental impacts of sin goods on health (e.g. Aronsson and Thunström, 2008; Cremer et al., 2012; Goulao and Pérez-Barahona, 2014; O’Donoghue and Rabin, 2003, 2006).

2A large literature suggests that the marginal efficiency of health should decrease as the stock of health increases. The subsequent implication is that marginal effects of health on the effective health care should be set to values which make the rate of return decrease in health in the long run. More discussion can be found in Section 3.2.
the relationship between health and labor suggested in Burns and Mullahy (2016).
Moreover, we can thus examine how the presence of labor income taxes in the second-
best setting influences the structure of optimal sin taxes (Bovenberg and Goulder, 1996).

We approach the optimal taxation problem by taking the perspectives from both
individuals and the government. By decomposing the structure of optimal sin taxes,
we find that the additive property between the Pigouvian element and the efficiency
element proposed by Sandmo (1975) is retained in our model. The corrective role
of optimal sin taxes can be justified by both the short- and long-term externalities.
Moreover, we calibrate the model on the UK economy to obtain quantitative im-
pclications for sin taxes. Our quantitative results show that, in line with Bovenberg
and Goulder (1996), the Pigouvian element is lower when the first-best policy is
unavailable in the economy. Nevertheless, it should be noted that the second-best
sin taxes are not necessarily lower than the first-best sin taxes due to the presence
of the efficiency element.

To further examine the property of sin taxes, we construct a revenue-neutral
“sin tax reform” which replaces labor income taxes with sin taxes. Our numerical
analysis shows that the implementation of sin taxes has double-dividends in terms of
not only improving population health but also enhancing both economic performance
and welfare.

This chapter is organized as follows. Section 3.2 introduces an economy with
individuals who internalize the detrimental effects of sin goods on health but not
necessarily the externalities on the fiscal budget. Section 3.3 formulates the optimiza-
tion problem from the government’s perspective (the Ramsey problem) and presents optimal taxes by comparing the optimal conditions from the two perspectives. Section 3.4 decomposes the structure of optimal sin taxes and provides quantitative implications regarding the properties of sin taxes. Section 3.5 shows the simulation results with revenue-neutral sin tax reforms, which further address the role of sin taxes in the economy. Conclusions are offered in Section 4.5.

3.2 The economy

The individual objective is to maximize the lifetime utility.

\[ U = \int_0^\infty e^{-\rho t} u(c, x, L) dt, \quad (3.2.1) \]

where \( \rho \) denotes the rate of time preference, \( c \) denotes numeraire goods, \( x \) denotes sin goods, and \( L \) denotes leisure. \( L \) can be specified as

\[ L \equiv (1 - l)H(h), \quad (3.2.2) \]

where \( l \) is the fraction of healthy time allocated to labor supply, and \( h \) is the stock of health. \( h \) generates healthy time through the \( H \) function, which is assumed to be concave as in Grossman (1972). The objective function is subject to the constraint of asset \( a \) and the law of motion of \( h \):

\[ \dot{a} = (1 - \tau_k)ra + (1 - \tau_l)wlH - c - (1 + \tau_x)x - T, \quad (3.2.3) \]

In Grossman (1972), individual time can be separated into two: sick time and healthy time. Individuals are only able to work during healthy time. Moreover, as in Hokayem and Ziliak (2014), we further assume that individuals can also enjoy leisure while being healthy.
\[ \dot{h} = M(m, \epsilon_x, \epsilon_h) - \eta(x) - \delta h, \]

where

\[ M_m > 0, \ M_x \leq 0, \ \eta_x > 0, \ \eta_{xx} \leq 0, \ \delta > 0. \]

In equation (3.2.3), \( \tau_k, \tau_l, \) and \( \tau_x \) represent taxes on capital income, labor income, and sin goods respectively. \( r \) and \( w \) represent the factor prices of capital and labor, and \( T \) represents lump-sum taxes. For simplicity, we normalize the after-tax price of \( c \) into unity without affecting the results of this chapter. Equation (3.2.4) shows that the stock of health can be accumulated through effective health care \( M \) and deteriorated with \( x \) via the \( \eta \) function and the natural depreciation \( \delta \). \( M \) is affected by the provision of public health care \( m \),\(^4\) individual consumption of \( x \), and the individual level of \( h \). The inclusion of \( x \) and \( h \) captures the short- and long-term externalities on \( M \). To understand the short-term externalities on \( M \): although individuals can use \( m \) to recover from short-term problems caused by \( x \), they simultaneously crowd out the resources available for other health problems and the opportunities to further improve \( h \). Therefore, \( M_x \leq 0 \). To understand the long-term externalities on \( M \): with given \( m \), the marginal efficiency of \( h \) decreases as individuals increase their own health (e.g. Galama et al., 2012; Galama and Van Kippersluis, 2018; Grossman, 1972). The implication for our model would then be \( d\dot{h}/dh < 0 \). Hence, \( M_h \) can be either positive or negative as long as it is less than \( \delta \). To model the internalization of the short- and long-term externalities, we include \( \epsilon_x \) and \( \epsilon_h \) to represent

\(^4\)It should be noted that the provision of \( m \) is exogenous from an individual perspective. However, it would be endogenous if we take the perspective from the government.
3.2. The economy

individual degrees of internalization of $M_x$ and $M_h$ respectively. It should be noted that $0 \leq \epsilon_x \leq 1$ and $0 \leq \epsilon_h \leq 1$. The extreme case where $\epsilon_h = \epsilon_x = 1$ indicates that individuals fully internalize the effects of $x$ and $h$ on $M$; on the other hand, the other extreme case where $\epsilon_x = \epsilon_h = 0$ indicates that individuals completely ignore the two effects.

The economy is constituted by two sectors: the goods sector $y$ and the health sector $m$. Both sectors require capital $k$ and labor supply $lH$ as inputs.

\begin{align*}
y &= f(sk, vlH), \quad (3.2.5) \\
m &= m((1 - s)k, (1 - v)lH), \quad (3.2.6)
\end{align*}

where $s$ and $v$ are the fractions of capital and labor supply devoted into the goods sector. The efficiency condition of the goods market implies $r = f_1$ and $w = f_2$, where $f_i$ denotes the derivative of production functions $f$ with the $i$th argument.

The government collects tax revenue to finance $m$ as below:

\begin{equation}
\tau_k r a + \tau_l w lH + \tau_x x + T + \dot{b} = m + rb, \quad (3.2.7)
\end{equation}

where $b$ is the government debt.

The Hamiltonian function for the individual maximization problem is formulated as

\begin{equation}
\mathcal{H} = u(c, x, L) + \lambda [(1 - \tau_k) r a + (1 - \tau_l) w lH - c - (1 + \tau_x) x - T] \\
+ q [M(m, \epsilon_x x, \epsilon_h h) - \eta(x) - \delta h],
\end{equation}

where $\lambda$ and $q$ are the co-state variables. The first-order conditions are

\begin{equation}
u_c = \lambda, \quad (3.2.8a)\end{equation}
\[ u_x = \lambda(1 + \tau_x) - q(M_x \epsilon_x - \eta_x), \quad (3.2.8b) \]

\[ u_L = \lambda(1 - \tau_l)w, \quad (3.2.8c) \]

\[ \lambda(1 - \tau_k)r = \rho \lambda - \dot{\lambda}, \quad (3.2.8d) \]

\[ u_L(1 - l)H_h + \lambda(1 - \tau_l)wlH_h + q(M_h \epsilon_h - \delta) = \rho q - \dot{q}, \quad (3.2.8e) \]

With equation (3.2.8c), equation (3.2.8e) can be

\[ \dot{q} = q(\rho + \delta - M_h \epsilon_h) - u_L H_h. \quad (3.2.9) \]

### 3.3 The Ramsey problem

We employ the primal approach, which enables us to maximize the social welfare directly through choices of allocations (see Atkinson and Stiglitz (2015)). The implementability constraint can be obtained through equations (3.2.3) and (3.2.8d)

\[ \lambda_0 a_0 = \int_0^\infty e^{-\rho t}[u_c - u_L lH + u_x x + q(M_x \epsilon_x - \eta_x)x + u_c T] dt. \quad (3.3.10) \]

With equations (3.2.3) and (3.2.7), we derive the feasibility constraint as

\[ \dot{k} = f(sk, vlH) - c - x. \quad (3.3.11) \]

The constraint for \( h \) from the government’s perspective is specified as

\[ \dot{h} = M(m((1 - s)k, (1 - v)lH), x, h) - \eta(x) - \delta h. \quad (3.3.12) \]

We then formulate the Hamiltonian function as

\[ H^g = u + \Omega[\lambda_0 a_0] + \gamma[\dot{k}] + \omega[\dot{h}] + \psi[\dot{q}] + \nu[m - \bar{m}], \]

\[ = u + \Omega[u_c c - u_L lH + u_x x + q(M_x \epsilon_x - \eta_x)x + u_c T + u_c a_0] \]
\[ + \gamma \left[ f(sk, vlH) - c - x \right] \]

\[ + \omega \left[ M(m((1 - s)k, (1 - v)lH), x, h) - \eta(x) - \delta h \right] \]

\[ + \psi \left[ q(\rho + \delta - M_h\epsilon_h) - u_LH_h \right], \]

\[ + \nu[m - \bar{m}], \]

where \( \Omega, \gamma, \omega, \psi, \) and \( \nu \) are the co-state variables, and \( \bar{m} \) indicates the required level of spending on \( m \).

The first-order conditions can be written as below.\(^5\)

\[ \frac{\gamma}{u_c} = 1 + \Omega \Delta_c - \frac{u_{cL}}{u_c} \psi H_h, \quad (3.3.13a) \]

\[ \frac{\gamma}{u_x} = 1 + \Omega \Delta_x + \frac{q}{u_x} \left[ M_x\epsilon_x - \eta_x + (M_{xx}\epsilon_x - \eta_{xx})x \right] + \frac{\omega}{u_x} \left[ M_x - \eta_x \right] \]

\[- \frac{\psi}{u_x} (qM_{xh}\epsilon_h + u_{xL}H_h), \quad (3.3.13b) \]

\[ \frac{\gamma f^2}{u_{L}} = 1 + \Omega \Delta_L - \frac{u_{LL}}{u_L} \psi H_h, \quad (3.3.13c) \]

\[ \dot{\gamma} = \gamma (\rho - f_1), \quad (3.3.13d) \]

\[ \dot{\psi} = \psi (M_h\epsilon_h - \delta) - \Omega \left( M_x\epsilon_x - \eta_x \right) x, \quad (3.3.13e) \]

\[ \dot{\omega} = \omega (\rho + \delta - M_h) - u_LH_h \left[ 1 + \Omega (\Delta_L - 1) \right] - \Omega qM_{xh}\epsilon_x x \]

\[ + \psi \left[ qM_{hh}\epsilon_h + u_LH_h + u_{LLH_h}^2 \right], \quad (3.3.13f) \]

where

\[ \Delta_c \equiv 1 + \frac{u_{cc}}{u_c} c - \frac{u_{cL}}{u_c} lH + \frac{u_{cx}}{u_c} x + \frac{u_{cT}}{u_c}, \quad (3.3.14) \]

\[ \Delta_x \equiv 1 + \frac{u_{xx}}{u_x} c - \frac{u_{xL}}{u_x} lH + \frac{u_{xx}}{u_x} x + \frac{u_{xT}}{u_x}, \quad (3.3.15) \]

\(^5\)The detailed derivation can be found in Appendix 3.A.1.
3.3. The Ramsey problem

\[\Delta_L \equiv 1 + \frac{u_{cL}}{u_L} - \frac{u_{LL}}{u_L} lH + \frac{u_{xL}}{u_L} x + \frac{u_{cL}}{u_L} T. \] (3.3.16)

To derive optimal taxes, we compare marginal rates of substitution (MRS) derived from the individual problem and those from the Ramsey problem. The optimal taxes are

\[\tau_l = \frac{u_L}{\gamma f_2} \left[ \Omega (\Delta_L - \Delta_c) + \psi H_h \left( \frac{u_{cL}}{u_c} - \frac{u_{LL}}{u_L} \right) \right], \] (3.3.17)

\[\tau_x = \frac{q}{u_c} (M_x \epsilon_x - \eta_x) + \frac{u_x}{\gamma} \left\{ \Omega \left[ \Delta_c - \Delta_x - \frac{q}{u_x} (M_x \epsilon_x - \eta_x + (M_{xx} \epsilon_x - \eta_{xx}) x) \right] - \frac{\omega}{u_x} (M_x - \eta_x) + \psi \left( \frac{q}{u_x} M_{xh} \epsilon_h + \left( \frac{u_{xL}}{u_x} - \frac{u_{cL}}{u_c} \right) H_h \right) \right\}. \] (3.3.18)

In the steady state, \(\dot{q} = \dot{\gamma} = \dot{\psi} = \dot{\omega} = 0\). Accordingly, equations (3.2.9), (3.3.13d), (3.3.13f), and (3.3.13e) imply that, in the steady state,

\[q = \frac{u_L H_h}{\rho + \delta - M_h \epsilon_h}, \] (3.3.19)

\[\tau_k = 0, \] (3.3.20)

\[\psi = \frac{\Omega (M_x \epsilon_x - \eta_x) x}{M_h \epsilon_h - \delta}, \] (3.3.21)

\[\omega = \frac{1}{\rho + \delta - M_h} \left\{ u_L H_h [1 + \Omega (\Delta_L - 1)] + \Omega q M_{xh} \epsilon_x x - \psi \left[ q M_{hh} \epsilon_h \right] + u_L H_{hh} + u_{LL} H_{hh} \right\}. \] (3.3.22)

The government can directly control quantities when the first-best policy \(T\) is implementable. In this case, the implementability constraint is nonbinding and thus \(\Omega = \psi = 0\). Referring to equation (3.3.17), the implementation of \(\tau_l\) is only justifiable in the second-best setting. On the other hand, optimal \(\tau_x\) are not necessarily zero in the first-best setting. A further exploration of the structure of \(\tau_x\) will be provided in Section 3.4.
3.4 Optimal sin taxes

Sandmo (1975) finds that the optimal tax on an externality-generating commodity can be decomposed into the Pigouvian element and the efficiency element additively. The Pigouvian element counteracts externalities, and the efficiency element satisfies the government revenue requirements under the efficiency principles of taxation. In line with this finding, we find that the optimal sin taxes can be written in the form of

\[ \tau_x = \tau_x^p + \tau_x^e, \]

(3.4.23)

where

\[ \tau_x^p = \frac{u_L}{u_c} \times \frac{H_h}{(\rho + \delta - M_h \epsilon_h)(\rho + \delta - M_h)} \left[-M_x(\rho + \delta)(1 - \epsilon_x) + M_h \eta_x(1 - \epsilon_h) + M_x M_h(\epsilon_h - \epsilon_x)\right], \]

(3.4.24)

and

\[ \tau_x^e = \frac{M_x - \eta_x}{\gamma} \left[ \frac{u_L H_h}{\rho + \delta - M_h} \left( \frac{\gamma}{u_c} - 1 \right) - \omega + \frac{u_L H_h}{\rho + \delta - M_h} \right] \]

\[ + \frac{u_L}{u_x} \left\{ \Omega \left[ \Delta_c - \Delta_x - \frac{q}{u_x} (M_x \epsilon_x - \eta_x + (M_x x \epsilon_x - \eta_x x) x) \right] \right\} \]

\[ + \psi \left[ \frac{q}{u_x} M_x \epsilon_h + \left( \frac{u_{xL}}{u_x} - \frac{u_{cL}}{u_c} \right) H_h \right]. \]

(3.4.25)

\( \tau_x^p \) denotes the Pigouvian element which corrects both the short- and long-term externalities when either \( \epsilon_x \) or \( \epsilon_h \) is not zero, and \( \tau_x^e \) denotes the efficiency element when the first-best policy \( T \) is not available in the economy. Referring to equation (3.3.21), it is clear that \( \tau_x^e \) is zero in the first-best setting. The observation regarding equation (3.4.23) leads to the following proposition:

**Proposition 3.1.** The structure of optimal sin taxes can be decomposed into the Pigouvian element and the efficiency element additively. The Pigouvian element is
3.4. Optimal sin taxes

zero when individuals fully internalize both the short- and long-term externalities. The efficiency element is present only when the first-best policy is not implementable.

The additive property between $\tau^p_x$ and $\tau^e_x$ as suggested in Sandmo (1975) is also found in our dynamic setting. Even in the first-best setting, $\tau^p_x$ could be non-zero when either $\epsilon_x$ or $\epsilon_h$ is below unity. This observation justifies the corrective role of $\tau_x$ when individuals do not fully internalize the short or long-term externalities (or both) on $M$. We quantify the model with the parameters and variables listed below.

<table>
<thead>
<tr>
<th>Table 3.1: Benchmark parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark parameters and observables</td>
</tr>
<tr>
<td>Share of physical capital in the goods sector</td>
</tr>
<tr>
<td>Share of physical capital in the health sector</td>
</tr>
<tr>
<td>Relative productivity in the goods sector</td>
</tr>
<tr>
<td>Effects of health care spending on effective health care</td>
</tr>
<tr>
<td>Effects of short-term externalities on effective health care</td>
</tr>
<tr>
<td>Effects of long-term externalities on effective health care</td>
</tr>
<tr>
<td>Production efficiency of healthy time</td>
</tr>
<tr>
<td>Initial production in the goods sector</td>
</tr>
<tr>
<td>Rate of time preference</td>
</tr>
<tr>
<td>Ratio of numeraire commodities to output production</td>
</tr>
<tr>
<td>Ratio of sin goods to output production</td>
</tr>
<tr>
<td>Natural depreciation of health</td>
</tr>
<tr>
<td>Capital income taxes</td>
</tr>
<tr>
<td>Labor income taxes</td>
</tr>
<tr>
<td>Sin taxes</td>
</tr>
</tbody>
</table>
The calibrated results are presented in Table 3.2. Detailed calibration can be found in Appendix 3.A.2.

<table>
<thead>
<tr>
<th>Calibration</th>
<th>$k_0/y_0$</th>
<th>$h_0/y_0$</th>
<th>$s_0$</th>
<th>$v_0$</th>
<th>$\theta$</th>
<th>$\psi$</th>
<th>$\eta$</th>
<th>$B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ratio of physical capital to output production</td>
<td>5.7112</td>
<td>11.4235</td>
<td>0.9324</td>
<td>0.9007</td>
<td>0.1057</td>
<td>2.4977</td>
<td>6.5121</td>
<td>1.8715</td>
</tr>
<tr>
<td>Ratio of health capital to output production</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fraction of physical capital invested in the goods sector</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fraction of labor supply invested in the goods sector</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Relative preference to sin goods</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Relative preference to leisure</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Detrimental effects of sin goods</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Relative productivity in the health sector</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 3.1: The first- and second-best $\tau_p^x$ when $\epsilon_x = \epsilon_h = 0$ with different levels of $\bar{m}$.
Figure 3.1 shows the first- and the second-best Pigouvian elements in the presence of different levels of $\bar{m}$. For the conciseness of the chapter, we focus on the benchmark case with $\epsilon_x = \epsilon_h = 0$ in this section. The analysis of other cases is provided in Appendix 3.A.3. The solid line represented changes in the first-best $\tau^p_x$, and the dotted line represents the changes in the second-best $\tau^p_x$ setting. This figure clearly shows that, $\tau^p_x$ in the second-best setting are generally lower than those in the first-best setting. The reason behind the lower $\tau^p_x$ is that individuals can tolerate higher levels of externalities because they value public goods less in the presence of $\tau_l$. This finding accords to Bovenberg and Goulder (1996) in that the implementation of income taxes distorts the corrective taxes downward. However, it should be noted that lower Pigouvian elements in the second-best setting do not imply lower $\tau_x$ in the second-best setting as well.

Figure 3.2: The first- and second-best $\tau_x$ when $\epsilon_x = \epsilon_h = 0$ with different levels of $\bar{m}$
Figure 3.2 shows the overall first- and second-best $\tau_x$ when $\epsilon_x = \epsilon_h = 0$ with our benchmark parameters. Referring to equation (3.4.25), the efficiency element could be non-zero in the second-best setting. Therefore, even though the Pigouvian element is lower in the presence of $\tau_l$, the second-best $\tau_x$ can still be higher due to the presence of the efficiency element.

### 3.5 Sin tax reform

In this section, we characterize the impacts of sin taxes by simulating changes in economic variables in response to a revenue-neutral “sin tax reform” which replaces $\tau_l$ with $\tau_x$.

![Figure 3.3: The economic impacts of a sin tax reform](image)

Figure 3.3: The economic impacts of a sin tax reform
3.6. Conclusion

Figure 3.3 shows the changes in \( h, y \) and \( U \) in response to the increases in \( \tau_x \) with the sin tax reform. By observing the quantitative results, we find that this revenue-neutral reform has the double-dividends which improve not only health but also economic output in the goods sector and welfare of the economy.

The concept of double-dividends is often used in environmental economics to point out that environmental taxes are beneficial not only to the environment but also to economic efficiency (e.g. Bovenberg and De Mooij, 1994; Goulder, 1995). As in Wang et al. (2017), the promotion of health can be achieved by using the revenue from \( \tau_x \) to cut discretionary taxes. In this chapter, less costly \( m \) can be achieved by utilizing the revenue from \( \tau_x \) to reduce the distortion from \( \tau_l \). To understand the benefits on \( y \) and \( U \): as \( \tau_l \) decrease in our sin tax reform, the resulting increases in the after-tax labor income further boost the steady state labor supply (as in equation (3.2.8c)). This increase eventually improves the economic performance in the goods sector and thus the consumption of \( c \) in the long run.

3.6 Conclusion

This chapter explores the structure of optimal sin taxes in the presence of income taxes and the provision of health care in a dynamic general equilibrium model. We contribute to the literature of sin taxes with the following findings. First, we justify the role of sin taxes in terms of the short- and long-term externalities on public funds. Second, we show that the additive property between the Pigouvian element and the efficiency element in the optimal sin taxes is retained in our dynamic setting. In addition, our simulation shows that the second-best Pigouvian taxes are distorted
downward by the implementation of labor income taxes. The reason behind this finding is that individuals can tolerate more externalities since they value public goods less in the second-best setting. However, with the presence of the efficiency element, optimal sin taxes in the second-best setting are not necessarily lower than those in the first-best setting. Third, we find that the implementation of sin taxes has double-dividends in terms of not only improving population health but also enhancing economic performance and welfare in the UK economy.

3.A Appendix

3.A.1 Optimal conditions of the Ramsey problem

As shown in Section 3.3, the Ramsey problem is

\[ \mathcal{H}^g = u + \Omega [\lambda_0 a_0] + \gamma [\dot{k}] + \omega [\dot{h}] + \psi [\dot{q}] + \nu [m - \bar{m}], \]

\[ = u + \Omega [u_c c - u_L lH + u_x x + q(M_x \epsilon_x - \eta_x) x + u_c T + u_c a_0] \]

\[ + \gamma \left[ f(s k, v lH) - c - x \right] \]

\[ + \omega \left[ M(m((1 - s) k, (1 - v) lH), x, h) - \eta(x) - \delta h \right] \]

\[ + \psi \left[ q(\rho + \delta - M_h \epsilon_h) - u_L H_h \right], \]

\[ + \nu [m - \bar{m}]. \]

The first-order conditions with respect to \( c, x, L, s, v, k, \) and \( q \) are then

\[ u_c + \Omega [u_c + u_{cL} lH + u_{cL} x + u_{cL} T] = \gamma + \psi u_{cL} H_h, \]

(3.A.1a)
\[ u_x + \Omega [u_{cx} c - u_{xL} H + u_x + u_{xx} x + q (M_x \epsilon_x - \eta_x) + q (M_{xx} \epsilon_x - \eta_{xx}) x] + u_{cx} T = \gamma - \omega (M_x - \eta_x) + \psi (q M_x \epsilon_h + u_{xL} H_h), \]  
\[ u_L + \Omega [u_{cL} c - u_{LL} lH + u_L + u_{xL} x - q M_{mx} \epsilon_x m_2 (1 - v) x + u_{cL} T] = \gamma f_2 v + \omega M_m m_2 (1 - v) + \psi (-q M_m \epsilon_h (1 - v) + u_{LL} H_h) + \nu m_2 (1 - v), \]
\[ \gamma f_1 = m_1 [\Omega q M_{mx} \epsilon_x x + \omega M_m - \psi q M_{mh} \epsilon_h + \nu], \]
\[ \gamma f_2 = m_2 [\Omega q M_{mx} \epsilon_x x + \omega M_m - \psi q M_{mh} \epsilon_h + \nu], \]
\[ \Omega q M_{mx} \epsilon - xxm_1 (1 - s) + \omega M_m m_1 (1 - s) - \psi q m_{mh} \epsilon_h m_1 (1 - s) + \gamma f_1 s + \nu m_1 (1 - s) = \rho \gamma - \dot{\gamma}, \]
\[ \Omega (M_x \epsilon_x - \eta_x) x + \psi (\rho + \delta - M_h \epsilon_h) = \rho \psi - \dot{\psi}. \]

With equations (3.A.1d) and (3.A.1e), equations (3.A.1c), (3.A.1f), and (3.A.1g) can be rewritten into the forms of equations (3.3.13c), (3.3.13d), and (3.3.13f).

### 3.3.2 Calibration

To calibrate the model, we employ the following functions:

\[ u = \ln c + \theta \ln x + \phi \ln L, \]  
\[ H = h^\alpha, \]  
\[ f = A (sk)^\alpha (v lH)^{1 - \alpha}, \]  
\[ m = B ((1 - s) k)^\beta ((1 - v) lH)^{1 - \beta}, \]  
\[ M = m^\kappa x^{-\kappa x} h^\kappa h. \]
We calibrate the model on the UK economy 2005-2015. The parameters are calculated by using the optimal conditions of the household problem. $\kappa_m$ is calibrated as 0.8 following Halliday et al. (2017). $\delta$ is calibrated as 0.43 in line with Rockwood and Mitnitski (2007). We set that $\tau_x = 0.16$, $\tau_l = 0.26$, and $\tau_k = 0.29$ in accordance with the updated data set of McDaniel (2007). We further choose that $\rho = 0.04$, $l = 0.25$, $\alpha = 0.3$, $\beta = 0.22$, and $\mu = 0.08$ as in Wang et al. (2017). We take $\kappa_x = 0.2$, $\kappa_h = 0.4$, $A = 1.5$, and $\epsilon_x = \epsilon_h = 0$ as the benchmark calibration.

The OECD stats shows that the average health spending as a share of GDP in the UK from 2005-2015 is around 0.09. Accordingly, we set that $\frac{m_0}{y_0 + m_0} = 9\%$.\(^6\)

Assuming $y_0 = 1$, we further obtain $m = 0.0989$. Observing that the share household spending on alcohol and tobacco being 4% and debt to GDP ratio being 7.7%, we find that $x_0 = 0.044$ and $b_0 = 0.8462$. With the market clearing condition $y = c + x$, we calculate that the $c_0 = 0.9560$.

With the efficiency condition of equalizing the marginal productions of $k$ and $lH$ across two sectors, we calculate that

$$s_0 = \frac{\alpha y_0}{\alpha y_0 + \beta m_0} = 0.9324, \quad (3.7)$$

and that

$$v_0 = \frac{(1 - \alpha)y_0}{(1 - \alpha)y_0 + (1 - \beta)m_0} = 0.9007. \quad (3.8)$$

With equations (3.2.8c) and (3.2.8a), we are able to calibrate that

$$\phi = \frac{y_0(1 - l_0)(1 - \tau_l)(1 - \alpha)}{c_0 l_0} = 1.6254. \quad (3.9)$$

---

\(^6\)Variables with a subscript 0 denotes the initial variables in our calibration hereafter.
From equation (3.2.8d), the initial $k$ is calibrated as

$$k_0 = \frac{(1 - \tau_k)\alpha y_0}{s_0 \rho} = 5.7112. \quad (3.A.10)$$

With the specific function of $y$, we obtain the initial $h$:

$$h_0 = \left\{ \frac{4^{v_0}}{v_0 l_0} \left[ \frac{m_0}{((1 - s_0)k_0)^\beta} \right]^{\frac{1}{\alpha}} \right\}^{\frac{1}{\beta}} = 11.4235. \quad (3.A.11)$$

With this initial $h$, we can use the $m$ function to obtain $B$:

$$B = \frac{m_0}{((1 - s_0)k_0)^\beta} \left[ \frac{1}{(1 - v_0)l_0 h_0^\rho} \right]^{1 - \beta} = 1.8715. \quad (3.A.12)$$

In the steady state, $\dot{h} = 0$. Accordingly,

$$\eta = \frac{M_0 - \delta h_0}{x_0} = 6.5121. \quad (3.A.13)$$

Referring to (3.2.8b), we find that

$$\theta = \frac{x_0}{c_0} (1 + \tau_x) + q_0 \kappa_x M_0 \epsilon_x + q_0 \eta x_0 = 0.1057. \quad (3.A.14)$$

With the parameters and initial values calculated above, we then calibrate $\Omega = 0.3585$ by using equations (3.3.13a), (3.3.13b), and (3.3.13c).

### 3.A.3 Sensitivity analysis

In this appendix, we compare the simulation results with different parameter combinations.
Figures 3.A.1 and 3.A.2 show the Pigouvian elements and optimal sin taxes with different levels of internalization of externalities on effective health care. As
discussed in Section 3.4, \( \tau^p_x \) are lower in the presence of income taxes because individuals can tolerate more externalities in the second-best economy. However, as shown in Figure 3.2, the second-best \( \tau_x \) can be higher than the first-best \( \tau_x \) due to the presence of \( \tau^e_x \). The extreme case of \( \epsilon_x = \epsilon_h = 1 \) indicates an economy where individuals internalize both the short- and long-term externalities perfectly. Referring to equation (3.4.24), the Pigouvian element and thus the first-best sin taxes are then zero. \( \tau^e_x \) is the only component in the second-best \( \tau_x \) in this case.

Figures 3.A.3 and 3.A.4 present the case where \( \kappa_h < 0 \). As discussed in Section 3.2, \( \kappa_h \) can be set with a negative value as long as \( M_h < \delta \).

Figure 3.A.3: The first- and second-best \( \tau^p_x \) with different \( \epsilon_x \) and \( \epsilon_h \) when \( \kappa_h < 0 \)

\[\text{In this Appendix, we use } \kappa_h = -0.2 \text{ as an example.}\]
Referring to equation (3.4.24), $\tau^p_x$ can be negative when $M_h < 0$. In this case, a higher level of $h$ is detrimental to the effectiveness of health care services. As shown in Figures 3.A.3 and 3.A.4, instead of implementing taxes on $x$, the government may have to subsidize on $x$ to lower the accumulation of $h$. This special case shows the difference between paternalistic and non-paternalistic approach to optimal taxation: A paternalistic government may discourage individuals from consuming $x$ due to the detrimental effects of $x$ on $h$; on the other hand, a non-paternalistic government may encourage individuals to consume more $x$ since it recognizes the fact that individuals derive utility from $x$. 

Figure 3.A.4: The first- and second-best $\tau_x$ with different $\epsilon_x$ and $\epsilon_h$ when $\kappa_h < 0$
Chapter 4

Labor supply and endogenous lifetime

In this chapter, we analyze the relationship between labor supply and public health care in a two-period model with endogenous survival probabilities. With higher survival probabilities, individuals generally offer more labor supply during old age. However, if the additional spending on health care is funded through taxes on old-age labor income, individuals would work more during young age but less as they reach old age. In the face of improvements in production technologies, individuals increase labor supply in both young and old age. On the other hand, in the face of improvements in medical technologies, individuals offer more labor supply during old age but less when they are young. These changes in labor choices result in lower survival probabilities in the economy. Furthermore, we discover that optimal taxes on labor supply depend on the coefficients of relative risk aversion with respect to labor and consumption.
4.1 Introduction

Most modern countries have witnessed a continuous increase in life expectancy over the past two centuries. In addition to the advance in medical sciences, the growing spending on health is usually regarded as the main source for the steady increase in life expectancy (e.g. Brown, 2014; Costa, 2015; Lichtenberg, 2004; Lubitz et al., 2003). Endogenizing decisions about health has consequently become one of the main strands in the research of health policies (e.g. Chakraborty, 2004; Hall and Jones, 2007). In response to the changes in health policies, individuals may alter their intertemporal decisions on consumption and labor supply. These changes have important implications for policy makers.

Figure 4.1 depicts the number of male survivors in England and Wales from 1850 to 2010, and Figure 4.2 plots the number of deaths across all age from 1850 to 2010 (Sources: Office for National Statistics (2012)).

![Figure 4.1: Number of male survivors in England and Wales across all ages](image-url)
In line with the observation of Cervellati and Sunde (2013), we do not detect major changes in the maximal life expectancy in Figures 4.1 and 4.2 even with advances in medical research during the past two centuries. Instead, the documented increases in life expectancy are mainly driven by increases in survivors within the working population. As shown in Börsch-Supan et al. (2014), increasing the number of survivors in the working population could have large impacts on the economy if the effects of individual behaviors are included in the model. Acemoglu and Restrepo (2017) also point out that an aging society does not necessarily imply a poor economy if we consider the contribution of old-age labor supply. Therefore, it is possible to benefit the economy with more provision of health care which raises survival probabilities within the working population. However, the government would have to impose more taxes to sustain additional provision of health care. This additional reliance on taxes (especially on income taxes) might discourage labor supply.
4.1. Introduction

In this chapter, we explore the relationship between public policies and labor supply with the consideration of endogenous lifetime. We extend the basic model of Fleurbaey et al. (2016) and Leroux and Ponthiere (2018) by endogenizing individual survival probabilities and including the role of taxes in the economy.\textsuperscript{1} In this chapter, survival probabilities depend on a government’s provision of health care. we discover that spending on health care generally encourages labor supply during old age. Nevertheless, if the spending is funded mainly through taxes on old-age labor income, it would discourage individuals from supplying labor in later life. In addition to the analysis of the impacts of policies, we examine the impacts of changes in production and medical technologies respectively. We find that improvements in production technologies encourage labor supply during both young and old age; on the other hand, improvements in medical technologies may encourage labor supply in young age but not in old age. Furthermore, we derive optimal taxes from a welfarist perspective. We find that these optimal taxes depend on the coefficients of relative risk aversion regarding consumption and labor supply.

A road map of this chapter is as follows. A two-period model with endogenous survival probabilities is constructed in Section 4.2. The equilibrium conditions of the model are derived in Section 4.3. To obtain quantitative implications for the

\textsuperscript{1}Both papers focus on the comparison between “ex ante egalitarianism”, where the social planner looks at the level of expected lifetime utilities, and “ex post egalitarianism”, where the social planner looks at the final distribution of the realized utilities. Their results suggest that, under ex post egalitarianism, a higher level of consumption in the first period is required to compensate the premature dead. We do not focus on the comparison between different forms of egalitarianism, so we will not discuss this issue any further.
economy, we calibrate the model by using the empirical data of the UK. In addition to the analysis around the impacts of taxes on the economy, we also derive optimal taxes in Section 4.4. A conclusion is provided in Section 4.5.

### 4.2 The model

In this section, we develop a two-period model for a small open economy with the spending on public health care, $h$. The duration of each period is normalized to unity. In the first period, individuals work for $l \in [0, 1]$ units of time with the wage rate $w$. In the second period, individuals work for $z \in [0, 1]$ units of time with the wage rate $\delta w$, where $\delta \leq 1$ denotes the depreciation in labor productivity during old age. Depending on survival probabilities $\pi$, individuals can live either one or two periods. Survival probabilities $\pi$ can be improved with the spending on $h$.

For simplicity, we assume additive separability in individual lifetime utility. The forms of individual preferences toward consumption and leisure are identical across the two periods. Consequently, the lifetime utility can be presented as

$$u(c) - v(l) + \pi(h)[u(d) - v(z)], \quad (4.2.1)$$

where $c$ is the consumption in the first period and $d$ is the consumption in the second period. The utility obtained from consumption satisfies $u'(\cdot) > 0$ and $u''(\cdot) < 0$. The disutility parts in equation (4.2.1) are increasing and convex: $v'(\cdot) > 0$ and $v''(\cdot) > 0$. The function of survival probabilities $\pi$ is assumed to satisfy $\pi'(h) > 0$ and $\pi''(h) < 0$ (as in Chakraborty, 2004; Dávila and Leroux, 2015; Leroux et al., 2011b).
4.2. The model

The budget constraint in the first period is

\[ c + s + b = (1 - \tau_l)wl, \]

(4.2.2)

where \( s \) denotes saving, \( b \) denotes debts, and \( \tau_l \) denotes taxes on labor income in the first period. The inclusion of \( b \) in the first period allows us to analyze the relationship between \( h \) and labor income taxes in the second period. The rationale behind this inclusion is that the government can issue \( b \) in the first period to balance taxes in the second period.

Let \( a \equiv s + b \) denotes the total assets in the first period. The budget constraint in the second period is therefore

\[ d = \frac{\bar{R}}{\pi}a + (1 - \tau_z)\delta wz, \]

(4.2.3)

where

\[ \bar{R} \equiv (1 - \tau_k)R. \]

In the above equation, \( R \) denotes the rate of returns on assets, \( \tau_z \) denotes taxes on labor income in the second period, and \( \tau_k \) denotes taxes on these returns. In the second period, individuals earn (or lose) from \( s \), \( b \) and labor supply \( z \) with depreciated labor productivity \( \delta w \).

The lifetime budget constraint is thus

\[ (1 - \tau_l)wl - c + \frac{\pi}{\bar{R}}(1 - \tau_z)\delta wz - \frac{\pi}{\bar{R}}d = 0. \]

(4.2.4)

To finance the spending on \( h \), the government collects taxes from labor income and returns on asset \( a \). For simplicity, we assume the before-tax returns \( R = 1 \) hereafter without loss of generality. Therefore,

\[ h = \tau_lwl + \tau_z\pi\delta wz + \tau_k a, \]

(4.2.5)
4.3 The equilibrium

Individuals maximize the lifetime utility (4.2.1) subject to budget constraints (4.2.2) and (4.2.3). The Lagrangian function is therefore

\[ \mathcal{L} = u(c) - v(l) + \pi (u(d) - v(z)) + \lambda \left[ (1 - \tau_l)wl - c + \frac{\pi}{R} (1 - \tau_z)\delta wz - \frac{\pi}{R} d \right]. \] (4.3.6)

The first-order conditions are

\[ u_c = \lambda, \] (4.3.7)
\[ u_d = \frac{\lambda}{R}, \] (4.3.8)
\[ v_l = \lambda (1 - \tau_l)w, \] (4.3.9)
\[ v_z = \frac{\lambda}{R} (1 - \tau_z)\delta w, \] (4.3.10)

where \( \lambda \) is the Lagrange multiplier associated with the lifetime budget constraint.

4.3.1 The effects of the taxes

In this subsection, we explore the impacts of adjusting \( \tau_l, \tau_z \), and \( \tau_k \) on the economy. We derive the variations in each endogenous variables by using the government budget constraint, the individual budget constraint, and the first-order conditions. To obtain quantitative results, we calibrate the model on the UK economy with the parameters listed in Table 4.1. The detailed illustration of the calibration can be found in Appendix 4.A.2.

\[ ^{2}\text{The full derivation can be found in Appendix 4.A.1.} \]
4.3. The equilibrium

Table 4.1: Benchmark parameters and calibration

<table>
<thead>
<tr>
<th>Benchmark parameters and calibration</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Utility weight of consumption</td>
<td>$T_1$</td>
<td>52</td>
</tr>
<tr>
<td>Utility weight of labor supply</td>
<td>$T_2$</td>
<td>47</td>
</tr>
<tr>
<td>Coefficient of relative risk aversion</td>
<td>$\gamma$</td>
<td>1.5</td>
</tr>
<tr>
<td>Labor income taxes in the first period</td>
<td>$\tau_l$</td>
<td>0.26</td>
</tr>
<tr>
<td>Labor income taxes in the second period</td>
<td>$\tau_z$</td>
<td>0.26</td>
</tr>
<tr>
<td>Capital income taxes</td>
<td>$\tau_k$</td>
<td>0.29</td>
</tr>
<tr>
<td>Depreciation in productivity</td>
<td>$\delta$</td>
<td>1</td>
</tr>
<tr>
<td>Wage rate</td>
<td>$w$</td>
<td>10</td>
</tr>
<tr>
<td>Consumption in the first period</td>
<td>$c$</td>
<td>1</td>
</tr>
<tr>
<td>Labor supply in the first period</td>
<td>$l$</td>
<td>0.25</td>
</tr>
<tr>
<td>Life expectancy</td>
<td></td>
<td>81.40</td>
</tr>
<tr>
<td></td>
<td>Life Expectancy</td>
<td>81.40</td>
</tr>
</tbody>
</table>

Calibration

<table>
<thead>
<tr>
<th>Calibration</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Asset accumulated in the first period</td>
<td>$a$</td>
<td>0.85</td>
</tr>
<tr>
<td>Consumption in the second period</td>
<td>$d$</td>
<td>0.7959</td>
</tr>
<tr>
<td>Labor supply in the second period</td>
<td>$z$</td>
<td>0.0475</td>
</tr>
<tr>
<td>Survival probabilities</td>
<td>$\pi$</td>
<td>0.4729</td>
</tr>
<tr>
<td>Utility measure of labor supply</td>
<td>$\alpha$</td>
<td>32.7489</td>
</tr>
<tr>
<td>Spending on health care</td>
<td>$h$</td>
<td>0.8973</td>
</tr>
</tbody>
</table>

We then apply the calibrated results to simulate the changes in economy in
response to the changes in policies. Figure 4.3 shows the impacts of adjusting $\tau_l$ from 0% to 100% on the economy.

Referring to equation (4.3.9), increases in $\tau_l$ raise the marginal costs of labor supply during young age. These higher costs discourage $l$. In response to the reduction in $l$, individuals have to decrease $c$ so that the first-period budget constraint can be held. Consequently, individuals have to decrease $d$ so that the MRS between $c$ and $d$ can be restored. The bottom-left panel in Figure 4.3 presents a Laffer curve where the government spending on $h$ increases until $\tau_l$ reach a certain level. Therefore, the implementation of higher $\tau_l$ could increase the provision of public health care as long as $\tau_l$ are kept on the left-hand side of the Laffer curve. With the subsequent higher survival probabilities, individuals have to supply more $z$ to compensate the resulting loss from the rate of returns (see equation (4.2.3)).

Figure 4.4 shows the impacts of adjusting $\tau_z$ on economic variables.
4.3. The equilibrium

As indicated by equation (4.3.10), marginal costs of labor supply during old age increase with higher $\tau_z$. Individuals hence supply less $z$ in response to higher $\tau_z$. As implied in equation (4.2.3), $d$ decreases in response to the reduction in the after-tax labor income. A reduction in $c$ is required so that the MRS between $c$ and $d$ can be restored (see equations (4.3.7) and (4.3.8)). To explain the increases in $l$: the implementation of higher $\tau_z$ can be viewed as a higher debt in the first period. Therefore, given $\tau_l$ and $w$, individuals have to work more during young age to equalize both sides of equation (4.2.2). The bottom two panels in Figure 4.5 indicate that survival probabilities could be raised with the implementation of higher $\tau_z$ as long as the taxes are keep at the left-hand side of the Laffer curve.

Figure 4.5 shows the effects of increasing $\tau_k$ on economic variables.
As shown in the second-period budget constraint (4.2.3), with \( a > 0 \) in our calibration, higher \( \tau_k \) reduce the rate of returns and therefore the consumption of \( d \). Referring to equation (4.2.2), individuals would rather consume more than save in young age since savings become less valuable. \( z \) increases because individuals have to work more to compensate the loss of returns on assets from higher \( \tau_k \). To sustain the MRS between \( l \) and \( z \), individuals have to reduce \( l \). The bottom two panels in figure 4.5 show that, as long as \( \tau_k \) are kept on the left-hand side of the Laffer curve, increases in \( \tau_k \) could raise \( h \) and thus \( \pi \).

Our analysis shows that additional spending on health care can improve survival probabilities as long as taxes are kept on the increasing part of the Laffer curve. In the face of increases in lifetime, individuals generally increase labor supply in old age. However, if the additional \( h \) is funded by \( \tau_z \), individuals would work more during young age but less as they reach old age.
4.3. The equilibrium

4.3.2 The effects of changes in technologies

In this subsection, we investigate the impacts of changes in production technologies and medical technologies respectively.

We simulate the impacts of positive technology shocks on this small open economy by raising the level of $w$ from 0 to 100. The rationale behind this simulation is that the improvements in production technologies enhance the levels of labor productivity in both young and old age; these changes in labor productivity would reflect on the level of wage rates in equilibrium. In our analysis, $R$ is not affected by the technology shocks in this small open economy because it is internationally determined. The economic impacts of changes in production technologies are depicted in Figure 4.6.

![Graphs showing the impacts of improving production technologies](image)

**Figure 4.6: The impacts of improving production technologies**

As shown in Figure 4.6, both $c$ and $d$ increase in response to the increases in $w$. This co-movement can be examined from equations (4.2.2) and (4.2.3). Increases
4.3. The equilibrium

in $w$ also raise the marginal returns on labor supply in both young and old age. Therefore, individuals are encouraged to raise both $l$ and $z$ in equilibrium (see equations (4.3.9) and (4.3.10)). In our calibration, the government consequently collects more revenue for $h$ from the given tax scheme. Therefore, $h$ and $\pi$ increase in response to the increases in $w$.

We then examine the impacts of variations in medical technologies, which are embodied in the changes of $\delta$. The intuition is that medical technologies could restore the loss in labor productivity from aging. We simulate these impacts by changing the value of $\delta$ from 0 to 1. The simulated results are plotted in Figure 4.7.

Figure 4.7: The impacts of improving medical technologies

With higher $\delta$, individuals are able to consume more commodities over time as implied in equation (4.2.4). Increases in $\delta$ raise the marginal benefits of supplying labor during old age (see equation (4.3.10)). Accordingly, individuals offer more $z$ in equilibrium. To maintain the MRS between $l$ and $z$, individuals have to reduce
4.4. Optimal taxation

In our calibration, $h$ decreases because the impacts of lowering $l$ outweigh the impacts of raising $z$ in the government budget. Therefore, $\pi$ decrease as $\delta$ increases.

4.4 Optimal taxation

In this section, we explore optimal taxes from a welfarist perspective. We employ the primal approach (see Atkinson and Stiglitz (2015)) to allow the government to directly control the quantities.

By applying the first-order conditions into equation (4.2.4), we form the implementability constraint as below

$$vl - uc - \pi ud + \pi vz - ucT = 0,$$

(4.4.11)

where $T$ is the lump-sum tax. The feasibility constraint is

$$wl + \pi \delta wz - c - \pi d - h = 0.$$

(4.4.12)

The maximization problem from a government’s perspective is thus

$$L^g = u(c) - v(l) + \pi(h) [u(d) - v(z)]$$

(4.4.13)

$$- \Omega [vl - uc - \pi ud + \pi vz - ucT]$$

(4.4.14)

$$+ \psi [wl + \pi \delta wz - c - \pi d - h],$$

(4.4.15)

where $\Omega$ is the Lagrangian multiplier associated with the implementability constraint and $\psi$ is the Lagrangian multiplier associated with the feasibility constraint. It should be noted that $\Omega = 0$ in the first-best setting where $T$ is implementable.
4.4. Optimal taxation

To examine the second-best optimal taxes, we set $T = 0$. The first-order conditions are thus

\[ u_c = \psi - \Omega [u_{cc}c + u_c], \]  
\[ u_d = \psi - \Omega [u_{dd}d + u_d], \]  
\[ v_l = \psi w - \Omega [v_{ll}l + v_l], \]  
\[ v_z = \psi \delta w - \Omega [v_{zz}z + v_z], \]  
\[ \pi_h [u(d) - v(z)] - \Omega [\pi_h v_{zz}z - \pi_h u_{ld}] + \psi [\pi_h \delta wz - \pi_h d - 1] = 0. \]

From equations (4.3.7), (4.3.8), (4.4.16), and (4.4.17), we know that

\[ \bar{R} = \frac{\psi - \Omega [u_{cc}c + u_c]}{\psi - \Omega [u_{dd}d + u_d]}. \]  

From equations (4.3.7) and (4.3.9), we can derive that

\[ \frac{u_c w}{v_l} = \frac{1}{1 - \tau_l}. \]  

We then know from equations (4.4.16) and (4.4.18) that

\[ \frac{\tau_l}{1 - \tau_l} = \Omega \left[ \frac{v_{ll}l}{v_l} - \frac{\tau_l}{1 - \tau_l} - \frac{u_{cc}c}{u_c(1 - \tau_l)} \right]. \]  

Consequently,

\[ \tau_l = \frac{\Omega (\sigma_l + \sigma_c)}{1 + \Omega (1 + \sigma_l)}, \]  

where $\sigma_l = v_{ll}/v_l > 0$ and $\sigma_c = -u_{cc}c/u_c > 0$ are the coefficients of relative risk aversion toward $l$ and $c$ respectively.

From equations (4.3.7) and (4.3.9), we find that

\[ u_d \delta w = v_z + \Omega [v_{zz}z + v_z - \delta w(u_{dd}d + u_d)]. \]
From equations (4.4.17) and (4.4.19),
\[
\frac{\tau_z}{1 - \tau_z} = \Omega \left[ \sigma_z + \frac{\sigma_d}{1 - \tau_z} - \frac{\tau_z}{1 - \tau_z} \right],
\]
(4.4.26)
where \(\sigma_z = v_{zz}/v_z > 0\) and \(\sigma_d = -u_{dd}/u_d > 0\) are the inverses of the coefficients of relative risk aversion with respect to \(z\) and \(d\) respectively. Therefore,
\[
\tau_z = \frac{\Omega(\sigma_z + \sigma_d)}{1 + \Omega(1 + \sigma_z)} > 0,
\]
(4.4.27)

Referring to equations (4.4.24) and (4.4.27), we find that optimal taxes depend on the coefficients of relative risk aversion. With the same forms of \(u(\cdot)\) and \(v(\cdot)\) across the two periods, equations (4.4.24) and (4.4.27) indicate an optimal tax scheme with uniform labor income taxes over lifetime. For conciseness of the chapter, we only present the variations in \(\tau_l\) hereafter.

Figure 4.8 depicts the simulated variations in the optimal \(\tau_l\) in response to the changes in \(\sigma_c\) and \(\sigma_l\) respectively.

Figure 4.8: Variations in optimal taxes in response to the changes in the coefficients of relative risk aversion
4.4.1 The effects of changes in technologies with optimal taxation

In this subsection, we simulate the economic impacts of changing production technologies and medical technologies when the optimal tax scheme is applied in the economy. By fixing the coefficients of relative risk aversion to the benchmark parameter set, we are able to acquire optimal taxes in response to the changes in the two technologies respectively. Figure 4.9 depict the variations in optimal $\tau_l$ in response to the changes in production technologies and the changes in medical technologies.

![Figure 4.9: Variations in optimal taxes in response to the changes in production technologies and medical technologies](image)

Referring to equation (4.4.24), changes in $\tau_l$ are mainly driven by the changes in $\Omega$ with the coefficients of relative risk aversion fixed in the calibration. The
4.4. Optimal taxation

variations in $\Omega$ are depicted in Figure 4.10.

Figure 4.10: The variations in $\Omega$ in response to the changes in production and medical technologies

As depicted in the left panel of Figure 4.9, optimal $\tau_l$ decrease in $w$ because a higher $w$ reduces the shadow price of the second-best constraint $\Omega$. The intuition behind the relationship between $w$ and $\Omega$ is that higher productivity makes the economy more resourceful. The marginal cost of public funds is hence lower. As depicted in the right panel of Figure 4.9, optimal $\tau_l$ increase in $\delta$ because a higher $\delta$ raises $\Omega$. Referring to Figure 4.7, increases in $\delta$ pose negative impacts on the public fund $h$ in our calibration. These negative impacts indicate the negative relationship between $\delta$ and $\Omega$.

In Figures 4.11 and 4.12, we compare the impacts of changes in production technologies on the economy with benchmark taxes and those on the economy with optimal taxes.
In our calibration, increases in $w$ pose similar effects on both economies in that
4.4. Optimal taxation

$c, d, l, z, h$, and thus $\pi$ all increase as $w$ improves. It should be noted that $\tau_k$ in our optimal tax scheme is zero. The implementation of zero $\tau_k$ contributes to a smoother consumption path with higher levels of consumption across both young and old age. Moreover, under the optimal tax scheme, individuals also obtain a smoother labor choice over time with lower $l$ but higher $z$ in equilibrium. As depicted in Figure 4.12, these variations in the economic variables transfer into higher levels of utilities in our calibration.

In Figures 4.13 and 4.14, we compare the impacts of changes in medical technologies on the economy with benchmark tax rates and those on the economy with optimal taxes.

Figure 4.13: The impacts of improving medical technologies under optimal taxation
4.4. Optimal taxation

It should be noted that the reason why the dotted lines in both Figures 4.13 and 4.14 look rather flat is because of the relatively large variations in the economy under the optimal tax scheme. In response to the increases in $\delta$, optimal labor income taxes increase as a higher $\delta$ raises $\Omega$. Higher taxes suppress the increases in $c$ and $d$ in the face of higher productivity during old age (see equations (4.3.9) and (4.3.10)). Nevertheless, the optimal tax scheme contributes to a smoother consumption path with higher levels of both $c$ and $d$. Individuals obtain a smoother path for labor choices by lowering $l$ and raising $z$. With the implementation of optimal taxes, the economy also reaches higher levels of utilities as shown in Figure 4.14.
4.5 Conclusion

We develop a two-period model with endogenous lifetime to analyze the relationship between labor supply and the provision of public health care. In our model, the provision of health care determines individual probabilities of survival into old age. An additional spending on health care can generally encourage individuals to offer more labor supply during old age. However, this finding is reversed if the additional spending is funded mainly through taxes on old-age labor income.

Consumption and labor supply would be raised in the face of improvements in production technologies. These increases transfer into higher spending on health care and thus higher survival probabilities in our calibration. On the other hand, with improvements in medical technologies which reduce the loss of labor productivity from aging, individuals offer more labor supply during old age but less during young age. In our calibration, the resulting survival probabilities are lower in response to the improvements in medical technologies. The reason is that the fiscal impacts of decreased young-age labor supply overshadow those of increased old-age labor supply.

Furthermore, we derive a welfarist optimal taxation and find that optimal taxes on labor income depend on the coefficients of relative risk aversion toward labor supply and consumption. The implementation of these optimal taxes contributes to a smoother consumption path with higher levels of consumption in both young and old age. These optimal taxes also bring a smoother labor choice over time by lowering young-age labor supply but raising old-age labor supply.

In this chapter, we assume that the depreciation of labor productivity in old age
is exogenous. This setting can be extended by including the negative impacts of health care services on depreciation rate. This extension can offer important policy implication regarding the recent global trends of lower child mortality and increases in chronic incapacitating diseases in adults. However, as this chapter focuses on the impacts of increasing survival probabilities, we will leave this potential extension to our future work.

4.A Appendix

4.A.1 Analytical analysis

We utilize the lifetime budget constraint and the first-order conditions to examine the effects of taxes on the economy. Equation (4.2.4) and the first-order conditions can be rewritten into:

\[ c = (1 - \tau_l)wl + \frac{\pi}{R}(1 - \tau_z)\delta wz - \frac{\pi}{R}d, \]

\[ \frac{u_c}{u_d} = \bar{R}, \]

\[ \frac{v_l}{u_c} = (1 - \tau_l)w, \]

\[ \frac{v_z}{u_c} = \frac{(1 - \tau_z)\delta w}{R}, \]

\[ h = \tau_lwl + \tau_z\pi\delta wz + (\bar{R} - \bar{R})a. \]

Taking full derivative of the above five equations, we obtain that:

\[ dc = -wld\tau_l - \frac{\pi}{R}\delta wzdr_z - \frac{\pi}{R^2}[(1 - \tau_z)\delta wz - d]d\bar{R} - \frac{\pi}{R}d\bar{d} + (1 - \tau_l)wdl \]

\[ + \frac{\pi}{R}(1 - \tau_z)\delta wd\bar{z} + \frac{\pi h}{R}[(1 - \tau_z)\delta wz - d]d\bar{h}, \]  

(4.A.1)
\[ dd = \frac{d}{\sigma_d \bar{R}} d\bar{R} + \frac{\sigma_c d}{\sigma_d c} dc, \quad (4.A.2) \]
\[ dl = \frac{-l}{\sigma_l (1 - \tau_l)} d\tau_l - \frac{\sigma_c l}{\sigma_l c} dc, \quad (4.A.3) \]
\[ dz = \frac{-z}{\sigma_z (1 - \tau_z)} d\tau_z - \frac{z}{\sigma_z \bar{R}} d\bar{R} - \frac{\sigma_c z}{\sigma_z c} dc, \quad (4.A.4) \]
\[ dh = \frac{1}{1 - \theta} \left\{ (1 - \tau_k)wl d\tau_l + \pi \delta wz d\tau_z - \pi d\bar{R} - \tau_k dc + \tau_l w 
+ \tau_k (1 - \tau_l w) d\tau_l + \tau_z \pi \delta wz d\tau_z \right\}, \quad (4.A.5) \]

where

\[ \theta \equiv \tau_z \pi h \delta wz \leq 1. \]

The condition of \( \theta \leq 1 \) should not be violated; otherwise, the government can keep generating revenue merely through increments in taxes.

Considering the changes in \( c, d, l, z, \) and \( h \) in response to the changes in \( \tau_l \), we construct the problem into the following matrix form:

\[
\begin{bmatrix}
-1 & -\frac{\pi}{\bar{R}} & (1 - \tau_l)w & \frac{\pi}{\bar{R}} (1 - \tau_z) \delta w & \frac{\pi h}{\bar{R}} \left[ (1 - \tau_z) \delta wz - d \right] \\
\frac{\sigma_c d}{\sigma_d c} & -1 & 0 & 0 & 0 \\
-\frac{\sigma_l l}{\sigma_l c} & 0 & -1 & 0 & 0 \\
-\frac{\sigma_z z}{\sigma_z c} & 0 & 0 & -1 & 0 \\
-\frac{\tau_k}{1 - \theta} & 0 & \frac{\tau_l + \tau_k (1 - \tau_l) w}{1 - \theta} & \frac{\tau_z \pi \delta w}{1 - \theta} & -1
\end{bmatrix}
\begin{bmatrix}
dc \\
d\bar{R} \\
d\tau_l \\
dl \\
d\tau_l \\
dw
\end{bmatrix}
\]

\[
= \begin{bmatrix}
dl \\
0 \\
\frac{l}{\sigma_l (1 - \tau_l)} \\
0 \\
-\frac{(1 - \tau_k)wl}{1 - \theta}
\end{bmatrix}
\]
We define the coefficient matrix above as \([A]\). The determinant \(|A|\) is
\[
|A| = \frac{1}{cR(1-\theta)\sigma_d\sigma_z}\left\{ -(1-\theta)[cR\sigma_d\sigma_z + \sigma_c(d\pi\sigma_1\sigma_z + w\sigma_d \\
\times (lR\sigma_z(1-\tau_l) + z\delta\pi\sigma_1(1-\tau_z))] + \pi_h\sigma_d(d - wz\delta + wz\delta\tau_z) \right. \\
\times \left. [-c\sigma_1\tau_l + w\sigma_c(l\sigma_z(\tau_k + \tau_l - \tau_k\tau_l) + z\delta\pi\sigma_1\tau_z)] \right\}. 
\]
(4.A.6)

We reform \(A\) by replacing its first column:
\[
\left| A_c \right| = \begin{vmatrix}
   wl & -\frac{\pi}{R} & (1-\tau_l)w & \frac{\pi}{R}(1-\tau_z)\delta w & \frac{\pi}{R}[(1-\tau_z)\delta wz - d] \\
   0 & -1 & 0 & 0 & 0 \\
   \frac{1}{\sigma_1(1-\tau_l)} & 0 & -1 & 0 & 0 \\
   0 & 0 & 0 & -1 & 0 \\
   \frac{-(1-\tau_k)wl}{1-\theta} & 0 & \frac{\tau_l+\tau_k(1-\tau_l)}{1-\theta} & \frac{\tau_k\pi\delta w}{1-\theta} & -1 \\
\end{vmatrix}
\]
\[
= \frac{wl}{\sigma_l} \left\{ -1 + \sigma_l + \frac{\pi\tau_k}{(1-\theta)(1-\tau_l)R} [\sigma_l(1-\tau_k)(1-\tau_l) + \tau_k(1-\tau_l) \\
+ \tau_l] [d - \delta wz(1-\tau_z)] \right\}.
\]

The derivative of \(c\) with respect to \(\tau_l\) is thus
\[
\frac{dc}{d\tau_l} = \frac{\left| A_c \right|}{|A|}.
\]

We obtain the following determinant by replacing the second column in \(|A|\):
\[
\left| A_d \right| = \begin{vmatrix}
   -1 & w\sigma_d & (1-\tau_l)w & \frac{\pi}{R}(1-\tau_z)\delta w & \frac{\pi}{R}[(1-\tau_z)\delta wz - d] \\
   \frac{\sigma_l\sigma_d}{\sigma_c} & 0 & 0 & 0 & 0 \\
   -\frac{\sigma_l}{\sigma_c} & \frac{l}{\sigma_l(1-\tau_l)} & -1 & 0 & 0 \\
   -\frac{\sigma_1}{\sigma_c} & 0 & 0 & -1 & 0 \\
   -\frac{\tau_l}{1-\theta} & \frac{-(1-\tau_k)wl}{1-\theta} & \frac{\tau_l+\tau_k(1-\tau_l)}{1-\theta} & \frac{\tau_k\pi\delta w}{1-\theta} & -1 \\
\end{vmatrix}
\]
\[
= \frac{\sigma_c wld}{cR(1-\theta)\sigma_d\sigma_l(1-\tau_l)} \left\{ R(1-\theta)(1 + \sigma_l)(1-\tau_l) - \tau_l \left[ -\sigma_l(1-\tau_k) \right] \right\}.
\]
\[
\times(1 - \tau_l) + \tau_k(1 - \tau_l) + \tau_l \left[ d - \delta wz(1 - \tau_z) \right] \}.
\]

The derivative of \( d \) with respect to \( \tau_l \) is thus

\[
\frac{dd}{d\tau_l} = \frac{|A_d^\tau_l|}{|A|}.
\]

We reform \(|A|\) by replacing the third column:

\[
|A_l^\tau_l| = \begin{vmatrix}
-1 & -\frac{\pi}{R} & (1 - \tau_l) & 0 & 0 & 0 \\
\frac{\sigma_d}{\sigma_{d c}} & -1 & 0 & 0 & 0 & 0 \\
\frac{\sigma_l}{\sigma_{l c}} & 0 & -1 & 0 & 0 & 0 \\
\frac{\sigma_z}{\sigma_{z c}} & 0 & 0 & -1 & 0 & 0 \\
\frac{\tau_k}{1 - \theta} & 0 & -\left(\frac{1 - \tau_k}{1 - \theta}\right) & \frac{\tau_k \delta w}{1 - \theta} & \frac{\tau_k \pi}{1 - \theta} & -1
\end{vmatrix}
\]

\[
= \frac{-1}{c R (1 - \theta) \sigma_d \sigma_l \sigma_z (1 - \tau_l)} \left\{ -(1 - \theta) \left[ c R \sigma_d \sigma_z + \sigma_c (d \pi \sigma_z + w \sigma_d \right.ight.
\]
\[
\times \left( -l^2 R \sigma_z (1 - \tau_l) + z \delta \pi (1 - \tau_z) \right) \} + \pi_h \sigma_d (d - \delta wz (1 - \tau_z)) \left[ -c \sigma_z \tau_k 
\]
\[
+ w \sigma_c (l^2 \sigma_z (1 - \tau_k) (1 - \tau_l) + z \delta \pi \tau_z) \} .
\]

The derivative of \( l \) with respect to \( \tau_l \) is thus

\[
\frac{dl}{d\tau_l} = \frac{|A_l^\tau_l|}{|A|}.
\]

We obtain the following determinant by replacing the fourth column in \(|A|\):

\[
|A_z^\tau_l| = \begin{vmatrix}
-1 & -\frac{\pi}{R} & (1 - \tau_l) & 0 & 0 & 0 \\
\frac{\sigma_d}{\sigma_{d c}} & -1 & 0 & 0 & 0 & 0 \\
\frac{\sigma_l}{\sigma_{l c}} & 0 & -1 & 0 & 0 & 0 \\
\frac{\sigma_z}{\sigma_{z c}} & 0 & 0 & -1 & 0 & 0 \\
\frac{\tau_k}{1 - \theta} & 0 & -\left(\frac{1 - \tau_k}{1 - \theta}\right) & \frac{\tau_k \delta w}{1 - \theta} & \frac{\tau_k \pi}{1 - \theta} & -1
\end{vmatrix}
\]
\[
\frac{w z \sigma_c}{c R (1 - \theta) \sigma_l \sigma_z (1 - \tau_l)} \left\{ -\bar{R} (1 - \theta) (1 + \sigma_l) (1 - \tau_l) - \pi_h \lbrack l \sigma_l (1 - \tau_k) (1 - \tau_l) - \tau_k (1 - \tau_l) - \tau_l [d - \delta w z (1 - \tau_z)] \right\}.
\]

The derivative of \( z \) with respect to \( \tau_l \) is thus
\[
\frac{d z}{d \tau_l} = \frac{|A_z |}{|A|}.
\]

We then reform \( |A| \) by replacing the fifth column:
\[
|A_h^z| = \begin{vmatrix}
-1 & -\frac{\pi}{\bar{R}} & (1 - \tau_l) w & \frac{\pi}{\bar{R}} (1 - \tau_z) w & \delta w & w l \\
\frac{\sigma_d}{\sigma_c} & -1 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & \frac{l}{\sigma_l (1 - \tau_l)} & \\
0 & 0 & -1 & 0 & \frac{l}{\sigma_l (1 - \tau_l)} & \\
\frac{\tau_k}{1 - \theta} & 0 & \frac{(1 - \tau_k) w}{1 - \theta} & \frac{\tau_k \delta w}{1 - \theta} & -\frac{(1 - \tau_k) w l}{1 - \theta} & \\
\end{vmatrix}
\]

\[
\frac{w}{c R (1 - \theta) \sigma_d \sigma_l \sigma_z (1 - \tau_l)} \left\{ -\sigma_z [d \pi \sigma_c (l \sigma_l (1 - \tau_k) (1 - \tau_l) - \tau_k (1 - \tau_l) - \tau_l) + \bar{R} \sigma_d (-\sigma_d w l^2 (1 - \tau_l) + c(2 \tau_k (1 - \tau_l) - \sigma_l (1 - 2 \tau_k) (1 - \tau_l) + \tau_l))] + w z \delta \pi \sigma_c \sigma_d [(\tau_k (1 - \tau_l) + \tau_l) (1 - \tau_z) - \bar{R} (1 - \tau_l) \tau_z - l \sigma_z (1 - \tau_l) (1 - \tau_k) \times (1 - \tau_z) + \bar{R} \tau_z] \right\}.
\]

The derivative of \( h \) with respect to \( \tau_l \) is thus
\[
\frac{d h}{d \tau_l} = \frac{|A_h |}{|A|}.
\]
The effects of changing in $\tau_z$ can be examined from the problem below:

\[
\begin{bmatrix}
-1 & -\frac{\pi}{R} & (1 - \tau)w & \frac{\pi}{R}(1 - \tau_z)\delta w & \frac{\pi h}{R}[(1 - \tau_z)\delta wz - d] \\
\frac{\sigma_{zd}}{\sigma_{dc}} & -1 & 0 & 0 & 0 \\
\frac{\sigma_{zd}}{\sigma_{dc}} & 0 & -1 & 0 & 0 \\
\frac{\sigma_{zd}}{\sigma_{dc}} & 0 & 0 & -1 & 0 \\
\frac{\tau_k}{1-\theta} & 0 & \frac{[\tau_l + \tau_k(1-\tau)]w}{1-\theta} & \frac{\tau_x \pi \delta w}{1-\theta} & -1
\end{bmatrix}
\begin{bmatrix}
\frac{dc}{d\tau_z} \\
\frac{dd}{d\tau_z} \\
\frac{dl}{d\tau_z} \\
\frac{dz}{d\tau_z} \\
\frac{dh}{d\tau_z}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\frac{\pi}{R}\delta wz \\
0 \\
0 \\
\frac{z}{\sigma_x(1-\tau_x)} \\
\frac{\pi \delta wz}{1-\theta}
\end{bmatrix}
\]

We reform $|A|$ into the following form by replacing the first column

\[
|A|^\tau_z = \begin{vmatrix}
\frac{\pi}{R}\delta wz & -\frac{\pi}{R} & (1 - \tau)w & \frac{\pi}{R}(1 - \tau_z)\delta w & \frac{\pi h}{R}[(1 - \tau_z)\delta wz - d] \\
0 & -1 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 \\
\frac{z}{\sigma_x(1-\tau_x)} & 0 & 0 & -1 & 0 \\
-\frac{\pi \delta wz}{1-\theta} & 0 & \frac{[\tau_l + \tau_k(1-\tau)]w}{1-\theta} & \frac{\tau_x \pi \delta w}{1-\theta} & -1
\end{vmatrix}
\]

\[
= \frac{\pi \delta wz}{R(1-\theta)\sigma_x(1-\tau_z)} \left\{ (1 - \theta)(1 + \sigma_x)(1 - \tau_x) - \pi h[-\sigma_x(1 - \tau_x) + \tau_x] \right\}
\times [d - \delta wz(1 - \tau_x)].
\]

The derivative of $c$ with respect to $\tau_z$ is thus

\[
\frac{dc}{d\tau_z} = \frac{|A|^\tau_z}{|A|}.
\]
By replacing the second column in $|A|$, we obtain the following determinant

$$|A'_{\tau}| = \begin{vmatrix} -1 & \frac{\bar{\pi}}{R}\delta wz & (1 - \tau_l)w & \frac{\bar{\pi}}{R}(1 - \tau_z)\delta wz & \frac{\bar{\pi}_h}{R} [(1 - \tau_z)\delta wz - d] \\ \sigma_c d \sigma_{dc} & 0 & 0 & 0 & 0 \\ \sigma_c l \sigma_{lc} & 0 & -1 & 0 & 0 \\ \sigma_c z \sigma_{zc} & \frac{z}{\sigma_z (1 - \tau_z)} & 0 & -1 & 0 \\ \frac{\tau_h}{1 - \theta} & -\frac{\pi \delta wz}{1 - \theta} & \frac{[\tau_l + \tau_h (1 - \tau_l)]w}{1 - \theta} & \tau_z \pi \delta w & -1 \end{vmatrix}$$

$$= -\frac{\pi \delta wz d \sigma_c}{cR(1 - \theta)\sigma_z \sigma_{zc} (1 - \tau_z)} \{ -1 + \theta - [-1 + \theta + \pi_h (\delta wz - d)\tau_z + \delta wz \pi_h \tau_z^2 \\
- \sigma_z (1 - \tau_z)[1 - \theta + \pi_h (d - \delta wz (1 - \tau_z))] \}.$$

The derivative of $d$ with respect to $\tau_z$ is thus

$$\frac{dd}{d\tau_z} = \frac{|A'_{\tau}|}{|A|}.$$

$|A|$ can be reformed into the following form by replacing the third column

$$|A'_{\tau}| = \begin{vmatrix} -1 & -\frac{\bar{\pi}}{R} & \frac{\bar{\pi}}{R}\delta wz & \frac{\bar{\pi}}{R}(1 - \tau_z)\delta wz & \frac{\bar{\pi}_h}{R} [(1 - \tau_z)\delta wz - d] \\ \sigma_c d \sigma_{dc} & -1 & 0 & 0 & 0 \\ \sigma_c l \sigma_{lc} & 0 & 0 & 0 & 0 \\ \sigma_c z \sigma_{zc} & 0 & \frac{z}{\sigma_z (1 - \tau_z)} & -1 & 0 \\ \frac{\tau_h}{1 - \theta} & 0 & -\frac{\pi \delta wz}{1 - \theta} & \tau_z \pi \delta w & -1 \end{vmatrix}$$

$$= \frac{\pi \delta wz d \sigma_c}{cR(1 - \theta)\sigma_z \sigma_{zc} (1 - \tau_z)} \{ -1 + \theta - [-1 + \theta + (\delta wz - d)\pi_h]\tau_z + \delta wz \pi_h \tau_z^2 \\
- \sigma_z (1 - \tau_z)[1 - \theta + \pi_h (d - \delta wz (1 - \tau_z))] \}.$$

The derivative of $l$ with respect to $\tau_z$ is thus

$$\frac{dl}{d\tau_z} = \frac{|A'_{\tau}|}{|A|}.$$
We then reform $|A|$ again by replacing its fourth column

$$|A^\tau_z| = \begin{vmatrix} -1 & -\frac{\pi}{\bar{R}} & (1-\tau_l)w & \frac{\pi}{\bar{R}}\delta wz & \frac{\pi}{\bar{R}} [(1-\tau_z)\delta wz - d] \\ \frac{\sigma_d}{\sigma_{dc}} & -1 & 0 & 0 & 0 \\ -\frac{\sigma_l}{\sigma_{lc}} & 0 & -1 & 0 & 0 \\ -\frac{\sigma_z}{\sigma_{zc}} & 0 & 0 & \frac{z}{\sigma_z(1-\tau_z)} & 0 \\ -\frac{\tau_z}{1-\theta} & 0 & \frac{[\tau_l+\tau_k(1-\tau_l)]w}{1-\theta} & -\frac{\pi\delta wz}{1-\theta} & -1 \end{vmatrix}$$

$$= \frac{z}{c\bar{R}(1-\theta)\sigma_d\sigma_l\sigma_z(1-\tau_z)} \{ (1-\theta)[c\bar{R}\sigma_d\sigma_l + \sigma_c(d\pi\sigma_l + w\sigma_d(l\bar{R}(1-\tau_l) - z\delta\pi\sigma_l(1-\tau_z))] + \frac{\pi\delta wz}{1-\theta} \\ \times [d - \delta wz(1-\tau_z)] \}. \]$$

The derivative of $z$ with respect to $\tau_z$ is thus

$$\frac{dz}{d\tau_z} = \frac{|A^\tau_z|}{|A|}. \]$$

By replacing the fifth column in $|A|$, we obtain the following determinant

$$|A^\tau_h| = \begin{vmatrix} -1 & -\frac{\pi}{\bar{R}} & (1-\tau_l)w & \frac{\pi}{\bar{R}}(1-\tau_z)\delta w & \frac{\pi}{\bar{R}} \delta wz \\ \frac{\sigma_d}{\sigma_{dc}} & -1 & 0 & 0 & 0 \\ -\frac{\sigma_l}{\sigma_{lc}} & 0 & -1 & 0 & 0 \\ -\frac{\sigma_z}{\sigma_{zc}} & 0 & 0 & -1 & \frac{z}{\sigma_z(1-\tau_z)} \\ -\frac{\tau_z}{1-\theta} & 0 & \frac{[\tau_l+\tau_k(1-\tau_l)]w}{1-\theta} & \frac{\tau_z\pi\delta w}{1-\theta} & -\frac{\pi\delta wz}{1-\theta} \end{vmatrix}$$

$$= \frac{\pi\delta wz}{c\bar{R}(1-\theta)\sigma_d\sigma_l\sigma_z(1-\tau_z)} \{ c\sigma_d\sigma_l[\sigma_z(\tau_k - \bar{R})(1-\tau_z) + \tau_k(1-\tau_z) + \bar{R}\tau_z] + \sigma_c[\pi\sigma_l\sigma_z(\tau_z - \sigma_z(1-\tau_z))] + w\sigma_d(-z\delta\pi\sigma_l(1-\tau_z) - \bar{R}(1-\tau_z)(1-\tau_z) + \sigma_z(\bar{R} + \tau_k(1-\tau_l) + (1-\bar{R})\tau_l)(1-\tau_z) - \bar{R}\tau_z - \tau_z + \tau_zl\tau_z)]. \}
The derivative of \( h \) with respect to \( \tau_z \) is thus

\[
\frac{dh}{d\tau_z} = \frac{|A_h\tau_z|}{|A|}.
\]

The effects of changing in \( \tau_k \) can be examined from the problem below:

\[
\begin{bmatrix}
-1 & -\frac{\pi}{R} (1 - \tau_l)w & \frac{\pi}{R} (1 - \tau_z)\delta w & \frac{\pi}{R} [1 - \tau_z] \delta w z - d \\
\frac{\sigma_d}{\sigma_d c} & -1 & 0 & 0 & 0 \\
\frac{\sigma_l}{\sigma_l c} & 0 & -1 & 0 & 0 \\
\frac{\sigma_p}{\sigma_p c} & 0 & 0 & -1 & 0 \\
\frac{\tau_k}{1 - \theta} & 0 & \frac{[\tau_l + \tau_k (1 - \tau_l)] w}{1 - \theta} & \frac{\tau_p \delta w}{1 - \theta} & -1
\end{bmatrix}
\begin{bmatrix}
d c \\
d d \\
d l \\
d z \\
d h
\end{bmatrix}
= \begin{bmatrix}
\frac{\pi [(1 - \tau_z) \delta w z - d]}{R^2} \\
-\frac{d}{\sigma_d R} \\
0 \\
\frac{z}{\sigma_z R} \\
\frac{a}{1 - \theta}
\end{bmatrix}.
\]

By replacing the first column in \( |A| \), we obtain that

\[
|A^R_c| = \begin{bmatrix}
\frac{\pi [(1 - \tau_z) \delta w z - d]}{R^2} & -\frac{\pi}{R} (1 - \tau_l)w & \frac{\pi}{R} (1 - \tau_z)\delta w & \frac{\pi}{R} [1 - \tau_z] \delta w z - d \\
-\frac{d}{\sigma_d R} & -1 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 \\
\frac{z}{\sigma_z R} & 0 & 0 & -1 & 0 \\
\frac{a}{1 - \theta} & 0 & \frac{[\tau_l + \tau_k (1 - \tau_l)] w}{1 - \theta} & \frac{\tau_p \delta w}{1 - \theta} & -1
\end{bmatrix}
\]

\[
= \frac{1}{R^2} \left\{ \frac{\pi d}{\sigma_d} + \frac{1}{(1 - \theta)\sigma_z} \left[ \left( (1 - \theta) \pi + a \hat{R} \pi h \right) \sigma_z (d - \delta w z (1 - \tau_z)) - \delta w z \pi \times (1 - \theta - (1 - \theta + (d - \delta w z) \pi h) \tau_z - \delta w z \pi h \tau_z^2) \right] \right\}.
\]
The derivative of \( c \) with respect to \( \bar{R} \) is thus
\[
\frac{dc}{d\bar{R}} = \frac{|A_c\bar{R}|}{|A|}.
\]

We then reform \(|A|\) by replacing its second column
\[
|A_d\bar{R}| = \begin{vmatrix}
-1 & \frac{\pi[(1-\tau_z)\delta w - d]}{\bar{R}^2} & (1 - \tau_l)w & \frac{\pi}{\bar{R}}(1 - \tau_z)\delta w & \frac{\pi_0[(1-\tau_z)\delta w - d]}{\bar{R}} \\
\sigma_{\bar{z}c} \sigma_{dc} & \sigma_{d\bar{R}} & 0 & 0 & 0 \\
-\frac{\sigma_{d\bar{R}}}{\sigma_{dc}} & 0 & -1 & 0 & 0 \\
-\frac{\sigma_{d\bar{R}}}{\sigma_{dc}} & \frac{\bar{z}}{\sigma_{d\bar{R}}} & 0 & -1 & 0 \\
-\frac{\tau_k}{1-\theta} & \frac{a}{1-\theta} & \frac{\tau_l - [\tau_l(1-\tau_z)\delta w - d]}{1-\theta} & \frac{\tau_l\delta \bar{w}}{1-\theta} & -1
\end{vmatrix}
\]= \frac{-d}{cR^2(1-\theta)\sigma_{d\sigma_{l\sigma_z}}} \times \{c\sigma_{l\sigma_z}[(1-\theta)\bar{R} + \pi_0\tau_k(d - \delta w(1 - \tau_z)) + \sigma_{dc}[(1-\theta)\pi \\
+ aR\pi_0\sigma_1(d - \delta w(1 - \tau_z)) + \sigma_\delta R(1-\theta)\delta(1 - \tau_l) - \pi_0(\tau_k(1 - \tau_l) + \tau_l) \\
\times (d - \delta w(1 - \tau_z))]|\}.
\]

The derivative of \( c \) with respect to \( \bar{R} \) is thus
\[
\frac{dd}{d\bar{R}} = \frac{|A_{d\bar{R}}|}{|A|}.
\]

By replacing the third column, we rewrite \(|A|\) into the following form
\[
|A_{d\bar{R}}| = \begin{vmatrix}
-1 & \frac{\pi[(1-\tau_z)\delta w - d]}{\bar{R}^2} & (1 - \tau_l)w & \frac{\pi}{\bar{R}}(1 - \tau_z)\delta w & \frac{\pi_0[(1-\tau_z)\delta w - d]}{\bar{R}} \\
\sigma_{\bar{z}c} \sigma_{dc} & \sigma_{d\bar{R}} & 0 & 0 & 0 \\
\sigma_{d\bar{R}} & 0 & -1 & 0 & 0 \\
\frac{\sigma_{d\bar{R}}}{\sigma_{dc}} & \frac{\bar{z}}{\sigma_{d\bar{R}}} & 0 & -1 & 0 \\
-\frac{\tau_k}{1-\theta} & \frac{a}{1-\theta} & \frac{\tau_l - [\tau_l(1-\tau_z)\delta w - d]}{1-\theta} & \frac{\tau_l\delta \bar{w}}{1-\theta} & -1
\end{vmatrix}
\]= \frac{-\sigma_{dc}l}{cR^2(1-\theta)\sigma_{d\sigma_{l\sigma_z}}} \times \{d(1-\theta)\pi \sigma_{d\sigma_z} - \sigma_{dc}[(1-\theta)\pi + aR\pi_0\sigma_1(d - \delta w(1 - \tau_z)) \\
\times (d - \delta w(1 - \tau_z))]|\}.
\]
\[-\pi \delta wz (1 - \theta - (1 - \theta + (d - \delta wz) \tau_z - \pi_h \delta wz \tau_z^2))\].

The derivative of \(l\) with respect to \(\bar{R}\) is thus

\[
\frac{dl}{d\bar{R}} = \frac{|A^R_l|}{|A|}.
\]

We reform \(|A|\) by replacing its fourth column and obtain

\[
|A^R_z| = \begin{vmatrix}
-1 & -\overline{\pi} & (1 - \tau_l)w & \frac{\pi[(1 - \tau_z)\delta wz - d]}{R^2} & \frac{\pi_h[(1 - \tau_z)\delta wz - d]}{R} \\
\sigma_d \sigma_{\delta c} & -1 & 0 & -\frac{d}{\sigma_d \bar{R}} & 0 \\
-\sigma_d \sigma_{\delta c} & 0 & -1 & 0 & 0 \\
-\sigma_z \sigma_{\delta c} & 0 & 0 & \frac{z}{\sigma_z \bar{R}} & 0 \\
-\frac{\tau_h}{1 - \theta} & 0 & \frac{\gamma + \tau_h (1 - \tau_l)}{1 - \theta} & \frac{a}{1 - \theta} & -1
\end{vmatrix}
\]

\[
= \frac{z}{c \bar{R}^2 (1 - \theta) \sigma_l \sigma_z} \{\sigma_l [(\bar{R} (1 - \theta) + \pi_h \tau_h (d - \delta wz (1 - \tau_z))] + \sigma_c [(1 - \theta) \pi \\
+ a \bar{R} \pi_h \sigma_l (d - \delta wz (1 - \tau_z))] + \sigma_c [(1 - \theta) \pi + a \bar{R} \pi_h \sigma_l (d - \delta wz (1 - \tau_z)) \\
+ wt (\bar{R} (1 - \theta) (1 - \tau_l) - \pi_h (\tau_h (1 - \tau_l) + \gamma) (d - \delta wz (1 - \tau_z)))\}.
\]

The derivative of \(z\) with respect to \(\bar{R}\) is thus

\[
\frac{dz}{d\bar{R}} = \frac{|A^R_z|}{|A|}.
\]

\(|A|\) can be reformed into the following form by replacing its fifth column

\[
|A^R_h| = \begin{vmatrix}
-1 & -\overline{\pi} & (1 - \tau_l)w & \frac{\pi[(1 - \tau_z)\delta wz - d]}{R^2} & \frac{\pi_h[(1 - \tau_z)\delta wz - d]}{R} \\
\sigma_d \sigma_{\delta c} & -1 & 0 & 0 & -\frac{d}{\sigma_d \bar{R}} \\
-\sigma_d \sigma_{\delta c} & 0 & -1 & 0 & 0 \\
-\sigma_z \sigma_{\delta c} & 0 & 0 & \frac{z}{\sigma_z \bar{R}} & 0 \\
-\frac{\tau_h}{1 - \theta} & 0 & \frac{\gamma + \tau_h (1 - \tau_l)}{1 - \theta} & \frac{\tau_z \pi \delta w}{1 - \theta} & \frac{a}{1 - \theta}
\end{vmatrix}
\]
\[ \frac{-1}{cR^2(1 - \theta)\sigma_d}\{ -c\sigma_1[d\pi\sigma_z\tau_k + \sigma_d(\pi\delta wz(\tau_k(1 - \tau_z) + \tilde{R}\tau_z) + \sigma_z \\
\times (a\tilde{R}^2 - \pi\tau_k(d - \delta wz(1 - \tau_z)))]) + \sigma_c[d\pi\sigma_z(-a\tilde{R}\sigma_l + \omega l(\tau_k(1 - \tau_l) + \tau_l)) \\
+ \sigma_d(-z\delta\pi\sigma_l(a\tilde{R} - (a\tilde{R} - (d - \delta wz)\pi)\tau_z + \delta wz\pi\tau_z^2) + \omega l(\pi\delta wz(\tau_k(1 - \tau_l) \\
\times (1 - \tau_z) - \tilde{R}\tau_z + \tau_l(1 - (1 - \tilde{R})\tau_z)) - \sigma_z(a\tilde{R}^2 + \pi\tau_k(1 - \tau_l)(1 - \tau_z) - \tilde{R}\tau_z \\
+ \tau_l(1 - (1 - \tilde{R})\tau_z)) - \sigma_z(a\tilde{R}^2 + \pi\tau_k(1 - \tau_l)(d - \delta wz(1 - \tau_z)) - \sigma_z(a\tilde{R}^2 \\
+ \pi\tau_k(1 - \tau_l)(d - \delta wz(1 - \tau_z)) - \tau_l(a\tilde{R}^2 - d\pi + \pi\delta wz(1 - \tau_z))))}\}. \]

The derivative of \( h \) with respect to \( \tilde{R} \) is thus

\[ \frac{dz}{d\tilde{R}} = \frac{\left| A_R^R \right|}{|A|}. \]

4.A.2 Calibration

This appendix calibrates our model on the UK economy. Referring to Leroux and Ponthiere (2018), we specify \( u(\cdot) \) and \( v(\cdot) \) as below

\[ u(c) = T_1c\frac{1}{1 - \gamma}, \quad (4.A.7) \]

\[ v(l) = T_2\alpha l\phi, \quad (4.A.8) \]

where \( T_1 \) and \( T_2 \) denote that individuals only work for a fraction of time. As in Leroux et al. (2011a), we set \( \phi = 2 \). In line with Leroux and Ponthiere (2018), \( T_1 \) and \( T_2 \) are calibrated as 52 and 47 to indicate the case where individuals have 5 weeks of holidays per year. The initial value of \( l \) is chosen as 0.25 in line with Prescott (2006).

We then calibrate the value of \( \alpha \) from equations \( 4.3.7 \) and \( 4.3.9 \):

\[ \alpha = \frac{T_1c\frac{1}{\gamma}(1 - \tau_l)w}{T_2\ell} = 32.7489. \quad (4.A.9) \]
Survival probabilities $\pi$ follow the form suggested in Chakraborty (2004):

$$\pi = \frac{h}{1 + h}. \quad (4.A.10)$$

To calibrate the value of $\pi$ and $h$, we first look at the data on life expectancy. Based on the data of Human Mortality Database (2018), the life expectancy at birth in the UK was 81.4 in 2016. In accordance with the data provided by the Office for National Statistics (2012), we assume that the maximal lifetime is 110 year. Therefore, each period would be 55 years long in our calibration. We can then derive survival probabilities with the following form:

$$\pi = \frac{(\text{Life Expectancy} - 55)}{55}. \quad (4.A.11)$$

Therefore, $\pi$ is 0.4720 and $h$ is 0.8973. $d$ and $z$ are calibrated as 0.7959 and 0.0475 by using equations (4.3.8) and (4.3.10).

### 4.A.3 The effects of taxes on utilities

![Figure 4.A.1: Changes in utilities in response to the adjustments in taxes](image-url)
Chapter 5

Conclusions and future works

This thesis explores the roles of public policies and endogenous health from three aspects: the mechanism underlying the health effects of taxes, the structure of optimal sin taxes, and the relationship between labor supply and public health care.

5.1 Conclusions

In Chapter 2, we develop a general equilibrium model with endogenous health to examine the effects of unhealthy commodity taxes in the economy. We contribute to the literature by providing the mechanism to explain a well-documented finding that taxes on unhealthy commodities alone could be ineffective in promoting health (as found by Fletcher et al., 2010; Mytton et al., 2012; Schroeter et al., 2008). In our model, the economy is comprised of the goods sector, which produces consumption commodities, and the health sector, which provides individuals with the stock of health. Following Grossman (1972), the stock of health in our model generates the so-called “healthy time” which is available for work. One novelty of our model is
found in how it embeds individual preference for health in that of leisure. This novelty allows us to detect the variations in labor supply in response to the changes in policies. Although unhealthy commodities provide individuals with utility, they pose detrimental effects on health. We find that, in response to the implementation of unhealthy commodity taxes, individuals reduce not only their consumption of unhealthy commodities but also their investment in health. Therefore, the beneficial effects of reducing consumption of unhealthy commodities are offset by the negative effects of reducing investment in health. In addition to the exploration of this underlying mechanism, we also investigate the policy which promotes both health and welfare more effectively. We find that, with revenue-neutral adjustments in taxes on labor income or capital income, the implementation of unhealthy commodity taxes can improve the level of health through income effects. Moreover, both reforms contribute to higher levels of welfare in the long run. The results offer important guidelines to policy makers: the introduction of a tax on unhealthy commodities, for example a “sugar tax”, should always be coupled with a reduction in other tax burdens in order to improve the level of population health and increase overall welfare effectively.

In Chapter 3, we move away from the traditional paternalistic view to explore the structure of optimal sin taxes based on the short- and long-term externalities on public funds.¹ We find that optimal sin taxes contain the Pigouvian element

¹Studies of sin taxes usually take the paternalistic view which assumes that the government knows better than individuals in terms of their own health (e.g. Cremer et al., 2012; Goulao and Pérez-Barahona, 2014; O’Donoghue and Rabin, 2003, 2006).
which corrects the short- and long-term externalities on public funds, and the efficiency element which only appear in the second-best setting. In line with Bovenberg and Goulder (1996), our calibration results show that the Pigouvian element in the second-best setting is generally lower. The reason behind this finding is that individuals can tolerate more externalities since they value public goods less in the presence of income taxes. However, the second-best optimal sin taxes are not necessarily lower than the first-best optimal sin taxes due to the presence of the efficiency element in the second-best setting. Furthermore, we calibrate the model on the UK economy and find that the implementation of sin taxes has double-dividends in terms of not only improving population health but also enhancing economic performance and welfare.

In Chapter 4, we explore the relationship between labor supply and public policies in an economy with endogenous lifetime. We extend the basic model of Fletcher et al. (2010) and Leroux and Ponthiere (2018) by including the roles of endogenous health and tax policies. In our model, individuals can live either one or two periods, depending on the survival probabilities which are determined by the provision of public health care. We find that, in the face of higher survival probabilities, individuals generally offer more labor supply during old age. However, this statement is reversed if the additional spending on health care is mainly funded through taxes on old-age labor income. In addition, we investigate the economic impacts of variations in production and medical technologies respectively. We find that, with higher levels of production technologies, individuals increase labor supply in both young and old age. These changes allow the government to collect more tax revenue. On the other
hand, with higher levels of medical technologies, individuals offer more labor supply
during old age but less during young age. Survival probabilities consequently de-
crease because the negative impacts on fiscal budget from reducing young-age labor
supply overshadow the positive impacts from raising old-age labor supply. Further-
more, we derive an optimal tax scheme from a welfarist point of view. Optimal
taxes on labor income depend on the intertemporal elasticities of substitution. The
implementation of these optimal taxes contributes to a smoother consumption path
with higher levels of consumption in both young and old age. Individuals also obtain
a smoother labor choice over time with less labor supply in young age but more in
old age.

5.2 Future works

Various aspects regarding the roles of public policies and endogenous health have
not been the focus of this thesis.

First, we do not specifically examine the dynamic properties of endogenous health
in Chapter 2, because we focus more on the long-run changes in the economy. A
controversial problem of studying the dynamic properties of health is that individual
health might grow without limits in the model. To avoid this problem, we can extend
our model to further include endogenous longevity (as in Ehrlich and Chuma (1990)
and Kuhn et al. (2015)), endogenous mortality at each point of time (as in Hall and
Jones (2007) and Halliday et al. (2017)), or endogenous health technologies (as in
Jones (2016) and Kuhn and Prettner (2016)).

Second, we treat the internalization of fiscal externalities as parameters in Chap-
5.3 Concluding remarks

In conclusion, the main contribution in this thesis include: (1) providing the underlying mechanism to explain why unhealthy commodity taxes may fail in promoting health, (2) justifying and decomposing the structure of optimal sin taxes from a non-paternalistic point of view, and (3) constructing a theoretical framework to analyze the relationship between labor supply and public policies in the economy. The additional consideration regarding this thesis include the dynamic analysis of endogenous health, the exploration of endogenous degrees of internalization, and the inclusion of the interactions between different generations. Moreover, we focus on the theoretical aspect of the relationship between health and policies rather than the empirical analysis in this thesis. We leave the empirical analysis in this topic to our future work.
Bibliography


Deschénes, O., Greenstone, M., and Shapiro, J. S. (2017). Defensive investments and


